THREE ESSAYS ON AGRICULTURAL PRICE VOLATILITY

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YIYONG YUAN

Dr. Joseph Parcell, Dissertation Supervisor

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The undersigned, appointed by the dean of the Graduate School, have examined the dissertation entitled

THREE ESSAYS ON AGRICULTURAL PRICE VOLATILITY

presented by Yiyong Yuan,

a candidate for the degree of doctor of philosophy of Agricultural Economics,

and hereby certify that, in their opinion, it is worthy of acceptance.

Professor Joseph Parcell

Professor Michael Monson

Professor Scott Brown

Professor Stanislav Kolenikov

To my Parents and Wife

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ABSTRACT

Price risk analysis is one of the central themes to understand price flows within the agricultural value chain and on the futures market. Price change across the agriculture market affects producers, processors, retailers and consumers. The price volatility that has been the primary indicator of price variation and risk is shaped by the underlying market structure. Each market participant has a need to better their understanding of the price uncertainty across different markets due to supply-demand shifters. This need is especially important when structural change is present or is perceived to be present.

The three essays of this dissertation cover issues of understanding and managing price uncertainty across the meat value chain and related futures market. The first essay discusses the implications of recent change in retailing industry's pricing strategy; the second essay describes a State Space Model approach estimation of the joint distribution of cash-futures prices and a simulation-based Conditional-VaR approach determination of optimal futures exposure determination in contrast with minimum variance hedge ratio; the third essay describes the empirical changes in the meat price volatility at the farm level in view of the recent industry structural change.

For the first essay, I investigated the impact of two coexisting retail price strategies for selling perishable products on the volatility of both the farm-level price and the retailer's margin. The two strategies included the traditional High-Low strategy and the Every-Day-Low-Price (EDLP) pricing strategy. In contrast to non-perishable consumer products, perishable products, which are often of very inelastic demand, obtain their price fluctuations mainly through supply side shocks. A two-retailer model was developed to

examine the volatilities of grocery retailers' margin and producer price due to supply shocks for a perishable product. Results indicated a volatility difference exists between EDLP and High-Low retailers' marginal revenue when the two pricing strategies coexist, and as the market share of EDLP format increases this margin volatility difference deepens and farm-level price volatility also increases.

For the second essay, I proposed a state space model based estimation of the cash-futures price joint distribution and a coherent C-VaR-approach optimal futures exposure determination based on simulated data in response to situations where the preference-free optimal hedge ratio no longer exists and the minimum variance hedge ratio is not appropriate. The State Space Model serves as an alternative method to other joint distribution estimation methods. The determined optimal futures exposure showed that the minimum variance hedge ratio discourages hedging. Parallel analyses using existing constant minimum conditional variance (MCV) hedge ratio models and a time-varying MCV ratio based on Multivariate GARCH models was also conducted for comparison. The C-VaR approach optimal futures position exposure reported different optimal futures positions for the "short hedge" and the "long hedge" situations.

For the third essay, I analyzed the historical change of the realized price volatility defined as the weekly hog price absolute return from 1973 to 2008 using long memory effect in the mean and variance process. The ARFIMA-FIGARCH/IGARCH Model results confirmed a significant long memory effect in the absolute return for a period around the end of the 1990s with documented structural change. I found no significant long memory effect for any other period. The model result also showed a significant ARCH-M effect that is explained as a fierce industry structural adjustment leading to a more dramatic price volatility change.

Chapter 1

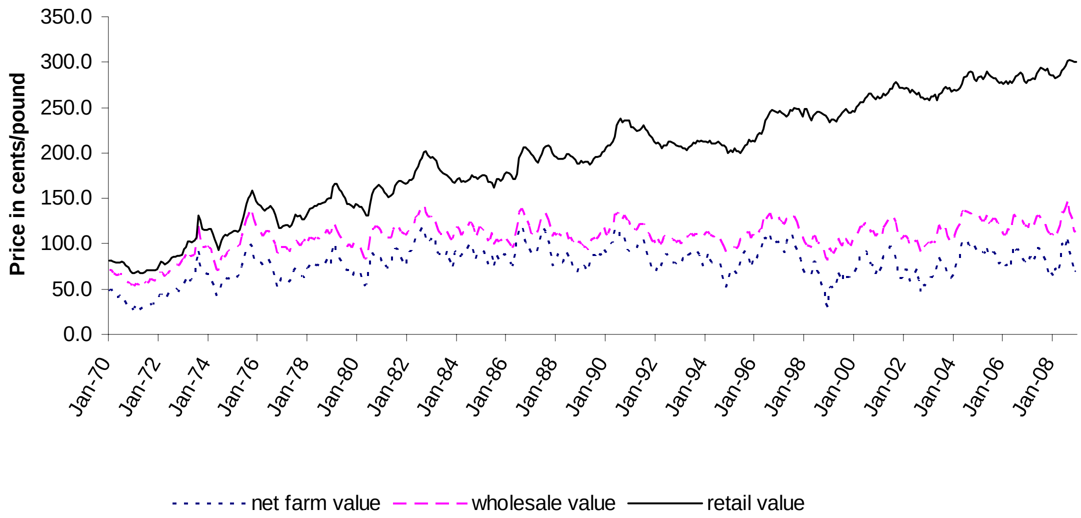
Introduction: a Note on Price Risk, Volatility and Return

Commodity spot price risk and related futures price risk is one of the most important topics of agriculture risk research. A good understanding of price risk is of great significance to farmers, traders and distributors, agriculture insurers and government policy makers.

Commodity price data is often available in daily frequency, with longer-interval data being derived from the daily data. While daily and derived/censored longer-interval data (e.g., nearby futures price) is often used for the futures price, high-frequency data is also used in the market microstructure research. Price variation at any frequency level is very interesting in and of itself. However, because short term price, e.g., daily price, is almost impossible to model and predict, structural equation models mainly focus on monthly or longer-interval data modeling. For short-term price fluctuation, only price volatility can be modeled and predicted. Analysis of price risk and volatility is therefore often the focus of short-term price studies. For risk analysis, price data is generally transformed into return data (discrete or log). This is because return data more conveniently fit into the decision theory under risk and possess at least equally desirable statistical properties when compared to price data.

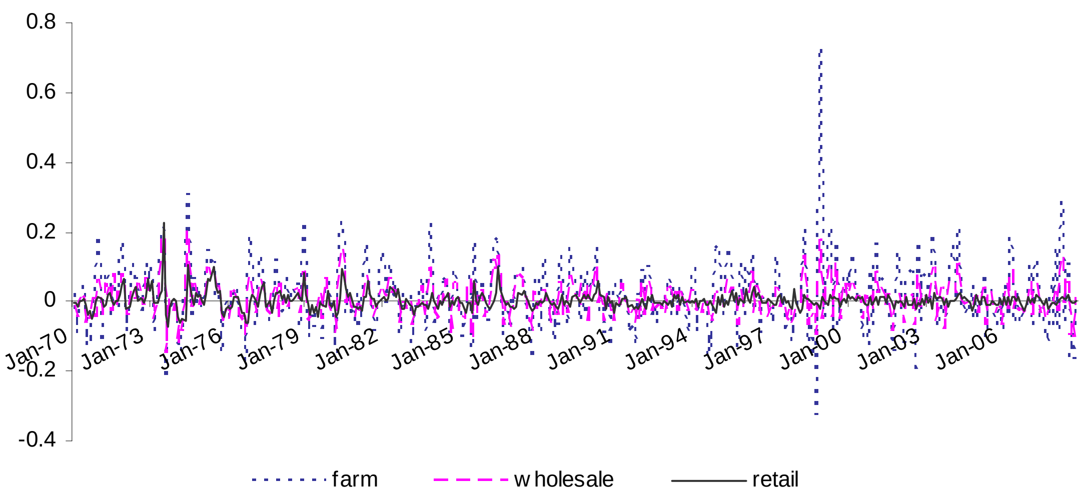
Figure 1 and Figure 2 demonstrate the monthly pork prices at the farm, wholesale and retail levels as well as the corresponding discrete return series for the period of 1970-2008. It is clear that the transformation into return data loses the price level information while retaining the information on the volatility.

Figure 1 U.S. Monthly Pork Price at Farm, Wholesale and Retail Level 1970-2008



Date source: USDA ERS

Figure 2 Return Based on U.S. Monthly Pork Price at Farm, Wholesale and Retail Level 1970-2008



Date source: USDA ERS

Because the return data is based on monthly prices, the month to month variation displays a seasonal variation. In contrast, the return data derived from shorter-term price is at a better position to measure price volatility.

It is obvious that risk is closely related to volatility. Frequently, price risk is often identified as price volatility or variability. These loosely defined concepts often confuse their significance in related studies. The following part of this chapter will clarify these concepts and motivate the topics of this dissertation research.

1.1 Volatility as an Imperfect Measure of Risk

Volatility is usually defined as the Standard Deviation (S.D). Focusing on the return's risk, risk measure has been centered on the concepts of variance/ S.D. as advocated by the Markowitz (1952) framework of mean-variance portfolio selection via quadratic optimization. Such definition does not attach more importance to losses than profits. For risk defined as S.D., it does not make much sense to distinguish a return's risk from a price or other economic variable's risk. Although one can justify such risk measure by assuming that a decision maker views S.D as the risk measure to be minimized, the decision maker's utility function is quadratic or returns of underlying assets follow an elliptically distribution (Ingersoll, 1987), researcher has been proposing risk measure with more desirable properties. Artzner et al. (1999) proposed "coherent" risk measures that are monotonic, sub-additive, linearly homogeneous, and translation invariant. Kusuoka (2001) suggested two additional properties: law-invariance and comonotonic-additivity. These coherent properties have sound appeal to both practitioners/regulators and decision theorists. Recent studies have proposed a risk measure, the expected shortfall or conditional VaR (C-VaR) and its application/estimation (Rockafellar and Uryasev (2000,2002), Quaranta and Zaffaroni (2008), Cai and Wang (2008)). Coherent risk measures (all the six axiomatic requirements including the four proposed by Artzner et al. (1999) and the two by Kusuoka (2001)) have been shown to be consistent with the so-called pessimistic Choquet expectation decision theory (Bassett et al. 2004). In particular, variance/S.D. of asset returns violates the sub-additivity, one of the central requirements for risk measures to be coherent. A detailed discussion on coherent risk measure can be found in Chapter 3.

1.2 Volatility as a Measure of Price Variation

Even though S.D. is not a coherent risk measure, except for investment return, volatility is still a meaningful measure for variability. It is hard to justify people should only be concerned with the "left tail distribution" or some special measures of a commodity price such as some coherent risk measures. Volatility provides less-detailed information of the distributional information yet remains a meaningful measure of variability on other occasions.

If a price series is viewed as a random process, as price discovery depends on market structure and the market participants' behavior, a change in market structure can be viewed as changes in the parameters of such a random process and can possibly lead to a volatility change. The perfect description of the volatility of a general random process is the ensemble variances at each time. Because it is not possible to study these ensemble variances for each time point of a time series, focus is placed on "asymptotically" stationary processes. Here stationary refers to stationary up to the second order, i.e. the weak stationary. "Asymptotically" is used here because the weak stationary means no volatility change at all. Indeed, the most popular models on volatility-the GARCH type models are only "asymptotically" stationary because their volatility process is not exactly stationary. There are other types of volatility models, e.g., the stochastic volatility model and long memory GARCH models inclusing FIGARCH, that allow for the long memory process in volatility. The long memory GARCH type models allow slowly decaying autocorrelation of the volatility process. The stochastic volatility model partially hides the efforts to model the volatility process in a "Markov-process fashion" by adding an extra and independent volatility randomness source that is not of any inter-temporal dependence structure. A more detailed discussion on volatility models is in Chapter 4.

1.3 Absolute Return as a Measure for Realized Price Volatility

One fact of great interest is that people usually (parametrically) model price processes to estimate a price's volatility, e.g., GARCH. Natural questions for such a choice are: what is the realized price volatility? Can we directly model the realized price volatility? Is there any difference between modeling the price and modeling the realized price volatility if we are only interested in price volatility in and of itself?

Conceptually, if price volatility is predictable, i.e. $Var(p_{t+1} | F_t)$ exists, because $Var(p_t + (p_{t+1} - p_t) | F_t) = Var(p_{t+1} - p_t) | F_t)$, then there is only one realization "$| p_{t+1} - p_t |$" to determine $Var(p_{t+1} - p_t) | F_t)$. Therefore, the absolute price change series can be used as a measure of the price volatility defined as S.D. The estimated volatility process, e.g., that of the GARCH model, is comparable to the price change series in terms of capturing the price volatility. To address the concern that price level change may obscure the price change, price change can be scaled by a nearby price, e.g., beginning period, to obtain a return series. Significant change in agricultural commodity price levels may include the seasonal price drop that exaggerates the down side price change or a constant price level increase due to the emerging demand of a commodity with slow supply adjustment. Importantly, such a measure is conceptually distinct from model-based volatility estimates and/or forecasts from traditional models such as GARCH because it represents the actual realized price variability assessed from ex-post data rather than ex-ante (conditional) return variances implied by a parametric model.

Directly modeling the (absolute) return series apparently does not deny the applicability of existing volatility models. Such an approach of direct modeling of realized volatility is related with the approach of modeling price to study price volatility in a similar way as an ARMA model approach of price is related with a supply/demand structural equation approach to study price. While for the later case research practice on annually determined price favors the structural equation approach, for the former case directly analyzing return series may prove favorable because the volatility structure handled in price models can now be handled in the mean process of a directly modeled return series. For the same reason, the volatility of "realized volatility" itself can now also be studied using existing econometric methods if it is desired.

1.4 Analysis of Return as the Theme of This Dissertation

The benefit of measuring realized price volatility with absolute return is not only limited to the improved investigation capacity of existing econometric methods for studies on volatility. The analysis of the realized price volatility is also convenient to fit into the traditional supply/demand framework of price determination so that the structural equation perspective can bear its impact on the price volatility.

The first essay of this dissertation focuses on the analysis of realized price volatility defined as the absolute return to investigate the evolution of the price volatility in a structure change perspective. The second essay determines the optimal futures position exposures determination using a coherent risk measure, the conditional Value at Risk (C-VaR) in a typical situation of hedging pork product with Lean Hog Futures. Underlying Data Generation Process is estimated using the daily data of futures and one pork product as an example. The third essay makes an empirical investigation of the absolute return series of the farm-level hog price for the period of 1973-2008 in view of the hog industry structural changes and suggests an ad hoc framework explaining the empirical findings.

Chapter 2

Essay I: Volatilities of Producer Price and Retailers' Margin of Perishable Flow Products with Coexisting Every-Day-Low-Price and High-Low Strategy

2.1 Introduction

There are two popular pricing strategies in the retailing industry: Every-Day-Low-Price (EDLP) and High-Low (Hi-Lo). Market share of food retailers that adopt EDLP strategy has been increasing with Wal-Mart leading the charge. EDLP pricing refers to the strategy of holding the prices fixed across production shocks, whereas Hi-Lo pricing refers to the strategy of running promotion or discount from time to time (Lal and Rao, 1997). The EDLP strategy motivates many questions for various stages of the supply chain. For durable goods and non-perishable products, the use of inventory management supplements production level shocks. Often with an inelastic demand, perishable products made from perishable or flow commodities derive their price fluctuation mainly through supply shocks. A flow commodity is a commodity that is in a state of constant harvest, processing, distribution and marketing. Livestock is a flow commodity. Breeding herd numbers, natural disasters, input costs change and seasonal factors contributes to supply changes, which can not be completely controlled. No prior work has explicitly analyzed how a supply shock impacts industry performance given adoption of EDLP format at the retail-level. What is the retailers' revenue effect of different pricing policies when facing a supply shock? How do the different pricing policies affect the way meat products move forward in the presence of supply gluts? [1] The objectives of this paper are to explore how the retailers' margin volatilities differ by pricing strategy and how farm-level price is impacted by the retail-level market share increase of EDLP strategy in the value chain. Volatility, as the wide-used concept in time series analysis, is defined as the (conditional) variance of the variable of interest. We utilize the example of the meat supply chain throughout the paper to enable practical examples, but our results are applicable for any perishable commodity.

Researchers developed supply-demand models such that a change in quantity supplied impacts prices throughout the value chain, by a price transmission mechanism, and in turn the retail price changes to induce or reduce consumer buying (e.g., Boyd and Brorsen (1988); Kinnucan, and Forker (1997); Marsh and Brester (2004); Miller and Hayenga (2001); Reed, Elitzak, and Wohlgenant (2002); Schroeder (1988); Schroeder and Hayenga (1987); and Ward (1988)).

If the use of EDLP pricing strategy has increased in market share, new studies of price transmission may indicate asymmetric price responses in the presence of a market pricing strategy change (structural change). Without considering such structural change, researchers may inappropriately interpret such a finding as "market power" rather than as a structural change. For instance, we often see seasonal meat price patterns at wholesale (Capps et al (1994); and Parcell (2000)) and retail (Capps, 1989) level that are opposite of the seasonal quantity of meat supplied. Yet, an increasing share of EDLP pricing strategy may begin to mitigate such price-quantity seasonal relationships in the future.

Whereas examining structural change in the meat supply chain is common (e.g., Goodwin and Brester (1995); McGuirk et al. (1995); Parcell, Mintert, and Plain (2004); Piggott et al. (1995); and Piggott and Marsh (2004)), these studies have not accounted for such exogenous market factors as EDLP pricing strategy at the retail level, and thus may bias results. Much prior research has estimated demand elasticities (e.g., Brester and Schroeder (1995); Brester, Marsh, and Atwood (2004); Hayes and Meyer (2003), Kinnicun et al. (1997); and Marsh, Schroeder, and Mintert (2004)). If EDLP pricing strategy accounts for a significant market share of retail-level meat, then the results of former studies may not reflect the new retail price-quantity demanded relationships used by today's decision maker. And, those studies investigating market power within the meat supply chain (e.g., Muth and Wohlgenant (1999); and Schroeter, Azzam, and Zhang (2000) may have overestimated the level of market power if not appropriately accounting for the presence of EDLP pricing strategy within the meat value chain.

Lal and Rao (1997) utilized a game theory approach to analyze losers and winners from the adoption of EDLP in the presence of two types of consumers: time constrained and cherry pickers. They found EDLP stores offering less service to be of interest to cherry picker consumers and Hi-Lo stores offering more service to be of interest to time constrained consumers. In conclusion, they point to a mix of EDLP and Hi-Lo pricing as a strong promotional (not pricing) platform to attract consumers. Kwong (2003) reached a similar conclusion.[2] Even for such a mixed pricing format, retail price fluctuations will be less than historical observations because of the presence of EDLP. However, if there exist food retailers using fixed pricing for certain products, i.e. if there is no change in retail price by which to alter consumer meat demand, then how must the rest of the retailers react in order to absorb a supply shock, and how will the margin of the two different retailers adjust?

Sexton, Zhang, and Chalfant (2003) simulated the welfare impacts of a two-channel pricing model on producers. As the market-share of the EDLP strategy increased, producer surplus decreased. Their theoretical model was extended by Boessen (2006) to analyze the comparative statistic results of changes in the market-share of the EDLP strategy on retail level normalized returns. He found that as the EDLP strategy market share increases, farm price flexibility increases (in absolute value) at an escalating rate. We build on prior research to investigate the impact of two co-existing strategy on volatilities of retailers' revenue and farm-level price in a more general model framework.

