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A game theory approach to negotiations in defense acquisitions in the context of value-driven design: an aircraft system case study

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A game theory approach to negotiations in defense acquisitions in the context of value-driven design: An aircraft system case study

by

Garima Vinay Bhatia

A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Aerospace Engineering

Program of Study Committee:
Christina Bloebaum, Major Professor
Ran Dai
Leifur Leifsson

Iowa State University
Ames, Iowa
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DEDICATION

To Aastha, my forever inspiration

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First and foremost, I would like to express my sincere gratitude to my advisor, Dr. Christina Bloebaum for her continuous support and guidance. I could not have imagined having a better advisor for my Masters program.

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ABSTRACT

The acquisition of weapon systems in Major Defense Acquisition Programs (MDAPs) is an extremely complex procedure involving hundreds of thousands of individuals, right from contracting through design and manufacturing to the sustainment and finally the disposal of the system. The complete acquisitions process involves a number of milestones spanning the entire life of the program. Traditionally, all defense acquisition programs follow a requirements-driven systems engineering approach, where requirements are formed by the buyer or the Department of Defense (DoD), and a cost-based method is generally used to award contracts and develop systems in a bid to minimize costs. However, even with an approach that focuses on cost, there usually exist tremendous budget overruns and time delays in the development of such large scale complex weapon systems, which has been a major concern for the government in recent times.

Recently, there has been a shift of focus from cost-based acquisitions to a price-based and performance-based approach, however, the underlying idea behind these methods is still the fulfillment of requirements. These approaches have their own shortcomings, and problems with MDAPs still persist. Value-Driven Design is a new design philosophy that intends to capture the true preferences of stakeholders by means of a meaningful mathematical function called value function as opposed to using requirements which only serve as proxies to the true preferences. Researchers have proposed the use of value-based approaches for the acquisition of weapon systems in recent times.

This thesis exploits the use of these new approaches in the negotiations phase of defense acquisition, which forms a crucial phase just before the final contract is written. The first part of this research looks at a transition from requirements to value, by proposing a price and performance-based value approach to defense acquisitions, whereas the second part is based completely on value. The aim of the research is to maximize the payoffs to both the government and the contractor developing the weapon system for the government. In this research, the ideas of bargaining from game theory have been proposed in an effort to provide a mathematical foundation to negotiations.

CHAPTER 1

INTRODUCTION

With the increase in the complex nature of systems, the design process has become more tedious than ever. The coordination and involvement of multiple organizations and a workforce of thousands are the crucial components in the design and development of LSCES. Complex interactions or tight couplings between the components of the systems, the long development times and the extremely large costs associated with this development are some of the other characteristics of LSCES [2]. The design process involves decision-making at each level of the organizational hierarchy as well as across organizations. These LSCES often interact with other LSCES in order to fulfill their operational purpose, thus adding to the complexity of these systems. A satellite and its launch vehicle are an example of such interaction. An example of an organization dealing with the design of large scale systems on a global level is The Boeing Company, that employed 162,715 people including both the commercial and defense sectors, as of 29th October 2015 [3].

A large number of commercial organizations are involved in producing weapon systems for national defense as are required by the Department of Defense (DoD) or the government [4]. These complexity associated with such weapon systems is usually greater than that of civil aircraft due to the state of art technology that goes into these systems and also keeping into mind the conditions under which these systems operate [5]. Another distinguishing feature between the realms of the commercial and the military world is the development times associated with these aircraft, with the military aircraft systems taking almost four times as long to develop as compared to the civil sector [6].

Needless to say, the process associated with the acquisition of these systems also differs considerably. The defense acquisition procedures are highly complex and extremely difficult to fathom. As quoted from a RAND report of 2009 on defense acquisition, according to senior policy researcher Jeffrey Drezner, “The products of the Department of Defense (DoD) acquisition process are perceived as becoming increasingly complex, emphasizing multifunction and multimission system configurations.… The management and oversight of these complex programs have similarly become more complex. Changes may be needed in the organizations and procedures used to manage the development, production, and sustainment of these complex weapon systems.” [7]

Traditionally, cost-based acquisition has been used as the method of acquiring weapon systems in which the DoD calls out for proposals from participating contractor, and the contractor offering the best system at the least cost is awarded the contract. However, this method has led to tremendous cost and schedule overruns, as will be discussed in a later section of the thesis [8]. Reforms to the traditional methods such as price and performance-based contracting have been introduced, but these methods have their own drawbacks, which also be explained elaborately in further chapters. The contracting process involves a number of stages to support the acquisition of the system [9]. One of the crucial ones of these is the negotiation that takes place between the buyer and the contractor just before the final contract is written. Negotiation may take place over the price or some part of the contract that the government may not be satisfied with. This thesis focuses on improving the negotiation procedure by the use of the theory of bargaining with an aim to achieve an optimal system design. The thesis comprises of two parts – Part 1 aims at combining price-based, performance-based and value-based acquisitions in order to improve the final design of the system, whereas part 2 proposes a new form of bargaining, that over the attributes of the system. The idea behind the latter part is that the attributes reflect the true preference of operational success of the government and the price is used just as a proxy. Both the aforementioned ideas are centered on the negotiation phase of the acquisitions process, and aim at studying the effect on the characteristics of the system obtained and its price by implementing these ideas. Both the ideas also try to address a transition from requirements to value, as value is used as a payoff evaluator in both the cases.

The following chapter concisely describes the two research questions developed for this research and the approach to addressing each of the questions, as well as the sub-tasks involved in both.

CHAPTER 2

RESEARCH QUESTIONS

This chapter describes the research questions that were formed to initiate the new approaches to improve the current defense acquisitions processes.

Research Question 1

"Can a game theory enhanced value approach to negotiations in a combined priced and performance-based contracting scenario lead to a better system design as compared to that obtained by using the traditional requirements-driven method?"

This research question will be addressed by creating a negotiation model that uses a performance based requirement stated by the DoD to determine the price associated with an aircraft example test case, to be designed by a contractor that represents a commercial organization, and then using a value model to create a game of bargaining between the players (government and contractor) over this established price to reflect the final price of the system and its characteristics.

Task 1

One of the tasks for this research question will be to investigate the effect of player order in the game of bargaining. This will be accomplished by changing the order of the player making the first offer from the main research question, and examining the effects on the player payoffs for doing this.

Research Question 2

"Can a value approach combined with negotiation over attributes be used to bridge the gap between the preferences of the government and the contractor in a defense acquisitions context for weapon systems?"

This research question will be used to investigate a bargaining over system attributes, a new form of bargaining, in order to explore the effects on the value to the stakeholders. No cost consideration on the part of the government will be taken into account initially, and two attribute sets will be found that will represent the attribute values yielding the maximum payoff (in this case the value) to both the stakeholders respectively. The attributes will be those that are common to the values of both the players. A bargain model will be constructed to study a negotiation over one of the attributes and the effect of this negotiation on the values to both the players. The final set of attributes achieved will be used to determine the payoffs (values) obtained by both the players, as well as check for the final price to be paid by the government.

Task 1

The task for this research question will be to investigate a game of random bargaining, where there is no definite sequence to making offers. The formulation of the bargain model will be similar to the main research question, however, in this model, a probability will be included that will associate the player payoffs to making offers in the game. The same value functions will be used to evaluate the payoffs as above.

Organization of Thesis

Chapters 1 and 2 provided an overview of the issues with the current defense acquisition methods and the motivation behind the research and also defined the specific research questions addressed in this thesis. Chapter 3 will provide the necessary background required for proceeding through the thesis and understanding the topics addressed in this research. Chapter 4 will focus on describing the aircraft model designed to be used as a test bed for this study. Chapter 5 will give details about the various value functions that can be used by the government and the company and also define in detail the value functions particularly used in this research. Chapters 6 and 7 will delve in to the core result and contain the results and discussions. Chapter 6 will specifically focus on the proposed new combined contracting or research question 1, whereas chapter 7 will describe negotiation over attributes or research question 2. The final Chapter, Chapter 8, will summarize the research and state some of the future work for the research and the possible areas that could be explored.

CHAPTER 3

BACKGROUND

The design of Large-Scale Complex Engineered Systems (LSCES) is extremely difficult to address as a whole as these systems are characterized by their extreme complexity and multidisciplinary nature. Numerous people belonging to different disciplines often spanning across various geographical locations work towards the design of a single system that takes long times (even decades) to develop and whose development costs often surpass the billion dollar mark [10, 11].

Traditionally, the requirements-driven Systems Engineering (SE) approach is used to design these LSCES. The needs of the customer are translated into requirements which are formed at the top level and then flowed down the hierarchy of the organization to assist each subsystem design team in the design process [12]. However, requirements only serve as proxies to the true preferences of the stakeholder. Value-Driven Design is a new design philosophy that intends to capture these true preferences by means of a meaning mathematical function called value function. This is a single unit function (usually monetary) which is decomposed down to the lowest level, enabling consistency in preferences and design decision-making so as to enable an optimal system design [13].

Value models will be used extensively in this thesis to highlight the advantages of designing for value in defense acquisitions over traditional requirements, and the combined contracting will use both requirements and value, which could well serve as a transition from requirements to value. The second research question will only focus on value. The aircraft model used to perform the study consists of three disciplines, and multidisciplinary analysis and optimization will be necessary to address the couplings in the model and to obtain system consistency, as has been investigated in multiple studies [14]. The content in this chapter describes the defense acquisition process in detail and sheds light on bargain modeling, as well as gives an overview about traditional systems engineering, value-driven design and multidisciplinary design optimization.

Defense Acquisitions

Procurement refers to the purchase of any good or service. The term 'acquisition' is broader and refers to the entire life cycle of the good or service, right from design and engineering to construction to sustainment and finally to disposal. The defense acquisition process is extremely complicated and with more than \$314 billion at stake annually, these programs attract too much attention as it is the taxpayers' money that is being put to use [15, 16]. There was an increased emphasis on cost cuts post the Cold War due to limited budgets allotted to defense. This caused a shift of focus from increased performance to reduced costs in defense acquisitions [17-19]. As a result, there have resulted various types of contracts that target to reduce the price of the system to the government. There exist different types of contracts that are followed by the DoD under different circumstances, and the policies for these contracts are dictated by the Federal Acquisition Regulation (FAR) [20, 21].

The dynamics of the acquisition program however change based on the market for the system being obtained and the government cannot always necessary control the price of the systems it purchases. For certain weapon systems, there may exist a monopsony (when there is a single buyer in the market), with the government playing the monopsonist. In such a case, one may assume that the DoD that represents the government in the defense acquisition programs will have complete control over the price of the system; this however, may not necessarily be true. The government's power to determine the price is dictated by external factors such as sudden national threats, cases of war or no immediate requirement of weapon systems during times of peace. In any caser, the quantity of systems desired by the government changes, which affects the cost to the contractor, thus taking away the government's ability to single-handedly influence the price [22].

In other cases of monopolies (single seller), the price determining power is withdrawn from the government as the sellers ability to influence the worth of the system is highly elevated, which again puts the government in a difficult situation [23, 24]. However, even in the case involving multiple contractors, an oligopoly may exist (contractors grouping together) which again is a disadvantage to the DoD as it cannot be the lone influencer of the price of the system [22]. Underbidding and software proprietary to the contractor are other issues that lead to cost overruns to the DoD in the long run even if the initial decided prices are within budget [25]. Due to the above reasons, there have been a number of attempts to reform the defense acquisition procedures. There has been an increased emphasis on the shift from cost and price based acquisitions to those that are more operationally focused. The concept of value in acquisitions over the traditional requirements-driven approach is also gaining momentum with the idea being to capture the true preference of the government rather than using the proxies of price or cost. The following section gives an overview of cost-based, priced-based and performance-based acquisitions as well as a description of the new value approach to defense acquisitions.

Cost-Based Acquisitions (CBA)

This is a traditional defense acquisition process that involves an in-depth cost analysis to be conducted on the part of the contractor, and a detailed report of this cost analysis is to be submitted to the DoD. The DoD reviews the proposals submitted by all the contractors participating in the acquisition program, and based on the best bid, usually a cost plus fee, generally called a cost plus contract, is written. A cost plus contract is one in where the fee is the supplemental amount awarded to the contractor over the cost incurred by him, and the fee is generally a pre-decided percentage of the cost [26, 27]. A major defense acquisition program (MDAP) as a whole is an exhausting process and involves a number of milestones, right from developing requirements to award of contract to development of the system and its sustainment to its disposal. Fig. 1 depicts a chart created by the Defense Acquisition University (DAU) that tries to show the entire defense acquisition process on a single page, and the complex nature of this process can be realized by looking at this chart.

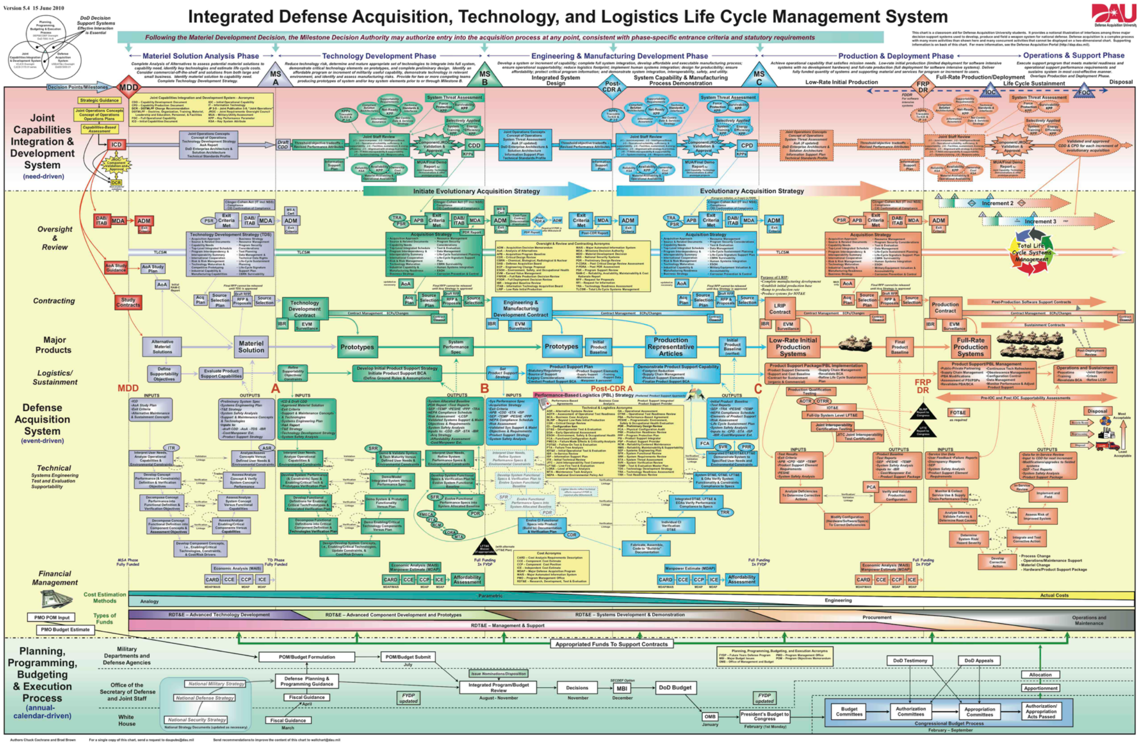


Figure 1. Integrated Defense Acquisition, Technology, and Logistics Life Cycle Management System [1].

Because this research is concerned with the process of award of contracts, particularly the negotiations phase, an overview of a typical CBA program and its various stages involved in the pre-award phase are described in Fig. 2. A brief description of each of the activities preceding the

System Development and Demonstration (SDD), where a contractor is selected to develop the weapon system, follows [28].

- Determine requirements: The DoD forms requirements to describe what the system is, it's expected behavior, a general idea of the cost and development time among others. This process may be quite elaborate and may involve a number of studies and analysis, which may result in this phase itself taking a number of years. Requirements are developed using the Joint Capabilities Integration and Development System (JCIDS) process [29, 30].
- Conduct market research: The DoD conducts a market research to determine if there already exist systems that fit the requirements or for the existence of current systems that may be modified to meet the requirements developed by the DoD. The extent of the market research depends on a number of factors and varies from system to system [31, 32].
- Develop acquisition strategy and write acquisition plan: These include developing business plans, identifying the type of contract including incentives and terms and conditions, and also detailed forecasting such as cost and schedule relationships, competition sought and budget, among other considerations. This is followed by source selection, which involves identifying the best offerors of services, including contracting, legal and technical expertise, etc.
- Publish Government Point of Entry (GPE) Notice and distribute solicitation: A notice is published on the GPE website before a solicitation is issued in order to increase awareness and competition as well as involve participation of smaller businesses. The official solicitation or requests from the federal government for contractors to submit their proposals is released a fortnight after the notice. For weapon systems that involve

negotiations, the solicitation is generally called a request for proposal (RFP). The contractors are then provided with a 30-day period to submit their proposals [33].

