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Essays in macroeconomics and labor markets

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ESSAYS IN MACROECONOMICS AND LABOR MARKETS

by

Lawrence F. Warren

A thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Economics
in the Graduate College of
The University of Iowa

August 2016

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CERTIFICATE OF APPROVAL

PH.D. THESIS

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ABSTRACT

This dissertation contributes to the current understanding of labor markets, focusing on the use of micro level data and computational modeling to study the interaction of unemployment with various aspects of the macroeconomy. I address the fact that frictions in the labor market carry over into other dimensions of firms' and workers' decisions, such as a firm's incentive to utilize its current labor force, workers' participation in the labor market, and the decision to acquire or discharge debt.

In Chapter 1, I study involuntary part-time employment over the business cycle. I document that the population at work part-time for economic reasons (*PTE*) is countercyclical, volatile, and transitory. Workers in *PTE* are nearly three times more likely than the unemployed to return to full-time work in a given month, and seven times more likely than full-time workers to become unemployed. Using household survey data, I demonstrate that cyclical fluctuations in *PTE* come from changes in the transition rates between full-time and part-time employment rather than between part-time and unemployment. Moreover, these movements are primarily due to within-job changes in hours. Accordingly, I model part-time work focusing on a firm's decision to hire, fire, or partially utilize its labor force. Firms in the model are heterogeneous in size and productivity, and are subject to search frictions. The model produces firm-level utilization of part-time employment which is consistent with observed worker flows, and varies across the size and age distributions of firms. Over the

business cycle, the model matches the observed relative volatility of unemployment and *PTE*. Part-time labor utilization by firms increases the volatility of vacancies and unemployment in the model relative to the case with only an extensive margin.

Chapter 2 studies the interaction of a participation margin in a labor market search model. Introducing a participation margin of whether or not to actively search for a job requires the use of large idiosyncratic shocks to workers' participation incentives in order to match monthly labor flows in the data. If we measure the participation transitions of workers outside of employment where search decisions are observable and apply this same transition process to employed workers, any search model will overstate the transition of workers out of employment to nonparticipation. Allowing the participation transition of workers to depend on their employment state fixes these flows, but this transition process is unobservable for employed workers. Taking advantage of the longer panel of the 1996 Survey of Income and Program Participants, I estimate the markov process for participation transitions of employed workers using their observed search behavior before and after an employment spell. The difference in the transition process measured for employed and nonemployed workers is consistent with an interpretation of attachment to the labor force. I build a directed search model with a labor force participation margin subject to employment-dependent shocks and show that it can match the labor market flows in US data.

Chapter 3, which is jointly authored with Chander S. Kochar, investigates the effects of student loans on labor market outcomes. The student loan market is the second largest source of household debt in the United States, with \$1.2 trillion

in outstanding debt. Unlike other sources of unsecured credit, student loans cannot be discharged in bankruptcy. Using data on college graduates from the 1993/03 Baccalaureate and Beyond Longitudinal Study, we first identify that student loan debt has a significant negative effect on students' earnings after graduation. We show that the inability to discharge debt in bankruptcy is critical to produce this result within a simple search theoretic framework. We propose a richer model with student loan debt and a delinquency/default decision to study the effects of recent changes to student loan policies on the labor market and delinquency outcomes of college graduates.

PUBLIC ABSTRACT

This dissertation contributes to the current understanding of labor markets, focusing on the use of micro level data and computational modeling to study the interaction of search frictions and unemployment with various aspects of the macroeconomy. I address the fact that frictions in the labor market carry over into other dimensions of firms' and workers' decisions, such as a firm's incentive to utilize its current labor force, workers' participation in the labor market, and the decision to acquire or discharge debt.

In Chapter 1, I study involuntary part-time employment over the business cycle. In addition to unemployment, workers may be underutilized through being employed part-time when they would desire full-time work. I document that the population at work part-time for economic reasons (*PTE*) is countercyclical and fluctuates with the business cycle with similar volatility to the unemployment rate. The popular narrative of the involuntary part-time worker involves the recently unemployed who, for lack of better alternatives, has settled on part-time work. However, I show using data from the Current population survey and Survey of Income and Program Participants that the cyclical fluctuation in the population of part-time employed for economic reasons comes from the movement of workers between full-time employment and part-time rather than between part-time and unemployment. Not only is the fluctuation in *PTE* from within-employment, but it is largely due to within-job changes in hours.

Since fluctuations in *PTE* are driven by changes in the hours demanded of existing employees rather than changes in workers' job search or acceptance behavior, I model part-time work focusing on a firm's decision to hire, fire, or partially utilize its labor force in the face of firm-level and aggregate shocks. This requires a search model where firms are heterogeneous in size and productivity. The model produces part-time labor movements which are consistent with the evidence from worker flows in the CPS and match the relative volatility of unemployment and part-time for economic reasons over the business cycle. Part-time labor utilization by firms increases the volatility in vacancies and unemployment in the model relative to the model with only an extensive margin. Due to heterogeneity in firm size and productivity, the model has implications for the size and age distribution of firms and the growth rates of hours and employment.

Chapter 2 studies the interaction of a participation margin in a labor market search model. Introducing a participation margin where workers decide whether or not to actively search for a job requires the use of large idiosyncratic shocks to workers' participation incentives to match observed monthly labor flows. If we measure the participation transitions of workers outside of employment where search decisions are observable and apply this same transition process to employed workers, any search model will overstate the transition of workers out of employment to nonparticipation. Allowing the participation transition of workers to depend on their employment state fixes these flows, but this transition process is unobservable for employed workers. Taking advantage of the longer panel of the 1996 Survey of Income and Program

Participants, I estimate the markov process for participation transitions of employed workers using their observed search behavior before and after an employment spell. The difference in the transition process measured for employed and nonemployed workers is consistent with an interpretation of attachment to the labor force. I build a directed search model with a labor force participation margin subject to employment-dependent shocks and show that it can match the labor market flows in US data.

Chapter 3, which is jointly authored with Chander S. Kochar, investigates the effects of student loans on labor market outcomes. The student loan market is the second largest source of household debt in the United States, with \$1.2 trillion in outstanding debt. Unlike other sources of unsecured credit, student loans cannot be discharged in bankruptcy. Using data on college graduates from the 1993/03 Baccalaureate and Beyond Longitudinal Study, we first identify that student loan debt has a significant negative effect on students' earnings after graduation. We show that the inability to discharge debt in bankruptcy is critical to produce this result within a simple search theoretic framework. We propose a richer model with student loan debt and a delinquency/default decision to study the effects of recent changes to student loan policies on the labor market and delinquency outcomes of college graduates.

TABLE OF CONTENTS

LIST OF TABLES	xii
LIST OF FIGURES	xv
CHAPTER	
1 PART-TIME EMPLOYMENT AND FIRM-LEVEL LABOR DEMAND OVER THE BUSINESS CYCLE	1
1.1 Introduction	1
1.2 Part-time Employment in Household Survey Data	6
1.2.1 Definitions	7
1.2.2 PTE is Countercyclical and Volatile	9
1.2.3 PTE is Not Persistent	11
1.2.4 PTE Status Fluctuates due to Within-job Movements	12
1.3 A Model of Part-time Employment	17
1.3.1 Firms	19
1.3.2 Matching and Contracts	20
1.3.3 The Worker's Problem	21
1.3.4 The Firm's Problem at Age 1	23
1.3.5 Planner's Problem	25
1.4 Dynamics of the Firm	29
1.5 Quantitative Exercises	34
1.5.1 Calibration Overview	34
1.5.2 Production	34
1.5.3 Matching Function and Vacancies	35
1.5.4 Home Production and Part-time	36
1.5.5 Permanent Firm Types and the Size/Age Distribution of Firms	36
1.5.6 Idiosyncratic Shocks	37
1.5.7 Aggregate Shocks	38
1.6 Results	39
1.6.1 Recruitment, Part-time, and Separations by Employment Growth of Firms	39
1.6.2 Part-time Employment by Firm Age, Size, and Productivity	41
1.6.3 Business Cycle Properties	43
1.7 Discussion	46
1.7.1 Ongoing Work with US Census Bureau Data	46
1.7.2 Conclusion	48

2	MATCHING LABOR FLOWS IN SEARCH MODELS WITH LABOR FORCE PARTICIPATION	62
2.1	Introduction	62
2.2	A Search Model	68
2.2.1	Environment	69
2.2.2	Worker's Problem	70
2.2.3	Flows	72
2.3	Matching Flows in the Data	74
2.3.1	Fitting the Model to Data	74
2.4	Transitions from SIPP Data: Estimating χ and δ	77
2.5	Robustness	84
2.5.1	Alternative Specifications of Participation	84
2.6	Business Cycle Implications	88
2.7	Discussion: Attachment and Discouragement	90
2.8	Conclusion	92
3	STUDENT LOAN DEBT AND LABOR MARKET OUTCOMES	94
3.1	Introduction	94
3.2	Institutional Details	100
3.2.1	Subsidized and Unsubsidized Loans	101
3.2.2	Repayment, Delinquency and Default	108
3.3	Data	110
3.3.1	Sample Selection	111
3.3.2	Descriptive Statistics	113
3.4	Empirical Framework	114
3.4.1	Evaluating the RK identifying assumptions	117
3.4.2	Estimation Strategy and Results	123
3.4.3	Discussion	128
3.4.3.1	Individuals in graduate school in 1994	129
3.4.3.2	Time to first job & number of interviews	133
3.4.3.3	Earnings in dollars amounts	134
3.4.3.4	Impact of debt on later years	137
3.5	Model	140
3.5.1	Environment	140
3.5.2	Non-dischargeable Loans	142
3.5.3	Dischargeable Loans	146
3.6	Conclusion	148

APPENDIX

A	APPENDIX TO CHAPTER 1	154
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A.1	Appendix: Decentralized Problem	154
A.1.1	Firm Entry	155
A.1.2	Competitive Equilibrium	155
A.1.3	Calibrated Firm Size and Age Distribution	156
A.2	Appendix: Proofs	157
A.3	Appendix: Data	167
A.3.1	Constructing the CPS sample and flow data	167
A.3.2	PTE workers by Origin	171
A.3.3	Cyclical characteristics of all flows	171
A.3.4	Distinguishing Slack Work and Failed Full-time Searchers	172
A.3.5	Aggregate Hours Fluctuations and Part-time for Economic Reasons	176
A.3.6	The Survey of Income and Program Participants	178
B	APPENDIX TO CHAPTER 2	185
B.1	Transitions from SIPP Data	185
B.2	Probability of Dropping out of Unemployment	185
C	APPENDIX TO CHAPTER 3	189
C.1	Robustness Checks	189
C.1.1	Including Independent Students	189
C.1.2	Standard Checks	190
C.2	Proof for Proposition 3	190
	REFERENCES	204

LIST OF TABLES

Table

1.1	Cyclical Properties of Labor Market Stocks	10
1.2	US Average Monthly Flow Probabilities	11
1.3	Cyclical Properties of Flows between <i>FT</i> and <i>PTE</i>	14
1.4	Parameters for Matching the Size and Age Distribution of Firms	37
1.5	Parameters in Benchmark Calibration	39
1.6	Part-time Percentage by Firm Age and Size	42
1.7	Business Cycle Statistics	44
2.1	Flow Breakdown	72
2.2	US Monthly Flow Data	74
2.3	Data	76
2.4	Calibration	76
2.5	SIPP Flows, '96-'99	78
2.6	Data	85
2.7	Using $\widehat{\chi_{Emp}}$ and χ_{Non}	85
2.8	Benchmark Calibration from Krusell et. al, 2009	86
2.9	Flow Breakdown	87
2.10	Simulated Flows: Expansions	88
2.11	Simulated Flows: Recessions	88
2.12	Simulated Flows with Procyclical <i>UN</i> : Expansions	90

2.13	Simulated Flows with Procyclical <i>UN</i> : Recessions	90
3.1	Characteristics of Respondents by Stafford Loan Eligibility	115
3.2	Relationship between <i>need</i> and Predetermined Characteristics	121
3.3	Impact of Stafford Loans on 1994 Earnings	127
3.4	Characteristics of Respondents by 1994 Graduate School Enrollment . . .	131
3.5	Characteristics of 1994 Graduate School Enrollers by Stafford Loan Eligibility	132
3.6	Months to First Job	134
3.7	Number of Jobs Interviewed for upon Completion of Degree	136
3.8	Impact of Stafford Loans on 1994 Earnings	151
3.9	Impact of Stafford Loans on 1996 Earnings	152
3.10	Impact of Stafford loans on 2002 Earnings	153
A.1	Firm and Employment Shares by Size and Age	157
A.2	CPS Variables Used In Producing Labor Flows	168
A.3	Values for PRWKSTAT	170
A.4	Cyclical Properties of PTE outflows	171
A.5	Cyclical Properties of U outflows	172
A.6	Cyclical Properties of Labor Market Statistics	173
A.7	US Average Monthly Flow Probabilities: CPS 1994:1-2014:12	174
B.1	Comparing Separation Rates	185
B.2	Duration Dependence	187
C.1	Relationship between <i>need</i> and Predetermined Characteristics: Dependent and Independent Students	194

C.2	Impact of Stafford Loan Eligibility on Stafford Loans Borrowed: Dependent and Independent Students	195
C.3	Impact of Stafford Loans on 1994 Earnings : Dependent and Independent Students	196
C.4	Relationship between <i>need</i> and Predetermined Characteristics: Linear . .	197
C.5	Impact of Stafford Loan Eligibility on Stafford Loans Borrowed: Linear .	198
C.6	Impact of Stafford Loans on 1994 Earnings : Linear	199
C.7	Impact of Stafford Loan Eligibility on Stafford Loans Borrowed: Excluding Exogenous Regressors	200
C.8	Impact of Stafford Loans on 1994 Earnings : Excluding Exogenous Regressors	201
C.9	Robustness Check: Varying Bandwidth	202

LIST OF FIGURES

Figure

1.1	Part-time Employment's Share of Aggregate Employment by Reason . .	49
1.2	Monthly Probability of Exiting Full-time Employment	50
1.3	Monthly Transition Probabilities out of Unemployment	51
1.4	<i>FT</i> – <i>PTE</i> Transition Probability by Job Transition Status	52
1.5	<i>PTE</i> – <i>FT</i> Transition Probability by Job Transition Status	53
1.6	<i>FT</i> – <i>PTE</i> Transition Probability by Job Transition Status	54
1.7	<i>PTE</i> – <i>FT</i> Transition Probability by Job Transition Status	55
1.8	Vacancies and Hiring Policies over the Life of a Firm	56
1.9	Part-time and Separation Rates in Response to a Productivity Shock . .	57
1.10	Dynamics of a Simulated Firm	58
1.11	Distribution of Firms by Age (in years): Firm Shares and Employment Shares	59
1.12	Vacancy Rates and Hiring Rates of Firms	60
1.13	Layoffs, Part-time, and Hours Growth	61
2.1	Separations of Workers by Initial Type Before Employment Spell	83
2.2	Separations of Workers by Type After Employment Spell	84
3.1	Distribution of Subsidized and Unsubsidized Loans by Dependency Status	105
3.2	Empirical Distribution of Stafford Loans in the Last Year of Schooling as a Function of <i>need</i>	107
3.3	Empirical Distribution of Cumulative Stafford Loans as a Function of <i>need</i>	116

3.4	Distribution of <i>need</i> and the Different Components of <i>need</i>	120
3.5	The Distribution of Baseline Characteristics	122
3.6	The Reduced Form Impact of <i>need</i> on log of 1994 Annual Earnings . . .	123
3.7	Probability of Attending Graduate School in 1994	133
3.8	The Reduced Form Impact of <i>need</i> on 1994 Annual Earnings	135
3.9	The Reduced Form Impact of <i>need</i> on 1996 Annual Earnings	138
3.10	The Reduced Form Impact of <i>need</i> on 2002 Annual Earnings	139
3.11	Model with Non-dischargeable Debt	145
3.12	Model with Non-dischargeable debt	148
A.1	Shares of PTE stock by Origin	179
A.2	Part-time Employment for Economic Reasons: Share of Aggregate Em- ployment	180
A.3	Sum of Net Hours Loss by Workers in Each Flow	181
A.4	Total hours worked by individuals classified in <i>PTE</i>	182
A.5	Sum of Net Hours Gain/Loss by Workers in <i>PTE</i> – <i>FT</i> and <i>FT</i> – <i>PTE</i> Flows	183
A.6	Sum of Net Hours Loss by Workers in Each Flow	184
C.1	Sample Consisting of Dependent and Independent Students	203

CHAPTER 1

PART-TIME EMPLOYMENT AND FIRM-LEVEL LABOR DEMAND OVER THE BUSINESS CYCLE

1.1 Introduction

Part-time employment for economic reasons (*PTE*) has recently attracted attention from policy-makers as an indicator of weakness in the aggregate labor market due to its unprecedented size during the last recession. *PTE* reached 7% of employment at its peak, affecting roughly 9 million workers in the US. Like the unemployment rate, the share of employment in *PTE* is volatile and countercyclical. Our understanding of part-time labor and its relation to the aggregate economy, however, is lacking relative to our knowledge of unemployment. This paper focuses on examining the changes in the population of *PTE* over the business cycle and developing a search model of involuntary part-time work.

I begin by using data from the Current Population Survey to document three important facts about *PTE*: first, more full-time workers become part-time employed for economic reasons each month than become unemployed, and this inflow is more volatile than is the flow of full-time workers to unemployment. During the last recession and subsequent recovery, the decrease in aggregate hours caused by individuals moving from full-time to *PTE* is on average 77% of the loss in hours caused by full-time workers separating to unemployment. Second, *PTE* is not a persistent state, and is characterized by high flow probabilities to full-time employment and unemployment. Workers in *PTE* are three times as likely as the unemployed to return

to full-time work, and about seven times as likely as full-time workers to separate to unemployment. Lastly, *PTE* fluctuates due to within-job changes in hours rather than to changes in unemployment status.¹ Using the dependent interview structure of the CPS as in Fallick and Fleischman (2004) to look at job transitions, I find that 82% of the transitions of employed workers into or out of *PTE* are due to within-job changes in hours.

I build upon the framework of Kaas and Kircher (2015) on the basis of these facts to model firm-level labor demand with a part-time employment margin in a frictional labor market. This focus on firm-level demand is appropriate because the movement of workers with respect to *PTE* is primarily within-job. Firms are heterogeneous in productivity, and firm size is determined through a decreasing returns to scale production technology in labor. Firms expand and contract their respective workforces in response to idiosyncratic and aggregate shocks, while facing frictions due to costly vacancy postings and competitive search. Firms can use part-time labor when facing a negative productivity shock in order to lower wage costs temporarily and reduce layoffs, avoiding the costly recruitment necessitated by future hiring when productivity increases.

Given that the facts outlined in CPS data concern worker flows and the model

¹This result is also found in work by Canon et al. (2014), who use a methodology similar to Shimer (2012) to show that within-employment flows contribute more to the rise in *PTE* in the last recession than the flow of workers to or from unemployment. In addition to this result, they document the differences in wages, occupations, and demographic characteristics of workers in full-time and part-time work by reason and link these properties of workers in *PTE* to the job-polarization literature. I use the dependent interview structure of the CPS to show that the majority of the fluctuations in *PTE* are due to within-job changes rather than from job-to-job transitions.

is about firms, I focus on the ability of the model to reproduce the qualitative features of *PTE* observed in the CPS data. Part-time utilization within firms is volatile in the model, implying that the flows of workers between full-time and part-time employment are large relative to the movement of workers between full-time employment and unemployment. This matches the first fact I outlined previously: workers in full-time employment are more likely to move to part-time than to separate to unemployment. The volatility also reflects the finding that workers in part-time employment have a high probability of returning to full-time work. Firms use part-time labor in conjunction with separation, so that workers placed in *PTE* also experience higher separation rates than do full-time workers. The high respective probabilities of either returning to full-time or separating to unemployment are consistent with the lack of persistence in *PTE* indicated by the CPS data. Just as in the data, the aggregate stock of *PTE* is as volatile as unemployment and negatively correlated with output over the business cycle.

In quantitative exercises, I investigate the distribution of part-time usage across firms of varying growth rates and age, and within different size categories. I find that part-time utilization is correlated with employment growth for shrinking firms, reflecting firms' usage of *PTE* along with layoffs in response to negative shocks. The model is consistent with the positive cross-sectional correlations of employment and hours growth outlined in empirical work by Cooper et al. (2004, 2007) and Trapeznikova (2014) at the aggregate level. Consistent with their empirical findings on firm-level correlations of hours and employment growth, firm-level usage of

part-time leads employment growth in my model. To extend the analysis of part-time usage to observable firm characteristics, I calibrate the model to match the firm size and age distribution found in Business Dynamics Statistics data from the Bureau of Labor Statistics. Heterogeneity in a permanent component of firm-level productivity matches the firm size distribution in the data, while the entrant share of firm productivity types and firm exit probabilities match the age distribution of firms. Very young firms use less part-time employment due to high employment growth, and part-time usage increases with firm age. Heterogeneity in the exit rates of firms plays a key role in the distribution of part-time utilization. If exit rates were equal for all firms, part-time use would decrease with permanent productivity (and, therefore, size). Lower exit probabilities for high-productivity firms increase the value of future labor in the same manner as does a lower discount rate, incentivizing productive (and large) firms to use part-time employment instead of layoffs. Part-time utilization increases by about 40% for the largest firms once exit rates are allowed to vary.

Over the business cycle, the model generates countercyclical and volatile movements in both part-time employment and unemployment. The incorporation of a part-time employment margin increases the volatility of both unemployment and vacancies relative to the case with only an employment margin. Part-time employment affects the cyclical properties of the model through two mechanisms. First, part-time utilization increases the volatility of output relative to average labor productivity. Aggregate shocks cause a reduction in output from firms' immediate use of part-time employment. Due to decreasing returns to scale in labor, this decrease

in labor utilization is accompanied by an increase in productivity per hour of labor. This effect is similar to that caused by time-varying effort in the labor-hoarding and factor-utilization literature, as exemplified by Burnside et al. (1993), Burnside and Eichenbaum (1996), and Bils and Cho (1994). Burnside et al. (1993) assume fixed hours per worker prior to the realization of productivity shocks in addition to fixed employment. Firms vary labor-utilization by demanding varying levels of effort, producing volatility in output and procyclical labor productivity. My model achieves similar effects by allowing for the adjustment of hours through the use of part-time employment by firms.

The second effect of the part-time margin in the model is an increase in the volatility of unemployment and vacancies from changes in the use of layoffs by firms. Firms use fewer separations in reaction to negative shocks when there is an operative part-time margin. Because idiosyncratic shocks generate the majority of endogenous worker separations, a model with part-time labor requires a less persistent shock process to generate the same unemployment rate as does a full-time only model. This increase in the prevalence of firm-level shocks produces more employment adjustment, increasing the volatility of unemployment and vacancies over the business cycle. The increased volatility in unemployment and vacancies is not accompanied by an increase in the volatility of the separation rate because layoffs are decreased in favor of part-time employment. The amplification of unemployment and vacancies over the business cycle in a model with large firms is also found in Elsby and Michaels (2013), but through a different mechanism. In their model, decreasing returns and

the Stole-Zwiebel bargaining process for wages implies that workers and firms split the marginal and infra-marginal surplus of a match, resulting in a low surplus for the marginal worker, akin to the recalibration of the standard search model in Hagedorn and Manovskii (2008).

Section 1.2 describes and reports the facts I outline in CPS data. Section 1.3 presents a search model which is consistent with the documented data on part-time work. In section 1.4, I characterize the dynamics of the firm through its policy functions on hiring, part-time utilization, and layoffs. In Section 1.5, I calibrate the model to match aspects of the aggregate labor market as well as the size distribution of firms. Section 1.6 discusses the implications of the calibrated model for part-time use over the firm size and age distribution, as well as the business cycle properties of the model. Section 1.7 outlines ongoing work using US Census Bureau data to document the co-movements of hours and employment growth over the distribution of firms and concludes.

1.2 Part-time Employment in Household Survey Data

In this section, I use data on part-time employment in the Current Population Survey (CPS) to highlight several facts about part-time work.

1. Part-time for economic reasons is volatile and countercyclical, and co-moves with unemployment.
2. Part-time for economic reasons is not persistent, and is characterized by high flow probabilities back to full-time employment and to unemployment.

3. Fluctuations in part-time employment arise largely from within-employment (to and from *FT* or *PTN*) instead of between *PTE* and non-employment (unemployment, *U* or nonparticipation, *N*). Further, part-time employment transitions primarily occur within job rather than from job-to-job transitions.

These facts are important for understanding the mechanism through which cyclical movements in *PTE* arise. Particularly, I make the case that these fluctuations in part-time employment for economic reasons are driven primarily by firm-level demand or “slack work and business conditions” as opposed to coming from workers’ search and job-finding behavior (“failure to find a full-time job”).

1.2.1 Definitions

The Current Population Survey gives detailed information at the individual level on labor market participation, either in employment or nonemployment, and the weekly hours worked by detailed reason. The definition of full-time and part-time employed by reason given by the Bureau of Labor Statistics (BLS) follows from questions asked of employed workers about working hours and availability. The BLS definition and terminology is “At work” by full- or part-time and reason. A worker is referred to in this paper as “Full-time” if they are classified by the BLS as “At work 35 or more hours.”² Workers who report working a total of 1-34 hours at all jobs during

²This is different from the typical definition of “employment” in that absent workers are not counted in this population. This definition relies only on actual hours worked, and does not necessarily coincide with being in the “usual” full-time labor force, which consists of those in the labor force who report usually working 35 or more hours per week (regardless of current employment or actual hours worked in the survey week). Similarly, the part-time labor force consists of employed and unemployed individuals who usually work less than 35

the reference week are classified as “At work part-time.” Part-time workers are also further classified by reason for working part-time. Workers who are classified as “At work part-time for economic reasons” (*PTE*) are individuals working part-time hours who report wanting and being available for full-time work and provide the reason for being at work part-time as either “Slack Work/Business Conditions,” “Failure to Find Full-time Work,” or “Seasonal Work/Changes in Demand.” Those who report being “At work part-time for non-economic reasons” (*PTN*) are individuals “At work part-time” who report reasons for working part-time that are not related to employer demand, or report that they are unwilling or unable to work full-time. In this paper, I refer to the BLS classifications of “At work full/part-time/part-time by reason” simply as “Part-time employed (for econ/non-econ. reasons)” and “Full-time employed.”³

hours per week.

³The sample time-frame considered for the gross flows analysis of Part-time employment is 1994-2014, after the CPS redesign. The sample time-frame initially studied for the gross flows analysis was chosen due to the fact that there are substantial changes in the 1994 redesign affecting the classification of workers into full-time and part-time employed, especially by reason. Polivka and Miller (1998) document that prior to the redesign, the CPS survey structure only asked those who reported working less than 35 hours what was their usual hours worked per week. This leads to an underestimation of the size of the part-time labor force by 9.8% relative to the redesign. The 1994 redesign also corrected an over-estimate of the part-time employed for economic reasons. The reported number of part-time employed for economic reasons is over-stated by 20% relative to the unrevised CPS. This difference is primarily due to the unrevised survey assuming desire for and availability to work full-time based on the reported reason given for being part-time employed, rather than asking this question directly. For details on the CPS variables and responses used to classify workers, see Appendix A.3.1

1.2.2 PTE is Countercyclical and Volatile

The volatility and cyclicality of part-time employment by reason is easily illustrated by considering each group’s share of total employment. Figure 1.1 shows that part-time employed for non-economic reasons displays a secular increase until about 1970, after which it makes up a roughly constant share of 14% of total employment.⁴ Although this group fluctuates cyclically, it does so in tandem with total employment. By contrast, part-time employment for economic reasons is markedly counter-cyclical, increasing to nearly 7% of employment after the 1980’s “double-dip” recession and again peaking at about 7% of employment in 2009. That is, part-time employment for economic reasons fluctuates significantly more than the stock of total employment over the business cycle.

To analyze the cyclical movements and volatility of the stocks of full and part-time labor, I compare them to the cyclical properties of quarterly aggregate output.⁵ The business cycle statistics of the stocks of full-time and part-time labor outlined in Table 1.1 confirm the cyclical patterns shown in Figure 1.1. Full-time labor and part-time employment for non-economic reasons have similar volatilities and are both pro-cyclical. The unemployment rate is counter-cyclical and much more volatile than output or employment. Part-time employment for economic reasons is nearly

⁴Except for the discrete jump due to the 1994 CPS redesign.

⁵I take the average of the monthly flows during the three months of each quarter. I then log and de-trend the quarterly series using a Hodrick-Prescott filter with a smoothing parameter of $\lambda = 10^5$ as in Shimer (2005), and compare the standard deviations of the cyclical components of each variable along with their correlation with the cyclical component of logged and detrended aggregate output (Y).

as volatile as the unemployment rate and also negatively correlated with aggregate output. Given that these statistics often motivate the interest in understanding the cyclical dynamics of the unemployment rate, it seems reasonable to devote attention to the cyclical dynamics of involuntary part-time labor.

Table 1.1: Cyclical Properties of Labor Market Stocks

	Y	FT	PTN	PTE	$u\ rate$
$std(x)$.024	.027	.024	.174	.190
$std(x)/std(Y)$	1	1.15	1.00	7.39	8.08
$corrcoef(Y, x)$	1	.906	.826	-.861	-.795

Cyclical components of quarterly and quarterly averaged series, logged and HP Filtered with $\lambda = 100,000$

Another measure of interest is the extent to which the transition of workers in and out of part-time for economic reasons accounts for the fluctuations in hours worked over the business cycle. While the total contribution to fluctuations in total hours from flows in and out of PTE is small, the changes in hours attributed to individual flows related to PTE can be large and cyclical. For instance, the total of net hours lost in a given month due to workers who moved from FT to PTE is nearly as large as the total hours lost by full-time workers who moved to unemployment. These patterns are explored further in A.3.5.

1.2.3 PTE is Not Persistent

The average monthly transition probabilities between employment states in the CPS show the lack of persistence in *PTE* relative to other employment states. in Table 1.2, I show the average of the seasonally adjusted monthly transition probabilities of workers between states of full-time employed, at work part-time for non-economic reasons, at work part-time for economic reasons, unemployment, and nonparticipation for the sample period 1994-2014.⁶

Table 1.2: US Average Monthly Flow
Probabilities

From	To				
	FT	PTN	PTE	U	N
FT	.880	.080	.015	.009	.014
PTN	.287	.582	.043	.019	.066
PTE	.301	.225	.361	.064	.046
U	.120	.071	.047	.522	.240
N	.015	.022	.003	.026	.934

From Current Population Survey,
1994:1-2014:12

Dis-aggregating employment into full-time and part-time categories highlights some key differences in the flow probabilities for employed workers. Within-employment transitions are large, especially for *PTE* workers. The transition probability of full-

⁶The CPS flow data is constructed from a modified version of code written by Robert Shimer. For additional details, please see Shimer (2007) and his webpage. <http://sites.google.com/site/robertshimer/research/flows>

time workers to part-time for economic reasons ($FT \rightarrow PTE$) is *larger* than the flow of full-time workers to unemployment ($FT \rightarrow U$). As seen in Table 1.2 or Figure 1.2, while about 1% of full-time workers become unemployed in a given month, 1.5% of workers transition to part-time for economic reasons. Although these transition probabilities seem small, the number of workers who move to both PTE and U from full-time work is large due to the size of the full-time employed population. More full-time workers move to PTE than to U .

Part-time for economic reasons is a very transitory state: only 36% of workers at work part-time for economic reasons remain in PTE the following month. By contrast, about half of unemployed workers remain unemployed in a given month. The flow of workers back to full-time employment is also much higher for the part-time employed than it is for the unemployed. While only 12% of unemployed workers move to a full-time job each month, nearly one-third of workers classified as PTE return to full-time employment. While part of the transitory nature of PTE is due to the high probability of returning to full-time work, workers in PTE also have a high probability of moving to PTN and nonemployment. 6.4% of workers in PTE become unemployed on average each month, a rate that is 7 times higher than for full-time workers.

1.2.4 PTE Status Fluctuates due to Within-job Movements

Here, I show that the fluctuation in the PTE stock comes from changes in the flow-probabilities of employed workers to and from PTE rather than changes in the

flows to or from non-employment. Looking at the reasons reported in the CPS for why a worker may be at work part-time highlights the two different ways in which workers may become underemployed. Workers can be classified as *PTE* due to slack work or business conditions, or they may be unable to find a full-time job. The first reason would indicate that firm-level demand for currently employed workers drives changes in *PTE*, and would coincide with $FT \leftrightarrow PTE$ (within employment) flows. The second reason of failure to find full-time work would coincide with $U \leftrightarrow PTE$ flows (across employment/nonemployment states). The fluctuation in the within-employment flows and the lack of any fluctuation in the flows across *PTE* and U/N indicate that firm-level demand and the “slack work” reason are the primary source of cyclical variation in part-time employment for economic reasons.

To establish that within-employment flows fluctuate, examine the cyclical properties of flows between *FT* and *PTE* in Column 1 of Table 1.3. The flows of full-time employed to *PTE* are strongly countercyclical and 6.4 times as volatile as aggregate output. Meanwhile, flows from *PTE* to *FT* are strongly procyclical. In contrast with the cyclical movements in the $FT \leftrightarrow PTE$ flows, the across-employment/nonemployment flows of workers between *PTE* and *U* in Column 2 of Table 1.3 have very little correlation with aggregate output. The difference in the cyclical variation in the within-employment and across-employment/non-employment flows is obvious if we plot the probabilities of unemployed workers to both full-time and part-time for economic reasons. Figure 1.3 shows the variation in the $U \rightarrow FT$ flow (the full-time job finding probability) in comparison with the nearly constant

$U \rightarrow PTE$ flow probability. If we suspected the popular narrative of the PTE worker as someone who failed to find a full-time job to be the source of the fluctuations in PTE , we would expect that in recessions, a higher fraction of the unemployed would move to PTE . While it is true that more workers in total arrive to PTE from U in recessions, this increase is entirely due to the size of the unemployment stock rather than an increase in the probability of a worker moving to PTE .⁷

Table 1.3: Cyclical Properties of Flows between
 FT and PTE

	Column 1		Column 2	
From	FT	PTE	PTE	U
To	PTE	FT	U	PTE
$std(x)$.150	.077	.092	.075
$corrcoef(Y, x)$	-.773	.855	-.255	-.245

Cyclical components of quarterly and quarterly averaged series, logged and HP Filtered with $\lambda = 100,000$

To establish that the cyclical movements in the PTE rate are not just within-employment but due to within-job hours changes, I use the dependent interview structure of the post-1994 CPS to classify which transitions in full-time and part-time

⁷As an additional check, I plot the shares of the stock of PTE workers by their status the previous month in Figure A.1 in Appendix A.3. The majority of workers reporting to be in PTE in a given month come from voluntary employment (either full-time or part-time) or were in PTE the month before. The percentage of the PTE stock which comes from the inflow of nonemployed (both U and N) individuals is also constant over the business cycle.

status coincide with job changes. In months 2-4 and 6-8 of a respondent's survey, the interviewer asks if the individual is still employed with the employer named in the previous month. As shown in Fallick and Fleischman (2004), this dependent interview makes it possible to track job-to-job transitions for workers who remain employed in consecutive months. I note which transitions between FT , PTN , and PTE correspond with a change in job. Especially with part-time and full-time status changes, one relevant concern is that the gain or loss of jobs for multiple job holders could be responsible for a substantial fraction of the $FT - PTE$ flow. Therefore, I include any change in the listed primary employer or any change in the number of jobs held for multiple job holders as my measure of a job transition. Figure 1.4 plots the separate transition probabilities of workers from FT to PTE while staying with the same employer for transitions to PTE while experiencing a job transition. Comparing the flow probabilities from FT to these states, it is clear that the majority of the $FT - PTE$ flow and its cyclical movements come from within-job changes in hours worked rather than from job-to-job transitions. Flow probabilities from PTE back to FT via the same job or a job transition (either a job-to-job transition or an increase to multiple jobs) are also similarly plotted in Figure 1.5.

It is also useful to observe the fraction of transitions which correspond with a job change. Looking at flows from FT to FT , PTN , and PTE in Figure 1.6, it is interesting to note that until the recovery from the Great Recession, the share of $FT - PTE$ transitions which come from a job change is declining. This decline is especially steep during the last recession. Looking at Figure 1.7, the fraction of flows

out of *PTE* that are due to job changes is also relatively small and declining. 80–90% of transitions from *PTE* back to full-time employment come from within-job. These figures also show that the majority of transitions into and out of *PTE* by individuals who remain in employment occur within-job. This fact is not specific to just the $FT \leftrightarrow PTE$ flows. In fact, of all job transitions of employed workers into or out of *PTE* (excluding $PTE - PTE$ flows with the same employer), 82% of all transitions occur within job.

One limitation of the CPS data is the inability to observe whether or not these movements of workers between employment states correspond with specific job transitions, especially for multiple job holders. While the CPS tracks the number of jobs for multiple job holders and job transitions are clearly identified for single job holders, The only job identity maintained is the primary job of the respondent. One cannot distinguish a job transition for a multiple-job holder who remains at their primary job if the number of jobs held remains constant in both months. For example, there is no way to tell if a multiple-job holder moved from *FT* to *PTE* because of a job-to-job transition in their secondary job. Fortunately, labor force participation data in the Survey of Income and Program Participants (SIPP) tracks all jobs held in each month, allowing for all job transitions to be tracked.⁸ I find that in the 1996

⁸The SIPP follows about 60,000 individuals over a time-frame spanning 36-48 months, and provides monthly responses of workers' labor force participation, weekly hours worked, and reported reason for working part-time. There are several differences between the structure of the SIPP and CPS surveys which are noted in Appendix A.3.6. Although the exact equivalent to the definition of "At work part-time for economic reasons" cannot be reconstructed in the SIPP, I classify workers as full-time or part-time based on their self-response as part-time or full-time, and infer *PTE* workers from the reason they provide for being employed part-time. Due to the panel structure and the availability of job-specific data in

panel of the SIPP, The fraction of flows into and out of *PTE* which correspond to job changes are similar to those found in the CPS data. 75% of all flows into or out of *PTE* do not coincide with any job transition.

Before moving on to the model, it is important to establish that although the cutoff for defining part-time vs. full-time work is somewhat arbitrary at 35 hours per week, the actual hours worked by full-time and part-time employees reflect the notion that the part-time workweek is significantly shorter than full-time work. This is especially important to note for the case of part-time work for economic reasons, as it is possible that within-job movements between *FT* and *PTE* could be an artifact of firms reducing workers' hours only slightly below the 35 hour cutoff. This is not the case, however, as both part-time for economic reasons (23.11 hrs/wk) and noneconomic reasons (21.61 hrs/wk) work roughly half of full-time hours (44.55 hrs/wk).

1.3 A Model of Part-time Employment

To study the implications of firm-level changes in demand in the aggregate labor market over the business cycle, I build a model of part-time employment. I incorporate a part-time margin in a competitive search environment with large firms and costly recruitment. The model builds on the framework of Kaas and Kircher (2015), with the extension of the firm's problem to account for a part-time employment decision. Firm size is determined by decreasing returns to scale production

the SIPP, I can observe whether a worker's change in status to or from *PTE* corresponds with a change in jobs, including the gain or loss of one or multiple jobs.

in labor, while firm growth is subject to search frictions and convex vacancy posting costs.⁹ Alternative frameworks incorporating firm size and search include the random search models of Elsby and Michaels (2013) and Moscarini and Postel-Vinay (2013) and the directed search model in Schaal (2012). These environments, however, all incorporate linear vacancy posting costs. The inclusion of convex recruitment costs produces rich firm dynamics over the life-cycle of the firm, rather than instantaneous firm growth as would occur under linear vacancy costs.

The model is in discrete time with a continuum of risk-neutral workers of mass one and an endogenous mass of firms. Workers and firms discount future payoffs at a rate $\beta < 1$. The timing of the model is as follows: first, aggregate productivity is revealed, new firms pay an entry cost, and idiosyncratic productivity for firms is revealed. Next, firms choose the fraction of their labor force to employ on a part-time basis, α . Firms then produce using their stock of employed labor, post contracts for new hires and vacancy postings, and choose the separation rate for workers. Firms exogenously exit at the end of recruitment with probability δ . Workers consume their wages, and unemployed or part-time employed workers consume leisure. Lastly, unemployed workers and vacancies are matched, and separations occur, changing the stocks of unemployment and employment for the next period.

⁹Another way to determine firm size is through wage-posting and competition over workers as used in Moscarini and Postel-Vinay (2013).

1.3.1 Firms

Firms operate a decreasing returns to scale production technology. A firm's output in one period is $xzF(L)$ when utilizing a mass $L \geq 0$ of labor in production. F is twice differentiable, strictly increasing, strictly concave, and satisfies the Inada conditions. $x \in X$ is the firm's level of idiosyncratic productivity, and $z \in Z$ is aggregate productivity: X and Z are finite state spaces. Because workers are identical, and hours and bodies are assumed to be perfect substitutes, a firm produces the same output from utilizing one unit of labor in production, regardless of this unit coming from two part-time workers (working half-time) or one full-time worker. I assume that the firm only has the ability to employ any individual worker either at full-time or at half-time. The firm's part-time utilization is decided by choosing what fraction $\alpha \in [0, 1]$ of its labor force to employ at half-time (henceforth part-time), while the fraction $1 - \alpha$ of its labor force is employed full-time and supplies one unit of labor per worker.

