Spring 2015

Essays on inequality and human capital

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ESSAYS ON INEQUALITY AND HUMAN CAPITAL

by

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A thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Economics
in the Graduate College of
The University of Iowa

May 2015

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CERTIFICATE OF APPROVAL

PH.D. THESIS

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To my loving parents.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my advisor, Prof. Martin Gervais, for his numerous discussions and continued supports throughout the course of my Ph.D. Without his encouragement and guidance, it might have been almost impossible to finish my Ph.D. program. Next, I acknowledge my committee members, Prof. Alice Schoonbroodt, Prof. Nicolas Ziebarth, Prof. David Frisvold, and Prof. Yu-Fai Choi for their valuable suggestions and comments on my research.

I also extend my special thanks to my friends and collegues, including Varun Kharbanda, Michael Andrews, Chander Kochar, Larry Warren, Lev Lvovskiy, Marilyn Pease, Caleb Floyd, Phil Erickson, and many others who provided insightful discussions. Especially I would like to thank Alan Gelder and Kevin Carlson for their continuous feedback on my writing. Lastly, I am sincerely grateful to my loving parents who have given huge support to me throughout.

ABSTRACT

This dissertation contributes to the current understanding of human capital and its importance for earnings inequality and taxation. Human capital is typically defined as the stock of knowledge or skills acquired through education and working experience. The first chapter analyzes student borrowing behaviors in postsecondary education in the United States, the second chapter studies cross-country differences in earnings inequality within an endogenous growth model of human capital accumulation, and the third chapter examines the impact of endogenous human capital formations over a life-cycle on optimal fiscal policy.

In Chapter 1, I document that new federal student loans for higher education in the United States have risen more than 5 times over the past 20 years. What caused this dramatic increase? I develop a heterogeneous life-cycle model of human capital accumulation to analyze individual college and borrowing decisions. Using this framework, I assess the quantitative contributions of changes in the college wage premium, college costs, maximum borrowing limits, and loan interest rates to explain the significant rise of federal student loans. I find that the calibrated model accounts for 57 percent of the actual increase in loans from 1990 to 2011. Increases in the college wage premium and college costs are important factors in generating the sharp rise in loans and, particularly, the increase in the fraction of borrowers and borrowing amounts. The expansion of credit availability and decreased loan interest rates have a relatively minimal impact on individual college and borrowing decisions.

Chapter 2 explores why earnings inequality has been substantially higher in the US than in European countries over the last 30 years. I focus on the role of differences in tax progressivity, intergenerational earnings persistence, returns to education investments, and public education spending. I develop a growth model of human capital accumulation, and show analytically how those factors affect the dynamics of earnings inequality. The calibrated model accounts for 31 percent of the observed differences in earnings inequality between European countries and the US for 2003-07. Differences in returns to education investments and intergenerational earnings persistence are quantitatively important, suggesting the potential role of educational policy in ameliorating rising earnings inequality.

Chapter 3, written jointly with Martin Gervais, analyzes the role of endogenous human capital accumulation in shaping optimal fiscal policy within a life-cycle growth model. We show that when investment in human capital is not verifiable—making the tax code incomplete—a non-zero capital income tax becomes optimal in order to alleviate the distortionary effects of the labor income tax on investment in human capital. This is true even if the government has access to a full set of age-dependent labor and capital income taxes. The main result is in sharp contrast to the finding in Jones et al. (1997) that all interest taxes are zero in infinitely-lived agent models with endogenous human capital formation.

PUBLIC ABSTRACT

This thesis adds to the current area of human capital and analyzes its importance for earnings inequality and taxation. Chapter 1 documents a dramatic increase in new federal student loans for higher education in the United States over the past 20 years, and addresses the question of what caused this sharp rise. The main finding is that increases in the college wage premium and college costs are important driving factors and that expansions of credit availability and decreased loan interest rates have a relatively minimal impact on the rise in federal student loans. Chapter 2 explores why earnings inequality has been substantially higher in the US than in European countries over the last 30 years. Using an endogenous growth model of human capital accumulation, I find that differences in education system and intergenerational earnings persistence are quantitatively important, suggesting the potential role of educational policy in ameliorating rising earnings inequality. Chapter 3 analyzes the role of endogenous human capital formation in shaping optimal fiscal policy within a life-cycle growth model. The key finding is that when investment in human capital is not verifiable to the government, a non-zero capital income tax becomes optimal in order to mitigate the distortionary effects of the labor income tax on investment in human capital. This main result is in sharp contrast to the finding of Jones et al. (1997) that all interest taxes are zero in infinitely-lived agent models with endogenous human capital formation.

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CHAPTER 1
ACCOUNTING FOR THE RISE IN FEDERAL STUDENT LOANS

1.1 Introduction

New federal student loans issued for postsecondary education in the United States have increased considerably, rising from 21 billion dollars in 1990 to 110 billion dollars in 2011 (see Figure 1.1). What accounts for this dramatic increase in federal student loans? Understanding factors that generated this explosive growth is a critical issue for policymakers who have concerns not only about student capacity to repay loans after college, but also about students' needs for more funds due to rising college costs. In particular, concerns about student loans have been greatly intensified in recent years as total outstanding student loan debt overtook total outstanding credit card debt, amounting to over 1 trillion dollars and becoming the largest non-mortgage household debt in 2012.

This sharp rise in federal student loans can be attributed to a combination of several factors. On the demand side, commonly discussed in previous literature, including Avery and Turner (2012) and Kane (2007), significant increases in college expenses and financial returns to college education are important driving forces that affect an individual's college and borrowing decisions. On the supply side, as pointed out by Ionescu (2009, 2011), policy changes in federal student loan programs such as increased maximum borrowing limits or decreased loan interest rates can potentially impact college attendance as well as student borrowing.

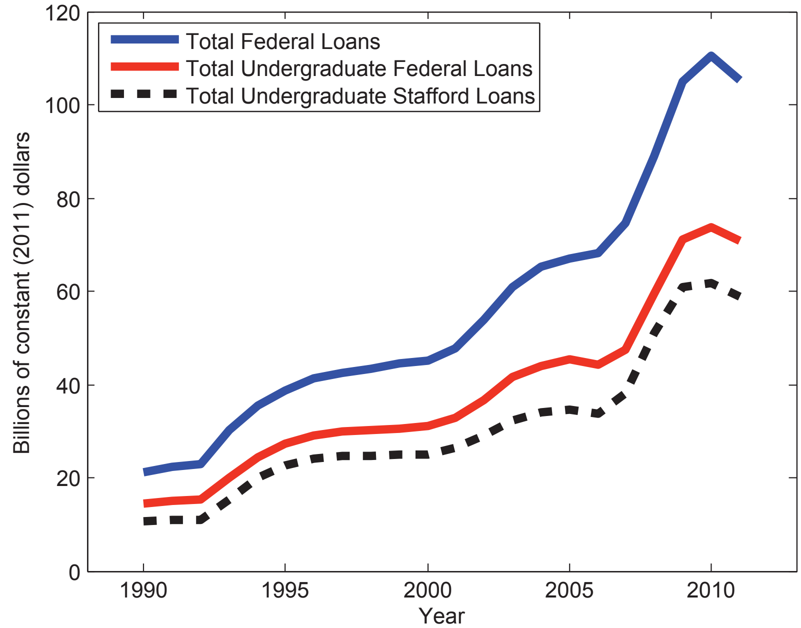


Figure 1.1: Trends in new federal student loans issued

Using aggregate undergraduate Stafford loans, the largest federal student loan program in the United States, I divide the total loans by high school graduates and decompose the loans into college enrollments, fraction of borrowers, and average borrowing amounts. To uncover a key component driving the substantial increase of loans, I take log differences of total Stafford loans per high school graduate and individual components of the loans between 1990 and 2011, calculating the contribution of each component to the growth in loans. Table 1.1 shows that the sharp rise of Stafford loans is largely due to a dramatic expansion in the fraction of borrowers, contributing 57 percent to the increase in total loans over the past 20 years. The increase in the borrowing amounts per borrowers is the second most important com-

Table 1.1: Growth Decomposition of Undergraduate Stafford Loans

	1990	2011	Changes	Growth
	(A)	(B)	$\ln(B/A)$	Decomp.
College Enrollment (%)	0.60	0.68	0.13	0.08
Fraction of Borrowers (%)	0.27	0.66	0.89	0.57
Average Borrowing ($2011)	3,831	6,629	0.55	0.35
Stafford Loans per HS Graduate ($2011)	621	2,975	1.57	1.00

ponent, followed by the modest increase in the college enrollment rate, contributing 35 percent and 8 percent, respectively, to the actual growth of Stafford loans from 1990 to 2011.

Motivated by Becker (1964) and Ben-Porath (1967), I develop a simple life-cycle model of human capital accumulation to analyze an individual's college and borrowing decisions. An individual is heterogeneous in terms of ability and initial wealth, and makes a discrete college-enrollment decision based on initial characteristics.[1] Once an individual chooses to work after high school graduation, his earnings evolve over time at a constant growth rate. If an individual decides to go to college, however, he pays college costs and receives grants which decreases with initial wealth but increases with student's ability. If a college student has insufficient funds, he can borrow from the government to finance college and living expenses while in college.[2]

[1]One can think of an individual's ability as his future earnings capacity.

[2]I assume that the government is the only source of student borrowing. This assumption sharply contrasts with Lochner and Monge (2011)'s article which focuses on both government student loan programs and private lending to explain a positive relationship between ability and human capital investment for credit-constrained students observed in data.

There is a maximum loan amount that students can borrow, which is either net full college costs or an exogenous loan limit imposed by the government, whichever is lower. The benefit of obtaining a college degree is the college wage premium as well as high growth rates of earnings. I abstract from uncertainty on loan interest rates and earnings after college, and I assume that all student loans will be repaid: there is no default in equilibrium.[3] Moreover, I assume that credit markets after college are perfect.

I analytically show how changes in the four driving factors—the college wage premium, college costs, maximum borrowing limits, and interest rates on student loans—affect an individual's college and borrowing decisions. Similar to Belly and Lochner (2007) and Restuccia and Vandenbroucke (2013)'s findings, there are cutoff levels of ability and initial wealth determining college attendance and borrowng decisions: these thresholds turn out to be a function of the four driving factors. A rise in the college wage premium directly increases college enrollments and causes more students to become borrowers and take out more loans due to increased incentives to smooth their lifetime consumption. On the other hand, a rise in college costs discourages more individuals from attending college and induces the students who do enroll in college to become borrowers and increase their borrowing levels to finance high college expenses. An increase in maximum borrowing limits relaxes the required

[3]Avery and Turner (2012) and Johnson (2011) argue that large variations in expected returns to college education affect an individual's college and borrowing decisions. That is, the increased uncertainty of financial returns to college education lowers both the value of investing in college and the borrowing level due to the increased risk of default. These authors' findings sharply contrast with observed increases in college enrollment and student borrowing.

level of initial wealth for college attendance, encouraging more individuals to go to college. Moreover, the expanded credit limit helps previously credit-constrained students smooth lifetime consumption by increasing their borrowing amounts. Lastly, the decrease in interest rates on student loans makes college students much easily get access to student loans, and more students become borrowers and increase their borrowing amounts.

I calibrate the benchmark model to reproduce salient features of the U.S. economy in 1990. Specifically, I assume that a joint distribution of ability and initial wealth is lognormal, and I calibrate parameters associated with the distribution to match a set of key statistics: college enrollment rate, fraction of borrowers, average borrowing amounts, and college attendance rates by family income in 1990. The parameter of the college wage premium is set to match the observed earnings ratio of college to non-college workers at ages 25-30 in 1990. Other parameters, including college costs, maximum borrowing limits, and grants, come from the College Board and the National Postsecondary Student Aid Study in 1990 (NPSAS90). I directly estimate earnings growth rates by education level from the National Longitudinal Surveys of Youth in 1979 (NLSY79).

Using this calibrated model, I implement a quantitative experiment to assess the role of changes in the college wage premium, college costs, maximum borrowing limits, and interest rates on student loans to account for the increase in Stafford loans from 1990 to 2011. I find that the model generates an increase of 213 percent, accounting for 57 percent of the actual rise in Stafford loans for the last 20 years.

Because of the counteracting effects of increases in the college wage premium and college costs, the model produces a modest rise in college enrollment rates, explaining 33 percent of the actual increase between 1990 and 2011. The model also generates large expansions in the fraction of borrowers and borrowing amounts, accounting for 59 percent and 84 percent, respectively, of their observed increases over the past 20 years.

Next, I conduct a set of counterfactual experiments to decompose the relative importance of the four driving forces for the rise of Stafford loans. The first experiment examines the role of the increase in the college wage premium; that is, what would have happened if the college wage premium rose from 1990 to 2011, with the other factors remaining constant at 1990 levels? Stafford loans would have increased by 44 percent from 1990 to 2011, and the college wage premium alone would have accounted for 21 percent of the observed rise of Stafford loans. This increase of loans is mainly driven by a large schooling response to college returns, which is consistent with Restuccia and Vandenbroucke (2013)'s finding. The fraction of borrowers and average borrowing amounts also rise due to increased consumption smoothing motives, but their increases are quantitatively small. The second experiment quantifies the effect of the increase in net college costs on the rise of Stafford loans. The rise in college costs alone would have accounted for 25 percent of the actual increase of Stafford loans over the last 20 years. This experiment shows that the rise in college costs is an important driving force generating rapid expansions in the fraction of borrowers and borrowing levels, despite its large negative effect on college enrollment rates. The third experiment investigates the impact of expansions of credit availability caused by both increased full college costs and maximum borrowing limits on the increase of Stafford loans. Holding the net tuitions and fees that students must pay fixed at their 1990 levels, the model predicts that the credit expansions alone accounts for only 5 percent of the observed increase of Stafford loans from 1990 to 2011. That is, without changes in market prices, the expansion of credit availability itself has negligible impacts on an individual's college and borrowing decisions, consistent with Keane and Wolpin (2001) and Abbott et al. (2013)'s findings. Lastly, I examine the role of the decrease in interest rates on student loans; that is, what would have occured if the loan interest rates dropped from 1990 to 2011, while the other driving forces remained constant at their 1990 levels? The model predicts that the decrease in interest rates on student loans alone would have accounted for only 2 percent of the observed rise of federal Stafford loans over the last 20 years. In summary, the main culprits behind the sharp increase in federal student loans are attributed to changes in market prices, not the changes in government policy.

To my knowledge, this paper is the first to quantitatively analyze the trends in federal student loans, contributing to the literature on college loans. Many existing papers focus on the effects of policy changes in federal student loan programs on college enrollment rates and default rates (Ionescu 2009, 2011), on the impacts of educational debts on academic outcomes and career decisions (Minicozzi 2005; Rothstein and Rouse 2011), and on the effects of student debts on marriage (Gicheva 2011) or puchasing homes (Chiteji 2007). However, in spite of the explosive increase of student loans and growing concerns about student capacity to repay loans, there has been no attempt to quantitatively analyze the evolution of student loans. This paper aims to fill that void.

The closest study is Avery and Turner (2012). They ask the question of whether college students borrow too much or too little, and examine various factors affecting students borrowing. Their conclusion is that the argument that students borrow too high can obviously be rejected, which is the same claim of this paper. However, they do not quantitatively estimate the effects of changes in market prices and government policies on the increase of student loans. My work is also similar to Lochner and Monge (2011) in terms of analyzing changes in student borrowing in responses to increases in college returns and costs. On the other hand, they focus on the role of private lending for how human capital accumulation responds to changes in policies or economic environments, while this paper focuses on quantitatively accounting for the sharp increase in federal student loans.

Lastly, this paper contributes to the growing literature on human capital and credit constraints in education. Studies using NLSY79 conclude that borrowing constraints have little effect on college-attendance decisions, even after controlling for ability and family backgrounds (Cameron and Heckman 1998, 2001; Keane and Wolpin 2001; Carneiro and Heckman 2002; Cameron and Taber 2004). However, using NLSY97, Belly and Lochner (2007) suggest that parental financial resources have become a critical determinant of college attendance in the 2000s. Consistent with these findings, my paper also shows the increased effect of family income on college attendance. Moreover, I find the increased importance of family income for student borrowing behaviors, arguing that credit constraints in postsecondary education in the United States have been more pronounced.

The rest of this paper proceeds as follows. Section 1.2 describes a model economy and analytically examines how changes in the college premium, college costs, and maximum borrowing limits impact an individual' college and borrowing decisions. Section 1.3 calibrates the model to match a set of key statistics of federal Stafford loans in 1990 and quantitatively evaluates the importance of the changes in the three driving forces to account for the dramatic increase of federal Stafford loans over the last 20 years. Section 1.4 concludes.

1.2 Model

I develop a simple life-cycle economy with heterogeneous agents that differ in their ability and initial wealth. I analytically characterize individuals' college and borrowing decisions, and show how increases in the college wage premium, college costs, and borrowing limits change individuals' incentives of attending college and borrowing.

1.2.1 Environment

The economy is populated by overlapping generations of individuals living for J+1 periods. One can think of j = 0 as the first year after high school graduation. Individuals are heterogeneous in terms of ability (a) and initial wealth (w), which are jointly drawn from distribution F(a,w). Ability represents future earnings capacity, and initial wealth corresponds to an income endowment received from parents.

I model a high school graduate's decision to invest in college education by maximizing his present value of utility over the life-cycle. At first period, given his ability and initial wealth, each individual decides whether to attend college or to directly go to the labor market to work. Once an individual chooses to work after high school graduation, his earnings evolve over time with constant growth rates. Meanwhile, once an individual decides to go to college, he pays college costs, and receives grants which decrease with initial wealth. In case that a college student might not have sufficient funds, he can borrow from the government to finance college expenses and consumption while in college.[4] After a college education, an individual's earnings jump up by the college wage premium, and grow faster than a high-school individual's earnings. Figure 1.2 summarizes time of an individual's decision.

I abstract from uncertainty on loan interest rates or earnings after college, and all federal student loans must be repaid after college: there is no default in equilibrium. Moreover, I assume that credit markets after college are perfect in order to focus on a college agent's borrowing decision. To simplify the analysis, the gross rate of interest (R) is exogenously given and equal to the inverse of the subjective discount factor (β), i.e. $\beta R = 1$, so that all individuals want to perfectly smooth consumption throughout their lifetime.

An individual's life-cycle problem can be backwardly solved in two stages.

[4]To focus on federal student loans, I assume that a college student is not allowed to get access to private lendings.

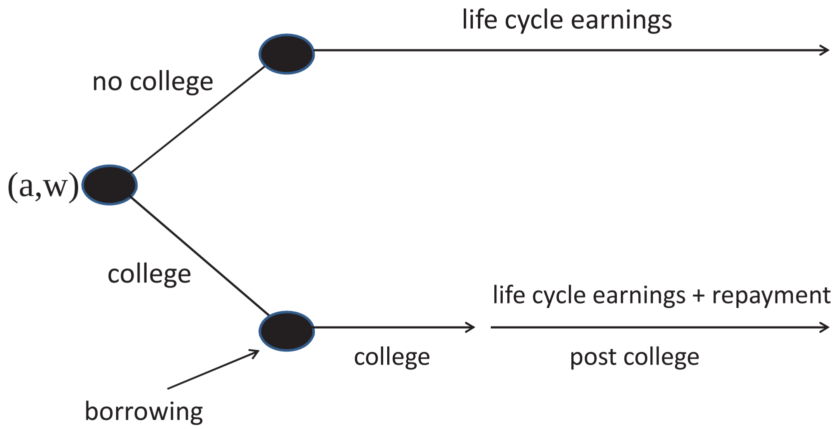


Figure 1.2: Timing of decisions

First, for each education choice, I solve for optimal path of consumption, and calculate an individual's lifetime utility. In the case of a college student, I also solve for his optimal borrowing from the government. An individual chooses between college versus no-college to maximize lifetime utility.

1.2.2 Non-College Agents

Given his ability (a) and initial wealth (w), a non-college agent optimally chooses his life-cycle consumption sequence to maximize his lifetime utility. Specifically, a non-college agent's lifetime utility maximization problem is given by:

$$V_{nc}(a, w) \equiv \max \sum_{j=0}^{J} \beta^j \frac{c_j^{1-\sigma}}{1-\sigma} \tag{1.1}$$

subject to

$$\sum_{j=0}^{J}\left(\frac{1}{R}\right)^{t} c_j = \zeta w + a\Phi_{nc}, \tag{1.2}$$

$$\Phi_{nc} = \sum_{j=0}^{J}\left(\frac{1}{R}\right)^{j} \exp\left(g_{nc} j\right), \tag{1.3}$$

$$\zeta = \sum_{j=0}^{S-1}\left(\frac{1}{R}\right)^{j}, \tag{1.4}$$

where σ denotes the coefficient of risk aversion, c_t represents consumption at time t, and g_{nc} is returns to potential labor market experience. Notice that $a\Phi_{nc}$ is the present value of lifetime labor income for a non-college agent with ability a.

1.2.3 College Agents

An individual who decides to go to college additionally chooses how much to borrow from the government to fund college expenses. Given his ability (a) and initial wealth (w), a college-agent's lifetime utility maximization problem is described by:

$$V_c(a, w) \equiv \max \sum_{j=0}^{J} \beta^j \frac{c_j^{1-\sigma}}{1-\sigma} \tag{1.5}$$

subject to

$$c_j + k = w + d + G(a, w), \quad j = 0, \dots, S-1, \tag{1.6}$$

$$\sum_{j=S}^{J}\left(\frac{1}{R}\right)^{j-S} c_j = ap\Phi_c - d\left(\sum_{i=1}^{S} \widetilde{R}^i\right) \tag{1.7}$$

$$\Phi_c = \sum_{j=S}^{J}\left(\frac{1}{R}\right)^{j-S} \exp\left[g_c\left(j-S\right)\right] \tag{1.8}$$

$$0 < d \leq \overline{d}(w, a) \quad , \quad \overline{d}(w, a) = \min\left\{\max\left\{0, f - G(w, a) - w\right\}, d_{max}\right\}, \tag{1.9}$$

where S represents years of schooling, d denotes federal student loan amounts, and $\widetilde{R}$ is gross interest rates on student loans. Consistent with current federal student loan programs, there is a maximum loan amount $\left(\overline{d}(w)\right)$ that students can borrow, which depend on an individual's initial wealth. This limit is determined by the minimum value of either full college costs (f) minus available grants minus parental contribution (w) to college or the exogenous loan limit (d_{max}) imposed by the government. A college education involves a time cost and resource cost (k), and benefits an individual with the college wage premium (p) and relatively high earnings growth rate. G(a,w) represents grants that are function of an individual's initial wealth and ability. This captures the fact that financial aid policies, including Pell Grants, are more generous to youth from low economic backgrounds and to youth with high ability. Notice that $ap\Phi_c$ represents the present value of lifetime earnings for the college agent with ability a.

Notice that a college individual who has a large initial wealth can save with the market interest rate (R), and achieves perfect consumption smoothing over his lifetime, because of the assumption $R\beta = 1$. The maximization problem of the agent is given by

$$V_c(a, w) \equiv \max \sum_{j=0}^{J} \beta^j \frac{c_j^{1-\sigma}}{1-\sigma} \tag{1.10}$$

subject to

$$\sum_{j=0}^{J} \left(\frac{1}{R}\right)^j c_j + \zeta k = \zeta w + \zeta G(w, a) + \left(\frac{1}{R}\right)^S ap\Phi_c, \tag{1.11}$$

$$\Phi_c = \sum_{j=S}^{J} \left(\frac{1}{R}\right)^{j-S} \exp\left[g_c\left(j - S\right)\right], \tag{1.12}$$

$$\zeta = \sum_{j=0}^{S-1} \left(\frac{1}{R}\right)^j. \tag{1.13}$$

Due to the assumption of the complete credit market, I can express his present-value lifetime budget constraint as in (1.11); On the left-hand side, we have the present-value lifetime expenditures, including college costs, and on the right-hand side, we have the present-value lifetime income, including initial income and grants received from the government.

