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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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To the Lighthouse

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ABSTRACT

This dissertation consists of three essays in microeconomic theory with an emphasis on consumer search and strategic experimentation. Among many issues, I shed some lights on search frictions, commitment and information.

In Chapter 1, together with Michael Choi and Kyungmin Kim, I consider an oligopoly model in which consumers engage in sequential search based on partial product information and advertised prices. We derive a simple condition that fully summarizes consumers' shopping outcomes and use the condition to reformulate the pricing game among the sellers as a familiar discrete-choice problem. Exploiting the reformulation, we provide sufficient conditions that guarantee the existence and uniqueness of pure-strategy market equilibrium and obtain several novel insights about the effects of search frictions on market prices. Among others, we show that a reduction in search costs increases market prices, but providing more pre-search information raises market prices if and only if there are sufficiently many sellers.

In Chapter 2, I study the effects of limited price commitment on consumer search and optimal pricing. I consider an environment in which consumers are uncertain about a seller's commitment to the advertised price. I characterize the set of pure-strategy equilibria and find that a higher degree of commitment is beneficial to the consumers. I evaluate the effects of regulation that limits the extent of a seller's deviation from the advertised price and demonstrate that stricter regulation may not be welfare improving. I also consider the case where sellers have heterogeneous levels

of commitment power and investigate how the difference in commitment power influences market outcomes. I find that a higher degree of commitment does not direct consumers' search order when all sellers have limited commitment. Conversely, full commitment allows a seller to dictate consumers' visit when his rivals have limited commitment. Finally, I show that the impact of search costs on prices depends on the level of commitment, the magnitude of the search cost and whether consumers have ex-ante heterogenous valuations of the product.

In Chapter 3, together with Kyungmin Kim, I consider a two-player exit game in which each player faces a one-armed bandit problem and the two players' types are negatively correlated. We provide a closed-form characterization of the unique (perfect Bayesian) equilibrium of the game. We show that, in stark contrast to the case of positive correlation, the players exit the game at an increasing rate over time and one player exits for sure before a deterministic time.

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CHAPTER 1
CONSUMER SEARCH AND PRICE COMPETITION

1.1 Introduction

We consider an oligopoly model in which consumers sequentially search for the best product based on partial product information and advertised prices. A key distinguishing feature from traditional consumer search models is the observability of prices before consumer search. Consumers still face a non-trivial search problem, because they do not possess full information about their values for the products. In this environment, prices affect each seller's demand not only through their effects on consumers' final purchase decisions, but also through their effects on consumer search behavior. We study how the presence of the latter channel affects sellers' pricing incentives and what its economic consequences are. In particular, we investigate the effects of search frictions on market prices.

Consumer search models with observable prices have been drawing growing attention in the literature. The Internet has significantly lowered the cost of collecting price information. Now it is common to check prices online and visit stores only to get hands-on information and/or finalize a purchase. In the meantime, the model captures some salient features of online marketplaces and price comparison websites. A consumer typically begins with a summary webpage displaying multiple items. She clicks a certain set of items, collects more detailed information, and then makes a final purchase decision. Our model captures such consumer behavior particularly well. The analysis of our model can produce meaningful insights about the role of the Internet in traditional markets and the working of online marketplaces.

Similar models have been studied in three recent papers, Armstrong and Zhou (2011), Shen (2015), and Haan, Moraga-González and Petrikaite (2015).[1] All three papers analyze a symmetric duopoly environment but consider different correlation structures for consumers' prior (known) and match (hidden) values. Both prior and match values are perfectly negatively correlated between the products in Armstrong and Zhou (2011), whereas both are independent in Haan, Moraga-González and Petrikaite (2015). Shen (2015) examines an intermediate case where each consumer's prior values are perfectly negatively correlated, while her match values are independent, between the two products. Our model adopts the same independence structure as Haan, Moraga-González and Petrikaite (2015) but allows for general market structure and asymmetric sellers.

It is well-recognized that such consumer search models do not admit tractable characterization. There are two main difficulties. First, consumer search behavior is complicated and hard to summarize. Each consumer undergoes sequential search, whose complexity grows fast as the number of sellers increases or new features are introduced into the model. This is likely to be the reason why all previous studies have restricted attention to the duopoly case. Second, the sellers' best response functions

[1]An early precursor to these papers is Bakos (1997), who studies several versions of a (circular) location model. One of his extensions considers the case where quality (value) information is significantly costlier than price information. The limit version where price information can be obtained at zero cost is equivalent to the case where prices are publicly observable.

do not behave well in general. There may not exist a pure-strategy equilibrium, and the model rarely produces sharp comparative statics results.

We overcome the first difficulty by identifying a necessary and sufficient condition that summarizes consumers' search outcomes.[2] We utilize an elegant solution by Weitzman (1979) for a class of sequential search problems and show that, although Weitzman's solution is necessary to fully describe optimal search *behavior*, the optimal search *outcome* (i.e., a consumer's eventual purchase decision) can be fully summarized by a simpler condition that is familiar in discrete-choice models. The condition pinpoints the extent to which search frictions distort consumers' purchase decisions (i.e., how a consumer's purchase decision under sequential search differs from that under perfect information) and allows us to reformulate the pricing game among the sellers as a discrete-choice problem.

For the second difficulty, we obtain sufficient conditions under which the seller's best response functions are well-behaved and, therefore, there exists a unique pure-strategy market equilibrium. We exploit the induced discrete-choice structure of our model and characterize sufficient conditions on the primitives of our model under which we can apply both general results in the literature on supermodular games and specific results in discrete-choice models. Despite certain limitations,[3] our charac-

[2]An effectively identical condition has been independently discovered by Armstrong (2016). See also Armstrong and Vickers (2015), who consider a more general problem of which demand systems have discrete-choice foundations and show that the demand system under consumer search belongs to the class.

[3]In particular, our sufficient conditions do not encompass a benchmark case where consumers are ex ante symmetric (i.e., do not possess any prior product information), for which it is known that there does not exist a pure-strategy equilibrium (see Armstrong and Zhou,

terization allows us to derive some sharp comparative statics results and, therefore, learn more about the working of the model, as we elaborate below. In addition, our analysis is likely to be informative for the environments that are not covered by our sufficient conditions.

We pay special attention to the relationship between search frictions and market prices. It was recognized early on that the Internet dramatically reduces market frictions and, therefore, should deliver more efficient market outcomes, by transforming traditional businesses as well as creating many new markets. This promise has been fulfilled in various ways by now, but several phenomena that are at odds with it still persist. In particular, it has been repeatedly reported that the Internet has neither significantly lowered markups nor reduced price dispersion (see, e.g., Ellison and Ellison, 2005; Baye, Morgan and Scholten, 2006) These suggest that search frictions are significant even in online markets and cast doubt on the conventional wisdom that a reduction in search frictions is necessarily beneficial to consumers. The following results provide some new insights for these important issues.

As a methodological contribution, we show that the effects of various changes in search frictions can be summarized by their effects on dispersion of the induced discrete-choice distributions. This is useful, because there is a systematic relationship between preference diversity and equilibrium prices in discrete-choice models: equilibrium prices increase as consumers' preferences become more diverse.[4] In other

2011).

[4]This is a classic idea in the literature on Bertrand competition under product differentiation. We contribute to the literature by providing an appropriate measure of preference

words, we derive several comparative statics results regarding search frictions, which are hard to obtain directly, by studying their effects on the induced distributions and utilizing a result that links between preference diversity and equilibrium prices in discrete-choice models.

We show that an increase in the value of search raises market prices. Specifically, we establish that, provided that the sellers are symmetric, the equilibrium price increases as search costs decrease or the distribution of match values becomes more dispersive (which increases the expected return of search).[5] Note that this is opposite to the standard result in the literature. As the value of search decreases, a consumer is less likely to leave for another seller and, therefore, more likely to purchase from the current seller. The sellers then have an incentive to extract more surplus from visiting consumers and, therefore, charge higher prices. This is the main mechanism behind the opposite result in the literature. However, it crucially depends on the assumption of unobservable prices (i.e., no price advertisement), which implies that the sellers cannot influence consumer search behavior. In our model, the sellers compete in prices to attract consumers. When the value of search falls, price competition becomes more severe, which induces the sellers to lower their prices.

In contrast, improving pre-search information quality has an ambiguous effect

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diversity (product differentiation). See Section 1.5.1 for a more comprehensive discussion and our result.

[5]We note that, whereas the first result regarding search costs has also been established by Armstrong and Zhou (2011) and Haan, Moraga-González and Petrikaite (2015), the second result regarding the distribution of match values is, to our knowledge, new to the literature.

on market prices. We show that providing more precise information for consumers before search increases market prices if and only if the number of sellers is above a certain threshold. There are two opposing effects. On the one hand, it reduces consumers' incentives to explore more products, which, as above, intensifies price competition among the sellers. On the other hand, consumers' preferences before search (prior values) become more dispersed, which relaxes price competition. We prove that the latter effect dominates the former, and thus providing more product information before search increases market prices if and only if there are sufficiently many sellers.

These results allow us to reinterpret various empirical findings in the literature, which, conversely, justifies the empirical relevance of our model. For instance, Lynch and Ariely (2000) run a field experiment with online wine sales and find that providing more product information lowers consumers' price sensitivity. Bailey (1998) and Ellison and Ellison (2014) report that online prices are often higher than off-line prices. This naturally arises in our model, given that search costs are significantly lower online than off-line. Ellison and Ellison (2009) report that markups are relatively higher for high-quality products than for low-quality products. Within our model, this can be understood as consumer preferences being more diverse, or the relative cost of search being lower, for high-quality products.

We also provide two novel insights for the case where the sellers are asymmetric. First, we study which sellers have a stronger incentive to post higher prices. We show that Weitzman index, which is the most natural candidate in the current sequential search context, does not provide enough guidance in general. We provide a sufficient condition under which the sellers' prices can be clearly ranked and also show that Weitzman index can be still useful to predict price rankings in some specific contexts. Second, we analyze the effects of search costs on asymmetric sellers. We show that when one seller has a higher marginal cost than the other, an identical increase in search costs raises demand for the low-cost seller but lowers demand for the high-cost seller. Intuitively, this is because consumers become more price-sensitive as search costs increase, and the low-cost seller posts a lower price. One noteworthy implication of this result is that the high-cost seller has a stronger incentive to lower his price than the low-cost seller as search costs increase. Since the former posts a higher price than the latter, this means that the price difference between the two sellers falls as search costs rise. In other words, an increase in search frictions may reduce price dispersion. This result contrasts well with a classical insight in search theory that price dispersion is a symptom of search frictions.

This paper joins a growing literature on ordered search, which investigates the effects of (both exogenous and endogenous) search order on market outcomes and various ways sellers influence consumer search behavior (order). See Armstrong (2016) for a comprehensive and organized introduction of the literature and several useful discussions. In light of this literature, we consider the case where each consumer's search order is fully endogenized and a seller influences search behavior through the choice of her price, which is arguably the most basic instrument.

One interpretation of our model is to introduce consumer search into a canonical model of Bertrand competition under product differentiation. Indeed, our model reduces to that of Perloff and Salop (1985) if consumers incur no search costs. We make it transparent how consumer search models with price advertisements are related to discrete-choice models (and what the former can learn from the latter). In addition, we show that dispersive order is an appropriate measure for preference diversity and explain how the result can be used to obtain several comparative statics results regarding search frictions.

As explained above, our model can be interpreted as a model of online marketplaces. In this regard, our paper is related to two strands of literature on electronic commerce. First, there are several theoretical studies that develop an equilibrium online shopping model. For example, Baye and Morgan (2001) analyze a model in which both the sellers and consumers decide whether to participate in an online marketplace, while Chen and He (2011) and Athey and Ellison (2011) present an equilibrium model that combines position auctions with consumer search. Our paper is unique in that the focus is on consumer search within an online marketplace. Second, a growing number of papers draw on search theory to study online markets. For example, Kim, Albuquerque and Bronnenberg (2010) develop a non-stationary search model to study the online market for camcoders. De los Santos, Hortaçsu and Wildenbeest (2012) test some classical search theories with online book sale data and argue that fixed sample size (i.e., simultaneous) search theory explains the data better than sequential search theory. Dinerstein, Einav, Levin and Sundaresan (2014) estimate online search costs and retail margins with a consumer search model based on the "consideration set" approach, and apply them to evaluate the effect of search redesign by eBay in 2011. Although empirical analysis is beyond the scope of this paper, we think that our equilibrium model is tractable and structured enough to be taken to data.

The rest of the paper is organized as follows. We introduce the environment in Section 1.2. We analyze consumers' optimal shopping problems in Section 1.3 and characterize the market equilibrium in Section 1.4. We study the effects of search frictions on market prices in Section 1.5 and provide two results, one about price rankings and the other about price dispersion, in Section 3.5. All omitted proofs are in the appendix.

1.2 Environment

The market consists of n sellers, each indexed by $i = \{1, ..., n\}$, and a unit mass of consumers. The sellers face no capacity constraint, while each consumer demands one unit among all products. The sellers simultaneously announce prices. Consumers observe those prices and search optimally.

Each seller i supplies a product at no fixed cost and a constant marginal cost c_i. We denote by $p_i \in \mathcal{R}_+$ seller i's price. In addition, we let $\mathbf{p}$ denote the price vector for all sellers (i.e., $\mathbf{p} = (p_1, ..., p_n)$) and $\mathbf{p}_{-i}$ denote the price vector except for seller i's price (i.e., $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$). Denote by $D_i(\mathbf{p})$ the measure of consumers who eventually purchase product i. Seller i's profit is then defined to be $\pi_i(\mathbf{p}) \equiv D_i(\mathbf{p})(p_i - c_i)$. Each seller maximizes his profit $\pi_i(\mathbf{p})$.

A (representative) consumer's random utility for seller i's product is given by $\tilde{V}_i = V_i + Z_i$. The first component V_i represents the consumer's prior value for product i, while the second component Z_i is the residual part that is revealed to the consumer only when she visits seller i and inspects his product. As for prices, we let $\mathbf{v} = (v_1, ..., v_n)$ and $\mathbf{z} = (z_1, ..., z_n)$ denote the realization of a consumer's value profile for each component.

The products are horizontally differentiated. We assume that V_i and Z_i are drawn from the distribution functions F_i and G_i, respectively, identically and independently across consumers and products (and independently each other), where both F_i and G_i have full support over the real line and continuously differentiable density f_i and g_i, respectively. Independence across products allows us to utilize the optimal search solution by Weitzman (1979), while independence between V_i and Z_i leads to a clean and easy-to-interpret characterization.[6]

Search is costly, but recall is costless. Specifically, each consumer must visit seller i and discover her match value z_i in order to be able to purchase product i. She needs to incur search cost $s_i (> 0)$ on her first visit. She can purchase the product immediately or recall it at any point during her search. Each consumer can leave the market at any point and take an outside option u_0.

A consumer's ex post utility depends on her value for the purchased product $\tilde{v}_i$, its price p_i, and her search history. Let N be the set of sellers a consumer visits.

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[6]We note that independence between V_i and Z_i is restrictive not by itself, but because of a joint additive-utility specification ($\tilde{V}_i = V_i + Z_i$). It is always possible to reinterpret (redefine) Z_i, so that it is independent of V_i (see, e.g., Eső and Szentes, 2007). In this case, a restriction is only due to the utility specification. On the other hand, Z_i can always be defined as $Z_i \equiv \tilde{V}_i - E[\tilde{V}_i|V_i]$ (see, e.g., Krähmer and Strausz, 2011). In this case, independence between V_i and Z_i imposes a restriction.

If she purchases product i (in N), then her ex post utility is equal to

$$U(v_i, z_i, p_i, N) = v_i + z_i - p_i - \sum_{j \in N} s_j.$$

If she does not purchase and takes an outside option, then her ex post utility is equal to

$$U(N) = u_0 - \sum_{j \in N} s_j.$$

Each consumer is risk neutral and maximizes her expected utility.

The market proceeds as follows. First, the sellers simultaneously announce prices $\mathbf{p}$. Then, each consumer shops (searches) based on available information $(\mathbf{p}, \mathbf{v})$. We study subgame perfect Nash equilibrium of this market game.[7] We first characterize consumers' optimal shopping behavior (given any price vector) and then analyze the pricing game among the sellers.

1.3 Consumer Behavior

In this section, we analyze consumers' optimal sequential search problems.

1.3.1 Optimal Shopping

Given prices $\mathbf{p}$ and prior values $\mathbf{v}$, each consumer faces a sequential search problem. She decides in which order to visit the sellers and, after each visit, whether to stop, in which case she chooses which product to purchase, if any, among those she has inspected so far, or visit another seller. Although this is, in general, a complex

[7] For notational simplicity, we do not formally define consumers' search strategies. See Weitzman (1979) for a formal (recursive) description of search strategy.

combinatorial problem, an elegant solution is known by Weitzman (1979). Independence between v_i and z_i leads to an even sharper characterization, as reported in the following proposition.[8]

Proposition 1.1. *Given* $\mathbf{p} = (p_1, ..., p_n)$ *and* $\mathbf{v} = (v_1, ..., v_n)$*, the consumer's optimal search strategy is as follows: for each* i*, let* z_i^* *be the value such that*

$$s_i = \int_{z_i^*}^{\infty} (1 - G_i(z_i))dz_i. \tag{1.1}$$

(i) Search order: the consumer visits the sellers in the decreasing order of $v_i + z_i^* - p_i$ *(i.e., she visits seller* i *before seller* j *if* $v_i + z_i^* - p_i > v_j + z_j^* - p_j$*).*

(ii) Stopping: let N *be the set of sellers the consumer has visited so far. She stops, and takes the best available option by the point, if and only if*

$$\max\{u_0, \max_{i \in N} v_i + z_i - p_i\} > \max_{j \notin N} v_j + z_j^* - p_j.$$

Weitzman's solution is based on a single index for each option (seller). Let r_i be the reservation value such that a consumer is indifferent between obtaining utility r_i immediately (which saves additional search costs s_i) and visiting seller i (which gives her an option to choose between r_i and $v_i + z_i - p_i$):

$$r_i = -s_i + \int \max\{r_i, v_i + z_i - p_i\} dG_i(z_i).$$

Weitzman (1979) shows that the optimal search strategy is to visit the sellers in the decreasing order of r_i and stop as soon as the best realized value by the point exceeds

[8] The measure of consumers who are indifferent over multiple choices is negligible, because F_i and G_i are assumed to be continuously increasing for all i. For notational convenience, we ignore those consumers throughout the paper.

all remaining r_i's. In our model, due to the additive-utility specification, Weitzman's index simplifies to $r_i = v_i + z_i^* - p_i$, where z_i^* is given by equation (1.1).

1.3.2 Shopping Outcomes

Despite its elegance, Weitzman's solution cannot be directly used to summarize consumers' shopping outcomes and derive the demand functions. Consider the simplest case where there are two sellers and there is no outside option (i.e., $u_0 = -\infty$). Even in this case, there are three different paths through which a consumer eventually purchases product i. First, a consumer may visit seller i first and purchase immediately. Second, a consumer may visit seller i first, try seller j as well, but recall product i. Third, a consumer may visit seller j first but purchase product i. Total demand for seller i is the sum of all these demands. The number of paths grows exponentially fast as the number of sellers n increases.

One of our main breakthroughs is to identify a necessary and sufficient condition for consumers' eventual purchase decisions and, therefore, provide a simple way to summarize shopping outcomes. In order to motivate the result, consider the same duopoly case as above. The three paths through which a consumer purchases product i correspond to each of the following conditions:

(i) $v_i + z_i^* - p_i > v_j + z_j^* - p_j$ (visit i first) and $v_i + z_i - p_i > v_j + z_j^* - p_j$ (stop at i).

(ii) $v_i + z_i^* - p_i > v_j + z_j^* - p_j$ (visit i first), $v_i + z_i - p_i < v_j + z_j^* - p_j$ (not stop at i), and $v_i + z_i - p_i > v_j + z_j - p_j$ (prefer i to j).

(iii) $v_i + z_i^* - p_i < v_j + z_j^* - p_j$ (visit j first), $v_i + z_i^* - p_i > v_j + z_j - p_j$ (not stop at

j), and $v_i + z_i - p_i > v_j + z_j - p_j$ (prefer i to j).

Notice that the first condition can be simplified to $v_i + \min\{z_i^*, z_i\} - p_i > v_j + z_j^* - p_j$, while the second and the third conditions together can be reduced to $v_i + \min\{z_i^*, z_i\} - p_i \leq v_j + z_j^* - p_j$ and $v_i + \min\{z_i^*, z_i\} - p_i > v_j + z_j - p_j$. Intuitively, a consumer purchases product i if she either does not visit seller j or finds a sufficiently low realized value of z_j. Combining these inequalities, we arrive at the following single inequality:

$$v_i + \min\{z_i, z_i^*\} - p_i > v_j + \min\{z_j, z_j^*\} - p_j.$$

This simple condition can be extended for the general case by considering each pair of sellers and accommodating the outside option, as formally reported in the following lemma.

Lemma 1.1 (Eventual Purchase). *Let $w_i \equiv v_i + \min\{z_i, z_i^*\}$ for each i. Given $\mathbf{p}$, $\mathbf{v}$, and $\mathbf{z}$, the consumer purchases product i if and only if $w_i - p_i > u_0$ and $w_i - p_i > w_j - p_j$ for all $j \neq i$.*

Lemma 1.1 suggests that consumer shopping behavior can be summarized as in canonical discrete-choice models.[9] The only difference is that consumers' purchase decisions are made based, neither on true values $\tilde{v}_i$ nor on prior values v_i, but on newly identified values w_i, which we call *effective values* from now on. Clearly, w_i is

[9]Lemma 1.1 holds even if prices are not observable to consumers before search, as long as consumers have correct beliefs about prices (i.e., in equilibrium). However, the result does not hold if a seller deviates, because consumers' search decisions are based on their expectations about prices, while their final purchase decisions depend on actual prices charged. That property makes Lemma 1.1 less useful in such a setting.

related to underlying values $\tilde{v}_i$ and v_i. In particular, w_i converges to $\tilde{v}_i$ as s_i tends to 0 (in which case z_i^* approaches ∞) and is determined only by v_i as s_i tends to infinity (in which case z_i^* approaches $-\infty$). Intuitively, search frictions prevent consumers from making fully informed decisions, and the problem becomes more severe, and consumers rely more on their prior information $\mathbf{v}$, as search frictions increase. The specific truncation structure is driven by a monotonicity property of Weitzman's solution. If a consumer visits seller i, Weitzman's indices for all remaining sellers are lower than $v_i + z_i^* - p_i$. Therefore, the consumer necessarily stops if z_i exceeds z_i^*, which implies that the probability that a consumer purchases product i stays constant above z_i^*.

In order to utilize Lemma 1.1, we let H_i denote the distribution function for the new random variable $W_i = V_i + \min\{Z_i, z_i^*\}$, that is,

$$H_i(w_i) \equiv \int_{-\infty}^{z_i^*} F_i(w_i - z_i) dG_i(z_i) + \int_{z_i^*}^{\infty} F_i(w_i - z_i^*) dG_i(z_i). \tag{1.2}$$

The distribution function H_i crucially depends on s_i. If s_i tends to 0, then z_i^* becomes arbitrarily large (see equation (1.1)) and, therefore, H_i becomes the convolution of F_i and G_i. If s_i explodes, then z_i^* approaches negative infinity, in which case H_i depends only on F_i.

1.4 Market Equilibrium

In this section, we consider the pricing game among the sellers and provide sufficient conditions under which there exists a unique pure-strategy equilibrium.

Lemma 1.1 implies that the demand function for each seller can be derived as in standard discrete-choice models. A consumer purchases product i if and only if his effective utility for product i, $w_i - p_i$, exceeds the outside option u_0 and the corresponding utility for each other product, $w_j - p_j$. Therefore, the measure of consumers who purchase product i is given by

$$D_i(\mathbf{p}) = \int_{u_0+p_i}^{\infty} \left(\prod_{j \neq i} H_j(w_i - p_i + p_j) \right) dH_i(w_i).$$

This demand system exhibits standard properties for imperfect substitutes: demand for seller i decreases in own price p_i and increases in competitors' prices $\mathbf{p}_{-i}$. However, the demand system does not behave well in general: $D_i(p_i, \mathbf{p}_{-i})$ may not be quasi-concave in p_i, and a seller's best response does not necessarily increase in $\mathbf{p}_{-i}$.

The literature has found that log-concavity is an appropriate restriction for the distribution functions. It not only guarantees the existence and uniqueness of equilibrium, but also generates intuitive comparative statics results (such as declining market prices as the number of sellers increases). The following result by Quint (2014), translated into our environment, is well applicable to our model.[10]

Theorem 1.1 (Quint, 2014)**.** *Suppose that for each i, both $H_i(w_i)$ and $1 - H_i(w_i)$ are log-concave. Then, $D_i(\mathbf{p})$ is log-concave in p_i, and $\log D_i(\mathbf{p})$ has strictly increasing differences in p_i and p_j. In addition, there exists a unique pure-strategy equilibrium in the pricing game among the sellers.*

Distributional log-concavity ensures log-concavity and log-supermodularity in

[10]Quint (2014) provides further relevant discussions, including common distribution functions that satisfy the log-concavity condition and weaker conditions sufficient for each result. For more thorough technical treatments of log-concavity, see, e.g., Bagnoli and Bergstrom (2005).

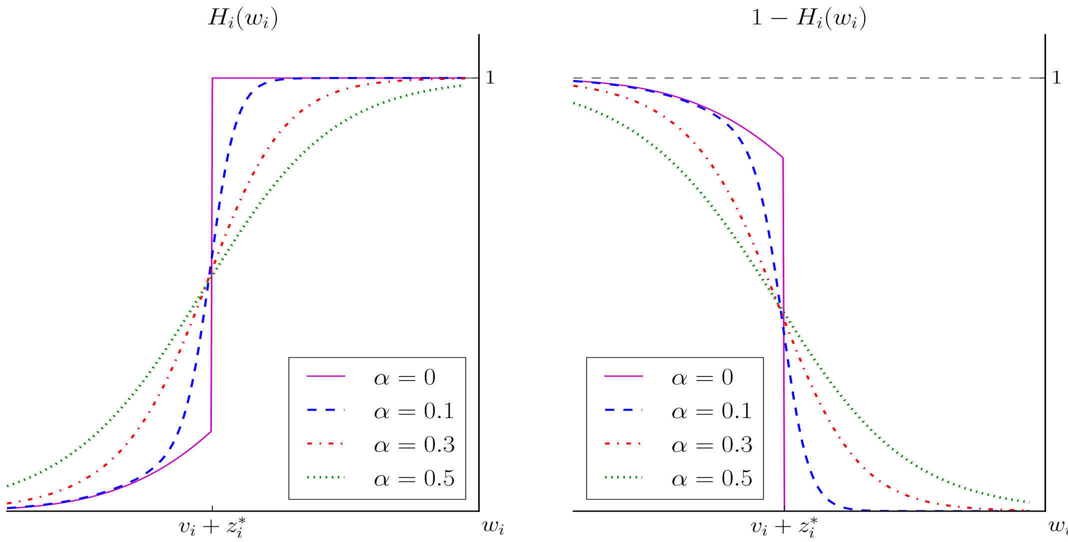
demand. These two properties imply that the pricing game is a supermodular game and, therefore, has a pure-strategy equilibrium, as an application of more general existence theorems (see Vives, 2005). Uniqueness is not implied by general theory, but driven by a specific structure of the model, namely that $D_i(\mathbf{p})$ is invariant when all prices, together with $-u_0$, increase by the same amount, which is not a general property in supermodular games.

As shown above, H_i in our model is not exogenously given but depends on F_i, G_i, and s_i in a specific way. Therefore, log-concavity cannot be directly imposed on H_i and $1 - H_i$. A natural assumption is that all primitive distribution functions F_i, $1 - F_i$, G_i, and $1 - G_i$ are log-concave. However, the assumption alone does not guarantee the log-concavity of H. In fact, even a stronger assumption that the density functions f_i and g_i are log-concave is not sufficient.[11]

In order to understand the origin of the problem, consider the case where F_i is degenerate at v_i. In this case, $H_i(\mathrm{w})$ jumps up at $v_i + z_i^*$ (see the solid line, corresponding to $\alpha = 0$, in the left panel of Figure 1.1) and, therefore, cannot be globally log-concave. This is driven by the upper truncation structure of the random variable W_i, which is, in turn, due to the sequential search nature of consumers' problems, as explained in the previous section. When F_i is continuously distributed over the real line, the atom at $v_i + z_i^*$ is continuously scattered, which ensures the continuity of H_i. However, if F_i is sufficiently concentrated around v_i, then the slope

[11] If the density function f is log-concave, then both distribution function F and survival function $1 - F$ are log-concave. See Bagnoli and Bergstrom (2005) for more details.

Figure 1.1: Log-Concavity of the Distribution Function



$H_i(w_i)$ and $1 - H_i(w_i)$ for different dispersion levels of F_i. For both panels, $F_i(v_i) = 1/(1 + e^{-v_i/\alpha})$ (logistic distribution), and $G_i = \mathcal{N}(0, 1)$ (standard normal distribution).

of H_i at $v_i + z_i^*$ can be arbitrarily large (see the dashed line, corresponding to $\alpha = 0.1$, in the left panel of Figure 1.1). Therefore, H_i may still fail to be log-concave.

We provide sufficient conditions under which this problem is not binding and Theorem 1.1 applies. We begin by imposing sufficiently strong log-concavity on the primitive distributions.

Assumption 1.1. *For each i, both density functions f_i and g_i are log-concave.*

Although this assumption does not guarantee the log-concavity of H_i, it suffices for $1 - H_i$, as formally stated in the following lemma.

Lemma 1.2. *Under Assumption 1.1, $1 - H_i$ is log-concave.*

Proof. Integrating by parts the first term in equation (1.2) leads to

$$1 - H_i(w_i) = \int_{-\infty}^{z_i^*} f(w_i - z_i)(1 - G_i(z_i))dz_i \tag{1.3}$$

The log-concavity of g ensures the same property for $1-G_i$. Since both f and $1-G$ are log-concave, the integrand is log-concave in (w_i, z_i). The desired result then follows from Prekopa's theorem, which states that if the integrand is log-concave, then the integral is also log-concave.[12] □

To understand the difference between H_i and $1 - H_i$, consider, again, the case where F_i is degenerate. Both H_i and $1 - H_i$ are discontinuous at $v_i + z_i^*$. However, $1 - H_i$ jumps down and, therefore, preserves log-concavity over the interval below $v_i + z_i^*$ (see the right panel of Figure 1.1). In addition, $1 - H_i(w_i)$ remains equal to 0 above $v_i + z_i^*$. These two properties ensure that $1 - H_i$ is log-concave when F_i is degenerate. When F_i is not degenerate, $1 - H_i(w_i)$ is continuous and stays positive. However, these properties do not disrupt log-concavity, and thus $1 - H_i$ is always log-concave under Assumption 1.1.

Our first result provides a sufficient condition under which H_i is globally log-concave. It states that if F_i is sufficiently dispersed, then H_i is log-concave under Assumption 1.1.[13]

Proposition 1.2. *Fix random variables V_i and Z_i with density f_i and g_i, respectively.*

[12] See, e.g., Caplin and Nalebuff (1991) and Choi and Smith (2016) for a formal statement of the theorem and its uses in related contexts.

[13] Haan, Moraga-González and Petrikaite (2015) conjecture this result and provide a set of confirming numerical examples. Our result formalizes their conjecture.

Define $V_i^\sigma \equiv \sigma V_i$ *and* $W_i^\sigma \equiv V_i^\sigma + \min\{Z_i, z_i^*\}$. *Let* H_i^σ *denote the distribution function for* W_i^σ. *Then, there exists* $\overline{\sigma} < \infty$ *such that the distribution function* H_i^σ *is log-concave whenever* $\sigma > \overline{\sigma}$.

To understand this result, recall that the failure of log-concavity of H_i is due to the probability mass at z_i^*. Now notice that, since $W_i = V_i + \min\{Z_i, z_i^*\}$ (also see equation (1.2)), dispersion on F_i scatters this atom through the real line, which makes H_i increase more slowly and, therefore, mitigates the main problem. When F_i is sufficiently dispersed, the effect of the mass point is small (i.e., H_i does not increase too fast at any point), and thus H_i can be log-concave (see the left panel of Figure 1.1).

Our second condition is based on the idea that global log-concavity is not necessary for Theorem 1.1. Specifically, it is clear that in equilibrium p_i exceeds c_i. Therefore, it suffices that H_i is log-concave only on the parameter region where $w_i \geq u_0 + c_i$.

Proposition 1.3. *Suppose Assumption 1.1 is satisfied. (i) Given* $u_0 > -\infty$, *there exists* $\overline{s}_i < \infty$ *such that if* $s > \overline{s}_i$, *then* $H_i(w_i)$ *is log-concave above* $u_0 + c_i$. *(ii) Given* $s_i > 0$, *there exists* $\overline{u}_0$ *such that if* $u_0 > \overline{u}_0$, *then* $H_i(w_i)$ *is log-concave above* $u_0 + c_i$.

Intuitively, if s_i is sufficiently large, then the value of visiting seller i is small. In this case, z_i^* lies in the irrelevant (sufficiently negative) region, while Assumption 1.1 ensures that H_i is log-concave in the relevant region. Similarly, if u_0 is sufficiently large, then consumers' effective values are relevant only when they are sufficiently large and, in particular, far exceed z_i^*. Again, Assumption 1.1 guarantees that H_i behaves well in the relevant region.

In the remaining sections, we restrict attention to the parameter space where H_i and $1 - H_i$ are log-concave at least over the relevant region and, therefore, there exists a unique pure-strategy equilibrium. Although restrictive, this allows us to go one step further and investigate sellers' pricing incentives in our model. In addition, we maintain Assumption 1.1. As in many existing studies, log-concavity allows us to derive clean and intuitive comparative statics results.

1.5 Symmetric Sellers: Search Frictions

In this section, we study how search frictions influence the sellers' pricing incentives.[14] For clear insights as well as tractability, we restrict attention to the case where the sellers are symmetric. Precisely, we assume that buyers' values for each product are drawn from identical distribution functions F and G, the sellers have an identical marginal cost c, and consumers face identical search costs for all sellers (i.e., for all i, $F_i = F$, $G_i = G$, $c_i = c$, and $s_i = s$). We let p^* and $\pi(p^*)$ denote the symmetric equilibrium price and profit, respectively.

1.5.1 Preference Diversity

We begin by establishing a result that is useful through this section. The result is, in fact, of interest by itself, in regard to the literature on Bertrand competition

[14]We omit some standard comparative statics results. For example, it is easy to show that more intense competition, such as introducing an additional seller or increasing the outside option, lowers market prices. See Quint (2014) for further results and illustrations.

under product differentiation. It is well-known that horizontal product differentiation provides a way to overcome the Bertrand paradox: each seller has some loyal consumers (who value the seller's product more than other products) and, therefore, can set a positive markup even under Bertrand competition. It is natural that the more differentiated consumers' preferences are, the higher prices the sellers charge. A challenge has been to identify an appropriate measure of preference diversity (product differentiation). In their seminal work, Perloff and Salop (1985) show that constant scaling of consumers' preferences necessarily increases the equilibrium price, but find that the result does not extend for mean-preserving spreads. Our result provides an answer to the long-standing open question.[15]

We utilize the following measure of stochastic orders, so called *dispersive order*.

