May 2012

# The Case for Quantum State Realism

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A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy

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Thesis format: Monograph

by

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A thesis submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

The School of Graduate and Postdoctoral Studies
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London, Ontario, Canada

THE UNIVERSITY OF WESTERN ONTARIO
School of Graduate and Postdoctoral Studies

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entitled:

**The Case for Quantum State Realism**

is accepted in partial fulfillment of the
requirements for the degree of
« Doctor of Philosophy»

Date

Chair of the Thesis Examination Board

## Abstract

I argue for a realist interpretation of the quantum state. I begin by reviewing and critically evaluating two arguments for an antirealist interpretation of the quantum state, the first derived from the so-called 'measurement problem', and the second from the concept of local causality. I argue that existing antirealist interpretations do not solve the measurement problem. Furthermore, I argue that it is possible to construct a local, realist interpretation of quantum mechanics, using methods borrowed from quantum field theory and based on John S. Bell's concept of 'local beables'.

If the quantum state is interpreted subjectively, then the probabilities it associates with experimental outcomes are themselves subjective. I address the prospects for developing a subjective Bayesian interpretation of quantum mechanical probabilities based on the Quantum de Finetti Representation Theorem. Epistemic interpretations of the quantum state can be divided into those that are epistemic with respect to underlying ontic states, and those that are epistemic with respect to measurement outcomes. The Pusey Barrett and Rudolph (PBR) theorem places serious constraints on the former family of interpretations. I identify an important explanatory gap in the latter sort of interpretation. In particular, if the quantum state is a subjective representation of beliefs about future experimental outcomes, then it is not clear why those experimenters who use quantum mechanics should be better able to negotiate the world than those who do not.

I then turn to the task of articulating a positive argument for the thesis of quantum state realism. I begin by articulating a minimal set of conditions that any realist interpretation must meet. One assumption built into the PBR result is that systems prepared in a given quantum state have a well-defined set of physical properties, which may be completely or incompletely described by the quantum state. Antirealist interpretations that reject this assumption are therefore compatible with the PBR result. A compelling case for quantum state realism must therefore be made on more general grounds. I consider two concrete examples of phenomena described by quantum mechanics that strongly suggest that the quantum state is genuinely representational in character.

**Keywords:**

Philosophy of physics, foundations of quantum mechanics, the measurement problem, philosophy of science, quantum state, realism, confirmation theory, philosophy of probability

*For Rick Tait and Meaghan Eastwood*

Acknowledgements

I would like to extend a very special thank you to Wayne Myrvold. This project would not have been possible without Wayne's constant intellectual support and encouragement.  Special thanks to Robert DiSalle, Chris Smeenk, William Demopoulos and Erik Curiel for valuable discussion and comments along the way. Thank you also to Ryan Samaroo, Mike Cuffaro, Molly Kao, Robert Batterman, William Harper, and Doreen Fraser. I would also like to thank my external examiners Robert Spekkens and Dan Christensen.

I owe a special debt of gratitude to Chris Fuchs, whose work in the foundations of quantum mechanics was an inspiration for this project, and who very graciously invited me to participate in discussions at the Perimeter Institute for Theoretical Physics in the fall of 2009.

Finally, I would like to gratefully acknowledge financial support from the Social Sciences and Humanities Research Council and the Foundational Questions Institute.

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# Introduction

## Why Does Quantum Mechanics Need an Interpretation?

If the success of a scientific theory is measured by its capacity to accurately predict the outcomes of physical experiments, then quantum mechanics is undoubtedly the most successful physical theory ever developed. Yet for all of its empirical success, quantum mechanics raises novel interpretive difficulties for the philosopher of science. It has often been remarked that any mature physical theory ought to contain within itself its own theory of measurement. This is not merely a consequence of the physicality of measurement; it is due to the fact that the assignment of values to physical quantities, a procedure which undergirds the evidentiary basis of any theory, is mediated by theoretical assumptions.

What theoretical assumptions are brought to bear on measurement processes in quantum mechanics? According to the standard view, the quantum wave function is a mathematical representation of the physical state of a quantum system. The state evolves in time according to the linear Schrödinger equation, and the state at any given time can be used to assign probabilities to potential experimental outcomes according to a law known as the 'Born rule'. The probabilistic predictions of the theory are then subject to empirical confirmation. The so-called 'quantum measurement problem' is concerned with the fact that if an experimental apparatus is treated as a quantum system, then a typical quantum state at the end of an experiment will be a sum or 'superposition' of terms corresponding to different experimental outcomes. If the quantum state description is taken to be complete and correct, then it seems that there are no definite experimental outcomes, which is problematic. To deal with this problem, the so-called 'collapse postulate' is introduced into the standard formalism; according to this postulate, when a measurement is performed, a quantum superposition 'collapses' into a definite state corresponding to one of the terms in the superposition, via a mechanism that is not governed by the Schrödinger equation. This state of affairs has led John S. Bell to famously remark that 'either the wave function, as given by the Schrödinger equation, is not everything, or it is not right'. The significance of this tension between the Schrödinger evolution and the determinacy of experimental outcomes, and the status of the collapse postulate, has been the subject of intense debate since the formalism of quantum mechanics was first developed.

One possible attitude to take is that despite the obvious difficulties posed by the measurement problem, there are still good grounds for adopting a realist interpretation of the quantum state. Realists share a commitment to the idea that the quantum state represents the real state of a system, both before and after a measurement. On the face of it, this is a difficult view to maintain, given the apparent role of collapse in the theory. Realists may choose one of the horns of Bell's dilemma, or, in the case of the many worlds interpretation, reject the dilemma altogether. According to the De Broglie-Bohm interpretation, the quantum mechanical description of reality is correct as far as it goes, but must be supplemented by further deterministic 'hidden variables' that ultimately guarantee, given initial conditions, that the outcomes of measurements will always be determinate. Other realist approaches have elected to seize the other horn of Bell's dilemma, modifying the dynamics of the Schrödinger equation to allow for the possibility that wave functions may undergo spontaneous collapses into definite post-measurement states. Finally, a third option that retains realism is provided by the 'many worlds' interpretation, which argues that every time a quantum experiment with different outcomes with non-zero probability is performed, all of those outcomes are realized, each in a different world.

All of the above interpretations share a commitment to quantum state realism, and attempt to address the measurement problem within a realistic framework. A completely different kind of strategy for dealing with the measurement problem involves embracing an antirealist interpretation of the quantum state. Antirealists avoid the measurement problem by denying that the quantum state description, whether of the measurement apparatus or any other phenomenon, is an objective representation of physical reality. If the quantum state does not track real features of physical systems, then the probabilities it furnishes are subjective. This makes it possible to describe the 'collapse of the wave function' in subjective terms, perhaps as an example of the sort of information updating or belief change we associate with information acquisition in non-quantum contexts.

## Motivations for Quantum State Antirealism

As with realist interpretations, the phrase 'quantum state antirealism' captures a number of interpretations. The idea that the formal apparatus of quantum mechanics does not describe an observer-independent quantum reality has a venerable history. The so-called Copenhagen interpretation of quantum mechanics, associated mainly with the Danish physicist Niels Bohr, but also with Wolfgang Pauli, Werner Heisenberg, and Max Born, was probably the first serious attempt to understand the relationship between quantum mechanics and the phenomena it describes in an antirealist framework, and remains highly influential to this day. Bohr, for example, is reported to have said:

> There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature.[1]

With recent developments in the field of quantum information theory, a new variation on this antirealist theme has emerged which tries to interpret the quantum state as a representation of the subjective information of agents interacting with quantum systems, rather than a description of physical reality. The most radical wing of this

[1] As quoted in Petersen [1963].

new school argues that the probabilities assigned to measurement outcomes by the quantum state are in all cases entirely subjective.[2]

If the case can be made that quantum states are states of information rather than states of reality, then the fact that the quantum state undergoes a discontinuous transformation on measurement is given a straightforward explanation: the so-called 'collapse of the wave function', in which a superposition is reduced to a state corresponding to a determinate measurement outcome, can be interpreted as a change of belief rather than a physical transformation in the system not governed by the Schrödinger equation.

Furthermore, a successful account of the collapse of the wave function in subjectivist terms would seem to have advantages that go beyond addressing the measurement problem. A second apparent difficulty posed by measurement events relates to the issue of locality and the problem of assessing the compatibility of quantum mechanics with the special theory of relativity. It is a peculiar feature of quantum mechanics that two systems with separate quantum states may interact in such a way that their states become 'entangled', meaning that the overall quantum state of both systems is no longer reducible to the individual states of the previously separate systems. Erwin Schrödinger famously described the phenomenon of entanglement as the essential feature of quantum mechanics that distinguishes it from classical physics:

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[2] See Fuchs [2003], Caves, Fuchs and Schack [2001].

> When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call [entanglement] *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled.[3]

Part of what enforces this departure from classical lines of thought is that entanglement may extend over arbitrary distances. This entails that the quantum state associated with an entangled system has a manifestly nonlocal character. Measurements of entangled systems will in general immediately transform the quantum state associated with those systems; consequently, interacting with one part of an entangled system will generally change the quantum state associated with the entire system, regardless of its spatial extent. This state of affairs famously led Einstein to conclude that the quantum mechanical description of physical systems must be incomplete:

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[3] Schrödinger [1935], p. 555.

> I incline to the opinion that the wave function does not (completely) describe what is real, but only a (to us) empirically accessible maximal knowledge regarding that which really exists [...] This is what I mean when I advance the view that quantum mechanics gives an incomplete description of the real state of affairs.[4]

Einstein' attitude is later cited with approval by Chris Fuchs, who writes:

> Einstein was the first person to say in absolutely unambiguous terms why the quantum state should be viewed as information [...]. His argument was simply that a quantum-state assignment for a system can be forced to go one way or the other by interacting with a part of the world that should have no causal connection with the system of interest.[5]

Harrigan and Spekkens [2010] echo this sentiment when they write that 'Einstein showed that not only is locality inconsistent with [the quantum state] $\psi$ being a complete description of reality, it is also inconsistent with $\psi$ being ontic'.[6]

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[4] A. Einstein, Letter to P. S. Epstein, 10 November 1945, extract from D. Howard [1990] p. 103. Einstein's argument will be taken up in greater detail in chapter 2.

[5] Fuchs [2002].

[6] I will consider Einstein's argument for the incompleteness of quantum mechanics in some detail in chapter 2.

Given these considerations, it might well be wondered why so many commentators have taken it for granted that the quantum state is ontic. In fact, antirealist arguments notwithstanding, the present work will argue that there are very good grounds for interpreting the quantum state realistically. Much of the evidentiary basis of quantum mechanics is derived from relative frequency data that is compared to the predictions of the theory. The role of the quantum state in this procedure is to furnish probabilistic predictions that are subject to test in the context of repeatable experiments. At a minimum, a realistic interpretation of the quantum state will have it that i) there exist states of reality which it is the business of our physical theories to describe; ii)  different quantum states correspond to different states of reality. The injective case takes into account the possibility of supplementary variables, as in the de Broglie-Bohm theory.

Some antirealists, such as Chris Fuchs, deny that states of reality are in principle amenable to a dynamical description, and so align themselves with the view expressed in the quotation from Bohr above. Other commentators, such as Rob Spekkens, accept that reality is amenable to physical description, but deny that quantum mechanical states can be mapped one-one into states of reality or ontic states. Fuchs' view, as we shall see shortly, is hard to square with the evidentiary basis of quantum theory. More modest antirealist or epistemic interpretations that maintain the descriptive role of physics face a different set of challenges. The recently discovered Pusey, Barrett and Rudolph (PBR) theorem places considerable constraints on the space of possible ontic models of quantum mechanics, on the assumption that the quantum state is epistemic.[7] These arguments will be considered in more detail in the chapters to follow. I turn now to a brief outline of the structure of the dissertation.

## Outline of Thesis Structure

If the case for a realistic interpretation of the quantum state is to be made compelling, arguments for the contrary view must be addressed; it is therefore a central burden of the present work to critically evaluate the arguments for an epistemic interpretation of the quantum state. In chapter 2, entitled 'Critical Examination of Two Arguments for Quantum State Antirealism', I consider two related arguments that have recently been put forward in defense of quantum state antirealism.

The first argument stems from the measurement problem, which has already been alluded to above. If we interpret the quantum state as representing information about a system rather than a state of reality, then quantum state 'collapse' no longer requires a physical explanation, but is perhaps better thought of as an instance of rational belief change in the context of information acquisition. I consider two recent proposals for addressing the measurement problem in a subjective Bayesian framework, the first due to Jeff Bub [2007], the second due to Chris Fuchs [2003,

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[7] Pusey, Barrett and Rudolph [2011].

2008]. I argue that neither of these proposals successfully addresses the measurement problem, on the grounds that the transition to post-measurement states cannot be modelled in entirely subjective terms, but represents a (non-unitary) physical process.

The second argument I consider is based on locality. As I have already suggested, one of the main challenges facing a realist interpretation of the quantum state is the apparent tension between taking the quantum state to be a real, on the one hand, and the locality assumption of special relativity. The worry is that quantum entanglement may extend over arbitrary distances, apparently implying that quantum state collapse, regarded as a physical process, is non-local in character. Proponents of an antirealist interpretation of the quantum state, including Harrigan and Spekkens [2010] and Fuchs [2010], [2003], have argued that locality is in fact inconsistent with a realist or ontic interpretation of the quantum state. The aforementioned authors all assume that locality presupposes a principle of *separability*: that systems can be regarded as having separate existence insofar as they occupy different regions of space.[8] One reason to regard separability as a necessary condition for locality derives from the principle of local interaction, according to which any adequate interpretation of quantum theory must include local beables in its ontology. It might be thought that nonseparability is incompatible with the existence of such local beables. This presents a *prima facie* difficulty, since

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[8] This view is also advocated in Norsen [2009].

the principle of local causality is a cornerstone of quantum field theory, and the existence of quantum entanglement shows that separability fails in general for quantum systems. Nevertheless, it can be shown that the requirement of local causality does not presuppose separability, and the case can be made that quantum field theory, as standardly formulated, is locally causal.

Chapter 3, entitled 'Evidence, Explanation and Ontic State Realism', addresses the prospects for developing a subjective Bayesian interpretation of quantum mechanical probabilities. I begin by reviewing the classical subjectivist interpretation of probabilities due to de Finetti [1937], focusing in particular on de Finetti's famous Representation Theorem. Crucial to de Finetti's original proof of the theorem is the assumption of exchangeability, which says that the probabilities associated with a series of trials such as coin tosses are permutation symmetric.[9] The de Finetti representation theorem guarantees that an agent who judges a sequence of coin tosses to be exchangeable will bet on the outcome of future coin tosses as if the coin has an objective but unknown probability. De Finetti famously interpreted this result as demonstrating the dispensability of the concept of objective chance.

It turns out that there is an analogous result pertaining to quantum mechanical experiments known as the 'Quantum de Finetti Representation Theorem'. The Quantum Bayesians Caves, Fuchs and Schack [2002a] argue that the latter theorem

[9] In fact, the exchangeability assumption can be relaxed to so-called 'Markov exchangeability', and the result still obtained. I will have more to say about the significance of this point in chapter 3.

shows that the concept of an unknown quantum state is itself dispensable. I identify an important explanatory gap in the Quantum Bayesian account. In particular, an important question presents itself: if the quantum state does not track physical states, then why are those experimenters who use quantum mechanics better able to negotiate the world than those who do not? According to Chris Fuchs, quantum mechanics places empirical constraints on the space of possible probability assignments, rather than determining those probabilities outright. Fuchs interprets the Born rule as an 'empirical addition to Bayesian coherence'.[10] From a realist perspective, the constraint on probability assignments is straightforward: it is derived from the fact that the objective quantum state assigns probabilities to experimental outcomes within that restricted set. This sort of explanation is obviously not available to the Quantum Bayesian; from an antirealist perspective, the constraint seems mysterious.

In chapter 4, 'The Case for Quantum State Realism', I attempt to articulate and defend the thesis of quantum state realism. I begin by defining the problematic as one of attempting to correctly identify the relationship between the mathematical formalism of quantum mechanics and the phenomena that the theory describes. An important recent result due to Pusey, Barrett and Rudolph [2011] places significant constraints on the space of epistemic or antirealist interpretations of the quantum state. While the PBR result does not show that antirealist interpretations are

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[10] Fuchs [2008].

incompatible with the predictions of quantum mechanics, it does show that surprisingly modest assumptions are required to bring epistemic interpretations into conflict with the predictions of the theory.

One of the assumptions built into the PBR result is that systems prepared in a given quantum state have a well-defined set of physical properties. Antirealist interpretations that reject this assumption are therefore compatible with the PBR result. I argue that a compelling case for quantum state realism must therefore be made on more general grounds. I consider two concrete examples of quantum phenomena that strongly suggest that the quantum state is real. The first example concerns the phenomenon of interference in the famous two-slit experiment. The second example is derived from quantum chemistry and quantum crystallography. The examples are intended to illustrate the fact that changes in the wavefunction representing quantum systems in general correspond to physical changes in the systems in question.

# Chapter 2: Critical Examination of Two Arguments for Quantum State Antirealism

Collapse is something that happens in our description of the system, not to the system itself. Likewise, the time dependence of the wavefunction does not represent the evolution of a physical system. It only gives the evolution of our probabilities for the outcomes of potential experiments on that system. This is the only meaning of the wavefunction.[11]

Chris Fuchs and Asher Peres

## Introduction

The purpose of the present chapter is to critically examine two kinds of argument that have been put forward in support of an antirealist interpretation of the quantum state. Broadly construed, quantum state antirealism is the thesis that the quantum state represents the beliefs or states of knowledge of observers, rather than objective features of a physical system. In the history of the debate over quantum foundations, antirealist interpretations have occupied a central position. Niels Bohr, for example, argues that quantum phenomena enforce a reexamination of the kinds of knowledge obtainable from physical systems. Such a reexamination is militated by the interaction between quantum objects and the macroscopic instruments used to conduct quantum

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[11] Fuchs and Peres [2000], p. 71.

experiments. Bohr notes that scientific knowledge can only be gained under 'reproducible and communicable conditions', and given that, according to Bohr, such conditions are necessarily described by classical concepts which are the 'refinement' of the everyday concepts that make communication possible, we must also describe the results of actual experiments using such concepts. Bohr concludes that

> Just this circumstance [the necessity of classical description] implies that no result of an experiment concerning a phenomenon which, in principle, lies outside the range of classical physics can be interpreted as giving information about independent properties of objects.[12]

Bohr's antirealism is motivated by epistemological considerations. In Bohr's view, the theoretical description of physical phenomena necessarily has recourse to classical concepts, despite the status of quantum phenomena as 'lying outside the range of classical physics'. Such phenomena are 'essentially determined by the interaction between the objects in question and the measuring instruments necessary for the definition of the experimental arrangement'.[13] Knowledge of quantum systems is not, in Bohr's view, divorceable from the experimental arrangements used to investigate them; in this sense such knowledge is inherently contextual.

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[12] Bohr [1938], p. 26.

[13] Ibid.

Contextuality of itself does not entail that experimental results cannot be interpreted as giving information about independent properties of objects; such properties may be essentially non-classical in character. The premise that classical concepts are necessary for the description of phenomena is essential to Bohr's conception of independent properties. The above quotation appears to imply that, for Bohr, classical concepts presuppose a sharp distinction between 'the behavior of objects and the means of observation'[14], and that such a distinction is incompatible with the ascription of independent properties to quantum systems.

Given that we have run up against phenomena that are essentially non-classical in character, we might wonder whether Bohr's epistemological stance with respect to the privileged status of 'classical' concepts is too restrictive. Does the existence of quantum phenomena not militate against the retention of a strictly classical conceptual framework? Bohr famously answered this question in the negative, and concluded that the description of quantum phenomena can never be interpreted as providing genuine information about independent properties of quantum systems. Other physicists were less dogmatic, and open to the possibility of adopting a new conceptual scheme to describe the new physics. It is possible to interpret the inadequacy of classical concepts for the description of quantum phenomena such as the uncertainty principle as good grounds for seeking out new, non-classical concepts. This non-classicality need not commit us to an antirealist attitude toward quantum

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[14] ibid.

mechanical descriptions of phenomena. In correspondence with Bohr, Erwin Schrödinger writes that

If you want to describe a system, e.g., a mass point by specifying its *p* and *q*, then you find that this description is only possible with a limited degree of accuracy. This seems to me very interesting as a limitation of the applicability of the *old* concepts of experience. But it seems to me imperative to demand the introduction of *new* concepts, in which this limitation *no longer* occurs. Since what is unobservable in principle should not at all be contained in our conceptual scheme, it should not be representable in terms of the latter. In the *adequate* conceptual scheme it ought no more to seem that our possibilities of experience are restricted through unfavorable circumstances.[15]

This suggests that the quantum state description, which does not ascribe a definite but unknown position and momentum to a quantum system, may nevertheless be complete  if we abandon the idea, held dogmatically by Bohr, that quantum systems must be given a classical description.[16] It is only the adherence to classical concepts that leads to the conclusion that there are features of reality not accessible to experience.  Bohr's response invokes once again the necessity of 'old experiential' concepts:

I am scarcely in complete agreement with your stress on the necessity of developing "new" concepts. Not only, as far as I can see, have we up to now no clues for such a re-arrangement, but the "old"

[15] Bohr [1985].

[16] This is the view defended in chapter 4 of the present work.

experiential concepts seem to me to be inseparably connected with the foundation of man's power of visualizing.

The invocation of 'man's powers of visualizing' might suggest a psychological justification for the "necessity" of experiential concepts that is surely on dubious epistemological ground. How exactly are the limits of man's powers of visualizing to be demarcated? Bohr was certainly aware of the existence of non-Euclidean geometries and their obvious utility in the formulation of relativistic space-time theories. The existence of such conceptual frameworks demonstrates that the generalization of concepts which are 'visualizable' (the concepts of Euclidean geometry) can be scientifically and explanatorily fruitful.[17] Whether Bohr would have regarded non-Euclidean geometries as visualizable in the relevant sense is perhaps unclear; in any case, the history of the development of the special and general theories of relativity, and the 19th century geometry which undergirds the mathematical structure of these theories, is an object lesson for those philosophers who championed the necessity of existing concepts.

An enduring feature of Bohr's treatment of the philosophy of quantum mechanics is the prominence given to the role of observation, understood as a macroscopic process receiving a classical description, in discussions of quantum

[17] I do not wish to prejudge the question of whether or not, according to Bohr, non-euclidean geometries can be visualized in the relevant sense.

phenomena. Observation is implicated, for example, in the necessity of the interaction between quantum systems and the macroscopic measuring instruments necessary for the 'definition of the experimental arrangement'.[18] The use of everyday concepts, 'perhaps refined by the terminology of classical physics', is mandated by the need to gain knowledge under reproducible and communicable conditions. The fact that there is an unavoidable interaction between quantum objects and the experimental apparatus implies that there is also an absolute limit, in Bohr's view, to the extent to which it is possible to describe quantum systems in a manner independent of the means of observation.

It is reasonable to expect that the result of an observation may depend not only on the state of the system but also on the overall disposition of the apparatus. 'Measurement' of an observable need not yield the same result independently of whether or not some other observable is measured simultaneously.[19]Nevertheless, Bohr's antirealist attitude towards quantum systems, and his focus on the possibilities and limitations of observation, finds a modern echo in contemporary antirealist

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[18] Bohr [1938], p. 26.

[19] John S. Bell famously discusses, and criticizes, the non-contextuality assumption in von Neumann's 'no hidden variables proof' in Bell [1966]. See also von Neumann [1932] and Mermin [1993].

interpretations of the quantum state.[20] Many contemporary supporters of antirealist interpretations of the quantum state have made important contributions to the field of quantum information theory, and the information-theoretic constraints implied by quantum mechanics figure heavily in contemporary foundational discussions. What are the arguments in favor of quantum state antirealism that have emerged from this program?

In this chapter, I will critically examine two prevalent arguments for an antirealist interpretation of the quantum state. The first argument draws inspiration from the problem of measurement, and is hinted at in the quotation from Fuchs and Peres at the beginning of this chapter. The second argument derives from the phenomenon of quantum entanglement, and builds upon the famous Einstein, Podolski and Rosen (EPR) argument for the incompleteness of quantum mechanics. Chris Fuchs calls this ‘‘the cleanest argument I know that the quantum state is solely an expression of subjective information- the information one has about a quantum system’.[21] It will be argued that neither of these arguments provides a compelling case against quantum state realism.

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[20] Cf. Fuchs [2003], Caves Fuchs and Schack [2002], Fuchs and Peres [2000], Spekkens [2007]. For reasons to be discussed in chapter 4 of the present work, I do not consider Rovelli's [1996] relationalist program to be antirealist.

[21] Fuchs [2003], p.

## 2.1 The Measurement Problem as Motivation for Quantum State Antirealism

Antirealist interpretations of the quantum state do not so much solve the measurement problem as dissolve it altogether; the problem simply does not arise if the quantum state is not taken to represent objective features of physical systems. The measurement problem arises if one attempts to treat a measurement apparatus, like the things it measures, as a quantum system. Given that laboratory equipment is composed of particles that obey the laws of quantum mechanics, it seems reasonable to expect that such a treatment should at least in principle be possible.[22] But if we describe the apparatus as a quantum system, its state at the end of an experiment will typically be a sum of terms corresponding to different experimental outcomes, with corresponding probabilities. To make this claim more precise, suppose that a quantum system is initially in the state $|S\rangle$. Typically, this state will be a linear superposition of different eigenstates of some observable. For simplicity, we assume that the observable in question can be represented by an operator in two-dimensional

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[22] Bohr would disagree that measuring devices can be treated as quantum mechanical, since the description of quantum phenomena that such devices provide necessarily has recourse to classical concepts . As we will see shortly, Bohr's (I think ultimately incorrect) attitude finds a contemporary voice in the writings of Jeff Bub.

Hilbert space. It is a condition of adequacy for measurement that the post-measurement state of the system correspond to an eigenstate of the observable in question:

$$\begin{aligned} |S_i\rangle \otimes |M_o\rangle &\Rightarrow |S_i\rangle \otimes |M_i\rangle \\ |S_k\rangle \otimes |M_o\rangle &\Rightarrow |S_k\rangle \otimes |M_k\rangle \end{aligned} \tag{1}$$

where $|S_i\rangle$ and $|S_k\rangle$ are eigenstates of the corresponding operator. The initial state of the measuring apparatus is given by $|M_0\rangle$. The initial state of the combination of system and apparatus is therefore represented by the product vector:

$$|initial\rangle = (a|S_i\rangle + b|S_k\rangle) \otimes |M_0\rangle \tag{2}$$

After a finite period of interaction, on the assumption of linear evolution of the combined system (2) according to the Schrödinger equation, the final state of the combined system and apparatus will in general be a superposition of different measurement outcomes:

$$|final\rangle = a|S_i\rangle \otimes |M_i\rangle + b|S_k\rangle \otimes |M_k\rangle \tag{3}$$

According to the standard collapse postulate, the actual outcome of the experiment will be either $|S_i\rangle \otimes |M_i\rangle$ or $|S_k\rangle \otimes |M_k\rangle$, with corresponding probabilities $|a|^2$ and $|b|^2$, respectively. Such a 'collapse' to one or the other eigenstate involves a non-linear projection in violation of the linear Schrödinger dynamics. On the other hand, if the dynamics is taken to be complete and correct, then it seems that there are no determinate measurement outcomes as in (3), a state of affairs that seems *prima facie* irreconcilable with our experience.

Quantum state antirealists avoid the measurement problem by denying that the quantum state description of the measuring apparatus is an objective representation of the setup. Anton Zeilinger, for example, writes that

> If we accept that the quantum state is no more than a representation of the information we have, then the spontaneous change of the state upon observation, the so-called collapse or reduction of the wave packet, is just a very natural consequence of the fact that, upon observation, our information changes and therefore we have to change our representation of the information, that is, the quantum state. [23]

If it is true that the quantum state is just a 'representation of information' in Zeilinger's sense, i.e., just a state of knowledge rather than a state of reality, then state collapse does not require a physical explanation, and the problem of measurement is reduced to the seemingly much more pedestrian problem of accounting for rational

[23] Zeilinger [1999], p. S291.

belief change in the context of information acquisition. An obvious candidate for modeling rational belief revision in this context is Bayesian conditionalization. I will now consider two related but distinct recent proposals for addressing the measurement problem explicitly in a subjective Bayesian framework, the first due to Jeff Bub, the second due to Chris Fuchs.[24]

*Bub on the 'Two Dogmas of Quantum Mechanics'*

Jeff Bub argues that the measurement problem is an artifact of what he and Itamar Pitowsky have identified as 'two dogmas of quantum mechanics'.[25]The first dogma, defended by John S. Bell, is that the concept of measurement should not be introduced as a primitive into the formulation of our most fundamental physical theories. Instead, measurement processes should be describable in terms of the dynamical laws of the theory, if only in principle.[26]The second dogma is quantum state realism: the view that the quantum state is a representation of physical reality. The second dogma leads to what Pitowsky has called the 'big measurement problem', which I have outlined above, namely the problem of reconciling determinate measurement outcomes with

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[24]See Fuchs [2003], [2008] and Bub [2007]. See also Bub and Pitowsky [2010] and Pitowsky [2002].

