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From Mirror to Mirage: The Idea of Logical Space in Kant, Wittgenstein, and van Fraassen

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A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy

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FROM MIRROR TO MIRAGE: THE IDEA OF LOGICAL SPACE IN KANT,
WITTGENSTEIN, AND VAN FRAASSEN

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by

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Graduate Program in Philosophy

A thesis submitted in partial fulfillment
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The School of Graduate and Postdoctoral Studies
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Abstract

This dissertation investigates the origin, intellectual development and use of a semantic variant of the idea of logical space found implicitly in Kant and explicitly in early Wittgenstein and van Fraassen. It elucidates the idea of logical space as the idea of images or pictures representative of reality organized into a logico-mathematical structure circumscribing a form of all possible worlds. Its main claim is that application of these images or pictures to reality is through a certain conception of self.

The first chapter presents a novel interpretation of Kant's semantic theory of schemata in the *Critique of Pure Reason*, showing that a structure of the imagination induced by the transcendental self informs an implicit idea of logical space. The second chapter offers an intellectual history of the idea through developments in the organization of images introduced by Helmholtz and Hertz. The third chapter reveals early Wittgenstein's idea of logical space to be his notion of the self, demonstrating how this serves to unify propositions of the *Tractatus Logico-Philosophicus* concerning solipsism, realism, ethics, aesthetics and mysticism with those pertaining to the picture theory of meaning. The fourth chapter provides a historical overview of the development of van Fraassen's empiricism in relation to his adaptation of logical space, and evaluates his recent proposal in *Scientific Representation: Paradoxes of Perspective* that the problem of coordination in the semantic view of theories is dissolved through self-location in logical space. After identifying a number of problems this proposal creates for his empiricism, a brief suggestion is made about how van Fraassen might improve upon his conception of logical space, and how an empiricist view of scientific representation might be understood as a result.

Key Words: logical space, schema, self, image, picture, model, semantic view of theories, Kant, Wittgenstein, van Fraassen, Helmholtz, Hertz, scientific representation, philosophy of science, semantics

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Introduction

The schema of time a line. (Kant)¹
[L]ogical space (time) is the real line (van Fraassen)²

This dissertation investigates the origin, intellectual development and use of a semantic variant of the idea of logical space – what might be called “image space” or “picture space” – found implicitly in Kant and explicitly in early Wittgenstein and van Fraassen.³ Among the most abstract ideas ever used in a philosophy, it sets the stage for highly influential and complex philosophical systems in historical and contemporary thought, which exemplify the traditions of transcendental idealism, metaphysics and empiricism. When approached through the idea of logical space, these systems are shown to be subtle variations in the way three major philosophers have understood the relations among imaginative representation, reality and the self.

For these philosophers⁴ the idea of logical space is the idea of images or pictures of reality organized into a logico-mathematical structure circumscribing a form of all possible worlds. As the form of imaginative representation, the idea of logical space functions as an arena where cognition engages reality and as an interface where images meet up with concepts. It tells us how applied mathematics encounters logic and how natural language receives its semantic content. And it enlightens us about the way we are in the world.

Notwithstanding its potential appeal to a wide range of philosophical interests, the idea of logical space common among Kant, Wittgenstein and van Fraassen has never been elucidated.⁵ Inspiration came with van Fraassen’s latest book, *Scientific Representation* (2008). There the application of scientific theories in the semantic view, what is sometimes called the “problem of coordination”, is accounted for by self-ascription of location in logical space. However, the idea of logical space did not feature prominently in van Fraassen’s writing since his early work on the causal theory of time and on the semantic approach to the analysis of the structure of scientific theories more generally. There he first adapted the idea of logical space from Wittgenstein’s *Tractatus*

(1921/1961) and used it to explicate the meaning of “time” on the basis of an argument that we conceive time as an “ideal entity” in which the structure of all actual events embeds. “Time”, he claimed, refers to this entity, an abstract structure of our choosing we use to represent all actual events – a logical space. In our contemporary setting that developed with Newtonian physics it is the real line. He had erred, however, in thinking all he had done was “to objectify [Kant’s pure] form of our intuition, and to describe it as a form, as opposed to a condition of sensible perception” (1970a, 101). As the quotations at the outset of this introduction suggest, the connection to Kant’s *Critique of Pure Reason* (1781-1787/1998) is through the “schema of time” produced in our spatiotemporal imagination.

This study was undertaken with the intention of elucidating the idea of logical space in order to properly evaluate van Fraassen’s account of self-location in logical space. It was also prompted by a vague suspicion that important and revealing connections among van Fraassen, Wittgenstein and Kant were to be had by seeking each philosopher’s answer to three simple questions: What is meant by an “image” or “picture”? How is it organized? How does it relate to reality? It was a genuine surprise, however, that such simple questions seemed to go to the heart of three philosophical systems, providing insight into difficult exegetical problems and positions, such as how to make literal sense of Kant’s chapter on the schematism of pure concepts in the *Critique*, how to unify Wittgenstein’s *Tractatus*, and how to understand Fraassen’s claim that we know empirical structure of real entities even though the structure is not real. What emerged, moreover, was a way to relate these three philosophers through developments in the organization of images introduced by Helmholtz and Hertz.

Perhaps the most interesting insight to come into view was how the idea of logical space is inextricably bound with a conception of self. Van Fraassen introduced his account of self-location into his philosophical system some forty years after adopting the idea of logical space. That he had simply *recovered* a notion of the self as a part of the idea was an unexpected surprise. What became clear to me first in Wittgenstein and then in Kant was that the idea of logical space is really the idea of a certain form of imagination whose relation to reality is through a unique conception of the self. It

appears that for Kant the form of imagination has the structure like that of a spherical *convex mirror* and is induced by the self in the synthesis of reality. For Wittgenstein, on the other hand, it seems the form of the imagination has the structure like that of a spherical *concave mirror* and is projected *as* the self in expressive thought to reflect reality. And for van Fraassen the self locates reality into a form of public imagination that has features of what he characterizes as a *mirage*: an inter-subjective image bearing a diffuse relation to reality that bewitches us into projecting it onto the world.

This dissertation is divided into four chapters and involves the analysis of five philosophical systems around their notion of an image or picture, its organization and its relation to reality. The first chapter, “Schemata and the Birth of Logical Space”, argues that the idea of logical space can be found in Kant’s *Critique* in the organization of schemata of pure sensible concepts, i.e., concepts such as those of time, space, and geometric figures, within the potential classificatory space Kant describes in his Appendix to the Transcendental Dialectic. It provides a new more literal interpretation of Kant’s schematism that takes the transcendental schemata of the categories to be constitutive of pure sensible schemata, and pure sensible schemata to function similar to regulative schemata. It shows that a regulative schema is the form of a regulative idea, and as such it is a logical outline (syllogistic inference pattern) correlated to a *focus imaginarius* and (through the major premise of the inference) to an original image. Building upon this pattern, it demonstrates that a pure sensible schema is the form of a pure sensible concept, and as such it is a spatiotemporal outline produced as a focal point in pure or empirical synthesis of a kind of image. The chapter then argues that a subset of points within Kant’s classification space corresponds to pure sensible schemata, and that this subset of points has a logico-mathematical structure circumscribing the form of all possible figurative experience. It concludes by claiming that this structure is induced by Kant’s conception of the transcendental self as a formal limit.

Divided into two parts, the second chapter, “From Schema to Model”, presents a novel intellectual history through the philosophical systems of Helmholtz and Hertz that illuminates the intellectual path taken to the development of Wittgenstein’s idea of logical space. The first part is a reconstruction of Helmholtz’s philosophical system that appeals

to Kant's architectonic to fill out the implications of Helmholtz's characterization of the concept of causality as a transcendental regulative idea. The substitution of a transcendental schema for a regulative one is shown to be the basis of Helmholtz's claim that an image of external causes is the major premise of an unconscious (inductive) syllogistic inference as well as his account of concept formation in the physical sciences as the interpretation of this inference. It provides evidence that Helmholtz understood full well that images in his system were organized under a regulative schema into a space converging towards a focal point, and that the major premise of the schema was an original image serving as an ideal for cognitive activity.

The second part of the chapter demonstrates how in his *Principles of Mechanics* (1900/1956) Hertz adopts two key features of Helmholtz's philosophical system. In rejecting Helmholtz's principle of causality, Hertz throws out his mentor's use of Kant's regulative idea and corresponding *focus imaginarius*. Hertz retains, however, the idea of images organized under an original image. He develops, moreover, his notion of an "image of things" around Helmholtz's view of a concept in the physical sciences, but in this case as having the form of a conscious (deductive) syllogistic inference. Bringing these two features together, Hertz conceives of a "general" and "simple" scientific image as having the form of a logical schema under which images of things are organized, and serving as an ideal for philosophical reconstruction. This reconstruction involves evaluating images of things, since he also takes a scientific image to constitute them through interpreted a priori forms of intuition and conventional definitions. Presenting what he takes to be a superior scientific image of mechanics, Hertz enhances the predictive power of its logical form by encasing it within a convenient mathematical schema. This schema is an outline for a multi-dimensional coordinate system, and it both organizes and constitutes idealized "dynamical models" that express the logical form of images of things. Significantly, Hertz speculates that the conformity between thought and reality is because mind and nature share the form of this schema.

It is from Hertz's suggestion of a thought schema organizing and constituting models of reality that we enter the third chapter, "From Model to Mysticism". This chapter argues that there is a simple "picture" that the propositions of the *Tractatus*

describe, and it is because this “picture” is not *in* logical space but *of* logical space that the *Tractatus* is ultimately treated as nonsensical. The chapter shows how this picture *of* logical space is an amalgam of Hertz’s speculation of a thought schema with Schopenhauer’s view of the self as will: in expressive thought the self projects itself as the completed whole of logical space, a spherical concave mirror reflecting reality to a *focus imaginarius*; in willing silence the self collapses into the *focus imaginarius* where it exists as the potential whole of logical space. The central claim of chapter three is that it is the shift in perspective in relation to the world that unifies the seemingly intractable propositions of the *Tractatus* concerning solipsism, realism, ethics, aesthetics, and mysticism with those concerning the picture theory of meaning.

The final chapter, “From Mysticism to Myth”, presents van Fraassen’s adaptation of Wittgenstein’s idea of logical space to a contemporary empiricist setting, one that attempts to sever the idea from its metaphysical underpinning and psychologistic overtone. In this chapter we encounter van Fraassen’s initial thought that an imaginative and conventional coordinate system of possible entities embedding the concrete structure of actual entities, i.e., a logical space, can solve issues in the interpretation of a scientific theory within the syntactic view of theories. Rather than appeal to modal realism, the suggestion is that a logical space can provide necessary “intuitive content” for modal notions of a theory involving basic physical relations. When the syntactic view of theories is rejected some time later, we see logical space take on a life of its own in a semantic view of theories as the form of models that have an unmediated relation to real entities. We become aware that it encounters its own problem of interpretation, one van Fraassen attempts to solve by introducing the notion of self-location. The final section of the chapter shows how self-location recovers three unities traditionally associated with the idea of logical space, and how these unities conspire against the viability of the idea of logical space within van Fraassen’s empiricism.

Based on insights gathered from Kant, Hertz and Wittgenstein, this dissertation concludes with a very brief suggestion of how van Fraassen might modify the idea, and how an empiricist view of scientific representation could be understood as a result.

Chapter 1: Schemata and the Birth of Logical Space

[The schema of an idea] lies in reason like a seed, all of whose parts still lie very involuted and are hardly recognizable even under microscopic observation.

This schematism of our understanding with regard to appearances and their mere form is a hidden art in the depths of the human soul, whose true operations we can divine from nature and lay unveiled before our eyes only with difficulty.

Kant, (A834/B862; A141/B180-181)

1.1 Introduction

Hume spurred Kant into action with the claim that

... reason completely and fully deceives herself with [the concept of cause], falsely taking it for her own child, when it is really nothing but a bastard of the imagination, which, impregnated by experience, and having brought certain representations under the law of association, passing off the resulting subjective necessity (i.e., habit) for an objective necessity (from insight). (Ak. IV, 257-258)⁶

Kant set out to show that the concept of cause was in fact the legitimate child, having developed from a *transcendental schema* in the womb of the intuitive imagination impregnated by pure understanding. And just like the concept of cause, the birth of idea of logical space can be traced to Kant's theory of schemata running through the heart of his *Critique of Pure Reason*.

This chapter identifies Kant's idea of logical space as the logico-mathematical organization of figurative pure sensible schemata within a space of classification, and is based largely upon a novel interpretation of Kant's theory of schemata. Inspired by Kant's account of a regulative schema as a logical outline executing a pure concept by directing a convergence of empirical concepts towards an imaginary focal point, it demonstrates that a (figurative) pure sensible schema is a spatiotemporal outline that serves as a focal point towards which an empirical synthesis of a certain kind of image occurs. It also serves as a criterion for the application of a pure sensible concept evident in empirical images of a sort. On this construal, a transcendental schema is *constitutive* of pure sensible schemata, serving as a criterion for the application of a category by specifying "schemata of sensibility" evident in images in general.

This new interpretation of Kant's theory of schemata is motivated by the general sense that existing scholarship has not been successful in coming to terms with Kant's chapter "On the Schematism of the Pure Concepts of the Understanding". Even though he regarded his chapter to be "one of the most important", Kant recognized that "[i]n general, the schematism is one of the most difficult points" (2005, R 6359 (1797, 18: 685-7)). It is so obscure that Schopenhauer characterized it as a curiosity "famous for its profound darkness, because nobody has yet been able to make sense of it" (as quoted in Schaper 1964, 270, n.4), a description that rings true even today. Schemata are variously described as rules governing imagination (Chipman (1972, 42); Longuenesse (1998, 116); Allison (2004, 210)), methods or procedures of the imagination (Pippin, (1976, 162); Young (1984, 128); Friedman (1992, 41)), concepts of understanding (Bennett (1966, 141); Walsh (1957/1958, 102); Guyer, (1987, 159)), and formal intuitions of sensibility (Paton (1936, 30); Allison (2004, 216)). Aside from their lack of consensus, these interpretations all attribute misstatements, inaccuracies or unintelligibility to Kant, even though he saw fit *not* to amend the chapter for the second edition of the *Critique*. Perhaps the hermeneutical problem posed by Kant's schematism requires an approach different from the one that seems have been taken: rather than try to reconcile Kant's schematism with an interpretation of the *Critique*, perhaps an attempt should be made to reconcile the *Critique* with an interpretation of the schematism that does not attribute misstatements, inaccuracies or unintelligibility to Kant. In taking the latter tack, this Chapter can be thought of as the first step of a larger project in Kant scholarship.

The body of this chapter is divided into six sections. The first section provides a brief overview of the development of Kant's notion of a schema from its pre-critical conception, as a divine *plan* for the structure of a monadic realm of simple substances underlying a spatial and temporal world, to its critical transformation into a regulative idea of reason and products of the spatiotemporal imagination. The next two sections place the faculty of the imagination symmetrically between the faculties of sensibility and understanding in the context of Kant's framework of representations. The following section works through Kant's chapter on the schematism, making literal sense of notoriously obscure and controversial passages. The fifth section considers the role of schemata in mathematical and philosophical cognition, identifying a potential logico-

mathematical structure bridging the two. And the final section considers Kant's view of the self in relation to this structure, one that allows it to be characterized as a potential "logical space" as Wittgenstein uses this term.

1.2 From Divine Schema to *Focus Imaginarius*

Schaper (1964) notes that

[s]chemata in general for Kant, as for many thinkers of his age, were something like plans or diagrams. For example, a blueprint for construction for a bridge stands midway between a general idea and its particular construction in steel, iron, or wood. Schemata provide rules for construction, but have no simple image character or necessary pictorial resemblance. (271-272)

Kant's schematism, or "the procedure of the understanding with ... schemata" (A140/B179), is the ascription to human cognition of a constructive process earlier attributed to the divine intellect. Kant's critical re-conception of God from creator of the universe to ultimate end of human reasoning led to a transformation of a divine schema from an outline constituting a "heavenly plan" to an outline of human reason bringing about a *focus imaginarius*. Paralleling this development was the re-conception of human schemata from appearances of the divine schema to focal points of empirical imagination. What remains constant throughout is the role of a schema as a rule-based *telos* in the execution of a pure concept, be it as a purposeful plan or as the aim of cognition.

In his first treatise on metaphysics, *A New Exposition of the First Principles of Metaphysical Cognition* (1755/2010), Kant modified Leibnizian-Wolffian metaphysics to accommodate Newtonian natural philosophy.⁷ While he maintained a conception of God as creator of the ultimate constituents of the universe, Kant rejected the notion of active force as an internal principle of change. In its place he proposed a view of physical influx whereby one substance can bring about change of the inner state of another. As a result, the basis of a single world of interconnected substances was no longer pre-established harmony, but the Newtonian doctrine of divine omnipresence: through a "*schema* of the divine intellect", "the divine idea" (Prop. XIII) presented a fundamental law of mutual interaction among simple substances. This interaction constituted phenomenal space in which the fundamental law manifested as Newton's laws of physics. Friedman points out

that the divine schema, as captured in Kant's *Universal Natural History and Theory of the Heavens*, also written in 1755, was Kant's notion of God's plan:

The matter which is the primitive material of all things is therefore bound by certain laws such that if these laws are allowed to operate freely matter must necessarily bring forth beautiful combinations. Matter has no freedom to deviate from this plan of perfection. Since it is thus subject to a supremely wise purpose, it must have necessarily been placed in such harmonious relations through a first cause ruling over it (1, 228.3-11, as quoted in Friedman (1992), 11)

Kant thus gave Newtonian physics a metaphysical foundation as the schematic *application* of a divine idea: God's plan directed his idea's determination of the structure of phenomenal interactions expressed by Newton's laws.

By the time of his *Dissertation* (1770/1929), it became apparent to Kant that his conception of space could not be sustained.⁸ At this point he rejected Leibniz's relational view according to which spatial properties are derived from simple substances, supplanting it with a claim that space is a form of sensible intuition through which an aspect of the fundamental law unifying the monadic realm into a single world appears to us (§16). Time also became a form of sensible intuition upon which the intuitive continuity (not distinguished from infinite divisibility or denseness) of space was said to depend (§14). Since the fundamental law continued to express a non-spatial (and non-temporal) divine schema, its appearance as an intuitive principle was taken to express the divine schema as it *appeared* to human cognition: space is "*subjective and ideal*"; and, as it were a schema, issuing by a constant law from the nature of the mind, for the coordinating of all outer *sensa whatsoever*" (§15.D). Human schemata thus originated as an intuitive reflection of the divine plan for the phenomenal world. Indeed, Kant added that if it were said that "the mind, along with all other things, is upheld by the infinite power of a single cause", the doctrine of divine omnipresence led to the position that "space, which is the universal and necessary condition, sensitively apprehended, of the co-presence of all things, can ... be entitled *omnipraesentia phaenomenon*" and "the concept of time as a single, infinite, and immutable, in which are and persist all things is the *aeternitatis phaenomenon* of the general cause" (§22, Scholium).

It seems, then, that Kant's idea of human schemata originated in the pre-critical period as an intuitive appearance of the divine plan for the phenomenal world. In

conceiving space and time as forms of sensible intuition, however, Kant removed the realm of simple substances from epistemic purview. Concomitantly, the gulf between intellectual and sensible faculties became such that pure concepts (other than those of space and time) had no application to objects of the phenomenal world, and were thus without meaning. In response, Kant absorbed metaphysics as a theoretical science into an intricate account of objective phenomenology. God as being became an illusion in the *Critique*, an “imagined object” (A671/B699). He was no longer creator of the ultimate constituents of the universe, but a regulative idea in a hypothetical use of reason representing a direction of inquiry that maximally organized objective experience into a systematic and purposive order:

Then it is said, e.g., that the things in the world must be considered *as if* they had gotten their existence from a highest intelligence. In such a way the idea is only a heuristic and not an ostensive concept and it shows not how an object is constituted but how, under the guidance of that concept, we ought to *seek after* the constitution and connection of objects of experience in general. (A671/B699)

The highest formal unity that alone rests on concepts of reason is the *purposive* unity of things, and the *speculative* interest of reason makes it necessary to regard every ordinance in the world as if it had sprouted from the intention of a highest reason. Such a principle, namely, opens up for our reason, as applied to the field of experience, entirely new prospects for connecting up things in the world in accordance with teleological laws, and thereby attaining the greatest systematic unity among them. (A686-687/B714-715)

Consequently, there was no longer room in Kant’s metaphysics for a divine schema that brings about the nexus of objects: “it is not from a highest intelligence that we derive the order of the world and its systematic unity” (A673/B701). But Kant maintained that “[f]or its execution the idea needs a *schema*, i.e., an essential manifoldness and order of the parts determined *a priori* from the principle of the end” (A833/B861). In the critical shift to God as a regulative idea, the original notion of a divine schema was transformed into a different kind of *telos*: no longer an organizing structure *from* which a purposeful universe developed, it became an organizing structure that served as the end *towards* which all empirical cognition aimed.

In the case of any regulative idea (e.g., God, a simple soul or a completed world), a “schema of reason” is “the idea of the *maximum* of division and unification of the understanding’s cognition in one principle” (A665/B693). More specifically, it is the *conceptual form* that presents a logical rule to organize and unify experience. It is in

relation to the *use* of a regulative idea, and thus its execution through a schema, that Kant alludes parenthetically to a “*focus imaginarius*”:

... [regulative ideas] have an excellent and indispensably necessary regulative use, namely that of directing the understanding to a certain goal respecting which the lines of direction of all its rules converge at one point, which, although it is only an idea (*focus imaginarius*) – i.e., a point from which the concepts of the understanding do not really proceed, since it lies entirely outside the bounds of possible experience – nonetheless still serves to obtain for these concepts the greatest unity alongside the greatest extension. Now of course it is from this that there arises the deception, as if these lines of direction were shot out from an object lying outside the field of possible empirical cognition (just as objects are seen behind the surface of a mirror); yet this illusion (which can be prevented from deceiving) is nevertheless indispensably necessary if besides the objects before our eyes we want to see those that lie far in the background, i.e., when in our case, the understanding wants to go beyond every given experience (beyond this part of the whole of possible experience), and wants to make the measure of its greatest possible and uttermost extension. (A644/B672)

This passage contains Kant’s only explicit use of the term “*focus imaginarius*” in the *Critique*. A close reading of the passage suggests that he is associating a regulative idea or, more precisely, a *regulative schema* with a “point” at the apex of “lines of direction” of a mirror-type structure of the imagination, a structure that serves to facilitate the *heuristic* role of the schema.

It may be tempting to interpret *focus imaginarius*, not as focal point of the imagination, but as a fiction of reason and dismiss the structure as a sort of metaphor.⁹ There are at least three reasons to resist this. Most obviously the passage does not say that a focal point is a fiction. It says that an *object* correlated to this point is illusory precisely because the focal point “lies entirely outside the bounds of possible experience”. Rather than a fiction, Kant seems to be telling us that a *focus imaginarius* is the end-point or *limit* of a mirror-type structure of the imagination.

To take the view that a *focus imaginarius* is in reason rather than in the imagination overlooks the role of the imagination in relation to reason. This role is evident in the following passage:

... it is precisely nature’s inadequacy to the ideas – and this presupposes both that the mind is receptive to ideas and that *the imagination* [emphasis mine] strains to treat nature as a schema for them – that constitutes what both repels our sensibility and yet attracts us at the same time, because it is a dominance that reason exerts over sensibility only for the sake of expanding it commensurately with reason’s own domain (the practical one) and letting it look outward toward the infinite, which for sensibility is an abyss. (Ak. XX, 265)

Kant never conceived the imagination as merely a synthetic one constrained within the bounds of possible experience. In the *Prolegomena to Any Future Metaphysics* (1786/2004), for example, he describes his critical project as one where the understanding must be relied upon to impose such bounds upon the imagination:

The imagination can perhaps be excused if it daydreams every now and then, that is, if it does not cautiously hold itself inside the limits of experience; for it will at least be livened and strengthened through such free flight, and it will always be easier to moderate its boldness than to remedy its languor. That the understanding, however, which is supposed *to think*, should instead of that, *daydream* – for this it can never be forgiven; for all assistance in setting bounds, where needed, to the revelry of the imagination depends on it alone. (Ak. IV: 317)

Indeed, Kant tells us that “*Imagination* is the faculty for representing an object even *without its presence* in intuition” (B151). This can be understood as capturing *two* things. The imagination can represent an object without matter (sensation). Also, the imagination can represent an object without form (space and time) and matter, i.e., as a formless point that “lies entirely outside the bounds of possible experience” correlated (as we saw in the case of the regulative idea *God*) with an “imagined object”.

The interpretation of a *focus imaginarius* as a fiction of reason and not as a focal point of the imagination also overlooks the implicit role of the imagination in the case of an “original image” (not to be confused with an “imagined object”) associated with (at least with some) regulative ideas:

But something that seems to be even further removed from an objective reality than the idea is what I call the *ideal*, by which I understand the idea not merely *in concreto* but *in individuo*, i.e., as an individual thing which is determinable, or even determined, through the idea alone. ... What is an ideal to us, was to *Plato* an *idea in the divine understanding*, an object in that understanding’s pure intuition, the most perfect thing of each species of possible beings and the original ground of all its copies in appearance.

Without venturing to climb as high as that, however, we have to admit that human reason contains not only ideas but also ideals, which do not, to be sure, have a creative power like the *Platonic* idea, but still have *practical* power (as regulative principles) grounding the possibility of perfection of certain *actions*. ... [E.g.] the sage (of the Stoics) is an ideal, i.e., a human being who exists merely in thoughts, but who is fully congruent with the idea of wisdom. Thus just as the idea gives the *rule*, so the ideal in such a case serves as the *original image* for the thoroughgoing determination of the copy, and we have in us no other standard for our actions than the conduct of this divine human being, which we compare ourselves, judging ourselves and thereby improving ourselves, even though we can never reach the standard. (A568-570/B596-598)

Associated with a regulative idea such as *wisdom* or *God* (A673/B701), is an original *image*.¹⁰ In saying reason “contains” original images, Kant seems to be identifying an imaginative counterpart to at least part of the conceptual form, i.e., the schema, contained within a regulative idea. In certain situations (we shall see) the original image correlates to the schema’s major premise.

Kant makes a distinction between an “imaginary object” and an “original image” of a regulative idea (e.g., A571/B579). How, then, are we to understand the relation between them? He gives us a hint when he says, parenthetically, that an illusory object is correlated with a focal point of converging lines “just as objects are seen behind the surface of a mirror”. Given the heuristic role that a regulative idea plays in drawing the understanding towards unity, the kind of mirror structure he seems to have in mind is a spherical convex mirror. See Figure 1.1:

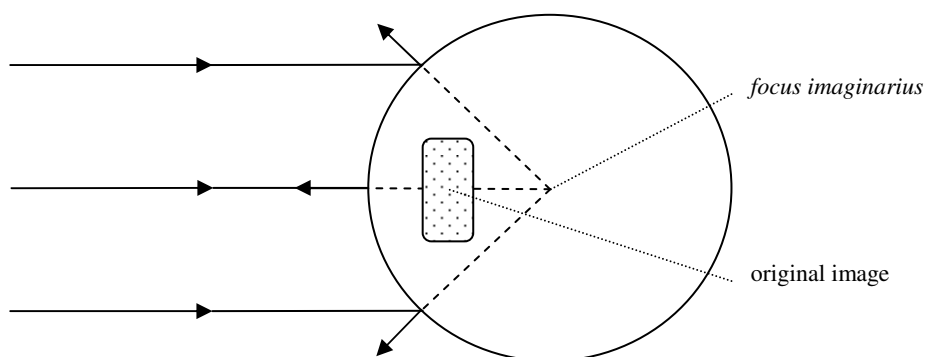


Figure 1.1: Regulative Idea as Spherical Convex Mirror

As we shall later, this basic structure of imagination in relation to reason enables the imagination to facilitate the discovery of empirical laws. I maintain that it is also the key to understanding Kant’s view of the imagination in relation to the understanding.

It is through the relation of imagination to the understanding that human schemata undergo critical development. No longer the appearance in intuition of a divine schema, Kant’s pre-critical human schema becomes fragmented and associated with forms of categories and pure sensible concepts. However, unlike regulative schemata these

schemata are not logical conceptual forms. I will argue that they are spatiotemporal structures presenting the same rule as conceptual forms governing imaginative synthesis. I will show that a figurative¹¹ pure sensible schema, distinguished from an image (A140/B179), is a focal point of the imagination for synthesis of a kind of image, one constituted by categorical schemata for syntheses of images in general. See Figure 1.2:

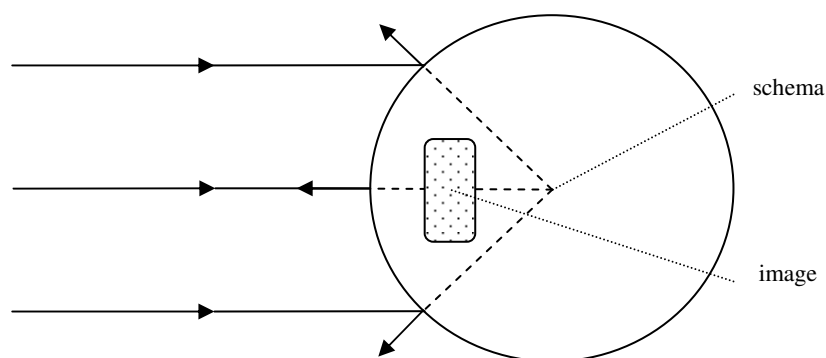


Figure 1.2: Pure Sensible Concept as Spherical Convex Mirror

Like Kant’s original notion of the divine schema, the notion of a human schema is transformed from the appearance of an organizing structure *from* which phenomenal reality develops to organizing structures *towards* which synthesis of phenomenal reality aims.

1.3 Critical Representations

The Analytic teems with intuitive, conceptual, and imaginative representations – generally “modifications of the mind” (A99) – unified by rule under the umbrella of conscious self-identity that accounts for the cognition of an object of experience. An object of experience comes about through a mixture of matter and form. Objective matter is sensation such as “impenetrability, hardness, color” (A21/B35) given a posteriori as an “effect” (A19/B34) on sensibility. Objective form arises from contributions of the forms of intuition by sensibility and connected manifolds of intuition by understanding (B129).

In virtue of conscious self-identity, the imagination synthesizes the separate forms and manifolds of space and time together with matter into an image that, in virtue of being thought through empirical concepts of the understanding, is cognized as an object of experience.

Considered independently of one another, each of the faculties of sensibility, imagination and understanding provides an impoverished representation an object of experience, the form of which can also arise a priori. See Figure 1.3:

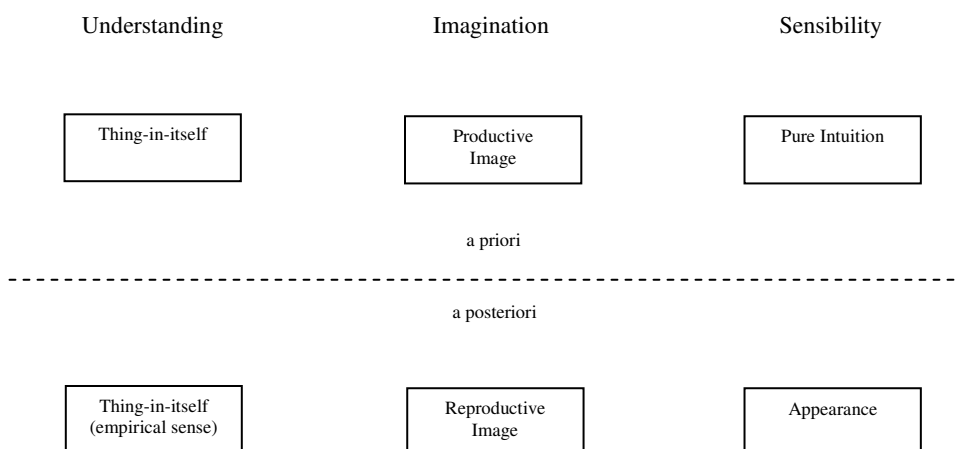


Figure 1.3: Object of Experience Represented by Faculty

Sensibility represents an object as singular intuition. With matter the intuition is an “appearance”;¹² without matter the intuition is simply “pure” (A100). The imagination represents an object as an image (A120-121), which may be synthesized reproductively or productively (B152). While reproductively synthesis generates only empirical images, productively synthesis generates either a pure image through the act of imagining or an empirical image through the act of drawing (A713/B741). The understanding represents an object a priori as a “thing-in-itself” (A254/B310), but also “in an empirical sense”.¹³

Cognition of an object takes place within a typology of concepts of the understanding, concepts that either serve as conditions for the possibility of experience or are derived by reflecting and abstracting from empirical images.¹⁴ “Borrowed” (A220/B267) from, or “derived from actuality in” (A223/B270), experience, an *empirical*

concept has “in it only some marks of a certain kind of objects of the senses” (A727/B755). A “mark” is a property, and in the case of empirical concepts “[o]ne makes use of certain marks only as long as they are sufficient for making distinctions; new observations, however, take some away and add some, and therefore the concept never remains within secure boundaries” (A728/B756). By contrast, a *pure* concept is one where “nothing empirical is intermixed” (B3). Pure concepts are further distinguished between notions and pure sensible concepts. A notion “has its origin solely in the understanding (not in a pure image of sensibility)” (A320/B377). Notions that are the conditions for the possibility of objects of experience are pure concepts of the understanding, i.e., the categories; otherwise, they are ideas of reason.¹⁵ That they have their origin solely in the understanding accounts for the fact that categories can be identified only by a metaphysical deduction from the forms of judgements. In virtue of the categories, “every concept” has “the logical form of a concept (of thinking) in general” (A239/B298).

A “pure sensible concept” (A140/B180) has its origin “in a pure image of sensibility” and thus “contains a pure intuition in itself” (A719/B748). For Kant one cannot think a pure sensible concept without instantiating an image, a cognitive process he calls “the *construction* of concepts”.¹⁶ In the case of the pure sensible concepts *line*, *circle*, *space*, and *time*, for example,

[w]e cannot think a line without *drawing* it in thought, we cannot think a circle without *describing* it, we cannot represent the three dimensions of space at all without *placing* three lines perpendicular to each other at the same point, and we cannot even represent time without ... *drawing* a straight line (which is to be the external figurative representation of time)... (B154)

In addition to geometrical concepts and concepts of the forms of intuition, pure sensible concepts include arithmetical concepts.¹⁷ Although sensibility contributes the forms of intuition to the construction, the understanding contributes its unity of object:

By contrast, that which determines space into the figure of a circle, a cone, or a sphere is the understanding, insofar as it contains the basis for the unity of the construction of these figures. The bare universal form of intuition called space is therefore certainly the substratum of all intuitions determinable upon particular objects, and, admittedly, the condition for the possibility and variety of those intuitions lies in this space; but the unity of the objects is determined solely through the understanding ...” (Ak. IV: 321-322).

Standard Kant scholarship readily attributes the unity of an object of experience to the categories as temporal conditions for the possibility of experience in general, neglecting the possibility that this unity is also attributed to pure sensible concepts of shapes as spatial conditions for the possibility of a *kind* of figurative experience. In my view much of the difficulty in coming to terms with Kant's schematism is due to this oversight.

It must be remembered that the *Critique* is a mere "outline" of transcendental philosophy, and does not "contain an exhaustive analysis of all human cognition a priori" (A13/B27). It is framed within Kant's general view that "all kinds of determinations of space can and even must be able to be represented *a priori* if [empirical] concepts of shapes as well as relations are to arise" (A29), and that determinations of parts of space "are only thought in it" (A25/B39) through pure concepts:

Thus the mere form of outer sensible intuition, space, is not yet cognition at all; it only gives the manifold of intuition *a priori* for a possible cognition. But in order to cognize something in space, e.g., a line, I must *draw* it, and thus synthetically bring about a determinate combination of the given manifold, so that the unity of this action is at the same time the unity of consciousness (in the concept of a line), and thereby is an object (a determinate space) first cognized. (A137/B138)

The tendency in the literature is to limit the significance of passages like this to mathematical reasoning. Not only does this ignore the *application* of geometrical propositions, however, it overlooks Kant's subtle point that *pure* sensible concepts for everyday objects can be *discovered* in experience. I will establish this important exegetical claim as the chapter progresses, but for now it should suffice to demonstrate its plausibility through the following passage:

[E]ven in concepts is an origin of some of them revealed *a priori*. Gradually remove from your experiential concept of a *body* everything that is empirical in it – the color, the hardness or softness, the weight, and even the impenetrability – still remains the *space* that was occupied by the body (which has now entirely disappeared), and you cannot leave that out. Likewise, if you remove from your empirical concept of every object, whether corporeal or incorporeal, all those properties of which experience teaches you, you could still not take from it that by means of which you think of it as a *substance* or as *dependent* on a substance (even though this concept contains more determination than that of an object in general). Thus, convinced by the necessity which this concept presses itself on you, you must concede that it has its seat in your faculty of cognition *a priori*. (B5-B6).

If one begins with the empirical concept of a (corporeal) *body* and removes "everything that is *empirical* in it" (i.e., matter), one is not left with space per se, but "the *space* that

was occupied by the body”. In other words, one is not left with space as the *form* of intuition, but with space as a *formal* intuition that involves “comprehension of the manifold” through understanding (B160n), i.e., the determination of the form of intuition through a *pure* concept. Likewise, if one begins with an empirical concept of a (corporeal) *body* (or even an incorporeal object) and removes “all those properties of which *experience teaches you*” (i.e., properties you discover through experience), one is *not* left with the space that was occupied by the body, but with substance. The obvious inference to make is that the space determined by a pure concept in relation to an empirical concept can be discovered through experience. In the case of the empirical concept *dog*, for example, experience teaches us that the space occupied by the body involves a comprehension of the manifold through the pure sensible concept *four-footed animal in general* (A141/B180), a *kind* of shape. As we shall see, a *literal* interpretation of Kant’s schematism just involves taking pure sensible concepts to be conditions for the possibility of a kind of figurative experience. But we need to first appreciate two symmetries of the synthetic imagination.

1.4 Symmetries of the Spatiotemporal Imagination

A point emphasized by Friedman (1992) is that the central distinction of the *Critique* between the faculties of sensibility and understanding is underwritten by a fundamental division between mathematics and logic. Kant’s logic is syllogistic, an essentially monadic rather than polyadic logic. This is problematic from the point of view of mathematical reasoning: in monadic logic infinity cannot be represented conceptually and therefore cannot represent the deduction of infinity of objects. Only in polyadic logic can quantifier-dependence arise, and it is the dependence of existential quantifiers on universal quantifiers that allows for infinite iterability. To account for mathematical cognition, Kant is forced to represent infinity intuitively in the forms of spatial (A25/B39-40) and temporal (A32/B48) intuition. He bridges the conceptually finite and the intuitively infinite through rules governing an infinitely iterable synthetic process housed within the imagination. It should be added that since imaginative synthesis is finite, infinite iterability is only *potential*.

More generally, the synthetic imagination plays the crucial role of reconciling the otherwise distinct faculties of sensibility and understanding. As the source of intuitions, the faculty of sensibility is the receptive faculty through which “objects are given to us”; as the source of concepts, the faculty of understanding is the spontaneous faculty through which “objects are thought” (A15/B29). Connecting concepts with intuitions is necessary to bring about cognition of objects, for, as Kant famously states, “[t]houghts without content are empty, intuitions without concepts are blind” (A51/B75). However, as representations intuitions are essentially different from concepts. Whereas an intuition “is immediately related to the object and is singular”, a concept “is mediate, by means of a mark, which can be common to several things” (A320/B377). The source of neither intuitions nor concepts, the faculty of the imagination is “a blind though indispensable function of the soul” (A78/B107) that resolves the singular non-discursive character of intuition and the general conceptual character of understanding.

The imagination effects this reconciliation through a synthesis. In general, a *synthesis* is “the action of putting different representations together with each other” (A77/B103). Friedman (1992, 40) emphasizes that imaginative synthesis is *spatiotemporal*, and that for Kant this synthesis involves “motion, which unites both elements”, “namely space and time” (A41/B58). Along with Stein (1977), Friedman goes so far as to say that Kant “comes close to making the idea of [Newtonian] space-time explicit” (161) in his *Dissertation*:

What is simultaneous is not made simultaneous simply by not being successive. For when succession is taken away there is indeed removed a certain conjunction of things within the temporal series, but there does not on that account at once arise another real relation, such as is the conjunction of all in the same moment. For simultaneous things are joined in the same moment of time just as successive things in different moments. Thus though time possesses only one dimension, yet the ubiquity of time (to use Newton’s manner of speaking), owing to which all things conceivably by sense are *at some time*, adds to the quantum of actuals a second dimension, so far as they hang, as it were, from the same point of time. For if you represent time by a straight line produced to infinity, and simultaneous things at any point of time by lines drawn perpendicular to it, the plane thus generated will represent the phenomenal world, both as to its substance and as to its accidents. (§14.5n.)

Indeed, implicit within Newtonian space-time is a function that identifies space-time points that have the same time coordinate, as well as a function that identifies space-time points that have the same space coordinate. It is thus endowed with *projections* of space-

time onto time and onto space giving rise to conceptions of absolute time and absolute space. The projection of space-time onto time “decomposes” space-time into an infinite number of three-dimensional Euclidean spaces, each representing “all of space at a given time”; the projection of space-time onto space “decomposes” space-time into an infinite number of point trajectories each representing “a given place over all time” (DiSalle 2009). See Figure 1.4:

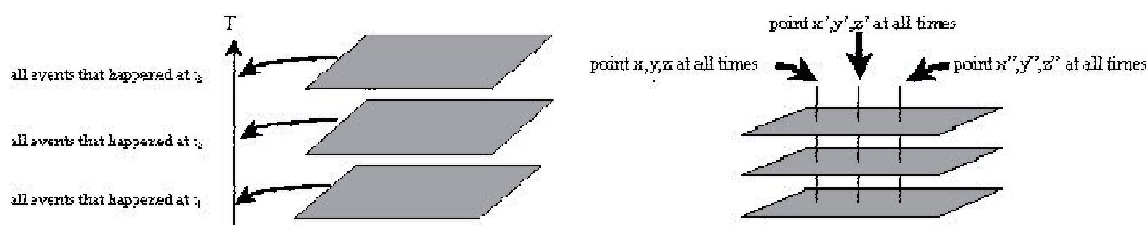


Figure 1.4: Projections of Newtonian Spacetime (From DiSalle (2009), Figure 6)

As the passage from his *Dissertation* suggests, Kant seems to picture the projection of Newtonian space-time onto time.

My working hypothesis is that Kant conceives imaginative synthesis in the *Critique*, which he describes in three synthetic steps as “apprehension in the intuition” (A98), “reproduction in the imagination” (A100) and “recognition in the concept” (A103), against this projection. Indeed, since his notion of *image* in the second step is a temporal succession of distinct apprehensions from the first step (A100-102, A120-121), its synthesis appears to be in the context of the former projection, not the latter.¹⁸ The value of this working hypothesis is that it facilitates the articulation of two symmetries that untangle Kant’s schematism.

The first spatiotemporal symmetry is what I call the *vertical* symmetry between productive and reproductive synthesis of a figure in space:

It may look, to be sure, as if the possibility of a triangle could be cognized from its concept in itself (it is certainly independent of experience); for in fact we can give it an object entirely *a priori*, i.e., construct it. But since this is only the form of an object, it would still always remain only a product of the imagination, the possibility of whose object would still remain doubtful, as requiring something more, namely that such a figure be thought solely under those conditions on which all objects of experience rest. Now that space is a formal *a priori* condition of outer experiences, that this very same formative synthesis by means of

which we construct a figure in the imagination is *entirely identical* with that which we exercise in apprehension of an appearance in order to make a concept of experience of it – it is this alone that connects with this concept the representation of the possibility of such of such a thing. (italics added, A223-224/B271)

I agree with Friedman (1996, 102) that this passage accounts for the application of mathematics. As I see it, these syntheses are identical because they are both determined by the same pure sensible concept of a figure in space, in this case the pure sensible concept *triangle*.

It is not obvious, however that the two syntheses should be “entirely identical”. As Friedman (1992, 126) also emphasizes, Kant’s notion of the synthesis of shapes in space is based upon Euclidean construction from points, lines, a compass and a ruler.¹⁹ Pure apprehension in the a priori construction of a figure in the imagination generates only part of the shape in the first instant: “motion, as *description* of a space, is a pure act of the successive synthesis of the manifold in outer intuition in general through productive imagination” (B155n) involving the motion of a mathematical point (Ak: IV, 489). However, empirical apprehension will “run through and then ... take together this manifoldness” of an appearance into a “*unity* of intuition” (A99) called a “perception” (A120n). Although this “action” (A99) of the imagination²⁰ is also described as motion and, by analogy, drawing (cf. B162)), it synthesizes the shape in the first instant. The passage thus equates the syntheses illustrated in Figure 1.5:

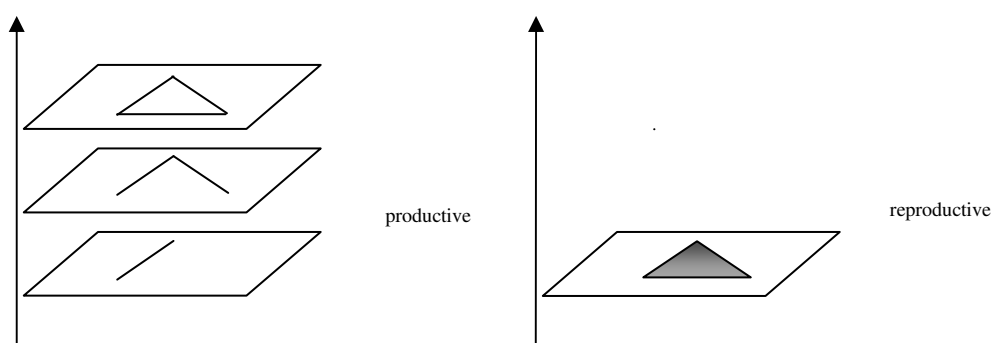


Figure 1.5: Productive and Reproductive Synthesis

As a succession of intuitions the first is an image, but the second is not (until it perdures in succession). The syntheses are “entirely identical”, I suggest, if we recognize that a figure productively constructed over time contains as much *four-dimensional* space-time

structure as that empirically apprehended at an instant of time, and that the form of a pure sensible concept presents a common rule governing both.

Keep in mind that this vertical symmetry only implies that the understanding determines *figurative* kinds; it does not derogate from the role Kant tells us reason plays in determining other empirical kinds. Moreover, even though the syntheses may be identical, the infusion of matter in the reproductive imagination changes the character of its product, making it less abstract and ideal by comparison. For this reason Kant claims that distinguishing whether “a case in *concreto*” belongs under “the universal in *abstracto*” (A134/B173) arises from “mother-wit”, “the power of judgment ... that cannot be taught but practiced” (A133/B172).

The vertical symmetry just identified is consistent with Kant’s account of rules of apprehension in the formation of empirical concepts. Every empirical concept is borrowed from experience via comparing, reflecting and abstracting a number of empirical intuitions (images):

I see, e.g., a spruce, a willow, and a linden. By first comparing these objects with one another I note that they are different from one another in regard to the trunk, the branches, the leaves, etc.: but next I reflect what they have in common, trunk, branches, and leaves themselves, and I abstract from the quantity, the figure, etc., of these; thus I acquire the concept of a tree. (Ak, IX: 94-95)

Kant is clear that in this case empirical intuitions are serving, not as schemata, but “examples” (Ak: XX, 351). To compare is to look for differences among the examples *selected* from among all other empirical intuitions, and so differences noted are among examples that already instantiate a kind of *form* (e.g., the shape of a tree). Only *after* such comparison does reflection and abstraction give rise to an empirical concept *tree* representing a kind of empirical image. Once formed, an empirical concept presents a “rule of apprehension” involving both form *and* matter for comparison with other empirical intuitions:

We compare only what is universal in the rule of our apprehension. For example, one sees a sapling, so one has the representation of a tree; an elongated rectangle makes one think of a square. (Ak. XVI: 557) (This passage and the prior one are as quoted in Longuenesse (1998, 116).)

Contrary to Longuenesse, just as rules of apprehension involving shapes presuppose categories, they also presuppose spatiotemporal rules presented by pure sensible concepts.²¹

The second spatiotemporal symmetry is the *horizontal* symmetry among the faculties of understanding, imagination and sensibility. The imagination bridges the singular non-discursive character of intuition with the general conceptual character of understanding through a spatiotemporal synthesis that “has as its aim no individual intuition but rather only unity in the determination of sensibility” (A140/B179). Where I depart from most scholars, even from Friedman (1992, 197-200), is in my emphasis that the “unity in the determination of sensibility” to which the spatiotemporal imagination aims is not a unity *in* sensibility. Unlike this traditional interpretation of Kant (e.g., Paton, (1936, 18)), I do not take Kant to be saying that the imagination *imposes* its synthesis upon time *or* space, but rather takes ingredients from both – specifically, a structure of the manifold of time determined by pure concepts of the understanding and (in certain cases at least) a structure of the manifold of space determined by a pure sensible concept – and brings them together (with or without matter) in the synthesis of a spatiotemporal object within the imagination itself. Although I see understanding and sensibility as distinct from the imagination, they are united with it in apperception through a common unity of spatiotemporal rule (i.e., the unity of an object of experience) *presented* by the form of pure concepts in understanding, the a priori affinity (pre-synthetic associability) of appearances in sensibility and pure schemata in imagination.

That a unity of spatiotemporal rule is presented by a pure sensible concept is evident from the following passage that involves an appeal to the pure sensible concept *triangle* to explain the unity of an object of experience:

Hence we say that we cognize the object if we have effected a synthetic unity in the manifold of intuition. But this is impossible if the intuition could not have been produced through a function of synthesis in accordance with a rule that makes the reproduction of the manifold necessary *a priori* and a concept in which this manifold is united possible. Thus we think of a triangle as an object by being conscious of the composition of three straight lines in accordance with a rule according to which such an intuition can always be exhibited. Now this *unity of rule* determines every manifold, and limits it to conditions that make the unity of apperception possible, and the concept of this unity is the representation of the object = X, which I think through those predicates of a triangle. (A105)

Kant's point is that the pure sensible concept *triangle* structures the predicates "three", "straight", "line" in virtue of a form presenting a rule for the construction all possible triangle images.²² Cognition of a triangle object in experience involves thinking the form of the pure sensible concept of a shape (and the form of the categories more generally) while empirically synthesizing an actual triangle image.

That the same unity of spatiotemporal rule is presented by an a priori affinity of appearances is more obscure. It is perhaps easiest to first *abstract* from the synthesis brought about by the imagination and consider the pre-synthetic unity attributable to the separate manifolds of space and time:

We have *forms* of outer as well as inner sensible intuition *a priori* in the representations of space and time, and the synthesis of the apprehension of the manifold of appearance must always be in agreement with the latter, since it can only occur in accordance with this form. But space and time are represented *a priori* not merely as forms of sensible intuition, but also as *intuitions* themselves (which contain a manifold), and thus with the determination of the *unity* of this manifold in them (see the Transcendental Aesthetic).*

*Space, represented as *object* (as is really required in geometry), contains more than the mere form of intuition, namely the *comprehension* of the manifold given in accordance with the form of sensibility in an *intuitive* representation, so that the *form of intuition* merely gives the manifold, but the *formal intuition* gives unity of the representation. In the Aesthetic I ascribed this unity merely to sensibility, only in order to note that it precedes all concepts, though to be sure it presupposes a synthesis, which does not belong to the senses but through which all concepts of space and time first become possible.²³ For since through it (as the understanding determines the sensibility) space or time are first *given* as intuitions, the unity of this *a priori* intuition belongs to space and time, and not to the concept of the understanding. (B160-161)

Kant's point in this controversial passage becomes clear in light of the horizontal symmetry relevant to space "represented as *object* (as is really required in geometry)". A pure sensible concept has its origin in an image in the sense that one cannot think the concept without simultaneously constructing it in space-time; equivalently, one cannot construct the concept in space-time without simultaneously thinking a pure sensible concept. Its possibility thus "presupposes a synthesis" that both *determines* and is *determined by* sensibility. If one were to abstract from this synthesis to the separate forms of space and time, what "precedes all concepts" is a structure of the manifold of space (i.e., space as a "formal intuition") determined by the pure sensible concept and a structure of the manifold of time (i.e., time as a "formal intuition") determined by the categories and presupposed by pure sensible concept.

Now think of formal intuitions of space and time as ingredients for synthesis. In relation to an empirical synthesis, they constitute a *potential* in sensibility, i.e., an affinity of appearances: “[t]he ground of the possibility of the association of the manifold, insofar as it lies in the object, is called the affinity of the manifold” (A113), and in relation to an object in general it is called “transcendental affinity” (A114). Kant claims that by means of their affinity, appearances “stand under constant laws and *must* belong to them” (A113). In other words, the affinity of appearances *presents* rules of spatiotemporal synthesis.

Since synthetic unity in the determination of sensibility is *effected* by the understanding, the understanding is “the legislation for nature” (A126). But the same unity is also *aimed* for by the spatiotemporal imagination in the production of a schema:²⁴

The schema is in itself always a product of the imagination; but since the synthesis of the latter has as its aim no individual intuition but rather only the unity in the determination of sensibility, the schema is to be distinguished from an image. ... [The] representation of a general procedure of the imagination for providing a concept with its image is what I call the schema for this concept. (A140/B179)

On a literal construal of this passage, a schema is neither rule nor procedure. In the *Critique*, representations only ever *present* rules, and if a schema is understood as a rule its distinction from other representations collapses.²⁵ To interpret a schema as the operations or procedures of the imagination that the rule governs is to attribute to Kant the elementary mistake of confusing product for activity.²⁶ The *only* interpretation that can do justice to this passage is this: a schema is, as Kant says elsewhere, an “outline”²⁷ serving as a *telos* that *presents* a rule of synthesis and thereby *represents* a general synthetic procedure that the rule governs. As I will now show, simply taking Kant at his word makes perfect sense of his schematism.

1.5 Schematism of Pure Concepts

Kant’s chapter on the schematism is the first chapter of the second book of the *Analytic* called “The Analytic of Principles” that introduces the faculty of judgement. “If the understanding in general is explained as the faculty of rules, then the power of judgement is the faculty of *subsuming* under rules, i.e., of determining whether something

stands under a given rule (*casus datae legis*) [“case of the given law”] or not” (A132/B171). The chapter on the schematism identifies *imaginative* conditions of subsumption of appearances under the categories. The following chapter then uses these conditions to derive a set of principles of the pure understanding that serve as the basis of all synthetic a priori propositions.²⁸

The key to understanding Kant’s schematism is to recognize *exactly* why he does not think it necessary to state these conditions of subsumption in the case of pure sensible concepts:

In all subsumptions of an object under a concept the representations of the former must be *homogeneous* with the latter, i.e., the concept must contain that which is represented in the object that is to be subsumed under it, for that is just what is meant by the expression “an object is contained under a concept. Thus the empirical concept of a *plate* has homogeneity with the pure geometrical concept of a *circle* for the roundness that is thought in the former can be intuited in the latter. (A137/B176)

In all other sciences, where the concepts through which the object is thought in general are not so different and heterogeneous from those that represent it *in concreto*, as it is given, it is unnecessary to offer a special discussion of the application of the former to the latter. (A138/B177)

Kant is saying that for object O to be subsumed under a pure representation PR, PR must contain some property X that *every* empirical representation ER represents of O, in which case ER is “homogeneous” with PR. The individual plate is subsumed under the pure concept *circle* because “the empirical concept [ER] of a plate [O] has homogeneity with the pure geometrical concept [PR] of a *circle* for the roundness [X] that is thought in the former can be intuited in the latter.”

The first thing to note is that the question of subsumption under pure concepts, and thus the question of their application to what is subsumed, is *evidential*. In this illustration, an object of experience such as a plate is the cognition through an empirical concept of the formal unity of connected perceptions in an empirically synthesized image. From the vertical symmetry of the imagination we know that this cognition is possible in virtue of the determination of the empirical concept by a pure sensible concept of a shape. But “distinct from all our representations,” an object “is nothing for us” (A105), and so application of the rule presented by the form of the pure sensible concept to the object – hence subsumption of an object under the concept itself – can only be made evident from

properties of its form shared with reproductively synthesized images. Now the properties of the form of a pure sensible concept are revealed in a productive synthesis of pure intuition (“we cannot think a circle without *describing* it” (B154)). And properties of reproductively synthesized images are revealed by empirical concepts arrived at through abstraction from and conscious reflection upon these images. Because one can discern the same property (e.g., roundness) in both pure intuition and empirical thought it is unnecessary to identify the conditions of subsumption of pure sensible concepts.

In the paragraph that follows Kant takes note of the fact that subsumption under pure concepts can not be taken as self-evident in the case of individual categories. Because homogeneity cannot be found between a category and actual perceptions, it cannot be said that appearances are evidently subsumed under the category; equivalently, it cannot be said that the category applies to appearances:

Now pure concepts of the understanding, however, in comparison with empirical (indeed in general sensible) intuitions, are entirely unhomogeneous, and can never be encountered in any intuition. Now how is the *subsumption* of the latter under the former, thus the *application* of the category to appearances possible, since no one would say that the category, e.g., causality, could also be intuited through the senses and is contained in the appearance? This question, so natural and important, is really the cause which makes a transcendental doctrine of the power of judgement necessary, in order, namely, to show the possibility of applying *pure concepts of the understanding* to appearances in general. (A137/B176-A138/B177)

The first thing to note is the move from the subsumption of objects to the subsumption of appearances. Presumably this is because causation is evident in a temporal series of perceptions. Appearances are like objects in the sense that “without the relation to at least possible consciousness” they “would be nothing at all” (A120). Accordingly, one must look to homogeneity with their representations in order to determine their subsumption under a pure concept. By “intuited through the senses” Kant identifies these representations as *perceptions*: “[s]ense represents the appearances empirically in *perception*” (A115). However, while a concept cannot be homogeneous with an object because the object of cognition is not itself a representation (it is the effect of “the formal unity of the consciousness” (A105)), it can nonetheless be homogeneous with appearances, for “appearances are ... but themselves only representations” (A109). This is because properties contained in the perception are, as a matter of inference, “contained

in the appearance”. Hence if subsumption of appearances involves homogeneity of a category with perceptions, it also involves homogeneity of a category with appearances. Given all this, Kant’s point is straightforward: unlike a pure sensible and empirical concept, no property is evidently shared between a category and perceptions, and thus between a category and appearances.²⁹

Especially in the older literature one encounters the view that Kant has lost sight of himself in his schematism and is tangled up in a pseudo-problem.³⁰ However, in a footnote found in his *Metaphysical Foundations of Natural Science* (2004a) Kant states that the transcendental deduction establishes “that [experience] is possible solely through [the categories], and, conversely, that these concepts are capable of meaning and use in no other relation than to objects of experience” (475n-476n). In the same note, he distinguishes the problem “how experience is now possible by means of these categories”, and, conversely, how categories have meaning and use.³¹ The solution to this second problem is generally understood to be the point of Kant’s schematism.³² Although Kant’s semantics and epistemology run together, it seems these two problems separate them. The first is more a semantic problem of establishing that pure concepts *can* relate to appearances; the second is more an epistemic problem of establishing how they *in fact* relate to appearances.

More perspicuously, without a *spatiotemporal* product of the imagination that presents the rule that governs its synthesis, there is no *criterion of evidence* by which to determine the application of pure concepts in the synthesis. Kant needs to correlate the form of a pure concept with a spatiotemporal *structure* that captures its unity of synthetic rule so he can identify properties of an imaginative synthesis to indicate the application of the rule. As it stands there is no such mark of the pure concept to indicate a determinate empirical use of it. The issue is rendered moot in the case of a pure sensible concept because the concept’s empirical application is evident whenever it is thought and an image productively constructed; that is, by productively constructing images of a spatiotemporal kind we come to *know* in mathematical cognition universal properties that serve as a criterion of what it is in appearances that counts as evidence for the concept’s application. In the case of the pure sensible concept *triangle*, for example, what counts as

evidence for its application is that the internal angles of a figure enclosed by three straight lines add to 180° .

But a category cannot be constructed since it does not originate in a pure image of sensibility, but rather in the understanding.³³ In virtue of the metaphysical deduction, the form of each category can be said to contain a logical function of the understanding. Even though the transcendental deduction of the categories that augments their form in principle by demonstrating that the categories as a collective relate to the transcendental affinity of appearances in virtue of a common unity of spatiotemporal rule,³⁴ he has yet to identify the spatiotemporal structure that presents this rule.³⁵ In a dispute with Hume over the application of the category *causality*, for example, Kant faces the objection that he has not offered anything better than his metaphysical predecessors. Perhaps the form of the category relates to the transcendental affinity of appearances that are the appearance in human cognition of the divine plan. Indeed, a reason Kant says he needs to address the problem of *how* categories make experience possible is so that

the perceptive reviewer may not be left with the necessity, certainly unwelcome even to himself, of taking refuge in a pre-established harmony to explain the surprising agreement of appearances with the laws of the understanding, despite their having entirely different sources from the former. (Ak. IV: 476n)

In other words, Kant recognizes he has yet to eliminate the possibility claimed in his pre-critical philosophy that it is God that necessitates imaginative synthesis through a *divine* schema that brings about the same spatiotemporal law as the form of the categories and the transcendental affinity of appearances.³⁶ A distinctly *human* “transcendental product of the imagination” (A142/B181) must be introduced that features the law so as to serve as *criterion* of evidence of the synthesis of the imagination that the law governs.³⁷

I have yet to come across literature that explicitly takes notice of the fact that Kant refers to *categorical* schemata in *two* distinct senses: “transcendental schemata” and “schemata of sensibility” (A145/B185). When he refers to a “transcendental schema” he is talking about a “representation [that] must be pure” (A138/B177) – i.e., a pure criterion of evidence. When he refers to “schema of sensibility” he is talking about a representation of “*how [things in general] appear*” (A147/B186) – i.e., the empirical evidence itself. The former represents a “pure synthesis” (A142/B181), while the latter

represents an empirical one (a harbinger of his theory of concept exhibition (hypotyposis) in the third *Critique* in which he claims “[s]chemata contain ... direct exhibitions of the concept” (Ak: V, 352)).³⁸

With one exception, Kant gives us only schemata of sensibility, setting aside “what is required for transcendental schemata” as a “dry and boring analysis” (A142/B181). We can understand why Kant would focus on schemata of sensibility: the infusion of matter in empirical synthesis in apprehension alters features of a rule of pure synthesis, and it is from schemata of sensibility that the principles of the understanding are derived. But we can also see how Kant’s failure to identify transcendental schemata generates difficulties. How could he, in principle, identify a pure criterion of evidence to, as he says, “indicate *a priori* the case” (A135/B175)³⁹ without first appealing the evidence itself? How is each criterion of evidence plausible or even unique?⁴⁰ How can a criterion of evidence explain *how* categories make experience possible and, conversely, *how* categories have meaning and use?

From the literature one is left with the impression that because Kant does need to account for the application of pure sensible concepts in terms of human schemata, the pure sensible schemata do no philosophical work. To the contrary, we cannot answer these questions about transcendental schemata without them.⁴¹ In the remaining part of this section, I will present what I take a pure sensible schema to be, appealing to this interpretation to make clear sense out of the infamous passages on pure sensible schemata, and then argue that transcendental schemata are constitutive of them. My final claim in this section will be that transcendental schemata are indeed “dry and boring”: they are just properties of a line as Kant understood them.

