

# Local knowledge in mathematics teaching

A product of professional action



Inger Nergaard

Local knowledge in mathematics teaching  
A product of professional action

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'Cause you know sometimes words have two meanings  
In a tree by the brook  
There's a songbird who sings  
Sometimes all of our thoughts are misgiving  
Jimmy Page & Robert Plant

I did not aim for the heaven, just for some enlightenment about a process significantly important to me: the facilitation for pupils' learning mathematics.

I have, through this work, experienced that words can have two meanings, sometimes even more, and I have learned to reconsider convictions because my thoughts were misgiving. I have also learned that norms for what entails a week of work does not apply to PhD-fellows.

However, I have also throughout this long process of frustration and joy, challenges and highs before reaching the point of writing these words, met with people who have become to mean a lot to me, both academically and personally; I hope we still keep in touch.

Girls in the PhD-barracks, thank you for being there when I needed to vent my frustrations. Moreover, Trude, I am grateful for you spending time reading the dissertation and suggesting improvements.

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Stephanie, "sure" was your response when I asked you to have a look at the language. I have attempted to follow your suggestions, what would remain of incorrect and inadequate language is my responsibility.

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Finally, Ragnar, thank you for patiently accepting that my life for some years has taken place behind a computer. I cannot promise never to use it again, but I can promise that it will not be to the extent it has occurred during these years.

I owe you!





# Abstract

The aim of this study is to investigate an experienced mathematics teacher's local knowledge for teaching the subject. Local knowledge is to be understood as the knowledge a teacher has developed in the process of her practice, her "classroom competence" or "wisdom of practice", also known as craft knowledge.

This thesis reports the results that emerged from analyses of the data gathered when observing mathematics lessons and having subsequent conversations with one lower secondary mathematics teacher. The data consists of twenty-three lesson observations and seventeen conversations with the teacher Tea.

A qualitative research design of a descriptive case study was chosen. Based on data extraction agreements, data was collected using video-recordings and field notes from classroom observations and stimulated conversations. Through the analysis of the classroom observations, each lesson was structured into openings, mathematics work, and closings, and the mathematics work into plenary and seatwork segments before analysing the parts for all activities going on in the lessons. The subsequent conversations formed the basis for analysing Tea's knowledge for and in mathematics teaching, and were categorized according to Ball and colleagues' framework Mathematical Knowledge for Teaching (MKT). To explain what appeared not to be covered by the MKT-framework, I used The Knowledge Quartet by Rowland and colleagues and Jaworski's The Teaching Triad.

Both localized knowledge and localized knowing were demonstrated. However, as indicated in the previous paragraph, my research also shows that there is more to the teaching of mathematics than what is elaborated in the MKT framework. For example, there appeared to exist a didactical dimension to horizon content knowledge; Tea knew methods for teaching used in primary school, methods on which she sometimes based her teaching. Moreover, Tea's beliefs and values appeared to influence her teaching. For example, she could have high achieving pupils solve many similar simple tasks because they then would experience the joy of saying, "I understand", thus feeling successful. A distinctive feature was her considerable care for pupils' well-being. This was evidenced by her involvement with her pupils both within and outside the classroom, and her effort attempting to create good learning environment. Her sensitivity, however, appeared sometimes to occur at the expense of the mathematics.

Finally, I experienced that the three frameworks I used in the analysis provide opportunity to get a holistic picture of what entails knowledge for and in mathematics teaching. I assert that all are equally important for student teachers to know, and suggest that they all should be included in the curricula for students who plan to become mathematics teachers.



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# 1 My way into the scholarly arena

New educators coming into our profession as teachers and principals bring with them great powers as observers. They bring new energy, ideas, and hope, and a deep capacity for learning. But it is the veterans and those who are *exiting* the profession who carry with them the abundance of craft knowledge (Barth, 2004, p. 59).

My study is about fusing theory and results of experience in mathematics teaching; i.e. exposing the knowledge experienced Norwegian lower secondary mathematics teachers' have developed in the process of their teaching. As such, it is most of all about the “massive collection of experiences and learnings that those who live and work under the roof of the schoolhouse inevitably accrue during their careers” (Barth, 2004, p. 56), which Ruthven (2002) refers to as craft knowledge.

The study thus focuses on a concept that for some researchers within our field may sound as an oxymoron: craft is for craftsmen, not academics. For clarification, in Section 1.3, I consider what *craft knowledge* entails and how it connects to the research area within mathematics education. In the same section, I also explain some of the background for my conceptual framework (Figure 1-3). In Section 1.4, I present the aim for the research, the research questions, and a summary of my conceptual framework, while I provide an overview of the thesis in Section 1.5. Towards the end of that section, I discuss what consequences my study might have for educating future teachers. I will now continue with discussing the background and motivation for conducting this study: in Section 1.1, I manifest my motivation for researching into mathematical knowledge for teaching, while in Section 1.2, I deal with my background for working with the theme.

## 1.1 Motivation

My interest for researching into the area of mathematics education evoked after publication of the results from the international study TIMSS 2003. The study exposed that Norwegian eighth grade pupils (lower secondary school) performed poorly in mathematics<sup>1</sup>:

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<sup>1</sup> It has to be noted that Norwegian pupils in TIMSS 2011 showed “progress, but long way to go” (Grønmo et al., 2012).

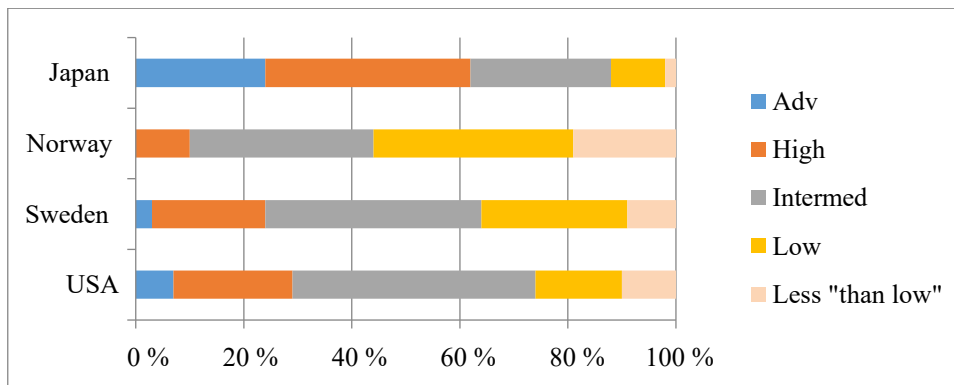


Figure 1-1. Norwegian eighth grade students' results<sup>2</sup> in mathematics compared to students from Japan, Sweden and USA<sup>3</sup> (Mullis, Martin, Gonzalez, & Chrostowski, 2004).

The study also showed that Norwegian pupils scored lower than the international average concerning attitude towards mathematics, while nearly half of the students claimed to have high self-esteem regarding the subject. Compared to the international average of 91.2%, all Norwegian pupils were provided with fully qualified teachers, however less access to teachers having studied mathematics for at least one year at university level (37.4%, international average 70.9%). Concerning access to teachers having studied mathematics education at university level for at least one year, Norwegian students were significantly underprovided; only 2.8 % of Norwegian students had access to such teachers compared to the international average on 53.7%.

Despite all Norwegian pupils had access to certified teachers; the eighth graders showed poor results. To me this raised a question: with what knowledge do teachers approach the mathematics classroom? In other words, what knowledge of mathematics did prospective teacher students hold when entering their education, and what were they offered in their education? Did the education prepare them for future work as mathematics teachers in lower secondary school? Alternatively, since the tested grade eight students have been in lower secondary school for only short time (lower secondary begins at grade eight), were teachers in primary school adequately prepared to teach mathematics?

This information combined with interest for what professional knowledge and competencies Norwegian mathematics teachers at lower secondary level held, made me conduct a small inquiry to find out

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<sup>2</sup>Advanced benchmark; 625, High international benchmark; 550, Intermediate international benchmark; 475, Low international benchmark; 400, "Less than" low international benchmark < 400 (Grønmo, Bergem, Kjærnsli, Lie, & Turmo, 2004; Mullis et al., 2004)

<sup>3</sup> These countries are chosen due to my growing interest for Japanese lesson study, that Sweden is our neighbor country, and that USA is a westerly country having almost equal GDP per capita (CIA, 2013).

whether the teachers themselves felt adequately prepared for working as mathematics teachers at that level. With the research question “Do lower secondary school teachers of mathematics who have graduated from Teacher University College (TUC) perceive themselves to be adequately prepared for their work?” and the auxiliary question; “what is the extent and nature of the mathematical education of teachers of mathematics in lower secondary schools?” I went out to interview practicing teachers at the level in focus. The study was the final part of the Master programme in Mathematics Education<sup>4</sup> at the University of Agder, and resulted in the thesis titled “Teacher Knowledge and Competencies in Mathematics; A study of Norwegian lower secondary school teachers” (Nergaard, 2007).

The study exposed that six out of seven of the novice teachers, who had all followed general teacher education programmes, did not feel adequately prepared for teaching mathematics at the lower secondary level. One even stated he did not dare to be a substitute teacher at grade ten; he did not know the mathematics at that level. Their lack of confidence, and dissatisfaction about the education they were offered, made me want to contribute to the education of future mathematics teachers. I know there are excellent teachers working in Norwegian lower secondary schools who possess knowledge that, if being research into, can contribute to development of scholarly knowledge within the teaching of mathematics, and consequently benefit future teacher students. The aim of this study is to make public that important and valuable local knowledge.

## **1.2 Background**

Having studied pure mathematics for one year and a half, and chemistry and pedagogy for one year each at Norges Lærerhøgskole<sup>5</sup> in Trondheim, I started my career as a lower secondary teacher in 1978. To be licenced as a teacher, I studied practical pedagogy<sup>6</sup> in 1984. Until 2005, when I attended the master programme at Agder University College<sup>7</sup>, I worked full time as a teacher in a lower secondary school in a rural area in the southern part of Norway. Most of these years I taught mathematics and natural sciences. When I started teaching in 1978, 75%

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<sup>4</sup> In Norwegian; matematikdidaktikk.

<sup>5</sup> A part of University of Trondheim, which, in 1996, became Norwegian University of Science and Technology.

<sup>6</sup> Pedagogisk seminar.

<sup>7</sup> It became University of Agder three months before I graduated.

of those teaching mathematics at my school had studied mathematics in addition to their teacher education. Due to that mathematics was an optional subject in teacher education until 1990, in 2003 the number of teachers having mathematics in their education had decreased to 25%. I was then transferred to teach only mathematics.

Because of my interest for the educational background held by the practicing lower secondary mathematics teachers I met during my career, which partly motivated me to start researching into the area, I now find it timely to take a short glance at the mathematics education these teachers normally held (Section 1.2.1). Moreover, since the research was carried out in lower secondary classrooms, I also write some words about the Norwegian compulsory school system (Section 1.2.2). However, I first make a short narrative about “my educational life” after finishing the master programme, which resulted in applying for a research fellowship on the doctoral programme at the University of Agder.

### **1.2.1 Mathematics in teacher education 1973 - 2016**

When I started my teaching career in 1978, it was common that teachers of mathematics in lower secondary school had studied the subject at either a University or a University College. Those who had a general teacher education, which at that time did not include mathematics as a compulsory subject, usually had an additional course in mathematics to be prepared for their teaching at that level. As indicated above, this changed over the years; the number of teachers having extended their regular teacher education by studying mathematics, and those having studied mathematics and practical pedagogy (as I did, cf. Footnote 6), decreased.

From 1973, teacher students could follow a subject specific course in mathematics didactics equal to a quarter of a year, and in 1990, mathematics was implemented as a compulsory subject (Birkeland & Breiteig, 2012). The teacher students were then required to study mathematics for one quarter of a year (15 ECTS<sup>8</sup>). From 1998, this was extended to half a year study. There was no separation on whether one should teach lower or higher grades in compulsory school.

The poor results in mathematics exposed in the international studies mentioned in Section 1.1, TIMSS 2003, alarmed the Ministry of Education, and requirement of having studied at least 60 ECTS mathematics for teaching at lower secondary level was implemented in 2008 (Kunnskapsdepartementet, 2008). A new and differentiated four year teacher education programme was initiated in 2010

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<sup>8</sup> European Credit Transfer and Accumulation System, where 60 credits equals to one year of study.

(Kunnskapsdepartementet, 2009); one route for those who planned to teach grades 1 – 7, and another for those searching for competence to teach grades 5 - 10. Students who follow the route for teaching grades 1 – 7 are all required to study mathematics for half a year (30 ECTS), while for those studying for teaching mathematics in grades 5 – 10, there is an requirement for 60 ECTS. There is also possible to follow a five-year programme for those wanting to have a master's degree in mathematics education for the grades 8 – 13. In addition, by extending their education with 30 ECTS pedagogy and 30 ECTS mathematics didactics, persons who have studied at least 60 ECTS mathematics outside teacher education programmes can get a licence for teaching the subject. From 2017, all teacher education in Norway will lead to a master's degree, however differentiated for the grades 1 – 7, and 5 – 10 (Regjeringen, 2014).

The research reported in this monograph was conducted in Norwegian lower secondary schools. I thus find it appropriate to write a few words about the compulsory school system as it operates today, which I do in the next section.

### 1.2.2 Norwegian compulsory school today

In Norway, all inhabitants have obligations and rights to ten years of education, usually starting the year the child turns six years. After completing primary and lower secondary education or its equivalent (the first ten years), they are, upon application, entitled to three years of full-time upper secondary education (Kunnskapsdepartementet, 2015b, § 2-1, § 3-1).

The lower secondary education lasts for three years and at least for 38 weeks per year, and should include the subjects arts and craft, food and health, foreign languages, mathematics, mother tongue teaching and basic Norwegian for language minorities, music, natural science, Norwegian, physical education, religion, philosophies of life and ethics, and social studies. The pupils are provided eleven hours per week of mathematics divided within these three years,

A central principle in the Norwegian education system is equity in education for all:

Equity in Education is a national goal and the overriding principle that applies to all areas of education. [...] [This] means to provide equal opportunities in education regardless of abilities and aptitudes, age, gender, skin colour, sexual orientation, social background, religious or ethnic background, place of residence, family education or family finances (Utdanningsdirektoratet, 2008).

To ensure pupils' needs for social belonging, they are to be organized into classes or basic groups. The sizes of the groups should not be larger than is justifiable for pedagogical and security reasons, and the

organization should not be due to gender, ethnicity or academic level. For parts of the education, however, the pupils can be divided in other groups when needed (Kunnskapsdepartementet, 2015b, § 8-2).

To give the reader some background information about the educational systems within which this research has been conducted, I have, in Sections 1.2.1 and 1.2.2, considered the mathematics in Norwegian teacher education, and the Norwegian compulsory education respectively. Next, I will write some words about what influenced my decision to apply for the research fellowship that resulted in the work reported in this thesis, namely the dialogic cycle (Ruthven, 2002) and concept craft knowledge.

### **1.3 The dialogic cycle and mathematics teachers' craft knowledge**

The interest for what kind of knowledge teachers deploy while teaching mathematics has followed me for more than thirty years. After finishing the Master Programme at the University of Agder, I wanted to learn more about teachers' knowledge, and continued to follow courses at the University. I then learned about Ruthven's (2002) *dialogic cycle*, a model which describes a knowledge conversion between the practice of teaching and the practice of research (cf. Figure 1-2). This triggered my interest for doing educational research, and served as an incitement for me to apply for a research fellowship on the doctoral programme at the University.

The use of the term craft knowledge in the academics was unknown to me until I was introduced to Ruthven's (2002) model. The word *craft* had for me, until then, meant work such as carpentry, pottery and sewing, and craft knowledge was the knowledge craftsmen deployed to teach apprentices. However, when being introduced to the dialogic cycle, I realised that this is what teachers all over the world are doing every day; crafting their teaching, thus developing craft knowledge.

It has to be noted that as I was searching for craft knowledge, for me it felt more natural to use the word *localized*, because that is what I did; as I observed the teaching, I *localized* knowledge that was deployed. Moreover, as it was observed, it became part of the actual teacher's *local knowledge*. However, while building an understanding of how teachers' *local knowledge* is to be understood in this study, I draw on existing research, and will in that respect use the original concept, namely *craft knowledge*.

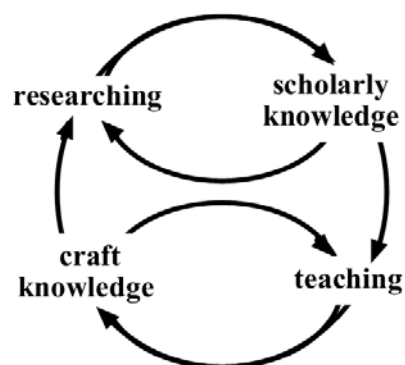


Figure 1-2. The dialogic cycle (Ruthven, 2002, p. 22).

The dialogic cycle was first published in the 2002 edition of the Handbook of international research in mathematics education with the title “Linking researching with teaching: Towards synergy of scholarly and craft knowledge” (Ruthven, 2002). Ruthven was leaning to Cooper and MacIntyre; “craft knowledge describes the knowledge that arises from and, in turn, informs what teachers do” (1996, p. 76), and Bromme and Tillema, who use the term “professional knowledge” about the knowledge that

is developed as a product of professional action, and it establishes itself through work and performance in the profession, not merely through accumulation of theoretical knowledge, but through the integration, tuning and restructuring of theoretical knowledge to the demands of practical situations and constraints (1995, p. 262).

Ruthven suggested a model that exemplifies this knowledge conversion. The model describes two minor ovals coupled in an overarching cycle; one oval (Figure 1-2, lower part) illustrates the practice of teaching where teachers through their practice develop craft knowledge that can be used in future teaching, whereas the upper oval illustrates research feeding into the scholarly arena and vice versa. The overarching cycle couples the scholarly and the teaching areas.

For this study, I had no intention to affect practitioners by feeding into their practices. Initially, I thus only considered the loop between the teaching and development of craft knowledge (cf. Ruthven’s dialogic cycle), hoping to make public the knowledge teachers develop through their process of teaching, and why they do so. Moreover, I assumed that there were more to this loop than the dialogic cycle shows, suggesting that there are two types of knowledge/knowing which impact on teachers’ actions in the classroom. Knowledge, I see as an understanding or information a person has about a certain topic, and is in either a

person's mind or known by people generally<sup>9</sup>. Knowing, however, is more intuitive, dynamic and subjective, and directs a person's actions in situations as response to events that arise. In Chapter 2.1, I further elaborate on the concepts knowledge and knowing as understood in this study.

## **1.4 Aims, conceptual framework, and tentative research questions**

What knowledge do teachers have, how do they deploy this knowledge in the classroom, and what do they do to promote students' learning? These, and many other similar questions, occupied my mind when starting the process that resulted in this thesis. That is, the work I did for my master thesis had illuminated for me that there were differences between my informants' self-esteem and view on what teaching mathematics entails. I met teachers who claimed they were not adequately prepared for working as mathematics teachers at lower secondary level and thus reluctant to teach grade ten pupils. I met with teachers who expressed self-confidence about their teaching, but were not able to find the midpoint between two fractions without using decimals. And I met with teachers who claimed algebra to be the easiest content to teach because it is only about having students learn to use formulas (Nergaard, 2007). However, I also met teachers who made me believe that there exists craft knowledge worthwhile exposing and reporting.

Teachers develop their teaching, and consequently their knowledge for teaching, during their entire career (Leinhardt, 1989), knowledge which is important to make public (Barth, 2004; Grevholm, 2010). It is not expected that teacher students learn all that is needed to know about mathematics and mathematics teaching during their education (Borko, Eisenhart, Brown, Underhill, & Agard, 1992). However, the more they can learn about the reality of what goes on in mathematics classrooms, the better they will be prepared for the day they, all by themselves, face their pupils. Through this study, I hope to draw implications for the development of further teacher education programmes.

After teaching mathematics for more than 30 years, a belief about how a teacher develops her teaching and knowledge for teaching has grown in my mind. I believe that, in addition to using knowledge coming from research (scholarly knowledge), courses, books, and colleagues (brought knowledge, which I assume is informed by scholarly knowledge), teachers develop their own craft knowledge (*local*

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<sup>9</sup> <http://dictionary.cambridge.org/dictionary/british/knowledge?q=knowledge>



*knowledge*). Local knowledge, I conjecture, develops as a result of teachers' experiences in the classroom, knowledge of their pupils, and pupils' responses to their actions in the classroom. In order to capture the complexity of my informant's local knowledge, I formulated three concepts, *localized knowledge* of mathematics teaching, *localized knowing* of mathematics teaching, and *general professional knowledge*.

The teacher develops her *localized knowledge* for example when reflecting on a lesson, such as a lesson that did not turn out as expected and/or when planning for a lesson. Her dynamic *localized knowing* is enacted as response to contingent situations as they arise in the classroom. Localized knowing is based on her knowledge of mathematics combined with knowledge about an individual student, a certain group of students, or a specific class. Some of this knowledge might be worthwhile to share with colleague and bring to future lessons, and might, if being studied, contribute to development of scholarly knowledge. The third category, *general professional knowledge*, concerns classroom routines such as small, socially shared and scripted pieces of behaviour, for example hand-raising and turn-taking in speaking (Leinhardt, 1983), i.e. what Yackel and Cobb (1996) refer to as classroom social norms. In expert teachers'<sup>10</sup> classrooms these were found to be beneficial for students' attainment, and helped facilitate class management and fluency in the classroom (Brown & McIntyre, 1993; Leinhardt, 1983). They are, in that sense, an important part of what goes on in a classroom.

The following figure illustrates the flow of information into the practice of teaching and the process of development of this practice as I see it. As such, it illustrates my conceptual framework:

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<sup>10</sup> Identified by reviewing the *growth* scores of students over a 5-year period and selecting the classrooms that appeared within the highest 15% of each grade (Leinhardt & Smith, 1985).

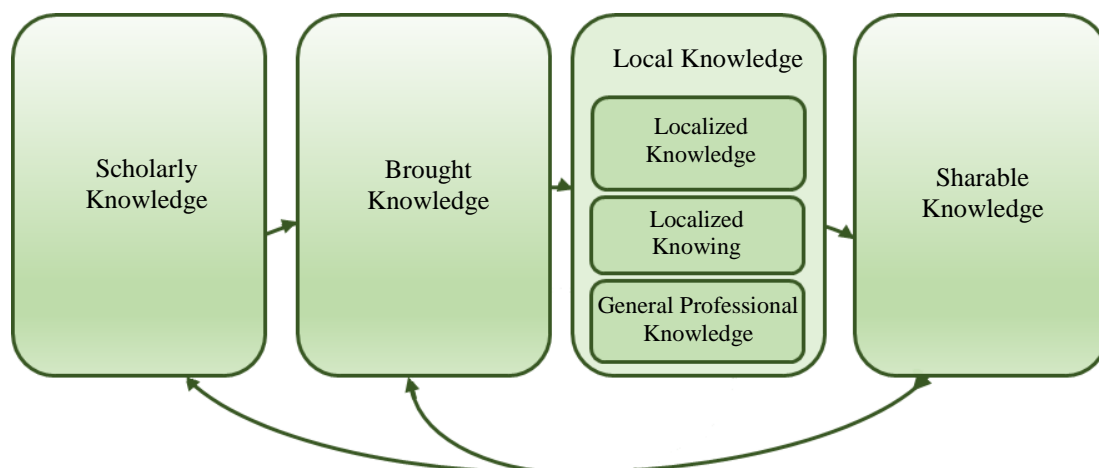


Figure 1-3. Illustration of my initial conceptual framework.

Barth (2004, p. 57) claims that teachers learn from practice, and that “These hard-won learnings are the gold nuggets we mine out from the gravel of our experience”. Suggesting that there are “gold-nuggets”, Barth indicates that some of these experiences are worthwhile to use in future teaching as well as being shared with others. I accept Barth’s indication. My focus is on the local knowledge teachers have developed in the course of their practice, I thus wanted to research into the practice of mathematics teaching to figure out if my assumption about the trisection of local knowledge in mathematics teaching existed. When starting on the project, I had one main and two auxiliary working questions guiding my first steps into the world of research:

- What is the evidence of local knowledge in mathematics teaching in the practice of an experienced Norwegian teacher?
  - What is the evidence of localized knowledge in mathematics teaching in the practice of an experienced Norwegian teacher?
  - What is the evidence of localized knowing in mathematics teaching in the practice of an experienced Norwegian teacher?

The auxiliary questions reflect two of the three conjectured concepts included in teachers’ local knowledge. In Chapter 2.1, I elaborate on how knowledge and knowing are to be understood in this study, while the concepts of localized knowledge and knowing are explained in Chapter 3.3.

After considering the literature referred to in Chapters 2 and 3, which confirmed *evidence* of craft, or local, knowledge, I changed the wording of the research questions, using characterization of the knowledge instead of evidence. In Chapter 3.4, I present the research questions that guided me through the process of analysing my data.

I conjectured that teachers’ local knowledge consists three branches of local knowledge: *localized knowledge*, *localized knowing*, and

*general professional knowledge*. One already knows that teachers' craft knowledge develops in the practice of teaching (Leinhardt, 1990; Ruthven, 2002). One also knows that general professional knowledge and classroom routines are part of the knowledge teachers develop in the classroom (Leinhardt, Weidman, & Hammond, 1987). This study has thus focused on finding evidence for the two remaining categories, *localized knowledge* and *localized knowing*.

## **1.5 Overview of the thesis**

This thesis consists of eight chapters; an introductory chapter (Chapter 1) followed by two theory chapters (Chapters 2 and 3) in which I describe theoretical positioning as underpinnings for my analysis respectively. In Chapter 2, I define concepts used in the study, and position the study theoretically, while I, in Chapter 3 consider some educational research, and present my conceptual framework and position it in the scholarly literature. I present the methodology in Chapter 4.

This study has been about searching for evidence for the local knowledge the teacher Tea deploys while teaching mathematics. In Chapters 5, 6, and 7, I discuss Tea's background, her practice, and her local knowledge for teaching mathematics. In Chapter 5, I first attend to comments from her principal and from some of her pupils before Tea herself makes some comments on her practice. In Chapter 6, I consider the analysis of the actual actions carried out in Tea's classroom, while I, in Chapter 7, deal with Tea's knowledge for teaching mathematics, based on her own comments and illustrated by episodes from her classroom. In Chapter 8, I discuss my findings related to the research questions and my conceptual framework. I also discuss how I had to combine three frameworks to explain Tea's knowledge for and in teaching mathematics: Jaworski's (1994) Teaching Triad (TT), Rowland, Huckstep, and Thwaites' (2005) Knowledge Quartet (KQ), and Ball, Thames, and Phelps' (2008) Mathematical Knowledge for Teaching (MKT). In Chapter 8, I also make some critical reflections on the process.

In the following chapter, I provide a theoretical perspective for the study in addition to presenting the literature about mathematics knowledge for teaching on which the study relies. I also here describe how I distinguish between the concepts *knowing* and *knowledge* as two types of sources for information teachers deploy in their teaching.



## 2 Theoretical perspective on teacher knowledge and knowing

By presenting my background and my motivation for doing this work, the previous chapter set the scene for this study. I also touched the core concept for the study, namely teachers' *craft knowledge*, or *local knowledge*, which is the term I prefer to use for the concept, and considered how lower secondary mathematics teachers have been educated since 1973. In this chapter, I discuss aspects of knowledge for teaching in general and for mathematics teaching in particular as theorized in educational research.

Having my conceptual framework in mind, I start with explaining the terms knowledge and knowing (Section 2.1), and continue with looking at how researchers in mathematics education describe/define the concepts scholarly knowledge (Section 2.1.2) and local knowledge (Section 2.1.3). Based on the discussions in the sections, each section ends with an explanation of how the concepts are to be understood in this study. In Section 2.2, I consider what knowledge is required for teachers in general. This is followed by a section in which I look into some aspects of mathematical knowledge teacher students are offered (Section 2.2.2). In Section 2.2.3, I consider some philosophical aspects of mathematics teaching, in Section 2.3, I discuss aspects of teachers' learning, while in Section 2.4, I provide a short summary of the theoretical stance I have taken.

### 2.1 Theorizing aspects of knowledge

In this section, I define five concepts that are crucial for the study; teachers' knowledge and knowing, scholarly knowledge, and craft knowledge. In Section 2.1.1, I consider common use of the concepts *knowledge* and *knowing*, and in Sections 2.1.2 and 2.1.3, I discuss different scholars' use of the concepts *scholarly knowledge* and *craft knowledge*. In each section, I define how the concepts are to be understood in this study.

#### 2.1.1 Theorizing teacher knowledge and knowing

Knowledge is a commonly used word, and its meaning seems for the most part to be "taken-as-shared" among its users. In this study, however, I propose a difference between the concepts of *knowledge* and *knowing*, which I clarify in this sub-section.

By definition, knowledge is the understanding, or information, a person has about a subject, either learned from experience or gained

from studying the particular subject, and is either in a person's mind, or known by people generally (cf. Footnote 9). This definition requires a discussion on where the "mind is located"; in a person's head or in the individual-in-social-action (Cobb, 1994). Is it actively constructed as the result of her/his adaptive process of coming to know (Lerman, 1989), or results from enculturation into concept practices (Cobb, 1994)? This question I discuss in a later section (2.3).

In this study, *knowledge* thus assumes an ontological dimension of something a person either possesses or does not. I see it as "a storable, deliberately treatable, and retrievable object-like item, called knowledge, from a loft, called memory" (Bauersfeld, 1994, p. 138). As such it is the subset of the structure of facts, concepts, principles, procedures and phenomena of a subject-matter domain which is "stored" in a person's mind<sup>11</sup> (Greeno, 1991; Shulman, 1986). Consequently, teacher knowledge is the understanding and information a teacher has, or is required to have, about the subject and how it is to be taught. This involves information and understanding about how to conduct her profession, and includes different aspects of professional knowledge as defined by for example by Shulman (1986, 1987, cf. Section 2.2.1.1), Steinbring (1998, cf. Section. 2.2.3.4), Rowland, Huckstep and Thwaites (2005, cf. Chapter 3.2.3.2), and Ball, Thames and Phelps (2008, cf. Chapter 3.2.3.3).

Knowledge can be either subjective or objective. A person's subjective knowledge is objective knowledge which has been reorganized and internalized, and remains subjective until being published or discussed and agreed on with others who "know" (Ernest 1991). Tacit knowledge, which Hegarty (2000, p. 453) describes as being "context-specific and not readily communicated other than by demonstration", is an integral part of a person's subjective knowledge. It is a resource ready to be used in practice, and is thus included in teachers' *knowing* (see below, this section). Its counterpart is objective explicit knowledge which can be described in general terms and be generally applied in for example teaching (Ernest, 1991; Hegarty, 2000).

Over the years, there have been more debates concerning what "types" or "level" of mathematical knowledge which are regarded as adequate for teachers in their engagement with pupils' learning (Hiebert & Lefevre, 1986; Moreira & David, 2008). How teacher students are taught, and what and why they are to learn, affect how they will teach; for example should they be taught for conceptual or procedural

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<sup>11</sup> Knowledge is also "stored" in books by means of intentionally organized symbols, however it is the reader who has to make sense of its meaning (Ernest, 1991).

knowledge? It is now widely accepted that teaching for conceptual knowledge is better for students' learning of the subject (Hiebert & Lefevre, 1986). Conceptual knowledge in this sense means knowledge rich in relations between discrete pieces of information, which the teacher is able to recognize and use when required, whereas procedural knowledge is characterized as prescriptive and mechanical.

*Knowing*, as distinct from *knowledge*, is deliberately used to distinguish two types of sources for information a teacher uses in the classroom, and “denotes the momentary activation of options from experienced actions” (Bauersfeld, 1994, p. 138). Thus, in addition to using knowledge as described in the previous paragraph, a person's actions may be directed by an intuitive, subjective “knowing”, which emerges in situations as response to events that arise (Lampert, 1986), for example for a teacher in her classroom. Knowing is in this sense an active process unfolded in the context of teaching (Cochran, DeRuiter, & King, 1993), and dependent on the teacher's knowledge of the specific learner in relation to the specific subject which is to be learned (Ball et al., 2008). It is a subconscious and tacit state of knowledge which is being induced and demonstrated (Hegarty, 2000) in the interaction between the teacher and the pupil, for example in relation to a problem a pupil might have, or an unexpected suggestion from a pupil. The pupil's need evokes the teacher's dynamic knowing, which makes her act accordingly. It also includes recognition of what resources that exists and can be used productively to support the ongoing activity (Greeno, 1991).

In this section, I have explained how knowledge and knowing are used in existing research. This study extracts their meaning, resulting with knowledge as an ontological dimension of something a person possesses or not, and knowing as an intuitive and dynamic entity directed towards pupil and action. My conceptual framework includes different “kinds” of knowledge; *brought knowledge*, *sharable knowledge*, *local knowledge*, and *scholarly knowledge*. The concept *scholarly knowledge* is inspired by Ruthven's dialogic cycle (Ruthven, 2002, p. 22) and will be defined in the next section. I explain the concepts brought knowledge, sharable knowledge and local knowledge in Chapter 3.3.

### 2.1.2 Scholarly knowledge

This research intends to inform development of the existing scholarly knowledge base within the area of mathematics education. Ruthven (2002) claims that by researching into the practice of teaching, which is what I have done, craft knowledge can be elicited and systematized into

*scholarly knowledge*. This section continues with discussing different scholars' views on how scholarly knowledge is understood.

Ruthven's (2002) dialogic cycle suggests that scholarly knowledge is created within the practice of researching. Indirectly, as I understand it, Ruthven thus suggests that scholarly knowledge does not include knowledge coming from (research-based) textbooks one has to interpret before using in practice. Hegarty (2000) suggests that there are three sources of knowledge which inform teaching: knowledge deriving from primary research (in line with Ruthven), knowledge deriving from scholarship and review, and knowledge embedded in material and procedures. The first two are created within the practice of research. Hegarty, however, imposes restrictions on the results of research before it can be a part of teachers' knowledge base. He suggests contributions from primary research to "be mediated through second-order processes" (Hegarty, 2000, p. 458), meaning that review processes have to follow explicit rules and guiding principles (exhaustiveness, transparency and replicability) before informing teacher practice. Ruthven does not impose such strong requirements for creating scholarly knowledge (however not contradicting it): "craft knowledge is elicited and codified through researching, stimulating (re)construction of scholarly knowledge" (Ruthven, 2002, p. 22).

White (1987), however, appears to use scholarly knowledge about the knowledge students gain from education. She asserts that many teacher education courses

do not systematically prepare teachers to transmit *scholarly knowledge* to their students [so] for a model of teacher preparation to be coherent it must link important skills such as classroom management, the ability to individualize instruction, and the transmission of *academic knowledge* (White, 1987, p. 19, my italics)<sup>12</sup>.

White appears to use scholarly knowledge and academic knowledge (of the school disciplines) interchangeably; it is the knowledge teacher students gain from their education, thus assuming that scholarly knowledge is knowledge students acquire in the academy. This appears consistent with Moreira and David (2008) who express that the academic *mathematics* required for prospective secondary teachers to a large part is a scientific body of knowledge produced and organized by professional mathematicians.

Leinhardt, McCarthy Young and Merriman (1995) suggest that teacher students acquire declarative, formal and universal knowledge at

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<sup>12</sup> These quotations are not to be read as a reference or critique of current teacher education programmes. I acknowledge that it is written years ago. The quotations are used to show White's interchangeable use of *scholarly* and *academic* knowledge.



the universities. For knowledge to be formal and universal, it has assumedly emerged from scholarly work or research in scrutiny to scholars' discussions, which then makes it scholarly.

To summarize: my framework stems from Ruthven's (2002) dialogic cycle where scholarly knowledge that informs teaching is developed from research on teachers' practice. The rigid rules Hegarty (2000) imposes for knowledge gained from research to be theoretically acceptable seems to extend what is indicated by Ruthven (2002). The declarative, formal and universal knowledge students get in the universities (Leinhardt et al., 1995) can be characterized both as scholarly and academic knowledge (White, 1987).

These examples indicate that there is not necessarily a unitary view among scholars on the meaning of similar words referred to in their work, in this case *scholarly knowledge*. I choose to understand scholarly knowledge as that which is created within the area of research (Ruthven, 2002). Teacher education courses are informed by such research-based knowledge; teacher educators being responsible for the courses then bring it along/present it to their students. In their future practice, student teachers, together with knowledge acquired from (research based) educational textbooks, pupils' textbooks, and school curriculum, bring this knowledge into the classroom, my framework conceptualises this as a part of teachers' *brought knowledge* (cf. Chapter 3.3).

In this section, I have mainly discussed the concept *scholarly knowledge*, and explained how the concept is to be understood in this research. Another main concept, which needs a broader description, is that of craft knowledge. According to Ruthven (2002), a teacher's craft knowledge might inform a teacher's further teaching as well as the base of scholarly knowledge if being researched into. The next section thus continues with considering different scholars' "definitions" of craft knowledge, focusing on mathematics teachers' craft knowledge. Towards the end, I clarify how I use the concept in my study.

### **2.1.3 Craft knowledge – a product of professional action**

Initially, educators attempted to solve practical educational problems by applying theories from the social sciences (Schwab, 1971). Each of these theories treated parts of the problems isolated from each other, and suggested different treatments. This led to poor solving of the problems (Schwab, 1971), and resulted in an understanding that theories for solving educational problems had to be derived from the practice of

education itself (McNamara & Desforjes, 1978)<sup>13</sup>. They asserted that scholars have to search for, and base theories on “classroom competence”, also called “wisdom of practice”(Shulman, 1986), i.e. teachers’ professional knowledge, their *craft knowledge*. Scholars seem to agree on what is meant by mathematics teachers’ craft knowledge. However, how and the extent to which they “define” the concept, appears to differ. This will be subject to discussion in what follows.

*Craft* is historically connected to the skilled practice of a practical occupation, and *craft knowledge* is what the master was passing on to their apprentices (Leinhardt, 1990). The apprentices usually received their formal training when working together with a master craftsman, for example in a bakery or in tailoring. McNamara and Desforjes (1978) extended the meaning of the concept, asserting that craft knowledge also was to be found in the classroom as what they called “classroom competence”; i.e. the professional knowledge of practicing teachers (Bromme & Tillema, 1995).

This section continues with considering the development of craft knowledge in mathematics teaching (Section 2.1.3.1), what it entails (Section 2.1.3.2), and how I use it in this study (Section 2.1.3.3), beginning with Schön (1983), who more than thirty years ago suggested how professional knowledge develops, however subjective craft knowledge.

### ***2.1.3.1 Development of craft knowledge***

Schön (1983) suggests two “ways” in which teachers develop their professional knowledge. Contingent reflection on specific episodes in the classroom can provide teachers with fresh and valuable subjective knowledge about the class, the individual student, or their relation to the content to be taught (reflection in action, Schön, 1983). Likewise, their knowledge base can be extended when a teacher subsequently reflects on a lesson or an episode, or when planning for a new lesson which Schön (1983) refers to as reflection on action.

Brown and McIntyre (1993, p. 17) assert that teachers acquire their professional knowledge “primarily through their practical experience in the classroom rather than their formal training”, indicating that it is first when entering the field of practice as a certified teacher that teachers actually learn to be professionals. In a later book McIntyre, in authorship with Cooper, expands its origin to include that it develops from reflection, and are thus in accordance to Schön (1983):

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<sup>13</sup> McNamara and Desforjes provided a suggestion for solving this problem. This will be considered in the next chapter (3.1).

Professional craft knowledge is the knowledge that teachers develop through the processes of reflection and practical problem-solving that they engage in to carry out the demands of their jobs (Cooper & McIntyre, 1996, p. 76).

All this feed into the base of teachers' subjective craft knowledge. I do not assert that knowledge emerging from reflections in or on actions lead to *objective* craft knowledge, which this study characterizes as shareable craft knowledge (cf. Chapter 3.3). Even if the teacher herself sees the knowledge as viable for her teaching, it is not recognized as objective sharable knowledge until being accepted by others who "know" (Ernest, 1991).

### **2.1.3.2 What is craft knowledge**

While McNamara and Desforges (1978) used the terms craft knowledge and "classroom competence" interchangeably for the practical competence and professional understanding developed in the classroom without any specific definition, Leinhardt (1990, p. 18) "defines" craft knowledge as

[T]he wealth of teaching information that very skilled practitioners have about their own practice. It includes deep, sensitive, location-specific knowledge of teaching, and it also includes fragmentary, superstitious, and often inaccurate opinions.

Leinhardt thus appears to limit craft knowledge to include only the information *very skilled* teachers have about their teaching, however not mentioning that it results from classroom experiences. She acknowledges that it can be fragmentary, superstitious or inaccurate, a view that coincides with the view I proposed at the end of the previous section; not all craft knowledge is worth sharing with others. She further suggests that teachers' craft knowledge, together with theory and empirical research, is an important component in the design and validation of national teacher assessments. I assume that Leinhardt's definition derives, among other sources, from the research she and her colleagues conducted by contrasting novice and expert teaching (Leinhardt, 1983, 1989; Leinhardt & Greeno, 1986; Leinhardt & Smith, 1985; Leinhardt et al., 1987). These studies showed significant differences in the competence novice and expert teachers exercised in the classroom, and demonstrated that craft knowledge in the practice of teaching can be recognized both in *practical* and *theoretical* issues. Practical craft knowledge reveals itself through the presence of routines such as small, socially shared, and scripted pieces of behaviour<sup>14</sup> (i.e. hand raising and

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<sup>14</sup> These would be included within social norms as described by Yackel and Cobb (1996) which will be discussed in 2.2.3.3.

turn taking in speaking). These were found to be beneficial for students' attainment, and helped facilitate class management and fluency in the classroom (Brown & McIntyre, 1993; Leinhardt, 1983, 1993; Leinhardt et al., 1987). Teachers' theoretical craft knowledge is manifested by expert teachers' presentation of more elaborate and deeper categories of mathematical problems than novices (Leinhardt & Smith, 1985). Moreover, skilled teachers have a large repertoire of routines and activities they perform fluently. This appears important since the combination of lesson structures and subject matter knowledge was reported to be fundamental to effective teaching (Leinhardt & Greeno, 1986). Brown and McIntyre describe this as developing and maintaining students' normal desirable state of student activity (NDS); "which are steady states of activity seen by teachers as appropriate for pupils at different stages of lessons" (1993, p. 67) (cf. Section 2.2.3).

Furthermore, they assert that professional craft knowledge guides teachers'

day-to-day actions in the classroom, which is for the most part not articulated in words and which is brought to bear spontaneously, routinely and sometimes unconsciously on their teaching (p. 17).

Brown and McIntyre were thus more specific and detailed, however not so restrictive, about what they considered as craft knowledge as Leinhardt (1990). Their research design ensured that their informants were competent teachers, however not highlighting that they had to be *skilled*. The second part of their definition, "brought to bear spontaneously, routinely, and sometimes unconsciously", includes what I, in Section 2.1, defined as an intuitive and dynamic knowing which propels teachers actions, and thus included in the category teacher's localized *knowing*.

Cooper and McIntyre add that

[C]raft knowledge describes the knowledge that arises from and, in turn, informs what teachers do. As such, this knowledge is to be distinguished from other forms of knowledge that are not linked to practice in this direct way. Craft knowledge is not, therefore, the kind of knowledge that teachers draw on when explaining the thinking underlying their ideal teaching practices. Neither is it knowledge drawn from theoretical sources. Professional craft knowledge can certainly be (and often is) informed by these sources, but it is of a far more practical nature than these knowledge forms (1996, p. 76).

Cooper and McIntyre appear as a source for Ruthven's (2002) (cf. Chapter 1.3) view on what entails craft knowledge in agreeing in that it in turn informs what teachers do. However, there seems to be a small distinction between what these scholars see as informing development of craft knowledge; while Cooper and McIntyre assert that craft knowledge *can be* informed by theoretical sources, Ruthven's dialogic cycle indicates that its development *is* indirectly informed by scholarly

sources. Ruthven (2002) adds to the description of craft knowledge that it

refers to the professional knowledge which teachers use in their day-to-day classroom teaching; action-oriented knowledge which is not generally made explicit by teachers, which they may indeed find difficult to articulate, or which they may even be unaware of using (2002, p. 14).

Ruthven's definition thus seems to summarize some of what scholars who have preceded him define as craft knowledge.

### ***2.1.3.3 Craft knowledge as understood in this study***

In addition to being the knowledge that arises from, and in turn, informs what teachers do (Cooper and McIntyre, 1996), my definition of mathematics teachers' craft knowledge includes the knowledge experienced and skilled mathematics teachers have about their practice (Leinhardt, 1990) which they use in their day-to-day classroom teaching (Brown & McIntyre, 1993; Ruthven, 2002). It is practice based, i.e. it is knowledge of mathematics teaching revised in the process of teaching (Brown & McIntyre, 1993), and is often informed by theoretical sources (Cooper & McIntyre, 1996). However, I differentiate between the knowledge teachers are able to articulate, and the part which they might find difficult to articulate (Brown & McIntyre, 1993; Ruthven, 2002). In this study, I refer to craft knowledge (or localized knowledge, which will be used in the forthcoming) as the knowledge teachers are able to articulate, while the last category I assert as included in teachers' craft *knowing*, hereafter described as localized knowing. It has to be noted that all observed localized knowledge/knowing is, in its outset, subjective knowledge, whether tacit or explicable. In the next paragraph, I discuss what can make it objective.

I primarily sought local knowledge in mathematics teaching that is worth sharing with others, such as students, colleagues or researchers. During the observations, however, I intended to expose as much as possible of the knowledge deployed in the lessons. What was local knowledge and brought knowledge, my informant and I somewhat talked about during our subsequent conversations. At that point, what we recognized as local knowledge was still subjective. To be able to characterize it as objective sharable knowledge it has, as asserted by Ernest (1991), to be agreed upon as viable by others who "know". It has to be discussed and reflected upon, because one cannot necessarily assume that because something worked in one particular situation, it will work in the next situation which may be somewhat different (Hegarty, 2000). This also required avoidance of the "fragmentary, superstitious, and often inaccurate opinions" which Leinhardt (1990, p. 18) claims is included in teachers' craft knowledge. This was a challenge I had to face when doing this research; sharing the knowledge

with me does not necessarily make it objective, a fact that I was aware of when analysing and particularly when presenting the data.

#### **2.1.3.4 Summary**

I have, in this section presented the foundation for, and definition of, the concept local knowledge as used in this study. I have indicated what aspects of local knowledge I see as worthwhile informing the base of scholarly mathematics knowledge for teaching. Since Shulman (1986, 1987) published his assertion about what a knowledge base for teaching at least should include, many succeeding scholars have based their research on his work. Starting with Shulman, I, in the next section discuss some aspects of knowledge and knowing seen as needed and adequate for teaching in general (e.g. Cochran et al., 1993; Hegarty, 2000; Shulman, 1987), and for my conceptual framework in particular. However, I will first make a working definition on what this research regards as entailing the work of teaching.

## **2.2 Theoretical perspectives on knowledge and teaching**

In the previous section, I considered different aspects of knowledge. As an approach to mathematical knowledge for and in teaching, I start this section with discussing knowledge for teaching (Section 2.2.1) and mathematics teaching (Section 2.2.2), before making some philosophical considerations of mathematics teaching (Section 2.2.3).

### **2.2.1 Knowledge for teaching**

Based on the lack of focus on subject matter in teacher education in the beginning of 1980, which Shulman and his colleagues refer to as “the missing paradigm” problem (1986, p. 6), he published two articles (1986, 1987) which have influenced later research on teacher knowledge significantly. In the introduction to his 1986 paper “Those who understand; Knowledge growth in teaching” Shulman quotes George Bernhard Shaw’s words; “He who can, does. He who cannot, teaches”, an aphorism he characterized as a “calamitous insult to our profession” (Shulman, 1986, p. 4). After asking whether Shaw should “be treated as the last word on what teachers know and don't know, or do and can't do” (Shulman, 1986, p. 4) he set out to inquire into what knowledge was required for teachers at that time. The paper ends with Shulman’s “refinement” of Shaw’s aphorism; “Those who can, do. Those who understand, teach” (1986, p. 14).

#### **2.2.1.1 Shulman’s knowledgebase for teaching**

What Shulman discussed in his 1986 paper to me appears to function as an introduction to his knowledge base for teaching, which was published in 1987. It includes seven categories of knowledge meeting the request

for “intellectual, practical, and normative basis for the professionalization of teaching” (Shulman, 1987, p. 4):

- content knowledge;
- general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;
- curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers;
- pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
- knowledge of learners and their characteristics;
- knowledge of educational contexts, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures; and
- knowledge of educational ends, purposes, and values, and their philosophical and historical grounds (Shulman, 1987, p. 8).

This knowledge base describes knowledge required for teaching in general, not particularly mathematics. Of particular interest to researchers within mathematics teaching are the categories content knowledge and pedagogical content knowledge, whereof the last category represents

the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue (Shulman, 1987, p. 8).

These categories will be subject to more attention in the next section.

Due to its inclusion of “all” dimensions of teacher knowledge, Shulman’s list of knowledge still appears as one of the professions’ most important knowledge bases for teaching. It asserts knowledge *for* teaching, but derives, among other sources, also from knowledge from *within* teaching. According to Shulman (1987) the base was drawn from four main sources; scholarship in content disciplines, material and settings for educational processes (curricula, textbook, institutional structures and finances), research on phenomena affecting teaching (teaching, learning, social and cultural factors), and the *wisdom of practice*.

A considerable amount of research has been based on the work of Shulman since it was published. Many researchers have pointed to missing aspects in his knowledge base, which would be of no surprise to

Shulman as he claimed that the included categories were a minimum of what teachers should know. From the scholars who have made contributions to Shulman's knowledge base for teaching, two will be considered here because they are relevant to my framework; Cochran et al. (1993) for expanding pedagogical content knowledge into pedagogical content knowing (2.2.1.2), and Hegarty (2000) for including embedded knowledge and insight into the educational knowledge base (2.2.1.3). Moreover, Rowland, Huckstep, and Thwaites' (2005) Knowledge Quartet, and Ball, Phelps, and Thwaites' (2008) Mathematical Knowledge for Teaching are based on, and have also made contributions to, Shulman's (1987) work. I use their work as analytical tool when analysing my informant's local knowledge for teaching, and partly for the explanation of my conceptual framework. These theories will thus be subject to further attention in Chapters 3.2.3.2 and 3.2.3.3 respectively.

### ***2.2.1.2 Pedagogical content knowing***

Arguing that Shulman (1987) does not pay sufficient attention to, or rather veils contextual factors of teaching, when focusing on “the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful” (Shulman. 1987, p. 15, my italics), Cochran et al. (1993) propose an extended version termed *pedagogical content knowing* (PCKg). In addition to content and pedagogical knowledge, PCKg includes knowledge about the target for the teaching, namely the students, and knowledge about their learning environment. It is thus a dynamic and continually developing category of knowing:

Increasingly strong PCKg enables teachers to use their teaching strategies for teaching specific content in a discipline in a way that enables specific students to construct useful understanding in a given context (Cochran et al., 1993, p. 266). This implies that teachers' PCKg have to be developed in the context of their understanding of, for example, pupils' learning strategies, developmental age, and environmental factors, including pupils' social, political, cultural and physical environment. Cochran et al. (1993) suggest that all four components of understanding develop simultaneously; they are not separately acquired and put together during teaching practice. They also claim that teacher students start with relatively limited understanding, which expands and becomes more elaborate as they gain experience. Related to my framework, I here see two levels of understanding; first, knowledge about pupils' environmental context as part of teachers' localized knowledge and its dynamic form, i.e. something happening in the moment, as localized knowing, and secondly, its developmental nature indicates that it, in the next stage, becomes part of teachers' local knowledge.



### **2.2.1.3 *Embedded knowledge and insight***

In addition to the categories of knowledge and knowing as proposed by Shulman (1987) and Cochran et al. (1993), Hegarty (2000) suggests a category embedded knowledge as a significant feature of teachers' knowledge base. Embedded knowledge are based on accumulated knowledge and prior investigation. It includes a range of intellectual resources, such as curricular materials, tests, assessment instruments and organizational routines, which the teacher can draw on in her teaching. In my framework, such knowledge feeds into the category brought knowledge.

Hegarty (2000) points to which knowledge base a teacher draws on and how she accesses it in the 'teaching moment', i.e. when she interacts with one or more learners so as to stimulate and direct their learning. He asserts that teachers draw on *theory* and *research* when it comes to general knowledge and understanding of children's developmental and cognitive level, for example, how socio-economic deprivation is associated with poor performance. This requires that teachers need to know something about pupils' personal background that might influence their learning. Hegarty (2000, p. 460) suggests *insight* into more facets of pedagogy and content knowledge as essential to behave coherently and intelligently in the "act of teaching", including insights in how to combine input from educational theories, pedagogical and content knowledge, own experience, teaching, and classroom management skills. He further asserts, that the insight by which teachers approach their teaching propels an intelligent action to be taken in a specific moment of interaction with one or more pupils, thus included in teacher's pedagogical knowing (Cochran et al., 1993), as discussed above, as well as contingency (Rowland et al., 2005, cf. Chapter 3.2.3.2) and localized knowing.

### **2.2.1.4 *Knowledge for teaching, a summary***

I have, in this section, discussed aspects of teacher knowledge and knowing "needed to promote comprehension among pupils" (Shulman, 1987, p. 8). Cochran et al. (1993) criticized Shulman for paying too little attention to the "target" for the teaching, i.e. the pupils, which made them suggest an extension of the category pedagogical content knowledge to also include knowledge about the pupils and their learning environments.

It has also considered the *insight* with which teachers have to approach their teaching (Hegarty, 2000). In addition to general knowledge/knowing, it is required that teachers have subject specific teaching knowledge. In the next section, I consider aspects of

mathematics for teaching, for example questions about what mathematics is necessary and appropriate for teaching at different levels (i.e. Borko, Eisenhart, Brown, Underhill, & Agard, 1992; Moreira & David, 2008), critique and further extension of Shulman's (1987) knowledge base for teaching related to teaching mathematics (Bromme, 1994; Peterson, Fennema, Carpenter, & Loef, 1989; Steinbring, 1998), and mathematics for teaching as a special branch (Ball & Bass, 2003).

### **2.2.2 Mathematical knowledge for teaching**

While both Ball et al. (2008) and Rowland et al. (2005) based their research on Shulman's (1986) subject matter knowledge (SMK) and pedagogical content knowledge (PCK), the result of their work were different. While Ball and her colleagues focused on mathematical knowledge *for* teaching, Rowland and his colleagues had their focus on mathematical knowledge *in* teaching. In Chapter 3.2, I discuss mathematical knowledge in teaching. This section thus continues with considering some aspects of knowledge *for* mathematics teaching.

Research demonstrates that it is not necessarily the number of courses in mathematics taken that is the most relevant for being a competent mathematics teacher (Borko et al., 1992; Even & Tirosh, 1995; Ma, 1999; Moreira & David, 2008). It thus invites speculation about what content within mathematics per se is appropriate for teaching at different levels (Davis & Simmt, 2006; Hill, Rowan, & Ball, 2005; Kahan, Cooper, & Bethea, 2003; Lim, 2007). Ma (1999) observed that the Chinese teachers in her study, on average, had less mathematics education than the US participants, nevertheless, they appeared to demonstrate a more profound understanding of the subject in their teaching. Ma's research resonates with Steinbring's (1998), and Ball and her colleagues' (2003; 2008) argument that a different kind of mathematical knowledge is required by teacher students than that required by those studying for occupations with high mathematical content, such as engineering, physics or accounting.

Researchers also question the adequacy of some content concepts as they are presented in the education of prospective teachers (e.g. Borko et al., 1992; Moreira & David, 2008). In the next section, I consider aspects of mathematical content offered in (some) teacher education programmes.

#### ***2.2.2.1 Mathematics offered in (some) teacher education programmes***

As mentioned in Chapter 1.2, in Norway it is generally required that, to teach mathematics at lower secondary level, one has to study for at least 60 credits in mathematics (Kunnskapsdepartementet, 2008). This

can be gained in the teacher education programme for grades 5 – 10<sup>15</sup>, following a master-degree programme in natural sciences that qualifies for teaching grades 8 – 13, or by studying mathematics or related subjects at a University or University College combined with a one-year (60 credits) practical-pedagogical course specializing on the teaching of mathematics. This raises a question about what mathematics should be included in the different routes. Three examples will be provided; the inadequate mathematics knowledge offered to the teacher discussed in Borko et al.'s (1992) work, the advanced mathematics offered to prospective Brazilian secondary teachers (Moreira & David, 2008), and one example from a Norwegian textbook for teacher students.

Borko et al. (1992) illuminated the inadequacy of the mathematics the informant in their research, Ms. Daniels, experienced in her teacher education programme. To become a mathematics teacher, Ms. Daniels completed three years of study at the university as a mathematics major, which included two-year courses in calculus and modern algebra, and introductory course in mathematical proofs. In addition, she had followed several computer science courses, and a mathematics method course. However, while reviewing the division of the fractions algorithm, a student asked why the algorithm works. Ms. Daniels provided a practical example, which turned out to be an example of multiplication rather than division of fractions, a mistake she discovered, but did not attempt to clarify to the pupils. Apparently, Ms. Daniels lacked conceptual understanding of fraction division despite her extended education in mathematics.

Ms. Daniels headed towards teaching at elementary and middle grades; a job she was apparently not adequately prepared for, evidenced by the above example. Focusing on the education of prospective *secondary* teachers, Moreira and David (2007) question the adequacy of academic mathematics courses taught by professional mathematicians. Acknowledging that teachers need to know about  $\mathbb{N}$  (natural numbers) and how they are extended to rational numbers and further to real numbers, they question the need for more abstract definitions of these, such as the formal construction of the set of rational numbers from integers (Moreira & David, 2008, p. 32). They assert that while it is important for a mathematician to consider different forms of the idea of rational numbers, for a schoolteacher it is more important to understand what a rational number is. For the construction of pupils' conception of

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<sup>15</sup> From 2017, the programme will be extended to a five-year programme leading to a master's degree in teaching for grades 5 - 10.

rational numbers, it is important for mathematics teachers to know several interpretations of these, for example as part-whole, decimal, ratio or a division. Definitions as those presented above do not particularly contribute to student teachers' understanding of how to make rational numbers comprehensible to their future pupils (Moreira & David, 2008).

The discipline courses at Universities still appear to offer traditional academic mathematics courses, such as more or less advanced courses in calculus, whereas those following teacher education programmes learn mathematics for teaching at the adequate level. In Norway that means that those studying for teaching in grades 1 – 7 and 5 – 10 respectively learn mathematics that is adapted to those levels. For example, while Brazilian teacher students had to learn abstract definitions of the different sets of numbers (Moreira & David, 2008), Norwegian teacher students heading at teaching lower secondary pupils might for example meet the definition: “The rational numbers,  $\mathbb{Q}$ , consists of all fractions  $\frac{a}{b}$  where a and b are whole numbers and  $b \neq 0$ .”<sup>16</sup> (Hinna, Rinvold, & Gustavsen, 2011, p. 161, my translation). These examples show that in Brazil, one seeks to educate teachers through an abstract approach to rational numbers (Moreira & David, 2008), while Hinna et al. (2011) provide an explanation that appears easier to grasp and understand for the teacher students, and also directly useful for their work as mathematics teachers.

In this section, I have considered aspects, and adequacy, of knowledge about numbers offered to teacher students. Several researchers have studied challenges teacher students might meet in their practice, provided what they are offered in their education. Even and Tirosh (1995), for example, point to domains of functions and undefined mathematical operations as problematic issues for teachers who are not able to explain these topics adequately. However, more important than looking into what does not work, is to look for what can help teacher students to be adequately prepared for being successful mathematics teachers. In the next section, I deal with some features of knowledge, which appear important for the teaching of mathematics<sup>17</sup>.

#### ***2.2.2.2 Knowledge adequate for teaching mathematics***

In the previous section, I provided an example of a teacher who was not adequately prepared for her work as mathematics teacher (cf. Chapter 1.1 for the result of the research I conducted for my master dissertation). Focusing on mathematics as a scientific discipline, only including for

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<sup>16</sup> De rasjonale tall,  $\mathbb{Q}$ , består av alle brøker  $\frac{a}{b}$  der a og b er hele tall og  $b \neq 0$ .

<sup>17</sup> A superficial review of some teacher education programmes in Norway reveals differences in what is offered to students. An in-depth review is not the focus of this study.

example the learning of rules and propositions, is likely to provide teacher students with instrumental understanding (Skemp, 1976) of the content. A problem could then arise when the teacher student or novice teacher is faced with a question such as the one Ms Daniels got from the pupil who wanted to know why inverting the divisor and changing the operation works for dividing fractions (Section 2.2.2.1). Bromme (1994, p. 74) claims:

The school subjects have a “life of their own” with their own logic; that is, the meaning of the concepts taught cannot be explained simply from the logic of the respective scientific disciplines.

In accordance to Shulman (1986, 1987) and Steinbring (1992, 1998), he thus asserts that mathematics as subject for teaching is different from its academic discipline; it has a philosophy of its own including epistemological foundations of the subject and its learning. In an attempt to make public how “the meaning of the concepts” *can* be explained, scholars have for some decades studied the work of mathematics teaching, and continues to do so. While studying teaching practice, they observe the mathematics *within* the teaching, which also is the focus of this study, and will be considered in the next chapter. Another reason for researching into teaching, is for making instruments to measure mathematical knowledge for teaching. This is not the focus of this study, and will thus not be further considered.

### ***2.2.2.3 Knowledge for teaching mathematics, a summary***

In this section, I have discussed aspects of the mathematics prospective teachers meet in their education: knowledge that is adequate for the teaching at secondary level, and some knowledge that is regarded as inadequate. Being responsible for facilitating a productive learning environment, the teacher needs to know and understand the mathematics the pupils are to learn (included knowledge of some mathematics beyond the actual level, horizon knowledge (Ball & Bass, 2003)), and understand how to support pupils to acquire that knowledge. This, I consider in Chapter 3.

Teaching is affected by teachers’ beliefs and values (Peterson et al., 1989). Some teachers believe in learning as a process of internalization of negotiated knowledge, while other see learning as a construction process where new knowledge is assimilated and adapted into existing knowledge structures. In the next section, I briefly look into these aspects, and conclude by expressing the view I take in this study about how knowledge is acquired.

### 2.2.3 Philosophy of mathematics teaching

As discussed in Section 2.2.1.2, some researchers have pointed to missing issues in Shulman's (1987) knowledge base for teaching. In addition to missing any focus on the learners and their environment (Cochran et al., 1993), Bromme (1994) and Peterson et al. (1989), for example, criticize Shulman for not describing qualitative features of teachers' professional knowledge, and how content knowledge and pedagogical content knowledge relate to each other (Cochran et al., 1993; Steinbring, 1998). This section continues with considering some philosophical aspects of the nature of mathematics teaching.

#### 2.2.3.1 Experience and beliefs influencing classroom actions

Mathematics teachers' view or conception of the nature of mathematics, their model or view of the nature of mathematics teaching and on the process of learning mathematics are key belief components that affect how they facilitate pupils' learning (Ernest, 1989). Further, Ernest asserts that, due to powerful influence of social context or their level of consciousness of own beliefs, there can be disparities between the mental models of teaching and learning the teachers have and what they enact in the classroom.

From reviewing literature about teachers' practice and mathematical beliefs published in the years 1975 - 2003, Handal (2003) reports that teachers' mathematical beliefs originates from their experiences as learners, and can mainly be categorized in three dimensions; beliefs about what mathematics is, how mathematics teaching and learning actually occur, and how it ideally should occur. He argued for that a large number of teachers, despite of educational reforms, still perceive mathematics as a discipline with rules and procedures, and that teachers holding progressive beliefs find it difficult to render their ideas into practice due to pressure concerning examinations, administrative demands, and parents' traditional expectations. Rowland et al. (2005), however related to prospective teachers, also report that teachers' beliefs are influenced by their experiences as learners, for example beliefs about under which conditions pupils will best learn mathematics.

Peterson et al. (1989) showed how teachers' beliefs about the subject, the curriculum, and instruction influenced classroom actions, and impacted on pupils' achievement. They suggest that teachers' *pedagogical beliefs in mathematics* is interrelated to pedagogical content knowledge. This resonates with Bromme (1994), who proposes that teachers have a philosophical perspective on their teaching, preferring the term philosophy to beliefs "in order to emphasize that it is a part of metaknowledge, soaked with implicit epistemology and ontology" (1994, p. 79). Bromme suggests that *the philosophy of school mathematics* includes ideas about epistemological foundations of

academic mathematics, of mathematics as a school subject, about how it is learned, and how it relates to other fields of life and knowledge. It is influenced and enriched by experience, and modified as they are informed by situations in the classroom, a component this study recognizes as *local knowledge* (cf. Section. 2.1.3). Bromme (1994) thus, as did Ball and Bass (2003) some years later, suggested a distinction between mathematics content knowledge as an academic discipline and mathematics knowledge for teaching, a distinction which later is clearly expressed by Ball, Thames and Phelps (2008)<sup>18</sup>.

Acknowledging that many scholars do recognize beliefs as part of the domain that combines the subject with teaching, pedagogical content knowledge (Shulman, 1986), Ball, Thames and Phelps (2008) did not express inclusion of the concept in their model, mathematical knowledge *for* teaching (MKT). Bromme (1994), however suggests that beliefs are included in his broad concept philosophy *of* school mathematics, and Peterson et al. (1989) suggest that pedagogical beliefs *in* mathematics is interrelated with pedagogical content knowledge. Rowland et al. (2005), observed that teachers' beliefs impact classroom actions, thus including beliefs as part of the domain foundation in the Knowledge Quartet, a model I have used as an analytical tool.

### **2.2.3.2 How teachers evaluate their work**

As indicated in Section 2.1.3.2, classroom activities also appear important to teachers. In addition to evaluating their work in terms of pupils' progress (understanding, production, accomplishment), teachers consider pupils' activity in the classroom as important (Brown and McIntyre, 1993). As long as the pupils continued to act in a desirable manner, which the authors term *Normal Desirable State of Pupil Activity* (NDS), the teachers were satisfied. What was considered as NDS varied from teacher to teacher, and could vary within different stages of a lesson. While one teacher describes her NDS in terms of pupils being quickly seated and getting on with their work, another highlights continuity, purposeful nature of activity, and the relationship between himself and the pupils as important. The mathematics teacher's NDS was characterized by providing highly structured mathematics tasks, and pupils looking back on previous work to meet challenges provided by the new work. Furthermore, for most secondary teachers, NDS fell into two categories; those characterizing interactivity between the teacher and the whole class, and those who wanted their pupils to work individually.

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<sup>18</sup> To be discussed in the next chapter.

### 2.2.3.3 Norms in the classroom

While the mathematics teacher in Brown and McIntyre's (1993) study apparently regarded his responsibility as providing pupils with appropriate mathematical tasks for individual work, Yackel and Cobb (1996) aimed at supporting teachers in establishing classroom norms characterized by mathematics discussions specific to pupils' mathematical activity, i.e. sociomathematical norms. As distinct from social norms, which foster social autonomy, sociomathematical norms foster intellectual autonomy.

