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Student effort, race gaps, and affirmative action in college admissions: theory and empirics

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STUDENT EFFORT, RACE GAPS, AND AFFIRMATIVE ACTION IN COLLEGE ADMISSIONS: THEORY AND EMPIRICS

by

Brent Richard Hickman

An Abstract

Of a thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
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in the Graduate College of
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Thesis Supervisors: Professor Srihari Govindan
Professor Harry J. Paarsch

ABSTRACT

In this dissertation, I develop a framework to investigate the implications of Affirmative Action in college admissions on both study effort choice and college placement outcomes for high school students. I model the college admissions process as a Bayesian game where heterogeneous students compete for seats at colleges and universities of varying prestige. There is an allocation mechanism which maps each student's achieved test score into a seat at some college. A color-blind mechanism ignores race, while Affirmative Action mechanisms may give preferential treatment to minorities in a variety of ways. The particular form of the mechanism determines how students' study effort is linked with their payoff, playing a key roll in shaping behavior.

I use the model to evaluate the ability of a given college admission policy to promote academic achievement and to minimize racial academic gaps—namely, the achievement gap and the college enrollment gap. On the basis of these criteria, I derive a qualitative comparison of three canonical classes of college admissions policies: color-blind admissions, quotas, and admission preferences.

I also perform an empirical policy analysis of Affirmative Action (AA) in US college admissions, using data from 1996 on American colleges, freshman admissions, and entrance test scores to measure actual AA practices in the American college market. Minority college applicants in the United States effectively benefit from a 9% inflation of their SAT scores, as well as a small fixed bonus of approximately 34 SAT points. I also estimate distributions over student heterogeneity and perform a series of counterfactual policy experiments.

This procedure shows that AA practices in the US significantly improve college placement outcomes for minorities, at the cost of discouraging achievement by the most and least talented students. The analysis also indicates ways in which AA could be re-designed in order to better achieve its objectives. As it turns out, a quota system produces a substantial improvement relative to either the current system or a color-blind system. However, quotas are illegal in the US and cannot be implemented as such. Nevertheless, I propose a variation on the AA policy already in place that is outcome-equivalent to a quota.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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This work is dedicated in loving memory to my grandfather, I. Reed Payne (1930–2009; Ph.D., Clinical Psychology, The Pennsylvania State University, 1963). Grandpa was the reason I started talking about getting a doctorate when I was eight. Grandpa was also why I still thought it a good idea as I grew old enough to know what that meant.

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CHAPTER 1
INTRODUCTION

Beginning with the administration of John F. Kennedy, the United States government has mandated Affirmative Action (AA) policies in various areas of the economy, including education, employment, and procurement. The objective of AA, as articulated by policymakers, is to counteract competitive disadvantages for racial minorities due to past institutionalized racism. As President Lindon Johnson stated in his commencement address at Howard University in June, 1965,

> You do not take a person who, for years, has been hobbled by chains and liberate him, bring him up to the starting line of a race and then say, 'you are free to compete with all the others,' and still justly believe that you have been completely fair... We seek not just freedom but opportunity.

Two persistent academic disparities among race groups are often cited as a rationale for AA in college admissions. The first is a widely documented phenomenon known as the *achievement gap*, which is typically measured in terms of standardized test scores. In 1996, the median SAT score among minority college candidates was at the 22nd percentile for non-minorities.[1]

The second academic disparity, which I shall refer to as the *enrollment gap*, involves placement outcomes in post-secondary education: among students who attend college, minorities are under-represented at selective institutions and over-

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[1] Here, the working definition of the term "minority" is the union of the following three race classifications: Black, Hispanic and American-Indian/Alaskan Native. See Chapter 3 for a more detailed discussion. An extensive study of the black-white test score gap is given in Jencks and Phillips (1998).

represented at low-tier schools.[2] Using institutional quality measures for American colleges, I show in Chapter 3 that minorities made up almost 18% of all new college freshmen in 1996, but they accounted for only 11% of enrollment at schools in the top quality quartile. In the bottom quartile, minorities accounted for nearly 30% of enrollment.[3] These circumstances are viewed by many as residual effects of past social ills, and race-conscious college admission policies have been targeted toward addressing the problem.

Despite its intentions, much debate has arisen over the possible effects of AA on the incentives for academic achievement. Supporters claim that it levels the playing field, so to speak. The argument is that AA motivates minority students to achieve at the highest of levels by placing within reach seats at top universities—an outcome previously seen by many as unattainable.[4] In this way, it makes costly effort investment more worthwhile for the beneficiaries of the policy. Critics of AA argue just the opposite: by lowering the standards for minority college applicants, AA creates adverse incentives for them to exert less effort in competition

[2]Ultimately, policy-makers care about AA presumably because of persistent racial wage gaps, which translate into economic well-being in a variety of ways. These wage gaps are related to the college admissions market in two ways: first, relatively few minorities enroll in college, and second, among minority college matriculants, relatively few end up at elite institutions. Although both are interesting aspects of the college admissions problem, in this paper the enrollment gap on which I focus concerns college placement outcomes *conditional on participation in the college market*. The implications of AA for college enrollment decisions is left for future research.

[3]Institutional quality measures are based on data and methodology developed by US News & World Report for its annual *America's Best Colleges* publication. For a more detailed discussion, see Chapter 3.

[4]Fryer and Loury (2005) have used this argument as a possible rebuttal to their "Myth 3: Affirmative Action Undercuts Investment Incentives."

for admission to college. By making academic performance less important for one's outcome, they argue, AA creates a tradeoff between equality and achievement. Some critics of AA go even further, bringing into question whether such policies are capable of improving outcomes for disadvantaged market players, or whether the benefits go disproportionately to economically privileged members of the targeted demographic group.[5]

While the arguments on both sides of the debate seem intuitively plausible, satisfying answers to the AA controversy require an economic framework that allows for rigorous quantification of the social costs and benefits involved. With that in mind, I propose such a model of college admissions in order to inform better the policy debate. I frame the model as a Bayesian game, where heterogeneous students compete in grades for seats at post-secondary institutions. Each student is characterized by a privately-known type that determines the marginal utility cost of working to achieve a grade. Students observably belong to different demographic groups, and I allow for costs to be asymmetrically distributed across groups.[6] For any student who wishes to go to college, there is a seat open at some institution, but no two seats are equally desirable. Allocations of college seats are determined by a mechanism that maps each student's grade into a college seat. The mechanism may include race as a consideration. Under the payoffs induced

[5]Sowell (2004) has expounded this argument in considerable detail; an extensive discussion of the opposite viewpoint was offered by Bowen and Bok (1998).

[6]My intention is *not* to suggest that the asymmetry reflects differences of inherent ability across differing demographic groups. The appropriate interpretation involves asymmetry arising from socioeconomic factors which affect a student's academic competitive edge. See Section 2 for a full discussion.

by a given allocation rule, students optimally choose effort, based on their own type and competition they face from other students.

This model of competition for college admissions is strategically equivalent to a multi-object all-pay auction with incomplete information. Using analytic tools from auction theory, I solve for equilibrium behavior in order to assess the implications of different admission policies. Grade distributions are the equilibrium objects of principal interest from a policy standpoint because, in the model, they allow for a complete characterization of the enrollment gap, the achievement gap, and overall academic performance.

A meaningful investigation must encompass settings where the number of competitors is large, but analysis of the game becomes unwieldy even for moderately sized sets of players.[7] I show that there is a well-defined and simple notion of a limiting strategic environment as the number of players grows without bound. The model equilibrium can be approximated to arbitrary precision for a large enough set of competitors by treating agents and prizes as continua, rather than finite sets.

A novel feature of the model is that it allows for comparisons of alternative implementations of AA, whereas the previous literature has focused primarily on color-blind versus race-conscious college admissions. I study and compare three canonical classes of AA. The first is a quota rule, where seats are reserved

[7] Typically, college markets involve enough competitors to make this a problem. For example, the US National Center for Education Statistics reported that in 2005 over 1.8 million recent high-school graduates enrolled in college.

for allocation to minorities, effectively splitting the competition into two separate competitions. The second variety is an American-style AA, commonly referred to as an "admission preference," where minority achievement is assessed a markup before deciding who gets to attend which school. Finally, I also consider a color-blind admission rule where no preferential treatment is given.

My objective is to address four research questions. First, what effect does AA have on effort incentives: does it encourage students to study more or less, and does it affect all students' effort decisions in the same way? Second, what effect does it have on the achievement gap: does it widen or narrow the difference in achievement across demographic groups? Third, how effective is AA at achieving proportional enrollment in college? In other words, does the intended allocative effect of a policy remain after factoring in the behavioral response produced by altering the rules of the competition? And fourth, are there differences among alternative AA policies in terms of the first three criteria?

Although a complete policy analysis is difficult for a researcher who cannot observe the social choice function, by making some light assumptions on the preferences of the policy-maker, one can still guide the debate in meaningful ways. Henceforth, I assume that the policy-maker has the following three objectives in selecting an admission policy: *(1)* narrowing the enrollment gap (*i.e.*, achieving parity in the profiles of colleges attended by different groups) *(2)* narrowing the achievement gap (*i.e.*, achieving parity in the profiles of academic achievement produced by different groups); and *(3)* preserving incentives which encourage high academic performance. My theoretical framework is useful for this purpose because the equilibrium grade distributions for each group are sufficient to gauge success along each objective. I make no assumptions concerning how the policy-maker weights the three objectives, so establishing a preference ranking between two policies will only be possible if one performs better along all three criteria.

The main contribution of this thesis is to highlighting the importance of comparisons across different AA formats. Certain implementations indeed perform poorly—for example, a simple fixed grade markup that was used formerly at the University of Michigan. Relative to color-blind admissions, a Michigan rule erodes effort incentives for minorities by uniformly subsidizing grades regardless of individual achievement. Rational students take at least some of the grade boost as a direct utility transfer, rather than using it to bolster their competitive edge. A Michigan rule also creates discouragement effects for non-minorities which diminish their performance as well. Moreover, in equilibrium, the policy simply re-shuffles admissions at the lowest-ranked colleges, leaving allocations of the best seats unchanged from a color-blind outcome. Thus, a uniform grade boost accomplishes little, but comes at a potentially high cost. More general admission preference rules can be designed to overcome some of the drawbacks of the Michigan rule.

Intuitively, minorities in a color-blind competition compete at the margin with non-minority counterparts of the same type. When a markup is assessed, they may end up competing with counterparts of higher ability levels, so students adjust behavior to the point at which they compete on the margin with students on the same competitive standing. The main shortcoming of a Michigan rule is that the marginal bonus for a little extra effort is zero, leading to the unambiguously negative behavioral response. A more sophisticated admission preference can do much better when a higher level of achievement results in a higher grade bonus. This can provide incentives for minority students to increase achievement and compete with non-minority counterparts of higher ability.

This thesis also produces new contributions to the policy debate by showing that there are meaningful ways in which both the advocates and critics of AA are correct. On the one hand, a tradeoff between equality and effort does exist, in the sense that there is always some segment of the population for which achievement diminishes under AA. Another shortcoming common to all forms of AA is that the achievement gap widens among top students, relative to colorblind allocations. Moreover, certain AA policies can be ineffective at producing intended changes to market outcomes. On the other hand, some varieties of AA can indeed overcome discouragement effects for disadvantaged minorities, potentially producing an increase in average achievement within the minority group, and even among the population as a whole. Moreover, it is possible to achieve academic performance gains while producing a more representative college admissions profile.

There is a substantial amount that can be said qualitatively with the model, as outlined in the above discussion, but a final answer to the questions I pose is empirical in nature. Accordingly, I structurally estimate the model in two parts. First, I estimate the American admission preference system from data on college enrollment and test scores via the generalized method of moments. The result is an affine markup rule where minority test scores are inflated by 9 percent, plus a small fixed bonus. In the second stage I estimate the distributions of student ability using tools from the structural auction econometrics literature developed by Guerre, Perrigne and Vuong (2000, henceforth GPV). These estimates facilitate a set of counterfactual experiments which quantify the differences between alternative policies.

The results of the empirical analysis indicate that actual AA practices in the United States significantly improve market outcomes for minority students. If AA were eliminated from college admissions decisions in the US, minority enrollment in the top two quartiles of colleges would decrease by a third and a quarter, respectively. Even more striking is the fact that the majority of the displaced minority enrollment would be absorbed by the bottom quartile, in a color-blind world. AA also narrows the gap between median SAT scores among minorities and non-minorities by 14 percent. It discourages achievement among minority students at the upper and lower extremes of the score distribution, while encouraging students in the middle to score higher. The two effects balance each other out, so that virtually no change occurs for average minority SAT scores.

As for policy comparisons, no clear ranking can be established between a color-blind scheme and the estimated US admission preference without knowing how the policy-maker's preferences weight the three objectives. The latter narrows the achievement gap, the former results in higher overall academic achievement, and neither is clearly better in terms of the achievement gap. On the other hand, it can be reasonably argued that a quota system is superior to *both* of the other two policies on all three objectives: it produces the highest academic performance, a substantial narrowing of the achievement gap, and, by design, it closes the enrollment gap completely.

Explicit quotas are illegal in the United States and cannot be implemented as such.[8] Nevertheless, using insights from the workings of a quota mechanism, I propose a simple variation on the AA scheme currently in place, which delivers the same performance along the three policy objectives, and can be implemented using only information on race and grades. Another interesting property of this alternative policy is that it is a self-adjusting AA rule that naturally phases itself out as the racial asymmetry diminishes.

The remainder of this thesis has the following structure. In Section 1.1, I briefly discuss the relation between this work and the previous literature on AA. In Chapter 2, I develop the theoretical framework of competition for college admissions. Section 2.1 contains an outline of the model, while in Section 2.2.1 I introduce the solution concept of an *approximate equilibrium* which adds tractability when the number of players is large. In Section 2.3, I show that the maximizers of a student's limiting objective function constitutes an approximate equilibrium of the finite college admissions game when the number of competitors is large. I derive approximate equilibria under color-blind admissions, quotas and admission preferences. Section 2.4, contains qualitative comparisons of achievement and race gaps under the different admissions policies for the special case where

[8]The US Supreme Court Ruling in *Regents of the University of California v. Bakke*, 438 U.S. 265 (1978) established the unconstitutionality of explicit quotas in the US.

costs are linear in achievement. I also illustrate the model by solving it for a special case where private types are Pareto distributed. In Section 2.5 I conclude the theoretical discussion and describe avenues for further research.

Chapter 3 contains the empirical component of the project. In Section 3.1 I provide description of the data that will be mapped into the model. In Section 3.2, I propose structural estimators for AA practices and for the college market model. The later estimator follows the method of Guerre et al. (2000), while incorporating techniques developed by Karunamuni and Zhang (2008) on boundary-corrected kernel density estimation, to overcome certain technical problems in the estimation. In Section 3.3, I discuss the results of estimation and the counterfactual exercise, while in Section 3.4 I outline the alternative policy proposal and conclude. In the Appendix, I collect the proofs of various theorems and technical details concerning the data.

1.1 Previous Literature

This is the first paper of which I am aware that attempts simultaneously to address all four questions posed in Section 1, but there are various papers in the literature which attempt to address some subset of the first three. Coate and Loury (1993) studied a bilateral matching model of skills acquisition in order to address the first question (effort/human capital investment decisions) and the second one (achievement gaps) too. Minority job applicants strategically interact with potential employers who have tastes for racial discrimination *à la* Becker, and workers decide whether to forego a fixed exogenous skill-acquisition cost. There are exactly two levels of achievement, namely, being "qualified" or "unqualified." The government mandates a minimal minority employment level—similar to a quota in my model—and skill investment decisions are given by a threshold rule: all workers with costs below a fixed cutoff choose to acquire skills. The threshold rule is such that the government mandate can be gradually increased over time so that all workers previously acquiring skills still choose to do so, and an additional set of minority workers also choose to acquire skills.

An important difference between the paper by Coate and Loury (1993) and this thesis is the agents' choice set. In the former model, agents face a binary choice of whether to acquire a fixed skill level at a fixed, exogenous cost. In contrast, I allow for agents to choose any skill level, which means that the exact cost incurred is at the agent's discretion. Heterogeneity among individuals exists in the form of differences among marginal costs of skill acquisition. When this is true, any AA policy changes every player's behavior. This creates a tradeoff between equality and effort, and outcome changes may no longer be unambiguously desirable.

However, there is a more fundamental distinction between bilateral matching models, in general, and the all-pay auction framework. The value added in the theoretical approach I employ for studying college admissions is that it includes the competitive interaction between two different groups that are unevenly affected by AA. As it turns out, this is a central concern when assessing how behavior responds to incentive changes under a given policy. For example, in a color-blind world, with heterogeneous students, each one intuitively competes on the margin with others of similar ability levels. However, when the test scores of one group are assessed a markup—as in an admission preference—it may be possible that minority students end up competing on the margin with non-minorities of differing ability levels. As I show later, a common theme arising from such policies is that students adjust their behavior so that, on the margin, they compete against other students who are on roughly the same competitive standing. This concept informs the researcher concerning the properties of markup functions that produce desired changes (*e.g.*, the marginal markup for an additional unit of achievement).

Various models in the contests literature also attempt to address the first two questions as well. This includes papers by Fain (2009) and Fu (2006) (see also Fu (2004)), which are two-player all-pay contests under complete information, so heterogeneity among competitors is commonly observable. In both models, an interaction between one advantaged player and one disadvantaged player competing for a single prize is studied: both authors find that an admission-preference-like AA rule benefitting the disadvantaged player increases effort exerted by both players. Each then uses these results to argue that colleges will admit a higher-quality body of students if the school gives preference to the minority students by weighting their grades more heavily. Schotter and Weigelt (1992) have performed an experimental analysis of a two-player model similar to that of Fain (2009), with similar results.

However, when extrapolating their results to a competition involving many students, these authors have implicitly assumed that *every* beneficiary of the AA policy is at a competitive disadvantage to *every* other student not benefitting from it. However, this assumption is inappropriate in the context of college admissions, where AA is based only on one's observable race, rather than one's unobservable characteristics which determine academic competitiveness. The current model produces very different results, due to the fact that there are both high-cost types and low-cost types in the minority group, all of whom benefit from AA. There are also high-cost types in the non-minority group who do not benefit from AA.

In January of 2008, presidential candidate Barack Obama famously stated in a television interview that his daughters should not be treated as disadvantaged in college admissions decisions, and that perhaps white children raised in poverty should benefit from AA. The results of this paper are consistent with the intuition behind Mr. Obama's assertion: a common feature in both classes of AA considered here is a reduction in effort among both low-cost minorities and high-cost non-minorities. For the former group, AA provides a competitive boost that was not needed; for the latter, AA exacerbates discouragement effects. In short, when there are both gifted and challenged students in each demographic group, the unambiguous benefits arising from AA are no longer a foregone conclusion.

A final related paper is Franke (2008), who analyzed the effect of an admission-preference-like AA policy in a contest with many players. Franke showed that when the policy-maker is fully informed on student heterogeneity, he can design a grade-weighting scheme that raises all players' effort, relative to a color-blind rule. While this is certainly an improvement over a simplistic two-player model, Franke still relied on the strong assumption of complete information to construct the beneficial policy. In that sense, his research can be thought of as a characterization of the "first-best" outcome, where no information is hidden from the policy-maker. By contrast, I evaluate the tradeoffs faced by a policy-maker who cannot observe individual characteristics other than race. A college admissions college board can see each student's grade, but it cannot observe the cost incurred to achieve that grade. In keeping with the *Wilson doctrine*, I constrain the current theoretical exercise to evaluating policies that are implementable without knowledge of model primitives like private cost types and the associated distributions.

As for the third question—characterizing the equilibrium enrollment gap—many models have been designed to characterize admissions outcomes at a single post-secondary institution under AA. The papers concerning contests mentioned above fit this description, as they all involve competition for a single indivisible good. Another paper, by Chan and Eyster (2003), involved investigating AA in a setting where a single college chooses what profile of students to admit, subject to a capacity constraint. However, if colleges and universities differ in meaningful ways in terms of quality, then these models cannot address the question of how admission policies affect racial composition among different segments of the quality spectrum. This issue requires a model where many heterogeneous college applicants are being matched with many heterogeneous colleges. In the current setup, this aspect of college admissions is captured by a set of distinct prizes for which students compete.

CHAPTER 2
GAME-THEORETIC ANALYSIS OF COLLEGE ADMISSIONS

2.1 Model

I model the competition among high-school students for college admissions as a Bayesian game. Students belong to two demographic groups—minorities and non-minorities—and each student is characterized by a privately-known study cost type. Students compete in grades for a set of heterogeneous prizes—seats at colleges/universities of differing quality—and private types determine the costliness of academic achievement. Students have single-object demands and prizes are allocated by a pre-specified mechanism, according to grades. AA enters the model if the mechanism bases allocations partially on race as well. Students can observe the set of prizes before making decisions, but they must incur a non-recoverable cost associated with academic achievement *before* learning which prize they will receive. A student's payoff at the end of the game is the utility derived from consuming a prize, minus the utility cost of his achieved grade. An equilibrium of the game is characterized by a set of achievement functions that prescribe each student's optimal effort level. A formal description of the components of the game is given below.

2.1.1 Costs and Benefits

The agents are a set $\mathcal{K} = \{1, \ldots, K\}$ of students who observably belong to a minority group $\mathcal{M} = \{1, 2, \ldots, M\}$ or a non-minority group $\mathcal{N} = \{1, 2, \ldots, N\}$, where $M + N = K$. Students are heterogeneous, and each is characterized by a privately-known study cost type $\theta \in [\underline{\theta}, \overline{\theta}]$. Agents view the types of their opponents as independent random variables, and there is a common prior on types within each group, $\Theta \sim F_i(\theta),\ i = \mathcal{M}, \mathcal{N}$. Students have access to a common strategy set $\mathcal{S} = \mathbb{R}_+$, comprising grades/test scores. In order to achieve grade level s, an agent must incur a cost $\mathcal{C}(s;\theta)$, which depends on his type.

This specification of costs lends itself to several interpretations: θ could arise from either cognitive or non-cognitive factors. Costs could reflect of an underlying labor-leisure tradeoff where students differ either by preferences for leisure, or by the amount of labor input required to produce a unit of s. Alternatively, it could reflect some psychic cost of exerting mental effort to learn new concepts, where the amount of effort required to produce a given grade differs among students. The cost type θ could also reflect many other external factors affecting students' academic performance such as home conditions, affluence, school quality, and access to things like health-care and tutors.

The rewards for academic achievement are a set of prizes

$$\mathbf{P}_{\mathcal{K}} = \{p_k\}_{k=1}^{K},$$

where p_k denotes the utility of consuming the k^{th} prize. The prizes are seats at distinct colleges and universities, and students have single-object demands: a student can at most attend one school. There are enough prizes for every student who competes (*i.e.*, there are enough seats open to serve anyone who wishes to go to college), but no two prizes render the same utility: $p_k \neq p_j,\ k \neq j$. At the end of the game, an agent's payoff is the utility from consuming a prize minus the cost of achievement, or

$$\Pi(s;\theta) = p - \mathcal{C}(s;\theta).$$

An alert reader will notice that I have implicitly assumed agents have identical preferences over differing colleges and universities. However, it is not essential to the model for all students to place the same value on a seat at a given college; the important assumption here is that students rank prize values the same.[1] Without this assumption, a policy discussion concerning admission outcomes is impossible, and the researcher is left with the unsatisfying conclusion that fewer minorities attend elite institutions simply because they prefer it that way. An alternative view of the homogeneous-ranking assumption is that students have similar preferences over school attributes such as per-pupil spending, graduation rates, student-faculty ratios, and so forth.

2.1.2 College Admission Policies

Grades are mapped into payoffs as the outcome of a matching market with three stages: students send reports of their achievement levels to various colleges/universities, admissions boards make acceptance/rejection decisions, and

[1]For certain specifications of the cost function (*e.g.*, $\mathcal{C}(s;\theta) = \theta h(s)$) each student's objective function can be normalized by his type to get a game where achievement is uniformly costly across all competitors, but where each derives different utility from occupying a given seat. In this equivalent model, all prizes still follow a uniform ranking, but the marginal utilities of upgrading to the next best prize are unique to each competitor.

students choose among the options given to them by the market. I assume that there are no frictions in the matching market, so that its outcome can be implemented by a centralized mechanism which uses the set of grades $\mathbf{s} = \{s_{\mathcal{M},1}, \ldots, s_{\mathcal{M},M}, s_{\mathcal{N},1}, \ldots, s_{\mathcal{N},N}\}$ achieved by all students to allocate prizes. In other words, the assumption here is that the market is efficient in the sense that it is effective at matching higher performing students (holding race constant) with higher quality schools.

A simple "color-blind" admission rule is one which assortatively matches prizes with grades. The student submitting the highest grade is awarded the most valuable prize, and so on. In what follows, it will be convenient to treat a competition with color-blind admissions as the baseline model.

As for AA, consider first a quota system similar to what's known as *Reservation Law* in India. This law mandates that a certain percentage of seats be set apart for allocation only to certain demographic groups. There are many possible quota rules indexed by a number $q \in \{1, 2, \ldots, M\}$ of prizes reserved for minorities. However, for simplicity I shall consider only the case of a full quota rule, where exactly M prizes are reserved for minorities. Under a full quota rule, students compete only with members of their own group. It is also necessary to specify *how* prizes are selected for reservation. There are many possibilities once again, but for simplicity I shall focus on the case where a *representative* set of M prizes is set aside. This can be accomplished by either randomly selecting M prizes from the set $\mathbf{P}_{\mathcal{K}}$, or by first ordering prizes by quality and then selecting out every m^{th} prize, where $m = \frac{M+N}{M}$. In what follows, it will be easiest to consider the random selection method, but this is without loss of generality: when the set of prizes is large, the overall effect will, on average, be virtually the same.

The form of AA as implemented in the United States is different, because of a 1978 Supreme Court ruling that explicit quotas—*i.e.*, earmarking seats for allocation only to students of a particular race—are unconstitutional.[2] Since then, American higher education institutions have been forced to seek other means by which to implement AA. The resulting system is commonly referred to as an *admission preference*, where test scores achieved by minority students are given more weight in admissions decisions. I model an admission preference rule as a grade transformation function $\tilde{S} : \mathbb{R}_+ \to \mathbb{R}_+$. This mechanism matches prizes assortatively with non-minority grades and *transformed* minority grades

$$\{s_{\mathcal{N},1}, \ldots, s_{\mathcal{N},N}, \tilde{S}(s_{\mathcal{M},1}), \ldots, \tilde{S}(s_{\mathcal{M},M})\}.$$

In other words, under an admission preference admissions boards view each minority student with a grade of s as if he had submitted a grade of $\tilde{S}(s)$ instead.[3]

Regardless of whether admissions are color-blind, or follow some form of AA, ties between competitors are assumed to be broken randomly. That is, in the event of a tie between two or more scores (some of which may be transformed), each student involved in the tie is assigned a random index for the purpose of

[2]See *University of California v. Bakke* (438 U.S. 265 1978).

[3]It should be noted here that this grade transformation rule defines a very general class of mechanisms. In fact, the color-blind and quota rules are both special cases.

ranking him with his competitors.

Before making decisions, agents observe the set of prizes $\mathbf{P}_{\mathcal{K}}$, the admission rule, $\mathcal{R} \in \{cb$ (color-blind), q (quota), ap (admission preference)$\}$, and the number of competitors from each group M and N. As mentioned above, students share a common prior on the type distributions $F_{\mathcal{M}}$ and $F_{\mathcal{N}}$. Under the payoff correspondence $\Pi(\mathbf{s};\theta)$ induced by a particular admission rule, students optimally choose grades based on their own type, taking into account their opponents' optimal behavior. A *(group-wise) symmetric equilibrium* of the Bayesian game $\Gamma(M, N, \mathbf{P}_{\mathcal{K}}, \mathcal{R})$ is a set of achievement functions $\gamma_i : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}_+,\ i = \mathcal{M}, \mathcal{N}$ which generate optimal grades, given that ones' opponents behave similarly. For the remainder of this thesis, I shall restrict attention to the class of symmetric equilibria.

2.1.3 Policy Objectives

Equilibrium achievement functions and private cost distributions induce a set of group-specific grade distributions, $G_{\mathcal{M}}$ and $G_{\mathcal{N}}$ and a population grade distribution G. These are ultimately the objects of interest from a policy standpoint, as they fully characterize achievement, achievement gaps, and enrollment gaps in equilibrium. In what follows, the achievement gap will be formally represented by a function $\mathcal{A} : [0, 1] \to \mathbb{R}$ defined by

$$\mathcal{A}(q) \equiv G_{\mathcal{N}}^{-1}(q) - G_{\mathcal{M}}^{-1}(q).$$

In words, $\mathcal{A}$ characterizes the difference between minority and non-minority achievement at each quantile of the grade distributions. Thus, to eliminate the achievement gap is to accomplish an outcome where $\mathcal{A}(q) = 0,\ \forall q \in [0,1]$.