2.2 The Two-Channel Model

In this model, the margin is modeled as the variable difference between downstream and upstream prices to investigate retailers' margin response to agricultural commodity supply shocks under a general cost function assumption. Without compromising the generality, the number of inputs in the processor technology is restricted to two: an agriculture commodity and all other processing services. To simplify the problem, any intermediate wholesaler in the supply chain is suppressed and the processor industry is assumed to be at least not a monopoly/monoposony situation and possibly faces a certain level of competition structure, i.e. oligopoly/oligopsony, perfect competition or a situation in between. A representative processor is assumed to receive a fixed margin.[3]

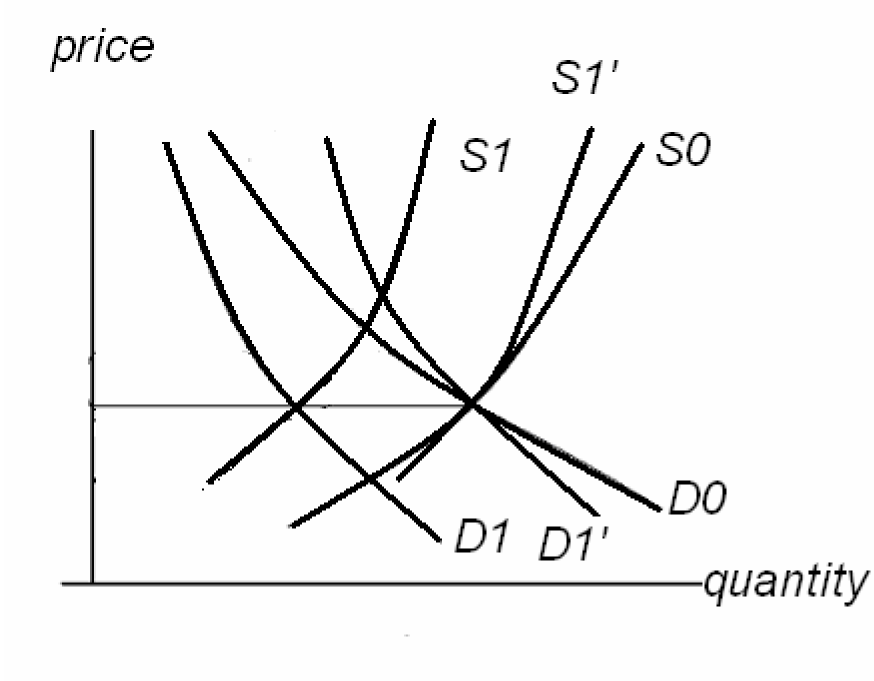
In addition to the assumption of a constant processor margin, we make the following three assumptions about the supply chain. First, the supply chain is composed of farm producers, one representative processor technology for underlying processors and two retailers: EDLP retailer and Hi-Lo retailer; Second, the EDLP retailer and the Hi-Lo retailer each faces a separate demand, i.e. each retailer has its own 100% loyal shoppers. The assumption of separated shoppers is based on the belief that consumers are represented by large-basket shoppers and small basket shoppers. Cherry pickers are of limited number and of a fixed percentage of shoppers. With one dominant EDLP store, these cherry pickers' visit rate to an EDLP store tends to be constant and therefore their purchases at the EDLP store can considered forming part of the fixed demand for EDLP retail. More importantly, Regular EDLP shoppers may not shop a Hi-Lo store simply to take advantage of the promotional price for a single product; Third, the margin values of both EDLP retailer and Hi-Lo retailer vary and are strategically independently of each other. Because of the limited information available on how margin is earned by retailers, we do not make a restrictive assumption and therefore limit the viability of the model. It is assumed that each of the EDLP and the Hi-Lo retailer has a variable margin denoted by r and R respectively. The margin of either retailer depends on many factors including W, where W is an agricultural commodity supply shift factor, like weather, with a larger value of W indicating larger production levels defined by the function $h(p_a, W)$. Variable definitions used in the development of the model are located in Table 9 of the Appendix.

In contrast to the approach used by Sexton, Zhang, and Chalfant (2003) and Boessen (2006), the market share enters the model differently and results in an increasingly steeper demand curve on the Hi-Lo/aggregate retail market in a general setting.

Increasing EDLP retail market share amounts to an increasing number of shoppers exiting the spot market and "entering" a contract with a "fixed" transaction price. Considering the change of the Hi-Lo retail demand curve in response to the exiting shoppers in the above sense, if all the shoppers are homogeneous in terms of their individual demand, which can be assumed true for the consideration of an aggregate demand, it is noted that the aggregate demand $QD = F(P)$ becomes the Hi-Lo demand $QD_{Hi-Lo} = A * F(P)$ since the new quantity demanded should be proportional to the total demand at each price levels, where $A \in (0,1)$ is the share of remaining Hi-Lo shoppers accounting for the total shoppers. It is apparent that a steeper Hi-Lo demand curve results when A becomes smaller. Similarly, aggregate supply $QS = G(P)$ becomes Hi-Lo supply $QS_{Hi-Lo} = B * G(P)$ with exiting retail supply. More importantly it is assumed that consumers can only shop in one type of store (EDLP or Hi-Lo) so that $A = B$ holds because consumers who are "committed" to shop the EDLP stores "enter" the fixed-price commitment at the same rate as the EDLP share expands. Apparently the resulting retail price remains unchanged if $A = B$ as shown in Figure 3 where Hi-Lo demand is subscripted as "1" such that $D1(p) = A * D0(P)$ and $S1(p) = A * S0(P)$ so that the resulting price for Hi-Lo market does not change. It is evident that the only change is the quantity that enters the fixed-price "contract" i.e the EDLP market channel. To obtain the new aggregate demand, the Hi-Lo demand $D1(p)$ is only needed to shift right by this quantity to arrive at $D1'(p)$ and result in a steeper aggregate demand curve. This is because separated and loyal shoppers are assumed.

How should we obtain the new aggregate supply curve? While we are tempted to shift the Hi-Lo supply curve $S1(p)$ in the same way as we do with $D1(p)$. It has to be noted that the new aggregate supply curve $S1'(p)$ rather than $S0(P)$ results only when exiting EDLP retail supply is no longer available for the aggregate retail market. While EDLP retail supply can be separated from the Hi-Lo market, this does not mean upstream suppliers of the EDLP stores---the processors can be separated accordingly. If no further production technology is assumed for retailers and the processors can be separated accordingly, this means the processors supplying the EDLP retailer will not exploit their production capacity beyond the “contracted” quantities any more. This situation may not be true because processors, not like the separated and loyal retail shoppers, can still make their extra capacity available to the aggregate retail market through the Hi-Lo retailer, so that the new aggregate supply will still be $S0(P)$ rather than $S1'(p)$. The model development will assume that the new aggregate supply will still be $S0(P)$.

Figure 3 Effects of Existing Hi-Lo Retail Demand/Supply on Retail Market



The implication of the alternative situation of new aggregate supply curve will be further tackled in the Discussion Section after the model development.

It is noted that the changing A or B denotes a retail market structural change in a relatively long term. It is entirely possible that the "fixed" price offered by EDLP retailer can vary with very different values of A or B during a sufficiently long interval. However, during a relatively short term that is experiencing fast EDLP share expansion, price offered by as well as retail quantity demanded of EDLP retailer can be treated as "fixed". The authors have observed a fixed pork chop price offered by Wal-Mart for a period as long as 33 weeks that is possible for a significant change of EDLP share to occur. Even if EDLP price can change within a very short period, EDLP retail price may not respond to a "short-term" farm-level supply shock unless such shock or shock from other sources is consistently affecting the price level.

The representative processors' cost function is specified as

(1) $c = f(Y, p_a, \overline{p}_b)$

Where c is total cost; $Y = y + \overline{x}$ is the aggregate quantity; $\overline{x}$ is fixed quantity sold in the EDLP channel; y is quantity sold in the Hi-Lo channel; $\overline{p_b}$ is fixed part of the price of "bundled" processing services; p_a represents the producer price of the agriculture commodity. By equation (1), the marginal cost function is

(2) $MC = f_Y(Y, p_a, \overline{p}_b)$

The processor's marginal cost function and a constant processor margin were used to model the price spread from producer price to processor price in the supply chain. Retailers incur variable margins on each marginal product decided by the processor price and retail price. The equality of agricultural commodity supply and the derived demand based on the processor's cost function will pass the impact of farm-level supply shock to the retail market.

Combining the representative processor's marginal cost equation (2) with processor's constant margin m_0 and EDLP retailer's margin r yields

(3) $f_Y(Y, p_a, \overline{p}_b) + m_0 + r$,

which refers to the retail level supply function. Thus,

(4) $\overline{p}_x = f_Y(Y, p_a, \overline{p}_b) + m_0 + r$,

where $\overline{p}_x$ is the fixed price in the EDLP marketing channel, so that the market clears (in the EDLP channel).

The Hi-Lo retailer's margin can be denoted by R, and then similarly the Hi-Lo retail level supply function can be specified as:

(5) $f_Y(Y, p_a, \overline{p}_b) + m_0 + R$.

and,

(6) $p_y = f_Y(Y, p_a, \overline{p}_b) + m_0 + R$

is the marketing clearing relationship..

Furthermore, suppose the demand for a Hi-Lo retailer's product can be represented by $y = J(p_y, N)$, where N denotes the Hi-Lo channel demand shift factor, like population, so that

(7) $\overline{p}_x = f_Y(\overline{x} + J(p_y, N), p_a, \overline{p}_b) + m_0 + r$

and

(8) $p_y = f_Y(\overline{x} + J(p_y, N), p_a, \overline{p}_b) + m_0 + R$.

The derived demand for the agricultural commodity, $f_{p_a}(\overline{x} + J(p_y, N), p_a, \overline{p}_b)$ is derived by Shepard's lemma, and will be set equal to the farm level supply $h(p_a, W)$. So the market clearing relationship can then be specified as:

(9) $f_{p_a}(\overline{x} + J(p_y, N), p_a, \overline{p}_b) = h(p_a, W)$.

We seek to analyze changes in both farm- and retail-level price, and we seek to examine retail-level margin change for the EDLP and Hi-Lo retailers due to a supply shock (ΔW). Rearranging equations (7), (8) and (9) and making log transformation, yields:

(10) $\ln(\overline{p}_x - m_0 - r) = \ln f_Y(Y, p_a, \overline{p}_b)$,

(11) $\ln(p_y - m_0 - R) = \ln f_Y(Y, p_a, \overline{p}_b)$

and,

(12) $\ln f_{p_a}(Y, p_a, \overline{p}_b) = \ln h(p_a, W)$.

In the following derivation, the symbol * denotes the collection of variables $Y, p_a, \overline{p}_b$.

Now, differentiating equations (10), (11) and (12) w.r.t W to obtain:

(10)’ $$\frac{1}{MC}\frac{-dr}{dW} = \frac{f_{Y^2}(*)}{f_Y(*)}J_{p_y}\frac{dp_y}{dW} + \frac{f_{Y,p_a}(*)}{f_Y(*)}\frac{dp_a}{dW},$$

(11)’ $$\frac{1}{MC}\left(\frac{dp_y}{dW} - \frac{dR}{dW}\right) = \frac{f_{Y^2}(*)}{f_Y(*)}J_{p_y}\frac{dp_y}{dW} + \frac{f_{Y,p_a}(*)}{f_Y(*)}\frac{dp_a}{dW},$$

(12)’ $$\frac{f_{p_a,Y}(*)}{f_{p_a}(*)}J_{p_y}\frac{dp_y}{dW} + \left(\frac{f_{p_a p_a}(*)}{f_{p_a}(*)} - \frac{h_{p_a}}{h}\right)\frac{dp_a}{dW} = \frac{h_W}{h}.$$

where $\overline{p}_x - m_0 - r = p_y - m_0 - R = MC$. Solving equations (10)' and (12)' for $\frac{dp_y}{dW}$ and setting equation (10)' equal to equation (11)’ yields $\frac{dR}{dW} = \frac{dr}{dW} + \frac{dp_y}{dW}$. Substituting $\frac{dp_y}{dW}$ into the relationship yields the following relationship:

(13) $$\frac{dR}{dW} = \frac{dr}{dW} + \frac{(-\frac{dr}{dW})\frac{1}{MC} - \frac{f_{Y,p_a}(*)}{f_Y(*)}\frac{h_W/h}{f_{p_a^2}(*)/f_{p_a}(*) - h_{p_a}/h}}{\frac{f_{Y^2}(*)}{f_Y(*)}J_{p_y} + \frac{\frac{f_{Y,p_a}(*)}{f_Y(*)}\left(-\frac{f_{p_a,Y}(*)}{f_{p_a}(*)}J_{p_y}\right)}{\frac{f_{p_a^2}(*)}{f_{p_a}(*)} - \frac{h_{p_a}}{h}}}.$$

Equation (13) can be re-specified as the following:

(13)' $$\frac{dR}{dW}=\frac{dr}{dW}\left[1-\ 1\Big/\ MC*J_{p_y}\left(\frac{f_{Y^2}(*)}{f_Y(*)}+\frac{\frac{f_{Y,p_a}(*)}{f_Y(*)}\left(-\frac{f_{p_a,Y}(*)}{f_{p_a}(*)}\right)}{(e_{ddp_a}-e_{hp_a})\frac{1}{p_a}}\right)\right]+Intercept\,.$$

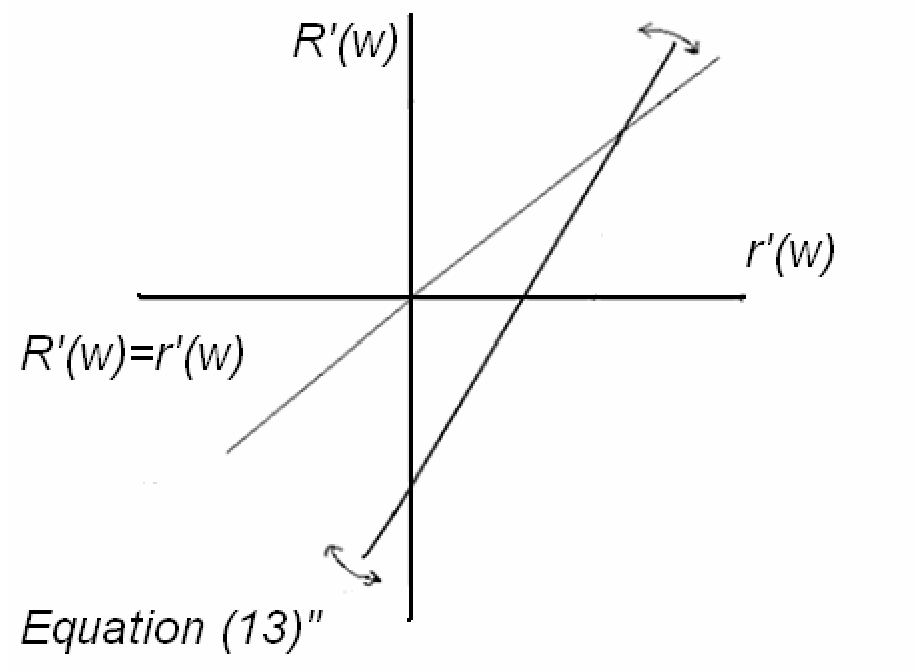
Applying Euler's theorem, (13)' yields the following:

(13)" $$\frac{dR}{dW}=\frac{dr}{dW}\left[1-Y\Big/MC*J_{p_y}*\left(k-1-k\frac{e_{MC,p_a}}{e_{ddp_a}-e_{hp_a}}\right)\right]+Intercept\,.$$

Where the $\text{Intercept}=-\frac{e_{MC,p_a}\frac{h_W}{h}}{(e_{ddp_a}-e_{hp_a})\frac{1}{p_a}}\Big/\left[Y\Big/J_{p_y}*\left(k-1-k\frac{e_{MC,p_a}}{e_{ddp_a}-e_{hp_a}}\right)\right]$, e_{MC,p_a} refers to the elasticity of marginal cost w.r.t p_a, e_{hp_a} refers to the supply elasticity, e_{ddp_a} refers to the elasticity of derived demand, and k refers to the degree of homogeneity of the cost function in output space. By assuming $k>1$, $e_{hp_a}\geq 0, J_{p_y}<0$ and $e_{ddp_a}<0$, the slope of equation (13)' or (13)" is always larger than 1 and the intercept is always negative. The condition $k>1$ states that the production level of the typical processor is at a reasonable stage with respect to its fixed investment. We estimated a model of Cobb-Douglas form which corroborates a decreasing return to scale of U.S beef production in the short term. In addition, by equation (13)", this condition can also be relaxed. In terms of structural change's impact on the slope, according to equation (13)", lower marginal cost, steeper Hi-Lo retail demand curve(i.e. bigger EDLP share, indicated by smaller $|J_{p_y}|$, because $|J_{p_y,new}|=A|J_{p_y,old}|$), less elastic response of marginal cost to agricultural commodity price, more elastic demand and supply of agriculture commodity will lead to larger slope of this straight-line.

In contrast, except for marginal cost being irrelevant, smaller $|J_{p_y}|$, smaller e_{MC,p_a} bigger $|e_{ddp_a}|$ and $|e_{hp_a}|$ also lead to bigger absolute value of the intercept.

Figure 4 Margin Adjustment to Supply Shock Varies with EDLP Market Share Change



By a similar argument, the intercept term is also negative. Furthermore, since J_{p_y} appears only in the denominator of equation (13), the intersection between equation (13) and the 45^o line remains unchanged when only EDLP share (embodied in J_{p_y}) changes ceteris paribus (see Figure 4).

Hence, this straight line rotates but is hinged through the intercept with the 45^o straight line when the EDLP market share changes, *ceteris paribus*.

In the above equations, $\frac{dR}{dW}$ and $\frac{dr}{dW}$ are measures of the change in margin relative to a farm-level supply shock. Each point $[R'(W), r'(W)]$ on equation (13)' corresponds to a margin adjustment response by EDLP and Hi-Lo retailers relative to a farm-level supply shock dW . When $W(t)$ follows a given stochastic process, for instance a Brownian motion process, $[R'(W(t)), r'(W(t))]$ becomes random but must satisfy the constraints specified by equation (13)'.

Proposition 1 (On Producer Price Volatility):

Producer price volatility becomes larger with an increasing EDLP market share.

Proof: Solving equation (12)' for $\frac{dp_a}{dW}$ using the same transformation as for equation (13)' yields,

(14) $$\frac{dp_a}{dW}\frac{1}{p_a} = \frac{\frac{h_W}{h} - \frac{f_{p_a,Y}(*)}{f_{p_a}(*)}\frac{dy}{dW}}{(e_{ddp_a} - e_{hp_a})}$$

where the denominator of the RHS is negative. To decide the effect of increasing EDLP share, ΔW is still assumed to denotes a positive supply shock in the analysis as before. In equation (14), $\frac{dy}{dW} > 0$ (equivalent to $\frac{dp_y}{dW} < 0$) always holds, because farm product supply shifting to the right leads to a lower producer price and therefore a lower processor marginal cost.

Furthermore the downward sloping farm-level demand curve means $\frac{dp_a}{dW} < 0$.

Since $\frac{h_W}{h} > 0$ and $\frac{f_{p_a,Y}(*)}{f_{p_a}(*)} > 0$, an increase of $|\frac{dp_y}{dW}|$ (equivalent to a decrease of $|\frac{dy}{dW}|$) leads to an increase of $|\frac{dp_a}{dW}|$ or $|\frac{dp_a/dW}{p_a}|$. This shows that the realized return will have an increasing absolute value if the price shock is derived only from the farm supply shock.

Next, the connection between increasing $\left|\frac{dp_a/dW}{p_a}\right|$ and larger volatility, i.e. the conditional coefficient of variation will be established. Considering the discrete case, suppose $\frac{\Delta p_{a,t}}{\Delta W_t p_{a,t}}$ takes a series n_t such that $|n_t|$ is an increasing series (this series certainly should be very short in the principle of the comparative static analysis).

Define $\Delta W_t = W_{t+1} - W_t$ and $\Delta p_{a,t} = p_{a,t+1} - p_{a,t}$, the conditional variance of $p_{a,t}$,

$Var(p_{a,t+1} \mid F_t) = Var(\Delta p_{a,t} + p_{a,t} \mid F_t) = Var(\Delta p_{a,t} \mid F_t)$
$= Var(\Delta W_t n_t p_{a,t} \mid F_t) = n_t^2 p_{a,t}^2 Var(\Delta W_t \mid F_t)$, so that the conditional coefficient of variation[4] of $p_{a,t}$,

$CV(p_{a,t+1} \mid F_t) = \sqrt{Var(p_{a,t+1} \mid F_t)} \Big/ p_{a,t} = |n_t| \sqrt{Var(\Delta W_t \mid F_t)}$ where F_t is the filtration denoting the information set available at time t. If the conditional variance of the supply shock ΔW, i.e. $Var(\Delta W_t \mid F_t)$ remains unchanged. With an increasing series $|n_t|$, this means the conditional coefficient of the producer price should increase over time. Note that $Var(\Delta W_t \mid F_t)$ being stable is a reasonable assumption since its change mainly reflects climate change or biological production process change.

Proposition 2 (On Retailers' Margin Volatility):

By equation (13)', with a slope larger than one, a Hi-Lo retailer' margin adjustment has a higher conditional variance than that of the EDLP retailer,

(15) $Var\left(R_{t+1} \mid F_t\right) = b_t^2 Var\left(r_{t+1} \mid F_t\right) + g_t^2 Var(\Delta W_t)$ *where* $b_t > 1$ is the series of slope and g_t is the series of the intercept.

Proof: Define $\Delta R_t = R_{t+1} - R_t$, $\Delta r_t = r_{t+1} - r_t$. $\left[\frac{\Delta R_t}{\Delta W_t}, \frac{\Delta r_t}{\Delta W_t}\right]$ follows linear equation (13)" with slope $b_t > 1$ and negative intercept g_t. Considering discrete case, equation (13)" can be written as (16) $\Delta R_t = b_t \Delta r_r + g_t \Delta W_t$. Since b_t and g_t are adapted to F_t, taking conditional variance of equation (16) given F_t obtains

(17) $Var(\Delta R_t \mid F_t) = b_t^2 Var(\Delta r_t \mid F_t) + g_t^2 Var(\Delta W_t \mid F_t)$. Because the conditional variances of the two retailers' margin are respectively

(18) $Var(R_{t+1} \mid F_t) = Var(R_t + \Delta R_t \mid F_t) = Var(\Delta R_t \mid F_t)$ and

(19) $Var(r_{t+1} \mid F_t) = Var(r_t + \Delta r_t \mid F_t) = Var(\Delta r_t \mid F_t)$.