Social norms in the classroom are constituted at two stages of interaction; for small-group collaboration and for whole class discussions (Yackel, Cobb, & Wood, 1991). Different groups might have different sets of norms, so when pupils change groups, a new set of norms has to be negotiated. These norms largely regulate the activity in the groups, which mean that the teacher can concentrate on interventions into groups that need assistance. For whole class discussions, there are typically explanations of a variety of methods and interpretations for the problems on which they have worked in the small groups. In the classroom Yackel, Cobb and Wood (1991) describe, pupils are expected to justify their answers, followed by discussions in which pupils challenge each other's solutions. These general norms describe regularities in patterns of social classroom interaction regardless of content.

Sociomathematical norms are interactively constituted and continually negotiated social normative aspects, though regulating pupils' *mathematical* argumentations in the classroom (Yackel & Cobb, 1996), thus different for different classrooms. For their classroom, Yackel and Cobb suggest that sociomathematical norms, as the social norms, can include pupils' competence to provide justification for their proposals, however expectation of *mathematical* reasoning, and *mathematically* acceptable and adequate explanations. For example, it is about what counts as mathematically *different* from what was proposed, as mathematically *sophisticated*, mathematically *efficient*, and mathematically *elegant* responses. Thus, when a pupil has provided a justified proposition, the teacher asks for different solutions. Discussions and negotiations then follow until a solution is accepted and taken-as-shared by all. Hence, the learning of the subject is a process of active individual construction of the new information based on a process of social negotiation in the class.

Sociomathematical norms include an understanding of what counts as acceptable mathematical explanations; the basis for explanations should be mathematical rather than status-based. Yackel and Cobb (1996) assert that explanations are relative to perceived expectations of others. For



example due to socially established cues for evaluation and authority-based rationales, pupils can interpret teachers' reaction to a response as an implicit indicator of how the teacher values the response mathematically. An enthusiastic "yeah!" indicates a favoured solution, while an "is that right?" might be interpreted as if the solution is incorrect (Yackel & Cobb, 1996, p. 464). The negotiations of sociomathematical norms gives rise to opportunities for establishing what counts as acceptable explanations and responses. For the teacher it could be about providing understandable adequate mathematical responses, while for the pupils it is about learning to focus on mathematical reasoning rather than social status when interpreting teachers' responses.

#### 2.2.3.4 Two models of teaching

Yackel and her colleagues (1996; 1991) and Brown and McIntyre (1993) particularly focused on *social* norms affecting pupils' learning, while Peterson et al. (1989) and Bromme (1994) discussed how *teachers'* personal beliefs and values, i.e. norms, affected their epistemological considerations and decisions. Similar to Peterson et al. (1989), Steinbring (1998) also focuses on epistemological aspects of teaching when questioning the relation between Shulman's (1987) categories content knowledge and pedagogical content knowledge. Steinbring (1998) suggested that without consideration of their relationship, teaching can be interpreted as a linear model:

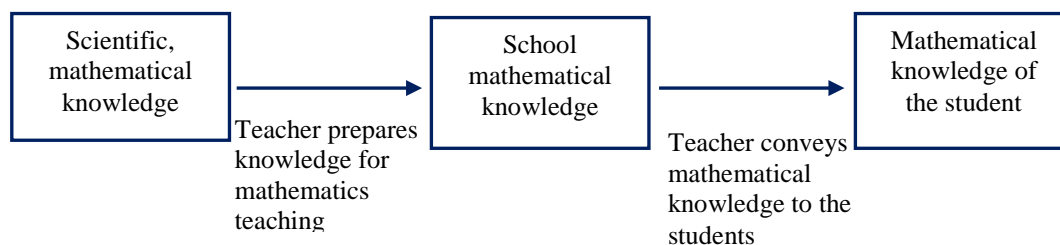


Figure 2-1. A linear model of the teaching-learning process (Steinbring, 1998, p. 158).

This model assumes that the teacher transforms her academic mathematical knowledge into mathematical knowledge adequate for pupils' learning, which she then conveys to the pupils. The first step (first arrow) requires content knowledge, while pedagogical content knowledge is related to the second step, the conveyance of the knowledge. The model exemplifies that the teacher offers learning environments for the pupils, but that she has no opportunity for observing and diagnosing pupils' learning, and hence no possibility to vary and adjust her teaching to pupils' needs. Steinbring (1998) argues that in order to be able to assist pupils, the teacher has to diagnose and

analyse pupils' construction of the mathematics to be learned. This requires an interactive relationship between the teacher and the pupil, which is not the case in the above model. He thus suggests the existence of two relatively independent autonomous systems, the pupils' learning process, and the interactive process between teacher and pupils, which both include their own specific epistemological status of mathematical knowledge:

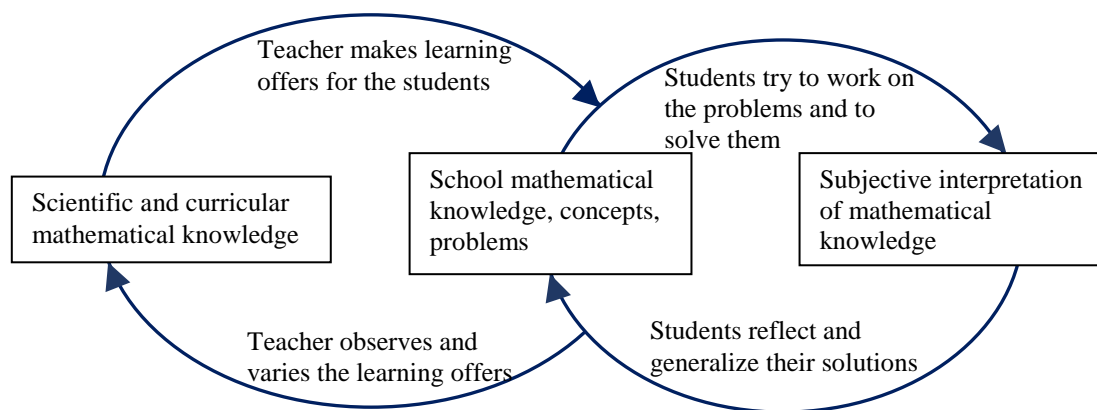


Figure 2-2. The pupil learning and teacher-student interaction as autonomous systems (Steinbring, 1998, p. 159).

The model in Figure 2-2 visualizes that the teacher prepares her academic and curricular knowledge to facilitate pupils' learning. The pupils try to solve the school mathematics problems offered by the teacher, thus constructing their own subjective interpretation of the mathematics that is to be learned. The teacher observes as the pupil tries to generalize the problems, a process that makes it possible for the teacher to adjust her teaching if necessary. Steinbring describes this combination of content knowledge and pedagogical knowledge as *epistemological knowledge of mathematics in social learning setting* (1998, p. 160). It differs from Shulman's (1987) pedagogical content knowledge in that it also includes epistemological knowledge as an analytical "tool" for teachers to use in their teaching and communication of mathematics.

Being able to find socially shared and adequate referents for concepts to be used in mathematics teaching might also be an issue concerning the special kind of mathematics for school teachers as mentioned above (e.g. Ball & Bass, 2003; Bromme, 1994). Thus, while I, in this section, have focused on epistemological aspects of mathematics teaching, in the next I discuss some features of mathematical knowledge that is particularly important and necessary for teaching the subject.

## 2.3 A theoretical perspective on the process of teachers' learning

Extension of a person's knowledge base implies that something is learned. Learning can occur while reading something, or from experience. This study focuses on the knowledge a teacher has developed in practice, i.e. what she has learned from her experiences while carrying out her profession. The question about how one comes to know has occupied scholars for centuries, and has led to extensive discussions, particularly between scholars holding differing views, such as constructivism based on Piaget's theories and socio cultural theories developed from the work of Vygotsky. Piaget and Vygotsky both focused on the "growing mind", however differing in how they perceived how one comes to know. While adherents of Piaget see the growth of the mind as a process of active knowledge construction, Vygotsky's successors emphasize that learning occurs through enculturation into a community.

Whereas Piaget and Vygotsky concentrated on children's learning, this study focuses on adults' learning, i.e. what *teachers* learn in their practice. Schön (1983) argues that teachers in the process of their teaching learn by reflecting on their actions or reflecting while in action. It is thus natural to assume that these processes occur resulting from an interaction between the teacher, the pupils and the content, an assumption that to a certain degree coincides with a socio-cultural view on how learning occurs. In Section 2.3.1, I thus consider Cobb's contention about the existence of complementarity between constructivist and socio-cultural perspectives. My research is situated in the practice of teaching, a social setting in which pupils are expected to learn. Wenger (1998) asserts that as human beings we live and learn in social settings, i.e. communities of practice. This, I further discuss in Section 2.3.2.

### 2.3.1 Co-existence of social and cognitive perspectives

Instead of discussing whether one should adopt one perspective rather than the other (i.e. constructivism or socio-cultural theories), Cobb (2007) claims that the focus should be kept on what contributes to improvement of learning, justified by its potential to address the issue at stake (Cobb, 1994). Further, he argues that the perspectives constitute the background for each other in classroom work; collective normative activities in the classroom emerge and are continually regenerated as the participants (pupils and teacher) interpret and respond to each other's actions. Likewise, their interpretations do not exist apart from their participation in the classroom. The *co-existence* of social and cognitive

perspectives thus makes sense of the complexity of classroom life and activity (Cobb, 2007).

Cobb (2007) thus takes the perspective that the relation between social and cognitive perspectives is reflexive, implying that neither exists without the other. Socio-cultural theory was seen as attractive because of its focus on mathematics as a complex human activity rather than as disembodied subject matter. However, the “socio-culturalist’s” assertion that practices exist prior to and independently of teacher’s and pupils’ activity does not fit into Cobb’s perspective. He defined classroom practice as an emergent phenomenon jointly established by the pupils and their teacher. The idea of learning as a process of reorganizing activity was adapted from the cognitive perspective, while the view of cognition as being distributed across minds, persons, and symbolic and physical environments, was adapted from distributed theories of intelligence. This last perspective sees cognition as extending out into the environment, thus including tools such as computers as a resource for reasoning (Cobb, 2007).

I sought for knowledge and knowing developed in the profession of teaching, which Wenger (1998) refers to as learning in a community of practice (see next section), i.e. happening in a social context. The teacher learns as she engages with her pupils. Likewise, my observations and conversations also occurred in a social context. Having my research questions answered, however, required that I considered both social and cognitive perspectives, it is a question about where the mind “is located” (Cobb, 1994). I thus sought support from Cobb’s suggestion about the *co-existence* of social and cognitive perspectives: I took the stance of asserting that teacher’s localized knowledge exists within her “head”, while her knowing, which depends on a social interaction is “located” in the individual-in-social-action” (Cobb, 1994, p. 13). Hence, for finding evidence for my first auxiliary research question, teacher’s localized knowledge, I took a cognitive perspective, while a social perspective supported my search for localized knowing.

### **2.3.2 Learning in the context of lived experience of participation**

Humans are social beings, which is a central aspect for learning (Wenger, 1998, p. 4). Further Wenger (1998, p. 4) asserts, “knowledge is a matter of competence with respect to valued enterprises”, that knowing is a “matter of participating in the pursuit for such enterprises”, and that learning is about producing ability to experience the world as meaningful.

As explained in the previous section, the knowledge I sought develops in the profession of teaching, i.e. in the interaction with pupils and the mathematics in a classroom. My informant’s knowledge is a matter of competence to teach mathematics, and her knowing a pursuit

for assisting particular pupils learn the mathematics, thus helping them to “produce ability to experience “ (Wenger, 1998, p. 4) the mathematics as meaningful. In this section, I provide a brief summary of some components Wenger asserts as necessary for characterizing social participation as a process of learning and knowing: forming of identity and experiencing meaning while participating in a community of practice. When considering these aspects, I situate the community in the classroom; focus is particularly on the teacher, not on the pupils, as the one learning in this community.

### ***2.3.2.1 Experiencing meaning***

Engaging in a practice is about experiencing what gives meaning to about whatever the practice is. I assume that in a mathematics classroom, what gives meaning for a teacher is firstly about helping pupils learn. However, when for example experiencing poor learning, the meaning could also include development of knowledge for teaching mathematics, the result from which is the focus in this study.

In the process of negotiating meaning, Wenger (1998, p. 52, 53) suggests an interaction between two constituent processes: participation and reification. Participation in this sense is a process that combines doing, feeling, thinking, talking, and belonging, provided this occur for members of social communities. Reification, Wenger suggests, covers processes of making, designing, representing, perceiving, interpreting, using, and decoding, to mention but a few. For a teacher of mathematics who has experienced failure in her teaching, the reification might mean to decode her experiences, interpreting and perceiving what went wrong and why, followed by designing new representations for, and approaches to, her teaching. The result of these processes will assumedly provide a new meaning to her pursuit for helping pupils to learn mathematics.

### ***2.3.2.2 The classroom as community of practice***

Wenger (1998) suggests that a community of practice constitutes three dimensions of practices: mutual engagement, joint enterprise, and shared repertoire, which exist because its members are engaged in negotiating its meanings, its enterprise, and developing a shared repertoire. When being brought together, either by employment in a job or in a classroom, the group of people, due to diversity, at first form an ill-defined group. What makes it a community of practice is when being included in mutual engagement for joint or complementary contributions to the enterprise, where complementary contributions come from people having different roles due to for example competence. For the majority of the members in a classroom community, the meaning constitutes the enterprise of learning. The complementary contributions then come from the teacher

as the more knowledgeable member, who at the outset is in the classroom to transform her knowledge into forms that are “pedagogically powerful and yet adaptive” (Shulman, 1987, p. 15) to the pupils. Over time, the participants in the community develop what Wenger (1998) refers to as a shared repertoire, for example routines, ways of doing things, actions, or discourse by which they create meaningful statements, i.e. a resource for negotiating meaning. In a mathematics classroom, “shared repertoire” might be somewhat similar to what Yackel and Cobb (1996) refer to as sociomathematical norms, which is also included in Leinhardt’s concept craft knowledge (Leinhardt, 1983). In Leinhardt’s classrooms, however, the routines appeared to be stated by the teachers, and not result of negotiation between teachers and pupils. Teachers extend their knowledge of teaching while in practice (Barth, 2004; Leinhardt & Smith, 1985; Leinhardt, 1989, 1990; Ruthven, 2002), i.e. as a member of a classroom community, thus developing what I categorize as local knowledge.

### ***2.3.2.3 Developing identity as a teacher***

Issues of identity are integral part of aspects of social theory for learning. It is inseparable from issues of practice, community and meaning, and is manifest in the way members engage in and relates to one another (Wenger, 1998, p. 145, 149). A person’s identity develops in the constant work of negotiating self in the interplay between participatory experience and reification. In a community of practice, one learns certain ways of interaction with other members; one develops expectations about how to treat each other, and how to work together. In a classroom there exist (at least) two forms of identities, that of “the” pupil<sup>19</sup> and that of “the” teacher. I argue that teacher students, resulting from their experiences of being pupils for many years, enter the teacher education having beliefs and assumptions about how it is to be a teacher and what the work of teaching entails. Through their education, they assumedly get more insight into the work of teaching. It is when entering their teaching practice, however, they start developing their own practice-based identity as a teacher, following what Wenger refers to as *inbound trajectories*; the novice teachers join the community with the prospect of becoming full members. Participation in the community, experiencing new demands, dealing with specific situations, and sorting out what matters and what does not, generate occasions for renegotiating one’s identity. A person’s identity is thus temporal. For a teacher this means that she may renegotiate her identity in the meeting with new

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<sup>19</sup> Within the group of pupils, I assume that there exist a multiplicity of sub-identities.

classes, new pupils, parents or colleagues as all these may influence her way of experiencing the meaning of her membership in the community.

## **2.4 Summary**

I have, in this chapter, discussed theoretical aspects I have considered in my study. I have considered a brief definition of the two related, but as used in this study, distinct terms *knowledge* and *knowing*, which, I argue, both constitute sources for supporting pupils to learn mathematics.

Because of its importance for the development of teacher's sharable knowledge, I have also discussed the concept craft knowledge as referred to by several mathematics education researchers. Moreover, in addition to some important epistemological considerations necessary for teachers' professional knowledge, I have provided a brief introduction to some main concepts in Wenger's (1998) theory of learning in communities of practice.

The theories I have used might be somewhat old, but in my opinion, they are still as adequate as they were at the time of their development. I argue that connecting some European views on teaching and teacher professional knowledge (e.g. Bromme, 1994; Steinbring, 1998) to US views (e.g. Ball & Bass, 2003; Shulman, 1987) is a strength because my research then has a broader international foundation than it would if I only considered one of them.





### 3 Theoretical foundation and conceptual framework

In the previous chapters, I situated the study in a Norwegian teacher education historical perspective as well as in a theoretical perspective of theories for mathematics teaching. In this chapter, I discuss some research concerning knowledge *in* the teaching of mathematics, particularly the part that I have used as foundation for my conceptual framework, which I discuss and explain towards the end of the chapter.

My study focuses on the knowledge mathematics teachers deploy in the course of their teaching, particularly the knowledge developed in the process of carrying out their profession. Such development might come from experiencing inadequate or insufficient existing knowledge of mathematics and mathematics teaching, or with the meeting with new pupils, parents, and school management. It thus focuses on teachers' learning arising from reflecting on their professional experiences. Based on the newly acquired knowledge, the teacher is then able to extend her knowledge base to include the results of these experiences, which can be, for example, about class management or approaches for teaching the subject. I now turn to some scholars who during the last thirty - forty years have studied the work of teaching mathematics, of which some will serve as the theoretical foundation for my conceptual framework.

The chapter is divided into two main parts; first the existing research which makes the theoretical basis for the study, and secondly, the explanation of my conceptual framework including the research questions. The first part starts with discussing reasons for *why* some scholars (e.g. McNamara & Desforjes, 1978; Schwab, 1971) saw it as necessary to engage in research directly pointed to education, and their suggestions about how to include the practical field of education. It ends with a presentation of the Mathematical Knowledge *for* Teaching framework (MKT, Ball et al., 2008), and two frameworks focusing on knowledge *in* mathematics teaching, the Teaching Triad (TT, Jaworski, 1994) and the Knowledge Quartet (KQ, Rowland, 2013; Rowland et al., 2005), which I all used when analysing my informant's knowledge for teaching the content.

When analysing the conversations with my informant Tea, Ball and colleagues' (2008) framework served as an analytical tool for exposing her knowledge *for* the teaching of mathematics. While analysing her knowledge *in* the teaching, however, I experienced the MKT framework

as insufficient for my purpose, thus using the KQ (Rowland et al., 2005) and TT (Jaworski, 1994) frameworks.

### **3.1 Educational research; a retrospect**

As mentioned in Chapter 2.1.3, forty years ago, much educational literature and textbooks appeared to be theoretical, educators borrowed theories from behavioural sciences to solve practical educational problems (McNamara & Desforges, 1978; Schwab, 1971). Each of these theories treated parts of the problems arising in the complex field of education isolated from one another, thus suggesting different treatments for the same subject. Schwab (1971) claimed that this led to poor solving of the problems, and suggested that student teachers should discuss existing theories related to actual teaching episodes. McNamara and Desforges (1978) suggested that educational theories should be based on teaching practice itself; teacher trainers and established teachers should co-operate in both the training of students and development of instructional knowledge. They proposed a framework for conducting classroom research that also can be used in teacher training, thus changing focus from theories developed in sciences outside schools to theories developed within the practice of teaching.

Also Shulman (1987, p. 11) argued for the importance of searching for theories from within the educational field: “one of the more important tasks for the research community is to work with practitioners to develop codified representations of the practical pedagogical wisdom of able teachers”. However, researching into teachers’ practice is not “straight forward”, teachers are individuals whose knowledge and skills depend on their personal histories, values and beliefs (Brown & McIntyre, 1993; Peterson et al., 1989). Even if teachers have followed the same educational programmes, there are diversities in their ways of thinking about the specific knowledge they bring into the classroom, which influences their teaching (Brown & McIntyre, 1993). However, Brown and McIntyre also assert that there are strong indications that experienced teachers, at a more abstract level, have much in common. Examples such as fluency and confidence in interpretations of classroom events, readiness to make inferences, and greater selectivity in focusing on instructionally important facets of classroom activities, are all competences achieved in the classroom, i.e. *craft knowledge*, as discussed in the previous chapter.

This section continues with looking into one strand of research conducted for the development of mathematical knowledge *for* teaching. In the next sections, I thus discuss some research conducted within teaching practice for the last two decades, research focusing on what

mathematics knowledge is important *for* teaching, whose results influence this study.

### 3.1.1 Researching into the work of teaching

Assuming that teaching is affected by the teachers' subject matter knowledge<sup>20</sup>, Ball and Bass (2003) as part of the research group at the University in Michigan, set out to identify knowledge mathematics teachers need to teach the subject effectively. By researching into the daily work of mathematics teachers, they illuminated many "challenges" teachers have to cope with in the course of their mathematical work. They then, for example, observed that teaching the subject involved substantial mathematical work; teachers have to unpack compressed and abstract mathematics, and they have to know how mathematical domains are connected at a given level. They also observed how important it is for teachers to know how mathematical ideas grow, for example when extending pupils' repertoire from working on natural numbers to integers. As examples of mathematical work teachers have to carry out in the course of their teaching, they suggested (2003, p. 11):

- Design mathematically accurate explanations that are comprehensible and useful for students
- Use mathematically appropriate and comprehensible definitions
- Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process
- Interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems, and insights (both predictable and unusual)
- Be able to respond productively to students' mathematical questions and curiosities
- Make judgments (sic) about the mathematical quality of instructional materials and modify as necessary
- Be able to pose good mathematical questions and problems that are productive for students' learning
- Assess students' mathematics learning and take next steps.

This list of "actions" and competencies might include some of what Steinbring (1998) asserted as missing in Shulman's (1987) knowledge base for teaching (cf. Chapter 2.2.3).

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<sup>20</sup> At that time this was statistically evidenced in the UK ( Rowland, Martyn, Barber, and Heal, 2001, referred to in Huckstep, Rowland, & Thwaites, 2003).

The Michigan group continued focusing on the work teachers do when teaching mathematics. As a result, they suggested sixteen mathematical tasks for teaching, a list somewhat coinciding with the list from Ball and Bass (2003) as reproduced above, however, developed and extended:

- Presenting mathematical ideas
- Responding to students' "why" questions
- Finding an example to make a specific mathematical point
- Recognizing what is involved in using a particular representation
- Linking representations to underlying ideas and to other representations
- Connecting a topic being taught to topics from prior or future years
- Explaining mathematical goals and purposes to parents
- Appraising and adapting the mathematical content of textbooks
- Modifying tasks to be either easier or harder
- Evaluating the plausibility of students' claims (often quickly)
- Giving or evaluating mathematical explanations
- Choosing and developing useable definitions
- Using mathematical notation and language and critiquing its use
- Asking productive mathematical questions
- Selecting representations for particular purposes
- Inspecting equivalencies (Ball et al., 2008, p. 400)

When analysing the mathematical demands for teaching, the Michigan group sought to identify knowledge required for teaching the subject. Their search resulted in six broad categories of knowledge: common content knowledge, specialized content knowledge, horizon content knowledge, knowledge of content and students, knowledge of content and teaching, and curricular knowledge. These were organized into the Mathematical Knowledge for Teaching-framework, MKT. The MKT is one of three frameworks I have used when analysing the knowledge my informant, Tea, deployed when teaching the subject. Along with the other two frameworks mentioned in the previous section, the Teaching Triad (Jaworski, 1994) and the Knowledge Quartet (Rowland et al., 2005), I further consider the MKT-framework in Section 3.2.3.

I have largely used the work by the Michigan group (i.e. Ball & Bass, 2003; Ball et al., 2008) for the articulation of my conceptual framework (Section 3.3) and their MKT-framework as an analytical tool for exposing Tea's knowledge for teaching mathematics. In this section, I have considered some of what the Michigan group's work shows as entailing the work of teaching mathematics over the years from 2003 to 2008, and what they assume as included in mathematics knowledge for

teaching. In the next section, I turn from considering mathematics knowledge *for* teaching to some research conducted for exposing different perspectives of knowing *within* teaching, starting with a section about culture and teaching.

## **3.2 Knowledge in mathematics teaching**

The phenomenon of mathematics knowledge in teaching reflects different perspectives about mathematics teachers' knowledge and different ways of "knowing" within teaching (Rowland & Ruthven, 2011, p. 2). This section continues with discussing research on some of these different ways of knowing, both practical and theoretical knowing, not focusing on subject matter knowledge<sup>21</sup>.

### **3.2.1 Culture and teaching**

There is nothing controversial in the assertion that subject matter knowledge is an essential component for the teaching of mathematics (Borko et al., 1992; Huckstep et. al., 2002). However, research demonstrates that it is not necessarily the number of courses in the subject that is the most relevant for being a competent mathematics teacher (Borko et al., 1992; Even & Tirosh, 1995; Ma, 1999; Moreira & David, 2007). As indicated in Chapter 2.2.2, researchers have speculated about what content within the field of mathematics is appropriate for teaching at different levels (Hill et al., 2005; Kahan et al., 2003; Lim, 2007). I then referred to Ma's (1999) observation that even if the Chinese teachers she studied, on average had less mathematics education than the US participants, they appeared to demonstrate a more profound understanding of the subject in their teaching. Ma described a scenario where a pupil came to her teacher telling that she had figured out a theory about a connection between the perimeter and area of a closed figure; she had discovered that if the perimeter increases, so does the area (Ma, 1999, p. 85). Out of the 23 US teachers, only two showed interest in investigating the claim, while for the Chinese teachers, 92% did the same. Even if the US teachers had knowledge of the formulas related to the task, they were weak in their general attitude towards mathematics, and behaved in, what Ma calls, an un-mathematical way.

Stiegler and Hiebert's (1999) video study of Japanese, German, and US teachers also showed cultural differences across continents concerning teachers' attitude towards mathematics; they exposed a "gap" in the effectiveness of teaching methods between these countries. Pepin

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<sup>21</sup> This will be discussed in the next section.

(2011) showed that there also exists cultural differences within the European countries; while the English teachers in her study focused on the content to become “digestible” to pupils, the French teachers emphasized development of the knowledge residing within the teacher. In German schools, there were demonstrated context specific approaches<sup>22</sup>; in Hauptschule the correctness of the mathematics was in focus, while in Gymnasium thinking logically was perceived as appropriate for the teaching of the subject.

As indicated in Chapter 1.1, one hundred percent of Norwegian pupils were in 2004 reported to be taught by certified teachers (Mullis et al., 2004), and still Norwegian pupils performed below the international mean on international tests. A question was raised; were Norwegian teachers qualified to teach *mathematics*? The Ministry of Education’s response to this question was to differentiate the education for those who plan to work in primary school (1 – 7), and those who plan to teach in higher compulsory grades (5 – 10) as can be read in Chapter 1.2.1, and to offer a five-year programme for teaching grades 8 – 13. From 2017, all who plan to teach compulsory school have to follow a five-year teacher education programme, still differentiated between primary (1 – 7) and higher (5 – 10) grades (Regjeringen, 2014).

### 3.2.2 Approaches to teaching

In 1976, Skemp advocated for a need for teaching for relational understanding, i.e. teaching pupils both how and why. Since then (and probably before) methods for teaching, which also reflects teachers’ knowledge in and view on teaching, have been widely debated; should one follow a traditional teacher-centred path, or work in more pupil-centred teaching? Traditional teaching is teacher centred, and its segments can be characterized by a three part process; initiation, reply and evaluation (IRE); the teacher poses some information/question, a pupil responds, and the teacher evaluates the response, a sequence which is repeated until a desired response is given (Borko et al., 1992; Wells, 1993).

Discussions about methods for teaching have particularly been subject to controversies in the US, where heated debates between those who believe in traditional teaching have raged; teachers explaining methods for solving tasks followed by pupils practicing the methods, and those who believe that pupils should be more involved in for example open and problem solving approaches (Boaler, 2008). As an example of these controversies, Boaler (2008) refers to a case in a Californian

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<sup>22</sup> In Germany students are divided into three schools; Gymnasium, Realschule and Hauptschule

secondary school where a teacher left teaching because she was not allowed to teach in the way she believed. She had been successful in enhancing pupils learning by using open approaches, but parents' beliefs about pupils having to learn mathematics using only traditional methods forced upon her a curriculum that made her leave school.

To study whether different approaches to teaching mathematics impact on pupils' learning, Boaler (2002), over a period of three years, followed two groups of English pupils attending schools having different approaches to teaching mathematics. "Phoenix Park" offered open-ended projects, mostly to mixed ability groups, and at "Amber Hill", they taught "traditional" mathematics to pupils placed in ability groups. Pupils from "Phoenix Park" scored significantly higher than "Amber Hill"-pupils on a range of assessments, including the national examination.

Also in Norway teacher centred teaching and individual seatwork still seems to be the most used form for approaching pupils learning. 91.2% of Norwegian pupils at the age 11 – 19 (grades 5 – 10 and 11 – 13) claim that teachers are at the board several times per week, 44.2% that discussion between pupils and teacher occur several times per week, and 88.2% claim that they work alone several times per week (Wendelborg, Røe, & Skaalvik, 2011, p. 165)<sup>23</sup>. Fifteen percent say that they can work in groups several times per week. The survey is obligatory for grades seven and ten in compulsory school and grade one in upper secondary school. Klette (2013) confirms that the teaching in Norwegian classrooms appears as having great emphasis on the teacher being at the board controlling the classroom conversation. Concerning mathematics, there is emphasis on solving tasks individually (Klette, 2013).

### 3.2.3 Three frameworks particularly important to this research

Several scholars describe teachers' knowledge *in* teaching. For example Jaworski (1994), suggests a theoretical construct connecting generalized characteristics of three "domains" of activity in which teachers engage, *The Teaching Triad* (TT, Section 3.2.3.1), while Rowland, Huckstep and Thwaites' (2005, Section 3.2.3.2) from their research developed *The Knowledge Quartet* (KQ), which asserts four observable dimensions that seemed to be informed by teachers' mathematical or pedagogical content knowledge. For identifying<sup>24</sup> the mathematical knowledge in teaching demonstrated by my informant during our conversations, I use both

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<sup>23</sup> It has to be clarified that the study concerns all subjects in school.

<sup>24</sup> For the record, they are used as identification and not explanation of her knowledge for teaching.

frameworks as analytical tools auxiliary to my main framework (see below). I describe these later in this section.

Today there is widespread consensus among scholars of the importance of teaching for understanding. To facilitate for pupils' learning, the teacher thus has to have profound understanding of the mathematics included in the curriculum. For a teacher, this is reflected in Ball and colleagues' (2008, Section 3.2.3.3) category specialized content knowledge. I use Ball and colleagues' framework as an analytical tool for identifying my informant's mathematical knowledge for teaching, which I consider towards the end of this section.

In an order based on year of publication, I now present some of what I perceive as main ideas of the three frameworks, thus starting with the teaching triad (Jaworski, 1994). After presenting the three frameworks, I describe some ideas that connects the three frameworks.

### **3.2.3.1 *The Teaching Triad***

The triad emerged from an ethnographic study on investigative teaching Jaworski (1994) undertook involving three mathematics teachers. In their naturalistic setting, the teachers engaged pupils in open-ended and problem-solving tasks for fostering their mathematical thinking and understanding. Through the study, Jaworski identified general characteristics of investigative teaching, particularly three domains of activities in which teachers engaged. These were linked together in a theoretical construct; *the teaching triad*; comprising *sensitivity to students*, *mathematical challenge* and *management of learning*.

*Sensitivity to students* concerns teachers' knowledge of pupils and attention to their needs. It is also about the ways in which the teacher interacts with individuals and groups of pupils, for example by offering tasks on which all pupils were able to start, and being encouraging with valuing pupils' contributions. The construct consists of two distinct parts, *cognitive* sensitivity, which refers to the teacher's appreciation and recognition of pupils' thinking, and *affective* sensitivity concerning pupils' wellbeing within the classroom.

*Mathematical challenge* relates to the challenges the teacher offers to their pupils to engender mathematical activity, including setting tasks and posing questions. For pupils who are able to guide their own learning, for example, this can mean very little input from the teacher, while for others, the teacher has to pose more individually tailored challenges by offering hints or clarifying questions.

The third domain of activity in the teaching triad is *management of learning*. Management of learning refers to the teacher's role in the constitution of the learning environment such as setting norms for the class, organizing the class in groups, and planning of activities, for example, how to introduce a task. It works on two planes; on an



individual plane where she interacts with individual pupils, and on a plane where she coordinates actions and decisions to meet needs for all pupils in the class.

A key factor for success of a teaching/learning episode is *harmony* between the three elements of the triad, and particularly between sensitivity to pupils and mathematical challenge. A teacher has to foster an atmosphere in the class where learning is appreciated, she has to value, and at the same time, challenge pupils' mathematical thinking. In addition, she has to support the pupils both emotionally and cognitively. In the complexity of the classroom interaction, the teacher is required to be aware of the challenge of combining these activities. Another, and complementary, tool for analysing teacher activities in the classroom is described and defined by Rowland and his colleagues (2005) in *the knowledge quartet*, which I consider next.

### **3.2.3.2 The Knowledge Quartet**

To locate ways in which teachers draw on their knowledge of mathematics and mathematics pedagogy, and influenced by Shulman's (1986) work, Rowland, Huckstep and Thwaites (2005) studied the teaching of novice, trainee elementary school teachers during their school-based placements following a one-year programme leading to a Postgraduate Certificate in Education. Through observing and analysing twenty-four lessons taught by the student teachers, they identified four main units/dimensions important for teaching. These were *foundation*, *transformation*, *connection*, and *contingency*, whereof the last three are activities/behaviour demonstrated in the planning for, and in the act of teaching, and which rest on foundational knowledge.

The category *foundation* is about theoretical background acquired through education, experience, and preparation for mathematical work in the classroom. It concerns the knowledge a teacher possesses, and enables her to approach pedagogical actions and choices. It thus includes knowledge and understanding of the subject, and how it is learned, and about use of a mathematical vocabulary. It also includes a belief-aspect that concerns teachers' views about the conditions under which pupils learn best, a view that is influenced by their experiences as learners. I assert that Ball and colleagues' framework (2008, Section 3.2.3.3), mathematical knowledge for teaching, coincides with ideas within this category.

The mathematics work in the classroom includes management of learning (Jaworski, 1994), which, amongst others, is about transforming one's knowledge, a second category in the knowledge quartet. *Transformation* refers to a teacher's capacity to transform possessed

mathematics knowledge into forms that facilitate pupils' learning. Rowland and his colleagues (2005) refer to Shulman's (1986) concept pedagogical content knowledge (PCK) when defining how transformation is to be understood; it is about re-presenting possessed mathematical knowledge and "make it comprehensible to others" in forms of "powerful analogies, illustrations, examples, explanations, and demonstrations" (Shulman, 1986, p. 9). It is thus, unlike foundation, directly directed towards pupils to assist their acquisition of the mathematical language and formation of subject concepts.

Coherence across lessons in the teaching of a topic is important if understanding is to be fostered. The third category in the knowledge quartet, *connection*, accounts for that. *Connection* concerns the coherence of the planning, teaching and sequencing of topics for mathematics instruction displayed within and across episodes and between a series of lessons. It thus binds together choices and decisions that are made for conceptual or procedural learning of a mathematical topic. The ordering of tasks, which also is an issue in that matter, entails awareness of the cognitive demands in different topics and tasks, as well as the structural connections within the subject, thus requiring sensitivity to pupils (Jaworski, 1994).

By its very nature, life in a classroom can be unpredictable, not all can be planned for. A teacher has to be prepared for unexpected inputs and responses, and in order to deal with such events, she is required to act contingently. The unit *contingency* includes teachers' readiness to respond to pupils' mathematical ideas, and when appropriate, to deviate from the planned agenda. This category requires that teachers are fluent in mathematics; the quality of the responses to pupil's unanticipated ideas depends partly on the teacher's knowledge resources (Rowland et al., 2005). To value pupils' mathematical contributions is about showing sensitivity to them (Jaworski, 1994), and appears important for their knowledge construction. To ignore such input can be perceived as lack of interest in the mathematical activity going on in the classroom, even if using time constraint as an excuse to not involve in unanticipated actions. It also requires that the teacher possesses specialized content knowledge (Ball et al., 2008), which is one of the domains in the Mathematical Knowledge for Teaching (MKT-framework), the third framework I used for my analysis.

### ***3.2.3.3 Mathematical Knowledge for Teaching***

Ball and her colleagues (2008) elaborated Shulman's (1986) categories *pedagogical content knowledge* and *content knowledge* when developing their framework; domains of mathematical knowledge for teaching. Through the Mathematics Teaching and Learning to Teach Project and the Learning Mathematics for Teaching Project (LMT), Ball and her

colleagues sought to contribute to a broad discussion about what teaching entails. Focusing on the teaching of mathematics (the work of teaching) and on the mathematics used in teaching, they investigated the demands of teaching (also referred to in 3.1.1). These studies resulted in the development of a set of hypotheses about the nature of mathematical knowledge for teaching, and in the development of survey instruments to investigate the nature, role, and importance of different types of mathematical knowledge for teaching. These instruments are today used in several countries, included Norway where a group of researchers at the University of Stavanger has translated the test items into the Norwegian language (Mosvold, Fauskanger, Jakobsen, & Melhus, 2009) and adapted the instruments into Norwegian cultural context (Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012).

The studies referred to at the beginning of this section led Ball and her colleagues (2008) to identify six domains of mathematical knowledge *for* teaching, and resulted in a refinement, subdivision and extension of Shulman's (1986) content knowledge and pedagogical content knowledge, each into three sub-domains. Content knowledge they divided into *common content knowledge*, *specialized content knowledge* and extended with *horizon content knowledge*, while pedagogical content knowledge they dissected into *knowledge of content and students*, *knowledge of content and teaching* and extended with *knowledge of content and curriculum*.

Knowledge of *content and curriculum* is consistent with the category curriculum knowledge in Shulman's (1986) knowledge base (Ball et al., 2008). The requirement for knowledge about the curriculum is because it contains important information for teaching a particular subject:

The curriculum is represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (Shulman, 1986, p. 10).

Shulman also suggests two additional aspects: *lateral* knowledge, relating to the curriculum being taught in other subject areas, and *vertical* knowledge, which is about familiarity with preceding and future content in the same subject area. Concerning the latter, vertical knowledge, it appears to be somewhat consistent with *horizon content* knowledge (Ball et al., 2008). The additional *lateral* knowledge (Shulman, 1986), I include in Ball and colleagues' (2008) category knowledge of *content and curriculum*.

Knowledge of *content and students* combines knowledge about the pupils and the mathematics. It concerns teachers' ability to predict what pupils will find interesting and motivating, anticipating what pupils are likely to think, and what they will find confusing. It is also about interpreting incomplete thinking as expressed in pupils' use of the mathematical language, and knowledge and understanding of common pupil conceptions and errors. Knowledge of content and students is both useful and necessary when managing for learning and posing adequate mathematical challenges (Jaworski, 1994), as well as being able to transform (Rowland et al., 2005) one's knowledge into what is "pedagogically powerful" (Shulman, 1987, p. 15).

Teachers' knowledge of *content and teaching* is about teachers' knowledge of teaching in relation to the content, and requires interaction between specific mathematical understandings related to pupils' learning. It thus includes competence to design instruction, sequencing the content, evaluating advantages and disadvantages of different representations, and identifying affordances of different instructional methods and procedures.

Content knowledge, which the Michigan group (Ball et al., 2008) terms *subject matter knowledge*, is dissected into three parts; *common content knowledge*, *specialized content knowledge* and *horizon content knowledge*. *Horizon content knowledge* is awareness about how mathematical topics are related over the span of mathematics included in the curriculum; it thus appears consistent with Shulman's (1986) category *vertical knowledge* (see previous paragraph). For mathematics teachers in lower secondary school this means that they need to know the mathematics pupils worked on prior to entering lower secondary school, and what they will meet when going to upper secondary school. They can then relate their teaching to, and prepare the pupils for, what will come. For consistency in the teaching, I include knowledge about teaching approaches in single topics of the mathematics across different grades as part of teachers' horizon content knowledge, referring to it as *extended horizon content knowledge*. I acknowledge that this extended understanding is not pure subject content knowledge, thus indicating that the horizon knowledge should be "situated" partly within PCK.

The distinction into *common* and *specialized* content knowledge distinguishes the knowledge of persons who teach mathematics from those who use the subject in other occupations where mathematics is required as a tool; however, teachers are required to have both "kinds" of knowledge. I particularly appreciate this differentiation of subject matter knowledge because an assumption in the study is that teachers' local knowledge in mathematics teaching in some instances can be characterized as specialized content knowledge.

*Specialized* content knowledge is knowledge and skills unique to teaching. It includes competence to make features of particular content visible to and learnable by pupils, i.e. to make, choose, and use mathematical representations effectively. In accordance to Boaler and Humphrey (2005), I elaborate mathematical representations to include arithmetic, algebraic, and geometric forms. For clarity, I also add a semantic form (cf. Figure 10-1 for an example of the different representations). It also includes knowledge about why a certain method can work, to explain and justify one's mathematical idea, and to recognize unfamiliar patterns in pupils' errors. Ability to find and remedy unexpected errors is a competence only necessary for those who are responsible for others' learning and understanding. Such knowledge is necessary if a situation of contingency (Rowland et al., 2005) occurs in the classroom, a situation which also may require cognitive and affective sensitivity to pupils (Jaworski, 1994).

*Common* content knowledge is mathematical knowledge used also in other settings than teaching. It is knowledge about how to solve certain tasks, using rules, using mathematical notations correctly, and recognizing wrong answers. It also includes knowledge about identifying inaccurate definitions in textbooks. Ball and her colleagues (2008) suggest, for example, knowing about the infinity of numbers lying between 1.1 and 1.11 as knowledge common to people using mathematics.

#### **3.2.3.4 Synthesis**

I have discussed three frameworks that derived from researching the practice of mathematics teaching. The *Teaching Triad* (TT) emerged from an ethnographic study of investigative mathematics teaching (Jaworski, 1994), the *Knowledge Quartet* (KQ) from investigating novice and trainee teachers in practice (Rowland et al., 2005), and *Mathematical Knowledge for Teaching* (MKT) from studying the teaching of a third grade public school classroom for an entire year (Ball et al., 2008). The two latter build on some of Shulman's (1987, p. 8) seven "categories of knowledge that underlie the teacher understanding needed to promote comprehension among students". Whereas the teaching triad and the knowledge quartet both consider mathematical activities *within* the classroom, the MKT is about knowledge *for* teaching mathematics. Moreover, while I use the KQ to identify classroom activities such as transformation of knowledge into forms pupils can understand, and how teachers connect discrete parts of the mathematics, the TT guides me for the identification of the facilitation for learning, and how mathematical challenges and sensitive aspects are

attended to. The frameworks then complement (and somewhat overlap) each other. I thus found it both useful and necessary to combine these frameworks for identifying the complexity of the knowledge my informant argued for deploying while teaching.

My interest and focus is on teachers' knowledge *for* teaching mathematics as well as knowledge *in* teaching mathematics, a fact that is reflected in my conceptual framework. The following section, which explains my framework, thus ends with a visualiation of how the three frameworks presented in this section overlap different parts in my framework.

### **3.3 My conceptual framework**

Coming into the field of educational research having more than thirty years of teaching experience from the level I research into is of both benefit and disadvantage. The benefit is about knowing the school system, knowing the pupils at that age, and having a thorough understanding of teachers' frustrations and joys. At the same time, I experienced it as a disadvantage because at the first sight one sees all this as "every-day stuff" not worth noting. I realized that to get a distance from my own experiences, it was important to have a framework for the research. For me, that would include the work of scholars within educational research, which I have considered in this and the previous chapter. My framework has been subject to changes over the years, changes mostly due to discussions with others. I return to that process in the methodology chapter (Chapter 4.8).

Informed by Ruthven's (2002) dialogic cycle, my framework describes a cycle which includes two categories of knowledge that informs teaching, a trisected category of local knowledge developed in the process of teaching, and a category that includes knowledge worth sharing with others. My framework thus forms four broad concepts; *scholarly knowledge*, *brought knowledge*, *local knowledge* and *shareable knowledge* as explained below. Two reasons made me decide to use the name *local knowledge* instead of *craft knowledge*; it describes that the knowledge is locally founded, and will hopefully not be subject to misunderstanding or discussion, which I experienced the first time hearing about craft knowledge in mathematics teaching. Discussions about whether the word "craft" was a suitable description of something developed in the mathematics classroom (cf. Chapter 1.3) occurred, hence, when I decided to look for such knowledge, I changed the name to *local knowledge*. The framework and its interacting character can be visualized as follows:

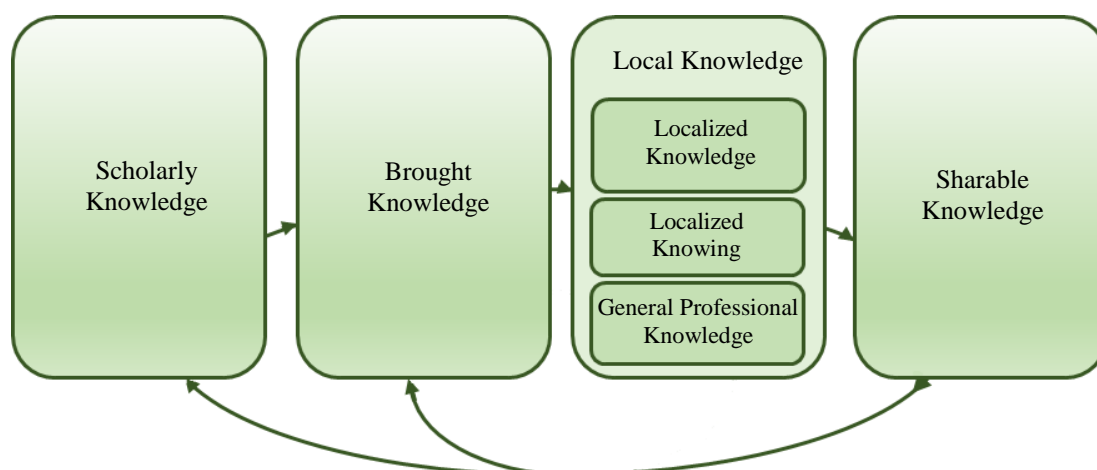


Figure 3-1. Visualization of my conceptual framework – dialectic between sharable local knowledge and scholarly knowledge.

*Scholarly knowledge (SK)* is knowledge that is created when researchers, who study the practice of teaching, recognize local (craft) knowledge worth elicit and systematize into a knowledge base, as defined in Chapter 2.1.2.

*Brought knowledge (BK)* describes the knowledge teachers bring to the classroom from formal education, textbooks or coursework, knowledge that should be informed by scholarly knowledge. It is knowledge brought from their own schooling as learners, from discussions with colleagues, or informal experiences through life itself. It is the *brought* repertoire, from which the teachers can draw for their planning and teaching, and does not necessarily include what teachers develop through their teaching. However, if it is agreed upon by colleagues, it is made sharable and can be a part of their brought repertoire (cf. Figure 3-1).

It is knowledge of the mathematics that is to be taught, knowledge of didactics related to that mathematics, and knowledge about the general characteristics of the variety of pupils who are to learn, all brought to the classroom from education etc. Included in this category is also knowledge of the teacher's role in the classroom, i.e. to be and behave as a teacher, learned from experience as learners, discussions and observation of teaching while in education; “*Look at (mathematics teaching) practice; Look at my practice*” (Adler & Davis, 2011, p. 150)<sup>25</sup>, and from observing colleagues. Rowland and his colleagues’ (2005) foundation is about the theoretical background teachers have

<sup>25</sup> Adler and Davis (2011) propose three models for teacher students learning to teach mathematics: Look at (mathematics teaching) practice (video), Look at my practice (looking at the lecturer) and Looking at your own practice (students reflecting on own practice).

gained through preparation for work in the mathematics classroom, thus included in this category. Common and specialized knowledge of the mathematics (Ball et al., 2008) required by the curriculum occupies a significant part of teacher's brought knowledge.

*Local knowledge* is dissected into three subcategories, *localized knowing*, *localized knowledge* and *general professional knowledge*. As explained in Chapter 2.1, the shift between “knowledge” and “knowing” is not accidental. To remind the reader: “Knowledge” assumes an ontological dimension of something a person either possesses or does not. The amount of “knowledge” that one possesses might be “measured” in some form of test. However, teachers' actions in the classroom may also be directed by a dynamic, unconscious and often implicit, tacit “knowing” that only emerges in the situation.

*Teachers' localized knowing* in mathematics teaching reflects teachers' particular perspectives and the distinct knowledge they have about the individual pupil related to the subject. It is about how they understand the same object for teaching differently depending on their knowledge of the individual. Knowing the individual pupil depends on the teacher's ability to identify the pupil's level of cognitive development, and their thinking and understanding (Marks, 1990; Carpenter et al., 1996). It is thus a dynamic application of knowledge to unique situations as well as demonstrated spontaneously as situations arise in the classroom (contingency, Rowland et al., 2005), and cannot necessarily<sup>26</sup> be transferred from one individual to another. The category connects the knowledge of the pupil and the subject (Ball et al., 2008), thus combining knowledge of pedagogy and knowledge of mathematics; the knowledge of the pupil directs the approach to the mathematics that is to be taught. Ball and colleagues' (2008) *knowledge of content and student*, as well as Jaworski's (1994) categories *sensitivity to students*, *management of learning*, and *mathematical challenge* occupy a considerable part of this category. When I asked Tea why she taught different pupils differently, she explained how her knowledge of the pupils directed her approaches:

Uum, and in a way it is like, right, [2] I do know them, I have had them for one year, so in a way it was an attempt to, [3] to reach them where I thought they would understand (conversation 2).

The school Principal was present in our third conversation, also she commented on how Tea managed her localized knowing, and on the importance of learning to know the individual pupil:

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<sup>26</sup> When considering the outcome from the event, the teacher can see that the approach also might suit another student, within the current class/group or in another class/group.



It is amazing to see how fast one has to think creatively to find solutions to each pupil, and that is what it is all about. And, if one does not have relations and if one does not know the pupils well and bothers about it then it is difficult to find methods (conversation 3).

*Teachers' localized knowledge* in mathematics teaching reflects the knowledge teachers have developed in the process of their teaching or in the planning for teaching. It can result from two different situations; for example when reflecting on (unsuccessful) prior lessons, or when planning a lesson for a distinct group or class (Anderson, 2011) related to the subject. It is thus about how they understand the same object for teaching differently due to prior experience, or depending on their knowledge of group characteristics and environmental context for learning (Cochran et al., 1993). It is socially constructed; the complexity of classes as social groups, the large variety of individuals and their combination in groups and locations require teachers to continually learn and adapt to new situations. It is thus dynamic, and cannot necessarily<sup>27</sup> be transferred from group to group, or from situation to situation. However, it can (a) be applied in preparation for teaching particular classes or groups due to the knowledge teachers have of the class/group. The category connects the knowledge of the group or class and the subject, and as such, it combines knowledge of pedagogy and knowledge of mathematics. Ball and colleagues' (2008) knowledge of *content and student* and knowledge of *content and teaching*, as well as Jaworski's (1994) categories *sensitivity to students*, *management of learning*, and *mathematical challenge* occupy a considerable part of this category. It also (b) connects experience and knowledge of the content; resulting from prior experiences, the teacher develops new approaches. This knowledge creation is included in Ruthven's (2002) *dialogic cycle*, and reflects the teacher's capacity to transform her knowledge to facilitate pupils' learning (Rowland et al., 2005; Shulman, 1987). I asked Tea why she decided to change approach when teaching functions (conversation 16, Table 10-55):

Tea Eee, that [2] I did because I felt it went so badly last time, [1] I think [3] I think they became so, I do not think [4] that they, that it became [interrupts herself], I then had a very, like formal and defined, that is, I do not think I made it. I do not think they understood it well enough. And then you actually have to, and then you actually have to, it is like you have to try, to look if, well, is it something wrong about me?

Inn Is this something you carry out to see if it works?

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<sup>27</sup> When considering the outcome from the event, the teacher can see that the approach also might suit another classes, within the current class/group or in another class/group.

Tea Yes, because I, I do not understand [1] or [2] I, I was so surprised last time [3] about why they did not learn [2] so I have like been thinking hm, then I have to try something else [the textbook<sup>28</sup> uses the function machine]

*Teachers' general professional knowledge* is about managerial decisions (Cooney, 1988) involving carrying out practical arrangements such as implementation of routines that may not be content specific, but specific to the class in focus, i.e social norms (Yackel et al., 1991). It can be about special arrangements for individual pupils in the class, or demands from pupils' parents. Knowledge of these arrangements is acquired from communication with parents, and application of imaginative common sense to pupils' practical needs in the classroom. This category includes managing pupils in available space resources, carrying out routines such as following-up pupils' absence, and communicating messages between home and school. There are more requirements and demands put on teachers that impacts upon the time available for mathematics teaching and learning. The mathematics curriculum is time consuming, thus having to deal with too much non-content work in the class puts an extra dimension to the daily workload that teachers experience. Through the analysis, another dimension of professional knowledge came forth, knowledge that necessarily did not have anything to do with requirements, however, according to Tea, important to pupils' learning: creating safe and predictable environment. I further discuss this in Chapters 4.8 and 7.7.

*Teachers' shareable knowledge:* Teachers often bring formal and procedural knowledge from their education (Borko et al., 1992; Even & Tirosh, 1995; Moreira & David, 2008). Shortcomings they experience when their *transformation* (Rowland et al., 2005; Shulman, 1987) of the content does not facilitate pupils' learning and understanding make them reflect about methods and approaches. Sometimes suggestions for solutions are to be found in the teachers' manuals, discussions with colleagues or other sources, which makes the knowledge *brought knowledge*. However, quite often, through reflecting on the social and interactive process of mathematics teaching (Steinbring, 1998), teachers themselves figure out approaches that facilitate pupils' understanding, thus developing *local knowledge*. Local knowledge includes localized knowing, localized knowledge, and general professional knowledge (see previous sections). The teachers will experience some of this local knowledge as worthwhile bringing forth to colleagues and new classes. If agreed on by colleagues, it might be accepted because its character is general, and will thus extend the teacher's base of *shareable* knowledge.

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<sup>28</sup> (Hagen et al., 2006)

Research into this knowledge can contribute to the development of scholarly knowledge. In classroom settings, didactical approaches that include common content knowledge can be developed by the teacher to be specialized content knowledge for teaching. Ball and colleagues' (2008) *knowledge of content and teaching* is included in this category, so are Jaworski's (1994) *management of learning* and *mathematical challenge*, and Rowland et al.'s (2005) *transformation*. Adler and Davis' (2011) "*Looking at your own practice*" play a distinct role in the development of shareable knowledge.

In this section, I have dealt with the conceptual framework for the study. The framework consists of four broad categories; *scholarly knowledge*, *brought knowledge*, *local knowledge* and *shareable knowledge*, all concerning mathematics teaching. The category *local knowledge* is subdivided into *localized knowing*, *localized knowledge*, and *general professional knowledge*. The focus of this research is the categories explained above. In the next section, I present the research questions, which ask for characteristics of *localized knowing* and *localized knowledge* in mathematics teaching. However, first I present Figure 3-2, which illustrates how the dimensions of the mathematical knowledge for teaching, **MKT** (Ball et al., 2008), the knowledge quartet, **KQ** (Rowland et al., 2005), and the teaching triad, **TT** (Jaworski, 1994) can be inserted into my conceptual framework.

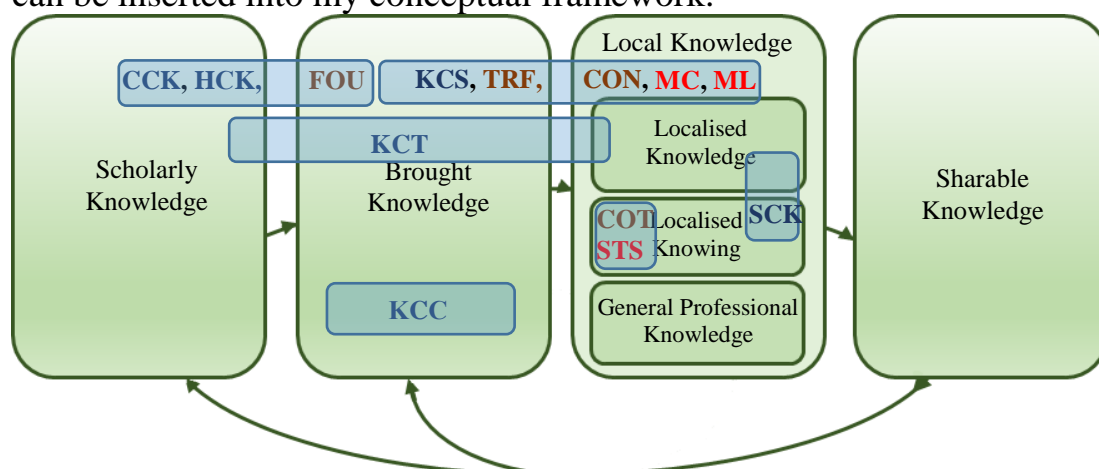


Figure 3-2. The domains of **TT**, **KQ**, and **MKT** inserted into my framework. The dimension sharable knowledge includes all domains. See p. 214 for explanation.

### 3.4 Research questions

Teachers develop local knowledge in their classroom practice as they reflect on current experience in the context of existing knowledge, which is an amalgam of scholarly knowledge, experiences as learners and accumulated local knowledge. The resulting competencies and skills are the substance of the teacher knowledge and knowing deployed by

teachers as they simultaneously manage the pupil, the class, and the subject to achieve the optimum outcome.

In Chapter 2, I referred to research arguing for the existence of craft knowledge, i.e. local knowledge, in mathematics teaching. Accepting that this knowledge exists and suggesting that my experienced informant hold such knowledge, I changed the wording of my tentative research questions as presented in Chapter 1.4. Thus, the research questions that has guided my research process are:

- What characterizes the local knowledge in mathematics teaching of an experienced Norwegian teacher?

Assuming that local knowledge consists of three distinct sources for facilitating pupils' learning of mathematics, whereof two are of particular interest, I posed two auxiliary questions:

- What are the characterizations of localized knowledge in mathematics teaching in the practice of an experienced Norwegian teacher?
- What are the characterizations of localized knowing in mathematics teaching in the practice of an experienced Norwegian teacher?

An important goal for me is to be able to inform teacher educators about what actually happens in an experienced mathematics teacher's classroom. In the search for the local knowledge, I thus observed the teaching of a teacher who, by the school administration, colleagues, pupils, and parents, was considered as successful, and subsequently had her comments on the teaching. In the next chapter, I discuss the methods and design I used to address my research questions.

## 4 Methodology and research design

In this chapter, I outline the methodological basis for my research, it is about how to get access to a teacher's knowledge for teaching mathematics (Section 4.1), my considerations for choosing a descriptive case study design while working in an interpretative paradigm (Section 4.2), and about the methods I used (Section 4.3). In Section 4.4, I consider ethical aspects for doing classroom research, while in Section 4.5, I describe the process of how I came to the design I followed for collecting my data. In Section 4.6, I consider the analysis of the data, i.e. how I analysed the classroom observations (4.6.1), and the process of developing the structure for analysing the conversations occurring subsequent to the observations (Section 4.6.2). In Section 4.7, I comment on how I reported the results from the analysis, and in Section 4.8, I explain the extension of my conceptual framework. While I, in Section 4.9, provide keys to the analysis, in Section 4.10, I consider the choice of episodes used to illustrate/illuminate my interpretations and assumptions.

### 4.1 Getting access to the knowledge/ knowing

There are several ways to go about to get access to the knowledge people possess. Hill, Rowan and Ball (2005) claim that some researchers typically focus on the number of courses taken or degrees attained to measure teachers' knowledge. However, many researchers argue that mathematical knowledge for teaching goes beyond that captured in courses taken, or basic mathematical skills (Borko et al., 1992; Even & Tirosh, 1995; Hill et al., 2005; Ma, 1999; Moreira & David, 2008). Attempts have been made to describe various components of the knowledge needed to teach mathematics. These are conceptualized by using different frameworks (Askew, 2008; Ball, Lubienski, & Mewborn, 2001). Moreover, several instruments for measuring teachers' mathematical knowledge for teaching have been developed, often using the format of multiple choice tasks (Hoover, Mosvold, Ball, & Lai, 2016). There appears, however, to be a lack of agreement on what different constructs entail within some instruments, for example differences in the operationalization of the categories content knowledge (CK) and pedagogical content knowledge (PCK) within mathematics (Kaarstein, 2014). The format of the measures have been subject to criticism; Petrou and Goulding (2011) point to the instruments' lack of acknowledging the role of teachers' beliefs, and Schoenfeldt (2007) criticizes the multiple choice-format arguing that opening up the items would be fruitful.

However, to get access to the mathematical knowledge *for* and *in* teaching and the mathematical knowledge developed *in* the process of teaching, which is my focus, one needs to take the classroom context of teachers' professional work into account (Rowland & Ruthven, 2011). Over a period of eight months, I thus visited one mathematics classroom to observe the practice of teaching, and subsequently having conversations with the practitioner.

## **4.2 Methodology; a qualitative research design**

The focus of this research is the knowledge and knowing mathematics teachers in lower secondary school deploy and have developed in their practice of bringing about learning. The arena for this practice is the classroom where the teacher activates her knowledge to facilitate pupils' learning and her subsequent reflections on the teaching. Peoples' behaviour are infused with intentions, we need to understand the interpretations they give for what they do, which cannot possibly be revealed by observation alone (Pring, 2007). I thus entered into that world making it visible by turning it into two kinds of representations, field notes and videos (Denzin & Lincoln, 2005) before using these representations in subsequent conversations with the teacher. My research is thus qualitative. In the following two sections, I discuss the paradigm as well as the tradition of qualitative research within which I situated the project.

### **4.2.1 Interpretative research paradigm**

This research set out to make public the knowledge an experienced mathematics teacher deployed while teaching the subject in lower secondary school, and attempted to understand the teacher's rationale for her actions and intentions<sup>29</sup>. It required my interpretations of what I observed in the lessons and learned from the open-ended interviews. I also needed to understand the interpretations the teacher provided for what she was doing, i.e. understand the world from the her perspective (Pring, 2007). The naturalistic and interpretive approach I took to the data, i.e. constructing the reality (ontology) from observations and conversations (Denzin & Lincoln, 2005), coincides with an interpretative paradigm, in which I positioned my study.

For a project involving direct cooperation, three forms of collaborations appear to be of significance: data-extraction agreements, clinical partnerships, and co-learning agreements (Wagner, 1997). In co-learning agreements, the researcher and the practitioner seek to learn something together as they are both engage in actions and reflections

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<sup>29</sup> I describe the methods for obtaining this in section 4.3, and will not be explained here.

about (each other's) practice, while in a clinical partnership the researcher and the practitioner work together to improve the practice. In data-extraction agreements the researcher is the agent of the inquiry and the reflections, and seeks to describe the work of the practitioner (Wagner, 1997). Since my research is about studying and reporting what knowledge a particular teacher deploys without any intention to influence the practice, I took the approach of a data-extraction agreement.

My research is based on a belief that there are multiple realities (relativist ontology) where my informant and I worked together to create common understandings through observations, dialogues and interpretations (subjectivist epistemology). As explained in Chapter 2.1, the *knowledge* I searched for is a deliberately treatable and retrievable object-like item that is "stored" in a person's mind, while I characterize *knowing* as being dynamic and activated in situations in response to events that arise. To have my first auxiliary research question addressed, the characterization of teachers' localized knowledge, I took the stance that a person's "mind" is located in the head, while teacher's localized knowledge is a social construct, localized in the individual-in-social-action (Cobb, 1994). This, I assert, is consistent with the constructivist-interpretivist paradigm.

In addition to being a researcher, at the time of the observations I also worked as a mathematics teacher in a lower secondary school. In the current case, that included parallel classrooms similar to those I taught myself. Choosing an interpretive perspective requires a neutral position on the part of the researcher, she cannot bring preconceived attitudes. "There are no objective observations, only observations socially situated in the world of-and between-the observer and the observed" (Denzin & Lincoln, 2005, p. 21). For me researching the world so close to, and to a certain extent connected to my own practice, turned out to be a challenge, a challenge it took some time to overcome. This I describe in 4.5.1.

The next stage of the research was to decide what research design to use. Since my focus is on a particular teacher's knowledge for teaching mathematics, I found it most appropriate to choose a single-case study.

#### **4.2.2 Case study design**

Several designs can be used in an interpretive inquiry. Denzin and Lincoln (2005) list for example ethnography, case study, grounded theory, action and applied research, and clinical research among others. Each of these strategies connects to specific literature and methods. What strategy is appropriate depends on the focus of the study and the

research questions, and comprises guidelines to employ while moving from paradigm to the empirical world (Denzin & Lincoln, 2005). My purpose was to study the teacher “down-to-earth” (Stake, 1978) and I aimed at understanding “the dynamics present within single settings” (Eisenhardt, 1989, p. 534), a process which coincides with the focus of a case study design.

The strength of the case study design is that it is able to deal with evidences such as documents, artefacts, observations and interviews, can take the forms of exploratory, explanatory, or descriptive studies, and can be undertaken as single-case or multiple-case studies (Yin, 2014). The purpose of exploratory case studies is to identify research questions or procedures to be used in a subsequent research, explanatory case studies aim to explain how and why certain conditions occurred, while the purpose of descriptive case studies is to describe a phenomenon. My purpose is to describe the local knowledge one experienced teacher has developed in the process of her teaching, thus taking on what Yin (2014) labels a descriptive single-case-study. Searching for a teacher’s local knowledge, the teacher is my *unit of analysis*. However, having two embedded subcases within the holistic case of local knowledge, my focus is on the characterizations of her localized knowledge and her localized knowing.

Statistical data are commonly used when generalizing from empirical studies. This is less relevant when doing case study research. In a case study, even a single-case study, one can, for example, by connecting the empirical findings to existing theories or theoretical propositions, make *analytic* generalizations (Yin, 2014). I have used three frameworks against which I have analysed my data (Ball et al., 2008; Jaworski, 1994; Rowland et al., 2005), and would thus be able to make analytic generalization. However, it is important to present the data with sufficient clarity, for example in separate texts or tables, so that readers are allowed to judge interpretations of the data. To substantiate my “claims”, I thus present my informant through the words from her Principal, some of her pupils, and herself in Chapter 5, her practice in Chapter 6, and her own comments on her knowledge for teaching in Chapter 7, all evidenced by direct statements or illustrations. Additional data can be found in the appendices.

### 4.2.3 Summary

I have situated my research within a constructive-interpretive research paradigm, explained that my ontological assumptions is that the localized knowledge I seek is to be found within my informant’s head while her knowing is localized in the individual-in-social-action (Cobb, 1994). As for the epistemology, localized knowledge assumes a deliberately treatable and object-like item, while knowing is dynamic and activated



as response to events occurring in the classroom. I have followed the design of a descriptive single-case study (Yin, 2012), however not straight forward as I experienced some constraints concerning my initial conceptual framework and the development of my research questions. Having decided on these issues, the next step was to decide which methods to follow. These I describe in the following section.