- Receive and evaluate proposals: Once the proposals are received from the competing contractors in the format desired by the government, each proposal is evaluated. The proposal must specify cost and technical details in the context of fulfilling the requirements specified in the RFP. The factors on which the evaluations are based must be pre-specified in the RFP. Based on the contractors' cost details and technical solutions, the government does its own cost analysis, and the government's estimated cost is then used instead of the costs provided by the contractors. The government evaluation teams also conduct thorough evaluations of various rates, such as labor, overhead, general, etc. in order to verify the authenticity of a contractor's proposed costs [34].
- Conduct fact findings and discussions: This involves a physical visit to the contractor facilities where the system is to be developed and discussions with the contractors for clearer understanding of the proposals on the part of the Contracting Office (CO). The CO also informs the contractors of deficiencies or ways of improvement at this point.
- Request Final Proposals: The contractors submit their modified, at times completely revised final proposals, called the best and final offer (BAFO) after discussion with the CO.
- Evaluate final proposals, negotiate and write contracts: Another evaluation follows post submission of the BAFO, and this time it is mainly the final price and cost that are the focus. A contract is written for the winner, and this is also the point where the profit percentage is decided in the cost plus contract scenarios [35].

These steps are generally followed by briefings, decision approvals and post award audits, etc. The next phase involving Sole-Source contracting, or awarding the contract to a single contractor, which is generally the case in acquisition of weapon systems, is the one in which negotiations take place. Negotiations are conducted on aspects of the proposal that the CO does not agree with. When negotiating over the price, the DoD asks contractors for the actual cost, which is then used as a basis for the negotiation. The Truth in Negotiations Act (TINA) ensures that the data provided by the contractor is accurate, or else the contractor is penalized heavily if found otherwise [36]. Following the negotiations, a document called Price Negotiation Memorandum is prepared to show how a fair and reasonable price for the weapon system was established [37].

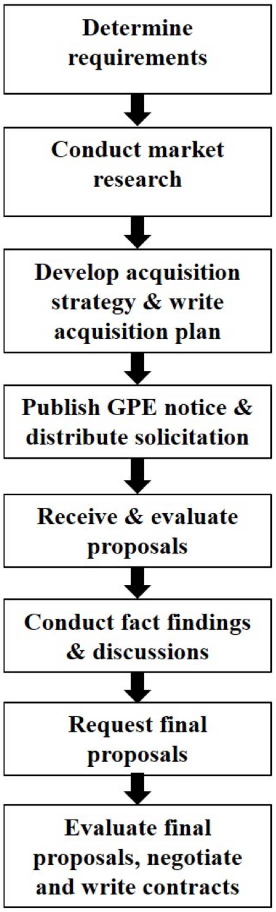


Figure 2. *An Overview of CBA*

Drawbacks of CBA:

Based on the available literature, certain drawbacks have been observed about CBA. In this form of acquisition, because the profit made by the contractor depends on the cost, there is no incentive on the part of the contractor to cut short the cost. This may lead to excessive cost overruns and may push the price paid by the DoD to the higher end of the spectrum. Also, because of the large costs involved in conducting a detailed cost analysis, a multitude of contractors may stay away from participating in the acquisition program, thus reducing competition and weakening the government’s hold over the price. Companies also consider cost data to be proprietary due to the competition involved, and may not be very pleased about sharing this data. Apart from these, under TINA, contractors are asked to provide cost data structures in specific government formats under a unique accounting system driven by the federal

Cost Accounting Standards (CAS), which may discourage civilian contractors lacking this government-unique accounting infrastructure from participating[28, 38, 39].

Due to the disadvantages of CBA mentioned above, price-based acquisitions (PBA) were introduced in a bid to improve the defense acquisition processes, and are described in detail in the following section.

Price-Based Acquisitions (PBA)

This form of acquisition was introduced as a reform to overcome the shortcomings of CBA stated in the previous section. At present, PBA is being advocated as a strong tool for improving cost and performance objectives, and is being considered for improvisation of MDAPs since a long time. The way that PBAs differ from CBAs is that they do not primarily require the contractor to supply cost data, and a contract is written based on a reasonable and fair price established without having knowledge of the cost [28, 40]. This, it is claimed, helps in saving overhead costs and also cuts down on the time required for the cost analysis, thereby promoting shorter schedules. This form of acquisition follows the same procedures as those described for the CBA process, however, it results in cost and schedule savings for the DoD in the proposal evaluation and fact finding phase as they do not have to go through the tedious process of reviewing the authenticity of extensive cost data supplied by the contractor. In PBA, the government conducts a broad market survey to determine the appropriate price of the system under consideration, which is followed by a negotiation between the DoD and the stakeholder to arrive upon a price that both agree with. The market research stage mentioned in the CBA process above takes a greater time and costs more in PBA because of the price market research involved. The company, however, ends up saving significant contracting costs as they do not have to carry out an in-depth cost analysis and submit a report in the government-approved documenting format. This results in promotion of wider competition among the contractors and a greater number of companies participating in the acquisition process, which works to the advantage of the government.

Drawbacks of PBA:

Even though the above theory dictate that PBA lead to significant cost reductions when compared to CBA, a study conducted by RAND involving interviews with actual government and company officials found that in certain cases, the cost involved in conducting the cost analysis and preparing the proposal was a very small fraction of the total contracting cost. There may also occur cases where no pre-defined market exists for a system under consideration, which may make the price determination an arduous task for the government. The absence of actual cost structures in the contracting process may also lead to underbidding by the contractors to win the proposal, and then lead to an increased price for the DoD in a later period, or the DoD may be denied access to some cutting-edge technologies if the price were to be kept at the original bidding value.

Another reform to the traditional acquisitions is the performance-based service acquisition (PBSA), where the emphasis is on the outcome of the service desired rather than the method by which the service is achieved, which is described in the next section.

Performance-Based Service Acquisitions (PBSA)

Under PBSA, there is a shift in the emphasis from contracting for resources, such as price or cost, to contracting for results. In essence, in these type of contracts, the government is only interested in the final payoff received rather than the process followed by the contractor to achieve the desired payoff from the system [41, 42]. This new acquisition reform is also sometimes referred to as Performance Based Logistics (PBL). The basic underlying framework of PBSA is that operational requirements are identified and laid down by the government, and the contractor is incentivized with economic rewards for fulfilling the objectives. Clauses are also written down for the award of incentives if a better performance than the minimum stated is achieved, which may motivate the contractor to deliver a superior system to the government. On the other hand, the contract also provides for including penalties if the performance goals are not met.

Although this type of contracting does give the contractor the freedom to pursue the development of the system in his desired way, there are certain drawbacks associated with PBSA. One of the key issues with these acquisition methods is the appropriate definition of the figures of merit that define the effectiveness of the desired system [43]. The other drawback is linking rewards or incentives to performance by means of a fair rewarding scheme, and further research is being put in to this sort of contracting.

Although each of the contracting methods described above have their own advantages and disadvantages, each of them still rely primarily on requirements for defining the system, which predominantly state what is not required from the system, and play substitutes to the true preferences of the stakeholders. A new reform to these traditional methods is value-based acquisitions (VBA), where the system is designed for the true value to the stakeholders. A brief description of VBA is given in the following section.

Value-Based Acquisitions (VBA)

The concept of VBA overlaps in certain ways with the idea of performance-based acquisitions, and is focused on the operational attributes that define performance. The underlying idea behind these type of acquisitions is the development of a value model that correctly captures the tradeoffs between cost and performance by capturing the attributes defining the two criteria accurately. Once a quantifiable value model has been developed, the price paid for the system is made a function of the value. This also provides as an incentive to the contractor to improve performance [44-46]. The use of value models helps in better preference communication and more informed decision-making, giving the designers more freedom to make design choices. Value models will be described in detail in a later section. VBA although much recent as compared to the traditional methods of contracting, is being realized as a form of acquisition holding tremendous potential for the future of defense acquisitions, and much research is being put in to it.

Even though the each of the forms of acquisition described above have their own pros and cons, the acquisition process is usually tailored according to the needs of each individual program, and usually characteristics from different methods overlap. This research proposes using a combination of the price-based and performance-based contracting principles in a way that the contractor finally designs for value. In essence, it is taking the best of each of the three methods and combining them in to a single price and performance-based value approach. Since it is after the negotiation phase that the final contract is written, this phase becomes crucial as it involves a lot of decision-making. This thesis thus bases its focus on this particular phase, and provides a mathematical foundation to negotiations by using the theory of bargaining, which forms a significant part of game theory.

Systems Engineering

The design of LSCES is traditionally done using the conventional Systems Engineering approach that evolved as a discipline during the latter half of the 20th century in order to tackle the problems associated with the design of large scale systems with ever-increasing complexity. This approach is based on requirements that are formed at the highest level in the organization [12, 47-49], and the organizations are generally decomposed in to hierarchies, which may either be component-based or discipline-based. An example of such a component-based hierarchical decomposition is the aircraft system that has been designed as a test case for this paper, depicted in Fig. 1. Systems engineering design practices are based on the V-model shown in Fig. 3 [47]. This model shows the steps associated with the development lifecycle of systems. The left arm of the V-model represents the 'Definition, Decomposition and Allocation' phase where the requirements are first identified at the top level based on customer needs. These requirements are then broken down and communicated to the subsystem level design teams, which further break down the requirements for the component level design teams. Once each design team has designed their respective subsystems occurs the 'Integration, Verification and Validation' phase represented by the right arm of the V-model. In this, the system is integrated and iterations are performed in case the final system is not consistent with stakeholder requirements. This approach however does not distinguish between designs, and does not consider if a potential design may be better than other designs. In short, any design that satisfies the stakeholder requirements is accepted as the final design.

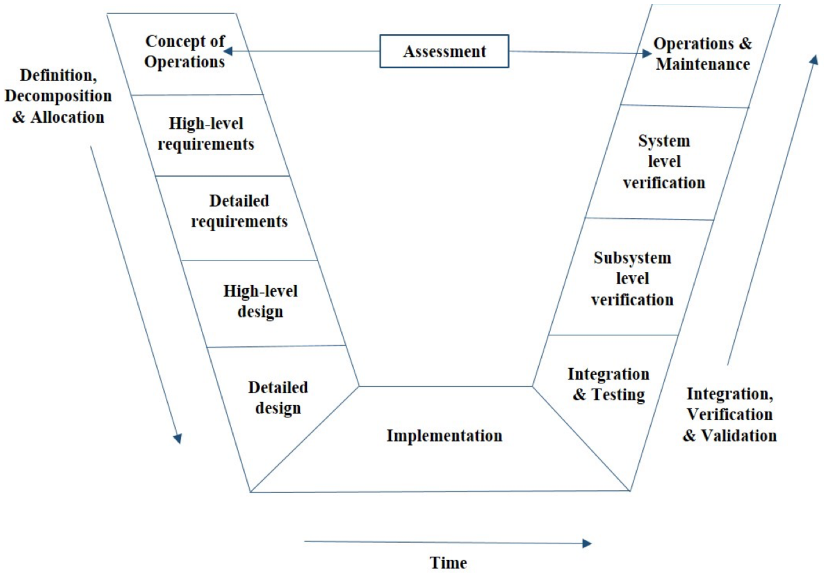


Figure 3. *Systems Engineering V-model*

The SE approach limits exploration of the design space due to the imposition of requirements, which serve as true proxies for the true preferences of the stakeholders. This approach does not consider the possibility that the best design may lie outside the design space bounded by requirements. For example, when a requirement on total cost of a system is put forth, the SE methodology leaves no possibility for a design to be selected that may cost more but also yield a much greater profit, which is what is actually desired by the stakeholders. It also doesn't consider optimization of the design within the feasible design space. Also, the physical interactions that exist in such large scale systems cannot be captured accurately using Interface Control Documents that are used in the SE process [12, 47-49]. The lack of a rigorous mathematical model to represent the couplings results in a system that is inconsistent in physics.

Multidisciplinary Design Optimization

MDO emerged as a field of optimization from structural optimization in the early 1980's [50, 51]. It addresses the couplings inherently present within the system such that the system consistency is obtained. The capturing of the interactions within the subsystems through couplings results in a system that is consistent in physics, an issue not addressed in the SE approach [12, 47-49]. Traditional MDO involves a design space representing the objective function that is to be optimized, and this design space is bound by the constraints that are levied on the objective function. Constraints essentially represent the requirements observed in traditional SE practices. However, it should be noted that

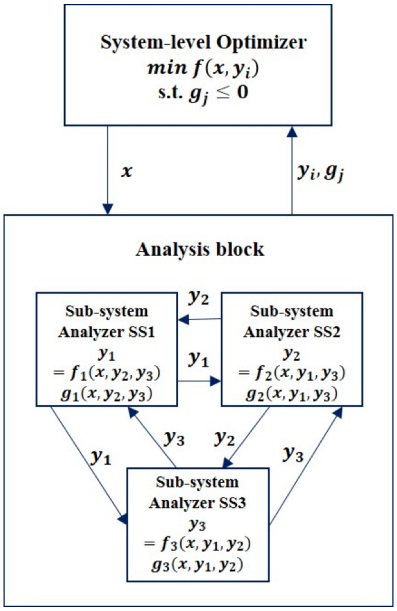


Figure 4. *Multidisciplinary Design Feasible (MDF) Framework*

MDO does not provide a means for creating an objective function but merely assumes that one already exists.

MDO provides for the capturing of couplings or behavior variables during both analysis and optimization through frameworks such as the Multidisciplinary Design Feasible (MDF) shown in Fig. 4 [14, 52]. The framework depicts how design variables denoted by X are initialized by the optimizer and fed into the system analysis block that performs iterations till convergence is obtained to determine a set of outputs. These outputs are then fed into the optimizer that performs optimization on the objective function using these outputs to determine a new set of design variables, which are again fed in to the analysis block. This process continues till convergence is obtained and the system is consistent at all levels for the final set of design variables. In the figure shown, three coupled subsystems (SS1, SS2 and SS3) have been considered. The variable 'y' represents the behavior variables that act as inputs and outputs between the subsystems. MDO is used to distinguish the best design from a wide range of design alternatives within the feasible design space. However, as the constraint-bound objective function merely serves as a surrogate for the true preferences of the stakeholder, MDO is used with a value function in this research to evaluate the optimal aircraft designs for both the government and the commercial organization.

Value-Driven Design

A new SE approach called Value-Driven Design (VDD) has been proposed recently that captures the true preferences of the stakeholders by means of a single function called Value Function [13, 53]. VDD minimizes the number of requirements placed on the system thereby offering better exploration of the design space so as to achieve an optimal design. Design optimization in the context of VDD is represented by Fig. 5 that shows the life cycle development of systems.

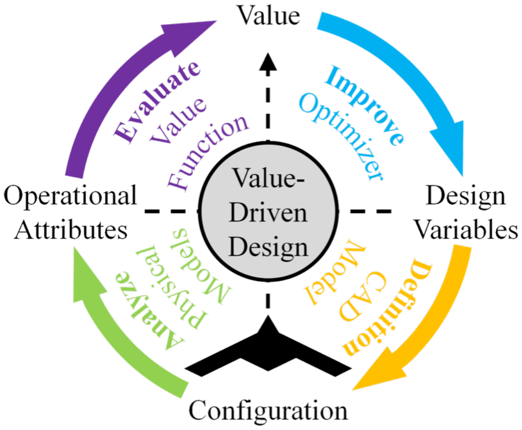


***Figure 5.** Value-Driven Design Process*

Value functions are functions of attributes which represent the characteristics of the system. These attributes in turn are function of design variables, or inputs given to the system that are altered to change the design of the system. VDD aims at minimizing constraints thereby enabling better exploration of the design space. The value function has a singular unit (usually monetary) that reflects the true preference of the stakeholder and allows for direct comparison between competing designs that have the same set of design attributes. The value function is formed such that it is consistent at all levels, with higher level attributes being functions of lower level attributes and design variables, and such that it captures tradeoffs with the help of a mathematical relationship. The decomposition of the value function to the lowest levels enables improved consistency in decision-making as decision-makers at all levels are designing for a single objective that is desired by the stakeholder at the topmost level [54].

Theory of Bargaining

Bargaining forms an extensive part of game theory and also incorporates principles of cooperative decision making [55]. In the economic model of bargaining, players try to divide a resource between each other by taking turns at making offers. Each player tries to have the largest share of the resource for himself. An offer made by the first player can either be accepted or rejected by the other player. If the offer is accepted, the game ends and the players divide the resource as per the offer made by player 1. If player 2 rejects the offer, he gets to make a counter offer, which in turn, can be either accepted or rejected by player 1. This process continues till one of the parties accepts an offer. A game of bargaining can theoretically have an infinite number of rounds; in practice however, it cannot be played indefinitely. Each player has a time discount factor δ that represents his impatience towards the game, where δ lies between 0 and 1, 0 representing a completely impatient and 1 a very patient player, respectively. After each round, the value of the system is reduced by a factor of δ. The greater the player patience, the better is the payoff received by the player. The subgame perfect equilibria for such a two-player bargaining game are shown, where x^* and y^* are the outcomes proposed by players 1 and 2, respectively, when each of them leads. The equilibria strategies and equilibria conditions are shown in Eqs. (1) and (2), respectively [56, 57].

$$\begin{gathered} x^* = \left(x_1^*, x_2^*\right) = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \delta_2\frac{1-\delta_1}{1-\delta_1\delta_2}\right) \\ y^* = \left(y_1^*, y_2^*\right) = \left(\delta_1\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right) \end{gathered} \tag{1}$$

$$\begin{gathered} x_2^* = \delta_2 y_2^* \\ \delta_1 x_1^* = y_1^* \end{gathered} \tag{2}$$

The following chapter describes elaborately the aircraft model that will be used as a test bed for this research.