By equation (17), equation (18) and (19) immediately means equation (15).

In equation (15), industry structure change is reflected in varying b_t and g_t. As indicated above, a lower marginal cost (only relevant for b_t), a steeper retail-level demand curve for the Hi-Lo retailer(the exogenous change interesting to us), a less elastic response of processor marginal cost to agricultural commodity price changes, a more elastic farm-level demand curve and supply curve each will lead to a larger b_t and larger $|g_t|$. Therefore, these changes will produce a bigger margin volatility difference between the two retailers.

Volatility (the conditional variance) in supply shock ΔW_t enters the equation by adding an extra volatility term to Hi-Lo retailer's margin multiplied by g_t^2. Although the volatility of supply shock ΔW_t is usually stable, it will adversely influence Hi-Lo retailer if $Var(\Delta W_t \mid F_t)$ does increase.

2.3 Discussion

As noted in the previous section, processors may not exploit their extra capacity. Despite of the low probability of such a situation, what would this specific situation mean for the model? Starting from equation (2), it is noted that such marginal cost function is consistent with the unchanged retail market aggregate supply $S0(P)$ as shown in Figure 3 but not consistent with the situation of $S1'(P)$. To accommodate the situation of $S1'(P)$, the new cost function becomes

(1)’ $C_F = F[Y, p_a, \overline{p}_b] = Af[(Y - \overline{x})/A, p_a, \overline{p}_b]$ and correspondingly, equation (2) is replaced by

(2)’ $MC_F = F_Y[Y, p_a, \overline{p}_b] = f_Y[(Y - \overline{x})/A, p_a, \overline{p}_b]$.

In equations (10)~(12) f is replace by F. In equation (10)’~(12)’, in addition to f being replaced by F, MC is replaced by MC_F. Solving (12)’ for (14) again, it can be shown that proposition 1 is no longer true since the effect of the steeper demand and that of the steeper supply on the retail market cancel each other out.

Equation (13) becomes

$$(13)' \qquad \frac{dR}{dW} = \frac{dr}{dW} + \frac{(-\frac{dr}{dW})\frac{1}{MC_F} - \frac{F_{Y,p_a}(*)}{F_Y(*)}\frac{h_W/h}{F_{p_a^2}(*)/F_{p_a}(*) - h_{p_a}/h}}{\frac{F_{Y^2}(*)}{F_Y(*)}\frac{1}{A}J_{p_y} + \frac{\frac{F_{Y,p_a}(*)}{F_Y(*)}\left(-\frac{F_{p_a,Y}(*)}{F_{p_a}(*)}\frac{1}{A}J_{p_y}\right)}{\frac{F_{p_a^2}(*)}{F_{p_a}(*)} - \frac{h_{p_a}}{h}}}$$

Continuing such substitution in solving subsequent equations, it can be shown that Proposition 2 will not hold either. It can be shown that the new equation corresponding to equation (13)' may not be hinged at its intersection with the 45^0 line. The variation with expanding the EDLP share of both the intercept and slope of this equation depends on more detail of the cost function.

Although the aggregate retail supply may well be more possible to remain unchanged, overall the model results depend on the assumption that the aggregate retail demand has to change to a steeper curve from $D0(p)$ in Figure 3. In reality, the situation might be that the aggregate demand curve becomes steeper but not 100% close to the $D1'(p)$ case and the aggregate supply curve largely remains unchanged. It is interesting to note such scenario also justifies the emphasis on supply side shock because demand side shock can not impact the supply chain so much when the supply curve changes little.

How realistic is this assumption of separated and loyal shoppers for the two types of retailers and to what extent will the model results be impacted? About the assumption of separated shoppers/retail demand, firstly, we note that there is really a geographic separation at the national level between large Hi-Lo grocery chain stores and EDLP stores in the U.S meat market. At local levels, different grocery stores are often located with proper distance. Secondly and more importantly, our model looks into the price at the aggregate market level and not a game theory model. Individual "cherry pickers" who shop the Hi-Lo store when there is Hi-Lo store promotion and shop EDLP store when there is no Hi-Lo store promotion will not affect the model when the Hi-Lo store's promotion is periodic. However, it also evident that a large market share of stores using a mixed pricing strategy will completely invalidate this model. About sensitiveness of the model to this assumption, it is noted that the model strictly requires the cost function of the representative processors to remain unchanged but only needs a steeper retail demand curve compared with the original one to obtain an increasing $|J_{p_y}|$ with an expanding EDLP sector. In other words, the model is more sensitive to the violation of the unchanged retail supply curve. Both Proposition 1 and Proposition 2 remain partly true even if the aggregate demand curve lies between $D1'(p)$ and $D0(p)$. In contrast, neither of the two propositions will not hold if supply on the retail market is allowed to vary with expanding an EDLP share as specified in equation (2)'.

2.4 Conclusion

As the EDLP pricing strategy first emerged, the concept of applying the EDLP pricing strategy to a flow or a perishable commodity was not fully explored. Previous research has not examined how farm-level supply shocks impact retailers and farmers considering coexistence of Hi-Lo and EDLP pricing formats. This study made an investigation about the revenue effect of the two pricing practices on retailer margin volatility and farm-level price volatility. As the level of the EDLP pricing strategy increase, the Hi-Lo retailer has more volatile margin than the EDLP. The Hi-Lo retailer will observe more fluctuation in its margin than EDLP retailer if all contracting and delivery is assumed to be finished instantly and costless. A different pricing strategy means different revenue risk types for the EDLP retailer and Hi-Lo retailer. More volatile revenue poses a difficultly for the Hi-Lo retailer and provides a possible explanation for the expansion of EDLP strategy that structurally means a lower revenue volatility. Also, an increasing EDLP market share was found to lead to a more volatile farm-level price. Such a result implies farmers will face greater price uncertainty as the level of EDLP market share grows.

Chapter 3

Essay II: Coherent Risk Measure, Hedge or Not and the Ratio: Optimal Futures Exposure—an Example with the Lean Hog Futures

3.1 Introduction

Theory on hedging was pioneered by Working (1953, 1953, 1962). Hedging a cash commodity position with futures contracts has been an important risk management tool for commodity buyers and sellers. With cash and futures prices being very volatile, a close and traceable correlation between the two price series allows for a reduction of the price risk. Hedging reduces risk because the futures price and cash price of the same commodity tend to move in close correlation, allowing the cash position to be offset by an opposite futures position.

Typically a realized imperfect correlation between the two prices often entails an “optimal” hedge ratio determination rule to act on. Since optimality has to be defined for a decision maker, the “optimal” hedge ratio needs to be based on some underlying assumptions about the decision making behavior. Most existing literature on hedge ratio determination has largely embraced the von Neumann-Morgenstern Utility function theory and often used the Mean-Variance approximation framework of Markowitz (1953). In the Mean-Variance framework, Kahl (1983) summarized the studies on hedge ratio determination with cash/future position being both endogenous and cash position given. Her results showed that, the minimum variance hedge ratio (Johnson, 1960; Stein, 1961;

and Ederington, 1979. (JSE)), is equivalent to optimal hedge ratio of Peck (1975) when the futures price is unbiased and cash position is given and the minimum variance hedge ratio is also equivalent to Heifner (1972, 1973) when the futures price is unbiased but cash position is not given. Meyers and Thompson (1989), also using the mean-variance approximation, proposed the application of flexible time series methods for incorporating the conditional information. Their suggestion can be viewed as the flexible function form of the error term in the regression price relationship discussed in Lence (1995).

Aiming for a preference-free optimal hedge ratio, Benninga, Eldor, and Zilcha, (1983)(BEZ), Lence (1995) and Rao (2000) made continuous efforts to derive sufficient and necessary conditions for the preference-free optimal hedge ratio that supports the use of the minimum variance hedge ratio suggested by JSE (1960,1961 and 1979). The sufficient and necessary conditions derived typically assume a relationship (BEZ, 1983 and Lence, 1995) between the cash price and the futures price or a join distribution (Rao, 2000) of them. These studies justify the current use of the two popular empirical hedge ratio models—the regression model and the GARCH type model–to derive the minimum variance hedge ratio.

According to Ingersoll (1987), when a decision maker's utility function is assumed to be quadratic or the underlying returns follow an elliptical distribution including normal, the risk measure, defined as variance, is equivalent to the expected-utility in the mean-variance framework. Therefore, those results on the equivalence of the minimum variance hedge ratio to the optimal hedge ratio (in the expected utility function framework) indicated by Kahl (1983) is consistent, particularly, with that of Rao (2000).

Conditional on an unbiased futures price, studies following this line have explored the time varying hedge ratio estimation and testing using the multivariate GARCH model (e.g., Moschinia and Myers 2002). There are also studies looking into hedge ratio estimation using long memory modeling, e.g., multivariate FIGARCH, with a focus on the decision makers facing long term exposure (e.g., Coakley, Dollery, and Kellard, 2007). Research on the minimum variance hedge ratio estimation without using a GARCH type model is also available, e.g., using a State Space Model (Vukina and Anderson, 1993).

Prior research indicates that the minimum variance hedge ratio is preference-free and desirable when the futures price follows a martingale process. But what if the futures price is not a martingale process or other necessary conditions suggested by Lence (1995) and Rao (2000) are violated? Under such situations, there is no longer a single-value preference-free optimal hedge ratio available. The joint distribution of the futures and cash prices/returns is needed since optimal hedge ratio determination in this case is not solely dependent on the moments up to the 2^{nd} order of the joint distribution. Under such conditions, solving for an optimal hedge ratio given a legitimate risk measure or preference (in terms of the Bernoulli utility function) is necessary, and the complexity of the problem to be solved depends on the joint distribution. If different risk measures/preferences are interesting, the joint distribution needs to be completely known to afford these choices. Since empirically deciding the analytical form of a joint distribution is a very challenging task, a simulation based method can be a good alternative.

This essay provides a new futures exposure determination framework for risk management. A GARCH model is used to estimate the futures return process and a State Space Model is used to estimate the price relationship between the return and futures return series. These two models together allow a calibration of the bivariate distribution of the two return series at the designated date for the position to be cleared. A coherent risk measure, the Conditional-VaR minimizing futures exposure is then obtained based on the simulated data according to Rockafellar and Uryasev (2000).

It is noted that very often in empirical hedge ratio studies the unbiased futures price assumption is true only when the data is averaged into weekly or longer periods for the regression model analysis or when the data is weekly or censored daily data (e.g., a mid-week data) for a GARCH model analysis. Many empirical studies typically do not test for the martingale hypothesis before estimating a minimum variance hedge ratio. Tests for the martingale hypothesis include the T-test, the variance ratio test of Lo and MacKinlay (1989) and the ranks and signs variance ratio tests of Wright's (2000). As Wright's test is nonparametric and more powerful, it was applied to nearby lean hog futures contracts and one individual contract[5]. The test results summarized in Table 10 of the Appendix favor a positive return for the lean hog futures contracts and hence will serve as a legitimate example for preference–dependent hedge ratio determination methods in this essay.

This essay will focus on the case of hedge ratio determination for pork products using the lean hog futures, as there has been no report available on the effectiveness of hedging using lean hog futures. Before the introduction of the lean hog futures, Hanyenga, Jiang and Lence, (1996) reported the hedging effectiveness of pork using live hog futures to be poor according to the minimum conditional variance hedge ratio method introduced by

Meyers and Thompson (1989). Ditsch and Leuthold(1996), using a simulation results based on lean hog index, reported effectiveness for hedging cash hog, loin and belly, but very poor results were found for hedging pork trimmings and ham. It is interesting to investigate the pork-lean-hog-future cross-hedge relationships if it is effective and to understand the reason if it is not. While the current research can be viewed as an empirical follow-up of the hedging effectiveness of pork products using lean hog futures, it more importantly contributes to the hedging/cross hedging problem using a futures contract in similar situations.

The following part of the paper consists of four sections: 3.2, a review of existing literature on current hedge ratio estimation methods, coherent risk measures, conditional-VaR-based portfolio selection/exposure determination and the State Space Model; 3.3, a section on the application of existing hedge ratio determination methods; 3.4, a section on State Space Model based simulation methods for the determination of optimum position exposure; 3.5, the conclusion.

3.2 Literature Review and Method Proposal

3.2.1 Existing Hedge Ratio Determination Methods

Most literature pertaining to hedge ratio determination uses the minimum variance hedge ratio. The development of time series econometrics has been the major propeller for the progress towards a “better” minimum variance hedge ratio estimation technique. Hedge ratio studies started with an unconditional hedge ratio that takes the minimum variance hedge ratio as the OLS regression of the form

$s_t = \alpha + \beta f_t + \zeta_t$, where s_t is the cash price for the product to be hedged and f_t is the futures instrument. Such models ignore the potential conditional information that helps predict the hedge ratio.

Meyers and Thompson (1989) suggested the Minimum Conditional Variance (MCV) hedge ratio model of the form

$s_t = \alpha + \beta f_t + \sum_{i=1}^{m} \gamma_i f_{t-i} + \sum_{j=1}^{n} \delta_j s_{t-j} + \sum_{k=1}^{q} \kappa_k x_{t-k} + \zeta_t$, where x_t is the other conditional information useful for determination of the hedge ratio. Usually, x_t may be the basis or related index price information. There are many variations of this specification. The price series can be replaced by price difference or return which may or may not fall into the Error Correction Model specification. Estimation of these models usually requires consideration of heterogeneity and autocorrelation to obtain reliable a test statistic for parameter estimates, e.g., Newey and West (1987). This method will be used as one of the two approaches in this study to estimate conditional hedge ratios for pork cuts. These methods can largely be classified as the regression relationship discussed in Lence (1995).

The bivariate GARCH model including the Diagonal VEC (Bollerslev, Engle and Wooldridge, 1988) model, the BEKK (Engle and Kroner, 1995) model, the extended DVEC (Ding, 1994 and Bollerslev, Engle and Nelson 1994) model, the time-varying correlation models by Tsay and Tsui (2002), and the popular DCC model (Engle, 2002) are all capable of estimating a hedge ratio by minimizing the conditional variance. Application of MGARCH model in agriculture commodity hedging includes Moschini and Myers (2001) and Manfredo, Garcia and Leuthold (2000). The most active developments of these models for application in financial products are usually concerned with a parsimonious parameter structure specification that guarantees the positive definiteness of covariance/correlation matrix and has a wide applicability. All these M-GARCH models can produce the MCV hedge ratio, which can be either time-varying or time-invariant. A time-varying hedge ratio is meaningful only when this time-varying ratio is predictable via a statistically reliable model. These models are very helpful for looking into the evolution trajectory of the minimum variance hedge ratio. Yet, it has to be noted that the optimality of these minimum variance hedge ratios depends on the martingale hypothesis of the futures price. These methods can not provide a single ratio, which is often of great applicability to decision makers.

Alternatively, time-varying hedge ratios can be derived by the flexible least squares (Kalaba and Tesfatsion, 1996) estimation or other filter-type methods such as the Kalman filter. The same concern follows for these models; what if the estimated hedge ratio is not the preference-free optimal ratio? Or, what if a simple rule-of thumb hedge ratio is beyond a time-varying estimator in terms of the applicability?

3.2.2 Coherent Risk Measures and Conditional-VaR Based Optimal Exposure Determination

Since the seminal work by Artzner et al. (1999), "coherent" risk measure has become a popular concept in risk management research. Since then, Value at Risk has been updated with the so called Conditional-VaR (C-VaR) to conform to the coherence standard.

For a given measure G defined on a σ-algebra V over a set X with $x \in V$, a risk measure $G(x) \to R$ is coherent if it is

(i). Monotonous, i.e. $x \geq y \Rightarrow G(x) \geq G(y)$;

(ii). Positive Homogeneous, i.e. $G(kx) = kG(x),\ for\ k \in R$ and $x \in V$;

(iii). Translation Invariant, i.e. $G(x+b) = G(x) + b,\ for\ b \in R$;

(iv). Sub-additive, i.e. $G(x+y) \leq G(x) + G(y),\ for\ x, y \in V$.

Kusuoka (2001) suggested two additional axioms: law-invariance and co-monotonic-additivity.

(v). Law Invariance, i.e. $G(x) = G(y), if\ \ x, y$ has the same probability law;

(vi).Co-monotonic-additivity, i.e $G(x+y) = G(x) + G(y),\ for\ x, y$ co-monotonic.

According to Ogryczak and Ruszczynski (2002) and de Giorgi (2005), if all the six properties are satisfied, the risk measure becomes consistent with the second order stochastic dominance principle and expected utility function. Recall that a random variable X first-order stochastically dominates a random variable Y if $E(U(X)) \geq E(U(Y))$, where $U(.)$ is any function nondecreasing and X second-order dominates Y if $E(U(X)) \geq E(U(Y))$ where $U(.)$ is any function nondecreasing and concave. Since $U(.)$ can denote the utility function of a risk averse agent, a risk measure satisfying all these regular properties is desirable.

The first four properties are the usual axioms cited to define a coherent risk measure. The monotonous property is simply based on the countable additivity of a σ-algebra. Positive Homogeneity refers to the requirement that repeating a risk asset multiple times will result in a risk that is of a multiple times magnitude. Translation invariance means a risk asset plus a risk-free asset value will result in a risk of the risky asset plus the risk-free value. Sub-additivity means the global risk of a portfolio is generally smaller than its sub-portfolio and the two values are equal only when the risk values of each of the sub-portfolio are concurrently occurring. This axiomatic property is the single most characterizing one to define the coherent risk measure. A risk measure violating this rule discourages the diversification of risk—the basic rule of risk management. Law-invariance property is also conforming to everyone' view of a sensible risk measure and is often used for convenience in some theory derivation. See example in Bassett et al. (2004). The co-monotonic additivity means the sub-additivity property becomes additivity when the two random variables are perfectly correlated.

It is interesting to note that neither variance nor the popular Value at Risk (VaR) is a coherent risk measure where the α-level VaR is defined as the minimum loss in the top $\alpha\%$ worst cases. Neither of these measures satisfies the most important axiomatic property, the sub-additivity.

In response to the theorists' proposal of the coherent risk measure, researchers have been suggesting coherent measures that are applicable in practice. The Conditional VaR (C-VaR), also called expected shortfall, has become a popular measure in recent years. Tasche (2002) discussed the theoretical properties of a generalized expected shortfall, and Rockafellar and Uryasev (2000) showed that the C-VaR based sample portfolio optimization can be solved via a linear programming problem. Bertsimas et al (2004) further showed the C-VaR based portfolio selection problem can always be efficiently solved as a convex optimization problem.

In this essay, the optimal futures position exposure problem will be solved using the simulated bivariate distribution of futures and spot returns following the sample optimization method by Rockafellar and Uryasev (2000).

For a risky return denoted as a random variable y and decision variable x, the *loss function* $f(x, y)$ is induced by y given the decision variable x. The α-level C-VaR given decision variable x is defined as

$\phi_\alpha(x) = (1-\alpha)^{-1}\int_{f(x,y)>\varsigma_\alpha(x)} f(x,y)\,dF(y)$ where $\varsigma_\alpha(x)$ is the α-level VaR of the random variable $f(x, y)$ induced by y and x, defined as $\varsigma_\alpha(x) = \inf\{z|\mathrm{P}[f(x,y) \le z] \ge \alpha\}$. It is noted that, while the integral interval $f(x,y) > \varsigma_\alpha(x)$ does not need a strictly larger sign for the continuous random variable, the C-VaR does need a strictly larger sign to distinguish it from the so-called "Tail Conditional Expectation" that takes an average over the values larger than or equal to the α-quintile for discrete or atomic probability case.

According to Rockafellar and Uryasev (2000), via the function $F_\alpha(x,\eta)$ defined as $F_\alpha(x,\eta) = \eta + (1-\alpha)^{-1}\int [f(x,y)-\eta]^+ dF(y)$ that is convex on η, where $[A]^+ = A$ *if* $A > 0$ and 0 otherwise, the C-VaR can be obtained according to $\phi_\alpha(x) = \min_{\eta \in R} F_\alpha(x,\eta)$. It is further shown $\min_{x \in X} \phi_\alpha(x) = \min_{(x,\eta)\in X \times R} F_\alpha(x,\eta)$ and $F_\alpha(x,\eta)$ is also convex on (x,η). The optimal decision variable x for given α can be solved by directly minimizing $F_\alpha(x,\eta)$ over x and η simultaneously since this will be a convex programming problem. Recall that, as shown in Lence (1995) and Rao (2000), when the preference-free optimal hedge position is not applicable, the optimal hedge ratio has to be dependent on the preference. Here α can be understood as one kind of characterization of the decision maker's preference, a pessimism parameter. It is noted, however, the analytical solution to x for given α is not easily available even for known joint distribution.

