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# Pathfinding by demand sensitive map abstraction

Sourodeep Bhattacharjee

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# Pathfinding by Demand Sensitive Map Abstraction

By  
Sourodeep Bhattacharjee

A Thesis Submitted to the Faculty of Graduate Studies  
through School of Computer Science in Partial Fulfillment of  
the Requirements for the Degree of Master of Science at the  
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2012

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PATHFINDING BY DEMAND-SENSITIVE  
MAP ABSTRACTION  
by  
SOURODEEP BHATTACHARJEE

APPROVED BY:

---

**Dr. Myron Hlynka**  
External Reader  
Department of Mathematics and Statistics

---

**Dr. Dan Wu**  
Internal Reader  
School of Computer Science

---

**Dr. Scott Goodwin**  
Advisor  
School of Computer Science

---

**Dr. Subir Bandyopadhyay**  
Chair of Defense  
School of Computer Science

Date: December 03, 2012

## **Author's Declaration of Originality**

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# Abstract

In this thesis, we present a new algorithm: Demand Sensitive Map Abstraction (DSMA). DSMA is a special kind of hierarchical pathfinding algorithm in which we vary the granularity of abstraction of the high-level map based on pathfinding request demand associated with various regions in the high level map and the search time of the last path request. Additionally, the low level A\* search is not restricted by the boundaries of the high level sectors. By dynamically varying the abstraction we are able to maintain a balance between path quality and search time. We compare DSMA with two variations where the granularity of abstraction is constant; one of those contains maximum granularity throughout (Dense HA\*) and the other contains the minimum (Sparse HA\*).

Our experimental results show that DSMA's performance is a balance between Dense HA\* and Sparse HA\*. Depending on the resources available DSMA can behave either as Dense HA\* or as Sparse HA\* or lie somewhere in between. Moreover we do not pre-cache paths at any level, which gives us the added benefit of working with a flexible abstract map without the necessity of changing the pre-cached paths if the low level map changes.

# Dedication

To my father, Late Debojit Bhattacharjee...

## Acknowledgments

First of all, I would like to thank my supervisor - Dr. Scott Goodwin for being patient with me and for helping me nurture the idea of Demand Sensitive Map Abstraction; always commending the strong ideas while providing suggestions on improving the weak ones. I would also like to thank Dr. Richard Frost for showing how and where to find dependable research materials and how to extract essential information from lengthy papers quickly. I would like to thank my friend Mirna Šečić, for her recommendations on academic writing style and improving the graphics presented in this thesis. I am grateful to the committee members -Dr. Myron Hlynka and Dr. Dan Wu and the Chair Dr. Subir Bandyopadhyay for finding time from their busy schedules to attend my thesis proposal and defense.

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# 1 Introduction

## 1.1 Problem Domain

In this thesis we try to solve the problem of pathfinding in game maps. Pathfinding is the problem of finding a route of desired quality from a given start location to a given goal location in a game map. It consists of two phases: path planning and path following; the latter encompasses the traversal of planned path and dealing with unpredicted dynamic obstacles or other mobile agents. In our thesis we restrict all our claims to path planning only.

We attempt to solve the problem of pathfinding in game maps using hierarchical pathfinding techniques. Hierarchical pathfinding involves the use of a high level abstract map created from the low level, actual, game map. This is employed to overcome the exponential search time of A\* search. We assume that the map is known a priori and hence exploration is not necessary.

All our maps use grid worlds at their lowest level and the grids have octile navigation which means a mobile agent can move north, south, east, west, north-east, north-west, south-east and south-west.

## 1.2 Contribution

In this thesis we present a new algorithm which we call Demand Sensitive Map Abstraction (DSMA). In this algorithm we vary the abstract map dynamically depending on the demand of pathfinding in a particular section and the last pathfinding time.

We claim that by varying the granularity of abstraction dynamically we can make optimal use of resources (CPU time and memory space) to find a suitable path, as opposed to keeping the granularity constant throughout the game-play. There are set, industry standard, upper and lower bounds on the time, that can be devoted to pathfinding when a game is being played which is one millisecond (1 ms) to three milliseconds (3 ms). DSMA attempts to keep the pathfinding time between these limits by varying the granularity of abstraction, associated with either high demand or low demand regions, depending on whether the last pathfinding time went above 3 ms (region is split into two, finely grained, regions) or below 1 ms (two low demand regions are coalesced to have a single, coarsely grained, region), respectively.

In order to prove our claim we compare DSMA to two other variations where we do not vary the granularity of the abstract map. This gives us an idea of the benefit we can get from dynamically varying the granularity of abstraction. We measure path quality, pathfinding time and number of cells explored to find a path. The experimental results support our claim, since the performance metric curves of DSMA lie between the curves of the two constant granularity variations.

Our work is significant because it is the first one to vary the granularity of abstraction associated with specific regions instead of adding more hierarchical levels to suit the resources available. So DSMA provides a new philosophy of interpreting hierarchical pathfinding and opens up avenues for further research in a new direction, within the same research area.

### **1.3 Organization**

This thesis contains four major sections: “Hierarchical Pathfinding” which discusses the previous related work, “Demand Sensitive Map Abstraction” in which we discuss our algorithm (DSMA) and explain why and how DSMA works, “Experimental Analysis” where we discuss our experimental setup, the results and discuss the graphs concisely and finally the “Conclusion.”

## 2 Hierarchical Pathfinding

The term “Hierarchical Pathfinding” was first penned in the paper “Hierarchical A\*: Searching Abstraction Hierarchies Efficiently” (Holte et al., 1996), which was the first paper that initiated this area of research. In large search spaces with many obstacles and pathfinding units, the execution time of A\* search (Hart et al., 1968) becomes prohibitively large. To overcome this problem, Hierarchical pathfinding is employed. The process involves splitting up the search space in regions and creating an abstract pathfinding map with these abstract regions by placing an edge between connected regions. Each region thus obtained can be further split into sub-regions. Each such connection of regions form one level of hierarchy in the collection of pathfinding maps. This process is carried out in a pre-processing step and all abstracted regions are cached, along with the distance between them.

Later, during game play, when any game agent requests a path, the algorithm begins searching for a path in the topmost abstract hierarchical map by running A\* search between regions. The path thus obtained is refined subsequently by running A\* search on lower level maps until the real game map is reached. Collectively, the pre-processing and the on-line search is called Hierarchical Pathfinding A\*. This process ensures search for the path is restricted only to those regions which is likely to contain the optimal path.

## 2.1 Near Optimal Techniques

In this section we will discuss some of the first papers on Hierarchical Pathfinding. The authors in these papers claim to have achieved results that are very close to optimal or the desired least cost (edge weight) path. While all of the papers discuss the idea of hierarchical pathfinding, their techniques are different from each other and hence demand a discussion.

### 2.1.1 Hierarchical A\*

The authors of the paper titled “Hierarchical A\*” (Holte et al., 1996) claim that this paper was the first one to address the problem of high execution time of A\* in large maps by applying Hierarchical Pathfinding techniques. The motivation of this paper was to break Valtorta’s Barrier (Valtorta, 1983), which is the number of nodes expanded when searching blindly. Valtorta’s theorem (Valtorta, 1983) states that this barrier cannot be broken by “embedding transformations” - abstracted state spaces. (Holte et al., 1996) is different from the other papers discussed in this thesis, as it deals with puzzles and not game world maps. Nonetheless it demands a discussion, since (Holte et al., 1996) is the first paper to explore the idea of Hierarchical Pathfinding.

The idea proposed by the authors involved selecting a state (node in the pathfinding graph) with maximum degree and grouping it together with its neighbors within a certain distance (abstraction radius), and forming a single node with them. The process was repeated until a level is created with only one state. The lowest level of this hierarchy represents the original graph.



When searching for a path in the lowest level, the algorithm proposed by the authors, searched for a high level path in the abstract levels until the highest level with a single state was reached. This algorithm was called Naive Hierarchical A\* (NHA\*).

In their preliminary experiments the authors found that NHA\* was expanding more states than A\*. They explained this observation by stating that A\* search using monotone heuristic will never expand the same state twice. NHA\*, on the other hand, expanded same states repeatedly in higher levels for same start-goal pair in the base level. To overcome the problem they introduced a technique they named h\*-caching. The idea involved caching h(s) values for states that have been expanded already and reusing these values in subsequent searches without expanding the state. Their experiment showed that NHA\* was expanding roughly the same number of states as A\*. This implied that Valtorta's Barrier was not yet broken and more enhancements were needed. The next improvement suggested by the authors relied on the idea that for every state X for which h(X) is known, a path from X to goal is also known. This precludes the necessity to expand X, if this path is cached along with h(X). The authors state that "knowing a path of length g(X) to X means knowing a path of length g(X) + h(X) to the goal". Instead of adding X to the open list, the goal state is added to it and the search terminates when the goal is on top of the list. They call this technique Optimal Path Caching. Thus NHA\* with Optimal Path caching was able to break Valtorta's Barrier; this is the claim made by the authors in (Holte

et al., 1996).

The paper concludes by mentioning two important future research areas: firstly, the authors state that a better method is needed to vary the granularity of abstraction and secondly, to find a better way to build abstracted states automatically.

### **2.1.2 Near Optimal Hierarchical A\***

The topic of Hierarchical Pathfinding was revived in the paper Near Optimal Hierarchical Pathfinding (Botea et al., 2004). Their paper addressed the problem of pathfinding on “large” maps where limited CPU and memory resources create severe bottlenecks. They achieved this by employing hierarchical pathfinding techniques. (Botea et al., 2004) is very well written and introduces many new ideas, algorithms and optimizations of those algorithms.

The previous work referred to by the authors include (but are not limited to) (Rabin, 2000) and (Holte et al., 1996). The authors state that their work is very similar to A\* Aesthetic Optimizations (Rabin, 2000) in that both employ hierarchical pathfinding by map abstraction. They differ in the respect that the algorithms proposed by the authors support multiple levels of hierarchy while the other supports only two. Another difference is that in A\* Aesthetic Optimizations, the optimal distances between two states are computed on-line while in this paper the authors pre-compute and cache this information. The authors state that their contribution is similar to Hierarchi-

cal A\* (HA\*) (Holte et al., 1996) in that both use hierarchical representation of the search space for reducing search effort. However, the two papers are different in the respect that Hierarchical Pathfinding A\* (HPA\*) proposed by the authors uses abstraction to enhance the representation of the search space while HA\* is used for “automatically generating domain independent heuristic state evaluations”.

The algorithm proposed by the authors consists of two phases: a pre-processing step and an on-line search. The pre-processing step defines a topological abstraction of the map. The map is divided into rectangular regions (“clusters”) as shown in the figure 1 below (Botea et al., 2004), p.24:

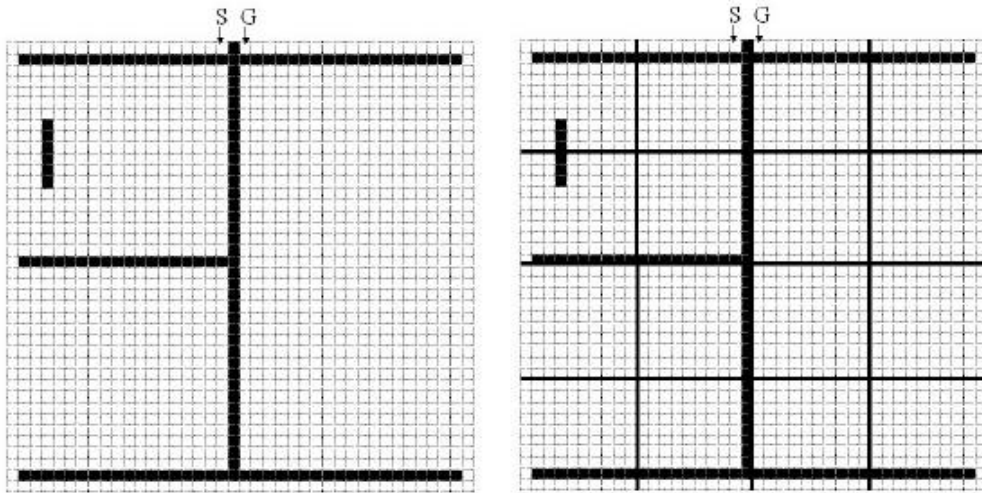


Figure 1: Illustration of Pre-Processing step: Near Optimal HPA\*

The authors then define a gateway between two clusters by identifying a set of entrances connecting them, where an entrance is the longest obstacle-free set of cells that lie in the border of two adjacent clusters. The transition point (the point where an agent goes from one cluster to another) is the midpoint of the entrance. The authors call this an inter-edge. For each pair of such points inside a cluster, the authors link them by edges and name them intra-edge. This forms a high level abstract graph. This is illustrated in the figure below (Botea et al., 2004), p.24:

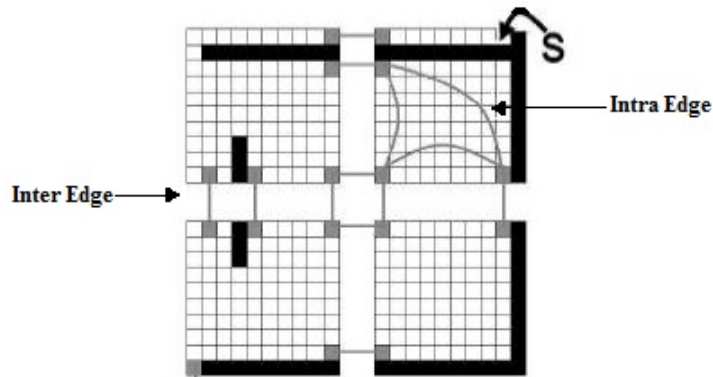


Figure 2: Illustration of Intra/Inter Edges in Near Optimal HPA\*

The next step consists of an on-line search. In this step the start and goal nodes are inserted into the abstract graph and use A\* search to find a path from start to goal in the abstract graph. This gives an abstract path which can be made concrete by mapping it to the low level graph and further refining the path.

For their experiments, the authors used 120 maps from a game (Baldur's Gate from Bioware) varying in size from 50x50 to 320x320. For each map the authors generated random start and goal states where a valid path between the two existed. The authors state that A\* is slightly better than HPA\* when the solution length is small (i.e., the search problem is easy). The authors explain the difference in performance by stating that the overhead of inserting start and goal states into the abstract graph and other such techniques becomes an unnecessary expense when the map is mostly empty or the path is possibly a straight line through the grid. However, given a real game scenario with standard number of obstacles and mobile agents, the authors claim that HPA\* is up to 10 times faster than a highly optimized A\*.

With respect to their contribution the authors state that their method of map abstraction is automatic and is independent of specific topology and works well with both random and real maps as well as static or dynamic environments. They also state that their technique is simple and easy to implement.

### 2.1.3 Partial Pathfinding using Map Abstraction and Refinement

The next paper we are about to discuss, titled “Partial Pathfinding using Map Abstraction and Refinement” (Sturtevant and Buro, 2005), is a significant one. The reason for its significance lies with the fact that this paper is the first one to explore the challenge of interleaving path planning with execution in the hierarchical pathfinding framework. In this paper the authors propose a set of algorithms they named - Path-Refinement A\* (PRA\*).