CHAPTER 4

AIRCRAFT MODEL

A high-level bomber aircraft model has been developed to use as a case study for this research [58-61]. The purpose of this aircraft is assumed to be the transportation of ammunition and personnel from one location to another. It should be noted that the model created for this study is approximate and based on past data and knowledge with some educated assumptions.

The mission profile selected to fulfill this purpose has been shown in Fig. 6. It is comprised of 4 mission segments, namely taxi, takeoff and climb, cruise before payload drop, cruise after payload drop (return segment) and the descent and land.

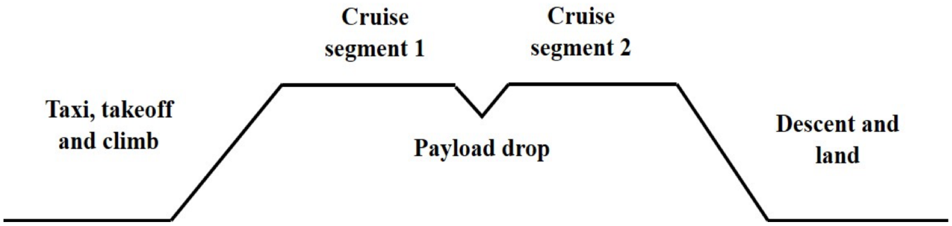


Figure 6. *Mission Profile for Aircraft*

The contractor's organization is assumed to be broken down hierarchically. It consists of teams that have been divided according to the main components of the aircraft, namely the wing, fuselage, tail, engine and landing gear. These form the first subsystem level. The wing and fuselage subsystems have been divided into further subsystems spanning one level down the hierarchy. The components for the second level for the wing are the spars, ribs and the skin whereas for the fuselage they are the frames, the longerons and the skin. Figure 7 depicts the hierarchical breakdown discussed above. Each of the individual subsystems of the aircraft and the associated equations for their analysis are described in Appendix A.

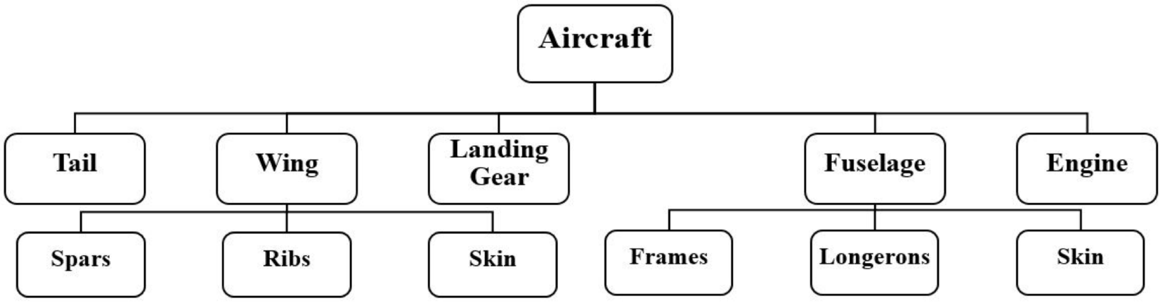


Figure 7. *Hierarchical Decomposition of Aircraft Model*

Since this is a high level model, only three disciplines and their corresponding interactions were considered in this research, namely structures, aerodynamics and performance. The three disciplines and the couplings between them has been represented by the Design Structure Matrix (DSM) shown in Fig. 8. Design variables are independent variables that are input externally. These define the design. The aircraft system considered in this research is comprised of 18 design variables out of which 14 are discrete and integers and the rest are continuous. Table 1 lists these design variables and their corresponding descriptions.

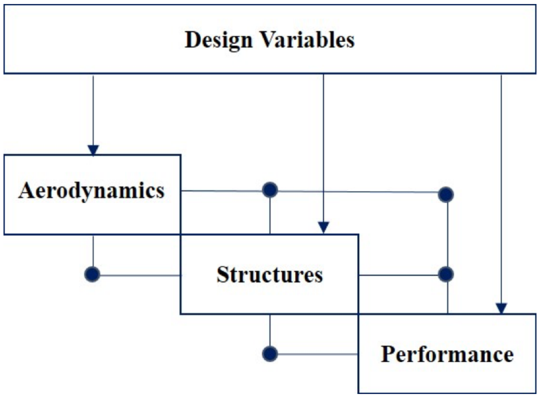


Figure 8. *Discipline-based DSM for Aircraft Model*

Table 1. *Description of Design Variables*

Design variable	**Description**
$Mass_{payload}$	Mass of payload in kg
$Type_{wing}$	Type of wing

Table 1 continued

l_{fuselage}	Length of the fuselage in m
$\text{Type}_{\text{tail}}$	Type of tail
$\text{Material}_{\text{tail}}$	Material of tail
$\text{Type}_{\text{landinggear}}$	Type of landing gear
n_{eng}	Number of engines
Type_{eng}	Type of engine
$\text{Material}_{\text{spar}}$	Material of spar
l_{wing}	Length of the wing in m
l_{chord}	Length of the chord in m
$\text{Material}_{\text{rib}}$	Material of rib
$\text{Material}_{\text{skin,wing}}$	Material of skin of wings
n_{frames}	Number of frames
$\text{Material}_{\text{frame}}$	Material of frames
$\text{Material}_{\text{longeron}}$	Material of longerons
$n_{\text{longerons}}$	Number of longerons
$\text{Material}_{\text{skin,fuselage}}$	Material of fuselage

Table 2 provides a list of the behavior variables of the aircraft system and their descriptions. Behavior variables are the outputs of the disciplinary analysis that represent the behavior of the system, whereas attributes are the outputs that characterize the subsystems.

Table 2. *Description of Behavior Variables*

Behavior variable	**Description**
M_{wing}	Mass of the wing in kg
M_{fuselage}	Mass of the fuselage in kg
M_{tail}	Mass of the tail in kg
M_{lndgear}	Mass of the landing gear in kg

Table 2 continued

M_{engine}	Mass of the engines in kg
M_{fuel}	Mass of the fuel in kg
Range	Total range in km
V_{cruise}	Cruise velocity in m/s
M_{spar}	Mass of spars in kg
M_{ribs}	Mass of the ribs in kg
M_{frames}	Mass of the frames in kg
$M_{longerons}$	Mass of the longerons in kg
M_{skin}	Mass of the skin in kg

Table A in Appendix A gives a detailed list of the attributes and behavior variables at each subsystem level and the design variables associated with each. The profit obtained by the company is the difference between the revenue generated by selling these aircraft and the cost to the company for manufacturing the aircraft. The total cost is assumed to be the sum of the costs of the individual systems and is given by Eq. (3). The revenue generated by the company is the product of the number of aircraft sold and the price of each aircraft, given by Eq. (4). The number of aircraft sold, in turn, is a function of the range, cruise velocity, stealth, and also the price per aircraft. The problem created follows the traditional design cycle shown in Fig. 1.

$$Cost_{total} = \left(Cost_{wing} + Cost_{fuselage} + Cost_{tail} + Cost_{landinggear} + Cost_{engine}\right) * N_{aircraft_{sold}} \tag{3}$$

$$Revenue = Price\ per\ aircraft * N_{aircraft_{sold}} \tag{4}$$

A set of design variables is initiated that feeds into the subsystems to define the physics. An analysis is then carried out to achieve system consistency and the output of this analysis are the attributes that feed into the value function to give the value associated with that set of attributes.

An optimization is then carried out to determine a new set of design variables that again feed into the subsystems, and the process repeats. This cycle goes on till a final optimum value is found, that corresponds to the maximum profit in the aircraft example considered in this research. Both the analysis and optimization performed for the aircraft model are multidisciplinary, in order to correctly capture all the subsystem interactions and disciplinary couplings for a consistent system [62]. Genetic Algorithm is used in this research as the method for optimization to account for the presence of discrete design variable choices [63].

The different value functions that can be used as per the preferences of the government and the contractor are described in the next chapter as are the details of the value functions used in this study.

CHAPTER 5

VALUE FUNCTIONS

The focus of this chapter is to discuss in detail the various value functions that can be used by the DoD and the contractor in different cases as per their preferences and also describes the values functions used particularly in this thesis for both the parties.

Value Functions for Company

It is considered that the contractor for designing the bomber aircraft desired for defense purposes by the government has been selected from amongst one of the many commercial companies bidding for the contract, and a negotiation is to take place between the contractor and the DoD before writing the final proposal. The primary preference of the contractor is to maximize the profit he can obtain by designing an aircraft that is consistent with the preferences of the government, at the same time ensuring that he himself isn't losing money. In this case, the value function used by the company should be one that correctly captures their true preference of maximization of profit for the organization. This value function yields a single dollar value that reflects the profit made by the company. The value function that can be used for such profit-seeking organizations is usually profit, which is the difference between the revenue generated by the company and the cost incurred by them in developing the system, as shown in Eq. (5) [64]. Here, $\pi_{i,j}$ represents the profit for a single company where $R_{i,j}$ is the company's revenue and $C_{i,j}$ is their cost. The indices "i" and "j" represent the company and the time period (e.g. a fiscal year), respectively.

$$\pi_{i,j} = R_{i,j} - C_{i,j} \tag{5}$$

A more practical modification of the profit value function is the Net Present Profit (NPP) metric, that accounts for the passage of time with respect to accumulation of profits as well as the discount associated with the net worth of future money. The NPP is given by Eq. (6). In this case, "r" represents the discount rate. A variation of the NPP represented in Eq. (6) is given by Eq. (7) [65]. This equation more specifically applies to LSCES and accounts for the profit obtained in the initial period (e.g. during the acquisition period) which is represented by $\pi_{i,0}$. This is simply the initial profit, and the profit during the later periods is represented by $\pi_{i,j}$. The variable "t" represents the number of future periods that are to be accounted for.

$$NPP_i = \sum_{j=0}^{t} \frac{\pi_{i,j}}{(1+r)^j} \tag{6}$$

$$NPP_i = \pi_{i,0} + \frac{1}{r}(1 - (1+r)^{-t})\pi_{i,j} \tag{7}$$

In this research, for simplicity of demonstration and understanding, the simple profit value function given by Eq. (5) is used. Specific descriptions of the profit value function are given in further chapters in the context to which the value function is used in each case. The other value functions will be considered in future work.

Value Functions for Government

The prime desire of the government is to have a system that offers the maximum probability of operational success. These systems are unique because there exists no defined market for such systems, and unlike other systems used for commercial gains, these systems do not generate any revenue for the government. These systems are mainly used for defense and research for technology improvement, among others. Thus, the value functions associated with such systems reflect the operational characteristics of the system rather than their monetary value. An example of such an operational value function is the probability of operational success, $p(OS)$ given by Eq. (8), that represents the success achieved from the operation when n number of systems are used involved in the operation. The operation succeeds only when each individual system succeeds $p(OS_i)$, and this success is independent of the success of the other systems. This probability of success of the operation is represented in terms of probability of operational failure, $p(OF)$, which represents the failure of the operation only if each individual system's operation $p(OF_i)$ fails, when n systems are considered [66]. Thus,

$$p(OF) = \left(p(OF_i)\right)^n$$

$$1 - p(OS) = \left(1 - p(OS_i)\right)^n$$

$$p(OS) = 1 - \left(1 - p(OS_i)\right)^n \tag{8}$$

The probability of operational success of an individual system is a function of the survivability and the effectiveness of the system, as has been established in past work [67]. The probability of survivability $p(S)$ is a measure of how survivable the aircraft is so that it can operate or perform its intended operations in hazardous environments. It is the probability that the aircraft will not be shot down during operation. Measures such as stealth technologies as well as control system redundancies are used to improve the probability of survivability of the aircraft. For the aircraft example used in this research, the survivability is taken to be a function of the velocity at which the aircraft can cruise and the stealthiness of the aircraft. The probability of effectiveness $p(E/F)$ is the chance that the system can successfully completes the operation, given that it already survives it. The effectiveness of the bomber aircraft considered in this case is considered to be a function of the amount of payload it can deliver and the range over which it can deliver the payload. The probability of operational success of a single aircraft is given by Eq. (9).

$$p(OS_i) = p(S \cap E) = p(S).p(E/S) \tag{9}$$

Cost per success

Although the primary desire of the government is to achieve the maximum probability of operational success for their system, there is only some defined amount up to which the government can spend on the system. The costs incurred for performing the desired operations cannot be overlooked. Thus, metrics have been developed to account for these costs along with the operational attributes. One such metric used traditionally is the 'cost-per-operation' (CPO), which is the ratio of the total cost to the buyer or the price paid to the contractor per aircraft $P_{aircraft}$ (assuming no other additional cost is incurred) and the number of operations, s, expected to be performed by the system over the course of its lifetime. The number of operations take into account the discount factor r to amount for the reduction in value due to the passage of time, the number of operations, m, performed by the system in a campaign, and y, the mean number of years between campaigns. s is given by Eq. (10) and the CPO is given by Eq. (11).

$$s = \frac{p(S)}{1-p(S)}\left(\frac{1-(p(S))^m}{1-(p(S))^m(1-r)^y}\right) \tag{10}$$

$$CPO = \frac{P_{aircraft}}{s} \tag{11}$$

A modified version of the CPO metric discussed above is the 'cost-per-kill' or the 'cost-per-success' (CPS), as it is generally described. This modified equation accounts for the operational costs along with the operational success of the system, which is the true preference of the stakeholder. It is the ratio of the cost-per-operation of a single system and the negative natural log of the probability of operational failure of that system. This accounts for the total number of attempts made by the system. The stakeholder using this metric aims to obtain a final system design that minimizes the cost-per-success. In short, the stakeholder aims to pay minimal price per aircraft and at the same time have the maximum operational success. The CPS is described by Eq. (12) [66].

$$CPS = \frac{CPO}{-\ln\left(1 - p(OS_i)\right)} \tag{12}$$

For this research, the probability of operational success described by Eq. 9 is used as the value function in order to examine the effects of not considering price as a part of the negotiations. The findings from this implication are described in detail in a later chapter. However, the use of cost or price-dependent value functions will definitely form an important part of future work for this research.

The next two chapters in this thesis will address the two research questions in particular and the results from the implementation of the proposed ideas will be discussed in Chapters 6 and 7.

CHAPTER 6

NEGOTIATIONS IN A COMBINED PRICE AND PERFORMANCE-BASED VALUE CONTRACTING SCENARIO

The focus of this research is the enhancement of the negotiations stage that occurs in typical defense acquisition programs in a bid to improve the design of the system and the value to both the government and the contractor. The first research questions aims to combine the traditional priced-based and performance-based contract structures and to use a value outlook to assess the final system design, the value to the contractor (profit) and the value to the government (an assumed measure of benefit for accepting an offer or rejecting it). The framework from the negotiation is based on the bargain model described above.

In the case where the three types of contracting have been combined, it is assumed that the government sets down a certain operational requirement that must be met by the aircraft system designed by the company and does not concern itself with the processes followed by the contractor as long as the requirement is met. They however, conduct an extensive market survey to determine the approximate price for the system under consideration. The survey could be based on prices of similar systems that may have existed in the past, or existing current technology, or talking to the experts in the field; and is totally dependent on the existing market for the system. The contractor on the other hand, on receiving the operational requirement from the government, does its own cost analysis to determine the price they will be putting out to the government. In order to achieve this, the company tries to come up with a design that maximizes their profit, which is indeed their true preference, at the same time making sure that the government’s operational is satisfied, even if minimally. Based on the cost obtained for the system (optimal) that maximizes their profit, the company establishes a price for the system that they intend on proposing to the government. This price is decided on the basis of a return rate that the company expects on their investment [68].