Fortunately, according to Rockafellar and Uryasev (2000), for the sample case this question will become a programming problem defined as

$$\min_{(x,\eta)\in X\times R}\left\{\eta+\frac{1}{m(1-\alpha)}\sum_{i=1}^{m}\left[f_i(x,y)-\eta\right]^{+}\right\}$$ where m is the number of sample size. For an optimal futures position determination of a "short hedge" problem, two risk returns/losses are considered. Denote the cash position loss as y_{cash}, futures position loss as $y_{futures}$, and normalize the cash position as "1", the programming problem becomes

$$\min_{(x,\eta)\in X\times R}\left\{\eta+\frac{1}{m(1-\alpha)}\sum_{i=1}^{m}\left[xy_{futures}+y_{cash}-\eta\right]^{+}\right\}$$. If the sample scenario properly represents the underlying joint distribution of the two returns, the solved minimum of this linear programming problem will give us a sample approximation to the expected loss for the worst α cases when taking x futures position and 1 cash position, i.e. the α level C-VaR and the solution x is the optimal futures exposure position or "hedge ratio" for given α. A "long hedge" problem is formulated similarly.

Because such determination of an optimal futures exposure position depends on the concept of expected shortfall that is different from the risk measure defined as VaR or variance. The determined optimal exposure and its explanations are also different. For the risk measure defined as variance, the optimal hedge ratio yields a single ratio that can not give a value of the probability associated with the loss incurred by such ratio. For an optimal futures position exposure using the VaR concept, while an estimate of the probability is provided, such determined exposure may discourage the hedge (diversification) because VaR violates sub-additivity.

This route of risk management strategy is a beautiful result, but its application still depends on the known DGP of the returns of the underlining assets/commodities. Without an easily applicable joint distribution of the returns, practical application can be based on the simulated sample. Popular choices for producing a sample scenario include the direct use of historical data or using an estimated Copula method to model the multivariate distribution. It is noted that these data generating methods generally assume the population process is stationary. As argued in the opening section, the nonparametric or non-structural model of the DGP may miss some fundamental relationship between prices. The following section will review the State Space Model (mainly linear) literature and studies on the futures-cash price relationship to motivate a structural perspective cash-futures price relationship model.

3.2.3 Cash-Futures Price Relationship and the State Space Model

In terms of asset price relationship, the “demand side” theory leads to the Capital Asset Pricing Model (CAPM) as opposed to the “supply side” theory or the Arbitrage Pricing Model (APM). Because the “demand side” theory approaches the problem within a utility maximization framework, particularly an explicit mean-variance form and can be easily parameterized, it receives much more attention in research literature compared to the Arbitrage Pricing Model approach.

As argued in the previous section, the applicability of the preference-free minimum variance hedge ratio needs restrictive conditions such as unbiased futures price. Although there is plenty of literature on the edge ratio that reports the general applicability of such an approach, it is mainly due to the a censoring effect that only surviving empirical hedge ratio studies were reported while those failed were simply labeled as due to the inefficient market price discovery process. There are presumably many empirical studies that only report viable models based on certain censored/averaged data, e.g., mid-week data or weekly data if we consider daily data as the original data set.

Those Econometric methods elaborating on the time-varying covariance specification and estimation of the return relationship is meaningful only when the existence of the preference-free optimal hedge ratio is justified. From this perspective, applied econometric methods targeted at testing for a time-varying versus a stable hedge ratio (co/variance) or other forms of variance complexity may only marginally improve the risky asset position determination problem.

It is well known that APM principle is simple and flexible but hard to apply. It is simple because the basic cash-future relationship is just $f_t = s_t e^{r_t \tau_t}$ where f_t and s_t are futures/cash price, r_t is interest rate, possibly combined with cost of carry/convenience yield and τ_t is the interval from t to futures position expiration. The relationship is flexible because it can fit into data of both low and high frequencies. This is in sharp contrast with the concept of basis provided by the “demand side” theory in which the basis has to be taken as the difference between the cash price and the “nearby” expiring futures price which is under a more stringent condition. Its flexibility is also due to the fact that additional information other than the interest rate can also be put into the model, e.g., storage cost for storable commodities. The other distinguishing features of the APM approach price relationship specification include: it tracks the information contained in a futures contract with a specific expiration time and might be able to glean information specific to that time window; and it explicitly incorporates the information of time to expiration into the price relationship.

With these many pros, this approach remains unpopular not by accident. One of the largest cons is its applicability using econometric methods. While State Space Models is capable of handling such a model and there are plenty of applications of State Space Models in Macroeconomics literature or finance literature, there has been very few applications of SSM specially applied to the cash-future relationship. One major reason is that with extra estimation cost, its multiplicative form can not easily fit into demand-side perspective concepts like risk premium and basis. For an example of applying SSM in the minimum variance hedge ratio framework, see Vukina and Anderson (1993).

The SSM approach is chosen to model the cash-futures relationship in the APM framework because it retains a sensible structural form of the price relationship that is true when arbitrage is efficient and can also be properly relaxed when arbitrage is not efficient. The competing methods for the joint distribution, e.g., multivariate GARCH type models or Copula method, give up the structural specifications of the price relationship. These two methods have other drawbacks. Particularly, the Copula method implicitly assumes the joint price process is strictly stationary. The M-GARCH model often fails to return consistent results especially for agricultural commodity cash prices when moving from averaged low-frequency data to higher-frequency data. There is not a rule to pick up data of different frequencies. In contrast, the SSM-APM specification of the price relationship always prefers a higher frequency (as high as daily but not the higher one used for microstructure) and is possibly more robust for unexpected price shock. e.g., observed data of a sustainable price increase in both futures and cash prices can still fit in the SSM-APM price form. Such properties is meaningful and desirable, especially for consumption commodities when market structures are experiencing substantial change, e.g., the constant failure of the convergence of the grain futures price to the cash price (Irwin, Garcia, Good, and Kunda, 2008) during the oil price surge of 2007-2008 . By specifying more detailed parameters, APM specification can be refined to reflect the price relationship in a much more precise and robust fashion using a SSM form.

The last note about the applicability of the APM specification of the cash-futures price relationship is that APM is not necessarily inconsistent with the necessary/sufficient conditions for a preference-free optimal hedge ratio suggested by Lence (1995) and Rao (2000). According to Lence (1995), the futures price is unbiased, and $s_T = hf_T + g(e_T)$, where $\mathrm{E}(f_T|e_T)=\mathrm{E}(f_T)$. In the APM form, $f_t = \alpha_t s_t e^{\beta_t \tau_t} \Leftrightarrow \log \frac{s_t}{f_t} = A_t + B_t \tau_t$ and can be understood as a more detailed relation of $\log \frac{s_t}{f_t} = \log(h + \frac{g(e_t)}{f_t})$ in Lence (1995).

The State Space Model (SSM) is originally created in a linear form by Kalman (1960) and Kalman and Bucy(1961). It has received vast follow-up because it has extensive applicability in both economics and engineering research. The State Space Model has been widely applied in macroeconomics and finance literature. Notable literatures include Harvey (1989), Harvey (1993), Hamilton (1994), West & Harrison (1997), Kim & Nelson (1999), Shumway &Stoffer (2000), and Durbin & Koopman (2001). Modern Kalman filter type models have become very versatile with certain forms of nonlinear, non-Gaussian models available (Jungbacker and Koopman, 2007). The linear form SSM can be derived by a normal assumption of the error term, a mixed estimator approach or a minimum mean square linear estimator.

A linear State Space Model using a first order Markov structure in the state variable can be represented in the following form

Measurement equation: $x_t = d_t + Z_t a_t + S_t \varepsilon_t$

State equation: $a_t = c_t + T_t a_{t-1} + R_t \eta_t$

where x_t represents a $m \times 1$ observable variable vector, a_t represents a $n \times 1$ unobservable state variable vector; ε_t of dimension $m \times 1$ and η_t of dimension $n \times 1$ are the zero-mean error term vectors for the measurement and the state equation respectively. The two error vectors are contemporarily and inter-temporarily uncorrelated and with each other with variance matrices $\sigma_{s,t}$ and $\sigma_{\eta,t}$ respectively; d_t and c_t represent the $m \times 1$ mean vector and the $n \times 1$ mean vectors for x_t and a_t respectively; Z_t of dimension $m \times n$ and T_t of dimension $n \times n$ are the two coefficient matrices.

Denoting $a_{t|t-1} = E(a_t \mid F_{t-1})$ and $\Sigma_{t|t-1} = COV(a_t \mid F_{t-1})$ where F_{t-1} is the filtration denoting information available up to period $t-1$. In the model, all the information available is contained in x_t , therefore,

The prediction equations $\begin{cases} a_{t|t-1} = T_t a_{t-1|t-1} + c_t \\ \Sigma_{t|t-1} = T_t \Sigma_{t-1|t-1} T_t^{'} + R_t \sigma_{\eta,t} R_t^{'} \end{cases}$ result by taking conditional expectation and variance respectively with respect to the transition equation given information set at $t-1$. With a_0 and Σ_0 available, prediction equations allows for the "prediction" of the state variable's conditional mean and variance. Because $a_{t|t-1}$ is only the conditional mean of the state variable, we expect it can not perfectly predict its value in period t and there should be a prediction error ς_t between a_t and $a_{t|t-1}$ so that $a_t = a_{t|t-1} + \varsigma_t$. The interest is to derive a "good" predictor of the state variable in period t. The previous equation only predicts the state variable with last period information but neglects the information available from the observable variable at period t as shown in the measurement equation. It is desirable to combine the information together to obtain a better prediction. Rewrite this equation as $a_{t|t-1} = a_t - \varsigma_t$ and combine it with the measurement equation to arrive at

Equation (*) $\begin{cases} a_{t|t-1} = a_t - \varsigma_t \\ x_t - d_t = Z_t a_t + S_t \varepsilon_t \end{cases}$. Now, notice that equation (*) reminds us of the familiar least squares regression. Because ς_t and $S_t \varepsilon_t$ can not allow a homoskedastic regression problem, the Generalized Least Square (GLS) estimator is used to obtain a "better" estimation for a_t considering information from both the last period state variable and the current period observable variable. Recall that the GLS estimator for the linear regression $Y = X\beta + e$ with known variance of e is

$\widehat{\beta} = (X'COV^{-1}(e)X)^{-1} X'COV^{-1}(e)Y$. A similar equation on the "better" estimator $a_{t|t}$ of a_t can be derived. Similarly, the GLS estimator's sampling variance formula

$Var(\widehat{\beta}) = (X'COV^{-1}(e)X)^{-1}$ gives the variance for $a_{t|t}$, denoted as $\Sigma_{t|t}$. These two equations, after some algebra and simplification, can be written as Equation

(**) $\begin{cases} a_{t|t} = a_{t|t-1} + K_t(x_t - x_{x|t-1}) \\ \Sigma_{t|t} = \Sigma_{t|t-1} - K_t F_{t|t-1} K_t' \end{cases}$, where $G_{t|t-1} = Var(x_t \mid F_{t-1}) = Z_t \Sigma_{t|t-1} Z_t' + S_t H_t S_t'$

and $K_t = \Sigma_{t|t-1} Z_t' G_{t|t-1}^{-1}$. Equation (**) is the update equation because it uses the predication equation's result to update the next period state variable values. This is the so-called Goldberg-Theil mixed estimator approach derivation of the Kalman filter. Other derivation methods can be found in the literatures e.g., Tanizaki (1996) and Hyndman et al. (2008).

The above derivation of the Kalman filter does not use any distributional assumptions on the error term. Assuming the error term in the measurement and transition equations is normal leads to a MLE estimation based on the Kalman filter. With known conditional variance $G_{t|t-1}$ for x_t, only the conditional mean $E(x_t \mid F_{t-1}) = Z_t a_{t|t-1} + d_t$ is needed to write out the conditional density $\prod_{t=1}^{T} P(x_t \mid F_{t-1})$ for the Gaussian MLE estimation problem. Unknown model parameters are in $\sigma_{s,t}$, $\sigma_{\eta,t}$, d_t, c_t, Z_t and T_t that are often very sparse. The MLE estimation of linear SSM can be executed conveniently in the free SsfPack (see http://www.ssfpack.com). For further detailed information on smoothing results, initialization of Kalman filter and more recent development see Commandeur and Koopman (2007) or Hyndman et al. (2008) for reference.

3.3 Hedge Ratio Determination Using Existing Methods

This section will derive the hedge ratios for pork products using the Lean Hog futures with data after the introduction of the Lean Hog Futures in CME. Methods include the conditional minimum variance ratio and time-varying hedge ratio using the bivariate GARCH model.

3.3.1 Data

Choice of data often raises concerns for hedging agricultural commodities using a related futures instrument. These concerns include the choices of data frequency and averaged data versus point data. This is often a critical issue for the hedge ratio determination due to the more complex relationship found between cash price and futures price for commodities compared to financial prices since futures contract prices for different expiration months should be treated differently. In this paper, difference between observation frequencies will also be studied to provide a more complete perspective for the hedge ratio determination. There are average cutout prices for pork and cut prices of Loin, Butt, Picnic, Rib, Ham and Belly with daily, weekly and monthly observations available for the analysis since the introduction of the lean hog futures contract in 1998.

3.3.2 Empirical Results

Conditional hedge ratio determination method is applied first. Because price series are not stationary and prices differenced are tested stationary, the following model is estimated for hog price and each pork product price.

$$\Delta s_t = \Delta\alpha + \beta\Delta f_t + \sum_{i=1}^{m}\gamma_i\Delta f_{t-i} + \sum_{j=1}^{n}\delta_j\Delta s_{t-j} + \sum_{k=1}^{q}\kappa_k\Delta x_{t-k} + \zeta_t$$

This model was applied to weekly nearby lean hog futures prices and cash prices of pork products. A low R^2 was obtained for each relationship with the highest R^2 being below 0.4 and hedge ratios being around 0.4 for the weekly models. Such a model applied to daily data returned an even lower R^2 and hedge ratios that are as low as 0.1. These results on daily data models may not be surprising because daily fluctuations of individual product cash prices and lean hog futures prices often have a weak connection, and the futures price is more closely related to the aggregate price i.e. the cutout price.

The low R^2 of these models is apparently due to a time-varying minimum variance hedge ratio that might not be stable over time. More importantly, as it is noted in the introduction section, the applicability of minimum variance hedge ratio using such regression type models needs to assume an unbiased futures price. For the current analysis, the variance ratio test rejected this assumption and therefore can not justify the optimality of the minimum variance hedge ratio. Will monthly data make more sense? Similar classical models of differenced price were applied to monthly data of pork cuts. The results are slightly better in terms of model significance compared to daily/weekly data. The estimated hedge ratios increased to around 1. However, the results based on monthly data are not very desirable or effective in practice because such results are essentially considering the trans-contract variability while any hedger should only be interested in variability for a contract with a specific definite expiration month.

The MGARCH models were applied to the same datasets and returned models that are not statistically acceptable. The MGARCH model failed for both daily data and the weekly data because the linear correlation in the mean/variance processes can not be exhausted in the model. However, an examination of the mid-week (it is Wednesday in this paper) return data (discretely calculated) with the removal of trans-contract return yielded a satisfactory model with linear correlation in the mean/variance process properly modeled and residuals tested to be close to white noise. The MGARCH model estimated is specified as

$R_t = \Phi_0 + \varepsilon_t$, $\varepsilon_t \mid F_{t-1} \sim N(0, \Sigma_t)$, where Φ_0 is zero in these models. Time to expiration is not considered here. $\Sigma_t = A_0 A_0' + \sum_{i=1}^{p} (A_i A_i') \odot (\varepsilon_{t-i}\varepsilon_{t-i}') + \sum_{j=1}^{q} B_j B_j' \odot \Sigma_{t-j}$, where A_0, A_i and

B_j is each a lower triangular matrix which can possibly be further simplified as a vector or scalar.

Such models estimated for cutout, loin and picnic turns out to be statistically satisfactory in the same sense of model check as indicated previously. In Table 1, B_1 is a vector for all 3 models while A_1 in cutout model is a lower triangular matrix and a vector for loin and picnic. Estimations of other pork products—Boston Butt, Rib and Belly—in the MGARCH framework, including the BEKK model, that allow dependence between the volatility processes can not produce statistically satisfactory results. Among the return series of the three products, the return series for Rib contains many zeros. However, that of the Boston Butt and Belly exhibit erratic behavior in that their observations show no autocorrelation, but the model residual shows autocorrelation. Using log return series (used simultaneously for the series pair if used) could not give satisfactory results either. It is possible that GARCH model specification may not be appropriate in this case. Table 1 gives the estimation results and the post-estimation model check statistics. It can be seen that the left-upper elements of these parameter matrices are reasonably close since they are estimates for the same parameter for the futures return series and the matrix diagonal model is actually three univariate GARCH models for the variances and covariance components respectively that impose weak restrictions on the parameter. By the estimated models, the unconditional covariance/variance as the limiting values of the conditional co/variance can be calculated like the unconditional mean of the ARMA model, and so does the "unconditional" hedge ratio, defined as the ratio of unconditional variance to the futures unconditional variance. These numbers are provided in Table 1. Figure 16 in the appendix shows the MCV hedge ratio, volatility of futures return, covariance and correlation between the futures return and the cutout return. Figure 17 in the appendix shows the estimated MCV hedge ratios for other products.

Table 1 QMLE Estimation of a Bivariate GARCH Model

Parameter\Model	Cutout	Loin	Picnic	Ham
A(1,1)	0.018** (0.0029)	0.013** (0.0029)	0.015** (0.0030)	0.015** (0.0033)
A(2,1)	0.017** (0.0047)	0.0044** (0.0013)	0.0038** (0.0018)	0.010** (0.0042)
A(2,2)	0.016** (0.0027)	0.013** (0.0030)	0.015** (0.0025)	0.036** (0.0069)
ARCH(1;1,1)	0.34** (0.058)	0.26** (0.048)	0.32** (0.055)	0.30** (0.054)
ARCH(1;2,1)	0.28** (0.097)	0.31** (0.045)	0.35** (0.033)	0.36** (0.061)
ARCH(1;2,2)	0.44** (0.070)	NA	NA	NA
GARCH(1;1,1)	0.83** (0.054)	0.91** (0.038)	0.88** (0.047)	0.88** (0.049)
GARCH(1;2,1)	0.62** (0.11)	0.92** (0.023)	0.92** (0.014)	0.79** (0.071)
Diagnostic Statistics (P-Value in Parenthesis)				
Q-Statistics χ2(12) of Standardized Residuals futures return	12.41(0.41)	13.56(0.33)	12.82(0.38)	13.03(0.37)
Q-Statistics χ2(12) of Standardized Residuals cash return	8.36(0.75)	13.73(0.32)	18.63(0.098)	18.53(0.10)
Squared Standardized Residuals **Q-Statistics χ2(12)** futures return	5.81(0.92)	6.47(0.89)	5.50(0.93)	5.96(0.91)
Squared Standardized Residuals **Q-Statistics χ2(12)** cash return	13.30(0.35)	11.82(0.46)	7.98(0.79)	6.26(0.90)
Squared Standardized Residuals **LM Test F(lag: 12)** futures return	0.59(0.93)	0.65(0.88)	0.55(0.95)	0.60(0.92)
Squared Standardized Residuals **LM Test F(lag: 12)** cash return	1.16(0.42)	1.25(0.36)	0.72(0.82)	0.63(0.90)
Unconditional Hedge Ratio	0.47	0.43	0.40	0.51

The M-GARCH models fitted to the pairs of product/futures returns are similar. By these figures, it can be seen that the MCV hedge ratio also exhibits a similar irregular pattern or at least can not be summarized by a simple quick reversion AR model. Such irregularity explains why the classical method returns low R^2. Interestingly, these hedge ratios are much below 1.The results based on weekly data showed that it is impossible to arrive at a single ratio in the minimum variance sense by considering the weekly price variability. What about the daily data applied to MGARCH? Actually, the daily data MGARCH models were estimated but turned out to be statistically unacceptable with the models failing to exhaust the linear correlation in either the mean or variance process. More importantly, the expected return of the futures is not close to zero, crucial to justify the use of a minimum variance hedge ratio. The regression methods applied to the daily data of spot price and nearby futures price resulted into parameter estimations far from statistical significance, and these minimum variance hedge ratios are not optimal either because the nearby future price series is tested to be away from a martingale assumption. In the next section, the SSM-APM analysis framework using daily data will be introduced.

3.4 State Space Model Based Simulation and Optimal Position Exposure Determination

According to the empirical models in the previous section, it is evident that the choice of the models and data is quite arbitrary. Many empirical studies report the valid models of data of a certain frequency with correspondent hedge ratio(s) and leave other choices of data frequency unattended, ignored or suppressed. It is often the case that no existing models can be fit to data of some frequencies. This is especially so for data that is of higher frequency in GARCH type models. For example, the weekly data for Boston Butt can not produce a valid GARCH model and hedge ratio using the mid-week data, while the same models work for other pork products.

The critical issue of the spot-futures price relationship is their join process or the joint distribution at the interested position close date. Unfortunately, such a task is very challenging. Popular methods like the Copula method has to assume a strictly stationary process to model the joint distribution. While many researchers consider the nonparametric approach as an advantage in modeling the process since it does not assume any parametric forms of the price relationship, it has to be noted that nonparametric methods may also miss the structurally implied relationship between the cash and futures prices. The following section motivates such an approach in a State Space Model framework.

3.4.1 The State Space Model of Cash-Future Relation

This section estimates a State Space Model of the time-varying relationship between the cash and futures prices.