Let us return to the illustration that opens the chapter. As noted, it is intended to show how homogeneity between pure and empirical representations underwrites subsumption under, and application of, a pure representation. We need to understand exactly what “homogeneity”, “subsumption” and “application” mean in this context and how the terms are integrated. Homogeneity for Kant means “sameness of kind” (A657-658/B685-686). In saying that “the empirical concept of a *plate* has homogeneity with the pure geometrical concept of a *circle*” Kant is saying that the “roundness” thought in

the former is of the same *kind* as the roundness intuited in the latter. Now the roundness thought in the empirical concept *plate* refers to a property shared by a posteriori image examples. But the roundness that can be intuited in a priori images refers to the property of a pure sensible schema:

... mathematical cognition considers the universal in the particular, indeed even in the individual [drawn figure], yet nonetheless *a priori* and by means of reason, so that just as this individual is determined under certain general conditions of construction, the object of the concept, to which this individual corresponds only as its schema, must likewise be thought as universally determined. (A714/B742)

The reason that a schema “is to be distinguished from an image” (A140/B179) is that it bears only properties of a universal. Hence, roundness thought in an empirical concept *plate* is the same *kind* as roundness that can be intuited in a pure sensible concept *circle* because both cases involve the instantiation of properties of a pure sensible schema.⁴²

For Kant homogeneity is a condition for the possibility of subsumption, which is to say that the pure concept “must contain” the sameness of kind as that which “is to be subsumed under it” (A137/B176). It is only by discerning the instantiation of a schema that judgement can distinguish whether or not something stands under the rule the schema presents, and, in virtue of the horizontal symmetry, whether or not it is contained under the pure sensible concept whose form presents the same rule. If we stay within the confines of the imagination, there is a picturesque way to understand the notion of subsumption at play here. As a “product of the imagination” a schema is an outline that represents “a general procedure of the imagination for providing a concept with its image” through a synthesis that “has as its aim no individual intuition but rather only the unity in the determination of sensibility” (A140/B179-180). What makes literal sense of this claim is that a schema is an outline that that comes into being during synthesis of an image as a *telos*: in pure synthesis it has the character of a purposeful *plan*; in empirical synthesis has the character of a unified end. The vertical symmetry of productive and reproductive synthesis of a figure is captured as follows: in productive synthesis it is *from* a schema that synthesis begins; in empirical or reproductive synthesis it is *towards* the schema that synthesis ends. A schema thus serves as a *focal point* of imagination. See Figure 1.6:

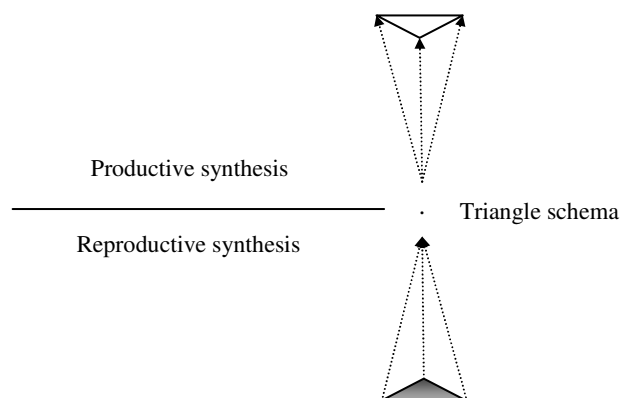


Figure 1.6: Pure Sensible Schema as Focal Point of the Imagination

In productive synthesis an image *evolves* from the schema; in reproductive synthesis an image *involves* towards the schema.

Given that Kant introduced his schematism relatively late in the preparation of the *Critique*,⁴³ it is plausible to think that he fashioned his conception of a schema of a pure sensible concept upon the schema of an idea of reason:

For its execution the idea needs a schema, i.e., an essential manifoldness and order of the parts determined a priori from the principle of the end. ... [The] schema contains the outline (monogramma) and the division of the whole into members in conformity with the idea, i.e., a priori [It] lies in reason like a seed, all of whose parts still lie very involved and are hardly recognizable even under microscopic observation. (A833-4/B861-2)

In the case of an idea of reason a schema has a *logical* form and as such belongs to reason as the form of the idea. But it is correlated to a *focus imaginarius*, a formless *limit* in the imagination that focuses it beyond the bounds of possible experience, giving rise to the *illusion* of an object of experience. A schema of a pure sensible concept, on the other hand, has a spatiotemporal form of an object of experience, and as such belongs to imagination. Much like a regulative schema, however, it is an “embryo” born in the imagination at the bounds of possible experience from the contribution of elements of the understanding and sensibility. It is not a limit of reason’s imagination; it is the focal point of understanding’s imagination.

This imaginative account of subsumption sheds light upon Kant's conception of the "application" of a pure sensible concept. A pure sensible concept applies to experience in virtue of an identical synthesis in pure and empirical intuition organized under a common schema produced in the imagination. By *realizing* the form of a pure sensible concept in a *focused* synthesis, a schema accounts for *how* the pure sensible concept makes experience possible. By serving as *criterion of evidence* gathered from pure syntheses, the exhibition of the schema in empirical synthesis is evidence of the application of a pure sensible concept that can be distinguished by judgement. In the exhibition of their form, schemata thus account for *how* pure sensible concepts have meaning and, at the same time, *how* their significance is *restricted* to an empirical use.⁴⁴

The strength of this interpretation of pure sensible schemata lies in its ability to make clear a notorious paragraph that appears just before Kant's discussion of transcendental schemata. The paragraph begins with a passage pertaining to productive synthesis of pure intuition:

In fact it is not images of objects but schemata that ground our pure sensible concepts. No image of a triangle would ever be adequate to the concept of it. For it would not attain the generality of the concept, which makes this valid for all triangles, right, or acute, etc., but would always be limited to one part of this sphere. The schema of the triangle can never exist anywhere except in thought, and signifies a rule of the synthesis of the imagination with regard to pure shapes in space. (A141/B180)

By "ground" Kant likely means "cause"⁴⁵ and so the point of the passage concerns concept formation. In particular, he wants to qualify his general claim that a pure sensible concept originates "in a pure image of sensibility" and say that it originates in a schema produced in the imagination. The first thing he says is that a productively constructed triangle image in pure intuition can only be an instance of a subclass of possible triangle images. Since the form of the concept *triangle* presents a rule that governs the productive synthesis all possible triangle images, the concept cannot originate in a triangle image, but in an outline of all possible triangle images. The second is that the schema "can never exist anywhere except in thought". For Kant "existence denotes the way in which an object of possible experience comes into being through the categorical structure of the understanding" (Caygill 1995, 183), and (I will show below) a pure sensible schema is *constituted* by categorical structure. The schema exists only "in

thought” in the sense that it does not come into being unless and so long as the pure sensible concept *triangle* is thought and an image simultaneously constructed.

Although this passage concerns pure intuition, its claims extend symmetrically to empirical intuition: “I put together in a pure intuition, just as in an empirical one, the manifold that belongs to the schema of a triangle in general and thus to its concept” (A718/B747). Accordingly, it is no surprise that what immediately follows in the *same* paragraph is a passage concerning empirical intuition:

Even less does an object of experience or an image of it ever reach the empirical concept, rather the latter is always related immediately to the schema of the imagination, as a rule for the determination of our intuition in accordance with a certain general concept. The concept of a dog signifies a rule in accordance with which my imagination can specify the shape of a four-footed animal in general, without being restricted to any single particular shape that experience offers me or any possible image that I can exhibit *in concreto*. (A141/B180)

This passage is the source of interpretive difficulties that are wholly unnecessary. A tradition seems to have become entrenched whereby Kant is taken to attribute to the empirical concept *dog* what is being said about a “general concept”.⁴⁶ But *clearly* the example is *not* about the form *and* matter of appearances, but only about their form (shape). What Kant is saying is just that the empirical concept *dog* is *determined* by “a certain general concept”, i.e., the pure sensible concept *four-footed animal in general*. Indeed, it is because the rule presented by the latter concept governs both a reproductive *and* productive synthesis that the passage contains the connections it does: “an object of experience or an image of it”; “any single particular shape that experience offers me or any possible image that I can exhibit *in concreto*”.

It must be remembered that Kant characterizes pure sensible concepts only by their *origin* “in a pure image of sensibility” or, more precisely, in a schema. This is evident by the fact that one cannot think a figurative pure sensible concept without constructing an individual image of a kind of shape. Given this characterization, it is possible for experience to tell us there are pure sensible concepts of figures that have yet to be (and may never be) used in geometrical proofs. We cannot *think* the concept *four-footed animal in general*, a concept concerning form *only*, without instantiating an image of a four-footed animal. (One cannot think the empirical concept *dog* without

instantiating an image of a dog either, but this concept is *determined by the understanding* and concerns not only form but also matter). By Kant's characterization, it is a pure sensible concept.

In this second passage, Kant is making a similar point about geometrical congruence and concept formation as the one that just precedes it. More so than in the case of a pure sensible concept, only a schema – not an image – can account for the origin of an empirical concept of a figure in space. How so? “Pure shapes” discussed in the first passage are images conjured in the productive imagination *without drawing*. Since they cannot be physically measured to establish their degree of congruence, Kant must appeal to the incongruence of *sub-classes* of images. Since empirical images can be measured, he does not need to make this appeal to establish their incongruence with one another. His point is that an outline of a kind of figure rather than particular incongruent figures of a kind is what accounts for the origin of empirical concepts (inasmuch as they concern figures in space).

This interpretation of pure sensible schemata is the *only* interpretation that can make *literal* sense of Kant's summary of both passages occurring next in the paragraph:

This schematism of our understanding with regard to appearances and their mere form is a hidden art in the depths of the human soul, whose true operations we can divine from nature and lay before our eyes only with difficulty. We can only say this much: the *image* is a product of the empirical faculty of productive imagination, the *schema* of sensible concepts (such as figures in space) is a product and as it were a monogram of pure *a priori* imagination, through which and in accordance with which the images first become possible, but which must be connected with the concept, to which they are in themselves never fully congruent, always only by means of the schema that they designate. (A141/B181)

That Kant intends to refer to *both* passages just discussed is indicated by the phrase “appearances and their mere form”, i.e., empirical intuition and empirical intuition without matter (pure intuition). The key to decoding this difficult passage is the symmetries of the spatiotemporal imagination. With these symmetries in mind, let us work through this passage line by line.

That we are to divine the schematism from *nature* highlights the import of the element of *discovery* of pure sensible schemata and thereby pure sensible concepts in use

in experience; that the schematism is a “hidden art in the depths of the human soul” highlights and the import of its relation to the *self*. Although the empirical faculty of the imagination carries out a reproductive synthesis of a figurative image, the *necessity* of this synthesis presupposes a *productive* synthesis of *that* image.⁴⁷ Now the term “sensible concepts” is rarely used by Kant,⁴⁸ but in keeping with the vertical symmetry just implied it seems he intends to refer to empirical concepts determined by pure sensible concepts. Accordingly, the “schema of a sensible concept (such as figures in space)” is the schema of the pure sensible concept, which is why Kant says it “is a product and as it were a monogram of pure *a priori* imagination”. It is because the schema focuses the reproductive imagination that images first become possible “through” it; it is because the schema presents the rule of construction that images first become possible “in accordance” with it. That images “designate” a schema refers to their schematic exhibition, and because an empirical concept arises from comparing images initially selected in virtue of exhibiting a common schema, incongruent images connect with the empirical concept “always and only by means of the schema”.

With this conception of pure sensible schemata, Kant ends the paragraph contrasting transcendental schemata:

The schema of a pure concept of the understanding, on the contrary, is something that can never be brought to an image at all, but is rather only the pure synthesis, in accord with a rule of unity according to concepts in general, which the category expresses, and is a transcendental product of the imagination, which concerns the determination of the inner sense in general, in accordance with conditions of its form (time) in regard to all representations, insofar as these are to be connected together *a priori* in one concept in accord with the unity of apperception. (A142/B181)

Transcendental schemata are like pure sensible schemata in that they are both products of the imagination and present a rule governing a pure synthesis. Where they differ is that a transcendental schema “can never be brought to an image at all” presumably because it “concerns the determination of the inner sense in general”. There is a strong temptation to *equate* a transcendental schema with determined inner sense (e.g., (Allison 2004, 216)) and account for its inability to bring about an image as due to a lack of spatial elements. But Kant does not say this exactly, nor can he. Since “time cannot be perceived in itself” (A183) it would follow that a transcendental schema is itself imperceptible; yet, the very

goal of Kant's schematism involves establishing the application of a category evident in perception.

Kant only says a transcendental schema "*concerns* the determination of the inner sense in general" and that such schemata are "nothing but *a priori time-determinations* in accordance with rules" (A145/B184). The phrase "in accordance with rules" is an important qualification, for it refers to governance of the *spatiotemporal* imagination. What Kant is saying, more precisely, is that a transcendental schema is a spatiotemporal criterion for what counts as the *perception* of a time-determination,⁴⁹ without which "one cannot grasp through an example what sort of thing is really intended by [categories]" (A241/B300). This is clarified in the second edition: "in order to understand the possibility of things in accordance with the categories, and thus to establish the *objective reality* of the latter, we do not merely need intuitions, but always *outer intuitions*" (B291). Since the "objective reality" of a category can be "exhibited only in outer intuition, and by means of that alone can it subsequently also be applied to inner sense" (B293),⁵⁰ to "*understand* the possibility of things in conformity with the categories" is just to be able to identify the property that the categories have in common with perceptions.

The temptation now is to swing the other way and equate a transcendental schema with a rule expressed by the category's form. Aside from the difficulty of understanding how a rule can serve as a criterion of evidence, this undermines the status of a schema as a "representation". Just like categories and appearances, a transcendental schema must have the status of a distinct representation in order stand between them:

Now it is clear that there must be a third thing, which must stand in homogeneity with the category on the one hand and the appearance on the other, and makes possible the application of the former to the latter. This mediating representation must be pure (without anything empirical) and yet *intellectual* on the one hand and *sensible* on the other. Such a representation is the *transcendental schema*. (A138/B177)

The horizontal symmetry brings clarity here. A transcendental schema is not a rule; like other representations in the *Critique* it *presents* a rule: as a distinct "modification of the mind" a transcendental schema is a "third thing" that presents the same rule as a category (through its form) and appearances (through a transcendental affinity), and can be

ascribed properties that stand in relations of homogeneity with these other representations.

What makes the Kant's transcendental schematism so obscure is that it is hard to see in what sense a schema is a *pure* representation. In the case of a pure sensible schema this is clear: it is a pure representation in the sense of a spatiotemporal outline of possible images of a kind that is a focal point in synthetic imagination. Yet we must think about a transcendental schema differently. Unlike a pure sensible schema, a transcendental schema does not come into being *through* categorical structure, for it *is* categorical structure *insofar* as it is a criterion for its perception. Transcendental schemata must thus be *constitutive* of pure sensible schemata, and the lack of sufficient *spatiotemporal* structure is why it cannot be brought to an image. Being constitutive of a focal point of the synthetic imagination is what accounts for *how* categories make experience possible and *how* they have meaning and use.⁵¹

Yet the “dry and boring analysis” Kant avoids still haunts us: How can Kant possibly identify pure criteria of evidence without appealing to impure evidence? How is each criterion plausible or even unique? A significant hint comes from Kant's own notes to the first edition: “The schema of time a line”.⁵² It may seem Kant is confusing a schema for an image. But recall that images contain schematic exhibitions. In saying the “schema of time a line”, we should understand Kant as referring to schematic exhibition of the pure sensible concept time. If productive and reproductive syntheses are indeed “identical”, then figures are constructed from curved or straight lines. Thus the pure sensible concept *time* is constitutive of all figures. But if transcendental schemata – as a collective – are constitutive of pure sensible schemata, they too are constitutive of all figures.⁵³ It seems, then, that transcendental schemata are related to a line.

Kant did not amend his chapter on the schematism for the second edition to incorporate notes he made to his copy of the first edition. But he did add a new section to the B-Deduction entitled “On the application of the categories to objects of the senses in general”. Here Kant tells us we *perceive* time-determination “under the name of the categories” whenever we construct the pure sensible concept *time* by drawing a line:

We also always perceive [“the determination of the manifold through the transcendental action of the imagination (synthetic influence of the understanding on inner sense)”] in ourselves. ... [W]e cannot even represent time without, in *drawing* a straight line (which is to be the external figurative representation of time), attending merely to the action of the synthesis of the manifold through which we successively determine inner sense, and thereby attending to the succession of this determination of inner sense. Motion, as action of the subject (not as determination of an object), consequently the synthesis of the manifold in space, if we abstract from this manifold in space and attend solely to the action in accordance with which we determine the form of *inner sense*, first produces the concept of succession at all. The understanding therefore does not *find* some sort of combination of the manifold already in inner sense, but *produces* it, by *affecting* inner sense. (B154-155)

The straight line synthesized a priori whenever the pure sensible concept *time* is thought is a spatial image constructed in time through a potentially infinite iterable process intuited as “motion of a point in space” (B292). Kant already tells us in the first edition that it is from the spatiotemporal properties of such a line that he *infers* properties of time:

[W]e ... represent the temporal sequence through a line progressing to infinity, in which the manifold constitutes a series that is of only one dimension, and infer from the properties of this line to all the properties of time, with the sole difference that the parts of the former are simultaneous but those of the latter always exist successively. From this it is also apparent that the representation of time is itself an intuition, since all its relations can be expressed in an outer intuition. (A33/B50)

From the last sentence Kant appears to be saying that inference from the properties of a line is to time, not as a form of intuition, but as a *formal* intuition (cf. B160). In executing this inference, “we *abstract from this manifold in space* and attend solely to the action in accordance with which we determine the form of *inner sense*”. In other words, the *collective* effect of the categories on inner sense accounts for time as a formal intuition we *perceive* in outer sense as a constructed line. It follows that the *individual* effect of a category on inner sense is a time-determination which we perceive in outer sense as the *property* of a line or as the property of a relation to a line.⁵⁴ Hence as a pure criterion of evidence, a transcendental schema is simply a certain property of, or in relation to, a line.⁵⁵ Can it get any more “dry and boring”?

Let us consider this interpretation of transcendental schemata in light of their grouping under “time-series”, “content of time”, “order of time” and “sum total of time” (A145/B184-185).⁵⁶ For each Kant provides a summary in the following form “the schema of each category contains and makes representable ...” (A145/B184) and identifies either a transcendental schema (criterion of evidence) or schemata of sensibility

(evidence), but not both. Under “time-series” Kant locates a single “pure” schema *number* for the categories of quantity: *unity*, *plurality* and *totality*. It “contains and makes representable ... the generation (synthesis) of time itself, in the successive apprehension of an object” (italics added, A145/B184). There are two characteristics of this transcendental schema. The first is that it “is nothing other than the unity of the synthesis of the manifold of a homogeneous intuition in general” (A143/B182). Here the schema *number* concerns the unity from which a line is generated, i.e., the unity of a *point* (from which is inferred time as an instant). The unity of a point is a criterion by which application of the category *unity* is evident in perception. It is evident, for example, in the unity of “*one representation*” (A99) (i.e., a potentially infinite three-dimensional slice of simultaneity) synthesized in apprehension:

Thus if, e.g., I make the empirical intuition of a house into perception through apprehension of its manifold, my ground is the *necessary unity* of space and of outer sensible intuition in general, and I as it were draw its shape in agreement with this synthetic unity of the manifold in space. This very same synthetic unity, however, if I abstract from the form of space, has its seat in the understanding, and is the category of the synthesis of the homogeneous in an intuition in general, i.e., the category of *quantity*, with which that synthesis of apprehension, i.e., the perception, must therefore be in thoroughgoing agreement. (B162)

Furthermore, “because I generate time itself in the apprehension of an intuition” (A142-143/B182) the schema *number* also serves as “a representation that summarizes the successive addition of one (homogeneous) unity to another”. It is thus not only a criterion of evidence for the category *unity*, it is a criterion by which application of the categories *plurality* and *totality* is evident in perception.

In the remaining groupings, Kant shifts from identifying a transcendental schema to specifying schemata of sensibility where the pure criterion is related to both form and matter. Under “time-content” one finds a single schema of sensibility for the categories of quality, i.e., *reality*, *negation*, and *limitation*. Kant does not give us a name for this schema, but refers to it as the “schema of a reality”, asserting that it “contains and makes representable ... the synthesis of sensation (perception) with the representation of time or the filling of time” (A145/B184). Sensation (matter) indicates “thing-hood, reality” and has “a degree or magnitude, through which it can more or less fill the same time” (A143/B182). Kant tells us that the category *reality* “indicates a being (in time)”,

negation “a non-being (in time)” (A143/B182) and (suggests) *limitation* indicates gradual approach to these two extremes.

Hence there is a relation and connection between, or rather a transition from reality to negation, that makes every reality representable as a quantum, and the schema of a reality, as the quantity of something insofar as it fills time, is just this continuous and uniform generation of that quantity in time, as one descends in time from the sensation that has a certain degree to its disappearance or gradually ascends from negation to its magnitude. (A143/B182-183)

It seems to me that the “schema of a reality” for all three categories makes evident the linear property of (Euclidean) *continuity* in the variation of sensations found in perception (from which is inferred the continuity of time).⁵⁷

We now come to a critical juncture with the other two groups of schemata of sensibility. All transcendental schemata, as *criteria* of evidence, are identified from the properties of a line constructed in the productive imagination. But recall Figure 1.5. Productive synthesis of a shape does not involve an instantaneous synthesis; it involves a synthesis over time into an image. In reproductive synthesis, by contrast, a shape is synthesized in an instantaneous perception that is reproduced over time as an image. Schemata of sensibility thereby fall into two groups: those that can exhibit the category in a single perception and in a series of perceptions; those that can exhibit the category in a series of perceptions only. The properties of a line denoted by schemata for “time-series” and “content of time” belong to the former, which makes them evident a priori in constructions.⁵⁸ Accordingly, their “manner of their evidence” (A180/B223) is said to be “immediate” (A161/B200) and “intuitive” (A162/B201), and can be exhibited a priori. It is because of this feature (A178-179/B220-222) that the schemata give rise to principles of the pure understanding Kant calls “mathematical” (A160/B199) or “constitutive” (A179/B222) in that the categories are evidently applied in the intuitive “composition” (B201n) of a *single* perception. On the other hand, the schemata of “time-order” and “sum total of time” (to be considered next) denote properties of, or in relation to, a line that cannot be exhibited in a construction. Although they are constitutive of experience (A664/B692), their manner of evidence is “mediate” (A160/B199) and “discursive” (A161/B201), for they must be exhibited in a *series* of individual perceptions. These are said to give rise to “dynamical” (A160/B199) or “regulative” (B179/B222) principles in

that the categories are evidently applied in a “connection” (B201n) of perceptions either among themselves or to the a priori faculty of cognition.

Under “order of time” one finds schemata for each of the categories of relation, i.e., *subsistence*, *causality* and *community*. Each “contains and makes representable the relation of the perceptions among themselves to all time (i.e., in accordance with a rule of time-determination)” (B145/B184). These schemata, it seems to me, are structural properties that govern the two-place relation $<$ of points on a line (from which is inferred a linear order of the manifold of time⁵⁹). The schema of sensibility of *substance*, persistence, is “the representation of the real as a substratum of empirical time-determination in general ... and in it alone can the succession and simultaneity of appearances be determined in regard to time” (A144/B183). That perceptions are connected to each other is evidence of the connectedness among points of a line (what today we would formally introduced through the axiom: $a < b \vee b < a \vee a=b$). The schema of sensibility of causality “consists in the succession of the manifold insofar as it is subject to a rule” (A144/B183). That perceptions would then stand in transitive relations is evidence of transitivity among points of a line (what we would today introduce through the axiom $a < c \ \& \ c < b \rightarrow a < b$).⁶⁰ Finally, the schema of sensibility of community “is the simultaneity of the determinations of the one with those of the other, in accordance with a rule” (A144/B183-184). That perceptions in reciprocal relations of causality happen at the same time is evidence of the property of irreflexivity individual points of a line (what we would today introduce through the axiom $\sim (a < a)$).⁶¹

Corresponding to each of the categories of modality, i.e., *possibility*, *existence* and *necessity* is a schema falling under the group, “sum total of time” that “contains and makes representable ... time itself, as the correlate of the determination of whether and how an object belongs to time” (A145/B184). Here the schema is not a property of a line, but a property of relating to a line. The schema of sensibility of possibility “is the agreement of the synthesis of various representations with the conditions of time in general” (A144/B184) – i.e., the application of the category *possibility* is evident in that the form of any perception is synthesized from a line or that any series of perceptions maps to a line. The schema of sensibility of actuality “is existence at a determinate time”

(A145/B184) – i.e., the application of the category *actuality* is evident in that perceptions map to a segment of a line. The schema of sensibility of necessity “is the existence of an object at all times” (Ibid.) – i.e., the application of the category *necessity* is evident in that perceptions that map to a segment of a line can map to any other segment of a line.

The distinction of empirical evidence from a pure criterion of evidence is what allows us to make sense of the following passage:

But it is also obvious that, although the schemata of sensibility first realize the categories, yet they likewise also restrict them, i.e., limit them to conditions that lie outside the understanding (namely in sensibility). Hence the schema is really only the phenomenon, or the sensible concept of an object, in agreement with the category. ([Translated from Latin: *Number* is the quantity of the phenomenon, *sensation* the reality of the phenomenon, *constancy* and the endurance of things the substance of the phenomenon, *eternity* the necessity of phenomena, etc.]). ... [S]chemata [are] merely representing [things in general] *how they appear...* (A146-147/B186)

Transcendental schemata are pure products of a focused imagination and identify what counts as intuitive evidence for the application of the categories to perceptions. But the evidence itself – sensible schemata – represent “*how they appear*”. Sensible schemata “realize” the categories by constituting the spatiotemporal form of objects of experience;⁶² in this sense a schema of sensibility is the “phenomenon”⁶³ or the “sensible concept of an object” in agreement with the category. At the same time, schemata of sensibility “limit” the categories by restricting their meaning to sensibility itself.

With this understanding of transcendental schemata, let us now return to Kant’s solution to the problem he set out to answer:

The concept of the understanding contains pure synthetic unity of the manifold in general. Time as the formal condition of the manifold of inner sense, thus of the connection of all representations, contains an *a priori* manifold in pure intuition. Now a transcendental time-determination is homogeneous with the *category* (which constitutes its unity) insofar as it is *universal* and rests on a rule *a priori*. But it is on the other hand homogeneous with the *appearance* insofar as *time* is contained in every empirical representation of the manifold. Hence an application of the category to appearances becomes possible by means of the transcendental time-determination which, as the schema of the concept of the understanding, mediates the subsumption of the latter under the former. (A138-139/B177-178)

By “every empirical representation of the manifold” (ER) I take Kant to mean every spatiotemporal perception. (As noted above, perceptions contain the properties of

appearances, so homogeneity with the former is homogeneity with the latter.) Kant wants to mediate the subsumption of appearances (A) under a category (PR) via subsumption under a schema (S). Given the example that opens the chapter, he needs to establish two things: 1) PR contains some X_1 that any schema S represents, in which case S is “homogeneous” with PR; and 2) schema S contains some X_2 that any representation ER represents of A, in which case ER (and A) is “homogeneous” with S. X_1 is identified as a universal property expressed by a (spatiotemporal) rule.⁶⁴ X_2 is identified as (the perception of) time.⁶⁵ In virtue of the latter, A is subsumed under S; in virtue of the former, A is further subsumed under PR.

Clearly, for Kant’s stated solution to work, a transcendental schema must be a pure *representation* – a modification of the mind – to which properties can be ascribed; it cannot be a rule or procedure. Moreover it must be distinct from both intellectual (concepts) and sensible (space and/or time) representations, yet common to them. As a *criterion* of evidence produced in the imagination, a transcendental schema presents the same rule as the form of the intellectual category and constitutes the spatiotemporal focus of sensible syntheses of the manifold of space and time. One might also say that as evidence, a schema of sensibility is distinct from and yet common to both sensible and intellectual representations. It is the “footprint” in the perception of appearances of the effect of a category on time. Together the two sides of categorical schemata tell us *how* categories apply appearances and *how* they have meaning and use.

Kant's schematism is summarized in Figure 1.7:

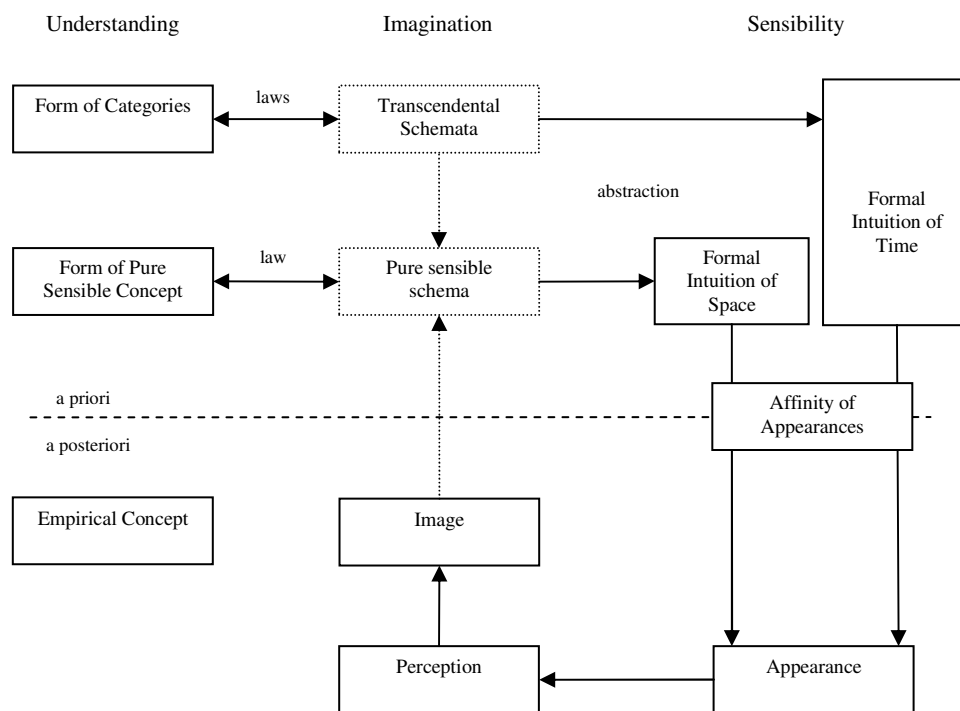


Figure 1.7: Kant's Schematism

Kant takes ownership of the imagination through schemata. A pure sensible schema is a spatiotemporal outline that guides the synthesis of images of a kind by focusing the imagination. It is a *telos* that presents the same rule governing a spatiotemporal synthesis as the form of a pure sensible concept and the affinity of appearances for a spatiotemporal kind from which Kant abstracts a formal intuition of space and the formal intuition of time. It serves as a criterion of evidence for the application of a pure sensible concept in that any image synthesized under its guidance will exhibit its properties and thus the properties of the form of the pure sensible concept. A transcendental schema is a spatiotemporal outline that guides the synthesis of images in general and constitutes every pure sensible schema of the imagination. It presents the same rule governing the spatiotemporal synthesis as the form of a category and the transcendental affinity of appearances from which Kant abstracts a property of the formal intuition of time. It serves as a criterion of evidence for the application of a category in that any series of

perceptions synthesized will exhibit its properties and thus the properties of the form of the category.

1.6 Schemata and Reasoning

What has stood in the way of this understanding of Kant's schematism is a failure to appreciate the role pure sensible concepts play as conditions for the possibility of a spatiotemporal kind of experience. It seems pure intuition is commonly given a Platonist construal in that an object constructed in pure intuition is taken to be essentially different from an object of experience, with the result that pure sensible concepts are relegated to mathematical cognition. In addition, empirical kinds are thought to be determined exclusively through reason. A literal construal of Kant's schematism tells against former, and Kant's account of mathematical and philosophical cognition, I will now show, tells against the latter. We will find that at the heart of both forms of cognition is a structure of schemata that constitutes a transcendental logical space.

In general, cognition is "objective perception" (A320/B377). "*Philosophical cognition is rational cognition from concepts, mathematical cognition that from the construction of concepts*" (A713/B741). In demarcating the two Kant is not distinguishing concepts; rather, by separating the "the discursive use of reasoning in accordance with concepts from its intuitive use through the construction of concepts" (A719/B747) Kant is distinguishing methods of cognitive reasoning. He illustrates mathematical cognition with the standard proof of Euclid's proposition that the sum of the angles of a triangle equals 180° :

Give a philosopher the concept of a triangle, and let him try to find out in his way how the sum of its angles might be related to a right angle. He has nothing but the concept of a figure enclosed by three straight lines, and in it the concept of equally many angles. Now he may reflect on this concept as long as he wants, yet he will never produce anything new. He can analyze and make distinct the concept of a straight line, or of an angle, or of the number three, but he will not come upon any other properties that do not already lie in these concepts. But now let the geometer take up this question. He begins at once to construct a triangle. Since he knows that two right angles together are exactly equal to all of the adjacent angles that can be drawn at one point on a straight line, he extends one side of his triangle, and obtains two adjacent angles that together are equal to two right ones. Now he divides the external one of these angles by drawing a line parallel to the opposite side of the triangle, and sees that here there arises an external adjacent angle which is equal

to an internal one, etc. In such a way, through a chain of inferences that is always guided by intuition, he arrives at a fully illuminating and at the same time general solution of the question. (A716-717/B744-745)

We construct the proof as follows (see Figure 1.8): Draw triangle ABC. Extend side BC by drawing a line from C to D. Draw line CE parallel to AB. Note that $\alpha = \alpha'$ and $\beta = \beta'$, so $\alpha + \beta + \gamma = 180^\circ$. Q.E.D.

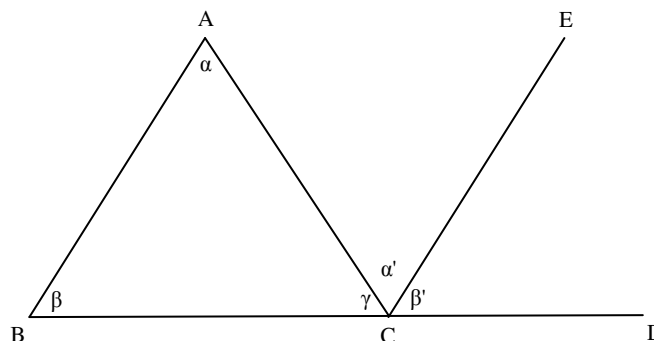


Figure 1.8: Proof of Euclid's Proposition 32

For Kant this proof is constructed by the productive imagination through a rule-governed iterative process that involves: 1) thinking a pure sensible concept (“triangle”); 2) making general inferences from the construction of a particular image (Figure 1.8); and 3) judging a synthetic a priori proposition (“the sum of the angles of a triangle equals 180° ”).

But how is this method of cognition an “objective perception” and thus empirical? Mathematical cognition is based on the presupposition that pure sensible concepts actually apply to experience:

Sensible intuition is either pure intuition (space and time) or empirical intuition of that which, through sensation, is immediately represented as real in space and time. Through determinations of the former we can acquire *a priori* cognitions of objects (in mathematics), but only as far as their form is concerned, as appearances; whether there can be things that must be intuited in this form is still left unsettled. Consequently all mathematical concepts are not by themselves cognitions, except insofar as one presupposes that there are things that can be presented to us only in accordance with the form of that pure sensible intuition. *Things in space and time*, however, are only given insofar as they are perceptions (representations accompanied with sensation), hence through empirical representation. (B146-147)

Reasoning to the synthetic a priori proposition by constructing the concept *triangle*, for example, is no cognition except insofar as one presupposes that the triangle form is synthesized in empirical apprehension. The basis of this presupposition is the vertical symmetry of imaginative synthesis:

... by means of mathematical and here indeed of geometrical construction ... I put together in a pure intuition, just as in an empirical one, the manifold that belongs to the schema of a triangle in general and thus to its concept, through which general synthetic propositions must be constructed. (A718/B747)

Of course, the vertical symmetry of imaginative synthesis does not *guarantee* that an empirical synthesis will *in fact* obtain, for the introduction of matter is entirely contingent. Indeed, I may construct pure sensible concepts that have yet to be (and may never be) encountered in any experience. That I construct a pure sensible concept of a hundred-sided figure in a geometrical proof, for example, does not entail there is such a *thing* – i.e., the figure mixed with sensation given in experience.⁶⁶ Experience must disclose whether we can regard the geometrical object as “real” (A720/B749). That is, we must *discover* not whether a pure sensible concept *can* apply to experience, but that it *actually* does.

Mathematical cognition differs from philosophical cognition in the type of inference involved in arriving at synthetic a priori propositions: mathematical cognition “considers the universal in the particular”; philosophical cognition “considers the particular only in the universal” (A714/B742). This difference turns on Kant’s distinction between form and matter of appearances:

There are thus two uses of reason, which, regardless of the universality of cognition and its *a priori* generation, which they have in common, are nevertheless very different in procedure, precisely because there are two components to the appearance through which all objects are given to us: the form of intuition (space and time), which can be cognized and determined completely *a priori*, and the matter (the physical), or the content, which signifies a something that is encountered in space and time, and which thus contains an existence and corresponds to sensation. (A723/B751)

Doing the work in mathematical cognition are pure sensible schemata that concern only the form of appearances. When the pure sensible concept is thought, a corresponding schema arises in the imagination that serves as a purposeful plan from which the productive synthesis of an image evolves. Objective perception is presupposed because

the schema *may* also serve as an end towards which the reproductive synthesis of a kind of image, i.e., kind of a temporal succession of appearances, involves.

The central role played by pure sensible schemata in mathematical cognition is clear. Kant tells us that mathematical reasoning “*make[s]* the [pure sensible] concept itself” (A730/B758) by assessing “the universal *in concreto* (in the individual intuition)” (A734/B762). The inference from a constructed image to properties of the form of the pure sensible concept is through a pure sensible schema (evident in the image) that presents the same unity of rule: “just as this individual is determined under certain general conditions of construction, the object of the concept, to which this individual corresponds only as its schema, must likewise be thought as universally determined” (A714/B742). The constructed image in Figure 1.8, for example, exhibits the schematic content of the form of the pure sensible concept *triangle*, enabling the inference of synthetic a priori propositions. In arriving at the proposition “the sum of the angles of a triangle equals 180°” a property of the form of the pure sensible concept *triangle*, and thus the object triangle, is demonstrated.⁶⁷ Since the truth of the mathematical judgement is made evident from Figure 1.8, the constructed image can be considered a *mathematical space of reasons*. As a potential kind of image, therefore, a pure sensible schema is also a potential space of reasons.

Now if, as I claim, a transcendental schema is constitutive of pure sensible schemata, two things should follow. Inasmuch as a category determines a pure sensible concept, there is no cognition without the presupposition of the latter’s applicability to experience.

The pure concepts of the understanding, consequently, even if they are applied to *a priori* intuitions (as in mathematics), provide cognition only insofar as these *a priori* intuitions, and by means of them also the concepts of the understanding, can be applied to empirical intuitions. Consequently the categories do not afford us cognition of things by means of intuition except through their possible application to *empirical intuition*. This, however, is called *experience*. The categories consequently have no other use for the cognition of things except insofar as these are taken as objects of possible experience. (B147-148)

Furthermore, because an individual transcendental schema lacks sufficient spatiotemporal structure to guide synthesis of a kind of image, derivation of a synthetic a priori proposition concerning the form of an individual category, or what Kant calls a principle

of the understanding, cannot directly involve a transcendental schema that represents a “pure synthesis”. Rather, its derivation must involve identifying properties of the corresponding schema of sensibility evident from experience in general that represents an empirical synthesis:

... if I am given the transcendental concept of a reality, substance, force, etc., it designates neither an empirical nor a pure intuition, but only the synthesis of empirical intuitions (which thus cannot be given *a priori*), and since the synthesis cannot proceed *a priori* to the intuition that corresponds to it, no determining synthetic proposition but only a principle of the synthesis of possible empirical intuitions can arise from it. A transcendental proposition is therefore synthetic rational cognition in accordance with mere concepts, and thus discursive, since through it all synthetic unity of empirical cognition first becomes possible, but no intuition is given by it *a priori*. (A722/B750)

This shift to schematic exhibition of the categories marks a decisive move from mathematical cognition to philosophical cognition.⁶⁸

In philosophical cognition one no longer assesses the universal in the particular, but rather the particular in the universal. Philosophical cognition “confines itself solely to general concepts” (A715/B743) and assesses “the universal *in abstracto* (through concepts)” (A734/B762), not in *concreto* (through construction of images). Deriving the synthetic a priori principle “all alterations occur in accordance with the law of the connection of cause and effect”, for example, one considers the schematic exhibition of the category *cause* in empirical intuition:

By means of the concept of cause I actually go beyond the empirical concept of an occurrence (that something happens) but not to the intuition that exhibits the concept of cause *in concreto*, rather to the time-conditions in general that may be found to be in accord with the concept of cause in experience. I therefore proceed merely in accordance with concepts, and cannot proceed through the construction of concepts, since the concept is a rule of the synthesis of perceptions, which are not pure intuition and which therefore cannot be *given a priori*. (A722/B750n)

To proceed “to the time-conditions in general that may be found to be in accord with the concept of cause in experience”, Kant tells us, is to “look around for some third thing” (A732-733/B761), i.e., a spatiotemporal schema of sensibility, upon which the principle is inferred.⁶⁹ One might say that in mathematical cognition properties of pure sensible schemata are *inferred*, while in philosophical cognition properties of schemata of sensibility are *recognized*.

In mathematical cognition derivation of synthetic a priori principles from pure sensible schemata involves judgement of pure intuition that is determinative (cf. A722/B750). In philosophical cognition, derivation of synthetic a priori principles from schemata of sensibility involves judgement of empirical intuition that is both determinative and reflective:

The faculty of judgement in general is the faculty of thinking the particular as contained under the universal. If the universal (the rule, the principle, the law) is given, then the faculty of judgement which subsumes the particular under it (even if, as transcendental faculty of judgement, it specifies a priori the conditions according to which alone [the particular] can be subsumed under this universal) is *determinative*. But if only the particular is given, for which the universal is to be found then the faculty of judgment is merely *reflective*. (Ak. V: 179, as quoted in Friedman (1992, 245n))

A quick reading of this passage can lead to confusion. *All* judgement of empirical intuition is reflective. When empirical intuition is determined by the understanding, judgement is *also* determinative:

With respect to the universal concepts of nature, under which an empirical concept in general (without particular empirical determinations) is first possible, reflection already has its instructions in the concept of a nature in general, and the faculty of judgement needs no particular principle of reflection, but *schematizes* these concepts a priori and applies these schemata to each empirical synthesis, without which absolutely no empirical judgement would be possible. The faculty of judgement is here in its reflection at the same time determinative, and the transcendental schematism serves here at the same time as the rule under which given empirical intuitions are subsumed. (Ak., XX, 212, as quoted in Friedman (1992, 245))

In this case judgement is not “*merely* reflective” because the category is “given” as a condition for the possibility of experience and thus is “not to be *found*” in cognition. This signals the “apodictic” use of reasoning in that “the universal is *in itself certain* and given, and only *judgment* is required for subsuming, and the particular is necessarily determined through it” (A646/B674). As we shall see, reflective and determinative judgement also occurs when pure sensible concepts apply to experience.

Contrasting the apodictic use of reasoning is the “hypothetical” or regulative use of reason discussed in the Appendix to the Transcendental Dialectic of the *Critique*. In a regulative use of reason “the universal is assumed only *problematically*”; “it is a mere idea, the particular being certain while the universality of the rule for this consequent is still a problem” (A646/B674). Since the rule is a “*logical principle*” (A648/B676), the

hypothetical use of reasoning is solely syllogistic. This use of reason involves “merely reflective” judgement: “it is only reflective judgement that uses this idea as a principle, for reflection rather than determination” (Ak. V: 180).⁷⁰ As we shall also see, regulative use of reason can be punctuated by episodes of apodictic use when reflective judgement is *augmented* by determinative judgement.

In its regulative use, reason extends the understanding beyond actual experience to all possible experience through a “purposive application” that systematically unites the manifold of the empirical concepts:

Reason never relates directly to an object, but solely to the understanding and by means of it to reason’s own empirical use, hence it does not *create* any concepts (of objects) but only *orders* them and gives them unity which they can have in their greatest possible extension, i.e., in relation to the totality of series; the understanding does not look to this totality at all, but only to the connection *through which series* of conditions always *come about* according to concepts. Thus reason really has as object only the understanding and its purposive application, and just as the understanding unites the manifold into an object through concepts, so reason on its side unites the manifold of concepts through ideas by positing a certain collective unity as the goal of understanding’s actions, which are otherwise concerned only with distributive unity. (A643-644/B671-672)

Reason effects this organization through a regulative idea that, in virtue of its form, is a schema serving as a purposeful plan and presenting a logical principle. Yet the idea (along with the schema and the logical principle) is only assumed, and so the organization “is only a *projected* unity” (A647/B675) towards a *focus imaginarius*. Without more, however, a regulative use of reason would fail to seek unity, for reason remains free to admit that the “idea ... entirely contradicts the arrangement of nature” (A651/B679).

The logical principle is thus “without sense or application” (A656/B684) unless the regulative idea is also accompanied by a “presupposition of ... an *object in the idea*” (A671/B699):

For the law of reason to seek unity is necessary, since without it we would have no reason, and without that, no coherent use of the understanding, and, lacking that, no sufficient mark of empirical truth; thus in regard to the latter we simply have to presuppose the systematic unity of nature as objectively valid and necessary. (A651/B680)

In virtue of this “transcendental presupposition”, logical principles are “synthetic propositions *a priori*” that act as “heuristic principles” (A663/B691) through which “we question nature according to these ideas, and we take our cognition to be defective as long

as it is not adequate to them” (A645/B673). Hence, it is a transcendental presupposition that correlates a focus imaginarius with the illusion of an object.

Kant sometimes refers to the regulative use of reasoning as philosophical “cognition *according to analogy*” (Ak. IV: 357), the main reason being that a schema of reason is an “analogue” of a schema of sensibility:

Yet although no schema can be found in *intuition* for the thoroughgoing systematic unity of all concepts of the understanding, an *analogue* of such a schema can and must be given, which is the idea of the *maximum* of division and unification of the understanding’s cognition in one principle. For that which is greatest and most complete may be kept determinately in mind, because all restricting conditions, which give indeterminate manifolds, are omitted. Thus the idea of reason is an analogue of a schema of sensibility, but with this difference, that the application of concepts of the understanding to the schema of reason is not likewise a cognition of the object itself (as in the application of the categories to their sensible schemata), but only a rule or principle of the systematic unity of all use of the understanding. (A665/B693)

Here Kant is drawing an analogy between the “collective unity” evident in application of an idea of reason and the “distributive unity” evident in the application of a category (noted in the passage before last paragraph). Although unity of the evidence is similar, the nature of the evidence is different.

We see this more clearly in the theory of the exhibition of a pure concept, or “hypotyposis”, found in the third *Critique*. Here Kant says a schema of reason gives rise to symbolic evidence of an idea:

Hence all intuitions supplied for a priori concepts are either *schemata* or *symbols*. Schemata contain direct, symbols indirect, exhibitions of the concept. Schematic exhibition is demonstrative. Symbolic exhibition uses an analogy (for which we use empirical intuitions as well), in which judgement performs a double function: it applies the concept to the object of a sensible intuition; and then it applies the mere rule by which it reflects on that intuition to an entirely different object, of which the former object is only the symbol. Thus a monarchy ruled according to its own constitutional laws would be represented as an animate body, but a monarchy ruled by an individual absolute will would be represented as a mere machine (such as a hand mill); but in either case the representation is only *symbolic*. For though there is no similarity between a despotic state and a hand mill, there certainly is one between the rules by which we reflect on the two and how they operate. (Ak. XX, 352)

Symbolic exhibition is analogical to schematic exhibition in that there is an agreement “in terms of the form of reflection rather than its content” (Ibid., 351). In schematic exhibition, reflective judgment determines a schema of sensibility according to a rule

presented by a transcendental schema. In symbolic exhibition, the reflective judgment analogously determines symbols of sensibility according to a rule presented by a schema of reason. But in symbolic exhibition there is “a transfer of our reflection on an object of intuition to an entirely different concept, to which perhaps no intuition can ever directly correspond” (Ibid., 353). In Kant’s example the perception of a state is organized under the ideas of rule of law or despotic rule. Application of either idea to perception is only symbolic: in the former the empirical concept *monarchy* is symbolically organized as the empirical concept *animate body*; in the latter the empirical concept *monarchy* is symbolically organized as the empirical concept *machine*.

Schemata of sensibility present synthetic mathematical and dynamical principles that, although constitutive and regulative (respectively) of intuition, are constitutive of empirical concepts in that they express conditions for their very possibility (A664/B692). Because schemata of sensibility also serve as evidence, they are involved in the cognition of an object such that “one judges in all strictness the truth of the universal rule” (A647/B675) it presents. In virtue of this “transcendental truth”⁷¹ the principle has a determinate meaning. By contrast, a schema of reason presents a logical principle regulative of empirical concepts by organizing them in virtue of a transcendental presupposition of applicability. Since it is not constitutive of empirical concepts, the principle lacks transcendental truth. In our investigations of nature, the projected unity serves as “as the standard for the empirical use of reason” (A675/B703) and thereby a “*touchstone of truth*” (A647/B675). Yet it is “not such that if one judges in all strictness the truth of the universal rule assumed as hypothesis thereby follows” (A647/B675); rather, any evidence only makes the principle more “probable” (A649/B677) at best “*approximating* the rule to universality” (A647/B675). But because it organizes empirical concepts, a logical principle has significance, though not a determinate sense (A665/B693).

A “touchstone of truth” is a euphemism for a criterion of evidence. As the form of a pure concept, a schema of reason is analogous to a schema of sensibility because of its analogy to a transcendental schema. Recall that a transcendental schema identifies a priori a criterion in relation to a distributive structure of empirical intuition – a line – as

evidence of the application of a category. By analogy, a schema of reason identifies a priori a criterion in relation to a collective structure of empirical concepts – what might be called a “space of reason” – as evidence of the application of an idea of reason. This is the occupation of a “place” within the structure:

If we survey the cognitions of our understanding in their entire range, then we find that what reason quite uniquely prescribes and seeks to bring about concerning it is the *systematic* in cognition, i.e., its interconnection based on one principle. This unity of reason always presupposes an idea, namely that of the form of a whole of cognition, which precedes the determinate cognition of the parts and contains the conditions for determining *a priori* the place of each part and its relation to the others. Accordingly, this idea postulates complete unity of the understanding’s cognition, through which the cognition comes to be not merely a contingent aggregate but a system of interconnected in accordance with necessary laws. (A645/B673)

Recall also that a transcendental schema is constitutive of a focal point; by analogy, a schema of reason is correlated to a “point” at the apex of this structure, i.e., a *focus imaginarius*, towards “which the lines of direction of all [the understanding’s] rules converge”.

However, whereas this focal point of reason’s imagination is a formless *limit* that “lies entirely outside the field of possible empirical cognition” (A644/B672), a focal point of understanding’s imagination is a spatiotemporal form that lies entirely inside the field of possible empirical cognition. Moreover, in the case of the former “there arises the deception, as if these lines of direction were shot out from an object” (A644/B672); in the latter case, the focal point *is* the spatiotemporal unity of an object of experience. And whereas a schema of reason may be associated with a non-synthetic “original image” that presents part of its principle, a transcendental schema is associated with a synthetic image that involves towards the principle it presents.

Kant allows for a variety of such spaces of reason, depending on the organizing idea. Significant for our purposes is his illustration of the organization of empirical concepts into a systematic unity of genera, species and sub-species through the interplay of three ideas and their principles: the idea *unity* contains the principle of the “homogeneity” of forms, “a principle of the *sameness of kind* in the manifold under higher genera”; the idea *manifoldness* contains the principle of the “specification” of forms, “a principle of the *variety* of what is same in kind under lower species”; and the

idea *affinity* contains the principle of the “continuity” of forms, a principle that “arises by uniting the first two”⁷² “which offers a continuous transition from every species to every other through a graduated increase of varieties” (A658/B686;A662/B690). Without these ideas, Kant tells us, philosophical cognition with empirical concepts would be impossible: without the principle of homogeneity “there could be no use of reason, because we can infer from the universal to the particular only on the ground of the universal properties of things under which the particular properties stand” (A652/B680); “if there were no lower concepts, then there would also be no higher ones” (A656/B684), and so the principle of specification is necessary; and without the principle of continuity, derivations of the particular from the universal could not “go much further than experience can reach” (A662/B690).

Together the three ideas organize the classification of “individual things” (A651/B679) based on their matter or content (A578/B606) – not their form. To help us grasp the systematic unity they bring about, Kant appeals to the structure of an organizing space of reason:

Systematic unity under the three logical principles can be made palpable in the following way. One can regard every concept as a point, which, as the standpoint of an observer, has its horizon, i.e., a multiplicity of things that can be represented and surveyed, as it were, from it. Within this horizon a multiplicity of points must be able to be given to infinity, each of which in turn has a narrower field of view; i.e., every species contains subspecies in accordance with the principle of specification, and the logical horizon consists only of smaller horizons (subspecies), but not of points that have no domain (individuals). But different horizons, i.e., genera, which are determined from just as many concepts, one can think as drawn out under a common horizon, which one can survey collectively from its middle point, which is the higher genus, until finally the highest genus is the universal and true horizon, determined from the standpoint of the highest concept and comprehending all manifoldness, as genera, species, and subspecies, under itself. (A658-659/B686-687)

It seems⁷³ that the *focus imaginarius* of this classification space correlates to the regulative idea *God* and illusory “*ens realissimum*” (“most real being”) (A576/B605). The form of the idea is a schema⁷⁴ presenting the “principle of *thoroughgoing determination*” (i.e., the principle that one of each pair of all possible contradictory predicates applies to each thing) (A572/B600) through a disjunctive syllogism:

The logical determination of a concept through reason rests on disjunctive syllogism, in which the major premise contains a logical division (a division of the sphere of a general

concept), the minor premise restricts this sphere to one part, and the conclusion determines the concept through this part. (A576-577/B604-605)

Kant seems to have in mind the following:

Major premise: All A's are either F or G.
Minor premise: Some A's are not G.
Conclusion: Therefore some A's are F.

where "A" is a genus (species of sub-species), "F" and "G" are contradictory predicates, and the conclusion determines a species (or sub-species) of "A". What makes this logical outline a form of the regulative idea *God* is that serving as the "transcendental major premise" (A577/B605) is an original image,⁷⁵ a universal representing "the sum total of all reality" (A577/B605) and serving (as we shall see) as an ideal for scientific discovery.

From the regulative idea *God* a cascading lattice of concepts and correlated points "can be derived only by repeated determination" (A652/B680) via disjunctive attribution of one of two contradictorily opposed predicates. For example:

Major premise: All absorbent earths are either chalky or muriatic.
Minor premise: Some absorbent earths are not muriatic.
Conclusion: Therefore, some absorbent earths are chalky.

Note that 1) the syllogistic inference *instantiates* the logical schema; 2) the major premise represents a *kind* of empirical image organized under the original image; and 3) the conclusion represents a kind of empirical image that is, in turn, represented by major premises of other determinations. This space of reason is a space of *disjunctive reasons*, i.e., a framework of disjunctive inferential relationships that justifies certain empirical propositions. Although the idea *God* is related to a thoroughly determined individual (*ens realissimum*) (A576/B604), "since each species is always a concept that contains within itself only what is common to different things, this concept cannot be thoroughly determined, hence it cannot be related to an individual" (A655-656/B683-684).^{76 77} For this reason the space must be continuous in Kant's sense that "a multiplicity of points must be able to be given to infinity". Yet the space is only *potentially* infinite⁷⁸ and actual places occupied in the space are discrete.⁷⁹

If too rigid a distinction is made between philosophical and mathematical cognition, one can lose sight of how the potential classification space, a potential space of

disjunctive reasons, connects with potential spaces of mathematical reasons. Kant is not oblivious to the fact that philosophical cognition makes use of mathematical cognition as a tool “to advance the essential ends of human reason” (A839/B867). Accordingly, “the regulative unity of experience rests not on appearances themselves (of sensibility alone), but on the connection of its manifold by *understanding* (in one apperception) (A583/B661). The structure of the classification space is moulded by determinations of the understanding that cuts the *form* of appearances into kinds through the application of a spatiotemporal principle before reason further cuts the *matter* of appearances into kinds through application of a logical principle. The necessity of the space for philosophical cognition is thus motivated by the content and *not* the form of appearances.⁸⁰

This is easy to overlook, since it is through reason that we *discover* empirical kinds, including spatiotemporal ones determined a priori. Kant provides an important example:

If we transpose the principles we have adduced, so as to put them in an order which *accords with their experiential* use, then the principles of systematic *unity* would stand something like this: *manifoldness, affinity, unity*, each taken, however, as idea in the highest degree of their completeness. Reason *presupposes those cognitions of the understanding which are first applied to experience* [emphasis mine] and seeks the unity of these cognitions in accordance with ideas that go much further than experience can reach. The affinity of the manifold, without detriment to its variety, under the principle of unity, concerns not merely the things, but even more the mere properties and powers of things. Hence if, e.g., the course of the planets is given to us as circular through a (still not fully corrected) experience, and we find variations, then we suppose these variations to consist in an orbit that can deviate from the circle through each of an infinity of intermediate degrees according to constant laws; i.e., we suppose that the movements of the planets that are not a circle will more or less approximate to its properties, and then we come upon the ellipse. The comets show an even greater variety in their paths, since (and far as observation reaches) they do not ever return in a circle; yet we guess at a parabolic course for them, since it is still akin to the ellipse and, if the major axis of the latter is very long, it cannot be distinguished from it in all our observations. Thus under the guidance of those principles we come to a unity of genera in the forms of these paths, but thereby also further to unity in the cause of all the laws of this motion (gravitation); from there we extend our conquests, seeking to explain all variations and apparent deviations from those rules on the basis of the same principle; finally we even add on more than experience can ever confirm, namely in accordance with the rules of affinity, even conceiving hyperbolic paths for comets in which these bodies leave our solar system entirely and, going from sun to sun, unite in their course the most remote parts of a world system, which for us is unbounded yet connected through one and the same moving force. (A662-663/B690-691)

The passage becomes perspicuous if we bear in mind the observational characteristics of orbits and their geometrical relation to one another:

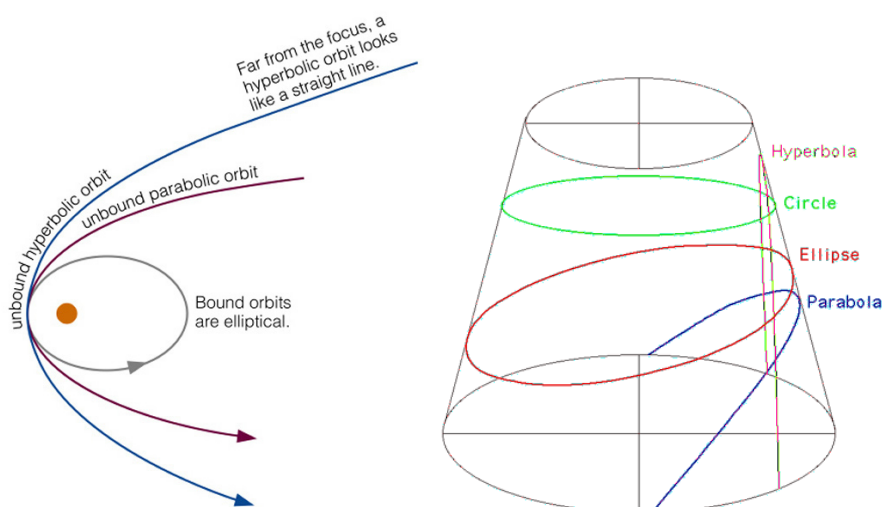


Figure 1.9: Astronomical Orbits and their Geometrical Relationship⁸¹

The passage illustrates how reason takes us beyond actual experience to all possible experience by facilitating discovery of the understanding's determination of spatiotemporal kinds through pure sensible concepts. Kant ascends the space of classification from our cognition of actual appearances (astronomical orbits around the sun) to a unifying causal genus (the empirical concept⁸² *gravity*) then descends to a species of possible (non-actual) experience (the empirical concept *hyperbolic orbit*). Doing the work is judgment, “a mediating link between understanding and reason” (Ak. V: 177), where reflective judgement in the use of reason can also be determinative judgement in the use of understanding.

More specifically, beginning with actual perceptions of circular orbits around the sun and simple laws that seem to govern them, we seek to unify these orbits under a more general empirical law in virtue of the ideas of *manifoldness*, *affinity* and *unity* organized under the idea *God*. Following the principle of specification, reason groups perceived orbits into two species: observationally bound orbits (planets) and observationally unbound orbits (comets). Following the same principle, reason specifies sub-species of bound orbits: either an observationally bound orbit is a circle or it is not, and those that are not circles we “suppose” are grouped as ellipses. Returning to the species of unbound orbits, two sub-species are further specified: either an observationally unbound orbit is parabolic or not. Again, we “guess” they are grouped as parabola. But what grounds these two suppositions?

There are two things: 1) “[r]eason presupposes those cognitions of the understanding which are first applied to experience”; and 2) in the case of the ellipse “we suppose ... an orbit that can deviate from the circle through each of an infinity of intermediate degrees according to constant laws” and we guess a parabola because “it is still akin to the ellipse and, if the major axis of the latter is very long, it cannot be distinguished from it in all our observations”. On the first ground reason is subject to the understanding that divides the shape of appearances into spatiotemporal kinds. Accordingly, inasmuch as the empirical concept *circular orbit* is determined by the pure sensible concept *circle*, we suppose that orbits that are not circles are nonetheless determined by pure sensible concepts. On the second ground, the principle of affinity dictates that the pure sensible concepts that determine the sub-species of orbits deviate minimally from one another. Hence we arrive at the empirical concept *elliptical orbits* and the empirical concept *parabolic orbit*.

From these orbits actually experienced, we “come to the unity of genera in the forms of these paths” and “thereby further” to the empirical concept *gravity*. Although Kant does not supply the details of these two moves, we can turn to the *Critique of Judgment* (1790-1793/1987)⁸³ and the *Prolegomena* to fill them in. Now Kant describes ascensions generally as a process of abstracting from the determination of empirical concepts (A656/B684), which involves removing a pair of contradictory predicates. In the first ascension this would involve removing the predicates “bound” and “unbound”. In doing so, the orbits are unified under the empirical concept *conic orbits*. That we arrive at this genus is through an earlier *mathematical discovery* of the organization of these curves by geometers who, “unwittingly working for posterity”, constructed the pure sensible concept *cone* (Ak. V: 363).

Organization of the orbits further under the empirical concept *gravity* is less obvious and more complex. The form of the empirical concept *gravity* presents the empirical law of gravity. According to this law, attraction between two bodies is proportional to their masses and inversely proportional to the square of their distance. Proportionality to mass alone makes the law empirical, and thus to be discovered. Kant took the view that some laws, such as those of chemistry, are “merely empirical” in that

they “are not receptive to the application of mathematics”, but only because there is “*no concept to be discovered that can be constructed*” (italics added, Ak. IV: 471). In relation to the inverse square law of the law of gravity, however, §38 of the *Prolegomena* tells us there *is* a concept to be discovered that can be constructed: the pure sensible concept *sphere*. Determination by a pure sensible concept needs to be discovered through reason not only because we are unaware whether it actually applies to experience, but also because we cannot grasp a priori how it may do so.⁸⁴

Kant tells us that under the guidance of the principles of manifoldness, affinity and unity we come to “unity in the cause of all the laws of this motion”. By “laws of motion” he is likely alluding to Kepler’s law of areas: if a body is observed to move in a circle, ellipse or parabola in a way that satisfies this law with respect to the focus of the orbit, we observe that the body is accelerated to that focus inversely proportional to the square of the distance from it. Kepler’s laws are independent of mass and concern only the kinematics of relative motion,⁸⁵ and for Kant they likely have the status of inductive generalizations of orbital motion.⁸⁶ Now ascension *from* the empirical concept *conical orbits* seems to be in virtue of reflective judgement that also determines the schema of sensibility evident in the acceleration of orbits and subsumes them under the category *cause*. In ascending *to* the empirical concept *gravity* judgement appears to have in hand a comparison of the intensive magnitude of other perceptions, for “the degree of reality as cause” is called “the moment of gravity” (A168/B210).⁸⁷ Here reflective judgement that also determines the shape of the orbits subsumes them under the pure sensible concept *sphere*, for the inverse square relation rests “merely on the relation of spherical surfaces with different radii” (Ak. IV: 321).

Having organized conical orbits under the empirical concept *gravity* as the “power of things”, we are in a position to extend understanding to all possible experience. If we “descend” to the empirical concept *conic orbit*, we know from previous mathematical work that there remains a sub-species of unbound orbits not actually experienced past, present or future determined by the pure sensible concept *hyperbola* and captured by the empirical concept *hyperbolic orbits*. It seems at this point reflective judgement

speculates on the mere possibility (“more than experience can ever confirm”) of inter-solar orbits of comets.