1.2.4 Analysis

1.2.4.1 Non-College Agent's Behavior

Because of the assumption of $\beta R = 1$, an agent's optimal consumption plan is constant throughout his lifetime. Using his lifetime budget constraint, I calculate the optimal consumption level in each period as follows:

$$c_{nc}(a, w) = (\zeta w + a\Phi_{nc}) \left(\frac{1 - \beta}{1 - \beta^{J+1}} \right). \tag{1.14}$$

Notice that his consumption level increases with his ability (a) and initial wealth (w). Hence, a non-college agent's lifetime value of the non-college agent is given by:

$$V_{nc}(a, w) = \left(\frac{c_{nc}(a, w)^{1-\sigma}}{1 - \sigma} \right) \left(\frac{1 - \beta^{J+1}}{1 - \beta} \right). \tag{1.15}$$

1.2.4.2 College Agent's Behavior

The behaviors of a college individual who does saving during college can be easily analyzed. Because of the assumption $\beta R = 1$, he can achieve perfect consumption smoothing over his lifetime. The per-period consumption level can be derived from his present-value lifetime budget constraint:

$$c_{col}(a, w) = \left(\zeta w + \zeta G(w, a) - \zeta k + \left(\frac{1}{R} \right)^{S} ap\Phi_c \right) \left(\frac{1 - \beta}{1 - \beta^{J+1}} \right)$$

Hence, the college agent's lifetime value is given by:

$$V_c^*(a,w) = \left(\frac{c_{col}\,(a,w)^{1-\sigma}}{1-\sigma}\right)\left(\frac{1-\beta^{J+1}}{1-\beta}\right). \tag{1.16}$$

I now analyze the behaviors of a college agent who borrows from the government. Notice that a college agent's life-cycle problem can be mapped into a two-period problem. That is, a simplified two-period college agent's problem is given by:

$$\max \left(\frac{c_s^{1-\sigma}}{1-\sigma}\right)\left(\frac{1-\beta^S}{1-\beta}\right) + \beta^S \left(\frac{c_w^{1-\sigma}}{1-\sigma}\right)\left(\frac{1-\beta^{J-S}}{1-\beta}\right) \tag{1.17}$$

subject to

$$c_s = w + d + G(w,a) - k, \tag{1.18}$$

$$c_w \left(\frac{1-\beta^{J-S+1}}{1-\beta}\right) = ap\Phi_c - d\left(\sum_{i=1}^{S} \widetilde{R}^i\right), \tag{1.19}$$

$$0 < d \le \overline{d}(w,a).$$

Using a college agent's budget constraint and the first order conditions, I can derive his optimal borrowing amounts as follows:

$$d^*(a,w) = \frac{ap\Phi_c C - (w + G(a,w) - k)\,D}{BC + D}, \tag{1.20}$$

where B $= \sum_{i=1}^{S} \widetilde{R}^i$, C $= \frac{1-\beta}{1-\beta^{T-S+1}}$, and D $= \left(\beta^S \left(\frac{1-\beta^{T-S}}{1-\beta^{J-S+1}}\right)\left(\frac{1-\beta}{1-\beta^S}\right) B\right)^{1/\sigma}$. Clearly, as a college agent is more able (high a) or the college premium (p) increases, he expects to have higher earnings after college, and is willing to borrow more to smooth his consumption over lifetime. Moreover, as a college agent is less wealthy (low w) or college costs (k) rise, his optimal borrowing level also becomes larger to pay for the increased college expenses. Also, notice that student borrowing is negatively correlated with interest rates on the borrowing.

I first describe a credit-unconstrained college agent's optimal consumption plan and his value function. Using his lifetime budget constraint, I can calculate his optimal consumption level in each period:

$$c_s^*(a,w) = w + d^*(a,w) + G(w,a) - k \tag{1.21}$$

$$c_w^*(a,w) = \left(ap\Phi_c - d^*(a,w)\left(\sum_{i=1}^{S}\widetilde{R}^i\right)\right)\left(\frac{1-\beta}{1-\beta^{J-S+1}}\right) \tag{1.22}$$

Therefore, a credit-unconstrained college agent's lifetime value is given by:

$$V_c(a,w) = \left(\frac{(c_s^*)^{1-\sigma}}{1-\sigma}\right)\left(\frac{1-\beta^S}{1-\beta}\right) + \beta^S\left(\frac{(c_w^*)^{1-\sigma}}{1-\sigma}\right)\left(\frac{1-\beta^{J-S}}{1-\beta}\right). \tag{1.23}$$

Clearly, this value rises in the agent's ability (a), initial wealth (w), and the college premium (p), while it decreases with education costs (k).

I turn to a credit-constrained college agent's behavior. In this case, a borrowing constraint precludes a college agent from achieving perfect consumption smoothing over lifetime. Since his borrowing amounts are simply the maximum borrowing limit, his consumption levels during and after college are respectively:

$$\bar{c}_s(a,w) = w + G(a,w) + \overline{d}(w,a) - k, \tag{1.24}$$

$$\bar{c}_w(a,w) = \left(ap\Phi_c - \overline{d}(w,a)\left(\sum_{i=1}^{S}\widetilde{R}^i\right)\right)\left(\frac{1-\beta}{1-\beta^{J-S+1}}\right). \tag{1.25}$$

Hence, a credit-constrained college agent's lifetime value is as follows:

$$\overline{V_c}(a,w) = \left(\frac{\bar{c}_s^{1-\sigma}}{1-\sigma}\right)\left(\frac{1-\beta^S}{1-\beta}\right) + \beta^S\left(\frac{\bar{c}_w^{1-\sigma}}{1-\sigma}\right)\left(\frac{1-\beta^{J-S}}{1-\beta}\right). \tag{1.26}$$

1.2.4.3 Who Goes to College?

An individual chooses his education level which gives him the higher lifetime utility. A college enrollment decision of an individual who does saving during college is:

$$V_c^*(a,w) \geq V_{nc}(a,w) \quad \Leftrightarrow \quad a \geq \frac{\zeta k + \zeta G(a,w)}{p\beta^S \Phi_c - \Phi_{nc}} \equiv a^*(k,p) \tag{1.27}$$

$$d^*(a,w) \leq 0 \quad \Leftrightarrow \quad w \geq \frac{ap\Phi_c}{D} k - G(a,w) \tag{1.28}$$

Equations (1.27) and (1.28) specify a set of agents who choose to enroll in college and do not borrow due to large initial wealth. Equation (1.27) is the condition of attending college, i.e. higher lifetime value of saving college agents than that of non-college agents. Notice that there exists a cut-off level of ability ($a^*(k,p)$) that increases in college expenses (k) and decreases in the college wage premium (p). That is, college attendance decision is independent of initial wealth. A rise in college costs lowers the value of attending college, so that only very able agents are willing to sacrifice the increased college expenses in order to obtain high market returns to college education. This implies that the cutoff level of ability increases as college costs rise. A similar interpretation applies to the college wage premium. That is, as financial returns to college education increase, lesser-able agents are willing to attend college and pay for college expenses in order to acquire higher benefit of obtaining a college degree. Equation (1.28) specifies the condition of college students saving. In other words, an individual's initial wealth is high enough to cover net college costs and present value of his earnings.

A college decision of an individual who is not borrowing-constrained in college is given by:

$$V_c(a,w) \geq V_{nc}(a,w), \tag{1.29}$$

$$0 < d^*(a,w) \leq \overline{d}(w,a). \tag{1.30}$$

Equations (1.29) and (1.30) determine a set of agents who choose to enroll in college and is not be borrowing-constrained. Equation (1.29) is the condition of attending college, i.e. higher lifetime value of credit-unconstrained college agents than that of non-college agents. Contrast to the case of saving college students, borrowing-unconstrained students cannot achieve perfect consumption smoothing due to relatively high interest rates of college borrowing, compared to risk-free market interest rates. Equation (1.30) specifies the condition of not being borrowing-constrained, implying that an individual's initial wealth is low enough for him to borrow from the government but not too low to be credit-constrained.

A college enrollment decision of an agent who is borrowing-constrained in college is:

$$\overline{V_c}(a,w) \quad \geq \quad V_{nc}(a,w), \tag{1.31}$$

$$d^*(a,w) \quad > \quad \overline{d}(w,a). \tag{1.32}$$

In similar, Equations (1.31) and (1.32) specify a set of agents who decide to attend college but is borrowing-constrained in college. That is, even if agents is borrowing-constrained in college due to relatively low initial resources, attending college is beneficial because of relatively high ability. Equation (1.31) is the condition of going to college, i.e. greater lifetime value of borrowing-constrained college agents than that of non-college agents. In this case, college attendance decision depends not only on agents' initial wealth but also maximum borrowing limits. So, a rise in maximum borrowing amounts increases college enrollment rates by providing more credits for borrowing-constrained students. Equation (1.32) implies the condition of being borrowing-constrained. That is, an agent's initial resources is too low to cover college expenses and present value of his earnings net maximum borrowing amounts, and his optimal borrowing level is larger than maximum borrowing limits, becoming credit-constrained.

1.2.4.4 Who Becomes a Borrower?

I now analyze which type of a college student becomes a borrower, and examine how increases in the college wage premium, college costs, and maximum borrowing limits affect thresholds of ability and initial wealth to determine a student becomes a borrower. To do so, let $\overline{w}$ be a cutoff level of initial wealth at which a college agent's optimal borrowing amounts are zero:

$$\overline{w} = ap\Phi_c\left(\frac{C}{D}\right) - G(a, \overline{w}) + k,$$

where C $= \frac{1-\beta}{1-\beta^{T-S+1}}$ and D $= \left(\beta^S\left(\frac{1-\beta^{T-S}}{1-\beta^{J-S+1}}\right)\left(\frac{1-\beta}{1-\beta^S}\right)\sum_{i=1}^{S}\widetilde{R}^i\right)^{1/\sigma}$. That is, a college agent with initial wealth less than $\overline{w}$ becomes a borrower. Notice that the threshold of initial wealth ($\overline{w}$) depends on a college agent's ability. As a college agent is more able, he expects his future earnings to be higher, and decides to borrow more because of consumption smoothing motives, implying that the threshold level of initial wealth increases. In addition, the cutoff level of initial wealth is positively related to the college wage premium (p) and education costs (k). The interpretation is straightforward. As market returns to college education increase, more college agents become borrowers to smooth their consumption over lifetime. Similarly, high college costs directly stimulate college agents' incentives to borrow, so that more college agents become borrowers. Hence, as the college wage premium or education cost rises, a measure of borrowers gets larger, implying that the cutoff level of initial wealth increases. However, notice that the threshold level of initial wealth is independent of maximum borrowing limits. That is, the decision to become borrowers has nothing to do with maximum loan amounts that students can borrow.

1.2.4.5 Comparative Statics

It is very helpful to visualize an individual's decision rules I have analyzed so far in ability and initial wealth space. Using parameter values described in the next section, Figure 1.3 graphically summarizes decision rules of different types of agents. As shown in Figure 1.3, an individual's ability plays an important role in determining college enrollment. In other words, even though an individual has large wealth, he choose not to attend college unless his ability level is high enough. Furthermore, notice that an individual is willing to enroll in college as long as he has high enough ability, even if he is borrowing-constrained due to low initial wealth. This is because going to college is so beneficial for him. Conditional on college going, an individual's ability and initial wealth still play a crucial role in determining whether he becomes a borrower. As a college agent is more able, he wants to borrow more because of

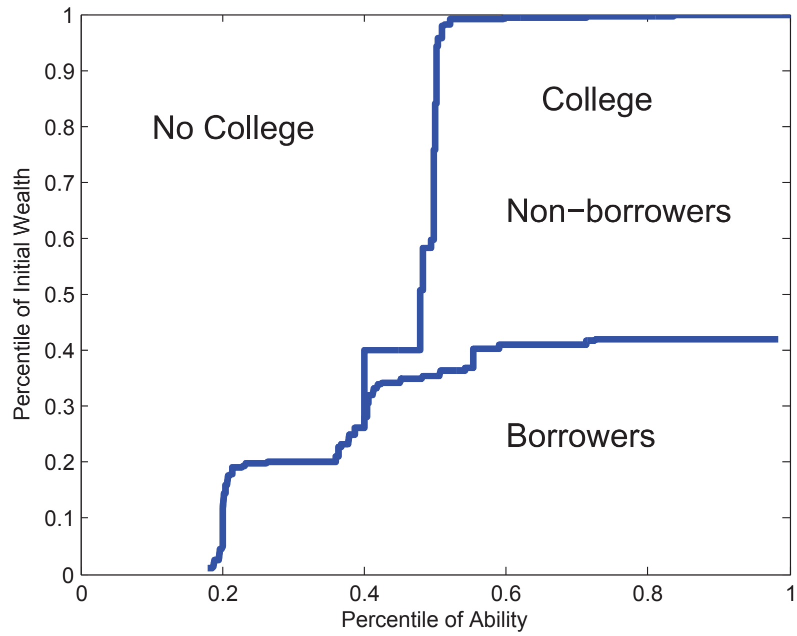


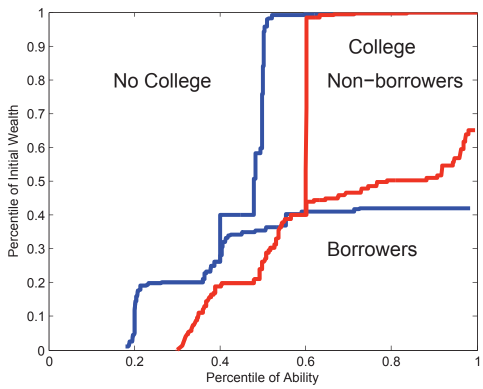
Figure 1.3: Decision rules of different types of agents

increased consumption smoothing motives, and needs large enough initial wealth to make himself not be a borrower. This is why the threshold of initial wealth increases as a college agent has higher ability. However, since a college student with high ability is more likely to have high initial wealth and grants increase in ability, the slope of the threshold is not that steep.

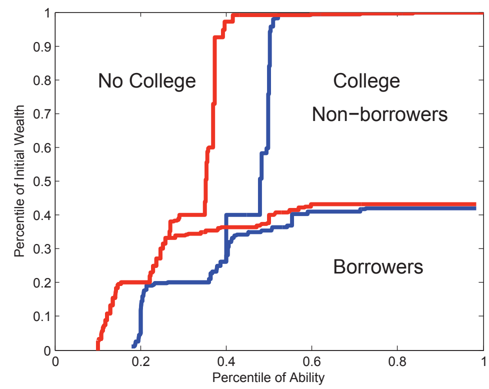
To examine how an individual's college and borrowing decisions change in a reponse to changes in the college wage premium, college costs, maximum borrowing limits, and loan interest rates, I introduce 2011 values of them one at a time. Figure 1.4 illustrates the results. The red line represents changed decision rules of an individual and the blue line is the original one as in Figure 1.3. As shown in Figure 1.4 (a), increases in college expenses lower an individual's lifetime value of attending college, and his college-enrollment thresholds of ability and initial wealth move to the northeast, implying that less individuals decide to go to college. That is, only a few students who are relatively able and rich are willing to pay for increased college costs to acquire college education. Clearly, a college student wants to borrow more in order to finance increased college costs, and the cutoff level of initial wealth to determine whether a college student become a borrower increases. Hence, the rise in college costs decreases college attendance rate, but increases the fraction of borrowers and average borrowing amounts.

Figure 1.4 (b) illustrates the case of an increase in the college wage premium. The rise in the college wage premium increases an individual's lifetime value of going to college, and the college-going thresholds move to the southwest, meaning that more agents decide to attend college. In other words, an individual who is less able and poorer is now willing to pay for college costs in order to acquire increased financial benefits of obtaining a college degree. Moreover, the rise in the college wage premium makes a college student borrow more due to increased consumption smoothing motives. Accordingly, the slope of the cutoff level of initial wealth to determine whether a college student becomes a borrower rises, meaning that a student needs more initial wealth not to become a borrower. Thus, the rise in the college wage premium increases college enrollment rate, the fraction of borrowers, and borrowing amounts.

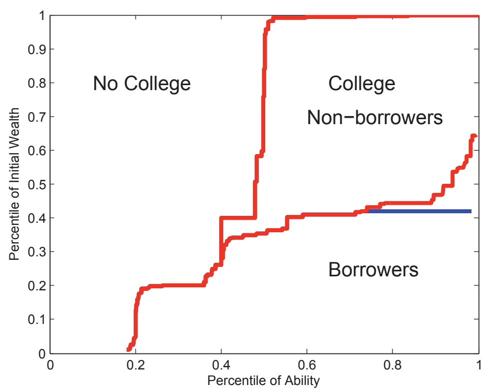
Figure 1.4 (c) shows the case of a rise in maximum borrowing limits. The increase in maximum borrowing limits encourages more individuals to go to college



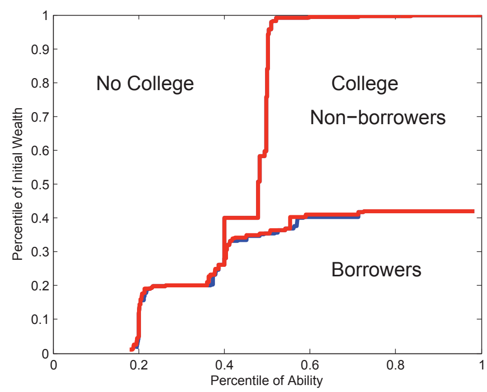
(a) Increase in college costs



(b) Increase in college premium



(c) Increase in borrowing limits



(d) Decrease in student loan rates

Figure 1.4: Changes in agent's decision rules

by relaxing the required level of initial wealth for college attendance. So, the cutoff line of college attendance for low initial wealth is expanded, but quantitatively not that much. Also, because of extended maximum borrowing limits, students from high income family who were not allowed to borrow from the government become getting access to student loans. Hence, the cutoff line to determine whether to borrow or not goes up for students with large initial wealth, implying that more college students become borrowers.

Lastly, Figure 1.4 (d) shows the case of a decrease in interest rates on student loans. Intuitively, the decrease in interest rates makes more college students become borowers, and increases their borrowing amounts. In this sense, the cutoff level of initial wealth to determine whether college students become borrowers goes up, implying that the measure of borrowers increases. However, the quantitative effect is negligible. The drop in loan interest rates may affect college-going decisions by relaxing student's borrowing burden, so that more students decide to go to college. This is represented by the extended cutoff line of college attendance, but its quantitative impacts are small.

1.3 Quantitative Analysis

The model is calibrated to match a set of key observations of federal Stafford loans as shown in Table 1.1 Using the calibrated model, I assess the quantitative contribution of increases in the college wage premium, college costs, and maximum borrowing limits in accounting for the dramatic rise of federal Stafford loans from

1990 to 2011. I also conduct counterfactual experiments to decompose the relative importance of these driving forces.

1.3.1 Calibration

The calibration strategy proceeds in the following two steps. First, using previous literature and data from the College Board, National Longitudinal Surveys Youth in 1979 (NLSY79), and National Postsecondary Student Aid Study in 1990 (NPSAS90), I exogenously assign values to parameters associated with preference, earnings, federal Stafford loan programs, and college costs without solving for the model. Next, I calibrate a joint distribution of ability and initial resources so that equilibrium properties of the model replicate salient features of federal Stafford loans in 1990. The parameter of the college wage premium is also calibrated to match average earnings ratio of college to non-college in 1990. Table 1.2 summarizes exogenously given parameter values, and Table 1.3 shows calibrated parameter values and corresponding target statistics.

Demographics and preference: The time period in the model represents one year. An individual lives 47 model periods, corresponding to a real life age of 19 to 65. The number of years of college education is $S = 4$. The subjective discount factor is set to $\beta = 1/1.05$ to match the real gross interest rate of 1.05. I set $\sigma = 2$, which is common in the literature.

Table 1.2: Exogenous Parameter Values for the 1990 Equilibrium

Parameter	Name	Value	Target/Source
β	Discounter factor	0.95	real avg rate (5%)
σ	Coefficient of risk aversion	2	Browning et.al (1999)
S	Years of schooling	4	college years
J	Life expectancy	46	real life age 19-65
g_{nc}	Work exp. of non-col.	0.025	NLSY79
g_c	Work exp. of col.	0.040	NLSY79
k	Published tuition and fees	$7,921	College Board
d_{max}	Maximum borrowing limit	$12,006	DOE-NCES
f	Full college costs	$14,080	College Board
$G(w_{q1})$	Avg. fed. grants for Q1 family inc.	$2,139	NPSAS90
$G(w_{q2})$	Avg. fed. grants for Q2 family inc.	$898	NPSAS90
$G(w_{q3})$	Avg. fed. grants for Q3 family inc.	$199	NPSAS90
$G(w_{q4})$	Avg. fed. grants for Q4 family inc.	$56	NPSAS90
$G(w_{q5})$	Avg. fed. grants for Q5 family inc.	$34	NPSAS90
$G(a_{q1})$	Avg. Non-fed. grants for Q1 ability	$1,942	NPSAS90
$G(a_{q2})$	Avg. Non-fed. grants for Q2 ability	$2,353	NPSAS90
$G(a_{q3})$	Avg. Non-fed. grants for Q3 ability	$2,759	NPSAS90
$G(a_{q4})$	Avg. Non-fed. grants for Q4 ability	$3,619	NPSAS90
$G(a_{q5})$	Avg. Non-fed. grants for Q5 ability	$5,146	NPSAS90

College costs, grants, and federal Stafford loans: Using the Trends in College Pricing (2013) provided by College Board, I estimate full college costs (f), including room and board, and published college tuition and fees (k) as an enrollment-weighted average for 4-year public and non-for profit private universities in 1990. I also use the NPSAS90 to estimate federal and non-federal grants. Since federal grants, including Pell grants, are typically need-based grants, I estimate their values across different family income quintiles. However, non-federal grants, such as institutional or privat grants are generally merit-based grants, so that I estimate them across different student's ability quintiles measured by SAT or ACT scores. Using the Trends in Undergraduate Stafford Loan Borrowing: 1989-90 to 2007-08 provided by U.S. Department of Education, I set the maximum borrowing limit (d_{max}) imposed by the government to be $12,006 in 2011 constant dollars, which is the average amount between the limit for dependent students and the limit for independent students in Stafford loan programs.

Estimation of earnings function: Using the NLSY79 (1979-2006) data on wage income, education, age, and the Armed Forces Qualifying Test (AFQT) quartile, I directly estimate an individual's earnings function. Similar to Lochner and Monge (2011), the specification for the individual (i) earnings (y_i) by education level (s) is a function of AFQT quartile (q_i) and working experience (x_i). Taking logs, I obtain the following regression equation with measurement error ($\epsilon_{i,s}$):

$$\ln(y_{i,s}) = \ln\left(a_{q_{i,s}}\right) + g_{1,s}x_i + \epsilon_{i,s}$$

The parameters that are of interest are the growth rates of earnings by different education groups ($g_{1,s}$). As shown in the Table 1.2, the earnings growth rate of college agents are 15 percentage points higher than that of non-college agents. Consistent with Gourinchas and Parker (2002), Guvenen (2009), and Ionescu (2009, 2011), there exists a systematic difference in earnings growth rates by education level.