[25] See Bub [2007], Bub and Pitowsky [2010].

[26] For a discussion of the problems raised by the word 'measurement' in discussions of quantum mechanics see Bell [1990].

the fundamental dynamics of quantum theory.[27]As Bub points out, the von Neumann collapse postulate does not provide a solution to the measurement problem if such a solution is to be given in essentially dynamical terms while also respecting the first dogma, prohibiting a split between accounts of measurement and non-measurement processes in the theory.[28]

According to Bub, the first, 'against measurement', dogma is called into question by the no-cloning theorem.

> Now, the first dogma is called into question if, as a contingent matter of fact, there is a limitation on copying information-if the dynamical implementation of a universal cloning machine is in principle excluded by structural features of events. In a 'no cloning' world, as I will show below, no complete dynamical account of a measurement process is possible in general: ultimately, a measuring instrument in a quantum measurement process…produces a probability distribution over distinguishable measurement outcomes, and how the individual outcomes come about is not subject to further dynamical analysis.[29]

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[27] Pitowsky [2006]

[28] Examples of theories that respect both Dogmas include the Bohm theory and the GRW theory. Both theories furnish a dynamical account of the objective evolution of quantum states that is universal in scope, i.e., includes measurement processes.

[29] Bub [2007], p.236.

According to Bub, if it is impossible to implement a universal cloning machine for quantum states, then no 'complete' dynamical account of measurement processes can in principle be given. A measuring instrument acts as a source of classical information by producing a probability distribution over distinguishable measurement outcomes. Quantum mechanics does not furnish an account of how individual measurement outcomes come about, and therefore the measurement process is at this stage not amenable to further dynamical description; in this sense measurement processes are 'irreducibly statistical'.

Two comments are immediately in order. The first is that, according to Bub, the unavailability of a dynamical implementation of a universal cloning machine is already enough to rule out the first dogma, that measurement should not be introduced as a primitive into quantum theory. The second comment concerns the notion of a 'complete' dynamical account. If the unavailability of such an account turns on the irreducibly statistical character of measurement events, as Bub suggests, then a dynamical account is incomplete just in case it fails to be deterministic. But this seems to leave open the possibility of a nondeterministic dynamical account of measurement.[30] I will return to this point shortly.

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[30] Note that I *do not* mean to suggest that moving to a stochastic dynamics that recovers the predictions of quantum mechanics might allow for the possibility of cloning unknown states. The point I wish to emphasize is that a dynamical account of measurement is not ruled out by the no-cloning theorem.

The no-cloning theorem is a result that forbids the creation of identical copies of an arbitrary unknown quantum state. Suppose that the initial state of a system we wish to copy is given by $|\psi\rangle_A$. In order to copy this system, we form a composite system $|\psi\rangle_A|r\rangle_B$ , where $|r\rangle_B$ is a second (independent) system with the same state space as $|\psi\rangle_A$. We wish to perform a unitary operation on $|\psi\rangle_A|r\rangle_B$ such that

$$U|\psi\rangle_A|r\rangle_B \rightarrow |\psi\rangle_A|\psi\rangle_B.$$

Suppose that the same unitary operation also clones a second vector $|\varphi\rangle_A$ :

$$U|\varphi\rangle_A|r\rangle_B \rightarrow |\varphi\rangle_A|\varphi\rangle_B.$$

for all possible states φ in the state space. Since *U* is unitary, it preserves inner products, and therefore

$$\langle r|_B\langle\varphi|_A|\psi\rangle_A|r\rangle_B = \langle r|_B\langle\varphi|_A U^\dagger U|\psi\rangle_A|r\rangle_B = \langle\varphi|_B\langle\varphi|_A|\psi\rangle_A|\psi\rangle_B. \qquad (4)$$

On the assumption that the states in question are normalized, we have $\langle\phi|\psi\rangle = \langle\phi|\psi\rangle^2$, implying that the quantum states $\varphi$ and $\psi$ are either identical or orthogonal. Hence performing a unitary operation cannot clone a general quantum state.

As an example, consider a quantum controlled-not gate for qubits. The controlled-not gate is able to perform unitary transformations on a given set of input quantum states delivered by some information source. The quantum controlled-not gate is able to produce copies of orthogonal input states $|0\rangle$ and $|1\rangle$, but will in general fail to reproduce superpositions of these basis states. This is because a unitary transformation performed on a quantum state in a superposition will in general lead to an entangled state:

$$|\psi \otimes 0 \;=\; c_1\ 0 \;+\; c_2\ 1 \quad 0 \quad \overset{U}{\ } \ c_1\ 0 \quad 0 \;+\; c_2|1\ |1 \;\neq |\psi\ |\psi$$

Unless the $c_i$ are equal to 0 or 1.

If quantum mechanics is taken to be both complete and correct, then it follows that no dynamical account of the measurement process is possible whereby the outcomes of measurements can be predicted with certainty. Bub concludes that 'there must always be some system involved in the process, taken as the ultimate measuring instrument, that functions simply as a classical information source in Shannon's sense'.[31]

The argument runs as follows. Suppose that an information source produces a quantum state. Call this state the 'source state'. Suppose further that there exists a device capable of unambiguously identifying the source state by means of some dynamical transformation. We can imagine a measurement device interacting with the source state to produce a distinguishable state that is itself a record of the source state. Call this latter state the 'measurement state'. If we assume that any known state can be produced from a (suitably chosen) reference state by means of some dynamical evolution, then we need only use the measuring device to identify the output of the information source, and perform a dynamical evolution on the resulting measurement state to clone the source state. The impossibility of such an unrestricted cloning device entails that it is impossible to produce an arbitrary measurement state by deterministic dynamical evolution, and therefore that a measurement device, understood as a device that produces distinguishable measurement outcomes, must do so stochastically.

Bub draws the following moral from the no-cloning theorem:

[31] Bub[2007], p. 242.

> To sum up: if ‘no cloning’ is accepted as a fundamental principle, then our world must be such that there is no dynamical account of the individual occurrence of the outcome of a quantum measurement, which is to say that the world is ‘irreducibly statistical’.[32]

This quotation suggests that a dynamical account of measurement must be deterministic. If the only available dynamics is unitary Schrödinger evolution, then this assimilation of dynamics to determinism might seem justified. Indeed, that a purely dynamical account of the measurement process that takes as its starting point the completeness and correctness of quantum mechanics has to fail is perhaps not surprising.[33] This state of affairs is precisely what led John Bell to famously remark that ‘either the wave function, as given by the Schrödinger equation, is not everything, or it is not right.’[34]Bub identifies the nexus of the difficulty not in the dynamics of the theory but in our attitude towards the dynamics. In particular, it is the dogmas of quantum state realism and the search for a dynamical explanation of all quantum mechanical phenomena, including the measurement process, that gets us into trouble with the ‘big measurement problem’. It is only if we insist on the possibility of cloning (i.e., that the wavefunction as given by the Schrödinger equation is both complete and

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[32] Ibid., p. 243.

[33] Except, perhaps, to an Everettian.

[34] Bell [2004], p. 201.

correct) that we are led to the impression of a foundational difficulty in the theory of quantum measurement.

Bub suggests that the first dogma, that measurement should never be introduced as a primitive into a fundamental physical theory, is called into question by the no-cloning theorem. If a dynamical account of measurement is in principle unavailable, then the concept of measurement must continue to occupy a privileged foundational role. Of course, one may accept this conditional statement while rejecting both antecedent and consequent. It can be interpreted as strong motivation to continue the search for a more adequate dynamical account of the measurement process.

The motivation for what Bub and Pitowsky call the 'first dogma of quantum mechanics' is not, at root, a predilection for dynamical explanations of all physical phenomena. One can accept the first dogma while also accepting that there are, for example, perfectly good examples of kinematical explanations. Rather, the goal is to try to find a formulation of our most fundamental dynamical theory which does not contain, within its own formulation, a privileged role for observers. Presumably the interactions between systems and observers are fundamentally amenable to a physical description, insofar as observers are themselves physical systems. But by introducing observation as a primitive concept into the formulation of the theory, we create a division of the world into 'systems' and 'observers' without providing a principled account of where this division is to be affected. This introduces an uncomfortable arbitrariness into the theory with respect to the question of scope. The point is expressed succinctly in the following quotation from John Bell:

> The concepts 'system', 'apparatus', 'environment', immediately imply an artificial division of the world, and an intention to neglect, or take only schematic account of, the interaction across the split…Einstein said that it is theory that decides what is 'observable'. I think he was right- 'observation' is a complicated and theory-laden business. Then that notion should not appear in the *formulation* of fundamental theory.[35]

Chapter 4 of the present work is concerned to defend the 'second dogma of quantum mechanics', namely quantum state realism. There are two comments that can be made immediately with respect to Bub's diagnosis of the measurement problem. The first is that Bub's rejection of the first dogma does not so much solve the measurement problem as elevate it to the status of a constitutive principle. Rejecting this dogma is not simply a matter of giving up on the possibility of a complete dynamical account of measurement in terms of the linear Schrödinger dynamics. One can hold the view that measurement processes are stochastic while also maintaining a realist attitude with respect to the quantum state. This is achieved, for example, by dynamical collapse models. The important point is that it is possible to furnish a physical description of measurement processes within a realist framework if we allow for a slightly less restrictive notion of dynamics than simply 'unitary Schrödinger

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[35] Bell [1989]. Reprinted in Bell [2004], p. 215.

evolution'. In other words, it is not the case that measurement processes are necessarily distinct from dynamical processes and must therefore be postulated separately. To accept the latter view would be tantamount to biting off much more than simply 'measurement processes are irreducibly statistical'.

Secondly, Bub compares the attempts to grapple with the measurement problem to the historical attempts to provide a dynamical account of length contraction in the context of special relativity. Bub writes:

> Putting it differently, a solution to the big measurement problem, say along the lines of Bohm's hidden variable theory, is simply an attempt to provide a dynamical explanation for 'no cloning'. It is analogous to Lorentz's attempt to provide a dynamical explanation for length contraction in terms of distortions that occur to bodies as they move through the ether.[36]

Just as Lorentz sought to explain length contraction in terms of dynamical distortions in bodies moving through a mechanical ether, so contemporary quantum foundations 'dogmatists' attempt to furnish a dynamical account of measurement. But the analogy is not apt. In the case of the special theory of relativity, the combination of the light postulate, according to which the speed of light is independent of the source, and the principle of relativity, according to which the laws of physics are Lorentz invariant, is

[36] Bub [2007], p. 242.

sufficient to explain the observed phenomena of time dilation and length contraction. What emerges from a conceptual analysis of these two postulates is that there is no need to postulate a relation of absolute simultaneity. It is not clear that the phenomena in question are explained on purely kinematical grounds, since the explanation draws upon, among other things, the behavior of light rays and the description of intermolecular forces within solid bodies. In any case, the theory is uncontroversially self-consistent; if the forces governing the behavior of solid bodies are Lorentz cavariant, then rods in motion relative to a given rest frame will contract.

By contrast, the attempts to find a solution to the measurement problem in quantum mechanics are motivated by an apparent inconsistency between the dynamics of the theory (unitary Schrödinger evolution) and observed determinate measurement outcomes. By making the apparatus part of the system and describing it quantum mechanically, we simply move the 'cut' between system and apparatus by subsuming the combined system+apparatus under a higher level of description. This is a state of affairs that requires interpretation; the presence of the cut in the formulation of the theory is problematic if we want to hold onto the idea that it is our fundamental theories themselves that determine what counts as a measurement event. If observation is not to be accorded a privileged status as a process not governed by the Schrödinger dynamics, then a dynamical account of measurement must be sought that addresses this apparent tension. None of these difficulties are present in the case of special relativity. It is possible to give a rigorous and consistent account of measurement processes in special relativity that is lacking in the standard formulation of quantum mechanics. It is clearly desirable to find a theory capable of accounting for both measurement processes and ordinary (unitary) dynamics, thereby providing a unified picture of the physical world.

*Fuchs on Quantum Measurement as Bayesian Updating*

Chris Fuchs also addresses the measurement problem explicitly in his "Quantum mechanics as quantum information (and only a little bit more)". According to Fuchs' account, quantum states represent the beliefs of agents interacting with quantum systems, and hence collapse is just a special case of belief updating:

> Up to an overall unitary 'readjustment' of one's final probabilistic beliefs-the readjustment takes into account one's initial state for the system as well as one's description of the measurement interaction-quantum collapse is *precisely* Bayesian conditionalization.[37]

If it were true that quantum collapse is 'precisely Bayesian conditionalization', and nothing more, then the problem of measurement would seem to be banished

[37] Fuchs [2002]. arXiv e-print quant-ph/0205039, p. 8. Later published as Fuchs [2003].

altogether.[38] Unfortunately, as Fuchs acknowledges, it is not. Fuchs writes about quantum measurement:

A quantum measurement is any "I know not what" that generates an application of Bayes' rule to one's beliefs for the outcomes of a standard quantum measurement-that is, a decomposition of the initial state into a convex combination of other states and then a final "choice" (decided by the world, not the observer) within that set.[39]

The decomposition of the initial state Fuchs refers to is a decomposition of the state $\rho$ into a weighted sum of normalized density operators $\rho_d$ :

$$\rho = \quad _{d} P(D)\, \rho_d \qquad (5)$$

We are asked to imagine an observer refining his initial state of belief by choosing a term from this sum corresponding to the "data" collected. This is followed by a 'mental readjustment' of the observer's beliefs which, according to Fuchs, 'takes into account details both of the measurement interaction and the observer's initial quantum state'.

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[38] Fuchs writes: 'The formal similarities between Bayes' rule and quantum collapse may be telling us how to finally cut the Gordian knot of the measurement problem. Namely, it may be telling us that it is simply not a problem at all! Indeed, drawing on the analogies between the two theories, one is left with a spark of insight: perhaps the better part of quantum mechanics is simply 'law of thought''.

[39] Ibid., p. 38.

This mental readjustment stage is unitary and hence information preserving, and leads to the post-measurement state $\rho_d$:

$$\rho_d \rightarrow \rho_d = V_D \rho_d V_D^{\dagger} \qquad (6)$$

Where $V_D$ is a unitary operation.

According to the standard treatment which Fuchs criticizes, mixed quantum states represent states of ensembles whose components are in different pure states, or states of individual systems about which we have only incomplete knowledge. In the latter case, the weight attached to a given pure state reflects the epistemic probability assigned to the proposition that the given system is in that pure state. In the standard representation, a mixed state is therefore associated with both objective and epistemic probabilities. When a measurement is performed, the state collapses to an eigenstate of the associated observable, and information is gained about the nature of the state in question. Measurement processes therefore involve a combination of updating and collapse.

In the case where $\rho$ is a pure state, knowledge of the state is complete, and the process of measurement involves a pure disturbance of the system into a new pure state. Fuchs comments on this standard collapse picture for pure states:

> Let us take a moment to think about this special case in isolation. What is distinctive about it is that it captures in the extreme a common folklore associated with the

measurement process. For it tends to convey the image that measurement is a kind of gut-wrenching violence: In one moment the state is $\rho = |\psi\rangle\langle\psi|$, while in the very next it is $\Pi_i = |i\rangle\langle i|$. Moreover, such a wild transition does not depend upon the details of $|\psi\rangle$ and $|i\rangle$; in particular the two states may be almost orthogonal to one another. In density operator language, there is no sense in which $\Pi_i$ is contained in $\rho$; the two states are in distinct places of the operator space. That is,

$$\rho \neq \sum_i P(i)\Pi_i .$$[40]

Fuchs now contrasts this standard measurement 'folklore' with the general framework for information gathering in Bayesian probability theory. Suppose Alice's initial state of belief is captured by $P(h)$ for some hypothesis $h$. Suppose that Alice subsequently acquires some new piece of information $d$. We can capture the idea that Alice conditionalizes on the new evidence by expanding $P(h)$ in terms of the joint probability distribution $P(h,d)$ and choosing the term corresponding to the new evidence $d$:

$$P(h) = \sum_{di} P(h|d)P(d) \xrightarrow{d} P(h|d) \qquad (7)$$

[40] Fuchs [2002], pp. 29-30.

The left side of equation (7) is just a partition of the event space, with $P(h|d)$ representing the new state of belief in light of the acquired evidence $d$. The decomposition of the initial state into a convex sum, combined with the picture of Bayesian updating as picking out the conditional probability from the sum, suggests a possible strategy for drawing a formal analogy between quantum measurement and Bayesian updating.

Fuchs now asks:

> Why does quantum collapse not look more like Bayes' rule? Is quantum collapse really a more violent kind of change, or might it be an artifact of a problematic representation?[41]

As a first step toward answering this question, we might attempt to represent the initial quantum state as a convex sum of post measurement states $\rho_m$:

$$_m P(m)\rho_m \qquad (8)$$

Performing a measurement on $\rho$ would then yield an outcome *m,* allowing us to 'pick off' the post-measurement state of the system $\rho_m$. Hence the transition from pre- to post-measurement state would take the form $\rho \rightarrow \rho_m$ in analogy with the transition from $P(h) \rightarrow P \;\; H \;\; D$ in a classical Bayesian setting.

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[41] Ibid., p. 30

Unfortunately the analogy with classical updating breaks down, because in general the initial state $\rho$ cannot be represented in the form $\sum_m P(m)\rho_m$, where $\rho_m$ is the post-measurement state. However, it is possible to express $\rho$ as a sum (though not generally of post-measurement states) by defining the state

$$\rho_m = \frac{1}{p(m)} \sqrt{\rho} E_m \sqrt{\rho} \tag{9}$$

Where the $E_m$ are effect operators.[42] Using this definition, Fuchs rewrites $\rho$ as

$$\rho = \sum_m p(m)\, \rho_m \tag{10}$$

Where we have used the fact that

$$\rho = \rho^{1/2} I \rho^{1/2} = \sum_m \rho^{1/2} E_m \rho^{1/2} \tag{11}$$

The picture of measurement that emerges from this framework is as follows. Suppose that Alice's initial (pre-measurement) quantum state is given by $\rho$. Upon

[42] Here the positive operator valued measure (POVM) formalism is being used. Each effect operator $E_m$ is associated with a measurement outcome $m$, and satisfies the completeness condition that the set of such operators sums to the identity. Furthermore, according to the (generalized) Born rule, the probability of obtaining a measurement outcome m is given by $p(m) = tr(E_m \rho)$.

measurement, Alice observes the outcome $m$. Alice updates her beliefs by selecting $\rho_m$ from the sum (10). In general $\rho_m$ will not, however, equal any post-measurement states $\rho_m$. This is where the analogy with classical conditioning breaks down.[43]In order to recover $\rho_m$, Fuchs introduces a second step into the picture. At this stage, Alice must perform a 'mental readjustment' on her 'refined belief' $\rho_m$ in order to arrive at the post-measurement state $\rho_m$. Fuchs represents this final adjustment by a unitary operator $U_m$ such that

$$\rho_m = U_m \rho_m U_m^{\dagger} \qquad (12)$$

Hence the overall picture that emerges of the measurement process consists of two parts. In the first stage, an agent refines their beliefs by selecting the state $\rho_m$ from the sum given on the right side of (10). Next, the agent performs a unitary adjustment on $\rho_m$, thereby transforming it into the post-measurement state $\rho_m$. Fuchs writes that 'quantum measurement is nothing more, and nothing less, than a refinement and a readjustment of one's initial state of belief.'[44] The latter mental readjustment is supposed to 'take into account details both of the measurement interaction and of the observer's initial quantum state'.[45]The justification given for this factorization of the

[43] The discussion that follows is indebted to Palge and Konrad [2008].

[44] Fuchs [2002], p. 22.

[45] Fuchs [2002], p.22.

measurement process is based on the observation that the input and output states are those of the standard quantum mechanical formalism.[46]

Does this picture succeed in demonstrating that the 'gut-wrenching violence' associated with collapse is an 'artifact of a problematic represention'? Certainly Fuchs has dressed quantum measurement in formal garb that more closely resembles classical Bayesian updating. What is initially surprising is the apparent lack of a formal analog of von Neumann collapse in the formalism given in equations (8) through (12). What we have instead is Bayesian style belief refinement followed by a unitary evolution. But of course collapse is non-unitary. How then is Fuchs' treatment able to recover the post-measurement state $\rho_m$? Formally, the trick is accomplished by associating a distinct unitary operator with every $\rho_m$ associated with a belief refinement (see equation (9)). This association is mandated by the fact that unitary transformations are bijective; if the final state $\rho_m$ is to be recovered from each distinct $\rho_m$ for which the probability of $\rho_m$ is non-zero, we will need a distinct $U_m$ corresponding to each of the possible $\rho_m$ to accomplish the task.

What is it about the details of the measurement interaction that guides this unitary 'mental readjustment' in the second stage of measurement? Since unitary transformations conserve the von Neumann entropy, there can be no increase in information at this stage.[47] Instead, we are asked to imagine the measuring device

[46] Fuchs calls the factorization a 'purely conceptual game'. (Fuchs [2002], p. 34).

[47] For a discussion of the von Neumann entropy, see Fuchs [2002], p. 31-32.

'enforcing a further 'back action' or 'feedback' on the measured system', although, since the factorization of measurement into refinement and readjustment is a purely conceptual game, nothing crucial is supposed to hang on this intuitive picture. Consider the special case where the observer's state of belief is represented by a pure state $\rho = |\psi\rangle\langle\psi|$. In this case, no measurement can refine the state of belief, since for any $E_d$, the terms in equation (11) simply reduce to

$$\rho^{1/2} E_d \rho^{1/2} = P(d)|\psi\rangle\langle\psi| \qquad (13)$$

Since pure states are states of maximal information, measurements on such states are purely of the mental readjustment sort. Fuchs writes:

> The only state change that can come about from a measurement [of a pure state] must be purely of the mental readjustment sort: We learn nothing new; we just change what we can predict as a consequence of the side effects of our experimental intervention. That is to say, there is a sense in which the measurement is solely disturbance.[48]

It deserves to be said that in the pure state case, belief refinement drops out altogether. But the state change brought about by measurement on a pure state is in

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[48] Ibid., Pg. 34

general not unitary.[49] On the other hand, on Fuchs' account, a measurement on a pure state is entirely of the 'mental readjustment' sort, the essential consequence of the measurement interaction is to change the predictions that can be made about the system, and involves no information gain. The measurement disturbs the system into a new pure state, and the mechanism of this disturbance is governed by the von Neumann collapse postulate. Crucially, the disturbance in question cannot be the result of a dynamical interaction governed by the Schrödinger equation between the system and the measuring apparatus, since such an interaction would lead to an entangled state, and not to the state obtained by a 'mental readjustment'. Nor can it be modeled as an instance of classical Bayesian belief refinement. Hence collapse has not been eliminated after all in Fuchs' representation; it remains as an 'uncontrollable disturbance' that is not merely an 'artifact of a problematic represention', as is seen most clearly in the case of a pure state. For pure states, Fuchs' 'mental readjustment' is *precisely* what is normally regarded as collapse. The collapse postulate captures formally what from a physical point of view appears to be a physical process not governed by the usual Schrödinger dynamics. [50] This is, of course, the measurement

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[49] Fuchs writes: 'In Particular, when the POVM is an orthogonal set of projection operators $\{\Pi_i = |i\rangle\langle i|\}$ and the state-change mechanism is the von Neumann collapse postulate, this simply corresponds to a readjustment according to the unitary operators $U_i = |i\rangle\langle\psi|$.' As Palge and Konrad [2008] point out, however, the latter operator is not unitary.

[50] I do not wish to beg the question against Everettian interpretations.

problem. Fuchs' picture of mental readjustment suggests, misleadingly, that the state transition in question can be regarded as a mere change of belief.

Hence there is a basic philosophical worry associated with Fuchs' picture. Fuchs introduces the above formalism, at least in part, to address the measurement problem in an antirealist setting. Fuchs writes:

> The formal similarities between Bayes' rule and quantum collapse may be telling us how to finally cut the Gordian knot of the measurement problem. Namely, it may be telling us that it is simply not a problem at all!

Modulo a final readjustment, quantum collapse is supposed to be Bayesian updating, and this is supposed to alleviate our worries about accounting for the pre- and post-measurement states of a quantum system, since 'quantum collapse is something that happens in our description of the system, not to the system itself'.[51] As we have seen, the problem with Fuchs' picture is that the post-measurement state is not generally available in the form of a term in equation (8) above. Nor is there a way to recover the post-measurement state using the resources of Bayesian updating alone. In the case of pure states, belief refinement drops out altogether and we are left once again with a picture of uncontrollable disturbance.

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[51] Fuchs and Peres [2000], p. 71

Fuchs' treatment of mixed states, involving information acquisition and corresponding belief refinement, is also problematic. If the post-measurement state were contained in (8), we would seem to be off to the races; quantum measurement could then be recovered as a special case of Bayesian 'refinement' of prior belief, updated in light of the new evidence associated with measurement outcomes. But since $\rho_m$ is not in general a post-measurement state, it is difficult to see how it could be 'decided by the world', to use Fuchs' phrase. The difficulty is only placed in sharper relief by consideration of the fact that the map taking $\rho$ into $\rho_m$ is nonlinear, and is therefore outside of the space of state transitions allowed by the standard Schrödinger evolution. If $\rho_m$ is neither a post-measurement state nor a state reachable by the standard formalism, then in what sense can it be said to be decided by the world? There does not appear to be any physical motivation for invoking the intermediary belief refinement state $\rho_m$. Furthermore, the final mental readjustment that takes us from $\rho_m$ to $\rho_m$ is left essentially unjustified. Since the relevant transformation is unitary it is also information-preserving, and so cannot be taken to contribute to learning. Instead we are invited to imagine a feedback mechanism in the measuring apparatus. Given the purely conceptual character of the factorization of the measurement process, it is doubtful that much weight can be attached to such a back-reaction picture.

We must therefore conclude that Fuchs has not succeeded in addressing the measurement problem by recasting the projection postulate as a special case of Bayesian updating. Measurement events result in state transitions that do not appear to be governed by the Schrödinger dynamics. This is a state of affairs that ought to trouble anyone who is convinced that measurement interactions, qua physical events, ought to be amenable to quantum mechanical description. Simply recasting quantum states as states of belief does not address the problem, since the agent who performs a measurement on a quantum system finds that the post-measurement state is, to use Fuchs' phrase, 'decided by the world'.[52] The limiting case of measurement on a pure state brings this point into particularly sharp relief.

In recent work (Fuchs [2010], [2009]), Fuchs describes the Born rule as an 'empirical addition to Bayesian coherence', acknowledging explicitly that Born rule probabilities obtained from the quantum state are constrained by empirical conditions that go beyond the purely rational constraints imposed by Dutch book coherence. I will have more to say about Fuchs' interpretation of the Born rule in chapter 3 below.

## 2.2 Is Locality Incompatible with Quantum State Realism?

*Einstein's Incompleteness argument and Quantum State Antirealism*

[52] I will have more to say about this point in chapter 3.

In *Albert Einstein: Philosopher Scientist*, Einstein presents an indirect argument for the incompleteness of quantum mechanics and the quantum state description of physical reality.[53] Many contemporary authors have drawn inspiration from Einstein's original argument, combined with the subsequently discovered 'no-go' theorems due to Bell, Kochen and Spekker, to conclude that the quantum state must be interpreted epistemically. Chris Fuchs identifies Einstein's incompleteness argument as 'the cleanest argument I know that the quantum state is solely an expression of subjective information-the information one has about a quantum system'.[54] According to Fuchs, the argument demonstrates both the incompleteness of quantum mechanics and the subjectivity of the quantum state.[55] In a recent paper, Nicholas Harrigan and Robert Spekkens build on this interpretation of the significance of Einstein's argument. The authors write that 'Einstein showed that not only is locality inconsistent with [the quantum state] $\psi$ being a complete description of reality, it is also inconsistent with $\psi$ being ontic.' The purpose of the present section is to evaluate the claim made by

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[53] The argument presented in the Schilpp volume simplifies significantly the argument put forward originally in Einstein, Podolsky and Rosen [1935]. I will refer to the latter paper as the EPR paper in subsequent discussion.