For the aircraft test case, it is assumed that the government desires that the aircraft be at least 72% successful in its operations, i.e. they wish that their probability of operational success from the mission, given by Eq. (13) be 0.72. The value was chosen arbitrarily to represent an aircraft which is fairly successful, so that no extreme was considered. However, this value can always be changed to reflect the government's desire and to see the effect of the changed value on the cost of the system.

$$Thus,\ p(OS_i) \geq 0.72 \tag{13}$$

This is passed on as a requirement to the contractor to incorporate in his design. In this case, the price is decided based on the total cost to the contractor and the investment that he expects on his investment. The profit thus, is taken to be a percentage of the total cost, which is the nothing but the sum of the costs of the individual subsystems as given by Eq. 3. Normally, the government offers a 15% return on investment for experimental, developmental, or research projects, but to obtain a wider margin for negotiation, we consider return rates between 10-20% [69]. The price or the revenue is then the sum of the cost and the profit, according to Eq. 5. In order to determine the price to be quoted, the contractor does an analysis for a design in which his aim is to maximize is profit. This is done by means of using MDO, in which the objective function is the profit given as a function of the cost and the return rate denoted by r, as discussed above, and the government's requirement is set as a constraint. In this form of contracting, we assume that the number of systems desired is pre-decided and is taken to be 100 in this example. Thus, the formal optimization statement for the contractor is given by Eq. (14)

$$find\ X = \left[X_{discrte}\ X_{integers}\ L_{wing}\ L_{chord}\ L_{fuselage}\ Mass_{payload}\right] \tag{14}$$

$$Min\ f(X) = -Profit\ per\ aircraft = -(r * Cost\ per\ aircraft)$$

$$s.t\ g_1: 0.72 - p(OS_i) \leq 0$$

For simplicity, the profit achieved due to a single aircraft is calculated and then the total profit attained by the contractor is simply the multiplication of the profit per aircraft and the number of aircraft sold, which in this case is 100. Thus, the total profit to the contractor is given by Eq. (15), whereas the price per aircraft to be quoted is given by Eq. (16).

$$Total\ profit = Profit\ per\ aircraft * No. of\ aircraft\ sold$$

$$Thus,\ Total\ profit = Profit\ per\ aircraft * 100 \tag{15}$$

$$Price\ per\ aircraft = Profit\ per\ aircraft + Cost\ per\ aircraft \tag{16}$$

The results from the optimization, i.e. the system attributes are given in Table 3. Table 4 lists the profit to the contractor for different return rates.

Table 3. *Results from Optimization in Combined Contracting*

Attribute	***Value***
Range (in km)	17,812
Mass of payload (in kg)	79,999
Cruise velocity (in m/s)	510
Stealth	0.9
$p(OS_i)$	0.72

Table 4. *Contractor Profit for Different Return Rates in Combined Contracting*

r	***Price per aircraft ($M)***	***Profit per aircraft ($M)***	***Total Profit ($B)***
10%	590	53.67	5.36
15%	616	80.4	8.04
20%	644	107	10.7

For the part involving priced-based acquisitions, a negotiation takes place between the two entities before writing the final contract. This occurs because the government would want to pay the lowest possible price for their desired system, whereas the contractor would want the highest possible price for the same system in order to maximize his profit as given by Eq. (16).

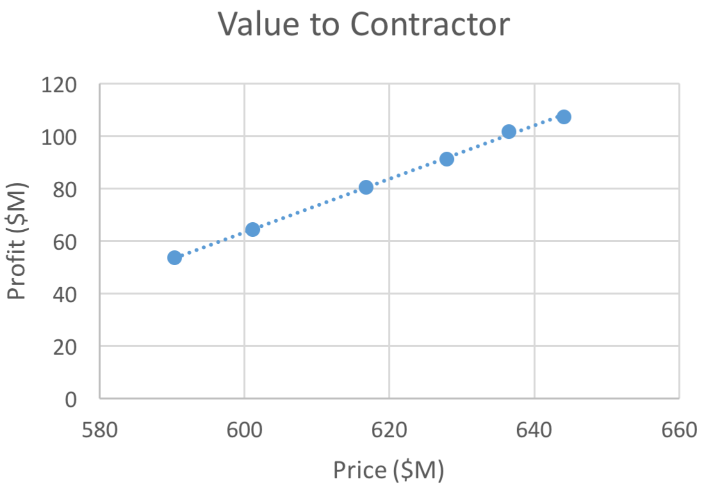


Figure 9. *Contractor's Profit as a Function of Price*

In this research, we propose an addition to the operation cum price-based contracting, the value to the stakeholders. We propose that each of the parties evaluate for their true preference or value in the bargaining game that ensues between them to decide when to accept or reject an offer. In this case, the value to the contractor (denoted by V_c) remains the profit they will receive from providing the DoD with its desired system, and is a function of the price, given by Eq. (17). The relationship between profit and price is linear in this case as is found from running the analysis for different return rates *r*, and can be seen from the plot in Fig. 9. The value to the DoD (denoted by V_g) is taken to be an arbitrary measure of the benefit to them depending on the final price paid for the system, i.e. the lower the price, the higher the value. This is described by Eq. (18).

$$V_c = Profit\ per\ aircraft = 1.0142 * Price\ per\ aircraft - 536.709 * 10^6 \qquad (17)$$

$$V_g = Value\ per\ aircraft = -0.0205 * Price\ per\ aircraft + 13.3225 * 10^6 \qquad (18)$$

The patience of the DoD in accepting an offer right away holding back till further rounds in a bid to reduce the price depends on the urgency with which the system is desired. The urgency may refer to times of sudden threats to national security, sudden outbreak or an ongoing war, times of peace, etc. In the same way, the patience of the contractor will depend on their perceived difference in value from accepting a proposed offer and waiting for the next round to put forth their bid. This has been captured by assigning a numerical value to the patience levels of both the entities, represented by δ, where δ is a number between 0 and 1. The value of the commodity decreases by a factor of δ with every round, and is used as a decision guideline by the players to wait for another round in the game of bargaining or accept the offer right away. The patience level increases as δ goes from 0 to 1. A value of $\delta = 1$ represents a very patient player whereas $\delta = 0$ represents a completely impatient player [70]. In this part of the research, a game of sequential bargaining is considered, where players take turns at making offers, and the game continues till an offer is accepted. A case of random bargaining, where there is no particular order in which players propose, is discussed in a later section of this chapter.

In this bargaining game, it is assumed that each of the respective players have a threshold price below or above which they will not accept an offer. In case of the DoD, it decides on a maximum price that it will be willing to offer for the system, based on its market research, and will never go above that price. In case of the contractor, they set a minimum price that they would be willing to accept for designing the system for the DoD. If the price offered to them goes below this price, it would be beneficial for them to simply quit the contracting process. The negotiation begins with one player offering a price. In this case, the government would start with the lowest reasonable price possible, whereas on the other hand, the contractor would start with the price they think would fetch them the highest profit, if they were to begin. The threshold and the starting

***Table 5.** Threshold Prices for Government and Contractor in Combined Contracting*

	Threshold price ($M)	***Starting offer ($M)***
Government	645	601
Contractor	590	644

prices for the bargain model constructed for this research for both the government and the contractor are given in Table 5.

Once an offer has been made, the offeree uses the price offered by the offeror to evaluate his payoff, in this case the value. He then uses this value to ascertain if his equilibrium condition, given by Eq. (2) has been met. If the offeree believes that the price being offered to him in the current round cannot yield him a better value than that he would receive by waiting for the next round to make an offer, he rejects the offer. After each round, the value to either of the parties decreases by a factor of their respective patience levels. If an offer is rejected by a player, the offeree in the previous round becomes the new offeror in the current round, and the previous offeror alters his offering price to be offered in the next round, did the game continue further. In case of the government, if a price proposed by them is rejected by the contractor in the current round, they increase their price to be offered in the next round, were the game to continue to further rounds. In the same way, the company would reduce their price. In this study, the prices were changed by 1% after every round. The negotiation continues to and fro till an offer is accepted, and the game ends. The bargain model in this study is tested for varying patient levels of both the players, and the results obtained are given in the Table 6. δ_g and δ_c represent the patience factors of the government and contractor, respectively. For this case, it is assumed that the government

Table 6. *Result of the Game Using Combined Contracting*

Patience factors (δ)		***Rounds***	***Offer accepted***		***Final price per aircraft ($M)***	V_g $(*10^3)$	V_c ***($B) (Profit from 100 aircraft)***
δ_g	δ_c		***Govt***	***Comp***			
0.1	0.95	2	✓	×	644.05	119	11.06
0.2	0.9	4	✓	×	637.61	251	8.84
0.9	0.1	1	×	✓	601.11	999	7.29
0.3	0.8	3	×	✓	607.12	262	7.90
0.5	0.5	1	×	✓	601.11	999	7.29
0.6	0.7	3	×	✓	607.12	525	7.90
0.98	0.98	8	✓	×	624.92	511	9.31

makes the first offer, and the implicit advantages of beginning the bargain are shown in a sub test case later in this chapter.

From the above table, it can be seen that when the contractor is extremely patient (0.95) and the government has very low patience (0.1), the game ends in the second round because the government accepts the offer right away. The value received by the government is thus very low on the arbitrary scale. The contractor, on the other hand, receives the highest value because the government accepts his first proposed offer itself, thus yearning the contractor the highest possible profit. As the contractor's patience reduces (0.9) and the government's increases (0.2) the government still accepts the offer, but the game goes on for a longer period and ends only after 4 rounds. The government's value improves due to the greater patience, but is still comparatively low. The contractor's profit on the other hand, drops by about $3 billion. In the case where the government is extremely patient (0.9), as in periods of global peace, when there is no urgent need for the system, and when the contractor is quite impatient (0.1), the contractor accepts the first offer and the game ends in the first round itself as the government begins the game. This yields the contractor quite a low profit of $7.29 billion, which is about a $4 billion lower than his highest attainable value. When both the players have a very high patience (0.98), the game goes on for a longer period, as intuition suggests, because neither of the players are willing to give up. In this case, the game ends after 8 rounds with the government finally accepting the company's offer. Even though the game goes on for a longer period, the value of the system to the players reduces by only 2% after each round, which does not result in a great reduction in the final values to the players, as can be seen from the table.

An interesting observation made in this model however, is that even though theory says that the patience factor should dictate players' decision to accept or reject an offer, the sensitivity of the players' value or payoff plays an important role, as can be seen from the obtained results. In this test case, the contractor's profit is extremely sensitive to price, and because he is only negotiating for prices offering return rates between 10% and 20%, his value drops or increases quite suddenly with price, causing him to accept the offer even when his patience is relatively high (0.8) as compared to the government's (0.3). The same holds true even when the contractor's patience changes to 0.7, and the government's increases to 0.6, and both the values result in the contractor accepting the offer in the third round yielding him the same profit. The government's earned value, however, varies in the two cases as their value changes by a different factor for either case. These observations can be seen clearly in Table 6. When both the players have a patience factor of 0.5, the contractor ends up accepting the offer in the first round itself due to the same reason as above.

The results obtained from this combined form of contracting are compared to a traditional form of acquisition, where the government puts forth a requirement for the desired system to be the most cost-effective. This is a traditional requirement that plays proxy to the true preference of both the stakeholders as the government wishes for maximum mission success whereas the contractor desires maximum monetary gains for his organization. This requirement is communicated to the highest level in the organization of the company, for example, the CEO. The requirement of minimum cost is then passed down the hierarchy of the organization in order for the designers to come up with a design and the cost estimate. The lower level teams break down the requirements and form their own requirements in order for the design to be feasible. The company then tries to come up with a design that minimizes cost and satisfies the other design requirements as well, which intuitively translates into an optimization problem, with the objective being minimum cost. For this study, it is assumed that the government puts out a requirement for an aircraft weapon system with minimum cost that also:

- Weighs less than 150,000 kg
- Flies a range of at least 9000 km

The design teams analyze these requirements and based on these, create their own requirements. The formal optimization statement for this test case is given by Eq. (19), where *i* represents the total number of subsystems.

$$find\ X = \left[X_{discrte}\ X_{integers}\ L_{wing}\ L_{chord}\ L_{fuselage}\ Mass_{payload}\right] \tag{19}$$

$$Min\ f(X) = Total\ cost\ per\ aircraft = \sum_{i=1}^{m} Cost_i$$

$$s.t\ g_1 : Mass_{total} - 150000\ kg \leq 0$$

$$g_2 : 9000\ km - Range \leq 0$$

$$g_3: 165\ m/s - V_{cruise} \leq 0$$

$$8\ m \leq L_{wing} \leq 12\ m$$

$$2\ m \leq L_{chord} \leq 4\ m$$

$$12\ m \leq L_{fuselage} \leq 20\ m$$

$$15000\ kg \leq Mass_{payload} \leq 50000\ kg$$

It is assumed that the requirement on the velocity and the 4 side bounds were imposed by the design team of the contractor to ensure a meaningful design. The attributes from the obtained final system and the corresponding probability of operational success for this system are given in Table 7. All the constraints were satisfied. The price per aircraft for such a system that minimizes cost and the corresponding profit to the contractor for selling 100 aircraft with three different return rates *r* are shown in Table 8.

Table 7. *Results from Optimization Using CBA*

Attribute	***Value***
Range (in km)	9001
Mass of payload (in kg)	49,999
Cruise velocity (in m/s)	257
Stealth	0.5
$p(OS_i)$	0.402

Table 8. *Contractor Profit for Different Return Rates Using CBA*

r	***Price per aircraft ($M)***	***Profit per aircraft ($M)***	***Total Profit ($M)***
10%	29	2.64	264
15%	30	3.96	396
20%	31	5.28	528

As can be seen from the above tables, the probability of mission success achieved by using the traditional requirements-driven approach is around 40%, whereas that obtained from the previous value-based approach was 72%. Also, all the operational attributes from this approach are much lower in value compared to the combined contracting process. The profit that can be obtained by the contractor from this system, even with a 20% rate of return is still 3 orders lower in magnitude than the lowest possible profit that he can get by using the previous approach. In any case, even after negotiating and based on the patience levels of both the players, the final payoff to both the players will be lesser than that obtained by using the combined performance, price and value based approach to contracting. In this traditional cost-based acquisition process, once the cost analysis is completed by the company, a detailed report about the cost breakdown is provided to the DoD by the contractor, and a bargain ensues between the two parties in a bid to maximize their respective payoffs. Here, there is no need for the government to conduct a market survey as an exact dollar amount is provided to them by the company. The payoff to the company or their profit in this game of bargaining is denoted by P_c whereas they payoff to the government, again an arbitrary measure of benefit as a function of the price paid is denoted by P_g. The equations for the payoffs to either of the players are denoted as Eq. (20) and Eq. (21), respectively. Again, the payoff equation for the government has a negative slope because the lower the price, the higher their payoff or benefit.

$$P_c = Profit\ per\ aircraft = Price\ per\ aircraft - 26.41 * 10^6 \tag{20}$$

$$P_g = Benefit\ per\ aircraft = -3.406 * Price\ per\ aircraft + 10.8961 * 10^6 \tag{21}$$

The same theory behind threshold prices for both the players described above applies to this case as well, and the values of the threshold and starting prices for the DoD and the contractor are given in Table 9

Table 9. *Threshold Prices for Government and Contractor Using CBA*

	Threshold price ($M)	***Starting offer ($M)***
Government	32.00	29.50
Contractor	29.05	31.69

The same logic about the equilibrium conditions from above applies to this game. The results for different patience levels of the players are given in Table 10 and is followed by an interpretation of the obtained results.

Table 10. *Result of the Game Using CBA*

Patience factors (δ)		***Rounds***	***Offer accepted***		***Final price per aircraft ($M)***	P_g $(* 10^3)$	P_c ***($M) (Profit from 100 aircraft)***
δ_g	δ_c		***Govt***	***Comp***			
0.1	0.95	2	✓	×	31.690	100	528.70
0.2	0.9	4	✓	×	31.380	208	447.30
0.9	0.1	1	×	✓	29.500	848	309.00
0.3	0.8	3	×	✓	29.790	224	338.50
0.5	0.5	1	×	✓	29.500	848	309.00
0.6	0.7	3	×	✓	29.795	448	399.72
0.98	0.98	9	×	✓	30.697	421	428.78

The results obtained from the cost-based acquisitions follow a similar trend with regards to the patience levels of the players as in the previous case, however, in this case also, the sensitivities of the player payoffs have an effect on the final result, as cab be clearly seen from Table 10 above.

The important point to be considered from this study is the notable difference in the obtained results when different forms of contracting are used. Table 11 gives a side by side comparison of the final system attributes and the prices after negotiations using the traditional CBA and the proposed new combined method of contracting.

Table 11. *Comparison of CBA and Combined Acquisitions*

	Cost-based acquisitions	***Combined acquisitions***
Range (in km)	9001	17,812
Mass of payload (in kg)	49,999	79,999
Cruise velocity (in m/s)	257	510
Stealth	0.5	0.9
$\boldsymbol{p(OS_i)}$	0.402	0.72
Total profit for lowest contractor patience ($)	309.00 million	7.29 billion

As can be seen from the above table, the system obtained from the proposed combined form of contracting has a much more superior design as compared to the one obtained from the traditional process. Similarly, the new form of contracting yields much better payoffs to both the government and the contractor, which is what they truly desire of any system being designed.

This concludes the proof to the first research question.

Task 1: Impact of Beginning the Bargaining Game on Payoffs

As an additional in this part of the research, the influence of the player making the first offer in the negotiation on the final player payoffs is tested. In order to achieve this, for the same system and the same final set of attributes as above, a bargaining game is simulated in which the contractor is the first player to make an offer, in order to observe how the final payoffs to the players are affected. This is done for both the traditional acquisitions and the new combined acquisitions processes. The results for the two games along with a side by side comparison with the above cases are given in Tables 12 and 13.