Firms choose separation and the fraction of their labor force to employ part-time as a function of contingent histories of x and z . A firm of age j at time t experiencing history (x^j, z^t) is subject to an exogenous exit probability δ , and chooses a separation probability $s(x^j, z^t) \in [s_0, 1]$, and a fraction of its labor force to employ part-time, $\alpha(x^j, z^t) \in [0, 1]$. The probabilities $\delta \geq 0$ and $s_0 \geq 0$ reflect the possibility of exogenous firm death and worker separation, respectively.

Firms choose history-contingent recruitment policies to hire workers. A firm with workforce L and productivity (x, z) that posts V vacancies pays recruitment

costs $C(V, L)$. I adopt a constant-returns specification for the firm's vacancy cost function $C(V, L)$, as in Merz and Yashiv (2007):

$$C(V, L) = \frac{c}{1 + \gamma} \left(\frac{V}{L} \right)^\gamma V.$$

With this specification, the average cost per vacancy for a firm depends on the firm's vacancy rate (V/L) . The flow cost per vacancy is equal for all firms recruiting the same percentage of their current labor force.

1.3.2 Matching and Contracts

The labor market features competitive search, under which firms compete for workers by posting long-term contracts. Unemployed workers direct their search to those postings that yield the highest expected utility, taking the fact that better contracts will have a lower probability of matching into account. With a standard matching function, at each contract type there is a corresponding queue length of λ unemployed per vacancy, yielding a matching probability for the firm's vacancy of m . If a firm is to fill a vacancy with probability m , it must offer a contract that attracts $\lambda(m)$ workers. With the usual assumptions on the matching function, the function $\lambda(m)$ is the inverse of the matching function, and satisfies $\lambda(0) = 0$, $\lambda'(0) \geq 1$, and $\lambda'(1) = \infty$. The worker's probability of matching is thus $m/\lambda(m)$.

Firms offer contracts to each cohort of workers recruited in a given period. These contracts specify a sequence of policies which apply to each worker in this cohort for every contingent history so long as the match exists. If the state of a contract offered by an age j firm in a particular history is defined as $\tau = \{j, (x^j, z^t)\}$,

then a contract is denoted as:

$$\mathcal{C}(\tau) = (w_f(\tau), w_p(\tau), \alpha(\tau), \phi(\tau))$$

The retention probability $\phi(\tau)$ is the firm's exit probability δ multiplied by the probability of match separation $s(\tau)$, so that the probability of separation for a worker is $1 - \phi(\tau)$. $\alpha(\tau)$ specifies the probability of a worker being placed on part-time work. The wage is a function of full-time or part-time employment: $w_f(\tau)$ or $w_p(\tau)$, respectively.¹⁰ Due to risk-neutrality, workers simply value the expected present value of being employed in a contract, $W(\mathcal{C}(\tau))$.¹¹

1.3.3 The Worker's Problem

A worker can be employed full-time, contributing one unit of time to labor, or part-time at 1/2 unit of labor. If unemployed, the worker receives utility $b \geq 0$, representing leisure or home production. Similarly, a part-time employed worker receives utility ℓb , where $\ell \in (0, 1)$ is the fraction of leisure or home production gained in part-time. The parameter ℓ reflects the fact that part-time work frees up only a fraction of time for leisure, as well as the possibility that some portion of b could come from transfers or unemployment insurance benefits that would not be available to a part-time worker.

¹⁰In this case, wage is total wage per period rather than wage per unit time worked.

¹¹Note that these contracts are general, and may be binding for both workers and firms, in that separation and part-time probabilities specified in the contract may bind either party. The assumption of commitment on either side may be relaxed in some cases. In general, however, the timing of payoffs to the worker depends on the ability to commit to certain actions.

Let $U(z^t)$ be the utility value of an unemployed worker in history z^t , and $W(\mathcal{C}(\tau))$ be the present expected value of employment in contract $\mathcal{C}(\tau)$ to a worker before production occurs. Because utility is linear, the worker treats the probability of part-time work as a lottery. The employed worker's value function satisfies the recursive equation:

$$\begin{aligned} W(\mathcal{C}(\tau)) &= [w_f(\tau)(1 - \alpha(\tau)) + w_p(\tau)\alpha(\tau)] + \alpha(\tau)(\ell b) \\ &+ \beta\{(1 - \phi(\tau))\mathbb{E}_{z'}U(z') + \phi(\tau)\mathbb{E}_{\tau'}[W(\mathcal{C}(\tau'))]\}. \end{aligned} \quad (1.1)$$

An unemployed worker's search problem involves maximizing the expected utility gain of employment, taking the matching probability and value of each contract into consideration. Potential contracts are observed, and parameterized by the tuple $(m, \mathcal{C}(\tau))$. Knowing that a contract yields a probability of matching for the worker of $m/\lambda(m)$, the worker's search value is:

$$\rho(z^t) = \max_{(m, \mathcal{C}(\tau))} \frac{m}{\lambda(m)} (1 - \delta) \beta \mathbb{E}_{\tau'} [W(\mathcal{C}(\tau')) - U(z^t)], \quad (1.2)$$

which reflects the expected probability of matching with a firm offering contract $\mathcal{C}(\tau)$ multiplied by the expected gain in the worker's value function derived from being employed with that contract during the next period. The Bellman equation for the unemployed worker then satisfies

$$U(z^t) = b + \rho(z^t) + \beta \mathbb{E}_{z'} U(z'). \quad (1.3)$$

The unemployed worker receives constant flow utility b from leisure or unemployment benefits, and the option value of searching, $\rho(z^t)$. Since workers can direct their

search to different contracts, their flow value from unemployment must be equal in any market that attracts workers. This implies that $\rho(z^t)$ is common to workers in any submarket; hence, $\rho(z^t)$ determines the contract value the firm must post to fill a vacancy with a positive probability m .

1.3.4 The Firm's Problem at Age 1

The firm's problem involves the recruitment of new workers through the posting of contracts for new vacancies, and the commitment to past contracts offered to its current labor force. The firm of age j in history (x^j, z^t) takes as given its current stock of workers hired up to the current period, $(L_i)_{i=0}^{j-1}$, and the contracts signed by previous labor cohorts, $(\mathcal{C}_i)_{i=0}^{j-1}$. The firm's separation rate and retention probability must satisfy $s(\tau) = (1 - \phi(\tau))/(1 - \delta)$ in order to avoid violating its prior commitments to past labor cohorts, and to be consistent with exogenous probabilities of separation and firm death. The firm then decides to post a contract \mathcal{C}_j in V vacancies. The contract \mathcal{C}_j achieves the firm's desired matching probability m , taking the worker's value of search $\rho(z^t)$ as given. To simplify, I describe the problem of a firm of age 1 with a single cohort of existing workers. The complete firm's problem and definition of competitive equilibrium appears in Appendix A.1.

Consider a firm of age 1 in history (x^1, z^t) . Letting $\tau = \{1, (x^1, z^t)\}$, the

problem of the age 1 firm is:

$$J[(\mathcal{C}_0), (L_0), \tau] = \max_{m, V, \mathcal{C}_1} x_1 z_t F[\tilde{L}(\tau)] - \mathcal{W}(\tau) - C(V, L_0) \\ + \beta(1 - \delta) \mathbb{E}_{\tau'} \{J[(\mathcal{C}_{0,1}), (L_{0',1}), \tau']\} \quad (1.4)$$

s.t.

$$\tilde{L} = L_0(1 - \alpha_0(\tau)/2), \quad L'_1 = mV, \quad L'_0 = L_0 \frac{\phi_0}{(1 - \delta)} \quad (1.5)$$

$$\mathcal{W}(\tau) = L_0[(1 - \alpha_0(\tau))w_{f0}(\tau) + \alpha_0(\tau)w_{p0}(\tau)] \quad (1.6)$$

$$\rho(z^t) = \frac{m}{\lambda(m)}(1 - \delta)\beta \mathbb{E}_{\tau'}[W(\mathcal{C}(\tau')) - U(z^t)] \text{ if } m > 0. \quad (1.7)$$

The firm enters the period with an existing labor force L_0 hired under contracts \mathcal{C}_0 . The firm produces using \tilde{L} in (1.5) and pays a wage bill \mathcal{W} in (1.6), with α and $\{w_f, w_p\}$ being consistent with contract \mathcal{C}_0 . The firm chooses contracts \mathcal{C}_1 to post in V vacancies at a cost $C(V, L)$. It chooses a matching probability m through the value of the contracts posted, knowing that achieving m requires a queue length of $\lambda(m)$ per vacancy. The minimum utility a contract \mathcal{C} must promise to attract a queue length of $\lambda(m)$ per vacancy comes from the worker's participation constraint, (1.7). At the end of the period, the firm's current cohort separates at rate $s = \frac{\phi}{1-\delta}$ leaving $L'_0 = L_0(1 - s)$ remaining in that cohort. The firm then gains a new cohort $L'_1 = mV$ from recruitment (1.5). This problem generalizes to the firm of age j , where each cohort and contract is indexed by the age of the firm at recruitment. The total labor force and wage bill are similar, except that they consist of the sum of all cohorts and their respective contracts. Lastly, the firm's policies must be consistent with its

commitment to past cohorts.¹²

Free entry of firms implies that the expected value of an entrant firm (before realizing x and with labor force $L = 0$) is less than or equal to the entry cost $K(z_t)$.

Letting $\tau_0 = \{0, x_0, z_t\}$:

$$\sum_{x_0 \in X} \pi(x_0) J[0, 0, \tau_0] \leq K(z_t). \quad (1.8)$$

If entry is positive, then this condition holds with equality.

1.3.5 Planner's Problem

The competitive equilibrium of the decentralized problem involves history-contingent and cohort-dependent policy functions for firms. Solving for the decentralized competitive equilibrium requires the tracking of the composition of worker cohorts within firms of different sizes and age-groups in all potential aggregate and idiosyncratic histories. The welfare theorems apply in this framework, so it is possible to solve an equivalent planner's problem. I show that the planner's problem simplifies to solving the maximization problem of the value of an individual firm given its current state, independent of cohorts and histories.

The sequential planner's problem is to maximize expected discounted output net of firm entry costs, opportunity costs of work, and recruitment and operating costs of the firm, subject to the constraint of having a unit mass of workers. Let $\psi(z^t)$ be the probability of aggregate history z^t given the transition matrix for aggregate states ψ , and $N(x^j, z^t)$ denote the measure of firms of a particular history. The planner's

¹²Section A.1 provides the general problem of the firm of age j , and the description of the free entry condition and definition of a competitive equilibrium.

problem is:

$$\begin{aligned}
& \max_{s, \alpha, V, M, N_0} \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) N_0(z^t) \right. \\
& + \sum_{j \geq 0, x^j} N(x^j, z^t) \left[x_j z_t F(L(x^j, z^t)(1 - \alpha_\tau(x^j, z^t)/2)) \right. \\
& \left. \left. - b(1 - \ell \alpha(x^j, z^t)) L(x^j, z^t) - C(V(x^j, z^t), L(x^j, z^t)) \right] \right\} \quad (1.9)
\end{aligned}$$

subject to

$$N(x^{j+1}, z^{t+1}) = (1 - \delta) \pi(x_{j+1} | x_j) \psi(z_t + 1 | z_t) N(x^j, z^t) \quad \forall (x^j, z^t) \quad (1.10)$$

$$L(x^{j+1}, z^{t+1}) = [1 - s(x^j, z^t)] L(x^j, z^t) + m(x^j, z^t) V(x^j, z^t) \quad \forall (x^j, z^t) \quad (1.11)$$

$$N(x_0, z^t) = \pi_0(x_0) N_0(z^t) \geq 0 \text{ and } L(x_0, z^t) = 0 \quad \forall z^t \quad (1.12)$$

$$\sum_{j \geq 0, x^j} N(x^j, z^t) [L(x^j, z^t) + \lambda(m(x^j, z^t)) V(x^j, z^t)] \leq 1 \quad \forall z^t. \quad (1.13)$$

Proposition 1. *A Competitive search equilibrium is socially optimal.*

Proof. See Appendix A.2 5 □

The planner's problem specifies history-contingent, cohort-contingent policies as in the competitive equilibrium. There exists, however, a solution to the planner's problem where policies are not only cohort-independent, but also independent of idiosyncratic and aggregate histories. Further, the planner's problem can be rewritten as the sum of recursive problems maximizing the social surplus of each firm. The value of a worker to the planner is given by the Lagrange multiplier on the aggregate feasibility constraint (1.13). Given a vector of multipliers for the aggregate resource constraint (1.13), $M = (\mu_1, \dots, \mu_n)$ for each $z \in (z_1, \dots, z_n)$, the social value of a firm with productivity x and z and labor stock L satisfies the following Bellman equation:

$$\begin{aligned}
G(L, x, z; M) = & \max_{s, \alpha, V, m} xzF(L(1 - \alpha(x, z)/2)) - bL(1 - \ell\alpha(x, z)) \\
& - \mu(z)[L + \lambda(m(x, z))V(x, z)] - C(V(x, z), L) \\
& + \beta(1 - \delta(x, z))\mathbb{E}_{x', z'} G(L', x', z'; M)
\end{aligned} \tag{1.14}$$

subject to

$$L' = (1 - s)L + mV, \alpha \in [0, 1], s \in [s_0, 1], m \in [0, 1] \text{ and } V \geq 0.$$

The result that the planner's problem is cohort-independent follows from the fact that workers are identical and have linear utility, so they are indifferent about the time-path of payment. Thus, the planner gains nothing by specifying cohort-specific policies. Further, any sequence of history-contingent policies in a decentralized equilibrium can be reproduced by the planner using policies that only depend on current states of idiosyncratic and aggregate productivity. This is due to the fact that the solution to the planner's maximization problem for the social surplus of a firm dictates the planner's optimal policy for the firm as a function of firm size, current productivity, and the social value of a worker, $\mu(z)$. If entry is positive, then $\mu(z)$ is pinned down by the entrant firm value that satisfies the free-entry condition (which is only a function of the aggregate state z_t) with equality. Since the social value of a worker is identical across all firms, and the social value of any firm can be solved for any size L and productivity realization x, z given $\mu(z)$, the planner's problem for each firm can be solved given $\mu(z)$. Thus, entry and firm-level policies and value-functions can be solved where the only aggregate state variable for the planner is the current

realization of z_t .

Efficient entry of firms requires that the entry condition is satisfied with equality:

$$\sum_{x \in X} \pi_0(x) G(0, x, z; M) = K(z). \quad (1.15)$$

Proposition 2. *(a.) Suppose that a solution of (1.14) and (1.15) exists with associated allocation $\mathbf{A} = (\mathbf{N}, \mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \alpha)$ satisfying $N(z^t) > 0$ for all z^t . Then \mathbf{A} is a solution to the sequential planning problem (1.9).*

(b.) If $K(z)$ and b are sufficiently small and if $z_1 = \dots = z_n = \bar{z}$, equations (1.14) and (1.15) have a unique solution (G, M) . If the transition matrix $\psi(z_j|z_i)$ is strictly diagonally dominant and if $|z_i - \bar{z}|$ is sufficiently small for all i , equations (1.14) and (1.15) have a unique solution.

Proof. See Appendix A.2 6 □

Conversely, the solution of the planner's problem with history-independent policies coincides with a solution of the competitive equilibrium. There are many decentralizations that yield a payoff-equivalent allocation to the planner's problem, though not necessarily the same allocation as the planner. This is due to the linearity of workers' utility so that the time-path of payments doesn't matter to the worker. Also important is the redundancy of cohort-specific policies for the planner. One simple example of a decentralization of the planner's allocation is if firms offer contracts which provide a "constant utility" contract by offering a constant full-time wage and a constant part-time wage. The firm provides the same utility for part-time

workers as it does for full-time work after accounting for the utility gained in leisure during part-time. Workers commit to employment until the firm destroys the match, and they are indifferent between full-time or part-time employment. In general, the nature of the contract and the ability for either workers or firms to commit to specific actions will affect the time-path of payments.

1.4 Dynamics of the Firm

The model's outcomes for firm-level labor demand through hiring, firing, and layoff can be illustrated through the optimal decisions of the planner's problem for a firm. These conditions highlight that firm expansion occurs gradually over time through variations in the number of vacancies posted by the firm and the matching probability of each vacancy.

The planner's problem can be solved given a vector $M = (\mu_1, \dots, \mu_n)$ of multipliers on the resource constraint (1.13), which correspond to the planner's value of a worker in n aggregate states. The first-order conditions determine optimal policies for s, α, m , and V for a given level of productivity and firm size L . The first order conditions for m and V relate the inter-temporal and intra-temporal tradeoffs of recruitment intensity for firms. The optimal recruitment policy in a stationary equilibrium features a declining match probability as L increases to the firm's optimal size.

Suppressing the state (x, z) for policy functions, the first-order condition with respect

to m gives:

$$\beta(1 - \delta)\mathbb{E}_{x', z'} \frac{\partial G(L', x', z')}{\partial L'} = \mu(z)m\lambda_m(m). \quad (1.16)$$

where $L' = L(1 - s_0) + mV$ if m is positive. Similarly, the first-order condition with respect to V gives:

$$\beta(1 - \delta)\mathbb{E}_{x', z'} \frac{\partial G(L', x', z')}{\partial L'} = \mu(z)\lambda(m) - C_V(V, L) \quad (1.17)$$

Combining (1.16) and (1.17) gives the intra-temporal optimality condition for hiring, describing the trade-off to the planner of increasing vacancies and increasing the matching probability for the firm. Intuitively, the planner equates the marginal cost to the firm of hiring another worker by opening more vacancies with the marginal cost of hiring an additional worker by increasing the matching probability of vacancies through more valuable contracts:

$$\mu(z)[m\lambda_m(m) - \lambda(m)] = C_V(V, L) \quad (1.18)$$

Using $\frac{\partial G(L', x', z')}{\partial L'}$ via the envelope theorem and (1.16), we can get the inter-temporal optimality condition, which governs hiring intensity of the firm over time.

$$\begin{aligned} \frac{\partial G}{\partial L'} &= x'z'F_{L'}(L'(1 - \alpha'/2)) - b(1 - \ell\alpha') - \mu(z') \\ &\quad - C_L(V', L') + (1 - s')\mu(z')\lambda_m(m') \end{aligned} \quad (1.19)$$

The planner hires if:

$$\beta(1 - \delta)\mathbb{E}_{x', z'} \frac{\partial G(L(1 - s_0), x', z')}{\partial L} > \mu_i\lambda'(\underline{m}),$$

where \underline{m} is the value of m for which $\lambda(m) = 0$. If this inequality is satisfied, (1.18)

characterizes the optimal vacancy postings $V^*(m(L, x, z))$ for a given matching probability m which satisfies (1.16) with (1.19) substituted inside the expectation term.

Figure 1.8 shows the vacancies posted and number of hires (mV) for a firm of a given productivity level as a function of its current workforce. The increasing gap between the number of vacancies posted and the number of hires with labor L reflects the declining matching probability used by the firm as it reaches maturity. Since vacancy posting costs are convex, the firm spaces hiring over time.

The optimal choice of α and s can also be solved from the first-order conditions of the firm's social surplus problem. The first-order condition with respect to α is:

$$xzF_L(L(1 - \alpha/2)) - 2\ell b = 0. \quad (1.20)$$

An alternative way to write this condition is to consider the planner's (firm's) ideal labor utilization today, \hat{L} . The optimal labor utilization solves:

$$\hat{L} = F_L^{-1}\left(\frac{2\ell b}{xz}\right)$$

. Since F_L is positive and decreasing, its inverse is also decreasing. Thus, labor utilization \hat{L} is increasing in productivity xz and decreasing in the share of leisure gained by moving from full-time to part-time work, ℓb . Intuitively, if the marginal product of labor in production today is high, the planner wants to allocate worker time to production and away from leisure. When the current marginal product of labor is low enough, the planner would like to move workers to part-time as the gains from added leisure are higher than the gains in output for the marginal worker. Writing \hat{L}

in terms of L and α and assuming an interior solution, the optimal part-time usage α^* is characterized by:

$$\alpha^* = 2 \left(1 - \frac{F_L^{-1}(\frac{2\ell b}{xz})}{L} \right)$$

If the optimal labor utilization \hat{L} is outside of the interval $(L/2, L)$, then α^* is a corner solution. When interior, part-time use is increasing in the leisure gained from part-time work, ℓb . Part-time use is decreasing in productivity, xz . We can also see that part-time usage is increasing in L , all else equal. It should be noted that the part-time decision is independent of future values of the firm, but the current size L for the firm is a function of past size and productivity as well as the future value of these workers to the firm. Thus, actual part-time utilization by firms depends on the difference between their actual size and their ideal current size, and it is not clear that part-time usage will be increasing or decreasing in firm size categories in the stationary distribution.

The first-order condition with respect to s is:

$$\beta(1 - \delta)\mathbb{E}_{x,z} \frac{\partial G(L', x', z')}{\partial L'} = 0. \quad (1.21)$$

Again, if the solution is interior then (1.21) holds with equality. If the expected value of labor tomorrow is high, then the planner will not separate with workers.

From the first order conditions for α and s , the planner will utilize part-time labor when the value of current productivity is low. If the marginal value of its future labor force is also low, the planner will separate with workers at the end of the period.

Figure 1.9 shows the part-time probability and separation rate for a firm at two different productivity levels (high and low). The part-time employment rate (red line) and separation probability (light gray line) of the firm are the two upward-sloping lines at each productivity level. The slope of the part-time percentage is steeper than the separation rate. For a given size, the part-time utilization rate is increasing faster than the separation rate as the size of the shock increases. In general, the part-time rate can be positive even if the firm is not separating (it may even be expanding), depending on parameters. A high productivity firm initially at size L which realizes a low productivity shock would have policy functions of the solid lines in Figure 1.9. The black points on the part-time and layoff lines indicate the firm's choice of part-time usage that period, as well as the fraction of its workforce it will layoff at the end of the period. Upon realization of the shock from high to low productivity, the firm would use part-time employment to decrease its labor utilization in the current period. After production, the firm would separate with 35% of its workers, starting the next period with 65% of its original labor force.

While the policy functions and first-order conditions give intuition on the factors that affect firm-level decisions for employment and part-time utilization, the actual dynamics of the firm requires tracking of a firm's state variable over time. The firm's state variables (L, x, z) move according to the transition processes for productivity and the law of motion for firm size ($L' = L(1 - s) + mV$) dictated by the policy functions. In Figure 1.10, I plot the simulation of an individual firm subject to idiosyncratic and aggregate shocks over a 20 year period using parameters

from the calibrated model. The firm grows upon entry and fluctuates with realized productivity shocks. Steep declines in the firm's size are the result of layoffs, while steady declines result from inaction in hiring or firing by the firm due to the exogenous separation of workers from the firm (at rate s_0). It is clear from the figure that part-time employment is volatile at the firm level and is used in conjunction with layoffs.

1.5 Quantitative Exercises

1.5.1 Calibration Overview

To calibrate the model, I first match the model to moments of the aggregate labor market with idiosyncratic productivity shocks, holding the realizations of aggregate shocks at their average value. Next, I set productivity, entry, and exit parameters to match the firm size and age distribution. The idiosyncratic shock process matches the average separation rate and the share of employment at firms with little to no employment growth. The aggregate shock process matches the quarterly autocorrelation and standard deviation of average labor productivity. Table 1.5 displays the values of parameters and the targets they match.

1.5.2 Production

I use a decreasing returns to scale function in labor:

$$xzF(L) = x_0x_1zL^\eta$$

where $\eta = 0.7$ is set to target a labor share of income of 0.66. Productivity of a firm is composed of an aggregate component z and idiosyncratic productivity $x = x_0x_1$.

Idiosyncratic productivity is a product of a fixed component x_0 which is drawn upon firm entry and a transitory component x_1 .

1.5.3 Matching Function and Vacancies

The matching function is of the form

$$m(\lambda) = (1 + k\lambda^{-r})^{(-1/r)},$$

where λ is the ratio of unemployed to vacancies and $m(\lambda)$ is the firm's probability of filling a vacancy. The cost function for vacancy postings is increasing in the vacancy rate of a firm, $\frac{V}{L}$:

$$C(V, L) = \frac{c}{1 + \gamma} \left(\frac{V}{L} \right)^{(1+\gamma)} L$$

where $\gamma = 2$ as in Kaas and Kircher (2015), so that the recruitment cost is cubic.

The model period is set to weekly, so the matching function parameter k is set to target a weekly job-finding rate of 0.129 (corresponding to a monthly rate of 0.45 as in Shimer (2005)) at the steady-state queue length. The parameter r is set to match an elasticity of the job-finding rate with respect to the tightness ratio ($\frac{V}{U}$) of 0.72. With multiple vacancies per firm and nonlinear posting costs, the rate of vacancy posting must be determined. The job-filling rate for vacancies is set to 0.3 to match the observed monthly vacancy yield of 1.3 as reported in Davis et al. (2013).¹³ The parameter c in the vacancy cost function is set to match this weekly job-filling rate

¹³The vacancy yield is the ratio of jobs created in a month to the observed stock of vacancies observed at a point in that month. Thus, a vacancy yield of 1.3 reflects that there are more hires in a month than the observed instantaneous stock of vacancies. This reflects time-aggregation as additional vacancies are posted as jobs get filled, and the vacancy yield is determined by the rate of vacancy postings and the job-filling rate.

of 0.3. In steady state, the queue length is the ratio of the job-filling and job-finding rate, yielding a steady-state queue length of $\lambda = 2.326$.

1.5.4 Home Production and Part-time

The benefit from home production b is set to match 70% of the average wage, corresponding with the calibrated value of non-market work in Hall and Milgrom (2008). To calculate the replacement ratio, I consider the special case of a constant wage per period for full-time and part-time workers. That is, the wage contract is a pair of constants $\{\bar{w}_f, \bar{w}_p\}$ for the duration of the match. The share of leisure gained by part-time workers, ℓ , is used to target a steady-state fraction of employment working part-time of 5%.

1.5.5 Permanent Firm Types and the Size/Age Distribution of Firms

What remains to be determined is the set of productivity parameters xz and their shock process. To match the broad size distribution of firms and the fact that a small fraction of very large firms retains a large share of total employment, the permanent component of idiosyncratic productivity x_0 is set to match the size distribution of firms. To match the firm share and employment distribution for 5 classes, x_0 takes on 5 values at entry, with entry share σ . The exit probability of firms, δ , is exogenous and also dependent on size classification. The vector δ is chosen by matching the observed job destruction rate for closing firms in each size class from the Bureau of Labor Statistic's Business Employment Dynamics data for 1992-2011. Using permanent firm types characterized by permanent productivity level x_0 , entry

share σ , and exit probability δ given in Table 1.4, the model can closely match the firm size and age distribution in the data, as seen in Figure 1.11.¹⁴ The vector x_0 matches the employment shares of firms in each size category, while σ and δ determine the share of firms in each size category.

Table 1.4: Parameters for Matching the Size and Age Distribution of Firms

	Permanent Productivity Types					Target/Source
Size class	1-49	50-249	250-999	1k-9,999	10k+	
(x_{0i})	0.363	0.736	1.168	2.03	4.138	Emp. Shares by firm size
(σ_i) , %	98.82	1.00	0.153	0.025	0.002	Firm Shares
(δ_i) , %	1.71	0.27	0.16	0.088	0.016	BED Data

Parameter values for permanent firm types used for matching the size and age distribution of firms.

1.5.6 Idiosyncratic Shocks

The shock process x_1 is evenly spaced between $[1 - \bar{x}, 1 + \bar{x}]$ and is redrawn with probability π each period. This shock process matches the fact that many firms in the data experience little to no net job growth in a given quarter. The idiosyncratic shock parameters match the average monthly separation rate and the share of employment at firms with monthly growth rates between -2% and 2% . The exogenous separation rate s_0 is set to match a monthly quit rate of 2% per month from the Job Openings

¹⁴Table A.1 in Appendix A.1.3 gives the share of firms and employment across firm size categories in the data and the model.

and Labor Turnover Survey (JOLTS) data.¹⁵

1.5.7 Aggregate Shocks

The aggregate shock process affecting z is a mean-reverting Markov process as in Appendix C of Shimer (2005). The aggregate shock process is parameterized by a persistence parameter ψ and range $[\underline{z}, 2 - \underline{z}]$. The parameters $(\psi, \underline{z}) = 0.015, 0.95$ are chosen to produce a quarterly standard deviation and autocorrelation of productivity shocks of $\sigma_z = 0.015$ and $\rho_z = .76$. The remaining parameter to be set is the entry cost $K(z)$. The entrant firm's value function is homogeneous in the vector $\{x_0, b, c, \mu, K\}$ in steady-state, so the stationary value of parameter $K(z)$ can be normalized arbitrarily. Tractability requires that there is positive entry of firms in every state, so that the value of μ for all workers is determined by the value at entrant firms. I allow $K(z)$ to vary with the aggregate state so that job creation at entrant firms is stable over the business cycle to ensure positive entry in all aggregate states.

I numerically solve for the values of $\mu, x_0, \ell, c, \bar{x}$, and π that minimize the target moments in steady-state (holding z at its average level). These moments are the job-finding rate, share of firms in each size category and share of employment by size categories, separation rate, ratio of b to average wages, and the employment share at firms with growth rate $< \pm 2\%$.

¹⁵The calibration strategy is taken from Kaas and Kircher (2015) to facilitate comparison with their model when the part-time decision of firms is absent.

Table 1.5: Parameters in Benchmark Calibration

Parameter	Value	Target/Source
β	0.999	Annual interest rate 5%
b	0.1	70% of average wages
ℓ	0.63	Average PTE share of employment = 5%
η	0.7	Labor share of income = 0.66
k	6.276	job-finding rate (JFR) = 0.129
r	1.057	Elasticity of JFR wrt tightness ratio = 0.72
c	11.44	Vacancy-filling rate = 0.3 (Davis et al., 2013)
γ	2.0	Vacancy cost function (Kaas and Kircher, 2015)
s_0	0.0048	Monthly quit rate = 0.02
π_x	0.933	Share of emp. in low-growth firms = 66%
\bar{x}	0.268	$x_1 \in [1 - \bar{x}, 1 + \bar{x}]$ Monthly separation rate = 4.2%
ψ	0.015	$\rho_z = .76, \sigma_z = 0.015$
\underline{z}	0.95	$\rho_z = .76, \sigma_z = 0.015$

Parameters used to match aggregate moments in the calibration of the model.

1.6 Results

The calibrated model matches the cross-sectional distribution of firms by size and age. Due to entry, exit, and productivity shocks, the simulated model produces varying employment growth rates within the distribution of firms.

1.6.1 Recruitment, Part-time, and Separations by Employment Growth of Firms

One test of the model is in its ability to match the vacancy rates, hiring rate, layoff rate, and part-time rate used by firms over the employment growth distribution. Figure 1.12 shows that the model is able to match the patterns of vacancies and hires across the growth distribution of firms. Similar to Kaas and Kircher (2015), the model matches the broad pattern that vacancies are posted by growing firms, though

the vacancy rate is higher than it is in the data. This can be matched more closely through a higher γ in the recruitment cost function. The data may also not entirely account for the increased recruitment of rapidly growing firms if multiple hires result from a single vacancy.

Figure 1.13 shows the growth rate of hours per worker, the layoff rate, and the part-time rate of firms by monthly employment growth rate. The model produces a positive correlation of layoff rates and part-time share of employment for contracting firms. This result is consistent with the evidence in CPS flow data that workers in part-time employment have high separation rates to unemployment. While layoffs are only used in contracting firms, some workers separate from all firms due to exogenous quits. Part-time usage, however, is exclusively used by shrinking firms. The likelihood of separating to unemployment given a worker has been placed into part-time work is therefore high, as some workers in the firm will also be laid off at the end of the period.

Although the monthly average part-time rate is lower than the layoff rate for shrinking firms, this is not necessarily contrary to the fact that full-time workers are more likely to move to *PTE* than to unemployment in the CPS. From the firm's individual policy function, Figure 1.9 in Section 1.4 showed that a contracting firm can use more part-time than separations in a period. While the layoff rate reflects total layoffs by a firm within the month, part-time percentage is an average of the weekly part-time utilization of a firm. The transitory nature of part-time usage and time-aggregation of total separations to a monthly rate can make the average part-

time rate low in Figure 1.13, even if firms are moving more workers to part-time than using layoffs.

This short-term nature of part-time utilization in the model is also reflected in the growth-rate of weekly hours per worker for shrinking firms. While part-time usage and layoffs increase steadily for shrinking firms, the average monthly change in weekly hours per worker is relatively stable except at very negative growth-rates. This is due to fluctuations in part-time usage within a given month. If part-time usage were very persistent, it would be reflected in a steadier decline in the monthly growth rate of average weekly hours per worker.

1.6.2 Part-time Employment by Firm Age, Size, and Productivity

In addition to the distribution of part-time employment in firms by growth rate, the model produces interesting results for the cross-section of firms which use part-time labor by firm size and age category, as displayed in Table 1.6. It is interesting to note that the part-time share is not monotone in size categories, with the largest and smallest firms utilizing more part-time labor. This is due to the entry and growth of large firms, and reflects the fact that these size categories are somewhat arbitrary. For example, firms in the range of 249-999 employees in the model move across size categories with large enough shocks. Looking at the part-time share of firms by permanent productivity level, we can see that part-time usage is roughly monotone and increasing in size except at the highest productivity. The variation seen in part-time usage across productivity levels comes partly from the heteroge-

neous exit rates of firms, which affects the long-run size of a firm. If exit rates were constant across firms, average part-time usage would actually be decreasing in the firm's permanent productivity level. Instead, it is increasing, since more productive firms also have much lower exit rates. This raises the future value of the firm, giving the firm more incentive to retain its current workforce through part-time work. Fixing the productivity level, part-time usage is increasing in age and reflects the fact that young, growing firms use less part-time labor.

Table 1.6: Part-time Percentage by Firm Age and Size

Size class	1-49	50-249	250-999	1k-9,999	10k+
PT %	4.80	5.07	1.25	6.41	4.99
PT % ≤ 2 years	3.34	4.08	1.03	4.06	3.21
PT % ≤ 5 years	3.82	4.40	1.23	6.73	5.45
PT % ≤ 10 years	3.99	4.83	1.28	6.81	3.95
Productivity (x_0)	$x_0 = 0.36$	$x_0 = 0.70$	$x_0 = 1.12$	$x_0 = 1.94$	$x_0 = 3.96$
PT %	3.71	4.71	5.28	6.41	4.99
PT % ≤ 2 years	3.21	3.50	4.82	4.03	2.92
PT % ≤ 5 years	3.56	4.33	5.11	6.71	5.32
PT % ≤ 10 years	3.65	4.56	5.61	6.80	3.91

The top half of the table gives the average part-time rate of firms in each size class in total, and in firms of less than 2, 5, and 10 years of age. The bottom portion of the table gives these statistics for firms by their permanent productivity type, regardless of their current size.

Although the model can reproduce the qualitative properties of worker flow data and has testable implications for the usage of part-time employment over the distribution of firms, what is lacking is data on the patterns of hours adjustments or

part-time usage of firms by age or size, both in the cross-section and over the business cycle. In future work, I plan to address this by documenting the hours and employment adjustments at the firm-level in the Longitudinal Business Database, Census of Manufactures/Annual Survey of Manufactures, and Quarterly Plant Capacity Utilization Survey data from the US Census Bureau.

1.6.3 Business Cycle Properties

I simulate the model with aggregate uncertainty to evaluate its ability to produce cyclical volatility in vacancies, unemployment, and part-time work over the business cycle. To evaluate the business cycle properties of the model, I focus on the relative volatility of part-time utilization, unemployment, vacancies, and worker flows in comparison to aggregate output. Two primary results arise from the business cycle analysis. First, the model is able to match the volatility and cyclical properties of PTE relative to U found in the data. That is, unemployment and part-time are countercyclical and similar in their relative volatility, as highlighted in Section 1.2.2. Second, the volatility of unemployment and vacancies is increased relative to the model without an operative part-time margin. Table 1.7 shows the business cycle moments for the model with and without part-time employment relative to the data. For the model without part-time labor, the model is recalibrated to the same target moments, but with $\ell = 0$. The primary change in parameter values between the two calibrations is the idiosyncratic shock parameters \bar{x} and π_x , which are .312 and 0.973 in the case without part-time (compared with $\bar{x} = .288$ and $\pi_x = .933$ with a

part-time margin). The results displayed are the standard deviation and correlation of the cyclical component of the logged and detrended variables relative to aggregate output (Y).

Table 1.7: Business Cycle Statistics

	$\frac{sd(x)}{sd(Y)}$	$\rho(x, Y)$	$\frac{sd(x)}{sd(Y)}$	$\rho(x, Y)$	$\frac{sd(x)}{sd(Y)}$	$\rho(x, Y)$
	Data		Model		Model (No PTE)	
Prod/Worker	0.67	.89	0.92	.97	0.93	.97
U	7.47	-.85	4.96	-.46	2.88	-.38
PTE	6.50	-.83	3.86	-.98	NA	NA
Vacancies	6.81	.43	2.28	.56	1.20	.39
JF Rate	3.86	.81	2.73	.66	1.15	.48
Sep. Rate	2.67	-.58	2.80	-.15	2.66	-.23

The model with an operative part-time margin improves the fit of the model to business cycle properties substantially. Unemployment becomes more volatile and countercyclical, though it is still not as volatile as it is in the data. The volatility of vacancies and the job-finding rate double. While the volatility in unemployment and vacancies increase, the separation rate remains close to the value in the data. Additionally, although the total volatility in PTE is higher in the data than in the model, the model matches quite well the relative volatility of PTE and U . The model also matches the strong negative correlation of PTE with aggregate output.

The change caused by part-time utilization on the business cycle properties in the model is twofold. First, the part-time margin changes the movement of measured labor productivity in reaction to a shock. A fundamental productivity shock causes

changes along the part-time margin for firms, which stabilizes the change in labor productivity for the firm because of decreasing returns to scale in the production function. This decrease in utilized labor means that output is more volatile for a given shock. The increase in volatility of output is similar to the effect of varying effort in real business cycle models with labor-hoarding, such as in Burnside et al. (1993). Similar to their model, productivity shocks are realized after employment is fixed. In the labor-hoarding literature, however, hours per worker are fixed during a period since workers supply a constant number of hours, and firms adjust utilization by contracting with workers over effort when the shock is realized. This produces procyclical movements in labor productivity and increases the volatility of output from a shock. In this model, variation in labor utilization occurs in the hours margin through part-time employment rather than in unobservable effort. Firms adjust to productivity shocks using part-time employment, changing the amount of output per worker. Productivity varies less than output since decreasing returns to scale increases the average productivity of firms when labor utilization decreases.

The second effect of part-time utilization is in the volatility of the model, and comes from the effect of part-time employment on the separation rate. Firms with the option of using part-time labor use fewer layoffs in the event of a large idiosyncratic productivity shock. Since the calibrated idiosyncratic shock process matches the average monthly separation rate, when firms use part-time instead of layoffs, a less persistent shock is needed to produce the same average monthly separation rate. This increases the volatility of unemployment and vacancies over the business cycle

as well, since aggregate shocks primarily change the magnitude of job destruction and creation in adjusting firms. More firms adjusting labor produces a larger response to aggregate changes in productivity. One interesting point to note is that the volatility of the separation rate remains low as a result of the part-time margin being used instead of layoffs, so that the increase in volatility affects job creation more than job destruction.

It should be noted that this increased volatility arises even with a conservative value of leisure. The volatility of unemployment and vacancies can also be increased by raising the value of leisure b to be close to the value of working, as in the recalibration of the model of Mortensen and Pissarides (1994) by Hagedorn and Manovskii (2008). The reason volatility increases with an increase in b is that the surplus created by a filled vacancy decreases as workers become indifferent between leisure and work. This makes vacancies and the job-finding rate very sensitive to changes in productivity. However, raising the benefit from non-market work in this model will also result in a counterfactually high volatility of the separation rate.