Definition 1.1. *The distribution function H_2 is more dispersed than the distribution function H_1 if $H_2^{-1}(b) - H_2^{-1}(a) \geq H_1^{-1}(b) - H_1^{-1}(a)$ for any $0 < a \leq b < 1$.*

Intuitively, a more dispersed distribution function increases more slowly (its inverse increases faster), as it density is more spread out. This order is location-free and, therefore, neither is implied by nor implies first-order or second-order stochastic dominance. Mean-preserving dispersive order, however, implies mean-preserving spread: if H_2 is more dispersed than H_1 with the same mean, then H_2 is a mean-

[15]Zhou (2017) studies the effects of bundling in the Perloff-Salop framework and independently discovers an almost identical result. Precisely, his Lemma 2 is equivalent to our Proposition 1.4, provided that there is no outside option (i.e., $u_0 = -\infty$). Our result is slightly more general than his, in that we account for the outside option. In addition, whereas his lemma is an isolated result in his paper, we fully utilize it for subsequent comparative statics.

preserving spread of H_1.[16]

The following result shows that there is a good sense in which dispersive order is an appropriate measure of product differentiation.

Proposition 1.4. *The equilibrium price p^* increases as H becomes more dispersive and $H(u_0 + c)$ weakly decreases.*

Proof of Proposition 1.4. The equilibrium condition for p^*, which stems from an individual seller's first-order condition and the symmetry requirement, can be rewritten as

$$\frac{1}{p^* - c} = \frac{-\partial D_i(p^*)/\partial p_i}{D_i(p^*)} = \frac{\int h(\max\{u_0 + p^*, w\}) dH(w)^{n-1}}{\frac{1}{n}(1 - H(u_0 + p^*)^n)}.$$

Letting $\phi \equiv H(u_0 + p^*)$ and changing the variable with $a = H(w)$, we get

$$\frac{1}{p^* - c} = \frac{h(H^{-1}(\phi))\phi^{n-1} + \int_\phi^1 h(H^{-1}(a)) da^{n-1}}{\frac{1}{n}(1 - \phi^n)}. \tag{1.4}$$

If H becomes more dispersive, $dH^{-1}(a)/da = 1/h(H^{-1}(a))$ increases (i.e., $h(H^{-1}(a)$ decreases) for each a. If, in addition, $H(u_0 + c)$ decreases, then $\phi = H(u_0 + p^*)$ also decreases for any $p^* \geq c$, because a distribution function crosses a less dispersive one only once from above. Notice that both of these lower the right-hand side. The desired result now follows from the fact that the left-hand side is strictly decreasing in p^*, while the right-hand side is increasing in p^* (see the appendix for a proof of this last claim). $\square$

[16] See Shaked and Shanthikumar (2007) for further details. Dispersive order has been adopted and proved to be useful in other economic contexts. See, for example, Ganuza and Penalva (2010).

The relevance of dispersive order is particularly transparent when there is no outside option (i.e., $u_0 = -\infty$), which is the case considered by Perloff and Salop (1985) and many subsequent studies. In that case, the second condition about $H(u_0 + c)$ is vacuous, and thus dispersive order alone dictates how market prices vary: market prices rise (fall) if H becomes more (less) dispersive.

1.5.2 Search Costs

The following result reports the effects of varying search costs s on the equilibrium price p^* and each seller's equilibrium profit $\pi(p^*)$.

Proposition 1.5. *Both equilibrium price p^* and equilibrium profit $\pi(p^*)$ decrease as s increases.*

Proof. We utilize Proposition 1.4 to prove the price result. Specifically, we show that the random variable $W = V + \min\{Z, z^*\}$ falls in the first-order stochastic dominance (which implies that $H(u_0 + c)$ increases) and becomes less dispersive as s increases. Notice that z^* decreases in s (see equation (1.1)). This immediately implies that W decreases in the sense of first-order stochastic dominance. For the dispersion result, let $\tilde{G}(z)$ denote the distribution function of the random variable $\min\{Z, z^*\}$. By its definition, $\tilde{G}(z) = G(z)$ if $z < z^*$ and $\tilde{G}(z) = 1$ if $z \geq z^*$, which implies that $\tilde{G}^{-1}(a) = \min\{G^{-1}(z), z^*\}$ for $a \in (0, 1)$. Clearly, the quantile function $\tilde{G}^{-1}(a)$ becomes weakly flatter at any $a \in (0, 1)$ as z^* decreases. This implies that $\min\{Z, z^*\}$ becomes less dispersive as s increases. The desired result follows once this result is combined with the fact that the density function f is log-concave.[17]

An increase in s affects each seller's profit $\pi_i(\mathbf{p}) = D_i(p_i, p_{-i})(p_i - c)$ through the following three channels:

$$\frac{d\pi_i(\mathbf{p})}{ds} = \frac{\partial p_i}{\partial s}\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} + \frac{\partial p_{-i}}{\partial s}\frac{\partial \pi_i(\mathbf{p})}{\partial p_{-i}} + \frac{\partial z^*}{\partial s}\frac{\partial \pi_i(\mathbf{p})}{\partial z^*}.$$

Each term represents the marginal effect of own price, that of the other sellers' prices, and that of consumer search behavior, respectively. In equilibrium, the first term is equal to 0 by the envelope theorem ($\partial\pi_i(\mathbf{p})/\partial p_i = 0$). The second term is negative because $\partial p^*/\partial s \leq 0$, as shown above, and $\partial\pi_i(\mathbf{p})/\partial p_{-i} \geq 0$, as the products are imperfect substitutes one another. The last term is also negative because $\partial z^*/\partial s < 0$ and $\partial\pi_i(\mathbf{p})/\partial z^* \geq 0$: the latter inequality stems from the fact that an increase in z^* increases the distribution function H in the sense of first-order stochastic dominance, induces less consumers to take the outside option and, therefore, increases each seller's demand $D_i(p^*)$. Overall, it is clear that $d\pi(p^*)/ds \leq 0$. $\square$

Both price and profit results are in stark contrast to those of most existing consumer search models where consumers discover prices through search. When prices are not observable before search, an increase in search costs decreases the value of additional search and, therefore, increases the probability that a consumer purchases from the current seller.[18] This induces the sellers to charge higher prices as search

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[17]See Theorem 3.B.8 in Shaked and Shanthikumar (2007). If a random variable X is the convolution of two random variables X_1 and X_2 (i.e., $X = X_1 + X_2$) and X_1 has log-concave density, then X becomes more dispersive as X_2 becomes more dispersive.

[18]To be precise, the result in consumer search models with unobservable prices crucially

costs increase. When prices are observable before search, they directly influence consumer search (see Proposition 1.3): the lower price a seller offers, the more consumers visit him first. As search costs increase, consumers search less and are more likely to purchase from their first visit. This intensifies price competition among the sellers and leads to lower prices.

Proposition 1.5 raises an interesting possibility that consumer surplus may increase when search costs increase. An increase in search costs has a direct negative effect on consumer welfare. However, if the sellers lower their prices dramatically in response, overall consumer welfare may rise. Indeed, there is an example in which an increase in search costs is beneficial to consumers. It arises when consumers' outside option is sufficiently unfavorable and there are sufficiently few sellers. In this case, the sellers possess strong market power and, therefore, charge a high price. An increase in search costs induces them to drop their prices quickly, up to the point where the indirect effect outweighs the direct effect and, therefore, consumer welfare increases.

The following proposition addresses a closely related question of how an increase in returns to search affects the equilibrium price p^*. To obtain clean insights, we restrict attention to the case where consumers have no outside option.

Proposition 1.6. *Provided that consumers have no outside option, the equilibrium price p^* increases as G becomes more dispersive.*

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depends on the log-concavity property of the relevant distributions. For example, in Anderson and Renault (1999) where prices are not observable and F is degenerate, the equilibrium price increases in s if $1 - G$ is log-concave but decreases in s if $1 - G$ is log-convex (and assuming that there exists a symmetric pure-strategy equilibrium). Our comparison applies only to the former case.

Proof. By the logic given for the price result in the proof of Proposition 1.5, it suffices to show that the random variable $\min\{Z, z^*\}$ becomes more dispersive as $G(z)$ becomes more dispersive. To this end, recall that the quantile function for the random variable $\min\{Z, z^*\}$ is given by $\tilde{G}^{-1}(a) = \min\{G^{-1}(a), z^*\}$ for $a \in (0,1)$. It suffices to show that the slope of $\tilde{G}^{-1}(a)$ increases for all $a \in (0,1)$. For $a < G(z^*)$, the result is immediate from $\tilde{G}^{-1}(a) = G^{-1}(a)$. For $a = G(z^*)$, the result follows from the fact that $G(z^*)$ rises as G becomes more dispersive: rewriting equation (1.1) with $b^* = G(z^*)$ and $b = G(z)$ yields $s = \int_{b^*}^{1}(1-b)\partial G^{-1}(b)/\partial b db$. If G becomes more dispersive ($\partial G^{-1}(b)/\partial b$ rises), the integrand rises, and thus the lower support b^* must rise in order to maintain the equation.[19] □

Notice that this is consistent with Proposition 1.5, as a decrease in search costs can be interpreted as a proportional increase in search returns. Proposition 1.6 demonstrates that the main insight in Proposition 1.5 extends beyond proportional changes and holds with any dispersive perturbations.

1.5.3 Pre-search Information Quality

In our model, consumers search because they have imprecise information about their values for the products. This means that search frictions can also be measured by the extent to which consumers are uncertain about their match values. We now examine the effects of improving pre-search information quality on the equilibrium price p^*.

[19] This argument is due to Choi and Smith (2016).

For tractability, we specialize our model into a Gaussian learning environment, where both F and G are given by normal distributions with mean 0. In addition, we assume that F has variance α^2, while G has variance $1-\alpha^2$, for some $\alpha \in (0,1)$ (i.e., $V \sim \mathcal{N}(0,\alpha^2)$ and $Z \sim \mathcal{N}(0,1-\alpha^2)$). Our choices of the variances are deliberate. Notice that $\tilde{V} = V+Z \sim \mathcal{N}(0,1)$ for any α. In other words, our variance specification ensures that the distribution for consumers' ex post values $\tilde{V}_i$ stays unchanged when α varies. The parameter α measures the quality of pre-search information: as α increases, consumers' ex post values $\tilde{V} = V + Z$ are influenced more by (known) V and less by (hidden) Z. We also assume that consumers have no outside option.

We find that, unlike Propositions 1.5 and 1.6, the equilibrium price may or may not increase as pre-search quality information improves. In particular, if the number of sellers is sufficiently small, then p^* decreases in α.

Proposition 1.7. *There exists an integer $n^*(\alpha)$ such that a marginal increase in α increases p^* if and only if $n \geq n^*(\alpha)$.*

Recall that the equilibrium price increases when H becomes more dispersive (Proposition 1.4). Importantly, the result depends only on H, not separately on F and G. This means that a decrease in α has two effects on p^*. On the one hand, it spreads out G, which, as shown in Proposition 1.6, tends to increase p^*. On the other hand, it reduces dispersion of F, which may translate into lower dispersion of H and, therefore, push down p^*.[20]

[20]Unlike G, H may not become more dispersive when F becomes more dispersive. This is, of course, because of asymmetry between F and G. In particular, the upper truncation

The following lemma, which establishes the relationship between α and dispersion of H, is useful to understand the specific pattern in Proposition 1.7.

Lemma 1.3. *There exists $w^*(< \infty)$ such that the slope of $H^{-1}(a)$ decreases in α if and only if $a > H(w^*)$.*

This lemma states that an increase in α has disproportionate effects on dispersion of H: the left portion of H becomes less dispersive, while the right portion grows more dispersive. Recall that $W = V + \min\{Z, z^*\}$. Since $\min\{Z, z^*\}$ is bounded above by z^*, if w is rather large, $H(w)$ is mostly determined by the behavior of F. Since F becomes more dispersive in α, $H(w)$ also does so for w large. If w is rather small, $H(w)$ is affected by all three V, Z, and z^*. The effects of the first two cancel each other out, because $V + Z \sim N(0, 1)$. The last effect through z^*, however, makes H less dispersive, because z^* decreases in α (see equation (1.1) and the proof of Proposition 1.5).

When there are many sellers, the effective value of a consumer's purchased product is likely to exceed w^*. This implies that the equilibrium price p^* mainly depends on the right side of H (i.e., the region above w^*). As shown in Lemma 1.3, H grows more dispersive in α over the region. The opposite reasoning holds if there are few sellers. In either case, Proposition 1.7 follows from Proposition 1.4.

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structure of Z generates a probability mass for each V. This does not interfere in dispersion of Z being translated into that of W, but may between V and W. The result still holds if the density function f is decreasing over the relevant region, but not in general.

1.6 Asymmetric Sellers: Prices and Price Dispersion

In this section, we return to the general setting with asymmetric sellers and study some questions that arise in the presence of seller asymmetry.

1.6.1 Who Post Higher Prices?

In the presence of seller asymmetry, the most natural question is which sellers post higher prices. This has not been addressed thoroughly in the literature, partly because most theoretical studies restrict attention to the symmetric case and partly because of its complexity. We take one step forward by providing a sufficient condition under which one seller posts a higher price than another. We demonstrate the usefulness of our condition with a series of corollaries.

Proposition 1.8. *If $W_i - c_i$ dominates $W_j - c_j$ in the hazard rate order and the reverse hazard rate order,*[21] *then $p_i - c_i \geq p_j - c_j$.*

For the intuition, consider the duopoly case with no outside option. In this case, seller i's profit function is given by

$$\pi_i(p_i, p_j) = (p_i - c_i) \int H_j(w_i - p_i + p_j) dH_i(w_i).$$

If, hypothetically, H_j were degenerate at $\overline{w}_j$, then the integral would be equal to $1 - H_i(\overline{w}_j + p_i - p_j)$, and thus seller i's first-order condition would reduce to

$$\frac{1}{p_i - c_i} = \frac{h_i(\overline{w}_j + p_i - p_j)}{1 - H_i(\overline{w}_j + p_i - p_j)}.$$

[21] A random variable X_1 with distribution function F_1 dominates another random variable X_2 with distribution function F_2 in the hazard rate order if $f_1(t)/(1 - F_1(t)) \leq f_2(t)/(1 - F_2(t))$ for all t. Similarly, X_1 dominates X_2 in the reverse hazard rate order if $f_1(t)/F_1(t) \geq f_2(t)/F_2(t)$ for all t.

As H_i increases in the hazard rate order, the right-hand side decreases, which implies that the optimal price p_i increases. Similarly, if H_i were degenerate at $\overline{w}_i$, then seller i's first-order condition simplifies to

$$\frac{1}{p_i - c_i} = \frac{h_j(\overline{w}_i - p_i + p_j)}{H_j(\overline{w}_i - p_i + p_j)}.$$

Applying a similar argument, it follows that p_i increases as H_j decreases in the reverse hazard rate order. In the appendix, we prove that these two conditions suffice for the result in general (i.e., without hypothetical degeneracy assumptions).

Our first application of Proposition 1.8 concerns the relationship between marginal costs and markups. We show that otherwise symmetric sellers with higher marginal costs charge lower markups.

Corollary 1.1. *If $F_i = F_j$, $G_i = G_j$, $s_i = s_j$ and $c_i > c_j$ for some i and j, then $p_j - c_j \geq p_i - c_i$.*

Proof. Given Proposition 1.8, it suffices to show that $W_i - c_i$ rises in the hazard rate order and the reverse hazard rate order as c_i falls. The result follows from the log-concavity of H and $1 - H$: the former implies that $h(w_i + c_i)/H(w_i + c_i)$ increases, while the latter implies that $h(w_i + c_i)/(1 - H(w_i + c_i))$ deceases, as c decreases. □

It is a tempting conjecture that Weitzman index based only on the value distributions and search costs (i.e., $v_i + z_i^*$) would be closely tied with prices. Specifically, if p_i is equal to 0 for all i, consumers visit the sellers in the decreasing order of $v_i + z_i^*$. Since a seller with a higher index would attract more consumers, it is plausible that the seller would post a higher price. Our second corollary of Proposition 1.8 shows that this conjecture does not hold in general.

Corollary 1.2. *Suppose $F_i = F_j$, $c_i = c_j$, and $z_i^* = z_j^*$ for some i and j. If z_j dominates z_i in the likelihood ratio order,*[22] *then $p_j \geq p_i$.*

Corollary 1.2 shows that even if two sellers have the same Weitzman index (based on the value distributions and search costs) and share other characteristics, one seller may post a higher price than the other. Intuitively, Weitzman index captures only the average behavior of a distribution above a certain point. However, a seller's optimal price depends on the entire behavior of the distribution, which cannot be summarized by a single index. To be more concrete, suppose $p_i = p_j$, so that consumers are equally divided between the two sellers. In this case, seller j has relatively fewer consumers on the margin and, therefore, faces a stronger incentive to increase her price than player i, which ultimately leads to the outcome $p_j > p_i$.

Our final application of Proposition 1.8 illustrates the relationship between associated search costs and prices. For the same reason as above, it is plausible that sellers with lower search costs would post higher prices. Unlike in the previous case, we present an affirmative result for this conjecture. Specifically, we provide a sufficient condition under which prices are inversely related to search costs (i.e., if $s_i < s_j$, then $p_i > p_j$). Notice that, since Weitzman index is decreasing in search costs, this result

[22] A random variable X_1 with distribution function F_1 dominates another random variable X_2 with distribution function F_2 in the likelihood ratio order if $f_1(x)/f_2(x)$ rises in x. The likelihood ratio order is equivalent to the monotone likelihood ratio property. Even if z_j dominates z_i in the likelihood ratio order, z_i^* can be equal to z_j^*, because z_k^* depends not only on G_k but also on s_k (see equation (1.1)).

also shows that the index, despite Corollary 1.2, may still provide useful guidance for price rankings.

Corollary 1.3. *Suppose all sellers are identical except that* $s_1 < ... < s_n$*, and the common density function* $f(v)$ *is such that* $-f'(v)$ *is positive and log-concave in* v *for all* $v > u_0 - z^*$*, where* $z^* \equiv \max_i z_i^*$*. Then,* $p_1 \geq ... \geq p_n$*.*

Intuitively, when the sellers differ only in associated search costs, the difference is unidimensional and, therefore, can be fully captured by a single-valued Weintzman index. The result, although clearly limited, is useful because various common distributions in the exponential family, including Gaussian, Gumbel, and Laplace distributions, have the right tails that satisfy the necessary distributional properties.

1.6.2 Search Costs

We now study the effects of search costs in the presence of seller asymmetry. We focus on two questions, who benefits from a reduction in search costs and what is the relationship between price dispersion and search costs. For tractability, we restrict attention to the simplest duopoly environment where there is no outside option and the two sellers differ only in their marginal costs. We assume that seller 1's marginal cost is strictly lower than seller 2's ($c_1 < c_2$), which implies that in equilibrium seller 1 charges a lower price than seller 2 ($p_1 < p_2$).[23]

Our first result shows that a reduction in search costs is beneficial to the disadvantaged seller (with a higher marginal cost).

[23] See the proof of Proposition 1.9 in the appendix.

Proposition 1.9. *Demand for seller* 1 *(*$D_1(\mathbf{p})$*) increases in* s*, while demand for seller* 2 *(*$D_2(\mathbf{p})$*) decreases in* s*.*

Notice that this result counters a common belief that more efficient firms will flourish, while less efficient firms will eventually vanish, as search costs decrease. Consumers search more actively (visit more sellers) when search costs are lower. In particular, more consumers make a purchase decision after visiting both sellers. This is more beneficial to seller 2, who charges a higher price and, therefore, attracts less fresh visitors.[24]

Proposition 1.9 suggests that the disadvantaged seller has a stronger incentive to lower the price as search costs increase. This generates a unique implication for the relationship between price dispersion and search costs, as formally stated in the following proposition.

Proposition 1.10. *The relative markup ratio* $(p_2 - c_2)/(p_1 - c_1)$ *decreases in* s*. If* $c_2 - c_1$ *is sufficiently large, then the absolute price difference* $p_2 - p_1$ *also decreases in* s*.*

The result indicates that an increase in search costs may reduce price dispersion. This is contrary to a well-established insight in search theory that price dispersion is a symptom of search frictions and market prices are more dispersed

[24] Armstrong (2016) finds a similar result in an environment where one seller is "prominent" and, therefore, visited by all consumers first. A reduction in search costs, which induces more consumers to visit both sellers, is beneficial to the non-prominent seller. Unlike our result, his result builds upon an asymmetric equilibrium in the symmetric environment, which exists because it is assumed that all consumers have an identical prior value (i.e., F_i is degenerate for each i) and prices are unobservable before search.

when there are more search frictions (see, e.g., Burdett and Judd, 1983; Stahl, 1989). Again, the result is driven by the fact that prices are observable to consumers and the role of search is only to gather more information about their values.

We conclude this section with another consequence of Proposition 1.9. When the sellers are symmetric, market prices necessarily decrease as search costs s increase (Proposition 1.5). If the sellers are asymmetric, the result may not apply to some sellers. In particular, the advantaged seller (seller 1) may increase her price when s increases. This occurs when an increase in search costs discourages lots of consumers from visiting seller 2 after seller 1, and thus demand for seller 1 increases sufficiently fast. In this case, seller 2 has an even stronger incentive to lower her price, while seller 1 may find it more profitable to increase her price. This also means that, unlike the symmetric case where all firms' profits fall as search costs increase (see Proposition 1.5), some firms may benefit from an increase in search costs and obtain higher profits.

1.7 Conclusion

We study an oligopoly model in which the sellers advertise their prices and consumers conduct optimal sequential search. We derive a simple condition that fully summarizes consumers' search outcomes and allows us to reformulate the pricing game as a familiar discrete-choice problem. We also provide sufficient conditions under which there exists a unique pure-strategy market equilibrium. Based on the characterization, we obtain a set of results that shed new light on the effects of search frictions on market prices. We show that a reduction in the value of search increases market prices, whereas providing more information before consumer search may or may not increase market prices. We also provide a sufficient condition under which one seller posts a higher price than another and demonstrate that a reduction in search costs may lead to more price dispersion in the presence of seller asymmetry.

Many interesting questions remain open. To name a few, we assume that all sellers are fully committed to their advertised prices. However, hidden fees, in various forms, are prevalent in reality. How does their potential presence affect consumer behavior and sellers' pricing incentives?[25] We consider the case where each seller sells only one product, but it is the exception rather than the rule. How should a multi-product seller price (or position) his products? Should the seller choose an identical price, or introduce difference prices, for ex ante symmetric products? If the products are asymmetric, which product should the seller make prominent and how?[26] We plan to address these and other related problems in the future.

[25]See Ellison (2005) and Dai (2016) for some developments along this line.

[26]See Gamp (2016) and Petrikaitė (2016) for some related problems.

CHAPTER 2
CONSUMER SEARCH AND OPTIMAL PRICING UNDER LIMITED COMMITMENT

2.1 Introduction

Advertised prices are often different from final prices in many markets. For example, stores may charge unexpectedly high shipping and installation fees when consumers shop for furniture and appliances. Moreover, online shopping usually involves shipping and handling fees, which may be observed only after adding a product to a shopping cart or filling out all the relevant shipping and payment information. According to Ellison and Ellison (2009), on Pricewatch.com, "shipping charges grew to the point that it was not uncommon for firms to list a price of $1 for a memory module and inform consumers of a $40 shipping and handling fee at check out." Likewise, a report in the Washington Post documents a case in which one consumer expected a $25 ride from Uber, but a "peak surcharge" led to a $120 bill. In a separate case, Airbnb listings included $25 cleaning fees were not disclosed until well into the booking process. These phenomena are the major reasons that consumers abandon shopping carts after expressing interest in purchasing a product. A 2009 Forrester survey finds that 44% of Web shoppers said that they did not complete an online transaction because shipping and handling costs were too high.

In this paper, I present a consumer search model in which there is uncertainty regarding a seller's commitment to the advertised price. Specifically, I analyze a game in which a seller posts his price and consumers decide whether or not to visit the seller after observing the posted price. Once a consumer visits the seller, she learns both the actual price and her valuation of the product. There are two types of sellers: the commitment type and the non-commitment type.[1] The commitment type seller always charges the advertised price, while the non-commitment type can deviate from the advertised price and charge a different price. The seller's type is his private information.

This model unifies two strands of consumer search literature. One studies environments in which prices are unobservable,[2] while the other considers cases in which prices are observable.[3] The former is the traditional approach, while the later has been receiving growing attention recently because the Internet makes price information more accessible. These two approaches can be partly interpreted as two extreme cases in my model. The first approach can be interpreted as the case where the seller is the non-commitment type for sure. In other words, it is common knowledge that

[1]Commitment power may come from several sources. First, the seller may interact with consumers repeatedly. If high surcharges over the advertised price deters consumers from purchasing, fear of building a bad reputation for not committing to the advertised price could contribute to the formation of commitment power. Second, with the popularity of online shopping and the sharing economy, sellers are increasingly likely to be individuals rather than large firms, and may therefore be subject to lie aversion, as documented in the behavioral and experimental literature (Gneezy (2005), Hurkens and Kartik (2009). Third, platform providers and consumer protection associations may have an incentive to punish sellers who are not committed to advertised prices, even if they cannot afford to monitor every transaction. Fear of being punished can also justify the commitment type seller in the market. Nevertheless, as the above examples of Pricewatch, Uber and Airbnb make clear, not all sellers are of the commitment type.

[2]See, e.g., Wolinsky (1986), Anderson and Renault (1999), Armstrong et al. (2009) and Chen and He (2011).

[3]See, e.g., Armstrong and Zhou (2011), Shen (2015), Haan et al. (2015) and Choi et al. (2016).

the advertised price is purely cheap talk and is therefore not related to the actual price. The second approach can be understood as the case where the seller is the commitment type for sure, in which case the advertised price surely coincides with the actual price.

There is a continuum of separating equilibria, but all of them are essentially identical to the market outcome where the seller has no commitment power. In any separating equilibrium, the advertised price fully reveals the seller's type. This means the non-commitment type does not benefit from limited commitment at all and therefore charges the same price as when his type is known to consumers. In the mean time, due to the non-commitment type seller's incentive to pool with the commitment type, the commitment type cannot credibly advertise a profitable price.

There is also a continuum of pooling equilibria with different pooling prices. Unlike separating equilibria, lower profitable market prices can be supported in pooling equilibria. Limited commitment may yield a even lower market price than full commitment, because of the seller's concern about consumers' inferences. Intuitively, such a price can be supported in equilibrium if consumers believe that the deviating seller is the non-commitment type for sure. Moreover, increasing the degree of commitment makes each advertised price more credible and thus allows a higher price to be supported as an equilibrium advertised price because even though the advertised price is higher, consumers are less likely to encounter surcharges over the advertised price.

A natural response to limited commitment power is to regulate the extent to which the actual price can deviate from the advertised price. Recently, several platform providers have adopted this form of regulation. For example, Pricewatch.com mandated that all firms offer UPS ground shipping for a fee no greater than an item-specific amount (i.e., $11 for memory modules). Ebay also has its own policy on maximum shipping and handling fees, while Airbnb does not permit cleaning fees to exceed a certain amount.

I find that stricter regulation (that is, allowing a smaller maximum price difference) may or may not improve market efficiency. On the one hand, stricter regulation limits the non-commitment type seller's deviation from his posted price and, therefore, has a direct effect of lowering the market price. On the other hand, stricter regulation relaxes the commitment type seller's fear of being perceived as a non-commitment type and induces the commitment type to charge a higher price and thus raise the market price. Consequently, there exists an interior optimal regulation level for consumer surplus and social welfare, at which the commitment type seller is willing to commit to a low advertised price, while the non-commitment type seller cannot surcharge arbitrarily.

I also investigate the impact of heterogenous levels of commitment on market outcomes. In particular, I examine if there are multiple sellers, in which order consumers visit the sellers and whether consumers prioritize visiting the seller who is more likely to be the commitment type. I show that if one seller has full commitment power, while the other seller has limited commitment, consumers unambiguously visit the seller with full commitment power first. In contrast, when both sellers have limited commitment, a higher degree of commitment does not dictate consumers' search order. The difference between this pair of results originates from the observation that the seller with full commitment power can credibly advertise a low price while the seller with limited commitment is unable to do so.[4]

I then demonstrate that the main insights carry through to an environment with ex-ante heterogenous consumers. The only difference lies in the effects of the search cost, on which the two existing frameworks on consumer search generate opposite results. When the seller is the non-commitment type for sure, the equilibrium price rises as search cost rises due to the "hold-up effect" and the "selection effect": the seller can not influence consumers' visiting decisions and, therefore, exploits visiting consumers as much as possible. In contrast, when the seller is the commitment type for sure, the equilibrium price falls as search cost rises due to the "directed search effect": when search cost rises, the seller can retain consumers by advertising a lower price.[5] In the current model with ex-ante homogenous consumers, conditional

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[4]This result links this paper to the literature on prominence in consumer search, where prominence refers to a seller who is sampled by all consumers first. See, e.g., Arbatskaya (2007), Zhou (2011), Armstrong et al. (2009) and Armstrong and Zhou (2011). The literature has focused on the environment in which a prominent seller has an incentive to lower his price below all other sellers to justify his prominent position. This paper contributes to this literature in two aspects. First, full commitment power endogenizes the prominent position in the market, while in the literature, prominence acts as a coordination device, and every seller can be made prominent. Second, commitment power provides an additional source of prominence other than the existing ones in the literature: commission payments, price comparison websites, and existing suppliers (Armstrong and Zhou (2011)).

[5]This result is also related to the price advertising literature where it is shown that firms must advertise lower prices to get consumers' visits when search cost rises. See, Butters (1977), Stegeman (1991), Robert and Stahl (1993), Konishi and Sandfort (2002) and Anderson and Renault (2006).

on visiting, a marginal increase in search cost neither influences consumers' purchasing decision nor provides additional information about match values. Hence both the "hold-up effect" and the "selection effect" vanish. The "directed search effect" from both types of seller works to ensure that consumers still visit the seller as search cost rises. It follows that prices decrease with respect to search cost. Opposingly, with ex-ante heterogenous consumers, a marginal increase in search cost reduces the measure of consumers visiting the seller and implies that the visiting consumers have higher ex-ante valuations of the product. Therefore, how the equilibrium price set varies with search cost is mainly governed by the "selection effect". Specifically, the commitment type seller faces a tradeoff between the "selection effect" when pooling with the non-commitment type and when being perceived as a non-commitment type seller. Which "selection effect" is more severe as search cost rises depends on the level of commitment and the magnitude of the search cost. Prices are often non-monotone with respect to the search cost.

One interpretation of this model is to regard the non-commitment type seller's capability to advertise a different price than the actual price as a tool of obfuscation. Janssen and Non (2008), Ellison and Wolitzky (2012), and Wilson (2010) model obfuscation as a deliberate attempt to increase consumers' search costs in order to avoid Bertrand competition. Their approach examines sellers' efforts to influence the accessibility of product information, while I investigate sellers' endeavors to manipulate the transparency of information. As the internet makes information more accessible than before, transparency of information is an aspect that should not be overlooked.

The literature contains other approaches to account for the difference between advertised prices and actual prices. Gabaix and Laibson (2006) point out that shrouded attributes by sellers include two mutually exclusive categories: avoidable add-ons and unavoidable surcharges. They analyze the case of avoidable add-ons with boundedly rational consumers and make a conjecture that regulating the price difference of add-on products may hurt consumers. Ellison (2005) also studies add-on pricing with consumers who differ in their sensitivity to price differences or likelihood of switching between firms. Price differences in the first category are caused by the quality difference between a product with the add-on and a product without the add-on. This paper addresses the second category in which price differences are caused by seller's deviation from the advertised prices. With the prevalence of online shopping, unavoidable surcharges like shipping and handling fees become an increasingly relevant issue in daily life.

The rest of the paper is organized as follows. Section 2.2 introduces the model and discusses the case of complete information. Section 2.3 analyzes the monopoly model with incomplete information regarding the sellers' type. Section 2.4 studies the effect of regulation on equilibrium prices and welfare. Section 2.5 investigates the duopoly model with sellers differ in levels of commitment. Ex-ante heterogenous consumers are introduced in Section 3.2. Section 3.6 concludes. All proofs and omitted details are presented in the Appendix.

2.2 Model

2.2.1 Environment

The market consists of one seller and a unit mass of consumers. The seller supplies one product to different consumers, each with no fixed cost and constant marginal cost $c(\geq 0)$. At the beginning of the market, the seller posts a price costlessly, denoted by p. The seller may charge a different final price, p', after a consumer visits. $D(p, p')$ is the measure of consumers who eventually purchase from the seller. The seller's profit is then defined as $\pi(p, p') \equiv D(p, p')(p' - c)$. The seller maximizes his profit $\pi(p, p')$.

The seller can be one of two possible types: commitment type and non-commitment type. The commitment type seller charges the posted price, i.e. $p' = p$. The non-commitment type seller is not restricted by the posted price. The price difference between the final price and the posted price can be interpreted as the seller's adoption of additional fees like shipping and handling fees. The seller is also allowed to offer a lower final price than the posted price, which can be understood as a discount or a coupon. The prior probability that the seller is of commitment type is given by μ. The seller's type is his private information.

Each consumer has unit demand. A (representative) consumer's random utility for the seller's product is given by Y, which is revealed to a consumer only after she visits the seller. Let y denotes the realization of a consumer's value for the product. Y is drawn according to the distribution function G over the interval $[\underline{y}, \bar{y}]$, independently across consumers. I allow each support to be infinite and I assume that G has continuously differentiable density g.

I maintain the following regularity assumption about the distribution function G through the paper.

Assumption 2.1. $1 - G$ *is log-concave.*[6]

It is well-known that log-concavity is satisfied with various well-behaved distributions (see, e.g., Bagnoli and Bergstrom, 2005) and is an appropriate distributional assumption in various contexts. I fully utilize Assumption 2.1 to ensure the existence and uniqueness of equilibrium in the complete information cases. Assumption 2.1 also guarantees that the problem is well-behaved in the general setup.

Each consumer needs to incur search cost $s(> 0)$ to visit the seller in order to gauge her match value and discover the actual price. This mainly represents the transportation cost or the opportunity cost of time spent visiting the seller and testing the product.

A consumer's ex post utility depends on her value for the purchased product y, its actual price p', and the search cost s if she decides to visit. Specifically, if a consumer visits the seller and eventually purchases from him, then her ex post utility is equal to

$$U(y, p') = y - p' - s.$$

Each consumer can leave the market at any point without making a purchase. A leaving consumer takes an outside option $\underline{u}$. Each consumer is risk neutral and maximizes

[6] g is log-concave is a sufficient condition for Assumption 2.1.

her expected utility.

The market proceeds as follows. First, the seller announces price p. Then, each consumer decides whether to visit the seller based on available information (p). If a consumer decides to visit, she observes (y, p') and makes her final purchase decision. I study the sequential equilibrium of this game. I first characterize consumers' behavior given any price vector and then analyze the seller's optimal pricing decision and market equilibrium.

2.2.2 Consumer Behavior

Each consumer makes two decisions: whether to visit and whether to purchase. The decision to visit depends on the belief of the final price upon observing the posted price. The belief is pinned down in equilibrium. The decision to purchase relies on the final price if the consumer visits the seller. Let $H_p(p')$ denotes the belief of the final price p' when a consumer sees posted price p.

Given the seller's posted price p and final price p', consumers' visiting and purchasing decisions are summarized by the following inequalities:

(i) Visit strategy: The consumer visits the seller if and only if $r \geq \underline{u}$, where r is the value such that

$$r = -s + \int_{-\infty}^{\infty} \int_{\underline{y}}^{\bar{y}} \max\{r, y - p'\} dG(y) dH_p(p'). \tag{2.1}$$

(ii) Purchase strategy: The consumer purchases from the seller if and only if $y - p' \geq \underline{u}$.