As for the enrollment gap, let $F_{P_i}(p),\ \ i = \mathcal{M}, \mathcal{N}$ denote the distribution of prizes awarded to either group in equilibrium. The enrollment gap is a function $\mathcal{E} : [0,1] \to \mathbb{R}$ defined by

$$\mathcal{E}(q) \equiv F_{P_N}^{-1}(q) - F_{P_M}^{-1}(q).$$

Once again, to eliminate the gap is to accomplish an outcome where $\mathcal{E}(q) = 0,\ \forall q \in [0,1]$.[4] Finally, the overall profile of academic achievement is represented by the population grade distribution,

$$G(s) = \frac{M}{M+N} G_{\mathcal{M}}(s) + \frac{N}{M+N} G_{\mathcal{N}}(s).$$

Measures of benefits and costs cited in the policy debate over AA are often related to or derived from $\mathcal{A}$, $\mathcal{E}$, or G. For example, a statement about the test score gap that "the median minority SAT score lags behind the non-minority median by 150 points," is equivalent to the statement $\mathcal{A}(.5) = 150$. The reason for defining race gaps and achievement in such general terms is to avoid imposing

[4]At the moment, this is an abuse of notation, given that F_{P_i} is a step function having no proper inverse, since the set of prizes is finite. However, in the limit as the number of prizes (and players) grows without bound, the problem disappears. The analysis hereafter will concentrate on the limiting case, as it meaningfully reflects the character of a large finite game while adding tractability to the model.

strong assumptions on what concerns policy-makers most. To wit, if preferences place the same weight on the enrollment gap at every point of the college quality spectrum, then $\mathcal{E}$ could be reduced to $\mathcal{E} = \int_0^1 \left[F_{P_N}^{-1}(q) - F_{P_M}^{-1}(q) \right] dq$. If the policy-maker cares more about the enrollment gap at elite schools, then this would be inappropriate.

Having formalized my notion of race gaps and achievement, I shall proceed under the following light assumptions regarding the policy-maker's preferences:

Assumption 2.1.1. For two achievement gap functions, $\mathcal{A}^*$ and $\mathcal{A}$,

$$\mathcal{A}^*(q) \leq \mathcal{A}(q) \; \forall q \in [0,1] \quad \Rightarrow \quad \mathcal{A}^* \succcurlyeq \mathcal{A},$$

and $\mathcal{A}^* \succ \mathcal{A}$ if in addition

$$\exists q^* \in [0,1] \;\; s.t. \;\; \mathcal{A}^*(q^*) < \mathcal{A}(q^*).$$

Assumption 2.1.2. For two enrollment gap functions, $\mathcal{E}^*$ and $\mathcal{E}$,

$$\mathcal{E}^*(q) \leq \mathcal{E}(q) \; \forall q \in [0,1] \quad \Rightarrow \quad \mathcal{E}^* \succcurlyeq \mathcal{E},$$

and $\mathcal{E}^* \succ \mathcal{E}$ if in addition

$$\exists q^* \in [0,1] \;\; s.t. \;\; \mathcal{E}^*(q^*) < \mathcal{E}(q^*).$$

Assumption 2.1.3. For two population grade distributions, G^* and G,

$$G^*(s) \leq G(s)\ \forall s \in \mathbb{R}_+ \quad \Rightarrow \quad G^* \succsim G,$$

and $G^* \succ G$ if in addition

$$\exists s^* \in \mathbb{R}_+ \ \ s.t.\ \ G^*(s^*) < G(s^*).$$

2.1.4 Model Assumptions

In order to guarantee existence of a pure-strategy equilibrium, it will be necessary to make the following assumptions concerning the form of the study cost function:

Assumption 2.1.4. $\frac{\partial \mathcal{C}}{\partial s} > 0$; $\frac{\partial \mathcal{C}}{\partial \theta} > 0$; $\frac{\partial^2 \mathcal{C}}{\partial s^2} \geq 0$; and $\frac{\partial^2 \mathcal{C}}{\partial s \partial \theta} \geq 0$.

In words, costs are assumed to be convex and increasing in achievement level s and type θ. Marginal costs are also assumed to be increasing in student types, so that a smaller θ not only means a lower cost of achieving grade level s, but also a lower marginal cost of increasing output from s to $s + \varepsilon$.

It is also necessary to make the following assumptions concerning beliefs:

Assumption 2.1.5. The private cost distributions $F_{\mathcal{M}}(\theta)$ and $F_{\mathcal{N}}(\theta)$ have continuous and strictly positive densities $f_{\mathcal{M}}(\theta)$ and $f_{\mathcal{N}}(\theta)$, respectively.

One aspect of the model is worth highlighting here. By assuming that private cost distributions are static and exogenous, I am implicitly taking a short-run view of policy implications. One could conceive of a broader model in which the policy-maker designs a mechanism today to affect the evolution of private costs for future generations—*i.e.*, the children of today's college freshmen. Such an undertaking is beyond the scope of the current exercise, and is left for future research. Instead, I shall concentrate on the implications of the policy-maker's choices for actions and outcomes of older school children and today's college candidates, whose private costs can reasonably be viewed as fixed.

Finally, when considering an admission preference rule, I shall restrict attention to policy functions $\widetilde{S}$ satisfying certain sensibility criteria.

Assumption 2.1.6. $\widetilde{S}(s)$ is a strictly increasing function lying above the 45°-degree line.

Assumption 2.1.7. $\widetilde{S}(s)$ is continuously differentiable.

Assumption 2.1.6 corresponds to the notion that the policy is geared toward assisting minorities, effectively moving each minority student with a grade of s ahead of each non-minority student with a grade of $\widetilde{S}(s) \geq s$. Moreover, it states that a policy-maker will not choose to reverse the ordering of any segment of the minority population, so that some students are awarded prizes of lesser value than other students within their own group whose grades were lower. Assumption 2.1.7 implies that the policy-maker does not make abrupt jumps in either the assessed grade boost, or the marginal grade boost. Aside from characterizing the behavior of a sensible policy-maker, Assumptions 2.1.6 and 2.1.7 also guarantee that introducing $\widetilde{S}$ into the model does not interfere with existence of the equilibrium.

2.1.5 Auction-Theoretic View of the Game

The model defined above is strategically equivalent to a special type of game known in the contests literature as an *all-pay auction*. An all-pay auction involves a strategic interaction in which agents compete for a limited resource by incurring some type of unrecoverable cost *before* learning the outcome of the game. In the present model of college competition, high school students cannot recover lost leisure time or disutility incurred by study effort if they discover that they did not make it into the college they had hoped for.

The centralized college admissions board is analogous to an auctioneer selling off a set of heterogeneous prizes according to a pre-determined mechanism. Students are similar to bidders, and the grades they work for are analogous to bids. The value here in recognizing the connection to auction theory is that I can import a well-developed set of analytic tools for characterizing the equilibrium. For example, as the following proposition shows, I can conclude *a priori* that a monotonic equilibrium exists. As I shall shortly demonstrate, existence and monotonicity provide an invaluable step toward analytic and computational tractability of the model when K is large, as it is in college admissions.

Proposition 2.1.8. *In the college admissions game* $\Gamma(M, N, \mathbf{P}_{\mathcal{K}}, \mathcal{R})$ *with* $\mathcal{R} \in \{cb, q, ap\}$, *there exists a unique symmetric pure-strategy equilibrium* $(\gamma_{\mathcal{M}}(\theta), \gamma_{\mathcal{N}}(\theta))$ *where achievement is strictly decreasing in private costs. Therefore,* $G_i(s) = 1 - F_i\left[\gamma_i^{-1}(s)\right]$.

A formal proof of Proposition 2.1.8 is left to an appendix, as it is fairly involved. Briefly though, proving existence and monotonicity is a straightforward application of results reported by Athey (2001). From there, continuity and differentiability follow directly (although the argument is somewhat complex) from the definition of an equilibrium. Once differentiability has been established, some well-known results from differential equations theory establish a unique solution to the first-order conditions of a student's decision problem. Any symmetric equilibrium prescribes optimal behavior (by definition), and therefore it must satisfy the unique solution to the first-order conditions.

2.2 Equilibrium Analysis

2.2.1 Approximate Equilibrium

In this section I introduce an alternative solution concept that I adopt for tractability. For large $K = M + N$, the model equilibrium is analytically and computationally intractable, because a decision-maker's objective function is a complicated sum of functions based on the order statistics of opponents' costs. Agents know that their *ex-post* payoff depends on their rank within the grade distribution, and under monotonicity this is the same as their rank within the realized cohort of opponents. Thus, expected equilibrium payoffs are a weighted average of the prizes, where the weight on the k^{th} best prize is one's probability of being the k^{th} lowest order statistic among K competing types.

For simplicity and tractability, I assume that the number of competitors is large enough so that a student has a very good idea of his rank within the realized sample of private costs. I approximate this large, finite model by considering the limiting case as $K \to \infty$, but in order to do so I must first introduce some additional notation. Let μ denote the asymptotic mass of the minority group: as each new agent is created, nature assigns him to group $\mathcal{M}$ with probability μ, and then he draws a private cost from the appropriate group-specific distribution. Given this assumption, with probability one the limiting sample of competitors is a dense set on the interval $[\underline{\theta}, \overline{\theta}]$, and each knows with certainty that his sample rank is the same as his rank in the unconditional private cost distribution $\mu F_{\mathcal{M}}(\theta) + (1-\mu)F_{\mathcal{N}}(\theta)$.[5]

For analytic convenience, I also assume that prizes are generated as independent draws from a compact interval $\mathcal{P} = [\underline{p}, \overline{p}] \subset \mathbb{R}_+$ according to a known prize distribution $F_P(p)$ satisfying

Assumption 2.2.1. F_P has a continuous density $f_P(p)$, which is strictly positive on $\mathcal{P}$; and

Assumption 2.2.2. (*zero surplus condition*) $\underline{p} = \mathcal{C}(0; \overline{\theta})$.

Even though prize values are observable *ex ante*, framing them in this way provides an intuitive view of the limiting set of prizes: as $K \to \infty$, $\mathbf{P}_{\mathcal{K}}$ becomes a dense set on $\mathcal{P}$, and the rank of a prize with value p converges to $F_P(p)$.

Assumption 2.2.2 is necessary because it provides a boundary condition that is used to solve the equilibrium equations. Although the current theoretical

[5]The fact that the limiting sample is a dense set can be seen by applying the following logic: given any two numbers $\theta, \theta' \in [\underline{\theta}, \overline{\theta}]$, where $\theta < \theta'$, the probability mass assigned to the interval (θ, θ') is strictly positive under my assumptions on F_Θ. Therefore, as the number of independent draws from F_Θ gets large, the probability of hitting the interval (θ, θ') at least once approaches one. Thus, a countably infinite random sample of agents will be everywhere dense on $[\underline{\theta}, \overline{\theta}]$.

exercise focuses solely on the competition among high-school students for college admissions, the zero surplus condition can be thought of as reflecting broader market forces not explicitly included in model. In the broader model, prize values are the additional utility one gains from going to college versus opting out, and $[\underline{\theta}, \overline{\theta}]$ is the set of individuals who demand a college seat, being a subset of a larger group of individuals, some of whom choose the outside option. If schools and firms can freely choose to enter the market and supply either college seats or jobs for unskilled laborers, the marginal college candidate—the highest private cost type opting for college, $\overline{\theta}$—will be just indifferent between attending college and entering the work force as an unskilled laborer. This point highlights a limitation of the current model: it attempts to characterize student behavior *conditional on participation in the post-secondary education market*, and it is not intended to provide insights into the decision of whether to acquire additional education. This aspect of the college admissions problem is left for future research.

With that out of the way, I can treat both agents and prizes as if they belong to a continuum, rather than a finite set. This allows me to avoid framing decisions in terms of the distributions of complicated order statistics, and it reduces each agent's decision problem to a simple objective function expressed in terms of θ, μ, $F_{\mathcal{M}}$, $F_{\mathcal{N}}$ and F_P. Given the well-behaved nature of the model primitives, the maximizers of the finite objective functions converge to the maximizer of the limiting objective function, which allows me to derive what I refer to as an *approximate equilibrium*.

Definition 2.2.3. Consider a generic game $\Gamma(\mathcal{K}, S, \Pi)$, where $\mathcal{K} = \{1, 2, \ldots, K\}$ is the set of players, $S_i \subseteq \mathbb{R}$ is the strategy space for the i^{th} player, and $\Pi(s_1, \ldots, s_K)$ characterizes payoffs on $S = S_1 \times \cdots \times S_K$. Given $\delta > 0$, a *δ-approximate equilibrium* is a K-tuple $\mathbf{s}^\delta = (s_1^\delta, \ldots, s_K^\delta)$, such that there exists an equilibrium $\mathbf{s}^* = (s_1^*, \ldots, s_K^*)$ of Γ, where $\|\mathbf{s}^\delta - \mathbf{s}^*\|_{\sup} < \delta$.

The approximate equilibrium concept is more relevant for my purposes than the ε-equilibrium introduced by Radner (1980), which is a profile of strategies generating payoffs that are ε close to payoffs in some equilibrium of Γ. The drawback of an ε-equilibrium is that it need not resemble the equilibrium strategies which generate the payoffs being approximated.[6] In my case, the strategies are a principal concern: I want to concentrate on the effects of admission policies on both payoffs *and* behavioral choices. However, there is a connection between the two concepts, as the following remark demonstrates.

Remark 2.2.4. For a Nash equilibrium $\mathbf{s}^*$ of Γ, if $\mathbf{s}^* \subset U \subset S$, where U is a neighborhood of $\mathbf{s}^*$, and if the payoff function Π is continuous on U, then the set of payoffs generated by ε-equilibria and δ-approximate equilibria form bases of neighborhoods of equilibrium payoffs. That is, given an ε-equilibrium associated

[6]Radner (1980) used an ε-equilibrium to resolve a dilemma in dynamic Cournot oligopoly games. For a fixed set of firms, as long as the number of periods is finite, the unique subgame perfect equilibrium involves static equilibrium strategies being played every period, whereas collusion suddenly becomes possible in the limit. Radner showed that there is a collusive ε-equilibrium of the finite-horizon Cournot game in which cartels are sustainable. Equilibrium payoffs can be replicated to arbitrary precision (*i.e.*, it is nearly optimal to collude if the time horizon is far enough away), even though the ε-equilibrium strategies are excluded from a neighborhood of equilibrium strategies.

with $\mathbf{s}^*$, there exists $\delta > 0$ such that for a δ-approximate equilibrium we have

$$\|\Pi(\mathbf{s}^\delta) - \Pi(\mathbf{s}^*)\|_{\sup} < \varepsilon.$$

Conversely, given a δ-approximate equilibrium associated with $\mathbf{s}^*$, there exists $\varepsilon > 0$ such that for an ε-equilibrium we have

$$\|\Pi(\mathbf{s}^\varepsilon) - \Pi(\mathbf{s}^*)\|_{\sup} < \|\Pi(\mathbf{s}^\delta) - \Pi(\mathbf{s}^*)\|_{\sup}. \square$$

In the context of the college admissions model, I seek to characterize a set of approximate achievement functions, denoted $\gamma^\infty_{\mathcal{M}}(\theta)$ and $\gamma^\infty_{\mathcal{N}}(\theta)$, such that given a fixed tolerance level $\delta > 0$, the functions approximate actual equilibrium achievement to δ-precision for large enough K.

My approximate equilibrium concept is similar to the *oblivious equilibrium* developed by (Weintraub et al., 2008, WBR) to approximate Markov Perfect Equilibria in dynamic oligopoly games. In such models, firms (and researchers) must compute a complex and intractable state transition process in order to exactly determine equilibrium strategies. Instead, WBR assume that firms make nearly optimal decisions based on a long-run average industry statistic which is inexpensive to compute. This issue of computational tractability leads to an alternative interpretation of approximate equilibria. Aside from being a useful approximation of equilibrium behavior, one could also view them as an exact characterization of the behavior of agents with bounded information processing ability. Rather than precisely tracking expected payoffs based on all of the order statistics of a large set of competitors, a cognitively constrained agent with private cost θ might find it more attractive to base decisions on his limiting rank $F_i(\theta)$ instead.

2.3 Approximate Equilibria Under AA

I shall proceed by deriving the maximizers of an agent's limiting objective function as the natural processes described in Section 2.2.1 generate increasingly large sets of competitors and prizes. I then prove that the resulting derivations satisfy Definition 2.2.3 above. From this point on, all discussion and derivations will be in terms of the approximate equilibrium, $\left(\gamma^{\infty}_{\mathcal{M}}(\theta), \gamma^{\infty}_{\mathcal{N}}(\theta)\right)$, so I shall drop the ∞ superscript for notational ease. Moreover, to avoid tedious verbosity, in what follows I shall henceforth refer to the approximate equilibrium and the approximate achievement functions simply as "the equilibrium" and "the achievement functions," unless the context requires me to be specific. If it becomes necessary to distinguish between the actual equilibrium of a game with K agents and the approximate equilibrium, I shall refer to the former as the "finite equilibrium" and I shall abuse the notation slightly and denote the former by $\gamma(\cdot; K)$, listing K as a parameter. Keeping in mind the processes generating agents and prizes, this notational abuse is not unreasonable: by the law of large numbers, any two randomly generated games with a large number of players K will be probabilistically very similar.

Superscripts shall be used below to keep track of the admission policy which defines payoffs in the game. Under policy $\mathcal{R} \in \{cb, q, ap\}$, the achievement functions and grade distributions are denoted by $\gamma^{\mathcal{R}}_{\mathcal{M}}(\theta)$, $\gamma^{\mathcal{R}}_{\mathcal{N}}(\theta)$, $G^{\mathcal{R}}$, $G^{\mathcal{R}}_{\mathcal{M}}$, and $G^{\mathcal{R}}_{\mathcal{N}}$. Finally, in what follows, it will sometimes be convenient to work with the inverse achievement function, which I denote by $\psi^{\mathcal{R}}_i(s) \equiv \left(\gamma^{\mathcal{R}}_i\right)^{-1}(s)$.

2.3.1 Color-Blind Game

Recall that a color-blind allocation rule means simple positive assortative matching of prizes with grades. I claim (proof to follow later) that in the limit, in equilibrium, this process is equivalent to using the following reward function for a student submitting a grade of s:

$$\begin{aligned}
\pi^{cb}(s) =& F_P^{-1}\left[G^{cb}(s)\right] \\
=& F_P^{-1}\left[\mu G^{cb}_{\mathcal{M}}(s) + (1-\mu)G^{cb}_{\mathcal{N}}(s)\right] \\
=& F_P^{-1}\left[1 - \left(\mu F_{\mathcal{M}}\left[\psi^{cb}(s)\right] + (1-\mu)F_{\mathcal{N}}\left[\psi^{cb}(s)\right]\right)\right].
\end{aligned}$$

Intuitively, the quantiles of the population grade distribution $G^{cb}(s)$ are mapped into the corresponding prize quantiles. Since individuals' limiting payoffs do not depend on race, it follows that $\gamma^{cb}_{\mathcal{M}}(\theta) = \gamma^{cb}_{\mathcal{N}}(\theta) = \gamma^{cb}(\theta)$; hence, the lack of subscripts on the inverse achievement functions in the third line.[7]

In equilibrium, the limiting net payoff for an agent with cost type θ sub-

[7] The theorist with experience in asymmetric auctions may find this statement puzzling, but one must keep in mind that it merely applies to *limiting* payoffs. In a two-player game, differing behavior arises from the fact that a minority and a non-minority with the same private cost type will view their likely standing in the distribution of realized competition differently, due to the asymmetry in the cost distributions. However, the likely difference between their expected ranks vanishes as the number of players gets large.

mitting grade s is

$$\Pi^{cb}(s;\theta) = F_P^{-1}\left[1 - \left(\mu F_{\mathcal{M}}\left[\psi^{cb}(s)\right] + (1-\mu)F_{\mathcal{N}}\left[\psi^{cb}(s)\right]\right)\right] - \mathcal{C}(s;\theta).$$

Differentiating, I get the following first-order condition (FOC):

$$-\frac{\mu f_{\mathcal{M}}[\psi^{cb}(b)] + (1-\mu)f_{\mathcal{N}}[\psi^{cb}(b)]}{f_P\left(F_P^{-1}\left[1-\left(\mu F_{\mathcal{M}}\left[\psi^{cb}(s)\right] + (1-\mu)F_{\mathcal{N}}\left[\psi^{cb}(s)\right]\right)\right]\right)}\frac{d\psi^{cb}(s)}{ds} = \mathcal{C}'(s;\theta). \quad (2.1)$$

Using the fact that $\frac{d\psi^{cb}(s)}{ds} = \frac{1}{(\gamma^{cb})'(\psi^{cb}(s))}$, and the fact that in equilibrium we have $\psi^{cb}(s) = \theta$, I can substitute to get

$$(\gamma^{cb})'(\theta) = -\frac{\mu f_{\mathcal{M}}(\theta) + (1-\mu)f_{\mathcal{N}}(\theta)}{f_P\left(F_P^{-1}\left[1-\mu F_{\mathcal{M}}(\theta) - (1-\mu)F_{\mathcal{N}}(\theta)\right]\right)\mathcal{C}'[\gamma^{cb}(\theta);\theta]}. \quad (2.2)$$

This differential equation partially solves for equilibrium achievement, but a boundary condition is also required.

By monotonicity, a student with cost type $\overline{\theta}$ is sure to be awarded the lowest quality prize, so the Assumption 2.2.2 implies the following boundary condition:

$$\gamma^{cb}(\overline{\theta}) = \mathcal{C}^{-1}(\underline{p};\overline{\theta}). \quad (2.3)$$

With that, I am ready to prove that the derivations above provide meaningful insights into the equilibrium of a finite college admissions games where the number of competitors is large. The proof is fairly involved, but it is based on simple ideas. I first prove that the finite objective functions converge pointwise in probability to the limiting objective function listed above. By viewing a K-player game as being randomly generated by the natural processes outlined in Section 2.2.1, one can think of a player's objective function as a random variable; hence the concept of convergence in probability. Using pointwise convergence, I can invoke Egorov's Theorem to deliver uniform convergence of the sequence of finite objective functions. Finally, using uniform convergence, I can invoke the theorem of the maximum to show that the finite maximizers are close to the solution of equation (2.2) and boundary condition (2.3) for large K.

Theorem 2.3.1. *Given $\rho,\ \varepsilon,\ \delta > 0$, there exists $K^* \in \mathbb{N}$, and a set $E \subset [\underline{\theta}, \overline{\theta}]$ having (Lebesgue) measure $m(E) < \rho$, such that for any $K \geq K^*$, on any closed subset of $[\underline{\theta}, \overline{\theta}] \setminus E$ we have the following:*

(i) $\gamma^{cb}(\theta)$ as defined by equation (2.2) and boundary condition (2.3) generates an ε-equilibrium of the K-player color-blind game, and

(ii) $\gamma^{cb}(\theta)$ is a δ-approximate equilibrium for the K-player color-blind game, or

$$\|\gamma^{cb}(\theta) - \gamma_i^{cb}(\theta; K)\|_{\sup} < \delta, \ \ i = \mathcal{M}, \mathcal{N}.$$

As the proof of Theorem 2.3.1 is fairly involved, I have left it to an appendix.

Before moving on, I should note that Theorem 2.3.1 can be strengthened slightly, to show that an ϵ-equilibrium and a δ-approximate equilibrium obtains on the entire set $[\underline{\theta}, \overline{\theta}]$, rather than on a subset with close to full measure. However, the proof of the stronger version of the theorem invokes results from the econometric theory literature less familiar to economic theorists, concerning uniform convergence in probability of stochastic functions. See the appendix for details on the alternative form of the proof.

2.3.2 AA: Quota Game

I now depart from the baseline color-blind model, and derive the approximate equilibrium in the presence of race-conscious admission policies, beginning with quotas. Recall that a quota system in the finite game can be thought of as randomly selecting M prizes and setting them aside for allocation to group-$\mathcal{M}$ agents. This effectively splits the single asymmetric competition apart into two separate, symmetric competitions. As K gets large, the sample distributions of prizes reserved for each group both converge in probability to F_P. Thus, the limiting quota rule is equivalent to a set of group-specific reward functions of the form

$$\pi_i^q(b) = F_P^{-1}\left[G_i^q(b)\right],\ i = \mathcal{M}, \mathcal{N}. \tag{2.4}$$

Intuitively, the quantiles of the group-specific grade distributions are mapped into the corresponding quantiles of the prize distribution.

In equilibrium, the utility for a group-i student with cost θ achieving a grade of s is

$$\Pi_i^q(s,\theta) = F_P^{-1}\left[G_i^q(s)\right] - \mathcal{C}(s;\theta) = F_P^{-1}\left(1 - F_i\left[\psi_i^q(s)\right]\right) - \mathcal{C}(s;\theta).$$

This is identical to payoffs in the color-blind game, except that the unconditional cost distribution has been replaced with F_i for group $i = \mathcal{M}, \mathcal{N}$. By symmetry then, equilibrium achievement will be determined by

$$(\gamma_i^q)'(\theta) = -\frac{f_i(\theta)}{f_P\left(F_P^{-1}\left[1 - F_i(\theta)\right]\right) C'\left[\gamma_i^q(\theta); \theta\right]} \tag{2.5}$$

and boundary condition (2.3).

Theorem 2.3.2. *Given $\rho,\ \varepsilon,\ \delta > 0$, there exists $K^* \in \mathbb{N}$, and a set $E \subset [\underline{\theta}, \overline{\theta}]$ having (Lebesgue) measure $m(E) < \rho$, such that for any $K \geq K^*$, on any closed subset of $[\underline{\theta}, \overline{\theta}] \setminus E$ we have the following:*

(i) $\gamma_i^q(\theta),\ i = \mathcal{M}, \mathcal{N}$ as defined by equation (2.5) and boundary condition (2.3) generates an ε-equilibrium of the K-player quota game, and

(ii) $\gamma_i^q(\theta)$ is a δ-approximate equilibrium for the K-player quota game, or

$$\|\gamma_i^q(\theta) - \gamma_i^q(\theta; K)\|_{\sup} < \delta,\ \ i = \mathcal{M}, \mathcal{N}.$$

Once again, the proof of Theorem 2.3.2is relegated to an appendix. As before, by using a slightly more complicated proof technique, this result can be strengthened to demonstrate that γ_i^{ap} generates an ε-equilibrium and a δ-approximate equilibrium on the entire set $[\underline{\theta}, \overline{\theta}]$, rather than on a subset with nearly full measure. See the Appendix for details.

2.3.3 AA: Admission Preference Game

In 1978, the Supreme Court of the United States ruled in the case of Regents of the University of California v. Bakke that explicit quotas are unconstitutional. Subsequently, American college admissions boards have been forced to seek other means by which to implement AA. These alternative implementations were referred to above as admissions preferences. An admission preference rule is modeled as a grade transformation function $\tilde{S} : \mathbb{R}_+ \to \mathbb{R}_+$, where $\tilde{S}(s)$ is increasing and $\tilde{S}(s) \geq s$. Here, prizes are matched assortatively with non-minority grades and transformed minority grades

$$\{s_{w1}, \ldots, s_{wW}, \tilde{S}(s_{m1}), \ldots, \tilde{S}(s_{mM})\}.$$

In what follows, it will be convenient to derive the equilibrium in terms of inverse equilibrium strategies. Under an admission preference, minority students are repositioned ahead of their non-minority counterparts with grades of $\tilde{S}(s)$ or less. Thus, the limiting gross payoff function for group $\mathcal{M}$ is

$$\begin{aligned} \pi^{ap}_{\mathcal{M}}(s) &= F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}\left(\tilde{S}(s)\right) + \mu G_{\mathcal{M}}(s)\right] \\ &= F_P^{-1}\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(\tilde{S}(s))\right] + \mu F_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(s)\right]\right)\right] \end{aligned} \tag{2.6}$$

and the gross payoff function for group $\mathcal{N}$ is

$$\begin{aligned} \pi^{ap}_{\mathcal{N}}(s) &= F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}(s) + \mu G_{\mathcal{M}}\left(\tilde{S}^{-1}(s)\right)\right] \\ &= F_P^{-1}\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(s)\right] + \mu F_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(\tilde{S}^{-1}(s))\right]\right)\right]. \end{aligned} \tag{2.7}$$

The intuition for the above expressions is similar to the intuition for the color-blind and quota reward functions: the limiting mechanism maps the quantiles of a (mixed) distribution into the corresponding prize quantiles. For non-minorities, it is a mixture of the distributions of non-minority grades and transformed minority grades. For minorities, it is a mixture of the distributions of minority grades and *inverse-transformed* non-minority grades. Thus, a student's standing with respect to members of his own group doesn't change, but standing with respect to members of the other group does. For minorities it changes in a positive direction according to $\tilde{S}$, and for non-minorities it changes in a negative direction, since $\tilde{S}^{-1}$ lies below the 45°-line.