1.7 Discussion

1.7.1 Ongoing Work with US Census Bureau Data

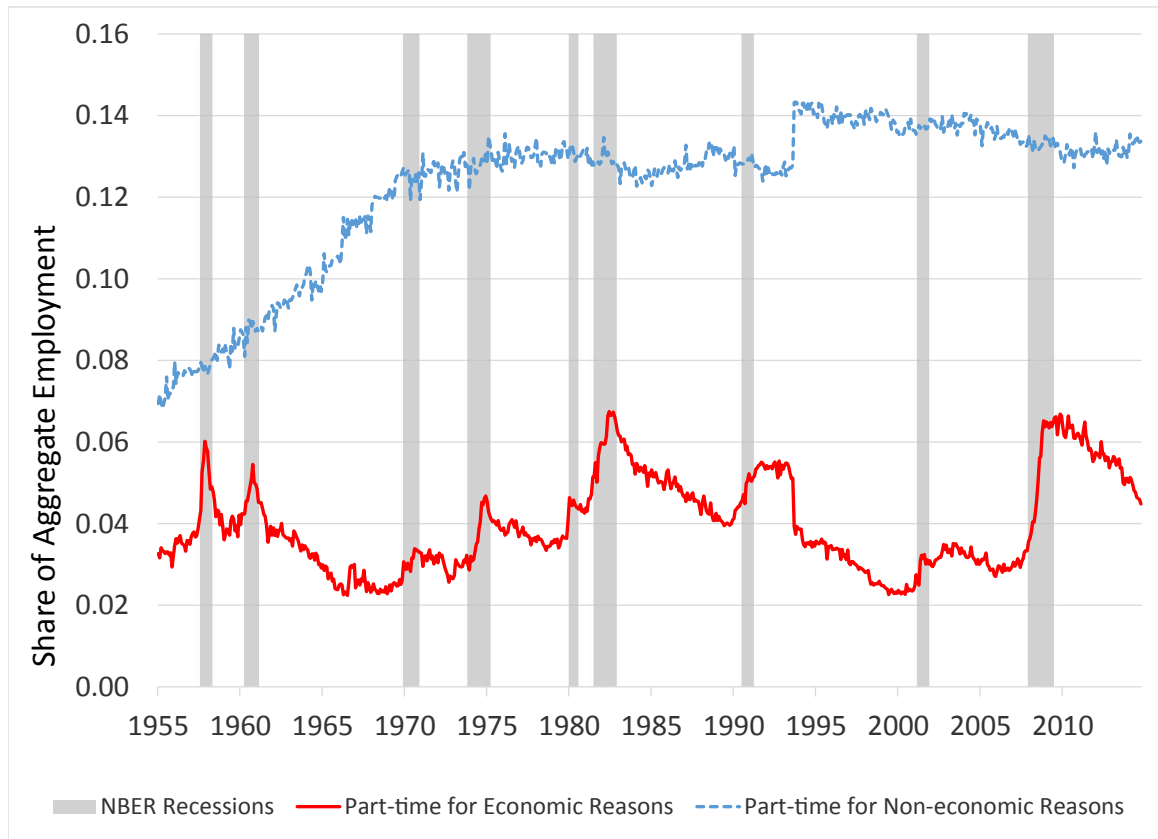
I show that the model can reproduce the qualitative properties of worker flow data at the firm level and over the business cycle. In addition, the model has implications for the cross-sectional distribution of part-time usage by firms. Firm growth causes young firms to use relatively little part-time labor, while the exit probability

of firms changes the usage of part-time employment over the firm-size distribution. However, what is lacking along these dimensions is data on the patterns of hours adjustments or labor utilization of firms, both in the cross-section and over the business cycle. In ongoing work, I address this by documenting the cyclical and cross-sectional properties of hours and employment changes at the firm-level in US Census Bureau Data. I plan to document the patterns in employment growth and hours growth in manufacturing firms using the Longitudinal Business Database (LBD) and Census of Manufactures/Annual Survey of Manufactures (CMF/ASM). The goal is to document the correlation of hours and employment growth over the cross-sectional distribution of firms by size and age category. Also of interest is the cyclical volatility of these moments at the firm level by firm and establishment size, age and productivity. This is motivated by the findings of Moscarini and Postel-Vinay (2012), who document that the employment growth rate for large firms varies more over the business cycle than it does for small firms. Fort et al. (2013) show that young and small firms are very volatile over the cycle, while small but older firms have less cyclical employment growth. I plan to extend these facts to cover the response of hours growth rates to changes in aggregate unemployment and output by firm-level observable characteristics such as size and age. Understanding the properties of firm-level growth rates in hours and employment over the firm distribution will provide valuable insight into the dynamics of unemployment, employment, and labor utilization at the firm level.

1.7.2 Conclusion

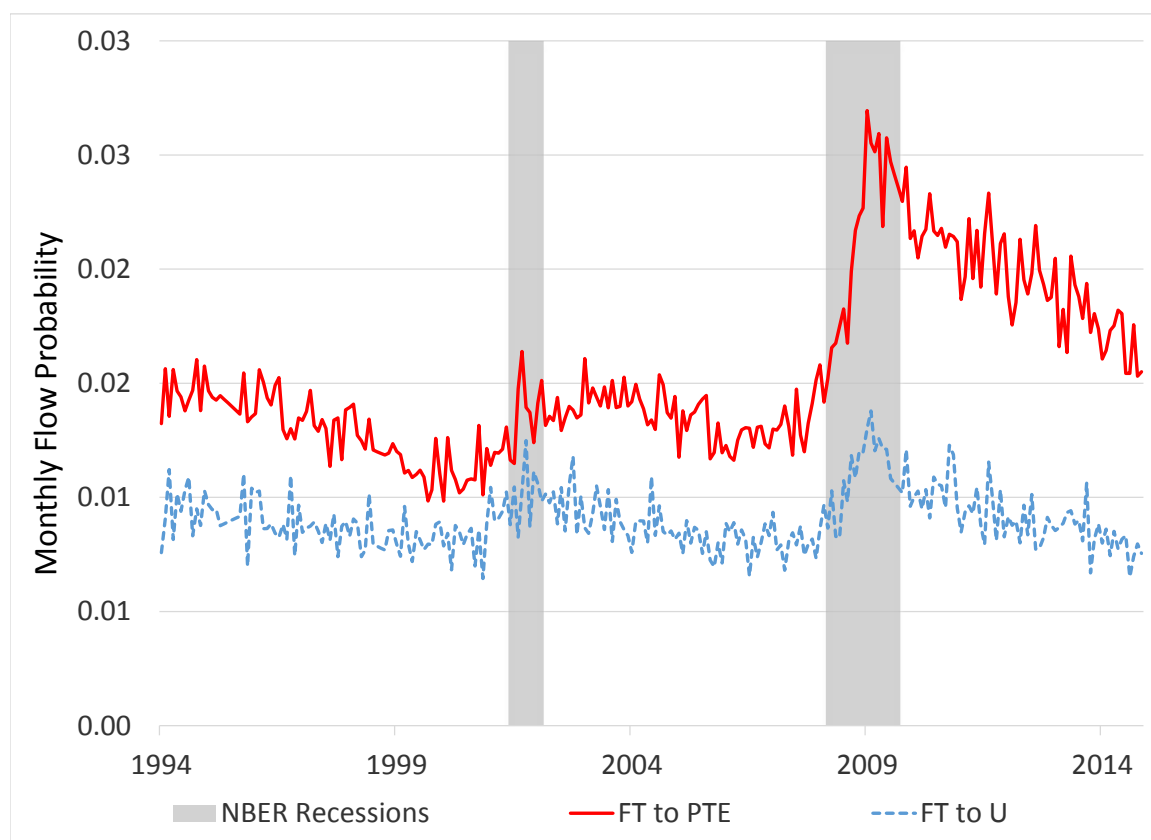
In this paper, I document that fluctuations in part-time employment for economic reasons arise from within-job changes in hours due to slack work or business conditions. Motivated by this finding, I build a model to explain the cyclical movements in part-time employment based on firm-level changes in labor demand. I show that the model is capable of matching the patterns of worker transitions for part-time employment found in CPS data. Particularly, part-time employment utilization is transitory and characterized by high transitions to and from full-time employment. Firms also use part-time with layoffs, producing a high probability of separating to unemployment for part-time workers. Part-time utilization in the cross-sectional distribution of firms is dependent on firm characteristics such as age, size, and productivity. Over the business cycle, part-time employment and unemployment are countercyclical and display similar volatilities. Relative to the case with no part-time margin, the model can account for a significant increase in business cycle volatility in unemployment and vacancies.

Figure 1.1: Part-time Employment's Share of Aggregate Employment by Reason



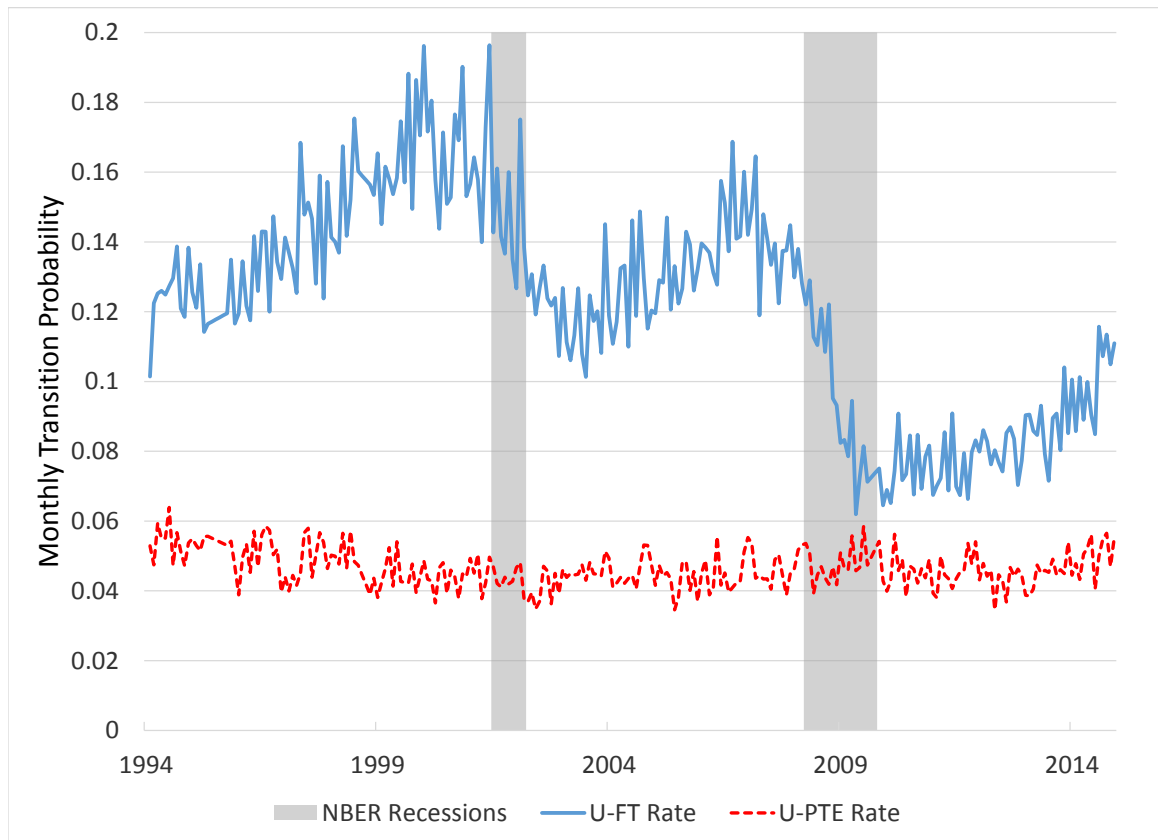
CPS, May 1955 - December 2014, from the Federal Reserve Economic Database of the Federal Reserve Bank of St. Louis. Gray bars in figures indicate NBER recession dates.

Figure 1.2: Monthly Probability of Exiting Full-time Employment

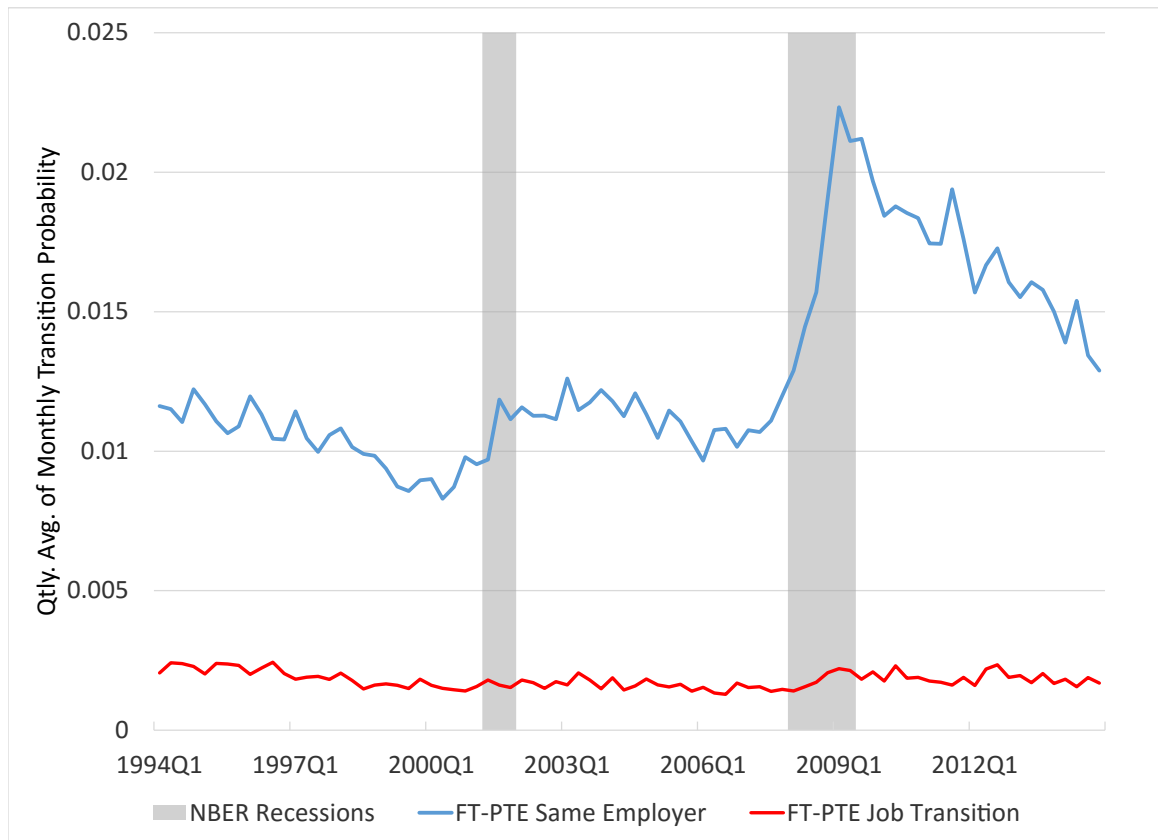


The dashed blue line plots the monthly transition probability of full-time workers to unemployment. The solid red line plots the monthly transition probability of full-time workers to part-time for economic reasons. The flow probabilities are constructed from CPS micro data from 1994-2014. Gray bars indicate NBER recession dates.

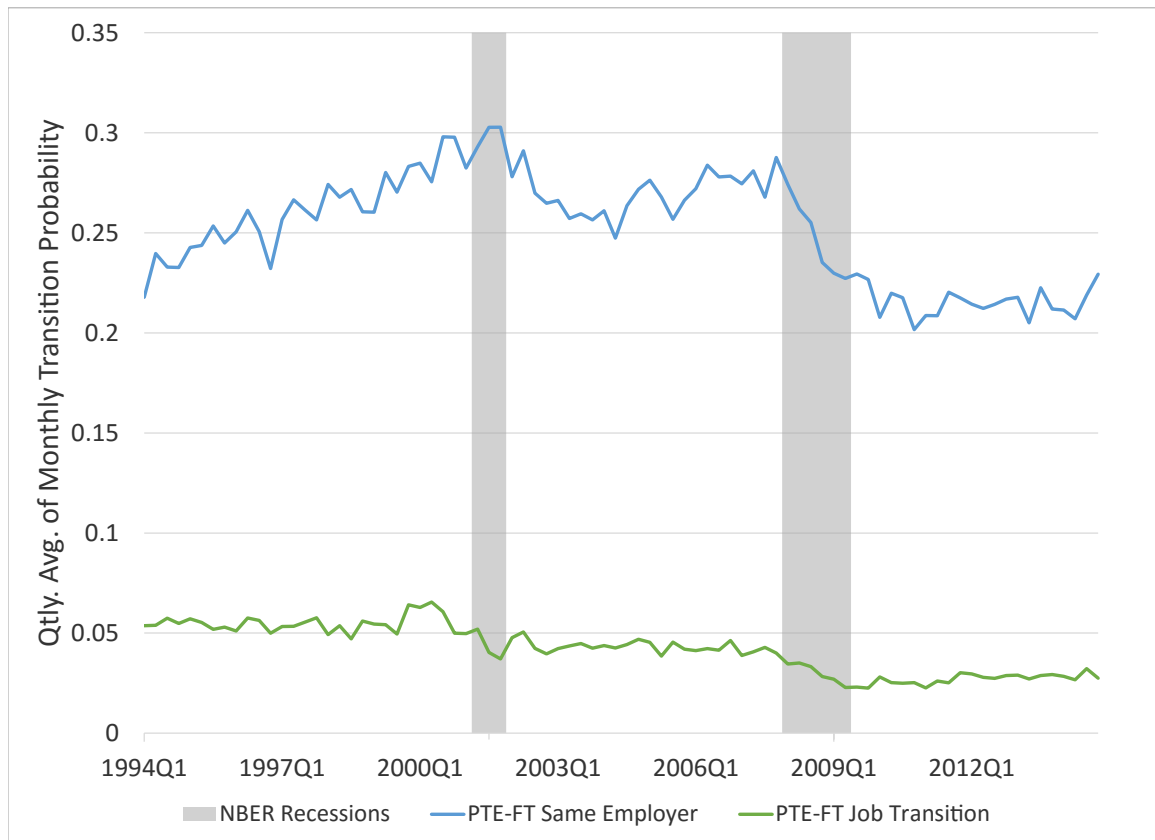
Figure 1.3: Monthly Transition Probabilities out of Unemployment



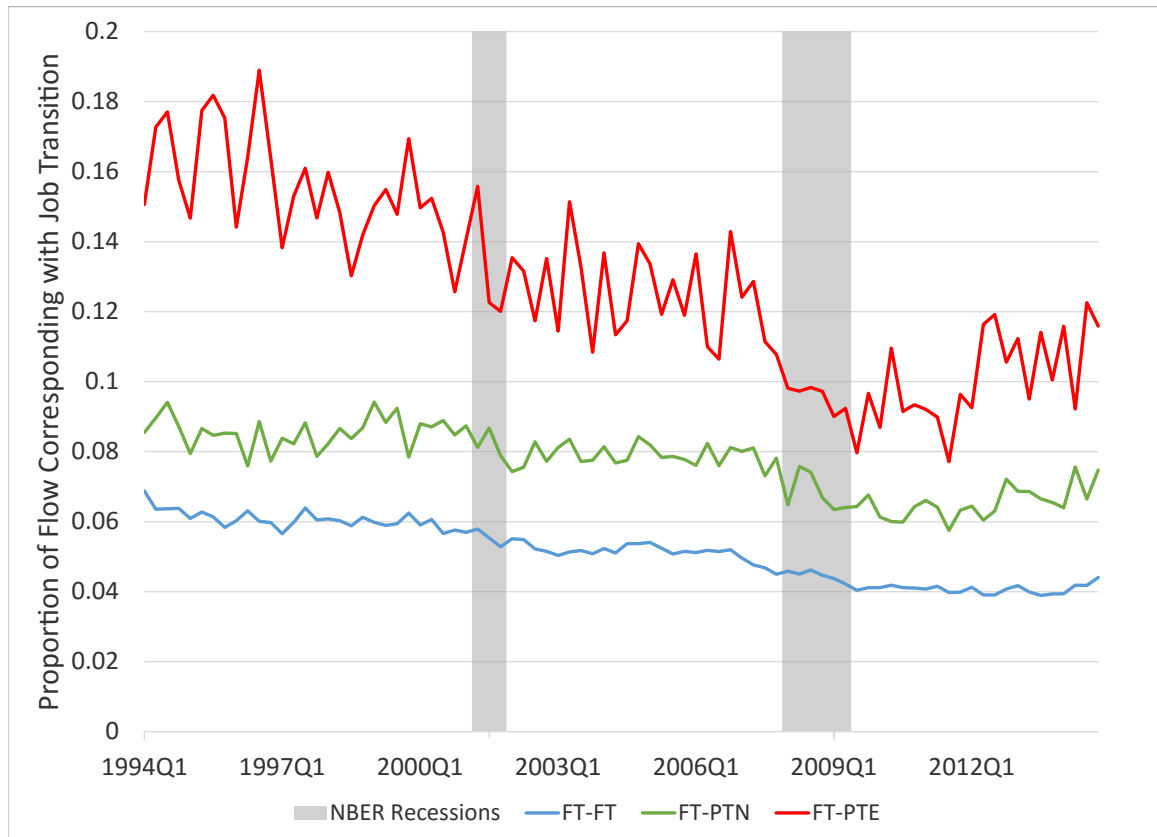
The solid blue line plots the fluctuating transition probability of a worker from unemployment to full-time work. The dashed red line plots the nearly constant probability of an unemployed worker moving to *PTE*.

Figure 1.4: $FT - PTE$ Transition Probability by Job Transition Status

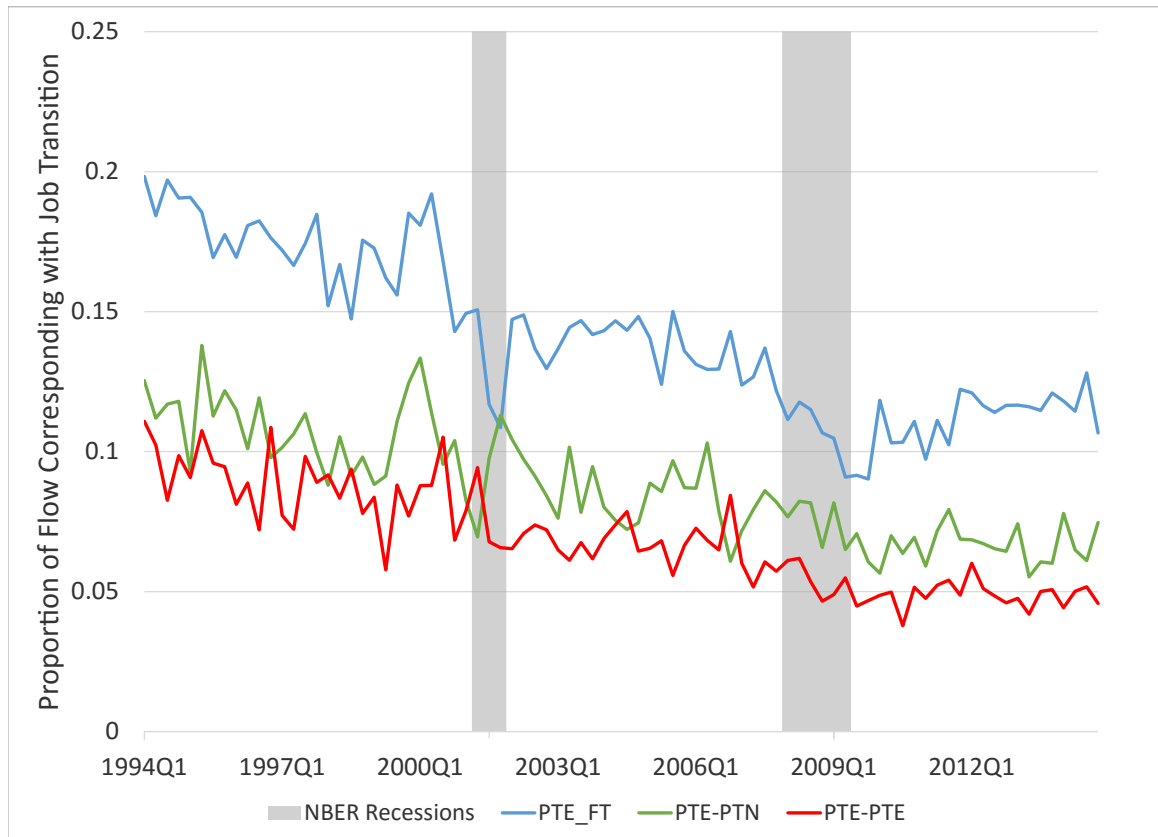
The blue line plots the quarterly averaged monthly probability of moving from full-time to part-time for economic reasons while remaining with the same employer. The red line plots the probability of the same transition while also experiencing a job transition.

Figure 1.5: *PTE* – *FT* Transition Probability by Job Transition Status

The blue line plots the quarterly averaged monthly probability of moving from part-time for economic reasons to full-time while remaining with the same employer. The green line plots the probability of the same transition while also experiencing a job transition.

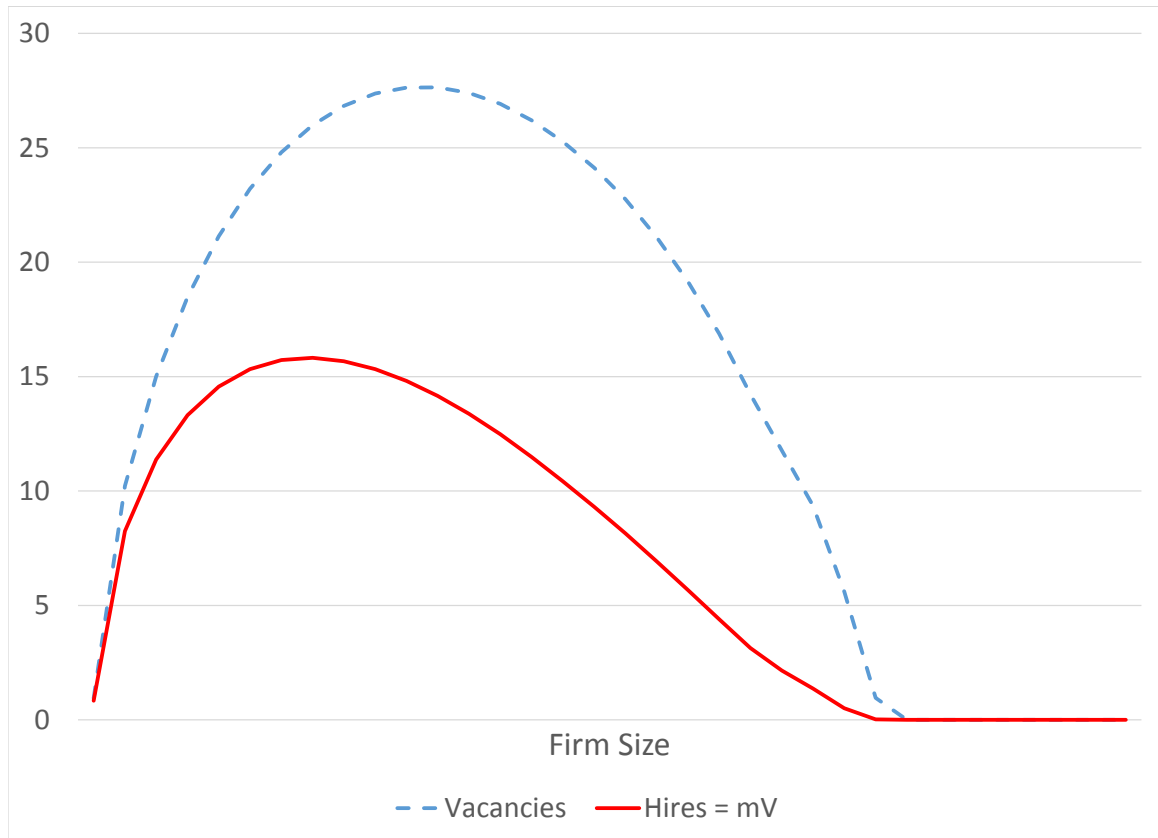
Figure 1.6: $FT - PTE$ Transition Probability by Job Transition Status

The quarterly averaged proportion of each transition probability out of FT which was accompanied with a job transition, defined as a change in the primary employer reported the previous month or a change in the number of jobs held if the respondent was a multiple job holder in either month.

Figure 1.7: *PTE* – *FT* Transition Probability by Job Transition Status

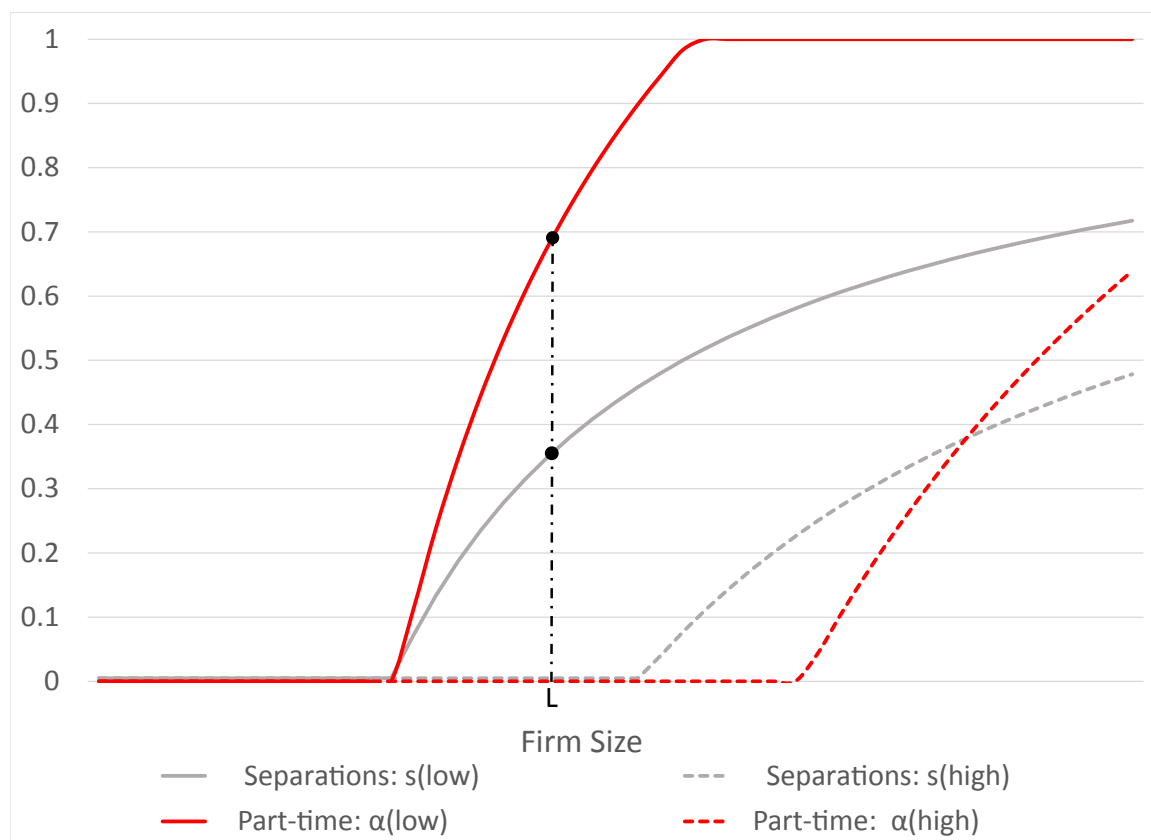
The quarterly averaged proportion of each transition probability out of *PTE* which was accompanied with a job transition, defined as a change in the primary employer reported the previous month or a change in the number of jobs held if the respondent was a multiple job holder in either month.

Figure 1.8: Vacancies and Hiring Policies over the Life of a Firm



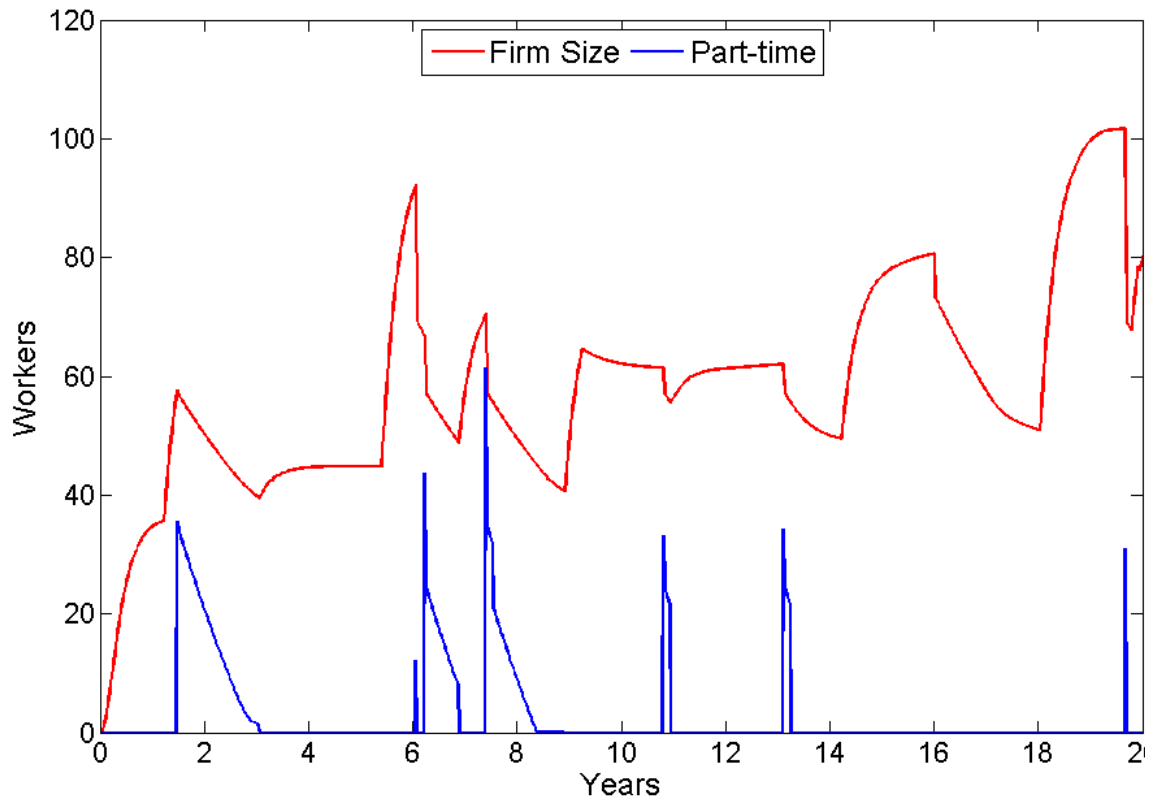
Vacancy posting and hiring policies of a firm as a function of current firm size. Vacancies are plotted with the blue dashed line. Total hires per period (in red) is the product of vacancies and the matching probability m per vacancy, which is declining in size.

Figure 1.9: Part-time and Separation Rates in Response to a Productivity Shock



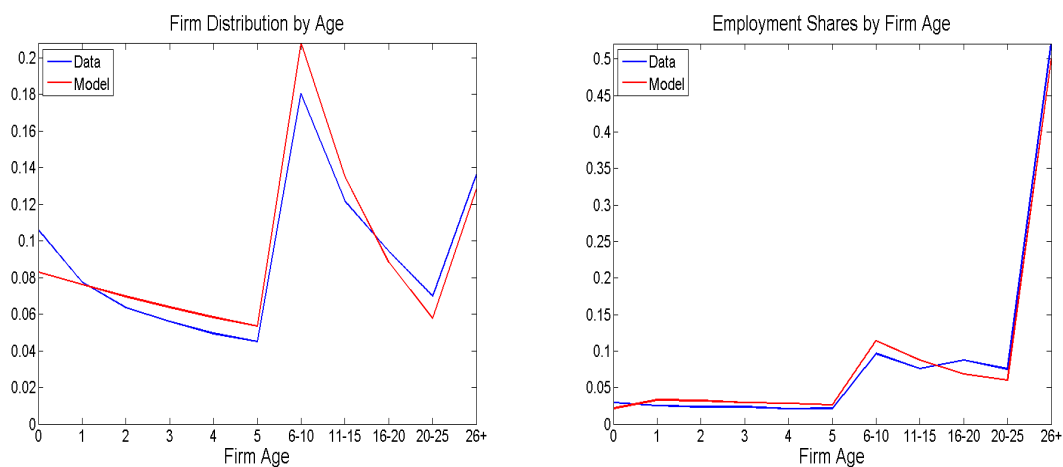
Dashed lines are the policies of a high productivity firm. Solid lines are the firm's policies after a low productivity shock. The black vertical line and dots indicate the choice of layoffs and part-time usage of a high productivity firm of size L after it receives a low productivity shock.

Figure 1.10: Dynamics of a Simulated Firm



The red line plots the firm's size L . The blue line plots the number of workers employed part-time, αL . Increases in firm size result from hiring, while sharp declines in size indicate layoffs. Smooth declines in firm size result from firm inaction and exogenous separations.

Figure 1.11: Distribution of Firms by Age (in years): Firm Shares and Employment Shares

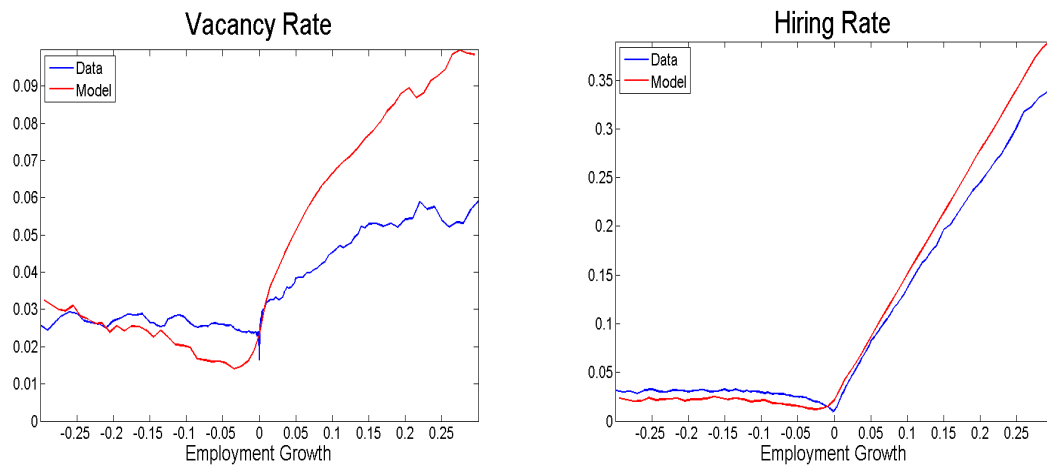


Cross-sectional relationship between firm age (in years) and firm shares/employment shares.

The blue lines are from Business Dynamics Statistics data by the Census Bureau for 2005.

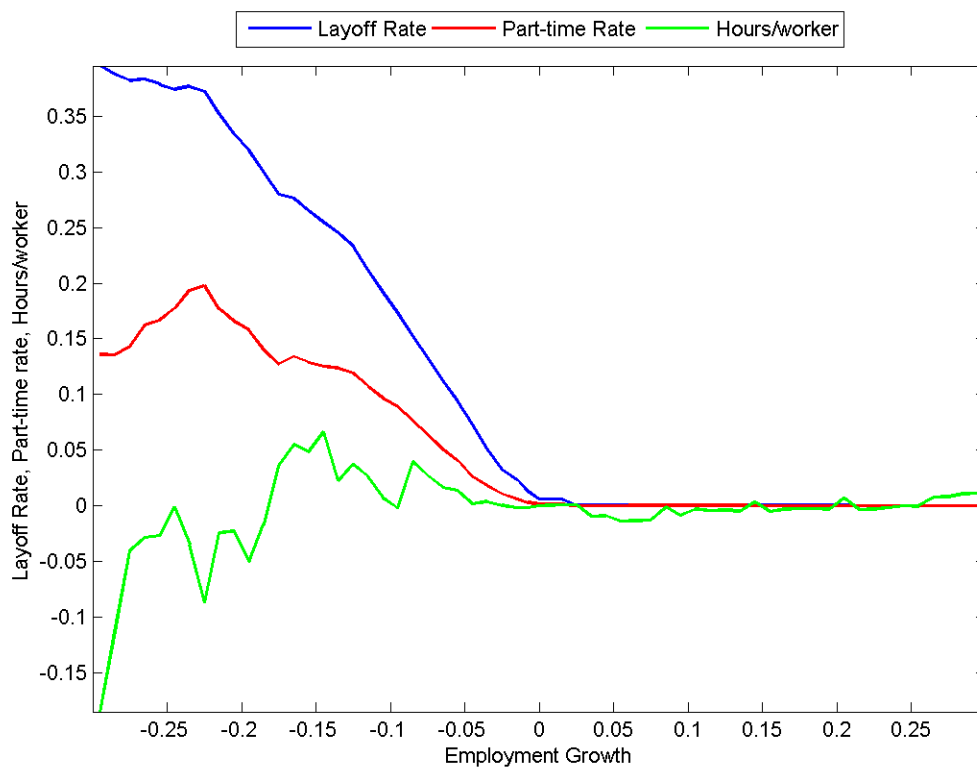
Model statistics are in red

Figure 1.12: Vacancy Rates and Hiring Rates of Firms



Monthly vacancy rates and hiring rates of firms, plotted by monthly employment growth rate. The blue lines are from data used in Davis et al. (2013). The red lines are model statistics.

Figure 1.13: Layoffs, Part-time, and Hours Growth



Layoffs, part-time, and weekly hours growth by monthly employment growth rate. The blue line is the layoff rate of firms, the red line indicates part-time utilization as a fraction of a firm's workforce, and the green line indicates the growth rate of average weekly hours for firms by employment growth.

CHAPTER 2

MATCHING LABOR FLOWS IN SEARCH MODELS WITH LABOR FORCE PARTICIPATION

2.1 Introduction

The study of labor market participation and its interaction with labor market frictions has been focused on matching the patterns in labor market flows between employment, unemployment, and nonparticipation over the business cycle. Achieving this goal has been somewhat problematic, as it is difficult to get flows into and out of nonparticipation to match the data. These difficulties become compounded when also trying to account for business cycle fluctuations, as the relative magnitudes of each flow must be consistent with data. Even small discrepancies in flows have large implications for stocks if the errant flow rates occur for a large population such as employment or nonparticipation. Accordingly, before attempting to model the co-movement of worker flows with the business cycle, it seems natural to understand first how to match the flow data in a steady state environment: this is the goal of this paper.

The motivation for incorporating a participation margin in labor market search models is that flows in and out of nonparticipation are quite large. CPS data over 1968-2007 show that about 4.7% of nonparticipants move directly to employment in the average month (the *NE* flow) and 2.4% begin searching, becoming unemployed (the *NU* flow). Given the size of the inactive population, the flow from non-participation is quite large and accounts for more new hires than those who find

work from unemployment. Another striking feature of these flows is that on average, 23% of unemployed workers will quit searching in a given month, moving from unemployment to inactivity.

This paper shows that in order to match flows between employment, unemployment, and nonparticipation in a search model, the transition process affecting workers' search decisions must differ by employment state. A single, employment-independent transition process for workers' search decisions will produce compositions of workers either in employment or nonemployment that will cause counterfactual flows. As this result comes from the identification of transition rates for participation and the stationary distributions they imply, it is not constrained to a particular model. The need for transitions to vary by employment state will be present in any three-state model of the labor market with search frictions, regardless of the mechanism used to create a participation margin. A better understanding of matching labor flows will help to guide future labor market models in matching these flows in both stationary and business cycle environments.