Joint initial distribution of ability and wealth: Assuming that a joint distribution of initial ability and wealth is lognormally distributed, I calibrate a vector of parameters of their means and variances ($\mu_a, \sigma_a, \mu_w, \sigma_w, \rho_{aw}$) so as to reproduce a set

Table 1.3: Calibrated Values and Targets

Parameter	μ_a	σ_a	μ_w	σ_w	ρ_{aw}	p
Value	7.23	0.61	9.45	1.09	0.68	1.45

Target	Data	Model
College enrollment rate	0.60	0.60
Fraction of borrowers	0.27	0.27
Average borrowing per year ($ 2011)	3,831	3,881
College wage premium 25-30	1.35	1.35
College enrollment rate in w_{q1}	0.46	0.45
College enrollment rate in w_{q2}	0.53	0.50
College enrollment rate in w_{q3}	0.65	0.64
College enrollment rate in w_{q4}	0.76	0.81

of key statistics of federal Stafford loans in 1990: college enrollment rate, fraction of borrowers, average borrowing amounts, and college attendance rates across different family income.[5] Each parameter has a direct effect on some data targets. That is, parameter values associated with initial wealth distribution play important roles to match student's borrowing behaviors, and those values with ability distribution have direct effects on college enrollment. Especially, the correlation parameter between ability and initial wealth plays a crucial role to replicate different college enrollment rate across family income. The parameter of the college wage premium is also calibrated to reproduce the average earnings ratio of college to non-college at age 25-30

[5]The fact that there are substantial variations in college enrollments across family income classes may be interpreted as an evidence that credit constraints may preclude low-income youth from attending college. However, previous studies based on the NLSY79 show that the borrowing constraint has little impact on an individual's college attendance (Cameron and Heckman 1998, 2001; Keane and Wolpin 2001; Carneiro and Heckman 2002; Cameron and Taber 2004).

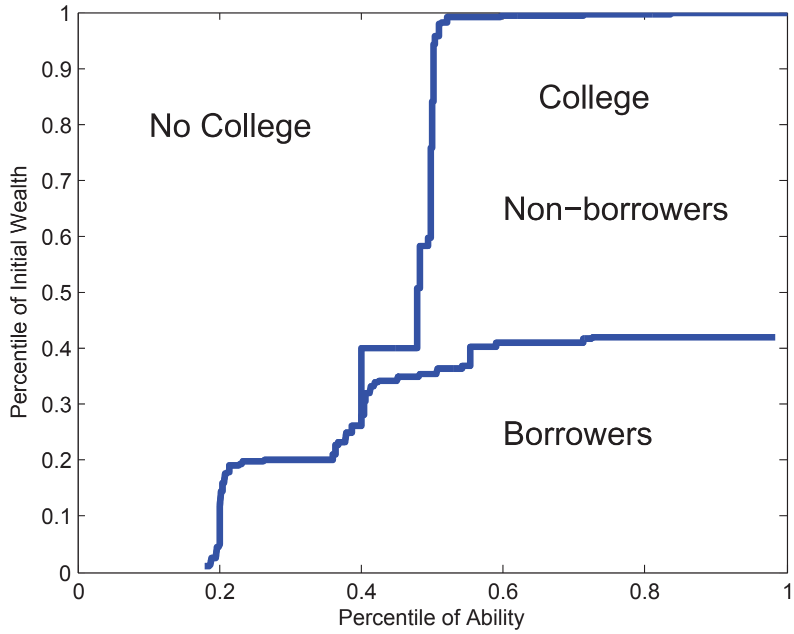


Figure 1.5: Distribution of agents in the calibrated economy

obtained from the Integrated Public Use Microdata Series (IPUMS) in 1990 (1.35). Table 1.3 shows the calibrated parameter values and corresponding target statistics. Overall, the model does a good job in matching a set of targets.

Given the calibrated joint initial distribution of ability and wealth, Figure 1.5 graphically describes cutoff levels of ability and initial wealth for college-attendance and borrowing decisions, and shows a measure of non-college and college students who are borrowers or non-borrowers in ability and initial wealth percentiles space in 1990. There is a college attendance cutoff level of ability above which agents decide to go to college, and the ability threshold level depends on an individual's initial resources. This is because grants vary across different family income quantiles, the threshold ability levels change accordingly. That is, the cutoff level of ability for college enrollments of students from poor family income is lower than that of students from rich family income, because grants for students with low initial wealth are more generous. Overall, 60 percent of total measure of agents decide to go to college. Moreover, there is another cutoff level of ability and initial wealth to determine whether students become borrowers, and the area below the thresholds is the measure of the fraction of borrowers, accounting for 27 percent of total college students. Notice that the cutoff level of initial wealth increases as individuals' ability is higher. This is because more able students want to smooth their consumption over lifetime so that they need more initial wealth not to become borrowers. In addition, college students who are relatively able but poor are more likely to be credit-constrained, accounting for 11 percent of college students in the calibrated model 1990.

1.3.2 Assessing the Model's Behavior

I investigate the benchmark economy along different dimensions that are not explicitly targeted in the parameterization. Specifically, using the NLSY79, I examine how an individual's college enrollment is different across disparate ability levels measured by AFQT, and compare the results to the corresponding model statistics.[6] Even though the data have an earlier cohort of students than the model, it is meaningful to compare the model predictions to the data in order to validate the calibrated model. Furthermore, I use the NPSAS90 to document how an individual's borrowing behaviors vary across different SAT scores and family income, and compare the model

[6] Recall that an individual's college enrollment rates by different family income quantiles are already used in the calibration as targets.

Table 1.4: College Enrollments across Different Ability

	AFQT				
	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Total
NLSY79	22%	45%	69%	88%	55%
Model	8%	35%	100%	100%	60%

predictions to the corresponding empirical counterparts. Moreover, I compare both a model-generated distribution of agents by borrowing amounts to the corresponding empirical data.

1.3.2.1 College Enrollment Rate

As presented in Table 1.4, an individual's ability is an important determinant for college attendance. In the model, an agent's ability determines his financial returns to participating in college, and there exists a cut-off level of ability above which an agent decides to go to college and below which he does not. Contrast to the model predictions, the data shows that there are still 12% of students in the top ability quantile who do not go to college, whereas there are 22% of students in the bottom ability quantile who do. This is partly because the AFQT is not a perfect measure of an agent's ability. Or, the model does not capture preference heterogeneity, such as different psychic costs across students, may play in the data.

Table 1.5: Fraction of Borrowers by Family Income and Ability

Family Income					
Year	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Total
NPSAS90	41%	35%	24%	10%	27%
Model	75%	25%	8%	1%	27%

SAT/ACT					
Year	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Total
NPSAS90	28%	28%	27%	26%	27%
Model	59%	23%	15%	9%	27%

1.3.2.2 Fraction of Borrowers

Table 1.5 displays a comparision of participation rates in federal Stafford loans generated by the model and the corresponding empirical data across different family income and ability. Clearly, consistent with the data, the model predicts that the fraction of borrowers decreases as family income increases. However, there is quantitatively a large difference in the bottom family income quantiles. Compared to the data, the model predicts that too many students in the bottom family income take out the loans. This is because the family income may not be a perfect measure for the initial endownment in the model. It could be that even if students' family income is low, they would not take out loans if they comes from wealthy family. The model also predicts that relatively less able students are more likely to take out federal loans. This is because they have relatively less initial wealth and less non-federal grants. In contrast, the data shows that the fraction of borrowers across different ability levels does not vary much. Maybe the ability in the model is not perfectly consistent with

Table 1.6: Borrowing Amounts by Family Income and Ability

Family Income					
	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Total
NPSAS90	$3,961	3,774	3,701	3,891	3,831
Model	$6,224	4,092	3,769	1,436	3,881
SAT/ACT					
	Quartile 1	Quartile 2	Quartile 3	Quartile 4	Total
NPSAS90	$3,796	3,839	3,737	3,962	3,831
Model	$4,494	3,978	3,645	3,409	3,881

the ability measured by SAT or ACT.

1.3.2.3 Average Borrowing Amounts

Table 1.6 compares the model's predictions on average borrowing amounts by family income and ability to their corresponding data. The model produces the clear negative relation between family income and borrowing amounts, while the data has U-shaped patterns of borrowing amounts as family income rises. In addition, compared to the data, the model quantitatively produces much more variations in borrowing amounts by family income. That is, the model predicts that students from the bottom family income borrow too much relative to those from the top family income, whereas the data does not have a large discrepency in borrowing amounts acorss different family income. The model generates a negative relationship between borrowing amounts and student's ability. This is because students with low ability are more likely to have less initial wealth and less grants. Even though the borrowing

Table 1.7: Distribution of Agents by Total Borrowing Amounts

	Total Borrowing Amounts				
	<$10,000	$10,000-15,000	$15,000-20,000	$20,000-25,000	>$25,000
NPSAS90	52%	20%	12%	8%	7%
Model	57%	31%	11%	1%	0%

amounts in the data do not have clear patterns with student's ability, overall there is not much difference in terms of magnitude.

1.3.2.4 Distribution of Borrowing Amounts

Table 1.7 shows the distribution of agents by borrowing amounts generated in the model and the corresponding empirical counterparts from the NPSAS90. Consistent with the data, the model produces a large proportion of college students borrowing less than 10 thousand dollars (2011 dollars) and relatively small share of students borrowing more than 20 thousand dollars. The model is also quantitatively similar to the distribution of agents in the data, even though the model relatively has more students borrowing less, compared to the data.

1.3.3 Accounting for the Rise of Federal Stafford Loans

I use the calibrated model to quantitatively assess the importance of increases in the college wage premium, college costs, and credit availability and decrease in loan interest rates to account for the significant rise of federal Stafford loans from 1990 to 2011. To do so, I allow for the parameter value of interest rates on student loans

Table 1.8: Changed Parameter Values

Parameter	1990 value	2011 value
$\widetilde{R}$	1.08	1.055
k	\$7,921	\$15,649
f	\$14,080	\$24,776
d_{max}	\$12,006	\$12,746
p	1.44	1.75
$G(w_{q1})$	\$2,139	\$4,700
$G(w_{q2})$	\$898	\$3,910
$G(w_{q3})$	\$199	\$870
$G(w_{q4})$	\$56	\$61
$G(w_{q5})$	\$34	\$20
$G(a_{q1})$	\$1,942	\$4,511
$G(a_{q2})$	\$2,353	\$5,327
$G(a_{q3})$	\$2,759	\$6,194
$G(a_{q4})$	\$3,619	\$6,972
$G(a_{q5})$	\$5,146	\$9,145

($\widetilde{R}$), college tuition and fees (k), total grants (G), full college costs (f), and maximum borrowing limits (d_{max}) to change as in their 2011 values. I also recalibrate the college premium parameter to match the 2011 value of the average earnings ratio of college to non-college at ages 25-30 (1.65) in 2011. Table 1.8 summarizes changed parameter values from 1990 and 2011.

Notice that the interest rates on student loans decrease by 2.5 percentage points for the past 20 years. Both published tuitions and fees (k) and full college costs (f), including living costs, rise a bit less than twice from 1990 to 2011. With the increase in collge costs, total grants also rise. Federal grants for students from less family income relatively increase sharply by more than twice, but those for students from rich family income decrease a bit. Moreover, non-federal grants for all

Table 1.9: Quantitative Results

	1990		2011	
	Data	Model	Data	Model
College enrollment rate (%)	0.60	0.60	0.68	0.63
Fraction of borrowers (%)	0.27	0.27	0.66	0.50
Average borrowing per year ($2011)	3,831	3,881	6,629	6,248
Stafford loans per HS graduate ($2011)	621	629	2,975	1,968

ability quintiles approximately increase twice for the last 20 years. Lastly, the college premium (p) at ages 25-29 also increases by 22 percent from 1990 to 2011.

Table 1.9 shows the model's implications for the level of federal Stafford loans as well as the corresponding empirical counterparts in 1990 and 2011. The model generates an increase of 213 percent from 1990 to 2011 and accounts for 57 percent ($=\frac{1{,}968-629}{2{,}975-621}$) of the actual rise in federal Stafford loans for the last 20 years. In each component aspect, because of the counteracting effects of increases in the college wage premium and college costs, the model produces a modest rise in college enrollment rates, explaining 33 percent ($=\frac{0.63-0.60}{0.68-0.60}$) of the observed rise in college enrollment rates from 1990 to 2011. Meanwhile, the model generates large expansions in the fraction of borrowers and borrowing amounts, accounting for 59 percent ($=\frac{0.50-0.27}{0.66-0.27}$) and 84 percent ($=\frac{6{,}248-3{,}881}{6{,}629-3{,}831}$), respectively, of their observed increases from 1990 to 2011.

Figure 1.6 graphically describes changed cutoff levels of ability and initial wealth for college-attendance and borrowing decisions, and shows how the distribution of agents has been changed from 1990 to 2011. The red line shows the 2011

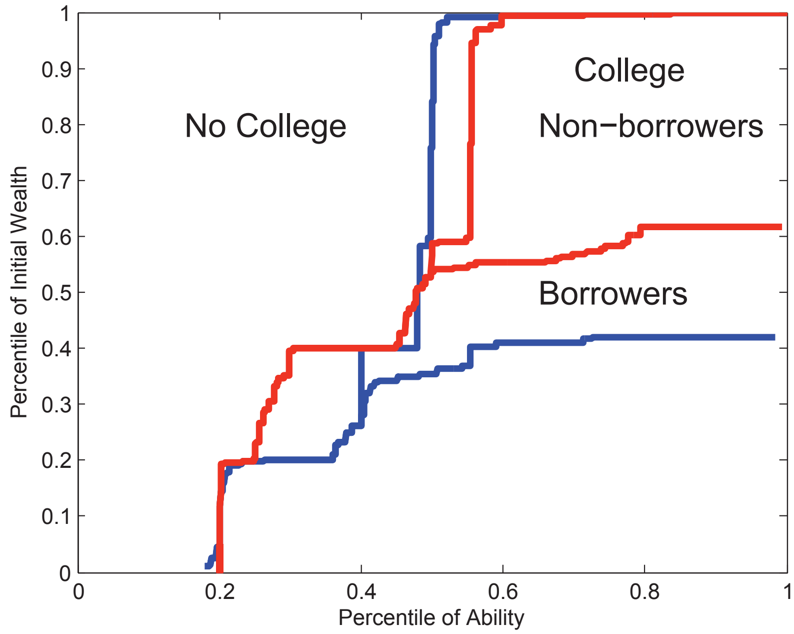


Figure 1.6: Changes in the distribution of agents

distribution of agents, and the black line is the original one in 1990. The changed cutoff level of college enrollment shows that the increase of students with relatively low initial wealth and ability is striking, because of the large increase in grants for those students. However, due to the huge increase in college costs but the small increase in grants for students with modest ability and high initial wealth, they decide not to go to college in 2011, even if overall college wage premium rises. Overall, college attendance rates 2011 are higher than in 1991. The threshold levels of initial wealth to determine whether students become borrowers sharply shifts up, meaning that college students become more borrowers. This dramatic rise mainly comes from the fact that individuals in 2011 need large initial resources to lower their strong desire to borrow caused by increased college costs and consumption smoothing motives.

Table 1.10: Counterfactual Experiments

	Counterfactuals			
	p	k & G(w)	d_{max} & f	$\widetilde{R}$
College enrollment rates	0.75	0.47	0.61	0.60
Fraction of borrowers	0.35	0.44	0.30	0.28
Average borrowing per year ($2011)	4,260	5,896	4,046	3,991
Stafford loans per HS graduate ($2011)	1,118	1,219	752	671

1.3.4 Counterfactuals

I quantitatively decompose the relative importance of the four different driving forces for the significant rise in federal Stafford loans. To do so, I conduct four counterfactual experiments. The first experiment is implemented to examine the role of the increase in the college wage premium; that is, what would have happened if the college wage premium rised from 1990 to 2011, while the other two driving factors remained constant at their 1990 levels? The row labeled "p" in Table 1.10 reports the result of such experiment. Federal Stafford loans would increase by 81 percent, and the college wage premium alone would have accounted for 21 percent $(=\frac{1,118-629}{2,975-621})$ of the observed rise of federal Stafford loans over the last 20 years. Notice that the increased returns to college education considerably boost college attendance rates, consistent with Restuccia and Vandenbroucke (2013) who show a large schooling response to college returns. The fraction of borrowers and average borrowing amounts also rise because of increased consumption smoothing motives, but not quantitatively large.

The second experiment is designed to quantify the effect of the increase in net college costs on the sizable growth of federal Stafford loans; in other words, what would have occured if the college costs increased from 1990 to 2010, while the other two driving factors remained at their 1990 levels? The row labeled "k and G(w)" in Table 1.10 reports the result. College costs alone would have accounted for 25 percent ($=\frac{1{,}219-629}{2{,}975-621}$) of the actual increase of federal Stafford loans from 1990 to 2011. This experiment shows that the rise in college costs is the most important driving force generating rapid expansions in the fraction of borrowers and borrowing levels, even though the increased cost of attending college markedly decreases college enrollment rates.

The third experiment is conducted to investigate the impact of the expansion in credit availability on the sharp rise of federal Stafford loans. That is, what would have happened if the borrowing limit rised from 1990 to 2011, while the other two driving factors remained constant at their 1990 levels? The row labeled "d_{max} & f" in Table 1.10 reports the result of such experiment. The expanded credit availability due to the increase in colleg costs including living expenses and the decline in average parental contributions to students has accounted for 5 percent ($=\frac{752-629}{2{,}975-621}$) of the observed rise of federal Stafford loans for the past 20 years. So, the expansion of credit availability itself has negligible impacts on an individual's college and borrowing decisions, consistent with Keane and Wolpin (2001) and Abbott et.al (2013).

The last experiment is implemented to examine the role of the decrease in interest rates on student loans; that is, what would have happened if the loan interest rates dropped from 1990 to 2011, while the other two driving factors remained constant at their 1990 levels? The row labeled "$\widetilde{R}$" in Table 1.10 reports the result of such experiment. Federal Stafford loans would increase by 7 percent, and the decrease in interest rates on student loans alone would have accounted for 2 percent $(=\frac{671-629}{2{,}975-621})$ of the observed rise of federal Stafford loans over the last 20 years.

In summary, increases in both the college wage premium and college costs are important factors in generating dramatic expansions on federal Stafford loans, especially on the fraction of borrowers and borrowing amounts. Furthermore, the decline in parental contributions to students play an important role for students to take out loans and borrow more. However, the rise of credit availability brought about by more generous eligibility requirements of student loans alone has relatively minimal impacts on an individual's college and borrowing decisions.

1.4 Conclusion

I have constructed a simple life-cycle model of human capital accumulation to quantify the importance of changes in returns to college education, college costs, borrowing limits, and loan interest rates in explaining the rise in federal student loans in the United States for the last 20 years. The model features discrete schooling choices, college loan decisions, and individual heterogeneity in terms of ability and initial resources. I analytically characterize an individual' college and borrowing decisions as a function of the college wage premium, college costs, and borrowing limits, and show how those decisions change in response to increases in the three driving factors. Quantitatively, the model accounts for 57 percent of the observed increase in federal Stafford loans from 1990 to 2011. Increases in the college wage premium and net college costs are important driving factors, explaining 21 percent and 25 percent, respectively, of the actual rise of federal Stafford loans for the last 20 years. Especially, the rise in net college costs plays an important role in generating sharp expansions on fraction of borrowers and borrowing amounts. However, the government's policy changes, such as increase of credit availability and decrease in interest rates on student loanshas relatively minimal impacts on an individual's college and borrowing decisions.

There are several issues that would be worth doing further. First, I have not considered the general equilibrium effects on an individual's college and borrowing decisions. For instance, it would be relevant to allow colleges or universities to optimally set their own prices which may be affected by government financial aids, and study how those prices impact college attendance and borrowing behaviors. Second, it would be interesting to assess the effects of changes in uncertainty on college graduations or idosyncratic shocks on earnings after college and default possibilities on students' borrowing decisions. That is, increased variations in expected returns at the time of college entry may impact the investment value of college and the borrowing level.

CHAPTER 2
TRENDS IN EARNINGS INEQUALITY: A CROSS-COUNTRY ANALYSIS

2.1 Introduction

Earnings inequality has been substantially higher in the US than in European countries for the past 30 years. Table 1 shows cross-country trends in earnings inequality, measured by log earnings differences between the 90th and 10th percentile for male workers (LP90-10) obtained from OECD statistics and Guvenen et al. (2014). According to Table 2.1, average difference in inequality between the US and other European countries in 1978-82 is 0.44, with the gaps further widening in recent years and reaching 0.58 by 2003-07. Why has this been the case? The objective of this paper is to provide a theory accounting for trends in earnings inequality and to assess quantitatively the importance of cross-country differences in tax structure, intergenerational persistence of earnings, returns to education investments, and public education spending. Understanding these driving forces is crucial in setting government policies to alleviate various economic and social issues caused by rising earnings inequality.

I construct an endogenous growth model of intergenerational transmission of human capital to provide a theoretical framework for the dynamics of earnings inequality. Based on Glomm and Ravikumar (1992) and Chen (2005), I develop a two-period overlapping-generation economy in which each individual is heterogeneous in terms of innate ability and parental human capital. A young agent accumulates human capital through public and private education investments; the amount of human

Table 2.1: Log Wage Difference between the 90th and 10th

	1978 - 1982		2003 - 2007	
	average	Δ from US	average	Δ from US
Denmark	0.77	0.52	0.99	0.63
Finland	0.89	0.40	0.94	0.68
Germany	0.93	0.36	1.10	0.52
Netherlands	0.84	0.45	1.05	0.57
Sweden	0.69	0.60	0.87	0.75
UK	0.99	0.30	1.30	0.32
Average	**0.85**	**0.44**	**1.04**	**0.58**
US	1.29	0.00	1.62	0.00

capital acquired determines labor income when old. To understand the mechanism of how the model works, I characterize trends in earnings inequality analytically, which is positively related to intergenerational persistence of earnings and returns to education investments, but negatively related to tax progressivity and the level of public education spending.

Using the US as a benchmark, I calibrate the model to reproduce trends in earnings inequality in the US from 1978-82 to 2003-07. All countries are assumed to share the same parameter values as in the US, except for the level of earnings inequality in 1978-82, tax structure, intergenerational earnings persistence, returns to education investments, and public education spending. That is, some key parameters associated with the dynamics of earnings inequality are country-specific, while others are common across countries. I then assess the quantitative implications of the calibrated model for cross-country differences in earnings inequality over 2003-

07. I find that the model explains 31 percent of the observed differences in earnings inequality between the US and European countries. I also conduct several counterfactual experiments to decompose the relative importance of these driving forces, and find that differences in returns to education investments and intergenerational persistence of earnings are the most crucial determinants, followed by differences in initial inequality, tax structures and public education expenditures.