The authors refer to HPA\* (Botea et al., 2004) as a related work. The authors state that HPA\* and PRA\* is similar considering the fact that both algorithms use hierarchical pathfinding. The two algorithms differ in the respect that HPA\* overlays an entire map with clusters/sectors and an interconnected set of such clusters form the high level graph, on the other hand PRA\*, instead of making a general abstraction of the entire map, studies the map topology and groups together tiles (from the grid based low level map) which are cliques forming one node and connects them with orphans.

The authors introduce their idea using a simple algorithm - QuickPath and extend the same to PRA\*, PRA\*( $\infty$ ) (one that finds complete paths) and PRA\*(k) (one that finds partial paths). The authors explain QuickPath using the figure 3 below (Sturtevant and Buro, 2005), p.3:

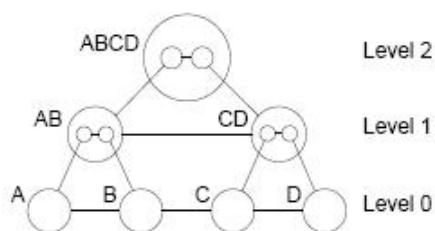


Figure 3: Illustration of QuickPath

After building the abstraction hierarchy, all the nodes in the graph resolve to a single high level node. To check if two nodes are connected, the algorithm checks if they ever converge into the same parent within the abstraction hierarchy. This provides a “quick check for path-ability” and can also find plausible paths in the game world, by tracking the parents of the nodes, without performing an extensive search, as claimed by the authors.

The PRA\* proposed by the authors apply four enhancements to the QuickPath algorithm. Firstly, the authors use a heuristic to make the search more directed. Secondly, it allows path refinement in areas outside the abstract path such as in corridors. Thirdly, it starts path-planning from the bottom of the hierarchy instead of the top. Finally, it finds only partial paths in the entire path planning step. Meaning the partial path thus found, is executed/traveled by the agent before further planning. The QuickPath algorithm ensures that planning and execution is interleaved. The authors also propose two more variations: PRA\*( $\infty$ ) which is a version of PRA\* in which partial refinement is not allowed and PRA\*(k) that refines k number of nodes from the abstract path at a time. For experimentation, the authors claim to have used 116 maps from Baldur’s gate and Warcraft 3 and scaled them to size 512 X 512. The maps were represented as square tile grids with octile relationship between tiles - an agent could move in eight directions from any tile. They generated random start and goal pairs for each map. They claim that PRA\*( $\infty$ ) has similar performance as HPA\* and both are faster than A\* while delivering close to optimal results. On the runtime of



PRA\*(k) the authors state that the algorithm is “super-linear in the path length”. They state that the paths derived from PRA\*(k) are longer compared to A\* and PRA\*(∞) but the algorithm proves to be faster than the other two.

In conclusion, the authors claim that their paper was the first one to have attempted partial pathfinding techniques and successfully interleaved planning with action. As future work, they plan to extend PRA\* to incorporate cooperative behavior.

#### 2.1.4 Cooperative Pathfinding

The paper titled Cooperative Pathfinding by Silver (Silver, 2005) addresses the problem of multi-agent cooperative pathfinding where agents must collaborate with each other to find optimal paths to their destinations. Although this paper is related to research on cooperative pathfinding, the reason we are reviewing its contributions is that the algorithms suggested by the author are inspired from and provide elegant extensions to HA\*. We will see later in our literature review how other scientists extended the work of Silver to derive more general and robust Hierarchical Pathfinding algorithms.

The author refers to a general, industry standard, algorithm called Local Repair A\* (LRA\*) which, the author claims, is often used in many video games to tackle the problem of cooperative pathfinding. LRA\* works by re-planning an agent’s path, using the A\* algorithm, when encountered with an unexpected obstacle such as another agent. The author criticizes LRA\*

by pointing out that in crowded situations, bottlenecks can take very long time to get resolved. He also states that frequent re-planning in order to avoid collisions creates “visually disturbing behavior that is perceived as unintelligent”.

Silver proposes three novel algorithms in his paper : Cooperative A\* (CA\*), Hierarchical Cooperative A\* (HCA\*) and Windowed Hierarchical Cooperative A\* (WHCA\*). A brief description of the three algorithms are as follows:

CA\*: In this algorithm the path for each agent is calculated individually in “three dimensional space-time” and a reservation table is maintained that contains entries about the location of an agent (on the way to its goal) at certain instances of time. An entry in the reservation table is avoided by other agents when planning their routes. An additional “wait move” is also incorporated in the table in which an agent waits for another agent to pass before moving.

HCA\*: This algorithm is inspired from HA\* (Holte et al., 1996). However the abstraction employed here simply consists of a high level map in which all mobile obstacles, other than the current pathfinding agent, are non-existent. The reservation table of CA\* is used in tandem with the abstraction when planning paths. Another feature of HCA\* is that it uses Reverse Resumable A\* (RRA\*) for calculating abstract distances between two locations. RRA\* works by running A\* in reverse, from goal to start.

WHCA\*: WHCA\* fixes a window to the look ahead of HCA\*. Thus

WHCA\* calculates partial paths up to a certain depth. At regular intervals, as the agent has followed a certain distance of the planned partial path, the window depth is shifted forward (toward the goal) and a new partial path is calculated. In order to guarantee that the agent is moving in the correct direction, the path planning in the abstract map is performed up to the goal (that is the complete path is calculated).

For the experiments, the author uses 10 randomly generated mazes. The mazes were of size 32 x 32 and obstacles were strewn over 20% of the tiles. The tiles were four-connected. Then random start and goal positions were generated for a number of agents such that for no two agents, the start or goal positions coincided. In the experiments, CA\*, HCA\*, WHCA\* and LRA\* were compared in terms of success rate (agents being able to reach goals within 100 turns), path length and cycle count ( an agent visiting an already visited grid).

The author claims that as the map gets more crowded, LRA\* begins to struggle for a higher success rate. CA\* and HCA\* perform better than LRA\*. WHCA\* performs the best in this regard, especially with a bigger window size. For measuring path length, the author fixed the shortest distance as a lower bound and claims that using LRA\*, the path lengths are almost twice the lower bound while CA\* and HCA\* finds paths that are only 20% above the lower bound. With regard to the cycle count, the author states that LRA\* produces 10 times the number of cycles produced by the algorithms proposed by the author.

In the concluding section, the author states that for “Simple environments”, LRA\* is sufficient to find optimal paths. But in more complex environments involving multiple moving obstacles, CA\* and its family perform better in finding optimal paths in a reasonable amount of time.

## 2.2 Improvements to Near Optimal Techniques

### 2.2.1 Improving Collaborative Pathfinding using Map Abstraction

In the paper titled “Improving Collaborative Pathfinding using Map Abstraction” (Sturtevant and Buro, 2006), the authors address the problem of cooperative pathfinding by combining partial pathfinding (Sturtevant and Buro, 2005) with WHCA\*(Silver, 2005) and cooperative pathfinding techniques suggested by Silver (Silver, 2005) with PRA\*(Sturtevant and Buro, 2005).

The authors refer to A\* (Hart et al., 1968), WHCA\*(Silver, 2005) and their previous work PRA\* (Sturtevant and Buro, 2005). While commenting on the shortcomings of the previous work, the authors state that A\* is inadequate for multi-agent pathfinding and WHCA\* assumes some restrictions such as four direction movements and constant speed of all units.

The authors propose two algorithms in (Sturtevant and Buro, 2006). The first algorithm, WHCA\*(w,a) is an extension of WHCA\*. The new algorithm accepts two parameters  $w$  (the window size) and  $a$  (the abstraction level used). This algorithm allows WHCA\* to operate on higher levels of the hierarchy instead of the base level. The second algorithm attempts to combine WHCA\* with PRA\* yielding Cooperative Path-Refinement A\* (CPRA\*). The algorithm uses PRA\* at all levels of hierarchy other than the lowest level, where WHCA\* is employed.

In order to compare the two new algorithms, the authors used the maps shown below (Sturtevant and Buro, 2006), p.4. In these maps the darker areas are not navigable, while the lighter are possible start and goal positions. Mobile agents are made to move from a random location on the left side of the maps to a random location on the right. The algorithms were compared in terms of nodes expanded - initially (“during the first second”) and in the subsequent seconds. Path quality was also compared by measuring the maximum of total distance traveled by all agents.

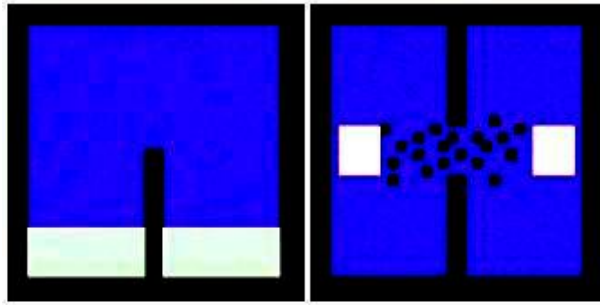


Figure 4: Experimental maps for  $WHCA^*(w,a)$  and  $CPRA^*$

From the results the authors obtained, it was deduced that  $WHCA^*(w,a)$  has a lower initial cost (expands less nodes in the first second) than  $CPRA^*$ . However in subsequent seconds  $CPRA^*$  expands less nodes than  $WHCA^*$ . In terms of path quality, the authors state that the two algorithms have similar performance.

Finally the authors conclude the paper by stating that if there are many units in the game scenario and there are limitations in memory, then  $CPRA^*$  is a better choice than  $WHCA^*(w,a)$ . On the other hand if higher initial cost is not a problem and there are no memory constraints, then  $WHCA^*(w,a)$  performs better. As future work, the authors plan to store information like traffic congestion in the abstracted map, so as to avoid those routes when planning. The authors also propose to dynamically vary the window size of  $CPRA^*$  according to the congestion level of the map.

### 2.2.2 HPA\* Enhancements

In the paper titled “HPA\* Enhancements” (Jansen and Buro, 2007), the authors address the problem of optimal pathfinding using hierarchical techniques. They do this by proposing three improvements to  $HPA^*$ .

The previous work referred by the authors includes  $HPA^*$  (Botea et al., 2004),  $PRA^*$  (Sturtevant and Buro, 2005) and Triangulation Refinement  $A^*$  ( $TRA^*$ ) (Demyen and Buro, 2006). While the authors do not find any shortcoming of  $HPA^*$ , they claim that the improvements suggested by them makes  $HPA^*$  faster and more optimal.

The improvements suggested by the authors are as follows:

**Faster Path Smoothing** The authors state that in HPA\* path smoothing is performed to make them shorter and more optimal. The smoothing method proposed by Botea et al. shoots imaginary straight lines in all eight directions from each node  $n$  on the path. When a line reaches another node  $m$  further up the path, the intermediate nodes between  $m$  and  $n$  are replaced by a straight line and the algorithm continues with two positions before  $m$  on the new path. The authors state that the computation involved is expensive, though the path received is close to optimal. The authors address this issue by placing a “bounding box” along the path to be refined. Place smaller bounding boxes makes the smoothing process faster although the path received is less optimal. The authors claim that the reduction in time has greater impact than the slightly longer paths received on the overall quality of the process.

**Using Dijkstra’s Algorithm** To find paths between entrances of a cluster, HPA\* uses A\* algorithm. The authors propose to use Dijkstra’s algorithm instead since the worst case running time of A\* is worse than the worst case running time of Dijkstra’s single source shortest path algorithm.

**Lazy Edge Weight Computation** HPA\*’s standard strategy to deal with dynamic environments is to recompute entrances and edge weights of affected clusters when changes occur in the game environment. The authors propose to compute edge weights on demand when an agent requests a path. The



authors state that in an optimistic situation the weights of some of the unnecessary edges will never be calculated during the game play.

In order to test the theories, the authors use two sets of maps: the first comprised of 116 real game maps from Baldur's Gate and Warcraft III and 80 artificial maps; both were of size 512 x 512. The percentage of blocked portion of the map was varied from 10 to 80. For each map, random start and goal positions were chosen. The authors compare standard HPA\* to improved HPA\* in terms of relative path length and time to find a path. The experimental results they obtained suggested that pathfinding time increases as the size of the bounding box is increased. They also find that sub-optimality decreases as the size of the bounding box increases. They also find that computing edge weights using Dijkstra's algorithm takes far less time than using A\*. When testing the algorithms for lazy edge weight computation, the authors claim to have found that using the lazy technique, the abstract map is built very quickly since no expensive pre-computation of edge weight is involved. They also state that the total time for calculating edge weights was found to be less than the total time using the eager approach. The authors attribute this to the fact that some of edge weights are never computed using the lazy approach.

In conclusion the authors state that the improvements to HPA\* suggested by them proves to be beneficial given the scope of the experiments performed. They state that although preliminary results are promising, further research is needed to judge the merits of the improvements in dynamic environments.

## 3 Demand Sensitive Map Abstraction

### 3.1 Motivation

HPA\* effectively solves the problem of pathfinding in large search spaces in a reasonable amount of time. The technique, however, has its own limitations which prevent it from being used in commercial games. The expensive pre-caching of intra edges step saves a lot of time on scenarios where most of the map is being used for pathfinding. On the other hand, the same feature becomes an artifact when only a part of the map is traversable and pre-caching intra edges of unused sectors of the map is entirely unnecessary (Jansen and Buro, 2007). A corollary of the same situation occurs when the map is dynamic and a change in Traversability in a portion of the map calls for an expensive update of the abstract map. Another interesting observation is that there is no scheme of varying the granularity of abstraction *non-uniformly* across the map in the HPA\* paper or any of the literature that followed it -varying the granularity of abstraction, non-uniformly, (at the single high level) depending on the last pathfinding time (time taken by the A\* search to find the previous path), is the central idea of our thesis.

The motivation for our thesis sprouts from the idea of tessellating terrain using triangular bin-trees (Samet, 1990) (Duchaineau et al., 1997) and the fact that it is reasonable to put a cap on the time required to find a path from start to goal. An ideal time range to find an initial path would be 1 millisecond to 3 milliseconds (ms) (Dalmau, 2003) (Bulitko et al., 2007)

given the current technology standards. In the following paragraphs we will emphasize the two ideas.

In the paper titled “ROAMing Terrain” (Duchaineau et al., 1997), the authors use spatial data structures called triangle binary trees (bin-trees) to dynamically vary the Level of Detail (LOD) in a map by using a grid of triangles and splitting or merging them as needed to increase or decrease the level of detail as needed, respectively. The dynamic LOD technique inspires our thesis in that, we use the same data structure to create our abstract map and dynamically vary the granularity of abstraction by joining or dis-joining neighbor triangles. The reason for choosing triangle bin trees is that it was found in (Demyen and Buro, 2006) that using triangles, instead of regular quadrilaterals as in HPA\*, to build the abstract map allows curved corners and irregularly shaped obstacles to be incorporated in the map and make use of extra space around them for pathfinding.