To illustrate this result, I introduce a general search model with three labor market states. The model provides an example for how transitions affecting labor force participation must depend on employment state, and will be convenient for identifying and estimating the shock process that affects a worker's search decision. What governs a worker's labor force participation status in this model is their 'type,' which determines whether they would choose to search when not employed. An interpretation of these types is a cost of searching. However, I allow for both 'types'

of workers to reach employment, though at different arrival rates to reflect their respective search intensities. Because of this environment, both types of workers may move directly to employment. Workers' idiosyncratic types are subject to a Markov transition process, producing the flows of workers between nonparticipation and unemployment. Finally, both types of workers separate from employment with some probability, returning to the nonparticipation or unemployed pool depending on their type.

Changes in a worker's 'Type', whatever types may be, can be observed by changes in a worker's reported job search decisions. This leads to a natural interpretation of type as heterogeneity in productivity, searching costs, or priors on such variables that would change a worker's search behavior when out of employment. The key factor is that a change in this 'type' affects a worker's search decision if not employed, but not necessarily their willingness to participate in the labor market. The fundamental difficulty with identifying any transition process for these types is the fact that they are difficult to identify in employment when search decisions are not observed. To overcome this problem of identifying types in employment, I use the 48 month panel of the 1996 Survey of Income and Program Participants (SIPP) to observe workers' search decisions prior to obtaining a job, and track their search decisions if they separate from the job during the survey. I estimate a Markov transition process for types from these employment spells. I find that the Markov process for employed workers I estimate from employment spells in the SIPP is consistent with what is needed in the model to match the average monthly flows in US labor market

data.

Many of the three-state labor market models can be mapped to different interpretations of the types I described above. The labor market models in the literature consider the heterogeneity of agents to be in their desire to participate, only allowing for changes between being employed or unemployed (participation), and nonparticipation to be driven by these transitions. If a participating ‘type’ of agent were to receive a transition shock, they would only move directly to nonparticipation from the change in ‘type’. Tripier (2004) and Veracierto (2008) find that in such a model, a positive aggregate shock causes a large spike in the unemployment rate through the NU flow as aggregate shocks move nonparticipants into unemployment, causing counterfactual unemployment movements. Krusell et al. (2011, 2012) allow for sufficiently large idiosyncratic shocks to labor market productivity to affect agents’ participation decisions, which allows them to partially match the flow data while avoiding a procyclical unemployment rate through the NU flow. The specification of a type change affecting participation in their models requires the transition rate of employed workers from participating to inactive types to drive both the UN flow and the EN flow simultaneously, since the separation rate for an employed worker who receives a bad shock to labor market productivity is 1. Since only one transition process governs all agents, this results in being unable to match the large UN flow without causing a counterfactually high separation rate to inactivity, making the EN flow too large.

The reason that a single transition process fails to match labor flow patterns

is due to the composition of worker types in employment that is generated from the transition process and flow probabilities. If the transition process is calibrated to match the flows to and from employment, that process will fail to match flows in nonemployment (namely, the large UN flow). If the transition process matches the observed transitions in search decisions in nonemployment, it will match the UN and NU flows but fail to match the composition of workers in employment. The transition process from matching the UN and NU flows results in a stationary distribution of workers with a ratio of 12 inactive types (who would choose not to search if out of employment) to 1 active type (who would search if out of a job). Although the number of workers moving to employment from nonparticipation and unemployment is roughly equal, having the same transition process as nonemployed workers, the employed converge to a similar ratio of types as in nonemployment. Since the separation rate of these workers is identical, this produces an EN flow that is about 10 times as large as the EU flow, which is at odds with the data. No matter how one chooses to identify the transition process, if it is assumed to affect all workers independent of employment status, it will always result in counterfactual flows either for the employed or the non-employed population. Allowing these transitions to be dependent on the worker's employment status allows one to exactly match flows in steady state.

I purposefully abstract from modeling heterogeneity in the population outside of this participation decision to focus on the transition process in this search decision necessary to match labor flows. Although it is very likely that heterogeneity, for

instance in the nonparticipant population plays a large role in the search and flow behavior of individuals, the goal is to understand what must change about the search behavior itself to match flows in the data. These changes could be driven by changes in heterogeneous characteristics of individuals. I provide evidence, however, from separation rates that workers seem to share the same separation rates regardless of their labor market status prior to a job or the labor market state they separate to. The only way for the employment to nonemployment (EU, EN) flows observed in the data to be consistent with a constant transition process would then require that the flows between nonemployment states such as the NU flow would be counterfactual.

The economic reasons for the difference in transition processes for employed versus nonemployed workers are difficult to identify, but have a natural interpretation. Those who are employed may become attached to the labor force through some mechanism like experience or networks that make search less costly over time spent in employment. Alternatively, increasing one's duration in nonemployment might make it tougher to search for a job because of discouragement or increasing costs of searching, and nonemployed agents increasingly choose not to search once out of a job.

In section 2.2, I introduce a simple model with participation and search, and outline how the model generates each of the monthly labor flows. I then show in section 2.3 that a transition process that is independent of an agent's employment status will fail to match the average flows in the data. In section 2.4, I use the 1996 panel of the SIPP to estimate this transition process for types in employment, and show that

allowing these transition processes to be employment-dependent allows one to match these flows in the data. In section 2.5, I show that this problem with matching flows with a single, invariant transition process is present in other specifications of search models with participation. Section 2.6 examines the model's ability to match average flow rates in a business cycle setting. Section 2.7 discusses the plausible interpretations of employment status affecting the transition process for agents as attachment to the labor force or a discouraged worker effect.

2.2 A Search Model

The model is a generalized search model in the spirit of Mortensen and Pissarides (1994). Although this model is used as an example, any search model can be written to produce arrival and separation rates for each type of worker and produce these flows. To incorporate a participation margin, I use a search decision when a worker is not employed. Agents are heterogeneous in that they consist of two types, dictating inactivity or unemployment when not employed. In this case, type completely dictates an agent's search behavior, but any mechanism such as a search cost can be used to generate such a search decision.

Workers match with vacancies in a frictional labor market, with probabilities dependent on their type. Once employed, workers earn a wage and separate at a uniform rate. Wages could be determined through any mechanism such as generalized Nash bargaining or directed search, so long as the separation rate remains exogenous in steady-state. An idiosyncratic shock process governs the transition of agents

between types.

2.2.1 Environment

The economy is populated by a continuum of workers with measure one and a continuum of firms of positive measure. Both workers and firms are infinitely lived. Time is discrete, and both workers and firms discount the future at $\beta \in [0, 1]$. Workers have linear utility functions over discounted future consumption.

$$\sum_{t=0}^{\infty} \beta^t c_t \text{ where } c_t \in \mathbb{R}_+$$

Firms maximize discounted lifetime profit $\pi_t \in \mathbb{R}$ in each period:

$$\sum_{t=0}^{\infty} \beta^t \pi_t$$

At the beginning of a period, an employed worker loses his job with probability $\delta \in [0, 1]$. If an employed worker loses his job, he cannot apply to the labor market in that period.

Agents face idiosyncratic uncertainty in their type, determining the probability s_i , $i \in \{n, u\}$ that they will be able to reach the labor market in a given period. If unemployed, they have probability s_u of reaching the labor market, and if a non-participant, they can reach the labor market with probability s_n , where $s_u > s_n$. The labor market in the economy matches workers who reach the labor market with vacancies randomly through a constant returns to scale matching function. If a worker reaches the labor market, they match with a vacancy with probability $p(\theta)$, where θ is the tightness ratio in the labor market. The probability that a worker of type i becomes employed is thus $\lambda_i = s_i p(\theta)$.

After matching, an agent of type i who is not employed receives b . Note that since all agents without a job get b , it can be thought of as home production, or that everyone receives an identical benefit from nonemployment. The classification of a non-employed worker as unemployed or inactive next period is completely dependent on an agent's type. To keep the type and labor force participation status of an agent distinguishable, I will refer to the *type* of the worker as a search cost type. Those who choose nonemployment are 'high search cost' types, and types who choose unemployment when not employed will be referred to as 'low search cost' types. Employed workers produce y units of output and consume w as specified in the labor market. At the end of the period, nature draws the type i' of each agent from probability distribution $\chi(i'|i)$. The expectation over the state tomorrow, \mathbb{E} , is over types.

2.2.2 Worker's Problem

The classification of unemployment vs. inactivity is completely dictated by an agent's type. If the agent is not employed and $type = u$, the agent searches and is classified as unemployed, and if $type = n$, the agent is classified as out of the labor force that period. Note that as long as $s_n > 0$, the job finding probability λ_n for type n workers will be positive since they can access the labor market without being classified as unemployed, and will move directly to employment if matched.

At the production stage, the type $i \in \{u, n\}$ non-employed worker's value

function is:

$$W_{non}(i) = \{b + \beta \mathbb{E}_i[\lambda_i W_{emp}(i') + (1 - \lambda_i) W_{non}(i')]\} \quad (2.1)$$

The employed worker's value function can now be characterized:

$$W_{emp}(i) = w + \beta \mathbb{E}[\delta W_{non}(i') + (1 - \delta) W_{emp}(i')] \quad (2.2)$$

Since there is no search in employment, the worker simply consumes his wage and separates with probability δ .

Firms and workers come together in the labor market through a reduced form, constant returns to scale matching function. Workers find a job with probability $p(\theta)$ where $p : \mathbb{R}_+ \rightarrow [0, 1]$ is twice continuously differentiable, strictly increasing, and strictly concave and satisfies the conditions $p(0) = 0$ and $p'(0) < \infty$. A firm matches with a worker with probability $q(\theta)$ where $q : \mathbb{R}_+ \rightarrow [0, 1]$ is twice continuously differentiable, strictly decreasing, strictly concave and satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$, and $\lim_{\theta \rightarrow \infty} q(\theta) = 0$.

The firm produces a constant output y while matched with a worker, paying wage w .

$$J = y - w + \beta \{(1 - \delta)J\} \quad (2.3)$$

Which can be simplified to:

$$J = \frac{y - w}{1 - \beta(1 - \delta)} \quad (2.4)$$

There is free entry of firms in the labor market, with a vacancy posting cost k . Thus,

$$k \geq q(\theta)J \quad (2.5)$$

and $\theta \geq 0$ with complementary slackness. Now I describe some properties of the model and its calibration, focusing on the characterization of the labor market flows to each state.

2.2.3 Flows

Flows between the three states determine the rates of unemployment, employment, and inactivity. These flows are determined by the arrival rates and separation rates in the model, as well as the transition of workers across types. Table 2.1 displays the mapping of components of the search model to each flow.

Table 2.1: Flow Breakdown

From	To		
	E	U	N
E	$1 - \delta$	$\delta * \% \text{“}u\text{” type in } E$	$\delta * \% \text{“}n\text{” type in } E$
U	$\lambda_u = s_u p(\theta)$	$(1 - \lambda_u)\chi(u' u)$	$(1 - \lambda_u)\chi(n' u)$
N	$\lambda_n = s_n p(\theta)$	$(1 - \lambda_n)\chi(u' n)$	$(1 - \lambda_n)\chi(n' n)$

1. Consider the flow from Unemployment (U) and Inactivity (N) to Employment (E), henceforth the UE and NE rates. The matching probability of workers in U or N is $\lambda_i = s_i p(\theta)$. The difference in the matching probability of nonpartic-

ipants and unemployed agents comes from the fact that $s_u > s_n$, reflecting that high search cost types have a lower chance of reaching the labor market than low search cost types, who are unemployed.

2. The UU , UN , NU , and NN flows for agents result from the probabilities of remaining non-employed and the transition matrix over types, χ . Note that for an agent in U , they can either move to E or N , or stay in U . We discussed the UE flow above. With probability $1 - \lambda_i$ the agent stays without a job. The agent's choice of search effort depends on the agent's type. If the agent's type changes, the agent will move between U and N the next period. The transition probability over agents' types affects the movement of agents between U and N in this way.
3. The EU and EN flows of workers from employment to unemployment and out of the labor force are determined by the separation probability δ of the worker. In this case with no aggregate uncertainty, it is innocuous to assume that separations are constant and exogenous at rate δ . The exogenous separation probability δ causes all workers to separate into non-employment at the same rate. The distribution of search cost types in the employment pool dictates the relative size of the EU vs. EN flow, while the magnitude of δ dictates the percentage of all employed workers who separate in a given month.

2.3 Matching Flows in the Data

The average monthly transitions from each state of the labor market in the US are well documented in the Current Population Survey (CPS) conducted by the Bureau of Labor Statistics. It is convenient to use for its consistency, high frequency and large sample size and is summarized in table 2.2.¹

Table 2.2: US Monthly
Flow Data

From	To		
	E	U	N
E	.954	.014	.031
U	.273	.495	.230
N	.047	.024	.928

From CPS 1967:6-2008:12

2.3.1 Fitting the Model to Data

To map flows from the model to the data, I choose an arrival rate for low and high search cost workers to employment to match the UE and NE flows. I then choose δ to match the total flow probability out of employment, so that $\delta = EU + EN$. To choose parameters for the transition matrix χ , I use the flow rates of workers between nonparticipation and unemployment. Since flows between U and N in steady

¹The CPS tables summarize flow data which was constructed by Robert Shimer. For additional details, please see Shimer (2007) and his webpage. <http://sites.google.com/site/robertshimer/research/flows>.

state only rely on the movement of workers between types, I can use the relative flows for UU, UN, NU , and NN to match the persistence of each type, u and n . If the only transitions from U to N come from transitions between types, this implies that an unemployed worker who did not gain employment has a probability of $\frac{UU}{UU+UN} = 0.6838$ of remaining a low search cost type (u) and staying unemployed. Similarly, with only two types, the probability that an inactive worker who fails to gain employment remains a high search cost type (n) and stays inactive is $\frac{NN}{NU+NN} = 0.9745$. This yields the following transition matrix for types:

$$\chi(i'|i) = \begin{vmatrix} pr(u'|u) & pr(n'|u) \\ pr(u'|n) & pr(n'|n) \end{vmatrix} = \begin{vmatrix} .684 & .316 \\ .026 & .975 \end{vmatrix}$$

This transition matrix yields a steady state population of types where the low search cost types who choose unemployment are about one-twelfth the population of inactive types. This is slightly higher, but similar to the data in the CPS over this time period, where average unemployment is about 6% of the *Labor Force*, or roughly 3.8% of the population in my sample, while the percentage of the population not in the labor force is on average 35.5%.

Recall that to match the flows out of employment, there is only one parameter, δ . The exogenous separation probability dictates the minimum rate of flows from employment to non-employment. It is the ratio of types in employment that controls the relative difference between the EU and EN flows. I fail to get variation in the relative magnitude of the EU and EN flows - they are always different by a magnitude of about nine, reflecting the relative sizes of the stocks of each type

of agent in employment based on the stationary distribution generated by χ .² It is this tension between the relative sizes of the EU and EN flows versus the stationary distribution of types from χ that makes matching flows in a 3 state model so difficult. The results of the initial calibration in Table 2.4 show that the calibrated model does not produce enough separations of employed workers to unemployment, and produces too many separations of workers to nonparticipation. This discrepancy in flow rates causes stocks to be off by a large factor as well. The fraction of the population in nonparticipation that is generated by the model is too high by about 8 percentage points.

Table 2.3: Data

From	To		
	E	U	N
E	.954	.014	.031
U	.273	.495	.230
N	.047	.024	.928
Pop:	.607	.038	.355

CPS 1967:6-2008:12

Table 2.4: Calibration

From	To		
	E	U	N
E	.950	.005	.045
U	.273	.497	.230
N	.047	.024	.929
Pop:	.544	.025	.431

Parameters: $\lambda_u = .273$,
 $\lambda_n = .047$, $\delta = .050$, χ

The result that the NE and UE flows vary by a factor of about nine in my calibration comes directly from the fact that the employed population is converging to

²Although χ produces a stationary distribution of 12 to 1, the roughly equal inflow of types into employment and separation rates prevent the employed pool from converging completely to the stationary distribution generated by χ .

the same stationary distribution of 1 low cost type to 12 high cost types. The separation rate to unemployment and nonparticipation is dependent on only one parameter, δ . In section 2.4, I show that a type-independent separation rate is consistent with data on outflows from employment. If the data is consistent with the same separation rate for both types of agents, it is the evolution of types in the employment pool that must be counterfactual. I now propose that I allow the persistence of agents' types to vary depending on whether they are employed or not employed, and estimate this transition process along with the separation rate for employed workers using the 1996 panel of the Survey of Income and Program Participation.

2.4 Transitions from SIPP Data: Estimating χ and δ

Incorporation of a transition process that depends on a worker's employment status should help the model to match the flows in the US labor market data. It would be ideal to use observed movements of agents in micro level data to verify that transitions between worker types in the data are consistent with what is needed to match flows in the model. As my framework allows both types of agents to be employed, transitions between types are only observable in nonemployment when agents' search behavior can be observed, making it difficult to pin down what the transition process for employed individuals would be in the data.

With longitudinal data, one can estimate this transition process by observing the search decisions of agents before and after complete employment spells of varying durations. Unfortunately, the rotation sampling and short duration of household

participation in the Current Population Survey makes it only possible to observe extremely short employment spells from match to separation. To overcome this problem, I use the Survey of Income and Program Participants (SIPP), which is a large panel survey of households with demographic, income, and labor force participation information for individuals over the span of 3 to 4 years. I use the 1996 panel of the SIPP data as it is the longest panel at 48 months. It also incorporates many of the redesigns used in the CPS after 1994. The following table shows the monthly labor flows for the SIPP from 1996:1-1999:12.

Table 2.5: SIPP Flows,
'96-'99

From	To		
	E	U	N
E	.980	.006	.014
U	.201	.692	.107
N	.025	.009	.966
Pop:	.620	.025	.355

I use the panel structure of the 1996 SIPP data to identify agents' types when entering employment by observing whether agents who move to employment came from inactivity or unemployment in 1996. I then follow these individuals over the course of the panel and observe the search decisions of workers once they separate from their job. In this way, I have the starting and ending type for every complete job

spell started in 1996. I choose to only observe spells that begin in 1996 to avoid oversampling short employment spells.³ I can estimate from this sample of employment spells a Markov transition process that will fit the observed transitions of type from before and after each spell.

To measure the transition rate of worker types in employment, I estimate a Markov transition matrix where not every state is observable in each period, as outlined in Sherlaw-Johnson et al. (1995). An ideal data set would have information on a worker's intention to search should they end up jobless in each period, so that a worker's type could be identified throughout an employment spell. Since this state can only be observed the month before and after a job spell by observing search activity when the worker is not employed, the employment spell is effectively a length of unobserved transitions over type. In this sense, each complete job spell can be seen as one observation of the outcome of the transition matrix for employed workers multiplied by the probability of remaining employed for each type for t periods. Besides the transition matrix, this also provides an estimate of the separation rate for each type. I then test the restriction that the separation rate is independent of type. Let the transition matrix $\chi_{1-\delta}$ include the probabilities of remaining employed and possible transitions between types. Then $\chi_{1-\delta}$ is:

$$\chi_{1-\delta}(i'|i) = \begin{vmatrix} (1 - \delta_u)pr(u'|u) & (1 - \delta_n)pr(n'|u) \\ (1 - \delta_u)pr(u'|n) & (1 - \delta_n)pr(n'|n) \end{vmatrix}$$

And the transition matrix χ_δ incorporating the probability of separating from em-

³Fujita and Moscarini (2013) use a similar approach when observing employer recalls in the 1996 panel of the SIPP.

ployment can be written as:

$$\chi_\delta(i'|i) = \begin{vmatrix} (\delta_u)pr(u'|u) & (\delta_n)pr(n'|u) \\ (\delta_u)pr(u'|n) & (\delta_n)pr(n'|n) \end{vmatrix}$$

The probability of observing an ij transition after an employment spell of length t is then:

$$pr(ijt) = ((\chi_{1-\delta})^{t-1} * (\chi_\delta))_{ij}$$

This is the ij_{th} component of the transition matrix and separation probabilities after t periods of transitions.

Let O_{ijt} denote the number of observed transitions from state $i \in \{u, n\}$ to state $j \in \{u, n\}$ occurring after spell length t and $(\chi^t)_{ij}$ the ij_{th} component of the matrix $(\chi_{1-\delta}^{t-1}) * (\chi_\delta)$, (the probability of a worker in state i being in state j and separating after t time units). The likelihood of observed data Y given transition matrix χ is represented as:

$$g(Y|\chi) = \prod_i \prod_j \prod_t ((\chi^t)_{ij})^{O_{ijt}}$$

And the log-likelihood is given by:

$$\log g(Y|\chi) = \sum_i \sum_j \sum_t O_{ijt} \log((\chi^t)_{ij})$$

I estimate the transition matrix χ from 5,786 job spells initiated in 1996 which terminate before the end of the sample. Each job spell has an initial and final state corresponding to the worker's search activity the month before and after

employment, and the length of the employment spell in months. It is important to note that this estimation ignores incomplete spells, as only complete employment spells yield enough information to estimate the transition process. The estimation does not include workers who entered the data sample in employment, as their initial type cannot be observed by whether they joined employment from nonparticipation or unemployment. Similarly, I must exclude workers who leave the sample still employed, as their type cannot be inferred until a worker separates. About 38% of employment spells started in 1996 do not separate before the end of the sampling period.

The resulting transition matrix is estimated from the complete employment spells, with standard errors in parentheses:

$$\widehat{\chi_{Emp}}(i'|i) = \begin{vmatrix} pr(u'|u) & pr(n'|u) \\ pr(u'|n) & pr(n'|n) \end{vmatrix} = \begin{vmatrix} .805 & .195 \\ (.033) & (.033) \\ .100 & .900 \\ (.030) & (.030) \end{vmatrix}$$

Compared to the transition matrix identified for the nonemployed population as outlined in Section 2.3:

$$\chi_{Non}(e'|e) = \begin{vmatrix} .880 & .120 \\ .017 & .983 \end{vmatrix}$$

The estimated δ_i parameters are:

$$\begin{array}{ll} \hat{\delta}_u = .117 & \hat{\delta}_n = .122 \\ (.040) & (.024) \end{array}$$

A likelihood ratio test produces the LR statistic of 0.172 compared to the critical value of $\chi^2_{0.05;1} = 3.841$, and thus fails to reject the null hypothesis that $\delta_n = \delta_u$. This

rules out the possibility that different separation rates by type can account for the different flow rates out of employment for unemployed workers and nonparticipants. The restricted estimate for δ that is identical for each type is:

$$\hat{\delta} = .121$$

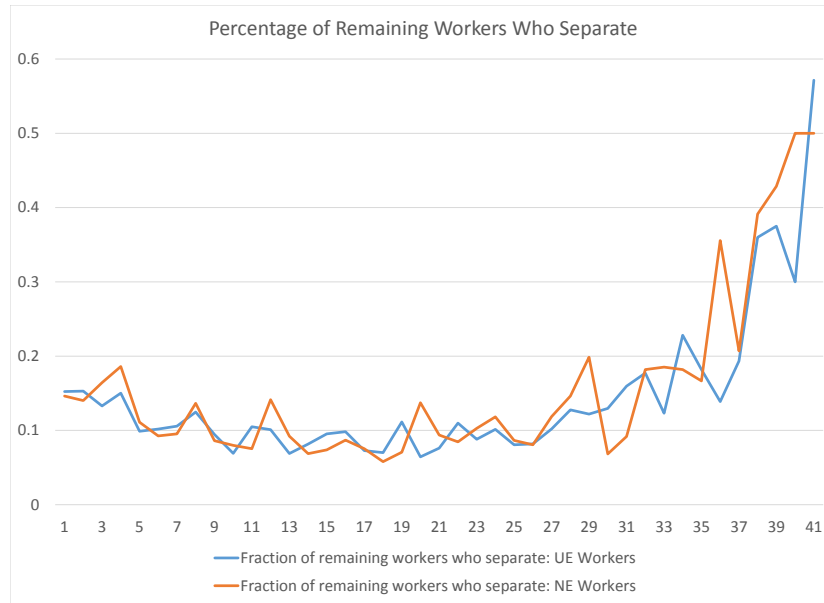
$$(.007)$$

Figure 2.1 also illustrates that the two groups experience roughly the same separation rates for the duration of the sample. I split the observed employment spells into those that originated from U and N . I repeat the same exercise but split the population by workers' types when they exit, regardless of their type upon entry in figure 2.2.

The fact that δ cannot account for the difference in separation rates to U and N implies that these differences must come from the evolution of these types in employment. The difference between the transition matrix for employed individuals and that of the nonemployment pool is that searching types are more persistent in the employment pool than in the nonemployment pool. In fact, the stationary distribution of the transition matrix produces the appropriate stock of 2/3 “high search cost type” workers and 1/3 “low search cost type” workers in the employment pool. Although there are an infinite number of Markov processes that generate a particular stationary distribution, estimating the transitions of types over employment spells identifies the unique transition process that best fits the data for this subset of employed workers.

In the examples in this paper, I have used the CPS flow rates as they are available for a longer time frame than for the SIPP. Using the transition process estimated

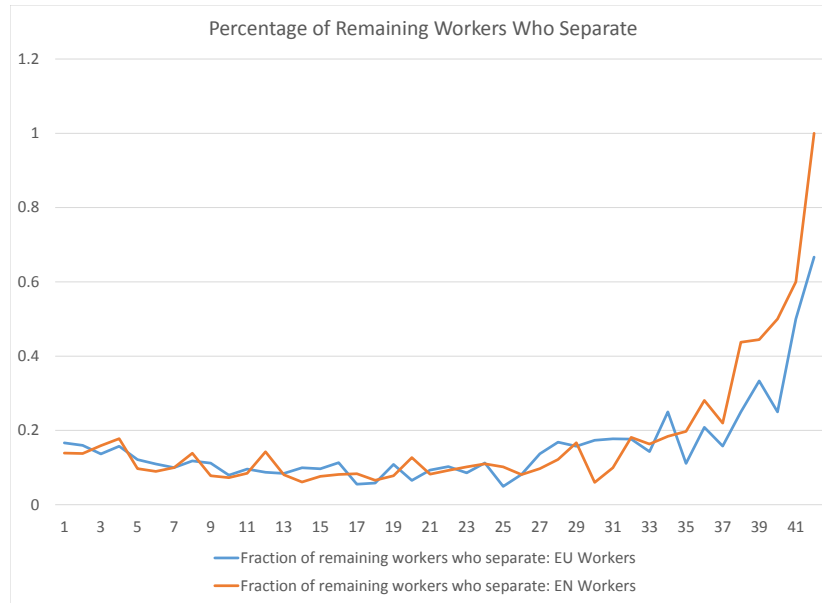
Figure 2.1: Separations of Workers by Initial Type Before Employment Spell



in SIPP data could be problematic for matching flows for the CPS, especially since the average monthly flows in the SIPP data are much more persistent than the flows in the CPS. However, the ratio of the *EU* and *EN* flows are very similar in both datasets. Using the transition process estimated from the SIPP data produces the same stationary distribution needed to match flows in the CPS. In the Appendix, I replicate this exercise using the average monthly flows in the 1996 SIPP as the data to match and achieve the same success.

I verify in Table 2.7 that the model can match average monthly flows in the data by calibrating the model while using $\widehat{\chi}_{Emp}$ as the transition process for types when workers are employed, and χ_{Non} when workers are not employed.

Figure 2.2: Separations of Workers by Type After Employment Spell



2.5 Robustness

2.5.1 Alternative Specifications of Participation

Now that it is shown that employment-dependence of the transition process over types can produce aggregate monthly flows consistent with data, I show that the need for different transition processes by employment state is robust to the specific characteristics of the model. It is necessary to show that the reason for the initially poor matching of flows in this model when all types have the same transition process independent of employment status is not a result of using a participation margin based on search decision only. Using standard alternatives where participation transitions

Table 2.6: Data

From	To		
	E	U	N
E	.954	.014	.031
U	.273	.495	.230
N	.047	.024	.928
Pop:	.607	.038	.355

CPS 1967:6-2008:12

Table 2.7: Using $\widehat{\chi}_{Emp}$ and χ_{Non}

From	To		
	E	U	N
E	.950	.016	.033
U	.273	.497	.230
N	.047	.024	.929
Pop:	.573	.038	.39

Parameters: $\lambda_u = .273$,
 $\lambda_n = .047$, $\delta = .050$, $\widehat{\chi}_{Emp}$,
 χ_{Non}

affect both search and employment will generate a similar result. This issue that flows out of employment conflict with the stationary distribution of worker types is robust to other specifications of labor force participation, and will result in poor matching of flows with any model with labor frictions.

To illustrate this, consider an alternative specification of labor force participation using a mechanism that affects both the employment and search decision, such as shocks to labor market productivity coupled with some disutility of working. In such a model, job offers arrive for all nonemployed individuals at the same rate, so the NE flow is the percentage of nonparticipants who receive a shock to become participants times the percentage of workers who receive a job offer. The remaining workers to receive the participation shock but do not receive a job offer comprise the NU flow. A shock to productivity inducing nonparticipation would show up as a flow from E to N or from U to N , depending on the employment status of the worker.

While the EU flow only comes from the separation rate δ , the EN flow comes from the participation shock hitting employed workers, causing them to drop out. This environment is similar to the one used in Krusell et al. (2011), but without assets. I abstract from assets for simplicity. The flows in their calibrated model are reported in Table 2.8:

Table 2.8: Benchmark Calibration from Krusell et al., 2009

From	To		
	E	U	N
E	.947	.021	.031
U	.407	.527	.066
N	.034	.044	.922

The mapping of model elements to flows using similar notation to Section 2.2 is displayed in Table 2.9.

If these flows were only accounted for by a shock to productivity such that high productivity workers choose to participate and low productivity workers choose inactivity, the transition process can be identified by the NE and NU flows and a population weighted average of the EN and UN flows. Doing so produces the following transition matrix:

$$\chi(e'|e) = \begin{vmatrix} 1 - (Epop * EN + Upop * UN) & Epop * EN + Upop * UN \\ NE + NU & NN \end{vmatrix}$$

Table 2.9: Flow Breakdown

From	To		
	E	U	N
E	$(1 - \delta) * \chi(u' u)$	$\delta * \chi(u' u)$	$\chi(n' u)$
U	$\lambda * \chi(u' u)$	$(1 - \lambda)\chi(u' u)$	$(1 - \lambda)\chi(n' u)$
N	$\lambda * \chi(u' n)$	$(1 - \lambda)\chi(u' n)$	$(1 - \lambda)\chi(n' n)$

$$= \begin{vmatrix} .966 & .034 \\ .078 & .922 \end{vmatrix}$$

This matrix produces the stationary distribution of participants to nonparticipants at a ratio of 2 to 1. However, doing so makes the size of the UN flow constrained to being too small at 0.066 in order to maintain the appropriate sizes of the EU and EN flows in the data. The point of this exercise is not to delve into a critique of their paper, but to outline that these tensions are present in any model with 3 labor market states. To generate any higher of a UN flow rate in this model in an attempt to match the data would require the increase of the rate that high productivity types become low productivity agents, but this would also generate counterfactually high separation rates from employment to nonparticipation. Even with the use of additional idiosyncratic shocks and considerable success in matching the comovement of flows over the business cycle, it is difficult for such a model to match the levels of the flows in the data.

2.6 Business Cycle Implications

The adjustment of transition processes for worker types to be dependent on employment state has been shown to generate steady state flows in line with the data. Now I consider if the same process can generate similar flows over the business cycle. Starting with the stationary distribution as the initial population, I generate series of flows and populations based on a series of draws of expansions and recessions, and measure the average flow rates in each state of the cycle.

To generate business cycle variation in the model, I draw a series of indicator variables for expansions and recessions from a Markov process which generates a fraction of time spent in recession of 0.156 and expected duration of recession of 9 months, consistent with the data from 1967-2008. I then measure the average arrival rates from U and N and the average separation rate from E in booms and recessions. I also measure the transition matrix for non-employed types using the whole sample.

Table 2.10: Simulated

Flows: Expansions

Data			
	E	U	N
E	.955	.014	.031
U	.275	.493	.230
N	.047	.024	.928
Model			
	E	U	N
E	.956	.015	.029
U	.275	.496	.230
N	.047	.024	.929

Table 2.11: Simulated

Flows: Recessions

Data			
	E	U	N
E	.952	.016	.032
U	.265	.510	.223
N	.046	.024	.929
Model			
	E	U	N
E	.953	.016	.032
U	.266	.502	.232
N	.046	.024	.930

The model is successful in matching average flows in expansions and recessions. The one discrepancy is that the UN flow is too large in the model during recessions. This is due to the UN flow being procyclical in the data, while the model will produce a countercyclical UN flow. The only mechanism that allows for changes in the UN flow in the model is the size of the Unemployment pool (through a decrease in the arrival rate and increase in the separation rate) and the constant Markov process for types. Since this produces a countercyclical unemployment stock, the UN flow will also be countercyclical in the model.

If we measure the average flow rates between U and N in expansions and recessions separately, producing a Markov process for types by business cycle state, we find that the changes in χ_{Non} between states is small, but it differs so that low search cost types become less persistent in recessions. That is, those who are in unemployment during recessions are less likely to move to inactivity than those who are unemployed during expansions. Incorporating this change in the UN transition probability generates a procyclical UN flow, and the model's simulated flow probabilities match the average flows in the data almost exactly.

$$\chi_{Non|Exp} = \begin{vmatrix} .6838 & .3162 \\ .0255 & .9745 \end{vmatrix}$$

$$\chi_{Non|Rec} = \begin{vmatrix} .6956 & .3044 \\ .0257 & .9743 \end{vmatrix}$$

The cyclical properties of average stocks is relatively consistent with the data. The properties of flows in the simulated model show that movements of workers from U and N require additional flows beyond simple transitions of binary types.

Table 2.12: Simulated

Flows with Procyclical

 UN : Expansions

Data			
	E	U	N
E	.955	.014	.031
U	.275	.493	.230
N	.047	.024	.928
Model			
	E	U	N
E	.956	.015	.029
U	.275	.495	.231
N	.046	.024	.929

Table 2.13: Simulated

Flows with Procyclical

 UN : Recessions

Data			
	E	U	N
E	.952	.016	.032
U	.265	.510	.223
N	.046	.024	.929
Model			
	E	U	N
E	.953	.016	.032
U	.266	.509	.225
N	.046	.025	.930

Specifying the heterogeneity that determines ‘type’ and introducing a distribution over this dimension should work to produce flows more in line with the data. Work by Krusell et al. (2012) demonstrates that such a model with participation based on idiosyncratic productivity shocks and search frictions is capable of matching the cyclical properties of stocks and labor flows over the cycle, though it fails to match the steady state levels of these flows.

2.7 Discussion: Attachment and Discouragement

I have so far outlined how the transition process of agents must be different in employment compared to nonemployment in order to match aggregate flows in the data. As mentioned earlier, the need for searching types to be more persistent or inactive types to be less persistent in employment has a natural interpretation as at-

tachment to or detachment from the labor force. The model is agnostic about the possible sources of shocks to search activity and the mechanisms through which agents' transitions change with employment state. I appeal to the literature on attachment to the labor force and detachment or discouragement to support this interpretation. As previously I established the nature in which participation decisions must change with employment, the question to try to answer from data is the presence and correlation with observable characteristics of such an effect in agents' decisions at the micro level.

Labor force attachment seems a fitting story for the difference between employed and nonemployed workers' transitions in types. There is some evidence from studies of employment and labor market data to suggest that past employment experiences help to dictate agents' future labor force participation. Ellwood (1982) finds that in labor force transitions of young men in the National Longitudinal Survey of Youth, unemployment and nonparticipation are particularly difficult to differentiate between for workers who are just entering the labor force. He attributes this difficulty to either discouragement or low attachment to the labor force, as many young workers spent extended periods of time in nonparticipation. After 4 years of being out of school, employment and unemployment increase (and nonparticipation decreases) dramatically. Abraham and Shimer (2001) show that the trend of increased unemployment duration is due to the increased labor force attachment of women. The topic of discouraged workers in unemployment is pervasive in the labor literature, though consistent evidence as to the size and importance of the phenomenon is less clear. Be-

nati (2001) summarizes the mixed findings of the empirical literature on discouraged worker effects and provides some evidence for a significant effect from analyzing the components of the out-of-labor force population in the Current Population Survey.

2.8 Conclusion

This paper illustrates the fundamental tension between stationary distributions of worker types and the matching of labor market flows in models with labor force participation and idiosyncratic transitions. I demonstrate how a single transition process for all workers' types fails to match the needed composition of types in and out of employment for matching flows. This leads to either counterfactual flows out of employment or the failure to match flows between nonparticipation and unemployment. I show that using a separately estimated transition process for employed workers and thus allowing these transitions to depend on employment state allows for the exact matching of labor flows between employment, unemployment, and nonparticipation. The necessity of idiosyncratic transitions to be dependent on employment status is not unique to the specification of a participation margin through any particular mechanism such as search costs or labor market productivity, but rather relies on any mechanism producing the right composition of workers in employment. The fact that separation rates are identical and independent of previous nonemployment state or future nonemployment state leaves the evolution of types in employment to produce a composition of workers that is consistent with these flows. Although the micro data available in the 1996 Survey of Income and Program Participants is not

sufficient to clearly identify one particular effect as the reason for these changes, plausible interpretations of this state-dependence include attachment to or detachment from the labor force.

CHAPTER 3

STUDENT LOAN DEBT AND LABOR MARKET OUTCOMES

3.1 Introduction

The federal student loan program has recently drawn the notice of policy groups, the media, and academia. A series of reports from the Federal Reserve Bank of New York (Haughwout et al. (2015), Lee (2013)) show that student loan debt is now the second largest source of household debt in the United States, surpassed only by home mortgages.¹ Hershbein and Hollenbeck (2014, 2015) report that 71% of bachelor's degree recipients in the class of 2012 graduated with student debt.² While the literature has focused primarily on studying the delinquency and default rates and designing optimal student loan programs (e.g. Lochner and Monge-Naranjo (2015), Looney and Yannelis (2015)), little is known about how student loans impact labor market outcomes: that is the focus of this paper.

Specifically, we establish empirically that student loan debt leads to lower earnings after graduation. To establish this result we use data from the Baccalaureate and Beyond Longitudinal Study: 1993/03 (B&B: 93/03). The B&B: 93/03 surveyed a representative sample of undergraduate students who received their bachelor's degree during the 1992-93 academic year, with follow-up surveys conducted in 1994, 1997

¹According to the data, total outstanding student loan debt is currently around \$1.2 trillion. This is more than total outstanding credit card debt and total outstanding car loan debt.

²Other papers that document the rise in student loans include Akers and Chingos (2014), Haughwout et al. (2015), Steele and Baum (2009), Woo (2014) and Arvidson et al. (2013).

and 2003. The study collects data at the individual, institutional, and administrative levels. The administrative data are linked to the National Student Loan Data System (NSLDS), which provides information on loan types and amounts disbursed throughout an individual's undergraduate study. The institutional-level data provide the relevant information necessary to determine eligibility for need-based Stafford loans (also known as subsidized Stafford loans) during the last year of schooling for each individual in the B&B: 93/03. From the individual-level data, we have information about earnings as well as standard individual characteristics.

Unconditional means show that earnings for those eligible to borrow in their last year of schooling were approximately 8.5% lower than those who were ineligible. Because some factors that affect borrowing, such as motivation and family support, also affect income, simple regression results will be biased. There also exist valid concerns of biased results due to parental income, given the persistence of intergenerational wealth. In order to estimate unbiased results, the variation in debt used to identify the effect should be exogenous to the outcome. Our identification strategy for estimating the causal effect of cumulative debt on earnings exploits a kink in Stafford loan program eligibility, specifically for subsidized Stafford loans.³

Subsidized Stafford loans are need-based loans; i.e., one must demonstrate financial need in order to be eligible. Financial need for any given academic year

³The two main components of the federal student loan program are subsidized and unsubsidized loans. For subsidised loans, the government pays interest while the individual is in school, whereas for unsubsidized loans, interest begins to accrue the moment the loan is disbursed.

is computed as the difference between annual cost of attendance (COA) and the sum of expected family contribution (EFC) and grants or scholarships the student receives from government or institutional sources. Students who cannot show financial need are still eligible for unsubsidized and parent PLUS loans, but will not receive subsidized loans.⁴ For students with financial need, the borrowing cap increases linearly with need.⁵ This results in a kink in the amount of permitted subsidized loans around the need threshold value of 0. To the left of the threshold, the slope of the amount of subsidized loans that can be borrowed is 0, to the right of the threshold, the slope for amount that can be borrowed is 1. This kink at the need threshold allows us to implement a regression kink (RK) design (Card et al. (2012)).⁶

We take advantage of the fact that dependent undergraduate students had access to only subsidized Stafford loans through the 1992-93 academic year.⁷ Because all students in our sample graduated in 1993, any dependent student with positive loan amounts could only have obtained them through subsidized loans. As such, our benchmark sample consists of only dependent students.⁸ While we only have

⁴Parent PLUS loans are unsubsidized loans taken out by parents of students, and they usually carry a higher interest rate than do Stafford loans.

⁵ The maximum amounts allowed have varied over time and also depend on the year of enrollment. In 1992-93, students enrolled in the third year or higher were allowed a maximum of \$4,000 in subsidized loans.

⁶Recently, Turner (2014) and Marx and Turner (2015) implement a RK design to study the impact of the federal Pell Grant program on educational attainment and student borrowing.