- Combined contracting

Table 12. *Result of Reversed Order Game Using Combined Contracting*

		$\boldsymbol{\delta_g = 0.1}$ $\boldsymbol{\delta_c = 0.95}$		$\boldsymbol{\delta_g = 0.9}$ $\boldsymbol{\delta_c = 0.1}$		$\boldsymbol{\delta_g = 0.6}$ $\boldsymbol{\delta_c = 0.7}$		$\boldsymbol{\delta_g = 0.98}$ $\boldsymbol{\delta_c = 0.98}$	
		Player 1=Comp	***Player 1=Govt***	***Player 1=Comp***	***Player 1=Govt***	***Player 1=Comp***	***Player 1=Govt***	***Player 1=Comp***	***Player 1=Govt***
Rounds		1	2	2	1	4	3	8	8
Offer accepted	***Govt***	✓	✓	×	×	×	×	×	✓
	Comp	×	×	✓	✓	✓	✓	✓	×
Price per aircraft ($M)		644.05	644.05	601.11	601.11	607.12	607.12	619.32	624.92
P_g (*10^3)		119	119	899	999	285	525	599	511
P_c ($M)		11640	11060	7290	7290	7900	7900	9140	9310

- Cost based contracting

Table 13. *Result of Reversed Order Game Using CBA*

		$\delta_g = 0.1$ $\delta_c = 0.95$		$\delta_g = 0.9$ $\delta_c = 0.1$		$\delta_g = 0.6$ $\delta_c = 0.7$		$\delta_g = 0.98$ $\delta_c = 0.98$	
		Player 1=Comp	***Player 1=Govt***	***Player 1=Comp***	***Player 1=Govt***	***Player 1=Comp***	***Player 1=Govt***	***Player 1=Comp***	***Player 1=Govt***
Rounds		1	2	2	1	4	3	8	9
Offer accepted	***Govt***	✓	✓	×	×	×	×	×	×
	Comp	×	×	✓	✓	✓	✓	✓	✓
Price per aircraft ($M)		31.69	31.69	29.50	29.50	29.79	29.79	30.39	30.69
P_g (*10^3)		100	100	763	848	245	448	520	421
P_c ($M)		528.70	502.26	309.00	309.00	338.50	338.50	398.38	428.78

From the two test cases above, it can be seen that making the first offer definitely yields a better payoff to the proposer if the offer is accepted by the other player and not himself. For example, in the two examples given above, when $\delta_c = 0.95$ and $\delta_g = 0.1$, the government accepts the offer in both cases. However, the profit or payoff to the company is greater when they make the first offer, as can be seen. In the same way, when $\delta_c = 0.1$ and $\delta_g = 0.95$, the company accepts the offer in both the test cases. In either case, the payoff received by the government is greater when they are the first player. The boxes highlighted in yellow indicate which player is at an advantage. The observations made above are quite intuitive, because if the accepting player makes the first offer, then the other player has to wait for another round before his offer is accepted, which amounts to a reduction in the value of the system by a factor of δ. Making the first offer averts this loss because the game ends in one lesser round, thereby giving the player a higher payoff.

However, in the case where both the players are extremely patient, it can be observed that the player making the first offer always receives the better payoff, irrespective of which player finally accepts the offer, as can be seen from both the test cases when δ for both the players is 0.98, which suggests that in an infinitely long game (almost), the player making the first offer always stands at an advantage.

With this, research question 1 as well as the additional study on player order have been completed. The next chapter will address the second research question and state the findings from the implication of the proposition.

CHAPTER 7

NEGOTIATION OVER ATTRIBUTES USING VDD

Where the conventional method is to bargain over the price of a system, in this research, we consider a new approach to the method of bargaining, that over the attributes of the system. This approach has been proposed in a previous study but according to the author's knowledge, no detailed work has been carried out on the topic so far [70]. It focuses to investigate if the direct bargaining over the system attributes would enable bridging the gap between the stakeholder preferences, and thereby lead to a better design of the final system. The common attributes that affect the values of both the government and the company are identified for the aircraft example and a bargain model is set up which is described in detail later in this section.

In this case, it is assumed that both the players are only concerned with their value, i.e. the DoD is not concerned with the budget. They desire a system that gives the maximum operational success. The contractor on the other hand, wishes for a system that maximizes his profit. As a result of these different preferences, there exists a value gap, which needs to be filled. This is where bargaining over attributes comes into picture. The common attributes affecting the values of both the players are identified, and the optimal values of these attributes (that maximize value) are calculated for each of the players. A negotiation then follows over these values, until an agreement point is reached and an offer is accepted, resulting in a design somewhere in between the optimal designs of both the players. A description of this bargain model and the optimal values to the players are given in the following section.

Bargain Model

The new method of bargaining is introduced in this research in a bid to achieve a better system design by capturing the attributes that reflect the true preferences of the stakeholders. In order to enable this, the common attributes that affect the values of both the government and the company are identified. In the case of the aircraft example considered in this research, there are 4 common attributes that impact the profit as well as the probability of operational success, namely Range, Mass of payload, Cruise Velocity, and Stealth. In order to test the effectiveness of the bargain model, it is first applied to an example problem before proceeding to apply it to the aircraft model. A simple system is designed which is hierarchically decomposed into two tiers for one party (assuming the company in this case), whereas for the other party it is just one level (assuming the government) which are shown in Fig. 10. It is assumed that the value functions of both the stakeholders are functions of three common attributes, A_1, A_2 and A_3. The value functions for the government and the contractor, denoted by V_g and V_c are represented by Eq. (22) and Eq. (23), where A_g represents the attribute set desired by the government and A_c represents the attribute set desired by the company. $A_{g1}, A_{c1}, A_{g2}, A_{c2}$, etc. correspond to the values of the attributes A_1, A_2 and A_3 desired by the government and the company, respectively. It is also assumed that the values of all three attributes lie between 0 and 10.

$$V_g = f(A_1, A_2, A_3) = A_{g1}A_{g2} - 2A_{g3} - A_{g2}^2 \tag{22}$$

$$V_c = f(A_1, A_2, A_3) = A_{c1}^2 - A_{c2}^2 + 3A_{c3} \tag{23}$$

In this case, a sequential bargain game of infinite horizon is considered. It was assumed that the government initiates the bargaining, hence they are *player 1* and the company is *player 2.* From intuition, it is clear that both the respective players' preferred attribute set for the design would be one that maximizes their individual values, i.e. their optimal set of attributes($A_g^* \ and \ A_c^*$). Thus, in the first round, player 1 always proposes A_g^*. Like player 1, player 2 also desires his optimal set A_c^* to be the final attribute set. Thus, when A_g^* is proposed, player 2 accepts the offer only if he believes that his value from A_g^* is greater than the value he would obtain by rejecting the offer and proposing his own attribute set A_c^* in the next period, discounted by a factor δ_c. If this equilibrium condition is met, player 2 accepts the offer, the game ends and A_g^* is the final attribute set to be used for design. However, if this is found to be not true, player 2 will reject the offer and the game will move to round 2. In this round, player 2 will propose A_c^* and player 1 will check for his equilibrium, i.e. he will accept A_c^* is greater than the value he would receive by waiting for the next round and proposing a new attribute set $A_{g_{new}}$, discounted by δ_g. If the condition is satisfied, it puts an end to the game and A_c^* is the final attribute set, if not, the game continues in a similar fashion till one player accepts the offer.

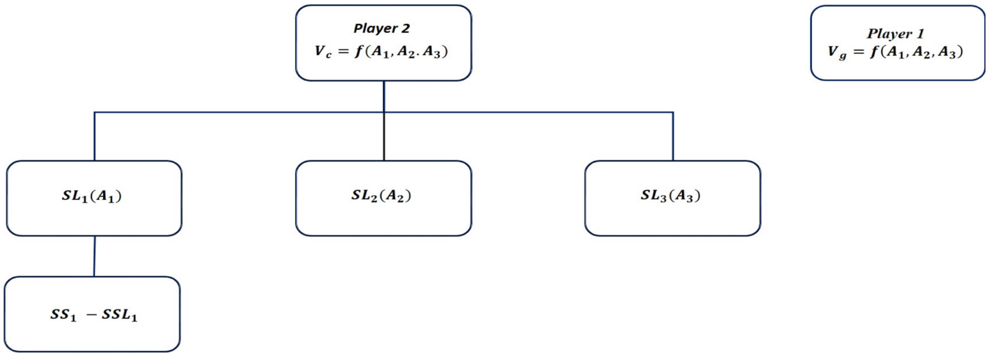


Figure 10. *Hierarchy for Example Problem*

The equilibrium conditions for players 1 and 2, assuming that player 1 makes the first offer, are given by Eq. (24) and Eq. (25). Here, V_{1_g} and V_{1_c} represent player 1's values due to his own attribute set and player 2's attribute set, whereas ${V_2}_c$ and ${V_2}_g$ represent player 2's values due to attribute sets $A_c$and A_g. The deltas in this case represent the discount factors, which in case of the company may represent the reduction in value to account for the money to be paid to the employees, the inflation in the future periods or the time lost while the negotiation is taking place. Thus, the longer the game lasts, the greater is the reduction in value. For the government, the delta may represent the losses in research or losses during the time of war associated with the delay in the design and manufacture of the aircraft system [71].

$$ {V_1}_c \geq \delta_g * {V_1}_g \tag{24} $$

$$ {V_2}_g \geq \delta_c * {V_2}_c \tag{25} $$

Optimal Values for Contractor

As mentioned earlier, the organization of the company in this paper is designed according to the 5 major components of the aircraft, and the hierarchy spans two levels. The profit obtained by the company is the difference between the revenue generated by selling these aircraft and the cost to the company for manufacturing the aircraft. Because the system is being designed for value, the number of aircraft sold in this case is not fixed, but depends on the operational attributes. The total cost and revenue to the company are the same as given by Eq. (3) and Eq. (4). The number of aircraft sold in this case is a function of the range, cruise velocity, stealth, and also the price per aircraft.

The formal optimization statement for determining the optimal attribute set and the optimal value for the contractor is given by Eq. (26).

$$ find\ X = \left[X_{discrte}\ X_{integers}\ L_{wing}\ L_{chord}\ L_{fuselage}\ Mass_{payload}\right] \tag{26} $$

$$ Min\ f(X) = -Profit\ = -(Revenue - Cost_{total}) $$

In this example of using VDD, all constraints are eliminated. The values of the 4 attributes common to the government and the company mentioned earlier at the optimal design point and the value to the contractor from these attributes (optimal value) are given in Table 14.

The following values of the attributes will be used at the beginning of the negotiation game between the government and the company as the first proposal made by the company. If given their way, the company would always want to have this attribute set to use for the design as it gives them the maximum payoff. An illustration for what the aircraft would look like using the company's optimal attribute set is shown in Fig. 11.

Table 14: *Optimal Attribute Values for Company*

Attribute	***Value***
Range (in km)	14054
Mass of payload (in kg)	79,999
Cruise velocity (in m/s)	454
Stealth	0.4
Value/Profit (in $B)	4.381

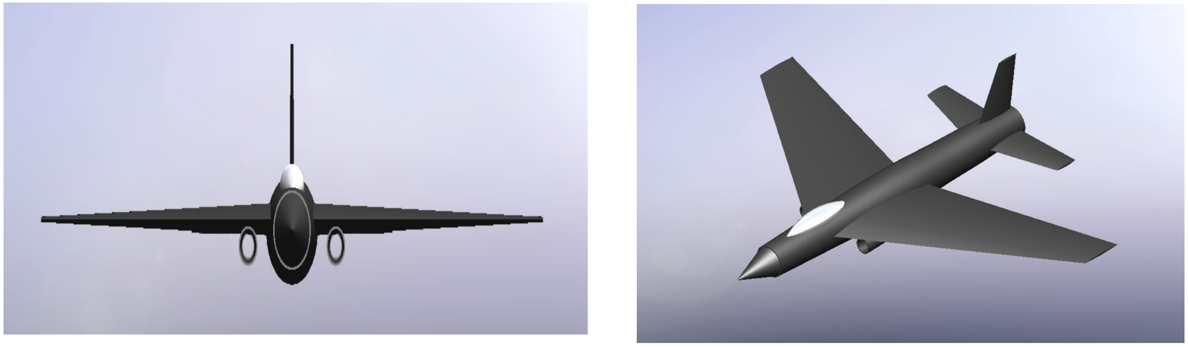


Figure 11. *Design of Aircraft Using Contractor's Optimal Attribute Set*

Optimal Values for Government

The government does its own analysis and brings forth a set of attributes that they think will fetch them the maximum value. In this case, the government wants to maximize its mission success. The optimal values of the 4 attributes for the government are described in Table 15.

Table 15: *Optimal Attribute Values for the Government*

Attribute	***Value***
Range (in km)	18000
Mass of payload (in kg)	99687
Cruise velocity (in m/s)	677.47
Stealth	0.9
Value/$p(OS_i)$	0.721

As can be seen from the obtained values, the government desires a system with exceptional characteristics that yields a fairly high chance of operational success. If given their way, the government would want this to be the final set of attributes. However, due to a difference in preferences of the players, a negotiation takes place to determine the final attributes to be designed for. An illustration for what the aircraft would look like using the company's optimal attribute set is shown in Fig. 12.

Figure 12. *Design of Aircraft Using Government's Optimal Attribute Set*

Bargaining over Attributes

Example problem

For the example problem, bargaining is done only over attribute A_3 for simplicity. To begin with, the optimal attribute set for each player 1 and player 2 are found, and the game begins with player 1 proposing his optimal solution. The blocks highlighted in yellow indicate the player making the offer in that round. Player 2 checks for his equilibrium condition using this attribute set as well as his own attribute set, using Eq. (25), and discovers that the equilibrium condition is not satisfied. The offer is rejected and the game continues to further rounds, till player 2 finally accepts the offer after 11 rounds of bargaining. After each round, the players modify their values of the attribute A_3 such that it lowers their own value obtained from this new attribute set and increases the other player's value. The game moves in such a way that it tries to achieve middle ground, i.e. the players reduce their respective values till an agreement point is reached, which is described in Fig. 13. The initial values of both players, the reduced values after each round, as well

Table 16: *Results for Bargaining over Attributes for Example Problem*

Round	A_g	A_c	δ_g	δ_c	V_{1_g}	V_{1_c}	V_{2_g}	V_{2_c}	***Status***
1	[10 5 1]	[10 1 10]	0.9	0.8	23.00	-11.00	78.00	129.00	Rejected by player 2
2	[10 5 2]	[10 1 10]	0.9	0.8	21.00	-11.00	81.00	129.00	Rejected by player 1
3	[10 5 2]	[10 1 9]	0.9	0.8	21.00	-9.00	81.00	126.00	Rejected by player 2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10	[10 5 6]	[10 1 6]	0.9	0.8	13.00	-3.00	93.00	117..00	Rejected by player 1
11	[10 5 6]	[10 1 5]	0.9	0.8	13.00	-0.99	93.00	114.00	Accepted by player 2

as the final set of values after the players reached a common ground are shown in Table 16. The shaded blocks represent the proposing player for that round.

Thus, the game ends with the final attribute set $A = (A_1, A_2, A_3) = (10,5,6)$. The final equilibrium value of player 1 drops from 23 to 13 whereas the value of player 2 drops from 129 to 93. Thus, the final solution after the bargaining game is as follows:

$$\boldsymbol{V^* = \left(V_g, V_c\right) = (13, 93)}$$

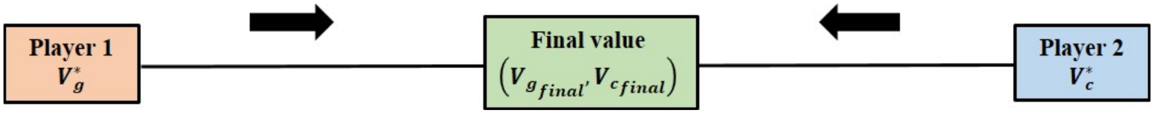


Figure 13. *Game of Bargaining for Example Problem*

Aircraft model

The attribute bargain model that was successfully created for the example problem is then applied to the entire aircraft. Again, it is assumed that the government begins the negotiation by making the first offer. In this case, the discount factors for the players are kept constant, as are shown in Table 17. This initial offer made by the government is their optimal attribute set A_g^*, given in Table 15 that fetches them the maximum probability of operational success of 0.721. However, the company refuses this offer in round 1 because it yields them a value of \$681.81M which does not meet their equilibrium condition as they believe that they can obtain a better value (\$4.31B) by waiting out till the next round and proposing their own attribute set. Thus, the game moves to round 2 where the company is the proposing player. The company proposes V_c^*as this attribute set fetches the company a maximum profit of \$4.318B. However, on plugging in the value of this proposed attribute set into their own value function, the government infers that they could do much better by waiting for the next round (Eq. (24)). Thus, the offer made by the company is rejected and the game proceeds.

In round 3, the government becomes the proposing player again. However, this time, the government proposes a new attribute set $A_{g_{new}}$ as their previous attribute set was rejected by the company. This new attribute set is attained by changing the value of range by 2% and then running an analysis using this new range to calculate the values of the other attributes. The new attribute set reduces the government's value by 0.007%, i. e. the probability of operational success drops from 0.721 to 0.716. However, for this new set of attributes, the contractor's profit sees a tremendous rise to $2.28B, from the initial $681.81M. The contractor compares this value to the value he could potentially obtain in the next round by proposing a modified attribute set.