Recall that the APM approach states that the two prices for the same underlying asset (not cross hedge) with a no-arbitrage argument should follow a basic relation $f_t = s_t e^{r_t \tau_t}$, where τ_t is the interval between t and futures contract expiration and interest r_t is often listed as the major part of the cost of carry. For commodities, explicit storage costs and the asymmetry of arbitrage of the futures/commodities due to the convenience yields leads to a price relationship $f_t = s_t e^{(r_t + R_t - c_t)\tau_t}$ in which R_t is the storage cost factor, and

c_t is convenience yields. Without prior information, the two factors R_t and c_t are assumed to vary over time as well as the interest rate r_t, and it can be modeled explicitly or treated implicitly by binding it with the interest rate r_t. It is noted that when the cash price is sustainable and substantially increasing e.g., corn price in 2008 due to the ethanol industry prospect, the convenience yield for storable may experience significant increase and vary the price relationship. With relatively stable cost of carry this change may means smaller values for the total effect in the three factors in the price relationship. For non-storable commodities, more complex cost of carry may complex the judgment for the change of the total effect of these three factors. While it is always more informative to identify more factors, for simplicity purpose, the three factors are modeled together as β_t in this paper for perishable pork products.

Adding a constant coefficient to the price relation to provide room for different cash products and taking logarithm yields $\ln f_t = \ln k + \alpha_t + \ln s_t + \beta_t \tau_t$ [1], in which α_t and β_t are time varying parameters. In addition, $\ln c$ is also treated as the mean of α_t so that $\ln f_t = \alpha_t + \ln s_t + \beta_t \tau_t$ with only two factors kept for the price relationship/basis risk.

With these factors revisited, we can model the time-varying relationship by explicitly considering 3 time-varying factors: α_t, hedging horizon τ_t, short rate and β_t.

Now the State Space Model can be set up as follows:

$\ln \frac{f_t}{s_t} = \alpha_t + \beta_t \tau_t + u_t$ (20),

$\beta_{t+1} = \beta_t + v_t$ (21),

[1] k is the ratio of live hog price to pork product and treated as constant.

$$\alpha_{t+1} = \alpha_t + w_t \qquad (22),$$

where α_t , β_t , c_t and b_t are unobservable time varying parameters describing the changing relationship between spot price and futures price,

$u_t \sim iid\ N(0, \sigma_u^2)$,

$v_t \sim iid\ N(0, \sigma_v^2)$,

$w_t \sim iid\ N(0, \sigma_w^2)$,

and $E(v_t u_t, w_t u_t) = 0$.

Without any prior information, the unobserved state variables α_t and β_t are first assumed to follow random walk processes first. In such a setting, the random walk assumptions can accommodate more complex process with higher variances estimates for the error terms. In other words, such specification is rough but help avoid possible misspecifications of the underlying data generating processes of these unobservable state variables. In addition, the SSM assumption of the uncorrelated error variances means the variation of the change of α_t should not be related to the change in β_t seeing that α_t denoting the essential price ratio between the cash price and futures price should not be related with β_t denoting the total effect of cost of carry and convenience yields.

In this model, change of these time-varying parameters may be tracked to study their evolution trajectory. Measurement equation (20) expresses observations in terms of state variables.

Because there is not a statistically viable model for the Boston Butt data in the multivariate GARCH framework, the SSM will be applied to Boston Butt daily data and correspondent Lean hog Futures data starting from 1998 after the introduction of the Lean Hog Futures at CME. The December contract is used as an example. The result of the model estimation is as follows:

Table 2 QMLE Estimation of a SSM Equation (20) ~ (22)

Parameters	Value	Std. Error	t value	Log-likelihood
$\ln(\sigma_v^2)$	-20.51	0.2147	-95.54	
$\ln(\sigma_w^2)$	-7.064	0.03161	-223.50	5074.16
$\ln(\sigma_u^2)$	-28.06	36.37	-0.7715	

It turns out the variance of the error term for the observation variable is not statistically significantly different from zero. Restricting it to zero and the re-estimation of MLE results indeed confirm this in the likelihood ratio sense because the log likelihood does not detect any change. This result is an expected result because the APM "structural" specification of the cash-futures relationship should lead to a small variance in the measurement equation.

For this APM-SSM with random walk specification for the stated variable, it is interesting to check if the price relationship is time-varying. A CUMSUM test is appropriate here based on Recursive Least Square (RLS) estimation of model: $\ln\frac{f_t}{s_t} = \alpha + \beta\tau_t + v_t \quad v_t \sim iid\ N(0,\sigma_v^2)$ (23). Analysis of this model in the form of State Space Model approach has the advantage that the RLS estimate of the coefficients is readily available from the Kalman filter (the SSM model without the restriction of $\sigma_u^2 = 0$ needs to be used). Figure 5 shows the result of the CUMSUM test of the RLS. The two straight lines give the 95% confidence bands for CUMSUM_t. Apparently, the parameters in model (4) are not constant. Time varying specification of parameters α_t and β_t is needed.

This is also a highly expected result. Figure 6 shows the Kalman filter estimation of the observable variable and the two state variables. It can be seen that β_t experiences a stable section within each of the years or only experiences a change of 1 or 2 times in a year (note that here one year consists of around 250 days excluding weekends) and α_t exhibits an apparent mean-reversion pattern.

Figure 5 CUSUM Test for Parameter Constancy

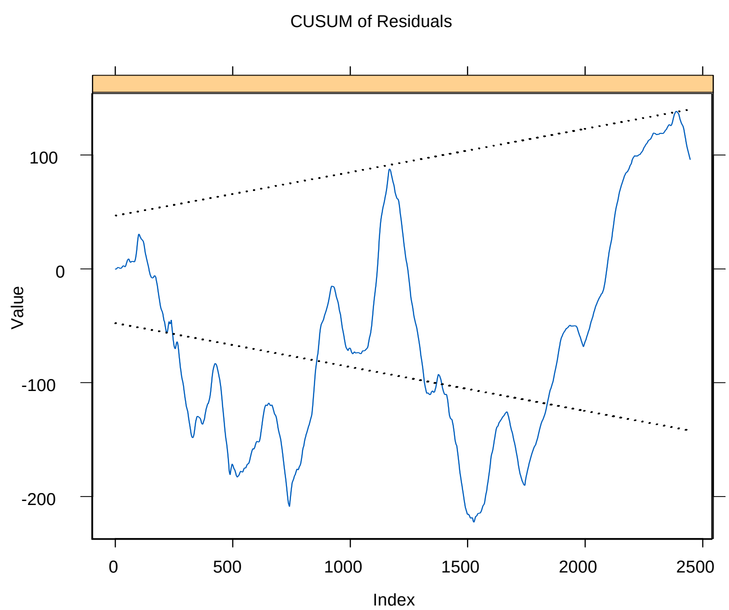
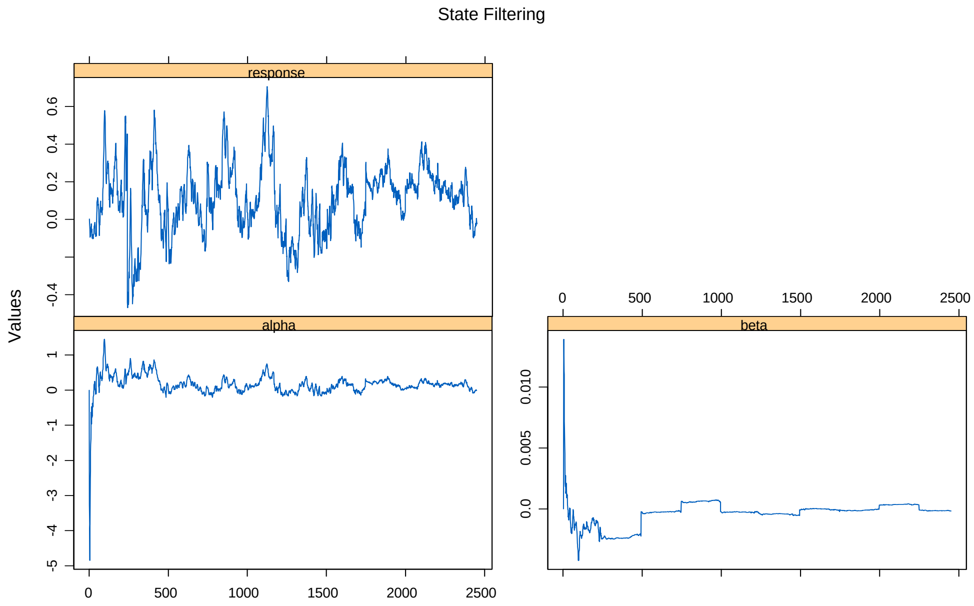


Figure 6 Filtered Moment of the Observable/State Variable for Equation (20) ~ (22)



More volatile α_t and less volatile β_t is also a well expected result: the essential ratio between the spot Boston Butt price and the lean hog futures price/hog price is supposed to be constant. But inefficiency of the price discovery across the cash market and futures market may make this value quite volatile. In contrast, β_t includes the cost of carry and convenience yield each of which is relatively stable. If the price discovery is not efficient across the two markets during some period it should be expected to resume the efficiency later, therefore α_t is supposedly to follow a mean-reversion pattern that is also recognizable in Figure 6.

Since α_t is showing a mean-reversion pattern, it follows the familiar auto-regression process. An AR (1) process for α_t is first considered.and then an AR (2) process to sufficiently exhaust the linear relation in the α_t process using a convenient likelihood ratio test. It turns out the AR (2) process is sufficient because a higher order is not necessary. For an AR (2) process for α_t .

The model becomes

$$\ln \tfrac{f_t}{s_t} = \alpha_t + \beta_t \tau_t \qquad (20')$$

$$\beta_{t+1} = \beta_t + v_t \qquad (21')$$

$$\alpha_{t+1} = \delta_\alpha + \phi_1 \alpha_{t-1} + \phi_2 \alpha_{t-2} + w_t \qquad (22')$$

In a more condensed way, the model can be written

$(z_t', y_t)' = \delta_t + \Phi_t z_{t-1} + \mu_t$, where

$\delta_t = (0, \delta_\alpha, 0, 0)$ $z_t = (\beta_t, \alpha_t, \alpha_{t-1})'$, $\phi_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \\ \tau_t & 1 & 0 \end{pmatrix}$, and $\mu_t = (v_t, w_t, 0, 0)'$.

β_t is not stationary and is therefore given a diffuse initialization meaning infinite variance given no prior information. The initial variance matrix of α_t, P is determined as follows.

$vec(P) = (I_4 - (F \otimes F)^{-1}) vec(V_w)$, where

$F = \begin{pmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{pmatrix}$ and $V_w = \begin{pmatrix} \sigma_w^2 & 0 \\ 0 & 0 \end{pmatrix}$. Therefore the initial variance matrix for β_t and α_t is

$\begin{pmatrix} \infty & 0 & 0 \\ 0 & P_{11} & P_{12} \\ 0 & P_{21} & P_{22} \end{pmatrix}$, where ∞ is usually taken as a “very big” number for the Kalman filtering.

There are 4 parameters estimated in such model using the MLE method with all parameters significant[2]. The initial values of ϕ_1 and ϕ_2 are important for the convergence of MLE computation. Since β_t only experiences a relatively small change, the auto-

[2] Keeping u_t in the model will again lead to an insignificant estimate of its variance.

regression order of series $\ln\frac{f_t}{s_t}$ is used as the initial value of the MEL estimation. Table 3 is used to show the parameter estimates, t values and log-likelihood:

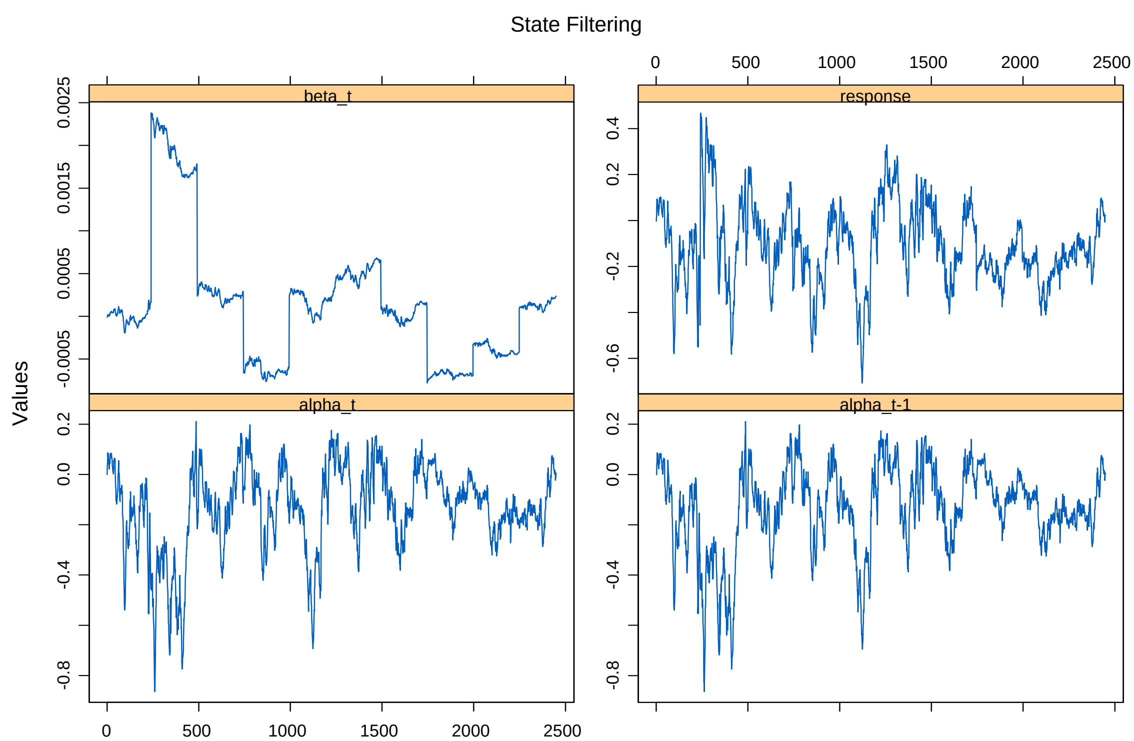
Table 3 QMLE Estimation of SSM Equation (20') ~ (22')

Parameters	Value	Std. Error	t value	Log-likelihood
AR(1) for state variable α_t				
δ_α	0.001385	0.0008023	1.726	5080.47
ϕ_1	0.9896	0.003434	288.2	
$\ln(\sigma_v^2)$	-20.47	0.2167	-94.5	
$\ln(\sigma_w^2)$	-7.069	0.03187	-221.8	
AR(2) for state variable α_t				
δ_α	-0.002267	0.0007539	-3.008	5141.06
ϕ_1	1.228000	0.0210600	58.310	
ϕ_2	-0.244600	0.0210500	-11.620	
$\ln(\sigma_v^2)$	-20.330000	0.1748000	-116.300	
$\ln(\sigma_w^2)$	-7.138000	0.0320200	-222.900	

It is possible that v_{t+1} and w_{t+1} might be correlated. For example in macroeconomics literature on State Space Model application, Morley et al. (2002) has shown that the difference between Beveridge-Nelson Decompositions and the Clark model trend-cycle decomposition is due to the independence assumption of trend and cycle innovations. The model allowing correlation between α_t and β_t is also explored, but report no significant estimates. This is a reasonable result as discussed previously since there should be no relation between the combined factor composed of convenience yields and cost of carry and the ratio between Boston Butt and lean hog futures. It is also noted that SSM specification of the form $\ln f_t = \alpha_t + \beta_t \tau_t + \gamma_t \ln s_t$ is not interesting to us because it is not a "structural" form. In fact, there are some authors who might argue γ_t is a time-varying hedge ratio since it reflects a time-varying ratio of the two returns. Figure 7 shows the results of filtered estimates for state moment and the observed variable.

Above Kalman filtering estimation shows that, much like those found in the original model β_t seems to follow a process with certain “jump” across years but usually remain relatively stable within one year and α_t follows a mean reversion process. Toward the determination of the spot-futures price relationship usually within a span less than a year, this implies that we can treat β_t as constant while model α_t with an AR(p) process with necessary order of auto-regression to make it stationary if no further model of the β_t process is desired.

Figure 7 Filtered Moment of the Observable/State Variable for Equation (20’) ~ (22’)



3.4.2 Simulation Based on State Space Model and Optimal Exposure Position Determination

With the SSM of the cash-futures price relationship, simulation of the joint distribution of the cash-futures price pair at a designated date only needs an extra model of one of the price series. A GARCH (1, 1) model of the Lean Hog futures return ($\Delta \ln f_t$) is estimated with the following estimation results in Table 3. The student-t distribution innovation is applied because the normality test and kurtosis reports the appropriateness of a heavy tail innovation. Of course, this is an ad hoc choice because there is other innovation available such as Generalized Error Distribution (GED). All the post estimation diagnostic statistics are also satisfying.

Table 4 QMLE Estimation of GARCH (1, 1) Model of the Lean Hog Futures Return (log)

Parameter	Estimate	Std.Error
A	0.00002468	4.06E-06
ARCH(1)	0.26724284	3.92E-02
GARCH(1)	0.68254721	2.75E-02
Student-t DF	3.473515	0.2078627

This model can be used to simulate n-period returns with a given starting value. With the n-period futures returns simulated from the GARCH model, $\ln f_t$ can also be obtained for the corresponding n periods.

It is noted that the SSM in last subsection can be rewritten as

$$\ln s_t = \ln f_t + \alpha_t + \beta_t \tau_t \qquad (23)$$

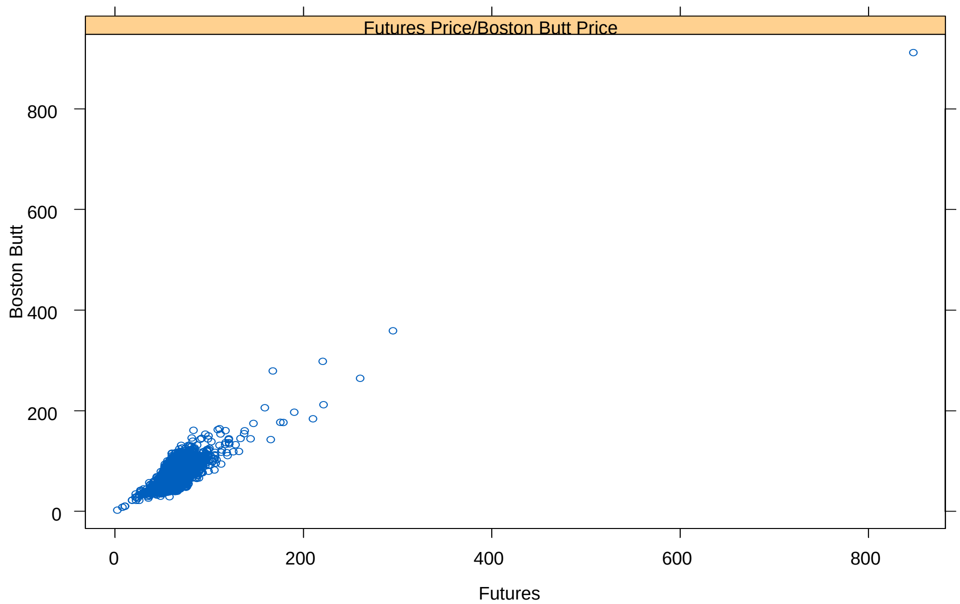
$$\alpha_{t+1} = -\delta_\alpha + \phi_1 \alpha_{t-1} + \phi_2 \alpha_{t-2} + w_t \qquad (24)$$

$$\beta_{t+1} = \beta_t + v_t \qquad (25)$$

Using the simulated $\ln f_t$ the above SSM is used to simulate $\ln s_t$ for the n periods. Repeating the two-step GARCH-SSM simulation gives us arbitrary large sample simulation for the n-th period of log price pair. Starting values needed include the filtered estimates of the state variables α_t and β_t. The simulated n-th period log cash price depends on each of the $n-1$ previous period's futures log price. These simulated price pairs are then used to calculate the discrete return by comparing them with the starting period price pair.

In this paper, a 49-days-ahead futures/cash log price pair is simulated 10,000 times with starting date of 10/05/2007, so that the simulated data forms a sample of size 10,000 for the log prices on 12/14/2007. The simulated 10,000 price pairs/return pairs are demonstrated in Figure 8, and the summary statistics are available in Table 11 of the Appendix. From the summary statistics, the minimum variance hedge ratio can be readily calculated at 0.5212. The two prices demonstrate close correlation since they are located around a straight-line, as is partially expected.

Figure 8 Scatter Plot of Simulated Futures and Boston Butt Price



Based on the calculated returns using these prices, it is possible to revisit the risk management task commonly found for agricultural commodity related business: determine the optimal futures position exposure given a cash position.

