Mathematics Teachers' Interpretation of the Curriculum Reform, L97, in Norway

Doctoral thesis

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Faculty of Mathematics and Sciences
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Preface

This doctoral study was financed by the Norwegian Research Council. I was given a three years doctoral scholarship as part of the KUPP program, Kunnskapsutvikling I Profesjonsutdanning og Profesjonsutøving, (Knowledge Development in Professional education and Professional practice). The goal of this program was to develop research based knowledge in relation with professional practice. The professional practice I have been studying is teaching of mathematics in lower secondary school. Before undertaking this study, I had long experience both as a mathematics teacher in lower secondary school and as a mathematics teacher educator. This study which has allowed me to study mathematics teaching and learning from a new angle, from an observer's perspective, has added yet another dimension to my professional experience which can contribute hopefully to "knowledge development" in the field more widely.

First of all I will express my appreciations to the teachers who let me into their mathematics classroom during the spring 2004. Without their willingness, this study had not been possible to carry out. I will also thank the teachers who participated in the focus groups who shared their experiences and contributed in discussions. Also a great thank to the students who shared their mathematics thoughts with me, and thanks to the schools' leadership for opening up the school for my research.

This doctoral study has been carried out at Agder University College and I am deeply grateful to my supervisor Professor Barbara Jaworski for sharing her skills and knowledge within the field of mathematical education with me and also for her critical eye and guidance in my research. Also a great thank to Associate Professor Gard Brekke for giving me encouragement during the application process for the scholarship and being a good support throughout the study. I will also express a deep gratitude to the Faculty of Mathematics and Sciences at Agder University College, for good support, especially during the last six months, and thank you, all fellow doctoral students and other people working at Agder University College for interesting discussions, encouragement and support.

Last but not least, thank you very much all my friends and colleagues at Oslo University College where I have my daily work.

Bodil Kleve
Oslo / Kristiansand, Norway
February 2007

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1. Introduction

"Mathematics Teachers' Interpretation of the Curriculum Reform, L97, in Norway" is a study of mathematics teachers' teaching practice in lower secondary school related to a curricular reform. I was interested to study teachers' responses to and interpretations of a new curriculum, L97. This was prompted by other research in which I had been engaged which suggested that students were performing less well in mathematics after the introduction of the new curriculum than before.

Before undertaking this study I had long experience both as a mathematics teacher in lower secondary school and as a mathematics teacher educator at a University College. My involvement in in-service training courses for teachers in relation with the curriculum reform created some of the background for why I wanted to undertake this research. My experience as a mathematics teacher and mathematics teacher educator has been of considerable value in enabling me to interpret and understand some of the complexity of mathematics teachers' daily work with students.

In this first chapter, I will introduce the background and context for my research, relating to international comparative studies. Next I discuss backgrounds and aspects of educational reforms and an account of the curriculum L97 as a written document. I then refer an evaluation carried out of the curriculum reform R97 of which L97 was part. Finally I introduce my study and research questions and present an overview of the chapters of the thesis.

Background and context

Mathematics in school and students' learning outcomes in mathematics have been widely discussed in Norway both politically and in research. Large scale comparative studies have taken place. The Third International Mathematics and Science Study, TIMSS (Lie, Brekke, & Kjærnsli, 1997) is one of several studies of the *International Association for the Evaluation of Educational Achievement.* The most important results in mathematics from Norway's participation in TIMSS in 1995 were:

- Norwegian students perform relatively poorly, especially in Mathematics.
- Norwegian students perform significantly lower than the mean of all countries, lower than in Sweden, about the same as Denmark and Iceland (Finland did not participate).
- There was no significant difference between genders with regard to performance in mathematics. However with regard to attitudes, boys in Norway were more positive to mathematics than girls and

boys showed a greater self-confidence with regard to mastering the subject than the girls did.

- Within an international perspective, mathematics teachers in Norway have very little education in mathematics. Teachers teach many subjects and have relatively poor qualifications in mathematics.

  (Lie, 2001; Lie, Brekke, & Kjærnsli, 1997).

In TIMSS 2003, also Norwegian students performed significantly lower than the international mean. However, the same countries did not participate in 2003 as in 1995 so it is not easy to compare directly. What gives more concern, is that Norwegian students' performances on the same tasks in 2003 were lower than in 1995 (Grønmo, 2004).

Norway also participated in PISA, *Programme for International Student Assessment*, another international comparative study, organised by OECD in 2000 (Lie, 2001) and in 2003 (Kjærnsli, Lie, Olsen, Roe, & Turmo, 2004). The students participating in PISA were 15 years old, and thus in lower secondary school in Norway. In these studies Norwegian students performed at an average level from an OECD perspective. However, studying the results from a Nordic perspective, Norway performed significantly lower than the other Nordic countries (Finland, Denmark, Iceland and Sweden). Comparing the Norwegian results in 2000 and 2003, the decline in absolute performance in Norway is modest, but compared to the other Nordic countries, the gap in performance is considerably higher in 2003 than in 2000 (Kjærnsli, Lie, Olsen, Roe, & Turmo, 2004).

Educational reforms, R97 and the Curriculum L97

Reform 97, (R97) was a nationwide educational reform which took place in Norway in 1997. Part of the reform started already in 1993 in upper secondary school into which I will not go further into detail here. Before I present the reform, there are two issues related to the background of R97 on which I want to comment, because I see them as essential in how teachers respond to a new curriculum:

First, R97 is a "top-down reform" (Gjone, 2003). This means that it was politically initiated and not initiated by the teachers. Hence teachers looked upon it as a directive from above.

Second, in Norway there have been several governmental shifts in relatively few years resulting in a reform initiated by one government being implemented by another. Hence, R97 which was prepared by a Labour Party government was implemented by a Minority Coalition Government representing the Christian Democratic, Conservatives and Liberal parties.

During their time with governmental power, the Minority Coalition Government started their work with yet another reform, "Kunnskapsløftet" (Knowledge Promotion), LK06, (Kunnskapsdepartementet, 2006; Utdannings- og forskningsdepartementet, 2005) and introduced national tests in the subjects Norwegian, English and Mathematics which were carried out in 2004 and 2005. The introduction of these national tests led to teachers concentrating more upon meeting the assessment criteria on the tests rather than upon the working methods encouraged in the curriculum. The government taking over in September 2005 (Majority Government representing the Labour, Centre and the Socialists Left parties) decided to stop these tests (temporarily, they said) because there were so many objections towards them, especially from teachers and teacher educators.[1]

The most important changes in the way education is organised to come out of Reform 97 compared to the previous curriculum, M87 (Kirke- og undervisningsdepartementet, 1987)[2] were that

1. School starts at the age of 6 instead of 7.
2. There should be 10 years of schooling instead of 9.
3. As part of the more wide-ranging Reform 97, which affected the whole of the compulsory education system, a new curriculum was implemented in August 1997.

This Curriculum, for grade 1-10 (age 6 to age 15), is called "Læreplanverket for den 10-årige grunnskolen", and abbreviated commonly to L97. I refer to the English version of the curriculum, "The Curriculum for the 10-year compulsory school in Norway" (Hagness & Veiteberg, 1999)[3]. L97 is thus the written document, the curriculum that is intended by curriculum writers and government to be implemented in classrooms. In the curriculum the learners are referred to as pupils. Following the custom in international research in mathematics education, I have chosen to refer to the learners as students in my research. However, to be accurate when quoting from L97 I use the term pupils.

The curriculum for the 10 years compulsory school in Norway, L97, consists of three main parts; a general part or the Core Curriculum for compulsory, upper secondary and adult education; Principles and Guidelines for compulsory education and Subject syllabuses (under which the

[1] These are issues only to be mentioned as background information, indicating that shifts in governments lead to "disturbances" within education. However, a further discussion of these issues is beyond the scope of this thesis.

[2]The Norwegian Ministry of Education and research was called: "Kirke og Undervisningsdepartemente" in 1987, "Kirke. Utdannings og Forskningsdepartementet (KUF)" in 1994, "Utdannings og Forskningsdepartementet" in 2005, and now (from 2006): "Kunnskapsdepartementet".

[3] I am quoting from the English version of the curriculum whenever possible, but sometimes I do not think the English translation is as good as it could be from the Norwegian so then I refer back to the Norwegian version in order to get a more accurate translation.

mathematical part belongs). All three parts are illustrated with colourful pictures.

The Core Curriculum (The General Part)

The core curriculum or the general part ("Generell del") of the curriculum L97, which became effective from September 1993 contains the overall aims for compulsory school (year 1-10), upper secondary school (year 11-13) and adult education. These aims are explicitly pointed out as something to work towards and according to which outcome can be evaluated. The core curriculum acknowledges that the point of departure for schooling is different for different children: social background, personal aptitude and local origin; and that education must be adapted to the needs of the individual. The overall aim is to expand the individual's capacity to perceive and to participate, to experience, to emphasise and to excel. To promote these aims, careful examination of basic values, view of mankind and nurturing tasks is considered necessary and is outlined with focus on seven different aspects of human beings. The seven aspects of mankind which are outlined with regard to education can be summarised as:

- The spiritual human being:
  Education shall be based on fundamental Christian and humanistic values and on the view that all persons are created equal;
- The creative human being:
  Education shall meet children on their own terms, entail thinking in making conjectures, train the ability to wonder and reflect and provide learners with the ability to acquire and attain new knowledge themselves;
- The working human being:
  Emphasis is placed on the teacher as part of the school's staff and that they together share responsibility for pupils' development. Collaboration with parents is also highlighted. The most important pedagogical task is said to be to convey to children that they are continuously making headway and to encourage them to gain trust in their own abilities;
- The liberally-educated human being:
  Emphasis is placed on schooling that will provide a multi-faceted and all round general education. Concrete knowledge about human beings; know-how to face life's different challenges and qualities and values facilitating collaboration with others are emphasised;
- The social human being:
  In the Norwegian version this headline is "Det samarbeidende mennesket" which I will translate as "The collaborative human being" rather than "the social human being" as in the official English

version. Weight here is put on the fact that a person's aptitude and identity develop in interactions with others and that human beings are formed by their environment and vice versa, that they contribute to forming it;

- The environmentally-aware ("allmenndannede") human being: Focus is on humans as part of nature and that education must provide awareness of the interconnections in nature and of the interplay between humans and nature;
- The integrated human being:
  In this section weight is put on what is written as "seemingly contradictory" (p.55) educational aims presented in L97 such as

  > [ ] concern for others - *and* to foster ability to plot one's own course;
  > to overcome self-centeredness [ ]–*and* to inspire strength to stand alone;
  > to develop independent and autonomous personalities –*and* the ability to function and work as a team (p. 55, emphasis in original).

  Education is encouraged to look upon these as dual aims to be balanced, especially to balance between individual needs and commitment to society.

These aspects of human beings are seen as the overall aims of the curriculum on which the subject syllabuses are based.

Principles and Guidelines for Compulsory Education

The Principles and Guidelines section of L97 is the bridge between the Core Curriculum (CC) which contains the overall aims as summarised above and the subject syllabuses (mathematical part, M, in my study) which are based on the core curriculum. The Principles and Guidelines (PG) describe the general aims, the subject related objectives and the content for each subject. In this section it is emphasised that there is one school for all pupils, regardless of where they live, their social backgrounds, their gender, and of their religions and ethnic origin. The school is supposed to be an important part in reducing social inequality and in promoting equality between people with different backgrounds.

In Norway, there is a long tradition having a comprehensive and compulsory school system, taking in both primary and lower secondary education. One of the most significant developments in this curriculum, L97, is that greater emphasis has been placed on a central curriculum. This is intended to ensure a nation-wide education system with a common content of knowledge, traditions and values for all pupils in the country.

In the principles and guidelines of L97 guidelines for common content of the subject syllabus are presented. This is to ensure that education shall promote the development of the seven aspects of human beings presented in the core curriculum (the spiritual, creative, working, liberally-educated, social, environmentally-aware and integrated human being). The organisation and structure of the presentation of the subject syllabuses (mathematics in my study) is the same for all subjects with an introduction, general aims for the subject, objectives for the main stages of education (primary 1-4, intermediate 5-7 and lower secondary 8-10), and main subject elements for each grade. Thus the levels of aims are:

- Overall aims presented in the core curriculum;
- General aims (for each subject presented in each subject's syllabus) for the subject which shall explain what the pupils are supposed to be working towards and
- Subject related objectives (presented in each subject's syllabus) for the main stages, primary, intermediate and lower secondary, which describe expected competence in each of the main areas.

The main subject elements show the subject contents for each grade and describe what subject matter should be covered in each main area. Each of the main stages: primary, intermediate and lower secondary is characterised:

- For the primary stage (grade 1-4) the syllabuses presuppose a varied interaction between play and other activities and learning shall move from the known to the unknown. Play is seen as a starting point for organised learning. "Learning nurtures play and play nurtures learning" (L97 p. 80). At this stage there will be a gradual move from thematic structure of the content to more subject-specific content.
- At the intermediate stage (grade 5-7) the different subjects become more distinct and working methods shall promote and develop pupils' abilities to concentrate and work on assignments given over a period of time.
- Lower secondary stage (grade 8-10) is based on and shall be further developed from the primary and the intermediate stage. In addition to that it shall ensure continuity also with regard to transition to upper secondary school. The study of subjects shall be more in depth and more advanced. Learning by doing is emphasised as a working method and new learning shall be based on what pupils already know.

The allocation of time for subjects is also presented in "Principles and Guidelines". In the primary stage mathematics has 532 periods out of a total of 3040, at intermediate stage 437 periods out of a total of 3078, and at lower secondary stage 418 out of 3420. Mathematics has thus relatively smaller space in lower secondary stage than in the two other stages.

The Mathematical Part of L97

The Committee writing the mathematical part of L97

There were 7 persons involved in developing a written curriculum for mathematics which made up the mathematical part of L97. The leader of the committee was the only academic and he did the writing. The other members were all experienced teachers (both genders). They were recruited from different parts of the country and represented all stages in school (primary, intermediate and lower secondary). Both a head-teacher and a representative from a teachers' trade union were in the committee. To try to find out what constraints the committee was given, together with the task to write the curriculum, thought processes they had to go through and what theoretical (if any) perspectives they had, I had a conversation with the leader, in June 2005 and with one of the other members in January 2006.

As constraints and guidelines in their work they had the "Core Curriculum", "Principles and Guidelines" and they were given some verbs[4] they could use. According to the two interviewed these were useful elements as guidance in their work. They also studied curricula and reports from other countries; Sweden, Scotland, the English and Welsh Cockroft Report (1982) and the American NCTM Standards (NCTM, 1989). They were influenced by the work of The Freudenthal Institute in The Netherlands and the Shell Centre in Nottingham, England. The committee's intention was to move school mathematics closer to research mathematics than what they perceived the subject had been in earlier curricula. This they had tried to mirror in the introduction where *play* at the primary stage, *play and games* at the intermediate stage and *practical situations and students' own experience* at the lower secondary stage are emphasised. The leader of the committee said they were "inspired by international trends about constructivism in mathematics education". How this is reflected in L97 is presented in Chapter 5 where I offer my theoretical analysis of the mathematical part of L97.

Mathematics in L97

According to common guidelines for content presented in Principles and Guidelines the subject syllabus for mathematics consists of three parts:

- Introduction
  - The subject and educational aims
  - Approaches to the study of mathematics
  - The structure of the subject
- General aims for the subject

[4] Examples of verbs: Express, reason, solve, analyse, experiment, explore, describe, define, interpret, apply, carry out, discover, gather, differentiate, organise, generalise, found, etc

- Objectives and main subject elements for each of the three main stages

In the introduction to the mathematical part, weight is put on practical application of mathematics, and on the links between school mathematics and mathematics in the outside world. It is emphasised that the working methods chosen shall ensure equal opportunities for all students to develop favourable attitudes which are looked upon as essential for the learning of the subject. Elements such as reasoning, imagination and experience are pointed out as important. Mathematics shall be looked upon as a science, an art, a craft, a language and a tool, and the learning of mathematics shall be looked upon as a process.

The main areas of the subject in all stages are: *Mathematics in everyday life*; *Numbers and Algebra* (only *Numbers* in primary and intermediate stages); *Geometry* (*shape and space* in primary stage). In addition *Handling of Data* is an area in both intermediate stage and in lower secondary stage and *Graphs and Functions* in lower secondary stage. Thus there are five main areas of the subject in the lower secondary stage.

The committee wrote an unpublished note to the Ministry of Education where they gave reasons for what they had written in the mathematical part of L97 both philosophically and with a discussion about the premises on which the mathematical part was based. In this note which is reported in Breiteig & Venheim (1999), weight is put on mathematics as a school subject which, partly because of introductions of technical tools such as calculators and computers, is going through a thorough change. They emphasised that, in this curriculum, they wanted to break down the division between school mathematics and mathematics in society outside the classroom, and that was one of the reasons for bringing in "Mathematics in everyday life" as a main subject area in the new curriculum. This main area was new with regard to prior curricula and a study of how this subject area has been dealt with in the classroom has been carried out by Mosvold (2006).

With regard to calculating with numbers the committee wanted to put more weight on the meaning of computational operations, mental calculations, strategies in learning tables and on judging and choosing methods. They wanted to put less weight on isolated drill, memorising rules and procedures and on exact answers without judgement.

With regard to geometry, which traditionally has focused on constructions with compasses and ruler, they expressed a wish for the students to gain a wider experience with other aspects of the topic and especially the relation between geometry and art. The students shall use fantasy, creativity and their imaginations of shape.

In Norway algebra is also called "bokstavregning" which can be translated into "letter-calculations". This has often been connected with meaningless manipulations with symbols, which the committee now wanted to be toned down. The committee's writing about algebra in the note to the Ministry of Education underpins this. They wrote:

> Believing that the students will get a functional conception of what it is all about by learning computational rules with "letter expressions" is nearly the same as leading them blindfolded into the "room of algebra". The result is that many will snuffle, getting many hits and hurts and their greatest wish will be to get out of it as soon as possible (Breiteig & Venheim, 1999, pp. 15-16, my translation).

In Chapter 5, I present my theoretical analysis of L97. There I discuss to what extent I see procedural and conceptual knowledge reflected in the curriculum, I discuss school mathematics versus research mathematics and how that is reflected in the curriculum and I argue how I see an investigative approach to mathematics in L97.

Evaluation of the Curriculum Reform

NFR (Norges Forsknings Råd), The Research Council of Norway was asked by UFD (Utdannings og Forskningsdepartementet) the Ministry of Education, to carry out a thorough evaluation of the reform R97. The evaluation process was organised as a research program funding 26 different projects. One of the research projects in the program was "*Change and development with R97 as a basis for further planning and adjustment – the case of mathematics*" (Alseth, Brekke, & Breiteig, 2003). This research was carried out during the years 2000-2003 and it studied the different levels of the curriculum reform with regard to mathematics: The intended, implemented and attained curriculum.

These terms: intended, implemented, and attained refer to a curriculum seen from three points of view (Goodlad, 1979; Howson & Wilson, 1986; Lie, Brekke, & Kjærnsli, 1997; Robitaille et al., 1993). On the *Intended level* are the written documents, guides and instructions and plans for in-service training and competence building. The next level, the *Implemented curriculum,* is the classroom level; what teachers teach, the teachers' teaching practice. The implemented curriculum can be viewed as an interpretation of the intended curriculum; it is the teacher's interpretation of the intended curriculum. Students' learning outcome or their performance is the *Attained curriculum*. This is the outcome of schooling both in terms of what concepts, processes and attitudes towards mathematics students have achieved in the course of their schooling years.

In their analysis of the *intended* mathematics curriculum Alseth et al. (2003) found that L97 puts an increased emphasis on five main areas: Mathematics as an area of discovery, as a tool, as a creative field, as reflective and meta-cognitive activities and mathematics in society and culture. Important elements in the mathematical part of L97 are to investigate, to generalise to justify and to make representations of results. According to their analysis, L97 recommends an explorative and experimental approach. They found that it emphasises five areas which they claim to be in accordance with international tendencies within mathematical education: Practical foundation; Conceptual development; Investigation and problem-solving; Communication and cooperation and Mathematics in a historical and cultural perspective. An analysis of the "Plan for in-service courses in mathematics" (Kirke- utdannings- og forskningsdepartementet, 1997) shows that important elements from L97 were taken into account in the plan. This plan emphasises communication and cooperation and exploring activities.

To investigate how L97 was *implemented*, a classroom study was carried out (Alseth et al. 2003). This study reveals that the teaching of mathematics still, mainly, follows a traditional pattern where the teacher starts the class by giving an introduction including revision of homework and presenting new material. Specified skills are often in focus, the subject is rarely presented holistically and skills are drilled rather than understood. This is in great contrast to what is intended in L97 where students are supposed to develop their own mathematical concepts and skills are supposed to be based on understanding and general concepts and principles within the subject.

Due to my interest in students' learning outcomes in mathematics with regard to the reform I became involved in this research project and, in collaboration with it, I did an in depth study of students' knowledge in mathematics. That was a study of the attained curriculum. I compared results on tasks given to students in grade 4 and grade 7 in March 2001 with results on the same tasks given to students in 1995 (Kleve, 2003). The comparative study shows that the students performed generally lower in 2001 than in 1995. This is especially visible within *Procedural knowledge* as computational skills. Also within calculating with decimal numbers and in reading decimal numbers into a number line, the students in 01 performed lower than the students in 95. There was no remarkable decline within what is described as students' *conceptual knowledge* within the topic of number. Tasks measuring students' understanding of the positional system indicate a slight improvement. Also when students were asked to choose the right operation of calculation, there was an improvement. These findings may be seen to reflect that L97 focuses more on conceptual knowledge than curricula prior to L97. Concerning dynamical geometry there was a better performance in 2001 than in 1995. L97 is focusing on this part of the geometry to a greater extent than earlier curricula.

Performances from 1994 and 2002 for students in 9th grade, were compared with especial focus on skills, understanding of numbers, calculations, statistics and probabilities (Alseth et al., 2003). A decline was found with regard to numbers and calculations while no significant change was found with regard to statistics and probabilities. Most of the decline with regard to numbers and calculations was in connection with carrying out procedures.

My study and research questions

My first intention when applying for a scholarship to undertake a doctoral study was to investigate students' learning outcome and attitudes to mathematics after the implementation of L97. At that time (2001/2002) I was involved in the research project referred to above (Evaluation of the reform). Finding that students performed generally lower made me wanting to find out how the teachers interpreted the reform and how it was implemented in the classrooms.

From 1997 until 2003 I was much involved in competence building of teachers. To ensure that students would receive an education compatible with the curriculum, plans for in-service-training of teachers and guides or instructions for how to implement the curriculum were developed (Kirke- utdannings- og forskningsdepartementet, 1997). On request from the Ministry of Education I was involved in the development of the "Plan for competence building in mathematics" (Brekke, Alseth, Botten, & Kleve, 1999). This plan was meant to follow up and further develop the plans for in-service training and to support mathematics teacher educators in developing and carrying out in-service training courses for mathematics teachers, and to have a long term perspective.

Part of my job as a teacher educator at that time was to organise, develop and teach in-service-training courses for mathematics teachers. The in-service-training courses I arranged had mainly a character of gathering several teachers, either at the teacher training college or locally in a community. Several activities mostly in small groups took place. I experienced that what most teachers wanted to gain from these courses, were methods and tips to carry out the recommendations of the curriculum. If I presented a certain investigative activity, some of the teachers went back to his/her class and carried out that activity right away. The feedback they gave me was that their students enjoyed such activities. What seemed to be less apparent was what learning outcome the students would have, even when that had been stressed and reflected upon in the course. Some teachers never tried out new activities. They participated in the course because they had to and stated clearly that they had no intention to change their teaching practices. Yet others reflected on the new ideas and adjusted them with regard to their own class and subject content. Soon I wanted to find out what was happening in the classrooms.

Thus my experience from the work with competence building of teachers together with the findings about the decline in students' achievement in mathematics referred to above, kindled my curiosity about how teachers interpreted the intended curriculum and how they implemented it in their classrooms. That created some of the background for wanting to carry out this research. Based on my long experience from mathematics classrooms, both as a mathematics teacher and as a teacher educator, I assert that it is easy to provide whole classes with drill and practice (traditional teaching). More challenging, or if at all possible, is to elicit genuine student thinking through investigative activities in a whole class (as encouraged in L97).

In my research I have worked with four mathematics teachers to explore how they responded to the curriculum, both how they interpreted it in terms of their thinking about it and expressing themselves through self estimation, in focus groups and interviews, and also how they implemented it in terms of what they did in the classroom, their instructional practice. As a preliminary outline the figure below illustrates this:

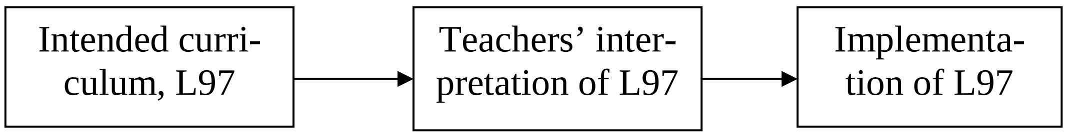

Figure 1, Levels of curriculum

These are the levels of the curriculum I have addressed through my doctoral work. This figure can thus be looked upon as a skeleton of my thesis, and is elaborated in Chapters 3, 4 and 5. My research questions are:

- How are teachers in their mathematics teaching practice responding to the L97's recommendations?
- What kinds of teaching practices are observable in the mathematics classroom?
- How are teachers' practices related to their beliefs about teaching and learning mathematics?

In the third research question I have introduced the term "beliefs". I see teachers' implementation of the curriculum being influenced both by their interpretation of the intended curriculum and also by their beliefs about teaching and learning mathematics. According to Thompson (1992) the study of teachers' beliefs is important in understanding how the teachers interpret and implement curricula. The relation between their expressed beliefs and their instructional classroom practice is important to consider in obtaining as broad a picture as possible of how teachers are responding to a curriculum reform.

In this part of the chapter I have offered an account of the Curriculum L97 as a written document to inform the rest of my study. To further inform my study, in the next chapter I will present a review of literature on research about mathematics teachers' teaching practice in connection with curriculum reforms and literature about how teachers' beliefs are influencing their teaching practice. Also in other countries than Norway there have been educational reforms in mathematics throughout the last 10-20 years which have been influenced by the Cockroft Report in England and Wales (Cockcroft, 1982) and the NCTM standards in the US (NCTM, 1989).

Relevant literature presented in the review is to a great extent related to the NCTM standards and associated teaching materials or to the Cockroft report. No similar studies in Norwegian classrooms as the one I have carried out have been done earlier. The findings from the review of literature both inform and support the study I have carried out. Many of the studies reported from the literature are about changes in teachers' instructional practices in connection with reform curricula. My study is not about teachers' change, since I did not study the teachers before L97 was implemented. Rather, through my work I want to guide the reader from the intended curriculum, which I have presented in this chapter and of which I will offer a theoretical analysis (in Chapter 5), through a study of teachers' teaching of mathematics related to the curriculum in their classrooms.

I have studied the teachers in focus groups, in individual conversations, interviews, and in their classrooms. I will point out differences between aspects of the curriculum as presented in the written documents and the ones taking place in the classrooms which are influenced by the interactions between teacher and students, among students and by the materials used. The latter is "the enacted curriculum" a term I will elaborate further later in the thesis. Through the study of mathematics teaching I will highlight factors influencing the course of the curriculum from the one as a written document to the ones enacted in the classrooms. Finally, from my study of the teachers' instructional practices and the enacted curricula, I will draw links back to the intended curriculum L97, by presenting some findings related to my research questions.

Overview of the thesis

The thesis has nine chapters. In this chapter I have presented an account of the curriculum L97, both how it is intended and how it has been evaluated, and I have illuminated the mathematical part through interviews with two of the members of the committee developing the written mathematics curriculum.

Chapter 2 is a review of literature on research about mathematics teachers' teaching practice in connection with curriculum reforms, and also a review of belief research in mathematics education. This review informs my study, and is organised in a way which addresses my research questions.

Chapter 3 concerns methodological issues where I address the methodological considerations in undertaking a qualitative study taking a case study approach, and the use of grounded theory analysis. I discuss theoretical perspectives in my study, including aspects of constructivism and socio-cultural theories and I offer a rationale for use of both theories to illuminate and inform the emphasis on individual and social perspectives on teaching and learning in my study.

In **Chapter 4**, Methods, I present the research design, the process of selecting teachers for my study and research methods used. I also give an account of the analytical process. The issue of trustworthiness is addressed in this chapter in relation with the presentation of the multiple methods I used. In the last part of this chapter I offer a discussion of ethical aspects in my research.

Chapter 5 is an introductory analysis chapter meant to inform and highlight key issues in my study and to form a bridge to the analysis of the three teachers and their teachings. I start by discussing aspects of mathematical knowledge which I apply together with the theoretical perspectives discussed in Chapter 3, in analysing L97. I present the analysis of focus group interviews, discuss how I see teachers' beliefs influencing their teaching practice and I discuss patterns of discourse in the mathematics classrooms which I use in the analysis of the teachers' teaching practice in the next chapters.

In the **Chapters 6, 7, and 8** I present my analysis relating to three teachers, Bent, Cecilie and David, their beliefs and classroom practices. Each of the chapters has three parts; analysis of the conversations with the teacher; analysis of his/her classroom practice, and a final part in which I offer a portrait and characteristics of the teacher's teaching. In the first of these chapters, which is the chapter on Bent, I give an account of how the categories used in the analysis of the three teachers developed.

Chapter 9 is the concluding chapter. In this chapter I present the constraints influencing teachers' implementation of a curriculum reform and how the enacted curricula differ from teacher to teacher and thus how students in different classrooms are provided with different opportunities for learning. In this chapter I present some findings related to my research questions in attempting to draw attention back to the intended curriculum L97. Finally, I discuss some strengths and limitations in my study and indicate some consequences my findings can have for mathematics teacher education and further research.

2. Literature Review

As indicated in the previous chapter, the background for my research was formed by: studies of students' mathematical performance before and after L97 was introduced; my experience from in-service-training courses with teachers in connection with the implementation of the L97 reform; and suggestions from an evaluation report of L97, that L97 was not implemented as intended. Starting with the first two research questions, *"How are teachers in their mathematics teaching practice responding to the L97's recommendations?"* and *"What kinds of teaching practices are observable in the mathematics classroom?"* I have searched for literature on research about teacher's teaching practice in connection with curriculum reforms. A review of literature about teachers' teaching practice in relation to reform movements makes up the first part of this chapter.

Teachers' beliefs are part of my third research question: *"How are teachers' practices related to their beliefs about teaching and learning mathematics?"* My focus here is on the teacher's role in the classroom and ways in which this relates to his or her stated beliefs. Thus the second part in this chapter is a review of literature on teachers' beliefs about teaching and learning mathematics.

Curriculum reform and teachers' teaching practice

Reform effort in mathematics education has its roots in the 1980s and 1990s when many countries reported a growing crisis in education and especially within education in mathematics and science. As referred in the previous chapter, and was the case for Norway, the TIMSS studies had pointed out differences across countries and thus been influential in making politicians be aware of and looking critically on the curriculum. Reform curricula have been developed in many countries which were inspired by the English *Cockcroft Report* (Cockcroft, 1982). The recommendations of the Cockcroft Report were widely accepted in both Great Britain and abroad. The curriculum content in the report was broad. It contained both a list of basic mathematical competencies which students were supposed to achieve and objectives concerning understanding and processes as well as skills were expressed (Millett & Johnson, 1996).

Reform curricula were also inspired by the *Curriculum and evaluation standards for mathematics* (NCTM, 1989), *American Professional Standards for teaching Mathematics* (NCTM, 1993) and later the *"Principles and Standards for school mathematics"* (NCTM, 2000) in the US. When the curriculum and evaluation standards came in 1989, NCTM called for proposals for curriculum materials which should reflect the visions in the "Standards", both with regard to the mathematical content and pedagogy. These instructional materials are the "Standards-based" materials from which American school children have been studying mathematics. Many studies have been carried out which report on how students' performance using "standards-based" materials differed from that of students using more traditional materials (Senk & Thompson, 2003). Much of the literature reported is from studies about how teachers in the US have responded to the curriculum and teaching materials developed according to the NCTM standards of which L97 was inspired.

Teachers' roles in the developmental process of a new curriculum have been subject of research, both in terms of what teachers express verbally about the reform and also if and how their teaching practice has changed. The literature on studies of teachers' teaching practice in relation to curriculum reforms reports both qualitative and quantitative studies and research methods varying from large scale surveys to small case studies. The studies

- describe a curriculum reform and focus on associated teaching materials' potential for teaching practice;
- identify obstacles, constraints and issues in teachers' decision making when implementing a reform;
- point to variation among teachers in how they respond to new reforms and implications for teachers' development.

I have organised the literature review on teachers' teaching practice according to the bullet points above. Thus these are issues discussed in the review which are related to my first two research questions. How findings in research more widely relate to findings in my research is indicated in the final chapter.

First however, under separate headings, I will refer to how the international TIMSS study, which is the largest international comparative study carried out in mathematics, created the background for large scale studies about teachers' reactions to reform movements. One of the main outcomes of the TIMSS study was the comparison of students' performance in mathematics across the participating countries. Taking the results from the TIMSS study as a starting point, Stigler and Hiebert (1999) in 1995, carried out a study of lessons in Japan, Germany and US. Based on videotaped lessons from randomly selected eighth-grade classrooms in the three countries, they described and compared mathematics teaching in the three countries. The authors revealed that there were differences in teaching practices within each culture and that the differences across cultures were enormous. They claimed that in America there is no system of improving teaching, they are always reforming, and that reforms in educational systems do not necessarily imply improvement. Furthermore they claimed that the issue that matters is *how* teachers can improve their teaching, because teaching might be an even more influencing factor on students' learning than earlier studies have suggested. Taking the Japanese "Lesson study" as a starting point they suggested a plan for how to improve "the act of teaching" in the US.

In their article "Improving Mathematics Teaching" Stigler and Hiebert (2004) argued that studying lessons from different cultures on a video gave both the researchers and the teachers ideas how to teach mathematics alternatively. Through their studies of the videos from 1995, they found that although teachers in all three countries said that they had read the curriculum documents and that they used the reform ideas in their teaching, there was great unevenness in how the reform ideas were interpreted among the teachers. Their studies revealed little evidence that the teachers' classroom practice reflected the goals of the reforms.

Expanding on the first, a second video study was carried out in 1999 (Stigler & Hiebert, 2004). Seven countries participated in this study: the US, Australia, the Czech Republic, Hong Kong, Japan, the Netherlands and Switzerland, of which the US was the country having performed the lowest on the TIMSS 1995 mathematics achievement test for 8^{th} grade. From this study Stigler and Hiebert concluded that the way in which teacher and students worked in the classroom was the most important issue. One of the main findings was that high achieving countries as Japan and Hong Kong did not transform what they called "making connections problems" into "procedural exercises" as most teachers in the US did.

This was also emphasised by Jacobs, Hiebert, Givvin, Hollingsworth & Wearne (2006) who focused on *teaching,* which they termed "one key element of classroom practice" (p.5), in the US through studying videos from the TIMSS 1995 and 1999 video studies. They wanted to find out if and how teachers' teaching practices had changed from 1995 to 1999. In studying the videos from classrooms in the US, they compared their findings with the recommendations of "Principles and Standards" (NCTM, 2000) and examined to what extent the mathematics teaching was more like that of traditional teaching than of the kind promoted in the Standards. Throughout the video study teachers who were participating had been given questionnaires for the purpose of investigating their knowledge of teaching and their perception of how they implemented the standards and "current ideas" from "Principles and Standards" in mathematics teaching and learning. In the samples from both 1995 video study and from 1999 video study a considerable number of teachers reported "some degree" of familiarity with current ideas from the principles and standards. In the questionnaire significantly more teachers in the 1999 study reported "a fair amount" or "a lot" in their answers of "To what extent do you feel that the lesson you taught today is in accord with current ideas about the teaching and learning of mathematics?" Thus the results from teachers' questionnaires gave the impression that a majority of teachers in the US were teaching in line with the ideas in the NCTM standards and the teachers reported that the videotaped lesson illustrated these ideas. However, the researchers found that the video-data allowed them to go beyond the teachers' self reports from the questionnaires. The videos were coded according to the recommendations in "Principles and Standards" with regard to: Problem-solving, Reasoning and Proof, Communications, Connections and Representations. All in all they found that classroom practice was not consistent with the Standards and that "the typical eighth-grade classroom displays teaching at odds in many respects with the recommendations" (p.28). They claimed that the changes they found in the US mathematics classrooms were at the margin, meaning that the features from "Principles and Standards" were implemented at the margin of the teaching rather than its core.

These studies demonstrate findings that teachers' teaching practice was different from what the teachers had reported. The relation between what the teachers said about their own practice and what I observed them do in the classroom is addressed in my 3rd research question.

Curriculum reform and associated teaching materials

Reform curricula challenge a long tradition in mathematics teaching which has been teacher centred and has been focusing on students exercising skills and procedures (Senk & Thompson, 2003).

It has been pointed out that it is not always inevitable that curriculum materials as such are sufficient to implement a new reform (Remillard, 2000). However, many researchers who have been investigating how reform curricula have been implemented have suggested potential for enhancement of teachers' teaching practice through the use of reform materials. Reys, Robinson, Sconiers & Mark (1999) suggested rather to focus on the potential in the curricula and associated teaching materials in the participating nations in the TIMSS study than on the comparison of students' achievements across the nations. Suggesting that the NCTM standards documents in the US provide specifications that call for change in current practice in the US, they outlined common features in different curricular interpretations of the NCTM standards within the US. The features in these interpretations were consistent with features in curricula in countries where students performed well on the TIMSS testing. By highlighting these common features they emphasised the potential the NCTM standards, both the curriculum and associated teaching materials, had to create change in current teaching practice in the US and for better learning for students.

Also discussing features in the NCTM standards, Trafton, Reys & Wasman (2001) identified common characteristics of NCTM standard based mathematics curriculum materials: *Comprehensive*, that there is a focus on core mathematics for all students; *Coherent*, that the core ideas are seen as an integrated whole; *Depth*, that they develop ideas in depth; *Sense making*, that they increase the ability to learn, remember and use mathematics; *Engage students* and *Motivate learning*. By emphasising these characteristics, which they regarded as important aspects in mathematics, they showed how they saw the potential for teacher development and thus for students' learning of mathematics through the use of NCTM standard based materials.

Attention has also been drawn to the educative potential in reform curriculum materials. Such materials have shown to be effective in teachers' development both when used in in-service-training and in pre-service courses for teachers. Lloyd (2002) reported how teachers both in pre-service training and in-service training experienced their work with reform curriculum materials, and how the materials had the potential for influencing teachers' beliefs about teaching and learning mathematics, about pedagogy and about the curriculum. In accounting for how the use of curriculum materials could increase professional development through the teachers' reflections on their use of such materials she claimed that "curriculum materials can provide the basis for conceptual exploration and knowledge" (Lloyd, 2002, p. 156). She showed how two teachers, with different conceptions about the reform curriculum who used the same kinds of curriculum materials in their teaching, developed deeper understanding of both traditional mathematics teaching (of which they had long experience) and of reform oriented teaching.

Collopy (2003) focused on the educative potential in curriculum materials indicating that such materials can be effective development tools for mathematics teachers. Through her use of interviews and observations of two upper elementary teachers' mathematics lessons she reported on the teachers' learning through their use of potentially educative mathematics curriculum materials. However, based on the findings from her study of the two teachers Collopy suggested that reform materials had shown to be effective for some teachers' development but not for all. One of the teachers she studied changed her teaching practice towards a more reform oriented direction, whereas the other teacher did not demonstrate any change: "The study cases illustrated the teachers' dynamic and divergent nature of opportunities to learn through reading materials and enacting lessons" (p. 287).

Since reforms in mathematics focus less on exercising skills and procedures and more on students' own investigations and explorations, group work and explanations of which both prospective teachers and teachers in schools have little experience themselves, the use of reform materials has been shown to be demanding and also time consuming for teachers. Spielman & Lloyd (2004) found that prospective teachers who had been using reform oriented curriculum materials valued group work and students' own investigation which are encouraged in reform materials over exercising procedures which is more common in traditional teaching. Therefore the use of such materials can be of decisive importance in teacher education for the purpose of preparing prospective teachers for teaching according to reform materials.

The studies reported here suggest that reform curricula challenge traditional conceptions about mathematics teaching, and that there is a potential for enhancement of teachers' teaching practice through the use of reform curricula teaching materials. Teaching materials as the kinds related to the NCTM standards are not developed accordingly in Norway with regard to L97. The teachers have the L97 written curriculum, in which working methods, mathematical focus and aims are described, and they have associated textbooks. However, the features described in the NCTM standards which are found to be corresponding to features in curricula in countries where students performed well on the TIMSS tests, are also found in the L97 curriculum.

Obstacles Constraints and Issues in teachers' decision making

In Cyprus, Christou, Eliphotou-Menon & Philippou (2004) pointed out evidence suggesting that educational reforms are not implemented within the timescale planners and policymakers have envisaged. Many issues and obstacles influence the teacher's decision making in implementing a curriculum reform and using reform materials in the teaching. There is a long way from the intended curriculum written in the curriculum documents to the teacher's teaching practice, and it is not sufficient only to place innovative materials at teachers' disposal for teachers to change (Manouchehri & Goodman, 1998).

The teacher's teaching practice – as it turns out in the classroom – has been an important subject of research in investigating how teachers have been responding to curriculum reforms. This part of the curriculum, the *enacted curriculum*, which is what actually takes place in the classroom (Ball & Cohen, 1996) is the curriculum jointly constructed by the teacher and the students and the teaching material (Remillard, 1999, 2005; Ross, 2003; Spielman & Lloyd, 2004; Tarr, Chávez, Reys, & Reys, 2006). Ross (2003) used "narrative inquiry" in her study carried out in Canada. The experience in the classroom became a focus in her research when she came to understand the curriculum as lived in the classroom, the enacted curriculum, as the interplay between the teacher and the students more than as written in the curriculum documents that come from outside the classroom.

There are many constraints and issues between the intended curriculum and the one enacted. The processes teachers have to go through from reading the curriculum documents (the intended curriculum) to their classroom practice (the enacted curriculum) have been demonstrated as complex in nature. The arenas in which teachers have to engage in this process are both "*designing*" (selecting and designing mathematical tasks), "*constructing*" (enacting the tasks and responding to students' work with them) and "*curriculum mapping*" (organisation of the entire curriculum and weekly and daily planning of lessons) (Remillard, 1999). Even when teachers have expressed their agreement in the principles lying behind the reform, the actual classroom practice, the enacted curriculum, turned out to be traditional in style (Broadhead, 2001; Norton, McRobbie, & Cooper, 2002).

Broadhead (2001) examined constructs from the Norwegian L97 more closely by interviewing those responsible for designing the subject-related curricula in all subjects and she also used teacher questionnaires. She recognised the difficult relation between the ideological underpinnings in L97, which are based on constructivist theory, and the reality of practice. As challenges she identified thematical work encouraged in L97, the extent of content associated with each subject, the weight L97 puts on active and participative learning, and also the fact that students in the same class have different abilities. Based on the questionnaires she found that teachers did not have any problems accepting these challenges, however, it was in their actual practice in the classroom that difficulties with L97's intentions occurred.

The extent of content to be covered in the curriculum and diversity in abilities among students in the same class were also reported as obstacles by Norton et al. (2002). According to their interpretation, the principles lying behind the reform they studied were also based on constructivist theory. They investigated the relationship between teachers' goals for their students and their actual teaching practices. For the less able students seven out of nine teachers in their study had "calculational" goals and eight out of nine had "show and tell" as the most preferred pedagogy for these students. For the able students all teachers had conceptual understanding as a goal, and three teachers had "show and tell" as the preferred pedagogy, three had "explain" as the preferred pedagogy and three had "investigative" as the preferred pedagogy for the able students. This study showed that teachers interpreted the curriculum differently according to their perceptions of their students in terms of focusing more on traditional goals and traditional style of teaching of less able students.

Most teachers have long experience with traditional mathematics curricula, both as students and also as teachers. This is the kind of mathematics they know well, and feel comfortable with. Williams and Baxter

(1996) reported that even when a teacher at first had made progress towards a reform, s/he returned quickly back to more traditional teaching. Prawat (1992) also reported that desired changes in classroom practice failed to appear although the teacher demonstrated willingness to teach more experimentally.

Traditional beliefs and practices regarding school mathematics are challenged by reform oriented curricula, and teachers' deeply held beliefs can serve as obstacles in implementing new reforms. The most established beliefs about mathematics are of a narrow view on the subject which focuses on memorisation of facts and formulae and mastery of procedures and the notion that students are best served by tracking or being grouped according to abilities (Reys, Reys, Barnes, Beem & Papick, 1998). In the previous section I referred to Lloyd (2002) who reported how experience with curriculum materials has the potential to change teachers' beliefs about teaching and learning mathematics. She discussed teachers' beliefs about teaching and learning mathematics and how they were related to "inquiry mathematics" which is mirrored in recent efforts to improve K-12 mathematics education in the US. She suggested that because teachers' views on and beliefs about mathematics often are of traditional school mathematics and that teachers have not experienced mathematics that departs from the traditional school mathematics, there are obstacles for teachers to implement "inquiry mathematics" in their own classrooms, even when they are supported with specially designed curriculum. Manouchehri & Goodman (1998) also pointed to teachers' beliefs as a factor constraining how reform curricula are implemented. They carried out ethnographic research to study the implementation of standards-based curricula and found that the ways in which the different teachers valued and implemented reform programs were highly influenced by what the teachers knew about mathematics, by what they knew about pedagogical practices and by their personal theories of teaching and learning mathematics. Their findings suggest the importance for teachers to have concrete images of what it is like to teach in accordance with reform movements. This was also emphasised by Smith Senger (1998/1999) who claimed that if such images were not present, efforts in implementing a reform were in vain.

Obstacles in implementing curriculum reforms are not always found in teachers' stated beliefs or in their beliefs in practice. Obstacles and constraints are reported as lying outside the teacher such as other colleagues, the school's discourse and parents' expectations, perceptions and concerns. Parents often question changes in curricula, and an outspoken group of parents, even if the group is small, "can serve as strong catalysts or formidable obstacles to reform at the classroom, school and districts level" (Reys et al., 1998, p. 45). Parents' resistance to change was yet another obstacle together with lack of time for planning and carrying out the activities suggested in the reform, difficulties in establishing a balance between teaching for mastery of basic skills and development of conceptual knowledge reported by Manouchehri & Goodman (1998).

Having considered teachers' beliefs and obstacles lying outside the teacher's control such as parental influence, there is still more to take into account for a successful implementation of the reform. The actual classroom context and the students' contributions during the lesson are of decisive importance for how the enacted curriculum turns out. Skott (2004) described changes in teachers' roles in mathematics classrooms in view of reform initiatives in school mathematics. He suggested that reform movement in mathematics education entails an apparent contradiction which reflects that "classroom practices compatible with the reform insert a certain planned unpredictability in the teaching-learning processes" (p.239). The expected classroom practices are formulated outside the classroom and the teachers are not offered a set of well defined methods to carry out the reform's expected classroom practices. Skott (2001a) described a novice teacher whose images of mathematics teaching were influenced by reform movements, how the teacher coped with the complexity of the classroom and how the complexity influenced the teacher's decision making. Skott introduced the construct "critical incidents of practice" (CIP) on the basis of how the teacher coped with the often conflicting demands in the classroom. He described the teacher's priorities in relation to mathematics, mathematics as a school subject and to teaching and learning mathematics and discussed a novice teacher's school mathematics images and his classroom practice. Skott found that the relationship between the two was very different in different situations and his findings indicated that Ernest's (1991) "institutional or contextual constraints" play different roles in different situations, even within the same classroom.

The actual classroom context was also pointed out as a constraint in Remillard's (1999) study. She carried out a case study of two teachers in elementary school and their use of the same reform-oriented material in the process of enacting the curriculum and studied the relations between the teacher, the textbook and the enacted curriculum. Due to the two teachers' different beliefs about mathematics and the teaching of mathematics, she found that they selected mathematical tasks for the lessons differently and different school and classroom contexts were factors pointed out as contributing to the teachers' beliefs and thus their actual classroom practice.

Variations among teachers and implications for developmental programs for teachers

In the previous sections, I have presented how reform curriculum materials have been found to serve as potential for teachers' development both in connection with in-service training and pre-service training and I have pointed out obstacles and constraints that reform materials do not automatically serve as potential for teachers' development. The picture of how curriculum reforms have served as potential for teachers' development and their teaching practice is multifaceted. In the literature, there is reported great variation in how reform materials have been implemented, even among teachers in the same school (Kilpatrick, 2003b; Stigler & Hiebert, 2004; Tarr, Chávez, Reys, & Reys, 2006). The teachers' (different) expectation of the curriculum materials has been presented as one possible explanation for the divergence in their responses to the reform ideas (Collopy, 2003). Rowan, Harrison & Hayes (2004) suggested that there exists an implicit and not well defined curriculum which is organised by deeply held beliefs about what is good mathematics teaching. These beliefs are fuzzy and are enacted differently by different teachers in US schools. Rowan et al. suggested these kinds of beliefs together with teaching at all grades being to some extent still teacher directed and follow a traditional pattern, as possible explanations for variations in curriculum coverage and teaching style both across schools and among teachers in the same school. (A review of literature on the role of teachers' beliefs in relation with their teaching practice is presented in the next section.)

How best to implement the NCTM standards and other curriculum reforms has been part of the outcome of research within the field. A common thread in these research recommendations is the emphasis on the teacher as the crucial factor in reform and to invest resources in teacher development. Based on the view that teaching is a cultural activity and the issue how teachers can improve their teaching, Stigler and Hiebert (1999; 2004) suggested that the teachers are the key for improving teaching, and that the crucial issue to improve students' learning is improvement among "average teachers". They claimed that teachers need theories, empirical research, and alternative images of what implementation looks like and suggested for teachers to share their knowledge and collaborate with each other.

Lloyd (1999) discussed how knowledge about similarities and differences in teachers' implementations of curriculum reforms could create the background for programme for teachers' development. Remillard (1999) presented a model suggesting that students' meetings with a new curriculum are mediated by a variety of teachers' decisions. Therefore she requested curriculum developers' attention to the teacher's role in developing curriculum which again requires knowledge of the arenas she outlined about the curriculum development and the factors that influence teachers in their decision makings. Also Broadhead (2001) emphasised that teacher development has to be sensitive to the issues that teachers are practicing in an actual classroom every day in their working life.

Based on their findings, Jacobs et al. (2006) suggested that recommended changes within mathematical education must address the realities of current practice. Realising that the implications of the NCTM standards for teaching are neither trivial nor quickly attained Trafton et al. (2001) suggested that continual ongoing support for teachers is necessary. Teachers have to work through the curriculum material, they have to confront issues with new teaching strategies and they will also have to develop their own mathematical knowledge. They therefore pointed out the importance of developmental programme for teachers. Collaboration among teachers in the same school or within the same school district has been suggested as part of such developmental programme. Reys & Reys (1997) claimed that to be successful in reforms which are intended to lead to increased knowledge, skills and understanding among students in mathematics, one has to start with the teachers. They suggested "Collaborative curriculum investigation" as a vehicle for teacher enhancement and conducted a project which supported teachers and administrators of schools in their implementation of the NCTM curriculum reform. Pointing to the potential the NCTM standards create for change, Reys et al. (1999) emphasised the importance of informing teachers and school administrators about the unique characteristics in reforms so they could learn about the new mathematics curricula, explore their implications for students' learning of mathematics in making curricula which best suit its needs.

Lloyd (2002) drew attention to and discussed how teachers' experiences with curriculum materials could be part of an educative process, eventually changing teachers' beliefs and developing reform oriented beliefs about teaching and learning mathematics. She suggested emphasis on professional development among teachers by using curriculum materials for the teachers to more effectively use textbooks and resource materials to teach both themselves and their students in the future. Spielman and Lloyd (2004) concluded that when prospective teachers learn mathematics in reform guided classroom settings, their instructional practice supports current reform efforts. The importance of the context in which the teaching of mathematics takes place was also emphasised by Remillard (2000). She claimed that curriculum materials and teacher development opportunities do not operate in isolation, but that the materials' effect on teaching is mediated by the teaching context. She found that teachers engaged in three types of learning when implementing a new curriculum: exploration of the content in the preparation to teach, investigating ideas underlying students' confusions and engaging in mathematical discussions with students. Thus the introduction of reform-oriented textbooks must occur together with supporting "teachers as curriculum developers rather than implementers" (Remillard, 2000, p. 348).

Although many teachers and educators have expressed their support for curriculum reforms, implementing curriculum reforms is a slow process. Smith Senger (1998/1999) suggested that it was not sufficient for teachers to verbalise reforms well without acting on them. Her study revealed that there was something more involved while teachers move towards a change. She investigated three teachers' inner reflections and exterior manifestations and how they struggled with issues of reform on the one side and traditional teaching on the other side. None of the teachers in her study changed drastically, but all of them demonstrated promise of long term change through their own reflections on current practice and practice recommended in the reform.

Christou et al. (2004) explored Cypriot teachers' concerns regarding the implementation of a reform curriculum and to what extent teachers' concerns varied according to their overall teaching experience. They found that overall most teachers accepted the decision to proceed with the reform curriculum and that they were more concerned about the planning, organising and time demands than about students' mathematical learning outcome and about their own ability to implement the new curriculum. However, teachers with not so much teaching experience seemed to view the latter as the most crucial factor (Christou et al., 2004). They thus emphasised the importance of paying attention to teachers' concerns and experiences when implementing a new reform, and they put the responsibility on educational leaders and policymakers to acknowledge and identify the concerns of teachers in the process of implementing curriculum reforms. They suggested that their findings could be used to inform planning and implementation in-service training of teachers.

Teachers' beliefs about teaching and learning mathematics

In my study I am investigating how teachers are implementing a curriculum reform and how teachers' practice in the classroom is related to their beliefs about mathematics and their teaching of mathematics. In mathematics educational research many studies have been carried out to investigate teachers' beliefs. I have reported research which has shown how teachers' work and experiences with innovative curriculum teaching materials relate to and can change teachers' beliefs and thus suggested a need for greater attention to teachers' beliefs about mathematics curriculum (Lloyd, 2002; Manouchehri & Goodman, 1998; Reys et al. 1998; Rowan et al. 2004). The reason why teachers' beliefs have been interesting to study as part of research into links between curriculum and teaching is the assumption that to understand teaching from teachers' perspectives, researchers have to understand the beliefs with which teachers understand their own work (Thompson, 1992). According to Thompson (1992) the study of teachers' beliefs or conceptions has made mathematics educators become aware of how teachers' beliefs about teaching and learning mathematics influence how they interpret and implement curricula. Teachers' beliefs, conceptions, cognitions or images are mental objects that cannot be seen from outside, so the challenge for a researcher is how to get insight into those objects (Thompson, 1992; Wilson & Cooney, 2002).

"Belief system" has been used in the research literature about teachers' beliefs as a metaphor to describe how an individual's beliefs are organised (Thompson, 1992). According to Scheffler (1965) a "belief is a cluster of dispositions to do various things under various associated circumstances" (p.85). Eisenhart, Shrum, Harding & Cuthbert (1988) described a belief system as a set of non contradictory beliefs which limit dissonance, contradiction and chaos. They claimed that if teachers shall change their practice, the desired change has to be related to teachers' beliefs about teaching and learning mathematics. According to Eisenhart et al. (1988) educational reform programs should take teachers' existing beliefs into account because educational reform programs "are unlikely to accomplish their goals unless they are first made compatible with or translatable into existing belief system" (p.52). This indicates that teachers' beliefs about teaching and learning mathematics will have implications for success or failure of educational reform programs.

Knowledge and beliefs

In the research literature in mathematics education about knowledge and beliefs, there seems to be diversity of views and approaches (Furinghetti & Pehkonen, 2002). Furinghetti and Pehkonen suggested that one reason for this diversity is that not all researchers take the distinctions between knowledge and beliefs seriously and that some argue that it is not so important to make this distinction. When talking about teachers' beliefs as premises for their classroom activities, it is difficult to distinguish between their knowledge, both their mathematical knowledge and pedagogical content knowledge, and their beliefs. I see a person's knowledge and his/her beliefs as intertwined. However, before I go further into theories about teachers' beliefs, I will discuss how a distinction between knowledge and beliefs, at least linguistically, can be made according to various sources.

The main difference between knowledge and beliefs is that beliefs can be held with different degrees of conviction and that one person's beliefs can differ from another person's beliefs and both can accept that. According to Wilson and Cooney (2002) a belief can be a statement of intent, "I believe I'll go to the movies" (p. 129). It can also be a claim based on evidence (a thermometer), "I believe it is cold outside" (p.129). Beliefs are often characterised by lack of agreement over how they can be evaluated or judged which means that one person's beliefs can be contradicted by that of another person (Thompson, 1992). Saying "I know" is an assumption that might have been verified by others. Thus a claim about knowing is stronger than a claim about believing. According to Thompson (1992) the degree of inter-subjective consensus and the types of arguments needed for acceptance distinguish knowledge and beliefs.

Wilson and Cooney (2002) who were working within a constructivist paradigm suggested that the borderlines between knowledge and beliefs have been "blurred" as a consequence of trying to understand what is lying behind a teacher's change. They suggested that as a reason why some researchers tend to use the terms cognitions or conceptions. According to Wilson and Cooney (2002) believing is a necessary but not sufficient condition for knowing and although beliefs sometimes may be associated with a cognitive component, it is a weaker condition than knowing. As Wilson and Cooney (2002) also pointed out, knowing is usually associated with truth and certainty, and has often independent factual references whereas beliefs are associated with doubts and disagreements. From a constructivist perspective we never know the reality as such and the notions of truth and right and wrong, and thus "knowing", become problematic issues. However, the notions fit and viability can replace the traditional conception of truth. The introduction of the concept of viability, which for constructivists means that concepts, models and theories are viable if they prove adequate in the contexts in which they were created, replaces the conception that there is only one ultimate truth describing the world. Constructivism and constructivist notions are discussed in the section about theoretical perspectives underpinning my study in the next chapter (page 53).

Leatham (2006) avoided linking knowledge to the traditional notion of truth and accounted for knowledge in terms of beliefs; that knowledge is stronger that beliefs. By taking a socio-cultural position he proposed using "sensible systems of beliefs" as a theoretical and methodological framework in the study of teachers' beliefs. Within this system he made the following distinction between knowledge and belief:

> Of all things we believe, there are some things that we "just believe" and other things that we "more than believe - we know". Those things we "more than be-

> lieve" we refer to as *knowledge* and those things we "just believe" we refer to as beliefs (p. 92, emphasis in original).

Through this distinction Leatham said nothing about truth, and looked upon beliefs and knowledge "as complementary subsets of the set of things we believe" (p. 92).

Beliefs and practice

There have been reported varying degrees of consistency between teachers' conceptions of mathematics and their instructional practice (Thompson, 1992). There seems to be higher degree of consistency when teachers report traditional conceptions about mathematics and its teaching than when teachers report a more reform oriented view. According to Thompson (1992) this alerts us to an important methodological consideration:

> Any serious attempt to characterise a teacher's conception of the discipline he or she teaches should not be limited to an analysis of the teacher's professed views. It should also include an examination of the instructional setting, the practices characteristic of that teacher, and the relationship between the teacher's professed views and actual practice (p.134).

The importance of the relationship between teachers' teaching practice and teachers' conceptions about mathematics and mathematics teaching in connection with the implementation of a curriculum reform was also emphasised by Cooney (2001). He viewed teachers' teaching practices as highly influenced by their views about mathematics and mathematics teaching. He reported several studies on teacher's change and suggested that "teachers' conceptions about mathematics and mathematical teaching strongly influence if not dictate their movement toward a reform oriented teaching" (p.18). He also reported studies where change in beliefs occurred simultaneously with change in behaviour, or even that change in classroom practice preceded change in beliefs.

Relationship between teachers' thinking and teachers' teaching practice is also suggested through Jaworski's (1998) study. She emphasised the necessity of teachers' "cycles of reflective activity through which knowledge grew and was refined" (p.26). In her work with the teachers, issues that the teachers had not been aware of or not thought about in much depth were opened up. She linked her research to the teachers' actual concerns for students' mathematical understanding, and thus the developments of teachers' teaching practice occurred as a result of their own research on and questioning of their own teaching and through the cyclic processes of reflective thinking.

Skott (2001b) challenged much of the underlying rationale and premises lying behind research about teachers' beliefs and he questioned research which has as an implicit premise that a teacher's beliefs can serve as explanatory principles for practice. Being inclined to take more cultural factors into account he claimed that what the teacher does in the classroom makes sense for the teacher based on the multiple motives for the present action even if those actions may seem inconsistent for an observer. “Students’ and teachers’ actions do make sense, [ ], teachers cannot be inconsistent” (Skott, 2001b, pp. 6-7). Instead of assuming that there is something lying behind a teacher’s practice which is called a teacher’s beliefs, he looked upon the motives determining a teacher’s practice not as predetermined beliefs but rather as entities emerging from the interactions with the students in the classroom. This underpins how the socio-cultural complexity of the classroom plays a role in research about teachers’ beliefs. He introduced the notion SMI, School Mathematical Images, “to describe teachers’ idiosyncratic priorities in relation to mathematics, mathematics as a school subject and teaching and learning mathematics in schools” (Skott, 2001a, p. 6). He thus limited the types of beliefs in his study by not investigating unconscious beliefs but only teachers’ explicitly described priorities in relation to school mathematics from interviews and questionnaires. Skott (2001a) focused rather on consistency between those expressed beliefs than inconsistencies. When to the researcher, there seemed to be inconsistency between a teacher’s beliefs and his actual practice, Skott tried to make sense of the inconsistency rather than viewing the inconsistencies as something being wrong and needing to be fixed (Leatham, 2006).

Taking Skott’s (2001b) claim that “inconsistency is an observer’s perspective” Leatham (2006) accounted for the problem of consistence or inconsistence between a teacher’s beliefs and practice by the introduction of the beliefs as a “sensible system”. Viewing teachers’ beliefs about teaching and learning mathematics as a “sensible system” Leatham (2006) suggested interpreting teachers’ beliefs not as inconsistent with their actions in the classroom, but rather as systems where certain beliefs have more influence over actions than others. He exemplified this perspective by accounting for how a teacher’s beliefs about classroom management had more influence over her classroom practice than that of her beliefs about the effect of group work. Therefore the students did not work in groups although the teacher had expressed her beliefs about group work as an effective learning activity.

Wilson and Cooney (2002) pointed out logistical circumstances as one kind of constraints preventing teachers from acting according to their beliefs. As opposed to Skott’s and Leatham’s socio-cultural perspective in accounting for the problem of (in)consistency, Wilson’s and Cooney’s account was constructivist oriented. If a teacher said that he believed problem solving was the best way to learn mathematics, but problem solving activities were not observed in his class, this did not necessarily indicate what Skott (2001b) termed inconsistency. Rather, they said, it could be accounted for by suggesting that there was not a viable interpretation of what that teacher meant by problem solving or that the teacher's beliefs about problem solving were peripheral to other contrasting beliefs.

Goos, Galbraith & Renshaw (1999) emphasised the role of the teachers and their beliefs about mathematics. Teachers' beliefs influence the features of the classroom environment they create. However, they acknowledged that there exist constraints and pressures that may prevent teachers from acting according to their beliefs. They identified three core beliefs from interviews with their teachers;

1. Students learn mathematics by making sense of it for themselves;
2. Teachers should model mathematical thinking and encourage students to make and evaluate conjectures;
3. Communication between students should be encouraged so they can learn from each other, sharpen their understanding, and practise using the specialist language of mathematics

(p.51-52)

According to Goos et al. these features correspond with key aspects of the notion of zone of proximal development (into which I go further into detail in the next chapter), and they were consistent with the interactions in the classrooms they had observed.

Askew, Brown, Rhodes, Johnson & Wiliam (1997a, 1997b) identified three models or sets of beliefs in characterising the approaches teachers took towards the teaching of numeracy:

> Connectionist –beliefs based around both valuing students' methods and teaching strategies with an emphasis on establishing connections with mathematics;
> Transmission – beliefs based around the primacy of teaching and a view of mathematics as a collection of separate routines and procedures;
> Discovery – beliefs clustered around the primacy of learning and a view of mathematics as being discovered by pupils (1997a, p. 2).

A teacher with a connectionist orientation emphasised the complexity of mathematics and that students and teachers were collaborating and sharing ideas. A teacher with a transmission orientation emphasised the role of the teacher as the main source of knowledge, weight was put on delivering of "knowledge in its final form". A discovery orientation put emphasis for the students to discover and construct knowledge themselves (Askew, Denvier, Rhodes, & Brown, 2000). These models of teachers' beliefs support the analysis and findings in my research. How they relate to my findings is explained in the final chapter.

Before concluding this chapter and reflecting back on the literature on teachers' teaching practice and teachers' beliefs I will emphasise the problematic features in trying to weave together the perspectives outlined above. Some of the authors have written from a constructivist perspective whereas others have written in a socio-cultural perspective. These treatments of literature beg for some discussion of theoretical perspectives in relation to how I will deal with the data in my study. When studying L97 I have identified both constructivist elements as well as socio-cultural elements. In my research I am trying to make use of both theoretical positions. These theoretical positions are addressed in the next chapter.

3. Methodology

The purpose of my study is to investigate how mathematics teachers are responding to our current curriculum, L97, which became effective in 1997, both in terms of what they are saying about it and what they do in the classroom. My research questions in this study are:

- How are teachers in their mathematics teaching practice responding to the L97's recommendations?
- What kinds of teaching practices are observable in the mathematics classroom?
- How are teachers' practices related to their beliefs about teaching and learning mathematics?

In this chapter I start with some methodological considerations in carrying out a qualitative study, taking a case study approach and using grounded theory analysis. Then I discuss aspects of constructivism and socio-cultural theories before I offer a rationale for my use of both constructivism and socio-cultural theories to illuminate and inform the emphasis on individual and social perspectives on teaching and learning in my study.

Methodological considerations and a case study approach

What teachers say about a curriculum, about their teaching, what they say about their students, their abilities and how they learn, what they say about how they prepare lessons together with observations of their classroom practice are important perspectives from which to study a teacher's interpretation of a curriculum.

Other researchers such as Simon and Tzur (1999) have looked upon teachers' practices both as everything they do in the classroom and everything that contributes to their teaching combined with everything they say, think and know about what they do. In my study I rather have chosen to distinguish between what the teachers say about L97 and about their practice, *what they say*, and what I observe they do in the classroom, *their teaching practice*. I look upon both as important components of the teacher's interpretation of the curriculum. Through the analysis of the data collected about what they say and what they do, together with an analysis of the relation between the two, I will present characteristics of each teacher's teaching which, in each case provides an example of how a teacher interprets L97. It is this interpretation of the curriculum that is my focus, not the teachers themselves.

The background for this study, teachers' interpretation of a curriculum, is, as outlined in the introductory chapter, a comparison of students' performances in mathematics before and after the implementation of the curriculum, based on a quantitative study of their performances. The outcome of that quantitative study inspired me to carry out a qualitative study with an interpretative approach investigating how teachers interpret the curriculum. However, the purpose of this study is not to find causal explanations for why the students performed as they did, but rather to present a characterisation of the mathematics teaching practice seen from the perspectives of L97 and through the theoretical lenses on which it is based.