### **4.3 Methods and data collection**

Frequently used qualitative observation methods are writing field notes and the production of video material, methods which often are combined with interviews or questionnaires (Rautaskoski, 2012). The immediate goal of the study was to illuminate the categories of knowledge/knowing which informs experienced lower secondary mathematics teachers' practice, especially the knowledge they have developed as a result of their interaction with the pupils and the subject; their local knowledge. Experienced teachers will often apply their professional knowledge in routine situations as, it seems, automatic reactions that are beneath a level of conscious reflection (Brown & McIntyre, 1993). Therefore, to stand any chance of demonstrating my informant's local knowledge, I considered it insufficient only talking to her, it was necessary to *observe* her in her regular classroom situations. To address my research questions, I thus saw it as necessary to observe regular classroom teaching as well as having subsequent conversations with my informant, thus following an ethnographic research design (Rautaskoski, 2012).

Before starting this project, my only experience with fieldwork was one visit to a school when collecting data for a course in the master programme and using a questionnaire and conducting some structured interviews to get empirical data for my master dissertation. These interviews followed a certain pattern where I had questions for the teachers, and tasks I wanted solved. Even if the teachers had the opportunity to intervene in the interviews and come up with their own thoughts, I had no experience with stimulated recall-interviews. Thus, when I attended the PhD-programme, I felt the need to do some initial fieldwork to get some experience of both classroom observation and subsequent conversations before starting the work for my main study. The work presented here thus comprises a pilot study, involving two mathematics teachers, and a main study involving one mathematics teacher. The pilot study I only consider as part of my development as a researcher, and will not be subject to considerations in respect to my research questions. However, in Section 4.5.1, I discuss what I learned from the study, including a few examples of teachers' local knowledge.

#### **4.3.1 Classroom observation**

As discussed in the previous section, Yin (2012) suggests that direct observations and interviews in the form of open-ended conversations with key participants, are common sources for case studies. I did not believe that I, as a researcher, was able to address my research questions from observations alone. I assumed that it also required access to teachers' subsequent reflection on, and rationalisation for their actions. I found it necessary to use methods that recorded teachers' actions in such a way that the knowledge informing these actions could be brought forth in the later conversation. I thus chose to use a digital video camera that recorded onto a memory card from which I downloaded to my computer once the observed lesson ended. The teacher and I could then together view episodes from the lessons, selected either by her or me, immediately after the lesson, i.e. while the lesson was still fresh in the teacher's mind.

Writing field notes from classroom observations in parallel to video recordings also provided valuable contributions to the amount of data. However, using a handheld camera made the making of notes a challenge, as will be further considered at a later stage. In the next section, I discuss some thoughts about the technique of stimulated recall.

#### **4.3.2 Stimulated recall**

Stimulated recall technique, i.e. to stimulate an informant's memory by showing a video-sequence, attempts to identify the cognitive processes involved in teaching and learning, and can be used to access the teacher's thinking while she was in class (O'Brien, 1993; Lyle, 2003). I thus video-recorded all lessons and conversations. However, there have been some critics to the method. Yinger (as cited in Lyle, 2003, p. 864), for example, suggests that stimulated recall may not provide "immediate retrospective probing" for accessing short-term memory or episodes stored in long-term memory. Looking at the videos may produce a "new view" that was not available to the individual at the time of the actual lesson. To increase the validity of the procedure, it is thus important to minimize the time delay between the event and the recall (Lyle, 2003).

#### **4.3.3 Open-ended interviews**

My plan for the subsequent conversations (open-ended interviews), was to look through the video together with the teachers, and that either of us could stop the film when we observed something on which we wanted to focus, the teacher by explaining what she saw, or I asking for clarification.

For my pilot study, Ann and Bea at Arnvik lower secondary school agreed to let me observe their teaching as well as talking with me about the lessons subsequent to their teaching. Unfortunately, we only once got the opportunity to have the conversation the very same day as the

observation. I will return to that issue in Section 4.5.1. Experiences from the pilot study led me to modify my approach to the stimulated conversations, which I discuss in Section 4.5.2.

## **4.4 Ethical issues**

Research entails ethical considerations. As a researcher, one experiences situations that impact on the ethical considerations. It is thus important, in advance, to have thought through and considered what to do if such situations arise. I will, in this section, deal with some of the ethical aspects researchers might meet when conducting classroom research.

In Norway, there are formal guidelines for research within which one operates. However, within the current area of research there also exists informal guidelines to follow, guidelines that also concern the researcher's own ethical view. This section continues with providing a brief overview on these issues.

### **4.4.1 National guidelines**

All research in Norway is regulated by national research policy and guidelines established by Norwegian Ministry of Education. The Research Council of Norway (RCN) acts as Norway's official body for development and implementation of national research strategy, it allocates funding, is strategic adviser for the government, and is concerned with ethical issues:

The effectiveness and credibility of research cannot be maintained without carefully weighed consideration and active implementation of ethical standards. Ethics in research encompasses two normative systems: one to ensure good scientific practice (i.e. researcher ethics) and one to safeguard individuals and society at large (i.e. research ethics) (Research Council of Norway, 2007).

The Ministry has appointed a national committee to investigate cases of research misconduct (researcher ethics), and national committees for research ethics working to enhance researchers' ethical awareness within the area of medical research (NEM), science and technology (NENT), and in social sciences and the humanities (NESH). Educational research comes under the guidelines of the latter, NESH (Personal contact with its Director, 2012.06.05).

These committees are independent, and appointed by the Ministry to acts as "a national watch-post, inform and advise upon research ethics within the relevant fields of research" (NESH, 2011). Detailed guidelines are developed, and include topics such as "informed consent" (Fossheim, 2009a), "responsibility for the individual" (Alver, 2016), "confidentiality" (Fossheim, 2009b) and "privacy" (Langtvedt, 2009).

In the next section, I discuss how I dealt with these issues.

#### 4.4.2 Dealing with individuals

A research project in Norway that includes information about individuals is likely to be subject to notification. Researchers are obliged to submit a notification form to Norwegian Social Science Data Services (NSD, 2012), the data protection authority for research which also was appointed by RCN. Before I contacted my prospective informants, I submitted the form and discussed with them the letter of consent to film in the classroom that was to be submitted to parents.

From the very moment a researcher gets in contact with informants, ethical issues might arise, be it related to the formal situations as “taken care of” by guidelines presented above or of a more personal character concerning the pupils and the teachers, or the researcher attitudes. As a researcher within a classroom, one might for example come in possession of sensitive information about a pupil or the teacher, or one could meet situations of some kind of abuse.

To get the permission to conduct the studies, I followed the formal road by contacting school principals asking for permission to conduct such a study in their school, which I got. For the first school I then contacted two teachers asking for their permission to observe their teaching, explaining that I *only* wanted to observe their teaching to learn about the knowledge they deployed. The teachers assisted me in getting permission from the parents to film their children by handing out a consent letter for the parents to sign. The letter included information about the study and about pupils’ right to withdraw from the study without any explanation. All parents approved, and that made it easier for me; I did not have to watch out for avoiding certain pupils. For the second school, I also first approached the Principal to ask for her permission, and she was positive. I did not know any of the teachers at that school personally; only by occasional professional contact (Section 4.5.3). She suggested a colleague, Tea, whom she assumed suited my criteria: she is experienced, regarded as an excellent teacher, and having studied mathematics. Contact with the pupils and their parents were conducted the same way as with the first school. Tea also informed the parents in a meeting she had with them before I started my observations. Tea and her colleague offered to take the responsibility to gather parents’ permissions, which I gratefully accepted.

Actually, it takes more than following rules to conduct an ethically sound study. When observing classrooms, one might meet with people of different political, ethnical and religious views, and different orientations and backgrounds than oneself. It is important for a researcher to take a neutral stance, not letting such differences impact on the work that has to be done. A second point of concern is what to do if I, during the observations, happen to experience some kind of abuse towards a pupil,

either from the teacher or from a peer. Fortunately, nothing happened that made me have to deal with issues of that kind.

#### 4.4.3 Publication of information

Two issues that have to be considered when doing research are *what* to do with results emerging from the research and *how* to publish these results. Participants offer time and information, and this requires the researcher to show respect for the individual, the data, and the results which emerges from the study. And the research has to be presented in a way which convinces an informed reader (Jaworski, 1997). Both are ethical considerations, and concerns slightly different aspects than those given by the national guidelines.

I have used pseudonyms to protect my informants' identity, as highlighted in the national guidelines. However, there is a tension between the need to report in detail and protection of the involved. When offering details sufficient to enable readers to judge about the validity of the findings, one may find that only changing names is not enough to protect informants' identity (Pirie, 1997). For example, specific details might be enough to identify schools involved in the project. I have, in that sense, tried to avoid information that could make any identification, be it the schools or the persons with whom I have interacted. I also want to make the reader aware that all episodes presented in this thesis are parts in a larger context of a complex classroom situation. I hope the reader will have that in mind while reading the analysis.

The amount of data and the "lengthy nature [...] of space taken to present an account" (Jaworski, 1997, p. 118) in qualitative research is also a matter of ethical concern. Hours of observations and conversations are condensed to some pages of writings representing a few episodes which report details enough to support the arguments sought, here teachers' mathematical knowledge for teaching (Jaworski, 1997) and enable the readers to judge about its validity (Pirie, 1997). The effort made by the participant is thus reduced to some minutes of reading, a situation that might not in full give credit to their effort.

It is in my interest to protect my informants and at the same time do justice to them for the time they willingly spent with me sharing their mathematical knowledge for teaching.

In the next sections, I provide details about my studies, both the pilot study (Section 4.5.1) and my main study (Section 4.5.3), including information about the schools and my informants. All names are pseudonyms, naming my pilot study informants by the first two letters in the alphabet, Ann and Bea, and the informant for my main study I

called Tea, a shortening for teacher. In Section 4.5.2, I consider changes made due to reflecting on the experiences with the pilot study.

## 4.5 Collecting data

The data, which consisted of some classroom notes, videos from classroom observations and subsequent conversation with the practicing teachers, were collected in two phases, a pilot study and a main study. As explained above, I was a novice in the art of observing teaching and conducting conversations for research. Being also an experienced mathematics teacher, it was a challenge to keep in mind that I was a researcher and not a mathematics teacher. This will be evident in the forthcoming discussion.

### 4.5.1 The pilot study – observations and conversations

The pilot study was conducted over a period of six months at Arnvik lower secondary school in the academic year 2009/2010. The school is situated in a rural area, and was chosen out of convenience; it was near my regular place of work, and I had some knowledge about it. At the time of the observations, the school had four parallel classes at each level. Two teachers, Ann and Bea, agreed to let me observe their teaching and to have stimulated-recall conversations subsequent to the lessons.

The study includes four observations and five subsequent conversations. The four observations consisted of three observations in one regular grade eight classroom (age 13 – 14) and one observation in a group of four low achieving grade ten pupils (age 15 – 16). The plan for the pilot study was to observe several lessons; we had not, prior to the observations, agreed on how many and when. Both Ann and Bea were positive towards the observations, however, lack of time and opportunities resulted in the observations as listed in Table 4-1:

Who	Class	Date	Duration	Observation
Ann	Grade 8	2009.09.17	43:28	1; Arithmetic
		2009.11.23	45:21	2; Geometry
		2010.03.16	01:23:12	3; Simple algebra, measurements [DR only due to damaged hard drive]

Bea	Grade 10	2009.11.13	01:08:04	1; Directed number, video damaged
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Table 4-1. Overview over observations in the pilot study.

The informants for this phase were two teachers that I knew slightly. I was allowed to observe their teaching as well as having subsequent conversations where the plan was to look through the videos together. My first observation was with Ann going through a test the class had the previous week. I took my place in the classroom several minutes before

the pupils entered the room, and turned on the camera when the school bell rang a couple of minutes before the pupils entered the room. I did not want to disturb the pupils by the sound of the camera being switched on. The teacher started with a short introduction of me before she asked the pupils to focus on what was going on at the board. Except from one boy who ran back and forth at the board a couple of times, the pupils seemed to follow the teacher's instructions. However, I noticed one comment from one pupil at the end of the lesson; "Now we learned more", and the teacher afterwards telling that one of the most competent pupils was more quiet than usual.

The intention was to talk to the teachers immediately after the observations, which would, I believe, give the best outcome. However, due to teachers' busy schedules, this was only possible after the very first conversation with Ann. I came to this first conversation with an open mind having some ideas about what to talk about, but I soon forgot about those ideas. Afterwards, I realized that I did not exactly focus on the mathematics; my attention was very much on how she managed the pupils. I experienced my first failure. Ann was very understandable, and agreed to have a second look at that particular video at a later stage.

Two periods of observation followed: the first three weeks later, firstly with Bea teaching a group of low ability grade ten pupils followed by a second observation of Ann and her regular grade eight class three days later. Neither of them had the possibility to talk with me immediately after class. I had to wait for about one month before we could have our conversations.

The fourth, and last, observation for my pilot study took place three months later. I then had the opportunity to observe Ann's class when some of the pupils worked with measurement and some reviewed simple algebra. She had time to talk with me three days later, but unfortunately, the videos of both observation and conversation were lost due to a damaged hard drive.

The schedule for the conversations in the pilot study is outlined in Table 4-2:

Who	Class	Date	Duration	Conversation
Ann	Grade 8	2009.09.17	44:48	Conversation 1
		2009.12.17	33:48	Conversation 1B
			31:30	Conversation 2
		2010.03.19		Conversation 3 [damaged hard drive]

Bea	Grade 10	2009.12.17	01:10:52	Conversation 1
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Table 4-2. Overview over the conversations in the pilot study.

#### ***4.5.1.1 What I learned from the pilot-study***

I went out to undertake the pilot-studies to learn about conducting fieldwork. Having the opportunity to follow two teachers over a period of six months, however not many lessons, I got some insight into the practice of the two teachers, Ann and Bea. I experienced the existence of local knowledge (Ann), and I observed a teacher who denied herself to learn from situations that arose in the classroom (Bea). What happened in Bea's classroom can be read in Nergaard (2012). In the next section, I present one episode from Ann's classroom.

In Ann's grade nine classroom, I observed a lesson where she reviewed a test, one lesson where the focus was on the construction of angles, and one lesson where the pupils could choose between reviewing simple algebra and measurements. While observing and talking to Ann, I identified some instances I interpret as evidence of local knowledge, of which I present two. The first episode happened when she asked Arne for his solution to which is the larger of 4.3 and 3.8. Arne, who was sitting at his desk throwing a key up in the air, provided the correct solution immediately, and he put away the key. He did not resume the activity for the rest of the lesson. For me, as an outsider, asking him seemed to be a conscious action for having him stop his unwanted activity. However, in our subsequent conversation Ann revealed that it was about posing him a question she knew he would manage. Moreover, after experiencing poor learning of the construction of angles the previous year when she had "taught very thoroughly how to construct the different angles", she this time really "engaged in that they should understand that constructing a ninety-degree angle actually is to bisect a straight angle" (conversation Ann 2).

Throughout a day at work, teachers might teach several subjects, meet with many pupils and thus be exposed to many experiences and impressions that have to be processed. This happens day after day, week after week, and while new experiences occur, old impressions might fade. Thus, and as suggested by Lyle (2003), it is important that the conversation occur as soon as possible after the lesson, ideally on the very same day as the lesson took place. As explained above (4.5.1), I was only once provided with such an opportunity. In addition, looking through the entire video together was time consuming, and did not seem necessary. This made me rethink how I should structure the fieldwork to ensure the best possible opportunity to collect the data I needed to address my research questions. I present the results of my thinking in the next section.

#### **4.5.2 Changes due to reflections on the pilot study**

Digging into the area of educational research also confronted me with some technical challenges. Except from doing a small scale study in my



master programme on mathematics education and interviews with teachers who teach mathematics in lower secondary schools for my master thesis (reported in Chapter 1.1), I have never been close to the role of researching teaching. I was, however, quite clear about what I was looking for; the knowledge teachers deployed while teaching the subject.

The fieldwork conducted for the pilot study, as reported in 4.5.1, made me rethink my approach. Firstly, I wanted to find an informant that would participate due to an interest for informing the research area of educational research. Given that my research is extractive (Wagner, 1997), not purposely offering anything in return, I did not expect there would be many such teachers around. I also needed a teacher who was willing to participate in a subsequent conversation within a few hours after the lesson had ended and would provide own opinions, not what could be regarded as “political correct”. A third aspect that evolved as a challenge during the pilot study was the time aspect. It is very time-consuming to look through the whole video together and discuss as events/situations show up. Thus, I wanted to have some time between the observation and the conversation to look through the video and mark episodes of interest for discussion.

### **4.5.3 Main study, preparation and observation**

The data for my main study was collected at Tunholt lower secondary school, which is situated in the outskirts of a small Norwegian town. I contacted the Principal at the school because I experienced both the management and the teachers as very positive to me as a researcher while I was collecting data for my master thesis. At the time of my observations, the school had a new principal, but she was just as positive as the previous one, and welcomed me to do my observations. Moreover, the teacher suggested by the Principal, Tea, expressed an interest and willingness when meeting her for the very first time. This made me realise that I, with her support, could be able to have the opportunity I needed for following my intentions. The school had at the time of observation five parallel classes on each level.

#### ***4.5.3.1 The class, teachers, textbook, and preparations.***

Data collection took place over a period of eight months in the academic year of 2010/2011, beginning in August 2010 and ending in April 2011. However, to prepare the pupils for what was coming, I visited the class to present myself and my project before the pupils went for summer holiday in spring 2010. The class was a regular Norwegian grade nine class consisting of 27 pupils at the age of 14 – 15 years. It includes three first and three second-generation Norwegians, and one bilingual pupil whose mother originates from an Asian country. Ten pupils follow an

extended individual plan for their learning, whereof five were provided extra resources due to their need for extra assistance. These were not present in the mathematics lessons. One of the twenty-two remaining pupils is strongly medicated, and two pupils suffer dyslexia but were not provided any extra support.

Together with Tom, Tea was the class-teacher for these pupils. In addition to teaching mathematics, she taught natural sciences, food and health, and Religion, Philosophy of life and Ethics. She thus had relatively good knowledge of her pupils and the subjects they were expected to learn.

The class used the textbook Tetra 9 (Hagen, Carlsson, Hake, & Öberg, 2006), treating six topics, number and algebra, equations, geometry, percent, probability and functions, whereof I observed lessons focusing on number and algebra, equations, percent, and functions. Each chapter consisted of four parts: a basic course, a test for deciding whether one was prepared for more challenge (following a red course) or needed more practice within the topic (following a blue course), and a summary. Tea did not follow the path suggested by the authors. This I further consider in Chapter 7.2.3.1.

Before I started my observation, I carried out the same procedure of getting parents' permission to film in the classroom as in Arnvik (cf. Section 4.4.2). I observed nineteen lessons during the period, whereof four lessons were observed "twice" as the class was divided into two equal groups and the teacher most often did exactly the same with both groups. This happened before Christmas. The class were provided a new schedule after Christmas, which is the reason why there were only four "twice-taught" lessons. Thus, my data consists of twenty-three lesson observations.

The subsequent conversations were planned to take place each day after Tea having finished her teaching.

#### **4.5.3.2 Observations**

While planning the observations for the main study, I considered adopting a pattern of regular periods of fieldwork: three periods of three weeks of observation having six weeks of analysis in between. However, I realised that the consequences could be that I might enter the classroom while they were in the middle of some work, thus missing the introduction. How teachers introduce a new topic was also of interest, so I agreed with Tea that I would follow her local schedule for when she planned to introduce a new topic. The plan was not completed when I started my observation, so for the first four weeks my visits followed the pace expected by the teacher based on her experience. In late September, I got the plan for the entire year, and I then decided what and when to observe during the school year. Some adaptations had to be made;

teaching of equations planned to be taught weeks 41 and 42 took place in weeks 43, 44, and 45, and functions was taught at an earlier stage than planned; weeks 14 and 15 instead of weeks 17 and 18. The adaptations did not cause any problems for me.

I visited Tunholt lower secondary school nineteen times; the lessons are thus identified by number according to the order of observations from 1 to 19, as noted above. Before Christmas, the class was divided into two equal groups once per week. These lessons occurred on the same day, and are thus named with the letters A and B in addition to the number of the visit. The topics for each of the 19 lessons, dates, time and duration are displayed in Table 4-3:

Obs	Date	Started	Duration	Topic
1	2010.08.30	10:56	41:58	Strategies for multiplying numbers of base ten
2A	2010.08.31	08:26	42:48	Strategies for dividing numbers of base 10
2B	2010.08.31	09:10	43:07	Strategies for dividing numbers of base 10
3	2010.09.06		40:20	Negative number
4A	2010.09.07	08:26	45:34	Negative number 10:32:25
4B	2010.09.07	09:15	38:22	Negative number
5	2010.09.20	10:56	42:05	Beginning algebra (11:03)
6A	2010.09.21	08:27	42:26	Algebra. Pupils evaluate TEA's test
6B	2010.09.21	09:11	43:48	Algebra. Pupils evaluate TEA's test
7	2010.10.12	10:56	45:45	Showed clips from video and discussed with pupils
8	2010.10.25	10:56	43:00	Beginning equations
9A	2010.11.02	08:26	44:36	Solving simple equations
9B	2010.11.02	09:11	46:37	Equations, look for mistakes in pupil's solution
10	2010.11.08	10:56	45:45	Individual work on equations
11	2010.11.09	09:14	34:21	Preparing for test, solving tasks (equations)
12	2011.01.03	10:55	43:47	Percent, the meaning and importance of (Gallup)
13	2011.01.04	09:14	40:29	Percent, the meaning and importance of (eclipse)
14	2011.01.10	10:55	42:56	Percent, three rules for calculation within percent
15	2011.01.11	09:15	38:24	Working on plan, doing some problem solving
16	2011.04.04		37:39	Introducing coordinate system, functions
17	2011.04.05	08:23	46:42	Functions
18	2011.04.11	10:56	40:35	Functions
19	2011.04.12	08:23	44:14	Drawing graphs

Table 4-3. Observation plan (open cells indicate loss of information due to damaged hard drive).

The topics were chosen due to interest for the development from calculating with numbers to algebra, equations and functions. Before Christmas, I followed the teaching of strategies in number calculation via beginning algebra to algebra and equations, and after Christmas following the work with percent at the beginning of the semester and functions, which was the last topic for the semester. During the observations, I tried to write some observation notes, at least while the teacher was at the board, but holding a camera made it somewhat problematic. It was impossible to write any notes when I followed Tea around in the classroom helping pupils. 16 hours and 15 minutes of observations are video recorded. The videos were transferred to a computer immediately after the lesson, and data reduction notes were written in between the observation and the subsequent conversation to the extent that I had time before the teacher showed up.

When I entered the classroom for the first observation, only one pupil directed a question to me regarding my work, it concerned who were to see the videos, a question the same pupil also asked when I visited the class in May. During the very first observation, I tried to keep a distance from the pupils to avoid stressing them. They seemed so relaxed that I already the next day began to follow Tea around in the classroom when the pupils worked individually or in groups. For the first three weeks (observations 1 – 4) I felt that the pupils unconditionally accepted my presence. They welcomed me when I entered the classroom, and most of them were eager to show me their work.

My intention was to observe the teacher and her teaching, but sometimes it was impossible to avoid that also the pupils were captured on the videos. However, I tried hard to avoid filming those who seemed shy, and totally avoided those who, for some reason, had told the teacher that they felt uncomfortable having the camera pointed towards them. This happened on one occasion during the first four weeks. Before my fifth observation, a father contacted Tea believing that my presence would influence his son's learning opportunity. Tea and I talked to the principal about the incident, but she encouraged me to continue as I had started. We also asked the class about their feelings; this resulted in that a few pupils said they felt uncomfortable having the camera in the classroom, while more did not care. I promised to let them have a look at one lesson so that they could see that the focus was on Tea. Immediately after we were done watching the video, we had a short discussion, but I was concerned the pupils would be reluctant to reveal their true feelings in my presence. We discussed how the camera was used, whether it would be better if it was situated in the back of the classroom. Tea suggested having the pupils discuss with her colleague before my next observation. The same result occurred; a few pupils had some objections, more said

they did not care, while the majority did not say anything. The fact that some pupils did not see my presence as a problem was revealed at the first observation following the discussion about my presence. A boy then commented to his peer towards the end of the lesson: “I did not know that Inger was here, [3], Theo, did you know she was here?” even if I, together with the teacher, said good morning to the class before the lesson began. For a while, I kept distance from the pupils who had expressed scepticism to my observation, but when experiencing that they sought Tea’s support at her desk even if she was filmed, I felt that the signal for acceptance was reinstated. Neither Tea, her colleague, nor the Principal believed that the protest came out of genuine concern for pupils’ learning. The rest of my observations occurred without any problem; they even welcomed me back when I returned after a one-month stay at a University abroad.

As explained earlier, in addition to taking video recordings, I also tried to write some field notes during the observations. Using a hand held camera, I met some challenges in writing the field-notes. I had my notebook on a table in front of the classroom. Following the teacher in the classroom, it was not easy noting what I should remember to ask for. However, as often as possible, I went to my notes, writing small comments and questions to be recalled when I immediately after the lesson looked through the videos preparing for our conversation (see next section), and also to be used during the conversations.

The first stage of analysing the data occurred immediately after the observed lessons. I was provided a room where I could sit and prepare for the conversation. In the next section, I explain how this first analysis was carried out.

#### ***4.5.3.3 A first stage of analysing the observations***

The time between the observations and our conversations lasted from twelve minutes to two hours and seventeen minutes, with seventy-three minutes as the mean time. To prepare for the conversation, I had a first looking through the video. During this time, I listened to explanations and conversations, and looked for shifts in the activities, adding some notes to my very brief observation notes. This gave me the opportunity to recall what I, due to using a handheld camera, could not write while being in classroom. These notes made the foundation for what I wanted Tea to clarify. To stimulate her memory, I chose episodes of interest, i.e. episodes that could help me identify her knowledge for teaching, for her to look at while clarifying and explaining events, activities, and actions that occurred in the classroom.

Having that information at hand, I was prepared to meet with Tea for the open-ended conversations (Yin, 2012). In the next section, I discuss details concerning these conversations that took place for searching for Tea's knowledge for and in teaching mathematics.

#### **4.5.3.4 Conversations**

Stimulated recall can be assisted in more ways; either a predesigned set of more or less structured questions, less structured interview situation or not structured at all (Lyle, 2003). As explained in the previous section, I brought questions for clarification of issues I wanted to know more about. Except for that, to avoid contamination of the research process, I tried to minimize my influence. As time passed, Tea took more and more responsibility for our conversations.

The conversations took place on regular basis throughout my observation. Only twice, I experienced that Tea had no time for a conversation, after observation eleven and nineteen she had to go to meetings. In our first four conversations, I started our talk by asking about something from my notes, but in the fifth conversation, Tea opened our conversation by telling about a father who had contacted the school because he was concerned that my presence in the class might disturb the teaching (see previous section). Tea explained that he was a kind of person who often complained, and that she believed that his comment really was not about my presence but more about the entire situation around his son. We agreed on that it was a case for the Principal.

After that particular conversation, I tried to make her start with asking her to tell about what was in top of her head. The intention for this was to ensure that her accounts were grounded in her own perception of the lesson (Cooper & McIntyre, 1996) before I came up with any questions which could have had impact on her memory. If I found it necessary, due to for example lack of understanding or a wish for elaboration, some of the aspects she highlighted were further explored. Sometimes these events were stimulated by using a video clip from the situations she mentioned. Because of the work I carried out in the period between the observation and the conversation, it was easy for me to find the clips that included situations about which Tea wanted to talk. Occasionally I experienced that Tea was very tired, or even appeared exhausted, when she came for our conversation. I then commented on what I noticed, which resulted in her talking about both personal and professional issues, a situation I interpreted as trust. Our conversations then followed a pattern of stimulated recall based on my notes (see above) and often ended by agreeing on my next visit.

Examples from the beginning of some conversations, the first one I initiated, and the second was initiated by Tea:

- Inn I have now visited you four times, actually six lessons. I see that you use certain time; of course, you have some practical stuff you need to go through, and then some time for instruction followed by a certain time on seatwork. Do you have any principle in that regard?
- Tea No, not really. It is somewhat dependent on the theme and what they know or do not know in advance. Eee, often it will be like, I talk in the beginning and they work towards the end, but if there are small topics then it happens that I talk a little, they work a little. [...] I try to see to that a lesson, it happens that a lesson is only [seat-] work, but it happens, I try make that a lesson is never only talking (conversation 4, Table 10-1).

The following excerpt is an example on how Tea initiated the talk. It comes from conversation seven, following the lesson where the pupils saw clips from earlier observations:

Interesting to see the class. That session reflects what one struggle with daily. It is a few who mean something [3] eee, and that of different motives. And then it is a bunch sitting there being tourists, and I have to say that I am very tired of it (conversation 7).

In the following table (Table 4-4) I present the dates, time of day, and duration of our conversations. To have a first overview, I titled each conversation by what I saw as the most elaborated or important aspect discussed:

Conv	Date	Started	Duration	Title
1	2010.08.30	12:12	38:37	Understanding most important
2	2010.08.31	12:10	36:24	Uses different strategies for different pupils
3	2010.09.06		51:17	Principal: Tea is contingent, reacts spontaneous.
4	2010.09.07	12:06	43:02	Structure of lessons
5	2010.09.20	13:09	44:25	Response from parent
6	2010.09.21	12:09	43:24	We agreed to show a video to the pupils
7	2010.10.12	12:00	35:10	Tea sad because pupils seemed focused on self
8	2010.10.25	13:05	42:24	Tea is resigned; many pupils reluctant to participate
9	2010.11.02	12:55	49:11	Uncertain on how to deal with algebraic fractions
10	2010.11.08	13:33	23:28	Education and reflection on pupils' work
11	2010.11.09			No conversation
12	2011.01.03	12:58	46:46	To say something is to convince oneself that one knows
13	2011.01.04	10:12	53:43	Real life context and lesson organization
14	2011.01.10		59:30	Lost file
15	2011.01.11	10:09	59:31	One learn better if topic are presented in familiar context
16	2011.04.04		38:11	Change of approach to functions, lost file information
17	2011.04.05	10:10	32:59	Hard to make pupils do what is not on the plan
18	2011.04.11	13:01	38:56	Change of approach, use of IT complicated
19	2011.04.12			No conversation

Table 4-4. Conversation plan (empty cells indicate no conversation or missing information due to lost files).

It was important to me that the teacher felt comfortable while I was in the classroom to observe her teaching. The way she opened up for discussing personal issues as discussed above lead me to believe she did. The conversations, I will say, occurred in a relaxed and informal atmosphere, and Tea was very cooperative. All questions asked were fully, and thoughtfully, answered; never did I experience short answers like a simple “yes” or “no”. During her often-supplementary comments, I felt like commenting with “umm” or “ok” just to indicate that I was following her reflections. To transcribe such conversations can be challenging, the list would be extensive if I included all my comments of attention, which I consider in Section 4.6, analysis of the data.

## 4.6 Methods of data analysis

In this section, I deal with how I analysed my data. As I was seeking for the teacher’s knowledge, I realized that it was her explanations and account for planning, actions and happenings that was the most important. To have a background for understanding her teaching, I thus first set out to present my methods for analysing the classroom



observations (4.6.1) before analysing our conversations (4.6.2), which is focus in the forthcoming two subsections.

I tried to approach my data with an open mind, following a grounded theory approach (Corbin & Strauss, 2008). I used notes from the fieldwork (classroom observations, first analysis (cf. Section. 4.5.3.3), and conversations), as I watched and re-watched the videos for further structuring and coding of my data. For the second analysis of the conversations, I started with using the MKT framework (Ball et al., 2008), and followed up using the TT framework (Jaworski, 1994), and the KQ framework (Rowland et al., 2005) as analytical tools.

#### **4.6.1 Analysing the observations**

When analysing the classroom observations, I started to organize the classroom interactions into four parts: openings, plenary segments, individual seatwork segments, and closings (cf. Figure 6-1). I used a spreadsheet software to get a schematic overview, noting the actions and the time spent in the different main parts of the lesson as mentioned above. I first made a rough division by theme in opening (cf. Figure 6-3), plenary (cf. Figure 6-4), and closing sections (cf. Figure 6-9), and by whom Tea visited in the individual seatwork segments (cf. Figure 6-6). It also included a schematic overview of the themes Tea brought to bear while teaching. In the next round, I classified and coded all activities, actions, conversations and interactions, and added to the scheme while further analysing the lessons.

##### **4.6.1.1 Openings**

The openings consisted of organizational matter at different levels, preparation for the lesson, pupils matter, and some issues of personal involvement. Seven sub-categories of organizational matter emerged, while there occurred three forms of preparation, and one category of pupil matter (cf. Figure 6-3). Organization, preparation, pupil matter, and other issues I organized as general professional knowledge, i.e. in accordance with my conceptual framework. The analysis showed that social life in the class appeared to be important to Tea; every day she spent some time on small talk, and being somewhat personal with her pupils. These categories I organized into a category *involvement*. Her engagement with the pupils was also evident in our conversations. This raised a question about whether knowledge of involvement with pupils should be included in the category general professional knowledge. Because of its apparent prominent importance for Tea's teaching, at this stage I decided to categorize involvement at the same level as general professional knowledge, an action that actualizes a discussion about whether I should extend my conceptual framework (see Section 4.8 for

my final decision). However, *involvement* makes the basis for Chapter 7.7 where I consider how social influences shape Tea’s knowledge for teaching mathematics. Table 4-5 shows the stages of coding. I first organized the codes (description of actions) into level 1 categories of like topics (cf. Figure 6-3). These I then organized into level 2 categories, which organizes the majority of the activities in the openings into the category *general professional knowledge*, according to my conceptual framework, however included the new categories *involvement* and *other* (Figure 6-3):

Level 2	Level 1	Codes
GPK	Organization	Administrative organization; naming books, use of pupils’ personal sites, five classes merged to four
		Organization of the day, changes due to organizational matter
		Organization of earnings for the planned school trip (economical)
		Management of pupils
		Organization related to parents
		Preparing for organization of social arrangements
		Organization of teaching
	Preparation	Pupils coming to rest
		Pupils getting their books and coming to rest
		Preparing for looking at some video clips
Pupil matter	Pupil matter; asking and looking for missing pupils	
INV	Personal relationship	Personal relationship; Showing respect, surrogate for parents, patience
	Small-talk	Small-talk; books they have borrowed, discussing French words written on the board, pupil observed in the schoolyard, a pupil having problems with her zipper, flowers freezing during the night, etc.
OTH		Other: Principal calling about a pupil, presentation of researcher, greeting other than pupils, talking about another subject, pupils ask about non-subject matter

Table 4-5. Categories of actions taken in the openings.

I present the results from the analysis of the openings in Chapter 6.2.

#### 4.6.1.2 Mathematics work; plenary and individual seatwork

Using the field-notes and data reduction notes, I made a first coding of the actions and activities occurring while working with the content. At an early stage, I experienced that I needed to review the classroom episodes. With the notes as a basis, I thus started two parallel processes; noting who were talking, what they were talking about, and for how long, while transcribing episodes of interest for supporting or confirming Tea’s claims and attitudes as expressed during our conversations. That work provided a second by second overview of all activities in the class: who and with what the different participants were contributing.

Since the focus of my study is to illuminate Tea's knowledge for teaching mathematics, when categorizing and coding the data, my focus was on her interaction with the content and the pupils. That work led to extracting and coding of twenty-two different actions/activities. These I organized into six level one categories of activities while in plenary (cf. Table 4-6), and five level one and three level two while doing individual seatwork (cf. Table 4-7).

Since the categories exposition and IRF are both about explanation of the mathematics, I organized them into the level 2 category Explanation. The two dimensions of preparation occurring at level 1, preparation for plenary work and preparation for seatwork, I organized into Preparations at level two. I placed the remaining two categories Subject and Pupil activity into level 2 (cf. Figure 6-4). Table 4-6 summarizes the result of the categorization:

Level 2	Level 1	Code	Examples
Explanation	Exposition	Explaining to the pupils	Then we see that multiplying with a half is the same as dividing with two
	IRF	Invites pupils to respond	It is three, x equals three because, why was that?
Subject		Engaging with mathematics	We are not such a small island that sits [2] calculating stupid tasks in books.
Pupil production		Pupil encouraged to produce	Draw a cheese [8, Thor draws], that is a cheese with holes, yes
Preparation	Prepare for plenary	Setting the goal	The next we will be working on is letter-calculation
	Prepare for seatwork	Prepare for individual seatwork	If that circle is the sun [5] try to draw the moon on your circle and look [2] how much of the sun would we then have seen? [2] Would we see much [1] or? [3] Approximately how much of the sun would we have seen?

Table 4-6. Activities occurring in plenary sections.

Concerning the activities occurring in individual seatwork segments, I organized the level one activities into three forms, activities related to mathematics, to issues of social interaction (involvement), and other actions, such as closing a window. At level 1, I exposed five forms of mathematical activity; Tea responded to lack of understanding, she confirmed solutions, clarified misunderstandings or concepts, intervened, or she just looked at what they were doing, cf. Table 4-7 and Figure 6-6:

Level 2	Level 1	Code	Example
Math	Lack of understanding	Understanding	But can you add a number having x as surname with a number without a surname?
	Confirming	Confirmation	Fifteen times point two yes, is that reasonable?
	Clarifying	Clarification	how many grams are zero point one kilo
	Intervening	Intervention	put the number with the highest number-value [absolute value] on the top
	Look and leave	Looking	
Involvement			How are you?
Other			Draw a cheese [8, Thor draws], that is a cheese with holes, yes

Table 4-7. Activities occurring in individual seatwork segments.

#### 4.6.1.3 Mathematics work; closings

The approach for analysing the closings occurred the same way as for the plenary and seatwork segments. The closings, if including more than just a “take a break” comment concerned either mathematics or non-mathematical stuff like reminding the pupils to go to another room. At level 2, I exposed three mathematics related issues: talking about further work, review, and goals:

Level 2	Level 1	Code	Example
Math	Further work	Further work	Make sure to get some math done for not blowing it this week too
	Review	Review	Turning the back one on the head [...] and multiply
	Goal	Goal	We did not come as far as we should in this lesson
Other			You can put away your books and go to the lab

Table 4-8. Activities occurring in the closings.

As can be seen in Figure 6-9, the category further work includes three dimensions, review includes three dimensions, and the category goal includes two dimensions of mathematical activities.

#### 4.6.2 Analysing the conversations

For analysing the conversations, I first started with following Corbin and Strauss’ (2008) method for writing the memos, initially by writing memos for six of our eighteen open-ended conversations. In the first attempt for analysing the conversations, I used my conceptual framework, a framework I had developed due to experience, however based on the work of the Michigan-group (i.e. Ball & Bass, 2003; Ball et

al., 2008). I thus only had a limited number of examples that could explain what Tea talked about, and I had to look for another framework. For my next attempt, I used Ball and Forzani's (2009, p. 501) list of work teachers must do, which gave me an insight into Tea's practice, her principles, and her knowledge/knowing for teaching mathematics. However, it did not provide sufficient evidence for me to be able to characterize the knowledge she deployed when teaching, particularly not her local knowledge, which was required for addressing my research questions. Thus, when starting all over again, I wrote memos from all the conversations, following the procedure illustrated in Figure 10-2, and exemplified in Figure 4-1, until I had memos covering everything about which we had been talking throughout our conversations. The example comes from conversation two where Tea responded to my question about what she regards as her job:

[8] It is [5], that's all [3] which is [1] between, that is in relation to pupils it is all that is between [1] they open their eyes in the morning to going to bed at night [2] and the parents till they settle at night (conversation 2).

The above excerpt is the first fifty seconds of Tea's response to my question. The talk on this particular topic lasted for three minutes and thirty-six seconds, and the following excerpt is the memo from that sequence:

Conversation 2, video 20100831121023, 01:12 – 04:50

1. M<sub>3</sub>

- a. Inger asks what Tea sees as her job, Answer: everything regarding school that goes on between the time pupils open their eyes in the morning and goes to bed in the evening. For parents until they go to bed [TEACHER: responsibility]. Parents shall be 100% sure about that their kids are in school when parents believe they are in school [TEACHER: trustworthy], and demands parents to tell if their kid are not coming [TEACHER: demanding]. This year she teaches the class in RPE, food and health, natural sciences and mathematics. She does not usually teach food and health; she follows her own class. Some of her pupils go to studio, a group for pupils with special needs.

TEACHER: responsibility / trustworthy / demanding

Talk is about that Tea sees all things connected to school for her pupils from early morning to late evening as her job, and that she and the parents always shall know where the pupils are when it is school.

2. M<sub>4</sub>

Categories of practice	Categories of principles	Categories of knowledge/knowing
Takes responsibility Follows own class (several subjects)	Parents have to know their children are in school	Knowledge of <b>pedagogy</b> (responsibility, trustworthy, demanding) Knowledge of <b>self</b> (trustworthy, demanding)

Figure 4-1. Example of a memo.

For each conversation, I summarized the categories from the column knowledge/knowing, had them transferred into excel because that would make it easy to collect like categories. Fifteen different categories evolved, for example didactics, pedagogy, content etc. as can be seen from the memo above (cf. Figure 4-1). Within these categories, I did a more thorough second categorization, also using practice, principle, however this time splitting the categories knowledge/knowing, as I search for both (cf. Table 4-9). To distinguish between knowing and knowledge, I used her comments on episodes where her knowing was activated and expressed (cf. Chapter 2.1.1 for explanation of the differences). The following number of occurrences then evolved:

Category	Practice	Principle	Knowledge	Knowing
Assessment	0	2	3	0
Content	22	16	39	2
Curriculum	8	3	11	0
Didactical devices	37	31	3	1
Didactics	40	43	50	3
Educational ends	7	0	0	0
Parents	0	0	3	0
Pedagogy <sup>30</sup>	53	57	11	11
Psychology	0	4	0	0
Pupils	8	37	81	5
School	1	2	2	0
Self	9	13	5	0
Social (norms +)	3	5	4	0
Teacher	1	1	2	0
Teaching	0	0	1	0
	189	214	215	22

Table 4-9. Number of occurrences of each category, second analysis.

Within each of these categories I looked for domains of knowledge for teaching (Ball et al., 2008). Common content knowledge I coded “1” (further dealt with in 7.4), specialized knowledge for teaching (“3”; further dealt with in 7.3), horizon content knowledge (“2”; 7.5), knowledge of content and student (“4”; 7.1), knowledge of content and teaching (“5”; 7.2), and knowledge of content and curriculum (“6”; 7.6). I categorized each topic into only one domain, but soon realized that it could be problematic when writing about my findings. I still, in the first place, decided to keep it that way. After the first categorization into these domains, several categories of knowledge/ knowing remained. These were knowledge of learning aids, didactics, pupils, pedagogy, norms, knowledge of colleagues, and school organization.

<sup>30</sup> Notice where this category comes from.

To ensure reliability of my coding, I had a colleague carrying out the same procedure. He knew that I used Ball and colleagues' (2008) six domains, and that I had identified some categories that I meant did not "fit" into these domains. As I used common terms (see previous paragraph), I did not operationalize these categories. To avoid affecting his coding, I did not tell him what these topics were. When comparing our results, the agreement was approximately 71%. We mostly agreed on which topic could be categorized into Ball and colleagues' domains (2008), however he suggested that some "belonged" to several domains, thus doing what I should have done initially when realizing that some of the topics actually could fit into more domains. He also categorized several of the remaining topics (see previous paragraph) into these domains.

We discussed our categorizations, agreeing on using his coding; firstly due to his more fine-grained coding on topics I coded as pedagogy, but also because of his more intuitive coding on other topics, such as "com" for communication, an aspect Tea regarded as crucial for learning. I first coded it as "smn", socio-mathematical norm, but realized that "com" more directly expressed what I wanted to communicate. My recoding resulted in 87% agreement. Usually our coding were not in disagreement, the discrepancies were rather due to one having connected a topic to more categories.

As suggested in the previous paragraph, I experienced Ball and colleagues' (2008) MKT-framework as insufficient for fully identifying the mathematics knowledge Tea deployed in her teaching. It guided my identification of her knowledge *for* teaching, but was of limited value when exploring her knowledge *within* mathematics teaching. Thus, as explained in Chapter 3.2.3, for the issues of actions occurring within her teaching, I used Jaworski's (1994) Teaching Triad and Rowland and colleagues' (2005) Knowledge Quartet as analytical tools, which will be evident amongst others in Chapter 7.7.

Table 4-10 and Table 4-11 show the number and percent of occurrences organized into domains of mathematical knowledge *for* teaching and mathematical knowledge *in* teaching respectively.

<u>Domains of MKT</u>	<u># occurrences</u>	<u>Percent</u>
Knowledge of students and teaching	257	34,
Knowledge of content and teaching	231	30.6
Specialized content knowledge	103	13.6
Common content knowledge	31	4.1
Horizon content knowledge	82	10.8
Knowledge of content and curriculum	52	6.9
Total	756	

Table 4-10. Number and percent of occurrences organized into MKT

<u>Domains of facilitation</u>	<u># occurrences</u>	<u>Percent</u>
Communication	14	5,7
Examination, evaluation, tests	6	2,4
Organizing	6	2,4
Personal confidence	14	5,7
School as part of life	7	2,8
Sensitivity to students	138	55,9
Time aspect	5	2,0
Value system	57	23,1
Total	247	

Table 4-11. Number and percent of mathematical knowledge in teaching

See also Figure 7-2 for the illustration of Tea's mathematical knowledge *for* teaching (Table 4-10), and Figure 7-3 for Tea's mathematical knowledge *within* teaching (Table 4-11).

#### 4.6.3 Categorization, a summary

From the classroom observations, I divided the data into three parts; openings, mathematics work, and closings. From the openings, six categories emerged. Three of these concerned practical issues teachers "must do", which I organized into *general professional knowledge* (GPK). The remaining two were of personal character, thus organized into the category *involvement*, a category that emerged from the analysis. From the mathematics work, the analysis exposed four main categories of activities within the plenaries (explanation, subject, pupil production, and preparation) and three types in the seatwork segments (mathematics, involvement, and other). The closings were about either further work, review, discussing goals, or comments of practical issues, such as where to go for the next lesson.

By writing memos from six of our open-ended conversations, I first categorized the data into whether she talked about her practice, stated her principles, or expressed knowledge/knowing concerning the teaching of mathematics. After rewriting the first six, and writing memos from the remaining conversations, I again categorized the data into domains of practice, principles, knowledge and knowing before organizing them into Ball and colleagues' (2008) domains within the MKT-framework.



The findings will be further considered in Chapters 6 (Tea's practice) and 7 (Tea's knowledge for and in teaching mathematics).

#### **4.7 Writing up the findings**

Through the observations and conversations, I got information about how Tea carried out her teaching, and her philosophy for doing what she did, both representing her knowledge for teaching. I thus found it reasonable to write two chapters about Tea's teaching, presenting her practice in Chapter 6 and her knowledge for teaching mathematics in Chapter 7.

When writing Chapter 6, Tea's practice, I started with presenting the organization of the lessons, including some statistics, characteristics of the lessons, and how the mathematics work was carried out. This is followed by consideration of what happened in the different parts of a lesson; opening, plenary segments, individual seatwork segments, and closings, all illuminated by excerpts from the observations.

The themes that characterizes Tea's knowledge for teaching mathematics resulted from the categories exposed in our conversations. As I used the MKT-framework (Ball et al., 2008) when analysing the conversations, I found it reasonable to present Tea's mathematical knowledge for teaching using their categories as headings. Tea's focus on pupils stands out as an influential characteristic of her as a teacher and her teaching, I thus found it natural to begin Chapter 7, Tea's knowledge for teaching mathematics, by outlining her knowledge of content and students (Chapter 7.1). The remaining five categories, I present in the following order: knowledge of content and teaching in Chapter 7.2, specialized content knowledge in 7.3, common content knowledge in Chapter 7.4, horizon content knowledge in Chapter 7.5, and finally knowledge of content and curriculum in Chapter 7.6.

Initially I claimed that I would only focus on Tea's knowledge and knowing for teaching mathematics (cf. Chapter 3.2), however, due to what Tea shared in our conversations, and justified by the subsequent analysis of the data, I found it reasonable to include a seventh section, knowledge of management and social norms (Chapter 7.7). Tea's focus on her pupils (mentioned earlier in this section) and her effort for making a fruitful milieu for the learning of mathematics stood out as particularly important to her; her awareness for the learning of the subject I interpret as included in her knowledgebase for teaching. I thus suggest an extension of my initial conceptual framework, as will be discussed in the next section.

For exemplification and justification, throughout Chapter 7, I illuminate my claims and interpretations with excerpts from our conversations.

#### **4.8 Extending my conceptual framework**

As explained in Chapter 1.3, I was informed by Ruthven's (2002) dialogic cycle when developing my own conceptual framework. My experiences as a mathematics teacher at lower secondary level for more than thirty years then informed my elaboration on the category craft knowledge, as discussed in Chapters 1.3 and 1.4. I suggested that one, when observing teaching, could find evidence of local knowledge that included localized knowledge, localized knowing, and general professional knowledge related to the teaching of the subject. When analysing the data, however, I realized that Tea had relatively great focus on the milieu in the classroom and on the pupils' well-being, which she believed had impact on their learning. In her lessons, this was illustrated by the way she involved in her pupils lives and sharing episodes from her own life. In our conversations, it was evidenced by the way she talked about her pupils and their learning (cf. Chapter 7.7). This made me realize that the category general professional knowledge includes more than requirements. In Tea's classroom, it had in it a didactical dimension, as described in the previous section. I thus changed its name to *Knowledge of Facilitating Mathematics Learning* (KFML).

#### **4.9 Key to the analysis and transcriptions**

All names in the analysis are pseudonyms, however, boys are given boys' names and girls are given girls names. To help keep in mind who are pupils and who are teachers, the teachers (Tea and Tom) have names with three letters, while all pupils have four letter names. At Tunholt lower secondary school, all pupils have names starting with T, such as Tina, Tone and Tuva for girls, and Thor, Trym, Tore and Tuan for boys. Written text include dots, commas, and exclamation marks. This is not always the case when presenting speech. However, to make the transcripts more readable, I have chosen to include such signs where the informants' tones makes it appear naturally. Words written by using *italic* letters indicate that the teachers put emphasis on the word. The participants in my study have a dialect close up to one of the official Norwegian languages. It was thus fairly easy to transcribe the talk. To enhance the experience of the activity in class, I wrote what happened in brackets, for example such as [writes  $2x + 3 = 11$ ] and [points to  $2x$  in both equations] in the below example:

Then we say that the equations we first are going to be good at now, they are for example those who says two x plus three equals eleven [writes  $2x + 3 = 11$ ] [1] for example. Eee [3] because it has both in it a number [points to 3 in both the

first and the new equation] that we have to move-change, and then it has in it a number in front of  $x$  [points to  $2x$  in both equations] so that we have to do more things (observation 9A).

Numbers in brackets indicates pauses in seconds, while dots in brackets ([..]) are used when the talk might be of limited relevance for what the excerpt illuminates. The following example comes from a conversation when I started with asking: “Quite spontaneously after two lessons with equations?” Tea responded by first telling about her plans for the lesson, then talking about some pupils, for example Turi who was allowed to use a calculator:

[She] cannot add and subtract, and multiply and divide. So I then made a decision [2] which I had not thought of in advance, and that was to say use the calculator.[1] Because [4] the aim was to practice equation-technique and it would be quite hopeless if they cannot add two numbers. [..] And now I said use calculator because [interrupts herself], to try to focus on what they should understand, right (conversation 9).

In our conversations, Tea several times talked about her concern for pupils’ use of calculators. Thus, when searching for how she dealt with use of such in her lessons, one of the episodes I found was the one presented above.

When making statements, Tea often interrupted her talk by providing some kind of background information or extra explanations. To get the statement as accurate as possible, I fully transcribed what she said, also the “extra” inputs. In such situations, I chose to leave out parts of the transcribed text I interpreted as irrelevant for the theme in focus. The [..] in the above text represents the following utterance:

And then it is like [2] that is, one ought to assume that the pupils know basic arithmetic before they start at lower secondary school [1] I think, [3] I get shocked when the pupils do not know how to divide twenty-four by three. [2] And I, yes, but right, then it becomes like, now we have stressed with that in the first part, right, first part of this year (conversation 9, Table 10-67).

I do not know whether pupils’ apparent lack of understanding has influenced Tea’s attitude towards calculators, however, when using the excerpt to explain why she let Turi use the calculator, I omitted the part about Tea’s feelings due to the length of the excerpt if writing the whole text.

The above excerpt also shows how I made some inputs to increase the readability and enhance the understanding; I added “She”. Tea started her utterance by saying, “cannot add and subtract”, and to indicate that the “she” was my input, I put brackets around the word.

## **4.10 The choice of episodes to illustrate my analytical findings**

The analysis exposed many episodes of similar character, episodes that all could all have been used to illustrate or illuminate certain aspects of Tea's practice or her knowledge for teaching mathematics. However, I had to choose. For Tea's practice (Chapter 6), I have either reproduced excerpts of episodes from lessons directly in the text, or referred to excerpts reproduced in the appendix (Chapter 10). I have, as far as possible, reproduced classroom episodes that stand as general examples of the categories. Moreover, I have referred to, or reproduced, episodes from all lessons, except lesson seven, where I shared some video clips with the pupils, and lesson ten when Tea sat at her table and the pupils came to her for help. When referred to, they can be read in the appendix (Chapter 10). Concerning Chapter 7, which deals with Tea's knowledge for and in teaching mathematics, I have, when discussing different aspects of her knowledge, used Tea's comments on her teaching as a starting point, and sometimes included excerpts from classroom episodes for illuminating her comments. I have also in chapter seven reproduced, or referred, to all our conversations.

I want to make clear that my purpose is not to judge Tea's practice or her knowledge. My purpose is to make public and present this as objectively as possible, and then synthesize this into an account of one teacher's local knowledge.

## **4.11 Summary methodology and methods**

In this research, I have been out searching for teachers' local knowledge in mathematics teaching, and how they, through practice, have developed shareable knowledge. In this chapter, I have outlined the methodology, methods and research design for the study. The ethical considerations are also discussed. The process of a longstanding development before reaching the final conceptual framework is also presented, as well as how I experienced the observations in the classes and conversations with the participating teachers.

I have been out observing and talking to three experienced teachers, thus getting certain insight into their practice and the knowledge they deployed while teaching, particularly focussing on one of them, namely Tea. The process of analysing the data resulted in an understanding of her knowledge for teaching, which I present in the three forthcoming chapters: the teacher Tea in Chapter 5, her practice in Chapter 6, and her knowledge for teaching mathematics in Chapter 7.

I wanted to observe an experienced teacher having formal competence for teaching mathematics at lower secondary level, and was, by school administration, colleagues, pupils, and parents, characterized as an

excellent teacher. Chapter 5 I devote to illuminate Tea as a colleague and teacher; I there let the voices of the Principal and some of the pupils as well as herself be heard.

The analysis of the conversations and observations provided several categories and dimensions of organizational and educational character, as I have commented on it this chapter. In Chapter 6, I focus on the organizational aspect of Tea's teaching, and presents some statistics as well as her management of the lessons and the content. In Chapter 7, I discuss the results of analysing Tea's knowledge for teaching based on her own account for that teaching. I mainly used Ball and colleagues' (2008) MKT-framework for analysing her knowledge, but found characteristics of Tea's knowledge that needed to be characterized otherwise, as will be evident.

In the chapter that follows, statements from her Principal, some of her pupils, and from Tea herself, provide an opportunity for the reader to learn to know the teacher Tea.



## 5 Tea, teaching a grade nine class

This chapter is about Tea, it is about her educational background as a teacher at lower secondary school, about how her pupils and her Principal see her, and how she looks upon herself as a teacher. The first section is about her education, which is spread over many years, from attending a teacher education programme in 1975, studying pure mathematics in 1977/78, to following different courses while practising as a teacher.

### 5.1 Tea – a versatile teacher

Tea began her education in the academic year 1974/75 by following a preparatory study for the studies at University level<sup>31</sup>. At the same time, she studied psychology and followed a course in statistics and psychological measurement methods. In autumn 1975, she joined a teacher education programme in the northern part of Norway. The very same year, the Norwegian Government implemented the “law about teacher education of 8. of June 1973” (Kunnskapsdepartementet, 1999). General teacher education then became a three-year study, and the schools were to be named Pedagogical Colleges<sup>32</sup>. These colleges should have the responsibility for educating teachers for kindergarten and general teachers for all subjects in compulsory school, which at that time consisted of levels 1 – 9. The colleges should also provide opportunity to specialize in specific subjects, as well as practical pedagogical education for students who had studied specific subjects at a University or University College and wanted a certificate as specialized subject teachers.

Tea followed what she calls an experimental teacher education programme, which in the first year included a basic course in so-called “modern” mathematics<sup>33</sup>. According to Tea, who had followed a mathematics-scientific programme at upper secondary school, the course did not add anything to her knowledge about mathematics. The experiment included that the students in the second year could choose their own programme. Tea chose to study the Norwegian language, Christianity, sports, pedagogy and social pedagogy. The third year was dedicated to subject specialization, and Tea moved to another institution

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<sup>31</sup> Examen philosophicum.

<sup>32</sup> In 1980 changed to Teacher University Colleges (Kunnskapsdepartementet, 1999).

<sup>33</sup> Due to clean up in connection to merging of institutions, it was impossible to obtain any documentation from the institution where she educated.

to study mathematics for one year<sup>34</sup>. This programme included Mathematical Analysis I and Linear algebra in the autumn, and Mathematical Analysis II, statistics, EDB and Effective Calculations in the spring. The EDB-course included different number systems, the function and construction of a computer, elementary numerical analyses, and the programming language FORTRAN. In the Effective Calculations course, the students learned about the mathematical foundation for the IT-technology, “effective algorithms”, and examples that showed limitations for the use of computers. The different courses were equivalent to mathematics studied for one year at a University College (personal communication with an employee at Høgskolen i Bø).

Tea began her teaching career in 1979, and has followed four different curricula: M74, M87, L97, and LK06<sup>35</sup> (Regjeringen, 2013). While working full time as a teacher in lower secondary school, she has followed courses in environmental education, school law, informatics, public administration and management, and a course in special needs focusing on ADHD, Asperger syndrome and Tourette syndrome.

As already explained (Chapter 4.5.3), I had Tea in my mind when visiting Tunholt lower secondary school to ask for permission to observe mathematics teaching. At that time, I did not know Tea, but had experienced her as an extraordinary teacher when I was called out to evaluate the mathematics exam for a boy who was able to see only a small part in front of him. The exam was conducted by Tea reading the tasks, and the boy telling Tea what to write, all while I observed the process. I barely met her a second time when I went to her school to conduct some interviews for my master thesis, and she asked if I wanted to interview her. I had to tell her that she was not on my list that time, however, I did not forget her, which led to that I contacted the school when I needed an informant to my study. I dedicate the next section to the Principal who will explain why Tea was her natural choice when I asked for an experienced teacher who she regarded as successful.

## **5.2 Comments from the Principal**

When I asked the Principal at Tunholt lower secondary school for a teacher who was experienced, and at the same time seen as successful by the administration, colleagues, pupils, and parents, the Principal immediately suggested Tea. The Principal was not aware that I had Tea in mind when asking for permission to observe teaching in her school,

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<sup>34</sup> Grunnfag.

<sup>35</sup> Mønsterplan av 74, Mønsterplan av 87, Læreplan av 98, and Kunnskapsløftet av 2006.



however still suggesting her. Tea was called to the principal's office, and she did not ask for any time to think about the question I posed; her immediate response was positive – she wanted to give a voice to practicing teachers. I did not videotape the first conversation with the Principal, so later I went back to her office and asked why Tea was the one she believed fulfilled my request for an experienced and excellent teacher (conversation Principal, Table 10-2):

Principal Because she is experienced and clever

Inger How do you know?

Principal Because I have been in her class and seen how she teaches, I have from pupils, from teachers and other who are in her class, from parents got, and get excellent critics [1] about how she teaches. And I see [interrupts herself] [1] know her view on humans. I know how she works in relation to the whole pupil, not only [interrupts herself]. And she is good at motivating; she does not give up what so ever, so it is everything. I have known her, I have been at this school for [2] twenty-seven years, and I have been colleague to Tea for twenty-seven years, so I have pretty good insight.

The Principal knew Tea very well; they have been colleagues for twenty-seven years. When I, three years earlier, visited Tunholt to conduct interviews for my master study, the school had another principal. Thus, at the time I did my observations, the present principal had been the school leader for only a few years. Prior to taking on the principal's role, she had been a teacher at the same school for about twenty-four years, thus claiming to have “pretty good insight” about Tea and her practice. She described Tea as a teacher who focuses on the whole pupil, not only the subject, that she is good at motivating pupils, and that she is stubborn; “she [Tea] doesn't give up, what so ever”.

The Principal also participated in one of our conversations looking through the episodes together with Tea and me. After about twenty-four minutes, I asked for her comments; she was particularly focused on how Tea is able to think creatively to adjust her methods for finding solutions for each individual pupil (conversation 3, Table 10-3):

Principal It is amazing to see how fast one has to think creatively to find solutions for each pupil, and that is what it is all about. And if one does not have relations, and if one does not know the pupils well and take the inconvenience with that, then it is difficult to find methods. And, and particularly in mathematics that is an unbelievable challenge. It was very fun to see ,Tea, but I know you as a person too, I know that you are very visual, very visual and eh bodily

Inger What importance do you think it has for the teaching that she is like that?

Principal It means everything, actually, because [interrupts herself], and then you are fast to change all the time; you see, and you do not think long before you carry on helping people to get understanding

The Principal highlighted Tea's competence to think creatively and quickly to find solutions for each pupil, an ability the Principal claimed is a consequence of Tea taking the inconvenience of learning to know each pupil. She furthered that Tea's way of being means everything for the process of teaching, and addressing Tea, she commented; "you see, and you do not think long before you carry on helping people to understanding".

The Principal focused on Tea's contingent actions and adaptations in the classroom, adaptations that she is able to carry out because she had learned to know every pupil. This competence is what this study recognizes as *localized knowing*. Tea uses her knowledge of the individual to find methodological approaches to help individual pupils understand.

The Principal described Tea in very positive terms on behalf of the colleagues, the parents and the pupils. I did not want to involve either parents or colleagues to give their comments of her teaching; however, I was very interested to hear what her pupils had to say. I asked Tea if she would mind if I talked to her pupils. It was not only quite okay for her, she also said that she did not need to know what the pupils told me; "They are free to say whatever they want about me, and I do not need to know".

### **5.3 Pupils about Tea's teaching**

I thus asked Tea to suggest six pupils, both boys and girls and at different levels of attainment, for the conversations. Three girls and three boys, who all agreed to contribute, were suggested; girls attaining 13% (Tale), 34% (Tone), 57% (Tina) on the semester-test before Christmas, and boys attaining 53% (Thom), 61% (Trym), and 84% (Tuan) on the same test. The conversations were carried out the second week of January, and all were individual.

I began all conversations with telling the pupils that I wanted to hear about Tea's teaching, highlighting that I was not interested in their opinion of Tea as a person. I also told that Tea said that she did not need to know what the pupils said, an information Tone met by saying; "It doesn't matter; Tea knows that I am very strong in my statements when I really want to".

The first question all pupils got was about what they liked/disliked about her teaching (conversation with pupils, Table 10-4):

- Tone I like her teaching because she, she makes it very easy for us and those, yes, those who have her as a substitute teacher, to understand because she stands out in a way from all the other teachers – in the way she does it
- Thom I think the way Tea teaches is easier than how I previously was taught, and I believe my grades have improved. [...] That is, like it is the way she explains things, and she makes it easier and not so complicated.
- Tina It is nothing I dislike, she kind of teach in a way so I actually understand it, [...] it is only the way she does it and like that, I do not quite know, but it is nothing silly about what she does.
- Trym I like it very much, the way she teaches is very nice, because she kind of shows us kind of, that is we repeat it many times so we really get what we do and not only shows a task, but she explains it properly
- Tale I think she is clever, I kind of understand what she means. My previous teacher was only at the board. I make it better now when I sit down and work with the tasks because I get help
- Tuan I think it is a bit simple [the teaching], but I have done mathematics for a long time so I understand it at once, I think she explains very simple so it is understandable

All, except Tuan, appreciate Tea's teaching; he experienced it as a "bit simple". I asked if he thought about asking for more challenging tasks, but then he replied; "No, I think it is quite okay, if I try more and more difficult [tasks] then I gradually forget what is simple". Tone and Thom asserted that her way of teaching makes it easier for them. Thom also added that she (Tea) does not make it so complicated as his previous teacher did:

My previous teacher made it complicated by doing a lot of tasks in different ways, while Tea does it in a special way that you get used to so I can do it more easy for myself (conversation with Thom).

Tina could not say what it is about Tea's teaching that makes her understand, while Trym explained that Tea repeats the content many times and explains properly. Earlier Trym felt that mathematics was boring because "it was very much numbers all the time", while he now experiences the subject positively because "Tea makes much of it fun", which motivates him.

Thom and Tale did not start their lower secondary education at Tunholt; Thom attended the school in January the previous year, thus having been in the class for one year, while Tale started at the beginning of the present academic year. While Thom was satisfied with Tea because she teaches in a way that makes it easier than his previous teacher did, Tale was happy about the fact that Tea left the board to help the pupils when they were doing tasks.

These conversations happened one week after they had started to work on percent. By that time, I had realised that Tea used a lot of energy to make the topics familiar to the pupils; she often contextualized the mathematics as will be evident in the following two chapters, Chapter 6, and Chapter 7. For introducing percent, Tea the first day read a story about young people who got their drivers licence for moped (Chapter 7.2.4), and the second day by focusing on a solar eclipse happening that day. I wanted to know how important these introductions had been for their learning, and asked if they remembered how she began these particular lessons. Only one of the pupils (Trym, 61%) could recall these introductions, so to the remaining pupils, I gave a short summary of the story about the driver's licence and the episode with solar eclipse. I then asked the pupils<sup>36</sup> if they had some thoughts about why she began the way she did:

- Tone I thought that it was impossible for me to remember so many numbers [laugh], not even how I round off. [...] I believe she used the moped because we are youths so it should be more interesting
- Thom It is okay that she brings in such things once in a while, it is also understandable, and it is because we shall learn more, learn to better understand the way she teaches
- Tina I do not know, but probably it was to explain a little about how percent can be used, that you have to use percent that way
- Trym Because one gets better relations to what one does, that it will be more personal (conversations with pupils, Table 10-5)

Tone thought that it was impossible for her to remember that many numbers, which actually was what Tea aimed at; she wanted them to be a bit confused about all these numbers, thus experiencing that using percent would help them to see connections (Chapter 7.2.4.2). Tone also believed that using a story about moped as context was to make it more interesting to them as youths, and Trym (61%) meant that by making it more personal they would get better relationships to what they do. Regarding the solar eclipse, Tone said "I do not know if it had that much to do with percent, but she turned off the light because it was a solar eclipse and that we should see it".

Thom (53%) thinks it is okay that she brings in "such things once in a while", and that she does it so they shall learn more. Tina, who performed 57%, however, had no opinion about why Tea chose to read these stories, but said that it probably was about explaining how percent can be used. Tea considers Tina to be an intelligent pupil: "to her I use the form of proofs [...] because she understands it, and in a way needs it" (conversation 3).

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<sup>36</sup> I unfortunately forgot to pose that question to the last two students.