Now let us consider the second point in which we mentioned that it is reasonable to keep the pathfinding time between 1-3 ms approximately. This time does not include the path following or the execution step in which a game agent travels the planned path. In the second chapter of the book (Dalmau, 2003), the author writes about real time game loop models; a comprehensive discussion of the same is not in the scope of this thesis. The author defines games as “time dependent interactive applications” consisting of a virtual game world, a simulator to make the world seem real, a presentation layer which displays the virtual world to the player and controllers to allow players

to interact with the virtual world.

To say games are time constrained, means a game must display information at a constant speed which is usually above 25 frames per second (modern games run 60 frames per second or more). More frames per second implies less time available for real time simulators to modify the world to display new states. Real time interactive applications such as games consist of three tasks running simultaneously. First the current state of the virtual world must be updated constantly, second the player must be able to interact with the world and finally the result should be presented to the player using visuals and sounds. When information is displayed at 60 frames per second, the simulator has only 16.67 ms (approximately) to modify the world (update the world and render the result). The 16.67 ms in hand is to be spent judiciously and hence only about 1-3 ms can be ideally devoted to pathfinding according to Bulitko in (Bulitko et al., 2007).

In the next section, we will show how the above facts and conjectures are woven together to derive Pathfinding by Demand Sensitive Map Abstraction (DSMA).

### **3.2 Abstract Terrain Representation: Triangle Bintrees**

The current section gives an insight into Triangle Binary Trees. Please note that the term “cell” and “grid” will be used interchangeably to refer to

a minimum square unit of the game world (grid-based) that an agent or obstacle can occupy (for our experiments each cell is 8 x 8 pixels) as shown in figure 5 below:

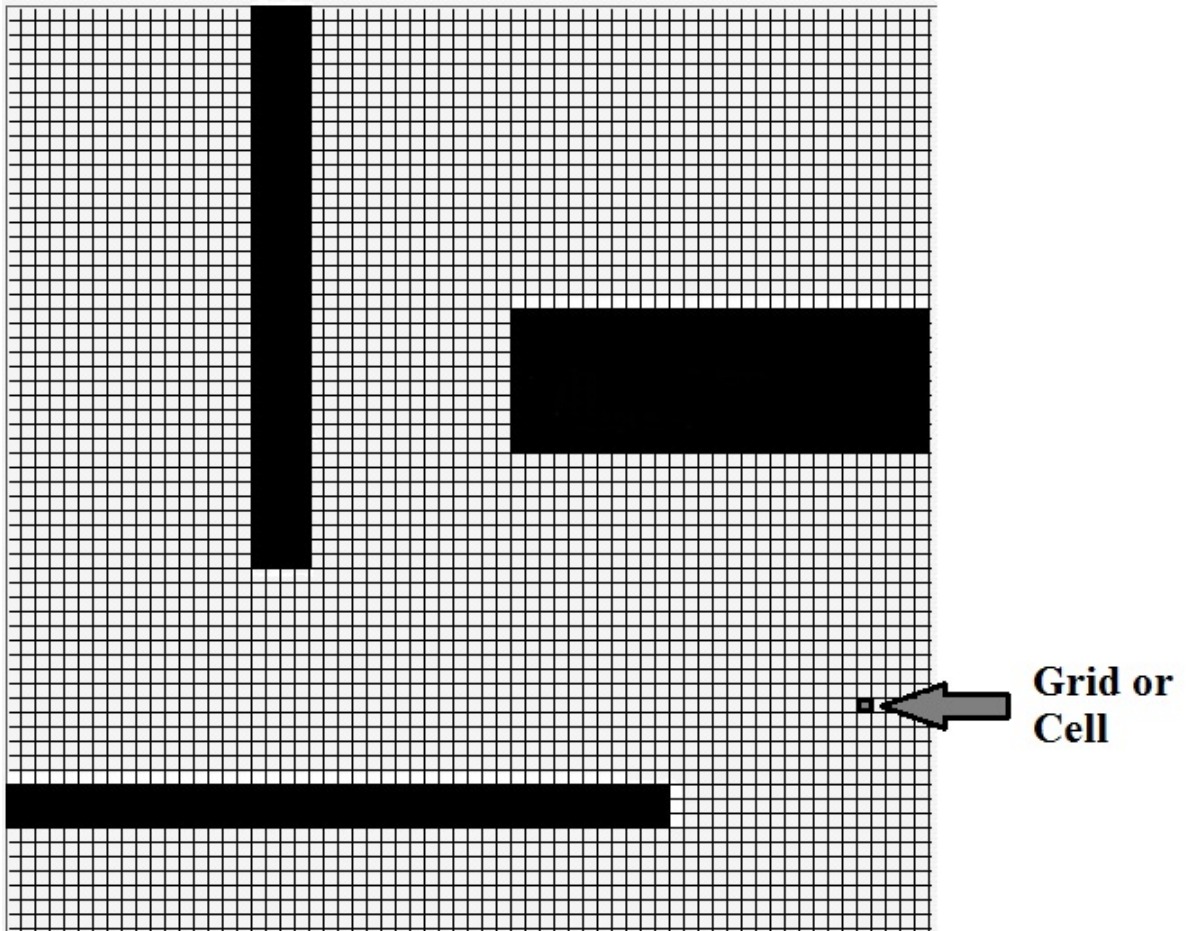


Figure 5: Grid/Cell

A triangle Binary tree (Lindstrom et al., 1996) (triangle bin-tree) is a spatial data structure. Triangle bin-trees are binary trees with space representational properties of Quad Trees. A triangle bin-tree is comprised solely of right isosceles triangles and hence never develops Cracks or T-junctions. Triangle bin-trees are mainly used for tessellating terrain (Duchaineau et al., 1997).

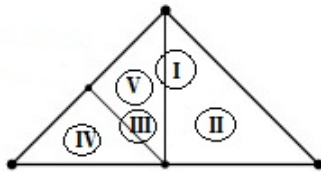


Figure 6: Illustration of Triangle Binary Tree

Let us consider the figure 6 above. The triangle bin-tree consists of a triangle (I) and two possible children- a left child (III) and a right child (II). When triangle I is decomposed, we obtain II and III. Similarly, when triangle III is decomposed, triangles IV and V are produced. Conversely, it is also possible to compose or merge two neighbor triangles to reduce granularity. Such decomposing and composing can be continued until a suitable granularity is reached for a given region or for the entire available area, uniformly or non-uniformly. Hence, in our thesis we use triangle bin trees to represent hierarchical abstract maps so as to vary the granularity of abstraction dynamically according to demand.

It is necessary to keep track of all the neighbors of a particular triangle so that cracks and T-junctions do not develop. In our implementation, we store the triangles as Triplets of three grid coordinates in a hash table to facilitate fast storage and retrieval. Henceforth, the term “triangle bin-tree” and “triplet” will be used interchangeably, referring to any arbitrary triangle in the abstract map. The neighbors are defined as triangles sharing two common low level cells. For the high level A\* search, triangles having one common cell are considered as neighbors if other strongly connected neighbors are not available. Every triangle has a level associated with it. When a triangle at level  $n$  decomposes, the resulting child triangles are of level  $n + 1$ . Similarly when two triangles both at level  $n$  merge, the resulting triangles are at level  $n - 1$ . Any given configuration of states in a triangle binary tree can be obtained from other states by series of composition and decomposition.



The illustration of level change is given below:

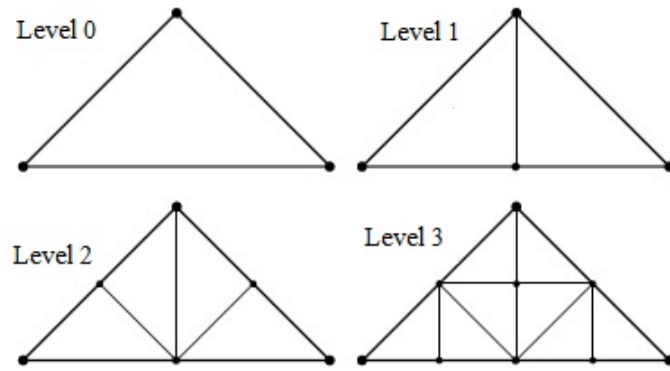


Figure 7: Levels in Triangle Binary Tree

The reason we are using triangle bin trees instead of quad trees as in HPA\*, is that in HPA\* the hierarchical paths are pre-computed and cached; while in our thesis we find hierarchical paths on-demand. Running A\* search on-demand in the hierarchal map using quad trees is, theoretically, an expensive process as there are more nodes to be processed (quads have more entrances/sides than triangles). Moreover decomposing a quad will produce four regular quads again making the hierarchical search expensive. On the other hand triangles when decomposed produce two new triangles and each of them can be decomposed if necessary.

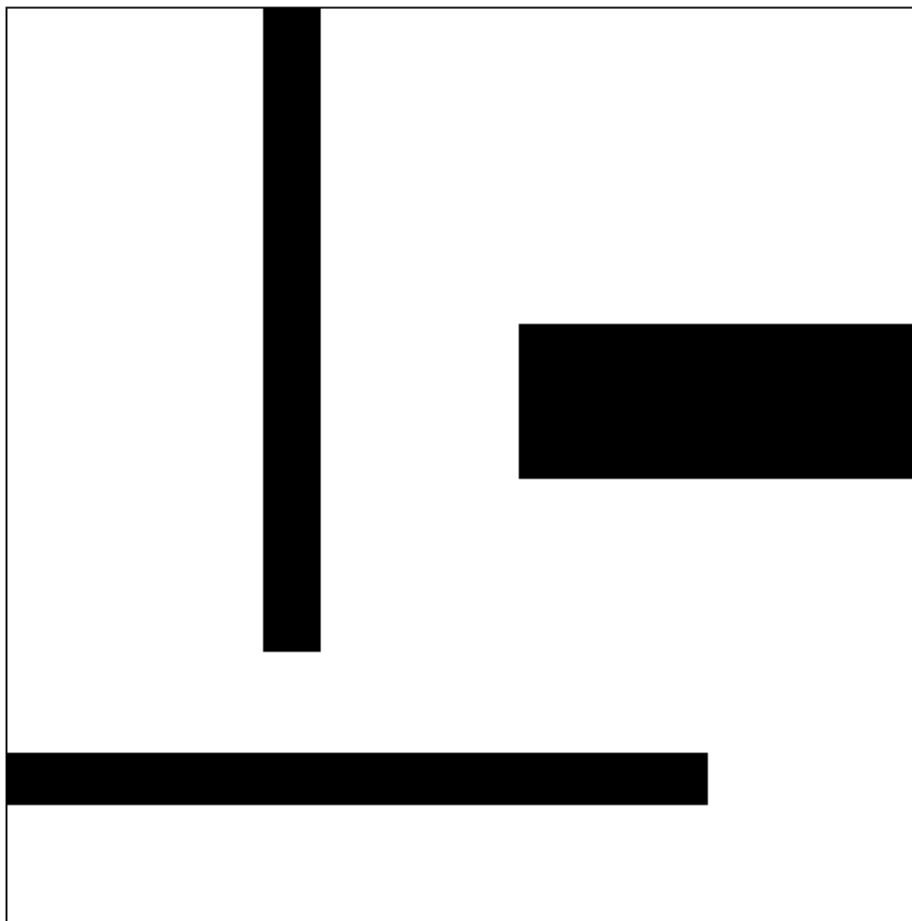


Figure 8: Sample Game Map

Let us consider a game map as shown in figure 8 above. The dark regions represent obstacles and the white region is the navigable space. Our first action is to lay grids on this map thereby getting the map as shown in figure 9 below. Each individual cell is 8 pixels by 8 pixels.

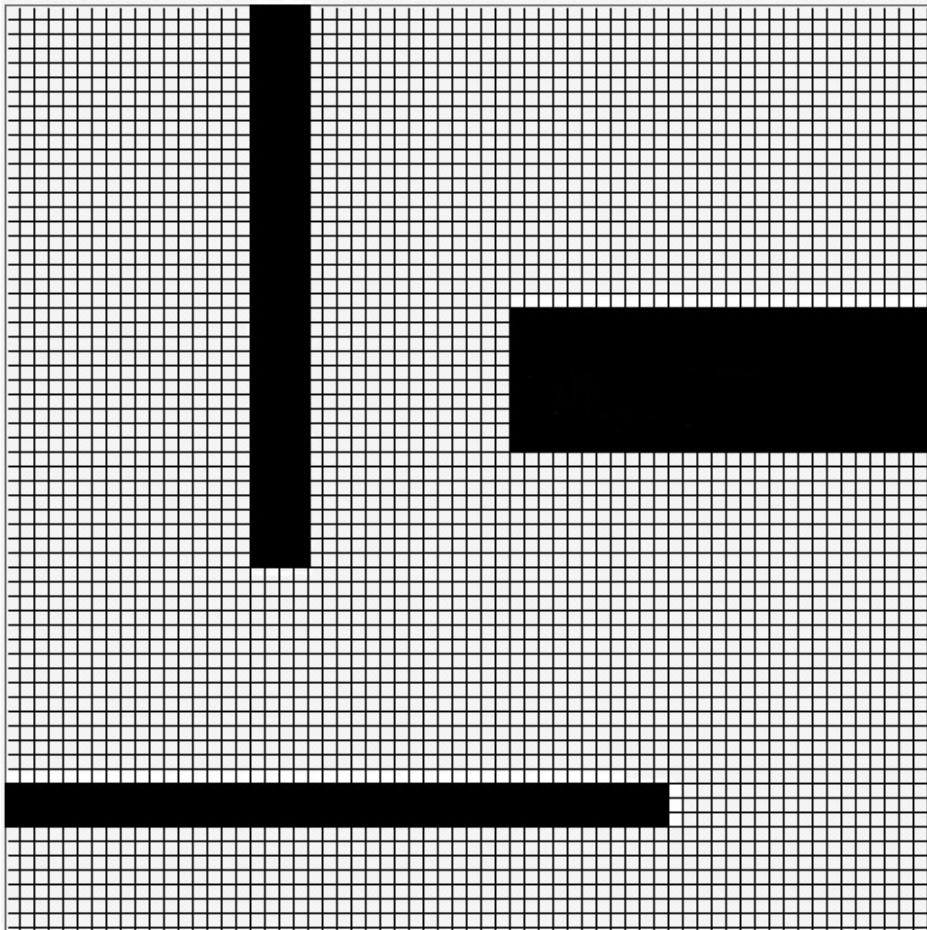


Figure 9: Sample Game Map with Grids

The initial hierarchal map is hard coded and will usually contain four clusters as shown in figure 10 below, we have marked the four clusters as T1, T2, T3, and T4 .