Choosing a qualitative study

To gain information about how teachers respond to a curriculum, carrying out a quantitative study using a questionnaire could have been one option. This would have made it possible to do a large scale survey with many teachers from different parts of the country. However, there were several reasons for me not to choose a quantitative study with questionnaire as a main research method. The role of theory in relation to quantitative and qualitative research is different in its principal orientation: in quantitative research theory is being tested and has thus a deductive orientation, whereas in qualitative research the orientation is inductive and theory is being generated. Quantitative research "embodies a view of social reality as an external objective reality", whereas qualitative research "embodies a view of social reality as a constantly shifting emergent property of individuals' creation" (Bryman, 2001, p. 20). My theoretical standpoint is the latter. I see the classroom as a *constantly shifting emergent property of individuals' creation* rather than as an *external objective reality*.

Other disadvantages of doing a large scale survey with a questionnaire were that it would have consisted of *my* questions and thus it could have been an assertion (or disproval) of *my* beliefs and attitudes; it would not give me the depth I wanted from the study; most questions would have had to be closed since people are reluctant to write much on questionnaires; on the open ended questions the teachers might have recalled episodes that would have been significant to them rather than representing a regular activity; and finally, responding to a questionnaire is rarely very binding for the respondent (Goodchild, 2002). I chose to give the four teachers in my study a questionnaire in addition to and as a supplement to the qualitative study I carried out, however, not to do a large scale survey (the use of questionnaire is further elaborated in Chapter 4 and Chapter 9). It is not my intention to dichotomise between quantitative and qualitative research paradigms. Educational research embraces a wide range of research questions, some of which require quantitative methods whereas others require qualitative methods (Pring, 2000).

I decided to carry out a case study with an interpretative approach and to use an "ethnographic style of inquiry" (Goodchild, 2002). The qualitative interpretative research paradigm works in an opposite direction from the positivist paradigm which adopts a top-down perspective, using the general to deduce predictions about particular instances. An interpretative research paradigm rejects the certainty the objectivists take for granted and explores the unique features and circumstances surrounding a particular case, exploring the richness that can serve as example of something general (Ernest, 1998b). The positivist research paradigm puts weight on prediction and control which in the interpretative paradigm are replaced by interpretative notions of understanding, meaning and action (Carr & Kemmis, 1986). The summaries of findings in qualitative research can be expressed as “fuzzy generalisations” (Bassey, 1999). My intention is from the outcome of the case study to make “fuzzy generalisations” meaning “based on the cases I have found” to say “in some cases it may be found” or: “I found that this teacher interpreted the curriculum such and such, others might implement it in similar ways”. Making fuzzy generalisation in educational research always involves an element of uncertainty, and it is a reminder that there are many variables that influence a teacher’s practice, and also students’ learning outcomes (Bassey, 1999).

Taking a case study approach

When starting my study, I was searching for illustrative cases of teachers’ interpretation of the curriculum which could serve as examples of something more general. A case study is “a research design that entails the detailed and intensive analysis of a single case” (Bryman, 2001, p. 501). A case study is a study of the singular but with attention to the importance of the context within which it acts (Bassey, 1999; Pring, 2000; Walford, 2001; Wellington, 2000). Yin (1994) emphasises that a case study investigates a phenomenon in its real-life context and that the boundaries between the context and the phenomena are not clear. I studied teachers in their real life contexts, the schools and classrooms. In my research I have chosen a case study approach which entails a detailed analysis of three cases for a comparative purpose.

Having selected a case study approach, I had to decide how to find cases for my study. How I did that is described in Chapter 4. Furthermore, having selected teachers did not provide me with data for the analysis. I still had to decide on methods through which I would collect data to address research questions. The research methods I am using have derived from an interpretative tradition of social science and are outlined in the next chapter. Research methods in case studies have several common features (Pring, 2000): Intensity of examining the particular; the language and terms used during investigation does usually have to be of a kind with which the participants are familiar; the researcher is responsive to the participants’ experience and the distance between the researcher and the participants is narrowed, therefore “the resulting study is more a negotiation than a discovery of what is the case” (Pring, 2000, p. 41). I experienced that throughout my field study, the teachers became more familiar with my presence, and especially one of the teachers, Bent (Chapter 6), offered more reflections with regard to his teaching practice in our conversations towards the end of the study than he did in the beginning.

Within the interpretative knowledge paradigm, the status of knowledge is ambivalent; truth is constructed both individually and in social settings and related to interpretations. Rather than providing scientific explanations of social practices, in the interpretative paradigm social practices are subjects of interpretations and understanding (Bryman, 2001; Carr & Kemmis, 1986).

My role as the researcher

When I am studying teachers’ interpretations of a curriculum I am interested in their actions in the classroom and in their subjective meaning which lies behind what they say. The actual physical movements are not of the main interest, it is the actor’s meanings lying behind these actions that become the subject of interpretations by the observer (Carr & Kemmis, 1986; Pring, 2000). The consequences of those actions are in the students’ interpretations of them and in students’ learning outcomes.

Important issue to be aware of is that the observations I am making are filtered through my understanding and preconceptions (Pring, 2000). When I refer to the teacher’s classroom practice, it is my interpretation of what I saw / heard / experienced in the classroom based on my presence, field-notes and audio recordings which sometimes were illuminated by the teacher’s comments in a later conversation. It is not possible to refer to the teachers’ real practices as an objective identity, because I as the researcher could never stand aside as though my presence had no influence upon the situation of the classroom. Also, I as the researcher and observer am influenced by my own experience from school. I am familiar with the school context from several perspectives; as a student, as a student teacher, as a teacher, as a school leader, as a parent, as a teacher educator, and now as a researcher in the mathematics classroom. Taking a critical stance to my research I realise that my own experience from the classroom can be seen both as strength but also as a bias to be considered in my research. This is further discussed in the next chapter where I address the issue of trustworthiness and in the final chapter where I address validity and rigour.

Being familiar with the classroom and teenagers and their behaviour made it possible for me to enter the classroom without causing much disturbance. However, being familiar with the classroom context was a challenge not to look only for what I expected, or to report only what I found significant. I have used both constructivist and socio-cultural lenses to analyse classroom observations and conversations with the teachers to provide me with insights from different perspectives. These theoretical perspectives are discusses in the next part of this chapter.

Still it is important to emphasise that in trying to make an account of each teacher's interpretation of the curriculum and present characteristics of the teachers, it will be characteristics and an account of the teacher's interpretation from my perspective and through the conceptual lenses I have used. I am doing a qualitative research with a case study approach and thus studying a few teachers in a few classrooms. Therefore the outcome of my study can be only statements with uncertainty and it is important to emphasise that these statements can only be credible when seen in conjunction with the whole research (Bassey, 1999). However, recognising the difficulty of generalising from qualitative research Delamont and Hamilton (1984) acknowledged the "generality from good ethnography are just as useful to both researchers and practitioners as those available from systematic observations" (p. 19). And they wrote:

> Despite their diversity, individual classrooms share many characteristics. Through the detailed study of one particular context it is still possible to clarify relationships, pinpoint critical processes and identify common phenomena. Later, abstracted summaries and general concepts can be formulated, which may, upon further investigation, be found to be germane to a wider variety of settings (p.19).

Thus despite the fact that I cannot generalise as one can do from quantitative statistical data, generic insights can be offered from what I have learned through my research.

Use of grounded theory

When analysing the data collected, I did not look upon the data as an objective reality to be discovered, but rather as an interactive process between me and the data. Thus the analytical account I present is an account from my perspective through the conceptual lenses I used. The codes and categories I used in the analysis emerged from my data through the way I dealt with them.

"Grounded Theory" is theory derived from the data (Strauss & Corbin, 1998). In such studies the researcher begins with a study and allows the theory to emerge from the data. Contrary to coding quantitative data where the codes are predetermined and fixed and coding is a way of managing data, coding data in qualitative research is the first step in generating theory (Bryman, 2001).

According to Strauss & Corbin (1998) qualitative analysis is not a way of quantifying qualitative data, but rather "a nonmathematical process of interpretation, carried out for the purpose of discovering concepts and relationships in raw data and then organising these into a theoretical explanatory scheme" (p. 11). This seems to suggest that there is an objective reality in the data to be discovered by the researcher. Grounded theory in the form presented by Strauss and Corbin has been criticised for being objectivist meaning that it aims to reflect a reality and thus reflecting a positivist stance (Bryman, 2001; Charmaz, 2000). Discussing "Objectivist Versus Constructivist Grounded Theory" Charmaz (2000) offered a constructivist approach to grounded theory which "recognised that the categories, concepts, and theoretical level of an analysis emerge from the researcher's interactions within the field and questions about the data" (p. 522). This means that although I do not look upon my data as representing an objective reality to be discovered, I can still use grounded theory in the sense offered by Charmaz. The theoretical analysis in my study emerged through my interactions with the data.

Theoretical perspectives underpinning my study

As outlined in the Introduction (Chapter 1) the background for my research was formed by: studies of students' mathematical performance before and after L97 was introduced; my experience from in service training courses with teachers in connection with the implementation of the reform and suggestions from an evaluation report of L97, that L97 is not implemented as intended.

Starting with my first research question: *How are teachers in their mathematics teaching practice responding to the L97's recommendations?* I found it valuable to make a theoretical interpretation of L97 as a preliminary study informing the analysis of the teachers' teaching. This theoretical analysis is presented in Chapter 5 together with an account of the mathematical knowledge reflected and how I see an investigative approach to mathematics being encouraged in the curriculum. In order to make the theoretical analysis of the curriculum, I will discuss the theories involved. In the Introduction I referred to the committee developing the written mathematical part of L97 saying explicitly that they had been inspired by constructivism. I therefore start with a discussion of constructivism.

The second research question: *What kinds of teaching practices are observable in the mathematics classroom?* is addressed through classroom observations. What the teachers in my study said about L97 and about teaching and learning of mathematics, together with their classroom practice form the basis for my interpretation of their thinking and beliefs.

In analysing the data about teachers' thinking and their classroom practice, I did not find the constructivist approach sufficient for my needs. Constructivism did not provide me with all the angles that I wanted for my analysis. I have also based my analysis on socio-cultural theories. My discussion of constructivism is therefore followed by a discussion of socio-cultural theories before I compare the two theoretical perspectives and then account for the use of the theories in my research.

Thus I have drawn on both constructivism and socio-cultural theories in the analysis of the curriculum, in the analysis of teachers' thinking and in the analysis of teachers' classroom practice. Towards the end of the previous chapter, Literature Review, I pointed out that the literature I had studied reflected both constructivist perspectives and views based on socio-cultural theories. This emphasises the importance of discussing how I will deal with the data from different theoretical perspectives in my study. Since I am drawing on some of the original scholars in these areas much of the literature is quite old. However, I have also drawn on recent literature wherever possible.

Constructivism

The committee said they were inspired by international trends in constructivism when developing the written part of L97, and they referred especially to the working methods suggested in the curriculum. Thus what they talked about was implications of a constructivist view for activities in the classroom.

In Chapter 1 I indicated that the committee wanted to tone down meaningless manipulations with symbols. A view on teaching mathematics which puts weight on unrelated routine mathematical tasks involving application of learnt procedures, stressing that every task has one right answer has its background in theories referred to in the literature as absolutist theories (Ernest, 1991). Disapproval and criticism of failures are often results of this view and it supports a view that mathematics can be transmitted from one person to another. This is what I refer as a traditional style of teaching mathematics.

From a constructivist point of view, knowledge cannot be true in the sense that it matches the real world. Constructivists therefore introduced the concept of *viability* (Glasersfeld, 1995), to mean that concepts, models and theories are viable if they prove adequate, they *fit*, in the contexts in which they were created. This view replaces the absolutists' belief that there is only one ultimate truth describing the world.

In this section I will address some key features in Piaget's constructivist theory. Furthermore, I will discuss the notion of misconception from a constructivist perspective, and I will indicate a constructivist perspective on teaching discussing miscommunication between teacher and student(s) as an issue. I will use the concepts discussed in this section in the analysis of the mathematical part of the curriculum in Chapter 5 and also in analysing the data from the teachers' teaching practices in the subsequent chapters.

Development of cognitive structures – assimilation and accommodation

According to a constructivist philosophy one supposes that the person herself constructs her knowledge, that it is not passively gained from the environment and that learning takes place through a process of adaptation which organises a person's world of experience. Building of concepts is done through reflection and abstraction and learning is not a stimulus – response phenomenon. The interpretation of constructivism as a cognitive position held by most constructivists in mathematics education is that cognitive structures are developmental constructions (Noddings, 1990). According to Piaget development precedes and is a prerequisite for learning. The individual discovers the world on her own and creates concepts herself that correspond with her experience in the real world. Piaget defined learning as cognitive reorganisation preceded by assimilation and accommodation.

Assimilation is a mental process where new elements are integrated into existing mental structures. In terms of learning mathematics the process of assimilation occurs when a mathematical challenge can be addressed by already existing cognitive structures. The new challenge is interpreted through already existing structures. New experiences are thus interpreted in terms of something already known.

> The general concept of assimilation also applies to behaviour and not only to organic life. Indeed, no behaviour, even if it is new to the individual constitutes an absolute beginning. It is always grafted into previous schemes and therefore amounts to assimilating new elements into already constructed structures (innate as reflexes are, or previously acquired) (Piaget, 1970, p. 707).

Hence Piaget indicated that a person's conceptual structures do not have a fixed starting point. Already constructed cognitive structures will always be in a person's mind, either as a result of previous experience or they are innate.

A prerequisite for learning is that a process of accommodation must take place. Sometimes new information differs so much from existing structures in a person's mind that they cannot be integrated or assimilated into existing structures. Perturbation occurs which through reflective abstraction leads to adapting existing structures in order to accommodate the new information. According to Piaget, reflective abstraction is essential in the development of cognitive mathematical structures. Thus there is a connection between mental activity and learning of mathematics. Referring to biological assimilation and accommodation Piaget says: "similarly in the field of behaviour we shall call accommodation any modification of an assimilatory scheme or structure by the elements it assimilates" (Piaget 1970, p. 708). A process of accommodation and thus a process of learning is a "process of continual revision of structures" (Noddings, 1990, p. 9).

Misconceptions in mathematics from a constructivist perspective

Accounting for the learning of mathematics through constructivist lenses, the process of accommodation implies that existing structures have to be revised and adapted to the new challenge. For example, many children's perceptions that division makes something smaller are viable within operations with whole numbers. They can do division tasks with whole numbers within an existing cognitive structure. However, when given a task where the divisor is less than one, the answer becomes bigger than the dividend, their perception does not fit and a revision of the existing structures is necessary. If a revision of existing structures does not take place, an apparent misconception results. A misconception can also be regarded as an over-generalisation: The concept a child has about division has developed within the set of whole numbers and is (over)generalised to numbers smaller than one. The judgement of misconception is not done by the person having it but by an external observer such as a teacher, comparing observed activity or expression with their own wider experience of the phenomena.

Mathematical knowledge, to be viable, has to fit the environment, may be the teacher, the textbook or other students or prior constructions in mathematics.

> It [i.e. construction of knowledge] is, however, constrained by conditions that arise out of the material used, which, be it concrete or abstract, always consists of the results of prior construction (Glasersfeld, 1984, p. 31).

According to this, construction of mathematical knowledge is constrained by prior constructions in mathematics. The constructions in mathematics which can be a mathematical problem to be solved put constraints on the mathematical ideas a student has.

Von Glasersfeld (1995) says that notions and rules that students can have which to the teacher seem to be misconceptions can be viable within the students' field of experience. He argues that it is not enough for the teacher to present a counterexample which lies outside the student's experiential world. "Only when students can be led to see as their own a problem in which their approach is manifestly inadequate will there be any incentive for them to change it" (p.15).

Jaworski (1994) questions the notion of misconception: "if there are 'mis'conceptions, what is then a 'conception'? Is this some form of knowledge which the 'mis'conception is not?" (p. 20). She thus points out that the term misconception is not a straightforward term to use. However, because the term is used such a lot in the literature and in the curriculum, L97, I am going to continue to use it, but with proviso of that I recognise the problematic nature of the word.

A constructivist perspective on teaching

One view which is agreed upon from a constructivist point of view is the rejection of the assumption that one can simply pass on information to learners. Research on pupils' misconceptions, the use of misconceptions to promote students' development of powerful constructions and the fact that students themselves are supposed to construct their understanding through a process of reflection, are all essential components in interpretations of a constructivist view (Confrey, 1990).

According to a constructivist theory, learning mathematics requires constructions and classroom strategies with emphasis on mathematical activity in a mathematical community as a common thread (Davis, Maher, & Noddings, 1990). The role of the community, other learners and teachers, to provide setting, pose challenges, and offer support to encourage construction of mathematical knowledge, is emphasised.

> Learning mathematics requires construction, not passive reception, and to know mathematics requires constructive work with mathematical objects in a mathematical classroom. Mathematics teachers, therefore, need to accept as a major task the responsibility for establishing a mathematical environment in their classrooms (Davis et al.1990, p.2).

Students' activities and the use of manipulatives are also emphasised from a constructivist point of view from which mathematics is seen as a "model of possible action, representation, explanation and justification" (Confrey, 2000). However, it is important to realise that mathematics and the learning of mathematics are not "captured" in the manipulatives, neither is mathematical meaning inherent in manipulatives but the manipulatives and the use of concrete materials can create a opportunity for reflective abstraction (Cobb, 1988; Confrey, 2000).

One issue which has been dealt with by several researchers within mathematics education is whether students perceive the teacher's intended meaning. Since students through assimilation put their meanings into the words they hear from the teacher, and those meanings are not necessarily the same as that of the teacher, who has experience beyond that of the student, teacher and students often talk past each other.

Cobb (1988) claims that it is more problematic to account for successful communication, which he characterises as "a dynamic changing fit between the meanin-making of active interpreters of language and action" (p. 89), than *miscommunication*. Jaworski (1994) writes that *closeness in perspectives* between the participants in a discussion is an important consideration from a constructivist view within a teaching-learning situation but that we can never know if we have attained match in meaning even if it is achieved. Von Glasersfeld (1995) puts weight on the teacher's being concerned with what goes on in the student's mind and not only focusing, like a trainer, on the trainee's performance. "The teacher must listen to the student, interpret what the student does and says, and try to build up a "model" of the student's conceptual structure" (p. 14). He thus puts the responsibility for *closeness in perspectives* and *successful communication* on the teacher who in order to build up a viable *model of student's conceptual structure* needs to bear in mind that whatever the student does or says in a context makes sense to the student in that context.

From a constructivist perspective, mathematical operations and objects are our personal constructs fitting our own experience. These experiences include interactions with other people who have their own constructs about the same mathematical operations and objects. Reinforcement of constructs occurs when there is accord between different people's construct of the same mathematical objects or operations (Jaworski, 1994).

Socio-cultural theories

Socio-cultural theories of learning are based on the work of Vygotsky and incorporate more than one theory (Daniels, 2001); Activity theory (Engeström, Miettinen, & Punamäki-Gitai, 1999); socio-cultural approaches (Wertsch, 1991); situated learning models (Lave & Wenger, 1991; Wenger, 1998) and distributed cognition (Salomon, 1993). All theories share the view that "L.S Vygotsky provides a valuable tool with which to interrogate and attempt to understand the process of social formation of mind" (Daniels, 2001, p. 70).

In this section I will discuss some of the key features in socio-cultural theory to which I refer in the analysis of L97 in Chapter 5 and in analysing the teachers and their teachings in the subsequent chapters.

The major focus in socio-cultural theories is how social discourse creates conditions for the development of the individual's mental functioning. A socio-cultural perspective emphasises participation in social settings as forming the minds of participants through mediational activity, using cultural tools, of which language is the most important. Thus classroom activity and the language associated with school settings and wider communities are central to formations of meanings and understandings.

The individual mental plane as part of the socio-cultural process

According to Vygotsky, learning is a result of the process of social formation of mind and thus manifests itself first in the social plane and then in the individual. The individual mental plane is constituted as part of the socio-cultural process. It is therefore important to understand the social relations in order to understand the individual who exists in those social relations. He writes:

> Any function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, *between* people (*interpsy-*

> *chological*), and then *inside* the child (*intrapsychological*) (Vygotsky, 1978, p. 57emphasis in original).

Just as children's development appears on the social level before it appears within the individual, teachers' knowledge about teaching can be seen to appear first in the social where the teaching practice takes place, influenced by the cultural environment, and then inside the individual teacher as their personal constructions. Where does the teachers' knowledge about their teaching come from? Some teachers have been teaching for many years, have long experience as teachers, and their thinking about their teaching is thus formed through their own practice and the cultural environment. Their practices are formed and also constrained by their socio-cultural participation, both in terms of what they do and what they do not do in their teaching practice.

Mediating tools and the role of language

Central in socio-cultural theories is the use of tools as auxiliary means in the learning process. Vygotsky (1978) draws an analogy between the role of a tool in labour and the sign as an instrument in psychological activity characterised by their mediating function. Vygotsky looks upon the use of signs and language as essential when solving a psychological problem. Lerman points to the analogy between physical tools and cultural tools and how they transform us internally.

> Just as one's thinking of acting in the world is transformed by learning about a hammer and its purpose, so too a ruler, the natural numbers, and the notion of drag in dynamic geometries become tools which transform us and how we act in the world mathematically (Lerman, 2000d, p. 57).

The teacher or a peer becomes central in the process of mediating the world through the use of tools for learning. According to Renshaw (2003) mediational means or cultural tools in the mathematical learning process do not only amplify the task but change the nature of the task; and he uses the graphical calculator as example.

According to the work of Vygotsky (1978) the development of knowledge in children is a process of internalising activities that have developed in social practice. Thus the attention is on the social activity and on socially mediated tools as signs and language, and a view that learning and development occur dialectically. The role of the language or the child's speech is central in Vygotsky's writings as he found it to be both necessary and natural for children to speak while they act. The language, use of signs and words in communication with others, is the most important tool in the process from social knowledge to personal knowledge. To be able to frame a question when not being able to carry out a mathematical task may sometimes be crucial to proceed further. Thus from a socio-cultural perspective, language and communication are crucial factors in the process of teaching and learning mathematics.

Misconceptions in mathematics from a socio-cultural perspective

On page 55 I discussed misconceptions in mathematics from a constructivist perspective. Here I will offer a brief account of misconceptions as conceptions that develop through activity in classrooms including classroom dialogue. For example, the concepts that division makes smaller is discussed, practiced and reinforced as a cultural tool over years of classroom activity. Participation in practices of using division with whole numbers over long time periods forms this concept as a meaningful object. Recognition that the concept does not extend for all rational numbers has to develop also through participation and mediation.

The zone of proximal development

In socio-cultural terms learning is seen as a process of participation and internalisation from the social plane to the individual. An individual learns through participating in social practice or an activity in the classroom and the mental plane is formed through such engagement. According to Vygotsky the whole nature of higher mental functioning is social (Wertsch, 1991).

As opposed to the function of a physical tool which is externally oriented, language is internally oriented. What kinds of instruction are then optimal to an individual child? How can teachers support students to gradually make sense of mathematical concepts? Realising that instruction within education is not seen as an end in itself, Vygotsky saw the need for a theory which could address the relationship between instruction and development (Chaiklin, 2003). Vygotsky offered the zone of proximal development, ZPD, as a tool through which the internal course of development can be understood:

> It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978, p. 86).

Vygotsky thus emphasised the interactions with more knowledgeable others in the ZPD. A key feature of Vygotsky's socio-cultural theory is that what a child can imitate to-day, they can do with assistance tomorrow and on their own the day after (Lerman, 1998). Vygotsky thus emphasised the role of the teacher and especially the role of social influence and the use of language. However, it is important not to look at the zone of proximal development as a fixed field the child brings with her/him to school but rather as a dynamic product of experiences of the individuals, both child and teachers and the teachers' goals for their students in mathematics (Lerman, 2000b, 2000d).

Mortimer and Scott (2003) look upon ZPD as an alternative approach to measuring the student's ability which traditionally has been through a system of formal examining. The ZPD offers a way of judging the student's ability through what s/he is capable of doing with assistance from a teacher. The challenge for the teacher is thus how s/he can help students to achieve a level of performance that they are not yet capable of achieving alone. Mortimer and Scott (2003)

> consider that the continuous monitoring of students' understandings and responding to those understandings, in terms of how they relate to the intended scientific point of view, must be central to the teacher's role (p. 20).

And they write:

> As the teacher is engaged in these linked processes of monitoring and responding, he or she is probing and working on the "gap" between an individual student's existing understanding and their potential level of unassisted performance. They are working with the students in the ZPD (p.21).

Bruner (1985) terms the teacher's engagement in ZPD as scaffolding:

> the tutor in effect performs the critical function of "scaffolding" the learning task to make it possible for the child, in Vygotsky's word, to internalise external knowledge and convert into a tool for conscious control (p. 25).

Bruner used the term "scaffolding" to describe a teacher's role in interacting with a student in the student's zone of proximal development. Scaffolding is the process in which the teacher supports the student until s/he achieves control over a new function or conceptual system. In the analysis of the classroom observations in my research I consider as scaffolding when the teacher challenges the student by questioning their thinking; highlights key aspects of the task; reminds the student what s/he has done so far and about previous knowledge. Since the focus in my research is on the teacher and the teaching, I have not studied collaboration between students. However, scaffolding takes also place when a student is working together with a fellow student.

It is important to remark that the student should not be considered passive when "scaffolding" takes place. Some students can even introduce their own scaffolds in the learning process. Students can also serve as scaffolds for each other, and the interweaving of everyday and scientific knowledge can also be looked upon as a form of scaffolding (Goos et al., 1999).

How teachers meet the challenge of how to work with students in the ZPD and thus scaffold students' learning are important issues in the analysis of their classroom practice. I look upon scaffolding as a range of teaching strategies used in creating conditions for students' possibilities of learning. Patterns of discourse in the mathematics classroom and what teaching strategies the teachers used in creating conditions for students' possibilities of learning are important foci in the analysis of my data which I address in Chapter 5.

The role of social practice and discourse in the classroom

From a socio-cultural point of view thought and language are seen as dialectical. The child brings thought to the process of interaction, whereas language pre-exists in the social and is external to the individual (Lerman, 2000a). Lerman places *social practices of mathematics* in the place of *objective reality of mathematics*. From a socio-cultural point of view, the child is not supposed to reach this objective reality of structures of mathematics on his / her own through reflective abstraction (which Lerman thus implicitly claims to be the case from a constructivist view). Instead Lerman writes that “Learning school mathematics is nothing more than initiation into the practice of school mathematics, hence the central role of the initiator” (Lerman, 2000a, p. 215). He thus places the teacher and the social practices in the classroom, the classroom discourse, in a pivotal role.

Cobb, Boufi, McClain &Whitenack (1997) emphasise the role of the classroom discourse and the relationship between a reflective discourse in the classroom and student’s mathematical development. They discuss how students’ participation in a reflective discourse in the classroom constitutes conditions for students’ possibilities of learning mathematics. According to Cobb et al. shifts in discourse can support students’ mathematical development. Teachers are usually the ones who through their teaching strategies can initiate shifts in discourses although shifts in discourse can also be initiated by students.

In my analysis of the teachers’ teaching practice I point out how shifts in discourse occurred together with shifts in mathematical focus in the lesson and thus conditions for students’ learning and support for their development were created.

Constructivism and Socio–cultural theories

The role of communication is central when doing educational research. Why do we communicate and by what means? From a constructivist point of view communication has been a theoretical problem because it has been linked with the transmission of thoughts and it has often been claimed that knowledge cannot be verbally transferred (Sierpinska, 1998). Sierpinska presents how the role of communication has been looked upon throughout some time and from different theoretical perspectives: From looking upon communication as a means to communicate notions *to* the students, through how communication could enhance cognition, a view on communication reflected from a constructivist position, to the recent attention around the processes of communication *among* students and *with* students and the question of the emergence of taken-as-shared meanings through communication in classroom cultures.

Kieran, Forman and Sfard (2001) point out differences between research in mathematics education reported ten to fifteen years ago and current reports. Whereas the language in the earlier research encompassed concepts as mental schemes, misconceptions and cognitive conflicts, newer research seems to address activities, patterns of interactions and communication. They write: “While the older texts speak of learning in terms of personal *acquisition*, the newer ones portray it as the process of becoming a *participant* in a collective doing” (p.1, emphasis in original). On this basis they see how research acknowledges the social nature of human thought, and that the learner becomes a part of a larger whole. If learning mathematics is conceptualised as developing a discourse, most natural units of analysis can be found in the discourse itself. Therefore, from a socio-cultural perspective, the discourse in the mathematics classroom of which the students are parts, is important unit of analysis.

Different perspectives in the two theories

From a socio-cultural point of view, learning is seen as interrelated historical, cultural, institutional and communicative processes (Wertsch, 2002). This view contrasts with an individualistic perspective on learning and rather provides a conception of learners as cultural and historical subjects in social practices. Differences between constructivism and socio-cultural theories as theories of learning can be seen in how relations between learning and development shift and concerning the location of mind, whether in the individual’s head (mathematical learning is constituted by active cognitive reorganisation), or in the individual-in-social-action (mathematical learning is constituted by enculturation within a community) (Cobb, 1994). Constructivists draw on Piaget’s genetic epistemology as theoretical background whereas socio-cultural theorists most often refer to work from Vygotsky. Differences in perspectives arise in the way we theorise the interaction between the individual and the social environment (Lerman, 1998, 2000c). Where Piaget inserted the active interpreting subject between stimulus and response, Vygotsky placed the “mediation of culture” in the sense that a cultural tool; a textbook, a teacher or another peer or other tools transform students and how they act in the mathematics world (Lerman, 1998, 2000c).

Learning in different terms

From a socio-cultural point of view, learning takes place as a result of participating in social practice. Vygotsky identifies learning as leading development, meaning that development is a result of several socially and culturally experienced situations. In socio-cultural terms learning is looked upon as integration or enculturation into a community of practice. Where Piaget sees equilibration as the mechanism for learning, Vygotsky introduced the concept of the zone of proximal development, ZPD, as a tool through which the internal course of development can be accounted for and where mediation of culture takes place (Lerman, 2000c).

Development and learning or learning and development?

A core issue between these two theories is whether development precedes learning or vice versa. Does a person have to be on a certain developmental stage for learning to take place or does learning lead to a person's development? Social factors are for constructivists the most common and significant interactions that can trigger disequilibrium in an individual's cognitive system. However, if the child is not ready, in terms of not being at a certain developmental stage or in terms of having constructed the necessary pre-formed structures, learning will not take place (Lerman, 1998; Säljö, 2005). If accounting for learning from this perspective "teaching cannot do other than exercise the child within her or his particular cognitive state" (Lerman, 1998, p. 335), and the pedagogical task will only be to match the content in the given task with the students' level of thinking (Säljö, 2005).

From a socio-cultural perspective it is through confrontation with new sets of reasoning the individual develops and becomes able to act in new ways in social practice (Säljö, 2005; Vygotsky, 1986). The socio-cultural perspective represents a social and collective view on how knowledge is created and developed (Säljö, 2001). Contrary to constructivist theories socio-cultural theories take social, cultural and historical factors to be *constitutive* of learning (Lerman, 2000d).

An individual and social perspective

Cobb (1989) warns against restricting oneself to one of the two positions, cognitive or socio-cultural in analysing mathematical teaching and learning. His suggestion is to coordinate analysis developed in the different contexts and to complement a cognitive constructivist perspective, from which meanings are assumed to be compatible, with an anthropological perspective, from which meanings are assumed to be shared. By contrasting and comparing the two theories - constructivism (students actively construct their mathematical ways of knowing) and socio-cultural theories (emphasis is put on the socially and culturally situated nature of mathematical activity), Cobb (1994) questions the assumption that initiates a forced choice between the two. His central issue is to "explore ways of coordinating constructivist and socio-cultural perspectives in mathematics education" (p.13). He argues that mathematical learning can be seen both from a constructivist view as a process of active individual construction driven by the mechanism of equilibration and from a socio-cultural view as a process of enculturation into mathematical practices.

Cobb (1994) contrasts and compares the two theories both with regard to activity, thought and teacher's role in the negotiation process. With regard to activity, constructivists focus on students' sensory-motor and conceptual activity, whereas socio-cultural theorists see activity linked to participation in culturally organised practices. Constructivists see conceptual processes as thoughts located in the individual and account for psychological development through the quality of the individual's interpretive activity. Socio-cultural theorists explain how the individual's participation in social actions influences psychological development and they give the culturally organised practices as an account for psychological development. Learning from a constructivist perspective occurs through negotiation of meanings in social interactions by continually modifying interpretations. The individual's construction is thus in the foreground, and assimilation and accommodation are used as metaphors to explain learning. Socio-cultural theorists use the term appropriation as a metaphor for learning. They see negotiation as a process of mutual *appropriation* in which the teacher and student use each other's contribution. The teacher's role is to appropriate the students' mathematical actions into wider system of mathematical practise.

The central claim in Cobb's (1994) article is that the socio-cultural and constructivist perspectives each constitute the background for the other:

> Rogoff's (1990) view of learning through acculturation via guided participation assumes an active constructing child. Conversely, von Glasersfeld's view of learning as cognitive self organisation implicitly assumes that the child is participating in cultural practices (p.17).

He exemplifies the complementarities between the two perspectives by analysing computational strategies both from a socio-cultural perspective and from a constructivist perspective. Participation in a new practice accounts for increased computational strategies from a socio- cultural point of view, whereas from a constructivist perspective cognitive forms created by the individual account for their new practice.

Cobb (1994) recommends for mathematical education to consider what the two perspectives have to offer relative to what shall be investigated:

> The challenge of relating actively constructing students, the local micro-culture, and the established practices of the broader community requires that adherents to each perspective acknowledge the potential positive contributions of the other perspective. [ ] The socio-cultural perspective gives rise to theories of the conditions for the possibility of learning, whereas theories developed from the constructivist perspective focus both on what students learn and the process by which they do so (p.18).

Taking the importance of social interactions in the classrooms she studied into account, Jaworski (1994) considers inter-subjectivity or taken-as shared knowledge as a product of social interaction and

> [ ] individuals as constructing meaning within the socio-cultural settings of the classroom and its surroundings – a constructive process that occurs while participating in a cultural practice, frequently while interacting with others (p. 211).

She thus sees individual construction of knowledge, the actively cognising subject, being influenced by the classroom and the interactions going on there.

Skott (2004) also argues for the two theoretical positions as conceptual lenses. He writes:

> [ ] a combination of individual and social perspectives is needed to account for the learning opportunities of mathematics classrooms, and [ ] constructivism and socio-cultural theory contribute significantly in these two fields respectively (p. 233).

Emphasis on both individual and social perspectives, importance of social interaction in individual constructions and thus the complementarities between the two perspectives suggested in this section, create the background for how I see the role of theories in my study which I address in the next part of this chapter.

The role of theories in my study

In my study I am investigating how mathematics teachers are interpreting the curriculum, L97, both in terms of what they say about it and what I observed them do in the classroom. Based on my long experience as a mathematics teacher and teacher educator and my study of research within the field, some of which I refer to in this thesis, I see the social dimensions in the classroom and the classroom culture as important factors in the learning process. I see interactions in the classroom and the individual's construction of knowledge as interrelated processes and individual students' mathematical activity and the culture of the classroom are reflexively related. I also look upon each individual student as an active cognising subject and as a constructor of knowledge. The individual student in these teachers' classes (whom I study) and the teacher him/herself can from a constructivist point of view be seen as an individual constructor of knowledge. For that purpose constructivism provides me with adequate theoretical lenses. Referring to Piaget, Von Glasersfeld (1995) says that "the most important occasions for accommodation arise in social interaction" (p.11). This emphasises how important discussion and interactions are both between students and between teacher and student(s) in the learning process and this I address through the second research question in my study: What kinds of teaching practices are observable in the mathematics classroom? And do teachers encourage social interactions, discussion and reflections?

The complexity of the classroom is a phenomenon well known to everybody who has been working in a classroom. Classrooms are often messy and complex places. They are messy physically; tables and chairs may be in any disorder, students do not find their books and they often sit in places not supposed to sit. Entering this "mess" as a teacher, means having to clear up which takes time from the actual mathematics lesson. Not only physically, but also on the mental plane the classroom is complex and may seem confusing and characterised by a youth culture (Cobb, 2001; Daniels, 2001; Lerman, 2000b). Lerman (2000b) points out that students often rather than responding to what the teacher is offering, find aspects of their peer interactions more important. That might be aspects of gender roles, ethnicity or body shape. Abilities valued by peers are often more important aspects for teenagers than those valued by the teacher or by parents. For example, it may be more important for the student being attractive to a classmate than paying attention to the teacher. Daniels (2001) writes that despite students being given ostensibly unambiguous tasks, when they have to be solved in the complex context of the classroom, the factors making up the complexity have to be addressed in the research of these classrooms. Realising the complexity of the classroom, Cobb (2001) expresses his concern for that complexity as an issue when developing an analytical framework for classroom research. He presents his experience from the classroom as a rationale for one of the criteria of the analytical framework which is to “enable us to document the developing mathematical reasoning of individual students as they participate in the practices of the classroom community” (Cobb, 2001, p. 463).

Skott (2001a) studied how teachers coped with the complexity of the classroom and introduced the concept “Critical Incidents of Practice”, CIP, an instance of teacher’s decision making where multiple and possible conflicting motives of the teacher’s activity evolved. He thus points out that considering the complexity of the classroom is crucial in the study of teachers’ beliefs about teaching and learning mathematics.

Teachers are also exposed to other conflicting contexts or visions. In their article “The multiple voice of a mathematics classroom community” Forman and Ansell (2001) refer to a broader community than that of the classroom and address the conflicting visions of, for example, an educational reform among parents and teachers. Realising that different communities in which teachers and learners participate, often espouse conflicting norms and values, they refer to Cole who adds an historical dimension to the participation metaphor:

> Only a culture-using human beings can “reach into” the cultural past, project it into the future, and then “carry” that conceptual future “back” into the present to create the socio-cultural environment of the newcomer (Cole, 1996, p. 186).

Forman and Ansell integrate the past and the future to understand the present to bridge the individual and the social context in which the individual participates.

In educational reforms, such as R97 (of which L97 was part), there are similarly many voices. Teachers’ voices are not unified; neither are parents’ and society’s voices; and the voices reflect conflicting goals within mathematical education. Teachers’ teaching practice is influenced by the way teachers have been taught themselves and their own experience as practicing teachers. Their practice is also influenced by expectations about students' learning in the future, sometimes expressed as mathematical aims in the curriculum.

So how do teachers create conditions for possibilities of learning in the classroom? I use the construct "common ground" which can be looked upon as taken-as-shared or a community of practice. The teachers often refer to and remind the students about their previous knowledge, which can be looked upon as the class's taken-as-shared in the sense that it is as if the students in the class share this knowledge. At least it is the teacher's intention that due to prior classroom activities this knowledge is shared. From a constructivist point of view, we can never know exactly what the knowledge we take-as-shared looks like from others' perspectives. But the value of the concept of inter-subjectivity or taken- as - shared can help us "to provide a bridge between individual construction and some consensus in mathematical understanding within a community" (Jaworski, 1994, p. 212). From a socio-cultural perspective common ground can be seen as a community of practice in which shared understanding is jointly constructed.

The socio-cultural setting can be seen as a way of forming a background from which I can look at individual constructions. I also see teachers' thinking and their classroom practice highly influenced by culture. To analyse that issue I did not find constructivism adequate. In the work with the teachers in my study I experienced that they did not only interpret the curriculum, L97, as constructivists. What they said about the curriculum was not only based on their reading and thus their personal constructions of what they had read. What they said was deeply embedded in their own experience as a teacher, in their own teaching practice and in the educational community. The social culture within which the teachers do their teaching is about being in a school community, it is about being in an educational community, and it is about society's and parents' expectations, including politicians' expectations, and also teachers' expectations. Seeing teaching as a cultural activity, implies that the act of teaching evolves over a period of time, in the cultural setting where it takes place (Stigler & Hiebert, 1999). Thus socio-cultural theories provide me with appropriate perspectives which according to Daniels (2001) are valuable in attempts to understand the process of social formation of mind. Looking through socio-cultural lenses is helpful when struggling to make sense of how teachers' thinking and their teaching practices are developed with regard to the cultural environment in which they exist.

Important issues for me to address in the analysis of teachers' classroom practice have been how teachers met these challenges of the complexity of the classroom: How did they deal with the mess they met when entering the classroom? How did they deal with students not paying attention? And on the mental plane, are the students ready to start thinking mathematically as the bell rings and the mathematics lesson is supposed to start? How did shifts in discourses occur? The teacher is usually ready to start working with mathematics when the lesson starts; however, that is rarely the case for all students. And what did the teacher do, what kinds of teaching strategies did the teacher use to capture students' *mathematical* attention?

I see teachers' past (their own experience) and the future (the goals for the students in the subject) as important factors in the analysis of teachers' present practice. Just as a teacher described in the study by Forman and Ansell (2001), expressed the conflict between showing the students the algorithm and their actual understanding of it, and Skott (2001a) considered the complexity of the classroom, I will point to how conflicting views from different contexts constrained the practice of the teachers' in my study. Thus when studying teachers' beliefs about teaching and learning mathematics, I recognise the importance of taking account of this complexity.

Summing up the chapter

In order to guide the analysis of L97 and the analysis of the empirical study I have presented some methodological considerations and theoretical perspectives. These considerations and perspectives informed the methods I used in the research process. In this chapter I have argued for selecting a case study approach. In order to answer my research questions I needed to find some teachers who were willing to participate in my research and for me to study their teaching. How I selected these teachers is outlined in the next chapter together with a discussion of the research methods I used which provided me with data which enabled me to answer the research questions.

4. Methods

Research design and research methods

If one wants to find out something, one "goes out and has a look" (Pring, 2000, p. 33). According to my research questions, I wanted to find out how teachers interpreted the curriculum and implemented it in their classrooms. I therefore decided to enter the mathematics classrooms to investigate teachers' practice, and to have focus group meetings and conversations with the teachers to find out what they said about L97 and mathematics teaching and learning.

Thus I have conducted an empirical study using research methods fitting largely into an ethnographic style of inquiry. In the previous chapter I discussed the methodological considerations I had to make in undertaking this study in which I am taking an interpretative approach and using constructivism and socio-cultural theories as conceptual lenses. The research methods I have used have derived from an interpretive tradition of social science. According to interpretivist assumptions, all human activity is social and meaning making activity, and meanings, actions, context and situation are closely linked and therefore make no sense in isolation from each other (Eisenhart, 1988). When doing ethnographic research, the goal for the researcher is to make sense of the world from the participants' perspectives in an attempt to understand the participants' shared meanings and taken-for-granted assumptions (Eisenhart, 1988; Wellington, 2000). Therefore being involved in the activity and then reflecting critically upon inter-subjective meanings are important. Several methods of data collections are suggested when doing research with an ethnographic approach (Bryman, 2001; Eisenhart, 1988; Walford, 2001; Wellington, 2000):

- Participant observation. The decision of degree of participation depends on the nature of the research question;
- Ethnographic interview. Informal, like a conversation;
- Search for artefacts and documents about the group (information produced by and used by participants, like transparencies from the teacher, hand outs, textbook, concrete materials, etc.);
- Researcher introspection. The ethnographer records and reflects critically the kinds of things that are happening in the research situation.

My study is a case study of mathematics teachers' interpretation of the curriculum reform L97 in Norway, both in terms of what they say about it and in terms of their classroom practice:

- How are teachers in their mathematics teaching practice responding to the L97's recommendations?

- What kinds of teaching practices are observable in the mathematics classroom?
- How are teachers' practices related to their beliefs about teaching and learning mathematics?

I chose methods of data gathering in line with the above suggested methods in research with an ethnographic approach. The methods I used are presented below. I also present the data resources these methods provided me with on which I have based the analysis of the teachers and their teaching:

- Focus groups
  - Audio recordings and transcriptions
  - My personal notes from the groups
- Conversations with the teachers
  - Audio recordings and transcriptions
  - My personal notes from the conversations
  - Copies of work plans
- Classroom observations
  - Audio recordings and transcriptions
  - Field notes
  - Copies of what was written on the board and of transparencies teacher used
  - Textbooks and copies of worksheet and hand outs
- Estimation form
  - Teachers were asked to estimate their own teaching, what they looked upon as ideal teaching and how they looked upon L97
- Teacher's own writing about ideal teaching
  - I asked the teachers to write about one page long what they looked upon as ideal mathematics teaching
- Questionnaire
  - See page 89 where I say more about the questionnaire

All data are naturally collected in Norwegian. The transcriptions of the data are thus in Norwegian and have been analysed in Norwegian. I have translated transcriptions and analysis of data into English for the purpose of writing about it. Some of my personal notes from the conversations and focus groups, field notes from the classroom and my reflections are written directly in English (Table 1. page 81). Overviews of lessons (Table 2 page 82) and summaries from classroom-observations (Table 3, page 86) are done in English based on the audio-recordings which are in Norwegian. When translating data from one language to another, there will always be a kind of interpretation involved. Thus it is important to be aware of the different layers of interpretations of my data. When, in the presentation of the analysis of the teachers' teaching, I sometimes found it difficult to be accurate in translating into English what was said in Norwegian I have written what was said in Norwegian in a footnote.

Figure 2 is an overview of how the research methods address the different parts of my research. In this overview I have included students' performance because this is important in addressing the attained curriculum. The comparative study I did as a preliminary study to my doctoral work which showed that students perform generally lower in 2001 than in 1995, formed part of the background for my research (see chapter 1). However, students' performances as such and thus the attained curriculum is not a part of my doctoral project and thus not a part of this thesis.

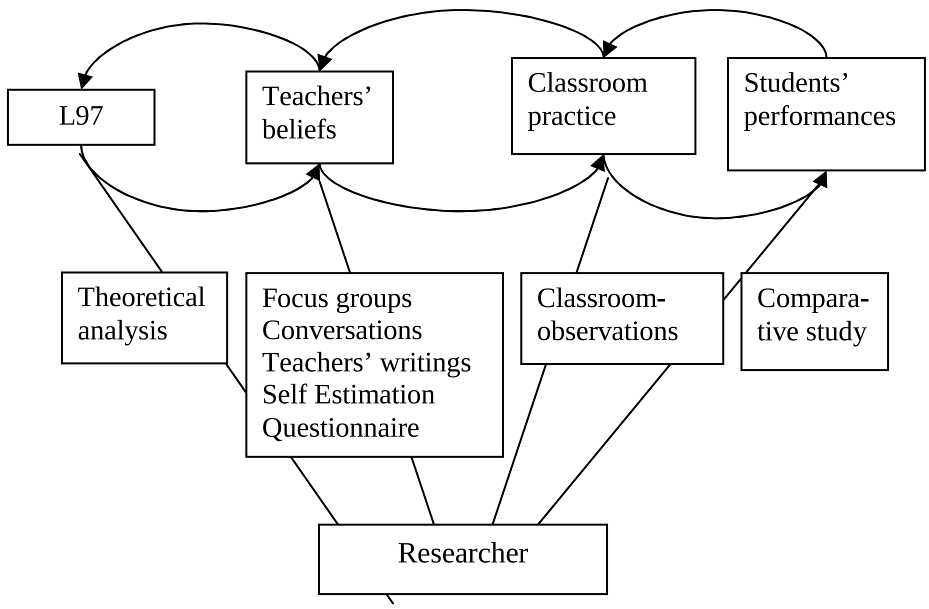

Figure 2, Research design and research methods

Research process

Focus groups and selection of teachers

In order to address my research questions I needed to talk with teachers and to study teachers' teaching. Thus my first challenge was how to find teachers who were willing to participate in research and thus become part of my study. I also had to make decisions about how many teachers I wanted to observe and at what stage (primary, intermediate or lower secondary stage). Reflecting on my experience with teachers in schools who were practice teachers (mentors) for my student teachers and my experience with in-service training courses with teachers in school in relation with the implementation of the new curriculum, L97, I was aware that teachers responded differently to the curriculum. Some responded reluctantly; some tried to ignore the new curriculum; some liked what was encouraged in L97 and tried out new exploring activities they had seen at courses right away; some "adjusted" the curriculum to what they had always been doing and some reflected on what L97 said and tried to take that seriously into account. As discussed in Chpater 1, this was part of the background for wanting to carry out this research. I decided to arrange focus groups with teachers, both to get information about teachers' thoughts about teaching and learning mathematics and their thoughts about L97, and also based on these meetings to select teachers for the further study.

Focus groups contain elements of two research methods: it is a *group interview* and the *interview is focused.* The members of a focus group are invited because they are known to have experience from a particular situation which in this case is teaching mathematics (according to L97). A focused interview is to ask open questions about a specific situation (Bryman, 2001).

According to Krueger (1994) focus groups are useful in obtaining information which is difficult or impossible to obtain by using other methods. Using focus groups generally means that the researcher can intervene into the conversation and pose questions to probe what somebody just has said. According to Bryman (2001) the use of focus groups has not only a potential advantage when a jointly constructed meaning between the members of the group is of particular interest. Participants' perspectives are revealed in different ways in focus groups than in interviews, for example through discussion and participants' questions and arguments. However, Bryman pointed out possible problems of group effects in a focus group situation that must not be ignored. I experienced such group effects and I realise the importance of treating group interaction as an issue when analysing data from the focus groups.

A challenge in using focus groups was to what extent I was able to interpret the meanings lying behind and looking through the words the participants were saying and from that make inference about the teachers' interpretation of the curriculum. In analysing the data from my focus groups it was important for me to be aware of the different levels of information the data give. On one level teachers speak from their inner thoughts and meanings, struggling to express what are really inside their heads, they speak from their individual constructions they have perceived viable in their own practice. On another level they speak from what they know as a teacher and what they say is deeply embedded in social practices of being a teacher, and thus very socio-culturally rooted.

A third level can be rhetoric: The teacher knew who I was, and could try either to express what s/he was thinking I wanted to hear or since s/he knew what the curriculum said, s/he could express that or s/he could challenge that. In such cases the teachers would respond to me and who I am rather than to who they are. When analysing what teachers said in focus groups it is important to be aware that the teachers' views were revealed in different ways than in individual conversations. What they said could be a way of positioning themselves rather than trying to express their inner thoughts. Krueger & Casey (2000) encourage use of questions leading persons to speak from experience; by "[asking] participants to think back" (p.58) rather than wishes for or what might be done in the future. That increases the reliability since it focuses on particular experience from the past.

I arranged 4 focus groups with a total of 15 teachers. The focus groups had different characters, purposes and foci. In this chapter I present how I used the three first focus groups as a method of selecting the teachers for my study. In the next chapter, Mathematics, L97 and Teachers, I present an analysis of these focus groups to highlight key issues for the study of the teachers' teaching. The fourth focus group meeting was held towards the end of my study, with the teachers participating in my research. In the final chapter I discuss findings from this last focus group.

Focus group 1, June 2003

In June 2003 I had one focus group meeting with 6 teachers in a community outside Oslo. This community had a collaborative project with regard to in service training of teachers with Oslo University College (in which I had not been involved). I made contact with the school leader of the community saying that I was undertaking a doctoral study and wanted to discuss L97 with mathematics teachers. She "picked out" 6 teachers, 2 from each of the three stages: primary, intermediate and lower secondary, to participate in a focus group meeting.

For this focus group I had prepared some general statements (L97 was not explicitly mentioned in these statements) about mathematics teaching and learning and I asked each member of the group to rank the statements with regard to importance.

Based on this meeting in which I learned that teachers valued statements about mathematics teaching and learning differently according to on what level they were teaching, I decided to narrow my study to teachers in lower secondary school. Since one of the two teachers in lower secondary school was going abroad the following school year, and the other expressed reluctance to participate in further study, none of the teachers from this first focus group meeting could participate further in the main study and I decided to arrange another focus group meeting only with teachers from lower secondary school.

Focus group 2, September 2003

In August 2003 I contacted head teachers in three different lower secondary schools in another community. My request was if they could ask mathematics teachers (at least two from each school) to participate in a focus group meeting. What I asked for at this stage was to have teachers for a focus group, with the option of being part of my study. Since these meetings were not part of the teachers' work and had to take place either right after end of the day at school or during the evening, I asked the head teachers if the teachers who participated could include these meetings as part of their working hours. The head teachers expressed interest in my project (which I had told them was a doctoral study on how mathematics teachers responded to the L97 curriculum), so they were positive to that.

The head teachers at the three schools, Toppen, Haugen and Dalen were eager to give me information about some of their mathematics teachers without my asking for it. I wrote down what they said: "s/he is a researchable teacher", "s/he is very traditional", "s/he is traditional but on the move", "s/he has been inspired through participating in in-service training courses and has started experimenting". Thus I was given images of the teachers before I met them.

I prepared the same statements to discuss in this focus group as for the first focus group, and invited the teachers to a focus group Sept. 24th. Because of another meeting coming up for the teachers at Dalen, only teachers from Toppen (Liv and Harald) and Haugen (Petter, Kari, Alfred and Bent) came to the first meeting which took place at Haugen School and lasted for 1 ½ hour with a coffee break included. All these teachers except one (he had worked as an economist for many years and this was his second year as a teacher), had long experience as teachers, they had different educational backgrounds; teacher training college, university, economics and engineer. Since it is possible to become a mathematics teacher in Norway in lower secondary school with different educational backgrounds, I wanted to take that into account when selecting teachers for the study. During a period of time in Norway (in the eighties) mathematics was not a compulsory subject for students in teacher education colleges. Although some teachers did not choose mathematics during their own education, they could nevertheless teach mathematics in schools. This was not the case for any of the teachers in this focus group.[5] They all had what corresponds with today's 60 ects in mathematics.[6]

After this meeting I asked the teachers if they were interested in participating in my study. The two from Toppen were not. All four from Haugen expressed uncertainty and said they "might". I decided to arrange a third focus group where the teachers from Dalen could participate together with the four teachers from Haugen who had said they could be interested in participating in the study. I did not invite the two teachers from Toppen to this focus group.

Focus group 3, October 2003

In addition to the four teachers from Haugen and the two teachers from Dalen, Cecilie and David, I invited one of my former student teachers, Tom, who was now working as a teacher in Oslo, to participate in this focus group. By the end of this focus group I wanted to make a decision as to which teachers would participate in my research. My aim was through classroom observations and interviews to do an in depth study of three teachers about their interpretation of L97. In case one of the teachers had to withdraw from the study, I decided to do the study with four teachers, and later I restricted the detailed analysis to three of them.

My intention was to do the classroom study during the spring term 2004. After the third focus group meeting, Kari and Petter from Haugen School were still ambivalent about letting me into their classrooms. The main reason they gave was that they had 10th grade and they had to focus on the final exam. They were afraid that my presence in the class would "disturb" them and their students in their preparations for the exam. The other reason they gave, also related to the exam, was that they were not going to do any "exciting" activities but mainly train the students for the upcoming exam.

Alfred who was teaching 8th grade and Bent who was teaching 9th grade at Haugen School expressed willingness to participate in my study. Neither Cecilie nor David from Dalen School had any objections, although they were teaching 10th grade, and expressed willingness to participate as well. For Tom to be part of the study there were too many practical hindrances both for him (he only had a temporary job at the school he was working at that time and he was sharing his classes with another teacher) and for me (his school was difficult for me to reach without spending too much time).

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[5] Although many mathematics teachers, especially those in primary and intermediate stages in Norway have poor education in the subject, most mathematics teachers in lower secondary school have at least 30 and sometimes 60 ects in mathematics. Teachers in schools in and around Oslo generally have more education than teachers in more rural places.

[6] 60 ects correspond to one year full study.

The four teachers in my study

I now had four teachers fitting the following criteria:

- Teachers in lower secondary school
- Both male and female teachers
- Teachers with different educational backgrounds
- Teachers who wanted to participate in the study

Alfred

Educational background from the university: He had studied Mathematics, Physics and Chemistry and afterwards taken Pedagogical Seminar, "Pedagogisk seminar, Ped. Sem."[7] to be a teacher. He had been a teacher for more than 30 years and was now teaching 8th grade at Haugen School.

Bent

Educational background from teacher training college: Had chosen "20 vekttall" (60 ects) mathematics.[8] Had been a teacher for 8 years and was now teaching 9th grade at Haugen School.

Cecilie

Educational background as an engineer from NTH[9]: Cecilie was the only female teacher in the sample. She was educated as an engineer, and had been working as "road planner" for many years. She decided to be a teacher 8 years ago and took Practical Pedagogical Education "Praktisk pedagogisk utdanning, PPU" to get a certificate of education. She was now teaching 10th grade at Dalen School.

David

Educational background from the university: He had studied Mathematics, Chemistry and Physics and had a Master degree in Biology. Afterwards he had taken "Pedagogisk Seminar" to get a certificate of education. He had been a teacher for nearly 30 years and was now teaching 10th grade at Dalen School.

Preparation for classroom research

Having made agreements with the four teachers, Alfred, Bent, Cecilie and David that they were going to participate in my study, I went to Haugen School and to Dalen School in December 2003 to have a preliminary talk with each of the teachers, to visit their classes and to make

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[7]"Pedagogisk seminar" is general pedagogical knowledge including didactics related to the subjects studied earlier. Together with subject knowledge this gives "certificate of education". When Alfred and David took "Ped. Sem", it was ½ a year of study. Later, when Cecilie took it, it was extended to 1 year of study, and it was then called "Praktisk pedagogisk utdanning," or PPU.

[8] Between 1992 and 2003 mathematics was compulsory with 5 "vekttall" which correspond to 15 ects in teacher education with an option to take 20 vekttall altogether, i.e. 60 ects.

[9] "Norges Tekniske Høgskole", a University College which became a university NTNU, Norwegian University in Science and Technology in 1996.

appointments for when I could be in their classes after Christmas and also for when we could talk before or after lessons.

Informed consent

One issue for this meeting was *informed consent* (see page 97, where informed consent is discussed as an ethical issue) and my promise to make the data unrecognisable so my writings cannot be tracked back to the teachers and individual students. I made written agreements with the teachers (Appendix 3). I also wrote a declaration of *professional secrecy* (Appendix 4). As a teacher educator I have a general *professional secrecy* with regard to what I eventually would come to know about students of confidential information.

Degree of participation

Another issue discussed was the *degree of my participation in the class while carrying out the observation*. There are several options in deciding which end of a continuum with regard to degree of participation during the observation depending on the nature of the research question (Bryman, 2001). The continuum goes from involvement as a complete participant to being a complete observer where the observer does not interact with the participants at all. With regard to my study neither of the two ends of the continuum was an option. I could not be a complete participant because then I would have been a covert observer which is impossible when being present.

On the other hand not interacting with the teacher and students in the classroom is not advisable in ethnographic research (Bryman, 2001) and it was not an option for me either. I moved around interacting with the students part of the time. Then I could talk with the students, look at their work, I had the possibility to ask them questions and also be available as a teacher when they needed help. I was an Observer-as-participant (Bryman, 2001). My role was overt to the students. I sometimes engaged in interactions between students and teacher and students. I never commented aloud on what the teachers and students said in plenary. This is an ethical issue discussed later in this chapter. However, I answered if the teacher asked me. I often took the role as a teacher during individual work. I sometimes discussed issues that arose during the lesson with the teacher in a conversation afterwards. (Student X had problem so and so, how would you have responded to that?). While I was moving around, I could not collect field notes. This I did when I was sitting down.

Audio-recording of data

A third issue during my first meeting with the teachers was to find out if they were willing to carry a *minidisk recorder* with a microphone during the lessons I was going to observe. None of the teachers had any objections to that. The teachers agreed to have a microphone attached to them so everything the teacher said can be heard on the minidisk.

Conversations with the teachers

The interviews I had with the teachers before and/or after the lessons were informal and conversational in style. I refer to them as pre-lesson conversations or post-lesson conversations or just as “pre” and “post”. Thus when writing “16/1 pre” or “Jan 16th pre”, I refer to pre-lesson conversation January 16th. These conversations were audio-taped and transcribed. The number of conversations with each teacher and the length of them differed from teacher to teacher (see Table 5, page 91)

When thinking about what to focus on in the analysis of the conversations, I had to go back to my research questions. “How are teachers in their mathematics teaching practice responding to the L97’s recommendations?” I did not only gain information about L97 when talking explicitly about it. I also had to analyse what the teachers said about their teaching and about their views on what and how students learn and relate that to L97. What the teacher said s/he intended to do in the lesson to create learning possibilities for students became an important focus since that also can be seen to illuminate how the teacher responded to the curriculum.

All information I gained through teachers’ own utterances was useful when answering my third research question: “How are teachers’ practices related to their beliefs about teaching and learning mathematics?” To answer this, I compared teachers’ beliefs about teaching and learning mathematics (what they say they ought to do and what they say is good mathematics teaching) with what I observed in the classroom. It was also important to consider constraints preventing the teacher from acting according to his/her beliefs. I found that these constraints sometimes were stated explicitly by the teacher him/herself during the conversations. I experienced that there were many issues I could code as constraints. Thus the code “constraints” occurred frequently when analysing the data.

Coding conversations with the teachers

I read through the transcripts and listened to the tapes several times to get ideas of what we were talking about and how that related to my research questions. Codes and categories emerged from the data through this work. I made notes in the margins of the transcripts every time I studied them. After a while I could see that several codes or categories occurred more frequently than others. Some of the codes were the same for all teachers and some were unique for each one of them.

To get a better overview when analysing the transcripts of the conversations I chose to import the transcriptions into NVivo[10] and to analyse the conversations and teaching practice from one teacher first, with the intention that the framework emerging for one teacher could be applied for the two others as well. I chose to code the conversations with Bent first, and a detailed outline of how I did this is presented in the analysis of the conversations with him in Chapter 6, “Bent”. I then used the framework from this analysis to analyse conversations with the other teachers.

Coding qualitative data is not a straightforward process. First of all, I soon realised that we talked about more than one aspect of teaching and learning mathematics at a time. The conversations had an informal style since my intention was that the teachers should feel comfortable during these conversations and not find the situation threatening. This was part of my ethical considerations in undertaking a qualitative research which I discuss in the last part of the next chapter. This informal style did not make it easy to get an overview and I also found that the sentences were not holistic and that the teacher often started a sentence talking about one aspect and ending up talking about something quite different because associations popped up while s/he was talking. Thus it was sometimes hard to keep track in the conversation.

Before starting the coding I was concerned about the reliability of the work I was doing; will my coding be consistent? Will what I do with regard to the coding to-day be the same tomorrow and the day after? Bryman (2001) presents some steps which ought to be born in mind during the coding process. One of his recommendations, is to do the coding several times which I did; first by reading through the transcripts without making notes (to be acquainted with the data) and then reading again while making notes in the margin. From these notes codes were created which I imported into NVivo together with the transcripts. I coded the transcripts three times in NVivo, and then again with paper and pencils. When first starting coding in NVivo, I was not very systematic in the process. I used the codes that had occurred to me from studying the transcripts. I used only free nodes (codes) so every node had equal status, ie none was subordinated to another. I coded mathematical focus, teacher’s intention for the next lesson, his/her reflection about a previous lesson or about teaching mathematics in general, disciplinary aspects, constraints, students’ difficulties and mastering, classroom culture in any disorder.

Altogether I had between 30 and 40 different codes when I did the first coding. Nvivo was useful in this process because I could easily print

[10] Nvivo is a qualitative research program for the purpose of analysing a large amount of qualitative data. This program offers a lot of tools for analysing qualitative data from which I only used a limited part in my analysis.

out the extracts I had coded according to these codes. For example, whatever I had coded "constraints" could come up in one document, and I could easily go into further detail about what the constraints mentioned were, and all passages coded as "classroom culture" could be studied further to see how the teacher referred to the culture in the classroom, did s/he appreciate students' contributions, etc. However, dealing with 30-40 different codes I found being too many, so I realised that I had to restrict myself to fewer codes. How I did this is outlined in the analysis of the conversations with Bent, Chapter 6.

Classroom observations

The purpose of doing classroom observations was first of all to answer my second research question: "What kinds of teaching practices are observable in the mathematics classroom?" and also the first research question: "How are teachers in their mathematics teaching practice responding to the L97's recommendations?" was addressed through what I observed in the classroom: Were students encouraged to work investigatively (as L97 suggests) or was the teacher directing them what to do? Was the teacher focusing on conceptual understanding (as L97 suggests) or exercising skills and procedures? Was the teacher encouraging the students to be active in the learning process as L97 suggests?

Collecting and handling data

All lessons I observed were audio-taped. I did not have any other microphones than the one the teachers carried in the room which means that I sometimes cannot hear what students were saying. It was also sometimes hard to distinguish between different students' comments and statements and I can therefore tell only approximately how many students were involved in a discussion. Here I could rely on my field notes. The field notes were helpful in reminding me what happened in the classroom that had not been recorded. I also copied what the teacher wrote on the board and I have copies of transparencies used and of hand-outs.

During the period while I was doing the classroom observations (Jan-April 04), I made an overview to help myself to keep track of what I had observed with each teacher and conversations I had had with them. Table 1, page 81, shows how I did this with each of the teachers' first lesson.

For each lesson I wrote down if we had a conversation before the lesson started, sometimes with a few words about what was said, then the topic of the lesson and how long it lasted, then if I had a post-lesson conversation. Sometimes I put my own reflections from the day in italics.

Alfred: 8/1, Geometry , construction	Conversation before the lesson (short) Lesson (60 min): Teaching from the board for about 40 minutes. (*Focusing on techniques)* Conversation after the lesson with reflections
Bent: 8/1, Surface and volume	Conversation before the lesson Lesson (60 min): Using concrete materials. Teaching from the board in discussion and interaction with students for about 30 minutes. The other half of the lesson he goes around and helps students in their work. He asks for pupils' suggestions when they ask for help. (*Focusing on using formulae, Links to algebra and the use of brackets)*
Cecilie: 14/1, Pythagorean Triples.	Lesson (60 min): Teaching /telling /asking from the board for 15 minutes before students get an exploring activity; does this happen always? Find more triples. Euclid's formula *Focusing on exploring activities and history; half of the students (or less) worked with mathematics*
David: 14/1, Equations with two unknowns	Conversation before the lesson. He tells what he is going to do Lesson (45 min): teaching from the board for 15 minutes. *Focusing on methods*. Students working on tasks practicing methods for 25 minutes. Teacher runs around helping. *Tells the students how to do it* Conversation after the lesson

Table 1, First lesson with each teacher

The recorded classroom observations provided me with a lot of data, principally in the form of sound. How to handle this amount of data became an issue while working with it. In the beginning I felt I needed to get a holistic overview of the classroom observations I had made so I put them all on one page in Table 2 below.

The table has five columns, one with the lesson number and one for each of my four teachers. Thus I have also here one cell for each lesson, but only with the date and topic (for example: 11/2, Geometry, Pythagoras). This helped me in selecting which lessons to transcribe and analyse into detail. I wanted lessons with different topics from each teacher, and I also wanted to have some of the same topics from the different teachers. (The shaded cells in Table 2 show lessons analysed in detail.)

After having finished the classroom observations and having transcribed some of the lessons fully, I made the decision that Bent, Cecilie and David who were teaching 9th and 10th grades were the three teachers whose teaching I wanted to analyse in detail. Alfred who was the oldest of the four teachers was talking about moving to another part of the country and /or to retire either the next year or the year after. Since I was planning to follow up the teachers in a later study I made the decision to omit the data from Alfred rather than one of the others. I still had three different educational backgrounds represented. I also had both genders represented.