⁷While independent students always had access to unsubsidized loans, dependent students could only avail of them from the 1993-94 academic year onward.

⁸In a sample that includes both dependent and independent students 85% of all borrowers

information on financial need in the last year of schooling, our focus is on cumulative debt. This is because we want to understand the impact that debt has on earnings. Borrowing in the last year alone need not be representative of total debt accumulated by a student. This is especially true given the transitory nature of financial need from year to year, which in turn determines eligibility for Stafford loans. As a result, the mapping between financial need and total amount borrowed (simply referred to as debt hereafter) is not a sharp mapping, but rather a “fuzzy” one: we therefore use a fuzzy RK estimation technique.

Estimates show that an additional \$100 of borrowing reduces income one year after graduation by approximately 0.1%. Extrapolating this result, earnings for an individual with the mean level of borrowing are on average 5% lower than earnings of an individual with no debt. Moreover, our result is robust to various specifications. For example, in a sample that consists of both dependent and independent students, earnings for an individual with the mean level of borrowing are on average 3.8% lower than earnings of an individual with no debt. Results show that debt continues to have an impact, although of lesser magnitude, on earnings 3 years after graduation, but the effect fades away 9 years after graduation. The impact of an additional hundred dollars of student debt on earnings reduces to 0.04% by 1996. Unconditionally, the difference in earnings is persistent over time, but our identification strategy, which relies on the kink does not capture this effect. Consequently, individuals with debt

had only subsidized loans. Our results are robust to including independent students in the sample.

not only make lower nominal wages, but also have to use part of those lower wages to make loan repayments.

We then turn to economic theory to show that there exists a simple mechanism consistent with the empirical finding, whereby more debt leads individuals to quickly find employment rather than wait for an ideal job. Specifically, we use a simple one-period directed search model, along the lines of Moen (1997), in which individuals enter the labor market with an exogenous amount of debt. The standard trade-off in such models is the inverse relationship between the target wage level and the probability of finding employment at that wage. The higher the wage an individual targets, the lower the probability of being successful and, therefore, the longer the expected duration of unemployment.

When debt is modeled along the lines of student debt; i.e., it cannot be discharged in bankruptcy, we show that individuals search for lower-wage jobs as debt increases. This is because the individual is liable for her debt regardless of the job search outcome. Failure thus becomes increasingly undesirable in the level of debt. Consequently, as debt increases, a successful job search at a lower wage is preferred over the risk involved with searching for a higher wage. Crucial to this result is that student debt is not dischargeable in bankruptcy. In fact, we show that if an individual is allowed to discharge her debt, higher debt causes her to search for higher wages at lower probabilities of success. The intuition behind this result is that as debt increases, the increase in utility from successfully finding a job at a given wage decreases due to higher debt payments. Since she knows she can declare bankruptcy

if she is unsuccessful, default and bankruptcy puts a floor on her outside option and causes her to search for higher paying jobs with a lower probability of success – the exact opposite of our observation.

The two scenarios emphasized above are arguably very stark: either debt cannot be discharged, or, when it can, individuals automatically declare bankruptcy when their job search fails. However, there is a large literature documenting and trying to account for the relatively high frequency of loan delinquency and default on student loans (e.g. Gross et al. (2009), Lochner and Monge-Naranjo (2014), and Looney and Yannelis (2015)). Extensions of our model that explicitly allows for these phenomena suggest that what is key for our result is that individuals must eventually pay their debt, rather than the exact path that leads to that outcome. However, preliminary results suggest that the low take-up rate of income contingent repayment schemes is difficult to rationalize. Indeed, Ionescu (2011) finds that a policy allowing for income-contingent repayments that is restricted to financially constrained borrowers induces a 0.8 percent increase in welfare. Additionally, grants can also increase welfare if they lead to a reduction in student debt. Abbott et al. (2013) determine that there are significant short-run impacts of expanding tuition grants, especially of the need-based component of such grants. Our model would lead to a similar conclusion as grants along this margin would reduce the amount of debt taken up by the students.

While the literature linking student debt to economic outcomes post graduation is rather thin, there are a few notable exceptions. For example, the notion that

individuals who graduate with student debt tend to value finding a job quickly rather than risk looking for extended periods of time is consistent with Baum (2015), who documents that student debt discourages entrepreneurship. She notes that in addition to the mechanism emphasized here, individuals with student debt may have limited access to credit. Similarly, student debt leads to delayed home ownership (Mezza et al. (2014)) as well as marriage and fertility (Gicheva (2013) and Shao (2014)).

The rest of the paper proceeds as follows: in Section 2 we describe the relevant institutional details on the federal student loan program. We describe our data and sample selection process in Section 3. In Section 4, we discuss and carry out our empirical approach. Section 5 presents the directed search model and related theoretical results, and we conclude in Section 6.

3.2 Institutional Details

In this section, we review various institutional details, including the difference between Stafford subsidized and unsubsidized loans and how eligibility for the two loan types has changed over time. This information explains why we focus initially on dependent students, and why the fortuitous timing of the B&B 93-03 survey permits us to take advantage of the regression kink design. The following institutional details are mostly obtained from a National Center of Educational Statistics (NCES) report (Berkner (2000)).

3.2.1 Subsidized and Unsubsidized Loans

Federal student loans can be categorized into Stafford subsidized and unsubsidized loans, Perkins loans and parent PLUS loans. Of these, Stafford subsidized and unsubsidized loans are the most prominent in terms of dollar volume and number of borrowers and we therefore focus our attention on just the Stafford loan program.⁹ The first serious difference between subsidized loans and unsubsidized loans is the time at which interest begins to accrue.¹⁰ For subsidized loans, students are not charged interest while they are enrolled at least half time and during the grace period that follows (usually six months). The federal government subsidizes the cost of such loans by paying the interest over the duration. The federal government does not pay the interest on behalf of students who carry unsubsidized loans. Interest begins to accrue upon disbursement, which is then added to the principal. As a result, the amount owed on a subsidized loan will be the same as the principal borrowed when repayment begins; the amount owed on an unsubsidized loan will be the original principal borrowed plus accrued interest. The second main difference between the two loan types is the respective eligibility requirements. The requirement to qualify for subsidized loans has been consistent over time: a student must demonstrate financial need (this will be discussed in detail below). The requirements to qualify for unsubsidized loans have varied over time, however, especially with respect to dependent status. Before

⁹Perkins loans are need-based subsidized loans that are disbursed by the educational institution and carry a fixed interest rate of 5%. One has to prove exceptional need to be eligible for Perkins loans. PLUS loans are taken out by the parents of a dependent student to help cover expenses remaining after having taken out subsidized and unsubsidized loans.

¹⁰They sometimes carry different interest rates as well.

proceeding further, let us first consider how the government distinguishes dependent from independent students for federal loan purposes.

Most undergraduates under the age of 24 are classified as dependent while enrolled.¹¹ Those undergraduates who are 24 years of age or older and all others who are married, have children or dependents for whom they provide more than half support, or are veterans, orphans or wards of the state (for example, children in foster care) are classified as independent. For dependent students, the income of the parents is a major consideration in determining the need for financial aid; for independent students, only spousal and student income is considered. Federal student loans are capped on an academic year basis. There are separate limits for subsidized loans and for unsubsidized loans and the combination of the two. These limits are also a function of the year of enrollment. Independent students are allowed to borrow more than are dependent students by combining subsidized and unsubsidized loans. The rationale for this provision is that additional loan funds are available to dependent students through their parents, while independent students are not expected to be able to rely on parental financial assistance. Having established the distinction between dependent and independent students, we move forward with how loan policies have varied over time for these groups.

The 1986 Higher Education Act (HEA), also known as the 1986 Reauthoriza-

¹¹More specifically, the student must be less than 24 years of age at the start of the academic year. This means that an average high school student entering college at the age of 18 years would remain a dependent for the first six years of his undergraduate education.

tion, established provisions for six academic years: 1987-88 to 1992-93.¹² The major federal student loan program consisted of guaranteed student loans called Stafford loans during this period. Banks and other lenders provided the funds for Stafford loans, which were guaranteed against default by the federal government through guaranty agencies. Between 1987 and 1993, all Stafford loans were subsidized and available to both dependent and independent students on the basis of financial need (*need* henceforth). Unsubsidized loans were also available through Supplemental Loans for Students (SLS), a separate federal guaranteed student loan program. SLS loans were primarily intended to allow independent students to supplement Stafford loans, though some dependent undergraduates with exceptional need could also qualify.¹³

The Reauthorization of 1992 and additional legislation made substantial changes to the structure of the federal student loan program, generally beginning with the 1993-94 academic year. The separate SLS program was phased out and replaced by unsubsidized Stafford loans, now available to both dependent and independent students. Loan limits were also increased. The existing loan program was renamed as the Federal Family Educational Loan Program (FFEL). In addition, the William D. Ford Direct Loan Program was established as an alternative system of processing

¹²Many of the changes of a Reauthorization go into effect on October 1 of that year (which is the beginning of the federal fiscal year). However, more than three-fourths of the student loans for a given academic year are processed before October, so the full effect of the changes is not evident until the following academic year.

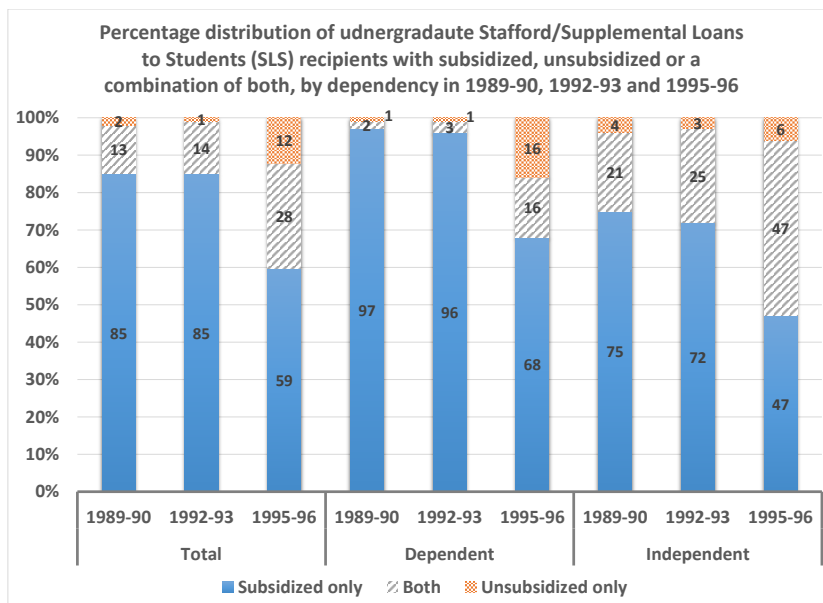
¹³Dependent undergraduates whose parents applied, but were rejected, for federal parent PLUS loans (which require meeting creditworthiness criteria) were able to qualify for SLS loans. This provision was continued after 1993 and is still in effect, allowing some dependent students to obtain unsubsidized loans at the independent student maximum amounts.

Stafford loans. In the direct loan program, funds are provided directly through the Department of Education. From a student perspective, the two loan programs were more or less equivalent. The 1998 Reauthorization of the Higher Education Act made no major changes to the structure of these programs, or their eligibility requirements and loan limits. FFEL ended with the enactment of the Health Care and Education Reconciliation Act of 2010. As of July 1, 2010, all FFEL loans ceased, and the direct loan program is the only source of student loans.

Figure 3.1 shows the percentage distribution of subsidized and unsubsidized loans by dependency status. Of all dependent borrowers, 97% and 96% had only subsidized loans in 1989-90 and 1992-93 respectively. This lends credence to the fact that most of the changes in the 1992 Reauthorization came into effect only with the 1993-94 academic year. This is further supported by the fact that in the 1995-96 academic year the proportion of dependent borrowers with only subsidized loans dropped to 68%; the rest had some combination of only unsubsidized loans or a mixture of subsidized and unsubsidized loans. As a result, any analysis of dependent students up to the 1992-93 academic year focusing on federal student loans will consist of subsidized loans almost exclusively. It is therefore important to understand how the government determines *need*, and how we might use variation in the *need* formula to establish a regression kink design.¹⁴

¹⁴We also note that over the years of interest, about 75% of independent students and 85% of the entire undergraduate student population only had subsidized loans. Therefore, it is conceivable that any pattern in borrowing for dependent students might still exist, though perhaps weakened somewhat, when including both dependent and independent students. We investigate the matter further when conducting robustness checks.

Figure 3.1: Distribution of Subsidized and Unsubsidized Loans by Dependency Status



Notes: This is Figure 4 reproduced from Berkner (2000).

Financial need is determined by comparing the cost of attendance (COA) to the ability to pay for them on the part of the student. COA is the sum of tuition, fees and other educational expenses (books, supplies, board, lodging, etc.). It is estimated for various categories of students by financial aid office at each institution based on factors such as attendance status, dependency etc.

The ability to pay is measured by an index called the expected family contribution (EFC). Students wishing to take out student loans for a given academic year fill out the Free Application for Federal Student Aid (FAFSA) in the previous spring. Since FAFSA requires tax information, it is usually filled out after filing tax returns.

The federal government determines EFC and provides the number to the student's institution, based on the information provided in FAFSA. The EFC is a complicated formula based on income and assets, with adjustments for family size and the number of family members enrolled in postsecondary education.

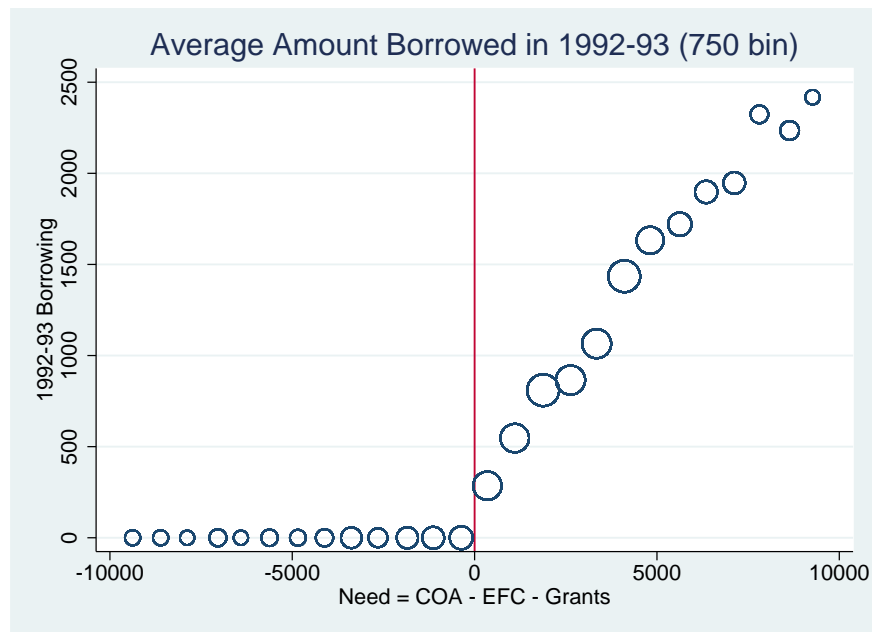
Need-based federal aid eligibility and amount is determined by comparing the COA and EFC. If the EFC is greater than the COA (negative *need*), the student is not eligible for need-based aid. If the EFC is less than COA, the amount of aid for which the student qualifies is equal to COA minus EFC. If the student receives any grants or other aid, the amount is subtracted from need. Any remaining need may be covered by a subsidized Stafford loan, up to the annual limit (as explained in footnote 5). Therefore, need can be quantified as:

$$need = COA - EFC - Grants.$$

Anyone with *need* less than or equal to 0 is ineligible for subsidized Stafford loans. Anyone with positive *need* is eligible for an amount equal to *need* up to the annual limit. Hence, the change in the slope of the amount that can be borrowed around the need threshold value of 0 (the slope is 0 for those below the threshold, and 1 for those above it) will prove key in the regression kink (RK) design studied later. Figure 3.2 shows the empirical distribution of loans taken out in the last year of schooling as a function of *need*. While our analysis in Section 3.4 uses cumulative amount borrowed rather than borrowing in the last year alone, Figure 3.2 nevertheless serves as a useful illustration of the policy.

Next we discuss the repayment, delinquency and default process in some detail.

Figure 3.2: Empirical Distribution of Stafford Loans in the Last Year of Schooling as a Function of *need*



Notes: Bin size is \$750. The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

Although these details are not immediately relevant to the empirical analysis, we believe that it is important to know these details in order to understand the underlying mechanism that is driving the empirical results.

3.2.2 Repayment, Delinquency and Default

Once a student graduates from school (or enrolls less than half time, or drops out), she receives a six month grace period before repayments begin.¹⁵ During this six month grace period a borrower can choose one of several repayment plans. The most popular is the ten year standard repayment plan. Under this plan, individuals make fixed monthly payments deemed actuarially fair. If the interest rate on the loans is fixed or the loans were consolidated, the monthly payments would be the same for every month. The monthly payments change annually if the interest rate is variable, but remain constant during the year (from July to the following June). Other variations of this scheme include the extended plan (for individuals who have cumulative borrowing above \$35,000) and the graduated plan, under which the payment amounts increase every two years over a period of ten years. There are also a host of income-based and income-contingent plans, where an individual pays a certain fraction of her adjusted gross income (usually between 10%-20%, depending on the plan) for 20-25 years, or until the loan has been paid off. Most of these income plans cap monthly payments to the corresponding amount in the standard ten year plan.

¹⁵Loans for individuals who enrol in graduate school automatically go into deferment, so no payments have to be made during this period. The grace period is nine months for Perkins loans, but they form a very small fraction of student loan program.

Any additional balance remaining at the end of the plan is forgiven, although it may be liable to taxation. Historically, loan servicing agencies have pushed the standard ten year plan, and it has been the most popular of all repayment plans by far. Only recently have income plans started to increase in popularity.¹⁶

An individual is deemed delinquent on her loans upon her first missed payment. At this point she must become current on her loans within nine months.¹⁷ The cost of remaining in delinquency increases with time. From a monetary viewpoint, the longer one remains in delinquency, the more one has to pay back due to interest accumulating on unpaid capital. If the loan is 30+ days delinquent, the individual can be charged additional late fees. Credit agencies get notified once it becomes 90+ days delinquent, negatively affecting access to credit. In addition, this can cause trouble when signing up for utilities, a cell phone plan, getting approval to rent an apartment and buying homeowner's insurance.¹⁸ Additional late fees can be added as delinquency continues. If the loan is 270 days delinquent (or more), it is termed as being in "default", and the consequences become more severe. At this point, the defaulted loans are assigned to a collection agency (collection fees are borne by the borrower); the government, moreover, has the power to garnish wages (up to 15%) or withhold tax refunds and social security benefits (up to 15%). The entire unpaid

¹⁶From our dataset, about 2% of respondents in repayment were in some form of income-based plan in 1994. According to the latest government data in Q2 of 2015, about 17% of individuals in repayment are in some income-based plan: <https://studentaid.ed.gov/sa/about/data-center/student/portfolio>.

¹⁷New government regulations are considering extending this period to twelve months.

¹⁸<https://studentaid.ed.gov/sa/repay-loans/default>.

balance of the loan and any interest is immediately due and payable. In addition, one loses eligibility for additional federal student aid as well as eligibility for deferment, forbearance, and income based repayment plans when in default.

In consequence, there is almost no means of getting out of student loan debt aside from repayment. Next, we proceed to describe the B&B 1993/03 data, as well as our sample selection process. The institutional details covered in this section lay down the foundation for the choices we make when selecting the sample. For example, why our focus on dependent students leads to a clean experimental design, and why we choose to drop those enrolling into graduate school immediately after obtaining their undergraduate degree.

3.3 Data

The data source for this study is the Baccalaureate and Beyond Longitudinal Study (B&B). The sample is derived from the 1993 National Postsecondary Student Aid Survey (NPSAS), a nationally representative cross-section sample of all postsecondary students in the US. The B&B is a nationally representative sample of students who received their bachelor's degree during the 1992-93 academic year. These students were then surveyed in 1994, 1997 and 2003. Data are collected from three sources: survey data, institutional data and student loan administrative data. Institutional data, in addition to describing institutional characteristics, contains information about expected family contribution, cost of attendance and grants or scholarships received by the student during the last year of school. Student loan administrative

data is obtained through the National Student Loan Data System (NSLDS): it contains information about the amount and type of loan disbursed during each year of undergraduate schooling.

3.3.1 Sample Selection

The initial data set derived from the 1993 NPSAS consists of approximately 11,200 observations.¹⁹ First, we drop respondents with missing information related to age, race, gender or citizenship, and those who dropped out of the survey before the 1994 round; this removed about 1,300 observations. We then remove those who are non-US citizens or disabled, approximately 600 observations. Next, we drop those enrolled in graduate school (master's/PhD/professional) during the 1994 survey, reducing the count by approximately 1,500. As indicated previously, being in graduate school automatically defers repayment for those who have student loans. As a result, the mechanism that drives the model does not apply to such individuals. This leaves us with a sample of approximately 7,800 observations. Dropping individual who chose to go to graduate school in 1994 might raise some concern if student loan debt has an impact on this decision. In the Section 3.4.3 we address this concern (along with other possible extensions). Our benchmark sample consists of only dependent students. As explained earlier while discussing Figure 3.1, this allows us to exploit the kink in the Stafford loan program and conduct a well-defined experiment in order to understand the impact of student debt on income. This leaves us with about 4,700

¹⁹Due to data security reasons, any reference to number of observations in this document is rounded to the nearest ten.

observations.²⁰ Finally, we only consider individuals who have need in the range (bandwidth) of $[-\$10,000, \$10,000]$. This is because a valid RK design considers only those values in the vicinity of the threshold value. We later conduct robustness checks to ensure that the results hold for different bandwidths. This leaves us with a sample of about 3,900 observations, all of whom graduated with a bachelor's degree in the same academic year and have similar experience (in terms of number of years in the labor force as well as the prevalent macro environment).

Given that we are trying to understand what affects income, education level, work experience, and the initial state of the economy would otherwise have to be controlled. A key feature of this data is therefore that they result in a fairly homogenous sample with amenable qualities for the question we are attempting to answer.

Given our mechanism, the primary outcome variable of interest is the respondents' annual earnings in 1994. Specifically, this is a measure of individuals annual salary at the job they held in April 1994. If it is indeed the case that individuals with student loans, knowing that they will have to start making repayments soon after graduation, have lower initial reservation wages, we should see this in the data soon after they graduate. Given that the students graduate in 1993, add the six month grace period before repayments being, repayments therefore being around the start of 1994.

²⁰A 60-40 split between dependent and independent students seems to be on par with the trends during that time period. See Digest of Education Statistics 1993 (<http://nces.ed.gov/pubs93/93292.pdf>) for more details.

3.3.2 Descriptive Statistics

Table 3.1 displays the characteristics of our sample. The first column shows characteristics for those respondents who were ineligible for a Stafford loan in the last year of their undergraduate degree program ($need < 0$). The second column displays characteristics for students who were eligible for Stafford loans in the last year of their undergraduate studies ($need > 0$) and column 3 reflects the entire sample. In addition, Table 3.1 is divided into three panels: student demographics and characteristics; educational related costs; annual earnings.

Under current trends, the race composition might seem highly skewed towards whites, a group accounting for 86% of the sample. This racial composition of the sample is fairly similar to the prevailing national statistic at that time however.²¹ The SAT scores were out of a maximum of 1600 (800 for the math section and 800 for the verbal section). For those students who had only an ACT score, we used an ACT-SAT conversion table to determine a comparable SAT score.²² We note that the average SAT scores of those who were eligible are not very different from those who were ineligible. To the extent that SAT score is a good proxy for ability, this strengthens our claim about the homogeneity of the sample. In addition, the average SAT score in the sample is very close to the national average, which was just over

²¹See Table 1 of the NCES report: <http://nces.ed.gov/pubs2006/2006156.pdf>.

²²This conversion table is the outcome of a joint study by the ACT and the College Board (the College Board conducts the SAT): <http://www.act.org/aap/concordance/pdf/reference.pdf>. All results are robust to using only using actual SAT scores.

1000 in 1992.²³

In Panel B the Avg. Borrowed (cum.) term is the unconditional cumulative average amount borrowed by individuals while obtaining their undergraduate degree. We observe non-negative values for those who were ineligible for loans in the last year of their study because they were eligible and borrowed at some point between enrolling and graduating. Take note that not every eligible individual borrows. Finally Panel C shows the average annual incomes across groups for 1994, 1996 and 2002 respectively. The trend in the means shows that those ineligible to borrow in the last year of schooling made more than those who were eligible to borrow in the final year, on average. In 1994, those eligible to borrow made about 8.5% less than those ineligible to borrow. In the subsequent years the gap between the two groups seems to lessen, where those eligible have earnings about 4%-5% lower than those ineligible.

3.4 Empirical Framework

We use the variation induced by the kink in the Stafford loan program for the 1992-93 academic year to identify the impact of cumulative student loans on income. Because the students in our study graduate in 1993 and we focus only on dependent students, we need not worry about the type of Stafford loan; they were all subsidized Stafford loans. Additionally, given that subsidized loans are need-based loans, a kink occurs where the slope of the eligible Stafford loan amount changes from 0 to 1. Figure 3.3 displays the empirical distribution of cumulative Stafford

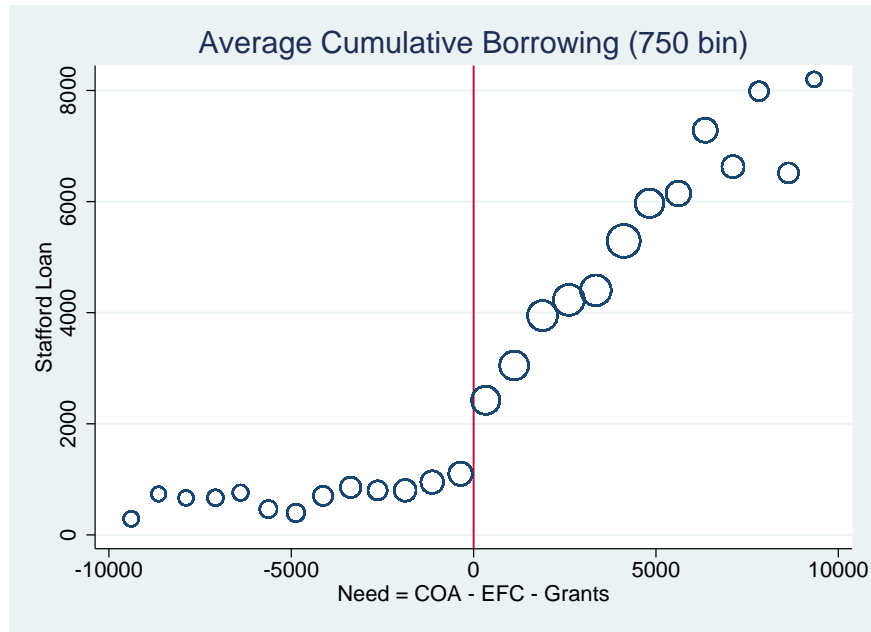
²³<http://www.infoplease.com/ipa/A0883611.html>

Table 3.1: Characteristics of Respondents by Stafford Loan Eligibility

	Ineligible	Eligible	Full sample
<i>A. Student demographic characteristics</i>			
Male	0.449	0.421	0.432
Age	22.17	22.13	22.15
Black	0.040	0.073	0.061
Hispanic	0.028	0.055	0.045
White	0.906	0.835	0.860
SAT	1015	980	992
Parent Income	\$60,650	\$44,640	\$50,400
Parent Education			
Less than bachelor	0.48	0.56	0.53
Bachelor	0.28	0.25	0.26
Master's and higher	0.24	0.19	0.21
Family Size	3.72	3.91	3.84
<i>B. Education related costs</i>			
EFC (92)	\$15,026	\$5,843	\$8,790
COA (92)	\$11,815	\$11,118	\$11,450
Grants (92)	\$876	\$1,432	\$1,216
Avg. Borrowed (cum.)	\$851	\$5,015	\$3,595
<i>C. Average Annual Earnings</i>			
1994	\$22,765	\$20,983	\$21,630
1996	\$28,765	\$27,532	\$27,980
2002	\$52,273	\$49,487	\$50,515
Observations	1,430	2,460	3,880

Ineligible if $need < 0$. SAT uses ACT-SAT conversion tables to compute SAT scores of students who have an ACT score but no SAT score. Parental income was reported for 1991. EFC is expected family contribution, and COA is cost of attendance. COA includes tuition and fees, books and supplies and living expenses. COA is provided by the institution. Both EFC and COA are for the final academic year (1992-93). All dollar values are in nominal terms. Avg. Borrowed is the unconditional cumulative average amount borrowed for undergraduate degree. Students with $need$ greater than \$10,000 and less than -\$10,000 are excluded.

Figure 3.3: Empirical Distribution of Cumulative Stafford Loans as a Function of *need*



Notes: Bin size is \$750. The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

loans borrowed as a function of *need* in the final year of schooling, where *need* is calculated as discussed above. There is a clear kink about the *need* threshold value of 0. Let

$$y = \tau Staf + g(need) + U$$

represent the causal relationship between the log of annual income in 1994, (y), and cumulative Stafford loans borrowed, ($Staf = Staf(need)$), where U is a random vector of unobservable, predetermined characteristics. The required identifying as-

sumptions for the RK design are: (1) the direct marginal impact of *need* on y is continuous (e.g., around the eligibility threshold, there are no discontinuities in the direct relationship between *need* and y); (2) the conditional density of *need* (with respect to U) is continuously differentiable at the threshold for Stafford loan eligibility (Card et al. 2012). The monotonicity assumption needed for the estimates to describe a local average treatment effect is satisfied by the policy determining loan eligibility.²⁴ Provided that the relationship between unobservable factors and *need* evolves continuously across the Stafford loan eligibility threshold, the RK design approximates random assignment in the neighborhood of the kink. The second assumption, moreover, generates testable predictions concerning how the density of *need* and the distribution of observable characteristics should behave in the neighborhood of the threshold.

If these assumptions hold, and with locally constant treatment effects, the RK estimator, τ_{RK} , will identify the causal impact of cumulative Stafford loans on log earnings in 1994. Next, we provide supporting evidence for the validity of the identifying assumptions.

3.4.1 Evaluating the RK identifying assumptions

We evaluate the RK identifying assumptions through two exercises. First we test for discontinuities in the level and slope of the density of *need* at the threshold.

²⁴This assumption requires that the direction of the kink be non-negative or non-positive for the entire population. In particular, it rules out situations where some individuals experience a positive kink at $need = 0$, but others experience a negative kink at $need = 0$. That the Stafford loan borrowing limit increases in *need* ensures this holds.

Then, we test for discontinuities in the level and slope of the distributions of observable characteristics including age, gender, race, log parental income, parental education and family size respectively.

The first condition is that the density of *need* should be smooth. The idea here is that students, while allowed to have certain control over *need*, should not be able to manipulate it precisely. A logical argument in defense of this assumption is as follows. *need* is a function of 3 distinct components, where the value of each component is revealed separately from the others. In order to precisely manipulate *need*, therefore, the student must have the ability to manipulate the three factors while simultaneously dealing with the timing problem, making it a very difficult task. The cost of attendance is determined by the institution. Individuals, by choosing which school to attend have a level of control over COA in the first year of schooling. After this point, however, to the extent that she does not change schools, the individual takes changes in COA as given. Therefore, under the assumption that the individual did not change her school in the last year of schooling, she cannot have precise control over COA in that year. As a result, using COA in the last year of an individual's studies strengthens the argument against the ability to precisely estimate *need*. The EFC formula is an extremely complicated formula which makes manipulating it a very difficult task. Two components of EFC over which the family might exert some control are income in the previous year and the number of dependents in college. Because the FAFSA is filled out after parents file their tax return, precise EFC manipulation would demand that parents adjust their entire income from the previous year. Grants, the third

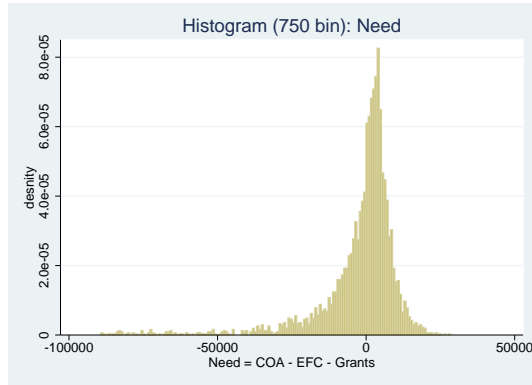
component of *need*, are usually determined by factors directly out of the control of the student. Most grant related factors, moreover, are not known at the time of filing for FAFSA, thereby making it an instrument hard to precisely manipulate. If having precise control over any one component of *need* is difficult, having the ability to do so in a coordinated fashion between all three components is certainly very unlikely. Figure 3.4 displays the distribution of *need* as well as its various components. We do not observe bunching about any single point over the range of *need*. Panel 3.4(e) shows the distribution of *need* focussing on the range of $[-\$10000, \$10000]$, the range of need over which we will focus our attention. We notice no sudden change in the density around the threshold.

We run the following set of regressions to formally test for discontinuities in the level and slope of the distribution of observables around the threshold:

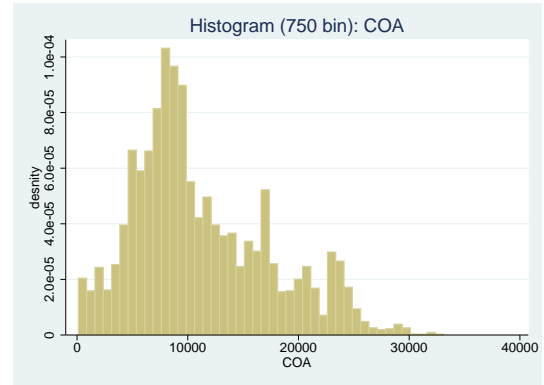
$$X_i = g(\text{need}_i) + \alpha_1 \mathbf{1}[\text{need}_i > 0] + \alpha_2 \text{need}_i \times \mathbf{1}[\text{need}_i > 0] + \epsilon_i$$

where X_i is a predetermined covariate of individual i . $g(\cdot)$ is a polynomial function of *need*, the degree of which is determined to minimize the Akaike Information Criterion (AIC). The hypothesis we want to test is, $H_0 : \alpha_1 = 0$ for no discontinuity in the level and $H_0 : \alpha_2 = 0$ for no change in the slope. Table 3.2 shows the results for age, gender (male), race (white vs non-white), parental income, family size, parental education (college vs less than college) and SAT score. $g(\cdot)$ is defined as quadratic in this specification, although we also test to make sure the results hold when $g(\cdot)$ is linear in robustness checks.²⁵ As we can see from the table, we fail to reject the null

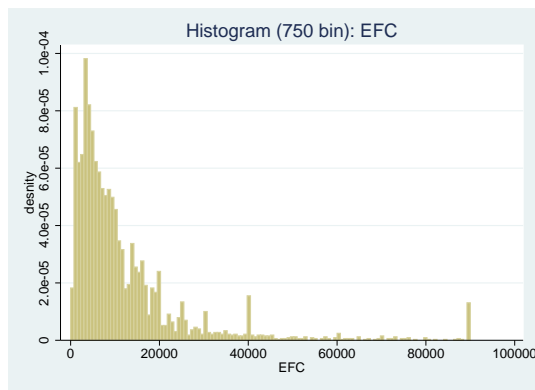
²⁵As Dong (2014) and Gelman and Imbens (2014) note, it is not ideal to use polynomials

Figure 3.4: Distribution of *need* and the Different Components of *need*

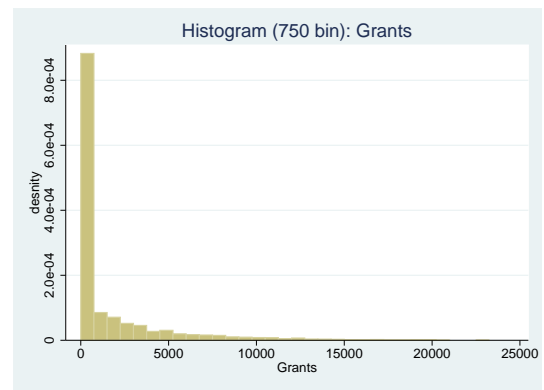
(a)



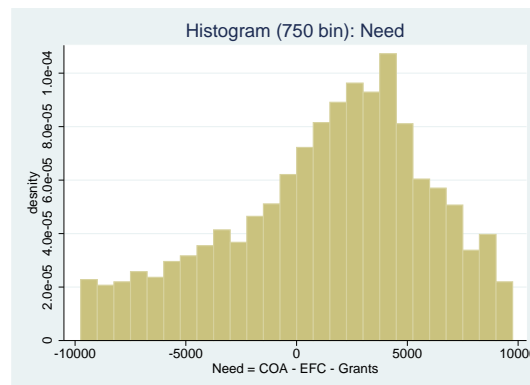
(b)



(c)



(d)



(e)

Table 3.2: Relationship between *need* and Predetermined Characteristics

X (co-variate)	α_1	α_2 (RK)
age	0.066 (0.057)	-0.004 (0.0417)
male	0.019 (0.033)	0.039 (0.024)
white	-0.047 (0.037)	-0.011 (0.017)
log parental income	-0.21 (0.13)	0.031 (0.033)
family size	0.100 (0.085)	0.012 (0.061)
College Ed. Parent	-0.115 (0.077)	0.008 (0.024)
SAT	-26.37 (23.19)	-5.55 (9.52)
Observations	3,880	3,880

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. Due to very small values in column 3, all numbers are scaled by 1000. The polynomial degree was set to 2. Each value is from a separate set of regressions. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

in all cases. Figure 3.5 shows the distribution of baseline characteristics as a function of *need*: we find no evidence of discontinuous changes in the level or slope of the distributions.

higher than two in degree. As a result we limit ourselves to only linear and quadratic polynomials.

Figure 3.5: The Distribution of Baseline Characteristics

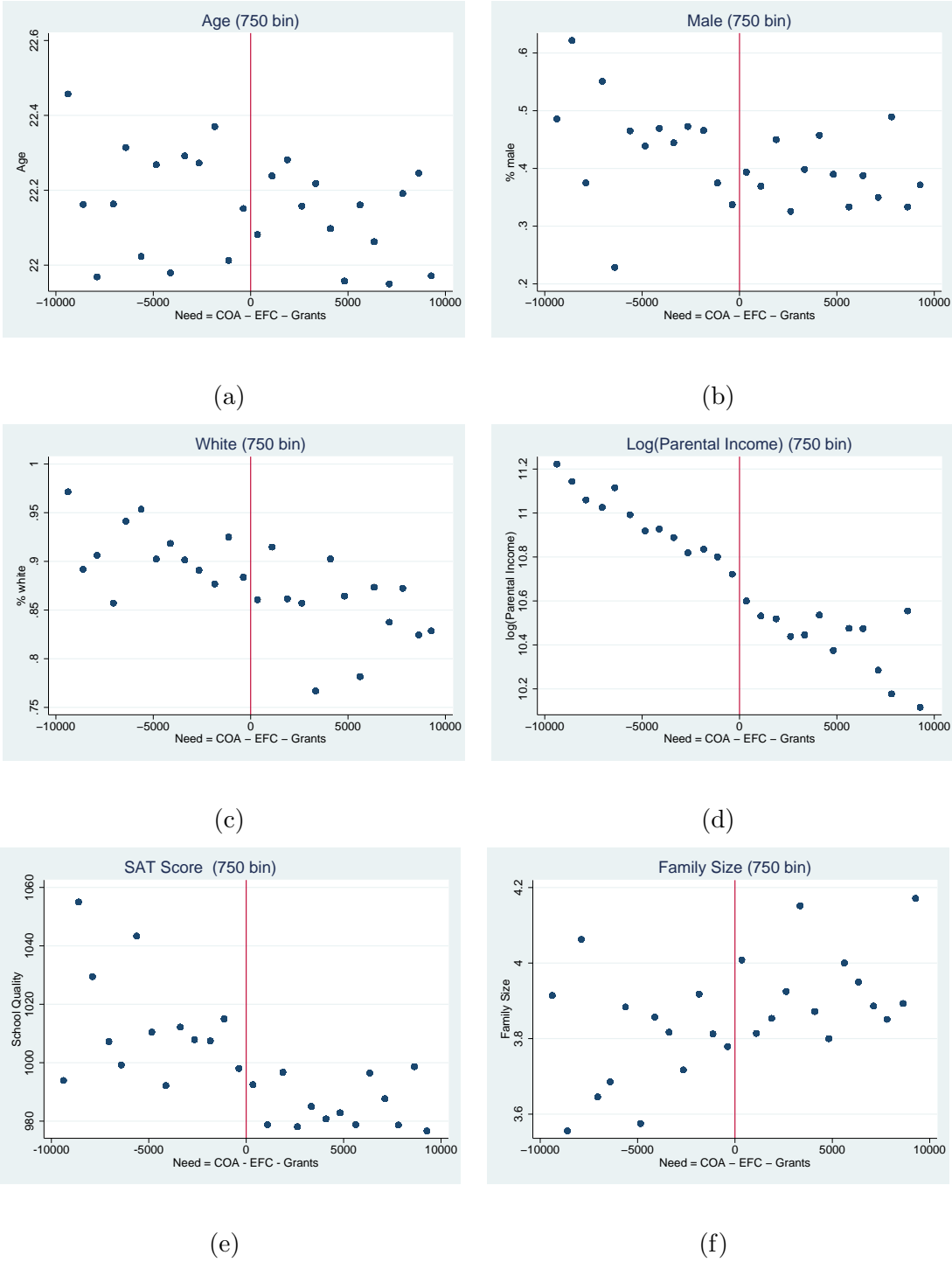
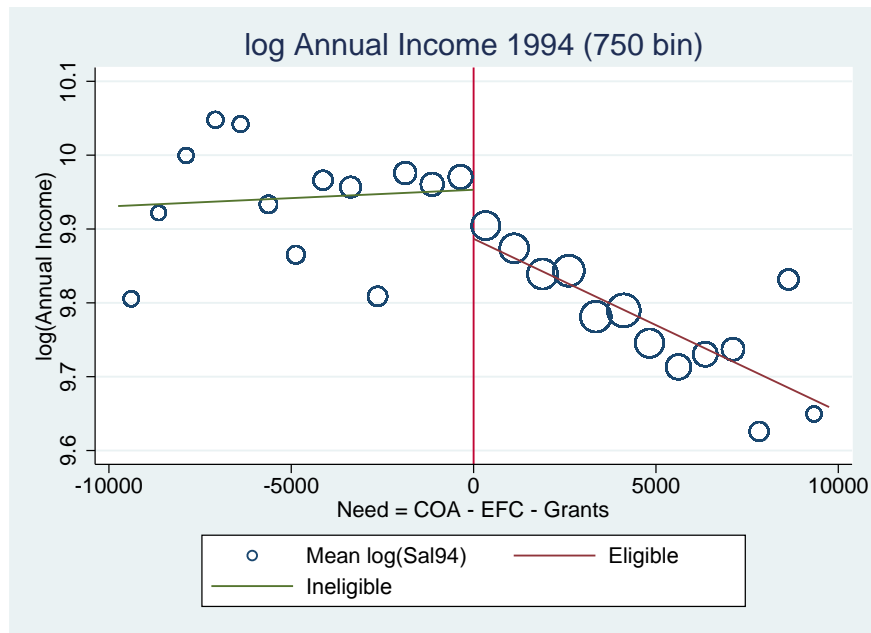


Figure 3.6: The Reduced Form Impact of *need* on log of 1994 Annual Earnings

Notes: The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

3.4.2 Estimation Strategy and Results

Figure 3.6 displays log annual earnings in 1994 as a function of *need*. There is a distinct downward slope to the right of the threshold point, indicating a negative relationship between amount borrowed and earnings. To the left of the threshold, the pattern is hard to discern. To help better understand the trends on either side of the threshold, the straight lines are fitted values of the raw data. To be noted is the change in slope around the threshold.