When a consumer makes her visiting decision, she compares the reservation value for seller's product and the outside option. The reservation value is defined such that a consumer is indifferent between obtaining utility r immediately and visiting seller (which requires paying search cost s and gives her an option to choose between r and $y - p'$). As the distribution of the match value, belief of the final price and the search cost are the same across consumers, all consumers make the same visiting decision. Equation (2.1) also shows that consumers tend to visit the seller less likely as the unit search cost s increases: as s increases, r decreases. In addition, if belief of the final price is degenerate at price p, the reservation value can be written explicitly as $r = y^* - p$ due to the additive-utility specification, where y^* is defined by $s = \int_{y^*}^{\bar{y}} [1 - G(y)]dy$. It follows that as s increases, y^* decreases.

2.2.3 Complete Information

When there is complete information over seller's type, the belief of the final price is degenerate. Consumers compare $y^* - p$ and $\underline{u}$ to make their visiting decisions. Once all consumers visit the seller, the optimal price to charge is given by $p_N \triangleq \operatorname{argmax}_p p[1 - G(p + \underline{u})]$, which is uniquely defined due to Assumption 2.1. This is also the price that is always charged by the non-commitment type seller. Here I focus on the interesting case where $y^* - p_N < \underline{u}$.[7] This condition implies that when the seller is the non-commitment type for sure, consumers do not visit the seller and he obtains zero profit. No matter what price the seller posts, consumers rationally expect him

[7] In the other parameter region ($y^* - p_N \geq \underline{u}$), consumers visit the seller for sure regardless of the seller's commitment power. It follows that both types of seller charge the same price and earn the same profit. The case of incomplete information is also equivalent to the two corner cases. Market inefficiency does not present under this parameter region.

to charge p_N, which exceeds their visiting threshold. There is a hold-up problem. On the contrary, when the seller is the commitment type for sure, it is optimal for him to post and charge a price such that consumers are indifferent between visiting or not: $p_C \triangleq y^* - \underline{u}$. I assume $p_C - \underline{u} \geq c$ to ensure that the seller obtains non-negative profit $\pi(p_C, p_C)$. Trade occurs and consumers earn surplus. Commitment power resolves the hold-up problem. A simple comparison yields $p_C < p_N$.

2.3 Equilibrium with Incomplete Information

In this section, I analyze the market equilibrium when the seller's type is uncertain. Under incomplete information, price advertisement serves as a signaling device. A distinction between two types of seller is that the posted price from the non-commitment type seller is a cheap talk message while the posted price from the commitment type seller is a "costly" signal since he charges the price he posts, which is payoff relevant. Depending on whether the posted price is informative, the pure strategy equilibrium naturally falls into two categories: separating and pooling equilibrium. I examine each in turn, and then investigate the effect of search cost, along with a brief discussion on equilibrium refinement. As is typical in signaling games, many equilibria arise here due to lack of restriction on consumers' beliefs off-the-equilibrium path. To simplify the analysis, I put a restriction on consumers' beliefs off-the-equilibrium path so that they consider any deviating price to be posted by the non-commitment type seller. This is one of the beliefs that support the largest set of equilibria. In section 2.3.4, I apply several refinements to see if some equilibria are more reasonable than the others.

2.3.1 Separating Equilibrium

In any separating equilibrium, because consumers can distinguish the commitment type and the non-commitment type from the posted price, the non-commitment type faces the same problem as in section 2.2.3. Hence the non-commitment type charges p_N and gets zero demand. It follows that the commitment type seller has to post and charge a price higher than p_C in a separating equilibrium. This is because if the commitment type seller posts a price lower than p_C, the non-commitment type can attract consumers and obtain higher profit by mimicing this price. The following proposition summarizes these insights.

Proposition 2.1. *There is a continuum of separating equilibria. In any separating equilibrium, the commitment type seller posts and charges $p > p_C$ and the non-commitment type seller posts $p' \neq p$. No matter what price the non-commitment type posts, he charges p_N.*

Despite the fact that there is a continuum of equilibria, all the equilibria are outcome equivalent to the one when the seller is non-commitment type for sure (i.e., $\mu = 0$). As in a typical separating equilibrium, the non-commitment type does not benefit from the uncertainty over seller's type (enhancing commitment power in the market), while the commitment type is hurt by the presence of the non-commitment type.

Proposition 2.1 argues that when the seller is commitment type for sure, introducing a small degree of uncertainty renders the market inefficient. It creates a discontinuity in the sense that market price jumps from p_C to p_N as $\mu = 1$ changes to $\mu = 1 - \epsilon$. The market with minimally limited commitment works as if there is no commitment power at all. This conclusion depends on the restriction of separating equilibria. As shown in the next section, unlike the inefficiency result from the separating equilibrium, the pooling equilibrium exhibits a different welfare implication.

2.3.2 Pooling Equilibrium

In a pooling equilibrium, upon observing posted price p, consumers know that there is μ chance that the seller is the commitment type and p is the final price, while there is $1 - \mu$ chance that the seller is the non-commitment type and charges a different final price p_N. The following result describes what happens in a pooling equilibrium.

Proposition 2.2. *There exists $\hat{\mu}$ such that for $\mu \geq \hat{\mu}$, a continuum of pooling equilibria exists, where $\hat{\mu}$ is defined by*

$$\underline{u} = -s + \hat{\mu}\ \mathbb{E}\max\{\underline{u}, y - c\} + (1 - \hat{\mu})\mathbb{E}\max\{\underline{u}, y - p_N\}. \tag{2.2}$$

For each μ, there exists $\bar{p}(\mu)$ such that, $\forall p \in [c, \bar{p}(\mu)]$, there exists an equilibrium in which both types of seller post p and the non-commitment type seller charges $p_N \geq p$, where $\bar{p}(\mu)$ is defined by

$$\underline{u} = -s + \mu\ \mathbb{E}\max\{\underline{u}, y - \bar{p}(\mu)\} + (1 - \mu)\mathbb{E}\max\{\underline{u}, y - p_N\} \tag{2.3}$$

In addition, $\bar{p}(\mu)$ increases in μ.

Minimum level of commitment, $\hat{\mu}$, is needed to ensure the existence of pooling equilibrium. Since consumers do not visit the seller when there is no commitment power, tiny amount of commitment is not enough to attract them to visit. $\hat{\mu}$ is defined such that consumers are indifferent between taking the outside option and visiting when they expect the commitment type to charge the marginal cost (lowest price that is acceptable to the seller) and the non-commitment type to charge the optimal price (p_N).

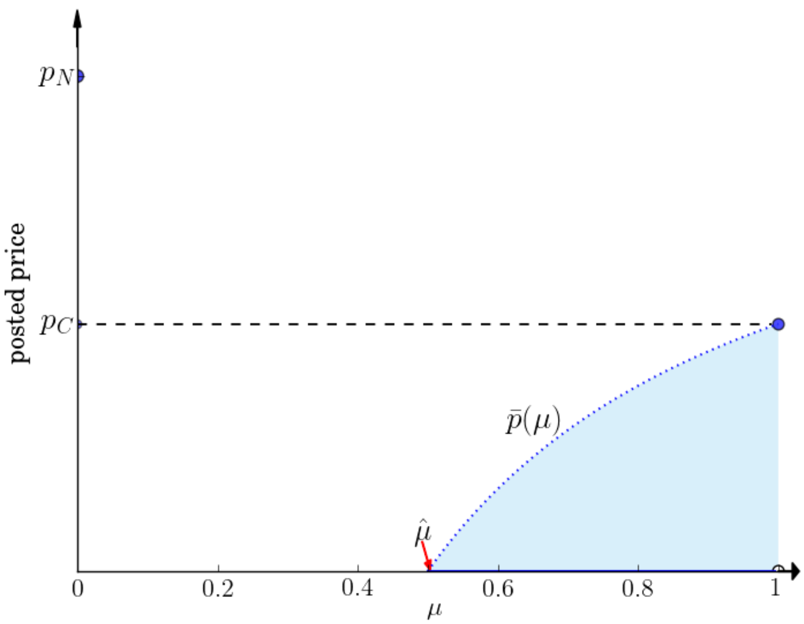
The pooling equilibrium price set is determined by comparing equilibrium profit and deviation profit for both types of seller. Deviation profit is zero for both types of seller as when deviation occurs, the seller is perceived as the non-commitment type.[8] The upper bound of the pooling price set, $\bar{p}(\mu)$, is pinned down by both types of seller's incentive to deviate. Any price above $\bar{p}(\mu)$ deters consumers to visit and delivers zero profit to both types of seller. The lower bound, c, comes solely from the commitment type's incentive constraint. Any price lower than c attracts consumers to visit but delivers negative profit to the commitment type. In contrast, the non-commitment type is free to deviate from this price once consumers visit him.

Higher level of commitment enables the seller to post and charge a higher price in the pooling equilibrium. As μ rises, each posted price is more credible and consumers are less likely to be charged additional amount. Consumers' reservation

[8]There are other off-equilibrium path beliefs that can support the pooling equilibrium. For instance, if a seller deviates to a price above p_N, then the belief associates with this deviating price is not restricted. This is because the price is so high that no consumer will visit the seller, no matter what type the seller is. As this is not the major concern of the paper, I focus on the belief stated in the main text for simplicity.

value for the seller thus increases given each posted price. $\bar{p}(\mu)$ increases to make consumers indifferent between visiting and taking the outside option (to keep the equality of equation (2.3)).

Figure 2.1: Pooling Equilibrium varies with Level of Commitment



The set of equilibrium posted prices given level of commitment, μ, with $F, G \sim N(0,1)$ (standard normal distribution), $\underline{u} = 0$ and $c = 0$. $y^* = 0.25$, $p_N = 0.5$ and $\hat{\mu} = 0.5$.

The non-commitment type seller benefits from pooling with the commitment type as partial commitment power mitigates the seller's hold-up problem. In contrast, the commitment type suffers from the presence of the non-commitment type.

Whatever price the commitment type posts, consumers expect chances to be charged some higher price. Therefore, the commitment type has to cut his price in order to attract consumers to come. In a pooling equilibrium all consumers visit the seller and the non-commitment type seller charges the optimal price. This indicates that the non-commitment type seller obtains higher profit when he pools with the commitment type than when he obtains full commitment power.[9] Consumer surplus stays constant at the upper bound of the pooling equilibrium. For each price within the equilibrium price set, consumer surplus increases as level of commitment rises since they are less likely to be charged additional amount. Consumers are better off when there is more commitment power in the set sense.

2.3.3 The Effect of Search Cost

In this section, I study the impact of increasing search cost on equilibrium prices. The following lemma summarizes the results when there is complete information of seller's type.

Lemma 2.1. *If the seller is non-commitment type for sure (i.e., $\mu = 0$), then the unique equilibrium price, p_N, stays constant as search cost, s, increases; if the seller is commitment type for sure (i.e., $\mu = 1$), then the unique equilibrium price, p_C, decreases as search cost, s, increases.*

Commitment power plays a crucial role in the direction of this comparative

[9]In a similar environment, Haan et al. (2015) shows that a seller whose price is hidden would like to advertise his price to the consumers in order to gain higher profit. It can be interpreted as the seller with no commitment power would like to gain full commitment power.

statics exercise. Conditional on visiting the seller, the decision to purchase is independent of the magnitude of search cost. Therefore, the optimal price to charge conditional on visiting is also independent of search cost. As the non-commitment type seller is unable to use price to dictate consumer's decision to visit, the equilibrium price p_N stays constant with respect to search cost. In contrast, the commitment type seller is able to reduce price to direct consumers' visiting decisions, mitigating the impact of rising search cost. I call this the "directed search effect".

The first part of Lemma 2.1 is related to the same comparative exercise in Wolinsky (1986) and Anderson and Renault (1999). They show that in an oligopoly environment with no commitment power, equilibrium price goes up when search cost rises. With multiple sellers, higher search cost increases the cost of exploring other options after visiting the current seller. Thus the current seller can extract more surplus from the visiting consumers. I call this the "hold-up" effect. This effect is absent in the current model with monopoly.[10] Haan et al. (2015) also conducts the same exercise in a duopoly environment with ex-ante heterogenous consumers. There another effect to drive the price up is the "selection" effect: when search cost rises, the visiting consumer has higher ex-ante valuation for the product and thus is less likely to leave. In section 3.2 I study a model with ex-ante heterogenous consumers under limited commitment and investigate how "selection" effect interacts with limited commitment.

[10]The "hold-up" effect can be also introduced into the current model by inserting quitting cost to make going back to the outside option more costly after exploring the seller.

The second part of Lemma 2.1 is related to the same comparative exercise in Armstrong and Zhou (2011), Haan et al. (2015) and Choi et al. (2016). They find that in an environment with full commitment power (observable price), equilibrium price always goes down with search cost. Even when the "hold-up effect" and the "selection effect" both present, the "directed search effect" dominates both of them.

The following lemma summarizes how search cost affects the equilibrium price set with uncertainty over seller's type.

Proposition 2.3. *For each μ, the upper bound of the pooling equilibrium price set, $\bar{p}(\mu)$, decreases as search cost, s, increases. The minimum level of commitment power, $\hat{\mu}$, increases in search cost.*

As the separating equilibrium is equivalent to the case of no commitment, the effective price stays constant in search cost according to Lemma 2.1. The lower bound of the pooling equilibrium is also independent of the search cost as it equals marginal cost. The upper bound of the pooling equilibrium is the highest posted price such that consumers are willing to visit the seller. Therefore, the upper bound decreases to compensate for higher search cost. Moreover, $\hat{\mu}$ increases in search cost because when search cost rises, visiting becomes less attractive than taking the outside option directly at the original level of $\hat{\mu}$. Higher level of commitment is required to make visiting desirable. With absence of the "hold-up effect" and the "selection effect", the "directed search effect" alone leads to this result.

2.3.4 Equilibrium Refinement

As in other signaling games, this game suffers from equilibrium multiplicity. In order to see whether one equilibrium is more prominent than the others, several refinement criteria are applied.

I first apply the Intuitive Criterion by Cho and Kreps (1987) and show that all the separating and pooling equilibria survive. In the current context, the Intuitive Criterion regards a proposed equilibrium posted price p as "intuitive" if there does not exist another price $\tilde{p}$ to deviate to for which the commitment type is better off while the non-commitment type is worse off, when consumers believe that the deviation comes from the commitment type. I first argue that any separating equilibrium satisfies the Intuitive Criterion. In any separating equilibrium, both types of seller obtain zero profit. If there is a price such that the commitment type is better off by deviating, then the non-commitment type is also better off by mimicing this price, as it must attract consumers to visit and deliver positive profit. Therefore, such a deviating price can never exist. I then argue that any pooling equilibrium is intuitive. Let $p^* \in [c, \bar{p}(\mu)]$ be an equilibrium posted price, then the range of tempting deviating price for the commitment type is $[p^*, \bar{p}(\mu)]$. None of the price in this range can make the non-commitment type worse off.

Conversely, applying the Undefeated equilibrium by Mailath et al. (1993) removes all the equilibria that are inefficient from the seller's point of view and delivers a unique outcome. In short words, the Undefeated equilibrium is equivalent to the notion of Pareto efficiency for both types of seller in the current setting. The idea of how this refinement works is as follows: consider an equilibrium price p^* and a deviating price $\tilde{p}$, if $\tilde{p}$ delivers higher profits for both types of seller than p^* does, then the refinement requires consumers to form the same beliefs as those in the deviating equilibrium. If this belief is different from the one in the proposed equilibrium, then the original equilibrium is defeated. In the current setup, $\bar{p}(\mu)$ delivers highest profit to the commitment type seller among all pooling equilibrium prices while the non-commitment type is indifferent among all prices. In addition, all pooling equilibrium deliver higher profits than all separating equilibrium. Therefore, the only equilibrium that is undefeated is the pooling equilibrium with $\bar{p}(\mu)$ as the posted price.

Even though the Undefeated equilibrium selects the unique equilibrium, it only considers the efficiency among both types of seller and ignores consumers' welfare. In fact, the unique equilibrium it selects delivers lowest consumer surplus. I thus focus on characterizing the whole equilibrium set in the following analysis, with paying special attention to the upper and lower bound of the posted price set, as the first one delivers highest profits to both types of seller and the second one generates the highest volume of trade and highest consumer surplus.

2.4 Regulation

In this section I study the impact of a platform provider's regulation on market behavior. A natural form of regulation to consider is a restriction on the gap between advertised prices and actual prices.[11] Intuitively, as the non-commitment type seller's

[11]Another form of regulation is information disclosure. For example, the platform provider could require the seller to display final price on the search page and inform the

ability to deviate from the advertised price induces market inefficiency, one might think that restricting this ability could fix the inefficiency. It is shown in this section that, however, regulation does not necessarily enhance market efficiency.

Regulation requires the maximum allowed difference between advertised prices and actual prices to be smaller than Δ; i.e., $p' - p \leq \Delta$. At the beginning of the game, the platform provider announces Δ, then the game proceeds as in previous sections, with common knowledge among the players that the seller will get unlimited punishment if the price difference exceeds Δ.[12] I normalize outside option, $\underline{u}$, and marginal cost, c, to 0 for simplicity.

When there is complete information over seller's type, the commitment type is not influenced by regulation, as he behaves as if he is under the strictest regulation, $\Delta = 0$. The non-commitment type seller, on the other hand, benefits from regulation. The following lemma summarizes equilibrium prices with regulation.

Lemma 2.2. *If the seller is the non-commitment type for sure (i.e., $\mu = 0$), then the seller posts price p such that $p + \Delta = y^*$ and charges price y^*.*

Lemma 2.2 shows that regulation grants full commitment power to the non-commitment type seller. In equilibrium, the seller posts the highest price such that consumers are willing to visit, as consumers know regulation prohibits the seller from

consumers about the policy. By doing this, the non-commitment type no longer exists in the market. The effect of this regulation depends on the original market condition, as discussed in section 2.3.2.

[12]Adopting other forms of regulation such as change absolute price difference to relative percentage price difference, or relate the amount of punishment to the magnitude of deviation does not change the results qualitatively.

charging more than the reservation value of the product. The seller then indeed finds it optimal to charge the reservation value given the regulation constraint.

This result depends on the fact that the posted price can be negative (lower than marginal cost). It allows situations, for instance, where $y^* = 10$ and $\Delta = \$20$. The non-commitment type seller posts $-\$10$ in equilibrium. Price promotion may be a common practice but large Δ is less realistic. It can be shown that if we restrict the posted price to be non-negative, then with $\Delta > y^*$, regulation is not effective, as the seller is unable to utilize it to dictate consumers' visiting decisions.

When the seller is the non-commitment type for sure, regulation acts as a commitment device and enhances market efficiency. When there is uncertainty over seller' type, however, the effect of a stricter regulation on welfare is mixed. The following lemma shows that strict regulation eliminates inefficient separating equilibria.

Lemma 2.3. *If there is uncertainty over seller's type (i.e., $\mu \in (0, 1)$), then*

(i) if $\Delta \leq y^*$ (strict regulation), separating equilibrium does not exist;

(ii) if $\Delta > y^*$ (weak regulation), the non-commitment type posts price p such that $p+\Delta = y^*$ and charges price y^* while the commitment type posts any price higher than y^*.

If a separating equilibrium exists under strict regulation, the non-commitment type behaves the same as under the case of $\mu = 0$ by posting $y^* - \Delta$ and charging y^* because the posted price is informative. It follows that the commitment type cannot be separated from posting a higher price, since the non-commitment type can post that price as well. Moreover, the commitment type is not willing to be separated by posting a lower price, since he gains more profit by posting $y^* - \Delta$, which is the highest price that guarantees consumers' visit. Therefore, there does not exist a pair of prices that do not induce deviation. The second part of Lemma 2.3 shows that weak regulation delivers full commitment profit to the non-commitment type but zero profit to the commitment type. The difference with strict regulation case is that under weak regulation the commitment type does not have incentive to mimic the non-commitment type anymore.

Proposition 2.4. *If there is uncertainty over seller's type (i.e., $\mu \in (0,1)$), then*

(i) there exist $\underline{p}^\Delta$ and $\bar{p}^\Delta$ such that $\forall p \in [\underline{p}^\Delta, \bar{p}^\Delta]$ can be supported as a pooling equilibrium posted price. Both $\underline{p}^\Delta$ and $\bar{p}^\Delta$ weakly decrease in Δ;

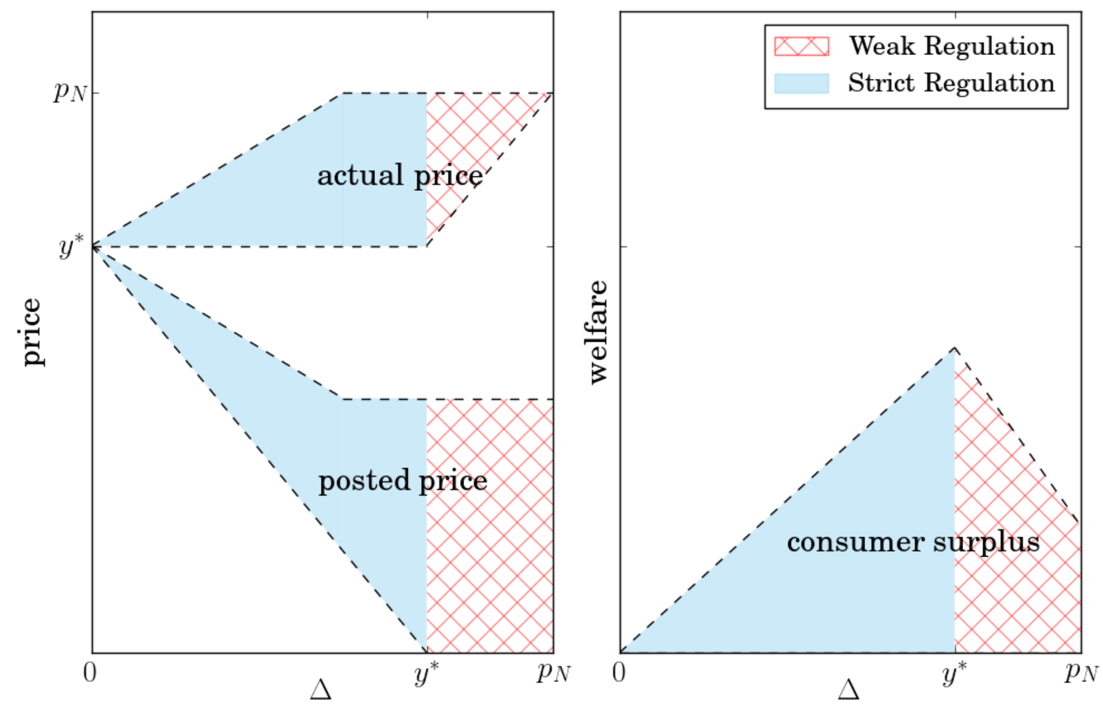
(ii) for each price $p \in [\underline{p}^\Delta, \bar{p}^\Delta]$, there is a unique optimal price that is charged by the non-commitment type, $\phi_{\mu\Delta}(p)$. Both $\phi_{\mu\Delta}(\underline{p}^\Delta)$ and $\phi_{\mu\Delta}(\bar{p}^\Delta)$ weakly increase in Δ;

(iii) the lower bound of the corresponding consumer surplus set stays constant while the upper bound first increases then decreases in Δ.

Figure 2.2 depicts the effect of regulation as described in Proposition 2.4. As can be seen from the graph, weak regulation and strict regulation deliver distinct welfare implications. I focus on the lower bound of the posted price set (upper bound of the consumer surplus set) since the upper bound is set such that consumer surplus is 0.

In the region of weak regulation ($\Delta > y^*$), deviation generates zero profit, as shown in Lemma 2.2. The commitment type seller is thus willing to charge marginal

Figure 2.2: The Effect of Regulation on Pooling Equilibrium



$G \sim U[0,1]$ (standard uniform distribution), $c = 0$ and $\underline{u} = 0$. $y^* = \frac{4}{11}$ and $p_N = \frac{1}{2}$. The left panel shows the equilibrium posted price set and the actual price set that is charged by the non-commitment type seller. The right panel displays the corresponding consumer surplus set.

cost in equilibrium. Correspondingly, the lowest price that is charged by the non-commitment type seller decreases as Δ decreases, since the seller is increasingly restricted by regulation to charge arbitrarily. Overall, consumer surplus increases when Δ decreases due to regulation's direct effect of restricting the price difference.

In the region of strict regulation ($\Delta \leq y^*$), deviation is profitable for the seller, as shown in Lemma 2.2. The non-commitment type obtains full commitment profit, which is invariant with respect to Δ. The best deviation profit for the commitment type is acquired by posting and charging $y^* - \Delta$, as it is the highest price that attracts consumers to visit, when consumers consider the seller to be a non-commitment type. It follows that deviation becomes more profitable as Δ decreases. Stricter regulation mitigates the commitment type's concern of being perceived as a non-commitment type and requires a higher posted price for the commitment type to stay in equilibrium. In addition, the price difference between two types of seller when deviation occurs is Δ. Therefore, regulation's direct effect on restricting the surcharge over the posted price is overshadowed by the indirect effect on providing commitment device and mitigating seller's concern to deviate. Altogether, consumer surplus decreases when Δ decreases.

Consequently, Proposition 2.4 implies that there exists an interior optimal regulation level ($\Delta = y^*$) for consumer surplus and social welfare in terms of set sense. At the optimal level of regulation, the commitment type seller adopts large price promotion while the non-commitment type seller is restricted to charge arbitrarily. Since all consumers visit the seller in equilibrium, social welfare is determined by volume of trade and moves along the same direction with consumer surplus. The seller, on the contrary, prefers extreme level of regulation. The commitment type benefits from stricter regulation while the non-commitment type gets hurt by stricter regulation. On the one hand, regulation's ability to limit the non-commitment type's surcharge over the posted price is advantageous to the commitment type but disadvantageous to the non-commitment type. On the other hand, regulation reduces the seller's concern of being perceived as a non-commitment type, which is beneficial for both types.[13]

2.5 Heterogenous levels of commitment

In this section, I investigate how limited commitment influences equilibrium price, consumer search order and welfare in a duopoly environment. With an additional seller, consumers face a more complex problem, as they need to decide which seller to visit first and whether or not to recall a previously visited seller. This complexity reflects back into sellers' problem and is further complicated by heterogenous levels of commitment. With heterogeneity among the sellers, a natural question to ask is which seller posts a lower price. More specifically, does a seller with higher level of commitment post a lower price and thus get visited by consumers first? It turns out that whether seller has full commitment or limited commitment plays a crucial role in determining market outcome.

2.5.1 Environment

There are two sellers, seller 1 and seller 2. Seller i has probability μ_i to be the commitment type. At the beginning of the market, sellers simultaneously post a price costlessly. I let p_i denote seller i's posted price and p_{-i} denote the other seller's posted price. Seller i may charge a different price p'_i once consumers visit him.

[13] A platform provider may also want to use regulation to maximize the expected profit. The highest expected profit along the lower bound of the equilibrium can be achieved either at no regulation level ($\Delta \geq p_N$) or at full regulation level ($\Delta = 0$). Which one is adopted depends on whether the monopolist profit from the non-commitment type solely or the full commitment profit from both types is higher. The highest expected profit along the upper bound of the equilibrium may not be achieved at extreme regulation level. It depends on the shape of the profit function.

Each consumer has unit demand. A consumer's value for seller i's product is given by y_i, which is revealed to the consumer after she visits seller i. y_i is drawn from a common distribution $G(y)$ which has a density function $g(y)$ over the interval $[\underline{y}, \bar{y}]$, identically and independently across consumers and products. Search is costly and with perfect recall. Each consumer incurs search cost s to investigate each seller's product. Outside option and marginal production cost are normalized to 0.

After sellers post prices, consumers engage in sequential search. Let $H_{i(p_i,p_{-i})}(p'_i)$ denotes the belief of the final price of seller i when a consumer observes posted price (p_i, p_{-i}). Given the beliefs of the final prices $(H_{1(p_1,p_2)}, H_{2(p_2,p_1)})$, actual prices (p'_1, p'_2), and match values (y_1, y_2), according to Weitzman (1979)'s Pandora rule, the consumer's optimal search strategy is as follows:

Seller i's reservation value of the product is defined similarly as in equation (2.1)

$r_i = -s + \int_{-\infty}^{\infty} \int_{\underline{y}}^{\bar{y}} \max\{r_i, y_i - p'_i\} dG(y_i) dH_{i\{p_i,p_{-i}\}}(p'_i).$

(i) Search order: the consumer visits the sellers in the decreasing order of r_i given r_i is non-negative.

(ii) Stopping: if the consumer decides to visit a seller, assume seller i, then she stops searching and purchases from seller i if $y_i - p'_i > \max\{0, r_{-i}\}$. If the consumer visits both sellers, then she purchases from the seller who yields highest surplus $\max\{y_1 - p'_1, y_2 - p'_2\}$ given it is non-negative.

2.5.2 Complete Information: Symmetric Sellers

When both sellers are the commitment type ($\mu_1 = \mu_2 = 1$), the seller who posts a lower price gets all consumers' visit first as they are ex-ante identical. Thus each seller has incentive to undercut the rival's price to get a discrete jump in demand. However, product differentiation prevents prices equal marginal costs to be an equilibrium. Therefore, the equilibrium necessarily involves mixed strategies.[14]

When both sellers are the non-commitment type ($\mu_1 = \mu_2 = 0$), there are three pure strategy equilibria: one symmetric and two asymmetric. The symmetric equilibrium is the focus of Wolinsky (1986) and Anderson and Renault (1999), where the two sellers set the same price and half of the consumers first visit each seller. The asymmetric equilibrium is the focus of Armstrong et al. (2009), where one seller (the "prominent" seller) is believed to charge a lower price than the other seller. In equilibrium, all consumers visit the prominent seller first and their beliefs are confirmed. The prominent seller indeed charges a lower price because his demand is more elastic than the non-prominent seller's: consumers who visit the prominent seller can choose to continue searching or to take the outside option, while consumers who visit the non-prominent seller must be disappointed by the prominent seller already. Equilibrium multiplicity makes it impossible to predict which equilibrium arises.

For simplicity, from now on suppose that y is uniformly distributed on the unit interval $[0, 1]$. Then the reservation value is given by $y^* = 1 - \sqrt{2s} \leq 1$. When

[14]The derivation of mixed strategy equilibrium is tedious, even for the case of exponential distribution. An example can be found in Choi et al. (2016).

both sellers are the non-commitment type, the symmetric equilibrium price is given by $p^* = \frac{-2(1+y^*)+\sqrt{(1+y^*)^2+4}}{2}$. $y^* - p^* \geq 0$ is required to guarantee an active market where all consumers engage in search. Thus, the range of search cost considered here is $0 \leq s \leq \frac{1}{8}$.

2.5.3 Complete Information: Asymmetric Sellers

Apart from the two complete information cases discussed above, there is a third case with complete information: one seller is the non-commitment type while the other seller is the commitment type. This case serves as a building block to study heterogenous levels of limited commitment. The following proposition describes what happens in equilibrium.

Proposition 2.5. *(i) If seller 1 is the non-commitment type ($\mu_1 = 0$) and seller 2 is the commitment type ($\mu_2 = 1$), there exists a unique equilibrium outcome where seller 2 charges a lower price than seller 1.*

(ii) The symmetric equilibrium price is ranked in between the asymmetric equilibrium prices:

$$p_2 \leq p^* \leq p_1.$$

The first part of Proposition 2.5 states that the seller with full commitment power necessarily charges a lower price and gets visited by all consumers first. In equilibrium, even if seller 1 posts a lower price than seller 2, all consumers visit seller 2 first, as they believe that seller 1 will charge a higher price than seller 2 upon their visits. In order to understand this result, suppose that equilibrium prices are equal for both sellers. Seller 2 can credibly lower the price by ϵ to get a discrete jump in demand, while seller 1 is unable to manipulate consumers' beliefs. Moreover, suppose that seller 1 charges a lower price in equilibrium. Then seller 2 has incentive to undercut seller 1, unless seller 1's price is low enough such that seller 2 is unwilling to commit to it. Nevertheless, seller 1 is unable to maintain such a low price due to lack of commitment. The second part of Proposition 2.5 comes from the observation that the demand function of the seller in the symmetric equilibrium is a weighted average of the demand function of the sellers in the asymmetric case and so does the elasticity.

The equilibrium prices with asymmetric sellers are identical to the asymmetric equilibrium prices in the case where both sellers are the non-commitment type and seller 2 is made to be the prominent seller. The equivalence comes from the fact that as long as the prominent seller is visited first by all consumers, belief of the prominent seller's final price no longer matters. Consequently, the prominent seller's type does not have impact on the equilibrium prices.

The equivalence among the equilibrium outcome allows me to borrow results from Armstrong et al. (2009), in which they discuss the impact of prominence on welfare and the effect of search cost. As full commitment power brings prominence to the seller, I investigate the implication of their results on injecting commitment power into the market. The detailed analysis and formal proof of these results can be found in their paper.

Corollary 2.1. *Compared to the symmetric equilibrium of the case where both sellers* *are the non-commitment type, when one seller gains full commitment power, welfare and consumer surplus is reduced. The commitment type seller gains more profit while the non-commitment type seller gains less profit. Industry profit is higher when search cost is small.*[15]

If the platform provider aims to extract sellers' profits by charging an entry fee, then whether to grant commitment power to one seller depends on the magnitude of search cost. On the contrary, if the goal is to maximize social welfare or consumer surplus, then granting commitment power to one seller is counterproductive. The welfare implication here is another example where injecting more commitment power into the market hurts consumers. In the mean time, a seller would like to gain full commitment power and prevent his rival from gaining it.

The next corollary states the comparative statics result with respect to search cost. This result is interesting in the sense that in the literature, as discussed in section 2.3.3, if all the sellers are the commitment type, equilibrium price decreases in search cost due to the "directed search effect"; while if all the sellers are the non-commitment type, equilibrium price increases with search cost due to the "hold-up effect" and "selection effect".

Corollary 2.2. *Both asymmetric equilibrium prices p_1 and p_2 increase with search*

[15] Armstrong, Vickers and Zhou(2009) argues that there are two reasons for welfare to fail. First, prominence induces nonuniform pricing, which misaligns consumer and social planer's tradeoff between search costs and match values. Second, total output is lower when one seller is made prominent. The result with individual sellers is not surprising. Industry profit only rises when the non-prominent seller does not lose too much by being placed later into the search order: when search cost is small and consumers still visit the non-prominent seller even if he charges a higher price.

cost s.

To see why this is the case, fix the commitment type seller's price and consider a marginal increase in search cost. It follows that the non-commitment type seller's price increases due to the "selection effect": consumers' decision to continue searching reflects that they are disappointed with the commitment type seller's product. Conversely, fix the non-commitment type seller's price, all consumers still visit the commitment type seller for a marginal increase in search cost. In addition, less consumers leave the commitment type seller and continue to search. Therefore, the "hold-up effect" raises the commitment type seller's price. The absence of "directed search effect" and the presence of both "hold-up effect" and "selection effect" drive both prices to increase.