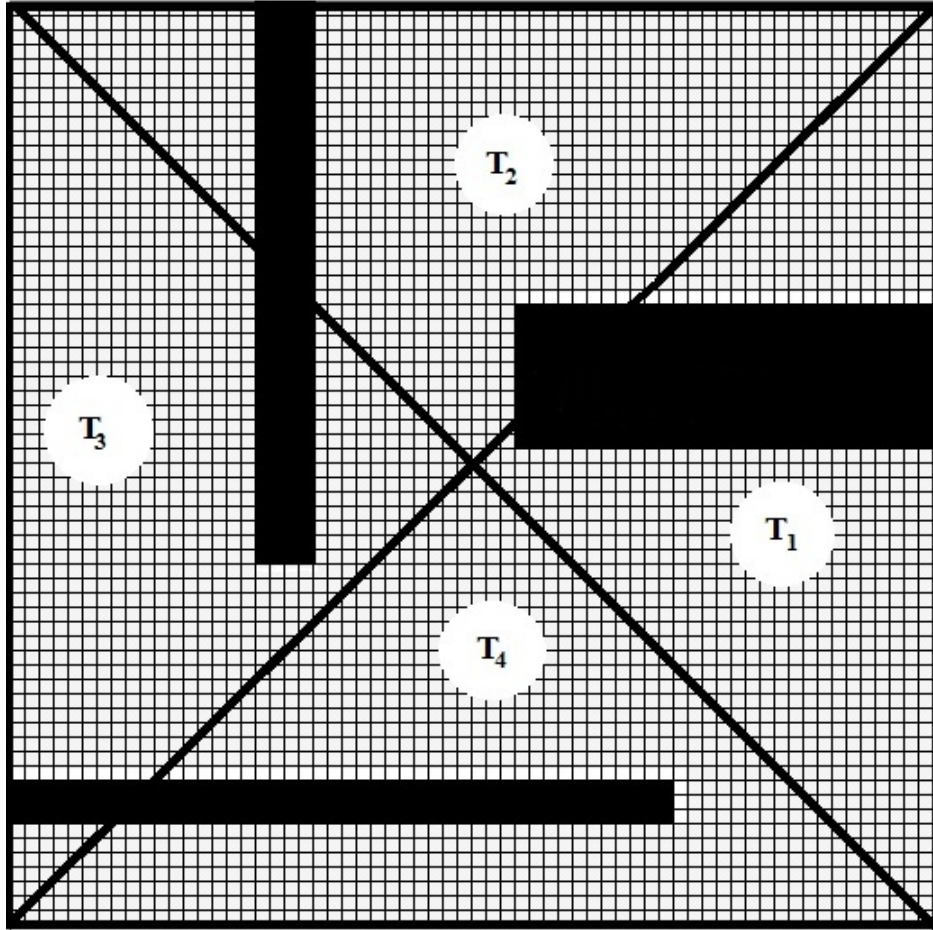


Figure 10: Sample Game Map showing Hierarchical Clusters

### 3.3 Measuring Demand

In our thesis, we vary the granularity of abstraction dynamically according to the demand associated with triplets. Hence, in addition to level we add another parameter- Demand, to triplets. A value associated with demand represents how many times a triplet was explored in previous hierarchical A\* searches. Demand is increased by one, for triplets containing start and goal nodes and for every triplet that gets expanded (explored) in hierarchical A\* search. We also decrease demand by one for every triplet that does not get expanded in the high level A\* search.

When the execution time of previous A\* search rises above three milliseconds, we decompose the highest demand triplet (breaking ties arbitrarily) thereby forcing the low level A\* search to expand less nodes so that the total search time is reduced. This step is expected to return less optimal path for lower execution time.

Conversely, if the execution time of previous A\* search falls below one millisecond, we merge two of the lowest (collective) demand neighbor triplets (breaking ties arbitrarily) since more time can be allocated to pathfinding. Combining two low demand triplet forces the low level A\* search to explore more nodes. This is expected to result in more optimal paths at the expense of higher execution time.

### 3.4 Composition and Decomposition Queues

Previously, we have discussed Triangle Bin Trees (Triplets) and how the demand of a triplet is manipulated. In this section we will discuss the process to keep track of high and low demand triplets and how we compose or decompose them when needed.

#### 3.4.1 The Decomposition Queue

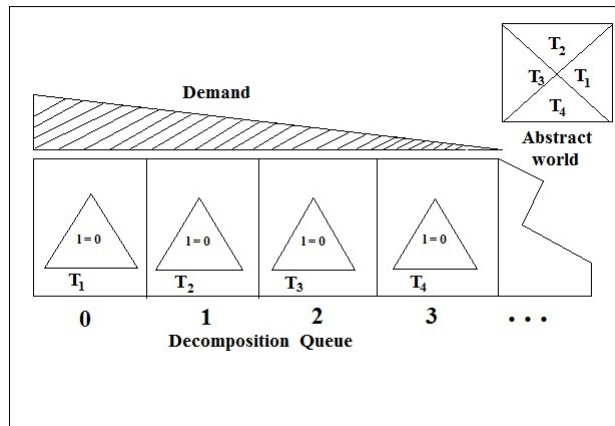


Figure 11: The Decomposition Queue

Initially this queue contains all triplets in the abstract map at level 0. A visual representation is shown in figure 11. We assume that initially all triplets in the abstract world (hard coded) are at their coarsest level; that is we do not decompose triplets at level 0. Figure 12 shows an initial world/game map and the initial decomposition queue alongside it.

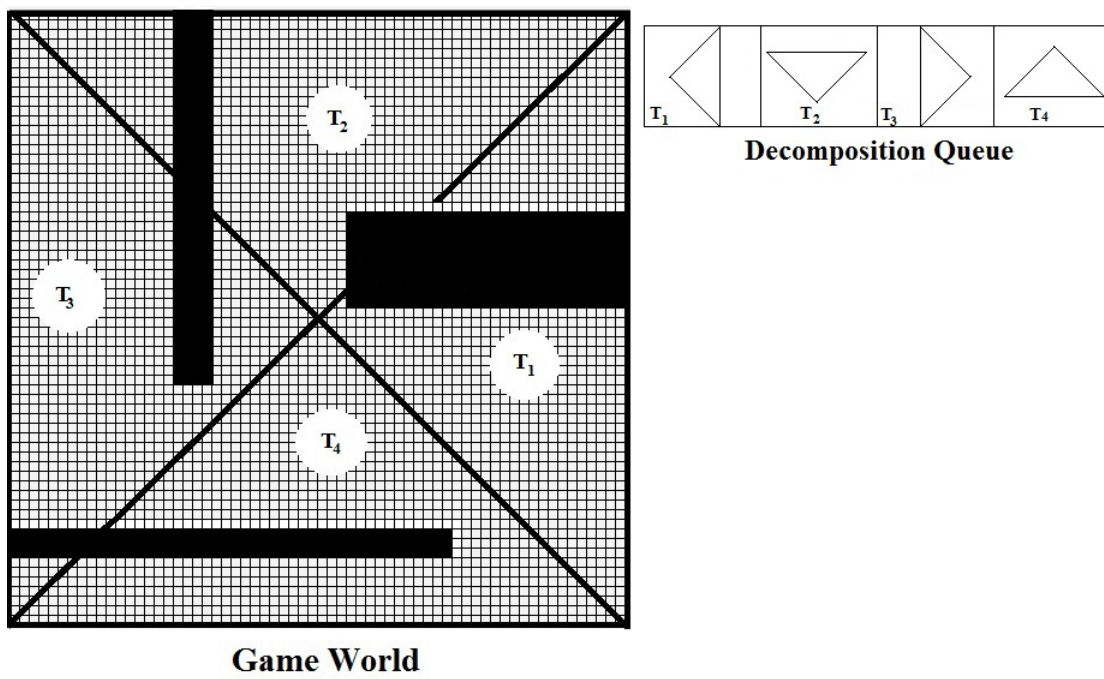


Figure 12: Initial Decomposition Queue



The interpretation of figure 12 is that the abstract map is composed of four clusters T1, T2, T3 and T4 and each of the clusters can be further decomposed to derive non-uniformly fine grained abstract map. If any of the triplets is decomposed, each of the children will have half the demand of the parent and level of the children will be one more than the one of the parents.

### 3.4.2 The Composition Queue

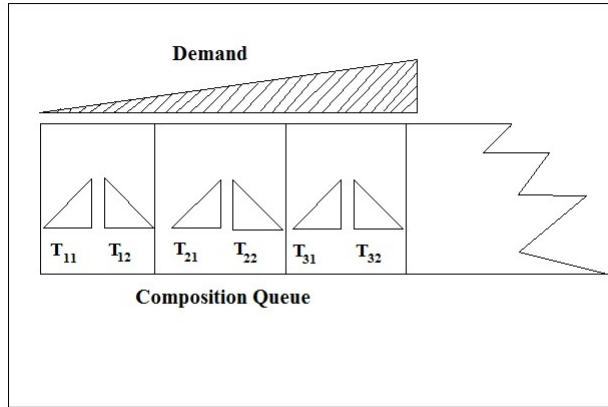


Figure 13: The Composition Queue

This queue contains pairs of triplets that can be composed or merged, with the lowest demand pair in the front of the queue. Initially this queue is empty. In our thesis and experiments we maintain a policy that triplets at level 0 cannot be composed or merged. That is, the composition queue of the initial map is empty. This policy can however be modified in future work related to this thesis. The trivial idea is shown in figure 14 below.

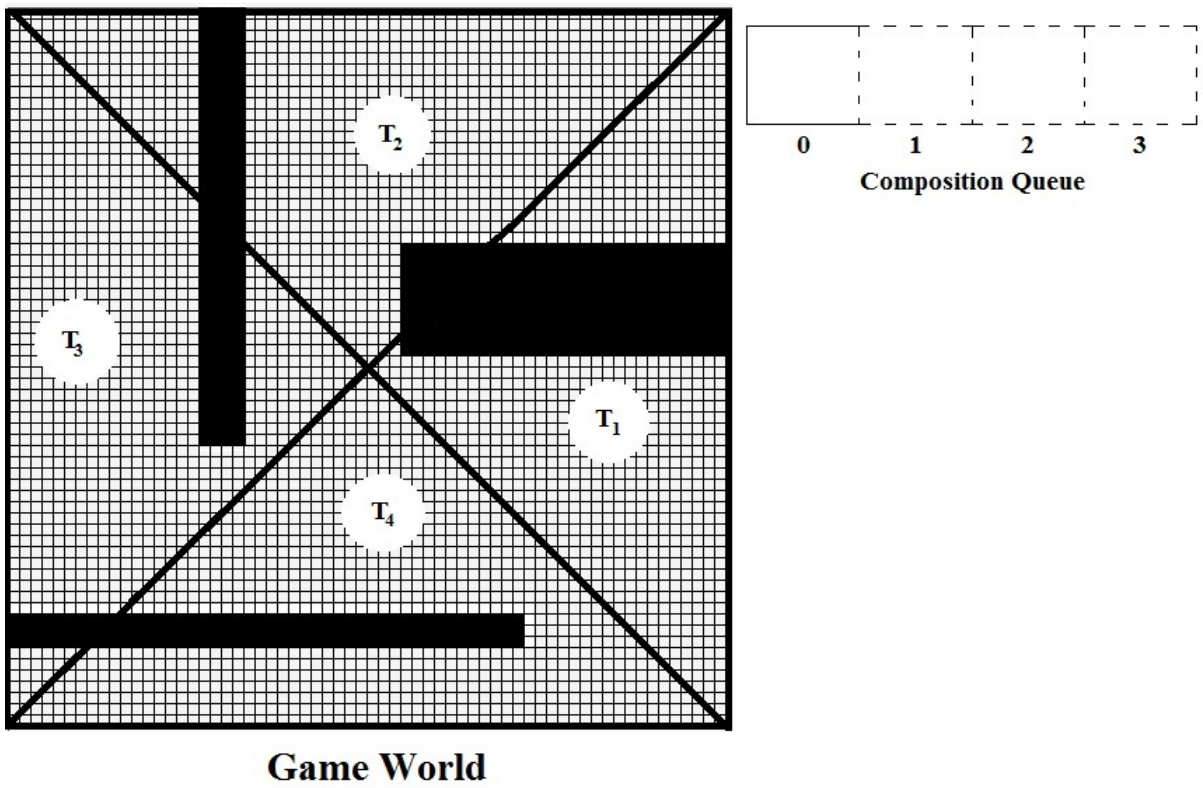


Figure 14: Empty Composition Queue

However if one of the triplets (say T1) was decomposed in a previous operation (into say T11 and T12), the composition queue would have one entry as shown in figure 15 below.

