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Essays on money, credit, and monetary policy

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University of Iowa

ESSAYS ON MONEY, CREDIT, AND MONETARY POLICY

by

Hyung Sun Choi

An Abstract

Of a thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy degree
in Economics in
the Graduate College of
The University of Iowa

August 2008

Thesis Supervisor: Professor Stephen D. Williamson

ABSTRACT

This dissertation studies the relationship between the existence of multiple means of payment and the effects of monetary policy.

Chapter 1 studies the endogenous choice of means of payment when holding money is risky. In steady state equilibrium, the marginal rate of substitution of cash goods for credit goods depends on the crime rate as well as the nominal interest rate. Credit may be used when the return on money is not positive. A positive money injection reduces the crime rate and transactions costs. When the crime rate is positive, welfare increase with inflation, and the Friedman rule is not necessarily optimal.

Chapter 2 discusses the risk-sharing role of monetary policy when the asset market is segmented. A fraction of households exchange money for interest-bearing government nominal bonds in the asset market. The government injects money through open market operations with only participating households. In equilibrium, money is nonneutral and there are distributional effects of monetary policy. With idiosyncratic endowment risk, monetary policy cannot perfectly insure households. The optimal money growth rate can be positive and the Friedman rule is not optimal in general.

Chapter 3 is built on the work of Chapter 1 and Chapter 2 in exploring distributional effects of monetary policy when individuals can choose means of payment among alternatives. In equilibrium, monetary policy has distributional effects. With a positive money injection, some households purchase a greater variety of goods with cash while others purchase a greater variety of goods with credit. Consumption may increase or decrease because household can choose alternative means of payment. Credit is used to dampen fluctuations in consumption arising from monetary policy. The liquidity effect arises under a certain condition.

Abstract Approved: ________________________________

Thesis Supervisor

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Title and Department

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Iowa City, Iowa

CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

Hyung Sun Choi

has been approved by the Examining Committee for the thesis requirement for the Doctor of Philosophy degree in Economics at the August 2008 graduation.

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To my Parents, my brother Joon Sun Choi, and my adviser Stephen D. Williamson.

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This dissertation studies the relationship between the existence of multiple means of payment and the effects of monetary policy.

Chapter 1 studies the endogenous choice of means of payment when holding money is risky. In steady state equilibrium, the marginal rate of substitution of cash goods for credit goods depends on the crime rate as well as the nominal interest rate. Credit may be used when the return on money is not positive. A positive money injection reduces the crime rate and transactions costs. When the crime rate is positive, welfare increase with inflation, and the Friedman rule is not necessarily optimal.

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CHAPTER 1
MONEY AND CRIME IN A CASH-IN-ADVANCE MODEL

1.1 Introduction

The main purpose of this paper is to develop a rich model to explain the coexistence of credit and money and to study its qualitative implications. Historically, currency has been a resilient medium of exchange because everyone accepts it without the user's identity being revealed. However, given its usefulness in anonymous transactions, money has been a target for theft. Credit has no such theft problem, ignoring the possibility of identity theft. However, using credit incurs transactions costs, for example, the time waiting to check credit history, or a fee paid to the credit card company. Therefore, people carry both cash and credit and choose a different means of payment to purchases goods as there are circumstances where either can be advantageous.

There have been several studies of the coexistence of multiple systems of payment such as Lucas and Stokey (1987), Prescott (1987), Ireland (1994), and Lacker and Schreft (1996). In Lucas and Stokey (1987), cash goods and credit goods exist and a consumer has to use cash to purchase cash goods and credit to purchase credit goods. The consumer's choice of means of payment is exogenously fixed and the use of credit does not create transactions costs. Thus, the model cannot explain a consumer's choice between cash and credit when purchasing goods. Further, in this model monetary policy does not affect the choice of means of payment and its implications for a consumer's behavior is limited. On the other hand, Prescott (1987), Ireland (1994), and Lacker and Schreft (1996) have developed a model where a consumer carries multiple means of payment and can choose a different means of payment in a goods market for higher utility. In addition to an opportunity cost of holding an asset as in Lucas and Stokey (1987), they introduce transactions costs of credit as a tradeoff. In equilibrium, a consumer purchases some goods with cash and other goods with credit. Cash and credit coexist because consumers substitute credit for money if there is a higher nominal interest rate and they use more cash if transactions costs increase. A consumer uses credit for larger purchases and cash for smaller ones. Furthermore, Ireland (1994) and Lacker and Schreft (1996) show how credit is substituted for money to purchase a greater variety of goods. In Prescott (1987), Ireland (1994), and Lacker and Schreft (1996), all the results are valid only if the nominal interest rate on assets is positive. A consumer uses credit to obtain interest from nominal assets. If the nominal interest rate is zero, then the credit market disappears and consumers carry only money.

However, in my model, money and credit can coexist with a nonpositive nominal interest rate by adding another opportunity cost of money, theft, in addition to the nominal interest rate. Consumers are likely to reduce cash in their pockets and use more credit if the risk of theft is high. The model is built on Ireland (1994). A unit mass of households exists and each household consists of a shopper and a worker. Households trade nominal bonds in the asset market. A worker can steal shoppers' money from other households before shoppers start to purchase consumption goods.

Unlike He, Huang, and Wright (2005, 2006), a worker never fails to steal cash from other shoppers. In steady state equilibrium, the marginal rate of substitution of cash goods for credit goods depends on not only the nominal interest rate but also the risk of holding money which is different from that in Prescott (1987), Ireland (1994), and Lacker and Schreft (1996). A consumer prefers spending on credit when the nominal interest rate is higher or when the crime rate is higher. Thus, consumers can spend on credit although the nominal interest rate is not positive. A change in the money growth rate affects both the nominal interest rate and the crime rate and welfare can increase with inflation.

Introducing the risk of holding money is not a new idea. In Prescott (1987), he points out that the risk of loss by theft or fire can be an additional feature of money. Also, He, Huang, and Wright (2005, 2008) discuss that cash is relatively riskier than other means of payment, for example checks, debit, or credit cards, to carry around. He, Huang, and Wright (2005, 2008) introduce a theft problem in a search framework. Buyers can lose their money while they search for consumption goods because sellers can be thieves at any time. A bank can endogenously arise and buyers can choose to deposit their money in it for safekeeping purposes. Once they deposit money in the bank, they carry checks as another means of payment in the meantime. In equilibrium, with endogenous theft, money can possibly disappear when the storage fee is smaller than the money they could lose. Buyers deposit even with a negative nominal interest rate and inflation can be optimal as in my model.

However, He, Huang, and Wright (2005, 2008) cannot explain why a consumer brings multiple means of payment together in the goods market because a buyer carries either money or checks, but not both. The model cannot distinguish consumption purchased with cash and checks, so monetary policy implications do not explain the effects on consumption with cash and with checks unlike in my model. Also, in their model the individual consumption level does not change smoothly with theft. The effects of monetary policy on individual consumption and theft do not arise in a continuous manner. For some parameters, individual consumption depends on inflation when there is no theft, but for other parameters individual consumption is independent of inflation when there is theft which depends on inflation. My model provides richer implications for both the behavior of consumers who bring cash and credit together and the relation between individual consumption and theft.

The remainder of the paper is organized as follows. Section 2 describes the basic setups. Section 3 discusses a money-only economy. Section 4 studies a cash-credit economy and its implications in steady state. Section 5 presents monetary policy and welfare implications. Section 6 concludes.

1.2 The Economic Environment and Timing

Time is discrete and indexed by $t = 0, 1, 2, ...$ The economy consists of a unit mass of infinitely-lived households. Each household consists of two agents: a worker and a shopper. A continuum of spatially separated markets indexed by $i \in [0,1]$ exists at each period. In each market i, workers produce and sell distinct, perishable consumption goods indexed by $i \in [0,1]$ in every period.

The household has preferences given by

$$U(\{c_t, x_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 ln(c_t(i))di - x_t \right\}$$

where β is the discount factor; $c_t(i)$ represents consumption goods purchased at market i at time t; x_t represents transactions costs.

At the beginning of each period t, each household enters the period with M_t units of currency and B_t units of one-period nominal bonds. Then, the household learns the money growth rate, θ_t. The government controls the nominal money supply through nominal lump-sum transfers, $P_t \tau_t$ and the government budget constraint is

$$M_{t+1}^s = M_t^s + P_t \tau_t$$

$$M_{t+1}^s = \theta_t M_t^s.$$

where P_t is the average price level of consumption goods.

The asset market opens and each household trades one-period nominal bonds, B_t, and money. Each bond sells for q_t units of money in period t and is a claim to one unit of money in period $t+1$. The asset market closes.

A worker and a shopper at each household leave their houses for a goods market. At the goods market, first, a worker decides how much time to devote to working, n_t^w, and stealing, n_t^s, given one unit of time:

$$n_t^w + n_t^s = 1.$$

Then, a worker steals a proportion of money from shoppers of other households before they start to exchange goods. A worker does not steal his/her own shopper's money.

The amount of money that a worker steals is given by $\phi(n_t^s)\bar{M}_t$ where $\bar{M}_t$ denotes the quantity of money held by other shoppers. The function, $\phi(n_t^s)$, denotes the yield from effort in stealing,

$$\phi(n_t^s) = \begin{cases} \pi n_t^s & \text{,if } n_t^s \in [0, \frac{1}{\pi}] \\ 1 & \text{,if } n_t^s \in (\frac{1}{\pi}, 1], \end{cases}$$

where $\pi > 1$ is the degree of stealing efficiency; a worker steals more with a higher π. A shopper cannot spend the stolen money, $\phi(n_t^s)\bar{M}_t$, within the period, but he/she can use it to pay off within-period IOUs at the end of the period. Note that nominal bonds are safe from theft because shoppers bring only cash in the goods market to purchase consumption goods.

After workers steal money, workers and shoppers start to exchange consumption goods. In the market, a worker produces consumption goods with a linear technology, $y_t = n_t^w$. While a worker produces goods, a shopper from each household travels from market to market purchasing good i for the household's consumption. When shoppers purchase goods, they have two ways of acquiring goods from market i. One is to use non-interest bearing currency that has two opportunity costs: the gross nominal interest rate, R_t, and the risk of holding money, $\bar{n}_t^s \in [0, 1]$. The other is to spend on credit, incurring transactions costs $\gamma(i) > 0$, to purchase good i where $\gamma(i)$ is increasing, differentiable on i, $\gamma(0) = 0$ and $\lim_{i \to 1} \gamma(i) = \infty$. Transactions costs appear as effort in the household's preferences,

$$x_t(i) = \int_0^1 \xi_t(i)\gamma(i)di$$

where $\xi_t(i)$ is an indicator variable: $\xi_t(i) = 1$ if shoppers use credit to buy good i at time t and $\xi_t(i) = 0$ if shoppers use currency to buy good i at time t. The cash-in-advance constraint in the goods market is

$$\int_0^1 P_t(i)(1-\xi_t(i))c_t(i)di \leq (1-\phi(\bar{n}_t^s))\left(M_t + P_t\tau_t - q_tB_{t+1} + B_t\right).$$

where $P_t(i)$ is the price of consumption good i.

At the end of each period, all agents return home. Workers bring wages, $P_tn_t^w$, and money they stole from other shoppers in the goods market, $\phi(n_t^s)\bar{M}_t$, back home. No further trade or barter is allowed. The household budget constraint is

$$\int_0^1 P_t(i)c_t(i)di + M_{t+1} = (1-\phi(\bar{n}_t^s))\left(M_t + P_t\tau_t - q_tB_{t+1} + B_t\right) + P_tn_t^w + \phi(n_t^s)\bar{M}_t.$$

Figure 1.1 summarizes the timing of events within a period.

1.3 Money-only Economy

Understanding a money-only economy with theft and studying its monetary policy implications will provide a useful background to a further discussion of a cash-credit economy with theft in Section 4. In a money-only economy, the household is not able to spend on credit and transactions costs disappear, or $x_t(i) = 0$. The household's optimization problem is then, for all i,

$$\max_{c_t(i),n_t^w,n_t^s,m_{t+1},b_{t+1}} \sum_{t=0}^{\infty} \beta^t \int_0^1 ln(c_t(i))di$$

subject to

$$\int_0^1 p_t(i)c_t(i)di \leq (1-\phi(\bar{n}_t^s))\left(m_t + p_t\tau_t - \theta_tq_tb_{t+1} + b_t\right) \tag{1.1}$$

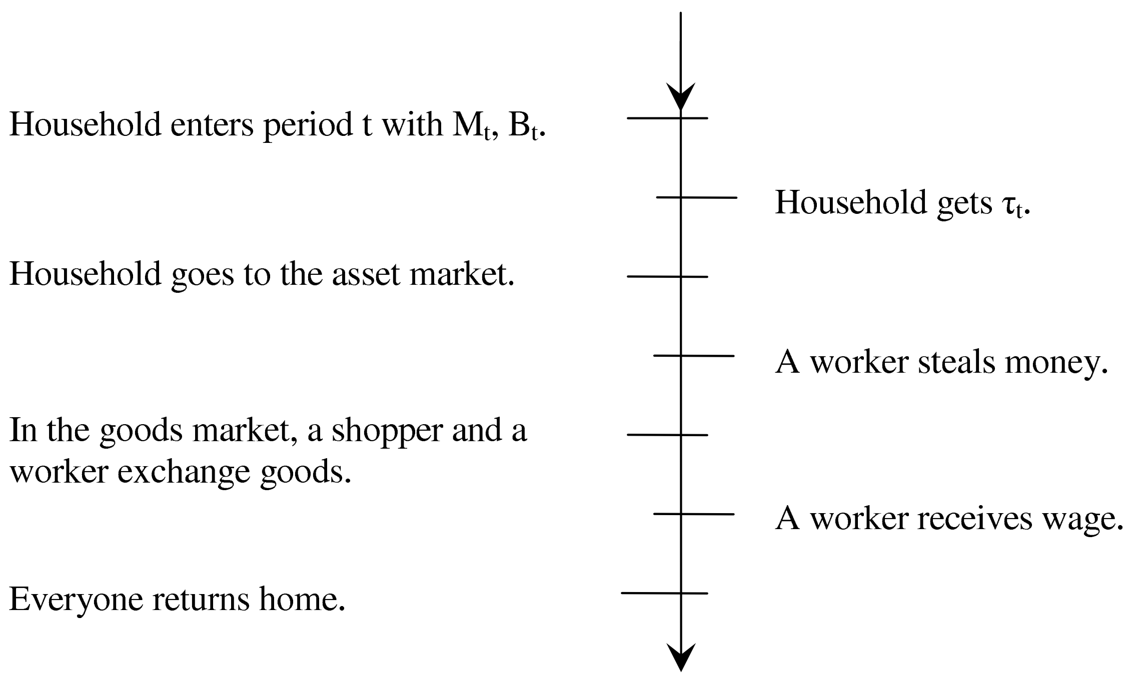

Figure 1.1: Timing in period t

$$\int_0^1 p_t(i)c_t(i)di + \theta_t m_{t+1} = (1-\phi(\bar{n}_t^s))\,(m_t + p_t\tau_t - \theta_t q_t b_{t+1} + b_t) + p_t n_t^w + \phi(n_t^s)\bar{m}_t \tag{1.2}$$

$$n_t^w + n_t^s = 1 \tag{1.3}$$

$$m_{t+1} \geq 0, \qquad n_t^s \geq 0, \qquad n_t^w \geq 0 \tag{1.4}$$

$$b_{t+1} \geq \bar{b} \tag{1.5}$$

where inequalities (1.4) are the nonnegativity constraint. Inequality (1.5) is the no Ponzi-scheme constraint and $\bar{b} \leq 0$ is a scalar. To make the household's dynamic optimization problem stationary, divide all the constraints by M_t^s.

Definition: A competitive equilibrium consists of the sequences $\{c_t(i), n_t^s, n_t^w,$ $m_{t+1}, b_{t+1}, M_{t+1}^s, p_t(i), \tau_t, R_t\}_{t=0}^{\infty}$ where $i \in [0,1]$ such that:

1. $\{c_t(i), n_t^s, n_t^w, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ solves the household's problem given $\{M_{t+1}^s, p_t(i),$ $\tau_t, R_t\}_{t=0}^{\infty}$.

2. Markets clear in every period:

   (a) Bond Market: $b_{t+1} = 0$,

   (b) Money Market: $m_{t+1} = 1$,

   (c) Goods Market: for each market i,

$$n_t^w = c_t(i),$$

$$n_t^w di = \int_0^1 c_t(i)di,$$

(d) By symmetry, $n^s_t = \bar{n}^s_t$,

(e) $m_t + p_t\tau_t = \theta = \bar{m}_t$,

From (a), no one will hold any nominal bonds in equilibrium since all households have the same preferences. Equation (e) represents that in a symmetric equilibrium every worker steals and brings home the same amount of money.

In steady state equilibrium, first, $p(i) = p$ for all market i holds because the marginal utility of consumption, the same linear production technology that is independent from i, and the goods market clearing condition in each market i imply a no-arbitrage condition. The marginal utility of consumption is identical across market i: for all $i, j \in [0, 1]$,

$$c(i)p(i) = c(j)p(j).$$

Since $c(i) = n^w$ for each market i and n^w is independent from market i, every market sells goods at the same price, $p(i) = p$.

Next, binding cash-in-advance and budget constraints in equations (1.1) and (1.2) imply

$$p\int_0^1 c(i)di = (1 - \pi n^s)\theta$$
$$\int_0^1 c(i)di = (1 - n^s),$$

The equations above imply the equilibrium solution is $n^s = 1$ and $c(i) = 0$ for all i. Suppose a worker steals all the cash that shoppers carry to the goods market before they purchase consumption goods. That is, $n^s \in [1/\pi, 1]$, where $\phi(n^s) = 1$. Since shoppers lose all of their cash, they cannot purchase anything and consume nothing although a worker produces goods. Thus, a worker will not produce any goods, i.e. $n^s = 1$, since no one is able to purchase any goods. In this circumstance, shoppers have to bring money into the goods market even though they know they will lose it anyway because money is the only medium of exchange for consumption goods. Money becomes a mere object of stealing because it does not play a role as medium of exchange. The price level, p, is indeterminate. The existence of stealing breaks down the nominal economy without alternative means of payment since money is no use for purchasing goods in equilibrium.

Furthermore, in the money-only economy, the solution that a worker always steals and no one produces is unique. That is, $n^s \in [0, 1/\pi)$ that includes interior solutions does not hold in equilibrium. First, suppose a worker decides not to steal at all, or $n^s = 0$. Then, a worker spends all time on working and producing consumption goods. However, this is not an equilibrium because the marginal value of stealing, $\phi(n^s)\theta$, is always bigger than the marginal value of working, p. Formally, in equilibrium, if $n^s = 0$, then the marginal value of stealing is zero, $\phi(n^s)\theta = 0$, since $\phi(n^s) = 0$. Next, from equations (1.1) and (1.2),

$$p \int_0^1 c(i)di = \theta$$
$$\int_0^1 c(i)di = 1,$$

and the marginal value of working is greater than zero, $p = \theta > 0$. Therefore,

$$p > \phi(n^s)\theta = 0$$

where stealing is relatively cheaper. That is, the marginal cost of not working is relatively cheaper in this circumstance and a worker always has an incentive to steal and in equilibrium $n^s = 0$ cannot hold.

Now, suppose $n^s \in (0, 1/\pi)$ which is an interior solution. Then, a worker steals some of cash hold by other shoppers. In equilibrium, the choice of n^s implies that the price level is

$$p = \pi\theta.$$

However, given the price level, markets do not clear. Total consumption is less than total output:

$$\frac{1}{\pi} - n^s < 1 - n^s,$$

from the binding cash-in-advance constraint in equation (1.1),

$$p \int_0^1 c(i)di = (1 - \pi n^s)\theta,$$

and the market clearing condition in equation (1.2),

$$\int_0^1 c(i)di = (1 - n^s).$$

A worker produces less and steals more. The marginal benefit from stealing is greater than the marginal benefit from working. In other words, the relative price of stealing is cheaper. A worker devotes his time to stealing other shoppers' money and a worker steals all the time and produces nothing. Thus, equilibrium does not exist when $n^s \in (0, 1/\pi)$.

1.4 Cash-Credit Economy

In this section shoppers can choose their means of payment as cash or credit when they purchase good i. Shoppers can spend using credit to avoid theft, but transactions costs, $x_t(i)$, arise as shoppers start to use credit rather than money.

1.4.1 Optimization

In a cash-credit economy, the household solves the following: for all i,

$$\max_{c_t(i),x_t,n_t^w,n_t^s,\xi_t(i),m_{t+1},b_{t+1}} \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 ln(c_t(i))di - x_t \right\}$$

subject to

$$\int_0^1 p_t(i)(1-\xi_t(i))c_t(i)di \leq (1-\phi(\bar{n}_t^s))\left(m_t + p_t\tau_t - \theta_t q_t b_{t+1} + b_t\right) \tag{1.6}$$

$$\int_0^1 p_t(i)c_t(i)di + \theta_t m_{t+1} = (1-\phi(\bar{n}_t^s))\left(m_t + p_t\tau_t - \theta_t q_t b_{t+1} + b_t\right) + p_t n_t^w + \phi(n_t^s)\bar{m}_t \tag{1.7}$$

$$x_t = \int_0^1 \xi_t(i)\gamma(i)di \tag{1.8}$$

$$n_t^w + n_t^s = 1 \tag{1.9}$$

$$m_{t+1} \geq 0, \qquad n_t^s \geq 0, \qquad n_t^w \geq 0 \tag{1.10}$$

$$b_{t+1} \geq \bar{b} \tag{1.11}$$

where inequalities (1.10) are the nonnegativity constraints and inequality (1.11) is the no Ponzi-scheme constraint with $\bar{b} \leq 0$ a scalar. Again, divide all the constraints by M_t^s to make the household's dynamic optimization problem stationary.