This paper contributes to the literature on education, human capital, and earnings inequality by analyzing cross-country differences in earnings inequality through the lens of a growth model of human capital accumulation. To my knowledge, most of the previous earnings inequality literature concentrates on accounting for rising earnings inequality in the US quantitatively (Autor et al. 2008, Guvenen and Kuruscu 2010, Heckman et al. 1998, Kambourov and Manovskii 2009, and Krusell et al. 2000), while little work has been done in a cross-country setting. However, in recent years, a few of papers has started to analyze earnings inequality across countries. Herrington (2014) examines the role of differences in labor income tax and public education expenditures across school districts in accounting for differences in earnings inequality between the US and Norway. Guvenen et al. (2014) attempt to explain differences in wage inequality between the US and continental European countries, emphasizing the role of differences in progressive labor income tax policy. In contrast with previous papers, this paper focuses on the transition of earnings inequality, rather than a steady-state equilibrium, and provides a transparent mechanism describing how each of the key determinants affects the dynamics of earnings inequailty in an analytically tractable equilibrium model.

My work is also related to previous papers that analyze the effects of taxation and education spending on income inequality and intergenerational persistence quantitatively, including Restuccia and Urrutia (2004) and Seshadri and Yuki (2004). Moreover, there are a few of papers that study an intergenerational earnings persistence in a cross-country setting, such as Bjorklund and Jantti (1997), Checchi et al. (1999), and Holter (2014). In recent years, Cordoba and Ripoll (2013) and Restuccia and Vandenbroucke (2014) have tried to account for cross-country dispersions of average schooling years quantitatively.

Investigating human capital accumulation through education investments also connects this paper to a strand of literature on growth and income inequality. In terms of modeling, my paper is closely related to Glomm and Ravikumar (1992) and Chen (2005), who examine the role of private versus public education systems for economic growth and income inequailty. Similarly, Glomm and Ravikumar (2003) explore the evolution of income inequality in an education regime in which the quality of schools depends on publicly-provided funding via income taxes. Those papers do not conduct quantitative exercises using real data, however, which sharply constrasts to my paper. In addition, several papers, including Benabou (2002), Arcalean and Schiopu (2010), and Blankenau and Simpson (2004), explore the relationship between growth and education spending in an endogenous growth model of human capital formation, studying optimal income taxations and education subsidies.

The rest of this paper is organized as follows. Section 2.2 describes the model and Section 2.3 derives the dynamics of earnings inequality analytically. Section 2.4 sets the model parameters, Section 2.5 presents quantitative results, and Section 2.6 concludes.

2.2 Model

I develop an endogenous growth model of intergenerational human capital transmission. The economy is populated by overlapping generations of individuals living for two periods. Each individual is ex-ante heterogeneous in innate ability and parental human capital. A young individual accumulates human capital by making education investments, and the human capital acquired when young determines labor earnings when old. The measure of each generation is normalized to one, and there is no population growth over time.

2.2.1 Individual's problem

In the first period, a young agent born at time t draws his innate ability (z_t) and parental human capital (h_t). As in Chen (2005), I assume that the young agent is born with an endowment which is a constant fraction (η) of parental after-tax earnings, and he decides how much to consume, save, and invest in human capital. In the second period, he earns labor income based on the amount of human capital acquired when young, and has a single child to form a household unit. At the end of the second period, the old agent dies and his child becomes a parent. Given a real wage rate per unit of human capital (w) and a gross real interest rate (R), the maximization problem of a young agent in cohort t is:

$$\max_{c_{i,t},e_{i,t},s_{i,t+1},c_{i,t+1}} \quad \ln(c_{i,t}) + \beta \ln(c_{i,t+1}) \tag{2.1}$$

$$s.t \quad c_{i,t} + s_{i,t+1} + e_{i,t} = \eta \tilde{y}_{i,t}, \tag{2.2}$$

$$c_{i,t+1} = (1-\eta)\, \tilde{y}_{i,t+1} + R s_{i,t+1}, \tag{2.3}$$

$$\tilde{y}_{i,t+1} = \lambda \left(w h_{i,t+1}\right)^{1-\tau}, \tag{2.4}$$

$$h_{i,t+1} = z_{i,t} \left[E_t^{\rho} + e_{i,t}^{\rho}\right]^{\frac{\gamma}{\rho}} h_{i,t}^{\delta} H_t^{1-\gamma-\delta}, \tag{2.5}$$

$$c_{i,t} > 0 \quad , \quad e_{i,t} \geq 0. \tag{2.6}$$

where $\eta, \gamma, \delta, 1-\gamma-\delta \in (0,1)$, $\gamma+\delta < 1$, and $\rho \in (-\infty, 1]$. β is a discount factor, $c_{i,t}$ and $c_{i,t+1}$ are consumptions when young and old, respectively, and $s_{i,t+1}$ represents savings. Additionally, ρ is the degree of elasticity of substitution between public and private education services, E_t represents a level of public education spending, and H_t is a measure of aggregate human capital.

2.2.2 Human capital

To accumulate human capital, the young agent uses public education (E_t), which is a constant fraction (d) of aggregate income (wH_t), and private education (e_t) to complement publicly-provided education services. Public and private inputs combine with innate ability, parental human capital, and the average level of human capital to produce a new stock of human capital used for future earnings. I assume that the human capital accumulation function in equation (2.5) has a constant returns to scale (CRS) functional form.[1] Parameters γ and δ govern the earnings elasticity of education investments and of parental human capital, respectively. In addition, I assume that public and private education expenditures are aggregated into total education investments through a constant elasticity of substitution (CES) production technology with an elasticity parameter (ρ), as in Bearse et al. (2005), Arcalean and Schiopu (2010), and Cordoba and Ripoll (2013).

Note that public and private inputs are imperfect substitutes in creating human capital. One interpretation of this is that most public education expenditures are used for primary and secondary education, which provide general skills. Private spending mainly finances college education or on-the-job training, and so provides more specific skills. The combination of general and specific skills acquired in primary, secondary, and tertiary education contributes to human capital production for future generations.

Following Epple and Romano (1998), Benabou (2002), and Chen (2005), I assume the time-invariant distribution of innate ability to be log-normal (Γ_z) with mean μ_z and variance σ_z^2. I also assume that the initial human capital (Γ_{h1}) is log-normally distributed with mean μ_{h1} and variance σ_{h1}^2. Moreover, a child's innate ability is assumed to be uncorrelated with parental ability and human capital.[2]

[1] The CRS production function of human capital is commonly used in the previous literature, as in Lucas (1988), Glomm and Ravikumar (1992), Glomm (1997), de la Croix and Doepke (2003), and Chen (2005).

[2] According to the empirical literature, including Sacerdote (2002) and Plug and Vijverberg (2003), there exists a positive relationship between a child's ability and his parent's ability and human capital. As such, I take a more conservative approach in terms of generating earnings inequality.

2.2.3 Progressive taxation

Following Benabou (2002) and Heathcote et al. (2014), I assume that an individual faces the following tax function, mapping from gross labor earnings (y_t) to disposable earnings ($\tilde{y}_t$):

$$\tilde{y}_t = \lambda y_t^{1-\tau}. \tag{2.7}$$

The parameter λ shifts the tax function and determines the average level of taxation, while the parameter τ defines the degree of tax progressivity and is the crucial object of interest. To see why τ captures the degree of progressivity of tax systems, we need to examine the ratio of marginal to average tax rates. In general, a tax schedule is considered to be progressive (regressive) if the ratio of marginal to average tax rates is greater (smaller) than one for all earnings levels. We define $T(y_t)$ as the tax paid by an agent with earnings y_t as follows:

$$T(y_t) = y_t - \lambda y_t^{1-\tau}. \tag{2.8}$$

The ratio is therefore $\frac{T'(y_t)}{T(y_t)/y_t} = \frac{1-\lambda(1-\tau)y_t^{-\tau}}{1-\lambda y_t^{-\tau}}$. Hence, when $\tau = 0$, the ratio is one, implying a flat tax rate of $1-\lambda$. $\tau > 0$ makes the ratio larger than one and the tax system consequently becomes progressive, and $\tau < 0$ yields a regressive tax system.

2.2.4 Government

The government spends a share d of aggregate income on public education expenditures. Additionally, a share g of aggregate income is spent unproductively. All government spendings are financed through taxes on labor income. Revenues and expenditures are balanced in every period:

$$\int_0^1 \left(y_i - \lambda y_i^{1-\tau}\right) di = (g+d) \int_0^1 y_i di. \tag{2.9}$$

2.2.5 Equilibrium

Given the initial distribution of human capital (Γ_{h1}), tax policies (τ, λ, g, d), and prices (w,R), the equilibrium consists of a set of sequences of aggregate quantity $\{H_t\}$, human capital distribution $\{\Gamma_{h_t}\}$, and decision rules $\{c_{i,t}, e_{i,t}, c_{i,t+1}, s_{i,t+1}, h_{i,t+1}\}$ such that:

1. The decision rules of agents $\{c_{i,t}, e_{i,t}, c_{i,t+1}, s_{i,t+1}, h_{i,t+1}\}$ solve for the agent's utility maximization probem (2.1) subject to (2.2)-(2.6).
2. Given Γ_{h_t}, the distribution of human capital at time t+1 $\left\{\Gamma_{h_{t+1}}\right\}$ evolves according to

$$h_{i,t+1} = z_{i,t}\left[E_t^{\rho} + e_{i,t}^{\rho}\right]^{\frac{\gamma}{\rho}} h_{i,t}^{\delta} H_t^{1-\gamma-\delta}.$$

3. The government budget constraint in (2.9) is balanced in each period.

2.3 Analytical Results

In this section, I characterize the dynamics of earnings inequality analytically. To do so, I focus on two special cases of aggregations of private and public education services; one is a Cobb-Douglas specification, i.e., $\rho = 0$, and the other case is a perfect substitute, i.e., $\rho = 1$.[3]

[3]Although previous papers have not yet reached a consensus on the value of elasticity of substitution between public and private education spending, many papers, including Bearse

2.3.1 Analysis for the Case of $\rho = 0$

I first examine the Cobb-Douglas specification case. The first order conditions of the agent's problem show that the optimal level of private education investment is:

$$e_{i,t} = \left[\frac{(1-\eta)\gamma(1-\tau)\lambda\left(wz_{i,t}E_t^{\gamma}h_{i,t}^{\delta}H_t^{1-\gamma-\delta}\right)^{1-\tau}}{R}\right]^{\frac{1}{1-\gamma(1-\tau)}}. \tag{2.10}$$

Because public education expenditures are simply a constant fraction (d) of aggregate income (wH_t), the optimal level of private education investments can be rewritten as follows:

$$e_{i,t} = \left[\frac{(1-\eta)\gamma(1-\tau)\lambda\left(wz_{i,t}\,(dw)^{\gamma}\,h_{i,t}^{\delta}H_t^{1-\delta}\right)^{1-\tau}}{R}\right]^{\frac{1}{1-\gamma(1-\tau)}}. \tag{2.11}$$

The optimal level of investment in private education reflects the well-documented fact that individuals invest more in human capital when they have higher innate ability or their parents have larger human capital stock. By substituting the optimal level of private education spending in equation (2.11) into the human capital production function, an agent's next-period stock of human capital is:

$$h_{i,t+1} = \left[z_{i,t}\,(dw)^{\gamma}\left(\frac{(1-\eta)\gamma(1-\tau)\lambda\left(w\,(dw)^{\gamma}\right)^{1-\tau}}{R}\right)^{\gamma}h_{i,t}^{\delta}H_t^{1-\gamma-\delta}\right]^{\frac{1}{1-\gamma(1-\tau)}}. \tag{2.12}$$

This implies that an individual's level of human capital becomes greater as his innate ability or parental human capital increases. Notice, however, that a high tax progressivity discourages an agent from accumulating his human capital, resulting in a low human capital stock.

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et.al (2005), Arcalean and Schiopu (2010), and Cordoba and Ripoll (2013), have concluded that it ranges from 0 to 1. I discuss the two extreme cases for $\rho = 0$ and 1 in this section, and address the analysis for the remaining cases in the quantitative section.

Taking logarithms on both sides of (2.12), I rewrite it in additive form,

$$\begin{aligned} \ln h_{i,t+1} &= C_1 + \left(\frac{1}{1-\gamma(1-\tau)}\right)\ln z_{i,t} + \left(\frac{\delta}{1-\gamma(1-\tau)}\right)\ln h_{i,t} \\ &+ \left(\frac{1-\gamma-\delta}{1-\gamma(1-\tau)}\right)\ln H_t, \end{aligned} \tag{2.13}$$

where $C_1 = \left(\frac{\gamma}{1-\gamma(1-\tau)}\right)\ln\left((dw)\left(\frac{(1-\eta)\gamma(1-\tau)\lambda(w(dw)^{\gamma})^{1-\tau}}{R}\right)\right)$. Note that the intergenerational persistence of human capital is negatively correlated with tax progressivity (τ), but positively with both earnings elasticity of education investments (γ) and parental human capital (δ). Moreover, the distribution of human capital at any point in time is log-normal.[4] Hence, given the lognormal distribution of human capital at time t with mean μ_t and variance σ_t^2, the average human capital at time t (H_t) is $\exp\left(\mu_t + \frac{\sigma_t^2}{2}\right)$, and the distribution of next period human capital is also lognormal with mean μ_{t+1} and variance σ_{t+1}^2, where

$$\begin{aligned} \mu_{t+1} &= C_1 + \left(\frac{1}{1-\gamma(1-\tau)}\right)\mu_z + \left(\frac{1-\gamma}{1-\gamma(1-\tau)}\right)\mu_t \\ &+ \left(\frac{1-\gamma-\delta}{1-\gamma(1-\tau)}\right)\frac{\sigma_t^2}{2} \end{aligned} \tag{2.14}$$

$$\sigma_{t+1}^2 = \left(\frac{1}{1-\gamma(1-\tau)}\right)^2\sigma_z^2 + \left(\frac{\delta}{1-\gamma(1-\tau)}\right)^2\sigma_t^2. \tag{2.15}$$

Observe that I used the assumption that the realization of innate ability is independent of parental human capital in (2.15).[5] The dynamics of the variance of human capital distribution, the key object of interest, can be interpreted as the trends in earnings inequality. Given the variance of ability distribution (σ_z^2) and level of earnings

[4]This is because both innate ability and initial human capital are assumed to be lognormally distributed, and the sum of the two normal distributions is also a normal distribution.

[5]If there exists a positive correlation between a child's innate ability and parental human capital, a covariance term should be included in the dynamics of the variance of human capital distribution.

inequality at period t (σ_t^2), the higher the intergenerational persistence of earnings (δ) or the returns to education investments (γ), the higher the level of earnings inequality for the next period. That is, as family background becomes more important, the rich get richer and the poor get poorer; social mobility becomes lower, resulting in higher earnings inequality. Similarly, increased earnings elasticity of education investments leads to more unequal human capital investments, contributing to higher earnings inequality. In contrast, the more progressive the tax system (τ), the lower the level of earnings inequality in the next period. The interpretation is straightforward. A highly progressive tax system discourages agents from investing in human capital; in particular, its effects are much more pronounced for agents from high-income backgrounds. With higher public education expenditures, furthermore, agents do not need to make additional private education investments. This makes the distribution of human capital less unequal, consequently resulting in lower earnings inequality.

In the long-run, earnings inequality in this economy converges or diverges, depending on the magnitude of (2.15). That is, under the condition $\left|\frac{\delta}{1-\gamma(1-\tau)}\right| < 1$, earnings inequality converges to the steady-state level

$$\sigma^2 = \frac{\sigma_z^2}{\left[1-\gamma\left(1-\tau\right)\right]^2 - \delta^2}. \tag{2.16}$$

2.3.2 Analysis for the Case of $\rho = 1$

Consider the case of perfect elasticity of substitution between public and private education inputs. From the first order conditions of the individual's problem, the optimal level of private education spending is:

$$e_{i,t} = \left[\frac{(1-\eta)\gamma(1-\tau)\lambda \left(w z_{i,t} h_{i,t}^{\delta} H_t^{1-\delta-\gamma} \right)^{1-\tau}}{R} \right]^{\frac{1}{1-\gamma(1-\tau)}} - E_t. \tag{2.17}$$

Notice that private education spending ($e_{i,t}$) should not be negative, due to constraint (2.6) in the agent's problem. So, private education investment can be positive or zero, depending on the parameter values. Obviously, as ability ($z_{i,t}$) or parental human capital ($h_{i,t}$) increases, returns to education investment get larger, and private education investments therefore increase. As public education spending (E_t) becomes larger, however, the agent invests less in private education.

For this analysis, I consider two extreme cases for tractability; one is the case in which all individuals make positive private education investments to supplement public education expenditures (mixed public-private education regime), and the other is that they use only public education services (pure public education regime).[6]

2.3.2.1 Case 1: Mixed public-private education regime

In this case, the level of public education spending is low enough that all individuals must supplement publicly-provided education services by making private education investments. Using the optimal level of private education investments in (2.17), I obtain the following agent's stock of human capital for the next period:

$$h_{i,t+1} = \left[z_{i,t} \left(\frac{(1-\eta)\gamma(1-\tau)\lambda w^{1-\tau}}{R} \right)^{\gamma} h_{i,t}^{\delta} H_t^{1-\gamma-\delta} \right]^{\frac{1}{1-\gamma(1-\tau)}}. \tag{2.18}$$

[6]The reality must be somewhere in between; in other words, some people use both private and public education expenditures and others use only public education spending.

To derive the trends in earnings inequality, I take logarithms on both sides of (18), and rewrite it as follows,

$$\begin{aligned}\ln h_{i,t+1} &= C_2 + \left(\frac{1}{1-\gamma(1-\tau)}\right)\ln z_{i,t} \\ &+ \left(\frac{\delta}{1-\gamma(1-\tau)}\right)\ln h_{i,t} + \left(\frac{1-\gamma-\delta}{1-\gamma(1-\tau)}\right)\ln H_t, \qquad (2.19)\end{aligned}$$

where $C_2 = \left(\frac{\gamma}{1-\gamma(1-\tau)}\right)\ln\left(\frac{(1-\eta)\gamma(1-\tau)\lambda w^{1-\tau}}{R}\right)$. Then, given the lognormal distribution of human capital at time t with mean μ_t and variance σ_t^2, the human capital distribution for next period is also lognormal with mean μ_{t+1} and variance σ_{t+1}^2, given by:

$$\begin{aligned}\mu_{t+1} &= C_2 + \left(\frac{1}{1-\gamma(1-\tau)}\right)\mu_z + \left(\frac{1-\gamma}{1-\gamma(1-\tau)}\right)\mu_t \\ &+ \left(\frac{1-\delta-\gamma}{1-\gamma(1-\tau)}\right)\frac{\sigma_t^2}{2} \qquad (2.20) \\ \sigma_{t+1}^2 &= \left(\frac{1}{1-\gamma(1-\tau)}\right)^2\sigma_z^2 + \left(\frac{\delta}{1-\gamma(1-\tau)}\right)^2\sigma_t^2. \qquad (2.21)\end{aligned}$$

Notice that the dynamics of earnings inequality in (2.21) are similar to that in (2.15) for the Cobb-Douglas specification case. That is, given the variance of ability distribution (σ_z^2) and level of earnings inequality at period t (σ_t^2), earnings inequality rises as the intergenerational earnings persistence (δ) or the return to education expenditures (γ) increases, while it declines as tax systems are more progressive (τ).

Similarly, under the stationarity condition, i.e., $\left|\frac{\delta}{1-\gamma(1-\tau)}\right| < 1$, earnings inequality eventually converges to the steady-state level defined in (2.16).

2.3.3 Case 2: Pure public education regime

In the case where no agents make any private education investments due to large public education spending, their total level of education investments is given by the level of public education spending, i.e., $E_t = dwH_t$. Here, the level of human capital for next period is:

$$h_{i,t+1} = z_{i,t} (dw)^{\gamma} h_{i,t}^{\delta} H_t^{1-\delta}. \tag{2.22}$$

Taking logarithms on both sides of (2.22), I rewrite it as follows:

$$\ln h_{i,t+1} = \gamma \ln dw + \ln z_{i,t} + \delta \ln h_{i,t} + (1 - \delta) \ln H_t. \tag{2.23}$$

Because a child's innate ability is assumed to be uncorrelated with parental human capital, given the lognormal distribution of human capital at time t with mean μ_t and variance σ_t^2, human capital for the next period is also lognormally distributed with mean μ_{t+1} and variance σ_{t+1}^2,

$$\mu_{t+1} = \gamma \ln dw + \mu_z + \mu_t + (1 - \delta) \left(\frac{\sigma_t^2}{2} \right), \tag{2.24}$$

$$\sigma_{t+1}^2 = \sigma_z^2 + \delta^2 \sigma_t^2. \tag{2.25}$$

Observe that the dynamics of earnings inequality under a pure public education regime are no longer dependent on returns to education expenditures (γ) or tax progressivity (τ). More importantly, the rate at which earnings inequality increases under pure a public education regime is lower than that under the mixed public-private education regime analyzed in the first case. In this sense, as the level of public education spending (E_t) increases, fewer individuals supplement publicly-provided education services, thus decreasing private education expenditures. The cross-sectional dispersion of human capital declines, therefore, leading to lower earnings inequality.

Notice that as the intergenerational persistence of earnings (δ) is less than one, the steady-state earnings inequality is given by:

$$\sigma^2 = \frac{\sigma_z^2}{1-\delta^2}. \tag{2.26}$$

The critical issue is now the degree to which tax structures, intergenerational persistence of earnings, returns to private education investments, and public education spending vary across countries. In the following section, I document empirical evidence on cross-country variations in these factors, and investigate how quantitatively important they are in explaining differences in earnings inequality across countries.

2.4 Calibration

This section presents my calibration strategy for the parameter values of the model. I first calibrate the model economy to reproduce the country-specific level of earnings inequality in 1978-82 by pinning down the variance of initial human capital in each country. Then, I use the US economy as a benchmark, and appropriately choose the variance of ability distribution to replicate trends in earnings inequality in the US from 1978-82 to 2003-07.[7] Moreover, I assume that the other countries have the same parameter values as the US, including preferences and calibrated ability distribution. In addition to different levels of initial earnings inequality across countries, however,

[7] In Section 2.5.3, I analyze the robustness of this choice by considering Finland which has experienced the smallest increase in earnings inequality to be a benchmark economy. It turns out that the explanatory power of the model is robust to whichever country is chosen as a baseline.

some key parameters associated with the evolution of earnings inequality are also country-specific: tax structures, intergenerational earnings persistence, returns to education investments, and public education spending.

2.4.1 Common Parameters across Countries

Table 2.2 shows common parameter values. One period of the model is 25 years. I assume the discount factor (β) to be $(0.96)^{25} = 0.38$, which corresponds to a real interest rate (R) equal to $(1.04)^{25} = 2.67$. Since the wage rate is a scale factor in this economy, it is normalized to 1. Following Laitner and Ohlsson (2001), I set the parameter capturing the fraction of inherited income (η) to 0.11.[8]

The previous literature has not yet reached a consensus on the degree of elasticity of substitution (ρ) between public and private education services. Arcalean and Schiopu (2010) consider a two-stage education framework (K-12 and college education), and calibrate the value of elasticity of substitution in each education stage. I take an average of their estimated values of substitutability (0.50) as a benchmark case. Moreover, I conduct a sensitivity analysis for this choice by considering alternative values for elasticity of substitution.