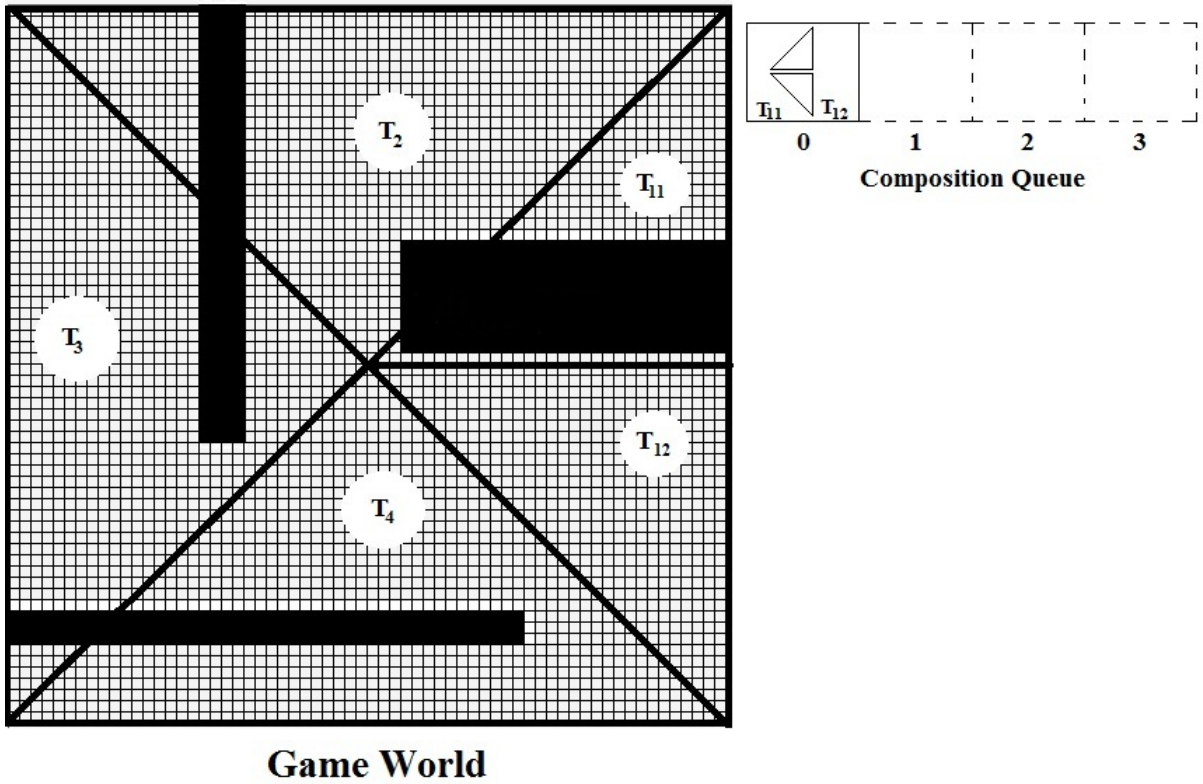
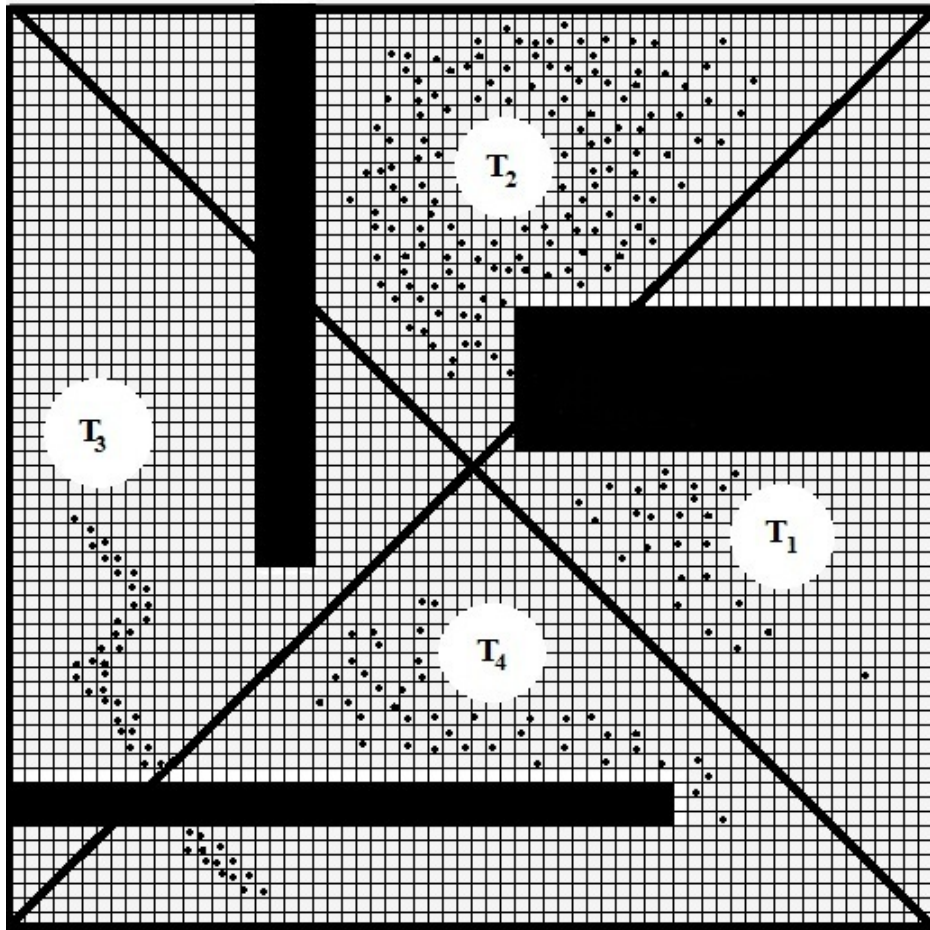


Figure 15: Initial Composition Queue

The interpretation of the figure 15 is that triplets T11 and T12 can be composed/merged later into the game-play to derive their parent triplet. If T11 and T12 are composed the parents demand will be the collective (sum of) demand of T11 and T12 and level of the parent will be one less than the level of T11 and T12.

### 3.5 Operations

#### 3.5.1 The Decomposition Operation



**Game World**

Figure 16: Relative Demand Levels

Consider the game map in figure 16; the dots represent the paths taken by game agents. In other words, the dots are the foot prints of the agents. As stated previously, we keep track of these footprints and use them to dynamically vary the granularity of abstraction of the triplets. In addition to that, we also keep track of the running time in A\* search . If the last running time of A\* search went beyond of 3 milliseconds, we apply the decomposition operation.

When the decomposition operation is applied, we extract the highest demand triplet from the decomposition queue (T2 for the map shown above) and we decompose it; thereby replacing T2 by its children T21 and T22 as shown in figure 17 below.

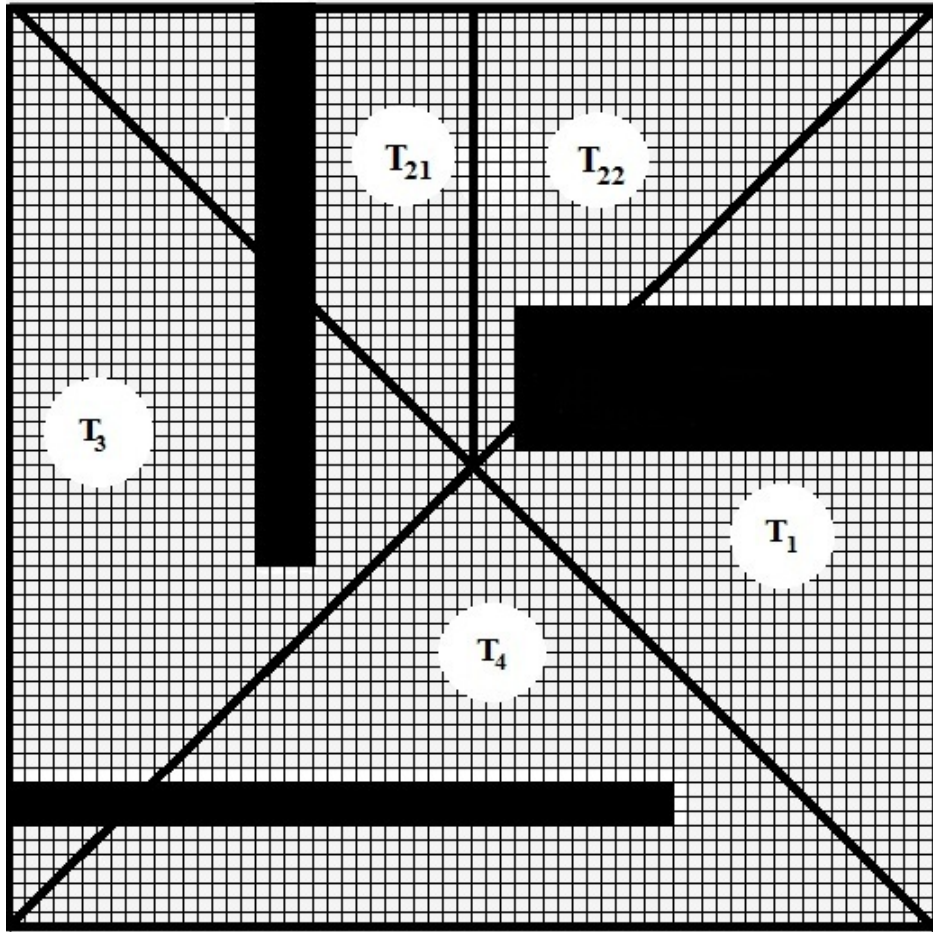


Figure 17: Decomposition

The way decomposition queue and composition queue is manipulated in a decomposition operation is illustrated in figure 18 below.

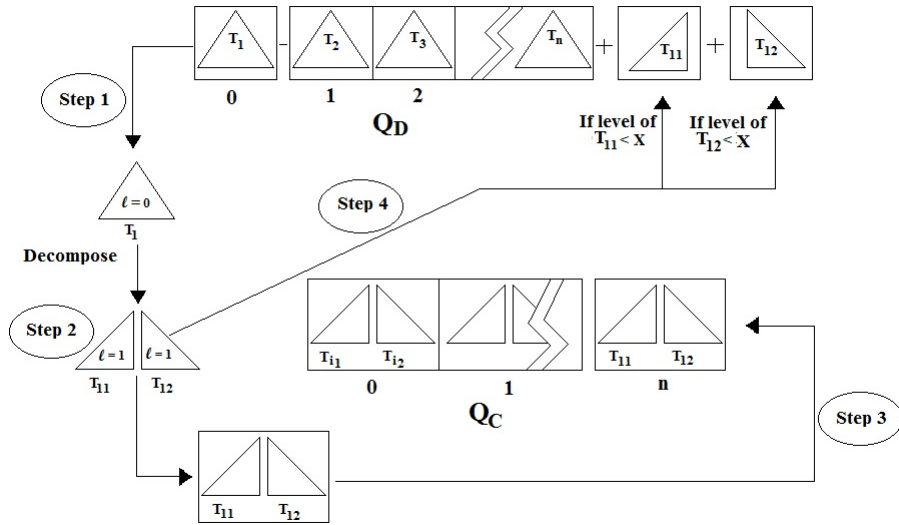


Figure 18: The Decomposition Operation

Illustrated in figure 18 above is the decomposition process. We pop the first triplet from  $Q_D$ , split the triplet as to form two isosceles triangles. Assign the demands of  $T_{11}$  and  $T_{12}$  as half demand of  $T_1$  for both. From the triplet that can be composed/merged and push it to  $Q_C$ . Additionally, it might be possible to decompose  $T_{11}$  and  $T_{12}$  unless they have reached the finest permissible granularity (say  $x$ ). So, we preform the check for granularity and push  $T_{11}$  and  $T_{12}$  to  $Q_D$ , if possible.

A simple algorithm for the decomposition operation is given below.  $TA$  is the last running time of  $A^*$  search and  $UTA$  is the Upper Threshold Time limit for  $A^*$  search which is 3 milliseconds in our thesis.  $Q_d$  is the decompo-



sition queue while  $Q_c$  is the composition queue.

---

**Algorithm 1** Decomposition Algorithm

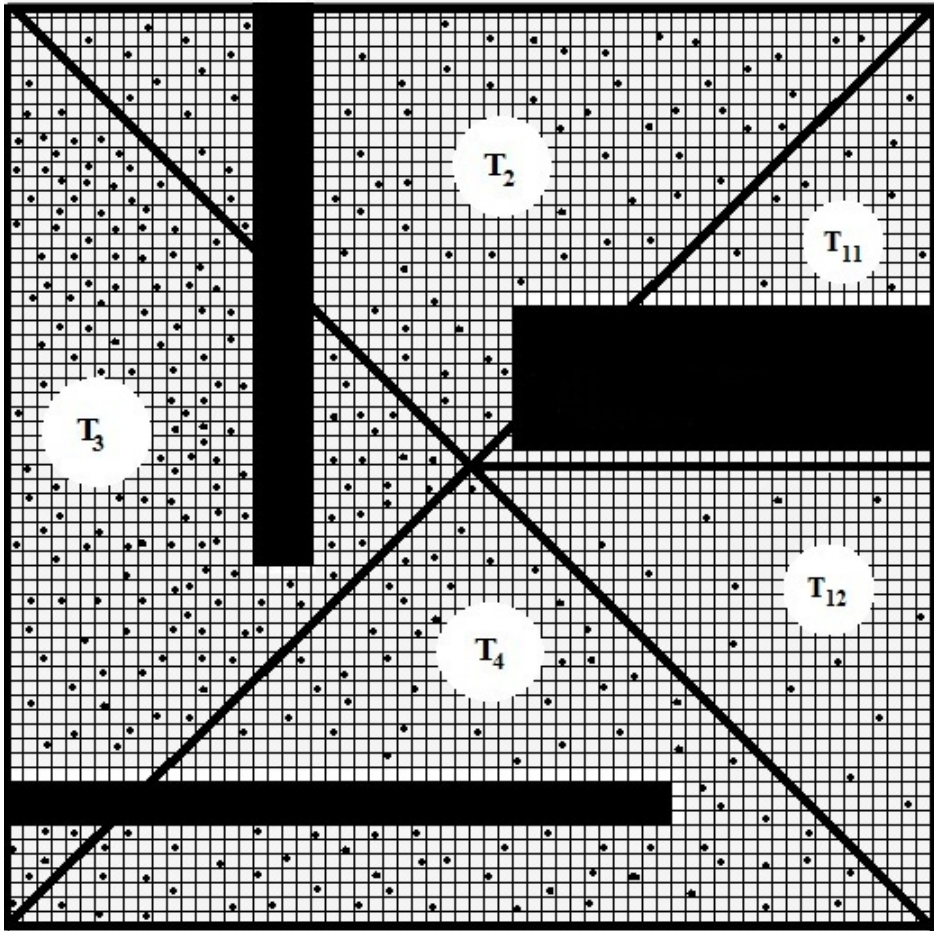
---

```
if  $TA \geq UTA$  then  
  Decompose{t}  
  Remove t from  $Q_d$   
  Add t11 and t12 to  $Q_d$   
  Add t11 and t12 pair to  $Q_c$   
end if
```

---

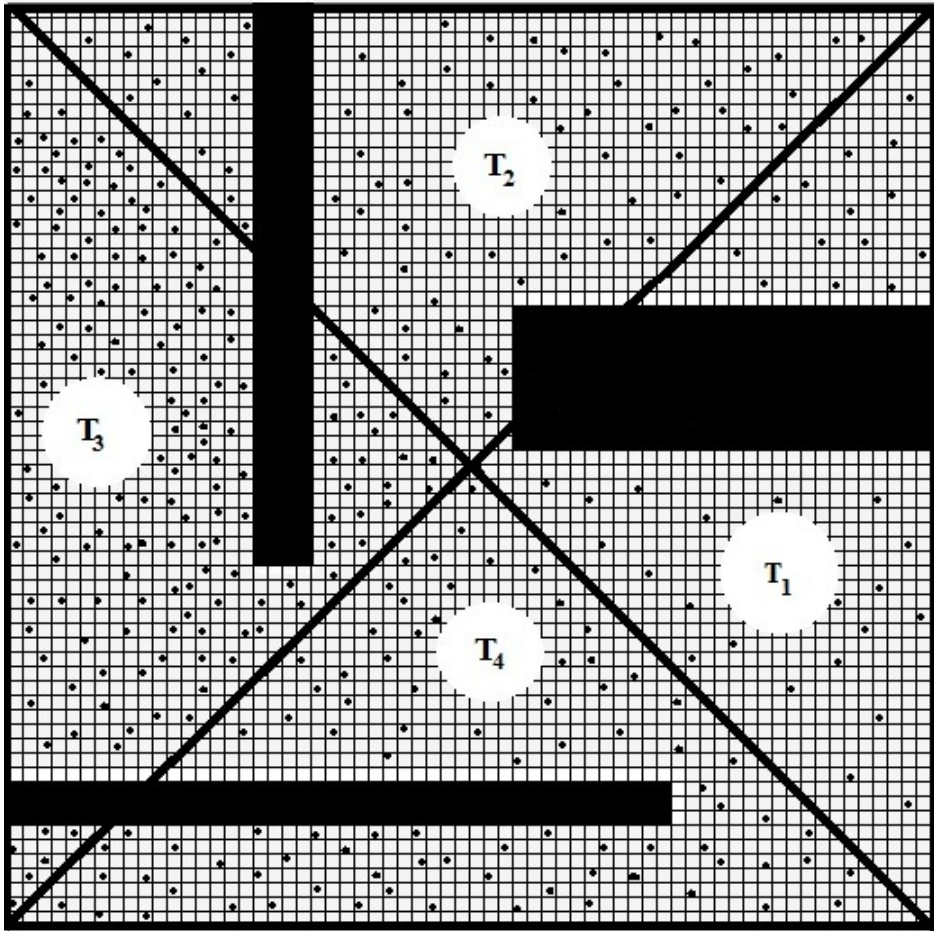
### 3.5.2 The Composition Operation

The reverse of the decomposition operation is the composition operation; which we apply when the last running time of an A\* search went below 1 millisecond. In this operation, we usually pick a triplet pair from the composition queue with lowest collective demand. Usually this is the first element of the queue since we sort the composition in ascending order of collective demand at regular intervals. For the game world depicted in figure 19, if the last A\* search time went below 1 millisecond, we would compose/merge T11 and T12 resulting in the game world shown in figure 20 below.