Definition: A competitive equilibrium consists of the sequences $\{c_t(i), n_t^s, n_t^w, x_t, \xi_t, m_{t+1}, b_{t+1}, M_{t+1}^s, p_t(i), \tau_t, R_t\}_{t=0}^{\infty}$ where $i \in [0, 1]$ such that:

1. $\{c_t(i), n_t^s, n_t^w, x_t, \xi_t, m_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ solves the household's problem given $\{M_{t+1}^s, p_t(i), \tau_t, R_t\}_{t=0}^{\infty}$.

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   (a) Bond Market: $b_{t+1} = 0$,

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$$n_t^w = \int_0^1 c_t(i)di,$$

   (d) By symmetry, $n_t^s = \bar{n}_t^s$,

   (e) $m_t + p_t\tau_t = \theta = \bar{m}$,

From (a), no one will hold any nominal bonds in equilibrium since all households have the same preferences. Equation (e) states that in a symmetric equilibrium every worker steals and brings home the same amount of money.

Drop t subscripts and let primes denote variables dated $t+1$. The household's optimization problem translates into the following a dynamic programming problem:

$$v(m, b; \theta) = \max_{c,x,n^w,n^s,\xi,m',b'} \int_0^1 ln(c(i))di - x + \beta v(m', b'; \theta)$$

subject to (1.6), (1.7), (1.8), (1.9), (1.10), and (1.11).

1.4.2 Equilibrium

The household determines n^s and n^w given the linear expected rate of return, $\phi(n^s)\bar{m}$, and the linear production technology, pn^w, where p is the price of working[1]. Therefore, corner solutions, $n^s = 0$ or $n^s \in [1/\pi, 1]$, are potentially possible. However, in equilibrium, the household will not choose corner solutions because the marginal benefit of stealing equals to the marginal cost of stealing only when $n^s \in [0, 1/\pi)$.

First, suppose a worker steals more than $1/\pi$ unit of time, $n^s \in [1/\pi, 1]$. Then, $\phi(n^s) = 1$ and shoppers lose all of their cash before entering the goods market. Without cash, shoppers purchase consumption goods only with credit for all markets $i \in [0, 1]$ and $i^* = 1$. A shopper puts infinite transactions costs, $\lim_{i\to 1} \gamma(i) = \infty$, and each household gets a utility of negative infinity in equilibrium. That is, the marginal benefit from working is clearly bigger than the marginal benefit from stealing. Therefore, a worker has an incentive to work more and spends less than $1/\pi$ of time for stealing.

Second, suppose a worker does not steal at all or $n^s = 0$. Then, in equilibrium, the cash-in-advance constraint in equation (1.6) and the budget constraint in equation (1.7) are

$$p \int_{i^*}^{1} c^0(i)di = \theta$$

$$p \int_{0}^{i^*} c^1(i)di = p - \theta,$$

[1] the real wage is normalized to one

and the resource constraint is

$$\int_{i^*}^{1} c^0(i)di + \int_0^{i^*} c^1(i)di = 1.$$

They imply the price level should be bigger than the money growth rate,

$$p \geq \theta$$

since

$$\int_{i^*}^{1} c^0(i)di = \frac{\theta}{p} \leq 1$$
$$\int_0^{i^*} c^1(i)di = 1 - \frac{\theta}{p} < 1.$$

When $p \geq \theta$, the household demands for cash exceeds the supply. In order to obtain more cash, each household can either work more or steal from other households. A worker cannot provide more labor since $n^w = 1$ and the marginal benefit from stealing is larger than the marginal benefit from working. Thus, a worker has an incentive to work less and starts to spend more time to steal cash.

In equilibrium, therefore, the household chooses $n^s \in [0, 1/\pi)$ and a set of interior solutions, the choices of $c(i)$, $\xi(i)$, n^s, m', and b' are as follows assuming λ_1 and λ_2 denote the Lagrange multipliers to the cash-in-advance constraint and the budget constraint:

$$\frac{1}{c(i)} - \lambda_1(1 - \xi(i))p - \lambda_2 p = 0 \tag{1.12}$$
$$\frac{1}{c^1(i)} - \lambda_2 p = 0 \qquad ,\text{if } \xi(i) = 1$$
$$\frac{1}{c^0(i)} - (\lambda_1 + \lambda_2)p = 0 \qquad ,\text{if } \xi(i) = 0$$

$$\xi(i) = \begin{cases} 1, & \text{if } ln(c^1(i)) - \gamma(i) - \lambda_2 c^1(i)p > ln(c^0(i)) - c^0(i)(\lambda_1 + \lambda_2)p \\ 0, & \text{if } ln(c^1(i)) - \gamma(i) - \lambda_2 c^1(i)p < ln(c^0(i)) - c^0(i)(\lambda_1 + \lambda_2)p \end{cases} \tag{1.13}$$

$$\lambda_2(-p + \pi\theta) \leq 0 \qquad (< 0 \text{ holds, if } n^s = 0) \tag{1.14}$$

$$\beta(\lambda_1' + \lambda_2')(1 - \pi n^{s\prime}) = \lambda_2\theta \tag{1.15}$$

$$\beta(\lambda_1' + \lambda_2')(1 - \pi n^{s\prime}) = (\lambda_1 + \lambda_2)\theta q(1 - \pi n^s). \tag{1.16}$$

First, in steady state equilibrium, $p(i) = p$ for all market i holds as of money-only economy. At the goods market, each shopper faces the same marginal utility of consumption in equation (1.12) across market i, so for all $i, j \in [0, 1]$,

$$c(i)p(i) = c(j)p(j).$$

Since $c(i) = n^w$ for each market i and n^w is independent from market i, every market sells goods at the same price, $p(i) = p$.

However, in equations (1.12), the marginal utilities of consumption purchased with credit and cash differ once a shopper choose a means of payment to purchase goods at market i. For example, if a shopper purchases good i with credit, then $c(i) = c^1(i)$. Shoppers do not have to hold a physical form of credit to purchase good

i and c^1 shows up only in the budget constraint. If a shopper purchases good i with currency, then $c(i) = c^0(i)$. Since shoppers require to hold cash in their hands to purchase good i, c^0 is subject to both the cash-in-advance and budget constraints.

In equation (1.13), the decision of $\xi(i)$ comes from a tradeoff between the transactions costs of credit, $\gamma(i)$, and the opportunity cost of holding money, $c^0(i)\lambda_1$. The household picks credit to purchase good i when the marginal benefit of using credit, $ln(c^1(i)) - \gamma(i) - \lambda_2 c^1(i)p$, is larger than that of using cash, $ln(c^0(i)) - c^0(i)(\lambda_1 + \lambda_2)p$ and vice versa.

In equilibrium, inequality (1.14), determines the crime rate, n^s, and $p = \pi\theta$ from the binding cash-in-advance constraint. The marginal benefit from stealing, $\pi\bar{m}$, is equal to the marginal cost of giving up wages for stealing, p. That is, the returns on stealing and working are identical, so a worker is indifferent between stealing and working. Finally, equations (1.15) and (1.16) imply that the intertemporal marginal rate of substitution matters for deciding m' and b'.

In steady state, for all t, the list of variables is constant. Then, equation (1.15) becomes

$$\frac{\beta}{\theta}(1 - \pi n^s)(\lambda_1 + \lambda_2) = \lambda_2 \tag{1.17}$$

and from equation (1.16), the nominal interest rate is

$$R = \frac{1}{q} = \frac{\theta}{\beta}.$$

They imply that the binding cash-in-advance constraint holds, $\lambda_1 > 0$, when

$$\frac{R}{(1 - \pi n^s)} > 1. \tag{1.18}$$

In equation (1.18), the risk of losing money, n^s, and the nominal interest rate, R, are opportunity costs of holding money while R is the opportunity cost of holding money in Lucas and Stokey (1987), Prescott (1987), Ireland (1994), and Lacker and Schreft (1996). With a positive n^s, the cash-in-advance constraint can bind and inequality (1.18) holds even though the nominal interest rate is zero, $\theta = \beta$, or negative, $\theta < \beta$. That is, in the case of a high crime rate, the household is willing to pay an implicit fee, i.e. a negative interest rate, for safekeeping purposes as for example, as of He, Huang, and Wright (2005, 2008). Without theft, $n^s = 0$, inequality (1.18) becomes $\theta > \beta$. The nominal interest rate should be positive and the discount rate, β, makes a lower bound of the money growth rate, θ, as of Lucas and Stokey (1987), Prescott (1987), Ireland (1994), and Lacker and Schreft (1996).

Now, equations (1.12) and (1.17) imply that

$$c^0(\lambda_2, n^s, \theta) = \left\{ c^* | c^* = \frac{\beta(1 - \pi n^s)}{\theta} \frac{1}{\lambda_2 p} \right\} \tag{1.19}$$
$$c^1(\lambda_2, n^s, \theta) = \left\{ c^* | c^* = \frac{1}{\lambda_2 p} \right\}.$$

First, in equations (1.19), consumption with credit, c^1, is greater than consumption with money, c^0. In other words, a shopper spends on credit for larger purchases and uses cash for smaller purchases. For large purchases, shoppers can use credit rather than cash to avoid theft. Second, a shopper using the same means of payment for some market i acquires the same amount of consumption goods, i.e. $c^0 = c^0(i)$ and $c^1 = c^1(i)$, because the marginal value of cash is the same across markets and so is the marginal value of wealth. The volume of one specific consumption good purchased by credit at market, i, is independent of transactions costs. Transactions costs increase only when the household purchases a greater variety of consumption goods with credit. Therefore, aggregate consumption and transactions costs simplify as follows:

$$\int_0^1 c(i)di = \int_0^{i^*} c^1(i)di + \int_{i^*}^1 c^0(i)di = i^* c^1 + (1-i^*)c^0,$$

$$\int_0^1 \xi(i)\gamma(i)di = \int_0^{i^*} \gamma(i)di,$$

and the goods market clearing condition is

$$i^* c^1 + (1-i^*)c^0 = 1 - n^s. \tag{1.20}$$

where $c^1 = c^1(\lambda_2, n^s, \theta)$ and $c^0 = c^0(\lambda_2, n^s, \theta)$. Also, binding constraints with equation (1.14) become

$$(1-i^*)c^0\pi = (1-\pi n^s) \tag{1.21}$$

$$i^* c^1 \pi = \pi - 1.$$

Last, from equation (1.19), the marginal rate of substitution of c^0 for c^1, c^1/c^0, depends on both the crime rate and the nominal interest rate unlike Lucas and Stokey (1987), Prescott (1987), Ireland (1994), and Lacker and Schreft (1996):

$$\frac{\theta}{\beta}\frac{1}{(1-\pi n^s)} = \frac{c^1}{c^0}. \tag{1.22}$$

Shoppers adjust their ratio of consumption with credit to consumption with cash with any change of the crime rate as well as with any change of the nominal interest rate. For example, shoppers can adjust the ratio of consumption with credit to consumption with cash not to obtain the nominal interest but to avoid theft. That is, if the crime rate, n^s, increases, then a shopper purchases larger amount of goods with cash in market i compared to purchases with credit.

Inequalities (1.13) determine a cut off, $i^*(\theta, n^s)$, between credit and cash choices,

$$i^* \in \left\{ i^* | \gamma(i^*) = ln\left(\frac{c^1}{c^0}\right) \right\} \tag{1.23}$$

where transactions costs are equal to the marginal rate of substitution of c^0 for c^1. Thus, a higher i^* implies a higher marginal rate of substitution of c^0 for c^1. A shopper replaces credit for cash and purchases a larger variety of goods with credit in the market. In other words, a shopper uses credit to acquire good i where $i < i^*$ and uses cash to acquire good i where $i > i^*$.

Finally, a set of solutions, (i^*, n^s, c^0, c^1), in steady state equilibrium follows. Equations (1.21) and (1.22) determine a closed-form solution for i^* and equations (1.22) and (1.23) determine the crime rate, n^s,

$$i^* = \frac{\pi - 1}{\frac{\theta}{\beta} + \pi - 1} \tag{1.24}$$

and

$$n^s = \frac{1}{\pi}\left(1 - \frac{\theta}{\beta} e^{-\gamma(i^*)}\right) \tag{1.25}$$

where $n^s \in [0, \frac{1}{\pi})$, $i^* \in [0, 1)$, and inequality (1.18) imply that

$$e^{\gamma(i^*)} > \frac{\theta}{\beta} > 0$$

$$n^s > \frac{1}{\pi}\left(1 - \frac{\theta}{\beta}\right)$$

and $i^* \in [0,1]$ is satisfied with any $\theta > 0$. Equations (1.21), (1.24), and (1.25) determine consumption with cash and credit,

$$c^0 = \frac{1}{\pi}\left(\frac{\theta}{\beta} + \pi - 1\right) e^{-\gamma(i^*)} \tag{1.26}$$

$$c^1 = \frac{1}{\pi}\left(\frac{\theta}{\beta} + \pi - 1\right)$$

and aggregate consumption with cash and credit is

$$(1 - i^*)c^0 = \frac{\theta}{\pi\beta} e^{-\gamma(i^*)}$$

$$i^* c^1 = \frac{\pi - 1}{\pi}.$$

1.5 Monetary Policy and Welfare

The effects of monetary policy on cash-credit choice, i^*, work differently with theft and without theft. In the case of existing theft, $n^s > 0$, equations (1.22) and (1.23) imply that a shopper is indifferent between using cash and credit when the marginal cost of using cash equals the marginal costs of using credit,

$$\frac{\theta}{\beta}\frac{1}{(1 - \pi n^s)} = e^{\gamma(i^*)}$$

where there are two opportunity costs of holding money, R and n^s. When there is a change in the money growth rate, the household makes an adjustment on both n^s and i^*. Thus, the monetary policy implications for n^s and i^* are not as straightforward as Prescott (1987), Ireland (1994), and Lacker and Schreft (1996). That is, inflation does not necessarily make a shopper use more credit and a worker steal more.

Besides, in equation (1.25), crime can disappear with large enough inflation given $p = \pi\theta$ because inflation taxes stealing. Once crime disappears, then a further increase in the money growth rate affects only i^* as in Prescott (1987), Ireland (1994), and Lacker and Schreft (1996). Shoppers increase their credit use for a larger variety of goods given the higher interest rate.

1.5.1 Monetary Policy Effects with Theft

In equation (1.24), i^* decreases when the money growth rate, θ, increases:

$$\frac{\partial i^*}{\partial \theta} < 0.$$

A shopper purchases a smaller variety of goods with credit and uses cash for a greater variety of goods, when θ increases. In Ireland (1994), a higher θ increases the nominal interest rate, $R = \theta/\beta$, and a shopper spends on credit for a larger variety of goods to obtain a higher interest though sacrificing higher transactions costs. However, in my model, given the additional opportunity cost of money, theft, the household has to take account of n^s in its adjustment with transactions costs, $\gamma(i^*)$. The nominal interest rate is higher, but the household is more concerned about transactions costs which affect directly the household's utility.

In equation (1.25), first, the crime rate, n^s, increases when cash-credit choice, i^*, increases:

$$\frac{\partial n^s}{\partial i^*} = \frac{\theta}{\beta}\frac{\gamma'(i^*)}{\pi}e^{-\gamma(i^*)} > 0$$

where $\gamma'(i^*) > 0$ because $\gamma(i^*)$ is increasing on i^*. If i^* increases, then transactions costs, $\gamma(i^*)$, increase. In order to compensate for more credit use, a worker needs to spend more time in stealing cash from other shoppers because the household needs more stolen cash to pay off its purchases with credit at the end of period. Next, n^s

decreases as θ increases:

$$\frac{\partial n^s}{\partial \theta} = -\frac{1}{\pi}\left(\underbrace{\frac{e^{-\gamma(i^*)}}{\beta}}_{\text{direct effect}} + \underbrace{\frac{\theta}{\beta}\frac{\partial e^{-\gamma(i^*)}}{\partial i^*}\frac{\partial i^*}{\partial \theta}}_{\text{indirect effect}}\right) < 0$$

where

$$\frac{\partial e^{-\gamma(i^*)}}{\partial i^*} < 0.$$

There are two monetary policy effects on n^s. One is the direct effect of θ which implies that the household can have a positive income effect by producing more consumption goods because the price of goods, $p = \pi\theta$, increases in equation (1.14). The other is the indirect effect of θ through i^* which implies that inflation reduces shopper's purchases with credit. Overall, the direct effect dominates the indirect effect when θ increases. If θ increases, then the price level increases and the value of cash decreases. The household needs more cash to purchase consumption goods. In order to get more cash, a worker can either work more for a larger income or steal more cash from other shoppers. The household chooses to work more because the marginal benefit from working is higher than the marginal benefit from stealing when the money growth rate increases. In other words, the household can steal more cash with a smaller amount of time due to inflation. Thus, the economy produces higher output and less transactions costs. Unlike a standard cash-in-advance model, inflation taxes stealing.

In equations (1.26), consumption with cash, c^0, and consumption with credit, c^1, stay positive with theft while the consumption level is zero in a money-only economy with theft. The household is better off with credit because a shopper chooses a different means of payment, credit or cash, where either is more advantageous to avoid transactions costs or theft and to get the nominal interest rate. First, the volume of consumption purchased with cash, c^0, increases with i^*. A shopper purchases a larger amount of same goods with cash, but he/she purchases a greater variety of goods with credit when i^* increases. Unlike c^0, c^1 is not directly subject to i^*. Next, both c^0 and c^1 increase with θ,

$$\frac{\partial c^0}{\partial \theta} = \frac{1}{\pi}\left[\frac{e^{-\gamma(i^*)}}{\beta} + \left(\frac{\theta}{\beta} + \pi - 1\right)\frac{\partial e^{-\gamma(i^*)}}{\partial i^*}\frac{\partial i^*}{\partial \theta}\right] > 0$$

$$\frac{\partial c^1}{\partial \theta} = \frac{1}{\pi\beta} > 0.$$

A higher money growth rate increases c^0 and c^1 in contrast to Ireland (1994) where c^0 increases and c^1 may increase or decrease. Inflation creates higher output, so the household consume more. Equation (1.26) also implies that aggregate consumption with cash, $(1 - i^*)c^0$, increases, but aggregate consumption with credit, i^*c^1, is constant given a higher money growth rate. There are two opposing effects of θ on i^*c^1 since i^* decreases and c^1 increases. One is a positive effect on output since n^s decreases and the other is a negative effect on i^*. These effects cancel out, so i^*c^1 stays constant. Finally, in equation (1.23), the marginal rate of substitution of c^0 for c^1 decreases with θ,

$$\frac{\partial\left(c^1/c^0\right)}{\partial \theta} = \frac{\partial e^{\gamma(i^*)}}{\partial i^*}\frac{\partial i^*}{\partial \theta} < 0,$$

and it implies that the relative proportion of c^1 to c^0 increases. A shopper is willing to replace cash for credit since a higher money growth rate decreases i^* in equations (1.23) and (1.24).

1.5.2 Monetary Policy Effects without Theft

When $n^s > 0$, inflation taxes stealing and can eliminate crime from the economy. In the case of no crime, a shopper is indifferent between using cash and credit if from equation (1.25)

$$\frac{\theta}{\beta} = e^{\gamma(i^*)}$$

where now R is the only opportunity cost of holding money. That is, inflation affects i^* only and it makes a shopper use credit for a larger variety of goods given the higher interest rate. Thus, without theft, equations (1.20) and (1.22) become:

$$i^* c^1 + (1 - i^*) c^0 = 1 \tag{1.27}$$

$$\frac{\theta}{\beta} = \frac{c^1}{c^0}, \tag{1.28}$$

where $\theta > \beta$ should hold to get $i^* > 0$.[2] In equation (1.28), the household's marginal rate of substitution of c^0 for c^1 solely depends on the money growth rate. The cash-credit choice, i^*, is also subject only to the nominal interest rate. From equations (1.23), (1.27), and (1.28), a set of solutions, i^*, c^0 and c^1, characterizes as follows:

$$\gamma(i^*) = ln\left(\frac{\theta}{\beta}\right), \tag{1.29}$$

$$c^0 = \frac{1}{1 + \left(\frac{\theta}{\beta} - 1\right) i^*} \tag{1.30}$$

$$c^1 = \frac{\frac{\theta}{\beta}}{1 + \left(\frac{\theta}{\beta} - 1\right) i^*}.$$

[2] The bound of the money growth rate $\theta \in [\beta, \beta e^{\gamma(i^*)}]$ is to be hold to bind the cash-in-advance constraint where $\gamma(i^*) \in \Re$.

where

$$\theta > \beta.$$

As in Ireland (1994), equation (1.29) shows that inflation increases i^* since $\gamma(i)$ is increasing on i,

$$\frac{\partial i^*}{\partial \theta} > 0.$$

Since there is no theft, the household simply decides to use credit for a larger variety of consumption goods to obtain a higher interest. In equations (1.30), consumption with cash, c^0, decreases,

$$\frac{\partial c^0}{\partial \theta} = -\frac{\left(\frac{\theta}{\beta} - 1\right)\frac{\partial i^*}{\partial \theta} + \frac{i^*}{\beta}}{\left(1 + \left(\frac{\theta}{\beta} - 1\right) i^*\right)^2} < 0,$$

but consumption with credit, c^1, may increase or decrease with the money growth rate,

$$\frac{\partial c^1}{\partial \theta} = \frac{\theta}{\beta}\left\{\frac{\frac{1-i^*}{\theta} - \left(\frac{\theta}{\beta} - 1\right)\frac{\partial i^*}{\partial \theta}}{\left(1 + \left(\frac{\theta}{\beta} - 1\right) i^*\right)^2}\right\} = \begin{cases} > 0, & \text{if} \quad \frac{1-i^*}{\theta} > \left(\frac{\theta}{\beta} - 1\right)\frac{\partial i^*}{\partial \theta} \\ < 0, & \text{otherwise.} \end{cases}$$

Since shoppers use credit more often, they purchase more consumption goods with credit and less consumption good with cash. However, if the inflation rate keeps increasing, then transactions costs increase as well. Thus, at some point inflation begins to push down consumption with credit. Similarly, aggregate consumption with cash, $(1 - i^*)c^0$, decreases and aggregate consumption with credit, i^*c^1, is ambiguous with inflation.