This reconstruction is thrown into relief when we examine §38 of the *Prolegomena* in detail. Taking the empirical law of gravity as exemplar, it tries to show

... that laws which we *discover* in objects of sensory intuition, especially if these laws have been cognized as necessary, are already held by us to be such as have been put there *by the understanding*, although they are otherwise in all respects like the laws of nature that we attribute to experience. (italics added, Ak. IV: 320)

More specifically, “nature rests on laws that the understanding cognizes *a priori*, and indeed chiefly from principles of the determination of space” for “that which determines space into the figure of a circle, a cone, or a sphere is the understanding, insofar as it contains the basis for the unity of the construction of these figures” (Ak., IV: 321-322). But rather than illustrate this through the use of reason in the classification of empirical concepts, here Kant works strictly with pure sensible concepts, providing a classification structure of astronomical orbits through the progressive generalization of schematic properties.

In the first paragraph Kant begins with a universal property of a circle:

If one considers the properties of the circle by which this figure unifies in a universal rule at once so many determinations of the space within it, one cannot refrain from ascribing a nature to this geometrical thing. Thus in particular, two lines that intersect each other and also the circle, however they happen to be drawn, nonetheless always partition each other in a regular manner such that the rectangle from the parts of one line is equal to that from the other.⁸⁸ Now I ask: “Does this law lie in the circle, or does it lie in the understanding?” i.e., does this figure, independent of the understanding, contain the basis for this law in itself, or does the understanding, since it has itself constructed the figure in accordance with its concepts (namely, the equality of the radii), at the same time insert into it the law that chords cut one another in geometrical proportion? If one traces the proofs of this law, one soon sees that it can be derived only from the condition on which the understanding based the construction of this figure, namely, the equality of radii. (Ak. IV: 320-321)

By “geometrical thing” Kant is alluding to the fact that because a circle’s universal properties are exhibited in experience, a circle is “immediately represented as real in space and time”. By “nature” he means “the first inner principle of all that belongs to the *existence* of a thing” (Ak: IV: 467) as well as that which “signifies the connection of determinations of a thing in accordance with an inner principle of causality” (A418/B446n). It is a mistake to interpret “causality” mechanically in terms of universal

laws⁸⁹ when Kant is clear in the third *Critique* that in this context he only means “causality in terms of purposes” (Ak. V: 359):

... suppose two lines are to intersect so that the rectangle under the two parts of the one line will be equal to the rectangle under the two parts of the other: solving this problem seems very difficult; but in fact all lines that intersect within a circle and are bounded by its circumference divide automatically in this proportion. The other curves in turn provide us with other purposive solutions that we did not think of at all in the rule for their construction. (Ibid., 362-363)

His first point then is that when we consider universal properties of a circle, they *seem* to be derived from the *circle's* inner purpose. But they originate in the understanding, for “when I draw a figure *in accordance with a concept*, I introduce the *purposiveness* into the figure” (Ibid., 365). I introduce this purposiveness when, beginning with an outline or “plan” in the imagination, I construct a circle whose schematic properties (e.g., the equality of radii) are attributable to the form of the pure sensible concept *circle*.⁹⁰

Following the same path as the Appendix, the paragraph then shifts from a circle to other conic sections:

If we now expand upon this concept so as to follow up still further the unity of the manifold properties of geometrical figures under common laws, and we consider the circle as a conic section, which is therefore subject to the very same fundamental conditions of construction as other conic sections, we then find that all chords that intersect within these latter (within the ellipse, the parabola, and the hyperbola) always do so in such a way that the rectangles from their parts are not indeed equal, but always stand to one another in equal proportions.⁹¹ (Ak. IV: 321)

Because it builds upon the first passage, we should take Kant to be referring to elliptical, parabolic and hyperbolic conic sections – indeed the cone figure itself – as “geometrical things”. The claim implied is that when we consider universal properties of non-circular conic sections, we realize they originate in the understanding, for their proof arises from the a priori constructions of conic sections displays schematic properties attributable to the form of pure sensible concepts, most generally to the pure sensible concept *cone*.

The final part of the paragraph marks a generalization from geometrical things in *actual* experience to *all possible* experience through the pure sensible concept *sphere*:

If from there we go still further, namely to the fundamental doctrines of physical astronomy, there appears a physical law of reciprocal attraction, extending to all *material* nature, the rule of which is that these attractions decrease inversely with the square of the

distance from each point of attraction, exactly as the spherical surfaces into which this force spreads itself increase, something that seems to reside as necessary in the nature of the things themselves and which therefore is customarily presented as cognizable *a priori*. As simple as are the sources of this law – in that they rest merely on the relation of spherical surfaces with different radii – the consequence therefrom is nonetheless so excellent with respect to the variety and regularity of its agreement that not only does it follow that *all possible* orbits of the celestial bodies are conic sections, but also that their mutual relations are such that no other law of attraction save that of the inverse square of the distances can be conceived as suitable for a system of the world. (italics added, 4: 321)

Here it becomes clear we are to understand previous conic sections as “geometrical things” in virtue of the orbits of celestial bodies given in experience. These orbits are observed to obey the inverse square law, and if one considers this universal property of celestial bodies orbiting the sun, one cannot refrain from ascribing an inner principle of causality to the things. And since the reciprocal attraction of bodies extends to “material nature”, i.e., “the whole of all appearances” (Ak. IV: 467), the same can be said of all bodies. Yet, the inverse square law does not originate in things but in the understanding, for it is based on the *a priori* constructions of spheres of different radii, which rests on schematic properties attributable to the form of the pure sensible concept *sphere*. Thus some properties of the empirical concept *gravity* are features of the schema of the pure sensible concept *sphere* that can be cognized through a mathematical space of reasons.

In summary, reason organizes experience under a schema in harmony with schemata of categories and pure sensible concepts. These schemata underwrite a logical *and* mathematical heuristic that guides judgment to discover empirical laws. The force of this heuristic arises from an imaginative structure, a potential classificatory space of points converging towards a *focus imaginarius*, some of which are themselves focal points of the synthetic imagination. In its relation to an image, each focal point has the structure of a *spherical convex mirror*. See Figure 1.10:

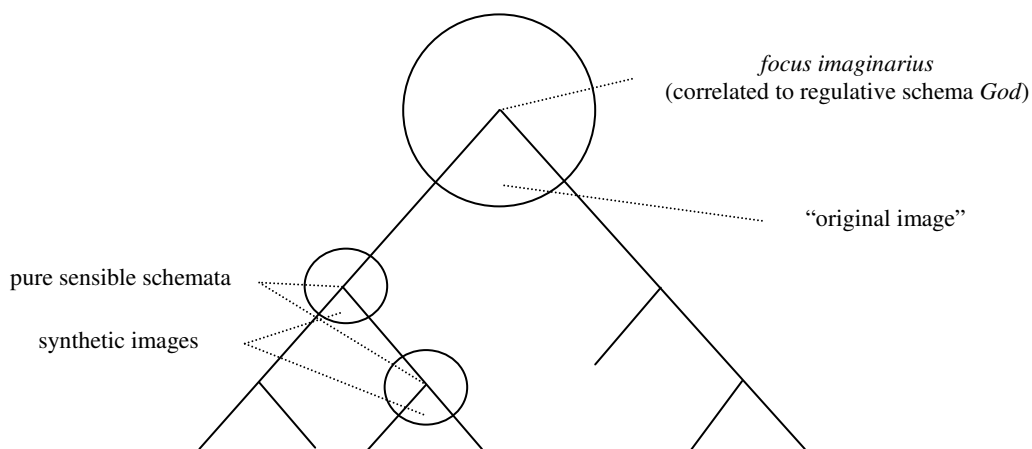


Figure 1.10 Kant's Logical Space

The *substructure* constituted by focal points of the synthetic imagination is logico-mathematical structure that lies at the heart of Kant's theory of cognition. Not only is it a potential logico-mathematical space of reasons, it circumscribes a form of all possible spatiotemporal experience. And because, as we shall see next, its particular structure appears to be due to Kant's conception of the self, it is a potential *logical space* (in Wittgenstein's sense of the term).

1.7 The Self

Kant distinguishes between two senses of 'I' from the nature of self-consciousness: "the I that I think [differs] from the I that intuits itself (for I can represent other kinds of intuition as at least possible)" (B155). The former is the transcendental I,

... the simple and in content for itself wholly empty representation *I*, of which one cannot even say that it is a concept, but a mere consciousness that accompanies every concept. Through this I, or He, or It (the thing), which thinks, nothing further is represented than a transcendental subject of thoughts = *x*, which is recognized only through the thoughts that are its predicates, and about which, in abstraction, we can never have even the least concept; because of which we therefore turn in a constant circle, since we must always already avail ourselves of the representation of it at all times in order to judge anything about it; we cannot separate ourselves from this inconvenience, because the consciousness in itself is not even a representation distinguishing of a particular object, but rather a form of representation in general, insofar as it is to be called a cognition, for of it alone can I say that through it I think anything. (A345-346/B404)

The transcendental I is a *limit* of the intellect: it is “that which remains after all accidents (as predicates) have been removed” (Ak. IV: 333) from the inner sense. As a limit “it is nothing more than a feeling of an existence without the least concept, and is only a representation of that to which all thinking stands in relation” (Ak. IV: 334 n.).

Schopenhauer understands Kant’s notion of the transcendental self in the following way:

If we summarize Kant’s utterances, we shall find that what he understands by the synthetic unity of apperception is, so to speak, the extensionless centre of the sphere of all our representations, whose radii converge on it. It is what I call the subject of knowing, the correlative of all representations. (1966, vol. I, 451-452)

Criticizing it, Schopenhauer asserts that it confers upon the intellect the unity of consciousness “at most only as much as a convex mirror has, whose rays converge at an imaginary point behind its surface” (1966, vol II, 139-140). In saying this he seems to be influenced by an understanding of the relation between the transcendental I and the second sense of self – the psychological I. In order to obtain “a systematic unity of all the appearances of the inner sense” reason “makes out” “a concept of reason (an idea)” “of a simple self-sufficient intelligence” (A682/B710). Although this “psychological idea” signifies “nothing other than the schema of a regulative concept” (A684/B712), the transcendental presupposition of an object in the idea correlates an illusory “soul” (A342/B400) with its *focus imaginarius*. Since “we intuit ourselves only as we are internally affected *by our selves*” (B166), Schopenhauer likely sees the *focus imaginarius* as brought about by the limit that is the transcendental *I*, prompting him to characterize the latter the way he does. Add to this Kant’s claim that the procedure of the understanding with pure sensible schemata is “a hidden art in the depths of the human *soul*” (italics added, A141/B180), the transcendental *I* might then be taken to affect a general structure of the imagination.

Indeed, what this chapter has shown is that a literal interpretation of Kant’s schematism is to be had by patterning the structure of understanding’s imagination upon the structure of reason’s imagination. Schopenhauer’s insight suggests that as the “form of representation in general”, the transcendental I *induces* a general structure of the imagination similar to that of a convex mirror. It is through this structure that the

imagination accommodates the formal proposition “I think”, i.e., “the form of apperception, which belongs to and precedes every experience” (A354) that must be able to accompany imaginative representation (B132). Moreover, it is through this structure that focal points arise in virtue of which regulative ideas, categories and pure sensible concepts apply to reality in the synthesis and organization of empirical images.

1.8 Conclusion: The Birth of Logical Space

Kant’s pre-critical notion of a schema was God’s plan for a structure of the universe that executes a divine idea as God imagines. In the *Critique* the divine schema is internalized in human cognition as regulative schemata along with transcendental and pure sensible schemata that execute pure concepts as the transcendental self imagines. Although fragmented, critical schemata are unified through a general structure of the imagination patterned on a spherical convex mirror. Through this structure a potential classification space of empirical concepts is organized under the regulative idea *God* where a subset of places converging towards the *focus imaginarius* are themselves focal points of spatiotemporal syntheses determined by the categories and pure sensible concepts.

This is Wittgenstein’s idea of logical space in its transcendental form: a logico-mathematical structure circumscribing a form of all possible worlds through which the self applies imaginative representations to reality. But in Kant’s case the space is only potential, the logic monadic and the mathematics limited to Newtonian space-time. The development of polyadic logic and the geometric treatment of mechanics that will give rise to the idea of logical space we see in Wittgenstein.

Chapter 2: From Schema to Model

For better and worse, almost every philosophical development of significance since 1800 has been a response to Kant.

Coffa (1991, 7)

2.1 Introduction

This chapter explores developments in the logical organization of images that occurred in 19th century philosophy of science leading from Kant to Wittgenstein's idea of logical space. These developments transpired in the philosophical systems of Helmholtz and Hertz, offering a glimpse into how these working scientists understood the relation between imaginative representation and reality.

This chapter is divided into two main parts. The first is a general reconstruction of Helmholtz's philosophical system that appeals to the architectonic of Kant's *Critique of Pure Reason* (1998) to articulate the implications of Helmholtz's adaptation of causality as a regulative idea. It demonstrates that Helmholtz possessed remarkable depth in his understanding of Kant, knowing full well the repercussions of substituting a transcendental schema for a regulative one in his notion of an image, its organization, and its relation to reality. The second part presents a novel exegesis of Hertz's *Principles of Mechanics* (1900/1956) that lays bare his basic move of replacing the original image associated with the major premise of a regulative idea with a scientific image associated with the logical schema of a set of fundamental ideas connected by a fundamental law. It brings to light Hertz's incorporation of mathematical developments in geometry and physics that capture his scientific image as an idealized multi-dimensional coordinate system. I shall argue that Hertz's suggestion that this mathematical form is shared with reality is the likely precursor to Wittgenstein's idea of logical space.

2.2 Helmholtz

The emergence of non-Euclidean geometries in the 19th century had a profound effect on philosophy of science. Helmholtz was among a number of philosophers who

came to reject Kant's view of the intuitive construction of pure sensible concepts and thus the expression of their schematic properties in synthetic a priori propositions. What Kant failed to realize, Helmholtz surmised, was that in "Euclid's method of constructive intuition" we have "involuntarily and unconsciously drawn in some very general results of experience, which the power of executing certain parts of the operation has already taught us practically" (1870/1971, 247). Although Helmholtz abandoned pure intuition in order to return to the empiricist tradition Kant had rejected, he nonetheless allowed Kant to dominate his philosophical and scientific thought:

I was faithful to Kant in the beginning of my career as I am now; or rather, I believed then that that which I wished to see altered in Kant was an insignificant side issue which, next to that which I still esteem to be his chief result, need not come up for consideration. ((*Vortäge und Reden*, 1884, I, vii) translated by Kahl (1971 xxii, n.9))

Kant's chief result, Helmholtz tells us in his well-known epistemological essay, *The Facts of Perception* (1878/1971a), was that "[t]he law of causality is in reality a transcendental law, a law which is given a priori" (390). Helmholtz surmised, however, that it is not a constitutive principle but a regulative one, conceiving the form of the pure concept *causality* as a regulative schema organizing a potential space of all images.

2.21 Physiological Forms of Intuition

Helmholtz adopts Kant's general position that philosophy and science grapple with common epistemological questions from different points of view: "[t]he laws of thought, after all, are the same for the scientist as for the philosopher" (1878/1971a, 369). As Hatfield (1990) points out, however, "Helmholtz was a scientist first and a philosopher second" (165). His scientific investigations in physiological acoustics and optics led him to reconceive Kant's epistemology around the psychology of perception and a physiology of empirical intuition. His starting point was Johannes Müller's principle of specific energies of nerves found in his *Treatise of Human Physiology* (1838-1840). Kahl (1971) summarizes this principle as follows:

Fundamental to this principle is the assumption that all we know about the external world enters our consciousness as the result of external causes. These causes affect the sense organs, producing excitations which are transmitted by the nerves to the brain. It is in the

brain that they first become conscious sensations, and there they are combined to produce perceptions of the objects around us. (xxvi)

Building upon this principle, Helmholtz makes a distinction between differences in “quality” of sensations (e.g., red, warm, sweet) that belong to a particular sense and differences in the “modality” of the five senses (1878/1971, 368-369). Following Fichte, each sense is said to define an exclusive “circle of quality” where qualities of one circle cannot transition or be compared to those of another. Circles of quality are accounted for on the basis of different types of nerves: “Every sensory nerve ... when excited by even the most varied stimuli, produces sensation only within its own specific circle of quality” (Ibid., 370). A circle of quality accounts for structure through the make-up of sensory nerves, such as that of colours in the combinations of red, green and blue-violet sensations and the patterns of consonance and dissonance that result from tone sensations that lie close together. For this reason Helmholtz says that “our physiological makeup incorporates a pure form of intuition insofar as the qualities of sensation are concerned” (Ibid., 373).

It seems that a collection of nerves from various circles of quality (e.g. sight and touch) give rise to a version of Kant’s forms of temporal and spatial intuition. The “ordering in the time sequence is the inescapable form of our inner intuition” (Helmholtz 1887/1971, 441); “[s]pace is an a priori form of intuition, necessarily prior to all experience, insofar as the perception of it is related to the possibility of motor volitions, the mental and physical capacity for which must be provided by our physiological make-up before we can have intuitions of space” (Helmholtz 1878/1971, 374). The former accounts for the direction of the natural number series⁹² and (presumably) for the one-dimensionality of time, while the latter accounts for the three-dimensionality of space.⁹³

As is well-known, Helmholtz argued that the necessity of three-dimensional spatial experience did not necessitate Euclidean space:

Our eyes see everything in the field of vision as a number of colored plane surfaces. That is their form of intuition. However, the particular colors that appear at any one time, the relationships among them, and the order in which they appear are the effects of external causes and are not determined by any law of our organization. Equally, the fact that space is a form of intuition implies just as little concerning the facts which are expressed by the axioms. (Helmholtz 1878/1971, 378)

He claimed that axioms of Euclidean geometry (1870/1971) and axioms of arithmetic (1887/1971) are discovered from regular sense impressions.⁹⁴ In the case of geometry, the specific structure of Euclidean space arises from perceptions that objects of finite shape can be moved without changing shape (free-mobility) and that the curvature of space is indiscernible.⁹⁵ In the case of arithmetic, repetitive counting gives rise to the regularity of the natural number series. Hence, the axioms of geometry and those of arithmetic “are not propositions given a priori, but propositions that must be confirmed or refuted by experience” (Helmholtz 1887/1971, 437).

2.22 Intuitive Images in a Theory of Representation

Helmholtz denied any relation of similarity between sensations and their causes on the basis that sensations are “signs” not “images” of external objects:

Our sensations are simply effects which are produced in our organs by objective causes; precisely how these effects manifest themselves depends principally and in essence upon the type of apparatus that reacts to the objective causes. What information, then, can the qualities of such sensations give us about the characteristics of the external causes and influences which produce them? Only this: our sensations are signs, not images, of such characteristics. One expects an image to be similar in some respect to the object of which it is an image; in a statue one expects similarity of form, in a drawing similarity of perspective, in a painting similarity of color. A sign, however, need not be similar in any way to that of which it is a sign. The sole relationship between them is that the same object, appearing under the same conditions, must evoke the same sign; thus different signs always signify different causes or influences. (Helmholtz 1878/1971, 372)

Unlike Kant, Helmholtz did not consider sensations to be merely matter for representational form. He appears to have adopted a two-world interpretation of the *Critique*, taking sensations to be effects of things-in-themselves conceived as objective causes, signals⁹⁶ that transmit information from particularized external sources. At the same time Helmholtz denied sensations are “images” because they lack the similarity relation we find in any *graven image*,⁹⁷ such as structural, perspectival or colour similarity. His implicit claim is that the notion of an “image” as a representation is to be understood in the context of a theory of representation that takes the relation between a representation and what is represented as similarity in some respect.

Just as Kant distinguishes between “form of intuition” and “formal intuition”, Helmholtz distinguishes between “form of intuition” and “intuition”. But for Kant, recall, formal intuition is a unity of the manifold of intuition determined by pure concepts. For Helmholtz, on the other hand, “intuition” is a posteriori, designating “the occurrence of ideas wherein only a sense impression is experienced, the idea of an object coming subsequently to consciousness without the mediation of any further conceptualization” (1894/1971b, 506). Sense impressions first give rise to a non-reflective and non-conceptual “idea” called “perception” (Ibid., 505-506). Through their impression into memory, regular perceptions give rise to an “intuitive image” beneath the surface of consciousness:

Among the traces which frequently repeated perceptions leave behind in the memory, the ones conforming to law and repeated with the greatest regularity are strengthened. In a receptive, attentive observer, intuitive images of the characteristic aspects of the things that interest him come to exist; afterwards he knows no more about how these images arose than a child knows about the examples from which he learned the meanings of words. (Helmholtz 1878/1971, 380)

An intuitive image is a relation among different kinds of signs, serving as the major premise of a pre-conceptual “inductive” (Helmholtz 1894/1971b, 508) inference:

In some of my earlier works I called the connections of ideas which take place in these processes *unconscious inferences*. These inferences are unconscious insofar as their major premise is not necessarily expressed in the form of a proposition; it is formed from a series of experiences whose individual members have entered consciousness only in the form of sense impressions which have long since disappeared from memory. Some fresh sense impression forms the minor premise, to which the rule impressed upon us by previous observations is applied. (1878/1971, 381)

This inference or intuition is a “fact of perception”⁹⁸ in virtue of which we are said to have pre-conceptual knowledge.⁹⁹

Generally speaking, Leroux (2001, 192) is right to say that Helmholtz follows Kant in identifying an empirical image with a lawful temporal succession. But this claim is apt to mislead. For Kant the temporal succession is lawful because empirical synthesis of an image is governed by an a priori rule of association of perceptions (1781-1787/1998, A100-102). In virtue of this rule, the *synthesis* has unity. I have argued, moreover, that in the case of figurative images (at least) it has the unity of a kind of image. For Helmholtz the temporal succession is lawful because (as we shall see) the

empirical synthesis of an image arises from a repeated association of distinct kinds of perceptions *believed* to be repeatable. In virtue of this belief, the *image* has unity in relating kinds of perceptions. Characterized by definite properties, an image is a structure of perceptions that arises from the interaction among empirical, physiological and mental elements and functions as an a posteriori rule of association of these perceptions. Helmholtz's theory of representation adds that as an adequate representation, an empirical image is isomorphic to a structure of causes:¹⁰⁰

To popular opinion, which accepts on faith and trust the complete veridicality of the images which our senses apparently furnish of external objects, this relationship [between sensations and objective causes] may seem very insignificant. In truth it is not, for with it something of the greatest importance can be accomplished: picturing¹⁰¹ the lawful regularities in the processes of the external world. All natural laws assert that from initial conditions which are the same in some specific way, there always follow consequences which are the same in some other specific way. If the same kinds of things in the world of experience are indicated by the same signs, then the lawful succession of equal effects from equal causes will be related to a similar regular succession in the realm of our sensations. If, for example, some kind of berry in ripening forms a red pigment and sugar at the same time, we shall always find a red color and a sweet taste together in our sensations of berries of this kind. (Helmholtz 1878/1971, 372)

He is more explicit about this in his earlier *Handbook of Physiological Optics* (1857-67): “the representations of the external world are images of the lawlike succession of natural events” (446, translated in (Schiemann 1998, 27)).

A key difference between Kant and Helmholtz concerns *when* images are organized. For Kant empirical images are synthesized from individual perceptions and then *further* organized under the regulative idea *God* into kinds of images *described* by the major premise of a conscious inference. For Helmholtz empirical images are formed *already* organized as a relation among kinds of perceptions under the regulative idea *causality*, and *function* as a major premise of an unconscious inference (e.g., “All red strawberries are sweet”).

2.23 Law of Causality as Regulative Principle

Having adopted Müller's principle, Helmholtz always adhered to a version of causal realism, but moderated it over time. Initially he presented a “hidden-causes

realism”, the view that changes are caused by a material world hidden behind the phenomena (Heidelberger 1998, 10-11). Images were said to be true if they enable us to predict the results of our actions:

Every image is the image of a thing merely for him who knows how to read it, and who is enabled by the aid of the image to form an idea of the thing. Every image is similar to its object in one respect, and dissimilar in all others, whether it be a painting, a statue, the musical or dramatic representation of a mental mood, etc. Thus the ideas of the external world are images of the regular sequence of natural events, and if they are formed correctly according to the laws of our thinking, and we are able by our actions to translate them back into reality again, the ideas we have are also the *only true* ones for our mental capacity. All others would be false. ((1857-67), as translated by Kahl (1971, xxxiv))

Later Helmholtz adopts the more sophisticated view that it is a “metaphysical hypothesis” that “the material world about us exists independently of our ideas”, no more than “an exceedingly useful and practical hypothesis” (1878/1971, 385-386). Although he found it to be highly improbable, he could “not see how a system of even the most extreme subjective idealism, even one which treats life as a dream, can be refuted” (Ibid.).

As other commentators have noted, this moderation in Helmholtz’s causal realism coincides with a refinement in his view of the principle of causality. Initially the principle was an a priori law of thought on par with the principle of sufficient reason (Hatfield 1990, 211). Later it developed into a regulative principle, “nothing but the presupposition of the lawful regularity or uniformity in all natural phenomena” (1881 appendix to 1847/1971, 49). In failing to appreciate Helmholtz’s understanding of Kant, however, it seems many commentators fail to appreciate the implications of adopting this view of the principle of causality.

It has been claimed, for example, that Helmholtz’s principle of causality is “constitutive of the real” (Hatfield 1990, 211) or “constitutive of [the] relationship of [sensations] to an external world” (Friedman 1997, 33). Whatever these claims may mean, they overlook the significance of Müller’s principle. Helmholtz tells us that “[a]ny epistemology which is based on the physiology of the senses must advise man to proceed to action in order to take possession of reality” (as quoted in Heidelberger (1998, 13)). More specifically,

[e]ach movement we make by which we alter the appearance of objects should be thought of as an experiment designed to test whether we have understood correctly the invariant relations of the phenomena before us, that is, their existence in definite spatial relations. (Helmholtz 1878/1971, 384)

If we must always test invariant appearances for illusion, then obviously Müller's principle implies a rejection of a constitutive view of the real or of our relation to the real.

Recall that for Kant a regulative principle is assumed "problematically". Helmholtz recognizes this in his claim that the regularity of sensations actually encountered only makes the principle more probable: "We have no justification other than its results ... for the application of the law of causality" (Helmholtz 1971a, 390), yet "all ... natural laws arrived at by induction ... should properly be thought of as only more or less probable" (Ibid., 398). He also recognizes this in his claim that the principle of causality is "our intellectual impulse to view everything that happens as conforming to law and thus as conceivable" (1878/1971c, 361), for we know from Kant that this impulse arises because the regulative principle is accompanied by the transcendental presupposition of an object in the idea. Indeed, it is only in virtue this presupposition that the regulative principle of causality can have the status Helmholtz attributes to it of being a "transcendental law" (Ibid., 390).¹⁰² If one adopts Müller's principle that sensations are effects of an independently existing world, then given the presupposition of an object in the idea *causality* there must also arise the presupposition that sensations are the effects of *objective* causes. It is precisely because such objects are *presupposed* that Helmholtz's mature causal realism is *hypothetical*.

For Helmholtz the regulative idea *causality* arises in virtue of our ever-present belief in the uniformity of regularity: "We can do no more than accept the proverb, "Have faith and keep on!" (1878/1971, 390). This departure from *Critique* does not preclude our bringing to light the significance of replacing a category with a regulative idea by examining what it means to replace a transcendental schema with a regulative one. The regulative schema Helmholtz seems to have in mind is the following:

<i>Major premise:</i>	All occurrences of A are followed by occurrences of B.
<i>Minor premise:</i>	A occurs.
<i>Conclusion:</i>	Therefore B is will occur.

In virtue of this logical schema, the regulative idea *causality* presents the principle of causality, “all changes in nature necessarily follow from causal conditions” (1894/1971b, 525). Now in Helmholtz’s case the impact of the schema of causality must be explored by considering the pre-conscious and conscious levels separately.

Let us begin with at the pre-conscious level of images. Associated with the presupposition of an object in the idea is a *focus imaginarius* that serves to obtain “the greatest unity alongside the greatest extension” (Kant 1781-1787/1998, A644/B672). Functioning as the major premise of the regulative schema is an original image, i.e., “something fundamental and unchanging which is the cause of the changes we observe” (Helmholtz 1878/1971, 390), serving as an ideal towards which our involuntary movements (e.g., of the eye during perception) aim in testing invariant appearances. Converging towards the *focus imaginarius* is a potentially infinite space of interconnected “places”, each of which is “occupied” by an image. Each image functions as the major premise of a syllogistic inference that instantiates the regulative schema *causality*, whose minor premise and conclusion are kinds of sensations. For example:

<i>Major premise:</i>	All red strawberries are sweet.
<i>Minor premise:</i>	Red strawberry.
<i>Conclusion:</i>	Therefore sweet.

The major premise is a lawful temporal succession, an image, representing hypothetical causal structure in the real world. This image is organized under an image functioning, say, as the major premise “All strawberry plants grow red strawberries”.

Helmholtz provides a more sophisticated illustration in his elucidation of the “idea of the material form”:

... the memory images of pure sense impressions can also be used as elements in combinations of ideas, where it is not necessary or even possible to describe those impressions in words and thus to grasp them conceptually. A large part of our empirical knowledge of the natural relations among the objects around us obviously originates in this way. The blending of many perspective images of an object into the idea of a three-dimensional form seems to me an especially clear example of the kind of combination of sensual intuitions which corresponds to an inference. The idea of the material form represents or stands for all the perspective images, which in turn can be derived from it by a sufficiently powerful geometric imagination. Even views not previously perceived, such as those which result when cross-sectional cuts are made in any one of a number of directions, are derivable from such an idea. Indeed, the idea of a three-dimensional figure has no

content other than the idea of the series of visual images which can be obtained from it, including those which can be produced by cross-sectional cuts. In this sense, we may rightly claim that the idea of the stereometric form of a material object plays the role of a concept, formed on the basis of the combination of an extended series of sensuous intuition-images. It is a concept, however, which, unlike a geometrical construct, is not necessarily expressed in a verbal definition. It is held together or unified only by the clear idea of the laws in accordance with which its perspective images follow one another. (1894/1971b, 507)

The idea of the material form “plays the role of a concept” in that it

... comprehends under itself an infinite number of particular intuitions following one another in time, all of which can be derived from it just as the generic concept ‘table’ in turn comprehends within itself all particular tables and expresses their common characters. ((1857-67), as quoted in Friedman (1997, 31)).

Like an image in Kant’s classification space, the idea of the material form occupies a place in structure under which is organized a potentially infinite number of places, the first level down being occupied by a potentially infinite number of perspectival images, the second level down, presumably, being occupied again by a potentially infinite number of less comprehensive images, and so on. Unlike Kant’s classification space, however, this structure is organized at an unconscious level.

This potential space of all images reconciles Helmholtz’s view that when it is taken to be an adequate representation, an image represents causal structure with his claim that it also represents a “cause”:

If we have fully investigated some regularity, have established its conditions completely and with certainty and, at the same time, with complete generality, so that for all possible subsequent cases the effect is unequivocally determined – and if we have therefore arrived at the conviction that the law is true and will continue to hold true at all times and in all cases – then we recognize it as something existing independently of our ideas, and we label it a *cause*, or that which underlies or lies behind the changes taking place. (Note that the meaning I give to the word *cause* and its application are both exactly specified, although in ordinary language the word is also variously used to mean antecedent or motive.) (1878/1971a, 387)

Our “conviction that the law is true” is our belief that certain repeated regularities investigated are repeatable, grounded in our belief that our perceptions are uniform. With this conviction we take the image to be an adequate representation. That it represents a “cause” is not a shortcoming in Helmholtz’s exposition of his views as some have suggested.¹⁰³ He recognizes that, organized under the regulative idea causality, images

are like sensations in that kinds of images can stand in a lawful succession that is a more general image. See Figure 2.1:

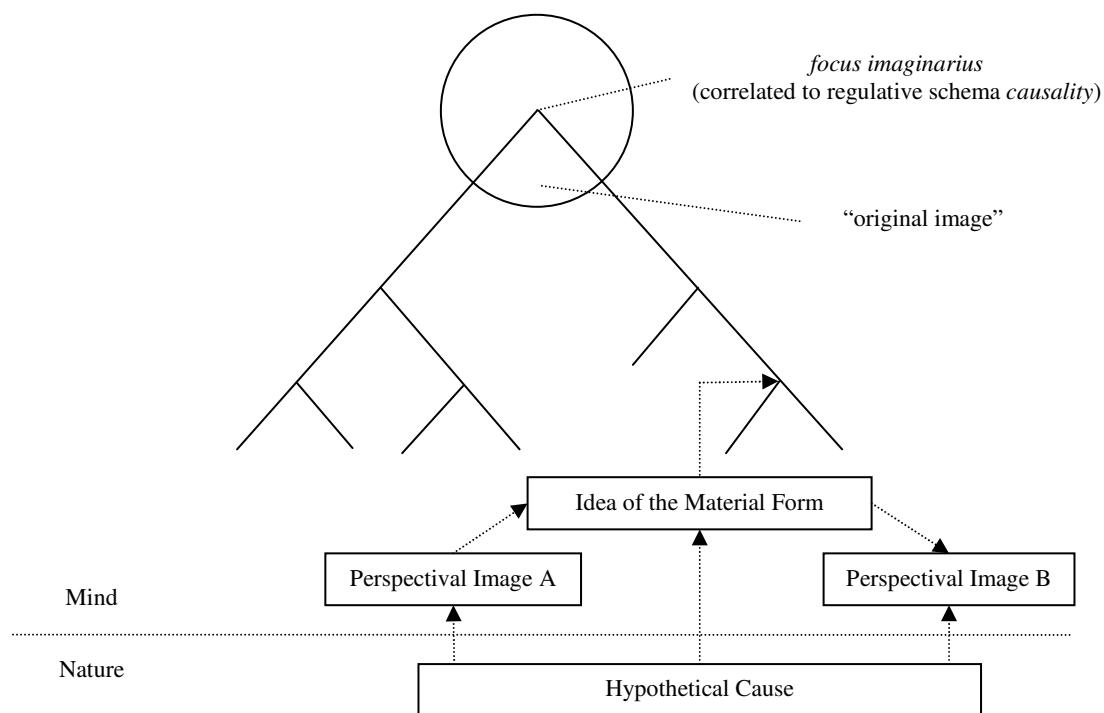


Figure 2.1: Helmholtz's Potential Space of All Images

We thus see how Helmholtz's philosophical system is based on adopting a view of the structure of imagination analogous to that of Kant.

From Helmholtz's potential space of all images can we specify a potential *logical space* as we did in Kant? The obvious contender is the subspace of ideas of the material form since it is a figurative space whose points are unities of objects. But I am disinclined to characterize this subspace as a logical space. Although the images may be mathematizable, the organizational structure would have been regarded as merely logical. More significantly, it does not bear any relation to a conception of self. Helmholtz's potential space of all images may be similar to Kant's potential classification space, but

this omission of Kant's transcendental self is no oversight. In adopting Müller's principle, Helmholtz wants to replace Kant's top-down synthesis of images under the transcendental unity of apperception with a bottom-up theory of pre-conscious image formation under a tacit belief in the uniformity of perceptions that, giving rise to the regulative idea *causality*, induces a structure of imagination.

Let us now shift to the conscious level of concepts. Belief in the uniformity of regularity – and the regulative idea *causality* it gives rise to – is a condition for the possibility of “conceptual understanding”, i.e., “the method by which the world is submitted to our thoughts, facts are ordered, and the future predicted” (Helmholtz 1878/1971, 390). This is the Kantian element of Helmholtz's system. Without our experience of repeated regularity “our conscious activities would cease” (Ibid.) and without our belief that repeated regularities are repeatable, the conscious activity we did have would not involve conceptual understanding:

Every inductive inference is based upon the belief that some given relation, previously observed to be regular or uniform, will continue to hold in all cases which may be observed. In effect, every inductive inference is based upon a belief in the lawful regularity of everything that happens. This uniformity or lawful regularity, however, is also the condition of conceptual understanding. Thus belief in uniformity or lawful regularity is at the same time belief in the possibility of understanding natural phenomena conceptually. If we assume that this comprehension or understanding of natural phenomena can be achieved – that is, if we believe that we shall be able to discern something fundamental and unchanging which is the cause of the changes we observe – then we accept a regulative principle in our thinking. It is called the law of causality, and it expresses our belief in the complete comprehensibility of the world. (Ibid., 389-390)

“*To comprehend ... means to form concepts*” (1894/1971b, 517), and so belief in the uniformity of perception is also a condition for the possibility of concept formation.

Helmholtz rejects the following account of concept formation presented by traditional logic:

First we combine objects which are the same in some respect to form a class. Then we specify the set of characteristics that will be used to distinguish the objects in the class; this is usually called the *definition* of the class. Giving a definition, therefore, consists in specifying the complex of properties which are necessarily present in all members of a class. (1894/1971a, 518)

Without more, empirical concepts would only specify properties “necessary and sufficient to form the definition, to delimit the class, and to fix the name” giving rise to

tautologically necessary propositions (e.g., “mammals are animals born alive and suckled by their mothers”) (Ibid.). Following Mill,¹⁰⁴ he adds that concepts also have “connotations” present to all members grouped under the concept, supporting universal statements that can be used in syllogistic inference (e.g. “all mammals are warm-blooded”) (Ibid.). Still, he retains a Kantian view of concept formation from the univocal interpretation of images:

The term *thought* should rightly be applied only to those combinations of ideas for which a person is able to formulate explicitly the individual propositions from which inferences¹⁰⁵ are drawn, to verify their reliability, and finally to connect them consciously in making inferences. (1894/1971a, 506)¹⁰⁶

As suggested in the last clause, empirical concepts formed this way are connected in virtue of connections among the interpreted images. One might say that at the level of consciousness Helmholtz’s potential space of all images manifests as a potential space of causal reasons: i.e., a potentially infinite space of interconnected “places”, each of which is occupied by a concept and described by a major premise of a syllogistic inference that instantiates the regulative schema of *causality*, conveying a notion of justification relative to network of causal inferences.

Compared to his potential space of all images, Helmholtz’s potential space of causal reasons has a number of distinguishing features. The first is that it has significantly less structure. Helmholtz recognizes that

[l]anguage is much too poor for the exact description of the many sense impressions which even a single object, especially one of somewhat irregular or complicated form, affords the eye and the hand. To describe such impressions in words, moreover, would be an enormously lengthy, time-consuming occupation – one which we obviously need not carry out if we have had the intuition-image of the object impressed upon us. Where this has been done, as well as in cases where no verbal description is possible, the sense impressions without any verbal expression are quite sufficient. (1894/1971b, 503)

Another feature is that it is an idealization: “natural bodies [never] in fact correspond exactly to the abstract conception we have obtained of them by induction” (1870/1971, 263). A third characteristic is that the principle of causality plays an expanded role. At the pre-conceptual or unconscious level, the principle functions as a heuristic whereby we seek out and test invariant sense impressions through involuntary bodily activity. At the conceptual level the principle also functions this way, but in relation to experimental

activity. Moreover, it impels us to “seek and, if possible, find still more inclusive laws under which many more individual circumstances are comprehended” (Helmholtz 1894/1971a, 528). In other words, it also serves as a standard of conceptual understanding: we take our comprehension to be defective so long as “places” in the potential space of causal reasons remain “unoccupied”. Notwithstanding the differences between these two spaces, the heuristic impulse associated with both is explained through the patterning of their structure on a convex mirror, one that draws both imagination and comprehension towards unity.

2.24 Science

Because the heuristic function of the principle of causality occurs at both conscious and the pre-conscious levels, Helmholtz regards the process of arriving at laws in physical sciences through the method of deliberately contrived experimental interposition to be similar to the process of arriving at intuitive images by means of everyday bodily action. This is implicit from the following passage:

Of the greatest importance, finally, for the fixity of our conviction in the correctness of our sensory perceptions are the tests that we undertake by means of the optional motions of our body. There thereby arises the same kind of fixed conviction, relative to merely passive observation, that we gain in scientific investigations through the experimental method. The proper ultimate ground, through which all our consciously executed inductions receive the power of conviction, is the law of causality. ((1857-67, 29) as quoted in (Friedman 1997, 32))

Because the principle of causality also functions at the conscious level as a standard of conceptual understanding, “[t]he goal of the physical sciences is to comprehend natural phenomena” (1894/1971a, 517).

In order to accomplish this goal,

... we seek in general to determine what changes occur, what external influences and causes must exist in order for these changes to take place, and what must exist in order to prevent these changes. To comprehend these processes of *change* which we observe among the objects in the external world, we must carry out in the physical sciences a procedure completely analogous to the procedure of concept formation with respect to natural *forms*. (Helmholtz 1894/1971a, 520)

Concept formation with respect to natural forms observed in natural sciences such as zoology and botany is the one mentioned above in which we “seek to include classes of bodies under concepts and to look for their connotations” (Ibid.). But there the task “is to describe static conditions” (Ibid.). In the physical sciences the task is “a description of the processes we actually perceive” (Ibid., 521).

This is really the same conceptual task; only the form of the linguistic product is different. We can give linguistic expression to a class encompassing the conditions and course of some process of change only in the form of a natural law. For example, “two ponderable bodies which are at a finite distance from each other in space undergo an acceleration, each in the direction of the other” – that is, they move with increasing velocities towards each other. ... Our task, in other words, is to form classes of changes or processes such that, in addition to the observed invariant relation which corresponds to the definition of the concept, there are other regular processes analogous to the connotations of the concept. ... [W]e can express all regularities in the processes of nature as laws, each with its specific factual content. To do this, we have only to state precisely the conditions under which a specific phenomenon takes place and then specify exactly how the process will continue. (Ibid., 520-521)

In Helmholtz’s example the law of gravitational acceleration arises from the concept *gravity*, and the concept is formed from 1) the conditions of “two ponderable bodies” and “finite distance from each other in space”; and 2) the invariant relation of changes in the position of these bodies.

Helmholtz’s point is that in the physical sciences concept formation is not merely a univocal interpretation of an image serving as the major premise of a certain unconscious inference that instantiates the regulative schema *causality*; rather, it is a univocal interpretation of the inference itself. For this reason a concept in the physical sciences is not static like the concept of an object. The concept of an object is formed from an image that implies another image from a given image. By contrast, a concept in the physical sciences is formed from the interpretation of all images functioning as the condition, the consequent and the major premise that connects the two. As we shall see, Hertz takes these as a single image.

Before turning to Hertz, a brief examination of Helmholtz’s linguistic account of force is in order.

As a rule, in the linguistic formulation of a law we deviate from the formulation of natural laws just indicated in that we form abstractions, and indeed introduce *verba substantia*. For

example, we express the first of the [law of gravity] by stating that between any two material bodies at a finite distance from each other in space there exists a continuous force of attraction of a certain magnitude. Instead of the simple description of the phenomenon of movement, we introduce an abstraction – the force of attraction. (1894/1971a, 521)

We inject this “empty abstraction” (Ibid.) into the descriptive formulation of physical laws when “we recognize a law as a power analogous to our will” (Helmholtz 1878/1971, 387). And since “the abstractions *material object* and *force* cannot be separated” (Helmholtz 1894/1971a, 524), the concept *force* is inseparable from the concept *cause*. Helmholtz summarizes:

A law, considered as an objective power, we call *force*. *Cause*, according to its original meaning, is the unchanging existent (that is, matter) which lies behind the changes of phenomena; the law of its effects is force. The impossibility ... of conceiving of these in isolation from each other thus follows simply from the fact that the law of an effect presupposes certain conditions under which it is realized. A force separated from matter would be the objectification of a law which lacked the conditions for its realization. (Appendix (written 1881) to 1847/1971, 50)

The point is that the regulative idea *causality* accounts, not only for the hypothetical concept *cause*, but also for the “hypothetical substantive” (Helmholtz 1894/1971a, 524) that is the concept *force*.¹⁰⁷ Helmholtz asserts that as long as we do not take the latter as denoting “a real thing having *independent* existence” (Ibid., 525) its use is legitimate, for “[o]nly by using this mode of expression can we state that a law which we have discovered is always ready to operate and may show its power at any moment” (Ibid.).

With Helmholtz’s adaptation of Kant’s transcendental law of causality the groundwork was laid for an epistemological tradition in Germany beginning with Hertz where “[a]ll of our ideas and concepts are ... inner thought-pictures or, when uttered, combinations of sounds” and the *Bildtheorie* view that “science is only an inner picture, a thought-construction” in virtue of a “thought schema”.¹⁰⁸

2.3 Hertz

Helmholtz was Hertz’s scientific and philosophical mentor; Hertz was undoubtedly Helmholtz’s most famous student. However, Hertz’s own experimental findings and philosophical reflections led him to abandon the notion of force in

electrodynamics (Heidelberger 1998, 20). The *Principles of Mechanics* (1900/1956)¹⁰⁹ is Hertz's attempt to integrate electrodynamic theory with a general theory of mechanics that takes effects of forces to be effects of a mechanical medium comprised of rigidly connected systems of hidden masses in fast cyclical motion.

Published a few months after Hertz's death in 1894, the *Principles* is a work of physics in its own right. Yet its author did not consider its import to be its contribution to science: "What I hope is new, and to this alone I attach value, is the arrangement and collocation of the whole – the logical or philosophical aspect of the matter" (Pref., iv). Indeed, its intellectual influence is mainly to be found in its lengthy introduction, which presents a novel image theory of scientific representation. It is here that an important link between Kant and Wittgenstein is forged.¹¹⁰

2.31 "Images of a Riotous Imagination"

The place to start is Hertz's return to a central motivation of Kant's critical project. Hertz's ultimate philosophical concern is the effect our imaginative grasp of scientific theories has on our images of external objects:

We see a piece of iron resting upon a table, and we accordingly imagine that no causes of motion – no forces – are there present. Physics, which is based upon the mechanics considered here and necessarily determined by this basis, teaches us otherwise. Through the force of gravitation, every atom of the iron is attracted by every other atom in the universe. But every atom of the iron is magnetic, and is thus connected by fresh forces with every other magnetic atom in the universe. Again, bodies in the universe contain electricity in motion, and this latter exerts further complicated forces which attract every atom of the iron. In so far as the parts of the iron themselves contain electricity, we have fresh forces to take into consideration; and in addition to these again various kinds of molecular forces. Some of these forces are not small; if only a part of these forces were effective, this part would tear the iron to pieces. But, in fact, all the forces are so adjusted amongst each other that the effect of the whole lot is zero; that in spite of a thousand existing causes of motion, no motion takes place; that the iron remains at rest. Now if we place these conceptions before unprejudiced persons, who will believe us? Whom shall we convince that we are speaking of actual things, not images of a riotous imagination? (13)

Hertz is expressing concern over how the concept *force* constitutes our images of things, and the goal of the *Principles* is to subdue what is evidently a "riotous imagination". As we saw in the last chapter, the goal of Kant's *Critique* can also be understood (in part) as an effort to discipline the imagination. The understanding, Kant says, must "think"

thereby “setting bounds” to the “revelry” of the imagination”, rather than “daydream” by indulging it. His description of unbridled imagination presages that of Hertz:

The understanding begins all this very innocently and chastely. First it puts in order the elementary cognitions that dwell in it prior to all experience but must nonetheless have their application in experience. Gradually, it removes these constraints, and what is to hinder it from doing so, since the understanding has quite freely taken its principles from within itself? And now reference is made first to *newly invented forces in nature*, soon thereafter to beings outside nature, in a word, to a world for the furnishing of which building materials cannot fail us, since they are abundantly supplied through fertile invention, and though not indeed confirmed by experience, are also never refuted by it. (italics added, (Ak. IV: 317))

The essential difference between the two projects is that for Kant the imagination is focused synthetically through constitutive concepts that give rise to transcendental and pure sensible schemata, whereas for Hertz the imagination is moderated through an appropriate choice of constitutive fundamental ideas (and principles connecting them), the idea *force* not being one of them.

Hertz rejects Helmholtz’s conception of force not only by rejecting the transcendental law of causality, but by more or less ignoring Müller’s principle and Helmholtz’s account of physiological intuition. As a result, Hertz forsakes Helmholtz’s invocation of hypothetical causes of sensations and perceptions, his notion of unconscious inferences and his empirical account of the origin of the axioms of arithmetic and geometry. He falls back onto Kant’s forms of intuition:

[Time] is the time of our internal intuition. It is therefore a quantity such that the variations of the other quantities under consideration may be regarded as dependant upon its variation; whereas in itself it is always an independent variable.

[Space] is space as we conceive it. It is therefore the space of Euclid’s geometry, with all the properties which this geometry ascribes to it. It is immaterial to us whether these properties are regarded as being given by the laws of our internal intuition, or as consequences of thought which necessarily follow from arbitrary definitions.¹¹¹ (45)

In keeping with Helmholtz, however, it is important to recognize Hertz continues to reject Kant’s notion of formal intuition, i.e., the combination of the manifold as determined by pure concepts of the understanding. As a result, forms of intuition do not apply to experience through Kant’s a priori synthetic imagination. Instead, they are said to be interpreted through conventional “laws of transformation” (§302) that connect them with a metric.¹¹² So interpreted, they become “symbols for objects of external experience in

that we settle by what sensible perceptions we intend to determine definite times [and] space-quantities” (§297). As interpreted symbols, forms of intuition are “fundamental ideas” (4) that “become parts of our images of external objects” (§302), i.e., they are *constitutive* of our images of external objects.

It is worth emphasizing that by “external objects” Hertz means “mind-independent objects” *not* “spatial objects”. Apart from a conventional law of transformation, space (and time) “are in no sense capable of being made the subjects of our experience” (§297). A law of transformation associates these conceptions with “external experience, *i.e.* concrete sensations and perceptions” (*Ibid.*). Here “external” has the sense of “given”. Following Helmholtz, Hertz is taking concrete sensations and perceptions to be given to us through a relation to things-in-themselves (though not the sign-cause relation his mentor hypothesized). It is through a law of transformation connecting internal intuitions of space and time with given sensations and perceptions that the former become constitutive of our internal images of external things-in-themselves.

While the fundamental ideas of space and time arise from a priori forms of intuition, other fundamental ideas such as that of mass, force and energy arise from a priori definitions. Interpreted through laws of transformation they too become constitutive of our internal images of external things-in-themselves. Accordingly, we always have “the possibility of modifying and improving our images” (2) through our choice of such definitions. The *Principles* argues that all conventional fundamental ideas except for a certain interpreted definition of mass typify a riotous imagination.

2.32 Images and Models

Notwithstanding his return to a central motivation of Kant’s critical project – disciplining the imagination – Hertz incorporates adaptations introduced by Helmholtz. He begins his introduction to the *Principles* with a statement Helmholtz could have written:

The most direct, and in a sense the most important, problem which our conscious knowledge of nature should enable us to solve is the anticipation of future events, so that we may arrange our present affairs in accordance with such anticipation. (1)

This problem is solved when consciousness, relying upon our knowledge of past events “obtained by chance observation” or “by prearranged experiment”, infers future events on the basis of our images of external objects:

We form for ourselves images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured. In order that this requirement may be satisfied, there must be a certain conformity between nature and our thought. Experience teaches us that the requirement can be satisfied, and hence that such a conformity does in fact exist. (Ibid.)

Although Müller’s principle does not enter Hertz’s system, Hertz admits that images are “built up” from “the results of experience” (3), which, as already noted, are external concrete sensations and perceptions.¹¹³

Although it is generally recognized that Hertz owes his notion of an “image of things” (4) to Helmholtz,¹¹⁴ the basis of the connection remains obscure. This because there is confusion over what a Hertzian image is. If one were to take Hertz as distinguishing “image” from “form”, the previous passage seems to suggest images are the conditions and consequents of a logical inference along the lines of Helmholtz’s signs.¹¹⁵ Alternatively, one might interpret Hertz as claiming “the relations between images of objects are images of relations between objects” (Leroux 2001, 192), suggesting there are not two but three related images: the condition, the consequent and their relation. Another option, one van Fraassen (2008, 196) suggests, is to say it is a single image evolving logically from initial conditions to a final consequent.

Van Fraassen’s interpretation of the Hertzian image as a logical process is the most natural reading of the passage. Furthermore, since Hertz is worried about how fundamental ideas from physical science constitute our images of things, it is not surprising he would begin with Helmholtz’s notion of a physical concept. As already discussed in the last section, such a concept has the form of a logical inference, and arises from the interpretation of images that serve as a major premise, a minor premise, and conclusion. Since Hertz does not provide an account of concept formation but simply takes it that “images are ... our conceptions of things” (1), it would be natural for him to adopt the notion of an image as having the logical form of Helmholtz’s physical concept.

However, for Hertz the form of logical inference is not inductive but deductive.¹¹⁶ This shift in logic accommodates Hertz's view that not only are images "built-up" from experience, they are "built-down" from fundamental ideas, i.e., internal intuitions and a priori definitions interpreted by laws of transformation. Fundamental ideas are symbols constitutive of internal images, making them "symbols for external objects of experience"¹¹⁷ and, since they are also concepts it seems, "the symbolic language" of experience (§302).¹¹⁸ A Hertzian symbol of an external object should not be confused with a Helmholtzian sign of an objective cause: it is not a signal transmitting information from the external cause, but (like a Helmholtzian image) a *representation* of things-in-themselves.

Hertz appears to incorporate Helmholtz's theory of representation as a theory of symbolic adequacy: as an adequate symbol for an external object, an image is taken to be similar to it. The similarity is not one of causal structure, however, but one of logical structure. Thus Helmholtz's theory of representation commits Hertz to a form of modal realism where an external object is taken to have the form of a syllogistic logical inference (cf. §428). Hertz adds that our epistemic access to things-in-themselves is limited to this shared structure:

With the things themselves [our conceptions of things] are in conformity in *one* important respect, namely, in satisfying the above-mentioned requirement. For our purpose it is not necessary that they should be in conformity with the things in any other respect whatever. As a matter of fact, we do not know, nor have we any means of knowing, whether our conceptions of things are in conformity with them in any other than this *one* fundamental respect. (1-2)

In light of the conventionality of Hertz's constitutive approach to image formation, however, it would be a mistake to equate "conformity" in this passage as a relation of isomorphism between the representation and the represented.

The reason is that Hertz also tells us that in our attempt to form a lawful image of the universe, our images of things (e.g., a piece of iron) must take on more structure (e.g., atoms subject to gravitational forces) than revealed in experience:

If we try to understand the motions of bodies around us, and to refer them to simple and clear rules, paying attention only to what can be directly observed, our attempt will in general fail. We soon become aware that the totality of things visible and tangible do not

form a universe conformable to law, in which the same results always follow from the same conditions. We become convinced that the manifold of the actual universe must be greater than the manifold of the universe which is directly revealed to us by our senses. If we wish to obtain an image of the universe we which shall be well-rounded, complete, and conformable to law, we have to presuppose, behind the things we see, other invisible things – to imagine confederates concealed beyond the limits of our senses. (25)

It is precisely because we can choose different presuppositions that “[v]arious images of the same objects are possible” (2), and that our efforts to create a lawful image of the universe should be evaluated in relation to their constitutive effect upon our images of external objects. Accordingly, “conformity” in relation to Hertz’s fundamental requirement should be understood only as isomorphism of real logical structure to a part of an image, or *embedding*. Hertz exemplifies what we would today call a species of epistemic structural realism: through the empirical (logical) substructure of our images, all we know about the external world is its empirical (logical) structure.

But Hertz’s epistemology is even more subtle than this. We access the structure things through properties of “essential relations” (2) of our images taken to “represent *observable* relations of things” (italics added, 9). This is to say that essential relations represent not merely repeated regularities, but repeatable ones. Yet Hertz also admits Hume’s problem of induction: representational or symbolic adequacy is always defeasible with “later and riper experience” (3). This feature of Hertz’s epistemology is puzzling to many commentators.¹¹⁹ But what he seems to want is to balance a constitutive account of image formation built upon properties and relations delivered through experience with a theory of representation that takes the representational relation to be one of structural similarity. The problem is that Hume shows that the empirical elements of images (actual senses and perceptions) deliver only *indefinite* structure, i.e., properties of properties and relations for which there is always the possibility of divergence. And so the deliverances of experience *permit* us at any given time to constitute images having a *definite* empirical structure, i.e., properties of *universal* properties and relations not subject to the possibility of divergence, through common forms of intuitions and a priori definitions that apply to *possible* senses and perceptions (cf. §302). The effect of adopting Helmholtz’s theory of representation is that this definite empirical structure is *projected* onto reality as the form of an external object. None of

this precludes experience from informing us at a later date that our projection was premature.

Helmholtz had grounded a similar projection of images in our belief in the uniformity of perceptions as a condition for the possibility of conceptual understanding. In rejecting Helmholtz's regulative idea *causality*, however, it seems Hertz is denying this belief (he never mentions it). Without more this leaves Hertz's claim of the conformity between our images and external objects unexplained. However, Hertz expresses the logical form of an image as an idealized mathematical structure, and, as discussed later, suggests that it is shared mathematical structure between mind and nature that provides this explanation, a remark picked up and developed by Wittgenstein.

Given the contemporary orientation that takes logic and mathematics to be intimately related, it is not uncommon for commentators to overlook Hertz's important distinction between the "physical content of the image" and the "mathematical form in which it [is] represented" (29).¹²⁰ It must be remembered that the intellectual milieu of the *Principles* continued to be dominated by Kant's view that logic and mathematics involve different types of reasoning. For Hertz the physical content of an image takes on the form of a syllogistic logical inference. When this inference is expressed in an ideal mathematical form, the physical content of an image is *represented* by a "model" (1). We can think of the image as "encased" within the model. See figure 2.2:

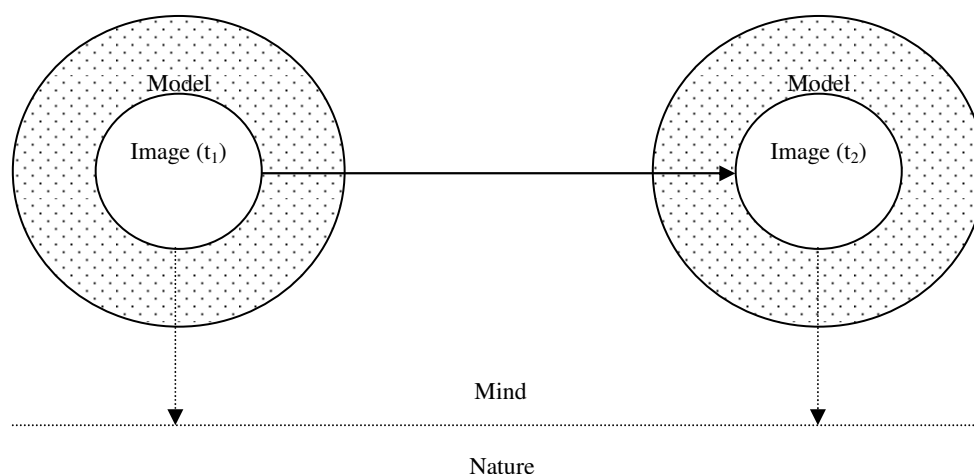


Figure 2.2: Relationship between Mathematical Model and Logical Image
(The top arrow stands for the logical evolution of the image expressed mathematically in the model; the downward arrows stand for the projection of logical structure as the form of an external object.)

Hertz realizes that encasing an image within a model is necessary to enhance the predictive power of its logical form because syllogistic logic cannot capture the notion of infinity implicit in Hertz's forms of intuition and explicit in his definition of mass (3).¹²¹ This is suggested at the two places in the introduction to the *Principles* where Hertz talks of models. The first occurs at the outset:

When from our accumulated previous experience we have once succeeded in deducing images of the desired nature, we can then in a short time develop by means of them, as by means of models, the consequences which in the external world only arise in a comparatively long time, or as the result of our own interposition. We are thus enabled to be in advance of the facts, and to decide as to present affairs in accordance with the insight so obtained. (1)

In this passage Hertz tells us that from images of things we have “succeeded in deducing” we “then in a short time develop by means of them” models capable of making complex predictions over extended periods of time or through experimentation. A number of pages later Hertz adds that his statements concerning mathematical form “refer without exception to concrete images of space as perceived by our senses”, that they “represent possible experiences” and that they can be “confirmed by direct experiments, viz., by measurements made with models” (30). Together the implication seems to be that a model is a representation of the physical content of an image that mathematically expresses its logical form, enabling sophisticated and detailed quantitative prediction. Indeed, as we shall see Hertz is of the view that the physical content of an image can be represented by any number of mathematical forms chosen as a matter of convenience.

2.33 Scientific Representation and its Mathematical Form

Hertz does not distinguish among images of things. What is usually overlooked, however, is that he makes a distinction between images of things and a “general image” (5).¹²² This distinction arises through Hertz's notion of a “scientific representation” (2). It appears that a scientific theory is a system of equations: “Maxwell's theory is Maxwell's system of equations” (Hertz 1893/1962, 21) and in the case of mechanics it is Lagrange's equations of the first form (D'Agostino 1998, 91). A scientific representation, on the

other hand, is a physical interpretation of these equations through a set of fundamental ideas connected by a set of fundamental principles (4). A scientific representation presents two things: 1) a “first system of principles” (5); and 2) a general image – what I will call a *scientific image*. Hertz claims that because a first system of principles “satisfies the requirement that the whole of mechanics can be developed from it by purely deductive reasoning without any further appeal to experience”, a scientific representation presents “the simplest image which physics can produce of things in the sensible world and the processes which occur in it” (4).

Because a scientific representation involves fundamental ideas derived from Kantian forms of internal intuition, the scientific image it presents must be taken to *constitute* our images of things. But because a scientific image also involves interpreted conventional elements, a plurality of scientific images is possible, the theory being their “minimum denominator” (D’Agostino 1998, 90). As Hertz famously says, “we can choose as we please” “the cut and color” of the “gay garment which we use to clothe” a theory with “our powers of imagination” (1893/1962, 28), and “we can thus obtain various images of things”. These choices are not mere academic exercises, for it is through our choice of scientific representation that we have “the possibility of modifying and *improving* our images”.

What, exactly, is Hertz’s scientific image? If an image of an external object is a syllogistic inference, then a “general” and “simple” image is a general and simple syllogistic inference. This is to say that the scientific image is a *logical schema* that presents principles of mechanics. This explains why Hertz “sketches” scientific images through “general outlines” each of which provides a “general plan” according to which a scientific representation is arranged (15, 26). This characterization of the scientific image is consistent with a view of the *Principles* put forth by Boltzmann (1905/1960) who understood it as presenting “an inner, mental picture” (249) or “thought schema” (251-252). More significantly, it is in accord with the account of mechanics afforded by Wittgenstein (1921/1961) who, mentioning Hertz (6.361), saw it generally as an attempt to construct true pictures “according to a single plan” (6.343).

As I understand Hertz, his notion of a scientific representation is a substitute for Helmholtz's regulative idea *causality*. Recall that a regulative idea has the form of a logical schema that presents a regulative principle. Hertz's scientific representation, which is a set of fundamental ideas connected by fundamental principles, has the form of a logical schema that presents the principles of mechanics. Unlike Helmholtz for whom the major premise of the logical schema is an original image, however, Hertz takes the logical schema as the original image, i.e., as the "general" yet "simple" scientific image. Indeed, Hertz's scientific image has at least two important attributes of Helmholtz's original image. In the Helmholtzian system the original image is an *ideal* of cognitive activity. In the Hertzian system our *choice* of a scientific representation, and therefore our choice of the scientific image it presents, is also an ideal of activity. But it is not an ideal of cognitive activity. Nor is it an ideal of scientific activity (that aims only to suit "practical applications or the needs of mankind" (40)). Rather, it is an ideal of *philosophical* activity that aims to reconstruct any scientific representation that develops in order to perfect it:¹²³ "[w]e cannot attain to perfection in any direction" (33), but "we should endeavour to show [logical clearness] by a representation so perfect that there should no longer be any possibility of doubting it" (9).¹²⁴ Second, in the Helmholtzian system the *concept* of an original image is an idealization of our images in virtue of abstraction and induction. In the Hertzian system where concept and image are the same a scientific image is an idealization in virtue of the laws of transformation that interpret fundamental ideas (§290-300) and the fundamental principles connecting them (cf. §315f.).¹²⁵

If a scientific image is a logical schema that presents the principles of mechanics, then deductions from these principles would correspond to our images of things.¹²⁶ This is to say that the scientific image constitutes images of external objects not only through interpreted forms of internal intuition and a priori definitions, but through the instantiation of a logical schema. Hertz's notion of a scientific representation can therefore be thought of as organizing a potential space of all images of things similar to what we saw in Helmholtz, though in this case having a deductive logical structure and occurring at level of consciousness.