[54] Fuchs [2003], p. 5.

[55] Although, as will become clear, Fuchs has in mind a different notion of completeness than the EPR criterion. See also chapter 3 of the present work.

these authors that locality is inconsistent with a realist or ontic interpretation of the quantum state.

Einstein's argument in the Schilpp volume attempts to derive a contradiction from the assumption of completeness and the claim that the state of a physical system $S_1$ does not depend on the kind of measurement performed on a spatially separated system $S_2$. Einstein asks us to imagine that $S_1$ and $S_2$ initially interact with one another and are subsequently separated:

> Now it appears to me that one may speak of the real state of the partial system $S_2$. To begin with, before performing the measurement on $S_1$, we know even less of this real state than we know of a system described by the $\Psi$ - function. But on *one* assumption we should, in my opinion, hold fast: The real situation (state) of system $S_2$ is independent of what is done with system $S_1$, which is spatially separated from the former. According to the type of measurement I perform on $S_1$, I get, however, a very different $\Psi_2$ for the second partial system…But now the real state of $S_2$ must be independent of what happens to $S_1$. Thus, different $\Psi$-functions can be found (depending on the choice of the measurement on $S_1$) for the same real state of $S_2$. (One can only avoid this conclusion either by assuming that the measurement on $S_1$ changes (telepathically) the real state of $S_2$, or by generally denying independent real states to things which are spatially separated from each other. Both alternatives appear to me entirely unacceptable.)[56]

Einstein concludes that the $\Psi$-function cannot be a complete description of the physical situation, since different $\Psi$-functions are associated with system $S_2$

[56] Schilpp, A. (ed.), [1949] p. 84-85

depending on the sort of measurement carried out on $S_1$. Einstein assumes that partial systems have real states, and that these states are amenable to physical description. It is clear that two distinct and correct quantum mechanical representations of the same physical state are implicated in the scenario Einstein describes depending on the type of measurement performed.[57] This seems to entail that the quantum mechanical description associated with either measurement is at best incomplete.

The incompleteness conclusion is not forced on us as a matter of logic, as Einstein acknowledges. We can avoid the incompleteness conclusion by assuming an instantaneous 'telepathic' influence of one system on the other upon measurement, or by denying 'independent real existence' to both systems insofar as they occupy different regions of space. In other words, in order to avoid the incompleteness conclusion, we must reject at least one of the following propositions:

i)Subluminal causation: there cannot be a faster-than-light causal interaction between spatially separated regions $S_1$ and $S_2$; or

ii) $S_1$ and $S_2$ can be regarded as having independent real states insofar as they occupy different regions of space.

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[57] Reacting to this consequence of the argument, Erwin Schrödinger writes: "It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it." (Schrödinger [1935], p. 555).

I will follow Howard (1985) in referring to (i) as the 'locality principle' and (ii) as the 'separability principle'. Einstein held that the rejection of either of these principles would involve a radical reformulation of physics as we know it, enforcing a reconsideration of basic suppositions about the formulation and testing of physical theory. Here is a quotation from a 1948 *Dialectica* article in which Einstein expresses serious reservations about abandoning separability:

> Without such an assumption of the mutually independent existence ... of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation.[58]

At the time of Einstein's writing, the available empirical evidence did not rule out a local hidden variable theory as the most obvious explanation of the statistical character of quantum mechanics. More crucially, it was not known at the time that the empirical predictions of quantum mechanics are in fact incompatible with such an explanation. Einstein therefore understandably chose to interpret quantum mechanics as an incomplete theory, awaiting supplementation by a hidden variable

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[58] Translation from Howard [1985], pp. 171-201.

framework that would explain the probabilities computed from the quantum state in terms of incompletely known degrees of freedom.

It is worth emphasizing that the locality and separability principles are logically independent of one another. This implies that it is possible to construct a local, nonseparable physical theory. Theories that admit entanglement as a basic feature of the physical world but which deny the possibility of superluminal causation are in this category. The EPR scenario Einstein presents above is amenable to such a theoretical description. If the quantum state is interpreted realistically, then entanglement between $S_1$ and $S_2$ implies nonseparability, even in the absence of a mechanical interaction between the two subsystems when a measurement is carried out on either.[59]

### *The Bell Factorizability Condition and Locality*

The possibility of interpreting the phenomenon of quantum entanglement as an instance of nonseparability raises the question of whether nonseparable theories can

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[59] This may (or may not) be what Bohr [1935] has in mind when he writes: 'Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system.*'

be made relativistic. Bell's theorem is often glossed as precluding the possibility of a 'local hidden variable theory'.[60] This does not rule out a relativistic interpretation of quantum theory which is not of the hidden variable sort. The locality assumption in Bell's Theorem can be expressed formally as the so-called 'Bell factorizability condition', which is a constraint on the probabilities associated with measurement outcomes performed at spacelike separation:[61]

$$p_m(s,t|a,b) = p_m^{S_1}(s|a)\, p_m^{S_2}(t|b) \tag{14}$$

Where *a* and *b* are measurements performed on $S_1$ and $S_2$, respectively, *s* and *t* are the outcomes of those measurements, and *m* is the 'complete state', containing all of the properties of the pair ($S_1$, $S_2$). This factorization condition can be used to derive a Bell-type inequality. What Bell's theorem and subsequent empirical tests show is that any empirically viable theory must violate condition (14) by predicting correlations between distant measurement outcomes not attributable to common causes in the past light cones of $S_1$ and $S_2$. Jarrett [1984] has shown that condition (14) is equivalent to the conjunction of two logically independent conditions, most commonly labeled 'outcome independence' and 'parameter 'independence'[62]. Outcome independence is

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[60] Cf. http://plato.stanford.edu/entries/bell-theorem/

[61] Adapted from Shimony [2009].

[62] This terminology is due to Shimony [1984].

the condition that, once all common causes have been screened off, the experimental outcomes on $S_1$ do not change the probabilities associated with those on $S_2$:

$$P_m^{s_1}(s|a,b,t) \equiv P_m^{s_1}(s|a,b) \tag{15}$$

Parameter (or remote context) independence is the condition that the probabilities associated with outcomes on $S_1$ do not depend on the setting of $S_2$:

$$P_m^{s_1}(s|a) \equiv P_m^{s_1}(s|a,b). \tag{16}$$

The equivalence of these independence conditions with the BF condition (14) entails that any theory that recovers the empirical predictions of quantum mechanics must violate at least one of these conditions.

If one regards relativity as implying that events at spacelike separation must be statistically independent of one another, in the sense that the probabilities associated with joint outcomes at $S_1$ and $S_2$ are just the product of their individual occurrences, the empirically well-established violations of Bell's inequality rule out a genuinely relativistic interpretation of quantum theory. It can be argued, however, that relativity does not entail such strong claims about separability. Instead, what is required for a genuinely relativistic theory is the imposition of a certain kind of spacetime structure, namely the structure of Minkowski space, along with the postulation of Lorentz invariant physical laws. Such a theory need not be separable, though it would have to obey Howard's locality condition.

Ghirardi and Grassi [1994,1996] have shown that it is impossible to construct a fundamentally Lorentz invariant theory (that is, one that does not contain a hidden preferred reference frame) that exhibits parameter dependence in the nonrelativistic limit. This result is interesting because it shows that theories exhibiting parameter dependence, such as the Bohm theory, must be nonlocal in Howard's sense. On the other hand, it is possible to show that a theory exhibiting only outcome dependence is incompatible with superluminal signaling.[63] It can be shown that the GRW and other dynamical collapse models exhibit only outcome dependence. There is therefore grounds for optimism that a genuinely relativistic quantum theory that recovers the predictions of quantum mechanics in the nonrelativistic limit can be constructed.[64] Furthermore, Roderich Tumulka has recently constructed a variant of the GRW collapse theory that is fully relativistically invariant.[65] In recent work, Daniel

[63] See Eberhard [1978], Ghirardi, Rimini and Weber [1980]. For a general discussion of these results, see Ghirardi [2008].

[64] Of course, what is ultimately required is a relativistic quantum field theory. For present purposes, however, we are concerned to show only that a realistic interpretation of quantum mechanics in the non-relativistic limit is compatible with special relativity.

[65] Tumulka [2006].

Bedingham has developed a relativistic collapse model that incorporates interacting particles.[66]

Given that it is possible to write down a Lorentz invariant theory, in Minkowski space, that recovers the empirical predictions of quantum mechanics and therefore violates (14), it is not clear that Bell inequality violations rule out compatibility with the special theory of relativity. This remains true even if the quantum state is interpreted realistically as furnishing an objective description of nonseparable, entangled systems. The Bell inequality violations imply that any theory will violate outcome independence, or parameter independence, or both. This rules out the possibility of a local hidden variable model that furnishes a complete description of individual systems. What light do these results shed on Einstein's incompleteness argument? Fuchs is persuaded by Einstein's argument for the incompleteness of the quantum state description:

> Rejecting the rigid connection of all nature-that is to say, admitting that the very notion of separate systems has any meaning at all- one is led to the conclusion that the quantum state cannot be a complete specification of the system. It must be information, at least in part.[67]

Fuchs goes on to draw the following lesson from the Bell inequality violations:

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[66] Bedingham [2011].

[67] Fuchs [2003], p. 7.

> The last 19 years have given confirmation after confirmation that the Bell inequality (and several variations of it) are indeed violated by the physical world. The Kochen-Specker no-go theorems have been meticulously clarified to the point where simple textbook pictures can be drawn of them. Incompleteness, it seems, is here to stay: the theory prescribes that no matter how much we know about a quantum system—even when we have maximal information about it—there will always be a statistical residue.

In Fuchs' view the chief lesson to be drawn from Bell's theorem is that quantum mechanics is 'necessarily incomplete', in the sense that the theory is irreducibly statistical.

For our purposes, there are two important points to be made with respect to Fuchs' remarks. The first is that Fuchs' use of the term 'incompleteness' is different than Einstein's. In Einstein's (EPR) sense of the term, a theory is complete just in case every element of reality is captured within the theory. If there are objective chances, perhaps encoded in the quantum state for a physical system, then a stochastic theory may be complete in the EPR sense while remaining incomplete in Fuchs' sense. Hence the fact that quantum mechanics is irreducibly statistical does not by itself constitute an argument for the incompleteness of the quantum state description in the EPR sense, that there are elements of reality (hidden variables) not represented in the quantum state.

The second point is that Fuchs does not distinguish clearly between the notions of separability and locality, at least in the quotation above. Rejecting the 'rigid connection of all nature' would seem to suggest the impossibility of non-local interactions. On the other hand, 'admitting that the very notion of separate systems has any meaning at all' suggests the denial of nonseparability.

Harrigan and Spekkens [2010], like Fuchs, see Einstein's incompleteness argument as an argument from locality to an epistemic interpretation of the quantum state.[68] While acknowledging the distinction between separability and locality, the authors explicitly state, in contradistinction to the view defended here, that separability is a necessary condition for locality.[69] While stating without argument that separability is essential to 'any sensible notion of locality', the authors also point out that it is not sufficient. To the separability condition they add a local causality condition derived from Bell [1981]. According to Harrigan and Spekkens' definition, a separable ontological model of the quantum state is 'locally causal' just in case the probabilities of events in a spacetime region B are independent of events in space-

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[68] However, it should be emphasised that Harrigan and Spekkens, unlike Fuchs, do not view the argument as conclusive in favor of an epistemic interpretation.

[69] The authors write: 'A necessary component of any sensible notion of locality is separability'. (Harrigan and Spekkens [2010], p. 140). The authors go on to provide a formal definition of separability in terms of a Cartesian product of distinct spatial regions, which is essentially equivalent to the informal definition due to Howard [1985].

time region A, once one has 'screened off' common causes in the intersection of the backward light cones of A and B.[70] This condition can be expressed formally as:

$$p(B|A,\lambda_c) = p(B|\lambda_c) \qquad (17)$$

where *A* and *B* are propositions about events occurring in regions A and B, and $\lambda_c$ is a complete specification of the properties of space-time region C.

Harrigan and Spekkens now define 'locality' as the conjunction of their local causality and separability conditions. Any theory that fails either conjunct will therefore fail to be local according to the authors. For example, for an entangled state such as the singlet state, a factorization of the composite system $\psi_{AB}$ into $\psi_{AB} = \psi_A \otimes \psi_B$ cannot be achieved, implying a failure of separability, and hence also of locality according to Harrigan and Spekkens' definition.

It is easy to see that according to Harrigan and Spekkens' definition, any theory that exhibits Bell inequality violations is necessarily 'non-local' (where 'locality' has been defined as the conjunction of separability and condition (17)) . This is true regardless of one's attitude toward the nature of the quantum state. Since equation (17) implies the Bell factorizability condition as it is defined in equation (14), it is easy to show that any theory that recovers the predictions of quantum mechanics, whether

[70] The term 'locally causal', as used by Harrigan and Spekkens, should not be confused with Howard's locality condition.

realist or otherwise, will exhibit 'nonlocality' in Harrigan and Spekkens' sense. The authors prove that 'any ψ-ontic ontological model that reproduces the quantum statistics (QSTAT) violates locality',[71] i.e,

$$\psi\text{-ontic \& QSTAT} \rightarrow \sim L. \qquad (18)$$

But this conditional is trivially satisfied, since the second conjunct in the antecedent already implies the consequent. Furthermore, the consequent is itself the negation of a conjunction one of whose conjuncts, local causality (as defined in (17)), implies the failure of QSTAT. Hence (18) can be rewritten:

$$(\psi\text{-ontic \& } \sim LC \rightarrow \sim(S \text{ and } LC)) \leftrightarrow (\psi\text{-ontic \& } \sim LC \rightarrow \sim LC \vee \sim S) \qquad (19)$$

where 'LC' indicates local causality and 'S' is separability.

Hence a failure of 'locality' in Harrigan and Spekkens' sense is no argument against a realist or ontic interpretation of the quantum state. Since any empirically adequate interpretation of quantum mechanics is constrained to predict Bell inequality violations, it follows that any empirically adequate interpretation of the quantum state must also be non-local in Harrigan and Spekkens' sense. As the authors acknowledge, Bell locality is neutral with respect to the question of the status of the

[71] This is theorem 11, ibid., p. 143.

quantum state; Bell inequality violations therefore cannot be harnessed to argue in favor of an epistemic interpretation. This shows that although the conditional

$$L \ \& \ QSTAT \rightarrow \sim(\psi\text{-ontic}) \qquad (20)$$

is logically equivalent to (18), as indicated by by Harrigan and Spekkens, it is also trivially satisfied, since its antecedent is logically false.[72]

Properly distinguishing between the logically independent concepts of locality, understood in the Einstein/Howard sense, and separability turns out to be very important if the task at hand is to try to formulate a relativistic theory that respects Bell's theorem. Bell's theorem does not demonstrate the incompatibility of relativity with any realistic (or 'psi-ontic') theory that recovers the empirical predictions of quantum mechanics. What is implied by the theorem is that any such theory must violate outcome independence or parameter independence, or both. Theories that violate outcome independence alone have been constructed; the GRW theory is an example of a such a theory. As has already been pointed out, the GRW theory does not require a preferred foliation, nor does it permit superluminal signaling. It is on this basis that John Bell remarks that the GRW theory 'takes away the ground of my fear

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[72] Since L is the conjunction of separability and local causality, and the latter condition implies the negation of QSTAT.

that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance.'[73]

*Local Beables and the Prospects for a Local, Realist Quantum Theory*

According to Chris Fuchs, interpreting the quantum state realistically is incompatible with locality. Fuchs contends that any viable interpretation of the quantum state ought to 'expel once and for all the fear that quantum mechanics leads to 'spooky action at a distance'. One reason to fear spooky action at a distance is that it is *prima facie* unrelativistic. If one is inclined to hold the view, explicit in Harrigan and Spekkens [2010] and arguably implicit in Fuchs' analysis, that 'any sensible notion of locality presupposes separability', then it might be supposed that separability is a necessary condition for us even to make sense of the principle that interactions are local.[74] This would be problematic, since the requirement of local interaction is a cornerstone of quantum field theory. If this is the worry of Fuchs, Harrigan and Spekkens, however, it is unfounded. It can be shown that the requirement of local

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[73] Bell [1987].

[74] The view that a failure of separability entails nonlocality is also arguably implicit in Norsen [2009]. Norsen remarks that a failure of Jarrett's 'completeness' condition (Jarrett [1984]; our 'outcome independence') 'indicates the presence of some nonlocal causation…in the candidate theory in question.' (288)

interaction does not presuppose separability for its formulation. Theories that contain both local and nonlocal beables can be constructed. If the theories in question can be made to satisfy the requirement of local interaction, and include fundamentally Lorentz covariant dynamics, then there is room for optimism that a realist interpretation of quantum mechanics compatible with the special theory of relativity can be found. As we will see shortly, the key point is that the denial of 'all beables are local beables' does not entail 'no beables are local beables'.[75]

In a relativistic collapse theory, for example, it is possible to explicate the notion of a system's possessing a local, intrinsic property in a natural way.[76] For example, in the 'mass density' formulation of the GRW theory, the local beables of the theory are mass densities on the past light cone of a spacetime region.[77] The GRW theory and related dynamical collapse models are both fundamentally Lorentz invariant and nonseparable. Such theories are also ψ-ontic and complete in the EPR sense: the description of the physical world in terms of the wave function does not leave out any

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[75] The term 'local beable' is due to John Bell [1984]. I will have more to say about local beables below.

[76] See Myrvold [2003], Ghirardi and Grassi [1994], and Ghirardi [2000]. It should be noted that I do not wish to presuppose that any realist interpretation that incorporates local beables will be a variant of the GRW collapse model.

[77] The mass density ontology associates a field $m(x,t)$ with every point $x$, given by the expectation value of the mass density operator $M(x)$ at $x$. The mass density operator is obtained by multiplying the mass of every particle type by the number density operator for that particle, and then summing over all possible particle types. (See Ghirardi [2008]).

elements of reality. Given the completeness of the theory, the probabilities derived from the Born rule are taken to represent objective features of physical chance setups.

To get a sense for how such a nonseparable, locally causal theory works, suppose that Alice and Bob each have a quantum system, located at spacetime points $S_1$ and $S_2$ respectively. Alice and Bob can each subject their systems to external fields. These fields are local in the sense that they can be represented by local operations, in a manner to be explained below. Alice and Bob can also couple their systems to devices that perform measurements. If we assume that the probabilities associated with measurement outcomes on each of $S_1$ and $S_2$ may be correlated, then we can consider the pair to be a spacelike-extended objective chance setup, with chances associated with local measurement outcomes on each member of the pair.

Given that entanglement extends over arbitrary distances, chance setups may become arbitrarily large. An entangled quantum state is therefore associated with such a spacelike extended system. According to special relativity, there is a continuous infinity of possible foliations of an extended four-dimensional object. Whether or not a state is entangled will in general depend upon the specification of a spacelike hypersurface of simultaneity. To introduce measurement operations into this picture, we need only specify that for each foliation of spacetime, collapses occur along those hypersurfaces that intersect measurement events. This has the consequence that expectation values associated with measurements are themselves foliation-relative. In other words, to quote David Albert, we must 'let go of the requirement that the situation associated with two intersecting space-like hypersurfaces in the Minkowski-space must agree with one another about the expectation values of local observables at points where the two surfaces coincide'.[78]

Albert considers this picture to be artificial, demonstrating only the formal compatibility of collapse theories with the structure of Minkowski space. According to Albert, any theory which exhibits nonseparability is 'metaphysically incompatible' with the special theory of relativity, whether or not it is 'dynamically compatible', as the picture sketched above is. While Harrigan and Spekkens focus on the concept of local causality considered as a general condition, Albert is concerned with a somewhat different issue, what he calls the 'nonnarratability' of nonseparable collapse theories. Nonnarratability is the condition that, in a relativistic spacetime, the complete specification of the quantum state history along one foliation does not uniquely determine the state history along another foliation.[79] A quantum mechanical state of the world at a time $t$ is just a specification of the expectation values of all of the local and non-local observables at $t$. Nonnarratability arises from the fact that the entirety of what there is to say about a relativistic quantum mechanical world cannot be specified as a one-parameter family $K$ of such instantaneous states of the world. In particular, it is not hard to show that such a specification will necessarily leave the

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[78]Albert [2000], pp. 5-6.

[79] This was shown in Albert and Aharanov [1984], and generalized to non-measurement contexts in Myrvold [2002].

expectation values of non-local quantum mechanical observables that are instantaneous along some other foliation *K'* unspecified.[80]

The issue, according to Albert, is that we have paid too dear a price in order to save Lorentz invariance in the relativistic collapse picture; we have 'let go of the idea of the world's having anything along the lines of a narratable story',[81]since the history of the world according to one foliation does not uniquely determine the history along any other. Nevertheless, despite Albert's worries, we can still tell a narratable story for any given spacelike region along a given foliation, and the story relative to one foliation can be obtained by means of local dynamical transformations from any other foliation.

The possibility of constructing an account of local causality in a quantum framework depends crucially on the role of what John Bell calls 'local beables'.[82] At the operational level, the description of measurement interactions ultimately reduces to the dispositions and behavior of objects located in space and time. Such an assumption of the localization of physical objects undergirds the operational meaning of such statements as 'the pointer is oriented to the right (or left)'. We must be able to talk about the spatiotemporal dispositions of measurement devices in a systematic way in order to secure, at the experimental level, the evidentiary basis of quantum

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[80] Cf. Albert [2008].

[81]Ibid., p. 6.

[82]Bell [1975].

theory. An adequate theory will be able to furnish predictions, whether deterministic or probabilistic, about the behavior of measurement devices that can then be tested. In other words, the theory must be able to furnish an account of local beables. One reason to suppose that separability is a necessary condition for local causality is that local causality requires local beables. However, separability is not a requirement for the formulation of local causality. Once again, the key point is that the denial of 'all beables are local beables' does not entail 'no beables are local beables'.

As Myrvold [2003], Ghirardi and Grassi [1994], and Ghirardi [2000] have pointed out, in a nonseparable, foliation-relative collapse model, it is possible to construct a picture of the dynamical evolution of states in such a way that the transition from one foliation to another foliation is brought about by means of entirely local operations, in a manner that does not privilege any one foliation, and which preserves the concept of a narratable history of state evolution along any given foliation.[83] On this picture, the differences between state histories given with respect to different foliations are attributable entirely to the fact that these foliations join up points of spacetime in different ways.

Suppose that the quantum state associated with some hypersurface $\sigma$ is given by the density operator $\rho(\sigma)$.[84] Kraus [1983] argues that any operation can be modeled as a completely positive linear mapping $\phi$ of the set of trace-class operators

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[83] See Myrvold [2003], Fleming [1989], and Woodcock [2007].

[84] The following discussion follows that presented in Myrvold [2003].

into itself, and shows that any such function can be represented by a countable set of operators $\{K_i\}$ such that

$$\phi(\rho) = \sum_i K_i \rho K_i^t \text{ with } \sum_i K_i^t K_i \leq I. \qquad (19)$$

Unitary evolution has a Kraus representation consisting of a single Kraus operator. An operation that takes pure states into pure states is called a *pure operation*. A mapping $\phi$ which preserves the trace of $\rho$ is said to be a *nonselective* operation, while an operation that is not trace preserving is said to be *selective*. Within this formalism, a typical measurement event can be thought of as a preparation of an ensemble followed by a selection of those members that yield a given outcome; if collapses are regarded as physical processes, they can be represented as pure, selective Kraus operations in which the selection is made by the system itself. Hence in this case the operation does not preserve the trace of the density operator; the post-collapse state is given by $\phi(\rho)/Tr(\phi(\rho))$ if the initial state is $\phi(\rho)$.[85]

Local evolution of a system is defined in terms of the notion of a *local operation*, by associating an algebra of trace-class operators having Kraus representation *R(O)*

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[85] The advantage of choosing a $\phi$ that does not preserve norm is that the function can be linear. For a discussion of this point see Ghirardi *et al* [1993].

with a bounded spacetime region *O*.[86] The self-adjoint members of *R(O)* correspond to observables measurable via operations confined to *O*. An operation *local* to *O* will have a Kraus representation consisting of operators belonging to *R(O)*. Suppose that the region *O* is bounded by two hypersurfaces $\sigma$ and $\sigma'$, which are coextensive everywhere except in the region *O.* The *local evolution condition* demands that if $\sigma'$ lies nowhere to the past of $\sigma$, then the state on $\sigma'$ be obtainable from $\sigma$ via an operation local to the region *O* between $\sigma$ and $\sigma'$. This condition ensures that any differences in the states $\sigma$ and $\sigma'$ be attributable to events in the region *O*. The local evolution condition is satisfied by the standard formulations of quantum field theory. Furthermore, a theory satisfying the local evolution condition has the property that state transitions between different hypersurfaces are attributable entirely to local operations. This makes it possible to describe the transition from the history of events along one foliation to that along another without picking out any preferred foliation, all the while against the backdrop of Minkowski spacetime.

The local evolution condition, combined with the assumption that operators associated with spacelike observables commute, has important consequences. Myrvold [2003] has shown that for any two spacelike hypersurfaces $\sigma$ and $\sigma'$, such that no part of $\sigma'$ lies to the past of $\sigma$, and $\sigma$ and $\sigma'$ coincide everywhere except in the region *O*, the eigenstates and eigenvalues of observables local to regions spacelike separated from *O* are shared completely by $\sigma$ and $\sigma'$. This in turn entails that if $|\psi(\sigma)\rangle$

[86] See Clifton and Halvorson [2001].

and $|\psi(\sigma')\rangle$ are both eigenstates of some observable $\Omega$ local to a region $\Lambda$ disjoint from *O*, with eigenvalues $\omega$ and $\omega'$, then $\omega = \omega'$.

This result allows to us construct a notion of 'property local to some spacetime region *O*' in a natural way. Consider some four-dimensional object *O* and some spacelike slice $\alpha$ of *O*. Assuming that observables satisfy the usual eigenstate-eigenvalue link, that is, the observable $\Omega$ has the definite value $\omega_k$ if and only if $\Omega|\psi\rangle = \omega_k|\psi\rangle$, then define *O* to have the property $\omega_k$ just in case, as of $\alpha$,

$$\Omega|\psi(\sigma)\rangle = \omega_k|\psi(\sigma)\rangle$$

for the state $|\psi(\sigma)\rangle$taken with respect to any hypersurface $\sigma$ containing $\alpha$. This entails that a system at a given spacetime point *P* has the property $\omega_k$ if the state on the past light cone is an eigenstate of $\Omega$.[87]

The present consideration of dynamical collapse theories and the notion of local properties that such theories employ is intended to show that it is possible to construct a formally exact theory that recovers the predictions of quantum mechanics,

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[87] Myrvold [2003] shows that this condition is equivalent (assuming a local evolution condition along with the usual microcausality condition that operators representing spacelike separated observables commute) to the following formulation given in Ghirardi [2000], p. 1364: 'A system at a spacetime point P possesses the objective property $\Omega = \omega_k$ if and only if the state on the past light-cone is an eigenstate of $\Omega$ belonging to $\omega_k$.'

is compatible with the special theory of relativity in the sense that it is fundamentally Lorentz invariant and incorporates local beables at the operational level, and is realist with respect to the quantum state. Some commentators have objected to such theories on metaphysical grounds, arguing that they are not fully relativistic despite their formal compatibility with the special theory of relativity. Given that theories can be formulated in Minkowski space with a clear notion of local causality based on the attribution of local properties, it is not clear what further metaphysical demands could be placed on such theories in order to render them relativistic. The existence of quantum entanglement gives us good reason to believe that nonseparability is a basic feature of the physical world. This state of affairs necessitates, to borrow Einstein's phrase, a 'departure from physical thought in the sense familiar to us'. But nonseparability does not preclude the possibility of furnishing an exact definition of local, intrinsic properties.