1.5.3 Welfare

Suppose that the monetary authority chooses the money growth rate to maximize the household's welfare in equilibrium. Then, the policy maker solves

$$W = \max_{\theta} \left\{ i^* ln(c^1(i^*)) + (1 - i^*) ln(c^0(i^*)) - \int_0^{i^*} \gamma(i) di \right\}.$$

Like i^* in Section 5.1, welfare behaves differently with crime or without crime. With crime, the welfare measure has a simplified form using equation (1.23),

$$W = \max_{\theta} \left\{ ln(c^1(i^*)) - (1 - i^*)\gamma(i^*) - \int_0^{i^*} \gamma(i) di \right\},$$

and it monotonically increases with inflation due to lower transactions costs, $\gamma(i^*)$, and higher output, $y = 1 - n^s$: in equations (1.24) and (1.26),

$$\frac{\partial W}{\partial \theta} = \frac{1}{c^1} \frac{\partial c^1}{\partial \theta} - (1 - i^*) \frac{\partial \gamma(i^*)}{\partial i^*} \frac{\partial i^*}{\partial \theta} > 0 \tag{1.31}$$

where

$$\frac{\partial c^1}{\partial \theta} = \frac{1}{\pi \beta} > 0, \qquad \frac{\partial i^*}{\partial \theta} < 0.$$

In contrast to Ireland (1994) and Dotsey and Ireland (1996), an inflationary monetary policy improves social welfare when theft exists because a worker produces more with less crime and a shopper use cash for a larger variety of goods implying smaller transactions costs. Welfare increases with inflation until the crime rate, n^s, goes to zero.

Without crime, the welfare measure is, given equation (1.29),

$$W = \max_{\theta} \left\{ ln(c^1(i^*)) - (1 - i^*) ln\left(\frac{\theta}{\beta}\right) - \int_0^{i^*} \gamma(i) di \right\}$$

and it monotonically decreases with inflation: in equations (1.30),

$$\frac{\partial W}{\partial \theta} = \frac{1}{c^1}\frac{\partial c^1}{\partial \theta} - \frac{(1-i^*)}{\theta} = \frac{-\left(\frac{\theta}{\beta}-1\right)\left(\frac{\partial i^*}{\partial \theta}+\frac{i^*(1-i^*)}{\theta}\right)}{1+\left(\frac{\theta}{\beta}-1\right)i^*} < 0, \tag{1.32}$$

where $\theta > \beta$,

$$\frac{\partial i^*}{\partial \theta} > 0,$$

and

$$\frac{\partial \int_0^{i^*}\gamma(i)di}{\partial \theta} = \left(\frac{\partial \int_0^{i^*}\gamma(i)di}{\partial i^*}\right)\frac{\partial i^*}{\partial \theta} = \gamma(i^*)\frac{\partial i^*}{\partial \theta}.$$

As in Ireland (1994), an inflationary policy increases a shopper's credit choice and the amount of consumption with credit increases. A shopper reduces the use of cash in response to the higher interest rate. Therefore, if the government increases the money growth rate further when n^s is zero, then welfare decreases because i^* increases. Overall, welfare is maximized when the government picks the money growth rate where the crime rate just becomes zero. The Friedman rule does not generally hold.

1.5.4 The Optimal Money Growth Rate

Welfare is maximized when the crime rate goes to zero. That is, the government wants to increase the money growth rate to eliminate crime, but once crime disappears the government does not want more inflation. Equations (1.24) and (1.29) determine the optimal money growth rate:

$$\gamma\left[\frac{\pi-1}{\frac{\theta}{\beta}+\pi-1}\right] = ln\left(\frac{\theta}{\beta}\right) \tag{1.33}$$

where $n^s = 0$; $\gamma(\theta, \pi, \beta)$ is a function of the money growth rate, θ, the coefficient of rate of return on stealing, π, and the discount factor, β. Given the relation above, the optimal growth rate can be greater than one depending on the values of π, and β, and the function $\gamma(.)$.

Figure 1.2 describes the characteristics of the optimal money growth rate, θ^*. Suppose $\theta_1 = 1$. Then, equation (1.33) implies that θ^* is greater than one. Shoppers put too much transactions costs,

$$\gamma(1, \pi, \beta) > \frac{1}{\beta}.$$

Thus, the government needs to increase the money stock in order to increase the nominal interest rate. Shoppers use credit for a smaller variety of goods since i^* decreases, so transactions costs decrease. Now, in Figure 1.2, suppose $\theta_2 = 1$. Then, in equation (1.33), i^* is less than one. This time, the nominal interest rate is too high,

$$\gamma(1, \pi, \beta) < \frac{1}{\beta}.$$

Thus, the government decreases the money stock in order to decrease the nominal interest rate rate. Shoppers use credit for a larger variety of goods since i^* increases, so transactions costs increase.

1.5.5 A Simple Example

A simple example will provide more clear understanding of a cash-credit economy. Assume $\gamma(i) = -ln(1-i)$ where $\gamma(0) = 0$ and $\lim_{i \to 1} \gamma(i) = \infty$. Then, equation (1.24) implies that

$$\gamma(i^*) = ln\left(1 + \frac{\pi - 1}{\frac{\theta}{\beta}}\right)$$

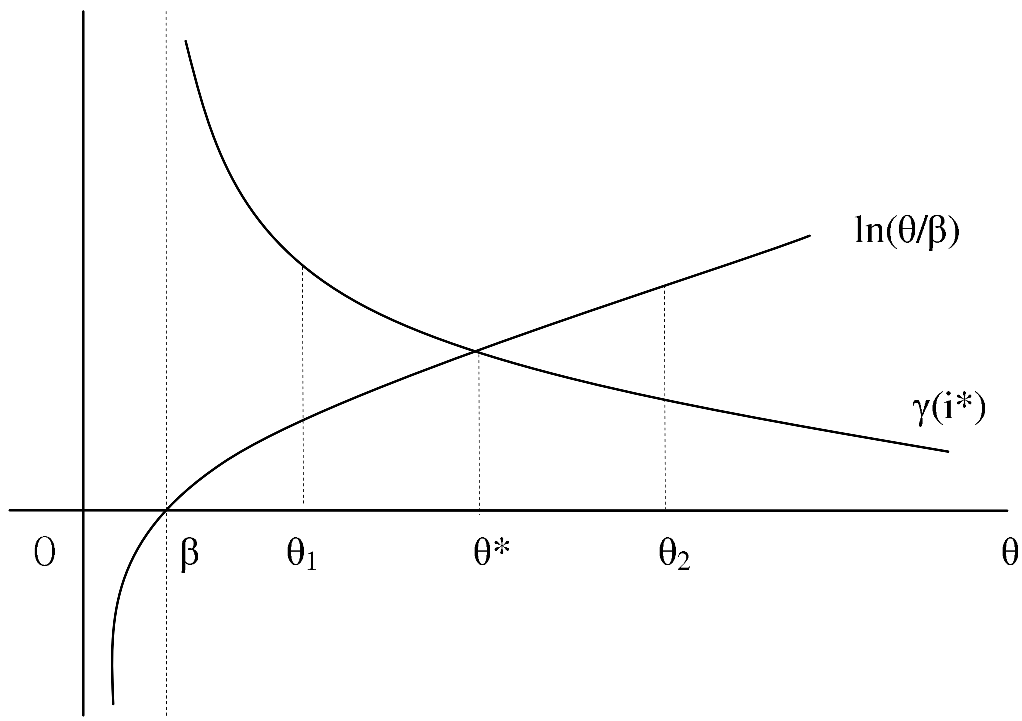

Figure 1.2: The Optimal Growth Rate

where transactions costs decrease with the money growth rate. Thus, equation (1.29) implies that

$$ln\left(1+\frac{\pi-1}{\frac{\theta}{\beta}}\right)=ln\left(\frac{\theta}{\beta}\right). \tag{1.34}$$

A set of solutions without theft are

$$i^{*}=1-\frac{\beta}{\theta},$$

where

$$c^{0}=\frac{1}{1+\left(\frac{\theta}{\beta}-1\right)i^{*}},$$

and

$$c^{1}=\frac{\frac{\theta}{\beta}}{1+\left(\frac{\theta}{\beta}-1\right)i^{*}}.$$

Finally, given $\gamma(i)=-ln(1-i)$, welfare reaches its maximum when the money growth rate[3] is

$$\theta=\frac{\beta}{2}\left(1+\sqrt{4\pi-3}\right) \tag{1.35}$$

where $\theta>\beta$ since $\pi>1$. The optimal inflation rate can be positive or negative depending on the values of π and β.

Figure 1.3 and Figure 1.4 provide a consistent graphical description of monetary policy effects on consumption with credit, c^1, and with cash, c^0, the cash-credit choice, i^*, the crime rate, n^s, and welfare. Assume $\beta = 0.99$ and $\pi = 1.059$. In

[3]In equation (1.34),

$$\theta^2-\beta\theta-\beta^2(\pi-1)=0.$$

The solution for the inflation rate is in equation (1.35) by using the quadratic formula.

Figure 1.3, the horizontal axis denotes the money growth rate, θ, and the vertical axis denotes i^*, n^s, and welfare. A blue line representing n^s decreases as θ increases and becomes zero when the money growth rate is greater than 1.045. A red solid line representing i^* decreases if θ increases and increases if θ is greater than 1.045. A black solid line represents welfare and it increases with inflation and start to decrease if θ is greater than 1.045. From equation (1.35), welfare reaches a peak without crime where the inflation rate is positive, $\theta = 1.045$. Thus, given $\beta = 0.99$ and $\pi = 1.059$, the inflationary monetary policy is desirable and the crime rate is zero. In this case, the Friedman rule is not optimal. If θ is greater than 1.045, then i^* and welfare behave like those in Ireland (1994) and crime disappears.

In Figure 1.4, the horizontal axis denotes the money growth rate, θ, and the vertical axis represents consumption with credit, c^1, and with cash, c^0. A blue solid line represents c^1 and a red solid line represents c^0. With crime, both c^1 and c^0 increase with inflation since the total production increases. Without crime, first, c^1 keeps increasing by a certain inflation rate and starts to decline after that. However, c^0 monotonically decreases.

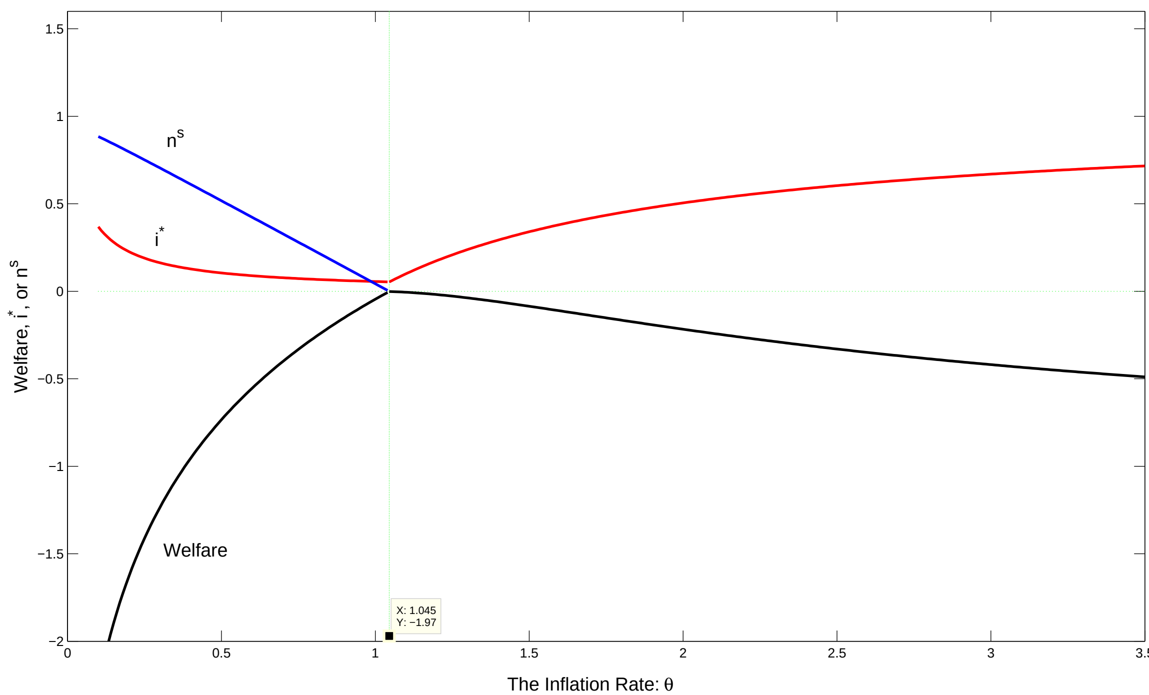

Figure 1.3: Welfare, i^*, and n^s

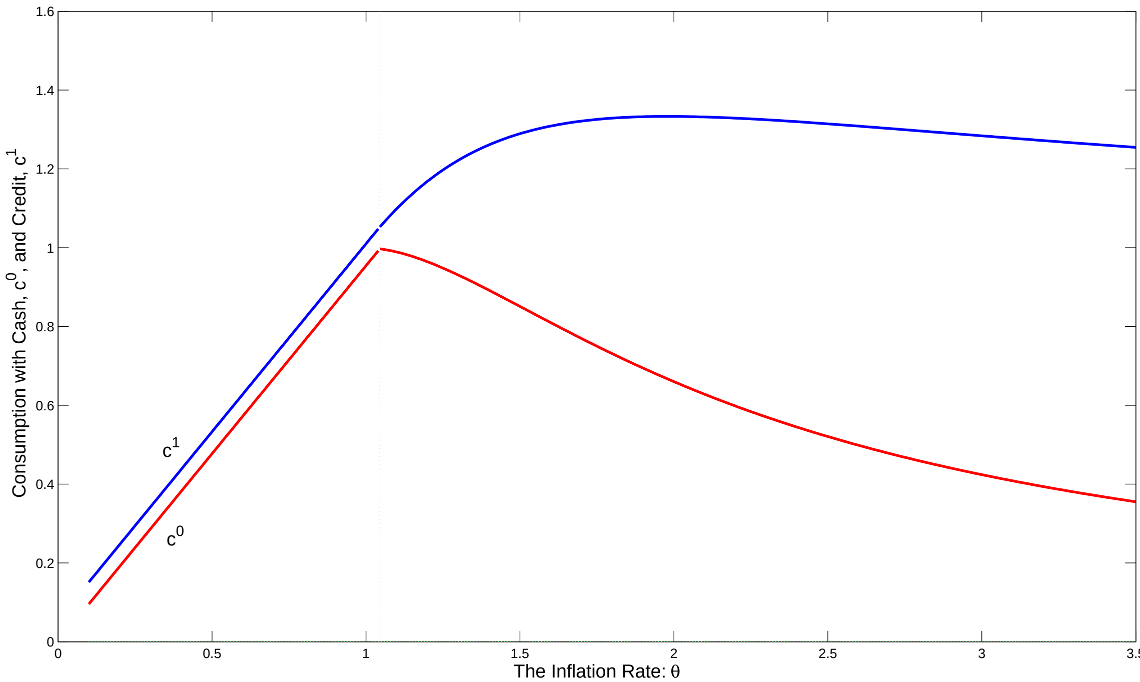

Figure 1.4: Consumption with credit and cash

1.6 Conclusions

This paper develops a rich model of money and credit and explores its monetary policy implications. Each household decides whether to use money or credit in transaction involving an array of goods. There exists the risk of holding money as another opportunity cost of holding money in addition to the nominal interest rate. The opportunity cost of credit is a transaction cost. In steady state equilibrium, both cash and credit coexist. The household spends on credit to avoid theft or avoid the opportunity costs of holding money, but uses money to reduce the transactions costs of credit. That is, the marginal rate of substitution of cash goods for credit goods depends on both the nominal interest rate and the crime rate. Furthermore, shoppers spend on credit although the nominal interest rate is not positive.

Monetary policy affects cash-credit choices and the crime rate which decrease as the money growth rate increases. Inflation taxes stealing. Once the crime rate hits zero, then the economy works as in Ireland (1994) where transactions costs increase with inflation. Therefore, welfare improves with inflation if the crime rate is positive unlike Dotsey and Ireland (1996), but it decreases in inflation once the crime rate becomes zero. Welfare is maximized when the crime rate hits zero. The government wants to increase the money growth rate to eliminate crime, but once crime disappears the government does not want more inflation. The Friedman rule is not optimal.

CHAPTER 2
RISK, LIMITED PARTICIPATION AND MONETARY POLICY

2.1 Introduction

The objective of this paper is to develop an asset market segmentation model and to explore the risk-sharing role of monetary policy. In the model, individuals face uninsurable endowment risk and there are distributional effects of monetary policy due to asset market segmentation. Monetary policy redistributes the consumption across households. Thus, monetary policy can play a risk-sharing role which provides crude insurance to economic individuals.

Several asset market segmentation models in which money is nonneutral have been developed recently including Lucas (1990), Fuerst (1992), Alvarez, Lucas, and Weber (2001), Alvarez, Atkeson, and Kehoe (2002), and Williamson (2007). These models have some features in common. First, there are *traders* who participate in the asset market and *nontraders* who do not. Next, the government controls the money supply through open market purchases of interest-bearing government bonds and only traders exchange money for nominal bonds in the asset market. Thus, traders initially receive a money injection from the central bank. In equilibrium, money is nonneutral and monetary policy creates distributional effects between traders and nontraders. For example, positive money injection increases traders' consumption and decreases nontraders' consumption. Last, asset market segmentation causes liquidity effects on the nominal interest rate. The nominal interest rate decreases as the result of a money stock increase. However, many asset market segmentation models are built to explain the behavior of asset prices and the exchange rate instead of discussing serious risk-sharing roles of monetary policy.

This paper will emphasize the active risk-sharing role of monetary policy affecting an aggregate economy and those who make up the economy by extending the model of Alvarez, Lucas, and Weber (2001). There are two types of households: traders and nontraders with the fractions of traders and nontraders are exogenously given. Costs to join the asset market are infinite while the costs are finite in Alvarez, Atkeson, and Kehoe (2002). Each period, traders receive same constant endowments, but nontraders receive an uninsurable endowment shock. In the asset market, the government injects money through open market operations and traders exchange money for interest-bearing government nominal bonds. Traders receive the money injection first, but nontraders do not receive it because they have no access to the asset market.

In equilibrium, money is nonneutral. Money has distributional effects on consumption and it redistributes consumption between traders and nontraders. If the government increases the money stock, then traders' consumption increases because traders receive the money injection in the asset market. However, nontraders' consumption decreases since they do not participate in the asset market. Inflation taxes nontraders' consumption. Also, there exist liquidity effects on the nominal interest rate. Next, the government can use monetary policy as a risk-sharing tool across households. If endowment shocks are identical across nontraders, then traders and nontraders consume equally and the allocation is Pareto optimal. Monetary policy provides perfect risk-sharing. The optimal money growth rate can be positive or negative and the Friedman rule is not optimal in general. On the other hand, if nontraders receive idiosyncratic shocks, then the limits of risk-sharing become apparent. Monetary policy only partially insures nontraders because money redistributes consumption goods between traders and nontraders as groups. Monetary policy is a rather blunt risk-sharing tool and it does not achieve a Pareto optimal allocation. The money growth rate maximizing welfare can be positive or negative, so the Friedman rule is not optimal.

The remainder of the paper is organized as follows. Section 2 describes the basic environments. Section 3 discusses the equilibrium dynamics. Section 4 and 5 study monetary policy implications for risk-sharing and discussions of welfare takes place. Section 6 concludes.