I calibrate the mean (μ_{h1}) of the initial distribution of human capital to match the US annual real earnings in 1980 ($18,516). Following Chen (2005), I choose the

[8]Using the Panel Study of Income Dynamics data, Laitner and Ohlsson (2001) estimate the average ratio of an individual's inherited income to his own income, which is approximately 6.49 percent. Consistently with Chen (2005), considering the average annual growth rate of per capita output for the US in the past 30 years (2 percent), I calculate the value of η (0.11). That is, I can solve for η such that $\frac{\eta w H_t}{w H_{t+1}} = 0.065$, where $\frac{H_{t+1}}{H_t} = (1.02)^{25}$.

Table 2.2: Common Parameters across Countries

Exogenous Parameters		
Parameter	Value	Source/Data
β	0.38	Annual interest rate (4%)
η	0.11	Laitner and Ohlsson (2001)
ρ	0.50	Arcalean and Schiopu (2010)
Calibrated Paramters		
Parameter	Value	Target
μ_{h1}	7.48	US median earnings in 1980
μ_z	0.66	US annual income growth rate

mean of the ability distribution (μ_z) such that the growth rate of income along the balanced growth path is 2 percent annually, which is the average annual growth rate of per capita output in the US over the last 30 years. Because the US is the benchmark economy, I calibrate the standard deviation of the ability distribution (σ_z) to replicate the earnings inequality of LP90/P10 for the US in 2003-07 (1.62) so that I reproduce the trends in US earnings inequality perfectly. I also assume that this ability distribution is shared across countries.

2.4.2 Country-specific Parameters

Each country starts at a different level of earnings inequality in 1978-82, and differs in tax structure, intergenerational persistence of earnings, returns to education investments, and public education spending.

Table 2.3: Standard Deviation of Human Capital Distribution

Denmark	Finland	Germany	Netherlands	Sweden	UK	US
0.30	0.35	0.37	0.33	0.27	0.39	0.50

2.4.2.1 Initial level of earning inequality

I calibrate the standard deviation of the initial human capital distribution (σ_{h1}) to match the country-specific earnings inequality of LP90/P10 in 1978-1982. Table 2.3 shows the calibrated values of the standard deviation across countries. Notice that the standard deviation in the US is larger than that in any other country; this replicates the relatively high level of earnings inequality.

2.4.2.2 Tax structure

I use country-specific tax schedules estimated by Guvenen et al. (2014) to extract a mapping between 2003 gross (y) and disposable ($\tilde{y}$) earnings.[9] Using their mapping from before-tax to after-tax earnings in each country, I estimate the country-specific tax progressivity (τ) by least squares as follows:

$$\log(\tilde{y}_i) = \log\lambda + (1-\tau)\log y_i. \tag{2.27}$$

[9]Using estimates of the gross labor income tax for every income level obtained from the OECD tax database, Guvenen et.al (2014) fit the following tax function to the available data points:

$$\tau(y/AW) = a_0 + a_1(y/AW) + a_2(y/AW)^{\phi},$$

where AW stands for average wage earnings, and a_0, a_1, a_2, and ϕ are country-specific parameter values. Refer to Section 3 of Guvenen et al. (2014) for more details on how they derive country-specific tax schedules.

Table 2.4: Tax Structure across Countries in 2003

	Progressivity	Average Tax	Tax Revenues % GDP
Denmark	0.24	6.24	48.02
Finland	0.21	4.79	44.11
Germany	0.24	7.39	35.78
Netherlands	0.15	2.95	36.86
Sweden	0.17	3.01	47.77
UK	0.14	2.68	34.44
US	0.11	2.28	25.47

Given the estimated cross-country tax progressivity (τ), I determine the country-specific average level of taxation (λ) using the government budget constraint,

$$\int_0^1 \left(y_i - \lambda y_i^{1-\tau}\right) di = (g+d) \int_0^1 y_i di = \mu \int_0^1 y_i di, \tag{2.28}$$

where μ is country-specific 2003 tax revenue as a percentage of GDP obtained from the OECD statistics. Table 2.4 shows the estimated tax progressivity (τ), average level of taxation (λ), and share of tax revenues relative to GDP (μ) across countries; consistent with Guvenen et al. (2014), I find that the tax systems in Denmark and Germany are the most progressive, while the US has the least progressive tax structures. These cross-country differences in tax progressivity have great effects on individual incentives to accumulate human capital, and consequently on pre-tax earnings distribution.

2.4.2.3 Intergenerational persistence of earnings

I calibrate the parameter for the intergenerational persistence of earnings (δ) to reproduce empirical values of intergenerational earnings elasticity. This is measured

Table 2.5: Intergenerational Persistence of Earnings across Countries

	Denmark	Finland	Germany	Netherlands	Sweden	UK	US
Data	0.15	0.18	0.31	0.22	0.27	0.50	0.47

as the slope coefficient obtained by regressing log earnings of children against log earnings of parents. Fortunately, Corak (2006) and D'Addio (2007) provide cross-country comparisons of intergenerational earnings elasticity, and show that there exist substantial differences across countries. Table 2.5 summarizes these values. Compared to other countries, the US and UK have relatively high values for intergenerational persistence of earnings, implying that parental earnings have relatively large effects on their children's future earnings.

2.4.2.4 Returns to educational investments

Following Cordoda and Ripoll (2013), I calibrate the earnings elasticity of educational investments (γ) so that the model replicates private education spending as a proportion of total education spending, obtained from the OECD Education Statistics.[10] Table 2.6 presents the country-specific empirical target and its corresponding calibrated value. Observe that there exist considerable cross-country differences in private education expenditures as the share of total education expenditures. For instance, almost all education services in Denmark, Finland, and Sweden are publicly provided, while private education spending in the US is very large. These substantial

[10]Because of data limitations, I cannot access to data originating before 1990, and use the average value from 1990 to 2000 as a target.

Table 2.6: Returns to Education Investments: Targets and Calibrated Values

Country	Target	Calibrated value
	Ratio of private edu. to total edu.	Returns to education inv.
Denmark	0.01	0.03
Finland	0.01	0.02
Germany	0.19	0.13
Netherlands	0.07	0.06
Sweden	0.02	0.02
UK	0.16	0.09
US	0.32	0.16

differences make the calibrated values of elasticity of educational expenditures vary a lot across countries

2.4.2.5 Public education spending

The parameter of public education expenditures as a share of GDP (d) is chosen from World Bank Data directly, which is summarized in Table 2.7.[11] As expected, Denmark and Sweden have relatively high levels of public education spending, but overall cross-country differences are small.[12]

[11] To be consistent with time periods reflected by the relative fraction of public education spending in Table 2.6, I calculate an average value of public education spending for 1990-2000.

[12] Although average public education expenditures per student are similar across countries, public expenditures across school districts may be quite different. For instance, according to Herrington (2014), there is twice as much variation in public education spending per student (relative to the average spending per student) across districts in the US as in Norway, implying that the variance of public education spending across districts is much larger in the US than Norway. In this sense, setting the quality of public education to be dependent on parental income would be an interesting extension, in that it captures the impact of dispersions in public education spending on explaining cross-country differences in earnings inequality.

Table 2.7: Public Education Spending % GDP

Denmark	Finland	Germany	Netherlands	Sweden	UK	US
7.08	5.76	4.55	5.57	6.62	4.88	5.00

2.5 Quantitative Results

This section investigates the quantitative implications of the calibrated model for cross-country differences in earnings inequality. I proceed to decompose the separate effects of tax structures, intergenerational persistence of earnings, returns to education investments, and public education spending on cross-country differences in earnings inequality to identify the relative importance of those determinants. Furthermore, I conduct a sensitivity analysis for the choice of benchmark economy and the elasticity of substitution between public and private education investments.

2.5.1 Model's fit

Table 2.8 shows key quantitative results for cross-country differences in earnings inequality. The second and third columns are the observed cross-country levels of earnings inequality and differences from the US in 2003-07, respectively. Earnings inequality for Sweden, say, is 0.75 (i.e. 75 log points) lower than the US, showing that Sweden has the largest gap from the US. On average, earnings inequality in other developed countries is 0.58 lower than that of the US. The fourth and fifth columns present the corresponding values generated by the calibrated model, and the sixth column gives the explanatory power of the model. For example, the model-simulated

Table 2.8: Earnings Inequality of LP90-10 in 2003-07

	Data		Model		Explained
	Level (a)	Δ from US (b)	Level (c)	Δ from US (d)	(d)/(b) (e)
Denmark	0.99	0.63	1.40	0.22	0.35
Finland	0.94	0.68	1.41	0.21	0.31
Germany	1.10	0.52	1.49	0.13	0.25
Netherlands	1.05	0.57	1.43	0.19	0.33
Sweden	0.87	0.75	1.40	0.22	0.29
UK	1.30	0.32	1.52	0.10	0.31
Average	**1.04**	**0.58**	**1.44**	**0.18**	**0.31**
US	1.62	0.00	1.62	0.00	-

inequality for Sweden is 0.22 below from the US; the model therefore accounts for 29 percent of the observed difference in earnings inequality between Sweden and the US in 2003-07. Overall, the model produces a 18 log percentage point gap from the US, explaining 31 percent of the observed inequality differences between the US and other developed countries.

2.5.2 Counterfactuals

I examine quantitatively the relative importance of initial inequality level, tax structure, intergenerational persistence of earnings, returns to education investments, and public education spending in accounting for cross-country variations in earnings inequality. I implement several counterfactual experiments by holding one factor to the country-specific level and equating all the other factors to the corresponding US value one at a time.

Table 2.9: Counterfactual Experiments

	Initial Level	Tax	Int. Per.	Ret. to Edu.	Pub. Edu.
	Δ from US	Δ from US	Δ from US	Δ from US	Δ from US
Denmark	0.05	0.04	0.08	0.10	0.02
Finland	0.04	0.03	0.08	0.11	0.01
Germany	0.02	0.02	0.05	0.02	0.00
Netherlands	0.04	0.01	0.08	0.10	0.00
Sweden	0.05	0.01	0.07	0.11	0.01
UK	0.02	0.00	0.00	0.05	0.00
Average	**0.03**	**0.02**	**0.06**	**0.08**	**0.01**

Table 2.9 presents the results of these experiments. I find that the strongest quantitative effect comes from returns to educational investments. Because the relative proportion of private education expenditures as a share of total education expenditures varies widely across countries, earnings elasticity with respect to educational investments itself generates a difference of 0.08. The second most important element is the variation in intergenerational earnings elasticity across countries. When equating all parameter values to the US levels except for intergenerational earnings elasticity, the model generates a 0.06 difference. The difference in initial level of earnings inequality itself contributes by producing a 0.03 inequality gap, followed by the differences in tax structure and public education spending, predicting 0.02 and 0.01 differentials, respectively.

Table 2.10: Sensitivity Analysis: Elasticity of Substitution

	Data	Model		Model	
		$\rho = 0.25$		$\rho = 0.75$	
	Δ from US (a)	Δ from US (b)	Explained (b)/(a)	Δ from US (d)	Explained (d)/(a)
Denmark	0.63	0.20	0.32	0.24	0.38
Finland	0.68	0.20	0.29	0.23	0.34
Germany	0.52	0.13	0.25	0.14	0.27
Netherlands	0.57	0.18	0.32	0.21	0.37
Sweden	0.75	0.20	0.27	0.23	0.31
UK	0.32	0.09	0.28	0.10	0.31
Average	**0.58**	**0.17**	**0.29**	**0.19**	**0.33**
US	0.00	0.00	-	0.00	-

2.5.3 Sensitivity analysis

2.5.3.1 Elasticity of substitution between public and private education

The elasticity of substitution is set to $\rho = 0.5$ as the baseline. To examine the robustness of this choice, I use different values of the elasticity of substitution and recalibrate the model economy as described in Section 2.4 in each experiment. Table 2.10 presents the results. Compared to the basline, using the low value of substitution elasticity ($\rho = 0.25$) produces lower inequality differences, while the model performance increases slightly for the higher value ($\rho = 0.75$). Overall, the predictions of the model remain robust under the alternative values of ρ.

Table 2.11: Sensitivity Analysis: Finland as a Benchmark

	Data		Model		
	Level (a)	Δ from US (b)	Level (c)	Δ from US (d)	Explained (d)/(b)
Denmark	0.99	0.63	0.95	0.26	0.41
Finland	0.94	0.68	0.94	0.27	0.40
Germany	1.10	0.52	1.04	0.17	0.33
Netherlands	1.05	0.57	0.97	0.24	0.42
Sweden	0.87	0.75	0.96	0.25	0.33
UK	1.30	0.32	1.07	0.14	0.44
Average	**1.17**	**0.30**	**1.44**	**0.18**	**0.38**
US	1.62	0.00	1.21	0.00	-

2.5.3.2 Finland as a benchmark economy

Recall that the US economy is chosen as the benchmark economy in the calibration, and that the high standard deviation of the ability distribution is required to replicate the large increase in earnings inequality in the US. To evaluate the sensitivity of this choice, I take Finland, which has experienced the smallest rise in earnings inequality, to be the benchmark economy, and I examine how robust the model predictions are to this change. Following the calibration procedures described in Section 2.4, I recalibrate the model economy, calculating cross-country differences in earnings inequality. As shown in Table 2.11, the explanatory power of the model (38 percent) tends to be higher than that of the baseline model. Overall, however, the quantitative results remain robust to which country is taken as the benchmark economy.

2.6 Conclusion

In this paper, I develop a simple endogenous growth model of human capital to study cross-country differences in earnings inequality. To understand the mechanism of how the model works, I analytically derive the dynamics of earnings inequality, which turns out to be related to tax progressivity, intergenerational earnings persistence, returns to education investments, and public education spending. In the quantitative analysis, I find that the calibrated model accounts for 31 percent of the observed cross-country differences in earnings inequality. According to counterfactual experiments, differences in returns to education expenditures and intergenerational earnings persistence quantitatively play critical roles in generating the cross-country variations in earnings inequality, followed by differences in initial earnings inequality, tax structures, and public education spending.

I have made many simplifying assumptions in the model for tractability. Future work would incorporate cross-country differences in wage rates and education funding systems within a general equilibrium model. In particular, rather than treating public education as a uniform transfer to all individuals, analyzing the importance of different funding systems of public education and expenditure dispersion across districts would be interesting to investigate. The differences in public funding systems would have strong effects on an individual's human capital accumulation, cross-sectional dispersion of earnings, and consequently on earnings inequality. In addition, it would be an interesting extension to allow a tax system to vary over time, and to explore its effect on cross-country changes in earnings inequality.

CHAPTER 3
OPTIMAL TAXATION IN LIFE-CYCLE ECONOMIES WITH ENDOGENOUS HUMAN CAPITAL FORMATION

3.1 Introduction

[1]In the context of overlapping generations models, an individual's life-cycle profile of labor productivity has important implications for optimal fiscal policy. A non-constant life-cycle productivity causes an individual's optimal consumption-work plan to vary over lifetime, so that the government finds it optimal to tax consumption and labor earnings at different rates over the life-cycle. Despite its crucial role in shaping optimal tax rates in life-cycle economies, most of previous Ramsey taxation literature largely takes the productivity profile as exogenous.[2] This paper fills that void by studying optimal fiscal policy in a life-cycle model in which the productivity profile emanates from individuals' decision to accumulate human capital.

To characterize optimal fiscal policy, we construct a standard Ramsey problem where the government maximizes a utilitarian welfare—function defined as the discounted sum of successive generation's lifetime utility—by choosing government debt as well as age-dependent proportional taxes on capital income, labor income, and human capital investment. As is common in the literature, our analysis uses the primal approach, according to which the government directly chooses an optimal allocation—rather than tax rates—subject to a series of constraints guaranteeing that

[1]This chapter is joint work with Martin Gervais.

[2]As discussed below, exceptions include Bovenberg and Jacobs (2005), Jacobs and Bovenberg (2010), and Peterman (2014).

the allocation is feasible and consistent with individual optimization.

While the set of fiscal instruments available to the government is always central to Ramsey-type analyses, it is particularly relevant in our context, as that set depends on one's assumption as to whether the government can or cannot observe individuals' human capital investment. That is, depending on whether human capital policy is available or not, optimal tax policy becomes radically different. Specifically, we show that when human capital investment is observable and can be taxed/subsidized, the government can design a tax system which mimics a lump-sum tax in the initial period of individuals' life, allowing the government to set all remaining tax instruments equal to zero. In other words, the government effectively uses the human capital policy to offset the distortionary effects of labor and capital income taxations in the first period. Evidently, this strategy becomes infeasible when the tax code is incomplete, i.e. when human capital investment is unobservable: a compelling assumption.[3] In this case, optimal income tax policy entails non-zero taxes on both capital and labor income, even if a full set of age-dependent tax rates are available.[4] That is, without human capital policy to offset any distortion on skill formation, optimal capital taxes should be positive in order to mitigate the distortionary effect of the labor income tax on investments in human capital.

The above results, in particular with respect to the non-zero taxation of in-

[3]The monetary investments in human capital can be thought of as the necessary inputs in the production of human capital, such as books, computers, traveling, or the cost of living over life-cycle, which are effectively non-verifiable.

[4]It should be noted that these results obtain under a specific, though widely-used, class of utility functions.

terest income, are in sharp contrast with Jones et al. (1997), who study optimal taxations in the context of an infinitely-lived agent model with human capital accumulation. With the assumption on non-observable investments in human capital, they find that labor income tax should also be zero in the long-run, in addition to Chamley (1986) and Judd (1985)'s zero taxation on capital income. This radically different result is driven by an individual's endogenous life-cycle structure of productivity. In other words, since the return to investments in human capital when young is higher than that when old, an individual's human capital investment decreases with ages, and the life-cycle productivity profile is never constant over the life-cycle. This causes the individual's optimal consumption-work plan to vary with age, and an optimizing government taxes consumption and labor earnings at different rates over lifetime.

Our paper is not the first work to study optimal fiscal policy within an overlapping-generation framework. Erosa and Gervais(2002) and Garriga (2003) show that an optimizing government will typically use non-zero capital and labor income taxes that depend on age. Moreover, Gervais (2012) shows that if the government does not have access to age-conditioned taxes, then it is possible to imperfectly mimic the optimal policy with progressive taxation on labor income together with a non-zero tax on capital. Conesa and Krueger (2006) study the optimal progressivity of the tax code in a life cycle model where heterogenous agents face uninsurable productivity risk, and Gorry and Oberfield (2012) characterize the optimal non-linear tax schedule by including an endogenous extensive margin of labor supply. In particular, Conesa et al. (2009) finds that the optimal tax policy entails both a progressive labor income tax and a high tax on capital income, and that the sizable optimal capital income tax is mainly driven by an individual's life-cycle structure. Even though these studies explore the optimal way in which to tax interest income in a life-cycle model, they all take the life-cycle productivity—a crucial element for optimal taxation—as exogenous. In this sense, this paper contributes to the literature in pubic finance by analyzing the effect of endogenous human capital formation on optimal fiscal policy.

Our work is also related to research on optimal income taxation jointly determined with some education policies. In a static model, Bovenberg and Jacob (2005) argue that education subsidies play a powerful role to eliminate distortions in human capital formation caused by distortionary income taxes. Subsequent papers, such as Maldonado (2008) and Jacobs and Bovenberg (2011), examine the importance of the complementarity between ability and human capital, while Costa and Maestri (2007) emphasizes the role of risky investments in human capital. Jacobs and Bovenberg (2010), which is closely related to our paper, analyze the effects of non-verifiable human capital investments on optimal income taxations and show positive taxes on capital income to alleviate the distortionary effects of the labor income tax on human capital investment. While many insights from this work also appear in our analysis, none of these papers consider a dynamics in a general equilibrium setting.

There are, however, recent papers on dynamic optimal taxation that also study human capital policies in a life-cycle model. Krueger and Ludwig (2013) quantitatively characterize the optimal policy mix of progressive income taxes and education subsidies where education takes the form of a discrete college-education choice: thereafter, individuals' life-cycle productivity profile is taken as given, which is the key difference from our paper. Our work is also closely related to Peterman (2014), who analyzes the effects of endogenous human capital accumulation on optimal tax policy. By contrast to our work, he assumes that the tax rates cannot be age-dependent, and that individuals accumulate human capital through learning-by-doing during working periods, not goods investments. Lastly, there are also several papers which use the Mirrlees approach to optimal taxation in the context of human capital accumulation, most prominently Bohacek and Kapička (2008), Grochulskia and Piskorskib (2010), Kapička (2015), Kapička and Neira (2015), and Stantcheva (2015).

The rest of this paper is organized as follows. Section 3.2 describes the economic environment and Section 3.3 formulates the Ramsey problem. Section 3.4 and Section 3.5 derives optimal tax policy under non-verifiable and verifiable human capital investments, respectively. Section 3.6 concludes and discusses future directions.

3.2 The Economy

We consider a life-cycle economy in which individuals live for a finite number of periods. Individuals endogenously accumulate their own human capital through goods investments, and make consumption/saving and labor/leisure choices in each period. The government collects taxes paid by individuals to finance a given stream of public spending. The set of fiscal policy instruments is government debt as well as proportional taxes on consumption, investments in human capital, labor and capital income, where all tax rates can be age-dependent.

3.2.1 Households

Individuals live (J+1) periods. A new generation is born every period and indexed by date of birth. The generations alive at date 0 are -J, -J+1, ..., 0. It will be convenient to denote the age of individuals alive at date 0 by $j_0(t)$. We set $j_0(t) = 0$ for all other generations, and for any generation t, $j_0(t) = \max\{-t, 0\}$.[5] We assume that the population grows at a constant rate n per period, and let μ_j be the share of age-j individuals in the population, which satisfies $\mu_j = \mu_{j-1}/(1+n)$, for j = 1, ..., J, where $\sum_{j=0}^{J} \mu_j = 1$.

Individuals derive utility from consumption and leisure. We denote $c_{t,j}$ and $n_{m_{t,j}}$ as consumption and time devoted to work in period (t+j) by an age-j individual born in period t, respectively. In each period, individuals are endowed with one unit of time, and optimally allocate their time between leisure and labor supply. We assume that initial asset holdings $a_{t,j_0(t)}$ of individuals are given and equal to 0 if t $\geq$ 0. Because tax rates are age-dependent, the after-tax prices that individuals face also are dependent on age. Let $q_{t,j}$, $s_{t,j}$, $w_{t,j}$ and $r_{t,j}$ be the after-tax prices of consumption, investments in human capital, labor services, and capital services, respectively.

The problem faced by an individual born in period t $\geq$ -J is to maximize lifetime utility subject to a sequence of budget constraints and law of motion of

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[5] One can think of $j_0(t)$ as the first period of an individual's life affected by the change in fiscal policy, which occurs at date 0.

human capital accumulation:

$$U^t(\pi) \equiv \max \sum_{j=0}^{J} \beta^j u(c_{t,j}, n_{m_{t,j}}) \tag{3.1}$$

$$\begin{aligned}
\text{s.t.} \quad q_{t,0}c_{t,0} + a_{t,1} + s_{t,0}x_{t,0} &= w_{t,0}n_{m_{t,0}} + (1+r_{t,0})a_{t,0}, && (3.2)\\
q_{t,j}c_{t,j} + a_{t,j+1} + s_{t,j}x_{t,j} &= w_{t,j}h_{t,j}n_{m_{t,j}} + (1+r_{t,j})a_{t,j}, j = 1, ..., J && (3.3)\\
h_{t,1} &= G_0(x_{t,0}), && (3.4)\\
h_{t,j+1} &= (1-\delta_h)h_{t,j} + G(x_{t,j}, h_{t,j}), j = 1, ..., J-1, && (3.5)
\end{aligned}$$

where $a_{t-j,j}$, $x_{t-j,j}$, and $h_{t-j,j}$ denote the asset holdings, goods investment in human capital, and human capital stock of an age-j individual at date t, respectively. The monetary investments in human capital can be thought of as the necessary inputs in the production of human capital, such as books, computers, traveling, or the cost of living. The utility function is assumed to be increasing in consumption and leisure, strictly concave, and satisfies standard Inada conditions. We define $U^t(\pi)$ to be the maximum lifetime utility obtained by an individual from generation t under fiscal policy π. The human capital production function $G(x, h)$ depends on goods investments and current stock of human capital stock, and is assumed to be constant returns to scale. Notice that to avoid the initial lump-sum taxations on exogenously given initial stock of human capital, we assume that individuals work without human capital at the initial period in (3.2), and the human capital production only depends on goods investment in (3.4).