I have experienced that teachers often try to make topics familiar and interesting by putting them in a context. Listening to the comments these pupils made, they appeared not to pay much attention to the stories Tea prepared for supporting their understanding of the importance of knowing percent. Particularly Tina, who is regarded as an intelligent pupil, appeared not to understand why Tea chose to contextualize the topic. To speculate about why Tina did not pay attention to Tea's contextual stories is not within the focus of this study. However, the fact that Tea chooses to use proofs when explaining topics to Tina, "because she needs it", might indicate that the pupil is more concerned about formality of the topic than the context in which it can be used.

Tea prepared for the lessons by making stories that could be connected to the topic on the agenda in addition to being interesting and/or familiar to the pupils. She planned for the lesson by making the topic relevant to that particular group of people, thus connecting her mathematical knowledge to the knowledge of the group. This study recognizes this competence as included within *localized knowledge*.

By talking to Tea's Principal and her pupils, I have been able to localize both categories of *local knowledge*. These I further discuss in Chapter 8. In the next section, however, I provide a short summary of how Tea looks upon herself, how the Principal knows Tea, and how the pupils experience her teaching.

#### **5.4 The teacher Tea, a summary**

I wanted Tea to tell with her own words why she decided to participate, about her practice, and her pedagogical ideas. However, I did not want to ask her for more favours than I already had, so I put together parts from our conversations and sent it to Tea for comments and approval, which she did. She there presents herself as a teacher with strong opinions about school politics as well as how to administer schools. Having been in the practice for so many years, she has experienced many pedagogical innovations, for example implementation of calculators, pupils being responsible for their own learning, and suggestions about throwing out the boards. It is her opinion that the first two out of them, implementation of calculators and pupils being responsible for their own learning, have served as obstacles for development of pupils' mathematics knowledge. She also experienced suggestions about removing boards from the classrooms. She refused to follow that "innovation"; no board was removed from her classroom. I have reproduced Tea's "story" in full in Chapter 10.3.1.

She became very happy when the result from TIMSS 2011 was published (Grønmo et al., 2012), experiencing that what she always had believed in was confirmed by research; the classroom and homework are important to pupils' achievement. Tea believes that, for many pupils, the school and their classroom is the only certainty in their lives, the only place where they feel safe. It is thus important to focus on the class and classroom as the unit for pupils' belonging. Moreover, she worked hard for creating an atmosphere of trust where the pupils can collaborate to the best for each other's learning.

Tea got positive critique from the Principal and the pupils. The Principal told about a teacher who is appreciated by the whole school community, and the pupils talked positively about her teaching. Two pupils, who began their lower secondary education elsewhere, experienced Tea as more focused (keeping to one method) and more attentive (leaving the board to help pupils) than they had earlier experienced. For both pupils this made a difference.

In this chapter, I have provided background information for the following two chapters, Chapter 6 in which I provide some episodes from Tea's classroom, and Chapter 7 where I provide her comments on some classroom episodes, from which I have attempted to extract her knowledge for teaching mathematics. As described in Chapter 3.3, my focus is on teachers' *local knowledge*, which I subdivided into teachers' *localized knowledge* and teachers' *localized knowing*. The next chapter is about Tea's practice as it evolved from the analysis of the observations carried out in her classroom. I present classroom episodes that stand as examples of Tea's knowledge for teaching mathematics. As earlier explained, my purpose is not to judge Tea's practice or her knowledge for teaching, it is about exposing and presenting these as objectively as possible, and synthesizing this into an account of a teacher's local knowledge. It is thus important to be aware that all episodes presented in this thesis are parts in a larger context of complex classroom situations, and I ask the reader to keep that in mind while reading the analysis.

## 6 Tea's practice

In this research, I focus on an experienced teacher's knowledge for teaching mathematics. It is not about the outcome of that teaching, i.e. pupils' learning. However, as the work of teaching aims at promoting pupils' learning, I find it important to, in addition to considering the knowledge directly connected to the teaching of mathematics, also look into how a teacher creates the learning environments for her pupils. My aim for reporting from the classroom observations is thus to provide an accurate and objective description of Tea's practice; her routines and structuring of the lessons (Section 6.1) and highlight the relationship between Tea and her pupils, a relationship that was particularly demonstrated during the openings to the lessons (Section 6.2). In the section about the closings of the lessons, I, for example, deal with how Tea encouraged the pupils to work with the subject outside of the classroom (6.5), which evidences parts of her local knowledge. I deal with details about the mathematics work in plenary segments in Section 6.3, and individual seatwork segments in Section 6.4.

The observations were carried out over a period of eight months. I had initially planned to observe three periods of three weeks each with six weeks of analysis in between (cf. Chapter 4.5.3). But, to get the opportunity to observe what interested me the most, I had to change plans due to the local implementation of the curriculum. In addition to the teaching of percent, I am explicitly interested in work on variables and unknown, algebra, equations, and functions. I also wanted to observe the introduction of new topics, i.e. the very first lesson the class focused on a topic. These topics were not scheduled according to my initial plan, so I had to change my plans accordingly.

The observations started August 30<sup>th</sup> 2010 and ended April 12<sup>th</sup> 2011. During these months, I visited the school nineteen times as illustrated in Table 4-3. In four of the visits, the class was divided into two groups, identified as A and B, in which both were engaged with the same topic, as can be read from Table 4-3. Three of these (lesson two, four, and six) were carried out in heterogeneous groups decided by the teachers, while for the fourth lesson (lesson nine) the class was divided into two groups related to pupils' wishes, due to some wanted "to work with some simple equations, while others wanted to start with the difficult equations" (observation 9A). On two occasions, the work was dedicated to other issues than mathematics teaching. Because the pupils were curious about my work, lesson seven was used to show some video clips from some of my observations. Data from that lesson is thus omitted in the analysis.

Tea used ten minutes of lesson fifteen to question me: twenty-eight minutes into the lesson, Tea asked for the camera, gave it to a pupil, and interviewed me about my work.

In this chapter, I first provide some background information concerning the twenty-two observed lessons that included mathematics work. This will be followed by four sections about “life” in class as explained above.

## **6.1 The organization of the lessons**

Grade nine at Tunholt lower secondary school was provided three 45 minutes mathematics lessons per week. This corresponds to one hundred and eleven lessons during an academic year, which in Norway accounts for thirty-seven weeks of schooling. I visited the class nineteen times, thus observing about 17% of the lessons dedicated to mathematics teaching at the level in focus. Every Tuesday during the autumn semester, two lessons were organized as working in “small” groups. The class was then normally split into two heterogeneous groups; one group (group A) began by working on mathematics while the other first had Norwegian (group B) with the other class-teacher, Tom, in another room. After about forty-five minutes of work, the groups exchanged rooms and subject. I refer to these lessons using the number of the visit followed by an uppercase A or B.

### **6.1.1 Some statistics**

The Norwegian mathematics subject curriculum for lower secondary school consists of five main subject areas: number and algebra, geometry, measuring, statistics and probability, and functions. Because I have a particular interest in algebra, equations and functions, I decided to concentrate on these topics. This study thus deals with only two of the five main areas, namely number (which included percent) and algebra (which included equations), and functions. Table 6-1 shows the dates and the duration of the mathematics work in the lessons.

I visited Tunholt lower secondary school over a period of eight months. During this period, I had, by choice, the opportunity to observe teaching of number, algebra, equations, percent, and functions:



	No	Min	No	Min	No	Min	No	Min	Sum
Date	08.30		08.31		09.07				
Number	1	35	2AB	66	4AB	70.5			171.5
Date	09.20		09.21						
Algebra	5	35	6AB	76.5					111.5
Date	10.25		11.02		11.08				
Equations	8	34	9AB	82	10	40.5			156.5
Date	01.03		01.04		01.10		01.11		
Percent	12	35	13	34	14	38.5	15	25.5	133
Date	04.04		04.05		04.11		04.12		
Functions	16	36.5	17	37	18	38	19	37	148.5
Total									721

Table 6-1. Minutes observed per topic.<sup>37</sup>

As mentioned above, the lessons were organized to last forty-five minutes. The video-clips showing the teacher-led activities in the twenty-two lessons ranged from 34.4 to 46.7 minutes ( $M = 42.3$  minutes,  $SD = 3.1$  min, Median = 43.0, cf. Table 4-3), whereof the mathematics activity varied between 25.5 and 41 minutes ( $M = 36.1$  minutes,  $SD = 4.2$  min, Median = 36.2 min, cf. Table 6-1). The gap between the time in the classroom and what was spent on mathematics work was for example caused by long opening sequences. It was also sometimes caused by waiting for pupils returning from lessons in another room, or, as the most outstanding, lesson fifteen when Tea stopped the mathematics work to have one pupil interview me about my work and my forthcoming trip to a university abroad.

### 6.1.2 Characterization of the lessons

The lessons normally included an opening, a period of mathematics work, and usually a closing (seventeen out of twenty-two). The mathematics work consisted of one or more sequences (cf. 6.1.4), which each included one plenary segment and one segment of individual seatwork. All over, the openings accounted for approximately 9.2% of the lessons, and the mathematics work for 82.9%. The closings accounted for approximately 3.6% of the time, while “other” aspects (management, disturbances from outside and the “researcher interview” as explained in the introduction to this chapter) accounted for 4.3%. On five occasions, the pupils worked individually until the end of the lesson. Tea then, for example, announced that it was time to go to another lesson, time to recess, or time to eat.

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<sup>37</sup> It has to be remembered that lessons no 2, 4, 6, and 9 are “twin”-lessons.

What	Mean	SD	Median	Range
Openings	4.1	2.4	4.0	0 – 11.4
Main part	36.1	4.2	36.2	24.3 - 44.5
Closings	2.1	1.9	1.4	0.3 – 5.9

Table 6-2. Minute variation within lessons.

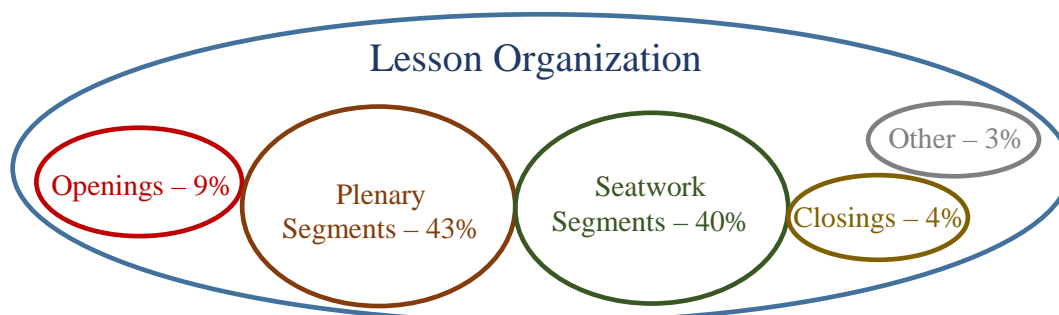


Figure 6-1. Percent of activities in Tea's mathematics lessons.

In nine of the twenty-two openings, Tea was standing at her desk waiting for the pupils to prepare for the lesson by getting their books before standing still at their desks without any instruction from Tea. The remaining openings also included other issues, for example Tea being somewhat private, which I consider in Section 6.2. The teacher-led plenary segments of the mathematics work occupied 42.6% of the time spent in the classroom, i.e. 45.7% of the mathematics work (Section 6.3), while the individual seatwork, which will be dealt with in Section 6.4, occupied 40.3% of the total time in class (48.6% of the mathematics work). The closings, which dealt with both mathematical and/or organizational issues, I consider in Section 6.5.

### 6.1.3 The mathematics work

As explained in the previous section, the main part of the lesson, i.e. the mathematics work, consisted of sequences of plenary work and individual seatwork. Table 6-3 summarizes the duration of the mathematics work that was carried out in Tea's class, both for the entire mathematics work as well as split into plenary segments and the segments of individual seatwork:

What	Mean	SD	Median	Range
Mathematics work	35,1	4.2	35.3	24.3 – 44.5
Plenary math work	18.0	8.4	20,9	0 – 29.8
Individual seatwork	17.1	8.1	16.3	3.6 – 35.3

Table 6-3. Minutes of mathematics work.

As can be read in the table, the mathematics work in Tea's class varied widely. It happened that a lesson was only individual seatwork (lesson

11), but there was no lesson that was “only talking”, i.e. plenary mathematical work (cf. Table 10-1).

In all, but one, of Tea’s lessons, the mathematics work began with a plenary segment followed by individual seatwork. Early in my observations, I experienced that she varied the mathematics work by sometimes having one such sequence, while at other times carrying out several of them. In the following section, I define and summarize the sequences of mathematics work observed in Tea’s classroom.

#### 6.1.4 Sequences of mathematics work - definition

A sequence of mathematics work comprises one segment of plenary work and one segment of individual seatwork. Figure 6-2 illustrates a randomly chosen example of transition between a plenary segment and a segment of individual seatwork. The excerpt is from the analysis of lesson 4B, which comprised two sequences of mathematics work, and illustrates the last part of the second plenary segment (3 out of 10.5 minutes) and the beginning of the second segment of individual seatwork (two out of nine minutes). In this example, the plenary segment consisted of half a minute exposition (Tea explaining the mathematics), one minute IRF (initiation, response, follow-up), and one and a half minutes of preparation for the following individual seatwork:

Time	What	Sequence	Action	
26 - 27	Rules (plus/minus)	Teacher led plenary (Section 6.3)	Exposition	Cycles in plenary segments
			IRF	
27 - 28	Preparation for seatwork		Preparation	
28 - 29				
29 - 30		Individual seatwork (Section 6.4)	Short stops monitoring pupils working	
30 - 31	Thor		Longer stops: helping individuals	

Figure 6-2. Part of a sequence of mathematics work (excerpt from lesson 4B).

The excerpt also exposes that Tea, during pupils’ individual seatwork, monitored pupils working and, when needed, made stops of various lengths.

All lessons, except lesson eleven, which only included individual seatwork, consisted of strings of a various number of plenary segments followed by a segment of individual seatwork. Twelve of the twenty-two lessons (54.5%) included one sequence of mathematics work, six (27.3%) included two sequences, and two lessons (9.1%) included strings of three sequences while one lesson (4.5%) consisted of five sequences of mathematics work.

### 6.1.5 Organization of the work, a summary

In Section 6.1, I presented some practical, statistical, and organizational issues that evolved from analysing the observations of Tea's mathematics classroom. It has shown how Tea's lessons routinely were following a three part structure of an opening, main mathematical part and most often also a closing that was more extensive than only announcing that the lesson was over.

The main mathematical part also normally followed a structure of one or more sequences of mathematical work, sequences comprising plenary and individual seatwork segments. In the remainder of this chapter, I provide a more detailed insight in the practice carried out in Tea's classroom. I here present the activities that went on in the mathematical part of the lessons as well as consider what went on in the openings and in the closings, cf. Figure 6-1. Three main categories of actions were observed in the openings (cf. Figure 6-3), five in the plenary segments (cf. Figure 6-4), three in the seatwork segments (cf. Figure 6-6), and two categories in the closings (cf. Figure 6-9). The percentages illuminate how much of the total time of the mathematics lessons that was spent on the different activities. It has to be noted the numbers do not add up to one hundred. This is due to occurring activities having no impact on the teaching, thus categorized as "other activities" and omitted in the presentation.

## 6.2 Openings

Two overarching top-level issues, *general professional knowledge* and *involvement* in pupil matter characterized the openings. The category involvement includes how Tea worked to build personal relationships, and how both class-teachers, Tea and Tom, involved the pupils in their personal lives. The general professional knowledge were recognized by two sub-categories, *organization* of for example pupil or administrative matter, and practical *preparation* for the lesson (cf. Table 4-5). In addition to these two, there were also irregularly occurring non-routine issues such as presenting the researcher, greeting "guests", or talking about subjects other than mathematics. These will not be considered further.

The openings accounted for 9.2% of the time spent in the mathematics classroom, of which 40.1% concerned pupils preparing for the lesson. Different issues were communicated during the openings; in 39.5% of the time it concerned organizational matter, 12.5% involvement, and 7.9% “other” matter.

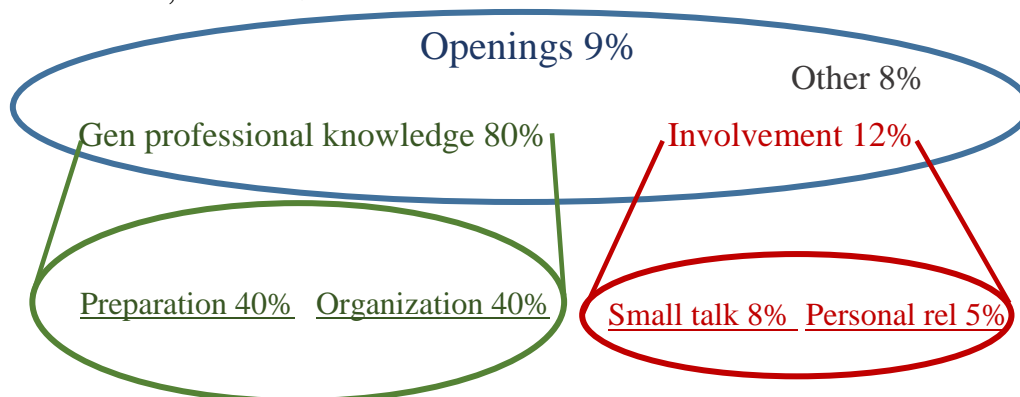


Figure 6-3. Activities in the openings (cf. Table 4-5 for explanation).

This section will continue with discussing what was observed during the openings. Excerpts that further supports the episodes can be found in appendix (Chapter 10), these are referred to in the text.

### 6.2.1 General professional knowledge

General professional knowledge (see the conceptual framework, Figure 1-3) is part of what teachers are expected to deal with in their ordinary lessons. Tea and Tom, who shared the responsibility as class teachers for the class, had been working together for more than thirty years. In addition to being class teachers, they both teach several subjects in the class. As class teachers, they have special responsibilities for their class:

The class or basic group shall have one or more teachers (class teachers) who have special responsibility for the practical, administrative and social-pedagogical tasks that concern the class or basic group and the pupils who are there, including contact with the home (Kunnskapsdepartementet, 2015b, § 8-2, translated by the author).

There is limited time allocated for communicating information to classes, thus teachers have to take time from their regular lessons when it becomes necessary. In the following section, two sub-categories of general professional knowledge, *preparation for the lesson* and *organization*, will be discussed, starting with the preparation.

#### 6.2.1.1 Preparation for the lesson

Every first time Tea and the pupils met for the day they greeted each other, a routine that also was carried out when others met the class for the first time. On eight occasions, I came to observe the fourth lesson, and in seven of these either Tea initiated a greeting, or, if Tea forgot, the pupils asked for it.

Pupils preparing for the lesson by getting their books and greeting each other accounted for 40.1% of the opening time. On occasions when the class would be separated into two groups, all pupils first met in their regular classroom and directly went to their desks quietly waiting for the teachers to greet them before they communicated messages as those being considered in this section. Ten times there were two “rounds” of pupils preparing by getting their books; when they had greeted Tea and Tom as explained above (four times), and six times when Tea, before announcing the mathematics work, experienced that the pupils had not got their books as she had asked them to.

### **6.2.1.2 Organisation**

As explained above, there was limited time allocated for communicating collective messages to the classes, so Tea and Tom had to take time from their regular lessons when this became necessary. Five categories of organizational matter were observed during the openings, administrative organization, organizing the teaching, organizing the day, organizing the pupils, and organizing for social events. Twenty-four percent of the opening time was spent on organizational matters.

#### Administrative organization

Messages from the administration or administrative matters such as naming books or checking passwords were communicated five times during the openings. In the beginning of lesson sixteen, for example, Thom, who represented the class in the pupil council, told the class that the administration planned to merge five classes to four for the coming school year. Tea’s comment to the message can be read in Chapter 10.5, Table 10-8.

#### Organizing the day

Three times Tea was observed communicating messages about the day. Once (lesson 4A) it was general information about what they would do that day, while two concerned rescheduling due to a concert (lesson 9A) and to an oral exam for another class in which their regular teachers would be sensors (lesson 17, Chapter 10.5, Table 10-9 ).

#### Organizing for social events

The class was involved in several social events, and the planning for these events were often scheduled in the openings to the observed lessons. On eleven occasions, in three different lessons (lessons 9A, 17, and 19), upcoming social events such as parents meeting, disco for next year’s eighth graders, visit from kindergarten, and the school trip were discussed. An excerpt illuminating Tea and Tom talking about how the parents and the pupils worked to raise money for the school trip, and one communicating a message from one parent can be read in Chapter 10.4, Table 10-10.

#### Organizing the pupils

This category, which embraces the organization of pupils, occurred eighteen times in twelve lessons. Six times it involved “missing” pupils, either Tea thought there were too few pupils in the room (three times), or she missed particular pupils (three times). It was also about asking the pupils to take off their coats, or for example ask them to stop tilting their chairs. Twice she had to remind a pupil that she should go to have another teacher in that particular lesson. She also had to remind some pupils that the final deadline for handing in a project was approaching (cf. Chapter 10.4, Table 10-11).

#### Organizing teaching in general

Six times Tea talked about how to organize the subject. It could for example be about informing the pupils about a coming test (lessons ten and nineteen) or telling that they, when finishing their work-plans, would get some “weird tasks” (lesson fifteen), meaning problem solving. In lesson eighteen she informed the pupils about plans she had for the following four weeks, weeks that included Easter (cf. Chapter 10.4, Table 10-12).

#### **6.2.2 Involvement**

In addition to discussing issues that related to the teaching, also other issues were dealt with in the openings. These issues were part of the 2.3 minutes on average not spent on pupils’ preparation. The category *involvement* was observed to consist of two sub-categories, *personal relationship*, where the teachers talked about some of their own personal experiences, and *small talk*, which included some occasional talk. These will be further considered in the two following sub-sections.

##### **6.2.2.1 Small talk**

When the pupils returned to the class after having Norwegian with Tom, Tea could for example ask if they had borrowed some books (lesson 6B), or talk a little about Erasmus Montanus (lesson 9B). French words that were written on the board (lesson 8), and a strange word she had heard (lesson 10) were also subject to some attention.

There also occurred other small episodes when Tea and Tom opened up for talking about issues that were not related to the teaching or to management of the class.

##### **6.2.2.2 Personal relationship**

On five occasions in three lessons, I observed Tea and Tom talking about private issues, concerning their own experiences and issues that concerned pupils. In lesson 4A, for example, it first concerned Tori whose zipper had fastened and they made some jokes about that, and secondly they talked with Thom about why he was so smartly dressed

that particular day. And in lesson 9A, which happened after the first night with frost, Tea was concerned about her chrysanthemums while Tom talked about his car that stood in the garage “being depressed” (Chapter 10.5, Table 10-13).

The opening segments ended with Tea starting to talk about the mathematics as she, for example, did in lesson nineteen: “today we will focus on drawing more graphs” (observation 19). What happened after the transitions, I communicate in the next section.

### 6.3 Plenary segments

The plenary segments reported in this section lasted for  $M = 19.0$  minutes, whereof 18.0 minutes consisted of cycles of mathematics communication: expositions and IRFs (Wells, 1993).

Analysis of the plenary segments exposed five categories of actions taken; in addition to *explanation* (Section 6.3.1), I also observed periods of *management* (Section 6.3.3), *subject matter* talk (Section 6.3.4), *pupil activity* (Section 6.3.5), and additional *preparation* (Section 6.3.2).

What	Mean	SD	Median	Range
Explanation	14.1	6.7	15.9	0 – 22.7
Preparation	3.5	2.1	3.1	1.2 – 9.4
Management	1.0	1.7	0.5	0 – 7.7
Subject matter	0.7	0.8	0.5	0 – 2.7
Pupil activity	0.4	1.0	0.0	0 – 3.5

Table 6-4. Minutes of activities in the plenary segments.

The categories *explanation* and *preparation* both included two sub-categories (cf. Figure 6-4). The *preparation* happened both as preparation to plenary segments (Section 6.3.2.2, additional to the preparation that occurred in the opening sessions) and preparation to the individual seatwork (Section 6.3.2.1). The *explanation* consisted of expositions (Section 6.3.1.1) and communicative interaction between Tea and her pupils (IRF: instruction, response, and follow-up, Section 6.3.1.2).

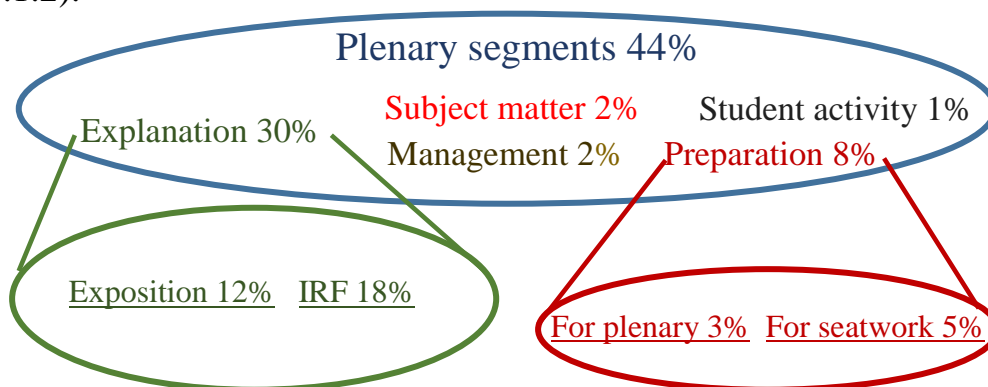


Figure 6-4. Activities included in plenary segments (cf. Table 4-6 for explanation).



### 6.3.1 Explanations

“Explanations are the process by which new material (i.e., concepts, procedures, connections) is connected to prior knowledge and located within the semantics of the particular discipline” (Leinhardt, 1993, p. 30). As explained above, in my study, I separate between expositions (teacher communicates new material, Section 6.3.1.1) and teacher involving the pupils (IRF, initiation, response, and follow-up, Section 6.3.1.2). An IRF was carried out when she, for example by posing questions, invited the pupils into the communication. Sometimes Tea posed seemingly rhetorical questions, which she did not follow up even if the pupils responded. Because they do not follow the IRF-pattern, I have categorized such episodes as expositions. I observed two patterns of IRFs: a triadic interaction between the teacher, the pupils, and the content (IRF-t, cf. Figure 10-5) and an “angled” interaction where the teacher took the responsibility for the response when the pupils did not provide a satisfactory explanation. She thus acted as a link between the pupils and the content (IRF-a, cf. Figure 10-6). This I further explain and exemplify in Section 6.3.1.2.

What	Mean	SD	Median	Range
Expositions	6.0	3.6	5.4	0 – 12.7
IRF-t	9.0	4.0	9.5	0 – 14.7
IRF-a	0.5	0.4	0.3	0 – 1.7

Table 6-5. Minutes of expositions and IRFs.

Each of the categories *expositions*, which accounted for 14.7% of the mathematics work, and *IRFs* (accounting for 21.8% of the time working on mathematics) will be illuminated by two examples from the classroom. Both excerpts will include sub-categories. I have provided additional excerpts in Chapter 10.5.2.

#### 6.3.1.1 Exposition

Expositions, i.e. Tea teaching the mathematics without being interrupted by the pupils (Figure 10-4), occurred as *expounding* of new content, or *review* of topic or tasks the pupils had worked on. One hundred and sixty expositions were observed in the twenty lessons,  $M = 0.8$  minutes, range 0.1 – 4.5 minutes.

To exemplify an *exposition* and its sub-categories *review*, *expounding*, and *contextualization*, excerpts from lesson nineteen will be presented. The lesson is chosen because all categories are represented; *review* of proportional functions (cf. Table 10-17), and a condensed illustration of both *expounding* and *contextualization* (cf. Table 10-18).

## Review

When they met for lesson nineteen, the class had worked on proportional functions for one week, and in the previous lesson, been introduced to linear functions. Tea started the lesson with reviewing what a proportional function is, and explained that they would use a “strategy”:

Tea When we shall draw a graph to a function, right, then there is a strategy we shall use. We are going to find [she interrupts herself], we have like a cake recipe almost like buns where it says what we shall do. [...] Right, often it is like that function can be, eee, four x [writes  $y = 4x$ ], [1] that is a proportional function, right, because it passes through

Boy [1] The origin (observation 19, Table 10-17).

The “strategy”, i.e. “a recipe” for using the coefficient and the constant to decide the graph, was first introduced to the pupils during the previous lesson. She now reminded the pupils about the formula for proportionalities, and by waiting for the pupils to complete the sentence, “it passes through”, she invited the pupils into the conversation, thus initiating an IRF, which I deal with in Section 6.3.1.2.

Two more excerpts illustrating review can be read in Chapter 10.5.2, Table 10-15, equations, and Table 10-16, multiplying decimals.

### Expounding and contextualization

The excerpt illustrating an *explanation* is also from lesson nineteen, approximately thirty-three minutes into the lesson. The pupils had worked for 17.5 minutes on drawing graphs of the functions  $y = 2x$  and  $y = 2x + 1$  in one coordinate system, and  $y = 3x$  and  $y = 3x - 1$  in another system. Tea had observed that some pupils had not yet understood how to draw graphs, while others were able to use the “strategy” mentioned in the previous paragraph. To help the pupils who did not know how to draw these graphs, she *contextualized* the function  $y = 2x$  suggesting that the coefficient could mean the price for a packet of gum. Tea first marked the coordinates in the system by telling that if they bought one pack of gum it would cost two crowns, and that it would be four crowns if she bought two packs:

Then it says y equals two times x. [6] Just like if this was two crowns per packet of gum [2] right? If I buy one packet of gum then it costs two crowns [marks (1, 2)], do you agree? [No response heard, 2] If I buy two packets of gum [3, while making dashed lines] it costs [1] four crowns. [3] Then we can think like this or we can put it in a table and say x, and y equals two times x [makes a table]. The table is a calculator, [1] it is supposed to help you if you cannot manage to imagine it in your head (observation 19, Table 10-18).

After marking the points (1, 2) and (2, 4) she drew a table she called a “counting machine” and calculated the y-values for  $x = 0$ ,  $x = 1$ , and  $x = 2$ . She then drew the graph for these values and continued with doing the same for the function  $y = 2x + 1$ , pretending that the “+ 1” was what one had to pay for the paper if one wanted to have the packs wrapped.

The exposition ended by Tea telling the pupils that the two graphs were parallel with the distance one "because both raises by two, and one over because it says plus one" (observation 19). Tea and the pupils interacted when working on the remaining two functions  $y = 3x$  and  $y = 3x - 1$ , thus carrying out IRFs (Section 6.3.1.2).

Another example of contextualization was observed in lesson twelve when Tea read a story called "the dream of the moped certificate". The story can be read in the Chapter 10.5.2, Table 10-22. More excerpts illustrating explanations and contextualization are also presented in Chapter 10.5.2, explanation of equations (cf. Table 10-19 and Table 10-20), and the contextualization of the task  $(-4) - (-6)$  (cf. Table 10-21).

### **6.3.1.2 IRF**

IRFs (initiation, response, and follow-up) occurred on average  $M = 9.7$  minutes per lesson in the eighteen lessons that included interaction between the teacher, the content, and the pupils. Two types of communication were recognized as an IRF, a triadic interaction between all the three actors in the classroom (IRF-t, cf. Figure 10-5) which, if Tea was not content with the responses, sometimes ended with her taking the whole responsibility for the topic (IRF-a, cf. Figure 10-6). Often Tea initiated an IRF by starting a sentence she expected the pupils to complete, but most often she invited the pupils to interact by posing closed questions ( $M = 24$  per lesson).

The following excerpt exemplifies an IRF that occurred in lesson five. The topic was algebra, and the focus was on parentheses. They had solved the tasks  $2 + (3 + 4)$ ,  $a + (a + b)$ , and  $4 - (1 + 2)$ , and were now to solve  $a - (a + b)$ . The interaction started with Tea asking what to do "to get further", which Tian was asked to explain. He responded by first explaining what happened with the "a"s (a minus a equals zero), then the complete solution (minus b) (observation 5, Table 10-23):

Tian A minus a equals zero, minus b

Tea Then there is a question, what, what, what did you do now? IRF-t

Trym Can't we only take it down, that is, what is

Tea Why did you think like that, Tian [points at him] IRF-t

Tian It will be plus and plus make plus and minus and plus

Tony It will be a, no it will be b

Tea Yes, because it was a rule we had [1] we said that when it was plus IRF-a  
and minus then it was minus. [1] We also said something else [2]  
minus in front of a parenthesis, what did we do with the signs in  
the parenthesis then? What did we do with the signs in the  
parenthesis?

Theo Calculated

Tea [2] With a minus in front of the parenthesis, what did we do with IRF-t  
all signs in the parenthesis?

Theo We first added them

Trym [4] Eee, we did something with them

Tea [2] What did Tian say IRF-t

Theo [2] Okay, we took a, for example a minus a and then we calculated  
it [Tea wrote  $a - a - b$ ] and then we took b and then it will be  
minus in a way

Trym We make it equal to the other

Tea We had to change signs within the parenthesis when it was minus IRF-a  
in front of it

Tea wanted Tian to explain how he solved the task. She appeared not to pay any attention to Trym's suggestion; rather she once more asked Tian why he thought "like that". Tea met his and Tony's response by reminding the pupils about "rules" they had, "rules" about what to do when having "plus and minus", and when having "minus in front of a parenthesis". Once more Tea asked what to do if there was "a minus in front of the parenthesis", and once more she appeared to pay no attention to suggestions from the pupils who were not asked to respond. In the third attempt, she asked for what Tian said, and when Theo then suggested "we took a, for example a minus a and then we calculated it" she wrote as he spoke, however adding the third operation before he mentioned it, making the task  $a - a - b$ . Neither this time did she respond to the inputs from the pupils, rather she herself told what the "rule" was about: "we had to change signs within the parenthesis when it was minus in front of it".

By first asking Tian how to solve the task, Tea initiated an IRF-t. Instead of having the pupils explain about the "rules" they "had", after Tony's response, she interrupted pupils' contributions, thus making the interaction an IRF-a. This was followed by a second IRF-t, also this ending with Tea explaining, thus ending the entire interaction with an IRF-a.

Additional examples of IRFs can be read in Chapter 10.5.2, Table 10-24 (equations), and Table 10-25 (number).

### **6.3.2 Preparation**

The plenary segments included *preparations* occurring at two distinct occasions: preparation for plenary work additional to the preparation carried out in the openings, and preparations for individual seatwork. The transitions between the openings and the mathematics work were triggered by Tea for example saying “okay we start” while the transitions between individual seatwork and the next plenary segment most often were initiated by Tea asking the pupils to put away their pencils. These transitions are categorized as preparation.

Preparation observed in the plenary mathematics work accounted for 17.8% of the time spent on plenary segments. It has to be noted that Tea sometimes, while preparing for individual seatwork, also reviewed some of the mathematics that was dealt with during the explanation (cf. Section 6.3.1). The preparation carried out in the openings and the plenary segments together account for 11.3% of the total time spent in the classroom.

#### ***6.3.2.1 Preparation for individual seatwork***

All lessons included one or several segments of individual seatwork, and were all, except for lesson fifteen, initiated by Tea explaining what the pupils should do during that work. On one third of the transitions between plenary and individual seatwork she only pointed to tasks on their work-plans or in the textbook (Hagen et al., 2006). Five times, it included a presentation of some handouts she had prepared, and once (lesson eight) she guided the pupils by doing the first task on the board.

Sometimes Tea wanted the pupils to carry out some individual seatwork to prepare for further plenary work as, for example, she did in lesson thirteen. The pupils had first drawn circles that represented the sun in their notebooks. She then wanted them to illustrate a solar eclipse covering 85% of that “sun” (Chapter 10.5.2, Table 10-26). After the seatwork, she had a pupil draw her suggestion on the board.

#### ***6.3.2.2 Preparation for plenary work***

Six times, there occurred additional preparation after the preparation carried out in the opening sessions (lessons 2A, 5, 12, 14, 16, and 17). In lessons five and fourteen, it was prompted by two pupils asking for a pencil and a book respectively. The remaining four concerned Tea preparing for the lesson as for example in lesson seventeen when reminding the pupils about the task they got the previous day as a preparation for the current lesson (Chapter 10.5.2, Table 10-27).

Tea used the board frequently. This sometimes led to her, in addition to drawing on it, also spending some time cleaning the board for further work. That work is also included in the category *preparation* for plenary segments.

### **6.3.3 Managerial issues in plenary segments**

The plenary mathematics work was sometimes interrupted by issues categorized as management, i.e. issues of non-mathematical character with which Tea dealt. Such issues occurred in sixteen of the lessons that included plenary segments, and occupied 5.5% of the time spent on plenary work. Three categories were observed: disturbances, pupil behaviour, and organizational matter.

#### **6.3.3.1 Disturbances**

Disturbances were observed to be initiated both from inside the classroom, and from someone coming in to communicate messages such as inviting to the canteen or asking for photos for the yearbook. The inside disturbances dealt with the milieu in the classroom such as talking about controlling the radiators or closing the curtains. Twenty-nine episodes of disturbances were observed,  $M = 1.2$  minutes, range 0.2 – 6.2 minutes. The maximum time of disturbances happened in lesson eight (introduction to equations): on nine occasions the mathematics work was interrupted, having the visit from the janitor as the most outstanding episode; for more than one minute and a half he was in the room looking at the sink.

#### **6.3.3.2 Pupil behaviour**

On fifteen occasions, Tea commented on pupil behaviour. Short messages such as “you have to look at the board, Tove” (observation4A) and “do you know, Troy, *out*, and have a lap around [the school] and back again. We do not sleep during the lesson, you know” (observation 19) were heard. In lesson eight, Tea took some time to talk about her worry for some of the pupils who did not participate in the plenary work (Chapter 10.5.2, Table 10-28).

#### **6.3.3.3 Organizational matter**

There also occurred a few episodes of organizational matter such as talking about how and what to work on next (lessons 2B, 6B, and 14), and that there were only a few lessons remaining before the semester-test (lesson 19). The organizational matters occurred in five lessons, total seven times and lasted for 3.6 minutes,  $M = 0.7$  minutes.

### **6.3.4 Background for learning mathematics**

Tea also spent some time in the plenary segments talking about mathematics that did not directly have anything to do with the content being considered. In lesson eight for example, introduction to equations, she started with reviewing what they learned in primary school:

$\square + 3 = 9$ . This led to a discussion about which level they learned this, and ended by Tea telling the pupils that they had to remember that she also had been a teacher in primary school. The background mathematics talk occurred  $M = 0.9$  minutes per lesson,  $SD = 0.8$  minutes, median = 0.7 minutes, and range 0.1 – 2.7 minutes.

### 6.3.5 Pupil activity

The category *pupil activity* is about pupils working on the board. This occurred in four lessons (lessons 9B, 13, 15, and 16), accounted for 2.1% of the total time spent on plenary segments, and was initiated either by pupils or by Tea.

Twice I observed that pupils asked for permission to come to the board to show their solutions, Tian who wanted to solve the equation  $4(3x - 2) - 2(x - 2) = 5(4 - 2x)$ , and Tone showing the solution to a matchstick puzzle, how many sticks to move to change the direction of the house:

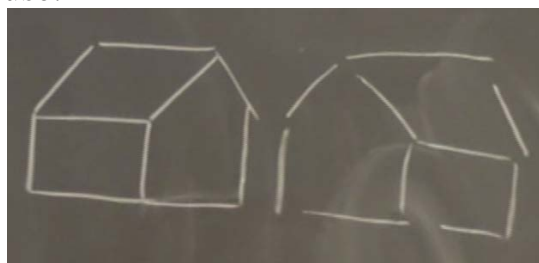


Figure 6-5. Tone solves a task on the board, video 20110111101527, 27:46.

In two lessons, Tea asked pupils to come to the board, in lesson thirteen Tara was asked to show how she illustrated the moon covering 85% of the “sun” as explained in Section 6.3.2.1, and in lesson sixteen some pupils were asked to mark coordinates in a coordinate system and to draw an arbitrary graph.

### 6.3.6 Plenary segments, a summary

I have, in this section, dealt with the plenary segments as I observed them in Tea’s lessons. I have outlined the different categories of actions taken, primarily by Tea, however also sometimes, but very rarely, by the pupils. Five main categories of activities were observed; preparation, explanation, management, pupil activity, and background mathematics talk.

The most frequent activity was explanation, accounting for 71% of the time spent on plenary segments and 30.3% of the total time spent in the mathematics classroom. The explanations occurred either as expositions where Tea explained the content (28.5% of the plenary work) or interactions between Tea, the pupils, and the content (IRF, 42.5% of the plenary work). The interactions occurred in two “forms”,

interactions between all three actors in the room (IRF-t) which sometimes, due to missing or poor responses, became an “angled” interaction (IRF-a).

Of the remaining plenary time, 17.8% was spent on preparations either in transition to plenary work or to individual seatwork segments. There also occurred episodes when Tea asked pupils to come to the board (2.1%), Tea talking about the content in general (3.6%), and she dealing with managerial issues (5.5%).

As preparations to the individual seatwork, Tea either asked the pupils to do certain tasks from their work-plans or pointing to tasks in the textbook. Sometimes she wrote tasks on the board, which she wanted them to solve, or they were asked to copy what was on the board. In the next section, I deal with individual seatwork, and refer from the segments where Tea monitored the pupils working on tasks that required knowledge about the mathematics.

## 6.4 Individual seatwork segments

The seatwork occupied 40.3% of the time spent in the mathematics classroom ( $M = 16.6$  minutes per lesson), whereof Tea was directly occupied with the pupils for 69.6% of the time ( $M = 11.6$  minutes per lesson). For the remaining time Tea cleaned the board, walked between the pupils, getting something from the closet, checking the radiators, answering the phone, or sitting at her desk.

Three ways of establishing contact were observed; the pupils raised an arm to indicate they wanted contact, pupils stopped Tea when passing, or Tea contacted a pupil herself. As shown in Table 6-6, the pupils contacted Tea for two reasons; they had a mathematics related question or wanted to discuss other issues. Except for lesson ten, Tea walked around monitoring the pupils working. On 19.8% of her stops she only took a glance before leaving, while for 19.5% she stopped for a while making some comments.

Who	What	Percent	Min/time	Min/lesson
Pupils	Mathematics	41.2	0.9	6.6
	Other (cf. Table 6-7)	9.2	0.3	0.6
Tea	Talking mathematics	19.5	1.0	3.5
	Only looking (math)	19.8	0.1	0.7
	Other (cf. Table 6-7)	10.3	0.2	0.6

Table 6-6. Categories of contact during seatwork related to total time in classroom.

This section will continue with reporting the mathematics (Section 6.4.1) and other issues (Section 6.4.2).



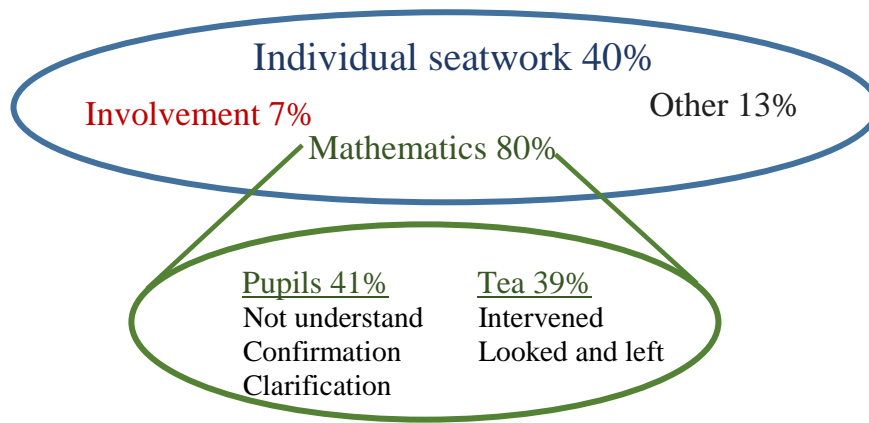


Figure 6-6. Work and activities included in individual seatwork segments.

#### 6.4.1 Mathematics work in seatwork segments

Four categories of mathematics-related requests were observed:

- Pupils expressed lack of understanding, 41.9%
- Pupil asking for confirmation, 17.1%
- Need for clarification, 14.0%
- Tea intervened, 27%

Need for clarification (third point) concerned short questions such as “how many grams are zero point one kilo<sup>38</sup>” (observation 2B), “is that Celsius” (observation 4A), “is the number of boxes important” (observation 17, Tom drawing the graph to  $y = 6.5x$  where  $x$  is number of boxes). These will not be further discussed. When Tea responded (first and second points), and intervened (fourth point), four different approaches were observed: explaining as she did in the plenary segment (repetition), showing to the board, confirmation with or without justification, and simplification.

##### 6.4.1.1 Lack of understanding

Three forms of responses were observed when a pupil indicated lack of understanding; Tea repeated what she had explained earlier, she anticipated her plans, or she pointed to what was written on the board.

##### 6.4.1.1.1 Repeating previous given explanation

For the individual seatwork, the pupils usually practiced tasks similar to what was explained in the plenary segments. In lesson one, multiplying decimals, Tea introduced two “strategies”: converting the decimals to fractions before multiplying, and “move-the-point”, which was about first making natural numbers then multiplying these numbers and place

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<sup>38</sup> The pupil left out “grams”, which Norwegians often do when referring to kg.

the point according to the number of decimals. The example used to exemplify the first strategy was  $4 \times 0.5$  which she converted to  $4 \times \frac{1}{2}$  before doing the calculation. Tron was the first to raise his hand saying he did not understand what to do with  $16 \times 0.5$ , and Tea responded:

Yes, if you instead of one-half had written, instead of point five had written one half [4], what would it then have been, sixteen times one half. [5] Would you try to write it? [3] One over two times, what did we do with the sixteen?

(Observation 1, Table 10-29.)

Tea repeated what was explained at the board, converting 0.5 to  $\frac{1}{2}$  and asking Tron to write it in his notebook. She also asked what they did with “the sixteen”, which, according to what was said in the plenary segment, meant  $\frac{16}{1}$ , making the task  $\frac{16}{1} \times \frac{1}{2}$ . The whole conversation can be read in Chapter 10.5.3, Table 10-29.

During pupils’ seatwork, Tea was observed several times responding similarly to the above and similar tasks. For six more lessons, such approaches were observed (2A, 2B, 13, 17, 18 and 19). For lessons 2A, 2B, and 13 it concerned practicing strategies almost similar to lesson one (see above), while in lesson nineteen they practiced a “strategy” for drawing graphs. In lessons seventeen and eighteen, the pupils copied what was on the board.

#### ***6.4.1.1.2 Anticipating the plan***

On a few occasions, Tea met pupils’ lack of understanding by explaining what she had planned to do later in the lesson. In lesson thirteen, for example, Thom asked for help to solve 20% of 800. Tea responded that one, according to the textbook (Hagen et al., 2006, p. 115), should write 20% as  $\frac{20}{100}$  and shorten it by ten:

Yes, right, then you suddenly get tasks that you in a way, that is, the textbook says [1] that you shall do it in a special way, it says that you shall [1] first divide [1] by ten and then multiply by two if it is twenty percent, right? (Observation 13, Table 10-31.)

She continued with explaining “a technique” she had planned to introduce next for the whole class: writing  $\frac{800 \times 20}{100}$  in Thom’s notebook she said, “it is no point in taking eight hundred times twenty divided hundred. You can shorten [...] and then the solution comes sailing” (observation 13, cf. Chapter 10.5.3, Table 10-31 for the entire instructional dialogue).

#### ***6.4.1.1.3 Showing to the board***

Lesson 14 started with Tea writing three examples of percent; “how much is 20% of 300”, “how many percent is 60 of 100”, and “what number gives 30 if you take 40% of it”. For the seatwork, she had

“picked tasks similar to those three types” (observation 14). The following excerpt, showing Tea guiding Tara to find what to pay for a 25% discounted swimming-suit, is randomly chosen from several similar conversations occurring between Tea and her pupils:

- Tea You can choose different methods, right, but, ee, you can do it like, like we did, ee, two hundred and sixty times
- Tara [3] I don't know
- Tea [1] The percent is [points to the board]
- Tara Twenty-five
- Tea Divided [4] with what should you divide, what fraction is percent
- Tara It is hundred
- Tea Yes, it is hundred
- Tara It is like what it costs is
- Tea Yes, and then you find what the [1] discount is, right, and then you have to remove to figure out what it costs. [inaudible]
- Tara Oh yes, how much I save
- Tea How much you save, yes (observation 14, Table 10-32).

At first Tea explained that Tara could do it “like we did“, which apparently did not help Tara. While pointing at the board she guided Tara through the IRF (thirteen turns).

Tea showing to the board was also observed in lessons one, five, eight, and seventeen. An example from lesson seventeen can be read in Chapter 10.5.3, Table 10-33.

#### **6.4.1.2 Asking for confirmation**

Seventeen percent of the requests concerned pupils asking if they had solved the task correctly. Three responses were observed; Tea confirmed and left, Tea confirmed and discussed the task further, and Tea helped pupils who failed. The last two responses will be exemplified in the forthcoming.

##### **6.4.1.2.1 Pupil failed**

When a pupil had failed, Tea often explained the same way as she did in the plenary segment. In lesson five, which focused on introductory algebra, however, Tea introduced a new “categorization” of a variable, “a surname”. Tone had solved  $8x + (3x + 4)$ , and called on Tea to ask if the answer was  $15x$ . Pointing to the task with her left index finger Tea asked if one can add a number having “x as surname with a number without a surname”:

- Tea But can you add a number having x as surname with a number without a surname?
- Tone [4] Yes
- Tea Can you? Are they the same family?
- Tone No, but they marry
- Tea Yes, but marrying, then it suddenly is multiplication, you know [3] tsj, tsj, so you cannot do that. [4] Then you have to take them separately, eight x plus three x equals [points at them]
- Tone Eight x plus three x equals [2] eee
- Tea Eleven x [1] plus [points at 4]
- Tone Four (observation 5, Table 10-34).

Tone had added all parts in the task, and suggested that she did so because the numbers “marry”. According to Tea “marrying” is multiplication, so in this case one had to take those “having x as surname” separately, reading and pointing to the x’s. She also added the two, read the operation, and pointed to the “4”, leaving to Tone to read the number before she left.

Using “surname” for distinguishing between numbers and variables apparently helped the pupils solve the tasks correctly, and was observed several times, both when working on algebra and on equations. Tea also suggested that the royal family could represent the number because “Märtha<sup>39</sup> had no surname before she married Ari Behn” (observation 5).

One example from lesson eight, equations, which includes use of surname as the unknown can be read in Chapter 10.5.3, Table 10-30.

#### ***6.4.1.2.2 Correct, but Tea wanted explanation***

In lesson 2A, Tina was working on multiplying decimals. She had solved  $15 \times 0.2$  when she stopped Tea asking if three was the correct solution. Tea responded by asking if the solution was reasonable, but Tina was not sure:

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<sup>39</sup> Märtha is a Norwegian princess.

- Tea Fifteen times point two yes, is that reasonable?
- Tina [1] I do not know
- Tea Why don't you know?
- Tina I am just unsure
- Tea You are unsure. [2] But, [1] well, it is correct. [2] Could we manage to explain this in some way? If you had converted it to, if you had converted it to [2] point two to a fraction; [points to the notebook] what would it then be?
- Tina [1] Two [1] tenth
- Tea Two tenths yes. You can write that, you know, fifteen [2] times [1] two tenth [1], yes, then you could put on the invisible one, right? Fifteen times two is [points to the numbers]
- Tina Thirty
- Tea Yes, [2] and one times ten is [Tina writes the answer], [2] thirty divided ten equals
- Tina It is three
- Tea Yes, can you see that? (Observation 2a, Table 10-35).

When Tina asserted that she was “unsure”, Tea confirmed that the solution was correct and asked if it was possible “to explain this in some way”. Tea pointed to Tina’s notebook asking what it would be if she converted “point two to a fraction”. Tina responded “two tenths”, which she, on Tea’s suggestion, also wrote it in her notebook along with the “invisible one”, making the task  $\frac{15}{1} \times \frac{2}{10}$ . While first reading and pointing at the numerators then at the denominators Tea guided Tina in the solving process, a process that appears similar to the one reported in Section 6.4.1.1.

### **6.4.1.3 Tea intervened**

Twenty-seven percent of the contact between Tea and individual pupils occurred when Tea observed something on which to comment. When she saw Turi struggled to calculate  $475 - 500$ , for example, she suggested that Turi could “put the number with the highest number-value [absolute value] on the top” (observation 4B), and to Tord, who drew all parallels with the axes when marking coordinates, she commented that it was not necessary to “draw all those lines” (observation 17).

#### **6.4.1.3.1 Simplification**

Longer stays as the one considered below, were also observed. It lasted for 7.5 minutes and exemplifies how Tea adapted her explanation to the pupil.

When monitoring Thor in lesson thirteen, Tea observed that he had only written the answer, 135g, to the task: “Jarlsberg-cheese contains 27% fat. How many grams of fat are there in 500 g cheese” (Hagen et al., 2006, p. 136, translated by the author). He was not able to tell how he

had solved the task<sup>40</sup>, so Tea made him draw the “cheese” (Figure 6-7, to the left) asking if he was able mentally to find the fat in a cheese that was twice that size, i.e. one thousand gram. Again he said no, so Tea drew the thousand gram “cheese”, marked an area indicating 27% fat, wrote “27% of 1000”, and  $\frac{27}{100}$  (Figure 6-7, to the right):

Tea And then the question is; twenty seven percent [1] of one thousand [writes 27% of 1000], [3] that is [3] if it had been hundred, then it would have been twenty-seven hundredths [writes  $\frac{27}{100}$ ]

Thor [2] Twenty-seven thousandths

Tea Do you believe that it would have been twenty-seven thousandths?

Thor No (observation 13, Table 10-36).

Tea’s example made Thor suggest twenty-seven thousands, a suggestion he withdrew when Tea asked if he believed it was so (cf. Chapter 7.2.5).

When Tea wrote  $= \frac{\quad}{1000}$  asking what should be on the top, Thor suggested “two hundred and seventy”. Tea agreed, adding, “two hundred and seventy grams are fat”, writing it in the “fat-area” and as the numerator in the fraction (cf. Figure 6-7).

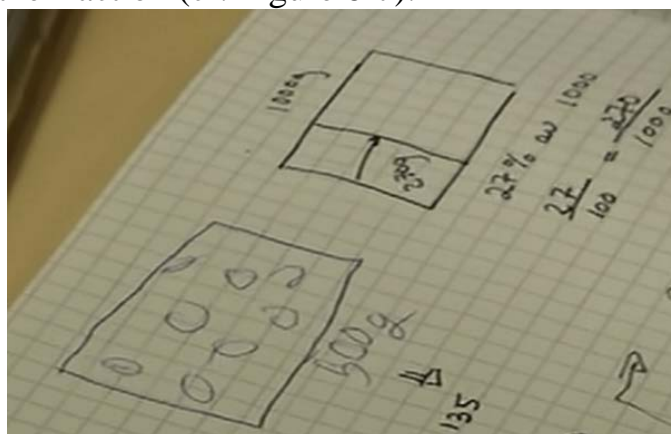


Figure 6-7. 500 g and 1000 g cheese.

She reminded Thor about his solution asking, “why did you get that”, but got the same response: “I do not know, I do not remember”.

Now Tea, by first pointing to the small and then to the big “cheese”, led Thor’s attention towards the relationship between the two:

Tea What, how, how big is this cheese related to that one

Thor Twice as little [points to the little one]

Tea Yes, or half the size, [1] half as big cheese, how will it then be with the fat?

Thor [1] Twice as big [points to the big one] (observation 13, Table 10-36).

<sup>40</sup> The whole conversation can be read in appendix 10.5.3, Table 10-36.

The first three utterances consider the relation between the sizes of the cheeses, and when Tea asked about the fat, Thor apparently thought about the fat as an area (recall the drawing).

Tea continued by asking if he then could tell what “half of two hundred and seventy” could be. Once more Thor responded negatively and added “I don’t know division”, so Tea changed approach; she drew two hundred-notes, one fifty and two tens (Figure 6-8) asking how that could be divided in two:

- Thor Hundred and ten  
 Tea [2] Many hundred would each get?  
 Thor Two hund [interrupts himself], [1] no  
 Tea Yes, two, if you should divide by two  
 Thor Two, twenty-five, hundred and sixty  
 Tea Hundred, they would have hundred each, do you agree?  
 Thor Mmm (observation 13, Table 10-36)

Tea did not comment on Thor saying “hundred and ten”, rather she asked how many hundreds each would get, apparently meaning that two persons should share “money”. Three more turns occurred before Tea provided the solution: “they would have hundred each”.

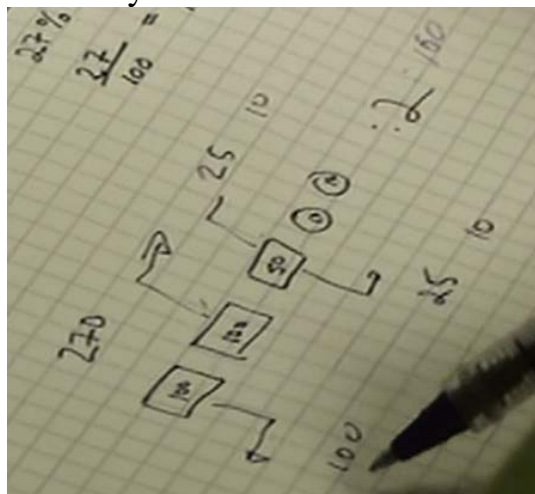


Figure 6-8. Sharing 270 crowns.

After Thor suggested dividing the fifty into two twenty-fives, (Tea drew two arrows) and that they should have one ten each, Tea asked how much “they” would get:

- Tea Much would they get, then?  
 Thor [1] Hundred and [1] thirty-five  
 Tea Well, you can divide (observation 13, Table 10-36).

Tea compared Thor’s suggestion with his initial answer and explained that since the original “cheese” was half the weight; the fat also had to be half of two hundred and seventy grams.

These excerpts reveal how Tea tried to make the problems less abstract; she doubled the weight of the cheese to make it an easier number with which to calculate, she changed to money, which is more concrete than weight, and she made drawings, which illustrated the concepts.

#### 6.4.2 Other requests than mathematics

For 2.9% of the time, either Tea approached a pupil or a pupil called on her to discuss non-mathematical issues. Five categories were exposed:

Who	Work-plan/tasks	Correction	Caring	Practical	Small talk
Pupils	29.2			16.7	1.4
Tea	12.5	25.0	6.9	4.2	4.2

Table 6-7. Percent of instances other than mathematics.

The most frequent issue concerned questions/clarifications about the work-plan, or pupils telling they had finished what was on the plan. Twenty-five percent of the instances were about Tea telling pupils to start working, while for 6.9% she approached a pupil to ask for how she/he was. The practical issues concerned, for example, asking for pencil or ruler, withdrawing curtains, requests for calculators, or messages from home.

#### 6.4.3 Summary

In thirteen lessons, the pupils worked on their work-plans during the seatwork segments. Five times, they got handouts, and in two lessons, the pupils copied what was on the board.

Contact between Tea and the pupils were established either on pupils' request, or that Tea approached the pupils. Four categories of issues were observed: pupils indicated lack of understanding, pupils wanted confirmation, pupils wanted clarification, or Tea intervened in pupils' work.

Tea met pupils' *lack of understanding* by explaining almost the same way as she did in the plenary segment, anticipating her plan, or drawing attention to what was written on the board. Tea's response to a request for *confirmation* depended on whether the solution was correct or not. If the pupil had failed, she responded as with lack of understanding, or, as exemplified in Section 6.4.1.2, inventing a new "concept", referring to the variable as "surname". If the solution was satisfactory, two outcomes were observed; she accepted and left or she asked for further clarification.

8.6% of the approaches concerned Tea *intervening* in pupils' work. This could be short comments, or more extensive work, as described in the episode above (Section 6.4.1.3 and Chapter 10.5.3, Table 10-36). The approach Tea chose when guiding Thor towards the solution of the task



about the fat in the cheese, I recognize as *localized knowing* as included in my conceptual framework.

If the pupils asked for other than the mathematics, it most often concerned the work-plan. Tea's most frequent non-mathematical comments were about reminding pupils to start working. I also observed that Tea on some occasions approached pupils to ask about their wellbeing.

## 6.5 Closings

In this section, I consider the third distinct part in Tea's lessons, the closings. As explained above, Tea's lessons followed a certain organizational structure that included an opening, a main mathematical part and often a closing. 77.3% of the lessons had a closing that included more than just for example a brief "then you have to take a break" (observation 6B), and occupied 3.6% of the time spent in the mathematics classroom.

Sometimes the closings included some attention to a non-mathematical topic, they will not be further considered, however included in Figure 6-9. Here I will only focus on the sixteen closings in the twenty-two observed mathematics lessons that included some attention to the mathematics. These varied in length, ranging from 0.3 to 5.0 minutes having  $M = 2.7$  minutes.

Three main categories of mathematical issues emerged from analysing the data; Tea communicated *further work* the pupils had to carry out, talking about *goal*, or *reviewing* the topic. In this section, I consider these three categories and their sub-categories, starting with the category *further work*. All excerpts that illuminate the considered episodes can be seen in Chapter 10.5.4, each referred to in the text.

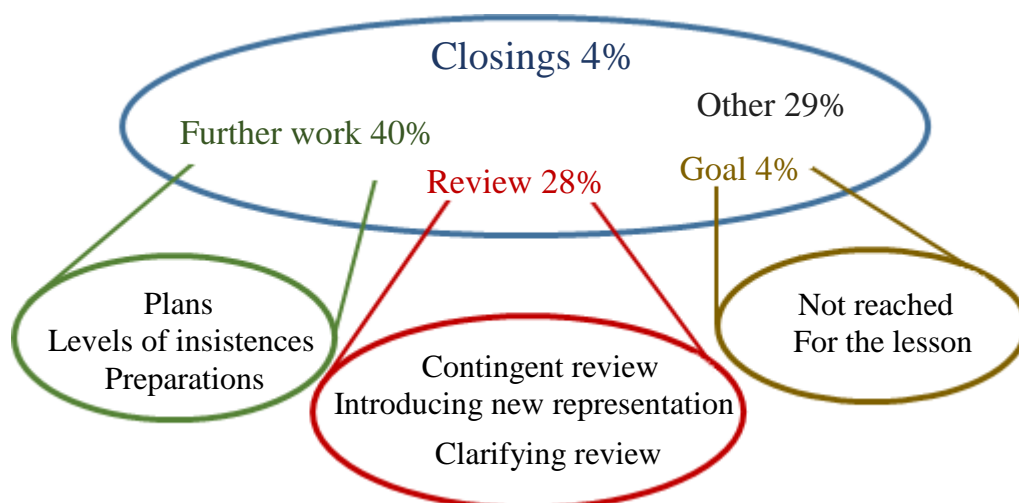


Figure 6-9. Overview of the work included in the closings.

### 6.5.1 Further work

This category embraces Tea communicating what *plans* she had for further work on an actual topic, *levels of insistences* for what they have to do to finish certain work during the week, and *preparation* for further work. It has to be noted that several of the episodes within these categories intersect. In this section, I first consider levels of insistences Tea communicated to make the pupils work, and deal with the plans she shared with her pupils before making a short presentation of preparation to further work.

#### 6.5.1.1 Levels of insistences

Tea communicated clear messages. Three levels of insistence for further work were observed in her classroom; levels differentiated by the meaning of the words she used and the pressure she put on these words. This sequence will present the three levels, referred to as *shall-do*, which was a message communicated in a soft tone, *smart-to-do*, also communicated softly, and *must-do*, which was communicated in a strict tone.

##### 6.5.1.1.1 Shall do

An example of shall do occurred in lesson one when Tea had observed that the pupils during the seatwork had been working on page ten in the textbook (Hagen et al., 2006), a page they were expected to do as homework the previous day. In the closing, Tea first pointed to that particular page before she told the pupils what other pages “that shall be done this week” (Chapter 10.5.4, Table 10-37). She also suggested a plan for how the pupils could carry out the work during the week. This

episode I also use as an example for the category *plan for the week* (Section 6.5.1.2).

#### **6.5.1.1.2 Smart to do**

Within the endings of five lessons, Tea used a *smart-to-do* level when she would encourage the pupils for further work. She could for example say that it would be smart to “make sure to get some math done for not blowing it this week too” (observation 4B), or “smart to solve some equations” (observation 11). Lessons 2A and 2B focused on dividing decimals. During the individual seatwork, however, Tea experienced that the pupils practiced multiplying decimals. When five minutes of the lesson remained, she asked the pupils to put down their pencils. She added that it would be “rather smart to spend time in the work session on Monday to work a little with maths so that none is left having done nothing”; she was worried that the topic for the day would “be like behind in your [the pupils’] heads“ (Chapter 10.5.4, Table 10-38). Twice per week, the pupils had the possibility to remain in school working on their work-plans after regular lessons. In this example, Tea pointed to the Monday work session. This I further deal with in Section 6.5.2.

#### **6.5.1.1.3 Must do**

“Must-do”-messages, putting pressure on the word “must”, were communicated towards the end of lessons 5, 6A, 16, and 17. In lesson 6A, which focused on algebra, she even communicated two “must-do”-messages: “this is a type of mathematics where it is of no use making an all-out effort. You *must* work a little everyday”, and “you *must* do this by tomorrow” (observation 6A).

For the individual seatwork in lesson five, Tea asked the pupils to do twelve small tasks about adding and subtracting parentheses. Some of the pupils were still working on the first task,  $8x + (3x + 4)$  when it was time for the break. Tea thus, in the closing, urged the pupils to finish the twelve tasks: “those tasks you *must* do by tomorrow”. The whole message can be read in Chapter 10.5.4, Table 10-39.

The episodes considered in this section include plans for what the pupils should do, however focusing on the level of insistence Tea communicated. I will next deal with the category plans, some of the episodes will thus overlap.

#### **6.5.1.2 Plans**

Tea often communicated plans at the beginning of the mathematics work. However, sometimes she also talked about plans in the closings. Once it concerned plans for the subject in general, five times Tea told

what they would do in the next lesson, and once she made a suggestion about how the pupils should work during the week.

#### ***6.5.1.2.1 Plan for the week***

As was explained in the “shall-do”-paragraph in the previous section, in the closing to lesson one, Tea read the pages the pupils were expected to work on during the week. She continued: “those are the pages we are going to work on, a little in school, little done at home, and a little in the work session” (Chapter 10.5.4 Table 10-38), thus helping the pupils to organize their work for the week. It will, however, be evidenced that the pupils did not always follow Tea’s suggestion (cf. Section 6.5.2).

#### ***6.5.1.2.2 Plan for the content***

Once, in lesson 9A, I observed that the closing only included a short message about the content. Tea then stopped the individual seatwork by only telling the group that it was time for them to go to the next lesson, and concluded by saying “there will be more equations” (lesson 9A).

#### ***6.5.1.2.3 Plan for tomorrow***

On six occasions, Tea talked about the plans she had *for tomorrow*. The example of this category comes from lesson sixteen when Tea urged the pupils to do a certain task “by tomorrow because we are going to talk about it” (Chapter 10.5.4, Table 10-40). Tea had made the task, which was about calculating the profit when selling soft drinks on a disco they were to hold for next year’s eighth graders.

#### ***6.5.1.3 Preparation for further work***

In lesson 6A, Tea was observed preparing for what would come next. The class had been working on adding and subtracting simple algebraic expressions that included parentheses. In the closing, Tea explained what they would meet next: “new tasks show up, tasks looking like this” (observation 6A), writing and solving  $2(x + 3)$ .