However, the introduction of a preference function $\tilde{S}$ introduces some complications into the analysis. Note that equation (2.7) only holds for s such that $\tilde{S}^{-1}(s) \geq 0$, because one can only invert grades in the range of the function $\psi^{ap}_{\mathcal{M}}$. If $\tilde{S}$ passes through the origin, then this condition is satisfied for every grade in the choice set. On the other hand, there is an interesting class of admission preference rules which do not pass through the origin.[8] An example is an affine rule of the form

$$\tilde{S}(s) = \Delta_1 + \Delta_2 s,$$

[8] An additive admission preference $\tilde{S}(s) = s + \Delta$ was explicitly used in undergraduate admissions at the University of Michigan. Admissions decisions were based on an index ranging from 0-120, with a bonus of 20 points being assessed to all students from underrepresented racial minority groups. This policy was in place until 2003 when the Supreme Court ruled in a joint opinion on *Gratz v. Bollinger* and *Grutter v. Bollinger*, that the bonus was unconstitutional. The opinion of the Court stated, somewhat vaguely, that while the Michigan rule was too "narrowly defined" and "mechanical," universities still have the right to consider race as a "plus factor" in admissions decisions. This abrogated a 1996 ruling to the contrary by the US Fifth-Circuit Court in the case of *Hopwood v. Texas*.

where minority students receive a fixed subsidy of Δ_1, regardless of their grade. In that case, non-minorities whose grades are less than $\tilde{S}(0)$ are placed behind *all* minority students, meaning that they compete only with other non-minority students whose grades are less than $\tilde{S}(0)$. This leads to the following proposition:

Proposition 2.3.3. *In the college admissions game, with an admission preference mechanism $\tilde{S}$, where $\tilde{S}(0) = \Delta > 0$, it follows that group-$\mathcal{N}$ players with equilibrium grades below $\tilde{S}(0)$ behave as they would under a quota rule.*

Proof: Let θ_Δ denote the non-minority private cost type who's equilibrium grade is Δ and let

$$p_\Delta = F_P^{-1}\left(1 - [(1-\mu)F_{\mathcal{N}}(\theta_\Delta) + \mu F_{\mathcal{M}}(\theta_\Delta)]\right)$$

denote the highest prize awarded to agents whose transformed bids are Δ or less. Also, let $\mathcal{P}_\Delta = [0, p_\Delta]$. On the interval $[\theta_\Delta, \overline{\theta}]$, group $\mathcal{N}$ agents know that they are competing only among themselves for the lowest mass

$$\nu = (1-\mu)F_P(p_\Delta) = (1-\mu)\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(\Delta)\right] + \mu F_{\mathcal{M}}\left[\psi_{\mathcal{N}}^{ap}(\Delta)\right]\right)\right]$$

of prizes since the grade markup necessarily places them behind every minority student (note that ν is also the mass of high-cost, group-$\mathcal{N}$ agents receiving prizes in $\mathcal{P}_\Delta$). It is as if they are playing a game where the prize distribution is

$$F_{P_\Delta}(p) = \frac{F_P(p)}{\nu},\ p \in [0, F_P^{-1}(\nu)]$$

and where the distribution over competition is

$$F_{\mathcal{N}_\Delta}(\theta) = \frac{F_{\mathcal{N}}(\theta) - (1-\nu)}{\nu},\ \theta \in [\theta_\Delta, \overline{\theta}].$$

Following a similar argument as in the proof of Theorem 2.3.2, the limiting objective function for high-cost agents from group $\mathcal{N}$ is

$$F_{P_\Delta}^{-1}\left(1 - F_{\mathcal{N}_\Delta}\left[\psi_{\mathcal{N}}^{ap}(s)\right]\right) - \mathcal{C}(s;\theta).$$

Since $F_{P_\Delta}^{-1}(r) = F_P^{-1}(\nu r),\ r \in [0,1]$, the objective can be rewritten and rearranged as follows:

$$\begin{aligned} & F_P^{-1}\left[\nu\left(1 - \frac{F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right] - (1-\nu)}{\nu}\right)\right] - \mathcal{C}(s;\theta) \\ & = F_P^{-1}\left(1 - F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right]\right) - \mathcal{C}(s;\theta), \end{aligned}$$

which is exactly the same limiting objective function as under a quota. Since the boundary condition is also the same it follows that on the interval $[\theta_\Delta, \overline{\theta}]$, we have $\gamma_{\mathcal{N}}^{ap}(\theta) = \gamma_{\mathcal{N}}^{q}(\theta)$ and $\theta_\Delta = \psi_{\mathcal{N}}^{q}(\Delta)$ from which the result follows. This also provides a boundary condition $\gamma_{\mathcal{N}}^{ap}\left[\psi_{\mathcal{N}}^{q}(\Delta)\right] = \Delta$ for the solution of $\gamma_{\mathcal{N}}^{ap}$ on the lower interval $[\underline{\theta}, \theta_\Delta]$. ■

Knowing how $\gamma_{\mathcal{N}}^{ap}$ behaves on the upper type interval (if there is one), I have a boundary condition for non-minorities on the lower interval $[\underline{\theta}, \theta_\Delta]$ for general $\tilde{S}$. Before proceeding, it will be useful to observe that the gross payoff functions satisfy $\pi_{\mathcal{M}}^{ap}(s) = \pi_{\mathcal{N}}^{ap}(\tilde{S}(s))$.

On the lower interval, the limiting objective functions for groups $\mathcal{M}$ and $\mathcal{N}$ are, respectively,

$$F_P^{-1}\left[1-\left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(\tilde{S}(s))\right]+\mu F_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(s)\right]\right)\right]-\mathcal{C}(s;\theta),\ s\geq 0$$

and

$$F_P^{-1}\left[1-\left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right]+\mu F_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(\tilde{S}^{-1}(s))\right]\right)\right]-\mathcal{C}(s;\theta),\ s\geq \tilde{S}(0)$$

and the FOCs for $\mathcal{M}$ and $\mathcal{N}$, respectively, are

$$-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(\tilde{S}(s))\right](\psi_{\mathcal{N}}^{ap})'(\tilde{S}(s))\tilde{S}'(s)+\mu f_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(s)\right](\psi_{\mathcal{M}}^{ap})'(s)}{f_P\left[\Pi_{\mathcal{M}}^{ap}(s)\right]}=\mathcal{C}'(s;\theta)$$

and

$$-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right](\psi_{\mathcal{N}}^{ap})'(s)+\mu f_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(\tilde{S}^{-1}(s))\right]\frac{(\psi_{\mathcal{M}}^{ap})'(\tilde{S}^{-1}(s))}{\tilde{S}'(\tilde{S}^{-1}(s))}}{f_P\left[\Pi_{\mathcal{N}}^{ap}(s)\right]}=\mathcal{C}'(s;\theta).$$

In equilibrium, it will be true that $\psi_i^{ap}(s)=\theta$ for group i, so by substituting and rearranging I get

$$\begin{aligned}(\psi_{\mathcal{M}}^{ap})'(s)=&-\frac{\mathcal{C}'\left[s;\psi_{\mathcal{M}}^{ap}(s)\right]f_P\left[\Pi_{\mathcal{M}}^{ap}(s)\right]}{\mu f_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(s)\right]}\\&-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(\tilde{S}(s))\right]}{\mu f_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(s)\right]}(\psi_{\mathcal{N}}^{ap})'(\tilde{S}(s))\tilde{S}'(s)\end{aligned}\tag{2.8}$$

and

$$\begin{aligned}(\psi_{\mathcal{N}}^{ap})'(s)\tilde{S}'(\tilde{S}^{-1}(s))=&-\frac{\mathcal{C}'\left[s;\psi_{\mathcal{N}}^{ap}(s)\right]f_P\left[\Pi_{\mathcal{N}}^{ap}(s)\right]\tilde{S}'(\tilde{S}^{-1}(s))}{(1-\mu)f_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right]}\\&-\frac{\mu f_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(\tilde{S}^{-1}(s))\right]}{(1-\mu)f_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right]}(\psi_{\mathcal{M}}^{ap})'(\tilde{S}^{-1}(s)).\end{aligned}\tag{2.9}$$

Equations (2.8) and (2.9), with boundary conditions complete the solution for $(\gamma_{\mathcal{M}}^{ap}, \gamma_{\mathcal{N}}^{ap})$. By evaluating equation (2.9) at $\tilde{S}(s)$ and substituting it into the FOC for minorities, equation (2.8) reduces to

$$\mathcal{C}'\left[s;\psi_{\mathcal{M}}^{ap}(s)\right] = \mathcal{C}'\left[\tilde{S}(s);\psi_{\mathcal{N}}^{ap}(\tilde{S}(s))\right]\tilde{S}'(s), \tag{2.10}$$

which provides a relation between grade selection in the two groups. As it turns out, equation (2.10) is important for characterizing the effects of $\tilde{S}$ on minority bidding. The solution for equilibrium grades under a general admission preference $\tilde{S}$ is given by Proposition (2.3.3); equations (2.9) and (2.10) and boundary condition $\psi_{\mathcal{N}}^{ap}(\Delta) = \psi_{\mathcal{N}}^{q}(\Delta) = \theta_{\Delta}$, where $\Delta = \tilde{S}(0)$. Of course, the following theorem is needed to validate this claim.

Theorem 2.3.4. *In the college admission game with an admission preference $\tilde{S}$ satisfying assumptions 2.1.6 and 2.1.7, given $\rho,\ \varepsilon,\ \delta > 0$, there exists $K^* \in \mathbb{N}$, and a set $E \subset [\underline{\theta}, \overline{\theta}]$ having (Lebesgue) measure $m(E) < \rho$, such that for any $K \geq K^*$, on any closed subset of $[\underline{\theta}, \overline{\theta}] \setminus E$ we have the following:*

(*i*) *an ε-equilibrium of the K-player admission preference game is generated by $\gamma_i^{ap}(\theta),\ i = \mathcal{M}, \mathcal{N}$ as defined by Proposition (2.3.3), equation (2.9), equation (2.10), boundary condition (2.3) for non-minorities and boundary condition*

$$\mathcal{C}'\left[0;\theta^*\right] = \mathcal{C}'\left[\Delta;\theta_{\Delta}\right]\tilde{S}'(0)$$

for minorities, where $\theta^* = \inf\left\{\theta\ :\ \gamma_{\mathcal{M}}^{ap}(\theta) = 0\right\}$, $\theta_{\Delta} = \psi_{\mathcal{N}}^{ap}(\Delta)$, and $\Delta = \tilde{S}(0)$;

and

(ii) $\gamma_i^{ap}(\theta)$ *is a δ-approximate equilibrium for the K-player quota game, or*

$$\|\gamma_i^{ap}(\theta) - \gamma_i^{ap}(\theta;K)\|_{\sup} < \delta, \;\; i = \mathcal{M}, \mathcal{N}.$$

Proof: The proof is similar to that for Theorem 2.3.1. ■

As before, by using a more complicated proof technique, this result can be strengthened to demonstrate that γ_i^{ap} generates an ε-equilibrium and a δ-approximate equilibrium on the entire set $[\underline{\theta}, \overline{\theta}]$, rather than on a subset with nearly full measure. See the Appendix for details.

2.4 Uniform Prizes and Linear Costs

As the above discussion demonstrates, the present model of market competition is flexible enough to handle a wide range of specifications. In this section I shall impose two simplifications in order to facilitate qualitative characterizations of model equilibria. First, since the objective of this research is to characterize the effects of policy changes, I shall abstract away from the intricacies of the "supply side" of the market, and assume that prizes (*e.g.*, college seats) are distributed uniformly on the unit interval. Second, in order to simplify the analysis I henceforth adopt a cost function that is linear in achievement:

$$\mathcal{C}(s;\theta) = \theta s.$$

In order to derive qualitative comparisons of behavioral responses for different groups that are unevenly affected by AA, it will be helpful to assume that the type distributions are ordered by likelihood ratio dominance, or

Assumption 2.4.1. $h(\theta) = \frac{f_{\mathcal{M}}(\theta)}{f_{\mathcal{N}}(\theta)}$ is a strictly increasing function on $[\underline{\theta}, \overline{\theta}]$.

Likelihood ratio dominance is essentially a strong form of first-order stochastic dominance.[9] In other words, the game is assumed to be asymmetric in the sense that minority study costs are higher on average.

The asymmetry assumption is not intended to imply that there are fundamental differences in inherent ability across the two groups, as types reflect a myriad of environmental factors as well. Rather, it is in keeping with arguments made by proponents of AA regarding systemic competitive disadvantages for minorities, due to various historical factors. For example, White children in the United States, on average, are more affluent and attend primary and secondary schools that are better funded than African-Americans. The idea behind cost asymmetry is that an average minority student must expend more personal effort to overcome the environmental obstacles—poverty, poor health-care, lower quality K-12 education, and so forth—eroding his competitive edge.[10]

[9] An excellent exposition of this topic can be found in Krishna Krishna (2002).

[10] There is some empirical evidence consistent with this view. Neal and Johnson Neal and Johnson (1996) found that for the Armed Forces Qualification Test, "family background variables that affect the cost or difficulty parents face in investing in their children's skill explain roughly one third of the racial test score differential" (pg. 871). Fryer and Levitt Fryer and Levitt (2004) analyzed data on racial test-score gaps among elementary school children in an attempt to uncover the causes. They found that by controlling for socioeconomic status and other environmental factors which vary substantially by race, test-score gaps significantly decrease, but not entirely. They test various hypotheses

2.4.1 Full Quota

Some interesting observations can be made about how behavior changes when moving from a color-blind policy to a quota. It turns out that with quota admissions, the highest performing minority students decrease their academic achievement and the lowest performing students increase it. For non-minorities the change is exactly the opposite: high performers increase their effort and low performers decrease it. This result is formalized in the following theorem.

Theorem 2.4.2. *If prizes are uniform, $C(s;\theta) = \theta s$, and $F_{\mathcal{M}}(\theta)$ dominates $F_{\mathcal{N}}(\theta)$ according to the likelihood ratio order, then there exists $\theta^* \in (\underline{\theta}, \overline{\theta})$ such that*

(i) *for minority competitors, $\gamma^q_{\mathcal{M}}(\theta) < (>) \gamma^{cb}(\theta)$ for each $\theta < (>) \theta^*$; and*

(ii) *for non-minority competitors, $\gamma^q_{\mathcal{N}}(\theta) > (<) \gamma^{cb}(\theta)$ for each $\theta < (>) \theta^*$.*

Proof: First note that with uniform prizes and linear costs, equations (2.2) and (2.5) simplify to

$$\begin{aligned} \gamma^{cb}(\theta) &= \int_{\theta}^{\overline{\theta}} \frac{\mu f_{\mathcal{M}}(u) + (1-\mu) f_{\mathcal{N}}(u)}{u} du \quad \text{and} \\ \gamma^q_i(\theta) &= \int_{\theta}^{\overline{\theta}} \frac{f_i(u)}{u} du, \quad i \in \{\mathcal{M}, \mathcal{N}\}, \end{aligned} \tag{2.11}$$

from which it can easily be seen that $\gamma^{cb}(\theta) = \mu \gamma^q_{\mathcal{M}}(\theta) + (1-\mu)\gamma^q_{\mathcal{N}}(\theta)$.

By likelihood ratio dominance, it follows that $f_{\mathcal{M}}$ and $f_{\mathcal{N}}$ have a single crossing at some $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$, and $f_{\mathcal{M}}(\theta) > f_{\mathcal{N}}(\theta)$ for each $\theta > \tilde{\theta}$. From this and

to explain the remainder of the gap, and find that disparities in school quality is the only one not rejected by the data.

equations (2.11) it follows that $\gamma^q_{\mathcal{M}}(\theta) > \gamma^q_{\mathcal{N}}(\theta)$ for each $\theta \in [\tilde{\theta}, \overline{\theta})$. Moreover, if the functions cross at some θ^* then it must be on the interval $[\underline{\theta}, \tilde{\theta})$. Note that

$$\gamma^q_{\mathcal{M}}(\underline{\theta}) = \int_{\underline{\theta}}^{\overline{\theta}} f_{\mathcal{M}}(u)\frac{1}{u}du < \int_{\underline{\theta}}^{\overline{\theta}} f_{\mathcal{N}}(u)\frac{1}{u}du = \gamma^q_{\mathcal{N}}(\underline{\theta}),$$

where the inequality follows from the fact that each side is an expectation and $f_{\mathcal{N}}$ places more weight on larger values of the function $\frac{1}{u}$ (*i.e.*, smaller values of u) on the interval $[\underline{\theta}, \overline{\theta}]$. Then by likelihood ratio dominance and continuity it follows that θ^* exists on the open interval $(\underline{\theta}, \tilde{\theta})$ and is unique. Finally, the result of the theorem follows from the fact that γ^{cb} is a convex combination of $\gamma^q_{\mathcal{M}}$ and $\gamma^q_{\mathcal{N}}$. ■

Theorem 2.4.2 highlights some interesting facts concerning a student's decision problem. The intuition derives from the fact that a quota mechanism splits the competition apart into two separate competitions. In doing so, it alters the distribution of competition that each minority student faces from $(1-\mu)F_{\mathcal{N}}(\theta) + \mu F_{\mathcal{M}}(\theta)$ to $F_{\mathcal{M}}(\theta)$. For a low-performing (*i.e.*, high-cost) student, the mass of competitors at an advantage to him is relatively large before the change. Since costs are sunk, it is not worthwhile for such a student to exert much effort when competing with the population at large. This phenomenon is commonly known in the all-pay auctions literature as the *discouragement effect*. However, if the student faces only minority competitors (with higher costs on average), the discouragement effect is mitigated and effort increases.

For high-performing (*i.e.*, low-cost) minorities, the effect is reversed: when the relative mass of similar low-cost opponents decreases there is less need to aggressively outperform the competition in order to win a highly valued seat. Thus, effort decreases among top minority students. The effects for non-minority students in each category are exactly the opposite, for the same intuition. The discouragement effect for high-cost non-minorities is exacerbated by moving to a quota system, and low-cost non-minorities must compete more aggressively against a set of competitors whose costs are on average lower.

An evaluation of a quota in terms of the policy objectives outlined in Section **??**, as compared to the baseline color-blind mechanism, is provided in the following corollary:

Corollary 2.4.3. *Maintain the assumptions of Theorem 2.4.2 and let $\theta^* \in (\underline{\theta}, \overline{\theta})$ be the cutoff type defined there. Then the following statements immediately follow:*

(i) *(minority achievement) $G^q_{\mathcal{M}}(s) > (<) G^{cb}_{\mathcal{M}}(s)$ for each $s > (<) \gamma^{cb}(\theta^*)$,*

(ii) *(non-minority achievement) $G^q_{\mathcal{N}}(s) < (>) G^{cb}_{\mathcal{N}}(s)$ for each $s > (<) \gamma^{cb}(\theta^*)$*

(iii) *(achievement gap) $\mathcal{A}^q(q) > (<) \mathcal{A}^{cb}(q)$ for each $q > (<) \mu F_{\mathcal{M}}(\theta^*) + (1-\mu) F_{\mathcal{N}}(\theta^*)$, and*

(iv) *(enrollment gap) $\mathcal{E}^q(q) < \mathcal{E}^{cb}(q)$ for all $q \in (0, 1)$.*

In words, the effect on the achievement gap is mixed because of how the different groups respond to the policy. A quota widens it among the best and brightest students, since low-cost minorities decrease achievement and low-cost non-minorities increase achievement, relative to the color-blind case. At the bottom end of the score distribution we see a narrowing of the achievement gap, as high-cost minorities work harder and their non-minority counterparts decrease output. Finally, an attractive quality of the quota system is that by design it achieves an enrollment gap of zero (this is true in the general case as well). Thus, when types are ordered by stochastic dominance, a quota is guaranteed to produce an improvement over a color-blind mechanism in terms of the enrollment gap.

2.4.2 Admission Preferences

When costs are linear, equation (2.10) reduces to

$$\psi_{\mathcal{M}}^{ap}(s) = \psi_{\mathcal{N}}^{ap}(\tilde{S}(s))\tilde{S}'(s). \tag{2.12}$$

As mentioned previously, this equation reveals much about minority grade selection under preference rules. For beginners, it indicates under what circumstances the admission preference rule will lead to a mass-point of students achieving a grade of zero, as outlined in the following proposition.

Theorem 2.4.4. *In the college admissions game, assume that study costs are of the form $\mathcal{C}(s;\theta) = \theta s$ and assume an admission preference mechanism $\tilde{S}$ satisfying assumptions 2.1.6 and 2.1.7. Moreover, define $\Delta \equiv \tilde{S}(0)$ and $\theta_\Delta \equiv \psi_{\mathcal{N}}^{ap}(\Delta) = \psi_{\mathcal{N}}^{q}(\Delta)$. Then the following results follow:*

(i) *If $\tilde{S}'(0) \geq (>) \frac{\overline{\theta}}{\psi_{\mathcal{N}}^{ap}(\Delta)}$ the grade achieved by a minority student with the highest possible cost type is non-negative (strictly positive).*

(ii) *If $\frac{\underline{\theta}}{\psi_{\mathcal{N}}^{ap}(\Delta)} < \tilde{S}'(0) < \frac{\overline{\theta}}{\psi_{\mathcal{N}}^{ap}(\Delta)}$ there is a positive mass $\zeta \in (0,1)$ of minority students* *who choose equilibrium grades of zero.*

(iii) *If* $\tilde{S}'(0) < \frac{\theta}{\psi_{\mathcal{N}}^{ap}(\Delta)}$ *all minority students choose equilibrium grades of zero.*

Proof: From equation (2.12) it follows that

$$\inf \psi_{\mathcal{M}}(0) = \psi_{\mathcal{N}}(\Delta)\tilde{S}'(0),$$

which solves for the lowest minority type who achieves a grade of zero. Statements *(i)*, *(ii)* and *(iii)* then follow from substituting the left-hand side to test whether

$$\overline{\theta} \gtreqless \psi_{\mathcal{N}}(\Delta)\tilde{S}'(0)$$

and

$$\underline{\theta} \gtreqless \psi_{\mathcal{N}}(\Delta)\tilde{S}'(0). \ \blacksquare$$

Note that Theorem 2.4.4 holds for general prize distributions, and does not depend on stochastic ordering of types.

At this point I shall further simplify the analysis by focusing on an additive admission preference of the form

$$\tilde{S}(s) = s + \Delta.$$

Aside from being a useful illustrative tool, this policy is of particular interest as it was previously used in undergraduate admissions at the University of Michigan. For that reason I shall refer to it below as the *Michigan rule*. Theorem 2.4.4 shows that for any level of Δ, a Michigan rule will lead to a mass point of minority students with zero achievement in equilibrium. This is because it has a slope of one, whereas $\frac{\overline{\theta}}{\psi^{ap}_{\mathcal{N}}(\Delta)} > 1$ for any positive Δ. However, if one were to consider a more general affine admission preference, say

$$\tilde{S}(s) = \Delta_0 + \Delta_1 s$$

then as Proposition 2.4.4 shows, the slope coefficient could be chosen so as to eliminate the mass point of minority students achieving grades of zero.

I shall now proceed to derive the equilibrium under a Michigan rule. In this case, equation (2.10) further reduces to

$$\psi^{ap}_{\mathcal{M}}(s) = \psi^{ap}_{\mathcal{N}}(s + \Delta), \tag{2.13}$$

which indicates that a minority student with type θ will achieve a grade of exactly Δ less than his non-minority counterpart with the same type. Recall that non-minority students whose grades are less than Δ will behave the same as under a quota rule, giving a boundary condition of

$$\psi^{ap}_{\mathcal{N}}(\Delta) = \psi^{q}_{\mathcal{N}}(\Delta) = \theta_{\Delta}, \tag{2.14}$$

and minority students with costs above θ_{Δ} choose a grade of zero.

Equation (2.13) can be substituted back into the decision problem of $\mathcal{N}$

agents to get

$$F_P^{-1}\left[1-\left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right]+\mu F_{\mathcal{M}}\left[\psi_{\mathcal{N}}^{ap}(s)\right]\right)\right]-\theta s,$$

which gives the familiar FOC:

$$(\gamma_{\mathcal{N}}^{ap})'(\theta)=-\frac{(1-\mu)f_{\mathcal{N}}(\theta)+\mu f_{\mathcal{M}}(\theta)}{f_P\left[F_P^{-1}\left(1-[(1-\mu)F_{\mathcal{N}}(\theta)+\mu F_{\mathcal{M}}(\theta)]\right)\right]\theta}. \tag{2.15}$$

Recall that this is the same as the differential equation arising from the FOC under a color-blind rule. The important difference here is that its solution depends on a different boundary condition, given by equation (2.14).

An evaluation of a Michigan rule in terms of the policy objectives outlined in Section **??**, as compared to the baseline color-blind mechanism, is provided in the following theorem:

Theorem 2.4.5. *Let prizes be uniform, let achievement costs be of the form $\mathcal{C}(s;\theta)=\theta s$, let $F_{\mathcal{M}}(\theta)$ dominate $F_{\mathcal{N}}(\theta)$ according to the likelihood ratio order, and assume a Michigan admission preference $\widetilde{S}$ with fixed grade markup $\Delta<\gamma_{\mathcal{N}}^{q}(\theta^*)$, where θ^* is the cutoff type in Proposition 2.4.2. Then the following statements are true:*

(i) *(minority achievement) $G_{\mathcal{M}}^{ap}(s)>G_{\mathcal{M}}^{cb}(s)$ for all s,*

(ii) *(non-minority achievement) $G_{\mathcal{N}}^{ap}(s)<G_{\mathcal{N}}^{cb}(s)$ for all s*

(iii) *(achievement gap) $\mathcal{A}^{ap}(q)>\mathcal{A}^{cb}(q)$ for all q, and*

(iv) *(enrollment gap) $\mathcal{E}^{q}(q)<(=)\mathcal{E}^{cb}(q)$ for all $q<(>)\mu F_{\mathcal{M}}(\theta_\Delta)+(1-\mu)F_{\mathcal{N}}(\theta_\Delta)$,* *where* $\theta_\Delta = \psi^{ap}_{\mathcal{N}}(\Delta)$.

Proof: The fact that non-minorities decrease their achievement relative to the color-blind case follows immediately from the above derivations. First, recall that $\gamma^{ap}_{\mathcal{N}}(\theta) = \gamma^{q}_{\mathcal{N}}(\theta) < \gamma^{cb}_{\mathcal{N}}(\theta)$ on the interval $[\theta_\Delta, \overline{\theta}]$ (see Theorem 2.3.3). Second, note that equations (2.15) and (2.14) imply that non-minority achievement parallels color-blind achievement, but from a lower initial condition. Note also that by Theorem 2.4.4 and equation (2.13) minority types below θ_Δ reduce achievement by exactly Δ more than their non-minority counterparts, and all other non-minority types achieve a grade of zero. From this it follows that the achievement gap is unambiguously widened at all quantiles of the grade distribution. Finally, by the same argument, the enrollment gap is unaffected for types above θ_Δ because in equilibrium a Michigan rule merely compensates for the minority behavioral response and does not alter relative standing between the two groups. ∎

This result is significant for several reasons. First, it highlights an important aspect of the behavioral response to admission preferences. Although the policy-maker may intend to bolster minority students' competitive edge with a grade subsidy, a rational student may simply treat the markup as a direct utility transfer and reduce achievement. When costs are linear and the markup does not depend on output, minority students scale back achievement by exactly the amount of the markup, relative to non-minority counterparts.[11] This picture becomes more

[11]When costs are convex in achievement, the effect is less straightforward (see equation (2.10)) and complete consumption of the grade boost will not obtain in general. However, it will generally be the case that the beneficiaries of a Michigan rule will use at least some portion of the grade markup as a direct utility subsidy, rather than using it only to bolster

bleak when one recognizes that non-minorities also reduce their achievement, relative to the color-blind case. In other words, when moving from color-blind allocations to a Michigan rule, a markup of Δ leads to a grade reduction of *more* than Δ for *all* minority students, and a smaller reduction for all non-minorities as well. The implication is that a Michigan rule is unambiguously detrimental to effort incentives.

Second, given the above facts about the relation between minority and non-minority achievement under a Michigan rule, it immediately follows that the policy will lead to a widening of the racial achievement gap at every point in the type support, relative to a color-blind policy. This is because like types have the same achievement under a color-blind rule, whereas the difference in achievement between a minority and a non-minority with type θ under the Michigan rule is $\min\{\Delta, \gamma_{\mathcal{M}}^{ap}(\theta)\}$, which is strictly positive for any $\theta < \overline{\theta}$.

Finally, these facts also imply that an admission preference rule is an ineffective means for helping minorities into better colleges. With linear costs, students choose grades so that once the fixed markup is assessed relative standings between minorities and non-minorities is the same as under a color-blind allocation mechanism. The only change that occurs is among minorities scoring zero and non-minorities scoring below Δ. Thus, a Michigan rule has *no allocational effect* except to re-shuffle allocations in the lower tail of the prize distribution. Enrollment at the most selective schools will be identical to enrollment under a

their competitive edge.

color-blind admission policy.

The intuition behind the behavioral response to an admission preference stems from equation (2.12), reproduced below for convenience:

$$\psi^{ap}_{\mathcal{M}}(s) = \psi^{ap}_{\mathcal{N}}\left[\tilde{S}(s)\right]\tilde{S}'(s).^{12}$$

In a color-blind world, minority students compete on the margin with non-minority counterparts of roughly the same cost type, but a grade markup alters this relationship. In general, students respond to the policy by adjusting behavior until once again they are competing on the margin with non-minority counterparts on the same competitive standing. As the above example illustrates, behavioral responses to the policy may be undesirable, and they can even nullify the ability of the policy to alter allocations by changing the relative standing between minorities and non-minorities. Expression (2.12) leads to a necessary condition for the minority behavioral response to be in a desirable direction.