On checking for these values, he infers that the value on the table is greater than what he would make by waiting out till the next round, implying that his equilibrium condition given by

Table 17: *Results for Bargaining over Attributes for Aircraft Example (Sequential)*

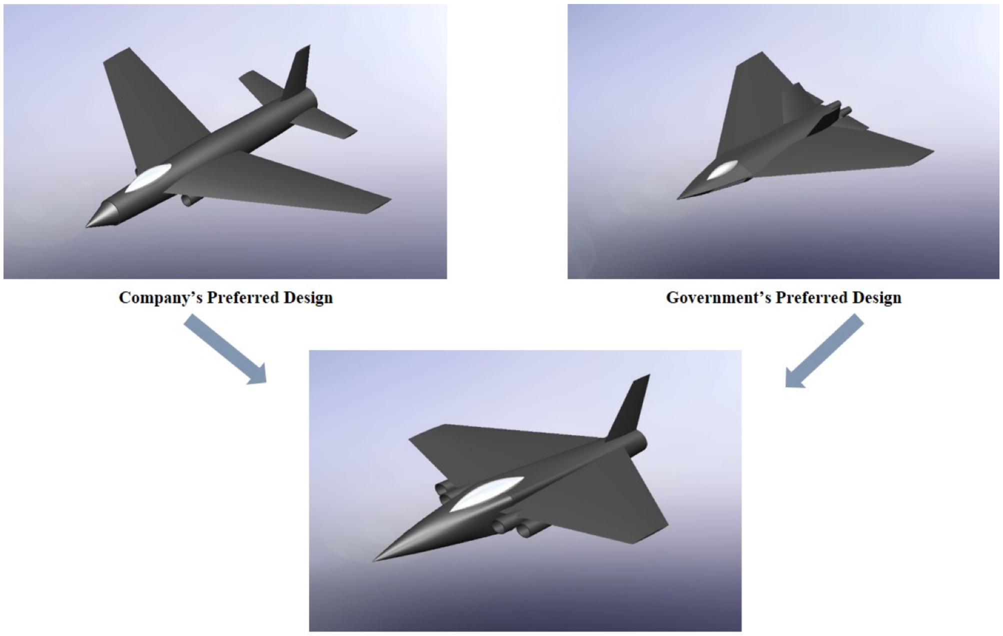
Round	A_g		A_c		δ_1	δ_2	V_{1_g} (%)	V_{1_c} (%)	V_{2_g} (in $)	V_{2_c} (in $)	**Status**
1	Range (in km)	18000	Range (in km)	14054	0.97	0.9	0.721	0.392	681.8 M	4.38 B	×
	$M_{payload}$ (in kg)	99687	$M_{payload}$ (in kg)	80000							
	Velocity (in m/s)	677.47	Velocity (in m/s)	454.89							
	Stealth (%)	0.9	Stealth (%)	0.4							
2	Range (in km)	17640	Range (in km)	14054	0.97	0.9	0.716	0.392	2.28B	4.38B	×
	$M_{payload}$ (in kg)	99687	$M_{payload}$ (in kg)	80000							
	Velocity (in m/s)	565.82	Velocity (in m/s)	454.89							
	Stealth (%)	0.9	Stealth (%)	0.4							
3	Range (in km)	17640	Range (in km)	14335	0.97	0.9	0.716	0.394	2.28B	1.34B	▪ ✓
	$M_{payload}$ (in kg)	99687	$M_{payload}$ (in kg)	80000							
	Velocity (in m/s)	565.82	Velocity (in m/s)	517.89							
	Stealth (%)	0.9	Stealth (%)	0.4							

Table 18: *Final System Design and Values (Sequential Bargaining)*

Values of attributes		***Value to government p(OS) (%)***	***Value to company (in $B)***	***Final price per aircraft ($M)***
$Range\ (in\ km)$	17640	0.716	2.28	184.71
$M_{payload}\ (in\ kg)$	99687			
$Velocity\ (in\ m/s)$	565.82			
$Stealth\ (\%)$	0.9			

Eq. (25) is satisfied. Thus, the contractor accepts the offer and $V_{g_{new}}$ is the final attribute set to be designed for. The sequential game described above is shown in Table 17. The highlighted blocks represent the proposing player in each round. Table 18 shows the final values of the attributes used for design and the corresponding values of the company and the government for this attribute set. Thus, the final system design will yield a collective profit of $2.28B to the company and will have a 71.6% chance of achieving mission success.

Fig. (14) pictorially demonstrates the negotiation game that takes place between the government and the contractor and depicts what the negotiated system may look like.



Company's Preferred Design

Government's Preferred Design

Negotiated Final Design

Figure 14. *Final System Design after Bargaining over Attributes*

An interesting observation that can be made from Table 18 is that even though an assumption was made in this part of the study that the price of the system isn't a factor that affects the decisions of the government, which may be unrealistic in the actual defense acquisitions world, it can be seen that negotiating solely over the attributes of the system does not drive the price paid by the government to an unrealistic amount, and yet results in a system with a good success probability and one that also results the contractor a commendable profit.

Task 1: Random Bargaining

Another variation of the bargaining game is one in which offers are made randomly by the players rather than following a fixed sequence shown in the above case. In this game, if an offer is rejected, there is no fixed order as to which player will make the offer in the following round. The player whose offer is rejected in the previous round can again make a new offer in the next round. These kind of bargaining games have as probability associated with the outcome that corresponds to the uncertainty associated with which player will make the next offer. The game begins by determining an expected attribute set *(EA)* that incorporates the probabilities of each player making the offer in the next round. The expected outcome can be calculated using Eq. (27), where *"q"* is the probability that player 1 (the government in this case) makes the proposing offer. Thus, the probability that player 2 (company) will offer in the following round is *(1-q)*.

$$EA = q * V_g + (1 - q) * V_c \tag{27}$$

The proposing player uses the expected attribute set to evaluate his own payoff due to the uncertainty associated with him proposing the offer in the first place, when his first proposing offer again is his optimal attribute set, A_g^* for the government and A_c^* for the contractor for the example considered here. The equilibrium strategies for the government and the company are given by Eq. (28) and Eq. (29) [70]. The notations used in the equations are the same as used previously.

$$g^* = \left(V_g(EA), V_c(A_g)\right) \tag{28}$$

$$c^* = \left(V_g(A_c), V_c(EA)\right) \tag{29}$$

The government prefers attribute set A_g^* as the set to be used for design as it maximizes their value from the system whereas the company prefers A_c^*. After an offer has been rejected, the player who made the offer changes his attribute set such that it lowers his value and increases the value of the other player, in a bid to have his offer accepted in a later period. The new expected attribute set is calculated again that depends on the probabilities of the players making the next offer. Once a new offer is made by a player, the other player uses this attribute set to determine if he could gain more by waiting and proposing her own attribute set in the next period, keeping in mind the probability of him making the next offer. If he finds this to be true, he rejects the offer and the game moves to the next period. This continues till one of the players' equilibrium condition is met and the offer is accepted. The equilibrium conditions for the government and the company for a game of random bargaining are shown in Eq. (30) and Eq. (31) [71].

$$V_g(A_c) \geq \delta_1 * V_g(EA) \tag{30}$$

$$V_c(A_g) \geq \delta_2 * V_c(EA) \tag{31}$$

The results from a game of random offers, when applied to the aircraft model, are shown in Table 19. The game goes on for 31 rounds till contractor finally accepts the offer because. For this study, the probability q is generated in each round using a random number generator. The table shows the value of the final attribute set and the payoffs of the government and the company from this game.

As can be seen from the table, even though the attributes of the system are very similar to those obtained from the game of sequential bargaining, the final values to the players is reduced because the game goes on for an extensive number of rounds, thereby reducing the value with every round.

Table 19: *Final System Design and Values (Random Bargaining)*

Values of attributes		***Value to government p(OS) (%)***	***Value to company (in $M)***	***Final price per aircraft ($M)***
$Range\ (in\ km)$	17892.30	0.620	930.59	185.44
$M_{payload}\ (in\ kg)$	99687			
$Velocity\ (in\ m/s)$	566.64			
$Stealth\ (\%)$	0.9			

This concludes the second research question. A summary of the thesis as well as the potential studies that can be carried out as a continuation to this work will be discussed in Chapter 8.

CHAPTER 8

SUMMARY, CONCLUSION AND FUTURE WORK

Summary and Conclusion

Taking into account the various cost overruns, schedule delays and under performance issues related with the current defense acquisition processes, an effort is being made to address the issues in the current methods in order to obtain a better system design as well as yield a higher value to the contractors. This is achieved by using the two new approaches to defense acquisitions proposed in this research that aim at introducing a value-based outlook to defense acquisitions

This research has shown how a combined price and performance-based value approach to contracting represents a transition from the traditional requirements-driven methods to a more value-based perspective. The research concentrated on only a small piece of negotiations in the otherwise tremendously complex defense acquisition processes and the negotiations were mathematically modeled using theory of bargaining in order to assist the decision-makers in making more informed decisions. The results obtained from the first approach, when compared to a traditional cost-based acquisition method, have shown that proposed idea yields exceedingly better results both in terms of the operational success of the aircraft that is the true preference of the government, as well as culminates into a much higher profit or value for the contractor.

The second approach proposed in this research concentrates purely on value, and deals with negotiating directly over the attributes of the system that define the operational characteristics of the system. This has also shown that a value-based approach to negotiation over attributes results in a superior system design in terms of operational characteristics as well as results in a high profit to the contractor. This research has also demonstrated that even if the cost is not considered as in important deciding factor for the government, the cost for the final system obtained after negotiation is not driven to an unrealistically high amount.

Future Work

This research holds tremendous potential for future work due to the possibility of exploration in numerous areas of defense acquisitions. With regards to the particular area of focus dealt with in this research, the incorporation of risk in design could be an interesting an important piece to study, because risk forms a major part in weapon system acquisitions.

Another area of focus for future research could be studying the effect of changing the requirements on the payoffs of the players after an initial system has been negotiated upon. The effect of incentives in the combined acquisitions scenario as well as in the pure value approach would also be a thought-provoking piece to see if an improved performance is obtained. As this research only used the simple profit value function for the contractor and did not include a cost component in the value function for the government, using different and more realistic value functions to see the effect on the results would be an interesting part to be considered in the future.

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APPENDIX A

AIRCRAFT MODEL

Appendix A will discuss the aircraft system subsystems, define the variables associated with these subsystems and describe the equations involved in each of the subsystem analysis. The entire analysis and model was programmed using Matlab [72].

Table A gives a list of the attributes and the corresponding design variables associated with each subsystem of the aircraft.

Table A. *List of design variables and attributes for aircraft*

Tiers for Company (Value function: Profit)			**Attributes**	**Design variables**
SYSTEM (Bomber Aircraft)			Cost, Revenue	Number of Aircraft (n)
Subsystem level 1	**(SS1)** Wing		C_{wing} M_{wing}	Type of wing, Mass of payload
	(SS2) Fuselage		$C_{fuselage}$ $M_{fuselage}$	Length of fuselage
	(SS3) Tail		C_{tail} M_{tail}	Material of tail, Type of tail
	(SS4) Landing gear		$C_{lndgear}$ $M_{lndgear}$	Type of landing gear (tail dragger, tricycle,etc)
	(SS2) Engine		C_{engine}, M_{engine} M_{fuel}, Range, V_{cruise}	n_{Eng}, Type of engine
Subsystem level 2	Wings	**(SS1)** Spar	C_{spar} M_{spar}	Material of spar, Length of wing, Length of chord
		(SS2) Ribs	C_{ribs} M_{ribs}	Material of rib, Length of wing Length of chord
		(SS3) Skin	C_{skin}	Material of skin, Length of wing, Length of chord
	Fuselage	**(SS1)**Frames	C_{frames} M_{frames}	n_{frames}, Material of frame
		(SS2) Longerons	$C_{longeron}$ $M_{longerons}$	$n_{longerons}$, Material of longerons, Length of fuselage
		(SS3) Skin	C_{skin} M_{skin}	Material of skin

Variable definitions

Variables and Parameters	**Description**	**Type**	Value
$\sigma_{\mathrm{radial}_{\mathrm{max}}}$	Maximum radial stress in N/m^2	Calculated	----
$\sigma_{\mathrm{tangent}_{\mathrm{max}}}$	Maximum tangential stress in N/m^2	Calculated	----
$\mathrm{Assembly}_{\mathrm{cost}}$	Assembly cost per tyre in $	Assumed	5000
$\mathrm{D}_{\mathrm{cruise}_1}$	Drag for first cruise segment	Calculated	----
d_{LG}	Diameter of landing gear in m	Calculated	----
$\mathrm{F}_{\mathrm{per}_{\mathrm{tyre}}}$	Force on each tyre in N	Calculated	----
$\mathrm{n}_{\mathrm{tyre}}$	No. of tires	Calculated	----
$\mathrm{V}_{\mathrm{cruise}_1}$	Velocity in first cruise segment in m/s	Calculated	----
$\mathrm{V}_{\mathrm{tyre}}$	Volume of tyre in m^3	Calculated	----
$\mathrm{W}_{\mathrm{cruise}_1}$	Total weight of aircraft at the beginning of first cruise segment	Calculated	----
$\mathrm{W}_{\mathrm{cruise1}_{\mathrm{end}}}$	Weight of aircraft at the end of first cruise segment	Calculated	----
$\mathrm{W}_{\mathrm{cruise2}_{\mathrm{end}}}$	Weight of aircraft at the end of 2^{nd} cruise segment in N	Calculated	----
$\mathrm{W}_{\mathrm{cruisefrac}_{\mathrm{new}}}$	Ratio of weights after and before cruise	Assumed	0.6
w_{LG}	Width of landing gear in m	Calculated	----
$\rho_{\mathrm{air}_{\mathrm{cruise}}}$	Density of air at cruise in kg/m^3	Assumed	0.1875
ρ_{tyre}	Density of tyre in kg/m^3	Referenced [73]	720
A	Location of pin 1 from the end of the spar	Calculated	----
a_1	Fractional distance between spar 1 and CG	Assumed	0.26

a_2	Fractional distance between spar 2 and CG	Assumed	0.73
A_{cross}	Cross-sectional area of skin in m^2	Calculated	----
$Area_{CS}$	Area of cross section of fuselage in m^2	Calculated	----
$Area_{long}$	Cross-sectional areal of longeron in m^2	Calculated	----
A_{rib}	Cross-sectional area of rib in m^2	Calculated	----
A_{spar}	Area of spar in m^2	Calculated	----
B	Distance between pins 1 and 2	Calculated	----
b_1	Fractional distance between spar 1 and CP	Assumed	0.5
b_2	Fractional distance between spar 2 and CP	Assumed	0.5
Base weight	Assumed base weight of each tail	Assumed	----
Cd_{to}	Coefficient of drag at takeoff	Assumed	0.015
Cl_{cruise}	Coefficient of lift ar cruise	Assumed	0.3
Cl_{to}	Coefficient of lift at takeoff	Assumed	1.5
$\mathrm{Cost}_{\mathrm{aluminum}_{\mathrm{perkg}}}$	Cost of aluminum per kg in \$	Assumed	1.8
$\mathrm{Cost}_{\mathrm{carbon}_{\mathrm{perkg}}}$	Cost of carbon per kg in \$	Assumed	140
$\mathrm{Cost}_{\mathrm{fuel}_{\mathrm{perkg}}}$	Cost of per kg of fuel in \$	Assumed	1000
$\mathrm{Cost}_{\mathrm{rubber}_{\mathrm{perkg}}}$	Cost of per kg of rubber in \$	Assumed	800
$\mathrm{Cost}_{\mathrm{titanium}_{\mathrm{perkg}}}$	Cost of titanium per kg in \$	Assumed	1800
$Cost_{engine}$	Cost of engines in \$	Calculated	----
$Cost_{frame}$	Cost of frame in \$	Calculated	----
$Cost_{fuselage}$	Cost of fuselage in \$	Calculated	----
$Cost_{long}$	Cost of longeron in \$	Calculated	----
$Cost_{manu,ellip}$	Cost of manufacturing an elliptical wing in\$	Assumed	25M

$Cost_{manu,rect}$	Cost of manufacturing a rectangular wing in$	Assumed	10M
$Cost_{manufacturing}$	Cost of manufacturing in $	Assumed	----
$Cost_{rib}$	Cost of ribs in $	Calculated	----
$Cost_{rib}$	Cost of rib in $	Calculated	----
$Cost_{skin}$	Cost of skin in $	Calculated	----
$Cost_{skin}$	Cost of skin in $	Calculated	----
$Cost_{spar}$	Cost of spars in $	Calculated	----
$Cost_{spar}$	Cost of spar in $	Calculated	----
$Cost_{tail}$	Cost of tail in $	Calculated	----
$Cost_{tyre}$	Total cost of tires in $	Calculated	----
$Cost_{wing}$	Cost of wing in $	Calculated	----
D	Drag in N	Calculated	----
$d_{fuselage}$	Assumed initial diameter of fuselage in m	Assumed	3
F_{horz}	Horizontal force on ribs in N	Calculated	----
$F_{individual}$	Force on a single rib in N	Calculated	----
flight $time_1$	Time taken to cover $Range_1$	Calculated	----
$Force_{LG}$	Force on landing gear during landing in N	Calculated	----
F_{para}	Net force acting horizontally in N	Calculated	----
F_{perp}	Net force acting vertically in N	Calculated	----
$Frame_{material}$	Material of frame	Calculated	----
$F_{section}$	Force on a section in a rib in N	Calculated	----
F_{vert}	Vertical force on ribs in N	Calculated	----
$Height_{rib}$	Height of rib in m	Assumed	0.24
$Height_{spar}$	Initial guess for height of spar	Assumed	0.2
I_{long}	Moment of inertia of longeron in m^4	Calculated	----