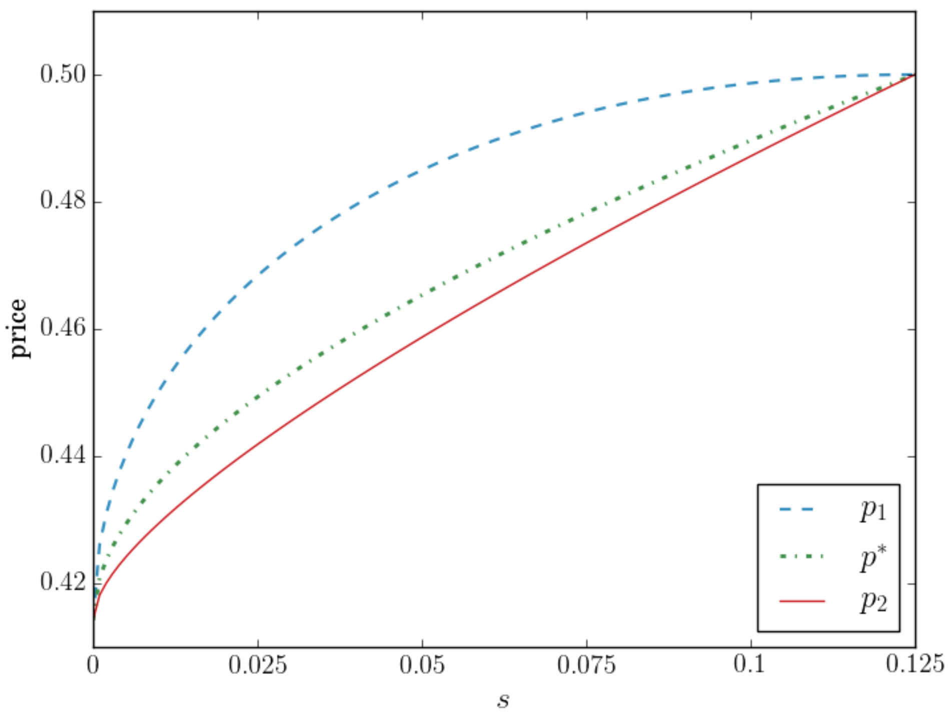
2.5.4 Incomplete Information

Proposition 2.5 states that seller 2, who has more commitment power than his rival, charges a lower price and becomes the prominent seller. This result continues to hold when seller 1, the non-commitment type seller, obtains partial commitment power. However, it breaks down when seller 2 loses full commitment power, as shown in the following proposition.

Proposition 2.6. *(i) If seller 1 has limited commitment ($0 < \mu_1 < 1$), and seller 2 has full commitment power ($\mu_2 = 1$), there does not exist any equilibrium where seller 1 posts a lower or equal price than seller 2.*

(ii) If both seller 1 and seller 2 have limited commitment ($\mu_1, \mu_2 < 1$), there exists

Figure 2.3: Asymmetric Equilibrium Prices



How equilibrium prices are ranked and change with respect to the search cost. p^*, the symmetric equilibrium price, is represented by the green dotted line. The blue dashed line represents p_1, the asymmetric equilibrium price by the non-commitment type seller. The red solid line represents p_2, the asymmetric equilibrium price by the commitment type seller.

equilibrium where seller 1 posts a lower price, where seller 2 posts a lower price and where both sellers post the same price, regardless of how μ_1 *and* μ_2 *are ranked. All three equilibria in the case where both sellers are the non-commitment type (*$\mu_1 = \mu_2 = 0$*) remain as equilibria.*

Proposition 2.6 can be illustrated by an example where seller 2 only has 1% probability to be the commitment type. If somehow consumers believe that seller 2 charges a lower price and visit him first, then seller 1 would like to cut price to redirect consumers' attention. If seller 1 has full commitment power, the price cut is effective. Equilibrium that seller 2 charges a lower price does not exist. However, if seller 1 has limited commitment (even if seller 1 has 99% probability to be the commitment type), he cannot use price cut to win everyone back because small amounts of uncertainty concerning the seller's commitment power takes away the seller's ability to use price to direct consumer search.

In order to understand why all three equilibria of the symmetric case carry through, consider any equilibrium among the three and fix the equilibrium price for seller 2. If seller 1 is the non-commitment type, he has no incentive to deviate, as this is the equilibrium in the case where both sellers are the non-commitment type. If seller 1 is the commitment type, he is believed to be the non-commitment type when he deviates from the candidate equilibrium, and is thus expected to charge the candidate equilibrium price. Consequently, it is optimal for the seller 1 to charge the equilibrium price. Deviation does not work for the commitment type as he loses his ability to influence consumers' belief due to limited commitment. Therefore, consumers' inferences determine the seller's pricing decision.

Proposition 2.5 and 2.6 together imply that only the seller with full commitment power can credibly post a low price and guarantee himself a prominent position. If a seller is able to engage in some costly investment to enhance his level of commitment, unless he is able to gain full commitment, the investment maybe wasteful.

2.6 Ex-ante Heterogenous Consumers

In this section, I briefly discuss the impact of limited commitment when consumers differ in their match values before they visit the seller. To be specific, I introduce ex-ante heterogeneity by assuming that a (representative) consumer's random utility for the seller's product is given by $V = X + Y$, where X is known to each consumer before she visits the seller, while Y is revealed to a consumer only after she visits the seller. The known component X is based on easily observable characteristics, such as brand and basic design. The hidden component Y captures more precise information about the product, which is available once a consumer inspects the product more carefully, such as consumer reviews. Here another effect due to lack of commitment power emerges: the fact that consumers visit the seller reveals information about their ex-ante valuations of the product. Given search cost and belief of the final price, only consumers who have high enough ex-ante valuations visit the seller. The non-commitment type seller uses this information to determine the final price upon observing the visits. This is the "selection effect".

The characterization is complicated with ex-ante heterogenous consumers. Nevertheless, most of the implications carry through: higher degree of limited commitment enhances market efficiency; stricter regulation may hurt consumers; higher degree of limited commitment does not bring prominence to the seller. The major difference regarding implications lies in the effect of the search cost.

Recall that in the benchmark model with ex-ante homogenous consumers, as discussed in section 2.3.3, the "directed search effect" requires the upper bound of the pooling equilibrium price set to decrease when search cost rises in order to keep consumers. The impact of search cost on the equilibrium price set is monotone.

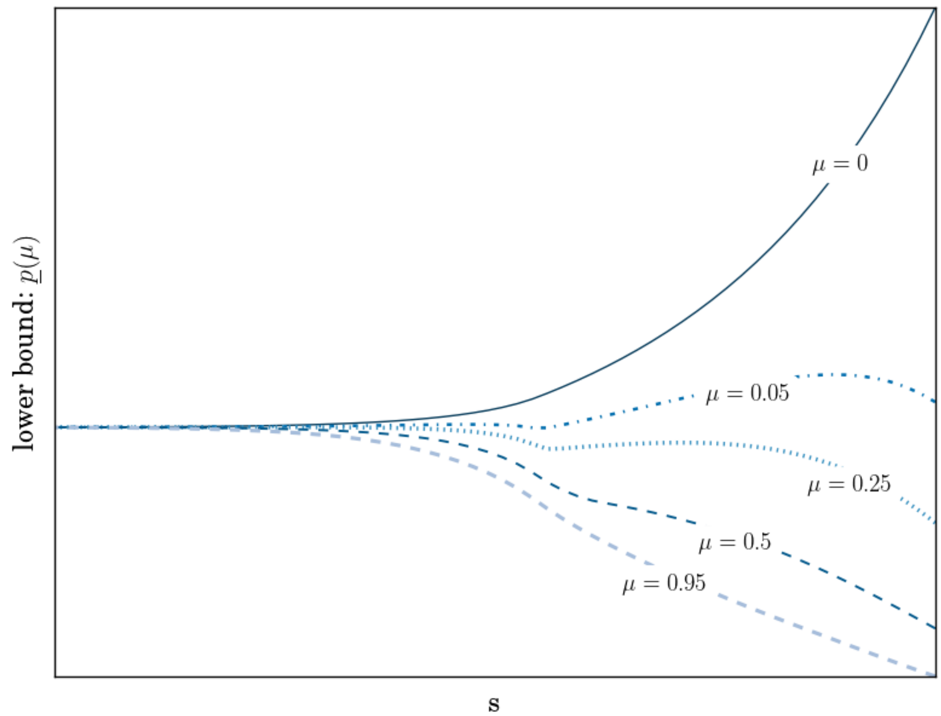
With presence of ex-ante heterogenous consumers, in contrast, the impact of the search cost is non-monotone. Simply put, the effect of the search cost on pooling equilibrium price set is determined by comparing how equilibrium and deviation profit vary with search cost. The "selection effect" reduces both equilibrium and deviation profit when search cost rises. Which profit decreases faster depends on the level of commitment and the magnitude of the search cost. As displayed in Figure 2.4, the effect of the search cost varies with different parameters. Nevertheless, examining the limiting cases provides clean results, which are discussed in detail in the appendix.

In this information age, reducing search cost and providing information to consumers can be relatively easy with modification of search engine and webpage design. The comparison of the search cost result with ex-ante homogenous and ex-ante heterogenous consumers suggests both seller and platform provider to be cautious in investing in such practices.

2.7 Conclusion

This paper examines the implications of limited commitment power of sellers' advertised prices, so that consumers are uncertain whether actual prices coincide with advertised prices. The key to drive all the results is that as long as consumers have doubt about the seller's type, the commitment type seller is unable to direct consumers' search decisions by posting a lower price, since consumers may assume

Figure 2.4: The Effect of the Search Cost



$F \sim \mathit{Laplace}(0,1)$, $G \sim N(0,1)$ (standard normal distribution), $c = 0$ and $\underline{u} = 0$. Different lines represent the lower bounds of the pooling equilibrium price set for various levels of μ. When $\mu = 0$, the solid line also represents the upper bound of the pooling equilibrium price set.

that this price comes from the non-commitment type seller. This driving force results in several consequences. I show that a higher degree of limited commitment leads to higher welfare in the pooling equilibrium. In order to address the inefficiency caused by lack of commitment power, I study the effect of regulation that restricts the seller's deviation from his advertised price. I find that stricter regulation can hurt consumers and an intermediate level of regulation maximizes consumer surplus and social welfare. In an environment where sellers have heterogenous levels of commitment, I demonstrate that only full commitment power can dictate consumers' search, while limited commitment, regardless of its level, is incapable of granting a seller the prominent position. I further check the robustness of these results with an environment where consumers have ex-ante heterogenous valuations of the product and find that the implication regarding the effect of search cost is different. With ex-ante homogenous consumers, the effect is monotone while with ex-ante heterogenous consumers, the effect is non-monotone and relies on level of commitment and magnitude of search costs.

The model relies on the assumption of fully rational consumers, who can perfectly foresee seller's incentive to deviate from the advertised price. Boundedly rational consumers can be introduced into this model by assuming that consumers believe that the posted price is the actual price before they search. Then the non-commitment type seller posts a low price to make sure that consumers visit him and charge the optimal price. Seller's type disappears from consumers' perspective. The commitment type's concern of being regarded as a non-commitment type also vanishes. A quick investigation shows that stricter regulation always helps consumers as the only role it left with is to restrict seller's ability to charge arbitrary. Another way of introducing bounded rationality is to regard the advertised price as a natural reference point for consumers. When consumers encounter sellers who charge higher price than the advertised price, they suffer from an increment of price and a deviation from reference point.

How level of commitment influences the interaction between seller's pricing decision and consumers' search behavior has important implications for platform providers and social planers. This paper suggests platform providers be cautious when they design online shopping environments, as the effects of search cost, regulation and access to commitment devices for various sellers are non-monotone. Depending on the goal of platform providers, there may exists different optimal combinations of these choice variables. For example, how many commitment devices are available in the market? How much does a seller need to pay to obtain commitment power? Whether the optimal market structure consists of sellers with various levels of commitment or not? The framework in this paper may have a wide range of applications in various contexts.

CHAPTER 3
SOCIAL LEARNING UNDER NEGATIVE CORRELATION IN AN EXIT GAME

3.1 Introduction

Learning comes in two ways, private and social. In other words, we learn not only from our own experiences, but also from others' behavior. Furthermore, the two forms of learning are intertwined, because others also learn from our behavior. Models of strategic experimentation, in which multiple players simultaneously engage in learning, offer a natural framework to study how private learning and social learning interact each other and what their economic consequences are.

Most papers in the literature study the case of positive correlation in which good news to a player is also good news to other players (more precisely, if a player's type is good, then the other player's type is also, or more likely to be, good). Bolton and Harris (1999), Keller, Rady and Cripps (2005), and Keller and Rady (2010) consider the case of perfect social learning in which both actions and payoffs are observable to other players.[1] Rosenberg, Solan and Vieille (2007) and Murto and Välimäki (2011) consider the case of imperfect social learning in which only actions are observable. Specifically, they analyze an exit game in which each player knows

[1]They adopt different learning processes. Bolton and Harris (1999) consider a Brownian learning model, Keller, Rady and Cripps (2005) employ an exponential learning model (in which Poisson signals arrive at a positive rate only at the good state), and Keller and Rady (2010) examine a Poisson learning model (in which Poisson signals arrive at different rates at different states).

only whether the other players stay in the game or not.[2]

Negative correlation, with which good news to a player is bad news to the other player, is equally plausible, but has received disproportionally less attention: to our knowledge, Klein and Rady (2011) is the only relevant contribution. The form of correlation is not crucial for social learning per se. In particular, learning is always valuable, and thus free-riding incentives arise whether correlation is positive or negative. However, the correlation structure determines the content of social learning and, therefore, dictates the equilibrium dynamics. Klein and Rady (2011) demonstrate this in the model with perfect social learning. In their baseline model (with two players and perfect negative correlation), if both players engage in experimentation, then the players' beliefs, conditional on no success, stay constant, which has significant implications for the equilibrium structure and efficiency.[3]

We consider negative correlation in the context of an exit game. Specifically, each player decides whether to stay in the game or not (exit). They can exit at any point in the game, but exit is irreversible.[4] Each player is either good or bad. If a player is good, then he receives lump-sum rewards at a positive Poisson rate. The

[2]Imperfect social learning raises a non-trivial inference problem regarding the types of the remaining players. Rosenberg, Solan and Vieille (2007) focus on perfect positive correlation, but incorporate a more general signal structure, while Murto and Välimäki (2011) consider a simple signal structure, but allow for imperfect positive correlation.

[3]Most notably, there exits an equilibrium in which each player plays a simple cutoff strategy (experimenting if and only if the probability that his type is good exceeds a certain threshold), and the resulting equilibrium is efficient for a range of parameter values. None of these results holds in the case of positive correlation (see Keller, Rady and Cripps, 2005).

[4]In our model, the player who exits first has an incentive to re-enter the game once the other player also exits. We provide a brief discussion on re-entry in Section 3.6.3.

other player is bad and never receives a lump-sum reward. Each player prefers staying if his type is good, but prefers exiting immediately if his type is bad. This model can be interpreted as an imperfect-learning counterpart to Klein and Rady (2011), or as a negative-correlation counterpart to Murto and Välimäki (2011).

We provide a closed-form characterization of the unique (perfect Bayesian) equilibrium of the model and contrast it to the unique equilibrium under positive correlation. In both cases, the equilibrium features two phases. In the first phase, no player exits, and thus social learning does not occur. Conditional on no success, a player becomes more pessimistic solely based on his private learning. Once the players become sufficiently pessimistic, the second phase begins, in which the players exit at a positive rate, and thus a player learns not only from his own experience, but also from the other player's behavior. In both cases, the second phase ends upon a player's exit.

One clear difference between positive correlation and negative correlation lies in the behavior at the end of the second phase. With positive correlation, a player's exit triggers the other player's exit: a player's exit reveals that he has not succeeded yet, which makes the other player more pessimistic and, therefore, willing to exit. With negative correlation, the same news is good news to the other player, who will then revise up his belief and stay longer in the game.

More importantly, the correlation structure affects the way private learning and social learning interact and, therefore, the resulting equilibrium dynamics in the second phase. For the players to remain indifferent between staying and exiting in the second phase (which is necessary for them to exit at a positive rate), the benefit of social learning must compensate increasing pessimism due to private learning. With positive correlation, the benefit comes in the form of positive information: a player is more likely to stay in the game when he is good than when he is bad. Therefore, the fact that a player stays in the game allows the other player to overcome his own pessimism and be willing to stay in the game. Formally, this translates into the players' beliefs, conditional on no success and no exit by the other player, staying constant and the second phase stretching without a limit. With negative correlation, the corresponding benefit comes in the form of an increasing amount (speed) of social learning. A player's staying is now bad news to the other player, and thus social learning makes the players more pessimistic. This implies that the players, conditional on no success and no exit by the other player, necessarily become more pessimistic over time. This growing pessimism can be compensated only through even more amount of social learning, which translates into the players, conditional on no success, exiting the game at an increasing rate over time. Growing pessimism and increasing (negative) social learning reinforce each other. As a result, under negative correlation, in stark contrast to the positive-correlation case, the second phase necesssarily ends by a finite deterministic time (by which one player exits for sure and, therefore, the players' conditional beliefs converge to 0).

The rest of the paper is organized as follows. We introduce the model in Section 3.2 and present two benchmark models, the single-player problem and the positive-correlation case, in Section 3.3. We analyze the symmetric case (in which the two players are ex ante identical) in Section 3.4 and the asymmetric case in Section 3.5. We discuss three relevant extensions in Section 3.6.

3.2 The Model

We set up the model in continuous time. Time starts from 0 and is indexed by $t \in \mathcal{R}_+$. There are two players, player 1 and player 2. Per each time unit $[t, t+dt)$, the players first decide whether to stay in the game or exit the game. Staying is costly: each player incurs flow cost c per unit time. Exit is costless but irreversible: once a player exits, he cannot reenter the game.

Each player is either good or bad. If player i is good, then he constantly receives lump-sum payoff $v(> 0)$ at a Poisson rate λ. If player i is bad, then he never receives a lump-sum payoff.[5] The payoff a player receives when he exits the game is normalized to 0. In order to avoid triviality, we assume that a player strictly prefers staying in the game to exiting if his type is good, while the opposite is true if his type is bad. Formally, we assume that $\lambda v > c > 0$.

The players are initially uncertain about their types. Denote by p_i the prior probability that player i's type is good. The players' types are perfectly negatively correlated: if player i is good, then player j is bad. It is necessary that $p_1 + p_2 = 1$. This information structure is common knowledge between the players.

[5]Notice that once a player receives payoff v, he becomes sure that his type is good. This is a common simplifying assumption in the literature (see, e.g., Keller, Rady and Cripps, 2005; Klein and Rady, 2011; Murto and Välimäki, 2011; Bonatti and Hörner, 2011). It is well-known that if this assumption is relaxed (i.e., a player may receive payoff v even if her type is bad), then the analysis becomes significantly more complicated (see, among others, Keller and Rady, 2010).

Each player's action is observable to the other player, but his payoff is not. This means that when player j stays in the game, player i is not sure whether player j has already succeeded or not. Once a player receives a lump-sum payoff, it is a dominant strategy for him to stay in the game. Therefore, player j's exit is good news to player i, as it reveals that player j has not succeeded, which is more likely when player j is bad (i.e., player i is good).

Denote by 0 "stay" and by 1 "exit". An action profile at time t is then a vector $a^t = (a_1^t, a_2^t) \in \{0, 1\}^2$. The public history of the game is then a sequence of action profiles. Let H^t denote the set of all histories until time t, and $H^0 \equiv \emptyset$. Finally, define the set of all histories $H \equiv \cup_t H^t$.

Each player's private history consists of the public history and his past realized payoffs. Since the optimal strategy of a player who has ever received payoff v is straightforward, it suffices to consider the private histories up to which each player has not received payoff v. This implies that each player's strategy can be defined as a function of the public history. Formally, player i's pure strategy, conditional on no success, can be defined as a function $s_i : H \to \{0, 1\}$, where $s_i(h^t)$ represents player i's exit decision following history h^t. Since exit is irreversible, player i's strategy s_i is admissible only when if $s_i(h^t) = 1$, then $s_i(h^s) = 1$ for any history h^s following h^t. Player i's mixed strategy is a probability distribution over the set of player i's admissible pure strategies.

Each player maximizes his expected discounted sum of payoffs and is risk neutral. We study perfect Bayesian equilibrium of this game: for both $i = 1, 2$, player

i's strategy is a best response to player j's strategy after any history $h^t \in H$, and the players' beliefs after each history are obtained by Bayes' rule whenever possible.

3.3 Two Benchmarks

This section provides the results for two benchmark models, one without social learning (i.e., the single-player problem) and the other with positive correlation.

3.3.1 No Social Learning

We first consider the case where a player does not observe the other player's action. This case is formally identical to the single-player experimentation problem, which is familiar in the literature. In particular, the following result is a straightforward modification of Proposition 3.1 in Keller, Rady and Cripps (2005).

Proposition 3.1. *In the absence of social learning, each player stays in the game if and only if he assigns a greater probability than p^* to the event that his type is good, where*

$$p^* \equiv \frac{c}{\lambda\left(v + \frac{\lambda v - c}{r}\right)}.$$

Conditional on no success, his belief (the probability that he is good) decreases according to $\dot{p}(t) = -\lambda p(t)(1-p(t))$. His expected payoff as a function of his belief is equal to

$$V(p) = \begin{cases} 0, & \text{if } p \leq p^*, \\ \frac{p\lambda v - c}{r} + \frac{c - p^*\lambda v}{r}\frac{1-p}{1-p^*}\left(\frac{1-p}{p}\frac{p^*}{1-p^*}\right)^{r/\lambda}, & \text{if } p > p^*. \end{cases}$$

The result implies that in the absence of social learning, each player's optimal strategy takes a simple form: he stays in the game only until his belief reaches p^*. The length of time player i stays in the game, denoted by t_i^*, can be explicitly calculated as follows: if $p_i \leq p^*$, then $t_i^* = 0$ (i.e., immediate exit). Otherwise, his belief must be p^* at time t_i^*. Therefore,

$$p^* = \frac{p_i e^{-\lambda t_i^*}}{p_i e^{-\lambda t_i^*} + 1 - p_i} \Rightarrow t_i^* = -\frac{1}{\lambda} log\left(\frac{1-p_i}{p_i}\frac{p^*}{1-p^*}\right). \tag{3.1}$$

For later use, define $t^* \equiv \min\{t_1^*, t_2^*\}$.

3.3.2 Positive Correlation

Now we consider the case where the players' types are positively correlated: if player i is good (bad), then player j is also good (bad). This case has been extensively studied in the literature. In particular, our model is a special case of Murto and Välimäki (2011), with two players and perfect correlation. Note that the players should have the same prior beliefs in this case (i.e., $p_1 = p_2$).

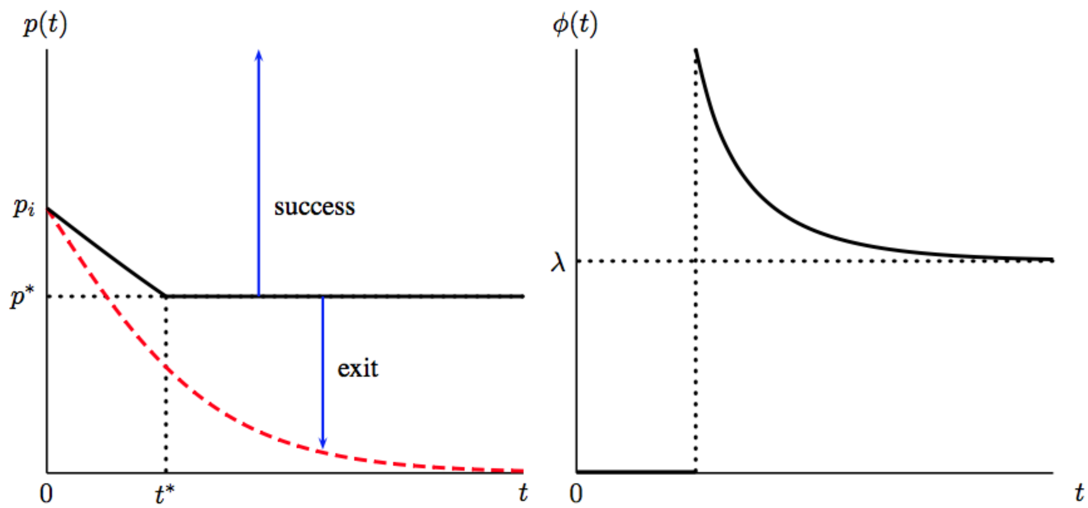
The following proposition provides a closed-form characterization of the unique equilibrium.

Proposition 3.2. *In the model with perfect positive correlation, there exists a unique equilibrium. Until time t^*, no player exits and the players' conditional beliefs $p(t)$ decrease according to $\dot{p}(t) = -\lambda p(t)(1-p(t))$. After time t^*, each player, conditional on no success, exits at a decreasing rate:*

$$\phi(t) = \frac{\lambda p^*(1-p_i)e^{2\lambda t}}{((1-p_i)e^{2\lambda t} + p_i)p^* - p_i},$$

and the players' conditional beliefs $p(t)$ stay constant at p^*. If one player exits, then the other player follows immediately.

Figure 3.1: Equilibrium Dynamics under Positive Correlation



Each player's belief $p(t)$ (left) and exit rate $\phi(t)$ (right) at time t, conditional on no success and no exit by the other player. The dashed line in the left panel represents the players' beliefs when they know that both of them have not succeeded by time t (i.e., $p_i e^{-2\lambda t}/(p_i e^{-2\lambda t} + 1 - p_i)$).

Proof. See the appendix. □

Figure 3.1 illustrates the resulting equilibrium dynamics. Until time t^*, no player exists. Therefore, each player learns only from his own experience (failure), updating his belief as in the single-player problem. Once the players' beliefs reach p^*, they randomize between staying and exiting at a well-defined rate: if player i stays for sure, then player j does not learn from player i's action and, therefore, exits immediately, unless he has already succeeded and, therefore, knows that his type is good. This delivers a significant amount of information to player i, who will then follow player j's action: conditional on no success, player i exits if player j exits and stays if player j stays. This, in turn, provides a lot of information for player j, which deters player j's exit in the first place, unraveling the given equilibrium structure. Following the same logic, it is also clear that no player exits with a positive probability at each point in time.

The players' beliefs $p(t)$, conditional on no success and no exit, stay constant once they reach p^* (see the left panel of Figure 3.1). There are two opposing effects. On the one hand, player i's own failure pushes down his belief: without social learning, his belief would keep decreasing as in the single-player problem. On the other hand, player j's staying is good news to player i: player j is more likely to stay when he is good than when he is bad. In equilibrium, these two effects are balanced, and thus $p(t)$ stays constant after t^*. The equilibrium exit rate $\phi(t)$ is strictly decreasing over time (see the right panel of Figure 3.1). This is because each player is more likely to know his type and, therefore, staying becomes an increasingly better indicator of the good type as t increases. The exit rate $\phi(t)$ converges to λ. This is because in the limit, player j knows his type for sure, and thus his exiting at rate λ (when his type is bad) suffices to compensate player i's own failure and restore player i's belief back to p^*.

3.4 Symmetric Negative Correlation

We now study the main model with negative correlation. We first consider the symmetric case where $p_1 = p_2 = 1/2$, which is directly comparable to the case of positive correlation.

Evolution of the players' conditional beliefs. We begin by characterizing how the players' beliefs evolve over time. Specifically, we derive how each player' belief $p(t)$, conditional on no success and no exit, changes over time when the other player's exit strategy is given by $\phi(t)$.

Conditional on player i being good, the probability that player i does not succeed until time t is equal to $e^{-\lambda t}$. Player j never succeeds and the probability that player j does not exit by time t is given by $e^{-\int_0^t \phi(y)dy}$.

Conditional on player j being good, player i never succeeds. Player j stays for sure if he has succeeded, but might have exited if success arrives rather late. To formally derive the relevant probability, let x denote player j's first success time, which is exponentially distributed with parameter λ. Player j stays until time t as long as he has not exited by time $\min\{x,t\}$, whose probability is equal to $e^{-\int_0^{\min\{x,t\}} \phi(y)dy}$. It follows that the probability that player j stays until time t is equal to $\int_0^\infty e^{-\int_0^{\min\{x,t\}} \phi(y)dy} d(1-e^{-\lambda x})$.

Combining the above two cases, by Bayes' rule, the probability that player i assigns to the event that his type is good at time t, conditional on no success and no exit, is given by

$$p(t) = \frac{p_i e^{-\lambda t} e^{-\int_0^t \phi(y)dy}}{p_i e^{-\lambda t} e^{-\int_0^t \phi(y)dy} + p_j \int_0^\infty e^{-\int_0^{\min\{x,t\}} \phi(y)dy} d(1-e^{-\lambda x})}.$$

Arranging the terms and using the fact that $p_1 = p_2 = 1/2$, the expression shrinks to

$$p(t) = \frac{1}{2 + \int_0^t e^{\int_x^t (\lambda + \phi(y))dy} \lambda dx}, \tag{3.2}$$

which implies

$$\dot{p}(t) = -\lambda p(t)(1 - p(t)) - \phi(t)p(t)(1 - 2p(t)). \tag{3.3}$$

The players' conditional beliefs $p(t)$ always decrease over time. There are two reasons. First, each player learns from his own failure. The first term in equation (3.3) represents this effect. Second, each player learns from the other player's action. Since player j is more likely to stay when his type is good, player j's staying is bad news to player i, further pushing down player i's belief. This effect is captured in the second term in equation (3.3).

The players' conditional value function.

In the symmetric equilibrium, each player's expected payoff as a function of his belief $p(t)$ is identical to that without social learning in Proposition 3.1.[6] Intuitively, player i learns from player j's action only when player j may exit (i.e., $\phi(t) > 0$). However, if $\phi(t) > 0$, then exit is an optimal strategy for player i. In other words, social learning occurs only when it is irrelevant to the players' expected payoffs.

Together with the fact that $p(t)$ is strictly decreasing, this significantly simplifies the analysis. No player exits until his belief reaches p^*. This implies that $p(t)$ reaches p^* at time t^*. If $t > t^*$, then each player's expected payoff, conditional on no success and no exit, stays constant at 0: recall that $V(p) = 0$ if $p \leq p^*$ in Proposition 3.1, and $p(t)$ always decreases.

[6] Murto and Välimäki (2011) prove this result in the model with positive correlation (Lemma 1 in their paper). Despite the difference in the correlation structure, their argument applies unchanged to our model with negative correlation. What is crucial for the result is the irreversibility of exit. The result does not hold if the players can reenter the game.

To utilize the fact that the players' conditional expected payoffs remain equal to 0 after time t^*, we calculate the probability that player i assigns to the event that player j has not succeeded, conditional on his staying until time $t(> t^*)$. If player i is good, then player j is bad and, therefore, has not succeeded for sure. If player j is good, then the probability that he stays and has succeeded before time t is equal to $\int_0^t e^{-\int_0^x \phi(y)dy} d(1-e^{-\lambda x})$, while the probability that he stays but has not succeeded until time t is equal to $e^{-\int_0^t (\lambda+\phi(y))dy}$. Therefore, the total probability that player j stays but has not succeeded by time t is equal to

$$\begin{aligned} & p(t) + (1-p(t)) \frac{e^{-\int_0^t (\lambda+\phi(y))dy}}{e^{-\int_0^t (\lambda+\phi(y))dy} + \int_0^t e^{-\int_0^x \phi(y)dy} d(1-e^{-\lambda x})} \\ = \quad & p(t) + (1-p(t)) \frac{1}{1+\int_0^t e^{\int_x^t (\lambda+\phi(y))dy} \lambda dx}. \end{aligned}$$

Applying equation (3.2), the probability simplifies to $2p(t)$.

Combining all the results so far, the following equation holds whenever $t > t^*$:

$$\begin{aligned} 0 = V(p(t)) \quad = \quad & -cdt + 2p(t)\phi(t)dt \cdot V(0.5) \\ & +(1-2p(t)\phi(t)dt)(p'(t)\lambda dt \cdot (v + e^{-rdt}V(1)) \\ & +(1-p'(t)\lambda dt)e^{-rdt}V(p(t+dt))) \end{aligned}$$

where

$$p'(t) = \frac{p(t)(1-\phi(t)dt)}{p(t)(1-\phi(t)dt) + (1-p(t))\frac{(1-\phi(t)dt)+\int_0^t e^{\int_x^t (\lambda+\phi(y))dy}\lambda dx}{1+\int_0^t e^{\int_x^t (\lambda+\phi(y))dy}\lambda dx}}.$$

The left-hand side is a player's expected payoff when he exits the game, while the right-hand side is his expected payoff when staying. The right-hand side consists of four terms: the first term is the flow cost of staying. The second term represents the possibility that the other player exits, in which case the player's belief jumps to 1/2 and he faces the same problem as in Proposition 3.1. The last two terms correspond to the case where the other player stays in the game. In that case, the player's belief updates to $p'(t)$, the player succeeds with probability $p'(t)\lambda dt$, and his continuation payoff becomes either $V(1)$ or $V(p(t+dt))$, depending on whether he succeeds or not.

Arranging the terms, the expression simplifies to

$$c = 2p(t)\phi(t)V(0.5) + p(t)\lambda(v + V(1)) \Leftrightarrow \phi(t) = \frac{\frac{c}{p(t)} - \lambda(u + V(1))}{2V(0.5)}. \tag{3.4}$$

Intuitively, the left-hand side is the marginal cost of staying an instant longer, while the right-hand side is the corresponding marginal benefit. The latter comes from the fact that the other player may exit, which occurs at rate $\phi(t)$ in case the other player has not succeeded yet (whose probability is equal to $2p(t)$), or the player may succeed, which occurs at rate λ when his type is good (whose probability is equal to $p(t)$). The value of $\phi(t)$ is well-defined whenever $t \geq t^*$, because $c = p^*\lambda(u + V(1)) \geq p(t)\lambda(u + V(1))$. It is strictly decreasing in $p(t)$ and, therefore, strictly increasing in t. Intuitively, player i becomes increasingly pessimistic as he continues to fail, while player j does not exit. Since this increases player i's incentive to exit, player j must exit at an increasing rate, so as to provide a stronger incentive for player i to stay.

Equilibrium characterization.

Combining equations (3.3) and (3.4) yields the following non-linear differential equation for $p(t)$:

$$\dot{p}(t) = -\lambda p(t)(1 - p(t)) - \frac{c - p(t)\lambda(v + V(1))}{2V(0.5)}(1 - 2p(t)).$$

Arranging the terms leads to

$$\dot{p}(t) = -\frac{c}{2V(0.5)} + \frac{2c + \lambda(v + V(1) - 2V(0.5))}{2V(0.5)} p(t) - \frac{\lambda(v + V(1)) - \lambda V(0.5)}{V(0.5)} p^2(t). \tag{3.5}$$

This is a quadratic first-order differential equation, known as a Riccati equation, with constant coefficients, and admits a closed-form solution, as reported in the following lemma for convenience.[7]

Lemma 3.1. *Suppose $p(t)$ is a deterministic function of time t, $p(t^*) = p^*$, and satisfies*

$$\dot{p}(t) = A + Bp(t) + Cp^2(t),$$

where A, B, and C are constant real numbers. The solution to the differential equation is[8]

$$p(t) = -\frac{k_2}{C} \frac{\frac{k_1}{k_2} \frac{k_2 + Cp^*}{k_1 + Cp^*} e^{(k_1 - k_2)(t - t^*)} - 1}{\frac{k_2 + Cp^*}{k_1 + Cp^*} e^{(k_1 - k_2)(t - t^*)} - 1}, \tag{3.6}$$

---

[7] This equation arises in various contexts in macroeconomics and finance. See, e.g., Ljungqvist and Sargent (2004) and Nawalkha, Soto and Beliaeva (2007). In most cases, the structure is either exogenously imposed or obtained as a solution to a linear quadratic dynamic programming problem (see Chapter 5 in Ljungqvist and Sargent, 2004). We are unaware of any other model in which a Riccati equation endogenously arises as in our model.