Theorem 2.4.6. *Let achievement costs be of the form $C(s;\theta) = \theta s$, let the admission preference policy $\widetilde{S}$ satisfy assumptions 2.1.6 and 2.1.7, and define $\widetilde{M}(s) \equiv \widetilde{S}(s) - s$ to be the score markup implied by the admission preference. Then for $\theta \in [\underline{\theta}, \overline{\theta}]$, we have*

$$\mathcal{A}^{ap}\left[F_{\mathcal{M}}(\theta)\right] \leq \mathcal{A}^{cb}\left[F_{\mathcal{M}}(\theta)\right]$$

[12] Recall that this expression results in part from the linear functional form of costs assumed in this section. I concentrate on the restricted version here for expositional purposes: it vastly simplifies the intuition. For the original expression which incorporates general forms of utility curvature, see equation (2.10).

only if the marginal markup for minority type θ is positive, or

$$\widetilde{M}'\left(\gamma^{ap}_{\mathcal{M}}(\theta)\right) > 0.$$

Proof: The result of the theorem follows straightforwardly from two facts. The first, $\widetilde{M}'(s) > 0 \;\Leftrightarrow\; \widetilde{S}'(s) > 1$; the second, $\mathcal{A}^{ap}\left[F_{\mathcal{M}}(\theta)\right] \leq \mathcal{A}^{cb}\left[F_{\mathcal{M}}(\theta)\right]$ only if $\gamma^{ap}_{\mathcal{M}}(\theta) \geq \gamma^{ap}_{\mathcal{N}}(\theta)$. This is because $\gamma^{cb}_{\mathcal{M}}(\theta) = \gamma^{cb}_{\mathcal{N}}(\theta)$ and all of the achievement gap in the color-blind game comes from asymmetry in the type distributions. Thus, in order to close the gap, it is necessary for minority students to achieve higher scores than their non-minority counterparts of the same type.

Fix $\theta \in [\underline{\theta}, \overline{\theta}]$ and let $s = \gamma^{ap}_{\mathcal{M}}(\theta)$. If $\widetilde{S}'(s) = 1$, then (2.12) reduces to

$$\psi^{ap}_{\mathcal{M}}(s) = \psi^{ap}_{\mathcal{N}}(\tilde{S}(s)),$$

from which it follows that $\gamma^{ap}_{\mathcal{N}}(\theta) - \gamma^{ap}_{\mathcal{M}}(\theta) = \widetilde{M}(s) > 0$. By monotonicity, a similar inequality holds when $\widetilde{S}'(s) < 1$. ■

Note that Theorem 2.4.6 holds for general prize distributions and does not require assumptions about stochastic ordering of types. The proof of the theorem highlights a tension between the level of the admission preference and it's slope. As equation (2.12) illustrates, an increase in the level of the markup (holding slope fixed) causes minorities to reduce output, relative to their non-minority counterparts. On the other hand, an increase in the marginal markup (holding the level of the markup fixed at a given point) has the opposite effect.

The critical flaw in a Michigan rule is that its slope is identical to a color-blind markup $\widetilde{S}^{cb}(s) = s$, and it simply offers a higher level of assistance, Δ.

Another way to think about this is that a Michigan rule fails to reward achievement: with a zero marginal markup, students achieving higher scores get the same boost as everyone else. In essence, Theorem 2.4.6 shows that in order for AA to be effective, the assistance it renders to minority students must be merit based. A multiplicative rule of the form $\widetilde{S}(s) = (1+r)s,\ r > 0$ is an example of an admission preference with a positive marginal markup, $\widetilde{M}'(s) = r$. As general results other than those above are difficult to prove, I shall defer discussion of a multiplicative preference to the following section, where I illustrate the model using numerical methods.

2.4.3 Example: Pareto Types

In this section I illustrate the model by computing approximate equilibria for a simple special case. Prizes are distributed uniformly on the interval $[0, 100]$, so that $F_P(p) = \frac{p}{100}$. The fraction of minority college candidates is $\mu = 0.25$ and private costs follow a Pareto distribution, with the upper tail truncated to the interval $[1, 5]$. For group i the private cost distribution is

$$F_i(\theta) = \frac{1 - \theta^{-\kappa_i}}{1 - \overline{\theta}^{-\kappa_i}},\ \kappa_i > 0,\ i = \mathcal{M}, \mathcal{N},$$

where $\kappa_{\mathcal{M}} = 0.1$ and $\kappa_{\mathcal{N}} = 1.5$, so that stochastic dominance holds. The further $\kappa_{\mathcal{M}}$ is from $\kappa_{\mathcal{N}}$, the more pronounced is the asymmetry across groups. All parameters in this section were chosen for purely illustrative purposes, with the exception of Δ, which is discussed below. With the above parameters specified, solving for the equilibrium is a simple mater of integrating differential equations.

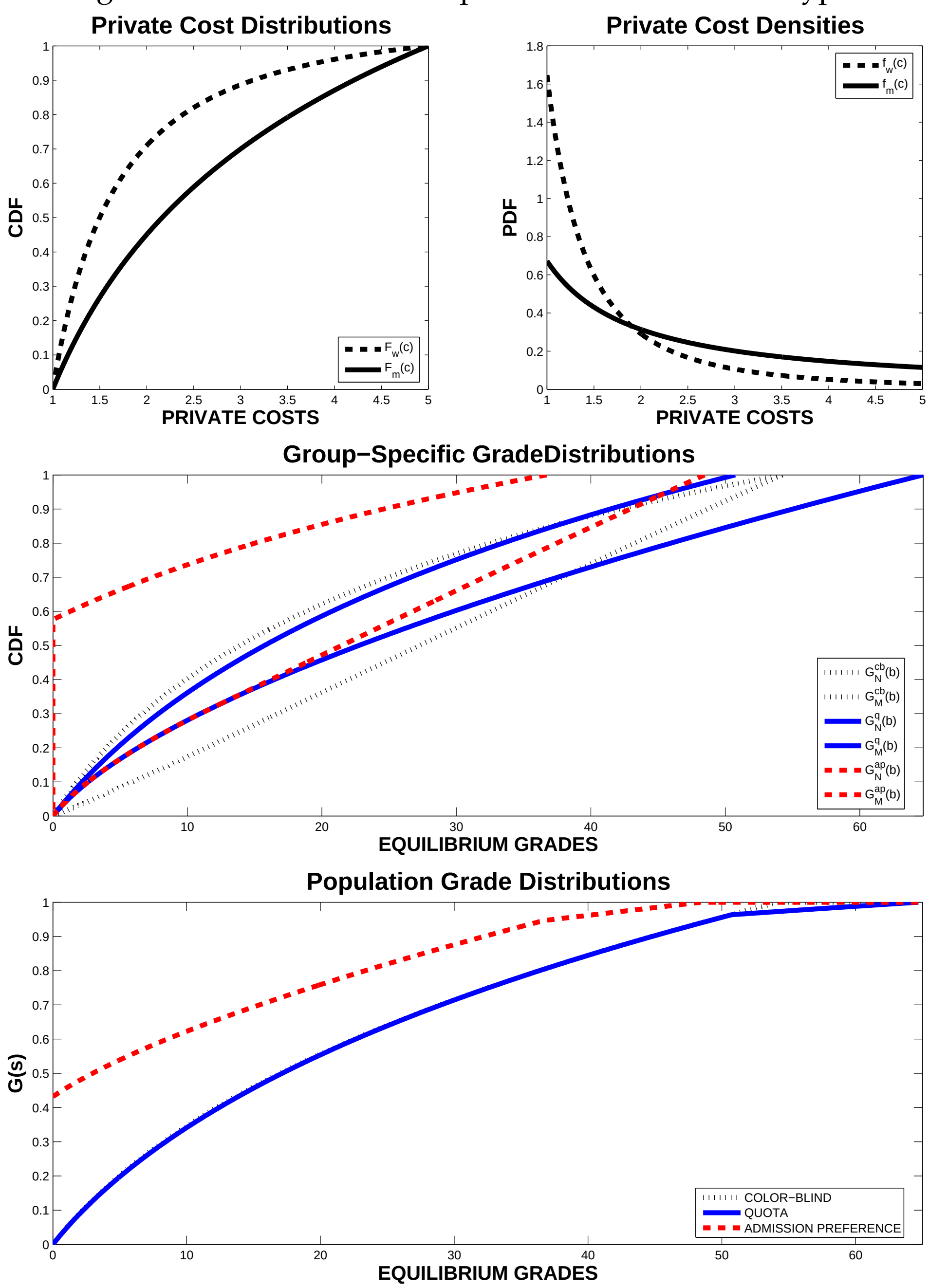
A value of Δ was chosen based on the above parameters to facilitate comparisons between a quota and a Michigan rule. A key characteristic of a quota is that it ensures that the average prize value allocated to members of each group is the same. Thus, I compute the fixed grade subsidy Δ^* that equates the average prize value awarded to each group. This provides some basis for comparison of the two different policies, as they are both designed to achieve a common objective.[13]

Computing Δ^* under uniformly-distributed prizes is fairly simple. Recall from Section 2.4.2 that when costs are linear, an additive grade subsidy only alters equilibrium allocations among agents who grade less than Δ. Once again, let θ_Δ denote the player type that submits a grade of Δ for group $\mathcal{N}$ and let $p_\Delta = F_P^{-1}\left[1 - \{(1-\mu)F_{\mathcal{N}}(\theta_\Delta) + \mu F_{\mathcal{M}}(\theta_\Delta)\}\right]$ denote the top prize allocated to players whose transformed grades are Δ or less.

Within the interval $[0, p_\Delta]$, the top mass μ of prizes are awarded to group $\mathcal{M}$ and the rest are given to group $\mathcal{N}$. Thus, the average prize given to candidates with costs $c \leq \theta_\Delta$ in group $\mathcal{M}$ and $\mathcal{N}$ are, respectively, $(p_\Delta + \mu p_\Delta)/2$ and $\mu p_\Delta/2$. The average prize awarded to players of either group with private costs above

[13]The defining characteristic of a quota is that it equates all moments of the group allocation distributions, but this is impossible to do with a Michigan rule in an asymmetric game. Here, I focus on the first central moment for illustrative purposes.

Figure 2.1: Numerical Example: Pareto Distributed Types



θ_Δ are the same–recall that transformed equilibrium grades are the same for a given θ in this interval–and are given by $(\overline{p} + p_\Delta)/2$. Therefore, the average prize allocated to group $\mathcal{M}$ candidates is

$$[1 - F_{\mathcal{M}}(\theta_\Delta)] \frac{p_\Delta + \mu p_\Delta}{2} + F_{\mathcal{M}}(\theta_\Delta) \frac{\overline{p} + p_\Delta}{2} \tag{2.16}$$

and the average prize for group $\mathcal{N}$ is

$$[1 - F_{\mathcal{N}}(\theta_\Delta)] \frac{\mu p_\Delta}{2} + F_{\mathcal{N}}(\theta_\Delta) \frac{\overline{p} + p_\Delta}{2}. \tag{2.17}$$

Thus, Δ^* is determined by the following equality

$$\begin{aligned} &[1 - F_{\mathcal{M}}(\theta_{\Delta^*})] \frac{p_{\Delta^*} + \mu p_{\Delta^*}}{2} + F_{\mathcal{M}}(\theta_{\Delta^*}) \frac{\overline{p} + p_{\Delta^*}}{2} \\ &\qquad = [1 - F_{\mathcal{N}}(\theta_{\Delta^*})] \frac{\mu p_{\Delta^*}}{2} + F_{\mathcal{N}}(\theta_{\Delta^*}) \frac{\overline{p} + p_{\Delta^*}}{2}. \end{aligned} \tag{2.18}$$

For the above parameters, Δ^* is about 24% of the maximal grade achieved by group $\mathcal{M}$. Of course, the size of Δ^* depends on the degree of asymmetry between the two groups, and is therefore an empirical question. For example, a $(\kappa_{\mathcal{M}}, \kappa_{\mathcal{N}})$ pair of $(0.6, 1)$ cuts Δ^* roughly by half. This example merely demonstrates that in order for a Michigan rule to deliver the same average allocative effect as a quota, the fixed markup potentially must be quite large.

Figure 2.1 plots several objects of interest. The top two panes are the distributions and densities of private costs. The middle pane displays group-specific grade distributions under each of the three admission policies. A color-blind rule is denoted by a dotted line, a Michigan rule is denoted by a dashed line, and a solid line denotes a quota. For pairs of lines with the same style, the one lying to the left is the grade distribution for minorities. The bottom pane displays population grade distributions, following a similar convention. When comparing two grade distributions, keep in mind that if distribution *i* lies to the right of distribution *j* in some region, it indicates an interval of students who are enticed to achieve a higher academic output under policy *i*.

The middle plot gives an idea of how within-group behavior changes, and also how the achievement gap changes under different policies. As Propositions 2.4.4 and 2.4.5 suggest, the picture for an additive markup is dismal. With any Michigan rule, the policy-maker must settle for a mass-point of zero achievement in order to equalize average outcomes for each group. As this example shows, the mass point can be potentially large. The general insight here is that as asymmetry increases, a Michigan rule becomes increasingly inadequate as a policy instrument. Notice also the substantial leftward shift in non-minority grades associated with the quota-comparable Michigan rule. Recall, too, that allocations in the upper tail of the prize distribution are unaffected, even with this extreme version of the policy. A more definitive analysis is ultimately an empirical exercise, but this example demonstrates how an ill-designed admission preference can lead to a substantial social loss, while producing little in the way of desired change.

As outlined in Proposition 2.4.2, a quota rule produces some interesting benefits, relative to a color-blind system. The middle pane shows that all minorities below the 80^{th} grade percentile and all non-minorities above the 70^{th} grade percentile increase their performance, relative to a color-blind system. Of course, there are also costs involved, as all other students decrease academic output. Although it is difficult to tell from the lower pane, it turns out that a quota produces a slight first-order dominance shift in the overall population grade distribution.

In contrast, the difference from Michigan-rule grades for both color-blind and quota admissions is quite stark. The results here highlight some inaccuracies in statements made by American policy-makers that, “at their core, the Michigan policies amount to a quota system” (see Bush Rosen et al.). In fact, the contrast between the two versions of AA could not be more stark: a Michigan rule in no way resembles a quota system in terms of either its behavioral or its allocative effects.

2.5 Discussion and Conclusion

In this chapter, I have explored the qualitative implications of different AA policies in college admissions. Design of the AA implementation can have significant effects on both effort choice and college placement. On the one hand, it appears that the critics of AA are correct in assuming that a tradeoff exists between equality and academic performance incentives, although the exact nature of the tradeoff—whether it results in a socially desirable change—cannot be resolved theoretically. On the other hand, proponents of AA are also correct in assuming that race-conscious admissions can potentially increase academic performance for some minorities by diminishing discouragement effects. However, in the process of leveling the playing field for some minority students, the situation is made worse for disadvantaged (*i.e.*, high-cost) non-minorities. Moreover, advantaged minorities find diminished incentives for academic performance in an environment where the competition they face is less fierce.

Although the model produces many useful qualitative insights into the college admissions problem, recovering the exact nature of the equity–achievement tradeoff for the purpose of enlightening policy decisions is an empirical exercise. Two reasons exist: first, a complete comparison between admission preferences, quotas, and color-blind admissions requires knowledge of type distributions. It may also require knowledge of the policy-maker's preferences. Second, a meaningful policy analysis requires empirical measurement of actual AA practices in order to compare them to alternatives. In the following chapter I present structural estimates of the model in order to produce quantitative comparisons between different college admissions practices.

If one takes the issues of achievement and enrollment gaps separately, then somewhat more can be said qualitatively. For example, the results proven here provide a possible theoretical explanation for some well-known empirical results on the predictive power of college entrance test scores. Vars and Bowen (1998) used data on SAT scores and subsequent academic performance at "highly selective" post-secondary institutions to investigate whether the entrance test score predicts college success equally well for different races. Their results indicate that

> "while SAT scores are related to college GPA for both blacks and whites, the relationship is weaker for blacks. More important, and

> more disturbing, at every level of SAT score, blacks earn lower grades than their white counterparts... Most sobering of all, the performance gap is greatest for the black students with the highest SATs. The reasons for this gap are not well understood; nevertheless, we believe that many gifted African-American students at academically selective institutions are not realizing their full academic potential."[14]

Propositions 2.4.2 and 2.4.5 may provide an explanation for this puzzle: they indicate that any type of AA based only on race (*i.e.*, both quotas and admission preferences) widens the achievement gap among the highest-performing students. This is illustrated in Figure 1, where the upper tails of the grade distributions for each group are further apart under AA than under color-blind admissions. Since these high-performing students are typically the ones who gain admission to selective institutions, it is plausible that the predictive disparity is the result of a behavioral response to AA. As illustrated in Figure 1, under a quota, achievement by top minority students is compressed to a tighter interval, relative to color-blind admissions, whereas the opposite is true for non-minority scores. For high-ability individuals, AA creates effort disincentives for minorities, relative to non-minorities, and it may also diminish separation among minorities while increasing separation among non-minorities.

Thus, to the extent that SAT scores are a meaningful measure of the human capital high-school students bring with them to college, and to the extent that the

[14](Bowen and Bok, 1998, Ch. 3, pp. 72–78) have reported similar findings concerning the relationship between SAT scores and subsequent college GPA.

subsequent labor market resembles the competitive setting outlined in Section 2.1, an explanation for Vars and Bowen (1998) may be that AA practices in the skilled labor market do not provide adequate incentives for top minority students to perform at their "full academic potential."

As for AA and enrollment, by construction quotas achieve 100 percent equal allocations in the sense that the racial makeup of student bodies at schools of all quality levels will be reflective of population proportions. On the other hand, the effectiveness of admission preferences in rearranging allocations is hampered by the rational behavioral response to this type of policy. Clearly, a policy-maker should *not* treat behavior as fixed when predicting equilibrium allotments of college seats to different groups under different policies. Indeed, in the case of a Michigan-type additive admission preference, such an assumption may result in a near total nullification of any intended change.

Two other interesting directions for further research exist: first, an important related question would be how AA might affect educational attainment decisions among minorities. The current model focuses on student behavior, conditional on participation in the college market, but there is another interesting group of individuals to consider as well: those whose college/work-force decisions may be affected by a given policy. This question could be addressed by formalizing the "supply-side," being comprised of potential colleges and firms who may enter the market and supply post-secondary education services or unskilled jobs. Such a model might illustrate how the marginal agent (*i.e.*, the individual indifferent between attending college and entering the workforce) is affected by a given college admission policy. This would help to characterize the effect of AA on the total mass of minorities enrolling in college.

Finally, the eventual goal for this line of research should be to answer the question of how AA helps or hinders the objective of erasing the residual effects of past institutionalized racism. This will require a dynamic model in which the policy-maker is not only concerned with short-term outcomes for students whose private costs are fixed, but also with the long-run evolution of the private cost distributions. Empirical evidence suggests that academic competitiveness is determined by such factors as affluence as well as parents' education. If AA affects performance and outcomes for current minority students in a certain way, then the next question is what effect it might that have on their children's competitiveness when the next generation enters high school? If a given policy produces the effect of better minority enrollment and higher achievement in the short-run, then one might conjecture that a positive long-run effect will be produced. However, given the mixed picture on the various policies considered in this paper, it seems evident that a long-run model is needed in order to give meaningful direction to forward-looking policy-makers. I hope the theory developed here will serve as a basis for answering these important questions in the future.

CHAPTER 3
SEMIPARAMETRIC STRUCTURAL POLICY ANALYSIS OF AMERICAN COLLEGE ADMISSIONS

3.1 Data

I now proceed with an empirical exercise, first describing the data that will be used to recover each component of the model.[1] Ultimately, the objects of principal empirical interest are the group-specific private cost distributions, $F_{\mathcal{M}}(\theta)$ and $F_{\mathcal{N}}(\theta)$; the demographic parameter μ; the prize distribution $F_P(p)$; and the cost function $\mathcal{C}(s;\theta)$. These objects will enable the counterfactual experiments, which are the ultimate goal of the policy analysis. However, it will first be necessary to obtain estimates of some intermediate objects: the group-specific grade distributions, $G_{\mathcal{M}}(\theta)$ and $G_{\mathcal{N}}(\theta)$; the distributions of prizes allocated to each group under the actual AA policy, $F_{P_{\mathcal{M}}}(p)$ and $F_{P_{\mathcal{N}}}(p)$; and the actual AA policy $\widetilde{S}(s)$, corresponding to the data-generating process. To identify the various model components, I use data on quality measures for colleges and universities in the US, freshman enrollment, and student-level college entrance test scores.

I use data for the academic year 1995-1996 primarily because one can reasonably assume that, prior to that year, AA policies determining payoffs were stable and known to decision-makers. In the summer of 1996 the outcome of a federal lawsuit *Hopwood v. Texas* (78 F.3d 932, 5$^{\text{th}}$ Cir. 1996) was finalized, mark-

[1] SAT test scores were derived from data provided by the College Board. Copyright ©1996 The College Board. www.collegeboard.com. ACT test scores and ACT-SAT concordances were provided by ACT. Copyright ©1996 ACT. www.act.org. The views expressed in this research are not the views of either The College Board, ACT or the US Department of Education.

ing the first successful legal challenge to AA in US college admissions since 1978, nearly two decades before.[2] Subsequently, other potentially important changes occurred, including state laws banning AA being passed in Texas, California, and Michigan.

3.1.1 Prize Data

Institutional quality measures are derived from data and methodology developed by US News & World Report (USNWR) for the purpose of computing their annual *America's Best Colleges* rankings; see Morse (1996). USNWR collects data on fourteen quality indicators for American colleges and universities each year; the sample size in 1996 was 1,314 schools. USNWR classified the fourteen indicators into six categories: *selectivity*, comprised of application acceptance rate, yield (% of accepted students who choose to enroll), average entrance test scores, and % of first-time freshmen in the top quartile of their high school class; *faculty resources*, comprised of % of full-time instructional faculty with a PhD or terminal degree, % of instructional faculty who are full-time, average faculty compensation, and student/faculty ratio; *financial resources*, comprised of education spending per student and non-education spending per student; *retention*, comprised of graduation rate and freshman retention rate; *alumni satisfaction*, comprised of % of living alumni contributing to annual fund drives; and *academic reputation*,

[2]On March 18, 1996 the US Fifth Circuit Court disallowed race-conscious admissions decisions at the University of Texas law school, but appeals continued for several months afterward. The outcome of the case was finalized in July when the Supreme Court declined to review the Fifth Circuit's ruling. The last successful legal challenge before *Hopwood* was in 1978, when the Supreme Court declared quotas unconstitutional in *University of California v. Bakke* (438 U.S. 265 1978).

comprised of a ranking measure taken from a survey of college administrators. A single index of quality is determined by computing empirical distributions for each indicator, and taking a weighted average of the fourteen empirical cumulative distribution function (CDF) values for a given school. In the Data Appendix, I summarize weights and descriptive statistics for each the quality indicators.

One drawback with using the USNWR method for my purposes is that it separates schools by Carnegie classification (*i.e.*, national/regional universities and national/regional liberal arts colleges) and geographic region (*i.e.*, northern, southern, midwestern and western; see Morse (1996) for more details). Therefore, I alter the method slightly by combining all schools into the same set. This does not pose a problem for most of the quality indicators, except one: the academic reputation score. This score is determined by asking college administrators to rank the schools in their Carnegie class and region. Since the reputation score loses its meaning when taken outside of these smaller subsets of schools, I drop it from the list and generate the quality index with the remaining thirteen indicators, uniformly spreading the reputation weight among the remaining five categories. This is of little consequence for the overall rankings, due to the high degree of correlation among the quality indicators.

With the modified USNWR quality measure in hand, I establish the uniform prize ranking by interpreting a school's quality index as a measure of prize value. More precisely, I assume that there is a linear relationship between the USNWR quality index for each school and the utility derived from occupying a seat there. I argue that interpreting the quality index as a meaningful measure of value is sensible for two reasons. First, acquiring information to rank schools and judge one's chances for admission is a costly exercise for an inexperienced high-school student, but USNWR solves this problem by providing large quantities of data on many schools, along with advice on how to interpret the data. Second, the validity of the USNWR rankings is presumably reinforced in the student's mind by the enthusiasm with which so many schools advertise their status in the *America's Best Colleges* rankings. One need not search long through undergraduate admissions web pages to find multiple references to USNWR.

The other relevant data on school characteristics is freshman enrollment, provided by the US Department of Education through the National Center for Education Statistics' Integrated Postsecondary Education Data System tool. For each school in the sample, I obtained a tally of all first-time freshman enrollment (including full-time and part-time), for the following seven racial classifications: White, Black, Hispanic, Asian or Pacific Islander, American Indian or Alaskan Native, non-resident alien, and race unknown. The data representing schools are $\{Q_u, M_u, N_u\}_{u=1}^{U}$, where for the u^{th} school Q_u is the modified USNWR quality index, M_u is the number of seats awarded to minorities, and N_u is the number awarded to non-minorities. There are $1,056,580$ total seats open at all schools; for individual schools the median is 451 seats, the mean is 804.09, and the standard deviation is 934.78.

The above data characterize the sample of prizes and the samples of prizes allocated to each group, given by

$$
\begin{aligned}
\mathbf{P}_{\mathcal{K},K} &= \{p_k\}_{k=1}^{K} = \left\{\{p_{ui}\}_{i=1}^{M_u+N_u}\right\}_{u=1}^{U}, \;\; p_{ui} = Q_u, \\
\mathbf{P}_{\mathcal{M},M} &= \{p_m\}_{m=1}^{M} = \left\{\{p_{uj}\}_{j=1}^{M_u}\right\}_{u=1}^{U}, \;\; p_{uj} = Q_u, \\
\mathbf{P}_{\mathcal{N},N} &= \{p_n\}_{n=1}^{N} = \left\{\{p_{ul}\}_{l=1}^{N_u}\right\}_{u=1}^{U}, \;\; p_{ul} = Q_u.
\end{aligned}
$$

The fact that there are multiple prizes in the data with the same value represents a departure from the theory, but it is a small one given that the largest school in the sample (in terms of enrollment) has a mass of only 6.6×10^{-3}, while the mean and median schools have masses of 7.61×10^{-4} and 4.268×10^{-4}, respectively. Another possible criticism of this approach is that the rankings are dependent upon an arbitrary weighting scheme. Critics sometimes accuse USNWR of manipulating the weights assigned to the different quality indicators, in order to alter the relative standings of elite schools. However, this objection is inconsequential if one takes the larger picture into account. Because of the high degree of correlation among the thirteen quality indicators, the overall prize distribution is remarkably robust to substantial changes in the weighting scheme. While it is possible that the relative rankings of the top ten schools are affected somewhat by such changes, the bigger picture is very stable.

Finally, I have yet to specify the distinction between groups $\mathcal{M}$ and $\mathcal{N}$. I shall define the minority group as the union of the race classes Black, Hispanic, and American Indian or Alaskan Native; non-minorities are all others. This corresponds to the notion that AA policies are targeted toward groups that

Table 3.1: Racial Percent Representation Within Different Academic Tiers

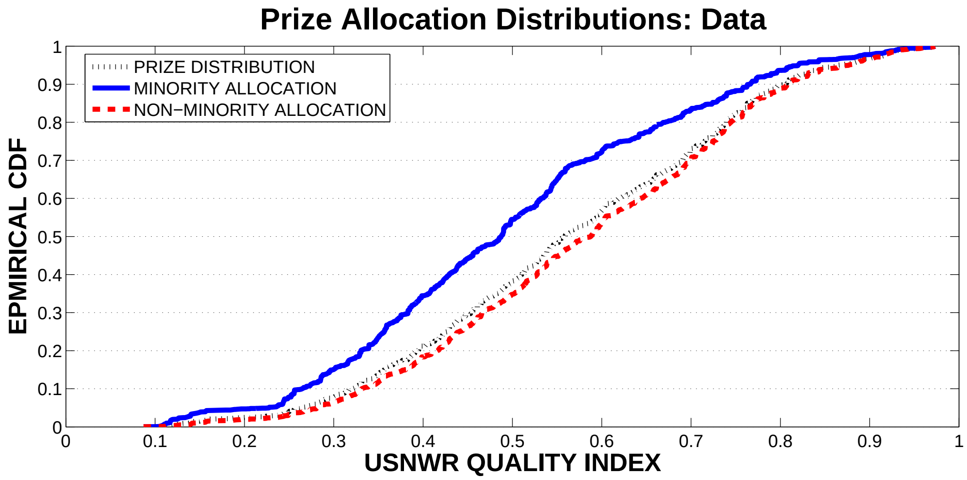
Tier	% of Total Enrollment in Tier	Black 11.2	Hispanic 5.7	American Indian/ Alaskan Native 0.8	White 72	Asian/ Pacific Islander 5.7	$\mathcal{M}$ **17.7**	$\mathcal{N}$ **82.4**
I	35.9	5.6	5	0.5	74.8	9.3	**11.1**	**88.9**
II	26.8	10.1	5.3	0.8	75.1	4.2	**16.1**	**83.9**
III	20.4	14.2	6.2	0.9	70	4.3	**21.3**	**78.7**
IV	16.8	21.3	7.3	1.2	63.4	2.2	**29.7**	**70.3**

are under-represented at elite universities and over-represented at lower-quality schools. Table 3.1 provides a clearer picture of this criterion. As is done in *America's Best Colleges*, I have sorted the schools in descending order of quality index and separated them into four tiers, each containing one quarter of the schools in the sample. Tier I comprises the schools with the highest quality indices, and so on. I compute the mass of each race group within each tier to show representation; I also list the population mass of each race group under its name. The final two columns contain figures for the aggregated race groups.[3] I also list the percentage of students in each tier, as quality quartiles are different from quartiles in terms of enrollment.

Each of the minority race classes is under-represented in the top two tiers and over-represented in the bottom two. For whites it is the opposite. For Asians/ Pacific Islanders the difference is even more pronounced: they are heavily under-represented in every tier except the top. Similar observations hold when the

[3]The table does not list the race unknown and resident alien groups, which is why the first five population masses do not quite sum to one. However, these groups are included in the calculations for group $\mathcal{N}$, so the final two masses do sum to one.