Neither Fuchs nor Harrigan and Spekkens explicitly argue that compatibility with special relativity, purely formal or otherwise, is a necessary feature of a viable interpretation of the quantum state. It may be that there are metaphysical intuitions motivating Fuchs' remark that 'admitting that the very notion of separate systems has any meaning at all, one is led to the conclusion that the quantum state…must be information'. The clear implication of this remark is that realism with respect to entangled states is incompatible with the meaningfulness of the concept of separate systems. Harrigan and Spekkens' remark that 'any sensible notion of locality presupposes separability' seems to express the same thought more explicitly. The consideration of hypersurface-dependent collapse theories illustrates that there is an important sense in which this statement is incorrect, or at least highly misleading, even in the context of a nonseparable theory. Fuchs' remark that the meaningfulness of the notion of separate systems depends upon giving up a realist interpretation of entangled states is therefore too quick. It is possible to construct theories in which entangled systems in Minkowski space have intrinsic properties in an absolute (observer and frame-independent) sense. Indeed, any theory which violates outcome independence but does not violate parameter independence is a candidate for such an interpretation. The existence of such theories, even if they are only interpreted as toy theories illustrating the logical relationships between relativistic and quantum mechanical constraints, demonstrates the possibility of an exact formulation of quantum mechanics that is realist and fundamentally Lorentz invariant. Hence more needs to be said in order for the case against a realistic interpretation of entangled quantum states to be made compelling.

## Conclusion

At first blush, quantum state antirealism is an attractive position. In principle, if one rejects the view that the quantum state is representative of physical reality, and can account for all quantum phenomena in an antirealist framework, then deep interpretive problems associated with the process of measurement and entanglement become less mysterious. Nevertheless, it has been argued that simply rejecting an ontic interpretation of the quantum state does not by itself solve the measurement problem. I have examined two explicit attempts to address the problem of measurement in an antirealist framework. In Fuchs' [2003] framework, the problem of accounting for the transition from pre- to post-measurement states remains. In Bub's [2007] analysis, on the other hand, a special place is accorded to the measurement process in the formulation of the theory, which arguably leaves the most pressing philosophical worry surrounding measurement unaddressed.

I have also argued that the phenomenon of quantum entanglement does not rule out an ontic interpretation of the quantum state in a relativistic framework. If there are good grounds for retaining a realist interpretation of the quantum state, and it will be argued in chapter 4 of the present work that there are such grounds, then one is faced with the problem of addressing the apparent tension between quantum nonseparability and the special theory of relativity. It has been argued that there is reason for optimism with respect to the prospects for a fundamentally realist and Lorentz invariant quantum theory. If such theories can be constructed, then much of the physical basis for worrying about 'spooky action at a distance' is arguably removed.

# Chapter 3: Evidence, Explanation and the Quantum State

[...]present quantum theory not only does not use – it does not even dare to mention – the notion of a "real physical situation." Defenders of the theory say that this notion is philosophically naive, a throwback to outmoded ways of thinking, and that recognition of this constitutes deep new wisdom about the nature of human knowledge. I say that it constitutes a violent irrationality, that somewhere in this theory the distinction between reality and our knowledge of reality has become lost, and the result has more the character of medieval necromancy than of science.
–E.T. Jaynes[88]

## Introduction: The Nature of Probabilities, Classical and Quantum

### *The Dualistic Conception of Probability*

It has often been remarked that the concept of probability can be understood in at least two distinct ways. According to the subjectivist or epistemic conception, probabilities represent the degrees of belief of agents, acting in the face of uncertainty. On this conception, the probability accorded to a proposition *P* can be thought of as the degree of confirmation accorded to *P*, given one's assessment of the available

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[88] Jaynes [1980].

evidence. The second concept of probability is what Ian Hacking [1975] has called the 'aleatory' conception of probability; this is the probability associated paradigmatically with games of chance, where one speaks of the chance of heads on a coin toss, or the chance of rolling three sixes on five rolls of a fair die.

The idea that these two conceptions of probability are not only compatible with each other, but actually complementary, is also not new. As long ago as 1837 Poisson remarked that 'an event will have, by its nature, a greater or less chance, known or unknown; and its probability will be relative to the knowledge we have, in regard to it.'[89] In the introduction to his classic *Logical Foundations of Probability* Carnap writes:

> We shall try to show that we have to distinguish chiefly two concepts of probability; the one is defined in terms of frequency and is applied empirically, the other is a logical concept and is the same as degree of confirmation. It will be shown that both are important for the method of science, and thus the controversy between the two "conceptions" of probability will be dissolved.[90]

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[89] Poisson [1837].

[90] Carnap [1962], p. 2.

According to Carnap's account,[91] the two notions of probability play distinct but equally important and complementary roles in the confirmation of scientific theories. The first, epistemic or "subjective" sense of probability, which Carnap calls probability$_1$, is a logical notion, representing the degree of confirmation of some hypothesis relative to some evidence. The second, factual or empirical concept of probability is derived from the observation of relative frequencies:

> (i) Probability$_1$, is the degree of confirmation of a hypothesis *h* with respect to an evidence statement *e,* e.g., an observational report. This is a logical, semantical concept. A sentence about this concept is based, not on observation of facts, but on logical analysis; if it is true, it is L-true (analytic).
>
> (ii) Probability$_2$, is the relative frequency (in the long run) of one property of events or things with respect to another. A sentence about this concept is factual, empirical.[92]

For Carnap, the objective-subjective dichotomy is actually something of a misnomer for what is essentially a contrast between logical and empirical notions. Just as deductive logic and mathematics have a perfectly objective role to play in the

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[91] Carnap speaks here of frequencies rather than objective chances. For our purposes, however, the relevant point is that relative frequencies provide factual evidence affecting the degree of confirmation accorded to some relevant class of propositions. This may include propositions about chances.

[92] Ibid., p. 19

methodology of science, so too does the inductive logical concept probability$_1$. Nevertheless, the degree of confirmation accorded to some hypothesis is clearly an epistemic notion.

The connection between the epistemic and objective notions of probability can be illustrated by considering the simple example of a coin toss. Suppose that Alice wishes to decide whether or not a given coin is fair. We will also assume that she has little to no prior knowledge of the properties of the coin, and that she is non-dogmatic in the sense that she is willing to change her beliefs about the properties of the coin in light of new evidence. The most obvious way for Alice to sharpen her beliefs about the probability $h$ of heads on any toss is to repeatedly toss the coin, observing the results and updating her beliefs in accordance with the observed relative frequency of heads.[93] This procedure presupposes that Alice has subjective beliefs about the properties of the coin, that her credences about these properties can evolve in light of frequency data, and that the frequency data in question is evidentiary with respect to Alice's beliefs about the coin.

A natural gloss to put on all of this is that Alice is using relative frequency data to learn about the chance of heads on any given trial.[94] If Bob, who has different background beliefs about the coin, performs a similar sequence of experiments, he too

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[93] I will have more to say about the mechanics of belief revision in light of new evidence below.

[94] See Myrvold [2011].

will come to have a posterior credence function that is sharply peaked around *h*.[95] In a situation like this, in which the dynamics of a system permits any rational agent with reasonable credences about the properties of the system to arrive at essentially the same probability *h* for some proposition *P* about the system, we seem to be justified in speaking of the common probability *h* as the *chance* of *P*.[96] The process of learning about chances builds in the distinct but closely connected notions of subjective degree of belief (credence), relative frequency and objective chance; we may have degrees of belief about chances, which are subject to revision on the basis of observed frequencies.

Despite its obvious explanatory value, it has sometimes been argued that the concept of objective chance is dispensable, or even incoherent. It is certainly possible to give an account of the sort of example outlined above, in which agents systematically adjust their beliefs about physical setups in light of frequency data, without recourse to the notion of objective chance. I turn now to Bruno de Finetti's classic treatment of this problem.

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[95] Assuming that his prior beliefs are also reasonable and non-dogmatic.

[96] I think that this is in fact the correct way to think about this scenario. Cf. Myrvold [2011]. I say 'seem to be justified' because this inference is not forced upon us as a matter of logic, and is in fact explicitly rejected by de Finetti and his followers.

*The Subjectivist Interpretation of Probability and Quantum State Tomography*

In his foundational essay "Foresight: its Logical Laws, its subjective sources", Bruno de Finetti famously articulates a radically subjectivist interpretation of probability. On de Finetti's account, all probabilities are personal, subjective degrees of belief rather than states of nature, allowing individuals to make quantitative judgments in the face of uncertainty. In the "Foresight" paper, published in 1937, de Finetti notes what Leonard Savage [1954] describes as a 'close mathematical parallelism' between the axioms of probability theory, on the one hand, and the notion of personal probability that can be constructed by analyzing rational decision making in the face of uncertainty.[97][98] If the set of all events can be partitioned into an arbitrarily large number of subsets, then the degree of probability assigned by a person to a given event is possible to define with arbitrary precision. The notion of personal probability is defined operationally, by making mathematically precise the 'trivial and obvious

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[97] As Savage notes, the existence of this parallelism does not imply the possibility of assigning an unambiguous probability to every event. See Savage [1972], p. 33.

[98] The text of de Finetti [1937] is an English transcription of a lecture originally delivered in French at the Institut Henri Poincare in 1935. At the time, de Finetti was not aware of the work of F.P. Ramsey [1926] which also develops an account of personal probability in terms of decisions made in the face of uncertainty.

idea' that the conditions under which someone would be disposed to bet on an event reveal the degree of probability assigned to the event by that individual.

A typical problem arising in statistical analysis is to estimate from a series of observations of some repeatable experiment the probability $p$ associated with a particular outcome. Such a procedure is usually couched in language that already presupposes an objectivist interpretation of probable events. For example, if an experimenter wishes to estimate the probability $p(\mathrm{H})$ that a coin toss will come up heads, a series of trials is performed, and the relative frequency of heads is then used to estimate $p(\mathrm{H})$. From the point of view that $p(\mathrm{H})$ is a property of the coin, this procedure is necessary to hone in on $p(\mathrm{H})$, the unknown probability associated with the coin.

According to the subjectivist account, probabilities can be determined only by interrogating the agents who believe them, rather than by investigating the properties of objects. Accordingly, the phrase 'unknown probability' is an oxymoron in this framework. The de Finetti representation theorem shows that it is possible to represent the coin toss example and other examples like it without making reference to unknown probabilities. The key to eliminating talk of unknown probabilities is to assume the equivalence of repeated trials with respect to probabilistic predictions, or equivalently, to judge that sequences of trials are permutation symmetric, a condition which de Finetti labels 'exchangeability'. Taking the coin-toss example, the exchangeability criterion implies that every sequence of $M$ coin tosses with $N$ heads is equiprobable. De Finetti shows that on the basis of this exchangeability assumption, an agent who performs a series of trials such as a coin toss will act as though there is some probability distribution over the outcomes to which her degrees of belief will converge, so long as she updates her beliefs in a coherent and non-dogmatic manner. Furthermore, our agent will expect that any other rational and non-dogmatic agents starting with exchangeable priors will likewise converge on this same probability distribution. In general, given the assumption of exchangeability, agents will behave *as if* there is some unknown objective probability distribution to which their beliefs will converge. Hence from a subjectivist point of view, 'unknown' probabilities can be defined operationally in terms of the (subjectively well-defined) notion of an exchangeable sequence of events.

According to many physicists working in the field of quantum information theory, the probabilities associated with experimental outcomes in quantum mechanics are themselves subjective or epistemic in character.[99] According to these authors, quantum states are states of knowledge, rather than states of reality. A *prima facie* problem presents itself for these authors with respect to the status of unknown quantum states that is analogous to the challenge posed to subjectivists by 'unknown probabilities' in probability theory. If quantum states are states of knowledge or belief, then they must be known or believed by someone; the question is whether or not talk of unknown quantum states can in principle be eliminated. A result known as

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[99] Cf. Fuchs [2003], Caves, Fuchs and Schack [2002a, 2002b], Fuchs and Peres [2000], and Spekkens [2007].

the ‘quantum de Finetti representation theorem’ shows that, given certain reasonable restrictions on prior beliefs and a sufficiently informative series of measurements on a quantum system, different agents will converge in their quantum state assignments for the system in question.

There are at least two important senses in which Born rule probabilities defined by the quantum state might be regarded as subjective. According to the first sense, articulated and defended by Chris Fuchs, probabilities derived from the Born rule are to be thought of as subjective representations of the attitudes of experimenters toward possible measurement outcomes, and nothing more. On this view, there is no ‘ontic state of the system’ to which quantum states refer, even in a statistical sense. In contrast to the Quantum Bayesian view, one might instead hold that systems actually do possess an ontic state, and that quantum mechanics is the statistical theory of such states in the same sense in which Liouville mechanics is the statistical theory of classical states. This latter view, which goes back to Einstein, is defended in Spekkens [2007]. The Pusey, Barrett and Rudolph (PBR) theorem, together with the earlier cluster of theorems due to Bell, Kochen and Specker, place serious constraints on the space of theories of this sort. One possible attitude to take towards these and other ‘no-go’ results, the attitude explicitly adopted by Chris Fuchs, is that the project of attempting to articulate a picture of preexisting dynamical states in a quantum mechanical framework is fundamentally misguided. If one adopts such an attitude, an obvious question presents itself: what is it that scientists are doing when they conduct experiments, if not learning about the properties of physical systems? In this chapter I critically examine the sorts of answer to this question that the quantum Bayesian might offer.

In section 3.1, the subjectivist account of probability articulated by de Finetti and applied to quantum mechanics by the quantum Bayesians Caves, Fuchs and Schack is reviewed. In section 3.2, the quantum de Finetti representation theorem is examined. Chris Timpson has recently argued that adopting the quantum Bayesian attitude towards the quantum state leads to problems with explanation.[100] According to Timpson, the quantum Bayesian is unable to give an adequate account of the role of data acquisition in grounding our beliefs about the outcomes of experiments. I will argue that there are indeed good grounds for worrying about an explanatory deficit in the quantum Bayesian account; however, I diagnose the source of the difficulty somewhat differently than Timpson does. Recently, Chris Fuchs has defended the idea that the Born rule is to be understood as an 'empirical addition to Bayesian coherence'. I argue that Fuchs' description of the Born rule is actually difficult to square with a radical subjectivist attitude toward quantum probabilities.

## 3.1 The Subjectivist Interpretation of Probability and Quantum Bayesianism

[100] I am referring in particular to what Timpson calls the 'means-ends objection'. See Timpson [2008].

*Quantum Bayesianism and the Space of Epistemic Interpretations of the Quantum State*

Epistemic interpretations of the quantum state hold that there does not exist a one-one mapping from quantum states to states of the world. One way to cash out this claim is to maintain that there are in fact ontic states of the world, defined in terms of physical properties, but that the quantum state is not among these properties. It may be that the same physical state of a system is compatible with more than one quantum state.[101] In this case, it is appropriate to interpret the quantum state as providing a statistical representation of the physical state of a system in analogy with the statistical Liouville representation of classical states.[102] On this view, the quantum state represents a state of incomplete knowledge. Characteristic of this view is the thesis of ontic state realism; while the quantum state is taken to be epistemic, it is nevertheless defined over a space of possible ontic states about which we have incomplete knowledge. This suggests that quantum mechanics might be supplanted by a more complete theory, and that the search for such a theory constitutes a worthwhile research program.

An epistemic interpretation that is nevertheless ontic state realist faces serious constraints. The Bell-Kochen Specker theorem implies that any such ontic model must

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[101]See also chapter 4 of the present work for a discussion of this point, and the corresponding notion of 'physical state'.

[102] I will have more to say about the possible analogy with Liouville mechanics below.

be contextual. Furthermore, the recently discovered Pusey, Barrett and Rudolph (PBR) theorem places further constraints on the space of possible ontic models. In particular, in light of PBR, individual systems cannot have well-defined properties in an epistemic framework that assumes that probabilities assigned to properties depend only on the quantum state.[103]

The 'QBist' interpretation, which Chris Fuchs describes as 'the perimeter of Quantum Bayesianism', sidesteps these difficulties by denying the thesis of ontic state realism altogether.[104] On this view, there is no ontic state of the system about which we have only incomplete knowledge. The probabilities associated with quantum state assignments are epistemic with respect to measurement outcomes rather than ontic states. We have incomplete knowledge not about the state of a system, but about the possible results of our interactions with such systems, i.e., measurement outcomes. Furthermore, it is an objective fact about the world that our interactions with quantum systems are necessarily unpredictable; the uncertainty associated with

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[103] See Pusey, Barrett and Rudolph [2011]. See also chapter 4 of the present work for a fuller discussion of the PBR result and its implications for the epistemic view of quantum states.

[104] Fuchs [2010] describes 'QBism' as a further development of Quantum Bayesianism: "Quantum Bayesianism, as it is called in the literature, usually refers to a point of view on quantum states originally developed by C. M. Caves, C. A. Fuchs, and R. Schack. The present work, however, goes far beyond those statements in the metaphysical conclusions it draws—so much so that the author cannot comfortably attribute the thoughts herein to the triumvirate as a whole."

measurement outcomes cannot be eliminated even if we have maximal information about the systems in question.

It is with this objective uncertainty in mind that the quantum Bayesians write that quantum mechanics is not only incomplete, but *necessarily* so. According to Caves, Fuchs and Schack [2002b], a theory is 'complete' just in case it is dispersion-free, yielding definite yes or no answers to all questions that can be asked of a system.[105] A theory is necessarily CFS-incomplete just in case it contains a 'necessary statistical residue'. CFS completeness should therefore not be confused with the 'completeness' employed in the famous 'EPR' argument of Einstein, Podolski and Rosen [1935]. According to the latter authors, a theory is complete just in case every element of physical reality is captured by the theory. The EPR argument is intended to show that quantum mechanics is incomplete in the sense of providing only a partial description of reality within an ontic state realist framework. The quantum Bayesians reject the notion that the quantum state description is incomplete in this sense. Nevertheless, Caves, Fuchs and Schack take the EPR argument,[106] combined with the Bell, Kochen and Specker (BKS) theorems to show that maximal information about a system is not complete information, and cannot be completed. This information-theoretic

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[105] Hereafter I will refer to such completeness as 'CFS-completeness'.

[106] What the EPR argument in fact shows is that if quantum mechanics is taken to be complete in the sense that every element of physical reality is represented in the theory, then one cannot maintain both the separability principle and the locality principle. More on this in chapter 2 of the present work.

incompleteness is characteristic of quantum theories as distinct from classical theories such as Liouville mechanics.

Given the irreducibly statistical character of quantum mechanics, we live in a world of *objective uncertainty*, but nevertheless, if the quantum Bayesians are right, not a world of *objective chance*. Instead, the quantum Bayesians represent idealized agents as having subjective numerical degrees of belief satisfying the axioms of probability theory. These include beliefs about the possible consequences of an agent's interactions with a quantum system. Where the quantum Bayesian departs from many other Bayesians, and indeed from other epistemic interpretations of the quantum state, is in his attitude towards quantum uncertainty. As we have seen, probabilities are not to be understood as measures of ignorance or imperfect knowledge, as evidenced by the following quotation from Chris Fuchs' 'QBism, The Perimeter of Quantum Bayesianism':

> Imperfect knowledge? It sounds like something that, at least in imagination, could be perfected, making all probabilities go to zero or one- one uses probabilities only because one does not know the true, preexisting state of affairs...QBism finds its happiest spot in an unflinching combination of "subjective probability" with "objective indeterminism."[107]

[107] Fuchs [2010], p. 8 n.

This objective indeterminism is rooted in the uncertainty associated with agent-system interactions. When an agent performs a measurement, something new comes into the world in an inherently unpredictable way:

> QBism says when an agent reaches out and touches a quantum system- when he performs a *quantum measurement*- that process gives rise to birth in a nearly literal sense.[108]

On this view, there is no 'true, preexisting state of affairs' of which the quantum state is a representation, whether complete or incomplete.[109]

A key pillar of Fuchs' analysis is his interpretation of the significance of the Born rule. Quantum states are used to calculate probabilities via the Born rule, and conversely, if one assigns probabilities to a well-selected set of measurements, this is mathematically equivalent to defining a quantum state. According to the QBist analysis, there is no sense in which the quantum state represents even a part of the external world. What then are we to make of the role of the Born rule? The correct way to think about Born rule probabilities according to Fuchs is that they represent an empirical addition to probability theory, imposing further normative constraints or

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[108] ibid.

[109] It is in this sense, according to Fuchs' account, that necessary information-theoretic incompleteness is characteristically quantum: the quantum state does not encode subjective probabilities in the sense of Liouville mechanics, associating multiple statistical states to a given (imperfectly known) ontic state.

rules of consistency that an agent should strive to satisfy. I will return to Fuch's interpretation of the Born rule below. Before considering Fuchs' interpretation of the Born rule in further detail, I will turn to a brief discussion of the de Finetti representation theorem and its quantum analog. The viability of the Quantum Bayesian interpretation of quantum mechanics ultimately rests on whether the view can do justice to the evidentiary basis of the theory within a subjectivist framework. The quantum de Finetti theorem shows that the notion of an 'unknown quantum state' can be accommodated within an operational framework that does not presuppose objective probabilities.

*The de Finetti Representation Theorem*

Suppose that an agent is about to toss a coin, which may or may not be fair.[110] The agent wishes to know what probability to assign to any given outcome *a*, given what she knows about the properties of the coin, and the overall physical setup. In an objectivist framework it is typically assumed that, in addition to the agent's personal probability function *P* for the outcomes of tosses, there exists a set of probability measures $P_r$ some member of which represents the actual probability of heads or tails. Tosses are assumed to be independent, so that if *p* is the probability of tossing heads

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[110] The following discussion of the general framework of subjective probability is indebted to Jeffrey [1983], [1996], Brathwaite [1957], and de Finetti [1937].

on the *n*th toss, then the probability that *i* tosses result in heads and *j* tosses result in tails is given by $p^i(1-P)^j$ . The real probability of heads is assumed to be some unknown number between 0 and 1.

If one adopts the dualistic framework in which subjective probabilities are assigned to objective chances, then our agent's personal probability *P* will be a weighted average over $P_r$, weighted according to her assessment of the likelihood that the objective probability falls within any given interval on [0,1]. The adjudged probability for any given hypothesis *P(H)*, such as the hypothesis that the next four tosses of the coin will come up heads, will be the expectation value associated with the weighted average of possible probability measures associated with this hypothesis:

$$P\ H\ = \mathbf{E}\ P_r\ \boldsymbol{H} \qquad (1)$$

Where $\mathbf{E}\ P_r\ \boldsymbol{H}$ is a weighted average of possible objective functions $P_r$. Assuming that there is some unknown probability associated with each possible outcome, then it can be shown that the strong law of large numbers holds, i.e., 'almost certainly',

*P*(limiting frequency of heads = the probability of heads) = 1.

The fact that the limiting frequency of heads converges to a definite value, the 'unknown probability' of heads, suggests that there is a property of the coin, namely the chance that any given toss will yield heads, that is discovered given enough time to experiment with it.

De Finetti rejects this interpretation of the sequence of events, and the corresponding assumption that coin tosses represent independent events of equal but unknown probability. In the place of this latter notion, de Finetti introduces the concept of *exchangeability*. A set *n* of events is exchangeable with respect to some property *A* if, for every *m* less than or equal to *n,* the probability of any *m* members of *n* having property *A* is independent of which members of *n* are chosen. Exchangeability is a weaker notion than independence; independence implies exchangeability, but also entails that the probability of *m* events having the property *A* is equal to the *m*th power of the probability for a single event's having that property.

In a subjectivist framework like de Finetti's, there is no objective probability measure associated with outcomes of die tosses, and hence no family of possible measures to which the actual objective measure belongs. Instead, agents adopt a set of measures relative to which all tosses of the die are exchangeable. De Finetti rejects the parallel that a frequentist or propensity theorist might draw between tossing a coin and drawing balls from an urn with replacement. There is a readily accessible objective fact about the urn, namely the proportion of balls of any given color contained within it, upon which to build conditional probability judgments. In this case, according to de Finetti, we are sufficiently familiar with the physical setup to be able to make meaningful attributions of probability:

> If we consider the case of an urn whose composition is unknown, we can doubtless speak of the probability of different compositions and of probabilities relative to one such composition; indeed the assertion that there are as many white balls as black balls in the urn expresses an objective fact which can be directly verified, and the conditional probability, relative to a given event, has been well defined.[111]

The conditional probability is 'well defined' provided some actual event occurs. Furthermore, a probability assignment reflects the judgment, subject to straightforward inspection, that there are a certain number of white or black balls in the urn. No such procedure of direct verification is available for judgments of probability regarding coin tosses. In the case of the coin toss,

> […] one does not have the right to consider as distinct hypotheses the suppositions that this imperfection has a more or less noticeable influence on the 'unknown probability', for this 'unknown probability' cannot be defined, and the hypotheses that one would like to introduce in this way have no objective meaning.[112]

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[111] de Finetti [1937], p. 141.

[112] Ibid., 141-142.

The introduction of the concept of exchangeability allows de Finetti to consider the elements common to the urn example and the coin tossing example, without presupposing that coin tosses constitute independent events of equal but unknown probability. Exchangeability is perfectly well defined within a subjectivist framework in terms of events and betting rates on them, with no need to refer to the 'nebulous and inexact' notion of an unknown probability. Exchangeability means that, for a set of *n* events (with respect to a property *A*), for every *m* less than or equal to *n*, an agent's betting rate on any *m* events will be the same, regardless of the order in which the events occur. Although weaker than independence, exchangeability is sufficient to ensure that with *n* repetitions of a coin toss, the cumulative probability distribution function *P(H)* of (1) will approach a unique limiting function. In particular, de Finetti shows that if an agent's personal probability function *P* is exchangeable with respect to a given set of events, then *P* is expressible as $P(H) = \mathbf{E}[P_S(H)]$ for some hypothesis *H*, where $P_S(H)$ is the personal probability measure for some limiting relative frequency *S* of heads. This condition, known as the *de Finetti representation theorem*, shows that exchangeability together with the law of large numbers entails that the probability distribution function $P_n$ for *n* tosses of a coin will approach, with increasing *n,* a unique limiting function $P_S$.

The probability distribution function $P_S$ is perfectly well defined within a subjectivist framework as a betting rate approached by conditioning on observable events taken to be exchangeable. This ensures that 'a rich enough experience leads us always to consider as probable future frequencies or distributions close to those which have been observed.'[113] The assumption of exchangeability ensures that the observation of relative frequencies enforces a convergence of belief among different agents towards a limiting distribution close to the observed relative frequency of events.

The de Finetti representation theorem guarantees that an agent who judges a sequence of coin tosses to be exchangeable will bet on the outcomes of future coin tosses as if the coin has an objective but unknown probability of heads. Furthermore, the agent's degrees of belief concerning the value of this chance will mesh with her betting rates on the outcomes of subsequent coin tosses in the manner prescribed by Lewis' Principal Principle.[114] An agent who judges such a sequence to be exchangeable will also update their beliefs diachronically as if they believe that they are learning about objective chances. De Finetti interprets this result as showing that the concept of objective but unknown chances is eliminable:

> The essential question, and the only one which is a little less elementary, is the justification and the explanation of the reasons for which in the prediction of a frequency one is generally guided, or at least influenced, by the observation of past frequencies. It is a question showing that there is no need to admit, as it is currently held, that *the probability of a phenomenon has a determinate value* and

[113] Ibid., 142.

[114] See Lewis [1980]. See also Greaves and Myrvold [2008].

> that it suffices to get to know it. On the contrary, the question can be posed in a way which has a perfectly clear sense from the subjectivistic point of view.[115]

An objectivist may grant that it is possible to furnish an account of belief revision guided by the observation of past frequencies in a purely subjectivist framework, without adopting that framework. It is also possible to interpret the de Finetti representation theorem in an objectivist framework, by interpreting the agent's degrees of belief about coin tosses as beliefs about chances. Then the limiting function $P_s$ is just the chance distribution in which the chance of heads is equal to $obj_r$, and $P \;\; H \;\; = \mathbf{E}[P_S \;\; H \;\; ]$ is equivalent to $P \;\; H \;\; = \mathbf{E}[P_R \;\; H \;\; ]$, a representation of the agent's degrees of belief about which of the functions $P_R \;\; H$ gives the actual chances. Nevertheless, de Finetti's result is important for our purposes because it shows how one can make sense of repeatable experiments involving relative frequencies in an operational framework that does not presuppose the existence of objective chances. Caves, Fuchs and Schack, building on the work of Hudson and Moody, have provided a mathematically elementary quantum analog of the de Finetti representation theorem.[116] The authors show that the concept of an 'unknown quantum state' can be dealt with in a framework in which quantum states are taken to be states of knowledge rather than states of belief. The result is analogous to de Finetti's result for

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[115] De Finetti [1937], p. 152.