2.2 The Model

Time is discrete and indexed by $t = 0, 1, 2, ...$ There is a continuum of infinitely-lived households with a unit mass. A fraction α of the households are traders who participate in the asset market every period and the rest, $1 - \alpha$, are nontraders who never participate in the asset market. Each household consists of a shopper and a seller. The preferences of a household are given by

$$U(\{c_t\}_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} \right),$$

where E_0 is the expectation operator conditional on information in period 0; $\beta \in (0, 1)$ is a household's discount factor; and c_t represents perishable consumption.

In each period, each trader receives the constant endowment, y. Each nontrader i gets the idiosyncratic endowment, $y_{i,t} \in [y_l, y_h]$ where $y_h > y_l > 0$, i.i.d. across nontraders' households, with a distribution $F_t(y)$ and its aggregation is as follows:

$$y_t^n = \int y_{i,t} dF_t,$$

and the aggregate output of the economy is

$$Y_t = \alpha y + (1 - \alpha) y_t^n$$

where aggregate output fluctuates depending on the realization of y_t^n.

At the beginning of each period t, traders enter the period with $M_{r,t}$ units of currency and B_t units of interest-bearing one-period government nominal bonds. Nontrader i enters the period with $M_{i,t}$ units of currency. At period 0, traders start with $M_{r,0}$ and B_0 and nontraders with $M_{i,0}$. Traders receive constant endowments, y, and nontraders receive endowment shocks, $y_{i,t}$. Traders and nontraders can not consume their own endowments.

Traders go to the asset market and exchange money for interest-bearing one-period government nominal bonds. Each bond sells for q_t units of money in period t and is a claim to one unit of money in period $t+1$. In the asset market, the government increases in the money stock, $\mu_t M_t^s$, through open market operations. At period t, the government budget constraint is

$$\bar{B}_t - q_t \bar{B}_{t+1} = M_{t+1}^s - M_t^s$$

$$M_{t+1}^s = (1 + \mu_t) M_t^s$$

where $\bar{B}_t$ denotes nominal bonds that mature in period t; $\bar{B}_{t+1}$ is newly issued nominal bonds with price q_t that mature in period $t+1$; $\mu_t > -1$ is the net money growth rate. Nontraders do not go to the asset market and they cannot acquire government nominal bonds. Therefore, when the government increases the money supply through open market operations, traders initially receive it and nontraders do not.

Traders and nontraders meet at the goods market and they exchange consumption goods for cash. The cash-in-advance constraints of traders and nontrader i are, respectively,

$$P_t c_{r,t} \leq M_{r,t} + B_t - q_t B_{t+1}$$

$$P_t c_{i,t} \leq M_{i,t}$$

where $c_{r,t}$ is the consumption of traders in period t; $c_{i,t}$ is the consumption of nontrader i in period t.

At the end of each period t, everyone returns with the revenue of sales. No further trade arises. The budget constraints of traders and nontrader i are

$$P_t c_{r,t} + M_{r,t+1} = M_{r,t} + B_t - q_t B_{t+1} + P_t y$$

$$P_t c_{i,t} + M_{i,t+1} = M_{i,t} + P_t y_{i,t}.$$

where $M_{r,t+1}$ is currency that traders transfer from period t to period $t+1$; $M_{i,t+1}$ is currency that nontrader i receiving $y_{i,t}$ transfers from period t to period $t+1$; $P_t y$ is traders' revenue from sales and $P_t y_{i,t}$ is nontrader i's revenue from sales.

2.3 Equilibrium Dynamics

2.3.1 Optimization

Traders solve the following problem,

$$\max_{c_{r,t},M_{r,t+1},B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{r,t}^{1-\gamma}}{1-\gamma} \right)$$

subject to

$$P_t c_{r,t} \leq M_{r,t} + B_t - q_t B_{t+1} \tag{2.1}$$

$$P_t c_{r,t} + M_{r,t+1} = M_{r,t} + B_t - q_t B_{t+1} + P_t y$$

$$M_{r,t+1} \geq 0, \qquad B_{t+1} \geq \bar{b}, \qquad \text{where } \bar{b} \leq 0.$$

Now, nontraders i solve the following problem,

$$\max_{c_{i,t},M_{i,t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{i,t}^{1-\gamma}}{1-\gamma} \right)$$

subject to

$$P_t c_{i,t} \leq M_{i,t} \tag{2.2}$$

$$P_t c_{i,t} + M_{i,t+1} = M_{i,t} + P_t y_{i,t}$$

$$M_{i,t+1} \geq 0.$$

Definition: A competitive equilibrium consists of the sequences $\{c_t, M_{t+1}, B_{t+1}, M_t^s, P_t, R_t\}_{t=0}^{\infty}$ such that:

1. $\{c_t, M_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ solves the household problems of traders and nontraders given $\{M_t^s, P_t, R_t\}_{t=0}^{\infty}$.

2. Markets clear in every period:

   (a) Bond Market: for each trader,

$$B_t - q_t B_{t+1} = \frac{\mu_t M_t}{\alpha}.$$

   (b) Money Market: for all t,

$$M^s_{t+1} = M_{t+1} = P_t Y_t$$

   (c) Goods Market:

$$\alpha c_{r,t} + (1-\alpha)\int c_{i,t} dF_t = Y_t$$

2.3.2 Equilibrium

In equilibrium, first, the cash-in-advance constraint of traders binds if

$$q_t = \beta E_t\left[\frac{u'(c_{r,t+1})}{u'(c_{r,t})}\frac{P_t}{P_{t+1}}\right] < 1. \tag{2.3}$$

In other words, the price of nominal bonds should be less than one, $q_t < 1$. Similarly, the cash-in-advance constraint of nontraders i binds if

$$\beta E_t\left[\frac{u'(c_{i,t+1})}{u'(c_{i,t})}\frac{P_t}{P_{t+1}}\right] < 1. \tag{2.4}$$

Next, traders exchange cash and nominal bonds in the asset market and they receive a change in money supply first. The flow of the money stock for each trader is

$$\frac{\mu_t M_t}{\alpha} = B_t - q_t B_{t+1} \tag{2.5}$$

and it is same across traders. At the goods market, assuming both cash-in-advance constraints bind, traders and nontraders bring cash acquired from their sales of endowments in the previous period,

$$M_{r,t+1} = P_t y \tag{2.6}$$

$$M_{i,t+1} = P_t y_{i,t}$$

where $M_{r,t+1}$ is identical across traders and $M_{i,t+1}$ is different across nontraders due to idiosyncratic $y_{i,t}$. The aggregate money stock of nontraders is defined as

$$M^n_{t+1} = P_t y^n_t.$$

Thus, from equations (2.6) the money market clearing condition is

$$M_{t+1} = \alpha P_t y + (1-\alpha) P_t \int y_{i,t} dF_t.$$

Letting

$$Y_t = \alpha y + (1-\alpha) y^n_t,$$

the average price level is

$$P_t = \frac{M_{t+1}}{Y_t} \tag{2.7}$$

and the inflation rate is

$$\frac{P_t}{P_{t-1}} = (1+\mu_t)\frac{Y_{t-1}}{Y_t}$$

where M_{t+1} is the aggregate money stock that goes to the next period and $P_t Y_t$ represents the aggregate income transferring to period $t+1$.

Equations (2.1), (2.2), and (2.5) - (2.7) imply

$$P_t c_{r,t} = P_{t-1} y + \frac{\mu_t M_t}{\alpha}$$

$$P_t c_{i,t} = P_{t-1} y_{i,t-1}$$

and the consumption of traders and nontraders is

$$c_{r,t}(\mu_t) = \left(\frac{y + \mu_t Y_{t-1}/\alpha}{1 + \mu_t}\right)\frac{Y_t}{Y_{t-1}} \tag{2.8}$$

$$c_{i,t}(\mu_t) = \left(\frac{y_{i,t-1}}{1 + \mu_t}\right)\frac{Y_t}{Y_{t-1}}. \tag{2.9}$$

In equation (2.8), traders consume the same amount of goods. However, in equation (2.9), nontraders consume different quantities of goods, depending on $y_{i,t-1}$. Money is nonneutral since the money supply affects the consumption of traders and nontraders. Besides, money redistributes consumption goods among households. First, traders' consumption goes up and nontraders' consumption goes down as the result of an increase in the money growth rate. Traders gain consumption goods because they receive the government money injection first in the asset market. However, nontraders do not receive the money injection in the asset market and their consumption decreases. Inflation taxes nontraders' consumption since the value of cash decreases. On the other hand, if the money growth rate decreases, traders' consumption goes down and nontraders' consumption goes up. Second, the impact of distributional effects of money on nontraders are different depending on $y_{i,t}$. That is, the nontraders who receive relatively larger endowments gain or lose more consumption goods with a change in the money growth rate. Current endowment shocks to nontraders, $y_{i,t}$, do not affect current consumption because their cash holdings depend on the previous endowments. Last, in equation (2.8), the price of nominal bonds, q_t, is

$$q_t = \beta E_t\left[\left(\frac{c_{r,t}}{c_{r,t+1}}\right)^{\gamma}\frac{1}{1 + \mu_{t+1}}\frac{Y_{t+1}}{Y_t}\right] \tag{2.10}$$

and the intertemporal marginal rate of substitution of traders determines the price. The price of nominal bonds embeds two opposing effects of the money growth rate, μ_t. One is the Fisherian effects where a higher money growth rate leads a higher interest rate. If the inflation rate, P_{t+1}/P_t, is to increase due to a higher money growth rate, μ_{t+1}, then the bonds price decreases and the nominal interest rate increases. The other is the liquidity effect where the nominal interest rate decreases with a higher money growth rate. In equation (2.8), $c_{r,t}$ increases with the money growth rate, μ_t. Thus, the price of nominal bonds increases with μ_t and the nominal interest rate decreases.

In inequalities (2.3) and (2.4), if both cash-in-advance constraints bind, then the price of nominal bonds price is always less than one and the money growth rate has the following bound[1]: for all i,

$$\frac{Y_{t-1}}{(1-\alpha)y^n_{t-1}}\left(1-\frac{\alpha}{(Y_t^{\gamma-1}\beta\Psi_r)^{\frac{1}{\gamma}}}\right) < \frac{1}{1+\mu_t} < \frac{Y_{t-1}}{y_{i,t-1}}\left(Y_t^{\gamma-1}\beta\Psi_i\right)^{\frac{1}{\gamma}}, \tag{2.11}$$

where $\Psi_r > 0$ and $\Psi_i > 0$ are constant,

$$\Psi_r = E_t\left[\left(\frac{1}{c_{r,t+1}}\right)^{\gamma}\frac{Y_{t+1}}{1+\mu_{t+1}}\right] \qquad \text{and} \qquad \Psi_i = E_t\left[\left(\frac{1}{c_{i,t+1}}\right)^{\gamma}\frac{Y_{t+1}}{1+\mu_{t+1}}\right].$$

2.4 Risk-Sharing without Idiosyncratic Shocks

Understanding the role of monetary policy without idiosyncratic shocks will provide a useful background for a further discussion of the role of monetary policy with idiosyncratic shocks in the later section. First, the social planner determines a

[1]The derivation is in Appendix A.1.

Pareto optimum by solving the following problem.

$$\max_{c_{r,t},c_{i,t}} \left\{ \alpha u(c_{r,t}) + (1-\alpha) \int u(c_{i,t}) dF_t \right\}$$

subject to

$$\alpha c_{r,t} + (1-\alpha) \int c_{i,t} dF_t = Y_t.$$

In a Pareto optimum, traders and nontraders consume equally, for all i

$$c^*_{r,t} = Y_t = c^*_{i,t} = c^{n*}_t,$$

where

$$c^n_t = \int c_{i,t} dF_t.$$

Now, suppose $y_{i,t}$ is identical across nontraders, i.e. $y_{i,t} = y^n_t$ for all i. Then, in equilibrium, without idiosyncratic endowment shocks, monetary policy achieves a Pareto optimum if the money growth rate is

$$\mu^*_t = \frac{\alpha}{Y_{t-1}} \left(y^n_{t-1} - y \right), \tag{2.12}$$

and the consumption of traders and nontraders is

$$c^*_{r,t} = Y_t = c^*_{i,t} = c^{n*}_t.$$

The optimal money growth rate, μ^*_t, achieves a Pareto Optimal allocation, but the nominal interest rate is not zero in general. First, μ^*_t in equation (2.12) should satisfy the boundary condition from inequality (2.11) and it is[2]

$$(Y^{\gamma-1}_t \beta \Psi_r)^{\frac{1}{\gamma}} < 1 < (Y^{\gamma-1}_t \beta \Psi^n)^{\frac{1}{\gamma}} \tag{2.13}$$

[2]The derivation is in Appendix A.2.

where $\Psi_i = \Psi^n$.

In equation (2.12), the optimal money growth rate, μ_t^*, can be positive or negative depending on the endowments of nontraders and traders, y_t^n and y, the fraction of traders, α, aggregate endowments, Y_{t-1}. First, the discrepancy of endowments between traders and nontraders matters to set the optimal money growth rate. The optimal growth rate can be positive or negative. Suppose nontraders and traders receive identical endowments, $y_{t-1}^n = y$. Then, every household receives identical endowments and the optimal growth rate is zero, $\mu_t^* = 0$. No redistributive monetary policy is necessary. If traders receive larger endowments than nontraders, $y_{t-1}^n < y$, then a decreasing money growth rate, $\mu_t^* < 0$, achieves perfect risk-sharing by redistributing endowments from traders to nontraders. Deflation increases the value of money, so nontraders can acquire more consumption goods. On the other hand, if nontraders have larger endowments than traders, $y_{t-1}^n > y$, then inflation, $\mu_t^* > 0$, achieves perfect risk-sharing by redistributing consumption goods from nontraders to traders. With inflation, the value of money decreases and nontraders acquire less consumption goods in the market.

Figure 2.1 depicts the relationship between the optimal money growth rate and the consumption of traders and nontraders when there are no idiosyncratic shocks. Traders and nontraders consume equally at μ_t^* and the optimal money growth rate can be positive or negative. Besides, if $\mu_t < \mu_t^*$, then nontraders consume more than traders. The government increases the money growth rate in order to to achieve an optimal consumption allocation and it redistributes consumption goods from nontraders to traders. On the other hand, if $\mu_t < \mu_t^*$, then traders consume more than nontraders. The government decreases the money growth rate and it redistributes consumption goods from traders to nontraders.

Next, in equation (2.12), an increase in the fraction of traders, α, may increase or decrease the optimal money growth rate, μ_t^*. Suppose $y_{t-1}^n > y$. Then, μ_t^* becomes positive and it increases with α. When traders receive lower endowments relative to nontraders and more households become traders, the optimal money growth rate should increase to redistribute consumption goods from nontraders to traders. However, if $y_{t-1}^n < y$, then μ_t^* is negative and it decreases with α. When traders hold more endowments relative to nontraders and more households become traders, the optimal money growth rate should decrease to redistribute consumption goods from traders to nontraders.

A change in aggregate output, Y_{t-1}, may increase or decrease the optimal money growth rate, μ_t^*, depending on the cause of changing Y_{t-1}. First, if Y_{t-1} increases because of an increases in y_{t-1}^n as the result of a change in distribution $F_{t-1}(y_i)$, then μ_t^* increases with y_{t-1}^n:

$$\frac{\partial \mu_t^*}{\partial y_{t-1}^n} = \frac{\alpha y}{(Y_{t-1})^2} > 0,$$

in order to redistribute consumption goods from nontraders to traders. On the other hand, if a change in $F_{t-1}(y_i)$ results in a decrease in $y_{n,t-1}$, then μ_t^* decreases. Next, if Y_{t-1} increases due to an increase in y or α given $F_{t-1}(y_i)$, then μ_t^* decreases with Y_{t-1}. If y or α increases, then the aggregate consumption of traders increases. A lower μ_t^* can redistribute consumption goods from traders to nontraders.

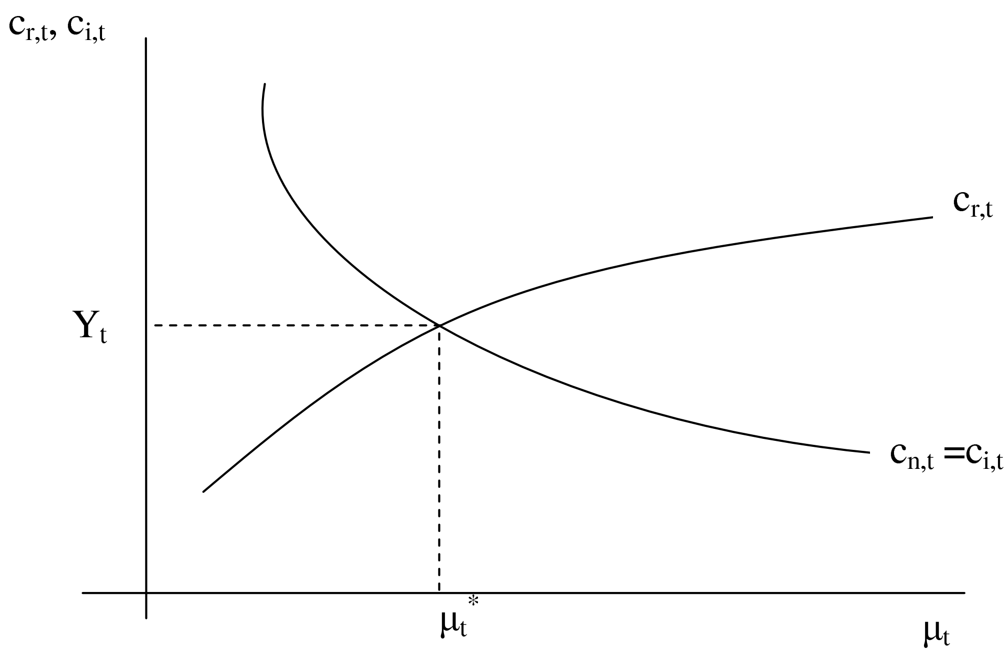

Figure 2.1: Optimal Money Growth without Idiosyncratic Shocks

2.5 Risk-Sharing with Idiosyncratic Shocks

2.5.1 Money Growth and Welfare

With idiosyncratic endowment shocks, monetary policy is a rather blunt tool for sharing risk across households. Suppose $y_{i,t}$ is idiosyncratic across nontraders. Then, in equation (2.9), each nontrader's consumption,

$$c_{i,t}(\mu_t) = \left(\frac{y_{i,t-1}}{1+\mu_t}\right)\frac{Y_t}{Y_{t-1}},$$

differs depending on his endowments in the previous period. Monetary policy cannot control individual consumption and cannot perfectly smooth consumption across nontraders. Perfect risk-sharing is not feasible because monetary policy only redistributes consumption between traders and nontraders as groups.

Suppose the government chooses the money growth rate to maximize the following welfare measure in equilibrium:

$$W_t = \max_{\mu_t}\left\{\alpha u(c_{r,t}) + (1-\alpha)\int u(c_{i,t})dF_t\right\}$$

given equations (2.8) and (2.9). Then, welfare reaches its maximum[3] when the money growth rate is

$$\hat{\mu}_t = \frac{\alpha}{Y_{t-1}}\left\{\left(\frac{y^n_{t-1}}{\widetilde{y}^n_{t-1}}\right)^{\frac{1}{\gamma}} - y\right\}, \tag{2.14}$$

where

$$\widetilde{y}^n_t = \int (y_{i,t})^{1-\gamma}\, dF_t.$$

[3] The derivation is in Appendix A.3.

The money growth rate, $\widehat{\mu}_t$, should satisfy the boundary condition in inequality (2.11) where

$$1 + \hat{\mu}_t = \frac{A_t}{Y_{t-1}} \left(\frac{y^n_{t-1}}{\widetilde{y}^n_{t-1}} \right)^{\frac{1}{\gamma}},$$

$$A_t = 1 + \alpha \left\{ 1 - \left(\frac{\widetilde{y}^n_{t-1}}{\left(y^n_{t-1}\right)^{1-\gamma}} \right)^{\frac{1}{\gamma}} \right\} \geq 1,$$

and by Jensen's inequality[4]

$$\left(\frac{\widetilde{y}^n_{t-1}}{\left(y^n_{t-1}\right)^{1-\gamma}} \right)^{\frac{1}{\gamma}} \leq 1.$$

In equation (2.14), y^n_t is the mean value of endowments; $\widetilde{y}^n_{t-1}$ represents the average of idiosyncratic endowments weighted by the coefficient of relative risk aversion, γ. Given equations (2.9) and (2.14), the consumption of traders and nontraders[5] are, for all i,

$$\hat{c}_{r,t} = \frac{Y_t}{A_t} \qquad \text{and} \qquad \hat{c}_{i,t} = \left(\frac{y_{i,t-1}}{y^n_{t-1}} \right) \left(\frac{\widetilde{y}^n_{t-1}}{\left(y^n_{t-1}\right)^{1-\gamma}} \right)^{\frac{1}{\gamma}} \frac{Y_t}{A_t}. \tag{2.15}$$

Traders and nontraders do not consume equally and the equilibrium allocation is not Pareto optimal. Especially, given $\widehat{\mu}_t$, every trader consumes less than the optimal consumption level, Y_t. For nontraders, some consume less than Y_t, but some consume more than Y_t depending on the distribution of $y_{i,t}$. Monetary policy does not provide perfect risk-sharing to all nontraders, but it can affect each nontrader to some extent.