I_{rib}	Moment of inertia of rib in m^4	Calculated	----
I_{spar}	Moment of inertia of spar in m^4	Calculated	----
L	Lift in N	Calculated	----
L/D	Lift to frag ratio	Assumed	15
l_{chord}	Length of chord in m	Calculated	----
$Longeron_{material}$	Material of longeron	Calculated	----
l_{wing}	Length of wing in m	Calculated	----
$Mass_{fuel_{cruise}}$	Total mass of fuel consumed during cruise in kg	Calculated	----
$Mass_{fuel_{cruise1}}$	Mass of fuel consumed in cruise segment 1 in kg	Calculated	----
$Mass_{fuel_{new}}$	Assumed new mass of fuel in kg	Assumed	10000
$Mass_{fuel_{old}}$	Assumed old mass of fuel in kg	Assumed	1000
$Mass_{long_{new}}$	Assumed new mass of longerons in $	Assumed	1000
$Mass_{long_{old}}$	Assumed old mass of longerons in $	Assumed	100
$Mass_{engine}$	Mass of engine in kg	Calculated	----
$Mass_{engine}$	Mass of engines in kg	Calculated	----
$Mass_{fittings}$	Mass of fittings in $	Assumed	5000
$Mass_{frame}$	Mass of frame in kg	Calculated	----
$Mass_{fuel}$	Mass of fuel in kg	Calculated	----
$Mass_{fuselage}$	Mass of fuselage in kg	Calculated	----
$Mass_{fuselage}$	Mass of fuselage in kg	Calculated	----
$Mass_{landinggear}$	Mass of landing gear in kg	Calculated	----
$Mass_{payload}$	Mass of payload in kg	Calculated	----
$Mass_{rib}$	Mass of rib in kg	Calculated	----
$Mass_{skin}$	Mass of skin in kg	Calculated	----

$Mass_{spar}$	Mass of spar in kg	Calculated	----
$Mass_{tail}$	Mass of tail in kg	Calculated	----
$Mass_{tail}$	Mass of tail in kg	Calculated	----
$Mass_{tyre}$	Mass of tires in kg	Calculated	----
M_{bend}	Maximum bending moment on spar	Calculated	----
M_{ij}	Bending moment at different points i on the two spars j	Calculated	----
M_{ij}	Bending moment at different points i on each longeron j	Calculated	----
n_{engine}	No. of engines	Calculated	----
N_{ribs}	No. of ribs in wing	Assumed	5
P_{atm}	Atmospheric pressure at an assumed maximum altitude of 50000 ft	Referenced [74]	12000
$P_{fuselage}$	Pressure on fuselage in N/m^2	Calculated	----
$P_{individual}$	Pressure on a single frame in N/m^2	Calculated	----
$P_{internal}$	Cabin pressure at 7000 ft in N/m^2	Referenced	80000
$P_{longeron}$	Pressure on a single longeron in N/m^2	Calculated	----
$P_{payload}$	Pressure due to payload in N/m^2	Calculated	----
P_{total}	Total internal pressure in fuselage in N/m^2	Calculated	----
R	Resultant force in N	Calculated	----
Range	Total range in km	Calculated	----
$Range_1$	Range covered during first cruise segment in m	Calculated	----
r_i	Initial assumed inner diameter of fuselage in m	Calculated	----
r_o	Initial assumed outer diameter of fuselage in m	Assumed	1.5
$R_{tensile}$	Tensile load in N	Calculated	----

SFC	Specific fuel consumption	Referenced [75]	----
$\text{Skin}_{\text{material}}$	Material of skin	Calculated	----
$\text{Spar}_{\text{material}}$	Material of spar	Calculated	----
Stress_1	Maximum stress on spar 1	Calculated	----
Stress_2	Maximum stress on spar 2	Calculated	----
$\text{Stress}_{\text{frame}}$	Maximum stress on frame in N/m^2	Calculated	----
$\text{Stress}_{\text{max}}$	Maximum of stresses on spars 1 & 2	Calculated	----
S_{wing}	Area of wing in m^2	Calculated	----
$\text{Tail}_{\text{material}}$	Material of tail	Calculated	----
t_{frame}	Thickness of frame in m	Assumed	0.4
TSFC	Thrust specific fuel consumption	Referenced [75]	----
t_{skin}	Thickness of skin in m	Assumed	0.0012
T_{stress}	Tensile stress on skin in N/m^2	Calculated	----
$\text{Type}_{\text{engine}}$	Type of engine	Calculated	----
U_{ij}	Shear force 1 at different points i for the two spars j	Calculated	----
$\text{Ulti}_{\text{frame}}$	Ultimate strength of material of frame in N/m^2	Referenced [73]	----
$\text{Ulti}_{\text{long}}$	Ultimate strength of material of longeron in N/m^2	Referenced [73]	----
Ulti_{rib}	Ultimate strength of material of rib	Referenced [73]	----
$\text{Ulti}_{\text{skin}}$	Ultimate strength of material of skin in N/m^2	Referenced [73]	----
$\text{Ulti}_{\text{spar}}$	Ultimate strength of material of spar in N/m^2	Referenced [73]	----
V_{ij}	Shear force 2 at different points i for the two spars j	Calculated	----
$\text{Volume}_{\text{frame}}$	Volume of frame in kg/m^3	Calculated	----
$\text{Volume}_{\text{long}}$	Volume of longeron in m^3	Calculated	----

$\text{Volume}_{\text{rib}}$	Volume of rib in kg/m^3	Calculated	----
$\text{Volume}_{\text{skin}}$	Volume of skin in m^3	Calculated	----
$\text{Volume}_{\text{spar}}$	Volume of spar in kg/m^3	Calculated	----
V_{to}	Takeoff velocity in m/s	Assumed	120
$\text{Weight}_{\text{fuel}_{\text{landing}}}$	Weight of fuel consumed during landing in N	Calculated	----
$\text{Weight}_{\text{fuel}_{\text{takeoff}}}$	Weight of fuel consumed during takeoff in N	Calculated	----
W_{final}	Weight at the beginning of landing segment in N	Calculated	----
W_i	Load perpendicular to the plane on resolving R in N	Calculated	----
$\text{Width}_{\text{longeron}}$	Initial assumed length of longeron in m	Assumed	0.05
$\text{Width}_{\text{rib}}$	Width of rib in m	Assumed	0.04
$\text{Width}_{\text{spar}}$	Initial guess for width of spar	Assumed	0.05
W_{ii}	Load along the plane on resolving R in N	Calculated	----
W_{landing}	Weight at the end of landing segment in N	Calculated	----
W_{to}	Total weight of aircraft at takeoff in N	Calculated	----
y_{long}	Distance from neutral axis in m	Calculated	----
y_{rib}	Distance from neutral axis to edge of rib in m	Calculated	----
y_{spar}	Distance from neutral axis to edge of spar in m	Calculated	----
Z_{long}	Section modulus for longeron	Calculated	----
Z_{rib}	Sectional modulus of rib	Calculated	----
Z_{spar}	Section modulus for spar	Calculated	----
α_{to}	Angle of attack at takeoff in degrees	Assumed	12

ρ_{air}	Density of air at sea level in kg/m^3	Approximated	1.125
$\rho_{aluminum}$	Density of aluminum in kg/m^3	Referenced [73]	2700
ρ_{carbon}	Density of carbon fiber in kg/m^3	Referenced [73]	1900
ρ_{frame}	Density of material of frame in kg/m^3	Referenced [73]	----
ρ_{long}	Density of material of longeron in kg/m^3	Referenced [73]	----
ρ_{spar}	Density of material of spar in kg/m^3	Referenced [73]	----
$\rho_{titanium}$	Density of titanium in kg/m^3	Referenced [73]	4500
ρ_{wood}	Density of wood in kg/m^3	Referenced [73]	160
σ_{bend}	Bending stress	Calculated	----
σ_{burst}	Bursting force in N	Calculated	----
σ_{shear}	Maximum shear stress	Calculated	----

Wing

Wing (SSL1)

$$S_{wing} = l_{wing} * l_{chord}$$

$$W_{to} = \left(Mass_{payload} + Mass_{fuselage} + Mass_{landinggear} + Mass_{tail} + Mass_{engine} + Mass_{fuel}\right) * 9.81$$

$$Cost_{wing} = \begin{cases} Cost_{spar} + Cost_{rib} + Cost_{skin} + Cost_{manu,rect}, & if\ rectangular\ wing \\ Cost_{spar} + Cost_{rib} + Cost_{skin} + Cost_{manu,ellip}, & if\ elliptical\ wing \end{cases}$$

$$Mass_{wing} = Mass_{spar} + Mass_{rib} + Mass_{fittings}$$

Wing (SSL2_SS1 – Spars)

$$S_{wing} = l_{wing} * l_{chord}$$

$$L = 0.5 * \rho_{air} * V_{to}^2 * Cl_{to} * S_{wing}$$

$$D = 0.5 * \rho_{air} * V_{to}^2 * Cd_{to} * S_{wing}$$

$$I_{spar} = \frac{width_{spar} * height_{spar}^3}{12}$$

$$A_{spar} = width_{spar} * height_{spar}$$

$$y = \frac{height_{spar}}{2}$$

$$Z = I/y$$

$$F_{perp1} = a_2 * L - b_2 * W_{to}$$

$$F_{para1} = a_2 * D$$

$$R_1 = \sqrt{F_{perp1}^2 + F_{para1}^2}$$

$$F_{perp2} = a_2 * L - b_1 * W_{to}$$

$$F_{para2} = a_1 * D$$

$$R_2 = \sqrt{F_{perp2}^2 + F_{para2}^2}$$

$$W_1 = R_1 \cos(\mathrm{\alpha_{to}})$$

$$W_{11} = R_1 * sin(\alpha_{to})$$

$$W_2 = R_2 \cos(\mathrm{\alpha_{to}})$$

$$W_{22} = R_2 * sin(\alpha_{to})$$

$$A = \frac{l_{wing}}{6}$$

$$B = \frac{l_{wing}}{3}$$

$$M1_1 = \frac{W_1 * A^2}{2} = M3_1$$

$$M2_1 = \frac{W_1 * B^2}{8}$$

$$M_{bend1} = \max(M1_1, M2_1)$$

$$\sigma_{bend1} = \frac{M_{bend1}}{Z}$$

$$U1_1 = W_1 * A$$

$$V1_1 = \frac{M2_1 - M1_1}{B} - \frac{W_1 * B}{2}$$

$$U2_1 = V1_1 + W_1 * B$$

$$V2_1 = \frac{M3_1 - M2_1}{B} - \frac{W_1 * B}{2}$$

$$U3_1 = V2_1 + W_1 * B$$

$$V3_1 = \frac{0 - M3_1}{A} - \frac{W_1 * A}{2}$$

$$\sigma_{shear} = \frac{\max(U1_1, V1_1, U2_1, V2_1, U3_1, V3_1)}{A_{spar}}$$

$$Stress_1 = \begin{cases} \sigma_{bend1}, & if\ \sigma_{bend1} > \sigma_{shear1} \\ \sigma_{shear1}, & if\ \sigma_{shear1} > \sigma_{bend1} \end{cases}$$

$$M1_2 = \frac{W_2 * A^2}{2} = M3_2$$

$$M2_2 = \frac{W_2 * B^2}{8}$$

$$M_{bend2} = \max(M1_2, M2_2)$$

$$\sigma_{bend2} = \frac{M_{bend2}}{Z}$$

$$U1_2 = W_2 * A$$

$$V1_2 = \frac{M2_2 - M1_2}{B} - \frac{W_2 * B}{2}$$

$$U2_2 = V1_2 + W_2 * B$$

$$V2_2 = \frac{M3_2 - M2_2}{B} - \frac{W_2 * B}{2}$$

$$U3_2 = V2_2 + W_2 * B$$

$$V3_2 = \frac{0 - M3_2}{A} - \frac{W_2 * A}{2}$$

$$\sigma_{shear2} = \frac{\max(U1_2, V1_2, U2_2, V2_2, U3_2, V3_2)}{A_{spar}}$$

$$Stress_2 = \begin{cases} \sigma_{bend2}, & if\ \sigma_{bend2} > \sigma_{shear2} \\ \sigma_{shear2}, & if\ \sigma_{shear2} > \sigma_{bend2} \end{cases}$$

$$Stress_{max} = \begin{cases} Stress_1, & if\ Stress_1 > Stress_2 \\ Stress_2, & if\ Stress_1 < Stress_2 \end{cases}$$

$\{ if\ Spar_{material} = Wood$

$$\rho_{spar} = 160 \frac{kg}{m^3}$$

$$Ulti_{spar} = 40 * 10^6 \frac{N}{m^2}$$

```
        $while\ Stress_{max} > Ulti_{spar}$
        $width_{spar} = width_{spar} + 0.001$
```

```
        $height_{spar} = 4 * width_{spar}$
        $Calculate\ Stress_{max}$
        $end$
$end\}$
$Volume_{spar} = l_{wing} * width_{spar} * height_{spar}$
$Mass_{spar} = \rho_{spar} * Volume_{spar}$
$Cost_{spar} = Cost_{spar_{pervolume}} * Volume_{spar}$
$\{\, if\ Spar_{material} = Aluminum$
$\rho_{spar} = 2700 \frac{kg}{m^3}$
$Ulti_{spar} = 483 * 10^6 \frac{N}{m^2}$
        $while\ Stress_{max} > Ulti_{spar}$
        $width_{spar} = width_{spar} + 0.001$
        $height_{spar} = 4 * width_{spar}$
        $Calculate\ Stress_{max}$
        $end$
$end\}$
$Volume_{spar} = l_{wing} * width_{spar} * height_{spar}$
$Mass_{spar} = \rho_{spar} * Volume_{spar}$
$Cost_{spar} = Cost_{spar_{perkg}} * Mass_{spar}$
```

Wing (SSL2_SS2 – Ribs)

$$S_{wing} = l_{wing} * l_{chord}$$

$$L = 0.5 * \rho_{air} * V_{to}^2 * Cl_{to} * S_{wing}$$

$$D = 0.5 * \rho_{air} * V_{to}^2 * Cd_{to} * S_{wing}$$

$$F_{perp} = L - W_{to}$$

$$F_{horz} = D$$

$$R = \sqrt{F_{perp}^2 + F_{horz}^2}$$

$$F_{vert} = R * \cos{(\alpha_{to})}$$

$$Fhorz = R * \sin(\alpha_{to})$$

$$F_{individual} = \frac{F_{vert}}{n_{ribs}}$$

$$F_{section} = \frac{F_{individual}}{3}$$

$$height_{rib} = 6 * width_{rib}$$

$$I = \frac{width_{rib} * height_{rib}^{3}}{12}$$

$$A_{rib} = width_{rib} * height_{rib}$$

$$y = \frac{height_{rib}}{2}$$

$$Z = \frac{I}{y}$$

$$\sigma_{bend1} = \frac{F_{section} * L_1}{Z}$$

$$\sigma_{bend2} = \frac{F_{section} * a * b}{(a + b) * Z}$$

$$\sigma_{bend3} = \frac{F_{section} * L_2}{Z}$$

$$\sigma_{shear} = \frac{F_{section}}{A_{rib}}$$

$$Stress_{max} = \max(\sigma_{bend1}, \sigma_{bend2}, \sigma_{bend3}, \sigma_{shear})$$

```
$\{if\ Rib_{material} = Wood$
$\rho_{rib} = 160 \frac{kg}{m^3}$
$Ulti_{rib} = 40 * 10^6 \frac{N}{m^2}$
    $while\ Stress_{max} > Ulti_{rib}$
    $width_{rib} = width_{rib} + 0.001$
    $height_{rib} = 6 * width_{rib}$
    $Calculate\ Stress_{max}$
    $end$
$end\}$
```

$$Volume_{rib} = l_{chord} * width_{rib} * height_{rib}$$

```
$Mass_{rib} = \rho_{rib} * Volume_{rib}$
$Cost_{rib} = Cost_{rib_{pervolume}} * Volume_{rib}$
$\{ if\ Rib_{material} = Aluminum$
$\rho_{rib} = 2700 \frac{kg}{m^3}$
$Ulti_{rib} = 483 * 10^6 \frac{N}{m^2}$
    $while\ Stress_{max} > Ulti_{rib}$
    $width_{rib} = width_{rib} + 0.001$
    $height_{rib} = 6 * width_{rib}$
    $Calculate\ Stress_{max}$
    $end$
$end\}$
$Volume_{rib} = l_{chord} * width_{rib} * height_{rib}$
$Mass_{rib} = \rho_{rib} * Volume_{rib}$
$Cost_{rib} = Cost_{rib_{perkg}} * Mass_{rib}$
```