[116] Caves, Fuchs and Schack [2002]. See also Hudson and Moody [1976].

unknown probabilities. Furthermore, the authors draw the same moral for the concept of an ‘unknown quantum state’ that de Finetti originally drew for unknown probabilities: the concept is dispensable. I turn now to a brief consideration of the quantum de Finetti representation theorem, before assessing its importance for the quantum Bayesian interpretation of the quantum state.

*The Quantum de Finetti Representation Theorem and Quantum State Tomography*

Given that Caves, Fuchs and Schack hold that quantum states are states of belief rather than states of knowledge, the term ‘unknown quantum state’ has no place in the quantum Bayesian vocabulary. Nevertheless, the concept of an unknown quantum state is ubiquitous in foundational discussions, and in particular in the field of quantum information theory. In their paper ‘Unknown Quantum States: The Quantum de Finetti Representation’, Caves, Fuchs and Schack are concerned to analyze a particular measurement technique known as *quantum state tomography.* In a typical quantum tomographic procedure, a device of some sort prepares several copies of a quantum system in some fixed but ‘unknown’ quantum state $\rho$. The state $\rho$ may be either pure or mixed. The goal of the procedure is to perform enough measurements on a large enough number of copies of $\rho$ to be able to ‘reconstruct’ the identity of the state. It may happen that the experimenter is able to access the systems prepared by the device more easily than the mechanisms governing the device, so that he may learn something about the operation of the preparation device by investigating the states it prepares. In any case, the most important point is that the process of performing measurements on the systems prepared, including carrying out the same measurement in series as well as different kinds of measurements, allows the experimenter to narrow in on the unknown quantum state associated with preparations.

The procedure of reconstructing the state by performing quantum state tomography seems to presuppose that there is in fact an unknown quantum state the identity of which is eventually revealed by experiment. Such a description of the procedure is anathema to the quantum Bayesian interpretation of quantum states. CFS therefore aim to provide a description of quantum state tomography that makes no reference to unknown quantum states. In quantum Bayesian terms, the end product of a tomographic procedure is a single quantum state that 'captures the describer's state of knowledge'.[117]This state of knowledge will include a description of the entire 'procedure that uses the idea of an unknown quantum state in its description' in the sense that the resultant state will incorporate the history of interactions with the system and measuring device without presupposing that the interactions in question constitute measurements of preexisting properties of the system.

CFS take their cue from de Finetti and his treatment of the concept of a definite but unknown probability. We have already seen that de Finetti was able to treat the

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[117] Ibid., p. 4538.

classical example of a repeated experiment involving coin tosses within a subjectivist framework. The assumption of exchangeability is sufficient to guarantee that non-dogmatic agents who update their beliefs via Bayes' rule will eventually come to agree in their probability assignments given enough time to gather information, and exchangeability makes no reference to unobservable physical quantities.[118] Within the context of quantum state tomography, the analogous judgment to that of exchangeability in the coin tossing case is that there is no distinction between the systems that a device is preparing. From an operational perspective, this amounts to the claim that 'all the systems are and will be the same as far as observational predictions are concerned.'[119] As CFS point out, it is possible to hold such a point of view regarding the states prepared by some preparation device without presupposing that the device is preparing definite unknown states.[120] This is equivalent to the statement that if an experimenter judges a collection $N$ of a device's outputs to have an overall quantum state $\rho^{(N)}$, he will make the same judgement regarding any permutation of the $N$ output states, regardless of the size of $N$.

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[118] Although the assumption of exchangeability is both material and 'psychological' in the sense that it is neither confirmed nor infirmed by the evidence.

[119] Caves, Fuchs and Schack [2002], p. 4540.

[120] Although, as I will argue below, this assumption is substantial; there ought to be physical grounds for making it.

From an objectivist point of view, the purpose of quantum state tomography is to try to determine the quantum state $\rho$ as precisely as possible. Within the CFS framework, the 'Bayesian experimenter' assigns a prior quantum state to the joint system consisting of the $N$ systems $\rho^{(N)}$, representing the experimenter's state of knowledge prior to performing any measurements. The experimenter judges that the entire sequence of states produced by a preparation device has the structure of permutation invariance. With this notion of permutation invariance for quantum states in place, CFS prove the quantum de Finetti representation theorem, which says that a sequence of states $\rho^{(N)}$ is exchangeable just in case it can be written in the form

$$\rho^{(N)} = \quad P(\rho)\rho^{\otimes N} d\rho \qquad (2)$$

where $\rho^{\otimes N} = \rho \otimes \rho \otimes \cdots \otimes \rho$ is an $N$-fold tensor product of the state $\rho$ with itself, and $P(\rho)$ is a cumulative probability distribution over the density operators. The theorem says that it is possible to represent an exchangeable quantum state assignment as a mixture characterized by the probability distribution $P(\rho)$ over the product states $\rho^{\otimes N}$. Note the formal analogy between this equation and equation (1) above, involving a weighted average of possible objective probability measures.

The quantum de Finetti representation theorem guarantees that non-dogmatic experimenters who judge a sequence of quantum states $\rho^{(N)}$ to be exchangeable will converge in their quantum state assignments given the outcomes of a sufficiently informative set of measurements. The mechanism that allows this convergence to take place is just Bayes' rule for updating probabilities applied to quantum measurements. If $K$ measurements yield the results $D_K$ then the state of additional systems is given by an updated probability on density operators:

$$P(\rho|D_K) = \frac{P(D_K|\rho)P(\rho)}{P(D_K)} \quad (3)$$

Here $P(D_K\ \rho)$ is the probability of obtaining the measurement results $D_K$, given the state $\rho^{\otimes K}$ for the first $K$ measured systems, and $P\ \ D_K\ \ =\ \ \ P(D_K\ \rho)P\ \ \rho\ \ d\rho$ is the unconditional probability for the measurement results. As more measurements are performed and K becomes large, the posterior probability distribution $P(\rho\ \ D_K)$ becomes highly peaked on a particular state $\rho_{D_K}$ dictated by the measurement results.

CFS draw the following moral from the quantum de Finetti representation. With regard to the purpose of quantum state tomography, 'it is not about uncovering some "unknown state of nature", but rather about the various observers' coming to agreement over future probabilistic predictions'.[121] This result is significant because it ensures that the observation of relative frequency data which forms much of the evidentiary basis of quantum mechanics is sufficient to guarantee, given the assumption of exchangeability (permutation invariance of $\rho^{\otimes N} = \rho\otimes\rho\otimes\cdots\otimes\rho$ ), that

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[121] Ibid., p. 4541.

different agents will converge in their quantum state assignments. The assumption of exchangeability is ‘modest’ in the sense that it can be defined operationally in terms of gambling commitments and does not make any reference to objective but unknown probabilities pertaining to physical setups.

Of course, it remains possible to interpret the agent’s judgments regarding the outcomes of coin tosses or quantum measurements as beliefs about objective chances. The Quantum de Finetti theorem does not settle the case in favor of a Quantum Bayesian interpretation of quantum states any more than the classical de Finetti theorem settles the question of the interpretation of general probability theory. But the result does show that the activity of investigating ‘unknown quantum states’ is consistent with a quantum Bayesian interpretation of such states. This provides part of the answer to the question: what is it that scientists are doing when they conduct experiments on quantum systems and thereby updating their quantum state assignments, if not learning about the properties of those systems? The answer provided by the Quantum Bayesians is that they are learning to adjust their beliefs about the future consequences of their interactions with quantum systems.

## 3.2 The Means/Ends Objection and the Role of the Preparation Device

The de Finetti representation theorem furnishes a subjectivist description of what from an objectivist standpoint would be described as the process of learning about chances. The assumption of exchangeability, together with the assumption will conditionalize on observed data, is sufficient to explain our belief in the stability of a frequency, since the probability of a trial, given the observation of a certain frequency, tends to coincide with the value of that relative frequency.[122] The quantum de Finetti representation shows that it is possible to tell a similar story in the context of quantum state tomography. Given enough time to experiment with quantum systems, different non-dogmatic observers with exchangeable beliefs will come to agree in their probability assignments for experimental outcomes, i.e., their quantum state assignments. De Finetti famously declared that the only criterion of admissibility for probability assignments is coherence. This implies that all coherent probability assignments are admissible; it should not be taken to imply the impossibility of a preference ordering on probabilities within a subjectivist framework. Writing about the process of learning from experience, de Finetti notes that

> […] the prediction of a future frequency is generally based on the observation of those past; one says "we will correct" our initial opinions if "experience refutes them." Then isn't this instinctive and natural procedure justified? Yes; but the way in which it is formulated is not exact, or more precisely, is not meaningful. It is not a question of "correcting" some opinions which have been "refuted": it is simply a

[122] The existence of generalizations of exchangeability show that the condition is not necessary; more on this point below.

> question of substituting for the initial evaluation of the probability the value of the probability which is conditioned on the occurrence of facts which have already been observed.

Note that de Finetti explicitly acknowledges that the prediction of a future frequency is based on the observation of those past, and that the procedure of updating one's beliefs about future events is justified by the observation of past ones. It is reasonable to expect that an agent who has observed a set of outcomes and adjusted her prior beliefs accordingly will not freely exchange her posterior probability assignment for her prior beliefs. Furthermore, unless our agent has zero degree of belief that learning the results of $N$ runs of an experiment will change her probability distribution function for the result of trial $N+1$, coherence alone requires her to strictly prefer having knowledge of the first $N$ trials in order to improve her betting situation.

Analogous considerations apply to the case of experiments involving an unknown quantum state. Experimenters will always behave as though there is some objective but unknown Born rule probability assignment toward which their beliefs will converge, given enough time to observe relative frequency data derived from tomographic analysis of identically prepared quantum systems. Furthermore, rational agents in the laboratory will value learning from experience whether or not their Born rule probability assignments are ultimately subjective in character. Indeed, Quantum Bayesian experimenters behave so much like agents who believe in objective chances that it is not surprising that the orthodox interpretation of quantum state tomography, according to which the latter procedure is uncovering an objective but unknown quantum state, has dominated the literature on the subject.

Perhaps because of the close operational link between beliefs about chances and exchangeability, Chris Timpson has recently argued that if the Quantum Bayesian interpretation of quantum mechanics is correct, then there is an important disconnect between the means through which quantum mechanical experiments are conducted and the ends of those experiments. In putting forward his 'means/ends objection, Timpson writes:

> The puzzle is this: if there are only subjective probabilities, if gathering data does not help us track the extent to which circumstances favour some event over another one (this is the denial of objective single case probability), then why does gathering data and updating our subjective probabilities help us do better in coping with the world (if, that is, it does so)? Moreover, why should it be expected to? Why, that is, should one even bother to look at the data at all?[123]

The trouble that Timpson is alluding to is that we are learning something about the world when we do quantum mechanical experiments. This learning is reflected in the probabilities we associate with experimental outcomes; furthermore, those agents who conduct experiments seem to be better equipped to negotiate the world

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[123] Timpson [2008], 604.

because they have acquired data that causes their expectations to peak around certain probability distributions. If these distributions are not representative of the actual physical state of the world, but are instead purely subjective, then how do we explain the relative predictive power of agents who know quantum mechanics? What exactly is it that we are learning when we gather data?

The observation of relative frequencies of events and comparison of these frequencies with the probabilities predicted by quantum mechanics provides evidence that quantum mechanics is correct. If the observed pattern of data is much more probable on the assumption that quantum mechanics is correct than it is on the assumption that some other theory is correct, this provides evidence that quantum mechanics is indeed tracking objective features of the world. I share Timpson's worry that these ideas are hard to square with the hard line subjectivism of the Quantum Bayesians. But the real source of the problem that Timpson is trying to pinpoint is not that it is impossible for the subjective Bayesian to explain why it is desirable to gather evidence in order to update our subjective probabilities. Rather, the problem is that the quantum Bayesian is unable to adequately explain why agents who perform quantum state tomography and use the Born rule ought to adopt any particular attitude at all toward the probabilities occurring in the theory.

Specifically, two important questions present themselves: i) Why should agents adopt a symmetry assumption such as (possibly generalized) exchangeability at all? ii) Why must agents who associate probabilities with some given experimental outcome restrict their probability assignments for other experiments, as demanded by quantum mechanics? Objectivists have a ready answer to both questions. If one has prepared identical copies of a system, then exchangeability follows straightforwardly from independence. Furthermore, the relations between the probabilities associated with experimental outcomes are, from the objectivist point of view, simply discovered by experiment, and represent objective features of quantum systems. Neither of these answers is available to the Quantum Bayesian.

Some light can be shed on Timpson's question, 'why should we bother to look at data if the quantum state is subjective?', by considering the case of Liouville mechanics, in which epistemic states associate subjective probabilities with 'ontic' states of a classical system. As alluded to in the introduction to this chapter, there are important disanalogies between the subjective probabilities associated with classical statistical mechanical states and the subjective Born rule probabilities in a quantum Bayesian framework.

Liouville mechanics is a dynamical theory describing states of knowledge of a classical system. Classical systems are assumed to be described by a phase space, and the real state of the system, i.e., its *ontic* state at any given time, is assumed to correspond to a point in the phase space. The phase space representation consists of possible configurations and momenta of the particles comprising the system and therefore represents the space of possible ontic states of the system. Assuming that we know the ontic state of the particles comprising a system, we can compute the state at another time using Newton's equations.

In practice, we are rarely able to ascertain the ontic state of a macroscopic system containing on the order of $10^{23}$ particles each with position and momentum degrees of freedom. We are more interested in tracking observable macroscopic features of the system, such as temperature and pressure. Our knowledge of the ontic state of the system is therefore incomplete, and represented by a probability distribution (sometimes called a 'Liouville distribution') over the possible ontic states of the system. This state is clearly epistemic in that it represents our knowledge of the ontic state of the system; multiple, and indeed infinitely many, epistemic states are compatible with the same ontic state.

For our purposes, the importance of considering Liouville mechanics is that it represents a concrete example of a physical scenario in which epistemic states are used to describe the physical or ontic state of a system about which we have incomplete knowledge. Agents interacting with such systems can be expected to update their epistemic states by performing measurements on them and observing the dynamical evolution of the systems. The epistemic states associated with a statistical mechanical description will evolve dynamically according to the Liouville equation, which is derived from the Hamiltonian evolution of the possible ontic states over which the Liouville distribution is defined.

Liouville mechanics thus provides a concrete example of a case in which epistemic states are used to describe real physical states of affairs, and in which the process of observing physical systems and gathering data will lead agents to update their subjective probability assignments. It may be that quantum mechanics is analogous to Liouville mechanics, in the sense that the quantum state is an epistemic state defined over an ontic state space. On such an interpretation, in analogy with Liouville mechanics, different quantum states are compatible with the same ontic state, and the probabilities calculated via the Born rule are therefore subjective. In this case, it would be reasonable to expect that the process of performing experiments and gathering data would lead agents to update their subjective probability assignments in such a way that their posterior quantum state assignments for a system, though not determined uniquely, are at least highly peaked in certain regions of the ontic state space.

The trouble is that there are severe restrictions on the possible ontic models for quantum theory expressible in this sort of framework. The Bell and Kochen and Specker theorems imply that such models will be non-local and contextual.[124] Furthermore, the Pusey, Barrett and Rudolph (PBR) theorem places further important constraints on the space of such models.[125] In any case, the Quantum Bayesians reject the notion that the quantum state represents a state of incomplete knowledge of ontic states. The issue is not that we do not have epistemic access, for whatever reason, to a complete description of the deeper (sub-quantum) ontic state of a system. Rather, there is no such sub-quantum state of reality: the quantum state

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[124] See Bell [1966] and Kochen and Specker [1967].

[125] See chapter 4 of the present work for a fuller discussion of this point.

is a state of incomplete knowledge not about the ontic state of a system, but about the outcomes of future interventions on the system.

The exact answer to the question ‘knowledge about what?’ provided by the Quantum Bayesians, namely, ‘the results of interactions with systems’, adds new urgency to the question posed by Timpson, namely how it is that gathering data and updating our beliefs helps us to do better in coping with the world. Denying that the quantum state represents knowledge about dynamical properties of a quantum system, while maintaining, as the Quantum Bayesians do, that quantum mechanics is nevertheless the optimal ‘user’s manual’ for negotiating the world, amounts to denying that there is any deeper underlying reality about which agents have beliefs, subjective or otherwise. So what is it that agents are doing when they perform measurements? The Quantum Bayesian answer is that they are observing the consequences, for them, of interacting with systems and thereby bringing something entirely new into existence that was not there before the measurement took place. While the macroscopic world of tables and chairs and laboratory equipment is perfectly objective and observer-independent, when the lab equipment is put to the task of interacting with the world at the level where quantum mechanical effects become significant, the particular act of measurement itself takes on a new meaning, and comes to occupy center stage. It is not just that quantum systems do not have preexisting properties revealed by measurements, a point that realists may readily accept. Crucially, the ‘creation event’ resulting from a quantum measurement is not only undetermined by the macroscopic arrangement of the laboratory, but the interaction cannot be modeled as a stochastic process (this is the denial of objective probability and quantum state realism). Nor are we licensed to infer that systems prepared in an eigenstate of some observable are in an ontic state corresponding to that eigenstate.

How does this picture fit with the picture of quantum state tomography provided by the quantum de Finetti theorem? Recall that according to the latter theorem, different agents are licensed to act as though each individual quantum system has some unknown quantum state, with a probability density $P(\rho)$ representing his ignorance of what the state is. When the agent performs quantum state tomography, he is simply updating his quantum state assignment via Bayes' theorem, with the expectation that as he gathers more data, his posterior state will converge on some $\rho_{D_K}$. The assumption of exchangeability is crucial to the derivation of this result. It says that any two systems that a 'preparation device' spits out could be interchanged without changing the statistics our agent expects for any measurement he might perform. What grounds might an agent have for making such an assumption?

Chris Fuchs calls the assumption of exchangeability a 'very minimal belief', suggesting that adopting it does not involve substantial physical assumptions.
But exchangeability is not an unsubstantial assumption. Take the example of draws from an urn with replacement. What grounds do we have for judging the draws from the urn to be exchangeable? In the first place, we believe that there exists what John

Vickers calls a 'foundation of stable causes' from one draw to the next.[126] These include such factors as the solidity, number and color of the balls and the constancy of the force due to gravity across trials. Furthermore, whatever variable and random causes exist must be independent of the trials themselves. An example of a repeated process in which exchangeability can be expected to fail is a basketball player practicing shooting baskets.[127] Since the basketball player's skill can reasonably be expected to improve over time, it would be rational to bet in favor of a sequence of shot attempts in which the successful attempts occur more frequently in later trials.

What grounds do we have to believe that the systems generated by a preparation device can be interchanged without changing the expected statistics? Given that such systems do not have underlying properties at all in a Quantum Bayesian picture, the attribution of exchangeability does not appear to be physically grounded. The trouble is that it is difficult to make sense of the operational role of the preparation device by the lights of Quantum Bayesianism. In more orthodox interpretations of quantum mechanics, the purpose of a preparation device (at least ideally) is to 'prepare' a system in a given, perhaps unknown, state. Under certain conditions, it is reasonable to speak as though we know what the actual state of the system is. For example, I can prepare a quantum system in the '$\sigma_x^+$ state' by running it through a Stern-Gerlach magnet oriented in the $\sigma_x$ direction and selecting the '+'

[126] See Vickers [2010].

[127] The example is due to Persi Diaconis (see Vickers [2010]).

states. Such a preparation corresponds to the situation in which an experimenter is prepared to offer a probability of 1 to the proposition that the system in question will yield the result '+' if it is subjected to a $\sigma_x$ experiment. Most experimenters would be inclined to view the state in question as having the property of being spin-up in the x-direction.

Furthermore, if we regard the state preparation procedure in objectivist terms, as preparing systems in a given objective state, it is easy to make sense of the activity of quantum state tomography. In this case, experimenters have exchangeable beliefs precisely because they take the systems generated by a preparation device to be independent of one another, generating identical (perhaps unknown) quantum states. The convergence in belief for different experimenters is easy to understand on physical grounds. It represents the fact that the experimenters take the prepared states to have some objective but unknown properties giving rise to observed relative frequencies of measurement results. These relative frequencies can then be interpreted as evidence about chances.

On the other hand, if we follow Fuchs *et al.* in denying that preparation procedures yield systems with dynamical properties, let alone real quantum states, then the judgment of exchangeability seems somewhat arbitrary. Perhaps the Quantum Bayesians take their cue here from de Finetti, who regarded judgments of symmetry and the intersubjective agreement they engender as ultimately psychological:

> Our point of view remains in all cases the same: to show that there are rather profound psychological reasons which make the exact or approximate agreement that is observed between the opinions of different individuals very natural, but that there are no reasons, rational, positive or metaphysical, that can give this fact any meaning beyond that of a simple agreement of subjective opinions.[128]

If this is indeed the attitude one takes, then it seems that much of the agreement in the physics community over the attribution of states to systems is ultimately grounded in psychology.

Another possible subjectivist response is to point out that more general de Finetti representations exist that don't assume exchangeability.[129] Consider the following sequence: 0010010001010100010… This sequence does not appear to be random. Although every 0 is followed by a 0 or a 1, every 1 is followed by a 0. If this pattern persists over a sufficiently long period of time, then it would seem to be rational to assign a high credence to the occurrence of a 0, given that a 1 has been observed. If *P* is an exchangeable probability, the conditional probabilities

$$P(X_{n+1} = j | X_1 = i_1, X_2 = i_2 \ldots X_n = i_n)$$

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[128] De Finetti [1937], p. 152.

[129] See Zabell [2005], pp. 11-13.

depend only on the number of 1's, and not on their order in the sequence. Hence exchangeability is a plausible assumption only when the order in which outcomes occur can be ignored. The sort of dependence illustrated in the above sequence, known as Markov dependence, can nevertheless be incorporated into a weakened exchangeability condition known as *Markov exchangeability*. According to this condition, all sequences with the same initial member and the same number of Markov transitions (in the above example, the transition from '1' to '0') are assumed to be equiprobable. It is possible to prove a generalized de Finetti representation theorem incorporating only Markov exchangeability.[130]

Whether or not a generalization of the quantum de Finetti representation theorem analogous to the classical case exists is an important open question.[131] Direct observation of a sequence of quantum states will certainly reveal any statistical dependencies between members of the sequence. The correct attitude to take is that whatever symmetry requirements are satisfied by a sequence of quantum states are revealed by experiment, or else derived from physically motivated assumptions. CFS's quantum exchangeability assumption amounts to the claim that 'all of the systems are and will be the same as far as observational predictions are concerned'. This assumption is easy to understand if quantum systems are taken to be prepared

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[130]See Diaconis and Freedman [1980].

[131] I would speculate that such representations can be found; I hope to explore this question in future work.

independently in identical states. Given that Caves, Fuchs and Schack do not have recourse to such an account, the grounds for imposing an (perhaps generalized) exchangeability requirement remains mysterious.

But a further explanatory gap alluded to in the Timpson quotation above also presents itself. If there is no such thing as a correct probability assignment or a an objective quantum state, then why are those experimenters who use quantum mechanics better able to negotiate the world than those who do not? It may be that agents who use quantum mechanics are more successful than agents who do not because the theory places empirical constraints on the means through which probabilities are *updated*, rather than determining right and true probability assignments. This is the view that Chris Fuchs has adopted in his most recent work. On Fuchs' view, the Born rule is to be understood as an 'empirical addition to Bayesian coherence'. I turn now to a consideration of Fuchs' interpretation of the Born rule and the notion that quantum mechanics is in some sense a 'user's manual' for agents negotiating the physical world.

*The Born Rule as an Empirical Addition to Bayesian Coherence*

From the point of view of the QBist, quantum mechanics constitutes an extra tool in addition to Bayesian coherence with which to navigate an unpredictable world. The theory is fundamentally different than any theory that has come before it; rather than being a picture or representation of the world as it is, it is an 'addition to probability theory itself'.[132] Like probability theory, quantum mechanics on this view is essentially normative in character. Rather than prescribing beliefs, it offers rules of consistency that agents ought to satisfy with respect to their beliefs. These rules go beyond bare consistency, in the sense that the possible probability assignments allowed by quantum mechanics are a proper subset of those satisfying the axioms of classical probability theory. It is in this sense that quantum mechanics is an 'empirical addition to Bayesian coherence'. This interpretation of quantum mechanics as an empirical refinement of Bayesian coherence raises an obvious question: what is it that we have learned about the world when we learn that the allowable probabilities for experimental outcomes belong to a certain restricted set?

To try to get a sense for how a Quantum Bayesian might attempt to answer this question, it is worth looking at Fuchs' analysis of the Born rule in some detail. As Fuchs notes, 'if quantum mechanics is a user's manual, one cannot forget that the world is its author.'[133] If one takes the view that quantum mechanics is intimately linked to probability theory, an obvious technical question presents itself: can the textbook formulation of the theory, with its states, complex amplitudes, Hilbert spaces, and so on, be re-written in terms of 'probabilities in and probabilities out?'[134]

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[132] Fuchs [2010], p. 9.

[133] Ibid.

[134] Ibid, p. 12.

In fact it can.[135]The novelty of the Quantum Bayesian approach to this question is that it starts from the mathematical framework of 'symmetric informationally complete positive-operator-valued measures', or SIC POVM's.[136]

To define a SIC POVM, we start with a set of $d^2$ rank-1 projection operators $\Pi_i = |\psi_i\rangle\langle\psi_i|$ on a $d$-dimensional Hilbert space $H$ such that

$$\left|\langle\psi_i|\psi_j\rangle\right|^2 = \frac{1}{d+1} \qquad \text{whenever } i \neq j. \tag{4}$$

Owing to the symmetry implied by condition (4), SIC POVM's have several nice properties.[137] Such operators are positive semi-definite, form a basis for the space of operators on $H$, and after rescaling they become a resolution of the identity operator

$$\Sigma_i \frac{1}{d}|\psi_i\rangle\langle\psi_i| = I. \tag{5}$$

The set of operators in this resolution of the identity forms a Symmetric Informationally Complete Positive Operator-valued Measure, or SIC POVM. It turns

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[135] Cf. Ferrie and Emerson [2009], Wooters [1986].

[136] For a formal discussion of SIC's and informationally complete measurements, see Fuchs and Schack [2009].

[137] In what follows, I will sometimes use the term 'SIC' as shorthand for 'SIC POVM'.

out that an arbitrary density operator $\rho$ can be expressed as a linear combination of the $\Pi_i$. Because the operators are positive semi-definite and form a resolution of the identity, they can be interpreted as labeling the potential outcomes of a quantum measurement device. The measurement device in question does not correspond to a standard von Neumann measurement whose outcomes are eigenvalues of some Hermitian operator. Instead, as we have seen, SIC's are examples of the more general positive operator-valued measures, or POVM's.[138] They can therefore be thought of as labeling the outcomes of a generalized quantum measurement device.[139]

Suppose that we think of a quantum state as a representation of an agent's beliefs about the potential outcomes of a SIC measurement. It turns out that this SIC representation can be used to express the Born rule probabilities for any other quantum measurement. Fuchs asks us to consider a SIC measurement 'in the sky' with outcomes $H_i$ along with any standard von Neumann measurement 'on the ground'. Suppose that the latter measurement has outcomes $D_j = |j\rangle\langle j|$ with the vectors *j* representing some orthonormal basis. We are asked to consider two paths to the measurement on the ground. 'Path 1' proceeds directly to the measurement on the ground. 'Path 2' proceeds first to the measurement in the sky, and subsequently to the measurement on the ground.

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[138] In what follows it is assumed that SIC's exist in all finite dimensions. Whether or not this is true remains an open question (see Fuchs [2010]). It is known that they exist in dimensions 2-67.

[139] For a discussion of the POVM formalism, see Nielson and Chuang [2000].