This helps us to understand Hertz's frequently overlooked *two*-step characterization of the evaluation of a scientific representation. The first step involves appraising the fundamental ideas and fundamental principles for "scientific completeness" (8): "We require of [a scientific representation] that it should lead us to a clear conception of what properties are to be ascribed to the images for the sake of permissibility, what for correctness, and what for appropriateness" (2). In particular, it must plainly show that: 1) "What enters into the images, in order that they may be permissible, is given by the nature of our mind"; 2) "What enters into images for the sake of correctness is contained in the results of experience, from which images are built up"; and 3) "What is ascribed to the images for the sake of appropriateness is contained in the notations, definitions, abbreviations, and, in short, all that we can arbitrarily add or take away" (3).

The second step involves evaluating the scientific representation by assessing the images of things it constitutes. There are three criteria. Of "greatest importance" (33) is that images of things must be "permissible" in accordance with "the laws of our thought" (2), which is to say "not only must its characteristics be consistent amongst themselves, but they must not contradict the characteristics of other images already established in our knowledge" (23). These laws of thought are "the laws of the internal intuition" and "logical forms" (§1), the latter being for Hertz patterns of syllogistic inferences. Whether this criterion is met is a "yes or no" question and its answer holds "for all time" (3).

The second criterion is Hertz's fundamental requirement mentioned already "that the consequents of the images must be the images of the consequents": images of things must be "correct" in that their "essential relations" that "represent observable relations of things" do not "contradict the relations of external things" (2). As mentioned already, meeting the criteria does not entail that an image is unique, for the necessity of introducing relations concerning what is unobservable means that various images of the same objects are possible. Moreover, although meeting this criterion is a "yes or no" question, a "yes" answer is defeasible.¹²⁷ Defeasibility means that an image correct by the strict deliverances of experience may not be a true one *vis a vis* its essential relations.

The third criterion is that images of things must be the “most appropriate” images in that they are more “distinct” than other images satisfying the first two criteria by picturing “more of the essential relations of the object”. The most appropriate images, moreover, are “simpler” than other equally distinct images in that they contain “the smaller number of superfluous or empty relations” (2). These empty relations unavoidably enter our images because images are, in addition to products of experience, products of the mind. Whether the criterion is met is not a “yes or no” question, but one answered “only by gradually testing many images” (3) and comparing them with each other.

As I understand Hertz, a scientific representation that meets these standards of evaluation is accepted as symbolically adequate, and the images it constitutes are accepted as adequate symbols of external objects. As adequate symbols, images are taken to be similar to reality. This similarity relation involves projecting the empirical structure of an image onto reality and taking the projected structure to be the logical form of an external object. But since it is a projection precisely because it goes beyond the strict deliverances of experience, symbolic adequacy of images is always defeasible by later experience. We will see something remarkably similar to this view of scientific representation in the case of van Fraassen.

Hertz’s notion of a scientific representation is that of a physical interpretation of a set of equations (a theory) through a general and simple image that both constitutes and logically organizes images of things. In addition, he informally “outlines” (cf. 26 and 33) the “mathematical form” (29) of the image. Since the image is a logical schema, the mathematical form is a *mathematical schema*. In saying that “[t]he physical content of the image is quite independent of the mathematical form” that represents it, Hertz does not purport to establish an integration of mathematics with logic. To the contrary, he only requires that the physical content and mathematical form of the scientific image “are so suited that they mutually assist one another” (29). Mathematical form assists the physical content of the scientific image by not suggesting fundamental concepts that would work against the symbolic adequacy of the scientific representation;¹²⁸ the physical content of the scientific image assists the mathematical form through an appropriate choice of

definitions.¹²⁹ But for this criterion, Hertz implies that choice of mathematical form is entirely a matter of convenience.

2.34 Hertz's Scientific Representation

After assessing images of things constituted by the scientific representations of Newtonian-Laplacian mechanics (the fundamental ideas of space, time, mass and force connected via Newton's laws of motion and d'Alembert's principle) (4) and energetics (the fundamental ideas of space, time, mass and energy connected via a principle of energy transformation such as Hamilton's principle), Hertz rejects these scientific representations for reasons of permissibility and appropriateness. Seeking to rid himself of the "oppressive feeling" that comes with "obscure and unintelligible" (33) scientific images, he purports instead to "sharply delineate" (39) a complete scientific representation (the fundamental ideas of space, time and mass connected by a fundamental law) that presents a scientific image meeting his criteria for representational adequacy.

To the fundamental ideas of space and time that arise from our forms of intuition, Hertz adds the a priori fundamental idea *mass*. This definition has two notions at play: "material particle" and "material point". A material particle is "a characteristic by which we associate without ambiguity a given point in space at a given time with a given point in space at any other time". It is, in other words, a property of a space-time location by which it can be counted.¹³⁰ Although a material particle is "invariable" and "indestructible" its mass is infinitely small. The mass of any space volume, on the other hand, is the ratio formed by the number of material particles of the space to the number of material particles of some reference space. When the volume is conceived as an infinitely small space, mass is called a "material point".¹³¹ "A number of material points considered simultaneously is called a system of material points" and its mass is the sum of the masses of the separate points (§3-§8). "A system of material points which is subject to no other than continuous connections is called a material system" (§121).

In virtue of a conventional law of transformation that stipulates that the mass of a tangible, i.e., observable, body is determined by weighing,¹³² the a priori definition of mass is transformed into a fundamental idea symbolic of external objects and thereby constitutive of our images of things:

The mass of a tangible body as determined by this [law of transformation] possesses the properties attributed to the ideally defined mass. That is to say, it can be conceived [i.e., *imagined*] as split up into any number of equal parts, each of which is indestructible and unchangeable and capable of being employed as a mark to refer, without ambiguity, a point of space at one time to a point of space at any other time” (§300).

If intangible, i.e., unobservable, bodies are presumed to be part of these images, their mass can “only be determined by hypothesis” through “properties which are consistent with the properties of the ideally defined mass” (§301).

At this point in the *Principles* a non-idealized image of things becomes a “material system”: “By a material system is henceforth understood a system of concrete masses, whose properties are not inconsistent with the properties of the ideally defined material system (§121)” (§305). By inference Hertz then introduces the term “natural material system” (Ibid.), which I understand to mean nature *as if* it were a material system.¹³³ If material systems are taken as adequate symbols of external objects, we are committed to the ontological position that there are relations constituting natural material systems having properties isomorphic to the properties of the essential relations of the material system (cf. §306). This projection of empirical structure onto nature also involves an extension of our “mode of conception” where we imagine these relations to be among unobservable masses or “confederates concealed beyond the limits of our senses” (25).

Hertz identifies essential relations by first specifying essential relations between the various pairs of ideas. In the case of space and time these relations are the subject of kinematics common to all scientific representations. Although there are no essential relations among the ideas of mass and time,

[e]xperience teaches us that between mass and space there exists a series of important relations. For we find certainly purely spatial connections between the masses of nature: from the very beginning onwards through all time, and therefore independently of time, certain positions and certain changes of position are prescribed and associated as possible

for these masses, and all others as impossible. Respecting these connections we can also assert generally that they only apply to the relative position of the masses amongst themselves; and further that they satisfy certain conditions of continuity, which find their mathematical expression in the fact that the connections themselves can always be represented by homogeneous linear equations between the first differentials of the magnitudes by which the positions of the masses are denoted. (27)

(To say experience “teaches” us this can be misleading. This passage must be read alongside Hertz’s admission that the impossibility of certain connections is speculative: “permissible connections are defined solely by continuity” and the proposition of *natura non facit saltus* is “of the nature of a tentatively accepted hypothesis” (37).) More specifically, experience teaches certain spatial connections are constant:

Suppose we find in any way that the distance between two material particles remains constant at all times and under all circumstances. We can express this fact without making use of any other conceptions than those of space; and the value of the fact stated, as a fact, for the purpose of foreseeing future experience and for all other purposes, will be independent of any explanation of it which we may or may not possess. (34)

What Hertz must mean by “fact” is similar to what Helmholtz meant by “fact of perception”: experience teaches us there are constant spatial connections, but in the deductive constitutive (vs. Helmholtz’s inductive regulative) process of image formation we amplify these observed relations to observable ones. At this point they have the definite property of being “fixed” or “rigid”.

Next, Hertz identifies the essential relation between the three ideas of time, space and mass to be the “fundamental law”: “Every natural motion of an independent material system consists herein, that the system follows with uniform velocity one of its straightest paths” (27).¹³⁴ Although this law is “the probable outcome of most general experience” having only the status a “hypothesis” (§315), in the constitutive process in which we form images of external objects the law is taken to introduce definite empirical structure.

Together the fundamental ideas of time, space and mass and the fundamental law give rise to a scientific representation. This scientific representation presents both a scientific image (logical schema) and first principles of mechanics. The scientific image is an idealized image of “an all-pervading medium whose smallest parts are subjected to rigid connections” (41), one along the lines of Kelvin who “in his theory of vortex atoms ... endeavoured to present an image of the universe in accordance with this conception”

(26).¹³⁵ This general mechanistic world-view constitutes images of external objects as systems of rigidly connected masses in motion (§323). But it presents only a single principle of mechanics – the fundamental law: “[t]he aggregate of inferences ... drawn from the fundamental law forms the content of mechanics” (§311). The appeal of this scientific representation is that in our images of external objects inferred through the fundamental law the effects of continuous cyclical motion account for the effects of causes: “[i]n any new system of bodies which conforms to the fundamental law there is neither any new motion nor any cause of new motion, but only the continuance of the previous motion in a given simple manner” (§320). It is only for the sake of mathematical convenience that force enters “as a middle term conceived only between two motions” where “the motion of the first body determines a force, and ... this force then determines the motion of the second body” (28). Hertz’s scientific representation thus purports to eliminate our conceptions of cause and force by disciplining our imagination to form images of external objects in continuous motion.

Following his two-step evaluative process, Hertz claims his scientific representation is superior to those of Newtonian-Laplacian mechanics and energetics. For one thing it is complete. It shows that laws of thought comprising laws of intuition and logical forms are “given by the nature of our mind” and enter “into the images, in order that they may be permissible”. It also shows that rigid connections and a certain inertial motion are “contained in the results of experience, from which images are built up” and enter “into images for the sake of correctness”. And it shows that mass and laws of transformation are arbitrary “notations, definitions, abbreviations” that are “ascribed to the images for the sake of appropriateness”. For another thing its scientific image constitutes images of things that are permissible and correct, as well as more appropriate than those constituted by the other scientific images. Yet in light of its conventional elements and speculative hypotheses Hertz is forced to acknowledge that his scientific image is “not the only image of mechanics, nor yet the best image”, that he “only sought to find an intelligible image and to show by an example that this is possible and what it must look like” (33).

Hertz's scientific representation logically organizes the images of external objects it constitutes through a potential space of images. But just as we saw with Helmholtz, characterizing this space or any part of it as a *logical space* (as Wittgenstein uses the term) is not justified. There is no obvious relation to the self. And although it is mathematizable, the space is not mathematical – it requires the addition of a specific mathematical form. To this form we shall now turn our attention.

2.35 Models

Hertz presents a mathematical form for his scientific image, one admired by successors, in particular Lorenz, as a development in its own right (Lutzen 1994, 106). Its key feature is that, rather than the motion of a single point in three-dimensional space it considers the motion of a system of rigidly connected material points in multi-dimensional coordinate system, i.e., “[a] system of n points presents a $3n$ -manifold of motion, – although this may be reduced to any arbitrary number by the connections of the system”, that shares “many analogies with the geometry of the space of many dimensions” (Hertz 1900/1956, 30). Although this is “the origin of the modern geometric treatments of mechanics” (Lutzen 1994, 120), the idea did not arise *ex nihilo*. It is largely indebted to the development of multi-dimensional geometry by Cayley, Grassmann and Riemann that had preoccupied Helmholtz, as well as the preliminary extension of geometry to mechanics by Lipschitz and Darboux (Ibid., 112-118). More generally, it is a precursor to the modern concept of phase or state space, an idea which had been fermenting within the intellectual milieu and which developed fully a few years later.¹³⁶

Hertz did not consider his mathematical form to be a univocal expression of the logical form of his scientific image. He chose it for reasons of convenience, i.e., for 1) its “great simplicity and brevity”; 2) its ability to present the fundamental law in a way that does not suggest conceptions that could impede the overall attempt to improve our images of things, and; 3) the fact that it “gives a simple and intelligible explanation of ... analogies” “between ordinary mechanics and the geometry of space of many dimensions” (31-33).

Still, Hertz thinks the analogy to multi-dimensional geometry is only a formal one, for unlike the “supra-sensible abstractions” (32) of the latter his mathematical form represents the physical content of his scientific image. His idea is that inasmuch as his scientific image constitutes images of things, its mathematical form constitutes models that “encase” them by expressing their logical form. This hierarchical relation between model and image is implicit in the following definition:

418. Definition. A material system is said to be a dynamical model of a second system when the connections of the first can be expressed by such coordinates as to satisfy the following conditions:

- (1) That the number of coordinates of the first system is equal to the number of the second.
- (2) That with a suitable arrangement of the coordinates for both systems the same equations of condition exist.¹³⁷
- (3) That by this arrangement of the coordinates the expression for the magnitude of a displacement agrees in both systems.

Any two of the coordinates so related to one another in the two systems are called corresponding coordinates. Corresponding positions, displacements, etc., are those positions, displacements, etc., in the two systems which involve similar values of the corresponding coordinates and their changes.

419. Corollary 1. If one system is a model of a second, then, conversely, the second is also a model of the first. If two systems are models of a third system, then each of these systems is also a model of the other. The model of the model of a system is also a model of the original system. All systems which are models of one another are said to be dynamically similar.

(Note that equations of conditions of two systems will share the same number of independent variables¹³⁸ or dimensions, a point picked up by early Wittgenstein.) It must be remembered that this definition occurs after §305 where Hertz says that “material system” is to be understood going forward as “a system of concrete masses, whose properties are not inconsistent with the properties of the ideally defined material system (§121)”. In the context of this definition, therefore, “material system” is to be understood as the image of an external object. The equations of conditions, on the other hand, tell us that a dynamical model is an idealization.¹³⁹ Hertz even says that a dynamical model can be an abstraction from a material system.¹⁴⁰ It seems to me, therefore, that the basic point of this definition is to introduce a mathematical representation of the physical content of a material system that can enhance the predictive power of its logical form.

The corollary to the definition has the effect of encasing projected natural material systems within what might be called “natural dynamical models”. Hertz makes this fairly explicit in the following observation:

427. Observation 1. If we admit generally and without limitation that hypothetical masses (§301) can *exist in nature* in addition to those which can be directly determined by the balance [i.e., by weighing], then it is impossible to carry our knowledge of the connections of natural systems further than specifying models of the actual systems. We can, then, in fact, have no knowledge as to whether the systems which we consider in mechanics agree in any other respect with the actual systems of nature which we intend to consider, than in this alone, – that the one set of systems are models of the other. (italics added)

Using the language of idealized models, this observation captures what Hertz says at the outset of his introduction using the language of concrete images: “As a matter of fact, we do not know, nor have we any means of knowing, whether our conceptions of things are in conformity with them in any other than this *one* fundamental [requirement]”: “the consequents of the images must be the images of the consequents” (2). However, Lutzen (2005) claims the observation is meaningless. Given that the whole idea of the image theory is to allow for a plurality of images of nature, “it makes no immediate sense to claim that the external world is an image of our mental image” (109). But this ignores the conditional admission that unobservable masses, arrived at by extending our mode of imagination from observable masses, can exist in nature. This is not to claim that reality *is* an image; it is only to *project* an image onto reality. Through idealization the image projected and the projection itself can each be represented by a dynamical model. If all we can know about an external object is that it has a logical structure isomorphic to the empirical structure of our image of it, then given this projection and idealized representation all we can know about an external object is that it has a mathematical structure isomorphic to our dynamical model of it.

This helps us make sense of the important observation that immediately follows:

428: Observation 2. The relation of a dynamical model to the system of which it is regarded as the model, is precisely the same as the relation of the images which our mind forms of things to the things themselves. For if we regard the condition of the model as the representation of the condition of the system, then the consequents of this representation, which according to the laws of this representation must appear, are also the representation of the consequents which must proceed from the original object according to the laws of this original object. The agreement between mind and nature may therefore be *likened* to the agreement between two systems which are models of one another, and we can even

account for this agreement by *assuming* that the mind is capable of *making* actual dynamical models of things, and of working with them. (italics added §428)

In this observation Hertz speculates about why there is “the agreement between mind and nature” or why there is “the conformity between nature and our thought” (2). Since by his own definition a model represents another model representing it, Hertz analogizes the agreement between mind and nature to the relation between two such models. But if we now presuppose “that the mind is capable of making actual dynamical models *of things*, and of working with them” then by definition there are models in nature representing these models. This presupposition allows Hertz to go beyond the analogy to *explain* the conformity between mind and nature in terms of shared structure. But notice that the presupposition is only that the mind can make or *construct* dynamical models of things. The explanation, then, is that the mind is a form from which dynamical models are constructed, i.e., a *mathematical schema*, and that nature is a *real form* out of which natural dynamical models actualize. See Figure 2.3:

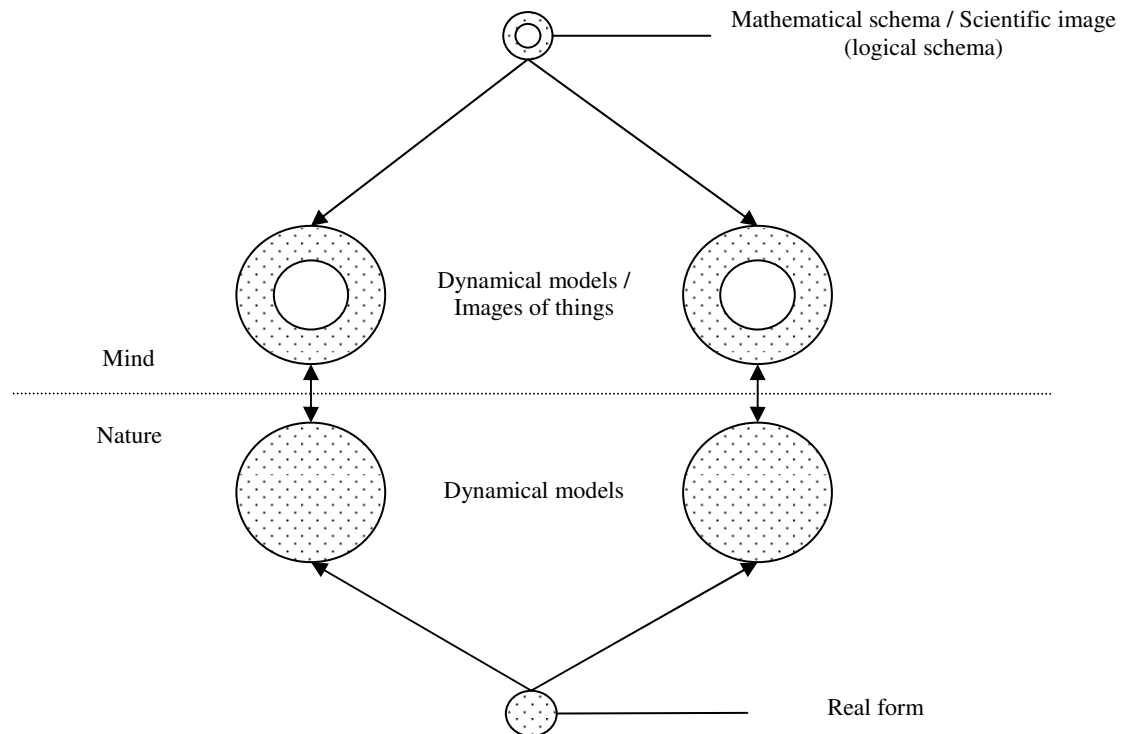


Figure 2.3 Explanation of Conformity between Thought and Reality

Since dynamical models express logical form, this explanation allows Hertz to account for his theory of representation where taking an image of an external object to be symbolically adequate involves projecting empirical (logical) structure onto nature and committing to its existence. Hertz's idea of a mathematical schema sharing the structure of real form out of which models are constructed that (in virtue of expressing a logical schema) logically organizes a potential space of all models is, I suggest, the intellectual precursor to Wittgenstein's idea logical space.

2.4 Conclusion: Shifting Mirrors

I shall conclude this chapter with a summary of the developments in the organization of images that Helmholtz and Hertz introduce leading to Wittgenstein's idea of logical space. Helmholtz's physiological investigations led him to reject Kant's notions of pure intuition and synthetic imagination. He nonetheless retained the notion of causality as a pure concept, replacing its transcendental schema with a regulative one presenting a transcendental law of causality. As a result, Kant's potential space of classification was transformed into a potential space of causal reasons, each place occupied by a concept formed in the interpretation of an image and described by the major premise of an inference that instantiates the regulative schema. At the pre-conscious level, the regulative idea causality organizes a much larger potential space of all images, each place being occupied by an image impressed into memory through repeated regularity of kinds of sense impressions. Each image functions as the major premise of a syllogistic inference that instantiates the regulative schema. Functioning as the major premise of the logical schema itself is an original image, an ideal of our investigative acts.

In his criticism of the notion of force, Hertz replaces Helmholtz's regulative idea *causality* (and its principle of causality) with fundamental ideas connected by a fundamental law, what he calls a scientific representation. Yet, he maintains Helmholtz's view of a concept in the physical sciences as a syllogistic inference and, in equating conceiving and imagining, takes an image of a thing to be a deductive inference. As a result, Hertz correlates Helmholtz's original image with the logical schema of his

scientific representation. Not only does the scientific representation organize a potential space of all images, it constitutes them through interpreted forms of internal intuition and an a priori definition. For reasons to do with predictive convenience, Hertz introduces the form of a multi-dimensional coordinate system that can express his scientific image, and from this mathematical schema dynamical models are said to be constructed that can express the syllogistic form of images of things. In turn these mathematical models are organized logically according to the organizational structure of these images. In order to explain the predictive conformity between thought and reality, Hertz speculates that mind and nature share the same (logico) mathematical structure: the former a thought schema;¹⁴¹ the latter real form.

Chapter 3: From Model to Mysticism

(The thought forces itself upon one): The thing seen sub specie aeternitatis is the thing seen together with the whole of logical space.

Notebooks (1914-1916/1961, 7.10.16; 83)

3.1 Introduction

The term “logical space” enters philosophy with neither definition nor elaboration in a few terse propositions found within Wittgenstein’s *Tractatus Logico-Philosophicus*. The period of time between Hertz’s *Principles of Mechanics* (1900/1956) and Wittgenstein’s *Tractatus* (1921/1961)¹⁴² is marked by the development of polyadic logic in Frege and Russell’s attempt to reduce mathematics to logic. The *Tractatus* is motivated by Wittgenstein’s puzzlement over the nature of this new logic given the problems which its founders encountered. A strange and brilliant philosophical work, it was written during Wittgenstein’s service in the Austrian army in World War I and first published in 1921 with the help of Russell. Notwithstanding its influence in the development of logical empiricism and philosophy of science in general, many would agree that the *Tractatus* “may well be the most difficult philosophical book written this century” (Coffa 1991, 142).

At least part of the difficulty with the *Tractatus* lies in the fact that, despite his admission to having made “no claim to novelty in detail” (pref. 3),¹⁴³ Wittgenstein was apparently reluctant to credit influences for his claim to novelty in general.¹⁴⁴ He does express a debt to Frege and Russell, but only for the “stimulation” (Ibid.) of his thoughts. With little else to go on, the general tendency in the commentary has been to situate the *Tractatus* mainly in the analytic tradition, approaching it as an extension of the logical considerations of Frege and Russell within a general philosophical movement to separate semantics from epistemology.¹⁴⁵ For many readers, however, this way of accessing the book is not very satisfying. “Part of the *Tractatus*’s fascination lies in its elusive unity” (Glock 1996, 365). Through a relatively small number of propositions it proffers “the final solution” (pref. 5) to the problems of philosophy. These include problems of

solipsism, realism, ethics, aesthetics, and mysticism that give rise to what most consider to be the book's most intractable claims.

As a result, philosophers have looked to influences other than Frege and Russell for insight. Two of them are Hertz and Schopenhauer. Hertz is mentioned at two points in the *Tractatus* (4.04, 6.361), and references to scientific “pictures of reality” appear in Wittgenstein's writing as early as 1913.¹⁴⁶ The substantial affect Hertz's *Principles* had on Wittgenstein's picture theory of meaning is now routinely acknowledged.¹⁴⁷ Unfortunately, Schopenhauer is not mentioned in the *Tractatus*, but he is referred to at one point in Wittgenstein's *Notebooks* (1961a, 2.8.16; 79). As a result, the extent of Schopenhauer's influence has come to be appreciated only relatively recently. Commentators who examine the influences of Hertz and Schopenhauer consider them separately, but the real significance of their influence upon the *Tractatus* becomes clear only when we consider them together.

In this chapter I elucidate the *Tractatus* around the idea of logical space as an autonomous development of Schopenhauer's conception of the transcendental self and Hertz's speculation about a (logico) mathematical thought schema that shares the form of reality. After exegesis of relevant propositions, I demonstrate that Wittgenstein's idea of logical space *is* his notion of the self, one that traces to Schopenhauer's transcendental self as will. I argue that this self has a perspectival relation to the world, shifting timelessly from a potential whole of logical space associated with “pure realism” to a completed whole of logical space associated with solipsism that has the structure of a concave mirror of reality. As the former, logical space is a development of Hertz's idea of a thought schema; as the latter, logical space is a development of Kant's spherical structure of the imagination. I justify my interpretation of logical space by extricating Wittgenstein's propositions concerning solipsism, realism, ethics, aesthetics and mysticism and bringing them seamlessly together with his picture theory of meaning.

3.2 The “Whole Sense” of the *Tractatus*

In the preface to the *Tractatus* Wittgenstein describes its central thesis:

The book deals with the problems of philosophy, and shows, I believe, that the reason why these problems are posed is that the logic of our language is misunderstood. The whole sense of the book might be summed up in the following words: what can be said at all can be said clearly, and what we cannot talk about we must pass over in silence.

Thus the aim of the book is to draw a limit [*Grenze*] to thought, or rather – not to thought but to the expression of thoughts: for in order to be able to draw a limit to thought, we should have to find both sides of the limit thinkable (i.e., we should have to be able to think what cannot be thought).

It will therefore only be in language that the limit can be drawn, and what lies on the other side of the limit will be nonsense. (pref. 3)

The “whole sense” of the book is captured more precisely by the following claims: 1) “What *can* be shown *cannot* be said” (4.1212); 2) what can be said can be said only by “means” (2.22) of what can be shown; and 3) what is said without means of what can be shown is nonsense. As I understand Wittgenstein, 1) what can be shown is logical space; 2) what can be said is a picture in logical space; and 3) the completed “whole of logical space” (3.42) circumscribes the *Grenze* (“limit” or “bound”¹⁴⁸) we run up against in the expression of our thoughts.

The point to recognize is that the whole sense of the *Tractatus* is said without means of what can be shown. Unlike the propositions of natural science (4.11), the propositions of the *Tractatus* are not pictures in logical space. Accordingly, Wittgenstein asserts at the end of the book that anyone who understands the propositions of the *Tractatus* “recognizes them as nonsensical” (6.54). But then any prefatory summary of its whole sense is nonsensical as well. Yet Wittgenstein still claims that “the *truth* of the thoughts that are communicated” are “unassailable and definitive” (pref. 5), and so there is a sense in which it has “sense” after all. To grasp the whole sense of the *Tractatus*, I suggest, is to recognize the coherent *method* behind its apparent madness.

The place to begin is with the phrase “whole sense” itself. Given the picture theory of meaning that the *Tractatus* presents, *prima facie* a summary of the whole sense of the *Tractatus* is a summary of a picture. “Thus the aim of the book is to *draw* a limit ... to the expression of thought”. To draw this limit is not to present a picture *in* logical space, however, but to present a picture *of* logical space. Notice the equivocation in the use of the word “picture” this generates. The first use of “picture” is in accord with its use in the *Tractatus*: a picture *in* logical space is a meaningful representation of reality. The second use of “picture”, however, is along the lines of our use of “model” in formal

semantics: a picture *of* logical space is an imaginative structure that interprets or satisfies the propositions of the *Tractatus*. Obviously such an interpretation does not make the propositions of the *Tractatus* meaningful representations of reality. They are, strictly speaking, nonsense. But it does give them a “sense” by which Wittgenstein can communicate to the reader a way to realize something of value.

What this chapter will show is that the method of the *Tractatus* is to take the reader along a journey of *self*-discovery of her existential relation to the world. The journey involves shifting the vantage point from which the reader orients her self to the world *as* logical space. Prior to proposition 6.4, the picture *of* logical space that satisfies the propositions of the *Tractatus* can be represented along the lines of Figure 3.1:

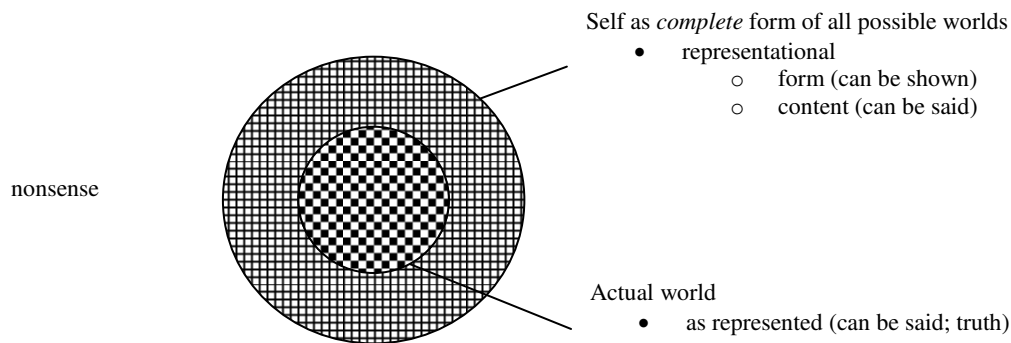


Figure 3.1: “Whole Sense” of the *Tractatus* prior to Proposition 6.4

Here logical space is a completed space of all possible worlds serving as the outer boundary of the actual world that is its own inner boundary. The outer boundary is the self of solipsism expressing itself imaginatively in representational thought. Like a great spherical concave mirror that surrounds the world, it reflects the world towards a *focus imaginarius* at its centre. This picture is not intended to convey information about the actual world, but only a “sense” of our representational relation to it in order to understand the logic of language and eliminate philosophical problems.

However, the goal of the *Tractatus* is not logical; rather “the book’s point is an ethical one”.¹⁴⁹ At proposition 6.4 there is a shift to aesthetics (6.41), ethics (6.42-6.43)

and mysticism (6.44). After proposition 6.4 the picture of logical space that satisfies the propositions of the *Tractatus* can be represented along the lines of Figure 3.2:

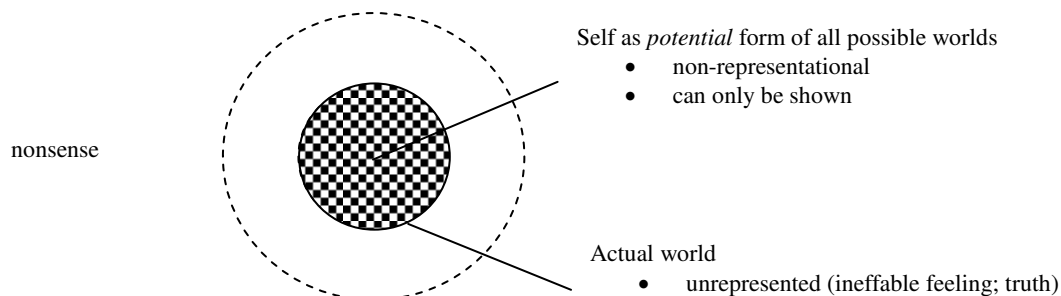


Figure 3.2: “Whole sense” of the *Tractatus* after Proposition 6.4

Here logical space is a potential space of all possible worlds serving as the inner limit at the centre of the actual world that is its own outer boundary. The inner limit is the self of pure realism willingly ceasing to express itself imaginatively in representational thought. From this vantage of the world the self exists together with reality as a form of potentiality. Again, this picture is not intended to convey information about the actual world, but only a “sense” of our existential relation to it.

When Wittgenstein tells us at proposition 6.54 that his own propositions are nonsensical, we realize that the ethical goal of the *Tractatus* is something only the reader can do. The *Tractatus* has led the reader to a sort of precipice where she must willingly discard the *Tractatus* along with all propositional thought in order for her *self* to experience existential truth. The aim of the *Tractatus*, that is, is to bring about a *feeling* of our existential relation to reality that is the basis of all representational truth, that *is* truth. To understand the *Tractatus* is thus to “transcend” it and “see the world aright” (6.54) through an act of self that experiences existence as ineffably mystical.

3.3 “Reality”, “World” and “Exist”

The *Tractatus* begins with our representational relation to reality and moves us towards our existential relation to it. In the process, it presents a picture of logical space in which the self shifts from outer boundary to inner limit of the world. This is not to say that one can conflate “reality” with “world”. Reality is both “form and content” (2.025). “Form is the possibility of structure” (2.033); content is the presence together with the absence of actual structure. Real content is the actualization of the potentiality determined by real form.

The content of reality given to us as the “totality of facts” (1.1) is the world. A “fact” is “the existence of states of affairs” (2.01). Since what is actual determines what is not actual, “[t]he totality of existing states of affairs also determines the states of affairs that do not exist” (2.05). What might be called an *elementary* fact is the existence of a single state of affairs.¹⁵⁰ Elementary states of affairs are independent of one another: “[f]rom the existence or non-existence of one state of affairs it is impossible to infer the existence or non-existence of another” (2.062). Hence the world is the totality of independent elementary states of affairs.

This leads us to Wittgenstein’s use of the term “exist”. In general there appear to be two kinds of existence: real existence and representational existence. For each there is existence that pertains to content and existence that pertains to form. In the case of real existence in relation to content, a state of affairs that obtains is said to exist because it “is a combination of objects (things)” (2.01). In an existing state of affairs there are “simple objects” (2.02) that “fit into one another like links of a chain” (2.03) producing “material properties” (2.0231). Note that though “[i]n a state of affairs objects stand in a determinate relation to one another” (2.031), a state of affairs is not “anything third that connects the links”.¹⁵¹

In addition to content, real existence pertains to form. Simple objects are substantive: “Objects are what is unalterable and subsistent; their configuration is what is changing and stable” (2.0271). And they have an essence: “the possibility of the states of affairs must be written into the thing itself” (2.012). This is to say a simple object has “internal properties” in contrast with the “external properties” produced through its

configuration with other objects (2.01231). These internal properties constitute the form that determines the potentiality of simple objects to combine with one another. Insofar as a simple object is part of an actual configuration of simple objects, its internal properties are said to “exist” (2.0121). But Wittgenstein also says, “What I once called ‘objects’, simples, were simply what I could refer to without running the risk of their possible non-existence; i.e., that for which there is neither existence nor non-existence”.¹⁵² What he seems to be saying is that insofar as a simple object is *not* part of an actual configuration of simple objects, its internal properties neither “exist” nor do not “exist”. (We shall see why he might say this when we examine his argument for simple objects.)

Representational elements configure to form a “picture”. Just like states of affairs, “[a] picture is a fact” (2.141). Wittgenstein attributes “existence” to this fact (3.4) even though it represents only “a possibility of existence and non-existence of states of affairs” (2.201). What might be called an *elementary* picture is an elementary configuration of representational elements. A central claim of the *Tractatus* is that the existence or non-existence of an elementary picture is independent of the existence or non-existence of any other elementary picture. In virtue of internal properties of its elements, pictures have a “pictorial form” (2.15), an essence to which Wittgenstein also attributes “existence” (5.131). (Later I will elaborate upon this notion of existence in relation to the existence of the self.)

3.4 Elementary Pictures, Propositions and Logical Spaces

With this understanding of “reality”, “world” and “exist”, let us turn to Wittgenstein’s idiosyncratic use of “picture”. For Wittgenstein “picture” and “proposition” are interchangeable. A proposition is a picture:

4.01 A proposition is a picture of reality. A proposition is a model of reality as we imagine it.

4.016 In order to understand the essential nature of a proposition, we should consider hieroglyphic script, which depicts the facts that it describes. And alphabetic script developed out of it without losing what was essential to depiction.

And a picture is a proposition:

When I wrote ‘A proposition is a logical picture of a fact’ I meant that I could insert a picture, literally a drawing, into a proposition and then go on with my proposition. I could accordingly use a picture in the same way as a proposition. How is that possible? The answer is, just because both agree in a certain respect, and what they have in common is what I call a *picture*. (December 9, 1931, as recorded by Waismann (1979, 185))

Note that Wittgenstein conceives a picture more generally as a mathematical extension of a drawn picture:

Here the expression ‘picture’ is already taken in an extended sense. I have inherited this concept of a picture from two sides: first from a drawn picture, second from the picture of a mathematician, which already is a general concept. For a mathematician talks of picturing in cases where a painter would no longer use this expression. (December 9, 1931, as recorded by Waismann (1979, 185))

What appears to underwrite the commonality between a picture and a proposition is that a mental element corresponding to reality is taken to correspond to an individual element in each of them:

I don’t know *what* the constituents of a thought are but I know *that* it must have such constituents which correspond to the words of language. Again, the kind of relation of the constituents of the thought and of the pictured fact is irrelevant. It would be a matter of psychology to find out. ... [A thought consists] of psychical constituents that have the same sort of relation to reality as words. (Wittgenstein 1961a-130)

In virtue of the relation between mental elements and reality, a proposition (expressive thought) and a picture (what can be understood as an “expressive image”) are both the same *model* of reality. A model is thus mental, and what is common between a picture and a proposition is the *form* that imaginative thought shares with reality.¹⁵³ This form is their “common logical pattern” and presents a rule or “law of projection” (4.0141). Driving Wittgenstein seems to be the intuition that just as geometrical analysis (or, more generally, analysis in the physical sciences) reveals elements of pictures, logical analysis will reveal the elements of propositions (4.221), and these analyses are essentially the same because psychological analysis will reveal common elements of imaginative thought relating to reality as the basis of both sorts of representational elements.

It is important to recognize that Wittgenstein does not share Hertz’s concern to discipline the imagination in our choice of a scientific image, nor is he hamstrung by monadic logic. Wanting more generally to understand the nature of the new polyadic logic Frege and Russell developed, Wittgenstein brings into his notion of a picture a

general feature from each of the Hertzian image and dynamical model. Although he does not take a picture mechanistically to be a material system of rigidly connected material points, it seems Wittgenstein does conceive the content of a picture to be a point or a collection of points *simpliciter*.¹⁵⁴ And while he rejects a notion of a model as a convenient mathematical form to express the logical form of an image, Wittgenstein adopts the form of a multi-dimensional coordinate system as part of the form of a picture.

The form of a Wittgensteinian picture is not exclusively mathematical, however. “Every picture is *at the same time* a logical one. (On the other hand, not every picture is, for example, a spatial one.)” (2.182). What makes a picture logical is not that it has the form of a syllogistic inference, but that it is constructed through the iterative operation of a “general propositional form” (4.5). This iterative operation also makes a picture mathematical (6.02f)¹⁵⁵ and, moreover, occurs within a multi-dimensional coordinate system of sensible properties. Together the general propositional form and the coordinate system constitute a picture’s “logico-pictorial” (2.2) form or “pictorial form” (2.15) for short. In the case of an elementary picture we might call this form an *elementary* logical space. The remaining part of this section will focus on an elementary coordinate system, leaving the general propositional form for the section that follows.

To understand this aspect of the form of the Wittgensteinian picture we need to bring together a few more propositions:

- 2.12 A picture is a model of reality.
- 2.2 A picture has logico-pictorial form in common with what it depicts.
- 2.201 A picture depicts reality by representing a possibility of existence and non-existence of states of affairs.
- 2.202 A picture represents a possible situation in logical space.
- 2.203 A picture contains the possibility of the situation that it represents.
- 4.04 In a proposition there must be exactly as many distinguishable parts as in the situation that it represents. The two must possess the same logical (mathematical multiplicity). (Compare Hertz’s *Mechanics* on dynamical models.)

As the last proposition tells us, a central connection to Hertz is found in the notion of multiplicity. In general, Wittgenstein conceives “multiplicity” to be the number of

independent measurements needed to specify a configuration of elements.¹⁵⁶ In its two related sentences, proposition 4.04 captures this notion in subtly different ways. The sense of multiplicity in the second sentence is one where “a particular mathematical multiplicity” is equivalent to “a particular number of dimensions” (5.475). This signifies the adoption of Hertz’s implicit view found in his definition of a dynamical model that a model represents only if what it is represented shares the same number of dimensions. To understand how this sense of multiplicity is related to the number of “distinguishable parts” in the first sentence of proposition 4.04, we must first recognize how propositions 2.202 and 2.203 capture pictorial form from two points of view.

We can think of a picture as “representing a possible situation *in* logical space”, in which case the number of independent measurements needed to specify the picture simply refers to the number of dimensions of the coordinate space. But we can also think of a picture as “*containing* the possibility of the situation it represents” in which case the number of independent measurements needed to specify the picture refers to the number of “distinguishable parts” in a picture that constitute its essence (cf. 3.34). Since “part” of a picture is a representational element, and since a part is “distinguishable” if it has unique internal properties (2.0233f), the number of independent measurements needed to specify a picture comes down to the number of different *kinds* of representational elements in the picture. The two points of view are equivalent: a picture *in* logical space *displays* the internal properties *in* a picture. Proposition 4.04 amounts to the claim that the number of different kinds of representational elements that configure into a picture must be the same as the number of different kinds of simple objects that configure into the state of affairs pictured.¹⁵⁷ That this can only be *shown* (cf. 4.041) is captured by a picture *in* logical space that *displays* this shared essence by means of the dimensionality of the space.

As I understand Wittgenstein, what such a display amounts to can be illustrated using a simple mirror. Imagine two different kinds of simple objects (i.e., simple objects that have distinguishable internal properties) that “fit into one another like links of a chain”. Imagine further that the real existence of this state of affairs is reflected as “speck” of light in an otherwise dark mirror. Now suppose two yardsticks make up the

left and bottom edges of the mirror. If we take the left edge of the mirror as the y axis and the bottom edge as the x axis, the speck of light is a point of reflection of the mirror-surface, say point $x = 2$, $y = 3$. This point is an *elementary* configuration of representational elements. The coordinate value $x = 2$ constitutes one link of the configuration, while the coordinate value $y = 3$ constitutes the other link.¹⁵⁸ The form of the representational element $x = 2$ is expressed (in part) by coordinate axis y since it can combine with any object represented by any value of the y axis; the form of representational element $y = 3$ is expressed (in part) by coordinate axis x since it can combine with any object represented by any value of the x axis. We could do away with the mirror and think of these coordinate values as having essences that constitute the essence of point $x = 2$, $y = 3$. But the mirror is a space of a kind of possibility that “displays” (2.172) pictorial form by expressing the essences of its representational elements, without which we cannot imagine them at all.

In displaying the essential form of its representational elements, the mirror also shows the essential form of the two linked objects it reflects. The value $x = 2$ corresponds to one simple object, while the value $y = 3$ corresponds to the other. This is why, I suggest, Wittgenstein gave “three knocks” and “6 feet” as examples of simple objects.¹⁵⁹ The mirror expresses the potential of each object to combine with the other kind of object as a complete two-dimensional space of possibility. Thus the mirror is also a space that *represents* a kind of *real* possibility: “[a] picture can picture any reality whose form it has. A spatial picture can depict anything spatial, a coloured one anything coloured, etc.” (2.171) But proposition 4.04 tells us that a picture can represent a configuration of simple objects only if its dimensionality is equal to the number of different kinds of simple objects that are linked together. And so even if the potential of the simple objects were infinite so that the mirror displaying their form is infinitely large, shared multiplicity between the representation and what is represented is nonetheless a *bound* of what can be shown.

In this example, a two-dimensional coordinate system is the “means” (2.22) by which a representational point “depicts” (2.201) the elementary configuration of simple objects. The point $x = 2$, $y = 3$ on the mirror’s surface is an elementary picture that

reflects an existing state of affairs. Not only does the state of affairs exist, however, so does the point:

If a point in space does not exist, then its coordinates do not exist either, and if the coordinates exist the point exists too. That is how it is in logic. (Wittgenstein 1961a, 21.6.15; 69)

3.032 It is impossible ... to give the coordinates of point that does not exist

In other words, a point exists if it is the configuration of specific coordinate values; specific coordinate values exist if they configure into a point.

For the sake of explanatory simplicity, I have used a mirror to illustrate an elementary logical space in a way that may be misleading. One might be left with a Helmholtzian “bottom-up” point of view that (like a mirror) a configuration of simple objects gives rise to a picture in an elementary logical space, generating a representation that is *ipso facto* true. The opposite is the case. More like Hertz’s “top-down” approach, a picture is constructed in an imaginative act and compared with reality. The constructive process is what Wittgenstein refers to as the “method of projection” involved in the expression of thought: “[t]he method of projection is to think the sense of a proposition” (3.11). It is the method by which the content of a picture is constructed *in* logical space (cf. 3.12, 3.41). By means of the form of a multi-dimensional coordinate system, pictorial content arises in the relation of “elements” (2.14), i.e., coordinate values, into a “nexus” or “concatenation” (4.22) that is a “structure” (2.15). The way this structure arises is expressed by a “law of projection” (cf. 4.0141).

Once constructed through the method of projection the picture has a “sense” (2.221) or truth-possibility (4.3). It “presents a situation in logical space” (2.11) and thereby presents its sense; it also “represents a possible [real] situation in logical space” (2.202) and thereby “represents ... its sense” (2.221). In this way a picture both “*shows* how things stand *if* it is true” and “*says that* they do so stand” (4.022) serving as a vehicle of information:

4.024 To understand a proposition means to know what is the case if it is true. (One can understand it, therefore, without knowing whether it is true.) It is understood by anyone who understands its constituents.

Even though a picture may not be true, as noted already its pictorial elements exist and, moreover, have objects that correspond to them (2.13). Yet independent from one another, pictorial elements have no meaning. Their meaning comes from their configuration with other pictorial elements into a structure (3.3) when the sense of the picture is thought.

In a picture “a situation is, as it were, constructed by way of experiment” (4.031) to be compared with existing states of affairs. In making this comparison the picture is “laid against reality like a measure” (2.1512). Through “feelers of the picture’s elements” (2.1515), coordinate values are correlated with simple objects: “[o]nly the end-points of the graduating lines actually *touch* the object that is to be measured” (2.15121). In virtue of these feelers a picture is a “tableau vivant” (4.0311) that “can be perceived by the senses” (3.1). What is perceived by the senses are “material properties” “produced” “by the configuration of objects” (2.0231). If the same configuration of objects obtains, the picture is felt as “true”; if not, it is felt as “false” (2.21). A picture in its “projective relation to the world” (3.12) is thus an “argument-place” (2.0131), and experience is the measurement of its truth or falsity. A picture is meaningful precisely because it is an *attempt* to measure the world *in* logical space (3.4).

It was indicated already that in this attempt each pictorial element of a representational configuration must correspond to a simple object whose real configuration is being assessed through the senses, otherwise it would not be the case that “one can understand [a picture] ... without knowing whether it is true a false”. But notice that the possibility of such a correspondence must be prior to the construction of the picture:

2.151 Pictorial form is the possibility that things are related to one another in the same way as the elements of the picture.

2.1511 *That* is how a picture is attached to reality; it reaches right out to it.

Wittgenstein is saying is that prior to the construction of a picture, i.e. prior to the projection of a configuration of mental elements common to both a proposition and a picture, there is already shared form between mind and reality. Moreover, this shared form is transcendental in character: it is a condition for the possibility of expressive

imaginative thought. What will be established later is that the method of projection presupposes that the “self” is “co-ordinated” (5.64) with reality. Wittgenstein is not specific about what he means by “coordination”, but implies that it pertains to shared form displayed in logical space (cf. 3.42).¹⁶⁰ It seems that it is in virtue of such prior coordination that “feelers” from pictorial elements (coordinate values) will reach out and “touch” only certain simple objects.

Wittgenstein’s radical development and generalization of Hertz’s image and model was probably influenced by and through Boltzmann.¹⁶¹ As Janik and Toumin (1993) summarize:

Boltzmann took Hertz’s account of mechanics as defining a system of “possible sequences of observed events,” and made it the starting point for a general method of theoretical analysis in *physics itself*. He did so by treating each independent property of a physical system as defining a separate coordinate in a multi-dimensional system of geometrical coordinates. (143)

Boltzmann’s method of physical analysis as an extension of geometrical analysis appears to have had an impact on how Wittgenstein understood logical analysis:

Only when we analyze phenomena logically shall we know what form elementary propositions have. Here is an area where there is no hypothesis. The logical structure of elementary propositions need not have the slightest similarity with the logical structure of propositions. Just think of the equations of physics – how tremendously complex their structure is. Elementary propositions, too, will have this degree of complexity. (December 22, 1929, as recorded in Waismann (1979, 42)

(By “degree of complexity” Wittgenstein likely means multiplicity, since this is determined by the number of independent variables in the equations of physics.) And in a limited way Boltzmann’s idea that analysis can reveal different observable properties corresponding to different dimensions of a form of representation seems to have influenced Wittgenstein’s characterization of elementary logical spaces.

“Space, time, and colour (being coloured) are forms of objects” (2.0251), and so there are “spatial objects” and “temporal objects” (2.0121) and (presumably) “colour objects” whose form is displayed in elementary logical spaces. In addition to space, time and colour, Wittgenstein presents elementary logical spaces for sound and touch (2.0131) even stomach-ache.¹⁶² Notice that all these spaces are associated with sensation, some of which are private and not all of which are involved in the physical sciences. Moreover,

notice there is no mention of spaces associated with measurable properties gathered through instrumentation, such as temperature and pressure that occupied Boltzmann. This suggests Wittgenstein's idea of logical space has one foot in the Kantian tradition exemplified by Helmholtz and Hertz that concerns itself primarily with representations in relation to ordinary and unmediated sensation.¹⁶³

As emphasized already, Wittgenstein takes the imaginative act of constructing a picture to be a propositional act of expressive thought. His account of elementary propositions is thus a linguistic elaboration of this account of elementary pictures. An elementary proposition “contains the form, but not the content, of its sense” (3.12-3.13), i.e., it is not a mere “set of names” (3.142) introduced as a matter of convention (4.002), but also an “essence” (4.013). In the method of projection the following arises: 1) the names “stand in a determinate relation to one another” (3.14) forming a “nexus” (3.3) or “concatenation” (4.22) constituting the content of sense, i.e., a “propositional sign” (3.12) that is “perceptible” (3.11) and that stands in a “projective relation to the world” (3.12); 2) in virtue of feelers, “elements of the propositional sign correspond to the objects of the thought” (3.2f); 3) a simple object is the “meaning” of a name (3.203) (now a “simple sign” (3.202) or “primitive sign” (3.26)); and 4) the propositional sign has “logical coordinates” (3.41) that “determines a place in logical space” (3.4). Logical space thus means by which a proposition says “*how* things are, not *what* they are” (3.221).

Wittgenstein makes a direct appeal to the idea of logical space in his account of the meaning of negated propositions. “The propositions ‘*p*’ and ‘ $\sim p$ ’ have opposite sense, but there corresponds to them one and the same reality” (4.0621). Yet the negation of an elementary proposition is not an elementary proposition (4.211). Rather, negating a proposition “determines a logical place *different* from that of the negated proposition” by describing “it as lying outside the latter’s logical place” (4.0641).

Let us end this section with a careful examination of the relation between an elementary logical space and reality. The form of a kind of object is displayed in an elementary space:

2.013 Each thing is, as it were, in a space of possible states of affairs. This space I can imagine as empty, but I cannot imagine the thing without the space.

2.0131 A spatial object must be situated in infinite space. (A spatial point is an argument – place.) A speck in a visual field, though it need not be red, must have some colour; it is, so to speak, surrounded by colour-space. Notes must have *some* pitch, objects of the sense of touch *some* degree of hardness, and so on.

It is not that each thing is in a “space of possible *things*”, but in a “space of possible *states of affairs*”, i.e., a space of possible *configured* things. A coloured speck, a spatial point, etc. are *not* things, but configurations of things: “objects are colourless” (2.0232), spaceless, etc. in that colour, position, etc. are material properties produced in configuration of certain kinds of objects. And these configurations are represented as true pictures by means of a kind of elementary logical space.

Since by definition a Hertzian model represents another model that represents it, one might be inclined to take proposition 2.013 as saying that an imaginative logical space represents a *real* logical space that, in turn, represents it. The proposition, “[t]he facts in logical space are the world” (1.13) might be cited in support.¹⁶⁴ Even if this were the way Wittgenstein understood reality, the claims made in this chapter would still stand with only minor qualifications. This interpretation, however, should be resisted for at least three reasons. The “as it were” in proposition 2.013 suggests that a thing “in a space of possible states of affairs” is simply a *projection* onto reality. In addition, while logical space is associated with the method of projection in which the sense of a proposition is thought, nowhere does Wittgenstein mention such a constructive process for reality. And the third reason is that Wittgenstein seems only to be saying that an elementary logical space of things is how we imagine the potentiality of reality:

2.0121 ... Just as we are quite unable to imagine spatial objects outside space or temporal objects outside time, so too there is *no* object that we can imagine excluded from the possibility of combining with others ...

What proposition 2.013 is saying, I suggest, is that I *can* imagine an elementary space as “empty” because I can imagine it without a picture in virtue of my prior coordination with reality. That I *cannot* imagine a thing without the elementary space is because I cannot imagine its essential properties apart from a space of possibility that displays them.

As I understand Wittgenstein, the method of projection involved in thinking the sense of a proposition is a projection by virtue of displaying the form of reality in the imagination and extending feelers from elements of the proposition to simple objects of a

certain kind. While we might regard the picture as *containing* an essence (just as we might regard a configuration of simple objects to be *in* real logical space), it is nonetheless the case that we think a picture *in* logical space (just as the configuration of simple objects *contains* an essence). It is the method of projecting or constructing a picture in logical space that makes an imaginative thought an *act* of expression.

3.5 Molecular Logical Space

It would seem that elementary pictures in elementary logical spaces are connected into composite pictures in combined logical spaces through the iterative operation of a general propositional form. This form is represented by a generalized version of the Sheffer stroke (cf. 5.1311) that takes elementary propositions as base, and connects them into composite propositions through “a finite number of truth-operations” (5.32). It applies even in the case of an elementary proposition, which “is a truth function of itself” (5). It is a structural component, in addition to a multi-dimensional coordinate system, constituting pictorial form that operates in the method of projection when the sense of a proposition is thought.

The general propositional form has a number of interesting features. Like the points it connects in the coordinate system, Wittgenstein attributes “existence” (4.5) to it. As we shall see later on, this is related to the timelessness of its iterative operation: “[i]n logic process and result are equivalent” (6.1261). The iterative operation of the general propositional form presupposes a multi-dimensional coordinate system:

5.475 All that is required [for logical operations] is that we should construct a system of signs with a particular number of dimensions – with a particular mathematical multiplicity.

And the coordinate system (as mentioned already and discussed more fully below) presupposes coordination between the self and reality. So when Wittgenstein denies that “negation, logical sum, logical product, etc. ... introduce more and more new elements – in coordination” (3.42), we should take him to mean that the addition of structure to a coordinate system by the general propositional form simply expresses the prior coordination between the self and reality.

With the general propositional form we understand why the Wittgensteinian picture integrates certain features from the otherwise distinct Hertzian notions of an image and model. Frege and Russell's development of polyadic logic enabled logic to express the infinity Kant thought necessary to relegate to forms of intuition, and so there no longer remained a compelling reason to keep mathematics separate from logic. While Kant had sought to distinguish mathematics from logic on the basis of temporal iterative operations of the imagination, Wittgenstein brings them together with the atemporal iterative operations of the general propositional form.¹⁶⁵ His idea is that "[a] number is the exponent of an operation" (6.021) and refers to a stage in the application of the iterative operation to a base. Understood this way, "[m]athematics is a method of logic" (6.234) shown to be an aspect of the general propositional form through the method of projection. Having been projected in a finite number of operations, a picture is thus a logico-mathematical integration presenting a certain number in logical space.

Following the terminology of a molecular proposition, let us call a composite picture a *molecular picture*, the essence of which is displayed in a *molecular logical space*. Whether a molecular picture is true or false – i.e., whether it represents a composite "fact" or "the existence of states of affairs" (2) – is a function of the truth of the elementary pictures that constitute it. A truth-function specifies a set of "truth-grounds" that "determine a range that it leaves open to the facts" (4.463). A "place" in molecular logical space shows the operation of the general propositional form as a truth table, which in turn shows the sense of a molecular proposition through its truth possibilities.

In the case of an elementary picture in elementary logical space, I maintain, a place is a point. Since a molecular picture is a truth function of elementary pictures, a molecular logical space is a truth function of elementary logical spaces. In the case of a molecular picture, a conjunction say, composed from different kinds of elementary pictures, a place would be single point in a composite molecular logical space (e.g., a speck of red dust is a point in a molecular logical space combined from the elementary logical spaces for colour and space). In the case of molecular picture composed from

elementary pictures of the same kind, a place must be a truth-functional connection of a set of points, i.e., a region, within the elementary logical space.

“My fundamental idea”, Wittgenstein says, “is that ‘logical constants’ are not representatives; that there can be no representatives of the *logic* of facts” (4.0312). Instead “[t]he propositions of logic are tautologies” (6.1), “analytic propositions” (6.11) that reflect the “logical form” (4.12, 6.12) of simple objects and show how propositions are interrelated. They describe the truth-functional structure of logical space, what Wittgenstein refers to as “scaffolding” in his claim that “scaffolding surrounding a picture” (3.42) represents the “scaffolding of the world” (6.124). It should be clear by now why he adds that these representations “presuppose that names have meaning and that elementary propositions have sense; and that is their connection with the world” (Ibid.): not only does operation of the general propositional form presuppose prior coordination between self and reality, it presupposes that its infusion of truth-functional structure occur within the structure of a multi-dimensional coordinate system in a method of projection.

That truth functional structure presupposes coordinate structure throws light on Wittgenstein’s view of time, one worth canvassing as a development of Kant’s pure sensible schema *time* and as a prelude to van Fraassen’s early view that time is a logical space. As mentioned Wittgenstein took time to be the form of a kind of simple object. Inasmuch as colour objects are “colourless”, temporal objects are be timeless. Since timelessness is not infinite temporal duration, but the *eternal present* (6.4311), temporal objects (in fact all simple objects) are sempiternalia.¹⁶⁶ A measurable present must be produced by a configuration of temporal objects, for it is the possibility of producing the sensible property of time that sets objects apart as essentially “temporal”. Given that that these temporal configurations are independent of each other, they are not ordered into past or future; they are too are sempiternalia. Now one can surmise that the form of temporal objects is displayed in an elementary one-dimensional logical space. In measuring a measurable present I construct a point in a one-dimensional line where feelers correlate *two* coordinate values of the line to two temporal objects. That more

than one coordinate value must be involved is implicit in this passage from Wittgenstein's *Notebooks*:

Having only one direction is a logical property of time.

For if one were to ask someone how he imagines having only one direction he would say: Time would not be confined to one direction if an event could be repeated.

But the impossibility of an event's being repeated, like that of a body's being in two places at once, is involved in the logical nature of the event. (Wittgenstein 1961a, 12.10.16; 84)

It seems Wittgenstein is saying that the direction of time expresses a structural property shown in a one-dimensional coordinate system. This would be the case if an elementary picture – a temporal point or instant – represented the configuration of two temporal objects, for then a '<' relation between two coordinate numbers would be the property of any elementary picture. At the time of the *Tractatus* the idea seems to have been that elementary temporal pictures are ordered in relation to one another in virtue of the '<' relation between the two representational elements. So ordered, a temporal instant exists, not as an eternal present, but as an *immediate present* in relation to a past and future present.

The obvious problem is that if elementary temporal pictures are independent of one another, there cannot be an a priori order among them either. A number of years later Wittgenstein amended his approach, claiming that the entire elementary logical space, rather than just the elementary picture, served as the "yardstick" that measured reality. This allowed certain inferences to be made not based on tautological form.¹⁶⁷ Laying down the entire time-scale now meant that in each case there is only one state of affairs that can exist as the immediate present, not several (simultaneous) ones. All this is, of course, a blow to the claim that elementary propositions are independent, for only elementary propositions of different kinds could now be said to be independent. Still it reveals something telling about what Wittgenstein had in mind when he wrote the *Tractatus*: in the method of projection elementary pictures are constructed *in* logical space.

3.6 The “Whole of Logical Space”

The idea of the “whole of logical space” (3.42, 4.463) is the idea of a general logico-mathematical form of all possible worlds given Wittgenstein’s understanding of “world”. It is the general form of imaginative representation, the boundary of our expressive thought:

3.02 A thought contains the possibility of the situation of which it is thought. What is thinkable is possible too.

3.03 Thought can never be of anything illogical, since, if it were, we should have to think illogically.

3.031 It used to be said that God could create anything except what would be contrary to the laws of logic. – The truth is that we could not *say* what an ‘illogical’ world would look like.

We cannot express what an illogical world would look like because the form in which we express thoughts is the logical space of all possible worlds. The whole of logical space accounts for the a priori character of logic: “[w]hat makes logic a priori is the *impossibility* of illogical thought” (5.4731). And (as we shall soon see) it accounts for its transcendental character as well.

When Wittgenstein speaks of “the whole – the infinite whole – of logical space” (4.463) he has in mind is a *completed* space of all possible pictures. The infinite whole of logical space does not commit Wittgenstein to an infinite number of (distinguishable) simple objects, and thus an infinite number of names with different meanings and (equivalently) an infinite number of elementary propositions. Because he appears to be agnostic about all three (4.2211, 5.5), it seems that he allows the possibility that more than one representational element can refer to a simple object.¹⁶⁸ What it does commit him to is the view that the number of coordinate values of any dimension is a completed infinity, one that displays the *potentially* infinite number of combinatorial possibilities that constitutes the essence of a kind of simple object.

Wittgenstein asserts that “[t]he logical scaffolding surrounding a picture determines logical space. The force of a proposition reaches through the whole of logical space” (3.42). What he seems to have in mind can be illustrated with an elementary proposition. That an elementary proposition is a truth function of itself “alludes to the

fact that in the truth-table notation every elementary proposition is expressed as a truth-function (conjunction) of itself and a tautology involving all other elementary propositions (e.g. ‘ $p. (q \vee \sim q)$, etc.’)” (Glock 1996, 141). In this sense “[i]f elementary propositions are given, then at the same time *all* elementary propositions are given” (5.524). But if all elementary propositions are given, then all elementary logical spaces are given, and so too are molecular propositions and molecular logical spaces. This is why Wittgenstein claims that “the whole of logical space must already be given by” “one place in logical space” (3.42).¹⁶⁹ In adding parenthetically that if it were otherwise “negation, logical sum, logical product, etc., would introduce more and more new elements – in coordination”, by “force of a proposition” Wittgenstein intends to capture a realization of the prior coordination between the self and reality through the *determination* of the whole of logical space. (As we will see, “force” refers to the wilful power of self-projection as the whole of logical space.)

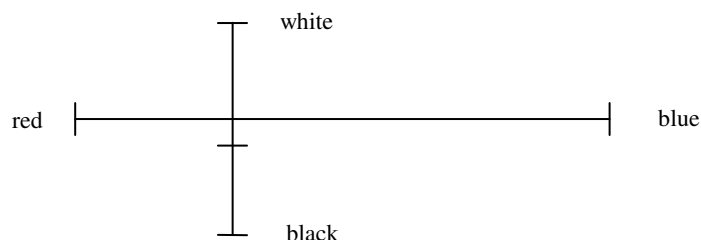
The force of a proposition in determining the whole of logical space is underwritten by the timeless iterative operation of the general propositional form that constructs an elementary proposition. But the operation constructs more than this. The general propositional form is “[t]his is how things stand” (4.5) and alludes to the fact that “[a] proposition constructs a world with the help of a logical scaffolding, so that one can actually see from the proposition how everything stands logically *if* it is true” (4.023). While a constructed proposition is “given” in the sense discussed last paragraph, it cannot be the case that every “given” proposition is constructed, for then a proposition would not only construct *a* possible world, but *all* possible worlds. How “everything stands logically *if* it is true” would be, trivially, how everything stands logically if any proposition is true.