Lesson-number	A Grade 8	B Grade 9	C Grade 10	D Grade 10
1	8/1 (60 min) **Geometry** Construction of quadrilaterals	8/1(60 min) **Geometry** Surface and Volume	14/1 (60 min.) Pythagorean triples	14/1 (45min) **Algebra** Equations with two unknowns
2	16/1 (60 min) **Geometry** Several constructions. Differentiated tasks.	16/1 (30 min) **Geometry** Overview of Volume	21/1 Figurtall (number patterns) Proofs, Angel sum in triangle	21/1 Graphical solution of equations with two unknowns
3	22/1 **Algebra** Starting on algebra as a topic	5/2 **Fractions** (Individual work or work in pairs)	28/1 **Algebra** Generalising Proofs and reasoning	28/1 **Statistics and probability** Misuse of statistics, Dice
4	5/2 **Algebra** + and – Teaching from the board	13/2 (30 min) **Fractions** Individual work	29/1 **Geometry** (Plato group) constructions	4/2 **Excel and probability**
5	13/2 **Algebra** Calculating with negative numbers	19/2 (60 min) **Fractions**	3/2 Reviewing a test (Socrates group)	11/2 **Geometry** Pythagoras
6	19/2 **Algebra** Reviewing home work	18/3 **Geometry**	4/2 **Excel** with girls from one class	18/2 **Geometry**. How to do constructions. Demonstrates
7	4/3 **Geometry** Area of quadrilaterals	24/3 **Geometry** Tasks on Pythagoras	18/2 **Excel**	3/3 **Measures** Length and area
8	18/3 **Geometry** Area of several figures	1/4 **Geometry** Constructions	10/3 **History** of equations and number systems	10/3 **Geometry** 30-60-90 triangles
9	24/3 **Geometry** Areas and volumes of several figures		17/3 **Statistics**	17/3 **Measurement** and relation

Table 2, **Overview of observed lessons**

I have geometry lessons from all teachers, and from Cecilie and David (grade 10) I have algebra and statistics as well. From Bent (grade 9) I have lessons with work on fractions. I selected the first three lessons and one of the two last ones and one in between from each teacher. I transcribed some of the lessons right after the observation, and some have been transcribed later. The process of transcribing 15 lessons, each lasting from 30 to 60 minutes together with the reflections I wrote while transcribing has been valuable in the process of analysis. Through that work I became acquainted with the data, I recalled my observations from the classroom so I got a more detailed picture of what had happened, and I felt I became familiar with each teacher and his/her class.

At the beginning, while transcribing, I wrote everything I heard as uniform text. However, analysing pages with text was not an easy task. I soon realised that it would be useful to structure the transcripts. I therefore made tables with four columns; in the first column I numbered the turns. The second column was for who was talking: I wrote the name of the teachers or A, B, C or D for the teachers respectively and names of students. If I knew the name of the student talking I have used their pseudonym names or first letter of that in the excerpts presented. I use S or Stud for unidentified students. The third column includes what is said. In the fourth I put my comments, reflections or something from my field notes. Later in the analytical process I put codes here.

Further, I divided the table into rows, one row for each turn. This gave me a better overview of the lesson, and it was through studying these overviews I could see patterns of discourse emerging. Also the lengths of each turn and the frequency of teacher's contributions in classroom interactions became visible using this table.

Having transcribed 2 lessons from each of the 4 teachers fully I realised that each transcription of one lesson contained so many pages that I still had difficulties getting the desired overview of the lesson although having organised it into the tables. At this stage I considered it useful to make some kind of data reduction to get a more concentrated overview, and also a holistic sense of each lesson. Therefore, from some of the lessons I wrote narrative summaries. These are summaries without any intended explicit interpretation. However, I am aware that there is always involved a kind of interpretation when dealing with data. For some of the lessons I wrote summaries directly by listening to the tape, whereas for the lessons I had already transcribed, I made the summaries based on the transcripts. I wrote down summaries of what was going on.

These summaries are all written in English and organised into sections and subsections. A shift between sections indicates some shift in the classroom from one form of activity to another form of activity, often between whole class activity and individual seat work. I often refer to the whole class activity in the beginning of a lesson as "lesson opening", or opening part of lesson. For each section I have put how long the section lasted and what the teacher was doing (teacher's role), for example teaching from the board, and what the students were doing, for example writing down what the teacher did on the board. The pattern of discourse and a brief description of the mathematical content of the section are also included here.

In the subsections I described the mathematical content and the interactions between teacher and students. A shift between two subsections indicates a shift in mathematical content or a shift between persons involved. The lengths of the subsections are also indicated; everything from less than a minute up to 20 minutes can be a subsection. So it is an actual shift in activity that determines the shift between two subsections and not the duration of it. For each subsection I have two columns; one for significant aspects and one for codes. Table 3, shows a narrative summary of the first section of the lesson with Bent Jan 8th, 2004.

In these data, I identified significant episodes which became a subject of analysis. An episode can be a subsection or part of a subsection. In presenting the data I have numbered the excerpt I am referring to. The episode is presented with the name of the teacher, date of lesson, from what section of lesson (Roman numeral) and number of episode within that section. Thus "*Excerpt 20, Cecilie, Jan 14th episode II-4*" means that this is number 20 of the presented data excerpts in the thesis and the presented episode is from Cecilie's lesson January 14th section II, episode 4. If not the whole episode is presented in the excerpt, I have put the turns included in the excerpts in brackets. The format of excerpts of the translated significant episodes presented in the analysis of each teacher is with four columns: Number of turn, who is talking, what is said and comments.

In presenting quotations or excerpts from the transcribed data, I sometimes use [ ] or [explanatory comment]. What is written within the brackets is meant as an explanatory comment (not expressed) and [ ] means that something said or written is omitted from the quotation.

Having written summaries of the whole class sections of five lessons from each of the teachers B, C and D, I soon realised that to be able to grasp the interactions between teacher and students, I needed fully transcribed episodes. The summaries gave me an overview, while I needed full transcriptions to capture details. I therefore decided to transcribe the lessons on which I base my analysis, and very soon codes and categorisations emerged from my data. Going back to my research questions, I was mostly interested in the pattern of discourse in the classroom and the teacher's strategies to create possibilities for students' learning. That would give me insight into how teachers responded to the L97 mathematics curriculum and the kinds of teaching practices being observable.

Geometry: Volume and Surface of solid blocks

Section	Subsection	Significant aspects	Codes
I 26 minutes Teaches from the board, in interaction with the students. Dialogic format. Going through volume and surface area of several solid blocks; cube, cylinder, prisms. Although discussions going on, teacher's voice is heard in between each student's comment	**I** **4 minutes** **Surface area of prism**. The teacher (B) asks student E if he has found a formula for surface of a prism. In a dialogue lasting for 1 ¾ minutes, teacher and student work out the formula. They are interrupted of a girl who wants to use brackets in the formula.	Teacher says that the process how they derived the formula is the important thing, not the formula itself	Managing class, RevPrioKnow, Textbook InvStudPart Process (SA-TQM-SA-TC-SS..) Interrupted
	II **6 minutes** **about use of brackets in formulae (the issue about brackets evolved among pupils)** A discussion between teacher and students if using brackets or not. Bent shows that if you put numbers for l and b the answer becomes the same both with and without brackets. Students "demand" brackets because that will give a better overview. Bent suggests that it will be most tidy without, however he accepts that some prefer using brackets. A student suggests putting 2*(1h+1b+bh) which is praised by the teacher; because what is in the bracket is each surface and there are two of each surface. Another student claims that multiplication is commutative. Tove refers to algebra, and talks about multiplying into the bracket. Another pupil (Camilla) claims that it makes a difference having brackets or not. She repeats what she means.	Teacher asks for alternatives Students come up with suggestions A mathematical discussion takes place as a result of students' questions. Rules for use of brackets A discussion is going on. The teacher's voice is heard in between each student's voice	Alternatives StructStudThink Miscommunication (twice) Students come up with suggestion
	III Purpose of use of formulae 10.50-12.20 (1 ½ min) The teacher asks the students about the purpose of having formulae. A pupil suggests that it is quicker to use a formula. The teacher concludes that it is time saving and that it gives you a better overview.	Purpose of using formulae Relation to other aspects of mathematics (formulae and equations)	StudentSugg
	IV **Formulae for surface of a cube. A dialogue between B and a student 12.20-13.45 (2 ½ minutes)** Teacher asks for other formula The formula for surface of a cube is told by a student. The teacher raises question about how many measures you need to use that formula.	A student tells how he has been thinking when developing the formula for surface of a cube. Teacher comments and confirms between each "step".	StructStudThink
	V **Surface area of cylinder** **(about 5 minutes):** A student tells how she has been thinking in developing the formula for surface of a cylinder. Other students interrupt to clarify. Teacher writes on the board and shows with a sheet of	Use of four sided (student) and square (teacher) Use of rounding (student) and circle (teacher) Illustrating a	StructStudThink Conceptual understanding

	paper how a cylinder can be unfolded. 2 issues are coming up: 1) There is no radius in a quadrilateral and 2) calling the other side of the rectangle which is the side surface for l or h	cylinder with a sheet of paper. Focuses on conceptual understanding	
	VI **Discussion about brackets in the formula for a cylinder (4 minutes)**: A student raises an issue: The way the teacher suggests to write the formula for surface of a cylinder (which is the same as in the textbook), is not very clear or easy to follow. Several suggestions using brackets in the formula are coming up. A girl is very confused about the use of the digit 2, one place it means r + r and in another place it means that you shall multiply everything with 2. The episode terminates with the teacher apologise for being rude with her when saying that she probably needs some more practice in this	Issue initiated by students Teacher is trying to "answer" their frustration. NB teacher tells a girl that she needs some more practice. Jfr L97: In a constructive and confident athmosphere…	Probing student's thinking Students suggestion StructStudThink Restates
	VII **Summary of the first part of the lesson** (2 1/2 min) The teacher sums up what they have done so far and tells the students what they are going to do next; working with their working program and measuring and calculating volume and surface area of different wooden blocks. They are supposed to fill in the results in a form where they also are asked to do perspective drawings of prism, cube etc. Students ask questions what to do and one student says she'll do her working program whereas the teacher says that he expects them to do the form as well.	Lots of noise	StructStudThink Classroom management Preparing activities in pairs or individually Encourages Collaboration

Table 3, Bent 8/1 narrative summary section 1

After having written narrative summaries and transcribed whole lessons or parts of lessons, coded and analysed, I made a lesson overview of each lesson (Table 4 is an example of such overview). The overview is between one and two pages long. It is divided in columns according to activity in class (whole class or individual seat work), and into rows in terms of the following parameters: Length of section, mathematical content, purpose of lesson stated by the teacher in pre conversation and teacher's given overview, tools used in the lesson, teacher's role, students' roles, significant aspects, significant episodes, pattern of discourse, teacher- students' interactions, common ground and disciplinary aspects.

These overviews allowed me to compare each teacher with him/herself across lessons to identify similarities and differences, and also to identify similarities and differences between the teachers.

Date: Jan 8th 2004, Topic: Geometry

	Whole class	Individual work
Time	26 minutes (10-15 different students involved.)	22 minutes 9 episodes, only 2 with more than 1 student involved.
Mathematical content	Volume and surface of cube, prism and cylinder	Volume and surface of cube, prism and cylinder
Purpose of lesson (According to the teacher)	Review what they have done as a task at home from the textbook. Relation between formulae and equation.	Work with solid blocks and working program. Students shall find out the formulae and fill in a form. Measure and calculate. Teacher wants the students to be “positively confused” when faced with concretes which is different from seeing drawings in the book.
Tools	Textbook, workbook, cube, prism, sheet of paper illustrating the surface area of a cylinder. Blackboard	Solid blocks for the students to investigate, textbook, workbook. Ruler, pencil
Teacher’s role	To see (not controlling) what students have done as their homework. B asks questions about the work they have done. Channelling a discussion (episode 2) initiated by a student.	Help students who ask for help. To give consent (or not) when they ask if what they have done is correct.
Students’ roles	Answer teacher’s questions and tell what and how they have done the task from the textbook. They also come up with issues that change the direction of the lesson	Do the given tasks- measure sides and calculate surface area and volumes. Work with the working program. Working in pairs.
Significant aspects	Students’ questions change the direction of the lesson. Miscommunication takes place.	
Significant episodes	Episode 2, 6 minutes. 6-7 students are involved in a discussion about brackets. Episode 6 about brackets in the formula for surface of cylinder.	II-2 where teacher explains to a student what to do. II-4 and II-7 typical answers from B when a student poses a question
Pattern of discourse	Dialogic and sometimes open as in episode 2 when B poses an open question which has a fruitful outcome. Circular pattern of discourse, IRF-RF-RF.	Students ask for consent or they ask how to do a task. Dialogic pattern of discourse. IRF pattern
Teacher - Student interactions	Teacher invites students to participate; he questions students’ thinking, Refers to prior knowledge, structures students’ thinking by highlighting key aspects and taking what students know as staring point. Also miscommunication (Episode I-2 and 6). Focuses on their conceptual understanding of volume and surface.	Teacher asks for student’s suggestion when they ask how to do it, refers to prior knowledge, he challenges their thinking.
Common ground	Working program. What is considered as their knowledge in this topic	
Disciplinary aspects	Students frequently talk when they are not supposed to. Arguing with the teacher. Blaming him when they don’t understand. Ask several times what they are supposed to do	

Table 4, **Lesson overview Bent Jan 8th**

Coding classroom observations

Common in the literature I have studied about coding qualitative data is the emphasis on that there is no unique way of doing the coding and analysis in a qualitative study and that there exists no prescriptions to be followed (Bryman, 2001; Strauss & Corbin, 1998; Walford, 2001; Wellington, 2000). This I experienced through my work with the data. Therefore, the way I have been doing it, as described, was not for me a predetermined method. It developed while I was working with the data. Through this work I have dipped in and out of my data, I have studied them holistically and also in detail while listening to the tapes and transcribing fully. I read and re-read through transcripts and summaries and made notes during this process. Thus I immersed myself into and acquainted myself with the data, and the categories I have been using emerged from my data.

A general issue when working with coding of data is whether categories or codes are derived from the data or if they are brought to the data a priori (Wellington, 2000). I did not start my study and I did not read through the transcripts with a blank mind. Before I started the coding process, I had had conversations with the teachers, I had studied L97, and I had been sitting in the classroom. All this is part of the analytical process. My own experience as a teacher and teacher educator has also influenced what codes I have been using. Some of the codes are derived from past research done by other researchers which I have studied (Goos, Galbraith, & Renshaw, 1999; Jaworski, 1994; Mortimer & Scott, 2003). As outlined in the methodology chapter I do not see the data in my study as representing an objective reality for me as the researcher to be discovered. Rather I interacted with the data as described above, and I was thus part of what was being observed. At all stages I found it important to reflect critically on the interpretations I was making and their source.

Self estimation, teachers' own writings and questionnaire

I also have information from the teachers obtained through *self estimation* and I have drawn on Pehkonen and Törner (2004). They used Dionne's (Dionne, 1984) perspectives of mathematics (traditional, formalist and constructivist perspective) in terms of toolbox aspect T (explained as mathematics is a toolbox, doing mathematics means working with figures, applying rules, procedures and using formulae), System aspect S (explained as mathematics is a formal, rigorous system, doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts) and process aspect P (explained as mathematics is a constructive process, doing mathematics means learning to think, deriving formulae, applying reality to Mathematics and working with concrete problems).

Pehkonen and Törner label these perspectives as similar or corresponding to Ernest's (Ernest, 1991) three views on mathematics; Instrumentalist, Platonist and Problem solving. My four teachers were asked to distribute 30 points corresponding to their estimation of the factors T, S and P in which they should value their real teaching, what they think is ideal teaching and what they think L97 reflects with regard to these aspects. To obtain more detailed information about their views on mathematics teaching, I also asked them to write down, about one page long, their personal opinion of what good or ideal mathematics teaching is.

Together with the estimation form (Appendix 2) I gave the teachers a *questionnaire* to answer (Appendix 1). I used the same questionnaire that was used about teachers' attitudes to mathematics in the KIM[11] study (Streitlien, Wiik, & Brekke, 2001). How the teachers in my study responded to the questionnaire is discussed in the final chapter. There I take the teachers' responses to the questionnaire, together with findings from Focus group 4, as a starting point for a cross case analysis and synthesis of findings based on analysis of classroom observations and conversations with the teachers. The information obtained from these three sources of data (estimation form, writings about ideal teaching and questionnaire) has enabled me to investigate the validity of the information obtained through the analysis of conversations, focus groups and classroom observations.

Different mathematical foci

Mathematics as a discipline is a central part in a study in mathematics education. L97 focuses on students' conceptual understanding and derivational knowledge (meaning developing new mathematical entities from existing knowledge). It focuses on the learning process, on investigations and exploring activities rather than exercising skills and procedures. The codes I have used to describe the mathematical focus in the lessons emerged both from the conversations I had with the teacher, from the lessons I observed and also from what is emphasised in L97. Aspects of mathematical knowledge generally and how aspects of mathematical knowledge are reflected in L97 are discussed together with a theoretical analysis of the curriculum in the next chapter. I have used the following categories with codes to describe the mathematical focus in my study:

- *Procedural* and method mastering *MethMast*
  Emphasis on exercising skills, procedures, methods, techniques, algorithms (reflecting traditional mathematics and the tool box aspect from the estimation form);
- Conceptual, *Concept*, and Derivational, *Deriv*

[11] "Kvalitet I Matematikkundervisningen" (Quality in Mathematics Teaching)

Emphasis on the development of concepts and new mathematical knowledge from existing knowledge (emphasised in L97);
- Structural, *Struct* Linking or making connections between different mathematical entities and concepts (emphasised in L97);
- Conventional, Use of symbols, *Symb* and conventions, *Conv;*
- Reflecting a fixed body of knowledge *FixedBodKnow* (reflecting the tool box aspect and part of the system aspect from the estimation form);
- Mathematics history *MathHist* (emphasised in L97);
- Real world or everyday problems (emphasised in L97), Invented or Genuine problem *Real/Invented;*
- Exploring *Explor*, Problem solving *ProbSolv* Conjecture, reasoning, generalisation *Conject* and reflection *Reflect* (encouraged in L97 and reflecting the process aspect from the estimation form).

These are codes I used when I analysed data from classroom observations and conversations with teachers. As I have indicated in brackets, some of the mathematical foci are emphasised in L97. How different mathematical foci were visible in the teachers' teaching in my study is discussed in the three chapters on the teachers, "Bent", "Cecilie" and "David".

Research questions and data resources available

I have done an in depth analysis of five lessons from Bent and David and of six lessons from Cecilie. I first did five from Cecilie's teaching too, but when listening through the tapes, and studying my field notes from the other lessons over again, I found that the lesson with Cecilie March 2nd had a different character than the other lessons with her and could provide additional features in characterising her teaching. I therefore analysed this lesson in detail as well.

With regard to the other lessons from the teachers not analysed in detail, I did not consider them to have added other aspects to my findings when studying them over again after having done the detailed analysis of the five (six) lessons from each. I had 6 conversations with Bent, 8 with David and 3 with Cecilie. For several reasons, Cecilie did not have more time for conversations than this. She did not write a page about ideal teaching either.

Table 5 is an overview of the conversations and lessons analysed in detail, which thus are the data sources forming the basis of my findings.

		B	C	D
I	Pre Lesson Post	1_B_8.jan 1_B_8.jan	1_C_14.jan	1_D_14.jan 1_D_14.jan 1_D_14.jan
II	Pre Lesson Post	2_B_16.jan 2_B_16.jan	2_C_21.jan 2_C_21.jan 2_C_21.jan	2_D_21.jan 2_D_21.jan 2_D_21.jan
III	Pre Lesson Post	3_B_5.feb 3_B_5.feb	3_C_28.jan 3_C_28.jan	3_D_28.jan 3_D_28.jan 3_D_28.jan
IV	Pre Lesson Post	5_B_19.feb 5_B_19.feb 5_B_19.feb	4_C_29.jan (Platon)	5_D_11.feb 5_D_11.feb 5_D_11.feb
V	Pre Lesson Post	8_B_1.april 8_B_1.april	5_C_3.feb (Socrates)	8_D_10.mars
VI	Lesson		9_C_17.mars	

Table 5, Overview of lessons and conversations analysed in detail

At first I imported all these data into NVivo, but I decided only to use NVivo to analyse some of the conversations and not to analysing the classroom observations. In an earlier section of this chapter, “Collecting and handling data” page 80, I have given an account of how I handled the data from classroom observations which gave me a desired overview which founded a base for the analysis. I preferred this way of analysing the data from the classrooms rather than analysing uniform text imported into NVivo because that provided me with the opportunity to be acquainted with the data.

When developing an analytical framework, I had to go back to my research questions and reflect upon how they could be answered through the data I had chosen to gather and now had available. What the teachers said both in focus groups, in conversations, what they wrote about ideal teaching and how they responded in the estimation form, give information about how they respond to the curriculum and therefore partly answer my first research question: *“How are teachers in their mathematics teaching practice responding to the L97’s recommendations?”* These methods of data gathering provided me with resources also for the analysis of what beliefs teachers have about teaching and learning mathematics, which is part of the third research question: “*How are teachers’ practices related to their beliefs about teaching and learning mathematics?”*

What I observed in the classroom provided data resources for analysing both how the teachers responded to the curriculum and also for the analysis of their teaching practices which is addressed in the second research question: "*What kinds of teaching practices are observable in the mathematics classroom?*" Since the third research question addresses the relationship between the teacher's beliefs about teaching and learning mathematics and their actual classroom practice, the analytical outcome of both teachers' utterances and of the classroom observations addresses that research question as well. An overview of how the methods used provided data to answer my research questions is presented in the table below.

Research methods **Research question**	Focus groups	Conversations	Estimation form	Teachers' writings	Classroom observations
How are teachers in their mathematics teaching practice responding to the L97's recommendations?	X	X	X	X	X
What kinds of teaching practices are observable in the mathematics classroom?					X
How are teachers' practices related to their beliefs about teaching and learning mathematics?	X	X	X	X	X

Table 6, Research questions and research methods

The issue of trustworthiness in my research

When undertaking qualitative research the issue of trustworthiness has to be considered (Bassey, 1999; Bryman, 2001; Goodchild, 2001; Walford, 2001; Wellington, 2000). How can I know that my analysis does not simply reveal the beliefs and attitudes I had before I started the research project? Based on the perspective that beliefs cannot be directly observed Leatham (2006) wrote:

> In order to infer a person's beliefs with any degree of believability, one needs numerous and various resources from which to draw those inferences. You cannot merely ask someone what their beliefs are (or whether they have changed) and expect them to know or not know how to articulate their answers (p. 92).

According to Leatham, a pitfall in research on teachers' beliefs is assuming that teachers can easily articulate their beliefs and that there is a one-to-one-correspondence between what teachers express and their beliefs. One way suggested by Leatham to avoid such pitfall is through the use of several methods of data gathering.

In this chapter I have outlined the different research methods being used. Cohen & Manion (1994) and Pehkonen and Törner (2004) emphasised triangulation in data gathering through the use of many simultaneously data gathering methods. I look upon the estimation form as a validation of the teacher's other utterances both with regard to how s/he looks upon own teaching with regard to the L97, how s/he looks upon L97 with regard to ideal teaching and also how s/he looks upon own teaching and L97 with regard to the three aspects, Toolbox, System and Process. In the final chapter, Synthesis and Conclusion, I compare my findings from the analysis of teachers' utterances and their classroom practice with how they responded in the questionnaire. This can also be seen as a validation of my findings. Having a teacher's utterances from both focus groups and conversations can also be looked upon as a triangulation of data and how what the teachers expressed in the different sources of data related to each other is important in the analysis.

But still, how can I know that my analysis of the data, my choice of codes and the coding I have been doing would have been the same if another researcher was going to analyse the same set of data? Since I have not had a researcher assistant who could review the transcripts and provide an independent analysis, recommended by Goodchild (2001), it has been important for me as the researcher to be aware of the "tautological danger". It has been in my mind throughout the whole research process.

Earlier in this chapter, under the heading "Coding conversations with the teachers", page 78, I elaborated how I dealt with this issue in the coding of the transcripts of the conversations. With regard to the analysis of the classroom observations I look upon the different sets of data (overviews, summaries, detailed transcriptions) as a possible condition for trustworthiness; the way I dipped in and out of the data, and immersed myself into them by listening to the tapes, transcribing, using my field-notes to support the transcriptions, organising the data in tables, writing summaries and writing overviews of lessons. Building on Lincoln and Guba (1985), Bassey (1999) presented a summary of stages to be done to enhance trustworthiness in a study. "Prolonged engagement" (Bassey, 1999, p. 76) in the data which is about immersing in its issues as I have done, is one recommendation. Keeping going back to the raw data searching for other features, as I did when I chose to analyse yet another lesson with Cecilie, is another condition for trustworthiness of the analysis recommended by Bassey (1999). I have also presented my study with preliminary findings at several conferences (PME 2004, Episteme 2004, Cerme 2005, Norma 2005) and I have had a 90% presentation of my doctoral work[12]. These were all occasions where I have been fortunate in having the possibility to get useful comments from fellow researchers on my analysis probing my interpretations. This is yet another recommendation suggested by Bassey (1999) for enhancing trustworthiness in research.

[12] A 90% seminar is an official presentation of doctoral work when approximately 90% complete. An opponent offers critical response.

Ethical aspects in the research process

In this research project, I have been using qualitative research methods. Participants in my research were teachers and students. The teachers participated in three different settings: in focus-groups, as mathematics teachers in the classroom and in conversations with me as the researcher. Students were participants of research in the classroom. Goodchild (2002) addresses the issue of researcher's disturbance in the class when doing classroom research. He writes that being an accustomed participant in a mathematics classroom made him confident that he could enter into the routine life of a class without causing much disturbance. Based on my experience from the classroom I felt the same. However, Goodchild (2002) raises an ethical issue about causing minimal disturbance. He writes:

> The major ethical dilemma is whether it is appropriate to enter a classroom for research purposes and intentionally withhold from the teacher information about the class that might contribute positively to the effectiveness of teaching and learning (p.55).

This I felt was an important issue to be considered and I had to make such choices several times. I did not intervene because it was not for that purpose I was there. I thus chose to cause minimal disturbance at the cost of (perhaps?) effective teaching and learning. In some of the cases I discussed the issue that had occurred with the teacher afterwards.

There are always personal elements implied in research. Using qualitative methods, personal meetings are taking place and we are situated in a field governed by right or wrong, in the field of ethics (Fog, 1993). It is not only within research we meet ethical problems. All value related choices we make when people are involved, raise ethical problems. It influences everything we do and we make many ethical decisions both in our lives as private persons and in our professional lives. Justice, respect, fairness and honesty are essential in our dealing with others in everyday life and thus in doing research and the issue of confidentiality is binding both when given to friends and to subjects of research.

Within qualitative research, there are interactions between the researcher and the people that are researched. It is an interpersonal endeavour (Sowder, 1998). Focus groups and conversations in my research were methods, tools which were unfolded by the way I as a person wanted to find out about the teachers' perspectives on their own beliefs about their teaching of mathematics. What teachers say is dependent on what they are doing, because in real life, cognition and action are interdependent. The only way to get information about the logic and meaning persons see in their own life is from the persons themselves. Within personal conversations and in focus groups discussions, I was dealing with people. The positivistic ideal of the non-involved spectator was impossible to fulfil. Therefore it was necessary for me to get involved to obtain the information I was seeking. The contact within the focus group went on in reciprocity and I as the observer was also observed. There was a relationship between human beings, and therefore issues about doing right or wrong. Conversations and focus groups were processes I as the researcher did together with the teachers that were participants in my research in order to get the results. The focus groups and conversations I had with the teachers were used as a means to an end which is contrasting the openness and reciprocity of a social conversation (Fog, 1993).

Soltis (1990) outlines 4 purposes of qualitative research: *description* (to describe the interpersonal, social and cultural contexts of education), *evaluation* (to evaluate with wide-ranging modes of assessments), *interventions* (bringing about change and understand its effect) and *critique* (political, cultural or social), and he address ethical issues to each one of them. The following purpose described by Soltis (1990) covers a large part of a descriptive purpose of my research.

> The classic and pervasive purpose of qualitative research has been to adopt, create, and use a variety of nonquantitative research methods to describe the rich interpersonal, social, and cultural contexts of education more fully than can quantitative research (p.249).

I am giving a narrative description of my understanding of mathematics classrooms and processes going on. However, my research project has also an evaluative character as it is an in depth study with the evaluation of the Curriculum reform in Norway (Alseth et al. 2003) as a starting point. I will attempt to "provide richer and wider-ranging assessments of educational processes, products and projects [and] add qualitative judgements of goal attainment to quantitative ones" (Soltis, 1990, p. 249).

Purpose of my research and related ethical issues

Is my research worth doing? This was a relevant question for me to ask before starting planning the research. A research not worth doing is waste of time for the people involved. Research should not be just a simple technical exercise, and being in a project you do not think is worth doing, would change much of the motivational and convictional quality (Smith, 1990). My intention was to do a study which can broadly outline how teachers interpret the mathematical part of L97 and how their implementations of it are related to their beliefs about teaching and learning mathematics.

One ethical issue to consider was if I ever can get access to teachers' beliefs. All information has to come from the teacher him/herself and in my case through conversations, focus groups, self-estimation, and their writings and from classroom observations. However, when teachers say things in a focus group or during a conversation, or in the classroom, or they write something, the beliefs that they have, that are behind what they say or write, are probably hugely more complicated than the words they express. So an ethical issue for me as a researcher was in trying to "use" the teachers' words to say something about their beliefs, to what extent I could make an interpretation of their beliefs from their utterances. This is also an issue of trustworthiness of my research and to meet this issue, I have used the different methods of data gathering.

One purpose of my research is descriptive. "Descriptive research in education has as its primary purpose a revealing of the human dimensions of some educational phenomenon" (Soltis, 1990, p. 251). The quality of the description must be of high priority here, and therefore problems regarding such things as privacy, deception and confidentiality can become key ethical concerns. It is also important to recognise that description is not neutral and doing a qualitative descriptive research as I did, placed me in a face–to–face relation with the teachers, both in focus groups, conversations and in classroom studies. Soltis (1990) emphasises the importance of the Kantian ethical imperative to treat persons as ends and not as means with regard to the principle of respect for persons. This was important for me to pay attention to throughout the study.

Various ethical issues need to be considered differently in the qualitative and quantitative domains. One reason why ethical issues in qualitative research are so different from ethics in quantitative research is described by Lincoln (1990):

> Qualitative research is a set of social processes characterised by fragile and temporary bonds between persons who are attempting to share their lives and create from that sharing a larger and wider understanding of the world (p. 287).

When the purpose of research is being evaluative, qualitative research being of evaluative character is more problematic where ethics are concerned than what is the case for research being quantitative. Whereas the measuring instruments used in quantitative research often are agreed forms of evaluation, standard tests or questionnaires, the measuring instrument used in my research which is qualitative, is myself. I therefore have had to consider honesty and fairness very carefully since they will be among the most relevant ethical principles in evaluation. I have had to be aware my own experience as a teacher and as a teacher educator as biases when trying to evaluate to what extent teachers implement L97. My project is not evaluative, and I will not contrast teachers or make a ranking list with regard to response /lack of response to the reform. I rather describe their teaching with regard to what they say and believe they are doing and also with regard to the recommendations of L97. However, in this there will necessarily be an evaluative aspect. It is of great importance to be careful not to provide negative information that can be harmful for those involved in my study.

Informed consent as ethical issue

> *Informed* means that the subject knows all that a reasonable person would want to know before giving consent – the facts and judgements or probabilities that would affect a reasonable person's decision whether to participate. *Consent* means explicit agreement to participate (Sieber, 1982, p. 15, emphasis in original).

Respect for persons and recognising the individual's dignity and autonomy are reasons for requiring informed consent: "[] the doctrine of informed consent reminds all involved in research of a categorical imperative against violations of autonomy" (Capron, 1982). However, informed consent is not a guarantee for autonomy; it can only partially protect it. Informed consent contains four elements all of which raise ethical issues: That the participant is fully informed, is competent to give consent, fully comprehends the condition of consent and gives it voluntarily (Sowder, 1998).

My teachers gave their consent voluntarily and I consider them being competent to give it. However, I can never be sure if they fully understood what subtle risks were associated with participating in this study. It was my responsibility to give full information about the research, including my research questions, why they were selected, what role they had, how I will use the data and the possibilities they had to always have the opportunity to withdraw.

I also gave the head teachers in the schools I was observing a letter where I informed about my research so they could inform the "samarbeidsutvalget" a committee consisting of representatives of parents, students and a local politician about my research.

Another issue I had to consider was that my research questions might not stay fixed and that they might change as a result of an evolvement throughout the study. It is a feature of qualitative research that the originally proposed research questions can become inadequate during the processes of the study, and that new questions can be uncovered. I discussed this with the teachers; however, my research questions did not change after I had started with classroom observations.

Students in the classroom were also subjects of my research. However, they were not put in an experimental position and they are not judged with regard to how they perform. Their experience and account of the mathematics lessons are in focus. Students were observed in their natural environment so informed consent was not an issue (Diener & Crandall, 1978). I had to apply to Norwegian Social Science Data Services, "Norsk Samfunnsvitenskapelig Datatjeneste", NSD, to undertake the research. An important issue to get permission was that no information about persons can be traced back to that person. I have therefore anonymised the data and all names used are pseudonyms. The letter of accept from NSD is enclosed in Appendix 5.

Issues related to the publication and reporting of my research

There might be conflicts between demands of science and demands of ethics (Fog, 1993). One such issue is the conflict of seeing and understanding more than I can say, both to the actual teacher and to the public. It is the difficulty being sensitive to the feelings of the teacher and yet give an accurate portrait. Ethical considerations have to be made because I as a researcher might interpret more from the data than the teacher's own understanding. The teacher has not asked me for my interpretations at all. It can cause serious ethical problems if I have to take responsibility for my understanding and write something about a teacher that diverges from her/his understanding of her/himself. This dilemma consists of the truthfulness that a scientific endeavour demands and the respect for a person's integrity and dignity on the other. This implies that I have to look critically at my scientific curiosity and my quest for knowledge (Fog, 1993).

It is important to have in mind that the data I will report have already been filtered through my theoretical position and biases (Sowder, 1998). As I have pointed out earlier, my background as a mathematics teacher and teacher educator will influence my interpretations and thus make them biased. The awareness of my presence probably influenced what was going on in the classroom and also the development of our conversations. Yet another issue to be aware in my study is the difficulty of being sensitive to the feelings of the teachers who were participants in my study and yet present an accurate portrait of the teachers.

While undertaking a qualitative study, anonymity is usually not possible. Many persons will know what school I was visiting and the teachers I was observing. Therefore the issue of confidentiality when reporting is of great importance and the real names of the participants are of course not written. Pseudonyms have been used both for the names of the schools of the teachers and of the students. Soltis (1990) writes "[] it is not hard to imagine scenarios in which the identity of those studied and reported on in articles and books can only be thinly disguised" (p.251). It has therefore only been possible for me to express thanks to the teachers and their students who participated in my research anonymously in the preface.

Some remarks

> Clearly, researchers need both cases and principles from which to learn about ethical behaviour. More than this they need two attributes: the sensitivity to identify an ethical issue and the responsibility to feel committed to acting appropriately in regard to such issues (Peshkin & Eisner, 1990, p. 244).

As human beings being part of a society we have all gained a fundamental sense or perception of moral facts which should help us to discern the moral issue in a given context. Empathy is an important concept both in daily life and in research. The research situation I have been putting myself into contained all the moral issues that I also meet in daily life. But in research they will have a specific twist that stems from the demands put on the situation by scientific standards (Fog 1993). I had to be aware that from a Kantian perspective my teacher participants might be wronged (not harmed) as they might have been treated as means to my ends. A challenge for me was to find out what I could do to compensate for the time they were spending on the research, what benefits I could offer them. I could offer them knowledge and help in reflecting on their own teaching practice. I also offered to teach lessons for the teachers if they needed some time off one day. Two of the teachers accepted my offer. However, I do not look upon that as part of my study.

Linking to the next chapters

In this chapter I have described the methods I have used in the research process; both the methods used in selecting the teachers for my study, and the methods I have used which have provided me with data to analyse in enabling me to provide some answers to the research questions. I have also discussed important ethical issues in undertaking research which involve persons. The next four chapters present my analysis. In the first, Chapter 5, I will offer a theoretical analysis of L97 and also present my analysis of the focus groups. This analysis demonstrates how the use of focus groups not only enabled me to select the teachers for my study as described in this chapter, but also provided me with data to analyse and thus highlighted issues in my study of the teachers. The three chapters on the teachers, Bent, Cecilie and David, show how the data obtained from the methods used and the way I handled the data which I have described in this chapter, enabled me to carry out the analysis of the teachers.

5. Mathematics, L97, and Teachers' Interpretations of L97

This is an introductory analysis chapter meant to inform, highlight key issues in my study and form a bridge to the analysis of the three teachers', Bent, Cecilie and David, responses to L97 and their teaching. Taking the title of this thesis, "Mathematics Teachers' Interpretation of the Curriculum Reform, L97, in Norway", as a starting point, I will address three themes: *Mathematics, L97,* and *Teachers' Interpretation of L97.*

In this chapter I offer a theoretical analysis of *L97* using the theoretical perspectives discussed in Chapter 3. Since this is a study of *mathematics* teaching, I start this chapter by discussing aspects of mathematical knowledge more generally and which I use in the analysis of the curriculum presented in next part of this chapter and also in the analysis of the teachers in Chapters 6, 7 and 8.

In the previous chapter, Methods, I described how I used Focus groups to select teachers for my study. The data collected from the focus groups, as well as enabling me to select teachers for the study, highlighted issues in the study of the teachers. In the second part of this chapter I present an analysis of the third Focus group meeting in which all teachers in my study participated. Thus this analysis of the teachers in a focus group preceded and informed the analysis of each individual teacher which I present in the successive chapters.

In order to analyse *teachers' interpretation* of a reform I will discuss aspects of teachers' beliefs and teachers' teaching practice. Studying the teachers' teaching strategies and the patterns of discourse in the teachers' classrooms has been valuable in the analysis of teachers' teaching practices. I provide here an introduction to my analysis of discourse.

Aspects of mathematical knowledge

Procedural and conceptual knowledge

For many persons knowing mathematics is being skilful in performing procedures and is related to basic mathematical concepts (Thompson, 1992). This view refers to what is described in the literature as *procedural knowledge*. Hiebert and Lefevre (1986) viewed *procedural knowledge* as made up of two parts. One part is the symbol representation system which is used to write mathematics in a certain way; the other part consists of the rules, algorithms and procedures which can be used to solve mathematical tasks. According to Brekke (1995) *procedural knowledge* focuses on exercising skills and procedures which can make students believe that mathematics is only a collection of isolated skills and rules.

Molander (1993) called the kind of knowledge, when students are supposed to apply a technique they have learnt rather than to think, *technical knowledge*. Technical knowledge is mainly a mastery of a technique. It does not lead or direct action. In a survey done of mathematics textbooks, he found that they to a large extent contained answers without questions, “the questions were there to fit answers” (p. 123). Skemp (1976) distinguished *instrumental understanding* from relational understanding, saying that students develop instrumental understanding if they are taught rules without any further explanations. Skovsmose (1994) distinguished between mathematical knowing, technological knowing and reflective knowing in mathematics. *Mathematical knowing* refers to the competence normally understood as mathematical skills (focused on in traditional mathematics education), which often is about correct use of algorithms.

Conceptual knowledge is not isolated but exists in a network rich in relationships (Hiebert & Lefevre, 1986). Creation of relationships between existing knowledge enhances conceptual knowledge and understanding. Skemp (1976) called this *relational understanding* and claimed that if students have developed relational understanding, they know for example *why* they solve an equation as they do, not only *how* to do it. Having relational understanding makes them motivated to explore further to find new relations. Molander (1993) used the term *directive knowledge* about knowledge that directs us, makes us see what is important and what we ought to do. He emphasised that a statement is not a piece of knowledge unless the question it is intended to answer is fully understood and the answer is understood as an answer to the question.

According to Skovsmose (1994) *technological knowing* refers to abilities in applying mathematics, the competence of selecting and applying the algorithm, using the right algorithm and *reflective knowing* refers to the competence in reflecting upon and evaluating the use of mathematics, evaluations of the consequences of technological enterprises. His terms, *technological and reflective knowing*, encapsulate the notion of conceptual knowledge. Referring to Piaget, Hiebert and Lefevre, taking a constructivist stance, wrote that regardless of what term we use, the process of creating conceptual understanding of new knowledge involves assimilating new knowledge into existing structures so that the new knowledge becomes part of an existing network. In his outline of relational mathematics and instrumental mathematics, Skemp (1976) claimed that there is not a discussion about better or worse way of teaching mathematics, but that there are two different subjects taught under the same name, mathematics.

According to von Glasersfeld (1995) the behaviourists succeeded in eliminating the distinction between training for performance and teaching for conceptual understanding which he claims has had unfortunate consequences for education: “it has tended to focus attention on students’ *performance* rather than on the *reasons* that prompt them to respond or act in a particular way” (p. 4, emphasis in original). Von Glasersfeld (1995) looked upon students’ eventual comprehension based on training rather as “fortunate accidents”. In solving problems within different contexts and in contexts differing from the ones in the textbooks, conceptual understanding is needed. Fifteen years of research on reasoning at the University of Massachusetts showed that students who were capable of giving right answers to standard questions in physics were not able to solve simple problems differing from the ones in the textbooks. Von Glasersfeld put part of the responsibility for this on the notion that competence in intelligent behaviour can be achieved by drilling performance. He wrote:

> The solving of problems that are not precisely those presented in the preceding course of instruction requires conceptual understandings, not only of certain abstract building blocks but also of a variety of relationships that can be posited between them. Only the student who has built up such a conceptual repertoire has a chance of success when faced with novel problems. Concepts cannot simply be transferred from teachers to students – they have to be conceived (Glasersfeld, 1995, p. 5).

Earlier, Cobb (1988) had noticed the same feature within mathematics educational research. He referred to

> an abundance of research [which] indicates that students routinely use prescribed methods to solve particular sets of tasks on which they have received instruction without having developed the desired conceptual knowledge (p.90).

Cobb thus questioned the instructions given which turned out to be successful only in the acquisition of specific skills, and he therefore suggested considering a constructivist perspective. He claimed that to be sure students have developed powerful conceptual structures they must demonstrate abilities to solve problems given in other situations than where the learning took place and also to be able to build on the conceptual structure in other domains.

Investigative mathematics and research mathematics

In the analysis of the mathematical aspects in L97 later in this section, I discuss how I see an investigative approach to teaching mathematics being encouraged and how the working methods described are closer to that of research mathematics than traditional school mathematics. I also argue for how I see this approach and working methods developing conceptual understanding. The discussion is based on the following researchers’ work:

In her study, Jaworski (1994) outlined common features of the mathematics classroom that for her seemed to be investigative: the tasks were inviting inquiry, and encouraging conjectures and justifications; the student’s thinking process was emphasised; the class was organised mainly in groups; many of the activities made use of physical objects; the teacher spent the time listening to and talking *with* the groups. Smith Senger (1998/1999) used the term “reform curricula” which emphasised problem solving and reasoning, use of concrete materials and technology, group work and communication. The role of the teacher is being a guide, listener and observer rather than a traditional authority and answer giver. Norton, McRobbie, & Cooper (2002) studied several teachers’ responses to an investigative mathematics syllabus, “Curriculum documents that are investigative reflect theories of learning consistent with major elements of social constructivist theory” (p.38).

Ernest (1998a) compared investigative mathematics, or reform mathematics, with research mathematics. He argued that the introduction of investigational work in school mathematics involves a major shift in rhetorical style:

> For instead of representing only formal mathematical algorithms and procedures, with no trace of the authorial subject, the text produced by the student may also describe the judgements and thought processes of a mathematical subject (p.257).

Ernest (1998a) compared school mathematics and research mathematics; how they are similar and how they differ. Both are intended to develop mathematical knowledge. His account of school mathematics describes the development of personal knowledge whereas the account of research mathematics describes development of public mathematical knowledge. He also identified how the culture of school mathematics differs from that of research mathematics. The school mathematics described consists of carrying out mathematical tasks, getting feed-back from the teacher and then carrying on with the same kinds of exercises. Research mathematics consists of sending a work (it might be a mathematical proof) off to a referee to have it judged.

Ernest (1998a) also described the nature of a classroom culture for mathematics in schools which can be seen as being close to research mathematics. Students work collaboratively with several mathematics books, exercise books, calculators, mathematical apparatus, instruments, pencils and coloured pens. Students discuss how to present their results of an exploratory activity in mathematics. After having presented their work, the rest of the class can pose questions. Later the teacher gives them a written assessment to which they shall respond and then discuss with the teacher. The teacher’s final assessment and grading are based on the students’ work, their responses and the discussion:

> A classroom like that described [ ] embodies a culture that is closer to that of the reform movement in mathematics education. The process of problem posing and solving; of representing, conjecturing, testing, and of mathematising in general, resemble the process of knowledge generation applied in the context of research

> mathematics. Many mathematics educators argue that this is the culture that it would be most productive to emulate in school mathematics (Ernest, 1998a, p. 258).

Cobb, Wood and Yackel (1990; , 1995) took such a view of mathematics learning as a basis for their constructivist whole class teaching experiment at first grade level, which sought to create opportunity for explanations and argumentations in mathematics to develop conceptual understanding.

The terms describing research mathematics, investigative work, reform curricula and teaching experiment in the mathematics classroom in the presentation above, can be compared with views reflected in L97 which I discuss further in my analysis of mathematical aspects reflected in L97 on page 108.

L97 – A theoretical analysis

Here I give a theoretical analysis of L97 which is based on a qualitative content analysis. Contrary to a quantitative content analysis which typically entails applying predetermined categories to the data, qualitative or ethnographic content analysis "employs some initial categorisation, but there is greater potential for refinement of those categories and generation of new ones" (Bryman, 2001, p. 381). According to Bryman, extracted themes are often illustrated with quotations. The themes from the analysis I did of L97 are illustrated with quotations from L97.

Since the committee which developed the written mathematical part of L97 said they were inspired by constructivist theory in their writings, I first present an analysis of L97 from a constructivist perspective before I present an account of how I see socio-cultural theories reflected in the curriculum. I then present an account of the mathematical focus comprising the kind of mathematical knowledge I see is reflected in L97 followed by a discussion of school mathematics versus research mathematics and, last, I discuss how I see L97 reflecting an investigative approach to mathematics.

L97 consists of three main parts, which I have outlined in the introduction, Core Curriculum, Principles and Guidelines and Subject Syllabuses (embracing the mathematics syllabus). When quoting from L97 I indicate from which part the quotation is taken with CC, PG and M together with the page number in the English version of L97.

L97 from a constructivist perspective

There are a number of quotations in the curriculum which suggest a constructivist origin and lead to associated questions related to constructivism. Here I present 4 quotations from L97 which I will discuss further below:

1. Pupils build up their knowledge, generate their skills and evolve attitudes largely by themselves (L97CC, p.34)

2. The subject syllabuses stress that pupils should be active, enterprising and independent. Pupils should learn by doing, exploring and experimenting, and in so doing acquire new knowledge and understanding (L97PG, p.83).
3. Mathematics has a variety of aspects, and learning takes place in a variety of ways. Pupils' experience and previous knowledge, and the assignments they are given, are important elements in the learning process (L97M, p.167)
4. Learners construct their own mathematical concepts. In that connection it is important to emphasise discussion and reflection. The starting point should be a meaningful situation, and tasks and problems should be realistic in order to motivate pupils. At times pupils may work with incomplete concepts, misconceptions, and they make occasional mistakes and misunderstand things. In a confident and constructive atmosphere such matters are grounds for further learning and deeper insight" (L97M, p.167).

The first two quotations are taken from the Core Curriculum and from Principles and Guidelines respectively. These together with the third quotation, which is taken from the Mathematical part, seem to indicate an individual approach to learning reflecting the central notions of assimilations and accommodation in Piaget's theory which I discussed in the section about constructivism in Chapter 3 (page 53). In the third quotation emphasis is put on students' experience and previous knowledge and thus the importance of what is already learned. This can be seen to indicate Piaget's view that conceptual structures do not have a fixed starting point and that already constructed cognitive structures will always be in a person's mind.

For the learning of mathematics the capability of recalling a past experience to make comparisons with a present experience is important. The Minister of Education at the time, Gudmund Hernes, told the committee to have the following in mind while they were working with L97: "In every lesson something new shall always be learned, and learning something that is new shall never take place"[13]. One of the members of the committee whom I interviewed cited this when I asked him about the emphasis in the mathematical part of L97 on students' previous knowledge. I interpret this as: "students shall learn something that is new in every lesson, however, nothing new should be introduced without it being possible to relate to existing knowledge." In my study I found that the teachers I observed frequently referred to students' experience and prior knowledge about which I go further into detail in the next three chapters.

In the fourth quotation from L97 above, reflection and discussion are emphasised as important in the learning process. According to the com-

[13] In Norwegian: "I enhver time skal det alltid læres noe nytt og det skal aldri læres noe som er nytt"

mittee developing the mathematical part of L97, it was Piaget's notion of "reflective abstraction" they had had in mind when formulating this. Discussion in the learning process is also emphasised. This quotation from L97 also deals with *incomplete concepts* and *misconceptions*. The emphasis on "a confident and constructive atmosphere" is in accordance with von Glasersfeld's (1995) writings about misconceptions that it is important for the students to "be led to see as their own problem" (p. 15) and for the teacher to build up a model of student's conceptual structure.

However, just as scholars within constructivist theory have pointed out the problematic use of the term misconception, which I discussed in Chapter 3 (page 55) the committee writing the mathematical part of L97 found the term problematic. According to my interview with the leader of the committee, they were unsure about the use of the term because they looked upon it as focusing on errors. Also, in terms of diagnostic teaching where misconceptions are supposed to be brought into view to provoke a cognitive conflict, they felt uncomfortable in "making traps" for the students. According to the fourth quotation from L97 above, a *misconception* is an *incomplete concept*. I see the misconceptions' "grounds for further learning" to be emphasised in that quotation: According to L97, a misconception is an incomplete concept but has the potential to develop into a complete concept through the work in the mathematics classroom.

L97 from a socio-cultural perspective

L97 encourages rooting mathematics in practical matters. It sees pupils' own experience to play an important role and it encourages the use of play and games in the teaching-learning process. "In play and games pupils can participate in making the rules themselves, learn to abide them, and see consequences of their choices" (L97 M, p. 167). According to Vygotsky (1978) play is a leading factor in a child's development and he emphasised the relation between action and meaning in real life and in play.

Attention is also given to the use of mathematics in society: "Mathematical knowledge and skills are an important foundation for participating in working life and leisure activities, and for understanding and influencing social processes" (L97 M, pp. 165-166). Thus the social and everyday uses of mathematics play a central role in L97. The variations between individuals, their cultural conditioning related to their different backgrounds are also emphasised. In the Core Curriculum it says: "The cultural baggage that learners carry with them, from the home, local community or earlier schooling, determines which explanations and examples have meaning" (L97 CC, p. 36). In the mathematical part the same issue is dealt with in the following way:

> The practical applications and methods chosen are meant to ensure that girls and boys alike, and pupils with different cultural and social backgrounds, have the opportunity to experience a sense of belonging and to develop favourable attitudes to the discipline (L97 M, p. 165).

Thus the curriculum encourages taking factors such as students' different social and cultural backgrounds into account when choosing examples and explanations to make sense for all students.

One of the general aims for mathematics in L97 is "For pupils to develop skills in reading, formulating and communicating issues and ideas in which it is natural to use the language and symbols of mathematics" (L97 M, p.170). Communication in the learning process and the use of language are thus emphasised. As discussed in Chapter 3, (page 58), the use of language in communication with others is according to Vygotsky the most important tool in the process from social knowledge to personal knowledge.

Important in Vygotsky's theory is his emphasis on the individual mental plane constituted as part of the social plane, which I discussed in Chapter 3 (page 57). The following quotation from the Core Curriculum emphasises the importance of the social plane and the interactions between teacher and student(s) and among students, and thus the classroom discourse, in the learning process.

> Teachers are the leaders of the pupils' community of work. Progress thus depends not only on how teachers function in relation to each pupil, but also on how they make each of the pupils relate to the others. In a good working team, the members enhance the quality of each other's work (L97, CC p. 39).

This is in accordance with Lerman (2000a) who placed the social practices and the teacher in pivotal roles in the learning process because he looked upon the child as not being able to reach an objectivity of mathematical structures on his/her own. I discussed this in Chapter 3 (see page 60).

L97 puts weight on the classroom discourse; "In *a confident and constructive atmosphere* such matters [i.e. incomplete concepts, misconceptions and occasional mistakes] are grounds for further learning" (L97 M, p.167, my emphasis). This quotation together with that "mathematics teaching must at all levels provide pupils with opportunities to [ ] – work cooperatively on assignments and problems" (L97 M, p.168), indicates collaboration and adult guidance and put weight on the social practice, the discourse and the role of the teacher in the classroom. According to my interpretation of L97, the teacher's role is to give adult guidance and to support students in becoming participants in classroom activities and to offer students learning possibilities in the zone of proximal development. Teachers are encouraged by L97 to help, maintain, develop and systematise the mathematical concepts children have when they start school.

In the analysis of the teachers in my study I focus on their use of language and how the discourse in the classroom, and shifts in discourse, creates possibilities for students' learning. Also the complexity of the classroom is an important aspect in my analysis.

All learning the child will meet in school has a previous history and all children have experiences with quantities, determination of size and arithmetical operations (addition, subtraction, and division) long before they start school (Vygotsky, 1978). Thus L97's emphasis on the knowledge children have before starting school can be accounted for from a socio-cultural perspective as it can from that of constructivist.

Mathematical focus in L97

In the first part of this chapter, I discussed aspects of mathematical knowledge generally, and how researchers within mathematics education have written about these aspects. I will now use the aspects discussed in analysing the mathematical knowledge I see reflected in L97.

Procedural and conceptual knowledge in L97

According to Thompson (1992), a common view on what knowing mathematics implies, is being skilful in performing procedures and identifying basic mathematical concepts. Unlike this common view, L97 is focusing more on conceptual knowledge and less on procedural and factual knowledge than earlier curricula. One of the general aims for mathematics in the curriculum which emphasises this is:

> for pupils to develop insight into fundamental mathematical concepts and methods and to develop the ability to see relations and structures and to understand and use logical chains of reasoning and draw conclusions (L97M, p.170).

This aim and the next quotation from "Approaches to the study of mathematics" emphasise conceptual understanding and relations among concepts and that students are encouraged to see structures within the subject:

> Pupils who have difficulties with memorising the basic multiplication facts must nevertheless be free to proceed to concepts and tasks involving the multiplication concept. Understanding multiplication and how to use it is more vital than memorising the tables (L97M, p.166).

The connections between development of procedural and conceptual knowledge, what is most important or what is a reasonable balance between them, have been discussed within research in mathematics education over the years (Hiebert & Lefevre, 1986). The above quotation from L97 emphasises the opposition between rote learning and conceptual understanding in mathematics and puts weight on the latter.

I suggest that experiences with students, who had only worked routinely with prescribed methods to solve particular sets of tasks and thus had not developed conceptual understanding, was partly the background for the focus on conceptual understanding and the expressed constructivist position among the members of the committee developing the mathematical part of L97. At the time L97 was written (mid nineties) constructivism was the dominant theory of learning. Based on what I discussed earlier in this chapter about procedural and conceptual knowledge and instrumental/relational understanding, I see the relation between constructivism as a cognitive perspective and teaching for conceptual understanding reflected in L97. Members of the committee were influenced by constructivism because they wanted to emphasise teaching for students' conceptual understanding rather than performance and to focus on the reasons why students respond in a particular way. I look upon the quotation from L97 above not only being valid for multiplication as it stands, but as an example more generally to focus on students' conceptual understanding rather than performance.

Investigative mathematics and research mathematics

As indicated in Chapter 1, both members of the committee which developed the written mathematical part of L97 whom I interviewed, confirmed my conjecture that the mathematics and the working methods reflected in L97 are closer to research mathematics than that of earlier curricula. While discussing this issue, they referred to the working methods described in the introduction to the mathematics part which they saw reflecting research mathematics, pointing to the following quotation:

> Pupils' own activities are of the greatest importance in the study of mathematics. The mathematics teaching must at all levels provide pupils with opportunities to
>
> - Carry out practical work and gain concrete experience;
> - Investigate and explore connections, discover patterns and solve problems;
> - Talk about mathematics, write about their work, and formulate results and solutions;
> - Exercise skills, knowledge and procedures;
> - Reason, give reasons, and draw conclusions;
> - Work cooperatively on assignments and problems.
>
> (L97M, p.168).

In the first part of this chapter I presented Ernest's (1998a) description of a classroom culture which is close to research mathematics and how he saw traditional school mathematics being different from research mathematics. Ernest's description of research mathematics presented on page 103, suggests a similar kind of culture in the mathematics classroom, as can be inferred from the quotation from L97 above.

Emphasising the approach to research mathematics discussed above, the members of the committee which wrote the mathematical part of L97, pointed to the working methods encouraged in the curriculum. They emphasised that the working methods described in the introduction to the mathematical part were closely linked to the mathematics to be taught. The working methods described in L97 are in line with Ernest's description of the culture of a classroom which is close to research mathematics and which mathematicians find most productive based on the reform movement in mathematics education. Based on the descriptions of investigative mathematics, reform mathematics and teaching experiment presented earlier in this chapter (Cobb, Wood, & Yackel, 1990; Ernest, 1998a; Jaworski, 1994; Norton, McRobbie, & Cooper, 2002; Smith Senger, 1998/1999; T. Wood, Cobb, & Yackel, 1995), I see an *investigative approach* as a collective term for what L97 suggests as being important for the teaching of mathematics:

- [], mathematics will also invite pupils to use their creativity and to experience its aesthetic aspects. Mathematics poses challenges to pupils' inventiveness and critical and analytical abilities. As they *experiment*, *experience*, *wonder* and *reflect*, the subject will help to develop the pupils' curiosity and urge *to explore* (L97M, p.165, my emphasis).
- In work on assignments and *problems involving problem solving and investigations*, calculators and other terms of information technology open up opportunities for new approaches. In works of this kind, it is especially important to understand numbers and operations, to be able to interpret diagrams and geometrical figures and to be able to make estimates and to *consider reasonable results* (L97M, p.167my emphasis).
- One of the general aims for mathematics is: for pupils to be stimulated to use their imaginations, personal resources and knowledge to find methods of solution and *alternatives* through exploratory and problem-solving activities and conscious choices of resources (L97M, p.170 my emphasis).

The verbs emphasised in the first quotation all indicate activities that are investigative. Investigative mathematics includes creativity, exploring activities and experimenting. The student him/herself has an active role in the learning-process. Work on open ended tasks including problem solving, justification and reflection are important. Students are working in groups rather than individually. Teaching is not only direct instruction from the board and not only individual seatwork on exercising skills and procedures. I see the investigative approach to mathematics being encouraged in L97 as to a large extent focusing on developing conceptual knowledge.

The terms describing an investigative approach in L97 can reflect elements of both constructivism and socio-cultural theories. Activities described by these terms generate understanding and they acknowledge both the importance of students' active constructions of knowledge and interactions with others in a classroom. This is the kind of mathematics referred to as research mathematics, investigative mathematics, reform curricula, and teaching experiment in the previous section.

Focus groups

Analysing focus groups

In my study I wanted to explore the interpretations teachers made of the L97 curriculum. To start this process, and to select teachers for my study I decided to arrange focus groups in which I would discuss with teachers their perspectives on L97. In the previous chapter, I described how, based on focus group meetings, I selected the teachers for my study. I also found it valuable to use data from the focus groups as part of the resource for analysis, particularly for the purpose of triangulation[14] and supporting the other sources of data from the teachers' utterances (conversations, self-estimation, writings and questionnaire). I audio recorded and transcribed the discussions that took place in these groups. In this part I present some findings from these meetings which highlighted issues from the perspectives of L97 discussed in the previous part of this chapter, in the study of the teachers' responses to L97 and their teaching.

I have chosen to present some of the analysis and extracts from the focus groups together with the analysis of the conversations I had with each teacher. This is when I see what the teachers said in focus groups in relation to the analysis of the conversations and classroom observations I had with them. The analysis of Focus group 4 (FG 4), which took place towards the end of my data collection, is presented in the concluding chapter and provides further evidence for my findings of the analysis of each teacher and serves the purpose of a cross case analysis.

Focus group 1 and Focus group 2

Before presenting a more detailed analysis of FG3, in which all teachers with whom I carried out classroom observations participated, I will briefly comment on FG1, which was held in June 03, and in which none of the teachers from the main study participated, and FG2, which was held in September 03, in which only two of the teachers whom I studied in their classrooms participated.

For FG1 and FG2 I had prepared some general statements for each member of the group to rank with regard to importance. Afterwards, I asked them to agree on a ranking of the statements based on the discussion in the group. L97 was not explicitly mentioned in any of the statements, however, throughout the discussion the teachers brought it up. In FG1, when one of the teachers from lower secondary school suggested that L97 was a "delusion" the other teachers agreed; the curriculum contained too much; it was nicely written with beautiful pictures. However, having students with all levels of abilities in one class made it impossible to implement L97. They also said that when studying the L97 curricu-

[14] The use of more than one method or source of data in the study of a social phenomenon so that findings may be cross-checked (Bryman, 2001, p. 509)

lum, they felt inadequate and that it imposed a bad conscience on them. This agreement among the participants in the focus group illustrates the issue of group effect discussed in the previous chapter. However, with regard to ranking the statements in this focus group which was with 6 teachers from all three stages in 1-10 schools I learned that the teachers in primary school valued the statements differently from teachers in lower secondary school. Teachers in primary school liked the working methods encouraged in L97 better than teachers in lower secondary school did. Based on interviews with teachers in all stages, the evaluation of the mathematical part of L97 reports similar findings (Alseth, Brekke, & Breiteig, 2003).

In the next focus group, FG2 in which the participants were all teachers in lower secondary school, in which the teachers from Dalen (Cecilie and David) did not participate, I experienced a tendency that each teacher mostly talked about and presented examples from own teaching practice and rarely responded to each other's contribution. However, there was one issue discussed of which they were all challenged; what to answer students' questions like: "What do we need this for?"[15] In that connection the teachers agreed upon the importance of the relation between school mathematics and the social and everyday use of mathematics emphasised in L97. I discussed this aspect of mathematics with regard to L97 in the section "L97 from a socio-cultural perspective" in the previous part of this chapter (page 106). There was also a common agreement in the group that it had been interesting to discuss issues about L97 and mathematics teaching and learning in a focus group.

Focus group 3, October 2003

Focus group 3 (FG 3) was also conducted before I started the case study of the teachers. This was the first meeting with some of the teachers who eventually became part of my case study. The teachers participating were the four teachers in my study: Alfred and Bent from Haugen School and Cecilie and David from Dalen School. In addition Petter and Kari from Haugen participated and I had also invited one of my former students, Tom, who was now working as a teacher in a lower secondary school in Oslo.

For this focus group I had prepared the following questions for discussion:

- What in your opinion is important competence for mathematics teachers?
- In what way do you relate your work to L97?
- Has L97 inspired you to try out new activities in your mathematics teaching?

[15] "In Norwegian students ask: "Hva skal vi med dette?"

- What do you think is the greatest challenge in your work as a mathematics teacher?
    - What have you succeeded with?
    - What do you think you have not yet accomplished?

I started with the first question explicitly, and aspects of other questions were addressed as part of the discussion. However, there was no time to discuss the last two parts of the fourth question. What the teachers felt they had succeeded with and what they found they had not accomplished, were issues explicitly discussed in the fourth focus group meeting later in my study. An analysis of this focus group meeting is presented in the final chapter.

Focus groups from a socio-cultural perspective

In analysing focus groups, from a socio-cultural perspective, I asked how does what participants say reflect meanings of the group or society more widely? How does what they say reflect aspects (including criticism) of the political and cultural society, of dominant groups influencing the official educational discourse (Lerman, 2000d), of their own school situation as a teacher or the one they had as a student themselves? Or how does what they say reflect aspects of the curriculum?

To illustrate this I will provide an example from FG3 which shows use of rhetoric. David knew who I was; he knew I was a teacher educator; he knew I had carried out courses for teachers in relation with the curriculum reform. Therefore, I conjecture David thought I wanted to hear nice things about the curriculum. David knew what L97 was saying (or he acted as if he did). Based on his understanding of what L97 said, he challenged it. This could have been because he wanted to position himself within the group, but it could also have been because he really meant that L97 is not a good curriculum for the mathematics subject. Yet another way to interpret what he said and why can be that he did not really know what the curriculum was saying, and he wanted to react reluctantly to it from the very beginning. In the quotation below, Petter (P) indicated he was sceptical to L97. David (D) then said (sarcastically?): "there are some nice pictures in it". That illustrated how teachers argued for or against a new curriculum, how they interpreted it. The language (also what was not said) was a mediating tool in the exchanges of meanings. Petter was the most experienced teacher in the group and had a special role here. He indicated something to which David responded and it illustrates how what they said was deeply embedded in the socio-cultural setting in the group and their experience.

B.K:	L97, how well do you know it? P, you seem dying to say something…
P:	Yes, I feel I am getting hot-headed when you mention L97.
D:	There are some nice pictures in it (sarcastic?)

[ ]

BK: Now we have talked very much about how L97 is weighting the mathematical topics. But what about the working methods it initiates? Do you have any opinions about that?

D: Read the newspaper, many interesting writings about it there. (There had been written many critical articles in the newspaper about L97 recent days)

BK: But what do *you* mean?

D: I am critical to the correct pedagogical view we are served from above. I am not sure if it is right.

BK: Can you say some more about it?

D: I believe that maybe pupils learn most if they have a teacher who knows their things, is enthusiastic, finds teaching being fun, who is a good motivator, and good in making the pupils function together. I really believe that the learning outcome becomes better then than if the students have lessons outdoor, working schedules and so on. I dare being that old fashioned, I think so.

P: One must be allowed to disagree with L97? Or?

D: Disagree, and say it over and over again, everywhere you are

BK: I want to know what your disagreement is about. Not only saying that you disagree with the pedagogical view from above. What is the pedagogical view coming from above?

D: I think it involves loss of flavour, things we are supposed to do nowadays. I think it implies knowledge's loss of flavour. Projects where pupils find something on the internet, print it out and read it with a few replacements of words in front of the whole class.

BK: Is that what L97 says?

D: No, but that is what happens.

My experience with Petter and David, and to a certain degree also Alfred (he was not so outspoken as the other two) in this focus group was that they were supporting each other with regard to a kind of ignorance towards L97. They had been teaching mathematics for many years, and they expressed their frustration of how the "old" kind of mathematics, especially algebra, was not in the curriculum any more to the extent they wished. Their mutual support in these views expressed in the focus group can be looked upon as communication of a rhetorical kind.

Next I will provide an example of how what teachers said in the focus groups reflected aspects of their experience as a teacher. Reflecting on the utterance from Bent below, he talked from a socio-culturally related everyday experience. Bent offered us something about the way he operated in the classroom. He spoke from his experience as a teacher, and what he had learned from this experience. From the quotation below it may be hard to understand what he meant, which demonstrates his struggle to express his experience. He said that teaching from the board could start off from a simple level. However, very soon what was presented from the board became too difficult for some students whereas others wanted to proceed even further. This illustrates the challenge of having students with different abilities in the same class. He said:

> I think a typical course, when you shall start with a new topic, is to teach from the board in the beginning and to start with something simple and then build it up to a certain level, and to work on tasks parallel to that. At a certain level you just have to stop the lecturing and separate. Some disappear far up and some remain on that level if they have at all reached the level they should. After that it is almost impossible to deal with teaching.