### **6.5.2 Review as a closing**

Review of the topic in focus of the day was observed in the closing of four lessons. Twice it happened as a consequence of what Tea experienced during the individual seatwork (lessons 2A and 2B), once Tea carried out a review because it led to bringing in another concept (lesson 12), and once the review occurred as a clarification of a task they had on a test before Christmas (lesson 13). These will be further dealt with in the following three paragraphs.

#### ***6.5.2.1 Contingent review***

In the individual seatwork in lessons 2A and 2B, as explained in Section 6.5.1.1.1, the pupils chose to continue practicing the topic they worked on the previous day, namely strategies for multiplying decimals. Tea was

concerned that what she had taught that day would “be like behind in your heads“ (cf. Section 6.5.1.1 and Chapter 10.5.4, Table 10-38), thus she reviewed the topic, repeating  $8 \div 0.1$  and  $24 \div 0.01$  focusing on converting the decimals to fractions before “turning the back one on the head [...] and multiply” (observation 2A).

#### ***6.5.2.2 Review for introducing a new representation***

The first lesson after Christmas focused on percent, and was the first time the pupils worked on percent in lower secondary school. As mentioned in Section 6.3.1.1, Tea started the plenary segment by reading a fictive story about a survey conducted on youths (Chapter 10.5.2, Table 10-22) which led to a discussion about percent and fractions as reasonable representations for magnitudes. Towards the end of the final plenary segment, Tea reviewed the connection between what the textbook called percent-form and fraction-form. She then also introduced the third representation, decimal-form (cf. Chapter 10.5.4, Table 10-43).

#### ***6.5.2.3 Clarifying review***

In lesson thirteen, the pupils met percent for the second time (cf. Section. 6.3.2.1). In the plenary segment, they had worked on representation of parts written as percent, fraction and decimals, while in the individual seatwork they solved tasks such as “how much is 25% of” or “how much is 75% of”. In the closing, she reviewed two tasks they had on the semester test at the end of the autumn semester. These were about how many percent of the pupils in a class had blue eyes if 60% of the boys and 60% of the girls had blue eyes, and how many percent had brown eyes if 30% of the boys and 30% of the girls had that colour. The majority of the pupils failed these tasks on the test, and Tea utilized their newly acquired knowledge to clarify their former misconceptions.

### **6.5.3 Goal**

Three times, I observed that she talked about goals, goals she had for the lesson (lessons 2B and 3) and goals they did not reach (lesson 8).

#### ***6.5.3.1 For the lesson***

In lesson three, which dealt with adding and subtracting positive and negative number, Tea explained that the goal for the lesson was to focus on thinking “whether the solution shall be positive or negative and not only having it coming out from the calculator” (observation 3). When working on multiplication and division of decimals, the goal was, in addition to learning how to convert decimals to fractions, also to learn strategies so that they could be independent of calculators (Chapter 10.5.4, Table 10-41).

### **6.5.3.2 Not reached**

Lesson eight focused on introduction to equations (cf. Section 6.3.4). Tea had apparently planned for more work, because at the closing she stated that they did not come as far as they should, “and for that, that we can thank very many, among others the janitor” (Chapter 10.5.4, Table 10-42). The mathematics work was interrupted several times; a pupil coming into the room asking for a sheet of paper, and some boys leaving the room to change for a school tournament. This last situation Tea utilized to tell that she was worried for the pupils that did not participate in the plenary work. In addition, the janitor showed up to have a look at the sink. All this slowed their progress, but as Tea concluded: they should be glad, at least for the janitor’s visit if that made the noise disappear (Chapter 10.5.4, Table 10-42).

## **6.6 Concluding remarks Chapter 6**

In Chapter 6, I have dealt with how Tea organized and taught her grade nine mathematics class. Statistical and organizational information were provided in Section 6.1, while the activity in the classroom were presented in Sections 6.2 (openings), 6.3 (plenary segments), 6.4 (individual seatwork), and 6.5 (closings).

Tea’s lessons normally consisted of three main parts, an opening, the mathematics work, and a closing. Five of her lessons, however, ended by Tea only telling that the lesson was over.

In the openings, Tea (and Tom) prepared for the lesson, carried out organizational matter, and communicated messages along with occasional “small-talk”. In these sections, Tea appeared as a caring teacher, devoted to her work as a class teacher. By involving the pupils in parts her private life, she opened up for a closer relationship to them than one can expect from a teacher.

The plenary and individual seatwork segments in Tea’s class accounted for 82.6% of the time spent in the classroom, whereof 45.7% was plenary *mathematics* work in which Tea was at the board carrying out expositions (14.7%) and IRFs (21.8%), while 48.6% occurred as individual seatwork segments where Tea guided her pupils (cf. Table 6-8). The remaining 5.7% occurred as “other” activities, such as managing the pupils and practical arrangements. In the individual seatwork segments, Tea monitored the pupils working. Contact between Tea and the pupils were established in three ways; pupils asked for help with the mathematics, pupils had other requests, or Tea intervening. Tea’s interventions most often concerned mathematics, however, sometimes also non-mathematical issues. While there occurred several instances that could be characterized as localized knowledge, only two examples of localized knowing were identified. This happened in

the seatwork segment in lesson thirteen, where Tea guided Thor towards the solution to a task about finding how much fat there were in a certain cheese, and lesson two when Tea halved six pencils to illustrate  $\frac{12}{2}$ .

From a study about how quality in teaching is understood, practiced, and experienced in school, KIO<sup>41</sup>, Haug (2012) reports that nine grade pupils were occupied with mathematics 76% of the time in class. On average Tea's pupils thus spent 24% more time on mathematics work than the pupils participating in Haug's (2012) study (cf. Section 6.1.2, and Table 6-8).

	Mathematics	Seatwork	Plenary segments <sup>42</sup>
Tea's pupils	82.6%	48,6%	45,7%
Pupils in Haug's study	7%	6%	30%

Table 6-8. Time spent on activities of mathematical work

Moreover, regarding the distribution of plenary mathematics work and the seatwork, Tea's pupils experienced significantly more plenary interactions (about 50%) than did the pupils in Haug's study. In addition, within the plenary segments, Tea and her pupils interacted in the form of IRFs, which accounted for 21.8% of the time they spent on mathematical work. The pupils in Tea's classroom were thus *actively* engaged with the mathematics in 70% of the time while focusing on the subject. Alexander (as cited in Haug, 2012) asserts that the probability for talk occurring in a lesson is  $\frac{2}{3}$ , of this the teacher talks for  $\frac{2}{3}$  of the time, of which  $\frac{2}{3}$  would be direct instruction or questions. The observations in Tea's class thus coincides with Haug's observations that teachers now offer more time on pupil activity than was normal, concretized by the  $\frac{2}{3}$  "rule"; it now rather concerns mathematical activity on the part of the pupils.

The closings were used to prepare for further work, some review, and talk about goals, discussing both goals she had for the lesson and goals they did not reach. Within what I refer to as *further work*, Tea expressed an insight into what would come next, both to prepare the pupils for what would come the next and the following lessons, thus connecting different parts of the mathematics (Rowland et al., 2005). She also suggested how to prioritize the work, and at what pace the pupils should approach the mathematics (management for learning, Jaworski, 1994). Moreover, her contingent reviews showed that she, during the lesson,

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<sup>41</sup> Kvalitet I Opplæringa translates as Quality In Education.

<sup>42</sup> The category "plenary" Haug refers to as collective teaching, while what I refer to as seatwork, he refers to as solving tasks.

had gotten an overview of what the pupils needed to hear more about. Leinhardt (1990) asserts that such location-specific knowledge develops in the course of teaching. I thus argue that the vast repertoire Tea conducts for her closing are characterizations of her local knowledge.

This study is about the teacher's knowledge for and in teaching mathematics, i.e. knowledge for facilitating pupils' learning mathematics. I therefore want to make a few comments about the understanding these pupils appeared to have about the mathematics. To learn mathematics one has to learn to solve mathematical problems or model situations mathematically (Watson, 2009). In this chapter, I illuminated that Tea worked hard to contextualize the mathematics; she made realistic stories, i.e. the moped certificate, and she connected the subject to practical situations in which the pupils were engaged, i.e. the school canteen and the school disco. I have reported from events where Tea modelled mathematical situations, which she, for example, did for Thor who was to find the amount of fat in a certain cheese. Despite Tea's efforts, excerpts provided in this chapter expose that many pupils lacked competence to decide what approaches to use. They sometimes applied procedures and rules without meaning, a difficulty Watson (2009) suggests appears due to having limited range of understanding of mathematical concepts, an understanding based on examples being similar to a prototype.

Having reported from the analysis of Tea's mathematics classroom it remains to report from our subsequent conversations. To get a clearer impression of the knowledge and knowing Tea deploys, in the next chapter, I consider Tea's comments on her actions and choices, analysed primarily using Ball and colleagues' (2008) framework Mathematical Knowledge for Teaching (MKT), however also the Teaching Triad (Jaworski, 1994) and the Knowledge Quartet (Rowland et al., 2005).



# 7 Tea's Knowledge For and In Mathematics Teaching

Within this chapter, I aim to provide an overview of the mathematical knowledge for and in teaching that my informant, Tea, holds. I use the analysis of our conversations as the major source for making her knowledge public. In the text, I use some examples to illuminate her knowledge, sometimes also illustrated by excerpts from her lessons. Sometimes, however, it was necessary to do it the other way around: showing examples of how she deployed her knowledge before drawing inferences about her knowledge/knowing.

As explained in Chapter 3.2.3, I used Ball and colleagues' (2008) framework Mathematical Knowledge for Teaching as an analytical tool to categorize Tea's knowledge *for* teaching mathematics. To specify particular aspects of Tea's knowledge deployed *within* the teaching, I used Jaworski's (1994) Teaching Triad and Rowland and colleagues' (2005) Knowledge Quartet, as will become evident in what follows.

The analysis of the conversations showed that the categories as described in Ball et al.'s (2008) framework were largely intertwined. For example, in any one particular statement and its following illustration, there could be evidence of Common Content Knowledge (CCK), Knowledge of Content and Student (KCS), and Knowledge of Content and Teaching (KCT, see for example 7.4.1 about a pupil providing a wrong answer). In another situation, one could recognize Tea's Knowledge of Content and Students (KCS), her Specialized Content Knowledge (SCK) as well as her Common Content Knowledge (CCK, cf. Section 7.3.2, introduction to negative number). Throughout our conversations, Tea spent eleven percent of the time characterizing her pupils; she was concerned about their lives both inside and outside school, about their competence in, and efforts for learning, the mathematics, i.e. pupils have a central place in her practice. I thus begin this analysis with first considering Tea's knowledge in relation to the category that has pupils in focus, Knowledge of Content and Students (Ball et al., 2008).

After discussing Tea's Knowledge of Content and Students (KCS) in 7.1, I deal with her Knowledge of Content and Teaching (KCT) in 7.2. In Section 7.3, I consider her Specialized Content Knowledge (SKS) and in Section 7.4 her Common Content Knowledge (CCK). I discuss the categories Horizon Content Knowledge (HCK) and Knowledge of Content and Curriculum (KCC) in Sections 7.5 and 7.6 respectively.

The analysis illuminated that there is more to Tea's knowledge for teaching mathematics than is included in the MKT-framework; her values and beliefs appeared to play a significant role in her practice as a mathematics teacher. Based on Rowland et al.'s (2005) component of beliefs in relation to the conditions under which pupils learn mathematics the best, in Section 7.7, I discuss how Tea facilitated the learning of mathematics.

Through the analysis of our conversations, I have been able to count the number of utterances/statements that could be organized into each of the categories in the MKT framework (cf. Table 4-10), and into categories of mathematical knowledge in teaching (cf. Table 4-11). Figure 7-2 illustrates the percent of comments that could be organized into the MKT framework, and Figure 7-3 those included in mathematics knowledge *in* teaching. However, I first provide an overview of the distribution between talk about mathematical knowledge *for* and *in* teaching (Figure 7-1):

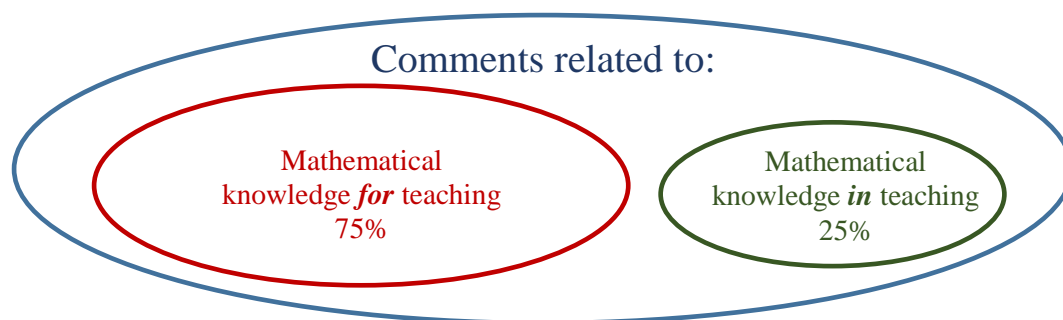


Figure 7-1. Comments on mathematical knowledge for and in teaching.

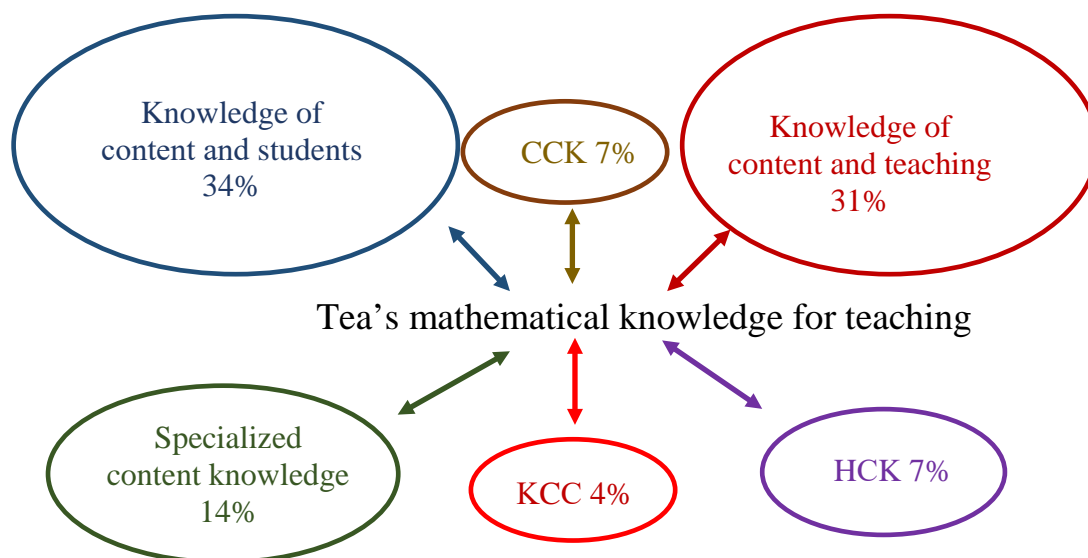


Figure 7-2. Percent of categories of mathematical knowledge for teaching.

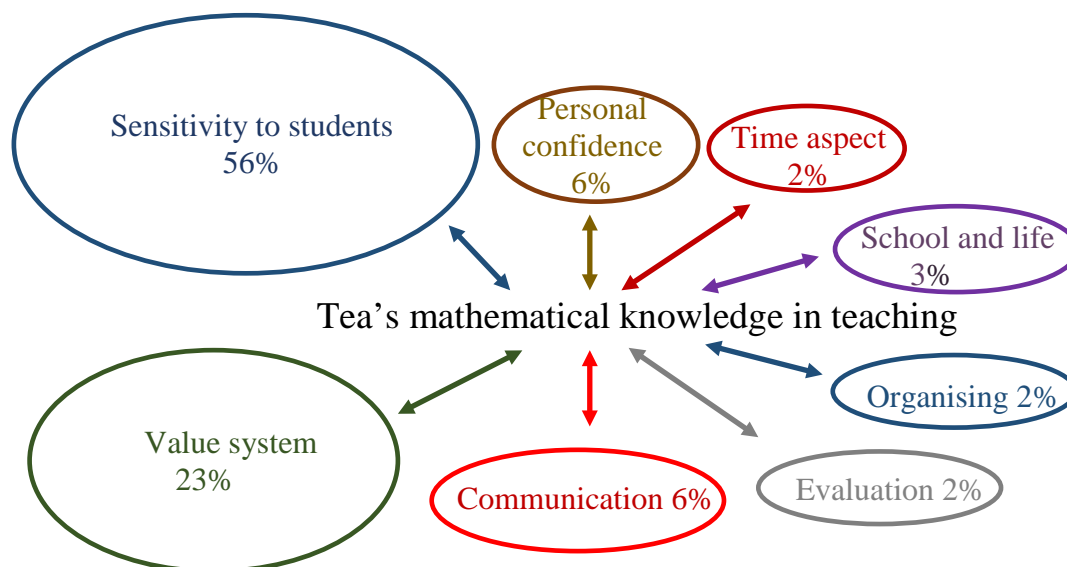


Figure 7-3. Percent of categories included in mathematical knowledge in teaching.

It has to be noted that several of Tea's statements were organized into more than one category (cf. Chapter 4.6.2).

In Chapter 3.2.3, I provided a short overview of the categories included in Mathematical Knowledge for Teaching (Ball et al., 2008), The Knowledge Quartet (Rowland et al., 2005), and the Teaching Triad (Jaworski, 1994). I also discussed how they relate to Shulman's articulation of a knowledge base for teaching (1986). For the reasons provided above, when presenting the excerpts illustrating each category in MKT, I experienced that some could be organised into several categories. I have attempted to clarify these within the text.

## 7.1 Knowledge of Content and Students

The domain Knowledge of Content and Students combines knowledge about pupils and knowledge about the content (Ball et al., 2008). In the pre-active phase (Cooney, 1988), this concerns predicting what pupils will find motivating, anticipating what they will find confusing, what they are likely to do, and whether they will find assigned tasks difficult or easy (Ball et al., 2008). For example, while in the mathematics lesson, it concerns interpreting pupils' incomplete thinking, and recognizing common student conceptions and misconceptions.

This section continues with focusing on what Tea believes motivate the pupils (Section 7.1.1) and her thoughts about pupils' competence (Section 7.1.2) before providing one example of how Tea interpreted and remediated a pupil's possible incomplete or incorrect mathematical thinking (Section 7.1.3), and considering how she resolved familiar errors (Section 7.1.4).

### 7.1.1 What motivates the pupils

Tea is concerned with pupils' understanding, which always will be the most important to her:

Primarily [1] I [2] want to make pupils understand, [2] that is, I [1] will never as long as I am doing this to [3], eee say that [1] there are any knowledge [1] or any way that is better than understanding (conversation 1, Table 10-45).

Based on the conversations with Tea, I interpret "understanding" in this situation as relational understanding (Skemp, 1976), an understanding she believes they will get when working on tasks they know will be useful to them:

I believe [1] that is, or I think it is very important that one always shows that what we are doing is no game. We do not play school, this is the reality, this we will be using, and that is how it is. We are not such a small island that sits [2] calculating stupid tasks in books (conversation 13, Table 10-46).

In an attempt to make pupils engage for learning mathematics, she thus tried to connect some of the topics to context she believed was interesting and motivating to them. This was for example accounting in the canteen run by the class when focusing on number (considered in 7.3.3, and Table 10-21), a story about youths having moped certificate when working on percent (cf. Section 7.2.4.2, and Table 10-22), and selling soda at a planned school disco when functions was the topic (Table 10-40).

Except for talking about the importance of making pupils understand what they do in school "is no game", Tea and I did not directly talk about pupils' anticipation, or what they liked. However, on one occasion she talked about efforts: she asserted that if they only had "bothered" (conversation 8, Table 10-72, cf. Section 7.5.2) they would experience the mathematics at upper secondary level as less time consuming. She also had some thought about pupils' competence for learning mathematics, which will be considered next.

### 7.1.2 Content and pupil competence

Tea was concerned that pupils have more or less ability for learning mathematics; there might be one percent talent, but learning requires "ninety-nine percent sweat" (conversations 1 and 12). To meet the requirements from the Ministry of Education about adapted and inclusive teaching (Kunnskapsdepartementet, 2015; Utdanningsdirektoratet, 2008), Tea differentiated by assigning tasks on pupils' work-plans based on their expected level of competence; what they would find easy or hard (Ball et al., 2008; Jaworski, 1994). Based on her knowledge of the pupils combined with knowledge about the challenge in the tasks (KCS), Tea thus differentiated by using pupils' work-plans, which is a common way of differentiating in Norwegian schools (Klette, 2007).

Tea's knowledge of the pupils also sometimes made her divide the class into homogenous groups, however within the regulations from the

Ministry of Education (Kunnskapsdepartementet, 2015b, also discussed in Sections 7.2.2 and 7.5.2). Concerning equations for example, the high-achievers got tasks such as  $\frac{x}{2} + \frac{x+1}{7} = x - 2$  (observation 9B), while the low-achievers were provided  $3x + 13 + 9x - 8 = 41$  (observation 9A) as the most challenging task. Her horizon content knowledge (cf. Section 7.5) combined with knowledge about the content and her pupils (KCS) made her let those who plan to study “theoretical mathematics” in upper secondary school learn about inverse proportions (conversation 16) and parabolas (conversation 18), while the remaining continued to draw graphs to linear functions (conversation 18).

### 7.1.3 Interpretation and remediation

On a few occasions, I observed her contingent “knowledge-in-action” (Rowland et al., 2005; Schön, 1983): when combining specialized content knowledge (Section 7.3) and knowledge about the pupils to adapt to their level of understanding (cf. Chapter 6.4.1.1.2), and to remedy for incomplete thinking or lack of knowledge. To illuminate this, I refer to one situation described in Chapter 6.4.1.3, where Tea for 7.5 minutes guided Thor towards the solution of a task about finding how many grams of fat there were in a piece of cheese (cf. Figure 6-7, Figure 6-8, and Table 10-36). She addressed Thor to check his homework, and the conversation reproduced in Table 10-36 occurred. To remind the reader; he was not able to tell how he had solved the task, and in her attempt to make him understand, they drew “cheeses”, worked on fractions, and at the end, exemplified the numbers by using money, which finally led them to the correct solution. Tea apparently here combined her knowledge of the content (CCK) and knowledge of the boy related to the content (KCS) to connect alternative ways of representing the content (Rowland et al., 2005). She even combined different concepts (percent and economy), which I interpret as an example of specialized content knowledge (SCK). She was, however, still not sure whether she had succeeded

I was a bit conscious at Thor, [1] a bit like to see if I was able to retrieve [...] Yes, for some reason [he had found the answer], but he had forgotten it already, but right, I thought I would be able to figure out how you found it, [1] have you used a calculator, [...] but it was like to test it [...] but right, it is far in there<sup>43</sup> [...] He made it [found the answer], and it seemed as, that is, you try somehow to see; "did you understand it", that is, it seemed like he was into at least [something], no, I do not know (conversation 13, Table 10-47).

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<sup>43</sup> A metaphor for being a low-achiever.

Thor was not able to recall how he had solved the task; however, at the outset Tea believed that she would be able to figure out how he did it. She made great effort to help him understand, but even if spending that much time guiding him, trying different approaches, she was not sure whether he understood the task. Even so, throughout my observations, I often observed her sensitivity to the low-achievers (Jaworski, 1994) by her spending much time with them.

I chose this episode as an example of Tea's KCS because it exposes more facets of Tea's knowledge for teaching mathematics. In addition to the CCK (understanding the mathematics involved in the task), it contains example of Tea's SCK and KCT (Ball et al., 2008) as well as connecting and contingently switching between different representations (Rowland et al., 2005), and sensitivity to students (Jaworski, 1994). I will thus, later in this chapter also refer to this episode, particularly in Section 7.3.2, in which I discuss representations.

This chapter continues with considering a second issue Ball and colleagues (2008) define as knowledge of content and students, namely resolving familiar pupils' errors.

#### **7.1.4 Resolving pupils' errors**

Ball and colleagues (2008) include knowledge of how to mathematically evaluate the nature of an error in the category SCK (cf. Section 7.3), while resolving an error pupils are likely to make are included in the category KCS. This section continues with considering how Tea met and resolved familiar errors (cf. Sections 7.1.4.2 and 7.1.4.3), however first considering Tea's attitude towards errors in general.

##### ***7.1.4.1 Tea's attitude towards errors***

Tea expresses a strong conviction that pupils learn better when expressing their knowledge aloud (conversations 1, 5, and 12), and that "one in a way grows a bit when daring to say this I know" (conversation 1). This made me ask how she thought about a pupil saying something incorrect, and how she would deal with the situation:

It is of course always a problem, [4] I clearly try [2] always, [2] or when it occurs, one tries to take it in, [3] tries to take it further. Not always sure it works [...] one says wrong things, right, [...] what I think is important then, it's a way to defuse it, "yes, yes, you are into something, but what is it, there is something here", that is, it is not so bad [2] because in a way, you've at least made some thoughts (conversation 1, Table 10-48).

In her preoccupation with having pupils expressing their knowledge aloud, Tea recognizes wrong answers/input as being problematic. When pupils do mistakes, she tries to defuse the situation and encourage the pupils by telling that they are onto something: "there is something here", that "it is not so bad" because they at least had "made some thoughts". Her sensitivity to the pupils (Jaworski, 1994) thus leads her to make

some general comments to ease the situation before commenting on the mathematics, an approach reminiscent of what Brousseau (1986) refers to as the Jourdain effect. Brousseau used that term for situations in which teachers can agree to recognize indication of a concept to avoid debating knowledge. In Tea's case, she would avoid telling that the pupils were wrong; rather she would respond in an encouraging manner, thus being at risk of misleading the pupils to believe they actually were into something.

In all lessons, I observed that she recognized and responded to lack of understanding, however sometimes also to errors (cf. Chapters 6.3.1.2, 6.4.1.2, and 6.4.1.3). She claimed to have "an idea that [3] all things that are wrong should in a way be rectified [2] so that [2] the last standing there at least is correct" (conversation 13). This section continues with first dealing with an example of how Tea remedied an error a pupil made in plenary (Section 7.1.4.2), then considering one example of how she met a pupil error while working individually (Section 7.1.4.3).

#### **7.1.4.2 Error in plenary segment**

The class had been practicing simple equations for about five minutes, and was now reviewing the solutions. They were to review  $x - 9 = 18$ , which was the last out of six tasks, and Tea waited for fourteen seconds before she asked Thao, whom she considers a timid boy, to read his solution. Thao, who "finally dared to say something" (conversation 8) first very quietly responded, "nine". Tea wrote the response on the board before recognizing the mistake, which she did without "extensive mathematical analysis or probing" (CCK, Ball et al., 2008, p. 401), and mildly (sensitivity, Jaworski, 1994) asked if it could be so, to which he responded "no, twenty-seven" (observation 8). Tea, who had her eyes on Thao, saw that he at once realized that it was incorrect, and he made the correction. In our subsequent conversation Tea's first comment was: "Finally Thao dared to say something [...] He is so gorgeous, and you saw that suddenly his face lit up" (conversation 8). To the actual happening, writing a wrong answer on the board, she commented:

In a way I think it is okay because the aim of doing like that, asking, that is what they should, right, the aim was [interrupts herself], what was so nice about this, well, we could see that it was the right answer. Thus, it was in a way quite all right (conversation 8, Table 10-49).

Resolving this error did not challenge Tea mathematically. Rather, I interpret her statement "mixing plus and minus [...] is easily done" (conversation 8) as an indication of the recognition of a common pupil error. The episode with Thao thus illuminates an example of her knowledge of content and student, KCS (Ball et al., 2008). After writing

the correct answer on the board, Tea deviated from her agenda by contingently (Rowland et al., 2005) utilizing the situation to introduce the concept of checking the solution to an equation. That action possibly eased the situation for Thao as well as accelerating the introduction of checking solutions, the latter exposing knowledge of content and teaching, KCT (Section 7.2).

As happening in the case with Thao, I observed that Tea sometimes responded to wrong answers by asking, “is it?” (For example observation 2b, 5, 8, 9A), a question the pupils apparently understood as meaning that their solution was incorrect, because the response was often “no” (cf. Section 7.3.4 for further consideration of such responses). Another way of meeting incorrect answers was sometimes to ignore the responses (cf. Trym and Tony in Chapter 6.3.1.2 and Table 10-23) and pose the same or rephrasing the question, a theme I comment on in Sections 7.2.4 and 7.4.1.

### ***7.1.4.3 Errors while working individually***

Due to the frequency of the contact between Tea and her pupils while doing individual seatwork, the pupils apparently spent more time waiting for clarification and help to solve the tasks (56%) than first trying before asking for confirmation (17%, cf. Chapter 6.4.1)<sup>44</sup>. Some of the pupils who asked for confirmation had incorrect answers. This was the case for Tone who wanted to know if her solution to  $8x + (3x + 4)$  was correct (cf. Section. 7.4.2). Tea’s first comment was “can you add a number having  $x$  as surname with a number without a surname”, which indicates that Tone had the solution  $15x$  (cf. Chapter 6.4.1.2 and Table 10-34). We did not actually discuss this as an error pupils are likely to make. Due to her quick response, however, I assume that Tea immediately recognized Tone’s (and some more pupils’) mistake as a common pupil error, which indicates KCS.

Tea’s remediation of this error was first to ask whether one could add numbers having a “surname” with a number without a “surname” (see previous paragraph). She then explained that one has “to add those who sort of have the same surname”, that one “always has to see if they are in the same family”, and if there are no “surname”, then “they are a special group, thus they have to be in a separate group” (observation 5). To help the pupils “understand” the differences between variables and constants, she “personalizes” the variables (having “surname”) and separates the concepts into two “groups”. For the pupils, however, it apparently became a “rule to remember”; when working on equations they used “surname” to separate constants and unknown.

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<sup>44</sup> The remaining 27% concerned Tea intervening.



### 7.1.5 Tea's KCS, summary

I have, in this section, considered how Tea's knowledge of the content related to knowledge of the pupils affected particular aspects of her teaching. Her belief in familiar context as motivation for pupils' learning led her to make great effort for finding and making interesting stories and tasks, for example the story about the moped certificate when introducing percent, and selling soda in a planned school disco when working on functions.

I have also discussed how her knowledge of a particular pupil made her contingently (Rowland et al., 2005) activate both specialized (SCK) and common content knowledge (CCK) in an attempt to help him understand how to find the amount of fat in a certain cheese. Tea here demonstrated what this study defines as localized knowing (see Chapter 3.3 for my conceptual framework). Moreover, contingently interchanging between percent and money evidences that she is able to make connections (Rowland et al., 2005) to other areas of the subject, thus facilitating relational understanding (Skemp, 1976).

The category KCS also includes familiarity to errors pupils are likely to make (Ball et al., 2008). In Section 7.1.4, I dealt with how Tea recognized (CCK) and resolved the mistake Theo made by mixing plus and minus when solving equations, and Tone's suggestion of adding constants and variables. For an experienced teacher as Tea, these are familiar errors, thus part of her KCS.

Tea's main concern is to help pupils understand, which she believes they do better if they understand that school is part of life, and not an "island" where one sits calculating "stupid tasks" (conversation 13, Table 10-46). In the next section, I consider more of how Tea managed the content to facilitate pupils' learning.

## 7.2 Knowledge of Content and Teaching

Knowledge of content and teaching is about deciding viable models for instruction, knowing how to deploy for example algorithms effectively, when to use pupils' remarks to highlight a mathematical point, and when to ask new questions or pose new tasks to further pupils' learning (Ball et al., 2008). The work of teaching includes planning for the teaching (Ball et al., 2008), thus integral to, however not explicitly mentioned as part of, categories in MKT<sup>45</sup>. Planning is important for facilitating pupils' learning, it is thus reasonable to consider Tea's planning in this section.

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<sup>45</sup> Planning is explicitly stated as included in Rowland et al.'s (2005) categories Transformation and Connection, and in Jaworski's (1994) Management for learning.

Moreover, I suggest that knowledge of sequencing the activity in the mathematics classroom, i.e. the combination of plenary and seatwork segments, as belonging to a teacher's KCT. These were considered in Chapter 6.1.3 (cf. Table 10-1), and will thus not be further dealt with, however somewhat used as foundation when considering classroom discourses (Section 7.2.5).

In this subchapter, I first deal with how Tea developed to be the practitioner she is today (Section 7.2.1) before discussing how she planned for teaching (Section 7.2.2). I also focus on different approaches to introducing a topic (conversations 3, 13 and 15, Section 7.2.4), structuring of the content (conversations 9, 15, and 18, Section 7.2.3), and on classroom discourses (Section 7.2.5).

### 7.2.1 Developing knowledge of content and teaching

When Tea started her career as a teacher, she was concerned about following suggestions from curriculum and textbooks: "I was very concerned for not doing as it was in the textbook and the curriculum, and also partly as I thought I had learned it myself" (conversation 5). Later she added "you are not very big when you come [start teaching], and you are afraid to do something wrong, and for not doing things well enough" (conversation 16). This has changed over the years; she has developed her own style:

I was very focused on the theoretical thinking all the time, that is [1] it should be set up as an equation; it should be done this way or that. Eee, while I somehow, as I got more confident in myself, eee, I have worked a lot more, that is a little more in different directions (conversation 5, Table 10-50).

As she got more confident, Tea changed her attitude towards teaching: from being dependent on textbooks and curriculum, to working "in more different directions". However, she has kept one idea she got from her University College teacher, an idea she valued: "he had such a philosophical approach to the content, it was so fun; why is it actually like this, why does one and one equal two" (conversation 16). "I always do that with my pupils too, why can't we divide by zero, then they say eee, and then I show how it is connected". By asking questions such as "why is that", "how did you do that", or "how can you know" she wanted her pupils to be aware of their solutions, not only providing the answer (observations 1, 5, 8, cf. Table 10-23 and Table 10-24 for examples).

An "old" idea adapted from her former University College teacher along with development of confidence in making own decisions have led to Tea carrying out the work of teaching as she does today. In the next four sections, I consider some results evolving from that development.

### 7.2.2 Planning for teaching

At Tea's school, planning for when to teach different topics occurred at three levels, all based on the national curriculum (Utdanningsdirektoratet, 2015). At the beginning of the academic year, the mathematics department sets the plan for the entire year. Tea and her colleagues then agree on when to teach different topics. They are at the outset "very concerned that it is the curriculum, and not the textbook, that steers" (conversation 2). When the curriculum at school level is set, Tea uses that curriculum to make the local mathematics curriculum for her class:

It takes place, that is, we then go down from all levels of planning and down to me, right, we skip all, because we have done that, we have agreed on when we shall have the different themes, and so on. And then I first look into the chapters in the textbook to look for what we are going to learn about that topic (conversation 1, Table 10-51).

For each chapter, Tea thus makes her local curriculum, first using the textbook to decide how much time to spend on each sub-topic presented in the chapter. Every Friday she decides what to do in the following week's lessons, and what pages in the textbook (Hagen et al., 2006) the pupils are supposed to work on during the week, either in school or at home. The plan, which occurs at two levels of difficulty (cf. Sections 7.1.2, 7.2.3.1, and 7.5.2 for further clarification), includes which tasks the pupils are supposed to solve, tasks primarily sourced in the textbook. At the end of the week, the pupils submit the week's homework before them, in plenary, sum up what they were supposed to learn that week.

In addition to using the textbook (Hagen et al., 2006), Tea also bases her planning on experiences from prior lessons, in particular lessons that did not turn out as wanted. She thus sometimes spends time going "around and think" (conversation 5, Section 7.2.4 and Table 10-56) about what she could have done differently.

As reported above (cf. Section 7.1.1), Tea primarily wanted to help pupils understand, however, she was also teaching for technique: "I think it always should be a possibility for understanding and a possibility for technique" (conversation 5). In our conversations, she particularly mentioned technique for working on algebra (conversation 6) and equations (conversation 9, observation 9A: "move-and-change-rule"). In lesson fourteen, she also provided three "recipes" for solving tasks with percent; finding the percentage, finding "the part", and finding "the whole". In the following section, I continue with considering how Tea structured the topics to facilitate pupils' learning.

### 7.2.3 Structuring the content

A teacher is responsible for structuring the lessons and the topic. In Chapter 6, I considered how Tea structured her lessons. They normally included an opening where they focused on general professional knowledge (see Section 7.7 for further information) and personal involvement (cf. Chapter 6.2), a midsection focusing on mathematics teaching and learning (cf. Chapters 6.3 and 6.4), and a closing that mostly concerned mathematics (cf. Chapter 6.5). The mathematics section included cycles of plenary and individual seatwork segments (cf. Chapter 6.1.4), dependent on the topic (cf. Table 10-1). In what follows, I consider her knowledge of structuring the topic to enhance pupils' learning.

Ball and colleagues (2008) argue for an interaction between specific mathematical understanding and an understanding of pedagogical issues as important for pupils learning, including knowledge of sequencing of particular content. I choose to expand this issue to include knowledge of structuring of the content, thus considering a broader aspect of planning than just sequencing particular topic. Hence, this section continues with dealing with Tea's use of work-plans (Section 7.2.3.1), and how she structured some of the topics (Section 7.2.3.2).

#### 7.2.3.1 Work-plans, *pro & con*

In Section 7.2.1, I dealt with how Tea developed as a teacher, hence her knowledge for teaching. There are also issues about school that has changed during her professional life, for example the introduction of work-plans, which also affected her knowledge about teaching the content. When Tea started teaching, the pupils had a small notebook where they, from one day to another, wrote their homework. Now teachers are required to make work-plans for a period, for example for one week. Tea normally welcomes the use of such plans, she is "a supporter of plans and goals and systems and all that stuff" (conversation 17), however sometimes she can get frustrated when pupils refuse doing what was not originally on the plan, she quotes the pupils: "It is not on the plan" (conversation 17), an attitude that irritates Tea:

It is not on the plan [4], and that, that is, so to speak, it is one of those things that, eee [3] like the old days when we had a note book for what to do when you get home. Thus, there are sometimes [1] that one actually [2] had needed [2] to have, so to speak [3] have such a book and say that those things you have to do.

Because, in a way, plans can always like, it is resistance in some sense [2] it was what was on the schedule, nothing else (conversation 17, Table 10-52).

Even if normally welcoming plans, Tea also experiences work-plans as problematic. Sometimes she would like to have the possibility to provide homework from one day to the other, but pupils' resistance towards doing what was not on the plan makes it troublesome:

One can say that this is the disadvantage with having weekly work-plans. Sometimes you wish you could give them homework from day to day because then you could better manage the homework (conversation 18, Table 10-53).

Tea claims that one could better manage homework if it was possible to assign it from one mathematics lesson to the next (KCT). There is no requirement from the school administration against having “from-day-to-day”-homework, but pupils’ resistance makes her reluctant to do it (KCS). Her KCT combined with her knowledge about the pupils thus results in that she is not always content with the requirement of using work-plans.

On the other hand, and as mentioned in Section 7.1.2, she uses the work-plans to individualize pupils’ work. This concerns management for learning (Jaworski, 1994), however still exposing KCT, as will be evident. To meet the Ministry of Education’s requirement for adapted teaching (Kunnskapsdepartementet, 2015), Tea differentiated by assigning tasks on pupils’ work-plans based on their expected level of competence, i.e. what she believed they would find easy or challenging. She then used the differentiation in the textbook (Hagen et al., 2006), however not following its suggested path, which was about all pupils first should follow a basic course (green course). A test would then decide whether one should continue practicing more basic tasks within the topic (following the blue course) or working on more challenging tasks (following the red course) (cf. Chapter 4.5.3.1). Tea, however, in the introduction assigned tasks coloured blue (easy tasks) for those who she considered as low-achievers, while the remaining first got tasks coloured green (basic tasks):

There are two work-plans; it is the one with the easy, those who are named the blue tasks in the book, right, which are the easiest tasks in the book. Those who have the easy edition, they have those on the work plan, [1] right, [2] while the other have the basic elements of the plan [3] now in introduction. [2] And afterwards they get a period when we sort of, [2] next week, for example, when [1] those who have the difficult schedule gets the red tasks, the difficult tasks, while I push some of the green [interrupts herself], the low-achievers, try to push them over to the green (conversation 4, Table 10-54).

Tea’s knowledge of the content and teaching (KCT), connected to her knowledge of content and pupils (KCS), led her to deviate from the textbook suggestions; she (and not the test) decided whom should follow the different courses. After the introductory work, she increased the challenge (Jaworski, 1994), trying “to push” some of the low-achievers to work with “green” tasks (basic course), while those following “the difficult schedule” continued with working on the tasks coloured red, “the difficult tasks”. Even if she mostly based her teaching on the textbook, Tea chose to deviate from the path suggested by the authors,

an approach not all teachers at the school followed (conversation 4). Her deliberate choices indicates that she possesses knowledge of the content and teaching (KCT) as well as knowledge of her pupils related to the content (KCS).

### **7.2.3.2 Structuring topics**

As mentioned in Section 7.2.2, Tea mostly followed the textbook (Hagen et al., 2006) and its organizing when teaching different topics, however not following the path for the work within the chapters as suggested by the authors (cf. previous section). She still had some objections, for example the order of when the textbook introduces Pythagoras' rule and congruence (conversation 16), how they deal with economy and velocity (conversation 15), and its use of "function machine" as an example of functions (conversation 18).

Concerning equations with fractions, her KCT was not yet fully developed; she has been "uncertain for thirty years" whether first to expand the expression to having a common denominator before multiplying with the denominator, or directly to multiply with the common denominator (conversation 9). This year she decided to teach the pupils to expand the fractions because many pupils "think it is okay to see that it is under all the way", however offering pupils to learn direct multiplication when she sees "they have control" (conversation 9).

Due to her experience of pupils' poor learning of functions in earlier years, this year she chose a practical approach, starting with a table where they wrote what one has to pay for different numbers of gum packages:

Eee, that [2] I did because I felt it went so badly last time, [1] I think [3] I think they became so, I do not think [4] that they, that it became [interrupts herself], I then had a very, like formal and defined, that is, I do not think I made it. I do not think they understood it well enough. And then you actually have to, and then you actually have to, it is like you have to try, to look if, well, is it something wrong about me? (Conversation 16, Table 10-55, also reproduced in Chapter 3.3 as an example of localized knowledge.)

Experiencing that the pupils did not learn when she used a "formal and defined" approach, including the use of function-machines (conversation 18), made her rethink, even thinking if it was something wrong about herself. This time she began with first drawing a graph, suggesting that it described the price for buying gum, then making a table that included the numbers of gums found on the x-axis and calculating the price (Figure 7-4), which corresponded to the values found on the y-axis.

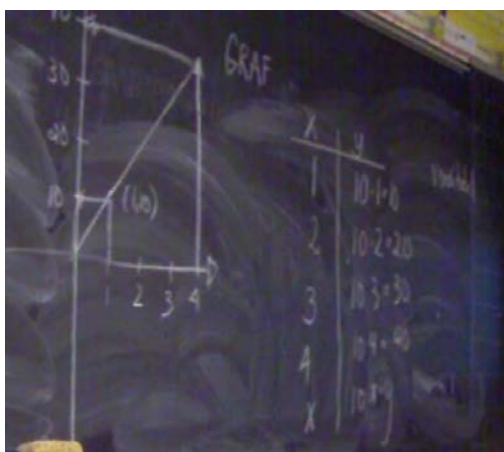


Figure 7-4. Graph and value table showing the price of buying gum

She would then introduce  $x$  “as part down” in the value table; “what if you do not know how many there are, you know, yes, we call them  $x$ . How will the math look like then?” (Observation 16).

Tea has never “reconciled with” that the textbook (Hagen et al., 2006) does not introduce Pythagoras’ rule and congruence until grade ten (conversation 16). Her knowledge about teaching and learning these topics suggests that they will not be sufficiently treated when working on them “over such a short time” (conversation 15). She was also concerned about economy and velocity. Her experience with teaching these topics made her focus more on them while teaching food and health and natural sciences respectively. In general, it appeared that Tea’s knowledge of the textbook combined with her knowledge of content and teaching made her rethink some approaches, which will be considered in the next section; she still however, chose to continue using the textbook.

#### 7.2.4 Approaches to teaching; viable models?

Over the years, Tea has changed some of her approaches to teaching, thus developed her KCT. This is, for example, due to experiencing poor learning, experiences that made her rethink her teaching:

[I]n a way you go around and think about how I could have done this differently. What is it they do not understand, and how should I have done it. That is, one does that, [2] how do I tell the pupils so they understand (conversation 5, Table 10-56).

Tea adjusted her teaching of topics in response to past successes or failures. For her this led to using a more practical approach to, for example, functions (conversation 18, see previous section), or having new ideas (conversations 5 and 13).

To illuminate this development, I will refer to a few examples resulting from her experiences: how she used errors to motivate for learning (Section 7.2.4.1), how she used context as motivation (Section 7.2.4.2), and context as “demystification” (Section 7.2.4.3).

#### **7.2.4.1 Using errors to motivate pupils for learning**

“I imagined that it would give another kind of engagement”

(conversation 6), Tea responded to my question about why she had the pupils evaluate a “test” she had “conducted”. The class had worked on some algebra while in grade eight and had, the previous day, started reviewing simple algebra. The mathematics work started with Tea saying: “Because I think it is fun having math-tests, I had one myself yesterday. The question is whether I did it correctly, and I want you to figure out” (observation 6A). The “test” included tasks with Tea’s “solutions” such as  $a + 2a - 3a + 2a = 2a$ ,  $a \times a \times 2 \times 3 = 5a^2$ , and  $6x - (6x - 2y) + 3y = y$ . In addition to solving the tasks to looking for errors, it was “another kind of engagement”; Tea’s reason for taking such an approach was for variation:

A little like for variation, really, firstly little to vary instead of giving them some additional tasks they should just calculate, ee. I think it was all right to give them that, partly because they would relate to a complete solution they had to think through, and also because then they, that is, [3] then they should first try it themselves, and then they should talk to someone, right. Because then they first in a way had to think through whether what I did was correct, decide themselves, and then they should argue with the neighbour. So, it was, it was an idea of trying to raise awareness, kind of [2] process around thinking about the answers (conversation 6, Table 10-57).

Tea made the “test” to vary her teaching and to raise awareness of processes of thinking. The “thinking” process included that each pupil should first evaluate the solutions individually for a while before discussing their results with a peer. The didactical intention was thus triadic: a) practicing algebra, b) deciding and defending solutions, i.e. practicing mathematical conversations, and c) encouragement for group work. I see all these as including processes of learning. Her KCT thus appears also to include variation in approaches as a means for learning as well as encouraging pupils to think before discussing with peers.

#### **7.2.4.2 Contextualization as motivation**

Tea was concerned about making pupils understand that one does “not play school”, but that school is “reality” (cf. Section 7.1.1 and Table 10-46). Her KCT thus included a belief that to motivate and facilitate learning, one had to make tasks she thought would be interesting to them. This was for example about using accounting in the school canteen when learning negative number (cf. Section 7.3.3 and Table 10-21), focusing on a coming school disco when introducing functions (observations 16 and 17, Table 10-40), and reading a story about youths having moped certificates when introducing percent, which will be consider next.

I illustrate how she contextualized by presenting “The dream of the moped certificate”, a story that should make the pupils understand the



importance of organizing measures into percent (also somewhat dealt with in Section 7.3.1):

A survey conducted by the Gallup Institute [1] exposed that two thousand four hundred and sixty-nine out of three thousand eight hundred and ten youths have a moped certificate before turning seventeen years. [1] Among these one thousand two hundred and thirty-eight have earned their own money to get the certificate. The others are divided into two groups, one has spent the money they got for their confirmation, and the other got the certificate as a present. During the first year having the certificate, six hundred and seventy-eight had different accidents while driving their moped, [1] three of those said that they did not use a helmet (observation 12, Table 10-22).

The story is about a fictive survey conducted by a Gallup Institute, and contains number of pupils having moped certificate, how many who had paid for the certificate themselves, how many who have had accidents, etc. Tea expected that the pupils would understand the importance of percent because they would “get confused when there are only numbers” (KCS, conversation 13), thus understanding that “if the numbers were systematized”, like in percent, they would be “easier to perceive” (KCT) (conversation 12).

#### **7.2.4.3 Contextualization as “demystification”**

When re-introducing algebra, she tried to make it less “creepy” (conversation 6, Table 10-66) by comparing variables (which she calls letters) to known concepts. She started with talking about a digit having different value according to its place in a number, and a number having different meaning depending on its notation (further considered in Section 7.4.2). She further exemplified variables by using a Norwegian word, “bønner”<sup>46</sup>, which “means different in different settings” (conversation 6). Thus, based on her experience of pupils seeing algebra as “creepy” (KCS), Tea attempted to “demystify” variables by highlighting that numbers and words could have different meanings dependent on the situation in which they occur. Knowing that the value of a digit depends on its position in a number, I interpret as Common Content Knowledge (CCK). However, using it to exemplify a key mathematical issue, here variable, I interpret as Specialized Content Knowledge (SCK, Section 7.3), while the entire idea of using this approach, appears to derive from her knowledge of content and teaching (KCT).

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<sup>46</sup> Beans and prayers.

She had a similar approach when explaining and visualizing the result of multiplying two negative numbers, as will be considered in 7.3.3.

### 7.2.5 Classroom discourse

Tea's KCT also includes knowledge about sequencing the mathematics activity in the classroom, the combination of plenary and individual seatwork segments:

It is somewhat dependent on the theme and what they know, or do not know in advance. Eee, often it will be like, I talk in the beginning and they work towards the end, but if there are small topics then it happens that I talk a little, they work a little (conversation 4, Table 10-1).

How teachers choose to start their mathematics work in a lesson, I also interpret as KCT. In Chapter 6.3, I considered the plenary teaching, while I, in Chapter 6.4, dealt with the individual seatwork segments. In this section, I consider some of the communication of mathematics happening in the classroom to the extent Tea commented on it. I interpret the teacher-led communication as sequencing for learning, thus part of KCT. Based on my observations in Tea's classroom, however, I also assert that some of it could be categorized as KCS.

As exemplified in Chapter 6.3.1, Tea invited the pupils to take part in plenary discussions; she believes that pupils learn better when formulating their knowledge aloud because they then assure to themselves that "this I can" (conversation 1, 5, 8, & 12). We did not explicitly discuss the different approaches she took; why the conversations took the form they did (the different IRF's). We did however talk about the wait-time before allowing pupils to respond to her questions, as for example in observation eighteen. Even if some pupils eagerly signalled that they wanted to respond, as often happened during my observations, she "did not want them to say something yet" (conversation 18), she rather wanted them to spend some more time on discussing the issue. I also, in the same lesson, observed the opposite, that she waited quietly for a discussion to stop before continuing. To my question about whether she considered following up the discussion, she responded: "No, just stop them, but without saying shush, because I did not want them to feel it was wrong, and it was not [wrong]" (conversation 18).

Tea thus welcomes communication in class, also when the pupils were practicing mathematics, but it had to follow some norms. For example, when a pupil needs some support from a peer or wants to discuss some subject matter, it has to have "some form, that is, they have to sit, if they will discuss, then they need to sit down" (conversation 10).

Some of what I observed appeared to follow certain "rules" in the class, for example that the discussion between the pupils stopped when

Tea stood quietly at the board. Another “rule” I observed, however probably tacit, was that the pupils appeared to believe they were wrong if Tea responded to their suggestion with a question like “is it?” (Cf. Sections 7.4.1.1 and 7.4.1.2). I assert that happened due to a classroom norm, which I further consider in Section 7.7.2.

### 7.2.6 Tea’s KCT, a summary

In this section, I have considered Tea’s Knowledge of Content and Teaching. I have dealt with how Tea’s knowledge of teaching mathematics has developed over the years, from reflecting suggestions in curricula and textbooks, to appearing confident with own decisions, often basing her teaching on own experiences.

I have also dealt with how school had changed during her career, and how that has affected her knowledge for teaching mathematics. For example, requirement for providing weekly work-plans, which, due to pupils’ reluctance to do tasks that was not on the work-plan, sometimes frustrated her. She still used it for what she meant was to the best for her pupils, namely to meet departmental requirement for adapted teaching. Every week she makes two work-plans; when introducing a new topic she makes one with easy tasks for the pupils she experiences as low achievers and one plan that includes basic tasks for the remaining pupils. After a while, her KCT leads her to make plans with increased challenge for the majority of her pupils.

The analysis exposed that her KCT also includes knowledge about how to vary the lessons between plenary and seatwork segments, and how to start working on the mathematics; either Tea carrying out expositions, they had plenary discussions (IRFs), or the pupils practiced mathematics (seatwork). Through the observations and analysis of the classroom discussions, I also got an impression about her KCT as including certain structures in her comments to pupils’ responses; if she followed up by posing a question, the pupils appeared to be certain that their suggestion was wrong.

Her Knowledge of Content and Teaching was also demonstrated when she deviated from the path suggested by the authors of the textbook, and making contextualized tasks to motivate for learning, tasks she believed would be of interest to the pupils. Examples were a survey about youths and their moped certificate (percent), about the school canteen (negative number), and about a coming school disco (functions). I also interpret her varying the teaching in an attempt to motivate pupils for learning as evidence of KCT. When orchestrating these approaches, one also recognized KCS, discussed in Section 7.1, as well as SCK, which I discuss next.

## 7.3 Specialized Content Knowledge

Specialized content knowledge is mathematical understanding and skill unique to teaching. For example, it is about choosing, making, and using mathematically appropriate representations, understanding different interpretations of the arithmetic operations, making features of particular content visible and learnable to pupils, analysing errors mathematically, and justifying own mathematical ideas (Ball et al., 2008).

In this section, I focus on Tea's specialized content knowledge, and consider three aspects as they were exposed in the data. It is about how Tea worked for making features of algebra, percent and functions visible to her pupils (Section 7.3.1), how she used mathematical representations (Section 7.3.2), and how she justified that the answer is positive when subtracting a negative number (Section 7.3.3).

### 7.3.1 Drawing attention to particular content features

For helping pupils understand the features of percent, Tea used two approaches, in the first lesson by drawing attention the "hassle" when one mentions many numbers in a row, and then in the following lesson by visualizing a percentage (solar eclipse).

As dealt with in Section 7.2.4.2, Tea read the "result" of a survey about youths having moped certificate to motivate for learning percent and understanding the usefulness of it (KCT). Attempting to make the pupils understand what percent is about, she read the text which included three four-digit, one three-digit, one two-digit, and two one-digit numbers (cf. Table 10-22), a story she believed would make pupils confused (conversation 13, KCS). The story gave rise to a discussion about the perception when one reads many numbers over a short period (here one minute) instead of for example "rounded numbers" (Tony, observation 12). Tea followed up by asking, "If this has been all youths in Norway, was it many or few who had the certificate?" to which Trym suggested "half the population", Tony "more than half", and Troy "a little more than two thirds", before Thom first suggested "seventy five percent", then "circa sixty percent". This initiated further discussions about percent related to fractions (connections between different meanings and descriptions of particular concept, Rowland et al., 2005).

The following day, Tea first had the pupils draw a circle and mark 85% of it, a part that should illustrate the solar eclipse occurring during the lesson. For Tea, using these two approaches, first making the pupils "confused when there are only numbers" (cf. Section 7.2.4.2) then having them visualize a percentage actually "is what percent is about" (conversation 13):

That is what [1] is the message with percent, you use this because it is such understandable amount, eighty-five percent, yes, then there are only fifteen again. Yes, but that is much, that is, teaching them to [2] that is, every time like

[2] when you can use that kind of thinking, then you much better communicate what you mean, that is sizes, yes. So therefore, I think it was perfect when they said on the news today, eighty-five percent. They did not say area, they did not say so and so, they said percent (conversation 13, Table 10-58).

Tea used this approach, drawing the sun (a circle) having the pupils marking how much of it would be invisible due to being covered by the moon (85%), to help them understand how a visualization can communicate the size of a percentage. Marking 85% of a circle, I interpret as an example of common content knowledge (CCK). However, in combination with the story she read the previous day, it becomes an important didactical tool for supporting the pupils to understand percent.

By reading the result of the “survey”, she attempted to make the pupils understand that “if the numbers were systematized” (CCK), as in percent, they would be “easier to perceive” (conversation 12), and then making the drawing which “communicates sizes better” (conversation 13), she attempted to make the feature of percent visible to them. Combining the two representations of percent as a “mental model that is easier to perceive [than only numbers]” (conversation 12) and a visual model for enhancing understanding the concept, is a competence “not typically needed for purposes other than teaching” (Ball et al., 2008, p. 400). It goes beyond the tacit understanding most people have of percent, thus representing what I interpret as Specialized Content Knowledge (SCK).

I have, in this section, barely dealt with different representations as a means of enhancing pupils learning. Since the competence of using different representations is regarded as specialized content knowledge, in the next section, I consider a few more examples of how Tea represented some mathematics topics in different ways.

### 7.3.2 Representations

In addition to the mathematics, the curriculum for common core subject of mathematics (Utdanningsdirektoratet, 2013) displays the purpose as well as some major competences and skills pupils are expected to learn in school, such as competence to read and write mathematics, and communicate mathematics orally and graphically. Hence, the teachers are also expected to be competent in these basic skills (Kunnskapsdepartementet, 2010). This section continues with considering how Tea made different representations, which either could be rooted in her Specialized Content Knowledge, or her Knowledge of Content and Students.

Tea was particularly concerned about making drawings, both as a general illustration, as a means for illustrating the mathematical idea, and a combination (observations 4A, 4B, 12, 13, 16, 17, and 18) in addition

to representing the topics in written or oral form. If the representations result from her knowledge about the pupils, they could be part of her Knowledge of Content and Students; otherwise, they could represent her Specialized Content Knowledge as will be exemplified below.

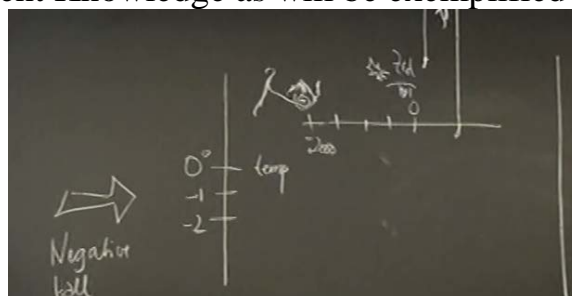


Figure 7-5. Illustration of negative number with decorations, observation 4A.

When reviewing negative number, she first focused on number lines because they were “very concerned about such in the class, you have a year-line with the birthdays there [on the classroom wall], and you have a time-line there, right” (conversation 4). She thus built on pupils’ interest when she, in the introduction, pinpointed that the point zero depends on the context. Starting with drawing a vertical line, illustrating a thermometer, she told that one then had decided a point “zero” at the temperature water freezes (observation 4A and 4B, cf. Figure 7-5). She also referred to natural sciences (lateral knowledge, Shulman, 1986) when highlighting that Fahrenheit and Kelvin scales have points zero defined at other temperatures compared to Celsius<sup>47</sup>. Moreover, the point zero on the horizontal number line in Figure 7-5 shows the birth of Christ, thus an example of the number line one usually uses in school. When introducing negative number, Tea activated more of the knowledge categories defined in Ball et al.’s (2008) model. She, for example used her knowledge about pupils’ interests in number lines (KCS) when selecting and using mathematically appropriate representations for explaining negative number (SCK). Considering these concepts separately, point zero defined as when water freezes, and when Christ was born, I interpret at Common Content Knowledge (CCK).

In Chapter 6.4.1.3 and Section 7.1.3 (cf. Figure 6-7, and reproduced in Table 10-36), I considered an episode of Tea helping Thor towards the solution to how many grams of fat there were in a five hundred gram “cheese” containing 27% fat. He did not understand how to approach the

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<sup>47</sup> 273.16 K equals 0 °C defined from the triple point of water (where solid phase, gas phase, and liquid phase occur in equilibrium) => 0K equals - 273.16 °C. (<http://www.bipm.org/en/publications/si-brochure/kelvin.html>). 0 °F = - 32 °C is the lowest temperature that has been possible to produce in a laboratory (<https://snl.no/fahrenheit>).

task, thus Tea suggested using drawings, first drawing the five hundred gram “cheese” which did not help him, then doubling the amount, drawing a thousand gram “cheese”. Thor then could tell that this last one included 270 grams of fat; the problem that followed was halving that number. Tea “converted” the 270 grams to 270 crowns, drawing two one-hundred-notes, one fifty-note, and two ten-coins to be shared (cf. Figure 6-8), which finally made Thor come to 135. As indicated in 7.1.3, this process appears to include more categories of teacher knowledge. Finding the appropriate representations and contingently being able to switch between different representations, here gram and crowns, I interpret as SCK. However, if she knows Thor as more familiar calculating with money than percent, this action is a result of her knowledge of content and students (KCS, Chapter 7.1). The episode also illuminates her competence to change approach to adapt to Thor, i.e. to contingently unify subject matter (Rowland et al., 2005,) <sup>48</sup> as well as her sensitivity to pupils’ level of competence (Jaworski, 1994). The mathematics per se, being able to calculate how much fat there is in the cheese, and dividing 270 in two, I assert is common content knowledge (CCK).

The above examples illustrate Tea’s competence to make different representations and connect different mathematical concepts. She thus acts as a role model for her pupils when urging them to “find different ways of doing things”, and also using different forms for representing solutions:

I want the pupils to find different ways of doing things, right. When we talk about solving problems and when we work on whatever it may be, like major tasks then I am concerned that they have to draw, [2] draw. All sorts of things can be drawn [1] and then I am concerned that they should [1] think, try to think through the problem and that they should try to find a solution to the problem and that they do not necessarily have to show calculation, but they must explain what they did. That is, there is always another way to do it and that, therefore [2, interrupts herself], and that is what I am trying to, I am trying to communicate, I think, I think myself that yes, there are methods [2] and yes, there are other ways (conversation 13, Table 10-59).

Knowing that pupils shall learn to communicate mathematics in different forms, including graphically, is curricular knowledge (Utdanningsdirektoratet, 2015). By inspiring her pupils to use different approaches when solving tasks, she thus uses her knowledge about the curriculum: “they do not necessarily have to show the calculations”, but

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<sup>48</sup> Even if it was Tea who addressed the pupil and not due to a contribution “made by her student during a teaching episode” (Rowland et al., 2005, p. 266).

they must “explain what they did”. How she made and presented different forms of representations, however, can be a result of her Specialized Content Knowledge (SCK), or her Knowledge of Content and Student (KCS) as illuminated in the example when introducing negative number (illustrated in Figure 7-5).

### 7.3.3 Justifying own mathematical idea

Competence in explaining and justifying own mathematical ideas are considered as SCK (Ball et al., 2008). Tea made great effort to make the mathematics familiar and understandable to her pupils (cf. Sections 7.1 and 7.2), which also included justifications for her ideas. One example is the story she made for finding a reasonable explanation to why subtracting a negative number makes an addition (conversation 4). She even talked to her former college teacher for a suggestion, but had to make the explanation herself (conversation 5). Starting from the task  $(-4) - (-6)$  (arithmetic representation, observation 4A), the story goes: the canteen having four in debt  $(-4)$ , they borrowed six  $(-6)$  from the Principal, who after a while forgave the debt (“putting a minus in front”:  $-(-6)$ ), Table 10-21). To my question about why she did not just tell the “rule” she responded:

Well, we did that as well, in the end, and you can do it, and that is also a way of teaching mathematics, this-is-how-it-is-mathematics, but it is not my way (conversation 4, Table 10-60).

Tea claims that “this-is-how-it-is-mathematics”, i.e. telling “rules”, is not her way of teaching (mathematical challenge, Jaworski, 1994). To help the pupils understand how to solve the above task, she thus made a story (semantic representation); in this case “putting a minus” in front of the debt, making the debt an “income”.

### 7.3.4 Tea discussing Tuan’s solution

Looking for patterns in, and mathematically analyse, pupils’ errors are considered as specialized content knowledge (Ball et al., 2008). I did not often observe pupils making errors, possibly due to them, instead of trying themselves, were waiting for guidance, as mentioned in Section 7.1.4. However, when they did, they usually did not argue when Tea questioned the solutions (see for examples in 7.4.1.1 and 7.4.1.2). Once, I observed that a pupil, who did not follow the suggested route to the solution, wanted to present how he had solved the task. To understand what Tea comments on, I first briefly explain his solving process (a full excerpt is reproduced in Table 10-61).

The episode comes from a lesson on equations, and the class worked on solving tasks, including  $\frac{x}{2} + \frac{x+1}{7} = x - 2$ . Tuan had come to a correct solution without using the common denominator as Tea had suggested, and wanted to explain what he had done. Tea allowed him to do so, but



only to her. First, Tuan had found a number that made the fraction  $\frac{x+1}{7}$  “one whole” (observation 9B), which is the case when  $x$  equals six. Putting the six into  $\frac{x}{2}$  resulted in an “even number”, e.g. three plus one. Having the left side equals four, made  $x$  equals six, which was the correct answer.

In our subsequent talk, Tea commented that she found the conversation interesting, but that she struggled to understand what he said and was aiming at, “it is not always easy to understand what he means, so it took some time before I understood” (conversation 9). She meant that he first had solved the task orally, and what he explained to her was some kind of “testing” of the solution. Unfortunately, Tea did not ask for further explanation, she ended the conversation by praising the boy: “Yes, but it was because you think so fast that you saw the answer. Smart guy” (observation 9B). In our subsequent talk, however, Tea commented on the importance of learning structures, for example that equations should be written “below each other”, and suggested that the boy “at some level is unstructured” (conversation 9), which caused his somewhat “odd” explanation to how he solved the task.

Tea’s SCK allowed her to understand how he had come to his answer, but she decided not to discuss his solution process further, which could be a result of her KCS; it was not always easy to understand what he meant (conversation 9).

Being able to solve this task is common content knowledge (Ball et al., 2008), while looking for patterns in pupil error, or sizing up whether a nonstandard approach would work in general, requires SCK. Tea understood Tuan’s nonstandard approach (SCK), however she did not follow up by discussing whether this approach would work in general.

### 7.3.5 Tea’s SCK, summary

I have considered parts of what is included in Ball and colleagues’ (2008) specialized content knowledge; making features visible and learnable to pupils, using different representations, and justification of own ideas. The above examples represent my interpretation of how some of Tea’s activities and actions in the classroom, as well as some parts of our conversations, were based on her specialized content knowledge. I have also considered an example of a pupil taking a nonstandard approach to solving an equation, an approach Tea understood; however not following up by discussing whether it would work in general. Moreover, the episodes also show how central the pupils are in Tea’s practice, how she, in the classroom, always showed sensitivity to them. And considering the depth of mathematics that is

presented, sometimes at the expense of mathematical challenge (Jaworski, 1994).

## **7.4 Common Content Knowledge**

Common content knowledge (CCK) is defined as “the mathematical knowledge and skill used in settings other than teaching”, i.e. “not special to the work of teaching” (Ball et al., 2008, p. 399). However, as teachers need to know the content they are teaching, they also need to know some of the mathematics used in these “other” settings. In addition to knowledge of the “material they are going to teach”, this include competence to recognize wrong or incomplete answers, which I discuss in Section 7.4.1, and correct use of mathematical notations, considered in Section 7.4.2.