**Game World**

Figure 19: Relative Demand Levels



**Game World**

Figure 20: Composition

The procedure in which the two queues are handled in a composition operation is illustrated in figure 21 below.

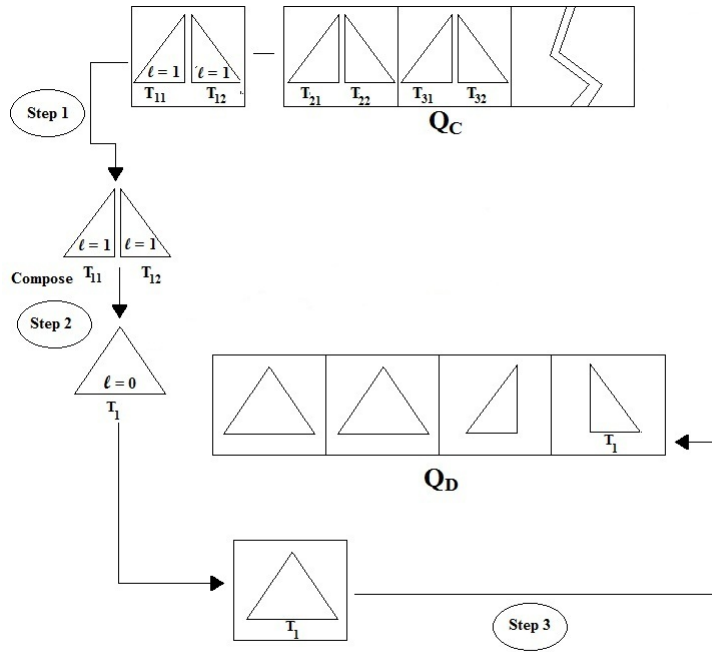


Figure 21: The Composition Operation

The illustration above shows the composition operation. A pair of triplets is popped out of  $Q_D$  and their parent triplet is pushed into  $Q_D$ . Demand of the parent is the combined demand of the child triangles. Since the demand for triplets is varied in subsequent  $A^*$  searches, it becomes necessary to organize  $Q_D$  and  $Q_C$  from time to time. Implying the content of the queues are sorted according to demand in ascending order for  $Q_C$  and descending order for  $Q_D$ . This operation, however is not performed very frequently to

save execution time.

Outlined below is a simple algorithm describing the steps involved in composition operation.  $TA$  is the last running time of  $A^*$  search,  $LTA$  is the Lower Threshold Time Limit for  $A^*$  search which is 1 millisecond in our thesis. We assume that at the front for the composition queue is an arbitrary triplet pair  $ti1$  and  $ti2$ . That is the pair has the lowest collective demand compared to all other triplet pairs.

---

**Algorithm 2** Composition Algorithm

---

```
if  $TA \leq LTA$  then
  Compose{ $ti1$ ,  $ti2$ } { This step results in parent triplet  $ti$ 
}
  Remove  $ti1$  and  $ti2$  from  $Qd$ 
  Add  $ti$  to  $Qd$ 
  Remove  $ti1$  and  $ti2$  pair from  $Qc$ 
end if
```

---

### 3.6 Pathfinding using DSMA

**Triplet Relationships** In this section we will discuss how DSMA finds path given a certain high level map. Two triplets in DSMA can have two kinds of relationships- they can be adjacent to each other or they can be neighbors. Adjacent triplets are those that have a side (or part of it) in common. Neighbor triplets have only one vertex (cell) in common. Hence all adjacent triplets are neighbors but not all neighbors are adjacent to each other. All neighbors are considered in the high level search as well. The relationships are demonstrated in the figure 22 below:

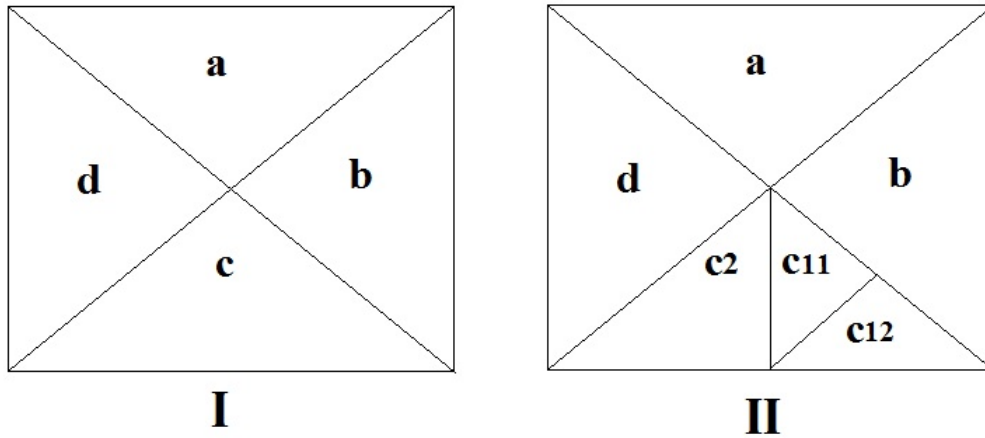


Figure 22: Adjacent and Neighbor Triplets

In I above all the neighbors are a,b,c and d while the adjacent triplets are (a,b), (b,c),(c,d) and (d,a). In figure II above both c11 and c12 are adjacent to b while only c11 is adjacent to c2.

**Pathfinding** Now let us consider the map in figure 23 below with the start and goal positions marked as  $S$  and  $G$ , respectively:

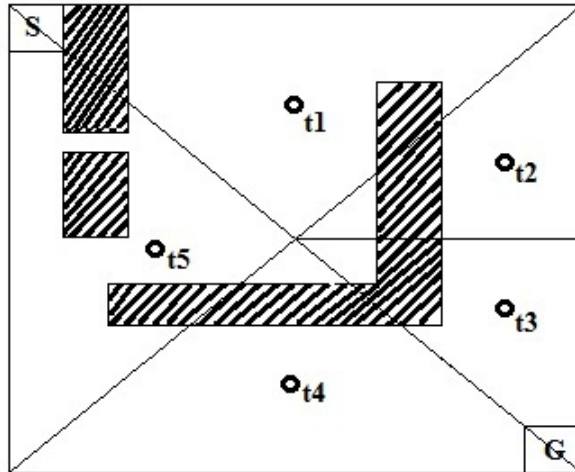


Figure 23: DSMA Map showing high level triplets and start and goal positions



As we can see above the start and the goal cells are ambiguously placed such that their centers lie on the border of the triplets. Since we use `IsVisible()` method in `GraphicsPath` in `C#`, the method resolves such conflicts arbitrarily. One could possibly, as future work, break the tie by placing the grids in a triplet whose centroid is closest to the center of the cell. With the `IsVisible()` method we have used, the high level  $A^*$  search can possibly take two directions as shown in figure 24 below:

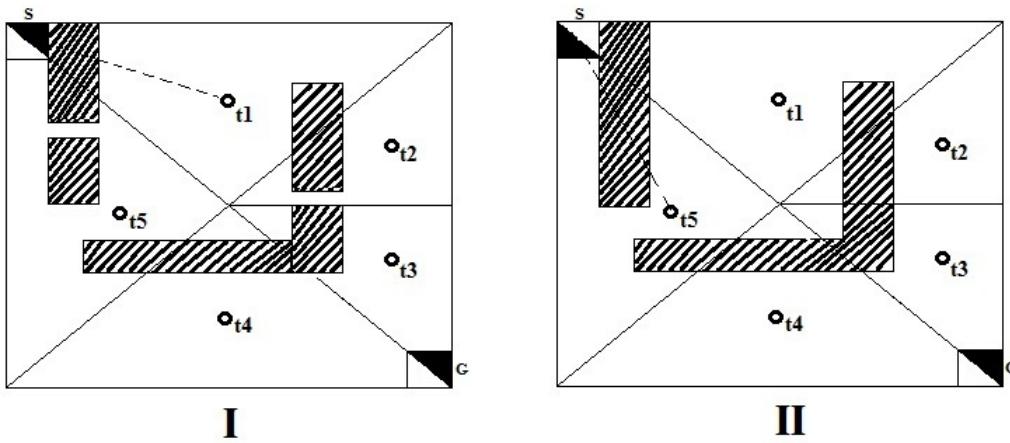


Figure 24: Two possible abstract paths

Let us proceed with case I above because it poses a new challenge which we will explore. The high level A\* search considers an abstract path from start to the centroid of t1 as shown in the dashed line below.

The high level A\* search does not consider obstacles, unless a triplet is over 90% full or an obstacle covers the centroid. If it is more than 90% full, a penalty is added to the triplet's f-value during high level A\* search. If an obstacle is covering a centroid, the low level A\* search attempts to connect the present way-point to the next way-point (centroid), after skipping the centroid being covered by an obstacle.

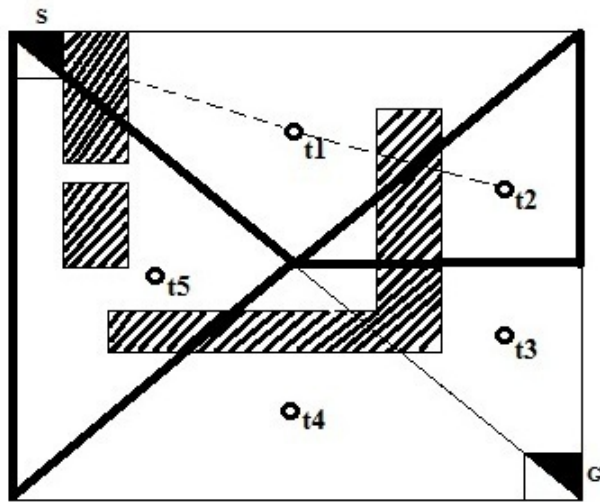


Figure 25: Choosing the Adjacent Triplets

The high level A\* search considers two of its adjacent triplets shown in bold lines in figure 25 above and selects t2 as it has a lower f-value. The tentative abstract path from t1 to t2 is shown in the dashed line above.

Now, let us assume that the goal is in t3. The only adjacent triplet to t2 is t3 and t3 contains the goal. Hence the high level search is successful and the complete abstract path is shown in figure 26 below:

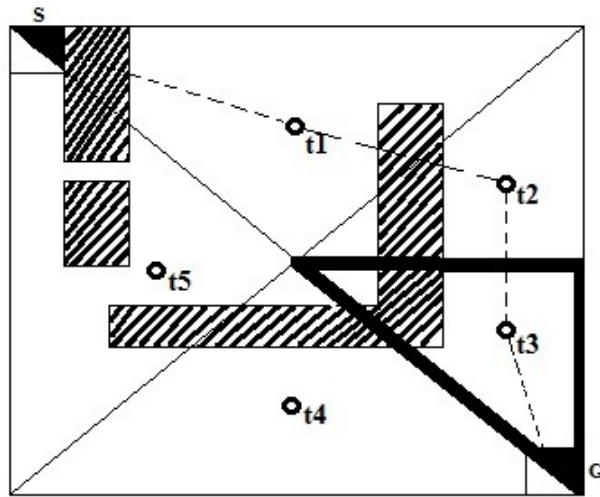


Figure 26: Complete Abstract Path

Thus the high level A\* search has discovered way-points which the low level A\* search must now connect to get the actual path. We are assuming that the abstract path is optimal since we have used A\* search and it is to be noted that the diagram above is not to scale. The challenge of crossing the obstacle, we mentioned above has been resolved by the low level A\* search, as shown in figure 27 below:

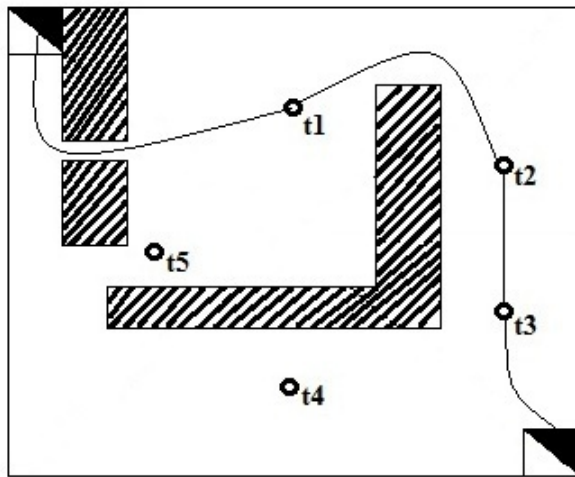


Figure 27: Actual Path

Another very important property of DSMA evident from the above is that the low level search is not restricted to the boundaries of the high level triplets. This is a departure from conventional hierarchical pathfinding techniques where the low level A\* search is restricted to the boundaries of the high level clusters, as mentioned in the introductory paragraph of section 2.