As discussed in the section 3.2.2, the "short hedge" problem in the sample case is equivalent to solving the linear convex programming problem

$$\min_{(x,\eta)\in X\times R}\left\{\eta+\frac{1}{m(1-\alpha)}\sum_{i=1}^{m}\left[xr_{futures}-r_{cash}-\eta\right]^{+}\right\}\ \text{s.t}\ \ x\geq 0\,.$$

Notice that here r_{cash} and $r_{futures}$ are returns, for $x \geq 0$, the *loss* for futures position is $xr_{futures}$ for a short futures position and the *loss* for \$1 cash position is $-r_{cash}$ for the long cash position. It is noted that the expected return for a short futures position x is $x(-\overline{r_{futures}})+\overline{r_{cash}}$. Because the sample mean returns are positive for both futures and cash commodity, adding a minimum expected return for the portfolio will lead to an upper bound for futures short position x if the expected return is lower than $\overline{r_{cash}}$. It is noted that for a risk-free rate 0.05,

$$[x(-\overline{r_{futures}})+\overline{r_{cash}}]/(1+x) \geq r_{riskfree} \Rightarrow x \leq (\overline{r_{cash}} - r_{riskfree})/(r_{riskfree} + \overline{r_{futures}})$$

$$x \leq (0.1686 - 0.05)/(0.05 + 0.07487) = 0.9498$$

$$\alpha = 0.99, \quad \eta = 0.2793, \quad x = 0.5769$$

$$\alpha = 0.95, \quad \eta = 0.1798, \quad x = 0.5459$$

$$\alpha = 0.90, \quad \eta = 0.1254, \quad x = 0.5148$$

The solved optimal futures exposure is also optimal when x is not required to be necessarily positive. This means "hedging," i.e. diversification, is advised. It is noted that here, bigger α may be interpreted as one kind of risk dislike, but may not be exactly matched with the risk-averse in the expected utility function framework. Bigger α means a decision maker is more pessimistic since s/he is more concerned with the expected loss of the worst α -above occasions. Like the efficient frontier in portfolio selection theory, smaller x, indicating higher expected return, should be associated with lower α, indicating a less pessimistic risk attitude. People who like single optimal exposure value for the two hedging situations should be assured by the relatively small range of the optimal exposure.

For the "long hedge" situation, the programming problem becomes

$$\min_{(x,\eta)\in X\times R} \left\{ \eta + \frac{1}{m(1-\alpha)} \sum_{i=1}^{m} \left[-x r_{futures} + r_{cash} - \eta \right]^{+} \right\} \text{ s.t } \; x \geq 0 .$$

Contrary to the "short hedge" problem, here the *loss* for the long futures position x is $-x r_{futures}$ and the *loss* for the short cash position is r_{cash}.

Again, for an expected return risk-free rate 0.05,

$$(x\overline{r_{futures}} - \overline{r_{cash}})/(1+x) \geq r_{riskfree} = 0.05 \Rightarrow x \geq (\overline{r_{cash}} + r_{riskfree})/(\overline{r_{futures}} - r_{riskfree})$$

$$\Rightarrow x \geq 8.7897$$

$\alpha = 0.99, \quad \eta = 0.5708, \quad x = 1.7058$

$\alpha = 0.95, \quad \eta = 0.5081, \quad x = 1.6986$

$\alpha = 0.90, \quad \eta = 0.3207, \quad x = 1.8032$

The solved optimal values are also optimal when x is not restricted to be positive. Similarly, here expected return shows longing more futures is desirable, but it has to be associated with a less pessimistic risk attitude. It is noted that $x=1.6986$ for $\alpha = 0.95$; it is supposed to be larger than 1.7058 when $\alpha = 0.99$. This is just an admissible computation precision error. Again the solved optimal exposures also have a small range.

One essential difference between the minimum variance hedge ratio and the C-VaR approach is obvious: there is only a single "hedge ratio" for the minimum variance approach, while the "ratios" in the C-VaR framework are different for the two situations: "short hedge" and "long hedge." The reason is obviously due to the fact that different tails of the distribution are used in the C-VaR optimizations. For the short hedge, the left tail of the cash return is considered, while for the "long hedge" the right tail of the cash return is considered. The SSM/C-VaR based optimal hedge ratio determined is compared with existing methods in Table 5.

Table 5 Optimal Hedge Ratio Determination Methods Comparison

Products	Data Frequency	Data Process Used	Risk Measure	Preference/Risk Measure Parameter	Hedge Ratios (Unconditional Ratios for Mid-Week GARCH Model)		Note
All Cuts	Daily/Weekly/Monthly	Regression Models	Variance	NA	Around 0.1/0.4/1 Respectively		Position Based
Cutout	Mid-Week	GARCH Models			0.47		Value Based
Loin					0.43		
Picnic					0.40		
Ham					0.51		
Boston Butt	Daily	GARCH/SSM Simulation			0.5121		
			C-VaR	α	Short Hedge	Long Hedge	
				99%	0.5769	1.7058	
				95%	0.5459	1.6986	
				90%	0.5148	1.8032	

Finally, the minimum variance hedge ratio estimated at 0.5212 is smaller than the optimal exposure solved for either the short hedge or long hedge situation. Does this mean the minimum variance hedge ratio discourages diversification? Note that, the minimum variance hedge ratio, according to the mean-variance approximation (Kahl, 1983), is the optimal hedge ratio if the decision maker is infinitely risk-averse, even though the futures price is not unbiased. Comparing it to the solved optimal ratios 0.5769 (short hedge) and 1.7058 (long hedge) for the pessimism parameter $\alpha = 0.99$, this confirms that variance as a risk measure violating sub-additivity indeed discourages diversification.

3.5 Conclusion

This essay investigated the optimal hedge ratio determination when the minimum variance hedge ratio is no longer the preference-free optimal hedge ratio. Such a situation may include the failure of the martingale hypothesis of the futures price or a “defective” statistical model using the regression models or a GARCH type model. With a GARCH model for the futures price and a State Space Model for the joint distribution of cash-futures prices, the preference-dependent optimal futures exposure based on a coherent risk measure, the conditional-VaR, is determined using simulate data.

This method provides a flexible framework for empirically estimating the cash-futures joint process as an alternative to existing historical/resample or copula methods and does not necessarily violate the suggested price relationship and martingale hypothesis of futures as detailed in Lence (1995) and Rao (2000). It also has the advantage of being capable of incorporating time to expiration information, short rate and other information that is detailed in the arbitrage pricing relationship. If its applicability is wide, this may help relieve the arbitrariness of picking up a viable empirical hedge ratio study among competing methods and data censorships.

The drawback to using a marginal process plus the SSM framework to estimate the joint distribution, much like using MGARCH model, is that the State Space Model with specified distribution may limit the joint distribution to a certain class. However, with the informative arbitrage pricing justification of the multiplicative price relationship, this limitation might raise less concern compared with models using even less “structural” specification e.g. the MGARCH model.

Chapter 4

Essay III: Structural Change and Long Memory in the Absolute Return Series of U.S. Hog Price 1973-2008

4.1 Introduction

This paper empirically investigates the U.S. hog price volatility from 1973 to 2008 and establishes a plausible connection between the found long memory in the weekly absolute return series and the structural changes that were documented in the 1990's: the shrinking spot transactions of hogs, the increasing capacity utilization and the expansion of meat retailers using the Every-Day-Low-Price strategy.

Economists have long been concerned with the possible structural instabilities in economic relationships. For a structural change defined as a "shifting parameter," there are the Chow-type tests for structural breaks with known timing and the development of theory for unknown timing and number of breaks by Bai and Perron (1998, 2003).

Long memory can be defined in terms of a decaying rate of autocorrelation in a time domain or the explosion of a low frequency spectral in the frequency domain. Since Granger (1980) derived the long memory process by considering a cross-sectional aggregation of data, there are a number of follow up studies using the idea of aggregation e.g., Lippi and Zaffaroni (1999) and Chambers (1998). These studies illustrate that aggregation is one reason for the observed long memory process found in empirical research. Alternatively, Diebold and Inoue (2001) showed that long memory process and regime switching can easily be confused with each other and hence, established that the long memory process can be generated by a structural change in the form of "regime switching" defined by the two authors.

The U.S meat supply chain has experienced significant structural change over the past twenty years with various implications for spot price variability. Two notable changes are found in the retail-level pricing strategy and the processing/farming organization. Studies on the expansion of market share for retailers using the Every-Day-Low-Pricing strategy indicate that farm-level price should have become increasingly volatile (Sexton, Zhang, and Chalfant, 2003, Boessen, 2006). For the change in the processing and farming sector, the arguments are two-fold. On one hand, the shift to larger-scale, year-round livestock production units may have caused smaller variation in production and thus lower price volatility. This is due to both a quick response associated with larger scale producers (Packers and Stockyards Programs, USDA, 1996) and the lower search cost associated with few sellers Stigler (1961). On the other hand, it is possible that marginal demand/supply by meat packers/producers in excess of their contract commitment may use the cash market as a buffering reservoir so that spot price volatility may have increased. This concern is increasingly important when the hog spot transaction share shrinks (Grimes and Plein, 2008) and the capacity-utilization ratio of pork processors increases (Parcell, Mintert, and Plain 2004). Since price volatility evolution responds to all kinds of structural changes, a natural question is what has empirically happened to farm-level price volatility, and what are the implications of these structural changes on price volatility? Should we expect a long memory effect in the price volatility and if so, why should we? In this paper, an empirical analysis of the U.S. hog price's volatility from 1973 to 2008 and a theoretical framework addressing the connection between empirical findings and structural change will be pursued to answer these questions.

To date, no research has extensively examined the long term dependence in spot hog price volatility, though there has been abundant research on the volatility of financial asset prices, including that of agricultural commodity futures prices (e.g., Crato and Ray 2000, Jin and Frechette 2004, Elder and Jin 2007, Baillie, Han, Myers, and Song 2007). Aradhyula and Holt (1988) examined retail meat price volatility using quarterly data and a GARCH model. Barkoulas and Labys (1997) studied the fractional price dynamics (but not volatility) of major commodities using GPH test and monthly data.

By the typical demand-supply equilibrium framework, a price shock may come from either demand side or supply side and so does the price volatility. When the livestock industry structure remains stable, price volatility is still not zero because of the endogenous uncertainty in the livestock production and the demand. Changes in the slopes of the demand curve and the supply curve can be interpreted as structural changes, shifting in the demand curve and the supply curve can be interpreted as the “regular” demand/supply shocks. If there is no such structural change as defined and no demand/supply shifting volatility change, price volatility should remain stationary over time. Short term volatility changes derived from the supply or demand side, such as the incidental bigger volatility in the supply curve shift, are transitory in terms of their impacts on the future volatility. On the contrary, changes in the slope of the demand or supply curve can lead to a persistent price volatility change. For example, The Oil Crisis in the 1970’s led to a higher price volatility level, but the impact on future price volatility should die out in the long term if it only shift the supply/demand curve. But if the demand curve becomes steeper, the same level of supply shock will result in a persistently larger price volatility.

Before performing any empirical investigation, price volatility is defined. Usually volatility is defined as the (conditional) variance. This definition is well supported by the popular conditional heteroscadastic models and stochastic volatility models. Despite the popularity of such models, because agricultural commodities often exhibit dominant cyclic and seasonal patterns, direct analysis of the price series needs a sophisticated model to simultaneously account for the seasonality, cycle and possible conditional heteroscadasticity in the model. Also making a more sophisticated model necessary is the possible long memory process which might be present in either the price series or the conditional variance of the price series. More importantly, for a study of price volatility spanning a long time period, the price level may experience substantial change that will lead to a price's conditional variance that is not comparable across periods. Finally, even if such sophisticate model simultaneously considering seasonal, cycle, long memory process in price level, and long memory process in price volatility can be applied, it is still impossible to further gauge the "volatility" of the price volatility and will miss the structural change process that is embodied in the very "volatility" of the price volatility.

As discussed in Chapter 1, the absolute return series $r_t = |(p_t - p_{t-1})| / p_{t-1}$ is a better measure for price variation, even if there is a strong seasonal and cyclic pattern in the price series. The absolute return series r_t can be interpreted as the observed volatility normalized by the nearby price level or observed coefficient of variation. In this paper, the absolute return series, rather than the price series, will be analyzed to examine the suspected long memory in both the mean process and variance process of the absolute return series.

The following part of this paper consists of a review of the relevant research methodologies, an empirical analysis, an ad hoc demand/supply framework explaining the empirical findings and conclusion respectively.

4.2 Literature Review and Methodology Proposal

Agricultural commodity price volatility, unlike financial assets, receives relatively little attention from economists. Agricultural commodity price series have their own distinct properties in comparison to that of speculative assets, as agricultural commodity prices often exhibit more manifest seasonal and cyclic behavior. The literature review includes three parts: long memory modeling, conditional heteroscadaticity modeling and cyclic and seasonal modeling. The literature survey of the seasonal modeling shows the difficulty to simultaneously model seasonality, cycle and long memory process in a conditional heteroscadasticity model framework.

4.2.1 Long Memory Model

Long memory process can be considered as a generalization of the ARIMA model in which the differencing order takes a fraction so that it is often called ARFIMA model. Its autocorrelations decays slow that $\sum_{-\infty}^{+\infty} \rho_k \to \infty$. The fractional difference filter is defined as $(1-L)^d = \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} L^k$, where d is the difference order and L is the lag operator. In frequency domain, the spectral density $f(\omega)=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\rho_k e^{ik\varpi}\to C_f\omega^{\alpha-1}$ as $\omega\to 0$,

where $C_f>0$ and $0<\alpha<1$, i.e. the spectral density goes to infinity as frequency approaches zero.

Tests relevant to long memory include the KPSS test, the R/S statistic and GPH test. The KPSS test is proposed by Kwiatkowski, Phillips, Schmidt and Shin (1992). In contrast with the ADF test and the Phillips-Perron test, the KPSS test tests against the null of I(0) of the time series. This test assumes the series can be written as a deterministic term, a random walk term and an I(0) process with possibly conditional heteroscadasticity. A test statistic is constructed as the Lagrange Multiplier statistic to test if the random walk term has a zero innovation variance.

The R/S statistic was developed by Hurst (1951), Mandelbrot (1975) and improved by Lo (1991). The modified R/S test for series y_t is defined as:

$\widehat{Q}=\frac{1}{\widehat{\sigma(q)}}\left[\max_{1\le k\le T}\sum_{i=1}^{k}(y_i-\overline{y})-\min_{1\le k\le T}\sum_{i=1}^{k}(y_i-\overline{y})\right]$, where $\widehat{\sigma(q)}$ is the long-run variance with bandwidth q. Under the null hypothesis of no long memory, robust to existing short memory, the statistic converges to the Brownian Bridge.

Geweke and Porter-Hudak (1983) proposed a semi-nonparametric test for the long memory process that is know as the GPH test. This test is based on the least square estimate of the difference order $\hat{d}$ in the Periodogram estimation of the spectral density. It can be shown that,

$\hat{d}\sim N\left(d,\pi^2\Big/6\sum_{i=1}^{n_f}(U_j-\overline{U})^2\right)$, where $U_j=\ln\left(4\sin^2(\frac{\omega_j}{2})\right)$ and $n_f=T^{\alpha}, 0<\alpha<1$.

4.2.2 Conditional Heteroscadaticity

Statistical models capable of measuring volatility change includes the well-known conditional heteroscadastic models, i.e. GARCH type model and stochastic volatility models pioneered by Melino and Turnbull (1990), Taylor(1994), Harvey, Ruiz, and Shepard(1994), Jacquier, Polson, and Rossi(1994).

GARCH volatility process can be modeled as follows

$P_t = \mu_t + a_t$, where

Conditional mean $\mu_t = E(p_t \mid F_{t-1})$ might be an ARMA process;

Conditional variance $\sigma_t^2 = Var(p_t \mid F_{t-1}) = E[(p_t - \mu_t)^2 \mid F_{t-1}]$;

Here the innovation $a_t = \sigma_t \varepsilon_t$ and squared innovation a_t^2 is further structured as an ARMA model with MA structure $a_t^2 - \sigma_t^2$ that is a martingale difference. The EGARCH model not only models the logarithm of σ_t^2, but also allows for a leverage effect, i.e the asymmetric response of conditional variance to the negative and positive innovation modeled by introducing a weighted innovation terms. When a fractional ARMA (p, d, q) model is allowed for a_t^2, a Fractionally Integrated GARCH model (FIGARCH) is derived according to Baillie, Bollerslev, and Mikkelsen (BBM, 1996) and Chung (1999). FIGARCH becomes Integrated GARCH (IGARCH) when d=1 to allow for persistence in the conditional variance. It is notable that the FIGARCH/IGARCH model is not covariance stationary, even if it is strictly stationary. Davidson (2001) further developed the Hyperbolic GARCH that nests FIGARCH model.

There is also a stochastic volatility model available, in which $a_t = \sigma_t \varepsilon_t$ is also assumed. A different structure lies in the log of conditional variance that follows an AR structure such that $(1 - \alpha_1 L - \cdots - \alpha_m L^m) \log \sigma_t^2 = \alpha_0 + v_t$, where $\varepsilon_t \sim N(0,1)$, $v_t \sim N(0, \sigma_v^2)$

and independent of ε_t . Compared to the GARCH type model, two independent sources of randomness from both ε_t and v_t exist here. It is important to note that the first-order linear correlation has to be properly accounted for before using these volatility models. This forms a difficulty for studying the price volatility when one has to model a price series that has a complex mean process.

4.2.3 Seasonality and Cycle

There has been a well-known research interest in the seasonality and cycle of agricultural commodity production and price, e.g., cattle cycle and hog cycle. Popular research methodology in this area is the Harmonic Regression, which is built on the Fourier theorem, using a series of fixed period sinosoid and cosinusoid as repressors to estimate the amplitude to analyze the change of the seasonality and cycle pattern.

Alternatively, the UCM (Unobserved Components Model) (Harvey, 1989), also known as STSM (Structural Time Series Model) is also widely applied. It is well known that the UCM can be represented in the SSM (State Space Model), see Harvey (1993), Hamilton (1994), Kim & Nelson (1999), Shumway and Stoffer (2000), Durbin and Koopman (2001), and Chan (2002). The UCM decomposes time series into trend, cycle, seasonal and irregular components. Different from the harmonic regression, the cyclic components in the UCM assumes time-varying amplitude and phase while it retains a fixed period.

Except for research intended for modeling the seasonal and cyclic behavior of the time series, rich seasonal adjustment methods are available to account for seasonality and cycle when the research purpose is to “transform” raw time series into comparable new series: moving average method or ARIMA multiplicative model suggested by Box Jenkins, and Reinsel. Systematic seasonal adjustment methods such as X11 and X12 have also been widely used for this purpose.

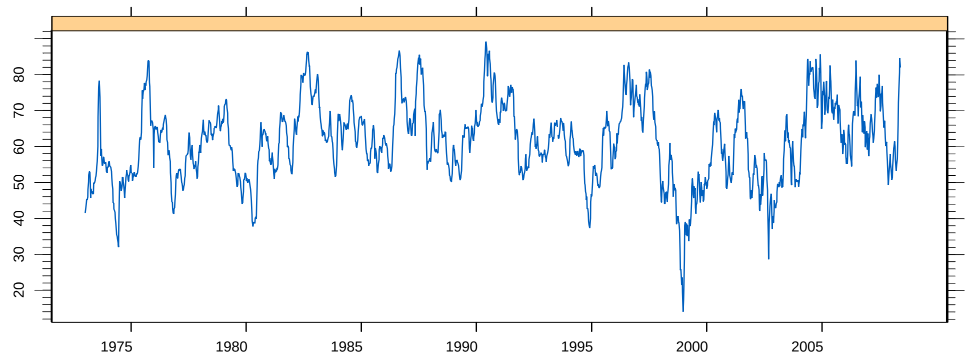
Most of these seasonal adjustment methods are developed for the purpose of removing seasonality and cycles that are of “secondary importance.” Developing a model based on an “adjusted” series to analyze the volatility change needs to properly address the effect of these “adjustments” on the volatility analysis. In contrast, developing seasonal models, e.g., a multiplicative seasonal model that also simultaneously incorporates conditional heteroscadastic effect, seems more appealing, e.g., the most recent Periodic Seasonal Reg-ARFIMA-GARCH model by Koopman, Ooms and Carnero (2007). However, it should be noted that even such models are still one “layer” away from our very first intention to investigate the long memory in the price volatility, as such a task will entail a “Periodic Seasonal Reg-ARFIMA-FIGARCH” model. However, direct analysis of the r_t series not only avoids the seasonal and cyclic pattern of the original price series, but also discloses more information about the volatility evolution with a model of given sophistication.

4.3 Empirical Analysis

To analyze the short term volatility change, relatively higher frequency data often bears more benefit. It is clear that quarterly data is only able to demonstrate the seasonality and cycle but completely suppress all the short term variance. Weekly data is used here due to the unavailability of daily hog price data for the interested historic period.

The data used is the weekly nominal price of carcass-based pork from Jan 1973 to May 2008. The summary statistics for the price series can be found in Table 12 of the Appendix. Figure 9 shows the price series.