In practice, depending on the program design and associated outcomes, there are two possible estimation techniques: a sharp estimation technique and a fuzzy one.

Consequently, in this section, first we explain why we use the fuzzy estimator: (τ_{RK}) . After that we show how the estimator can be deduced by estimating two separate sets of regressions, known as the first stage and reduced from regression equations. Finally we discuss the results as well robustness.

We now explain the reasoning behind using a fuzzy technique in more detail. Although *need* is unambiguously defined, we observe that there is in some sense an imperfect compliance of the rules, as is usually so in practice.²⁶ Specifically, in our case we map cumulative student loans on *need* in the final year of school. The mapping to *need* for individuals may vary from year to year depending on changes in COA, parental income, grants received and number of siblings in college to name a few. This is so, despite the fact that the threshold rule does not change over time. Also, not all students eligible in the last year do borrow – the take-up rate of the program is not 100%. Finally, if the treatment is a continuous variable, as in our case – cumulative Stafford loan amount – the fuzzy estimator has intuitive interpretation analogous to an IV specification.

Now, we can define the fuzzy RK estimator (τ_{RK}) as:

$$\tau_{RK} = \frac{\lim_{\varepsilon \downarrow 0} \left[\frac{\partial y |_{need=need_0+\varepsilon}}{\partial need} \right] - \lim_{\varepsilon \uparrow 0} \left[\frac{\partial y |_{need=need_0+\varepsilon}}{\partial need} \right]}{\lim_{\varepsilon \downarrow 0} \left[\frac{\partial Staf |_{need=need_0+\varepsilon}}{\partial need} \right] - \lim_{\varepsilon \uparrow 0} \left[\frac{\partial Staf |_{need=need_0+\varepsilon}}{\partial need} \right]},$$

where $need_0$ represents the Stafford Loan eligibility threshold. Note that the RK estimator is based upon the change in slope about the threshold. The numerator

²⁶In general, this occurs due to factors such as imperfect take-up of the program or the existence of factors other than the threshold rule that affect the probability of program participation.

measures the change in log income (y) as a function of $need$, and the denominator measures the change in cumulative loan amounts ($Staf$) as a function of $need$. Given the above definition, the next step is to estimate τ_{RK} .

The denominator can be determined by estimating what is known as the first stage equation, the numerator can be determined by estimating the reduced form equation. Consider the following first stage equation (3.1) and reduced form equation (3.2):

$$Staf_i = f(need_i) + \beta_1 \mathbf{1}[need_i > 0] + \beta_2 need_i \times \mathbf{1}[need_i > 0] + \eta \mathbf{X}_i + \nu_i \quad (3.1)$$

$$y_i = g(need_i) + \gamma_1 \mathbf{1}[need_i > 0] + \gamma_2 need_i \times \mathbf{1}[need_i > 0] + \phi \mathbf{X}_i + v_i, \quad (3.2)$$

where i indicates an individual, and $f(\cdot)$ and $g(\cdot)$ are polynomial functions of $need$, the degree of the polynomials is chosen such that the AIC is minimized. $\mathbf{1}[need_i > 0]$ is an indicator function taking on the value of 1 if $need$ is positive and 0 otherwise. \mathbf{X} is a vector of predetermined demographic characteristics. β_2 measures the change in the slope of cumulative Stafford loans around the threshold. It is interpreted as the change in cumulative borrowing for every dollar increase in $need$, by an individual barely eligible for Stafford loans. Similarly, γ_2 measures the change in the slope of log earnings around the threshold. In this framework, $\hat{\tau}_{RK} = \frac{\hat{\gamma}_2}{\hat{\beta}_2}$.

Equations (3.1) and (3.2) show in a very clear manner how τ_{RK} can be estimated. In addition, they provide an understanding of how cumulative debt and log earnings are impacted by $need$. One concern with this approach, however, is that determining the standard error for $\hat{\tau}_{RK}$ is not as straightforward. Fortunately, τ_{RK} can also be estimated using standard 2SLS estimation techniques, which in turn allows us

to determine robust standard errors. In order to do so we regress log earnings in 1994 on cumulative Stafford loans borrowed, and estimate 2SLS model where the second stage takes on the form:

$$y_i = \tau_{RK} \widehat{Sta}f_i + g(need_i) + \lambda \mathbf{X}_i + \delta_i, \quad (3.3)$$

using the kink as an instrument for debt. Here, τ_{RK} represents the impact of an additional dollar of cumulative Stafford loans on log annual earnings in 1994.

Table 3.3 shows the estimation results where we use a quadratic in *need* for all specifications and Stafford loans are in 100's of dollars. Panel *A* shows the result for the first stage and reduced stage estimations. The interpretation of these estimates are as discussed above. For example, from $\hat{\beta}_2$ we determine that for students who are barely eligible for Stafford loans, every dollar increase in *need* increases their cumulative borrowing by 46 cents on average. Panel *B* displays 2SLS estimates of the impact of Stafford loan amount on log earnings. The interpretation of the 2SLS estimator is that an additional hundred dollars of Stafford loan reduces 1994 annual income by about 0.1%.²⁷ Extrapolating linearly, at the average level of borrowing, an individual's earnings are 5% lower than an average debt free individual in 1994.

Additional robustness checks are carried out in Appendix C.1 from which we conclude that the overall patterns and results remain qualitatively similar. The first test we conduct is to verify that our results are robust to including independent students in the sample. The experiment is not as clean, because independent students

²⁷For small values of $\hat{\beta}$, $e^{\hat{\beta}} \approx 1 + \hat{\beta}$. That is, $\hat{\beta}$ gives the percentage change in income for a unit change in cumulative Stafford loans, where a unit is 100s of dollars.

Table 3.3: Impact of Stafford Loans on 1994
Earnings

<i>A. OLS estimates</i>	
$\hat{\beta}_2$	0.0046***
	(0.0011)
$\hat{\gamma}_2$	-0.00465**
	(0.00223)
<i>B. 2SLS estimates</i>	
Stafford Loan ($\hat{\tau}_{RK}$)	-0.00101**
	(0.00042)
Observations	3,880

$\hat{\beta}_2$ is the coefficient on the Stafford loan eligible \times distance from threshold in the first stage regression equation. Stafford loans are in 100's of dollars. γ_2 is the coefficient on the Stafford loan eligible \times distance from threshold in the reduced stage regression equation. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, family size and quadratic in need. $\hat{\gamma}_2$ term in Panel A are scaled by a factor of 1000. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

were also eligible to borrow unsubsidized federal loans. As noted earlier, however, the majority of independent students had only subsidized loans. We observe in this sample that there is more borrowing at the lower end of *need* and not as much on the higher end. The change in slope about the threshold is also not as sharp. The estimates for this sample show that earnings for an individual with the mean level of borrowing are on average 3.8% lower than earnings of an individual with no debt.

Various other tests are conducted to determine the robustness of our estimates. For example, we determine that excluding the vector of observable characteristics has minimal impact on the magnitude of τ_{RK} . It lowers the efficiency of the estimate, but the p-value remains lower than 0.05. The estimate is also robust to other polynomial specifications as well as smaller bandwidths.

Next we will present further supporting evidence and discuss a number of possible extensions. For example, we investigate the impact of dropping individuals enrolled in graduate school in 1994, we document number of months between graduation and joining of first job as well as number of interviews until first job, as well as estimate a model using dollar earnings instead of log earnings and look at longer term impacts of debt on earnings. Having established the empirical results and tested its robustness, we will then turn to theory to provide the intuition behind our finding.

3.4.3 Discussion

The goal of this section is two fold. First, we would like to determine the impact of some sample selections decisions we made. Specifically, the decision to drop

students who were in graduate school in 1994 and the decision to use log earnings instead of earnings (log earnings would automatically drop individuals that report zero earnings). Second, we would like to determine the long-term impact of student loan debt on earning, by estimating the model using earnings in 1997 and 2003.

3.4.3.1 Individuals in graduate school in 1994

We explained our reasoning to drop graduate students from the benchmark sample in Section 3.3.1. In this section, with the help of descriptive statistics we will attempt to establish whether: a) students that chose to go to graduate school are different from those that chose not to. If so, in what dimension? and b) did being eligible for student loans in the last year of schooling have an impact on the decision to go to graduate school?

The sample selection now is as follows. Instead of dropping individuals who were in graduate school in 1994, we keep them in the sample. Then after accounting for independent students and those outside the specified need bandwidth (students with *need* greater than \$10,000 and less than -\$10,000), we are left with 4,460 observations (i.e. original benchmark sample plus 760 individuals who went to graduate school in 1994, are dependent and within the specified need bandwidth). Therefore, in this new sample, 16.5% of the individuals were in graduate school in 1994 (in the raw sample 16.1% or 1,500 out of 9,300 individuals were in graduate school in 1994).

Table 3.4 summarizes the differences between individuals that chose to go to graduate school in 1994 (Grad94) and those that did not go to graduate school in

1994 (non-Grad94). The non-Grad94 sample is the benchmark sample outlined in Table 3.1. As the table shows, the composition of Asians in the Grad94 sample is almost twice as large as in the non-Grad94 sample. As would be expected, individuals that chose to go to graduate school had considerably higher SAT scores, came from richer households, had better educated parents and slightly bigger families. In terms of student loan debt, those that chose to go to graduate school had taken on slightly less debt than those who chose not to. Next we will investigate whether eligibility for student loan debt had an impact on graduate school decision.

In Table 3.5 we focus entirely on individuals that chose to go to graduate school in 1994. They are now broken into two groups, those that were ineligible for student loan debt in the last year of schooling and those who were eligible for debt in the last year of schooling. There is a significant difference in parental income between those ineligible and those eligible. This difference in parental income translates to a difference in average amount borrowed between the two groups. It is interesting to note that the difference in the average amount borrowed between eligible and ineligible students is lower for those that chose to go to graduate school than those that chose not to.

Figure 3.7 plots the probability of attending graduate school as a function of need in the last year of undergraduate study. As the figure shows, there is not much difference in the probability, especially around the threshold.

Table 3.4: Characteristics of Respondents by 1994 Graduate School Enrollment

	Grad94	non-Grad94	Full sample
<i>A. Student demographic characteristics</i>			
Male	0.477	0.432	0.439
Age	21.92	22.15	22.08
Asian	0.063	0.034	0.039
Black	0.056	0.061	0.060
Hispanic	0.035	0.045	0.043
White	0.842	0.860	0.855
SAT	1075	992	1007
Parent Income	\$54,814	\$50,400	\$51,189
Parent Education			
Less than bachelor	0.42	0.53	0.49
Bachelor	0.27	0.26	0.26
Master's and higher	0.29	0.21	0.22
Family Size	4.01	3.84	3.87
<i>B. Education related costs</i>			
Avg. Borrowed (cum.)	\$3,442	\$3,595	\$3,569
Observations	760	3,880	4,640

Grad94 indicates individuals who were in graduate school in 1994. non-Grad94 indicates individuals who were not in graduate school in 1994. The sample is the benchmark sample plus an additional 760 individuals who enrolled in graduate school in 1994, were dependents and within the specified need bandwidth. SAT uses ACT-SAT conversion tables to compute SAT scores of students who have an ACT score but no SAT score. Parental income was in 1991. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

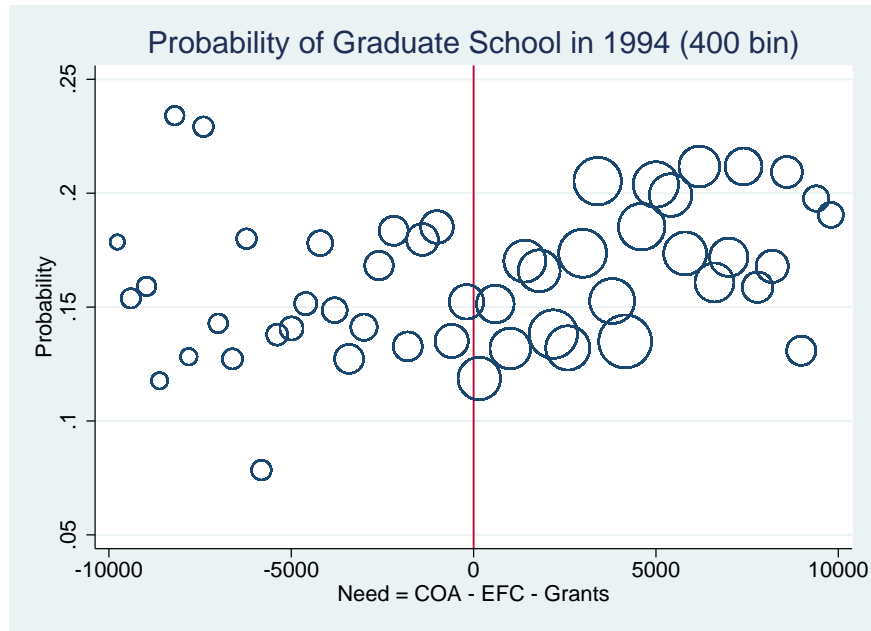
Table 3.5: Characteristics of 1994 Graduate School Enrollers by Stafford

Loan Eligibility

	Ineligible	Eligible	Grad94
<i>A. Student demographic characteristics</i>			
Male	0.517	0.455	0.477
Age	21.99	21.88	21.92
Asian	0.053	0.082	0.063
Black	0.048	0.061	0.056
Hispanic	0.031	0.037	0.035
White	0.837	0.845	0.842
SAT	1098	1062	1075
Parent Income	\$67,703	\$48,084	\$54,814
Parent Education			
Less than bachelor	0.42	0.43	0.42
Bachelor	0.27	0.26	0.27
Master's and higher	0.30	0.28	0.29
Family Size	3.99	4.05	4.01
<i>B. Education related costs</i>			
Avg. Borrowed (cum.)	\$1,080	\$4,520	\$3,442
Observations	230	530	760

Grad94 has the same definition as in Table 3.4. Ineligible if *need* < 0. The sample is the additional 760 individuals who enrolled in graduate school in 1994, were dependents and within the specified need bandwidth. SAT uses ACT-SAT conversion tables to compute SAT scores of students who have an ACT score but no SAT score. Parental income was in 1991. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Figure 3.7: Probability of Attending Graduate School in 1994



Notes: The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

3.4.3.2 Time to first job & number of interviews

Given that for each individual we know the date of graduation and date at which they started working, we can determine the number of months between graduation date and joining date.

Table 3.6 shows that while those that borrowed seem to take about a quarter of a month less (approximately a week) to start working, we do not observe much difference in starting times when controlling for debt eligibility. This means that we will not be able to use our identification strategy to test our hypotheses that those with debt are in a hurry to find a job. Table 3.7 shows that those that never

Table 3.6: Months to First Job

	Non-Borrow	Borrow	Ineligible	Eligible
	2.15	1.91	1.98	2.06
Observations	1910	1970	1,430	2,460

This table presents number of months between date of graduation and joining of first job. The data is separated into 2 mutually exclusive groups using a couple of different criteria. First the data is broken down into two groups based on if the student ever borrowed (Borrow) or not (Non-Borrow). Then the data is broken down into two groups based on eligibility for student loan program in the last year of schooling. Ineligible if $need < 0$. Students with $need$ greater than \$10,000 and less than -\$10,000 are excluded.

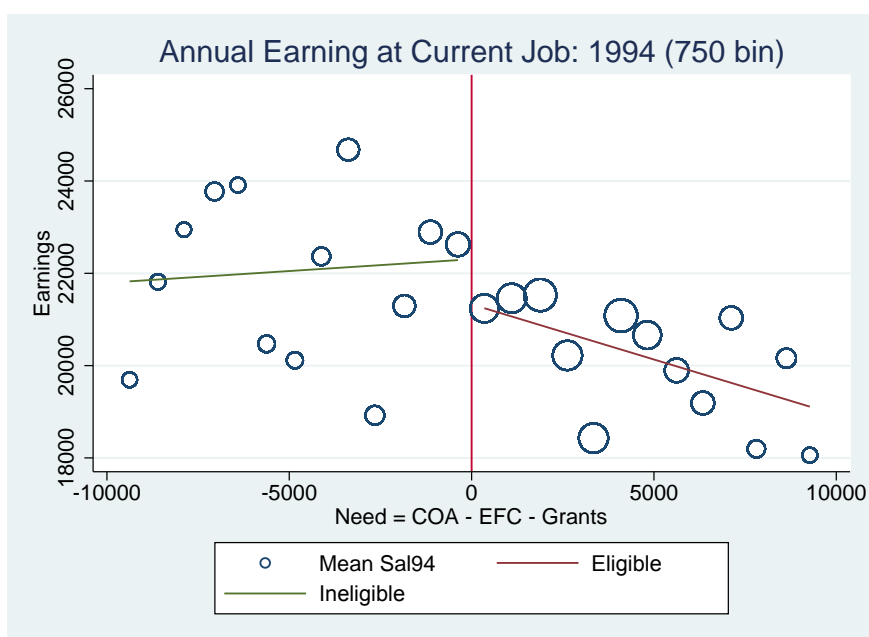
borrowed took fewer interviews than those that ever borrowed. Given that they also joined work at a later date, this would imply that non-borrowers were more selective in the type of jobs they interview for.

3.4.3.3 Earnings in dollars amounts

In this section, instead of using log of earnings in 1994, we use earnings in dollar amounts so as not to exclude individuals who reported zero earnings. Figure 3.8 shows the reduced form impact of $need$ on 1994 annual earnings. As we can see from the figure, earnings to the right of the threshold maintains a downward slope similar to the case where we used log earnings.

Table 3.8 shows the estimation results where we use a quadratic in $need$ for

Figure 3.8: The Reduced Form Impact of *need* on 1994 Annual Earnings



Notes: The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

Table 3.7: Number of Jobs Interviewed for upon Completion
of Degree

	Non-Borrow	Borrow	Ineligible	Eligible
	5.37	5.64	5.53	5.50
Observations	1910	1970	1,430	2,460

This table presents number of interviews. The data is separated into 2 mutually exclusive groups using a couple of different criteria. First the data is broken down into two groups based on if the student ever borrowed (Borrow) not (Non-Borrow). Then the data is broken down into two groups based on eligibility for student loan program in the last year of schooling. Ineligible if $need < 0$. Students with $need$ greater than \$10,000 and less than -\$10,000 are excluded.

all specifications. There are two main differences between between the model used to generate the results for Table 3.3 and the model used to generate the results for Table 3.8. The first difference, as already pointed out, is that we use earnings in dollar amounts instead of log earnings. The second difference is that for Table 3.8, Stafford loans are in dollars instead of 100's of dollars. Panel *A* shows the result for the first stage and reduced stage estimations. The interpretation of these estimates are as discussed above. For example, from $\hat{\beta}_2$ we determine that for students who are barely eligible for Stafford loans, every dollar increase in $need$ increases their cumulative borrowing by 46 cents on average. Panel *B* displays 2SLS estimates of the impact of Stafford loan amount on earnings. The interpretation of the 2SLS estimator is that

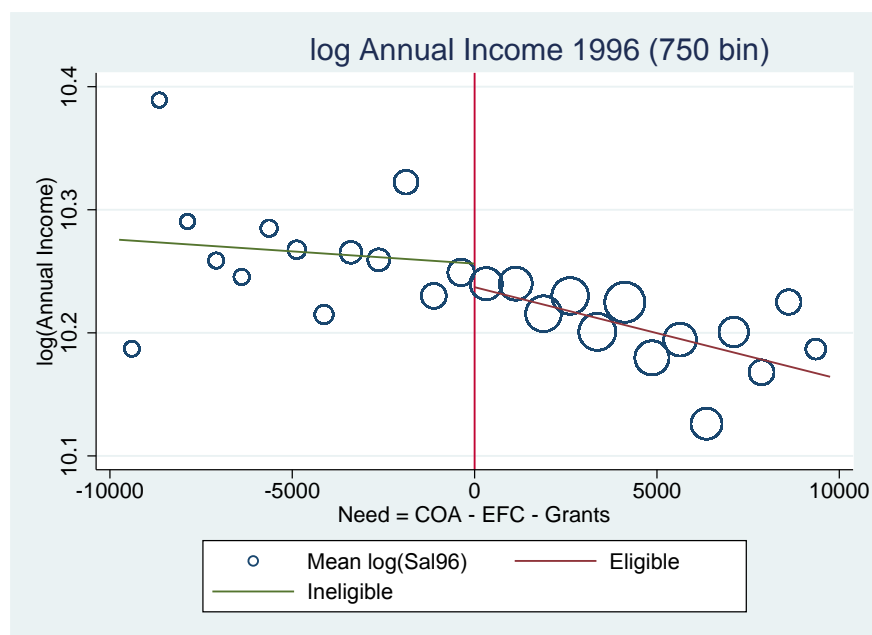
an additional dollar of Stafford loan reduces 1994 annual income by about 17 cents. Extrapolating linearly, at the average level of borrowing, an individual's earnings are 4.1% lower than an average debt free individual in 1994.

3.4.3.4 Impact of debt on later years

Given that the survey was conducted again in 1997 and 2003, we have information on individuals earnings in 1996 and 2002. Hence, in this section we determine the impact of student loan debt on earnings 3 years and 9 years post graduation. Figure 3.9 shows the reduced form impact of need on log 1996 earnings. We notice that there is a kink at the threshold value, though it does not as sharp as what we observed in 1994. Table 3.9 shows the estimation results where we use a quadratic in *need* for all specifications and Stafford loans are in 100's of dollars. Panel *A* shows the result for the first stage and reduced stage estimations. The first stage does not change, but the results for the reduced stage change since we are using earnings in 1996 for this table. The numbers for the Panel *B* displays 2SLS estimates of the impact of Stafford loan amount on log earnings. The interpretation of the 2SLS estimator is that an additional hundred dollars of Stafford loan reduces 1996 annual income by about 0.042%.

Figure 3.10 show the reduced form impact of need on log 2002 earnings. We notice that there is now no longer a kink at the threshold value. Table 3.10 shows the estimation results where we use a quadratic in *need* for all specifications and Stafford loans are in 100's of dollars. Panel *A* shows the result for the first stage and reduced

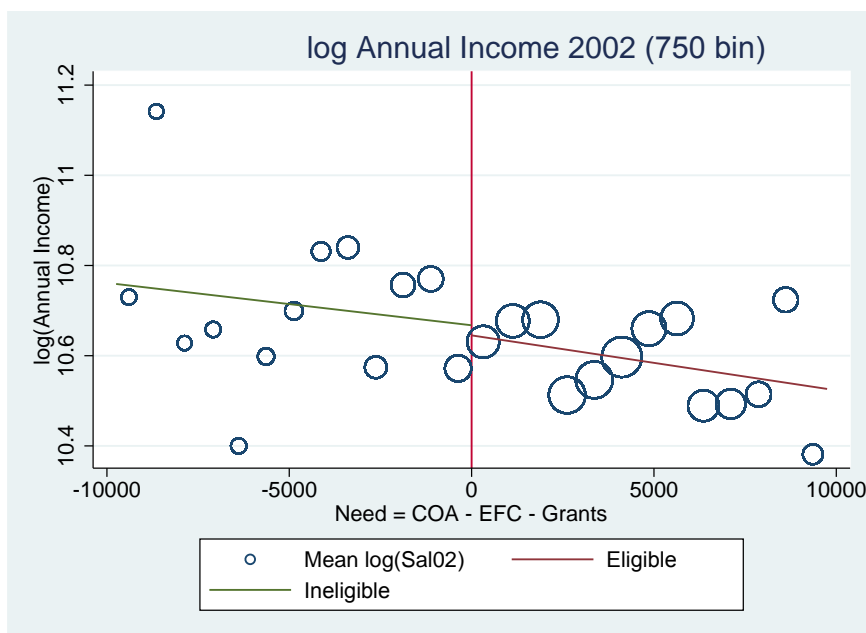
Figure 3.9: The Reduced Form Impact of *need* on 1996 Annual Earnings



Notes: The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

stage estimations. The first stage does not change, but the results for the reduced stage change since we are using earnings in 2002 for this table. As one would expect the change in slope is no longer significant. The numbers for the Panel *B* displays 2SLS estimates of the impact of Stafford loan amount on log earnings. Since the reduced stage showed insignificant results, we do not find any significant impact of debt on earnings in 2002. These results seem to imply that while individuals with debt are initially in a hurry to find a job, once having secured the job they are able to eventually find more optimal jobs over time.

Figure 3.10: The Reduced Form Impact of *need* on 2002 Annual Earnings



Notes: The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

3.5 Model

We use a single-period, competitive search framework along the lines of Moen (1997) with an exogenous amount of debt at the beginning of the period. What is crucial is whether or not the debt can be discharged at the end of the period. We first show that when debt cannot be discharged (as is the case with student loans), higher debt causes individuals to target lower wages at higher probabilities of finding work. The opposite happens when debt is dischargeable, i.e., higher debt causes individuals to target higher wages with lower probabilities of success.

3.5.1 Environment

The labor market consists of a continuum of workers and firms. The measure of workers is normalized to 1. The measure of jobs is endogenously determined through free entry, and there is a fixed cost $\kappa > 0$ associated with opening a vacancy.

Let $m(u, v)$ denote the number of new worker-firm matches, where u is the measure of unemployed fresh graduates searching for a measure v of vacancies. The matching function $m(u, v)$ captures the frictions in the market for workers and vacancies, which cause some workers and vacancies to remain unmatched in a period.

It is standard to assume that $m(u, v)$ is continuous, differentiable, non-negative, increasing, concave in both arguments and constant returns to scale (CRS), with $m(u, 0) = m(0, v) = 0$ for all (u, v) . Let $\theta = \frac{v}{u}$ denote the market tightness ratio. Due to CRS, θ is sufficient to determine job finding probabilities. Let p denote the

probability that an unemployed worker finds a job, then

$$p = \frac{m(u, v)}{u} = \frac{u m(1, v/u)}{u} = m(1, \theta) = p(\theta).$$

Similarly, if q is the probability that a firm fills a vacancy, then

$$q = \frac{m(u, v)}{v} = \frac{m(u, v)}{u \frac{v}{u}} = \frac{p(\theta)}{\theta},$$

where $p(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ . These conditions are also sufficient to ensure that the elasticity of $p(\theta)$, $\varepsilon_p = \frac{\theta p'(\theta)}{p(\theta)} \in (0, 1)$. In addition, we restrict our attention to matching functions where ε_p is either constant or decreasing in θ .²⁸

As noted above, firms must pay a vacancy posting cost κ . Any match within the period produces output y , which is divided between the worker and firm according to the posted wage, w . Free entry gives us the relationship between θ and w

$$q(\theta)(y - w) = \kappa. \tag{3.4}$$

As θ increases, the probability that a firm fills a vacancy, $q(\theta)$, decreases. It follows that as θ increases, $(y - w)$ must increase for (3.4) to hold and w must therefore decrease. That is, jobs that provide a high probability of matching for the worker (high θ) offer lower wages, and vice versa. This establishes the negative relationship between θ and w , which we will use later.

Individuals enter the labor market with a debt amount d corresponding to the student loan debt accumulated. To ensure that consumption remains positive,

²⁸This includes the most widely used forms of the matching function. For example, with urn ball matching functions, ε_p is decreasing and with Cobb-Douglas ($p(\theta) = \theta^\beta$ for $\beta \in (0, 1)$) ε_p is a constant.

we endow them with b . Individuals choose wages to maximize their expected utility. Matched workers receive wage w , while unmatched workers have different outcomes depending on the model specification, which will be covered in detail in the following sections. Finally, the model ends at the end of the period.

Below, we cover the case where the debt is modeled along the lines of student loan debt so that it cannot be discharged in bankruptcy. We do this by imposing that the individual must always pay off her debt at the end of period. We then model the case where loans can be discharged in bankruptcy and show the contrasting result this produces. These two specifications show that the ‘cannot discharge in bankruptcy’ condition is crucial to the observed behavior.

3.5.2 Non-dischargeable Loans

In this section, we model debt along the lines of a stylized student loan repayment scheme. Specifically, the individual cannot discharge her debt in bankruptcy and must pay it off, even if she is unsuccessful in finding a job. Note the problem below:

$$\max_{\theta, w} p(\theta) u(w + b - d) + (1 - p(\theta)) u(b - d), \quad (3.5)$$

subject to the constraint

$$\frac{p(\theta)}{\theta}(y - w) = \kappa. \quad (3.6)$$

The objective function is derived from the agent’s problem. The agent is successful in

finding a job with probability $p(\theta)$, in which case she receives utility from consuming the wage plus her endowment, net of the debt payments. With probability $(1 - p(\theta))$ she is unable to find a match, in which case she consumes her endowment net of debt payments; i.e., debt is always paid off at the end of the period irrespective of the outcome of the job search. The problem is subject to the free entry condition, where we substitute $q(\theta)$ with $\frac{p(\theta)}{\theta}$.

The first order condition (F.O.C.) for this problem is:

$$\underbrace{p'(\theta)[u(w + b - d) - u(b - d)]}_{\text{gain from higher probability}} = \underbrace{p(\theta)u'(w + b - d)}_{\text{gain from higher wage}} \underbrace{\frac{\kappa}{p(\theta)} \left[1 - \frac{\theta p'(\theta)}{p(\theta)} \right]}_{\text{exchange rate between } w \text{ and } \theta} . \quad (3.7)$$

This equation represents the trade-off between the gains from searching for a higher probability job and that of a higher wage. The left-hand side represents the increase in utility from searching for a job with marginally higher probability of success. The right-hand side can be divided into two parts as shown above. The first part shows the gain in utility from obtaining a marginally higher wage. The second part gives the trade-off, or exchange rate between the wage and the tightness ratio, as can be derived by totally differentiating the free entry condition.

In order to further understand the relationship between θ and w , we re-write (3.7) as

$$\underbrace{\frac{u'(w + b - d)}{u(w + b - d) - u(b - d)}}_{\text{decreasing in } w} = \frac{1}{\kappa} \underbrace{\frac{p'(\theta)}{1 - \frac{\theta p'(\theta)}{p(\theta)}}}_{\text{decreasing in } \theta} . \quad (3.8)$$

First, let us focus on the left-hand side of the equation. When w increases, the term on the left-hand side decreases. This is because as w increases, the denominator increases and the numerator decreases. κ is a constant, so for equality to hold, the

remaining part on the right-hand side must also decrease. Given the assumptions on $p(\theta)$, in particular, the concavity and the constant (or decreasing) elasticity assumption, it is easy to see that the term is decreasing in θ . Therefore, as w increases, the term on the left-hand side decreases, implying that the term on the right-hand side must decrease and θ must increase. This tells us that w and θ are positively sloped in the F.O.C. (3.7). We have already established in the Environment section that they are negatively sloped in the free entry condition. As shown in Figure 3.11, there exists a unique w_d^* and θ_d^* , for a given level of debt, d . Next, we establish how the level of debt impacts the search behavior.

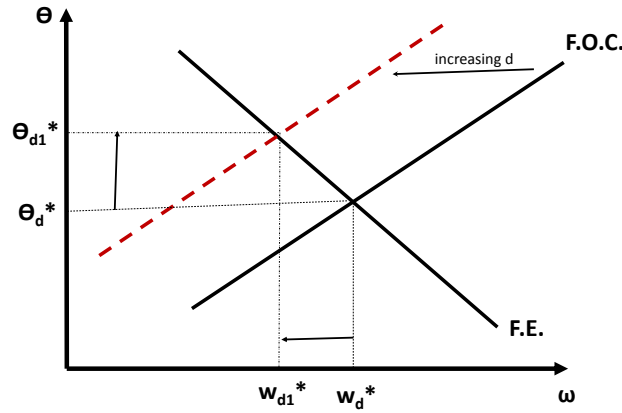
The utility function is of the class Decreasing Absolute Risk Aversion (DARA):

$$u(c) = \frac{1-\gamma}{\gamma} \left(\frac{\alpha c}{1-\gamma} + \beta \right)^\gamma \quad \text{where } \alpha > 0, \beta > -\frac{\alpha c}{1-\gamma}, \gamma \in (0, 1);$$

Proposition 3. *Under Assumption 3.5.2 higher debt causes individuals to search for lower wage jobs with a higher job-finding probability.*

Proof. While the proof is fairly mechanical and the details can be found in Appendix C.2, we will provide a quick outline here. The basic idea behind the proof is to first determine the condition under which the left hand side of (3.8) is decreasing in the debt level, d . As d increases, and the left hand side of (3.8) decreases, this cause the right hand side of (3.8) to decrease. Since it has already been established that the right hand side is decreasing in θ , an increase in d results in a corresponding

Figure 3.11: Model with Non-dischargeable Debt



Notes: F.E. represents the downward sloping free entry condition, F.O.C. represents the upward sloping first order condition. w^* and θ^* are the equilibrium values at a given debt level d . w_1^* and θ_1^* show the equilibrium values at a higher debt level d_1 . Individuals search for a lower wage with higher probability of success at d_1 compared to d .

increase in θ . As shown in Figure 3.11, when $d_1 > d$, the agent searches for a lower paying job with a higher probability of success.

Once we have determined the above condition, the next step is to determine the class of utility functions for which this is true. Finally note that both CRRA and log utility are special cases of DARA. For CRRA, set $\alpha = 1 - \gamma$ and $\beta = 0$, for log, $\gamma \rightarrow 0$ and $\beta = 0$. □

The intuition behind this result is as follows. Due to the non-dischargeability, as debt increases, the outcome of being unsuccessful becomes increasingly undesirable in the level of debt. As a result, as debt increases being successful in obtaining a job at a low wage is more desirable and provides higher utility than taking the increased

risk of being unsuccessful at targeting a higher wage. In other words, it eliminates the incentive to ‘gamble’ on targeting higher wage jobs. Next will show the impact of allowing debt to be discharged in bankruptcy and how this changes the behavior of individuals. In the proof we also show that the commonly used utility functions, namely CRRA and log are special cases of DARA.

3.5.3 Dischargeable Loans

Note the problem below:

$$\max_{\theta, w} p(\theta) u(w + b - d) + (1 - p(\theta)) u(\phi b), \quad (3.9)$$

subject to the constraints

$$\frac{p(\theta)}{\theta}(y - w) = \kappa. \quad (3.10)$$

The objective function is derived from the agent’s problem. The agent is successful in finding a job with probability $p(\theta)$, in which case she receives utility from consuming the wage plus her endowment, net of the debt payments. In the case that she is unable to find a match, she files for bankruptcy instead of paying off her debt. Since filing for bankruptcy is costly, she is only able to consume a fraction $\phi \in (0, 1]$ of her endowment. The magnitude of ϕ is not important for the qualitative results derived here. The problem is subject to the free entry condition, where we substitute $q(\theta)$ with $\frac{p(\theta)}{\theta}$.

The first order condition (F.O.C.) for this problem is:

$$\underbrace{p'(\theta)[u(w+b-d) - u(\phi b)]}_{\text{gain from higher probability}} = \underbrace{p(\theta)u'(w+b-d)}_{\text{gain from higher wage}} \underbrace{\frac{\kappa}{p(\theta)} \left[1 - \frac{\theta p'(\theta)}{p(\theta)} \right]}_{\text{exchange rate between } w \text{ and } \theta} . \quad (3.11)$$

This equation represents the trade-off between the gains from searching for a higher probability job and that of a higher wage. The left-hand side represents the increase in utility from searching for a job with marginally higher probability of success. The right-hand side can be divided into two parts as shown above. The first part shows the gain in utility from obtaining a marginally higher wage. The second part gives the trade-off, or exchange rate between the wage and the tightness ratio, as before.

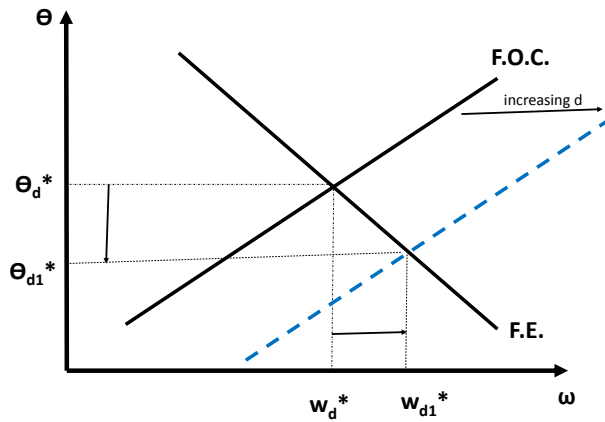
Proposition 4. *Higher debt causes individuals to search for higher wage jobs with a lower job-finding probability*

Proof. The free entry condition remains unchanged with an increase in d , as it is independent of the level of debt. The F.O.C. is a function of debt. We re-write (3.11) as

$$\underbrace{\frac{u'(w+b-d)}{u(w+b-d) - u(\phi b)}}_{\text{decreasing in } w, \text{ increasing in } d} = \frac{1}{\kappa} \underbrace{\frac{p'(\theta)}{1 - \frac{\theta p'(\theta)}{p(\theta)}}}_{\text{decreasing in } \theta} . \quad (3.12)$$

The left-hand side of (3.12) depends on d . As d increases, the denominator decreases and the numerator increases, causing the left-hand side to increase. In order to maintain equality, and with κ being constant, the remaining term on the right-hand side must increase. Because we have already shown that the remaining term is decreasing in θ , this means that θ must decrease. In other words, as debt goes

Figure 3.12: Model with Non-dischargeable debt



Notes: F.E. represents the downward sloping free entry condition, F.O.C. represents the upward sloping first order condition. w^* and θ^* are the equilibrium values at a given debt level d . w_1^* and θ_1^* show the equilibrium values at a higher debt level d_1 . Individuals search for a higher wage with lower probability of success at d_1 compared to d .

up, the individual starts searching for higher paying jobs with lower a probability of success. This is shown in Figure 3.12. □

The intuition behind this result is that as debt increases, the increase in utility from successfully finding a job at a given wage decreases due to higher debt payments. Because she knows she can declare bankruptcy if she is unsuccessful, default and bankruptcy puts a floor on her outside option and causes her to search for higher paying jobs with a lower probability of success.

3.6 Conclusion

While the recent rise in federal student loan debt is well documented, economists are only now examining the implications of these loans on borrowers. Consequently,

the goal of this paper is to understand the impact of student loan debt on labor market outcomes. In order to do so we first empirically establish the relationship between cumulative student loan debt and earnings less than a year after graduation. Using data from the Baccalaureate and Beyond Longitudinal Study: 1993/03 (B&B: 93/03), a nationally representative sample of undergraduates that received their bachelor's in 1993, we implement a regression kink (RK) design to determine the causal impact of debt on earnings. Key to this design is that up to 1993, dependent students could only borrow need-based Stafford loans. Estimates show that an additional hundred dollars of Stafford loan reduces 1994 annual income by about 0.1%. Extrapolating this result, earnings for an individual with the mean level of borrowing are 5% lower than those of an individual with no debt.

We then show that our empirical finding is consistent with economic theory. Specifically, using a simple one-period directed search model along the lines of Moen (1997) with exogenous amounts of debt, we show that as the level of debt increases, individuals will search for lower-wage jobs, with the accompanying higher probability of success. Crucial to this result is the fact that debt is modeled along the lines of student debt; i.e. it cannot be discharged in bankruptcy and therefore the individual is liable for her debts irrespective of the outcome of the job search. Once this condition is removed so the debt can be discharged in bankruptcy, the complete opposite result is observed. Individuals now search for higher-wage jobs with the accompanying lower probability of success.