We also assume that we are given the agent's personal probabilities $P(H_i)$ for the outcomes $H_i$ of the measurement in the sky, along with her conditional probabilities $P(D_j|H_i)$ for the outcomes on the ground, given the measurement in the sky. These are just the probabilities that the agent would assign on the assumption that the quantum system follows path 2. If our agent has coherent beliefs, then the law of total probability implies that she should assign the probabilities $P(D_j)$ for the measurement on the ground given by

$$P(D_j) = \sum_i P(H_i)P(D_j|H_i) \qquad (6)$$

How does this probability assignment relate to the probabilities for measurement outcomes on the ground, assuming that path 1 is followed instead of path 2? The Born rule implies that a direct measurement on the ground should have outcomes with probabilities given by

$$Q(D_j) = tr(\rho H_j) \qquad (7)$$

for some quantum state $\rho$.[140] Clearly, we may have in general

[140] Note that $D_j$ is used to denote an outcome of an experiment and $H_j$ to denote an operator in this equation.

$$P(D_j) \neq Q(D_j) \tag{8}$$

given that path 2, involving a 'measurement in the sky', is not a unitary process. Nevertheless, given the SIC formalism, we can represent $Q(D_j)$ as a linear function of $P(D_j)$. In particular, Fuchs and Schack [2009] are able to show that

$$\begin{aligned} Q(D_j) &= (d+1)P(D_j) - 1 \\ &= (d+1)\sum_{i=1}^{d^2} P(H_i)P(D_j|H_i) - 1 \end{aligned} \tag{9}$$

This is a particularly simple representation of the Born rule in terms of probabilities.[141] Note that equation (9) is just a linear function of $P(D_j)$. Furthermore, (9) makes no direct reference to quantum states. The equation says that we can represent the probabilities for the outcomes of any experiment in terms of the potential consequences of an explicitly counterfactual action (the 'measurement in the sky').

[141] Fuchs writes: 'The Born Rule is nothing but a kind of Quantum Law of Total Probability! No complex amplitudes, no operators—only probabilities in, and probabilities out.' (Fuchs [2010], p. 12).

What exactly is the nature of the empirical constraint imposed by the Born rule? Fuchs interprets equation (9) as encoding the additional 'impact of counterfactuality', metered by the Hilbert space dimension of the system:

> Seemingly at the heart of quantum mechanics from the QBist view is a statement about the impact of counterfactuality. The impact parameter is metered by a single, significant number associated with each physical system—its Hilbert-space dimension *d*. The larger the *d* associated with a system, the more *Q(Dj*) must deviate from *P (Dj )*. Of course this point must have been implicit in the usual form of the Born Rule, [Eq. (7)]. What is important from the QBist perspective, however, is how the new form puts the significant parameter front and center, displaying it in a way that one ought to nearly trip over.[142]

From Fuchs' perspective, the Born rule is to be seen as an addition to probability in the sense that it enforces certain relations between the possible probability assignments for different experiments. More specifically, Fuchs' novel representation of the Born rule assumes that nontrivial algebraic relations must hold between the operators representing different quantum mechanical experiments. For example, the requirement that $Q(D_j)$ take on values between 0 and 1 implies that

[142] Ibid.

$$\frac{1}{d+1} \leq P(D_j) \leq \frac{2}{d+1} \quad . \qquad (10)$$

Since any positive operator can be written as a linear combination of effects, any quantum state can be represented uniquely in terms of the probabilities associated with some SIC measurement. From an operational standpoint, this entails that an agent's probability assignments for actual experiments are constrained, as in equation (9), by her probability assignments for counterfactual experiments.

Let us take stock of where we are. If the program of rewriting the Born rule entirely in terms of 'probabilities in and probabilities out' is to be successful, then the demand that $Q(D_j)$ be a proper probability distribution places necessary restrictions on $P(D_j)$. What is it exactly that one is accepting when one accepts that these restrictions must hold? And why should a subjective Bayesian accept such restrictions on her probability assignments at all? The obvious answer is that we have done the experiments, and we know as a matter of experimental fact that the algebraic relations between operators that imply these restrictions on probability assignments *do* hold. The expectation values associated with different experiments exhibit the same algebraic relations as the corresponding operators. What kind of experimental fact is this? We have clearly learned something about the world by conducting these experiments. From a realist perspective, what we have learned is that the relations between the probabilities associated with different experiments reveal structural features of the world, encoded in the quantum state. What would the Quantum Bayesian say we have learned?

In their recent paper "Quantum Bayesian Coherence", Fuchs and Schack anticipate this sort of objectivist push. Under the subheading '*Why "Coherence-Plus" Instead of Objective Quantum States*?' the authors write:

> Why would a personalist Bayesian accept any a priori restrictions on his probability assignments? And particularly, restrictions supposedly of empirical origin? The reply is this. It is true that through an axiom like [Eq. (9)] one gets a restriction on the ranges of the various probabilities one can contemplate holding. But that restriction in no way diminishes the functional role of prior beliefs in the makings of an agent's particular assignments [$P(H_i)$ and $P(D_j|H_i)$].

The authors go on to write that

> This is the key difference between the set of ideas being developed here and the dreams of the objectivists: added relations for probabilities, yes, but no one of those probabilities can be objective in the sense of being any less a pure function of the agent. A way to put it more prosaically is that these normative considerations may narrow the agent from the full probability simplex to the set of quantum states, but beyond that, the formal apparatus of quantum theory gives him no guidance on which quantum state he

> should choose. Instead, the role of a normative reading of the Born Rule is as it is with usual Dutch book.[143]

But the analogy with 'usual Dutch book' coherence is not apt. From a Bayesian perspective, the role of probability theory is to provide a framework for evaluating belief that allows agents reasoning on the basis of uncertainty to detect potential logical inconsistencies in their beliefs, and to correct those inconsistencies. This is an entirely *a priori* exercise; the theory offers no guidance with respect to the question of which factual beliefs to maintain, so long as they are consistent with one another.

But what of an agent whose probability assignments do not satisfy equation (9)? There is no necessary logical inconsistency involved in such probability assignments, although agents who hold such beliefs will of course find those beliefs to be inconsistent with the predictions of quantum mechanics. This shows that a perfectly rational agent who has no knowledge of quantum mechanics will find herself at a distinct disadvantage relative to those familiar with the predictions of the theory. The agent will be prone to probability assignments that the theory does not allow. Of course, given enough time to interact with quantum systems, our agent may come to learn that the restriction (9) holds, and will no doubt conclude that she has learned an interesting fact about the world. All parties to the debate would agree that we are learning about objective features of the world when we learn this fact. But what are

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[143] Fuchs and Schack [2009], pp. 25-26.

these objective features? Fuchs and Schack acknowledge that the task of specifying the 'undesirable consequences' of ignoring restrictions like those in equation (9) remains unfinished in their framework:

> [S]pecifying those "undesirable consequences" [of assigning personal probabilities that do not satisfy equation (9)] in terms independent of the present considerations is a significant part of the project of specifying the ontology underlying the quantum-Bayesian position. But that is a goal we are not yet prepared to tackle head on.[144]

But this is a goal that has already been tackled head on, and met, in a realist framework. On the latter view, the reason that our probabilities must fall within a certain restricted set is that the objective chances happen to fall within this set. One's credences should be mixtures of candidate chances, i.e., the set of possible quantum states; if an agent's credences aren't in this set, then he or she will be certain that there is a better betting strategy that has a higher expectation value.

The Born rule can be viewed, in textbook terms, as a rule for computing probabilities for measurement outcomes from quantum states. If we wish to remove all mention of quantum states from the formalism, we can certainly do so. Ultimately, the evidentiary basis of quantum theory is derived from the analysis of repeatable experiments, and the formalism of the theory is answerable to these observations. If

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[144] Ibid., p. 27.

the predictions of the formalism are susceptible to confirmation, then the experimental data can be interpreted as providing evidence for the empirical adequacy of the theory. This evidence is checked against the probabilistic predictions of the theory, and so on the level of confirmation it is not surprising that the standard formalism can be rewritten without explicit reference to quantum states. This point is recognized by Bill Wooters, one of the early pioneers of efforts to provide a reformulation of quantum mechanics starting from probabilities:

> It is obviously possible to devise a formulation of quantum mechanics without probability amplitudes. One is never forced to use any quantities in one's theory other than the raw results of measurements.[145]

On the level of interpretation, the fact that we can rewrite the Born rule, as in equation (9), in terms of 'probabilities in' and 'probabilities out', thereby eliminating all reference to quantum states, does not constitute an argument for the eliminability of quantum states. Instead, it shows that whatever is objective about the quantum state representation can be transformed into a statement of the form of equation (9).

It is true that equation (9) does not impose a 'right and true' set of probability assignments for measurement outcomes, but only places additional empirical constraints on these outcomes beyond those implied by coherence alone. But different

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[145] Wooters [1986].

agents with different expectations for measurement outcomes will eventually converge on a limiting probability assignment for the outcomes of any repeatable experiment, given enough trials and assuming that they take any sequence of outcomes to be exchangeable. This procedure looks for all the world like it is uncovering objective features of physical setups, and the availability of a formulation of the theory in terms that begin and end with probabilities will not convince the '$\psi$-ontologist' to dispense with his interpretive commitments.

Furthermore, the great advantage of a realist interpretation of the quantum state, whether formulated in terms of probability amplitudes and Hilbert spaces or strictly in terms of probabilities via SIC measurements, is that it furnishes a compelling explanation of the activity of experimenting on quantum systems. If preparation devices really do prepare independent copies of identical quantum states, then the exchangeability assumption is trivially justified, and the widespread intersubjective agreement among scientists receives a straightforward explanation in terms of publicly accessible data. Furthermore, the observed restrictions on probability assignments implied by quantum mechanics receive a straightforward explanation as a structural feature of objective quantum states. More importantly, if the probabilities assigned to measurement outcomes are taken to be objective, then it becomes possible to explain many of the observed features of the physical world that would otherwise be mysterious, such as the organization of the periodic table and the lattice structure of crystals, as revealed by scattering experiments and crystallography. These considerations are taken up in greater detail in the next chapter.

## Conclusion

Quantum Bayesianism is a relatively well worked-out interpretation of quantum mechanics that starts from the premise that all probabilities, including Born rule probabilities, are inherently subjective. The Quantum Bayesians are able to address many of the obvious objections that have been raised against taking a subjectivist attitude toward quantum states. Not least among these objections is that the Quantum Bayesians are seemingly unable to make logical sense of quantum state tomography, involving experiments on 'unknown quantum states'. Such experiments seem to presuppose that there are such objects, and include preparations of 'identical copies' of quantum systems in order to perform multiple experiments.

While the Quantum Bayesians are able to provide a consistent account of quantum state tomography, or of the process of learning about 'unknown quantum states' within a subjectivist framework, I have suggested that the existence of such an account does not tell in favor of a subjectivist interpretation of the quantum state. To be fair, the Quantum Bayesians have not argued that technical results such as the quantum de Finetti theorem and the SIC POVM reformulation of the Born rule are sufficient to rule out quantum state realism. Nevertheless, the picture that Caves, Fuchs and Schack adopt can be evaluated on its own merits, and in comparison to rival interpretations. I have argued that such an evaluation already suggests looking elsewhere for an alternative interpretation of Born rule probabilities along realist lines. In Chapter 2 of the present work I have addressed two explicit arguments put forward in favor of an antirealist interpretation of the quantum state, and found these arguments wanting. The purpose of chapter 4 will be to provide positive arguments for quantum state realism. It is hoped that the present chapter has provided some indication of the explanatory holes in the Quantum Bayesian account that will be addressed in some measure by the work to follow in chapter 4.

# Chapter 4: The Case for Quantum State Realism

A knowledge of ψ enables us to follow the course of a physical process insofar as it is quantum-mechanically determinate: not in a causal sense, but in a statistical one…These probabilities are thus dynamically determined. (*Max Born, 'Physical Aspects of Quantum Mechanics', 1927*).

## Introduction

In the following chapter I attempt to articulate and defend the case for quantum state realism. I begin with an introductory section designed to motivate the problem of accounting for the nature of the quantum state, and to clearly define the scope of the problematic as one of adequately accounting for the relation of quantum mechanical phenomena to the formalism of the theory. I then argue that the proper interpretation of the quantum state must respect the evidentiary basis of quantum mechanics, which includes physical phenomena that are systematically predicted and explained by the quantum state function. It is ultimately the evidentiary basis of the theory, which includes relative frequency data described by the quantum state, which gives us good grounds for accepting quantum mechanics. This relative frequency data provides evidence for the existence of objective chances, encoded in the quantum state. I therefore conclude that the wave function is indeed genuinely representational.

Nevertheless, the relation of the mathematical formalism of quantum theory to the physical world has no analog in classical physics.

## 4.1 Yes, but…

Any attempt to interpret quantum theory realistically must immediately confront the obvious question, 'realism about what?' It is hardly an exaggeration to suggest that the various interpretations of the theory can be categorized according to their answers to this question. From the perspective of the de Broglie-Bohm theory, for example, we ought to be realists both about quantum states and the existence of hidden variables ultimately responsible for the observed statistical quantum phenomena. In the Everettian picture, on the other hand, the slogan 'nothing but the wavefunction' encapsulates the notion that Everettian quantum theory, unlike any other interpretation, follows from taking the formalism of the theory to be literally true and complete. That 'realism' can be marshaled as a motivation for adopting either of these interpretations at least suggests that this concept may be too vague or ambiguous to be used decisively in addressing questions of interpretation in the theory. Most parties to the debate seem to agree that being 'realistic', or at least identifying those elements of the theory that are real, is a desideratum for any interpretation. The conditions under which this desideratum can be met, however, seem to change according to which interpretation of the theory one is concerned to defend.

It should be stated from the outset that in advocating for a realist interpretation of the quantum state, I am not attempting to impose a diffuse realism desideratum on any interpretation of quantum theory. Nor am I attempting to defend the general thesis of scientific realism in the context of quantum theory. I am skeptical of the feasibility of such a project not only because of the unique difficulties posed by quantum mechanics, but also because it is not clear to me that the distinction between realism and instrumentalism is of general interest at the metatheoretical level. The goal of an interpretation of quantum theory, first and foremost, ought to be to furnish a correct and adequate account of the formalism of the theory in relation to the phenomena.[146] I take it that this is a basic assumption shared by those commentators that argue for an epistemic or subjective interpretation of the quantum state.[147] The characterization of the relationship between the formalism and the phenomena in quantum mechanics faces unique challenges not met in classical contexts, which may partially explain the widespread disagreement over the interpretation of the quantum

[146] In Carnap's terminology, having chosen a formal language, as embodied in the postulates of standard quantum theory, we ought to settle the internal questions posed by that particular choice of language. See Carnap [1950].

[147] Cf. Fuchs [2010, 2002], Bub and Pitiwsky [2010], Bub [2007], Fuchs and Schack [2008], Spekkens [2007], Caves, Fuchs and Schack [2002a,b], Fuchs and Peres [2000], Zeilinger [1999].

state. I will have more to say about these challenges and how they can be met in sections 4.3 and 4.4.

Empirical adequacy is an obvious regulative ideal for a scientific theory regardless of one's scruples with respect to the realism/antirealism debate. Even the staunch instrumentalist will demand that her theoretical representation be adequate to the observed phenomena. What does it mean for a theory to be empirically adequate? Bas van Fraassen defines a theory to be empirically adequate if 'what is says about the observable things and events in the world, is true-exactly if it 'saves the phenomena'.[148] Few philosophers of science would deny that according to this criterion quantum theory is highly successful. Yet the question of what the theory actually says about the world depends crucially upon how one interprets the quantum state. The latter question must be posed at the object level; it is a question of the physical content of the theory. Hence the status of the quantum state in quantum mechanics must ultimately be settled with reference to empirical methods. In particular, an examination of these methods will reveal whether the probabilities employed in the theory should be taken as evidence for the existence of objective chances, as the present work is concerned to argue, or whether they should be interpreted epistemically.

In what follows, therefore, I am not concerned with the question of how to evaluate the semantic status of scientific theories as such. I take it as given that

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[148] Van Fraassen [1980]

quantum theory is a paragon of successful physics. I do not intend to argue for or presuppose an attitude towards the metaphysical status of terms occurring in the theory. I do not know how one ought to address such concerns regarding the reference of theoretical terms.

Presumably the “scientific realist” may argue that an interpretation of the theory that does not address questions of reference is explanatorily inadequate, and that this referential inadequacy can be removed by requiring that the theory in question not only furnish adequate representations of the phenomena, but also be “true”. I would respond to such an argument, which I consider only to distinguish it from my own project, with a rhetorical question: confronted with two theories that correctly describe the phenomena, one of which is false and the other true, how are we to judge between them? I align myself with a long tradition in the philosophy of science that contends that this sort of question exposes a confusion regarding the relation between theories and the phenomena they describe.[149] Carnap’s ‘Empiricism, Semantics and Ontology’ argues persuasively that questions like ‘how do I know that the properties of phenomena actually inhere in substances?’ are in fact pseudo-questions.[150] (Of course, in Carnap’s view, we are always free to adopt a ‘thing

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[149] Cf. Stein [1989], Carnap [1950]. This general attitude can be traced back at least as far as Kant, who argued that we must clearly distinguish between questions of empirical reality (the purview of scientific enquiry) and those of transcendental reality (the purview of dogmatic metaphysics).

[150] Carnap, [1950].

language' or a 'phenomenal language' according to convenience). Within a given theoretical framework, internal questions can be posed and answered by means of logico-mathematical and empirical methods. These sorts of questions concern the existence of certain entities *within* a given theoretical framework. An example of such an internal question, germane to the present work, is 'are the probabilities occurring in the theory of the logical or empirical variety?' Of course we can also ask about the existence or non-existence of a system of entities taken as a whole, but the framework within which these entities are described will be powerless to adjudicate such external questions. As Carnap puts it, 'to be real in the scientific sense means to be an element of the system; hence this concept cannot be meaningfully applied to the system itself.'[151]

The basic point has its roots in the Kantian distinction between empirical reality and transcendental reality, which was originally marshaled against early modern dogmatic metaphysics. Given that, according to Kant, we have access only to the world of empirical phenomena, as described within a given conceptual framework, and given that this framework, in conjunction with experience, is the context of justification of all of our knowledge claims, it is useless to hypostatize entities (the 'objects of reference') that are beyond our capacity to experience. Such noumenal objects are by definition outside the bounds of human knowledge. What we *can* do is employ the conceptual resources at our disposal to better understand the phenomena

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151 Carnap [1950], 28.

themselves. I do not wish to presuppose that any attempt to utilize scientific concepts to justify metaphysical claims amounts to Kantian subreption; rather, I wish to argue that the theoretical framework within which the quantum state is defined in relation to physical phenomena, involving repeatable experiments that reveal relative frequencies predicted by the quantum wave function, strongly supports an objective interpretation of the quantum state.[152]

In articulating the case for quantum state realism, therefore, I am not attempting to reify or hypostatize the quantum state. In the Carnapian language, I am attempting to formulate and answer an internal question concerning the relationship between a certain mathematical structure and the phenomena that this structure allows us to make quantitative predictions about. Part of the task involves getting clear about how the theory generates probabilistic predictions, what constitutes our grounds for accepting the theory as empirically adequate, and in light of our answers to these questions, what attitude we should adopt towards the probabilities occurring in the theory. Part of what is 'non-classical' about the quantum theoretical description of nature is the manner by which the mathematical apparatus of the theory is interpreted in relation to observable experimental events. In a classical framework, one attempts to measure the attributes of entities like particles and fields whose properties are taken to inhere in the entities themselves regardless of whether a

[152] In the Kantian language, the quantum state is 'empirically real', whether or not it or any other aspect of physical theory is also 'transcendentally real'. Cf. CPR A373-374,

measurement is carried out. Even if the system in question is too complex to be modeled directly, it is still possible to draw conclusions about the average behavior of the system.

This picture of property measurement is highly dubious in the case of the quantum theory of measurement. To borrow another Carnapian turn of phrase, the 'thing language' is arguably appropriate to classical descriptions of phenomena, but less so in regimes where Planck's constant becomes significant. Nevertheless, the quantum state does contain genuinely physical information about the system it describes. While the description is in general probabilistic in character, these probabilities are (at least sometimes) perfectly objective. It is this latter claim that I have labeled 'quantum state realism', and which I wish to describe and defend in more detail in this chapter.

## 4.2 What is Quantum State Realism?

Having settled on the scope of the question as an internal question regarding quantum theory and its relation to phenomena, we may now ask: what exactly is affirmed by the proposition that the quantum state is real or objective? As a first step toward an answer, I will sketch a preliminary definition of quantum state realism due to Harrigan and Spekkens [2010]. In order to forestall misinterpretations of the definition, I will also briefly canvass a set of possible interpretations of quantum state realism that I do *not* have in mind.

Harrigan and Spekkens [2010] provides a convenient definition of minimal quantum state realism, which the authors characterize as a '$\psi$-ontic' interpretation of the quantum state. At a minimum, any ontological model of a quantum system, whether realist or antirealist, should assign properties to the system.[153] In an operational framework, we assume that a preparation procedure prepares a system in a particular *ontic state*, understood as a complete assignment of values to various physical parameters associated with the system. A point in the space of possible ontic states will then represent the actual ontic state of a system. With this definition in hand, we can ask about the relationship between the ontic state of a system and the quantum state used to represent it.

The simplest possible relation between the ontic state and the quantum state is an isomorphic relation. In such a model, the quantum state provides a complete description of the ontic state of a system. A $\psi$-complete model is obviously $\psi$-ontic or

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[153] At this point we assume very little about these properties. For example, they may be relational in character. I will have more to say about what these properties might be below. Certain psi-epistemic interpretations, such as that defended in Spekkens [2007] and Spekkens and Harrigan [2010] are willing to grant properties to systems that are nevertheless associated with multiple wavefunctions. Chris Fuchs' QBist interpretation is not willing to attribute dynamical properties to quantum systems, and therefore rejects the framework of ontological models for quantum states. For a criticism of the QBist framework, see chapter 3 of the present work as well as section 4.4 below.

realist. Another possibility is that a complete description of the physical state of a system requires supplementing the quantum state with additional variables. Such variables are usually described as 'hidden' because their value is unknown to someone who knows the quantum state. In such models, the quantum state provides only an incomplete description of reality. The complete ontic state is given by the quantum state $\psi$ together with supplemental variables $\omega$. The de Broglie Bohm theory is an example of such a $\psi$-supplemented model. The Bohm theory is $\psi$-ontic, since distinct $\psi$'s describe distinct physical states in that theory. Nevertheless, there is more than one ontic state corresponding to a given $\psi$, parameterized by the supplementary 'hidden' variables $\omega$.

A third possibility investigated by Harrigan and Spekkens is that $\psi$ is not only an incomplete description of the ontic state, but actually represents an epistemic state of knowledge. Whereas in the Bohm theory different states correspond to different states of reality, on this view, $\psi$ is not a variable in the ontic state space at all. Instead, $\psi$ represents a probability distribution over the ontic states. Variations in $\psi$ therefore represent different states of knowledge of the ontic states rather than different states of reality. Crucially, different $\psi$'s may have overlapping support, so that the same ontic state may be associated with multiple (indeed, infinitely many) quantum states.

This last situation is analogous to the situation encountered in classical statistical mechanics. In the latter theory, it often happens that we know some properties of a large collection of systems, such as mean kinetic energy or pressure. In such cases we are interested in how these properties vary dynamically, and many different microscopic trajectories are compatible with the macroscopic behavior of the system. If we associate a phase space with the possible states of the system, then our knowledge of the system is represented by a probability distribution over possible states of the system. Any given state of the system, represented by a point in phase space, will be compatible with multiple probability distributions. Similarly, it may be that the probabilities associated with quantum states are epistemic in character.

I will adopt the Harrigan and Spekkens taxonomy by categorizing $\psi$-complete and $\psi$-supplemented models as '$\psi$-ontic', or realist interpretations of the quantum state. What the two classes of models have in common is that they associate different quantum states with different states of reality. It should be stressed from the outset, however, that the space of $\psi$-epistemic interpretations is not exhausted by those interpretations that attribute properties to systems compatible with multiple quantum states. There is an important result, due to Pusey, Barrett and Rudolph (PBR), that shows that an ignorance interpretation that attributes definite, intrinsic properties to quantum systems prepared independently of one another is incompatible with the predictions of quantum mechanics. One can escape this conclusion by denying that quantum systems have definite properties prior to measurement, or by rejecting the idea that quantum systems can in principle be prepared in isolation (or in near isolation) from one another.[154] The former route is taken by the so-called quantum Bayesian school of Schack, Caves, Fuchs and Appleby. Quantum Bayesianism is epistemic about experimental outcomes rather than ontic states. The $\psi$-epistemic view adopted by Rob Spekkens, on the other hand, is epistemic about ontic states, but denies that they can be prepared in isolation from one another.

I will have more to say about the scope and significance of the PBR theorem below. The theorem is important because it makes clear, by means of a remarkably simple argument, what assumptions one has to give up in order to endorse an epistemic view of the quantum state. To the knowledge of the present author, all of the $\psi$-epistemic views that are currently represented in the literature reject one or more of these assumptions explicitly, independently of the PBR result. If the case for quantum state realism is to be made compelling, therefore, additional arguments are needed. The arguments against quantum state realism put forward in the present work are intended to weigh against all antirealist or $\psi$-epistemic interpretations on general philosophical grounds.

Before proceeding to the positive arguments, and in order to forestall possible misconceptions, I will briefly canvass a set of possible 'realist interpretations' of the quantum state that I do not wish to defend.

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[154] A further escape route is offered by the possibility of retrocausation. I will not consider this possibility here.

i) *The quantum state is not a local beable*. It is clear that the quantum state is not a 'local beable' in John Bell's [1975] sense of the term. The state transition associated with measurement, if it is regarded as a real physical process, occurs instantaneously over arbitrarily large spatial distances, and is therefore manifestly non-local. But the problem with attributing local beable status to the wavefunction runs deeper than this. The problem is that the wavefunction does not exist in spacetime; it does not have values at particular spacetime points. Rather the wavefunction exists in configuration space, with each point in the configuration space corresponding to a possible spacetime configuration of the system of interest.

Whether the wavefunction is some sort of non-local beable is perhaps less clear. Bell himself characterizes beables as objects which can be described in classical terms, and out of which observables are comprised. Examples include settings on pointer dials, instrument readings, etc. Bell also harnesses the beable concept to distinguish those elements of a theoretical formalism which are physical from those which are merely artifacts of gauge freedom. For example, the Lorenz and Coulomb gauges in classical electrodynamics exploit the fact that the **E** and **H** fields are unchanged if we take the vector and scalar fields **A** and $\phi$ and transform them simultaneously according to

$$A \to A + \nabla \psi$$
$$\varphi \to \varphi - \frac{\partial \psi}{\partial t}$$

where ψ is any function of x and t.[155] The **E** and **H** fields are 'physical' in the sense that they contain only physical degrees of freedom; a particular choice of vector and scalar potentials is a gauge. Therefore we are not overly concerned if, for example, the potentials fail to propagate locally in one or another gauge; imposition of the Coulomb or Lorenz gauges merely facilitates the description of the real dynamical evolution of electromagnetic fields.

For similar reasons, the instantaneous state transition over arbitrary distances need not bother us if we regard the wave function, in analogy with the scalar potential, as essentially a mathematical tool for correlating the results of experimental outcomes. However, the analogy with the Coulomb gauge in Maxwell's theory should not be pushed too far. In the case of gauge invariance, we possess some freedom to set the values of the potentials for ease of calculation without influencing the local beables of the theory, in particular the values of the **E** and **H** fields. The same cannot be said of the wavefunction; different values for the wavefunction correspond, in general, to different physical situations.

The wavefunction is an abstract mathematical object living in an 3N-dimensional configuration space which exhibits surprisingly non-local behavior when

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[155] See http://au.arxiv.org/abs/physics/0204034

projected down onto real space. As Bell points out, in contrast to the **E** and **H** fields, it only makes sense to ask for the amplitude of the wavefunction once several points in *physical* space have been specified.[156] Restricting attention to non-relativistic quantum mechanics, the wavefunction $\Psi(\mathbf{x}, t)$ is typically thought of as a time-dependent 'field' on configuration space, defining a probability density on physical space. The integral of $|\Psi(x, t)|^2$ over a given volume $V$ gives the probability that a particle will be found within the volume $V$. However, once a measurement is carried out, $\Psi(\mathbf{x}, t)$ undergoes a non-unitary transformation, and if the particle is found to be within $V$, the probability that it will be found anywhere else vanishes instantaneously.

When one attempts to incorporate relativistic considerations into this account, further complications are introduced.[157] One can meaningfully ask about the quantum state in a given region of spacetime only after a given Cauchy surface has been specified. If one wishes to provide a complete description of the evolution of the state of a spatially extended region, (and not merely of some extensionless point within it), it is necessary to first define a foliation consisting of spacelike hypersurfaces along which this state history is defined. This is true of any relativistic description of an extended region of space. But if one is prepared to entertain the possibility that the quantum state *at a given point* in spacetime constitutes part of the complete description of that point, then a complete description of *points* in space, and not

[156] See Bell [1987]. Reprinted in Bell [2004], p. 204.