3.2.2 Technology and Feasibility

There is a single produced good in our economy used for capital or consumption (private or government). We assume that the technology is represented by a neoclassical production function with constant returns to scale,

$$y_t = f(k_t, z_t), \tag{3.6}$$

where y_t, k_t, and z_t denote the aggregate (or per capita) levels of output, capital, and effective labor, respectively. Profit maximization by firms implies that capital and labor services are paid their marginal products: before-tax prices of capital and labor in period t are given by $\hat{r}_t = f_k\,(k_t, z_t) - \delta_k$ and $\hat{w}_t = f_z\,(k_t, z_t)$, where $0 < \delta_k < 1$ is the depreciation rate of capital.

Feasibility requires that total consumption plus investments (human and physical capital) be less than or equal to aggregate output:

$$c_t + x_t + (1+n)k_{t+1} - (1-\delta)k_t + g_t = y_t, \tag{3.7}$$

where c_t and g_t denote aggregate private and government consumption at date t, respectively, and all aggregate quantities are expressed in per capita terms. Specifically, the date-t aggregate levels of consumption, goods investment in human capital, physical capital, and labor input (expressed in efficiency units) are given by

$$c_t = \sum_{j=0}^{J} \mu_j c_{t-j,j} \quad , \qquad x_t = \sum_{j=0}^{J} \mu_j x_{t-j,j}$$
$$k_t = \sum_{j=0}^{J} \mu_j a_{t-j,j} \quad , \qquad z_t = \mu_0 n_{m_{t,0}} + \sum_{j=1}^{J} \mu_j h_{t-j,j} n_{m_{t-j,j}}.$$

3.2.3 The Government

To finance a given stream of public spending, throughout, we assume that the government has access to a set of fiscal policy instruments and a commitment technology to implement its fiscal policy. The set of policy instruments consists of government debt and age-dependent proportional taxes on consumption, goods investment in human capital, labor income, and capital income. The date-t tax rates on labor and capital income of an age-j individual born at period (t-j) are denoted by $\tau^w_{t-j,j}$ and $\tau^k_{t-j,j}$, respectively. Similarly, the date-t tax rates on consumption and goods investment in human capital are defined by $\tau^c_{t-j,j}$ and $\tau^s_{t-j,j}$, respectively. Moreover, the government can issue its debt (b_t) to match imbalances between revenues and expenditures every period. In per capita terms, the government budget constraint at period t is given by

$$\begin{aligned}
(1+\hat{r}_t)b_t + g_t &= (1+n)b_{t+1} + \sum_{j=0}^{J}\left(q_{t,j}-1\right)\mu_j c_{t-j,j} \\
&+ \sum_{j=0}^{J}\left(\hat{r}_t - r_{t-j,j}\right)\mu_j a_{t-j,j} \\
&+ \left(\hat{w}_t - w_{t,0}\right)\mu_0 n_{m_{t,0}} + \sum_{j=1}^{J}\left(\hat{w}_t - w_{t-j,j}\right)\mu_j h_{t-j,j} n_{m_{t-j,j}} \\
&+ \sum_{j=0}^{J}\left(s_{t,j}-1\right)\mu_j x_{t-j,j}. \qquad (3.8)
\end{aligned}$$

In the spirit of Ramsey, the government takes individual's optimizing behavior as given and chooses a fiscal policy to maximize social welfare defined as the discounted sum of individual lifetime welfare. That is, the government's objective function is given by

$$\sum_{t=-J}^{\infty} \gamma^t U^t,$$

where $0 < \gamma < 1$ is the intergenerational discount factor and U^t is the indirect utility function of generation t as a function of the government tax policy.

3.3 The Ramsey Problem

The Ramsey problem is typically defined as choosing optimal tax rates to maximize a given welfare function. In this section, we formally show an equivalent construction of the Ramsey problem where the government chooses allocations rather than tax rates. To do so, we first define the set of allocation that the government can implement.

Definition. *(Implementable Allocation) Let $\{g_t\}_{t=0}^{\infty}$ be a given sequence of government expenditures. Given initial aggregate endowments $\{k_0, b_0\}$ and initial individual asset holdings $\{a_{-j,j}\}_{j=1}^{J}$ such that*

$$k_0 + b_0 = \sum_{j=1}^{J} \mu_j a_{-j,j},$$

an allocation $\left\{ \{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}\}_{j=j_0(t)}^{J}, k_{t+J+1} \right\}_{t=-J}^{\infty}$ is implementable if there exists a fiscal policy $\left\{ \{q_{t,j}, s_{t,j}, w_{t,j}, r_{t,j}\}_{j=j_0(t)}^{J}, b_{t+J+1} \right\}_{t=-J}^{\infty}$ and a sequence of asset holdings $\left\{ \{a_{t,j}\}_{j=j_0(t)}^{J} \right\}_{t=-J}^{\infty}$ such that:

D1a. Given prices from the fiscal policy, $\left\{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}, a_{t,j+1}\right\}_{j=j_0(t)}^{J}$ solves the consumer's problem given by (3.1)-(3.5) for t = -J,...

D1b. Factor prices are competitive: $\hat{r}_t = f_k(k_t, z_t) - \delta_k$ and $\hat{w}_t = f_z(k_t, z_t)$, t = 0,1,...

D1c. The government budget constraint (3.8) is satisfied at t = 0,1,...

D1d. Aggregate feasibility (3.7) is satisfied at t = 0,1,...

Notice that the set of allocation from which the government can implement consists of the allocation chosen by individuals as a competitive equilibrium. In addition, it should be emphasized that the set of implementable allocations is critically dependent on the set of fiscal instruments available to the government. That is, an allocation which can be implementable with age-dependent taxes may not be implementable with age-independent taxes. Moreover, for a given set of tax instruments, many different tax policies can implement the same allocation. In proposition 3.1, we show that age-dependent consumption taxes are redundant fiscal instruments, implying that even without consumption taxes, a given allocation can still be implemented by a set of fiscal policies.

Proposition 1. *Let* $\{g_t\}_{t=0}^{\infty}$ *be a given sequence of government expenditures and let* $\left(k_0, b_0, \{a_{-j,j}\}_{j=1}^{J}\right)$ *be initial endowments such that*

$$k_0 + b_0 = \sum_{j=1}^{J} \mu_j a_{-j,j}.$$

If the fiscal policy $\left\{\{q_{t,j}, s_{t,j}, w_{t,j}, r_{t,j}\}_{j=j_0(t)}^{J}, b_{t+J+1}\right\}_{t=-J}^{\infty}$ *and the sequence of asset holdings* $\left\{\{a_{t,j}\}_{j=j_0(t)+1}^{J}\right\}_{t=-J}^{\infty}$ *implements the allocation* $\left\{\{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}\}_{j=j_0(t)}^{J}\right\}_{t=-J}^{\infty}$ *and* $\{k_{t+J+1}\}_{t=-J}^{\infty}$*, then any other fiscal policy* $\left\{\{\tilde{q}_{t,j}, \tilde{s}_{t,j}, \tilde{w}_{t,j}, \tilde{r}_{t,j}\}_{j=j_0(t)}^{J}, \tilde{b}_{t+J+1}\right\}_{t=-J}^{\infty}$

and sequence of asset holdings $\left\{\{\tilde{a}_{t,j}\}_{j=j_0(t)+1}^{J}\right\}_{t=-J}^{\infty}$ *satisfying*

$$\frac{1+r_{t,j_0(t)}}{q_{t,j_0(t)}} = \frac{1+\tilde{r}_{t,j_0(t)}}{\tilde{q}_{t,j_0(t)}}, \quad t=-J,\ldots, \tag{3.9}$$

$$\frac{w_{t,j}}{q_{t,j}} = \frac{\tilde{w}_{t,j}}{\tilde{q}_{t,j}}, \quad t=-J,\ldots, \quad j=j_0(t),\ldots,J, \tag{3.10}$$

$$\frac{s_{t,j}}{q_{t,j}} = \frac{\tilde{s}_{t,j}}{\tilde{q}_{t,j}}, \quad t=-J,\ldots, \quad j=j_0(t),\ldots,J-1, \tag{3.11}$$

$$\frac{(1+r_{t,t+1})\,q_{t,j}}{q_{t,j+1}} = \frac{(1+\tilde{r}_{t,j+1})\,\tilde{q}_{t,j}}{\tilde{q}_{t,j+1}}, \quad t=-J,\ldots, \quad j=j_0(t),\ldots,J, \tag{3.12}$$

$$\frac{a_{t,j+1}}{q_{t,j}} = \frac{\tilde{a}_{t,j+1}}{\tilde{q}_{t,j}}, \quad t=-J,\ldots, \quad j=j_0(t),\ldots,J, \tag{3.13}$$

also implements the allocation.

Proof. The goal is to show that any alternative fiscal policy and sequence of asset holdings satisfying (3.9)-(3.13) also satisfy (D1a)-(D1d) in Definition. Notice that (D1b) (factor prices) and (D1d) (feasibility) are trivially satisfied under the alternative fiscal policy.

First, we show that alternative fiscal policy and sequence of asset holdings satisfying (3.9)-(3.13) are consistent with (D1a). Using equations (3.10) and (3.12), we can replace the after-tax prices of labor and capital services in the consumer's budget constraint under the initial fiscal policy:

$$q_{t,j}c_{t,j} + a_{t,j+1} + s_{t,j}x_{t,j} = \tilde{w}_{t,j}\frac{q_{t,j}}{\tilde{q}_{t,j}}h_{t,j}n_{m_{t,j}} + (1+\tilde{r}_{t,j})\frac{\tilde{q}_{t,j-1}}{\tilde{q}_{t,j}}\frac{q_{t,j}}{q_{t,j-1}}a_{t,j},$$

Multiplying the above equation by $\frac{\tilde{q}_{t,j}}{q_{t,j}}$ and using condition (3.13), we obtain

$$\tilde{q}_{t,j}c_{t,j} + \tilde{a}_{t,j+1} + \tilde{s}_{t,j}x_{t,j} = \tilde{w}_{t,j}h_{t,j}n_{m_{t,j}} + (1+\tilde{r}_{t,j})\tilde{a}_{t,j},$$

So, any allocation $\left\{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}\right\}_{j=j_0(t)}^{J}$ which satisfies the consumer's budget constraint under the initial fiscal policy also satisfies the budget constraint under the alternative fiscal policy. We can also easily show that the opposite is true, and the two budget constraints are identical. In addition, because the consumer's decision problem is the same under both fiscal policies, we reach to the conclusion that $\left\{ \{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}, \tilde{a}_{t,j+1}\}_{j=j_0(t)}^{J} \right\}_{t=-J}^{\infty}$ solves the consumer's problem under the alternative fiscal policy. Hence, (D1a) is satisfied.

We now show that the government budget constraint (3.8) is satisfied under the alternative fiscal policy. Using the feasibility (3.7) and the property that the production fuction is constant returns to scale, we can rewrite the government budget constraint as follows:

$$\begin{aligned}\sum_{j=0}^{J} (1+\tilde{r}_{t-j,j})\,\mu_j \tilde{a}_{t-j,j} &= (1+n)\tilde{a}_{t+1} + \sum_{j=0}^{J} \tilde{q}_{t-j,j}\mu_j c_{t-j,j} + \sum_{j=0}^{J} \tilde{s}_{t-j,j}\mu_j x_{t-j,j} \\ &- \tilde{w}_{t,0}\mu_0 n_{m_{t,0}} - \sum_{j=1}^{J} \tilde{w}_{t-j,j}\mu_j h_{t-j,j} n_{m_{t-j,j}}.\end{aligned}$$

Using conditions (9)-(13), the above equation is rewritten as

$$\begin{aligned}&\sum_{j=0}^{J} (1+r_{t-j,j}) \left(\frac{\tilde{q}_{t-j,j}}{\tilde{q}_{t-j,j-1}} \frac{q_{t-j,j-1}}{q_{t-j,j}} \right) \mu_j a_{t-j,j} \frac{\tilde{q}_{t-j,j-1}}{q_{t-j,j-1}} \\ =\;& (1+n) a_{t-j,j} \frac{\tilde{q}_{t-j,j-1}}{q_{t-j,j-1}} + \sum_{j=0}^{J} \tilde{q}_{t-j,j}\mu_j c_{t-j,j} \\ +\;& \sum_{j=0}^{J} s_{t-j,j} \frac{\tilde{q}_{t-j,j}}{q_{t-j,j}} \mu_j x_{t-j,j} - w_{t,0} \frac{\tilde{q}_{t,0}}{q_{t,0}} \mu_0 n_{m_{t,0}} - \sum_{j=1}^{J} w_{t-j,j} \frac{\tilde{q}_{t-j,j}}{q_{t-j,j}} \mu_j h_{t-j,j} n_{m_{t-j,j}}.\end{aligned}$$

Multiplying both sides of the above equation by $\frac{q_{t-j,j}}{\tilde{q}_{t-j,j}}$, we get the government budget constraint under the initial fiscal policy holidng at all dates. Thus, the government budget constraint also holds under the alternative fiscal policy, and (D1c) is also satisfied. $\square$

The key implication of Proposition 3.1 is that we need to consider a fiscal policy arrangement as a whole rather than considering tax instrument individually. In particular, even if either consumption taxes or human capital investment taxes or labor income taxes can be removed from a given fiscal policy, the implementable allocation will not be affected. That is, we can always achieve these changes in tax policy by simply redefining the other tax instruments such that the conditions (3.9)-(3.13) are satisfied. In this sense, without any loss of generality, we set $\tau^c_{t,j}$ to be zero for all t and j as long as a tax instrument on human capital investments is available to the government.

Taking the primal approach, we construct a Ramsey problem where the government directly chooses allocations. To do so, we must impose restrictions on the set of allocations, so that any allocations chosen by the government can be decentralized as a competitive equilibrium. To derive a sequence of implementability constraint, we use the consumer's budget constraint and optimality conditions.

We denote $p_{t,j}$ and $\eta_{t,j}$ as the Lagrange multiplier associated with the consumer's budget constraint and law of motion of human capital faced by an age-j individual born in date t, respectively. The first-order conditions for the consumer's problem are given by

$$\beta^j u_{c_{t,j}} = p_{t,j}, \qquad j = 0, .., J \tag{3.14}$$

$$p_{t,j} = p_{t,j+1}(1 + r_{t,j+1}), \qquad j = 0, .., J-1 \tag{3.15}$$

$$u_{n_{m_{t,0}}} = p_{t,0} w_{t,0}, \tag{3.16}$$

$$\beta^j u_{n_{m_{t,j}}} = p_{t,j} w_{t,j} h_{t,j}, \qquad j = 1, .., J \tag{3.17}$$

$$p_{t,0} s_{t,0} = \eta_{t,0} G_0'(x_{t,0}), \tag{3.18}$$

$$p_{t,j} s_{t,j} = \eta_{t,j} G_{x_{t,j}}, \qquad j = 1, .., J-1 \tag{3.19}$$

$$\eta_{t,j} = \eta_{t,j+1} \left(1 - \delta_h + G_{h_{t,j+1}}\right) + p_{t,j+1} w_{t,j+1} n_{m_{t,j+1}}, j = 0, .., J-2 \tag{3.20}$$

$$\eta_{t,J-1} = p_{t,J} w_{t,J} n_{m_{t,J}}, \tag{3.21}$$

where $u_{c_{t,j}}$ and $u_{n_{m_{t,j}}}$ are the derivative of u with respect to $c_{t,j}$ and $n_{m_{t,j}}$, respectively. Similarly, $G_{x_{t,j}}$ and $G_{h_{t,j}}$ are the derivative of G with respect to $x_{t,j}$ and $h_{t,j}$, respectively. Using the equations (3.14), (3.18) and (3.19), we can rewrite the consumer's optimality conditions by substituting out the Lagrange multiplier $p_{t,j}$ and $\eta_{t,j}$:

$$u_{c_{t,j}} = \beta u_{c_{t,j+1}}(1 + r_{t,j+1}), \qquad j = 0, .., J-1 \tag{3.22}$$

$$u_{n_{m_{t,0}}} = u_{c_{t,0}} w_{t,0} \tag{3.23}$$

$$u_{n_{m_{t,j}}} = u_{c_{t,j}} h_{t,j} w_{t,j}, \qquad j = 1, ..., J \tag{3.24}$$

$$\frac{s_{t,0}}{G_0'(x_{t,0})} = \frac{1}{1 + r_{t,1}} \left[w_{t,1} n_{m_{t,1}} + \frac{\left(1 - \delta_h + G_{h_{t,1}}\right) s_{t,1}}{G_{x_{t,1}}} \right], \tag{3.25}$$

$$\frac{s_{t,j}}{G_{x_{t,j}}} = \frac{1}{1 + r_{t,j+1}} \times \left[w_{t,j+1} n_{m_{t,j+1}} + \frac{\left(1 - \delta_h + G_{h_{t,j+1}}\right) s_{t,j+1}}{G_{x_{t,j+1}}} \right], j = 1, .., J-2 \tag{3.26}$$

$$\frac{s_{t,J-1}}{G_{x_{t,J-1}}} = \frac{w_{t,J} n_{m_{t,J}}}{1 + r_{t,J}}. \tag{3.27}$$

We can obtain the implementability constraints of an individual born at period t by using his present-value budget constraint, optimality conditions (3.22)-(3.27), and the assumption that G is homogeneous of degree one:[6]

$$\begin{aligned}\sum_{j=0}^{J}\beta^j u_{c_{t,j}} c_{t,j} &= u_{c_{t,0}}\left[a_{t,0}(1+r_{t,0}) - \left(s_{t,0}x_{t,0} - \left(\frac{u_{n_{m_{t,0}}}}{u_{c_{t,0}}}\right)n_{m_{t,0}}\right) + \frac{G_0(x_{t,0})s_{t,0}}{G_0'(x_{t,0})}\right] \\ &= u_{c_{t,0}}\left[a_{t,0}(1+r_{t,0}) - s_{t,0}\left(x_{t,0} - \frac{G_0(x_{t,0})}{G_0'(x_{t,0})}\right)\right] + u_{n_{m_{t,0}}} n_{m_{t,0}} \\ &\equiv A_{t,0}. \end{aligned} \quad (3.28)$$

In the following Proposition 3.2, we show that any competitive equilibrium allocaton has to satisfy the implementability constraint, and any feasible allocation satisfying the implementability constraint can also be decentralized as a competitive equilibrium.

Proposition 2. *An allocation* $\left\{\left\{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}\right\}_{j=j_0(t)}^{J}, k_{t+J+1}\right\}_{t=-J}^{\infty}$ *is implementable with age-dependent taxes if and only if it satisfies feasibility (3.7) and the implementability constraint (3.28).*

Proof. By definition, implementable allocations satisfy feasibility as well as implementability constraint. We focus on showing that the converse is also true.

Assume that $\left\{\left\{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}\right\}_{j=j_0(t)}^{J}, k_{t+J+1}\right\}_{t=-J}^{\infty}$ satisfies the feasibility (3.7) and implementability constraint (3.28). We define before-tax prices as $\hat{r}_t \equiv f_k(k_t, z_t) - \delta_k$ and $\hat{w}_t \equiv f_z(k_t, z_t)$ and the sequence of after-tax prices $\left\{\left\{w_{t,j}, r_{t,j}, s_{t,j}\right\}_{j=j_0(t)}^{J}\right\}_{t=-J}^{\infty}$

[6] See Appendix B.1 for more detail derivations.

as

$$
\begin{aligned}
w_{t,j_0(t)} &\equiv \frac{U_{n_{m_{t,j_0(t)}}}}{U_{c_{t,j_0(t)}}}, \\
w_{t,j} &\equiv \frac{U_{n_{m_{t,j}}}}{h_{t,j}U_{c_{t,j}}}, \quad j = 1, \ldots, J, \\
1 + r_{t,j} &\equiv \frac{U_{c_{t,j}}}{U_{c_{t,j+1}}}, \quad j = 1, \ldots, J, \\
s_{t,J-1} &= \frac{G_{x_{t,J-1}} w_{t,J} n_{m_{t,J}}}{1 + r_{t,J}}, \\
s_{t,j} &= \frac{G_{x_{t,j}}}{1 + r_{t,j+1}} \left[w_{t,j+1} n_{m_{t,j+1}} + \frac{\left(1 - \delta_h + G_{h_{t,j+1}}\right) s_{t,j+1}}{G_{x_{t,j+1}}} \right], \quad j = 1, .., J-2, \\
s_{t,j_0(t)} &= \frac{G'_{j_0(t)}(x_{t,j_0(t)})}{1 + r_{t,1}} \left[w_{t,1} n_{m_{t,1}} + \frac{\left(1 - \delta_h + G_{h_{t,1}}\right) s_{t,1}}{G_{x_{t,1}}} \right].
\end{aligned}
$$

Then, by construction, $\left\{c_{t,j}, n_{m_{t,j}}, x_{t,j}, h_{t,j+1}\right\}_{j=j_0(t)}^{J}$ satisfies the consumer's first order conditions (3.22)-(3.27) for all t $\geq$ -J.

We now show that the consumer's budget constraint and the transversality condition are satisfied. Given $a_{t,j_0(t)}$, define $a_{t,j+1}$ recursively as follows:

$$
a_{t,j+1} = w_{t,j} h_{t,j} n_{m_{t,j}} + (1 + r_{t,j})\, a_{t,j} - c_{t,j} - s_{t,j} x_{t,j}, \quad j = j_0(t), \ldots, J.
$$

Notice that given the after-tax prices defined above, the implementability constraint implies that $a_{t,J+1} = 0$ for all t $\geq$ -J.