In the next section I will discuss how Bent went beyond his experience and offered us some of his reflections on his teaching.

Aspects of teachers' confidence

When studying the transcripts, which I had imported into NVivo, I noticed how the teachers expressed differing degrees of confidence throughout the discussion. Bent suggested the ability to motivate the students, and the importance of having mathematical knowledge to get an overview of the subject oneself, as competencies for a mathematics teacher. He used the expression "I am trying to …" when relating these competencies to his own practice: "I am trying to relate to practical issues, trying to make a relation to real life in a way, however I don't always manage". He was "trying to" make the students see the relevance in what they worked with; he was "trying to" convey the mathematics' intrinsic value, especially when it was not so easy to relate the mathematics to students' everyday life. He also said that he was trying to be enthusiastic. His use of words when speaking from his classroom practice revealed that he was not sure if he succeeded in doing what he thought was important, but he was trying. Continuing the quotation from Bent above, he went beyond his everyday experience in saying something about the issues that arose for him when he operated in certain ways, and his thoughts about it. Bent also revealed some of the "weaknesses" he perceived in himself as a teacher. He had tried out something but through what he said he demonstrated awareness that this might not have been the right thing.

> Then you have to walk around giving tasks. Last year I optimistically tried MUST tasks, OUGHT tasks and MAY tasks, that they should try to stretch themselves, but I didn't succeed in making it work. It turned out to be that they did what they had to (MUST) (agreement in the focus group), and some just tried OUGHT. But if they had homework in other subjects, they chose the less challenging way. So then it was easier to do as P says, give many tasks and rather reduce for those who need it. It is easier to put pressure on those who need challenges.

By saying this Bent also demonstrated that he had reflected on his own practice as a teacher. Being able to put his weaknesses as a teacher on the spot like this and sharing it with me and the other teachers in the group, I do not interpret as lack of self confidence but rather as reflecting a teacher who had faith in himself and had self confidence enough to be able to see his own teaching from more than one point of view. He had been able to step aside to consider his own teaching.

Bent also offered us his reflections on different levels of students' learning of mathematics, in which the other teachers consented, but without any further discussion. Bent said: "I have a feeling that they learn on different levels". He said that on one level they learn to solve a problem theoretically and perhaps manage to solve a similar problem in a same kind of context: "you have learned it in one setting on one level". The next level he said:

> is being able to carry out what you have learned theoretically for example about symmetries, and applying that when searching for and finding symmetrical patterns in a carpet: Going out looking in math-morning [which was the project work he talked about], having to apply it, then you learn and experience on a higher level.

He called this an "application competence". On yet another level you learn by expressing a problem orally. He said: "Formulating a problem for others is yet one level of learning".

When Tom said he felt that he did not know how to make students understand, especially those with "two"[16] in mathematics, David responded:

> I believe you'll have to live with that as a teacher. It is classical. You can work with some students throughout three years and they do not see /understand /remember the difference between $2x+2x$ and $2x \cdot 2x$. Even if you stand on your head and invent all possible variations you can think about there will still be some I believe [who will never manage], regardless of how clever you are as a teacher.

By saying this David demonstrated confidence as an experienced teacher. He spoke from his own experience as a teacher, an experience he knew that Tom did not have. This utterance also reflects a view that not all mathematics is for everybody, and that you cannot put the responsibility for this (the "two-students" not understanding or remembering) on the teacher. Through his long experience as a teacher, David had learned to accept this and he was now telling that to Tom who was a younger and less experienced teacher.

Cecilie also demonstrated self-confidence when telling about how she was handling the issue that students with different abilities in mathematics were placed in the same class. She had mixed two classes and grouped them according to interest in mathematics. I describe this further in the Cecilie chapter. She expressed her disagreement with Tom who had said that clever students will always manage, and she recommended the other teachers to group the students according to abilities ("interests") the way she was doing.

[16] He referred to getting the grade (mark) 2 in mathematics which is the lowest passing grade. 6 is the best grade.

The above discussion about aspects of teachers' confidence demonstrates how such information can be obtained through the use of focus groups. The way in which teachers expressed their confidence in own teaching practices highlighted issues of their teaching practices and informed my investigation of how they responded to a curriculum reform.

Mathematical focus

To highlight issues of my study of teachers' mathematics teaching, it was useful to study what aspects of mathematics they talked about in the focus group. One significant aspect throughout the conversation in the focus group was that *algebra* was the mathematical focus teachers mentioned most frequently when expressing their meanings and exemplifying from their teaching. David referred to algebra several times and was very concerned about algebra having been toned down in the new curriculum and said that he put more weight on algebra, equations and functions than L97 suggests. He also said that he would keep doing it because some students would need it for further studies. David said he was not so eager to force all work within mathematics into an everyday context: "I am more concerned that mathematics is a logical and playing subject[17]. When the students have done a huge algebra task and say YES 'I have managed', that makes me happy".

Bent also referred to algebra when expressing the importance of the mathematics' intrinsic value[18]. He expressed the value in itself of having the knowledge to solve an algebraic task or equation. However the focus on algebra was not so characteristic in what Bent nor Cecilie expressed as in what the other teachers expressed. The main mathematical focus in Bent's talks in this focus group was about having carried out a project work in mathematics which had been very successful. L97 encourages interdisciplinary project work and also project work within each subject. It was one of the latter in mathematics Bent referred to.

Cecilie mentioned algebra together with mathematics history as exciting topics to work with in her teaching of mathematics. However, she did not contribute very much to the discussion in this focus group (her proportion of talk was the lowest in the whole group).

The rulebook and the exam

Other topics coming up during the course of the discussion in the focus group were *the exam and the rulebook*. According to the reform, students should produce their own rulebook where they could write down all solved tasks, rules, formulae, methods, etc. as they wished. This is a book they can bring and use on the final exam in mathematics. I noticed a common scepticism to the rulebook among the teachers. Alfred said

[17] He said: "et logisk lekende fag"

[18] He said: "matematikkens egenart"

that he did not think the rulebook served any good for the mathematics. He did not think the rulebook was good for the students in their learning of basic mathematics. If it had been up to him, he would not have had the rulebook. Petter also expressed his scepticism. Issues concerning the rulebook were: Who was going to write it, what was supposed to be in it and what constraints should the teacher put on the writing and use of it. One constraint from the educational authorities on the rulebook was that it was supposed to be self made. David clearly stated that this would give the clever student a good rulebook and the weakest student a less good book. He found this unfair and challenged this demand: “I make as good summary as I can and hand that out to my students so they can paste it into their rulebook, with blank pages in between where they can make their own notes”. His reason for doing it this way and claiming that he legally did so was:

> If I had taken the pages I hand out and put it on an overhead and told them to copy it, then it would probably have been legally done. And that is quite comic. They have to do it with their own handwriting instead of me ensuring that everybody gets a good book.

By saying this and doing it this way, David told us that he only looked upon the rulebook as a product to be used in an exam. He did not consider the learning outcome from the process of making it, which is part of the intention of the rulebook. What David said also reveals his sincere care for all students. He said: “it is unfair with regard to the weaker students who are not able to make a good and useful book as possible”.

After the introduction of the use of rulebook on exams, which was part of the reform, R97, the nature of the tasks on the exams had changed. With regard to the new exam David said: “I think many of the tasks on the new exam are very OK. What can be discussed is how they are weighting the types of tasks”. This is the only favourable expression about the reform, R97 of which L97 was part, David offered in this focus group. He realised that the tasks for the exam had to be different from before the students could bring the rulebook. The tasks now contain a great deal of text which the students have to read The mathematical tasks are often presented within an everyday context; having to interpret a timetable, or other tables of information, a map, a recipe or patterns. Alfred was sceptical about this kind of exam since it disadvantages weak readers. Bent also offered some reflections around this new kind of exam. He felt that he now “got more out of each student”[19] and that everybody could manage something. His experience from earlier exams was that more students gave up on an earlier stage. He now felt that more students succeeded, at least a little.

[19] Meaning that what each student was capable of doing was better revealed through the new kind of exam

Summing up

As in FG2, there was a common agreement among the teachers in this focus group that it had been interesting participating in a focus group. The discussions which took place were of kinds that indicated a mutual acceptance of their interpretation of L97. The disagreements I noticed, for example about the clever students, were not on deep levels but rather on the surface. They agreed about how L97 weighted the different topics in mathematics (too little weight on algebra), and that they took their freedom to weight it as they wanted, according to their own judgements. They agreed on their scepticism to the rulebook, they partly agreed that the new kind of exam was acceptable and they agreed that having students with different abilities in the same class was one of the greatest challenges. The teachers were also interested in listening to each others practices, but only to a certain extent. They asked questions about Cecilie's grouping of the students and there was also some interest for the "Math-morning" project work Bent had carried out.

With regard to my research questions, what the teachers said in this focus group and how they said it gave me information about how the teachers responded to L97 in terms of what they were saying about it and what they were saying about their own classroom practice. This was useful information together with the individual conversations I had with each teacher in answering the first research question: "How are teachers in their mathematical practice responding to L97's recommendations?" The focus groups highlighted key issues and gave me a starting point for working with each of the teachers, Alfred, Bent, Cecilie and David, who became part of my further study. In the next part of this chapter I discuss teachers' beliefs and how I can infer conceptions about teachers' beliefs from what teachers say.

Teachers' beliefs and teachers' teaching practice in my research

In my study I use the term belief, and I look upon teachers' beliefs about teaching and learning mathematics and about L97 as cognitive constructions highly influenced by socio-cultural factors such as teacher's own experience and the school context, and also influenced by the teacher's knowledge in mathematics and about mathematics teaching. The insight I can get in my research into teachers' beliefs is through what the teachers say and write and through my interpretations of what I have observed in their classroom. I do not look upon beliefs as something that can be directly observed. Through the use of different theoretical lenses, my conceptions about teachers' beliefs have to be inferred from what they say about what they are doing in the classroom; what they say they think about their practice; what they say they think is good mathematics teaching and what they say about L97.

It has been important for me both to study teachers' beliefs about teaching and learning mathematics and also what I observed them doing in their classrooms. Thompson (1992) wrote that in order to understand teachers' teaching practices from the teachers' own perspective, understanding teachers' beliefs with which they understand their own work is important. I do not see a teacher's beliefs and his/her practice as a cause-effect issue, but rather as a reflexive process. A teacher's beliefs are influenced by his/her practice and the interactions in the classroom are again influenced by the teacher's beliefs. A teacher's practice can both act as a reinforcement of his/her beliefs but also as an incitement for change.

This is illustrated in Figure 3 where also the reflexive process between L97 and the teacher's beliefs is indicated. The teacher is interacting with the curriculum. What is in the curriculum is being understood by the teacher who keeps going back to the curriculum, thus the reflexive process between L97 and teacher's beliefs. When I reflect on my second research question: "What kinds of teaching practices are observable in the mathematics classroom", I recognise that what can be observed is influenced by students' contributions and their interactions with each other and with the teacher. The figure illustrates how I see socio-cultural factors, which sometimes act as constraints, influencing both teachers' beliefs and his/her classroom practice. These constraints which I found influencing a teacher's classroom practice were both in the teacher's beliefs about L97, in the classroom practice and in socio-cultural factors such as parents' and students' expectation. *Constraints* in my study are *cultural factors influencing teachers' beliefs and practice which may conflict with the recommendations of the curriculum and internally with each other.* The constraints I identified are detailed in my analytical accounts of the teachers' practice (Chapters 6, 7 and 8) and in the final chapter.

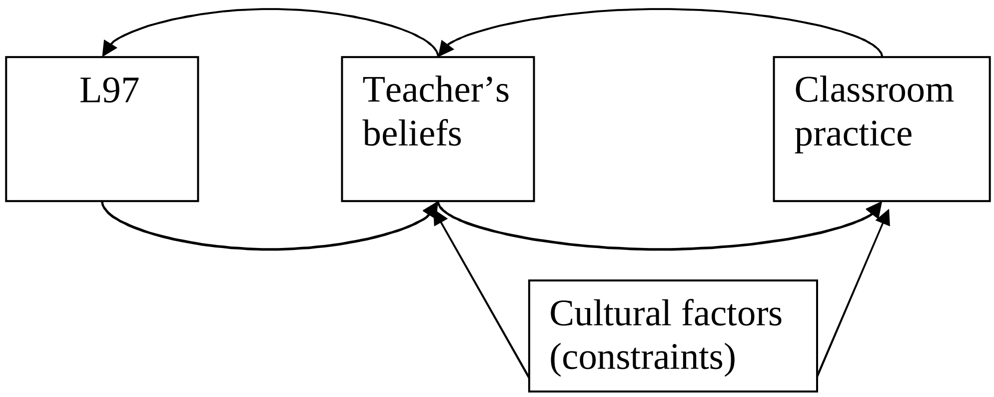


Figure 3, Teacher's interaction with the curriculum

In my research I am investigating how mathematics teachers interpreted L97, both in terms of what they said about it and in terms of their practice in the classroom. In the third research question I address the relation between the two, which I look upon as an important aspect of teachers' interpretation of the curriculum. One component of the teacher's interpretation is what s/he did in the classroom, the enacted curriculum (which is also influenced by incidents in the classroom, students' interactions, behaviour, and so on). The other component is what the teacher said in focus groups and in conversations, what s/he wrote and his/her responses to an estimation form. It is the relation between these two components, what I observed the teacher do and what s/he expressed in conversations, focus groups, self estimation and writing, I address through my third research question and it is the latter (what s/he expressed in conversations, focus groups, self estimation and writing) I term teachers' beliefs.

In doing belief research Lester (2003) warns against making a circular argument: claiming that people behave in a certain manner because of their beliefs and then infer a person's beliefs from the person's behaviour. One way to address the problem is according to Lester to use several research methods in investigating teachers' beliefs and not just infer the beliefs from the teacher's behaviour. In my study I did not think that it was sufficient for me to ask teachers about their beliefs and then expect them to be able to express their beliefs about teaching and learning mathematics and about L97. I have therefore used several sources of data in investigating teachers' beliefs. As described in the Methods chapter, I have used an estimation form which is a version of the one Pehkonen and Törner (2004) used in their study. They investigated how well information from different methodological sources to investigate teachers' beliefs in mathematics fit together. One of the sources they used was an estimation form in which teachers were asked to estimate both their real teaching with regard to three aspects (system, toolbox and process) and also what they looked upon as ideal teaching with regard to the same three aspects. I have drawn on their estimation form in my study. Asking teachers to estimate both *real* (from their perspective) and *ideal* was designed to focus their attention on potential differences between ways of seeing their teaching.

Since I address the relation between a teacher's beliefs and his/her teaching practice, studying the teachers in the classroom is an important part of my research. In the discussion of the theoretical perspectives underpinning my study I referred to patterns of discourse in the classroom and teaching strategies used to create conditions for possibilities of learning for students. I address my focus on teaching strategies and discourse in the next part of this chapter.

Teaching strategies and discourse in the mathematics classroom

Teaching strategies

When analysing the transcripts from classroom observations I identified several teaching strategies the teachers used when teaching mathematics. The identification I have made of the teaching strategies is influenced by Pimm (1987) who labels such strategies as “gambits”, Skott (2001a) who identifies teachers’ CIP’s – Critical Incident of Practice and Rowland’s (2000) outline of the use of pronouns in mathematics talk. A gambit is according to Pimm a conscious teaching strategy including gains and sacrifices. Instances of a teacher’s decision making are according to Skott critical to the teacher’s school mathematical images and often critical to further development of classroom interaction. Jaworski (1994) writes about teaching knowledge and teaching wisdom, and identifies some of the teaching actions as “decision points” indicating the development of the teacher’s ability to recognise that s/he is making a decision.

Drawing on Burton (1999) and Goos et al. (1999) I use the term *“authorship of knowledge”* as a code when either the teacher or students articulate knowledge which is already socially validated or has the potential to be so. It can be used by the teacher to state a convention or symbolic representation of mathematics that has been shown to be socially validated. I have used it when a teacher “cuts off” a discussion and presents what is conventionally correct and also when a student articulates mathematics that is already validated.

In Chapter 3 I discussed scaffolding as work within the zone of proximal development, and that in the analysis of the teachers’ teaching I consider as scaffolding when the teacher challenges the student by questioning their thinking; highlights key aspects of the task; reminds the student what s/he has done so far and about previous knowledge. In this section I discuss ways in which questions were asked and answered and thus could serve the act of scaffolding.

The teaching strategies I have identified in my analysis sometimes caused shifts in discourse or a change of direction of the lesson. The strategies include how the teacher responded to a student’s answer, either right or wrong; the teacher’s use of the pronouns we, you and it; how the teacher reprimanded students; how the teacher dealt with students’ (possible) errors or misconceptions; strategies for getting students’ attention; and strategies for “activating” students’ prior knowledge.

Teachers' ways of asking questions

Asking questions is perhaps the most common teaching strategy in mathematics. The purpose for the teacher to ask questions can be manifold; it can be to ensure students' attention, to assess students' knowledge, to encourage students' thinking, to control the line of students' thoughts (Pimm, 1987). I adapt Pimm's text (pp.50-59) to produce the following framework for my study. These are perspectives I have used in the analysis of my teachers and hence have influenced the analysis of the teachers' teaching in my study.

- <u>Closed questioning</u>:
  This is a verbal strategy used by the teacher often with the purpose of ensuring students' attention and to get students to talk mathematically. Sometimes this is not an explicitly formulated question, but a slight rise in intonation in teacher's talk followed by a pause or a gap for the student(s) to fill before the teacher carries on. There is often only one right answer to a closed question.
  Advantages (Strengths):
  - Allows the teacher to control the discourse and the mathematical focus and also to shape the allowed answers;
  - It is a way to break up the teacher's monologue, and to ensure students' attention.

  Disadvantages (Limitations):
  - The mathematical areas for such questions are often narrow and the students' answers can usually only consist of few words;
  - Such questioning does not allow students to formulate whole sentences or longer explanations. They have to follow the teacher's lines of thoughts. The interactions between the teacher and the students can be on the level of "guess what I am thinking about", and thus result in students' guesses rather than trying to work out a reasonable answer;
  - The teacher is often so focused on what word(s) s/he is thinking about that other correct answers may be overlooked;
  - The questions are not asked to discover something the teacher does not know, but rather to ascertain if the students know it or not. The kinds of answers one can expect are not lengthy mathematical explanations but rather fragments of such.
- <u>Open questioning</u>
  An open question is a question to which there is not only one predetermined correct answer. For example it can be asking students

about how they are thinking, or giving grounds for an answer. It can be asking for (other) ways of solving a mathematical task or asking a question like “How can we know that this is applicable in all cases”, a question involving generalisation or elaborating a proof in mathematics.

Advantages:

- Encourages students to give a holistic mathematic explanation and thus be able to express him/herself mathematically
- Often if students do not understand something in mathematics, and are asked to put words on it, they understand more after having formulated the problem into words

Disadvantages:

- Can lead the mathematical focus in a direction that was not intended and the other students can lose track

Ways of answering students' questions

Possible ways of answering questions from the students depend on the nature of the question, the content of it, how it is asked and why the teacher thinks it is asked (Pimm, 1987).

- <u>Answer students' questions directly</u>

One option is to answer the question directly; either by giving the information asked for or elaborating on the topic and to show and tell how to work out a mathematical task

Advantages:

- One advantage of this can be that if a student asks a direct question, and gets an answer to that, this might help to get over a minor hindrance to be able to proceed further
- It might be that if the teacher shows and tells how to do a task, this can serve as a template for the student to use to solve other similar tasks

Disadvantages:

- The students do not have to “struggle” and work through the difficulties him/herself
- The teacher cannot be sure that the student understands what the answer implies
- The student uses the given answer there and then and does not understand how to use it in other contexts, or will forget it because of not having struggled to find a solution him/herself

- <u>Not answering the question directly</u>

This is an alternative way of response to questions from students. Pimm (1987) outlines different possibilities of deflecting a question; The teacher asks the student to elaborate what (s)he thinks, or asks other students what they think, or the teacher overtly or covertly refuses to answer the question.

Advantages:

- Deflecting a direct request for information is a strategy often used by teachers who want to work investigatively making the students explore things themselves.
- Not answering a question directly can cause the students to become less reliant on the teacher and thus they learn to rely more on themselves
- By not answering a question directly Pimm (1987) writes that

  the teacher allows him/herself to escape from the tyranny of I(nitiation)-R(esponse)-F(eedback) framework [ ]. Tyranny because it locks the teacher into "centre stage", acting as controller of the communication as well as heavily influencing the types and range of spoken pupil contribution in class (p.56)
- Not answering a question directly but rather ask a question back like: "can you say more" and thus ask the students to articulate what the actual difficulty is, often result in student saying: "Oh, now I understand".

Disadvantages:

- By not answering a question from students directly the teacher relinquishes the possibility to give a clear explanation of a mathematical topic.
- If the teacher just says "I am not going to tell you" the students might lose faith in that teacher. It is therefore important for the teacher to explain why s/he is not going to tell.

Asking and answering questions are well known aspects of teaching practices which also were recognised in my study. How teachers' ways of asking and answering questions created patterns of discourse in the classroom, is discussed in the next section,

Pattern of discourse in the mathematics classroom

In order to discuss the general issues in relation to patterns of discourse, I have to explain why this became necessary as part of my analysis. Therefore I will take data from my study as a starting point, and discuss some of the constructs which have been guiding the analysis of my data. I will also clarify some of the terms used in the classes I observed. Phrases or words in italics here indicate codes used in analysis.

All lessons I observed had a kind of introductory part, a lesson opening. The lesson openings were of different lengths. The lengths did not only differ from teacher to teacher, but each teacher's lesson openings had also different lengths. One common feature with the opening parts of the lessons was that the teacher gave an overview of the lesson of the day. The given overview often included reference to the *work program*- a weekly plan made by the class's teachers with an overview of topics to be covered and work to be done during the week- or to tasks in the *textbook*. *The rulebook* was also often mentioned during the given overview. The *final exam* or how much work they had left to finish the syllabus (*time pressure/constraints*) was mentioned. What students were supposed to know about the lesson's topic was also presented during the overview (*previous knowledge*). *Disciplinary aspects* like calming the students' talk and movements and teacher's complaints about mess in the classroom took place before the presentation of the overview and sometimes continued during the overview.

Another common feature in the lessons was that very soon after the lesson had started, after the lesson opening, the teacher usually *invited students to participate* by asking questions. Sometimes the questions apparently had obvious answers and it rather seemed like asking a question was a strategy the teacher used to get students' attention; for example when the teacher in grade 10 asked what a triangle with a right angle was called (Right angled triangle). Other times the question did not have such an obvious answer. Then the answer one or more student gave - *student's answer* or *response* – became the subject of the teacher's evaluation or feedback. The teachers' evaluatory responses to students' responses could be either *consenting* or *refusing*, or the teacher could elaborate the student's answer and/or ask the student to explain his or her thinking, *the teacher questions student's thinking*.

Hence, in the lecturing part of the lessons I could see patterns of discourse and communicative approaches emerging from my data; the teacher asked a question, student answered, the teacher evaluated the answer or s/he gave feedback before asking a new question. I use the notion "communicative approach" to describe the *nature* of the interactions between teacher and students, how the teacher addressed students and responded to their contributions and ideas which emerged (Mortimer & Scott, 2003). As a tool in analysing the patterns of discourse in the lessons, the *patterns* of interactions in the lessons, I found Mehan's (1979) "triadic I-R-E interaction" useful. I, Initiation, was often a question asked by the teacher, R, Response, a student's (or students') answer and Evaluation, the teacher's evaluation of the response. Similar patterns of discourse are reported by Mortimer and Scott (2003). In their study of secondary science classrooms they referred to the triadic I-R-E pattern and the alternative I-R-F-R-F, which indicated chains of interactions between the teacher and the student(s), where F was the teacher's feedback to a response. Each of the I, R, E or F are phases in the course of interactions between the teacher and student(s). Mortimer and Scott (2003) labelled F "elaborative feedback" (p. 41).

The communicative approach which is the *nature* of the interactions between the teacher and student during the lesson, is central in my analysis of the classroom. I see the feedback phase, F, as the core of the teaching-learning process. The nature of the feedback the teacher gave illuminates an aspect of the teacher's teaching. Feedback from the teacher can be very powerful, and one outcome can be to bridge students' knowledge and school knowledge. In the presentation of a general overview of how I experienced the patterns of discourse in the lessons I observed, I have numbered the phases I-R-F-E as 1-2-3-4 respectively. The shifts or alternations between Phase 2, the response phase or *Student's contribution* (SC) and Phase 3, *Teacher's Feedback* (TF) could occur several times before Phase 4, The *Evaluation* took place. This last phase (Evaluation) includes both the teacher's explicit evaluation of students' contributions in terms of right and wrong, and/or that some kind of agreement between teacher and student(s) took place. I therefore also call this phase a *consolidation* or *consent* phase. In some episodes, there was no feedback phase, the conversation or discussion proceeds directly from students' contribution to the evaluation or consolidation phase.

Drawing on Goos et al. (1999) I use the term "*The teacher structures students' thinking* " as a teacher action based on the assumption that "Mathematical thinking develops through teacher scaffolding of the processes of inquiry" (p. 44).

Below I present a pattern of discourse with the indicated teacher–students interactions which is meant to capture what I saw in the lessons I observed. This pattern is meant to be a general overview of what I observed and it can be looked upon as a skeleton to be filled in by the observations from each teacher's classroom. It differed from teacher to teacher and from lesson to lesson.

1. Teacher invites the students to participate (Invitation or Initiation phase) *Teacher Invites, T I*. This includes:
    a. Teacher asks a question. The question can be closed or open, it can be related to prior knowledge (common ground), it can be related to homework and it can be related to conceptual understanding. The purpose of questions seemed sometimes to ensure students' attention.
    b. Teacher demands participation from the students
2. *Students' contribution, S C* (Response phase). This includes:
    a. A comment
    b. A statement - can be authorship of knowing - "knowing" that is authored, (articulated knowledge which is already socially validated. See page 122)
    c. A question
    d. An answer

3. *Teacher gives feedback, T F* (Feedback phase). This includes:
    a. With the intention to structure students' thinking by
        i. illustrating
        ii. questioning students' thinking
        iii. highlighting key information
        iv. pose guiding questions
    b. By reviewing or linking to prior knowledge. Interweaving familiar and new knowledge. Teacher links technical terms to everyday language and commonsense meaning. This is encouraged in L97.
    c. By sharing ideas, making ideas available to all students in class
        i. Teacher repeats or rephrases aloud what a student has suggested
        ii. Student is told to tell the rest of the class
    d. Checking and encouraging understanding
        i. Teacher asks for clarification
        ii. Encouraging and praising students, gives temporarily consent
4. *Consolidation – C* (Evaluation phase). This includes:
    a. Consent, teacher praises student(s) for giving a right answer of they agree on a solution
    b. Convention. A discussion terminates with a convention, for example name of a geometrical figure
    c. Authorship of knowing. In this phase the authorship of knowing is usually assumed by the teacher. Teacher explains how to work out the mathematics, or expresses a rule, formula or convention.
    d. Method mastering. A method of solving a mathematical challenge is either agreed upon or the teacher is presenting it.

One typical triadic pattern of discourse observed during whole class lessons was either *TQuestMath-StudAnsw-TCons or TQuestMath-StudAnsw-TRef*, meaning that teacher asks a mathematical question, student answers and the teacher either approves the answer or disapproves it (consents or refuses). This triadic pattern is often followed by teacher explaining mathematics further before a new question is posed. The communicative approach was often characterised by the teachers often referred to students' prior knowledge (what students now were supposed to know about this or another topic, a common ground for further learning) followed by teacher explaining mathematics further. These patterns miss the feedback phase which I claim is essential in the teaching learning process. There was a dialogue going on, however, sometimes in my analysis of the teachers' teaching I claim the dialogue being closed. That is when I suggest evidence for the teachers being content with only one predetermined answer.

Another pattern of discourse and communicative approach emerge when the feedback phase is present and alternations take place between students' contributions, SC and teacher's feedback, TF, where in the latter the teacher often structured students' thinking, referred to prior knowledge, shared ideas and encouraged understanding (3 a,b,c,d above). This is coded as *TquestMath-StudContribute--TStructStudThink-Consent.* There is often a circular movement between student's contribution and teacher's structuring of students' thinking. The interactions taking place here are subject for further analysis and are different from teacher to teacher.

Overall I saw different patterns of discourse during individual seat work parts of lessons from those in whole class. During individual seatwork, interactions between a student and teacher most often were initiated by the student; either the student asked for help in working on a task, or said that s/he did not understand something, or the student asked for consent, "have I done this correctly?" I found that these interactions differed from teacher to teacher.

One pattern of discourse observed during individual seatwork was: *StudUndNoth-TExplMath* (student says he understands nothing followed by teacher's explanation) or *StudQuestMath-TExplMath* (student has formulated a mathematical question and asks for help). There were also occasions when the teacher took initiative and asked if students understood or if they managed, if they were OK or if they have problems: *TQuest- StudAnswerOk,* or *TQuest-StudQuestMath-TEplMath,* or *TQuest-StudUndNoth-TExplMath.* Also here a feedback from the teacher could be characterised by structuring student's thinking as described in Phase 3 above. Like in whole class situations, the kinds of interactions between the teacher and the student that I observed here are crucial issues that can illuminate the characteristics of the teacher's teaching.

How the teacher responded when a student needed help is significant. Sometimes I experienced that the teacher was questioning the student's thinking and encouraging constructions from student's ideas and questions. Important foci are the ways in which the teacher challenged the students, the use of mathematical conventions and symbolism and whether and how the teacher was able to distinguish between the students' points of view and needs on the one hand and a school mathematics'/teacher's point of view on the other. Were the students expected to see what was obvious to the teacher? When explaining whys, did they elicit their own ideas? All these are issues which illuminate the characteristics of the teacher's teaching.

Forman and Ansell (2001) offer an alternative version of the traditional I-R-E pattern of discourse in which the student has a more active role and the role of the teacher is more of orchestrating a class discussion. Students often provide the mathematical explanations which the teacher revoices or restates to clarify or elaborate for the student and for the other students who are the (un)intended listeners. The student's contributions become legitimated by the teachers through his/her revoicing or restating. Other times the teacher takes student's contribution as a starting point and elaborates further on it. The teacher can also challenge the student's contribution by asking for reasons, how s/he has been thinking and why. Teachers' responses to students' contributions are not always deliberate.

Teachers' use of personal pronouns when teaching mathematics

When teaching from the board, teachers often use the personal pronoun "we" as grammatical subject in sentences when referring what is done. How to interpret this use of the pronoun "we" is a challenge in the analysis of the transcripts from the classroom observations. One interpretation of "we" can be the same as the use of that as pronoun when talking to small children or a nurse/doctor talking to a patient, what is talked about as social practice "this is how we do it here". The use of "we" when working with mathematics may emphasises the conventional aspect of mathematical practices and can imply that the way the teacher does the mathematics represents the community of mathematicians and thus the way the students are supposed to do mathematics as well. Rowland (2000) points to the use of "we" for the purpose of drawing the listener into complicity. However the opposite may also be the case when the effect of the use of "we" is to associate the teacher with a powerful group (the mathematicians) from which the students are excluded. All four teachers I observed used "we" more or less frequently when teaching from the board. It is hard to know how deliberately they did it. When the use of "we" indicate a social convention, no explanation is needed and constructions with the use of "we" convey the way things are or are not being done and the students' points of view become devalued. Thus the mathematical practice reflects the conventional aspect of the subject (Pimm, 1987).

The use of "you" occurs frequently in my data. In Norwegian the singular you is "du" and plural you is "dere". It is therefore possible to distinguish between the two. The Norwegian word "man" often takes the place of the use of "du" and it is hard to distinguish between those. "Man" is an unnamed third person, which I have translated into singular "you" or "one". In using "you" a person is addressed in second order. However, according to Rowland (2000) children rarely address the teacher directly by the use of "you". Their use of "you" is rather to express generalities or “anyone”. The use of “you” or the Norwegian “man” instead of “I” indicates a detachment from what is said. For students it may seem more binding to use “I” when presenting a mathematical statement in class than using “man” or “you” which will make the student more detached from what s/he is asserting.

The use of it

A striking feature in the conversations between teacher and students is the use of “it” when describing a mathematical relation, property or entity. Throughout a mathematical conversation the use of “it” occurs quite frequently. The interpretation of “it” can be problematic and the problem is whether there is a shared meaning in class (between the teacher and the students) about what “it” is.

Pimm (1987) points out the use of it as an expression of vague and half finished utterances, and often as an expression of a generalisation. Building on Pimm’s comments on the ambiguity of “it” Rowland (2000) suggests that “ ‘it’ is a distinctive and important feature of maths talk, to the extent that it acts as a linguistic pointer, invariant at the surface level” (p.101-102). He illustrates the variable character of the referent of “it” and how the use of “it” can be deictic – pointing to an already introduced concept or referring to a mathematical entity, and also how “it” can point to a statement or question to be formulated. Rowland illustrates how a child’s deictic use of “it” enables the child to talk about concepts which might be mathematical operations that the child has as a meaningful abstract concept but is not yet able to name. His analysis shows that “the beauty of the deictic “it” lies in its function as conceptual variable” (p. 108).

In the analysis of the teachers’ teaching in my research I have used teachers’ use of “it” as an analytical tool to account for occurrence of lack of closeness in perspectives between the teacher and the students, and I also suggest how the use of “it” as a pointer to something not yet agreed upon or not yet introduced, could cause confusion among the students about on what they were supposed to work.

Summing up the chapter

In this chapter I have analysed L97 from both constructivist and sociocultural perspectives drawing on literature about mathematical knowledge on which I have drawn in accounting for the aspects of mathematics reflected in the curriculum. These theoretical and mathematical aspects of the curriculum inform the analysis of the activity and thinking of the teachers in my study. The analysis of the discussions in focus groups informed and highlighted key issues in the study of each individual teacher.

The last two parts of this chapter, “Teachers’ beliefs and teachers’ teaching practice” and “Patterns of discourse in the mathematics classroom” form a bridge to the analysis of each teacher. Here I have addressed concepts in discourse central to my analysis of dialogue in the classrooms. The next three chapters, Bent, Cecilie and David, make up the core of my research, and I will present a detailed analysis of each of the teachers’ beliefs and his/her classroom practice in which I use concepts addressed in this chapter. Towards the end of each chapter I present a portrait of the teacher and I sum up the characterisation in an overview relating characteristics in beliefs and practice to L97.

6. Bent

Bent was the youngest teacher in my study. He had 10 years of teaching experience from lower secondary school. He had taken his teacher education at a teacher training college and had chosen extra study in mathematics since that was the subject he was most interested in teaching. Before going to teacher training college, he had taken 20 "vekttall" biology, 5 "vekttall" chemistry and 5 "vekttall" mathematics at the University[20].

When I first contacted Haugen School for the purpose of having teachers for a focus group, the head teacher gave me some information about the mathematics teachers at the school (without my asking for it). Bent was said to be "a traditional teacher who was on the move".

In my analysis of the focus groups in the previous chapter, I discussed how Bent talked frankly about his weaknesses as a teacher, rather seeing his being able to talk about his weaknesses as a demonstration of confidence and strength. In the focus groups and in our conversations, he offered his reflections about his teaching and of students' learning of mathematics, and he expressed a wish to improve his teaching. I can see that he offered more reflections during our conversations towards the end of my work with him than in the beginning. I do not know if that was because he then had become more conscious about his own teaching and actually had started reflecting more upon it, or if it was that he then felt more comfortable in the situation with me. This indicates researcher influence and that the distance between the researcher and a participant is narrowed throughout the research, which I discussed under the heading "taking a case study approach" in Chapter 3. Bent said that he liked having me in class and that he felt that I supported him. It was important to me that the teachers I observed felt that I treated them with respect and Bent seemed to acknowledge my respect (ethical aspects were discussed in Chapter 5).

According to Bent, his class was a "demanding" class. One teacher had given up being their form master, and Bent had been asked by the school's leadership to take over[21]. Bent was a well respected teacher at the school and also the teachers' representative in weekly meetings with the school's leadership. There was a friendly atmosphere between Bent and his students.

Analysis of conversations with Bent

As outlined in the Methodology and Methods chapters, I had a great amount of data and I used the qualitative research program NVivo as an aid in coding and analysing the data from the conversations. When I

[20] 20 vektall correspond to one year full study
[21] Classes in Norway have their form master ("klassestyrer") who is one of the class's teachers.

started the coding of the data from the conversations with Bent, the intention was to see if I could develop an analytical framework that I could use in the analysis of all teachers. Since Bent was the first teacher of whom I did the coding I used the categories which emerged from this work in coding data from other teachers.

Before I analyse the conversations with Bent, I will show how I dealt with the codes and categories which emerged from the data from the conversations with Bent.

Coding and Categorising, three emergent categories

In Chapter 4 I outlined how I coded data from conversations with teachers generally. Now I will show how the main categories which I used in the analysis of all teachers and their teaching emerged from the analysis of the conversations I had with Bent.

I had six conversations with Bent, four pre-lesson conversations, i.e conversations before a lesson, and two post-lesson conversations, i.e conversations after a lesson. After having coded the conversations I had with Bent twice in NVivo, I found it valuable to be more systematic in the coding process and I saw a possibility of grouping the codes I had used into six categories:

- Reflections about mathematics teaching and learning
    - o Teacher's reflections about mathematics and how students learn mathematics;
    - o Reflections on a previous lesson; what was good /not so good;
    - o Reflections about what he ought to have done that presumably would have been better;
    - o Reflections about classroom culture including discipline and issues related to individual students.
- Mathematical focus
    - o Conceptual: Emphasis on development of concepts and students' conceptual understanding;
    - o Procedural: Emphasis on exercising skills and procedures;
    - o Conventional: Emphasis on formulas, rules, use of symbols;
    - o Structural: Emphasis on connections between mathematical entities and concepts.
- Students' abilities
    - o Teacher comments on students' difficulties and challenges;
    - o Teacher comments on what students master;
    - o About teaching according to students' different abilities, differentiating;
- Conditions for possibilities of learning (including intentions for the next lesson)

- Individual work;
- Work in whole class;
- Collaboration;
- Illustrate with drawings etc;
- Use of concrete materials;
- Confusion (positive that will enhance learning and negative which might be an obstacle for learning);
- Show and tell. Teacher says he will show and explain to the students how to do mathematics.

- Common ground
  - Common habits and tools like work-plan[22] (work program), workbook, rulebook, textbook;
  - Common knowledge in class, or taken-as-shared.
- Constraints
  - Time constraints;
  - Parents' and students' demands putting constraints on the teacher's teaching;
  - Exam putting constraints on what tasks to work on;
  - Other constraints, like colleagues, curriculum, illness, work plan, classroom culture.

In addition to these codes I had L97 as a free code. Everywhere L97 was mentioned explicitly during the conversations, I used this code.

I found that these categories could give me relevant information with regard to my research questions. Very roughly, the first three give information about teachers' beliefs about teaching and learning mathematics, the next two about what the teacher intended to do in class and how he looked upon and talked about the classroom as a culture. The last one, constraints, are issues the teacher referred to as preventing him from acting according to what he thought was best and they illuminate how the teacher's practices in the classroom are related to his beliefs about teaching and learning mathematics.

I created a third document in NVivo and imported the data from conversations with Bent, created new nodes according to the categories above and coded the conversations again. Every time I coded a passage or a paragraph, I used at least one of the six main codes and one or more of the others. I found it quite problematic restricting myself to one code, and I still had difficulties to get a holistic overview. I found that not all codes gave me any useful information. I still had too many codes and I had used some of the codes too many times. The code "Reflections" was used so much that it did not give me much information. I had used it

[22] A work-plan or work program, is a weekly plan developed by the class's teachers in which the work to be done during a week is stated.

more or less whenever the teacher was saying something about what had happened in the classroom and what he was going to do. I found that it often overlapped with the other codes. I also tried to refine the code "Students' abilities". I asked myself: what does he say about students' different abilities? Is it only a constraint? Is it a challenge that he uses in the teaching? How does he respond to the fact that students have different abilities? Answers to these questions were needed to give the characterisation of Bent that I was seeking.

Reading through the transcripts with the coding indicated above, and studying them over again several times, three main categories emerged: Drawing on Cobb et al. (1997) who discussed how students' participation in a reflective discourse in the classroom constitutes *Conditions for possibilities of learning*, I have used the term as a category *(CPL)* including teaching strategies, aspects of classroom culture and discipline. The other two main categories I used are *Mathematical focus (MF)*, including conceptual, structural, procedural or conventional focus, and the actual mathematics topic studied, and *Students' abilities (SA)*, both including aspects of differentiating, how different students learn and issues about different students' mathematical knowledge told by the teacher. I decided to go through the transcripts again, with the purpose of pulling out what I saw with regard to three main categories from each of the conversations. I soon realised that the categories were not mutually exclusive but that the categories sometimes *overlapped*; meaning that what was said would fit into more than one category. For example Bent made a worksheet which the students should fill in for the purpose of developing conceptual understanding of the formula for the area of a trapezoid. That he made the worksheet provided a condition for possibilities of learning, which I categorised as CPL. His purpose of making it was to develop conceptual understanding and I therefore categorised that as MF.

In this work, the coding I had done in NVivo was useful. I printed out the documents with the codes I had made in NVivo, studied them over again and found that CPL, MF and SA captured what I found with regard to the conversations I had with Bent.

I started with CPL (conditions for possibilities of learning) and went through the transcripts for each conversation and wrote down what I saw in each conversation with regard to how Bent said he created possibilities for students' learning and I included aspects of classroom culture in it too. At this stage I did not include what I had explicitly coded as Bent's reflections. I did the same with MF (mathematical focus) and SA (students' abilities). Having finished the work, categorising into the three categories as described above, I decided to go through the print outs of the coded transcripts to investigate Bent's reflections about teaching and learning mathematics. I found that Bent's reflections during the conversations also could be grouped with regard to the main categories above. I used different colours for the three categories to mark the print-outs and I could add Bent's reflections into the three categories CPL, MF and SA as well.

I will now offer two examples to illustrate my interpretations of how one or more of the three categories were reflected in the data before providing further detail of the three categories.

Through our conversations Bent demonstrated that he offered a wide range of teaching strategies in creating conditions for possibilities of learning. Preparing for individual work he made worksheets, selected tasks from the textbook and made forms/tables for the students to fill in. He prepared for collaboration in pairs for example, to measure sides and to calculate volumes and surface areas of solid block. Preparing work in whole class sections for example, he brought concrete materials to illustrate and present overview of volumes and he prepared illustrations of calculations with fractions by drawing grids. Our conversations reflected conventional, structural, procedural and conceptual aspects of mathematics. Bent expressed the *conceptual* to be the most important.

The first illustration I present shows that Bent created conditions for possibilities of learning by preparing for an activity with concrete materials to address concepts of surface area and volume. He had also made a table for the students to fill in to help them keep a better overview. The mathematical focus was the relevant formulae and thus *conventional*. The purpose was to develop formulae. The mathematical focus was also *structural* (emphasis on connections between mathematical entities and concepts) by making links between formulae and equations. He also emphasised the importance for students to *understand* the formula – hence, also, a conceptual focus. In our pre lesson conversation Jan 8th he said:

B: I will start the lesson with surface area and volume. I think it is important with formulae, the concept of formula. That is something I will spend some time on. Because we use some formulae they shall try to understand these formulae, make them and use them.

B.K: You want them to derive the formulae themselves?

B: Not all but some. [ ] Derive formulae, use formulae, use formulae in relation with equations, all formulae are equations. They can exchange all links in the formula. And solid blocks; used them yesterday. Wooden solid blocks; cylinder, prisms, cube. We shall work with the work program; shall fill in this table.

And he showed me a table with the columns: "name of the figure", "formula for surface area", "formula for volume", "perspective sketch".

In the same pre-lesson discussion, Bent demonstrated knowledge about students' different abilities and how different students performed differently on traditional tasks and problem-solving tasks. He presented opinions about what students mastered and what kinds of difficulties they had. According to students' different abilities he discussed the necessity of conceptual understanding and he expressed different conditions for how conceptual understanding could be developed. Evidence for these interpretations can be found in the section "Mathematical Focus and Students' Abilities" later in this chapter. I see Bent's reflections around students' different abilities to be in accordance with L97's recommendations: "The teaching of mathematics must be attuned to the abilities of individual pupils who must be given tasks which they find meaningful and are capable of carrying out" (p. 166).

The second illustration (see quotation below) shows how Bent wanted to take students' abilities in mathematics into account when preparing a lesson. The task he had written on the board was: "A rope which is 5 ¼ metres shall be divided into lengths of 1 ¾ metres. How many lengths do you get?" He wanted the clever students[23] to look upon this as a complex fraction when working it out, even though work with complex fractions is not a topic in the curriculum. Thus he wanted to challenge the clever students. Looking through the perspective of the CPL category, the conditions he prepared for learning included the task and how to illustrate it by drawing. The mathematical focus here was the relation between conceptual understandings of the division of fractions and the mastering of doing it.

> It is for the clever ones. But I would like to proceed with the rope – with more difficult fractions, to illustrate that it functions then as well. I have a rope which is 5 ¼ metres and shall be divided into lengths of 1 ¾ metres. How many lengths do you get? One can see how many times one and a quarter goes in five and three quarters. One can count how many times one and three quarters go into it and then proceed to dividing. One will get a complex fraction. [ ] Then you get three as an answer and then you come to the rule they use and then you can turn the second upside down (19/2 pre).

These illustrations just start to indicate ways in which I saw aspects of Bents' expressed thinking and classroom teaching to fit the three categories. I now provide further details of these categories and show how they link to areas of theory expressed in Chapter 3.

Categorisation of Bent's teaching

In making this categorisation I recognise strong links between my analysis of Bent's teaching and theoretical concepts expressed in Chapter 3. Theory has both guided my analyses and emerged from them. By this, I mean that the theoretical thinking underpinning the curriculum has been available to me as I have analysed and categorised data, and that the categorisations themselves have enabled me to see more clearly how theory is linked to my characterisation of teaching and learning processes for each teacher. Through my analytical process, I have developed

[23] Bent referred to "flinke elever" which I have translated into English is as "clever students"

a stronger understanding of how theory and practice can be seen to be related.

Conditions for possibilities of learning

In this category, I include all aspects of Bent's classroom activity and expressed thinking that I see as contributing to his creation of a learning environment for his students. Both constructivist and socio-cultural lenses shed light on this characterisation. In Chapter 3 I discussed building of concepts through reflection and abstraction according to a constructivist epistemology. In Chapter 5 I referred how the authors of the mathematical part of L97 had drawn on Piaget's notion of reflective abstraction when formulating the statement, "Learners construct their own mathematical concepts. In that connection it is important to emphasise discussion and reflection" (L97 p.167). I also discussed L97 from a socio-cultural perspective, referring to quotations from the curriculum which emphasise the importance of social interactions and discussions in the classroom and how students' participation in a classroom discourse creates conditions for possibilities of learning. Elements of these two areas of theory can be seen in my characterisation below and I will discuss theoretical considerations later in this part.

One style of teaching evident in Bent's classroom was what he called *teaching from the board*. Bent said that he felt that teaching from the board was useful only for less than one third of the students in class. He would teach from the board mainly "in conversation with the students" (8/1 pre). He also expressed awareness of not making conversations too long when only a few students participated, because that was "misuse of the other students' time". However, he said he liked the conversations since the students involved benefit from their participation, and also because it leads to reflections among the students. It was clear that he saw advantages and disadvantages in this style, and needed to compromise on its use.

It was not only in such conversations that Bent wanted the students to be active. He wanted them to work on their own or to collaborate in pairs with concrete materials: for example, in measuring sides of polyhedra and calculating volumes and surface areas. He designed tasks for the students to find out what they had mastered and not mastered: for example, with regard to calculations with fractions.

My analysis (from classroom observations and conversations before and after lessons) that Bent wanted students to be active in the learning process is emphasised further in the following extracts from what he wrote about ideal teaching:

- I think teaching ought to be experimental. If the student can find out the knowledge / the rules / the formulas through own activities, I believe that it will last longer and that they get a greater ownership to the knowledge.

- The student often wants the teacher to show as many solutions as possible. It is a challenge to motivate students to work hard to overcome difficulties.
- Parents also want the teacher to teach from the board. However that is doing them disfavour. They must experience the difficulties on their own
- Teaching from the board has to be followed by students' own activity, either at school or at home. The student him/herself must work hard.
- To learn mathematics, the student has to work as much as possible on his/her own.

(From Bent's writing about ideal mathematics teaching).

By saying that presenting as much as possible on the board is doing the students disfavour, Bent seemed to suggest that he did not believe that knowledge can be transferred from him as a teacher to the students. Bent's claim that students will get a greater ownership of the knowledge and that it will stay longer if they find it out themselves, indicates a view that learning takes place at least partially, in the individual's mind.

For Bent, discussions and reflections were important in the learning process. L97 says that students' own activities are of the greatest importance in the study of mathematics and that the teaching must provide students with opportunities for the kinds of activities which Bent brought out in his classroom. We can see in these respects that Bent is following the curriculum in his design of teaching.

Mathematical focus and Students' abilities

As pointed out in the outline of the three emergent categories, Bent discussed the necessity of conceptual understanding and/or exercising procedures according to students' different abilities. Therefore I find it natural to present the categories "Mathematical focus" and "Students' abilities" under the same heading in the presentation of Bent. He seemed to express a relation between students' abilities and the weight he wanted to put on different aspects of mathematics. Generally Bent expressed a view which reflected more emphasis on the conceptual aspect (weight is put on relations between concepts and the ability to use the knowledge in other contexts than where it is learned) than on a procedural aspect (emphasis on memorising rules, exercising skills and procedures and mastery of skills) of mathematics.

Earlier I have argued that the L97 curriculum focuses more on conceptual understanding and less on procedural knowledge than prior curricula and that I see a relation between constructivism as a cognitive position and teaching for conceptual understanding in L97. Bent's intention to focus on the conceptual aspect of mathematics is thus in line with the curriculum. Reflections around students' conceptual understanding were often in focus during the conversations with Bent. When working with the volume and surface areas of polyhedra, Bent said that he intended to focus on students' conceptual understanding by using concrete materials:

> The transition of knowledge from seeing drawings in the textbook, knowing the formula, to sitting with it in their hands, doing the right measurements and calculating the surface area and volume, I'll spend some more time on. They know how to do it with drawings in the textbook, however, from that to have it in their own hand - -(8/1 pre).

Bent indicated that students might become confused when working on this activity. However he said that he wanted the students to master what he called "*positive confusion*", a confusion he wanted to provoke because it could enhance conceptual understanding through reflection for example upon how big a square centimetre was and if the answer they had got was reasonable. Thus I perceive a relation between Bent's focus on conceptual understanding and a constructivist perspective, and a view that learning takes place through the revision of existing structures in the learner's mind. Very tentatively, I suggest that Bent's term "positive confusion" can be accounted for by using the constructivist term "cognitive conflict" which occurs when a person recognises his/her own misconception and a reflection is necessary to adapting existing structures in order to accommodate the new information.

During the pre conversation we had 19/2 we talked about conceptual understanding of dividing fractions opposed to the method of mastering the procedure. Bent claimed that the brightest students automatically will gain conceptual understanding from mastering the method but that more students will understand *why* you have to turn the second fraction upside down if you focus on the *why* and explain *why*. However, he was afraid that some students would be frustrated when focusing on the why and that it would take too much time:

> The time I have got to spend on it might rather lead to a certain extent of frustration among some who are happily living with just using the rule and are managing well and not wanting to be an engineer or mathematics teacher. They know the rule and are happy with that. May be some more will understand it but some will be frustrated. [ ] I haven't reached any conclusion what is the smartest to do (19/2 post).

He argued for different levels of understanding. According to their abilities, for about one third or one fourth of the students it would be sufficient just to know the rule and how to use it. To enhance conceptual understanding of calculations with fractions, Bent wanted to focus on the relation between the methods they had available to carry out the calculations and an obvious answer. "The point was that they should see the answer and then use the method they knew and see that it fit the answer" (19/2 post). He thus put the relation between computational methods and conceptual understanding in focus. Bent gave a reason for why conceptual understanding is important in the learning process:

> It is to be able to solve more types of problems and to be able to manipulate the different concepts. They will benefit more from the tool in a way, be able to use

> it in other contexts, make connections within the mathematics, and be able to use it in a more functional way (19/2 post).

This is in accordance with von Glasersfeld's (1995) claim that when focus is put on students' performance rather than on conceptual understanding they will only be able to solve problems precisely like those presented and not be able to solve problems presented in other contexts. Cobb (1988) also wrote that students have developed conceptual understanding only when they can demonstrate abilities to solve problems in other situations than in those the learning took place. From Bent's point of view this activity could be seen as developing a classroom culture in which use of tools and making connections promote conceptual understanding.

What Bent said suggests that he wanted to focus on students' conceptual understanding; that it was important for the students to understand why the method of multiplying fractions is as it is. Looking at what Bent expressed here through constructivist lenses and using Piaget's notions of assimilation and accommodation, I see a teacher who indicated a view that students construct their own knowledge through the process of assimilation and accommodation. His saying he was afraid that some students might be frustrated or confused suggests that bright students may assimilate the *why* (conceptual understanding) into existing structures (the method) and a revision of the existing structures can take place, i.e accommodation, and on the other hand that some students are not in the position to assimilate the *why* into the *how* and that lack of assimilation will rather lead to confusion than to revision of existing structures and conceptual development. Thus he suggested that the weight to be put on conceptual understanding as opposed to emphasis on the procedural aspect depended on the student's abilities. Norton et al. (2002) reported similar findings. The teachers in their study expressed different goals with regard to conceptual understanding or procedural focus for students according to their perceptions of students' abilities, a difference which also was demonstrated in the teachers' actual classroom practice.

Through what Bent wrote about ideal teaching he gave the impression that he looked upon it as important for students to find out things through own activities and that knowledge they gain through exploring activities probably will last longer. However, in our conversations he did not give the impression that his students were doing exploring activities in his lessons. He rather reflected on it and gave several reasons for not doing it. The main reasons he gave were: Not having time, not knowing how, parents and students want him to teach from the board. I go further into detail in discussing this under the heading "Constraints" in the next section.

Bent's beliefs about teaching and learning mathematics

From the analysis of the conversations with Bent I found that

- he appreciated students' contributions and discussions in class and he wanted the students to be active in the learning process;
- he expressed a wish to do more exploring activities as L97 recommends, however, there were constraints preventing him from doing it.

According to students' different abilities he discussed the necessity of conceptual understanding and he expressed different conditions for how conceptual understanding could be developed.

Constraints

I have argued earlier that I see an "investigative approach to mathematics" reflected in L97 and that this approach promotes developing conceptual understanding. Alternatively what I have referred to as "traditional mathematics" focuses on correct use of algorithms, on drill and practice and thus often on errors and failures which again supports the view that mathematics consists of a fixed body of knowledge that can be transferred from the teacher or a textbook to the learner. According to what Bent said in conversations and focus groups he rather believed in an "investigative approach", (which I discussed with regard to L97 in Chapter 5) to the learning of mathematics than in a "traditional" focus on mathematics.

It soon struck me when I started the analysis process of the conversations that Bent very clearly stated what he ought to do with regard to teaching activities, and also with regard to how students learn mathematics in a best possible way. However, he gave me several reasons for not always acting according to this. I label these reasons as *constraints.* As shown earlier, constraints were one of the main codes when coding in NVivo. I printed out transcripts coded as constraints from NVivo. I soon realised that *time* was something Bent mentioned most often as a reason for not having acted as intended or as he wanted. However, there were different aspects of "lack of time"; He chose not to spend more time on a certain discussion:

> Some students were engaged in that discussion and at least half of the students were not, then I thought that okay, let me follow the discussion to a certain extent and then stop, if not I'll misuse the other students' time (16/1 pre).

In this case it was his choice not to go on with a discussion. Because of lack of time, he consciously knew that not all students gained conceptual understanding in mathematics. He said: "If I had an infinite amount of time, I could have spent lots of time on it and maybe got everyone to understand it, but I haven't" (19/2 pre).

In other cases, he gave grounds not controlled by him, for not having time for an activity; illness (either his children or himself) so he had been absent; his participation in a course outside school; loss of lessons because of other planned activities at school like project work, activity day (sports) etc. The most frequent reason he gave for not acting according to his ideals was that he felt it took too much time, and then he was afraid he would not have time for other important aspects of the subject. During the conversations he talked about the constant "*time pressure*" (Norwegian: "tidsklemma") both with regard to his time as a teacher preparing lessons, and also for the whole class that things were taking too much time. With regard to L97 and how to respond to its recommendations about exploring activities, he said:

> I notice if the students shall be more exploring like L97 encourages, I think it will take more time. However, I believe they learn better that way. So I haven't found a good way to make it effective for them to learn better. That doesn't mean that my opinion is that teaching from the board is the best way, but if I have to spend so much time doing exploring activities, will we then make it?, I'm thinking. Possibly we will, maybe (April 1st, post).

Indirectly Bent said that the *work plan* also was a constraint in his work with students. He said that in one class the students were more interested in ticking off in the work plan what exercises they had done rather than participating in the work he had prepared with the concrete materials. I think one reason for this is that the students looked upon the work plan as a job contract where the goal for the students was nothing more than doing what is in that contract. And having done that, the students had fulfilled their part of the contract and nobody could blame them for not having done their job. Thus the responsibility for learning was put on the content in the work plan and not on the student him/herself.

Another issue Bent said he had to compromise on was *parents' expectations*. He referred to parent-teacher meetings where parents had been saying they wanted him to teach more from the board. He had tried to argue for not teaching so much from the board by saying that some of the students would not understand, for some it would be too easy and that only about one third of the students would benefit from it. He had told the parents that he believed that students learn better if they find out things themselves. Despite his arguments, they still wanted him to teach more from the board, and he responded to these demands by doing it "at least for a while", he said, to "calm them down". Also students wanted him to teach from the board as they kept blaming the teacher for not having taught them if it was something they did not manage. They expected him to give examples of tasks on the board before they started working on similar tasks in their own books. These views are very culturally rooted and mirror a view on mathematics consistent with a transmission or absolutist view on knowledge (Reys et al., 1998) or they reflect practices that have been common in mathematics teaching for generations. A third constraint preventing Bent from teaching more according to L97's recommendations was "*lack of methods*". He actually did not know how to do it. He said:

> I believe it is good mathematics to combine exploring mathematics and mathematics bound by rules in a way. I think that is good mathematics because the students will get more, well, he understands the background in a way. However, there I have a way to go myself, and I really want to go that way because [ ] I believe it is an important part of the mathematics but I don't manage that so often, in a way, I like the old way of doing mathematics too. I like it, but I can see that the more exploring way is more useful for some students. But to do that, I need some more practice in presenting it from that point of view (post 5/2).

This shows that also for him the traditional way of dealing with mathematics was deeply rooted. However, he wanted to learn and he wanted to change his way of doing mathematics. What he said here reminds us that changing a practice is not an overnight-action. Implementing a new curriculum takes time, and it is hard, even when the teacher believes in it and wants to do it. The gap between the intended curriculum and the teachers' classroom practice, the enacted curriculum, even when the teachers have expressed their agreements with the principles lying behind the curriculum, has been pointed out as an issue by other researchers too, on which I have reported in Chapter 2 (Broadhead, 2001; Norton, McRobbie, & Cooper, 2002). Bent said that he had been taught mathematics traditionally and he knew how to teach "traditional" mathematics. However, he believed that exploring activities would be good but "I have to improve myself where exploring mathematics is concerned" (pre 5/2). What Bent said during the last conversation we had two months later emphasises this:

> Yes, I think I agree more with L97 than I practise. It is a challenge to find good methods. I feel I don't have the methods to be able to do that kind of process. What methods shall I use and what kinds of tasks shall I give to get into that process? (Post, April 1st)

What he said here is consistent with how he estimated his own teaching, ideal teaching and L97 in the estimation form which I present below.

Similar issues and obstacles constraining teachers' teaching practice, (parents' resistance to change, lack of time, tension between exploring activities and traditional teaching) have been reported in other research to which I have referred in Chapter 2 (Manouchehri & Goodman, 1998; Reys et al., 1998).

Bent's Estimation form

Just as what Bent wrote about ideal teaching emphasises the outcome of my analysis of the conversations I had with him and thus can be viewed upon as a validation of the analysis, Bent's estimation form also underpins and validates the outcome of my analysis. From this form we see that Bent evaluated L97 very close to how he looked upon ideal teaching. However, he estimated his own actual teaching far from that, especially with regard to the toolbox aspect (explained as: mathematics is a toolbox, doing mathematics means working with figures, applying rules, procedures and using formulae) and the process aspect (explained as: mathematics is a constructive process, doing mathematics means learning to think, deriving formulae, applying reality to mathematics and working with concrete problems). He admitted that he did not focus so much on the process aspect which includes exploring activities as he ought to and wished to do, and that his teaching was more traditional (closer to the tool box aspect) than he wanted it to be (ideal teaching) and than L97 recommends. This is in accordance with the outcome of the analysis of the conversations I had with him.

Bent	Mathematics as a toolbox	Mathematics as a system	Mathematics as a process
My real teaching	18	5	7
Ideal Teaching	10	7	13
L97's view on teaching mathematics	13	5	12

Table 7, Bent's estimation form

As all teachers in my study, Bent had been asked to distribute 30 points corresponding to his estimation of the toolbox, system and process aspects regarding ideal teaching and he valued the process aspect highest with 13 points. The fact that he gave the toolbox aspect 10 points which is a fair share of the 30 points tells us that he still looked upon the toolbox aspect (explained as working with figures, applying rules and procedures and using formula) as an important aspect of school mathematics. This is consistent with my analysis of the conversations. Thus this estimation form can be looked upon as a validation of the analysis of the data from the conversations. I found that especially *procedural*, exercising skills and procedures and *conventional*, focusing on formulas and conventions occurred throughout the conversations. However, the fact that he expressed a wish to switch towards a more process oriented way of teaching reflects that his work with L97 influenced this teacher's professional development in connection with implementing a curriculum reform.

As discussed in Chapter 2, Smith Senger (1998/1999) reported similar findings among the teachers in her research, that they struggled with reform issues on the one side and traditional teaching on the other. Although these teachers did not change their teaching practice drastically throughout her study, Smith Senger, based on the teachers' own reflections, suggested a promise of long term change.

Analysis of classroom observations with Bent

The lessons I observed with Bent were either 30 minutes lessons or 60 minutes lessons. The actual part of the lesson was always shorter because of disciplinary and organisational aspects. Every lesson started with chairs and desks in any disorder in the classroom; students talking or walking around and some students coming late. Bent had to calm the students before he could start teaching. An overview of Bent's lessons from which I have presented data excerpts, is presented in the table below.

Excerpts	Date	Mathematical topic
1, page 151, 2, page 155 3, page 156, 4, page 158 5, page 160, 6, page 161 12, page 176,13, page 177	Jan 8th	Geometry Surface area and Volume
7, page 163, 8, page 165 9, page 169,	Jan 16th	Geometry Overview of volume
14, page 179,	Feb 5th	Fractions, Individual work
10, page 171, 11, page 172 15, page 181,	Feb 19th	Fractions

Table 8, Overview of data excerpts from Bent's lessons

It struck me in the beginning and throughout the lessons the students asked several times in what book they were supposed to do the work, in the workbook or in the rulebook. As described in the previous chapter, the rulebook is a book in which the students are allowed to write rules, formulae, conventions, whatever they wish, and they are allowed to bring this book with them when having tests and also for the final exam. The rulebook is thus meant to be a mediating tool for the students to use both in the learning process and as a reference book on the final exam and other tests. The workbook is the book in which the students are supposed to do the homework and also the work at school.

The textbook was frequently used in Bent's lessons and he referred to explanations and examples in the textbook for the students to study and he gave them exercises from the textbook both to work with at school and for homework. The students were sitting in pairs which the class's teachers[24] had decided. According to Bent the students could choose whether to collaborate with their partner or not.

I have based the analysis of the classroom observations on the three categories "*Conditions for possibilities of learning (CPL)*", "*Mathematical focus (MF)*" and "*Students' abilities (SA)*" which, as I have indicated in the previous part, emerged from the analysis of the conversations I

[24] The teachers who were teaching different subjects in the class

had with Bent. The structure of this part is that there will be one "Whole class section" in which I address "Conditions for possibilities of Learning" and "Mathematical focus", and one section on "Individual work" in which I address "Students' abilities". In both sections I emphasise the teaching strategies Bent used with regard to how he created conditions for possibilities of learning, to mathematical focus and also to how he supported students according to their individual needs. An overview of his wide range of teaching strategies is presented in the "Portrait of Bent" which is the final part of this chapter.

Whole class sections of lessons

In Chapter 5, in the part about "Pattern of discourse in the mathematics classroom", I have written that the lessons I observed had a kind of lesson opening. In his lessons, Bent gave a short overview of the work of the day and he referred to the work plan, the rulebook and textbook. After having calmed the students, he either asked a mathematics question directly to get students attended to mathematics followed by an overview of the lesson of the day, or he told them what they should work with (topic and working methods) during the lesson, before he invited them to participate through an opening question. Some lessons had only whole class work and no individual work / group work or work in pairs, some had both whole class work and individual work, and some had only individual work (or work in pairs).

Conditions for possibilities of learning

First in this section, under the heading "Overview, opening question and tools", I will exemplify, using excerpts from 5 lessons in order to show how Bent in the opening of each lesson created conditions for possibilities of learning through

- illustrating with concrete materials which thus acted as a mediating tool in the learning process;
- presenting overviews of the course of the lesson and/or of what the students so far were supposed to know about the topic;
- the use of singular "you" addressed each individual student;
- how an opening question directed students' attention and invited them to participate. To show what I refer to as opening question, I have put them in italics in the quotations I am presenting.

I am also pointing out how, by analysing the teacher's use of the pronoun "it", I can account for confusions among students and how the use of personal pronouns emphasises the role of the teacher and that of the students in the lesson.

Next, under the heading "Pattern of discourse and communicative approach", I analyse the discourse and communicative approach in the lesson Jan 8th to show

- how learning can be seen as participation in the discourse of the lesson;
- how Bent dealt with the complexity of the classroom situation;
- how contributions from a student changed the course of the lesson;
- how Bent demonstrated teaching strategies in creating conditions for possibilities of learning by
    - challenging students' thinking,
    - testing a conjecture and
    - sharing a contribution from a student with the whole class.

I use the notion "authorship of knowing" when students or the teacher articulated knowledge which has been socially validated. In pointing out how I perceived that there were miscommunications or lack of closeness in perspectives between the teacher and the students I suggest that studying the use of "it" as a pointer to something not yet agreed upon, partly can account for such lack of closeness in perspectives.

To further underpin the analysis of how Bent created conditions for possibilities of learning, under the heading "Triadic pattern of discourse and the issue of revoicing", I present an episode from the lesson Jan 16th in which I show how

- Bent invited the students to participate;
- he took their contributions into account and commented on them and
- he challenged students by questioning their thinking.

In the analysis of this lesson I

- use the construct "mediating tool" from socio-cultural theory in accounting for how I interpret both the teacher's contribution, the language he used, and also contributions from students as mediational means;
- highlight how Bent summarised a discussion and symbolised the findings;
- point out how the use of "it" as a conceptual variable can account for miscommunication and
- I point to the complexity of the situation in the classroom with which Bent had to deal.

Finally I refer to significant aspects of how Bent dealt with the complexity of the classroom and how I saw Bent created conditions for possibilities of learning by revoicing and challenging students' thinking and illustrating with drawings in yet two more lessons, Feb 5th and Feb 19th.