[157] For a detailed discussion of this point, see, chapter 2 of the present thesis.

merely of extended regions, becomes foliation-relative.[158] The expectation values of an observable at a given point in spacetime will in general depend on the foliation chosen, and for each point there will be an uncountable infinity of possible foliations. What qualifies as a complete physical description of a given spatiotemporal point therefore hinges on whether beable status is granted to manifestly non-local structures like the quantum state in a physical theory. The claim that the quantum state is an objective representation of the physical state of a system necessarily involves a rejection of the so-called 'separability principle', according to which spatially separated systems can be regarded as having independent real states.[159]

If this account is correct, then it is not the case, to quote David Albert [2000], that 'the situation associated with two intersecting space-like hypersurfaces in the Minkowski-space must agree with one another about the expectation values of local observables at points where the two surfaces coincide'.[160] In what follows I will attempt to defend an interpretation of the quantum state that entails giving up on an ontology consisting entirely of local beables.[161] Nevertheless, the presence of non-

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[158] For a discussion of this point see Myrvold [2003]. See also Albert [2000].

[159] See Howard [1985] for a useful discussion of the distinction between locality and separability.

[160] Albert [2000], p. 6.

[161] The question of what such an ontology might look like, and what its implications are for the compatibility of quantum theory with relativity, are discussed in chapter 2, where I discuss the so called 'steering argument'.

local beables does not entail the non-existence of local beables; this is a point taken up in detail in chapter 2 of the present work.

ii) *The wave function is not a physical object.* Taken at face value, the interpretation of the wavefunction as a physical object, or a representation of a physical object, is indefensible. Quantum state realism entails that the wave function is genuinely representational. However, the physical interpretation of this representational structure requires a sophisticated set of correspondence rules that are part of the non-classical structure of quantum theory. For example, the wave function of the four outer electrons in the ground state of the carbon atom produces a tetrahedral structure that undergirds the actual observed tetrahedral structure of the diamond crystal. However, the wave function associated with this system is 12-dimensional. This example shows that the physical interpretation of the wavefunction involves some degree of subtlety. The twelve-dimensional wave function must be projected down onto physical space, and the probability of finding an electronic charge in a given volume must be evaluated in accordance with the Born rule.

Higher dimensional representations are already familiar in the classical regime. I have already mentioned the example of classical gauge freedom. The classical Hamiltonian framework provides an example of such a scheme. In this framework, the state of a closed classical system is represented by a 6N-dimensional phase space in which each of the N particles in the system possesses 3 position and 3 momentum degrees of freedom. The dimensionality of the phase space representation obviously does not correspond to the dimensionality of the system it represents.

Nevertheless, it would be a mistake to argue that this dimensionality is without physical significance.

In the case of the classical Hamiltonian framework, specifying the values of each of the position and momentum variables captures the state of a system at any given time. The Hamiltonian function then determines all subsequent states of the system. In the case of quantum mechanics, the Schrödinger equation governs the evolution of the state evolving in Hilbert space. Complete certainty with respect to both position and momentum parameters is in principle unattainable. Complete knowledge of the state in general yields only probabilities for measurement outcomes. The shape of the probability distribution in the 12-dimensional Hilbert space representation of the valence electrons of diamond crystal contains information about the expected charge density in the carbon atom when projected onto 3-dimensional physical space. The phase of the wave function in turn determines how this probability density changes with time. The fact that the wave function predicts the observed tetrahedral structure of the carbon atom is itself physically significant; I will argue that this predictive power cannot be adequately accounted for by the quantum Bayesian or any other antirealist interpretation of the quantum state. I will return to this example shortly.

iii) *The quantum state is not a relation between token observers and physical systems.* It is a minimal requirement for quantum state realism that the quantum state be understood as a representation of the state of a system, and not merely of someone's knowledge or beliefs about a system. This requirement clearly rules out

Quantum Bayesian interpretations. It might be wondered whether this minimal requirement is also incompatible with more moderately 'non-absolutist' programs such as the relational interpretation of Carlo Rovelli [Rovelli 1996]. Rovelli rejects the idea that there exist absolute or observer-independent quantum states. The crucial observation in support of this claim is that different observers may give different maximally complete accounts of the same sequence of events. The word 'observer' is used here in the same way that it is used in the description of Galilean relativity, where one speaks of velocities 'relative to some observer'. In a Galilean spacetime it makes no sense to speak of the absolute velocity of a body. Instead we speak of velocities relative to some observer. For analogous reasons, according to Rovelli, it makes no sense to speak of the quantum state of a system without first specifying a framework within which to define the state in question relative to some observer. The observer need not be a human being with a PhD, or even a computer or a cat, but could be any physical system capable of recording information, i.e., with more than one possible state.

Rovelli argues that there exist no observer-independent quantum states by invoking what he calls the 'third person problem'. We are asked to consider an observer $O$ (any sort of macroscopic measuring apparatus will do) that makes a measurement on (or interacts with) a system $S$. Rovelli assumes that the quantity $q$ being measured by $O$ can take on the two values 1 and 2. The states of $S$ are described in the standard way by rays in a 2-D Hilbert space $H_S$. Let the two eigenstates of the operator corresponding to a *q* measurement be $|1\rangle$ and $|2\rangle$. In general, at time $t=t_1$ prior to measurement, *S* will be in a state

$$|\Psi\rangle = \alpha|1\rangle + \beta|2\rangle$$

with $\alpha, \beta$ complex and $|\alpha^2| + |\beta^2| = 1$. *O* can measure either of the two values 1 or 2, with respective probabilities $|\alpha^2|$ and $|\beta^2|$.

If we assume that the result of a given measurement made by *O* is 1, then at time $t=t_2$ after the measurement the state of the system is $|1\rangle$. Now suppose that a second observer *P* describes the interacting system formed by *S* and *O*. We assume that *P* does not perform any measurements before $t_2$, but that *P* knows the initial states of both *S* and *O. P* describes *S* by means of a Hilbert space $H_s$ and *O* by means of a Hilbert space $H_o$. The *S-O* system is then described by the tensor product state $H_{so} = H_s \otimes H_o$. We assume, following convention, that *P* labels the state of the observer *O* before the measurement as $|init\rangle$. *O*'s measurement of *S* implies an interaction event between *O* and *S.* This interaction brings about a change in the state of *O*. The initial state of the *O-S* system is

$$|\Psi\rangle \otimes |init\rangle = (\alpha|1\rangle + \beta|2\rangle) \otimes |init\rangle$$

Linearity then implies that, after measurement, we have

$$|\Psi\rangle \otimes |init\rangle = (\alpha|1\rangle + \beta|2\rangle) \otimes |init\rangle \rightarrow \alpha|1\rangle \otimes |O1\rangle + \beta|2\rangle \otimes |O2\rangle$$

Where *O*1 and *O*2 correspond to 'the measuring apparatus points to 1(or 2)'. At $t_2$ the system is in the state $\alpha|1\rangle \otimes |O1\rangle + \beta|2\rangle \otimes |O2\rangle$. Thus, depending on where we draw the line between system and observer, we get a different description of the same sequence of events, although *P* is assumed not to interact with the *O*-*S* system prior to measurement. At $t_2$ the system is in state $|1\rangle$ according to O, i.e., the quantity *q* has value 1. According to *P*, the quantity is in a superposition $\alpha|1\rangle \otimes |O1\rangle + \beta|2\rangle \otimes |O2\rangle$. These are two distinct, correct descriptions of the same events. Notice that the correlation between the *q* variable of *S* and the pointer variable of *O* expresses the fact that the pointer variable has information about *S*.

Hence the framework within which any quantum state ascription is made is determined by the data or information available to the observer. For any given system *S* there is a definite probability that a given question *q* will yield a yes or no answer. What is really absolute or observer-independent is the probability of a sequence $A_1 \ldots A_n$ of property ascriptions, given a measurement framework. After observer *O* observes measurement outcome 1, he or she is no longer free to adopt a framework in which *q* is not 1. Furthermore, there is an important sense in which P's description of the *O*-*S* system contains the *information* that *O* has performed a measurement on *S*; this information is contained in the correlations $\alpha|1\rangle \otimes |O1\rangle + \beta|2\rangle \otimes |O2\rangle$. Here the four possible configurations that the *q* variable and the pointer variable can take have been reduced to two. Rovelli expresses the idea that the system *S* has a definite value relative to observer *O* with the locution: *O* has the “information” that $q$=1.

Information is a perfectly objective property of a physical system. Roughly speaking, it quantifies the relationship between the total number of possible configurations of a system and the system’s actual configuration. Intuitively, the greater the total number of possible configurations, the more informative the actual configuration of a system becomes. The concept of information expresses the fact that the configurations of different physical systems can be correlated. It is on this basis that Rovelli is able to define a physically possible ‘observer’ quite generally as any physical system capable of assuming more than one state or configuration. Two observers $O_1$ and $O_2$ will assign the same quantum state to a system *S* provided that $O_1$ and $O_2$ are themselves systems of the same type, i.e., belonging to the same equivalence class of possible configurations. These configurations determine the set of questions that O can ask of *S*. Nevertheless, the information capacity of *S* is intrinsic to *S* and does not depend on any observers. This information capacity is not only discrete but also finite. Hence it is possible to completely characterize a system by means of a finite string of yes/no experimental questions.

Such a binary string is essentially an alternative representation of the quantum state of a system. What is novel about Rovelli’s characterization is that it places the emphasis on the questions that can be asked of a system, and thereby on the observers asking the questions. Two observers may furnish different descriptions of the same set of events because they have different data (frameworks) for the same events. Once the data are specified, however, all probabilities are well defined and perfectly objective. Hence there is no conflict with the ascription of an objective quantum state, given some measurement context or observer type.

I have argued that the proper scope of the problem of interpreting the quantum state should be limited to questions of adequacy to the phenomena. In this context, I have suggested that the quantum wave function is a mathematical representation of quantum systems whose exact mathematical structure is physically significant, in the sense that different wavefunction representations correspond to different states of reality. I have also suggested that the quantum state is some sort of non-local beable, furnishing objective probabilities for measurement outcomes. I will now turn to the task of fleshing out these claims.

I first consider the scope and significance of a result due to Pusey, Barrett and Rudolph that rules out a certain class of antirealist interpretations. It will be seen that none of the antirealist views considered in the present work, including the quantum Bayesian approach favored by Caves, Fuchs, Schack, and Appleby, as well the framework defended in Spekkens and Harrigan [2010], are directly undermined by the PBR theorem. Nevertheless, the theorem clarifies the assumptions made in order to maintain an antirealist interpretation of the quantum state. It is these latter assumptions that I hope to call into question by means of specific examples drawn from the evidentiary basis of quantum theory.

## 4.3 The Pusey-Barrett-Rudolph Theorem

In their important paper 'The quantum state cannot be interpreted statistically', Matthew Pusey, Jonathan Barrett, and Terry Rudolph show that a certain class of epistemic interpretations of the quantum state is incompatible with quantum mechanics.[162] The sort of view in question is gleaned from some of the later writings of Albert Einstein, who believed that the wave function could be interpreted as an incomplete description of physical reality:

> I incline to the opinion that the wave function does not (completely) describe what is real, but only a to us empirically accessible maximal knowledge regarding that which really exists […] this is what I mean when I advance the view that quantum mechanics gives an incomplete description of the real state of affairs.[163]

The case for a statistical interpretation of the quantum state is perhaps best seen by means of a classical analogy. Consider the Newtonian description of a simple system, such as a particle on a wire. The behavior of the particle can be determined by the initial conditions (in this case, the initial position of the particle and its initial

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[162] Pusey, Barret and Rudolph [2011].

[163] Letter to P.S Epstein, 1945. Reprinted in Howard [1990], p. 103.

momentum) together with knowledge of all of the forces acting on it. These two pieces of information together suffice to determine the subsequent behavior of the system. We can model this system using a simple 2-dimensional phase space representation. Each point in the phase space corresponds to a particular position and momentum, and every point can be thought of as a possible instantaneous state of the system. Given the framework of Newtonian mechanics, the position and momentum of the particle will change deterministically with time, and this dynamics will be represented in the phase space by a trajectory through different points in the space.

The phase space representation can easily be generalized to systems with arbitrary degrees of freedom by moving to higher dimensional representations. In practice, it is impossible to model the evolution of a classical system with a macroscopic number of degrees of freedom by tracking each particle. Fortunately, for most systems, we are interested in the dynamical evolution of empirically accessible macroscopic properties such as temperature and pressure. The evolution of such macroscopic states is compatible with many different microscopic particle trajectories. We assign a probability distribution over phase space, representing our uncertainty about the actual position in phase space occupied by the system, and weighted according to how likely it is that any given microconfiguration will give rise to the observed macroscopic properties of the system. This probability distribution is constrained to evolve in time in a manner that respects the macroscopic properties of the system accessible to the experimenter.

It is clear that in the classical case the state assigned to the system is *epistemic*; the state represents our knowledge of the actual point in phase space occupied by the system. Crucially, the same *ontic* state of the system (the actual positions and momenta of all of the particles comprising the system) is compatible with more than one, and indeed infinitely many, epistemic states. Pusey, Barrett and Rudolph ask whether this conception of an epistemic state can be used to interpret the quantum state and the probabilities it furnishes for macroscopic events. By means of a very simple argument, the authors are able to show that it cannot.

*The PBR Argument*

PBR assume from the outset that quantum systems exist, and that they at least sometimes have definite properties. These properties need not include things like definite position and momentum; PBR assume only that the properties in question are sufficient to distinguish systems from their environment such that measurements can be performed on them. Such systems are assumed to be amenable to preparation in the sense that they possess definite properties after being prepared in a given state; for example, when a preparation procedure is carried out that yields states that assign probability zero to some experimental outcome, this is taken to be a fact about the system in question.

PBR first consider two different methods of preparing a quantum system. If the first method is used, the system is prepared in the state $|\phi_0\rangle$. If the system is prepared using the second method, the state is given by $|\phi_1\rangle$. $|\phi_0\rangle$ and $|\phi_1\rangle$ are assumed to be distinct, non-orthogonal states. The mathematical object $\lambda$ specifies completely the physical properties of the system after preparation. It need not be assumed that the preparation procedures yield pure states. The preparation may yield mixed states $|\phi_0\rangle_m$ and $|\phi_1\rangle_m$ that are sufficiently 'close' to being pure, in a sense to be described below.

PBR now adopt the taxonomy presented in Harrigan and Spekkens [2010]. If the quantum state is a physical property of the system, then either $\lambda$ is (close to) identical with $|\phi_0\rangle$ or $|\phi_1\rangle$ after preparation, in which case the quantum state furnishes a complete description of the system, or $\lambda$ consists of $|\phi_0\rangle$ or $|\phi_1\rangle$, supplemented with some additional parameters. Crucially, the quantum state is determined uniquely by $\lambda$, with or without additional variables.

If, on the other hand, the quantum state is epistemic in character, then a full specification of the properties $\lambda$ of a system need not determine the quantum state associated with the system.[164] This situation is analogous to that found in classical

[164] The authors use the term 'statistical', rather than 'epistemic', which is potentially misleading since it suggests that they are rejecting the probabilistic interpretation of the quantum state. In fact the

statistical mechanics, where the ontic state of the system is compatible with multiple epistemic (statistical mechanical) representations. A particular value of $\lambda$ will in this case be compatible with either preparation method, that corresponding to $|\phi_0\rangle$ or to $|\phi_1\rangle$.

PBR now show that this statistical interpretation of the quantum state is incompatible with quantum theory. First we choose a basis for the Hilbert space representation of $|\phi_0\rangle$ and $|\phi_1\rangle$ such that $|\phi_0\rangle = |0\rangle$ and $|\phi_1\rangle = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.[165]PBR immediately derive a contradiction from the assumption that the complete physical state $\lambda$ of the system is compatible with the preparation corresponding to $|\phi_0\rangle$ as well as that corresponding to $|\phi_1\rangle$.

Assume that the probability that, given $\lambda$, the preparation yields either $|\phi_0\rangle$ or $|\phi_1\rangle$ is at least $q > 0$. We will also assume that two states can be prepared in isolation from one another, such that their physical states are distinguishable, and upper bounds can be placed on the correlations between these states. Let the physical states associated with the two preparations be $\lambda_1$ and $\lambda_2$ respectively. It will happen that $\lambda_1$

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opposite is true. The term 'statistical' should be thought of in connection with statistical mechanics, which assigns epistemic probabilities to ontic states.

[165] This entails that $|\langle\phi_0|\phi_1\rangle| = 1/\sqrt{2}$. We are here adopting the conventional qubit notation: $|0\rangle \equiv (1,0)$ and $|1\rangle \equiv (0,1)$.

and $\lambda_2$ are compatible with both preparations $|\phi_0\rangle$ and $|\phi_1\rangle$ $q^2 > 0$ of the time if the states are pure and independent of one another. In other words, the physical state of the joint system comprised of $\lambda_1$ and $\lambda_2$ is compatible with all four possible quantum states $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |+\rangle$ and $|+\rangle \otimes |0\rangle$.

The two systems are now brought together and measured. PBR assume that the performance of measurements on the joint system will give results with probabilities determined entirely by the disposition of the apparatus along with the physical state of the system. The following is a joint measurement of the two systems, which projects onto four orthogonal post-measurement states:

$$|\xi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle)$$
$$|\xi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |-\rangle + |1\rangle \otimes |+\rangle)$$
$$|\xi_3\rangle = \frac{1}{\sqrt{2}}(|+\rangle \otimes |1\rangle + |-\rangle \otimes |0\rangle)$$
$$|\xi 4\rangle = \frac{1}{\sqrt{2}}(|+\rangle \otimes |-\rangle + |-\rangle \otimes |+\rangle)$$

Where $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$. It is easily seen that $|\xi_1\rangle$ is orthogonal to $|0\rangle \otimes |0\rangle$, $|\xi_2\rangle$ is orthogonal to $|0\rangle \otimes |+\rangle$, and similarly for the final two post-measurement states. This leads immediately to a contradiction with the assumption of an epistemic interpretation of the quantum state. At least $q^2 > 0$ of the time, according to our assumptions, the measuring device will yield a post-measurement state compatible with any of the four preparations $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |+\rangle$ and $|+\rangle \otimes |0\rangle$. But given the orthogonality relations between these preparations and the states $|\xi_i\rangle$, this result is incompatible with the predictions of quantum mechanics. The result is straightforwardly generalized to arbitrary pairs of states $|\phi_0\rangle$ and $|\phi_1\rangle$.

The contradiction derived above starts from the assumptions that the states $|\phi_0\rangle$ and $|\phi_1\rangle$ are pure, and that they are prepared independently of one another. In fact both of these assumptions, which are really idealizations, can be relaxed and the contradiction still obtained. Suppose that we relax the assumption of pure states, allowing instead that preparations may yield mixed states. PBR assume a non-zero probability *q* that the ontic state of the system is compatible with both preparation procedures $|\phi_0\rangle_m$ and $|\phi_1\rangle_m$. If this assumption is relaxed, so that the prepared states $|\phi_0\rangle_m$ and $|\phi_1\rangle_m$ are mixed but 'almost' pure, then the value for *q* predicted by quantum mechanics can be made arbitrarily small by preparing states that are arbitrarily close to pure states. The independence assumption can be similarly relaxed. We need assume only that the probability that the preparation is compatible with any of the states $|0\rangle \otimes |0\rangle$, $|0\rangle \otimes |+\rangle$, $|+\rangle \otimes |+\rangle$ and $|+\rangle \otimes |0\rangle$ is some real number $r > 0$ with *r* not necessarily equal to $q^2$. This is sufficient to derive the contradiction.

The PBR argument shows that the assumption that a given quantum state is compatible with more than one physical state $\lambda$, along with a small set of auxiliary assumptions, leads immediately to a contradiction with the predictions of quantum mechanics. This result forces any epistemic interpretation of the quantum state to clarify which, if any, of these assumptions should be rejected. In the spirit of John Bell, we might well wonder whether the fact that so much follows from such innocent assumptions should lead to us to question their innocence.

The authors identify three main background assumptions that go into their result. The first is that a quantum system can be prepared in isolation from the rest of the universe in a pure quantum state, and that such a system, on preparation, has a definite set of physical properties. We have already pointed out that these assumptions can be somewhat relaxed, and the result still obtained. That a system is capable of having definite physical properties is an assumption necessary to make sense of the central question addressed by the authors, namely whether or not the quantum state is among the physical properties possessed by a system. It is worth emphasizing that very little is assumed about the properties in question; they may be dispositional in character, and certainly they need not include such familiar properties as position and momentum, spin, etc. Indeed, what is crucial to the result is just that if a preparation procedure yields a probability of 0 for some experimental outcome, then this is a fact about the system. Nor do the authors assume that the quantum state exhausts the properties possessed by a system.

One might reject the assumption of property attribution on the grounds that individual prepared systems do not have well-defined properties. It may be that only relational properties are ontic. It is also clear that the quantum Bayesians Caves, Fuchs, and Schack would reject this assumption of property attribution. While the latter authors defend an epistemic view of the quantum state, they maintain that the state encodes our beliefs about the outcomes of measurements rather than the intrinsic properties of physical systems, relational or otherwise. Consider the following quotation from Fuchs [2010]:

> QBism says when an agent reaches out and touches a quantum system—when he performs a quantum measurement—that process gives rise to birth in a nearly literal sense. With the action of the agent upon the system, the no-go theorems of Bell and Kochen-Specker assert that something new comes into the world that wasn't there previously: It is the "outcome," the unpredictable consequence for the very agent who took the action.[166]

This quotation clearly suggests that Fuchs would reject the notion that systems have properties, even when they have been 'prepared' in certain states by preparation devices. The PBR framework does not presuppose that measured properties preexist measurements; nevertheless, the authors do assume that there are physical facts of the matter ('physical properties') that distinguish a system as having been prepared in a certain state. It is a virtue of the PBR result that it brings into sharper relief how heavy a price the Quantum Bayesian is willing to pay to hold onto an epistemic interpretation of the quantum state. Caves, Fuchs and Schack do not simply reject the

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[166] Fuchs [2010], p. 8.

idea that systems have properties revealed by measurement; this is an assumption that most realists would also reject. Crucially, the Quantum Bayesians also reject the basic assumption that whether or not a system has been prepared in a given state is really a fact about the system:

> […] a quantum state prepared by some physical device always depends on an agent's prior beliefs, implying that the probability-1 predictions derived from that state also depend on the agent's prior beliefs. Quantum certainty is therefore always some agent's certainty. Conversely, if facts about an experimental setup could imply agent-independent certainty for a measurement outcome, as in many Copenhagen-like interpretations, that outcome would effectively correspond to a preexisting system property. The idea that measurement outcomes occurring with certainty correspond to preexisting system properties is, however, in conflict with locality.[167]

In order for the PBR theorem to tell against a quantum Bayesian interpretation of the quantum state, the case must be made that systems really do possess physical properties after preparation. It is unlikely that any mathematical result analogous to the PBR theorem could make such a case. Therefore the case must be made on more general grounds.

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[167] Caves, Fuchs and Schack [2007]. For a criticism of the charge of nonlocality, see chapter 2 of the present work.

A second assumption is that it is possible to prepare multiple copies of a system in such a way that the physical properties associated with each individual system are uncorrelated. As we have seen, this assumption can also be relaxed; what is crucial is that the *n* preparations are jointly compatible with the complete physical state $\lambda$. This ensures that there is a non-zero probability that the measuring device is unsure which of the preparation procedures was used, entailing that an outcome prohibited by quantum mechanics has a non-zero probability of occurring.

One motivation for rejecting the assumption of independence is apparently provided by the Bell-Kochen Spekker (BKS) theorem, which shows that a viable hidden variables theory must exhibit nonlocality and contextuality. The properties associated with such theories will themselves be nonlocal and contextual. It is worth stressing, however, that the BKS theorem does not rule out the attribution of 'local beables' to quantum systems, a point taken up in chapter 2 of the present work. Furthermore, there are good grounds for thinking that quantum systems do indeed have localized properties at least some of the time. The existence of such properties helps to explain and undergird, among other things, the science of crystallography and the organization of the periodic table. If such local, independent properties do exist, then a defender of the $\psi$-epistemic interpretation must reject some other premise of the PBR argument.[168] I turn now to an examination of two case studies designed to

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[168] A third assumption for the PBR theorem is that measuring devices respond solely to the system they measure. Such devices respond solely to the complete physical state of the systems they measure in a

motivate the idea that the quantum state does indeed track objective properties of physical systems.

## 4.4 Two Case Studies in Support of Quantum State Realism

*Example: The Two-Slit Experiment*

In the simplest version of the classic two-slit experiment, a series of electrons are sent one-by-one through a system consisting of a barrier with two slits and a detection screen to the right of the barrier.[169] If we keep track of which slit each electron goes through as it encounters the barrier, a pattern is observed on the detector screen which suggests that the electrons are behaving like particles. However, when we do not track the path of each electron through the barrier, but simply observe detection events on the screen, an interference pattern is observed. This pattern suggests that the electron is wavelike, with a wave front passing through both slits and interfering with itself beyond the barrier. This interference effect is described quantitatively by means of the wave function $\psi(x,t)$. According to quantum theory, each individual

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manner dependent only on the physical disposition of the measuring device. Since anyone who rejects the first two assumptions will also reject this assumption, I do not consider it explicitly here.

[169] The following discussion is indebted to Bohm [1951].

electron's wave function at either slit determines the value of the function anywhere to the right of the slit. Furthermore, the probability of finding any given electron at a given location to the right of the barrier is proportional to $\psi^*(x,t)\psi(x,t)$.

After many electrons have passed through the barrier, the probability of finding an electron at any point to the right of the barrier is proportional to $|\psi(x,t)|^2$. In the two-slit experiment, all contributions to the wave function come from one of the two slits, which we can label 'slit A' and 'slit B'. Hence,

$$\psi(x) = \psi_A(x) + \psi_B(x) \tag{1}$$

Where the subscript indices represent the contributions to the wave function at a point x coming from slits A and B, respectively.[170]If only one of the two slits were open, we would expect (1) to imply that the probability of finding an electron in any location is proportional to the absolute value squared of the wave function at the slit. What happens when both slits are open is much more interesting, and illustrates the seemingly 'wavelike' behavior of the electron. In this case, the probability function is given by

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[170] I have implicitly assumed that the wave function obeys the Huyghens-Fresnel principle. See Bohm [1951], 121.

$$P(x) = |\psi_A(x) + \psi_B(x)|^2 = P_A(x) + P_B(x) + \psi_A^*(x)\psi_B(x) + \psi_B^*(x)\psi_A(x) \qquad (2)$$

The final two terms of equation (2) represent interference effects that would not be present if the electrons were behaving as an ensemble of classical particles. Suppose now that we decide to carry out a measurement of the position of the electron as it passes through the slit system. Such a measurement interaction changes the electron wave function in an unpredictable and uncontrollable way by introducing phase factors:

$$\psi = \psi_A(x)e^{i\alpha A} + \psi_B(x)e^{i\alpha B} \qquad (3)$$

Here $\alpha A$ and $\alpha B$ are definite but unknown phase constants whose uncertainty is at least $2\pi$, corresponding to complete uncertainty in the phase. There is no deterministic relation between the phase before and after the measurement interaction, nor are there any physical constraints on the phase difference $\alpha A - \alpha B$.

The phase factors $\alpha A$ and $\alpha B$ affect the interference terms but not the separate slit terms in equation (2). Since equation (2) is a probability equation describing the expected behavior of an ensemble of electrons, it has meaning only insofar as it characterizes the expected results of a set of similar experiments carried out under equivalent initial conditions. Since the interference terms fluctuate in a random way, we would expect that these random effects would average out, so that only the first two terms in (2), corresponding to the contributions from each of the separate slits, remain. In other words, if we perform a measurement of the position of the electron as it passes through the slit system, the probability that a particle can be found at a point x to the right of the slits just becomes

$$P(x) = P_A(x) + P_B(x) \tag{4}$$

which is just what we would expect for a distribution of classical particles coming through each slit separately. This fact illustrates part of the subtlety involved in interpreting the quantum formalism in relation to the phenomena. If a position measuring device is placed at the slits, the electron subsequently behaves, for all practical purposes, as though it went through either slit A or slit B with probabilities

$$P_A = \int \psi_A^*(x)\psi_A(x)dx \tag{5}$$

and

$$P_B = \int \psi_B^*(x)\psi_B(x)dx \tag{6}$$

respectively.