In future work we plan to extend this study along both the empirical and mod-

eling dimensions. Empirically, we plan to investigate if the impact of loans on wages is persistent by using the follow-up surveys in 1997 and 2003. This will also provide key insights for developing a richer model. In reality even though bankruptcy is not an option, default and delinquency can still be used to provide a certain measure of insurance. The well documented trends in delinquencies and defaults for student loans shows that this is indeed true. In addition, income-based plans provide considerably more insurance and impact occupation choice. With the steady increase in enrollments for income-based plans, they need to be modeled and studied as well. In our ongoing work, we propose a richer model placing student loan debt, a delinquency or default decision and a stylized income-based plan in a directed search framework to study the effects of recent changes to student loan policies on the labor market and delinquency outcomes of college graduates.

Table 3.8: Impact of Stafford Loans on 1994
Earnings

<i>A. OLS estimates</i>	
$\hat{\beta}_2$	0.46***
	(0.11)
$\hat{\gamma}_2$	-0.078**
	(0.0379)
<i>B. 2SLS estimates</i>	
Stafford Loan ($\hat{\tau}_{RK}$)	-0.1699**
	(0.0915)
Observations	3,930

$\hat{\beta}_2$ is the coefficient on the Stafford loan eligible \times distance from threshold in the first stage regression equation. γ_2 is the coefficient on the Stafford loan eligible \times distance from threshold in the reduced stage regression equation. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, family size and quadratic in need. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table 3.9: Impact of Stafford Loans on 1996
Earnings

<i>A. OLS estimates</i>	
$\hat{\beta}_2$	0.0046***
	(0.0011)
$\hat{\gamma}_2$	-0.00194**
	(0.00061)
<i>B. 2SLS estimates</i>	
Stafford Loan ($\hat{\tau}_{RK}$)	-0.00042**
	(0.00042)
Observations	3,880

$\hat{\beta}_2$ is the coefficient on the Stafford loan eligible \times distance from threshold in the first stage regression equation. Stafford loans are in 100's of dollars. γ_2 is the coefficient on the Stafford loan eligible \times distance from threshold in the reduced stage regression equation. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, family size and quadratic in need. $\hat{\gamma}_2$ term in Panel A are scaled by a factor of 1000. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table 3.10: Impact of Stafford loans on 2002

Earnings

<i>A. OLS estimates</i>	
$\hat{\beta}_2$	0.0046***
	(0.0011)
$\hat{\gamma}_2$	-0.00005
	(0.00497)
<i>B. 2SLS estimates</i>	
Stafford Loan ($\hat{\tau}_{RK}$)	-0.00002
	(0.00065)
Observations	3,880

$\hat{\beta}_2$ is the coefficient on the Stafford loan eligible \times distance from threshold in the first stage regression equation. Stafford loans are in 100's of dollars. γ_2 is the coefficient on the Stafford loan eligible \times distance from threshold in the reduced stage regression equation. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, family size and quadratic in need. $\hat{\gamma}_2$ term in Panel A are scaled by a factor of 1000. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

APPENDIX A APPENDIX TO CHAPTER 1

A.1 Appendix: Decentralized Problem

The firm of age j in history (x^j, z^t) takes as given its current stock of workers hired up to the current period, $(L_i)_{i=0}^{j-1}$, and the contracts signed by the previous labor cohorts, $(\mathcal{C}_i)_{i=0}^{j-1}$. To not violate its prior commitments to past labor cohorts and be consistent with exogenous probabilities of separation and firm death, the firm's current exit probability must satisfy $\phi(\tau) \leq (1 - s_0)(1 - \delta)$ as well as $\delta \leq 1 - \phi(\tau)$, $\tau \leq j - 1$. This means that $s(\tau) = (1 - \phi(\tau))/(1 - \delta)$ when $\delta < 1$ (when $\delta = 1$, s can be arbitrary). The firm then decides the contract \mathcal{C}_j to post in V vacancies, with \mathcal{C}_j consistent with attracting desired matching probability m . When J_j is the value of a firm of age j , the firm's problem is:

$$J_j[(C_\tau)_{\tau=0}^{j-1}, (L_\tau)_{\tau=0}^{j-1}, x^j, z^t] = \max_{m, V, \mathcal{C}_j} x_j z_t F(L(1 - \alpha_\tau(x^j, z^t)/2)) - \mathcal{W} - C(V, L) \\ + \beta(1 - \delta)\mathbb{E}_{x^j, z^t} J_{j+1}[(C_\tau)_{\tau=0}^j, (L_{\tau+})_{\tau=0}^j, x^{j+1}, z^{t+1}] \quad (\text{A.1})$$

s.t.

$$L_{j+} = mV, \quad m \in [0, 1], \quad V \geq 0, \quad L_{\tau+} = L_\tau \frac{\phi_\tau(x^j, z^t)}{(1 - \delta)} \quad \forall \tau \leq j - 1 \quad (\text{A.2})$$

$$s_0 \leq 1 - \phi_\tau(x^j, z^t)/(1 - \delta), \quad (\text{A.3})$$

$$\mathcal{W} = \sum_{\tau=0}^{j-1} [w_{p,\tau}(x^j, z^t)\alpha_\tau(x^j, z^t) + w_{f,\tau}(x^j, z^t)(1 - \alpha_\tau(x^j, z^t))] L_\tau, \quad L = \sum_{\tau=0}^{j-1} L_\tau, \quad (\text{A.4})$$

$$\rho(z^t) = \frac{m}{\lambda(m)}(1 - \delta)\beta\mathbb{E}_{x^j, z^t}[W(\mathcal{C}_j, x^{j+1}, z^{t+1}) - U(z^{t+1})] \text{ if } m > 0 \quad (\text{A.5})$$

The last condition is the workers' participation constraint, specifying the minimum utility a contract \mathcal{C}_j must promise to attract a queue length of $\lambda(m)$ per vacancy.

A.1.1 Firm Entry

Free entry of firms implies that the expected value of an entrant firm (before realizing x and with labor force $L = 0$) is less than or equal to the entry cost K .

$$\sum_{x \in X} \pi(x_0) J_0[0, 0, x^0, z_t] \leq K(z_t) \quad (\text{A.6})$$

If entry is positive then this condition holds with equality.

A.1.2 Competitive Equilibrium

A Competitive search equilibrium is a list

$$[U(z^t), W(\cdot), \rho(z^t), \mathcal{C}_j(x^j, z^t), V(x^j, z^t), J_j(\cdot), L_\tau(x^j, z^t), N(x^j, z^t), N_0(z^t)]$$

for all $t \geq 0, j \geq 0, x^j \in X^{j+1}, z^t \in Z^{t+1}, 0 \leq \tau \leq j$ and given initial firm distribution, such that:

- Firms' exit, hiring, part-time, and layoff strategies are optimal. That is, J_a is the value function and $\mathcal{C}_j(\cdot), \phi(\cdot) = (1 - \delta)(1 - s(\cdot)), \alpha(\cdot), m(\cdot)$, and $V(\cdot)$ are the policy functions for problem (A.1) subject to its constraints (A.2)-(A.5).
- Employment evolves according to

$$L_\tau(x^j, z^t) = L_\tau(x^{j-1}, z^{t-1}) \frac{\phi(x^j, z^t)}{1 - \delta}, 0 \leq \tau \leq j - 1,$$

$$L_j(x^j, z^t) = m(x^j, z^t)V(x^j, z^t), j \geq 0$$

.

- Firm entry is optimal. That is, the complimentary slackness condition

$$\sum_x \pi_0(x) J_0(x, z^t) \leq K(z_t), N_0(z^t) \geq 0, \quad (\text{A.7})$$

holds for all z^t , and the number of firms evolves according to their laws of motion (1.10) and (1.12).

- Workers' search strategies are optimal. That is, (ρ, U, E) satisfy equations (1.2), (1.3), and (1.1).
- Aggregate resource feasibility holds for all z^t :

$$\sum_{j \geq 0, x^j} N(x^j, z^t) \left[\lambda(m(x^j, z^t))V(x^j, z^t) + \sum_{\tau=0}^{j-1} L_\tau(x^j, z^t) \right] = 1. \quad (\text{A.8})$$

A.1.3 Calibrated Firm Size and Age Distribution

Table A.1 provides the distribution of firm shares and employment share by size classification in the Business Dynamics Statistics data and in the calibrated

model. The share of firms in each size category less than 2, 5, and 10 years old is also provided for the model and data.

Table A.1: Firm and Employment Shares by Size and Age

Size class	1-49	50-249	250-999	1k-9,999	10k+
Data					
Firm Shares	.956	.0364	.0054	.0017	.0002
Emp. Shares	.293	.162	.109	.176	.259
Share ≤ 2 years	.247	.072	.044	.023	.011
Share ≤ 5 years	.397	.169	.102	.054	.037
Share ≤ 10 years	.578	.313	.202	.120	.071
Model					
Firm Shares	.956	.0365	.006	.0014	.0002
Emp. Shares	.329	.166	.119	.152	.236
Share ≤ 2 years	.161	.029	.022	0.02	.015
Share ≤ 5 years	.355	.083	.068	.062	.046
Share ≤ 10 years	.583	.169	.137	.128	.098

A.2 Appendix: Proofs

Proposition 5. *A Competitive search equilibrium is socially optimal.*

The proof proceeds in two steps: First, the participation constraint is substituted into the firm's problem, and using the workers' recursive equations, we show that the firms' objective function is identical to the planner's objective function for the maximization of the social surplus of the firm when $\rho(z^t) = \mu(z^t)$. Second, we show that the choice sets of the firm and planner coincide. We can then show that the firm's entry decision coincides with optimal entry of firms when $\rho(z^t) = \mu(z^t)$.

We then show that the competitive equilibrium allocation satisfies the conditions for Lemma 1. Thus, the competitive search equilibrium is socially optimal since it is a solution to the sequence problem of the planner.

Proof. Define the social surplus of the firm of age j with history (x^j, z^t) , and pre-terminated employment and contracts as

$$\mathcal{G}_j[(C_\tau)_{\tau=0}^{j-1}, (L_\tau)_{\tau=0}^{j-1}, x^j, z^t] \equiv J_j[(C_\tau)_{\tau=0}^{j-1}, (L_\tau)_{\tau=0}^{j-1}, x^j, z^t] + \sum_{\tau=0}^{j-1} L_\tau [W(C_\tau, x^j, z^t) - U(z^t)] \quad (\text{A.9})$$

Using (1.1) and (1.3), the worker surplus satisfies:

$$\begin{aligned} W(C_\tau, x^j, z^t) - U(z^t) &= [w_{f,j}(x^j, z^t)(1 - a_\tau(x^j, z^t)) + w_{p,\tau}(x^j, z^t)a_\tau(x^j, z^t)] \\ &\quad - (1 - \ell a_\tau(x^j, z^t))b - \rho(z^t) \\ &\quad + \beta \phi_\tau(x^j, z^t) \mathbb{E}_{x^j, z^t} [W(C_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \end{aligned}$$

Substituting this equation and (A.1) into (A.9) and using $\sigma \equiv (C_\tau, x^j, z^t)$ and $\sigma_+ \equiv (C_\tau, x^{j+1}, z^{t+1})$ along with $L_{\tau+} = L_\tau \phi_\tau / (1 - \delta) \forall \tau \leq j - 1$ and $L = \sum_{\tau=0}^{j-1} L_\tau$ to get:

$$\begin{aligned} \mathcal{G}_j(\sigma) &= \max_{\delta, m, V, C_j} \left\{ x_j z_t F(L(1 - a_\tau(x^j, z^t)/2)) - C(V, L(1 - a_\tau(x^j, z^t)/2), x z_t) - f \right. \\ &\quad - \sum_{\tau=0}^{j-1} L_\tau [[w_{f,\tau}(x^j, z^t)(1 - a_\tau(x^j, z^t)) + w_{p,\tau}(x^j, z^t)a_\tau(x^j, z^t)]] \\ &\quad \left. + \beta(1 - \delta) \mathbb{E}_{x^j, z^t} J_{j+1}(\sigma_+) \right\} \end{aligned}$$

$$\begin{aligned}
& + \sum_{\tau=0}^{j-1} L_{\tau} \left[[w_{f,\tau}(x^j, z^t)(1 - a_{\tau}(x^j, z^t)) + w_{p,\tau}(x^j, z^t)a_{\tau}(x^j, z^t)] \right. \\
& - (1 - \ell a_{\tau}(x^j, z^t))b) - \rho(z^t) \\
& \left. + \beta \phi_{\tau}(x^j, z^t) \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_{\tau}, x^{j+1}, z^{t+1}) - U(z^{t+1})] \right] \tag{A.10}
\end{aligned}$$

$$\begin{aligned}
& = \max_{\delta, m, V, \mathcal{C}_j} \left\{ x^j z^t F(L(1 - a_{\tau}(x^j, z^t)/2)) \right. \\
& - \left[\rho(z^t) + (1 - \ell a_{\tau}(x^j, z^t))b \right] L \\
& - C(V, L(1 - a_{\tau}(x^j, z^t)/2), xz_t) - f \\
& + \beta(1 - \delta) \mathbb{E}_{x^j, z^t} J_{j+1}(\sigma_+) \\
& \left. + \beta \sum_{\tau=0}^{j-1} L_{\tau} \phi_{\tau}(x^j, z^t) \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_{\tau}, x^{j+1}, z^{t+1}) - U(z^{t+1})] \right\} \tag{A.11}
\end{aligned}$$

$$\begin{aligned}
& = \max_{\delta, m, V, \mathcal{C}_j} \left\{ x_j z_t F(L(1 - a_{\tau}(x^j, z^t)/2)) - [(1 - \ell a_{\tau}(x^j, z^t))b] L \right. \\
& - \rho(z^t)[L + \lambda(m)V] - f - C(V, L(1 - a_{\tau}(x^j, z^t)/2), xz_t) \\
& + \beta(1 - \delta) \mathbb{E}_{x^j, z^t} J_{j+1}(\sigma_+) \\
& \left. + \beta(1 - \delta) \sum_{\tau=0}^j L_{\tau+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_{\tau}, x^{j+1}, z^{t+1}) - U(z^{t+1})] \right\} \tag{A.12}
\end{aligned}$$

$$\begin{aligned}
&= \max_{\delta, m, V, \mathcal{C}_j} \left\{ x_j z_t F \left(L \left(1 - a_\tau(x^j, z^t)/2 \right) \right) - \left[(1 - \ell a_\tau(x^j, z^t)) b \right] L \right. \\
&\quad - \rho(z^t) [L + \lambda(m)V] - f - C(V, L(1 - a_\tau(x^j, z^t)/2), x z_t) \\
&\quad \left. + \beta(1 - \delta) \mathbb{E}_{x^j, z^t} G_{j+1}(\sigma_+) \right\}
\end{aligned} \tag{A.13}$$

Where (A.11) makes use of the fact that the wage bill of the firm and the total wages of all cohorts are equal, and the sums of search value and leisure utility for all cohorts can be aggregated to be expressed in terms of L .

Step (A.12) makes use of the fact that $L_{\tau+} = L_\tau \phi_\tau / (1 - \delta)$ to get the last line of (A.11) to be

$$\begin{aligned}
&\beta \sum_{\tau=0}^{j-1} L_\tau \phi_\tau(x^j, z^t) \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \\
&= \beta(1 - \delta) \sum_{\tau=0}^{j-1} L_{\tau+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \\
&= \beta(1 - \delta) L_{j+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})] \\
&\quad + \beta(1 - \delta) \sum_{\tau=0}^{j-1} L_{\tau+} \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_\tau, x^{j+1}, z^{t+1}) - U(z^{t+1})]
\end{aligned} \tag{A.14}$$

Then, using the participation constraint (A.5) in the firm's problem in the LHS of (A.14), we can write

$$\rho(z^t) \frac{\lambda(m)}{m} L_{j+} = \rho(z^t) \lambda(m) V = \beta(1 - \delta) \mathbb{E}_{x^j, z^t} [W(\mathcal{C}_j, x^{j+1}, z^{t+1}) - U(z^{t+1})]$$

to get to (A.13).

This shows that the objective function maximized by the firm subject to (A.5) coincides with the objective function of the planner for maximizing the social surplus

of the firm, (A.13), when $\rho(z^t) = \mu(z^t)$. Since the firm and the planner have the same objective function, it remains to show that the firm's choice set with cohort-specific contracts is equivalent to the planner's choice set using identical policies for all workers. That is, each feasible allocation for the planner using identical policies for all workers is attainable by a combination of cohort-specific contracts, and vice versa. Since there is no time-inconsistency problem, it is sufficient to show that the choice sets of both sequence problems are equivalent at the time of firm entry.

First, I show that any allocation feasible for the planner to achieve for a given firm at entry is also feasible for the firm. Consider the sequential formulation of the planner's problem as specified in (1.9), and let a feasible allocation for the planner for a firm at entry be a list of sequences $(\mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \mathbf{a}, \delta)$ with $\mathbf{L} = (L(x^j, z^t))_{j,t \geq 0}$ and similar notation for the other variables, where $m(x^j, z^t) \in [0, 1]$, $V(x^j, z^t) > 0$, $s(x^j, z^t) \in [s_0, 1]$, $a(x^j, z^t) \in [0, 1]$, $\forall x^j, z^t$ and \mathbf{L} and \mathbf{N} evolve according to their laws of motion for all $j \geq 0, t \geq 0$. Note that the planner's policies are identical for the entire stock of workers for a given firm. Let the firm's allocation be described by a list of sequences $((\mathbf{L}_\tau)_{\tau=\mathbf{0}}^{j-1} (\mathcal{C}_\tau)_{\tau=\mathbf{0}}^{j-1}, \mathbf{V}, \mathcal{C}_j, \delta)$, with $\mathbf{L}_\tau = (L_\tau(x^j, z^t))_{j,t \geq 0}$ and similar notation for contracts and exit probabilities. Note that the firm chooses a history-dependent and cohort-specific sequence of contracts. To show that the firm can reproduce any allocation feasible for the planner, we can set the vector of firm's policies \mathbf{V} , and δ identical to the planner for each history, and set for each cohort τ and history (x^j, z^t) , $s_\tau(x^j, z^t) = s(x^j, z^t)$, $a_\tau(x^j, z^t) = a(x^j, z^t) \forall j, \tau \geq 0$. The value of new contract \mathcal{C}_j is set to attract the queue length that produces the same matching

probability as the planner, $m(x^j, z^t)$ in each history. Then for each cohort τ , L_τ evolves so that $L = \sum_{\tau=0}^{j-1} L_\tau$ for the firm's total labor for all cohorts in each history, which is identical to the planner's stock of labor. Note that the firm can mimic the planner's allocation because it has no need to commit to separation rates or part-time employment rates in future histories to reach any value of contract \mathcal{C}_j . This is because the firm can use wages $w_{p,\tau}(x^j, z^t)$ and $w_{f,\tau}(x^j, z^t)$ in any history to alter the value of contract \mathcal{C}_j to attract the appropriate queue length without committing to a or s that would be inconsistent with the planner's choice in any history.

To show that any feasible allocation for the firm to achieve at entry is also feasible for the planner, we must show that the planner can achieve in any history an identical total labor force, hiring and exit policies, and total separation and part-time labor rate as the firm achieves through cohort-specific policies. To do this, we simply can set the planner's choice of $m(x^j, z^t)$ and $V(x^j, z^t)$ to match the vacancies $V(x^j, z^t)$ chosen by the firm in all histories, as well as the matching rate m chosen implicitly through the value of contract $\mathcal{C}_j(x^j, z^t)$. Similarly, the planner can set $\delta(x^j, z^t)$ to be identical to the firm's choice in every history. Lastly, if the planner's policy satisfies $s(x^j, z^t)L(x^j, z^t) = \sum_{\tau=0}^{j-1} s_\tau(x^j, z^t)L_\tau(x^j, z^t)$ and $a(x^j, z^t)L(x^j, z^t) = \sum_{\tau=0}^{j-1} a_\tau(x^j, z^t)L_\tau(x^j, z^t)$ at each history (x^j, z^t) , then the labor stock of the planner will be identical to the sum of labor stocks over all $j - 1$ cohorts of the firm. Since any allocation of labor in each history can be replicated by the planner, the choice set of the planner and the firm are identical. Since we showed they also maximize the same objective function when $\rho(z^t) = \mu(z^t)$ in (A.10), the maximization problem of

the entrant firm and the planner's social value of the firm at entry are identical.

It remains to show that firm entry is also socially efficient when $\rho(z^t) = \mu(z^t)$. Since the social value of an entrant firm coincides with the firm's value function at entry, we can show that this value coincides with the planner's value of an entering firm, $G_0(0, x, z^t)$ as specified in (A.15) when $\rho(z^t) = \mu(z^t)$. So the free entry condition in competitive equilibrium (A.7) coincides with the condition for socially optimal firm entry (1.15). Because of aggregate resource feasibility in equilibrium (A.8), the planner's resource constraint (1.13) is also satisfied. Since the allocation of a competitive search equilibrium satisfies all the conditions of 1, it is a solution of the planning problem (1.9) and is thus socially optimal. \square

Proposition 6. • *Suppose that a solution of (1.14) and (1.15) exists with associated allocation $\mathbf{A} = (\mathbf{N}, \mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \mathbf{a}, \delta)$ satisfying $N(z^t) > 0$ for all z^t . Then \mathbf{A} is a solution to the sequential planning problem (1.9).*

- *If $K(z)$, f , and b are sufficiently small and if $z_1 = \dots = z_n = \bar{z}$, equations (1.14) and (1.15) have a unique solution (G, M) . If the transition matrix $\psi(z_j|z_i)$ is strictly diagonally dominant and if $|z_i - \bar{z}|$ is sufficiently small for all i , equations (1.14) and (1.15) have a unique solution.*

Proof. Let $\beta^t \psi(z^t) \mu(z^t) \geq 0$ be the multiplier on the resource constraint (1.13) in history z^t . Then $\mu(z^t)$ is the social value of a worker in history z^t . For the vector of multipliers $\mu = (\mu(z^t))$, Let $G_t(L, x, z^t)$ denote the value of an existing firm with employment L and productivity x in aggregate productivity history z^t . The sequence

G_t obeys the recursive equations

$$\begin{aligned}
G_t(L, x, z^t) = & \max_{\delta, s, a, V, m} xz_t F(L(1 - a/2)) - (1 - \ell a)bL - f - \\
& \mu(z^t)[L + \lambda(m)V] - C(V, L(1 - a/2), xz_i) \\
& + \beta(1 - \delta)\mathbb{E}_{x, z^t} G(L_+, x_+, z^{t+1})
\end{aligned} \tag{A.15}$$

s.t.

$$L_+ = (1 - s)L + mV, \delta \in [\delta_0, 1], a \in [0, 1], s \in [s_0, 1], m \in [0, 1] \text{ and } V \geq 0.$$

It first must be shown that (A.15) is equivalent to the planner's problem (1.9) via Lemma (1). Then it must be shown that the reduced problem (1.14) solves (A.15) if entry is strictly positive in all states. \square

Lemma 1. • *For given multipliers $\mu(z^t)$, there exist value functions $G_t : \mathbb{R}_+ \times X \times Z^{t+1} \rightarrow \mathbb{R}$, $t \geq 0$, satisfying the system of recursive equations (A.15).*

- *If $\mathbf{X} = (\mathbf{N}, \mathbf{L}, \mathbf{V}, \mathbf{m}, \mathbf{s}, \mathbf{a}, \delta)$ is a solution of the planning problem (1.9) with multipliers $\mu = (\mu(z^t))$, then the corresponding firm policies also solve problem (A.15) and the complementary-slackness condition*

$$\sum_{x \in X} \pi_0(x) G_t(0, x, z^t) \leq K(z_t), \quad N_0(z^t) \geq 0 \tag{A.16}$$

is satisfied for all z^t .

Conversely, if \mathbf{X} solves for every firm problem (A.15) with multipliers μ , and if conditions (A.16) and the resource constraint (1.13) hold for all z^t , then \mathbf{X} is a solution of the planning problem (1.9).

Proof. Part a): Same as in Kaas and Kircher (2015). Show the operator T has a unique fixed point which is a sequence of bounded functions.

Part b): Taking a solution \mathbf{X} of the planning problem and writing $\beta^t \psi^t \mu(z^t) \geq 0$ for the multipliers on constraints (1.13), then the solution \mathbf{X} maximizes the Lagrangian:

$$\begin{aligned} \mathcal{L} = \max \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \Bigg\{ & -K(z_t)N_0(z^t) \\ & + \sum_{j \geq 0, x^j} N(x^j, z^t) \left[x_j z_t F((1 - a(x^j, z^t)/2)L(x^j, z^t)) \right. \\ & - b(1 - \ell a(x^j, z^t))L(x^j, z^t) - C(V(x^j, z^t), (1 - a(x^j, z^t)/2)L(x^j, z^t), x_j z_t) \\ & \left. - \mu(z^t)[L(x^j, z^t) + \lambda(m(x^j, z^t))V(x^j, z^t)] \right] \Bigg\} \end{aligned} \quad (\text{A.17})$$

Which is the sequential formulation of the recursive problem (A.15) with multipliers $\mu(z^t)$. This implies that the firm policies from the planner's problem yielding allocation \mathbf{X} also solve the recursive problem. The Lagrange function can be re-arranged to be the sum of the social values of entrant firms and the social values of existing firms of idiosyncratic history x^j at $t = 0$ (and aggregate history z^0).

$$\begin{aligned} \mathcal{L} = \max_{N_0(\cdot)} \sum_{t, z^t} \beta^t \psi(z^t) N_0(z^t) \Big[& -K(z^t) + \sum_x \pi_0(x) G_t(0, x, z^t) \Big] \\ & + \sum_{z \in Z} \psi(z^0) \sum_{j \geq 1, x^j} N(x^j, z^t) G_0(L(x^j, z^0), x_j, z_0) \end{aligned} \quad (\text{A.18})$$

This rearrangement makes clear that condition (A.16) describes optimal entry of firms. Since the solution to the planner's problem yields an allocation \mathbf{X} which is also the maximum of the sequential sum of recursive firm-level problems for all histories, the solution to (1.9) is also a solution to (A.15) with (A.16) satisfied for all aggregate histories z^t .

To prove the converse, suppose \mathbf{X} solves for every firm the recursive problem (A.15) with given multipliers $\mu(z^t)$, and that (A.16) and the resource constraint (1.13) are satisfied. The proof that \mathbf{X} solves the original sequential planning problem (1.9) subject to (1.13) follows by contradiction: Suppose there is an alternate allocation $\hat{\mathbf{X}}$ for the sequential planning problem under constraint (1.13) that strictly dominates \mathbf{X} . Denoting the net output created by firm (x^j, z^t) in \mathbf{X} as

$$\begin{aligned} O(x^j, z^t) \equiv & x_j z_t F\left((1 - a(x^j, z^t)/2)L(x^j, z^t)\right) - b(1 - \ell a(x^j, z^t))L(x^j, z^t) \\ & - C\left(V(x^j, z^t), (1 - a(x^j, z^t)/2)L(x^j, z^t), x_j z_t\right) \end{aligned}$$

and $\hat{O}(x^j, z^t)$ for $\hat{\mathbf{X}}$ Then total surplus \hat{S} in allocation $\hat{\mathbf{X}}$ satisfies

$$\begin{aligned} \hat{S} &= \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) \hat{N}_0(z^t) + \sum_{j \geq 0, x^j} \hat{N}(x^j, z^t) \hat{O}(x^j, z^t) \right\} \\ &= \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) \hat{N}_0(z^t) + \mu(z^t) - \mu(z^t) + \sum_{j \geq 0, x^j} \hat{N}(x^j, z^t) \hat{O}(x^j, z^t) \right\} \\ &\leq \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \left\{ -K(z_t) \hat{N}_0(z^t) + \mu(z^t) \right. \\ &\quad \left. + \sum_{j \geq 0, x^j} \hat{N}(x^j, z^t) [\hat{O}(x^j, z^t) - \mu(z^t) (\hat{L}(x^j, z^t) + \lambda(\hat{m}(x^j, z^t)) \hat{V}(x^j, z^t))] \right\} \\ &\leq \sum_{t \geq 0, z^t} \beta^t \psi(z^t) \hat{N}_0(z^t) \left[-K(z_t) + \sum_x \pi_0(x) G_t(0, x, z^t) \right] \\ &\quad + \sum_{z \in Z} \psi(z^0) \sum_{j \geq 0, x^j} N(x^j, z^0) G_0(L(x^j, z^0), x_j, z^0) + \sum_{t, z^t} \beta^t \psi(z^t) \mu(z^t) \\ &\leq \sum_{t \geq 0, z^t} \beta^t \psi(z^t) N_0(z^t) \left[-K(z_t) + \sum_x \pi_0(x) G_t(0, x, z^t) \right] \\ &\quad + \sum_{z \in Z} \psi(z^0) \sum_{j \geq 0, x^j} N(x^j, z^0) G_0(L(x^j, z^0), x_j, z^0) + \sum_{t, z^t} \beta^t \psi(z^t) \mu(z^t) = S. \end{aligned}$$

The first equality follows from adding and subtracting μ . The first inequality comes

from the resource constraint (1.13). The second inequality follows from the discounted sum of surplus values for an individual firm of age j at t being

$$\sum_{\tau \geq t} \beta^{\tau-t} \sum_{x^{j+\tau-t}, z^\tau} \psi(z^\tau | z^t) \pi(x^{j+\tau-t} | x^j) \prod_{k=t}^{\tau-1} [1 - \delta(x^{j+k-t}, z^k)] [O'(x^{j+\tau-t}, z^\tau) - \mu(z^\tau) [\hat{L}(x^{j+\tau-t}, z^\tau) + \lambda(\hat{m}(x^{j+\tau-t}, z^\tau)) \hat{V}(x^{j+\tau-t}, z^\tau)]]$$

being bounded above by $G_t(0, x_0, z^t)$ (for new firms, $a = 0$) or by $G_0(L(x^j, z^0), x^j, 0)$ (for firms of age j existing at $t = 0$) by the definition of G_t . The third inequality follows from the complementary slackness condition (A.16). Either $(-K(z_t) + \sum_x \pi_0(x) G_t(0, x, z^t))$ is zero, in which case the first sum is zero on both sides of the inequality, or the term is negative, in which case $N_0(z^t) = 0$ and $N'_0(z^t) \geq 0$, making the final expression weakly larger. The equality follows from the definition of the surplus value S and the original assumption that allocation \mathbf{X} solves (A.15).

This proves $S' \leq S$ and hence contradicts the hypothesis that $S' > S$. This completes the proof of Lemma 1. \square

A.3 Appendix: Data

A.3.1 Constructing the CPS sample and flow data

I extract the following variables from the CPS raw data, downloaded from NBER.

The variables needed to reproduce the gross flows analysis is listed in Table A.2, including the variable name and description.¹ Consecutive months of the CPS are merged by matching household identifier, state, and person number. Additional

¹The variable name in the STATA code is in parentheses, if changed.

checks are used to ensure an actual match such as age, race, and gender.² Once the merged files are created, the labor market transition of each matched individual is created from *PEMLR* and *PRWKSTAT*. Flows are then constructed and weights applied (*fweight*) to produce monthly flow probabilities for each possible labor market state, differentiating employed individuals between full-time employment, part-time employment for noneconomic reasons, and part-time employment for economic reasons. The weighted flows are then seasonally adjusted. I do not make any correction for time-aggregation as is used in Shimer (2005) or Elsby et al. (2009).

Table A.2: CPS Variables Used In Producing Labor Flows

HRHHID	1-12	Household identifier
HRMIS	63-64	Month-in-sample (mis)
HRLONGLK	69-70	Longitudinal link indicator (llind)
HRERSUF	75-76	Serial suffix
GESTFIPS	93-94	FIPS State code
PEAGE	122-123	Person's age as of the end of survey week (age)
PESEX	129-130	Sex (sex)
PTDTRACE	139-140	Race (race)
PULINENO	147-148	Person's line number (line)
PEMLR	180-181	Monthly labor force recode (status)
PEHRUSLT	224-226	How many hrs/week do you usu. work at your job/jobs?
PEHRACTT	247-249	Last week, how many hrs did you actually work?
PRPTREA	405-406	Detailed reason for part-time
PRWKSTAT	416-417	Full/part-time work status
PWSSWGT	613-622	Final weight (fweight)

²For dates where consecutive months cannot be linked, the flows are linearly interpolated. This occurs in Jun-Sept. 1995, Sept-Oct. 2009, and Sept.-Oct. 2009. The September dates are where Labor Day corresponds with the reference week in the survey, causing a counterfactual spike in the number individuals reporting "at work part-time for noneconomic reasons."

The main variables used to classify workers by labor force and part-time or full-time status are *PEMLR*, *PRFTLF*, *PRPTREA*, and *PRWKSTAT*. The variable *PEMLR* gives the individual's labor force status, whether employed, unemployed, or not in the labor force. *PRFTLF* gives the individual's part-time or full-time labor force status by whether they usually work 1 – 34 or 35+ hours per week, regardless of current employment or hours actually worked in the survey week. *PRPTREA* gives the reason that any worker classified as full-time who worked 1-34 hours or any worker classified as part-time or usually part-time ($PEHRUSLT = 0-34$ OR $PEHRACTT = 1-34$) gives for why they are working part-time, along with their usual part-time or full-time status. Those that report “Slack Work/Business Conditions”, “Could Only Find PT Work”, and “Seasonal Work” are considered part-time (or usually part-time) for economic reasons. Those who choose any other reason for working part-time (such as “Child Care Problems” or “Health/Medical Limitations”) are classified as part-time (or usually part-time) for non-economic reasons. *PRWKSTAT* then classifies individuals as full-time or part-time work status based on actual hours worked, usual part-time or full-time status, and economic or non-economic reasons. Those who list economic reasons for part-time work but report not being available for or wanting full-time work are also classified as part-time for non-economic reasons. The actual BLS definition I use to separate employed workers is the classification of “At work” by full- or part-time and by reason. This corresponds to those who actually worked 35 or more or less than 35 hours the past week. If 1-34 hours was worked, those who gave economic reasons and reported availability and desire to work full-time are

classified as “At work part-time for economic reasons”, and those who are not able or willing to work full-time or give non-economic reasons for working less than 35 hours in the reference week are classified as “At work part-time for noneconomic reasons.” Note that this classification leaves out absent workers who count as employed, and that part- vs. full- time is determined by actual hours worked, not usual part- or full-time status. The values for variable *PRWKSTAT* are listed in Table A.3. Following

Table A.3: Values for *PRWKSTAT*

-1	Not in Universe
1	Not in Labor Force
2	FT Hours (35+), Usually FT
3	PT for Economic Reasons, Usually FT
4	PT for Non-economic Reasons, Usually FT
5	Not at Work, Usually FT
6	PT Hours, Usually PT for Economic Reasons
7	PT Hours, Usually PT for Non-economic Reasons
8	FT Hours, Usually PT for Economic Reasons
9	FT Hours, Usually PT for Non-economic Reasons
10	Not at Work, Usually PT
11	Unemployed FT
12	Unemployed PT

BLS definitions in Statistics (1997), I classify $PRWKSTAT = 2, 8, 9$ as “at work full-time”, $PRWKSTAT = 3, 6$ as “at work part-time for economic reasons”, and $PRWKSTAT = 4, 7$ as “at work part-time for non-economic reasons”.³ Absent

³The following results are also robust to classification using classification by usual hours worked, ($FT:PRWKSTAT = 2$, $PTE:PRWKSTAT = 3, 6, 8$ and $PTN:PRWKSTAT = 4, 7, 9$)

workers are not included in “At work” full-time or part-time.⁴

A.3.2 PTE workers by Origin

Figure A.1 plots the shares of the current stock of *PTE* workers by their status the prior month. The majority of workers reported as at work part-time for economic reasons come from employment (the red and green lines). Only about 10% of *PTE* workers flowed from *U* or *N* in a given month. It is clear that this flow of workers from nonemployment is also constant over the business cycle.

A.3.3 Cyclical characteristics of all flows

Table A.4: Cyclical Properties of PTE outflows

From To	<i>PTE</i> <i>FT</i>	<i>PTE</i> <i>PTN</i>	<i>PTE</i> <i>PTE</i>	<i>PTE</i> <i>U</i>	<i>PTE</i> <i>N</i>
<i>std(x)</i>	.077	.073	.097	.092	.105
<i>std(x)/std(Y)</i>	3.25	3.11	4.12	3.91	4.45
<i>corrcoef(Y, x)</i>	.855	.693	-.873	-.255	.458

Table A.6 gives the standard deviation and relative volatility to that of aggregate output, and correlation with aggregate output of the cyclical component of the logged and H-P filtered stocks and flow probabilities for all labor market flows.

⁴For stocks, the monthly, seasonally adjusted series for total employment, part-time for economic reasons, and part-time for non-economic reasons for nonagricultural industries are pulled from FRED (Series LNS12035019, LNS12032197, and LNS12032200).

Table A.5: Cyclical Properties of U outflows

From To	U FT	U PTN	U PTE	U U	U N
$std(x)$.171	.185	.075	.080	.068
$std(x)/std(Y)$	7.26	7.88	3.18	3.41	2.90
$corrcoef(Y, x)$.808	.711	-.245	-.718	0.579

A.3.4 Distinguishing Slack Work and Failed Full-time Searchers

The reasons reported by workers as to why they are employed part-time that are classified as “economic reasons” are “Slack Work/Business Conditions,” “Failed to Find a Full-time Job,” and “Seasonal Work.” One concern about lumping these reasons into one population is that the behavior and flows for workers by detailed reason may be very different, and hence be important to distinguish. For instance, one might reasonably expect many failed full-time searchers to have come from unemployment rather than full-time employment the previous month. Similarly, a part-time employed worker due to slack work/business conditions might be more likely to have come from previously being employed full-time rather than having come from nonemployment. It could also be the case that the types of jobs held by failed full-time searchers is different than those who are in PTE due to slack work/business conditions. If the jobs held by the slack work group were once full-time positions with their current employer, it could be that the worker is more likely to gain full-time status from increased hours demanded in the same job, while a failed full-time searcher may only reach full-time status by switching jobs. In this section, I look at the stocks and flows of part-time and full-time employment, but further disaggregated into part-time

Table A.6: Cyclical Properties of Labor Market Statistics

	Y	FT	PTN	PTE	$u\ rate$
$std(x)$.024	.027	.024	.174	.190
$std(x)/std(Y)$	1	1.15	1.00	7.39	8.08
$corr(Y, x)$	1	.906	.826	-.861	-.795
	$FT - FT$	$FT - PTN$	$FT - PTE$	$FT - U$	$FT - N$
$std(x)$.009	.075	.150	.110	.070
$std(x)/std(Y)$	0.39	3.19	6.39	4.68	2.96
$corr(Y, x)$.380	-.135	-.773	-.525	.598
	$PTN - FT$	$PTN - PTN$	$PTN - PTE$	$PTN - U$	$PTN - N$
$std(x)$.052	.022	.147	.068	.055
$std(x)/std(Y)$	2.19	0.96	6.24	2.89	2.36
$corr(Y, x)$.137	.218	-.830	-.597	.512
	$PTE - FT$	$PTE - PTN$	$PTE - PTE$	$PTE - U$	$PTE - N$
$std(x)$.077	.073	.097	.092	.105
$std(x)/std(Y)$	3.25	3.11	4.12	3.91	4.45
$corr(Y, x)$.855	.693	-.873	-.255	.458
	$U - FT$	$U - PTN$	$U - PTE$	$U - U$	$U - N$
$std(x)$.171	.185	.075	.080	.068
$std(x)/std(Y)$	7.26	7.88	3.18	3.41	2.90
$corr(Y, x)$.808	.711	-.245	-.718	0.579
	$N - FT$	$N - PTN$	$N - PTE$	$N - U$	$N - N$
$std(x)$.102	.063	.140	.125	.003
$std(x)/std(Y)$	4.34	2.66	5.93	5.30	0.11
$corr(Y, x)$.786	.764	-.604	-.777	.258

by detailed reason. I focus on the two subsets of “Slack Work” and “Failed Full-time Search” for PTE . I make the argument that the two groups behave similarly enough that the distinction between SW and FFT is not very large, especially relative to the differences between PTN and PTE .

The stocks are again taken from FRED, while flow probabilities are constructed

from the basic monthly CPS files from 1994-2014. To produce the labor market states, the variables PEMLR and PRWKSTAT are used to categorize FT, PTN, U , and N . If a worker is classified as PTE , PRPTREA is used to create one population of slack work SW and one group of failed full-time searchers FFT .

The main difference in the stocks of SW and FFT are their relative size and cyclicalities. Looking at stocks before the redesign, the two groups are very similar in size. After the redesign, SW seems to increase substantially as the reason for part-time employment, especially during the Great Recession. Figure A.2 plots the shares of employment of both groups. What is noticeable in the graphs is the sharper increases and decreases in the slack work group during recessions. When looking at flows, however, the distinction between the two groups is not so clear.

Table A.7: US Average Monthly Flow Probabilities: CPS 1994:1-2014:12

From	To					
	FT	PTN	SW	FFT	U	N
FT	.829	.077	.011	.002	.009	.014
PTN	.278	.531	.025	.013	.017	.061
SW	.333	.199	.248	.050	.053	.038
FFT	.190	.220	.112	.296	.065	.046
U	.115	.067	.020	.019	.492	.225
N	.015	.021	.001	.001	.025	.901

From the average monthly flow probabilities, we can see that the flows of part-

time employed due to slack work and the flows of failed full-time searchers are similar. The main differences in the outflows involves the flow probability from each group to full-time employment. The persistence and separation rates to nonemployment are very similar. The second major difference between the two groups is the flow probability of full-time and voluntary part-time workers into *SW* is larger than the flow probability from these groups to *FFT*. This reflects the idea that slack work is largely comprised of workers who did not switch employers, but rather had their hours with their current employer cut below 35 hours per week.