### **7.4.1 Recognizing wrong or incomplete answers**

In all lessons, I observed that she recognized and responded to errors and lack of understanding. In Section 7.1.4, I considered how she helped pupils to correct errors (cf. Chapters 6.3.1.2, 6.4.1.2, and 6.4.1.3). In this section, I discuss how I experienced her way of recognizing (CCK) and responding (KCT, KCS or SCK) to wrong or incomplete answers.

As explained earlier, Tea’s lessons normally consisted of various number of plenary and individual seatwork segments. I experienced that she responded differently in the two segments, which will be evident in the coming two sections, plenary segments in Section 7.4.1.1, and seatwork segments in Section 7.4.1.2.

#### ***7.4.1.1 Recognizing errors in plenary segments***

In the plenary sections, which occupied approximately half of the time the class focused on mathematics (cf. Chapter 6.1.3, and Figure 6-4), Tea carried out explanations, consisting of expositions and IRFs (initiation, response, follow-up, cf. Chapter 6.3.1). The expositions consisted of pupils following Tea’s explanations, while the IRFs arose when she posed questions. I suggest that she carries out these activities due to her KCT. In this section, I focus on Tea recognizing errors (CCK). In an attempt to make it more understandable, I present a few episodes to illustrate how she responded to some of them.

I observed several forms of responses to erroneous suggestions, for example “is it?” (as illustrated in Section 7.1.4.2), rephrasing the question, or asking if it was so, as in the first example below, responses I experienced that the pupils apparently recognized as saying their suggestion was wrong. I here present two excerpts of Tea recognizing and responding to erroneous suggestions from her pupils. The first

excerpt is from lesson one, when the pupils should calculate  $4 \times \frac{1}{2}$  and

Tea asked “what to do first”:

Tony We took them upside-down or something

Tea Was it then we took upside down? Mmm

Tara We made a fraction

Tea We made a fraction of the four (observation 1, Table 10-62).

By asking what to do *first*, Tea indicated that there were “more” steps to take. She met Tony’s suggestion by questioning it (CCK), and, as I interpret the situation, by using the follow-up question (KCT), signalled that it was incorrect. Tara provided the answer Tea sought, and the IRF continued with Tea approving and clarifying that it was about making a fraction of “the four”, meaning  $\frac{4}{1}$  thus making the task  $\frac{4}{1} \times \frac{1}{2}$ . Apparently, her KCT includes that when multiplying a natural number with a fraction, one makes both factors a fraction.

In the above excerpt, Tea turned the wording of the incorrect response, making it a question. This was also the case when she asked Thor about how to write 0.5 as a fraction:

Thor Mmm [2] one fifth

Tea Is it one-fifth? [Points to 0.5]. Therefore, if you get point five parts of my millions, then you get one fifth of them?

Thor [2] No

Tea Then, how much of them do you get?

Thor I do not know (observation 2b, Table 10-63).

Thor suggested that 0.5 is the same as  $\frac{1}{5}$ , an assertion Tea questioned, “is it?” As in the episode discussed in Section 7.1.3 (Thor and the cheese, cf. Table 10-36), Thor responded to Tea’s rephrased question by “no”, apparently believing that her question indicated that his response was incorrect. In the above episode, Tea even posed an auxiliary question: “if you get point five parts of my millions, then you get one fifth of them”, which apparently did not give him any clue. The follow-up question, however, could be rooted in her KCS; when guiding Thor towards the solution to finding how many grams of fat it was in a certain cheese, as considered in Section 7.1.3, she also used money in the attempt to help him understand. I observed similar responses in lesson 5, 8, 9A, whereof Thao (discussed in Section 7.1.4.2) also managed to immediately provide the correct answer after first responding “no” to her “is it?”. The episode with Thor is thus another example of Tea’s KCT including a particular form for responses, thus underpinning my suggestion that there exists a discursive norm in the classroom, as discussed in Section 7.2.5.

### 7.4.1.2 Recognizing errors in individual seatwork segments

While working individually, I observed three main reasons for pupils addressing Tea, they uttered lack of understanding the task (42%), they asked for confirmation (17%), or they needed clarifications (14%). The remaining 27% was Tea who, for some reason, addressed the pupils, as she did with Thor, already considered in 7.1.3.

Within pupils asking for confirmation, I observed some having come to a wrong solution, as for example Tone who had come to  $15x$  as the solution to  $8x + (3x + 4)$  as discussed in 7.1.4.3, and Tage who got the graphs to the functions  $y = 2x$  and  $y = 2x + 1$  intersect (observation 19). The class was practicing drawing graphs, and visiting Tage, Tea saw what he had done (CCK), and asked “what strange thing” he had, a question followed by Tage’s “I am sure it is quite wrong”, which might indicate that he assumed he was wrong (as Thor did, cf. Sections 7.1.3 and 7.4.1.1). She did not immediately confirm that it was wrong, she rather “wrapped it”, continuing, “I asked what it was about”, thus being an example of how she preferred to meet errors, as discussed in Section 7.1.4.2<sup>49</sup>. This time, however, she continued solving the task, doing two of the three calculations.

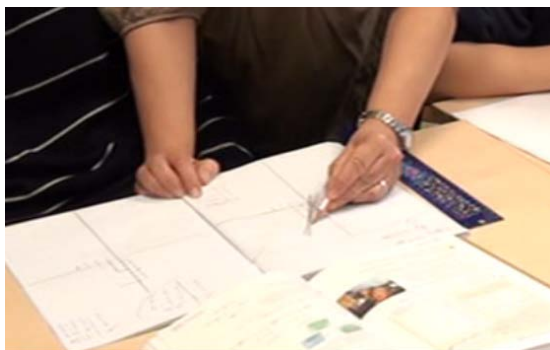


Figure 7-6. Tea helps Tage draw graphs (observation 19).

The conversation developed to be a step-by-step instructional dialogue (reproduced and analysed in Table 10-64), where Tea asked closed questions to which Tage responded by simple “yes”, “okay”, or answering simple calculations such as “if  $x$  equals two, then  $y$  equals”, thus performing an instructional dialogue Brousseau (1986) referred to as an example of a “Topaz” effect.

The episode with Tage illuminated how Tea could activate her KCT when guiding pupils whom she saw had made an error (CCK), and that her knowledge base, in addition to step-by-step instruction for remedying mistakes, apparently also includes belief about the teacher as responsible for “feeding” into pupils’ notebooks.

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<sup>49</sup> A reminder; she attempts to defuse the mistake.

### 7.4.2 Use of mathematical notations

Teachers need to be fluent in using mathematical notations and language (Ball et al., 2008). With an exception of Tea's use of notations for variables and unknown (in equations), we did not actually talk about her use of notations especially adapted to her teaching (continues below).

However, she used the common Norwegian words “plus”, “minus”, “times” and “share” for the arithmetic operations, and described what is inside parentheses like “something that belongs together just like a bag of candies” (observation 4A, Table 10-65, observations 5 and 6A, conversation 4, and 8). For example, having three bags, which all included the same amount of candies, for instance three chocolates, they all together had nine pieces of chocolate (observation 6A). She thus used “bag” as a metaphor for parentheses.

What follows is a special example of words Tea used when teaching algebra. It has to be clarified that she normally followed what is regarded as formal words for variables and unknown; however, she also sometimes used “homemade” words for the concepts. When re-introducing algebra (the class had worked on algebra the previous year), Tea started with drawing attention to words and numbers that could mean different things. For example, the number “1” has different values depending on what place it has in a number, or what it physically represents: one bag of potatoes or one crate of soda. She used that approach for making pupils understand that many things, such as numbers and words, can mean different things; hence, “x's and y's” were not “so awfully creepy”:

There are many things that can mean different things, [3] numbers can do it, [4] words can do it, and therefore are not a's and b's and x's and y's so awfully creepy, and so much different from much else. [...] That is, I try to make them to think about what we do, that is [4] eee [3], we use letters and numbers interchangeably; we know that things mean different in different settings. That is how it is with these numbers too (conversation 6, Table 10-66).

As an attempt to make pupils understand the use of variables, Tea tried to remind them that different things can, dependent on the setting, have different meanings, and that one can use letters and numbers interchangeably. While teaching, Tea used the notations “letters” and “letter-numbers” for variables (observation 5, conversation 6) and “letter-counting” for algebra (conversation 4, observation 5). I thus assume that she, in the quotation above, means that one can change between using letters, here variables, and numbers.

In addition to these expressions, Tea also used “numbers of which we do not know the value, such as a and b's” (observation 5) and “surname” as representations for variables (observations 5 and 6A, cf. Chapter 6.4.1.2)

and unknown (observation 9A); for example “can you add a number having  $x$  as surname with a number without a surname?” (Observation 5). Tea thus chose to use a known concept, “surname”, to separate a constant (a number without a surname) from a variable (number having  $x$  as surname), see Table 10-30 and Table 10-34 for examples. It is not to say that Tea did not know the formal notations for these concepts, she did, however, in the case of working on algebra and equations, choose to use familiar metaphors instead of their formal mathematical notations, variable and unknown.

Normally Tea used formal mathematical notations, such as positive and negative number, functions, graphs, etc., but also common Norwegian expressions as she did for the operations, “letters” for variables, and “letter-counting” for algebra. I know, by experience, that these expressions sometimes are used in Norwegian schools. Concerning “surname” as representation for variables or unknown (in equations), however, is a notation I have only met in Tea’s classroom. She sometimes also used representations she believed the pupils used in primary school. She thus connected what she believed was familiar to the pupils with representations/symbols they use in lower secondary school, actions which indicate a combination of KCS and horizon content knowledge as will be considered in the next section.

## **7.5 Horizon Content Knowledge**

This category includes knowledge about how topics related over the span of mathematics included in the curriculum, and being able to see connections to preceding and later mathematics education (Ball et al., 2008). I thus interpret this category to coincide with Shulman’s vertical knowledge as included in his category curriculum knowledge:

[F]amiliarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school, and the materials that embody them (1986, p. 10).

My analysis, however, exposed that there exists a didactical dimension to horizon knowledge; knowledge about teaching approaches carried out in preceding or later years in school. In this section, I therefore consider how Tea uses that suggested didactical dimension of horizon knowledge in her teaching. It has to be noted that I interpret Ball et al.’s (2008) category Horizon Content Knowledge as not including a didactical aspect. For Tea, horizon content knowledge concerns the

mathematics taught in the years both preceding and later to grade nine. In addition to knowing the mathematics included in the mathematics curriculum for the entire compulsory school, it thus includes familiarity with the mathematics the pupils will meet in upper secondary school. In the following subsections, I provide some insight into what I interpret as Tea's horizon knowledge. First, I consider knowledge she, due to her knowledge of the primary mathematics and experience of teaching, expected her pupils to bring from primary school (Section 7.5.1), and then how she prepares them for what she expects they will meet in upper secondary school (Section 7.5.2).

### 7.5.1 What pupils bring from primary school

The Norwegian curriculum provides requirements for the competences pupils are expected to have after grades 2, 4, 7, and 10 (Utdanningsdirektoratet, 2015). Tea's pupils were in grade nine, hence working towards competence requirements for grade 10. Due to her knowledge of the content taught in primary school (grades 1 – 7), Tea had some expectations about what the pupils should know when they started in grade nine (conversation 9). She also had some assumptions about how they were taught (conversations 8, 9 and 13). Thus, in addition to knowledge about what pupils normally should hold at the time, she sometimes also based her own teaching on these assumptions.

A significant part of the primary curriculum is about working with basic arithmetic (Utdanningsdirektoratet, 2015), which Tea knew. However, she experienced that some of her pupils still did not know how to do simple calculations, to which she had a rather strong reaction:

I think one should expect pupils to master the basics in arithmetic before they enter lower secondary school, [3] I get shocked when I experience pupils who do not manage to divide twenty-four by three (conversation 9, Table 10-67).

This statement fell as a comment to a pupil who asked for permission to use a calculator for dividing twenty-four by three when solving  $3x = 24$  (observation 9). Knowing that the pupils have worked with division for several years in primary school (HCK), this request from the pupil "shocked" Tea. One reason for such poor mathematics knowledge, Tea believes, is because "we have not managed to signal how important it is for pupils to learn" (conversation 13), and that "the focus on learning in primary school have not necessarily been the same all the time" (conversation 13). Hence, her horizon content knowledge was extended to also include an assumption about the focus on learning in primary school; it has not necessarily been the same all the time (see introduction to Section 7.5 for my extension of the category Horizon Content Knowledge). Even if she was not always particularly content with the

focus on learning in lower grades, she still occasionally connected her own teaching to concepts and procedures she believed the pupils had experienced in primary school, as will be considered next.

When introducing equations and percent, Tea presented the topics the way she believed the primary teachers made the introduction: “Then I think it is fine to present the introduction like, that is, like I believe it might have been” (conversation 13, Table 10-68). For example, when introducing equations, she used a box to represent the unknown ( $\square + 3 = 9$ , earlier considered in 6.3.4): “I introduce these boxes at the beginning because it is often how they practice addition in primary school, and afterwards multiplication” (conversation 8). Another indication of her extended horizon content knowledge (cf. Chapter 3.2.3.3 for definition on “extended” horizon content knowledge) was observed when introducing the expression “spine” when working on equations. She knows that many pupils learn to write the tasks with a “straight back”<sup>50</sup> when solving tasks, however she prefers to use “spine” when working on decimals:

I have used spine, we have used it in slightly different contexts because when they went to primary school [1] or, I do not know how it was with these pupils here then, but many pupils in primary school learn that the tasks should be straight in the back. And that is very stupid when they start on decimals, [1] because they then continue writing the numbers “straight in the back” [1] and then you have to introduce the concept of spine (conversation 9, Table 10-69).

Even if she was not sure whether the pupils she taught at the time had learned about tasks being “straight in the back”, she asserts that many pupils do learn to present their tasks like that in primary school.

Experiencing that “straight in the back” can be problematic for pupils when working on decimals (KCS, for example adding 2.7 and 4.35), she introduced the concept “spine” (KCT), a metaphor she used both with decimals (point under point) and equations (equal signs under each other). She thus used her knowledge of prior education (HCK) to also connect teaching approaches to her present lower secondary teaching.

I have considered the knowledge Tea has about the mathematics education her pupils experienced prior to lower secondary school. It includes knowledge of content, of teaching approaches, and focus on learning, which she deliberately used to enhance pupils’ knowledge and understanding of mathematics. The two latter issues, teaching approaches and focus on learning, I interpret as being dimensions within didactics. I have thus decided to extend Ball et al.’s (2008) category

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<sup>50</sup> Being “straight in the back” means that when writing the solving process, the different steps should be written under each other having the last digits straight beneath one another.



Horizon Content Knowledge to include a didactical dimension, as I described in the introduction to this section as well as in Chapter 3.2.3.3.

### 7.5.2 Preparing for upper secondary school and for life

Several times, I experienced Tea talking to her pupils about future life and future education. She was concerned about showing the pupils that what they do in school is for real; it is not a game (conversation 13, Table 10-46). She also spent more time on topics she saw as particularly important for their future life, for example when working on loans and interest because that will be “one of the worlds they will face very, very soon” (conversation 15, Table 10-70).

As explained in Chapter 1.2.2, a central principle in Norway is equity in education for all. This means inclusion in basic groups and adapted education at all levels (Kunnskapsdepartementet, 2015; Utdanningsdirektoratet, 2008). When teaching equations and functions, she took advantage of an opportunity to temporarily deviate from what was normal organization (Kunnskapsdepartementet, 2015b, §8-4) to group her pupils based on which educational program they plan to follow after ending lower secondary school:

Going to split them for what they think they should do in upper secondary school. Say to pupils that this is a theme, so if you are following general studies and have theoretical mathematics then you actually have to know a lot, right, know quite many things that even are difficult, I think, and then I think they should know a little on proportion, and slope and intersections, and everything. I mean they need to know that (conversation 16, Table 10-71).

Apparently, her horizon content knowledge includes some of the mathematics included in the upper secondary curriculum: “I do mathematics with my nephew who is in first grade in upper secondary school” (conversation 8). Knowing that “quite many things that are difficult” (conversation 16) in upper secondary mathematics, made her divide the class into two groups, based on whether they plan to follow a path providing “theoretical mathematics” or not. Regarding functions, the group of pupils planning to follow a theoretical path got the opportunity to learn about proportion, slope, intersection, and inverse proportion (conversations 16 and 17) while the remaining pupils should continue practicing to draw graphs (conversation 17).

Similarly, after the introduction of simple equations, the pupils could choose whether to practice more simple equations, or work on equations with fractions, including factorization and shortening (mathematical challenge, Jaworski, 1994). The latter topics were aimed at those who were “going to have a five or a six”, i.e. aiming at one of the two highest grades. In addition to opportunity for higher grades, Tea meant that

learning the topic at this level would ease the workload when coming to upper secondary school:

Then I am very clear [2] with some pupils I know had made it if they had bothered and say that you, you realize that these are [1] the two highest levels. If you are going to have a five or a six, then you actually need to know this. [...] This here, ridiculously stupid if you do not learn it now, then there will be twice as much to do in a few years (conversation 8, Table 10-72).

Her knowledge of some of the topics included in the upper secondary mathematics curriculum made her encourage (some of) the pupils to work hard at the present level. The result would then be twofold: in addition to having the opportunity to reach the grades they were aiming at for lower secondary mathematics, it would make mathematics easier when attending upper secondary school.

Tea admitted to having a rather strong conviction towards organisational differentiation: “I do not believe in classes being divided due to how smart or stupid they [pupils] are” (conversation 9A), and “to divide into good and weak pupils, it is in a way to lose a little at the outset. [...] It does not coincide with my humanity” (conversation 17). However, her knowledge about content included in curricula (horizon content knowledge) the pupils will meet later, combined with her knowledge about the pupils, caused her to deviate from her conviction (cognitive sensitivity to students, Jaworski, 1994): on a few occasions she let pupils work in groups so they could prepare for what she knows will come at later stages.

### **7.5.3 Tea’s horizon content knowledge, a summary**

I have, in this section, considered Tea’s horizon content knowledge, and exposed that she has knowledge of some of the mathematics included both in elementary and upper secondary curricula. These coincide with what Ball and colleagues (2008) include in the category horizon content knowledge.

She utilized her knowledge of some concepts the pupils would meet in upper secondary mathematics to prepare for what they would meet there, sometimes taking advantage of an opportunity to temporarily deviate from what was normal class organization (Kunnskapsdepartementet, 2015b, §8-4), differentiating the class based on where the pupils plan to go after finishing lower secondary school.

Concerning primary mathematics education, in addition to content knowledge, she also expressed some knowledge of teaching approaches as well as some thoughts about poor focus on learning throughout the years in primary school. These observations led me to extend the category Horizon Content Knowledge, as explained earlier, and further discussed in Chapter 8.3.1.1.

## 7.6 Knowledge of Content and Curriculum

Ball and her colleagues (2008) do not particularly elaborate on Shulman's category Curricular Knowledge which is:

represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (1986, p. 10).

In addition, Shulman suggests a lateral and a vertical curriculum, whereof the lateral curriculum is about teachers' ability to relate the content to topics or issues the pupils focus on simultaneously in other classes. The vertical curriculum concerns familiarity with the topics and issues that is included in prior and subsequent curricula, which I interpret as Horizon Content Knowledge, thus discussed in Section 7.5. Moreover, in Section 7.2.2, I considered Tea's knowledge of the mathematics included in the curriculum for lower secondary school. I there presented some of the approaches she used to facilitate pupils' learning, particularly how she and her colleagues used the national curriculum to plan for their teaching, hence not necessary to discuss further. What remains, is what Shulman (1986) terms lateral curriculum knowledge. This section, thus, continues with dealing with how her knowledge of lateral curricula was expressed in her mathematics teaching.

### 7.6.1 Lateral Curriculum knowledge

Lateral curriculum concerns other subjects the pupils meet simultaneously in school. In addition to mathematics, Tea taught food and health, natural sciences, and RLE<sup>51</sup> to the class, an opportunity I observed that she used deliberately. I here present (review) a few examples of how this was brought to bear in her classroom.

As explained in Section 7.3.2, Tea used two number lines for illuminating point zero while introducing negative number; a vertical line illustrating a thermometer and a horizontal line illustrating a year-line (cf. Figure 7-5). While talking about point zero as when water freezes, she also asked the pupils whether they remembered that, when having natural sciences, they had talked about two other temperature scales defining point zero at other natural measures (which the pupils did not). On the horizontal line, she drew a figure at point zero to indicate the birth of Christ, and a Stone Age man to the left of point zero.

She also used her lateral curriculum knowledge when comforting herself that her pupils learned to understand concepts better when "they

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<sup>51</sup> Religion, Philosophy of Life, and Ethics.

get the same things [...] in a more concrete way”, pointing to food and health and natural sciences:

I comforted myself in a way that, well it is just to comfort myself; I have these pupils in some other situations too, have them in natural science by weighing and measuring, and all those things right. Have them in food and health, [2] and then I end up to comfort myself with the fact that they get the same things in a way in those subjects, very many things [2] in a more concrete way (conversation 5, Table 10-73).

As she was responsible for teaching several subjects to the class, Tea had a particular opportunity to connect these subjects to highlight the importance and utility of the mathematics in these subjects, and vice versa. In addition, I also experienced that she sometimes connected to subjects she was not responsible for by using what was on the board when they entered the classroom, for example French. One day we met in the classroom for introduction to equations, there were three French words written on the board, and Tea spent about two minutes translating and talking about these words before erasing them. One of the boys then asked for the French word for equation, which led to another minute talking about the French Revolution and the national motto for France: *Liberté, égalité, fraternité*.

As indicated in Chapter 6.2, Tea spent some time on building relations with her pupils, and the above situation exemplifies how she wanted to help the pupils meet existing challenges: “I think challenges are fine. We begin every mathematics lesson on Mondays [...] with French because it is on the board, [...] it is almost like a game” (conversation 9A). She asserts that it is fine to do this because “if there is something you should be able to recognize in mathematics too, it is the language”.

Connecting different subjects is lateral knowledge; however, using such opportunities because it concerns mathematics and its language, I interpret as being part of her KCT. I have, in these sections, illuminated Tea as a versatile teacher; by including other subjects into her teaching of mathematics, she demonstrated knowledge beyond mathematical knowledge for teaching. She also, as will be evident in the next section, makes great effort to facilitate learning of the mathematics.

## **7.7 Knowledge of Facilitating Mathematics Learning**

Throughout our conversations, Tea expressed strong beliefs and meanings that affected her teaching, issues that appear not to be included in the MKT-framework (Ball et al., 2008). In this sub-section, I thus lean to Rowland and colleagues’ (2005, p. 261) *foundation* as part of the knowledge quartet when “considering under which conditions pupils will best learn mathematics”; Tea’s practice is founded on her beliefs and values.

In addition to localized knowledge and knowing, the category local knowledge in my suggested conceptual framework included general professional knowledge, GPK (cf. Chapter 1.4, Figure 1-3), a category that was initially not necessarily directed towards *mathematics* teaching. Tea, however, believes that the safe and predictable environment the school represents is important for pupils' learning (cf. Section 7.7.3). Facilitating learning, here learning of mathematics, then includes knowledge about how to make that environment in a way that helps pupils to be able to concentrate on their work. According to this, I suggest to include more into the initial category GPK than initially suggested, naming it *Knowledge of Facilitating Mathematics Learning* (KFML, cf. Chapter 4.8). What follows in this section will substantiate my suggestion.

Classroom norms, which I believe rests on expectations Tea had of herself, the pupils and of the parents, I discuss in Section 7.7.2. I consider Tea's involvement in her pupils' lives and well-being in Section 7.7.3, while I, in Section 7.7.4, provide a short summary; however, I start this section with considering how Tea's belief system appeared to impact on her teaching.

#### 7.7.1 Tea's belief system

Tea has worked as a teacher for more than thirty years. During these years, she has experienced four national curricula, M74, M87, L98, and LK06<sup>52</sup> (Regjeringen, 2013). Moreover, she has experienced new pedagogical ideas and implementations, initiated either by the national curriculum plans or by "radical" teachers (cf. 10.3.1). These could be implementation of calculators, grouping of the pupils, responsibility for own learning, and removing the boards from the classroom. No board was removed from her classrooms.

Tea asserts that the idea of responsibility for own learning and the implementation of calculators have both served as obstacles for development of pupil's mathematical knowledge. Tea claims that mathematics is a subject that requires directed management (cf. 10.3.1), an attitude that was reflected in the way she guided her pupils. In addition, her belief about calculators serving as "sleeping pillow" and "initiative to escape work" made her reluctant to allow the pupils to use calculators. However, she allowed some low-achievers to use one if the aim, for example, "was to practice equation-technique" (cf. Chapter 4.9).

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<sup>52</sup> Mønsterplan av 74, Mønsterplan av 87, Læreplan av 98, and Kunnskapsløftet av 2006.

When Tea started to teach, her work was subject to the national curriculum of 74 (Kirke- og undervisningsdepartementet, 1974), a plan that allowed organisational differentiation. A central principle today is that pupils normally belong to a class or basic group, but can be divided in other groups when needed (Kunnskapsdepartementet, 2015b, § 8-2) (cf. Chapter 1.2.2). Tea admitted to have a strong conviction towards organisational differentiation, she does not believe in classes being divided due to whether the pupil is “smart or stupid” (cf. Section 7.5.2). Tea differentiated by assigning tasks of different difficulty on pupils’ work-plans (cf. Section 7.2.3). Working in the normal groups<sup>53</sup> led to that the pupils sometimes had to solve many similar tasks, a situation Tea believed would make them feel successful, because they would experience “the joy of saying I understood” (conversation 18).

Tea’s beliefs and convictions thus made her reluctant to let pupils use calculators, and resist “radical” suggestions such as removing the boards from the classrooms. However, they also sometimes made her refrain from challenging her pupils mathematically. Her affective sensitivity to the pupils thus sometimes occurred at the expense of mathematical challenge (Jaworski, 1994).

### 7.7.2 Classroom norms

Brown and McIntyre (1993) suggest that teachers evaluate their lessons in terms of pupils’ progress (understanding, production, accomplishment) and activity; the teachers were satisfied as long as the pupils continued to act as desired. In this section, I discuss what norms that exist, or Tea wants to exist, as part of her facilitation for pupils’ learning of mathematics. Moreover, I show that for fulfilling all this, Tea had some expectations, mostly of herself, but also of her pupils and their parents.

When I entered the classroom, I experienced that there existed some social norms, or routines, for example to greet each other when meeting for the first time during a day: “I say to the pupils that I will see you all in the face before we sit down starting the day [...] because I think it is a nice way to show each other respect” (conversation 1). This was evidenced by one of her pupils who stated, “she brings us up in a way like a mom, I get a feeling of up-bringing” (conversation with Tone). There also appeared to exist some norms concerning dialogues, as first indicated in Section 7.1.4.2, then briefly mentioned in 7.2.5 and 7.4.1.1.

In the following, I will first present some of what I interpret as norms for the social life in Tea’s classroom; norms I suggest exist due to Tea’s expectations of herself, of her pupils and of the parents.

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<sup>53</sup> Tea’s class was sometimes divided into two homogenous groups (cf. Chapter 4.5.3).

### 7.7.2.1 *Expectation of self*

Concerning her job as the class teacher, Tea looks upon herself as responsible for both parents and pupils throughout the whole day:

It is [5], it is everything [3] which is [1] between, that is, in relation to the pupils it is all between [1] they open their eyes in the morning to going to bed at night [2] and the parents until they settle for the evening (conversation 2, Table 10-74).

She is “extremely concerned that the parents shall be hundred percent sure that when they have sent their child to school, then they are there” (conversation 2). This puts some demands on Tea, for example having a mobile phone in the classroom; a call could come from some parents, or she needed to search for a pupil that did not show up to the lesson.

Apparently, she also took responsibility for what the pupils had in their notebooks before leaving for home. I once experienced that she interrupted the individual seatwork to see to that the pupils had adequate examples to look at when doing homework:

It is somewhat important to me that they, that they eee, that they in a way shall [3] transform what they hopefully have heard [1] before they go home. [...] Because, eee, then you have the opportunity to stop [1] misunderstandings, and I, [1] you have opportunities to make such [interrupts herself], that they have some examples in their books [2] that they in a way have worked on before continuing at home (conversation 4, Table 10-75).

By providing the pupils with examples before going home, Tea hoped that they, when doing homework, would be able to “transform” what they have heard and thus have the opportunity to avoid misunderstandings.

In addition to working hard for supporting pupils learning the mathematics, Tea also strived to make good learning environment for them. She had arranged school trips, they have been out fishing, and she had baked cakes for them (conversations 7 and 8). She thus made considerable effort both in facilitating learning and in establishing a nice atmosphere in the class. This, I consider in the next section.

### 7.7.2.2 *Expectations of pupils*

Tea worked hard for establishing good social atmosphere within the class, but did not feel that she succeeded: “some of them do not *bother* to engage. [...] The world gets more and more egoistic, we point at people: you and you haven’t done anything, why should I *do* anything for you” (conversation 7), “one tries to bring it forward all the time, [4] right, but I feel again and again and again that I do not reach them” (conversation 8).

She was thus disappointed with pupils’ lack of interest in, and engagement for, maintaining good and caring atmosphere in the class. Once I experienced (observation 8) that she waited for some particular

pupils to leave the room before she talked to the remaining pupils about some worries she had due to lack of participation in plenary discourse: “I chose it as a golden moment. [...] I chose this deliberately because there were so many of the boys who left” (conversation 8). Tea considered that some of the boys had an “arrogant attitude” (conversation 8), that they “have like blinkers<sup>54</sup> for who is something and not and who is able to do something and who cannot” (conversation 9) and “laughed” (conversation 8) when some made mistakes. This made some pupils avoid responding in the classroom. Thus, when these boys left the classroom, she used the opportunity to talk about her worries, because she believes that “to learn requires that you dare to say things aloud” (observation 8, cf. Section 7.2.5).

As exemplified above, some of the pupils did not dare to participate in plenary discussion due to anxiety for being laughed at. This might indicate that there were not incorporated sound interpersonal socio-mathematical norms (Yackel & Cobb, 1996) in the classroom. Concerning the mathematics work per se, however, Tea had clarified what she expected of her pupils:

They know that eh, I expect that they are quiet, [2] they know that I wait, that is, right, they know they are getting very clear message, eh, probably sometimes too clear, that is, yes, they know I do not give up. They know that I never give up when it comes to content [...] because I know they need it (conversation 1, Table 10-76).

I experienced that the pupils normally accepted Tea’s requests for silence, and that they worked on the mathematics when she asked them to; they knew about her expectations of them.

Tea also organized the seating in the class, often built on pupils’ wishes. However, after Christmas she placed girls and boys in separate groups for making “the girls work harder” (conversation 12), while for some of the boys, it was “so they will not be lonely and kind of motivating each other for working” (conversation 13). After a few weeks, she reorganized the girls because she was not quite content with all of them: “I moved those girls because I did not want to see their backs, particularly Tove, she is so very distant” (conversation 15).

The above examples indicate that Tea, largely, was the authority in the classroom; she was very clear about how she wanted them to work and behave. However, she was still not entirely content with their effort and behaviour, they did not always fulfil what she tried to establish as their normal desirable state of student activity (NDS, Brown & McIntyre, 1993). She also had some expectations of the parents, which I consider in the next section.

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<sup>54</sup> Metaphor for being selective for who they accept.



### 7.7.2.3 *Expectations of parents*

As mentioned in Section 7.7.2.1, Tea required of herself that the parents should feel safe that their children were in school if they had sent them (conversation 2). She thus had her mobile telephone in the classroom so that parents could contact her if they had something to comment on, for example to tell that their child was ill. Actually, she “set very stringent requirements to parents that they should tell if their child is sick or away” (conversation 2). If the parents still did not call, Tea contacted them to check why the pupil was not present.

Tea also wanted the parents to take some responsibility for pupils’ work: “And the eternal pressures, the eternal nagging, [2] I must admit that I think that we to a greater extent should have the pleasure of sharing with parents” (conversation 4). Tea here pointed to the homework; she would welcome some effort from the parents to see to that their children actually did their homework.

Moreover, she was disappointed with parents’ lack of interest in working for the community. She experienced that some of them had the same attitude as their children: “They are also a bit like, we are not going to make money for *your* kid going to the city, that you actually have to do yourself” (conversation 7).

The above examples indicate that Tea had expectations of her pupils and of the parents, but most of all she had expectations of herself. Actually, she opened for being somewhat private, as will be evident in the next section.

### 7.7.3 **The parent surrogate**

In addition to knowing her pupils well, I also experienced Tea as very caring; sometimes taking the role as a surrogate parent, and being somewhat private (cf. Chapter 6.2.2):

I believe in that, and it is of course an eternal discussion me talking about private thing. Because I do that, what we did yesterday and, yesterday we shovelled the roof, that is speaks a little with them. [3] I believe in it, for I, that it is because we are so incredibly, we are the more with them than their parents are these days (conversation 15, Table 10-77).

Even if discussing the issue, she believes that teachers, due to spending more time with the pupils than their parents do at this time of life, ought to share something from their private life. In the openings (cf. Chapter 6.2), I often heard that she talked about private things, such as mentioned in the above excerpt, or for example commenting on a pupil who came extra well dressed.

She also knew about the pupils’ home conditions, and was concerned that some have it “completely utterly awful” (conversation 1) at home, a situation she believed led to they, at times, were not able to acquire any

knowledge. For these, school is the only predictable point they have in their lives:

Because it is Bolla<sup>55</sup> the hedgehog, right, who potters off and then she comes to her stump and then the stump is there (conversation 1, Table 10-78).

Tea believes that many pupils come to school just to experience the safe and predictable conditions they meet there; they have their own classroom and desk, and they experience “a form of rest compared to the unpredictable things around” (conversation 1). She claims that “many can, even if it is a bit messy around them, put that away because the frames of the class are so safe” (conversation 1), thus believing that the predictability offered in school makes some of these pupils able to concentrate on learning, in this case learning of mathematics.

#### **7.7.4 Tea's KFML, a summary**

In this chapter I have considered Tea's knowledge of managing for learning (Jaworski, 1994), and discussed her effort for creating what I interpret as her normal desirable state of pupils activity (Brown & McIntyre, 1993); classroom norms.

Tea suggests that unpredictable environment hinders pupils' learning. She thus puts great demands on herself attempting to create good social relations in the class, and to make parents confident that their pupils are in school when they have sent them. In return, she expects them to call and report if their child has a reason for not coming to school. Moreover, she sees herself as responsible for her pupils the whole day, not only when in school.

Apparently, Tea had set the norms in the class, norms concerning both content discourses (cf. Section 7.3.4) and behavioural matter. Her normal desirable state of student activity (NDS) includes that the pupils should be quiet while working on the mathematics, that they participate in work for a common benefit for the class, and that they show each other respect. She was sorry that a few of the boys made some peers avoid responding in the classroom due to fear for being laughed at.

I observed that Tea sometimes acted as a parent surrogate; she talked about private matters, and in a positive way noted particularities, such as commenting on a boy one day coming extra well dressed. She was also concerned about pupils having difficult home conditions; she meant that the school was the only predictable point in their lives. Thus, it appears that Tea sees herself as more than their mathematics teacher, even as class teacher, which she is for this class.

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<sup>55</sup> Bolla is the main character in a Norwegian children's song about the hedgehog that potters to school.

In line with Rowland et al. (2005), what is discussed in this section indicates that there is more included in mathematics teachers' knowledge base than suggested by Ball and colleagues' (2008). What Tea has demonstrated, thus justifies my suggestion for a category that includes knowledge about facilitating learning of the subject, as indicated at the beginning of this section.

## **7.8 Tea's Knowledge For and In Mathematics Teaching, a summary**

I have, in this chapter, considered Tea's knowledge for teaching mathematics, mostly related to Ball and colleagues' framework MKT (Ball et al., 2008). Where I found evidence of activities which were not explicitly defined within one of these categories, I sought "assistance" in Rowland and colleagues' (2005) *Knowledge Quartet* (KQ) and Jaworski's (1994) *Teaching Triad* (TT). Both the MKT and the KQ frameworks are outcome from empirical research, however based on the work of Shulman (1987). The TT evolved from Jaworski's (1994) inquiry into mathematics teaching some ten years in advance to the development of the MKT and KQ frameworks. I find it interesting that there was a need for all three frameworks to properly describe Tea's mathematical knowledge for teaching. That issue, I discuss in Chapter 8.3.

As showed in Sections 7.1 through 7.6, Tea's mathematical knowledge for teaching includes all knowledge categories within the MKT-framework (Ball et al., 2008), and also to some extent all dimensions in the Knowledge Quartet (Rowland et al., 2005). Regarding elements in the Teaching Triad (1994), the data exposed a biased focus; I experienced an extensive sensitivity to the pupils which appeared to occur at the expense of the mathematical challenge usually offered to nine grade pupils. This, I further consider in Chapter 8.3.3.

As explained in Section 7.7, Tea saw norms and safe environment as important for pupils' learning of mathematics, thus justifying the practical dimension in my conceptual framework, knowledge of general professional knowledge. I will not claim that all professional knowledge I observed concerned learning of the mathematics. However, I am suggesting that her engagement for establishing safe learning environment and social norms is part of her knowledge for the teaching of mathematics, a slightly different concept than mathematical knowledge for teaching. I thus suggested rename the initial category GPK to *Knowledge of Facilitating Mathematics Learning* (KFML, cf. Chapter 4.8 and Section 7.7).

In addition to the issues mentioned above, I end this chapter by listing some other aspects of knowledge evolving from my analysis, which I find particularly interesting:

- Knowledge of planning and structuring should be explicitly included in the category content and teaching.
- Specialized content knowledge appears to be influenced by sensitivity to pupils.
- Should *informal* use of mathematical notations be included in a teacher's common content knowledge?
- There appears to exist a didactical dimension of horizon content knowledge.

In the next chapter, I will further discuss these, and other, issues which evolved through the process of analysing my data.

## 8 Discussion and conclusion

The aim for this study was initially to make public the existence of two dimensions of local knowledge for teaching mathematics at lower secondary level. A search in the literature exposed the existence of craft knowledge, which I refer to as local knowledge, (Brown & McIntyre, 1993; Cooper & McIntyre, 1996; Leinhardt, 1990; McNamara & Desforjes, 1978). I thus changed focus to studying what characterized my informant's local knowledge.

Through analysing and interpreting the classroom observations and subsequent conversations with one experienced teacher, I have searched for evidence for addressing my research questions (revisited in Chapter 8.1).

I visited Tea nineteen times during the academic year 2010/2011, collecting written and videotaped data. The result of the data analysis are presented in Chapters 5, 6, and 7. In Chapter 5, I provide some general background information about the teacher Tea, in addition to how her Principal experiences her as her colleague, and how some of her pupils experience her as a teacher. In Chapter 6, I present the analysis of Tea's practice. I organized the classroom activities into four parts, openings, plenary and individual seatwork segments, and closings. Within those parts, I classified and coded all actions taken and topics presented and discussed in the classroom. In Chapter 7, I present the results of the analysis for exposing Tea's local knowledge for and in teaching mathematics.

This chapter starts with revisiting my research questions and reviews what local knowledge entails (Section 8.1). In Section 8.2, I discuss Tea's local knowledge, in Section 8.2.1, I consider how it was developed, and in Section 8.2.2, I describe two broad aspects which I assert stand out as characterising Tea's localized knowledge: motivating for learning (Section 8.2.2.1), and approaches to teaching (Section 8.2.2.2). I had the opportunity to observe only two episodes where I found evidence that Tea activated her localized knowing, one episode in lesson three, and one episode in lesson thirteen. I lost the video from lesson three, thus in Section 8.2.3, I have only one episode as a foundation when characterizing Tea's localized knowing. The category that evolved from the analysis, Knowledge of Facilitating Mathematics Learning, I discuss in Section 8.2.4, while in 8.2.5, I write some concluding remarks concerning the characterization of Tea's local knowledge.

I used three frameworks for analysing the data from the conversations with Tea; Ball and colleagues' Mathematical Knowledge

for Teaching (2008), Rowland and colleagues' Knowledge Quartet (2005), and Jaworski's Teaching Triad (1994). In Section 8.3, I discuss the impact of these frameworks, and make a modified visualisation of how their constructs overlap my conceptual framework based on the findings in my research. In Section 8.4, I summarise some implications my study might have for teacher education, in Section 8.5, I provide some suggestions for further research, while I, in Section 8.6, make some critical reflections on the process. In the final Section 8.7, I discuss my own learning before providing some final concluding remarks.

## **8.1 Revisiting the research questions**

The main research question asks what characterizes the local knowledge that informs the practice of one experienced mathematics teacher:

- What characterizes the local knowledge in mathematics teaching of an experienced Norwegian teacher?

This question has followed me during my fieldwork and analysis. I have made public local knowledge for mathematics teaching, as reflected in my conceptual framework (cf. Chapter 3.3) through analysing classroom observations and subsequent conversations with the teacher. As I also made the hypothesis that there are two dimensions of teacher knowledge for teaching mathematics, localized knowledge and localized knowing, I articulated two auxiliary research questions:

- What are the characterizations of localized knowledge in mathematics teaching in the practice of an experienced Norwegian teacher?
- What are the characterizations of localized knowing in mathematics teaching in the practice of an experienced Norwegian teacher?

Local knowledge, in the literature referred to as craft knowledge (Brown & McIntyre, 1993; Leinhardt, 1990; Ruthven, 2002) includes the knowledge experienced and skilled mathematics teachers have about their practice, which they use in their day-to-day classroom teaching, and is revised in the process of their teaching (cf. Chapter 2.1.3). I have, in this study, localized practice-based knowledge/knowing, i.e. knowledge of mathematics teaching revised in the process of teaching. However, I differentiate between the knowledge teachers are able to articulate, categorized as localized knowledge, which I discuss in Section 8.2.2, and what they might find difficult to articulate, categorized as localized knowing (discussed in Section 8.2.3). For the distinction between knowledge and knowing as to be understood in this study, I refer to Chapter 2.1.

## **8.2 Tea's local knowledge in mathematics teaching**

In this section, I consider what characterizes Tea's local knowledge in mathematics teaching. I start with how she developed to be the teacher

she was at the time of the observations (Section 8.2.1), addressing my research questions, i.e. considering the characteristics of her localized knowledge in Section 8.2.2, and her localized knowing in Section 8.2.3. Due to Tea's focus on facilitating learning, in Section 8.2.4, I consider her activity for managing and maintaining a sound learning environment, while in Section 8.2.5, I provide a summary of her local knowledge.

### **8.2.1 Development of local knowledge**

When Tea started her career as a mathematics teacher, she was very concerned about following requirements from the curriculum, the textbook, and how she assumed she had been taught herself (cf. Chapter 7.2.1). Over the years, she got more confident, and developed her own style; from being dependent on textbooks, she decided herself how to teach different topics, which is evident in the two previous chapters. Her confidence was also evident in her insistence to keep the blackboard in the classroom when the "trend" was to throw them out.

Cooper and McIntyre (1996, p. 76) assert that professional craft knowledge is "the knowledge that teachers develop through the processes of reflection and practical problem-solving that they engage in to carry out the demands of their jobs". Leinhardt (1990) suggests that expert teachers possess craft knowledge, defined as information very skilled practitioners have about their teaching, including deep, sensitive, local-specific knowledge of teaching. Leinhardt and Smith recognize expert teachers as teachers having a consistent growth scores of their students over a five-year period (Leinhardt & Smith, 1985). I do not know Tea's test scores, but I assert that, through engaging in "processes of reflections" on her teaching, she has developed the knowledge Leinhardt (1990) suggests expert teachers possess about their practice, as for example evidenced in the suggestions for further work and judgements about pace and prioritizations provided in the closings to the lessons (cf. Chapter 6.5). The fact that the Principal, the pupils, and parents consider Tea to be an excellent teacher should also indicate that she is in possession of such knowledge. In the following sections, I discuss further characterizations of Tea's local knowledge.

### **8.2.2 Tea's localized knowledge**

Localized knowledge in mathematics teaching is the knowledge teachers deliberately deploy when teaching the subject resulting from (at least) two situations: reflecting on prior lessons or planning for a distinct class or group (see Chapter 3.3 for further explanation). As is evident in Chapter 7, her localized knowledge can be characterized by several actions and activities, whereof two main issues predominate; motivating

for learning mathematics (Section 8.2.2.1), and locally developed approaches to teaching (Section 8.2.2.2).

### **8.2.2.1 *Motivating for learning***

Tea was concerned about motivating her pupils, which is a competence included in the category Knowledge of Content and Students (Ball et al., 2008). She believes that it is important for pupils' learning to connect the mathematics to real life situations (conversations 4, 12, 13, 15, and 16). When introducing new topics she thus connected to issues that concerned their daily life:

- Negative number – accounting in the school canteen (lesson 4A and 4B)
- Percent – Youths and their moped certificate and the solar eclipse (lessons 12 and 13)
- Functions – Suggested income in a planned school disco (lessons 16 and 17).

The grade nine pupils were responsible for the school canteen and the disco for next year's eighth graders. Tea thus suggested that the mathematics related to these activities would be interesting for the pupils to learn.

Moreover, using what was in the news was both for inspiring to learn the topic (i.e. percent) and for the pupils to be committed to what goes on outside the school: "One always do that, yes, one do that. [...] I think it is very important to all the time in a way inspire them to follow and be committed and register when they say on the TV that tomorrow is the eclipse" (conversation 13, cf. Chapter 7.3.1).

As indicated in Chapter 7.7.3, she opened up for introducing issues from her private life, which also concerned her teaching; in addition to pupils' real life situations, she connected her teaching to issues of personal interest. For example, she chose to use the survey on youths' moped certificate (cf. Chapter 7.3.1), because a person close to her was conducting such surveys, which she told the pupils. In addition, she utilized the situation to talk about surveys in general, thus connecting to other school subjects (lateral knowledge, Shulman, 1986), which she also did when teaching negative number as evidenced in Chapter 7.3.2 (different temperature scales, history, and Religion, Philosophy of Life, and Ethics).

Tea's competence for motivating the pupils were valued by her pupils, and recognized by her Principal: Trym, for example, who had experienced mathematics as boring before attending Tea's class, claims that Tea makes much of the mathematics fun (cf. Chapter 5.3). Tone asserts that Tea's aim for reading the story about the youth and moped certificate was for making the mathematics (percent) more interesting.



The Principal, who had worked together with Tea for many years, valued Tea's competence to never to give up:

[S]he does not give up what so ever, so it is everything. I have known her, I have been at this school for [2] twenty-seven years, and I have been colleague to Tea for twenty-seven years, so I have pretty good insight (Conversation with Principal, Table 10-2, also reproduced in Chapter 5.2).

I thus assert that competence of motivating pupils is one issue that characterises her localized knowledge, and so is how she approached her teaching, which I consider next.

### **8.2.2.2 Approaches to teaching**

As was evident in the previous section, a distinct characterisation of Tea's approach to teaching was her use of context. While planning she adjusted her teaching in response to past successes or failures: "[I]n a way you go around and think about how I could have done this differently" (conversation 5, cf. Chapter 7.2.3, and Table 10-56). This "thinking" led to development of new approaches to the teaching of particular topics, for example making stories as the one considered in the previous section, reorganizing the order suggested by the textbook, and vary the lessons. In the following three paragraphs, I consider one example from each of these categories, starting with the story she made for facilitating learning of subtracting negative numbers.

Concerning the accounting in the canteen, as mentioned in the previous section and considered in Chapter 7.3.3, Tea presented three tasks to which she wanted the pupils "to make good canteen-explanations" (observation 4A):  $1 + (-2)$ ,  $(-4) + 2$ , and  $(-4) - (-6)$ . She accepted Thom's explanation to the first task: "you eat two yoghurts and sell one", however suggesting that the "1" could stand for earning one "ten-crown", and that one has to buy for two "tens". To the next task, Theo then suggested, "you have forty crowns in debt and sell for twenty". For the last one, however, she had made the story about borrowing "six moneys", a loan that was forgiven (cf. Table 10-21). In addition, when teaching algebra, she made "stories", exemplifying variables by talking about words that could mean different things, and that a number could mean many things and a digit could have different values dependent on its place in the number (cf. Chapter 7.2.4.3). I referred to that idea as "demystification" of variables. Reflecting on her effort for contextualising all topics, which I assert, also is about "demystification", I suggest that her localized knowledge thus includes ideas about "demystifying" the mathematics.

The mathematical part of her lessons normally consisted of segments of plenary instruction and individual seatwork where pupils solved

tasks from the textbook (cf. Chapter 6.1.3). However, for variation she sometimes let the pupils practice mathematics the entire lesson (observations 10 and 11), and for providing “another kind of engagement” (conversation 6), sometimes assigned tasks that were not in the textbook (observations 6, 9B, and 15). For lesson six, Tea had carried out a “test” the pupils should evaluate and discuss to raise awareness to thinking processes (cf. Chapter 7.2.4.1), in lesson 9B the pupils should figure out who of two pupils had solved a certain task correctly, while in lesson 15, the pupils got some problems or “nuts” (cf. Chapter 6.3.5, Figure 6-5). Variation, both concerning how the lessons were organized and by providing sources for tasks other than the textbook thus appears to be localized knowledge Tea utilizes to enhance pupils’ learning.

The content in the textbook Tea’s class used was organized into three levels of difficulty, blue, green and red courses consisting easy, basic, and challenging tasks respectively. While the authors of the textbook (Hagen, et al., 2006) suggested that all pupils should start with the basic course followed by a test that should decide with which course they should continue, Tea decided to follow her own path (cf. Chapter 7.2.3). Tea’s expertise was evidenced in that she, as the only teacher in her school, started with assigning tasks from the blue course for low-achievers and from the green course for the remaining pupils, trying to push them further after finishing the assigned level. She also had some objections concerning the order of topics in the series of textbooks; the Pythagoras’ rule and congruence are introduced in grade ten, a fact with which she had never reconciled, however done nothing to change.

In addition to Tea’s effort for motivating pupils as discussed in the previous section, I assert that contextualizing all topics, deciding own curriculum, i.e. deviate from suggested agendas, variation in the lesson organization, and how to practice the mathematics, can be pointed to as characterizations of Tea’s localized knowledge.

### **8.2.3 Tea’s localized knowing**

When elaborating on the craft knowledge as implemented in Ruthven’s (2002) dialogic cycle, I had a strong conviction that there exists a dynamic dimension of knowledge which informs teaching; localized knowing. I did observe it, however not to the degree I had assumed. Localized knowing is “about how they understand the same object for teaching differently depending on their knowledge of the individual” (cf. Chapter 3.3). As mentioned in Chapter 7.3.2, Tea activated her localized knowing when guiding Thor finding the fat in the cheese (cf. Table 10-36). A reminder: she wanted Thor to tell how he had found that 27% of 500 grams equals 135 grams, which he could not recall. Tea then guided the boy from the 500 grams piece via the double, 1000 grams, which made him suggest that there were 270 grams fat. A problem raised

when going back to the original piece, i.e. halving that amount; he did not know how to do that, and Tea changed her approach to using sharing of 270 NOK. Another example I interpret as localized knowing was demonstrated when she, in lesson 2A, pretended to halve six of Tord's pencils to help him understand how 6.5 could be written as a fraction (observation 2a).

The episode with Thor demonstrated Tea's competence to contingently (Rowland et al., 2005) combine her knowledge of the pupil and his level of cognitive development (sensitivity to the student, Jaworski, 1994) with discrete parts of the mathematics (Rowland et al., 2005), a competence this study recognizes as localized knowing.

Unfortunately, I did not get any chance to observe more episodes that exposed examples of similar dynamic knowing. It is not to say that Tea's dynamic knowing was limited. As a response to an episode occurring in lesson 3, her Principal characterized Tea as being contingent: "it is amazing to see how fast one has to think creatively to find solutions for each student" (conversation 3). This indicates that there were episodes that illustrated such knowing. From the notes from the lesson<sup>56</sup>, I can see that Tea helped a girl to understanding by making drawings, which could be the situation the Principal commented on, but this is just speculation. The Principal claimed that this knowing results from engaging with, and learning to know, one's pupils:

[I]f one doesn't have relations, and if one doesn't know the students well and take the inconvenience with that then it is difficult to find methods. And, and particularly in mathematics that is an unbelievable challenge (conversation 3).

#### 8.2.4 Knowledge of facilitating mathematics learning

I will, in this section consider what characterized Tea's facilitation for learning, which includes expectations of self, individualization, managing and maintaining sociomathematical norms, and reluctance to use calculators. Towards the end of the section, I consider the impact it had on the category local *general professional knowledge* as suggested in my initial conceptual framework.

The MKT-framework (Ball et al., 2008) focuses on mathematical knowledge *for* teaching, and does not take into account all knowledge that is deployed *in* teaching. As discussed in Chapter 7.7, more issues than could be categorized into the MKT-framework thus evolved from the analysis. To explain what happened within the teaching, I then sought support in the Teaching Triad (Jaworski, 1994) and the Knowledge Quartet (Rowland et al., 2005). It was particularly the

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<sup>56</sup> I lost the video from lesson 3, cf. Table 4-3.

domain *foundation* in the KQ framework that allowed me to consider Tea's espoused beliefs about the subject, and how it is learned, i.e. her beliefs about what entails good learning environments for her pupils. The extent to which she worked for making good learning environments made me realize that it was necessary to reconsider the third category in my framework, general professional knowledge, to be more important for the teaching of mathematics than initially assumed. To substantiate my assertion and remind the reader, the following three paragraphs recall the issues mentioned at the beginning of the section, expectation of self, individualization, and attitude towards calculators.

In Chapter 7.7, I discussed norms in Tea's classroom, including expectations of herself: being responsible for the pupils the entire day, making sure parents know their children are at school, and having the pupils write adequate examples in their textbooks before going home. Moreover, I there considered how Tea worked hard to manage and maintain a sound learning environment, which included that she involved the pupils in parts of her private life.

Tea's humanity and beliefs makes her avoid using organisational differentiation (cf. Chapters 7.5.2 and 7.7.1). In line with the majority of Norwegian teachers (Klette, 2007), she usually differentiates by assigning tasks of different difficulty on pupils' work-plans. For high achievers, this sometimes meant they had to solve many similar tasks, such as drawing many graphs of linear functions, a situation Tea believed would make them feel successful, they would then experience "the joy of saying I understood, I can this" (conversation 18), however occurring at the expense of mathematical challenges (Jaworski, 1994). On the other hand, sometimes she offered opportunities for the pupils to choose whether they would be in a group learning about more challenging topics. Despite her strong conviction that pupils should not be differentiated according to level of competence (Tea used the words "smart" or "stupid"); she sometimes let high achievers get opportunities to learn more challenging topics, such as equations with fractions, inverse proportionalities, and slope. This inconsistency in her theoretical orientation coincides with Sosniak et al.'s (as cited in Handal, 2003) assertion that within each teacher's belief system there are beliefs that appears to be ideologically incompatible with the others.

Moreover, her belief system made her being reluctant to allow pupils to use calculators; "they in a way become sleeping pillows [...] because you can always use it" (conversation 5), thus serving as a way to escape work (cf. Chapter 10.3.1). She claimed that they serve as obstacles for development of mathematical knowledge, and that they, for many pupils, have served as an initiative to escape some work. But it happened that she allowed some of the low-achievers to use one when the aim for

practicing tasks was to learn strategies, for example as she did with Turi who was not able to divide twenty-four by three when “the aim was to practice equation-technique” (conversation 4, entire excerpt reproduced in Chapter 4.9).

In addition to what is considered above, I have comments from the school Principal that substantiates rearticulating the category *general professional knowledge*. Tea’s Principal, who had been her colleague for twenty-seven years, confirmed Tea’s concern for, and involvement with, her pupils. She considered Tea as a person who was more than a subject teacher, “I know her view on humans. I know how she works in relation to the whole pupil” (conversation with the Principal, cf. Table 10-2), and added that Tea was good at motivating pupils: “she doesn’t give up what so ever”. The Principal also characterized Tea as one who takes “the inconvenience” of learning to know and making relations to her pupils, so that she could “find methods” adapted to each pupil, an effort the Principal characterized as “an unbelievable challenge” in mathematics (earlier reproduced at the end of Section 8.2.3). These statements indicate that Tea’s effort for making good relations with her pupils is about facilitating mathematics learning.

The pupils, in particular Tone (cf. Chapter 7.7.2), confirmed the Principal’s words concerning Tea’s relations the pupils. Tone regarded Tea as a caregiver, or more like a mother: “she brings us up in a way like a mom; I get a feeling of up-bringing”. Based on these considerations and the analysis as described in Chapter 7.7, I believe it is reasonable to suggest an elaboration of the category general professional knowledge to be about *knowledge*, namely *Knowledge of Facilitating Mathematics Learning*.

### **8.2.5 Tea’s local knowledge – concluding remarks**

In this section, I have considered characteristics of Tea’s local knowledge. It has also discussed and justified the requirement for broadening and extending the initial category GPK, which due to its content, I renamed from general professional knowledge to Knowledge of Facilitating Mathematics Learning (Figure 8-1).

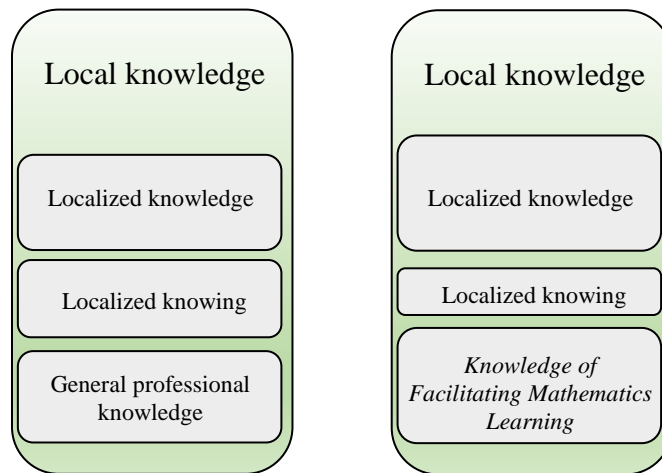


Figure 8-1: Suggested model to the left, Tea's local knowledge for teaching mathematics to the right

In addition to presenting an illustration of what local knowledge for teaching contains after the analysis, Figure 8-1 also depicts an idea of the distribution of these categories within Tea's local knowledge base. Her localized knowledge as well as her knowledge of facilitating learning each stood out as a much larger part of her knowledge base than her localized knowing. They were both strongly affected by her beliefs and values; beliefs about contextualization as incentive and motivation for learning, believes in her own responsibility, and her philosophy on humanity.

Tea's localized knowing was evident, but not demonstrated, to the extent that I had expected. My data include one lesson<sup>57</sup> where she contingently used different representations, and connected discrete parts of the mathematics (Rowland et al., 2005). Thus, I do not know whether such activities were reserved to "special" pupils (Jaworski, 1994), which was how she characterized Thor: "he is a special boy" (conversation 13), or it was a result of coincidence; my observations did not coincide with more pupils in need for such individual support. Tea initiated the episode with Thor. Concerning the pupils, I did not experience any of them initiating situations that required Tea to act contingently as understood in the Knowledge Quartet (Rowland et al., 2005). This could reflect the methods I used. I followed Tea with a handheld camera, which, for some pupils, could have made them refrain from providing inputs or seeking situations of uncertainty that required contingent action on the part of the teacher.

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<sup>57</sup> Maybe lesson 3 was the other exception, as mentioned in section 8.2.3.

As explained above, I believed that dynamic knowing would be demonstrated more often. I still believe that the competence to interact individually the way Tea did with Thor exists in classrooms, and is an important competence for teachers to have. I thus keep the category in my framework; however, in this case it does not exist to the extent I initially believed, as I have indicated in Figure 8-1.

As explained above, I utilized three frameworks when analysing my data, frameworks I initially used for explaining the categories in *my* conceptual framework. The analysis exposed dimensions of activities and actions I suggest as missing in two of the frameworks, the KQ and the MKT frameworks. In addition to discussing findings related to each of the frameworks, in the next section I thus suggest elaboration of the KQ and the MKT frameworks. At the end of the section, I integrate the dimensions of the Teaching Triad, the Knowledge Quartet, and the Mathematical Knowledge for Teaching frameworks into my own conceptual framework.

### **8.3 Integrating four frameworks**

My conceptual framework grew out from my experience within mathematics teaching, and not from research. Thus, I had not many good teaching episodes that could explain the suggested categories (cf. Chapter 4.6.2 and Section 8.2.4). I thus had to find frameworks that were developed on the basis of research, the Mathematical Knowledge for Teaching framework (Ball et al., 2008), the Knowledge Quartet (Rowland et al., 2005) and the Teaching Triad (Jaworski, 1994), and use them as analytical tools for working with my data. In the following, I discuss the results of using the three frameworks, my experiences with, and the suggested development of the MKT in Section 8.3.1, the importance and elaboration of the KQ in Section 8.3.2, and the contribution from the TT in Section 8.3.3. Each sub-section ends with an illustration of the *idea* of the extent to which the different dimensions of Tea's knowledge for teaching mathematics were observed relative to the respective frameworks. I end the section by considering how the different domains in these three frameworks can be integrated into a rearticulated framework (Section 8.3.4), thus beginning the creation of a framework that can be used as an analytical tool for analysing knowledge for working as a mathematics teacher, i.e. mathematical knowledge both *for* and *in* teaching.

#### **8.3.1 Investigating findings; the MKT-framework**

The MKT-framework (Ball et al., 2008) includes six domains of knowledge *for* teaching. They acknowledge that there are some

boundary problems; it is not always easy to discern how the definition of distinct categories differ from each other, for example, pupils' understanding of decimals can occur across more domains. My analysis confirms these challenges; I found that in a major number of episodes Tea's knowledge could be organized into more than one category. For example, in Chapter 7.2.4.3, contextualization as "demystification", and Chapter 7.3.2, using different representations, the knowledge she deployed could be organized into both KCS, CCK, and SCK. For those who want to use the framework for measuring different dimensions of knowledge, this may be a problem. For my study, it was a strength and a necessity to cross the boundaries, which I have indicated in the illustration of Tea's MKT (Figure 8-2).

The MKT-framework suggests different categories of mathematical knowledge for teaching, and is used in the development of a survey instrument for investigating these categories of knowledge. Based on my experiences, the MKT-framework can also be used to *analyse* teachers' knowledge *for* teaching the subject, however limited in relation to knowledge deployed *within* the teaching of mathematics.

### **8.3.1.1 Extended Horizon Content Knowledge**

Horizon content knowledge is about having awareness about how mathematical topics are related over the span of the mathematics included in the curriculum (Ball et al., 2008). In Ball and colleagues' illustration of the framework, they suggest horizon content knowledge as part of subject matter knowledge, but add that they are not sure whether it may run across other categories.

My study shows that the dimension horizon content knowledge crosses the boundary between subject matter and pedagogical content knowledge; I experienced Tea as having knowledge about approaches the pupils have met in primary school. When teaching equations, for example, Tea started with reviewing tasks like  $2 + 3 = 5$  and  $6 + 2 = 8$ , and continued:

Then you became very good on those things. Finally you were so good [2] that one day when you came to school then you got new calculations [3, writes  $3 + 3 = 9$  on the blackboard], then did the tasks look like that (observation 8, cf. Table 10-79).

I thus suggest extending the category horizon content knowledge to be included in both content knowledge and pedagogical content knowledge, making a crossover as illustrated in Figure 8-2.

### **8.3.1.2 Tea's Mathematical Knowledge for Teaching**

The excerpt reviewed in Section 8.3.1.1 and similar episodes provide evidence that there exists a didactical dimension to horizon content knowledge. Based on this overlap, the exposed transparency between the different dimensions, and the extent to which the different dimensions of



Tea's knowledge for teaching mathematics were observed, I suggest the following illustration of Tea's MKT-framework (cf. Figure 7-2):

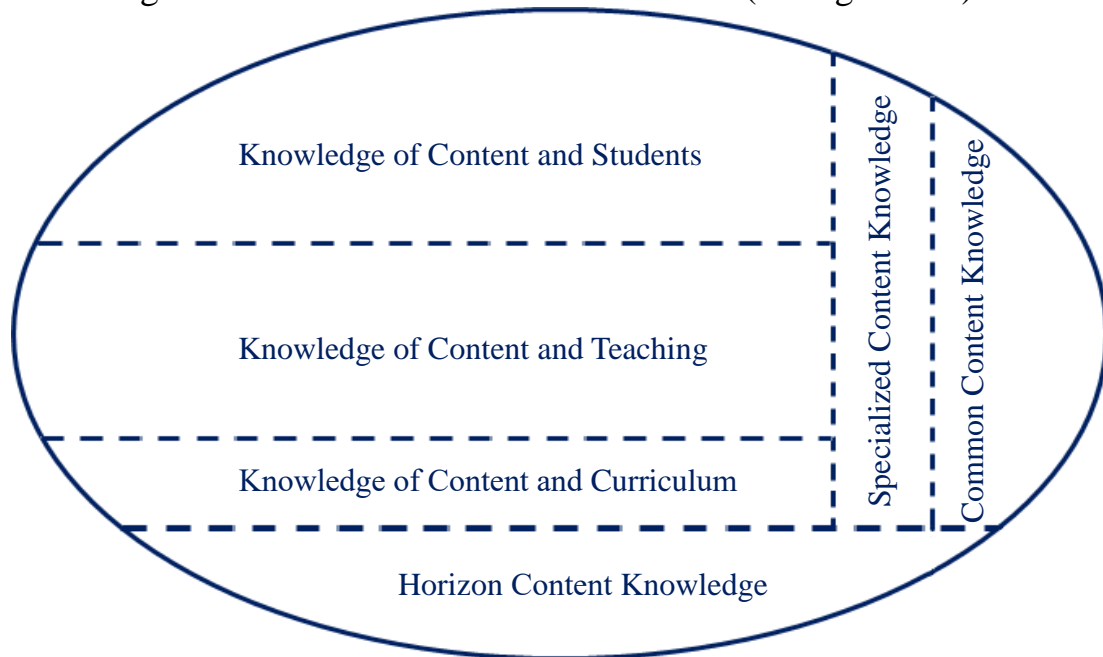


Figure 8-2. Illustration of Tea's Mathematical Knowledge for Teaching

It has to be noted that the MKT-framework serves as a foundation for items made for measuring teachers' knowledge through written tests. I did not "test" Tea's knowledge that way. I assume that if I had used such tests, her CCK and SCK would have become more prominent, and there would have been more balance between the categories.

### 8.3.2 Importance of the Knowledge Quartet

The knowledge quartet (Rowland et al., 2005) consists of four dimensions of knowledge: foundation, transformation, connection, and contingency. In Chapter 7.7, I discussed its importance for explaining Tea's beliefs as important to her mathematics teaching. In addition, I used all domains in the quartet to underpin my analysis and interpretations of the activities taken in the lessons. Foundation, I assert, strongly coincides with the dimensions in the MKT-framework. Of the remaining three, I provide some examples from Tea's classroom.

I interpret the KQ's dimension *connection* as connections within the subject, i.e. coherence across episodes and lessons, and connections with other topics within mathematics. I often experienced that Tea connected lessons by starting with a review of the work they carried out in the previous lesson (cf. Chapter 6.3.1.1, Table 10-15, Table 10-16, and Table 10-17). She also connected "discrete" parts of the mathematics, for example when decimals was on the agenda at the beginning of the

semester, she had pupils convert these to fractions even if the textbook focused on fractions at a later stage (conversation 12).