From an operational standpoint, it is impossible to distinguish physical processes governed by equation (3) from physical processes governed entirely by $\psi = \psi_A(x)e^{i\alpha A}$ or $\psi = \psi_B(x)e^{i\alpha B}$ with probabilities $P_A$ and $P_B$ that each of these equations is the correct wave function. When an observer checks to see which slit the electron actually went through, equation (3) is replaced by one of these reduced wave functions, depending on the result of the experiment. Given the initial measurement interaction, which changes the wave function from the form (2) to form (3), it is possible to interpret the subsequent measurement of the reduced wave function as revelatory of a pre-existing but unknown value, rather than as a physical disturbance that changes the state of the electron. Hence the probabilities (5) and (6) can be treated as classical probability functions representing incomplete information. The measurement interaction appears to transform the electron from a 'wavelike' object to a 'particle-like' object by eliminating the interference effects that would be expected were no measurement made.[171]

The insertion of a measurement device at the slits immediately transforms the mathematical representation of the probabilities associated with subsequent measurement outcomes, and does so in a discontinuous fashion. It is tempting to conclude that the probabilities involved are subjective, or perhaps represent incomplete information about the actual behavior of the electron. We routinely update probability assignments in light of new information at the classical level of description. For example, an agent's subjective probability that a race horse will win tomorrow's derby is changed immediately, and presumably decreased, if it is learned that the horse has in fact never won a race. Such a sudden change in the description of

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[171] For a detailed treatment of the physical process through which interference is destroyed by measurement processes, see Bohm [1951], 588-608.

the horse's race record obviously does not reflect a change in the horse, but merely an improvement in our information about it.

Can something similar be said about changes in the wave function? It is certainly true that the insertion of a photographic plate or other detection device at the slits makes it possible to interpret the subsequent behavior of the electron as particle-like. The probability that the electron will be found anywhere to the right of slit A, given that it was initially detected at that slit, is given by $\psi = \psi_A(x)e^{i\alpha A}$. If we choose to actually locate the electron to the right of the slits, this probability immediately collapses to 1 wherever the electron is found and 0 everywhere else. This discontinuous change is indeed analogous to the horse race case, except that the resultant probability distribution is infinitely peaked due to the presence of complete information.

Nevertheless, the analogy with a classical probability distribution function cannot be extended in the absence of a measurement interaction, which destroys interference effects and causes the electron to behave subsequently as if it were a particle. The final two terms in equation (2), corresponding to these interference effects, have consequences for our predictions regarding the subsequent behavior of the system. While the amplitude of the wave function determines the probabilities of measurement outcomes, the phase relations between the various parts of the wave function are responsible for the evolution of this amplitude, and in particular they predict the interference effects obtained on the screen to the right of the slits. As long as definite phase relations exist between $\psi_A(x)$ and $\psi_B(x)$, the possibility of interference effects remains, and the electron behaves as though it went through both slits simultaneously and interfered with itself to the right of the slits.

Since the electron is capable of demonstrating interference effects as long as phase relations exist between the parts of the wave function associated with it, it is appropriate to interpret these phase relations as encoding information regarding the behavior of the electron. Once the photographic plate at either slit detects the electron, the wave function immediately transforms into an interference-free representation. If this transformation in the wave function represented a mere updating of information, not accompanied by a physical disturbance in the system, then the physical state of the electron would be identical before and after the placement of the photographic plate. Yet once the plates are placed at the slits, a different pattern is observed on the scintillation screen. The presence of these plates obviously affects the pattern on the scintillation screen whether or not the plates are actually consulted to determine the initial trajectory of the electron. Of course the observer may choose not to consult the plates and thereby determine the paths of the electrons. In this case, the subsequent behavior of the electrons will be governed by equations (5) and (6), corresponding to a particle-like distribution on the screen.

If the photographic plates are not placed at the screen, however, the electrons behave as through they went through both slits simultaneously. A subsequent transformation of the wavefunction therefore seems to represent a genuinely physical transformation of the system. The subjectivist who denies that the wavefunction encodes genuinely physical information is placed in the difficult position of having to justify the presence of the interference terms in equation (2) at all[172], prior to a measurement event, or of asserting that, in principle, equations (2) and (3) are just alternative representations of the same physical situation. This is possible only if we are prepared to deny the physical significance of the phase relations between the interference terms in (2). But to ignore these phase relations is to ignore much of the predictive and explanatory power of quantum mechanics, and much of the evidentiary basis upon which the theory is built.[173] The phase relations between $\psi_A(x)$ and $\psi_B(x)$ contain information, which we might characterize, following Bohm [1951], by saying that the wave function (2) is in closer correspondence with the actual state of the electron than a classical probability distribution, specifying the probabilities that the electron will pass through either slit, would be. This information furnishes an account of the evolution of quantum probabilities that would be lost if all of the information regarding a system were limited to probability amplitudes alone.

How might a subjectivist account for the observed phenomena? The Quantum Bayesian might point out that the above discussion seems to presuppose that the electron is in some physical state or other, which the wave function is supposed to

[172] The hidden variable theorist may be able to do so. HVT's agree that the physical situation before the interaction is different than the physical situation immediately following, and are realist with respect to the quantum state in this sense.

[173] I will have more to say about this evidentiary basis in the next section.

describe. Chris Fuchs, in particular, denies that quantum systems have ontic states, even when they have been prepared in a given quantum state, and so would argue that it is inappropriate to characterize the electron passing through the slits as having a physical state at all. Hence, the spontaneous state change associated with measurement, on the QBist view, represents a mere change in the beliefs of the observer, rather than a change in the system itself. How then would the QBist account for the change in the quantum state description before and after measurement? It seems that the only account available to the QBist is to acknowledge that, over time, experimenters have learned to expect that the consequences of their interventions on physical systems will take on a certain form described by the wave function. This form changes depending on the overall macroscopic physical setup, for example in response to whether or not a detector is placed at the slits in the two-slit experiment.

Nevertheless, the intersubjective agreement that exists regarding the exact form of the wavefunction is, according to the Quantum Bayesian account, just a brute psychological fact about physicists who conduct experiments and the kind of assumptions, such as exchangeability defined over sequences of quantum state preparations, which they make about experimental setups; it receives no physical explanation.[174] While logically consistent, this interpretation of the existing intersubjective agreement appears to strain credibility.

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[174] For a further discussion of the this point, see chapter 3 of the present work.

On the other hand, a subjectivist who holds that quantum states are states of knowledge about hidden variables can accommodate the picture of physical disturbance described above, since such interpreters do not deny that the quantum state describes (statistically) an underlying ontic state, which may interact with a measurement device. For such authors, such as Spekkens [2007], a direct measurement is not purely Bayesian updating, but is always accompanied by a physical disturbance.

The trouble with adopting such an epistemic account is that we know, in light of the Bell, Kochen and Specker and PBR 'no go' theorems, that there exist serious constraints on the space of possible ontic models compatible with the predictions of quantum mechanics that nevertheless interpret the quantum state epistemically. The epistemicist would likely deny that individually prepared quantum states have isolable properties susceptible of measurement*, even when the prepared states are product states*. This means that a device that detects an electron at either slit in the two-slit experiment is not registering properties that can be localized at the slit, even when there is no entanglement with environmental degrees of freedom. Perhaps the underlying properties associated with the ontic states of the system are relational and therefore nonlocal in character, preventing the attribution of independent properties to such systems and thereby circumventing the PBR result. But in the absence of evidence to suggest that product states actually fail to satisfy the independence condition in the PBR result, it seems more reasonable to take the experimental results, which suggest that the systems in question do have localized properties, at face value.

*Example: Crystallography and the Periodic Table*

Much of the evidentiary basis of quantum mechanics comes from the field of quantum chemistry. The configuration and spatial distribution of electrons around atomic nuclei, which determines the chemical properties of atoms and molecules, is predicted and explained by quantum mechanics. The Schrödinger equation of the hydrogen atom can be solved analytically to obtain energy eigenstates corresponding to the allowable orbitals of the hydrogen electron. The wave function $\psi$ governing the electron contains both spatial and spin degrees of freedom, with the probability of locating the electron at any point in space given by $\psi^2 d\tau$, where $d\tau$ is the volume element. It is well known that the energy eigenstates obtained from the Hamiltonian operator for hydrogen predict and explain the existence of the Balmer series of atomic spectra in spectacular fashion. From the perspective of the quantum state realist, this empirical success already strongly suggests that the eigenstates of the hydrogen atom represent real features of the systems in question. But it is the combination of degeneracy in the allowable eigenstates of an electron along with the Pauli exclusion principle that is able to provide a basis for attributing a 'shell structure' to the electronic configurations of all of the atoms in the periodic table, and through these configurations the observed chemical properties of the elements.

Owing to the spherical symmetry of the electrostatic field surrounding the hydrogen atom, the operators used to characterize the state of a single electron, $\mathbf{H}$, $\mathbf{L}^2$, $\mathbf{L}_Z$, and $\mathbf{S}_Z$, form a commuting set so that the state of the hydrogen electron is specified completely in terms of the four eigenvalues (quantum numbers) *n*, *l*, $m_l$, and $m_s$, respectively.[175] By employing the central field approximation, which assumes that each electron feels an isotropic electrostatic field, it is possible to extend this model to atoms containing more than one valence electron and obtain approximate solutions to the Schrödinger equation for more complicated atomic systems.

According to the Pauli exclusion principle, the wavefunction for a system of identical fermions is antisymmetric. Consider a system of two electrons given by $\psi(1,2)$. If we label the position and spin of each electron $r_1$, $s_1$ and $r_2$, $s_2$, respectively, then the Pauli principle entails that

$$\psi(r_1,S_1;r_2,S_2) = -\psi(r_2,S_2;r_1,S_1) \tag{7}$$

If both particles have the same coordinates, we have

$$\psi(r_1,S_1;r_1,S_1) = -\psi(r_1,S_1;r_1,S_1) \tag{8}$$

which is satisfied if and only if $\psi(r_1,S_1;r_1,S_1) = 0$. In other words, there is zero probability of finding two electrons at the same point in space with the same spin.

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[175] These four operators are the Hamiltonian operator **H**, the angular momentum operator **L,** (the axial projection $\mathbf{L_z}$ commutes with $\mathbf{L}^2$) and the spin operator $\mathbf{S_z}$**.**

This entails that a system of two electrons orbiting an atomic nucleus is precluded from assigning the same set of atomic numbers $n$, $l$, $m_l$, and $m_s$ to each electron.

For each principle quantum number $n$, there are $2n^2$ wavefunctions corresponding to a given allowable energy.[176] Hence for n=1 there are two wave functions which give the same energy, entailing that the two available electron states must have opposite spin. In general, the exclusion principle forces electrons to occupy higher energy states further from the nucleus when the available lower energy states have been filled, which in turn is responsible for the fact that atoms containing three or more electrons exhibit spatially extended charge densities arranged periodically in 'shell' structures.

The electronic Hamiltonian and the associated wave function are able to predict observed atomic and molecular structure, which provides direct evidence for the correctness of the quantum mechanical description of chemical elements in terms of wave functions. In particular, the molecular structure, which is susceptible to direct observation, is encoded in the wave function representation. The wave function is able to predict and explain the shape of the electronic charge density associated with an atom or molecule in a systematic way. While analytic solutions of the wavefunction are available only in the special case of the hydrogen atom, approximate many-body solutions can be obtained for more complicated systems by means of numerical

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[176] For a detailed justification of this claim, see for example Liboff [1998], p.620.

methods and physically reasonable approximations.[177]These solutions can then be tested directly by means of scattering experiments and crystallography.

The most common techniques for determining the arrangements of atoms in solids include x-ray, neutron and electron diffraction crystallography. All three techniques exploit the fact that incident particles interact with the spatial distribution of valence electrons.[178]The periodic arrangement of crystalline structures acts as a diffraction grating, scattering incident x-rays or electrons in a predictable manner. The observed diffraction patterns are typically subjected to a Fourier analysis which yields information about the spatial distribution of charge density in valence electrons. The development of sophisticated experimental techniques such as electron lensing has allowed experimenters to systematically vary the geometry of diffraction experiments, allowing for more robust observations of crystalline structure.

Observation of the crystalline structure of diamond using x-ray crystallography reveals that diamond crystals exhibit a face-centered cubic, tetrahedral structure. Each of the four valence electrons of the carbon atoms forms a covalent bond with a single valence electron of a neighboring atom, with the four bond angles defining the

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[177] Examples of such approximations include the Born-Oppenheimer approximation, which exploits the high ratio between nuclear and electron masses, and the Harmonic approximation.

[178] Each of these techniques is useful for different kinds of crystallographic study. Incident photons interact exclusively with valence electrons; electrons, being charged particles, also interact with atomic nuclei and have wavelengths shorter than those available in the EM bandwidth, while neutrons feel both nuclear forces and magnetic fields due to their non-zero magnetic moment.

vertices of a tetrahedron. This observed crystal structure is explained by the charge density of the valence electrons of carbon, the geometry of which characterizes the possible crystal configurations of carbon atoms. This geometry in turn is described by the higher dimensional wave function of the valence electrons when projected down onto three dimensions. The structure of a crystal is responsible for various observable macroscopic properties of solids, such as hardness and melting point. Since this structure is determined by the wavefunction, it follows that these macroscopic properties of solids ultimately have a quantum mechanical explanation in terms of the wavefunction of orbital electrons.

How might an antirealist account for these phenomena? In their paper 'Quantum Theory Needs No interpretation' Chris Fuchs and Asher Peres argue that the wave function is not objective. In a reply to this article, Stanley Sobotka writes:

> The wavefunction of the four outer electrons in the ground state of the carbon atom produces a tetrahedral structure in Euclidean three-dimensional space that undergirds the observed tetrahedral structure of the diamond crystal. This is an objective fact about the physical world . . .[179]

This comment is in line with the point made above, that the wave function of the valence electrons essentially determines the chemical properties of the elements of

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[179]Sobottka *et al*. [2000].

the periodic table. Furthermore, the quotation clearly implies that Sobotka takes the three-dimensional charge density to be real, and representative of the physical arrangement of atoms in a crystal. Fuchs and Peres' reply to Sobotka's objection is instructive. The authors write:

> The truth is that the wavefunction of the four outer electrons lives in a 12-dimensional space, while our tangible physical world has only three dimensions. This example (contrary to its intended purpose) is an excellent one for showing that the wavefunction is a mathematical tool, not a physical object.[180]

Several comments are in order:

First, the purpose of the example is not to show that the wavefunction is a physical object living in physical space. I have already suggested that it is not necessary to construe the quantum state as a physical object in order to endorse quantum state realism. All parties to the debate would readily concede that the wave function is a mathematical object living in a higher dimensional Hilbert space. What is at stake is the representational capacity of this abstract object.

Nor is the point to be able to describe quantum phenomena using concepts derived from the 'tangible physical world'. The electronic charge density defined by the wave function should not be construed as an 'electron cloud' or some other

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[180] Fuchs and Peres [2000].

physical phenomenon readily susceptible to a description in terms borrowed from our experience of more familiar macroscopic phenomena. Indeed the phrase 'charge density' is itself potentially misleading, since it suggests localized particles distributed in a volume of space. But the electronic wavefunctions responsible for the 'charge density' are not classical waves. What is true is that the probability of finding an electron in a given region of space is equal to the amplitude squared of the wavefunction, and that this probability evolves in time according to a wave-mechanical mathematical formalism. In the words of John Bell,

> It is the mathematics of this wave motion, which somehow controls the electron, that is developed in a precise way in quantum mechanics. Indeed the most simple and natural of the various equivalent ways in which quantum mechanics can be presented is called just 'wave mechanics'. What is it that 'waves' in wave mechanics? In the case of water waves it is the surface of the water that waves…In the case of wave mechanics we have no idea what is waving…and do not ask the question. What we do have is a mathematical recipe for the propagation of the waves, and the rule that the probability of an electron being seen at a particular place when looked for there (e,g, by introducing a scintillation screen) is related to the intensity there of the wave motion.[181]

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[181] Bell [1986]. Reprinted in Bell [2004], p. 187.

What the carbon example is intended to illustrate is that the relationship of the mathematical formalism to the physical phenomena is one of genuine representation; this mathematical representation in turn requires interpretation via the Born rule.

Finally, the fact that a 3-dimensional charge distribution requires a 12-dimensional representation does not show that the representation contains excess structure beyond what is physically significant. This would follow if it were true that every functional representation that yields the same charge density when projected down to 3-space has the same physical content. But the Pauli exclusion principle implies that different electrons occupying the same orbital shell (such as the electrons orbiting a helium atom) must occupy different quantum states. This in turn shows that the same physical charge distribution may be associated with different quantum states. These differences are reflected in the different quantum numbers assigned to the orbital electrons, which in turn determine the possible physical interactions of the charge density with, for example, an inhomogeneous magnetic field.

Denying the reality of the quantum state introduces new questions about the phenomena that the quantum state is able to predict and explain. Is the charge density surrounding an atom real? Are the projections onto 3-dimensional space themselves real? Fuchs and Peres do not explicitly address the status of charge densities in relation to the wave function.

If pressed, Fuchs in particular might argue that it doesn't make sense to talk about the atomic structure of diamonds at all. Such talk seems to presuppose that the systems in question possess underlying ontic states, which the QBist denies. Instead, a

Quantum Bayesian might tell the following kind of story. Carbon atoms, like all quantum systems, are studied by means of experimental interventions whose consequences are inherently unpredictable. Nevertheless, agents who have experience interacting with diamond crystals (whether through crystallography experiments or otherwise) may come to be confident that performing certain kinds of experimental interventions, such as bombarding crystals with x-rays, will elicit certain kinds of results (like observable diffraction patterns). Over time, agents may come to believe that they have discovered properties of the diamond crystal, such as charge densities, that are useful for modeling and predicting the outcomes of future experiments. Nevertheless, these beliefs are never anything more than that; and the wave function is nothing more than a representation of these beliefs.

This sort of picture suggests that crystallographic experiments, which give rise to observable diffraction patterns, do not amount to observations of underlying atomic structure. It is true that the structures in question are not observed directly, but are reconstructed on the basis of a mathematical analysis of the observed data, which are obtained with the aid of highly sophisticated laboratory equipment. There is room to argue on this basis that the inference to atomic structure is not forced upon us as a matter of logic by the available evidence. If one is inclined to view the possibility of direct observation, unmediated by instruments and mathematical reconstruction, as a necessary condition for property attribution, this may seem like a reasonable stance to take.

While there is no inconsistency involved in maintaining such an attitude, I think the case for a realist interpretation of the quantum state is far more compelling. If the wave function is real, then the task of interpreting the charge densities, spin properties, and so on of atoms is relatively straightforward. These properties of atoms are publicly accessible and subject to experimental verification. It may be possible to admit these facts while still making the case for a subjective interpretation of the wave function, perhaps in line with the attitude adopted in Spekkens [2007]. I am not aware of such a case having yet been made, and the local structure of crystal configurations at least suggests a tension with the sorts of constraints on hidden variable interpretations implied by the PBR and Bell-Kochen Spekker theorems. But to deny the reality of charge densities altogether is an option that seems too heavy a price to pay for any interpretation.

In his article 'Yes, but…Some Skeptical Remarks on Realism and Anti-Realism' Howard Stein remarks:

> I do not claim to have a definitive formulation of the metaphysics of quantum mechanics; but I believe rather strongly that the difficulties it presents arise from the fact that *the mode in which this theory 'represents' phenomena* is a radically novel one. In other words, I think the live problems concern the relation of the forms- indeed, if you like, of the instrument- to phenomena, rather than the

> relation of (putative) attributes to (putative) entities, and that the ideological motives of ‘realism’ have here served as a kind of scholastic distraction, turning attention away from what is ‘real’ in the subject.[182] (author’s italics)

The Carbon atom example illustrates concretely Stein’s point that the mode in which quantum theory represents phenomena is a radically novel one. It is novel because the representation does not have a direct analog in classical physics. Interpreting the formalism is not simply a matter of separating out the gauge or artifactual elements of the formalism from those elements which are physically significant. Nor is the novelty of quantum theory to be found in the high degree of abstractness in its mathematical formulation. The classical Hamiltonian framework, out of which the Schrödinger picture was historically developed, is already highly abstract. So too is the Lagrangian framework out of which Feynman’s path integral formulation of the theory was developed.

A complete account of the means through which quantum theory represents phenomena will ultimately have to confront the measurement problem. If an experimental apparatus is treated as a quantum system, then a typical quantum state at the end of an experiment will be a sum of terms corresponding to different experimental outcomes. If the quantum state description is taken to be complete and

[182] Stein [1989], 59.

correct, then it seems that there are no definite experimental outcomes, which is problematic. I do not claim to offer a solution to the measurement problem. Nevertheless, I claim that the solution does not lie in an epistemic interpretation of the quantum state. My task in the present chapter, and the point to which my examples have been directed, is to argue that the correct interpretation of the quantum state must respect the evidentiary basis of quantum theory. It is true that we do not observe superpositions of measurement outcomes. Nevertheless, the probabilities (or branch weights) assigned to different outcomes can be confirmed or disconfirmed by experimental comparison with relative frequency data. The amplitude and phase of the wavefunction determine probability densities and currents, respectively, that can be tested against empirical observations. Any interpretation of the theory will ultimately have to confront this representational character, which constitutes part of what Stein describes as 'what is real in the subject'.

**Conclusion**

In the present chapter I have attempted to articulate and defend the case for quantum state realism. Along the way, I have tried to articulate what I take to be, following Howard Stein, the most important issue regarding the interpretation of the quantum state: the relationship of the mathematical formalism to the phenomena it describes.

Borrowing examples from electron diffraction experiments and quantum chemistry, I have suggested that the correct interpretation of this relationship is that it is one of genuine representation, in the sense that different quantum states correspond to different states of reality. I have also attempted to anticipate and accommodate certain obvious objections that might be leveled against this program, both on a physical and a philosophical level. While the measurement problem remains to be solved, I hope to have made the case for a treatment of this difficult issue of interpretation that sees quantum state realism as a point of departure rather than a point of controversy.

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# Curriculum Vitae

## Research Interests

**Area of Specialization:** philosophy of physics, philosophy of science

**Areas of Competence:** early modern philosophy (especially Kant), history of analytic philosophy, logic

## Education

**The University of Western Ontario**
London, Ontario, Canada

- PhD., Philosophy — September 2007- April 2012
  Advisor: Wayne Myrvold
  Thesis title: *Realism, Subjectivism, and the Interpretation of the Quantum State*
  Expected completion date April 23, 2012

- M.A., Philosophy — 2006 - 2007

**The University of Toronto**
Toronto, Ontario, Canada

- Honors B.A., Philosophy — 1999 - 2004
  *Degree conferred with high distinction*

**The University of Edinburgh**
Edinburgh, Scotland

- Honors exchange — 2002 - 2003
  *First class standing*

## Teaching and Academic Work Experience

**Instructor** January 2010 - May 2010
The University of Western Ontario

- PHL2020 Basic Logic (predicate logic)

**Visiting Fellowship** October 2009 - November 2009
Perimeter Institute for Theoretical Physics

- Visiting Fellow - Hosted by Christopher Fuchs

**Teaching Assistant**

- PHL021 Critical Thinking September 2008- May 2009
  Instructor: Ryan Robb

- PHL2500 Global Business Ethics May 2008-September 2008
  Instructor: Dean Proessel

- PHL023 Contemporary Moral Issues January 2008- May 2008
  Instructor: Dean Proessel

- PHL246 Philosophy of Mind September 2007- January 2008
  Instructor: Ausonio Marras

- PHL162F/W Business Ethics September 2006- May 2007
  Instructor: Dean Proessel

## Scholarships and Awards

**Scholarship Competitions**

- Foundational Questions Institute (FQXI) research grant in collaboration with Wayne Myrvold, 2011-2012. Total value $56,650; personal award value $32,000, plus teaching release

- Social Sciences and Humanities Research Council Doctoral Fellowship, 2010-2011
  Value $20,000

- Ontario Graduate Scholarship, 2010
  Value $15,000 (declined)

- Ontario Graduate Scholarship, 2007, 2008, 2009

Total value $45,000

- Arts and Humanities Alumni Graduate Award, May 2010
  Value $1150 (internal UWO scholarship competition)

- Western Graduate Thesis Research Award, January 2010
  Value $1150 (internal UWO scholarship competition)

- J.S. Maclean Scholarship (for outstanding achievement in undergraduate philosophy), University of Toronto, 2001, 2002, 2003, 2004
  Total value $2000

- St. George Scholarship for outstanding candidate studying abroad, 2002-2003 University of Toronto
  Value $3000

**Visiting Fellowships**

- Perimeter Institute for Theoretical Physics, Fall 2009.
  Funding provided by Christopher Fuchs through the Perimeter Institute

**Entrance Scholarships and Funding**

- University of Western Ontario Faculty of Graduate Studies Scholarship, 2007 - 11 Total value $28,524 (declined)

- Western Graduate Research Scholarship, 2006 - 2007
  Value $3036

- St. George Scholarship for outstanding candidate studying abroad, 2002-2003 University of Toronto
  Value $3000

- University of Toronto Undergraduate Entrance Scholarship, 1999

**Other Academic Distinctions**

- Dean's List, University of Toronto, 2003, 2004

- First class standing, University of Edinburgh, 2002, 2003

## Conference Presentations, Workshops and Publications

### Invited Workshops

Participant, “Quantum Theory Without Observers” workshop, Sexten Centre for Astrophysics, Sesto, Italy, August 2011. Workshop organizers Giancarlo Ghirardi, Sheldon Goldstein, Nino Zanghi. Also invited for the 2012 workshop.

Participant, ”New Frontiers in Quantum Foundations” workshop, workshop coordinator Antony Valentini, collaboration between the Perimeter Institute for Theoretical Physics and Clemson University, Clemson, March 2011.

### Peer-Reviewed Conference Presentations

“The Case for Quantum State Realism” 14th Congress of Logic, Methodology and Philosophy of Science, Nancy, France, July 2011

“Why I am not a Quantum Bayesian”, Canadian Philosophical Association (CPA) annual congress, Concordia University, Montreal, May 2010

‘Is Synthesis Psychologistic?’ Canadian Philosophical Association (CPA) annual congress, Concordia University, Montreal, May 2010

“Why I am not a Quantum Bayesian: the Steering Argument” Canadian Society for the History and Philosophy of Science (CSHPS) annual congress, Concordia University, Montreal May 2010

“Is Synthesis Psychologistic?” York University Philosophy Graduate Conference, York University, Toronto, April 2010

“Psychologism and the Transcendental Deduction” University of Waterloo PGSA Conference, University of Waterloo, May 2009

"The Second Analogy and A priori Causal Laws: A Critique of Michael Friedman", De Philosophia Conference, University of Ottawa, April 2008

"Where Does the Burden of Proof Lie? An Interpretation and Defense of Kripke’s Modal Argument in Naming and Necessity" 'Identity and Difference' Conference, Concordia University, Montreal, May 2008. Later published in the Journal *Gnosis* [2008].

“Comments on Fine-Grained Functionalism”, Philosophy of Mind, Language and Cognition Conference, University of Western Ontario, May 2007

**Non-Refereed Presentations**

“Why I am not a Quantum Bayesian”, Graduate Philosophy Student’s Association Conference, University of Western Ontario, March 2010

"The Second Analogy and A Priori Causal Laws: A Critique of Michael Friedman", Graduate Philosophy Student's Association Conference, University of Western Ontario, January 2008

Referee, Logic, Mathematics and Physics Graduate Conference, University of Western Ontario, May 2010, May 2009, May 2008

Conference Coordinator and Referee, Logic, Mathematics and Physics Graduate Conference, University of Western Ontario, May 2008

Conference Coordinator and Referee, Philosophy of Mind, Language and Cognition Graduate Conference, University of Western Ontario, 2007, 2008.

## Memberships and Affiliations

**Affiliations**

Canadian Philosophical Association (CPA)
Canadian Society for the History and Philosophy of Science (CSHPS)
Member, Rotman Institute for Science and Values, Fall 2008- present

**Fellowships**

Doctoral Fellow, Social Sciences and Humanities Research Council, 2010-2011
Visiting fellow, Perimeter Institute for Theoretical Physics, Fall 2009