Figure 3.1: Empirical Enrollment Gap



groups are aggregated. The difference in allocations is captured graphically in Figure 3.1. The dotted line is the empirical distribution of $\mathbf{P}_{\mathcal{K},K}$, the lower dashed line is the empirical distribution of $\mathbf{P}_{\mathcal{N},N}$, and the upper solid line is the empirical distribution of $\mathbf{P}_{\mathcal{M},M}$. As Figure 3.1 shows, the non-minority allocation dominates the minority allocation in the first-order sense. For example, roughly one half of minority students attend schools with a quality index of 0.5 or less, while the fraction of non-minorities in that same lowest segment is only one third.

3.1.2 Academic Achievement Data

The remaining data used to estimate the model are college entrance test scores. For the 1996 graduating seniors cohort, I have individual-level data on composite SAT scores, race, and other characteristics for a random sample of 92,514 students, with 73,361 non-minority observations and 19,153 minority observations. SAT scores range between 0 and 1,600 in increments of 10, but I drop the final digit and treat them as ranging from 0 to 160 in increments of 1. Before moving on, it will be necessary to address a preliminary technical concern: the interpretation of an achievement level of "zero."

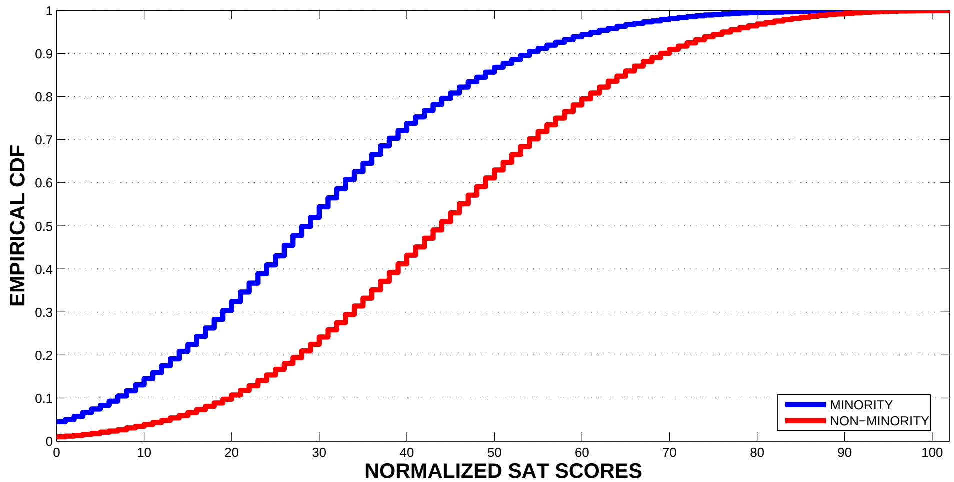
Theoretically, it is possible for a student to score zero by answering all test questions incorrectly. However, such a feat is extremely difficult unless one knows enough to achieve a nearly perfect score: with a probability of virtually one, an uninformed student will get a positive score, due to the multiple-choice format of the test. A reasonable interpretation of a student with an academic achievement of zero is one who engages in random responding to all test questions. As it turns out, a randomized test-score simulation exercise indicates that the SAT score one can expect from such an uninformed student is 58 (See appendix for details).

For the remainder of this thesis, SAT scores will be normalized by subtracting 58 (observed scores below 58 are normalized to zero), and the samples of normalized test scores are denoted by

$$\mathbf{S}_{\mathcal{M},T_{\mathcal{M}}} = \{s_{\mathcal{M},t}\}_{t=1}^{T_{\mathcal{M}}}, \text{ and } \mathbf{S}_{\mathcal{N},T_{\mathcal{N}}} = \{s_{\mathcal{N},t}\}_{t=1}^{T_{\mathcal{N}}},$$

where T_i is the number of grade observations on group $i = \mathcal{M},\ \mathcal{N}$. The academic achievement gap is illustrated in Figure 3.2 where the empirical distributions of normalized SAT scores are displayed. The median for non-minorities is 44, and the median for minorities is 29, which corresponds to the 22$^{\text{nd}}$ percentile for non-minorities. Figure 3.2 also suggests a small mass-point of minorities with scores of zero. This will serve as a partial specification test later on. The theory indicates

Figure 3.2: Test Score Distributions



that a necessary condition for mass-points in minority achievement is $\widetilde{S}(0) > 0$.

3.2 Empirical Model

Recall that the theoretical model outlined in Chapter 2 is strategically equivalent to an all-pay auction. An all-pay auction involves a strategic interaction in which agents compete for a limited resource by incurring some non-recoverable cost before learning the outcome of the game. In the college admissions model, the Board is analogous to an auctioneer, who auctions off a set of heterogeneous prizes according to a pre-determined mechanism. Students are similar to bidders, and the grades they achieve are analogous to sunk payments tendered to the auctioneer, since they cannot recover lost leisure time or disutility incurred by study effort.

Empirically, this is an attractive framework since the econometrics literature concerning auctions has emerged as one of the foremost successes in empirical industrial organization over the past two decades. Since the founding work of Paarsch (1992), auction econometricians have exploited the parsimonious, one-to-one link between observable behavior and private information to recover empirically the distributions over bidder heterogeneity. The key assumption underlying the *structural approach* to estimating these models is that the theoretical equilibrium is consistent with the data-generating process. Said differently, the assumption is that observed behavior was purposefully generated by rational decision makers. This assumption shall form the basis of my estimation strategy as well.

Another landmark paper in empirical auctions is by Guerre et al. (2000) (GPV), who devised an estimation strategy for auctions which is computationally inexpensive and does not rely on distributional assumptions. The main idea of the paper comes from an observation about equilibrium equations in auction models which express bids as functions of private information and the (unobserved) distribution of private information. GPV recognized that these equations could be rearranged so as to express a bidder's private information as a function of his observable behavior and the (observable) distribution over all bidders' behavior.

For the reader's convenience, I have reproduced the relevant equilibrium equations below. Under policy function $\widetilde{S}(s)$, minority achievement is characterized by

$$\begin{aligned}&\mathcal{C}'\left(s;\psi^{ap}_{\mathcal{M}}(s)\right)\\&\quad=-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(\widetilde{S}(s))\right](\psi^{ap}_{\mathcal{N}})'(\widetilde{S}(s))\widetilde{S}'(s)+\mu f_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(s)\right](\psi^{ap}_{\mathcal{M}})'(s)}{f_P\left(F_P^{-1}\left[1-\left((1-\mu)F_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(\widetilde{S}(s))\right]+\mu F_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(s)\right]\right)\right]\right)}.\end{aligned} \tag{3.1}$$

For non-minorities, achievement is given by

$$\begin{aligned}&\mathcal{C}'\left(s;\psi^{ap}_{\mathcal{N}}(s)\right)\\&\quad=-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(s)\right](\psi^{ap}_{\mathcal{N}})'(s)+\mu f_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(\widetilde{S}^{-1}(s))\right](\psi^{ap}_{\mathcal{M}})'(\widetilde{S}^{-1}(s))\frac{d\widetilde{S}^{-1}(s)}{ds}}{f_P\left(F_P^{-1}\left[1-\left((1-\mu)F_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(s)\right]+\mu F_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(\widetilde{S}^{-1}(s))\right]\right)\right]\right)},\end{aligned} \tag{3.2}$$

for $s \geq \widetilde{S}(s)$, and

$$(\gamma_{\mathcal{N}})'(\theta)=-\frac{f_{\mathcal{N}}(\theta)}{f_P\left(F_P^{-1}\left[1-F_{\mathcal{N}}(\theta)\right]\right)\mathcal{C}'\left[\gamma_{\mathcal{N}}(\theta);\theta\right]} \tag{3.3}$$

otherwise.

As I shall shortly demonstrate, these equations can be manipulated according to the GPV method to allow the econometrician to recover a sample of pseudo types implied by observed test scores and the distributions over test scores. However, the form of the policy function $\widetilde{S}$ plays a crucial role in defining those equations and determining how estimation should proceed (recall that equation (3.3) applies only if there is a positive grade boost for a minority score of zero). Therefore, I shall begin by proposing an estimator for $\widetilde{S}$.

3.2.1 Estimating $\widetilde{S}$

The rules of college admissions as set forth by the Board are exogenous to the model which I have defined. I shall assume that some function $\widetilde{S}$, as described in Sections 2.1.2 and 2.1.4, is consistent with the data-generating process. This is an empirically attractive construct because it nests a broad range of policies as special cases, including a quota and a color-blind rule. From the policy-maker's perspective, grades and race are mapped into outcomes via the following reward functions for each group:

$$\begin{aligned} \pi_{\mathcal{M}}(s) &= F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}(\widetilde{S}(s)) + \mu G_{\mathcal{M}}(s)\right],\ s \geq 0, \text{ and} \\ \pi_{\mathcal{N}}(s) &= F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}(s) + \mu G_{\mathcal{M}}(\widetilde{S}^{-1}(s))\right],\ s \geq \widetilde{S}(0). \end{aligned} \tag{3.4}$$

One key observation here allows for identification of the policy function:

$$\pi_{\mathcal{M}}(s) = \pi_{\mathcal{N}}(\widetilde{S}(s)). \tag{3.5}$$

Using this fact, one can recover $\widetilde{S}$ by determining what rule could have produced allocations $\mathbf{P}_{\mathcal{M},M}$ and $\mathbf{P}_{\mathcal{N},N}$ from the observed grade distributions. More specifically, for $r \in (0,1)$ let $s_{\mathcal{N}}(r) \equiv G_{\mathcal{N}}^{-1}(r)$ denote the r^{th} quantile in the non-minority grade distribution. For minorities, let $r_{\mathcal{M}}(r) \equiv G_{\mathcal{M}}\left(\widetilde{S}^{-1}(s_{\mathcal{N}}(r))\right)$ denote the quantile rank of the *de-subsidized* version of $s_{\mathcal{N}}(r)$ within the minority grade distribution. By Assumption 2.1.6 and by observation (3.5), it immediately follows that

$$
\begin{aligned}
F_{P_{\mathcal{M}}}^{-1}\left[r_{\mathcal{M}}(r)\right] &= F_{P_{\mathcal{N}}}^{-1}(r) \\
\Rightarrow \quad G_M\left(\widetilde{S}^{-1}\left[s_{\mathcal{N}}(r)\right]\right) &= F_{P_{\mathcal{M}}}\left[F_{P_{\mathcal{N}}}^{-1}(r)\right] \\
\Rightarrow \quad G_{\mathcal{N}}^{-1}(r) &= \widetilde{S}\left[G_M^{-1}\left(F_{P_{\mathcal{M}}}\left[F_{P_{\mathcal{N}}}^{-1}(r)\right]\right)\right],
\end{aligned}
\tag{3.6}
$$

where the second and third lines follow from substituting and from monotonicity. Equation (3.6) above provides a moment condition that forms the basis of a simple policy function estimator. For a given specification of $\widetilde{S}$ one can choose a set of quantile ranks and pick the parameters of the policy function so as to minimize the distance between the left-hand side and right-hand side.

More formally, assume that

$$
\widetilde{S}(s) = \sum_{i=0}^{I} \Delta_i s^i,
$$

(*i.e.*, the true policy function is linear in parameters) and estimate the parameter vector $\mathbf{\Delta} = (\Delta_0, \Delta_1, \ldots, \Delta_I)$ semiparametrically by the generalized method of moments as follows.[4]

Step 1: Choose the largest set of quantile ranks that can be gleaned from the data, or $\mathbf{r} = \{r_u\}_{u=1}^{U}$, where $r_u = \widehat{F}_P^{-1}(p_u),\ u = 1, \ldots, U$, and $\widehat{F}_P$ is the Kaplan-Meier

[4]The assumption that $\widetilde{S}$ is a polynomial of order I need not be a strong restriction on the empirical model. By Assumption 2.1.7, the Weierstrauss Approximation Theorem implies that the true policy function can be expressed as an infinite polynomial series $\widetilde{S}(s) = \sum_{i=0}^{\infty} \Delta_i s^i$. Alternatively, one could choose a truncation point I to grow at a rate no faster than the data; this would eventually allow for recovery of the true, unrestricted $\widetilde{S}$ as the number of available sample moments grows.

empirical distribution function for the set of prizes $P_{\mathcal{K},K}$.

Step 2: For $I \leq U$, let $\widetilde{S}(s) = \sum_{i=0}^{\infty} \Delta_i s^i$, and define

$$\widehat{\boldsymbol{\Delta}} = \arg\min \left\{ \sum_{u=1}^{U} \left[\widehat{G}_{\mathcal{N}}^{-1}(r_u) - \widehat{\widetilde{S}} \left(\widehat{G}_{M}^{-1} \left[\widehat{F}_{P_{\mathcal{M}}} \left(\widehat{F}_{P_{\mathcal{N}}}^{-1}[r_u] \right) \right] ; \boldsymbol{\Delta} \right) \right]^2 \right\},$$

where $\widehat{G}_{\mathcal{M}}$, $\widehat{G}_{\mathcal{N}}$, $\widehat{F}_{P_{\mathcal{M}}}$, and $\widehat{F}_{P_{\mathcal{N}}}$ are the Kaplan-Meier empirical distributions of $\mathbf{S}_{\mathcal{M},T_{\mathcal{M}}}$, $\mathbf{S}_{\mathcal{N},T_{\mathcal{N}}}$, $\mathbf{P}_{\mathcal{M},M}$, and $\mathbf{P}_{\mathcal{N},N}$, respectively.

Step 3: Using the standard errors from Step 2, test $H_0 : \Delta_0 = 0$. If H_0 is rejected, remove from $\mathbf{r}$ any r_u such that $H_0^* : \Delta_0 \geq \widehat{G}_{\mathcal{N}}^{-1}(r_u)$ is rejected and repeat Step 2. ■

Step 3 in the above process comes from the fact that Step 2 is defined by equation (3.5), which is only valid for $s \geq \widetilde{S}(0)$.

There are two senses in which this estimator is semiparametric. First, there are no assumptions imposed on the form of the distributions of grades and prizes. Empirical CDF inverses can be recovered via "nearest neighbor" interpolation of the Kaplan-Meier empirical distributions. Second, the polynomial specification allows for the order of the policy function to be chosen as high as desired, given enough data. In that sense, the above proposal could be classified under the broad umbrella of estimation by the *method of sieves*.[5] Said differently, this flexible form for $\widetilde{S}$ allows for a simple estimation procedure within a finite parameter space,

[5] A sieve is a sequence of nested, finite-dimensional parameter spaces whose limit contains the true parameter space. For an in-depth discussion on estimation by the method of sieves, see Chen (2005).

while virtually avoiding imposition of *a priori* restrictions on the behavior of the grade markup rule. If I is chosen to be large enough so that numerical stability is an issue, $\widetilde{S}$ could alternatively be specified as a weighted sum of orthogonal basis polynomials, rather than the standard polynomial basis.

Another advantage is that minimization in Step 2 is greatly simplified by the polynomial specification of $\widetilde{S}$, since $\widehat{\mathbf{\Delta}}$ can be found by simply regressing $\mathbf{Y} = \left(\widehat{G}_{\mathcal{N}}^{-1}(r_1), \ldots, \widehat{G}_{\mathcal{N}}^{-1}(r_U)\right)^{\top}$ on on the matrix of explanatory variables,

$$\mathbf{X} = \begin{pmatrix} 1 & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_1)\right)\right] & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_1)\right)\right]^2 & \ldots & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_1)\right)\right]^I \\ 1 & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_2)\right)\right] & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_2)\right)\right]^2 & \ldots & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_2)\right)\right]^I \\ \vdots & \vdots & \ddots & \vdots & \\ 1 & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_U)\right)\right] & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_U)\right)\right]^2 & & \widehat{G}_{\mathcal{M}}^{-1}\left[\widehat{F}_{P_{\mathcal{M}}}\left(\widehat{F}_{P_{\mathcal{N}}}^{-1}(r_U)\right)\right]^I \end{pmatrix}.$$

This implies the familiar estimator $\widehat{\mathbf{\Delta}} = (\mathbf{X}^{\top}\mathbf{X})^{-1}\mathbf{X}^{\top}\mathbf{Y}$, along with the familiar variance-covariance matrix for linear regression models.[6] Using well-known results, it follows that the above GMM estimator is consistent, asymptotically normal, and converges at rate $\sqrt{U}$.

3.2.2 Estimating Private Types

I now turn to the primary task of estimating the distributions over heterogeneity among competing students. Throughout this section, I shall consider

[6]For improved efficiency, one could incorporate an optimal weighting matrix $\mathbf{W}$ into Step 2 and minimize

$$(\mathbf{Y} - \mathbf{X\Delta})\mathbf{W}(\mathbf{Y} - \mathbf{X\Delta})^{\top}$$

instead. Using the current data set it will become clear later that there is little to be gained in this case.

the case where $\widetilde{S}(0) = \Delta_0 > 0$ since estimation in the opposite (simpler) case is similar, but with fewer caveats. Recall from Section 2.3.3 that in this case non-minority achievement is given by a piecewise differential equation. For minorities with equilibrium grades $s \in [0, \Delta_0]$, equilibrium achievement is characterized by differential equation (3.3). By monotonicity of the equilibrium, I have the following two identities,

$$G_{\mathcal{N}}(s) = 1 - F_{\mathcal{N}}\left[\psi_{\mathcal{N}}(s)\right], \text{ and}$$

$$g_{\mathcal{N}}(s) = -f_{\mathcal{N}}\left[\psi_{\mathcal{N}}(s)\right]\psi'_{\mathcal{N}}(s) = -f_{\mathcal{N}}(\theta)/\gamma'_{\mathcal{N}}(\theta).$$

Using this, I can re-write equation (3.3) to get the following

$$C'(s;\theta) = \frac{g_{\mathcal{N}}(s)}{f_P\left(F_P^{-1}\left[G_{\mathcal{N}}(s)\right]\right)} = \overline{\xi}_{\mathcal{N}}(s). \tag{3.7}$$

For non-minorities submitting grades above Δ_0, something similar can be done using differential equation (3.2). Recall that for a random variable S distributed according to $F(s)$, the distribution of $Z = \zeta(S)$ is simply $F(\zeta^{-1}(Z))$. Minority grades are distributed

$$S_{\mathcal{M}} \sim G_{\mathcal{M}}(s) = 1 - F_{\mathcal{M}}\left[\psi_{\mathcal{M}}(s)\right],$$

from which it follows that subsidized minority grades are distributed according to

$$\widetilde{S}(S_{\mathcal{M}}) \sim \widetilde{G}_{\mathcal{M}}(s) = G_{\mathcal{M}}\left[\widetilde{S}^{-1}(s)\right] = 1 - F_{\mathcal{M}}\left(\psi_{\mathcal{M}}\left[\widetilde{S}(s)\right]\right).$$

Note that $\widetilde{G}_{\mathcal{M}}$ and its derivative show up in equation (3.2), along with $G_{\mathcal{N}}$ and its derivative. Therefore, the differential equation for non-minority achievement above grade level Δ_0 can be re-written as

$$\mathcal{C}'(s;\theta) = \frac{(1-\mu)g_{\mathcal{N}}(s) + \mu\widetilde{g}_{\mathcal{M}}(s)}{f_P\left(F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}(s) + \mu\widetilde{G}_{\mathcal{M}}(s)\right]\right)} = \underline{\xi}_{\mathcal{N}}(s),\ s \geq \Delta_1. \tag{3.8}$$

Similarly, for minority achievement (conditional on positive output), equation (3.1) can be re-written as

$$\mathcal{C}'(s;\theta) = \frac{(1-\mu)\widetilde{g}_{\mathcal{N}}(s) + \mu g_{\mathcal{M}}(s)}{f_P\left(F_P^{-1}\left[(1-\mu)\widetilde{G}_{\mathcal{N}}(s) + \mu G_{\mathcal{M}}(s)\right]\right)} = \underline{\xi}_{\mathcal{M}}(s),\ s \geq 0, \tag{3.9}$$

where $\widetilde{G}_{\mathcal{N}}(s) = G_{\mathcal{N}}\left(\widetilde{S}(s)\right)$ is the distribution of *de-subsidized* non-minority test scores and $\widetilde{g}_{\mathcal{N}}$ is its derivative. Equations (3.7), (3.8), and (3.9) provide a simple basis for an estimator of the private cost distributions, as they express a student's unobservable private cost type in terms of objects which are all observable to the econometrician. This will allow for recovery of sample of *pseudo-private costs* for each group, which in turn facilitate estimation of the underlying distributions.

The advantages of this method are two-fold. First, the resulting estimation procedure is computationally inexpensive, since equilibrium equations need not be repeatedly solved as in, say a maximum likelihood routine. Second, estimation requires no *a priori* assumptions on the form of the distributions $F_{\mathcal{M}}$ and $F_{\mathcal{N}}$. However, there is one drawback: without parametric assumptions, it is impossible to identify private cost types for the potential mass point of minorities whose equilibrium achievement is zero. Under circumstances one might consider to be reasonable, this concern will only apply to a small portion of the sample, but it must be addressed. The policy function estimate and equations (2.10) and (3.7) can be used to recover the minority boundary condition

$$\theta^* = \inf\{\theta : \gamma_{\mathcal{M}}(\theta) = 0\}$$

by computing the solution to

$$\mathcal{C}'(0;\theta^*) = \mathcal{C}'(\Delta_0;\theta_{\Delta_0})\widetilde{S}'(0), \tag{3.10}$$

where θ_{Δ_0} solves

$$\mathcal{C}'(\Delta_0;\theta_{\Delta_0}) = \overline{\xi}_{\mathcal{N}}(\Delta_0).$$

By comparing the resulting estimate of θ^* with the estimate of $\overline{\theta}$ recovered from equation (3.7) (where $s = 0$), if the interval $[\theta^*, \overline{\theta}]$ has a non-empty interior, then the empirical model implies a mass point, and minority private costs corresponding to a grade of zero are non-identified.

One way of dealing with the non-identification problem is to parameterize the upper tail of the distribution. If the upper tail is sparsely populated, a reasonable option would simply be to spread the mass of minorities uniformly over $[\theta^*, \overline{\theta}]$.[7] With this modification, the equations above allow for recovery of a

[7]The specification error introduced by this parameterization can be assessed by comparing the results with alternative estimates obtained by mapping all zero-score observations for minorities onto either θ^* or $\overline{\theta}$.

sample of *pseudo-private costs*

$$\widehat{\boldsymbol{\Theta}}_{\mathcal{N},T_{\mathcal{N}}} = \{\widehat{\theta}_{\mathcal{N},t}\}_{t=1}^{T_{\mathcal{N}}} \text{ and } \widehat{\boldsymbol{\Theta}}_{\mathcal{M},T_{\mathcal{M}}} = \{\widehat{\theta}_{\mathcal{M},t}\}_{t=1}^{T_{\mathcal{M}}}$$

corresponding to each SAT score observation for minorities and non-minorities, respectively. From these, the underlying private cost distributions can be recovered, given some specification of the cost function $\mathcal{C}$. This leads to the next section.

3.2.3 Cost Function Estimation

Another advantage to the GPV method is that it provides for a partial specification test of the theoretical model. In any pure-strategy equilibrium, the theory requires that mappings (3.7), (3.8), and (3.9) must reflect a monotonic decreasing relation between private costs and academic achievement in order for the FOC to constitute an equilibrium. Given some specification of costs $\mathcal{C}$, if the data do not produce monotone decreasing mappings, the model is rejected on the grounds that the data are not consistent with a monotone equilibrium in the specified model. To begin, one might be inclined to consider a simple linear specification, say $\mathcal{C}(s;\theta) = \theta s$, as this would avoid introducing additional parameters into the model. However, this specification of costs leads to a non-monotone empirical mapping being recovered from equations (3.7), (3.8), and (3.9). As it turns out, there must be curvature in students' utility in order for the model to be consistent with the data.

I assume that achievement costs take the form

$$\mathcal{C}(s;\theta) = \theta \exp(\alpha s), \alpha > 0.$$

This choice is motivated by several factors, the most important being that it satisfies the regularity conditions required for existence of a monotonic, pure-strategy equilibrium (see assumption 2.1.4 in Section 2.1.4). Aside from that, it has other attractive properties as well. Note that the cost of submitting a grade of zero is strictly positive. This corresponds to the notion that students must forego some minimum cost to graduate high school as a prerequisite for participation in the college admissions market. As it turns out, the above cost function allows for a tight fit between the empirical model and the data (at the optimal value of α).

With this specification of private costs, equations (3.7), (3.8), and(3.9) become

$$\theta = \frac{g_{\mathcal{N}}(s)}{f_P\left(F_P^{-1}\left[G_{\mathcal{N}}(s)\right]\right)\alpha \exp(\alpha s)} = \frac{\overline{\xi}_{\mathcal{N}}(s)}{\alpha \exp(\alpha s)},\ s \leq \Delta_1, \tag{3.11}$$

$$\theta = \frac{(1-\mu)g_{\mathcal{N}}(s) + \mu \widetilde{g}_{\mathcal{M}}(s)}{f_P\left(F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}(s) + \mu \widetilde{G}_{\mathcal{M}}(s)\right]\right)\alpha \exp(\alpha s)} = \frac{\underline{\xi}_{\mathcal{N}}(s)}{\alpha \exp(\alpha s)},\ s > \Delta_1, \text{ and} \tag{3.12}$$

$$\theta = \frac{(1-\mu)\widetilde{g}_{\mathcal{N}}(s) + \mu g_{\mathcal{M}}(s)}{f_P\left(F_P^{-1}\left[(1-\mu)\widetilde{G}_{\mathcal{N}}(s) + \mu G_{\mathcal{M}}(s)\right]\right)\alpha \exp(\alpha s)} = \frac{\xi_{\mathcal{M}}(s)}{\alpha \exp(\alpha s)},\ s \geq 0, \tag{3.13}$$

respectively. The zero surplus condition and equation (3.11) imply a relation between the curvature parameter α, and the value of the lowest prize, $\underline{p}$:

$$\begin{aligned} \mathcal{C}(0;\overline{\theta}) = \overline{\theta} = \frac{\overline{\xi}_{\mathcal{N}}(0)}{\alpha} = \underline{p} \\ \Rightarrow \alpha = \frac{\overline{\xi}_{\mathcal{N}}(0)}{\underline{p}}. \end{aligned} \tag{3.14}$$

As discussed in Section 2.2.1, the zero surplus condition is analogous to broader market forces (not included in the model) that determine participation in the higher-education market. If students have a choice between going to college or some outside option, then the marginal college candidate will be indifferent between going to college and opting out. If prize values represent the additional utility from going to college over the outside option, then the result is equation (3.14). This condition places structure on the relative link between the utility of consumption and the disutility of work.

In related work, Guerre et al. (2009) and Campo et al. (2009) extended the GPV method to first-price auctions where agents' utility functions display some form of curvature. They show that such models are unidentified without imposing additional structure, due to the weak restrictions that the game-theoretic model places on observed bids. In fact, simply parameterizing either utility or the distribution of private information alone does not necessarily provide identification. Fortunately, the prize distribution in the college admissions model provides some additional structure, so here it is sufficient to parameterize just the cost function. Campo et al. (2009) used information on heterogeneity across auctioned objects to identify the utility function. This is conceptually similar to the role that the sample of prizes $\mathbf{P}_{\mathcal{K},K}$ plays, only instead of dealing with many (single-unit) auctions for heterogeneous items, I have a single "auction" with many heterogeneous objects.

Proposition 3.2.1. *If the cost function is restricted to the parametric class* $\mathcal{C}(s;\theta) = \theta \exp(\alpha s), \quad \alpha > 0$*, then there exists a unique curvature parameter* α *and a unique set of cost distributions* $F_{\mathcal{M}}$ *and* $F_{\mathcal{N}}$ *which rationalize a given set of grade distributions,* $G_{\mathcal{M}}$ *and* $G_{\mathcal{N}}$*, a policy function* $\widetilde{S}$*, and a prize distribution* $F_{\mathcal{P}}$*.*

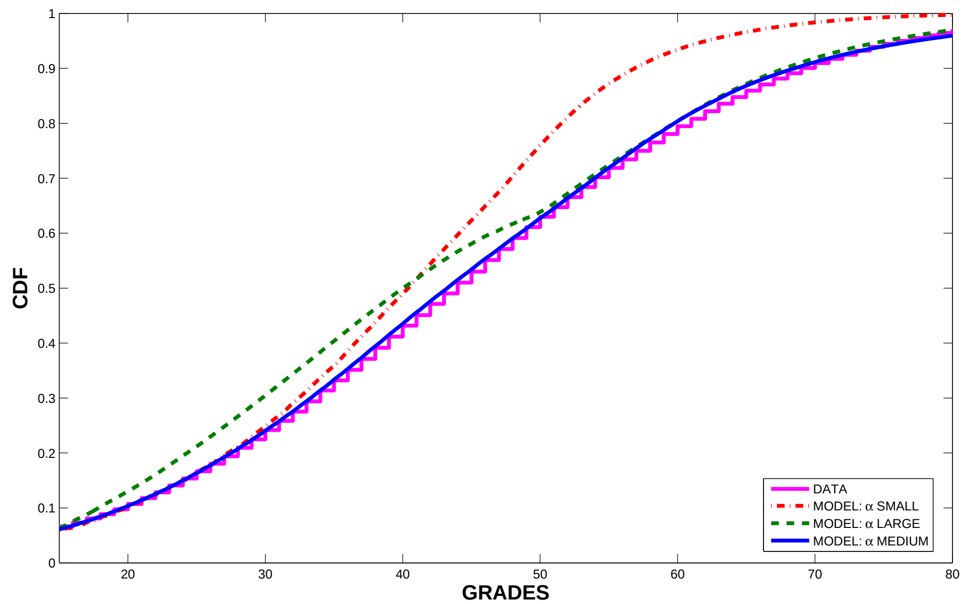
Heuristic Proof: Intuitively, the role of α and θ is to reconcile the prize utilities with the levels of observed achievement. At each grade quantile s, α must be such that the prize value allocated to a student with a score of s justifies the resulting cost $\mathcal{C}\left[s;\theta(s;\alpha)\right]$, where $\theta(s;\alpha)$ is defined by the inverse equilibrium equations (3.11), (3.12), and (3.13).