Lastly, we show that the government budget constraint holds. To do so, the budget constraint of the age-j consumer born in period (t-j) is multiplied by μ_j and added up for $j \in \{0, \ldots, J\}$:

$$\begin{aligned}\sum_{j=0}^{J} \mu_j \left(c_{t-j,j} + a_{t-j,j+1} + s_{t-j,j} x_{t-j,j}\right) &= \sum_{j=0}^{J} \mu_j \left(w_{t-j,j} h_{t-j,j} n_{m_{t-j,j}} + (1 + r_{t-j,j})\, a_{t-j,j}\right) \\ \Leftrightarrow c_t + (1+n) a_{t+1} + s_{t-j,j} x_{t-j,j} &= a_t + \\ &+ \sum_{j=0}^{J} \mu_j \left(w_{t-j,j} h_{t-j,j} n_{m_{t-j,j}} + r_{t-j,j} a_{t-j,j}\right). \quad (3.29)\end{aligned}$$

Using the fact that the production function is constant returns to scale, we can rewrite the feasibility constraint as follows:

$$c_t + x_t + (1+n) k_{t+1} + g_t = (\hat{r}_t + \delta)\, k_t + \hat{w}_t \sum_{j=0}^{J} \mu_j h_{t-j,j} n_{m_{t-j,j}} + (1-\delta) k_t. \quad (3.30)$$

Combining (3.29) and (3.30), we obtain

$$\begin{aligned}& g_t + \sum_{j=0}^{J} (1 - s_{t,j})\, \mu_j x_{t-j,j} - (1 + \hat{r}_t)\, k_t + a_t \\ = \; & (1+n)\,(a_{t+1} - k_{t+1}) + \sum_{j=0}^{J} \mu_j h_{t-j,j}\,(\hat{w} - w_{t-j,j})\, n_{m_{t-j,j}} - \sum_{j=0}^{J} \mu_j r_{t-j,j} a_{t-j,j}.\end{aligned}$$

Adding $\hat{r}_t a_t$ on both sides of the above expression and defining $b_t \equiv a_t - k_t$, we can rewrite the previous expression as

$$\begin{aligned}(1 + \hat{r}_t)\, b_t + g_t &= (1+n) b_{t+1} + \sum_{j=0}^{J} \mu_j h_{t-j,j}\,(\hat{w} - w_{t-j,j})\, n_{m_{t-j,j}} \\ &+ \sum_{j=0}^{J} \mu_j\,(\hat{r}_{t-j,j} - r_{t-j,j})\, a_{t-j,j} + \sum_{j=0}^{J} (s_{t,j} - 1)\, \mu_j x_{t-j,j},\end{aligned}$$

which is the government budget constraint without the consumption tax rates. □

3.4 Optimal Fiscal Policy

In this section, we formulate a Ramsey problem and characterize analytically the optimal fiscal policy by solving for the problem at steady state. We first show that under a complete tax system, all tax rates are optimally set to zero throughout an individual's lifetime, except for some age-0 tax rate which are set in such a way as to imitate a lump-sum tax, thereby achieving a first-best allocation. We then study a Ramsey problem in which the government cannot observe human capital investment, which renders the tax system incomplete and forces the government away from the above trivial solution.

3.4.1 Verifiable Investments in Human Capital

To begin, we define the pseudo welfare function (W_t) to include generation t's implementability constraint (3.28) in addition to its lifetime utility as follows:

$$W_t = \sum_{j=j_0(t)}^{J} \beta^j \left[u(c_{t,j}, 1 - n_{m_{t,j}}) + \lambda_t u_{c_{t,j}} c_{t,j} \right] - \lambda_t A_{t,0}, \tag{3.31}$$

where λ_t be the Lagrange multiplier associated with the implementability constraint. Notice that the multiplier λ_t will be strictly positive if it is necessary for the government to use distortionary taxation.

The Ramsey problem in this life-cycle economy consists of choosing an allocation to maximize the discounted sum of pseudo welfare function, subject to law of motion of human capital and the feasibility constraint for t = 0,. . . :

$$\begin{aligned}
\max_{\{c_{t,j}, h_{t,j+1}, x_{t,j}, n_{m_{t,j}}, k_{t+J+1}\}} \quad & \sum_{t=-J}^{\infty} \gamma^t W_t, \\
\text{s.t.} \quad c_t + x_t + (1+n)k_{t+1} + g_t \quad &= \quad f(k_t, z_t) + (1 - \delta_k)k_t, \\
h_{t,1} \quad &= \quad G_0(x_{t,0}), \\
h_{t,j+1} \quad &= \quad (1 - \delta_h)h_{t,j} + G(x_{t,j}, h_{t,j}), \quad j = 1, ..., J-1.
\end{aligned}$$

Notice that the government budget constraint is omitted in the Ramsey problem because it is automatically satisfied by Walras's law as long as the individual's present-value budget constraint holds.

Let $\gamma^t \phi_t$ be the Lagrange multiplier associated with the feasibility constraint and $\gamma^{t+j}\hat{\eta}_{t,j}$ with the law of motion of human capital. The first-order conditions for the Ramsey problem with age-dependent taxes are

$$
\begin{aligned}
\gamma^t W_{c_{t,j}} &= \gamma^{t+j}\phi_{t+j}\mu_j, \quad j = 0, ..., J, \\
\gamma^t \phi_t \mu_0 &= \gamma^t W_{x_{t,0}} + \gamma^t \hat{\eta}_{t,0} G_0'(x_{t,0}), \\
\gamma^{t+j}\phi_{t+j}\mu_j &= \gamma^{t+j}\hat{\eta}_{t,j} G_{x_{t,j}}, \quad j = 1, ..., J-1, \\
\gamma^t W_{n_{m_{t,0}}} &= \gamma^t \phi_t \mu_0 f_{z_t}, \\
\gamma^t W_{n_{m_{t,j}}} &= \gamma^{t+j}\phi_{t+j}\mu_j h_{t,j} f_{z_{t+j}}, \quad j = 1, ..., J, \\
\gamma^t \phi_t (1+n) &= \gamma^{t+1}\phi_{t+1}\left(1 + f_{k_{t+1}} - \delta_k\right), \\
\gamma^{t+j}\hat{\eta}_{t,j} &= \gamma^{t+j+1}\phi_{t+j+1}\mu_{j+1} f_{z_{t+j+1}} n_{m_{t,j+1}} \\
&+ \gamma^{t+j+1}\hat{\eta}_{t,j+1}\left(1 - \delta_h + G_{h_{t,j+1}}\right), j = 0, ..., J-2, \\
\gamma^{t+J-1}\hat{\eta}_{t,J-1} &= \gamma^{t+J}\phi_{t+J}\mu_J f_{z_{t+J}} n_{m_{t,J}},
\end{aligned}
$$

where $W_{c_{t,j}}$ and $W_{n_{m_{t,j}}}$ are the derivatives of W_t with respect to $c_{t,j}$ and $n_{m_{t,j}}$, respectively. Then, the steady-state solution is characterized by the following conditions:

$$W_{c_j} = \gamma^j \phi \mu_j, \qquad j = 0, ..., J, \tag{3.32}$$

$$\phi \mu_0 = W_{x_{t,0}} + \hat{\eta}_0 G_0'(x_0), \tag{3.33}$$

$$\gamma^j \phi \mu_j = \hat{\eta}_j G_{x_j}, \qquad j = 1, ..., J-1, \tag{3.34}$$

$$W_{n_{m_0}} = \phi \mu_0 f_z, \tag{3.35}$$

$$W_{n_{m_j}} = \gamma^j \phi \mu_j h_j f_z, \qquad j = 1, ..., J, \tag{3.36}$$

$$1 + n = \gamma \left(1 + f_k - \delta_k\right), \tag{3.37}$$

$$\hat{\eta}_j = \gamma^{j+1} \phi \mu_{j+1} f_z n_{m_{j+1}} + \hat{\eta}_{j+1} \left(1 - \delta_h + G_{h_{j+1}}\right), \quad j = 0, ..., J-2, \tag{3.38}$$

$$\hat{\eta}_{J-1} = \gamma^J \phi \mu_J f_z n_{m_J}. \tag{3.39}$$

Notice that the previous set of equations consists of $4J+3$ equations. Moreover, we have the steady state feasibility constraint, the implementability constraint, as well as J human capital law of motion, implying that we have a total of $5J+5$ equations. At the same time, we have the following variables to solve for: $\{c_j, n_{m_j}\}_{j=0}^{J}$, $\{x_j, h_{j+1}\}_{j=0}^{J-1}$, and k. That amounts to $4J+3$ variables. In addition, we have the multipliers ϕ, λ, and $\{\hat{\eta}_j\}_{j=0}^{J-1}$, for a total of $5J+5$ variables.[7]

Substituting out the multipliers ϕ and $\{\hat{\eta}_j\}_{j=0}^{J-1}$, we can rewrite the previous

[7] This is very convenient, as it means that the final steady state is self-contained, i.e. it is independent of the transition. This is a nice property of Ramsey problems with overlapping generations, by contrast to Ramsey problems with infinitely-lived agents, which feature steady state allocations that are a function of the transition (in particular the initial conditions.)

steady-state necessary conditions as follows:

$$\gamma W_{c_j} = W_{c_{j+1}}(1+n), \qquad j = 0, ..., J-1, \tag{3.40}$$

$$1+n = \gamma\left(1+f_k-\delta_k\right), \tag{3.41}$$

$$W_{n_{m_0}} = W_{c_0} f_z, \tag{3.42}$$

$$W_{n_{m_j}} = W_{c_j} f_z h_j, \qquad j = 1, ..., J, \tag{3.43}$$

$$\frac{W_{c_0} - W_{x_0}}{G_0'(x_0)} = W_{c_1}\left[f_z n_{m_1} + \frac{(1-\delta_h + G_{h_1})}{G_{x_1}}\right], \tag{3.44}$$

$$\frac{W_{c_j}}{G_{x_j}} = W_{c_{j+1}}\left[f_z n_{m_{j+1}} + \frac{\left(1-\delta_h + G_{h_{j+1}}\right)}{G_{x_{j+1}}}\right], j = 1, ..., J-2, \tag{3.45}$$

$$\frac{W_{c_{J-1}}}{G_{x_{J-1}}} = W_{c_J} f_z n_{m_J}. \tag{3.46}$$

To derive the optimal tax policy, we compare the optimality conditions of the Ramsey problem to those of the consumer's problem. Let's first characterize optimal tax rates on capital income. Combining the equation (3.40) and (3.41), we obtain the following government's optimality condition of equating its marginal rate of substitution between consumption today and tomorrow to the return on capital (net of depreciation):

$$\frac{W_{c_j}}{W_{c_{j+1}}} = \frac{u_{c_j}\left[1+\lambda(1+H_j^c)\right]}{\beta u_{c_{j+1}}\left[1+\lambda(1+H_{j+1}^c)\right]} = 1+\hat{r}, \tag{3.47}$$

where H_j^c is defined as follows:

$$H_j^c \equiv \frac{u_{c_j c_j} c_j}{u_{c_j}}, \qquad j = 1, ..., J,$$

$$H_0^c \equiv \frac{u_{c_0 c_0} c_0 - u_{c_0 n_{m_0}} n_{m_0} - u_{c_0 c_0} s_0\left(\frac{G_0(x_{t,0})}{G_0'(x_{t,0})} - x_{t,0}\right)}{u_{c_0}}.$$

The individual counterpart to (3.47) is given by (3.22),

$$\frac{u_{c_j}}{\beta u_{c_{j+1}}} = 1 + r_{j+1}. \tag{3.48}$$

Observe that the government's marginal rate of substitution takes the implementability constraint into account, whereas that of an individual does not. Moreover, the government considers before-tax prices, while individuals face after-tax prices. Dividing (3.47) by (3.48), we can characterize the optimal tax rates on capital income,

$$\frac{1+\hat{r}_{j+1}}{1+r_{j+1}} = \frac{1+\lambda(1+H_j^c)}{1+\lambda(1+H_{j+1}^c)}, \qquad j = 0, ..., J-1. \tag{3.49}$$

Notice that capital income tax rates are different from zero unless $H_j^c = H_{j+1}^c$.

Similarly, we derive the optimal tax rates on labor income. Combining the equation (3.32), (3.35), and (3.36) for a positve labor supply, we obtain the government's optimality conditions for leisure and labor supply:

$$\frac{W_{n_{m_0}}}{W_{c_0}} = \frac{u_{n_{m_0}}\left[1+\lambda H_0^{n_m}\right]}{u_{c_0}\left[1+\lambda(1+H_0^c)\right]} = \hat{w}_t, \tag{3.50}$$

$$\frac{W_{n_{m_j}}}{W_{c_j}} = \frac{u_{n_{m_j}}\left[1+\lambda H_j^{n_m}\right]}{u_{c_j}\left[1+\lambda(1+H_j^c)\right]} = h_j\hat{w}_t, \qquad j = 1, ..., J, \tag{3.51}$$

where $H_j^{n_m}$ is defined as follows:

$$H_j^{n_m} \equiv \frac{u_{c_j n_{m_j}} c_j}{u_{n_{m_j}}}, \qquad j = 1, ..., J \tag{3.52}$$

$$H_0^{n_m} \equiv \frac{u_{c_0 n_{m_0}} c_0 - u_{n_{m_0}} - u_{n_{m_0} n_{m_0}} n_{m_0} - u_{c_0 n_{m_0}} s_0 \left(\frac{G_0(x_{t,0})}{G_0'(x_{t,0})} - x_{t,0}\right)}{u_{n_{m_0}}}. \tag{3.53}$$

Because any optimal fiscal policy has to satisfy the consumer's optimality conditions, we can compare (3.50) and (3.51) to its analogue (3.23) and (3.24) from the consumer's optimization problem. By doing so, the optimal tax rates on labor income are given by

$$\tau_j^w = \frac{\lambda\left(H_j^{n_m} - H_j^c - 1\right)}{1 + \lambda H_j^{n_m}}, \qquad j = 0, ..., J. \tag{3.54}$$

Notice that the labor income tax rates will be zero only if $H_j^{n_m} - H_j^c = 1$ or $\lambda = 0$.

Finally, we derive optimal human capital policies by combining the government's optimality conditions (3.44)-(3.46) and consumer's counterparts (3.25)-(3.27) as follows:

$$\begin{aligned} 1 + \tau_0^s &= \left(1 - \left(\frac{W_{x_0}}{W_{c_0}}\right)\right)\left(\frac{1+\hat{r}}{1+r_1}\right) \\ &\times \left(\frac{w_1 n_{m_1} G_{x_1} + (1-\tau_1^s)(1-\delta_h + G_{h_1})}{\hat{w} n_{m_1} G_{x_1} + (1-\delta_h + G_{h_1})}\right) \end{aligned} \tag{3.55}$$

$$\begin{aligned} 1 + \tau_j^s &= \left(\frac{w_{j+1} n_{m_{j+1}} G_{x_{j+1}} + \left(1-\tau_{j+1}^s\right)\left(1-\delta_h + G_{h_{j+1}}\right)}{\hat{w} n_{m_{j+1}} G_{x_{j+1}} + \left(1-\delta_h + G_{h_{j+1}}\right)}\right) \\ &\times \left(\frac{1+\hat{r}}{1+r_{j+1}}\right), j = 1, ..., J-2 \end{aligned} \tag{3.56}$$

$$1 + \tau_{J-1}^s = \left(\frac{1+\hat{r}}{1+r_J}\right)\left(1-\tau_J^w\right). \tag{3.57}$$

Notice that the optimal tax rate on investment in human capital at age j will be equal to zero only if both the labor and capital income tax rates are zero.

We summarize the previous results in the following propositions.

Proposition 3. *The optimal tax rate on capital income is different from zero unless* $H_j^c = H_{j+1}^c$ *and the optimal tax rate on labor income is different from zero unless* $H_j^{n_m} - H_j^c = 1$. *The optimal tax rate on investment in human capital is different from* *zero unless* $H_j^c = H_{j+1}^c$ *and* $H_j^{n_m} - H_j^c = 1$*. Finally, if* $\lambda = 0$*, all tax rates are equal to zero by definition.*

Notice that the government should also choose an initial tax rate on investment in human capital ($s_{t,0}$) (See the implementability constraint in (28)). Taking the first-order condition, we get the following equation:

$$\gamma^t \lambda_t u_{c_{t,0}} \left(x_{t,0} - \frac{G_0(x_{t,0})}{G_0'(x_{t,0})} \right) = 0.$$

Because γ^t and $u_{c_{t,0}}$ are non-zero and $x_{t,0} \neq \frac{G_0(x_{t,0})}{G_0'(x_{t,0})}$ in general, $\lambda_t = 0$ should hold at the optimum. Therefore, under a complete tax system, all taxes are zero except for age-0 taxes, achieving a first-best allocation.

3.4.2 Non-verifiable Investments in Human Capital

In this section, we assume that the government cannot observe an individual's investment in human capital, and thus cannot directly tax or subsidize human capital investment. We view this assumption as reasonable, although we realize that some human capital investments (e.g. tuition) is indeed verifiable. Nevertheless, we proceed by assuming that the government relies on (age-dependent) capital and labor income taxes and explore how the non-verifiability assumption affects the optimal tax prescription.[8]

[8]It should be noted that we still assume consumption taxes away here: evidently, this is not without loss of generality, as with consumption taxes, the solution remains a first-best allocation following Proposition 3.1.

The individual's budget constraint is given by

$$c_{t,0} + a_{t,1} + x_{t,0} = w_{t,0} n_{m_{t,j_0(t)}} + (1 + r_{t,0}) a_{t,0}, \tag{3.58}$$

$$c_{t,j} + a_{t,j+1} + x_{t,j} = w_{t,j} h_{t,j} n_{m_{t,j}} + (1 + r_{t,j}) a_{t,j}, \quad j = 1, ..., J, \tag{3.59}$$

$$h_{t,1} = G_0(x_{t,0}), \tag{3.60}$$

$$h_{t,j+1} = (1 - \delta_h) h_{t,j} + G(x_{t,j}, h_{t,j}), \quad j = 1, ..., J-1, \tag{3.61}$$

Note that after-tax prices on investment in human capital over lifetime are one. The solution to the consumer's problem can be characterized by the following first-order conditions:

$$u_{c_{t,j}} = \beta u_{c_{t,j+1}} (1 + r_{t,j+1}), \qquad j = 0, .., J-1, \tag{3.62}$$

$$u_{n_{m_{t,0}}} = u_{c_{t,0}} w_{t,0}, \tag{3.63}$$

$$u_{n_{m_{t,j}}} = u_{c_{t,j}} h_{t,j} w_{t,j}, \qquad j = 1, ..., J, \tag{3.64}$$

$$\frac{1}{G_0'(x_{t,0})} = \frac{1}{1 + r_{t,1}} \left[w_{t,1} n_{m_{t,1}} + \frac{\left(1 - \delta_h + G_{h_{t,1}}\right)}{G_{x_{t,1}}} \right], \tag{3.65}$$

$$\frac{1}{G_{x_{t,j}}} = \frac{1}{1 + r_{t,j+1}} \left[w_{t,j+1} n_{m_{t,j+1}} + \frac{\left(1 - \delta_h + G_{h_{t,j+1}}\right)}{G_{x_{t,j+1}}} \right], j = 1, .., J-2, \tag{3.66}$$

$$\frac{1}{G_{x_{t,J-1}}} = \frac{w_{t,J} n_{m_{t,J}}}{1 + r_{t,J}}. \tag{3.67}$$

Observe that the tax or subsidy on investments in human capital no longer appears in the above conditions. Using the conditions (3.62)-(3.64), we can substitute out prices from the individual's present-value budget constraint, and obtain the following generation-t's implementability constraint:[9]

$$\sum_{j=0}^{J} \beta^j \left(u_{c_{t,j}} \left(c_{t,j} + x_{t,j} \right) - u_{n_{m_{t,j}}} n_{m_{t,j}} \right) = u_{c_{t,0}} a_{t,0} (1 + r_{t,0}).$$

Notice that the condition (3.62) defines $\tau^k_{t,j+1}$ for j=0, ..., J-1 and (3.63) and (3.64) determine $\tau^w_{t,j}$ for j=0, ..., J. Hence, as in Jones et al. (1997), the conditions (3.65)-(3.67) need to be additionally imposed as constraints in the Ramsey problem: this is where the incompleteness of the tax code manifests itself. Following Jones et al. (1997), we use (3.62)-(3.64) to eliminate prices from (3.65)-(3.67), which results in the following:

$$\begin{aligned}
\psi\left(t+1\right) &= \psi\left(\kappa_{t,0}, \kappa_{t,1}\right) = \frac{u_{c_{t,0}}}{G'_{t,0}(x_{t,0})} - \beta \frac{u_{c_{t,1}}}{G_{x_{t,1}}} \left[\frac{G_{x_{t,1}} u_{n_{m_{t,1}}} n_{m_{t,1}}}{u_{c_{t,1}} h_{t,1}} + \left(1 - \delta_h + G_{h_{t,1}}\right) \right], \\
&= u_{c_{t,0}} h_{t,1} G_{x_{t,1}} - \beta G'_{t,0}(x_{t,0}) \left[G_{x_{t,1}} u_{n_{m_{t,1}}} n_{m_{t,1}} + u_{c_{t,1}} h_{t,1} \left(1 - \delta_h + G_{h_{t,1}}\right) \right], \\
\psi\left(t+j+1\right) &= \psi\left(\kappa_{t,j}, \kappa_{t,j+1}\right) \\
&= \frac{u_{c_{t,j}}}{G_{x_{t,j}}} - \beta \frac{u_{c_{t,j+1}}}{G_{x_{t,j+1}}} \left[\frac{G_{x_{t,j+1}} u_{n_{m_{t,j+1}}} n_{m_{t,j+1}}}{u_{c_{t,j+1}} h_{t,j+1}} + \left(1 - \delta_h + G_{h_{t,j+1}}\right) \right], \\
&= u_{c_{t,j}} h_{t,j+1} G_{x_{t,j+1}} \\
&- \beta G_{x_{t,j}} \left[G_{x_{t,j+1}} u_{n_{m_{t,j+1}}} n_{m_{t,j+1}} + u_{c_{t,j+1}} h_{t,j+1} \left(1 - \delta_h + G_{h_{t,j+1}}\right) \right], \\
& \quad j = 1, ..., J-2, \\
\psi\left(t+J\right) &= \psi\left(\kappa_{t,J-1}, \kappa_{t,J}\right) = \frac{u_{c_{t,J-1}}}{G_{x_{t,J-1}}} - \beta \left(\frac{u_{n_{m_J}} n_{m_J}}{h_J} \right), \\
&= u_{c_{t,J-1}} h_J - \beta G_{x_{t,J-1}} u_{n_{m_J}} n_{m_J},
\end{aligned}$$

where $\kappa_{t,0} = (c_{t,0}, n_{m_{t,0}}, x_{t,0})$, $\kappa_{t,j} = (c_{t,j}, n_{m_{t,j}}, h_{t,j}, x_{t,j})$, $j = 1, ..., J-1$, and $\kappa_{t,J} = (c_{t,J}, n_{m_{t,J}}, h_{t,J})$.

[9]See Appendix B.2 for more detail.