[4]The derivation is in Appendix A.4.

[5]The derivation is in Appendix A.5.

2.5.2 A Further Discussion about Money Growth

In equation (2.14), the money growth rate maximizing welfare, $\hat{\mu}_t$, depends on the ratio of y_t^n to $\widetilde{y}_{t-1}^n$, the traders' endowments, y, the coefficient of relative risk aversion, γ, the fraction of traders, α, and aggregate endowments, Y_{t-1}. First, since each nontrader receives idiosyncratic endowments, the government should pick $\hat{\mu}_t$ considering not only redistribution effects between traders' endowments, y, and nontraders' endowments, y_{t-1}^n, but also individual effects on consumptions across nontraders,

$$\left(\frac{y_{t-1}^n}{\widetilde{y}_{t-1}^n}\right)^{\frac{1}{\gamma}}. \tag{2.16}$$

For example, suppose the distribution $F_{t-1}(y_i)$ changes, but it preserves the mean value, y_t^n. Then, if endowment gaps across nontraders, $\widetilde{y}_{t-1}^n$, increases, then $\hat{\mu}_t$ decreases. The government can improve welfare by decreasing the money growth rate, $\hat{\mu}_t$, because it redistributes more consumption goods to nontraders. On the other hand, if the gaps, $\widetilde{y}_{t-1}^n$, decreases, then $\hat{\mu}_t$ increases and the government redistributes more consumption goods to traders.

Next, $\hat{\mu}_t$ can be either positive or negative depending on the sign of

$$\left(\frac{y_{t-1}^n}{\widetilde{y}_{t-1}^n}\right)^{\frac{1}{\gamma}} - y. \tag{2.17}$$

If $\left(y_{t-1}^n/\widetilde{y}_{t-1}^n\right)^{1/\gamma} > y$, then $\hat{\mu}_t$ is positive. A positive $\hat{\mu}_t$ redistributes consumption goods from nontraders to traders since nontraders receive relatively larger endowments on average. On the other hand, $\hat{\mu}_t$ becomes zero or negative if $\left(y_{t-1}^n/\widetilde{y}_{t-1}^n\right)^{1/\gamma} \leq y$. A negative $\hat{\mu}_t$ redistributes consumption goods from traders to nontraders since traders receive relatively larger endowments.

However, money redistributes consumption goods between traders and nontraders as groups and monetary policy cannot achieve optimal consumption allocation, Y_t, across households. In other words, $\hat{\mu}_t$ transfers too much consumption goods for some nontraders and too little for others. Suppose μ_t^{i*} is the optimal money growth rate for nontrader i who receive $y_{i,t}$. Then, μ_t^{i*} is similar to equation (2.12) as

$$\mu_t^{i*} = \frac{\alpha}{Y_{t-1}} \left(y_{i,t-1} - y \right) \tag{2.18}$$

and in equations (2.9) and (2.18), every nontrader consumes equally,

$$c_{i,t}^{*} = Y_t. \tag{2.19}$$

However, $\hat{\mu}_t$ in equation (2.14) is not equal to μ_t^{i*} for all i when $y_{i,t}$ is stochastic across nontraders:

$$\hat{\mu}_t - \mu_t^{i*} = \frac{\alpha y_{t-1}^n}{Y_{t-1}} \left\{ \left(\frac{\left(y_{t-1}^n\right)^{1-\gamma}}{\widetilde{y}_{t-1}^n} \right)^{\frac{1}{\gamma}} - \frac{y_{i,t-1}}{y_{t-1}^n} \right\},$$

where by Jensen's inequality

$$\left(\frac{\left(y_{t-1}^n\right)^{1-\gamma}}{\widetilde{y}_{t-1}^n} \right)^{\frac{1}{\gamma}} \geq 1.$$

Given $\hat{\mu}_t$, some nontraders i consumes larger than Y_t when $\hat{\mu}_t > \mu_t^{i*}$ and other nontraders i consumes less than Y_t when $\hat{\mu}_t < \mu_t^{i*}$. Thus, some nontraders lose more than they should and others gain more than they should be in optimum. Monetary policy can not play a perfect risk-sharing role to nontraders and it is a rather blunt risk-sharing tool. Figure 2.2 describes it on a graph.

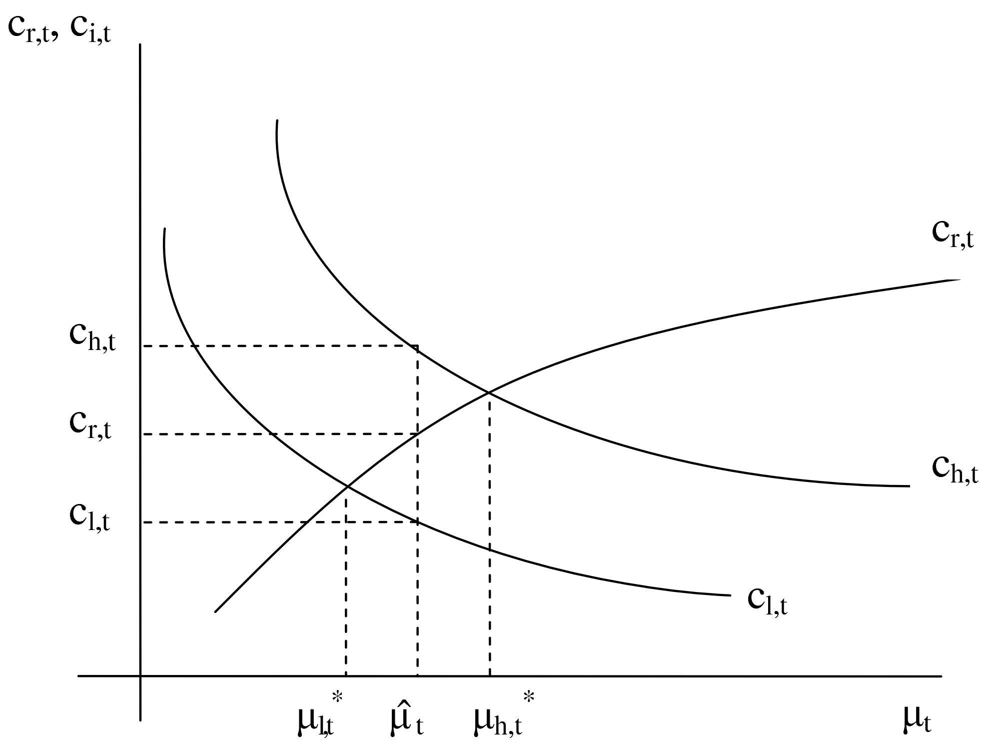

Figure 2.2: Optimal Money Growth with Idiosyncratic Shocks

Furthermore, by Jensen's inequality, the money growth rate, $\hat{\mu}_t$, with idiosyncratic shocks in equation (2.13) is also greater than the optimal money growth rate, μ_t^*, in equation (2.12)[6],

$$\hat{\mu}_t \geq \mu_t^*$$

where the equality holds if the coefficient of relative risk aversion is unity, $\gamma = 1$. When $\gamma = 1$, however, the money growth rate is at optimum, $\hat{\mu}_t = \mu_t^*$, but $\hat{\mu}_t$ does not perfectly insure nontraders. Nontraders still consume different amount of goods depending on $y_{i,t-1}$,

$$\hat{c}_{i,t}(\mu_t) = \left(\frac{y_{i,t-1}}{y_{t-1}^n}\right) Y_t,$$

although aggregate consumption is in optimum, $\hat{c}_{r,t} = Y_t = \hat{c}_t^n$. In the model, the optimal money growth rate does not have to be negative unlike Friedman (1960), but the optimal money growth rate, μ_t^*, should be lower than the money growth rate, $\hat{\mu}_t$. That is, when the economy is not efficient, the money growth rate tends to be larger.

The effect of the coefficient of relative risk aversion, γ, on $\hat{\mu}_t$ is ambiguous from equations (2.14) and (2.16)[7]:

$$\frac{\partial \hat{\mu}_t}{\partial \gamma} = \frac{\alpha}{Y_{t-1}} \frac{\left(y_{t-1}^n\right)^{\frac{1}{\gamma}} \left(\widetilde{y}_{t-1}^n\right)^{\frac{1}{\gamma}+1}}{\gamma} \int \left(y_{i,t-1}\right)^{1-\gamma} ln(y_{i,t-1}) dF_t.$$

If endowments $y_{i,t-1}$ happen to be less than one for some nontraders, then $ln(y_{i,t-1})$ can become negative for some nontraders i. Thus, overall, $\hat{\mu}_t$ can increase or decrease with γ.

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[6]The derivation is in Appendix A.6.

[7]The derivation is in Appendix A.7.

In equation (2.14), a change in the fraction of traders, α, affects the money growth rate, $\hat{\mu}_t$. Suppose $\left(y_{t-1}^n/\widetilde{y}_{t-1}^n\right)^{1/\gamma} > y$ from equation (2.17). Then, $\hat{\mu}$ is positive and it increases with α. When traders receive smaller endowments relative to nontraders and more households become traders, the government increases the money growth rate to redistribute more consumption goods from nontraders to traders. However, suppose $\left(y_{t-1}^n/\widetilde{y}_{t-1}^n\right)^{1/\gamma} < y$. Then, $\hat{\mu}$ is negative and it decreases with α. When traders hold more endowments relative to nontraders and more households become traders, the government reduces the money growth rate to redistribute more consumption goods from traders to nontraders.

Last, $\hat{\mu}_t$ may increase or decrease when Y_{t-1} fluctuates. The fluctuation in y_{t-1}^n affects Y_{t-1} and a change in $\hat{\mu}_t$ depends on the realization of $F_{t-1}(y_i)$ which determines y_{t-1}^n, $\widetilde{y}_{t-1}^n$, and Y_{t-1}.

2.6 Conclusion

This paper studies the risk-sharing role for monetary policy when money is nonneutral due to asset market segmentation. There exist heterogenous households, traders and nontraders, and they receive uninsurable income shocks each period. In the asset market, the government injects money through open market operations and traders exchange money for interest-bearing government nominal bonds. Traders receive the money injection first, but nontraders do not receive it because they do not participate in the asset market.

In equilibrium, money is nonneutral and it redistributes consumption goods between traders and nontraders as groups. Liquidity effects arise. If each type of household receives the same endowments, then monetary policy plays a perfect risk-sharing role. Given optimal policies, traders and nontraders consume equally and the equilibrium allocation is Pareto optimal. The optimal money growth rate can be positive or negative depending on the size of endowments of each type. The Friedman rule is not optimal in general

However, suppose nontraders receive idiosyncratic shocks. Then, monetary policy only redistributes consumption goods across groups and efficiency cannot be achieved with monetary policy. The money growth rate that maximizes welfare cannot achieve a Pareto optimal allocation because money redistributes consumption goods between traders and nontraders as groups. Therefore, monetary policy is a blunt tool for sharing risk across households. The money growth rate can be positive or negative and the Friedman rule is not optimal.

CHAPTER 3
MONEY, CREDIT, AND LIMITED PARTICIPATION

3.1 Introduction

An asset market segmentation model is constructed to explore distributional effects of monetary policy when there exist multiple means of payment. My model allows individuals to choose alternative means of payment, either cash or credit, to purchase consumption goods. Monetary policy has distributional effects due to asset market segmentation. The effects of monetary policy on the choice of means of payment are different between $Traders$ who participate in the asset market and $nontraders$ who do not. Thus, this paper provides implications for the interaction of multiple means of payment when money is nonneutral.

There have been several studies of the coexistence of multiple means of payment such as Lucas-Stokey (1987), Prescott (1987), Ireland (1994), and Lacker and Schreft (1996). Lucas-Stokey (1987) constructed a model of exogenous choice of means of payment. In Prescott (1987), Ireland (1994), and Lacker and Schreft (1996), cash and alternative means of payment coexist and there are many markets. Consumers substitute other means of payment for money if there is a higher nominal interest rate. On the other hand, they prefer to using cash if transactions costs increase. Next, a consumer uses alternative means of payment for larger purchases and cash for smaller ones. Furthermore, Ireland (1994) and Lacker and Schreft (1996) enable to show the monetary policy effect on a variety of goods purchased with cash and other means of payment. However, in these studies, money does not have distributional effects and furthermore, they do not explore the possibility that households use alternative means of payment to ease monetary policy shocks.

Several limited participation models where money is nonneutral have been developed recently including Lucas (1990), Fuerst (1992), Alvarez, Lucas, and Weber (2001), Alvarez, Atkeson, and Kehoe (2002). These models have some features in common. First, there are two types of households: traders and nontraders. Next, in the asset market, traders go to the asset market and exchange government nominal bonds and money. The government controls the money supply through open market purchases of interest-bearing government bonds. The money injection is initially received by only traders. In equilibrium, money is nonneutral due to asset market segmentation. Monetary policy creates distributional effects between the consumptions of traders and nontraders. For example, positive money injection increases traders' consumption and decreases nontraders' consumption. Also, asset market segmentation causes liquidity effects on the nominal interest rate. The nominal interest rate decreases as the result of a money stock increase. However, many of these models are built to explain the behavior of asset prices and the exchange rate instead of discussing an active role for monetary policy through the payments system.

This paper will extend the existing asset market segmentation model of Alvarez, Lucas, and Weber (2001) by using the approach of Ireland (1994), to obtain endogenous choice of credit and cash. In equilibrium, money is nonneutral and it has distributional effects on consumption goods between traders and nontraders. Monetary policy affects the choice of means of payment of traders and nontraders in an opposite way. Unlike Alvarez, Lucas, and Weber (2001), consumption of traders and nontraders may increase or decrease because there are two distributional effects of money: a direct effect and an indirect effect via the choice of means of payment. Suppose the government injects money. Then, the direct effect implies that traders can increase their consumption with a larger amount of money which comes from the asset market while nontraders can not. The indirect effect comes via the choice of means of payment. Traders purchase a larger variety of consumption goods with cash and nontraders purchase a larger variety of goods with credit. Thus, consumption of traders and nontraders may increase or decrease. Thus, credit can be used to compensate for fluctuations of consumption against the money injection. Liquidity effects disappear when the money growth rate is constant, but it may appear when the money growth rate is stochastic.

The remainder of the paper is organized as follows. Section 2 describes setups of the model. In section 3, it explains the equilibrium dynamics. Section 4 and 5 study monetary policy implications. Section 6 concludes.

3.2 The Environment and Timing

Time is discrete and indexed by $t = 0, 1, 2, ...$ There is a continuum of infinitely lived households with unit mass indexed by $i \in [0, 1]$. Each household consists of a shopper and a worker. A fraction α of the households are *traders* who participate in the asset market every period and the rest, $1 - \alpha$, are *nontraders* who do never trade in the asset market. There is a continuum of spatially separated markets indexed by $i \in [0,1]$ in each period and in each market i. There is a distinct, perishable consumption good. The household has preferences given by

$$U(\{c_t, x_t\}_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 ln(c_t(i))di - x_t \right\}$$

where E_0 is the expectation operator conditional on information in period 0; β is the discount factor; $c_t(i)$ represents consumption goods purchased at market i in period t; x_t represents transactions costs.

At the beginning of each period t, traders enter the period with $M_{r,t}$ units of currency and B_t units of one-period nominal bonds. Nontraders enter the period with $M_{n,t}$ units of currency. Traders and nontraders receive endowments, y, and they do not consume their own endowments.

Traders go to the asset market and exchange one-period government nominal bonds and money. Each bond sells for q_t units of money in period t and is a claim to one unit of money in period $t+1$. In the asset market, the government controls the money supply, M_t^s, through open market operations where nominal bonds that are issued in period $t-1$ and mature in period t are denoted by $\bar{B}_t$. Thus, the government budget constraint takes the form

$$\bar{B}_t - q_t \bar{B}_{t+1} = M_{t+1}^s - M_t^s \tag{3.1}$$

$$M_{t+1}^s = (1 + \mu_t) M_t^s$$

where $\bar{B}_{t+1}$ is newly issued nominal bonds with price q_t that matures at period $t+1$; $\mu_t > -1$ is the net money growth rate.

Shoppers and workers of households go to the goods market. Workers sell endowments, y, to shoppers from other households and shoppers travel from market to market purchasing consumption goods. When shoppers purchase goods, they have two ways of acquiring goods from market i. One is to use non-interest bearing currency which has the gross nominal interest rate, R_t, as an opportunity cost. The other is to use credit incurring transactions costs, $\gamma(i) > 0$, to purchase good i. The transactions costs function, $\gamma(i)$, is increasing, differentiable on i, $\gamma(0) = 0$ and $\lim_{i \to 1} \gamma(i) = \infty$. Transactions costs take the form of effort,

$$x_t(i) = \int_0^1 \xi_t(i)\gamma(i)di,$$

where $\xi_t(i)$ is an indicator variable: $\xi_t(i) = 1$ if shoppers use credit to buy good i at time t and $\xi_t(i) = 0$ if shoppers use currency to buy good i at time t. The cash-in-advance constraints in goods market of traders and nontraders are

$$\int_0^1 P_t(i)(1 - \xi_{r,t}(i))c_{r,t}(i)di \leq M_{r,t} + B_t - q_t B_{t+1}$$
$$\int_0^1 P_t(i)(1 - \xi_{n,t}(i))c_{n,t}(i)di \leq M_{n,t}$$

where $c_{r,t}(i)$ is consumption of good i purchased by a trader at period t; $c_{n,t}(i)$ is consumption good i purchased by a nontrader at period t; $P_t(i)$ is the price of consumption good i at period t.

At the end of each period, all agents return home. Workers receive the revenue from sales, $P_t y$. No further trade or barter is allowed. The budget constraints of traders and nontraders are

$$\int_0^1 P_t(i)c_{r,t}(i)di + M_{r,t+1} = M_{r,t} + B_t - q_t B_{t+1} + P_t y$$

$$\int_0^1 P_t(i)c_{n,t}(i)di + M_{n,t+1} = M_{n,t} + P_t y$$

where P_t is the average price level of consumption goods.

3.3 Equilibrium Dynamics

3.3.1 Optimization

Traders solve the following problem,

$$\max_{c_{r,t},x_{r,t},\xi_{r,t},M_{r,t+1},B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 ln\left(c_{r,t}(i)\right) di - x_{r,t} \right\}$$

subject to

$$\int_0^1 P_t(i)(1-\xi_{r,t}(i))c_{r,t}(i)di \leq M_{r,t} + B_t - q_t B_{t+1} \tag{3.2}$$

$$\int_0^1 P_t(i)c_{r,t}(i)di + M_{r,t+1} = M_{r,t} + B_t - q_t B_{t+1} + P_t y \tag{3.3}$$

$$x_{r,t} = \int_0^1 \xi_{r,t}(i)\gamma(i)di$$

$$M_{r,t+1} \geq 0, \qquad B_{t+1} \geq \bar{b}, \qquad \text{where } \bar{b} \leq 0 \tag{3.4}$$

where inequalities (3.4) are the nonnegativity constraints and the no Ponzi-scheme constraint.

Now, nontraders solve the following problem,

$$\max_{c_{n,t},x_{n,t},\xi_{n,t},M_{n,t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 ln\left(c_{n,t}(i)\right) di - x_{n,t} \right\}$$

subject to

$$\int_0^1 P_t(i)(1-\xi_{n,t}(i))c_{n,t}(i)di \leq M_{n,t} \tag{3.5}$$

$$\int_0^1 P_t(i)c_{n,t}(i)di + M_{n,t+1} = M_{n,t} + P_t y \tag{3.6}$$

$$x_{n,t} = \int_0^1 \xi_{n,t}(i)\gamma(i)di$$

$$M_{n,t+1} \geq 0 \tag{3.7}$$

where inequalities (3.7) are the nonnegativity constraints.

Definition: A competitive equilibrium consists of the sequences $\{c_{j,t}(i), \xi_{j,t}, M_{j,t+1}, B_{t+1}, M_t^s, P_t(i), R_t\}_{t=0}^{\infty}$ where $i \in [0,1]$ and $j \in \{r, n\}$ such that:

1. $\{c_{j,t}(i), \xi_{j,t}, M_{j,t+1}, B_{t+1}\}_{t=0}^{\infty}$ solves the household problems of traders and non-traders given $\{M_t^s, P_t(i), R_t\}_{t=0}^{\infty}$ for all market i.