The model's ability to reconcile the prize utilities with observed behavior hinges on the cost curvature parameter through the term

$$\alpha \exp(\alpha s) \tag{3.15}$$

in the denominators of the three GPV equations. Specifically, suppose one were to fix a value of α, recover the associated GPV estimates of $F_{\mathcal{M}}$ and $F_{\mathcal{N}}$, and then compute the implied model-generated bid distributions. If α is too high, then the exponent of (3.15) becomes very important and the model cannot produce the high grades observed in the data because prize values are not enough to compensate for the cost of achievement. In other words, the marginal rate of

Figure 3.3: Rationalizing a Grade Distribution



substitution of prize value for work is too low to rationalize a fixed grade distribution from a fixed prize distribution. If α is very small, then the exponent is unimportant for low grades (because $\exp(\alpha s)$ is close to 1), and the effect of the coefficient in equation (3.15) dominates. Costs become nearly linear for low α, and when this happens the behavioral separation in the model diminishes among low-performing students—in fact, equations (3.11), (3.12), and (3.13) eventually become non-monotonic just as under linear costs—and the observed low-score frequencies cannot be rationalized. However, in the middle there is a balance between the two extremes and the whole empirical grade distribution can be rationalized. Figure 3.3 provides an illustration. This concept motivates the proposed estimator below. ■

The estimator I propose for the utility function parameter is motivated by the fact that the model's ability to rationalize the empirical grade distributions $\widehat{G}_i,\ i = \mathcal{M}, \mathcal{N}$ vanishes as α approaches the two limiting extremes of 0 and ∞. For fixed α, the restricted GPV estimates of the cost distributions can be recovered from equations (3.11), (3.12), and (3.13). These and the equilibrium equations from Section 2.3.3 imply a set of model-generated grade distributions, $\ddot{G}_i,\ i = \mathcal{M}, \mathcal{N}$. The goal in choosing α, as with any parametric estimation routine, is to minimize the distance between the data and the model. I chose the Euclidean distance metric, which leads to the following nonlinear least squares (NLLS) estimator for the utility parameter:

$$\widehat{\alpha} = \arg\min \left\{ \sum_{j=1}^{J} \left[\ddot{G}_{\mathcal{M}}(s_j;\alpha) - \widehat{G}_{\mathcal{M}}(s_j) \right]^2 + \left[\ddot{G}_{\mathcal{N}}(s_j;\alpha) - \widehat{G}_{\mathcal{N}}(s_j) \right]^2 \right\}, \tag{3.16}$$

where $\mathbf{S} = \{s_1, s_2, \ldots, s_J\}$ is the set of all grades observed in the data, $\ddot{G}_i(\cdot;\alpha)$ is the model-generated grade distribution for group i given α, and $\widehat{G}_i$ is the Kaplan-Meier empirical CDF.[8]

While this is an intuitive criterion function, optimization is complicated by the fact that the derivatives $d^n\ddot{G}_i/d\alpha^n,\ i = \mathcal{M}, \mathcal{N},\ n = 1, 2, \ldots$ of the model-implied grade distributions are not readily available due to a lack of closed-form solutions for the equilibrium equations in Section 2.3.3. The lack of closed-form solutions also necessitates repeated solution of the model equations during opti-

[8]An alternative criterion one could adopt is to choose $\widehat{\alpha}$ so as to minimize the sup-norm distance between $\ddot{G}_i$ and $\widehat{G}_i$, for $i = \mathcal{M}, \mathcal{N}$.

mization for each guess of the cost curvature parameter. To address these problems, I use the *golden search method*, a derivative-free optimization algorithm.

Golden search begins with an initial guess on the search region, $[\underline{\alpha}, \overline{\alpha}]$ and evaluation of the objective function at two interior points $\alpha < \alpha'$. After comparing the functional values, the sub-optimal interior point is used to replace the nearest endpoint of the search region, and the process is repeated until the length of the search region collapses to a pre-specified tolerance, τ. The algorithm has some unique and attractive characteristics because the interior points are chosen as

$$\alpha = \varphi\underline{\alpha} + (1 - \varphi)\overline{\alpha}, \text{ and } \alpha' = (1 - \varphi)\underline{\alpha} + \varphi\overline{\alpha},$$

where $\varphi = (\sqrt{5} - 1)/2$ is the inverse of the golden ratio, a number famously venerated by ancient Greek philosophers (hence, the name "golden search"). By choosing the interior points in this way, with each successive iteration one of the interior points is carried over from the previous iteration, necessitating only one new objective function evaluation. Moreover, at each step the length of the search region contracts by a factor of exactly φ ($\approx$ .62), meaning that convergence obtains in a known number of steps equal to $[\log(\tau) - \log(\overline{\alpha} - \underline{\alpha})] / \log(\varphi)$.

Although the proposed semiparametric utility function estimator requires repeated computation of the model equilibrium, this last property of golden search gives the researcher an *a priori* idea of the magnitude of the problem. As for estimation of α with the current data set, it is easy to identify an appropriate search region of length less than 10, so convergence obtains in roughly 33-43 iterations for $\tau \in [10^{-8}, 10^{-6}]$. Of course, there are the usual problems of locating the global minimum as opposed to local minima, but this is not unique to derivative-free optimization methods. The final point left to discuss is the nonparametric density estimates that will be used in equations (3.11), (3.12), and (3.13). This is covered in the next section.

3.2.4 Boundary-Corrected Kernel Smoothing

Established asymptotic theory on GPV-type estimators is based on obtaining kernel-smoothed density estimates, which are known to exhibit excessive variance and bias near the extremes of the sample. GPV-type econometric routines typically address this issue by trimming elements from the sample of pseudo-private information based on kernel density estimates close to the extremes of the sample. However, addressing the problem in this way would cause problems here for several reasons. First, boundary conditions are needed for computation of the model equilibrium; second, the relation between α and $\underline{p}$ is pinned down precisely at the boundary—see equation (3.14)—and third, the boundary of the minority grade distribution plays a role in estimating interior values of non-minority private costs (see equation (3.12)). Fortunately, there is a well-established set of tools from the statistics literature for improving the performance of kernel density estimators when the underlying random variables live on a bounded support.

Let f denote a density function with support $[a, b]$ and consider nonparametric estimation based on a random sample $\{Z_1, Z_2, \dots Z_T\}$ using the standard kernel density estimator $\widehat{f}(x) = \frac{1}{Th} \sum_{t=1}^{T} \kappa \left(\frac{x - Z_t}{h} \right)$, where κ is a unimodal density function and h is a bandwidth parameter strategically chosen to approach zero at a rate no faster than $\frac{1}{T}$. It is well-known that on the set $[a+h, b-h]$ this estimator has bias of order $O(h^2)$, but on the complement of this set, the bias is $O(h)$. In particular, the standard method tends to underestimate density values on the set $[a, b] \setminus [a+h, b-h]$ for an intuitive reason: since it cannot detect data outside the boundaries of the support, it penalizes the density estimate near those boundaries. This is commonly referred to as the *boundary effect*.

Various methods have been developed to address the problem.[9] Two common coping techniques are known as the *reflection method* and the *transformation method*. The former is a simple technique in which the data are "reflected" outside the support near the boundaries, resulting in the following estimator: $\widehat{f}(x) = \frac{1}{Th}\sum_{t=1}^{T}\left\{\kappa\left(\frac{x-Z_t}{h}\right) + \kappa\left(\frac{x+Z_t}{h}\right)\right\}$. Transformation methods map the data onto an unbounded support via $\lambda : [a, b] \to \mathbb{R}$, resulting in $\widehat{f}(x) = \frac{1}{Th}\sum_{t=1}^{T}\kappa\left(\frac{x-\lambda(Z_t)}{h}\right)$.

While these methods reduce the bias due to boundary effects, they come at a cost of increased variance in the density estimate. However, Karunamuni and Zhang (2008) (KZ), overcome this problem by constructing a kernel estimator that is a hybrid of the reflection and transformation techniques. Formally, the

[9]See Karunamuni and Alberts (2005) for a more in-depth discussion of the various correction methods, as well as for a comparison of their performance.

boundary-corrected KZ density estimator is given by

$$
\begin{aligned}
\widehat{f}_T(x) &= \frac{1}{Th}\sum_{t=1}^{T}\left\{\kappa\left(\frac{x-Z_t}{h}\right)+\kappa\left(\frac{x+\widehat{\lambda}(Z_t)}{h}\right)\right\},\\
\widehat{\lambda}(y) &= y+\widehat{d}y^2+A\widehat{d}^2y^3,\\
\widehat{d} &= \log\left(f_T(h_1)\right)-log\left(f_T(0)\right),\\
f_T(h_1) &= f_T^*(h_1)+\frac{1}{T^2},\\
f_T(0) &= \max\left\{f_T^*(0),\frac{1}{T^2}\right\},\\
f_T^*(h_1) &= \frac{1}{Th_1}\sum_{t=1}^{T}\kappa\left(\frac{h_1-Z_t}{h_1}\right),\\
f_T^*(0) &= \frac{1}{Th_0}\sum_{t=1}^{T}\kappa_0\left(\frac{-Z_t}{h_0}\right),
\end{aligned}
\tag{3.17}
$$

where κ is a symmetric kernel with support $[-1,1]$; $A > \frac{1}{3}$; $h_1 = o(h)$; $\kappa_0 : [-1,0] \to \mathbb{R}$ is an optimal boundary kernel, given by $\kappa_0(y) = 6+18y+12y^2$; and $h_0 = \beta h_1$, with

$$
\beta = \left\{\frac{\left[\int_{-1}^{1}x^2\kappa(x)dx\right]^2\left[\int_{-1}^{0}\kappa_0^2(x)dx\right]}{\left[\int_{-1}^{0}x^2\kappa_0(x)dx\right]^2\left[\int_{-1}^{1}\kappa^2(x)dx\right]}\right\}^{1/5}.
$$

Interestingly, this estimator reduces to the standard kernel density estimator on the interior of the set $[a,b]\setminus[a+h,b-h]$. Most importantly, KZ show that if f is strictly positive and has a continuous second derivative within a neighborhood of the boundary, then $\widehat{f}_T$ as defined above has $O(h^2)$ bias and $O(\frac{1}{Th})$ variance everywhere on the support.[10]

---

[10]The assumption that the true density is strictly positive at the boundary is not neces-

The above boundary correction technique applies to the current empirical model of college admissions. A key assumption of the theory is that prizes and private cost types live on compact intervals, which in turn leads to bounded achievement. However, one can reasonably argue that these assumptions correspond to natural characteristics of the data. In the case of achievement, a student cannot put forth negative effort, so a grade of zero naturally forms a lower bound on the support of grades. By design, there is also a maximum attainable SAT score.[11] As for the prize distribution, an argument similar to the logic behind Assumption 2.2.2 establishes bounds on the support. Once again, the set of realized prizes is assumed to be the result of a broader market equilibrium including entry and exit of firms supplying post-secondary education and unskilled jobs to high school graduates. Therefore, $[\underline{p}, \overline{p}] = [\min\{\mathbf{P}_{\mathcal{K},K}\}, \max\{\mathbf{P}_{\mathcal{K},K}\}]$, and the upper and lower bounds arise naturally from the exogenous private cost distributions (including high-cost types who opt out of higher education) and the interaction between employment suppliers and universities.

I can now summarize the structural estimator of the college admissions

---

sary for boundary correction in general, just for the hybrid KZ estimator. If it is known *a priori* that the density attains a value of zero at the boundary, a suitable replacement with similar performance is the locally adaptive—meaning that the bandwidth is adjusted as domain points get closer to the boundary—kernel density estimator of Karunamuni and Alberts (2005). The cost associated with this alternative is that it is more difficult to implement.

[11]SAT scores are actually a proxy for overall academic achievement, so assuming that the maximal score forms a natural upper bound is an approximation to the truth. However, the data suggest it is a reasonable approximation: the number of students who manage a perfect SAT score make up less than three thousandths of a percent of the overall population.

model in the following three-step process:

Step 1: Obtain the following preliminary estimates

(i) the population demographic parameter $\widehat{\mu} = \frac{\sum_{u=1}^{U} M_u}{\sum_{u=1}^{U}(M_u+N_u)}$;

(ii) $\widetilde{S}$ as outlined in Section 3.2.1;

(iii) the boundary-corrected KZ prize density $\widehat{f}_P(p)$, and its integral, $\widehat{F}_P(p)$ from the sample of prizes;

(iv) the boundary-corrected KZ grade densities $\widehat{g}_{\mathcal{M}}(s|S > 0)$ and $\widehat{g}_{\mathcal{N}}(s)$, and the corresponding integrals, $\widehat{G}_{\mathcal{M}}$ and $\widehat{G}_{\mathcal{N}}$ from the samples of SAT scores.

Step 2: (i) For a given guess of α, estimate samples of pseudo-private costs $\widehat{\mathbf{\Theta}}_{\mathcal{N},T_{\mathcal{N}}}$ and $\widehat{\mathbf{\Theta}}_{\mathcal{M},T_{\mathcal{M}}}$ from equations (3.11), (3.12), (3.13), and (3.10), where the grade and prize distributions are substituted for the estimates from Step 1. In the event of a mass point at a score of zero for minorities, map minority scores of zero uniformly onto an evenly-spaced grid on $[\theta^*, \overline{\theta}]$, where the spacing between grid points is smaller than h, the bandwidth parameter for minority private costs, conditional on positive achievement.

(ii) Given part (i) of Step 2, estimate the study-cost parameter $\widehat{\alpha}$ via NLLS as outlined in Section 3.2.3. ■

Step 3: Obtain boundary-corrected KZ density and distribution estimates for private costs, $\widehat{f}_{\mathcal{M}}$ and $\widehat{f}_{\mathcal{N}}$, using the samples of pseudo types from Step 2.

3.2.5 Asymptotic Properties

In a related setting, Campo, Guerre, Perrigne, and Vuong (Campo et al., 2009, henceforth, CGPV) develop a similar semiparametric estimator of a first-price auction model where competitors' utility exhibits curvature. They parameterize bidder utility and use variation in observable auction characteristics to estimate it via a NLLS routine. After that, they recover type distribution estimates similarly as in GPV. CGPV prove asymptotic normality and show that the utility curvature estimator converges at rate $K^{(R+1)/(2R+3)}$, where R is the number of continuous derivatives of the (true) type distributions. Type distribution estimates converge at the optimal rate for kernel-based estimators.

The estimators I have proposed for α, $F_{\mathcal{M}}$ and $F_{\mathcal{N}}$ are conceptually the same as CGPV. Like them, I exploit variation in objects being auctioned to identify utility curvature, which I estimate via NLLS. Moreover, my type distribution estimates are conditioned on the utility curvature parameter, just as in CGPV. $\widehat{F}_{\mathcal{M}}$ and $\widehat{F}_{\mathcal{N}}$ are otherwise nonparametric and estimated via a two-step kernel smoothing procedure which involves analytically inverting the equilibrium equations from the theoretical model.

Henceforth, discussion pertaining to estimates and inference shall assume the asymptotic theory proven by CGPV. The standard error I report for $\widehat{\alpha}$ shall reflect the conservative assumption that type distributions have a single continuous derivative, or $R = 1$. This implies that the rate of convergence is $K^{2/5}$.

As a precaution, I also perform a bootstrap exercise to evaluate the role of sampling variability for the estimates (see the appendix for details and diagrams). The histogram of bootstrapped estimates for α appears fairly normal, with variance slightly smaller than the estimate I get by assuming $R = 1$. Moreover, ninety-five percent confidence bands for the type distributions are fairly tight (see appendix), suggesting that the large sample size eliminates concerns about sampling variability. Effectively, estimation amounts to an exercise in curve fitting. This will simplify the discussion on the counterfactuals considerably, as one can reasonably focus on policy changes under the estimated distributions while ignoring inferential concerns.

3.2.6 Practical Issues

Choice of A is generally inconsequential, as long as $A > \frac{1}{3}$, so I have selected $A = .55$ as suggested by KZ. By definition of the boundary-corrected estimator, the Gaussian kernel is not an option, so I have chosen the biweight kernel (also known as the quartic kernel) $\kappa(x) = \frac{15}{16}\left(1 - x^2\right)^2 \mathbb{I}[-1 \leq x \leq 1]$, where $\mathbb{I}$ is an indicator function. As proposed by KZ, I have selected bandwidth h via Silverman's optimal global bandwidth rule

$$h = \left\{ \frac{\int_{-1}^{1} \kappa^2(x) dx}{\left(\int_{-1}^{1} x^2 \kappa(x) dx\right)^2 \int_{[0,a]} \left(\frac{d^2 f}{dx^2}\right)^2 dx} \right\}^{1/5} T^{-\frac{1}{5}},$$

where the second term in the denominator is substituted by

$$\int_{[0,a]} \left(\frac{d^2 f}{dx^2}\right)^2 dx \approx \frac{3}{8}\pi^{-\frac{1}{2}}\sigma^{-5},$$

and where σ is the sample standard deviation.[12] Finally, there are many ways to choose $h_1 = o(h)$, but I use $h_1 = hT^{-\frac{1}{20}}$ as proposed by KZ.

In order to obtain estimates of the distributions, I numerically integrate the boundary-corrected, KZ densities via Simpson's rule. This method has the advantages of being both accurate and easy to implement. Moreover, I strategically choose the grid of points on which the densities are estimated so that the spacing is $\delta = \min\{h, .01\}$; this ensures that the resulting numerical error is of higher order than the statistical bias. The approximation error of Simpson's rule depends on the product of δ^5 and the fourth derivative of the actual integrand. Since the biweight kernel has a constant fourth derivative, the numerical error is actually $c\delta^5$, where c is fixed across domain points.

There are two final practical issues concerning numerical performance during estimation of $\widehat{\alpha}$. I reconcile $\underline{p}$ and α via an additive shift using equation (3.14), but before doing so, I treat $\widehat{\underline{p}} = \min_t\{Q_t\}$ as the numeraire good and divide all prize values by it. This has the effect of scaling up the length of the interval on which the optimal $\widehat{\alpha}$ lives (roughly by a factor of 10), to allow for finer adjustments. In order to compute $\ddot{G}$ at each golden search iteration, I solve the model equilibrium equations using a fourth-order Runge-Kutta algorithm. I also

[12]This comes from Silverman (1996), equation (3.27).

Table 3.2: Summary Statistics for Normalized SAT Scores and Prizes

Sample	# of Obs	Median	Mean	StDev	Min	Max
Minority Grades	18,407	29	29.86	17.76	0	101
Non-Minority Grades	73,361	44	44.3	19.1	0	102
Minority Prizes (Raw USNWR Quality Index)	186,507	0.4875	0.4958	0.189	0.087	0.973
Non-Minority Prizes (Raw USNWR Quality Index)	870,073	0.5877	0.5792	0.1826	0.087	0.973

take measures to ensure a finer grid of domain points in regions of the function marked by a high degree of curvature. The maximal grid-point spacing for the Runge-Kutta integration is approximately 0.019, resulting in a numerical error on the order of 10^{-6}. Each iteration required 51 seconds, on average, and convergence with a tolerance of 10^{-6} obtained in 28 iterations.

3.3 Results and Counterfactuals

3.3.1 Estimation Results

For the 1996 freshmen enrollment data, there were a total of 1,056,580 seats, with 186,507 going to minority students. This results in a demographic parameter estimate of $\widehat{\mu} = .17652$, with a standard error of .000141. Table 3.2 displays summary statistics on normalized grades for each group. It also displays summary statistics for prizes awarded to each group. These figures are for USNWR quality indices prior to performing the affine transformations discussed in the previous section. See Figures 3.2 and 3.1 for graphical representations of the grade and prize distributions.

I selected an affine specification of the grade transformation function, or $\widetilde{S}(s) = \Delta_0 + \Delta_1 s$. As it turns out, higher-order terms are unimportant, and this simple specification is enough to achieve a remarkably tight fit for the data.[13] In Table 3.3, I summarize the results of the policy function estimation. The regression R^2 value is 0.99789 and both the slope and the intercept are statistically significant. The high R^2 value is not necessarily surprising, given the nature of the sample: I have observations on virtually the entire universe of colleges, and the sample size for SAT scores constitutes a non-trivial fraction of the actual freshman population. More remarkable is the fact that such a tight fit is achieved with only a very simple SAT markup.

The estimated policy function assesses a grade inflation factor of 9.17 percent, along with an additive boost of 34 points (in the original SAT score units). For example, a minority student with an SAT score of 1000 would see his grade increased to

$$\widehat{\widetilde{S}}(1000) = 1.0917(1000) + 34 \approx 1126.$$

Combining these figures with the sample of normalized minority scores results in an estimated average grade boost of about 62 points in the original SAT score units.

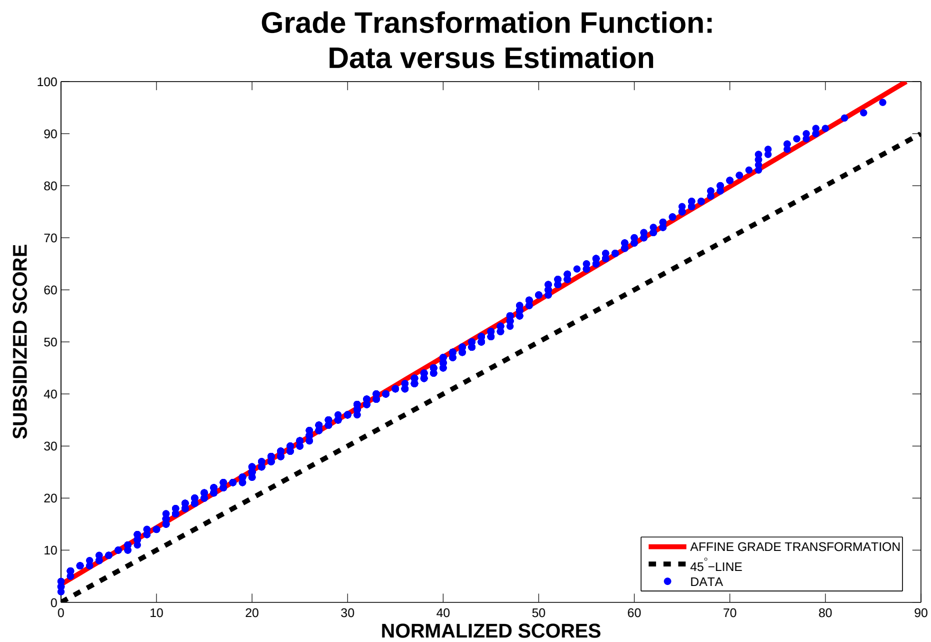
In Figure 3.4, I graphically compare the estimated policy function with the data. The solid line is the regression line, and the dots are a scatter plot of the

[13]Higher order polynomial specifications produce coefficients that are statistically significant, but they do not improve the fit of the model in any practical sense. Moreover, the affine estimate and the polynomial estimates differ the most toward the upper extreme of the sample where the data are very sparse.

Table 3.3: Estimated US AA Policy Function

	Δ_0	Δ_1	R^2	Implied Avg. Grade Boost
	3.4218	1.0917	0.99789	6.1611
	(0.00277)	(0.00000199)		
95% CI:	$[3.3187, 3.5251]$	$[1.089, 1.0945]$		

Figure 3.4: Grade Markup Estimation



X_us versus the Y_us from Section 3.2.1. The dashed line is the 45°-line, where the policy would lie under color-blind admissions. The dispersion of the data-points around the regression line represents the mis-specification error introduced by the assumption that individual college admissions boards can be treated as a single entity. The data suggest that there was a remarkable degree of coordination on AA practices among different colleges and universities in 1996.

This view of admission preferences is consistent with previous empirical work on AA. Chung et al. (2004), estimated the average SAT-equivalent grade boost received by minority students at elite universities. They use individual-level data on applications and acceptance decisions at three undisclosed institutions from "the top tier of American higher education" to estimate the admission preference assessed to minority students. Chung, *et al.* fitted a logistic regression model to the data in order to determine how different factors affect a student's probability of being accepted. They found that minority students receive a substantial SAT-equivalent boost in admission decisions—230 points for African Americans and 185 points for Hispanics. While these figures are not directly comparable to my measure of the admission preference—Chung, *et al.* measured the added probability of being accepted at a particular college, whereas $\widetilde{S}$ measures the increase in school rank for the final placement outcome associated with race—I also find that race plays a significant role in how college seats are allocated. Moreover, Chung, *et al.* found that the admission preference is highest for minority applicants with high scores. This is also consistent with my positive grade inflation estimate $\widehat{\Delta}_1 = 1.0917$ which implies a larger bonus for higher scores. Among minority SAT scores in the top 5% (*i.e.*, a score ≥ 1200), the average grade boost is 98 points.

Finally, in related work Chung and Espenshade (2005) found that the opportunity cost of admission preferences at selective institutions tends to be borne primarily by Asian students. According to their study, AA practices at elite universities are such that Asians receive a significant SAT-equivalent penalty, whereas whites do not. The current study offers some explanation as to why. First, recall that Asian students are under-represented in every tier except the top. Moreover, the SAT data suggest that the distribution of Asian SAT scores has a higher mean and a fatter upper tail than that for Whites. Both score distributions are roughly normal, with the former having mean and variance of 1039 and 213, respectively, and the latter having mean and variance 1030 and 183, respectively. Since the estimated policy function rewards high minority scores more, by extension it also penalizes high non-minority scores more (see equation 2.7). This is the reason why Asian applicants are negatively impacted the most: their score distribution has the fattest upper tail.

On the other hand, if one measures the opportunity cost of AA in terms of allocations of college seats, then it may actually *not* be the case that Asian students are most adversely affected. Inasmuch as the college admissions market is consistent with two key assumptions—namely, that *(i)* the market is effective at matching higher-performing students (of the same demographic class) with higher-quality schools, and *(ii)* the policy-maker does not attempt to rearrange the relative orderings of students within the same demographic group when devising an AA policy—then it will be the marginal non-minority students that are eliminated from elite institutions due to AA. For example, if a given admission preference produces a 10 percent reduction in non-minority enrollment within the top quartile of colleges, then only the lowest-scoring 10 percent within the top segment will be bumped down to schools in the next quartile. Since the best of the best non-minority students are disproportionately Asian—in fact, condi-

Figure 3.5: Estimated Private Types

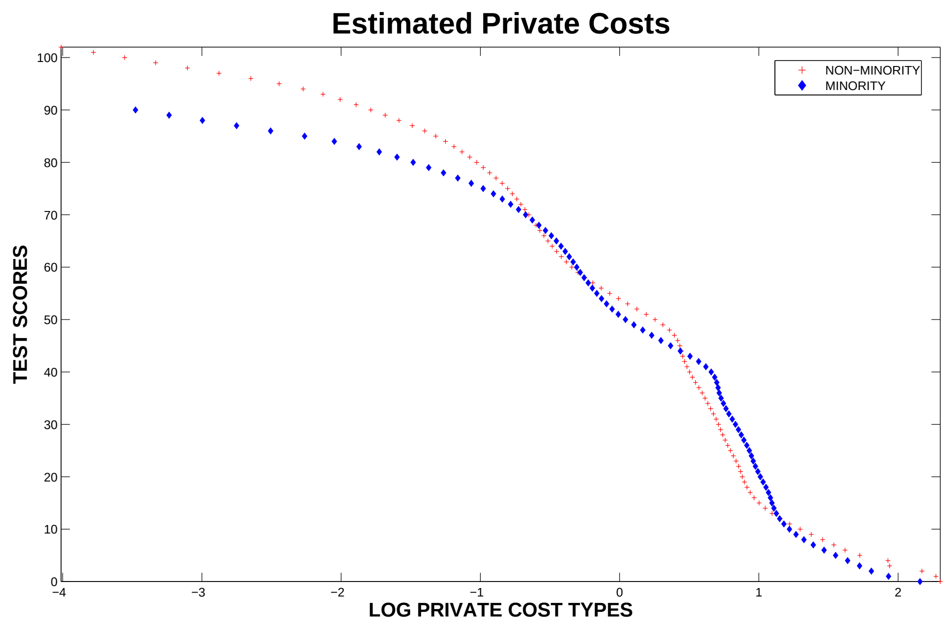


Table 3.4: Summary Statistics for Pseudo Types

Sample	Min	Max	Median	Mean	StDev
Minorities	.0005255	4.8754	0.5149	0.7452	0.6586
Non-Minorities	.0005255	4.8754	0.2241	0.3698	0.6251496

tional on scores above the median, Asian SAT results stochastically dominate all other groups—the negative allocational effect of AA would tend to be born predominantly by other non-minorities. This is true despite the fact that Asians are typically assessed the highest effective penalty by admission preferences.