3.7 Logical Space and Objects

One may have the impression that Wittgenstein presents simple objects as a metaphysical postulate, claiming their form is displayed in logical space. The opposite is the case: elementary logical spaces are revealed by physical analysis and simple objects that contain the form of these spaces are established as a matter of *reductio*

argumentation. An example of the kind of analysis Wittgenstein has in mind is revealed in a recorded conversation with Waismann in 1929:¹⁷⁰

Sign for a colour:



Every statement about colours can be represented by means of such symbols. If we say that four elementary colours [i.e., red, yellow, green, blue] would suffice [to generate all colours], I call such symbols of equal status *elements of representation*. These elements of representation are the ‘objects’.

The following question now has no sense: Are objects something thing-like, something that stands in subject-position, or something property-like, or are they relations, and so forth? It is simply where we have elements of representation of equal status that we speak of objects.

Figure 3.3: Physical Analysis and Simple Objects

Although Waismann’s notes are not all that clear, their gist is that the colour signified by the line intersecting the white-black axis is a *shade* of the elementary colour *yellow*. The *point* of intersection stands for an elementary picture in colour-space that is the configuration of two elements of representation: the coordinate value “yellow” and the coordinate value of the particular shade. In this conversation each coordinate value is simply taken to be correlated to a simple object, the configuration of which is represented by the elementary picture.

That the two elements of representation are indeed correlated to two simple objects is established through the following set of propositions:

2.02 Objects are simple.

2.021 Objects make up the substance of the world. That is why they cannot be composite.

2.0211 If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.

2.0212 In that case we could not sketch any picture of the world (true or false).

2.022 It is obvious that an imagined world, however different it may be from the real one, must have *something* – a form – in common with it.

2.023 Objects are just what constitute this unalterable form.

The argument found in these propositions is actually three sub-arguments that build upon one another.

The first establishes substance and occurs in propositions 2.0211 and 2.0212. It can be reconstructed as follows:

- P1: The whole of logical space is a multi-dimensional form of all possible worlds.
- P2: A picture presents a situation in the whole of logical space.
- P3: A picture represents a possible situation.
- P4: A picture has sense only if the situation it represents is structure that 1) is isomorphic; and 2) can make the picture true.
- P5: Assume there is no substance that could produce a situation in the world.
- P6: It follows that a picture can only represent an isomorphic situation presented by another picture, which can only represent an isomorphic situation presented by another picture, and so on in infinite regress.
- P7: But since no situation can make a picture true, no picture has sense.
- P8: Yet we routinely sketch pictures in multi-dimensional space that convey information.
- C: P5 is false.

What this first argument establishes is that there must be isomorphic situations in the world that can make a picture true and a supporting substrate that produces them.

The second argument found in propositions 2.022 and 2.023 builds upon the first and concerns the form of substance. We picture all situations of the real world as located in a form of all possible worlds, i.e., the whole of logical space. Given this form, there is a possible world that agrees with a particular picture. Since a picture represents a possible situation in the real world that can make it true, the real world must share the form of this possible world. Since substance is what produces a situation in the world, substance must also have the same form of the possible world.

The third argument concerns ontology and is mentioned in 2.021 and 2.023. That the form of substance must be a constitution of simple objects follows straightforwardly. If, as physical analysis shows, one elementary logical space is independent of another (e.g., the logical space for touch is independent of the logical space for sound), then it follows that the form of substance must be divided into kinds of substance. And if, as physical analysis also shows, there are elementary logical spaces that have more than one coordinate dimension, then substance must be divided (at most) into objects corresponding to individual coordinate values of a certain axis with an essence that can be represented by another a coordinate axis.

Note that nothing in the forgoing arguments establishes that there are simple objects that are bachelors. They merely establish simple objects insofar as they configure with other simple objects to produce material properties that can make a picture true. This is probably why Wittgenstein is uncommitted one way or the other to their existence or non-existence apart from these configurations. Furthermore, nothing establishes that elementary logical spaces we imagine simple objects to be situated in are real, only that reality shares its form. Finally note that if we combine these arguments with the argument that if elementary propositions are given we are given all elementary propositions or, equivalently, that if names with different meanings are given we are given all names with different meanings, then it follows that “[i]f all objects are given, then at the same time all possible states of affairs are also given” (2.0124) or, equivalently, that “[i]f objects are given, then at the same time we are given *all* objects” (5.524).

3.8 Logical Space and Science

The *Tractatus* was motivated by problems concerning the nature of logic with the overall the goal of solving all philosophical problems. Compared to the logic of language, philosophy of science does not seem to preoccupy Wittgenstein any more than ethics or aesthetics. Still natural science does occupy a special place in the *Tractatus*: “[t]he totality of true propositions is the whole of natural science (or the whole corpus of the natural sciences”) (4.11). It is difficult to deny that Wittgenstein’s overall

philosophical orientation might have been influenced by the view we find in the *Principles* (1900/1956) that certain problems in philosophy of science arise from “illegitimate questions” (8). And there is a strong Hertzian sway in Wittgenstein’s own philosophy of science evident in his inductive view of laws, schematic and conventionalist approach to mechanics, and the proposal of simplicity as a criterion of theory choice. Although the second area of influence will be briefly outlined in what follows, the real significance of Hertz’s influence will become clear in the next section.

Wittgenstein presents a view of science only to meet certain objections to the pre-eminence he places on the idea of logical space. The proposition that opens the discussion of science points out his concern:

6.3 The exploration of logic means the exploration of *everything that is subject to law*.
And outside logic everything is accidental.

Wittgenstein’s central claim that elementary propositions are independent requires supporting the position that the existence of a state of affairs is independent of the existence and non-existence of other state of affairs. This involves establishing that “[t]he only necessity that exists is *logical necessity*” (6.37). In other words, the only necessity is in virtue of the real form displayed in the whole of logical space.

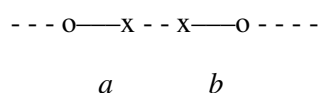
Wittgenstein sees his challenge as one of explaining away the so-called laws of induction and causality. The law of induction is the proposition that inductive generalizations that have accommodated facts in the past will continue to do so in the future. “The procedure of induction”, Wittgenstein says, “consists in accepting as true the *simplest* law that can be reconciled with our experiences” (6.363). However, this procedure has “only a psychological” justification (6.3631). The law of induction “is obviously a proposition with sense” (6.31), for it is an a posteriori generalization of psychological facts. But this is to say that it does not have the status of necessity, unlike an a priori law of logic.

The law of causality, on the other hand, is “the form of a law” (6.32). What this means is that the rule that all that happens has a cause is a structural feature of our form of expressing of thoughts; in particular, it falls out of the specific multiplicity of the logical space of space:

6.361 One might say, using Hertz's terminology, that only connexions that are *subject to law* are *thinkable*.

6.3611 We cannot compare a process with 'the passage of time' – there is no such thing – but only with another process (such as the working of a chronometer). Hence we can describe the lapse of time only by relying on some other process. Something exactly analogous applies to space: e.g. when people say that neither of two events (which exclude one another) can occur, because there is *nothing to cause* the one to occur rather than the other, it is really a matter of our being unable to describe *one* of the two events unless there is some sort of asymmetry to be found. And *if* such an asymmetry *is* found, we can regard it as the *cause* of the occurrence of the one and the non-occurrence of the other.

6.36111 Kant's problem about the right hand and the left hand, which cannot be made to coincide, exists even in two dimensions. Indeed, it exists in one dimensional space



in which the two congruent figures, *a* and *b*, cannot be made to coincide unless they are moved out of this space. The right hand and the left hand are in fact completely congruent. It is quite irrelevant that they cannot be made to coincide. A right-hand glove could be put on the left hand, if it could be turned round in four-dimensional space.

6.362 What can be described can happen too: and what the law of causality is meant to exclude cannot even be described.

Wittgenstein's point is that the law of causality captures asymmetries that arise from the dimensional boundary of the logical space of space that places limits on expressive thought. The bound of dimensionality precludes thought from "stepping outside" the logical space of time through which a temporal process is measured in order to compare that process with the logical space itself. Analogously in the case of the logical space of space, the boundary of dimensionality precludes thought from "stepping outside" the logical space to imagine the right and left hand as coinciding. More generally, it precludes thought from describing the occurrence of one event and the non-occurrence of another event as coinciding through a common cause, which is why we think only that the occurring event is caused.

Like geometrical laws, causal laws presuppose the logical space of space. The law of causality is a form of a law because it captures the boundary of the expression of spatial thought; equivalently, it is a limit to spatial language presupposed by any law of physics. As a feature of the dimensionality of a coordinate system, the law of causality is not an "an *inner* necessity like that of logical inference" (5.1362) that allows for

deductions to be made from what is known to what is unknown, but an “outer” necessity in virtue of circumscribing the possibility of spatial language. As such it does not derogate from the contingency of a state of affairs.

Wittgenstein describes mechanics as follows:

6.343 Mechanics is an attempt to construct according to a single plan all the *true* propositions that we need for the description of the world.

6.3431 The laws of physics, with all their logical apparatus, will speak, however indirectly, about the objects of the world.

The characterization of mechanics as an *attempt* to construct true pictures from a single *plan* obviously follows Hertz’s characterization of mechanics as an attempt to construct images of things from a schematic scientific image. Wittgenstein appears also to be influenced by Hertz’s position that mechanics involves conventional elements. He mentions both Hertzian (6.3432) and Newtonian mechanics (6.341) but does not commit to either view; rather, he sees both as imposing “unified forms” or “nets” that “correspond to different systems for describing the world” (6.341). In addition to the law of causality, each net imposes additional forms of laws. Wittgenstein mentions “minimum-principles” (6.321), the form of a law of conservation (6.33), and forms for the principle of sufficient reason and laws of continuity (6.34). As a result laws of mechanics “are about the net and not about what the net describes” (6.35); i.e., they are not explanations of natural phenomena (6.371), but “a priori insights about the forms in which the propositions of science can be cast” (6.34) that allow us to derive “axioms of mechanics” (6.341). To the latter Hertzian point Wittgenstein adds another: though different systems of mechanics may be empirically adequate, “we are told something about the world by the fact that it can be described more simply with one system of mechanics than with another” (6.342), i.e., certain structures are products of the mind.

It should be emphasized that although a “net” of mechanics can be understood as constituted in part by elementary logical spaces, one should not confuse this attempt to recover the whole of logical space with the whole of logical space itself. Wittgenstein says that a “state of affairs that would contravene the laws of physics can be represented by us spatially, one that would contravene the laws of geometry cannot” (3.0321) (and

one that contravenes the law of causality cannot either). Although the laws of geometry (and the law of causality) are constitutive of physical laws, the laws of physics are also constituted by conventional elements. It is because of these elements that violation of laws of physics may still be represented spatially.

Although a strong Hertzian influence is evident in Wittgenstein's view of science, it pales in comparison to the impact upon the *Tractatus* of the idea that the mind and nature share the same form. What will become clear in the next section is that Hertz's idea of a thought schema having the form of real potentiality inspires the unifying idea of the *Tractatus*.

3.9 Logical Space as the Metaphysical Self

In this section I will motivate the claim that the whole of logical space is Wittgenstein's metaphysical subject, leaving its justification for the section that follows. The place to begin is with Wittgenstein's appraisal of his own philosophical efforts. "A philosophical work", he says, "consists essentially of elucidations"; "its task is to make [thoughts] clear and to give them sharp boundaries" (4.112). The elucidations in the *Tractatus* that give sharp boundary to all thoughts is the proposition "[t]he facts in logical space are the world" (1.13) and, indirectly, the proposition "the world is the totality of facts" (1.1). However, Wittgenstein also states:

6.54 My propositions serve as elucidations in the following way: anyone who understands me eventually recognizes them as nonsensical, when he has used them – as steps – to climb up beyond them. (He must, so to speak, throw away the ladder after he has climbed it.) He must transcend these propositions, and then he will see the world aright.

7 What we cannot speak about we must pass over in silence.

The central exegetical task of the *Tractatus*, therefore, is to determine how it is that "the facts in logical space are the world" and "the world is the totality of facts" serve as final steps towards understanding the *Tractatus* and seeing the world aright.

The metaphor of throwing away the ladder one has climbed is used by Schopenhauer in a slightly different way: "the man who studies to gain *insight*, books and studies are merely rungs of the ladder on which he climbs to the summit of knowledge.

As soon as a rung has raised him one step, he leaves it behind” (vol II, 80).¹⁷¹ Schopenhauer’s influence on early Wittgenstein is now generally acknowledged. G. E. M. Anscombe writes:

As a boy of sixteen Wittgenstein had read Schopenhauer and had been greatly impressed by Schopenhauer’s theory of the ‘world as idea’ (though not of the ‘world as will’); Schopenhauer then struck him as fundamentally right, if only a few adjustments and clarifications were made. ... If we look for Wittgenstein’s philosophical ancestry, we should ... look to Schopenhauer; especially his ‘solipsism’, his conception of ‘the limit’ and his ideas on value will better be understood in light of Schopenhauer than of any other philosopher. (1959, 11-12)

However, the extent of Schopenhauer’s influence is also generally understated. A notable exception is Weiner’s *Genius and Talent* (1992), a book devoted to drawing out connections between Schopenhauer’s work and the *Tractatus*.¹⁷²

Schopenhauer’s theory of ‘world as idea’ or ‘world as representation’ is a version of transcendental idealism found in the first book of *The World as Will and Representation* (1966). Weiner summarizes:

Schopenhauer, like Kant, believes that the world is structured by the subject’s a priori forms of cognition, including time, space, and causality. But in contrast to Kant, Schopenhauer reduces all a priori categories and forms of intuition to one overarching structure, which he calls “the principle of sufficient reason”. ... In other words, each representation is the necessary consequence of another, which in turn is the necessary consequence of another. (1992, 48-49)¹⁷³

Schopenhauer’s more radical theory of ‘world as will’ is found in the second book. Based on the idea that the external body correlates with the subject’s internal will, it develops Kant’s philosophy into an immanent metaphysics whereby the phenomenal world is said to be a manifestation of noumenal will:

Schopenhauer thinks he can show “what the world is” by elaborating man’s immediate perception of his own inner nature. Through inner experience, man “already knows” that he is a blind, striving will objectified as a physical body. My intuition tells me that, internally I am a will, and externally, I am a bodily representation. (Ibid., 56-57)

The dead, mechanistic view of nature expressed in natural science gives way to a very different picture. Nature is animated by a multitude of wills, each striving blindly to live and reproduce. Each species or natural kind has its own character, that is, its own characteristic will; but there is also a family resemblance among the various wills in nature. All the different wills in nature participate in an unindividuated world-will, a formless, aimless, insatiable energy that infuses the universe as a whole. Consequently, the various

forces and species in nature are variations on a single theme, the noumenal will-to-life. (Ibid., 58)

Schopenhauer's motivation for this second book is captured in the following passages:

If we summarize Kant's utterances, we shall find that what he understands by the synthetic unity of apperception is, so to speak, the extensionless centre of the sphere of all our representations, whose radii converge on it. It is what I call the subject of knowing, the correlative of all representations. (1966, vol. I, 451-452)

Kant's proposition: "The *I think* must accompany all our representations," is insufficient; for the "I" is an unknown quantity, in other words, it is itself a mystery and secret. What gives unity and sequence to consciousness, since, by pervading all the representations of consciousness, it is its substratum, its permanent supporter [i.e., the will]. ... Without it the intellect would have no more unity of consciousness than a mirror, in which now one thing now another presents itself in succession, or at most only as much as a convex mirror has, whose rays converge at an imaginary point behind its surface. ... Fundamentally it is the will that is spoken of whenever "I" occurs in judgement. Therefore the will is the true and ultimate point of unity of consciousness, and the bond of all its functions and acts. It does not, however, itself belong to the intellect, but is only its root, origin, and controller. (1966, vol II, 139-140)

His analogy of Kant's transcendental self to the focal point of a spherical convex mirror is insightful, and was justified in the first chapter of this thesis as recovering Kant's structure of the imagination. Although Schopenhauer's reasons for rejecting Kant need not concern us, what is important for present purposes is his proposal that knowing consciousness is an activity stimulated by willing consciousness.

Wittgenstein's *Notebooks* reveal an active engagement with Schopenhauer,¹⁷⁴ and even suggests he may have been studying him again.¹⁷⁵ Here we find a crucial passage:

The thinking subject is surely mere illusion. But the willing subject exists.

If the will did not exist, neither would there be that centre of the world, which we call the I, and which is the bearer of ethics. (Wittgenstein 1961a, 4.8.16; 80)

But for the explicit claim that the transcendental subject "exists", the entire content of the passage is found in the *Tractatus*. There we find "[t]here is no such thing as the subject that thinks or entertains ideas" along with the implication that the subject is "my will" (5.631), a bearer of ethics (6.43) that is (sometimes) at the centre of the world (5.64). Yet the *Tractatus* also shows a marked departure from Schopenhauer in its rejection of the objectification of the will: "[t]he subject does not belong to the world" (5.632).

An interesting claim Schopenhauer makes is that genius has the ability to separate knowing from willing consciousness whereby “the knowing part of consciousness becomes purely objective and the clear mirror of the world” (1966, vol. II, 206). In its rejection of the objectification of the will, the *Tractatus* does not intimate any such separation of consciousness. What it does state is that logic is “a mirror-image of the world” (6.13). If as suggested willing consciousness *is* knowing consciousness, then perhaps in the *Tractatus* willing consciousness, i.e., the self, is “the clear mirror of the world”. Indeed, there is much to motivate such a view.

If logic is “a mirror image of the world” then thought and the world share structure. If we think of structure the way Kant does as a boundary or limit to chaos,¹⁷⁶ then the idea of shared structure between thought and the world is captured through the idea of a shared “boundary” or “limit”. This seems to have been Schopenhauer’s position:

[The forms of representation] belong only to the *object*, yet because they are essential to the object *as such*, they can be found also from the subject, in other words, they can be known *a priori*, and to this extent are to be regarded as the boundary common to both. (1966, vol. I, 25)

Rather than appeal directly to a shared limit between the self and object, Wittgenstein concerns himself in the first instance with shared limits between the whole of logical space and the world. His position is that the limits of the whole of logical space are boundaries of the world can be unpacked from the following proposition:

5.5561 Empirical reality is limited by the totality of objects. The limit also makes itself manifest in the totality of elementary propositions. ...

The totality of (distinguishable) simple objects correlates to the totality of names with different meanings. Since a name does not have meaning apart from its nexus with other names, the totality of simple objects is a boundary shown in the totality of elementary propositions. (If elementary propositions are independent they must have names with different meanings.) But this is not the totality of the world, for “[t]he world is the totality of facts, not of things” (1.1). The totality of facts is a boundary of real *actuality*, while the totality of things is a boundary of real *potentiality*. In saying “empirical reality is limited by the totality of objects”, Wittgenstein is claiming that real actuality is limited

by real potentiality. Since the whole of logical space (the totality of elementary propositions) displays real potentiality, Wittgenstein is claiming that actuality is limited by the whole of logical space. This is obviously so, for if the whole of logical space is the form of all possible worlds through its dimensionality and its general propositional form, then real actuality, the “world” according to Wittgenstein, must be limited by this form if it is to be a possible world at all.

Wittgenstein’s claim that the limits of the world are boundaries of the whole of logical space arises as follows. The limits of the world also include the limit of contingent actuality, i.e., the world conceived as “the *whole sphere* of what happens and is the case” where “all that happens and is the case is accidental” (italics added, 6.41). On this point Wittgenstein writes:

5.61 Logic pervades the world: the limits of the world are also its limits. So we cannot say in logic, ‘The world has this in it, and this, but not that.’ For that would appear to presuppose that we were excluding certain possibilities, and this cannot be the case, since it would require that logic should go beyond the limits of the world; for only in that way could it view those limits from the other side as well. We cannot think what we cannot think; so what we cannot think we cannot *say* either.

He is saying that just because actuality is a possibility, the form of all possible worlds must allow for the actual contingencies of the world. Thus, the limits of the whole of logical space are threefold: 1) dimensionality; 2) general propositional form; and 3) the independence of elementary propositions. The limits of the world are the same, though we can describe them in different terms: 1) multiplicity; 2) logical form; and 3) independence of elementary states of affairs.

When Wittgenstein says logic is “a mirror-image of the world” his assertions concerning contradiction and tautology tell us that this mirror has an outer and an inner limit. “Contradiction, one might say, vanishes outside all propositions: tautology vanishes inside them. Contradiction is the outer limit of propositions: tautology is the unsubstantial point at their centre” (5.143). There are two parts to this claim. A contradiction “vanishes outside all propositions” in the sense that every proposition follows from it, but it does not follow from any proposition; a tautology “vanishes inside” all propositions in the sense no proposition follows from it, but it follows from every proposition (Wittgenstein 1961a, 3.6.15; 54). More significantly, a “contradiction is the

outer limit of propositions” in the sense that “a contradiction fills the whole of logical space leaving no point of it for reality” (4.463); a “tautology is the unsubstantial point at their centre” in that “[a] tautology leaves open the reality to the whole – the infinite whole of logical space” (Ibid.). Altogether this suggests that the outer limit of the whole of logical space is the limit of a contradiction, while the inner limit of the general propositional form is the limit of a tautology. In other words, the dimensionality of the whole of logical space serves as the limit of a contradiction (since a contradiction cannot fill a space of greater dimension than that of the whole of logical space) and the general propositional form serves as the limit of a tautology (since a tautology arises from the iterative operations of the general propositional form).¹⁷⁷

How does this notion of an outer and inner limit of a mirror image of the world fit with the claim that the whole of logical space and the world share the same limits? If we think of the whole of logical space as the surface of a spherical *concave mirror* within which is “the whole sphere of what happens and is the case”, then we can imagine it as an *outer* boundary of expressive thought and thereby an *outer* limit of meaningful language, while we can imagine the world that shares this limit as the *inner* boundary of expressive thought and thereby the *inner* limit of meaningful language. Now Wittgenstein also writes “[t]he middle point of a circle can be conceived as its inner boundary” (1961a, 3.6.15; 54). Presumably this is because equal radii projected from the middle point of a circle delimit the points of the circle; equivalently, the radii from points of the circle converge towards the middle point. If we replace the circle with our spherical concave mirror, then the middle point is its *focus imaginarius*. If our mirror were to collapse into the *focus imaginarius*, then the whole of logical space would be an inner limit and the world would be an outer limit. What does logical space as an inner limit then have to do with the general propositional form? Since the iterative operations of the general propositional form accompany the method of projection, the inner limit can be taken as the point where the projection begins. As an inner limit the whole of logical space can be understood as a *thought schema*, a form of potentiality out of which the completed whole of logical space is projected.

In light of Wittgenstein’s claim that the willing self exists at the “centre of the world” we can regard the projection of the completed whole of logical space as an act of the self in relation to this thought schema. “One cannot will without acting” (Wittgenstein 1961a, 4.11.16; 87) and one cannot express thought without an act of projecting a proposition with sense onto the world, which suggests expressive thought is a projective act of will. If our notion of force is that of a wilful power it would follow that one can not (as Wittgenstein says) express a thought without the force of a proposition. Since “[t]he force of a proposition reaches through the whole of logical space” (3.42), it would then follow (as Wittgenstein also says) that one cannot express a thought without an act of projecting a proposition situated in the whole of logical space. What the *Tractatus* seems to be telling us is that in thinking the sense of a proposition the whole of logical space is projected through the timeless iterative operations of the general propositional form, operations underwritten by a wilful act of self.

In the projection of the whole of logical space pictorial form mirrors the form of reality and, if true, pictorial content mirrors the content of reality. Knowing consciousness can thus be thought of as a “clear mirror of the world”, as Schopenhauer would say. The mirrored form and content of reality would be reflected back towards a *focus imaginarius* at the centre of the world from which the act of self originates. This is a picture *of* logical space, one represented in Figure 3.4:

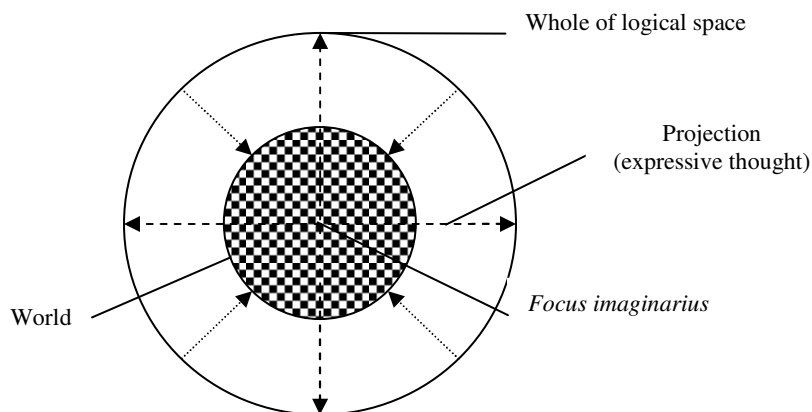


Figure 3.4: Picture *of* Logical Space as Concave Mirror

I argued above that what Wittgenstein means by “the facts in logical space are the world” is not that existing states of affairs are contained in a real multi-dimensional coordinate system. This picture *of* logical space shows us what this proposition does mean. “A logical picture of facts is a thought” (3) and “[t]he totality of true thoughts is a picture of the world” (3.01). If “[a] picture is a model of reality” (2.12) then the totality of true thoughts is also a true model of the world. And if “[a] picture presents a situation in logical space” (2.11) then a true model of the world presents a true situation in the whole of logical space. Since a true model of the world is a set of representational facts identical to the represented facts of the world, “[t]he facts in logical space are the world” just because in any projection the world *ipso facto* embeds in logical space, the form of all possible worlds. What the proposition really means, then, is that in expressing thought we surround the world in an imaginative “halo of possible worlds” that is the whole of logical space.

So far I have motivated a picture *of* logical space where the whole of logical space is a mirroring projection by the self, not one where the whole of logical space *is* the self. Perhaps the self is correlated to the *focus imaginarius* existing throughout as an inner limit to the expression of thought and logical space is a mere mental projection. However, such a view is at odds with what Wittgenstein says elsewhere. We are told in proposition 5.64 that when the self is “a point without extension” that “reality [is] coordinated with it”. As noted above, this seems to mean that reality and the self share the same form, one attributable to the internal properties of simple objects. But we are also given a principle of identity by which simple objects with indiscernible internal properties are the same (2.0233f), the implication being that only discernible properties are displayed in logical space. If both the self and the projected whole of logical space share the same form as reality, then by this principle the self is indiscernible from the projected whole of logical space, i.e., the self is the same as logical space.

One might reply that Wittgenstein distinguishes between the self and reality by rejecting the objectification of the will. Perhaps, then, the projected whole of logical space is not will, i.e., it is discernible from the self but not from reality. However, this would introduce a Schopenhauerian distinction between knowing and willing

consciousness, one at odds with the statement in the *Tractatus* that “[t]here is no such thing as the subject that thinks or entertains ideas”. Furthermore, the *Tractatus* speaks of a self that “shrinks to a point without extension (5.64), suggesting that the self also exists as an outer limit, oscillating between the two. One might add that without the existence of the self as the whole of logical space Wittgenstein’s notion of representational existence is a mystery.

The picture *of* logical space that is now motivated can be summarized as follows. The completed whole of logical space is the willing self that projects itself in expressing thought, leaving behind a *focus imaginarius* as an “echo” of its alternative mode of existing as the potential whole of logical space. In just the same way the potential whole of logical space is the willing self that “shrinks” itself in an act of silence leaving behind a “shadow” of its alternative mode of existing as the completed whole of logical space. Here the self exists as the centre of the world, not as the bearer of thought, but “as the bearer of ethics”. The sense in which the self exists as outer or inner limit is just the sense in which two different perspectives of the world are possible: one where the world is viewed as an inner limit and one where it is viewed as an outer limit. (See Figures 3.1 and 3.2.)

Where could this picture *of* logical space have come from? It seems to be the result of a synthesis of Hertz with Schopenhauer. Last chapter we saw Hertz speculate that the conformity between mind and nature can be explained through the shared structure of a thought schema and the form of reality, taking both to determine potentialities from which, respectively, mathematical models are constructed (and organized logically) and reality is actualized. In adopting Schopenhauer’s conception of the self as will and denying his distinction between willing and knowing consciousness, it seems Wittgenstein equates the willing self with Hertz’s thought schema and with the knowledge product wilfully constructed from it.

Motivating a picture *of* logical space is one thing, justifying it is quite another. It might be objected that figurative language intended to be taken metaphorically is being given too literal an interpretation. Indeed, what I am proposing is that the propositions of the *Tractatus* are, in effect, describing a picture *of* logical space, and that it is this

“picture” that confers “sense” upon them sufficient to communicate something of value. The best way to justify this picture *of* logical space, therefore, is to demonstrate how it unifies propositions of the *Tractatus* and how it brings to light its ethical goal.

3.10 Unifying the *Tractatus*

The strength of the picture *of* logical space just presented comes from its ability to make sense of some of Wittgenstein’s most obscure claims, i.e., those concerning solipsism, realism, ethics, aesthetics and mysticism. It is from the adaptation of Schopenhauer’s notion of a limit to the common boundary between the whole of logical space and the world that we approach the infamous solipsism of the *Tractatus*:

5.62 This remark provides the key to the problem, how much truth there is in solipsism. For what the solipsist *means* is quite correct; only it cannot be *said* but makes itself manifest. The world is *my* world: this is manifest in the fact that the limits of *language* (of the only language that I understand)¹⁷⁸ mean the limits of *my* world.

What the solipsist means – “the world is *my* world” – cannot be said; rather, it is shown in the “fact” that “*the limits of my language* mean the limits of my world” (5.6). The most natural reading of this proposition is that it refers to proposition 5.61 that builds upon proposition 5.5561 (quoted last section) to establish that the limits of the world and logical space are one and the same. The claim seems to be that the fact that the limits of my language *refer* (“what the solipsist *means* is quite *correct*”) to the limits of my world shows that the world is my world.

But prior to proposition 5.62 the claims have only concerned the limits of the world and the limits of language. The hurdle to overcome is that it appears neither that the limits of the world are the limits of *my* world nor that the limits of language are the limits of *my* language, for “[t]he world is independent of my will” (6.373)¹⁷⁹ and so independent of my self, and the limits of language – the whole of logical space – is common to all languages (cf. 5.512). The former is the most difficult to see, for the *Notebooks* reveals an inherent tension in Wittgenstein’s conception of the willing self in relation to the world. On the one hand there is the muscular feeling that accompanies acts of will in relation to movement of the body:

Does not the willed movement of the body happen just like any unwilled movement in the world, but that it is accompanied by the will? Yet it is not accompanied by a *wish!* But by will. We feel, so to speak, responsible for movement. My will fastens on to the world somewhere, and does not fasten on to other things. Wishing is not acting. But willing is acting. (My wish relates, e.g., to the movement of the chair, my will to a muscular feeling.) (Wittgenstein 1961a, 4.11.16; 88)

On the other hand all bodies in the world must be equivalent if elementary states of affairs are independent:

For the consideration of willing makes it look as if one part of the world were closer to me than another (which would be intolerable). But of course, it is undeniable that in a popular sense there are things that I do, and other things not done by me. In this way then the will would not confront the world as its equivalent, which must be impossible. (Ibid.)

This tension surfaces in the *Tractatus* with the claim that the world includes “my body” constituted by parts “subordinate to my will” (5.631)¹⁸⁰ together with the claim that “the world is independent of my will”.

The best approach, it seems to me, is to tackle this problem second after first reconciling Wittgenstein’s claim that the limits of language are the limits of *my* language or, equivalently, his claim that the limits of the whole of logical space are the limits of *my* whole of logical space. How to do so is suggested in the following recorded conversation:

Now it is possible to construct many different languages, each of which has a different man at its centre. Imagine for instance you were a despot in the Orient. All men were compelled to speak the language whose centre you are. If I spoke this language, I should say, ‘Wittgenstein has toothache. But Waismann is behaving as Wittgenstein does when he has a toothache. In the language whose centre you are it would be expressed just the other way around, ‘Waismann has toothache, Wittgenstein is behaving like Waismann when he has a toothache.’ All these languages can be translated into one another. Only what they have in common mirrors anything. Now it is noteworthy that *one* of these languages has a distinctive status, namely that one in which I can as it were say that I feel *real* pain[,] ... namely the language whose centre I am. The distinctiveness of this language lies in its application. It is not expressed. (December 22, 1929, as recorded by Waismann (1979, 49-50))

Wittgenstein’s point can be captured as follows. Any whole of logical space has the same limits, i.e., dimensionality, general propositional form and independence of elementary propositions. By means of the whole of logical space both you and I measure the pain of parts of the world. This involves measuring symptoms of pain such as wincing, but also aches. Concerning the former, you measure pain of parts of the world subordinate to my

will as I measure the pain of parts of the world subordinate to your will. Concerning the latter, however, both you and I have *private* elementary logical spaces, e.g., an elementary logical space for stomach-ache or toothache, that make the whole of logical space *unique* to each of us. By means of *my* elementary logical space for tooth-ache I do not measure configurations of objects that constitute parts of the world subordinate to your will and *vice versa*. Accordingly I cannot *say* I experience or do not experience the stomach-ache you experience; by the same token, you cannot *say* that you experience or do not experience the stomach-ache I experience (Ibid.). This difference is a difference in language. Strictly speaking, language is not *our* language, but “my language”, i.e., “the only language that I understand”, for my language is our language only in the sense that my language *includes* our language. Thus it is the limits of *my* language that limit the world and it is the limits of the world that limit of *my* language. The limits of my language and the limits of the world are common boundaries.

However, Wittgenstein’s claim is that the limits of my language refer to the limits of *my* world. This follows. Any world has the same limits, i.e., multiplicity, logical form and independence of elementary states of affairs. However, there are states of affairs that obtain that I can measure, which you cannot; there are states of affairs that obtain that you can measure, which I cannot not. Although these states of affairs are correlated to private mental elements, they do not derogate from the general independence of elementary states of affairs. Strictly speaking, then, the world is not *our* world, but *my* world. My world is our world only in the sense that my world *includes* our world. The limits of my language therefore *refer* to the limits of my world, which shows that the world is my world. We thus see how much truth there is in solipsism.

I argued that the proposition “the facts in logical space are the world” is elliptical for the claim that the world embeds in the projection of logical space by the metaphysical self. From what has just been established, one sees that “facts in logical space are the world” means that “the facts in *my* logical space are *my* world”, which is elliptical for the claim that my world embeds in the projection of my logical space by me. Now Wittgenstein makes an even stronger claim that “I am my world” (5.63) in the sense that “[t]he subject does not belong to the world: rather, it is a limit of the world” (5.632).

This is unintelligible unless it is also the case that *I* am *my* logical space, the *outer* limit of the world into which the world embeds, and in this sense the subject is *a* limit of the world.

This interpretation of Wittgenstein's solipsism is borne out by his equally infamous "pure realism":

5.64 Here it can be seen that solipsism, when its implications are followed out strictly, coincides with pure realism. The self of solipsism shrinks to a point without extension, and there remains the reality co-ordinated with it.

5.641 Thus there really is a sense in which philosophy can talk about the self in a non-psychological way. What brings the self into philosophy is the fact that 'the world is my world'. The philosophical self is not the human being, not the human body, or the human soul, which psychology deals, but rather the metaphysical subject, the limit of the world – not a part of it.

If the whole of logical space *is* the self of solipsism, then solipsism would indeed coincide with pure realism when the self of solipsism "shrinks" from the outer limit to the inner limit of the world. Like the self of solipsism, the self of realism is thus *a* limit of the world. This occurs when the willing self acts in a way other than expressing thoughts. In the absence of the operations of the general propositional form, the scaffolding surrounding pictorial representation collapses and the multi-dimensional coordinate system shrinks to a point. The transcendental subject would then be a *potential* whole of logical space at the centre of the world. Because there continues to be a shared multiplicity and general propositional form, the reality *remains* "co-ordinated" with the metaphysical self. But notice that with pure realism the world is not *my* world embedded *inside* my *completed* whole of logical space, but *the* world *outside* my *potential* logical space.¹⁸¹ The "whole sphere of what happens and is the case" is now the outer limit of the world and the metaphysical self is its inner limit. Notice also that as both an outer or inner limit the metaphysical subject is "the limit" of the world referred to in proposition 5.641.

If the self of pure realism is a "point without extension", i.e., a spaceless inner limit, then it is also a timeless inner limit. It is from the vantage of the self of pure realism that Wittgenstein makes the following claims concerning our existence:

6.431 So too at death the world does not alter, but comes to an end.

6.4311 Death is not an event in life: we do not live to experience death. If we take eternity to mean not infinite temporal duration but timelessness, then eternal life belongs to those who live in the present. Our life has no end in just the way which our visual field has no limits.

6.4312 ... Is not this eternal life [i.e., the eternal survival of the human soul after death] itself as much of a riddle as our present life? The solution of the riddle of life in space and time lies *outside* space and time. (It is certainly not the solution of any problems of natural science that is required.)

6.521 The solution of the problem of life is seen in the vanishing of the problem. ...

As the self of solipsism I am my whole of logical space in which my world embeds. And as the outer limit of the world “I want to report how *I* found the world”, and so “*I* have to judge the world, to measure things” (Wittgenstein 1961a, 2.9.16; 82). I judge the world, including those parts subordinate to my will, as positions in a three-dimensional coordinate system and as a succession of immediate presents in a one-dimensional coordinate system. As measured, my life is part of the world *inside* the forms of objects that are space and time, i.e., my life is part of the world embedded within elementary logical spaces that display such forms and *through* which *I* perceive it. With the collapse of the self of solipsism I am the spaceless and timeless centre of the “whole sphere” of the world. My life is now *outside* the forms of objects that are space and time. No longer a succession of instants perceived through a one-dimensional coordinate system, “[my] life has no end”. It has no end “in just the way which our visual field has no limits” because my perspective of it is one of looking outwards from inside a “whole sphere”. Accordingly, the problem of life is thus not answered through expressive thought, but in its very abeyance.

There is more to this passage. The iterative operations of the general propositional form in the method of projecting a picture in the whole of logical space are timeless and, for the same reasons, spaceless. If the self of pure realism is the spaceless and timeless inner limit of the world, then the projected self of solipsism must also be a spaceless and timeless outer limit of the world. Wittgenstein’s pure realism is not the position that “only the world exists”, nor is his solipsism the position that “only I exist”. Rather, they are two sides the general position that existence is *eternal present*:¹⁸² “What has history to do with me? Mine is the first and only world!” (Wittgenstein 1961a,

2.9.16; 82). It is because eternal present is undifferentiated present that “the world and life are one” (5.621). If I cease (or the world ceases) then existence ceases: “at death the world ... comes to an end”.

In his *Notebooks* Wittgenstein makes the transcendental claim that “the subject is not a part of the world but a presupposition of its existence” (1961a, 2.8.16, 79). This even stronger claim arises because the metaphysical self shares the same form as reality presupposed by real content or states of affairs. In the *Tractatus* logic (6.13), ethics and aesthetics (6.421) are transcendental because their basic notions are made possible by this fundamental coordination between reality and the metaphysical. Like the relation between solipsism and pure realism, ethics and aesthetics involve shifting between the metaphysical self as outer and inner limit.

Let us begin with ethics:

6.43 If the good or bad exercise of the will does alter the world, it can alter only the limits of the world, not the facts – not what can be expressed by means of language. In short the effect must be that it becomes an altogether different world. It must, so to speak, wax and wane as a whole. The world of the happy man is a different one from that of the unhappy man.

In its acts the will expresses “an attitude of the subject to the world” (Wittgenstein 1961a, 4.11.16; 87). The unhappy man looks at the world from the outer limit of the world as *his* world. He sees the world through logical space as one among possible worlds of his will, viewing it as a source that can satisfy the wants of his unique self. The happy man, by contrast, sees the world from the inner limit of the world as *the* world. He sees it “*sub specie aeternitatis* ... together with the whole logical space” and thus “*together with space and time instead of in space and time*” (Wittgenstein 1961a, 7.10.16; 83). The world is no longer viewed as “the facts in logical space” (1.13), but as the “totality of facts” (1.1) against a timeless “shadow” of projected logical space. Concomitant with this shift of the metaphysical self from outer to inner limit of the world, the world no longer wanes as the reciprocal limit inside a halo of possible worlds but waxes to the fore as the reciprocal limit outside a central point. Hence, the world is no longer viewed as a one among others, but “as a whole sphere of what happens and is the case”, i.e., a unique world that satisfies wants by “favour granted by fate” (6.374). The happy life thus

involves adopting a stoic attitude of equanimity¹⁸³ by “renouncing any influence on happenings” (Wittgenstein 1961a, 11.6.16; 73), foregoing “the amenities of the world” and accepting “the life of knowledge” (Ibid., 13.8.16; 81).

The shift that sees the world wax from inner to outer limit is even more dramatic in aesthetics. Ethics and aesthetics are “one and the same” (6.421), for “the good life is the world seen *sub specie aeternitatis*” and “[t]he good work of art is the object seen *sub specie aeternitatis*” (Wittgenstein 1961a, 7.10.16; 83). In “good art” (Ibid.) an object waxes to the fore:

As a thing among things, each thing is equally insignificant; as a world each one equally significant. If I have been contemplating the stove and then am told: but now all you know is the stove, my result does indeed seem trivial. For this represents the matter as if I had studied the stove as one among the many things in the world. But if I was contemplating the stove *it* was my world, and everything else colourless by contrast with it. (Something good about the whole, but bad in details.) For it is equally possible to take the bare image as the worthless momentary picture in the whole temporal world, and as the true world among shadows.¹⁸⁴ (*italics added, Ibid.*)

Hence, objects “acquire “significance” only through their relation to my will” (Ibid., 15.10.16; 84), which collapses from outer to inner limit of the world and even further to inner limit of a particular object in the shifting attitude of the transcendental subject.

A pleasant feeling is the reward for the good exercise of the will (6.422). This is the mystical feeling that accompanies willing as inner limit of the world:

6.432 *How* things are in the world is a matter of complete indifference for what is higher. God does not reveal himself *in* the world.

6.4321 The facts all contribute only to setting the problem, not to its solution.

6.44 It is not *how* things are in the world that is mystical, but *that* it exists.

6.45 To view the world *sub specie aeterni* is to view it as a whole – a limited whole. Feeling the world as a limited whole – it is this that is mystical.

6.522 There are, indeed, things that cannot be put into words. They *make themselves manifest*. They are what is mystical.

If I am right, to *view* the world as a limited whole is to orientate a *potential* whole of logical space that is *I* as inner limit of the world *together with* the world as outer limit. To *feel* the world as a limited whole is to experience its existence together with the existence of the self as eternal present. And this seems to be a development via Schopenhauer of

Kant's position that as a formal limit the transcendental self "is nothing more than a feeling of an existence without the least concept" (Ak. IV: 334 n.).

To feel the world as a limited whole would arise in virtue of the prior coordination between the metaphysical self and reality. If so, this belies the common claim that Wittgenstein presents a sophisticated version of the correspondence theory of truth.¹⁸⁵ In addition to representational truth there is also existential truth. The basis of representational truth is the correspondence between the content of my representation and the content of reality, but the basis of existential truth is my prior co-ordination with the form of reality. Since coordination is a condition for the possibility of representation, existential truth is constitutive of representational truth.

In summary, the aim of the *Tractatus* is achieved by first guiding the reader to the solipsistic viewpoint that "the facts in logical space are the world". By altering the limit of the world from inner to outer limit, it then guides the reader to the realistic viewpoint that "the world is the totality of facts", which is the proper ethical and aesthetic orientation. From here it tells the reader that, unlike the propositions of science, its philosophical expressions of thought are nonsense because they are said without the means of logical space, leaving the final step to the reader's own volition. For the reader to then "throw away" the *Tractatus* and "see the world aright", to grasp "the *truth* of the thoughts" it expresses as "unassailable and definitive", is not to express thought but to wilfully feel the world in silence as existing *sub specie aeternitatis*. In the end all that *truly* matters in the *Tractatus* is an ineffable *act* of philosophical wisdom that *I* must do.¹⁸⁶

3.11 Conclusion: A Reflection of Ourselves

This chapter has attempted to motivate and, as Wittgenstein suggests in his preface, "draw" the picture of logical space described by the propositions of the *Tractatus* that can be said to give them their "whole sense". This picture appears to be patterned on a spherical concave mirror where the metaphysical self wilfully exists as either the completed whole of logical space reflecting the world in imaginative representation

towards a *focus imaginarius*, or as the potential whole of logical space looking outwards from this point onto the world. The potential whole of logical space represents an intellectual development of Hertz's notion of a thought schema, while the completed whole of logical space represents an intellectual development of Kant's logical space when the mirror structure shifts from convex to concave. By adopting Schopenhauer's view of the self as will and denying that willing consciousness is separate from knowing consciousness, Wittgenstein naturally associates the self with both potential and completed logical space. The picture *of* logical space is thus the picture *of* the self.

This picture *of* the self is justified through its ability to capture in a clear way the "elusive unity" underlying the propositions of the *Tractatus*, bringing together Wittgenstein's picture theory of meaning with his solipsism, realism, ethics, aesthetics and mysticism. Logical space is the imaginative form of all possible worlds in virtue of which we project in representational thought the potentiality of the world we feel in silence. Only what is said by means of logical space has sense, for we convey information through the prior coordination between the self and reality displayed in logical space. Yet the ethical point of the *Tractatus* is not the value of what we say. It is the significance of silence where we do not attempt to measure the world, but experience it instead as an existing whole in the eternal present with the self. To understand Wittgenstein, I suggest, is to discover this for our selves.

Chapter 4: From Mysticism to Myth

Myth has a strong grip on the human imagination; it supplies the classification and the categories, the pigeon-holes and concepts, the *categorical framework* within which every subject is placed and understood. *So does science.*

(van Fraassen 1994b, 129)

4.1 Introduction

Van Fraassen's striking claim "[s]cience is bridled superstition" (1999, 11) is built upon an empiricist adaptation of Wittgenstein's metaphysical idea of logical space. Committed to the conceptualist and nominalist traditions of the middle ages that take the common properties and relations of things to be human creations, van Fraassen develops the thesis that these creations are imaginative logical spaces, categorical frameworks of possibility representing actuality located within them. His general orientation is that the logical spaces of our mythical world-picture have been replaced by ones within which accepted mathematical models of are mapped, allowing them to more adequately save the phenomena and giving rise to a "theoretical world-picture" that is, at times, equally bewitching (1980, 81).¹⁸⁷

To my knowledge, no one has explored and evaluated this thesis. Yet it is the idea of logical space that underwrites van Fraassen's empiricist view of structure, bringing together his semantic view of scientific theories with his equally important views of language and experience. The final chapter of this dissertation is an attempt to rectify this deficiency. After providing an overview of van Fraassen's idea of logical space and key features of his early philosophical method, it presents a brief synopsis of the relation between his semantic view of scientific theories and his views of science known as constructive empiricism and empiricist structuralism. This sets the stage for an exegesis of van Fraassen's idea of logical space as it progressed through these views. After bringing together its interpretive motivation, the problem of coordination it encountered and the solution of self-location proposed, the last section evaluates his idea of logical space against the backdrop of three unities traditionally associated with it: the unity of all possible worlds, the unity of existence and the unity of divine consciousness. The conclusion will be that van Fraassen's use of the idea of logical space is inconsistent with his empirical stance.

4.2 Overview

4.2.1 What is a Logical Space?

Like Kant and Wittgenstein before him, van Fraassen's idea of logical space is a somewhat elusive. Following Wittgenstein he takes a logical space to be "a general form of any possible world" (1970a, 100) that imagination presents as a form of representation. However, he dramatically liberalizes the idea of logical space, almost to the point of trivialization. A logical space need not be a multi-dimensional set of n-tuples of real numbers. It may also be "an algebra, a lattice, or even more rudimentary, a posit" (2008, 172). It need not even be a mathematical object. "The kinds of things which make up the membership of logical space is essentially arbitrary: they may be chairs, points, vectors, cabbages, kings, or bits of sealing wax" (1967, 175). This is because the space represents simply through the possibility that a member will correspond to an individual entity. He provides the following examples:

The color spectrum is the logical space for colored things. ... Porphyry's tree is the logical space of everything, the Library of Congress classification is the logical space of all books. We conceive of any extended opaque object as determinately located in the color spectrum and of any possible book that might eventually be published as somehow placed in our beloved LC classification. (1991b, 34)

These examples suggest that in its most general sense a logical space for van Fraassen is an array of possibilities that functions for us as a classification framework organizing reality according to our interests. It is a "space" only in the sense that it is a bounded form "open" to actuality it organizes.¹⁸⁸ He is not invoking the metaphor of, or even analogy to, physical space, for we shall see that space (as well as time and space-time) is also a logical space. Van Fraassen's idea of logical space is, generally speaking, an extension of our idea of physical space.

Invariably it seems that in science a logical space is mathematical. To characterize its role as a representation, van Fraassen invokes the metaphor of a concrete model. What we saw characterized in Wittgenstein as elementary logical spaces, i.e., forms of representation mirroring basic forms of reality such as time, space and colour, van Fraassen identifies as representations or "models"¹⁸⁹. Here the idea of logical space

is easier to understand as a form of imagination. Moving in the direction of Boltzmann, van Fraassen takes these spaces to concern not only the perception of everyday entities, but also perceptions involving instrumentation. Here the idea of logical space is easier to grasp as a form of representation. Basic logical spaces combine into more complex spaces within which a “model”, understood as a solution to mathematical equations,¹⁹⁰ is mapped. A logical space such as phase space or Hilbert space is “a space common to a whole family of models provided by that theory” (van Fraassen 2008, 164). As a model or as a form common to models, use of a logical space in science extends beyond simple classification to research design, prediction and explanation.

To characterize its conditions of representational use, van Fraassen has as of late shifted the base of the metaphor to a concrete map. He introduces his idea of self-ascription of location or simply *self-location* by analogy: to use a model *I* must locate the reality of my situation through measurable properties in the relevant logical space just as *I* have to in order to use a map. Self-location in relation to logical spaces used in science is distinctive in that it is in accord with a set of operational rules that yield a genuine measurement relative to the theory (van Fraassen 2008, 165-166).

As an entity, a logical space (of models) is *sui generis* in being characterized variously as an “abstract entity”, an “artifact”, and an “ideal entity”. Considered apart from what it represents, a logical space (in science) is an abstract mathematical object. As an abstract entity, a logical space is described through – or, as we shall see later, can be thought of as governing our expression of – a structure of universal concepts of measurable properties such as *red*, *green*, *position*, *momentum*, etc. This intimate relation between a logical space and a conceptual framework of universals is the basis upon which van Fraassen provides an account of universals to rival traditional *in re* and *ante rem* interpretations:

... the colour spectrum is a mathematical structure – part of the real line – in which we postulate all colored entities to be locatable in a way that reflects their color relations and which thus provides the picture that guides our thinking and discourse about colors. (van Fraassen 1985a)

... an abstract entity such as the color spectrum is not one of the things in the world, nor is the *whale*, nor the fall of night – although there are colored things, which match or clash

with each other, and individual whales, which give birth to other whales, and the paradoxical deepening and fading of colors at sunset. (van Fraassen 1991b, 24)

The color spectrum is the logical space for colored things. There is no need to add: and the color spectrum exists, eternally and at peace, in its own transcendent beauty, and so forth. (Ibid., 34)

Early on van Fraassen (1975) characterized his view of abstract entities as one of conceptualism, though without elaboration. If we understand conceptualism as a theory of universals that “sees them as shadows of our grasp of concepts” (Blackburn 2005), then it is useful to think of a logical space as just this shadow. If my “concept of *radio*, the sense of my word “radio”, is such that nothing can instantiate it unless there are electromagnetic waves” (van Fraassen 2006a, 126), the reason is that I do not classify something as a radio unless I locate it in a logical space of a scientific theory that guides a structure of universal predicates implicit within my concept *radio*.

Van Fraassen also characterizes a logical space (of models) as a “cultural object” (1970a, 107) or “artifact”. He takes an artifact to be any entity upon which we bestow a role and function. For example, a stone is an artifact once given use as a hammer. Apart from this bestowal by us, the stone has no representational content, for we bestow a representational role upon it by giving it “about-ness” (van Fraassen 2008, 25). Similarly, van Fraassen is of the view that “theories are artefacts” (Ibid., 238) in that we give “about-ness” to an abstract entity. But in order to bestow a representational role, we must first classify the entity as capable of carrying out this function. To avoid the obvious regress of classifying an entity as a logical space by locating it in another logical space, van Fraassen appeals to a “space of reasons”:

... to call, classify, something as a map or a model is to locate it in what Wilfrid Sellars called “the space of reasons” – at least as this phrase is now broadly understood. By itself this is not yet self-location. It is just to classify the item as having semantic content, and as having a role in reasoned discourse and in practices subject to norms of rationality. We can to some extent separate our understanding of the item, in the sense of grasping its semantic content, from the understanding of our own situation that comes with locating oneself ‘in’ or ‘with respect to’ the item. But the latter comes in train, so to speak. (Ibid., 84)

(As we saw in Chapter 1, this idea of locating a mathematical space within a conceptual framework is prefigured in Kant’s location of pure sensible schemata within a potential classificatory space of reasons.) It is significant that for van Fraassen more than one

abstract entity can play the representational role of a particular logical space; apart from this element of convention, no abstract entity is a logical space.

As an artifact, van Fraassen also refers to logical space an “ideal entity”, alluding specifically to Leibniz’s phrase (1991b, 34). What Leibniz meant by “ideal entity”, however, is not without controversy. According to Friedman (1992), Leibniz saw time and space as ideal phenomena representing the pre-established harmony. In particular,

... space is ideal because relations between substances are ideal: each substance mirrors the entire universe internally due to its own inner principle, and space is an ideal representation of the underlying order of monads expressed in the pre-established harmony. Indeed, since each simple substance by itself already expresses completely the order of the entire universe, nothing but the mere existence of substances is necessary to constitute phenomenal space. (Ibid., 7-8)

Following Leibniz, Wolffians reconciled monadic reality with the infinite divisibility of space by claiming geometry deals only with “creatures of the imagination by which we confusedly represent phenomenally the true monadic reality” (Ibid., 4, n.6). Where van Fraassen differs from the Wolffians is not in denying that an ideal entity is a “confused” representation, but in refusing to make the further claim that it represents the form of reality. In saying logical space is an “ideal entity” it seems he means only that it is a form of imaginative representation that accommodates all possibilities of a certain sort including real actuality (cf. 1970a, 102). As an ideal entity, a logical space (of models) is *completed* structure: it is an image or “picture” (1985a, 276) that “outstrips its *fundamentum in re*” (1972, 329).

It is important to recognize that van Fraassen’s notion of an ideal entity differs from Wittgenstein’s idea of logical space in that it is not an entity of private imagination associated with mental elements. A “mental representation”, van Fraassen claims, is an “oxymoron” (2008, 345, n.1), for one cannot bestow a representational role on mental entities. “There is no representation except in the sense that some things are used, made, or taken, to represent things as thus or so” (Ibid., 23), and here “use” “implies community: there is no such thing as an essentially private representation any more than private language, except in the sense in which private uses can exist as derived or parasitic on communal practices” (Ibid., 248, n. 24). As we shall see shortly, van Fraassen’s rejection of logical space as a private image is related to his rejection of

private language in that he introduces the idea of logical space within a project of analyzing our common linguistic framework. An ideal entity is thus an entity of *public* imagination enshrined in certain “coding conventions extant in the community” (Ibid., 23).

A logical space of models constituting our “theoretical world picture” is probably what van Fraassen refers to in the title of his book *The Scientific Image* (1980). It is instructive, therefore, to explore what kind of image it is. Reminiscent of the Wolffians’ view of an ideal entity as a confused representation, a basic claim of van Fraassen’s later empiricist structuralism is that a picture is a type of image that trades for success in use by us on selective similarity and distortion: the similarity is empirical structure (Ibid., 34-35); the distortion is surplus structure (Ibid, 30), abstraction and idealization (Ibid., 39-49). This accounts for van Fraassen’s view of science “that accepts the appearances alone as real, and all the rest as a unifying myth to light up our path” (van Fraassen 1994b, 133). It also accounts for our metaphysical impulse: “[o]ur language and reasoning is guided by a picture, a picture which bewitches us, the picture described by extreme realism” (1978, 15). Scientific realists make the mistake of projecting conventional elements of the scientific image into reality much like a confused observer makes the mistake of projecting a mirage, rainbow or *fata morgana* onto reality as a material thing. The latter are types images van Fraassen characterizes as “public hallucinations” (2008, 104). Since both the scientific image and a public hallucination are inter-subjective, bear a diffuse relation to reality, and bewitch us into projecting more onto reality than there is, I suggest that for van Fraassen the scientific image is a sort of conventionally produced public hallucination.

4.2.2 This is Who We Are

Van Fraassen’s appeal to the idea of logical space is in the context of questions not about what there is in reality, but “of how things are *for us*, of what it is to exist in this world *in the way we are there*” (2009, 471). His general philosophical approach appears to emerge from a synthesis of insights gathered through the method of phenomenology as exemplified in both continental and analytic traditions.

The term “phenomenology” emerged in the 18th century in the writings of Lambert and Kant to denote the description of experience or consciousness apart from considerations of intentional content (Blackburn 2005). The former is found in van Fraassen’s embrace of Hume’s “world of empiricism”:

... it is a world in which anything is possible, and whatever happens merely happens, and not because some thing greater is making it happen. ... There are no necessary connections in nature, no laws of nature, no real bounds on possibility. Those ideas all resulted when philosophers projected familiar models on the natural world. Really, nothing is necessary, and everything is possible.

I mean this. All of the above is true. (1994b, 123-124)

To embrace Hume’s world of empiricism is to fully appreciate the phenomenology of uninterpreted experience, i.e., experience as reflectively deconstructed from its classificatory, lawful and modal implications.¹⁹¹ This is, he tells us, the bare experience of Antoine Roquentin in Sartre’s novel *Nausea* (1964):

I went to the window and glanced out ... I murmured: *Anything* can happen, *anything*. ... Frightened, I looked at these unstable beings which, in an hour, in a minute, were perhaps going to crumble: yes, I was there, living in the midst of these books full of knowledge describing the immutable forms of the animal species, explaining that the right quantity of energy is kept integral in the universe; I was there, standing in front of a window whose panes had a definite refraction index. But what feeble barriers! I suppose it is out of laziness that the world is the same day after day. Today it seemed to want to change. And then, *anything, anything* could happen. (77)

The objects, events and processes that happen to us have properties and relations, and the properties of such properties and relations are concrete structures. But our experience of structure is not how we conceive structure in experience: concrete structure is *indefinite*, whereas we conceive structure as *definite*.

This is evident in our disinclination to even use the word “structure” for properties and relations mutable from one moment to the next; indeed, the potential for chaos is antithetical to our common-sense meaning of the term. At the very least what we normally mean by “structure” in experience are properties of *universal* properties and relations that particulars of bare experience instantiate. As Goodman’s (1955) new riddle of induction shows, it seems we *spontaneously* project not only relations (e.g., A happens after B) but also properties (e.g., green) of experience into the future, making definite the

properties of these relations (A will always happen after B) and properties (e.g., green is exclusive of blue).

To explicate what we normally mean by “structure” in relation to experience,¹⁹² van Fraassen turns from the phenomenology of bare experience to “how we conceive the world” (1985b, 206), invoking the method of conceptual phenomenology he observes in Kant. A general and “very simple-minded presentation of the method”, he claims, is one that amounts to

... accepting the principle that what we can and cannot imagine are indications of conceptual interconnections – put more grandiosely, of the structure of our conceptual framework. And this does not seem unreasonable so long as we are merely inquiring into our own conceptual framework. For one can hardly be said to have a concept of *X* unless one can imagine *X* or think about *X*; conversely, if I can imagine *X* and think of *X*, then I have a concept of *X*. (1970a, 44)

Kant’s insight in the *Critique* (1781-1787/1998), van Fraassen tells us, is that the principles of understanding determine a conceptual framework indicated by “a certain general form that any possible world must have; a possible world is just this necessary general form filled out by certain contingent contents” (1970a, 99). But whereas for Kant conceptual phenomenology serves to illuminate “the phenomenology of consciousness” (1985b, 206) facilitating description of cognitive faculties and their representations, for van Fraassen it serves “to elucidate and further articulate the conceptual scheme of accepted scientific theories” (1970a, 105).¹⁹³

Citing Carnap’s *Meaning and Synonymy in Natural Languages* (1956a), he sees the phenomenological method discussed in the analytic tradition as “primarily in connection with the subject of intension” (1970a, 45). In this paper Carnap responds to criticisms levelled by Quine, arguing for a viable theory of intension in addition to that of extension. Here “intension” is understood to be the undefined meaning component relevant for truth:

The technical term ‘intension’, which I use here instead of the ambiguous word ‘meaning’, is meant to apply only to the cognitive or designative meaning component. I shall not try to define this component. It was mentioned earlier that determination of truth presupposes knowledge of meaning (in addition to knowledge of facts); now cognitive meaning may be roughly characterized as that meaning component which is relevant for the determination of truth. (Carnap 1956a, 236-237)

To account for Kant's insight into our conceptual phenomenology it appears van Fraassen ventures further than Carnap and takes the cognitive meaning component to be the imaginative form of all possible worlds Wittgenstein identifies as (the whole of) logical space (1970a, 100-101). Notice that its semantic function in cognition is less like Kant's principles of the understanding derived from transcendental schemata, and more like the structure of schemata identified in chapter 1 as Kant's logical space.

To factor in Sartre's insight, van Fraassen presents a logical space (of models) as the "intentional correlate" of a conventional conceptual framework through which we perceive and conceive real entities and their concrete relations:

After all, what is this world in which I live, breathe and have my being, and which my ancestors of two centuries ago could not enter? It is the intentional correlate of the conceptual framework through which I perceive and conceive the world. But our conceptual framework changes, hence the intentional correlate of our conceptual framework changes – but the real world remains the same world. (1980, 81)

As I understand van Fraassen, our *spontaneous* experience of structure is a synthesis of actual relations that happen to us with imaginative forms of possible relations through which we conceive the relations as having definite structural properties.¹⁹⁴

Perhaps the best way to understand this idea is through the eyes of Helmholtz and Hertz. For Helmholtz our experience of structure reflects our belief that what is actual and observed is actual and *observable*. Although van Fraassen agrees, he does not take the position that this belief gives rise to a regulative idea. He rejects "recourse to a metaphysics pretending to knowledge beyond the reach of experience" (1985b, 206) by drawing from the traditions of nominalism and conceptualism that take universals to be human creations rather than transcendental conditions. In creating a logical space of models we create universals as sets of elements representing possible entities, and our *acceptance* of them involves our belief that they correspond to entities that are actual and *observable*. And so when we locate entities delivered in experience within these universals, we conceive the indefinite structure of properties and relations among what is actual and observed to be the definite structure of properties and relations among what is actual and observable. This is to say that the self represents itself as having located in a logical space concrete structure in a definite way, then *projects* this structure back onto

reality. This projection need not be thought of as unconstrained, for while fulfilled predictions instantiate the common properties and relations, disappointed expectations bite back. But if we allow leeway for choice among universals as van Fraassen does, then the element of convention precludes the metaphysical position that in creating universals we are just “carving nature at its joints”. All this echoes Hertz: a logical space of models is a representation, and to assert that it is similar in empirical structure to what it represents is to say that it involves a projection onto reality defeasible by later experience.

4.2.3 A View of Scientific Theories Between Two Views of Science

For van Fraassen the question “What is Science?” elicits several responses, due mainly to the fact that “[t]he word “science” displays a typical ambiguity between activity and product” (2002a, 155). This ambiguity finds expression in his two views of science known as constructive empiricism and empiricist structuralism as presented, respectively, in *The Scientific Image* (1980) and in *Scientific Representation* (2008). Constructive empiricism construes the question “What is Science?” “as asking what is the point, the *telos*, of that activity” (2007a, 342), while empiricist structuralism takes it to be asking “how a science relates to its domain of application” (2008, 2). Constructive empiricism is the view that, as activity, science aims primarily to produce theories that “save the phenomena” (1980, 12), i.e., real entities that are observable objects, events and processes. Empiricist structuralism is the view that, as product, science represents phenomena as embeddable in abstract structures that can only be described up to isomorphism (2008, 238). These two views of science are related: empiricist structuralism tells us what “saving the phenomena” amounts to in constructive empiricism if one adopts a structuralist account of scientific theories.