2. Markets clear in every period:

   (a) Bond Market: for each trader,

   $$B_t - q_t B_{t+1} = \frac{\mu_t M_t}{\alpha}.$$

   (b) Money Market: for all t,

   $$M_{t+1}^s = M_{t+1} = \alpha M_{r,t+1} + (1-\alpha) M_{n,t+1}$$

   (c) Goods Market: for each market i,

   $$\alpha c_{r,t}(i) + (1-\alpha) c_{n,t}(i) = y$$

   and in aggregate,

   $$\alpha \int_0^1 c_{r,t}(i)di + (1-\alpha)\int_0^1 c_{n,t}(i)di = y.$$

3.3.2 Equilibrium

In equilibrium, suppose $\lambda^1_{j,t}$ and $\lambda^2_{j,t}$ where $j \in \{r, n\}$ denote the Lagrange multipliers associated with the cash-in-advance constraint and the budget constraint respectively for traders and nontraders at period t. Then, at period t, the equilibrium choices of traders and nontraders for $c_{j,t}(i)$, $\xi_{j,t}$, $M_{j,t+1}$, and B_{t+1} are as follows:

$$\frac{1}{c_{j,t}(i)} - \lambda^1_{j,t}(1 - \xi_{j,t}(i))P_t - \lambda^2_{j,t}P_t = 0 \tag{3.8}$$

$$\frac{1}{c^1_{j,t}(i)} - \lambda^2_{j,t}P_t = 0 \qquad ,\text{if } \xi_{j,t}(i) = 1 \tag{3.9}$$

$$\frac{1}{c^0_{j,t}(i)} - (\lambda^1_{j,t} + \lambda^2_{j,t})P_t = 0 \qquad ,\text{if } \xi_{j,t}(i) = 0 \tag{3.10}$$

$$\xi_{j,t}(i) = \begin{cases} 1, \text{ if } ln(c^1_{j,t}(i)) - \gamma(i_{j,t}) - \lambda^2_{j,t}c^1_{j,t}(i)P_t > ln(c^0_{j,t}(i)) - c^0_{j,t}(i)(\lambda^1_{j,t} + \lambda^2_{j,t})P_t \\ 0, \text{ if } ln(c^1_{j,t}(i)) - \gamma(i_{j,t}) - \lambda^2_{j,t}c^1_{j,t}(i)P_t < ln(c^0_{j,t}(i)) - c^0_{j,t}(i)(\lambda^1_{j,t} + \lambda^2_{j,t})P_t \end{cases} \tag{3.11}$$

$$\beta E_t\left[\left(\lambda^1_{j,t+1} + \lambda^2_{j,t+1}\right) \mid \mu_t\right] = \lambda^2_{j,t} \tag{3.12}$$

$$\beta E_t\left[\left(\lambda^1_{r,t+1} + \lambda^2_{r,t+1}\right) \mid \mu_t\right] = q_t\left(\lambda^1_{r,t} + \lambda^2_{r,t}\right) \tag{3.13}$$

First, $P_t(i) = P_t$ for all market i holds because the marginal utility of consumption, the constant endowments, and the goods market clearing condition in each market i imply a no-arbitrage condition. In equation (3.8), the marginal utility of consumption is identical across market i: for all $i, k \in [0,1]$ and $j \in \{r, n\}$, $c_{j,t}(i)P_t(i) = c_{j,t}(k)P_t(k)$. Since $c_{j,t}(i) = y$ for each market i, every market sells goods at the same price, $P_t(i) = P_t$.

In equations (3.10) and (3.12), the cash-in-advance constraints of traders and nontraders bind, $\lambda^1_{r,t} > 0$ and $\lambda^1_{n,t} > 0$, if for $j \in \{r, n\}$,

$$\beta E_t \left[\frac{c^0_{j,t}}{c^0_{j,t+1}} \frac{P_t}{P_{t+1}} \right] < 1.$$

Thus, in equation (3.13), traders face the price of nominal bonds that is less than one, $q_t < 1$. Also, if both cash-in-constraints bind, then, in equations (3.9) and (3.10), both traders and nontraders use credit for larger purchases and use cash for smaller purchases:

$$\frac{c^0_{r,t}}{c^1_{r,t}} = \frac{\lambda^2_{r,t}}{\lambda^1_{r,t} + \lambda^2_{r,t}} < 1, \tag{3.14}$$
$$\frac{c^0_{n,t}}{c^1_{n,t}} = \frac{\lambda^2_{n,t}}{\lambda^1_{n,t} + \lambda^2_{n,t}} < 1.$$

Next, in equilibrium, money and credit coexist at period t. The cutoffs of the credit-cash choices of traders and nontraders are positive,

$$i^*_{r,t} > 0$$

and

$$i^*_{n,t} > 0$$

because equation (3.9) implies (3.10), and (3.11),

$$\gamma(i^*_{r,t}) = ln \left(\frac{c^1_{r,t}}{c^0_{r,t}} \right) > 0 \tag{3.15}$$

$$\gamma(i^*_{n,t}) = ln\left(\frac{c^1_{n,t}}{c^0_{n,t}}\right) > 0.$$

The cutoff, $i^*_{j,t}$, is determined where the transactions costs are equal to the marginal rate of substitution of $c^0_{j,t}$ for $c^1_{j,t}$ where $j \in \{r, n\}$. Since transaction costs increase as shoppers buy a greater variety of goods with credit, shoppers use credit to acquire good i where $i < i^*_{j,t}$ and they use cash to acquire good i where $i > i^*_{j,t}$. A bigger $i^*_{j,t}$, i.e. a higher marginal rate of substitution of $c^0_{j,t}$ for $c^1_{j,t}$, describes that shoppers purchases a larger variety of goods with credit in the market.

From equations (3.9) and (3.10), consumption with credit is equal among traders, $c^1_{r,t} = c^1_{r,t}(i)$, and nontraders, $c^1_{n,t} = c^1_{n,t}(i)$, because the marginal value of wealth is the same across markets. Similarly, consumption with cash is equal among traders, $c^0_{r,t} = c^0_{r,t}(i)$, and nontraders, $c^0_{n,t} = c^0_{n,t}(i)$, because the marginal value of cash is the same across markets. Therefore, the total resource constraint is

$$\alpha\left\{i^*_{r,t}c^1_{r,t} + (1 - i^*_{r,t})c^0_{r,t}\right\} + (1 - \alpha)\left\{i^*_{n,t}c^1_{n,t} + (1 - i^*_{n,t})c^0_{n,t}\right\} = y, \tag{3.16}$$

where $c^1_{r,t}$ is consumption with credit by a trader; $c^0_{r,t}$ is consumption with cash by a trader; $i^*_{r,t}c^1_{r,t}$ is the aggregate consumption purchased with credit by a trader; $(1 - i^*_{r,t})c^0_{r,t}$ is the aggregate consumption purchased with cash by a trader. Similarly, $c^1_{n,t}$ is consumption with credit by a nontrader; $c^0_{n,t}$ is consumption with cash by a nontrader; $i^*_{n,t}c^1_{n,t}$ is the aggregate consumption purchased with credit by a nontrader; $(1 - i^*_{n,t})c^0_{n,t}$ is the aggregate consumption purchased with cash by a nontrader.

The binding cash-in-advance constraint and the budget constraint of traders in equations (3.2) and (3.3) are

$$P_t(1 - i^*_{r,t})c^0_{r,t} = M_{r,t} + \frac{\mu_t M_t}{\alpha}, \tag{3.17}$$

$$P_t i^*_{r,t} c^1_{r,t} + M_{r,t+1} = P_t y. \tag{3.18}$$

Similarly, equations (3.5) and (3.6) imply that nontraders' are

$$P_t(1 - i^*_{n,t})c^0_{n,t} = M_{n,t}, \tag{3.19}$$

$$P_t i^*_{n,t} c^1_{n,t} + M_{n,t+1} = P_t y. \tag{3.20}$$

In equations (3.17) and (3.19), money is nonneutral because the money stock affects the consumption with cash of traders and nontraders. Traders initially receive the government money injection in the asset market. Nontraders do not receive it because they do not participate in the asset market. Traders can use the money injection in the goods market while nontraders cannot and it distinguishes their consumption with cash. At the end of the goods market, the distributional effect of money disappears because every household receives the same revenue from sales, $P_t y$.

In equilibrium, traders and nontraders receive only a fraction of their endowments, y, as cash at the end of period because they buy some varieties of goods with credit. Suppose, for all t, $\phi_{r,t} \in (0, 1)$ denotes the fraction of y that traders bring to the next period in the form of cash and $\phi_{n,t} \in (0, 1)$ denotes the fraction of y that nontraders bring to the next period in the form of cash. Then, the real money balances of traders and nontraders are

$$\frac{M_{r,t+1}}{P_t} = \phi_{r,t} y \qquad \text{and} \qquad \frac{M_{n,t+1}}{P_t} = \phi_{n,t} y \tag{3.21}$$

where $\phi_{r,t}$ and $\phi_{n,t}$ are determined by equation (3.12). Aggregate money demand is, for all t,

$$\frac{M_{t+1}}{P_t} = \Phi_t y \tag{3.22}$$

where

$$\Phi_t = \alpha\phi_{r,t} + (1-\alpha)\phi_{n,t} \in (0,1).$$

The inflation rate is

$$\frac{P_t}{P_{t-1}} = (1+\mu_t)\frac{\Phi_{t-1}}{\Phi_t}$$

3.4 Constant Money Growth

This section analyzes the choices of means of payment and the optimal monetary policy when the money growth rate, μ_t, is constant. Suppose $\mu_t = \mu$ for all t. Then, it is clear that each household carries the same fraction of income, $\phi_{r,t} = \phi_r$ and $\phi_{n,t} = \phi_n$, to the next period. Also, other choices of trades and nontraders are constant over periods. In equations (3.12), (3.17) - (3.22), traders' choices are

$$(1-i_r^*)c_r^0 = \left\{\frac{\phi_r + (\mu\Phi/\alpha)}{1+\mu}\right\} y \tag{3.23}$$

$$i_r^* c_r^1 = (1-\phi_r)y,$$

$$\frac{i_r^*}{(1-i_r^*)}e^{\gamma(i_r^*)} = \frac{(1-\phi_r)(1+\mu)}{\phi_r + (\mu\Phi/\alpha)}, \tag{3.24}$$

$$1 = \frac{\beta}{1+\mu}\left(\frac{c_r^1}{c_r^0}\right). \tag{3.25}$$

where equation (3.23) implies that

$$\mu \geq -\frac{\alpha\phi_r}{\Phi},$$

and nontraders' choices are

$$(1-i_n^*)c_n^0 = \frac{\phi_n}{1+\mu}y, \tag{3.26}$$

$$i_n^* c_n^1 = (1-\phi_n)y,$$

$$\frac{i_n^*}{(1-i_n^*)}e^{\gamma\left(i_{n,t}^*\right)} = \frac{(1-\phi_n)(1+\mu)}{\phi_n}. \tag{3.27}$$

$$1 = \frac{\beta}{1+\mu}\left(\frac{c_n^1}{c_n^0}\right). \tag{3.28}$$

First, when the money growth rate is constant, the liquidity effect disappears and only the Fisherian effect remains:

$$q = \frac{\beta}{1+\mu}$$

where the nominal interest rate is positive if $\mu > \beta - 1$.

In equations (3.25) and (3.28), the cutoffs of cash and credit purchases are identical between traders and nontraders,

$$i^* = i_r^* = i_n^*, \tag{3.29}$$

and

$$\gamma(i^*) = ln\left(\frac{1+\mu}{\beta}\right),$$

because the intertemporal marginal rates of substitution are the same. The real money balances of traders, ϕ_r, and nontraders, ϕ_n, are determined by equations (3.24), (3.27), and (3.29) as

$$\gamma\left(\frac{1}{\frac{\phi_r+(\mu\Phi/\alpha)}{\beta(1-\phi_r)}+1}\right)=ln\left(\frac{1+\mu}{\beta}\right) \tag{3.30}$$

$$\gamma\left(\frac{1}{\frac{\phi_n}{\beta(1-\phi_n)}+1}\right)=ln\left(\frac{1+\mu}{\beta}\right) \tag{3.31}$$

Equations (3.29) - (3.31) show that monetary policy has distributional effects on the cash-credit choice, real money balances, and consumption. Suppose the government injects money. Then, the nominal interest rate increases and traders receive the money injection in the asset market. Traders use credit for a larger variety of goods because the marginal cost of using money, the nominal interest rate, increases. However, nontraders do not receive the money injection and nontraders use credit for a larger variety of goods in order to ease the effect of inflation. Thus, both prefer using credit over money with a positive money injection:

$$\frac{\partial i^*}{\partial \mu}=\frac{1}{(1+\mu)\gamma'(i^*)}>0. \tag{3.32}$$

Although i^* increases with the money growth rate, the real money holdings of traders and nonraders are different. In equations (3.30) and (3.31), ϕ_r may increase or decrease with μ and ϕ_n decreases with μ:[1]

$$\frac{\partial \phi_r}{\partial \mu}>0 \qquad \text{or} \qquad <0, \tag{3.33}$$

[1]The derivation is in Appendix B.1.

$$\frac{\partial \phi_n}{\partial \mu} < 0. \tag{3.34}$$

Traders receive the money injection in the asset market, but they purchase a greater variety of goods with credit. There are two distributional effects on traders' choices: a direct effect and an indirect effect via the cash-credit choice. For traders, a direct effect increases real money balances, but an indirect effect decreases real money balances. In equation (3.23), aggregate consumption with cash may increase or decrease as well. On the other hand, nontraders simply decreases real money holdings, ϕ_n. The money injection reduces the value of money that nontraders hold and they start to purchase a larger amount of goods with credit because they cannot adjust their money holdings in the asset market. Thus, both direct and indirect effects imply that nontraders hold less cash, so in equation (3.26), aggregate consumption with cash clearly decreases. Nontraders use credit to compensate for the loss of consumption with cash due to inflation.

3.5 Stochastic Money Growth

In the previous section, when the money growth rate is constant, traders and nontraders choose credit and cash for the same variety of consumption goods. However, traders and nontraders respond to monetary policy in a different way due to direct and indirect effects. Liquidity effects do not appear in the nominal interest rate. This section studies the distributional effects of money when the household faces a stochastic money growth. Unlike the previous section, liquidity effects may appear.

Suppose that the money growth rate, μ_t, is independent and identically distributed. Then, from equations (3.17) - (3.22), traders' aggregate consumption choices are

$$(1-i^*_{r,t})c^0_{r,t} = \frac{\phi_{r,t-1} + (\mu_t \Phi_{t-1}/\alpha)}{1+\mu_t}\left(\frac{\Phi_t}{\Phi_{t-1}}\right) y \tag{3.35}$$

$$i^*_{r,t}c^1_{r,t} = (1-\phi_{r,t})y, \tag{3.36}$$

where

$$\mu_t > -\frac{\alpha\phi_{r,t-1}}{\Phi_{t-1}}.$$

Equations (3.35), (3.36), and (3.12),

$$1 = \beta E_t\left[\frac{c^1_{r,t}}{c^0_{r,t+1}}\left(\frac{\Phi_{t+1}}{\Phi_t}\frac{1}{1+\mu_{t+1}}\right)\right],$$

determine $\phi_{r,t}$ and $i^*_{r,t}$:

$$\left(\frac{i^*_{r,t}}{1-i^*_{r,t}}\right) e^{\gamma\left(i^*_{r,t}\right)} = \frac{(1-\phi_{r,t})(1+\mu_t)}{\phi_{r,t-1} + (\mu_t \Phi_{t-1}/\alpha)}\left(\frac{\Phi_{t-1}}{\Phi_t}\right), \tag{3.37}$$

and

$$i^*_{r,t} = \beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right) y\Psi_r \tag{3.38}$$

where Ψ_r is constant,

$$\Psi_r = E_t\left[\frac{\Phi_{t+1}}{c^0_{r,t+1}(1+\mu_{t+1})}\right].$$

Similarly, nontraders' aggregate consumption choices are

$$(1-i^*_{n,t})c^0_{n,t} = \frac{\phi_{n,t-1}}{1+\mu_t}\left(\frac{\Phi_t}{\Phi_{t-1}}\right) y, \tag{3.39}$$

$$i^*_{n,t} c^1_{n,t} = (1-\phi_{n,t})y. \tag{3.40}$$

Equations (3.39), (3.40), and (3.12),

$$1 = \beta E_t \left[\frac{c^1_{n,t}}{c^0_{n,t+1}} \left(\frac{\Phi_{t+1}}{\Phi_t} \frac{1}{1+\mu_{t+1}} \right) \right],$$

determines $\phi_{n,t}$ and $i^*_{n,t}$:

$$\left(\frac{i^*_{n,t}}{1-i^*_{n,t}} \right) e^{\gamma\left(i^*_{n,t}\right)} = \frac{(1-\phi_{n,t})(1+\mu_t)}{\phi_{n,t-1}} \left(\frac{\Phi_{t-1}}{\Phi_t} \right) \tag{3.41}$$

and

$$i^*_{n,t} = \beta \left(\frac{1-\phi_{n,t}}{\Phi_t} \right) y\Psi_n \tag{3.42}$$

where Ψ_n is constant,

$$\Psi_n = E_t \left[\frac{\Phi_{t+1}}{c^0_{n,t+1}(1+\mu_{t+1})} \right].$$

Money is nonneutral and it has distributional effects on real money balances, the cash-credit choice, and consumption. First, for traders, the following relation appears by inserting equation (3.38) into equation (3.37)[2]:

$$\frac{e^{\gamma\left(\beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r\right)}}{\dfrac{1}{\beta y \Psi_r} - \dfrac{1-\phi_{r,t}}{\Phi_t}} = \frac{1+\mu_t}{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}}. \tag{3.43}$$

Similarly, for nontraders, the following relation appears from equations (3.41) and (3.42)[3]:

$$\frac{e^{\gamma\left(\beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)y\Psi_n\right)}}{\dfrac{1}{\beta y \Psi_n} - \dfrac{1-\phi_{n,t}}{\Phi_t}} = \frac{(1+\mu_t)\Phi_{t-1}}{\phi_{n,t-1}}. \tag{3.44}$$

[2]The derivation is in Appendix B.2.

[3]The derivation is in Appendix B.3.

The effects of monetary policy on $\phi_{r,t}$ and $\phi_{n,t}$ are[4]

$$\frac{\partial \phi_{r,t}}{\partial \mu_t} > 0, \tag{3.45}$$

$$\frac{\partial \phi_{n,t}}{\partial \mu_t} < 0. \tag{3.46}$$

If the government injects money, traders receive it in the asset market, so the real money balances of traders, $\phi_{r,t}$, increase. However, the real money balances of non-traders, $\phi_{n,t}$, decrease. Inflation reduces the value of money and nontraders can not receive the money injection in the asset market, so their real money balances go down. In other words, total consumption with credit of traders in equation (3.36) decreases and that of nontraders in equation (3.40) increases.

3.5.1 Distributional Effects on Traders

In equation (3.43) and inequality (3.45), traders spend cash for a larger variety of goods if the money growth rate increases[5]:

$$\frac{\partial i^*_{r,t}}{\partial \mu_t} < 0 \tag{3.47}$$

because they receive the money injection in the asset market.

Traders hold larger real money balances and use cash more intensively and there are two effects of money on $c^0_{r,t}$ and $c^1_{r,t}$. One is a direct distributional effect and the other is an indirect effect via a change in $i^*_{r,t}$ on $c^0_{r,t}$ and $c^1_{r,t}$ in equations (3.35)

[4]The derivation is in Appendix B.4.

[5]The derivation is in Appendix B.5.

and (3.36). First, the effect on consumption with cash, $c^0_{r,t}$, is

$$\frac{\partial c^0_{r,t}}{\partial \mu_t} = \frac{1}{\left(1 - i^*_{r,t}\right)^2} \left\{ \underbrace{\frac{\partial \left((1 - i^*_{r,t})c^0_{r,t}\right)}{\partial \mu_t} \left(1 - i^*_{r,t}\right)}_{\text{direct effect}} \underbrace{+(1 - i^*_{r,t})c^0_{r,t}\frac{\partial i^*_{r,t}}{\partial \mu_t}}_{\text{indirect effect}} \right\}$$

and it may increase or decrease with μ_t.[6] The direct distributional effect may be positive or negative in equation (3.35) and inequality (3.45) and the indirect effect is negative in inequality (3.47). If the direct effect is greater than the indirect effect, then

$$\frac{\partial c^0_{r,t}}{\partial \mu_t} > 0.$$

Traders hold larger real money balances and purchase more consumption goods with cash. Also, a liquidity effect arises in the nominal interest rate. The bond price in equation (3.13) is

$$q_t = \beta E_t \left[\frac{c^0_{r,t}}{c^0_{r,t+1}} \frac{P_t}{P_{t+1}} \right]. \tag{3.48}$$

If the money growth rate increases, then $c^0_{r,t}$ increases and the bond price goes up. Thus, the nominal interest rate decreases with the money growth rate.

Now, the effect on consumption with credit, $c^1_{r,t}$, is

$$\frac{\partial c^1_{r,t}}{\partial \mu_t} = \frac{1}{\left(i^*_{r,t}\right)^2} \left\{ \underbrace{\frac{\partial \left(i^*_{r,t} c^1_{r,t}\right)}{\partial \mu_t} i^*_{r,t}}_{\text{direct effect}} \underbrace{-(1 - \phi_{r,t})y\frac{\partial i^*_{r,t}}{\partial \mu_t}}_{\text{indirect effect}} \right\} \tag{3.49}$$

and it also may increase or decrease with μ_t.[7] The direct distributional effect is negative in equation (3.36) and inequality (3.45) and the indirect effect is positive in

[6]Proposition 1 in Appendix B.9 implies it.

[7]The derivation is in Appendix B.6.

inequality (3.47). If the direct distributional effect is greater than the indirect effect, then

$$\frac{\partial c_{r,t}^{1}}{\partial \mu_t} < 0.$$

Traders hold larger real money balances and consume a smaller amount of consumption goods with credit.

3.5.2 Distributional Effects on Nontraders

For nontraders, in equation (3.44) and inequality (3.46), nontraders use credit for a larger variety of goods if the money growth rate goes up[8]:

$$\frac{\partial i_{n,t}^{*}}{\partial \mu_t} > 0 \tag{3.50}$$

because they do not receive the money injection in the asset market.