Wing (SSL2_SS3 – Skin)

$$S_{wing} = l_{wing} * l_{chord}$$

$$Volume_{skin} = t_{skin} * S_{wing}$$

$$L = 0.5 * \rho_{air} * V_{to}^2 * Cl_{to} * S_{wing}$$

$$D = 0.5 * \rho_{air} * V_{to}^2 * Cd_{to} * S_{wing}$$

$$F_{perp} = L - W_{to}$$

$$F_{horz} = D$$

$$R = \sqrt{F_{perp}^2 + F_{horz}^2}$$

$$R_{tensile} = R * \sin(\alpha_{to})$$

$$A_{cross} = l_{chord} * t_{skin}$$

$$T_{stress} = \frac{R_{tensile}}{A_{cross}}$$

```
$\{if\ Skin_{material} = Aluminum\ alloy$
```

```
$Ulti_{skin} = 310 * 10^6 \frac{N}{m^2}$
    $while\ T_{stress} > Ulti_{skin}$
    $t_{skin} = t_{skin} + 0.0001$
    $Calculate\ T_{stress}$
    $end$
$Mass_{skin} = Volume_{skin} * \rho_{aluminum}$
$Cost_{skin} = Cost_{aluminum_{perkg}} * Mass_{skin}$

$else\ if\ Skin_{material} = Carbon\ fiber$
$Ulti_{skin} = 600 * 10^6$
    $while\ T_{stress} > Ulti_{skin}$
    $t_{skin} = t_{skin} + 0.0001$
    $Calculate\ T_{stress}$
    $end$
$Mass_{skin} = Volume_{skin} * \rho_{carbon}$
$Cost_{skin} = Cost_{carbon_{perkg}} * Mass_{skin}$
$elsi\ if\ Skin_{material} = Titanium\ alloy$
$Ulti_{skin} = 950 * 10^6$
    $while\ T_{stress} > Ulti_{skin}$
    $t_{skin} = t_{skin} + 0.0001$
    $Calculate\ T_{stress}$
    $end$
$Mass_{skin} = Volume_{skin} * \rho_{titanium}$
$Cost_{skin} = Cost_{titanium_{perkg}} * Mass_{skin}$

$else\ if\ Skin_{material} = Carbon\ fiber$
$Ulti_{skin} = 600 * 10^6$
    $while\ T_{stress} > Ulti_{skin}$
    $t_{skin} = t_{skin} + 0.0001$
    $Calculate\ T_{stress}$
```

```
    $end\}$
```

Fuselage

$$Mass_{fuselage} = Mass_{frame} + Mass_{longerons} + Mass_{skin_{fuselage}}$$

$$Cost_{fuselage} = Cost_{manufacturing} + Cost_{frame} + Cost_{longerons} + Cost_{skin_{fuselage}}$$

Fuselage (SSL2_SS1 – Frames)

```
$Area_{CS} = \pi * \left(r_o^2 - r_i^2\right)$
$P_{payload} = \frac{Mass_{payload} * 9.81}{Area_{CS}}$
$P_{total} = P_{internal} + P_{payload}$
$P_{individual_{frame}} = \frac{P_{total}}{n_{frames}}$
$\sigma_{radial_{max}} = P_{individual}$
$\sigma_{tangent_{max}} = \frac{P_{individual} * \left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$
$Stress_{frame} = \max\left(\sigma_{radial_{max}}, \sigma_{tangent_{max}}\right)$
$\{if\ Skin_{material} = Aluminum\ alloy$
$\rho_{frame} = 2700$
$Ulti_{frame} = 483 * 10^6$
    $while\ Stress_{frame} > Ulti_{frame}$
    $d_{fuselage} = d_{fuselage} - 0.01$
    $Calculate\ Stress_{frame}$
    $end$
    $Volume_{frame} = 2 * \pi * (r_o - r_i) * t_{frame}$
$Mass_{skin} = Volume_{frame} * \rho_{frame}$
$Cost_{frame} = Cost_{aluminum_{perkg}} * Mass_{frame}$
```

```
$else\ if\ Skin_{material} = Carbon\ fiber$
$\rho_{frame} = 1600$
$Ulti_{frame} = 600 * 10^{6}$
    $while\ Stress_{frame} > Ulti_{frame}$
    $d_{fuselage} = d_{fuselage} - 0.01$
    $Calculate\ Stress_{frame}$
    $end$
$Volume_{frame} = 2 * \pi * (r_o - r_i) * t_{frame}$
$Mass_{skin} = Volume_{frame} * \rho_{frame}$
$Cost_{frame} = Cost_{carbon_{perkg}} * Mass_{frame}$
$end\}$
$Mass_{frame} = Mass_{frame} * n_{frames}$
$Cost_{frame} = Cost_{frame} * n_{frames}$
```

Fuselage (SSL2_SS2 – Longerons)

```
$\{while\ abs\left(Mass_{long_{old}} - Mass_{long_{new}}\right) > 10^{-6}$
    $Mass_{long_{old}} = Mass_{long_{new}}$
    $P_{fuselage} = \left(M_{wing} + M_{empannage} + M_{landinggear} + M_{payload} + Mass_{fuel} + Mass_{long_{new}}\right) * 9.81$
    $P_{longeron} = \frac{P_{fuselage}}{n_{longerons}}$
    $I_{long} = \frac{Width_{long}^{4}}{12}$
    $y_{long} = \frac{Width_{long}}{2}$
    $Z_{long} = \frac{I_{long}}{y_{long}}$
    $Area_{long} = Width_{long}^{2}$
    $A_{long} = \frac{L_{fuselage}}{6}$
    $B_{long} = \frac{L_{fuselage}}{3}$
```

```
$M1_{1_{long}} = \frac{P_{longeron} * A_{long}^2}{2} = M3_{1_{long}}$
$M2_{1_{long}} = \frac{P_{longeron} * B_{long}^2}{8}$
$M_{bend_{long}} = \max\left(M1_{1_{long}}, M2_{1_{long}}\right)$
$\sigma_{bend_{long}} = \frac{M_{bend_{long}}}{Z_{long}}$
$\{if\ Longeron_{material} = Aluminum\ alloy$
$\rho_{long} = 2700$
$Ulti_{long} = 483 * 10^6$
        $while\ \sigma_{bend_{long}} > Ulti_{long}$
        $Width_{long} = Width_{long} + 0.001$
        $Calculate\ \sigma_{bend_{long}}$
        $end$
$Volume_{long} = Area_{long} * L_{fuselage}$
$Mass_{long_{new}} = Volume_{long} * \rho_{long} * n_{longerons}$
$Cost_{long} = Cost_{aluminum_{perkg}} * Mass_{long_{new}}$

$else\ if\ Longeron_{material} = Carbon\ fiber$
$\rho_{long} = 1600$
$Ulti_{long} = 600 * 10^6$
        $while\ \sigma_{bend_{long}} > Ulti_{long}$
        $Width_{long} = Width_{long} + 0.001$
        $Calculate\ \sigma_{bend_{long}}$
        $end$
$Volume_{long} = Area_{long} * L_{fuselage}$
$Mass_{long_{new}} = Volume_{long} * \rho_{long} * n_{longerons}$
$Cost_{long} = Cost_{carbon_{perkg}} * Mass_{long_{new}}$

$else\ if\ Longeron_{material} = Titanium$
```

```
        $\rho_{long} = 4500$
        $Ulti_{long} = 950 * 10^6$
            $while\ \sigma_{bend_{long}} > Ulti_{long}$
            $Width_{long} = Width_{long} + 0.001$
            $Calculate\ \sigma_{bend_{long}}$
            $end$
        $Volume_{long} = Area_{long} * L_{fuselage}$
        $Mass_{long_{new}} = Volume_{long} * \rho_{long} * n_{longerons}$
        $Cost_{long} = Cost_{titanium_{perkg}} * Mass_{long_{new}}$
        $end\}$
    $end\}$
```

Fuselage (SSL2_SS3 – Skin)

```
    $Area_{CS} = \pi * \left(r_o^2 - r_i^2\right)$
    $P_{payload} = \frac{\left(Mass_{payload} * 9.81\right)}{Area_{CS}}$
    $P_{total} = P_{internal} + P_{payload}$
    $\Delta P = P_{total} - P_{atm}$
    $\sigma_{burst} = \frac{\Delta P * d_{fuselage}}{2 * t_{skin_{fuselage}}}$
    $\{if\ Fuselage_{skin_{material}} = Aluminum\ alloy$
    $\rho_{long} = 2700$
    $Ulti_{skin} = 310 * 10^6$
        $while\ \sigma_{burst} > Ulti_{skin}$
        $t_{skin_{fuselage}} = t_{skin_{fuselage}} + 0.0001$
        $Calculate\ \sigma_{burst}$
        $end$
    $Volume_{skin_{fuselage}} = \pi * L_{fuselage} * t_{skin_{fuselage}} * (d_{fuselage} - t_{skin_{fuselage}})$
    $Mass_{skin_{fuselage}} = Volume_{skin_{fuselage}} * \rho_{skin}$
```

```
$Cost_{skin_{fuselage}} = Cost_{aluminum_{perkg}} * Mass_{skin_{fuselage}}$

$else\ if\ Fuselage_{skin_{material}} = Carbon\ fiber$
$\rho_{long} = 1900$
$Ulti_{skin} = 600 * 10^6$
        $while\ \sigma_{burst} > Ulti_{skin}$
        $t_{skin_{fuselage}} = t_{skin_{fuselage}} + 0.0001$
        $Calculate\ \sigma_{burst}$
        $end$
$Volume_{skin_{fuselage}} = \pi * L_{fuselage} * t_{skin_{fuselage}} * (d_{fuselage} - t_{skin_{fuselage}})$
$Mass_{skin_{fuselage}} = Volume_{skin_{fuselage}} * \rho_{skin}$
$Cost_{skin_{fuselage}} = Cost_{carbon_{perkg}} * Mass_{skin_{fuselage}}$

$if\ Fuselage_{skin_{material}} = Titanium\ alloy$
$\rho_{long} = 4500$
$Ulti_{skin} = 950 * 10^6$
        $while\ \sigma_{burst} > Ulti_{skin}$
        $t_{skin_{fuselage}} = t_{skin_{fuselage}} + 0.0001$
        $Calculate\ \sigma_{burst}$
        $end$
$Volume_{skin_{fuselage}} = \pi * L_{fuselage} * t_{skin_{fuselage}} * (d_{fuselage} - t_{skin_{fuselage}})$
$Mass_{skin_{fuselage}} = Volume_{skin_{fuselage}} * \rho_{skin}$
$Cost_{skin_{fuselage}} = Cost_{titanium_{perkg}} * Mass_{skin_{fuselage}}$
$end$
```

Engine

Engine (SSL1)

```
$For\ Engine_{type} = 1, 2\ or\ 3$
```

$$Mass_{engine} = Mass\ per\ engine * n_{engines}$$

$$Cost_{engine} = Cost\ per\ engine * n_{engines}$$

$$\{while\ \left(abs\left(Mass_{fuel_{old}} - Mass_{fuel_{new}}\right)\right) > 10^{-6}$$

$$Mass_{fuel_{old}} = Mass_{fuel_{new}}$$

$$W_{to} = \left(Mass_{wing} + Mass_{payload} + Mass_{fuselage} + Mass_{landinggear} + Mass_{tail} + Mass_{engine} + Mass_{fuel_{new}}\right) * 9.81$$

$$W_{cruise_1} = W_{to} * 0.95545$$

$$V_{cruise_1} = \sqrt{\frac{2*W_{cruise_1}}{\rho_{air_{cruise}}*Cl_{cruise}*S_{wing}}}$$

$$D_{cruise_1} = \frac{W_{cruise_1}}{\left(\frac{L}{D}\right)}$$

$$Weight_{fuel_{takeoff}} = \left(W_{to} - W_{cruise_1}\right)$$

$$Range_1 = \left(\frac{V_{cruise_1}}{SFC}\right) * \left(\frac{W_{cruise_1}}{D_{cruise_1}}\right) * \log\left(\frac{1}{W_{cruisefrac_{new}}}\right)$$

$$flight\ time_1 = \frac{Range_1}{V_{cruise_1}}$$

$$Mass_{fuel_{cruise1}} = TSFC * flight\ time_1 * D_{cruise_1}$$

$$W_{cruise1_{end}} = W_{cruise_1} - Mass_{fuel_{cruise1}} * 9.81$$

$$Range = 2.05 * Range_1$$

$$Mass_{fuel_{cruise}} = 2 * Mass_{fuel_{cruise1}}$$

$$W_{cruise2_{end}} = W_{cruise_1} - \left(Mass_{fuel_{cruise}} + Mass_{payload}\right) * 9.81$$

$$W_{final} = W_{cruise2_{end}} * 0.995$$

$$Weight_{fuel_{landing}} = W_{cruise2_{end}} - W_{final}$$

$$Mass_{fuel_{takeoff}} = \frac{Weight_{fuel_{takeoff}}}{9.81}$$

$$Mass_{fuel_{landing}} = \frac{Weight_{fuel_{landing}}}{9.81}$$

$$Mass_{fuel_{new}} = Mass_{fuel_{takeoff}} + Mass_{fuel_{cruise}} + Mass_{fuel_{landing}}$$

$$Cost_{fuel} = Cost_{fuel_{perkg}} * Mass_{fuel_{new}}$$

$$Cost_{engine} * Cost_{engine} + Cost_{fuel}$$

$$end\}$$

$$Mass_{return} = \frac{W_{cruise2_{end}}}{9.81}$$

Tail

Tail (SSL1)

$For\ tail = Conventional, H - tail\ or\ V - tail,$

$\{if\ tail_{material} = Aluminum\ alloy$

$Mass_{tail} = Base\ weight * 2.5$

$Cost_{tail} = Cost_{aluminum_{perkg}} * Mass_{tail}$

$else\ if\ tail_{material} = Carbon\ fiber$

$Mass_{tail} = Base\ weight * 1.25$

$Cost_{tail} = Cost_{carbon_{perkg}} * Mass_{tail}$

$end\}$

$Cost_{tail_{total}} = Cost_{tail} + Cost_{manufacture}$

Landing gear

Landing gear (SSL1)

$W_{landing} = 0.995 * Mass_{return}$

$Force_{LG} = W_{landing} * 9.81$

$\{if\ Type_{LG} = Bicycle$

$n_{tyre} = 2$

$F_{per_{tyre}} = \frac{Force_{LG}}{n_{tyre}}$

$d_{LG} = 0.0163 * F_{per_{tyre}}^{0.315}$

$w_{LG} = 0.01043 * F_{per_{tyre}}^{0.315}$

$V_{tyre} = \left(\frac{\pi}{4}\right) * (d_{LG}^2 - w_{LG}^2) * w_{LG}$

$Mass_{tyre} = V_{tyre} * \rho_{tyre} * \frac{n_{tyre}}{100000}$

```
$Assembly_{cost} = Assembly_{cost} * n_{tyre}$
$Cost_{manu_{tyre}} = Cost_{rubber_{perkg}} * Mass_{tyre} * n_{tyre}$
$Cost_{tyre} = Assembly_{cost} + Cost_{manu_{tyre}}$

$else\ if\ Type_{LG} = Tricycle$
$n_{tyre} = 3$
$F_{per_{tyre}} = \frac{Force_{LG}}{n_{tyre}}$
$d_{LG} = 0.0163 * F_{per_{tyre}}^{0.315}$
$w_{LG} = 0.1043 * F_{per_{tyre}}^{0.480}$
$V_{tyre} = \left(\frac{\pi}{4}\right) * (d_{LG}^2 - w_{LG}^2) * w_{LG}$
$Mass_{tyre} = V_{tyre} * \rho_{tyre} * \frac{n_{tyre}}{100000}$
$Assembly_{cost} = Assembly_{cost} * n_{tyre}$
$Cost_{manu_{tyre}} = Cost_{rubber_{perkg}} * Mass_{tyre} * n_{tyre}$
$Cost_{tyre} = Assembly_{cost} + Cost_{manu_{tyre}}$

$else\ if\ Type_{LG} = Quadricycle$
$n_{tyre} = 4$
$F_{per_{tyre}} = \frac{Force_{LG}}{n_{tyre}}$
$d_{LG} = 0.0163 * F_{per_{tyre}}^{0.315}$
$w_{LG} = 0.01043 * F_{per_{tyre}}^{0.480}$
$V_{tyre} = \left(\frac{\pi}{4}\right) * (d_{LG}^2 - w_{LG}^2) * w_{LG}$
$Mass_{tyre} = V_{tyre} * \rho_{tyre} * \frac{n_{tyre}}{100000}$
$Assembly_{cost} = Assembly_{cost} * n_{tyre}$
$Cost_{manu_{tyre}} = Cost_{rubber_{perkg}} * Mass_{tyre} * n_{tyre}$
$Cost_{tyre} = Assembly_{cost} + Cost_{manu_{tyre}}$
$end\}$
```