It is important to recognize that constructive empiricism and empiricist structuralism, though obviously designed to work together, are fundamentally different kinds of philosophical theories. Constructive empiricism is formulated to contrast with scientific realism, if scientific realism is formulated as a doctrine of aim:¹⁹⁵

Science aims to give us, in its theories, a literally true story of what the world is like; and acceptance of a scientific theory involves the belief that it is true. This is the correct statement of scientific realism. (1980, 8)

Science aims to give us theories which are empirically adequate; and the acceptance of a theory involves as belief only that it is empirically adequate. This is the statement of the anti-realist position I advocate; I shall call it constructive empiricism. (Ibid., 12)

In both positions van Fraassen uses “belief” in a descriptive *not* a normative sense, i.e., in the sense that “the only belief that is *ipso facto* involved in acceptance is that the active criterion of success is met” (2007a, 342). In contrast to truth *in toto* (truth of what is observable, unobservable and possible), a theory is empirically adequate if, roughly speaking, “what it says about the observable things and events in this world, is true – exactly if the ‘saves the phenomena’ (van Fraassen 1980, 12). In addition, acceptance involves a pragmatic commitment to use the resources of the theory to describe phenomena, answer why or how questions, and design a research programme (Ibid.).

Notwithstanding frequent misunderstanding, constructive empiricism is not a theory of scientific knowledge: “Constructive empiricism is not a doctrine about epistemology, but about the aim of science” (Monton and van Fraassen 2003, 419-420); “It is not part of constructive empiricism to dictate a particular epistemic position” (Ibid., 407-408).

... I see core realist and anti-realist views of science as answers to “What *is* science?” which are logically independent of any epistemology. In this sense one could have an anti-realist view of science while believing in the complete literal truth of currently accepted science. And similarly one could have a realist view of science while maintaining that success in the scientific enterprise so far has been mainly illusory and is perhaps forever beyond human reach. (van Fraassen 2003, 481)

Hence constructive empiricism does not purport to justify agnosticism regarding unobservable entities.¹⁹⁶ In a move unnoticed by many scientific realists, van Fraassen shifts the debate from the rationality of scientific belief to the rationality of scientific *activity*: “[b]y taking them as central ingredients of our view of science we place ourselves in a position to *make sense of* those activities which we all agree are part of science” (1994a, 190).

As I understand van Fraassen’s formulations of them, scientific realism and constructive empiricism proffer *rational reconstructions of scientific activity*.¹⁹⁷ The

intentional activity philosophers classify as scientific is presumed to have an internally defined central or “basic” criterion of success. Both scientific realism and constructive empiricism postulate what this criterion is. They attempt to recover this activity within a framework that preserves characteristic features and distinctions agreed upon by philosophers¹⁹⁸ as relevant and significant. They try to show how in principle intentional activities classified as scientific can be made intelligible when understood as exemplifications of a strategy that best promotes the central aim identified.

Accordingly, van Fraassen’s realism is not epistemic. Rather, it is semantic: “[t]o say what is true is to say, of what is, that it is, and, of what is not, that it is not” (2003, 482). Beyond the strict deliverances of experience, i.e., beyond what is actual and observed, past and present:

[t]he question whether anything exists has to be replaced by the question: does the status we accord our best theories *require its existence?* It is not true that the former question is unintelligible or does not arise, it just does not arise for us directly: we can only sidle up to it *via* the latter question. (van Fraassen 1995, 142)

From its characterization of the belief involved in acceptance, constructive empiricism tells us that the status we accord our best theories requires the existence of what it says to be actual and observable past, present and future. Constructive empiricism contrasts with scientific realism in its claim that the rationality of scientific activity gives us “no reason to assert that our descriptive terms have determinate reference behind the phenomena” (van Fraassen 2010a, 470).

Why reject scientific realism in favour of constructive empiricism? For those who adopt an empirical stance, van Fraassen maintains, science “is a paradigm of rationality” (2002a, 63), and so “regarding how to best make sense of science ... one finds a central motivation, arguably the main motivation, for constructive empiricism” (Monton and van Fraassen 2003, 421). Moreover, “[e]mpiricists ... wish for epistemic modesty in their paradigms of rational inquiry” (Ibid., 407), and therefore seek to identify the “bottom line” (van Fraassen 1981, 673) aim of science that brings with it the least amount of belief in theory acceptance. In so doing empiricists demonstrate that “as far as the enterprise of science is concerned, belief in the truth of its theories is supererogatory” (van Fraassen 1985a, 255). Thus empiricists should adopt constructive empiricism rather than scientific

realism because it makes as good, if not better sense of science with less *de facto* belief about what there is.

One should recognize that as formulated, constructive empiricism does *not* presuppose the semantic view of scientific theories: “[t]he choice between the semantic approach and its rivals is entirely independent of the controversies between scientific realism and anti-realism” (van Fraassen 1991a, 15).¹⁹⁹ In *addition* to constructive empiricism, van Fraassen is of the view that a theory consists of a theoretical definition that presents a set of models and a theoretical hypothesis that asserts that certain types of real systems are related to these models (1987, 109). “When the equations formulate a scientific theory, their solutions are models of that theory” (van Fraassen 2008, 310) mapped in logical spaces. Truth *in toto* and empirical adequacy are then explicated in terms of correspondence between models.²⁰⁰ This is, of course, not to say that constructive empiricism was not formulated with the semantic view in mind.

The semantic view of scientific theories presents only a vehicle for scientific representation; it is not a view of what a scientific representation is (van Fraassen 2008, 348, n.26). Van Fraassen’s view of science known as empiricist structuralism is a theory of scientific representation that presupposes a structuralist view of scientific theories such as his semantic view. Constructive empiricism is only relevant to empiricist structuralism in that use of structural representation involves purpose and aim (Ibid., 28), and by identifying what an adequate structural representation amounts to, constructive empiricism serves to “highlight” (Ibid., 23) the selective resemblance we bestow upon our scientific representations.²⁰¹ Accordingly, the antagonist of empiricist structuralism is not scientific realism, but structural realism (Ibid., 3). Structural realism sees the depiction of a scientific representation as one of simple resemblance (Ibid., 198), while empiricist structuralism views it in terms of resemblance (in part) as well as distortion through abstraction, idealization and surplus structure.

4.3 Circa 1970: The Semantic View of Scientific Theories

4.3.1 The Causal Theory of Time

Van Fraassen introduced his conception of logical space in his dissertation under Grunbaum; it was also the innovative contribution behind his first book, *The Philosophy of Time and Space* (1970). The book's general concern is the analytic one of elucidating and articulating "the idea of time as it appears in our common conceptual framework and in the conceptual framework of the physical sciences" (Ibid., 104). This involves taking into account that "'time' is a singular term, purporting to carry singular reference" (1985b, 207) and adopting the approach of a causal theory of time.

A causal theory of time is a variant of the relational theory of time. As van Fraassen sees it, a relational theory of time views time, not the way Newton did as a real entity of infinite temporal magnitude into which are placed "[a]ll things ... as to order of succession" ((Newton 1960, 8) as quoted in (van Fraassen 1991b, 24)), but as an order "constituted by means of, or on the basis of, relations between the events and processes to be so ordered" (Ibid.). In line with the relational theory, van Fraassen claims "[t]ime is not real, time does not exist, there is no such thing as time" (Ibid.). He takes it instead that "there do exist physical entities, events and processes, which stand in various relations to each other, and thus constitute a complex relational structure" (1985b, 200).²⁰² A causal theory of time goes beyond a relational theory in its claim that the temporal order can be defined in terms of certain basic physical relations. Its basic task is to correctly describe these relations in non-temporal terms, and then use these descriptions to define the temporal order.

In proposing the first relational theory of time, Leibniz was also the first to propose a causal theory of time.²⁰³ However there were difficulties, notably the application of Hume's critique to Leibniz's rationalist theory of causation.²⁰⁴ With the advent of the special theory of relativity, the causal theory of time resurfaced in the early 20th century in the work of Reichenbach and Grunbaum. Reichenbach's hope was that if the basic physical relations could be described in non-spatiotemporal terms, the causal theory of time could be extended to a successful relational theory of Euclidean space and space-time implicit in the special theory of relativity. The term "causal" can be

misleading in this new setting, for it does not involve any general notion of causality. Rather, the physical relations are viewed pre-philosophically as the basic relations of accepted scientific theories. Moreover, “theory” denotes a *reconstruction* of scientific language. “The claim of the causal theory of time is not that spatiotemporal terms are *defined*, but that they are *definable*, in terms of [physical relations]. ... Formulations of theories are, in a sense, artificial, since they must rely on a choice of primitive terms (and of axioms) that is to some extent arbitrary” (van Fraassen 1970a, 195). One way or another, however, the theories of Reichenbach and Grunbaum had relied upon spatial or spatiotemporal concepts. To overcome this problem, van Fraassen sought to simplify the theory by providing “intuitive content for its notions” (Ibid., 198).

He identifies the basic physical relations as that of genidentity and light signals among events: two events are genidentical if they belong to the history of the same object; two events are related by a light signal if they are the emission, absorption or reflection of the same light signal. These relations of “causal connection”²⁰⁵ are taken as primitive:

Genidentity and signal connection are relations too basic in the conceptual scheme of physics and too empirical in their significance to be denied the status of physical relations, it seems to me, even in the absence of necessary and sufficient criteria for the applicability of the term “physical relation”. From this we draw the following conclusion: “causally connected,” and hence “causally connectible,” have a meaning that is not specifically spatiotemporal. Therefore, we are not guilty of the sleight of hand of developing a causal theory of time by giving a new name to a basic spatiotemporal relation. (1970a, 194)

At this point it is helpful to note that for van Fraassen observable entities we classify as “events” “processes” and “objects” (i.e., what he later refers to the “phenomena”) are related to one another through physical relations of genidentity and signal connections. An “event” is a “short-lived state” (Ibid., 32), a “process” is a “series of successive events” (Ibid.) and an “object” is a process in relation to aggregates of simultaneous events constituting a class of events that can be defined as a “world-line” (Ibid., 34-35; 183). Following Reichenbach he claims that it is a matter of preference whether we adopt event or object language (1991a, 454).

As the foregoing passage suggests, van Fraassen relies first and foremost upon relations of causal *connectibility* in order to define the temporal order. The reason is,

[t]o put it baldly, the structure of actual physical connections does not determine, as far as we can see, the spatiotemporal relations among actual events – as these are usually conceived.²⁰⁶ (van Fraassen 1970a, 197)

This point of our conceptual phenomenology is attributed to Kant:

... we conceive of time as one, as an individual, in which we conceive all events, possible as well as actual, as being located. If we think of different world histories, we think of them as unfolding in the same time. And when we chart different courses of events we chart them in our own time: Hugo could have murdered Hoederer this morning, he may do so tonight. We conceive of all events as necessarily located in time, as necessarily having some definite location there, whatever it may be. (van Fraassen 1985b, 206)

In a move that will define his empiricism, van Fraassen honours our pre-analytic intuition of structure over the strict deliverances of experience. But the challenge then becomes one of explicating the modality of causal connectibility without appealing to spatiotemporal terms: “two individual events e and e' are connectible, is, in the absence of information about their relative spatio-temporal location, not deducible from *any* general fact” (van Fraassen 1985b, 207). It is in order to meet this challenge van Fraassen appeals to the idea of logical space. Let us look at this move more closely.

To simplify the causal theory of time developed by Reichenbach and Grunbaum van Fraassen’s strategy was to turn their approach on its head. Generally speaking, both had attempted to first define the temporal order on each world line individually, and then explicate the temporal order of all events by using the relation of not being causally connectible to correlate the separate orderings. In this context connectibility refers to the finite velocity of light, and such a correlation concerns events connected by signals faster than light. Van Fraassen, on the other hand, explicates the time order of events on any world line (defined as a class of events, any two of which are genidentical) in part through relations of non-connectibility to events on other world lines. These relations are used to define “simultaneity classes” and then “continuous parts” of a world line, followed by relations of “temporal separation” among events and then the basic temporal relations *simultaneous*, *before* and *between* (1970a, 182-186; cf. 66-69).

Effectively, the relation of causal connectibility is being used to induce a continuum isomorphic to the real or extended real number system:

So we use a relation of connectibility to define [spatiotemporal relations as we conceive them], after having laid down suitable postulates on the relational structure of connectibility. But these postulates are calculated to make the structure of temporal relations, as defined, isomorphic to the (extended) real number system, for example. (van Fraassen 1970a, 197)

A function t can then be defined in such a way as to map all events onto either real number systems, preserving (inter alia) temporal separation and recovering the temporal order within a coordinate system (Ibid., 188). Either coordinate system can be thought as a structure of all possible events representing all actual events by embedding their structure (cf. Ibid., 104). But the possibility of both of these coordinate systems means that the temporal order of all possible events may be topologically open with or without beginning, or topologically closed. Yet we conceive time as topologically open and without a beginning. This feature of our conceptual phenomenology noted already by Kant was explained (at this time) by appeal to the conceptual framework of our accepted scientific theories:

The necessity, which Kant perceived, of time having the structure of the real line is only the necessity of a conceptual scheme that developed with the success of Newtonian physics. But this necessity is still with us in the sense that we have not accepted an alternative; only, recent cosmological speculation, and the violent demise of the classical framework (in some important respects) have greatly increased our tolerance of ambiguity at this point. (van Fraassen 1970a, 107)

Van Fraassen's basic idea was that our conceptual framework "schools" the form of our imagination (cf. Ibid., 45), and the relation of causal connectibility recovers this schooling (in part) in relation to how we imagine time.

Here it is helpful to recognize that van Fraassen had yet to reject the syntactic view that scientific theories are formal axiomatic systems, and that acceptance of a theory "consists in believing that the axioms and all their consequences are true" (van Fraassen 1985b, 200). In reconstructing the temporal language of our conceptual framework, it seems his causal theory of time was taken to be at the heart of accepted scientific theories. Accordingly, the notion of causal connectibility can also be understood to correlate reconstructed temporal relations with physical relations among events, accounting for the theory's interpretation. The theory's postulates on causal connectibility can be thought as constituting a coordinative definition by which the notion causal connectibility *interprets* a relational theory of time.

But if it is to bestow sense on a theory, the notion of causal connectibility must be *meaningful*. “From an empiricist point of view, there are besides relations among actual matters of fact, only relations among words and ideas” (van Fraassen 1987, 122), and so the challenge is to explicate the meaningfulness of causal connectibility without appealing to more than actual physical relations. It is interesting and perhaps telling that van Fraassen did not face this challenge head-on at this time. The notion of causal connectibility was simply put forth as a counterfactual conditional that met standards for linguistic austerity in the philosophical use of language provided by Quine (van Fraassen 1970a, 196-197). Van Fraassen’s real thoughts were left to an *obiter dictum*:

It seems to me, therefore, that, as presently formulated, the causal theory of time meets the standards of clarity currently imposed. But after having said this, I would like to argue that we can look upon the use of the counterfactual notion of causal connectibility as a dispensable convenience rather than as a necessity. ... My proposal is ... that we look upon the use of the connectibility relation as simply having the purpose of describing the logical space in which, we assert, all relational structures of actual connections can be embedded. This means that we think that the relation of connectibility is not needed to describe the actual world. It means also that the postulates on connectibility that we lay down just express a belief concerning the actual connections we may encounter, and nothing more. ... The postulates on connectibility only helped to single out the mathematical structure in question in a heuristic manner. (Ibid., 197-198)

His idea is that the notion of causal connectibility has “intuitive content” (Ibid.) in that it describes a pre-analytic structure through which we imagine the concrete structure of causal connections to be embedded. The structure that it describes is a logical space, the real line in this case, that arises from our immersion in a conceptual framework. Logical space thus serves as an ideal referent in a *reconstruction* of natural language, some elements of which correlate to individual real entities.

Van Fraassen had understood Wittgenstein’s idea of logical space very generally as the idea of a form of all possible worlds.²⁰⁷ He interpreted relevant provisions of his *Tractatus* (1961b) as follows:

Thus the assertion that something is of a certain kind entails that there is a set of families of properties such that this thing is characterized by one member of each family:

X is a medium-large physical object *entails* that *X* is somewhere in space, has some colour, some texture, some shape and.... *X* is an event *entails* *X* is somewhere in time, and....

The collection of these families of properties determines the logical space of that kind of thing. (Each family by itself, or each subcollection of these families, determines a subspace of that logical space, which may itself be called a logical space. Thus, Wittgenstein speaks of the “colour-space”.) (1970a, 100)

Since “members of a single family are mutually incompatible” and jointly exhaustive predicates of any degree that form a kind (Ibid., 13), van Fraassen associated the idea of logical space with a structure of conceptual interconnections. But Wittgenstein understood logical space to be a form of imaginative *and* linguistic thought, while van Fraassen had introduced the idea into an analytic project that takes it to be a form of imagination *representing* a form of linguistic thought:

We characterize the notion of *logical space* by saying that a logical space is a certain mathematical construct used to represent certain conceptual interconnections. By representing real things (instances of those concepts) by means of elements of this mathematical construct (their “locations”) we also represent relations among those things. (Ibid., 104)²⁰⁸

By directly representing our language in use (which is interpreted, if anything is) it seems van Fraassen considered logical space to be interpreted, which allowed it to provide meaningful intuitive content to modal notions in a reconstruction of language.

In providing intuitive content to the notion of causal connectibility in the causal theory of time, logical space gave van Fraassen a Kantian answer to the question “What is time?” Unlike Wittgenstein for whom time is a form of simple objects and more like Kant for whom time is a form of intuition, van Fraassen claims

... time is a logical space, a subspace of the total logical space of events[;] (1970a, 101)

... logical space (time) is the real line being used to represent all possible temporal relations among events and the conceptual interconnections among these relations. (Thus simultaneity is represented by identity of location on the real line, and the fact that temporal precedence is incompatible with simultaneity is reflected by the incompatibility of $<$ and $=$.) (Ibid., 102)

In rejecting the Newtonian claim that time is real van Fraassen clings to the view that time is a “container” of sorts, only it is an “ideal frame” (1991b, 23) that can change with revolutionary shifts to our conceptual framework. As it stands, time is the real line. It represents conceptual interconnections among temporal relations by representing the mutual incompatibility of these relations. In virtue of the irreflexivity of $<$ and the reflexivity of $=$, the real line represents structural properties of relations of temporal

precedence and simultaneity. The statement “no event is earlier than itself” can thus be said to analytic because it concerns the meaning structure of natural language explicated in the causal theory of time; it can be said to be synthetic a priori because, in encompassing all possible events, it is about what is actually the case.²⁰⁹ Once a metric is introduced, time serves as a universal to classify entities according to the measurable property *duration*.

The innovation behind van Fraassen’s adaptation of Wittgenstein’s idea of logical space is his view of it as synthesis of existing indefinite concrete structure with non-existing definite abstract structure. Duns Scotus argued that time is an aspect of movement and a measure, thus both mind-independent and mind-dependent, and van Fraassen makes a very similar claim:

... the view that time is a logical space allows a “Scotist” synthesis on the question whether time is a mind-dependent entity. A logical space is a mathematical construct *used to represent ...*; and that means, of course, *used by us*. If we users and representors did not exist, neither would there be something being used to represent. The real line cannot be used to represent the actual temporal structure of events unless the latter can be embedded in it. This is purely and entirely an objective question of empirical fact. But neither can the real line thus be used unless there are those capable of using it. Hence, in that case the logical space *time* (which is something used to represent something else) could not then exist.²¹⁰ (1970a, 107)

Time is a logical space, which is to say that it is a representation. In representing certain conceptual interconnections time is an idealized “model” (Ibid., 192) representing real events by embedding their structure. If we abstract from this use by us, there is just a set of real numbers, an abstract entity with no greater claim to existence than any other in nominalist eyes. This is behind van Fraassen’s cryptic conclusion “that it is not necessary to say that there is such a thing as time, but that if we do, the best possible answer to the further question what kind of thing it is, is that it is a logical space” (Ibid., 106). If what we refer to by “time” is taken *de dicto* then it does not exist; if what we refer to by “time” is taken *de re* then it does. Hence, it is not necessary to say that time exists, but if we do say that it exists then it is an imaginative structure embedding concrete structure (cf. van Fraassen 2010a, 464).

This view of time dovetails with van Fraassen’s Sartrean view of the phenomenology of bare experience. Unlike the order of all possible events, the order of

actual events is not settled in one way or another: “the definiteness of the relation between the order [of events] and what is ordered resides mainly in how the matter is to be conceived, and is underdetermined by the facts” (van Fraassen 1991b, 19). But in claiming time is a form of all possible worlds, van Fraassen must pay a price: “I have, I suppose, given up that demarcation between the real and the imagined worlds in terms of determinacy” (Ibid., 34). He argues, accordingly, that imaginative construction of the definite structure of real events that happen to us and that we are aware cannot be distinguished from that of the construction of the definite structure of fictional events encountered reading a novel.²¹¹

After introducing a time metric and the Euclidean space metric van Fraassen transitions from the causal theory of time to a theory of Euclidean space and a theory of the space-time implicit in the special theory of relativity (1970a, 188-191). Inasmuch as he claims time is a representation through the set of real numbers (the real number system), he claims space is a representation through “the set of all triples of real numbers” (the Cartesian coordinate system) and space-time is a representation through “the set of all quadruples of real numbers” (Minkowski space-time) (Ibid., 167); in other words, space and space-time are also logical spaces. Van Fraassen’s *obiter dictum* extends here as well, and becomes perspicuous in the context of his theory of Euclidean space.

Reichenbach identified principles he called “coordinative definitions” to transform a mathematical geometry – understood as “a purely abstract, deductive system, with nothing to say about physical relations” (van Fraassen 1970a, 132) – into a physical geometry:

- (1) The path of a light ray *in vacuo* is a straight line.
- (2) A rigid body free of distorting influences remains the same size when transported.

However, in addition to the fact that these principles are not in proper definitional form, van Fraassen claims they are also not in proper modal form:

... the question which is the correct geometry has no answer unless the interpretation through coordinative definitions is a total interpretation: since there is quite possibly a certain paucity of actual lightrays and other physical connections, there would in principle be infinitely many geometries that fit the actual facts equally well. Hence it is necessary to

say, for example, that the geodesics are the possible lightray paths. (van Fraassen 1972, 325)

His point is that if there is to be an interpretation of mathematical geometry, then principles purporting to connect theoretical relations with physical correlates must single out a structure. A problem in singling out Euclidean geometry, for example, is that “any two points lie on a straight line, but not necessarily on a light ray” (van Fraassen 1991a, 6); i.e., Euclidean geometry gives us more straight lines than there are actual light ray paths because the latter are absent in the dark and do not pass through opaque objects. And so we cannot say a coordinative definition *interprets* Euclidean mathematical geometry unless correlates for straight lines in the dark and straight lines passing through opaque objects are identified in a “complete” (Ibid.) translation.

Van Fraassen concludes that to do the interpretive job Reichenbach presupposes them to do, principles (1) and (2) must be amended to the following:²¹²

(1a) The path ABC is a straight line if and only if it *could* be the path of a light ray.

(2a) An object is 1-meter long if and only if it *could* be brought into exact coincidence with the meter standard kept in Paris.

But now it appears a complete interpretation presupposes modal realism. Since “an empiricist position must entail that the philosophical exploration of modality, even where it occurs in science, is to be part of the theory of meaning” (van Fraassen 1987, 122), the challenge in rejecting modal realism is to substitute a meaningful extension of e.g., actual light ray paths to all possible light ray paths, that can be stated without recourse to the geometrical language to be interpreted through this extension. This is where van Fraassen thinks the idea of logical space can do work in the syntactic view: Euclidean mathematical geometry interpreted through proper coordinative definitions has “intuitive content” in that it describes a representation of a conceptual framework of natural language, i.e., the Cartesian coordinate system presenting a structure of all possible light ray paths embedding the structure of actual light ray paths.

This appeal to the Cartesian coordinate system to solve an interpretive problem is not *ad hoc*. It is justified as a matter of conceptual phenomenology. We do not imagine space as possibly having been different from what it is, but only the order of positions *in*

space as having been possibly different (van Fraassen 1970a, 99). We thus imagine actual entities *through* a logical space.

Tentatively we may conclude the following: A mathematical geometry [construed as an axiomatic deductive system] describes what we have previously called a logical space. The coordinative definitions place or map physical objects and relations into this space. But they cannot do this with complete definiteness unless we allow them to rely on counterfactual assertions. (van Fraassen 1970a, 132-133)

The tentative conclusion is that our imagining of actual entities in space can be reconstructed as the mapping of real entities onto points of the Cartesian coordinate system in a way that preserves their spatial relations. This mapping relation is described by principles (1) and (2).²¹³ Definitions (1a) and (2a) merge principles (1) and (2) with counterfactual conditionals that, in addition, describe elements of the coordinate system to which non-actual but possible entities would be mapped if counter to the fact conditions obtain. By “filling-in” the Cartesian coordinate system in this way the structure of Euclidean space is singled out and the mapping of actual physical correlates occurs to definite regions. Definitions (1a) and (2a) thus effect a “total interpretation” without modal realism, since interpreted Euclidean mathematical geometry can then be taken to describe an embedding structure that is part ideal and part real and that represents a conceptual framework of natural language. See Figure 4.1:

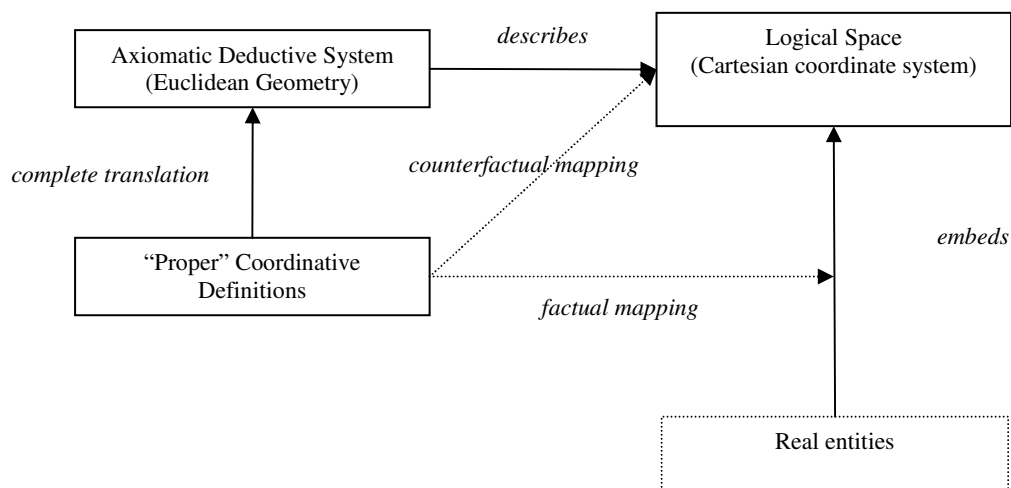


Figure 4.1: Proper Coordinative Definitions and Logical Space

Since he took physical geometry (and the causal theory of time) to be a “rudimentary physical theory” (1970a, 132) and Reichenbach’s coordinative definitions as the “paradigm ... case of correspondence rules linking a theory to physical facts” (1972, 325), it seems van Fraassen thought this account of interpretation extended to scientific theories in general.²¹⁴ But it was only “tentatively” presented in the form of *obiter* comments at this time because without “a tenable theory of counterfactuals in general” (Ibid., 198) it was regarded as “somewhat audacious” (Ibid, 197).

4.3.2 The Semantic Approach to Scientific Theories

Apparently motivated by considerations of quantum logic,²¹⁵ van Fraassen’s causal theory of time was part of a larger and more ambitious enterprise of retrenching the formal semantics of scientific language in logical space. His starting position for this project was Carnap’s general view of scientific language in *Meaning and Synonymy*:²¹⁶

It is today still mainly a natural language (except for its mathematical part), with only a few explicitly made conventions for some special words or symbols. It is a variant of the pre-scientific language, caused by special professional needs. (1956a, 241)

In this paper, Carnap wanted to provide “a practical motivation and justification for the introduction of corresponding concepts in pure semantics with respect to constructed languages” (Ibid., 247). He argued that pure semantical concepts like intension, synonymy, and analyticity are explicata for corresponding pragmatic concepts that can be determined by a linguist, and thus have a “good scientific status” (Ibid.). Relevant for our purposes is that he defined the intension of a predicate “as its range, which comprehends those possible kinds of objects for which the predicate holds” (Ibid., 238), claiming that “[t]he intension of a predicate for a speaker *X* is, roughly speaking, the general condition which an object must fulfil for *X* to be willing to apply the predicate to it” (Ibid., 246). This is determined by a linguist who conducts an interview of the speaker by presenting a range of possible cases to apply or not apply the predicate.

Van Fraassen’s retrenchment of formal semantics in logical space is based on Carnap’s concept of the intension of a predicate insofar as it appears in connection with meaning relations, what van Fraassen calls “relations of intent”²¹⁷ (1967, 163). Earlier we

saw the meaning relation “no event is earlier than itself”. In general, relations of intent are relations of exclusion, inclusion and equivalence among the range of predicates: e.g., “red excludes green”; “red includes scarlet”; “red matches red”. Although these examples appeal to the range of monadic predicates, relations of intent extend to predicates of various degrees. Like Carnap, van Fraassen claims that to be significant and relevant to the study of language relations of intent must have a pragmatic counterpart that is not abstracted from user and context of use, but determined with less intervention through a questionnaire assessing linguistic commitment (Ibid., 168-170).

Van Fraassen describes his semantic approach to the language of science as follows:

Our view, to put it succinctly, is that in natural and scientific language, there are meaning relations among the terms which are not merely relations of extension. When a particular part of natural language is adapted for a technical role in the language of science, it is because its meaning structure is especially suited for this role. And this meaning structure has a *representation* in terms of a *model* (always a mathematical structure, and most usually some mathematical space). This language game then has a natural formal reconstruction as an artificial language the semantics of which is given with reference to this mathematical structure (called a “semi-interpreted language”...). ... [T]he meaning relations referred to above are such that certain logically contingent statements will always be true, in virtue of the meanings of the terms which occur in them. In other words, the mathematical structure with reference to which the language is partly interpreted plays a role in determining validity, and we may say in such a case that a statement is analytic or holds a priori in a broad sense. (1970b, 327)

Here we see the idea of logical space generalized to such spaces as PVT space in elementary gas theory, phase space in classical mechanics, and Hilbert space in quantum mechanics. In these sophisticated cases a logical space is usually not a model. Here a “model” is a solution to equations that formulate a theory mapped in a logical space. The logical space is the form of representation that serves to classify phenomena according to their measurable properties. Even so, van Fraassen’s claims concerning logical space as model extend to a logical space of models.

It is important to recognize the distinction being made at this point between a logical space as a *representation* of meaning relations in natural language and a logical space as a *reconstruction* of these relations in formal semantics. The latter is abstracted from the user and involves the transformation of a mathematical entity²¹⁸ into to a

linguistic one. Typically in formal semantics an artificial language comprises syntax (vocabulary and grammar) and models (a family of interpretations of the syntax). In van Fraassen's formal semantics, reconstructive logical space adds structure to syntax through a function mapping the grammar of the language to regions of logical space in a way that recovers the topological structure²¹⁹ of representative logical space and thereby the meaning relations of the natural language it represents. See Figure 4.2:

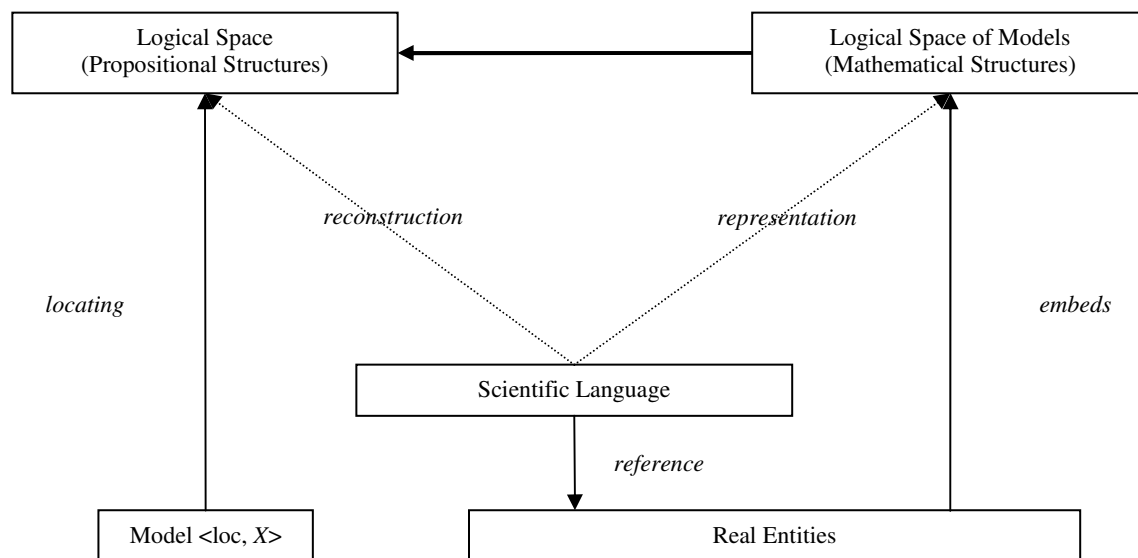


Figure 4.2: Logical Space as Reconstruction and Representation

In reconstructive logical space, explication of meaning relations of natural language is based on three functions. The first is an *interpretation* function f that maps predicates to regions of a nonempty set of elements H . Given H and f , a reconstructive logical space is “identified with the couple $\langle H, \{ f(P): P \text{ a predicate} \} \rangle$ ” (van Fraassen 1969, 160). A *satisfaction* function h maps a grammar attributing states or values to regions of logical space structured by the interpretation function.²²⁰ The grammar is a set of well formed formulas called “elementary statements”, sentences about measurable physical magnitudes that are reconstructions of observation reports. In virtue of describing logical space the language is said to be “semi-interpreted” (van Fraassen 1967, 173). A function loc maps a physical system X via its state to points in these regions.²²¹ In the context of reconstructive logical space, a “model” is a couple $M = \langle loc, X \rangle$. There

arises a truth-definition of an elementary statement: an elementary statement is true in the model if and only if $loc(X)$ belongs to the region of the elementary statement. Van Fraassen calls this region of reconstructive logical space the “proposition” of the statement (1991a, 30). A “model structure” is the logical space together with a domain of physical systems and a function assigning a location in logical space to each system, while a family of such model structures comprises the “possible world model structures” of a theory formulated in that language (van Fraassen 1980, 198-201).

“The family of propositions in general inherits a certain amount of structure from the geometric character of the state-space. To explore that structure is *ipso facto* to explore the *logic* of elementary statements” (van Fraassen 1991a, 30). A reconstructive logical space is thus a *propositional framework* constituted by reconstructions of natural meaning relations as reflected in certain of the valid (true in every model) statements of the artificial language. Since meaning relations are relations among the intension of predicates, i.e., their range which comprehends those possible kinds of real entities for which the predicate holds, the framework concerns universal properties and relations that can also recover the laws²²² of the theory as well as its taxonomy.

Van Fraassen’s semantic approach “gives a formal representation to a certain view of analyticity and synthetic a priori (essentially that of W. Sellars...)” (1967, 156).²²³ The meaning relations are expressed by synthetic a priori propositions such as “whatever is red is not green”; “whatever is scarlet is red”; “whatever is red matches whatever is red”. It seems he regards these propositions as a priori because they are descriptive of our conceptual framework and synthetic because what they describe is a form of all possible worlds defeasible with experience. It should be emphasized that with his formal semantics van Fraassen is not (at this point) rejecting the syntactic approach, only hoping to analyze the structure of physical theories in a way that “will also prove fruitful” (1970b, 338; cf. 326).

4.4 Circa 1980: Constructive Empiricism

4.4.1 Carving-Up Logical Space

The 1970's appears to have been a particularly creative time for van Fraassen as he was realizing the full implications of his semantic approach to scientific theories and battling the new onslaught of scientific realism. As he recalls (1985b, 199-203), absolutism had re-emerged in the philosophy of space and time with advances in understanding the general theory of relativity. He had not specifically considered the theory in his early work, but it had become clear that “[t]he models of this theory do not have a common space-time; they *are* space-times” (Ibid., 209). His continued commitment to the idea of logical space in space-time became more or less an article of philosophical faith.²²⁴

Ongoing problems with the syntactic view of theories prompted van Fraassen to reject it altogether and to move towards adopting the *obiter dictum* he had included in his first book: “the causal theory should say only that the structure of actual causal connections can be embedded in the relevant logical space” (van Fraassen 1985b, 207). But his philosophical narrative also shifted noticeably from talk of logical spaces to talk of models *simpliciter*. This was likely due to having replaced the syntactic view with Suppes’ semantic view that “to present a theory, we define the class of its models directly” (van Fraassen 1987, 109).²²⁵

While models took to the centre, the idea of logical space still set the stage for van Fraassen’s overall approach to philosophy of science. One might surmise that reconstructive logical space provided an intuitive clue how the debate with scientific realism could be recast. Suppe summarizes:

Ontological commitments are left unconstrained, being a matter of which logical space points one wishes to ontologically commit. Such commitments are via individual mapping functions (*Loc* functions) from real-world objects to points in logical space ... [A] *realist* has *Loc* functions onto every state variable and maintains that a theory is empirically true just in case theory-structure-allowed state transitions are identical to those possibly occurring in the actual world. *Antirealists* do not commit ontologically to all state variables. They only require countenancing *Loc* functions from observables and that theories be *empirically adequate*: If *W* is that portion of reality to which one attaches *Loc* functions, the image M^* of *W* is among the models comprising the theory. ... *Antirealism*

thus is just realism attenuated to the range of one's ontological commitment. (2000, S106-S107)

Shifting back to representative logical space involves recognizing that M^* corresponds to a set of points M^{**} satisfying the measurable parameters of a set of equations. Since the class of mathematical models that a theory presents satisfies all parameters of the equations, M^{**} is the “empirical substructure” of at least one these models. The debate with anti-realists can simply be re-cast as disagreement over which set of points in representational logical space corresponds to real entities or, in other words, what an *adequate* representation amounts to. The realist maintains it amounts to “an exact correspondence between reality and one of its models” (van Fraassen 1980, 197), while the anti-realist takes it that the total representation “has some model such that all appearances are isomorphic to empirical substructures of that model” (Ibid., 64).²²⁶ For van Fraassen this difference in representational adequacy is just a difference between truth *in toto* and empirical adequacy in the semantic view of theories.

With this clue in mind, perhaps, van Fraassen saw his task as one of distinguishing realism and anti-realism other than by fiat. His first step was to point out that science itself carves up logical space: “theory draws a picture of the world. But science itself designates certain areas in this picture as observable” (1980, 57). This is because science describes the human organism as “a certain kind of measuring apparatus” (Ibid., 17):

... science itself delineates, at least to some extent, the observable parts of the world it describes. Measurement interactions are a special subclass of physical interactions in general. The structures definable from measurement data are a subclass of the physical structures described. It is in this way that science itself distinguishes the observable which it postulates from the whole it postulates. The distinction, being in part a function of the limits science discloses on human observation, is an anthropocentric one. But science displays human observers among the physical systems it means to describe, it also gives itself the task of describing anthropocentric distinctions. (Ibid., 59)

His second step was to highlight only the observable area of logical space as relevant to representational adequacy. Ingeniously he does this through a doctrine of the aim of science:

If I say, without qualification, that I accept a theory, I certainly convey my belief that it is successful. But what counts as success depends on the aim; indeed, the aim of an enterprise is to be identified through the relevant criteria of success. And I take the aim of

science to be empirical success – success in the representation of empirical phenomena. (1985b, 201)

As discussed already, to “*convey* my belief” does not mean to justify or warrant it. It only means to assert that the representation is adequate: “[t]he scientist, in accepting the theory, is asserting the picture to be accurate in those areas” (van Fraassen 1980, 57). Constructive empiricism simply claims that the aim of accurately representing the phenomena is *sufficient* to recover the intentional activity of science as rational, which is to say that the aim of science is to correlate all actual and observable entities with a set of points within a logical space of models. To say the activity of science aims for anything *more* is not irrational, just supererogatory to answering the question “What is science?”.

4.4.2 A New Theory of Natural Language

Constructive empiricism brought with it a different perspective on the relationship between logical space and natural language. It could no longer be said that a logical space *represents* the conceptual framework of a scientific language. A logical space is accepted along with a class of models in the acceptance of a scientific theory, and must therefore be understood as somehow *determining* the conceptual framework. This marks a return towards the Wittgensteinian conception. Van Fraassen responds accordingly by attributing to representational logical space features of his reconstructive logical space, namely the feature of a propositional framework.

“I cannot pretend that we have a theory of language which is satisfactory or anywhere near complete” (van Fraassen 1980, 199),²²⁷ van Fraassen admits, but the general idea behind it is fairly clear. A key to understanding it is to realize that if we equate imagining and conceiving as van Fraassen does, we can think of the relation between a representational logical space of models and the conceptual framework of scientific language in two equivalent ways: an *interpretive framework* and an *imaginative guide*. See Figure 4.3:

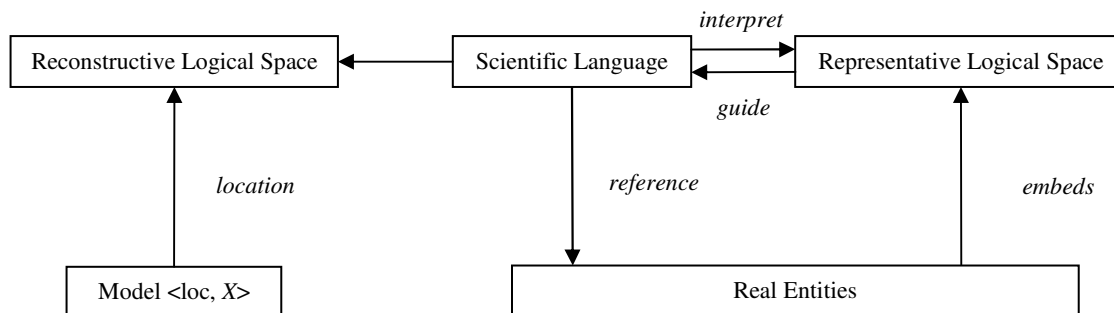


Figure 4.3: Logical Space as Interpretive Framework and Imaginative Guide

The new notion of a logical space of models as an interpretive framework is evident in the following passage:

The idea of logical space: that our language is governed by certain models, and that primary interpretation of language is in these models (with reference sometimes established via further relations of these models to reality).... (van Fraassen 1985b, 207)

More perspicuously:

If the link between language and reality is mediated by models, it may be a very incomplete link – without depriving the language of a complete semantic structure. The idea is that the interpretation of language is not simply an association of a real denotata with grammatical expressions. Instead, the interpretation proceeds in two steps. First, expressions are assigned values in the family of models and their logical relations derive from relations among these values. Next, reference or denotation is gained indirectly because those model elements *may* correspond to elements of reality. (1987, 122)

What van Fraassen is suggesting is that the truth conditions of a natural scientific language are regions of ideal representational elements in a logical space of models. These regions are propositions in the sense that to understand the language is *ipso facto* to imaginatively grasp these regions. In the case of a true observation report an element in its proposition corresponds to the real entity actually observed. Whether elements in other types of propositions are taken to correspond to elements of reality (e.g., to dinosaurs, distant galaxies, electrons, and possible universes) depends on whether the status we accord accepted theories requires it. Constructive empiricism tells us that the rationality of scientific activity requires only that elements representing the actual and observable be taken as corresponding to elements of reality. In a remarkably Hertzian

way it also tells us that scientific realists make the mistake of projecting additional elements onto the world.

The idea of a logical space of models as an interpretive framework is behind van Fraassen's position that the language of science is to be literally construed (1980, 10). This position has a number of interesting characteristics. We see that in the case of an observation report *the meaning* of the sentence is determined by truth conditions through a region of the logical space in which *what is meant* is located if and only if the sentence is true. This illustrates how the meaning of a true statement is a "Scotist synthesis" of elements that are both conventional and real.²²⁸ But while an object, event or process "*is logically implied* by a direct report of the seeing that" (van Fraassen 2011a, 438), it appears that in the case of a statement that is *not* an observation report but nonetheless expresses a proposition concerning what is actual and observable, an object, event or process is *rationally implied* by virtue of the status we accord the proposition itself. Constructive empiricism also illuminates that our discourse about unobservable entities concern *fictions*,²²⁹ expressing propositions under the supposition of the adequacy of their representation.²³⁰ Similarly, "it is as typical of modal and causal discourse that it receives its full interpretation *only* in the logical space (or somewhat more sophisticated models) and need not be understood in terms of reference and extension" (1985b, 207).²³¹

The other way to think of the relation between a logical space of models and the conceptual framework of scientific language is as an *imaginative guide*. This is a view of language in which "discourse is guided by models or pictures, and ... the logic of discourse is constituted by guidance" (van Fraassen 1987, 122). Van Fraassen probably owes the most to Sellars (1963) for this position. Rejecting his earlier traditional empiricist view that concepts are formed from particulars, Sellars proposes an alternative account of concept formation in terms of learning a conceptual framework. This involves becoming habituated to conform to a conceptual structure (learning "intra-linguistic moves") (Ibid., 314). It also involves being conditioned to respond to kinds of situations with specific kinds of verbal structures (learning "language entry transitions") (Ibid.), incorporating his earlier view that our concepts capture our belief in natural regularities by carrying general propositions as presuppositions.²³² Since he takes our conceptual

framework to be guided by a logical space of models, van Fraassen views Sellarsian conditioning generally as one of our “immersion in the theoretical world-picture” (1980, 81) informing assumptions and suppositions in our use of language (see *Ibid.*, 200). Our concepts are laden with predictions and classifications of observable properties in virtue of the epistemic dimension of theory acceptance, while the pragmatic dimension of acceptance accounts for our commitment to theoretical explanations appealing to unobservable entities as well as counterfactual, causal and modal locutions (see *Ibid.*, 202).²³³

Because a logical space of models is taken to be an imaginative guide, its topological structure is thought to determine (in part at least) the logical structure of a language. This is evident from van Fraassen’s understanding of logic as a philosophical discipline:

The phenomena, for us, are patterns of assertion and inference. This means that we do not have the task of representing the way the world is, but only the way it is thought of: the picture that guides reasoning. If this point is ignored, then we find ourselves doing a particularly naïve sort of pre-Kantian metaphysics, trying to correlate all aspects of our models to reality. ... What we should hold is only that we can save the common phenomena of inference, if we regard the agents as thinking of the world as surrounded by a halo of possible alternative worlds, actuality but an arbitrary fragment of an intricately woven web of possibilities (1978, 19-20)

To “regard the agents as thinking of the world as surrounded by a halo of possible alternative worlds” is just to look upon our cognition of reality as structured through logical spaces of models, accounting for the “form” (van Fraassen 1980, 198) or “shape” (van Fraassen 1992, 13) of natural language that can be recovered in the formal semantics of a reconstructive logical space.

4.4.3 Literate Experience and Our Knowledge of Structure

The idea of a logical space of models as an imaginative guide to discourse is behind van Fraassen’s view that experience is literate and his account of how perception is theory laden. His view of experience involves distinguishing between observing an entity and observing that something or other is the case (1980, 15). He later explains it this way:

As I see it, the main ambiguity in the philosophical notion of experience is between, on the one hand, what happens to us that we are aware of, and on the other hand, our immediate and spontaneous response to what happens to us. What happens to us, and which of the events that happen to us are noticed by us, those are factual questions whose answers depend on theory-independent factors. But how we respond – and here I include the very first, spontaneous response to those events, prior to any discursive thought – is clearly conditioned by the language in which we live. Any judgement involved in that response (such as “Lo! phlogiston escaping!”) always involves some implicit description of the event. This description is historically conditioned – and in general, theory-laden – to a very large extent. (van Fraassen 2000, 1658)

From the phenomenology of experience van Fraassen argues that experience is “Janus-faced” (2002a, 134)²³⁴ in the sense that it is constituted by both by a *real* aspect, i.e., what happens to us and that we are aware of, and a *textual* aspect, i.e., our perceptual judgment of what befalls us.²³⁵ In this way experience is said to be *literate*. Seeing is not believing;²³⁶ “*seeing is reading*”: if a person “sees a rat, he also ‘reads’ the word “rat” – the natural object has become for him a different inscription of that same word. He has been conditioned to respond to his experience of a rat with the observation report “Lo, a rat!” (2002b, 6). But the conditioning may such as to involve concepts laden with “implications that go far beyond the immediate deliverances of experience” as when I judge something I fall over to be a radio and spontaneously infer electromagnetic waves (van Fraassen 2006a, 126). A perceptual judgement is expressible as an observation report (“language entry transition”)²³⁷ that carries these implications (“intra-linguistic move”).

This multi-faceted view of experience allows for “a quite sharp separation between epistemology and semantics” (van Fraassen 2000, 1658). Our theory-independent awareness of what happens to us is the subject of semantics where we encounter empirical realism and a deflationary conception of truth. Van Fraassen subscribes to a “common sense realism in which reference to observable phenomena is unproblematic: rocks, seas, stars, persons, bicycles ...” (2008, 3).²³⁸ Adopting the Tarskian equivalence “‘Snow is white’ is true if and only if snow is white”, whether a statement is true depends on what the entities it describes are like.²³⁹ This conception of truth does not imply the existence of *in re* structure. Following Quine, “all that is needed for ‘Snow is white’ to be true is that snow be white, and *that* does not imply the existence of anything but snow” (Ibid., 248). “I do not see properties” (1993, 440), van Fraassen

asserts, and “[a]pplying the predicate to a subject is simple attribution or predication, and does not bring along an extraneous existential commitment” (2010b, 553).

But since the determination of truth presupposes knowledge of meaning, what is implied for “Snow is white” to be true is that the statement is assigned a region of colour-space corresponding to the predicate “white”. This just follows from van Fraassen’s view of colour space as an interpretive framework. The truth of the assertion becomes a correct classification of a real entity by “assigning it a location in that [region of the colour] spectrum” (van Fraassen 1980, 201). (Note that at this early stage “assigning a location” was not described as in virtue of the *self*.)

By contrast, the subject of epistemology is our theory-laden response to what happens where we attribute properties and relations to real entities and acquire knowledge of their structure. But the knowledge we acquire is not what it seems:

What nature is like does not depend on what our experience or representation of nature is like. *But we must not confuse this point with naïve realism about how we ourselves exist in this world.* As for us, we discern structure in nature; but the structure we discern there, we discern in the same way as we discern a story when looking at ink marks on a page. (van Fraassen 2002b, 16)

Recall, for example, that we discern the temporal structure of events in nature in the same way that we discern the temporal order of events reading a novel:

... only the structure of all events taken as a whole is set in time, since correct “placing” of events is determined by their mutual relations. And there may remain in principle more than one way to determine the placing. Rather than postulate some transcendent criterion of correctness – whether through counterfactual facts or in any other way – I suggest we accept the same imminent vagueness for the order of real events, as we do for the order of narrated events, underdetermined by the text. In both cases the world is conceived as determinate, but the necessity in how things are to be conceived does not engender a necessity in how they must be. (van Fraassen 1991b, 34)

Unlike Antoine Roquentin’s reflective deconstruction of the textual aspect of experience that reveals the structure of bare experience as indefinite, our spontaneous experience of concrete structure is an interpretation of reality in virtue of locating it in a determinate structure guiding perceptual judgement. But for van Fraassen to claim as Antoine would that the “reality of which I have a robust sense need not itself be all that robust, solid,

definite” (Ibid.) is not to claim as Kant would that we are the source of empirical structure:

If I know, as I do, that snow is white, that many apples are red, that there are nine planets orbiting our sun, does it not follow that I know something about the structure of the world? In saying these things are true, I attribute structure to the world, but my attributing does not make them true. ... We can say that we attribute structure to nature whenever we describe something in nature – that is innocuous, if all we mean by “attribute structure” is that we *say that* some things are thus or so, and related to other things in such or such manner. To attribute structure in this sense – which seems quite in order to me – is not to create structure, nor to be the source of structure – it is simply to describe. (van Fraassen 2006a, 157)

The attribution of empirical structure involves a Scotist synthesis of mind-dependent and mind-independent entities that comes close to the Hertzian projection of empirical structure we saw in the second chapter. Our attribution of structure – equivalently, our knowledge of structure – is “built up” from properties and relations delivered in experience and “built down” from a logical space of models that extrapolates them to properties and relations of all possible worlds. To the extent that we take our attributions of empirical structure to be definite or determinate, the ultimate source of structure is our value-based decisions to privilege certain properties and relations through acceptance of a logical space of models:

No wonder our nouns and adjectives speak loudly of what is pertinent to our tasks – our language grew up in praxis. In that sense we furnish the grounds for how our descriptions are structured. Even the words “world” and “nature” and “thing” belong to the vocabulary that grew up with us in this way. Therefore all we can really say is that there is structure in our descriptions of what we describe – nothing new is added if we replace “what we describe” by “world” or “nature”. The divisions marked in our descriptions – and what other divisions are you asking about? – are those which are important to us. This importance is entirely summarized by saying ‘they are ours’ – ‘a poor thing but our own’. (van Fraassen 2009, 470-471)

But having been built up from experience our attribution structure is constrained by reality, for it is always possible that our expectations will disappoint us.

From the historical content of van Fraassen’s writing it appears his epistemology developed around his idea of logical space and doctrine of the aim of science.²⁴⁰ In claiming that “[r]ationality is only bridled irrationality” in that “what is rational to believe includes anything that one is not rationally compelled to disbelieve (1989, 173), our privileging of properties and relations in the determination of structure is not taken to be a

matter of justification or warrant, but only coherence (2009, 469-470). Until there is reliable evidence to tell us otherwise, it is rational to allow a logical space of models to condition our perceptual judgements:

An accepted theory may be wrong. If we attend critically to our experience and we have the proper ration of epistemic luck, this falsity will manifest itself in the disappointment of expectations shaped by that theory. Until that happens, however, all those expectations may well be legitimate, warranted, entitled, rational, reasonable, what have you. The grounds adduced for them will be reports on our experience themselves shaped by that very theory, couched in its terms, and implying counterfactuals and predictions via that theory. (2000, 1658)

In the practice of science disappointed expectations arise through observation reports expressing perceptual judgement, serving as “*evidence* that a certain phenomenon occurred” (2004a, 3). Although they are non-representational,²⁴¹ these reports are used to construct data models²⁴² that are. A logical space of models is taken to be empirically inadequate if it lacks an empirical substructure that is isomorphic to a relevant and (presumably) accepted data model.²⁴³

4.5 Circa 1990: Empiricist Structuralism

4.5.1 The Problem of Coordination and the “Wittgensteinian Turn”

Recall that one of van Fraassen’s insights from his causal theory of time was that the syntactic view of theories cannot account for the interpretation of theoretical terms without proper coordinative definitions that single (or at least significantly narrow down) out a structure. But if modal realism is to be rejected, proper coordinative definitions can bestow meaning only through “intuitive content” understood as ideal elements of a representational mathematical structure some of which correspond to real entities. It seems that because this intuitive content was thought in the first instance to represent natural language, van Fraassen simply took the position that “coordinates are assigned to entities in such a way that the mathematical relations among the coordinates reflect those relations among the entities that we mean to describe (van Fraassen 1970a, 5-6).

Along with the advent of constructive empiricism the syntactic view of theories was rejected, and van Fraassen's concern over proper coordinative definitions was cast-off as a pseudo-problem:

If ... we translate 'straight line' as 'possible light ray path' the theoretical element is not absent, and the criteria of application are indefinite; is this possibility relative to laws and circumstances which could be stated without recourse to geometric language? The opposite view is to consider mathematics in use an abstraction from the science that uses it, and to leave the reconstruction of language along formalist lines to a different philosophical enterprise. (van Fraassen 1991a, 6)

But just like the *loc* function in van Fraassen's formal semantics, the mathematical function assigning coordinates to real entities now stood naked, abstracted from the user and context of use. Without knowing it van Fraassen fell squarely within the scope of a devastating objection Newman had levelled against Russell's structuralist program.

In *The Analysis of Matter* (1927) Russell presented a causal theory of perception that claimed our knowledge of the external world is only structural, i.e., we know only the properties of the properties and relations of external causes that stand in a one-one correspondence with our individual percepts. Newman (1928) objected that on this view all we know about the external world is the cardinality of causes, for given any number of individual percepts we can always impose a structure on the external world compatible with that number. In other words, the same cardinality of percepts and causes implies the existence of a correspondence whereby there is an implicit transfer of the relations among the percepts to among those causes. Under Russell's program, our knowledge of structure of the external world is therefore trivial: but for cardinality, a structure of percepts cannot pick out anything in the causes. Unfortunately for van Fraassen Newman's objection lay dormant for many years, resurfacing much later in philosophy of science in a paper by Demopoulos and Friedman (1984).

If we replace individual percepts and external causes with individual points in logical space and real entities then the objection will obviously apply to van Fraassen's idea of logical space. An assertion that a structural representation is adequate is only an assertion that there are properties and relations instantiating certain general properties. But the latter leave the former unspecified, and so the assertion is trivial. But for an empirical assumption concerning cardinality, embedding is a logical fact,²⁴⁴ one leaving a

representative logical space of models practically devoid of empirical content. Given the centrality of the idea of logical space the consequences overwhelm van Fraassen's view of science: 1) the notion theoretical truth, whether *in toto* or in part, is trivial; 2) the aim of constructing an adequate structural representation is too easy to achieve to make sense of the scientific activity such as theory-testing; and 3) how theoretical terms receive empirical reference is left unexplained.

Before examining van Fraassen's general response, it is instructive to compare him against Wittgenstein in the case of an elementary logical space. For Wittgenstein, recall, a coordinate *value* correlates to an *essential* simple object, i.e., an object that can potentially combine with other objects of the same kind. Whether it does or does not is a contingent matter, but if it does there arises an elementary state of affairs. In propositional thought the self projects itself as the whole of logical space where a point of the space is an elementary proposition representing a possible state of affairs. In virtue of "feelers" reaching out from its coordinate values to simple objects, the point may or may not correspond to a state of affairs. But coordination with reality is prior to this determination of truth or falsity and, I have argued, is the prior coordination of the self with reality. For van Fraassen logical space is not projected; it is just "there". An elementary proposition is a *set* of points in logical space, each of which can serve as an element onto which an *inessential* real entity is mapped. In and of itself, a coordinate value does not correlate with anything real. The mapping is thought to exist simply as a matter of representational adequacy highlighted by a doctrine of aim. What Newman's objection shows is that shared cardinality alone is sufficient for this mapping to exist.

From the point of view of van Fraassen's idea of logical space the problem of coordination is just one of assigning locations to real observable entities in a way that preserves their properties and relations without presupposing a description guided by that space. But in *Scientific Representation* van Fraassen sees the problem manifested in a cluster of interrelated issues plaguing structural representation since the *Bildtheorie* view of science exemplified by Hertz. One problem he identifies Wittgenstein already canvassed: limited to representational content alone, the application of structure leads to infinite regress. Wittgenstein made use of this to justify his claim that there is an

essential reality that shares the same form as pictures. Van Fraassen's (1992) response is to shift the base of the metaphor for logical space from a concrete model to a concrete map. He turns to Kant for insight into what this regress is telling us:

... the most precise map of the heavens ... would not enable me... to infer ... on which side of the horizon I ought to expect the sun to rise if it did not, in addition to specifying the positions of the stars relative to each other, also specify the direction by reference to the position of the chart relative to my hands. ((Kant 1992), as quoted in (van Fraassen 2008, 80))

Van Fraassen takes Kant to pointing out the “*inevitable indexicality of application*” (Ibid.) and that pertinent information for application cannot be found in the content of a structural representation:

It is not as if there is an object or event that is indescribable, ineffable, beyond the reach of objective or impersonal description. This act of self-description too can be described and the information can be included on a bigger map (with the label “location of vF's map-reading at time t”). But then what I need to use *this new* map is still a self-ascription of location with respect to it. ... An attempt to replace or eliminate these self-ascriptions leads to an infinite regress, using an infinite series of maps. (Ibid., 79)

Use of a map requires infusion of non-structural information expressible in an indexical judgement such as “I am *here*” or “*this is there*” on a map. As a condition of use, self-ascription of location, or simply “self-location”, is a condition of representation, for we do not bestow the status of a representation upon maps we cannot use. But if I must be able to locate the reality of my situation on a map for it to be a representation, then the assertion that it is adequate to its domain of application cannot be said to be trivial, for we bestow a representational status upon the map only if it respects the properties and relations we discriminate when using it.

Claiming “[t]here is here a *precise and perfect analogy between theory, model, and map*” (van Fraassen 2008, 80) here, the problem of infinite regress is taken to justify self-location as a solution to the problem of coordination. To use a model that a scientific theory presents, I must locate phenomena in the model's logical space. A key move van Fraassen makes is to say that self-location is involved in the theoretical classification of entities in perceptual judgement and to make clear that this use of a theory is logically independent of, and prior to, the assertion and assessment of the representational adequacy of the theory.²⁴⁵ Thus the assertion that a theory is empirically adequate is not

trivial. The only relevant models in relation to which such an assertion is made must respect properties and relations I can discriminate among phenomena when I locate them in the logical space common to these models. Except for the special role given to the *self*, this solution is, in the main, just a pragmatic elaboration of a position already found in *The Scientific Image*.²⁴⁶ More precisely, it is the identification of the pragmatic counterpart to the semantic concept of a *loc* function suggested in van Fraassen's formal semantics,²⁴⁷ but determined by philosophical reflection rather than questionnaire.

Self-location is a “linguistic act” (van Fraassen 1992, 10) of perceptual judgement through which *I* can describe real entities *I* am presently witnessing, the relevant parts of mathematical models, and how they are related to each other (see van Fraassen 2008, 388, n. 28). It is something I know to be case that is not part of the representational content of a scientific theory, and it is expressible in an indexical judgment in such terms as “I am *now here* in the space of possible states” (van Fraassen 2008, 78) or, more generally, “this, in front of me, is *there* in the [logical] space” (Ibid., 212). Van Fraassen refers to his response to the problem of coordination as an “indexical turn” (2009, 470) in his philosophy of science. He also refers to it as a “Wittgensteinian turn” “that focuses on us, on our use of theories and representations, and brings to light the impasses we reach when we abstract obliviously from the use to use-independent concepts” (2008, 235). As irony would have it, however, what van Fraassen seems to have done is to recover early Wittgenstein's idea of logical space as inextricably connected with the self.

4.5.2 Constructing a Logical Space

Van Fraassen's early view appears to have been that a logical space of models is constructed through a conventional elevation in status of attributes and regularities:

To make sense of our world in a *convenient* fashion ... we raise certain regularities to the status of laws²⁴⁸ and (not independently!) certain attributes to the status of natures. In the formal mode, this means that some statements assume the office of assumptions which may be tacitly used in all reasoning, and certain predicates are chosen to form a classificatory scheme. (1978, 14)

Rather than simple convenience, van Fraassen would now characterize this elevation more generally in terms of values and coherence:

Privileging relative to us involves value judgement as well as factual judgement. ... [A]ny privileged status which a property has it receives in our value judgements. It is true that forming opinions that go beyond our evidence typically involves selective attention to some properties rather than others – but the rationality of this practice will consist in its coherence (including coherence with the rest of our practices). (van Fraassen 2009, 469-470)

It appears that our privileging is captured in conventions of theoretical form, which are enshrined in norms evident in the guarding activity of journal editors and peer review (2007b, 416).

Concomitant with construction of a new logical space is the introduction of at least one theoretical term to be interpreted by way of this space. Here we encounter van Fraassen “hermeneutic circle” (2008, 116) in which “the measured parameter – or at the very least its concept – is *constituted* in the course of [its] historical development” (Ibid., 138). Viewed from *within* the historical process, a measurement procedure is introduced by an emerging theory within a background of prior theory and measurement. There is “*no presuppositionless starting point for coordination*” (Ibid., 137) – “*prior meaningful discourse relating to what will eventually be delineated within the theory’s domain*” (Ibid., 184) is required to construct a new logical space. Initially the measurement procedure identifies new (unstable) empirical regularities in the interaction between the measuring instrument and measured phenomena as a measured parameter. Although the procedure is required to satisfy certain coherence conditions, there remains much leeway for choice based on “a cluster of pragmatic desiderata” (Ibid., 134). Idealization is involved in both the stabilization of regularities (i.e., when judgements guided by the measurement procedure become well-calibrated) and the choice made (Ibid., 116). When accepted, the new theory presents a view from *above* the historical process. At this point the theory is taken to describe the measurement procedure, providing “operational rules” of measurement (Ibid., 165-166) and identifying the measured parameter with a new theoretical term. More specifically, “the parameter, identified by the eventually stabilized procedures for its measurement, is now classified by the theory as one aspect of the logical space that the theory provides for location of items in its domain” (Ibid., 128).