Nontraders hold smaller real money balances and use credit more intensively and there are two effects of money on $c_{n,t}^{0}$ and $c_{n,t}^{1}$. One is a direct distributional effect and the other is an indirect effect via a change in $i_{n,t}^{*}$ on $c_{n,t}^{0}$ and $c_{n,t}^{1}$ in equations (3.39) and (3.40). First, the effect on consumption with cash, $c_{n,t}^{0}$, is

$$\frac{\partial c_{n,t}^{0}}{\partial \mu_t} = \frac{1}{\left(1 - i_{n,t}^{*}\right)^2} \left\{ \underbrace{\frac{\partial \left((1 - i_{n,t}^{*}) c_{n,t}^{0}\right)}{\partial \mu_t} \left(1 - i_{n,t}^{*}\right)}_{\text{direct effect}} \underbrace{+ (1 - i_{n,t}^{*}) c_{n,t}^{0} \frac{\partial i_{n,t}^{*}}{\partial \mu_t}}_{\text{indirect effect}} \right\}$$

and it may increase or decrease with μ_t.[9] The direct distributional effect may be positive or negative in equation (3.39) and inequality (3.46) and the indirect effect

[8] The derivations is in Appendix B.7.

[9] Proposition 1 in Appendix B.9 implies it.

is positive in inequality (3.50). If the direct distributional effect is greater than the indirect effect, then

$$\frac{\partial c^0_{n,t}}{\partial \mu_t} < 0.$$

Nontraders hold smaller real money balances and they purchase less consumption goods with cash.

The effect on consumption with credit by nontraders, $c^1_{n,t}$, decreases with the money growth rate,

$$\frac{\partial c^1_{n,t}}{\partial \mu_t} = \frac{1}{\left(i^*_{r,t}\right)^2} \left\{ \underbrace{\frac{\partial \left(i^*_{n,t} c^1_{n,t}\right)}{\partial \mu_t} i^*_{n,t}}_{\text{direct effect}} \underbrace{-(1-\phi_{n,t}) y \frac{\partial i^*_{n,t}}{\partial \mu_t}}_{\text{indirect effect}} \right\} \tag{3.51}$$

and it may increase or decrease with μ_t.[10] The direct distributional effect is positive in equation (3.40) and inequality (3.46) and the indirect effect is negative in inequality (3.50). If the direct distributional effect is greater than the indirect effect, then

$$\frac{\partial c^1_{n,t}}{\partial \mu_t} > 0.$$

Nontraders hold smaller real money balances and consume a larger amount of consumption goods with credit.

3.5.3 Discussion

When the money growth rate is stochastic, in inequality (3.47), a positive money injection implies that traders purchase consumption goods for a greater variety of goods with cash in contrast to the result in inequality (3.32). With a positive money

[10] The derivation is in Appendix B.8.

injection, the marginal cost of using money, i.e. the nominal interest rate, in equation (3.48) may increase or decrease. Thus, traders purchase a greater variety of goods with cash that they receive in the asset market. However, nontraders use credit more intensively with a positive money injection to ease the effect of inflation although the money growth rate is stochastic.

A stochastic money injection results in two different distributional effects on traders' and nontraders' consumption. The money injection redistributes consumption goods from nontraders to traders, but the indirect effect on consumption via the change of $i^*_{r,t}$ and $i^*_{n,t}$ alleviates the direct effect of money on consumption. The effects on consumption with cash of traders, $c^0_{r,t}$, and of nontraders, $c^0_{n,t}$, are ambiguous and so are the effects of consumption with credit of traders, $c^1_{r,t}$, and of nontraders, $c^1_{n,t}$. To do understand it in a clear way, the following example with one means of payment will be useful. Suppose cash is the only means of payment in the economy. Then, $\phi_{r,t} = 1$, $\phi_{n,t} = 1$, $\Phi_t = 1$, $i^*_{r,t} = 0$, $i^*_{n,t} = 0$, and consumption with credit is zero, $c^1_{r,t} = 0 = c^1_{n,t}$. Thus, in equations (3.35) and (3.39), consumption of traders and nontraders are as in Alvarez, Lucas, and Weber with a unit velocity:

$$c^0_{r,t} = \left(\frac{1+\mu_t/\alpha}{1+\mu_t}\right) y$$

and

$$c^0_{n,t} = \left(\frac{1}{1+\mu_t}\right) y.$$

If the government injects money, then only traders receive it in the asset market. Money loses its value with inflation. Since household do not have alternative means of payment, traders hold more cash and consume more and nontraders hold less cash and consume less. Inflation taxes nontraders' consumption. Only the direct distributional effect is present and the indirect effect of money disappears. Without credit, consumption of traders and nontraders are highly affected by monetary policy and households do not have a device to alleviate the monetary policy shock.

Therefore, it is clear that the alternative means of payment, credit, enables households to ease monetary policy shocks. By switching from one to another means of payment, traders and nontraders can control their consumption more effectively by using credit against a money shock. On net, the effects of monetary policy on consumption of traders and nontraders are ambiguous. This result is very interesting in the sense that households hold multiple means of payment not simply to vary their means of exchange but to partially compensate for fluctuations of consumption if monetary policy moves against them.

3.6 Conclusion

An asset market segmentation model is constructed to study the distributional effects of monetary policy when there are multiple means of payment in the short run. There are traders, who participate in the goods market, and nontraders, who do not. In the asset market, when the government injects money through open market operations, traders receive the money injection first. In the goods market, traders and nontraders can use either credit or cash. By using cash, they forego nominal interest, and by using credit, they bear transactions costs.

In equilibrium, money is nonneutral and it has distributional effects on consumption goods between traders and nontraders. Monetary policy affects the choice of means of payment of traders and nontraders in an opposite way. Unlike Alvarez, Lucas, and Weber (2001), consumption of traders and nontraders may increase or decrease because there are two distributional effects of money: a direct effect and an indirect effect via the choice of means of payment. Suppose the government injects money. Then, the direct effect implies that traders can increase their consumption with a larger amount of money which comes from the asset market while nontraders can not. The indirect effect comes via the choice of means of payment. Traders purchase a larger variety of consumption goods with cash and nontraders purchase a larger variety of goods with credit. Thus, consumption of traders and nontraders may increase or decrease. Thus, credit may be used to dampen fluctuations in consumption arising from monetary policy. Liquidity effects disappear when the money growth rate is constant, and they may appear when the money growth rate is stochastic.

This paper shows that multiple means of payment can be used as a buffer against monetary shocks when monetary policy has distributional effects due to a segmented asset market. However, an unrealistic feature is the absence of financial intermediations. An interesting future extension would be to introduce default or the possibility of counterfeiting when only traders are allowed to access financial intermediaries in the model and then studying the distributional effects of monetary policy with multiple means of payment.

APPENDIX A
SELECTED PROOFS AND DERIVATIONS IN CHAPTER 2

A.1 Derivation of Inequality (2.11)

First, in equations (2.3) and (2.8),

$$\begin{aligned} q_t &= \beta E_t\left[\left(\frac{c_{r,t}}{c_{r,t+1}}\right)^\gamma \frac{1}{1+\mu_{t+1}}\frac{Y_{t+1}}{Y_t}\right] \\ &= \beta\left(\frac{\alpha y/Y_{t-1}+\mu_t}{1+\mu_t}\right)^\gamma \left(\frac{Y_t}{\alpha}\right)^\gamma \frac{\Psi}{Y_t} \\ &= \left(1-\frac{1}{1+\mu_t}\frac{(1-\alpha)y^n_{t-1}}{Y_{t-1}}\right)^\gamma \left(\frac{Y_t^{\gamma-1}\beta\Psi}{\alpha^\gamma}\right) \end{aligned}$$

where Ψ_r is constant and

$$\Psi_r = E_t\left[\left(\frac{1}{c_{r,t+1}}\right)^\gamma \frac{Y_{t+1}}{1+\mu_{t+1}}\right].$$

A positive return on nominal bonds, $q_t < 1$, implies

$$\frac{1}{1+\mu_t} > \frac{Y_{t-1}}{(1-\alpha)y^n_{t-1}}\left(1-\frac{\alpha}{(Y_t^{\gamma-1}\beta\Psi_r)^{\frac{1}{\gamma}}}\right) \tag{A.1}$$

where $(Y_t^{\gamma-1}\beta\Psi_r)^{\frac{1}{\gamma}} > 0$, $(1-\alpha)y^n_{t-1} < Y_{t-1}$ and $\mu_t > -1$.

Next, in equations (2.4) and (2.9), for all i,

$$\begin{aligned} \beta E_t\left[\left(\frac{c_{i,t}}{c_{i,t+1}}\right)^\gamma \frac{1}{1+\mu_{t+1}}\frac{Y_{t+1}}{Y_t}\right] &= \beta\left(\frac{y_{i,t-1}}{1+\mu_t}\right)^\gamma \left(\frac{Y_t}{Y_{t-1}}\right)^\gamma \frac{\Psi_i}{Y_t} \\ &= \beta\left(\frac{y_{i,t-1}}{Y_{t-1}(1+\mu_t)}\right)^\gamma (Y_t^{\gamma-1}\beta\Psi_i) \end{aligned}$$

where Ψ_i is constant and

$$\Psi_i = E_t\left[\left(\frac{1}{c_{i,t+1}}\right)^\gamma \frac{Y_{t+1}}{1+\mu_{t+1}}\right].$$

Inequality (2.4) implies

$$\frac{1}{1+\mu_t} < \frac{Y_{t-1}}{y_{i,t-1}} \left(Y_t^{\gamma-1}\beta\Psi_i\right)^{\frac{1}{\gamma}} \tag{A.2}$$

where $(Y_t^{\gamma-1}\beta\Psi_i)^{\frac{1}{\gamma}} > 0$, $(1-\alpha)y_{t-1}^n < Y_{t-1}$ and $\mu_t > -1$. Therefore, inequalities (A.1) and (A.2) imply inequality (2.11),

$$\frac{Y_{t-1}}{(1-\alpha)y_{t-1}^n}\left(1 - \frac{\alpha}{(Y_t^{\gamma-1}\beta\Psi_r)^{\frac{1}{\gamma}}}\right) < \frac{1}{1+\mu_t} < \frac{Y_{t-1}}{y_{i,t-1}} \left(Y_t^{\gamma-1}\beta\Psi_i\right)^{\frac{1}{\gamma}}.$$

A.2 Derivation of Equation (2.13)

When $y_{i,t} = y^n$ for all i, $\Psi_i = \Psi^n$ in inequalities (2.11) and

$$\frac{Y_{t-1}}{(1-\alpha)y_{t-1}^n}\left(1 - \frac{\alpha}{(Y_t^{\gamma-1}\beta\Psi_r)^{\frac{1}{\gamma}}}\right) < \frac{1}{1+\mu_t} < \frac{Y_{t-1}}{y_{t-1}^n} \left(Y_t^{\gamma-1}\beta\Psi^n\right)^{\frac{1}{\gamma}}.$$

Therefore,

$$1+\mu_t^* = \frac{y_{t-1}^n}{Y_{t-1}}$$

implies

$$(Y_t^{\gamma-1}\beta\Psi_r)^{\frac{1}{\gamma}} < 1 < (Y_t^{\gamma-1}\beta\Psi^n)^{\frac{1}{\gamma}}.$$

A.3 Derivation of Equation (2.14)

The first order condition is

$$\frac{\partial W_t}{\partial \mu_t} = 0 = \alpha\left(\frac{Y_{t-1}}{\alpha} - y\right)\left(\frac{y+\mu_t Y_{t-1}/\alpha}{1+\mu_t}\right)^{-\gamma} - (1-\alpha)\left(\frac{1}{1+\mu_t}\right)^{-\gamma}\widetilde{y}_{t-1}^n$$

and it implies that equation (2.13) holds. The second order condition comes negative:

$$\begin{aligned}\frac{\partial^2 W_t}{\partial \mu_t^2} &= -\alpha\gamma\left(Y_{t-1}-\alpha y\right)^2\left(y + \frac{\mu_t Y_{t-1}}{\alpha}\right)^{-\gamma-1}\left(\frac{1}{1+\mu_t}\right)^{1-\gamma} - (1-\alpha)\gamma\left(\frac{1}{1+\mu_t}\right)^{1-\gamma}\widetilde{y}_{t-1}^n \\ &= -\left(\frac{1}{1+\mu_t}\right)^{1-\gamma}\left\{\alpha\gamma\left(Y_{t-1}-\alpha y\right)^2\left(y+\frac{\mu_t Y_{t-1}}{\alpha}\right)^{-\gamma-1} + (1-\alpha)\gamma\widetilde{y}_{t-1}^n\right\} < 0,\end{aligned}$$

given positive consumption levels on both traders and nontraders.

A.4 Proof of $\left(y_{t-1}^{n}\right)^{1-\gamma} \geq \widetilde{y}_{t-1}^{n}$

Since $u(c)$ is strictly concave, Jensen's inequality, $E[u(c)] \leq u(E[c])$, holds. Given CRRA prefernces, $E[u(c)]$ and $u(E[c])$ are as follows:

$$E[u(c_{i,t})] = \frac{1}{1-\gamma}\left(\frac{Y_t}{Y_{t-1}}\right)^{1-\gamma}\int\left(\frac{y_{i,t-1}}{1+\mu_t}\right)^{1-\gamma}dF_t,$$

$$u(E[c_{i,t}]) = \frac{1}{1-\gamma}\left(\frac{Y_t}{Y_{t-1}}\right)^{1-\gamma}\left(\int\frac{y_{i,t-1}}{1+\mu_t}dF_t\right)^{1-\gamma}.$$

Therefore, $E[u(c)] \leq u(E[c])$ if and only if, for all t,

$$\widetilde{y}_{t-1}^{n} \leq \left(y_{t-1}^{n}\right)^{1-\gamma} \tag{A.3}$$

where

$$\widetilde{y}_{t-1}^{n} = \int\left(y_{i,t-1}\right)^{1-\gamma}dF_t,$$

$$y_{t-1}^{n} = \int y_{t-1}^{n}dF_t.$$

Therefore, equation (A.3) implies

$$\left(\frac{\widetilde{y}_{t-1}^{n}}{\left(y_{t-1}^{n}\right)^{1-\gamma}}\right)^{\frac{1}{\gamma}} \leq 1.$$

A.5 Derivation of Equation (2.15)

Given equations (2.9) and equation (2.14), the consumption levels of traders and nontraders are

$$\hat{c}_{r,t} = \frac{\left(\frac{y_{t-1}^{n}}{\widetilde{y}_{t-1}^{n}}\right)^{\frac{1}{\gamma}}}{\alpha\left(\frac{y_{t-1}^{n}}{\widetilde{y}_{t-1}^{n}}\right)^{\frac{1}{\gamma}}+(1-\alpha)y_{t-1}^{n}}Y_t$$

$$\hat{c}_{i,t} = \frac{y_{i,t-1}}{\alpha\left(\frac{y_{t-1}^{n}}{\widetilde{y}_{t-1}^{n}}\right)^{\frac{1}{\gamma}}+(1-\alpha)y_{t-1}^{n}}Y_t$$

where

$$1 + \hat{\mu}_t = \frac{1}{Y_{t-1}} \left\{ \alpha \left(\frac{y^n_{t-1}}{\widetilde{y}^n_{t-1}} \right)^{\frac{1}{\gamma}} + (1-\alpha)\, y^n_{t-1} \right\}.$$

Now, divide $\hat{c}_{r,t}$ and $\hat{c}_{i,t}$ by $\left(\frac{y^n_{t-1}}{\widetilde{y}^n_{t-1}} \right)^{\frac{1}{\gamma}}$. Then, rearrange the denominator with respect to α and equations (2.15) hold.

A.6 Proof of $\hat{\mu}_t \geq \mu^*_t$

Assume $\hat{\mu}_t$ do the money growth rate in equation (2.14) and μ^*_t represent the money growth rate in equation (2.12). Then, inequality (A.3) implies that

$$\hat{\mu}_t - \mu^*_t = \frac{\alpha}{Y_{t-1}} \left\{ \left(\frac{y^n_{t-1}}{\widetilde{y}^n_{t-1}} \right)^{\frac{1}{\gamma}} - y^n_{t-1} \right\} = \frac{\alpha y^n_{t-1}}{Y_{t-1}} \left\{ \left(\frac{\left(y^n_{t-1}\right)^{1-\gamma}}{\widetilde{y}^n_{t-1}} \right)^{\frac{1}{\gamma}} - 1 \right\} \geq 0.$$

where the equality holds if $\gamma = 1$.

A.7 The Effects of γ on $\hat{\mu}_t$

From equation (2.15), the effects of γ may be positive or negative depending on the distribution of $y_{i,t-1}$:

$$\frac{\partial \left(y^n_{t-1} / \widetilde{y}^n_{t-1} \right)^{\frac{1}{\gamma}}}{\partial \gamma} = \frac{\left(y^n_{t-1}\right)^{\frac{1}{\gamma}} \left(\widetilde{y}^n_{t-1}\right)^{\frac{1}{\gamma}+1}}{\gamma} \int \left(y_{i,t-1}\right)^{1-\gamma} ln(y_{i,t-1}) dF_t,$$

where

$$\frac{\partial \widetilde{y}^n_{t-1}}{\partial \gamma} = - \int \left(y_{i,t-1}\right)^{1-\gamma} ln(y_{i,t-1}) dF_t.$$

If some endowments are less than one, then $ln(y_{i,t-1})$ becomes negative. Therefore, the effects of γ on $\hat{\mu}_t$ can be positive or negative in equation (2.14):

$$\frac{\partial \hat{\mu}_t}{\partial \gamma} = \frac{\alpha}{Y_{t-1}} \frac{\partial \left(y^n_{t-1} / \widetilde{y}^n_{t-1} \right)^{\frac{1}{\gamma}}}{\partial \gamma}.$$

APPENDIX B
SELECTED PROOFS AND DERIVATIONS IN CHAPTER 3

B.1 Derivation of Inequalities (3.33) and (3.34)

In equation (3.29),

$$\gamma(i^*) = ln\left(\frac{1+\mu}{\beta}\right)$$

and insert it into equations (3.24) and (3.27). Then, after arranging them, equations (3.30) and (3.31) hold and

$$i^* = \frac{1}{\frac{\phi_n}{\beta(1-\phi_n)}+1} \tag{B.1}$$

$$i^* = \frac{1}{\frac{\phi_r+(\mu\Phi/\alpha)}{\beta(1-\phi_r)}+1} \tag{B.2}$$

and in inequality (3.30),

$$\frac{\partial i^*}{\partial \mu} > 0.$$

First, equation (B.1) has the following relation with respect to μ:

$$\frac{\partial \phi_n}{\partial \mu} < 0 \tag{B.3}$$

where

$$0 < \frac{\partial i^*}{\partial \mu} = \frac{-\left(i^*\right)^2}{\beta(1-\phi_n)^2}\left(\frac{\partial \phi_n}{\partial \mu}\right). \tag{B.4}$$

Now, equation (B.2) with inequality (B.3) implies that ϕ_r may increase or decrease with μ_t

$$\frac{\partial \phi_r}{\mu}\left(1+\mu+\frac{(1-\alpha)\phi_n\mu}{\alpha}\right)+\left(\frac{1-\alpha}{\alpha}\right)\frac{\partial \phi_n}{\mu}\frac{\mu}{\alpha}(1-\phi_r) < -\frac{\Phi}{\alpha}(1-\phi_r)$$

from

$$0 < \frac{\partial i^*}{\partial \mu} = \tag{B.5}$$
$$\frac{-\left(i^*\right)^2}{\beta(1-\phi_r)^2}\left\{\frac{\partial \phi_r}{\mu}\left(1+\mu+\frac{(1-\alpha)\phi_n\mu}{\alpha}\right)+\frac{\Phi}{\alpha}(1-\phi_r)+\left(\frac{1-\alpha}{\alpha}\right)\frac{\partial \phi_n}{\mu}\frac{\mu}{\alpha}(1-\phi_r)\right\}.$$

B.2 Derivation of Equation (3.43)

Insert equation (3.38) into equation (3.37). Then,

$$\left(\frac{\beta(1-\phi_{r,t})y\Psi_r}{\Phi_t-\beta(1-\phi_{r,t})y\Psi_r}\right)e^{\gamma\left(\beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r\right)} = \left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\left(\frac{1+\mu_t}{\frac{\phi_{r,t-1}}{\Phi_{t-1}}+\frac{\mu_t}{\alpha}}\right),$$

where

$$\frac{i^*_{r,t}}{1-i^*_{r,t}} = \frac{\beta(1-\phi_{r,t})y\Psi_r}{\Phi_t-\beta(1-\phi_{r,t})y\Psi_r}$$

and they imply

$$\frac{e^{\gamma\left(\beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r\right)}}{\frac{1}{\beta y\Psi_r}-\frac{1-\phi_{r,t}}{\Phi_t}} = \frac{1+\mu_t}{\frac{\phi_{r,t-1}}{\Phi_{t-1}}+\frac{\mu_t}{\alpha}}.$$

B.3 Derivation of Equation (3.44)