Throughout my observations, I experienced Tea transforming her knowledge for facilitating pupils' learning. In the introduction to percent, for example, her understanding of the concept made her use two approaches. In the first lesson, she read a story that should make pupils aware of the importance and usefulness of percent (cf. Chapter 7.2.4.2, the dream of the moped certificate), while in the next she had them make a drawing because "it communicates much more easily what you mean, that is sizes" (cf. Chapter 7.3.1, the size of the solar eclipse).

I observed some episodes of contingent actions, for example when guiding Thor finding the amount of fat in the cheese as considered in several sections above. Another contingent action occurred when Tea grabbing and pretending to halve six of Tord's pencils when he did not understand how 6.5 could be written as a fraction (observation 2a). Due to the occurrences of contingent actions, however, I would not say they stand out as characterization of Tea's knowledge in mathematics teaching.

### ***8.3.2.1 Extending the domain connection***

The knowledge quartet (Rowland et al., 2005) was developed after studying preservice teachers in their practices, i.e. its domains evolved from analysing the work these teachers carried out while teaching. They had not started their professional career, thus one assumes they do not yet have the overview over other school subjects, which experienced teachers may have. Tea is an experienced teacher; she knows several school subjects, and she connected her mathematics teaching to subjects such as natural sciences, history, food and health, and RLE. Tea possesses what Shulman (1986) terms lateral knowledge, a dimension not included in the KQ. I believe that teachers' ability to connect mathematics to other school subjects is important both concerning motivation for learning the subject, and for understanding its importance in society.

### ***8.3.2.2 Tea's Knowledge Quartet***

Based on what I experienced as Tea's actions and activities in the classroom, as discussed at several points in the dissertation, I suggest that her knowledge related to the dimensions in the KQ can be illustrated as in Figure 8-3, its proportions resulting from the amount of appearances, however not calculated in percent:

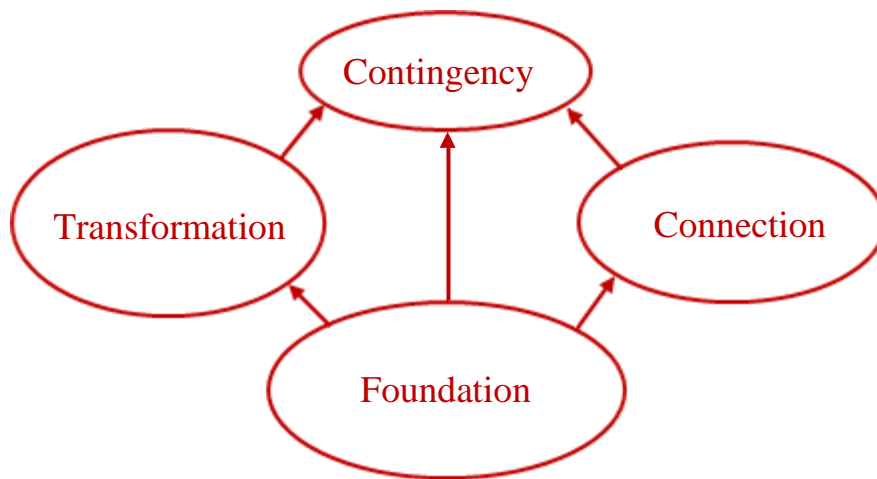


Figure 8-3. Illustration of Tea's Knowledge Quartet.

The figure illustrates the dimension contingency as smaller than the other categories. I observed a few instances of contingent actions (for example as with Thor and the cheese, and Tord and his pencils as mentioned over), however not many. That made me draw the contingency-oval smaller than the ovals illustrating the other dimensions, which I experienced did not differ in frequency of occurrences. A question arises, is this due to experience? Are experienced teachers prepared for all eventualities? I am not able to answer this question right now, but maybe it is something into which to research?

In Section 8.3.2.1, I mentioned how Tea connected mathematics to other school subjects. If knowledge of the lateral curriculum would be included in the current KQ, the oval illustrating Tea's "connection" would be somewhat bigger than illustrated in Figure 8-3.

### 8.3.3 Contribution from the Teaching Triad

When analysing Tea's practice, I frequently experienced her sensitivity to the pupils on both cognitive and emotional levels. To account for her caring, I sought assistance in Jaworski's (1994) Teaching Triad. Her sensitivity was evident both in the lessons and in our subsequent conversations (reported in Chapters 7.1.3, 7.1.4, 7.3.2, and 7.5.2). This, however, sometimes appeared to occur at the expense of the mathematical challenge (see for example Section 8.2.4). In addition, facilitating learning, and managing and maintaining learning environments were distinct activities that characterized Tea's practice.

Based on the results of the analysis of my observations and our conversations, my illustration of Tea's relations to the teaching triad would then be:

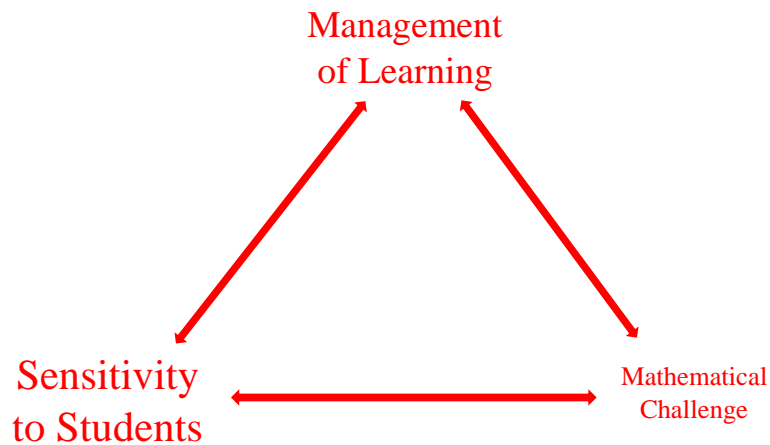


Figure 8-4. Illustration of Tea's Teaching Triad.

### 8.3.4 Three frameworks informing a fourth

In this section, I have considered Tea's knowledge for teaching related to the three analytical tools I have used. I have discussed how Tea's knowledge for teaching consisted of several categories of knowledge and dimensions of these. While I used the MKT-framework (Ball et al., 2008) for analysing the mathematical knowledge *for* teaching Tea brought into the classroom, I used the KQ (Rowland et al., 2005) for analysing the mathematical activities Tea enacted *in* her classroom. The Teaching Triad (Jaworski, 1994) served as a tool for analysing two dimensions of sensitivity to her pupils as well as the mathematical challenges they were offered.

The three frameworks are complementary. The KQ and the MKT are somewhat interconnected as both are based on Shulman's (1986) work, however the MKT focusing on mathematical knowledge for teaching while the KQ is about mathematical knowledge *in* teaching. The Teaching Triad (Jaworski, 1994), which stems from a study on investigative teaching, also concerns dimensions of activities undertaken within teaching. In Section 8.3.2, I suggested that the MKT is somewhat included within the KQ's dimension foundation. However, the dimension *connection* in the KQ, I interpret as related to the curricular knowledge in the MKT, thus "stretching" outside the foundation. Moreover, I interpret the third dimension of the teaching triad, management of learning, as included in all three frameworks: in the dimension transformation in the KQ and the knowledge of content and teaching in the MKT-framework. The frameworks very well served as analytical tools for my search for the characteristics of Tea's local knowledge for teaching mathematics.

The discussion above exposes that it is difficult to make a clear-cut and distinct implementation of the three discussed frameworks into my conceptual framework. Several of their dimensions stretch over more of

my categories. However, by making a rough subdivision of the different dimensions, an illustration might look like the following:

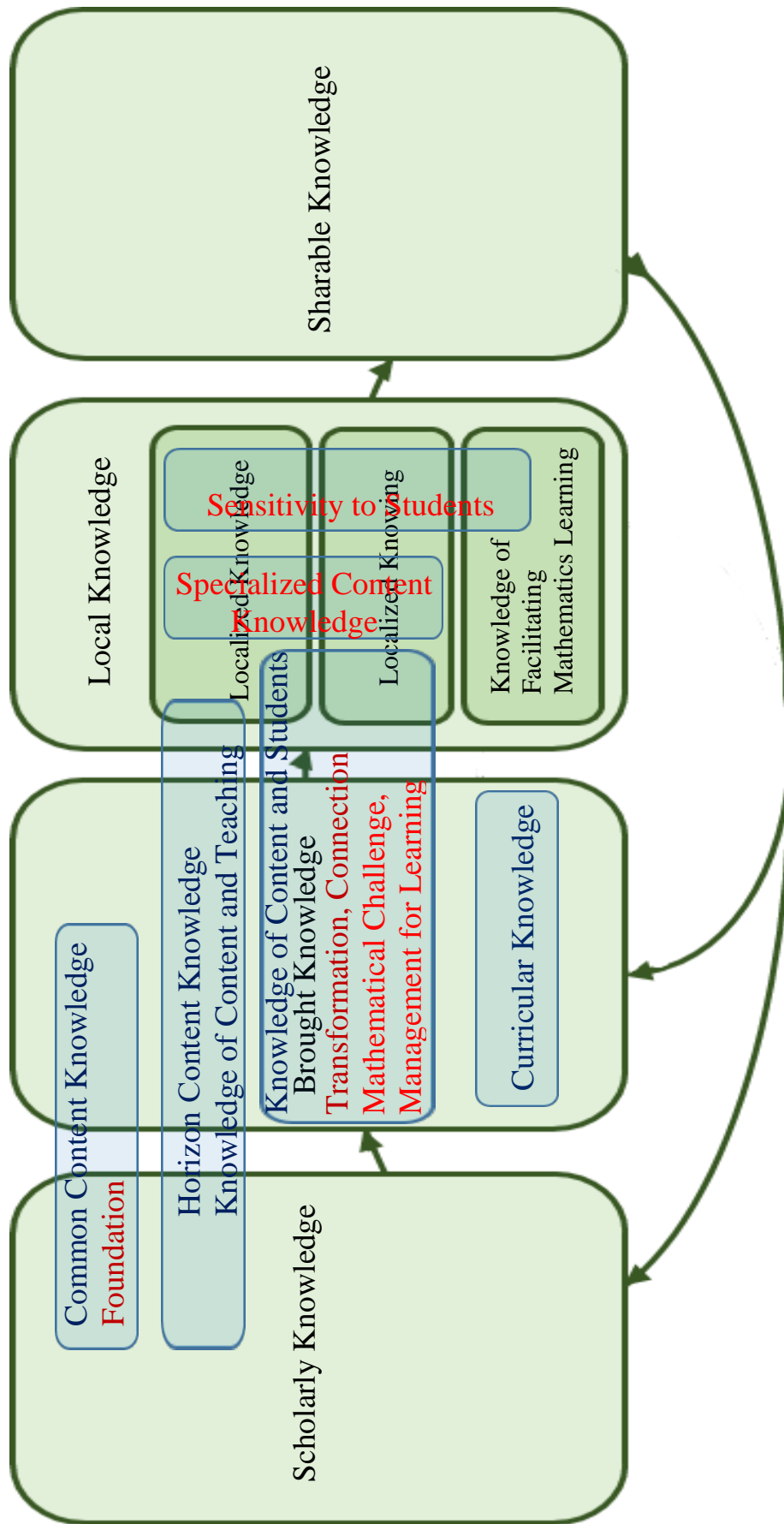


Figure 8-5. Dimensions from **MKT**, **KQ**, and **TT** inserted into my framework

The categories in the frameworks I used as analytical tools helped me to define my own framework more clearly. I see that **Common Content Knowledge** and **Foundation** stretch over the categories scholarly and brought knowledge, that **Horizon Content Knowledge** and **Knowledge of Content and Teaching** stretch over scholarly, brought, and localized knowledge, and that **Knowledge of Content and Curriculum** belongs in the category brought knowledge. **Knowledge of Content and Students**, **Transformation**, **Connection**, **Mathematical Challenge**, and **Management of Learning** belong to both brought and localized knowledge and knowing, while **contingency** is included in localized knowing. **Specialized Content Knowledge**, I recognized as both localized knowledge and knowing, but may well come as scholarly and brought knowledge. **Sensitivity to students**, I recognized in all three dimensions of local knowledge.

As illustrated in Figure 8-5, my analysis has exposed that all three analytical frameworks are required when analysing knowledge for and in teaching, at least in Tea's case. This research has thus provided information that enables me to explain my conceptual framework thoroughly, thus providing a unified tool that can be used to inform prospective and in-service teachers about what the work of teaching mathematics entails.

#### **8.4 Implications for teacher education**

I have observed a teacher who has been in the teaching practice for more than thirty years. It may thus seem somewhat peculiar, on this basis, to suggest implications her knowledge could have for teacher education. Barth (2004, p. 57, 59) invites to "dig" for what experienced teachers possess of "hard-won learnings", i.e. "the abundance of craft knowledge". I assume that he did not mean that the exposed craft knowledge was just for sharing with other experienced teachers. Based on Barth's suggestion and on my analysis, I allow myself to suggest some points I assert as important for novice teachers to be aware of before they enter their professional practice, hence for teacher educators.

My research does not provide any grand novel discovery. We know that teachers develop their knowledge in the process of teaching (Leinhardt, 1990). We know that there exist knowledge specific for mathematics teaching (Ball et al., 2008), and we know there exist knowledge in mathematics teaching (Jaworski, 1994; Rowland et al., 2005). I also know that many Norwegian teacher education courses use one or two of these as sources for students to learn about mathematics teaching. I do not know of any that uses all three. My research shows that one or two is not sufficient. I thus assert that all three are required if

students should get a thorough theoretical insight into the work of teaching mathematics.

My analysis exposed that teachers' values and belief-systems affect their teaching. In mathematics this can occur at the expense of offering mathematical challenges (Jaworski, 1994). In addition to the frameworks, I thus suggest that teacher education programmes aiming at educating mathematics teachers include discussions about the impact beliefs may play out in their work as mathematics teachers.

## 8.5 Further research

For years, there have been discussions about what knowledge is required for mathematics teachers, a discussion I believe will continue for years to come. I believe that those who attempt to make instruments for measuring mathematical knowledge for teaching (e.g. Ball et al., 2008) still have work to do. In accordance to Kaarstein (2014), the current research suggests that it is a challenge to make distinct definitions of the categories, which also Ball and her colleagues (2008) acknowledge.

As indicated in my conceptual framework, I believe in *knowing* as a dynamic kind of knowledge teachers deploy as situations arise in the classrooms. I did not observe this dynamic entity to the extent that I would suggest it as being an equal part of teacher's local knowledge as localized knowledge, which I indicated in Figure 8-3. However, I did observe a few episodes; examples that make me want to look further into the phenomenon, particularly how it applies to experienced teachers. I very much believe in the phenomenon and its importance, and hope to be able to get some more information about what it entails. As discussed in Section 8.2.5, the limited number of occurrences may be due to the method I used, for example, following Tea with a handheld camera could have made some pupils choose not to call on her. I thus need to consider using a different approach if I get another opportunity to expose local knowledge in mathematics teaching.

A second point of interest is the mathematics included in the education of teachers. Tea had extended her education by studying mathematics for one year; however, the mathematics was not particularly aimed at teaching. Mathematics was implemented as a compulsory subject in teacher education in 1990 (Birkeland & Breiteig, 2012), however for the quarter of a year, equivalent to 15 ECTS. One can then conclude that there assumedly are teachers who teach the subject without any formal education in the subject. It has to be noted; in 2008 a requirement for at least 60 ECTS points in mathematics was introduced for those who would teach the subject in lower secondary school (Kunnskapsdepartementet, 2008). While analysing my observations, I thus often wondered how the teaching of an experienced certified teacher



without having studied mathematics would have looked like. Having Ma's (1999) research in mind, I would very much like to study a teacher who was educated before 1990 and reported as having the same qualities as Tea, however without having studied mathematics to the same extent. Tea extended her teacher education with one year of additional studies in mathematics. Such an opportunity would assumedly provide some more local gold-nuggets (Barth, 2004) developed in the practice of mathematics teaching in lower secondary school.

## **8.6 Critical reflections**

There is an agreement within the qualitative research in education area that an evaluation of the quality of the research is required (Freeman, DeMarrais, Preissle, Roulston, & St. Pierre, 2007; Lester & Lambdin, 1998). There are, however, some disagreement over the terms to be used (Freeman et al., 2007). Lester and Lambdin (1998) assert that the criteria for evaluating research in mathematics education must resonate with questions and issues about the teaching and learning of mathematics. They (Lester & Lambdin) thus suggest seven criteria for identifying the quality of research conducted within our area: worthwhileness, coherence, competence, openness, ethics, credibility, and lucidity. In this section, I use these criteria to discuss the quality of my research.

### **8.6.1 Worthwhileness**

Does my research add anything, and does it deepen our understanding of issues associated with mathematics teaching and learning?

My research set out to identify and characterize the knowledge for mathematics teaching one experienced and appreciated mathematics teacher holds. I chose this theme because my main interest within mathematics education is teachers' knowledge for teaching the subject, and that Barth (2004, p. 57) urged to "mine out" gold-nuggets from "the gravel" of teachers' experiences. Knowledge created within the practice is thus sought (Barth, 2004; Grevholm, 2010; Ruthven, 2002). My research aimed to contribute both to expose the gold-nuggets, and, through exposing experienced mathematics teachers' knowledge for teaching, to illuminate the practice of mathematics teaching.

The result from a case study cannot be generalized to populations, however they are generalizable to theoretical propositions (Yin, 2014). Yin thus suggests that the data, under specific conditions (see next section) can lead to generalizations. My study does not represent the generality of Norwegian lower secondary mathematics teachers, but it has provided some insight into the knowledge one experienced and appreciated mathematics teacher deployed while teaching her grade nine

pupils. I see it as worthwhile to make such knowledge public, and so did my informant: “it could be a contribution to the ongoing debate in the media regarding Norwegian teachers’ competence. Media does not present Norwegian mathematics teachers particularly positive, which makes me sad” (cf. Chapter 10.3.1).

Initially, my research questions asked for evidence of locally developed knowledge and knowing (i.e. craft knowledge as defined in Chapter 2.1.3.3 and further explained in Chapter 3.3). When studying the literature, I read about its existence. This, I consider next.

### 8.6.2 Coherence

When I started this research, my intention was to find evidence of the craft knowledge Barth (2004, p. 60) asserts that experienced teachers hold, knowledge which is “taken to the grave” if not being researched into. As an experienced teacher myself, I know such knowledge exists, but I wanted to have it evidenced by research. I have earlier referred to the process of the development of my final research question (cf. Chapters 1.4 and 3.4, and Section 8.1), changing from looking for evidence of such knowledge to searching for its characteristics. To remind the reader: After beginning the research process, I was early convinced by the literature (e.g. Bromme & Tillema, 1995; Leinhardt, 1990; McNamara & Desforjes, 1978; Ruthven, 2002) about the existence of such knowledge. Even if none of the projects I read about were carried out in Norway, I had no reason to believe that it was otherwise for Norwegian teachers. Hence, I decided to look for its characterization and how it was deployed in teaching, thus changing my research question to: “What characterizes the local knowledge in mathematics teaching of an experienced Norwegian teacher?”

Throughout the process, I had the research questions in mind; but I have to admit that I sometimes lost focus. As has been evidenced, Tea was a very caring teacher; she was concerned about her pupils, and very eager to talk about them. In such situations, I could forget what I was out for, which led to missing comments on some mathematical activities I wanted to discuss. Sometimes, however, I marked such episodes for discussion next time we met.

My focus was thus on a “case”, i.e. the characteristics of the knowledge a teacher holds and deploys in the teaching process, my intention was to “retain a holistic and real-world perspective” (Yin, 2014, p. 4), being concerned with a rigorous and fair presentation of empirical data. This coincides with the case study method, which I followed throughout my work. Given that the data can shed light upon some theoretical concepts or principles, here a teachers’ local knowledge, a case study can allow *analytic* generalization (Yin, 2014). Analysing the data from a knowledge base which over thirty years

has grown to become “an inch wide and a mile deep” (Barth, 2004, p.55), i.e. Tea’s local knowledge, against three theories (Ball et al., 2008; Jaworski, 1994; Rowland et al., 2005), provides an indication of how these frameworks can be integrated (cf. Section 8.3.4). To meet the criteria for analytic generalization, however, I need to look to other research who have considered the integration of these frameworks, which is not the focus of this study.

To get access to the requested knowledge, I found it important to observe Tea’s teaching followed by subsequent conversation. Her knowledge would not be “visible” through classroom activities alone, and I would not be able to understand her stance without having episodes that I could connect to her explanations and utterances. When analysing the data, I categorized the classroom activities into lesson parts (openings, plenary segments, seatwork segments, and closings) before organizing all activities into the categories described in Chapter 4.6.1. I have, in Chapter 4.6.2 described the challenging process I experienced when analysing the data from our conversations. I thus here only refer to the final round of analysis. To deepen my understanding of her knowledge, I followed an open, however thorough, analysis of our conversations by first writing memos from all of them, then dividing the data into three rough categories, practice, principles, knowledge and knowing. Within each of these categories, fifteen categories representing pedagogical and didactical concepts evolved (cf. Table 4-9), which in the next phase were organised into the Mathematical Knowledge for Teaching (Ball et al., 2008), the Knowledge Quartet (Rowland et al., 2005), and the Teaching Triad (Jaworski, 1994) frameworks.

As explained earlier in this section, to address my research question, I needed both classroom observations and subsequent conversations with the teacher. This gave me the opportunity to compare what she enacted in the classroom with her mental model of learning and teaching. According to Ernest (1989), the powerful influence of social context or teachers’ level of consciousness of own beliefs can lead to disparities between their intentions and what they enact. I observed a high degree of consistency between these two elements in Tea’s practice. However, I also observed some elements I interpret as disparities between the two, for example her attitude towards, and enactment of, differentiation (cf. Chapter 7.5.2) and teaching for conceptual understanding (Hiebert & Lefevre, 1986; Jaworski, 1994), cf. Chapters 6.3.1 and 7.1.1. Even if such disparities are known within the area of mathematical education research (Ernest, 1989), it put some ethical demands on me as the researcher. I will return to that topic in Section 8.6.5.

To summarize, I have followed a descriptive single-case study design to make public the local knowledge my informant, Tea, deployed in her teaching. Even if I, due to Tea's eager for talking about her pupils, sometimes lost focus on the mathematics, I assert to have sufficient information to get my research questions addressed. Moreover, using a different method could have exposed more incidents of localized knowledge and knowing. I have not, through this study, been able to make any analytic generalizations (Yin, 2014) concerning the integration of the three frameworks (cf. Section 8.3.4). However, it is the beginning of a process where the product can be *one* framework that provides a holistic insight into mathematical knowledge *for* and *in* teaching.

### 8.6.3 Competence

Does my research include effective application of appropriate data collection, analysis, and interpretation techniques?

To the first part, effective application of appropriate collection of data, I would say two things. First, given the research question, which asked for the knowledge of one teacher, I felt that I could not ask for more of my informant's time. She was welcoming, positive, and helpful, and I would assume that I could have spent more time with her than I actually did, but I did not want to put more demands on her than what I initially asked for (cf. Chapter 4.5.3). This also concerns her planning; I did not want to ask for observing when she planned for the lessons. Second, I could have had a research question asking for the knowledge of several teachers. This would have provided more data, and possibly produced different results. However, it is important not to take on too much within the limits of such a study, but rather to deepen in the data one can gather within the time limits.

Concerning analyses, I first looked into the classroom activities, dividing the lessons into openings, plenary and individual mathematics work, closings, and the activities within each of these before taking on the analyses of our conversations. I then had an overview of all episodes, and it was easy for me to find examples with which to illustrate Tea's comments. As mentioned above, I met some challenges when analysing the conversations, challenges of personal character; I, what is popularly called "hit the wall", followed by a period experiencing lack of self-confidence, and faith in my data and myself. I was then ready to terminate my project. Fortunately, I got some help from a good friend and colleague; he spent time convincing me that I had something important to bring to our research area. With his help, I gradually "recovered" and continued the analysis, which resulted in what I have presented in Chapter 7 and further discussed in Section 8.2.

When taking on the research, my only experience with fieldwork was observation and some simple interviews in connection to a course

included in the master programme, and using a questionnaire and a few interviews connected to my master thesis. I was a novice in the area, thus I conducted a pilot study to experience classroom observations and subsequent interviews with the teachers. As explained in Chapter 4.5.1, this gave me some, but not sufficient, insight into what fieldwork entails. I thus entered the fieldwork with limited experience. To prepare the pupils for my observations, I visited the class to talk about my plans before they left for summer holiday. It was important to me that the pupils knew they were not the focus of my study. Thus, when entering the classroom for my first observation, the pupils knew about my mission. I had my handheld camera focussing on Tea, and followed her in the classroom. I avoided following her to pupils who were shy and appeared reluctant to the camera and me. However, it did not take long before the pupils appeared not to notice my presence.

For the conversations, I had prepared questions that were rooted in classroom episodes, and about other issues such as her education, how she experienced working as a mathematics teacher, and her beliefs about learning and teaching, etc. Our first conversations were characterized by firmly following the questions, but soon developed to be a conversation where Tea was free to talk about her experiences with the lessons. This led to her often talking about the pupils, not particularly focussing on the mathematics, as explained in the previous section. I let her do that, believing that it made her feel confident and more free and open for the questions I would pose, both about the mathematics and other issues. I thus believe that the information she provided during our conversations were genuine and honest, and not a result from attempts to act “politically correct”. There could, and probably should, be more mathematic-specific discussions in our conversations, but I do not know how the outcome of the conversations then would be. The comment I got when meeting her some time after my visits, “I miss our conversations” made me realize that I probably made the right choice, certainly I did for her.

#### **8.6.4 Openness**

I researched into a practice similar to the one within which I taught myself. I studied a teacher having almost similar education, and being at the same age as myself, having practiced teaching about as long as I have. Even if I had that particular teacher in mind when asking for an informant, I did not mention her. Based on my criteria, an experienced and valued teacher who had studied mathematics, the Principal suggested her.

Entering a classroom similar to my own, studying a teacher who could be myself, must lead to challenges and biases that I should expect could be hard to overcome. I did not. I believed that my knowledge of the subject, the school system, and the work as a mathematics teacher would be an invaluable advantage, which they were. However, researching into “my own” practice without being the teacher was far more challenging than I ever believed it would be. For a long time, the challenges overshadowed the benefits. The observations and conversations took place as planned, and I got the information I asked for, but when trying to see a bigger picture, I was stuck; I was not able to replace the teacher Inger with the researcher Inger. I had no problems analysing the lessons and its parts, it was about the knowledge she deployed while teaching, but it was problematic to take on a distanced and neutral view. It was first after making memos from all conversations, and categorizing the comments as pedagogical and/or didactical, I really felt that I was able to take an (almost) neutral stand. From that moment on, I have felt that I have seen, and reported about Tea’s knowledge for and in teaching mathematics as open and honest as possible, not making comments as if she was Inger.

Another aspect of biases while doing observations and conversations concern presence in the classroom. What influences would that make on the pupils and the teacher? As explained above, I visited the classroom beforehand, and attempted to avoid approaching pupils who had told Tea they felt uncomfortable having the camera pointed towards them, and those who appeared shy, which I interpreted as signalling reluctance to my presence (cf. Chapter 4.5.3). This did not concern many pupils. After being questioned by a father who feared my presence would influence his son’s opportunity to learn mathematics, we showed some clips from the lessons and discussed how the pupils felt about it. There were no comments that indicated that I should stop the observations. One week later I overheard Thor saying, “I did not know that Inger was here, [3], Theo, did you know she was here?”

I did not know Tea before I began the observations. I had met her twice, first when I evaluated the exam of a boy she examined, and once when I saw her on my way to interview one of her colleagues for my master thesis and she asked if I would interview her. Meeting regularly over a period of nine months, having good and relaxed atmosphere in the conversations makes your meetings somewhat like meeting with a friend; she opened up for being private. Sometimes she started our conversations by talking about family matters, which I interpreted as trust and indication of honesty, which I also apply to her comments on the mathematics. However, such close relationship may also compromise the research. I have tried to be as honest as possible. Even if the content

in Chapters 6 and 7 result from my interpretations, I have attempted to substantiate all suggestions and assertions by pointing to classroom episodes and excerpts of her statements.

### 8.6.5 Ethics

All I have considered so far in Section 8.6 concerns ethics: how to go about to conduct the research, how to meet and handle the informants, how to present persons and the information they provide, how to be honest to one's research and to oneself, and how to rapport the result. I have, all the way tried to keep this in mind, but it has not always been easy. I will here, in seven points list what I did for meeting general ethical issues and national ethical requests:

- I wanted to enlighten our research community about knowledge developed in the practice of teaching (worthwhileness).
- I have attempted to follow regulations set by the Research Council of Norway, including submitting the required notification form to Norwegian Social Science Data Services.
- Informed the participating school, pupils (and pupils' parents) and informant about my objectives, which was to get insight into the informant's knowledge for teaching mathematics without any deliberate influence on my part.
- Attempted to act respectfully during observations, conversations, and in the meeting with the informant in the school.
- In the analysis, finally looking for what was there, and not what I believed was there as I did in the beginning.
- In the presentations, trying to avoid information that could disclose the persons and the school involved in the study, for example by using pseudonyms.
- In the report, provide accurate, and objective, descriptions of Tea's practice and her knowledge, without any deliberate attempts to do any harm.

The first five of these bullet points have not been that challenging to follow. The last two points, however, one will not know until the thesis is published: will anyone recognize the school? Have I written something that discloses my informant's identity? Certainly, Tea and her closest colleagues will recognize "the story" of this dissertation. This work has lasted for many years. At the beginning of the project, I worked closely with my informant. I wanted "her" story, but did not want to put more workload on her. Based on our conversations, I thus wrote the "story" for her and sent it to her for corrections and/or confirmation. She confirmed its content, on which Chapter 5.1 is based

(cf. Chapter 10.3.1). I have not had any contact with her for the last two years, so she has not seen the latest version of the thesis.

As earlier indicated, this work has been going on for many years. Much has happened in the Norwegian school system, particularly concerning teaching approaches. If I had asked Tea today, it is likely that she would have used other approaches than those she used when I visited her. The story I have told, is thus about the knowledge for mathematics teaching anno 2011. I will not assert the story as old-fashioned, but it might very well not be about Tea's knowledge anno 2016. Still, it is a story of knowledge for and in mathematics teaching which is worthwhile telling.

#### **8.6.6 Credibility**

Credibility concerns justification of claims and conclusions. I have, throughout Chapters 6 and 7, where I present the result of my analysis, tried to illuminate and justify all my claims by examples or illustrations. Moreover, I have, in the previous sections, considered issues of credibility. I have also provided extra excerpts in the appendices. I thus hope that thoughtful and open-minded readers find my evidences believable, and that they meet requirements for analytic generalizations (Yin, 2014).

#### **8.6.7 Lucidity, clearness and organization**

The last point on Lester and Lambdin's (1998) list of criteria concerns the report's lucidity, clearness, and organization. My research is about a person's craft knowledge for and in teaching mathematics. For me it was thus important to define these concepts, both the known concept of knowledge (cf. Chapter 2.1), my use of the concept knowing (cf. Chapter 2.1.1) and the concept craft knowledge (cf. Chapter 2.1.3), which I later renamed to local knowledge. I also found it important and necessary to describe knowledge for teaching mathematics (Chapter 2.2.1) and knowledge in teaching mathematics (Chapter 3.2).

Having these concepts explained, and my tentative research questions defined, I decided the research design (Chapter 4.2) and research methods (Chapter 4.3). Reading the literature made me adjust my research questions, changing from looking for the existence of local knowledge, to searching for its characteristics.

The fieldwork was carried out over a period of eight months, a period in which I also did some quantitative analysis of the observations. During the years after finishing the fieldwork, I worked with the analysis of the observations and the conversations at the same time as writing the first four chapters of this thesis. The results from the analysis were then presented in Chapter 6 (Tea's practice) and Chapter 7 (Tea's knowledge for and in teaching mathematics).



This research is conducted in Norway, consequently, all talk in the videos of the observations and the conversations occur in Norwegian. I also used the Norwegian language when writing data reduction notes and transcripts. I had decided to write the thesis in English, thus using the English language when writing the memos. There is an underlying challenge when switching between two languages, particularly when translating the discussions in the classroom and Tea's comments. For an English-speaking person, some of the statements might occur unnatural. However, I wanted to stay "true" to what was said, and decided to keep the somewhat Norwegian-English that can be read in the text.

### 8.6.8 Summary

I have, in this section attempted to address Lester and Lambdin's (1998) criteria for judging the quality of my research. Educational theories should be based on the teaching practice itself (McNamara & Desforges, 1978; Schwab, 1971; Shulman, 1987). Thus, I believe it is both worthwhile and important to research into the practice to learn about the knowledge teachers deploy in their mathematics classroom. Throughout, I have, at the same time kept an openness that justifies my claims, attempted to treat my informant and the information I got from her ethically.

## 8.7 Concluding remarks

This research set out to identify characteristics of gold-nuggets within the gravel of an experienced mathematics teachers' knowledge base for teaching the subject, from which this dissertation reports. It has been a long-standing process to arrive at this final stage. I first conducted a pilot-study, which provided information I utilized when carrying out the main study. The twenty-three lesson observations (nineteen days) of the teaching of a grade nine teacher took place as planned. Moreover, I got the opportunity to get seventeen conversations with the teacher Tea. This happened within the planned time.

In the process of the analysis, however, I met some challenges, both of academic and personal issues. The work of analysis thus took more years than anticipated, however I finally reached a stage where I could write about my findings. When I now look back, I have some thoughts about what I would have done differently (Section 8.7.1), and some of what I have learned so far (Section 8.7.2).

### 8.7.1 What I would do differently

When starting the project, my focus was on observing teachers who have worked many years as mathematics teachers, that they were regarded as successful, and that they had studied mathematics *in addition* to what

was offered in their teacher education programmes. As explained in Chapter 1.2.1, ten years ago there was not much mathematics included in the teacher education curricula; I thus had to seek for a teacher that had extended her education in mathematics, which Tea had<sup>58</sup>.

I could, for example, also have asked for a teacher whose pupils show high scores in mathematics. I asked for a well-regarded experienced teacher. After observing and talking to Tea, I understand why the Principal suggested her; she was a focused and caring teacher, a quality both pupils and their parents, consequently a principal, appreciate. Throughout our conversations, Tea often talked about her pupils (considered in Section 8.6.3), however mostly about their wellbeing and not their understanding/learning of mathematics. Even if I directed the conversation towards the mathematics, I know that I today would put more focus on the categories in the frameworks I used when analysing the data. I maybe then would have access to aspects of knowledge for and in mathematics teaching which I did not expose.

In her study of Chinese and US teachers, Ma (1999) identified better understanding for teaching mathematics among Chinese teachers having less education in mathematics than the teachers in USA. As indicated in Section 8.5, when analysing my data, I started to wonder how it would look like if the participating teacher had no formal education of mathematics beyond what was offered in the teacher education programme. I somewhat missed an element for comparison. Comparison was not in my mind when starting the process, it just stood out as an element of interest when analysing the data: would it turn out differently, would I find more or less gold-nuggets?

I certainly, at some point, meant that it would be interesting to have a parallel case to consider, and I still think it would be, as I suggested in the section about further research (Section 8.5). However, when writing these final words, looking back, I feel that what I had to report was sufficient for the time being.

### **8.7.2 What I have learned**

In the foreword, I mentioned that this longstanding process has brought both frustration and joy. It has had its challenges both academically and personally, as mentioned above. I will here first comment on the importance of the work I have carried out, then comment on a few constraints I experienced through the process, however ending the thesis with the feelings I have after having worked through these constraints.

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<sup>58</sup> So had the teachers in my pilot study.

### **8.7.2.1 Academic importance**

I worked as a teacher in lower secondary for more than thirty years before starting this work. I have always regarded my work as very important, teaching our future workers is an *important* job. However, I admit that I did not really understand *how* important it is until I undertook this study, and later began to educate mathematics teachers myself. Observing how teachers transform their mathematical knowledge to pupils (my observations) at the same time as educating practicing teachers (my current work), has served as a wake-up call for me. I now really understand the important role of teaching, firstly the teaching of teachers, both prospective and in-service, and secondly their teaching of pupils in compulsory school as well as at higher grades. It is almost scary to think of me certifying teachers for teaching children for the future Norway.

### **8.7.2.2 Personal constraints**

When I entered the PhD-programme, I was very well aware of that it required a lot of work, however not really HOW much work. Even if my supervisor suggested about fifty-five hours per week, none can in advance imagine how much work one has to do within the expected three years of time for earning a PhD, four years if in addition teaching 25%. I believe that I, in the eight years that have passed since I started, have worked that much, and even more, except from the periods when I was called sick, and the year when I started my new work at a University College.

### **8.7.2.3 Final personal words**

This has been an amazing journey, from the greatest of joy to the deepest of depressions. I have to admit that I, during this journey, often have felt not competent to complete the research I started. The greater is then the feeling I have when writing these final words. I really hope that my work can contribute, and that I, next time I meet challenges, have learned from this work: keep on, inform prospective and in-service teachers about the knowledge they need for teaching the mathematics, and convince them about the joy of learning and teaching mathematics!



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# 10 Appendices

## 10.1 Appendix Chapter 3

An example of three representations:

*Semantic* representation: making an understandable explanation of the task

They are redecorating the school gym, and the floor will have new parquet. How many square metres do they need to cover the 48 m long and 22 m wide floor?

*Arithmetic* representation expanded to *algebraic* representation:

$$48 \times 22 = (40 + 8)(20 + 2)$$

*Geometric* representation:

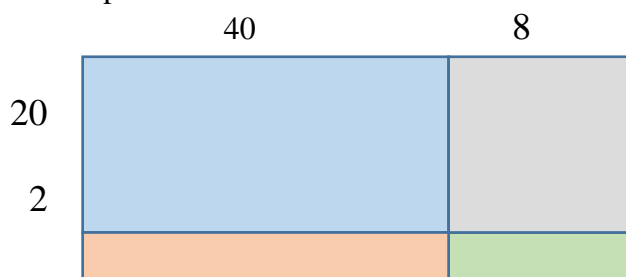


Figure 10-1: Three mathematical representations

## 10.2 Appendix Chapter 4

### Chapter 4.5.3, main study

Who	What is said (translation)	What is said (original)
Inn	I have now visited you four times, actually six lessons. I see that you use certain time; of course, you have some practical stuff you need to go through, and then some time for instruction followed by a certain time on seatwork. Do you have any principle in that regard?	Nå har jeg vært hos deg fire ganger, seks økter faktisk. Så ser jeg at du bruker ei viss tid, selvfølgelig har du noen praktiske greier som du må gjennom, og så ei viss tid på gjennomgang og så ei viss tid som dem bruker til å løse oppgaver. Har du noe prinsipp med forholdet der?
Tea	No, not really. It is somewhat dependent on the theme and what they know, or do not know in advance. Eee, often it will be like, I talk in the beginning and they work towards the end, but if there are small topics then it happens that I talk a little, they work a little. [...] I try to see to that a lesson, it happens that a lesson is only [seat-] work, but it happens, I try make that a lesson is never only talking.	Nei, ikke egentlig. Det er litt avhengig av hva som er tema og hva de kan fra før og ikke kan fra før. Eeeh ofte så blir det sånn at du, jeg snakker til å begynne med, og så jobber på slutten, men hvis det er veldig sånn korte ting så hender det jeg snakker litt og de jobber litt. [...] Jeg prøver å sørge for at en time, det hender at en time er bare å jobbe, men det hender, jeg prøver at en time aldri er bare å snakke.

Table 10-1. Conversation 4, video 201000907120626, 01:01.

## Chapter 4.6.2, writing memos

1. Read whole transcript carefully
2. Repeat, this time fairly quickly and choose one extract/turn by the teacher that strikes you as interesting
3. Write down why you find this interesting ( $M_1$ )
  - a. What is the extract about
  - b. Why does it grab your attention
  - c. What is interesting
4. Spend 20 minutes studying this extract and note ( $M_2$ )
  - a. What is the teacher talking about? Describe the topic in own words
  - b. What does the extract/turn reveal about her own practice? (Uncontroversial)
  - c. What knowledge/knowing is exposed by the teacher in this extract? (Interpretation)
5. List ( $M_3$ ):
  - i. Categories of practice exposed (a)
  - ii. Categories of teacher's views exposed (b)
  - iii. Categories of knowledge/knowing exposed (c)
6. Return to the whole transcript and look for further evidence of 5 i; ii, & iii. Note these and explain why they provide further evidence ( $M_4$ ).
7. Repeat steps 2 – 6 until all turns by the teacher have been considered in the same depth ( $M_{5-n}$ ).
8. Summarise ( $M_{n+1}$ )

Figure 10-2. Example of a memo

## 10.3 Excerpts Chapter 5

### Chapter 5.2, Comments from the Principal

Principal	Because she is experienced and clever	Fordi hun er erfaren og dyktig
Inger	How do you know?	Hvordan vet du det?
Principal	Because I have been in her class and seen how she teaches, I have got, and get excellent critics [1] about how she teaches from students, from teachers and other who are in her class, from parents. And I see [interrupts herself] [1] know her view on humans. I know how she works in relation to the whole student, not only [interrupts herself]. And she is good at motivating; she doesn't give up what so ever, so it is everything. I have known her, I have been at this school for [2] twenty-seven years, and I have been colleague to Tea for twenty-seven years, so I have pretty good insight.	Fordi jeg har vært i klassen og sett hvordan hun underviser, jeg har fått, og får, gode tilbakemeldinger [1] på hvordan hun underviser, fra elever, fra lærere og andre som er inne i klassen, foreldre. Og så ser jeg [avbryter seg selv], [1] kjenner menneskesynet hennes. Jeg vet hvordan hun arbeider i forhold til hele eleven, ikke bare [avbryter seg selv]. Og hun er flink til å motivere, hun gir seg ikke på tørre møkka, så det er alt. Jeg har kjent a, jeg har jo vært på skolen i [2] syv og tyve år, og jeg har vært kollega med Tea i syv og tyve år, så jeg har rimelig god innsikt.

Table 10-2. Conversation with Principal, video 20110411114218, 00:15.

Principal	It is amazing to see how fast one has to think creatively to find solutions for each student, and that is what it is all about. And if one doesn't have relations, and if one	Det er utrolig å se hvor kjapt man må tenke kreativt for å finne løsninger til hver enkelt elev, og det er jo det det dreier seg om. Og hvis man ikke har relasjoner og
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	doesn't know the students well and take the inconvenience with that then it is difficult to find methods. And, and particularly in mathematics that is an unbelievable challenge. It was very fun to see ,Tea, but I know you as a person too, I know that you are very visual, very visual and eh bodily	hvis man ikke kjenner elevene godt og tar seg bryet med det så er det vanskelig å finne metoder altså. Og, og spesielt i matematikk er jo det en utrolig utfordring. Det var veldig gøy å se, Tea, men jeg kjenner deg jo som menneske også, jeg vet at du er veldig visuell, veldig visuell og eh kroppslig
Inger	What importance do you think it has for the teaching that she is like that?	Hvilken betydning tror du det har i undervisningssituasjonen, at hun er med på den måten?
Principal	It means everything, actually, because [interrupts herself], and then you are fast to change all the time; you see, and you do not think long before you carry on helping people to get understanding	Det betyr alt, egentlig, fordi [avbryter seg selv], og så er du rask til å stille deg om, hele tiden; du ser, og så tenker du ikke lenge før du er i gang med å hjelpe folk til forståelse

Table 10-3. Converation 3, video 20100906125827, 24:14.

### Chapter 5.3, Pupils about Tea

Tone	I like her teaching because she, she makes it very easy for us and those, yes, those who have her as a substitute teacher, to understand because she stands out in a way from all the other teachers – in the way she does it	Jeg liker undervisningen hennes fordi at hun gjør det, hun gjør det veldig lettvtint for oss og de som, ja, de som har henne som vikar å forstå for hun skiller seg ut på en måte fra alle de andre lærerne – på måten hun gjør det på
Thom	I think the way Tea teaches is easier than how I previously was taught, and I believe my grades have improved. [...] That is, like it is the way she explains things, and she makes it easier and not so complicated.	Jeg synes måten Tea underviser på er litt lettere enn det jeg ble lært før, og jeg synes jeg er gått opp i karakter. [...]Altså, det er liksom måten hun forklarer tingene på, hun gjør det lettere og ikke så komplisert.
Tina	It is nothing I dislike, she kind of teach in a way so I actually understand it, [...] it is only the way she does it and like that, I do not quite know, but it is nothing silly about what she does.	Det er ikke noe jeg misliker, hun liksom lærer bort på en måte som jeg faktisk skjønner det [...], det bare måten hun gjør det på og sånn, jeg vet ikke helt jeg, men det er ikke noe som er dumt med det hun gjør
Trym	I like it very much, the way she teaches is very nice, because she kind of shows us kind of, that is we repeat it many times so we really get what we do and not only shows a task, but she explains it properly	Jeg liker det veldig godt, måten hun lærer bort på er veldig fin, for hun liksom viser oss liksom, altså vi går gjennom det mange ganger så vi får ordentlig inn hva vi gjør og ikke bare at vi går gjennom et stykke, men hun forklarer det ordentlig

Tale	I think she is clever, I kind of understand what she means. My previous teacher was only at the board. I make it better now when I sit down and work with the tasks because I get help	Jeg synes hun er flink, jeg skjønner hva hun mener, liksom. Den forrige læreren min sto bare på tavla. Jeg får det til bedre nå når jeg setter meg ned og jobber med oppgavene nå fordi jeg får hjelp
Tuan	I think it is a bit simple [the teaching], but I have done mathematics for a long time so I understands it at once, I think she explains very simple so it is understandable	Jeg synes det er litt enkelt [undervisninga], men jeg har jo regnet lenge, så jeg forstår det med en gang, jeg synes hun forklarer ganske enkelt at det blir forståelig

*Table 10-4.* Students about Tea's teaching (more videos)

### **Chapter 5.3, Pupils about Tea's examples**

Tone	I thought that it was impossible for me to remember so many numbers [laugh], not even how I round off. [...] I believe she used the moped because we are youths so it should be more interesting	Jeg tenkte at det var jo umulig for meg å huske så mange tall, hehe, ikke engang hvordan jeg runder det av. [...] Hun tok nok moped fordi vi er ungdommer fordi det skulle være mer interessant.
Thom	It is okay that she brings in such things once in a while, it is also understandable, and it is because we shall learn more, learn to better understand the way she teaches	Greit at hun kommer med noe sånt en gang i blant, det er jo forståelig det også, og det er jo for at vi skal lære mer, lære litt bedre å skjønne måten hun underviser på
Tina	I do not know, but probably it was to explain a little about how percent can be used, that you have to use percent that way	Jeg vet ikke jeg, men det var kanskje for å forklare litt hvordan prosent kan brukes, at du kan bruke prosent på den måten.
Trym	Because one gets better relations to what one does, that it will be more personal	Fordi man får litt bedre forhold til hva man driver med, at det blir mer personlig

*Table 10-5.* Students about Tea's contextualizing (more videos).

### **10.3.1 Chapter 5.4, Tea about Tea, a narrative**

When I was asked to participate in this research, two reasons made me come along; in addition to having some feedback on my teaching, it could be a contribution to the ongoing debate in the media regarding Norwegian teachers' competence. Media does not present Norwegian mathematics teachers particularly positive, which makes me sad. I want to raise my voice in that debate, and I feel that the best way to do that is by showing that we are hardworking competent practitioners. I also like the idea of discussing my own practice with others. I mean, I believe in what I do, but what do others mean about it?

I started my education by studying pedagogy for one year at the university located in our capital before getting my teacher's license at a teacher education college in the northern part of the country. Interest in mathematics made me study the subject for one more year before entering practice.

During my thirty-three years as a teacher in lower secondary school, I have been teaching subjects such as natural sciences, food and health, and religion, philosophies of life and ethics in addition to mathematics. While teaching I have

also studied school administration, school law, and pedagogy for special needs. Still, my favourite teaching subject is mathematics; I feel that I am familiar with the subject, and really love it when I experience that students like and understand the content.

In these thirty-three years, I have experienced many new pedagogical ideas, initiated either by governmental curriculum plans or by “radical” teachers like those who some years ago got the idea of throwing boards out from the classrooms. None was removed from my classroom! In these periods, the students were expected to do seatwork while the teachers only should work as facilitators. We experienced that the students needed a lot of supervision, thus this worked out as long as there were enough teachers available. In my school, economical reasons put an end to the period of having teachers practice as supervisors.

Concerning political initiatives, there are, in my opinion, two decisions that have served as obstacles for development of mathematical knowledge among our students; the idea concerning responsibility for own learning, and use of calculators. The extensive use of calculators has for many students served as an initiative to escape some work. In addition, the pedagogical idea about students’ responsibility for their own learning was devastating; especially for a subject like ours which, I believe, require directed management and hard work.

I strongly believe in Piaget’s stages of development, but also on stages of hard work before reaching understanding. It is like climbing a steep staircase; I say to my students that it sometime takes 99% hard work before reaching next stage of understanding. The remaining percent is talent. I believe that some are more gifted within one particular area, while others succeed in other areas. Still it is rare to meet talented students who do not need to work. However, hard work can make you reach far.

School development has always been of great interest to me, both teaching and school organization, but teaching has been a core concern; what is the “best” way to teach the subject? At the beginning of my career I focused on having all “theoretically” correct; word problems *should* be solved by using equations, and one *had* to use certain procedures. My teaching thus also to some extent reflected the way I had been taught myself. Fortunately this has changed; I now teach in a way that I find reasonable, trying to offer possibilities for both understanding and technique.

I do believe in the classroom as a unity where all students have their own seats; it is predictable and safe. Unfortunately, there are too many students living under unpredictable conditions, and for some of them, coming to school with its outspoken rules and frames might be the only basis in their lives. Thus, I was very happy about the results from PISA advanced<sup>59</sup> saying that the classroom and homework are important to students’ achievement. It is like going back to traditional ways of managing teaching; we have to focus on the class and classrooms as the unit for students’ belonging, and hence learning.

I do not believe that learning can happen in “vacuum”; the best learning takes place when the students articulate their knowledge to each other and thus

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<sup>59</sup> (Grønmo & Onstad, 2012)

confirming to themselves that they know and understand the content, because there is nothing like understanding. Thus, it is very important to create an atmosphere of trust where all students feel confident to expose their meanings as well as answering mathematical tasks without being afraid of unpleasant comments from their peers.

## 10.4 Additional information Chapter 6

### 10.4.1 Sequences of mathematics work

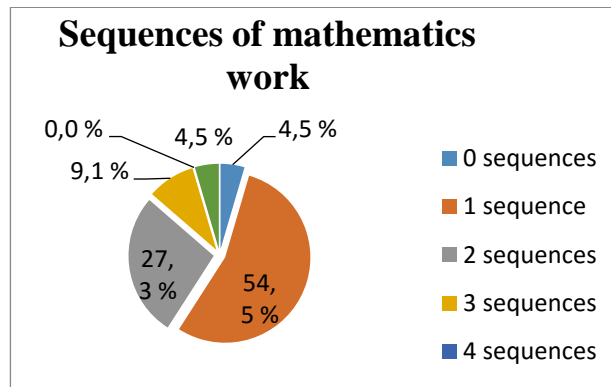


Figure 10-3. Sequences of mathematics work.

### 10.4.2 Forms of plenary interaction

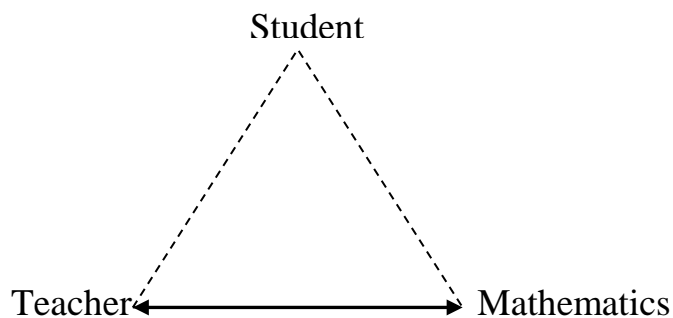


Figure 10-4. Illustration of an exposition in mathematics teaching.

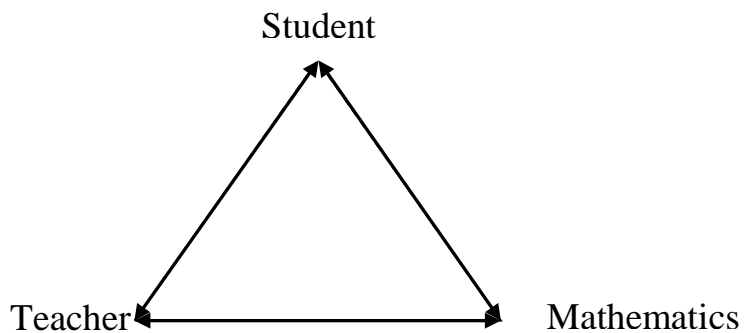


Figure 10-5. Triadic interaction between students, teacher and content (IRF-t).

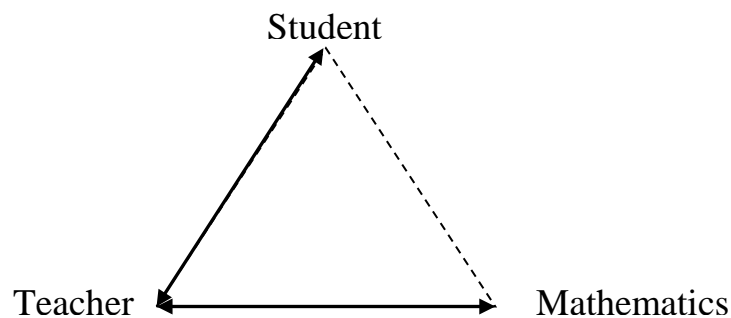


Figure 10-6. The teacher interacts with the students and the content (IRF-a).

### 10.4.3 Overview of additional excerpts

Section	Category	Lesson	Theme	Ref
Openings	adm. organization	16	merging classes	Table 10-7
	organizing teaching	17	change of schedule	Table 10-8
	org. for social events	9A	raising money	Table 10-9
	organizing students	14	solve group problem	Table 10-10
	organizing teaching	18	Before semester test	Table 10-11
	involvement	6A	personal issues	Table 10-12
	small talk	15	woman wanting baby	Table 10-13
	review	8	Intro equations	Table 10-14
	review	2A	positioning system	Table 10-15
	exposition	19	Strategy for drawing	Table 10-16
	exposition	9A	Price of gum	Table 10-6
	explanation	8	x instead of $\square$	Table 10-18
	explanation	8	$\square + 3 = 9$	Table 10-19
	contextualization	4A	$(-4) - (-6)$	Table 10-20
	contextualization	12	Dream of certificate	Table 10-21
	IRF	5	solving $(a + b) - (a + b)$	Table 10-22
	IRF-t with justification	8	$x + 4 = 13$	Table 10-23
	IRF-a with context	4B	$1 + (-3)$	Table 10-24
	preparation to seatwork	13	solar eclipse	Table 10-25
	preparation to plenary	17	mission	Table 10-26
student behaviour	8	teacher worried	Table 10-27	
Seatwork	repetition	1	$16 \times 0.5$	Table 10-28
	repetition	5	$2x + 3a + 4x - 2a$	Table 10-29
	anticipating plan	13	strategy for percent	Table 10-30
	showing to the board	17	mark points	Table 10-32
	asking for confirmation	5	x as surname	Table 10-33
	asking for confirmation	2A	What is $15 \times 0.2$	Table 10-34
	simplification	13	fat in the cheese	Table 10-35
Closings	shall do	1	this week	Table 10-36
	smart to do	2A	in the work session	Table 10-37
	must do	5	for tomorrow	Table 10-38
	plans	16	for tomorrow	Table 10-39
	goal	2B	remember method	Table 10-40
	goal	8	not reached	Table 10-41
	review	12	to prepare	Table 10-42
	review	13	clarifying	Table 10-43

Table 10-7. Overview of excerpts from Tea's lessons.

## 10.5 Excerpts from class

### 10.5.1 Excerpts Tea, openings

#### Chapter 6.2.1.2, administrative organization; lesson 16, merging classes

Tea	So it is very important, right [1] it is known as democracy, and it is very important to use [1] the democratic channels. [1] Now it	Så det er veldig viktig, ikke sant [1] det heter demokrati, og det er veldig viktig å bruke [1] de demokratiske kanalene. [1] Nå blir
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will be like everyone is going to talk about this, five classes will be merged to four [2] and then one can walk around yelling and [1] being dissatisfied and all sorts of thing, but that does not lead to anything. But one has, the school has a democratic system, [1] and every thought and such stuff about this you give to Thom [1] and he will take it further to that group

det jo sånn at alle kommer til å snakke om dette, fem klasser skal bli til fire [2] og så kan en gå rundt og kjefte og smelle og [1] og være misfornøyd og alle mulige ting, og så kommer man ikke noen vei med det. Men man har jo, skolen har jo et demokratisk system, [1] og alle tanker og ting og tang om dette her det gir dere til Thom [1] og så tar han det videre til den gruppa

*Table 10-8.* Observation 16, video IMAG0001, 00:01<sup>60</sup>.

**Chapter 6.2.1.2, organizing teaching; lesson 17, change of schedule**

Tea It is like that this day will be a little different because [2] there are very many lessons, or in three of the lessons you will have substitute teacher because in the next lesson I shall be sensor in natural sciences in 9D

Det sånn at denne dagen her den blir litt sånn annerledes fordi [2] fordi det blir veldig mange timer, eller tre av timene får dere vikar for det er sånn at i neste time så skal jeg være naturfagsensor i 9 D

*Table 10-9.* Observation 17, video 20110405082318, 01:55.

**Chapter 6.2.1.2, organizing for social events; lesson 9A, raising money**

Tom And it is, it is only a small part of Tunkskogen so it is possible to continue earning some thousands

Så er det jo, det er jo bare en liten del av Tunkskogen så det er jo mulig å fortsette å tjene noen tusenlapper

Tea It is relatively easy money compared to many other things. A message from Tony's father is that people who are going to sell waffles must bring table and chairs because there are none where you are going to sell

Det er jo relativt lett tjente penger i forhold til mange andre ting. En beskjed fra faren til Tony er at folk som skal selge vafler må ha med seg bord og stoler for det er det ikke på det stedet dere skal selge

*Table 10-10.* Observation 9A, video 2010.11.02/M2U00231, 02:34.

**Chapter 6.2.1.2, organizing students, lesson 14, solving a group problem**

Tea Sister sisters, [1] in the next lesson it is so that you do not have any more opportunities [1] to solve your group problem [2] then I solve it. I do not want you to sit another whole lesson

Søstrene sisters, [1] i neste time så er det sånn at da her dere ikke flere muligheter [1] til å løse [1] gruppeproblemet deres [2] da løser jeg det. Jeg vil ikke at dere skal sitte en hel time til

*Table 10-11.* Observation 14, video 20110110115525, 02:17.

**Chapter 6.2.1.2, organizing teaching in general**

Tea It is as if we only have a few [3] actually lessons left of the school

Det er sånn at vi har bare noen ganske få [3] timer egentlig igjen

<sup>60</sup> Thom had performed his message before the camera was turned on.

year. Or at least to the semester test, [1] it is May the fifth [1] and before we get there we will make us finish this topic, functions, [1] and then we will try to make some repetition

av skoleåret. Eller hvert fall fram til heldagsprøven, [1] den er femte mai [1] og før vi kommer dit så skal vi gjøre oss ferdig med dette temaet, funksjoner, [1] og så skal vi prøve å få repetert litt

*Table 10-12.* Observation 18, video 2011041105613, 02:18.

**Chapter 6.2.2, involvement; lesson 6A, personal issues**

Tea	At our place it was only four degrees and my large fine chrysanthemum which is so big, it hung with all its heads	Hos oss var det bare fire grader, og den store fine krysantemummen min som er så stor, den hang med alle hodene
Tom	The car stands in the garage and is depressed. Was on control yesterday and drove through <i>that</i> deep ponds	Bilen står i garasjen og er deppa. Var på kontroll i går og kjørte gjennom <i>så</i> dype vanddammer
Tea	And it was not accepted	Og den ble ikke godkjent

*Table 10-13.* Observation 6A, video 20100921082655, 01:01.

**Chapter 6.2.2.1, small talk; lesson 15, woman wanting a baby with high IQ**

Tea	Once I saw on the TV about a woman who should make a baby, not being helped by a man, but by using bought sperms, and she should [interrupts herself], then there is a sperm bank having sperms from people with fairly high IQ because she could not think of having a baby that was not on the level of Mensa	Jeg så på TV en gang om en kvinne som skulle lage seg barn, ikke ved hjelp av en mann, men ved hjelp av innkjøpt sæd, og hun skulle [avbryter seg selv], da finnes det en sædbank med sæd fra folk med ganske høy IQ for hun kunne ikke tenke seg å ha et barn som ikke var på Mensanivå
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*Table 10-14.* Observation 15, video 20110111101527, 02:20.

**10.5.2 Excerpts Tea, plenary segment**

**Chapter 6.3.1.1, review; lesson 8,  $\square + 3 = 5$**

Tea	And then you became very good at those things. At the end you were so good [2] that one day when you came to school you got new tasks [wrote $\square + 3 = 5$ on the board], tasks that looked like this	Og så ble dere kjempegode på å kunne de tinga der. Til slutt var dere så gode [2] at en dag når dere kom på skolen så fikk dere nye regnestykker [3, skriver $\square + 3 = 5$ på tavla], sånn så regnestykkene ut
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*Table 10-15.* Observation 8, video 2010.10.25/M2U00207, 09:09.

**Chapter 6.3.1.1, review; lesson 2A, positioning system**

Tea	What did we do yesterday? [1] We said something about that [1] there were some strategies we can use to multiply numbers which are less than [1] one. [2] We said something about [2] the positioning system. [...] There were actually two things, right, we talked about [...] multiplying and we said that [3] we can use the positioning system	Hva var det vi gjorde i går? [1] Vi sa noe om at [1] det var noen strategier vi kan bruke for å gange tall som er mindre enn [1] en. [2] Vi sa noe om [2] posisjonstallsystemet. [...] Det var egentlig to ting, ikke sant, vi snakka om det med [...] gange og så sa vi det at [3] vi kan bruke posisjonstallsystemet
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*Table 10-16.* Observation 2A, video 20100831082544, 04:58.



### Chapter 6.3.1.1, Expositions; strategy for drawing a graph

Tea	When we shall draw a graph to a function, right, then there is a strategy we shall use. We are going to find [she interrupts herself], we have like a cake recipe almost like buns where it says what we shall do. [...] Right, often it is like that function can be, eee, four x [writes $y = 4x$ ], [1] that is a proportional function, right, because it passes through	Når vi skal tegne en graf til en funksjon, ikke sant, så er det en strategi vi skal bruke. Vi skal finne [avbryter seg selv], vi har en sånn kakeoppskrift nesten som boller hvor det står hvordan vi skal gjøre. [...] Ikke sant, ofte så er det sånn at den funksjonen kan være, eee, fire x [skriver $y = 4x$ ] [1] det er en proporsjonal funksjon, ikke sant, den går gjennom
Boy	[1] The origin	[1] Origo

Table 10-17. Observation 19, video 20110412082256, 06:48.

### Chapter 6.3.1.1, Expositions

Tea	Then it says y equals two times x. [6] Just like if this was two crowns per packet of gum [2] right? If I buy one packet of gum then it costs two crowns [marks (1, 2)], do you agree? [No response heard, 2] If I buy two packets of gum [3 while making dashed lines], it costs [1] four crowns. [3] Then we can think like this or we can put it in a table and say x, and y equals two times x [makes a table]. The table is a calculator, [1] it is supposed to help you if you cannot manage to imagine it in your head	Så står det y er lik to ganger x. [6] Akkurat som om dette var to kroner per tyggispakke [2] ikke sant? Hvis jeg kjøper ei tyggispakke så koster den to kroner [markerer (1, 2)], er dere enige i det? [2] Hvis jeg kjøper to tyggispakker [3, mens hun stipler markering] så koster de [1] fire kroner. [3] Det kan vi tenke sånn, eller vi kan sette det inn i verditabellen, og si x, og y er lik to ganger x [tegner tabell]. Verditabellen det er en regnemaskin, [1] den skal liksom hjelpe deg hvis du ikke klarer å tenke det i hodet
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Table 10-18. Observation 19, video 20110412082256, 33:02.

### Chapter 6.3.1.1, explanation; lesson 8, replacing $\square$ by x

Tea	What separates that task from the other task [1] it is that we have, instead for the empty box or bag, then one has decided that [1] it shall look [2] like that [writes x in the box], [2] but actually what they ask about is only [1] what should be the content of that box so that this [equation] can be correct	Det som skiller det regnestykket fra det andre regnestykket [1] det er at vi har, i stedet for den tomme ruta eller påsan, så har man bestemt seg for at [1] det skal se [2] sånn ut [setter kryss i ruta], [2] men egentlig så er jo det de spør etter bare [1] hva skal være innholdet i den boksen der for at dette [uttrykket] kan stemme
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Table 10-19. Observation 8, video 2010.10.25/M2U00207, 13:55.

### Chapter 6.3.1.1, explanation; lesson 8, $\square + 3 = 5$

Tea	Here we are looking for a number, a value, a weight, something that	Her er vi på jakt etter et tall, en verdi, en vekt, et eller annet som
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makes [3] that what stands there [points to □ + 3] and what stands there [points to 5] are equal. Because it is actually like [1] that the important message [3, points to =] is that it is

gjør [3] at det som står der [peker på □ + 3] og det som står der [peker på 5] er likt. For det er nemlig sånn [1] at der er den viktige beskjeden [3, peker på =] det er at det er

*Table 10-20.* Observation 8, video 2010.10.25/M2U00207, 15:60.

**Chapter 6.3.1.1, contextualization; lesson 4A, contextualizing (−4) – (−6)**

Tea	Have no money to buy for, go to the Principal, plead and pray, and borrow six moneys. Borrow to shop. Are already in debt, borrow. And then the Principal says: it is okay, you have to pay back. Then they go down to the canteen, right, and they sell, and at the end they take one, two, three, four, five, six moneys they owe the Principal [exemplified by taking six books from the desk], and they walk up to the Principal, right? [6] They knock on her door [5], then they say hello Principal, we come to pay our debt. And then the Principal says no, you know what; we are going to put a [1] minus in front of that debt. What did the Principal say?	Har ikke noe penger å kjøpe for, går til rektor, trygler og ber og låner seks penger. Låner for å handle. Har gjeld fra før, låner. Og så sier rektor; det er greit, dere må nå sørge for å betale tilbake. Så går de ned i kantina, ikke sant, og så selger de og så til slutt tar de en, to, tre, fire, fem, seks pengene som de skylder rektor [tar opp seks bøker fra kateteret], og så går de opp til rektor, ikke sant? [6] Så banker de på hos henne [5], så sier de god dag rektor, vi kommer for å betale gjelda vår. Og så er det at rektor sier, nei vet dere hva, sier rektor, den gjelda skal vi jammen sette en [1] minus foran. Hva sa rektor da?
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*Table 10-21.* Observation 4A, video 20100907082547, 27:58.