In the section "Collecting and handling data" in Chapter 4 (page 80) I explained how I labelled episodes from the lessons of which I present excerpts from my data. Excerpt 1, Bent Jan 8th, episode I-1 is the label of the first excerpt presented from the classrooms, the teacher is Bent, the date of the lesson is Jan 8th, it is from the first section of the lesson (I) and the first episode.

Overview, opening question and tools

In the opening part of the lessons, Bent always referred to previous knowledge. Sometimes he started the lessons by referring to what they had worked with in a previous lesson, or what they at this stage ought to know about a topic, whereas other times he presented an overview of the day before referring to previous knowledge. In analysing what the teacher did in creating conditions for possibilities of learning, I have used the concept of *common ground* when referring to what the students in class were supposed to know about a topic or what they had been working on in a previous lesson. Also when referring to common habits and tools like work program, rulebook and workbook I use the construct "common ground". This construct has been useful for me as a tool in interpreting what I observed in Bent's classroom and it incorporates the meaning of "shared practice" meaning that there is an agreement in class how to practice the use of a book (rulebook, workbook).

For example in the opening part of Bent's lesson **Jan 8th** which lasted for 4 minutes, (see Excerpt 1, Bent Jan 8th, episode I-1, page 151) the teacher referred to previous work right away without giving an explicit overview of the whole day's work. By referring to yesterday's lesson in turn 1, the teacher reminded the students what they currently had been working on, the class's "common ground", and he asked an "opening question", a question related to their previous mathematical knowledge and thus a question to which the answer is supposed to be part of the class's common ground. This was a typical teaching strategy observed in Bent's classroom. In turn 3 he referred to what they now were supposed to know and what they had recently been working with; the name of the polyhedron which he illustrated by drawing on the board. He referred to the working program and he said what he wanted to focus on as a start of this lesson. In turn 7 he emphasised the purpose of this task and referred to the textbook where the task was presented.

Excerpt 1, Bent Jan 8th, episode I-1

Nr	Who	What is said	Comments
1	Bent	Yesterday we looked at a prism. Do you remember that we had the solid blocks? Does anybody remember the name? Sigurd?	Teaching strategy: ref prior knowledge Opening question
2	Sigurd	No, it is just a,…no wait..	
3	Bent	A right, a four sided right prism. Now you have your books, don't you? Have the book open and be ready to start. I will focus on a special activity from the work program on which you should have started.	Bent gives the answer without asking another student
4	Studs	On what page?	
5	Bent	It is on page 172	Some noise, 40 sec pass. Bent waits for the students to open the books
6	Bent	Page 172. There is an activity there which you are asked to do and which you hopefully have prepared at home?	Repeats. Questions homework (new pause)
7	Bent	Page 172, There is an activity there. Let us find it, because there are some things I'll do together with you. Here is a right quadrangular prism. You are asked to find surface area and you are asked to find a formula. It is the formula we are out for. Not the formulae itself, but how you have been thinking in working out the formula.	Repeats page number Illustrates by drawing a right quadrangular prism on the board

Through these first turns Bent created conditions for possibilities of learning by reminding the students about the name and shape of a prism with four sided base, what task they should work on and the purpose of this task. He emphasised that the purpose was on how to *derive* the formula and it was not on the formula itself he would focus (turn 7). This episode also demonstrated disciplinary aspects. Students had forgotten to bring their books and had to go and get them (it took 40 seconds) and they did not listen carefully to the teacher when he said what page it was on, so he told them three times. According to my field notes Bent spent 4 minutes calming the class before he started teaching mathematics in this lesson.

The next lesson, **Jan 16th**, was a 30 minutes lesson in which there was no individual work. The first 2 ½ minutes of this lesson Bent motivated for participation in "Kapp Abel" which is a mathematics competition between lower secondary classes from all over Norway. After this he directed students' attention to their common ground telling them to have their workbook, rulebook and textbook ready and that he was going to give a "summing up" of what they had been working with lately; he would focus on the formulae for the volume and surface area of solid blocks and that they were not going to do any calculations. He had brought concrete materials to the lesson to illustrate the similarities in the formulae for volumes of the different blocks. He also used a piece of paper to illustrate a cylinder which unfolded demonstrated that the surface area turned out as that of a rectangle. Thus he used concrete materials as mediating tools in the learning process. The "opening questions" which I have put in italics in the quotation below, were asked after some discussion with the students about where to write all this, whether in the rule-book or in the workbook. He said:

> We shall now do a quick summing up of how you think about the volumes of these figures we have been working with. And then you reach, [ ] I will not calculate with concrete measures, but rather talk a little about formulae and the way of thinking about that. *This cube, what is it standing on in a way? What is the surface standing on the table? How can you describe the surface which is lying on the table?*

Through these words, Bent told the students what they were going to focus on in this lesson, on which he invited them to participate through the opening questions. These opening questions emphasised the ground base of a solid which is essential in the formulae for volumes of solid blocks. Conditions for students' possibilities of learning were thus created both by illustrating with concrete materials, inviting students to participate through the opening questions and by focusing on the base of the solid.

The third example of an opening part is from the lesson with fractions, **Feb 5th**, there was no opening question and there was no whole class lecturing either. Bent had prepared two sets of tasks, worksheet I and II. He said:

> We shall work with calculation with fractions, and after a while *we* shall have got a relation to fraction. *You* already have. I have made a work sheet. I think *you* will do that quite easily. It is meant to give an indication of how *you* are doing with regard to the technical calculation with fractions. Can *you* do these, add, multiply and divide? And then *we* have another sheet afterwards which reveals more of the understanding of fractions. I want *you* to fill in these, use the workbook, and calculate through them.

I have put the personal pronouns he used here in italics to emphasise that he talked directly to the students using singular you[25]. He thus addressed this directly to each student which can be a wish to emphasise that this is meant for each individual to find out how s/he is doing with regard to the work with fractions. After these words Bent spent some time encouraging the students to collaborate in pairs and he spent some time organising those who did not have anybody to work with. Bent thus created possibilities for learning by preparing these worksheets, telling the students

[25] In Norwegian "du" is singular you, and "dere" is plural you. The ones in italics in this quotation are "du"

about the work of the day, encouraging them to find out what they were mastering and organising them into pairs.

The next illustration of an opening part is from the lesson **Feb 19th**. They were still working with fractions and Bent created conditions for possibilities of learning through the first turn by telling the students what they should work with during this lesson and by asking the opening question which is put in italics in the quotation below. He had also drawn a grid on the board. He said:

///	

> I can point out what to put in the rulebook if *it* is a rule. I think you have written *it* earlier. *It* is about multiplying fractions. [There is a pause because of some problems with the tape recorder]. Draw that quadrilateral. There I have made a small square with four smaller squares. And I have shaded one of them [ ], and *that one I can write as one fourth, can't I? Do you agree? If I shall draw, let me see, shall multiply by four like that, what part would I have got then?*

The opening questions in this lesson were thus both a request for agreement that he can write one of the little squares as ¼ and also that if you multiply it by four you will get the whole. These should not be difficult tasks, however, several questions came up from the students; what the drawing was and why and what they should write in the rulebook. As an aid to find a possible explanation why so many questions revealing students' confusion came up, I studied the use of "it" and how "it acts as a linguistic pointer, invariant at the surface level" (Rowland, 2000, p. 102). I have italicised the pronoun "it" in this quotation to suggest the use of "it" as a pointer to something not yet introduced or agreed upon. I interpret this as one reason for students' confusion about what they were supposed to do. In the third sentence Bent indicated that "it is about multiplying fractions". However, this was not the whole part of what "it" could be. "It" was pointing to what he was going to say in this lesson that the students should write in their rulebook, and "it" was also pointing to something the students had written earlier.

The last example of opening part of a lesson is from **April 1st**. The mathematical topic was geometry, Bent started by saying what they were going to do in this lesson. In this quotation, I have put the personal pronouns in italics. The study of the personal pronouns helped me in analysing how Bent through the use of "I" and "you" (plural) emphasised what his role was going to be in this section of the lesson and what the students' roles were. Later on in this section of the lesson, Bent explicitly told the students not to interrupt. This was the only time he did not invite the students to participate while lecturing. He wanted to show how to do a construction task on the board without interruptions and the students were told to copy from the board.

> The first part of this lesson *I* will do an exercise on the board. *I* will start with a sketch, then *I* will construct and then *I* will calculate the unknown sides and then *I* shall end up with a quadrilateral in the end. And there are some different things *I* want *you* to learn through this. *You* shall [ ] how to work out this type of a task, therefore *everybody* is supposed to write it down. Then there will be an application of Pythagoras, and we shall build a, make a triangle first and then extend it to a quadrilateral. It is a task on the work plan, so you have to find your book.[26]

By studying how Bent used the personal pronouns this quotation clearly shows that the teacher's role in this section was to do and to show a construction task on the board, while the students' roles were to copy the teacher's work.

So far I have presented how Bent in the beginning of each lesson referred to the class's common ground, presented an overview of the work of the day, and invited the students to participate, and also how he through opening questions early in each lesson created conditions for possibilities of learning. I will now proceed to how the nature of the discourse and the communicative approach revealed how Bent created conditions for possibilities of learning throughout the lesson.

<u>Pattern of discourse and communicative approach Jan 8th</u>

In the previous section I looked upon the openings of Bent's lessons. I will now look at what comes next in his lessons. In the analysis I have used the triadic I-R-E (F) pattern of discourse which I have described in Chapter 5 as a concept helping me interpreting the classroom discourse. The triadic pattern of discourse, I-R-E (F) has been useful as a framework to account for how students were active contributors in the "R"- the response phase and how Bent restated students' contributions and thus legitimated them in the feedback (F) phase. I will also account for how Bent dealt with the complexity of the classroom, how he was probing for students' conceptual understanding through his questioning and how I perceived that miscommunication took place. I will then provide further evidence for my findings by showing how these aspects of discourse and communicative approach occurred in the others of Bent's lessons.

On page 151 I presented turns 1-7 from Episode I-1 Jan 8th to illuminate an opening of a lesson. I will again use Episode I-1 to illustrate how the analysis of the pattern of discourse and communicative approach in the continuation of the opening of the episode can illuminate how Bent created CPL for the students in his lessons.

In the continuation of the opening part of Episode 1, Jan 8th, (from turn 7, see Excerpt 2, page 155) there was a dialogue between one student, Einar, and Bent. Bent asked a question which as an initiation could

[26] The italicised "you" in this quotation are singular

have opened up for Einar's elaboration of how he had done the task without interruptions from the teacher. In turns 10 to 20, 25 to 27 and 34, Einar demonstrated *authorship of knowing*. As discussed earlier, authorship of knowing is articulated knowledge which has been socially validated. The teacher controlled the course of this part of the lesson by commenting and consenting in between Einar's contribution although Einar demonstrated that he knew the formula for the surface area of the prism and how it could be worked out. This part had a triadic I-R-F R-F pattern of discourse. There was a dialogue going on between the student and the teacher, where the student demonstrated authorship of knowing and the teacher contributed with clarifying comments and consent. (Have you multiplied by two (turn 19)? Did you put any sign in between here (turn 26)?) Bent expanded Einar's explanation in turns 11 and 15 through the use of "*yes?*" and "*and?*" In this sequence Einar took actively part in the I-R-F-R-F pattern of discourse when providing the explanation which Bent restated, expanded and challenged. Einar's explanation was thus legitimated by the teacher and the other students were the addressed listeners. Possibilities for learning were created through participating in this discourse of the classroom.

Excerpt 2, Bent Jan 8th, episode I-1 (turn 7-21)

Nr	Who	What is said	Comments
7	Bent	How did it go? Einar, have you come to any formula there?	Asks a student directly
8	Einar	yes, ehm..	
9	Bent	How did you think about it?	Asks for student's thinking (I)
10	Einar	I have thought that two and two sides have the same size	Student's answer (R)
11	Bent	Yes?	Feedback +Probing (I)
12	Einar	The one over and under and side and side	Student's answer
13	Bent	Okay, the top and the bottom and?	Restates with other words and probes further
14	Einar	Front and back and the side	
15	Bent	Front and back and the end in a way, yes!?	Restates and probes
16	Einar	The top is length multiplied by breadth,	
17	Bent	So you have taken length multiplied by breadth?	Restates and probes further
18	Einar	And there are two of them	
19	Bent	Have you multiplied by two there?	Clarifies
20	Einar	I have written two *l b*.	
21	Bent	*2 l b* yes like that, yes. Mmm	*Writes 2lb on the board. Siv, has raised her hand*

Then another student, Siv, had raised her hand to ask a question. (see Excerpt 3, below). Bent said he did not want to answer her question right there, although she said it was about the formula (turn 23). Bent demonstrated his control of the course of the lesson by letting Einar finish first. Not letting Einar present his way of doing the task without commenting/restating/asking clarifying questions, emphasises the teacher's control of the course of the lesson and also a legitimating of Einar's contribution. A reason for not letting Siv interrupt could have been to keep the control and that he then thought that some of the other students could have lost track.

This shows how the teacher had to deal with the complexity of this classroom situation: Einar who was providing the explanation, Siv who interrupted and the other students who might have lost track and started talking with classmates. However, Siv did not give up. In turn 28 she interrupted again. This time Bent responded by asking a clarifying question (31) back, and again he demonstrated that he wanted to finish the course of the lesson by promising to answer the question later.

Excerpt 3, Bent Jan 8th, episode I-1 (turn 22-35)

Nr	Who	What is said	Comments
22	Bent	Is it a comment on what he is doing now?	Referring to Einar's explanation
23	Siv	It is about the formula	
24	Bent	Yes, let him finish before you…	Controls the course of the lesson
25	Einar	Then I have done the same for the sides	
26	Bent	Have you put any sign in between here?	Clarifies
27	Einar	Yes, plus and then I have done the same so it makes two *l h*	
28	Siv	Can I just ask a question? (interrupts)	
29	Bent	Yes?	Lets her ask a question
30	Siv	Why doesn't he put brackets?	Interrupts
31	Bent	What would you have put in brackets?	Questions student's thinking
32	Siv	I would have put l multiplied by b in brackets.	
33	Bent	Ok, brackets like that? (yes). I can take that afterwards because there are several possible ways to put brackets here. But let us carry on, we can take it later. Do you take the last part?	Expresses control of lesson
34	Einar	And then plus and 2 b h which is the last one.	
35	Bent	Yes, and that will equal- Surface which we are supposed to find out here. Has anybody else arrived at that?	Consolidates (Bent counts raised hands)

Siv's initiation about using brackets in turns 30 and 32 above, together with another student, Sigurd's response (turn 37 in episode I-2 page 158) to Bent's opening up for other ways of thinking about "it" which points back to how to solve the task (turn 36 in Episode I-2), changed the direction of the lesson and resulted in a discussion about use of brackets. This episode (episode I-2), which followed directly from episode I-1, illuminates how Bent took students' contributions into account which in turn changed the direction of the lesson. Bent listened to students, he let them come up with their thinking about use of brackets and he questioned their thinking. That he took students' contributions into account caused a shift in discourse from turn 36. Such shift in discourse can enhance conditions for possibilities of learning. It was not only a shift in discourse but also a shift in mathematical focus, from the formula for the surface area of the prism to algebra and use of brackets in formulae.

From turn 36 to 48 in Episode I-2 (page 158) I see another kind of discourse than in the first episode. This episode shows how Bent through his questioning was probing students' thinking. Structuring and challenging students' thinking by asking probing questions was one of the teaching strategies Bent demonstrated. The sequence started with Bent's invitation to the students to present how they had been thinking. This is an example of several shifts between the students' contributions and the teacher's feedback described in Chapter 5. After having asked *if anybody had been thinking differently* (36), he asked *if it matters* (38) or *if it makes any difference* (40) followed by *in what way* (42) will it make difference or not, before he *suggested to put values in the formula* (46) to see if that made a difference or not. What I have put in italics illustrates the different understandings of the use of brackets in the formula which he tried to challenge. Bent was not only content with if it mattered to use brackets, he wanted to know in what way, and finally suggested how they could test a conjecture if it mattered or not by putting values in the formula. Hence, suggesting trying out a conjecture by putting values in a formula was yet another teaching strategy Bent demonstrated.

Then three contributions from students came up: The answer will be the same (43), Eva who had put her hand up, but withdrew, and Tove who mixed with $2(l+h)$ (turn 47). Bent said that he thought he followed what Tove meant. However, he did not take it into account, he demonstrated authorship of knowledge (48) and did not clarify the difference in the use of brackets in $2(l+h)$ and $2(l \cdot b)$ which Tove through referring to algebra, demonstrated that she thought was the same as $2l \cdot 2b$. Bent said he thought he followed what Tove said, however, his "clarifying" comment (48) did not clarify Tove's conception of $2(l \cdot b)$. One conjecture is that Bent chose deliberately not to focus on the distributive law which they had been working with in algebra because that could cause confusion among other students or it would have taken too much time away from the work with formulae.

To account for this I have used the concepts “miscommunication” and “lack of closeness in perspective” from constructivist theory. In this case there seemed to be a lack of closeness in perspectives or a miscommunication between the teacher and the student in this episode. Tove’s conception of the use of brackets could be viable within her field of experience, however, as I have discussed in Chapter 3, (Theoretical perspectives underpinning my study, page 52), von Glasersfeld (1995) emphasises that it is not enough for the teacher to present a counterexample which lies outside the student’s experiential world. Bent’s comment in turn 48 is not an “answer” to Tove’s conception and can thus be viewed as lying outside her current experiential world. The teacher did not build up what von Glasersfeld (1995) calls a “model of the student’s conceptual structure” (p. 14).

In turns 38-41 I have put the pronoun “it” in italics. Again this shows the deictic use of “it” (Rowland, 2000), how it is used as a pointer to concepts not yet agreed upon. “It” was pointing to the (use of) brackets, however there was not yet a common agreement in class where to put the brackets which I see as one reason for the lack of closeness in perspective which occurred in this episode. The second “it” in 39 seems to be pointing to the formula and not to the brackets.

Excerpt 4, Bent Jan 8th, episode I-2 (turn 36-48)

Nr	Who	What is said	Comments
36	Bent	Has anybody done it in another way? Been thinking differently? Sigurd?	Teacher asks an open question
37	Sigurd	Haven’t been thinking differently, just two times l times h, I mean two and l times h in brackets.	Student suggests to use brackets
38	Bent	You have put brackets there (2lh) yes do you need those brackets there? You mentioned *it*, ought *it* to be there or not, or does *it* matter?	Teacher’s questions increase focus on whether brackets are needed or not
39	Siv	*It* gives a better overview. I didn’t understand when I saw *it* in the book. However, if there are brackets there, *it* would have been easier to understand, because we have learned algebra in a way.	Student links to algebra. “It” as a pointer to something in the book
40	Bent	Does *it* make any difference with or without brackets?	Teacher is probing students’ contributions
41	Students in class	Yes, no, yes	(I can hear uncertainty among stu-

			dents)
42	Bent	In what way will *it* make difference or not? The brackets will they change anything?	Teacher continues probing
43	Stud	The answer will be the same	
44	Bent	Eva?	Eva had her hand up
45	Eva	No, nothing	
46	Bent	You would say something else. The question is if the formula will change. Will you get another answer if we put values into it and you'd used brackets? What is *it* that, Tove?	Teaching strategy: Imagine using values in the formula (*Silence can be "heard")*
47	Tove	We think that we shall multiply what is within the brackets first, that *it* is l times b and then you multiply that by two, or you take two times l and then two times b as we learned when we had algebra, we multiply two into the brackets.	Tove's use of we. I can hear her "partner" comment. They have collaborated in pairs
48	Bent	I think I follow what you say. To be very accurate here you'd have to put $2 \cdot l \cdot b$ (writes on the board). In a way there are multiplication signs in between here, but one usually doesn't write them. What you say, Einar is that you write two times length times breadth. (Can hear students discussing this) Say it aloud, Jens!	Teacher concludes that 2lb is the same as $2 \cdot l \cdot b$, which is not an answer to Tove's problem. Teaching strategy: Sharing:Say it aloud

In the end of turn 48 Bent must have heard that Jens said something which he wanted him to share with the rest of the class since he asked him to say it aloud. This is yet another teaching strategy Bent used, *sharing*, so the rest of the class could take part in Jens' suggestion which was that "it doesn't matter how you multiply, in what order, the answer will be the same, anyway". From turn 48 to 54 Bent and Jens had a consenting dialogue and they both demonstrated authorship of knowledge. Bent concluded the dialogue by consenting to Jens:

> Yes it will be the same if I put a bracket there and then we have learned how to open up brackets, and I can open up the brackets here afterwards. I agree with you Jens, it doesn't make any difference if you put brackets or not. It is up to you if you wish to have it for a better overview.

This indicates that Bent's intention now was to conclude the discussion about brackets, however, another girl, Camilla, demonstrated that she had the same conception as Tove with regard to $2(l \cdot h)$. She explained what she meant in turn 56 (see excerpt page 160); however, Bent was still seeing $2(l \cdot h)$ and $2 \cdot l \cdot b$ as the two alternatives discussed (turn 57).

Excerpt 5, Bent Jan 8th, episode I-2 (turn 55-62)

Nr	Who	What is said	Comments
55	Bent	Let me hear, Camilla?	
56	Camilla	If you take two times l times b or two times l and two times b. It makes a difference!	Tries to explain what she means
57	Bent	Let us spend some more time on it. If you'd say, if you had the expression $2 \cdot l \cdot b$ or $2(l \cdot b)$ Those are the alternatives we are discussing.	Resigned and laughing. Writes on the board
58	Student	Can put it all into brackets.	
59	Bent	Yes, you can put all into brackets like that, yes. Then we say that the length is 10 cm and breadth is 5 cm. Can we exchange with values and see if it makes a difference? (*Can hear students say, "yes, it will")*. Two times length which is ten, times breadth which is five is two times ten, makes twenty times five, hundred. Two times $(l \cdot b)$ two times brackets again fifty, two times fifty is hundred. Any difference here?	Calculating on the board while talking: l=10 b=5 $2 \cdot l \cdot b =$ $2 \cdot 10 \cdot 5 =$ $20 \cdot 5 = 100$ $2 \cdot (l \cdot b) =$ $2 \cdot (10 \cdot 5)$ $2 \cdot 50 = 100$
60	Camilla	That was not what I meant. I meant two times 10 and two times five.	
61	Student	Now you are far out..	Stud comments
62	Bent	The answer is the same. You can think (ponder) through that and we can look at it individually afterwards. However, I want to recommend not using brackets at all. The answer becomes exactly the same. I mean the calculations will be the same. Personally I think that use of brackets there is too much, it becomes too much. Don't need the sign there then, I think	

This shows that he interpreted Camilla's contribution from his point of view, in the same way as he did with Tove in the previous episode. Bent did not make any clarification of the students' conceptions here. I conjecture that both Tove and Camilla mixed *2(l+h)* and $2(l \cdot h)$ and believed they could use the distributive law in the second expression as they could in the first. Camilla very clearly told Bent that he had not caught what she meant; and she articulated very clearly, also with values what she meant (60). This contribution however, was not taken into account by the teacher. I therefore indicate that there was a lack in closeness in perspectives between the teacher and the student; that the teacher did not build up a viable model of the students' thinking. However, it seemed as if the students became aware that their way of thinking was not the same as that of the teacher, however, as long as the teacher did not express that he had understood how the student was thinking, it became difficult for the student to make sense of the teacher's ideas. After turn 60 we did not hear any more from Camilla as, after turn 47 we did not hear any more from Tove, which suggests that they had given up.

The final excerpt (episode I-2, turns 71-76, page 161) from this episode shows how Eva demonstrated authorship of knowing. She introduced herself quietly into discussion, and Bent calmed the class and asked her to repeat. This is yet another example of how the teacher used the strategy of revoicing or restating what the student had said and thus legitimated the student's explanation. Eva was the author of this mathematical outline, Bent was the animator, and the other students were listeners. However, the teacher did not use this to clarify Camilla's and Tove's misunderstanding above.

Excerpt 6, Bent Jan 8th, episode I-2 (turn 72-76)

Nr	Who	What is said	Comments
71	Eva	(inaudible) Can put two before the brackets?	
72	Bent	Stop a minute, Eva is talking (*Interrupts other students*)	B deals with the complexity of the classroom
73	Eva	If you write two and then make brackets and then write lb plus lh plus bh and then brackets, does it work?	Student explains mathematics
74	Bent	Like this?	B writes what the student says on the board: 2(lb+lh+bh)
75	Eva	Yes	
76	Bent	Yes, it works, because there is one of each, one of each side of which there are two. Yes, it works very well, so that is Okay. That is surface. Good! Okay!	

In a conversation I had with Bent the week after this lesson (Pre lesson conversation 16/1) Bent expressed time pressure as a constraint for not wanting to carry further on with the discussion. He also expressed concern for the other students in the class who did not take part in the discussion. Thus cultural factors in a complex classroom constrained his decision making. These factors (time pressure and concern for students not taking part in the discussion) can be seen as being more central to Bent in this situation than his concern for one or two students (in this case Tove and Camilla) to develop conceptual understanding. However, some students continued arguing further about the use of brackets, and Bent then opened up for further discussion about brackets. This shows possibilities for learning through participation in a classroom discourse in which students were taking actively part. It also illustrates how the teacher had to deal with different students' contributions and demands, both students' explicit contribution and also the fact that some students became passive, had lost track and started talking to a class mate. I also perceived two different issues about brackets going on; one was Tove's and Camilla's conceptions about $2(l+h)$ and $2(l \cdot h)$, the other discussion was about $2(l \cdot h)$ being the same as $2 \cdot l \cdot h$ or not.

In this episode, Bent invited the students to participate and he let them come up with suggestions on which he commented, and he probed their thinking. That way conditions for possibilities of learning were created. Several times during this episode miscommunications between the teacher and student(s) took place. According to what Bent told me before the lesson and what he said to the students (turn 7, Excerpt 1, Bent Jan 8th, episode I-1, page 151) his intention was to focus on students' *thinking* when developing the formula for surface area of a cylinder. However by asking and open question (Excerpt 4, turn 36) the students started a discussion about the use of brackets in the formula. This episode shows how Bent dealt with conflicting issues in a complex classroom and the issue of students' different demands. I have pointed out how the teacher let a student explain the mathematics and through restating the student's contribution both kept control of the course of the lesson and legitimised student's mathematical explanation.

So far I have presented different aspects of communicative approach and pattern of discourse from one lesson. To provide more evidence for my findings, I will show how some of the same aspects occurred in the lesson on Jan 16th.

<u>Triadic pattern of discourse and the issue of revoicing</u>

To illuminate further how Bent created CPL I will present an analysis of Episode II-3 from the lesson Jan 16th (see page 163). The excerpt from the episode is presented below. Prior to this episode, the teacher had in conversation with the students developed the formula for volume of a cube and of a rectangular prism. Now he would generalise *Volume= ground base · height* to also be applicable to a prism with a triangle shaped ground base. Bent had asked: "Can I transfer this to the other ones I have here, the one with the triangle shaped base and the cylinder? Is it possible to carry out the same principle?"

In turns 3-10 one student, Andy, actively contributed in the mathematical discourse and the teacher restated, and thus legitimised and also expanded Andy's contributions. After Andy had suggested multiplying height with baseline (turn 4) Bent emphasised that that was the area of the triangle, which had not been explicitly asked for, but was part of the whole task. The teacher's contribution in turn 5 thus acted as a mediating tool between Andy's contribution and the task to be solved so the other students in the class could follow. Through this Bent kept the focus on the generality of the formula which was his expressed intention of the lesson. Conditions for possibilities of learning were created through students' participation in this classroom discourse. Andy's contribution in turn 6 can in the same way be looked upon as a mediating tool between Bent's emphasis on the original task and how to solve it. In turn 7 Bent restated what Andy said in turn 6, however slightly differently; Andy said *height times baseline* whereas Bent said *baseline times height*, which according to my experience with school mathematical textbooks is how the formula for the area of a triangle usually is expressed. Bent did not say *divided by two* but he included that when he wrote on the board.

Excerpt 7, Bent Jan 16th, episode II-3 (turn 3-11)

Nr	**Who**	**What is said**	**Comments**
3	Bent	Let us carry on with the triangle Andy!	
4	Andy	Has to multiply the height with the base line	
5	Bent	Yes, because it is the ground base, isn't it? We just have to include that it is the ground base multiplied with the height. It is the triangle there multiplied with the height that still is the volume.	Bent gives consent and highlights the goal of the task.
6	Andy	To find the ground base, you have to multiply the height with the base line and divide by two.	Andy authors the mathematics
7	Bent	Okay, base line multiplied with the height in the triangle. How can you separate between the height in the triangle and the height in the whole solid block, A?	*(Can hear that a student asks if base line and ground base is the same, however, the teacher either does not hear it or he ignores the question)*
8	Andy	Hmm?	
9	Bent	Can you separate between the height in the triangle and the height in the whole triangle shaped prism? Can you separate between that?	Bent highlights key aspects. Challenges the student
10	Andy	Because the height that is the height in the triangle shaped, that is that on the side there, because the ground base. Then there is the height of, from the one side of the base to the other side of the base, it becomes like breadth.	Student explains mathematics
11	Bent	Can label it h-one and h-two	Symbolises. W*rites* h^1 *and* h^2 *on the board. Not* h_1 *and* h_2

This contribution from Bent can be accounted for in several ways. First, it could have been done unconsciously. Second, Bent wanted to write the formula the way students were used to see it. Third, he did not say explicitly that he wanted *baseline* before *height* which could have been either not to put Andy on the spot or that he did not want to encourage a discussion about it because he looked upon it as a minor task.

In turns 7-10 Bent challenged further by asking for the difference between the height in the triangle and the height in the whole solid block. Andy still contributed with mathematical explanations which were restated by the teacher and thus legitimised. The teacher also "summarised" and turned the difference between the two heights into a symbolic expression h-one and h-two.

In the continuation of this episode (Excerpt 8 page 165) I perceive a miscommunication or lack of closeness in perspectives. Andy from the first sequence (see Excerpt 7 page 163) did not take part, but two other students did. Since I did not identify their names I have called them Stud 2 and Stud 3 in the excerpt. The communicative approach was dialogic, meaning that there was a dialogue between the teacher and the students going on. According to what Bent had said, his goal was to generalise the formula for volume of all prisms, and now to derive the formula for the volume of a prism with a triangular ground base. The teacher let students come up with comments and contributions, he commented on them, but he focused only on the triangle as the ground base in the prism. According to my field notes, some students had found out that the triangle the teacher presented as the ground base in the block was right angled. They suggested one of the rectangular sides of the solid as ground base. The length in the rectangle was the height (the one Bent labelled h2) of the prism with triangular ground base, and the breadth of the rectangle was the height (h1) in the right angled triangle. The other smaller side of the triangle now became height of the solid block. The volume of the block with rectangular ground base then was: $\frac{length \cdot breadth \cdot height}{2}$.

Stud 2 took up the concept "breadth" in turn 12 which Andy had used in turn 10, where he had explained the height of the solid block as the breadth of it (if you lay it down). However, Stud 2 seemed to mean that the smaller side of the rectangle becomes the breadth if you lay the solid block down. Bent was concerned not to use the term breadth about the side in a triangle. This shows that Andy had one conception of "breadth" (the distance from the triangular base to the triangular top, the height of the solid block), Stud 2 another conception which presumably was shared with Stud 3 (the smaller side of the rectangle side). Bent dealt with this complexity by suggesting using values and thus facilitating the task. Putting values into a formula to clarify was a teaching strategy he also used Jan 8th, when he put values into the expressions to show that it did not matter with brackets or not (Jan 8th, episode I-2, turn 59, p.160).

Excerpt 8, Bent Jan 16th, episode II-3 (turn 12-27)

Nr	**Who**	**What is said**	**Comments**
12	Stud2	Can call it breadth?	
13	Bent	Breadth, how?	Asks for clarification
14	Stud2	The breadth of the triangle	
15	Bent	Okay, the breadth there. However, I want to stick to what you have learned about ground base and height in a triangle.	Refers to previous knowledge. Conventional
16	Stud3	*It* will only be height if you lay *it* down? Because the height,… *it* might be discussed	
17	Bent	*It* can be discussed, but when does *it* become a height? Isn't **it** a height if *it* lies down in a special way?	What is "it"?
18	Stud3	It is okay saying that if you lay *it* down then you say that *it* is the height. The height upwards and h2 is then the breadth	
19	Bent	Could have called *it* h1 and h2, the height in the triangle and the height in the whole thing, do you agree?	Teacher suggests and asks for consent
20	Stud3	Can say breadth in the triangle	
21	Bent	*I* don't want to introduce breadth in the triangle, because then *you*'ll become confused. Shall see that h1, the first height in the triangle and multiply that with the base line.	Conventional
22	Stud3	h1, you mean the height within the triangle?	
23	Bent	The height within the triangle, yes	Restates
24	Stud3	And h2, that is the height …	
25	Bent	That is the height in the whole figure. And then you have to divide by two, don't you?	Structures student's thinking
26	Stud3	Is it h1 down there now?	
27	Bent	It says h1. Then you multiply with h2. It will be easier when we shall put numbers on it.	Conventional. Encourages

The different use of "it" in the turns 16, 17, 18, 19 and 21 emphasises that there was something not being quite clear in this sequence (Pimm, 1987). In this sequence, "it" functioned as a "conceptual variable" (Rowland, 2000). The first "it" in turn 16 pointed to the smaller side in the triangle, while the second pointed to the whole solid block and the third to the issue being discussed. The first "it" in 17 pointed also to the issue discussed. However, the second "it" in 17 seemed to point to an imaginary height, or to a definition of a height. Pointing to something not obvious, "it" had a deictic function. The third and the fourth "it" in turn 17 pointed to the height and the whole solid block respectively. The first "it" in turn 18 pointed to the solid block while the second pointed to the height. In turn 19 I interpret Bent's use of "it" as "what to call it", and thus as pointing more generally to the issue, for which he asked the class for consent. This unclear use of "it", can account for part of the reason why a miscommunication took place in this episode. Another reason could be that Bent treated the solid block as if there was only one possible ground base, which he had chosen to be the triangle while students saw other possibilities for ground bases in the solid block.

As an aid in analysing this episode I have studied the teacher's use of the personal pronouns "I" and "you" in turn 21. I see Bent's use of "I" here as a demonstration of authorship of knowing, imposing the students not to use breadth when working on triangles. The "you" (plural) was directly addressed to the students. This can be looked upon as a teaching strategy Bent decided to use and a way of dealing with the different contributions about the breadth so far. This strategy cut off a further discussion.

In a later conversation, Bent reflected upon this episode and said: "I remember we had to spend more time than I had expected because *they* didn't know, they got caught up in the concepts, what sides *I* actually meant. I had expected them to get that quicker"… (5/2 post).
In this case, it seemed to me that the teacher did not succeed in building up a model of students' conceptual thinking (Glasersfeld, 1995), and by giving "*they* didn't know what *I* meant" as a reason for this miscommunication might reflect a transmission view of teaching. However, it also illustrates the way Bent was reflecting upon why they did not know what he meant. It can also seem as if he put the responsibility for the miscommunication on the students.

This view contrasts L97's view and a constructivist epistemology where the teacher is supposed to make extensive accommodations to students' understanding if they shall not talk past each other as they did in this case (Cobb, 1988). According to constructivism the teacher and students are active meaning makers and they give meaning to each others' words and actions during the process of interaction. Therefore teachers and students often talk past each other like they did in this episode. According to Jaworski (1994) sharing of meaning is a crucial issue from a constructivist point of view. In this case the words the teacher used carried the teacher's meaning and not that of the students' and vice versa. As I have pointed out, the use of "it", and the three students' conceptions about breadth together with the other students' participation in this discourse, show the complexity, with which Bent had to deal. There were too many issues demanding his attention at the same time, so it was difficult to deal with each one on its own merits.

The triadic pattern of discourse was not so prominent in the lessons Feb 5th and Feb 19th but I identified the same aspects of Bent's teaching as pointed out so far. In the lesson Feb 5th, in his summing up, Bent challenged the students to express the difficulties they had with calculations with fractions. The teacher started by asking each pair of students directly to express their difficulties. When students just said: "dividing mixed numbers" Bent revoiced and structured their contributions. He asked clarifying questions and suggested reformulation of what they had said to highlight what their problem exactly was and promised to offer examples to clarify. Although Bent started by asking one pair of students at a time, many students spoke all at the same time. He thus had to deal with different contributions at the same time and he tried to sort out some of the common problems.

I did not find a typical triadic pattern of discourse in the fraction lesson Feb 19th either. As significant aspects of the whole class section of the lesson I noticed:

- The teacher used a grid to illustrate.
- During the first 10 minutes Bent asked for consent 10 times. (Isn't it? Is it reasonable?)
- Teacher focused on reasonable answers. Some students focused on the methods and seemed not to understand why they should "see" what the answer was when they could use the method to work out the tasks.
- The teacher and students seemed to be talking past each other.

Since this lesson (Feb 19th) so clearly demonstrated the tension between conceptual understanding and technical skills, I have chosen to present an episode which illuminates the above characteristics under the heading "mathematical focus" below.

Mathematical focus

In the conversations I had with Bent, he focused on the relation between computational methods and conceptual understanding. He gave the development of the ability to solve problems in different contexts as a reason for wanting to focus on conceptual understanding in his teaching. In the presentation of Bent's lessons so far, I have discussed *Conditions for possibilities of learning*; patterns of discourse and communicational approach and I have pointed to instances where there seemed to be miscommunications between the teacher and the students and also how students' contributions changed the direction of a lesson.

In Chapter 4 I presented the codes I used to identify aspects of mathematical knowledge in the lessons. In Chapter 5, I discussed "Aspects of mathematical knowledge" more thoroughly and how these aspects were mirrored in L97. In this section I will show how the different aspects of mathematics were focused in Bent's lessons. Both what aspects Bent intended to focus on and also aspects which became evident as a result of students' contributions in the lesson. I have divided this section in two subsections.

- First I refer to the lesson Jan 8th in order to point out how Bent focused on the derivational aspect of mathematics, how to derive the formulae for volumes and surface areas. In the lesson Jan 16th Bent followed this up and focused on the structural aspect and the generalisation of the formulae for volumes and surface areas of polyhedra. I refer how students' contributions caused an emphasis on other aspects. First the need for algebra in order to express formulae in geometry which mirrors a structural aspect (relations between different entities in mathematics); second, the conventional aspect in order to clarify a student's mixture of the terms *square* and *cube*.
- Next I discuss how Bent's intention with students' procedural work with fractions was for the students to become conscious about their own knowledge (Feb 5th) and how I perceived a tension between the teacher's focus on conceptual understanding of multiplying fractions and students' focus on the method of calculation (Feb 19th).

The mathematical focus in the lesson Jan 8th was the formulae for volumes and surface areas of solid blocks. However, it was not the formula itself that was the intended focus, but how to work the formula out. According to what Bent said in the quotation below, his intention was not to focus on the *conventional* aspect of mathematics. He told the students in the beginning of the lesson that he would focus on how to *derive* the formula and not the formula itself:

> Here is a right four sided prism. You are asked to find the surface area and you are asked to find a formula. It is the formula we are out for, not the formula itself but how you have worked to arrive at that formula (Jan 8th Episode I-1, turn 7).

Bent encouraged the students to derive the formula for surface area of a prism based on what they knew about area of a rectangle. Thus he focused on the *derivational* aspect of mathematics. He focused on the structure of the formulae and for the students to derive them to gain *conceptual* understanding of how the formulae were built up. He said: "[ ] they shall try to understand these formulae, make them and use them" (Jan 8th pre). However, due to students' contributions, they also dealt with algebra, the use of brackets and the distributive law (Excerpt 4, page 158, Excerpt 5, page 160 and Excerpt 6 page 161 from episode II-2 Jan 16th). This illustrates how both geometry and algebra were focused and the link between different entities and hence the *structural* aspect of mathematics.

Following up how to derive the formulae for volumes and surface areas of polyhedra, the focus in the lesson Jan 16th was to *generalise* the formulae for volumes of solid blocks; *Volume = ground base times height*. Bent emphasised the pattern or structure of how the formulae are built up, and the connections between a cube, prisms, and a cylinder. The mathematical focus of the lesson was thus the *structural* aspect together with focus on students' *conceptual* understanding of volumes. According to the conversation I had with Bent before this lesson, his intention was that students should get a holistic view of volumes of solid blocks: cube, prisms with rectangular, triangular and trapezoid shaped ground bases and cylinder. He wanted them to see the similarities in the formulae, that they are all ground base times height and emphasise understanding rather than cramming (Norwegian: "pugge") the formula.

I want to point out two instances in this lesson where the *conventional* aspect of mathematics had to be emphasised due to students' contributions. In the beginning of the lesson Jan 16th, when Bent had started with a cube and had developed the formula for the volume of the cube together with the students, Siv asked a question:

Excerpt 9, Bent Jan 16th, significant episode

Nr	Who	What is said	Comments
1	Siv	Isn't a cube the same as a square?	
2	Bent	About the cube? What is it with the cube that is a square?	He holds the cube up
3	Siv	All sides are the same	
4	Bent	Because you think in a way that all sides are squares? But would you call the whole thing a square?	*Some comments from other students, Siv did not answer*
5	Bent	It has square sides, however, we call it a cube.	Authorship of knowing

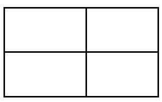
Siv's question suggests that she mixed the *conventional* terms *square* and *cube*. Bent did not tell her the difference right away, nor did he tell her that she was wrong. He rather questioned her thinking, and focused on her *conceptual* understanding of what a square is and what a cube is. Siv's answer in turn 3 *(all sides are the same)* is valid for both a square and a cube, however, Bent still challenged her thinking by first revoicing what she had said in a slightly different and more mathematical way and then he appealed to her judgement by questioning her thinking. Finally after a few comments from other students who had been thinking as Siv, Bent affirmed what the polyhedron was called and how it looked (turn 5).

Towards the end of this lesson the mathematical focus changed due to students' contributions and a discussion about "breadth" in a triangle took place. Then Bent used his position as a teacher and cut off the discussion and referred to the *conventional* aspect; not to use "breadth" when dealing with triangles (see Excerpt 8, Bent Jan 16th, episode II-3 page 165).

In the first lesson I observed when they were working with fractions (Feb 5th) Bent handed out two worksheets; Technical calculations with fractions (worksheet I) and understanding fractions (worksheet II). The mathematical foci in this lesson were thus both *procedural* and *conceptual*. However, the purpose of the procedural work was for the students to become conscious about what they mastered at this stage, and what they not yet had accomplished. The computational tasks were meant to be used as a means to judge their own knowledge and not to just exercise the procedures. Bent said that they could do a reasonable selection based on how much practice they thought they needed. That was a strategy Bent carried out in whole class to deal with the issue that students have different abilities in mathematics.

In the lesson Feb 19th I noticed a shift in focus concerning the work with fractions. According to the conversation I had with Bent before the lesson, the purpose was to enhance students' *conceptual* understanding of division and multiplication with fractions. I have identified significant episodes both from the whole class section of the lesson and from the individual seatwork section which illuminate the tension between conceptual understanding and technical skills of fraction arithmetic.

The teacher had illustrated on a grid that ¼ multiplied by 4 is 1.



$$\frac{1}{4}\cdot 4 = 1$$

And then the task was $\frac{1}{4}\cdot 2 =$

Bent wanted the students to "see" from studying on the grid that the answer was one half. However, Camilla's focus was how to work the task out according to an already learned method. According to my field notes she expressed her confusion why they had to do it this "complicated way" and not just use the method. She tried to get the teacher's consent that the method she expressed was right, while Bent tried to prove the method by considering a "reasonable answer." I have illustrated this in the excerpt below by writing her comments about the method in italics (turn 16- "*it is just to take four times one, isn't it?*" and in turn18- "*you* *take one times four and one times two"*) and Bent's focus on "seeing" a reasonable answer from the grid, in italics (turn 13 –"*does that sound reasonable?"*, turn 15 "*Do you think it sounds reasonable?*" and turn 17 "*But you see what it worked out to be?*). In turn 18 I have written out the calculation Camilla expressed (one times four in the denominator and one times two in the numerator). In turn 19, from looking at the grid, Bent encouraged the students to *see* that the method was right. I have studied the use of pronouns to account for Bent's focus and that of the students. Bent's use of "I" and "you" (singular) in turn 19 illustrated what *he* focused on and what *the students* focused on. "*And you have seen **it** already"*, he said (19). This suggests that he meant that the students could see the rule. His use of "it" was deictic; pointing to a concept. In this case "it" seemed to be the rule or the relation between the rule and what they could "see" from the grid.

Excerpt 10, Bent Feb 19th, episode I-1 (turn 13-19)

Nr	**Who**	**What is said**	**Comments**
13	Bent	In the same way, one fourth times two and then I would have got two squares within the picture. Two fourths or one half, *does that sound reasonable?* Andy!	Addresses Andy for disciplinary reasons?
14	Andy	Hmm?	
15	Bent	*Do you think it sounds reasonable?* Then I would get one half there? Yes, Camilla?	Camilla has put her hand up
16	Cam	You say that one fourth times four **–** *it is just to take four times one, isn't it?* You didn't say anything about *how* you worked it out?	Focus on the method
17	Bent	*But you see what it worked out to be?*	Focus on understanding
18	Cam	Yes you take *one times four and one times two*	$\frac{1}{4}\cdot 2=\frac{1\cdot 2}{4\cdot 1}$ method
19	Bent	Mmmm, Yes so that is what we had done to take four times one there. And *my* point is now that *I* have focused on what the answer is, and *you* want to know what *I* have done. And *you* have seen *it* already. Do you see that there *I* have taken one times two and that becomes two fourth? And *you* are out for the rule? But is **it** okay to multiply with a fraction?	Teacher tries to make the relation between a reasonable answer and the method for multiplying fractions

Bent continued working with multiplications of two fractions after the sequence presented above (turns 13-19). He drew a new grid to illustrate ¼ times ½ and asked the students what happens when you multiply a fraction by one half. They answered that it was the same as dividing by two. He exemplified with whole numbers and went on with fractions. The task was now to multiply ¼ by ½. I have studied the occurrence of and use of pronouns to account for the tension between the teacher's focus on "seeing" a reasonable answer and the students' focus on "the method". In turn 26 (episode I-1, Excerpt 11, page 172), a student Siv, put up her hand and asked explicitly about the method. Her use of "you" was not to address the teacher as "you", but to indicate the generality in multiplying fractions (is this how it is done?) (Rowland, 2000). Looking at Bent's use of pronouns in turn 27, the first "you" was addressed directly to Siv. Through his use of "I" he pointed to himself as the one doing this and trying to explain to the students the relation between the rule, the grid's illustration and that multiplying by one half is the same as dividing by two. In turn 27, I have put his request for consent in italics *("That is your logical answer?" [ ] "doesn't it?" [ ] "Is that logical?")* His use of "we" in turn 29: *"And then* ***we*** *have the experience from other numbers"*, addressed the attention to students' prior knowledge. Thus he tried to "bridge" between students' conceptions of a fraction multiplied by a whole number, which they already knew, and multiplying two fractions. In turn 29, I have also italicised "it" which in this case acts as a pointer to the rule: *Multiplying by ½ is the same as dividing by 2*.

Excerpt 11, Bent Feb 19th episode I-1 (turn 26-33)

Nr	Who	What is said	Comments
26	Siv	I just wanted to know, when you multiply fractions, do you multiply both the tops and the bottoms?	She wants to have the method confirmed
27	Bent	Because you want to get one eighth? *That is your logical answer?* I will give you an answer in a little while. Now I have put two extra verticals. Now there are two eighths. Okay. I did have one fourth and I multiplied by one half. That means that my shaded square has to be the half, *doesn't it?* It means that when I multiply by one half, I have to get the half left. And that is one eighth. *Is that logical*?	Teaching strategy: not answer right away. He uses the grid and puts two extra verticals Asks for consent twice
28	Stud	That one fourth …..	inaudible
29	Bent	Here *I* had one fourth. Then *I* multiplied by one half. And then *we* have the experience from other numbers that when *you* multiply by one half *I* get the half which is the same as dividing by two. But does *it* work with fractions? Does *it* work when you shall multiply one fourth by one half? And then *I* only get one eighth? This means that this square *I* have to divide in two, divide into two here. And then *I* have this bit left. *It* has to be that? Siv?	Tries to explain with the use of the grid and referring to whole numbers what the answer will be without using the method. Siv has raised her hand again
30	Siv	You have two fractions; do you then have to multiply both the tops and the bottoms? You multiply one by one, that makes one. Then I	Siv rephrases her question from earlier

		multiply four by two and that makes eight. One eighth.	
31	Bent	Top and bottom. That times that or that times that?	Clarifying
32	Siv	No, one times one and four times two.	Clarifying
33	Bent	You said, equals one eighth. And that is logical when one thinks that one has to bisect it, bisecting the square I had earlier? And then I have this single square left because that is half of the one fourth.	Writes on the board, Consolidates Siv's request and the illustration on the grid

In this sequence Siv asked three times to have the rule explicitly confirmed: turns 26, 30 and 32. Bent did not answer her question in turn 26 immediately, but said explicitly that he did not yet want to answer (but in a little while). Not answering a question directly was a teaching strategy Bent often used, but not always as explicitly as here. In this case, the strategy allowed him to elaborate further and to illustrate on the grid. Bent demonstrated reluctance just to confirm the rule. By illustrating on the grid, making a link to whole numbers and appealing to was "logic", he tried to bridge between previous and new knowledge and to build up mathematical structures and conceptual understanding.

However, these two sequences suggest that the students became confused when the teacher started focusing on the reasonable answer when they already knew the method for working it out. The teacher was doing something they were not used to. One student asked: *"Why do we have to do this when we know how to do it?"* This illustrates Skemp's (1976)

two kinds of mathematical mismatches which can occur;

- Pupils whose goal is to understand instrumentally taught by a teacher who wants them to understand relationally.
- The other way about (p.21).

In these sequences, the teacher focused on student's conceptual understanding of multiplication and division with fractions, while the students were focusing on the instrumental use of a method. In socio-cultural terms this can be accounted for as there is a culture of rule use among students and also parents whose assumptions Bent was trying to break, however he was finding it hard.

In this section I have analysed whole class sections of Bent's lessons. I have addressed the category CPL (Conditions for possibilities of learning) in which I discussed Bent's teaching strategies, the pattern of discourse, and that students became learners through participating in the classroom discourse. I have also addressed MF (Mathematical focus) and how different aspects of mathematics (structural, conventional and conceptual) became visible throughout the lessons. As I wrote in the beginning of "Analysis of classroom observations with Bent", the category

"Students' abilities", SA will be addressed in the next section, Individual work sections of lessons.

Individual work sections of lessons

The category "Students' abilities" emerged as described in the beginning of this chapter from the conversations with Bent, including students' difficulties and challenges, what they had mastered or not mastered and teaching according to students' different abilities. The category thus includes aspects of differentiating, aspects of how different students learn and issues about different students' mathematical knowledge expressed by the teacher and how he supported individual students according to their needs. The focus in this section is the latter, how Bent supported individual students according to their needs. I look upon the experiences the teacher gave the students in their work as support to develop their abilities to conceptualise the mathematics they were struggling with.

As pointed out both in the analysis of conversations with Bent and whole class sections of the lessons, the conceptual aspect of mathematics was often focused in Bent's teaching. My account of how I saw Bent supporting individual students, by focusing on the conceptual aspect of mathematics and thus creating possibilities for students' learning is based on four episodes; two episodes from the geometry lesson Jan 8th, one from the fraction lesson Feb 5th and one from the fraction lesson Feb 19th. These three lessons were the ones I observed which had individual work. The lengths of the individual work sections were 22, 29 and 7 minutes in the three lessons (Jan 8th, Feb 5th and Feb 19th) respectively, and I identified 10, 9 and 5 episodes respectively with an individual student or pairs of students in the three lessons. The lengths of the episodes varied between less than one minute and 6 minutes.

Bent prepared for individual work either in the opening part of the lesson or after the whole class section of the lesson. He thus motivated the students for the work by telling them what to do, what tools to use, and he encouraged them to collaborate in pairs. The individual work was linked to the work they had done in whole class in the same lesson. Also at this stage of the lesson, as in the beginning of each lesson, Bent had to deal with disciplinary aspects. The students in his class always took the opportunity to mess around when they were not strictly controlled by the teacher. A break between two kinds of activities was such a chance. Questions were asked about what book to use, rulebook or workbook and how to deal with the concrete materials and about the table to fill in (Jan 8th) and the two different work sheets (Feb 5th).

During individual work the students sat in pairs, however, the teacher did not demand that they should collaborate. Their roles were to do the assigned tasks Bent had prepared for them to do, and the teacher's role was to assist those who asked for help. He sometimes addressed students who had not asked for his help.

In all episodes, I noticed the following significant aspects: Bent encouraged and motivated by appealing to students' abilities and previous knowledge by saying "this you will manage quite well". When a student asked for help, either "How shall I do this" or "I do not understand", he never gave an answer or told them how to do it right away. He always either asked them to clarify their question: "What is it you do not understand?" or he asked the student(s) for suggestions "Do you have any suggestion?", or he asked if they had seen something similar before. Not only in the beginning of an episode but also throughout the episode in the dialogue with a student did Bent also ask for students' thinking or for their suggestions, and he always asked them to reason their suggestions. He thus took the student's thinking as a starting point and through his questions and comments he structured their thinking by reminding them of what they had done earlier on which they could build new knowledge or making a bridge between familiar and new knowledge.

In the analysis of how Bent worked with his students I have been using the notion of ZPD (zone of proximal development) from sociocultural theory as a tool to interpret how Bent as a more knowledgeable other supported individual students. I have used the term "scaffolding" when the teacher engaged in the zone of proximal development (Bruner, 1985). I have illustrated how Bent in his dialogues with individual students provided a bridge between the student's new and previous knowledge. He pointed out significant aspects of the tasks, and he also spaced out help according to the student's need. I will present exemplifying episodes from the lessons to illustrate this.

Geometry, Jan 8th, episode II-4

As described earlier in this chapter, the students worked with solid blocks (prisms, cube, and cylinder) and should fill in a table with name of the figure, a perspective drawing, and calculation of surface and calculation of volume in the lesson Jan 8th. The table had four columns and there should be one row for each figure. Some of the problems that occurred in this lesson dealt with the design of the task and not with the mathematical content. Many questions were asked by students about what to do and what the table was for. Bent's intention with this design of the lesson was for the students to get an overview, and also to measure sides and use the measures to calculate surface area and volume.

In the first episode below a student asked how to find the volume of a cube. It seemed as if he knew how to calculate the volume of a prism and Bent highlighted the similarities and differences between a cube and a prism and thus created a bridge between new (cube) and previous (prism) knowledge. As I have pointed out, one of Bent's teaching strategies in whole class sections of lessons was to ask for a student's suggestions. In turn 6 he encouraged transfer of the student's way of thinking of prism to the cube which made the student suggest a solution (7). In turn 8 Bent first emphasised the similarities (pointed to length, breadth and height) and then he asked for the differences (what is it that was special with this one) between a cube and a prism which made the student suggest what was special with a cube (9). In turn 10 Bent "summed up" by restating slightly more mathematically and confirming what the student had said.

Excerpt 12, Bent, Jan 8th episode II-4

Nr	Who	What is said	Comment
1	Stud	How to find the volume of this?	
2	B	Volume of the cube?	
3	Stud	Yes	
4	B	It looks like that. How would you have done it? Do you have any suggestion?	Teacher asks for suggestion
5	Stud	Other times when I have calculated volume, it has three sides	
6	B	There you have taken, for the prism you have taken length times breadth times height. Can you transfer that way of thinking to the cube?	Highlights similarities
7	Stud	Could multiply all of them with each other?	
8	B	Yes, sounds good. You have in a way length there, breadth there and height there. Makes length times breadth times height on that as well. But what is special with this?	Highlights similarities and asks for difference
9	Stud	All sides are equal sized.	
10	B	Yes. It is right what you say that you shall multiply all sides. Then you get the volume. Do you see the difference when calculating volume and surface? Do those being two easy concepts?	Teacher restates that multiplying all three sides makes the volume.
11	Stud	Yes	

Geometry Jan 8th, episode II-7

This episode (Excerpt 13, page 177) illustrates how a student was "struggling" with the prism which had a triangle as a ground base (which also was an issue in whole class). It was the area of the triangle that was the issue in this episode. In turn 2 Bent encouraged the student to "think back" on previous knowledge, which made the student suggest "divide by two" (turn 3). Bent challenged him further by asking what to divide by two and to recall what he had done with another triangle (turn 4). In turn 6 Bent expressed the formula explicitly for the student based on what the student said. He restated the student's answer in a more mathematical way and thus acknowledged the student's answer. The student was producing the mathematics while the teacher “scaffolded” by structuring the student’s thinking. Challenging the students to recall what he had done to solve a similar task, the teachers encouraged the students to “use the same way of thinking” (turn 10). The student still seemed to be unsure about the formula in this case. This could be an example of the “positive confusion” Bent had said he intended to provoke when students had to measure and calculate with actual solid blocks and not only with drawings in the book. Bent illustrated the area of a triangle by drawing a quadrilateral and explained that the area of the triangle had to be half of that of a quadrilateral.

Excerpt 13, Bent Jan 8th, episode II-7

Nr	Who	What is said	Comments
1	Stud	Area of this?	
2	Bent	Area of that? Actually it is a triangle. Have you ever learned about area of triangle?	
3	Stud	Yes, you just have to divide by two.	
4	Bent	What do you have to divide by two? Have you found out on the area of the right angled triangle there? What did you do there?	Points to another task where the student has calculated the area of a triangle
5	Stud	Three times four divided by….	Is interrupted
6	Bent	Yes, base line times height divided by two	Conventional
7	Stud	Three point nine	
8	Bent	And what is the height in the triangle?	
9	Stud	three	
10	Bent	Yes, and there you can use the same way of thinking	
11	Stud	But cannot just divide by two?	
12	Bent	Yes, you can because. Here *you* can see that there is a triangle, a right angled triangle and there *you* say baseline times height and then you divide by two. If it is a quadrilateral and the right-angled triangle equals that. If *you* say baseline times height *you* will get the whole quadrilateral, and then you divide by two to just get one of the triangles. However, the triangle you have got is approximately like this… (drawing?). But if *you* take baseline times height, *you* could have thought it was a quadrilateral.	From what is said I suppose that Bent shows by drawing how a right-angled triangle is half of a quadrilateral and then that is the case for any triangle.
13	Stud	It is still not the half.	
14	Bent	Yes, because the triangle there equals the triangle there, and the triangle there is the triangle there. That is why you can take baseline times height divided by two. Just a minute…	A lot of noise in the background. Bent has to calm the class
15	Stud	Okay. I see	
16	Bent	That is the way of thinking. In a triangle you	Generalising the

		can always take the baseline times height divided by two. So that is what you shall do. Baseline times height divided by two.	formula for area of triangle

In turns 12, 14 and 16 Bent explained *why* the formula for area of a triangle is as it is and he did not give up his explanation until the student said he understood. This illustrates how Bent focused on the student's conceptual understanding of the formula and that he not only told the student how to solve the task technically by using the formula.

Fractions Feb 5th, episode II-1: ½+3/4=

In the individual work section of the lesson Feb 5th there were 10 different episodes with a single student or pair of students. Bent had prepared two sets of tasks, worksheet I and worksheet II. In the analysis of whole class sections of lessons earlier in this chapter, on page 152, I quoted what Bent told the class was the purpose of these worksheets. Worksheet I had computational tasks with fractions; adding and subtracting tasks with both common denominators and with different denominators, and multiplication and division of fractions and mixed numbers. The purpose of this set of tasks was for the students to find out in what area of technical skills concerning fractions they had problems. The other worksheet had tasks which according to Bent revealed if the student had conceptual understanding of fractions. There were no episodes from work with worksheet II. A reason might have been that the students did not finish the first worksheet during the lesson. I will present extracts from some episodes to provide further evidence for the significant aspects described above.

In Episode II-1 (Excerpt 14, page 179), a boy (Ole) asked what to do when the denominators were different. He used the Norwegian "man" which is an unidentified third person often translated to "one" or "you". I emphasise this because it indicates that he asked for "the way of doing it", which emphasises the conventional aspect of mathematics. He did not remember which one was numerator and which one was denominator, he referred to them as "they" or "the number under". Bent did not tell him the names nor which was denominator. He structured Ole's thinking by reminding him about his knowledge (6), and by making links to adding fractions with similar denominators. Ole's classmate, Harald was included in the conversation (8 and 9). In turn 10 Bent asked Ole if he had met a similar problem earlier and thus encouraged him to think back. In 12 Ole is asked to reason why he did not add the denominators when doing 2/3 + 1/3. In this way he encouraged the student to reflect on his knowledge and to express what he knew. In turn 14 Bent made the link between 2/3 + 1/3 and ½+ ¾, and in turn 16 he consolidated Ole's thinking so far, and asked for his suggestion (16), and again he included

Harald. When he also refused (17) Bent reformulated his question and asked for another way of writing one half (18) and thus elicited Ole's answer. They agreed on 2/4. Bent probed Ole further in turn 20 and encouraged him to reflect on his answer. In turn 22 Bent challenged him by asking how he could say it was the same. In turn 24 Bent elaborated his question further by simplifying it and even further in 26. Finally Bent summed up what Ole had done, and why that was right (turn 28).

The task was: ½+3/4=

Excerpt 14, Bent Feb 5th episode II-1

Nr	Who	What is said	Comments
1	Ole	What do you (man) do when the number under is, when they are different?	The use of "man" and "they"
2	Bent	Meaning you have different?	
3	Ole	When those two are different	
4	Bent	*Who are those*?	Asking for accuracy
5	Ole	Those, two and four	
6	Bent	Two and four. *Do you remember their names*?	Probing to recall
7	Ole	Numerator and denominator, however I don't remember which is which.	
8	Bent	Harald?	
9	Harald	That is denominator and that is numerator	
10	Bent	Yes, numerator is the upper and denominator the lower. *Have you seen the problem before?*	Activates prior knowledge
11	Ole	Yes, but I don't remember what to do	
12	Bent	Don't remember. *But what is the problem, what kind of problem is it you have discovered here?* Here you have added two plus one, two third plus one third, and then you have taken two plus one is three. There you have three and three. Why did you not add the denominators?	Probes Ole to express what the difficulty is. Relates Ole's problem to what he has already done.
13	Ole	Because that wouldn't be right.	
14	Bent	No that is right. It had been wrong. So you got one. And then you did this (6/8+1/8) six plus one is seven. It is very good that you don't add the denominators. Because some try doing that. And then you discover a problem…	Consolidation
15	Ole	(Interrupts) because it doesn't make four sixth.	Expresses what it cannot be
16	Bent	There you have them similar, and then you know that they have to be similar to be added. Your challenge is to make them similar. *Do you have any suggestion for how to make them similar?* Do you know about a fraction making them? What about Harald?	Highlights key aspect
17	Harald	No	
18	Bent	Don't have a clue? *Can you write one half in another way?*	Eliciting

19	Ole	Yes, you can write it as two fourth.	
20	Bent	Does it help?	Encourages reflection
21	Ole	Yes, yes	
22	Bent	So try that. So you mean it is the same, two fourth plus three fourth is the same as one half plus three fourth. And then the answer? Mmm I agree, however, how can you say that this and that is the same, that one half and two fourth are the same? *What have you done with it to get that? Meaning with one half to make two fourth?*	Expresses and structures student's thinking Probing
23	Ole	Ehh	
24	Bent	*What do you have to do with two to get four?*	Simplifies the task
25	Ole	Expand it, I don't know?	
26	Bent	Yes, you have to expand the fraction. *What do you have to do to expand it to become four?*	Revoices and probes further
27	Ole	Multiply by two	
28	Bent	Yes, and then you have done the same on the top? So what you have done is to multiply both the upper and the lower. Because you know or you want that four has to be the common denominator, and then you have multiplied both the top and the bottom, and then you get four as a common denominator and then you can add.	Summing up what they have done, and presents it as student's work and thinking

I have put some of the questions and comments Bent made in italics. These are questions made to structure the student's thinking, by encouraging him to clarify the problem (turns 4 and 12), to think back (turns 6 and 22), to recall a similar problem (turn 10), to point out the significance of the problem. Bent asked for suggestion (turn 16), he simplified the problem (turns 18 and 24) and he challenged him to think through what he was doing.

I see this as a typical example of how a student could do this task with the help of the teacher who posed questions and thus structured his thinking. Ole was not able to solve the task on his own. They were working within the zone of proximal development and Bent's questions and comments acted as a scaffold. In turn 28 Bent summed up how to solve the task by repeating the process they had been going through and thus indirectly gave the student credit for having solved the task.

Fractions Feb 19th, episode II-5

The last episode I will present from individual seatwork with Bent is from Feb 19th. He wrote two tasks on the board, and according to what he said in the pre lesson conversation his intention was for the students to "see" the answer which Kari did (5, 7, 9, and 11) and then use the method they knew and to see that the method fitted the answer they had "seen". In the following episode a typical misconception is revealed. Kari said: "*Why do you divide? You are supposed to get more?*" (19).

According to L97 misconceptions ought to be "grounds for further learning and deeper insight" (page 167). In Chapter 3 I accounted for misconceptions from both constructivist and socio-cultural perspectives. A misconception can occur if a process of accommodation and thus revisions of existing structures do not take place. Furthermore a misconception can be looked upon as an over-generalisation; a concept which is viable within one context may not be viable within other contexts and becomes a misconception. In this case Kari's conception about division which was viable when working within the set of whole number and became a misconception when working with fractions. Bent provoked Kari's misconception by simplifying the task; asking the same question but with whole numbers instead of with fractions and asked for what arithmetical operation to use (12). That way of simplifying a task was a teaching strategy Bent used. He did the same when Kari later asked how to solve the next task. Throughout the whole episode he asked the student for suggestions and to reason her answer (8, and 10). In turn 16 Bent referred to what Kari had said and asked directly why she not could do that with fractions. He did not give Kari the answer; she had to find out with the help of his questioning. The two tasks were:

> Task 1: *20 litres with juice shall be filled into bottles that can take ½ litres. How many bottles do you need?*
> Task 2: *A rope which is 5 ¼ meters shall be divided into lengths of 1 ¾ meters. How many lengths do you get?*

Excerpt 15, Bent Feb 19th, episode II-5

Nr	Who	What is said	Comments
1	Kari	I didn't understand anything	
2	Bent	Not that one and not the other? But can you start with that one? Did you understand the grid?	Bent tells her to start with the simplest task
3	Kari	Yes, almost, yes, but…	
4	Bent	But..?	
5	Kari	I know how many bottles it will be	
6	Bent	How many?	
7	Kari	Forty?	
8	Bent	Yes, why?	Probes justification
9	Kari	It is obvious!	
10	Bent	Yes, it is obvious?	Probes justification
11	Kari	Because if you have twenty bottles with one litre each, then it would have been twenty bottles because you should have twenty litres in them. But it is a half bottle, and then it'll be	Explains how she can "see" the right answer

		twice as many.	
12	Bent	If I had said that it wasn't bottles with half litres, but said it was with two litres, how would you then find out?	Teaching strategy: Simplifies the task links to whole numb
13	Kari	Then I would just divide by two	
14	Bent	Twenty divided by two? And when I say there is half litres?	Structures student's thinking
15	Kari	Then I multiply by..	
16	Bent	Does it matter what kind of number that is there? You said that if it is two, you'd divide twenty by two, but you cannot do the same with twenty divided by one half?	Questions student's thinking
17	Kari	Twenty divided by one half?	
18	Bent	Twenty divided by one half? You agreed, you were quite clear that if there were two litres..	Links to whole numbers again
19	Kari	Why do you divide? You are supposed to get more?	Expressed Misconception
20	Bent	You said that if there were bottles taking 2 litres, then you have to divide twenty by two, and you would get ten bottles. It is logic in a way that you get ten bottles. And it is logic to get forty if the bottles take half a litre?	Restates what student has already said, thus structures her thinking
21	Kari	Yes?	
22	Bent	You used a method: twenty divided by two is ten, twenty divided by one half is forty.	
23	Kari	Is it?	Still not sure
24	Bent	Try to put it up as a division task.	
25	Kari	But how can you manage that?	
26	Bent	It is twenty and it is o point five, isn't it?	Relates to decimal fractions
27	Kari	Oh yes, but	
28	Bent	If you divide by a number which is less than one, then the answer becomes bigger than in this case twenty. Then it becomes bigger.	Authorship of knowing. Summarises
29	Kari	But the other one? How do you calculate that? Do you have to convert the whole numbers?	Asks about task 2
30	Bent	Think dividing that as well	Relates to previous task
31	Kari	But how can you divide there?	Still not sure
32	Bent	Simplify your numbers. Say that you had six metres rope and should divide it into lengths of two metres, what would you then have done?	Simplifies to whole numbers
33	Kari	Six divided by two?	
34	Bent	Yes, why making fuss about it? Here is a number divided by another number. Don't bother about fraction.	