Insert equation (3.42) into equation (3.41). Then,

$$\left(\frac{\beta(1-\phi_{n,t})y\Psi_n}{\Phi_t-\beta(1-\phi_{n,t})y\Psi_n}\right)e^{\gamma\left(\beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)y\Psi_n\right)} = \left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\left(\frac{1+\mu_t}{\frac{\phi_{n,t-1}}{\Phi_{t-1}}}\right),$$

where

$$\frac{i^*_{n,t}}{1-i^*_{n,t}} = \frac{\beta(1-\phi_{n,t})y\Psi_n}{\Phi_t-\beta(1-\phi_{n,t})y\Psi_n}$$

and they imply

$$\frac{e^{\gamma\left(\beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)y\Psi_n\right)}}{\frac{1}{\beta y\Psi_n}-\frac{1-\phi_{n,t}}{\Phi_t}} = \frac{(1+\mu_t)\Phi_{t-1}}{\phi_{n,t-1}}.$$

B.4 Derivation of Inequalities (3.45) and (3.46)

First, in equation (3.43), the right-hand side of equation (3.43) is

$$RHS = \frac{1+\mu_t}{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}},$$

and the effect of the money growth rate on RHS is negative:

$$\frac{\partial RHS}{\partial \mu_t} = \frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} - \dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}\right)^2} < 0 \tag{B.6}$$

where

$$\frac{1}{\alpha}\left(\frac{\alpha\phi_{r,t-1}}{\Phi_{t-1}} - 1\right) < 0.$$

Next, the left-hand side of equation (3.43) is

$$LHS = \frac{e^{\gamma\left(\beta A_t y \Psi_r\right)}}{\dfrac{1}{\beta y \Psi_r} - A_t}$$

where

$$A_t = \frac{1-\phi_{r,t}}{\Phi_t}$$

and

$$\Phi_t = \alpha\phi_{r,t} + (1-\alpha)\phi_{n,t}.$$

Since RHS decreases with μ_t, the effect of the money growth rate on LHS should be negative:

$$\begin{aligned}\frac{\partial LHS}{\partial \mu_t} &= \frac{\left\{\gamma'\left(\beta A_t y \Psi_r\right)\left(1-\beta A_t y \Psi_r\right)+1\right\} e^{\gamma\left(\beta A_t y \Psi_r\right)}}{\left(\dfrac{1}{\beta y \Psi_r} - A_t\right)^2}\left(\frac{\partial A_t}{\partial \mu_t}\right) \\ &= \frac{\left(\beta y \Psi_r\right)^2\left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\} e^{\gamma\left(i^*_{r,t}\right)}}{\left(1-i^*_{r,t}\right)^2}\left(\frac{\partial A_t}{\partial \mu_t}\right) < 0,\end{aligned} \tag{B.7}$$

where in equation (3.38), $i^*_{r,t} = \beta A_t y \Psi_r$ and

$$\frac{1}{\beta y \Psi_r} - A_t = \frac{1 - i^*_{r,t}}{\beta y \Psi_r}.$$

Therefore, the following should hold

$$\begin{aligned} \frac{\partial A_t}{\partial \mu_t} &= \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} \\ &= \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\left(\Phi_t + \alpha(1-\phi_{r,t})\right) + (1-\alpha)(1-\phi_{r,t})\frac{\partial \phi_{n,t}}{\partial \mu_t}\right\} < 0. \end{aligned} \tag{B.8}$$

Second, in equation (3.44), the right-hand side of equation (3.44) is

$$RHS = \frac{(1+\mu_t)\Phi_{t-1}}{\phi_{n,t-1}},$$

and the effect of the money growth rate on RHS is negative:

$$\frac{\partial RHS}{\partial \mu_t} = \frac{\Phi_{t-1}}{\phi_{n,t-1}} > 0. \tag{B.9}$$

Now, the left-hand side of equation (3.44) is

$$LHS = \frac{e^{\gamma\,(\beta B_t y \Psi_n)}}{\dfrac{1}{\beta y \Psi_n} - B_t}$$

where

$$B_t = \frac{1-\phi_{n,t}}{\Phi_t}$$

and

$$\Phi_t = \alpha\phi_{r,t} + (1-\alpha)\phi_{n,t}.$$

Since RHS increases with μ_t, the effect of the money growth rate should be positive:

$$\begin{aligned} \frac{\partial LHS}{\partial \mu_t} &= \frac{\left\{\gamma'\left(\beta B_t y \Psi_n\right)\left(1-\beta B_t y \Psi_n\right)+1\right\} e^{\gamma\,(\beta B_t y \Psi_n)}}{\left(\dfrac{1}{\beta y \Psi_n} - B_t\right)^2}\left(\frac{\partial B_t}{\partial \mu_t}\right) \\ &= \frac{(\beta y \Psi_n)^2\left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\} e^{\gamma\,\left(i^*_{n,t}\right)}}{\left(1-i^*_{n,t}\right)^2}\left(\frac{\partial B_t}{\partial \mu_t}\right) > 0, \end{aligned} \tag{B.10}$$

where in equation (3.42), $i^*_{n,t} = \beta B_t y \Psi_n$ and

$$\frac{1}{\beta y \Psi_n} - B_t = \frac{1 - i^*_{n,t}}{\beta y \Psi_n}.$$

Therefore, the following should hold

$$\begin{aligned}\frac{\partial B_t}{\partial \mu_t} &= \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} \\ &= \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\left(\Phi_t + (1-\alpha)(1-\phi_{n,t})\right) + \alpha(1-\phi_{n,t})\frac{\partial \phi_{r,t}}{\partial \mu_t}\right\} > 0.\end{aligned} \tag{B.11}$$

Overall, inequalities (B.8) and (B.11) imply that

$$\frac{\partial \phi_{r,t}}{\partial \mu_t} > 0,$$

$$\frac{\partial \phi_{n,t}}{\partial \mu_t} < 0,$$

where

$$\frac{\partial \phi_{n,t}}{\partial \mu_t}\left(\frac{\Phi_t}{1-\phi_{n,t}}\right) < -\frac{\partial \Phi_t}{\partial \mu_t} < \frac{\partial \phi_{r,t}}{\partial \mu_t}\left(\frac{\Phi_t}{1-\phi_{r,t}}\right). \tag{B.12}$$

B.5 Derivation of Inequality (3.47)

Equation (3.38) and inequality (B.8) in appendix D,

$$\frac{\partial A_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} < 0,$$

implies that

$$\frac{\partial i^*_{r,t}}{\partial \mu_t} = \beta \frac{\partial A_t}{\partial \mu_t} y \Psi_r < 0. \tag{B.13}$$

B.6 Derivation of Inequality (3.49)

First, in equation (3.36) and inequality (3.45),

$$\frac{\partial\left(i_{r,t}^{*}c_{r,t}^{1}\right)}{\partial\mu_t} = -\frac{\partial\phi_{r,t}}{\partial\mu_t}y < 0. \tag{B.14}$$

Now, from equation (3.49) and inequalities (B.8), (B.13) and (B.14),

$$\begin{aligned}
\frac{\partial c_{r,t}^{1}}{\partial\mu_t} &= \frac{1}{\left(i_{r,t}^{*}\right)^2}\left\{\frac{\partial\left(i_{r,t}^{*}c_{r,t}^{1}\right)}{\partial\mu_t}i_{r,t}^{*} - (1-\phi_{r,t})y\frac{\partial i_{r,t}^{*}}{\partial\mu_t}\right\} \\
&= \frac{1}{\left(i_{r,t}^{*}\right)^2}\left\{-\frac{\partial\phi_{r,t}}{\partial\mu_t}yi_{r,t}^{*} - (1-\phi_{r,t})y\beta\frac{\partial A_t}{\partial\mu_t}y\Psi_r\right\} \\
&= \frac{1}{\left(i_{r,t}^{*}\right)^2}\left\{-\frac{\partial\phi_{r,t}}{\partial\mu_t}yi_{r,t}^{*} + \left(\frac{y}{\Phi_t}\right)\beta\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)y\Psi_r\left(\frac{\partial\phi_{r,t}}{\partial\mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial\Phi_t}{\partial\mu_t}\right)\right\}.
\end{aligned}$$

Equation (3.38) implies the indirect effect is greater than the direct effect:

$$\begin{aligned}
\frac{\partial c_{r,t}^{1}}{\partial\mu_t} &= \frac{1}{\left(i_{r,t}^{*}\right)^2}\left\{-\frac{\partial\phi_{r,t}}{\partial\mu_t}yi_{r,t}^{*} + \left(\frac{y}{\Phi_t}\right)i_{r,t}^{*}\left\{\frac{\partial\phi_{r,t}}{\partial\mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial\Phi_t}{\partial\mu_t}\right\}\right\} \\
&= \frac{y}{i_{r,t}^{*}}\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\frac{\partial\Phi_t}{\partial\mu_t}.
\end{aligned} \tag{B.15}$$

Finally, inequalities (B.12),

$$-\frac{\partial\phi_{r,t}}{\partial\mu_t} < \left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\frac{\partial\Phi_t}{\partial\mu_t},$$

implies that $c_{r,t}^{1}$ may increase or decrease with μ_t:

$$\frac{\partial c_{r,t}^{1}}{\partial\mu_t} > -\left(\frac{y}{i_{r,t}^{*}}\right)\frac{\partial\phi_{r,t}}{\partial\mu_t},$$

where

$$\frac{\partial\phi_{r,t}}{\partial\mu_t} > 0.$$

B.7 Derivation of Inequality (3.50)

Equation (3.42) and inequality (B.11) in appendix D,

$$\frac{\partial B_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\} > 0,$$

implies that

$$\frac{\partial i^*_{n,t}}{\partial \mu_t} = \beta \frac{\partial B_t}{\partial \mu_t} y \Psi_n > 0. \tag{B.16}$$

B.8 Derivation of Inequality (3.51)

First, in equation (3.40) and inequality (3.46), total consumption with credit increases with inflation,

$$\frac{\partial \left(i^*_{n,t} c^1_{n,t}\right)}{\partial \mu_t} = -\frac{\partial \phi_{n,t}}{\partial \mu_t} y > 0. \tag{B.17}$$

Now, from equation (3.51) and inequalities (B.11), (B.16) and (B.17),

$$\begin{aligned}
\frac{\partial c^1_{n,t}}{\partial \mu_t} &= \frac{1}{\left(i^*_{n,t}\right)^2}\left\{\frac{\partial \left(i^*_{n,t} c^1_{n,t}\right)}{\partial \mu_t} i^*_{n,t} - (1-\phi_{n,t}) y \frac{\partial i^*_{n,t}}{\partial \mu_t}\right\} \\
&= \frac{1}{\left(i^*_{n,t}\right)^2}\left\{-\frac{\partial \phi_{n,t}}{\partial \mu_t} y i^*_{n,t} - (1-\phi_{n,t}) y \beta \frac{\partial B_t}{\partial \mu_t} y \Psi_n\right\} \\
&= \frac{1}{\left(i^*_{n,t}\right)^2}\left\{-\frac{\partial \phi_{n,t}}{\partial \mu_t} y i^*_{n,t} + \left(\frac{y}{\Phi_t}\right)\beta\left(\frac{1-\phi_{n,t}}{\Phi_t}\right) y \Psi_n \left(\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right)\right\}.
\end{aligned}$$

Equation (3.42) implies

$$\begin{aligned}
\frac{\partial c^1_{n,t}}{\partial \mu_t} &= \frac{1}{\left(i^*_{n,t}\right)^2}\left\{-\frac{\partial \phi_{n,t}}{\partial \mu_t} y i^*_{n,t} + \left(\frac{y}{\Phi_t}\right) i^*_{n,t}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\}\right\} \\
&= \frac{y}{i^*_{n,t}}\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t}.
\end{aligned} \tag{B.18}$$

Finally, inequalities (B.12),

$$-\frac{\partial \phi_{n,t}}{\partial \mu_t} > \left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t},$$

implies that $c^1_{n,t}$ may increase or decrease with μ_t:

$$\frac{\partial c^1_{n,t}}{\partial \mu_t} < -\left(\frac{y}{i^*_{n,t}}\right)\frac{\partial \phi_{n,t}}{\partial \mu_t},$$

where

$$\frac{\partial \phi_{n,t}}{\partial \mu_t} < 0.$$

B.9 Find out $\partial \Phi_t / \partial \mu_t$

In Appendix D, the effects of μ_t on equation (3.43) follows from equations (B.6) and (B.7):

$$\frac{(\beta y \Psi_r)^2 \left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\} e^{\gamma\left(i^*_{r,t}\right)}}{\left(1-i^*_{r,t}\right)^2}\left(\frac{\partial A_t}{\partial \mu_t}\right) = \frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} - \dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}\right)^2},$$

where in equation (B.8)

$$\frac{\partial A_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{r,t}}{\partial \mu_t}\Phi_t + (1-\phi_{r,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\}.$$

Therefore, they imply

$$\begin{aligned}
&\alpha\frac{\partial \phi_{r,t}}{\partial \mu_t} + \alpha\left(\frac{1-\phi_{r,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t} \\
&= \frac{-\alpha\Phi_t\left(1-i^*_{r,t}\right)^2}{(\beta y \Psi_r)^2\left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\} e^{\gamma\left(i^*_{r,t}\right)}}\left\{\frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} - \dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}} + \dfrac{\mu_t}{\alpha}\right)^2}\right\}.
\end{aligned} \tag{B.19}$$

Next, in Appendix E, the effects of μ_t on equation (3.44) follows from equations (B.9) and (B.10):

$$\frac{(\beta y \Psi_n)^2 \left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\} e^{\gamma\left(i^*_{n,t}\right)}}{\left(1-i^*_{n,t}\right)^2}\left(\frac{\partial B_t}{\partial \mu_t}\right) = \frac{\Phi_{t-1}}{\phi_{n,t-1}}.$$

where in equation (B.8)

$$\frac{\partial B_t}{\partial \mu_t} = \frac{-1}{(\Phi_t)^2}\left\{\frac{\partial \phi_{n,t}}{\partial \mu_t}\Phi_t + (1-\phi_{n,t})\frac{\partial \Phi_t}{\partial \mu_t}\right\}$$

Therefore, they imply

$$\begin{aligned}
&(1-\alpha)\frac{\partial \phi_{n,t}}{\partial \mu_t} + (1-\alpha)\left(\frac{1-\phi_{n,t}}{\Phi_t}\right)\frac{\partial \Phi_t}{\partial \mu_t} \\
&= \frac{-(1-\alpha)\Phi_t\left(1-i^*_{n,t}\right)^2}{(\beta y \Psi_n)^2\left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\}e^{\gamma\left(i^*_{n,t}\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right).
\end{aligned} \tag{B.20}$$

Now, add equations (B.19) and (B.20) and it gives

$$\begin{aligned}
\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} =& \frac{-\alpha\Phi_t\left(1-i^*_{r,t}\right)^2}{(\beta y \Psi_r)^2\left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\}e^{\gamma\left(i^*_{r,t}\right)}}\left\{\frac{\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}-\dfrac{1}{\alpha}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}+\dfrac{\mu_t}{\alpha}\right)^2}\right\} \\
&- \frac{(1-\alpha)\Phi_t\left(1-i^*_{n,t}\right)^2}{(\beta y \Psi_n)^2\left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\}e^{\gamma\left(i^*_{n,t}\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right) \\
=& \frac{(1-\alpha)\Phi_t\left(1-i^*_{r,t}\right)^2}{(\beta y \Psi_r)^2\left\{\gamma'\left(i^*_{r,t}\right)\left(1-i^*_{r,t}\right)+1\right\}e^{\gamma\left(i^*_{r,t}\right)}}\left\{\frac{\dfrac{\phi_{n,t-1}}{\Phi_{t-1}}}{\left(\dfrac{\phi_{r,t-1}}{\Phi_{t-1}}+\dfrac{\mu_t}{\alpha}\right)^2}\right\} \\
&- \frac{(1-\alpha)\Phi_t\left(1-i^*_{n,t}\right)^2}{(\beta y \Psi_n)^2\left\{\gamma'\left(i^*_{n,t}\right)\left(1-i^*_{n,t}\right)+1\right\}e^{\gamma\left(i^*_{n,t}\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right).
\end{aligned} \tag{B.21}$$

Proposition 1. *Given $\phi_{r,t-1}$ and $\phi_{n,t-1}$, equation (B.21) satisfies*

$$\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} = \begin{cases} = 0 & \text{if } \mu_t = \widehat{\mu}_t, \\ < 0 & \text{if } \mu_t < \widehat{\mu}_t, \\ > 0 & \text{if } \mu_t > \widehat{\mu}_t, \end{cases}$$

where

$$\widehat{\mu}_t = \alpha\left\{\left(\frac{1-\phi_{r,t}}{1-\phi_{n,t}}\right)\phi_{n,t-1} - \phi_{r,t-1}\right\}. \tag{B.22}$$

Proof. In equations (3.37), (3.38), (3.41) and (3.42), if $\mu_t = \widehat{\mu}_t$ as of equation (B.19), then

$$i^*_{r,t} = i^*_{n,t} = \widehat{i^*_t}$$

and

$$\frac{1-\phi_{r,t}}{1-\phi_{n,t}} = \frac{\Psi_n}{\Psi_r}.$$

Therefore, equation (B.22) implies that in equation (B.21),

$$\begin{aligned}
&\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} = 0 \\
&= \frac{(1-\alpha)\Phi_t\left(1-\widehat{i^*_t}\right)^2}{(\beta y)^2\left\{\gamma'\left(\widehat{i^*_t}\right)\left(1-\widehat{i^*_t}\right)+1\right\}e^{\gamma\left(\widehat{i^*_t}\right)}}\left\{\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\left(\frac{1}{\Psi_n}\right)^2-\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right)\left(\frac{1}{\Psi_n}\right)^2\right\},
\end{aligned}$$

where

$$\frac{\phi_{r,t-1}}{\Phi_{t-1}} + \frac{\widehat{\mu}_t}{\alpha} = \frac{\phi_{n,t-1}}{\Phi_{t-1}}\left(\frac{1-\phi_{r,t}}{1-\phi_{n,t}}\right) = \frac{\phi_{n,t-1}}{\Phi_{t-1}}\left(\frac{\Psi_n}{\Psi_r}\right).$$

Next, if μ_t decreases below $\widehat{\mu}_t$, then inequalities (3.47) and (3.50) imply that $i^*_{r,t}$ increases and $i^*_{n,t}$ decreases. Thus, $i^*_{r,t} > \widehat{i^*_t} > i^*_{n,t}$ holds if $\mu_t < \widehat{\mu}_t$. In equation (B.21), the effect of μ_t is negative because the first term decreases and the second term increases given $\phi_{r,t-1}$ and $\phi_{n,t-1}$:

$$\begin{aligned}
\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} =& \frac{(1-\alpha)\Phi_t}{(\beta y \Psi_r)^2\left\{\frac{\gamma'\left(i^*_{r,t}\right)}{1-i^*_{r,t}}+\frac{1}{\left(1-i^*_{r,t}\right)^2}\right\}e^{\gamma\left(i^*_{r,t}\right)}}\left\{\frac{\frac{\phi_{n,t-1}}{\Phi_{t-1}}}{\left(\frac{\phi_{r,t-1}}{\Phi_{t-1}}+\frac{\mu_t}{\alpha}\right)^2}\right\} \\
&-\frac{(1-\alpha)\Phi_t}{(\beta y \Psi_n)^2\left\{\frac{\gamma'\left(i^*_{n,t}\right)}{1-i^*_{n,t}}+\frac{1}{\left(1-i^*_{n,t}\right)^2}\right\}e^{\gamma\left(i^*_{n,t}\right)}}\left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right) < 0.
\end{aligned}$$

Last, if μ_t increases above $\widehat{\mu}_t$, then inequalities (3.47) and (3.50) imply that $i^*_{r,t}$ decreases and $i^*_{n,t}$ increases. Thus, $i^*_{r,t} < \widehat{i^*_t} < i^*_{n,t}$ holds if $\mu_t > \widehat{\mu}_t$. In equation

(B.21), the effect of μ_t is positive because the first term increases and the second term decreases given $\phi_{r,t-1}$ and $\phi_{n,t-1}$:

$$\begin{aligned}\frac{1}{\Phi_t}\frac{\partial \Phi_t}{\partial \mu_t} =& \frac{(1-\alpha)\Phi_t}{(\beta y \Psi_r)^2 \left\{ \frac{\gamma'\left(i^*_{r,t}\right)}{1-i^*_{r,t}} + \frac{1}{\left(1-i^*_{r,t}\right)^2} \right\} e^{\gamma\left(i^*_{r,t}\right)}} \left\{ \frac{\frac{\phi_{n,t-1}}{\Phi_{t-1}}}{\left(\frac{\phi_{r,t-1}}{\Phi_{t-1}} + \frac{\mu_t}{\alpha}\right)^2} \right\} \\ &- \frac{(1-\alpha)\Phi_t}{(\beta y \Psi_n)^2 \left\{ \frac{\gamma'\left(i^*_{n,t}\right)}{1-i^*_{n,t}} + \frac{1}{\left(1-i^*_{n,t}\right)^2} \right\} e^{\gamma\left(i^*_{n,t}\right)}} \left(\frac{\Phi_{t-1}}{\phi_{n,t-1}}\right) > 0.\end{aligned}$$

$\square$

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