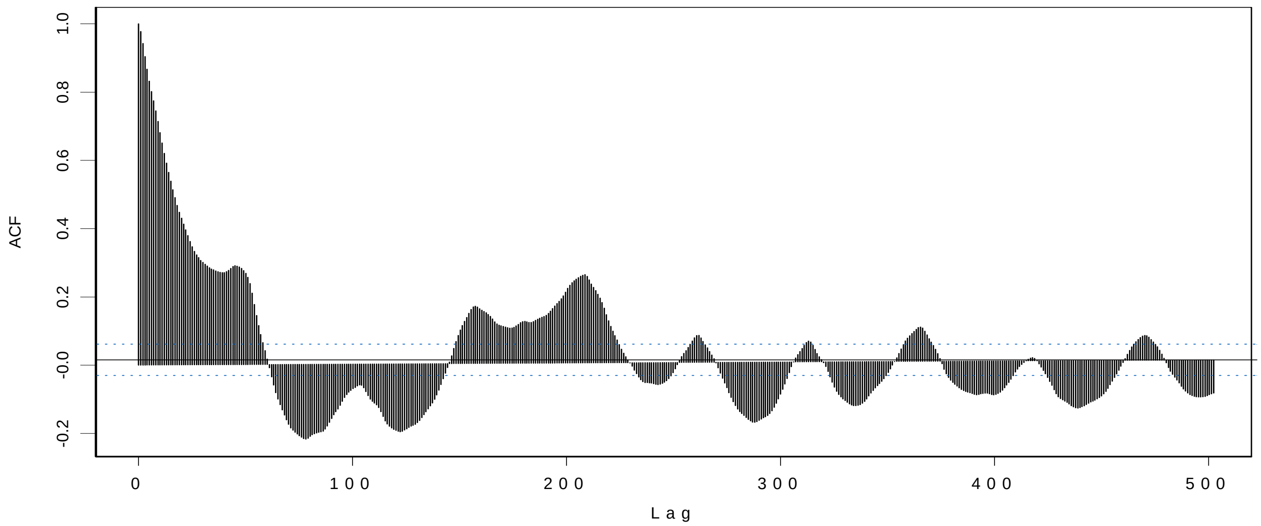
Figure 9 Nominal Weekly Pork Carcass Price Series



It is interesting to note that the nominal price does not experience any substantial increase in its mean over the 35 years span of the data. The price series indicates the well-documented seasonality and cyclic pattern.

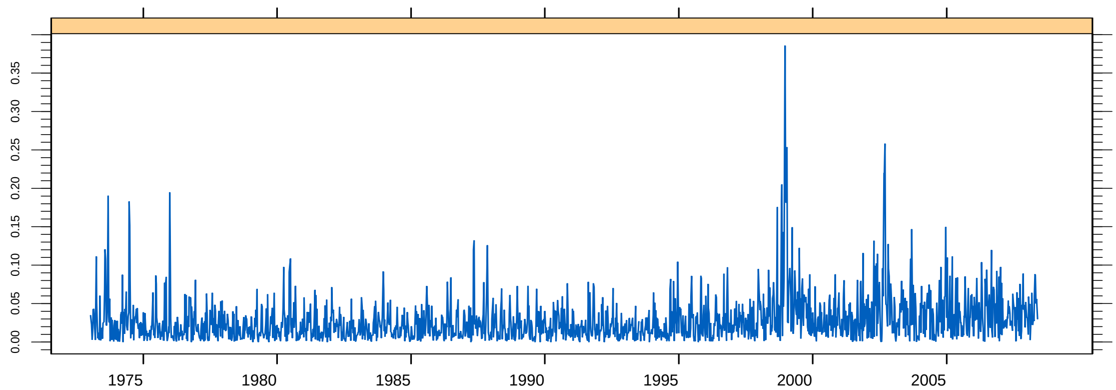
Tentative exploration of the price series shows its ACF is decaying slowly and detects the presence of a cyclic pattern. Without removing seasonality and cycles from the series, this is a well expected phenomenon. It turned out the ARIMA model or FARIMA model can not arrive at a white noise process before removing the cyclic component of the series. Tentative tests also showed no evidence for the existence of a unit root for the series and the differenced series.

Figure 10 Autocorrelation of Weekly Pork Carcass Price Series



As mentioned, direct modeling the price series may not be able to account for the detail of the long memory in the “variance process” of the hog price volatility (defined as absolute return in this essay). The absolute return series r_t is calculated and demonstrated in the following graph. According to the r_t series, it seems U.S. hog price volatility does not experience a large change before the end of the 1990’s, except for a minor spike in the 1970’s, likely due to the oil crises. However the price volatility does increase after the 1990’s. This observation will be tested in the empirical models with a dummy variable for the two periods.

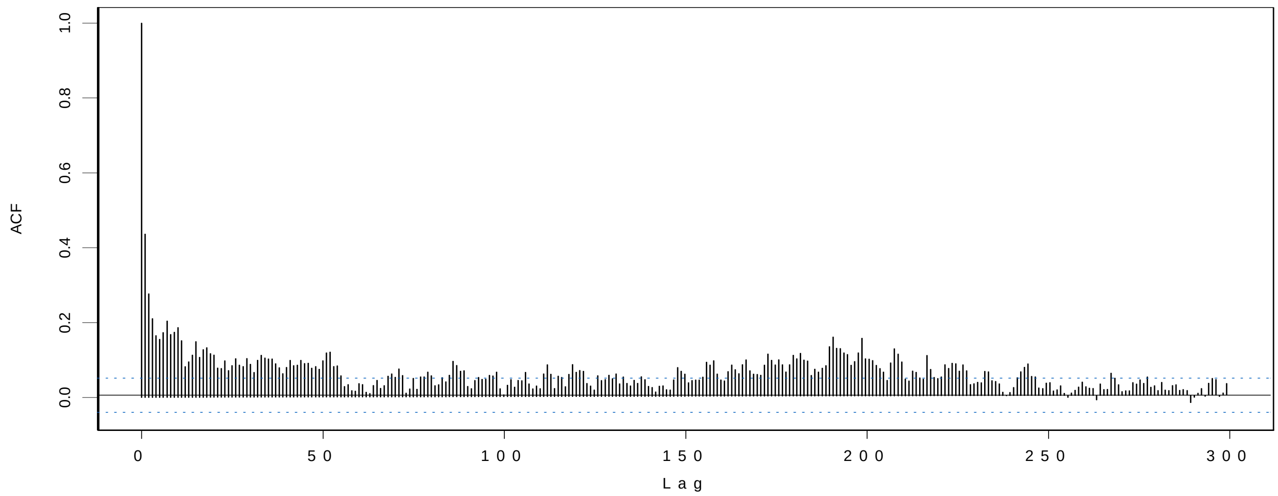
Figure 11 Absolute Return Series Based on Weekly Hog Price



Before analyzing the long memory process in the r_t series, the KPSS test for stationarity against unit root process and long memory process with a specification of a constant deterministic term was carried out. The KPSS test statistic is 3.42, indicating the rejection of the null that the r_t series is I (0) at the 0.01 significance level. An ARIMA or ARFIMA model is therefore appropriate.

The autocorrelations of the r_t series are shown in Figure 12. The figure demonstrates a slowly decaying autocorrelation series. For assurance, the ADF test was also applied to the series and the two sub-series. Using the automatic lag length selection procedure as in Ng and Perron (2001), the ADF test statistics were calculated at -8.078, -10.01 and -4.65 and rejected with P-Values at the scale of 10^{-13}, 10^{-18} and 10^{-4} for the entire series and the two sub-series.

Figure 12 Autocorrelation of Absolute Return Series



These KPSS and ADF tests statistics suggested that an ARFIMA model is appropriate for the r_t series. Because of the long time span, interesting structural change (e.g., increasing EDLP share or shrinking hog cash market) may not be effectively represented in every part of the period. Before estimating any AFRIMA model using the data for the entire period, it is interesting to know if the long memory effect is uniformly observed for the whole period or if it exhibits certain asymmetry from the beginning to the end of the absolute return series due to the uneven timeline of the structure change. It is also interesting to see if the existence of the short-term Oil price surge can impact the absolute return persistently.

Using the GPH test and the modified R/S test, the entire r_t series exhibited a significant long memory effect. The GPH test statistic was estimated to be 2.9682 while the modified R/S test statistic 3.5868. Both tests indicate a 0.01 level of significance of long memory effect. Next, a sub-series from the end of the r_t series with size 800 was formed. Shifting this 800-observation interval backward along the r_t series produced a group of sub-series. These sub-series were also tested for the long memory existence sequentially. The results are found in Table 6.

Table 6 Long Memory process Tests of Absolute Return Series and Sub-series

Modified R/S	GPH	Sub-Series by Observation	Actual Date Spanned
1.9423*	2.3664*	[1047,1847]	[1/23/1993, 5/24/2008]
2.5554**	2.3603*	[847,1647]	[3/25/1989, 7/24/2004]
2.245**	3.0906**	[647,1447]	[5/25/1985, 9/23/2000]
2.0436*	1.449	[547,1347]	[6/25/1983,10/24/1998]
1.1158	-0.0981	[447,1247]	[7/25/1981,11/23/1996]
0.8702	0.4112	[247,1047]	[9/24/1977, 1/23/1993]
1.1742	-0.1363	[47, 847]	[11/24/1973,3/25/1989]

*denotes .05 significance, ** denotes .01 significance

According to Table 6, the long memory effect is significant for the period spanning the last 25 years. The long memory tests' significance levels are not symmetric from the beginning of the series to the end of the series. The middle-end part of the series shows more significant long memory effect.

Since the entire r_t series exhibits an overall long memory effect, the ARFIMA model was first applied to the whole time period. An ARFIMA model was estimated using the ML estimation method by Beran (1995). His framework specifies the ARFIMA models with $d > -1/2$ as: $\phi(L)(1-L)^m[(1-L)^n r_t - \mu] = \theta(L)\varepsilon_t$,

where $-1/2 < m < 1/2$ and $d = m + n$.

Using AIC and BIC criterion, an ARFIMA (1, d, 0) model with mean 0.029 was estimated with all estimates significant as shown in Table 7.

Table 7 A Tentative Long Memory Model of the Realized Absolute Return

Parameter	Estimate/Std.Error
d	0.24**(0.032)
AR(1)	0.11**(0.041)

A residual check of this model showed a strong ARCH effect. The ARCH test (Engel, 1982) statistic $\chi^2(32)$ was estimated 301.56 while the Ljung-Box test (McLeod and Li, 1983) of the squared residual $\chi^2(32)$ statistic returned 640.53. Therefore, considering the conditional heteroscadasticity in the model will improve the model fit. The modified R/S test statistic, at 2.23 and significant at 1% level, indicated the squared residual exhibits a significant long memory effect. The GPH test, however, did not detect the long term dependence. The results of the long memory tests of the squared residual series will be further complemented by simultaneously modeling the long memory in r_t and its variance process.

In view of the fact that long memory effect does not exist uniformly in the entire period from 1973 to 2008, a sub-series from 05/28/1988 to 05/24/2008 is first taken to pursue further analysis. Note that using a series that is 1-4 years longer, as long as there is long memory effect indicated by the previous two long memory tests, basically returns similar results.

As an interesting exploration about the impact of the volatility of the absolute return series on its mean—the GARCH-M effect that allows for the impact of r_t's conditional variance's on r_t is also considered. It seems an AFRIMA-FIGARCH (Baillie, Bollerslev and Mikkelsen (BBM), 1996 and Chung, 1999) model with GARCH-M effect, either BBM or Chung specification might be appropriate. In fact, it turned out that either the BBM FIGARCH or an ARFIMA-IGARCH model returns a significant result. It is known that the conditional variance of an integrated GARCH model follows a straight line and indicates an even stronger persistence than fractional conditional heteroscadastic models (see discussion in Tsay, 2005). In contrast, Chung's AFRIMA-FIGARCH model specification did not return significant estimates on the GARCH parameters that model the short-term linear dependence in the squared residual series when the model's standardized residuals are free of conditional heteroscadastic effect. Forgoing the short term linear dependence in Chuang's model led to the standardized residual that is not free of conditional heteroscadasticity. Therefore the BBM model was used.

The following ARFIMA-FIGARCH BBM model with ARCH-M was estimated,

$(1-L)^d r_t^2 = \mu + a_t + b * dummy + c * \sigma_t^2$, $a_t = \sigma_t \varepsilon_t$, $\varepsilon_t \sim iid.N(0,1)$,

$\phi(L)(1-L)^{d_v} a_t^2 = \mu_v + [1-\theta(L)](a_t^2 - \sigma_t^2)$ and the ARFIMA-IGARCH model was of the same form except $d_v = 1$. Note that both of the two models do not have an ARMA structure in their means because the fractional integration structure was sufficient to model their dependence. Compared to the mean process, the variance process not only needs the long memory structure, but also needs the short term dependence structure. In contrast to the 1988-2008 period, the r_t series in the period 1973-1988 only needs an AR-ARCH model. Either its mean or conditional variance process tested free of long memory effect, while the model still showed significant ARCH-M parameter estimates.

Table 8 was created to show the estimation and residual diagnostic results of the models for the two periods: AR-ARCH models for 01/06/1973-05/21/1988 and ARFIMA-FIGARCH (BBM) models/ARFIMA-IGARCH models for 05/ 28/1988-05/24/2008.[3] Each model has an ARCH-M structure. To highlight the price volatility (defined as the absolute return series r_t) difference in the models for the period 05/ 28/1988-05/24/2008, a dummy for the period of 05/ 28/1988-05/ 23/1998 was applied in

[3] Note that the parameter estimates and t value for the constant term in the variance is multiplied by 104 in Table 8

those models. The ARCH-M effect is also contrasted across the models within each of the two periods. These models are estimated using the G@RCH package 4.2 authored by S. Laurent and J.P. Peters on the Ox console.

Table 8 Long Memory Models of the Absolute Return of Different Periods

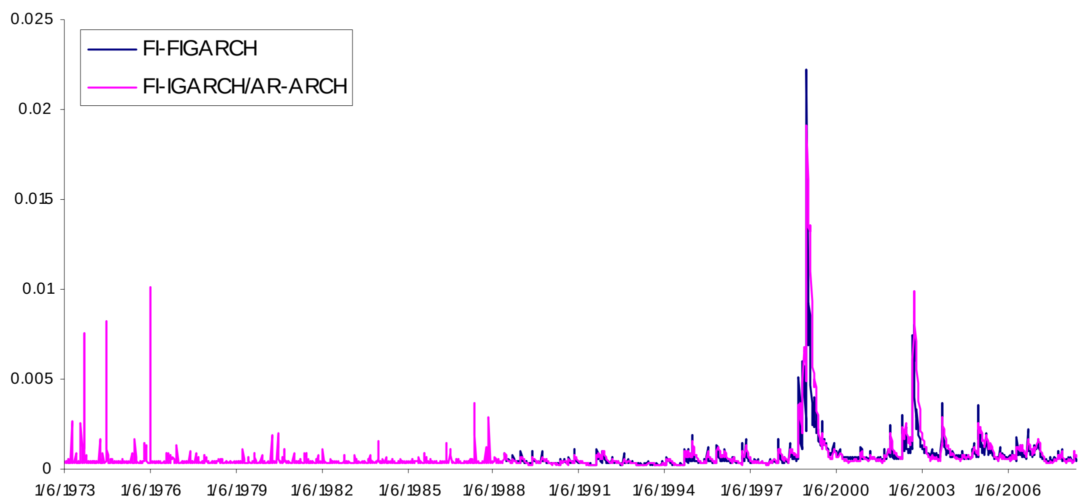
QMLE Estimation

(Robust Standard Error of Sandwich Formula, t-Value in Parenthesis)

Data Series	**05/28/1988-05/24/2008**				**01/06/1973-05/21/1988**	
Parameter\Model	ARFIMA-FIGARCH-BBM Benchmark	ARFIMA-FIGARCH-BBM	ARFIMA-IGARCH Benchmark	ARFIMA-IGARCH	AR-ARCH Benchmark	AR-ARCH
Constant(M)	0.023** (0.0028)	0.019** (0.0019)	0.026** (0.0024)	0.021** (0.0019)	0.021** (0.0013)	0.018** (0.0011)
Dummy (M) for Period 05/ 23/98-05/24/08		0.010** (0.0039)		0.012** (0.0038)		
AR					0.26** (0.036)	0.24** (0.04)
d-Arfima	0.23** (0.024)	0.15** (0.038)	0.21** (0.023)	0.14** (0.032)		
Constant(V)*10^4	0.96** (0.30)	0.99** (0.31)	0.19* (0.081)	0.19* (0.080)	3.34** (0.65)	3.36** (0.66)
ARCH			0.16** (0.37)	0.16** (0.037)	0.32** (0.12)	0.31** (0.12)
d-Figarch	0.26** (0.088)	0.25** (0.070)				
ARCH-in-mean		6.87** (2.61)		5.29** (1.42)		5.52* (2.35)
Diagnostic Statistics (P-Value in Parenthesis)						
Q-Statistics χ2(10) of Standardized Residuals	14.01(0.17)	15.28(0.12)	11.66 (0.31)	16.20 (0.09)	14.54 (0.10)	13.30 (0.15)
Q-Statistics χ2(10)	18.28 (0.050)	18.14 (0.053)	9.09 (0.33)	6.96 (0.54)	3.08 (0.96)	2.82 (0.97)
LM Test F (5, 1033)	0.41 (0.84)	0.30 (0.91)	1.11 (0.35)	0.30 (0.91)	0.15 (0.98)	0.14 (0.98)
LM Test F(10, 1023)	1.82(0.054)	1.82(0.053)	0.82 (0.61)	0.66 (0.76)	0.30 (0.98)	0.27 (0.99)
Squared Standardized Residuals **Tse RBD(10)**	16.05 (0.098)	15.94 (0.10)	9.08 (0.52)	7.64 (0.66)	3.10 (0.98)	2.83 (0.99)
Akaike Information	-4.5837	-4.589	-4.5748	-4.5836	-4.91	-4.92

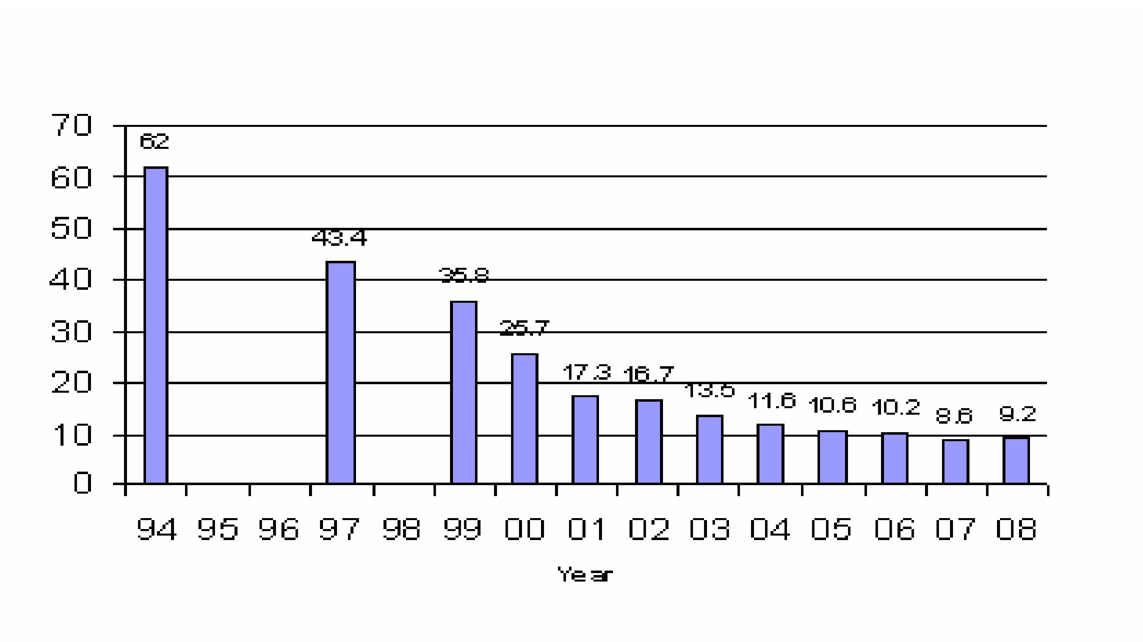
Because the model is the about the absolute return, i.e. the realized and standardized measure of the price volatility, it is affordable to examine the information on the "fierceness" of structural change and abrupt factors that affect the absolute return. The estimated conditional variance of r_t is demonstrated for the ARFIMA-IGARCH/AR-ARCH model and the ARFIMA-FIGARCH-BBM model in the following graph. The estimated volatility of AR-ARCH model for the first period is combined with that of the ARFIMA-IGARCH model for the second period. By the estimated volatility, it is clear that the spikes are found for the periods with oil price shocks and for the period roughly between 1998 and 2005.

Figure 13 Estimated Volatility of the Absolute Return



It is interesting to note that those spikes caused by oil crises are not associated with the long memory effect, while those in the period 1998 to 2005 are associated with significant long memory effect—both in an absolute return's mean process and its variance process. As documented by Grime and Plain (2008), these years also experienced drastic structural change—shrinking of cash hog market (See Figure 14). Such a coincidence suggests that a certain connection between the structural change and above empirical findings might exist. The all-around ARCH-M effect across all the models is also intuitive since more volatile structural changes should lead to higher realized price volatility, i.e. absolute return—the subject under investigation in this paper.