[8] Equivalently,

$$p(t) = \frac{p^* + \frac{2A + Bp^*}{\sqrt{4AC - B^2}} \tan\left((t - t^*)\sqrt{4AC - B^2}/2\right)}{1 - \frac{2Cp^* + B}{\sqrt{4AC - B^2}} \tan\left((t - t^*)\sqrt{4AC - B^2}/2\right)}.$$

Notice that the solution is not well-defined if $B^2 = 4AC$. In that case, the solution is

$$p(t) = \frac{1}{\frac{2C}{2Cp^* + B} - C(t - t^*)} - \frac{B}{2C}$$

where

$$k_1 = \frac{B + \sqrt{B^2 - 4AC}}{2}, \text{ and } k_2 = \frac{B - \sqrt{B^2 - 4AC}}{2}.$$

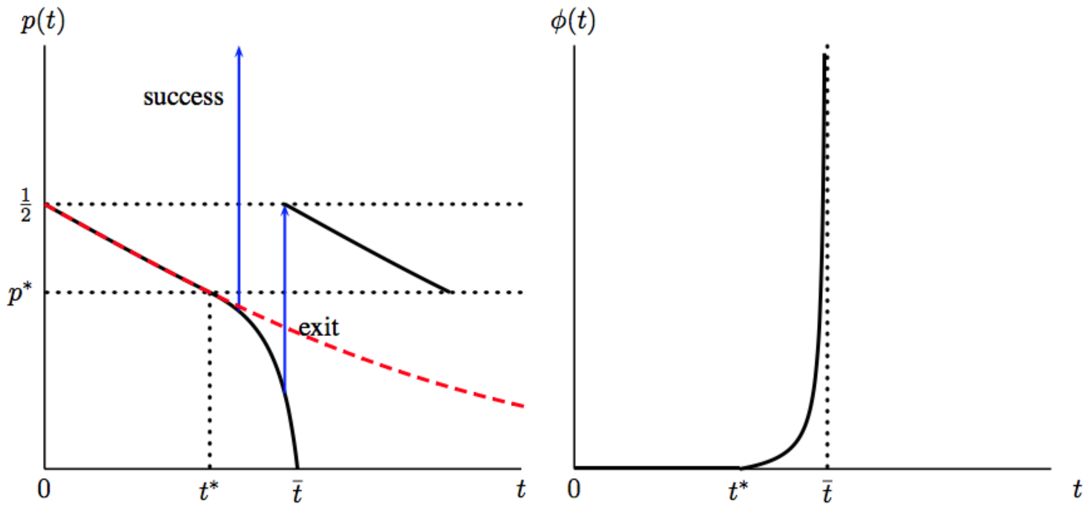
Applying Lemma 3.1 to equation (3.5) and the resulting solution $p(t)$ to equation (3.4), we obtain the following result.

Proposition 3.3. *In the symmetric case with negative correlation, there exits a unique equilibrium. Until time* t^**, no player exits and the players' conditional beliefs* $p(t)$ *decrease according to* $\dot{p}(t) = -\lambda p(t)(1 - p(t))$*. After time* t^**, the players' conditional beliefs* $p(t)$ *decrease according to equation* (3.5) *(whose solution can be obtained through Lemma 3.1), and each player exits at rate* $\phi(t)$*, as given in equation* (3.4)*. If one player exits, then the other player updates his belief to* $1/2$ *and behaves as described in Proposition 3.1.*

Proof. See the appendix. □

Figure 3.2 depicts the resulting equilibrium dynamics. As in the positive correlation case, the players, conditional on no success and no exit, exit at a positive rate from time t^*. Unlike in the positive correlation case, the players' beliefs $p(t)$ decrease, while their exit rate $\phi(t)$ increases, over time. As explained above, under negative correlation, both no success (private learning) and no exit by player j (social learning) make player i more pessimistic about his type, whereas under positive correlation, they work in the opposite direction and, in equilibrium, exactly cancel each other out. In order to compensate increasing pessimism, the players exit at an increasing rate (see the right panel of Figure 3.2). In other words, despite the fact that their

Figure 3.2: Equilibrium Dynamics under Negative Correlation with Symmetric Players



The players' beliefs $p(t)$ (left) and exit rate $\phi(t)$ (right) at time t, conditional on no success and no exit by the other player. The dashed line in the left panel represents the players' beliefs in the absence of social learning (i.e., $p_i e^{-\lambda t}/(p_i e^{-\lambda t} + 1 - p_i)$).

beliefs fall below p^*, they are willing to stay in the game, because they expect to learn more from the other player's action. These two effects reinforce each other: as $p(t)$ decreases, $\phi(t)$ must increase. This pushes down $p(t)$ even further, which in turn leads to even larger $\phi(t)$. These effects grow exponentially fast and result in the players' beliefs $p(t)$ converging to 0 and the exit rate $\phi(t)$ converging to infinity in finite time (time $\bar{t}$). Notice that this does not mean that the players' beliefs would indeed become equal to 0. It simply means that one player exits with probability 1 before time $\bar{t}$.

3.5 Asymmetric Negative Correlation

We now consider the asymmetric case where the players assign different prior probabilities to the event that their type is good. Without loss of generality, we assume that player 1 is more likely to be good than player 2, that is, $p_1 > p_2 (= 1 - p_1)$. In order to avoid triviality, we also assume that $p_2 > p^*$: otherwise, player 2 exits immediately.

For each $i = 1, 2$, we denote by $F_i(t)$ the probability that player i exits by time t, conditional on no success and no exit by player j. We also let $\phi_i(t)$ denote, whenever possible, player i's exit rate at time t (i.e., $\phi_i(t) = dF_i(t)/(1 - F_i(t))$) and $p_i(t)$ denote the probability that player i assigns to the event that his type is good at time t, conditional on no success and no exit by player j.

As in the symmetric case, in equilibrium the two distribution functions F_1 and F_2 have a common convex support: otherwise, each player's best response is a pure strategy. In addition, the minimum of the support must be equal to $t^* = t_2^* (< t_1^*)$: it is the point at which player 2's belief $p_2(t)$ reaches p^*, and thus social learning must occur for him not to exit immediately. Unlike in the symmetric case, player 1's belief $p_1(t^*)$ exceeds p^*. This raises a subtle issue. Player 1 strictly prefers staying to exiting whenever $p_1(t) > p^*$, while player 1's exit rate must be positive (i.e., player 1's action must be informative) in order for player 2 to be willing to stay whenever $p_2(t) \leq p^*$. As shown shortly, this issue can be resolved by player 2's exiting with a positive probability at time t^*, as it makes player 1's belief $p_1(t)$ drop below p^* instantly, conditional on no exit by player 2.

We first solve for player 2's conditional belief $p_2(t)$ and player 1's exit strategy $F_1(t)$ (equivalently, $\phi_1(t)$). Since player 1 never exits with a positive probability (i.e, the distribution function F_1 has no atom anywhere), these two can be derived just as in the symmetric case. We then characterize player 1's conditional belief $p_1(t)$ and player 2's exit strategy $F_2(t)$.

Player 2's equilibrium belief and player 1's equilibrium exit strategy.

Given player 1's exit strategy $\phi_1(t)$, conditional on no success and no exit by player 1, player 2's belief evolves as follows:

$$p_2(t) = \frac{p_2 e^{-\lambda t} e^{-\int_0^t \phi_1(y)dy}}{p_2 e^{-\lambda t} e^{-\int_0^t \phi_1(y)dy} + (1-p_2)\int_0^\infty e^{-\int_0^{\min\{x,t\}} \phi_1(y)dy} d(1-e^{-\lambda x})}.$$

Arranging the terms as in the symmetric case, it follows

$$\dot{p}_2(t) = -\lambda p_2(t)(1-p_2(t)) - \phi_1(t)p_2(t)\frac{p_2 - p_2(t)}{p_2}. \tag{3.7}$$

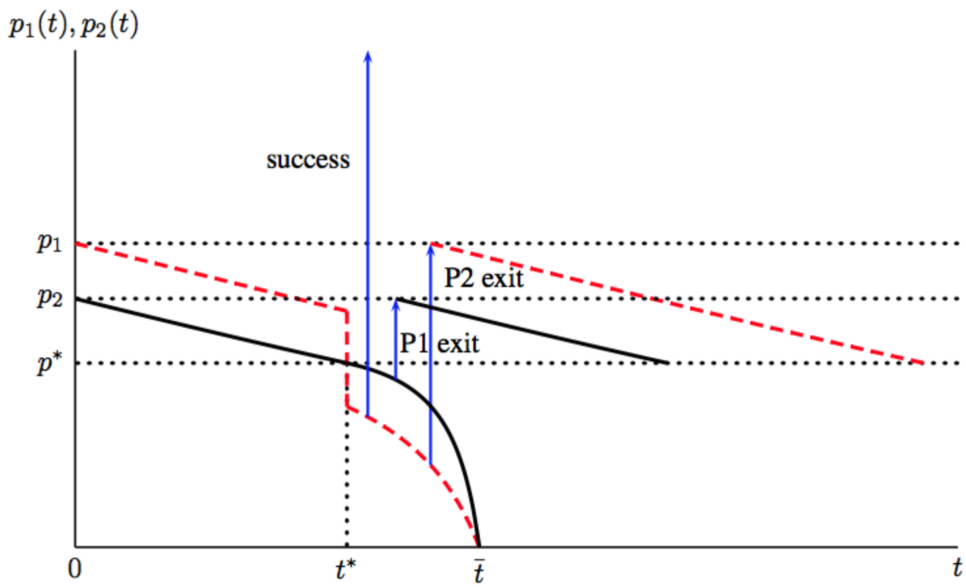
Observe that equation (3.3) is a special case of this equation, with $p_2 = 1/2$.

As in the symmetric case, player 2 must remain indifferent between staying and exiting whenever $t \in (t_2^*, \bar{t})$. Therefore,

$$c = \frac{p_2(t)}{p_2}\phi_1(t)V(p_2) + p_2(t)\lambda(v + V(1)). \tag{3.8}$$

This equation corresponds to equation (3.4) in the symmetric case. The only differences are that the probability that player 2 assigns to the event that player 1 has not succeeded by time t is equal to $p_2(t)/p_2$, instead of $2p_2(t)$, and that once player 1 exits (and thus player 2 knows that player 1 has not succeeded as well), player 2 updates his belief to p_2, instead of $1/2$.

Figure 3.3: Equilibrium Dynamics under Negative Correlation with Asymmetric Players



The players' beliefs conditional on no success in the asymmetric case. The dashed line is for player 1, while the solid line is for player 2.

Combining equations (3.7) and (3.8) leads to the following Riccati equation:

$$\dot{p}_2(t) = -\lambda p_2(t)(1 - p_2(t)) - \frac{c - p_2(t)\lambda(v + V(1))}{V(p_2)}(p_2 - p_2(t)). \tag{3.9}$$

As in the symmetric case, a closed-form solution can be obtained by applying Lemma 3.1, together with the boundary condition $p_2(t^*) = p^*$. Given the solution $p_2(t)$, player 1's equilibrium exit strategy $\phi_1(t)$ can also be explicitly derived. As in the symmetric case, $p_2(t)$ converges to 0 in finite time.

Player 1's equilibrium belief and player 2's equilibrium exit strategy. Following the same steps as above, we obtain the following equations: whenever $t \in (t^*, \bar{t})$,

$$\dot{p}_1(t) = -\lambda p_1(t)(1 - p_1(t)) - \phi_2(t) p_1(t) \frac{p_1 - p_1(t)}{p_1}, \tag{3.10}$$

and

$$c = \frac{p_1(t)}{p_1} \phi_2(t) V(p_1) + p_1(t) \lambda (v + V(1)). \tag{3.11}$$

Therefore, we again obtain a Riccati equation for $p_1(t)$:

$$\dot{p}_1(t) = -\lambda p_1(t)(1 - p_1(t)) - \frac{c - p_1(t)\lambda(v + V(1))}{V(p_1)} (p_1 - p_1(t)). \tag{3.12}$$

The difference from equation (3.9) lies in the boundary condition, because, as explained above, $\lim_{t \to t^*-} p_1(t) \neq p^*$. A necessary condition comes from the initial observation that the distribution functions F_1 and F_2 must have a common support. In particular, denote by $\bar{t}$ the upper bound of the support. Then, it must be that $p_1(\bar{t}) = p_2(\bar{t}) = 0$. Intuitively, if player j, conditional on no success, exits with probability 1 by time $\bar{t}$, then player i has no reason to stay beyond $\bar{t}$ and must assign probability 1 to his type being bad (player j's type being good), conditional on no success.

The exact value of $\bar{t}$ can be calculated from the solution to equation (3.9) (i.e., the value such that $p_2(\bar{t}) = 0$). Then, the condition that $p_1(\bar{t}) = 0$ can be applied to explicitly solve equation (3.12). Given the solution $p_1(t)$ over the interval $(t^*, \bar{t})$, player 2's equilibrium exit rate $\phi_2(t)$ can be obtained from equation (3.11). A necessary condition for $p_1(t)$ is that $p_1(t) < p^*$ for any $t \in (t_2^*, \bar{t}]$: otherwise, player 1 strictly prefers staying. This condition is guaranteed because $p_1(\bar{t}) = p_2(\bar{t}) = 0$, while $p_2(t)$ decreases faster than $p_1(t)$ before t^*: if $p_1(t) = p_2(t)$, then $|\dot{p}_1(t)| < |\dot{p}_2(t)|$, because $V(p_1) > V(p_2)$. Therefore, $p_1(t)$ must always stay below $p_2(t)$. Intuitively, player 1, due to the difference in prior beliefs, obtains more from player 2's exit than player 2 does from player 1's exit. Therefore, for them to be simultaneously indifferent between staying and exiting, player 1 must remain more pessimistic than player 2.

It remains to pin down player 2's exit probability at time t^*. Let $p_1^-(t^*) \equiv \lim_{t\to t^*-} p_1(t)(> p^*)$. Player 1's belief must jump down from $p_1^-(t^*)$ to $p_1(t^*)$. Conditional on player 1 being good, player 2 never succeeds and, therefore, exits with probability $F_2(t^*)$. Conditional on player 2 being good, player 2 exits only when he does not succeed by time t^* and, therefore, with probability $e^{-\lambda t^*}F_2(t^*)$. It follows that the probability that player 2 exits at time t^*, $F_2(t^*)$, must satisfy

$$p_1(t^*) = \frac{p_1^-(t^*)(1 - F_2(t^*))}{p_1^-(t^*)(1 - F_2(t^*)) + (1 - p_1^-(t^*))(1 - e^{-\lambda t^*}F_2(t^*))}. \tag{3.13}$$

We summarize all the results in the following proposition.

Proposition 3.4. *In the model with negative correlation (and $p_1 \geq p_2$), there exits a unique equilibrium. No player exits until time $t^* = t_2^*$. At time t^*, player 2 exits with probability $F_2(t^*)$ (as calculated in equation* (3.13)*), which lowers player 1's belief $p_1(t)$ below p^*. After time t^*, the players' beliefs, conditional on no success and no exit, evolve as in equations* (3.9) *and* (3.12)*, and each player exits at a positive rate as given in equations* (3.8) *and* (3.11)*. Player 2's expected payoff is identical to that in Proposition 3.1, while player 1 obtains a strictly higher expected payoff than in Proposition 3.1 whenever $p_1 > p_2 > p^*$.*

Proof. See the appendix. □

The result shows that social learning can be valuable under asymmetric negative correlation. More importantly, it illustrates when, and to whom, social learning is beneficial. As shown in the positive correlation case and the symmetric case, social learning does not improve the players' welfare if it occurs gradually. Intuitively, gradual social learning causes excessive delay, which offsets the benefit of social learning. If social learning occurs fast (as at time t^* in the asymmetric case), then it allows a player to enjoy the benefit, without incurring any delay cost. The player who enjoys the benefit is the one who is more willing to stay in the game and, therefore, relies less on social learning.

3.6 Discussion

In this section, we explain how to extend our analysis in various dimensions. For simplicity, we restrict attention to the symmetric case where the players begin with an identical probability of being good.

3.6.1 Imperfect Negative Correlation

We first consider the case of imperfect negative correlation, in which both players may be bad. In other words, now there are three possibilities: (i) player 1 is good, while player 2 is bad. (ii) player 1 is bad, while player 2 is good. (iii) both players are bad.[9] We denote by p_0 the prior probability of the last event. Since we consider the symmetric case, this means that the prior probability that each player

[9] One can consider the opposite case where both players may be good. The analysis is almost identical to the one here and, therefore, omitted.

is good (and the other is bad) is given by $p_i = (1 - p_0)/2$.

The equilibrium structure is similar to that of perfect negative correlation in Section 3.4. Let t^* be the point at which the players' beliefs reach p^* in the absence of social learning (that is, $p^* = p_i e^{-\lambda t^*}/(p_i e^{-\lambda t^*} + 1 - p_i)$ for $i = 1, 2$). Then, the players do not exit until time t^*, while they exit at a positive rate over the interval $[t^*, \bar{t})$.

There are two differences regarding buyers' conditional beliefs. First, player i's belief, conditional on no success and no exit by player j, decreases faster than under perfect negative correlation. This is because player i's own failure indicates not only the possibility that player j is good, but also the possibility that both players are bad. Formally, if $t \in (t^*, \bar{t})$, then player i's conditional belief evolves according to

$$p(t) = \frac{1}{2 + \int_0^t e^{\int_x^t (\lambda + \phi(y))dy} \lambda dx + \frac{2p_0}{1-p_0} e^{\lambda t}},$$

which implies that

$$\dot{p}(t) = -\lambda p(t)(1 - p(t)) - \phi(t)p(t)\left(1 - 2p(t)\left(1 + \frac{p_0}{1-p_0} e^{\lambda t}\right)\right). \tag{3.14}$$

Notice that equation (3.14) coincides with equation (3.3) when $p_0 = 0$, and the right-hand side is decreasing in p_0.

Second, player i's belief following player j's exit depends on the time player j exits: recall that player i's belief always goes back to p_i after player j's exit under perfect negative correlation. This is because the probability that both players are bad, conditional on no player's success, is strictly increasing over time. Formally, if player j exits at time t, then player i's conditional belief updates to

$$\frac{p_i e^{-\lambda t}}{p_0 + p_i e^{-\lambda t} + p_j e^{-\lambda t}} = \frac{1}{\frac{2p_0}{1-p_0} e^{\lambda t} + 2} < \frac{1}{2}.$$

Combining this with the fact that player i must remain indifferent between staying and exiting whenever $t \in (t^*, \bar{t})$,

$$c = \left(2 + \frac{2p_0}{1-p_0}e^{\lambda t}\right) p(t)\phi(t)V\left(\frac{1}{2 + \frac{2p_0}{1-p_0}e^{\lambda t}}\right) + p(t)\lambda(v + V(1)). \tag{3.15}$$

Notice that if $p_0 = 0$, then this equation reduces to equation (3.4).

As in Section 3.4, combining equations (3.14) and (3.15) yields a Riccati equation for $p(t)$. By standard arguments, there exists a unique solution to the equation. However, we are not able to derive an explicit solution. Technically, this is because, although the equation is similar to equation (3.5), its coefficients are time-varying, in which case closed-form solutions are known only for a limited class of Riccati equations.

3.6.2 More Players

We have restricted attention to the case where there are only two players. As explained below with the case of three players, the analysis becomes significantly more complicated in the general N-player case. This is a severe limitation of our analysis, given that Murto and Välimäki (2011) provide a characterization for the general case. A major difference is that buyers' conditional beliefs in the second phase remain constant even in the general case under positive correlation, which simplifies the analysis significantly as demonstrated in Section 3.3.2, but vary over time in an intricate way under negative correlation.

To be concrete, consider the case of three players in which only one of them is good. Let t^* be the point at which the players' conditional beliefs become equal to p^*

in the absence of social learning (i.e., t^* is the value such that $p^* = e^{-\lambda t^*}/(2 + e^{-\lambda t^*})$). Then, no player exits until time t^*. After time t^* until one player exits, they exit at a positive rate. Denote by $\phi(t)$ the symmetric exit rate of the players. Then, their conditional beliefs evolve according to

$$p(t) = \frac{1}{3 + 2\int_0^t e^{\int_x^t (\lambda + \phi(y))dy} \lambda dx}.$$

Suppose one player exits at time t. This reveals that the player has not succeeded yet. Then, the remaining players update their conditional beliefs to

$$p(t) = \frac{1}{3 + \int_0^t e^{\int_x^t (\lambda + \phi(y))dy} \lambda dx}.$$

Once a player exits, the game turns to a two-player game. Importantly, this subgame involves imperfect negative correlation, because there is a positive probability that the player who exited is actually good. As explained above, this problem is technically a lot more challenging than our baseline model, which also points to the difficulty of further characterizing the three-player case.

Nevertheless, it is possible to infer the limiting equilibrium outcome as the number of players tends to infinity. Suppose there are N players and it is common knowledge that $M(< N)$ players are good. Now let both M and N tend to infinity, while keeping its ratio M/N constant. In the limit, the problem becomes trivial, because, by the law of large numbers, correlation among players' types disappears, and thus there is no social learning. In other words, each player's problem reduces to the single-player problem in Section 3.3.1. The problem becomes non-trivial if there is aggregate uncertainty about the ratio M/N, as in Murto and Välimäki (2011).

However, again by the law of large numbers, the problem becomes identical to that of Murto and Välimäki (2011) in the limit as N tends to infinity.

3.6.3 Re-entry

Negative correlation gives rise to an incentive for the player who exits first to re-enter the game later. To be specific, consider the symmetric case in Section 3.4 and suppose player j exited first. As shown in Section 3.4, there is a positive probability that player i exits later. At that point, player j's belief is equal to $1-p^*$, which is even above 1/2: although it is common knowledge that both players have not succeeded, player i has experimented longer than player j, and thus player j is more likely to be good than player i. Therefore, player j is willing to re-enter the game as long as the re-entry cost does not exceed $V(1-p^*)$. Full characterization is fairly involved, mainly because the exiting player's expected payoff is now strictly positive (i.e., Lemma 1 in Murto and Välimäki (2011) no loner applies). However, the following result is straightforward to establish. Suppose the players can repeatedly re-enter the game if they pay a fixed cost $e > 0$ each time. If $e \geq V(1 - p^*)$, then the equilibrium without re-entry in Proposition 3.3 remains as an equilibrium. If $e < V(1-p^*)$, then the players alternately re-enter the game until one player eventually succeeds and, therefore, stays forever.

APPENDIX A
APPENDIX TO CHAPTER 1

Proof of Lemma 1.1. Sufficiency: $w_i - p_i > u_0$ implies that the consumer never takes an outside option u_0, because she is willing to visit at least one seller ($v_i + z_i^* - p_i > u_0$) and make a purchase ($v_i + z_i - p_i > u_0$). Given this, it suffices to show that if $w_i - p_i > w_j - p_j$, then the consumer never purchases product j.

- Suppose $z_j^* \leq z_j$, which implies that $w_j = v_j + z_j^*$. The consumer visits seller j only after seller i because $v_i + z_i^* - p_i \geq w_i - p_i > v_j + z_j^* - p_j$. Once she visits seller i, however, she has no incentive to visit seller j because $v_i + z_i - p_i > v_j + z_j^* - p_j$.
- Suppose $z_j^* > z_j$, which implies that $w_j = v_j + z_j$. In this case, even if she visits seller j, she either recalls a previous product ($v_i + z_i - p_i > v_j + z_j - p_j$) or continues to search ($v_i + z_i^* - p_i > v_j + z_j - p_j$) and finds a better product ($v_i + z_i - p_i > v_j + z_j - p_j$).

Necessity: if $w_i - p_i < u_0$, then the consumer does not visit seller i ($v_i + z_i^* - p_i < u_0$) or does not purchase product i even if she visits seller i ($v_i + z_i - p_i < u_0$). If $w_i - p_i < w_j - p_j$ for some $j \neq i$, then, for the same logic as above, the consumer never purchases product i. $\square$

Proof of Proposition 1.2. Since

$$(logH_i^\sigma(w_i^\sigma))'' = \frac{(h_i^\sigma)'(w_i^\sigma)H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2}{H_i^\sigma(w_i^\sigma)^2},$$

it suffices to show that $(h_i^\sigma)'(w_i^\sigma)H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2 \leq 0$ for all w, provided that σ is sufficiently large. Integrate equation (1.2) by parts, we have

$$H_i^\sigma(w_i^\sigma) = \int_{w_i^\sigma - z_i^*}^{\infty} G_i(w_i^\sigma - v_i^\sigma) dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*).$$

By straightforward calculus,

$$\frac{h_i^\sigma(w_i^\sigma)}{H_i^\sigma(w_i^\sigma)} = \frac{\int_{w_i^\sigma - z_i^*}^{\infty} g_i(w_i^\sigma - v_i^\sigma) dF_i^\sigma(v_i^\sigma) + (1 - G_i(z_i^*)) f_i^\sigma(w_i^\sigma - z_i^*)}{\int_{w_i^\sigma - z_i^*}^{\infty} G_i(w_i^\sigma - v_i^\sigma) dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*)} \tag{A.1}$$

and

$$\frac{(h_i^\sigma)'(w_i^\sigma)}{h_i^\sigma(w_i^\sigma)} = \frac{\int_{w_i^\sigma - z_i^*}^{\infty} g_i'(w_i^\sigma - v_i^\sigma) dF_i^\sigma(v_i^\sigma) + (1 - G_i(z_i^*))(f_i^\sigma)'(w_i^\sigma - z_i^*) - g_i(z_i^*) f_i^\sigma(w_i^\sigma - z_i^*)}{\int_{w_i^\sigma - z_i^*}^{\infty} g_i(w_i^\sigma - v_i^\sigma) dF_i^\sigma(v_i^\sigma) + (1 - G_i(z_i^*)) f_i^\sigma(w_i^\sigma - z_i^*)} \tag{A.2}$$

Changing the variables with $a = F_i^\sigma(v_i^\sigma)$ and $r = F_i^\sigma(w_i^\sigma - z_i^*)$, equation (A.1) becomes

$$\frac{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{\int_r^1 g_i((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*) da + (1 - G_i(z_i^*)) f_i^\sigma((F_i^\sigma)^{-1}(r))}{\int_r^1 G_i((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*) da + r}.$$

Since $V_i^\sigma \equiv \sigma V_i$, we have $F_i^\sigma(v_i^\sigma) = F_i(v_i^\sigma/\sigma)$, $(F_i^\sigma)^{-1}(r) = \sigma F_i^{-1}(r)$, $f^\sigma((F_i^\sigma)^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma$, and $(f_i^\sigma)'(F_i^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma^2$. Using these facts and arranging the terms in the right-hand side above yield

$$\frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) da + (1 - G_i(z_i^*)) f_i(F_i^{-1}(r))}{\int_r^1 G_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) da + r}.$$

Since $F_i^{-1}(r) - F_i^{-1}(a) \leq 0$, the denominator converges to r as σ explodes. Integrating $\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) da$ in the numerator by parts yields

$$G_i(z_i^*) f_i(F^{-1}(r)) + \int_r^1 G_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) df(F_i^{-1}(a)).$$

Again, since $F_i^{-1}(r) - F_i^{-1}(a) \leq 0$, the second term vanishes as σ tends to infinity, and thus the numerator converges to $G_i(z_i^*) f_i(F_i^{-1}(r))$. Therefore,

$$\lim_{\sigma \to \infty} \frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{f_i(F_i^{-1}(r))}{r}.$$

Similarly, changing the variables with $a = F_i^\sigma(v_i^\sigma)$ and $r = F_i^\sigma(w_i^\sigma - z^*)$ in equation (A.2) and following a similar procedure, we have

$$\lim_{\sigma\to\infty} \frac{\sigma(h_i^\sigma)'(F_i^{-1}(r) + z_i^*)}{h_i(F_i^{-1}(r) + z_i^*)} = \frac{(1 - G_i(z_i^*))f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))}.$$

Altogether,

$$\begin{aligned} & \lim_{\sigma\to\infty} \sigma \left[\frac{(h_i^\sigma)'((F_i^\sigma)^{-1}(r) + z_i^*)}{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} - \frac{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} \right] \\ = & \frac{(1 - G_i(z_i^*))f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r} \\ = & (1 - G_i(z_i^*)) \left[\frac{f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r} \right] - \frac{G_i(z_i^*) f_i(F_i^{-1}(r))}{r} < 0. \quad \text{(A.3)} \end{aligned}$$

For any $s_i \in (0, \infty)$, $G_i(z_i^*) \in (0, 1)$ by equation (1.1). The square bracket term is weakly negative because F is log-concave, thus the entire expression is weakly negative. Now we show the strict inequality (A.3) holds for all $r \in [0, 1]$. For $r \in (0, 1)$, the strict inequality (A.3) is true because $f_i(F_i^{-1}(r))/r > 0$ by the full support assumption. Since $f_i(F_i^{-1}(r))/r$ falls in r by the log-concavity of F_i, $f_i(F_i^{-1}(r))/r > 0$ at $r = 0$, and thus the strict inequality (A.3) also holds for $r = 0$. For $r = 1$, since f_i has full support, $f_i(F_i^{-1}(r))$ falls in r when r is large. Therefore $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ for some $r \in (0, 1)$. Since $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r))$ falls in r by the log-concavity of f_i, $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ when $r = 1$ and thus the strict inequality (A.3) holds when $r = 1$. Altogether, for each $r \in [0, 1]$ there is a $\bar{\sigma}_r$ such that if $\sigma > \bar{\sigma}_r$, then $(h_i^\sigma)'(w)/h_i^\sigma(w) - h_i^\sigma(w)/H_i^\sigma(w) < 0$ where $w = F^{-1}(r)$. Since $[0, 1]$ is a compact convex set, there exists $\bar{\sigma} = \max_{r\in[0,1]} \sigma_r \leq \infty$ such that if $\sigma > \bar{\sigma}$, then $(h_i^\sigma)'/h_i^\sigma - h_i^\sigma/H_i^\sigma < 0$ for all $r \in [0, 1]$, or equivalently $H_i^\sigma(w)$ is log-concave for all w. $\square$

The following Lemma is useful for proving Proposition 1.3.

Lemma A.1. *For any $a \in (0,1]$, there exists $s_a < \infty$ such that $h_i(F_i^{-1}(a))/H_i(F_i^{-1}(a))$ falls in s_i whenever $s_i \geq s_a$.*

Proof. Suppose $a \in (0,1)$, and let $w_i = F_i^{-1}(a)$. We show that $h_i(w_i)/H_i(w_i)$ falls in s_i if and only if s_i is large. By equation (1.1), $\partial z_i^*/\partial s_i = -[1 - G_i(z_i^*)]$. Then by equation (1.3), $\partial H_i(w_i)/\partial s_i = f_i(w_i - z_i^*)$. Therefore,

$$\frac{\partial}{\partial s_i} log\left[\frac{h_i(w_i)}{H_i(w_i)}\right] = \frac{f_i(w_i - z_i^*)}{h_i(w_i)}\left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{h_i(w_i)}{H_i(w_i)}\right]. \tag{A.4}$$

Suppose the square bracket term in the right-hand side is equal to 0 at some $s_i = s_a$. As s_i rises from s_a, $f_i'(w_i - z_i^*)/f_i(w_i - z_i^*)$ falls, because z_i^* falls in s_i and f_i is log-concave. The derivative of the second term in the square bracket with respect to s_i is equal to 0 at $s_i = s_a$. Thus $\partial[h_i(w_i)/H_i(w_i)]/\partial s_i \leq 0$ for all $s_i \geq s_a$. Equivalently, $\partial\left(h_i(w_i)/H_i(w_i)\right)/\partial s_i$ is reverse single-crossing in s_i.

To show $s_a < \infty$, it suffices to show $\partial(h_i(w_i)/H_i(w_i))/\partial s_i < 0$ as $s_i \to \infty$. If $s_i \to \infty$, then $z_i^* \to -\infty$ and the sign of $\partial(h_i(w_i)/H_i(w_i))/\partial s_i$ is the same as

$$\begin{aligned}\lim_{z_i^* \to -\infty}\left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{h_i(w_i)}{H_i(w_i)}\right] &= \lim_{z_i^* \to -\infty}\left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{\int f_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)dz_i}{\int F_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)dz_i}\right] \\ &= \lim_{z_i^* \to -\infty}\left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{f_i(w_i - z_i^*)}{F_i(w_i - z_i^*)}\right] < 0.\end{aligned}$$

The last inequality is true as $\lim_{z_i^* \to -\infty} f_i(w_i - z_i^*)/F_i(w_i - z_i^*) = 0$ and $\lim_{z_i^* \to -\infty} f_i'(w_i - z_i^*)/f_i(w_i - z_i^*) < 0$ by the log-concavity of f_i. Since $\partial(h_i(w_i)/H_i(w_i))/\partial s_i$ is reverse single-crossing in s_i and is strictly negative as s_i explodes, there exists $s_a < \infty$ such that $h_i(w_i)/H_i(w_i)$ falls in s_i for all $s_i > s_a$.

Finally, assume $a = 1$ and let $w_i = F_i^{-1}(a) = \infty$. In this case the right-hand side of equation (A.4) is strictly negative because $\lim_{w_i \to \infty} f_i'(w_i - z_i^*)/f_i(w_i - z_i^*) < 0$ by the log-concavity of f and $\lim_{w_i \to \infty} h_i(w_i)/H_i(w_i) = 0$. Therefore, $h_i(w_i)/H_i(w_i)$ falls in s_i when $a = 1$. $\square$

Proof of Proposition 1.3 . Proof of (i): To show that $H_i(w_i)$ is log-concave when s_i is large, it suffices to show the reverse hazard rate $h_i(w_i)/H_i(w_i)$ falls in w_i when s_i is large. Recall that $\partial z_i^*/\partial s_i = -1/[1 - G_i(z_i^*)]$ by equation (1.1) and $H_i(w_i) = \int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i) + F_i(w_i - z_i^*)$ by equation (1.2). Thus $\partial log(H_i(w_i))/\partial s_i = f_i(w_i - z_i^*)/H_i(w_i)$. Therefore $h_i(w_i)/H_i(w_i)$ can be written as

$$\frac{h_i(w_i)}{H_i(w_i)} = [1 - G_i(z_i^*)]\frac{\partial log(H_i(w_i))}{\partial s_i} + \frac{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)}{\int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i) + F_i(w_i - z_i^*)}.$$

We argue that the right-hand side falls in w_i when s_i is sufficiently large. To see this, note that an immediate corollary of Lemma A.1 is that $\partial log(H_i(w_i))/\partial w_i$ falls in s_i for all $w_i \geq u_0 > -\infty$ when s_i is sufficiently large. Equivalently, $\partial log(H_i(w_i))/\partial s_i$ falls in w_i for all $w_i \geq u_0$ when s_i is sufficiently large. Therefore, the first term in the displayed equation falls in w_i when s_i is large. It remains to show that the second term falls in w_i. To this end, consider the inverse of the second term

$$\begin{aligned}
&\left[\frac{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)}{\int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i) + F_i(w_i - z_i^*)}\right]^{-1} \\
= \quad & \frac{\int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i)}{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)} + \frac{F_i(w_i - z_i^*)}{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)} \\
= \quad & \frac{\int_{-\infty}^{z_i^*} G_i(z_i)f_i(w_i - z_i)dz_i}{\int_{-\infty}^{z_i^*} g_i(z_i)f_i(w_i - z_i)dz_i} + \frac{F_i(w_i - z_i^*)}{f_i(w_i - z_i^*)}\frac{f_i(w_i - z_i^*)}{\int_{-\infty}^{z_i^*} g_i(z_i)f_i(w_i - z_i)dz_i}.
\end{aligned}$$

The second line applies a change of variable $z_i = w_i - v_i$. The first term rises in w_i by the log-concavity of f_i and G_i. The second term rises in w_i by the log-concavity of F_i and f_i. Altogether, the entire expression rises in w_i. Since all elements in the expression are positive, its inverse falls in w_i.