The utility function parameter estimate is $\widehat{\alpha} = 0.054099$, with a standard error of 0.001339.The empirical mappings implied by $\widehat{\alpha}$ between private costs and achievement are displayed in Figure 3.5, where log private costs are on the ab-

Figure 3.6: Type Distributions

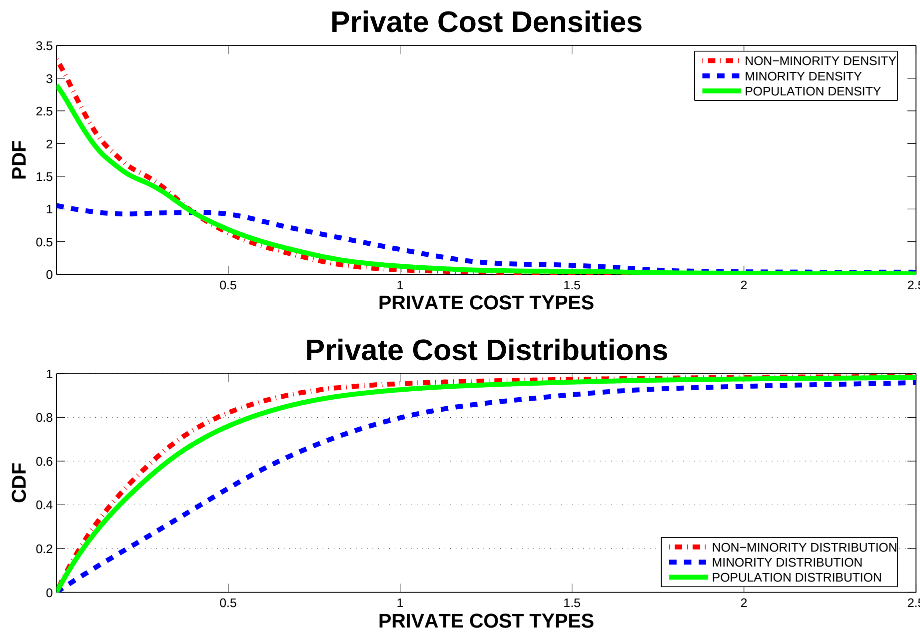


Figure 3.7: Goodness of Fit: SAT Score Distributions

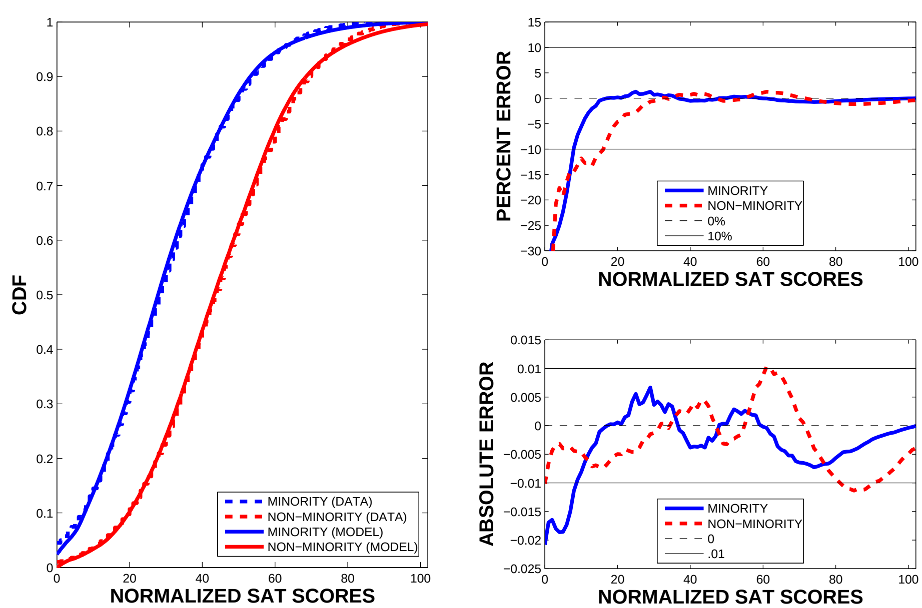
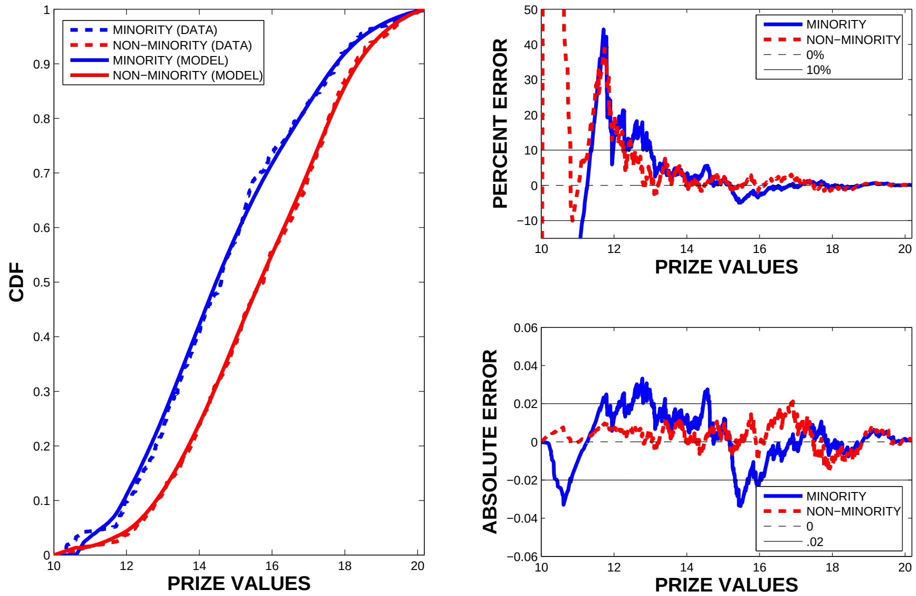


Figure 3.8: Goodness of Fit: Prize Distributions



scissa and SAT scores are on the ordinate. The fact that these mappings are monotonic—as required by theory—indicates that the data do not reject the empirical model. Summary statistics for pseudo-private costs are displayed in Table 3.4, and the private-cost distributions and densities are displayed in Figure 3.6. The model suggests that minority private costs stochastically dominate non-minority costs in the first-order sense.

Figure 3.7 illustrates the fit between the model and the data. The dashed step functions represent empirical grade distributions, and the solid lines represent model-generated analogs under the above parameter estimates. The right two panes display the percent error of the model from the data, as well as the absolute error. As another way of gauging goodness of fit, Figure 3.8 displays a similar plot for prize distributions, which were not directly targeted in the NLLS criterion function. These graphs provide suggestive evidence that the parameterizations introduced into the estimation scheme did not impose substantial mis-specification errors.

3.3.2 Counterfactual Policy Experiments

It is worth emphasizing that the standing assumption in the model is that the cost distributions $F_{\mathcal{M}}$ and $F_{\mathcal{N}}$ are invariant to policy changes. In that sense, the appropriate interpretation of this work is a short-run model of policy implications. It is reasonable to assume that individual characteristics which determine academic competitiveness are fixed for children born prior to a policy change. One could certainly conceive of a broader model in which the Board designs a policy today so as to affect the private costs of future generations (*i.e.*, the children of today's college freshmen), but such an undertaking is beyond the scope of the current exercise, and is left for future research. Instead, I shall concentrate on the effects of the policy-maker's choices on actions and outcomes for individuals such as today's college candidates, whose private costs are reasonably viewed as fixed.

With the structural estimates in hand, I am now ready to address the main objective of assessing policy implications. In particular, I wish to compare the effects of three separate policies: the status-quo admission preference, a quota rule, and a color-blind admission scheme. The maintained assumption on the policy-maker is that he cares primarily about three objectives: (i) facilitating academic

Table 3.5: Pct. Change in Group Achievement, Rel. to Status-Quo Policy

	Quantile:	10^{th}	25^{th}	Median	75^{th}	90^{th}
$\mathcal{M}$ Grades	Color-Blind:	+23.2	-2.4	-5.9	-1.9	+4.1
	Quota:	+65.8	+28.4	+13.2	+3.7	-1.8
$\mathcal{N}$ Grades	Color-Blind:	+7.4	+2.9	+1.2	+0.4	+0.7
	Quota:	-0.1	+1.4	+1.8	+2.3	+2.3

achievement, (*ii*) narrowing the racial achievement gap, and (*iii*) narrowing the enrollment gap. In terms of the objects associated with model equilibria, this means that he prefers a population grade distribution over another if it first-order dominates; a situation in which the separation between group-specific grade distributions is minimized; and similarly, a minimal separation between distributions of prizes allocated to each group in equilibrium. With these objectives in mind, I compute model equilibria and allocations under the three distinct policies, and I display the results below.

In Figures 3.9 and 3.10, I graphically present the results of the counterfactual experiments. Dashed lines denote distributions associated with the status-quo policy, solid lines denote distributions arising from a quota, and dash-dot lines denote a color-blind outcome. Between two lines with the same style, the one lying to the left pertains to minorities. When comparing two grade distributions, keep in mind that if distribution i lies to the right of distribution j in some region, that indicates an interval of students who achieve higher SAT scores under policy i. Table 3.5 displays percentage-changes in achievement under the two unobserved policies, relative to the status quo. The changes are measured at various quantiles, including the upper and lower deciles, the intermediate quartiles, and the median.

This information produces some intriguing insights into AA. Figure 3.9 and the first line of Table 3.5 characterize the effect of an admission preference on academic output. Relative to color-blind admissions, both the highest- and lowest-performing minority students decrease their effort, whereas students in the middle increase it. Although the policy-maker might hope that students will use a grade bonus solely to bolster their competitive edge, in some situations a rational student will react by treating the bonus as a direct utility transfer. In the case of high-performing students, the bonus is not needed and they achieve lower grades.

For low performers, the fixed grade boost Δ_0 improves their standing (the inflation factor is insignificant for scores close to zero), but in so doing, it adversely alters the marginal costs and benefits of achievement. In order to improve their payoff beyond what the grade subsidy achieves, they would have to compete with students whose costs are significantly lower than theirs. Thus, the marginal cost of competing is too high relative to the potential benefits. It is only for students whose costs are low enough—but not too low—that the admission preference entices additional investment in effort. As for non-minorities, an admission preference creates discouragement effects which discourage achievement among all types of students, relative to the color-blind case.

Figure 3.9 also illustrates some interesting insights on the effects of a quota.

It increases output among low-performing minorities, relative to a color-blind rule, and decreases effort among high-performers. The intuition is simple. A high-cost minority competing against the population at large is subject to a substantial discouragement effect since there is a large amount of competitors with lower costs. On the other hand, if he competes only against his own group (as with a quota) where costs are on average higher, then it is more worthwhile to invest in costly effort, since his relative standing with regards to the competition is improved. For low-cost minorities, the opposite effect occurs: when they only compete against other minorities, there is less need to outperform the competition as aggressively as before. For non-minorities with high- and low-cost types, the reverse effect applies (low-performers back off effort, high-performers increase it) by similar reasoning.

In January of 2008, presidential candidate Barack Obama famously stated in a television interview that his daughters should not be treated as disadvantaged in college admissions decisions, and that poor white children should be given extra consideration. The current empirical model seems to support the intuition behind Mr. Obama's assertion. It is interesting to note that both types of AA are detrimental to effort incentives for low-cost minorities *and* high-cost non-minorities.

With conflicting changes in academic output for different segments of the population, one might ask how the overall population grade distribution is effected. Table 3.5 answers this question: population grade distributions under each policy can be ordered by stochastic dominance. Color-blind admissions dominate

Table 3.6: Pct. Change in Enrollment, Rel. to Status-Quo Policy

	Tier:	I	II	III	IV
Minorities	Color-Blind:	-33.3	-24.8	+4.5	+43.4
	Quota:	+52.8	+14.3	-14.9	-42
Non-Minorities	Color-Blind:	+4.3	+4.6	-1.2	-19
	Quota:	-6.9	+2.6	+3.9	+18.5

the status quo, and a quota dominates the color-blind policy.

The model also shows that race-conscious admissions have a significant impact on college placement outcomes for minority students. Figure 3.10 displays the distributions of prizes allocated to each group under each policy. Note the substantial first-order dominance shift that occurs under either AA policy, relative to color-blind admissions. Table 3.6 numerically displays the percentage changes in enrollment for each college tier. By shutting down American AA (as in color-blind admissions) minority enrollment within the top quartile would decrease by a third, and within the upper middle quartile it would decrease by a quarter. Another striking feature of the table is that the majority of the displaced minority enrollment resulting from elimination of AA would end up in the lowest tier. The cost imposed on non-minorities amounts to roughly 4 percent and 5 percent of enrollment in each of the top two quartiles, respectively. Of course, on an individual level the benefits and costs to each group exactly balance out: any quality units reallocated to one student are necessarily transferred from another. Whether such transfers are justified is beyond the scope of economic reasoning.

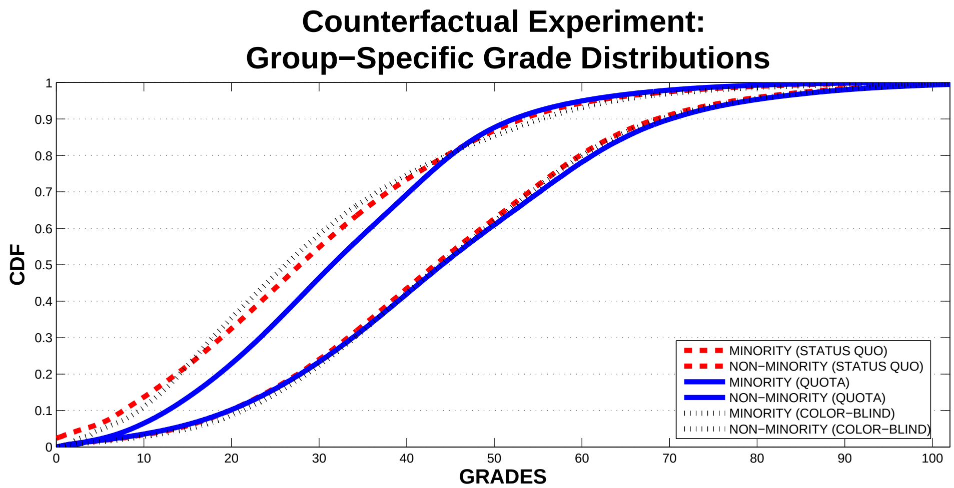
Table 3.7: Pct. Change in Policy Objectives, Rel. to Status-Quo Policy

	Quantile:	10^{th}	25^{th}	Median	75^{th}	90^{th}
Population Grades	Color-Blind:	+4.5*	+1.5*	+1*	+0.8*	+0.9*
(Objective I)	Quota:	+9.2**	+4.2**	+2.2**	+1.9**	+2**
Achievement Gaps	Color-Blind:	-2.5*	+9.1	+14.1	+6.3	-11**
(Objective II)	Quota:	-41.7**	-29.8**	-18.8**	-1.2**	+16.3
Enrollment Gaps	Color-Blind:	+56	+66.6	+80	+99.9	+106.2
(Objective III)	Quota:	-100**	-100**	-100**	-100**	-100**

However, it does appear that the AA policies implemented in real-world settings are effective at improving market outcomes for minorities, as intended by policy-makers. On the other hand, they do not eliminate the enrollment gap completely. For example, under a quota minority enrollment in the top tier would increase by an additional fifty percent. Loosely speaking, the US admission preference eliminates roughly $\frac{2}{3}$ of the enrollment gap in the top tier.

For a comparison of admissions rules along each of the policy objectives, I turn to Table 3.6, which tracks changes along objectives $I - III$. Once again, all figures are stated in terms of percentage changes, relative to the status-quo policy. For example, switching to a color-blind policy would increase the gap between prizes awarded to the median student from each group by 80%. In the table, asterisks are used to denote the preference ranking among the three policies. Two asterisks denote the most preferred outcome, one asterisk denotes the second-most preferred, and no asterisks indicates the least preferred. Interestingly, the status-quo admission preference never achieves the best outcome in any cate-

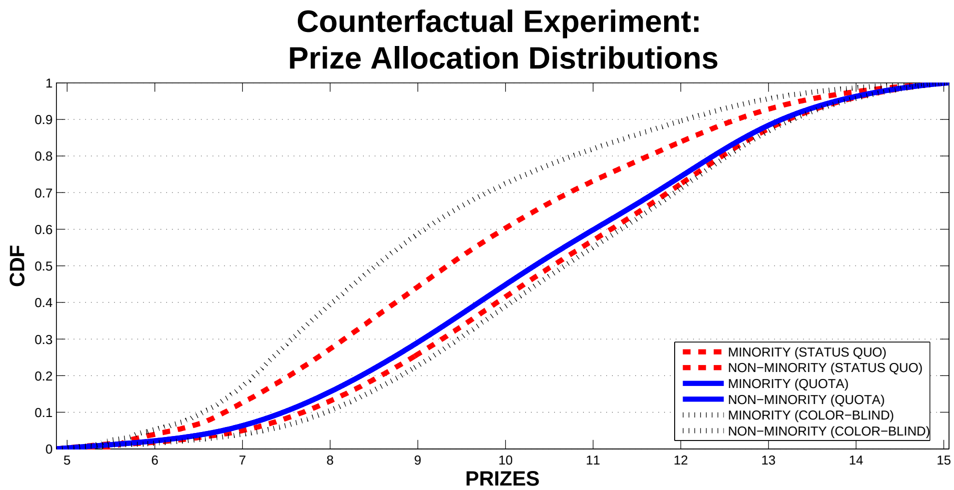
Figure 3.9: Achievement Counterfactual Results



gory. These figures also demonstrate that no ranking between a color-blind rule and the status quo can be established without knowing how the policy-maker's preferences weight objectives $I - III$. The former does strictly better in terms of academic performance, as mentioned above. The latter does strictly better in terms of enrollment gaps (see also Figure 3.10), and in terms of the achievement gap, the result is a toss-up.

On the other hand, a striking feature of the table is that a quota rule does strictly better than both other policies in nearly *every* category. Not only does it induce the highest academic output from the overall population of competitors, but it also shuts down the enrollment gap completely, by design. A quota also produces a substantial increase in minority achievement, as well as a narrowing of the achievement gap among the majority of the population. The lone drawback to a quota rule is that it produces the widest achievement gap in the upper tail of the grade distribution. However, one can argue that a quota rule appears

Figure 3.10: Allocations Counterfactual Results



to be the clear winner among college admission policies for reasonable social choice functions that do not place an extreme amount of weight on minimizing the achievement gap specifically in the upper tail of the grade distribution.

3.3.3 Alternative Policy Proposal

The counterfactual exercise has produced some valuable insights into the costs and benefits of AA. However, the value in knowing that a quota is a substantially superior policy choice would seem to be diminished by the fact that quotas are illegal in the US, because of a 1978 Supreme Court ruling. One might then ask whether an admission preference system can be modified so as to improve its performance, but without requiring an unreasonable level of information on the part of the policy-maker. As it turns out, the insights gained from the properties of a quota mechanism can be used to design a simple admission preference that performs similarly along the three policy objectives.

Formally, this alternative policy is defined by

$$\widetilde{S}^*(s) \equiv G_{\mathcal{N}}^{-1}\left[G_{\mathcal{M}}(s)\right].$$

In words, the college admissions board simply announces that it will map quantiles of the minority grade distribution into the corresponding quantiles of the non-minority grade distribution. For example, the median minority score is reassigned a value equal to the median non-minority score, and so on. The fact that this mechanism is outcome-equivalent to a quota immediately follows from plugging $\widetilde{S}^*$ or $(\widetilde{S}^*)^{-1}$ into equations (2.7) and (2.6), which then become equation (2.4).

Aside from its superior performance, this alternative admission preference has two other advantages worth mentioning. The first is its relative simplicity. In keeping with the *Wilson doctrine*, it does not require the policy-maker to have knowledge of students' individual abilities, or their beliefs about the competition they face. Rather, $\widetilde{S}^*$ allows the policy-maker to implement an improved outcome using only information on grades and race. The second advantage is that this mechanism is a self-adjusting grade markup rule: the bonus it assesses to minority test scores is proportional to the amount of asymmetry between demographic groups. In fact, if the competition is symmetric, $\widetilde{S}^*$ is also equivalent to a color-blind mechanism. This concept is formalized in the following Theorem.

Theorem 3.3.1. *For a sequence of cost distributions* $\{F_{\mathcal{M},k}, F_{\mathcal{N},k}\}_{k=1}^{\infty} \to (F_\Theta, F_\Theta)$*, let* $\widetilde{S}_k^*$

be defined by

$$\widetilde{S}_k^* \equiv G_{\mathcal{N},k}^{-1}\left[G_{\mathcal{M},k}(s)\right],$$

where $G_{i,k},\ i = \mathcal{M}, \mathcal{N}$ *are the equilibrium grade distributions. Then it follows that the induced sequence* $\{\widetilde{S}_k^*\}$ *converges to a color-blind rule, or* $\widetilde{S}^* = s$ *.*

Proof: As shown in Hickman (2010b), for each k, achievement under the mechanism defined by $\widetilde{S}^*$ is given by the following differential equation:

$$(\gamma_{i,k}^*)'(\theta) = -\frac{f_{i,k}(\theta)}{f_P\left(F_P^{-1}\left(1 - F_{i,k}(\theta)\right)\right)\mathcal{C}'(\gamma_{i,k}^*(\theta);\theta)},\ \ i = \mathcal{M}, \mathcal{N}, \tag{3.18}$$

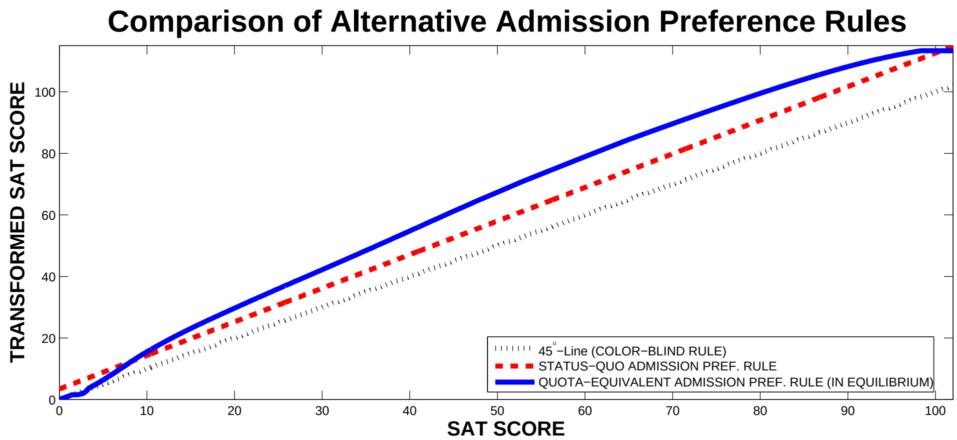
with a boundary condition given by the zero surplus condition. Moreover, achievement under a color-blind mechanism is characterized by

$$\begin{aligned}(\gamma_{\mathcal{M},k}^{cb})'(\theta) &= (\gamma_{\mathcal{N},k}^{cb})'(\theta)\\ &= (\gamma_k^{cb})'(\theta)\\ &= -\frac{\mu f_{\mathcal{M},k}(\theta) + (1-\mu) f_{\mathcal{N},k}(\theta)}{f_P\left(F_P^{-1}\left[1 - \mu F_{\mathcal{M},k}(\theta) - (1-\mu)F_{\mathcal{N},k}(\theta)\right]\right)\mathcal{C}'[\gamma_k^{cb}(\theta);\theta]},\end{aligned} \tag{3.19}$$

with the same boundary condition. Note that as $\{F_{\mathcal{M},k}, F_{\mathcal{N},k}\}_{k=1}^{\infty} \to (F_\Theta, F_\Theta)$, the right-hand sides of equations (3.18) and (3.19) above both converge to

$$\begin{aligned}(\gamma_{\mathcal{M}}^*)'(\theta) &= (\gamma_{\mathcal{N}}^*)'(\theta)\\ &= (\gamma^*)'(\theta)\\ &= -\frac{f_\Theta(\theta)}{f_P\left(F_P^{-1}\left[1 - F_\Theta(\theta)\right]\right)\mathcal{C}'(\gamma^*(\theta);\theta)},\ \ i = \mathcal{M}, \mathcal{N}.\end{aligned} \tag{3.20}$$

Figure 3.11: Policy Comparisons



Given this fact, we have $\{G_{\mathcal{M},k}, G_{\mathcal{N},k}\}_{k=1}^{\infty} \rightarrow (G, G)$, from which the result follows.

■

Figure 3.11 depicts a comparison of the status quo AA policy function with $\widetilde{S}^*$ (as generated by equilibrium grade distributions) and a color-blind policy under the 1996 cost distribution estimates. Several interesting observations arise from the plot. First, $\widetilde{S}^*$ overcomes the incentive problem at the lower end of the achievement distribution by closely resembling a color-blind rule for students whose academic output is low. Second, $\widetilde{S}^*$ encourages higher test scores for low and mid-range students (recall Figure 3.9) by awarding them an increasing marginal grade markup for low and mid-range scores. Third, the marginal grade markup eventually decreases as achievement increases (roughly around a grade of 60), corresponding to the notion that lower cost types need less help. Finally, the lone drawback of a quota rule—recall that it results in the lowest minority achievement and the widest achievement gap above the 90$^{\text{th}}$ percentile (see Table

3.6 and Figure 3.9)—arises from the fact that it gives too much assistance to high-performing students; in fact, it awards a larger grade boost than the status quo. This comes as a result of the information constraints that the policy-maker faces. Once he announces the policy $\widetilde{S}^*$, agents' behavior partially determines the shape of the grade transformation, making it impossible to improve incentives for all students, without observing their private information.

3.4 Conclusion

In this thesis I have provided some useful empirical insights into the costs and benefits of Affirmative Action in college admissions. I have documented that significant gaps exist among different races in terms of academic performance and academic outcomes. I have also demonstrated that a policy-maker's choice of what admission rule to implement can have a large impact on both performance and outcomes. Some policies are difficult to compare, while others emerge as being superior in terms of a set of general policy objectives. In particular, a quota rule promotes higher academic performance, and gives rise to a narrower achievement gap than an admission preference or a color-blind policy. By construction, it also shuts down the enrollment gap completely.

Future progress along this line of research can be achieved by studying a dynamic version of the model to explore the implications of college admissions policies in a setting where the policy-maker attempts to affect the long-run evolution of private-cost distributions. This will help to uncover how/whether AA helps or hinders the ultimate objective of erasing the residual effects of past institutionalized racism. The insights developed here will hopefully serve as a basis for answering these important questions in the future.

APPENDIX A
PROOFS APPENDIX

A.1 Proof of Proposition 2.1.8

Proof: Demonstrating existence and monotonicity is a straightforward application of Athey (2001) who proves existence and monotonicity of a pure-strategy equilibrium in a general class of auction-related games. The relationship between the grade distributions and the achievement functions follows immediately from the fact that achievement is a strictly decreasing function of private cost types. A formal proof of uniqueness is a bit more involved and is under construction. Briefly though, it follows the same logic as Hickman (2010a), Proposition 3.3 and Theorem 3.4. Given the well-behaved nature of the private cost distributions, it can be shown that any symmetric, monotonic equilibrium must also be differentiable. From differentiability, it follows that the equilibrium achievement functions must satisfy the first-order conditions of an agent's objective function. The first-order conditions define a standard initial value problem, and the fundamental theorem of differential equations can be invoked to show that a unique solution exists. Since any symmetric equilibrium of the college admissions game must be consistent with the unique solution of the first-order conditions, it follows that the symmetric equilibrium is unique. ■

A.2 Proof of Theorem 2.3.1

Proof: For notational ease, I shall drop the "*cb*" superscripts for the duration of the proof. Also, recall that finite functions are denoted by the presence of a parameter K, whereas limiting functions lack the extra argument. I begin by ordering the sample of K prizes from lowest quality to highest, denoting the k^{th} order statistic by $p_{(k:K)}$. Since $\gamma_i(\theta; K)$ is monotonic for $i = \mathcal{M}, \mathcal{N}$, the equilibrium expected payoff function in the K-player game can be written as

$$\begin{aligned}
&\Pi_i(s, \theta; K) \\
&\quad = \sum_{k=1}^{K} p_{(k:K)} \sum_{\substack{k_i \le \min\{k, K_i\}, \\ k_j = k - k_i}} \left[\binom{K_i - 1}{k_i - 1} F_i\left(\gamma_i^{-1}[s; K]\right)^{K_i - k_i} \left[1 - F_i\left(\gamma_i^{-1}[s; K]\right)\right]^{k_i - 1} \right. \\
&\qquad\qquad\qquad \left. \times \binom{K_j}{k_j} F_j\left(\gamma_j^{-1}[s; K]\right)^{K_j - k_j} \left[1 - F_j\left(\gamma_j^{-1}[s; K]\right)\right]^{k_j} \right] \\
&\qquad\qquad\qquad - \mathcal{C}(s; \theta).
\end{aligned}$$

In order for a player from group i to win the k^{th} prize, it must be the case that exactly $k - 1$ of his opponents have private costs above his own. For each opponent in his own group, this occurs with probability $1 - F_i\left(\gamma_i^{-1}[s; K]\right)$, and for each opponent in the other group, this occurs with probability $1 - F_j\left(\gamma_j^{-1}[s; K]\right)$. The binomial coefficients and the second summation operator in the expression above are designed to cover all the possible ways in which exactly $k - 1$ opponents have higher costs. Thus, the term within the inner summation is the probability of winning the k^{th} prize, and the overall objective function is a weighted sum of all K prizes, giving the expected prize won in equilibrium.