Although this may seem like evidence that these two populations are distinct, if we look at the workers who flow into the failed full-time search category in a given month, only about 18% of these workers transitioned from nonemployment. On average, 50% of workers in *FFT* came from *FT*, *PTN*, or *SW* the previous month. For workers who were part of the *SW* inflow, about 10% came from nonemployment and 64% came from *FT*, *PTN*, or *FFT*. The flow probabilities of workers from unemployment and nonparticipation to either *PTE* group is also nearly identical.

Another cause for concern in making a distinction between *PTE* workers by detailed reason is that the flow probabilities between all three part-time categories is quite large. It is very likely that there is significant misclassification in these part-time categories, given the high flow probabilities between groups. One possibility is that workers reporting failed full-time search is more an indicator of whether they are actively engaged in on-the job-search rather than the origin of the part-time employed worker or the nature of the job. The idea that the *FFT* group consists of workers

who came from nonemployment and ended up settling for a part-time job is not entirely supported by the flows data. One final concern is that further disaggregating *PTE* by detailed reason adds additional noise to the CPS flow probabilities due to a very small sample size for some flows, especially to and from *FFT*. Due to the lack of clear differences in the inflow or outflow probabilities of these *PTE* groups by detailed reason, I focus only on the categories of part-time employment by economic vs. non-economic reasons in this analysis.

A.3.5 Aggregate Hours Fluctuations and Part-time for Economic Reasons

The business cycle fluctuations in aggregate hours worked are similar in magnitude to real output per capita, while employment is roughly two-thirds as volatile over the business cycle. Hours per worker is only about one-third as volatile as per capita output.⁵ Of interest is the extent to which these cyclical fluctuations in hours can be accounted for by the transition between full-time and part-time labor, or if hours fluctuations are largely within these categories. The total of aggregate hours accounted for by workers in *PTE* does fluctuate cyclically as expected, as seen in Figure A.4. However, the net change in aggregate hours due to flows in and out of *PTE* is a function of the size of these flows and their originating stocks. The sum of hours changes experienced by workers flowing into and out of *PTE* are substantial and cyclical, but offsetting. Figure A.3 shows that the total of hours changes experienced by workers who transition from *FT* to *PTE* is nearly as large as the total loss

⁵See Andolfatto (1996)

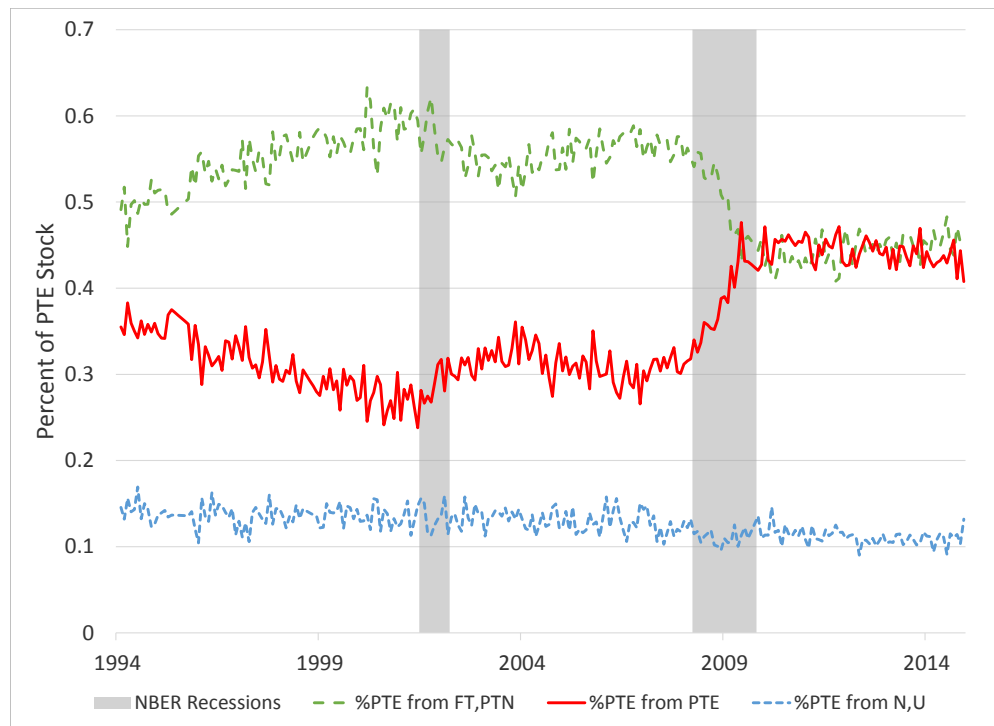
in hours from workers who move from FT to U .

Since aggregate hours worked is a stock whose changes are governed by both hours changes and the size of each flow, it is important to note that the change in hours due to a flow may be cyclical even if the flow probability itself is constant. If the stock of unemployment increases, the hours loss from the $PTE - U$ flow increases along with the number of workers moving to PTE from U due to the larger pool of unemployed. Thus, the fluctuation in hours may be cyclical even if the flow probability from PTE to U is nearly constant. Similarly, cyclical flow probabilities may produce only a small contribution to aggregate hours fluctuations if their stocks or their hours contributions are offsetting. For example, while the $FT - PTE$ flow and $PTE - FT$ flow are negatively correlated, The countercyclical $FT - PTE$ flow reduces hours and the procyclical $PTE - FT$ flow increases hours. If stocks were held constant, both flows should work to make aggregate hours more countercyclical. Figure A.5 shows that the hours changes contributed by workers in each flow offset to produce a much smaller net change. Since the PTE stock grows substantially in recessions, the total number of workers (and hours gained) moving to FT increases even though the transition probability decreases. This offsets much of the hours loss due to the increase in the $FT - PTE$ flow. Figure A.6 shows the relative magnitude of the gain in hours from the flow of workers to FT . Interestingly, the most cyclical flow probability (UE) is the least cyclical in terms of its contribution to the change in aggregate hours.

A.3.6 The Survey of Income and Program Participants

The Survey of Income and Program Participants consists of multiple panels of household surveys over varying lengths of time. In this study, I focus on the 1996 panel of the SIPP. This panel surveys about 60,000 individuals every four months for 48. The survey asks individuals retroactively their labor force participation/employment, work hours, and earnings at monthly frequency during each survey period. Due to the retroactive nature of the data, the SIPP responses to labor market questions differ slightly from their counterparts in the CPS. In CPS surveys, individuals are asked about labor market behavior for a specific reference week. Responses to questions on part-time work capture only those who report less than 35 hours of work in the reference week. In SIPP surveys, the respondent is asked whether they worked less than 35 hours in any week during the month. Respondents in the SIPP who report having worked part-time are not asked whether they were available for or desired full-time work at the time. They are asked to provide a reason for working part-time, including “Slack work or material shortage” and “Could not find a full-time job.” Similar to the CPS before the redesign, I classify as *PTE* any respondent who reported working part-time during the reference month and provided either of the afore-mentioned reasons for working part-time. Respondents who report any other reason for part-time work are classified as *PTN*.

Figure A.1: Shares of PTE stock by Origin



Share of *PTE* in a given month by reported labor force status the prior month. The green line indicates workers in *PTE* who reported being in *FT* and *PTN* the prior month. The red line indicates the share of *PTE* workers who reported being *PTE* the prior month. The blue line indicates workers who reported being unemployed or nonparticipants the prior month.

Figure A.2: Part-time Employment for Economic Reasons: Share of Aggregate Employment

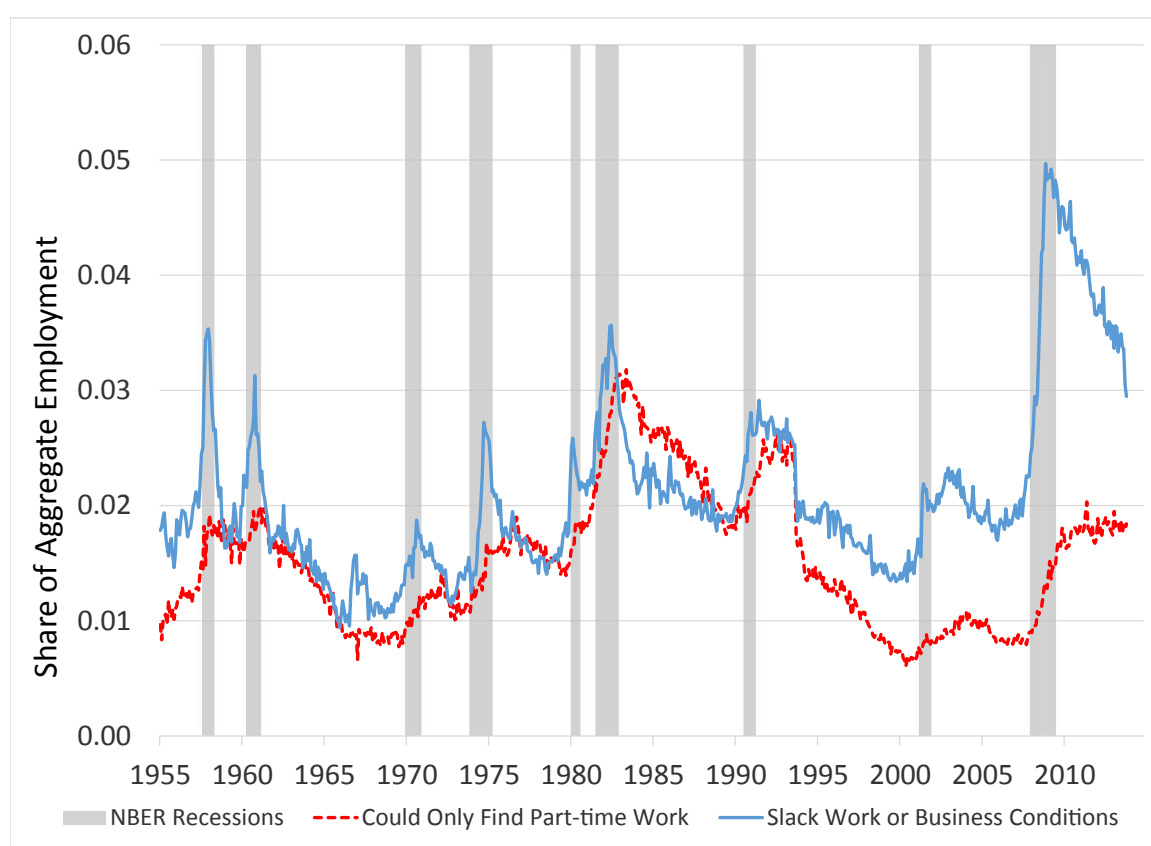
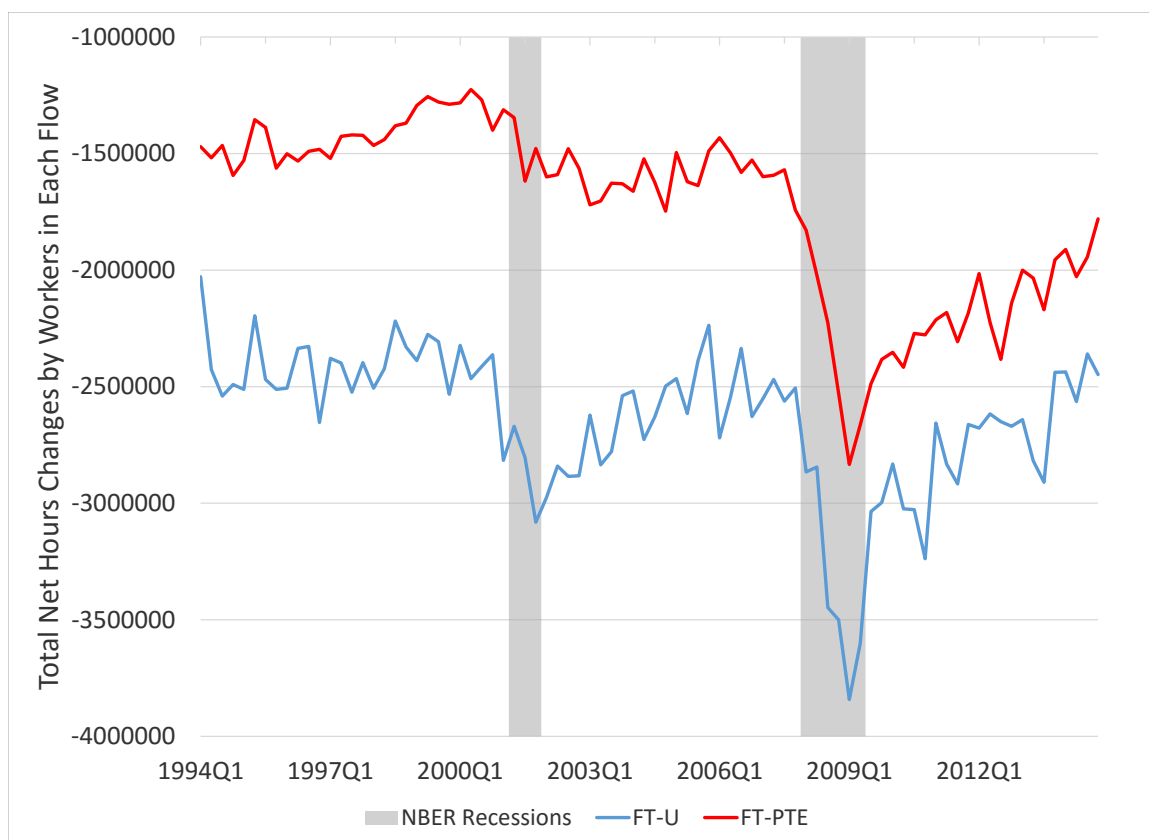
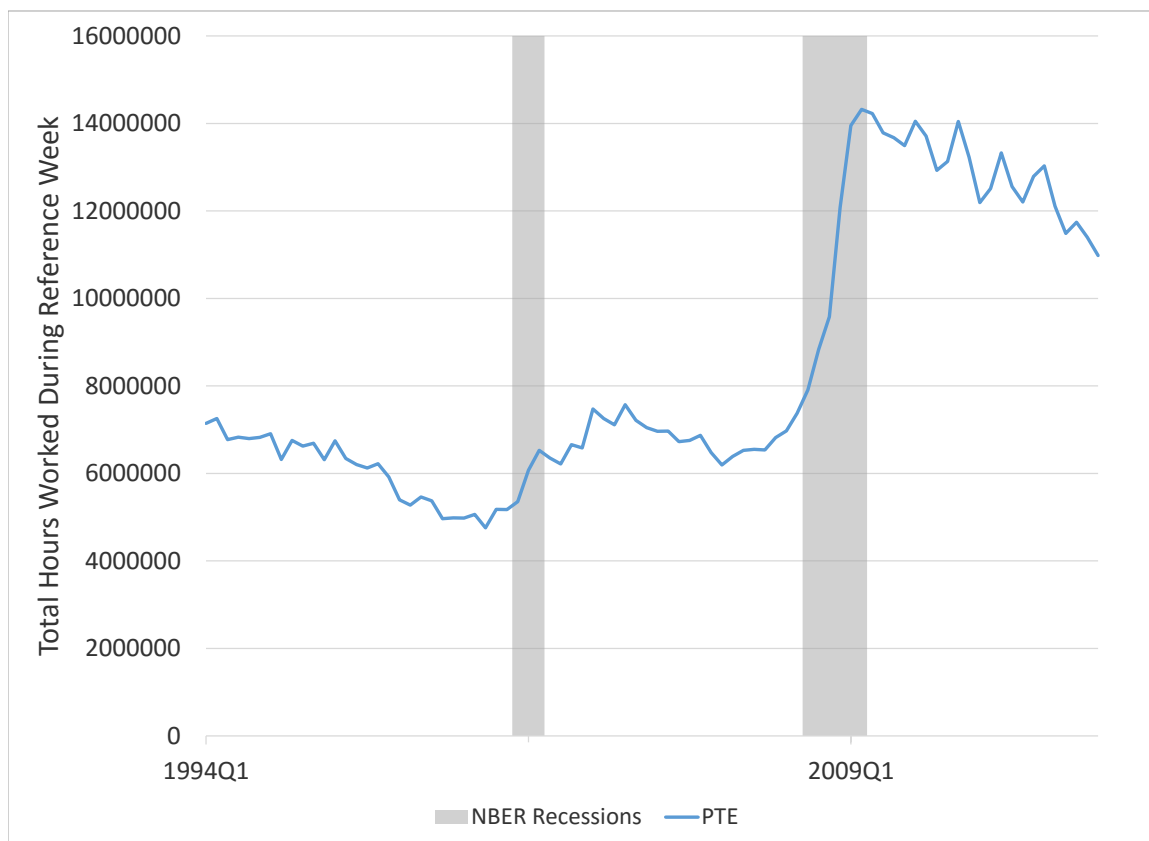


Figure A.3: Sum of Net Hours Loss by Workers in Each Flow

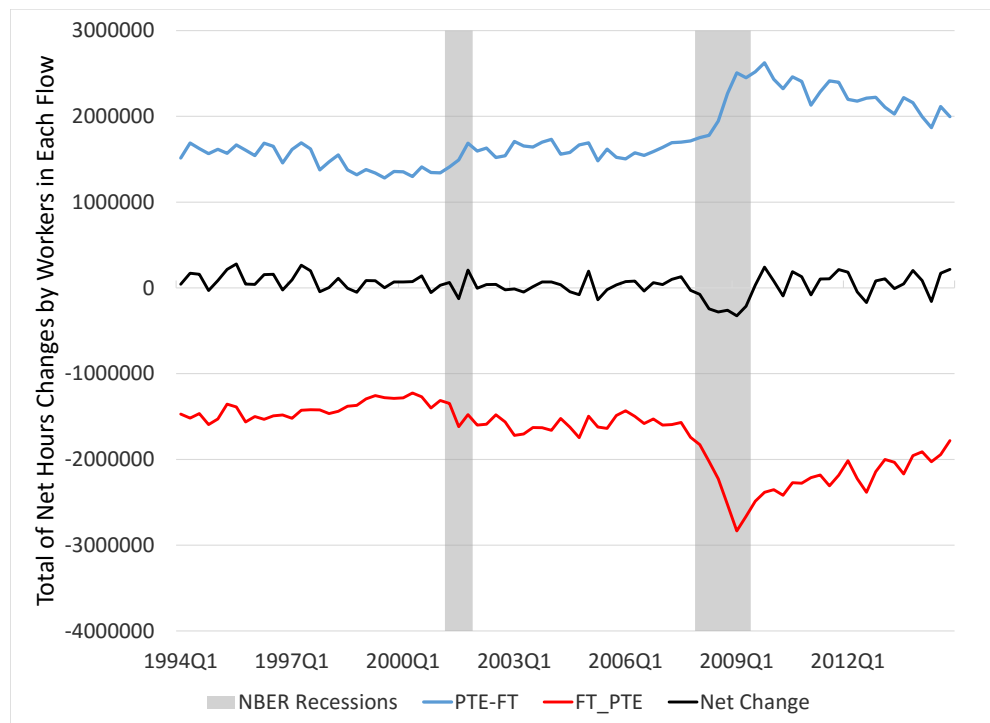


Each solid line plots the quarterly averaged total of the month-to-month change in reported weekly hours worked by all workers who experienced a transition from full-time employment to either *PTE* or *U*. The dotted line plots the total hours lost by all workers who either transitioned from *FT* to *PTE* or separated from *PTE* to *U*.

Figure A.4: Total hours worked by individuals classified in *PTE*

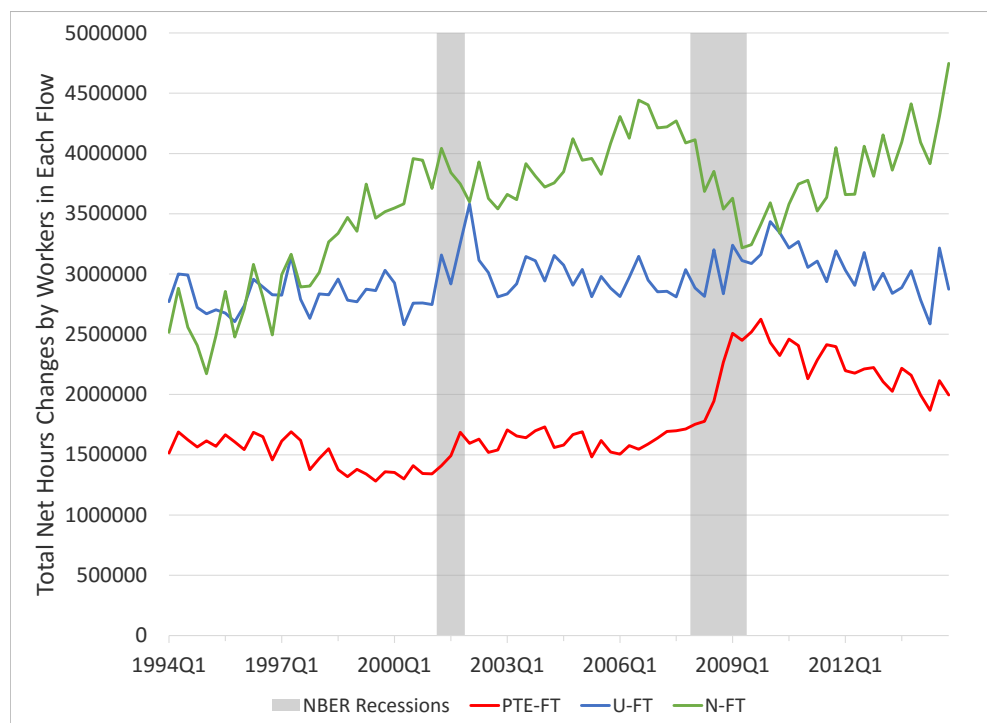
Quarterly averaged total of the weekly hours worked by all workers who report being employed part-time for economic reasons.

Figure A.5: Sum of Net Hours Gain/Loss by Workers in $PTE - FT$ and $FT - PTE$ Flows



Each solid line plots the quarterly averaged total of the month-to-month change in reported weekly hours worked by all workers who experienced a transition between PTE and FT .

Figure A.6: Sum of Net Hours Loss by Workers in Each Flow



Each line plots the quarterly averaged total of the month-to-month change in reported weekly hours worked by all workers who experienced a transition to full-time employment from either PTE , U , or N .

APPENDIX B

APPENDIX TO CHAPTER 2

B.1 Transitions from SIPP Data

I check to see if the separation rates from employment differ systematically between types. To do so, I split all workers for whom I observe a complete employment spell into two groups by their search behavior before they entered employment. The following table shows that the separation rates for both pools of workers coming from unemployment and nonparticipation is roughly equal.

Table B.1: Comparing Separation Rates

Year	Workers from U		Workers from N	
	Yearly	Monthly (δ)	Yearly	Monthly (δ)
1997	.370	.038	.370	.038
1998	.195	.018	.184	.017
1999	.129	.011	.116	.010

B.2 Probability of Dropping out of Unemployment

I consider the sample of unemployed workers and estimate a linear probability model with unemployment duration and plausible observable shocks to individuals as regressors. The regression is as follows:

$$Prob(UtoN)_{i,t} = \beta_0 + \beta_1 U duration_{i,t} + \beta_2 U duration_{i,t}^2 + \beta_3 X_{i,t} + \varepsilon_{i,t}$$

Where X consists of age, quadratic on age, education, female, race, change in spouse's employment status, change in collection of unemployment benefits, change in income of rest of household, change in number of children in household under 18, change in marital status, change in student status, and changes in an individual's self-reporting of disability. The first column is a simple cross-sectional regression. The second column includes individual fixed effects.

Duration of unemployment seems to increase the probability that an agent drops out of unemployment into inactivity in a given month. Duration seems to play a relatively small role compared with other factors that affect changes in search decisions such as changes in student or disability status, or the loss of unemployment benefits. An additional month spent in unemployment increases the probability an agent drops out by about 4%. This seems to indicate that discouragement does play a role in the difference between transitions of employed and nonemployed workers over types.

It is important to note that although both attachment and discouragement are plausible explanations for the observed difference between employed and nonemployed workers' transitions, the empirical test outlined above cannot satisfactorily identify one effect over the other. That is, both discouragement in nonemployment and increased attachment in employment serve to produce the same effect in transition rates, and a satisfactory comparison of transition rates between employed and

Table B.2: Duration Dependence

	Probability of exiting U to Nonparticipation Cross-section	Ind. Fixed Effects
Unemployment duration	0.00197** (3.26)	0.0194*** (20.77)
Age	-0.00145 (-1.50)	0.000718 (0.06)
Female	0.0316*** (9.24)	-1.769*** (-6.71)
Spouse gains job	-0.0219 (-1.29)	-0.0336 (-1.89)
Spouse loses job	-0.00818 (-0.46)	-0.00397 (-0.21)
Gain unemp. ben.	-0.0389*** (-4.35)	-0.0374** (-3.19)
Lose unemp. ben.	0.0295 (1.70)	0.0299 (1.68)
Gain in HH income	0.00779*** (11.22)	0.0106*** (12.68)
Loss in HH income	0.00756*** (10.59)	0.0104*** (12.07)
Married	-0.0115 (-0.29)	0.000123 (0.00)
Separated	-0.0445 (-1.65)	-0.0176 (-0.56)
Report as disabled	0.347*** (11.93)	0.342*** (11.13)
No longer disabled	0.187*** (6.34)	0.201*** (6.59)
Enrolled student	0.171*** (8.61)	0.122*** (6.11)
Leave student status	0.0629*** (3.69)	0.0790*** (4.13)
Observations	25964	25964
R^2	0.046	0.448

t statistics in parentheses, using heteroskedasticity robust standard errors
 * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

nonemployed workers is only possible with no conditioning for observable heterogeneity. The issue is open to other interpretations of these changes as well, such as the role of unemployment benefits in the participation decision.

APPENDIX C

APPENDIX TO CHAPTER 3

C.1 Robustness Checks

C.1.1 Including Independent Students

The first robustness check we conduct is to determine what happens when we include independent students as part of the sample. Given that independent students were also eligible for SLS loans, which were unsubsidized, a concern might be that we do not observe the kink in the data when considering the impact of total federal student loans. When one refers to Figure 3.1, we note that even among independent students, about 75% had only subsidized loans. Figure C.1(a) displays the empirical distribution of cumulative Stafford loans borrowed as a function of *need* in the final year of school when we include independent students in the sample.¹ We notice that qualitatively it is very similar to Figure 3.3: there exists a kink at $need = 0$ even when we include independent students to the sample. Quantitatively, the figures differ in terms of average amounts borrowed on either side of the threshold. To the left of the threshold, the average amount borrowed is higher in Figure C.1(a) than in Figure 3.3, but to the right of the threshold, it does not go as high in Figure C.1(a) as it does in Figure 3.3.

Table C.1 checks for discontinuities and kinks in the observable characteristics at the kinks using a quadratic in *need*. As can be seen, we can safely reject the null

¹Of the 7,800 observations, about 5,900 observations fit into the $need \in (-\$10,000, \$10,000)$ range.

for all observable characteristics. Figure C.1(b) plots log of 1994 annual income as a function of *need* for this sample. We notice that for Figure C.1(b), the change in the slope around the threshold seems less than what we observed for the case with just dependent students in Figure 3.6. Table C.2 shows the first stage results and Table C.3 shows the second stage results and the IV RK estimate. In general the results still hold, though the magnitude is lower.

C.1.2 Standard Checks

Next we conduct a standard series of robustness checks. First we make sure that the results are robust to the linear specification, then we check for robustness when excluding exogenous parameters from the regression analysis. In theory the exogenous parameters should only affect the efficiency of the *RK* estimator. Finally we check to make sure the results are robust to various bandwidth values. Overall, the results seem to hold across the various robustness checks.

C.2 Proof for Proposition 3

Proof. From equation (3.8), we know that the right-hand side of the equation is decreasing in θ . For a higher θ , we need the term on the right-hand side to decrease, which means the term on the left-hand side should be decreasing in d . Therefore, we need to determine the conditions under which the term on the left hand side is decreasing in d . Define

$$F(d) = u'(w + b - d) [u(w + b - d) - u(b - d)]^{-1},$$

then:

$$\begin{aligned}
F'(d) &= u''(w+b-d)[u(w+b-d)-u(b-d)]^{-1} \\
&\quad - u'(w+b-d)[u(w+b-d)-u(b-d)]^{-2}[-u'(w+b-d)+u'(b-d)] \\
&= -[u(w+b-d)-u(b-d)]^{-1} \left[u''(w+b-d) \right. \\
&\quad \left. - \frac{u'(w+b-d)[u'(w+b-d)-u'(b-d)]}{u(w+b-d)-u(b-d)} \right] \\
&= -[u(w+b-d)-u(b-d)]^{-1} u'(w+b-d) \left[\frac{u''(w+b-d)}{u'(w+b-d)} \right. \\
&\quad \left. - \frac{u'(w+b-d)-u'(b-d)}{u(w+b-d)-u(b-d)} \right]
\end{aligned}$$

Given the assumptions, both of the terms outside of the big square brackets are positive. This means that for $F'(d) < 0$, the term inside the big square bracket must be positive, since there is already a minus sign at the front of the expression. That is,

$$\frac{u'(w+b-d)-u'(b-d)}{u(w+b-d)-u(b-d)} < \frac{u''(w+b-d)}{u'(w+b-d)}$$

Now consider the class of utility functions with Decreasing Absolute Risk Aversion (DARA):

$$u(c) = \frac{1-\gamma}{\gamma} \left(\frac{\alpha c}{1-\gamma} + \beta \right)^\gamma \quad \text{where } \alpha > 0, \beta > -\frac{\alpha c}{1-\gamma}, \gamma \in (0, 1);$$

First, note that

$$u'(c) = \alpha \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-1}, \quad u''(c) = -\alpha^2 \left(\frac{\alpha c}{1-\gamma} + \beta \right)^{\gamma-2}.$$

Let $c_w = w+b-d$, $c_a = b-d$ and $\beta > -\frac{\alpha c_a}{1-\gamma}$;

then

$$\frac{u'(w+b-d) - u'(b-d)}{u(w+b-d) - u(b-d)} < \frac{u''(w+b-d)}{u'(w+b-d)}$$

can be written as

$$\begin{aligned} &\Rightarrow u'(c_w)[u'(c_w) - u'(c_a)] < u''(c_w)[u(c_w) - u(c_a)] \\ &\Rightarrow \alpha \left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{\gamma-1} \left[\alpha \left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{\gamma-1} - \alpha \left(\frac{\alpha c_a}{1-\gamma} + \beta \right)^{\gamma-1} \right] \\ &\quad < -\alpha^2 \left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{\gamma-2} \left[\frac{1-\gamma}{\gamma} \left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{\gamma} - \frac{1-\gamma}{\gamma} \left(\frac{\alpha c_a}{1-\gamma} + \beta \right)^{\gamma} \right] \end{aligned}$$

dividing both sides by α^2 and expanding the term

$$\begin{aligned} &\Rightarrow \left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{2\gamma-2} - \left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{\gamma-1} \left(\frac{\alpha c_a}{1-\gamma} + \beta \right)^{\gamma-1} \\ &\quad < -\frac{1-\gamma}{\gamma} \left[\left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{2\gamma-2} - \left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{\gamma-2} \left(\frac{\alpha c_a}{1-\gamma} + \beta \right)^{\gamma} \right], \end{aligned}$$

dividing both sides by $\left(\frac{\alpha c_w}{1-\gamma} + \beta \right)^{2\gamma-2}$

$$1 - \left(\frac{\frac{\alpha c_a}{1-\gamma} + \beta}{\frac{\alpha c_w}{1-\gamma} + \beta} \right)^{\gamma-1} < -\frac{1-\gamma}{\gamma} \left[1 - \left(\frac{\frac{\alpha c_a}{1-\gamma} + \beta}{\frac{\alpha c_w}{1-\gamma} + \beta} \right)^{\gamma} \right],$$

let $\frac{\frac{\alpha c_a}{1-\gamma} + \beta}{\frac{\alpha c_w}{1-\gamma} + \beta} = x$. Given our assumptions, $x \in (0, 1)$ and the inequality can be written

as:

$$\begin{aligned}
&\Rightarrow 1 - x^{\gamma-1} < -\frac{1-\gamma}{\gamma}[1 - x^\gamma] \\
&\Rightarrow \gamma(1 - x^{\gamma-1}) < -(1-\gamma)(1 - x^\gamma) \\
&\Rightarrow \gamma - \gamma x^{\gamma-1} < -(1 - x^\gamma) + \gamma(1 - x^\gamma) \\
&\Rightarrow 1 - \gamma x^{\gamma-1} - (1-\gamma)x^\gamma < 0;
\end{aligned}$$

multiply both sides by $x^{1-\gamma}$

$$\begin{aligned}
&\Rightarrow x^{1-\gamma} - \gamma - (1-\gamma)x < 0 \\
&\Rightarrow x^{1-\gamma} - (\gamma + (1-\gamma)x) < 0 \\
&\Rightarrow G(\gamma) - H(\gamma) < 0.
\end{aligned}$$

Note that $H(0) = G(0)$ and $H(1) = G(1)$. That is, G and H cross at 0 and 1. Also, $G'(\gamma) = -x^{1-\gamma} \log(x)$, $H'(\gamma) = 1 - x$, $G''(\gamma) = x^{1-\gamma} (\log(x))^2 > 0$, and $H''(\gamma) = 0$. This shows that $G(\gamma)$ is convex, while $H(\gamma)$ is linear. As a result, the inequality holds for $\gamma \in (0, 1)$

Also, note that CRRA is DARA where $\alpha = 1 - \gamma$, $\beta = 0$ and $\gamma \in (0, 1)$ and log utility is DARA with $\gamma \rightarrow 0$ and $\beta = 0$.

□

Table C.1: Relationship between *need* and Predetermined Characteristics: Dependent and Independent Students

X (co-variate)	α_1	α_2 (RK)
age	0.207 (0.55)	-0.035 (0.023)
male	0.027 (0.026)	0.019 (0.018)
white	-0.027 (0.020)	0.006 (0.013)
log parental income	-0.21 (0.14)	0.031 (0.033)
family size	0.150 (0.114)	0.021 (0.051)
College Ed. Parent	-0.029 (0.025)	0.004 (0.018)
SAT	-27.59 (21.43)	-4.39 (7.82)
Observations	5,910	5,910

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. Due to very small values in column 3, all numbers are scaled by 1000. Standard errors in parenthesis. The polynomial degree was set to 2. Each value is from a separate set of regressions. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.2: Impact of Stafford Loan Eligibility on Stafford Loans Borrowed: Dependent and Independent Students

β_2	0.0035*** (0.0016)
Observations	5,910

β_2 is the coefficient on the Stafford loan eligible \times distance from threshold. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, indicator for dependent, interaction term between dependent and parental income, quadratic in need. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.3: Impact of Stafford Loans on 1994
Earnings : Dependent and Independent Students

<i>A. OLS estimates</i>	
γ_2	-0.00266**
	(0.00136)
<i>B. 2SLS estimates</i>	
Stafford Loan (RK)	-0.00076**
	(0.00037)
Observations	5,910

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, indicator for dependent, interaction term between dependent and parental income, quadratic in need. *RK* terms in Panel A are scaled by a factor of 1000. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.4: Relationship between *need* and Pre-determined Characteristics: Linear

X (co-variate)	α_1	α_2 (RK)
age	0.077 (0.056)	-0.014 (0.011)
male	0.016 (0.032)	0.004 (0.006)
white	-0.052 (0.045)	0.005 (0.004)
log parental income	-0.243 (0.201)	0.04* (0.025)
family size	0.091 (0.082)	0.005 (0.017)
College Ed. Parent	-0.121* (0.068)	0.020 (0.031)
SAT	-34.75 (30.14)	-1.91 (2.66)
Observations	3,880	3,880

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. Due to very small values in column 3, all numbers are scaled by 1000. The polynomial degree was set to 1. Each value is from a separate set of regressions. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.5: Impact of Stafford Loan Eligibility on Stafford Loans Borrowed:
Linear

β_2	0.0052*** (0.0025)
Observations	3,880

β_2 is the coefficient on the Stafford loan eligible \times distance from threshold. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, indicator for dependent, interaction term between dependent and parental income, quadratic in need. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.6: Impact of Stafford Loans on 1994

Earnings : Linear

<i>A. OLS estimates</i>	
γ_2	-0.00385**
	(0.00192)
<i>B. 2SLS estimates</i>	
Stafford Loan (RK)	-0.00074**
	(0.00036)
Observations	3,880

γ_2 is the coefficient on the Stafford loan eligible \times distance from threshold. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, indicator for dependent, interaction term between dependent and parental income, quadratic in need. *RK* terms in Panel A are scaled by a factor of 1000. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.7: Impact of Stafford Loan Eligibility on Stafford Loans Borrowed:
Excluding Exogenous Regressors

β_2	0.0051***
	(0.0018)
Observations	3,880

β_2 is the coefficient on the Stafford loan eligible \times distance from threshold. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, indicator for dependent, interaction term between dependent and parental income, quadratic in need. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.8: Impact of Stafford Loans on 1994
Earnings : Excluding Exogenous Regressors

<i>A. OLS estimates</i>	
γ_2	-0.0055**
	(0.00268)
<i>B. 2SLS estimates</i>	
Stafford Loan (RK)	-0.00107**
	(0.00052)
Observations	3,880

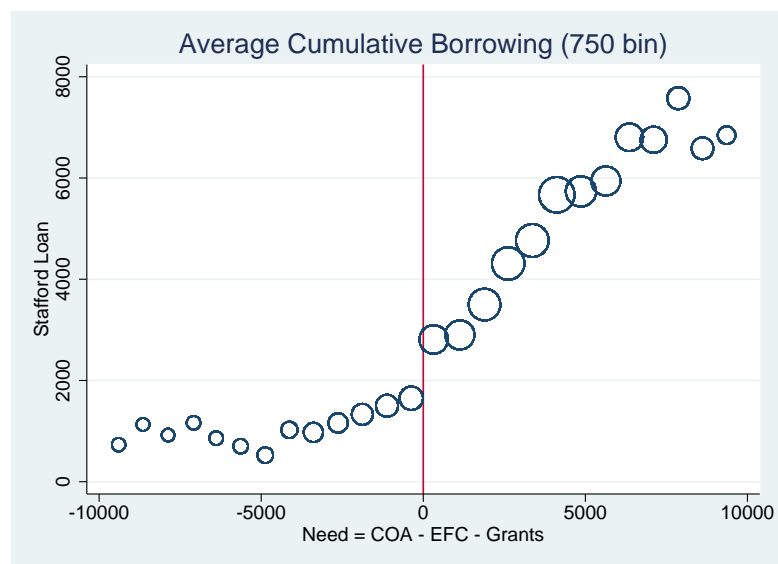
γ_2 is the coefficient on the Stafford loan eligible \times distance from threshold. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, indicator for dependent, interaction term between dependent and parental income, quadratic in need. *RK* terms in Panel A are scaled by a factor of 1000. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Table C.9: Robustness Check: Varying Bandwidth

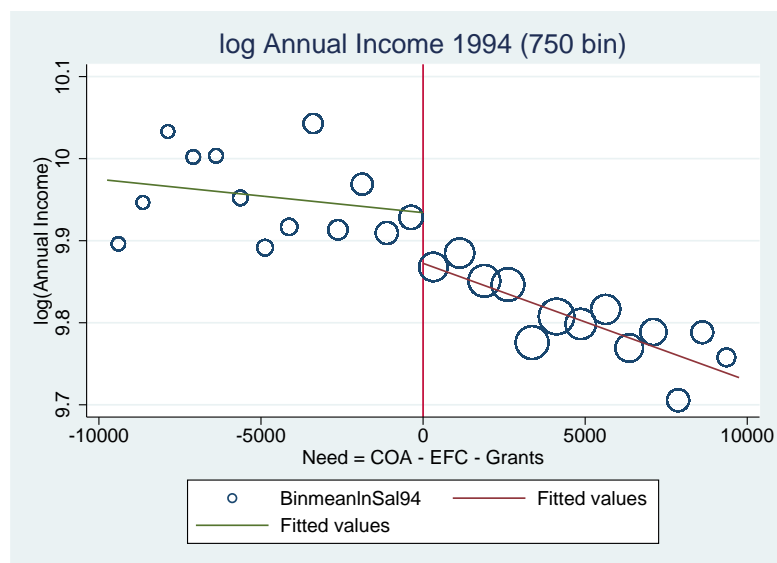
Bandwidth	(1) \$8,000	(2) \$5,000
<i>A. 2SLS estimates (polynomial order 1)</i>		
Stafford Loan (RK)	-0.0009** (0.00044)	-0.0011** (0.00053)
<i>B. 2SLS estimates (polynomial order 2)</i>		
Stafford Loan (RK)	-0.0012** (0.00061)	-0.0015** (0.00076)
Observations	3,110	2,360

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis. All regressions also include controls for age, gender, race, SAT score, major, parental education level, parental income, quadratic in *need*. Students with *need* greater than \$10,000 and less than -\$10,000 are excluded.

Figure C.1: Sample Consisting of Dependent and Independent Students



(a)



(b)

Notes: (a) Empirical distribution of Stafford loans as a function of *need*; (b) The reduced form impact of *need* on log 1994 annual earnings. The center of each circle represents the average amount borrowed in the bin. The size represents the number of people in the bin.

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