Proof of (ii): Since we assume the density function $f_i(v_i)$ is log-concave, it is single-peaked in v_i. Since f_i has full support and is a probability density function, it cannot be monotone and thus must rises and then falls as v_i rises. Thus there exists $\bar{u}_0$ such that $f_i'(w_i - z_i^*) \leq 0$ for all $w_i > \bar{u}_0$. It follows that for all $w_i > \bar{u}_0$, $h_i(w_i) = \int f_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)dz_i$ falls in w_i. Thus $h_i(w_i)/H_i(w_i)$ falls in w_i for $w_i \geq \bar{u}_0$. $\square$

Proof of Proposition 1.4. We prove the last claim in the proof of Proposition 1.4 in the main text. Index $D_i(\mathbf{p}, u_0)$ by the outside option u_0 and let $\mathbf{p}^* = (p^*, \ldots, p^*)$. Then, the right-hand side of equation (1.4) can be rewritten as $-\partial Log[D_i(\mathbf{p}, u_0)]/\partial p_i|_{\mathbf{p}=\mathbf{p}^*}$. Due to the additive utility specification, $D_i(\mathbf{p}, u_0) = D_i(\mathbf{p} + u_0, 0)$, that is, demand for each seller stays constant if all prices and $-u_0$ increase by the same amount. This implies

$$\begin{aligned} &\frac{\partial}{\partial p^*}\left[\frac{-\partial Log[D_i(\mathbf{p}, u_0)]}{\partial p_i}|_{\mathbf{p}=\mathbf{p}^*}\right] = \frac{\partial}{\partial p^*}\left[\frac{-\partial Log[D_i(\mathbf{p} + u_0, 0)]}{\partial p_i}|_{\mathbf{p}=\mathbf{p}^*}\right] \\ =&\frac{\partial}{\partial u_0}\left[\frac{-\partial Log[D_i(\mathbf{p} + u_0, 0)]}{\partial p_i}|_{\mathbf{p}=\mathbf{p}^*}\right] = \frac{-\partial^2 Log[D_i(\mathbf{p}, u_0)]}{\partial p_i \partial u_0}|_{\mathbf{p}=\mathbf{p}^*}. \end{aligned}$$

Since $D_i(\mathbf{p}, u_0)$ is log-submodular in (p_i, u_0) by the proof of Theorem 1 in Quint (2014), the right-hand is positive and thus $-\partial Log[D_i(\mathbf{p}, u_0)]/\partial p_i|_{\mathbf{p}=\mathbf{p}^*}$ rises in p^*. $\square$

For the next proof, recall from Section 5.3 that $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$ in the accuracy model.

Proof of Lemma 1.3. It suffices to show there exists $a' \in (0,1)$ such that $\partial h(H^{-1}(a))/\partial \alpha < 0$ if and only if $a > a'$. Let Φ denote the standard normal distribution function and ϕ denote its density function. Since $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$, $F(v) = \Phi(v/\alpha)$ and $G(z) = \Phi(z/\sqrt{1-\alpha^2})$. Inserting these into equation (1.2) and differentiating $H(w)$ with respect to α yield

$$H_\alpha(w) \equiv \frac{\partial H(w)}{\partial \alpha} = -\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\left(\frac{w-z^*}{\alpha^2}\right)\phi\left(\frac{w-z^*}{\alpha}\right),$$

where $\partial z^*/\partial \alpha$ can be obtained from equation (1.1) by applying the implicit function theorem. Differentiating again with respect to w gives

$$h_\alpha(w) \equiv \frac{\partial h(w)}{\partial \alpha} = -\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\left[1 - \left(\frac{w-z^*}{\alpha}\right)^2\right]\frac{1}{\alpha^2}\phi\left(\frac{w-z^*}{\alpha}\right).$$

Now observe that

$$\frac{\partial h(H^{-1}(a))}{\partial \alpha} = h_\alpha(H^{-1}(a)) - H_\alpha(H^{-1}(a))\frac{h'(H^{-1}(a))}{h(H^{-1}(a))}.$$

Let $w = H^{-1}(a)$ and apply $H_\alpha(w)$ and $h_\alpha(w)$ to the equation. Then,

$$\frac{\partial h(H^{-1}(a))}{\partial \alpha} = \frac{-1}{\alpha^2}\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\phi\left(\frac{w-z^*}{\alpha}\right)\left[1 - \frac{(w-z^*)^2}{\alpha^2} - (w-z^*)\frac{h'(w)}{h(w)}\right].$$

Since $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$, the density of $W = V + \min\{Z, z^*\}$ is

$$\begin{aligned} h(w) &= \frac{1}{\sqrt{1-\alpha^2}}\int_{-\infty}^{\infty}\phi\left(\frac{w - \min\{z, z^*\}}{\alpha}\right)\phi\left(\frac{z}{\sqrt{1-\alpha^2}}\right)dz \\ &= \frac{1}{\sqrt{1-\alpha^2}}\int_{-\infty}^{\infty}\phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr \end{aligned}$$

where the second line changes variable $r = (z^* - z)/\alpha$. Since $\partial\phi(x)/\partial x = -x\phi(x)$,

$$\frac{h'(w)}{h(w)} = -\frac{w - z^*}{\alpha^2} - \frac{\int_{-\infty}^{\infty} \max\{r, 0\}\phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}{\alpha\int_{-\infty}^{\infty}\phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}.$$

Applying this to the above equation leads to

$$\begin{aligned}
\frac{\partial h(H^{-1}(a))}{\partial \alpha} &\propto -1 + \left(\frac{w - z^*}{\alpha}\right)^2 + (w - z^*)\frac{h'(w)}{h(w)} \\
&= -1 + \frac{(z^* - w)}{\alpha}\frac{\int_{-\infty}^{\infty} 1_{\{r\geq 0\}} r\phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}{\int_{-\infty}^{\infty}\phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}.
\end{aligned}$$

The last expression is clearly negative if $w > z^*$. In addition, it converges to ∞ as w tends to $-\infty$. For $w \leq z^*$, it decreases in w because $(z^* - w)$ falls in w and the density $\phi((w - z^*)/\alpha + \max\{r, 0\})$ is log-submodular in (w, r). Therefore, there exists w' less than z^* such that the expression is positive if and only if $w < w'$. The desired result follows from the fact that $w = H^{-1}(a)$ is strictly increasing in a. □

Proof of Proposition 1.7. Given that there is no outside option, the condition for the equilibrium price p^* is given by

$$\frac{1}{p^* - c} = n\int h(w)dH(w)^{n-1} = n\int_0^1 h(H^{-1}(a))da^{n-1}.$$

By the implicit function theorem,

$$\frac{\partial p^*}{\partial \alpha} = -(p^* - c)^2 n\int_0^1 \frac{\partial h(H^{-1}(a))}{\partial \alpha}da^{n-1}.$$

The desired result follows by combining Lemma 1.3 above with the fact that for any real value function $\gamma : \mathcal{R} \to \mathcal{R}$, if $\int_0^1 \gamma(a)da^n \leq 0$ and there exists a' such that $\gamma(a) < 0$ if and only if $a > a'$, then

$$\int_0^1 \gamma(a)da^{n+1} = \frac{n+1}{n}\int_0^1 \gamma(a)a da^n \leq 0.$$

The last inequality is due to the fact that a is positive and strictly increasing and, therefore, assigns more weight to the negative portion of $\gamma(a)$ in the integral (Karlin and Rubin (1955)). The result follows by letting $\gamma(a) = \partial h(H^{-1}(a))/\partial\alpha$. $\square$

Proof of Proposition 1.8. Let $\bar{p}_i = p_i - c_i$ be product i's markup. We prove the claim by contradiction — Assume $\bar{p}_i < \bar{p}_j$ and show that seller i would deviate and choose $\bar{p}_i \geq \bar{p}_j$. Let $X_i = -\max_{\ell\neq i}\{W_\ell - c_\ell - \bar{p}_\ell, u_0\}$. Seller i's demand is $D_i(\mathbf{p}) = P(W_i - c_i - \bar{p}_i > -X_i) = P(W_i - c_i + X_i > \bar{p}_i)$. Let R_i be the distribution function of the random variable $W_i - c_i + X_i$ and r_i be its density function. Then $D_i(\mathbf{p}) = 1 - R_i(\bar{p}_i)$ and thus seller i's FOC is

$$\frac{1}{\bar{p}_i} = \frac{-\partial D_i(\mathbf{p})/\partial \bar{p}_i}{D_i(\mathbf{p})} = \frac{r_i(\bar{p}_i)}{1 - R_i(\bar{p}_i)}.$$

To derive a contradiction, it suffices to show $r_i(\bar{p}_i)/[1 - R_i(\bar{p}_i)] \leq r_j(\bar{p}_j)/[1 - R_j(\bar{p}_j)]$ whenever $\bar{p}_i < \bar{p}_j$. Recall that $D_i(\mathbf{p})$ is log-concave in $\bar{p}_i$ and log-supermodular in $(\bar{p}_i, \bar{p}_j)$ by Theorem 1.1, thus $r_i(\bar{p}_i)/[1 - R_i(\bar{p}_i)]$ rises in $\bar{p}_i$ and falls in $\bar{p}_j$. Similarly, $r_j(\bar{p}_j)/[1 - R_j(\bar{p}_j)]$ rises in $\bar{p}_j$ and falls in $\bar{p}_i$. Therefore, it suffice to show $r_i/[1 - R_i] \leq r_j/[1 - R_j]$ whenever $\bar{p}_i = \bar{p}_j$. Fixing $\bar{p}_i = \bar{p}_j$ and all other markups, if $W_i - c_i$ and $W_j - c_j$ have the same distribution, then clearly $r_i/[1 - R_i] = r_j/[1 - R_j]$. To show $r_i/[1 - R_i] \leq r_j/[1 - R_j]$ when $W_i - c_i$ dominates $W_j - c_j$ in the hazard rate and reverse hazard rate order, it suffices to show (a) $r_i/[1 - R_i]$ falls as $W_i - c_i$ rises in the hazard rate order and (b) $r_j/[1 - R_j]$ rises as $W_i - c_i$ rises in the reverse hazard rate order.

Proof of (a): First, note that $r_i/[1-R_i]$ falls if $W_i - c_i + X_i$ rises in the hazard rate order. By Lemma 1.B.3. in SS,[1] if the survivor function of X_i is log-concave, then $W_i - c_i + X_i$ rises in the hazard rate order when $W_i - c_i$ rises in the hazard rate order. To see why the survivor of X_i is log-concave, observe that

$$P(X_i > x) = P(\max_{j\neq i}\{W_j - c_j - \bar{p}_j, u_0\} < -x) = \prod_{j\neq i} H_j(c_j + \bar{p}_j - x)1_{\{u_0<-x\}}$$

$$log(P(X_i > x)) = \sum_{j\neq i} log(H_j(c_j + \bar{p}_j - x)) + log(1_{\{u_0<-x\}}) \tag{A.5}$$

where $1_{\{u_0<-x\}}$ is an indicator function of the event $\{u_0 < -x\}$. The left-hand side of the second line is concave in x because each element in the right-hand side is. This proves (a).

Proof of (b): Similar to (a), $W_j - c_j + X_j$ falls in the hazard rate order as X_j falls in the hazard rate order by Lemma 1.B.3. in SS because we have assumed the survivor of W_j is log-concave. It remains to show that X_j falls in the hazard rate order when $W_i - c_i$ rises in the reverse hazard rate order. As $W_i - c_i$ rises in the reverse hazard rate order, the ratio $h_i(c_i + \bar{p}_i - x)/H_i(c_i + \bar{p}_i - x)$ rises for all x and thus the slope of $log(H_i(c_i + \bar{p}_i - x))$ with respect to x falls at all x. Hence the slope of $log(P(X_j > x))$ with respect to x falls for all x by equation (A.5), which implies X_j falls in the hazard rate order. Altogether, $W_j - c_j + X_j$ falls in the hazard rate order as $W_i - c_i$ rises in the reverse hazard rate order. $\square$

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[1] Lemma 1.B.3. in SS: Assume the random variables X and Y are such that X dominates Y in the hazard rate order. If W is a random variable independent of X and Y and has log-concave survivor function, then $X + Z$ dominates $Y + Z$ in the hazard rate order.

Proof of Corollary 1.2. We show W_j dominates W_i in the hazard rate order and the reverse hazard rate order and then use Proposition 1.8 to prove the claim. Since the likelihood ratio order implies both the hazard rate order and the reverse hazard rate order by Theorem 1.C.1. in SS, it suffices to show W_j dominates W_i in the likelihood ratio order. Since $F_i = F_j = F$ and $z_i^* = z_j^* = z^*$, the likelihood ratio is

$$\frac{h_j(w)}{h_i(w)} = \frac{\int f(w - \min\{z, z^*\})g_j(z)dz}{\int f(w - \min\{z, z^*\})g_i(z)dz} = \int \frac{g_j(z)}{g_i(z)} \frac{f(w - \min\{z, z^*\})g_i(z)}{\int f(w - \min\{z, z^*\})g_i(z)dz} dz$$

The right-hand side can be interpreted as $E[g_j(X)/g_i(X)]$ where the random variable X has density $f(w - \min\{x, z^*\})g_i(x)$. The random variable X rises in the first-order stochastic dominance sense in w because f is log-concave. The function $g_j(X)/g_i(X)$ rises in X because Z_j dominates Z_i in the likelihood ratio order. Therefore, $h_j(w)/h_i(w)$ rises in w, or equivalently W_j dominates W_i in the likelihood ratio order. □

Lemma A.2 below shows that H_i falls in the likelihood ratio order as s_i rises under the premises of Corollary 1.3. Therefore, the conclusion of Corollary 1.3 follows from its premises by (i) Lemma A.2, (ii) the fact that the likelihood ratio order implies both the hazard and reverse hazard rate order, and (iii) Proposition 1.8.

Lemma A.2. *If $-f_i'(v)$ is positive and log-concave for all $v > u - z_i^*$, then $h_i(w_2)/h_i(w_1)$ falls in s_i for all $w_2 > w_1 \geq u_0$.*

Proof. Differentiating equation (1.3) yields

$$h_i(w) = \int_{-\infty}^{z_i^*} -f_i'(w - z)[1 - G_i(z)]dz.$$

To prove $h_i(w_2)/h_i(w_1)$ falls in s_i for all $w_2 > w_1 \geq u_0$, it suffices to show $(\partial h_i(w)/\partial s_i)/h_i(w)$ falls in w. Recall that $\partial z_i^*/\partial s_i = -1/[1-G_i(z_i^*)]$. Thus

$$\frac{\partial h_i(w)/\partial s_i}{h_i(w)} = -\frac{f_i'(w-z_i^*)}{\int_{-\infty}^{z_i^*} f_i'(w-z)[1-G_i(z)]dz}.$$

Since we assume $-f_i'(v)$ is positive and log-concave for all $v > u_0 - z^*$, $-f_i'(w-z)$ is log-supermodular in (w,z) for all $z \leq z_i^*$ and $w \geq u_0$. Therefore, the ratio $f_i'(w - z_i^*)/f_i'(w-z) > 0$ rises in w for all $z \leq z_i^*$ and $w \geq u_0$, and thus $[\partial h_i(w)/\partial s_i]/h_i(w)$ falls in w for $w \geq u_0$. Equivalently, $h_i(w_2)/h_i(w_1)$ falls in s for all $w_2 > w_1 \geq u_0$. □

The following lemma is useful for proving Proposition 1.9.

Lemma A.3. *Assume $F_1 = F_2 = F$, $G_1 = G_2 = G$ and $s_1 = s_2 = s$. The difference $W_2 - W_1$ grows less dispersive as the search cost s rises.*

Proof. Consider $W_2 - W_1 = V_2 - V_1 + \min\{Z_2, z^*\} - \min\{Z_1, z^*\}$. By Theorem 3.B.7 in SS, $W_2 - W_1$ grows more dispersive if (a) $V_2 - V_1$ has log-concave density and (b) the difference $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ grows more dispersive. Since we have assume V_2 and V_1 have log-concave density, so does $V_2 - V_1$. Thus (a) is satisfied. To see (b), let T be the distribution function of the absolute difference $Y = |\min\{Z_2, z^*\} - \min\{Z_1, z^*\}|$:

$$T(y) = P(y \geq |\min\{Z_2, z^*\} - \min\{Z_1, z^*\}|) = 1 - 2\int G(\min\{z, z^*\} - y)dG(z).$$

By the definition of dispersive order, Y grows less dispersive in s if and only if its quantile function T^{-1} grows flatter as s rises, namely $\partial T^{-1}(a)/\partial a$ falls in s for all

$a \in (0,1)$. Equivalently, $\partial T^{-1}(a)/\partial s = -[\partial T(y)/\partial s]/t(y)$ falls in $y = T^{-1}(a)$. Differentiating T with respect to y and s yields

$$\frac{\partial T^{-1}(a)}{\partial s} = -\frac{\partial T(y)/\partial s}{t(y)} = \frac{-g(z^* - y)}{\int g(\min\{z, z^*\} - y) dG(z)}.$$

The right-hand side falls in y by the log-concavity of g.

Therefore, Y grows less dispersive as s rises. Since Z_1 and Z_2 have the same distribution, the distribution of the random variable $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ is symmetric around 0, and its quantile function grows steeper as Y grows more dispersive.[2] Therefore, $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ also grows less dispersive as s rises. $\square$

Now we prove $p_1 \leq p_2$ and Proposition 1.9.

Proof of Proposition 1.9. When $n = 2$ and $u_0 = -\infty$, the demand function is $D_i(\mathbf{p}) = \int 1 - H_i(w - p_j + p_i) dH_j(w)$ for $i = 1, 2$. The first-order condition for seller 1 and 2 are

$$p_1 - c_1 = \frac{\int (1 - H_1(w - p_2 + p_1)) dH_2(w)}{\int h_1(w - p_2 + p_1) dH_2(w)} \text{ and } p_2 - c_2 = \frac{\int (1 - H_2(w - p_1 + p_2)) dH_1(w)}{\int h_2(w - p_1 + p_2) dH_1(w)}.$$

Define the price difference $\Delta \equiv p_2 - p_1$. Seller 1 and 2's first-order conditions imply

$$c_2 - c_1 - \Delta = \frac{\int (1 - H_1(w - \Delta)) dH_2(w)}{\int h_1(w - \Delta) dH_2(w)} - \frac{\int (1 - H_2(w + \Delta)) dH_1(w)}{\int h_2(w + \Delta) dH_1(w)}. \tag{A.6}$$

[2] To see this, let $\tilde{T}$ be the distribution function of $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$. Since the distribution of $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ is symmetric around 0, $\tilde{T}(y) = [1 - T(-y)]/2$ for $y < 0$ and $\tilde{T}(y) = [1 + T(y)]/2$ for $y \geq 0$. Thus $\tilde{T}^{-1}(a) = -T^{-1}(1 - 2a)$ for $a < 1/2$ and $\tilde{T}^{-1}(a) = T^{-1}(2a - 1)$ for $a \geq 1/2$. Recall that a random variable grows more dispersed if and only if its quantile function grows steeper at all quantile. Clearly, $\partial \tilde{T}^{-1}(a)/\partial a$ rises $\forall a \in (0,1)$ if $\partial T^{-1}(a)/\partial a$ rises $\forall a \in (0,1)$.

Easily, the left-hand side falls in Δ. The right-hand side rises in Δ by the log-concavity of the demand functions. Therefore, equation (A.6) has a unique solution for Δ.

Now we show $p_2 - p_1 \geq 0$ in equilibrium. Since W_1 and W_2 have the same distribution, $H_1 = H_2 = H$. Hence $\int h_1(w - \Delta)dH_2(w) = \int h_2(w + \Delta)dH_1(w)$. Therefore, the right-hand side of (A.6) is $[D_1(\mathbf{p}) - D_2(\mathbf{p})] / \int h_1(w - \Delta)dH_2(w)$. Since $c_2 - c_1 - \Delta \geq 0$ in equilibrium by Corollary 1.1, $D_1(\mathbf{p}) \geq D_2(\mathbf{p})$ in equilibrium. Since the distribution of W_1 and W_2 are the same, $D_1(\mathbf{p}) \geq D_2(\mathbf{p})$ implies $p_1 \leq p_2$.

Next, we show $D_1(\mathbf{p})$ falls and $D_2(\mathbf{p})$ rises as H grows more dispersive. Let Q and q be the distribution function and the density function of the absolute difference $|W_2 - W_1|$. Since $H_1 = H_2 = H$, the probability $P(|W_2 - W_1| \geq \Delta) = 2\int[1 - H(w + \Delta)]dH(w)$. Hence

$$Q(\Delta) = 1 - 2\int(1 - H(w + \Delta))dH(w) \quad \text{and} \quad q(\Delta) = 2\int h(w + \Delta)dH(w). \tag{A.7}$$

Since $D_1(\mathbf{p}) = 1 - D_2(\mathbf{p})$ and $Q(\Delta) = 1 - 2D_2(\mathbf{p})$, $Q(p_2 - p_1) = D_1(\mathbf{p}) - D_2(\mathbf{p})$. Therefore, $\partial D_1/\partial p_1 = \partial D_2/\partial p_2 = -q(p_2 - p_1)/2$. Thus, equation (A.6) can be written as

$$c_2 - c_1 - \Delta = \frac{2Q(\Delta)}{q(\Delta)} \iff c_2 - c_1 - Q^{-1}(a) = \frac{2a}{q(Q^{-1}(a))} \tag{A.8}$$

where the second equation applies a change of variable $a = Q(\Delta)$.

Since $Q(\Delta)/q(\Delta)$ rises in Δ by the log-concavity of the demand functions, the fraction $a/q(Q^{-1}(a))$ in equation (A.8) rises in a. As H grows more dispersive, the absolute difference $|W_2 - W_1|$ rises in the first-order stochastic dominance sense by

Theorem 3.B.31 in SS and $|W_2 - W_1|$ grows more dispersive by Lemma A.3.[3] Thus, $Q^{-1}(a)$ rises and $q(Q^{-1}(a))$ falls for all $a \in [0, 1]$. Since the left-hand side falls and the right-hand side rises in a, the solution of a falls as H grows more dispersive. Since $a \equiv Q(p_2 - p_1) = D_1(\mathbf{p}) - D_2(\mathbf{p})$ and $D_1(\mathbf{p}) + D_2(\mathbf{p}) = 1$, $D_2(\mathbf{p})$ rises and $D_1(\mathbf{p})$ falls as H grows more dispersive. □

Proof of Proposition 1.10 . Recall that $\partial D_1(\mathbf{p})/\partial p_1 = \partial D_2(\mathbf{p})/\partial p_2$ by $D_1(\mathbf{p}) + D_2(\mathbf{p}) = 1$. Since $p_i - c_i = D_i(\mathbf{p})/[\partial D_i(\mathbf{p})/\partial p_i]$, we have $(p_2 - c_2)/(p_1 - c_1) = D_2(\mathbf{p})/D_1(\mathbf{p})$. Therefore, $(p_2 - c_2)/(p_1 - c_1)$ decreases in s by Proposition 1.9.

Next, recall from equation (A.8) that the price difference $\Delta = p_2 - p_1$ solves

$$c_2 - c_1 - \Delta = \frac{2Q(\Delta)}{q(\Delta)}$$

and $Q(\Delta)/2q(\Delta)$ rises in Δ by the log-concavity of the demand functions. Thus, there is a unique solution for Δ, call it Δ^*. Clearly Δ^* rises in $c_2 - c_1$ by the displayed equation. We have seen in the last part of the proof of Proposition 1.9 that $Q(\Delta)$ rises in s for all $\Delta \geq 0$. Therefore, if $\partial q(\Delta)/\partial s \leq 0$ at $\Delta = \Delta^*$, then $\partial \Delta^*/\partial s \leq 0$. Therefore, to prove $\partial \Delta^*/\partial s \leq 0$ when $c_2 - c_1$ is large, it suffices to show $\partial q(\Delta)/\partial s|_{\Delta=\Delta^*} \leq 0$ when $c_2 - c_1$ is large. Since $c_2 - c_1$ affects $\partial q(\Delta)/\partial s|_{\Delta=\Delta^*}$ only through Δ^*, and Δ^* rises in $c_2 - c_1$, it suffices to prove $\partial q(\Delta)/\partial s \leq 0$ when Δ is large.

---

[3]Let Ω and $\tilde{\Omega}$ be the distribution function of $W_2 - W_1$ and $|W_2 - W_1|$ respectively. It is easy to show $\tilde{\Omega}^{-1}(a) = \Omega^{-1}((a + 1)/2)$ for all $a \in (0, 1)$. Hence $\tilde{\Omega}^{-1}$ becomes steeper when Ω^{-1} becomes steeper. Therefore Lemma A.3 implies $|W_2 - W_1|$ grows more dispersive.

To this end, let $\tilde{f}(\Delta) \equiv \int f(v)f(\Delta+v)dv$. Then by equation (A.7)

$$q(\Delta)/2 = \int\int \tilde{f}(\min\{z,z^*\} - \min\{\tilde{z},z^*\} - \Delta)g(z)g(\tilde{z})d\tilde{z}dz.$$

Differentiate with respect to s and uses $\partial z^*/\partial s = -1/[1-G(z^*)]$, then

$$\begin{aligned}\frac{\partial q(\Delta)}{\partial s} =& 2\int_{-\infty}^{z^*}[\tilde{f}'(z-z^*-\Delta) - \tilde{f}'(z^*-z-\Delta)]g(z)dz \\ =& 2\int_{0}^{\infty}[\tilde{f}'(-r-\Delta) - \tilde{f}'(r-\Delta)]g(-r+z^*)dr\end{aligned}$$

This expression is negative when Δ is large because

$$\begin{aligned}\lim_{\Delta\to\infty}[\tilde{f}'(-r-\Delta) - \tilde{f}'(r-\Delta)] &= \lim_{\Delta\to\infty}[\tilde{f}'(\Delta-r) - \tilde{f}'(\Delta+r)] \\ &= \lim_{\Delta\to\infty}\int f(v)[f'(\Delta-r+v) - f'(\Delta+r+v)]dv \\ &= \int f(v)\lim_{\Delta\to\infty}[f'(\Delta-r+v) - f'(\Delta+r+v)]dv \le 0.\end{aligned}$$

The first equation is true because $\tilde{f}'(v) = -\tilde{f}'(-v)$ by the definition of $\tilde{f}$. The second equation uses $\tilde{f}(\Delta) \equiv \int f(v)f(\Delta+v)dv$. The third equation is by the Dominated Convergence Theorem, as the absolute value of the integrand is bounded. The inequality is true as $f'(\Delta-r+v) - f'(\Delta+r+v) \le 0$ when Δ is large, because:

$$\lim_{\Delta\to\infty}\left[1-\frac{f'(\Delta+r+v)}{f'(\Delta-r+v)}\right]f'(\Delta-r+v) = \lim_{\Delta\to\infty}\left[1-\frac{f(\Delta+r+v)}{f(\Delta-r+v)}\right]f'(\Delta-r+v) \le 0.$$

The equation is by L'Hospital's rule. Since $f(v)$ falls in v when v is big, the fraction $f(\Delta+r+v)/f(\Delta-r+v) \le 1$ when Δ is big. Moreover $f'(\Delta-r+v) \le 0$ if Δ is big. Altogether the inequality is true. $\square$

APPENDIX B
APPENDIX TO CHAPTER 2: EX-ANTE HETEROGENOUS CONSUMERS

B.1 Environment

In this chapter, I analyze the environment with ex-ante heterogenous consumers. All the proofs are contained in Appendix C. Following section 3.2, let x and y denote the realization of a consumer's value profile for the product. I assume that, for each consumer, X is independently and identically drawn according to the distribution function F and Y is independently and identically drawn according to the distribution function G. In addition, X and Y are independent of one another. Finally, both F and G have full support over the real line and have continuously differentiable density f and g, respectively.

I maintain the following regularity assumption about the distribution functions F and G through the rest of paper.

Assumption B.1. *Both density functions f and g are log-concave.*

A consumer's ex post utility depends on her value for the purchased product $v = x + y$, its actual price p', and the search cost s if she decides to visit. Specifically, if a consumer visits the seller and eventually purchases from him, then her ex post utility is equal to

$$U(x, y, p') = x + y - p' - s.$$

The model with ex-ante heterogenous consumers encompasses the situation where the seller has uncertainty over consumers' outside option $\underline{u}$, which can be incorporated into the dispersion of the known component X.

The market proceeds as follows. First, the seller announces price p. Then, each consumer decides whether to visit based on available information (x, p). If a consumer decides to visit, she observes (y, p') and makes her final purchase decision.

B.2 Consumer Behavior

Each consumer makes two decisions: whether to visit and whether to purchase. The decision to visit depends on the belief of the final price upon observing the posted price. The belief is pinned down in equilibrium. The decision to purchase relies on the final price if the consumer visits the seller. Let $H_p(p')$ denotes the belief of the final price p' when a consumer sees posted price p.

Given the seller's posted price p and final price p', consumers' visiting and purchasing decisions are summarized by the following inequalities:

(i) Visit strategy: The consumer visits the seller if and only if $x \geq x^*$, where x^* is the value such that

$$\underline{u} = -s + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max\{\underline{u}, x^* + y - p'\} dG(y) dH_p(p'). \tag{B.1}$$

(ii) Purchase strategy: The consumer purchases from the seller if and only if $x + y - p' \geq \underline{u}$.

When a consumer makes her visiting decision, she compares the reservation value for the seller and the outside option. As the hidden component, belief of the final price and the search cost are the same across all consumers, these consumers with higher known component x are more likely to visit the seller. Therefore, there is a cutoff of the known component x^*. A consumer with known component x^* is indifferent between taking the outside option directly (the left-hand side of equation (B.1)) and visiting the seller to have the option of purchasing the product (the right-hand side of equation (B.1)). Equation (B.1) also shows that consumers tend to visit the seller less frequently as the unit search cost s increases: as s increases, x^* increases.

If belief of the final price is degenerate at price p_b, the cutoff of the known component can be written explicitly as $x^* = -y^* + p_b$ due to the additive-utility specification, where y^* is defined by $s = \int_{y^*}^{\infty}[1 - G(y)]dy$. Consumers visit the seller if $x + y^* - p_b \geq \underline{u}$. It follows that as s increases, y^* decreases.

B.3 Equilibrium with Complete Information

This section analyzes the two benchmark cases when there is no uncertainty regarding the seller's type and makes a comparison between the two cases in terms of equilibrium price and welfare. The results in this section will be useful for later analysis for comparison purpose. This section also discusses the underlying issues caused by lack of commitment power and the role of full commitment power.

B.3.1 Non-commitment Type: $\mu = 0$

When the seller is the non-commitment type for sure, posted price serves as a cheap-talk message, since consumers rationally foresee the seller's incentive to deviate from the posted price. The untrustworthy price posting reduces the model to the traditional consumer search model with unobservable price.

Here I abuse notation by denoting the expected price by p, since posted price is merely babbling. Given the expected price p and the final price p', the seller's demand is given by

$$\begin{aligned} D_N(p,p') &= P(\ x+y^*-\underline{u}\geq p\ \&\ x+y-\underline{u}\geq p') \\ &= \int_{p+\underline{u}-y^*}^{\infty}[1-G(p'+\underline{u}-x)]dF(x) \end{aligned}$$

and profit is given by $\pi_N(p,p')=(p'-c)D_N(p,p')$. Then $p'^*(p)=\arg\max_{p'}\pi_N(p,p')$ is the optimal price charged by the seller given consumers' expected price p. The equilibrium price p_N is obtained by applying the consistency requirement $p'^*(p_N)=p_N$ since consumers are fully rational.

Proposition B.1. *If the seller is the non-commitment type for sure (i.e., $\mu=0$), then there exists a unique equilibrium, in which the seller charges price p_N, where p_N is defined by*

$$\frac{1}{p_N-c}=\frac{\int_{p_N+\underline{u}-y^*}^{\infty}g(p_N+\underline{u}-x)dF(x)}{\int_{p_N+\underline{u}-y^*}^{\infty}[1-G(p_N+\underline{u}-x)]dF(x)}. \tag{B.2}$$

This equation can be interpreted as a monopoly pricing formula with an endogenous cutoff which takes into account consumers' rational expectation of the price. Since p_N is derived by a fixed-point argument, p_N is the optimal final price to charge if consumers believe that it is the final price.

Lack of commitment power leads to a hold-up problem. As the seller is unable to utilize posted price to direct consumers' visit, he exploits the visiting consumers by charging a relatively high price p_N. Consider the limit case where consumers know their match values before visiting (i.e., G is degenerate). If the price is p, the seller knows exactly the consumers who are going to visit him: those whose values are above $p+s$. The seller will charge the consumers at least $p+s$ and consumers rationally anticipate this price increase. This process drives the price to increase unboundedly. The hold-up problem is severe and there is no trade in equilibrium.

In the current setup, p_N does not grow unboundedly due to the presence of the hidden component. When the hidden component is dispersed, those consumers with high known components may end up with low match values while those who barely decide to visit may find their match values to be high. Consumers' private information of the hidden component prevents the seller from increasing the price unboundedly. Nevertheless, the hold-up effect is still present. In the next section, it is shown that commitment is one way to mitigate this hold-up problem.

B.3.2 Commitment Type: $\mu = 1$

When the seller is the commitment type for sure, consumers know exactly what price they are going to be charged before they search and thus they visit the seller only to learn their private match values. The credible price posting shrinks the model to the consumer search literature with observable price.

Consumers use the same price when they make decisions to visit and to purchase as the posted price equals the final price. They visit when $x + y^* - p \geq \underline{u}$ and purchase when $x + y - p \geq \underline{u}$. The seller's demand is given by $D_C(p) =$

$P(x + \min\{y, y^*\} - \underline{u} \geq p)$ and profit is given by $\pi_C(p) = (p - c)D_C(p)$.

Lemma B.1. *If the seller is the commitment type for sure (i.e., $\mu = 1$), then the unique profit maximization price is denoted by p_C. The market price is lower in the commitment type case than in the non-commitment type case (i.e., $p_C \leq p_N$). p_C is defined by*

$$\frac{1}{p_C - c} = \frac{[1 - G(y^*)]f(p_C + \underline{u} - y^*) + \int_{p_C+\underline{u}-y^*}^{\infty} g(p_C + \underline{u} - x)dF(x)}{\int_{p_C+\underline{u}-y^*}^{\infty}[1 - G(p_C + \underline{u} - x)]dF(x)}. \quad \text{(B.3)}$$

p_C is defined by the monopoly pricing formula where the seller has uncertainty about consumers' value, which is distributed according to the random variable $x + \min\{y, y^*\}$. The equation that defines p_C differs from the equation that defines p_N by the term $[1-G(y^*)]f(p_C+\underline{u}-y^*)$ in the numerator, which reflects the commitment type seller's capability of manipulating the price to dictate consumers' visiting decision.

To understand how the prices are ranked, compare the price elasticity of demand for each case. The commitment type seller's pricing decision influences both consumers' visiting and purchasing decision, while the non-commitment type's pricing decision only enters consumers' purchasing decision, thus the commitment type seller faces more elastic demand than the non-commitment type seller. From another perspective, as discussed in the previous section, the non-commitment type seller's high price is driven by the hold-up effect, while full commitment power prevents price from being driven up by eliminating this problem.