Figure 14 Percent of Hog Sold on Negotiated Market



Grimes and Plain (2008)

4.4 An Ad-hoc Discussion on the Structural-Change Driven Long Memory in the Absolute Return

The following framework provides an interpretation for such empirical findings. Suppose: farm-level demand f is determined by price p and a shifter u such that $f = f(p,u)$; farm-level supply h is determined by price p and a shifter w such that $h = h(p,w)$.

First, suppose price shocks only come from a supply shifter. Equaling the demand change and supply change produces (26) $\frac{dp}{p} e_{QD,p} = \frac{dp}{p} e_{QS,p} + e_{QS,w} \frac{dw}{w}$

where e stands for the elasticity for the noted subscripts. Rearrange (26) derives

(27) $\frac{dp}{p} = e_{QS,w} \frac{dw}{w} \Big/ (e_{QD,p} - e_{QS,p})$.

Second, symmetrically, suppose price shocks only come from the demand shifter, the equation

(28) $\frac{dp}{p} = e_{QD,u} \frac{du}{u} \Big/ (e_{QS,p} - e_{QD,p})$ is obtained.

It is noted that if equation (27) and (28) are combined together to arrive at equation

(29) $\frac{dp}{p} = (e_{QS,w} \frac{dw}{w} - e_{QD,u} \frac{du}{u}) \Big/ (e_{QD,p} - e_{QS,p})$

Writing (29) in discrete form and adding subscript leads to

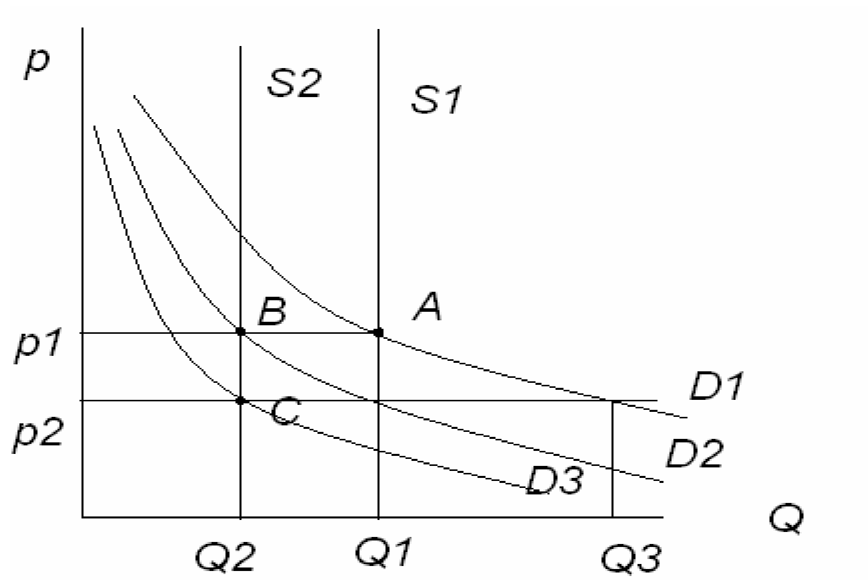
(30) $\frac{\Delta p}{p}|_t = (e_{QS,w}|_t \frac{\Delta w}{w}|_t - e_{QD,u}|_t \frac{\Delta u}{u}|_t) \Big/ (e_{QD,p}|_t - e_{QS,p}|_t)$

Because livestock supply is very inelastic in the short term, assume the supply curve is perfectly inelastic such that

(31) $\frac{\Delta p}{p}|_t = (e_{QS,w}|_t \frac{\Delta w}{w}|_t - e_{QD,u}|_t \frac{\Delta u}{u}|_t) \Big/ e_{QD,p}|_t$

To motivate the assumptions to be made for the explanation of the long memory effect in the absolute return series, I first investigate what happens if the buyers and sellers are exiting at the same rate. Suppose $QD_t = \alpha_t F(p,u)$ and $QS_t = \beta_t G(p,w)$ where $F(.)$ and $G(.)$ are the original demand/supply function, it can be observed that the price remains unchanged and the clear-out quantity decreases at the same rate if $\alpha_t = \beta_t$. Under this situation, $e_{QD,p}|_t$ remains unchanged because the slope and the clear-out quantity change at the same rate of $\alpha_t = \beta_t$ from period to period. If a random shock $\frac{\Delta w}{w}|_t$ from the supply side and $\frac{\Delta u}{u}|_t$ the demand side are uncorrelated from each other and from period to period, the absolute return series will simply be a white noise process. In Figure 11, the point "B" is the new equilibrium position when $\alpha_t = Q_2/Q_1$.

Figure 15 Exiting Demand and Supply



The above scenario contradicts the empirical finding that the absolute return is not a white noise process. Now suppose $\alpha_t \neq \beta_t$? When $\alpha_t < \beta_t$, the resultant price (point "C" in Figure 15) level is lower compared with the previous period because exiting demand outweighs exiting supply in the spot market.

Particularly, $e_{QD,p}|_t$ must shrink in absolute value *for a straight-line demand curve*.

Similarly, when $\alpha_t > \beta_t$, the opposite outcome happens. Since the price declined and long memory effect in the absolute return is observed, $\alpha_t < \beta_t$ seems more plausible.

It is noted that such a claim that $\alpha_t < \beta_t$ might be true, not only because actual hog price is declining, but also because the monotonic change of $|e_{QD,p}|_t|$ explains the long memory effect found in the absolute return and the increased absolute return's mean for the period 05/ 23/98-05/24/08 compared with the period from 1988-1998.

For the long memory effect, in equation (31), $e_{QS,w}|_t \frac{\Delta w}{w}|_t - e_{QD,u}|_t \frac{\Delta u}{u}|_t$ is stable because it reflects the short term shock of demand/supply curve. Denote it as b_t and assume it is I.I.D so that its absolute value is a white noise process. Denote $1/|e_{QD,p}|_t|$ as z_t, since $|e_{QD,p}|_t|$ is decreasing, assume z_t is a geometric series such as $x_0, kx_0, k^2x_0, k^3x_0 \cdots$ where $k > 1$ is constant, then the autocorrelation of resulting power series $|b_t|z_t$ decays very slow if $|b_t|$ has small variance relative to z_t.

Increased absolute return probably occurs because the EDLP retailer driven hog market restructure manifested itself during the period 05/ 23/98-05/24/08, so that decreasing $|e_{QD,p}|_t|$ leads to a high absolute return level. This increasingly higher absolute return might be better captured in a more sophisticated structure rather than a simple dummy.

Above framework only focus on the supply shock because only the change of demand curve slope is considered while the supply curve is assumed to be perfectly inelastic. However, the emphasis on the supply shock and correspondingly the demand curve slope change can only be justified if the supply shock is of a larger magnitude compared to the demand shock.

It is noted that a symmetric argument about the demand shock can also be made. According to USDA NASS, there is a sharp drop of the number of hog farms between since 1984. In 2008, 2.9% of the operations marketing 60% of the hogs and 12.2% of the operations marketed 60% of the hogs in 1995. Accordingly, pigs per litter increased from 8.4 to 9.48 from 1995 to 2008. If such supply side structural change is formed into the above analysis framework, a more inelastic supply curve results so that a similar argument for the long memory effect due to the demand shock is also viable.

4.5 Conclusion

The empirical results largely confirm the following 4 facts: long memory effect was manifested only after 1980’s, both in the mean process and the variance process of the absolute return; when the conditional variance of the r_t series was observed to be high, the value of r_t was also high for any period since ARCH-M effect turned out statistically significant for any one of the estimated models; the Price volatility level for period 05/28/1998-05/24/2008 was higher than the prior period; The price volatility defined as the absolute return has long term persistence in the recent 25 years and seems more persistent for the period after the mid-1990s.

While these empirical results do not tell if the change in price volatility is due to the production organization change (smaller number of hog farms or shrinking negotiated hog transaction), retailer pricing strategy shift (EDLP market share increase) or increased processor capacity utilization, it was shown by a supply demand determination framework that there was possibly a connection between the documented structural changes and the empirical findings including the increased mean and long term dependence in the realized price volatility, the absolute return.

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Appendices

A. Endnotes

[1] While the period of length of the fixed price strategy is unknown, casual observation suggests meat prices at least remain fixed across seasonal production cycles within the livestock industry, i.e., price does not adjust seasonally. It has to be noted that price will adjust in the long run in response to other economic forces, such as production cost, marketing cost etc. Every-Day-Low-Price cannot hold in the long run. The occasion on which we are looking into these two strategies is the "short run" in which a fixed price in EDLP channel holds.

[2] Product price elasticity has an impact on whether a product is better suited for EDLP or Hi-Lo pricing strategy. A product with a large, in absolute value, price elasticity is better suited for a Hi-Lo pricing strategy. For example, Lusk et al. (2001) report a larger, in absolute value, elasticity for Select beef than for Choice beef. A retailer using a mixed pricing strategy may use EDLP for higher value beef cuts, like Choice steak, and use the Hi-Lo pricing strategy for lower value beef cuts, like Roast.

[3] According to Rhodes, Dauve, and Parcell (2007), the processor is referred to as a margin taker and not as a margin maker. Because of the farm-level market influence of large scale processor, it is possible for them to earn a target margin if the kinked demand situation is true. Even if kinked demand does not exist and the processor's margin is completely variable in both ends, either one of the variable margin values for retailers, i.e. R and r can be regarded as the total margin claimable by both the retailer and the part of

business line of the representative meat processor that is supplying this retailer. In other words, the retail margin will be the margin for a particular part of meat chain consisting of one retailer and part of the representative processor. The entire framework about the two retailers' margin can be applied to that of these two particular sub-meat-chains.

[4]Strictly speaking, conditional coefficient of variation of variable x_t should be defined as $CCV(x_{t+1} | F_t) = Var(x_{t+1} | F_t) / E(x_{t+1} | F_t)$ that is equal to $Var(x_{t+1} | F_t) / x_t$ only when x_t is a martingale. However this definition is well geared to exclude the effect of seasonality in meat prices.

[5] The result is in the Appendix Table 9

B. Extra Tables and Figures

Table 9 Definition of Variables Used in Deriving the Model of First Essay

Variable	Definition
c	Total cost of representative processor
$\bar{x}$	Fixed quantity sold in Every-Day-Low-Price channel
y	Quantity sold in Hi-Lo channel
$\overline{p_b}$	Fixed price of other inputs
p_a	Producer price of agriculture product
p_y	Retail price in Hi-Lo channel
J	Demand for Hi-Lo channel
h	Supply of agriculture product
N	Shifting factor of demand for Hi-Lo channel
W	Shifting factor of agriculture product supply
r	Variable margin of Every-Day-Low-Price channel
R	Variable margin of Hi-Lo channel
m_0	Fixed margin of Representative Processor
e_{hp_a}	Supply elasticity of agricultural product
e_{ddp_a}	Derived demand elasticity of agricultural product
e_{MC,p_a}	Elasticity of processor's marginal cost w.r.t p_a

Table 10 Wright's Ranks and Signs Variance Ratio Tests of the Lean Hog Futures

Test for Individual Holding Period (day) and The Joint Test							
Nearby Contract	30	40	50	60	80	100	Joint Test by Kim
R1	2.6**	2.0479*	1.6995*	1.3816	1.1792	1.1781	2.6*
R2	2.0804*	1.4397	1.0664	0.6542	0.3535	0.3498	2.0804
S1	1.7551*	1.447	1.3354	1.3352	1.4788	1.4961	1.7551
Dec Contract	30	40	50	60	80	100	Joint Test
R1	0.2421	0.3319	0.5122	0.5635	0.4908	0.6107	0.6107
R2	0.7578	0.7514	0.8919	0.9027	0.7574	0.8507	0.9027
S1	0.4624	0.5611	0.6145	0.6689	0.6907	0.8258	0.8258
95% Critical Values	30	40	50	60	80	100	Joint Test
R1	1.6108	1.5533	1.6037	1.5676	1.5205	1.4276	2.1623
R2	1.5783	1.6021	1.5985	1.5812	1.5987	1.4973	2.1092
S1	1.5112	1.5441	1.5148	1.4621	1.5988	1.5893	2.0851

* indicates .05 significance and ** indicates .01 significance

Table 11 Summary Statistics of the Simulated Return Data of Futures and Boston Butt

Variable	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Covariance Matrix	
Cash Return	-0.95949	-0.00413	0.06368	0.074874	0.135452	13.34918	0.043751	0.046803
Futures Return	-0.95547	-0.00381	0.138758	0.168614	0.306903	14.12286	0.046803	0.089797

Table 12 Summary Statistics of the Pork Carcass Weekly Price Series 1973-2008

Min	1st Qu	Mean	Median	3rd Qu	Max	Total N	Std Dev
14.18919	53.78378	61.34782	61.72738	68.24324	89.27027	1847	10.99701

Figure 16 MCV Hedge Ratio/Covariance, Volatility, and Correlation for Cutout V.S. Lean Hog Futures

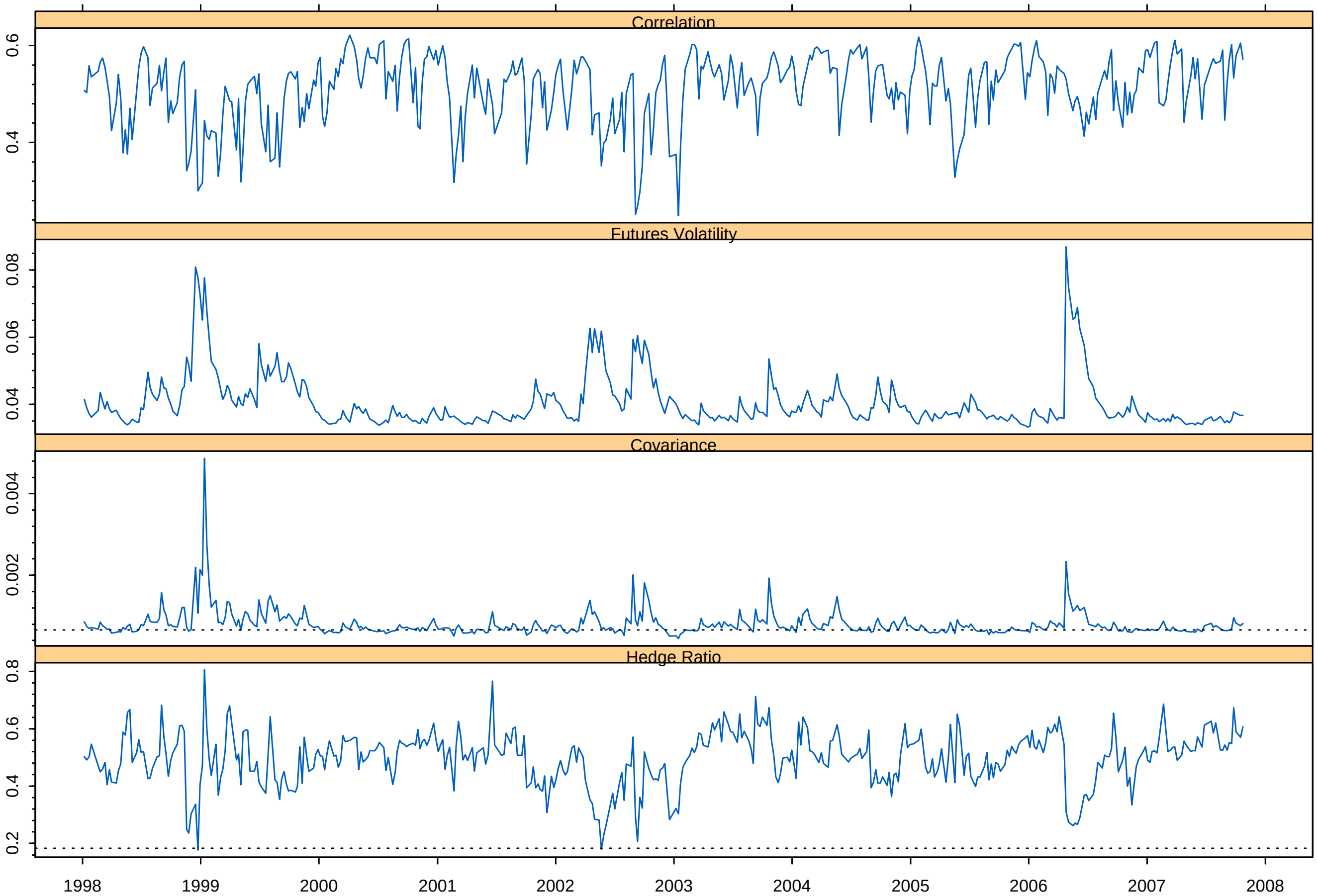
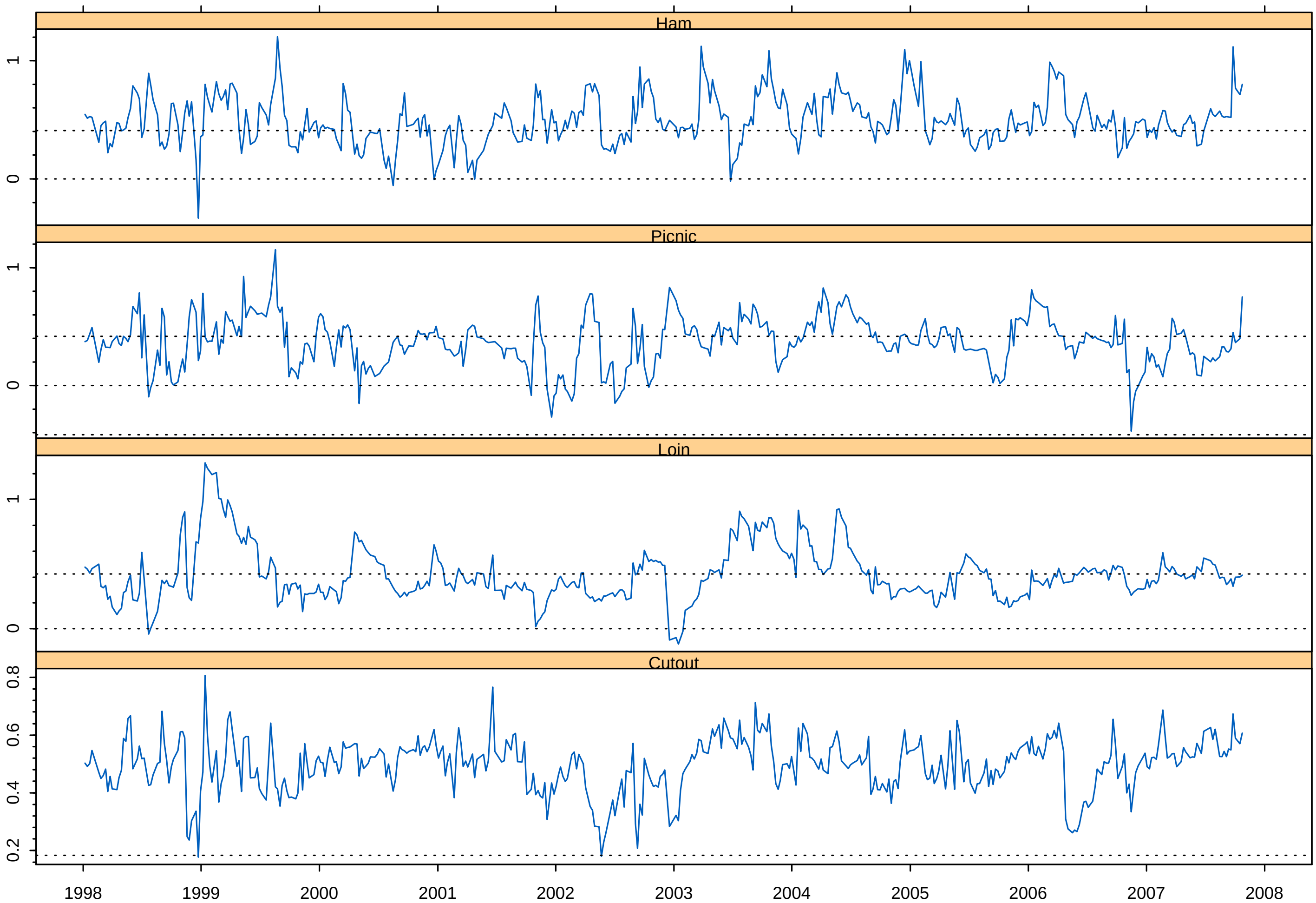


Figure 17 MCV Hedge Ratios for Pork Products



VITA

Yiyong Yuan earned a B.S in Horticulture (1999) and a M.S in Agricultural Economics and Management (2003) from China. He came to the University of Missouri in 2004 to study for a PhD in Agricultural economics. He also obtained a M.A of statistics at MIZZOU in 2007. He is interested in commodity/futures market, price and risk with a focus on empirical and statistical methods.