4.5.3 Logical Space as Map

In relation to the inevitable indexicality of application van Fraassen proposes that the analogy of a logical space of models to a concrete map is “precise and perfect”. He pushes the analogy even further, arguing that just like a concrete map it bears a certain relation to perspectivity. Abstracted from a specific context of use, a logical space of models presents what observable entities *are like* from a “Gods-eye-view” (2008, 70).²⁴⁹ Perspectivity enters in its use “in practical applications or even to test it or use it to explain something, or add to it through research” (Ibid., 82). It enters, that is, through the user’s identification of certain regions of its representational elements with specific phenomena. At this point there is a lawful translation from what the phenomena are like to what they *look like* in the observation or measurement set-up. Nonetheless what is the “most important in the concept of perspectivity” is the explicit or implicit reference to “a ‘horizon’ of other perspectives on the same objects” (Ibid., 39), for it is this invariant information that underwrites the possibility of “God-like reflections” upon empirical reality (Ibid, 71).²⁵⁰

An example of this type of reflection is taken from the special theory of relativity. Here logical space is a set of four-tuples of real numbers that has been bestowed the status of a form of representation. The theory presents a family of models mapped within this space that displays the space-time interval ($s^2 = t^2 - d^2$), a coordinate-free magnitude invariant from one frame of reference to another. It thereby depicts what the phenomena *are like* across frames of reference where magnitudes of time or distance differ. When an observer uses this logical space of models “choice of a coordinate system correlated to a defined physical frame of reference is required” (van Fraassen 2008, 71). This involves assigning coordinates to user-specific content arising from measurements of time and distance, and in a way that respects the space-time interval. In this manner “we introduce a coordinate system, a frame of reference” (van Fraassen 2007c, 50). The special theory of relativity thus describes what phenomena are like by showing in its use what they *look like* in a lawful translation of the space-time interval to the perspectival measurement information of a particular frame of reference.

In this example the assignment of coordinate values to the content of the measurements of time and distance can be thought of as an act of locating measured phenomena in logical space expressible as “*this is there*”. The self thus bestows meaning upon a logical space of models through an act of ostensive reference to real entities that is *meaningful*. The extra information – which region of the logical space of models corresponds to the observed entity – is not included in the content of the representation, but added in use. I *know* something not implied by a logical space of models, something that arises when I measure empirical reality and classify it through the measured property: I know *where* in logical space it is located.

4.5.4 Measuring as Self-Location

In the context of the semantic view of theories *The Scientific Image* holds that a theory is empirically adequate “if has some model such that all appearances are isomorphic to empirical substructures of that model” (64). Van Fraassen now admits to having used “appearances” ambiguously, meaning both “concrete observable entities” (2008, 386 n. 8) and abstract “data models” (Ibid., 391 n.24).²⁵¹ In *Scientific Representation* he disambiguates the term in two important steps. The first involves separating the mapping of observable entities and data models into a logical space of models by characterizing the former in terms of self-location and the latter in terms of an embedding function. The second involves equating “appearance” with the *information content* of perceptual judgment gathered in the context of an observation or measurement set-up in which observable entities interact, i.e., in the context of phenomena together with the human organism or, in addition, together with a measuring instrument. Accordingly appearances are “the contents of observation or measurement outcomes” (Ibid., 8). They are perspectival in that they are “*what [phenomena] look like in given measurement or observation set-ups*” (284). While self-location involves locating phenomena in a logical space of models through their appearance, embedding involves mapping of data models worked up from appearances into a logical space of models.

To understand measuring as self-location one should begin with an observation outcome before turning to the more complex case of a measurement outcome.

Unfortunately *Scientific Representation* does not say what an observation outcome is, leaving the reader to piece it together from van Fraassen's other writings. In general, measurement is a case of grading that serves "to classify items as in a certain respect greater, less, or equal" (Ibid., 172). In making an observation we are "in effect *performing a measurement without instruments*" (Ibid., 93). The content of the observation – i.e., the textual aspect of experience that constitutes perceptual judgement – is an appearance, a mathematizable "datum" (Ibid., 376, fn.14) through which we classify the observed entity – i.e., the real aspect of experience. What is particularly interesting is that appearances do not reside in the body or mind as Galileo and Descartes placed them; rather, they are as inter-subjectively accessible as reality:

"Appearance" does not refer here to subjectively experienced impressions. All those colourful, noisy things are public, and so are the appearances: the dictate of repeatability ensures that scientifically admissible experimental results are public. (Ibid., 276)

By "observation outcome" what van Fraassen has in mind, I believe, is what he refers to elsewhere as a "real image" (1999, 10). As he describes it, a real image is a synthesis of appearance and reality: it is a real entity we perceive as having the perspectival structure of an image. Unlike "private images" (2008, Ibid., 104) such as after-images or dreams, real images belong to *public* imagination. They are more like "graven images" (Ibid.) such as paintings or photographs in that our common experience does not distinguish them from real entities. But unlike a graven image, a real image does not represent real entities, which probably explains its omission from *Scientific Representation*.

To explore the relation between the scientific image (logical space of models) and a real image (observation outcome) is to attempt to grasp what seems to me to be the most basic intuition at work in van Fraassen's philosophical system. Unfortunately he is not all that explicit about it. However his new account of appearances purports "to achieve a synoptic vision in which the manifest and the scientific images both receive their due" (2008, 276). From his other writings van Fraassen is clear that this vision is intended to contrast the dichotomy Sellars (1963) had introduced. Van Fraassen's notion of a "manifest image" appears to be just that of an image of bare experience, i.e., an image deconstructed from theoretical implications:

... theories depict the ideal extensions of the experienced world. I would sooner think of the experienced ice cube that it is an incomplete object. ... It seems to me that the manifest pink ice cube, being itself neither definitely continuous nor definitely granular, can *consistently* be identified with the objects of various alternative theories, including aggregates of H₂O molecules. (van Fraassen 1976, 340)

It seems that the “radical incompleteness” of the manifest image is related to the indefinite structure of bare experience in that both the completeness of the image and the definiteness of its structure arise in the locating of observed entities within the scientific image. What the “synoptic vision in which the manifest and the scientific images both receive their due” hints at is a constitutive relation in our immediate perception that undermines any dichotomy. Because van Fraassen uses the phrase “synoptic vision” elsewhere in the context of a view “from within” and “from above” (2008, 139), I suggest that a real image is a perspectival view of what a real entity *looks like* “from within” a scientific image, while a scientific image itself is an aperspectival view “from above” of what the real entity *is like*.

Indeed, if to classify an observed entity with respect to its appearance is to *ipso facto* locate it in a region of a logical space of models, then to perceive a real image is just to perceive a real entity as located in the scientific image. (This is why when I classify an object as a radio I may conceive – equivalently, *imagine* – electromagnetic waves.) The Hertzian nature of this claim rings loudly (compare van Fraassen’s ice cube with Hertz’s piece of iron).²⁵² Like van Fraassen, Hertz recognizes that empirical structure is projected onto the world together with a scientific image. In Hertz’s case, however, he undertook the critical project of ameliorating negative effects accompanying this projection through a philosophical project of perfecting an ideal scientific image. Van Fraassen, on the other hand, seems to accept all aspects of this projection as part of our existential condition: “to exist in this world *in the way we are there*” is to experience the world in the way we understand it, in the way *we imagine it to be*.

It seems that the scientific image constitutes real images through what van Fraassen refers to vaguely as “conditioning”. “By means of conditioning – whatever that process may be, we don’t care – a correlation is established that makes the person in effect a reliable measuring instrument or detector of the conditions to which he is conditioned to react in that way” (van Fraassen 1992, 15). Being conditioned to react to

reality in a certain way involves carrying along the implications of a conceptual framework, what Sellars refers to as an “intra-linguistic move”, guided in part by a scientific image.

Here is an illustration. Suppose I perceive what I judge to be a billiard ball.²⁵³ In judging its appearance to be red and circular I classify the entity as a kind of colour and a kind of shape by locating it in the region of colour-space assigned to “red” and the region of the “logical space of shapes” (van Fraassen 2008, 179) assigned to “sphere”. The reason that I locate it in these particular spaces is that, in having accepted them, they imaginatively guide my conceptual framework for colours and shapes that condition my perceptual judgment. The real image is the outcome of what the entity *looks like* in the observation set-up viewed “from within” the composite logical space. In addition to the real entity itself, part of what constitutes this real image is the implication that the world is, as Roquentin says, “lazy”: the ball will not spontaneously turn green or cube-shaped at any point in time since the ball is located in an exclusive region of logical space. Abstracted from conditions of use, the composite logical space represents what the ball *is like*: it appears red and circular from any point of view. See Figure 4.4:

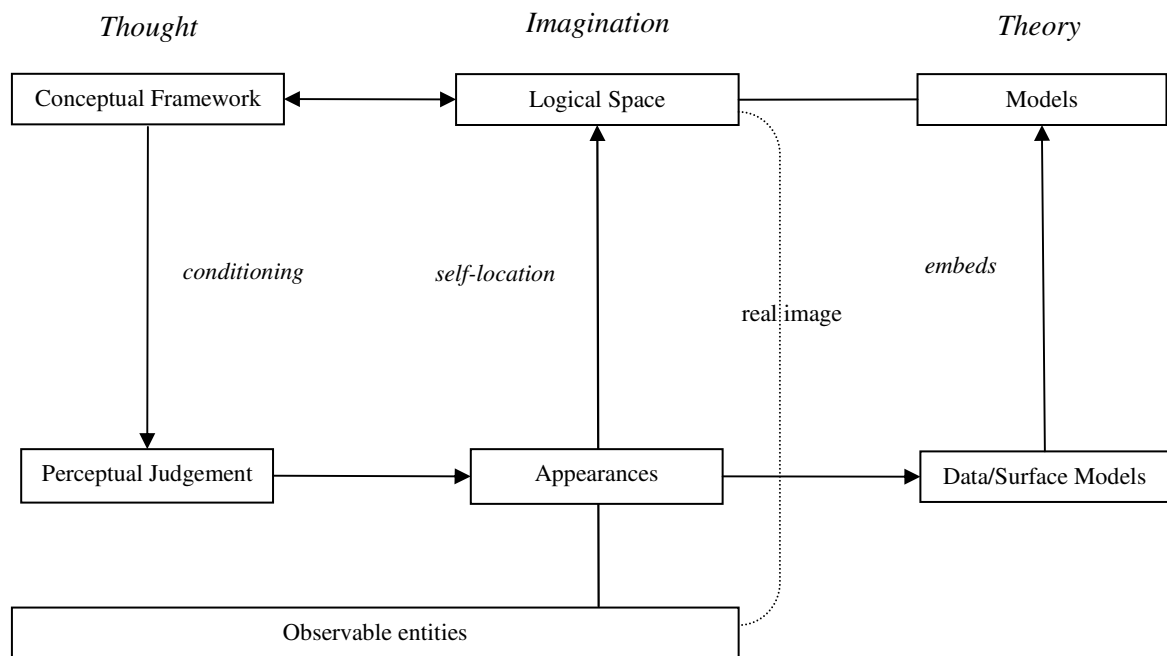


Figure 4.4: Empiricist Structuralism

This illustration extends naturally to van Fraassen's account of more theory-laden perception of real entities such as phlogiston and radios (cf. van Fraassen 2006a, 125-128).

Notice that the conception of logical space invoked here is that of an imaginative guide. But the datum of the perceptual judgement can be expressed as the observation report, "Lo! Red billiard ball", what Sellars refers to as a "language-entry transition". Accordingly self-location is taken to be implicit in this assertion,²⁵⁴ one that serves as evidence of the ball.²⁵⁵ In relation to the observation report we can also think of logical space as an interpretive framework. On this view the truth of the report implies that elements of regions of logical space attributed to "red" and "sphere" correspond to the observed ball.

It seems, then, that a real image constituted by the scientific image is the observation outcome of an observation set-up. The notion of an "observation set-up" is new for van Fraassen, and points to an extension of the term "phenomenon":

... "phenomenon" has both a generic and a specific use – a specific effect produced in a laboratory at a particular time is a phenomenon, but so are oxidation, ebbing, planetary motion, and so forth. In the generic use, as I understand the term, it refers to classes of observable entities. (van Fraassen 2008, 392, n.26)

A "produced" phenomenon is the result of observable entities in physical interaction with one another in an observation set-up: e.g., "the strange happening [a glowing sulphur ball] that involved a relationship between luminescence, rotation, friction, and sulphur was a new phenomenon" (Ibid., 95). An example outside of the laboratory is a rainbow. Because of certain invariance in the relations between the sun, cloud, rain and the observer in an observation set-up,²⁵⁶ a naturally produced phenomenon arises that the observer (also a real entity in this relationship) experiences as an arch of multiple colours. This observation outcome in this case, I take it, is the rainbow image, the content of which is the perspectival appearance of a naturally produced phenomenon.²⁵⁷

Here things get complicated relative logical space and its relation to reality. Van Fraassen wants to say that in appearing as an arch of multiple colours, my perceptual

judgement classifies entities by locating them in logical space. But the entities I locate are the entities I measure, i.e., the sun, cloud and rain. Hence, the rainbow image is what these three entities *look like* in the observation set-up; it is a view of these entities “from within” a scientific image. But because “rainbow” is a count noun we tend to think what we locate in logical space is the naturally produced phenomenon itself, i.e., we *reify* the result of the physical interaction among real entities as a material *thing*. In addition to a naturally produced phenomenon, therefore, the observation outcome is not a real image but a “public hallucination” (van Fraassen 2008, 104).²⁵⁸ (Notice for later that the status of a rainbow image as a public hallucination lies precisely in our propensity to construct an individual entity *out of* appearances in logical space.)

What is significant about van Fraassen’s account of rainbows is that the human observer can be replaced with another measuring instrument such as a camera. Here the camera, not the observer, is a part of the physical interaction that produces the phenomenon, and talk of “observation” now shifts to talk of “measurement”. The measurement outcome of the measurement set-up is a photograph. A photograph is itself a real entity we perceive as a real image (we classify it as a photograph). However, it is also a graven image, which is to say that it has representational content: we bestow upon it the status of a *representation* of the measured entities, i.e., the sun, cloud and rain. When I look upon the graven image and perceive what I classify as a rainbow, I perceive what these phenomena look like in the measurement set-up. Once again, in measuring we do not locate the produced phenomena in logical space; rather, “measuring locates the *target* in a theoretically constructed logical space” (van Fraassen 2008, 2).

This is behind van Fraassen’s challenge to the common view that instrumentation in physical interaction with observable entities *extends* our senses. Typically a measurement outcome is like the glowing sulphur ball in that it is an artificially produced phenomenon:

The heart of an experiment is typically a sort of measurement: the set-up produces or lends itself to a phenomenon that is meant to provide information about the character of some target object, event or process. The artificially produced or isolated phenomenon is treated as providing data about the target, to provide us with a view of it. (van Fraassen 2008, 66)

Here

... the appearance is determined jointly by the measurement set-up (involving both apparatus and system to which it is applied), the experimental practice, and the theoretical conceptual framework in which the target and measurement procedure is classified, characterized, and understood. (Ibid., 284)

The simplest case of instrumentation involved in measurement set-ups is the microscopic lens.²⁵⁹ In general “images produced by lenses are themselves (artificially produced) phenomena” (van Fraassen 2008, 97), and microscopes “imitate the ability of nature to create public hallucinations” (Ibid, 104). He admits that, unlike a rainbow, all invariants of an observable object are present so that “it is certainly the case that we can represent the images produced by the microscope to ourselves as images of real things (with the same structure as those images)” (Ibid., 108). However, he says that it is still possible to represent to ourselves that what is being located in logical space guiding, say, the classification of an image as a “paramecium” is not an unobservable entity, but the measured phenomena (slide, water sample) with which the microscope is in interaction.²⁶⁰ His idea, I take it, is that constructive empiricism claims the status we accord our best theories does not require that elements of logical space correspond to unobservable entities, and so does not require we represent to ourselves that we are locating anything but observable entities in logical space.²⁶¹ (Again notice again for later that van Fraassen must admit every microbiologist who believes she is looking *through* the lens of a microscope at a real entity (which is probably all of them) is using logical space to construct an individual entity *out of* appearances.)

Van Fraassen extends the microscopic case generally to all cases of measurements involving instrumentation. “A measurement outcome is something physical: an event, the end-state of the apparatus, or an object (photo, graph, list of numbers) produced by the measurement process” (van Fraassen 2008, 157). It is “most certainly real” (Ibid., 276). At the same time, “measurement procedures produce representations – *images*” (van Fraassen 2010b, 512).²⁶² Unlike an observation outcome, however, a measurement outcome is “a representation of what is measured” (van Fraassen 2008, 156): it represents “the object as it appears in that measurement set-up” (Ibid., 176) and “the target as located in a certain logical space” (Ibid., 176). It is a pictorial representation that trades for success in use by us on structural resemblance and distortion (Ibid., 182-183),

the content of which is perspectival information expressible in an indexical judgement of the form “this is how it is *from here*” (Ibid., 92).

4.5.4 Empirical Adequacy Clarified

Use of a logical space implies only the pragmatic commitment to use the theory’s classificatory framework in perceptual judgement:

That a claim is theory-laden does not mean that it presupposes the truth of a theory, only that theoretical terms are used in its formulation. For example, ‘this powder is classified in chemistry as sodium nitrate’ is theory-laden but does not imply anything about whether the chemical theory is true or false. (van Fraassen 2008, 371, n.3)²⁶³

Hence self-location does not usurp the kind of function referred to by “embedding”, which van Fraassen invokes in order to explicate the notion of empirical adequacy in the semantic view of theories. Compared to embedding, self-location is not mathematically precise: “[i]n general, measurement of an item classified as being in the domain of a particular theory will locate that item indefinitely” (Ibid., 164). More importantly, it is *prior* to assessing a theory’s empirical adequacy (Ibid., 256-257).

A theory’s empirical adequacy is assessed through abstract measurement outcomes constructed on the basis of appearances gathered through observation or, more typically, measurement outcomes: “In practice, the level at which a theory confronts experience is not that of raw data taken from individual measurement outcomes, but of the ‘data models’ constructed on their basis, and further [surface models]” (van Fraassen 2008, 153-154). Data models are built up from the appearances (data) of observation reports, i.e., they are built up from individual acts of self-location in logical space.²⁶⁴ A data model “summarizes the relative frequencies found”, while a “surface model” further “idealizes” “this summary ... with a continuous range of values” (Ibid., 167). In the progression from individual concrete measurement outcomes to a data, appearances are presented “in progressively more abstract ‘outcomes’” (Ibid, 251). In virtue of the appearances from which it is constructed, a data (or surface) model is perspectival and also contains information pertaining to invariances across perspectives. A data or surface model is an “*official* measurement outcome” (Ibid., 166) that encounters the theory’s

abstract models. A theory's empirical adequacy consists in the isomorphism of these outcomes to parts of its models called "empirical substructures" (Ibid., 289).

A measurement model is an artefact representing both measured phenomena and the appearances from which it is constructed (van Fraassen 2008, 252). Like the observation reports from which it is constructed, it is taken to be a "description" (Ibid., 257) of the phenomena. Van Fraassen admits Newman's trivialization objection applies here: the model cannot be said to be "a unique, single way of representing" (Ibid., 244) the measured phenomena. The objection's significance, he takes it, is that *we* selectively divide up the measured phenomena and "choose" (Ibid., 243) the structure of the model²⁶⁵ in virtue an accepted theory that guides its construction. Against the obvious reply that the theory could then only be said to represent phenomena "as described" or "as represented" and not the phenomena themselves, van Fraassen appeals to the prior acts of self-location implicit in the observation reports to invoke a pragmatic tautology: the claim that "the theory is adequate to the phenomena" is the same as the claim that "it is adequate to the phenomena as represented by us" (Ibid., 259).

Van Fraassen understands a pragmatic tautology to be a statement that (as understood by us) could be false, but cannot be denied: "paradigmatic examples" are "'cat' denotes cats" and "'Paul is a cat' is true if and only if Paul is cat".

They are undeniable by me, exactly because I acknowledge "cat" to be a word in my language. The semantic content, however, of these (to me undeniable) assertions are not necessary propositions, and most certainly not tautologies in the sense of semantics. If our language had developed differently in a certain way, then "cat" would have denoted gnats, rats or bats. Under such circumstances, uses of "cat" would not have been acts referring to cats, and "Paul is a cat" would have been used to state that Paul is (not a cat but) a gnat, rat or bat. (van Fraassen 1997, 23)²⁶⁶

A pragmatic tautology is illuminated through its correlation to a pragmatic incoherence, i.e., a statement that (as understood by us) could be true, but cannot be coherently asserted. The form of incoherence that illuminates these pragmatic tautologies is that of Moore's paradox: "It's raining in Peking, but I don't believe it". Unlike the logical contradiction "It is raining in Peking, but it is not raining in Peking", both conjuncts of Moore's sentence could be true simultaneously: there is a logically possible world where it is raining in Peking and I don't know it. What is wrong in asserting it, however, is that

“the expression of a judgement is immediately followed by the statement that I do not have *that* judgement” (van Fraassen 2011b, 20). In other words, incoherence arises because my second linguistic act (stating) undoes the content (having a certain judgement) of my first act (expressing).

Although van Fraassen is not perspicuous on how self-location in logical space can support a pragmatic tautology, we can illustrate what he probably has in mind with our simple example of the billiard ball. For van Fraassen, the sentence “‘The ball is red’ is true if and only if the ball is red” is a pragmatic tautology. The possibility of its falsehood trades on the distinction between mention and use (see van Fraassen 1997, 41 n.15). There is a logically possible world where language developed differently, where part of the colour spectrum we label “blue” is labelled “red”. The word “red” we use to describe that world in the second conditional of the sentence has its normal everyday meaning, but the word “red” that is mentioned in the first conditional, we say, has a different meaning in that world. However, if instead I acknowledge that the word “red” mentioned in the first conditional is a word in *my* language, I cannot coherently deny the sentence.

This tautology is illuminated by a pragmatic incoherence. Suppose I say that “The ball is red, but the sentence ‘The ball is red’ is not true for all I know or believe”. Van Fraassen would say this is a form of Moore’s Paradox (cf. 2008, 256). The possibility of its truth trades on the difference between use and mention just discussed. There is a logically possible world where language developed differently, where part of the colour spectrum we label “blue” is labelled “red”. The word “red” we use to describe that world in the first conjunct has its normal everyday meaning, but the word “red” that is mentioned in the second conjunct, we say, has a different meaning in that world. However, if instead I acknowledge that the word “red” mentioned is a word in *my* language, I cannot coherently assert the sentence. Why? Van Fraassen points out assertability or deniability “is a concept pertaining to use” and that “[a]ssertion, denial, calling into question, and the like are actions by a language user” (2008, 259). The content of my assertion “the ball is red” in the first conjunct is that the appearance of the ball as red is to be taken seriously as veridical, but then my denial “the sentence ‘The ball

is red' is not true for all I know or believe" expresses my doubt that the appearance of the ball as red is correct. I am pragmatically incoherent because the second act (denial) undoes the content of the first act (assertion), both of which are mine.

Since it concerns actual and observed entities, this incoherence also has an explanation in terms of logical space. My assertion that the ball is red describes a region of colour space, gaining its reference through the correspondence of an element of this region with the ball; equivalently, through its appearance I locate the ball in the red region of colour space which guides my assertion that the ball is red. Van Fraassen claims that self-location "is still properly called an assertion, it is a linguistic act" (1992, 10). This is because the act of locating the ball in the red region of colour space is implicit in the act of asserting that the ball is red. Hence, the content of my assertion is expressible by the indexical judgement "*this is there* in colour-space" (2008, 212). When, after the act of asserting the ball is red I immediately add "but the sentence 'The ball is red' is not true for all I know or believe", my denial is an act that also undoes the content of my act of locating the ball in the red region of colour space.

As I understand him, van Fraassen's position that a pragmatic tautology precludes the objection that theories are only adequate to data models and not to the phenomena is just an extension of this simple case. Clearly, the claim that "the theory is adequate to the phenomena" is the same as the claim that "it is adequate to the phenomena as represented by us" could be false: there is a logically possible world where the data model is incorrect or inaccurate, in which case the theory could be adequate to the phenomena but not adequate to the phenomena as represented by us. But why is it something I cannot coherently deny? We have to turn to the pragmatic incoherence that illuminates it and its explanation in terms of logical space.

Suppose that under the guidance of colour-theory I gather colour information from a number of billiard balls by measuring the wavelength of reflections from their surface. Suppose further that from the collection of appearances I construct a data model. "What I cannot do is to both present [the data model] as representing [the objects] and then say that perhaps it does not represent [them] at all" (van Fraassen 2008, 257). My presentation of the data model, van Fraassen tells us, is a linguistic act whose content is

that the data model be taken seriously as veridical, for it was constructed from measurement results I take seriously as veridical. These measurement results are acts of classification whereby I locate the measured balls in colour space common to the theoretical models (Ibid., 256). Because “the ability to self-attribute a position with respect to the representation is *the condition of possibility for use* of that representation” (van Fraassen 2008, 257), presentation of the data model is also an act of locating the billiard balls in colour space: when I present it I am “saying ‘we are *here*’ in a logical space” (Ibid., 257). My act of denial or doubt that the data model – or a theoretical model that shares its structure – represents the billiard balls opposes the content of my act of locating them in colour space expressible as “*this is there* in colour space”, landing me in pragmatic incoherence.

4.6 Unities that Conspire

The transcendental and metaphysical history of the idea of logical space as a relation among imaginative representation, reality and the self reveals at least three unities. The first is the *unity of possible worlds*: a logical space is a form of all imagined worlds that includes the real one. Kant achieves this unity through the identity of pure and empirical synthesis, while Wittgenstein realizes it through shared forms of logical space and reality. The second is more esoteric and might be called the *unity of existence*: given a certain conception of existence, both reality and logical space exist. For Kant pure sensible schemata and, derivatively, their organizing structure exist in virtue of coming into being through the categories; for Wittgenstein our form of representation exists though the self that is the eternal present. The third is what I call the *unity of divine consciousness*: through logical space the self has a “God’s-eye” view of reality. For Kant this falls out of the internalization of human schemata from a divine schema for the structure of the universe. Wittgenstein’s claim that “[t]here are two godheads: the world and my independent I” (1961a, 74) comes from shifting immanent and transcendent perspectives of the world by the self of pure realism and the self of solipsism.

What I will establish in this section is that one way or another each of these unities works against van Fraassen’s conception of logical space insofar as it is taken to be

consistent with his empirical stance. We get an intuitive sense for why this is the case when we compare his characterization of phenomena, logical space and the self:

Phenomenon	Self	Logical Space
Mind-Independent	Mind	Mind-Dependent
Existing	Existing	Non-Existing
Entity	Non-Entity	Entity

The unities implicit in the idea of logical space conspire to eradicate differences among the columns. With logical space no longer a mirror of reality, the unity of all possible worlds pushes against the mind-independence of phenomena and moves van Fraassen's philosophical system into phenomenalism. The unity of existence gives rise to a platonic²⁶⁷ commitment to the status of logical space. And the unity of divine consciousness undermines the plausibility of the claim that the self is not a transcendent entity.

4.6.1 The Unity of All Possible Worlds

The unity of all possible worlds inherent in the idea of logical space drives van Fraassen's claim that we discern structure in nature "in the same way as we discern a story when looking at ink marks on a page". His position that "seeing is reading" is based on the presupposition that the same logical spaces are involved in our experience of the actual world as in our imagining the possible world of a novel: e.g., "the constitution of time in our construction of the real world is not different in essential character from the constitution of time by the reader in his construction of the narrated world as he reads the text" (van Fraassen 1991b, 24). This is so notwithstanding that the former involves real events, while the latter involves merely fictional ones.

His claim that "*this* text is not so different in kind from other texts" (van Fraassen 1991b, 35), i.e., that the textual aspect of our experience of reality is not so different from the text of a novel, betrays in its emphasis this point. For self-location, a linguistic act expressible as "*this is there* in logical space", to bestow meaning upon pure mathematical

structure, “*this*” must refer to a real entity. But when I read a story it cannot be said that I locate *this* fictional entity *there* in logical space for the simple fact that there is no entity *to be* located. Suppose I am reading my novel and I come across the sentence “I hear scratching in the wall, the patter of little feet at midnight, my cheese disappears”. As I read the sentence I imagine a mouse (just as you did). The best that can be said is that I locate information content from a set of letters I interpret, i.e., *appearances*, in the same logical space I use to locate a real entity when I classify it as a mouse. But whereas it may be the case that in seeing an actual mouse I locate a real entity in logical space via appearances, in reading the sentence it can only be said that I construct a fictional entity *out of* appearances in logical space to form a fictional image.

Now consider the following passage from *The Scientific Image*:

And surely there are many telling ‘ordinary’ cases [of inference to the best explanation]: I hear scratching in the wall, the patter of little feet at midnight, my cheese disappears – and I infer that a mouse has come to live with me. Not merely that these apparent signs of mousely presence will continue, not merely that all the observable phenomena will be as if there is a mouse; but that there really is a mouse. ... [But] the mouse *is* an observable thing; therefore ‘there is a mouse in the wainscoting’ and ‘All observable phenomena are as if there is a mouse in the wainscoting’ are totally equivalent; each implies the other (given what we know about mice). (van Fraassen 1980, 19-20, 21)

Van Fraassen would now update this passage by replacing “observable phenomena” with “appearances”:

Very often, we do have to infer the reality of putative phenomena from the appearances in measurement outcomes. But that does not mean that they are not observable entities. If I see black droppings and infer the presence of a mouse, the mouse’s reality is inferred, but the mouse is not unobservable. (2011a, 437)

With this later passage in mind, the point of the earlier one is to confront philosophers who argue we infer real entities as the best explanation of ordinary appearances with an equally compelling hypothesis: a real entity is implied from a complex of appearances located in a logical space. But then why is it that are not simply *constructing* a real entity (e.g., a real mouse in the wainscoting) *out of* the appearances just like we construct fictional entities *out of* appearances when we read a story? If logical space is the form of all possible worlds, *prima facie* there should be no difference in use in relation among possible worlds. Van Fraassen thus has the burden of showing his conception of logical

space belongs to common sense realism (i.e., realism about observable entities existing independently of ourselves) not phenomenalism.

I can envisage three possible responses one might try to make. The first would be to say the possibility of fictional images is precluded in van Fraassen's claim that "mental images aren't images at all" (1999, 10). Mental images, he argues, are a reification based on an analogy to real images. But as Wittgenstein's (2001, xi) duck-rabbit picture shows, a real image can be seen in different ways, while the very idea of a mental image is that it cannot.²⁶⁸ The analogy thus destroys itself: unlike mental images, real images are irremediably ambiguous. One cannot simply dismiss this response with the reply that the duck-rabbit image is atypical, for van Fraassen also claims real images are ambiguous through misclassification: "[t]he item in front of us can be seen, so to speak, in different ways. It is a candy wrapper; I see it as a flower" (van Fraassen 2002b, 7).²⁶⁹ The problem is that van Fraassen also claims locating in logical space that occurs when reading a book is equivocal,²⁷⁰ and so one can just as easily say fictional images are not mental images because they are ambiguous too.

Rather than attempt to preclude the possibility of fictional images, another response would attempt to distinguish fictional images from real images. Van Fraassen acknowledges the possibility of "private images" such as dreams or after-images (2008, 104), and so it might be said that, unlike real images, fictional images are private images. The problem is that real images are not private because, according to van Fraassen, the repeatability of appearances ensures their inter-subjectivity. Clearly fictional images that arise from reading a book are repeatable simply by rereading the sentence, and thus public as well.

The third and best response is as follows. Van Fraassen claims the text of our spontaneous judgments "is already divided into dreaming, waking, thinking, musing, and so on" (van Fraassen 2002a, 137), which suggests that perhaps spontaneous judgement involved in reading a book is divided against spontaneous judgement involved in experiencing the real world. How so? He claims that "the difference between [the real and fictional worlds] we cannot formulate but only show: I can only touch *this* flesh, *these* stones, *this* wood" (1991b, 37). I suspect his position would be that spontaneous

judgement in the case of the real world involves self-location expressible as “*this is there* in logical space”, which includes the very same act of ostensive reference that shows the world is real rather than fictional: i.e., *this* refers to more than appearances (text). But such a response would require justification in order to discharge the burden van Fraassen carries, and in van Fraassen’s case it would have to come from the phenomenology of experience.²⁷¹

As hinted already van Fraassen’s characterization of spontaneous judgement in the case of naturally and artificially produced public hallucinations works against any such justification. When I look at a paramecium image my spontaneous judgement is not that it is a representation, but that it is a real entity. Upon *reflection* I might convince myself that as a *thing* a paramecium is a fictional entity I project onto reality, that the image is a representation produced in the physical interaction of the water sample, slide and microscope. Still, it cannot be said that my spontaneous judgement involves locating the measured water sample and slide in logical space. The best that can be said is that it involves locating appearances *out of* which I constructed a fictional entity. Because van Fraassen claims public hallucinations of this very sort are pervasive in our use of measuring instruments, the implication is that an act of ostensive reference to a real entity is not a condition of use of logical space. To use logical space all I have to do is locate appearances within it and construct an entity, fictional or real, out of them. Any act of ostensive reference van Fraassen claims will show a difference between the real world and a fictional world is added to our act of using logical space in immediate perception.

4.6.2 The Unity of Existence

Unlike Kant and Wittgenstein, van Fraassen wants the benefit of the idea of logical space without any commitment to its existence:

[Observation reports] have as complete text an indexical assertion that locates the speaker, on that occasion, in some definite part of his own general, “objective”, world picture. I realize that I am leaving this somewhat metaphorical or analogical: it is not literally true that we carry our “general” opinion with us in the form of a representation that encodes it, like a picture or a map. But we can reasonably represent ourselves as doing that, for present purposes. (van Fraassen 1992, 18)

For the purposes of a pragmatic theory of observation, that is, we can account for our immersion in theory-infected language if we conceive ourselves as cognizing the world through a scientific image. The idea of logical space is supposed to serve the aspirant conceptualist or nominalist with a theory of universals without invoking platonism in relation to mathematical objects:

... I am a nominalist. Of course, one can be a nominalist only in the way Saint Paul held one could only be a Christian, namely, in the sense of trying to be one. I do not really believe in abstract entities, which includes mathematical ones. Yet I do not for a moment think that science should eschew the use of mathematics, nor that logicians should, nor philosophers of science. I have not worked out a nominalist philosophy of mathematics – my trying has not carried me that far. Yet I am clear that it would have to be a fictionalist account, legitimizing the use of mathematics and all its intratheoretic distinctions in the course of that use, unaffected by disbelief in the entities mathematical statements purport to be about. Within mathematics, the distinction between structure of different cardinalities and the nonisomorphism of real number continuum and natural number series are objective. I cannot spell this out further. But I feel sufficiently clear on what the antirealist strategy must be to resist as irrelevant disputes within philosophy of mathematics while I cooperate in the task of an account of science, of its content, its structure, or its methodology. (1985a, 303)²⁷²

A unique challenge that van Fraassen faces in working out his fictionalism is to reconcile the claim that mathematical language is not to be literally construed with his position that scientific language describing describes real entities located within mathematical entities is to be given a literal construal. There is another more obvious problem. It is evident that over time the idea of logical space has come to do more philosophical work. What started as a representation of our conceptual framework useful in formal semantics to analyze the structure of scientific theories became the intentional correlate of the conceptual framework guiding natural language and ended up as something into which the self locates the reality of its immediate environment. I will now show that in this last step (at least) van Fraassen has reified logical space as an existing thing.

Van Fraassen's appeal to self-location in logical space in order to establish a link between the models that a theory presents and reality is based on his claim of "*a precise and perfect analogy between theory, model and map*". More specifically, the inevitable indexicality of application expressible as "*this is there*" is what is identical. If we reflect upon the indexicality of application in relation to a map, we realize that the act of self-location is a unification of three separate acts: ostensive reference to a *real* entity in the

user's environment, the content of which is expressible as "this"; ostensive reference to the *real* map, the content of which expressible as "there", and an act that connects the two. Since all three acts underwrite van Fraassen's elaboration of a pragmatic tautology, then in principle pragmatic incoherence can result if the content of any *one* of these three acts is opposed. If the analogy of self-location is indeed "a precise and perfect" one, the same incoherence must arise if we replace the concrete map with an abstract logical space.

Suppose (contrary to our misgivings last section) that spontaneous judgement involved in the experience of reality involves more than locating appearances, that the linguistic act of self-location in logical space involves ostensive reference to real entities. Suppose further that upon observing a certain billiard ball I say that "The ball is red, but the sentence 'There is such a thing as the ball' is not true for all I know or believe". Since van Fraassen equates "... exists' with 'There is such a thing as ...'" (2004b, 454), this is equivalent to saying "The ball is red, but the sentence "The ball exists" is not true for all I know or believe". As understood by me, both conjuncts of this sentence could be true simultaneously: I can envision a logically possible world where the ball is not a material thing but a hologram, i.e., a public hallucination. But in my world where the ball is not a hologram, my act of denying that the ball exists opposes the content of my act of ostensive reference to a real entity expressible as "this". Following van Fraassen's reasoning I am pragmatically incoherent.

Now suppose like van Fraassen I am committed to the view that abstract entities do not exist. And suppose further that apart from functioning for us as a representation, abstract entities do not exist. The issue is whether, having bestowed a representational role upon on an abstract entity, the idea of self-location in that entity gives rise to a commitment to its existence. Upon observing the same billiard ball I could say instead "The ball is red, but the sentence "There is such a thing as a part of the real line" is not true for all I know or believe". As understood by me both conjuncts of the sentence could be true simultaneously. I can envision a logically possible world where no part of the real line has been bestowed a representational role. In my world, however, parts of the real line have been bestowed a representational role. So am I pragmatically incoherent? Well

if colour space were a concrete map – say a drawn line – and if I said “The ball is red, but the sentence “There is such a thing as a drawn line” is not true for all I know or believe”, I would be pragmatically incoherent, for my act of denying that the drawn line exists opposes the content of my act of ostensive reference to a real entity expressible as “there”. By virtue of the “perfect and precise analogy” it follows that the same pragmatic incoherence arises if colour space is not a drawn line but a part of the real line.²⁷³ By conceiving logical space as a *completed* space and accounting for its application to reality by locating real entities in that space, van Fraassen crosses the threshold into platonism.

4.6.3 The Unity of Divine Consciousness

Van Fraassen’s appeal to self-location recovers Wittgenstein and Kant’s conception of logical space as inextricably linked with the self. Whereas Wittgenstein saw the self as the whole of logical space, van Fraassen understands it in a way closer to Kant: “We *conceive of ourselves as* located and oriented in this [scientific] world-picture *in one precise, definite way*. ... But remember Kant’s *Paralogisms*: there is no argument even from how we necessarily represent ourselves to what we are” (van Fraassen 2010a, 471).

Van Fraassen (2004b) provides the outline of his preliminary view of the self that seems to be based upon two presuppositions similar to those of his common sense realism. The first is that the self is the whole person I refer to when I say ‘I’. The second is that “most things we ordinarily say about ourselves are true” (Ibid., 455). Accordingly, I exist and it is true that I act, I bear properties such as thought, I am the terminus of relations to what is mine and (contrary to Hume) “I am a continuant” (Ibid.). The latter seems to be essential for van Fraassen’s view that time is a logical space, for if I were not a continuant I might not construct the temporal order of events that happen to me by *successively* locating them in the real number line.

And just like a real entity, I *appear* to others. I manifest in what is *mine*: “I *manifest* myself in nature, through my body, my movements, my words...” (van Fraassen 2004b, 466). “You do encounter me directly in experience; but that encounter has certain

physical relations involving our bodies as a precondition” (Ibid., 471). Also “I manifest ... in the social world” through “[t]he relationships that define my social position” (Ibid., 469). Although I am the enduring subject in which inhere physical events that manifest to others as incompatible, events others can represent as a world line (cf. van Fraassen 1991b, 26), manifestation of the self is not the self: “any person’s appearance is to be distinguished from that person” (van Fraassen 2008, 285).

“The self *transcends* all such appearances but that does not imply that the self is a *transcendent object*” (van Fraassen 2004b, 456, n.8). “I am in this world, but not of this world” (van Fraassen 2004b, 453) because I am not an entity: “‘entity’ is another synonym for ‘object’ or ‘thing’, and I am not one” (Ibid., 471). Yet the self “*is not nothing*” (Ibid., 455). “I exist, but I am not a thing among things, I am neither a physical object nor a mental substance or abstract entity, nor a combination thereof” (Ibid., 453). Thus I am neither my physical body nor the abstract logical space in which I locate my body when I observe it.

“Not being an object, when I wish to act I just do it” (Ibid., 468). The most illuminating assertion van Fraassen makes is that persons “*are what they will have been*, and what they will have been is still in the future, held jointly in their hands and the hands of fate” (2004b, 470, n.28). The import of time is characteristically Kantian; the centrality of willing existence is arguably Wittgensteinian. Perhaps the best we can say at this point in the development of van Fraassen’s view of the self is this: I am a willing existent necessary for the possibility of constructing a temporal order of events that happen and that I am aware by continuous projective acts of locating them in a logical space chosen to represent these events.

Self-location is a condition for the possibility of a relation between the self and a logical space:

The activity of representation is successful only if the recipients are able to receive that information through their ‘viewing’ of that representation. But what are the conditions of possibility for this reception? The recipient must be in some pertinent sense able to relate him or herself, his or her current situation, to the representation. (van Fraassen 2008, 80)

On one side of this relation is a feature of logical space:

... as Hermann Weyl put it graphically, there will be, even in the most theoretical sciences, an “ineliminable residue of the annihilation of the ego” to provide the conditions for relating the theoretical models to specific empirical situations. All the revolutionary developments in the theories of space and time as well as the upheavals in atomic physics testify to that. The former brought frames of reference to the center stage, the latter engendered what is in fact precisely called “the measurement problem” as fulcrum for philosophical interpretation. (Ibid., 87-88)

On the other side of this relation is a feature of mine:

We will just have to admit a non-pejorative sense of “subjective”. It is true that this solution [of self-location] gives a special role to consciousness in science. But it does so only on the premise that there is applied science, i.e., there is conscious use of science. The solution entails no more about consciousness than is contained in that premise. (van Fraassen 1992, 10)

This passage expresses a worry. If time, space, and spacetime are logical spaces, then the self-location is outside time, space and space-time. Obviously self-location along with the consciousness it entails are outside science. In limiting himself to conscious use in science, it seems van Fraassen does not want to speculate as to the nature of consciousness. Indeed, it seems he wants to stop here and say “[t]here is a mystery of consciousness. But it is not among the mysteries that the sciences confront, which they so fortunately and habitually address and solve” (van Fraassen 2004b, 476).

Yet clearly his idea of self location entails much more about consciousness than is contained in the premise that there is conscious use of science: what the idea of self-location in completed logical space entails about consciousness is different than, say, what the idea of self-projection of a potential logical space entails about consciousness. The relation between consciousness and the “ineliminable residue of the annihilation of the ego” forms a sort of interface where I meet up with a completed logical space. At this interface I can shift my view of reality from within logical space to a view of reality from above expressible in a law-like translation. With this shift perspectival consciousness elevates itself to a mode of consciousness capable of making “God-like reflections” upon phenomena. It should be noted that van Fraassen really has no choice but to make this sort of claim. His idea of logical space is that of an abstract entity bestowed a representational role as a matter of convention, and thus a product of public imagination. In ascribing my location in logical space, therefore, *I* must be able to transcend *my* consciousness to a level of *our* consciousness.

In the tradition of Haraway (1988), Okruhlik (2009) has expressed concern that empiricist structuralism is another god-trick of seeing everything from nowhere. Indeed to claim that there is more to meaning than “meets the eye”, that consciousness can transcend *situated understanding* and *imagine* the empirical world invariant across alternate perspectives, is to characterize consciousness as having the sort of unity traditionally attributed to the mind of God. This unity is implicit in the idea of logical space itself, which develops from the notion of a divine schema. Faced with the shadow of divine consciousness, Kant conceived the transcendental self as a formal limit of imagination. Wittgenstein was willing to simply admit a metaphysical notion of the self as a godhead that can imaginatively embed the entire world or, alternatively, shrink to a point in mystic union with it. In recovering the self against a conception of logical space as conventional, van Fraassen is forced into the position that at a certain plane consciousness is inter-subjective. As an empiricist who disavows metaphysics, he owes us an account why this shadow of the divine in his the idea of logical space does not give rise to a conception of the self as a transcendent entity that can grasp the completed infinite whole of logical space.

4.7 Conclusion: A Call to Rethink Logical Space

Van Fraassen transports Wittgenstein’s conception of logical space into a nominalist theory of meaning in order to solve a problem of interpretation without recourse to modal realism. In doing so, he severs the idea from its transcendental and metaphysical underpinnings without working through the consequences. It is perhaps no surprise that he should encounter a problem of interpretation that goes to the very heart of his novel conception. But his reply of self-ascription of location triggers unities traditionally implicit in the idea of logical space, unities that manifest as the spectres of phenomenalism, platonism and a metaphysics of self. To build van Fraassen’s empiricism around the idea of logical space requires subtle rethinking of the idea itself. As I will suggest in the conclusion to this dissertation, there are insights he might draw from Kant, Hertz and Wittgenstein.

Conclusion

This thesis has been about the origin, intellectual development and contemporary use of a semantic variant of the idea of logical space. This is the idea of a logico-mathematical framework of imaginative representation that delimits a form of all possible worlds. It arises from a philosophical orientation that images or pictures relate to reality through the self. In general four things have been demonstrated: 1) there are highly influential and difficult philosophical systems in diverse traditions that unify around the idea of logical space; 2) major differences among these systems spring from minor variations in this idea; 3) these philosophical systems have an intellectual history that runs through 19th century German philosophy of science; and 4) we can compare, evaluate and perhaps even develop these philosophical systems in a straightforward way. I shall conclude by explaining why illuminating the idea of logical space worked well to dissolve difficult exegetical problems, and by providing an outline for a future research project.

It should be clear that in relation to Kant and Wittgenstein, use of “mirror” is not intended as a metaphor but as a structural analogy for a form of imaginative representation that explains Kant’s theory of schemata and the underlying unity of Wittgenstein’s *Tractatus*. These explanations owe their strength to a method of inquiry that went to the heart of Kant and Wittgenstein’s philosophical systems. What this method reveals is that when a philosopher such as Kant or Wittgenstein makes a significant appeal to images or pictures in an attempt to construct a unified and comprehensive philosophical system, he has one in mind when he tries to communicate that system to others. Asking that philosopher “What is an image or picture?” “How is it organized?” and “How does it relate to reality?” brings to the fore content that provides a glimpse of what this is. This is to say that as presented by Kant, Wittgenstein and even van Fraassen, the idea of logical space reduces to a philosopher’s model of cognition. This is no more evident than in the *Tractatus* where its propositions seem to be satisfied by a picture of the self in shifting perspectives of the world. It is just as plain in Kant’s infamous schematism where a line by line interpretation shows a description in highly dense and complex prose of a simple structure of the imagination. And it is also apparent in van Fraassen’s empiricist structuralism in the attempt to solve fundamental problems

associated with his picturesque grasp of conceptual phenomenology. And yet it is more than a philosopher's model of cognition. It is the insight of a great mind into how we are in the world.

In my estimation the problems identified with van Fraassen's conception of logical space arise from his fictionalism. There appears to be nothing in his stance towards mathematical objects to constrain conceiving logical space as a completed space. He thus naturally draws inspiration by analogy to a concrete map, invoking acts of self-ascription of location in logical space to account for its non-trivial relation to reality. Yet there are insights to be gathered from Kant, Hertz and Wittgenstein that can point van Fraassen towards a more subtle way of thinking about the idea of logical space.

If the empiricist argument for the idea of logical space appeals to Kant's observations about conceptual phenomenology and then goes on to equate conceiving with imagining, perhaps it ought also to respect Kant's view that completed infinities are never images for us. It might recognize, in other words, his implicit view that a logical space is a potential space, that it is only a general schema from which images are constructed and organized. Given the close affinity between their philosophical systems, van Fraassen might take further inspiration from Hertz. A key insight Hertz presents is that a schematic scientific representation is what is "identical in essence" as "contained in almost all textbooks which deal with the whole of [a science] ... and in almost all lectures which cover the whole of this science" (1900/1956, 4). His implicit claim is that we are (as van Fraassen would say) "conditioned" to project images through learning schematic properties, that a scientific representation is just what is essential to this conditioning. And van Fraassen might also follow Wittgenstein by characterizing the application of a scientific representation to reality in terms of self-projection rather than self-location.

The advantage to thinking of a logical space of models as a schema rather than as an image is that its application is no longer thought of as self-location *in* a scientific representation, but as self-projection *conditioned by* a scientific representation. Rather than locating real entities in a scientific representation and then projecting a perspectival image back onto the world, application is understood more simply as the projection of a perspectival image. This seems to get around two of the problems identified. By

disassociating the scientific representation from the self, no issue of divine consciousness arises. And because the content of the act of self-projection is expressible merely by the indexical judgement “*this*”, platonic commitment to the existence of a logical space of models brought about by pragmatic tautology is avoided. However, the spectre of phenomenalism remains in that it seems we spontaneously project images *as* real (one “gets lost” in every meaningful story so to speak). Perhaps the idea of logical space together with the claim that experience is *literate* requires admitting that it is only through a non-spontaneous act of ostensive reference that we distinguish fictional entities.

This approach is a way to a different empiricist theory of scientific representation in relation to the semantic view of theories. It implies, for example, a characterization of representational adequacy as projected unity, one where a data model embeds within a logical space of models by involving towards an essential focal point of our conditioning. It also gives rise to a different way of thinking about how we are in the world, for now we encounter reality, and each other, in situated understanding.

Endnotes

¹ Note inserted in Kant's own first edition of the *Critique of Pure Reason* as indicated on p. 271 of the translation by Guyer and Wood (1998).

² *The Philosophy of Time and Space* (1970a, 102).

³ The term "logical space" is much abused and common usage has trivialized its meaning. By "semantic variant" I intend to distinguish the epistemic variant of "logical space" known as the "space of reasons" found in Sellars (1997), i.e., the idea of justification relative to a network of inferential relations. This is not to say that the semantic and epistemic variants are unrelated. Friedman (2001) presents a synthesis of the two he takes to be implicit within logical positivism.

⁴ The following conception of logical space comes from my understanding of the use of the term "logical space" by early Wittgenstein (1921/1961), which also appears to be van Fraassen's use of the term in empiricist structuralism. The first chapter of this dissertation shows where in the *Critique of Pure Reason* this conception is to be found.

⁵ Hyder (2002) attempts to connect Wittgenstein's conception of logical space to Helmholtz's perceptual manifolds. This dissertation presents Wittgenstein's idea of logical space very differently, and justifies the interpretation by its ability to unify the *Tractatus*.

⁶ Throughout I will follow the standard practice of quoting the *Critique of Pure Reason* by citing pages from the first and second original editions (A and B). Kant's other writings will be referred to the *Akademie Ausgabe* (indicated by Ak., with volume and page). Unless otherwise indicated, I use the translation of the *Critique of Pure Reason* (1781-1787/1998) by Paul Guyer and Allen W. Wood, the translation of the *Prolegomena to Any Future Metaphysics* (1783/2004) (Ak. IV) by Gary Hatfield, the translation of the *Metaphysical Foundations of Natural Science* (1786/2004) (Ak. IV) by Michael Friedman, and the translation of the *Critique of Judgment* (1790-1793/1987) (Ak. V and XX) by Werner S. Pluhar.

⁷ The following account of Kant's pre-critical conception of schemata is indebted to Friedman (1992, 5-34).

⁸ Friedman (1992, 25-28) identifies three problems: 1) it is unclear how the properties of space (e.g., continuity) can be derived from the external relations of non-spatial substances; 2) even if it were clear, our knowledge of these properties would not derive a priori, but from our empirical knowledge of the law of universal gravitation; and 3) since material and immaterial substances are said to be co-present in a single world, their distinction is undermined by their co-presence and interaction in space.

⁹ E.g., Allison (2004, 426).

¹⁰ "*Idea* properly means a rational concept, and *ideal* the representation of an individual being as adequate to an idea" (Ak. V: 232).

¹¹ As noted below, there are non-figurative pure sensible concepts. Claims in this chapter should be understood as limited to pure sensible concepts pertaining to shapes in space.

¹² "The undetermined object of an empirical intuition is called appearance" (B34/A20).

¹³ "[T]hat which is originally itself only appearance, e.g., a rose, counts in an empirical sense as a thing in itself, which yet can appear different to every eye in regard to colour. The transcendental concept of appearances *in space*, on the contrary, is a critical reminder that absolutely nothing that is intuited in space is a thing in itself, and that space is not a form that is proper to anything in itself, but rather that objects in themselves are not known to us at all" (A29-30/B45).

¹⁴ The following typology is taken from a note inserted in Kant's own copy of the first edition preceding the table of the categories: "On a characteristic of concepts; of intellectual, empirical, and pure sensible representations" (1998, 212n.).

¹⁵ An idea is a pure concept of reason "made of up notions, which goes beyond possible experience" (A320/B377).

¹⁶ "[T]o *construct* a concept means to exhibit *a priori* the intuition corresponding to it" (A713/B741).

¹⁷ "Now of all intuition none is given *a priori* except the mere form of appearances, space and time, and a concept of these, as *quanta* [i.e., magnitudes (A717/B745)] can be exhibited *a priori* in pure intuition, i.e., constructed, together with either its quality (its shape) or else merely its quantity (the mere synthesis of the homogeneous manifold) through number" (A720/B748).

¹⁸ An earlier version of this chapter provided an extensive reconstruction of the A-Deduction as a projection of space-time onto time that also plainly showed the motivation for Kant's schematism. Unfortunately, considerations of length preclude its inclusion into the final version.

¹⁹ Cf. Ak. XX: 198n.6. Friedman (1992) notes that this marks the fissure for Kant between mathematics and logic: “the existential proposition corresponding to the construction – [e.g.,] that for any point and any line there is a circle with the given point as center and the given line as radius – cannot be conceptually expressed for Kant. In mere syllogistic logic this existential proposition cannot, strictly speaking, even be stated” (126).

²⁰ Kant is clear that the synthesis of apprehension is an act of the imagination: “There is thus an active faculty of the synthesis of this manifold in us, which we call imagination, and whose action exercised immediately upon perceptions I call apprehension. For the imagination is to bring the manifold of intuition into an *image*; it must therefore antecedently take up the impressions into its activity, i.e., apprehend them” (A120). Also, cf. A120n.

²¹ To fit Kant’s schematism within her particular interpretation of the *Critique*, Longuenesse (1998) concludes Kant’s chronological presentation of comparison, reflection and abstraction is “implausible”. She asserts that the rule of apprehension is generated in its entirety in the operation of comparison and, though acknowledging that it pertains to properties of *sensation*, identifies this rule with a schema (116-117). Aside from the incoherence such an interpretation makes of Kant’s claim that “the schema of sensible concepts (such as figures in space) is a product and as it were a monogram of *pure a priori* imagination” (italics added, A141-142/B181), on my interpretation Kant’s chronological presentation is spot-on.

²² Kant gives us the rule at A164-165/B204: If I say: “[w]ith three lines, two of which are taken are greater than the third, a triangle can be drawn”, then I have here the mere function of the productive imagination, which draws the lines greater or smaller, thus allowing them to abut at any arbitrary angle.”

²³ The passage of the Aesthetic Kant is alluding to is probably the following:

I call all representations pure (in the transcendental sense) in which nothing is to be encountered that belongs to sensation. Accordingly, the pure form of sensible intuitions in general is to be encountered in the mind *a priori*, wherein all of the manifold of appearances is intuited in certain relations. This pure form of sensibility itself is also called pure intuition. So if I separate from the representation of a body that which the understanding thinks about it, such as substance, force, divisibility, etc., as well as that which belongs to sensation, such as impenetrability, hardness, color, etc., something from this empirical intuition is still left for me, namely extension and form. These belong to pure intuition, which occurs *a priori*, even without an actual object of the senses or sensation, as a mere form of sensibility in the mind. (A20-21/B34-35)

What Kant is clarifying in his footnote is that inasmuch as the “extension and form” that “belong to pure intuition” *determine* a pure sensible concept, they are also *determined* by a pure sensible concept in a synthesis.

²⁴ Friedman (1992, xiv) also construes the schematism as essentially spatiotemporal.

²⁵ Thus, I disagree with Bennett (1966, 141) when he says that the “nasty phrase ‘representation of a [general] procedure’ just means ‘rule’”. Following Longuenesse’s (1998, 50) account of a “rule of sensible synthesis”, Allison (2004, 210) introduces a non-discursive “perceptual rule” as schemata of pure sensible and empirical concepts that functions “to process the sensible data in a determinate way”. But there are exegetical problems with this interpretation. Contra Allison (and Longuenesse), Kant does *not* “explicitly” say a schema as representation *is* a rule, only that it “signifies” a rule, or is “related” to a concept “as a rule”, which suggests merely that it *presents* the same rule as a concept does; conversely, Kant never says in the *Critique* that a rule *is* a representation, only that a representation (e.g., a concept) “serves as” (A106) or “expresses” (A142/B181) a rule. In the passage just quoted, Kant does *not* distinguish schemata. Yet Allison does not adopt a view consistent with this, opting to recognize a categorical schema, not as a perceptual rule, but as a formal intuition (215). Longuenesse’s elaborate and complex position is more consistent in this respect. However, she does justice to the notion of the schema as a “product” of the imagination only by claiming that it is “generated” in universal comparison guided by a pre-discursive capacity to judge (115-122). This commits her to the view that “schemata are acquired *before* the concepts, which reflect them” (116 n.29). Kant never says this, and the main difficulty with this interpretation is that Kant’s account of non-transcendental schemata is only in relation to *shapes*, whereas the “perceptual rule” that both she and Allison conceive to be a schema extends to sensation.

²⁶ I thus depart from Friedman (1992) who argues this: schemata are “general procedures for constructing each and every figure of a particular kind (lines, circles, triangles, and so on)” (41). I reject a similar interpretation by Young (1984).

²⁷ Elsewhere Kant refers to a schema as a “monogram” (A142/B181) and states that a “schema contains the outline (*monogramma*)” (A833/B862).

²⁸ “[A]ll synthetic a priori propositions are related to [principles of the pure understanding] alone, indeed their possibility itself rests entirely on this relation” (B294). Principles of the pure understanding are “synthetic judgements that flow *a priori* from pure concepts of the understanding under [sensible] conditions” (A136/B175). And since these sensible conditions are transcendental schemata, “[t]he principles of the pure understanding ... contain nothing but only the pure schema, as it were, for possible experience” (A236-237/B296).

²⁹ This point is made even more clearly in the case of the category *substance*: without more, the category “shows me nothing at all about what determinations the thing that is to count as such a first subject is to have” (A147/B187).

³⁰ Kemp-Smith (2003, 334-342) dismisses Kant’s schematism as raising a pseudo-problem, Bennett (1966, 150) describes the problem it seeks to address as “hopelessly confused” and Warnock (1949, 80) suggests the problem is a “silly question”.

³¹ Indeed, Kant makes a similar claim about the point of the schematism at B167.

³² In his introduction to the *Critique*, for example, Guyer states “Even if the transcendental deduction does establish *that* the categories do apply to all possible data for experience, or (in Kant’s terms) all manifolds of intuition, it does so only abstractly and collectively – that is, it does not specify *how* each category applies necessarily to the objects given in experience or show that all of the categories must be applied to those objects” (italics added, 9-11).

³³ “The shape of a cone can be made intuitive without any empirical assistance.... I cannot exhibit the concept of a cause in general in intuition in any way except in an example given to me by experience, etc.” (A715/B743).

³⁴ In Kant’s own words: the transcendental deduction shows that “pure concepts *a priori*, in addition to the function of the understanding in the category, must also contain a priori formal conditions of sensibility (namely of the inner sense) that contain the general condition under which alone the category can be applied to any object” (A139-140/B178-179).

³⁵ Krause makes a very similar point to my own when he says that a schema “specifies ... the mark for the application of [a] category” (185), but differs in his construal of the mark as purely temporal.

³⁶ “Hence it is also requisite for one *to make* an abstract concept *sensible*, i.e., display the object that corresponds to it in intuition, since without this the concept would remain (as one says) without *sense*, i.e., without significance” (A240/B299)

³⁷ That a schema is a criterion of evidence is strongly suggested in the following comment: “But what must be remembered about all synthetic principles ... [is that they] ... have their sole significance and validity not as principles of the transcendental use of the understanding but merely as principles of empirical use, hence they can be *proven* only as such; consequently the appearances must not be subsumed under the categories per se, but only under their schemata” (italics added, A180-181/B223);

³⁸ It is in the sense of schemata of sensibility that Kant remarks that “the senses do not supply pure concepts of the understanding *in concreto*, but only the schema for their use, and that the object appropriate to this schema is found only in experience (as the product of the understanding from materials of sensibility)” (Ak. IV: 316).

³⁹ Like mathematics, “the peculiar thing about transcendental philosophy is this: that in addition to the rule (or rather the general condition for rules), which is given in the pure concept of the understanding, it can at the same time indicate *a priori* the case to which the rules ought to be applied” (A135/B174-175).

⁴⁰ See Chipman (1972) and Walsh (1957/1958) for criticism along these lines.

⁴¹ I am sympathetic to Chipman’s (1972) general claim that transcendental schemata are to be “understood derivatively” (42) from other schemata and that the mechanism of subsumption is “essentially “constructive” in character” (43).

⁴² I thus agree with Allison (2004, 212) that the homogeneity in this case is between pure and empirical intuition, not between a class concept and a member of that class.

⁴³ Its first mention is found Kant’s notes (2005) at R5552 (1778-79, 18:218-21): “That we must underlie all of our pure concepts of understanding with a schema, a [*crossed out*: relation] way of establishing composition in the manifold in space and time. That this schema is merely in the sensible representation of the subject”. Its late introduction speaks to its import in unifying the *Critique*.

⁴⁴ “Even space and time, as pure as these concepts are from everything empirical and as certain as it is that they are represented in the mind completely *a priori*, would still be without objective validity and without sense and significance if their necessary use on the objects of experience were not shown; indeed, their representation is a mere schema, which is always related to the reproductive imagination that calls forth the objects of experience, without which they would have no significance; and thus it is with all concepts without distinction” (A156/B195).

⁴⁵ Cf. A670/B698: “[T]he concept of a highest intelligence ... is only a schema ..., which serves only to preserve the greatest systematic unity in the empirical use of our reason, in that one derives the object of experience, as it were, from the imagined object of this idea as its ground or cause.”

⁴⁶ I have yet to come across an interpretation of this passage that deviates from this tradition. But then making sense of Kant without distorting his claims seems to be impossible. Through a painstaking analysis of this example Bennett (1966, 150) ends up dismissing the chapter as incoherent. Guyer (1987, 159) ends up collapsing Kant’s distinction between an empirical concept and a schema. Maintaining the distinction, Allison (2004) attributes to Kant the mistake of using “concept of dog” instead of “schema of dog” and, following Longuenesse (1998), finds himself forced to appeal to a “perceptual rule” (208-210) that extends what Kant is saying only about *form* to matter.

⁴⁷ Kemp-Smith (2003, 337) takes Kant to be making some sort of mistake here.

⁴⁸ It occurs in the following passage: “concepts certainly permit of division into sensible and intellectual ones; for one cannot determine any object for the latter, and therefore also cannot pass them off as objectively valid” (A255/B311).

⁴⁹ Thus, I depart from Guyer’s view in his introduction to the *Critique* that each category is associated with a “temporal schema” (10) and from Franswa’s (1978) claim that developments in the second edition suggest Kant’s temporal schemata ought to be supplemented with spatial schemata. Rather, I agree with Friedman’s recognition that “schematization in general necessarily involves what we now call *space-time*” (161n.) and that Kant had some sense of this notion.

⁵⁰ Another argument that transcendental schemata are essentially spatiotemporal arises in the following way: “All time-determination presupposes something *persistent* in perception” (B275), and that “in order to give something that *persists* in intuition ... we need an intuition *in space* (of matter), since space alone persistently determines, while time, however, and thus everything that is in inner sense, constantly flows” (B291). (As we will see later, space persistently determines because its parts are simultaneous.) Since the schema of the category *substance* is “the persistence of the real in time” (A144/B183), it follows that it is essentially spatiotemporal.