As the commitment type seller can always choose to post and charge price p_N, he gains higher profit than the seller with no commitment power. If we compare the two scenarios further, the market with full commitment power Pareto dominates the market with no commitment power. Commitment power generates a lower price, larger volume of trade and higher profit for the seller.

B.4 Equilibrium with Incomplete Information

In this section, I analyze the market equilibrium when the seller's type is uncertain. Depending on whether the posted price is informative, the pure strategy equilibrium naturally falls into two categories: separating and pooling equilibrium. I examine each in turn, along with a brief discussion on equilibrium refinement, and then investigate the effect of search cost. I assume that once the seller deviates from the candidate equilibrium, he is considered as a non-commitment type. This belief supports the largest equilibrium set.

B.4.1 Separating Equilibrium

In any separating equilibrium, because consumers can distinguish the commitment type and the non-commitment type from the posted price, the non-commitment type faces the same problem as in section B.3.1 and charges p_N. The commitment type seller will post and charge p_N. If the commitment type seller posts a price lower than p_N, the non-commitment type can attract more consumers and obtain higher profit by mimicing this price. In the meantime, the commitment type seller is unwilling to separate himself by posting a price higher than p_N since $p_C \leq p_N$: a higher price is further away from his ideal price. The following proposition summarizes these insights.

Proposition B.2. *There is a continuum of separating equilibria. In any separating equilibrium, the commitment type seller posts p_N and the non-commitment type seller posts $p \neq p_N$. No matter what price the non-commitment type posts, he charges p_N.*

Despite the fact that there is a continuum of equilibria, all the equilibria are equivalent to the one when the seller is non-commitment type for sure (i.e., $\mu = 0$). The only difference among these equilibria is non-commitment type's posted price, which is merely a cheap-talk message. As in a typical separating equilibrium, the non-commitment type does not benefit from the uncertainty over seller's type, while the commitment type is hurt by the presence of the non-commitment type. Both forces result in an inefficient outcome.

Proposition B.2 argues that when the seller is commitment type for sure, introducing a small degree of uncertainty renders the market inefficient. It creates a discontinuity in the sense that market price jumps from p_C to p_N as $\mu = 1$ changes to $\mu = 1 - \epsilon$. The market with minimally limited commitment works as if there is no commitment power at all: consumers pay a higher price and seller's profit decreases. This conclusion depends on the restriction of separating equilibria. As shown in the next section, unlike the inefficiency result from the separating equilibrium, the pooling equilibrium exhibits a different welfare implication.

B.4.2 Pooling Equilibrium

In a pooling equilibrium, upon observing posted price p, consumers know that there is μ chance that the seller is the commitment type and p is the final price, while there is $1-\mu$ chance that the seller is the non-commitment type and charges a different final price, denoted by $\phi_\mu(p)$. Since the posted price is no longer a cheap talk message and carries partial commitment power to the non-commitment type seller, the price that the non-commitment type seller charges is closely connected to the posted price. As the posted price increases, only consumers with higher known components visit, therefore the non-commitment type seller charges them an even higher price. Conversely, price difference $\phi_\mu(p)-p$ decreases in posted price p, because the non-commitment type seller is unwilling to deter too many consumers with high known components from purchasing. The following result summarizes the above discussion.

Proposition B.3. *There is a continuum of pooling equilibria. For each μ, there exists $\underline{p}(\mu)$ such that, $\forall p \in [\underline{p}(\mu), p_N]$, there exists an equilibrium in which both types of seller post p and the non-commitment type seller charges $\phi_\mu(p) \geq p$. In addition, the actual price charged by the non-commitment type seller $\phi_\mu(p)$ increases in p, while price difference $\phi_\mu(p)-p$ decreases in p.*[1]

The pooling equilibrium price set is determined by comparing equilibrium profit and deviation profit for both types of seller. When deviation occurs, the seller is perceived as the non-commitment type, and thus the best deviation profit is obtained by charging p_N. The upper bound of the pooling price set, p_N, is pinned down by both types of seller's incentive to deviate. Any price above p_N is undesirable to both types of seller as p_N generates little demand. Even in the case where the

[1]The equilibrium posted price set may not be convex. As the main analysis in the following sections focus on the lower bound of the posted price set, the discussion on the possibility of non-convexity is relegated to the appendix.

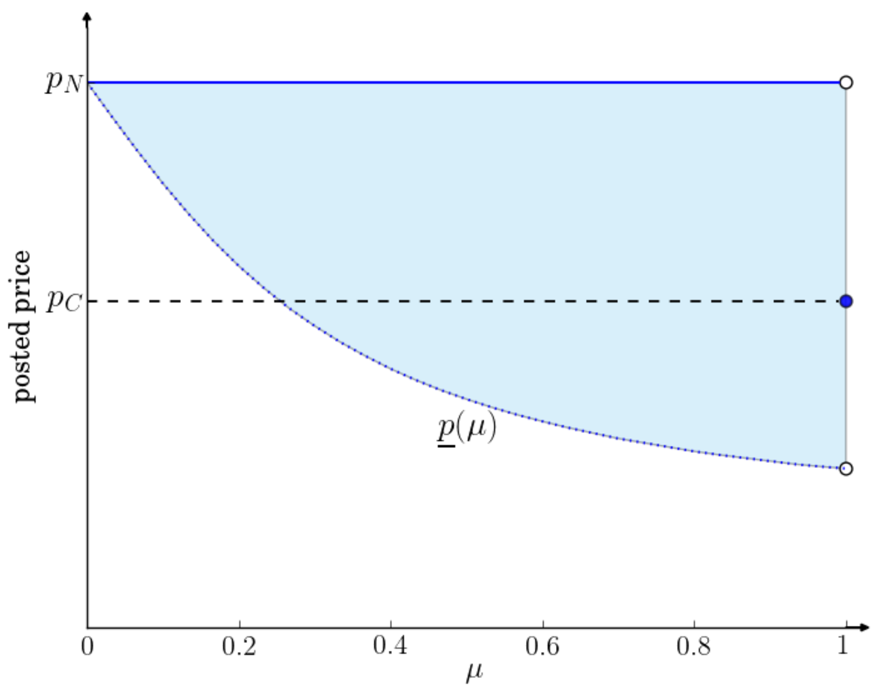
seller is non-commitment type for sure, the seller does not have incentives to charge a price higher than p_N. Since the commitment type seller always earn less profit than the non-commitment type for the same posted price, any price above p_N is undesirable to him as well. The lower bound, $\underline{p}(\mu)$, comes solely from the commitment type's incentive constraint. Consider the case where the pooling equilibrium posted price is close to c. The presence of the non-commitment seller raises consumers' expected price and deters their visit. A low posted price does not generate large demand for the commitment type seller, and he obtains a profit close to 0. Under these circumstances, he prefers to deviate even though he will be regarded as a non-commitment type. In contrast, the non-commitment type seller's equilibrium profit is a decreasing function of the posted price: the non-commitment type can charge visiting consumers arbitrarily, and thus he can replicate at least the same profit from more visiting consumers.

The following proposition completes the characterization of the pooling equilibrium.

Proposition B.4. *The lower bound of the pooling equilibrium price set, $\underline{p}(\mu)$, decreases as the prior probability of being a commitment type seller, μ, increases. The upper bound, p_N, remains unchanged.*

In order to understand why the lower bound of the price set decreases, consider the effect of increasing the level of commitment on the commitment type seller's equilibrium and deviation profit. Intuitively, higher level of commitment increases consumers' chance of visiting for a given posted price, since the advertised price is more credible and they are less likely to be charged additional amount. Additionally, higher level of commitment mitigates the non-commitment seller's hold-up problem; for a given posted price, the non-commitment type seller charges a lower price, which further lowers consumers' expected price. Therefore, a higher chance of visiting turns into a higher equilibrium profit for the commitment type seller. Since deviation profit is invariant with respect to the level of commitment, as level of commitment rises, a lower posted price is sustainable in equilibrium.

Figure B.1: Pooling Equilibrium varies with Level of Commitment under Ex-ante Heterogenous Consumers



The set of pooling equilibrium posted prices given level of commitment, μ, with $F \sim N(0,1)$, $G \sim N(0,1)$ (standard normal distribution), $\underline{u} = 0$ and $c = 0$.

As illustrated in Figure B.1, as $\mu \to 0$, $\underline{p}$ converges to p_N and the pooling equilibrium set shrinks to a singleton. As $\mu \to 1$, $\underline{p}$ converges to a price which is lower than p_C. Therefore, the pooling equilibrium set is lower hemicontinuous at $\mu = 1$ but not upper hemicontinuous. This implies that small degree of uncertainty regrading the seller's type being the commitment type (i.e., $\mu = 1 - \epsilon$) may benefit the consumers, since a price lower than p_C can be supported in equilibrium, and thus consumer surplus can be greater than in a market with full commitment power.

The non-commitment type seller benefits from pooling with the commitment type as partial commitment power mitigates the seller's hold-up problem. In contrast, the commitment type suffers from the presence of the non-commitment type. Whatever price the commitment type posts, consumers expect to be charged some higher price, and thus visit less frequently. This indicates that the non-commitment type seller may prefer pooling with the commitment type over obtaining full commitment power, especially when the level of commitment is high and the posted price is low.

B.5 Equilibrium Refinement

As in other signaling games, this game suffers from equilibrium multiplicity. In order to see whether one equilibrium is more prominent than others, several refinement criteria are applied.

I find that applying the Intuitive Criterion by Cho and Kreps (1987) eliminates at least the lower part of the pooling equilibrium set. Suppose that the equilibrium posted price is the lower bound of the posted price set, $\underline{p}(\mu)$, then when consumers observe off-path posted price $p_N - \epsilon$, they believe it is posted by the commitment type. On the one hand, for the non-commitment type seller, the benefit from pooling with the commitment type at a lower price overweights the benefit from direct commitment power at a higher price. On the other hand, the commitment type seller receives deviation profit at $\underline{p}(\mu)$. Deviating to $p_N - \epsilon$ while being perceived as a commitment type is more profitable than staying in equilibrium. Therefore, the Intuitive Criterion rules out the lower bound. As posted price rises, the unsent message set that is not preferable to the non-commitment type shrinks while the unsent message set that is preferable to the commitment type first shrinks then expands. The curvatures of distribution functions determine how the intersection of two sets changes.

Conversely, applying the Undefeated equilibrium by Mailath et al. (1993) removes the upper part of the pooling equilibrium set. The Undefeated equilibrium is equivalent to the notion of Pareto efficiency in this model. Since the lower bound, $\underline{p}(\mu)$, is the most preferable posted price by the non-commitment type seller while the commitment type seller prefers a posted price, $\hat{p}(\mu)$, in between the lower and upper bound, Undefeated equilibrium rules out the posted price set $[\hat{p}(\mu), p_N]$.

As there is no refinement criteria that selects the unique equilibrium and different criteria eliminate different subsets of the equilibrium set, I focus on characterizing the whole equilibrium set in the following analysis. I pay special attention to the lower bound of the posted price set, as it is the price that generates highest volume of trade. It is also the price that is most preferred by consumers and the non-commitment type seller. Separating equilibria are rule out by the Undefeated equilibrium but not by the Intuitive Criterion and thus are still discussed briefly in the analysis.

B.6 The Effect of Search Cost

In this section, I study the impact of increasing search cost on equilibrium prices. The following lemma summarizes the results when there is complete information of seller's type.

Lemma B.2. *If the seller is non-commitment type for sure (i.e., $\mu = 0$), then the unique equilibrium price, p_N, increases as search cost, s, increases; if the seller is commitment type for sure (i.e., $\mu = 1$), then the unique equilibrium price, p_C, decreases as search cost, s, increases. In both cases, profits decrease as s increases.*

It turns out that commitment power plays a crucial role in the direction of this comparative statics exercise. As search cost rises, consumers are less likely to visit unless they have high known components. The non-commitment type seller exploits the visiting consumers as much as possible, since he is unable to use price to dictate consumer's decision to visit. I call this the "hold-up effect". In contrast, the commitment type seller is able to reduce price to direct consumers' visiting decisions, mitigating the impact of rising search cost. For this reason, I call this the "directed search effect".

As the search cost rises, seller's profits decrease in both cases, but in different ways. The non-commitment type suffers from a more severe hold-up problem, while the commitment type compensates consumers by lowering his price. When the seller is the non-commitment type, consumers are made worse off with the rise of both search cost and price. When the seller is the commitment type, however, there are cases when an increase in search cost is beneficial to consumers as it is possible for the seller's price promotion to dominate the rising search cost.

The two opposite comparative statics results raise an interesting question regarding how search cost effects the equilibrium price set with incomplete information of seller's type. As the separating equilibrium is equivalent to the case where the seller is non-commitment type for sure, the effective price increases in search cost. Similarly, since the upper bound of the pooling equilibrium price is captured by p_N due to both types of seller's incentive constraints, it increases in search cost. This implies as search cost rises, higher posted price can be supported in equilibrium.

At the lower bound, however, the effect of increasing search cost is in general non-monotone, and depends on the level of commitment and the magnitude of the search cost. Since the lower bound is determined by the commitment type seller's incentive constraint, comparing how his deviation and equilibrium profit changes with search cost is crucial to this exercise. Deviation profit is obtained by being perceived as a non-commitment type seller and charging p_N:

$$(p_N - c) \int_{p_N + \underline{u} - y^*}^{\infty} [1 - G(p_N + \underline{u} - x)] dF(x). \tag{B.4}$$

As the search cost rises, deviation becomes less profitable for two reasons: the direct effect of rising search cost and the indirect effect of the hold-up problem. Equilibrium profit at the lower bound is given by

$$(\underline{p} - c) \int_{x^*(\underline{p})}^{\infty} [1 - G(\underline{p} + \underline{u} - x)] dF(x). \tag{B.5}$$

Fixing the lower bound $\underline{p}$, equilibrium profit also decreases due to the direct effect of rising search cost and the indirect effect of its impact on the non-commitment type seller's actual price; $\phi_\mu(\underline{p})$ increases in search cost s due to the hold-up effect as well. As both profits move in the same direction, the speed of each profit function's decreasing rate determines how the lower bound moves with search cost.[2] If equilibrium profit decreases faster, then the lower bound increases. Conversely, if deviation profit decreases faster, then the lower bound decreases. In general, which profit decreases faster is ambiguous. There is an example in the appendix in which the lower bound changes direction several times. Nevertheless, examining the limiting cases provide clean results, as some of the effects vanish.

Proposition B.5. *(i) As $\mu \to 0$, the lower bound of the pooling equilibrium $\underline{p}$ increases with search cost s; (ii) As $\mu \to 1$, the lower bound of the pooling equilibrium $\underline{p}$ decreases with search cost s.*

When probability of the seller being the commitment type is small, consumers' visiting decisions are mainly dictated by the non-commitment type seller's price: as $\mu \to 0$, $x^* \to p_N - y^* + \underline{u}$. Hence, the direct effect of the search cost and the indirect effect of the hold-up problem are similar for both deviation and equilibrium profits of the commitment type seller. The difference between deviation and equilibrium profit originates from the fact that p_N is the optimal price to charge if p_N is consumers'

[2]The speed of how profit decreases is related to the curvature of the profit function, which can be traced back to the curvature of the primary distribution functions F and G. Whether p_N or $\phi_\mu(\underline{p})$ increases faster is unclear, because $p_N = \phi_\mu(p_N)$ and the curvature of ϕ_μ can be concave or convex.

expected price. Therefore, equilibrium profit decreases faster than the deviation profit and the lower bound of equilibrium price set increases.

When probability of the seller being the commitment type is large, $x^* \to \underline{p} - y^* + \underline{u}$, the hold-up effect vanishes on the equilibrium path and equilibrium profit is only effected by the direct effect of search cost. Consequently, deviation profit decreases faster than equilibrium profit and a lower price can be sustained in the pooling equilibrium.

Proposition B.6. *(i) As $s \to 0$, $\underline{p}$ stays constant as search cost s increases; (ii) As $s \to \infty$, $\underline{p}$ decreases as search cost s increases.*

When search cost is negligible, the difference in commitment power and the difference between deviation and equilibrium profit vanishes. Thus the lower bound is invariant with respect to search cost as both the direct effect of search cost and indirect effect of the hold-up problem disappear. When search cost is sufficiently large, the lower bound of the equilibrium price set eventually decreases as search cost increases. Deviation profit decreases fast when search cost is large, as the hold-up problem is severe. On the equilibrium path, however, commitment power mitigates the hold-up problem and equilibrium profit does not decrease as fast as deviation profit. Therefore, a lower pooling price can be sustained in equilibrium.

In summary, when there is uncertainty over seller's type, the directed-search effect, as observed in the case of $\mu = 1$, disappears since the commitment type seller loses his ability to manipulate consumers' visiting decisions. The hold-up effect, as observed in the case of $\mu = 0$, remains and drives the results presented in this section.

The commitment type seller faces a tradeoff between the hold-up problem both when pooling with the non-commitment type and when deviating and being perceived as a non-commitment type seller. Which hold-up problem is more severe depends on the level of commitment and the magnitude of the search cost.

APPENDIX C
APPENDIX TO CHAPTER 2: OMITTED PROOFS

Proof of Proposition 2.1. The proof follows directly from the discussion above the proposition. □

Proof of Proposition 2.2. Assumption 2.1 ensures that the profit function is single-peaked at p_N. It follows that for any price between c and p_N, conditional on consumers' visits, the seller earns positive profit. $\hat{\mu}$ and $\bar{p}(\mu)$ are defined such that for each price within the set $[c, \bar{p}(\mu)]$, both types of seller obtain positive profit, which is higher than the deviation profit, 0. In addition, as $\bar{p}(\mu) < p_N$ and $\mathbb{E}\max\{\underline{u}, y - \bar{p}(\mu)\} > \mathbb{E}\max\{\underline{u}, y - p_N\}$ from equation (2.3) we know that when μ rises, we need $\mathbb{E}\max\{\underline{u}, y - \bar{p}(\mu)\}$ to be smaller to keep the equality. Therefore, $\bar{p}(\mu)$ increases in μ. □

Proof of Lemma 2.1. The first part follows from the definition of p_N. Since $p_C = y^* - \underline{u}$ and y^* decreases in search cost, p_C also decreases in search cost. □

Proof of Proposition 2.3. From equation (2.3) we know that when s rises, we need $\mathbb{E}\max\{\underline{u}, y - \bar{p}(\mu)\}$ to be larger to keep the equality. Therefore $\bar{p}(\mu)$ decreases in s. From equation (2.2) we know that as $c < p_N$, $\mathbb{E}\max\{\underline{u}, y - c\} > \mathbb{E}\max\{\underline{u}, y - p_N\}$. When s rises, we need $\mathbb{E}\max\{\underline{u}, y - c\}$ to increase to keep the equality. Therefore $\hat{\mu}$ increases in s. □

Proof of Lemma 2.2. The maximum expected price that let consumers to visit the seller is y^*. The non-commitment type seller want to charge as much as possible given consumers visit him. His profit for different posted prices p is $(p+\Delta)[1-G(p+\Delta)]$ if $p+\Delta \leq y^*$ and 0 if $p+\Delta > y^*$. Given $p_N > y^*$, it follows that the optimal posted price is $p = y^* - \Delta$. □

Proof of Lemma 2.3. The proof of the first part follows the discussion below the Lemma. For the second part, the non-commitment type posts p such that $p+\Delta = y^*$ and charges y^* as he does in the case of $\mu = 0$. The commitment type cannot post any price within the set of $[y^* - \Delta, y^*]$ because the non-commitment type can post that price as well and obtain higher profit. The commitment type is unwilling to post any price lower or equal than $y^* - \Delta$ because it delivers negative profit to him. Therefore, the commitment type posts any price higher than y^* and obtains zero profit in equilibrium. □

Proof of Proposition 2.4. Recall that there exists $\hat{\mu} \in (0,1)$ such that $0 = -s + \hat{\mu}\mathbb{E}\max\{0,y\} + (1-\hat{\mu})\mathbb{E}\max\{0, y-p_N\}$. Without regulation, pooling equilibrium exists for $\mu > \hat{\mu}$. The equilibrium pooling posted price set is $[0, \bar{p}(\mu)]$. The exact bounds for pooling equilibrium under regulation differ across parameters, which can be divided into the following cases.

(i): $\mu < \hat{\mu}$, pooling equilibrium exists when $\Delta \leq \Delta^*$ where Δ^* is defined by $0 = -s + \mu\mathbb{E}\max\{0,y\} + (1-\mu)\mathbb{E}\max\{0, y-\Delta^*\}$. It follows that $\Delta^* \in [y^*, p_N]$.

If $\Delta \in [y^*, \Delta^*]$, since deviation profit for the commitment type is 0, the lower bound of the pooling posted price is 0 while the upper bound of the pooling posted price is $\bar{p}^\Delta$, which satisfies $0 = -s + \mu\mathbb{E}\max\{0, y-\bar{p}^\Delta\} + (1-\mu)\mathbb{E}\max\{0, y-\bar{p}^\Delta-\Delta\}$. The corresponding set for actual price charged by the non-commitment type seller is $[\Delta, \bar{p}^{\Delta} + \Delta]$. From the equation that defines $\bar{p}^{\Delta}$ we know that as Δ increases, $\bar{p}^{\Delta}$ decreases while $\bar{p}^{\Delta} + \Delta$ increases. In this region, consumers surplus stays at zero at the upper bound while consumer surplus decreases with Δ at the lower bound. Social surplus decreases with Δ at the lower bound as well. The commitment type seller gets worse off as Δ increases while the non-commitment type seller gets better off as Δ increases.

If $\Delta \in [0, y^*]$, since deviation profit is by charging $y^* - \Delta$ to the commitment type seller, the lower bound of the pooling posted price is $y^* - \Delta$ while the upper bound of the pooling posted price is $\bar{p}^{\Delta}$. The corresponding set for actual price charged by the non-commitment type seller is $[y^*, \bar{p}^{\Delta} + \Delta]$. In this region, consumers surplus stays at zero at the upper bound while consumer surplus increases with rate Δ at the lower bound. Social surplus increases with rate Δ at the lower bound as well. The commitment type seller gets worse off as Δ increases while the non-commitment type seller gets better off as Δ increases.

(ii): $\mu > \hat{\mu}$, pooling equilibrium always exists.

When Δ is large, the upper bound of the pooling posted price is $\bar{p}(\mu)$, which is defined by $0 = -s + \mu \mathbb{E} \max\{0, y - \bar{p}(\mu)\} + (1-\mu)\mathbb{E}\max\{0, y - p_N\}$. $\bar{p}^{\Delta}$ stays constant at $\bar{p}(\mu)$ as Δ decreases until $\Delta = \tilde{\Delta}$, which is defined by $0 = -s + \mu \mathbb{E}\max\{0, y - p_N + \tilde{\Delta}\} + (1-\mu)\mathbb{E}\max\{0, y - p_N\}$. If $\Delta > \tilde{\Delta}$, the upper bound of the actual price charged by the non-commitment type seller stays at p_N.

If $\Delta < \tilde{\Delta}$, $\bar{p}^{\Delta}$ satisfies $0 = -s + \mu\mathbb{E}\max\{0, y - \bar{p}^{\Delta}\} + (1-\mu)\mathbb{E}\max\{0, y - \bar{p}^{\Delta} - \Delta\}$. The upper bound of the actual price charged by the non-commitment type seller is

$\bar{p}^{\Delta} + \Delta$. As Δ increases, $\bar{p}^{\Delta}$ decreases and $\bar{p}^{\Delta} + \Delta$ increases.

If $\Delta \in [y^*, p_N]$, the lower bound of the pooling posted price is 0 while the lower bound for the actual price charged by the non-commitment type seller is Δ.

If $\Delta \in [0, y^*]$, the lower bound of the pooling posted price is $y^* - \Delta$ while the lower bound for actual price charged by the non-commitment type seller is y^*.

The welfare results are the same as in Case (i) for different regions. $\square$

When $\mu_1 = \mu_2 = 0$, suppose p^* is the equilibrium price and seller 1 deviates to $p^{*'} > p^*$. As consumers believe that both sellers will charge p^*, they visit each seller with half probability. With probability $\frac{1}{2}$, seller 1 gets visited first and the demand is

$$1 - (y^* - p^* + p^{*'}) + \int_{p^{*'}}^{y^*-p^*+p^{*'}} y - p^{*'} + p^* dy.$$

With probability $\frac{1}{2}$, seller 2 gets visited first and the demand for seller 1 is

$$y^*[1 - (y^* - p^* + p^{*'})] + \int_{p^{*'}}^{y^*-p^*+p^{*'}} y - p^{*'} + p^* dy.$$

The aggregate demand for seller 1 is

$$\frac{1}{2}(1 + y^*)[1 - (y^* - p^* + p^{*'})] + \frac{1}{2}y^{*2} - \frac{1}{2}p^{*2}.$$

First order condition with $p^* = p^{*'}$ gives $p^* = \frac{-2(1+y^*)+\sqrt{(1+y^*)^2+4}}{2}$. It can be checked that large deviation is not profitable.

Proof of Proposition 2.5. In equilibrium, it will never be the case such that $p_1 = p_2$. Since if it is the case, and consumers visit each seller with equal probability, then the commitment type seller can always deviate to $p_1 - \epsilon$ and induce all consumers to visit him first.

Suppose in equilibrium $p_1 < p_2$ and let $\Delta \triangleq p_2 - p_1$. Seller 1's demand is

$$1 - (y^* - p_2 + p_1) + \int_0^{y^* - p_2 + p_1} y - p_1 + p_2 \; dy,$$

provided $y^* - p_2 + p_1 \in [0, 1]$. Later will check this is indeed satisfied and larger deviation is not profitable. Seller 2's demand given his actual price being p_2' is

$$y^* - p_2 + p_1 \text{ if } p_2' < -y^* + p_2$$

$$(y^* - p_2 + p_1)[1 - (y^* - p_2 + p_2')] + \int_0^{y^* - p_2 + p_2'} y - p_2' + p_1 \; dy \text{ if } -y^* + p_2 < p_2' < p_1$$

$$(y^* - p_2 + p_1)[1 - (y^* - p_2 + p_2')] + \int_{p_2' - p_1}^{y^* - p_2 + p_2'} y - p_2' + p_1 \; dy \text{ if } p_1 < p_2' < 1 - y^* + p_2$$

$$\int_{p_2' - p_1}^{1} y - p_2' + p_1 \; dy \text{ if } 1 - y^* + p_2 < p_2' < 1 + p_1$$

$$0 \text{ if } 1 + p_1 < p_2'$$

It can be checked that the maximum occurs at the interval $p_1 < p_2' < 1 - y^* + p_2$.

Seller 1's first order condition gives

$$p_2 = \frac{1 - y^* + \frac{{y^*}^2}{2} + 2\Delta - \frac{3}{2}\Delta^2}{1 - \Delta}$$

or

$$p_2 = 2p_1 - \sqrt{{p_1}^2 + {y^*}^2 - 2y^* + 3} + 1.$$

Seller 2's first order condition gives

$$p_2 = 1 - \frac{1}{2}y^* - \frac{1}{2}\Delta$$

or

$$p_2 = \frac{p_1 - y^* + 2}{3}.$$

Combine the two first order conditions deliver the equilibrium prices:

$$\Delta = \frac{7 - y^* - \sqrt{17y^{*2} - 30y^* + 49}}{8}$$
$$p_1 = \frac{-5 - 5y^* + 3\sqrt{17y^{*2} - 30y^* + 49}}{16}$$
$$p_2 = \frac{9 - 7y^* + \sqrt{17y^{*2} - 30y^* + 49}}{16}$$

Seller 1's second order condition is

$$-3p_1 + 2p_2 - 2$$

Seller 2's second order condition is

$$-2p_1 + 2p_2 - 2y^*.$$

It can be shown that $\Delta \geq 0$ and $y^* - p_2 + p_1 \in [0, 1]$. Δ first increases then decreases in y^*. Does seller 1 has incentive to deviate to some price $y^* + p_2 > p_1' > p_2$? The demand for seller 1 becomes

$$(y^* - p_1' + p_2)(1 - y^*) + \int_{p_1' - p_2}^{y^*} y - p_1' + p_2 \, dy,$$

Seller 1's profit decreases over this interval. Thus seller 1 has no incentive to deviate. Both p_1 and p_2 are higher than p^*. Both p_1 and p_2 decrease in y^*. Seller 1's profit is higher than the profit in the case where both sellers are the non-commitment type. Seller 2's profit is lower than the profit in the case where both sellers are the non-commitment type.

Suppose in equilibrium $p_1 > p_2$ and let $\Delta \triangleq p_1 - p_2$. Seller 1's demand is

$$(1 - y^*)(y^* - p_1 + p_2) + \int_{p_1 - p_2}^{y^*} y - p_1 + p_2 \, dy,$$

provided $y^* - p_1 + p_2 \in [0,1]$. Seller 2's demand given his actual price being p_2' is

$$1 \text{ if } p_2' < -y^* + p_1$$

$$[1 - (y^* - p_1 + p_2')] + \int_0^{y^*-p_1+p_2'} y - p_2' + p_1 \, dy \text{ if } -y^* + p_1 < p_2' < p_1$$

$$[1 - (y^* - p_1 + p_2')] + \int_{p_2'-p_1}^{y^*-p_1+p_2'} y - p_2' + p_1 \, dy \text{ if } p_1 < p_2' < 1 - y^* + p_1$$

$$\int_{p_2'-p_1}^{1} y - p_2' + p_1 \, dy \text{ if } 1 - y^* + p_1 < p_2' < 1 + p_1$$

$$0 \text{ if } 1 + p_1 < p_2'$$

We are looking for maximum to occur at the second range. Seller 1's first order condition gives

$$p_1 = \frac{y^* - \frac{y^{*2}}{2} - \Delta + \frac{1}{2}\Delta^2}{1 - \Delta}$$

Seller 2's first order condition gives

$$p_1 = \frac{1 - y^* + \frac{y^{*2}}{2} + 2\Delta - \frac{3}{2}\Delta^2}{1 - \Delta}$$

Combine the two first order conditions deliver the tentative equilibrium prices:

$$\Delta = \frac{2 + \sqrt{8y^{*2} - 16y^* + 17}}{4} > 0$$

gives negative prices. There cannot exist any equilibrium where $p_1 > p_2$.

Proof of the second part comes from Armstrong et al. (2009). □

Proof of Proposition 2.6. Proof of part (i).

A). Separating Equilibrium:

Let p_1 be posted price for the commitment type seller 1, p_1' be posted price for the non-commitment type seller 1, p_1'' be final price for the non-commitment type seller 1, p_2 be seller 2's price.

1). There exists separating equilibrium in which $p_1 = p_1'' > p_2$.

The equilibrium in the case of $\mu_1 = 0$ and $\mu_2 = 1$ remains to be an equilibrium here. The commitment type seller 1 posts p_1. The non-commitment type seller 1 posts any other price than p_1. Consumers believe the non-commitment type seller 1 to charge p_1. Given consumers' belief, both types of seller 1 has no incentive to deviate from this equilibrium. So does seller 2.

2). There does not exist any separating equilibrium in which $p_1 > p_1'' > p_2$ or $p_1'' > p_1 > p_2$.

In the first case the commitment type will deviate to the non-commitment type's price while in the second case the non-commitment type will deviate to the commitment type's price.

3). There does not exist any separating equilibrium in which $p_1 = p_2$ or $p_1'' = p_2$.

If such equilibrium exists, then seller 2 can always undercut to price $p_2 - \epsilon$ and get all consumers to visit him at first place.

4). There does not exist separating equilibrium in which p_1 & $p_1'' < p_2$.

If this is the case, then both types of seller 1 charge the same price. Since seller 1 is visited first, his profit is independent of consumers' belief. When seller 1 deviates from his posted price, consumers will still visit him first. Both types of seller 1 face same problem. Then the situation is identical to the one when $\mu_1 = 0$ and $\mu_2 = 1$. There does not exist such equilibrium from previous discussion.

5). There does not exist separating equilibrium such that $p_1'' < p_2 < p_1$ or $p_1 < p_2 < p_1''$.

In the first case the commitment type will deviate to the non-commitment type's price while in the second case the non-commitment type will deviate to the commitment type's price.

Therefore, the only separating equilibrium exists is equivalent to the equilibrium in the case of $\mu_1 = 0$ and $\mu_2 = 1$.

B). Pooling Equilibrium:

Let p_1 be the pooling posted price for seller 1, p_1' be the final price that is charged by the non-commitment type seller 1, p_2 be seller 2's price.

1). There exists pooling equilibrium in which $\mu p_1 + (1-\mu)p_1' > p_2$.

The equilibrium in the case of $\mu_1 = 0$ and $\mu_2 = 1$ remains to be an equilibrium here. Given consumers' belief, both types of seller 1 has no incentive to deviate from this equilibrium. So does seller 2.

2). There does not exist any pooling equilibrium in which $\mu p_1 + (1-\mu)p_1' = p_2$.

If such equilibrium exists, then seller 2 can always undercut to price $p_2 - \epsilon$ and get all consumers to visit him at first place.

3). There does not exist any pooling equilibrium in which $\mu p_1 + (1-\mu)p_1' < p_2$.

If such equilibrium exists, there are several cases.

1'. p_1 & $p_1' < p_2$

If this is the case, then both types of seller 1 charge the same price. Since seller 1 is visited first, his profit is independent of consumers' belief. When seller 1 deviates from his posted price, consumers still visit him first. Both types of seller 1 face same problem. Then the situation is identical to the one when $\mu_1 = 0$ and $\mu_2 = 1$. There does not exist such equilibrium from previous discussion.

2'. $p_1' < p_2$ and $p_1 > p_2$

As seller 1 is visited first, the non-commitment type seller 1 charges the optimal price. Since $p_1' < p_2$, when the commitment type seller deviates and is considered as the non-commitment type seller, he still gets visited first. Then the commitment type seller 1 strictly prefers to deviate to price p_1'. Thus this can not be an equilibrium.

3'. $p_1 < p_2$ and $p_1' > p_2$

Consider the non-commitment type seller 1. His demand is $1 - (y^* - p_2 + p_1') + \int_{p_1'}^{y^*-p_2+p_1'} y_1 - p_1' + p_2 \; dy_1$. First order condition gives

$$p_1' = \frac{1 - y^* + \frac{y^{*2}}{2} + p_2 - \frac{{p_2}^2}{2}}{2}.$$

Consider seller 2, with probability μ, his opponent is the commitment type while with probability $1-\mu$, his opponent is the non-commitment type. His demand is

$$\begin{aligned} & \mu[(y^* - p_2 + p_1)(1 - y^*) + \int_{p_2}^{y^*} y_2 - p_2 + p_1 \; dy_2] \\ & +(1-\mu)[(y^* - p_2 + p_1')(1 - y^*) + \int_{p_2}^{y^*} y_2 - p_2 + p_1' \; dy_2] \\ = & [y^* - p_2 + \mu p_1 + (1-\mu)p_1'](1 - y^*) + \int_{p_2}^{y^*} y_2 - p_2 + \mu p_1 + (1-\mu)p_1' \; dy_2 \end{aligned}$$

First order condition gives

$$\mu p_1 + (1-\mu)p_1' = \frac{y^* - \frac{y^{*2}}{2} - 2p_2 + \frac{3{p_2}^2}{2}}{2p_2 - 1}.$$

In equilibrium,

$$\mu p_1 = \frac{y^* - \frac{y^{*2}}{2} - 2p_2 + \frac{3p_2{}^2}{2}}{2p_2 - 1} - (1-\mu)\frac{1 - y^* + \frac{y^{*2}}{2} + p_2 - \frac{p_2{}^2}{2}}{2}.$$

Check $p_1 < p_2$, $p_1' > p_2$ and $\mu p_1 + (1-\mu)p_1' < p_2$. It is shown in the following that the intersection of these three inequalities is empty. Thus there does not exist such an equilibrium.