We can now define the Ramsey problem in which the government choose an allocation in order to maximize the discounted sum of pseudo welfare function (W_t), subject to law of motion of human capital, feasibility constraint, and the consumer's Euler equation of human capital investment ψ for t = 0, . . . ,

$$\begin{aligned}
\max_{\left\{c_{t,j},h_{t,j+1},x_{t,j},n_{m_{t,j}},k_{t+J+1}\right\}} \quad & \sum_{t=0}^{\infty}\gamma^t W_t \\
\text{s.t.} \qquad W_t &= \sum_{j=0}^{J}\beta^j\left[u(c_{t,j},1-n_{m_{t,j}})+\lambda_t\left(u_{c_{t,j}}\left(c_{t,j}+x_{t,j}\right)-u_{n_{m_{t,j}}}n_{m_{t,j}}\right)\right] \\
& \quad -\lambda_t u_{c_{t,0}}a_{t,0}(1+r_{t,0}), \\
c_t+x_t+g_t+(1+n)k_{t+1} &= f\left(k_t,z_t\right)+(1-\delta_k)k_t, \\
h_{t,1} &= G_0(x_{t,0}), \\
h_{t,j+1} &= (1-\delta_h)h_{t,j}+G(x_{t,j},h_{t,j}), \quad j=1,...,J-1, \\
\psi\left(t+j+1\right) &= 0, \quad j=0,...,J-1
\end{aligned}$$

Recall that $\gamma^t\lambda_t$ is the Lagrange multiplier associated with the implementability, $\gamma^t\phi_t$ with the feasibility constraint, and $\gamma^{t+j}\hat{\eta}_{t,j}$ with the law of motion of human capital. Additionally, let $\gamma^{t+j}\nu_{t,j}$ be the multiplier associated with the constraint of the consumer's Euler equation of human capital, i.e. $\psi\left(t+j+1\right)=0$. The first-order conditions for the problem are

$$\begin{aligned}
\gamma^t W_{c_{t,0}} &= \gamma^t \phi_t \mu_0 - \gamma^t \nu_{t,0} \psi_{c_{t,0}}(t+1), \\
\gamma^t W_{c_{t,j}} &= \gamma^{t+j} \phi_{t+j} \mu_j \\
&- \gamma^t \left(\nu_{t,j-1} \psi_{c_{t,j}}(t+j) + \nu_{t,j} \psi_{c_{t,j}}(t+j+1) \right), \quad j = 1, ..., J-1 \\
\gamma^t W_{c_{t,J}} &= \gamma^{t+J} \phi_{t+J} \mu_J - \gamma^t \nu_{t,J-1} \psi_{c_{t,J}}(t+J) \\
\gamma^t \phi_t \mu_0 &= \gamma^t W_{x_{t,0}} + \gamma^t \hat{\eta}_{t,0} G_0'(x_{t,0}) + \gamma^t \nu_{t,0} \psi_{x_{t,0}}(t+1) \\
\gamma^{t+j} \phi_{t+j} \mu_j &= \gamma^t W_{x_{t,j}} + \gamma^t \hat{\eta}_{t,j} G_{x_{t,j}} \\
&+ \gamma^t \left(\nu_{t,j-1} \psi_{x_{t,j}}(t+j) + \nu_{t,j} \psi_{x_{t,j}}(t+j+1) \right), \quad j = 1, ..., J-1 \\
\gamma^t W_{n_{m_{t,0}}} &= \gamma^t \phi_t \mu_0 f_{z_t} + \gamma^t \nu_{t,0} \psi_{n_{m_{t,0}}}(t+1) \\
\gamma^t W_{n_{m_{t,j}}} &= \gamma^{t+j} \phi_{t+j} \mu_j h_{t,j} f_{z_{t+j}} \\
&+ \gamma^t \left(\nu_{t,j-1} \psi_{n_{m_{t,j}}}(t+j) + \nu_{t,j} \psi_{n_{m_{t,j}}}(t+j+1) \right), \quad j = 1, ..., J-1 \\
\gamma^t W_{n_{m_{t,J}}} &= \gamma^{t+J} \phi \mu_J h_{t,J} f_{z_{t+J}} + \gamma^t \nu_{t,J-1} \psi_{n_{m_{t,J}}}(t+J), \\
\gamma^t \phi_t (1+n) &= \gamma^{t+1} \phi_{t+1} \left(1 + f_{k_{t+1}} - \delta_k \right) \\
\gamma^t \hat{\eta}_{t,j} &= \gamma^{t+j+1} \phi_{t+j+1} \mu_{j+1} f_{z_{t+j+1}} n_{m_{t,j+1}} + \gamma^t \hat{\eta}_{t,j+1} \left(1 - \delta_h + G_{h_{t,j+1}} \right) \\
&+ \gamma^t \left(\nu_{t,j} \psi_{h_{t,j+1}}(t+j+1) + \nu_{t,j+1} \psi_{h_{t,j+1}}(t+j+2) \right), j = 0, ..., J-2 \\
\gamma^t \hat{\eta}_{t,J-1} &= \gamma^{t+J} \phi_{t+J} \mu_J f_{z_{t+J}} n_{m_{t,J}} + \gamma^t \nu_{t,J-1} \psi_{h_{t,J}}(t+J)
\end{aligned}$$

The previous first-order conditions at the steady state can be expressed by:

$$W_{c_0} = \phi\mu_0 - \nu_0\psi_{c_0}(1), \tag{3.68}$$

$$W_{c_j} = \gamma^j\phi\mu_j - \left(\nu_{j-1}\psi_{c_j}(j) + \nu_j\psi_{c_j}(j+1)\right), \quad j = 1, ..., J-1 \tag{3.69}$$

$$W_{c_J} = \gamma^J\phi\mu_J - \nu_{J-1}\psi_{c_J}(J), \tag{3.70}$$

$$\phi\mu_0 = W_{x_0} + \hat{\eta}_0 G_0'(x_0) + \nu_0\psi_{x_0}(1), \tag{3.71}$$

$$\gamma^j\phi\mu_j = W_{x_j} + \gamma^j\hat{\eta}_j G_{x_j} + \left(\nu_{j-1}\psi_{x_j}(j) + \nu_j\psi_{x_j}(j+1)\right)$$

$$, \qquad j = 1, ..., J-1 \tag{3.72}$$

$$W_{n_{m_0}} = \phi\mu_0 f_z + \nu_0\psi_{n_{m_0}}(1), \tag{3.73}$$

$$W_{n_{m_j}} = \gamma^j\phi\mu_j h_j f_z + \Big(\nu_{j-1}\psi_{n_{m_j}}(j) + \nu_j\psi_{n_{m_j}}(j+1)\Big), \quad j = 1, ..., J-1 \tag{3.74}$$

$$W_{n_{m_J}} = \gamma^J\phi\mu_J h_J f_z + \nu_{J-1}\psi_{n_{m_J}}(J), \tag{3.75}$$

$$1 + n = \gamma\left(1 + f_k - \delta_k\right) \tag{3.76}$$

$$\hat{\eta}_j = \gamma^{j+1}\phi\mu_{j+1} f_z n_{m_{j+1}} + \hat{\eta}_{j+1}\left(1 - \delta_h + G_{h_{j+1}}\right) + \left(\nu_j\psi_{h_{j+1}}(j+1) + \nu_{j+1}\psi_{h_{j+1}}(j+2)\right)$$

$$, \qquad j = 0, ..., J-2 \tag{3.77}$$

$$\hat{\eta}_{J-1} = \gamma^J\phi\mu_J f_z n_{m_J} + \nu_{J-1}\psi_{h_J}(J) \tag{3.78}$$

Combining the equation (3.68)-(3.70), we get the government's optimality conditions of equating its marginal rate of substitution between consumption today and tomorrow to the return on capital (net of depreciation):

$$\frac{W_{c_0}}{W_{c_1}} = \frac{u_{c_0}\left[1+\lambda(1+M_0^c)\right]}{\beta u_{c_1}\left[1+\lambda(1+M_1^c)\right]} = (1+\hat{r})\left(\frac{1-\frac{\nu_0\psi_{c_0}(1)}{\phi\mu_0}}{1-\frac{\left(\nu_0\psi_{c_1}(1)+\gamma\nu_1\psi_{c_1}(2)\right)}{\gamma\phi\mu_1}}\right), \quad (3.79)$$

$$\frac{W_{c_j}}{W_{c_{j+1}}} = \frac{u_{c_j}\left[1+\lambda(1+M_j^c)\right]}{\beta u_{c_{j+1}}\left[1+\lambda(1+M_{j+1}^c)\right]}$$

$$= (1+\hat{r})\left(\frac{1-\frac{\left(\nu_{j-1}\psi_{c_j}(j)+\gamma\nu_j\psi_{c_j}(j+1)\right)}{\gamma\phi\mu_j}}{1-\frac{\left(\nu_j\psi_{c_{j+1}}(j+1)+\gamma\nu_{j+1}\psi_{c_{j+1}}(j+2)\right)}{\gamma\phi\mu_{j+1}}}\right), j=1,\ldots,J-1 \quad (3.80)$$

$$\frac{W_{c_{J-1}}}{W_{c_J}} = \frac{u_{c_{J-1}}\left[1+\lambda(1+M_{J-1}^c)\right]}{\beta u_{c_J}\left[1+\lambda(1+M_J^c)\right]}$$

$$= (1+\hat{r})\left(\frac{1-\frac{\left(\nu_{J-2}\psi_{c_{J-1}}(J-1)+\gamma\nu_{J-1}\psi_{c_{J-1}}(J)\right)}{\gamma\phi\mu_{J-1}}}{1-\frac{\nu_{J-1}\psi_{c_J}(J)}{\gamma\phi\mu_{J+1}}}\right), \quad (3.81)$$

where M_j^c is defined as follows:

$$M_j^c \equiv \frac{u_{c_jc_j}(c_j+x_j)-u_{n_{m_j}c_j}n_{m_j}}{u_{c_j}}, \qquad j=0,...,J. \quad (3.82)$$

The individual counterpart of (3.79)-(3.81) is given by

$$\frac{u_{c_j}}{\beta u_{c_{j+1}}} = 1+r_{j+1}, \quad j=0,\ldots,J-1. \quad (3.83)$$

Dividing (3.80)-(3.82) by (3.83), we can characterize the optimal capital income tax rates,

$$\frac{1+\hat{r}}{1+r_1} = \frac{1+\lambda(1+M_0^c)}{1+\lambda(1+M_1^c)}\left(\frac{1-\frac{\left(\nu_0\psi_{c_1}(1)+\gamma\nu_1\psi_{c_1}(2)\right)}{\gamma\phi\mu_1}}{1-\frac{\nu_0\psi_{c_0}(1)}{\phi\mu_0}}\right), \quad (3.84)$$

$$\frac{1+\hat{r}}{1+r_{j+1}} = \frac{1+\lambda(1+M_j^c)}{1+\lambda(1+M_{j+1}^c)} \quad (3.85)$$

$$\times \left(\frac{1-\frac{\left(\nu_j\psi_{c_{j+1}}(j+1)+\gamma\nu_{j+1}\psi_{c_{j+1}}(j+2)\right)}{\gamma\phi\mu_{j+1}}}{1-\frac{\left(\nu_{j-1}\psi_{c_j}(j)+\gamma\nu_j\psi_{c_j}(j+1)\right)}{\gamma\phi\mu_j}}\right), j=1,...,J-2, \quad (3.86)$$

$$\frac{1+\hat{r}}{1+r_J} = \frac{1+\lambda(1+M_{J-1}^c)}{1+\lambda(1+M_J^c)}\left(\frac{1-\frac{\nu_{J-1}\psi_{c_J}(J)}{\gamma\phi\mu_J}}{1-\frac{\left(\nu_{J-2}\psi_{c_{J-1}}(J-1)+\gamma\nu_{J-1}\psi_{c_{J-1}}(J)\right)}{\gamma\phi\mu_{J-1}}}\right), \quad (3.87)$$

Observe that capital income tax rates are almost always non-zero. That is, even if we find a class of utility function satisfying the condition of $M_j^c = M_{j+1}^c$, it is hardly ever for the remaining terms to be zero, especially for the Lagrange multiplier associated with the constraint of the consumer's Euler equation of human capital, i.e. $\psi(j+1)$. In addition, notice that even in the non-verifiable human capital investments case, the class of additive log utility and multiplicative CRRA does not satisfy the condition of $M_j^c = M_{j+1}^c$.

Similarly, we combine equations (3.68)-(3.70) with (3.73)-(3.75) to derive optimal tax rates on labor income,

$$\frac{W_{n_{m_0}}}{W_{c_0}} = \frac{u_{n_{m_0}}\left[1+\lambda M_0^{n_m}\right]}{u_{c_0}\left[1+\lambda(1+M_0^c)\right]} = \frac{\hat{w}+\nu_0\psi_{n_{m_0}}(1)}{1-\nu_0\psi_{c_0}(1)}, \tag{3.88}$$

$$\begin{aligned}\frac{W_{n_{m_j}}}{W_{c_j}} &= \frac{u_{n_{m_j}}\left[1+\lambda M_j^{n_m}\right]}{u_{c_j}\left[1+\lambda(1+M_j^c)\right]} \\ &= \frac{h_j\hat{w}+\frac{\left(\nu_{j-1}\psi_{n_{m_j}}(j)+\gamma\nu_j\psi_{n_{m_j}}(j+1)\right)}{\gamma\psi\mu_j}}{1-\frac{\left(\nu_{j-1}\psi_{c_j}(j)+\gamma\nu_j\psi_{c_j}(j+1)\right)}{\gamma\psi\mu_j}}, j=1,...,J-1,\end{aligned} \tag{3.89}$$

$$\frac{W_{n_{m_J}}}{W_{c_J}} = \frac{u_{n_{m_J}}\left[1+\lambda M_J^{n_m}\right]}{u_{c_J}\left[1+\lambda(1+M_J^c)\right]} = \frac{h_J\hat{w}+\frac{\nu_{J-1}\psi_{n_{m_J}}(J)}{\gamma\psi\mu_J}}{1-\frac{\nu_{J-1}\psi_{c_J}(J)}{\gamma\psi\mu_J}}, \tag{3.90}$$

where $M_j^{n_m}$ is defined as follows:

$$M_j^{n_m} \equiv \frac{u_{c_j n_{m_j}}(c_j+x_j) - u_{n_{m_j}n_{m_j}}n_{m_j} - u_{n_{m_j}}}{u_{n_{m_j}}}, \qquad j=0,...,J \tag{3.91}$$

Because any optimal fiscal policy has to satisfy the consumer's optimality conditions, we can compare (3.88)-(3.90) to their consumer's counterpart, that is,

$$\frac{u_{n_{m_0}}}{u_{c_0}} = w_0 \tag{3.92}$$

$$\frac{u_{n_{m_j}}}{u_{c_j}} = h_j w_j, \qquad j=1,...,J \tag{3.93}$$

By combining (3.88)-(3.90) with the above equations, the optimal tax rates on labor income are given by

$$1-\tau_0^w = \left(\frac{1+\lambda(1+M_0^c)}{1+\lambda M_0^{n_m}}\right)\left(\frac{\hat{w}+\frac{\nu_0\psi_{n_{m_0}}(1)}{\psi\mu_0}}{\hat{w}\left(1-\frac{\left(\nu_{-1}\psi_{c_0}(0)+\gamma\nu_0\psi_{c_0}(1)\right)}{\gamma\psi\mu_0}\right)}\right), \tag{3.94}$$

$$1-\tau_j^w = \left(\frac{1+\lambda(1+M_j^c)}{1+\lambda M_j^{n_m}}\right)$$

$$\times \left(\frac{h_j\hat{w}+\frac{\left(\nu_{j-1}\psi_{n_{m_j}}(j)+\gamma\nu_j\psi_{n_{m_j}}(j+1)\right)}{\gamma\psi\mu_j}}{h_j\hat{w}\left(1-\frac{\left(\nu_{j-1}\psi_{c_j}(j)+\gamma\nu_j\psi_{c_j}(j+1)\right)}{\gamma\psi\mu_j}\right)}\right), j=1,...,J-1 \tag{3.95}$$

$$1-\tau_J^w = \left(\frac{1+\lambda(1+M_J^c)}{1+\lambda M_J^{n_m}}\right)\left(\frac{h_J\hat{w}+\frac{\nu_{J-1}\psi_{n_{m_J}}(J)}{\gamma\psi\mu_J}}{h_J\hat{w}\left(1-\frac{\nu_{J-1}\psi_{c_J}(J)}{\gamma\psi\mu_J}\right)}\right). \tag{3.96}$$

Similar to optimal tax rates on capital income, notice that tax rates on labor income will be non-zero almost surely. Although we find a class of preference that satisfies $M_j^c = -1$ and $M_j^{n_m} = 0$, the remaining terms are almost unlikely to equal zero, in particular the Lagrange multiplier associated with the constraint of the consumer's Euler equation of human capital.

3.5 Conclusion

We study the impact of endogenous human capital accumulation on optimal fiscal policy within a life-cycle growth model. By contrast to the findings of Jones et al. (1997) that all interest taxes rates are zero in the long run of an infinitely-lived agent model with endogenous human capital formation, we show that all taxes are almost never zero in a life-cycle framework. While the key assumption to obtain this non-zero taxation result in our framework is that investment in human capital is not verifiable to the government, the same assumption is made in Jones et al. (1997). In our context, without human capital policy to offset any distortion on skill formation, capital income taxes are used in order to mitigate the distortionary effects of the labor tax on investments in human capital. Future work would be to supplement our analytical results by quantitatively characterizing optimal labor and capital income tax rates and exploring how different they are from optimal tax policies in a life-cycle model with exogenous life-cycle productivity.

APPENDIX A
APPENDIX TO CHAPTER 1

I describe sample selection criterions for the NLSY79 and NPSAS90 used for the model calibration and assessments.

A.1 National Longitudinal Surveys of Youth in 1979

The NLSY79 reflect a random survey of American youth ages 14-21 at the beginning of 1979, and keep track of the youth for 20 years. Since the data contain detailed information on family background, school attendance, and labor market outcomes, it is ideal for my study. I use this data to estimate earnings growth rates by education level and document different college attendance rates across disparate students' family income and ability.

For differences in college enrollment across family income and ability, I follow the same sample selection criterions as in Belly and Lochner (2007). First, I exclude the military and the nonblack, non-Hispanic disadvantaged subsamples. Also, I confine the sample to youths between ages 14 and 17 at 1979 to have reliable information on schooling attendance and parental income. So, I use average family income when youth are ages 16-17 right before college attendance, and drop out those not living with their parents at these ages. Any missing data on schooling attendance and family income are eliminated.

The NLSY data include comparable measures of ability embodied in AFQT scores, a composite derived from tests of arithmetic reasoning, word knowledge, paragraph comprehension, and numerical operations. These four tests are a subset of full set of tests taken by all respondents in the Armed Forces Vocational Aptitude Battery (ASVAB). I use the AFQT scores as a proxy for a student's ability, and categorize individuals according to their AFQT score quartiles.

A.2 National Postsecondary Student Aid Study in 1990

To explore college students' borrowing across different family income and ability, I use the National Postsecondary Student Aid Study in 1990 (NPSAS90). The NPSAS90 data contain rich information on the characteristics of students in college education and their finaning.

I first restrict the sample to full-time full-year students in college. The full-time full-year students are defined those enrolled for 12 or more semester credits for 9 or more months. I use a parental income as a measure of family income for dependent students. For independent students, I use total income of the students (and spouse, if married). As a proxy for students' ability, I use their SAT scores. If students only have ACT scores, I transform them to comparable SAT scores by following concordance tables provided by ACT and the College Board. Missing data on Stafford loan amounts, family income, and SAT or ACT scores are eliminated.

APPENDIX B
APPENDIX TO CHAPTER 3

B.1 Derivation of implementability constraints under verifiable human capital investment

The implementability constraints can be obtained by using the consumer optimality conditions to substitute out prices from the consumer's present-value lifetime budget constraint. The present value budget constraint of the consumer is given by:

$$\begin{aligned}\sum_{j=0}^{J}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right)(q_{t,j}c_{t,j}+s_{t,j}x_{t,j}) &= \sum_{j=1}^{J}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right)w_{t,j}h_{t,j}n_{m_{t,j}} \\ &+ w_{t,0}n_{m_{t,0}}+a_{t,0}(1+r_{t,0})\end{aligned}$$

Using the assumption that G is constant returns to scale, we can show that the consumer's budget constraint in equilibrium can be greatly simplified. Specifically, consider the term $\sum_{j=1}^{J}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right)\left(s_{t,j}x_{t,j}-w_{t,j}h_{t,j}n_{m_{t,j}}\right)$. Using the law of motion of human capital and the consumer's optimality conditions in (3.25)-(3.27), we can rewrite this term as follows:

$$\begin{aligned}&\sum_{j=1}^{J}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right)\left(s_{t,j}x_{t,j}-w_{t,j}h_{t,j}n_{m_{t,j}}\right)\\ =\ &\sum_{j=1}^{J}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right)\left[s_{t,j}\left(\frac{h_{t,j+1}-(1-\delta_h)h_{t,j}-G_{h_{t,j}}h_{t,j}}{G_{x_{t,j}}}\right)-w_{t,j}h_{t,j}n_{m_{t,j}}\right]\\ =\ &-h_{t,1}\left(\frac{1}{1+r_{t,1}}\right)\left(w_{t,1}n_{m_{t,1}}+\frac{s_{t,1}\left(1-\delta_h+G_{h_{t,1}}\right)}{G_{x_{t,1}}}\right)\\ +\ &\sum_{j=2}^{J}h_{t,j}\left[\left(\prod_{i=1}^{j-1}\frac{1}{1+r_{t,i}}\right)\frac{s_{t,j-1}}{G_{x_{t,j-1}}}-\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right)\left(w_{t,j}n_{m_{t,j}}+\frac{s_{t,j}\left(1-\delta_h+G_{h_{t,j}}\right)}{G_{x_{t,j}}}\right)\right]\\ =\ &-\frac{G_0(x_{t,0})s_{t,0}}{G_0'(x_{t,0})}\end{aligned}$$

Lastly, using the condition (3.22) to substitute out interest rates, we finally derive the generation-t's implementability constraint:

$$\begin{aligned}\sum_{j=0}^{J}\beta^j u_{c_{t,j}} c_{t,j} &= \left(\frac{u_{c_{t,0}}}{q_{t,0}}\right)\left[a_{t,0}(1+r_{t,0}) - \left(s_{t,0}x_{t,0} - \left(\frac{u_{n_{m_{t,0}}} q_{t,0}}{u_{c_{t,0}}}\right) n_{m_{t,0}}\right) + \frac{G_0(x_{t,0}) s_{t,0}}{G_0'(x_{t,0})}\right] \\ &= \left(\frac{u_{c_{t,0}}}{q_{t,0}}\right)\left[a_{t,0}(1+r_{t,0}) - s_{t,0}\left(x_{t,0} - \frac{G_0(x_{t,0})}{G_0'(x_{t,0})}\right)\right] + u_{n_{m_{t,0}}} n_{m_{t,0}} \\ &\equiv A_{t,0}\end{aligned}$$

Notice that each generation has its own implementability constraint in a life-cycle economy.

B.2 Derivation of implementability constraints under non-verifiable human capital investment

We can derive the individual's present-value budget constraint as follows:

$$\begin{aligned}\sum_{j=0}^{J}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right)(c_{t,j}+x_{t,j}) &= \sum_{j=1}^{J}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t,i}}\right) w_{t,j} h_{t,j} n_{m_{t,j}} \\ &+ w_{t,0} n_{m_{t,0}} + a_{t,0}(1+r_{t,0})\end{aligned}$$

Using the individual's first order conditions (3.62)-(3.64), we can substitute out prices from the present-value budget constraint,

$$\begin{aligned}\sum_{j=0}^{J}\left(\frac{\beta^j u_{c_{t,j}}}{u_{c_{t,0}}}\right)(c_{t,j}+x_{t,j}) &= \sum_{j=1}^{J}\left(\frac{\beta^j u_{c_{t,j}}}{u_{c_{t,0}}}\right)\left(\frac{u_{n_{m_{t,j}}}}{u_{c_{t,j}} h_{t,j}}\right) h_{t,j} u_{n_{m_{t,j}}} \\ &+ \left(\frac{u_{n_{m_{t,0}}}}{u_{c_{t,0}}}\right) n_{m_{t,0}} + a_{t,0}(1+r_{t,0})\end{aligned}$$

So, the implementability constraint is given by

$$\sum_{j=0}^{J}\beta^j\left(u_{c_{t,j}}(c_{t,j}+x_{t,j}) - u_{n_{m_{t,j}}} n_{m_{t,j}}\right) = u_{c_{t,0}} a_{t,0}(1+r_{t,0}).$$

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