**An Alternate Case:** Let us consider a case in which the goal was found (arbitrarily) to be in t4 instead of t3. The resulting high level abstract path and actual path is shown in figure 28 below as I and II respectively:

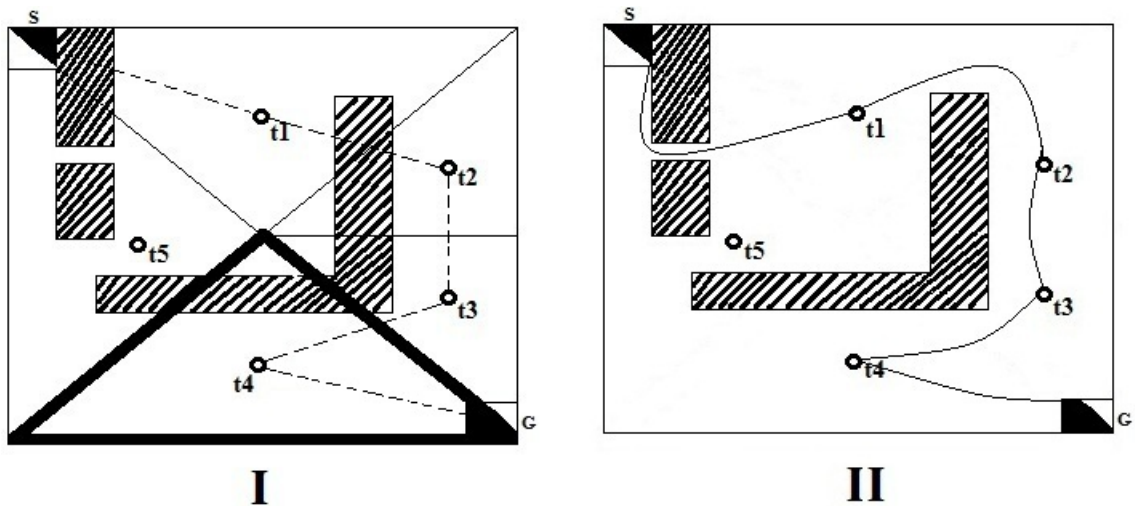


Figure 28: An Alternate Case

The resulting path has a detour via the centroid of t4. There are two solutions to resolve the problem, assuming that A\* does not find the path via t5 and t4 to be more optimal. One has already been mentioned above - to resolve the conflict by Euclidean distance from the centroid. The other solution is expensive but widely used to achieve better path quality- path smoothing and refinement techniques (Botea et al., 2004). The techniques were used in HPA\* to remove undesirable detours. Once path smoothing is applied, we can expect to receive as path as shown below:

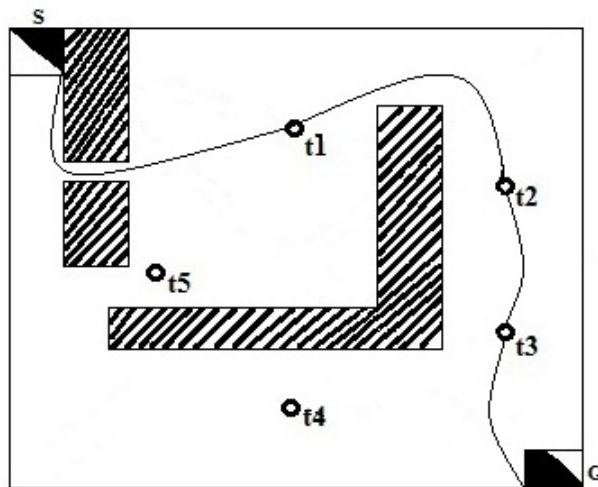


Figure 29: After Path Smoothing

## 4 Experimental Analysis

### 4.1 The Setup

In this section we perform a set of experiments. This section describes the experimental setup. All experiments are inspired from (Jansen and Buro, 2007) and (Sturtevant and Buro, 2005).

**Maps and Obstacles** We use ten different maps inspired from commercial RTS games. Each of the maps are scaled to three different pixel sizes: 256 by 256, 512 by 512 and 1024 by 1024. This gives us 30 different maps. All maps are grid worlds with octile navigational freedom. We conduct all experiments with one agent only, as our claims and experiments encompass the domain of path planning only and not the nuances of path following. In addition to hard coded obstacles in the maps, we also introduce random obstacles into the map (between calls to the respective algorithms; that is without modifying the map when a certain instance of path planning is in progress). The random obstacles are varied in density from 20% to 40%; always ensuring that the start and goal points are connected. That is, a path exists between the start goal points, for the algorithms to discover. We ensure this by running A\* search first and if the search fails we discard the start-goal pair. All obstacles are made to fit to cells, that is, an obstacle cannot occupy a cell partially.

**Procedure** In order to compare the algorithms we generate 500 random start and goal locations for every map. We compare DSMA to two versions of a generic hierarchical A\* search. The first version has a sparse and constant (single level) abstract map, containing eight triplets. We call this algorithm sparse HA\*. The second version has a denser and constant (single level) abstract map, containing 64 triplets. We call this algorithm dense HA\*. The sparse map has 8 triplets because that is the minimum number of triplets DSMA is allowed to have and the dense has 64 because that was maximum number of triplets used in HPA\* experiments and that is the maximum number of triplets we allow DSMA to create. It is to be noted that we are using some of the experimental standards of HPA\* and not comparing DSMA to HPA\*. By sparse or dense we are referring to the number of clusters in the abstract map of the generic HA\*, as shown in the diagrams below:



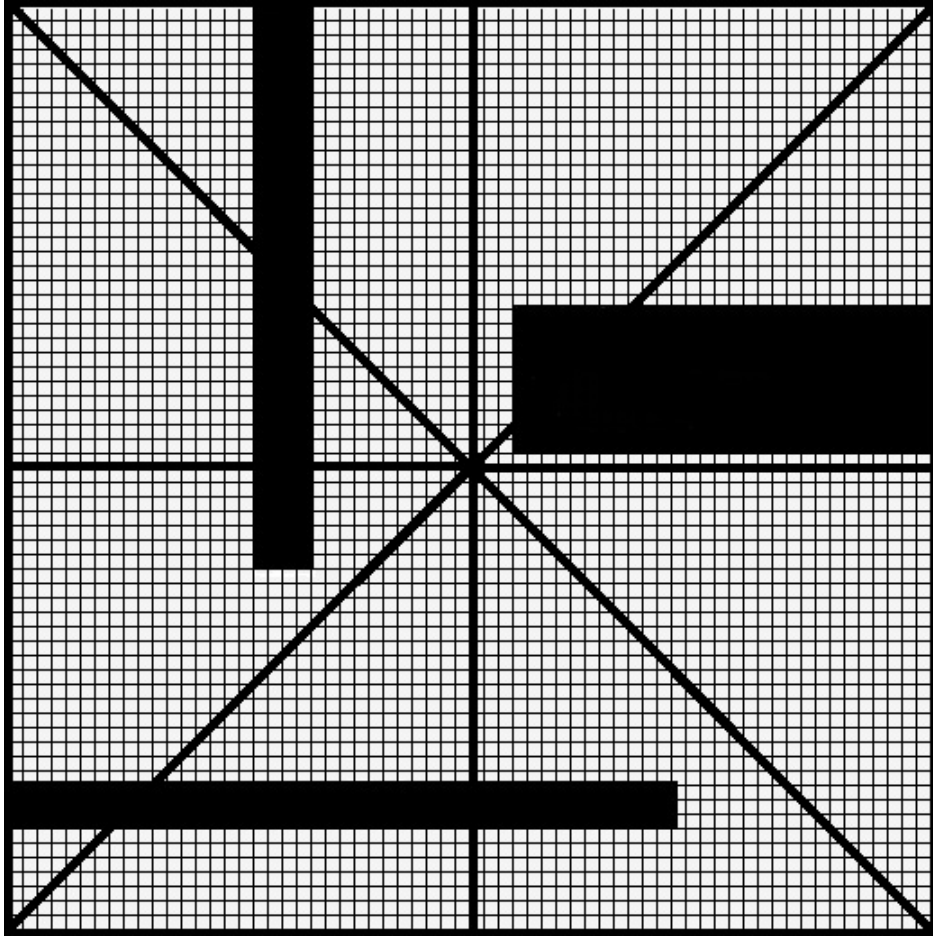


Figure 30: Sample Sparse HA\* Map

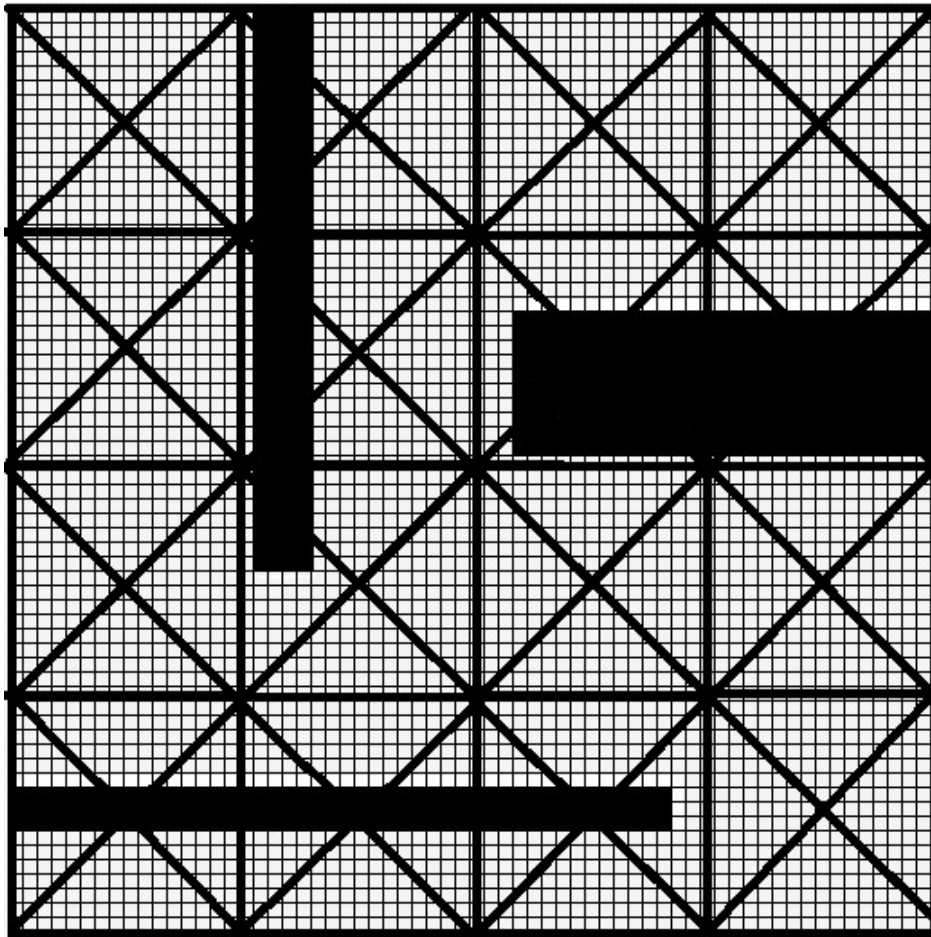


Figure 31: Sample Dense HA\* Map

The clusters are regular and hard coded. We are comparing DSMA to sparse and dense HA\* to see if we can benefit from dynamically varying the granularity of abstraction, as opposed to having a static abstract map. We also compare DSMA to a naive A\* search, to get a theoretical comparison to HPA\*. The lower level A\* search for all algorithms uses a hybrid heuristic (Chebyshev distance) for diagonal movements and Manhattan distance for straight line moves, whereas the high level A\* search for all algorithms uses Euclidean distance as a heuristic. All heuristics mentioned above are consistent and hence admissible as well.

**Performance Metrics and Presentation of Results** While carrying out the experiments we record the number of nodes expanded by the high level A\* and the low level A\* search combined, the time taken to find a path (including the overhead time for sorting the queues and decomposition/composition) and the length of path for each algorithm for the same map and same start-goal points. We do not use any form of path smoothing or refinement to keep our focus strictly on the behaviour of DSMA. We present highlights of the raw data in graphs in the next section.

## 4.2 Results and Discussion

### 4.2.1 Path Quality

Firstly, let us consider the graph below depicting relative path lengths of Dense HA\*, DSMA and Sparse HA\* against the path length given by A\*

in the x axis. This graph is taken from the maps of size 256 by 256 pixels. The points are average path lengths returned for a given A\* path length. The tables are sorted in ascending order by A\* path lengths, before the averages are determined. So it must be noted that the points plotted are not in sequential time.

We observe that the DSMA algorithm balances the path length according to the time taken by the respective last A\* search. The DSMA graph has sharp hills and valleys due to the reason that the map is small and the random start-goal generator very often produces two close points. This results in the search time being less than 1ms and subsequently a composition. A sequence of such frequent compositions raises the search time again. So, it can be said that DSMA responds well to the dynamic changes. The sensitivity of DSMA can be adjusted by modifying the upper and lower time limits.

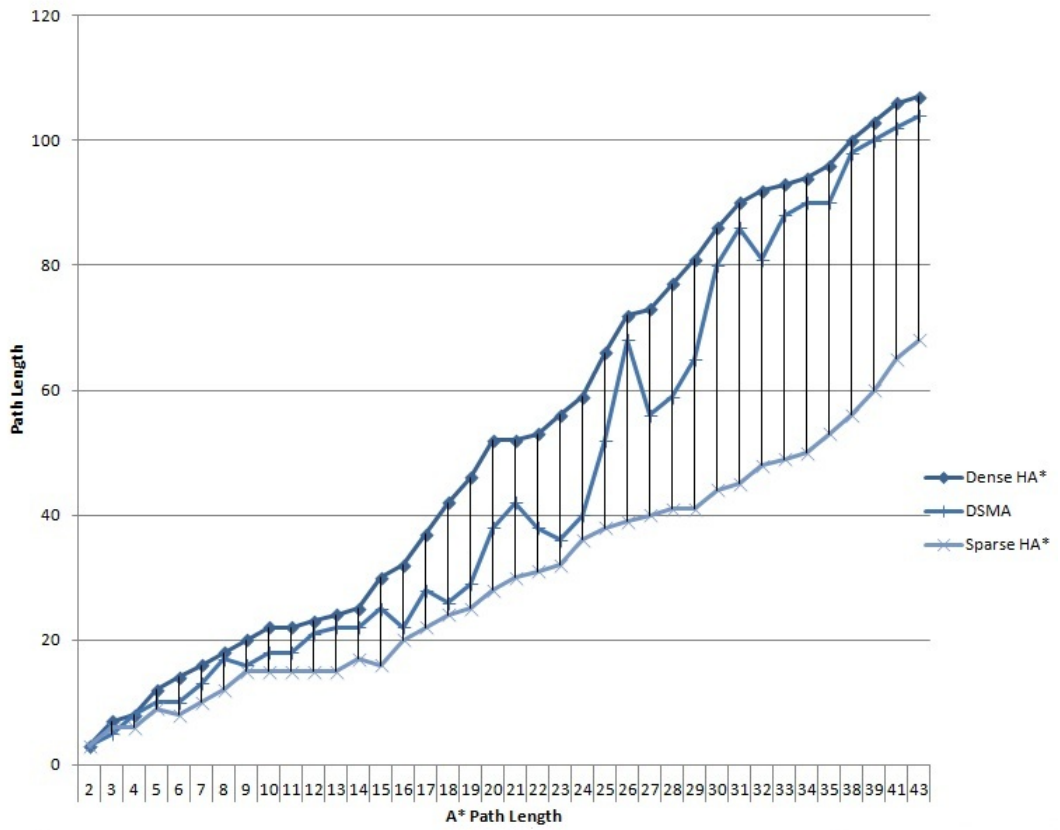


Figure 32: Path Length Graph

The graph below is for maps of size 1024 by 1024 pixels. The graph for map size 512 by 512 is similar and hence is not shown to avoid redundancy. This graph is smoother than the one above since the map size is large and the probability of the randomly generated start-goal points being far away is high. So we get a wide range of values from the path length of A\* search and hence the number of corresponding values being averaged is small with low standard deviation; while the number of points being plotted is high.