$p_1' > p_2 \Leftrightarrow -1 - \sqrt{y^{*2} - 2y^* + 3} < p_2 < -1 + \sqrt{y^{*2} - 2y^* + 3}$

$\mu p_1 + (1-\mu)p_1' < p_2 \Leftrightarrow p_2 > -1 + \sqrt{-y^{*2} + 2y^* + 1}$

Combine these two inequalities gives $-1+\sqrt{-y^{*2} + 2y^* + 1} < p_2 < -1+\sqrt{y^{*2} - 2y^* + 3}$.

Notice that $-1 + \sqrt{y^{*2} - 2y^* + 3} \leq \frac{1}{2}$ given $y^* \in [\frac{1}{2}, 1]$.

$p_1 < p_2 \Leftrightarrow 2(1-\mu)p_2^3 - (3\mu - 1)p_2^2 + 2[\mu(y^{*2} - 2y^* + 3) - y^{*2} + 2y^* - 5]p_2 - \mu(y^{*2} - 2y^* + 2) - y^{*2} - 2y^* + 2 > 0$, the left hand side of the inequality first increases then decreases then increases with respect to p_2.

Maximum occurs at $p_2 = \frac{\sqrt{12\mu^2 y^{*2} - 24\mu^2 y^* + 45\mu^2 - 24\mu y^{*2} + 48\mu y^* - 102\mu + 12y^{*2} - 24y^* + 61} - 3\mu + 1)}{6(\mu - 1)} < 0$, minimum occurs at $p_2 = \frac{-\sqrt{12\mu^2 y^{*2} - 24\mu^2 y^* + 45\mu^2 - 24\mu y^{*2} + 48\mu y^* - 102\mu + 12y^{*2} - 24y^* + 61} - 3\mu + 1)}{6(\mu - 1)} \geq -1+\sqrt{y^{*2} - 2y^* + 3}$. Thus the left hand side of the inequality decreases on the range $-1 + \sqrt{-y^{*2} + 2y^* + 1} < p_2 < -1 + \sqrt{y^{*2} - 2y^* + 3}$. It can be shown that the sign is negative for $p_2 = -1 + \sqrt{-y^{*2} + 2y^* + 1}$ when $y^* \in [\frac{1}{2}, 1]$.

Therefore, there only exists pooling equilibrium where seller 2 is visited first.

Proof of part (ii).

1). Suppose seller 1 is believed to charge p_1 and seller 2 is believed to charge p_2. Both types of sellers do not have incentive to deviate because once they deviate, they are believed to be the non-commitment type seller while p_1 and p_2 are the equilibrium prices when both types of seller are the non-commitment type. Therefore, the optimal prices given these beliefs coincide with p_1 and p_2.

2). Suppose both seller 1 and seller 2 are believed to charge p^* and visited with equal chance, then both types of seller 1 and 2 has no incentive to deviate as they are believed to charge p^*. p^* is the optimal price to charge when p^* is the expected price and when both sellers are visited with equal chance. $\square$

Proof of Proposition B.1. The first order condition is characterized by equation (B.2). The right-hand side of (B.2) can be rewritten as

$$\frac{\int_{\underline{u}-y^*}^{\infty} \frac{g(\underline{u}-k)}{1-G(\underline{u}-k)}[1-G(\underline{u}-k)]f(k+p_N)dk}{\int_{\underline{u}-y^*}^{\infty}[1-G(\underline{u}-k)]f(k+p_N)dk}$$

by change of variable: $k = x - p_N$. $g(\underline{u}-k)/(1-G(\underline{u}-k))$ falls in k by the log-concavity of G. Therefore the right-hand side rises in p_N as the random variable induced by the probability density $[1-G(\underline{u}-k)]f(k+p_N)$ falls in p_N in the first order stochastic dominance sense. This is because of $[1-G(\underline{u}-k_2)]f(k_2+p_N)/[[1-G(\underline{u}-k_1)]f(k_1+p_N)]$ falls in p_N for all $k_2 > k_1$, or equivalently, $f(k_2+p_N)/f(k_1+p_N)$ falls in p_N, which is implied by the log-concavity of f. $\square$

Proof of Lemma B.1. Demand function $\int_{p+\underline{u}-y^*}^{\infty}[1-G(p+\underline{u}-x)]dF(x)$ can be rewritten as $\int_{\underline{u}-y^*}^{\infty}[1-G(\underline{u}-k)]f(k+p)dk$, which is log-concave in p due to the log-concavity of f and g. Thus the profit function is single-peaked and maximizer is uniquely defined.

Comparing two first order conditions (B.3) and (B.2), notice that both the left-hand side of (B.3) and (B.2) are decreasing while both the right-hand side of (B.3) and

(B.2) are increasing. The right-hand side of (B.3) is always higher than that of (B.2) for the same price. Thus we have $p_N \geq p_C$. □

Proof of Proposition B.2. It only need to be shown that the non-commitment type seller's profit function is a decreasing function of the expected price. Then it follows that the non-commitment type would always mimic the commitment type's posted price if the posted price is lower than p_N. The non-commitment seller's profit function is given by $(p' - c) \int_{p+\underline{u}-y^*}^{\infty} [1 - G(p' + \underline{u} - x)] dF(x)$, where p is the expected price and p' is the actual price. Take derivative with respect to p gives $(p' - c)(-1)[1 - G(p' + y^* - p)] f(p + \underline{u} - y^*) < 0$. □

Proof of Proposition B.3. In a pooling equilibrium, the actual price charged by the non-commitment type seller is denoted by $\phi_\mu(p)$ given posted price p. As consumers cannot distinguish seller's type, upon observing the posted price, they assign prior belief to the final price. Then the cutoff of the known component x^* can be expressed as:

$$\underline{u} = -s + \mu \int_{-\infty}^{\infty} \max\{\underline{u}, x^* + y - p\} dG(y) + (1 - \mu) \int_{-\infty}^{\infty} \max\{\underline{u}, x^* + y - \phi_\mu(p)\} dG(y),$$

which is equivalent to

$$s = \mu \int_{p+\underline{u}-x^*}^{\infty} 1 - G(y) dy + (1 - \mu) \int_{\phi_\mu(p)+\underline{u}-x^*}^{\infty} 1 - G(y) dy.$$

First, $\phi_\mu(p)$ is uniquely defined. The non-commitment type seller's profit in a pooling equilibrium is $\pi_{PN}(p) = \phi_\mu(p) \int_{x^*}^{\infty} [1 - G(\underline{u} + \phi_\mu(p) - x)] dF(x)$. $\phi_\mu(p)$ is obtained by applying consistency to the first order condition:

$$\int_{x^*}^{\infty} [1 - G(\underline{u} + \phi_\mu(p) - x) - (\phi_\mu(p) - c) g(\underline{u} + \phi_\mu(p) - x)] dF(x) = 0.$$

It can be rewritten as

$$\frac{1}{\phi_\mu(p)-c}=\frac{\int_{x^*-\phi_\mu(p)}^{\infty}\frac{g(\underline{u}-k)}{1-G(\underline{u}-k)}[1-G(\underline{u}-k)]f(k+\phi_\mu(p))dk}{\int_{x^*-\phi_\mu(p)}^{\infty}[1-G(\underline{u}-k)]f(k+\phi_\mu(p))dk}, \tag{C.1}$$

which is obtained by rearranging the terms and change of variable: $k = x-\phi_\mu(p)$. The left-hand side of equation (C.1) is decreasing in $\phi_\mu(p)$. The right-hand side of equation (C.1) is increasing in $\phi_\mu(p)$, which guarantees the uniqueness. To see this, notice that $\frac{g(\underline{u}-k)}{1-G(\underline{u}-k)}$ is falling in k. The induced random variable with density $[1-G(\underline{u}-k)]f(k+\phi_\mu(p))$ with support $[x^*-\phi_\mu(p),\infty)$ falls in $\phi_\mu(p)$ in the first-order stochastic dominance sense. This is because (i) $\frac{[1-G(\underline{u}-k_2)]f(k_2+\phi_\mu(p))}{[1-G(\underline{u}-k_1)]f(k_1+\phi_\mu(p))}$ decreases as $\phi_\mu(p)$ increases for $\forall k_2 > k_1$, by log-concavity of f. (ii) $x^*-\phi_\mu(p)$ falls in $\phi_\mu(p)$, as

$$\frac{\partial x^*}{\partial\phi_\mu(p)}=\frac{(1-\mu)[1-G(\phi_\mu(p)-x^*+\underline{u})]}{(1-\mu)[1-G(\phi_\mu(p)-x^*+\underline{u})]+\mu[1-G(p-x^*+\underline{u})]}\leq 1.$$

Second, $\phi_\mu(p)$ increases in p. As the induced random variable with density $[1-G(\underline{u}-k)]f(k+\phi_\mu(p))$ with support $[x^*-\phi_\mu(p),\infty)$ rises in p in the first-order stochastic dominance sense, the right-hand side of equation (C.1) falls in p and results in a higher $\phi_\mu(p)$. By doing change of variable $\psi_\mu(p)=\phi_\mu(p)-p$ and repeating the same argument we can show that $\psi_\mu(p)$ decreases in p. Combining the fact that $\phi_\mu(p)$ increases in p and $\psi_\mu(p)=\phi_\mu(p)-p$ decreases in p, notice that $\phi_\mu(p)$ only cross p once, at p_N. $\phi_\mu(p)>p$ if $p<p_N$ while $\phi_\mu(p)<p$ if $p>p_N$.

Third, the non-commitment type seller's equilibrium profit is decreasing in the posted price p.

$$\begin{aligned}
\frac{\partial \pi_{PN}(p)}{\partial p} &= \frac{\partial \phi_\mu(p)}{\partial p} \int_{x^*}^{\infty} [1 - G(\underline{u} + \phi_\mu(p) - x)] dF(x) \\
&+ (\phi_\mu(p) - c)[-\frac{\partial x^*}{\partial p}][1 - G(-x^* + \underline{u} + \phi_\mu(p))] f(x^*) \\
&+ (\phi_\mu(p) - c)\frac{\partial \phi_\mu(p)}{\partial p} \int_{x^*}^{\infty} -g(\underline{u} + \phi_\mu(p) - x)] dF(x) < 0
\end{aligned}$$

since the summation of first term and the third term is 0 by first order condition and the second term is negative. As p rises, if fixing $\phi_\mu(p)$, then x^* decreases. Since $\phi_\mu(p)$ rises as p increases, x^* further decreases. Thus the non-commitment type's pooling equilibrium profit is a decreasing function of posted price. As deviation profit is given by $\pi_{PN}(p_N)$, the admissible posted price set to the non-commitment type seller is $[c, p_N]$.

Last, the lower bound of the pooling price set, $\underline{p}(\mu)$, is determined by the commitment type seller's incentive constraint. The commitment type seller's equilibrium profit is defined by

$$\begin{aligned}
\pi_{PC}(p) &= (p - c) \int_{x^*}^{\infty} [1 - G(\underline{u} + p - x)] dF(x). \\
&= (p - c) \int_{-\infty}^{\infty} [1 - G(\underline{u} + p - x)] 1\{x \geq x^*\} dF(x)
\end{aligned}$$

Both $(p - c)$ and $[1 - G(\underline{u} + p - x)]$ are log-concave in p. $1\{x \geq x^*\}$ is log-concave in x^* and decreases in x^*. Thus if x^* is convex in p, then log-concavity is preserved by multiplication and integration. It can be shown in various numerical examples including common distribution as Gaussian, Laplace and Gamma distributions that x^* is convex in p. However, for the case of uniform distribution, it is violated. Mollification of the uniform distribution to the entire real line can preserve this example.

Simulation results show that the profit function of the commitment type seller is single-peaked within the log-concave family (including uniform distribution). Even though the admissible posted price set to the commitment type seller may not be convex, existence of this set can still be shown. First notice that $\frac{\partial \pi_{PC}(p)}{\partial p}|_{p \geq p_N} \leq 0$ by using the first order condition of p_N. Along with the fact that $\phi_\mu(p_N) = p_N$, we know that the commitment type's equilibrium profit intersects with deviation profit at p_N. Second, notice that $\frac{\partial \pi_{PC}(p)}{\partial p}|_{p=c} \geq 0$ and equilibrium profit at $p = c$ is 0. Thus there must exists $\underline{p}(\mu)$ such that the $\pi_{PC}(\underline{p}(\mu)) = \pi_N(p_N, p_N)$ and $\frac{\partial \pi_{PC}(p)}{\partial p}|_{p=\underline{p}(\mu)} \geq 0$ by continuity. Thus the largest admissible posted price set to the commitment type seller is $[\underline{p}(\mu), p_N]$. The intersection of both type seller's admissible posted price set gives $[\underline{p}(\mu), p_N]$. Simulation results are available upon request. □

Proof of Proposition B.4. As the lower bound of the pooling posted price set is where the commitment type seller is indifferent between deviation or not, let $K(p) = \pi_{PC}(p) - \pi_N(p_N, p_N)$, then $\underline{p}$ is the solution to $K(p) = 0$. By implicit function theorem,

$$\frac{\partial \underline{p}}{\partial \mu} = -\frac{\partial K}{\partial \mu} / \frac{\partial K}{\partial p}|_{p=\underline{p}}.$$

Notice that

$$\frac{\partial K}{\partial \mu}|_{p=\underline{p}} = -(\underline{p} - c)\frac{\partial x^*}{\partial \mu}|_{p=\underline{p}}[1 - G(\underline{p} - x^* + \underline{u})]f(x^*),$$

$$\frac{\partial K}{\partial p}|_{p=\underline{p}} = \frac{\partial \pi_{PC}}{\partial p}|_{p=\underline{p}} \geq 0.$$

The last inequality follows by the fact that the derivative of π_{PC} with respect to p is positive at the lower bound. Thus the sign of $\frac{\partial \underline{p}}{\partial \mu}$ only depends on the sign of the term $\frac{\partial x^*}{\partial \mu}|_{p=\underline{p}}$. First, if $\phi_\mu(\underline{p})$ is fixed, then as μ rises, x^* decreases.

$$\frac{\partial x^*}{\partial \mu}|_{p=\underline{p}} = \frac{\int_{\phi_\mu(\underline{p})-x^*}^{\infty} 1 - G(y)dy - \int_{\underline{p}-x^*}^{\infty} 1 - G(y)dy}{\mu[1 - G(\underline{p} - x^*)] + (1-\mu)[1 - G(\phi_\mu(\underline{p}) - x^*)]} \leq 0$$

since $\underline{p} < p_N$ and $\underline{p} \leq \phi_\mu(\underline{p})$.

Second, as μ rises, $\phi_\mu(p)$ decreases. This can be seen from equation (C.1), which defines $\phi_\mu(p)$. As the random variable with density $[1 - G(\underline{u} - k)]f(k + \phi_\mu(p))$ with support $[x^* - \phi_\mu(p), \infty)$ falls in the first-order stochastic dominance sense. The right-hand side of equation (C.1) rises as μ rises. Thus $\phi_\mu(p)$ decreases.

Third, as already shown that $\phi_\mu(p)$ rises, x^* increases. The conclusion is obtained by combining all three facts. □

Proof of Lemma B.2. Equation (B.2) defines p_N. Following proof of proposition B.1, as s rises, y^* decreases, the random variable induced by the probability density $[1-G(\underline{u}-k)]f(k+p_N)$ with support $[\underline{u}-y^*, \infty)$ rises in y^* in the first order stochastic dominance sense. Thus the right-hand side of (B.2) decreases in s and p_N increases with s. The sigh of the derivative of profit function for the non-commitment type seller is same as $-(\partial p_N/\partial y^* - 1) \geq 0$, as it can be shown that $\partial p_N/\partial y^* \leq 1$ by doing a change of variable.

Proof of the second statement follows from proposition 5 in Choi et al. (2016). They show that the random variable $X + \min\{Y, y^*\}$ falls in the first-order stochastic dominance and becomes less dispersive as s increases, which results in p_C to fall with s. □

Proof of Proposition B.5. Proof of part (i):

The difference between equilibrium profit and deviation profit is defined by $K(p) = \pi_{PC}(p) - \pi_N(p_N, p_N)$ and lower bound $\underline{p}$ is such that $K(\underline{p}) = 0$. By implicit function theorem,

$$\frac{\partial \underline{p}}{\partial s} = -\frac{\partial K}{\partial s} / \frac{\partial K}{\partial p}|_{p=\underline{p}}.$$

Notice that $\lim_{\mu\to 0}\phi_\mu(p) = p_N, \forall p$ and $\lim_{\mu\to 0} x^*(p, \phi_\mu(p)) = p_N - y^*, \forall p$. It follows that $\lim_{\mu\to 0}\frac{\partial K}{\partial p}|_{p=\underline{p}} = 0$ and $\lim_{\mu\to 0}\frac{\partial K}{\partial s}|_{p=\underline{p}} = 0$ because $\lim_{\mu\to 0}\phi'_\mu(p) = 0$ and $\lim_{\mu\to 0}\partial x^*(p, \phi_\mu(p))/\partial s = \partial p_N/\partial s - \partial y^*/\partial s$. Thus we use L'Hospital rule to determine the sign of $\lim_{\mu\to 0}\frac{\partial \underline{p}}{\partial s}$. In the proof we use the fact that $\lim_{\mu\to 0}\frac{\partial \phi_\mu(p)}{\partial \mu}|_{p=\underline{p}} = 0$ and $\lim_{\mu\to 0}\frac{\partial \underline{p}}{\partial \mu} = 0$ from the proof of proposition B.4. First,

$$\begin{aligned}
\frac{\partial\{\frac{\partial K}{\partial p}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0} &= -p_N[1-G(y^*)]f(p_N-y^*)[\frac{\partial\{\frac{\partial x^*(p,\phi_\mu(p))}{\partial p}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0}].\\
\frac{\partial\{\frac{\partial x^*(p,\phi_\mu(p))}{\partial p}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0} &= 1
\end{aligned}$$

Therefore,

$$\frac{\partial\{\frac{\partial K}{\partial p}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0} < 0.$$

Second,

$$\begin{aligned}
\frac{\partial\{\frac{\partial K}{\partial s}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0} &= -p_N[1-G(y^*)]f(p_N-y^*)[\frac{\partial\{\frac{\partial x^*(p,\phi_\mu(p))}{\partial s}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0}].\\
\frac{\partial\{\frac{\partial x^*(p,\phi_\mu(p))}{\partial s}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0} &= \{\frac{A}{A+B}\}^2 \times [-\frac{1}{1-G(y^*)}] < 0
\end{aligned}$$

where $A = [1-G(y^*) - p_N g(y^*)]f(p_N-y^*)$ and $B = \int_{p_N-y^*}^{\infty} 2g(p_N-x) + p_N g'(p_N-x)dF(x)$.

Therefore,

$$\frac{\partial\{\frac{\partial K}{\partial s}|_{p=\underline{p}}\}}{\partial \mu}|_{\mu=0} > 0.$$

Thus $\lim_{\mu\to 0}\frac{\partial \underline{p}}{\partial s} > 0$.

Proof of part (ii):

(0) By implicit function theorem,

$$\frac{\partial \underline{p}}{\partial s} \quad = \quad -\frac{\partial K}{\partial s} / \frac{\partial K}{\partial p}|_{p=\underline{p}}.$$

Since

$$\frac{\partial K}{\partial p}|_{p=\underline{p}} = \frac{\partial \pi_{PC}}{\partial p}|_{p=\underline{p}} \geq 0,$$

by the definition of $\underline{p}$. The sign is determined by

$$\begin{aligned}
\frac{\partial K}{\partial s}|_{p=\underline{p}} \quad &= \quad \frac{\partial \pi_{PC}(\underline{p})}{\partial s} - \frac{\partial \pi_N(p_N)}{\partial s} \\
&= \quad -\underline{p}\frac{\partial x^*(\underline{p},\phi_\mu(\underline{p}))}{\partial s}[1 - G(\underline{p} - x^*(\underline{p},\phi_\mu(\underline{p})))]f(x^*(\underline{p},\phi_\mu(\underline{p}))) \\
&\quad\quad +p_N[\frac{\partial p_N}{\partial s} - \frac{\partial y^*}{\partial s}][1 - G(y^*)]f(p_N - y^*).
\end{aligned}$$

(1) Let $\pi_C(p)$ denotes the profit function for the case $\mu = 1$. As $\pi_C(p)$ is log-concave, it is single-peaked at $p = p_C$. Since $\pi_C(p_C) \geq \pi_C(p_N)$ and $\pi_C(c) = 0$, there exists $p_L \leq p_C$ such that $\pi_C(p_L) = \pi_C(p_N)$ and $\frac{\partial \pi_C(p)}{\partial p}|_{p=p_L} \geq 0$.

(2) As $\mu \to 1$, the profit function for the commitment type seller $\pi_{PC}(p) \to \pi_C(p), \forall p$. Since $\underline{p}$ is the point that delivers same profit as p_N does and generates positive derivative for π_{PC} with respect to p, $\underline{p} \to p_L$ as $\mu \to 1$.

(3) As $\mu \to 1$,

$$\begin{aligned}
\frac{\partial K}{\partial y^*}|_{p=\underline{p}} \quad &\to \quad -p_L[-1][1 - G(y^*)]f(p_L - y^*) \\
&\quad\quad +p_N[\frac{\partial p_N}{\partial y^*} - 1][1 - G(y^*)]f(p_N - y^*).
\end{aligned}$$

Since $\pi_C(p_L) = \pi_C(p_N)$, taking derivative with respect to y^* to both sides leads to

$$
\begin{aligned}
&-p_L[-1][1-G(y^*)]f(p_L-y^*) + p_N[\frac{\partial p_N}{\partial y^*} - 1][1-G(y^*)]f(p_N-y^*) \\
= &-\frac{\partial p_L}{\partial y^*}[\int_{p_L-y^*}^{\infty}[1-G(p_L-x) - p_L g(p_L-x)]dF(x) - p_L[1-G(y^*)]f(p_L-y^*)],
\end{aligned}
$$

with the second component of the product being positive since $\frac{\partial \pi_C(p)}{\partial p}|_{p=p_L} \geq 0$.

(4) We first argue here that $\pi_C(p)$ is log-supermodular in (p, y^*).

$$
\begin{aligned}
\frac{\partial \log \pi_C(p)}{\partial y^*} &= \frac{[1-G(y^*)]f(p-y^*)}{\int_{p-y^*}^{\infty}[1-G(p-x)]dF(x)} \\
&= \frac{1}{\int_0^{\infty} \frac{1-G(y^*-k)}{1-G(y^*)} \frac{f(p-y^*+k)}{f(p-y^*)} dk}
\end{aligned}
$$

The right-hand side increases as p increases.

(5) Then we show that $\frac{\partial p_L}{\partial y^*} \geq 0$. If search cost decreases and y_1 increases to y_2, and at y_1 we have relevant prices p_L and p_N. Log-supermodularity guarantees that $0 = \log \pi_C(p_N, y_1) - \log \pi_C(p_L, y_1) \leq \log \pi_C(p_N, y_2) - \log \pi_C(p_L, y_2)$. Thus $\pi_C(p_N, y_2) \geq \pi_C(p_L, y_2)$. Suppose p_N decreases to p'_N as y^* increases and $\frac{\partial \pi_C(p,y_2)}{\partial p}|_{p=p'_N} \leq 0$, $\pi_C(p_N, y_2) \leq \pi_C(p'_N, y_2)$. Therefore $\pi_C(p'_N, y_2) \geq \pi_C(p_L, y_2)$. Since $\frac{\partial \pi_C(p,y_1)}{\partial p}|_{p=p_L} \geq 0$ and $\frac{\partial \log \pi_C(p)}{\partial p}$ increases in y^*, $\frac{\partial \pi_C(p,y_2)}{\partial p}|_{p=p_L} \geq 0$. To keep $\pi_C(p'_N, y_2) = \pi_C(p'_L, y_2)$, $p'_L \geq p_L$.

(6) Thus $\lim_{\mu \to 1} \frac{\partial K}{\partial y^*}|_{p=\underline{p}} < 0$ and $\lim_{\mu \to 1} \frac{\partial K}{\partial s}|_{p=\underline{p}} > 0$, $\frac{\partial \underline{p}}{\partial s} < 0$. $\square$

Proof of Proposition B.6. Proof of part (i):

As $s \to 0$, $y^* \to \infty$ and $x^* \to -\infty$. All prices converge to the same price: $p_N, p_C, \underline{p} \to p^*$ where p^*, the optimal price to charge without search friction, is defined by $\int_{-\infty}^{\infty}[1 - G(p^*-x) - p^* g(p^*-x)]dF(x) = 0$.

As $\lim_{s\to 0}\frac{\partial K}{\partial p}|_{p=\underline{p}} = -p^*\frac{\partial x^*}{\partial p}[1-G(y^*)]f(p^*-y^*)$,

$$\lim_{s\to 0}\frac{\partial \underline{p}}{\partial s} = \lim_{s\to 0}(-\frac{\partial x^*}{\partial s}+\frac{\partial p_N}{\partial s}-\frac{\partial y^*}{\partial s})/\frac{\partial x^*}{\partial p} = 0$$

as the denominator goes to μ and the numerator goes to 0.

Proof of part (ii):

$$\lim_{s\to\infty}\frac{\partial \underline{p}}{\partial s} = \lim_{s\to\infty}\frac{-\underline{p}\frac{\partial x^*(\underline{p})}{\partial s}[1-G(\underline{p}-x^*(\underline{p}))]f(x^*(\underline{p}))+p_N[\frac{\partial p_N}{\partial s}-\frac{\partial y^*}{\partial s}][1-G(y^*)]f(p_N-y^*)}{\int_{x^*(\underline{p})}^{\infty}1-G(\underline{p}-x)-\underline{p}g(\underline{p}-x)dF(x)-\underline{p}\frac{\partial x^*(\underline{p})}{\partial p}[1-G(\underline{p}-x^*(\underline{p}))]f(x^*(\underline{p}))}$$

As $\lim_{s\to\infty}\frac{[1-G(y^*)]f(p_N-y^*)}{[1-G(\underline{p}-x^*(\underline{p}))]f(x^*(\underline{p}))} = 1$ and $\lim_{s\to\infty}\frac{\int_{x^*(\underline{p})}^{\infty}1-G(\underline{p}-x)-\underline{p}g(\underline{p}-x)dF(x)}{[1-G(\underline{p}-x^*(\underline{p}))]f(x^*(\underline{p}))} = \lim_{s\to\infty}-\frac{\partial x^*(\underline{p})}{\partial s}[1-\underline{p}\frac{g(\underline{p}-x^*(\underline{p}))}{1-G(\underline{p}-x^*(\underline{p}))}] = \lim_{s\to\infty}-\frac{\partial x^*(\underline{p})}{\partial s}$, it follows that

$$\lim_{s\to\infty}\frac{\partial \underline{p}}{\partial s} = \lim_{s\to\infty}\frac{-\underline{p}\frac{\partial x^*(\underline{p})}{\partial s}+p_N[\frac{\partial p_N}{\partial s}-\frac{\partial y^*}{\partial s}]}{-\frac{\partial x^*(\underline{p})}{\partial s}-\underline{p}\frac{\partial x^*(\underline{p})}{\partial p}}.$$

As (i) $\lim_{s\to\infty}\frac{\partial x^*(\underline{p})}{\partial s} = \lim_{s\to\infty}\frac{\partial p_N}{\partial s}-\frac{\partial y^*}{\partial s}$, (ii) $\frac{\partial x^*(\underline{p})}{\partial s}\geq 0$ and $\frac{\partial p_N}{\partial s}-\frac{\partial y^*}{\partial s}\geq 0$, (iii) $\underline{p}<p_N$, (iv) $\lim_{s\to\infty}\frac{\partial x^*(\underline{p})}{\partial p} = \mu$, when s is sufficiently large, $\lim_{s\to\infty}\frac{\partial \underline{p}}{\partial s}\leq 0$. $\square$

The following proposition shows that regulation can benefit consumers. Numerical results where regulation can hurt consumers are available upon request.

Proposition C.1. *If $\Delta \geq \tilde{\Delta} \triangleq \max\{p_C, \Delta_3, \Delta_8\}$, there is a continuum of pooling equilibria. There exists β such that $\forall p \in [\underline{p}, \beta]$ can be supported as a equilibrium pooling price. β increases as Δ increases and $\beta \leq p_N$.*

Proof can be found on online appendix.

Several remarks are in order. There is competition of two kinds of commitment power. On the one hand, the non-commitment type benefits from the commitment power by pooling with the commitment type; on the other hand, the fact that regulation is effective off-equilibrium path enables the non-commitment type to obtain partial commitment power. These two forces compete with each other in a non-trivial way to guarantee that each type has no incentive to deviate from the pooling price equilibrium.

To understand this result better, let us take $\Delta = p_N - \epsilon$. With $\Delta = p_N - \epsilon$, the most profitable deviation for the non-commitment type seller is to post $p = 0$ and charge $p_N - \epsilon$. Equilibrium profit is much less impacted by this change of regulation and his incentive to deviate is stronger. Thus p_N can no longer be in the pooling equilibrium price set and the upper bound of the price set is smaller. At the same time, there is negligible change in commitment type's incentive. Therefore the consumers benefit from the introduction of regulation policy. Both types of seller also benefit from regulation since their obtain higher profit with lower prices.

APPENDIX D
APPENDIX TO CHAPTER 3

Proof of Proposition 3.2. Given player j's strategy $\phi_j(t)$, player i's belief, conditional on no success and no exit by player j, evolves according to

$$\begin{aligned} p_i(t) &= \frac{p_i e^{-\lambda t}\int_0^\infty e^{-\int_0^{\min\{x,t\}}\phi_j(y)dy} d(1-e^{-\lambda x})}{p_i e^{-\lambda t}\int_0^\infty e^{-\int_0^{\min\{x,t\}}\phi_j(y)dy} d(1-e^{-\lambda x}) + (1-p_i)e^{-\int_0^t \phi_j(y)dy}} \\ &= \frac{\int_0^t e^{\int_x^t(\lambda+\phi_j(y))dy}\lambda dx + 1}{\int_0^t e^{\int_x^t(\lambda+\phi_j(y))dy}\lambda dx + 1 + \frac{1-p_i}{p_i}e^{2\lambda t}}. \end{aligned}$$

Differentiating with respect to t and arranging the terms,

$$\dot{p}_i(t) = -p_i(t)(1-p_i(t))\lambda + \phi_j(t)(1-p_i(t))\frac{((1-p_i)e^{2\lambda t}+p_i)p_i(t) - p_i}{(1-p_i)e^{2\lambda t}}.$$

Applying Lemma 3 in Murto and Välimäki (2011), $p_i(t)$ should stay constant once it reaches p^*. Therefore, it follows that

$$\phi_1(t) = \phi_2(t) = \frac{\lambda p^*(1-p_i)e^{2\lambda t}}{((1-p_i)e^{2\lambda t}+p_i)p^* - p_i}.$$

The result that if player j exits, then player i immediately follows comes from the fact that player i's belief, conditional on no success, jumps down to

$$p_i(t) = \frac{p_i e^{-2\lambda t}}{p_i e^{-2\lambda t}+1-p_i} \le \frac{p_i e^{-2\lambda t^*}}{p_i e^{-2\lambda t^*}+1-p_i} < \frac{p_i e^{-\lambda t^*}}{p_i e^{-\lambda t^*}+1-p_i} = p^*.$$

□

Proof of Propositions 3.3 and 3.4. (i) No player exits until time $t^* \equiv \min\{t_1^*, t_2^*\}$.

Define $\tilde{t}_i \equiv \inf\{t : p(t) \le p^*\}$, and $\tilde{t} \equiv \min\{\tilde{t}_1, \tilde{t}_2\}$. Since no player exits until $\tilde{t}$, $p_i(t) = p_i e^{-\lambda t}/(p_i e^{-\lambda t}+1-p_i)$ for any $t < \tilde{t}$. It is then immediate that $\tilde{t} = t^*$.

(ii) If $t > t^*$, then $p_i(t) \leq p^*$ for both $i = 1, 2$.

Suppose $p_i(t) > p^*$ for some $t > t^*$. Since $p_i(t)$ is always decreasing over time and player i never exits when $p_i(t) > p^*$ (Lemma 1 in Murto and Välimäki (2011)), this means that player j does not learn from player i's behavior over the interval $[t^*, t]$. Given this, player j, conditional on no success, prefers exiting immediately at time t^*: formally, $p_j(t') < p^*$ whenever $t' \in (t^*, t)$, and thus $c = p^*\lambda(v + V(1)) > p_j(t)\lambda(v + V(1))$ (see equation (3.4)). But this implies that $p_i(t^* + dt) = 0$ (note that $p_i(t)$ is player i's belief conditional on no success and no exit by player j), which is a contradiction.

(iii) The two distribution functions F_1 and F_2 have a common support of the form $[t^*, \bar{t}]$ for some $\bar{t}(> t^*)$ and are continuous. Finally, $F_1(t^*)F_2(t^*) = 0$.

Let $\underline{t}_i$ and $\bar{t}_i$ denote the lower bound and the upper bound of the support of F_i. Applying the same reasoning as in (ii), $\underline{t}_1 = \underline{t}_2 = t^*$. Now suppose $\bar{t}_i > \bar{t}_j$. In this case, player i does not learn from player j's behavior after $\bar{t}_j$. Since $p_i(t) \leq p^*$, conditional on no success, he exits immediately, which is a contradiction.

Now we show that the common support of F_1 and F_2 is the interval $[t^*, \bar{t}]$. Suppose $F_i(t)$ is constant on $[t^1, t^2) \subset [t^*, \bar{t})$. In this case, by the same reasoning as in (ii), player j, conditional on no success, exits immediately at t_1, which is a contradiction. Now suppose $F_i(t)$ has an atom at $t \in (t^*, \bar{t})$. In this case, player j has no incentive to exit close to t, that is, there exits $\varepsilon > 0$ such that $F_j(t)$ is constant on $[t - \varepsilon, t)$, which is a contradiction.

The no-atom result above does not apply to t^*. However, if $F_i(t^*) > 0$, then player j clearly strictly prefers waiting an instant more than exiting immediately, and thus $F_j(t^*) = 0$.

(iv) All the results here imply that an equilibrium necessarily takes the structure employed in the main text. The equilibrium uniqueness then follows follow from an explicit equilibrium construction in the main text.

(v) Denote by $V_1(t)$ player 1's expected payoff at time t. Whenever $p_1 > p_2$ (equivalently, whenever $F_2(t^*) > 0$), player 1's expected payoff $V_1(0)$ exceeds $V(p_1)$.

After time t^*, since exit is always an optimal strategy, player 1's expected payoff remains equal to 0. Therefore,

$$V_1(t^*) = \left(p_1^-(t^*) + (1 - p_1^-(t^*))e^{-\lambda t^*}\right) F_2(t^*)V(p_1).$$

Since the value function $V(\cdot)$ is convex, $V_1(t^*) > V(p_1^-(t^*))$. The desired result then follows from the fact that both $V_1(t)$ and $V(t)$ decrease according to the same law of motion over the interval $t \in [0, t^*)$ ($rV_1(t) = -c + \lambda p_1(t)(v + V(1) - V_1(t)) + \dot{V}_1(t)$ and $rV(t) = -c + \lambda p_1(t)(v + V(1) - V(t)) + \dot{V}(t)$ with $p(t) = p_1 e^{-\lambda t}/(p_1 e^{-\lambda t} + 1 - p_1)$) and, therefore, cannot cross each other.

$\square$

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