35	Kari	If you transfer that one to that one then it is like five multiplied by four is twenty, twenty-one quarters and then four, five, six, seven quarters, and then you multiply then, not then…	Kari converts mixed numbers to fractions
36	Bent	Then you have to take the length and divide by the other length, I'd nearly said. Try it up.	Highlights key aspect
37	Kari	Then I understand	

In the post lesson conversation Feb.19th Bent told me about this episode. He referred to Kari who at first did not understand how you could divide when the answer was supposed to be bigger. Bent's comment to this episode was:

> She was very concerned that it could not be forty if you divided twenty by a number, it wouldn't work to divide a number and get a bigger one. She got an "a-ha" experience, that it actually was possible. She is the type who, she is a type of student who really needs to gain a deeper understanding to proceed further (19/2, post).

This shows that the way Bent challenged Kari was based on his knowledge about her mathematical abilities which he took into consideration in his teaching. That Kari had to divide did not fit into her mental representations of division. Therefore a revision of existing structures (Noddings, 1990) had to take place through an assimilation process and a process of accommodations (Piaget, 1970).

Looking at this episode through socio-cultural lenses, the occurrence of the misconception can be accounted for as a conception developed through activity outside and in school including classroom discussions. The concept that division makes smaller has developed and been reinforced through focus the algorithm and on division by partition (sharing), and not working with division by measure. The latter is seldom dealt with in school.[27] The student's learning can be explained as work within the zone of proximal development in the same way as I explained in the other fraction episode. Bent structured the student's thinking by simplifying the task, by questioning her thinking and probing for justification of answers. Through this work with the teacher Kari said she understood (turn 37). I look upon Bent's suggestion to replace the fraction with a whole number as an act of scaffolding, or making a bridge between existing knowledge and new knowledge.

[27] In Norwegian we distinguish between the two concepts "målingsdivisjon", division by measure, and "delingsdivisjon", division by partition. The two tasks in this lesson were "målingsdivisjon", division by measure. "Delingsdivisjon", division by partition, sharing, has traditionally been most dealt with in textbooks and schools (Brekke, 1995)

> Some students were engaged in that discussion and at least half of the students were not, then I thought that Okay, let me follow the discussion to a certain extent and then stop. If not I'll misuse the other students' time" (Bent 16/1 pre lesson conversation).

I think the above quotation characterises much of Bent's teaching; the complexity of the class and classroom with which he dealt in every lesson. He included his students in his teaching in every lesson, he invited them to participate and let them come up with suggestions which sometimes changed the intended direction of the course of the lesson. I saw a triadic pattern of discourse in his whole class lessons. However, his types of questions encouraged the students to express mathematical knowledge which the teacher through his revoicing legitimated. Through the quotation above, Bent demonstrated that he reflected on the complexity of the classroom and how to deal with the often conflicting demands.

Bent's teaching strategies

Bent demonstrated a wide range of teaching strategies, both in whole class and during individual seatwork. He

- reminded the students about their previous knowledge in the beginning of a lesson
- invited the students to participate
- did not answer students' questions directly, he
    - either asked them to clarify
    - or to elaborate the question
    - or for a suggestion
    - or to express their thinking
- put values in formulae to confirm a conjecture
- facilitated a task by simplifying the numbers (whole numbers instead of fractions)
- challenged students' thinking
- structured students' thinking by
    - reminding them what they had already done
    - highlighting key aspects of the task
    - challenging them to recall if they had solved a similar task earlier

Characteristics of Bent's teaching

In the analysis of the conversations I had with Bent, I outlined how Bent expressed that he wanted students to be active in the learning process and that he wanted to focus on the conceptual aspect of mathematics. He also expressed his concern for how students with different abilities needed different levels of conceptual understanding. When encouraging the students for the Kapp Abel competition he told them that students with different abilities were good at solving different kinds of tasks. Bent also expressed a wish to focus more on the process aspects than he was achieving currently. However, he pointed out several constraints which prevented him from doing it. In the table below I present these characteristics in the left column. In the next two columns I present examples from the conversations and from classroom observations which illuminate these characteristics. In the row beneath each characteristic I present a relevant quotation from L97.

Bent

<table>
<tr><th>Characteristics</th><th>Conversations</th><th>Classroom observation</th></tr>
<tr><td rowspan="3">Bent wants students to be active in the learning process</td><td>When preparing lecturing from the board, he said he would do it “in dialogue” with the students (Jan 8th pre)</td><td>He invites students to participate in every lesson and takes their contributions into account</td></tr>
<tr><td>“To learn mathematics students have to work as much as possible on his/her own” (from his writings)</td><td>The students worked with concrete materials (Jan 8th and Jan 16th)</td></tr>
<tr><td></td><td>The students worked on worksheet to reflect on their own knowledge (Feb 5th)</td></tr>
<tr><td colspan="3">L97: Pupils’ own activities are of the greatest importance in the study of mathematics (p. 168)</td></tr>
<tr><td rowspan="5">Bent focuses on the conceptual aspect of mathematics</td><td rowspan="5">He wants the students to gain conceptual understanding to be able to use the concepts in different contexts (Feb 19th post)</td><td>Bent challenges and structures students’ thinking in all lessons, both in whole class and individual work.</td></tr>
<tr><td>He focuses on how to derive a formula and not only on the use of it (Jan 8th)</td></tr>
<tr><td>He focuses on the relation between the volumes of different solid blocks (Jan 16th)</td></tr>
<tr><td>Bent encourages reflection on own knowledge (Feb 5th)</td></tr>
<tr><td>Bent makes tasks to provoke misconceptions (Feb 19th)</td></tr>
<tr><td colspan="3">L97: Pupils who have difficulties with memorising the basic multiplication facts must nevertheless be free to proceed to concepts and tasks involving the multiplication</td></tr>
</table>

concept (p. 166)		
Bent demonstrates that he has reflected on how students have different abilities in mathematics	According to students' different abilities he discussed the necessities for students' conceptual understanding (Feb 19th post)	When motivating for the Kapp Abel competition he told the students that different students performed different on problem-solving tasks (as Kapp Abel tasks were) than on more traditional tasks (Jan 16th)
	He reflected on how a task (with the rope Feb 19th) could challenge clever students (Feb 19th pre)	Bent always questioned student's thinking when giving support in individual work and in whole class. Thus he "adjusted" the help according to students' thinking
L97: *The teaching of mathematics must be attuned to the abilities of individual pupils, who must be giving tasks which they find meaningful and capable of doing* (p.166)		

Table 9, Characteristics of Bent's teaching

According to what Bent said, he thought that students learn more and better through exploring activities than through traditional teaching from the board. This is in accordance with some of L97's recommendations. However, he gave several reasons for why he did not do that (constraints): He did not know how (*lack of methods*); *parents* wanted him to teach from the board; *students* expected him to present examples on the board so they could solve similar tasks on their own. The most frequent reason he gave for not having exploring activities was *time*; he thought it took too much time and that it would prevent him from "coming through" all the topics he is supposed to.

Bent also said that he appreciated classroom discussions and that students learn mathematics through active participation in the classroom discourse. This is in accordance with what I saw in the classroom: he invited students to participate during plenary sections of lessons and he encouraged students to come up with their views and he encouraged discussions. This is in line with L97.

Through what he said Bent gave the impression that he thought students' conceptual understanding was important and that he intended to put focus on that in the learning process. This is in accordance with what I saw in the classroom where he "structured students' thinking". He often referred to student's prior knowledge so they could "accommodate new knowledge into existing structures". He also took students' misconceptions as grounds for further learning.

From a socio-cultural point of view, the social practice in the classroom and the relation between classroom discourse and students' mathematical development are essential factors in the learning process. Moreover, knowledge grows as part of the social interactions and their cultural underpinnings: the individual mental plane is constituted as part of the socio-cultural process (Vygotsky, 1978). This I have outlined in the Chapter 3 under the heading "Theoretical perspectives underpinning my study". The extent to which Bent's thinking and beliefs might reflect such a theoretical position is impossible to say from the available data.

7. Cecilie

Cecilie's educational background was in engineering. However, after having worked as an engineer for many years, she decided to become a teacher, something she said she had always wanted to be. However, there was a campaign recruiting girls to engineering the year she finished upper secondary school. She said:

> I had very good grades from upper secondary school and it was that campaign, so I thought I had to use my good grades for something, and it was quite difficult to get in by that time, so when I was accepted, I had to start. If that campaign had not been, I would have studied physics and mathematics at the University. I had decided that, but then it was prestigious with the Technical University, something one should not take into account. One is not mature when one has to choose (Jan 21st, post).

Cecilie had been a teacher for only eight years, and was thus the least experienced teacher of the three in my study. She had been teaching only one year with the prior curriculum M87, a year she looked upon as her "trial year of teaching" (post-lesson conversation Jan 21st). She indicated that she had liked the "mathematical didactics" which had been part of the practical pedagogical education (PPU) she had to undertake to become a teacher. The mathematical didactics course was at that time influenced by the reform R97 and thus by the curriculum L97. Cecilie's experience of that course was "In retrospect I look upon it as an in-service training course with regard to L97" (post-lesson conversation Jan 21st).

Before I analyse the conversations with Cecilie with regard to the three aspects outlined in the previous chapter on Bent, Conditions of possibilities of learning (CPL), Mathematical focus (MF) and Students' abilities (SA), I will present some excerpts from what Cecilie said explicitly about L97. She consciously related her work to L97, and reading the transcripts from the conversations with her, I can see that she mentioned L97 relatively more often compared to the other teachers. Cecilie (C) said open-heartedly that she liked L97 and that she used it in preparing lessons, both with regard to working methods from "Approaches to the study of mathematics" and with regard to topics to be studied in "Main subject elements for grades 8-10".

C: When I start a new topic, I read L97 and I have made notes from books I have read that I can use.

BK: How did you react when L97 first came?

C: I liked it, but I didn't like the textbooks following it. I think they fitted the old curriculum.

BK: What do you especially like in it?

C: That it focuses on methods and deriving formulas. The way I see it, students are supposed to explore things themselves, using play for example. It becomes more exciting that way and I believe they learn

> some mathematics they won't learn by cramming (Norwegian: "pugge") the formulae (Post-conversation Jan 21st).

Cecilie emphasised that she believed that students learn better when they have to find out things themselves through exploring activities rather than learning ready made results.

Analysis of conversations with Cecilie

I only had three recorded conversations with Cecilie. She was so busy that she did not find time for more conversations with me, neither did she write the one page on ideal teaching I asked the teachers to do. I had pre- and post-lesson conversations Jan 21st and pre-lesson conversation Jan 28th. David participated both in pre-lesson conversation the 21st and pre-lesson conversation the 28th. So I have one conversation with only Cecilie which is the post-lesson conversation Jan 28th. Since the amount of data was small I found it better not to use Nvivo in analysing the conversations with her.

Conditions for possibilities of learning

Cecilie had two 10th grade classes, 10A and 10B, in mathematics at Dalen School. In Focus group 3 she told us that when they were in 8th grade, she had started discussing their interest for mathematics with them, different working methods and what area of mathematics they liked the most. She had given her students an option to choose between three groups: One group was going to have as "useful a purpose" as possible, in the next group the level of difficulty should be average; "a common lower secondary school class, like we usually teach for" she said, and finally a group for "those who wanted to be challenged". Thus in grade 9 she made three groups out of two classes for half a year. Since that did not work organisationally with regard to other subjects and the school's schedule and timetable, her two classes had been divided into only two groups from half way through grade 9 with 28 students in the group who were most interested and 23 in the other. Now her two classes 10A and 10B were mixed and divided according to "interest for mathematics" one double-lesson a week. Thus one double lesson a week she had the *Platon* group with those from both 10A and 10B who were interested in mathematics and one double lesson a week she had the *Socrates* group with those from both 10A and 10B who were not so interested in mathematics. She still had two lessons a week with the classes 10A and 10B separately. In Norway streaming students is not permitted which means that you cannot permanently segregate students according to their abilities. Doing it once a week is not looked upon as permanently, and calling it "according to interest" is a way of avoiding acknowledging that it was according to ability. According to what Cecilie said in the focus group, the working methods were the same and equally varied in both groups, but the subject content was different especially with regard to level of difficulty.

Based on what Cecilie said in our conversations, she prepared her lessons well. She used other resourses than the textbook when preparing the subject content to be worked with during the lessons. She said:

> In addition [to study L97] I read a lot. And I make notes in a separate book. And if I find something on the internet I write it down and sort it out in a file. One file is called prime numbers, another is called - [something else]. If not I would never have found back the exciting things just when I needed it. And then I leaf through my notebook to see if I can use something, so it turns out being quite varied (Jan 21st, post).

Thus Cecilie created Conditions for possibilities of learning both through the way her students were organised in different groups and also with regard to how she prepared the subject content to be taught in her lessons. Through the grouping the students could be challenged on a level according to the teacher's perception of their abilities. Through her use of varied approaches to the subject she wanted to provoke the students' interests for mathematics. How I observed Cecilie creating CPL in her mathematics lessons, in her classroom practice, is described in "Analysis of classroom observations with Cecilie" below.

Mathematical focus

The lessons I observed regularly with Cecilie, which I refer to as Wednesday lessons, were in an ordinary class, 10A, and thus with students with mixed abilities and interests. Cecilie emphasised that she had a separate program for the "Wednesday-lessons" during which she focused on mathematics history and mathematical proofs on which L97 puts weight. These topics were usually not reflected in the final written exams. However, Cecilie said both to me and to her students that this was something they could be asked about in an eventual oral exam[28]. She thus used a possible exam to motivate her students. In the Platon and Socrates groups they worked with topics like geometry, algebra, etc. and on kinds of tasks they could expect to get on the final written exam. They practiced tasks from earlier exams in these lessons.

During our conversations (Jan 21st and Jan 28th) we talked about what she had done and what she intended to do in the lessons. Reflecting on the lesson Jan 21st in which the students had "not concentrated enough"[29] she said she would take the work in that lesson as a starting point and "then I will do some more proofs too, and then use algebra in doing the proofs, for example Euclid's proof that there are an infinite number of

[28] In Norway the first official exam is at the end of year 10. Regions are then selected for written exams in one of the subjects Norwegian, English or Mathematics, which implies that only some students have written exam in mathematics each year. In addition some students are selected for oral examination in one subject which also can be mathematics.

[29] She said that the students had been "ukonsentrerte".

prime numbers" (Jan 21st post). Cecilie followed this up in our pre-conversation the week after. She said:

> [To-day] I will do a couple of classical proofs and then I will do a couple of examples from what students have been working on showing that it can be reasoned in another way, and the difference between that it works in one case and showing that it works in all cases (Jan 28th pre).

By taking what the students had been working on as a starting point and then show the difference between that "it works in some case and that it works in all cases" (a proof), Cecilie linked students' work with mathematics in school to historical aspects of mathematics. And when I asked her for what the learning goal for the lesson was she said: "More about the distinct character of mathematics;[30] proofs and algebra" (Jan 28th pre). This suggests that Cecilie saw mathematics in a wider perspective than that of traditional school mathematics and her interest for mathematics history was prominent. She demonstrated an interest for mathematics as a science which was also mirrored by her teaching. As I pointed out in the previous section (CPL) she used several resources when preparing her lessons: "I search many places trying to find tasks fitting in with the topic we are working on" (Jan 21st post).

In our conversations I asked her about how she looked upon the relation between the subject content in the Wednesday-lessons (in which she said she focused on mathematics history and proofs) and the lessons in which her students were divided according to interests, Platon and Socrates groups, in which they prepared for the final exam. First she said that she looked upon that as two independent parts of the subject, but reflecting further she said she looked upon the Wednesday lesson Jan 21st (Proofs and angle sum in a triangle) as algebra review. She said:

> I look upon them as independent parts, meaning that this [the work in Wednesday lessons] is history and about the characteristics of mathematics. It is a bit on the side, but at the same time it becomes a review and they exercise algebra and they get exercise with number patterns (Norwegian: "figurtall"). Thus they will review several things while working with another topic [mathematics history]. I find this a better way of doing it than having one month with mathematical history and thus having finished it (Jan 21st, post).

Cecilie's conscious use of L97 was apparent with regard to the Wednesday lessons. When I asked her if she used L97 to the same extent when planning the other lessons, she hesitated a little when saying "yes, but in a different way; more with regard to knowledge and skills and not so much with regard to the working methods" (Jan 21st post). In preparing the Wednesday lessons she said she used the general formulations from L97. In preparing the other lessons (Platon group and Socrates group) she looked more on what L97 said about knowledge and skills for the students to learn; for example in geometry, what they should learn to

[30] In Norwegian "Matematikkens egenart"

construct and what skills they should have. Thus for the preparation of Wednesday lessons she used "The subject and educational aim" and "Approaches to the study of mathematics" from the "Introduction" to the mathematical syllabus. In this part of L97 weight is put on the link between school mathematics and mathematics in the outside world, elements such as reasoning, imaginations and experience as well as mathematics as a science, art, craft, language and tool are emphasised. In preparing the Platon and Socrates lessons she said she used "Main subject elements for grades 8-10" in which the five main areas for lower secondary school (Mathematics in everyday life, Numbers and Algebra, Geometry, Handling of Data and Graphs and Functions) are outlined. Furthermore she said that she used the text-book neither in the Wednesday lessons nor in the Platon lessons, "but in the Socrates group we use it because they need just like, they use more traditional tasks" (Jan 21st post). This suggests that she had different goals for the students according to what group they were in. What Cecilie said about the issue having students with different abilities in the same class is discussed below.

Students' abilities

Already the first time I met Cecilie, in the third focus group meeting in October 2003, she asserted her concern with the *clever students*[31]. She said that she found it "very unsatisfactory" having students with different abilities in the same class, especially because it caused the clever students to suffer, not getting sufficient challenge. Having listened to the others in the focus group saying that the clever students will always manage, she said:

> I find it so unsatisfactory having all levels in one class and the clever ones, I do not agree with you saying they will always manage. I think that is wrong. They are used to being used as assistant teachers, not getting challenges, or having the possibility to work on their own (Cecilie in Focus group 3).

In the conversations I had with Cecilie after the lesson Jan 21st, in which they had worked with tokens to show generalities (for example that the sum of two odd numbers is an even number) which they also should show algebraically, she said:

> I thought I had spent so much time on the introduction, I looked at my watch. I thought I should have shown some examples before I started, but that would have bored some of the clever ones. So that is very difficult in a compound class. I thought it was right when I stopped the introduction that they could start with a task and then you can say it was an offer. The summing up afterwards was also difficult because they were not concentrating during the work with the tokens (Jan 21st, post).

[31] The Norwegian expression she used was "flinke elever", which is usually translated as *clever students.* "Flink" can also be referred to as *able, skilful* or *gifted.* In Norwegian being "flink" does not necessarily mean being hardworking. Those students getting good grades are called "flinke elever".

This suggests how she pointed out "students' different abilities" as a difficulty and a challenge ("very difficult in a mixed ablity[32] class"). She consciously had made the choice not to show some examples of the proofs before they started to work individually with the tokens and to solve the tasks algebraically because "the clever ones would have become bored". The introduction she referred to had lasted for 20 minutes, and according to my field notes, many students did not pay attention, however, the clever students did. This shows how factors such as time pressure and the classroom culture (some students had fallen off, some could be bored, and for some it would have been an aid for further work) were complex factors the teacher had to consider in her decision making. In this case she decided not to show some examples from the tasks they were going to work on.

In the analysis of Bent I reported similar findings, and such findings are also reported in the literature. Skott (2001a) termed instances where a teacher's decision making was critical for further development of the classroom interaction as "Critical Incidents of Practice". Jaworski (1994) identified such instances as "Decision Points" indicating the teacher's ability to recognise that she had made a decision, which Cecilie did. In this case Cecilie's concern for the clever students not to be bored was more central than her care for those for whom some examples from the tasks would have been an aid in their further work.

For the last focus group meeting, FG4 (March 13th) I had given the teachers the task to tell me and the others what they felt they had succeeded in as a mathematics teacher and what they felt they not yet had accomplished. Cecilie volunteered to start that round, with something she felt she had succeeded, by saying:

> Cecilie: I am very content that I manage to inspire the clever students, those who I believe will choose mathematics further. They get enough challenges and they are inspired. I feel that I succeed in that.
>
> BK: Can you say more about how you have managed, what you have done?
>
> Bent: That we are very interested to know
>
> Cecilie: A little because of the setting [Platon and Socrates group], that I have the possibility to choose a higher degree of difficulty because it is very important that they get enough challenges. But also that I am quite versatile, that I take many different topics, that I take a little history, Pythagorean triples, for examples, on which many clever students find it nice to do research. [ ] And I spend lots of time searching for good exploratory tasks, ponders, and tasks from the Abel competition[33]. That is very stimulating (Focus group 4, March 13th).

[32] In Norwegian she used the expression "sammensatt klasse".which also may be translated into compound class.

[33] The Abel or Kapp Abel competition is a competition in mathematics between classes in lower secondary school. The tasks in this competition are often referred to as tasks on which you have to pon-

She thus expressed three reasons for her success in challenging and inspiring the clever students: (1) The setting as a result of the streaming, (2) that she searched for good exploratory tasks and (3) her versatility and that she thus was able to challenge the clever students with topics from for example mathematics history. This emphasises her concern for the clever students and that she consciously searched for exploratory tasks for them in creating conditions for possibilities of learning. Answering questions from the other teachers in the focus group about pro and cons with regard to the "streaming", she said that students in the Platon group benefited most from the setting in which the two classes were divided according to interest, but she expressed that students in the Socrates group did not lose anything from it. This suggests that Cecilie did not look upon having students with different abilities in the same group as an advantage for the learning potential in that classroom. But rather that students are best served by tracking or being grouped according to abilities which is one of the most established beliefs that teachers have shown to have about mathematics teaching and learning (Reys et al., 1998).

According to my interpretation of what Cecilie said, she cared for the clever students in the following way:

- Her goal for them was to increase and maintain their interest for mathematics through the work in Wednesday lessons where they worked with mathematics history, proofs and had exploring activities,
- Through the work in Platon groups and not having to be ("*used as*") assistant teachers for the others (less able) or being bored because of *"slow"* students. In our conversations Cecilie pointed out especially two students, Baard and Svend, as *research types"* and also "*enthusiastic*" students.

Working with those in the Socrates group, Cecilie said that her goal for them was to get them through the exam. During the Wednesday lessons, she said she did not challenge the students from the Socrates group in the same way as she did with those from the Platon group. This is in accordance with my findings in the analysis of classroom observations. Reflecting on the "streaming" or "tracking" Cecilie did once a week "according to interest", but which turned out as "according to ability", a student's ability can be seen as something predetermined and that students have different abilities to conceptualise and thus they need different learning experience in the classroom. Based on what Cecilie said about students' ability and the streaming she did, I suggest that she was

der (Norwegian: "gruble") to solve. I know that "Ponders" is not an English word, neither is "grublis" in Norwegian. The Norwegian verb "gruble" from which "grublis" is created is to ponder. I have therefore freely translated "grubliser" into "ponders".

seeing ability this way, as something students have to different extent and that each student's individual ability constrains the experience through which s/he conceptualises mathematics.

Cecilie's beliefs about teaching and learning mathematics

To sum up what is written above I identify three core beliefs Cecilie has with regard to teaching and learning mathematics:

- She considered that students learn most by exploring things themselves
- She expressed her awareness of students' different abilities in mathematics, and said it was important to challenge the clever ones so their interest for the subject was maintained and further developed. Based on her own words, I suggest that she looked upon the individual student's ability as constraining the support she offered for them to conceptualise mathematics.
- Cecilie said, she had a special interest[34] in mathematics as a science. She thought it was important to link school mathematics to mathematics history and also to focus on mathematics that could enhance students' interest and thus motivate them for further studies in the subject. She said it was important not only having the written exam from the authorities as a goal in the mathematics lessons.

Cecilie's estimation form

According to her estimation form, Table 10, Cecilie valued the process aspect, which was explained in the estimation form as "mathematics is a constructive process, doing mathematics means learning to think, derive formulae, applying reality to mathematics and working with concrete problems", highest with 15 points with regard to ideal teaching. She evaluated the process aspect with 5 points less with regard to L97 which makes 10 points. Her estimation of the process aspect with regard to her own teaching was lying between her evaluation of ideal teaching and of L97. This tells us that according to Cecilie, L97 does not reflect the process aspect in mathematics as highly as it ideally should, and that her teaching had more of the process aspect than L97 but not as much as she ideally thought it should.

She valued the toolbox aspect, (explained as: mathematics is a toolbox, doing mathematics means working with figures, applying rules, procedures and using formulae) which mirrors the traditional view of mathematics, with 5 points with regard to ideal teaching. With regard to her own teaching she gave the toolbox aspect the same value as she gave to it with regard to L97. This shows that she was thinking that her teach-

[34] She expressed an interest in mathematics as science and mathematics history which was not only related to her work as a mathematics teacher but it had become a hobby.

ing mirrored the traditional aspect to the same extent as L97 does, but more than what she according to the estimation form saw as ideal teaching.

Cecilie	Mathematics as a toolbox	Mathematics as a system	Mathematics as a process
My real teaching	10	8	12
Ideal teaching	5	10	15
L97's view on teaching mathematics	10	10	10

Table 10, Cecilie's estimation form

So far these findings tell us that there is coherence between what Cecilie said she believed in and her estimation form. Hence, Cecilie's estimation form can be looked upon as a validation of some of the issues about L97 and her teaching discussed in our conversations. However, the estimation form did not offer her the opportunity to express her concern for the clever students which I found striking from what she said. Neither did it give her the chance to express her special interest for mathematics history nor how she used what she had read in other sources than the textbooks which she did both for the purpose of preparing lessons but also because she was interested.

On the intentional level Cecilie was a teacher who as a response to L97 wanted to put weight on the process aspect, which she thought she did (estimation form) and do investigative work since she believed that was the best way for students to learn mathematics. In the analysis of the conversations with Bent in the previous chapter I pointed out some reasons he gave me for not teaching according to his beliefs about the best ways students learn mathematics. I labelled these reasons as constraints. Cecilie did not point out similar reasons or constraints with regard to her teaching. She had grouped the students according to interest and thus partly "solved" the issue that students have as she said "different premises to learning mathematics". According to what she said, she prepared investigative work and exploring activities for the lessons which means that that was what she did on the intentional level. It remains to be seen how the classroom practice, the enacted curriculum, turned out. This I will address in the next part.

Analysis of classroom observations with Cecilie

Before presenting the analysis of classroom observations with Cecilie, I present an overview of analysed lessons from Cecilie to which I refer in this part.

Excerpts	Date	Group	Mathematical topic
16, page 204 18, page 212 20, page 229 21, page 231	Jan 14th	Wednesday group	Pythagorean triples
22, page 233 23, page 236 24, page 237 25, page 237 26, page 238	Jan 21st	Wednesday group	Number patterns, Proofs and Angle sum in triangles
17, page 208	Jan 28th	Wednesday group	Algebra. Generalising, Proofs and reasoning
	Jan 29th	Platon group	Geometry, constructions
	Feb 3rd	Socrates group	Reviewing a test
19, page 223	March 17th	Wednesday group	Statistics

Table 11, Overview of data excerpts from Cecilie's lessons

When analysing the classroom observations I had with Cecilie, I can see different kinds of interactions between the teacher and the students in whole class sections of lessons from what I perceived in the individual work sections of the lessons. In the whole class sections of lessons the teacher was either lecturing without inviting the students to participate, or she invited the students to participate by asking short closed questions, ensuring their attention. The teacher was thus controlling the course of these parts of the lessons. By lecturing I mean that she presented the mathematics, sometimes by using ready-made transparencies or by writing directly either on transparencies or on the board. Thus I experienced the discourse in the whole class sections of her lessons as traditional in style.

Also when working with topics I will call "untraditional topics" within school mathematics (exploring Pythagorean triples, proofs and reasoning, aspects of mathematics history) the way it turned out in the classroom was traditional in style; the questions she asked seemed to be for the purpose of control and to ensure students' attention. When studying the episodes from individual seat work, I can see that Cecilie supported students in their work. By challenging the students with comments and questions, a bridge was created between what a student already knew and what s/he was supposed to learn. I also found that the level of support Cecilie gave differed from student to student according to the individual's need. By challenging the students differently she demonstrated that she took each student's mathematical ability into account when working individually with them. In this relation the student's ability can be seen as having a potential to be developed through the experience the teacher gave him/her as support to conceptualise the mathematics they were working on.

Both in whole class sections of lessons and during individual seat work Cecilie gave much attention to the clever students. This is in line with what she said in conversations and in focus groups and I look upon that as a significant aspect in her teaching. Other significant aspects in her lessons were that many students did not pay attention during whole class sections of lessons and neither did they work with mathematics during individual work. These were aspects in the classroom I noticed and wrote down in my field notes. The level of noise and that the students were talking about things other than mathematics I can hear from my audio-recordings. These, together with students' different interests for working with mathematics and other disciplinary aspects were a range of factors in a socio-cultural setting, in which the teacher had to make her decisions while teaching mathematics.

To provide evidence for what I have written so far about Cecilie's classroom practice, I will first present analysis of whole class sections of lessons in which I address the three categories CPL, MF and SA and then of individual seat work in which I present analytical accounts of episodes with individual students.

Whole class sections of lessons

All lessons I observed with Cecilie had one or more whole class section(s) which had different lengths; from six minutes to the whole lesson. In these parts of the lessons Cecilie was teaching from the board and was in charge of the course of the lesson and also of the content. I calculated the amount of time Cecilie spent teaching from the board in the six lessons (Jan 14th, 21st, 28th 29th, Feb 3rd, March 17th) which I have analysed in detail and I found that she spent between 70 and 75% of the time of the lessons by lecturing from the board. In a lesson about mathematics history March 10th, which is not part of my detailed analysis, Cecilie lectured from the board for 90 minutes. Spending so much time lecturing is not in accordance with Cecilie's expressed view on how students learn mathematics. She said that what she liked about L97 was that the "students are supposed to explore things themselves, using play for example" (Jan 21st, post). Spending so much time lecturing also seems to contradict Cecilie's estimation form on which Cecilie valued the process aspect (explained as "mathematics is a constructive process, doing mathematics means learning to think, deriving formulae, applying reality to mathematics and working with concrete problems") highest both with regard to ideal teaching and how she estimated her own teaching. This suggests that the enacted curriculum, the classroom practice, which was the result of the interplay between the teacher and the students and the teaching materials, turned out differently from what the teacher said she intended.

In Chapter 2 I referred to the same kinds of findings reported in the literature, that even when teachers expressed their agreements in the principles lying behind reforms, the way the enacted lessons turned out were often more traditional in style (Broadhead, 2001; Norton, McRobbie, & Cooper, 2002).

Skott (2001b) warns against calling this “inconsistency”, because what teachers do, makes sense to them in the present situation. According to him teachers’ beliefs ought not to serve as explanatory principles for practice in research. Rather than using predetermined beliefs as explanations for practice he pointed to the motives determining the teacher’s practice as entities which emerged from interactions with students in the classroom.

Leatham (2006) suggests interpreting teachers’ beliefs as systems where certain beliefs have more influence over actions than others. This emphasises the importance of taking the complexity of the classroom into account when analysing the enacted lesson. I have analysed the whole class parts of the lessons with respect to the three categories Conditions for possibilities of learning, CPL, Mathematical Focus, MF, and Students’ abilities, SA, and I will address each category separately. As a synthesis (page 226) I present an overview of findings from the analysis of Cecilie’s whole class lessons where I draw together the relations I found between conditions for possibilities of learning she created, different mathematical foci and also how that was related to students’ different abilities or interests for mathematics.

Conditions for possibilities of learning

In this section, CPL, I will first present examples from whole class parts of lessons with Cecilie in order to show how she created conditions for possibilities of learning by

- directing students’ attentions in the opening parts of some lessons
- using resources other than the textbook in preparing the lessons
- using concrete materials and transparencies when teaching

Under the heading “Opening parts of lessons, resources used and concrete materials” I will analyse the openings of each lesson and I focus on Cecilie’s use of personal pronouns which can account for the different roles of the teacher and that of the students in the whole class sections. I will also point out how she used teaching materials to illustrate, which thus acted as mediating tools in the learning process.

Next, under the heading “Pattern of discourse, communicative approach and use of pronouns” I will analyse the discourse and communicative approach in the lesson Jan 14th which provided me with key sources of examples of how

- Cecilie was either lecturing from the board without inviting the students to participate, or
- she was leading the students through the mathematics by posing easy manageable and simple calculation questions.
- The use of the pronouns “I”, ”you” and “we” reflected who was doing or expected to do the mathematics and the use of “we” conveyed a conventional aspect of mathematics.
- When she used “it” to refer to a mathematical issue there seemed to occur uncertainty among the students what “it” was.
- A shift in discourse was initiated through her asking an open question.

I have pointed to the triadic pattern of discourse I-R-E in the analysis in order to account for how that pattern of discourse constrained the course of the lesson, and I have studied the teacher’s use of the personal pronouns to highlight the teacher’s role and the students’ roles in these parts of lessons. The teacher’s deictic use of “it” as a pointer to something not yet articulated or agreed upon, is also pointed out in accounting for students’ confusions about what they were supposed to do. For further evidence of my findings from Jan 14th I present excerpts of lessons Jan 28th and Jan 29th (the Platon group) and in each of them I emphasise how I noticed shifts in discourses in the lessons and at the same time shifts in mathematical focus with which I deal in the next section.

Opening parts of lessons, resources used and concrete materials

As pointed out in the analysis of the conversations I had with Cecilie, she prepared her lessons well. She often had ready-made overhead transparencies, including things she had read in different books which were not typical school mathematics and she often started the lesson by presenting what they should work with that day. For example, *Jan 21st* she started by saying:

> To-day **I** will talk about proofs in mathematics and how mathematics differs from other sciences. In Physics for example, *you* develop a theory which is supposed to cover a broad area, and which is supposed to explain what is observed and can predict what can happen in the future. *You* can try out a theory, and as long as trials fit with theory, the theory will not be rejected. Mathematics is built quite different. **You** can prove things. And now **I** will show **you** how mathematics is built up (Cecilie to her students Jan 21st).

I have empahsised the personal pronouns in this quotation because I have used the study of the use of personal pronouns as a tool in the analysis of what the teacher said. Her use of “I” and “you” (plural) in the first and last sentences (in bold) tells us what roles she and the students were going to have in the lesson: The teacher would show and tell the students. The two singular “you” (in italics) were not addressed to the students but mirrored the conventional aspect of mathematics (this is how it is done).

The "*you*" is an unaddressed third person (out there). She was going to show and tell about an aspect of mathematics, mathematics as a science and not as the school subject the students were familiar with.

Cecilie had made transparencies of *definitions* of point, line, equilateral triangle, isosceles triangle, similar triangles, congruent triangles and parallel lines. She had photocopied the *axioms* that through two points we can draw one straight line, and that we can draw line one parallel to another line through a point outside the line. In addition she had photocopied *sentences* to be proved from the definitions and axioms. She referred to Euclid and showed the students the book "Elements". She thus created conditions for students' possibilities of learning from a different perspective than that of textbooks by having read other books on the topics for the intention of using what she had read in the lessons. To illustrate number patterns on the overhead she had brought tokens which thus served as visual aids for the students to conceptualise notions such as odd-, even- and square numbers. The transparencies and the tokens served as mediating tools in the learning process.

The lesson I observed in the *Platon group* (those interested in mathematics) *Jan 29th*, in which they worked with topics and tasks taken from previous exams, the teacher started the lesson by saying:

> What *we* shall do to-day is to work with constructions. However, before *we* start with that *I* will show how *I* work out a task properly and nicely. The point doing it nicely is not only that somebody else shall read it and understand what *you* have done; it helps *you* to keep overview of the task. Now *I* will show an example. Copy it into your workbook.

In saying this she motivated students for the work they should do in the lesson and why they should work out a task properly[35]. Also here I have put the personal pronouns in italics to highlight the roles in this section. First, the use of "we" indicated that everybody should work and the use of "I" and "you" highlighted the roles of the teacher and the students respectively in this part of the lesson. Her use of you was singular. In that way she could address each student individually and thus make them more responsible for their own work. The teacher was going to show in order to help the students to get an overview, and each student was supposed to copy the teacher's work into their workbooks. This indicates a "show and tell" aspect of mathematics which is a more traditional aspect than the process aspect Cecilie valued highly with regard to the ideal teaching and also in estimating her own teaching.

In the statistics lesson *March 17th*, Cecilie started by saying: "There were two issues on the front pages of the newspapers yesterday and both had something to do with us". The students' engagements in the discus-

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[35] How they were supposed to work out a similar task on the exam. Cecilie suggested in the last focus group "how to get students to work out a task properly" as an issue she had not yet accomplished.

sion that followed showed that she had motivated the students to participate and that she had captured their attention by making them curious as to what it was about. Also for this lesson she brought transparencies. She had photocopied newspapers' presentations of issues in which statistics had been used to provide evidence, and took that as starting points for the discussion in her class.

In the opening part of the lesson *Jan 28^th^*, she started right away by referring to a task they had had on a test saying: "*The length of a rectangle is increased by 15% and the breadth is reduced by 20%. By how many percent does the area of the rectangle change?"* Thus there was no opening or overview of the day, but Cecilie took a task which was supposed to be known to the class and thus part of the class's common ground as a starting point. The students who had solved this task had done so by choosing values for the sides without proving a general change in area. Cecilie invited the students to participate from the very beginning by asking the question, (*by how many percent…*), and the purpose of the lesson was to show generally how much the area of the rectangle changed. This lesson had no individual work. As preparation for this lesson she had made transparencies ready with formulae for and patterns of prime numbers which were dealt with in the second part of the lesson.

Cecilie had also prepared transparencies for the lesson about mathematics history *March 10th*. She had transparencies with Babylonian, Egyptian, Greek, Arabian and European ways of solving equations.

Cecilie's first sentence in the lesson *Jan 14th* was: "I hope *you* have got *your* calculators, *you* will need them in doing this task" (plural you). This emphasised the roles of the students; that they were going to use their calculators doing the calculations. Cecilie had prepared for exploring activities[36] in terms of searching for Pythagorean triples in this lesson, into which I go in detail below.

<u>Pattern of discourse, communicative approach and use of pronouns</u>
To illustrate the pattern of discourse and the communicative approach indicated in the beginning of this section and which I found was typical in Cecilie's whole class lessons after she had invited the students to participate, I am going to use the lesson Jan 14th because it provides a good example of pattern of discourse, communicative approach and the use of pronouns. This was a Wednesday lesson in 10A (mixed abilities) in which they were exploring Pythagorean triples. The lesson, to which I refer throughout this section, had several whole class sections and two sections with individual work:

[36] Cecilie termed learning activities in which students should find out things themselves as "exploring activities".

Section I	Whole class	6 minutes (Pythagoras' theorem and triples)
Section II	Individual work	11 ½ minutes (Pythagorean triples)
Section III	Whole class	9 minutes (Pythagorean triples and Euclid's formula)
Section IV	Individual work	8 minutes (Exploring Pythagorean triples)
Section V	Whole class	8 minutes (summing up Pythagorean triples)
Section VI	Whole class	12 minutes (Mathematics history, Fermat's last theorem)

In the preceding section I emphasised how in the opening section of the lesson Jan 14th, Cecilie invited the students to participate. I will now present an episode from the first section of the lesson Jan 14th, Episode I-1 (see Excerpt 16, Cecilie Jan 14th episode I-1page 204**)** to emphasise the following:

- The class's knowledge about Pythagoras' theorem was taken as a starting point for the lesson and thus served as a *common ground*[37].
- How the triadic pattern of discourse, I-R-E constrained the course of the lesson.
- Studying the teacher's use of the personal pronouns "I" and "you" (plural) suggests the teacher's role and that of the students in this episode.
- Studying the teacher's use of we/us, can point to a conventional aspect of mathematics.
- Studying the use of "it" can offer a possible explanation for why some students did not "catch" the issue to be investigated.
- How a question which mirrors a generalisation in mathematics indicates a shift also in the discourse.
- How Cecilie linked students' knowledge about Pythagoras' theorem to aspects of mathematics history which is not usually studied in a lower secondary school in Norway.

For this lesson, which was the first lesson I observed with Cecilie, she had prepared to explore Pythagorean triples. She did not start with an overview of the lesson for the students but started right away telling the students that they would need their calculators. Then she drew a triangle on the board with the smaller sides 3.6 and 4.8 and asked how to find the third one (turn 4). She had thus invited students to participate. The topic, which was to explore Pythagorean triples, included finding Pythagorean triples and to identifying groups of such triples. Cecilie took Pythagoras' theorem, which the students at this stage were supposed to know, as a

[37] I refer to the class's prior knowledge, what has been taught in the class as the class's "common ground". The common ground in a class can also be common cultural tools, as rule-book, work-book or habits and also tests.

starting point by using Pythagoras' theorem in calculating sides in right angled triangles. The students' knowledge about Pythagoras' theorem was thus a *common ground* for this lesson.

The pattern of discourse in this episode was a typical I-R-E, I-R-E, I-R-E, which I have described in Chapter 5 and also used in the analysis of Bent. The teacher asked a question (I), a student answered (R); the teacher evaluated the answer, (E) approved it (consent) and posed a new question (I). By using a teaching strategy of closed questioning, the teacher controlled the discourse and the mathematical focus. She was in charge of the mathematics and of the course of the lesson. The turns with students' talk were short and fragile and none of them involved holistic mathematical explanations or even whole sentences, except for turn 5 - *you have to use Pythagoras* - which is a whole sentence, but still an answer to a closed question. The turns including the teacher's talk were longer and the questions she asked seemed to be to ascertain students' attention and the class's common ground. The questions she asked were simple calculation questions to be done with the calculator or they could be answered in few words (see turns 2, 4, 8, 10, 12). The teacher was leading the students through the mathematics.

Excerpt 16, Cecilie Jan 14th episode I-1

Nr	Who	What is said	Comments
1	Cecilie	I hope you've got your calculators, you'll need them in doing this task	Cecilie has drawn a right angled triangle on the board
2	Cecilie	What kind of triangle do we have there, Mikkel?	Closed question
3	Mikkel	Right angled triangle.	
4	Cecilie	Then we know the lengths of two sides. And now I don't use unit. We are only interested in the numbers. How can I find the third side, Leif?	
5	Leif	You have to use Pythagoras	
6	Cecilie	Yes. Have to use Pythagoras. Let us try to do that with this triangle. If we call this side x, Leif?	Asks for Pythagoras' theorem
7	Leif	Must take $x^2 = 3{,}6^2 + 4{,}8^2$ (*C writes on the board)*	
8	Cecilie	Yes, let's calculate that. Three point six squared is?	Simple calculation
9	studs	12,96	
10	Cecilie	Four point eight squared is?	Simple calculation
11	studs	Twenty three point o four	
12	Cecilie	Twenty three point o four (*she writes it on the board*) The sum of these numbers is?	Simple calculation
13	Stud	Thirty six	
14	Cecilie	It is thirty six	writes on the board

15	Stud	It makes six 6	Simple calculation
16	Cecilie	Yes. Okay. It became 6 long. This was lots of calculations. If we look at the numbers here we might have simplified it. Is it like, here I have added one point two, and if I add another one point two I'll get he third side? *Is that a rule which always works?* Let us take another example. New triangle (She draws a new triangle on the board with sides like 7.5 and 10). If that is 7.5 and that is 10, will that one be 12.5? Can you check if it works?	Indicates a shift in discourse by asking an open question but also a shift in mathematical focus
17	Baard	Yes	
18	Cecilie	That worked as well. Your exploratory task is now: *Does it always work? Does it work for any length?* Find new lengths on the smaller sides of a triangle and check if it works on any side	Indicates an exploration for a generalisation
19	Morten	*But I didn't understand what you did?*	*NB!*
20	Cecilie	Okay, once more, if we see that the third side is unknown. We don't know that one yet, but we claim that it is 12.5, because that one is 7.5, I add 2.5, and I claim that I can add 2.5 there and get the hypotenuse. And that is easier than taking that one squared and that one squared and add and take the square root. Then Baard said it worked. What is 7.5 squared?	Presents a similar example
21	Stud	56.25	
22	C	56.25. And what is the square root of that? (*points to 156,25*)	writes on the board
23	Stud	Twelve point five	
24	C	Twelve point five. Then my claim was right for these numbers. You can add 2.5 there and 2.5 there and then get the hypotenuse. So my claim worked in this case. Now you all check Choose two lengths of the shorter side in a right angled triangle. Take the difference and add to the larger. Then I claim you'll get the hypotenuse. You shall check if it works. Try that Did you understand now, Morten?	Restates student's answer Encourages students to find a counterexample
25	Morten	No, not really	
26	C	Then you explore that	

Although the teacher was doing the mathematics, she used *we* and *us* when lecturing. It is difficult to know why she used "we", it could have been to include the students and that she wanted to convey to the students that they were part of this and thus to ensure their attention, or it could reflect a conventional aspect of mathematics. In turns 6 and 8 she used "we" and "us", which could indicate that "this is how we calculate the third side in a right angled triangle". Studying both her further use of pronouns and that of the students can point to the roles of the teacher and the students. Cecilie asked: "How can *I* find the third side, Leif?" to which Leif answered: "*you* have to use Pythagoras". This suggests that Leif looked upon the teacher as the one doing the mathematics. This is also indicated through Morten's turn 19: "but *I* didn't understand what *you* did*"*- the student did not understand what the teacher did which suggests that the student looked upon this as the teacher's work and not on his.

In turn 16 Cecilie used both "we" and "it" and also "you": "Can *you* check if *it* works". I have emphasised "*Is that a rule which always works?*" because it indicates the generalisation aspect of mathematics. These questions together with the next: "Does *it* always work? Does *it* work for any length?" indicated a shift in discourse. These were open questions and the teacher initiated a possible generalisation. In these questions the use of "it" was introduced. What is this "it"? Morten had not caught what "it" was. My interpretation of what "it" was here is: *"If you take the difference between the two smaller sides in a right angled triangle, you can add the difference between them to the largest and then get the hypotenuse"*.

The teacher responded to Morten by taking a similar example with other numbers and explained again rather than finding another way to engage the student. However, she now started turn 20 by using "we" and continued: *…we claim…* In turn 16, when she first initiated the claim, she used "I". The switch to "we" suggests that the claim now was made public for the class. Cecilie started to ask a new set of closed questions (turns 20 and 22). After having drawn a new right angled triangle on the board with the smaller sides 7.5 and 10 she explained once more that the third side was 12.5 which the students checked by using Pythagoras' theorem. The students' roles were still to answer calculation questions (turns 21 and 23). In turn 24 in the final sentence she referred to "it" – the same "it" (if you take the difference between the two smaller sides in a right angled triangle, you can add the difference between them to the largest and then get the hypotenuse). According to Morten in turn 25, he had still not understood what she had been doing and what "it" was, even when Cecilie had emphasised her claim by restating the student's answer in turn 24.

The teacher's presentation was in a dialogic format as she invited students to participate. There were interactions between teacher and students, however, the interactions were within the teacher's system of understanding, and the students did not catch the entirety of the task as they only had to answer the closed questions and to carry out simple calculations. That the task was broken down into many pieces together with the introduction of “it” suggests why the students did not catch a holistic overview of what the “it” was. In one of the episodes from the first individual work section Jan 14th, I analyse in detail, the issue was “what is it?”

In the same way as Pythagoras’ theorem was referred to as the class’s common ground in the lesson Jan 14th, Cecilie took a task they had had on a test three weeks earlier as a starting point for the lesson *Jan 28th*. The task (*The length of a rectangle is increased by 15% and the breadth is reduced by 20%. How many percent does the area of the rectangle change?)* acted thus as the class’s common ground. This was the final task on the test and a 3-points task which is supposed to be one of the most difficult ones. Therefore only those who usually mastered the difficult tasks had tried to solve it. In the following analytical account of the first episode of this lesson (Excerpt 17, Cecilie Jan 28th, episode I-1) I will show how the students can be seen as learners through participating in the classroom discourse, and I will provide further evidence for some of the findings from the lesson Jan 14th such as:

- Cecilie invited the students to participate by asking them to do some calculations.
- Through a typical I-R-E pattern of discourse she kept control of the discourse in the lesson.
- She was leading the students through the mathematics.
- Also in this lesson there was a shift in discourse when initiating a generalisation.

When studying the transcript of this lesson I see that in the first turns (1-10), the teacher was breaking the task down into easy manageable closed questions and I find the same kind of discourse as in the one presented from the Jan 14th lesson above. The teacher asked closed and simple calculation questions (turns 1, 3, 5, 7) and thus kept control of discourse and mathematical focus. The students’ answers were short, often one word, and the students were not presenting any holistic mathematical explanations, or sentences.

Referring to what the students who had solved the task had done, choosing a length and a breadth of a rectangle, adding 15% to the length and reducing 20% on the breadth calculating the new sides and the new area, Cecilie did the same by choosing a length of 20 and a breadth of 5. She drew a rectangle on an overhead transparency and put 20 on the length, 15 on the breadth and asked the students for the area (turn 1) before she wrote A=100 beside it. She restated the student’s answer (turn 3) before she asked a new question. Again she restated and also praised Leif’s answer, drew a new rectangle with new sides and proceeded further with a new question; ”when the breadth is reduced by 20 % what is the new breadth then?” (turn 5). Another new simple calculation question (*what is the area*) was asked in turn 7. This shows that from turn 1 through 10 the discourse was very similar to the one in the whole class section Jan 14th: The teacher invited the students to participate by asking easy manageable questions for them to answer and was thus leading the students through the mathematics.

Excerpt 17, Cecilie Jan 28th, episode I-1

Nr	Who	What is said	Comments
1	Cecilie	How big is the area of that rectangle?	Simple Calculation
2	studs	Hundred	
3	Cecilie	It is hundred. Don't mind the units. The area is hundred. And then the task was: The length is increased by 15%, how much will the new length be?	Restates, elaborates the task
4	Leif	Twenty three	
5	C	Very good, Leif, very good mental calculation. The new length becomes twenty-three, and when the breadth is reduced by 20%, what is the new breadth then?	Praises, restates, and elaborates the task. Asks a new question
6	Baard	Four	
7	C	Yes, and what is the area?	Confirms, asks a new question
8	Baard	It is ninety two, right? It's a guess	
9	C	Right. It is not a guess, it is mental calculation. How many percent is the area reduced?	Confirms, asks a new question
10	Baard	Eight percent	
11	C	Yes it is. If it was hundred percent earlier, now it is ninety-two percent, an eight percent reduction. *Then the question is: are you sure it is applicable for other rectangles as well? This was for one special rectangle*.	Confirms and elaborates

In turn 11 Cecilie first confirmed, and then she restated Baard's answer with an additional explanation about why the reduction was eight percent. Baard's answer had thus been legitimated by Cecilie's restating and by emphasising how the answer became eight percent, the other students in the class, who had been listeners, were reminded about the calculation which had been done. Conditions for possibilities of learning were thus created by Cecilie for the students who were participating in this classroom discourse. I have underlined the last part of turn 11 which indicates a shift in discourse. The question she asked was not an easy manageable one but of a kind which invited for several suggestions. I discuss this further in the section "*Mathematical focus*" with regard to the shift in mathematical focus which also took place in the lesson Jan 14th.

By presenting these extracts from whole class sections from two of the lessons with Cecilie, I have shown how a typical triadic pattern of discourse constrained the course of the lesson and how Cecilie thus controlled both the course of the lessons, the students' attention and the mathematical content and progression by asking easy manageable closed questions for the students to answer.

I observed the same kind of discourse in the beginning of the whole class section of the *Platon group* lesson, Jan 29th (the group with students interested in mathematics, which also involved the clever students) as in the two lessons described above. The mathematical topic was geometry. The task in this lesson was to find the volume of a cylinder with a cone on top of it, and Cecilie worked the task out on the board. When presenting the opening of the Platon group lesson (page 201) I pointed out how the study of personal pronouns indicated the teacher's role and that of the students.

These roles were visible throughout the first section of this lesson. Like in the other whole class lessons described above, she ensured students' attention by asking short-answered questions. She did the mathematics while the students were answering easy manageable questions such as: *How can we divide the figure; what is the formula for the volume of a cylinder; what is the base in the cylinder; what is the formula for the volume of the cone? Now you can do the calculations on your calculator*. She thus controlled the discourse and the mathematical focus in the same way in this lesson as she did in the previous ones. The teacher worked out the formula for the whole figure with letters in a teacher controlled interactive style. She told the students to copy what she was doing on the board. She illustrated by drawing the figure and she used the personal pronoun "I" when telling what she was doing on the board. After having worked out the formula with letters she exchanged the letters with numbers and asked the students to plot the numbers into their calculators (as their task was Jan 14th).

Then there was a shift in discourse also in this lesson, as I pointed out within the two other lessons. The teacher asked a question of a different kind from the easy manageable closed ones so far, she used "we" in the question and in the follow up comments she said "you". Until this shift which took place after 12 ½ minutes and while doing the task on the bard she had used "I". She asked an open question by asking for students' opinions: "Then the question is: can *we* give the answer like this (1339, 73) Does anybody have an opinion about that?" Thus I found a shift in discourse also in the Platon lesson as I did in the two Wednesday lessons. The shifts in discourse indicated also shifts in mathematical focus, which I discuss further on page 218. In this lesson, the mathematical focus shifted from giving a properly written presentation of how a geometry task was solved and worked out to a discussion about number of digits in an answer. The students contributed actively in this discussion in which the conventional aspect (number of digits) of mathematics was focused.

In the Wednesday lessons (Jan 14th and Jan 28th) the shifts in discourse were followed by shifts in mathematical focus from emphasis on procedural aspects to emphasis on generalisations and proofs. In the presentation of the mathematical focus in Cecilie's lessons below I will go into further detail how "it" was used when she suggested conjectures about generalisations in mathematics.

Mathematical focus

According to what Cecilie said in Focus group 4, the mathematical foci in her lessons varied. She gave that as one ground for having succeeded in challenging and inspiring the clever students. In the previous section (CPL) I pointed out how there were shifts in discourse in the lessons Jan 14th, Jan 28th and Jan 29th, and that a shift in discourse and shift in mathematics focus occurred together. I will now show how the mathematical foci varied in Cecilie's lessons, and also point out Cecilie's versatility. First in this section, I will analyse the mathematical focus from each of the lessons Jan 14th, Jan 28th and Jan 29th in detail:

- Under the heading "Procedural – generalisations - exploring – mathematics history" I will show how the mathematics focus in the lesson Jan 14th shifted from procedural to generalisations and then the teacher encouraged for students to engage in an exploring activity. Towards the end of the lesson, the teacher made a link between what they had been working on in the lesson and the same aspect of mathematics in an historical perspective.
- As pointed out in the preceding section there was also a shift in discourse in the lesson Jan 28th. From a typical I-R-E pattern of discourse, Cecilie now asked if "it" was applicable for all cases, and thus there was a shift from a procedural aspect of mathematics to generalisations. Under the heading "Generalisations, Proofs and Conjectures" on page 215, I will show how Cecilie took generalising from a task they had had on a test as a starting point for dealing with proofs. She linked to work they had done in prior lessons and she presented historical conjectures and proofs.
- Under the heading "Conventional focus and a student explains mathematics", I will first show that also in the lesson Jan 29th, Platon group, there was shift in discourse when Cecilie asked an open question and hence a shift in mathematics focus. When analysing the conventional aspect which occurred at the same time as the shift in discourse, I have focused on the teacher's use of "we" which raises questions about the nature of mathematical knowl-

edge and can be "intended as a clue to generality" (Pimm, 1987, p. 71). Then, later in the lesson, Cecilie invited the students to explain their best way to construct a parallel line which a student did. I present an account of that episode because it was the only time while I observed Cecilie's class that I experienced a student presenting a holistic mathematical explanation without any interruptions.

Next I will present an account of the lesson March 17th in which they worked with statistics. This account shows how Cecilie not only had searched in other mathematical books and resources in preparing her lessons but also in newspapers, magazines and the internet. Neither in this lesson did she use examples from the text-book, but she used real life examples and thus linked school mathematics to society and real life which is encouraged in L97. Thus yet another aspect of mathematics and of Cecilie's versatility in her teaching was demonstrated. As a consequence of having made the relation between the two domains school mathematics and real life, I will point out how students' attention remained in the domain of real life throughout the discussions which took place.

Finally in this section I present an account of the lesson I observed in the Socrates group, both with regard to the CPL category and also with regard to Mathematical Focus. This to show how I found both the discourse and the mathematical focus different in this lesson than I did in the other lessons I observed with Cecilie.

Procedural – generalisation – exploring – mathematics history

The lesson *Jan 14th* provides me with a key source of examples of how I saw shifts in mathematical foci in Cecilie's lessons; Cecilie's versatility with regard to mathematical focus in her teaching; how she used what she had read in other resources than the textbook in her teaching; how she linked to mathematics history and how she took students' previous knowledge as a starting point for exploratory work. The lesson had four whole class sections, section I, III, V and VI (see overview of lesson page 203) with individual work (section II and IV) in between.

Looking at the shift in discourse I described in the previous section in the Jan 14th lesson (see Excerpt 16 page 204), there was a shift for the students from answering easy manageable closed questions to finding out, or as Cecilie put it, *explore*, if her claim was applicable for any triangle. Thus the mathematical focus shifted from *procedural*, calculating the third side in a right angled triangle showing that *it* (the claim) worked with the values she had used, to a *conjecture of generalisation*, a conjecture they were going to refute by finding a counter example. Cecilie asked the students to find out if they could take the difference between the two smaller sides in any right angled triangle and add to the largest to find the hypotenuse (turns 16, 18 and 24). Both Svend and Baard, the two, according to the teacher, cleverest and "exploring types" of students in class, answered "no" right away (they had found counterexamples). The task then became to find out why *it* ("*If you take the difference between the two smaller sides in a right angled triangle, you can add the difference between them to the largest and then get the hypotenuse*") worked in the cases she had shown on the board. Thus the mathematical focus shifted further; *from finding a counterexample to exploring* what was special with the sides in the triangles for which the claim worked. This exploring was done individually or in pairs while the teacher walked around and asked the students if they had found out. I will refer some of the episodes from this individual work section of the lesson in the "Individual work section" of this chapter in order to show how through analysing the use of "it", I could account for why a student started out wrongly and also how some students extended the task.

In the next whole class section (section III), see Excerpt 18 below, Cecilie first referred to the "solution" which indicated that there was one right answer to the task. This indication is emphasised in turns 5-7 where Cecilie did not approve 1.333 (4/3 was never suggested) as the right solution, but the other way around which was ¾. Also in this episode I see the exploring aspect of mathematics being reflected. Cecilie asked the students for suggestions (turns 7, 9 and 11) based on the exploring they had done individually and used that as a link for further work with Pythagorean triples. In turn 11 Cecilie drew the attention to mathematics history, and a relation between the work the students had done and the work of "Pythagoras and his mates" was expressed. She credited Tove for having switched to the area on which "Pythagoras and his mates"[38] had been working (highlighted in turn 11).

Excerpt 18, Cecilie Jan 14th, episode III-1

Nr	Who	What is said	Comments
1	Cecilie	Then we carry on. Several have found the solution here. Many have considered the ratio. It has something to do with the ratio between the smaller sides. Who will explain this?	There is a solution.
2	Students	Svend	Students in class suggest Svend
3	Cecilie	Mikkel!	Cecilie addressed Mikkel who has his hand up
4	Mikkel	Let's explain. The ratio between them is 1.3333… Ten divided by seven point five is 1.3333 which the	

[38] She said: "Pythagoras og gjengen hans"

		other one is too.	
5	Cecilie	Yes, it is, no…. … and Svend, you were thinking the other way around?	Cecilie addressed Svend to tell the "right" solution
6	Svend	The ratio between them is …. The larger side is …. I mean the smaller side is three fourth of the larger	
7	Cecilie	Yes. It is. So if *you* (singular) had taken the other way around, been thinking like this (writes $\frac{3.6}{4.8}$ on the board). *You*'d got the ratio. Yes! Three fourth. The smaller side is three fourth of the larger. Has anybody found other examples on sides giving the same solution, so you can just add to get the third? Baard?	Adressed Mikkel Restated Sven's answer and elaborated further
8	Baard	Ninety three and one third and seventy	
9	Cecilie	Ninety three and one third and seventy. And then the difference is twenty three and one third which gives hundred and sixteen and one third. Very good. Tove, did you have another example?	Writes $93\frac{1}{3}$ and 70 on the board, and then $116\ \frac{1}{3}$ Cecilie restated and elaborated Baard's answer
10	Tove	Yes, six and eight which makes the hypotenuse 10	
11	Cecilie	Yes, very good. The smaller side is three fourth of eight and then you have added two to get the third. Very good. *Now you (singular) have switched to another area on which Pythagoras and his mates worked.* That was to find whole number solutions to Pythagoras [theorem]. Pythagoras [theorem] works for all smaller sides or for any numbers in a right-angled triangle. But they [Pythagoras and his mates] searched for whole number solutions. That is what you (plural) have found an example of. Has anybody found another example? Baard?	Cecilie restated and elaborated Tove's answer
12	Baard	Three, four and five. And twelve, thirteen and fifteen. Not sure about the last one	
13	Cecilie	MMmm. Very good. If you look at 3, 4, 5 and 6, 8 and 10, they are whole number solutions of Pythagoras which we call "Pythagorean triples". And the question is how many solutions are there? How many whole-number solutions are there?	

Before proceeding further with the mathematical aspect in this section I will comment on some aspects of the discourse and thus how CPL was created in this part of the lesson. It was not a typical I-R-E pattern of discourse as I found in the first section of this lesson (see page 204), but Cecilie asked the students what they had found out and she invited them to explain. Students suggested Svend (one of the, according to the teacher, "exploring students") however, Mikkel had put his hand up and was addressed by the teacher. That they pointed out Svend, demonstrated part of the culture of the class and it suggests both Svend's position and his role in the class and thus the socio-cultural expectations of him.

The students' contributions were not only short answers to calculation questions as they were in the first section, but they were results of their own exploring and were taken as starting point for further work. Cecilie praised, restated and elaborated the students' contributions (turns 7, 9 and 11). She thus legitimised the contributions and the other students became learners through participating in the classroom discourse.

In turn 13 another not yet addressed aspect of mathematics was initiated; *infinity*. After some more suggestions from the students about triples Cecilie asked (in turn 21): "Are there limited possibilities, or can you, for how long can you carry on like that?" One student answered: "until you have filled up the board", and others expressed uncertainty. Cecilie had drawn a triangle on the board showing the extension from a 3-4-5 triangle to a 6-8-10 triangle to a 9-12-15 triangle etc… and said:

> This can continue as long as I wish. So with the triangle 3-4-5 as a starting point I can make infinite Pythagorean triples. This was with one ratio between numbers. There are other ratios between numbers also giving Pythagorean triples, which was found out already 300 B.C. Then a person with name Euclid, a great mathematician who published some books called the "Elements" which were used as textbooks for a long time until 16-1700 I think, and they are still used because he created the foundation for proofs in mathematics. We'll come back to that. He made a formula for how one could find Pythagorean triples and the formula is like this: You choose two whole numbers p and q and you get a Pythagorean triple if you square one of them and subtract the square of the other. That makes the length of one of the shorter sides. Then you get the other: two times p times q, and the third side is given by p squared plus q squared. (On the board: p^2-q^2, $2pq$, p^2+q^2).

She showed how to get the 3-4-5 triple by using p=2 and q=1. In the next section (section IV) students should work on their own to find more triples by using Euclid's formula. This was followed up in the next whole class section of this lesson, section V, (see overview of the lesson page 203) where students were asked to suggest more Pythagorean triples. Four or five students participated in suggesting triples. They were now exploring different Pythagorean triples and how to extend the triples by multiplying the sides with a whole number. Also in this section Cecilie took students' contributions into account and used them as starting point for discussions.

In the last section of this lesson Cecilie lectured about mathematics history. But before she started she asked the students for suggestions how to extend the work with Pythagorean triples. Relating the work of Pythagoras to find whole number solutions to $x^2+y^2=z^2$ she asked if anybody could think of an extension of the task. In that way she linked the work they had been doing with the Pythagorean triples to “Fermat’s last theorem”, that there is not found any whole number solution to $x^3+y^3=z^3$. The students were now not invited to participate. However, Baard demonstrated his interest for mathematics history by commenting in between the teacher’s lecturing. Cecilie told them about Diophantus and his work and she also told the students that there was a person now living, Andrew Wiles, who had proved that the third degree equation does not have any solutions. Thus mathematics history and present work of mathematicians were part of the mathematical focus in this lesson and Cecilie demonstrated her interest for mathematics as a science and for mathematics history which she shared with her students.

This lesson showed the versatility Cecilie referred to in Focus Group 4. It demonstrated how Cecilie had searched in other books to find aspects of mathematics history to fit with the topic they were working with. The lesson also demonstrated how she took what they knew about Pythagoras (his Theorem), the students’ common ground as a starting point for exploratory work and aspects of generalisation and infinity were included in the lesson together with mathematics history. However, only few students were engaged in the work and in the last section only Baard was engaged.