⁵¹ That a transcendental schema is constituted by contributions of the understanding and sensibility illuminates Kant’s characterization of the necessary agreement of experience with the categories as “a system of the *epigenesis* of pure reason” (B167), an allusion to the biological theory where an embryo is understood to be a new product constituted by germ cells of two parents (as noted in the translation by Guyer and Wood (1998), p. 727n.).

⁵² As noted in the translation by Guyer and Wood, p. 271.

⁵³ Paton (1936) is right that “[e]very object must exhibit all the transcendental schemata, and must fall, as regards to their different aspects, under all the categories. We must not be misled into supposing that Kant describes a whole series of synthesis which take place at different times” (44).

⁵⁴ Allison (2004, 215-216) simply equates a transcendental schema with a time-determination in the sense of a formal intuition of time. I depart from his view on two points. I reject his claim that a time-determination is a formal intuition of time, for Kant is only saying that a formal intuition of time arises from the *collective* effect of the categories. I also depart from his view that a time-determination is a product of a figurative synthesis; the product of a figurative synthesis is the *perception* of a time-determination.

⁵⁵ That the property is spatiotemporal is evident from the following passage: “Establishing that our concepts have reality always requires intuitions. If the concepts are empirical, the intuitions are called *examples*. If they are pure concepts of the understanding, the intuitions are called *schemata*” (Ak: ..., 351). If examples are spatiotemporal images, then schemata too must be spatiotemporal.

⁵⁶ I intend only to make out the plausibility of this interpretation. A future project would involve correlating the relation of this conception of transcendental schemata to the logical functions of the categories, on the one hand, and the principles of the understanding on the other.

⁵⁷ Although the property of continuity is evident in the “intensive” magnitudes of sensation, Kant is clear that the property is identical to that of the “extensive” magnitudes of space and time: “The property of

magnitudes on account of which no part of them is the smallest (no part is simple) is called their continuity. ... All appearances whatsoever are accordingly continuous magnitudes, either in their intuition, as extensive magnitudes, or in their mere perception (sensation and thus reality), as intensive ones” (A169-170/B211-212).

⁵⁸ For example, the property of unity of point and succession of points is evident in the construction the pure sensible concept *five* (cf. A140/B179). As to continuity, Kant does state that “mathematics also occupies itself ... with the continuity of extension as a quality of [spaces]” (A715/B743). Friedman, noting that the Euclidean conception of continuity is not distinct from denseness or infinity, outlines an iterable construction that captures Euclid’s conception of continuity:

... Euclid *generates* the necessary points [otherwise formally secured for constructions by an axiom of *continuity*] by a definite process of construction: the procedure of construction with a straight-edge and compass. We start with three basic operations: (i) drawing a line segment connecting any two given points (to avoid complete triviality we assume two distinct points to begin with), (ii) extending a line segment by any given line segment, (iii) drawing a circle with any given point as center and any given line segment as radius. We are then allowed to iterate operations (i), (ii), and (iii) any finite number of times. Euclid’s Postulates 1-3 give the rules for this iterative procedure, and the points in our “model” are just the points that can be so constructed. In particular, then, the infinity of this set of points is guaranteed by the infinite iterability of our process of construction. (61)

⁵⁹ Kant is explicit here: “we must order the determinations of inner sense as appearances in time in just the same way as we order those of outer sense in space” (B156).

⁶⁰ The link between causality and transitivity is well known. On this interpretation, transitivity is at the core of Kant’s theory of causality.

⁶¹ Not limited by monadic logic, the modern polyadic theory of order includes two additional axioms: $\Box b \Box a$ ($a < b$) (no endpoints) and $\Box a \Box b \Box c$ ($a < b \rightarrow (a < c < b)$) (denseness) (Friedman 1992, 62).

⁶² In general, I agree with Walsh (1957/1958) that realizing the categories means “giving them reference to features of concrete experience” (101).

⁶³ “Appearances, to the extent that as objects they are thought in accordance with the unity of the categories, are called *phaenomena*” (A248/249).

⁶⁴ I understand Kant’s assertion that “a transcendental time determination ... *rests* on a rule a priori” to be a claim that a time-determination is an inference from a spatiotemporal representation that presents a rule.

⁶⁵ This is implied from Kant’s statement that “pure intuition (with regard to it as representation, time, the form of inner intuition) grounds the totality of perception *a priori*” (A115).

⁶⁶ In the context of first passage “empirical intuition” does not mean intuition from constructing a concept by drawing an image, for the image that results is not “immediately represented as real in space and time”.

⁶⁷ “[T]hat which follows from the general conditions of the construction must hold generally of the object of the constructed concept” (A716/B744).

⁶⁸ There is no distinction in reasoning either to mathematical or dynamical principles (A724/B752). But why should derivation of mathematical principles necessitate an empirical synthesis, for properties of the transcendental schemata *number* and *continuity* are clearly evident in the constructions of pure sensible concepts? The answer is that schematic properties have a different character in an empirical synthesis (e.g., continuity of a line vs. continuity of sensation), and only in this context do they pertain to experience *in general* (rather than to merely figurative experience).

⁶⁹ Hence synthetic a priori principles are inferred from spatiotemporal properties. In this case “in order ... to make even inner alterations thinkable, we must be able to grasp time, as the form of inner sense, figuratively through a line, and grasp the inner alteration through the drawing of this line (motion)” (B292).

⁷⁰ Here Kant appears to contradict Longuenesse (1995) who argues that reflective judgment is inseparable from determinative judgement.

⁷¹ “All of our cognitions, however, lie in the entirety of all possible experience, and transcendental truth, which precedes all empirical truth and makes it possible, consists in the general relation to this” (A146/B185).

⁷² “For from the concept signifying a genus it can no more be seen how far its division will go than it can be seen from space how far division will go in the matter that fills it” (A655/B683); “there are no species or subspecies that are proximate (in the concept of reason), but intervening species are always possible, whose

difference from the first and the second species is smaller than their difference from each other” (A659/B687).

⁷³ In characterizing this space of classification as organized under the regulative idea *God* I have drawn inspiration from Wood’s (1978) identification of an “ontological space” (33, 50-55) in the *Critique* organizing individual things under the same idea. However, there are at least three differences between the two spaces. As we shall see, Kant’s space of classification is a *potential* infinity, whereas Wood’s ontological space is conceived (like the form of space) to be infinite. Because of this Wood is forced to view his ontological space as a “metaphor” (33 n. 16). On the other hand, I take Kant’s potential space to be a product of the non-spatiotemporal imagination under the guidance of reason. Significantly, Wood’s ontological space is structured only with predicates of sensation, whereas Kant’s space of classification includes intuitive predicates.

⁷⁴ The idea *God* “is only a schema, ordered in accordance with the conditions of the greatest unity of reason, for the concept of a thing in general, which serves only to preserve the greatest systematic unity in the empirical use of our reason, in that one derives the object of experience, as it were, from the imagined object of this idea as its ground or cause” (A670/B698).

⁷⁵ Kant is clear that although closely related to the original image, “the object of reason’s ideal”, *ens realissimum*, is the *focus imaginarius* of the space of reason: “[i]t is self-evident that with this aim – namely, solely that of representing the necessary thoroughgoing determination of things – reason does not presuppose the existence of a being conforming to the ideal, but only the idea of such a being, in order to derive from an unconditioned totality of thoroughgoing determination the conditioned totality, i.e., that of the limited” (A577-578/B605-606).

⁷⁶ “Every *concept*, in regard to what is not contained in it, is indeterminate, and stands under the principle of *determinability*: that of *every two* contradictorily opposed predicates only one can apply to it, which rests on the principle of contradiction and hence is a merely logical principle, which abstracts from every content of cognition, and has in view nothing but the logical form of cognition” (A571/B599).

⁷⁷ “For reason the ideal is ... the original image (*protoypion*) of all things, which all together, as defective copies (*ectypa*), take from it the matter for their possibility, and yet although they approach more or less nearly to it, they always fall infinitely short of reaching it” (A578/B606).

⁷⁸ “[T]he logical principle asserting indeterminacy of the logical sphere in regard to possible division would give no occasion” to “demand an actual *infinity* in regard to the varieties of things that can become our objects” (A656/B684).

⁷⁹ “[S]pecies in nature are really partitioned and therefore in themselves have to constitute *quantum discretum*, and if the graduated progress in their affinity were continuous, they would also have to contain a true infinity of intermediate members between any two given species, which is impossible” (A661/B689).

⁸⁰ The following passage is found in the Appendix:

If among the appearances offering themselves to us there were such a great variety – *I will not say of form (for they might be similar to one another in that)* [emphasis added] but of content, i.e., regarding the manifoldness of existing beings – then even the most acute human understanding, through comparison of one with another, could not detect the least similarity (a case which can at least be thought), then the logical law of genera would not obtain at all, no concept of a genus, nor any other universal concept, indeed no understanding at all would obtain, since it is the understanding that has to do with such concepts. The logical principle of genera therefore presupposes a transcendental one if it is to be applied to nature (by which I here understand only objects that are given to us). According to that principle, sameness of kind is necessarily presupposed in the manifold of a possible experience (even though we cannot determine its degree *a priori*), because without it no empirical concepts and hence no experience would be possible. (A653-654/B681/682)

The first sentence tells us that a transcendental presupposition is *unnecessary* if the principle of homogeneity is taken to concern only the *form* of appearances. The implication is that only because the principle of homogeneity extends to the content, i.e., the *matter*, of appearances that reason is free to admit (“can at least be thought”) that appearances are not divided into (non-spatiotemporal) kinds.

⁸¹ The illustration on the left is taken from Clayton, D. (2010), “The Way of Beauty”, Thomas Moore College of Liberal Arts (<http://thewayofbeauty.org/2010/10/is-there-a-place-for-celtic-art-today/>) and that on the right is from Bourke, P. (2002) “Conic Sections” (<http://paulbourke.net/geometry/conic/>).

⁸² A moment of gravity is weight, and weight is an empirical concept (A173/B215).

⁸³ In the third *Critique* Kant is concerned to explain the discovery and purposive systematization of particular empirical laws through reflective judgment that finds a universal for a given particular. Although empirical laws prior to investigation “are nonetheless contingent as far as we can see (i.e. we cannot cognize them a priori)” (183), no where does Kant explicitly deny that in its discovery we might come to know an a priori basis for a law in a pure sensible concept. This is not to say, of course, that *all* empirical laws will have an a priori basis in pure sensible concepts.

⁸⁴ This is consistent with Kant’s account of the relation between empirical laws and principles of the pure understanding:

Although we learn many laws through experience, these are only particular determinations of yet higher laws, the highest of which (under which all others stand) come out of the understanding itself *a priori*, and are not borrowed from experience, but rather must provide the appearances with their lawfulness and by that very means make experience possible. (A126)

The pure faculty of understanding does not suffice, however, to prescribe to the appearances through mere categories *a priori* laws beyond those on which rests a *nature in general*, as lawfulness of appearances in space and time. Particular laws, because they concern empirically determined appearances, *cannot be completely derived* from the categories, although they stand under them. Experience must be added in order to come to know particular laws *at all*; but about experience in general, and about what can be cognized as an object of experience, only those *a priori* laws offer instruction. (B165)

⁸⁵ Friedman (1996) points this out at pp. 191-192, noting that this is “a purely mathematical result concerning the kinematics of relative motion and does not depend, in particular, on the Newtonian laws of motion”.

⁸⁶ This point is made by Friedman (1992, 177).

⁸⁷ It is called a “moment” because “the degree [of reality] designates only that magnitude of the apprehension of which is not successive but instantaneous” (A168-169/B210).

⁸⁸ Friedman (1996) notes that Kant is referring to Proposition 35 of Book III of Euclid: “if two straight lines intersect one another within a circle at point E, and meet the circle at A, C, and B, D respectively, then $AE \times EC = BE \times ED$ ” (186).

⁸⁹ Contra. Friedman (1992, Ch. 5).

⁹⁰ Hence, Kant tells us that rather than a nature a circle has an “essence” or “the first inner principle of all that belongs to the possibility of a thing” (Ak. IV: 467n).

⁹¹ Friedman (1996) notes, “This property of conic sections is the natural generalization of Propositions 35-36 of Book III of Euclid ...: suppose that the members of two given pairs of straight lines intersect one another at E, E’ respectively and meet the conic at A, C, and B, D and at A’C’ and B’, D’ respectively; then if the lines AC, A’C’ and BD, B’D’ are respectively parallel to one another, $(AE \times EC)/(BE \times ED) = (A’E’ \times E’C’)/(B’E’ \times E’D’)$ ” (191).

⁹² “Moving forward and moving backward in the number series are not equivalent to each other; on the contrary, like the sequence of perceptions in time, they are actually quite different procedures” (Helmholtz 1887/1971, 441).

⁹³ “As all our means of sense perception extend only to space of three dimensions, and a fourth is not merely a modification of what we have but something perfectly new, we find ourselves by reason of our bodily organization unable to represent a fourth dimension” (Helmholtz 1870/1971, 262).

⁹⁴ “In earlier essays I have tried to establish that the axioms of geometry are not propositions given a priori, but propositions that must be confirmed or refuted by experience. I emphasize here once more that I do not reject Kant’s conception of space as a transcendental form of intuition. ... Since the empirical theory, which I have advocated, interprets axioms of geometry not only as not undemonstrable but indeed as requiring justification, it must clearly take the same position concerning the origin of the axioms of arithmetic, which are related to the form of intuition of time in the same way that the axioms of geometry are related to the form of intuition of space” (Helmholtz 1887/1971, 437).

⁹⁵ I am following Carrier’s (1994) interpretation of Helmholtz. For an alternative and interesting claim that Helmholtz regards free mobility as part of his form of intuition, see Friedman (1997).

⁹⁶ Because they are effects, Leroux (2001, 192) suggests Helmholtz’s sensations are really “signals”.

⁹⁷ My use of the term “graven image” comes from van Fraassen (2008, 104).

⁹⁸ “What we unquestionably can find as a fact, without any hypothetical element whatsoever, is the lawful regularity of phenomena. From the very first, in the case where we perceive stationary objects distributed

before us in space, this perception involves the recognition of a uniform or lawlike connection between our movements and the sensations which result from them. Thus even the most elementary ideas contain a mental element and occur in accordance with the laws of thought. Everything that is added in intuition to the raw materials of sensation may be considered mental” (Helmholtz 1878/1971, 386).

⁹⁹ “It would obviously be false, however, to try to maintain that we have no knowledge other than that which is developed from sense perception by logical or conceptual thought (Helmholtz 1894/1971b, 506).

¹⁰⁰ We now know from Newman’s (1928) objection to Russell that this claim of adequacy is trivially satisfied: because the number of signs is taken to be equal to the number of causes, it is just a point of logic that the structure of causes is the same as the structure of signs.

¹⁰¹ In accord with Friedman’s (1997, 21) translation of *Abbildung* in the passage, I have replaced Kahl’s “we can discover” with “picturing”. This brings it closer to Cahan’s translation of “imaging” found in Helmholtz (1995, 348).

¹⁰² Contra Hatfield (1990, 215-216), I fail to see the “tension” in Helmholtz’s view for failing “truly to attribute objective validity to the causal law”. Hatfield assumes that a transcendental law must be established by transcendental deduction in order to have objective validity. But in the *Critique* Kant tells us regulative principles “also have objective validity in regard to this object [i.e., the object of experience], yet not so as to *determine* something in it, but only to indicate the procedure in accordance with which the empirical and determinate use of the understanding in experience can be brought into thoroughgoing agreement with itself, by bringing it *as far as possible* into connection with the principle of thoroughgoing unity; and from that it is derived” (A666/B694).

¹⁰³ E.g., Schiemann (1998, 27).

¹⁰⁴ This appeal to Mill should not engender confusion. Even though Mill opposed Kant, Helmholtz and Hertz synthesize elements of empiricism with elements of transcendental idealism. Kantian overtones are also present in van Fraassen who says, “I have much sympathy for the transcendentalist tradition and will admit to flirting with it” (2010a, 463). He flirts with it through the idea of logical space.

¹⁰⁵ We must not forget that Helmholtz still lives in the world of logic prior to Frege and Russell: “Logic, or the study of scientific thought, after it had been developed by Aristotle, was handed down through scholastic philosophers of the Middle Ages. For the most part it has remained unchanged down to our own time” (Helmholtz 1894/1971a, 516).

¹⁰⁶ With this account of concept formation Helmholtz in hand, was in a position to challenge Kant’s view that it was impossible to conceive spatial relations that contradict the axioms of Euclidean geometry on the basis of what is possible to imagine. Recognizing “‘to conceive’ means ‘to form concepts’” (Helmholtz 1878/1971, 386) from pre-conceptual images, Helmholtz writes:

In his assertion that it is impossible to conceive spatial relations which contradict the axioms of geometry, as well as in his general interpretation of intuition as simple, irreducible mental process, Kant was influenced by the mathematics and the physiology of the senses at this time.

In order to try to conceive something which has never been seen before, it is necessary to know how to imagine in detail the series of sense impressions which, in accordance with well-known laws, would be experienced if the thing in question – and any changes in it – were actually perceived by any of the sense organs from all possible positions. Further, these impressions must be such that all possible interpretations of them except one can be eliminated. If these series of sense impressions can be specified completely and uniquely in this way, then in my opinion one must admit that the object is clearly conceivable. (Ibid., 378-379)

¹⁰⁷ “We must make use of the law [of causality] in order to arrive at the ideas of force and cause in the first place” (Helmholtz 1894/1971a, 526).

¹⁰⁸ Boltzmann (1905/1960, 245, 247, 251-252).

¹⁰⁹ Unless otherwise indicated, all references in this section are to the *Principles* (1900/1956).

¹¹⁰ Hyder’s (2003) attempt to link the *Principles* to Kant’s schematism is a source of inspiration for the first two chapters of this thesis. However, I found myself disagreeing with Hyder’s conception of a schema as an “operational rule” and began to think that his attempt to link the *Principles* directly with Kant’s *Metaphysical Foundations of Natural Science* (2004a) was not all that promising (cf. Lutzen 2005, ch.10). Christiansen (2006) builds upon Hyder’s suggestion of connecting Hertz to Kant’s schematism. As this chapter shows, the connection to Kant is not through schemata of the understanding, but through regulative schemata.

¹¹¹ Hertz's qualification has led to charges of inconsistency (e.g., Schiemann (1998, 12)). However, Hertz probably had Helmholtz in mind here (cf. Helmholtz 1870/1971, 263). As I see it, Hertz's point is that the issue of whether axioms of Euclidean geometry are intuitively synthetic or conceptually analytic a priori propositions is moot for *his* purposes, which is to evaluate competing scientific representations. Scientific representations arise from fundamental ideas, and fundamental ideas arise from laws of transformation in relation to internal intuitions *or* a priori definitions.

¹¹² "We determine the duration of time by means of a chronometer, from the number of beats of its pendulum. The unit of duration is settled by arbitrary convention" (§298); "We determine space-relations according to methods of practical geometry by means of a scale. The unit of length is settled by arbitrary convention. A given point in space is specified by its relative position with regard to a system of coordinates fixed with reference to the fixed stars and determined by convention" (§299). The conventional aspect of these rules is emphasized in §304.

¹¹³ Commentators often overlook the generality of Hertz's claim, i.e., that images of objects can arise from passive observation (e.g., Heidelberger 1998, 21). A notable exception is Schiemann (1998, 31).

¹¹⁴ E.g., Heidelberger (1998, 21) and Lutzen (2005, 86).

¹¹⁵ E.g., Lutzen (2005, 85-86) and Christiansen (2006, 5).

¹¹⁶ This shift in logic is noted by Schiemann (1998, 30).

¹¹⁷ Hertz's reference to "images or symbols" is a source of interpretive issues. For example, Graßhoff (1997, 101) takes this as a cue to interpret Hertz's images along the lines of Wittgenstein as containing symbols representing objects.

¹¹⁸ Given the import of this point, it is worth quoting the relevant section in full:

The three foregoing rules are not new definitions of the quantities time, space, and mass, which have been completely defined previously. They represent the laws of transformation by means of which we translate external experience, *i.e.*, concrete sensations and perceptions, into the symbolic language of the images of them which we form (*vide* Introduction), and by which conversely the necessary consequents of this image are again referred to the domain of possible sensible perceptions. Thus, only through these three rules can the symbols time, space, and mass, become parts of our images of external objects. Again, only by these three rules are they subjected to further demands than are necessitated by our thought. (§302)

¹¹⁹ E.g., Schiemann (1998, 33) and Lutzen (2005, 87).

¹²⁰ A notable exception is Lutzen (1994) and (2005), although he fails to distinguish between a model and an image.

¹²¹ Although the systems considered in the Principles consist of finitely many material points, no upper limit was assigned to their number and no lower limit to their mass (§7). Presumably this was in anticipation of the book's extension to the ether, which Hertz conceived as a continuum (Lutzen 2005-77).

¹²² Somewhat misleadingly, Lutzen (2005, 83) identifies a distinction between a "local" and a "global" image.

¹²³ "Our representation of mechanics bears towards the customary one somewhat the same relation that the systematic grammar of a language bears to a grammar devised for the purpose of enabling learners to become acquainted as quickly as possible with what they will require in daily life. The requirements of the two are very different, and they must differ widely in their arrangement if each is to be properly adapted to its purpose" (40). It does not "prevent us from understanding that mechanics could and must have been developed in the manner in which it actually has developed" (§735).

¹²⁴ The philosophical activity of reconstructing a scientific representation is characteristic of "mature knowledge":

Mature knowledge regards logical clearness as of prime importance; only logically clear images does it test as to correctness; only correct images does it compare as to appropriateness. By pressure of circumstances the process is often reversed. Images are found to be suitable for a certain purpose; are next tested as to their correctness; and only in the last place purged of implied contradictions. (10)

¹²⁵ As Schiemann (1998) puts it, "[t]he certainty that those sequences of thought at a remove from the world can be in contact with nature at all may be called the Platonic element of Hertz's conception of science" (32).

¹²⁶ Boltzmann (1905/1960) suggests that a less sophisticated version of Hertz's idea is found in the Euclidean method where "certain pictures were deduced from ... axioms only with the help of the laws of thought" (248).

¹²⁷ "[W]ithout ambiguity we can decide whether an image is correct or not; but only according to the state of our present experience, and permitting an appeal to later and riper experience" (3).

¹²⁸ See section 2.35 below.

¹²⁹ Lutzen (2005, ch. 12) argues that Hertz's definition of mass was motivated by his intention to deduce a multi-dimensional geometry of material points from Euclidean geometry.

¹³⁰ This point is made by Graßhoff (1997, 105).

¹³¹ Although Hertz worked mostly with material points, the distinction between material points and material particles appears to have been introduced out of considerations of fluid and continuum mechanics (Lutzen 2005, 140).

¹³² "The mass of bodies that we can handle is determined by weighing. The unity of mass is the mass of some body settled by arbitrary convention" (§300).

¹³³ This is implicit in §306 and explicit in §427 where Hertz refers to natural material systems in terms of masses that "exist in nature".

¹³⁴ "The law condenses into one single statement the usual law of inertia and Gauss's Principle of Least Constraint" (27-28). Lutzen (2005) points out: "1. Newton's first law of inertia [states] that in a system consisting of free points (with no connections) the points will move uniformly in straight lines ..., and 2. Gauss's principle of least constraint [states] that the natural motion of a connected system will minimize the *constraint* among all motions that have the same position and velocity" (199). Given Hertz's notion of "uniform motion" and "straightest path" one can reformulate the law as follows: "A free system moves with a constant speed along a path that is a straight as it can be without breaking the connections of the system" (198).

¹³⁵ In the second last paragraph to his introduction to the Hertz's *Principles*, Helmholtz claims to depart from Hertz on just this point:

English physicists – e.g. Lord Kelvin, in his theory of vortex-atoms, and Maxwell, in his hypothesis of systems of cells with rotating contents, on which he basis his attempt at a mechanical explanation of electromagnetic processes – have evidently derived a fuller satisfaction from such explanations than from the simple representation of physical facts and laws in the most general form, as given in systems of differential equations. For my own part, I must admit that I have adhered to the latter mode of representation and have felt safer in so doing; yet I have no essential objections to raise against a method which has been adopted by three physicists of such eminence.

¹³⁶ According to Nolte (2010), development of phase space has been traced to papers written by Liouville in 1838, Jacobi in 1842, and Boltzmann in 1871, but its mature form makes its debut in a reprint of a paper published by Poincare in 1889 and appears only to take hold in published articles by 1913. Even though Boltzmann never held the full concept of phase space before his death in 1906, a similar analogy between the mathematics of mechanical systems and geometry can be found in his *Lectures* of 1896 (1905/1960) where he acknowledges the influence of the *Principles*.

¹³⁷ In order for the model to make predictions, the equations of condition must be derived from given equations of condition of the system that capture the fixed connections of bodies and the equations expressing the fundamental law:

When equations result from the given equations of condition of a system and the fundamental law, which have strictly the form of equations of condition, then for the determination of the motion of the system it is indifferent whether we consider the original equations alone, or instead of them the derived equations, as a representation of the connections of the system. (§327)

¹³⁸ Myrvold (1990, 42).

¹³⁹ It is an idealization in that the "fixed connections of bodies" we imagine "are represented mathematically by equations of conditions" (11) as part of the "analytical representation" (§124, §131).

¹⁴⁰ "425. Corollary 1. In order to determine beforehand the course of the natural motion of a material system, it is sufficient to have a model of that system. The model may be much simpler than the system whose motion it represents."

¹⁴¹ Boltzmann (1905/1960) correctly understood Hertz to be saying that equations of a theory are interpreted through a thought schema, i.e., a basic and general "thought-picture"(250) from which specific thought

pictures are constructed, but offered his own logico-mathematical schema as part of a description of the scientific method:

In my book entitled *Lectures on the Principles of Mechanics*, I have also attempted a purely deductive representation of the fundamental principles of mechanics, but in quite a different way and much more closely tied to the usual treatment of mechanics. Like Hertz, I begin with pure thought-objects: exact material points. I relate their position to a rectangular coordinate system that is also thought and I imagine a mental picture of the motion which, at first is constructed in the following way: Every time that two material points are at a distance, r , from each other, each of them is to experience an acceleration in the direction of r , which is a function $f(r)$ of this distance. Later, this function can be disposed of at will. Furthermore, the accelerations of both points are to have a numerical relationship that remains unchanged at all times and which defines the relation of the masses of the two material points. How we are to imagine the motion of all material points is then unambiguously determined by the indication that the actual acceleration of each point is the vector sum of all accelerations found for it by means of the previous rule. This sum is then also added, as vector quantities are added, to the velocity of the point which is already given. Where these accelerations come from and just why I give the instruction that the picture should be constructed in this way is not further discussed. It suffices that the picture is a perfectly clear one which, by means of calculations, can be worked out in detail for a sufficient number of cases. It finds its justification only in the fact that the function $f(r)$ can in all cases be determined, such that the thought motion of the imagined material point becomes a faithful copy of actual appearances.

By means of this mode of treatment which we have called the purely deductive one, we have of course not solved the question of the nature of matter, mass, and force. However, we have avoided these questions by making their initial posing completely superfluous. In our thought schema these concepts are fully determinate numbers and directions for geometric constructions. We know how we are to think and execute them, so that we may obtain a useful picture of the world of appearance. (251-252)

With the exception of its relevance to the development of the modern notion of phase space and its approach to physical analysis discussed in Chapter 3, I have not thought it necessary to elaborate upon Boltzmann's system in this dissertation. Boltzmann thought philosophy was metaphysics and saw his *Bildtheorie* as descriptive of the methodology of science (Blackmore 1995, 69). What he took from the *Principles* was a presentation of the method of "purely deductive representation". On this method a conventional logical and mathematical thought schema encounters experience in a test for its predictive accuracy; it is not constitutive of our images of external objects. Indeed, Boltzmann explicitly rejects Hertz's notion of a priori laws of thought (i.e., laws of internal intuition and logical forms), taking them to be subject to development and modification through experience (1905/1960, 246).

¹⁴² Unless otherwise indicated all references to the *Tractatus* are to this translation by Pears and McGuinness. But for page numbers of the preface, I will follow the standard practice of citing proposition numbers.

¹⁴³ Wittgenstein later wrote "I believe that I have never invented a new line of thought" (McGuinness (1988, 84).

¹⁴⁴ Weiner (1992) suggests (perhaps uncharitably) that Wittgenstein suffered from "the anxiety of influence" and "projected the misleading image of a supergenius whose philosophy emerged *ex nihilo*" (15).

¹⁴⁵ E.g., Coffa (1991, ch. 8).

¹⁴⁶ In *Notes of Logic* (1961a) Wittgenstein wrote:

In philosophy there are no deductions; it is purely descriptive. The word 'philosophy' ought always to designate something over or under, but not beside, the natural sciences. Philosophy gives no pictures of reality, and can neither confirm nor confute scientific investigation. It consists of logic and metaphysics, the former its basis. (93)

¹⁴⁷ The first commentator to do so appears to have been Griffin (1964), and Janik and Toulmin (1993) created the current surge of interest. There is an obvious connection in their critical projects that is frequently commented upon: "When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions" (Hertz 1900/1956, 8); "If a question can be framed at all, it is also *possible* to answer it" (Wittgenstein 1961b, 6.5).

¹⁴⁸ As translated in Wittgenstein (1921/1961, 159).

¹⁴⁹ This was said by Wittgenstein to a prospective publisher as quoted in Weiner (1992, 109).

¹⁵⁰ “The structure of a fact consists of the structures of states of affairs” (2.04). This is also explained by Wittgenstein in a letter to Russell (1914-1916/1961, 129).

¹⁵¹ Wittgenstein made this clear in a letter to Ogden commenting on his translation of the *Tractatus*:

2.03: Here instead of ‘hang on one another’ it should be ‘hang one in another’ as the links of a chain *do!* The meaning is *that there isn’t anything third* that connects the links but that the links *themselves* make connexions with one another. ((1973, 23), as quoted by Graßhoff (1997, 114))

¹⁵² Wittgenstein (1964, 72) as quoted in Coffa (1991, 393, n.8). This is also suggested in the *Tractatus*: “In logic nothing is accidental It would seem to be a sort of accident, if it turned out that a situation would fit a thing that could already exist entirely on its own” (2.012f).

¹⁵³ Unlike Helmholtz and Hertz, Wittgenstein has no general theory of representation. As we shall see, this similarity of form is accounted for by the prior the coordination of the self with reality.

¹⁵⁴ “A proposition, a picture, or a model is, in a negative sense, like a solid body that restricts the freedom of movement of others, and, in a positive sense, like a space bounded by solid substance in which there is room for a body” (4.463).

¹⁵⁵ Bell and Demopoulos (1996) characterize Wittgenstein’s elementary propositions meta-mathematically through the notion of a free generator.

¹⁵⁶ This was conveyed to Waismann (1979, 75-80) in a meeting January 2, 1930.

¹⁵⁷ Coffa puts it this way: “The requirement that a symbolic system and a corresponding reality have the same multiplicity demands that the symbolic system and its objective correlate have exactly the same numbers of elements and that these are capable of exactly the same the same structural arrangements” (Coffa 1991, 156). This is not quite right, since it is not the number of elements *per se*, but the number of different *kinds* of elements that is pertinent to what is “distinguishable”. Elements that do not differ in their internal properties are not “distinguishable parts”.

¹⁵⁸ I am not the first to point out that pictorial elements are coordinate values in logical space. It is made by Griffin (1964, 103-104) and implied by Waismann (1979) in his “Thesis” (that Wittgenstein apparently read):

“Let us imagine a white sheet of paper covered with a network of lines. I can describe every mesh of the network by specifying two point-numbers. The elements in a state of affairs correspond to the point-numbers and the states of affairs themselves to the meshes of the network. If a state of affairs then exists in reality, we imagine the corresponding mesh filled in black. The distribution of black patches on the white sheet of paper then is a picture of reality in logical space. (261)

¹⁵⁹ (Wittgenstein 1964, 72) as noted by Coffa (1991, 393, n.8).

¹⁶⁰ I will discuss Wittgenstein’s notion of self below.

¹⁶¹ Wittgenstein studied engineering only because his plan to study with Boltzmann in Vienna was thwarted by Boltzmann’s suicide in 1906 (Glock 1996, 13). It is likely he was led to Hertz through Boltzmann’s own writings which show influence by the *Principles* (see Boltzmann 1905/1960).

¹⁶² “If I say ‘I have got stomach-ache,’ then this presupposes the possibility of a state of stomach-ache. My present state and the state of stomach-ache are in the same logical space as it were” (December 25, 1929, as recorded in Waismann (1979, 67)).

¹⁶³ As discussed in a footnote to the conclusion of last chapter, Boltzmann (1905/1960) was concerned with the “deductive method of representation” in science: from a thought schema pictures are constructed to test the schema against appearances. On a sufficiently broad construal of “appearances” it is inconsequential to this approach whether appearances arise through instrumentation. As we will see in chapter 4, after adopting Wittgenstein’s idea of logical space van Fraassen only develops a theory of measurement involving instrumentation some 40 years later.

¹⁶⁴ One might also cite “[t]he propositions of logic describe the scaffolding of the world, or rather they represent it” (6.124).

¹⁶⁵ Friedman (1992, 85, n.51) makes a similar point.

¹⁶⁶ Glock (1996, 270) also makes this claim.

¹⁶⁷ Consider the following proposition:

6.3751 ... the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour.

If elementary pictures are the independent yardsticks by which reality is measured, there is nothing from the logical structure of colour to preclude the possibility that two elementary colours can arise at the same place at the same time. By 1930 Wittgenstein was forced to concede that the yardstick was not the picture but its logical space:

It is not the individual graduating lines that are laid against it, but the entire scale. If I know that the object extends to graduating line 10, I also know immediately that it does not extend to graduating lines 11, 12, and so forth. The statements describing for me the length of an object form a system, a system of propositions. Now it is such an entire system of propositions that is compared with reality, not a single proposition. If I say, for example, that this or that point in the visual field is blue, then I know not merely that, but also that this point is not green, nor red, nor yellow, etc. I have laid the entire colour-scale against it at one go. This is also the reason why a point cannot have different colours at the same time. For when I lay a system of propositions against reality, this means that in each case there is only one state of affairs that can exist, not several – just as in the spatial case. All this I did not yet know when I was writing my work: at that time I thought that all inference was based on tautological form. At that time I had not yet seen that an inference can also have the form: This man is 2m tall, therefore he is not 3m tall. This is connected with the fact that I believed that elementary propositions must be independent of one another, that you could not infer the non-existence of one state of affairs from the existence of another. But if my present conception of a system of propositions is correct, it will actually be the rule that from the existence of one state of affairs the non-existence of all the other states of affairs described by this system of propositions can be inferred. (Recorded December 30, 1929 by Waismann, 63)

¹⁶⁸ Wittgenstein says later that any question about the number of objects “is without sense” (December 22, 1929, as recorded in Waismann (1979, 43).

¹⁶⁹ “A proposition reaches through the whole of logical space. Otherwise negation would be unintelligible” (January 5, 1930, as recorded in Waismann (1979, 91).

¹⁷⁰ (December 22, 1929, as recorded by Waismann (1979, 43))

¹⁷¹ This has been pointed out by Weiner (1992, 42-43).

¹⁷² However, Weiner may be accused at various times of going too far and overstating Schopenhauer’s influence.

¹⁷³ Corresponding to four forms of the principle are four different types of necessity: logical, mathematical, physical and moral (Schopenhauer 1974, 226-227).

¹⁷⁴ E.g., pp. 79-80.

¹⁷⁵ Wiener (1992) makes a persuasive case at pp. 68-72.

¹⁷⁶ Without rule-governed reproduction there are no images for Kant, only “unruly heaps” of representations (1781-1787/1998, A121).

¹⁷⁷ “Tautologies and contradictions lack sense” in that “[a] tautology has no truth-conditions, since it is unconditionally true: and a contradiction is true on no condition” (4.461).

¹⁷⁸ Here I depart from both the translations of Pears and McGuinness (“of that language which alone I understand”) and Ogden (Wittgenstein 1922) (“the language which I understand”) and follow Wittgenstein’s own correction to a copy of the first edition of the *Tractatus* (as quoted in Weiner (1992, 65).

¹⁷⁹ “6.374 Even if all that we wish for were to happen, still this would only be a favour granted by fate, so to speak: for there is no *logical* connexion between the will and the world, which would guarantee it, and the supposed physical connexion itself is surely not something that we could will.”

¹⁸⁰ “5.631 ... If I wrote a book called *The World as I found it*, I should have to include a report on my body, and should have to say which parts were subordinate to my will, and which were not, etc. this being a method of isolating the subject, or rather of showing that in an important sense there is no subject; for it alone could *not* be mentioned in that book.”

¹⁸¹ It is interesting to note that in the *Notebooks* Wittgenstein writes:

This is the way I have travelled: Idealism singles men out from the world as unique, solipsism singles me alone out, and at last I see that I too belong with the rest of the world, and so on the one side *nothing* is left over, and on the other side, as unique, *the world*. In this way idealism leads to realism if it is strictly thought out. (1961a, 15.10.16; 85)

Yet the *Tractatus* does not consider idealism as an option:

4.0412 For the same reason the idealist's appeal to 'spatial spectacles' is inadequate to explain the seeing of spatial relations, because it cannot explain the multiplicity of these relations.

By contrast, both solipsism and pure realism explain multiplicity as a shared limit between the metaphysical self and the world prior to experience.

¹⁸² Wittgenstein's idea of the eternal present is likely taken from Schopenhauer (Weiner 1992, ch.3).

¹⁸³ "That is why what happens, whether it comes from a stone or from my body is neither good nor bad" (Wittgenstein 1961a, 12.10.16; 84).

¹⁸⁴ Again, the influence is probably Schopenhauer for whom aesthetic consciousness "stops the wheel of time" (Schopenhauer 1966, vol I. 185).

¹⁸⁵ E.g., Glock (1996, 365).

¹⁸⁶ In the *Prototractatus* Wittgenstein wrote "My work consists of two parts: the one represented here plus all that I have *not* written. And it is precisely this second part that is the important one" ((1971, 15) as quoted in Coffa (1991, 142)).

¹⁸⁷ Van Fraassen distinguishes science from myth by its primary commitment to a method of thinking "with its ideal of constant reevaluation and self-critique" (1994b, 132) and distinguishes logical spaces of science from those of myth in that they develop from the introduction of new measurement procedures (2008, 165-166).

¹⁸⁸ Elgin (2010) puts van Fraassen's conception this way: "A logical space is a multidimensional array of possibilities open to the items that occupy the space. To represent an item in a logical space is to represent it as having a particular position in the array of possibilities the space marks out" (443).

¹⁸⁹ For use of the term "model" in relation to time and space see van Fraassen (1970a, 191-193); in relation to space-time and colour space see his (1985a, 276).

¹⁹⁰ A clear sense of this use of "model" is found in van Fraassen's (2008, 310).

¹⁹¹ This account of the phenomenology of experience obviously carries through van Fraassen's later (2002a) rejection of the factual thesis associated with traditional empiricism that experience is the sole source of information.

¹⁹² Van Fraassen introduces the idea of logical space to explicate the temporal order of events as we conceive them, or "time", but acknowledges (2010a, 463) that the basic structure of this argument is general, and does not depend on any special feature of the philosophical problems of time or space. I have taken the liberty of generalizing his initial approach.

¹⁹³ More specifically, "the extant sciences provide a kind of 'given' for philosophy: for a philosopher, the conceptual framework of the science of his day provides a subject more appropriate for analysis than for criticism. Other philosophical systems, of course, are fair game" (1970a, 52).

¹⁹⁴ "Phenomenal reality need not be fragmentary in itself, but its chaotic nature vis-à-vis human understanding forces us to treat it, conceive of it, as fragmentary" (van Fraassen 1985a, 276).

¹⁹⁵ Van Fraassen acknowledges his formulation of scientific realism is different from other formulations, such as that of Psillos (2010b, 547-548).

¹⁹⁶ In (Monton and van Fraassen 2003), van Fraassen acknowledges that although agnosticism is a "natural epistemic attitude" to have (420), "as far as logical consistency goes" (408) a particular constructive empiricist need not be agnostic about unobservable entities or about real non-actual possibility. If agnosticism is adopted, a person "would never say that 'we do not know about electrons merely because they are unobservable'" (420). Rather, "she would say she is agnostic about the very existence of electrons" (Ibid.). A doctrine of aim only imputes belief about existence in theory acceptance in order to make sense of the intentional activity of scientists – it says nothing about whether such belief is warranted or justified. Constructive empiricism tells us scientists act as if accepted theories inform them that the empirical world is uniform. Van Fraassen (1989) develops a liberal epistemology where it is rational to believe whatever one is not compelled to disbelieve, which allows him to say that belief in theory acceptance that goes beyond the strict deliverances of experience is rational.

¹⁹⁷ As an answer to "What is Science", constructive empiricism is a view that is "somewhat speculative in the interpretation it puts on what happens" that tries to describe "what is really going on" (1991a, 1-2). It is said to be slightly different than Cartwright's "more phenomenological description of scientific activity" (1991, 13). My claim that it is a *reconstruction* of scientific activity captures this difference in a way consistent with van Fraassen's claim that acceptance as successfully achieving the aim of scientific activity is in the *ideal*: it is "unqualified acceptance; in practice, acceptance will come with restrictions and qualifications, and belief will come in degrees" (2008, 345, fn.3).

¹⁹⁸ See van Fraassen (1994a, 191).

¹⁹⁹ Giere recognizes the point: “Van Fraassen’s arguments for [constructive empiricism] are acknowledged to be independent of his commitment to the semantic view of scientific theories” (2000, 521).

²⁰⁰ A theory is true if “there is an exact correspondence between reality and one of its models”; a “theory is empirically adequate if it has some model such that all appearances are isomorphic to empirical substructures of that model” (van Fraassen 1980, 197, 64). In his (2008) van Fraassen clarifies that “appearances” in this context means “data models” (391, n.24).

²⁰¹ “Given that the aim of science is to provide empirically adequate theories about what the world is like, we should conclude that wherever the representation does trade on likeness, the general rule of selectivity targets the observable phenomena. A model often contains much that does not correspond to any observable feature in the domain. Then, from an empiricist point of view, the model’s structure must be taken to reveal structure in the observable phenomena, while the rest of the model must be serving that purpose indirectly” (van Fraassen 2008, 87).

²⁰² Although at this point he occasionally uses the term “real structure” ((1970a, 104); (1985b, 205)), later van Fraassen prefers the less problematic notion of “concrete structure” (2010a).

²⁰³ Van Fraassen (1970a, 35-44) provides the following historical summary. Having perceived that the analysis of temporal order underlies that of temporal magnitude, Leibniz proposed that “[t]ime is the order of non-contemporaneous things”. This order was said to be constituted by qualitatively incompatible events related to each other as an “antecedent” that “contains the ground for” a “consequent”. Basic relations among states of affairs were those of *contrariety*, *qualification* and *causality*, which defined the basic temporal relation of *before*, and which in turn could be used to define the relations of *temporally between* and *simultaneity*.

²⁰⁴ “My earlier state of existence contains the ground for the existence of the later. And since, because the connection of all *things*, the earlier state in me also *contains* the earlier state of the other thing, it also contains the ground of the later state of the other thing, and is thereby prior to it” ((Leibniz 1951), as quoted in (van Fraassen 1970a, 38).

²⁰⁵ “...“X is causally connected with Y” is used as equivalent to “Either X and Y belong to the history of one and the same object, or belong to the history of one and the same signal, or are coincident with some pair of events thus connected.”” (van Fraassen 1970a, 194). The term “causal” is due to Reichenbach: “A light signal ... is a *causal chain*, because in Reichenbach’s terminology the emission of such a signal is one of the causes of its eventual reflections and final absorption; each reflection is also one of the causes of later reflections and final absorption” (Ibid., 172).

²⁰⁶ “[I]t is purely contingent whether there are any actual signal and genidentity connections in any given part of the universe. One might postulate that there are enough such connections to define temporal order for all events (given, one must assume, some other relations). And this postulate might be made plausible by accepted physical theory. In a philosophical account, however, one prefers to make as few empirical assumptions as possible” (van Fraassen 1970a, 195-196).

²⁰⁷ Van Fraassen sees his use of the term “logical space” to be “fully in accordance with the use of that term by Wittgenstein” and conjectures that its historical origin “is the use of vector spaces in physics (“phase space”, “configuration space”)” (1967, 172, n.). But he also views the idea quite generally as that of “the general form of any possible world” (1970a, 100) tracing it back to Kant. In particular, it is the form made necessary by the principles of the understanding in the *Critique* (1998) and also the essential form characterized in the principle of possible interactions found earlier in the *Dissertation* (1929). As noted in the first chapter of my dissertation, the latter principle expresses the divine schema. It should be evident by now, however, that there is a closer connection to Wittgenstein and a less general relation to Kant. As we saw last chapter, Wittgenstein views time to be the form of a kind of simple object displayed in the imaginative expression of a one-dimensional logical space, and this relates directly to Kant’s critical claim that “the schema of time a line”. There is no need, therefore, for van Fraassen to “to objectify ... [Kant’s temporal] ... form of our intuition and to describe it as a form, as opposed to a condition of sensible perception” (1970a, 101).

²⁰⁸ This is clearly evident in the following passage:

But what is logical space? Wittgenstein gives the example of the color spectrum: the logical space of colored things. But what exactly is the colored spectrum? It is merely a strip or line segment with markings, whether drawn on paper, merely imagined, or produced on a scale on the wall by means of a light source and prism. What it does is to give a picture, to a desired degree of

accuracy, of the part of our conceptual scheme that concerns colors. (“Why can’t a thing be red and green all over?” “Because ‘red’ and ‘green’ are tags of different parts of the spectrum, and an evenly colored surface has a unique location on the spectrum.”) To put it more generally: The color spectrum is a segment of the real line being used to represent the meaning relations among color words. (van Fraassen 1970a, 101)

²⁰⁹ The relation between logical space and language will be discussed in more detail next section.

²¹⁰ In saying that it “is purely and entirely an objective question of empirical fact” whether the real structure of actual events embeds within a mathematical structure, van Fraassen’s takes embedding to be a condition of representation that bestows meaning independent of us. But it is because this embedding can occur within a number of abstract structures that choice of structure is also a condition of representation that bestows upon an embedding structure the status of a “cultural object” (van Fraassen 1970a, 106). As became apparent some years later, however, the notion of an objective embedding relation is trivial, and so the condition of representation easily met. As we will see below, van Fraassen’s response will be to emphasize our use of logical space, drawing in pragmatic conditions of representation and retrenching meaning bestowal in acts of self-location necessary for use. He will then reconceive the Scotist synthesis in slightly different terms:

A scientific, technical, or artistic representation is an artifact. As such, it is both an object or event in nature, that we can regard purely through the physicist’s or chemist’s or mathematician’s eyes. But it is at the same time something constituted as a cultural object, through which its role or function, bestowed upon it in practice. Just what the representation is, or what is represented and how, is not determined entirely – and often enough, hardly at all – either by what is ‘in’ the natural object or by its physical or structural relations to other things. (2008, 30)

While it is still the case that apart from meaning there is nothing in a representation (Ibid., 25), in our use of it *we* bestow meaning on a representation by relating it to real entities and by bringing into play a host of pragmatic factors.

²¹¹ “I believe the constitution of time in our construction of the real world is not different in essential character from the constitution of time by the reader in his construction of the narrated world as he reads the text” (van Fraassen 1991b, 24).

²¹² Cf. van Fraassen (1970a, 132).

²¹³ Cf. van Fraassen (1989, 277-228).

²¹⁴ Cf. van Fraassen (1991a, 6).

²¹⁵ Cf. van Fraassen (1987, 105).

²¹⁶ This is evident at (van Fraassen 1970b, 326).

²¹⁷ These are contrasted with “contensive” meaning relations among predicates definable in terms of each other (van Fraassen 1967, 163).

²¹⁸ “The impact of Suppes’ innovation [re: the semantic view of theories] is lost if models are defined, as in many standard logic texts, to be partially linguistic entities, each yoked to a particular syntax. Here models are mathematical structures, called models of a given theory only in virtue of belonging to the class defined to be the models of that theory” (van Fraassen 1987, 122 n.2).

²¹⁹ “A topological feature is one that is preserved by a one-to-one continuous transformation”, e.g., dimensionality (van Fraassen 1970a, 134). Topological structure of a representative logical space includes not only the structure of the space itself, but also the structure of models that are mapped within it.

²²⁰ Depending on whether the state of the physical system changes with time, the satisfaction function is or is not time-dependent (if the former the satisfaction function is not time-dependent; if the latter the satisfaction function is time-dependent) (van Fraassen 1970b, 329).

²²¹ In this paper van Fraassen does not refer to a “physical system” but to individual *X* in the case of monadic predicates and set *D* in the general case of *n*-ary predicates. Neither does he refer to “satisfaction function” in the preceding sentence. I am anticipating the formal semantics of scientific language as presented in his (1970b).

²²² Note that for Carnap (1956b) meaning relations are made explicit through a set of “meaning postulates” laid down alongside axioms of a scientific theory, presupposing a distinction between meaning and empirical postulates. Van Fraassen’s semantic approach obliterates this distinction: in certain cases laws can be incorporated into the definition of logical space so all points of the space represent physically possible points, and none represent merely logically possible points. For him the difference between

empirical and meaning postulates is only the historical difference between laws made explicit by the scientist and principles implicit in the language game (1970b, 328, 331).

²²³ In *Science, Perception and Reality* (1963, ch.10) Sellars proposes a view of analyticity construed broadly as truth *ex vi terminorum*, arguing that the analytic propositions that constitute our conceptual framework include logically true and logically contingent propositions. Van Fraassen's connection to Sellars runs fairly deep, I think. Although it is beyond the scope of this present chapter, it would be useful to explore the influence Sellars had upon van Fraassen's early ideas since many carry through his later empiricism.

²²⁴ "In some easily recognizable way, our concepts of time and space have become ever more abstract. ... May it not be possible to fashion a concept of space-time from which all the variable features, differing from model to model, have been abstracted? ... Perhaps the adequate new conception has not yet been born, perhaps in retrospect these discussions in the second half of the twentieth century will be seen as contributing to its creation, and perhaps it will at the level of the most basic philosophical questions, be relevantly like earlier conceptions" (van Fraassen 1985b, 210-211).

²²⁵ Van Fraassen is not settled on an official definition of what a theory is, but emphasizes that "it must be the sort of thing that can be believed, disbelieved, doubted and so forth" (2008, 310).

²²⁶ Notice the tension in the notion of "appearances" when one compares a reconstructive logical space with a representational one. In the former, an appearance is a *physical system* mapped to a region of logical space; in the latter it is a *mathematical structure* isomorphic to part of a model. As van Fraassen would later admit, he used "appearance" at the time to mean both phenomena and data models (van Fraassen 2008, 391, n.24). He now uses the term "appearance" to mean perspectival information content, distinguishing it from both phenomena and data models.

²²⁷ A useful project in philosophy of language would be to critically evaluate this theory. Arntzenius (1991) has taken first steps by raising questions for van Fraassen's approach to meaning relations.

²²⁸ "Whether the apple is red depends on the meanings of "apple" and "red" in one sense, but not in the sense that makes us unrealistic. The dependence is just this: If the meanings of these words were different then so would be the truth conditions for the sentence "the apple is red". But the relevant independence is this: If the apple is red, then it would still have been red if the meanings of these words had been different" (van Fraassen 2006a, 151).

²²⁹ According to fictionalism, "discourse is to be interpreted literally or "at face value"; so our theories are true only if the problematic objects exist. We skirt commitment to those objects simply by denying that the theories are true" (Rosen 1994, 168). Literally construed, van Fraassen claims "theoretical [unobservable] entities are fictions" (1976, 335).

²³⁰ "Suppose *T* entails that statement ["*B* is not observable by humans"]. Then *T* has no model in which *B* occurs among the empirical substructures. Hence, if *B* is real and observable, not all observable phenomena fit into the model of *T* in the right way, and then *T* is not empirically adequate. So, if I believe *T* to be empirically adequate, then I also believe that *B* is unobservable if it is real" (van Fraassen 1985a, 256).

²³¹ "A graphic, in some what inaccurate way to put this would be: causal and modal discourse describes features of our models, not features of the world" (van Fraassen 1987, 122). Following Collingwood, van Fraassen sees causal discourse in science generally as "a metaphorical and analogical extension of the discourse of applied science and everyday life, i.e., human agency" involving intentional action, goals and choices (1993, 442).

²³² I am referring to Sellars' (1948). In his (1976) van Fraassen appears to accept its basic idea, citing its connection with Duhem.

²³³ Van Fraassen invokes the idea of colour-space to illustrate the language-governing feature of logical space:

Let us close with an example of how the logical structure of a language can be determined by an accepted theory: Wittgenstein's familiar example of the colour spectrum as a 'logical space'. A person uses a language in which he asserts such sentences as

1. *X* is green, *X* is not red, *Y* is red, ...
2. Nothing that is green is red
3. There is no possible object which is both green and red.

Sentences of type 1 he has been trained or conditioned to assert under certain experiential conditions. Sentences of type 2 still express assertions which are merely about what is actually the

case. But 3 goes well beyond that; it says something like: “there *could not be* something which is both red and green.

The explanation is that this person is guided by the idea of a simple abstract structure, the colour spectrum. We can think of this as a line segment, or an interval of real numbers (the wave lengths). He associates with each colour predicate, such as ‘green’, a part of that spectrum; he associates disjoint parts with ‘red’ and ‘green’; and when he says that an object is green or red, he is *classifying it*, that is, assigning it a *location* in that spectrum. So sentence 2 amounts to: no *occupied* location belongs at once to the parts associated with ‘red’ and with ‘green’, while 3 says that no point of the spectrum at all belongs to both parts. (Every model structure of this simple language consists of that spectrum plus a domain of entities and a function that assigns a location in the spectrum to each of those entities.) It will be clear how the logical connections among sentences in this language are determined by the structure of the colour spectrum. Blatantly modal sentences (such as 3) do occur; but this person evaluates them as true or false by reflection on the structure of the spectrum that guides all his uses of colour terms. His linguistic commitments can be summed up by referring to his use of this spectrum; his theory of colour consists in the family of models each of which is a classification of objects through location in this spectrum. (1980, 200-201)

²³⁴ It is warranted (inter alia) as follows: 1) interpreting what one is experiencing is part of experience (e.g., the experience of meeting a saint involves realizing that one is meeting a saint); and 2) self-attribution is also part of experience (e.g. the experience of winning the lottery requires self-attributing the named winner) (van Fraassen 2002a, 134-136); and 3) the fact we make mistakes interpreting experience (e.g., seeing a yellow candy wrapper and taking it to be a flower) (van Fraassen 2006a, 126).

²³⁵ The former is extensional (1992, 19) while it would be fair to say the latter is intensional in that it is propositional.

²³⁶ Cf. van Fraassen (2000).

²³⁷ Van Fraassen (1992) uses this Sellarsian language.

²³⁸ The justification for van Fraassen’s realism seems to be that presupposition-less discourse is “a view from nowhere”, and a presupposition of common sense realism is the basis of discourse in philosophy of science (2009, 479).

²³⁹ Similarly “‘Electrons are negatively charged’ is true if and only if electrons are negatively charged” (van Fraassen 2006b, 545 n.)

²⁴⁰ Cf. van Fraassen (1985a).

²⁴¹ Van Fraassen (2010b, 553) denies that a predicative statement is a representation.

²⁴² “The reports issued by institutions convey the data already reduced, summarized and corrected by statistical methods. But those summary reports are based on individual reports by trained observers – and *those* are the paradigm for observation reports” (van Fraassen 1992, 16).

²⁴³ It seems van Fraassen’s critics have not noticed that he does not offer a criterion of success for the construction of a data model in the practice of science. Surely there is such a criterion in the reproducibility of results. But if he were to offer such a criterion, I suggest, it would erode his ability to defend empirical adequacy as the criterion of success for theory production against more modest characterizations such as manifest adequacy (cf. Monton and van Fraassen 2003) or merely “saving the appearances”. It would go beyond the scope of this dissertation to elaborate on this point any further.

²⁴⁴ Ladyman (2009) puts it this way: “according to the extensional characterisation of relations defined on a domain of individuals, every relation is identified with some set of subsets of the domain. The power set axiom entails the existence of every such subset and hence every such relation”.

²⁴⁵ See van Fraassen (2008, 371, n.3).

²⁴⁶ Elaborating upon Duhem’s (1962) view that a theory provides us with a taxonomy to organize its empirical generalizations, *The Scientific Image* claims “[e]verything in the world has a proper classification within the conceptual framework of modern science. And it is this conceptual framework which we bring to bear when we describe any event, including an observation” (van Fraassen 1980, 58). It also claims that classifying an actual and observed phenomenon involves “assigning it a *location*” (Ibid., 201) in a logical space. One gets the sense van Fraassen sees himself as having fallen prey to the problem of coordination through the strategy employed in constructive empiricism of attempting to tackle scientific realism on its own terms (cf. (van Fraassen 2006a, 153) and (van Fraassen 2009, 470)).

²⁴⁷ Cf. van Fraassen’s (1967) and (1969).

²⁴⁸ “There is nothing in the idea of universality that should make philosophical hackles rise, nor would there be in the idea of law if a law stated merely what happens always and everywhere” (van Fraassen 1989, 26); “everything that can be said about the world, can be said in purely *general* statements, without modalities” (van Fraassen 1978, 14).

²⁴⁹ “But on the other hand, general scientific theories, in their ‘official’ formulation, are not perspectival descriptions, and their models – if we consider the entire range of models for a given theory – are not generally perspectival representations” (van Fraassen 2008, 86).

²⁵⁰ That there are invariances across perspectives in the content of perspectival measurement outcomes is illustrated in the case of cross-ratios read-off drawings or photos (van Fraassen 2008, 75).

²⁵¹ Cf. van Fraassen (1989, 227-228).

²⁵² Van Fraassen understands his account of the scientific image to be an “updated version of *Bildtheorie*, the ‘picture theory’ of science” (2010c, 514) originating with Hertz, and sees an affinity between empirical adequacy as a symmetry requirement to be satisfied by a structural representation and Hertz’s fundamental requirement (2008, 196, 306).

²⁵³ The example extends generally to quantitative predication:

In models, the terms ‘temperature’, ‘mass’, ‘force’, etc. do have denotations: they denote functions that assign numbers, vectors,... to other elements of the model. But in describing the bodies measured or represented, they provide a convenient *façon de parler* for predication:

The patient has a high temperature	The patient is hot
The patient’s temperature is 37.3°C	The patient is 37.3°C-hot

The predicate ‘37.3°C-hot’ is related to ‘hot’ as ‘scarlet’ is to ‘red’. (van Fraassen 2010b, 553)

²⁵⁴ “[O]bservation reports are indexical, acts of self-location” (van Fraassen 1992, 16).

²⁵⁵ “[A]n observation report is only symptomatic of its own truth, and symptoms don’t guarantee more than high probability” (van Fraassen 2004a, 3).

²⁵⁶ “The subtended angle is always 42 degrees, with that location (of eye or camera) between sun and cloud” (van Fraassen 2008, 103).

²⁵⁷ Van Fraassen’s (2008, 101-105) account of rainbows does not distinguish between a rainbow as a “naturally produced phenomena” and as the image type he calls a “public hallucination”.

²⁵⁸ In this case, the observation set-up also lacks the invariance of a real object since different positions of observation will locate the coloured arch in a different place. However, as we shall see next this is not essential to its status as a “public hallucination”.

²⁵⁹ Lenses such as in binoculars or telescopes have a mimetic function, producing phenomena taken to represent *observable* entities (van Fraassen 2008, 97). The “rough guide” in this context is “X is observable if there are circumstances which are such that, if X is present to us under those circumstances, then we observe it” (van Fraassen 1980, 16).

²⁶⁰ “[T]he appearances of objects are the contents of outcomes of measurements performed on those objects. But on what objects are those measurements by means of microscopes made? *On observable objects*. Leeuwenhoek inspected samples of pond water, others inspect specially prepared slides with samples of tissue. The microscope images are *how these observable objects appear* in those measurement set-ups. And these appearances are informative *about those observable objects*, not in and by themselves, but relative to the (presumed) empirical adequate theories that accommodate them” (van Fraassen 2011a, 411).

²⁶¹ Though not compelled, van Fraassen *chooses* to remain agnostic about unobservable entities in accordance with the commitments and values of his empirical stance. Citing quantum mechanics, he claims science rejects as a criterion of completeness that a theory must explain how appearances are produced from a theoretically postulated reality behind the phenomena (cf. 2008, ch.13).

²⁶² Here is the full passage:

Measurements occur in experimental procedure, under artificially designed conditions for empirical investigation. Under these conditions, measurement procedures produce representations – *images* – with complex, theory mediated relations to the entities on which the measurements are made. The content of the measurement outcomes [e.g., the end-state of the measuring instrument] – that is, the *appearance* of the measured object in the measurement set-up – locates the object in a theory-provided logical space.

²⁶³ See also van Fraassen (1980, 82).

²⁶⁴ “Our entire discussion of self-locating self-ascriptions was *a fortiori* a discussion of observation reports. They have as complete text an indexical assertion that locates the speaker, on that occasion, in some definite part of his own general, “objective”, world picture” (van Fraassen 1992, 18).

²⁶⁵ “That is, the phenomenon, what it is like taken by itself, does not determine which structures are data models for it – that depends on our selective attention to the phenomenon, and our decisions in attending to certain aspects, to represent them in certain ways and to a certain extent” (van Fraassen 2008, 254).

²⁶⁶ Pragmatic tautologies are not limited to observable entities: e.g., ““The sentence ‘Electrons are negatively charged’ is true if and only if electrons are negatively charged” is a pragmatic tautology” (van Fraassen 2006b, 545, n.10).

²⁶⁷ By small “p” platonism I have in mind Shapiro’s (2000, 28) characterization of a realist in ontology of mathematical objects who “defends something like a Platonic ontology for mathematics, without Platonic epistemology” that involves an appeal to mathematical intuition.

²⁶⁸ Compare this to early Wittgenstein position that such a shift in the perspective of a picture amounts to seeing “two different facts” (1961b, 5.5423).

²⁶⁹ Presumably, reproducibility of appearances would reduce the ambiguity in the second case, for the calibration of judgements would make accurate classification, i.e., correct location of the real entity in logical space, more probable. But even in science this ambiguity cannot be eliminated altogether. Following his distinction between the epistemology and semantics of experience, van Fraassen distinguishes a theory’s relation to appearances from its relation to reality, claiming that “the data we have may not be true. The relation between theory and data is independent of the truth of the data” (1981, 664). His epistemology of science defends the traditional view that “what we take as evidence itself is not indubitable, and we may later come to regard it as having been false”; yet, we do not “think ourselves irrational for engaging in this cognitive enterprise” (1984, 236).

²⁷⁰ “We have an ideal frame – we conceive of the events as in some definite order – but no rigid frame of reference. The locational function of each clue in the narrative, whether it refers to other parts of the narrated episodes or to episodes such as the World War in the readers’ presumed common history, must thus remain fragile, equivocal, and undermined by the rights of future narration. Therefore, the construction of the narrative time is always essentially internal to the text, even when the text gives every sign of wanting to be related to extratextual reality” (van Fraassen 1991b, 23).

²⁷¹ A previous note points out that van Fraassen’s argument experience is literate is supported by appeal to the phenomenology of experience. It goes without saying we are talking about the phenomenology of experience in relation to our *spontaneous* judgement, for if logical space is to function for us as an imaginative guide then its conditions of use must pertain to immediate non-discursive thought.

²⁷² Rosen (1994, 164-169) argues that the notion of theory acceptance and empirical adequacy in the context of the semantic view of theories commits van Fraassen to the existence of abstract entities. Monton and van Fraassen (2003) say that the attribution of tension between constructive empiricism and nominalist mathematics “presupposes that the practice and use of mathematics is intelligible only given realism or platonism in the philosophy of mathematics. That is in effect the supposition that mathematics is intelligible only if we can view it as a true story about certain kinds of things – not, for example, a supposition shared by intuitionism. Without offering a rival philosophy of mathematics, we may proceed in philosophy of science in the conviction that any satisfactory philosophical account of mathematics must imply that the sorts of applications of mathematics needed in philosophy of science are acceptable, correct, and intelligible” (412, n.).

²⁷³ Lately, van Fraassen is willing to countenance that “*mathematically speaking*” mathematical objects “exist” (2008, 310). But he offers no philosophy of mathematics to shore up what that means, and at the very least it must mean something different than real existence.

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