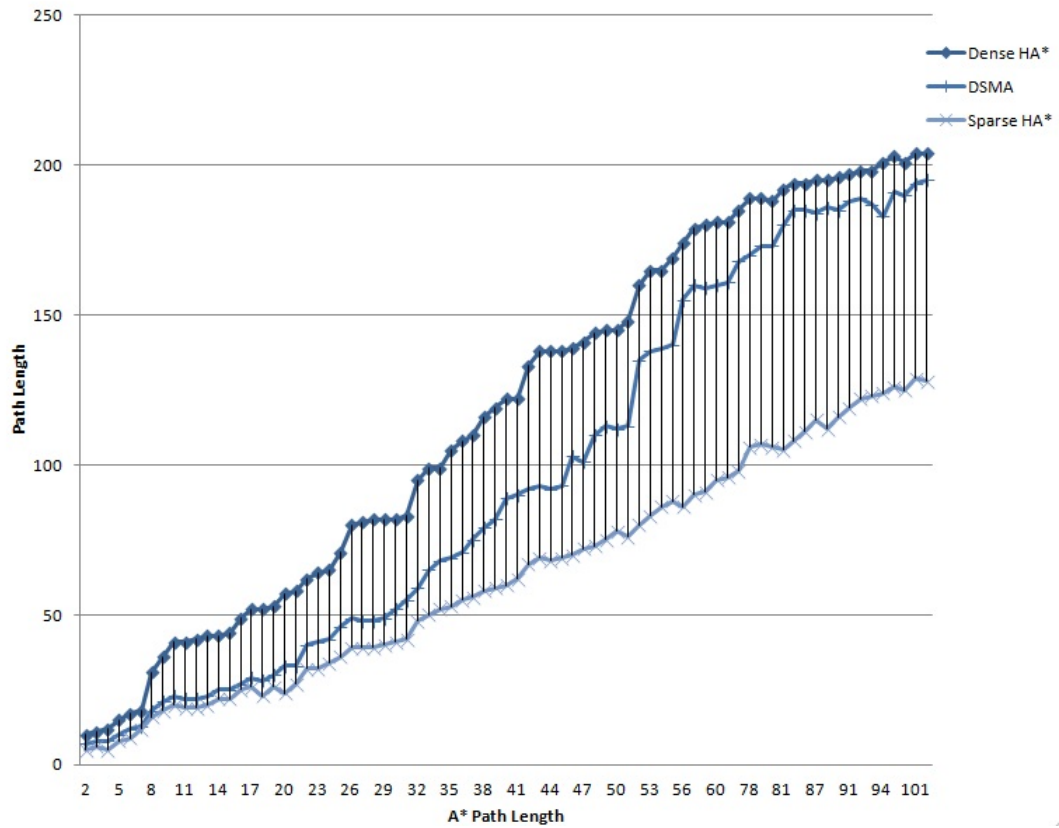


Figure 33: Path Length Graph

#### 4.2.2 Nodes Expanded

The graph below shows the relative number of nodes expanded by Dense HA\*, DSMA and Sparse HA\* against the optimal path length for map size 1024 by 1024 pixels. It is evident that DSMA can keep the nodes expanded between those given by the dense and sparse configurations. Another corollary observation is that depending on the situation, DSMA can behave like the Dense HA\* or Sparse HA\* there by striking a balance between search time and path quality. The graphs for 256 by 256 and 512 by 512 have similar trends and hence not presented to maintain brevity.

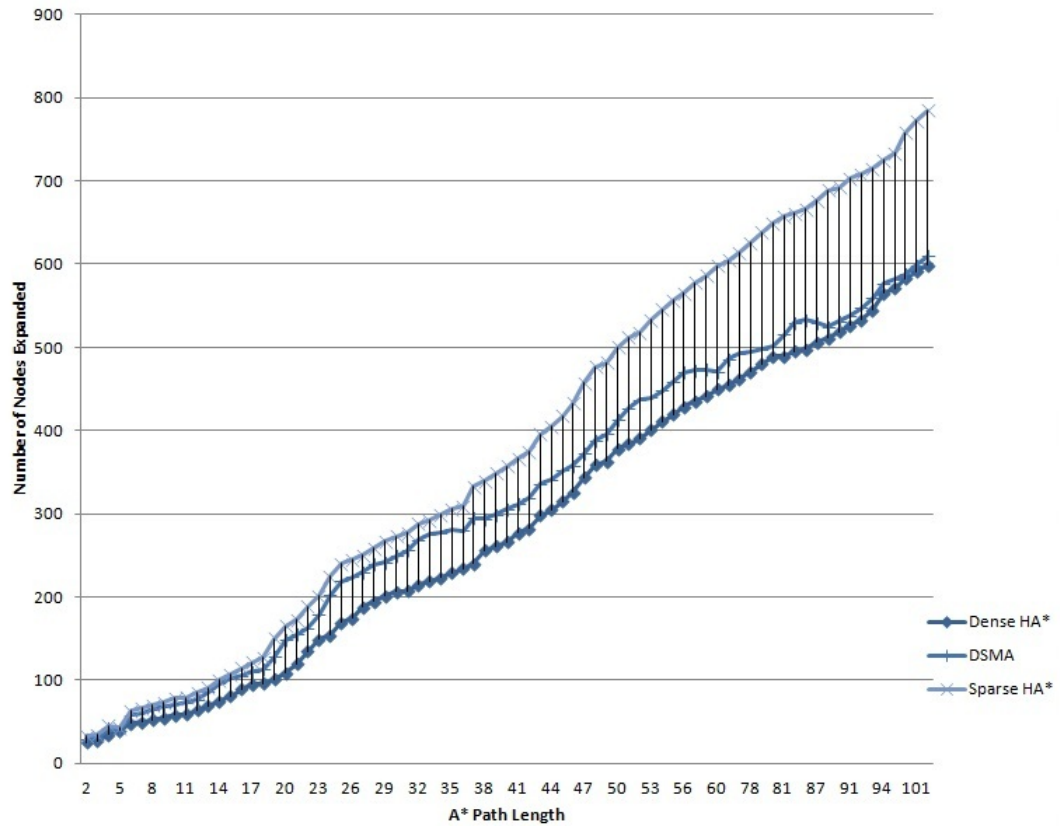


Figure 34: Nodes Expanded Graph



### 4.2.3 Relative Time Consumption

The bar chart below shows the relative time consumption of Dense HA\*, DSMA, Sparse HA\* and A\*. We can say that for smaller maps Dense HA\*, DSMA and Sparse HA\* have very similar performance, so using DSMA is not very advantageous. However as the map size grows, the differences increase and DSMA is a better choice especially when the map is dynamic. The results presented below are to give a general idea of the time differences and have high standard deviations especially for DSMA as it keeps adjusting the time.

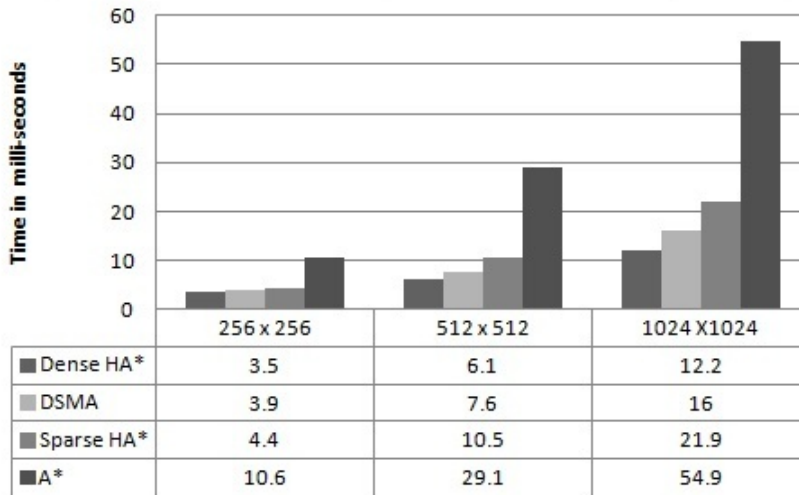


Figure 35: Path Length Graph

#### 4.2.4 Comparison on Map

The explanation of the results is provided in the self-explanatory diagram below where the start and goal are marked as  $S$  and  $G$ . The dots represent the closed list of the low level A\* search and the circles are waypoints provided by the high level search. The line connecting the start and goal represents the final path.

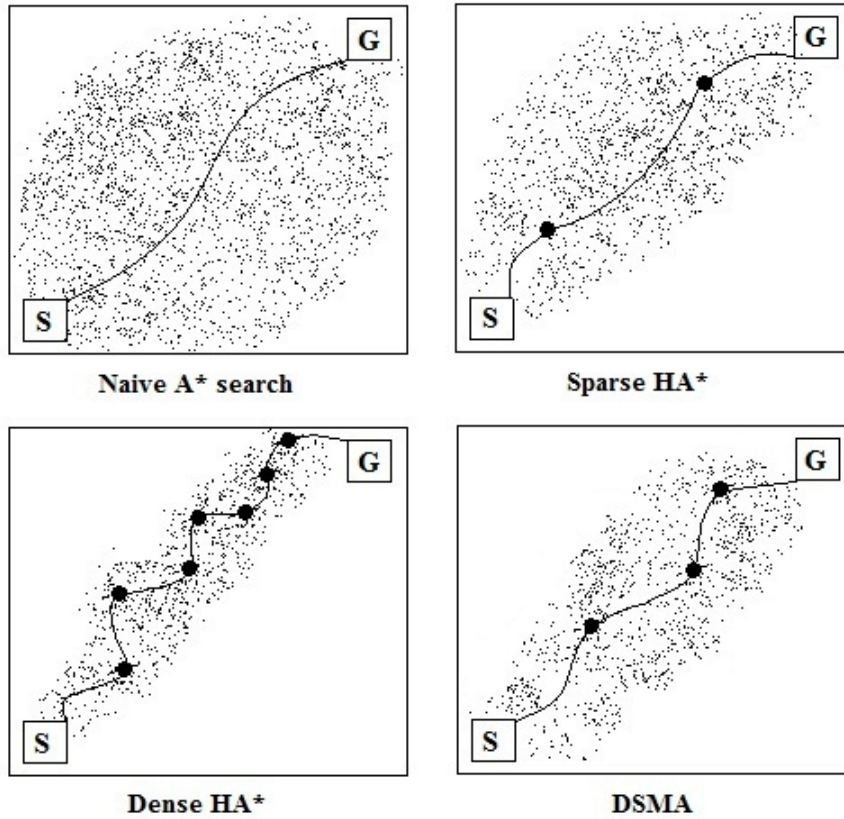


Figure 36: Visual Comparison

## 5 Conclusion

In this thesis we presented a new hierarchical pathfinding algorithm- Demand Sensitive Map abstraction in which we vary the granularity of abstraction dynamically depending on the pathfinding demand associated with various regions of the high level map and the last pathfinding time. DSMA is an alternative to the HPA\* (and other related work), as HPA\* pre-caches all intra-edges and DSMA does not have intra or inter edges. Instead DSMA uses a hybrid abstract edge that is computed on the fly. Moreover, the low level A\* search in DSMA is not restricted to the boundaries of high level triplets and this makes DSMA a special kind of hierarchical pathfinding algorithm.

DSMA is compared it to two cases: a highly detailed abstract map and a low detail abstract map. The results we derived are promising as DSMA is successful in balancing the path quality and search time and continuously evolves the abstract map to keep the balance. The highly detailed abstract map used in Dense HA\* has similar configuration (number of high level clusters) as those used in experiments for HPA\* and we found that if resources permit, DSMA can perform as efficiently (time-saving) as the dense HA\*. On the other hand, if the search time permits, DSMA can be made to resolve into sparse HA\* (approximately similar performance to naive A\* search) . Moreover we do not pre-cache paths, so DSMA can be applied to dynamic maps without any modification.

DSMA is better suited to be applied with maps of resolutions 512 by 512

pixels or higher to gain maximum advantage from varying the abstractions dynamically.

## 6 Future Work

As future work, we plan to implement forced composition and decomposition. A forced composition is one where the two triplets being composed do not come from the same parent and a forced decomposition is one in which we have to perform an additional decomposition in order to decompose a particular triplet or diamond. It would be interesting to experiment with different values of upper and lower threshold values of search time as well. Similarly it is also possible to experiment with different maximum and minimum permissible levels of decomposition.

## Glossary

**Chebyshev distance** In the context of A\* search, Chebyshev distance heuristic returns a value of estimated path cost to goal such that a diagonal move (North-East, North-West, South-East or South-West) is different from a straight line movement (North, South, East or West). The factor by which the two moves differ can be freely modified as desired.

65

**Clique** A clique in an undirected graph is a subset of its vertices such that every two vertices in the subset are connected by an edge. 11

**Cooperative Pathfinding** Cooperative pathfinding refers to a multi agent path planning problem in which agents must collaborate with each other to find optimal paths to their destinations, given complete information about the paths of each other. 14

**Cracks** These are defects resulting from polygonal mesh irregularities. The name comes from the discontinuity in terrain arising due to mis-alignment of mesh vertices. 28

**Level of Detail (LOD)** A term related to computer graphics, controlling level of detail involves diminishing the detail of an object representation as it moves away from the camera or according to other reference points such as object importance, eye-space speed or position. 24

**Manhattan distance** In the context of A\* search and pathfinding, Manhattan Distance is a heuristic that returns a path cost estimate to the goal equal to the sum of the required displacement horizontally and vertically. 65

**Orphan** A node in a graph that can be reached by a single operator. 11

**Quad Trees** Quad Tree is a type of spatial tree data structure which when decomposed produces four regular quads. They are used to segment two dimensional spaces by recursively decomposing or splitting the initial space into four parts. 28

**T-junctions** These are caused by irregular intervals between mesh vertices. The phenomenon is described as presence of a vertex in higher level that is not connected to any vertex in the lower levels. 28

**Traversability** Every position in a given map is associated with a value. This value indicates the traversability of that position. A game agent can access that position until the value reaches a certain threshold, beyond which the position is said to be untraversable or blocked. 23

## Acronyms

**CA\*** Cooperative A\*. 15–17

**CPRA\*** Cooperative Path-Refinement A\*. 18, 20

**DSMA** Demand Sensitive Map Abstraction. 25, 62, 65

**HA\*** Hierarchical A\*. 7, 8, 14, 15, 62, 65

**HCA\*** Hierarchical Cooperative A\*. 15, 16

**HPA\*** Hierarchical Pathfinding A\*. 8, 10, 11, 13, 20–22, 31, 62, 65

**LRA\*** Local Repair A\*. 14, 16, 17

**NHA\*** Naive Hierarchical A\*. 6

**PRA\*** Path-Refinement A\*. 11, 13, 14, 18, 20

**RRA\*** Reverse Resumable A\*. 15

**TRA\*** Triangulation Refinement A\*. 20

**WHCA\*** Windowed Hierarchical Cooperative A\*. 15, 16, 18, 20

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## Vita Auctoris

**NAME:** Sourodeep Bhattacharjee

**PLACE OF BIRTH:** Howrah, India.

**YEAR OF BIRTH:** 1987

**EDUCATION:**

2006-2010

Bachelor of Technology

Computer Science and Engineering

Asansol Engineering College

Asansol, India.