Recall my claim that the limiting equilibrium payoff function is given by

$$\Pi(s,\theta) = F_P^{-1}\left[1 - \left(\mu F_{\mathcal{M}}\left[\gamma_{\mathcal{M}}^{-1}(s)\right] + (1-\mu)F_{\mathcal{N}}\left[\gamma_{\mathcal{N}}^{-1}(s)\right]\right)\right] - \mathcal{C}(s;\theta).$$

I wish to show that for large K, it is nearly optimal to act as if one were maximizing $\Pi(s,\theta)$, rather than $\Pi(s,\theta;K)$. Since costs never change with K, I shall drop the cost terms and focus solely on convergence of the gross payoff functions $\pi(s,\theta;K)$ to their limit $\pi(s,\theta)$.

For $l \in [0,1]$, define

$$p(l;K) \equiv \left\{ p_{(t:K)} : t = \underset{k\in\{1,\ldots,K\}}{\operatorname{argmin}} \left| l - \frac{k}{K} \right| \right\},^{1}$$

Intuitively, $\{p(l;K)\}_{K=1}^{\infty}$ can be thought of as a random sequence of the t^{th} order statistic in the sample of prizes, where for each K, t is chosen so that $p_{(t:K)}$ approximates the l^{th} sample quantile as closely as possible. Since $\left|l - \frac{i}{K}\right| \leq \frac{1}{2K}$ for all $l \in (0,1)$, in the limit the t^{th} order statistic will be precisely at the l^{th} quantile within the sample of K prizes. Furthermore, since the sample distribution converges to F_P by the law of large numbers, it follows that $\underset{K\to\infty}{\operatorname{plim}}\, p(l;K) = F_P^{-1}(l)$.

For some $i = \mathcal{M},\mathcal{N}$, fix $\theta \in [\underline{\theta},\overline{\theta}]$ and let

$$l = 1 - \mu F_{\mathcal{M}}\left[\gamma_{\mathcal{M}}^{-1}(s)\right] - (1-\mu)F_{\mathcal{N}}\left[\gamma_{\mathcal{N}}^{-1}(s)\right],$$

[1] If there are multiple maximizers (there can be at most two) then choose t to be the lesser.

where $s = \gamma_i(\theta)$. Notice that

$$\underset{N \to \infty}{\text{plim}}\, p(l;K) = F_P^{-1}\left(1 - \mu F_{\mathcal{M}}\left[\gamma_{\mathcal{M}}^{-1}(s)\right] - (1-\mu)F_{\mathcal{N}}\left[\gamma_{\mathcal{N}}^{-1}(s)\right]\right) = \pi(s,\theta).$$

Moreover, For each K I can rewrite the finite expected gross payoff function as

$$\begin{aligned}
\pi_i(s,\theta;K) = p(l;K) & \sum_{\substack{k_i \le \min\{t,K_i\},\\ k_j = t-k_i}} \left[\binom{K_i-1}{k_i-1} F_i(\theta)^{K_i-k_i}\left[1-F_i(\theta)\right]^{k_i-1}\right. \\
& \left. \times \binom{K_j}{k_j} F_j\left(\gamma_j^{-1}[s;K]\right)^{K_j-k_j}\left[1-F_j\left(\gamma_j^{-1}[s;K]\right)\right]^{k_j}\right] \\
\sum_{\substack{k=1,\dots,N,\\ k\neq t}} p_{(k:K)} & \sum_{\substack{k_i \le \min\{k,K_i\},\\ k_j = k-k_i}} \left[\binom{K_i-1}{k_i-1} F_i(\theta)^{K_i-k_i}\left[1-F_i(\theta)\right]^{k_i-1}\right. \\
& \left. \times \binom{K_j}{k_j} F_j\left(\gamma_j^{-1}[s;K]\right)^{K_j-k_j}\left[1-F_j\left(\gamma_j^{-1}[s;K]\right)\right]^{k_j}\right].
\end{aligned} \tag{A.1}$$

Let

$$k_i^* \equiv \underset{1 \le k \le \min\{t,K_i\}}{\text{argmin}} \left|(1-F_i(\theta)) - \frac{k}{K_i}\right|^2$$

and note that the following can be extracted from the first term in (A.1):

$$\begin{aligned}
p(l;K) & \left[\binom{K_i-1}{k_i^*-1} F_i(\theta)^{K_i-k_i^*}(1-F_i(\theta))^{k_i^*-1}\right] \\
& \times \left[\binom{K_j}{t-k_i^*} F_j\left(\gamma_j^{-1}[s;K]\right)^{K_j-t-k_i^*}\left[1-F_j\left(\gamma_j^{-1}[s;K]\right)\right]^{t-k_i^*}\right].
\end{aligned}$$

The second and third components of the above product represent the probability

[2] If there are multiple maximizers (there can be at most two) then choose k_i^* to be the lesser.

that exactly $K_i - k_i^*$ group-i players have costs below θ and exactly $K_j - t - k_i^*$ group-j players achieve grades below $\gamma_i(\theta; K)$. Letting

$$\mu_i = \begin{cases} (1-\mu) & i = \mathcal{N} \quad \text{and} \\ \mu & i = \mathcal{M}, \end{cases}$$

this can be restated as the probability that fraction

$$\frac{K_i - k_i^*}{K_i} = 1 - \frac{k_i^*}{K_i} \xrightarrow{K} F_i(\theta)$$

of group-i players have costs below θ and fraction

$$\begin{aligned} \frac{K_j - t + k_i^*}{K} &\xrightarrow{K} \mu_j - l + \mu_i(1 - F_i(\theta)) \\ &= \mu_j - 1 + (1-\mu)_i F_i(\theta) + \mu_j F_j(\gamma_j^{-1}[s; K]) + \mu_i(1 - F_i(\theta)) \\ &= \mu_j F_j(\gamma_j^{-1}[s; K]) \end{aligned}$$

of all agents come from group j and achieve equilibrium grades below $\gamma_i(\theta; K)$. In each of the previous two expressions, the convergence over K term follows from the law of large numbers. Since the probability associated with this event is one in the limit, it follows that the pointwise probability limit of (A.1) is $\pi(s, \theta)$, for $i = \mathcal{M}, \mathcal{N}$.

Given that $\{\Pi(s, \theta; K)\}_{K=1}^{\infty}$ is a sequence of measurable functions converging pointwise to $\Pi(s, \theta)$ on a measurable set of finite measure, by Egorov's Theorem it follows that for any $\rho > 0$ there exists a set $E \subset [\underline{\theta}, \overline{\theta}]$ having measure

$m(E) < \rho$, such that $\{\Pi(s,\theta;K)\}_{K=1}^{\infty} \to \Pi(s,\theta)$ uniformly on the set $[\underline{\theta},\overline{\theta}] \setminus E$.

This is the same as saying that on the set $[\underline{\theta},\overline{\theta}] \setminus E$, it is nearly optimal to choose one's bid as if one's opponents were adopting a strategy of $\gamma(\theta)$, rather than $\gamma(\theta;K)$. Thus, given $\varepsilon > 0$, there exists K_ε such that for any $K \geq K_\varepsilon$, $\gamma(\theta)$ generates an ε-equilibrium of the K-player finite game. Furthermore, since all of the model primitives are well-behaved–θ is strictly bounded away from zero; $\mathcal{P}$ is compact; $F_{\mathcal{M}}$, $F_{\mathcal{N}}$, and F_P are absolutely continuous; and for each θ the set of undominated bids is compact-valued–I can invoke the Theorem of the Maximum on any compact subset of $[\underline{\theta},\overline{\theta}] \setminus E$ to show that the maximizers of $\Pi(s,\theta;K)$ and $\Pi(s,\theta)$ are close for large K. That is, given $\delta > 0$, there exists K_δ such that for any $K \geq K_\delta$, $\gamma(\theta)$ is a δ-approximate equilibrium of the K-player finite game, or

$$\|\gamma(\theta) - \gamma(\theta;K)\|_{\text{sup}} < \delta.$$

Finally, given $\varepsilon > 0$ and $\delta > 0$, then for any $K \geq K^* \equiv \max\{K_\varepsilon, K_\delta\}$, $\gamma(\theta)$ is a δ-approximate equilibrium which generates an ε-equilibrium of the K-player finite game on any closed subset of $[\underline{\theta},\overline{\theta}] \setminus E$. ■

A.3 Proof of Theorem 2.3.2

Proof: The logic of the proof is very similar to that of Theorem 2.3.1, but it is simpler because there is only one distribution to work with. Once again, I drop the "q" superscripts for the remainder of the proof and I begin by ordering the random sample of K prizes from lowest quality to highest, denoting the k^{th} order statistic by $p_{(k:K)}$. Since $\gamma(\theta;K)$ is monotonic, the equilibrium expected payoff function in the K-player game can be written as

$$\Pi(s,\theta;K) = \sum_{k=1}^{K} p_{(k:K)} \left[\binom{K-1}{k-1} F_i(\theta)^{K-k}(1-F_i(\theta))^{k-1} \right] - \mathcal{C}(s;\theta).$$

The first term is a weighted average of the order statistics, where the weights are the probabilities of winning each prize.[3] Recall my claim that the limiting equilibrium payoff function is given by

$$\Pi(s,\theta) = F_P^{-1}\left(1 - F_i(\gamma_i^{-1}(s))\right) - \mathcal{C}(s;\theta).$$

I wish to show that for large K, it is nearly optimal to bid as if one were maximizing $\Pi(s,\theta)$, rather than $\Pi(s,\theta;K)$. Since the cost of submitting a given bid never changes, I drop the second term from each payoff function and focus on convergence of the reward function sequence $\{\pi(\theta;K)\}_{K=1}^{\infty}$ to its limit $\pi(\theta)$.

For $l \in [0,1]$, define

$$p(l;K) \equiv \left\{ p_{(t:K)} : t = \underset{k\in\{1,\ldots,K\}}{\operatorname{argmin}} \left| l - \frac{i}{K} \right| \right\},^4$$

[3] In order for a player to win the k^{th} prize, there must be exactly $K-k$ competitors with lower costs and $k-1$ with higher costs. The probabilities of these two events are $F_i(\theta)^{K-k}$ and $(1-F_i(\theta))^{k-1}$, respectively. Finally, there are $\binom{K-1}{k-1}$ ways in which the intersection of the two events can occur. Thus, the probability of winning the k^{th} prize is $\binom{K-1}{k-1}F_i(\theta)^{K-k}(1-F_i(\theta))^{k-1}$.

[4] If there are multiple maximizers (there can be at most two) then choose t to be the lesser.

and once again, $\{p(l;K)\}_{K=1}^{\infty}$ can be thought of as a random sequence of the t^{th} order statistic in the sample of prizes, where for each K, t is chosen so that $p_{(t:K)}$ approximates the l^{th} sample quantile as closely as possible. Note that by the same logic as in the proof of Theorem 2.3.1, we have $\operatorname*{plim}_{N\to\infty} p(l;K) = F_P^{-1}(l)$. Fix $\theta \in [\underline{\theta}, \overline{\theta}]$ and let $l = 1 - F_i(\theta)$. Notice that

$$\operatorname*{plim}_{K \to \infty} p(l;K) = F_P^{-1}(1 - F_i(\theta)) = \pi(s,\theta).$$

Moreover, For each K I can rewrite the finite expected gross payoff function as

$$\begin{aligned}\pi(s,\theta;K) =& p(l;K) \left[\binom{K-1}{t-1} F_i(\theta)^{K-t}(1-F_i(\theta))^{t-1} \right] \\ &+ \sum_{k=1}^{t-1} p_{(k:K)} \left[\binom{K-1}{k-1} F_i(\theta)^{K-k}(1-F_i(\theta))^{k-1} \right] \\ &+ \sum_{k=t+1}^{K} p_{(k:K)} \left[\binom{K-1}{k-1} F_i(\theta)^{K-k}(1-F_i(\theta))^{k-1} \right].\end{aligned} \tag{A.2}$$

Note that

$$\begin{aligned}\left[\binom{K-1}{t-1} F_i(\theta)^{K-t}(1-F_i(\theta))^{t-1} \right] &= \Pr[K-t \text{ competitors have costs less than } \theta] \\ &= \Pr[\text{fraction } \frac{K-t}{K} \text{ have lower costs}] \\ &= \Pr[\text{fraction } \left(1 - \frac{t}{K}\right) \text{ have lower costs}] \\ &\xrightarrow{K} \Pr[\text{fraction } (1-l) \text{ have lower costs}] \\ &= \Pr[\text{fraction } F_i(\theta) \text{ have lower costs}] \\ &=1,\end{aligned}$$

where the convergence over K follows from the law of large numbers. Since probabilities must sum to one, equation (A.2) reveals that $\pi(\theta;K)$ increasingly resembles $p(l;K)$ as K gets large. Furthermore, since $\operatorname*{plim}_{N\to\infty} p(l;K) = F_P^{-1}(l)$, it follows that the pointwise probability limit of $\pi(\theta;K)$ is $\pi(\theta)$.

With pointwise convergence out of the way, the remainder of the proof is identical to the second half of the proof of Theorem 2.3.1. ■

A.4 Alt. Proof of Equilibrium Approximation

As mentioned in the body of the paper, the various results on equilibrium approximation can be strengthened to demonstrate that the derivations accurately reflect equilibrium actions and outcomes on the entire support of private costs. The cost of the stronger result is application of a more complicated proof which invokes results that may be less familiar to economic theorists. The logic is very similar under all three cases of color-blind admissions, quotas and admission preferences, so here I merely state and prove the alternative claim in the case of a quota rule, where the notation is simplest.

Theorem A.4.1. *Given ε, $\delta > 0$, there exists $K^* \in \mathbb{N}$, such that in the college admission game with a quota rule, we have the following:*

(i) $\gamma_i^q(\theta)$, $i = \mathcal{M}, \mathcal{N}$ *as defined by equation (2.5) and boundary condition (2.3) generates an ε-equilibrium of the K-player quota game, and*

(ii) $\gamma_i^q(\theta)$ *is a δ-approximate equilibrium for the K-player quota game, or*

$$\|\gamma_i^q(\theta) - \gamma_i^q(\theta;K)\|_{\sup} < \delta, \ \ i = \mathcal{M}, \mathcal{N}.$$

Proof: The first part of the proof involves showing that the finite objective functions converge pointwise in probability to the proposed limiting objective. The argument is identical to the one in the first half of the proof of Theorem 2.3.2

Using pointwise convergence, I can then invoke (Newey, 1991, Theorem 2.1) uniform convergence theorem to show that $\tilde{\Pi}(\theta; K)$ converges uniformly in probability to $\Pi(\theta)$ on the entire interval $[\underline{\theta}, \overline{\theta}]$. In order to do so, I must first verify a regularity condition and *stochastic equicontinuity* of the sequence $\tilde{\Pi}(\theta; K)$. For this part of the argument, it will be easier to think in terms of l, rather than θ. The regularity condition is that $l(\theta) = 1 - F_i(\theta)$ must live on a compact interval. Since θ lives on a compact interval and since F_i is continuous and monotonic, $l(\theta)$ attains values of 0 and 1 for finite values of θ.

At this point, all that remains is to verify equicontinuity of the sequence of functions $\tilde{\Pi}(\theta; K)$. For deterministic functions, it is known that pointwise convergence on a compact interval to a continuous limit implies uniform convergence if the sequence is equicontinuous. There is also an analogous condition for sequences of random functions, known as stochastic equicontinuity. In the context of my model, it basically means that for any point θ, $\tilde{\Pi}(\theta; K)$ must be continuous at θ at least with probability close to one for large K.[5] More precisely, $\{\tilde{\Pi}(\theta; K)\}_{K=1}^{\infty}$ is stochastically equicontinuous if for any $\epsilon, \epsilon' > 0$ there exists $\tau > 0$

[5]For a more detailed discussion on stochastic equicontinuity, see (Andrews, 1994, Section 2.1).

such that

$$
\begin{aligned}
&\limsup_{K\to\infty} \Pr\left[\sup_{l\in[0,1],\ l'\in B_\tau(l)} \left|\tilde{\Pi}(l;K) - \tilde{\Pi}(l';K)\right| > \epsilon'\right] \\
&= \limsup_{K\to\infty} \Pr\left[\sup_{l\in[0,1],\ l'\in B_\tau(l)} \left|\sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1}(1-l)^{K-i}(l)^{i-1}\right]\right.\right. \\
&\qquad\qquad \left.\left. - \sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1}(1-l')^{K-i}(l')^{i-1}\right]\right| > \epsilon'\right] < \epsilon,
\end{aligned}
$$

where $B_\tau(l)$ is an open ball centered at l with radius τ.

By similar arguments as above, it is apparent that

$$
\begin{aligned}
&\sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1}(1-l)^{K-i}(l)^{i-1}\right] \to F_P^{-1}(l) \text{ and} \\
&\sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1}(1-l')^{K-i}(l')^{i-1}\right] \to F_P^{-1}(l').
\end{aligned}
$$

Therefore, I can satisfy stochastic equicontinuity by choosing τ^* so that for all $l \in [0,1]$ and $l' \in B_{\tau^*}(l)$, the following is true:

$$
\left|F_P^{-1}(l) - F_P^{-1}(l')\right| < \epsilon'.
$$

Since F_P is continuous and $\mathcal{P}$ is compact, such a τ^* indeed exists. Thus, by Newey's uniform convergence theorem, it follows that for all $\epsilon > 0$, I have

$$
\lim_{K\to\infty} \Pr\left[\|\tilde{\Pi}(\theta;K) - \Pi(\theta)\|_{\sup} > \epsilon\right] = 0.
$$

In other words, When K is large, the equilibrium grade distribution under a monotonic equilibrium is such that it is nearly optimal to maximize as if one's (equilibrium) objective function were $\Pi(\theta, s) = F_P^{-1}(1 - F_i(\theta)) - \mathcal{C}(s;\theta)$.

This is the same as saying that it is nearly optimal to choose one's grade as if one's opponents were adopting a strategy of $\gamma(\theta)$, rather than $\gamma(\theta;K)$. Thus, given $\varepsilon > 0$, there exists K_ε such that for any $K \geq K_\varepsilon$, $\gamma(\theta)$ generates an ε-equilibrium of the K-player finite game. Furthermore, since all of the model primitives are well-behaved–θ is strictly bounded away from zero and lives in a compact set, $\mathcal{P}$ is compact, F_i and F_P are absolutely continuous and for each θ the set of undominated grades is compact-valued–the Theorem of the Maximum implies that the maximizers of $\tilde{\Pi}(s, \theta; K)$ and $\Pi(s, \theta)$ are close for large K. That is, given $\delta > 0$, there exists K_δ such that for any $K \geq K_\delta$, $\gamma(\theta)$ is a δ-approximate equilibrium of the K-player finite game, or

$$\|\gamma(\theta) - \gamma(\theta;K)\|_{\text{sup}} < \delta.$$

Finally, given $\varepsilon > 0$ and $\delta > 0$, then for any $K \geq K^* \equiv \max\{K_\varepsilon, K_\delta\}$, $\gamma(\theta)$ is a δ-approximate equilibrium which generates an ε-equilibrium of the K-player finite game. ■

APPENDIX B
DATA APPENDIX

B.1 USNWR Data and Methodology

Table B.1 contains descriptions and descriptive statistics of the quality measures used to compute the USNWR quality index. Column 1 contains variable descriptions and column 2 displays the weights placed on each category (within-category weights are uniform). Columns 3-5 display descriptive statistics. Column 5 displays total sample size for each variable. In cases where USNWR lacks a certain datum for some school, it replaces the datum with the lowest value observed for schools within the same region and Carnegie classification. Columns 3 and 4 display means and sample standard deviations for the schools where the variable value is observed. One final note is also worth mentioning: in computing the quality index, USNWR maps average SAT and ACT scores into the corresponding cumulative distribution values within the SAT and ACT score distributions. This allows for comparisons of scores on different tests. The mean and standard deviation for average test scores in the table reflect this transformation.

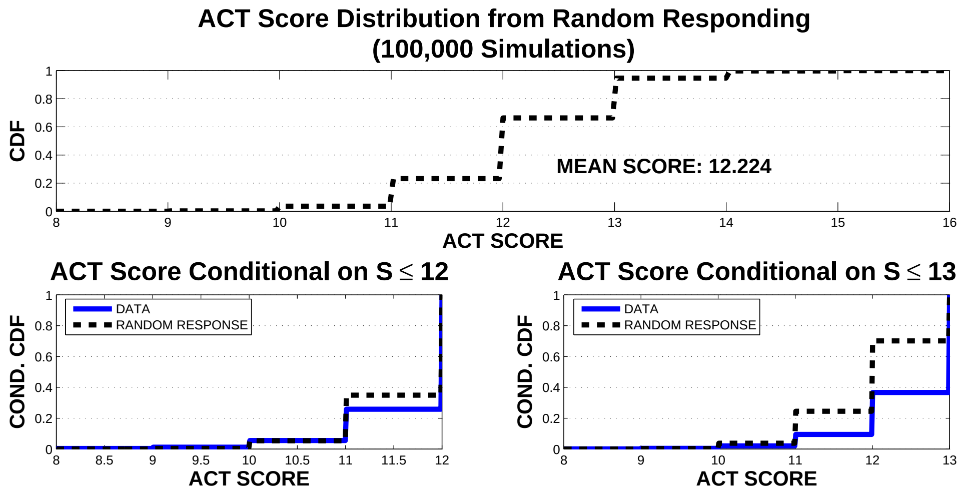
B.2 Zero Achievement Cutoff

Recall that the working interpretation of a student with zero academic achievement is one who simply engages in random responding to test questions. In order to uncover the distributions over random outcomes, I simulated 100,000 random responses to a published practice test for the ACT—another standardized

Table B.1: USNWR Quality Indicators:

Variable Description	Weight	Mean	StDev.	Obs.
Total Sample				1,314
SELECTIVITY	**15%**			
Acceptance Rate		.7597	.1553	1,226
Yield (% accepted students who enroll)		.4428	.1518	1,226
Avg. SAT/ACT Scores of Enrolled Students		.5515	.2101	1,152
% First-Time Freshmen in Top HS Quartile		.5227	.2038	1,008
FACULTY RESOURCES	**20%**			
% Full-Time Instructional Faculty w/Terminal Degree		.7622	.1665	1,221
% Full-Time Instructional Faculty		.6505	.1891	1,231
Avg. Faculty Compensation		$52,409.23	$12,982.11	1,291
Student/Faculty Ratio		14.99	4.2	1,245
FINANCIAL RESOURCES	**10%**			
Education Spending/Student		$9,494.56	$5,283.01	1,193
Non-Education Spending/Student		$5,951.12	$8,321	1,292
RETENTION	**25%**			
Avg. Graduation Rate		.5353	.6581	1,154
Freshman Retention Rate		.7396	.1146	1,224
ALUMNI SATISFACTION	**5%**			
Alumni Giving Rate		.2105	.1237	1,165
ACADEMIC REPUTATION	**25%**			
College Administrator Ranking Poll		N/A	N/A	N/A

Figure B.1: Defining Zero Effort



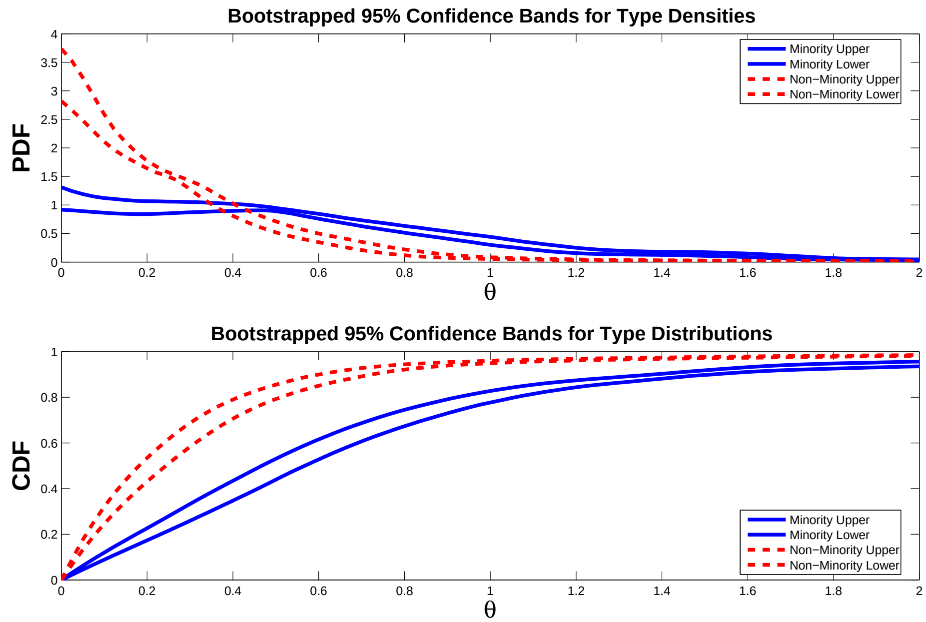
test widely used in US college admissions. The results are plotted in Figure B.1. The upper pane is the unconditional distribution of simulated random responses. The mean of the distribution is 12.1224, with a standard deviation of .9224.

The question of whether 12 or 13 is the appropriate zero-achievement cutoff is addressed in the lower two panes. On the left is a comparison of the simulated distribution and the distribution from the data, conditional on a score of 12 or less; the right pane is the same for a cutoff of 13. On the right side the two distributions are close, with a single crossing at a score of 11; the data distribution stochastically dominates on the left, and the distributions are not as close. Using these insights, I interpret an ACT score of 12 as corresponding to zero academic achievement, or in other words, $S = 0 \Leftrightarrow$ ACT score $= 12$.

I use score concordance tables to determine the equivalent zero achievement cutoff on the SAT test. Score concordances are jointly computed by the designers of the ACT and SAT using data on students who took both tests. The result is an interval of SAT scores being mapped into each outcome-comparable ACT score (since SAT scores occur on a finer grid). These indicate typical outcomes one can expect on the SAT for a student with a given score on the ACT, and vice versa. The SAT-equivalent range for an ACT score of 12 is 520-580

The alert reader may wonder why the random responding exercise was not performed using an SAT practice test instead. As it turns out, the mean score from random responding on the SAT is 450, significantly lower than the 520-580 range predicted by the concordance study. Moreover, conditional score distributions for the SAT do not render a similar fit as in the lower left pane of Figure B.1: random responding and actual data distributions conditional on low scores differ significantly in shape. However, this is not surprising considering that the SAT is designed to test one's academic aptitude (*i.e.*, ability for abstract reasoning), whereas the ACT is designed to test one's achievement (*i.e.*, acquisition of knowledge). Although study effort undoubtedly plays a major role in determining scores on both tests, the distinction between achievement versus aptitude becomes more pronounced near the lower extreme. As the concordance study suggests, individuals who choose to acquire low levels of knowledge—*i.e.*, individuals with scores statistically indistinguishable from random responding on the ACT—typically have aptitudes that allow them to beat random responding on the SAT. For more information on the distinction between the ACT and SAT tests, see http://www.act.org/aap/concordance/understand.html.

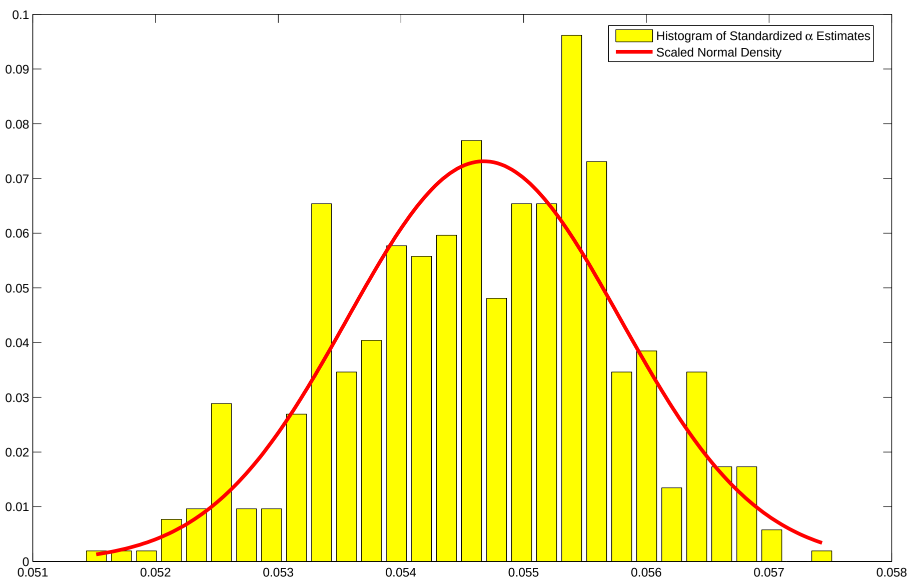
Figure B.2: Boostrapped Confidence Bands



B.3 Bootstrapped Standard Errors

For this exercise, I resampled the data 520 times and computed $\widehat{\alpha}$, $\widehat{F}_{\mathcal{M}}$, and $\widehat{F}_{\mathcal{N}}$ each time. Figure B.2 displays 95% confidence bands for the distribution and density estimates. Figure B.3 displays a histogram of cost curvature estimates. The bootstrapped mean and standard deviation of $\widehat{\alpha}$ are $\mu_{\widehat{\alpha}} = 0.054675$ and $\sigma_{\widehat{\alpha}} = 0.001112$, respectively. A $N(\mu_{\widehat{\alpha}}, \sigma_{\widehat{\alpha}})$ density (scaled by histogram bin width) has been superimposed on the histogram for comparison.

Figure B.3: Histogram of Bootstrapped $\hat{\alpha}$ Estimates



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