Generalisations, Proofs and Conjectures

In L97 weight is put on proofs and generalisations, and it encourages using algebra to generalise and prove and also to link to history. In the subject related objectives for lower secondary stage L97 says: “They should learn to interpret and use letters as symbols for unknown and variable quantities and to generalise and prove” (L97 p. 178). And in “Main subject elements in numbers and algebra for grade 10” it says:

> Pupils should have the opportunity to [ ]
>
> - experience how expressions with letters representing variable quantities can be used to formulate and prove general relationships
>   [ ]
> - see examples of numbers and algebra in cultural and historical contexts
>
> (L97 p. 182)

Thus Cecilie’s weight on generalisations, proofs and conjectures was in accordance with L97. According to what Cecilie told me before the lesson Jan 28th, she would focus on generalisations and mathematical proofs as she had done the week before; the difference between something being applicable for some cases but not all and on the other hand something being applicable for all cases. I have chosen to present examples from the lesson Jan 28th, because it is particularly representative of where generalisations, proofs and conjectures were dealt with. Cecilie continued the work from the week before (Jan 21st, see page 200 for an account of the opening part of that lesson) when the topic for the day was proofs and reasoning and she had said that mathematics is different from other sciences because you can prove things which she followed up in this lesson. Furthermore I will show how Cecilie had structured the lesson by taking a task they had had on a test as a starting point; that the work from Jan 14th and Jan 21st were related to this lesson and how she again linked to mathematics history. Finally I comment on Cecilie's versatility and how she dealt with the complexity of the classroom

Cecilie started by taking a task they had had on a test as a starting point, and she related to algebra which could be used to prove "it" (see Excerpt 17, Cecilie Jan 28th, episode I-1, page 208) generally. In that presented excerpt from the lesson *Jan 28th* I highlighted the sentence which indicated a shift in discourse. This was where there also was a shift in mathematical focus after the introductory part with easy calculation questions. This shift was of the same kind I pointed out in the turns 16 and 18 in the lesson Jan 14th (see excerpt page 204). The mathematical focus shifted from procedural to generalisation, and I could sense a request to make a conjecture: *"Are you sure [ ]?"* And although Baard claimed that he had shown three examples, Cecilie kept asking if "it" was applicable for all rectangles. This way she emphasised the characteristic of a proof. The use of "it" occurred in the same way here in turn 11 as in the Jan 14th lesson. According to my interpretation the "it" here was: *If you increase the length of a rectangle with 15% and reduce the breadth by 20% the area is reduced by 8%.* She asked for a method of doing it: *"Is there a method showing that "it" is applicable for all?"* Through this question she gave a hint that there was a way to prove this and when Svend suggested algebra, she challenged him to explain how to do it to which he answered "can use a and b, may be".

Cecilie recalled "the rule" from two weeks before (that when the ratio between two sides in a right-angled triangle is ¾, the difference between the two smaller sides can be added to the largest and give the hypotenuse) and used algebra to prove that in interaction with the students. Using Pythagoras' theorem they showed that $\left(\frac{4}{3}a\right)^2 + (a)^2 = \left(\frac{5}{3}a\right)^2$.

Furthermore she showed historical conjectures which had turned out not to work in all cases:

- Searching for primes and referring to conjectures that had not turned out to be right.
    - "*Funny pattern*": Searching for prime numbers and showing on a finished written transparency that 31 is prime, 331 is prime and 33…31 is prime when the number of 3's is 7 or less. However,

- 333333331 is not a prime because that can be written as $17 \cdot 19604873$
- "*Pattern with factorials*": Pattern with factorials giving prime numbers. 3!-2!+1!=5 is a prime, 4!-3!+2!-1! =19 is a prime, starting on 5,6,7,8 with the same pattern are all primes but coming to 9!-8!+7!-6!+5!-4!+3!-2!+1!= 326981= $79 \cdot 4139$ which is not a prime.

- She referred to Fermat who searched for primes using the formula (2 to the power 2^n)+1 and that all such numbers were primes for n=0,1,2,3,4. But in 1732 Euler found that when n=5 then 2^{32}+1=4294967297 is not a prime because it can be written as 641*6700417 and that neither 2^{64}+1 (Landry in 1880) nor 2^{128}+1 (Brillhart and Morrison in 1975) nor 2^{256}+1 (Brent and Pollard in 1981) are primes either. When showing these examples Cecilie asked the students what 2 to the power zero was, 2 to the power one, 2 to the power two and so on. Thus she ensured some students' attention and Baard answered.

She then proved algebraically that when *x* is even, *x* squared is even and, if *y* is odd, then y squared is odd. These were known generalities for the students however; proving them algebraically was not straightforward. Cecilie did this in interaction with the students by asking the following questions:

- *How can we write an even number with a formula?*
- *If x is 2n then x squared is ???*
- *Is a number 4n squared divisible by two?*
- *What is y squared if y is (2n-1)? What are the two brackets multiplied together?*

Finally Cecilie showed the students "*the proof known as the most elegant proof*" (Euclid 300 B.C), that there exist an infinite number of primes. Thus she added an historical dimension to the day's work as she did in the Jan 14th lesson and which is encouraged in L97.

Except for Baard's contribution in this subsection (he told us about a mathematics researcher he had read about in the newspaper who had made a computer program searching for prime numbers), Tove asked if it was possible to find a formula for prime numbers, or if one is just hoping to do it. Another student, Marte, asked what was so exciting about what they were now working with. In this section (which lasted for 35 minutes) Cecilie had been lecturing from the board all the time. It ended with Cecilie saying (as a response to Marte): "*Many people find this exciting. It is like climbing a mountain. Why climb on the top of a mountain? Yes, because the mountain is there. It is the same with the prime numbers. It is very exciting*". By saying this Cecilie conveyed her view of mathematics and her excitement for the subject.

According to my field notes most of the students (more than half the class) did not pay attention and there was a lot of noise which can be heard on the audio recordings. Svend and Baard were quite active throughout the lesson and 7 others said something as well, but they only contributed with one or two comments. Some students had started eating their lunch, some were working with other subjects, and some were chatting. One student fell asleep during the lesson (which caused a lot of laughter in class). This shows the complexity of the classroom with which the teacher had to deal. The way Cecilie dealt with it was to teach in interaction with the few ones who were engaged. Despite Cecilie's versatility, that she had prepared the lesson well by having studied several resources, brought ready written transparencies, related to prior work (both the task from the test, the work from Jan 14th and the known generalities referred above) as starting points (students' common ground), few students were engaged in the lesson and also few paid attention. Before the lesson Cecilie said she was going to lecture from the board and the learning goals for the lessons were mathematics as a science, proofs and algebra. Thus the course of the lesson was as intended, however, as pointed out students' engagement and attention were rather poor, and I suggest that the complexity of the classroom pointed out above, can partly account for that.

Conventional focus and a student explained mathematics

In the lesson I observed in the Platon group, there were two issues I will present accounts of. The first issue was typical in Cecilie's lessons, a shift in discourse and mathematical focus which occurred simultaneously took also place in this lesson; second, an untypical feature which occurred only in this lesson, that a student presented a holistic mathematical explanation without being interrupted by the teacher or by other students.

In this lesson the task was to work out the volume of a solid block which was a cylinder with a cone on the top, and Cecilie had presented how to work it out on the board. The students contributed doing the calculations on their calculators. They had worked out $\pi \cdot 8{,}0 \cdot 8{,}0 \cdot 4{,}0m^3 = 803{,}84m^3$ and $\frac{\pi \cdot 8{,}0 \cdot 8{,}0 \cdot 8{,}0m^3}{3} = 535{,}89m^3$ separately and then summarised to 1339.73 m^3.

The indicated shifts in discourse (discussed on page 209) and in mathematical focus in the *Platon* lesson were through Cecilie's question: "Then the question is: can *we* give the answer like this (1339.73) Does anybody have an opinion about that?" The use of "we" in this question indicated a request for a mathematical convention: How do we usually present answers? How many digits should be included? This was an open question and she invited contributions from the students by asking for their opinions. Thus the shift in mathematical focus was from procedural, to conventional. Cecilie referred to the numbers of digits given in the task, and asked for students' suggestions. She added that since the number with the least number of digits had two digits, only two digits should be in the answer. A girl suggested 1300, something Cecilie approved, however at the same time suggesting that it was a very big round off, and that when the number starts with one and hence has just exceeded a decadal unit, it would be more reasonable to round off with three digits. Another student, Svend suggested one thousand three hundreds and forty which was approved by Cecilie. When Svend asked if you would get an error or if that depended on the person doing the assessment if you did not have the right number of digits in the answer, Cecilie said:

> You are not supposed to know this to your fingers' tips. *It says that there shall be a reasonable use of digits in the answer*. So that means that you shall make a judgement. You shall not give an answer like this (points to 1339, 74). You shall make a judgement yourself of how accurate your answer is.

I do not know what Cecilie referred to saying: *"It says that there shall be a reasonable use of digits in the answer"*. L97 does not say anything about reasonable use of digits. One reason for the teacher to say it this way could be to convey a conventional aspect of mathematics, When the question about use of digits was raised, she used "we" which emphasised that it was the conventional aspect of mathematics she conveyed here ("can *we* present an answer like this?"). Or it could be as Pimm (1987) stated it: "a means of spreading responsibility, while at the same time deriving weight and authority from (large) number" (page 71).

In my presentations of the pattern of discourse and communicative approach I discussed how I saw Cecilie either was teaching from the board asking simple calculation questions to ensure students' attention or that she was lecturing without inviting the students to participate at all. In the following account I will present one episode from the Platon group Jan 29th which was different. In this episode a student gave a holistic mathematical explanation of how she had solved a task they were given to work with individually. This episode is also an example of that although the student, Kari, did present a fully understandable explanation of a way of how to (correctly) construct a line parallel to a given line segment through a given point, the teacher revoiced the student's explanation and presented her own version of the student's answer while showing the construction on the board. She thus legitimated Kari's explanation and made it more mathematical.

The task they were given was to find the best way for him/her to construct a parallel to a given line segment through a given point and Cecilie invited students to tell their way of doing it in plenary. Kari started. (The teacher had drawn a triangle ABC on the board and the task was to construct a parallel to AB through C)

> I took the C, a perpendicular from C, and then I constructed a perpendicular from A and straight up, and I still had the perpendicular from C. Then I took, I took the height from C and down to the point where it crossed the lower line and then I measured that height and in the height I had there I put the tip of the compasses and then I marked it up there too. (Laughter in class) It became right, but I am not so good to explain.

This was a relatively long turn from a student about mathematics in whole class. This was the only holistic mathematical elaboration done by a student in whole class I found during my classroom observations with Cecilie. The teacher did not interrupt her. The other students started laughing, and I suggest that was because the explanation was in such great detail and some of them lost track. I want to remark the mixture between everyday language and mathematical language Kari used when explaining for the rest of the class how she had done it. The teacher's response was very positive she praised her saying: *"Very good, very well explained, Kari. I shall take it slowly now. Ehh, yes, I understood everything Kari said"*. To which students in class responded: *"Then you were the only one"*. Thus the teacher's presented version of the student's explanation can be looked upon as a mediating tool of what the student said. The teacher's language which was more precise and more mathematical than that of the student, served as a mediating tool. The other students became learners through participation in the discourse.

Cecilie then started elaborating Kari's explanation and reformulated it while doing the construction on the board. One of the reasons why the teacher elaborated Kari's explanation could have been that the other students had started laughing and expressed their frustration of not understanding what Kari meant. Another suggestion is that teachers often feel that something is not taught or explained well enough if the teacher has not done it her/himself and in a conventionally correct mathematics language. This suggests the role of the teacher as being the owner of the mathematics being taught in school.

School mathematics and the outside world

L97 encourages to link mathematics to society outside school; "The syllabus seeks to create close links between school mathematics and mathematics in the outside world" (L97, p.165). How Cecilie linked school mathematics to the outside world was especially visible in the lesson with statistics, March 17th and is the main focus in this section. As I wrote in the beginning of this chapter, (see page 201) Cecilie captured the students' attention by linking the topic directly to issues from the newspaper. I will also present accounts of sections of this lesson to show

how the way she linked between school mathematics and society turned out in the lesson by pointing out

- that students became engaged in discussions from real life from which the examples the teacher used were taken;
- that the teacher and the students talked past each other when the teacher wanted to focus on the mathematical aspect of the issue discussed while the students' attention remained in the social domain;
- how Cecilie used surveys from real life to provide examples of and review statistical concepts from a test they had had.

When analysing this lesson I divided it into sections. The first bullet point is from section I, the second bullet point from section III while the third bullet point is from section II.

These sections lasted for about fifty minutes and were all about statistics. Cecilie was teaching from the board. She spent two minutes calming the class before the lesson started. In the first section (Section I, 15 minutes) she related the topic, statistics, to what can be read in the newspaper; first how statistics were used to bring about claims that children from well-off parents get better grades in school. This was presented as unfair in the newspaper and teachers were partly blamed for it in the article since one of the most important roles of school is to prevent inequality. As a reaction to this a discussion took place in the class, however, not about statistics but about plausible explanations why children with well off parents were getting better grades in school. The other topic, also taken from the newspaper the day before, was about an investigation finding that men were better drivers than women which resulted in yet another discussion, this time about plausible explanations why and if that was the case. In this section the teacher opened for students' contributions by letting them come up with suggestions on which she commented. However, the discussion was not mathematically focused. The discussed issues were about society; why children with well off parents perform better than others, and afterwards if and why men are better drivers than women.

In section III (20 minutes) they dealt with the topic "cheating with statistics". Cecilie had picked 5 different examples of how it was possible to use statistics to present a desired view. I will go into detail on the first one where Cecilie showed a picture of two men, one American and one other. This episode provides me with an example of how the students remained in the social domain while the teacher focused on the mathematics. The American had twice the income of the other which was illustrated with two bags of money. The American bag was doubled in three dimensions compared to the other. Cecilie asked the class if that was a correct presentation. The answers she got were like:

- It depends what they are working with.
- It depends how much money is in each bag.
- It depends on the living costs in the two countries they come from.

These answers from the students show that the students were not thinking mathematically and that their focus was quite different from that of the teacher. Students' attentions were in another domain than that intended by the teacher. Just as was the case in the first section, these students' attentions were still in the social domain, whereas the teacher's attention was on mathematics. Her focus was on the mathematical fact that if you double all sizes in a three-dimensional figure, the size of the figure is enlarged by $2^3=8$. Thus there was a conflict or a tension between the students' attention and that of the teacher. Drawing on already discussed aspects of CPL I will present an account of how Cecilie dealt with this issue.

Cecilie tried to emphasise what she meant: "I will show you yet another figure so you then can understand *what I mean*". This suggests that she did not want to tell the students directly what she meant, she wanted the students to conceptualise it themselves based on her illustrations and account. It also suggests that she was seeing the task from her perspective. She was leading the students towards the answer she had in her mind to the question by presenting a new figure where the bags of money were shaped as prisms. Her question in turn 3 (Excerpt 19, Cecilie March 17th episode III-1, page 223) which was a closed question since there was only one right answer to it (eight), resulted in five suggestions from different students about some kind of a relation between the sizes of the two bags: turns 6, 7, 9, 10 and 11, but none of them were right. Only one of the suggestions was not serious (11). The others can be accounted for. In turn 6 (four time the breadth), I conjecture that the student looked at two dimensions of the figure. The student's suggestion in turn 7 (three times the volume), I suggest, was that s/he was thinking three times as big since there were three dimensions. The student in turn 9, who suggested four, gave the right answer to the question Cecilie asked in turn 8 where she only mentioned two dimensions, "twice the height, [and] twice the breadth". The student who said six times in turn 10 could have been thinking two times three, the double of three dimensions. However, Cecilie did not elaborate on these suggestions and she did not ask the students to explain their answers either. Svend put his hand up and gave the right answer. Cecilie then elaborated on his correct answer (turn 14). His suggestion was thus legitimated by the teacher and shared with the others through the teacher's elaboration; learning could take place for the students as a result of participating in the class's discussion.

Excerpt 19, Cecilie March 17th episode III-1

Nr	Who	What is said	Comments
1	C	The next figure presents the same as the first and the person to the right has twice the salary as the other and the bag to the right has twice the height as the other. Is that a good presentation?	Apparently and open question together with (3)
2	Stud	No	
3	C	Reason?	
4	Stud	He cannot lift it	
5	C	He has so much salary and he has twice as much, Sturle?	Highlighting key issue of the task
6	Sturle	The other bag has at least four times the breadth too	
7	Stud	It has three times the volume, at least!	Reasonable suggestion.
8	C	You are into volume now. If the bag to the right has twice the height, twice the breadth, how big is the volume compared to it then?	
9	Stud	Then it will be four times	Reasonable
10	Stud	Six times	Reasonable
11	Stud	Twelve thousands	Not serious
12	C	Svend?	
13	Svend	Eight times as big	Right answer
14	C	Yes because here there are three dimensions doubled, two times two times two. That makes eight	Revoiced student's suggestion

This episode had a teacher controlled interactive style. The discourse was dialogic, she asked open questions, but there was only one right answer to the issue discussed. The mathematical focus was conceptual and structural. The focus was for the students to judge and compare the sizes of two three-dimensional figures based on the relation between their three measures. Svend had seen this relation, but my question is if the other students did see this relation based on what was said in turn 13 and in turn 14. The mathematical focus was also structural in the sense that geometry and statistics were linked; that geometric figures were used to illustrate statistical quantities.

Baard presented a remark right after this episode, but the recording is not good enough for me to hear what he said. However, I can hear Cecilie praising his remark.

In section II of this lesson, (which lasted for 15 minutes) Cecilie used the overhead while teaching statistics. She used examples from actual surveys which she had found on the internet and in the newspaper. That way she linked school mathematics to mathematics in society. She did not present any examples from the textbook, but she took students' performance on a test they had had as a starting point for what further work needed. Cecilie was teaching from the board and invited students to participate. It was a review of bar graph, histogram, sector diagram and central tendency (mode, mean and median).

All these are topics listed in L97. While teaching she ensured students' attention by asking questions like: "What kind of diagram should we use here?" The questions were closed and she was thus controlling the discourse and the mathematical focus. In this section of the lesson, when Cecilie was doing the review of different diagrams and central tendency, Baard and Svend did not participate. This suggests that the subject content presented could have been trivial for them, and that they therefore did not bother to participate either. This emphasises the diversity of students' interests which again underpins the complexity of the classroom and how difficult it is to engage all students in a mixed ability whole class in meaningful activities

All through this lesson I find the way Cecilie linked mathematics to life outside school significant. She had taken examples from the newspaper and from the internet. She did not use the textbook and her intention had been to exemplify the mathematics they should work on with examples from the real world and not with invented examples for school mathematics. However the way that worked out in the classroom was that the students did rather focus on the social issue discussed than the mathematics. This shows how the enacted curriculum, the way it turned out in the classroom, was possibly different than that of Cecilie's intention which was more mathematically focused.

<u>Discourse, communicative approach and mathematical focus in the Socrates group</u>

I found the lesson I observed in the Socrates group (with students not so interested in mathematics) quite different from the Wednesday lessons. This difference is presented schematically in the overview on page 226. When I observed the Socrates group, the program for the day was to review a national given test they had had. The teacher had marked it and she handed it out and spent most of the time of the lesson conveying to the students some of the right answers. For some of the tasks she showed and told how to do them. Occasionally she asked the students for the right answers, but not how they had solved the task. From my perspective the mathematical focus was procedural, focusing on rules and methods how to solve the tasks. There had been some estimation tasks (mental calculation) on the test and when she was going through them I noticed that she three times said: "*You have to think like this…..*" She thus tried to convey her way of thinking to the students. The learning environment was not of a kind for the teacher to ask: How did you think? It was very noisy and from my perspective the students were not very interested in the mathematical content, but only in how many points they had got on the test and the corresponding grade.

The discourse in this lesson was characterised more by show and tell than that of any of the other lessons I observed with Cecilie. With regard to discipline, there was a lot of noise and the students were not paying attention. This was the only lesson I observed with Cecilie where she asked students to open their textbooks. She told the students to study the coordinate system on page 354 in the textbook. Many students did not have their textbook since they usually did not use it. In the individual work section of the lesson the teacher was either out of class and the students fooled around or the students worked on a self evaluation form (2-3 minutes). This evaluation was part of the test. The purpose of this evaluation was for the students to reflect on their own knowledge in mathematics. What they did not manage on the test and what they thought they had to learn to not make the same kind of errors another time.

Students' abilities, CPL and MF

In the accounts from Cecilie's whole class lessons presented so far, I have several times suggested that many of the students in the class did not pay attention and that the clever students were challenged. This suggests that in Cecilie's whole class lessons there was a mismatch between the mathematical focus and students' abilities. These were issues in the classroom which made Cecilie's intentions for the lessons difficult to implement and resulted in an enacted curriculum different from what she intended. I will try to express how I saw the relations between CPL, MF and SA in Cecilie's whole class lessons.

When studying Cecilie I saw a relation between Conditions for possibilities of learning and Mathematical focus; or more precisely, I saw a relation between the pattern of discourse and the mathematical focus. I have suggested that shifts in discourse occurred together with shifts in mathematical foci. When leading the students through easy manageable closed and simple calculation questions, the mathematical focus was procedural or conventional. In this discourse and mathematical focus most students in the class paid attention (except the cleverest students as I have indicated was the case in the statistics lesson, page 224). When the discourse shifted as she asked an open question, the mathematical focus shifted to conjecturing generalisations. Now fewer students participated, and according to my field notes, more than half of the students did not pay attention. However, those participating were challenged and contributed with suggestions which the teacher followed up. When the mathematical focus shifted to mathematics history or mathematics as a science the teacher was lecturing and did not invite the students to participate. Sometimes one or two students contributed with comments. However, in these parts of the lessons very few students paid attention. In my discussion of the Socrates group I suggested that the teaching was characterised by a more “show and tell” than in the other lessons. She was showing how to solve the tasks she reviewed and she was telling the students how to think. The above account can be presented in the overview below.

Conditions for Possibilities of Learning	**Mathematical Focus**	**Students’ Abilities**
Lecturing or dialogue with two or three students	Mathematics history and mathematics as science	Students with special interest in mathematics participate
Open questions	Exploring, conjecturing generalisations	The cleverest students are challenged and engaged in discussions
Leading students through easy manageable, closed questions	Simple calculations, procedural focus, initiating exploring activities	More students participate and have the possibility to understand what is going on
Showing and telling	Procedural focus	Students not so interested in mathematics in the Socrates group

Table 12, Discourse and mathematical focus in Cecilie’s lessons

In my account of Cecilie I have pointed out two clever students, Svend and Baard who participated and commented both when Cecilie asked open questions and challenged for generalisations and also when she lectured mathematics history. However, these two students did not participate when the topics discussed were trivial as when Cecilie asked for names of bar diagrams and when they reviewed the task from the test about the cone and the cylinder. It was not until discussing the number of digits in the final answer that Svend’s voice was heard.

This emphasises what Cecilie had pointed out as the greatest difficulty in teaching mathematics; the issue of mixed ability classes. Although she had “solved” it once a week by grouping the students in a Socrates group and a Platon group, this seemed to be a complexity in the classroom constraining how her visions about teaching mathematics were implemented in the classroom and that the enacted curriculum as the curriculum jointly constructed by the teacher and the students turned out differently than that of her intentions. Her way of dealing with the complexity was to keep control in the lessons. She orchestrated discussions (to the extent discussions took place) and used a teacher controlled style when teaching and carried out the “exciting” mathematics on the overhead or on the board.

Individual work sections of lessons

There was not much individual work in Cecilie's lessons which all started with plenary work. In the lesson Jan 14th there was individual work in between whole class sections (see overview page 203). In the other lessons the individual work was towards the end of the lesson. According to my field notes few students worked with mathematics during these sections of the lessons, except in the first individual work section (II) Jan 14th. From the lesson Jan 21st I identified eight episodes with individual students. Five of these episodes were with either Baard or Svend. In the lesson Jan 14th more students were involved in a dialogue with the teacher. However, many of the "dialogues" were short. In the lesson Jan 29th the individual work took place after Cecilie had been teaching from the board for 45 minutes. There was much noise during individual seatwork, and from my field notes I see that less than half the class worked with mathematics. I can also hear the noise on the audio-recorder and that Cecilie several times was interrupted while being occupied with individual students. Thus it has been difficult to identify holistic individual episodes with the students. Towards the end of this section I present a summary of my findings from individual seatwork in Cecilie's lessons.

In this section on individual work in Cecilie's lesson I address the following which I have identified as significant aspects:

- Under the heading "*What is "it"?*" I will first present an episode to show that although Baard had taken actively part in the whole class section in which the task was introduced, he started off in a wrong direction. In my account of this episode I follow up the whole class lesson Jan 14th in which "it" was introduced. As a tool in analysing the episode, I have studied the use of "it" to account for why the student started off in a wrong direction. Second I will refer to how Cecilie addressed other students while they were working on this task to indicate that there was a tension between Cecilie's intention for the work with the task to be one of exploratory and how the work turned out in the classroom. She praised and encouraged students, but also eagerly wanted to show and tell what "it" was.
- One issue with which Cecilie said she felt she had succeeded was to challenge the clever students. Under the heading "*The clever students were challenged*" I present another episode with Baard from the same lesson (after he had found "it" out) where he demonstrated how he extended the same task and hence was challenged by the task itself and also that Svend was challenged by the same possible extension of the task. For these students the task

turned out to be an exploring activity which had been the teacher's intention.

- Finally, from individual work in Cecilie's lessons, I present 5 episodes. The purpose is to show how Cecilie provided different students with different levels of support. In the analysis of the episodes, presented under the heading *"Students need different levels of individual support"*, I have used concepts from socio-cultural theory as tools to account for how I saw the students with support from the teacher became able to solve the tasks. I account for
    - How students' thinking was structured through the teacher's support and thus a bridge between existing and new knowledge was created;
    - how the teacher was breaking the task into meaningful subgoals (Rogoff, 1990);
    - how the teacher spaced out the amount of help according to the student's need (Wood, 1998);
    - occurrence of "miscommunication". The teacher and student talked passed each other.

In using these terms I describe how I see the dialogue between the student and the teacher as work within the zone of proximal development. The teacher and the students were working in the "gap" between the actual student's present understanding and what s/he could perform without assistance (Mortimer & Scott, 2003). This is what Bruner termed "scaffolding", which according to Vygotsky was "to internalise external knowledge and convert into a tool for conscious control" (Bruner, 1985, p. 25).

What is it?

As a response to my discussion of "it" in the first section of Cecilie's whole class lesson Jan 14th, I will first refer to how Baard, although being one of the "clever students" in class, did not see the task holistically. The presented excerpt from the episode (Excerpt 20 page 229) shows how he argued for his solution while Cecilie argued for the correct solution. The task the students were asked to solve was: "Does *it* always work, does *it* work for any lengths of the sides in the triangle?" (See Excerpt 16, Cecilie Jan 14th episode I-1 page 204). Cecilie presented two triangles, one with the sides 3.6 – 4.8 – 6 the other with sides 7.5 -10 - 12.5 where *it* worked. According to my understanding I interpret the "it" as: "If you take the difference between the two smaller sides in a right angled triangle, you can add the difference between them to the largest and then get the hypotenuse". Indicating that "it" had something to do with the ratio between the sides, Cecilie asked the students to find out when "it" worked. Thus the task turned out to be a "guess what it is".

Baard started off by examining the relation between corresponding sides in the two triangles. He called for Cecilie's attention complaining that he although having written a whole page he had not found out anything. Cecilie told him to look at one triangle at a time: "Just investigate the numbers in one triangle" she said. This suggests that Baard was investigating something with the triangles, but he did not know what to investigate and why. He had taken the difference between the smallest sides in the two triangles and got 3.9 (7.5-3.6=3.9). Cecile gave a hint (investigate the numbers within each triangle) so he could find out the right thing for why "it" worked; 3-4 minutes later Baard called for Cecilie's attention again:

Excerpt 20, Cecilie Jan 14th episode II-14

Nr	Who	What is said	Comments
1	Baard	Cecilie, is it right that it is the ratio between the sides	
2	C	Yes	
3	Baard	I have found that it is two and one twelfth or 2,083333	He had found the ratio between corresponding sides
4	C	Two and one third?	
5	Baard	One twelfth or 2,08333	
6	C	Between the two?	
7	Baard	Between the two	
8	C	Calculate that once more	
9	Baard	If you take 7.5 and divide by 3.6 I get it. And then I can take..	
10	Cecilie	But Baard!	
11	Baard	It's me	
12	Cecilie	Rather compare the ratio within one triangle, and then you'll find the solution	
13	Baard	I cannot have three numbers in one fraction. It makes chaos	He still did not understand
14	Cecilie	No but you can say that an internal ratio counts here	
15	Baard	Yes, this and that. Those are the ones I have compared, I thought	
16	Cecilie	Those two, you shall compare. You shall compare those two.	Now Cecilie told him directly what to do
17	Baard	Oh dear. 10 divided by 7.5 and 4.8 divided by 3.6. Then I get the same because that is one and one third	
18	Cecilie	Yes,	
19	Baard	Then I understand	

The pattern of discourse here was different from that of whole class episodes which had a typical I-R-E pattern. In this episode there was no such triadic pattern of discourse. It was not the teacher initiating a question, but the student asked for consent for what he had done. When this tended to be wrong, an argumentation between the student and the teacher took place. The student saw this from his point of view (comparing the two different triangles), and the teacher saw it from her mathematical point of view; to find the correct solution which was that the ratio between the sides in each individual triangle is ¾ or 4/3. The mathematical focus was the "it", which was not explicitly agreed upon, and Baard was still doing the comparison across the two triangles (turn 9) even when Cecilie had told him to look at the sides within one triangle. However, now he looked at the ratio and not at the difference as he first did. In turn 12 Cecilie told him directly what to do to "*find the solution*". This shows that there was one way to find the correct solution to the task she had given the class to explore, and when Baard did not find out what he had to look for, Cecilie told him directly. Even when she told him to compare the ratio within one triangle he did not understand, because he could not have three numbers in a ratio (turn 13). The "it" was still not clear and he argued for what sides to compare (turn 15). He did not understand (turns 17 and 19) until Cecilie pointed out for him the two sides he had to compare to get the solution (turn 16).

During the individual seatwork in this section (section II) of the lesson Jan 14th when they were asked to find out why "it" worked. Cecilie walked around and asked students if they had found out or if they were "*on the track*". She praised those who had found that it had to do with the ratio between the two shorter sides being ¾. She also praised those who were on the track and encouraged them to find out further. A student asked: "It has something to do with the triangle, is that right?" Based on Cecilie's answer to this, I conjecture that the student was studying the angles. Cecilie answered: "It has something to do with the sides. It must [inaudible] yes, you can say it has to do with the angles, but you don't need to bring the angles in. However, you are right".

This shows how she praised and encouraged them to proceed further. With regard to the students who had not yet found out, a significant aspect was that she offered to show or tell them although they had not asked her to. This suggests that the teacher was keen to tell the students the solution and that there was one solution to the problem. Thus it did not turn out to be an exploring activity for most of the students but rather to be to "discover" that the ratio between the two smaller sides were ¾ . Thus I perceive a tension between the teacher's conceptions of an exploring activity and how it turned out in the classroom.

The clever students were challenged

To emphasise one of the characteristics that evolved from the analysis of whole class lessons with Cecilie, that the clever students were being challenged, I will present an episode from the next individual seatwork section which shows how clever students extended a task they were working on and thus were challenged by the mathematics within the task and their work turned out to be an exploring activity. The task was to find Pythagorean triples, both those that had the ¾ relation (in the 3-4-5 group) and also triples possible to find by choosing any prime-numbers p and q to get the triple (p^2-q^2, 2pq and p^2+q^2, Euclid's formula). The episode I present was with Baard (Excerpt 21, Cecilie Jan 14th episode IV-7, below). He captured much of the teacher's attention, which the teacher let him do, also when he walked over to her with his questions while she was talking with other students. In this episode Baard had found a Pythagorean triple by using Euclid's formula with p=7 and q=5, which gave the triple 24- 70- 74. He realised that this triple was a multiple of another triple, and thus "not the beginning of a row of triples" as he said. By dividing this triple by two he had got another triple in the same row or group: 12, 35, and 37. It had frustrated him that he did not find out how to find the p and q which would give him 12, 35 and 37, and he "excitedly" told Cecilie about his frustration. Cecilie then challenged him to try to find out what p and q in Euclid's formula would give him the beginning of the row of triples. The pattern of discourse was not a typical I-R-E. The interaction between the teacher and the student was characterised by the student explaining how he had been challenged by the task itself. He was trying to find out how to find the starting point of a row of triples. The teacher praised (2) what he had found out and asked a question in turn 4 to catch what his issue was. Baard explained (5) which resulted in Cecilie formulating that he could try to find what p and q were giving the 12-35-37- triple (turns 6 and 8). The teacher praised him for having found out how to extend the task she had asked them to do (10).

Excerpt 21, Cecilie Jan 14th episode IV-7

Nr	Who	What is said	Comments
1	Baard	If I use the primes 7 and 5, I still don't come to the starting point of a row. To call it a new row I need at least one prime in it. In this row (points to 12, 35, 37) the prime is 37.	He had realised that there was no prime in the 24-70-74
2	C	Yes, smart	
3	Baard	I multiplied up to that by using 7 and 5. They are both primes however I did not come to the beginning of a row	
4	C	What is that, because 12, 35, 37?	
5	Baard	Everything can be divided by two and that makes	Baard suggests

		12 and 35 and 37 which make my row	that 12-35-37 is the beginning of the row
6	C	Check what numbers that are giving this starting point. Because this is a starting point, isn't it? This is the smallest in that triple row there?	
7	Baard	Yes	
8	C	Can you find out what p and q are giving those numbers?	Cecilie challenged Baard
9	Baard	Okay, I'll try	
10	Cecilie	This was very well thought, Baard. Good development of the task, how to find the starting point in a row	Praising for extending the task

Eight minutes later in the lesson, in plenary (section V of the lesson, see overview page 203), Svend, another "clever" and interested student, asked: *"Is it possible to find out what p and q that have been used or do we have to guess?"* Cecilie became excited and referred to what Baard had worked on and asked Baard to explain what his challenge had been and what he had tried to find out. He had not found the p and q that gave the 12-35-37 triple which Cecilie left as a challenge. Svend took this as a challenge as well. None of the other students commented on it or asked questions about this challenge. I suggest that was because this was outside their system of interest and understanding.

Students need different levels of individual support

Based on the data I have from individual seatwork, I suggest that Cecilie took students' mathematical abilities as she perceived them, into account when giving support during individual seatwork. Analysing these episodes through socio-cultural lenses I see a teacher who supported the students according to their individual needs, and that her help was a support for them to work further with the tasks they were struggling with. The evidence I provide for this are from five episodes from the lesson Jan 21st.

- Under the heading <u>Show that the sum of two successive triangle numbers is a square number</u>**,** I will show how the level of challenge and the way Cecilie gave support demonstrate that she took the student's abilities into account.
- In presenting the next episode which is with the same student as in the first, Svend, the task was to <u>show that 1+3+5+7+………(2n-1) is a square number</u>, I will show how Cecilie also here offered the student the necessary support for him to solve the task and that she provided a bridge between this task and a task he had already solved earlier.

- The third episode I present is to illustrate how Cecilie supported a student who was not at the same level as Svend differently from how she supported Svend. She used tokens as concrete materials to show that the sum of two even numbers is an even number.
- The last two episodes deals with formulae for odd and even numbers, and in both episodes Cecilie exemplified with numbers in structuring the students' thinking. This way she provided a bridge between what the students could express orally or arithmetically and expressing generalities.

Show that the sum of two successive triangle numbers is a square number

In this episode with Svend (Excerpt 22, Cecilie Jan 21st episode II-2, page 233) Cecilie provided the necessary support for the student to solve the task which he did not manage on his own. In the analysis of this episode which is presented after the excerpt, I will discuss whether Cecilie was breaking this task into easy manageable bits which were the tasks Svend solved while she was the one exploring the mathematics and thus handing the solution over to the student, or did Cecilie in this episode give Svend the necessary support for him to solve the task?

When studying the excerpt below, I see a teacher who spaced out the amount of help needed for him to do the task with her help. She structured student's thinking by referring to prior knowledge (7) and what he had just done with numbers (13). By referring to the sum of the numbers one to hundred that Gauss did when he was little,[39] she linked the mathematics they were now doing to history. The general proof of what the sum of two successive triangle numbers was linked to arithmetic in turn 11. By relating to what he had done with numbers (13), Svend suggested a preliminary incomplete formula for a random triangle number. In turn 20 Svend tried to tell Cecilie how he understood this by linking to numbers, however, Cecilie interrupted (21). Apparently Svend succeeded in solving the task on his own after the help he had received until turn 25 because the next time Svend asked Cecilie for help he was working with another task.

Excerpt 22, Cecilie Jan 21st episode II-2

Nr	Who	What is said	Comments
1	Cecilie	Now you can go on with triangle numbers.	
2	Svend	I thought I did…	
3	Cecilie	There it is, triangle numbers are built up as 1+2+3+4	Highlights key aspect of the task

[39] They had done the sum 1+2+3+…..+98+99+100 in class the year before

4	A girl	We had it in grade 8	
5	Cecilie	Yes, We have had some of it before	
6	Svend	Oh yes, it is like…	
7	Cecilie	It is the row of numbers Gauss solved when he was little. How adding 1 to 100 for example?	Reminds bout prior work
8	Svend	It is like hundred times hundred and one...	
9	Cecilie	Yes, divided by?	
10	Svend	Divided by two	
11	Cecilie	Yes, What if the last number in the row is n?	Structures student's thinking
12	Svend	Hm?	
13	Cecilie	If the last number is a hundred, it makes hundred times hundred and one divided by two as you said, but if the last number is n, a random number n, how is the formula then?	Links to numbers Structures student's thinking
14	Svend	It becomes……………… $n \cdot (n+1)$	
15	Cecilie	Yes?	
16	Svend	Divided by two	
17	Cecilie	Yes!	
18	Svend	Is that a triangle number?	
19	Cecilie	Yes, that is a random triangle number	
20	Svend	I think I remember it because if we have triangle-number number four…	Thinking aloud
21	Cecilie	Yes, and then you have,… But how will you formulate, one of the tasks have two successive triangle numbers. If that is one triangle number, what is the triangle number before that?	Structures student's thinking by highlighting key aspect
22	Svend	It is one less, it is n-1.	
23	Cecilie	Correct! Then you put in that instead of the n?	
24	Svend	Then it becomes $(n-1) \cdot n$	
25	Cecilie	Yes, very good. If you add those two, then you can solve the task algebraically. And then you can do it by making a figure too to see it.	Praising and synthesising the work

Since Svend probably solved the rest of the task on his own (he did not ask for further help), I suggest that the way Cecilie interacted with Svend here was spacing out of the amount of help needed for him to solve the task. The way she linked to what he had done on his own already and the way she structured his thinking provided a bridge between Svend's existing knowledge and skills and the demands of this task. Svend succeeded in solving the task based on the support from Cecilie. When teachers serve to provide such a bridge Rogoff (1990) called it "Guided participation" emphasising the difference between structuring the task into meaningful sub-goals which does "not focus on breaking the task into minutely ordered steps to be mastered in a lockstep fashion" (p.94). This can also be seen as the aspect of tutoring termed as "contingent" instruction in Wood (1998) which means spacing out the amount of help needed (according to the student's ability) for the student to solve the task. Both contingent teaching and guided participation are forms of "scaffolding" which in socio-cultural terms means how the teacher makes the task manageable for the child within the zone of proximal development. Jaworski (1990) sees scaffolding from two points of view; one encouraging the student's dependency of the teacher (a crutch) and another providing support. In this episode I see the ways in which the teacher related to previous work (turn 7), structured student's thinking by relating to numbers (turns 11and 13) and highlighted key aspect (turn 21) as support rather than a crutch.

<u>Show that 1+3+5+7+…… (2n-1) is a square number</u>
About 15 minutes later, Cecilie interacted with Svend again, now it was about the next task on the list to be solved. (Since Svend did not ask about the general formula for the sum of two successive triangle numbers, I assume he had finished that task on his own). In the analysis of this episode which is presented below (Excerpt 23, Cecilie Jan 21st, episode II-7, page 236), I will show how a bridge was made between what Svend already could do arithmetically with numbers and proving generally with algebra that the sum of successive odd numbers is a square number.

In turn 2 Cecilie referred to what Svend had done when adding the numbers from 1 to hundred and encouraged him to do the same with the sum of odd numbers (turn 6: can you think similarly..?). She thus started to provide a bridge between this task and one he had already solved. In turn 8 she pointed out the difference between the two tasks. In turns 11-13, there seemed to be a miscommunication. Svend was thinking $\frac{(1+7)}{2}$ is four, "*then I know how many numbers I have used*…" (Was he thinking I have four fours?). Cecilie interrupted saying "no" because she was thinking $\frac{(1+7)\cdot 4}{2}$. In this case Cecilie did not try to catch what Svend meant because she was so focused on her method of solving it. However, through what she said, she structured Svend's thinking and *he* apparently caught what *she* meant and thus carried on. Turn 14 created the link between operating with numbers and generalising with algebra. The bridge between what Svend already knew and solving the task algebraically had started being built. Through turns 18 to 25, even if it might seem that Cecilie spaced out too much help, Svend kept saying *yes, yes* both in turn 21 and turn 23. That he answered in terms of numbers in 27 shows that he still was thinking in terms of numbers, but based on the teacher's words his thinking was structured and he switched to algebra and "saw" how the solution was (turn 29).

Excerpt 23, Cecilie Jan 21st, episode II-7

Nr	Who	What is said	Comments
1	Svend	I didn't understand how I ..	
2	Cecilie	What did you do? Did you turn that number row around?	Reminds him about the previous task
3	Svend	Yes, yes	
4	Cecilie	So that you got hundred, ninety nine, ninety eight, and then you summed like that?	Structures students' thinking
5	Svend	Hundred times, ..and then you get	
6	Cecilie	Can you think similarly with that row too, one, three, five, seven? What about trying the same way with one, three, five, seven up to 2n minus one?	Links to what he has done
7	Svend	Because it will become the same, but	
8	Cecilie	Just that now you don't have all natural numbers, you skipped the even ones. Try with for example 1+2+3+4+5, no sorry, 1+3+5+7	Highlights similarity and difference
9	Svend	It becomes eight	
10	Cecilie	Yes,	
11	Svend	And then divided by two. It becomes four. Then I know how many numbers I have used	Student's suggestion
12	Cecilie	No, you have eight, how many pairs of eights?	
13	Svend	Oh yes, like that, four, like thirty two, divided by two	
14	Cecilie	Can you think the same way, just that the last number is 2n minus one? If the last number is 2n minus one?	Links to previous work
15	Svend	Then the first number is (inaudible)	
16	Cecilie	Then the first number will always be one. What is the sum of the two?	(2n-1)+1
17	Svend	It is 2n	
18	Cecilie	Yes and what is the number before 2n minus one?	
19	Svend	2n minus one? Eh 2n…	
20	Cecilie	How much less is it?	Structures student's thinking
21	Svend	Three, oh yes, two, three, three?	
22	Cecilie	Yes! Two n minus?	
23	Svend	Three? Yes, yes	
24	Cecilie	Yes, right, and that is to be added to the second odd number which is?	
25	Svend	Which is two? Then it becomes… no, three I mean. Then it becomes 2n for all!	
26	Cecilie	Yes, so each pair becomes 2n. eh. How many pairs do you have?	Highlights key aspect
27	Svend	Four, four pairs	
28	Cecilie	But if you have n, 2n minus one?	Generalising
29	Svend	Then I have times n, n squared.	
30	Cecilie	Yes, and that shall be divided by	

31	Svend	Two and then I get	
32	Cecilie	Yes, then you have proved it. Very good!	Consent and praising

Show that the sum of two even numbers is an even number

The third episode (Excerpt 24, Cecilie Jan 21st episode II-3) I am presenting illustrates how Cecilie used tokens to show that when you add two even numbers, you can make two piles of tokens with the same height. She thus demonstrated that she made judgements of students' need. This time she showed with concrete materials whereas Svend was challenged to do it algebraically. I also see the lengths of Cecilie's turns in this episode compared to the lengths of her turns in the episodes above with Svend as significant. In the episode with Leif she explained a lot more and she showed him more too. This is in accordance with the findings from whole class parts of lessons where I referred to how the teaching was more characterised by showing and telling in the Socrates group (those not so interested in mathematics) than in the other lessons.

Excerpt 24, Cecilie Jan 21st episode II-3

Nr	Who	What is said	Comments
1	Cecilie	Leif, shall I help?	Offers to help
2	Leif	Yes, what were we supposed to do?	Student does not understand the task
3	Cecilie	Ha, ha. You shall add even numbers and see if you get an even or an odd number. This is an even number. How to show that these two make an even number? Yes, if you pile them on each other like that it will be an even number. And regardless how big of an even number you take, the column will have the same height, meaning that even numbers added will always make an even number	Teacher shows the student that two piles with an even number of tokens in each make two even piles
4	Leif	And then if you add an odd number and another odd number it won't be, I don't know, it becomes quite high, it makes and even number won't it?	Stud thinking aloud
5	Cecilie	Yes, two odd numbers make an even number. You can show it like this way for example. Eh, this is an odd number and this is an odd number, together it will make an even number because they fit into each other. But if you take another odd number, what will it then be?	Shows the student with tokens.
6	Leif	It makes even plus odd it makes….	Thinking aloud
7	Cecilie	Yes, what did you say?	clarifying
8	Leif	Even number plus odd number is odd number	Right answer
9	Cecilie	mmmm	

Formulae for odd and even numbers

Several students had difficulties expressing even and odd numbers algebraically, which also was a task Jan 21st. In one episode dealing with that, (Excerpt 25, Cecilie Jan 21st episode II-1) Cecilie demonstrated that she spaced out the amount of help needed for the student to proceed with the task. The student suggested *n* to be an even number and thus *n-1* or *n+1* to be an odd number. By presenting an example with numbers for which it did not turn out to be right (2), and then give a more in depth explanation of what a formula is supposed to be in turn 4 (*if you put in 1, you will have 2,…*) Cecilie structured the student's thinking and thus opened up for her to find the formula.

Excerpt 25, Cecilie Jan 21st episode II-1

Nr	Who	What is said	Comments
1	Student	If n is even, then an odd number is n-1 or n+1	
2	Cecilie	But imagine that you put one in here (in the formula). If you put 1 you'll get 1, if that is your formula for even numbers. You need a formula	Exemplifies that it becomes wrong
3	Student	Oh yes, you need a formula for even numbers first	
4	Cecilie	Yes, you must have a formula so the formula gives 2-4-6-8. If you put 1, into it, you'll have 2.	Explains how a formula shall work
5	Student	Oh yes	

The last episode I will present was with two girls who had showed with tokens that the sum of even numbers was an even number and the sum of odd numbers was an even number. Having problems expressing the results algebraically, Cecilie offered to support them (Excerpt 26, Cecilie Jan 21st episode II-4). In turn 1 Cecilie presented the same account of a formula as she did in the episode (II-1) presented above, and when girl 1 gave a wrong answer (2), Cecilie exemplified with numbers why it was wrong. Then in turn 4 she emphasised what the formula should give. The girl suggested that it was doubled (5), however she still had difficulties expressing that algebraically as she suggested *n squared*. Again Cecilie exemplified with numbers why that was wrong before the first girl suggested *n times two*.

Excerpt 26, Cecilie Jan 21st episode II-4

Nr	Who	What is said	Comments
1	Cecilie	Can you make a formula giving these numbers? If you put in 2 it makes 4?	Highlights key aspects
2	Girl 1	n+2	
3	Cecilie	No, because if you take, *if n is three you would have got five.* So what are you supposed to do with the n instead of adding two?	Counterexample
4	Girl 1	If n?	

5	Cecilie	If n is one, it makes two, if n is two, it makes four, if n is three you will have six. What has happened to the n?	Explains how the formula shall work
6	Girl 1	You have doubled it!	
7	Cecilie	Yes, and what is then the formula for even numbers?	Asks for algebraic expression
8	Girl 1	n times n?	
9	Girl 2	n squared!	
10	Cecilie	No, *it is not multiplied by itself, that had made nine*	
11	Girl 1	No, n times two!	
12	Cecilie	Yes! That is what an even number is	

These last two episodes demonstrate that students on this level had difficulties expressing generalities algebraically. Even when a girl expressed in words, *you have doubled it,* she still did not express it correctly with algebra. As I referred in the section from whole class lessons with Cecilie, one of the main subject elements for algebra is that “Pupils should have the opportunity to [ ] experience how expressions with letters representing variable quantities can be used to formulate and prove general relationships” (L97 p. 182).

Based on the above discussion from individual seat work parts of Cecilie’s lessons, I can summarise my findings as follows:

- How a student worked on the “it” demonstrated that there was not a common agreement of what “it” was.
- An exploring activity turned out differently from what was intended for most students in the enacted lesson.
- Cecilie took students’ mathematical abilities into account when giving support during individual seatwork. “Clever” students were given different support (challenged) than those “not so clever”.
- Cecilie structured students’ thinking by illustrating, questioning, presenting counterexamples and highlighting key aspects of the task. A bridge was created between what the students had done earlier and the task to be solved.

A portrait of Cecilie

> Many people find this exciting. It is like climbing a mountain. Why climb on the top of a mountain? Yes, because the mountain is there. It is the same with the prime numbers. It is very exciting (Cecilie about mathematics to her class Jan 14th 2004).

I have chosen this statement as one that really characterises the teacher Cecilie I met during my period of research. She demonstrated a genuine interest for mathematics and the students gave the impression of looking upon her as a teacher who had mathematics as her prime interest. Her interest for mathematics influenced some of the students’ views on mathematics; that mathematics is a lot more than traditional school mathematics.

Characteristics of Cecilie's teaching

In the table "Cecilie" I present the characteristics I found most prominent with Cecilie's teaching after having done the fieldwork and analysed the data collected. The characteristics are presented in the left hand columns. The columns in the middle contain extracts from conversations as evidence illuminating the characteristics. The columns to the right contain examples from the classroom observations which illuminate the characteristics I am presenting. Since my first research question (How are teachers in their mathematics teaching practice responding to L97's recommendations?) is to investigate how teachers are interpreting L97– both in terms of what they say about it and what they do in the classroom, quotations from L97 dealing with the same topic as the characteristic are written in italics in the row below the characteristic presented.

Cecilie

<table>
<tr><th>Characteristics</th><th>Conversations</th><th>Classroom observation</th></tr>
<tr><td rowspan="3">Cecilie expressed a genuine interest for mathematics as a science and for mathematics history. This interest she shared with her students</td><td rowspan="3">Cecilie said she read books about mathematics history and mathematics as a science. She used notes from these readings when preparing lessons (Jan 21st, post). She said that L97 gives her freedom to teach more according to her interest.</td><td>She linked school mathematics to mathematics history. Euclid and his elements, Fermat, Euler and Gauss (among others) are referred in her lessons (Jan 14th, Jan 21st, Jan 28th).</td></tr>
<tr><td>In the lesson Jan 21st she talked about how mathematics as a science differs from other sciences.</td></tr>
<tr><td>March 10th she gave a lecture about "history of equations and number systems".</td></tr>
<tr><td colspan="3">L97: It [mathematics] is a science, an art, a craft, a language and a tool. [ ]. Mathematics as a school subject seeks to mirror this breadth and this development</td></tr>
<tr><td>Cecilie linked school mathematics to students' everyday life, to the outside (real) world and to social science.
She rarely used textbooks, and the topics she was teaching were often not topics being those of typically school mathematics.</td><td>She said she liked L97 but not the textbooks following it (Jan 21st, post). She searched on the internet to find examples from real life.</td><td>In the statistics lesson March17th she used only examples from real life which she had found in the newspaper and on the internet.
In connection with leap year (Feb 29th) she had a lecture about the calendar in the lesson March 3rd.</td></tr>
<tr><td colspan="3">L97: The syllabus seeks to create close links between school mathematics and</td></tr>
</table>

<table>
<tr><td colspan="3">mathematics in the outside world.</td></tr>
<tr><td rowspan="3">Cecilie expressed her concern for clever students and she thought it was important to give them sufficient challenge.</td><td>Cecilie said that the clever students had been neglected and that they need to be more challenged (FG 3)</td><td>Cecilie kindled the clever students’ curiosity about mathematics (Jan 21st and Jan 28th)</td></tr>
<tr><td>She felt that she had succeeded in inspiring the clever students (FG 4)</td><td rowspan="2">Cecilie’s teaching in whole class was often in dialogue with one or two of the clever students (Jan 14th, Jan 21st, Jan 28th) and many of the other students did not pay attention</td></tr>
<tr><td>She consciously did not show with tokens because then the clever students would be bored (Jan 21st, pre)</td></tr>
<tr><td colspan="3">L97: Pupils who are capable of tackling difficult problems can be given assignments which go beyond what the curriculum indicates. All pupils must be given opportunities to participate in interesting activities</td></tr>
<tr><td>Cecilie gave students support according to their abilities</td><td>Cecilie said it was unsatisfactory having students with all levels of abilities in the same class</td><td>During individual work Jan 21st Cecilie was providing a bridge between students’ existing knowledge and the demands of a task to be solved by spacing out the amount of help needed</td></tr>
<tr><td colspan="3">L97: The teaching of mathematics must be attuned to the abilities of individual pupils who must be given tasks which they find meaningful and are capable of carrying out.</td></tr>
</table>

Table 13, Characteristics of Cecilie’s teaching

The characteristics presented in this table are consistent with regard to what Cecilie said in conversations and what I observed in her classroom practice. Thus with regard to these characteristics I found a coherence between Cecilie’s beliefs about mathematics teaching and her classroom practice which is part of an answer to my third research question: *How are teachers’ practice in the mathematics classroom related to their beliefs about teaching and learning mathematics?*

In addition to these characteristics for which I found coherence between beliefs and practice by presenting evidence both from conversations and classroom observations, I found utterances that were not consistent across what she expressed as her belief and what I saw in the classroom. One such utterance Cecilie expressed was her belief that *students learn mathematics better when they can find out things themselves through exploring activities rather than learning ready made results.* This belief is in accordance with L97’s recommendations. In Chapter 5 I argued for L97 encouraging exploring activities and investigative work and that it puts weight on the process aspect in mathematics rather than on the tool-box aspect (traditional aspect). Cecilie expressed her agreement with this in conversations which is also emphasised by her estimation form. Based on what she said in conversations and on her estimation form this was an expression of that Cecilie believed in the reform and that she also wanted to teach according to it. She prepared lessons with exploring activities. However, from the analysis of her classroom practice I claim that there was a mismatch between the mathematical activities Cecilie prepared and students' abilities, and that only the cleverest students did some exploring while Cecilie was mostly leading the students through the exploring activities. She did the exploring mathematics while the students were answering easy manageable closed questions. Out of the six lessons I have analysed in detail, she was teaching this way from the board 70-75% of the time. The enacted lessons turned out differently from what she intended. Her classroom practice with the students in her class having different abilities was a *constraint* for implementing this part of the reform.

Based on her claim that clever students have often been neglected in school, one of the characteristics presented in the table above is her concern for the clever students. My analysis of the classroom practice suggests that she did succeed in inspiring the clever students and I also claim that her concern for the clever students seemed to be at the expense of those not so clever. This emphasises Cecilie's expression that it is *unsatisfactory having all levels in one class*. I noticed that Cecilie during individual seat work offered students help according to their abilities. But I also saw that many students in whole class did not pay attention while she was lecturing "exciting" mathematics from the board, and I have indicated earlier that there was a mismatch between mathematical focus and some of the students' abilities. The question to be asked about this, which is beyond the scope for my thesis to answer, is how it at all is possible to implement L97's recommendations in mixed abilities classes as we have in Norway. Cecilie met this challenge partly by dividing her two classes into two groups according to their "interest for mathematics" once a week. I observed one lesson in each of these groups and I found that the discourse in the lesson with the group of students not so interested in mathematics was characterised by more showing and telling than in the other lessons. Both in the Wednesday lessons and in the lesson with the interested students Cecilie was leading the students through the exploring activities by asking easy manageable closed questions. This can be an indication that it is not so easy to elicit genuine student thinking and exploring activities in a whole class, and that implementing a curriculum reform is not a straight forward process although the teacher believes in the reform, that she wanted to implement it and prepared her lessons according to it. It shows that implementing a curriculum reform is a hard process despite the teacher's intention doing it. In

Chapter 2 I have reported similar findings from the literature (Broadhead, 2001; Norton, McRobbie, & Cooper, 2002; Prawat, 1992; Remillard, 1999; Williams & Baxter, 1996).

Stigler and Hiebert (1999) gave some issues to be considered in studying teachers' implementations of curriculum reforms: Teaching, not teachers, is the crucial factor; Teaching is a cultural activity, and therefore resistant to change; Although teachers claim that they are responding to curricular reforms, they only do it superficially. It is therefore necessary to invest far more than we now do in generating and sharing knowledge about teaching. This I discuss further in the final chapter, Synthesis and Conclusions.

8. David

David very generously put a lot of his time into my project. He came to school half an hour early every Wednesday I was there to have a conversation with me before school started. Very often we also talked during lunch. The analysis of the conversations which is presented below is based on eleven conversations: Pre-lesson and post-lesson conversations Jan 14th, Jan 21st, Jan 28th, Feb 11th, March 3rd and post-lesson conversation March 10th. David's educational background was from the University, with a master degree in Biology, and he had been a teacher for more than thirty years. He gave the impression of enjoying talking about mathematics, about his students and about his teaching. According to what he said, he liked mathematics; he liked his students and enjoyed teaching. He demonstrated a professional pride both as a mathematician and as a teacher. David also said that he liked the company of his students, and especially the students in the class he had this year. They had been on several trips together from which he had many nice experiences. He excited told me about a trip he was going to take with them in June this year, something he was really looking forward to

Based on what Cecilie and David said, I had the impression that it was Cecilie who had persuaded David to come with her to the focus group meeting (FG3) in October 2003. In the analysis of this focus group I indicated that David positioned himself in the group by expressing scepticism and reluctance to L97. When I asked David if I could do classroom observations with him, he said yes very positively and that I was very welcome. I got the impression that he felt my request as an acknowledgement of him as a teacher based on what he had said in the focus group despite his expressed reluctance to L97.

Analysis of conversations with David

Based on the coding and analysis process of the conversations with Bent, I wanted to see if the codes *Conditions for possibilities of learning*, CPL, *Mathematical focus, MF* and S*tudents' abilities* SA, could be used also in the analysis of the conversations with David. I started reading through the transcripts of the conversations making marks in the margin. I did this twice before I wrote down keywords, key-sentences and key issues from each conversation in English in a new document. When doing this I put CPL, MF and SA in brackets for each issue. However, it was not that what he said either could be placed in CPL or MF or SA. I found that the categories were not mutually exclusive but that they overlapped in the analysis of David as they did in the analysis of the two other teachers. What was said would fit into more than one category. One example was when David said, in the first conversations we had Jan 14th, that he tried to challenge all students according to their abilities. He told me about the preparation for the lesson which was about equations with two unknowns and that he was aware that there were four or five ("weaker") students in the class who "should rather use their energy on other tasks than learning equations with two unknowns". He had therefore prepared a work sheet with an answer sheet (fully solved tasks) for them to work with. These students, he said "will be happy to get [the grade] "three" in mathematics". This way he created conditions for possibilities for learning (CPL) for students who according to their abilities (SA) should rather practice on tasks with different mathematical focus (MF) which they were more likely to manage on the final exam.

As within the analysis of the other two teachers, the theoretical thinking which is underpinning the curriculum has guided my analysis of David. David very explicitly expressed his view on L97, both with regard to mathematical focus and working methods emphasised in it. Critical comments about the curriculum like the ones he presented in the focus group were also expressed in the conversations I had with him. For example during our pre-conversation Jan 28th David showed me some transparencies with statistics that he had taken from magazines that he was going to present in class to illustrate use and misuse of statistics. Statistics is a topic on which more weight is put in L97 than in prior curricula, I therefore said:

BK:	Did you use L97 when you.. ( )
D	(interrupting): No, I never leaf through that book
BK:	You never leaf through it?
D:	No, that I can't stand.
BK:	But this was not in the M87?
D:	No, that is right, but I do this rather because it is fun and not because it can be read in L97 (Jan 28th pre).

This emphasised the impression he gave in the focus group of his reluctance towards L97. Even when some of his opinions about mathematics teaching and learning coincided with L97, he did not reason his point of view from L97 but rather that it was reasonable from his point of view. He said he should have used more play and exploring activity, however, not because L97 encouraged that. David said that he challenged his students according to their abilities, but not because L97 said so. His scepticism to the curriculum was also reflected in his expressions about mathematical focus.

In this part I will start with mathematical focus, since this was so central to David's teaching. In the preceding chapters (Bent and Cecilie), I started with CPL. David very explicitly stated his view on mathematics and on mathematics as a subject in school. I have therefore chosen to start this part with "Mathematical focus" where I present an account of how David expressed his view on mathematics to be taught in school. Then I discuss how he said he created conditions for possibilities of learning the mathematics he emphasised through his teaching style. Finally, I address the category dealing with students' abilities including aspects of differentiating, how students' learn mathematics and issues about different mathematical focus according to students' different abilities.

Mathematical focus

As indicated in the introductory chapter, the authors behind the mathematical part of L97 wanted to break down the division between school mathematics and mathematics in society outside school; they wanted to put more weight on the meaning of computational operations, use of strategies and ability to choose and judge different methods, and less weight on exercising skills and procedures. With regard to algebra, they warned against meaningless calculations with symbols.

In Chapter 5 I offered a theoretical interpretation of L97, both with regard to working methods and the mathematical focus reflected in it. There I argued that L97 emphasises conceptual understanding and relations among concepts and that students are encouraged to see structures within the subject. According to research done within mathematics education, conceptual knowledge exists in a network which is rich in relationship whereas procedural knowledge focuses on rules and exercising skills (Brekke, 1995; Hiebert & Lefevre, 1986).

I will now present an account of David's expressed view on mathematics both with regard to what to him seemed to be important aspects of the subject and also his view on the mathematical aspect which according to him was reflected in L97.

"Classical Mathematics"

When studying the key issues that emerged from the transcripts of the eleven conversations on which this analysis is based, I noticed that David was very focused on mathematics as a subject matter. Already, in the focus group (FG3) in which he participated, he expressed his concern that L97 had not so much "classical mathematics"[40] as earlier curricula. His concern for classical mathematics was apparent in the conversations too. According to David, classical mathematics incorporated algebra, equations and functions which had not necessarily to be presented in a practical context and also classical geometry tasks where you have to do a construction with compasses and ruler. He expressed a dilemma how much time to spend working with algebra and was concerned that he might be putting too much effort into algebra compared to how it is weighted in the final exam, in which is has little weight. But because of

[40] David's words: "Klassisk matematikk"

the demands from upper secondary school with regard to algebra, he considered it necessary to emphasise algebra in lower secondary school the way he did. Also the textbook he used, which was approved according to L97, put much weight on algebra[41]. Therefore he looked upon the relations between L97, the final exam and textbooks as if there was a kind of lack of consistency. He said:

> I do not have anything against algebra. And the students mastering algebra find it all right. There is something fascinating with what is abstract too. I manage to make the students experiencing organic chemistry and formulae fun (Jan 14th pre).

He thus expressed both his own and some of the students' joy in working with algebra and formulae. The weight he put on algebra and that "the classical mathematics is neglected on the final exam" was a recurrent theme through all conversations we had. In the post conversation Feb 2nd, he said: "If it had been to teach only for the exam, most of the algebra could have been dropped. However, they will get it in upper secondary school ".

Since the exam lasts for only one day out of three years in lower secondary school he said, he would not take the exam too much into account when deciding what kinds of mathematics to focus on. This is not consistent with what he said about the exam when we talked March 3rd about exploring activities and play in mathematics. He said: "I think we are too much steered by the exam", and he gave the exam as one reason for not playing and experimenting in his mathematics lessons in grade 10, which he said he did in grade 8 and 9. Thus he used the final exam as a constraint for not having time for play and exploring activities. However, when talking about the weight he put on algebra he said: "the exam is only one day".

One reason I might suggest for his concern for algebra is his long experience as a teacher; that he knew algebra very well. He used "old" tasks from earlier exams, and he had experienced success in teaching algebra. Another reason I suggest is that algebra is "real mathematics" and traditionally it has been prestigious teaching mathematics and especially algebra, which he liked: "It is the play with algebra and equations that is so fascinating" (Jan 14th pre). This suggests that he mastered algebra very well and had adequate knowledge in mathematics for the purpose of teaching the subject. Yet another suggestion is that because he did not have so much experience preparing exploring activities he chose the easiest way and rather dealt with algebra and reasoned his choice by referring to the demands students will meet in upper secondary school. All these suggestions mirror that David was influenced by the socio-

[41] The teachers can choose what textbook to use. Other textbooks than the one David used, do not have so much "classical mathematics".

cultural setting in which he had been a teacher for many years. Teaching is a cultural activity and changing cultural activities is a long process. Therefore, teaching is found to be resistant to change (Stigler & Hiebert, 1999).

Procedural mathematics and the focus on the method

David focused much on the procedural aspect of mathematics and the importance of using the right methods and rules when solving mathematical tasks. He frequently used the expression "falling into the trap" [42] about students who made presupposed mistakes. The two first lessons I observed with David were about equations with two unknowns to be solved algebraically and/or graphically. Thus the mathematical focus discussed in our conversations Jan 14th pre and post and Jan 21st pre and post, was equations with two unknowns. David was very concerned that the students should learn the method and he clearly outlined the levels of difficulties in sets of equations with two unknowns:

> They will get equations served[43] where they can just add and then one of the letters disappears right away. That is the simple level most students now manage. [ ] The next step is that you have to multiply one of the equations with something to make a letter disappear (Jan 14th pre).

He also said: "The goal for this lesson is to master the addition method on equations with two unknowns" (Jan 14th pre). He said that he found the substitution method cumbersome so he would not focus on that. With regard to subtracting he said he actually did not care, however, he preferred them to add because then they did not have to worry about changing signs. His expressed preference to adding and not subtracting is consistent with an episode in his class: a student suggested multiplying the two equations in a way which would have led to subtracting being the right thing to do for "a letter to disappear". However, David recommended multiplying in a way that would lead to having to add the two equations to make a letter disappear (see Excerpt 37 page 285).

Before the lesson Jan 14th I asked him if he thought that the students would have any special problems with these tasks, and he answered:

> No, I think they will do all right because the method is so clear. It is obvious what to do. The job is just to find out what to do with the two equations to get rid of one of the letters. So when they have exercised that for a while, I believe because the numbers are so simple, that those I expect to master it will do so. However those who are weak in mathematics will also manage in the classroom because they do it over and over again, but they will lose it. They won't digest it (Jan 14th pre).

Saying this David demonstrated that he believed that learning the method (which is clear) was the way to learn how to solve equations with two unknowns. In the same conversation he said that it is valuable

[42] Norwegian: "gå I fella"
[43] Norwegian: "servert", indicating that the students passively receive something

to exercise methods and skills, and that L97 puts too much weight on practical application of mathematics. The last part of the quotation above demonstrated awareness of students' abilities and the relation between working methods and students' different abilities. He expected the weaker students to manage in the classroom because of exercising the method. However, he did not expect them to gain conceptual understanding ("they won't digest it") and thus forget it ("they will lose it"), and he expressed an acceptance of that. Thus the exercise of methods would not act as scaffold since they would lose it.

On Jan 21st David's class was going to work with graphical solutions of equations with two unknowns and David said he would put weight on the method: "They shall learn how to solve equations with two unknowns without any mishmash"[44]. In this lesson he wanted to relate to functions which the students had seen before. He said he would start by writing the two equations on the board as functions "*y=*" to see if they would recognise what he wrote as a function. Then, he said, he wanted to see if the students could see that to solve the same functions algebraically they would have to move the *x'es* to the same side as the *y's* and then multiply and add to get rid of one of the letters. His focus was on the method and thus technical and procedural; however here I also see a structural aspect (link between different mathematics entities and concepts), because, according to what he said in the pre conversation, his intention was to make connections between equations and functions in this lesson.

The conversations we had both before and after the lesson Feb 11th, when reviewing Pythagoras' theorem was the topic, were also characterised by talk about exercising skills and procedures. However David was concerned that those who knew Pythagoras' theorem should not have to exercise standard tasks where you get two sides "served" and then shall calculate the third. This indicates an awareness of students' needs according to their abilities. He said the clever students should rather practice tasks where for example one side was three times the other; the third was given and then they could calculate the sides. "And they need to understand that 3x within a bracket squared is 9 x squared and not 3 x squared". He said that when students made such an error, they were "falling into the trap".