**Chapter 6.3.1.1, contextualization; lesson 12, dream of moped certificate**

Tea	A survey conducted by the Gallup Institute [1] exposed that two thousand four hundred and sixty-nine out of out of three thousand eight hundred and ten youths have a moped certificate before turning seventeen years. [1] Among these have one thousand two hundred and thirty-eight earned their own money to get the certificate. The others are divided into two groups, one has spent the money they got for their confirmation, and the other got the certificate as a present. During the first year having the certificate, six hundred and seventy-eight had different <i>accidents</i> while driving their moped, [1] three of those said that they did not use helmet	I en undersøkelse utført av et gallupinstitutt [1] viste det seg at to tusen fire hundre og sekstini av tre tusen åtte hundre og ti ungdommer har mopedlappen innen de fyller sytten år. [1] Av disse har ett tusen to hundre og trettiåtte tjent penger til denne lappen sjøl. De andre er delt i to like grupper, den ene har brukt konfirmasjonspengene, den andre har fått lappen i gave. I løpet av det året, det første året med lappen har seks hundre og syttiåtte stykker hatt ulike <i>uhell</i> under mopedkjøring, [1] tre av disse sa at de ikke brukte hjelm.
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*Table 10-22.* Observation 12; video 20110103115516, 08:12.

### Chapter 6.3.1.2, IRF, solving $(a + b) - (a + b)$

Tian	A minus a equals zero, minus b	A minus a er null, minus b
Tea	Then there is a question, what, what, what did you do now?	Da var det et spørsmål, hva, hva, hva gjorde du nå?
Trym	Can't we only take it down, that is, what is	Kan vi ikke bare ta ned, altså, det som blir
Tea	Why did you think like that, Tian [points at him]	Hvorfor tenkte du sånn Tian [peker på han]
Tian	It will be plus and plus make plus and minus and plus	Det blir to pluss og pluss det blir pluss og minus og pluss
Tony	It will be a, no it will be b	Det blir a, nei det blir b
Tea	Yes, because it was a rule we had [1] we said that when it was plus and minus then it was minus. [1] We also said something else [2] minus in front of a parenthesis, what did we do with the signs in the parenthesis then? What did we do with the signs in the parenthesis?	Jo, for det var en regel vi hadde, [1] vi sa at når det var pluss og minus så blei det minus. [1] Vi sa noe annet også [2] minus foran parentes, hva gjorde vi med alle tegna inni parentesen da? Hva gjorde vi med tegna inni parentesen
Theo	Calculated	Regna ut
Tea	[2] With a minus in front of the parenthesis, what did we do with all signs in the parenthesis?	[2] Med minus foran parentesen, hva gjorde vi med alle tegna inni parentesen
Theo	We first added them	Vi plussa de først
Trym	[4] Eee, we did something with them	[4] Eee, vi gjorde noe med de
Tea	[2] What did Tian say	[2] Hva var det Tian sa da?
Theo	[2] Okay, we took a, for example a minus a and then we calculated it [Tea wrote $a - a - b$ ] and then we took b and then it will be minus in a way	[2] ok, vi tok a for eksempel, a minus a og så regna vi det ut [Tea skrev $a - a - b$ ] og så tok vi b så det blir på en måte minus
Trym	We make it equal to the other	Vi tar og gjør den lik som den andre
Tea	We had to change signs within the parenthesis when it was minus in front of it	Vi måtte bytte tegn inne i parentesen når det var minus foran den

Table 10-23. Observation 5, video 20100920105614, 24:19

### Chapter 6.3.1.2, IRF-t with justification; lesson 8, $x + 4 = 13$

Tea	Now are we going to review the tasks you have done. [4] Let's have a look. Raise your arm those of you who have an answer [4], Theo	Nå skal vi se på de oppgavene dere har gjort. [4] eee, skal vi sjå [skriver $x + 4 = 13$ på tavla]. Rekk opp handa de som har et svar [4], Theo
Theo	Nine	Ni
Tea	[writes $x = 9$ on the board] How can you know?	[Skriver $x = 9$ på tavla] Hvordan kan du vite det?

Theo	Ee, I take thirteen minus four and that will be nine	Ee, jeg tar tretten minus fire så blir det ni
Tea	Yes, or you can say it in another way, I know it because	Ja, eller, du kan si det på en annen måte, jeg vet det fordi
Theo	Nine plus four equals thirteen	Ni pluss fire er tretten
Tea	I know it because nine plus four equals thirteen, [1] quite right. You can then be sure that you have solved it correctly	Jeg vet det fordi ni pluss fire er tretten [1] helt riktig. Du kan være helt sikker på at du har regnet riktig du da.

Table 10-24. Observation 8, video 2010.10.25/M2U00207, 21:39.

### Chapter 6.3.1.2, IRF-a; lesson 4B, contextualizing 1 + (-3)

Tea	"I shall buy for three hundred" and you had one hundred, what would the debt be then?	"jeg skal handle for tre hundre kroner" og du hadde hundre fra før, hva ville gjelda være da?
Theo	Two hundred	To hundre
Tea	Then the debt would have been two hundred, right, you can use such an explanation [1] to understand why this will be minus two, [1] one minus three is minus two because we have one, as you said in the beginning, and uses three	Da ville gjelda være to hundre, ikke sant, du kan bruke en sånn forklaring [1] til å forstå hvorfor dette blir minus to, [1] en minus tre er minus to fordi vi har en, som du sa til å begynne med, og bruker tre

Table 10-25. Observation 4B, video 20100907091504, 20:44.

### Chapter 6.3.2.1, preparation to individual seatwork; lesson 13, solar eclipse

Tea	If that circle is the sun [5] try to draw the moon on your circle and look [2] how much of the sun would we then have seen? [2] Would we see much [1] or? [3] Approximately how much of the sun would we have seen?	Dersom den sirkelen er sola [5] prøv og tegn månen på sirkelen deres og se [2] hvor mye av sola ville vi sett da? [2] Ville vi sett mye [1] eller? [3] Sirka hvor mye av sola ville vi sett?
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Table 10-26. Observation 13, video 20110104101344, 05:07.

### Chapter 6.3.2.2, preparation to plenary segments

Tea	Yesterday you got a mission. You were told to do it till today, and if you haven't done it you have to lie very good	I går fikk dere et oppdrag. Dere fikk beskjed om at dere skulle gjøre det til i dag, og hvis dere ikke har gjort det så må dere jule veldig godt
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Table 10-27. Observation 17, video 20110405082318, 07:44.

### Chapter 6.3.3, student behaviour

Tea	But I am extremely worried for the rest of you [5] not that worried about Tuan either, you <i>never</i> say, [interrupts herself, 2] you <i>never</i> in any way show that you [made it], you never raise your hands and tell that you have got an answer [3] and that worries me [3] because I believe that to learn, it takes that you dare say things aloud	Men jeg er kjempebekymret for alle dere andre [5] ikke så bekymret for Tuan heller, dere sier <i>aldri</i> [avbryter seg selv, 2] dere viser <i>aldri</i> på noen måte at dere [har fått det til], dere rekker ikke opp handa og forteller at jeg har fått et svar [3] det bekymrer meg [3] fordi jeg tror at det å lære, det krever at dere tør å si en ting høyt.
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Table 10-28. Observation 8, video 2010.10.25/M2U00207, 29:05.

### 10.5.3 Excerpts, individual seatwork

#### Chapter 6.4.1.1.1, repetition

Tea	Yes, if you instead of one-half had written, instead of point five had written one-half [4], what would it then have been, sixteen times one half. [5] Would you try to write it? [3] One over two times, what did we do with the sixteen?	Jo hvis du i stedet for en halv hadde skrevet, i stedet for null komma fem hadde du skrevet en halv, [4] hva hadde det stått da, seksten ganger en halv. [5] Skal du prøve å skrive det? [3] En todel ganger, hva var det vi gjorde med den der seksten
Tord	We made it sixteen ones	Vi tok seksten en deler
Tea	We converted it to fraction, we wrote sixteen ones, then we multiplied, one times sixteen equals	Vi gjorde den om til brøk, vi skrev seksten endeler, så ganga vi en gange seksten er
Tron	Sixteen	Seksten
Tea	Sixteen, [2] two times one equals	Seksten, [2] to ganger en er
Tron	[2] two times one equals two	[2] to ganger en er to
Tea	Yes, like that, then how much is sixteen divided two?	Ja, sånn ja, hvor mye er seksten delt på to da?
Tord	Like that	Sånn, ja
Tron	Eight, it is eight then	Åtte, det blir åtte da
Tea	Yes, the answer is eight	Ja, da blir svaret åtte
Tron	Oh yes, like that	Å ja, sånn ja

Table 10-29. Observation 1, video 20100830105635, 21:55.

#### Chapter 6.4.1.1.1, repetition

Tea	What we have done now is that we have defined some new numbers [2] namely the numbers of which we do not know the value. It can be a, or x and b and such. And then we said that if you have similar then you can add them, just like that, [8] in a way you count how many you have of that type. And if you have x'es you do it that way, right	Det vi har gjort nå, det er at vi har definert noen nye tall [2] nemlig de tallene vi ikke kjenner verdien av. Det kan være a eller x og b og forskjellig. Og så sa vi det at hvis du har like så kan du legge dem sammen, sånn som det [8] du teller dem på en måte opp hvor mange har jeg av det slaget. Og hvis du har x'er så gjør du det på samme måten, ikke sant
Tron	I understand if it is equal and such	Nå skjønner jeg hvis det er likt og sånn
Tea	Then you understand that. But sometimes it is like	Da skjønner du det. Men så er det sånn at noen ganger
Tron	Like with negative numbers	Sånn som det med negative tall
Tea	Yes, right, but that one, they suddenly implement more different types. And it is clear that you have not been explained that. [1] It is, sometimes last year the tasks	Ja, ikke sant, men den der, der innfører de plutselig at det er flere forskjellige typer. Og det er klart at det er ting som du ikke har gått imellom. [1] Det er, noen ganger

looked like that, right [2] like that, and then the message was that you should, eee, here, here it also is so that you have to put together those who have the same surname, [2] they have x for surname and those have a for surname. [2] And those that have x can only be added to x'es and those who have a for surname can only be put together with other a's. Then you say two x [1] and then you go here and see that you have four x

Tron

Six x

Tea

That is six x [1] and then you go here and then you say three a [2] take away [1] two a

Tron

Yes, six x plus one a

*Table 10-30.* Observation 5, video 20100920105614, 29:52.

#### **Chapter 6.4.1.1.2, anticipating her plan for Thom**

Tea

Yes, right, then you suddenly get task that you in a way, that is, the textbook says [1] that you shall do it in a special way, it says that you shall [1] first divide [1] by the and then multiply by two if it is twenty percent, right?

Thom

Yes

Tea

Then you can do it in another way, [1] you can put it in another way, [1] you can write it as a [] percent. Right, that technique which [interrupts herself]. That is, you can, as I said, calculate it mentally that way, or you can introduce the technique, which we [1] are going to introduce now, later in the lesson. We say that you when you calculate twenty percent of something, [1] then it is the same as [1] you calculate, right, twenty percent is the same as twenty hundreds, right? And if, to write twenty percent here, then you can write twenty hundreds like this, and if you then see, then it no point in taking eight hundred times twenty divided hundred. You can shorten like this and this, and then the solution come sailing, right?

så regnestykkene sånn ut i fjor, ikke sant [2] sånn som det, og så fikk du beskjed om at du skulle, eee, her, her er det også sånn at du må bare slå sammen de som har samme etternavn liksom, [2] de har x til etternavn og de har a til etternavn. [2] Og de som har x kan bare slås sammen med x-er, og de som har a til etternavn kan bare slås sammen med andre a-er. Da sier du to x [1] og så går du hit og så ser du at her står det fire x

Seks x

Det er seks x [1] og så går du hit og så sier du tre a [2] ta bort [1] to a

Ja, seks x pluss en a

Ja, og ikke sant, da kommer du plutselig over på oppgaver som du på en måte, altså, boka sier [1] at du skal gjøre det på en spesiell måte, den sier at du skal [1] først dele [1] på ti og så gange med to hvis det er tjue prosent, ikke sant? Ja

Du kan gjøre det på en annen måte, [1] du kan sette det opp som en [] prosent. Ikke sant, den teknikken som [avbryter seg selv]. Altså, du kan som sagt regne det i hodet på den måten, eller du kan innføre den teknikken som vi [1] for så vidt skal innføre på, etter hvert i timen. Vi sier at når du regner tjue prosent av noe, [1] så er det det samme som [1] at du regner, ikke sant tjue prosent er det samme som tjue hundredeler, ikke sant? Og hvis, for å skrive tjue prosent her, så kan du skrive tjue hundredeler, sånn som det, og hvis du ser da, så er det sånn at, det er jo ikke noe vits i å ta åtte hundre ganger tjue og dele på hundre. Du kan forkorte sånn og sånn, og så kommer svaret seilende, ikke sant?

Thom	Should that be the solution?	Er det liksom svaret da?
Tea	That is the solution; twenty percent of eight hundred is one hundred and sixty.	Da er det svaret, tjue prosent av åtte hundre er hundre og seksti

Table 10-31. Observation 13, video 20110104101344, 21:19.

#### Chapter 6.4.1.1.3, showing different methods for percent for Tara

Tea	You can choose different methods, right, but, ee, you can do it like, like we did, ee, two hundred and sixty times	Du kan velge forskjellige måter, men, ee, du kan stille det opp sånn som, sånn som vi gjorde, ee to hundre og seksti ganger
Tara	[3] I don't know	[3] Jeg vet ikke
Tea	[1] The percent is [points to the board]	[1] Prosenten [peker på tavla], det er
Tara	Twenty-five	tjuefem
Tea	Divided [4] with what should you divide, what fraction is percent	Dele på [4] hva er det du må dele på, hva slags brøk er prosent
Tara	It is hundred	Det er hundre
Tea	Yes, it is hundred	Ja, det er hundre
Tara	It is like what it costs is	Det er sånn hva det koster er
Tea	Yes, and then you find what the [1] discount is, right, and then you have to remove to figure out what it costs. [inaudible]	Ja, og da finner du ut hva som er [1] rabatten, ikke sant, [1] og da må du ta bort for å finne ut hva det koster. [Utydelig]
Tara	Oh yes, how much I save	Å ja, hvor mye jeg sparer
Tea	How much you save, yes	Hvor mye du sparer ja

Table 10-32. Observation 14, video 20110110115525, 19:50.

#### Chapter 6.4.1.1.3, showing to the board

Tea	Don't you remember how you did it? But now I have done like that on the board, right, I have made such a table, [1] and then you think, [2] then you calculate, right, if you sell ten sodas you are there [points to his notebook], if you sell that for the cheapest then it will be sixty-five crowns, right, ten times [1] six point five	Husker du ikke åssen du gjorde det? Men nå har jeg gjort sånn der oppe på tavla, ikke sant, så har jeg laget en sånn verditabell, [1] og så tenker du [2] så regner du ut, ikke sant, hvis du selger ti brus er du der [peker i skriveboka hans], hvis du selger det på den billigste da så blir det sekstifem kroner, ikke sant, ti ganger [1] seks komma fem
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Table 10-33. Observation 17, video 20110405082318, 24:05.

#### Chapter 6.4.1.2, asking for confirmation, x as surname

Tea	But can you add a number having x as surname with a number without a surname?	Men kan du legge sammen et tall med x til etternavn med et tall uten etternavn da?
Tone	[4] Yes	[4] Ja
Tea	Can you? Are they the same family?	Kan du det? Er det samme familie?
Tone	No, but they marry	Nei, men de gifter seg
Tea	Yes, but marrying, then it suddenly is multiplication, you know [3] tsj,	Ja, men gifter seg, da er det gangestykke med en gang vet du

	tsj, so you cannot do that. [4] Then you have to take them separately, eight x plus three x equals [points at them]	[3], tsj, tsj, så du kan ikke det. [4] Da må du ta de hver for seg, åtte x pluss tre x er [peker på de to]
Tone	Eight x plus three x equals [2] eee	Åtte x pluss tre x er jo [2] eee
Tea	Eleven x [1] plus [points at 4]	Elleve x [1] pluss [peker på 4]
Tone	four	fire

Table 10-34. Observation 5, video 20100920105614, 37:10.

#### Chapter 6.4.1.2, asking for confirmation

Tea	Fifteen times point two yes, is that reasonable?	Femten ganger null komma to ja, er det rimelig?
Tina	[1] I do not know	[1]Jeg vet ikke jeg.
Tea	Why don't you know?	Hvorfor vet du ikke det da?
Tina	I am just unsure	Jeg er bare usikker
Tea	You are unsure. [2] But, [1] well, it is correct. [2] Could we manage to explain this in some way? If you had converted it to, if you had converted it to, [2] point two to a fraction; [points to the notebook] what would it then be?	Du er bare usikker. [2] Men, [1] altså, det er riktig [2], kunne vi klart å forklare det på noen måte da? Hvis du hadde gjort om den til, hvis du hadde gjort om den til, [2] null komma to til brøk [peker i kladdeboka], hva hadde det stått da?
Tina	[1] Two [1] tenth	[1] To [1] tiendeler
Tea	Two tenths yes. You can write that, you know, fifteen [2] times [1] two tenth [1], yes, then you could put on the invisible one, right? Fifteen times two is [points to the numbers]	To tiendeler ja. Det kan du skrive vet du, femten [2] ganger [1] to tideler [1], ja, da kunne du slengt på den usynlige eneren, ikke sant? Femten ganger to er [peker på tallene]
Tina	Thirty	Tretti
Tea	Yes, [2] and one times ten is [Tina writes the answer], [2] thirty divided ten equals	Ja, [2] og en ganger ti er [Tina skriver svaret], [2], tretti delt på ti er
Tina	It is three	Det er tre
Tea	Yes, can you see that?	Ja, ser du det?

Table 10-35. Observation 2A video 20100831082544, 28:47.

#### Chapter 6.4.1.3, simplification

Thor	Multiplied that one with that one [pointed at 27 and 500]	Ganga den med den [pekte på 27 og 500]
Tea	Did you multiply this one with that one? [did also point at the numbers]	Ganga du den med den? [pekte også på de to]
Thor	No, I added, no [3] I [unclear speech], I had it on a sheet of paper. [Shows what is written on the previous page] It looked, there were at least	Nei, jeg pluss, nei [3], jeg [utydelig], jeg hadde det på ark. [Blar opp på foregående side] Det så, det ble i hvert fall
Tea	Hundred and thirty five	Hundre og trettifem
Thor	Point four	Komma fire
Tea	Point four what?	Komma fire hva da?
Thor	Gram	Gram



Tea	Gram fat?	Gram fett?
Thor	Yes	Ja
Tea	But how did you find that out?	Men hvordan fant du ut det?
Thor	I do not remember	Det husker jeg ikke
Tea	Maa, [3] mmm	Maa, [3] mmm
Thor	I multiplied, divided, or added or subtracted	Jeg ganga, dela, eller plussa eller minus
Tea	Multiplied, divided, added <i>or</i> subtracted?	Ganga, dela, plussa <i>eller</i> minus?
Thor	Yes	Ja
Tea	Then I do not believe that you have really come to how to do that. [1] Hm, maybe we could draw that cheese? Draw a cheese [8, Thor draws], that is a cheese with holes, yes	Da tror jeg ikke du <i>helt</i> har kommet til hvordan du skal gjøre det der. [1] Hm, kanskje vi kunne tegnet den osten? Tegn en ost [8, Thor tegner], det er ost ja, med hull i
Thor	Uum	Uum
Tea	It is five hundred gram	Den er 500 gram
Thor	Yes	Ja
Tea	But, [2] if that cheese had been [1] twice that size?	Men [2] hvis den osten hadde vært [1] dobbelt så stor?
Thor	[3] Then it would have been one thousand grams, one kilo	[3] Da hadde den vært tusen gram, en kilo
Tea	Would you be able to find out how many, what proportion of the cheese, which would have been fat then, mentally?	Hadde du klart å finne ut hvor mange, hvor stor del av osten som hadde vært fett, i hodet da?
Thor	[6] No [shakes his head]	[6] Nei [rister på hodet]
Tea	Then we could draw, then we could draw the cheese [inaudible] then it says that [4] twenty-seven percent of that cheese [2] is fat [marks the fat on her drawing, writes 27%], how big part is that then? [2] And then the question is; twenty seven percent [1] of one thousand [writes 27% of 1000], [3] that is [3] if it had been hundred, then it would have been twenty-seven hundredths [writes 27/100]	Da kunne vi jo tegnet, da kunne vi jo tegnet osten [utydlig prat] så står det at [4] tjuesju prosent av den osten [2] er fett [markerer en del av osten og skriver 27 %], hvor stor del er det da? [2] Og så blir spørsmålet; tjuesju [1] prosent [1] av tusen [skriver 27 % av 1000], [3] det er, [3] Hvis det hadde vært hundre, så hadde det vært tjuesju hundredeler [skriver 27/100]
Thor	[2] Twenty-seven thousandths	[2] Tjuesju tusendeler
Tea	Do you believe that it would have been twenty-seven thousandths?	Tror du at det hadde blitt tjuesju tusendeler da, da?
Thor	No	Nei
Tea	How much do you believe that would have been if it should be thousandths [writes = , a fraction line, and denominator 1000], what	Hvor mye har det blitt da, tror du [2] hvis dette skulle bli tusendeler [skriver = og brøkstrek med nevner 1000], hva måtte det stå

	should it then stand on the top there [points to the top of the line]?	oppå der da [peker på oversiden av brøkestreken]?
Thor	[3] seven, no, [1] two hundred and seventy	[3] syv, nei [1] to hundre og sytti
Tea	Two hundred and seventy, yes [1] it is quite right. Two hundred and seventy grams of that cheese is fat. [3] What was your answer?	To hundre og sytti, ja [1] ja, det er helt riktig. To hundre og sytti gram av den osten er fett. [3] Hva var det du hadde fått igjen?
Thor	[2] Two hundred and seventy, no one thousand three hundred, no hundred and thirty-five point seven	[2] To hundre og sytti, nei et tusen tre hundre, nei hundre og trettifem komma syv
Tea	Yes, I think you got approximately hundred and thirty-five. [3] Why did you get that?	Ja, hundre og trettifem har du fått sånn cirka, tenker jeg. [3] Hvorfor har du fått det?
Thor	[5] I don't know	[5] Jeg vet ikke
Tea	You don't know?	Vet du ikke?
Thor	[1] I do not remember	[1] Jeg husker ikke
Tea	What, how, how big is that cheese related to that one	Hva, hvor, hvor stor er den osten i forhold til den?
Thor	Twice as little	Dobbelt så liten
Tea	Yes, or half the size, [1] half as big cheese, how will it then be with the fat?	Ja, eller halvparten så stor, [1] halvparten så stor ost, hvordan blir det med fettene da?
Thor	[2] Twice as big [points to the big one]	[2] Dobbelt så stor [peker på den store]
Tea	Twice as big on this one then it will be half on that [points first to the big one, then to the little cheese]	Dobbelt så stor på den da blir det halvparten på den [peker først på den store, så på den lille osten]
Thor	Uum	Uum
Tea	What is half of two hundred and seventy?	Hva er halvparten av to hundre og sytti?
Thor	[6] I don't know	[6] Har ikke peiling
Tea	Ohh, you don't know?	Ååå, har du ikke peiling? [Lys stemme]
Thor	No [shakes his head], don't know	Nei [rister på hodet], har ikke peiling
Tea	No, but look at two hundred and seventy, you must have found that number somewhere? [2] Two hundred and seventy, and if you had split them in two hundreds, [3] he? [2] hundred, sort of, and then we could have, if this had been money then we could take hundred and then we could take fifty and then we could take ten and ten, that could be something, how could you divided in two then?	Nei men se på to hundre og sytti da for du må jo ha funnet det tallet et sted? [2] To hundre og sytti, og hvis du hadde tatt og delt opp dette i hundrelapper da [3] hø? [2] hundre, liksom, og så kunne vi tatt, hvis dette hadde vært penger så kunne vi tatt hundre og så kunne vi tatt femti og så kunne vi tatt ti og ti, det kunne vært noe, hvordan kunne du delt det på to da?
Thor	Hundred and ten	Hundre og ti

Tea	[2] Many hundred to each?	[2] Mange hundre hadde de fått hver
Thor	Two hund [interrupts himself], [1] no	To hund [avbryter seg selv], [1] nei
Tea	Yes, two, if you should divide by two	Jo to, hvis du skulle delt det på to
Thor	Two, twenty-five, hundred and sixty	To tjuefem, hundre og seksti
Tea	Hundred, they would have hundred each, do you agree?	Hundre, de får en hundrings hver, er du enig i det?
Thor	Mmm	Mmm
Tea	He? One for this one and one to that one, [3] what should we do with that one?	He? En til den og en til den, [3] den der da hva skulle vi gjort med den?
Thor	Divided it on two	Delt den på to
Tea	What would that have been, then?	Er det det hadde blitt da?
Thor	Twenty-five	Tjuefem
Tea	It is twenty-five [8] [how] much left then?	Det blir tjuefem [8] mye hadde vi hatt igjen da, da?
Thor	[3] mm	[3] Mm
Tea	[How] much left to divide then?	Mye er det igjen å dele på da?
Thor	[1] Twenty crowns	[1] Tjue kroner
Tea	Uum, [how] much would each have then?	Uum, mye hadde de fått hver da, da?
Thor	[1] Ten	[1] Ti
Tea	[How] much would they get, then?	Mye hadde de fått da, da?
Thor	[1] Hundred and [1] thirty-five	[1] Hundre og [1] trettifem
Tea	You can divide	Kan jo dele jo

Table 10-36. Observation 13, video 20110104101344, 24:56.

#### 10.5.4 Excerpts Tea, closings

##### Chapter 6.5.1.1, shall do

Tea	Right, if you put away your pencil now, [2] then this is one of the pages that are in your work-plan that shall be done this week. Yess, [2] it is ten, eleven, twelve, thirteen, fourteen, those are the pages we are going to work on, a little in school, little completed at home, and a little in the work session	Ikke sant, hvis dere legger fra dere blyanten nå, [2] så er dette en av de sidene som står på arbeidsplanen som skal være gjort denne uka. Jess, [2] det er ti, elleve, tolv, tretten, fjorten, det er de sidene vi skal jobbe med, litt på skolen, litt skal dere gjøre ferdig hjemme, og litt i arbeidsøkta
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Table 10-37. Observation 1, video 20100830105635, 40:55.

##### Chapter 6.5.1.1, smart-to-do

Tea	It is rather smart to spend time in the work session on Monday to work a little with maths so that no one is left not having done anything. All the maths we worked on yesterday, right, then it will be	Det er nok veldig lurt å bruke tid på arbeidsøkta på mandag til å jobbe litt med matte sånn at ikke noen sitter igjen og ikke har gjort noe. All matta som vi jobbet med i går, ikke sant, da blir det litt sånn etter
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like behind in your heads. [2] And that is a bit silly, so look to that, make sure to come some way.

oppi huet deres. [2] Og det er litt dumt, så passe på det, sørge for å komme et stykke i vei.

Table 10-38. Observation 2A, video 20100831082544, 37:33.

#### Chapter 6.5.1.1, must-do; lesson 5, must do for tomorrow

Tea Message before you take a break [4], those who, these tasks you *must* do for tomorrow. You must absolutely do these tasks till tomorrow. I will look at the answers. It is the first thing I will do in the math lesson tomorrow. [3] It is on your work-plan as well, so it is not that bad. But, right, it is of no use to say that you will do it one day later because I *shall* see them tomorrow. We *got* to have these in place

Beskjed før dere tar friminutt[4]; de som, de oppgavene her, de *må* dere ha gjort til i morra. Dere er dønn nødt til å ha gjort oppgavene. Jeg skal se på de svara. Det er det første jeg skal gjøre i mattetimen i morgen. [3] Dette står på arbeidsplanen også, så dere dauer ikke av det. Men ikke sant, det nytter ikke å si at de skal dere gjøre en dag seinere for jeg *skal* se dem i morra. Vi er *nødt* til å ha disse på plass

Table 10-39. Observation 5, video 20100920105614, 41:23.

#### Chapter 6.5.1.2, Plans for tomorrow

Tea The problem about the soft drinks you *shall* have done till tomorrow because we are going to talk about it. You *have* to do it at home, that [3, points to the task] one, that problem got to be done

Det problemet med brusgreiene *skal* dere ha gjort til i morgen for det skal vi snakke litt om. Dere er *nødt* til å gjøre det hjemme, den [3, peker på oppgaven] der, det problemet må være gjort

Table 10-40. Observation 16, video IMAG0001+IMAG0002, 36:59.

#### Chapter 6.5.3, goal for the lesson

Tea What you should be left with here that is to [1] try to remember that there are methods to thinking easily instead of sitting down thinking that ohh, I cannot do this, I need a calculator. It is possible to think easily

Det dere skal sitte igjen med oppi dette her, det er å [1] prøve å huske på er at det finnes måter å tenke lettvint på i stedet for å sette seg ned å tenke at ååå, jeg kan ikke dette her jeg, jeg må ha kalkulator. Går an å tenke lettvint

Table 10-41. Observation 2B, video 20100831090959, 42:15.

#### Chapter 6.5.3, goal not reached; lesson 8, did not come as far as expected

Tea We did not come as far as we should in this lesson, and for that that we can thank very many, among others the janitor. But if we can get sound on [interrupts herself] if we can get off the noise from the sink, then we should be glad for it

Nå kom vi ikke så langt som vi skulle denne timen her, og det kan vi vel takke veldig mange for, blant annet vaktmesteren, men hvis vi kan få lyd på, [avbryter seg selv] sleppe å høre på den vasken en annen gang, så skal vi jo være glad for det.

Table 10-42. Observation 8, video M2U00207, 41:02.

#### Chapter 6.5.2, review for introducing new content

Tea How do we write a fraction on decimal-form, Thom?

Hvordan skriver vi en brøk på desimalform, Thom

Thom We use point

Vi bruker komma

Tea	Use point, and how was it, we learned that in the autumn, when we divided with hundred, what, where did we place the point then? [3] then we moved the point	Bruke komma, og hvordan var det, det lærte vi i høst, når vi delte på hundre, hva, hvor satte vi komma da? [2] Det lærte vi i høst, [3] da flytta vi komma
Tony	First	Forrest
Tian	Two places	To plasser
Tea	Two places yes, because there are two zeros	To plasser, ja, fordi det er to nuller

*Table 10-43.* Observation 12, video 20110103115516, 42:55.

### Chapter 6.5.2, clarifying review; lesson 13, percent on semester-test

Tea	On the test we had before Christmas there was a task about percent. It was a question about percent, and actually it was like, to be quite honest, the most of you [2] failed [..] I do not remember the exact numbers, but it was a similar task, but it said, it said to be sure like this [wrote 30% under both] [4] and what did you answer?	På juletentamen, så var det et spørsmål om prosent. Det var et spørsmål om prosent, og det var faktisk sånn, for å være helt ærlig, at de aller, aller fleste av dere [2] gjorde feil, [..] jeg husker ikke akkurat hva det var, men det var en tilsvarende oppgave, men det sto, da sto det for sikkerhets skyld sånn da [skriver 30 % under begge] [4] og hva var det dere svarte?
Stud	Sixty	Seksti
Tea	You answered sixty, yes, [5] thirty percent boys and thirty percent girls that gives sixty, you answered, but you did not answer that now, [2] how many percent of boys and girls [1] have blue eyes? [1] Yes, because when I said sixty there and sixty there [writes 60% under both eyes] [2] then you answered sixty [1], you did not answer hundred and twenty, because that would have been quite interesting that it was one hundred and twenty percent students in the class	Dere svarte seksti ja, [5] tredve prosent gutter og tredve prosent jenter det blir seksti svarte dere, men det svarte dere ikke nå, [2] hvor mange prosent av guttene og jentene [1] har blå øyne? [1] Jo for når jeg sa det var seksti der og seksti der [skriver 60 % under øynene] [2] så svarte dere seksti [1], dere svarte ikke hundre og tyve, for det hadde jo vært ganske interessant at det var hundre og tyve prosent elever i den klassen

*Table 10-44.* Observation 13, video 20110104101344, 38:57.

## 10.6 Excerpts Chapter 7

### 10.6.1 Excerpts Chapter 7.1, KCS

#### Chapter 7.1.1

Primarily [1] I [2] want to make the pupils understand, [2] that is, I [1] will never as long as I am doing this to [3], eee say that [1] there are any knowledge	Jeg [1] ønsker [1] primært å få elevene til å forstå, [2] altså det, jeg [1] kommer aldri så lenge jeg holder på med dette her til å [3] eee si at [1] det finnes noen
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[1] or any way that is better than understanding

*Table 10-45.* Conversation 1, video 20100830121151, 04:30.

I believe [1] that is, or I think it is very important that one always shows that what we are doing is no game. We do not play school, this is the reality, this we will be using, and that is how it is. We are not such a small island that sits [2] calculating stupid tasks in books.

*Table 10-46.* Conversation 13, video 20110104111215, 18:04.

kunnskap [1] eller noen måte som er bedre enn forståelse

[J]eg tror at [1] det er, eller jeg tror at det er så viktig at en hele tiden viser at det vi holder på med det er ikke lek altså. Vi leker ikke skole, dette er virkeligheten, dette skal vi bruke, sånn er det, vi er ikke en sånn liten øy som sitter å [2] og regner teite oppgaver i bøker

### Chapter 7.1.3

I was a bit conscious at Thor, [1] a bit like to see if I was able to retrieve [...] Yes, for some reason [he had found the answer], but he had forgotten it already, but right, I thought I would be able to figure out how you found it, [1] have you used a calculator, [...] but it was like to test it [...] but right, there is far in [...] [...] He made it [found the answer], [1] and it seemed as, that is, you try somehow to see; "did you understand it", that is, it seemed like he was into at least, no I do not know.

*Table 10-47.* Conversation 13, video 20110104111215, 05:28

#### Chapter 7.1.4.1

It is of course always a problem, [4] I clearly try [2] always, [2] or when it occurs one tries to take it in, [3] tries to take it further. Not always sure it works [...] one says wrong things, right, [...] what I think is important then, it's a way to defuse it, "yes, yes, you are into something, but what is it, there is something here", that is, it is not so bad [2] because in a way, you've at least made some thoughts.

*Table 10-48.* Conversation 1, video 20100830121151, 18:56.

In a way I think it is okay because the aim of doing like that, asking, that's what they should, right, the aim was [interrupts herself], what was so nice about this, well, we could see that it was the right answer. Thus, it was in a way quite all right.

*Table 10-49.* Conversation 8, video M2U00208, 23:39.

Jeg var litt bevisst da på Thor, [1] litt sånn for å se om jeg klarte å hente [...] Ja, av en eller annen grunn [hadde han funnet svaret], men han hadde glemt det allerede, men ikke sant, jeg tenkte jeg skulle klare å finne ut hvordan har du funnet det, [1] har du regnet på kalkulator, [...] men det var litt for å teste da [...] men ikke sant, der er det langt inn [...] [...] Han gjorde det [fant svaret], [1] og det virket på en, altså du prøver liksom å se; "skjønnte du det", altså det virket som han var inne på i hvert fall, nei jeg vet ikke jeg.

Altså, det er jo selvfølgelig alltid et problem, [4] det er klart at jeg prøver [2] alltid å, [2] eller når det skjer så prøver man å lure [3] lure det videre. Ikke alltid det går, [...] man sier jo feil [...] det som jeg tror er viktig da, det er på en måte å ufarliggjøre det, «ja, jo, du er inne på noe, men, hva er det da, det er et eller annet her», altså det er ikke så farlig det [2] fordi på en måte så, så har du i alle fall tenkt.

På en måte så synes jeg det er helt greit fordi målet med å gjøre sånn som dette, spørre, det er jo det at de skal, altså, målet var jo at, [avbryter seg selv] hva var det som var så fint med dette her, jo, vi kunne se at det var riktig svar. Så den var jo på en måte helt grei

## 10.6.2 Excerpts Chapter 7.2, KCT

### Chapter 7.2.1, developing knowledge of content and teaching

I was very focused on the theoretical thinking all the time, that is [1] it *should* be set up as an equation; it *should* be done this way or that. Eee, while I somehow as I got more confident in myself, eee, I have worked a lot more, that is a little more in different directions. I have tried, trying to, or I think that there shall always be an opportunity to understanding and an opportunity for technique, [2] right?

Jeg var veldig fokusert på den teoretiske tankegangen hele tiden, altså, [1] det *skal* stilles opp som likning, det *skal* gjøres sånn og sånn. Eee, mens jeg liksom etter hvert som jeg ble tryggere på meg sjøl, eee, så har jeg jobbet mye mer, altså litt mer i forskjellige retninger. Jeg har prøvd, prøver å, eller jeg tenker at det skal alltid være en mulighet til å forstå og en mulighet til teknikk, [2] ikke sant?

Table 10-50. Conversation 5, video 20100920130847, 16:01.

### Chapter 7.2.2, planning for teaching

It takes place, that is, we then go down from all levels of planning and down to me, right, we skip all, because we have done that, we have agreed on when we shall have the different themes, and so on. And then I first look into the chapters in the textbook to look for what we are going to learn about that topic.

Den foregår, altså da går vi ned fra alle plannivåer og så går vi ned til meg, ikke sant, vi hopper over alle, for vi har jo det, og vi har jo blitt enige om når vi skal ha de forskjellige temaene og så videre. Og da tar jeg først for meg kapitlene i altså matteboka for å se hva vi skal lære om dette temaet.

Table 10-51. Conversation 1, video 20100830121151, 34:50.

### Chapter 7.2.3, structuring

It is not on the plan [4], and that, that is, so to speak, it is one of those things that, eee [3] like the old days when we had a note book for what to do when you get home. Thus, there are sometimes [1] that one actually [2] had needed [2] to have, so to speak [3] have such a book and say that those things you have to do.

Because, in a way, plans can always like, that is resistance in some sense [2] it was what was on the schedule, nothing else

Det står ikke på planen [4], og det, det holdt jeg på å si, det er jo en av de tingene som, eee [3] som i gamle dager når vi hadde lekseboka for det skal du gjøre når du kommer hjem. Altså det er noen ganger [1] at en faktisk [2] hadde trengt [2] å kunne holdt på å si [3] ha en sånn bok og si at det der skal du gjøre. Fordi, på en måte så kan alltid sånne planer bli, altså en svøpe også på en måte [2] det var det som sto på planen, ikke noe annet

Table 10-52. Conversation 17, video 20110405101018, 02:02.

One can say that this is the disadvantage with having weekly work-plans.

Sometimes you wish you could give them lesson from day to day because then you could manage the homework much more.

Det er noe med, det er ukeplanens lite årlighet [1] kan du si, altså noen ganger så skulle du ønske at du kunne gi dem lekse fra dag til dag fordi at da ville du styrt leksene mye mer.

Table 10-53. Conversation 18, video 20110411130122, 30:39.

There are two work-plans; it is the one with the easy, those who are named the

Det er to arbeidsplaner; det er en med de lette, altså de som heter de blå

blue tasks in the book, right, which are the easiest tasks in the book. Those who have the easy edition, they have those on the work plan, [1] right, [2] while the other have the basic elements of the plan [3] now in introduction. [2] And afterwards they get a period when we sort of, [2] next week, for example, where [1] those who have the difficult schedule gets the red tasks, the difficult tasks, while I push some of the green [interrupts herself], the low-achievers, try to push them over to the green.

*Table 10-54.* Conversation 4, video 201000907120626, 04:42.

Eee, that [2] I did because I felt it went so badly last time, [1] I think [3] I think they became so, I do not think [4] that they, that it became [interrupts herself], I then had a very, like formal and defined, that is, I do not think I made it. I do not think they understood it well enough. And then you actually have to, and then you actually have to, it is like you have to try, to look if, well, is it something wrong about me?

*Table 10-55.* Conversation 16, video IMG0003, 20:45.

### Chapter 7.2.4, Approaches to teaching

[I]n a way you go around and think about how I could have done this differently? What is it they do not understand, and how should I have done it. That is, one does that, [2] how do I tell the pupils so they understand.

*Table 10-56.* Conversation 5, video 20100920130847, 38:31.

Yes, a little like for variation, really, firstly little to vary instead of giving them some additional tasks they should just count, ee. I think it was all right to give them that, partly because they would relate to a complete solution they had to think through, and also because then they, that is, [3] then they should first try it themselves, and then they should talk to someone, right. Because then they first in a way had to think through whether what I did was correct, decide themselves, and then they should argue with the neighbour. So, it was, it was an idea of trying to raise awareness,

oppgavene i den boka, ikke sant, som er de letteste oppgavene i boka. De som har den lette utgaven, de har de på arbeidsplanen, [1] ikke sant, [2] mens de andre de har grunnstoffet på planen [3] nå i gjennomgangsfasen. [2] Og så får de en periode nå etterpå når vi liksom, [2] neste uke for eksempel, hvor [1] de som har den vanskelige planen får de røde oppgavene, de vanskelige oppgavene, mens jeg skyver noen av de grønne, svake, prøve å skyve dem over på de grønne.

Eee, det [2] gjorde jeg fordi jeg synes det gikk så dårlig sist, [1] jeg synes [3] jeg synes de ble så, jeg synes ikke [4] at de, at det fikk [avbryter seg selv]. Da kjørte jeg veldig sånn derre [2], holdt på å si sånn formelt og definert, altså jeg synes ikke jeg fikk noe dreis på det. Jeg synes ikke de skjønnte det bra nok. Og da blir det faktisk sånn at du er du jo faktisk nødt til å, da må man jo prøve, se om det, ja, er det meg det er noe galt med?

[P]å en måte så går du rundt og tenker på kunne jeg gjort dette på en annen måte? Hva er det de ikke forstår, og hvordan skulle jeg ha gjort det. Altså, man gjør jo det, [2] hvordan skal jeg fortelle til elevene sånn at de forstår?

Jo, litt sånn for å variere, egentlig, for det første så litt for å variere istedenfor å gi dem noen flere oppgaver de bare skulle regne, ee. Så synes jeg det var ål right å gi dem det, delvis fordi de skulle forholde seg til et ferdig svar som de måtte tenke gjennom, og også fordi de da, altså, [3] for da skulle de først prøve sjøl, og så skulle de snakke med noen, ikke sant. Fordi da skulle de på en måte først tenke gjennom hvorvidt det jeg hadde gjort var riktig, bestemme seg sjøl, og så skulle de argumentere med naboen. Så det var, det var en sånn tanke på det med å prøve å bevisstgjøre, altså



kind of [2] process around thinking about the answers.

Table 10-57. Conversation 6, video 20100921120844, 25:36.

sånn [2] prosess rundt det med å tenke gjennom svare da.

### 10.6.3 Excerpts Chapter 7.3, SCK

#### Chapter 7.3.1, drawing attention to particular features

That is what [1] *is* the message with percent, [] it is that you use this because it is such understandable amount, eighty-five percent, yes, then there are only fifteen again. Yes, but that is much, that is, teaching them to [2] that is, all the times like [2] where you can use that kind of thinking, then you communicate much better what you mean, that is sizes, yes. So therefore, I think it was perfect when they said on the news today, eighty-five percent. They did not say area, they did not say so and so, they said percent

Table 10-58. Conversation 13, video 20110104111215, 16:04.

Det er jo noe som [1] *er* budskapet med prosent, [] det er jo at dette bruker du fordi det er så forståelig mengde, åttifem prosent, ja da er det bare femten igjen. Ja, men det er møte det, altså lære dem til å [2] altså, alle de gangene liksom [2] hvor du kan bruke den type tenkning, så kommuniserer du mye lettere hva du mener, altså størrelser, ja. Så derfor synes jeg det var helt perfekt når de sa det på nyheten i dag, åttifem prosent. De sa ikke et areal, de sa ikke så og så, de sa prosent

#### Chapter 7.3.2, Representations

I want the pupils to find different ways of doing things, right. When we talk about solving problems as and when we work on whatever it may be, like major tasks then I am concerned that they have to draw, [2] draw. All sorts of things can be drawn [1] and then I am concerned that they should [1] think, try to think through the problem and that they should try to find a solution to the problem and that they do not necessarily have to show calculation, but they must explain what they did. That is, there is always another way to do it and that, therefore [2, interrupts herself], and that is what I am trying to I am trying to communicate, I think, I think myself that yes, there are methods [2] and yes, there are other ways.

Table 10-59. Conversation 13, video 20110104111215, 27:27.

Jeg ønsker at elevene skal finne fram til forskjellig måte å gjøre ting på, ikke sant. Når vi snakker om å løse oppgaver så, og når vi jobber med uansett hva det måtte være av litt sånn større oppgaver så er jeg opptatt av at de skal tegne [2] tegne. Alle mulige ting kan tegnes [1] og så er jeg opptatt av at de skal [1] tenke, prøve å tenke gjennom problemet, og at de skal prøve å finne en løsning på problemet og at de ikke nødvendigvis behøver å vise utregning, men at de må forklare hva de gjorde. Altså at det finnes alltid en annen måte å gjøre det på og at den, altså [2, avbryter seg selv], og det prøver jeg å formidle, tror jeg da, synes jeg sjøl at ja, det finnes metoder [2] og ja, det finnes andre måter.

#### Chapter 7.3.3, justifying own ideas

Well, we did that as well, in the end, and you can do it, and that is also a way of teaching mathematics, this-is-how-it-is-mathematics, but it is not my way.

Table 10-60. Conversation 4, video 201000907120626, 14:37.

Jo, vi gjorde jo det også, til slutt, og du kan gjøre det og det er også en måte å drive matteundervisning på, sånn-er-det-bare-matematikken, men det er ikke min måte.

#### Chapter 7.3.4, Tea discussing Tage's solution

Tuan It is not how I thought, as I did there, but

Tea No

Tuan Can I show the other way I thought about it instead of

Tea You can show me

Tuan I did, I first tried to find one, what made his one to one whole [points to  $\frac{x+1}{7}$ ]. [3] And when I found that it was one whole, then you know that this one was even [points to 6]. And I used the even number over [points to  $\frac{x}{2}$ ].

Tea Then, how did you make that to one whole?

Tuan No, because, eee, six plus one, you see

Tea No, no, no, no, no

Tuan No [removes the paper with the task]

Tea Oh, oh, oh, oh, oh, you must remember, we cannot, yes, that is, you tried to find the solution?

Tuan No, I just tried to, I tried to make this one whole [points to  $\frac{x+1}{7}$ ].

Tea But then you have decided that x equals six?

Tuan Yes, but I just wanted to see first. And then it became one whole, and then this one became six, and then it became four, and then

Tea Four?

Tuan The answer is four. [4] The answer is four

Tea It is so if you have decided that x equals six

Tuan Yes, then this one will be one [points to  $\frac{x+1}{7}$ ], and if you take six there, then the answer is one there. Three plus one equals four

Tea Yes, you have found the answer. Yes, you have found the correct solution, but you have to find a way to calculate it

Tuan Made it one whole

Tea Yes, but that was because you think so fast that you saw the answer. Smart guy.

Det er ikke samme måten som jeg tenkte, som jeg gjorde der, men.

Nei

Kan jeg vise den andre måten jeg tenkte på i stedet for å

Du kan vise den til meg

Jeg gjorde, jeg prøvde å finne en, det som gjorde den her en hel først [peker på  $\frac{x+1}{7}$ ]. [3] Og da jeg fant ut at det var en hel, så vet du at det var partall her [peker på 6]. Og jeg brukte partallet over [peker på  $\frac{x}{2}$ ].

Hvordan gjorde du den til en hel da?

Nei, for at eh seks pluss en, skjønner du

Nei, nei, nei, nei, nei

Nei [tar bort lappen]

Åh, åh, åh, åh, åh, du må huske på det, vi kan ikke, ja, altså du prøvde å finne løsningen?

Nei jeg bare prøvde å, jeg prøvde å gjøre den her til en hel [peker på  $\frac{x+1}{7}$ ].

Men da har du jo bestemt deg for at x er lik seks da?

Ja, men jeg skulle bare se først. Og så ble det en hel, og så ble det her seks, og da ble det fire. Og så

Fire?

Svaret ble fire. [4] Svaret ble fire

Det er hvis du har bestemt at x var seks

Ja, da blir det en her [peker på  $\frac{x+1}{7}$ ], og så hvis du tar seks der så blir svaret en der. Tre pluss en blir fire

Ja, du har funnet svaret. Ja, der er riktig svar du har funnet, men du skal finne en måte å regne det på

Gjorde den til en hel

Ja, men det var fordi du var så kjapp i huet at du så svaret. Luringen.

Table 10-61. Observation 9B, video M2U00232, 30:25.

### 10.6.4 Excerpts Chapter 7.4, CCK

#### Chapter 7.4.1, recognizing wrong or incomplete answers

Tony We took them upside-down or something Vi tok de opp ned eller noe

Tea Was it then we took upside down? mmm Var det da vi tok de opp ned? Mmm mmm

Table 10-62. Observation 1, video 20100830105635, 12:35.

Thor Mmm [2] one fifth Mmmm [2] en femdeler

Tea Is it one-fifth [points to 0.5]. Therefore, if you get point five parts of my millions, then you get one fifth of them. Er det en femdel? [peker på 0,5] Så hvis du får null komma fem deler av min millioner, så får du en femdel av de

Thor [2] No [2] Nei

Tea Then, how much to you get of them? Mye får du av de da?

Thor I do not know Jeg vet ikke

Table 10-63. Observation 2b, video 20100831090959, 03:59.

What is said (E)	What is said (N)	What happens
Tea What strange thing do you have there [pointing at something in Tage's book]	Hva har du for noe rart der [peker på noe i boka til Tage]	Asking about "a strange thing" she saw
Tage I am sure it is quite wrong	Det er sikkert helt feil	Did he say this because Tea commented the way she did?
Tea That was not what I asked, about. [2] I asked about what it was. Have the two graphs crossed each other	Det var ikke det jeg spurte om [2] Jeg spurte om hva det var. Har de der grafene krysset hverandre?	She did not ask, however she knew it was wrong
Tron So have mine	Ja det har mine og	Tron's had also crossed
Tea I think that was odd	Det synes jeg var snodig	
Tage I began to think of that, but I do not know	Jeg begynte å lure på det, men jeg veit ikke	
Tea Because you, because if you take that one, does it raise by two? Yes, it rises by two, [1] one away and two up	Fordi du, fordi hvis du tar den der, stiger den med to? Ja, den stiger med to, [1] en bort og to opp	Still looking in Tage's book, commenting on $y = 2x$
Tage Yes	Ja	Tage agrees
Tea The next one [1] it should start in one [1], it should also rise by two [5] one along and two up	Den neste der [1] den skulle starte i en [1] den skal også stige med to [5] en bort og to opp	Commenting on $y = 2x + 1$ , who should "start in one"
Tage Okay	Okei [4]	
Tron [4] I do not understand anything of it	Jeg skjønner ingen ting av det der	Tron does not understand
Tea If you put it into a coordinate system, no in a	Hvis du setter det inn i et koordinatsystem, nei i en	Still turning to Tage, she first talks

graph, right, then you have y is equal two x [2] right [1] this was that one. And then you have figured out the coordinates up there [leads her hand upward y-axis] [2] If you have x, and y is equal two x plus one, right, and then insert the same numbers at the coordinates here, null, two and four. And then you say if x is zero then zero times zero zero, plus	graf ikke sant, så har du y er lik to x [2] ikke sant [1] det er den. Og så har du regnet ut koordinatene oppover der [fører handa oppover y-aksen] [2] Hvis du har x, og y er lik to x pluss en, ikke sant, og så setter du samme tall på koordinatene her, null, to og fire. Og så sier du hvis x er null så er null ganger null, pluss	about $y = 2x$ , which he had drawn correctly. She then suggested that x could take the values zero, two, and four, and started to insert the numbers in $y = 2x + 1$ . Tea reads the task, but leaves to Tage to say what should be added
Tage One	En	
Tea One, is?	En er	Tea confirms, and asks for the solution
Tage Three, no	Tre, nei	Wrong answer
Tea One, hehe	En, hehe	Tea corrects
Tage No, one	Nei, en	Tage confirms
Tea Yes, if x equals two then y equals	Ja, hvis x er to så er y	Tea asks for y if x equals 2
Tage Two times two plus one	To ganger to pluss en	Tage inserts $x=2$ into the task
Tea Plus one equals	Pluss en er	Tea repeats some of it
Tage Equals, ee [2] five	Er, ee [2] fem	Tage thinks for some seconds
Tea Yes, if you look here [points to the coordinate system], if x equals one [2] right, of x equals zero, the y equals one, then you are there [points to (0,1)], if x equals two [2] then y equals five, and then you have one, two, three, four, five [counts upwards the y-axis and marks (2,5)]	Ja, hvis du ser her da [peker på koordinatsystemet], hvis x er en [2] ikke sant, hvis x er null så er y en, da er du der [peker på (0,1)], hvis x er to [2] så er y fem, og da har du en, to, tre, fire, fem [teller oppover y-aksen og markerer (2,5)]	Tea inserts the coordinates into the coordinate system (see
Tage Okay	Okay	
Tea Look at it and calculate	Se på den, regn den ut	

Table 10-64. Observation 19, 20110412082256, 27:41

#### Chapter 7.4.2, use of mathematical terms

What did the parenthesis say? [2] The parenthesis said something like "do the same with all of us" [writing on the blackboard] said in a way the parenthesis, right. Parenthesis said that everything which is inside here [1] they

Hva var det parentesen sa? [2] Parentesen sa noe med at "gjør det samme med alle oss" [skriver på tavla] sa på en måte parentesen, ikke sant. Parentesen sa at alt som er inni her [1] det hører på en måte sammen akkurat

belong in a way together just like a bag of candies. The entire bag is yours, lucky pig

som en påsa med smågodt. Hele påsan er din, heldiggris

*Table 10-65.* Observation 4A, video 20100907082547, 20:45.

There are many things that can mean different things, [3] numbers can do it, [4] words can do it, and therefore are not a's and b's and x's and y's so awfully creepy, and so much different from much else. [...] That is, I try to get them to think about what we do, that is [4] eee [3], we use letters and numbers interchangeably; we know that things means different in different settings. That is how it is with these numbers too.

Det er mange ting som kan bety forskjellige ting, [3] tall kan gjøre det, [4] ord kan gjøre det, og derfor så er ikke a'er og b'er og x'er og y'er så forferdelig skumle, og så mye annerledes enn så mye annet. [...] Altså jeg prøver å få dem til å tenke rundt det vi gjør, altså [4] eee [3], vi bruker bokstaver og tall om hverandre, vi vet at ting betyr forskjellig i forskjellige settinger. Sånn er det med disse tallene også.

*Table 10-66.* Conversation 6, video 20100921120844, 04:36.

### 10.6.5 Excerpts Chapter 7.5, HCK

#### Chapter 7.5.1, what pupils bring from primary school

I think one should expect pupils to master the basics in arithmetic before they enter lower secondary school [1] I think, [3] I get shocked when I experience pupils who do not manage to divide twenty-four by three

En burde jo kunne forutsette at elevene kan grunnleggende regning før de begynner på ungdomsskolen [1] synes jeg [3] jeg blir rystet når elever ikke kan dele fir og tyve på tre.

*Table 10-67.* Conversation 9, video M2U00233, 05:35.

Then I think it is fine to present the introduction like, that is, like I think that it might have been.

Da synes jeg det er greit å presentere den innføringa sånn som, altså sånn som jeg tenker at den kanskje har vært.

*Table 10-68.* Conversation 13, video 20110104111215, 23:59.

Yes, I have used spine, we have used it in slightly different contexts because when they went to primary school [1] or, I do not know how it was with these pupils here then, but many pupils in primary school learn that the tasks should be straight in the back.

Ja, jeg har brukt ryggraden, vi har brukt det i litt forskjellige sammenhenger fordi da de gikk på barneskolen [1] eller, jeg vet jo ikke hvordan det var med disse elevene her da, men mange elever på barneskolen de lærer seg at stykker skal være rette i ryggen.

*Table 10-69.* Conversation 9, video M2U00233, 19:33.

#### Chapter 7.5.2, preparing for upper secondary school and for life

However, we shall in a way [3] over to the interests. Then we in a way over to loans and interest and. Umm, to spend some time on it [1] because it is so extremely [1] important, it is like it is to, it is one of the worlds they are going to face very, very soon.

Men så skal vi jo på en måte [3] over på renteregningen. Da skal vi jo på en måte over på lån og renter og. Uum, å bruke litt tid på det [1] fordi det er så kolossalt [1] viktig, det er jo sånn at det å, det er jo en av de verdenene de kommer til å møte veldig, veldig fort.

*Table 10-70.* Conversation 15, video 20110111110846, 42:25.

Going to split them for what they think they should do in upper secondary school.

Kommer til å dele dem etter hva de tenker de skal gjøre på videregående.

Say to students that this here is a theme, so if you are following general studies and have theoretical mathematics then you actually have to know a lot, right, know quite many things that even are difficult, I think, and then I think they should know a little on proportionalities, and slope and intersections, and everything. I mean they need to know that.

*Table 10-71.* Conversation 16, video IMG0003, 02:50

Then I am very clear [2] with some students that I know had made it if they had bothered and say that you, you realize that these are [1] the two highest levels. If you are going to have a five or a six, then you actually need to know this. [...] This here, ridiculously stupid if you do not learn it now, then there will be twice as much to do in a few years.

*Table 10-72.* Conversation 8, video M2U00208, 12:57.

## 10.6.6 Excerpts Chapter 7.6, KCC

### Chapter 7.6.1, Lateral curriculum

I comforted myself in a way that, well it is just to comfort myself; I have these pupils in some other situations too, have them in science by weighing and measuring, and all those things right. Have them in food and health, [2] and then I end up to comfort myself with the fact that they get the same things in a way in the those subjects, very many things [2] in a more concrete way.

*Table 10-73.* Conversation 5, video 20100920130847, 36:05.

## 10.6.7 Excerpts Chapter 7.7, KFML

### Chapter 7.7.2, classroom norms

It is [5], it is everything [3] which is [1] between, that is, in relation to the pupils it is all between [1] they open their eyes in the morning to going to bed at night [2] and the parents until they settle for the evening.

*Table 10-74.* Conversation 2, video 20100831121023, 01:26.

It is somewhat important to me that they, that they eee, that they [at home] in a way shall [3] transform what they hopefully have heard [1] before they go home so they do not start with a name on A, [1] hehehe, eee [1] in their own books [1]

Sier til elevene at dette her er et tema, så dersom du skal gå på allmenn og ha teoretisk matematikk så må du faktisk kunne mye, ikke sant, kunne ganske mange ting som er vanskelig til og med, synes jeg, og da synes jeg de skal kunne litt om proporsjonale, og stigningstall og skjæringstall, og alle mulige. Det må de kunne, mener jeg.

Da er jeg veldig tydelig [2] med noen elever som jeg vet hadde klart det hvis de hadde giddet, og si det at, at du skjønner det at dette her, det er [1] de to høyeste nivåene. Hvis du skal ha en femmer eller sekser så må du faktisk kunne dette. Dette her, lidderlig dumt hvis du ikke lærer deg det nå, da blir det dobbelt så mye å gjøre om et par år

Jeg trøster meg på en måte med at, det er bare for å trøste meg sjøl da; jeg har disse elevene i en del andre situasjoner og, har dem i naturfag med veiing og måling og alle de tinga der ikke sant. Har dem i mat og helse, [2] og så ender jeg opp med å trøste meg sjøl med det at de får jo de samme tinga på en måte i de faga, veldig mange ting [2] på en mer konkret måte.

Det er [5], det er alt [3] som er [1] mellom, altså i forhold til elevene så er det alt som er mellom [1] at de slår opp øya om morran til de legger seg om kvelden [2] og foreldrene til de legger seg om kvelden.

Det er på en måte viktig for meg at de, at de eee, altså de skal på en måte [3] omforme det de forhåpentligvis har hørt [1] så de ikke begynner på et navn med A, [1] hehehe, eee [1] i sin egen bok [1] etterpå, før de går hjem. Fordi, eee, da

afterwards, Because, eee, then you have the opportunity to stop [1] misunderstandings, and I, [1] you have opportunities to make such [interrupts herself], that they have some examples in their books [2] that they in a way have worked on before continuing at home

Table 10-75. Conversation 4, video 201000907120626, 02:07.

They know that eh, I expect that they are quiet, [2] they know that I expect, that is, right, they know they are getting very clear message, eh, probably sometimes too clear, that is, yes, they know I do not give up. They know that I *never* give up [2] when it comes to content [...] because I so incredibly sure that they need it.

Table 10-76. Conversation 1, video 20100830121151, 23:26.

### Chapter 7.7.3, parent surrogate

I believe in that, and it is of course an eternal discussion me talking about private thing. Because I do that, what we did yesterday and, yesterday we shovelled the roof, that is speaks a little with them. [3] I believe in it, for I, that it is because we are so incredibly, we are the more with them than their parents these day.

Table 10-77. Conversation 15, video 20110111110846, 05:27.

Because it is Bolla the hedgehog, right, who potters off and then she comes to her stump and then the stump is there.

Table 10-78. Conversation 1, video 20100830121151, 21:02.

## 10.7 Excerpts Chapter 8

### 10.7.1 Excerpt Chapter 8.3

And then you became very good on those things. Finally you were so good [2] that one day when you came to school then you got new calculations [3, writes  $+ 3 = 9$  on the blackboard], then did the tasks look like that.

Table 10-79. Observation 8, video M2U00207, 09:09.

### 10.7.2 Excerpt Chapter 8.3.3

No, [3] I do not think so, I think now that they are at a point that it is good to show that they have understood [...] And that's

har du mulighet til å stoppe [1] misforståelser, og jeg, [1] du har muligheter til å lage en sånn, altså at de har noen eksempler i boka si [2] som de på en måte har jobbet med før de fortsetter hjemme

De vet at eh, jeg forventer at de er rolige, [2] de vet at jeg venter, altså, ikke sant, de vet at de får veldig klar beskjed, ehh, sikkert noen ganger for klar, altså, ja, de vet at jeg ikke gir meg på tørre møkka. De vet at jeg gir meg *aldri* [2] når det gjelder fag [...] fordi jeg er så innmari sikker på at dette trenger de.

Jeg tror på at, og det er jo en evig diskusjon selvfølgelig, at jeg forteller om private ting, for jeg gjør jo det, hva gjorde vi i går og, i går måket vi taket, altså snakker litt med dem. [3] Jeg tror på det, for jeg, altså det er fordi vi er så innmari, vi er jo mer sammen med dem enn foreldrene deres i denne tiden her

For det er jo Bolla pinnsvin, sant, som [rister på hodet] tasler avgårde og så kommer a til stubben sin og så er den stubben der.

Og så ble dere kjempegode på å kunne de tinga der. Til slutt var dere så gode [2] at en dag når dere kom på skolen så fikk dere nye regnestykker [3, skriver  $+ 3 = 9$  på tavla], da så regnestykkene sånn ut.

Nei, [3] jeg tror ikke det, jeg tror nå at de er på et punkt at det er deilig å vise at de har skjønt [...] Og det er jo noe jeg

something I want, the joy of saying that I  
understood it, I can, and then, then I do  
not think it matters. [2] It is better to feel  
successful [2] I would say a bit too long  
[1] than frustrated

*Table 10-80.* Conversation 18, video 20110411130122, 11:30.

ønsker, gleden over å si at jeg forsto det,  
jeg kan det, og da, da tror jeg ikke det  
gjør så mye. [2] Det er bedre å føle seg  
vellykket [2] holdt jeg på å si litt for  
lenge [1] enn frustrert



## 10.8 Confirmation from Norwegian Social Data Services

Norsk samfunnsvitenskapelig datatjeneste AS  
NORWEGIAN SOCIAL SCIENCE DATA SERVICES



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Vår dato: 04.09.2009

Vår ref: 22445 / 2 / RKH

Deres dato:

Deres ref:

### KVITTERING PÅ MELDING OM BEHANDLING AV PERSONOPPLYSNINGER

Vi viser til melding om behandling av personopplysninger, mottatt 20.08.2009. All nødvendig informasjon om prosjektet forelå i sin helhet 03.09.2009. Meldingen gjelder prosjektet:

22445	<i>Exposing Teacher Craft Knowledge: A case study</i>
Behandlingsansvarlig	Universitetet i Agder, ved institusjonens overste leder
Daglig ansvarlig	Inger Norunn Nergaard

Personvernombudet har vurdert prosjektet og finner at behandlingen av personopplysninger er meldepliktig i henhold til personopplysningsloven § 31. Behandlingen tilfredsstiller kravene i personopplysningsloven.

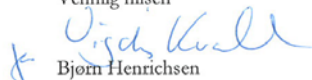
Personvernombudets vurdering forutsetter at prosjektet gjennomføres i tråd med opplysningene gitt i melde skjemaet, korrespondanse med ombudet, vedlagte prosjektvurdering - kommentarer samt personopplysningsloven/-helseregisterloven med forskrifter. Behandlingen av personopplysninger kan settes i gang.

Det gjøres oppmerksom på at det skal gis ny melding dersom behandlingen endres i forhold til de opplysninger som ligger til grunn for personvernombudets vurdering. Endringsmeldinger gis via et eget skjema, [http://www.nsd.uib.no/personvern/forsk\\_stud/skjema.html](http://www.nsd.uib.no/personvern/forsk_stud/skjema.html). Det skal også gis melding etter tre år dersom prosjektet fortsatt pågår. Meldinger skal skje skriftlig til ombudet.

Personvernombudet har lagt ut opplysninger om prosjektet i en offentlig database, <http://www.nsd.uib.no/personvern/prosjektoversikt.jsp>.

Personvernombudet vil ved prosjektets avslutning, 31.12.2020, rette en henvendelse angående status for behandlingen av personopplysninger.

Vennlig hilsen

  
Bjørn Hennrichsen

  
Ragnhild Kise Haugland

Kontaktperson: Ragnhild Kise Haugland tlf: 55 58 83 34  
Vedlegg: Prosjektvurdering

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Utvalget omfatter to ungdomsskolelærere og noen av deres elever.

Forsker kontakter selv lærerne, og lærerne informerer elevene om prosjektet og videresender informasjon til elevenes foreldre.

Opplysningene samles inn gjennom personlig intervju (bare lærerne) og observasjon av undervisning (lærerne og noen av elevene). Det gjøres lydopptak av intervju og videoopptak under observasjon, og lyd- og videoopptakene skal behandles på PC (jf. telefonsamtale med prosjektleder 31. august 2009).

Det gis skriftlig informasjon om prosjektet til lærerne, elevene og elevenes foreldre. Lærerne samtykker muntlig til deltakelse, mens det innhentes skriftlig samtykke fra foreldrene til at deres barn kan filmes i forbindelse med prosjektet. Personvernombudet mottok revidert informasjonsskriv til foreldrene 3. september 2009 og finner skrevet tilfredsstillende. Det forutsettes at lærerne får tilsvarende informasjon.

Prosjektet skal fullføres innen september 2013, men videoopptak og transkripsjoner skal oppbevares til 31. desember 2020 med tanke på mulige oppfølgingsstudier. Datamaterialet skal lagres på en passordbeskyttet server ved Universitetet i Agder. Deltakerne informeres om lagringen slik den er beskrevet over.

Senest innen utgangen av 2020 skal datamaterialet anonymiseres ved at videoopptak og eventuelle lydopptak slettes, og eventuelle indirekte identifiserende bakgrunnsopplysninger slettes eller omskrives slik at det ikke lenger er mulig å føre opplysningene tilbake til enkeltpersoner.

Dersom datamaterialet skal inngå i nye prosjekter sendes melding til personvernombudet før oppstart.