During this lesson one of the students asked me for help. The task was a right angled triangle where one side was 8 and another was three times the hypotenuse. The student had written: $8^2+x^2=3x^2$, and had thus "fallen into the trap" described above. I asked this student how *long* the sides were. She answered 8^2, x^2 and $3x^2$. Again I asked about the length

[44] He said "mix max" in Norwegian, which can be interpreted as: without any fuss

of the 3x side and she hesitantly tried x squared. Referring this episode to David in the post-lesson conversation Feb 11th and asking him what he would have said to this student, he said that obviously this student had not understood:

> You cannot take the squares and show them every time either[45]. In a way there is a balance between understanding and the mechanical here. [ ] It is obvious that if you had taken the squares even more thoroughly and more times, may be a few more had understood, but it is *a balance* how much time to spend on it. And it does not matter that much if some do it mechanically (Feb 11th, post, my emphasis).

The suggestion he thus came up with as an answer to my question was to draw the squares on the three sides and explain Pythagoras' theorem that way. However, he was not sure if it helped. Based on his long experience as a mathematics teacher, he meant that somebody would never understand. That is in line with what he said about students "not being able to digest it" (quotation page 248) with regard to solving equations with two unknowns. This suggests that he sometimes considered that doing mathematics mechanically was better than spending too much time trying to make students understand. His expressed "balance" in the quotation above between methods and conceptual understanding reveals the view that, for the weaker students, it is better to focus on the method than to spend a lot of time working for conceptual understanding which might not be achieved.

Conditions for possibilities of learning and David's teaching style

The most striking feature of how David talked about his own teaching was his belief that students would learn best when he explained. This is coherent with what he said in Focus Group 3. He referred to a survey he had had in his class where most students had said that they learn best when David explains followed by individual work on similar tasks (Jan 14th pre). Throughout our conversations the focus on explaining came up several times:

> I think that generally students learn best and quickest if they have things well and clearly explained; in a manner and in a language which make them follow and understand. I refuse to acknowledge that that is an outmoded teaching/learning method. People around me can mean what they wish about that really, but I believe that if you are good in explaining you can make people learn that way. I am quite convinced about that. I am sceptic of that students shall find out things by themselves. Some will probably find out very little (Jan 14th post)
> I like being a conveying teacher. That is not politically correct now, but I think that is fun and I feel that I succeed in it and that I get the students to follow me (March 3rd post).

Also implicitly, when David told me what he was going to do in a lesson his conveying style of teaching came through. Expressions as "show and

[45] Showing by drawing a square on each side of a right angled triangle

tell", "I will manage to get them to understand", "I have managed to sort out for them", "it is easy to show them" were frequently used. These expressions indicate that he looked upon the responsibility for students' learning as his. Thus these were ways in which he created conditions for possibilities of learning. When referring to a test on which his students had performed well he said: "I think I have managed to sort out the different types of percent calculations so they can really see what numbers to be compared" (Jan 21st pre) which reflects a view that he had made the students learn, but also that he had created conditions for their possibilities to learn. Continuing, he said that some students "still fall into the trap and do not see what to take the percentage of ", which indicated a right/ wrong view on mathematics.

When talking about individual seatwork David said that he wanted to explain to the students how to solve a task when they asked for help. When his students were working on their own I observed that David was busily rushing around to meet the students' constant calls for his help. When I asked him if he sometimes thought that students asked too early for help, he quite proudly said that he wanted to give them good service and he thought he managed that. "I keep it going for them, you know"[46] he said, and he expressed that he liked that.

This style of teaching is in line with the findings of Stigler and Hiebert (1999) in US classrooms. The teachers they studied indicated a view that they wanted to avoid confusions among students. If the students had problems, the teacher gave them support as soon as possible. The teachers in their video study sometimes pointed out possible problems before they even had occurred, so the students could avoid them. This was also the case for David; he did not want to focus on the subtracting method when solving equations with two unknowns because he wanted to avoid the problem with negative signs and he pointed out possible traps for the students to fall into before they had done so.

During our post lesson conversation Feb 11th I asked him about his "show and tell style". I had been studying some of the recorded lessons and had an impression that he was showing and telling the students what to do very quickly when they asked for help. We were talking about the episode described earlier when a girl had squared $3x$ and got $3x^2$ and asked me for help and I posed a question back asking her about the *length*. I told David that I like to challenge students when they ask and not to give them the answer right away. He then said: "yes, I try that too, leading them forward in a way".

Through the quotations above and also by referring to his survey in the class, David explicitly said that he was aware of his conveying style

[46] In Norwegeian: Jeg står på for dem, vet du

as a teacher and that this style was probably not what he was supposed to do ("not politically correct", "people around me can mean what they wish"). The quotes indicate that he felt the socio-cultural pressure from the reform ideas; however, he also demonstrated his confidence in this teaching style. In the analysis of the focus group (FG3), in Chapter 5, I discussed the different teachers' personal and professional confidences, indicating that David demonstrated his confidence with regard to his teaching through what he said in the focus group. The following quotation from one of our conversations (which was an answer to my question if my presence in the classroom would make him feel uncomfortable or disturbed) emphasises his level of personal and professional confidence:

> No, no. But of course I am not perfect, nobody is, but I have the feeling that my students throughout the years have been content with what I have been doing. I have received clear feedback for that. Not only what I am doing from the board as a math teacher, but also my behaviour and attitude towards the students. So I am quite confident that I am doing well (Jan 14th post).

He said that he tried to lead the students forward towards a solution. This expression indicates that he looked upon giving students the amount of help needed so they could carry on, as a teaching strategy and thus as a condition he created for students' possibilities of learning. The question is if his help acted as funnelling rather than support for the students to proceed further.

When we talked about equations with two unknowns he said that his experience was that some of the weakest students only managed with his help. The question is if it is possible to support in a way which makes the student not forget when s/he shall carry out mathematics on his own. L97 focuses on the students as knowledge producers and the teacher as the person who facilitates knowledge production. The challenge for the teacher is then where to draw the line between what is leading or funnelling the students towards a solution and guiding students' production of knowledge.

Students' abilities

Based on what David said in our conversations about his lessons, he demonstrated that he took students' different abilities into account when planning his lessons. In the discussion of "Mathematical Focus", I indicated David's awareness of students' different needs according to their abilities, and he expressed knowledge about what individual students would master: "those I expect to master it will do so" (Jan 14th pre, for whole quotation see page 248). He often prepared other tasks for the weaker students, and thus he created possibilities for them to learn something as he expressed it, more useful, which could help them getting the grade "three" in mathematics. He said that to get a "three" in mathematics they needed to be able to solve tasks like: "reading x and y from a graph, extend or reduce a recipe, find out how much something costs and simple percentage calculations". He showed me some tasks on which the 4-5 weaker students should work and said that it was no use for them trying to solve the most complicated algebra tasks which they would not manage anyway.

David said that when he was reviewing a test from the board, he consciously mainly addressed the cleverest students and not going through the easiest tasks because that was too boring. He expressed his concern for the clever students and that they often become losers in the kind of school one is supposed to have today. Through this I see a built in criticism of our school system. He referred to an article he had read in the newspaper the day before which expressed the same view.

David used the expression "falling into the trap" when students used a method wrongly or when they added 4 and 5 before multiplying by 3 in the expression $\frac{4+5\cdot 3+(10+4)}{2}$. Also when they were taking the percentage of the wrong proportion or wrote cm instead of cm^2 he said that they fell into the trap. It surprised him that they could write cm instead of cm^2 because he thought that cm^2 ought to pop up automatically when they saw the word "area". But he believed that it had something to do with being mature, and that some students would always fall into traps because they were not mature enough to understand. This is in line with what he said to Tom (a younger and less experienced teacher) in the focus group (FG3) – "you have to live with that as a teacher that some students will never understand the difference between $2x+2x$ and $2x\cdot 2x$". This indicates a view on a students' ability as being fixed and not as subject for development as teaching goes on. By giving students tasks according to their abilities David demonstrated a view that students have different abilities to conceptualise and thus need different learning experiences.

David's beliefs about teaching and learning mathematics

Based on the above analysis of the conversations I had with David, an overall impression of his beliefs about teaching and learning mathematics on the one side was that students' learning is dependent on the teacher's ability to show and tell or to explain. On the other hand he believed that no matter how well the teacher explains, some students will never learn. He related this to being mature. I suggest that this indicated a view on students' abilities as something constraining the experience through which they learn.

In my theoretical interpretation of L97 in Chapter 5, I argued that L97 reflects a constructivist view on teaching and learning mathematics and also a view reflecting socio-cultural theories. One pedagogical consequence of constructivism is that the use of telling has been toned down

(Lobato, Clarke, & Ellis, 2005). David explicitly said that he thought telling and explaining was a good teaching method. According to Lobato et al. the use of telling is undesirable from a constructivist view when it minimises the opportunity to learn about students' mathematical ideas and strategies, emphasises the teacher's authority, only focuses on the procedural aspect of mathematics, communicates that it is only one solution and closes down any attempts to explore. Hence David's commitment to telling is not in line with the aspect of telling Lobato et al. found desirable from a constructivist view.

David was aware that telling or conveying was not the "politically correct" thing to do. Although by saying this and thus demonstrating that he knew that telling was not the most encouraged teaching strategy in L97, he kept on telling and explaining because he believed in it as an effective teaching strategy. This is reflected in what he wrote about ideal teaching which can thus be seen as a validation of what he said in our conversations. One striking feature about what he wrote about ideal teaching was his emphasis on its independence from curriculum. That is in line with the kind of ignorance I claim that he showed to L97, both in the focus group and in the conversations. He wrote:

> Ideal teaching in mathematics requires first and foremost that (independent of curriculum):
>
> - The teacher knows the subject (meaning having mathematical content knowledge)
> - The teacher can lecture /explain / help students on a level hitting the students in a language they understand
> - The ability to motivate the students, create a classroom situation where they feel confident and want to learn
> - Get the students to understand that mathematics has something to do with reality
> - Make the students curious on logic, systems and what is abstract

Through the first three bullet points he characterised an ideal teacher: his/her knowledge, his/her ability to lecture and his/her ability to motivate students and to create a confident learning environment. The third point (the ability to motivate the students, create a classroom situation where they feel confident and wanting to learn) indicates a socio-cultural awareness and a view that learning takes place through participation in a classroom discourse. The last two points are about the students; they shall understand that mathematics has something to do with reality and they shall become curious. David showed that he put the responsibility for students' learning on the teacher.[47] He expressed the importance of creating a learning environment in the classroom.

[47] He used the Norwegian expression "få elevene til å forstå" and "gjøre elevene nysgjerrige" which I have translated into "get the students to understand" and "make the students curious"

When reflecting on David’s expression “*falling into the trap*” when students made errors in mathematics I see a teacher who was asserting a right/wrong view on mathematics and a view that somebody (those who are making the tasks for exams) as rather fooling the students to do a task wrongly than trying to find out what knowledge the student actually has. This view is in great contrast to L97’s where students’ errors and misconceptions are supposed to be grounds for further learning in the subject. Through this expression, falling into the trap, I also see a view that mathematics is remembering certain facts and procedures and not conceptual understanding. This is also in contrast to L97 where conceptual understanding is emphasised rather than exercising skills and procedures.

David’s estimation form

Based on what David said in focus-groups in conversations and in his writing, I perceive consistence. He did not use L97 in his planning, he looked upon his explanation of mathematics as crucial in the students’ learning process and he looked upon himself as a conveying teacher.

His estimation form tells us that he looked upon his own actual teaching as very close to what he looked upon as ideal teaching. The process aspect, explained in the estimation form as “mathematics is a constructive process, doing mathematics means learning to think, deriving formulae, applying reality to mathematics and working with concrete problems” was given 15 points with regard to both his real teaching and ideal teaching, whereas he evaluated L97 20 points with regard to the same aspect. This again emphasises that he believed that L97 puts too much weight on the process aspect and too little on the system aspect which was explained as: “mathematics is a formal rigorous system, doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts”. That he estimated his real teaching with 2 points more to the toolbox aspect (explained as: mathematics is a toolbox, doing mathematics means working with figures, applying rules, procedures and using formulae) than he estimated ideal teaching (and also L97) can be seen as an indication of that he was aware that he focused a little bit too much on rules and procedures and method mastering.

David	Mathematics as a toolbox	Mathematics as a system	Mathematics as a process
My real teaching	7	8	15
Ideal teaching	5	10	15
L97’s view on teaching mathematics	5	5	20

Table 14, David’s estimation form

One interesting feature about David's estimation form is the estimation of both his real teaching and ideal teaching with regard to the process aspect. Estimating his real teaching with regard to the process aspect with 15 points out of 30, and valuing ideal teaching the same with regard to the same aspect, shows that he saw this aspect as the most important out of the three aspects. Valuing L97 with 20 points with regard to the process aspect suggests that he found that L97 put too much weight on that aspect. Estimating his own teaching and valuing ideal teaching both with 15 points with regard to the process aspect is not in accordance with what he said in the conversations. There the focus on the procedural aspect and the method and thus the toolbox aspect was more characteristic. One interpretation of this can be that he looked upon the students' learning of mathematics as a process which he initiated through his lecturing. Another interpretation of the seeming inconsistency between what David said in the conversations and his estimation form can be that he may not have thought about what mathematics as a process implies. As discussed in Chapter 3, inconsistency is seen from my perspective as a researcher. David's practice was consistent with what he believed in, however, the question is what he saw as process-, toolbox- and system-aspects.

Wilson and Cooney (2002) suggested several possibilities why teachers sometimes expressed a belief about teaching mathematics which was not mirrored in that teacher's practice. As an example they presented a teacher who was claiming that problem solving was the essence of mathematics, however, in his classroom only procedural knowledge was emphasised. Rather than claiming inconsistency they recommended considering some possibilities: First, that there was not a viable way of interpreting what the teacher meant with problem solving. In the same way I indicate that there was not a viable interpretation of what David meant by the process aspect in mathematics, even though it was explained in the estimation. A second possibility suggested by Wilson and Cooney was that the teacher did not act according to his expressed beliefs because of "*logistical circumstances*". This was evident in the case of Bent in my study, who explicitly expressed logistical circumstances, the upcoming exam, and time pressure as constraints for not doing more exploring activities. David also expressed time as a constraint in saying that it was more efficient (with regard to time) to demonstrate a certain method for the students to use, than for the students to find out on their own. Also the expressed balance "In a way there is a balance between understanding and the mechanical here" (see page 250 for whole quotation), indicated an issue constraining his decision making. This leads to Wilson and Cooney's third possibility to consider, that the teacher's belief about problem solving could be peripheral to the importance of the procedural focus. In the case of David, the process aspect was not in balance with other aspects found in his teaching practice and can therefore be seen as peripheral to beliefs about aspects as explaining or showing and telling. I will discuss this further in the final chapter, Synthesis and Conclusions, where I relate my findings to how the teachers responded to the questionnaire.

Analysis of classroom observations with David

The lessons I observed with David were 45 minutes lessons. It always took some time for the students to be calmed so the actual mathematics lessons lasted for about 40 minutes. David was very enthusiastic when teaching. All lessons had an opening part which was in whole class and lasted from 5 to 17 minutes. In this part of the lesson, David introduced the topic of the day and described the organisation of the lesson. He often pointed to the final exam and how much time was left and how much of the subject syllabus they had left. He started by presenting examples of what they should work with on the board in interaction with the students. An overview of the lessons from which I present data excerpts is presented below.

Excerpts	Date	Mathematical topic
1, page 261, 2, page 263 6, page 274, 10, page 284 11, page 285, 12, page 286 13, page 287	Jan 14th	Algebra, equations with two unknowns
3, page 264, 14, page 288	Jan 21st	Graphical solutions of equations with two unknowns
4, page 267, 7, page 276 8, page 277, 15, page 290	Feb 11th	Geometry,
5, page 268, 9, page 278 16, page 291	March 10th	Geometry, 30-60-90 triangle

Table 15, Overview of data excerpts from lessons with David

During whole class sections of lessons, when David was teaching in interaction with the students, he was talking most of the time. He posed questions to the students and when they answered, he elaborated the answer before he asked another question. He often linked to previous knowledge and he linked technical terms to common sense. The last parts of the lessons were individual work sections where students worked either individually or in pairs on similar exercises as presented from the board in the whole class section, either from the textbook or from a work sheet handed out by the teacher. During individual work sections, David went over to the students when they called for his help. I noticed a great regularity in David's lessons of which I give a more detailed account in the presentation of "A portrait of David" which is the last part of this chapter. This part of the chapter is divided in two sections: The first on whole class sections of lessons with David, and the second on individual seatwork in his lessons.

Whole class sections of lessons

When analysing the whole class sections of David's lessons I identified several significant aspects of which I will now give accounts provided with evidence from the classroom. I have grouped the aspects according to the categories *Conditions for possibilities of learning*, *Mathematical Focus* and *Students' abilities*. In the first category, CPL, I show how David created possibilities for learning by starting with a monologue in which he presented an overview of the day, followed by a transition in which he invited students to participate, to the lecturing part in which I have identified several teaching strategies. In the next category, MF, I show how David focused on rules and methods both when dealing with algebra and with geometry and that the mathematical focus in his lessons was procedural. Finally concerning students' abilities, SA, in David's lesson, I show how he addressed students differently, and that he encouraged the students to make judgements themselves with regard to what they should practice.

Conditions for possibilities of learning

Here, I include aspects of David's teaching to show how he created conditions for possibilities for learning. First I point out how David started with a *monologue* in every whole class sections of lessons before he invited the students to participate. Next, I refer what I have identified and termed as a *transition question*. Such questions initiated a transition from the monologue to the lecturing part in which he invited the students to participate. Third, I present an analysis of the *lecturing* parts of David's whole class sections of lessons in which I focus on the teaching strategies David used to create conditions for possibilities of learning.

Starting with the *monologues* from the opening parts of the lessons Jan 14th, Jan 21st, Jan 28th, Feb 11th and March 10th (which lasted from half a minute to three minutes) I will point out some common features, and then illustrate them from David's own words.

- David started with a presentation of the agenda of the day including the mathematical topic they were going to work with and how the lesson would be organised. Thus he directed the students' attention.
- He sometimes had to call on some students who did not pay attention, which he characteristically did in a subordinate aside. This way he managed disciplinary aspects while introducing to-day's work and assignments.

- Studying his use of personal pronouns, which I have italicised, in these monologues allows me to make suggestions about his intentions for the students' and the teacher's roles in the lessons.

We now have to finish these equations with two unknowns. [ ] So far *we* shall now know equations with two unknowns [to a student who is not yet seated]: (you have to hurry up!). *We* shall take a repetition of that type, that you shall do, and then *I shall show* two things. It is in a way the next step, and that is the last step, and that is that *we* have to do some more work than *we* have to do here. Then *I shall show* an example from reality where *we* shall solve problems using equations with two unknowns. These are the two things you shall work with to-day (Jan 14th).

To-day *I shall say* the very, very last with regard to algebra. *We* have in a way come ashore [are seeing the end]. This last part has been quite lengthy. (Yet another having forgotten that we started an hour ago?) [to a student coming late]. Now *we* shall learn the very, very last thing *we* have done with x and y. However, *I claim we* have done it before. *We* shall just use what *we* have done before in a slightly different way (Jan 21st).

Now when *we* are in the position of having finished something, *we* shall do something in common and *you* shall carry on with your work-sheet. Most of *you* are dealing with making diagrams. When the diagrams are finished, it might be a little boring making the diagrams over again, however, now *we* are making one of each kind and those are the kinds *we* can get on the exam which *we* shall know, these four. Afterwards *we* shall have a look at misuse of statistics, which was what *I had planned to show* here [the overhead started burning when he switched it on, so he could not show the transparencies he had prepared. See analysis of conversation before the lesson Jan 28th] and then it is probability. Probability, then there are many exercises with use of dice. And then *I* was thinking that *we* first of all shall play with a dice conjuring trick. And *I* suppose everyone will try and you actually have to work in pairs (Jan 28th).

Two general things first (chatterbox!) [to a student who was talking]. From tomorrow you must manage to bring your compasses and ruler. [ ] In this chapter *I will not follow* the subsequent order in the textbook. [ ] What you shall work with *when I have finished* saying something now are these angles which *we* looked upon last time; adjacent angles, vertical angles, corresponding angles, straight angle. If you don't remember, look up the examples in the textbook. [ ] Then *we* skip some pages in the textbook, and take a quick round on Pythagoras (Feb. 11th).

The main job now is to finish the geometry chapter. And *we* have several things *we* have not learned there (stop that Jenny!) [to a girl who was doing something she was not supposed to] and then there are some things *we* have to repeat and *we* have to finish it after all. Page 165 and what you shall work with now, you shall get a sheet of paper from me where *I have made and written some exercises* on the sheet which you shall do, and some refer to the textbook (March 10th).

I have selected to focus on these quotations from the opening of the lessons for several reasons: By starting the lesson saying what they should do, David immediately included the students in the plans for the lesson so they could know what to expect of it, and he thus motivated them with reference to how far they had come and what they had left of the topic with regard to the subject syllabus. He often referred to the textbook, both with reference to what they had left and what exercises they should do/not do. He also encouraged the students to use the textbook as a reference book if there was something they did not remember. This way he made the students feel comfortable with regard to the work they should do.

These quotations also demonstrate how David dealt with disciplinary issues, how he managed the class. He did not stop the lecturing to reprimand a student, thus he did not make a big deal out of it. He told them in a subordinate aside so the interruption of his lecturing was minimised. In the lecturing part of the Jan 14th lesson from which I present excerpts below, he reprimanded Jenny in the same way. Also when he was helping a student during individual seatwork, he could tell another student in the other end of the room to calm down. He always addressed his students in a spirit of good fellowship and in a friendly way. This emphasises his experience as a teacher and his expressed feeling of success in his work as a teacher.

I have put the pronoun "we" in italics in the quotations from the monologues above. David's intentional use of "we" here seems to be not only to draw the students into his plans for the lessons but also for the students to feel complicity in these plans. Also when he used "we" in terms of what they had done and what work was left to do, the effect can be that the students felt that they as a class were working together as a team and that they had a job to do collaboratively. It can be seen as a tacit agreement of what they had in common both in terms of their knowledge; in terms of what they had done and what work they had left. David's use of "we" included himself together with the students. The use of we when saying "we have several things we have not yet learned" can seem odd because David as a teacher had learned this a long time ago. However, I suggest it as an indication that he looked upon learning as a joint enterprise in the class, and that he was one of the participants in this enterprise, with a special role. He put himself in a special role in his use of "I" when saying: "I shall show, I had planned to show, I shall say, I will not follow, I claim", which I also emphasised in the quotations above. His use of "you"[48] in these quotations referred to what the students should do when he had finished lecturing from the board. This

[48] "Dere" in Norwegian which is plural form of "you"

suggests that he looked differently upon his role during whole class work than when the students were working individually.

In David's lessons there was a transition from the monologue to the lecturing part of the whole class lessons. This is characterised by his invitation to the students to contribute to his lecturing. The *transition questions* were very similar in the different lessons and had the effect of drawing the students' attention to work with the topic of the day. They were easy questions and could often be answered in one word. Below I present the transition questions from the lessons Jan 21st, Feb 11th and March 10th. I have put the transition questions in italics.

- Continuing the monologue from Jan 21st quoted above David had written two equations on the board and said: *What does what is on the board look like? What type of a task is it natural to call this, Solveig?*
- The transition from monologue to lecturing Feb 11th was the question: *For what kinds of figure does Pythagoras' theorem work?*
- March the 10th David had drawn an equilateral triangle on the board and the transition question was: *How big is the angle on the top?*

For the lesson, Jan 14th, I present the transition question in the context within which it occurred. David invited a student, Jacob, to come to the board to do a task there. However, Jacob did not do any work on his own initiative; it was all directed by the teacher. David made him do what he would have done himself to solve this task. The exercise Jacob was invited to do was written on the board:

6x+3y=9
3x-3y=27

Excerpt 27, David Jan 14th, episode I-1 (turn 1-4)

Nr	Who	What is said	Comments
1	David	I would like one of you to do this first. It is the level we looked upon on Monday. It is readily served really. *Can Jacob take it? You must come here and take the whole thing. Must draw a line under that one (David draws the line). What you shall do first is the logical thing to do!*	On the board: $6x+3y=9$ $3x-3y=27$
2	Jacob	Add	
3	David	Yes, why do you want to add the two equations? (To a girl not paying attention: Jenny, you have to pay attention here now!) What happens when you add the two equations?	Teaching strategy: How he reprimanded a student
4	Jacob	The y's disappear	

After the transition question, “what you shall do first is the logical thing to do”, which in this case was more like an imperative than a question, the lecturing part of the lesson started.

In the *lecturing parts* David demonstrated use of several teaching strategies through which he created conditions for possibilities of learning. In the following I will present a detailed analysis of how he did this from the lessons Jan 14th and Jan 21st. From these lessons I show how

- A typical discourse in the lecturing part was that of an I-R-E (Initiation, Response, Evaluation) and
    - the teacher was in charge of and controlled the course of the lesson
    - he was showing and telling
    - the proportion of the teacher’s talk was enormous compared to the proportion of students’ talk
    - he asked closed questions
- David demonstrated several teaching strategies
    - he pointed out typical errors so the students could avoid making them
    - he restated students’ answers and elaborated on them, thus students in class became learners through participating in the discourse
    - he popularised the mathematical language to facilitate the meaning for the students
    - he took what students were supposed to know at this stage, students’ common ground, as a starting point for further learning
- I have studied the use of “we” which emphasises both the conventional aspect of mathematics and also for the use as a collective term for the class.

To provide further evidence for these findings I supplement with excerpts from the lessons Feb 11th and March 10th. In the right columns in the presentations of the transcripts I point to the teaching strategies and I comment on them in connection with the analysis of the transcripts. In the portrait of David (the last part in this chapter) I sum up the teaching strategies that I identified and I present a schematic overview of the course of his lessons.

The main difference between the monologue parts and the lecturing parts was that in the latter there were gaps in the teacher’s talk for the students to fill in or closed questions for the students to answer. Like the transition questions presented above, the “gap questions” in the lecturing part can often be answered in a few words. A typical feature was that David restated the student’s answer and elaborated it further. The lecturing part of the episode above continued in the following way:

$6x+3y=9,$
$3x-3y=27$

Excerpt 28, David Jan 14th, episode I-1 (turn 5-11)

Nr	Who	What is said	Comments
5	David	Yes, the *y's* disappear. And always in these tasks our goal is to make one letter disappear. And here it is quite simple because we have just as many *y's* in the upper as in the lower and then it is just to add right away [pause] we add!	Referred to students' common ground Jacob had written $9x = -18$ on the board while David was talking
6	Jacob	Thirty-six	while changing -18 to 36, thus $9x=36$
7	David	Yes, right and then it is not difficult to find x when you already have managed 9x=36 And then one ("man[49]") is half way through, aren't we? He has managed to find out that the x, behind that x a digit 4 is hiding. And then the question is: what is y? What number do we have to find to be suitable for y? and how do we find y?	Jacob wrote: $\frac{9x}{9}=\frac{36}{9}$ $x = 4$ Teaching strategy: Popularising
8	Jacob	Shall I show… ?	
9	David	Yes, you shall calculate y. You shall use one of the equations to calculate y. And then you can step aside and stop for a little while. What Jacob now did, he chose the upper equation. It does not matter which one you choose because y has the same value in both. However, I actually mean that it is easiest to take the one where there is plus in front of the y's. Very many if they take the one with minus 3y they just delete the minus, it just disappears on its way. That happened on the test we just had about an equation too. So perhaps it is safer to choose the upper not to get so many minus problems. And now Jacob has made a simple calculation which you can calculate--- Like that, yes. And then you have to move over as we always do in all equations. Do you manage that one? Nine minus twenty four?	Jacob wrote: $6 \cdot 4 + 3y = 9$ Teaching strategy: Elaborating for the rest of the class what Jacob was doing on the board (Sharing) Teaching strategy: Telling what to do to avoid errors Jacob wrote: $24+3y=9$ $3y=9-24$
10	Jacob	Minus fifteen	
11	David	Minus fifteen. Yes, Good Y is minus five	Jacob wrote: $\frac{3y}{3}=-\frac{15}{3}$ $y = -5$

[49] In Norwegian we have the word "man" which is an unidentified third person which can be translated into singular "you" or "one" as I have done here.

By presenting this excerpt from the lecturing part of the lesson Jan 14th, I want to show what I found being a typical discourse in David's lecturing part in whole class lessons. Neither Jacob, nor the other students said very much and they were not encouraged to further contribution either. The teacher was in charge of the lesson and did most of the talking. David restated Jacob's answers, both what he said and what he wrote on the board, and elaborated on that before he asked a new question. Turn 9 was a typical example of that. Thus the students in class became learners through participating in the classroom discourse. In turn 7 David said: "behind that x a digit 4 is hiding". I see this utterance as a popularising of the mathematical language. This is a teaching strategy David used and thus a way of making the language of mathematical equations closer to everyday language and thus equations more understandable. Then the challenge was to find y and when Jacob in turn 8 said: "shall I?" David told him what to do, how to do it and why. David pointed to possible mistakes students could make and how to avoid them: it is safer to choose the upper… he said (9). His request to avoid "the minus" can be a reason for Jacob's wrong calculation in turn 5.

The I-R-E pattern of discourse was typical in David's lessons as it was in the lessons with the two other teachers. In his lessons David was showing and telling, almost in a monologue, but with gaps for the students to fill in with answers to his questions. Starting with the transition question (turn 1 in the excerpt below) in the lecturing part of the lesson Jan 21st below, the discourse of this lesson was very similar to the one Jan 14th. David asked some questions (I), a student answered (R), David restated (E), elaborated and proceeded further by asking a new question (I). He used "we" in two ways; both as a collective term for members in the class (7), (9), (11) and (15) and also for a mathematics conventional purpose, that there are some conventional rules to be followed in our work with mathematics as in turn 5 below (what do *we* call it when there are two equations?). In turn 7 I interpret "we" used both to emphasise the conventional aspect (a set of equations, two of them where *we* shall find both x and y) in addition to the use of it as a collective term

Excerpt 29, David Jan 21st, episode I-1

Nr	**Who**	**What is said**	**Comments**
1	David	*What does what is on the board look like? What type of a task is it natural to call this, Solveig?* (transition question)	On the board: $y = 7 - x$ $y = x - 1$
2	Solveig	An equation	
3	David	**One** equation?	In Norwegian "an" and "one" is the same word "en". Teaching strategy: Emphasises "one"

4	Solveig	Two	
5	David	Two equations. What do *we* call it when there are two equations with two different letters in them?	Conventional use of we
6	Solvieg	Unknown equation	
7	David	Yes, rather equations with two unknowns or a set of equations or an equation set. You see[50], an equation set[51] is in a way a set of equations, two of them, where *we* shall find both x and y. What have *we* been doing to find x and y in the equations *we* have been dealing with so far? Need not saying it in detail but approximately how have *we* proceeded to find the *x* and the *y*, June?	Use of we as a collective term and also emphasising the conventional aspect
8	June	Add and subtract	
9	David	*We* have added equations and *we* have subtracted equations. And if *we* are not so lucky that *we* can add or subtract right away, *we* have had to help a little and what have *we* had to do before we could add or subtract, Eva?	"We" as a collective term for the whole class
10	Eva	Multiply	
11	David	We must multiply one or two of the equations. And if *we* master that method *we* can solve all sets of equations. If you should solve this set, you will probably see that it is quite simple. It is displayed a little differently than we are used to but that is on purpose. If you take a look, it is quite easy to make a piece of work here to get rid of a letter? Sindre?	Teaching strategy: Popularising: "get rid of"
12	Sindre	Then one gets rid of the *x*-es.	
13	David	Yes. Do you have to add or subtract the equations?	Structures students' thinking
14	Sindre	Must subtract (– pause) no, add.	(Could hear students in class protesting)
15	David	Yes, this *we* have said wrongly many times, but if I add the two it becomes zero. Then I have got 2*y* there and I have got 6. Meaning 2*y* equal 6. But this looks like something else. If not only looking upon it as equations with two unknowns as it also is, does it look like something else? Christian?	Related to something they already knew, to their previous knowledge Wrote $2y=6$
16	Christian	Graphical	

[50] I have translated the Norwegian "ikke sant" into "you see"
[51] I have translated the Norwegian "likningssett" with "equation set" and "et sett av ligninger " with "a set of equations"

17	David	Yes, it looks like, however, what did we call the expression if we only look at the upper? What did we call an expression with y equals something containing an x? What did we call that that kind of expression Jon?	Highlights key aspects. Structures students' thinking. Links to functions
18	Jon	A function?	Response
19	David	A function. And our last topic is called "graphical solution of equation set"	Revoiced

This excerpt demonstrates a kind of transition, or a bridge, from solving equations with two unknowns algebraically to solving them graphically and David did this by referring to functions. The excerpt shows how David took what the students already knew (solving equations algebraically and functions) as a starting point for what was new, graphical solution of two equations. Taking what the students already know as a starting point for learning is encouraged in L97: "Pupils' experience and previous knowledge, and the assignments they are given, are important elements in the learning process" (L97 p. 167). David related his lecturing from the board to what the students had done before. This way he structured students' thinking and created conditions for possibilities of learning.

When lecturing David asked closed questions which enabled him to keep control of the course of the lesson so it could proceed according to his plans. The question in turn 7 above, seemed at first to be an open question: "What have *we* been doing to find x and y in the equations *we* have been dealing with so far?" However, David very soon added that they did not need to give a detailed elaboration which suggests that he did not want the students to say very much at a time. He controlled a detailed mathematical elaboration by letting the students say only a few words at a time. Thus I do not look upon his restating as a legitimising of students' mathematical explanations. Because the students did not provide any explanations, they only filled in the gaps in the teacher's lecturing.

David's conveying style when lecturing continued throughout the lesson Jan 21st. The students' contributions were suggestions for values for x while David calculated y and drew the graphs on the board and was explaining or showing and telling mathematics. The number of utterances from the students during the first 14 minutes from the lesson was very small compared to that of the teacher. This became visible to me after having studied the data by organising the transcripts in different ways and also in a table with one row for each turn.

This lack of balance in proportion of talks was also the case in the lecturing parts Feb 11th and March 10th. I will show this by presenting an excerpt from each of these lessons. The first three turns in Excerpt 30 below, is a typical example of the I-R-E (F) pattern of discourse in David's lessons: The teacher started by asking a question, (I), which in this case it is the transition question, a student answered, (R) the teacher restated (E), elaborated further and/or reminded the student about what knowledge they were supposed to have (F) and asked a new question (I). Turns 1-3 from the excerpt below, show a typical discourse in the lecturing part of David's lesson. I start the excerpt with the transition question: *For what kinds of figure does Pythagoras' theorem work?*

Excerpt 30, David Feb. 11th, episode I-1 (turn 1-3)

Nr	Who	What is said	Comments
1	D	*…for what kind of figures does Pythagoras' theorem work?* That we can't mess up, because then we get in trouble, Dag?	Teacher asked a question
2	Dag	Right angled triangle	Stud answers
3	D	Right angled triangle. So every time there is a triangle with ninety degrees, regardless of what you do know or not, we know that in a way that yes, I can use Pythagoras. And what did Pythagoras find out about right angled triangles that we use to find sides, Tove?	Restated Reminded the students what they already knew (prior knowledge) Asked a new question

David's lecturing style by asking the students closed questions which can be answered mostly in one or a few words was a teaching style or strategy I saw him use in all lessons I observed. This strategy ensured students' attention, and at the same time allowed him to keep control of the course of the lesson. By asking these questions David also reminded the students about their prior knowledge, what they so far were supposed to know and he built on that. Through this strategy he created possibilities for students to learn what was new. This became very visible in the first part of the lecturing part March 10th. The students should learn that in a triangle with angles of 30, 60 and 90 degrees the smallest side was half the hypotenuse. David said this was a very important rule, and that they would have to know it and use it in all upcoming tests and on an eventual final exam. Thus he motivated the students to pay attention and to learn the rule. In Excerpt 31 page 268, I start with the transition question from the monologue.

Excerpt 31, David March 10th, episode I-1 (turn 1-19)

Nr	Who	What is said	Comments
1	David	*That triangle, how big is the angle on the top, Per?* (transition question)	An equilateral triangle was drawn on the board
2	Per	Sixty	
3	David	Sixty. What kind of triangle is it? Sindre?	Restated. Asked a new question
4	Sindre	Equilateral	
5	David	Equilateral. What do you know about an equilateral triangle?	Restated. Asked a new question
6	Sindre	All sides have the same length	
7	David	All sides have the same length. And the angles?	Restated. Asked a new question
8	Sindre	All angles have the same length	
9	David	Yes, yes. All sides have the same length and all angles have the same length One rule being valid here for these, yes, *the same size*. One rule being valid here is that the height, if we draw the height, what do you think the height does with the top sixty degrees? Jenny?	Restated and carried on. Students in class protested to "Angles have the same length" He drew the height
10	Jenny	Divides it	
11	David	Then I erase that one, and write thirty there and thirty there. What we do now is and now we concentrate upon one of the triangles I have made. You see, now I have made in a way, I have divided one triangle into two triangles. What about the angle to the height on the baseline. How is that angle, Dag?	Erased 60 from the first triangle and wrote 30 on each of the new angles
12	Dag	Ninety	
13	David	It is ninety. Then we have that. Where does the height hit the baseline, meaning the height in an isosceles triangle, the height in an equilateral triangle, where does it hit the baseline, Jacob?	Restated and went on
14	Jacob	In the middle	
15	David	In the middle. Can anybody try to say something about the length of that side compared to that side since it hits in the middle? Fritjof?	Restated. Pointed to the smaller side and the hypotenuse. New question
16	Fritjof	The half	
17	David	The half. Because the point is, you see, that the side there has the same size as that side there, it is an equilateral triangle. When that hits in the middle, the half, then it is the half of that and obviously that half of that. And the rule approaching now is that when we have a triangle with thirty, sixty and ninety degrees, the smallest side is half the longest, the hypotenuse.	Restated and went on

		That rule we have to know *and it comes in the rulebook too* and we have to be able to use it. It ought to be written. Meaning you must use the rule, you must reason this. I will show how the tasks will look like. Sindre?	Sindre had his hand up
18	Sindre	Is it when it is thirty, sixty and ninety or is it when it is sixty first and then dividing?	**NB!!**
19	David	No, it is in all triangles in the world where there are thirty, sixty and ninety and never otherwise. If it is thirty one there and fifty nine there it is not valid. Some students mix it up with Pythagoras. When is Pythagoras valid? What is the demand to use Pythagoras' theorem?	Teaching strategy: Pointed to possible mistake: mixing the 30-60-90 rule with Pythagoras

In the turns 3, 5, 7, 9, 13, 15 and 17 David restated a student's answer, elaborated and posed a new question. Even in turn he 9 restated a student's answer which contained a use of a wrong word ("lengths" of angles). This suggests that he did this automatically and that it was part of the course of the lesson. However after comments from the class (which shows that the students were paying attention) he corrected himself in a subordinate aside (put in italics in the transcript). The restating of a student's answers followed by his elaboration and a new question indicates that he did not get any unexpected answers. They all fit into his speech. In turn 18 there was a break. Sindre had raised his hand and asked a question. This question indicated that Sindre had not seen the relation between what David had done to show and prove the rule and the rule as such. I go further into detail about this issue when discussing the mathematical focus in David's lessons (page 270).

Three times during the lecturing part of the lesson March the 10th, David said: "the rule (about triangles with 30, 60, 90 degrees) will come in your rulebook". This means that David would write down the rule, copy it to everybody so they could paste it into their rulebook. David was concerned that everybody should have a minimum of what they needed of rules and methods in their rulebooks and he put the responsibility for that on him as a teacher. He told me this in a conversation we had and he also said it in focus groups.

So far I have been focusing on the course in the whole class lessons and how David started with a monologue, posed a transition question followed by lecturing where he invited students to participate, and that the typical pattern of discourse in the lecturing part was that of I-R-E. Before ending the "conditions for possibilities for learning" and the discussion of David's teaching strategies in the lecturing parts of David's whole class lessons I will highlight two more teaching strategies that I saw. One of these is how David was *sharing* with the rest of the class by reformulating a question from one of the students and thus making the whole class take part of it instead of answering it right away. An example of this was in turn 38, Feb 11th. Then a student asked "*shall we not take the square root of plus minus?*"[52] and David immediately shared this question with the rest of the class: "*Why shall we not take the minus solution here?*" and thus activated other students' conceptual thinking of the issue as well, so learning could take place through the participation in the classroom discourse. Another teaching strategy David used to "activate" or "capture" a student's thinking or attention, was to ask a student a question directly when s/he was not paying attention. An example of that was when he asked a student what six squared was and the student could not answer.

In the following I will discuss how David in his teaching was focusing on rules and procedures rather than on students' conceptual understanding and that his teaching style which I have outlined so far emphasises the procedural aspect of mathematics.

Mathematical focus

When studying the excerpts from David's lessons, I found the mathematical focus being highly procedural. The main foci are on factual knowledge, methods, rules, conventions and procedures.

When David gave an overview of the lessons in his monologues I see the mathematics they were working with being referred to as a fixed body of knowledge and the students were often encouraged to remember it.

- Jan 14th David referred to what they should work with as "the last step" (page 259).
- Jan 21st he said: "Now we shall learn the very, very *last thing* we have done with x and y" (page 259).
- Jan 28th he talked about a mathematical topic as something being finished (page 259).
- Feb 11th he referred to different angles and "If you don't *remember*, look it up in the textbook" (page 259).
- March the 10th he referred to *things* they not yet have learned (page 259).

The first three transition questions David asked between the monologue and the lecturing part reflect factual knowledge and the fourth is a question about how to do a procedure:

- What is it natural to call this?
    - Equations with two unknowns (Jan 14th)
- For what kinds of figures does Pythagoras' theorem work?
    - Right angled triangles (Feb 11th)

[52] The student said: "roten av pluss minus" which means both the negative and positive square root

- How big is the angle on the top?
    - Sixty degrees (March 10th)
- What you shall do first is the logical thing to do
    - Add (Jan 14th)

In the excerpts presented from lecturing parts I see the foci on methods, rules and procedures as characteristic. I will first go into detail of the two lessons dealing with algebra and then the two lessons dealing with geometry.

Algebra

- Jan 14th the focus was on the method of solving equations with two unknowns algebraically (Excerpt 27, page 261, and Excerpt 28, page 263). In the analysis of this lesson I show how David's focus was on the conventional and procedural aspects of mathematics.
- Jan 21st the focus was on the method of both algebraical and graphical solution (Excerpt 29, page 261), thus the focus was on the procedural aspect in this lesson as well.

In both these lessons I noticed a confusion among students whether to add or subtract to "get rid of" either x or y. I will provide evidence for this with examples from both lessons. Furthermore

- I will account for how David's use of the term "trick" emphasised the procedural aspect of mathematics
- I have studied David's use of personal pronouns to account for a view on school mathematics as something being invented by somebody
- Jacob (a student) expressed a misconception; however, David did not invite discussion of the student's understanding of the issue discussed. Teacher and student talked passed each other.
- I indicate a coherence between David's style of teaching and the procedural focus of mathematics

In the episode from Jan 14th (Excerpt 27, page 261 and Excerpt 28, page 263) the focus was on the procedure how to solve equations with two unknowns where adding the two equations, (6x+3y=9 and 3x-3y=27) would make one of the letters disappear.

When Jacob first wrote

$$\begin{array}{l} 6x+3y=9 \\ \underline{3x-3y=27} \\ 9x=-18 \end{array}$$

on the board (turn 5), David did not ask him how he got that, he told him right away: "we add" (turn 6). Also students in class objected when Jacob wrote -18, so he corrected it very fast. The fact that Jacob got -18 was not necessarily a careless mistake. It could have been a misconception; since he subtracted 3y from 3y when he added them, he should do the same with the numbers on the other side of the equal sign too. David's use of "we" when saying "we add", emphasises the conventional aspect of mathematics: "this is how we are doing this mathematics." My interpretation of the episode is that David's focus was not on developing students' conceptual understanding, but rather to work out the procedure or method correctly.

The same uncertainty was also demonstrated later in the same lesson in turns 27– 34 when continuing solving the two equations:

$$5x-4y=21$$
$$\underline{3x+2y=-5}$$
$$5x-4y=21$$
$$\underline{6x+4y=-10}$$

and a student, Dag, suggested 11x=31. David corrected and said: twenty one plus minus ten is the same as twenty one minus ten. Dag said yes, however another student said she did not understand. David took her comment into account by restating what he had already said: *"If I have twenty one and shall add minus ten that is the same as subtracting ten. So twenty one and minus ten is eleven all together"*.

Studying the mathematical focus in Excerpt 29, David Jan 21st, episode I-1, page 264, they were working with both algebraic and graphical solutions of equations with two unknowns and the procedural aspect of mathematics was still in focus. In turn 11 David encouraged the students to learn the method because then they would be able to solve all sets of equations. As well as in the lesson Jan 14th, also in the lesson Jan 21st (Excerpt 29, page 264) an uncertainty whether to add or to subtract was expressed (turn 14, "must add- no, subtract"). Sindre suggested subtracting first but changed his mind quickly. It is not possible to know if subtracting was a guess which he changed when he heard other students' protests or if it was a careless mistake or if he actually thought that subtracting was the right thing to do. (This again shows that students were attending) In turn 15 when Sindre first had suggested to add but changed to subtract, David pointed to the indicated wrong answer as a kind of error they had made, "Yes, this we have said wrongly many times", turn 15. However, he did not go into depth elaborating it, neither did he ask the students about how they were thinking. These episodes from work with algebra demonstrate that there was confusion among students about when to add and when to subtract. However David did not take this as starting points or grounds for further learning; Jan 21st he just reinforced adding and Jan 14th he told the students that adding minus ten is the same as subtracting ten.

A student who asked me for help during individual seatwork also demonstrated uncertainty whether to add or to subtract. This suggests that the focus on the method rather than on conceptual understanding caused uncertainty among the students. In the analysis of the individual seatwork sections of the lessons I present more evidence for my claim that students had not gained conceptual understanding of whether to add or subtract when solving equations with two unknowns.

David's focus on the procedural aspect of mathematics rather than on conceptual understanding is emphasised when they continued with the following exercise which David wrote on the board:

$5x-4y=21$ and
$3x+2y=-5$

and he suggested doing a "trick". He said:

> And then *they* can make it a bit harder for us. And then *we* shall take one as such and then *we* shall exercise that and then **I** will take a task from reality. And it is a *teeny-weeny trick* and the task is the same as the one we just had. (David Jan 14th, turn14)

Indicating that by doing a "teeny-weeny trick" they could make the task the same as the previous one, reflects a view that if they remembered the trick, they would manage working the task out.[53] David's use of "they" in this quotation was referring to some unidentified persons out there. It may be "those out there who are inventing mathematics" or it may refer to "those who are making the exam". I have highlighted this because it initiates a school mathematics view that somebody (out there somewhere) had decided what has to be learned and how to try out if it has been learned. By focusing on this David wanted, together with his students, to accomplish this challenge. His use of "we" emphasises the difference between "they out there" and "we in this class".

After this introduction David told his students that they were allowed to multiply and divide equations and he said: "In this case it is sufficient to trick with one"[54]. He solved the task on the board while asking questions to the class for suggestions what to do. A girl suggested multiplying the lower by two, a suggestion he appreciated and he was about to go on with that when he was interrupted by Jacob who said: The equations were

$5x-4y=21$ and
$3x+2y=-5$

[53] The "trick" was to multiply one or both equations so one letter disappears when the equations were added.

[54] Meaning that in this case, only multiplying one of the equations was needed to "get rid of one letter"

Excerpt 32, David Jan 14th, episode I-1 (turn 20-27)

Nr	Who	What is said	Comments
20	Jacob	But if you add, you get minus two and if you divide by minus two then…	Was interrupted by David
21	David	But if you add here you will get 8x minus 2y, you'll get two unknowns!	8x-2y=-16 J would divide by -2
22	Jacob	But you get -4x equals …	David interrupted
23	David	You cannot solve one equation with two unknowns. Then you have only one equation	David did no catch to what the student meant
24	Jacob	But you take it away if you divide!	NB! Misconception
25	David	You may divide one equation as well if you wish. Would you have divided the upper with two?	David focused on rules
26	Jacob	Yes, when you have taken them together, oh just forget it!	He gave up. David did not understand what he meant
27	David	No, not after having, then you must have done it right away. It is legal to divide too, however one has to do that very rarely. I don't think I will do that…But I hope everybody sees that if I manage to get that one to be four, then they disappear. Sometimes one has to multiply with a negative number. If both had been plus, then it is not sufficient to multiply by two because both are plus. Then I probably would have had to multiply by minus two. That you have to see out of the task. We shall exercise that afterwards.	David focused on the procedure and that they were going to exercise the procedure later

David did not catch what Jacob tried to say here. When sitting there in the classroom as an observer, I think I captured what Jacob was trying to say: After adding the two equations he would get 8x-2y=16 and he thought that dividing by minus two would make minus two y disappear and he would get - 4x = -8. Jacob demonstrated a misconception. However, David did not catch what he meant.

This suggests that David focused on the method and procedure and he did not see what the student was trying to express. David saw the mathematics from his point of view which was to show the students what he meant was the method to solve the equations. He did not see the issue from the student's point of view, which demonstrated the misconception that you get zero when dividing minus 2y by minus 2.

After the lecturing part of the Jan 21st lesson presented above, David did two exercises including graphical solution on the board in the same teaching style. I find coherence between this teaching style, *teacher ex-*

plains mathematics, asks a question (most often closed), student answers, teacher restates, elaborates and asks a new question, and the procedural mathematical focus. The teaching style emphasised the procedural aspect of mathematics. The teacher demonstrated the procedure of how to solve equations graphically and the level of difficulty of the questions he asked was such that students most often answered correctly. While doing the graphical solution David invited the students to contribute to suggesting variables while he was doing the mathematics. This means that the teacher could carry on with the procedure, method or technique he demonstrated on the board without being interrupted by students' eventual wrong answers. David rather told the students how to avoid making errors, than probing their misconceptions and taking them as a starting point for learning as acknowledged in L97.

Geometry

In the geometry lessons as well as in the two algebra lessons the procedural aspect was focused:

- Feb 11th the focus was on the procedural use of Pythagoras' theorem. (Excerpt 30, page 267)
- March the 10th the focus was on the rule that in a triangle with angles like thirty, sixty and ninety degrees, the shortest side is half of the hypotenuse (Excerpt 5, page 268).

In these geometry lessons from which I present excerpts, I noticed an uncertainty among the students about the relation between the process of developing a rule and the use of the rule. With regard to Pythagoras' theorem, some students did not see the relation between the theorem (in a right angled triangle the sum of the squares on the smaller sides equals the square on the hypotenuse) and the use of the theorem to calculate lengths of sides. The other issue was the relation between the process of developing the rule that in a triangle with angles 30, 60 and 90 degrees, the smallest side is half the length of the hypotenuse, and the application of the rule. I also show that when David invited students' contributions he put constraints on to what extent he wanted them to contribute. Furthermore I have studied the deictic use of "it". First a student's use of "it" as pointing to a concept not yet developed, second the teacher's use of "it" or "that" (Norwegian "det") pointing to different aspects of the rule; first *the general rule as such* and the *why that is a rule.*

On Feb 11th, when working with Pythagoras' theorem, the focus was on the technical use of the theorem and not on conceptual understanding. David's challenge to Tove in turn 5 (see excerpt below) could have been an indication that he wanted to go into deeper understanding of Pythagoras' theorem. However, the follow up dialogue, in turns 6-11, reflected a focus on how to use the method to find a side. Why the method was right and what it involved were issues with which, according to the teacher, they had worked thoroughly earlier. What David said in turn 11: “We have to add the two short ones to get the long one” shows clearly that he focused on the method and not on conceptual understanding. This can be one reason why a student who asked me for help during individual seatwork later in this lesson thought that $3x^2$ was the *length* of the side. The student did not seem to have developed conceptual understanding of Pythagoras’ theorem. I discussed this with David in the post lesson conversation Feb 11th where he expressed his concern for balance between time to spend on conceptual understanding and the mechanical (page 250). In the analysis of the individual seatwork I present episodes to provide further evidence of students’ lack of conceptual understanding of the theorem.

Excerpt 33, David Feb 11th, episode I-1 (turn 1-12)

Nr	Who	What is said	Comments
1	D	…for what kind of figures does Pythagoras’ theorem work? That we can’t mess up, because then we get in trouble, Dag?	Teacher asked a question
2	Dag	Right angled triangle	Stud answered
3	D	Right angled triangle. So every time there is a triangle with ninety degrees, regardless of what you do know or not, we know that in a way that yes, I can use Pythagoras. And what did Pythagoras find out about right angled triangles that we use to find sides, Tove?	Restated Reminded the students what they already knew (prior knowledge). Asked a new question
4	Tove	Found one side	
5	D	Yes, how?	
6	Tove	Take something squared	
7	D	Yes we shall take something squared?	
8	Tove	The other sides and then you get the long one	
9	D	Yes, what sides did we have to add squared and what did we have to put alone squared?	
10	Tove	The hypotenuse	
11	D	Yes, And that is logic, isn’t it? We have to add the two short ones to get the long one. That is not so strange. What we looked at when we learned this thoroughly earlier, was if we put on squares here the two small ones [squares] were the same size as the big one every single time. That is what he found out more than two thousand years ago. And that is what we use today. If we shall calculate this figure with Pythagoras, then the side here which to me is unknown, I call x. BC is unknown. Now I chose not to take the easiest type, those who need it can ask afterwards. However it is not a	David drew a triangle ABC on the board where B was the right angle and AC was 8cm and AB was 6cm

		big difference. If we shall put up Pythagoras with the numbers, with the information we have got here, not taking cm into account - that is just confusing. Per, what do we do then?	
12	Per	Six squared plus x squared is eight squared	David wrote on the board: $6^2+x^2=8^2$

David used his mathematical knowledge and the knowledge he had gained through many years of teaching mathematics in telling the students what kinds of strategies he preferred them to use when solving mathematical tasks. After turn 12 from the episode above, when solving the $6^2+x^2=8^2$, he invited the students to take part. However, he constrained their suggestions by saying that he preferred to square first and then move. Thus he had decided the course of how to solve the equation. When the equation was solved and David concluded that BC was 5.3 cm, a student called on David and asked:

Excerpt 34, David Feb 11th, episode I-1 (turn 36-43)

Nr	Who	What is said	Comments
36	Stud	But David	Student "took" the word
37	David	yes	
38	Stud	Shall *we* not take the square root of plus minus?	The use of "we": "we in the class"
39	David	Why shall *we* not take the minus solution here?	Teaching strategy: Sharing a question with the whole class The teacher's use of "we" emphasised the conventional aspect of mathematics
40	Stud	I don't know…	
41	David	Camilla?	
42	Camilla	Can not be minus a length	
43	David	No, a length in a triangle can not be below zero, true. But if you had an equation, if you had only had that equation not knowing what the numbers and the x were, then *we* obviously should have two solutions. But when *we* know that the x is a line in a triangle, then the negative solution is uninteresting.	

The question in turn 38 indicates that the student did not see the relation between Pythagoras' theorem, the method - which is an equation of second order, and what the method is used for - which is to find a length.

The focus was on the actual procedure solving the equation of second order and not for what the Theorem was to be used.

The same confusion, not seeing the relation between the proof of a rule and the actual rule was visible March 10th with regard to the statement that in a triangle where the angles were 30, 60 and 90 degrees the shortest side was half the hypotenuse. After having proved the rule by dividing an equilateral triangle, a student asked: "*Is it when it is thirty, sixty and ninety or is it when it is sixty first and then dividing*?" (turn 18, Excerpt 35). This question indicates that the student had not really understood how the proof of this statement developed. His question shows that he did not see the relation David had tried to show between dividing an equilateral triangle into two congruent 30-60-90 triangles, and any triangle with 30-60-90 degrees. The student did not yet see the process–product relation. She did not see the relation between the process of developing a rule and the general rule. David had focused on the rule. Although he had proved the rule by dividing an equilateral triangle, it seemed that the students had not gained conceptual understanding of the proof of the rule. This is emphasised in turns 32-39 starting with a student's question: "*is it always when it is ninety, sixty…?*"

Excerpt 35, David March 10th, episode I-1 (turn 32-39)

Nr	Who	What is said	Comments
32	Stud 1	Is it always when it is 90, 60..	Was interrupted by the teacher
33	D	Yes, BC equals AB divided by two. Meaning BC is half of AB. But I will have some more. Because one cannot just write this because **it** is not always valid. And then you must be bothered to write a sentence because the angles are thirty degrees, sixty degrees and ninety degrees. If you don't write *that*, you will not get full score when I grade the test neither on the exam.	
34	Stud 2	Is *that* valid as a reason?	
35	D	Yes, *that* is a reason which is good enough	
36	Stud 2	But you only tell what is standing there. You don't give a reason?	
37	D	Yes, I reason that that is half of that because of the sizes of the angles	
38	Stud 2	Oh, Okay	
39	D	And *that* is a reason. And the point is that we must know the rule which is saying that the shortest side is half of the longest and then we must say why in the world we are allowed to claim *it* and use *it*. And *it* is because the angles are thirty, sixty and ninety degrees. *That* we must highlight. *That* must be in the answer	

This excerpt shows that there was confusion as to what “it” and “that” were[55]. Turn 32 indicates a lack of students’ conceptual understanding of what the teacher had just shown on the board. Turns 34 and 36 indicate that yet another student did not see the relation between what David had shown and the general rule. “It” was used as a pointer, but there was uncertainty what “it” pointed to. When the student asked “is that valid as a reason?” she was pointing to the “the angles are thirty sixty and ninety”. However, having followed the lesson so far, she was not sure what to have to include from the proof in the rule so her claim about the lengths of the sides could be a valid claim. The relation between the actual proof of this sentence and the rule they should refer to was not yet clear for the students.

In this section I have discussed mathematical focus in David’s whole class sections of lessons. I have presented excerpts from both algebra and geometry lessons, suggesting that David focused on rules and methods and thus the procedural aspect of mathematics. I have also indicated that some students did not see the relation between the development of a rule and how to apply the rule (Pythagoras’ theorem and the 30-60-90 degrees rule). With regard to algebra, I have suggested that the teacher’s focus on “a trick” and the addition *method*, as reasons for that some students demonstrated uncertainty when to add and when to subtract.

Demonstrating such kinds of difficulties, suggests that these students had gained instrumental rather than relational understanding (Skemp, 1976). Skemp pointed out as advantages with teaching instrumental mathematics: More students will understand because ”instrumental mathematics is usually easier to understand”[ ] “The rewards are more immediate and more apparent”[ ]“because less knowledge is involved, one can get the right answer more quickly” (Skemp, 1976, p. 23). For the students to get as many tasks as possible correct on an eventual exam, David’s choice to focus on instrumental mathematics is in line with this as when he encouraged the students to learn the method of solving equations with two unknowns so they could solve all sets of equations (turn 11 Excerpt 32 page 261).

In connection with Skemp’s claim above that relational mathematics is easier to understand, I will emphasise that the students paid attention in these lessons. I have pointed out several instances which provide evidence for that. Thus the mathematical focus in David’s lessons was of a kind which made many students attending and listening and thus participating in the discourse.

[55] In Norwegian “it” and “that” is the same word “det”

I will also emphasise that I only refer to some students having developed instrumental understanding rather than conceptual understanding. There were of course students in David's class who did not ask for his help during individual work, and did not ask questions because they did not understand during whole class sections. It is not possible for me to know whether these students had developed conceptual understanding or not.

Students' abilities

In the analysis of the conversations I had with David I refer to what he said about speaking mostly to the clever students when reviewing a test. This is emphasised with what I found him saying in the monologue part of the lesson February 11th. He used you (plural, 2nd order) when talking about and thus *to* the clever students, and they (3rd order) when talking *about* the weak students.

> I will write a list of tasks to be done, and that is mostly with a view to **those** who struggle and are content if **they** manage so and so and so. And then I will take the chance that **you** who feel that **you** master math will skip many tasks in the textbook (Feb 11th).

This can indicate that he consciously spoke to the clever ones or that he believed that the weaker students did not listen, or that he pretended as if there were no weak students in the class. Also later in the same lesson (turn 11) he talked about those who did not understand in third person: "Now I chose not to take the easiest type, *those* who need it can ask afterwards". According to what David said, there were only a few students who would not understand. He talked about the weak students as a separate group (four or five students). When talking about the clever ones, he talked about most students in his class. The students I referred to who had difficulties with the conceptual understanding of Pythagoras' theorem, the 30-60-90 rule in a triangle and when to add/subtract when solving equations with two unknowns were not the students David called a weak student.

David also told the students that he found it ridiculous[56] that those who had the grades "four" and "five" in mathematics should work with simple tasks calculating the area of triangles and squares. Since they knew how to do it, they should rather work with tasks that challenged them more. He encouraged students to make judgements themselves about what they could skip and what they needed to exercise more. However, when students asked him for advice, he had clear opinions about each individual what s/he ought to work on. This indicates that, throughout the nearly three years he had been teaching this class in mathematics, he had developed a sense of each individual student's mathematical abilities.

[56] He used the Norwegian word "vanvittig"

According to my field notes more than half the students in class had their hands up to answer his questions in the lecturing parts, and those were the ones the teacher asked. David only addressed students who did not have their hands up if they demonstrated not paying attention. Thus he did not ask students who not were able to answer his questions, and the "flow" in his lecturing was thus ensured. The way David related what was new to what the students already knew and that he often said "this is only slightly different from what we have done before" he made the mathematical topic to be within the reach for his students.

Nearly all students participated in the whole class section, but he encouraged students with different abilities to work on different tasks and he sometimes prepared another sheet with tasks for "weaker" students. "Weaker" students were, according to David, those who may manage a "three" in mathematics.

According to Skemp (1976) the focus on the procedural aspect and instrumental understanding involves less knowledge. Thus also the less knowledgeable students could participate in most activities in David's class.

Individual work sections of lessons

Having finished the lecturing part of the whole class lessons, David had prepared some exercises for the students to work with individually or in pairs. The exercises were of the same kind as the teacher had been doing on the board in the lecturing part, and they were mostly taken from the textbook. David had also written some tasks on a sheet of paper which he handed out or he had copied some exercises from older textbooks. Between the monologue and the lecturing parts of the whole class lessons I identified "transition questions" (see page 261). Between the lecturing in the whole class sections and individual seatwork sections I noticed the same kinds of *transition utterances*:

Referring to the method of solving equations with two unknowns which he had just shown on the board David said:

> That method over there, will always give us the answer. I will now write the exercises on the board that you shall work with which are about this. All of them are about this (Jan 14th).

The next week, Jan 21st, David had written some exercises on the board. His use of the Norwegian "*man*", which I have translated into "*one*" indicates a conventional aspect of mathematics; this is how we do mathematics. He said:

> This is the very last thing we shall learn about algebra, it is solving equations by drawing graphs. And I really think everybody can manage the first exercise. And it might become a bit more difficult when *one* has to start turning things around. But what *one* shall do, regardless where *one* is, is doing the four exercises there (Jan 21st).

In the lesson February 11th time pressure was part of the transition utterance and he used that as part of the motivation for the students to work efficiently:

> I will now put a list of exercises, and there are three different topics we shall throw ourselves onto. We are far out in the school year, and time will pass very fast now. Winter vacation in a while, project work, and national tests and all of a sudden it is Easter (Feb 11th).

March 10th he had made some exercises on a sheet of paper which he handed out. There were also numbers of exercises from the textbook on the paper which they should do. He said:

> Everybody starts on the top here. Some of the exercises are in brackets, and this you will all manage quite well.

Thus a typical pattern in David's lessons was that he first demonstrated some mathematics exemplifying on the board and afterwards the students worked individually with similar tasks. This emphasises the focus on the exercise of skills and procedures and also on the conventional aspect of mathematics.

There was always some noise between whole class work and individual work; the students had to find their workbooks and textbooks and some students very soon called for David's attention, sometimes asking questions about practical matters or about a test or about the topic for the day.

During individual work David eagerly helped students when they asked for his help, which they frequently did. David rushed around from one student to another answering their questions by explaining to them how to solve the task. In the lessons I observed, David was helping between 10 and 30 different units of students (mostly single students but also pairs of students) during 20 to 30 minutes. The average time with each student or pair of students, one unit of help, was slightly more than one minute. All units of help were initiated by the student who asked for help with questions like: *"is this right?"* or *"I don't understand this, can you help?"* or a concrete mathematical question like: "*When I shall find the angle-sum is it just multiplying hundred and eighty with twelve?*"

The nature of the answers David gave was characterised by explaining, mostly showing and telling, giving consent or detecting an error the student(s) had made which had caused a wrong answer. Combining the students' requests for help and the nature of the teacher's help gave me the following linked categories in the analytical process:

- When student said s/he understands nothing, asked what to do (student understands nothing, SUN) or asked a mathematical question (student questions mathematics, SQM), the teacher's responded by explaining mathematics (teacher explains mathematics, TEM). Most often the nature of the explaining was showing

and telling. The teacher rarely asked the student for how s/he had been thinking.
- When a student asked for consent, the teacher responded by either acknowledging what the student had done (consent) by going through the exercise s/he was working with, or discovering possible errors. In the latter cases David most often showed the students how to do the exercise.

Furthermore, the nature of the difficulties the students asked for help from the teacher to "overcome", demonstrates that to a certain degree, they had gained instrumental understanding and not relational or conceptual understanding.

From the class's work with *equations with two unknowns* I will present episodes to highlight and provide evidence for the following aspects:

- Under the sub heading "SUN-TEM-SQM-TEM" (student understands nothing - teacher explains mathematics -student questions mathematics-teacher explains mathematics) I present an episode which illustrates the nature of how David responded when a student did not know how to proceed further (SUN).
- Under the sub heading "Multiply with what, add or subtract?" I present episodes from the work on equations with two unknowns, both Jan 14th and Jan 21st to further emphasise the nature of the dialogues between the teacher and the student and to show that students' questions when asking for help suggests that they did not understand when to subtract and when to add.

From the class's work with geometry, I present and discuss episodes to provide evidence for that

- In their work with Pythagoras' theorem the errors students made suggest that they had developed instrumental understanding but not relational or conceptual understanding of the theorem.
- When the task was to construct triangles, for which they could not start with AB as the horizontal line, students had to ask what to start with. This indicates that their images of constructing triangles were bound to certain contexts.

They also demonstrated that they did not "see" what the equal sides in an isosceles triangle were when it was not that AC=BC.

Equations with two unknowns

<u>SUN-TEM-SQM-TEM</u>

To illustrate the first bullet point above, I will present an episode in which the student had found the unknown x and said he did not know how to proceed to find the other unknown. The teacher did not ask him what he had done so far, but looked in the student's workbook and explained to the student how to solve the rest of the task. He challenged the student by asking “what do we do to find y?” However, the student answered wrongly and the teacher did not question the wrong answer but told him how to do it correctly. The teacher was showing and telling. The student had already found x=2 in

x+y=4

4x-3y=2, and did not know how to proceed further.

Excerpt 36, David Jan 14th episode II-13

Nr	Who	What is said	Comments
1	Stud	What shall I do now?	Student asked what to do *SUN*
2	D	Let me see. You have multiplied that one by four, and then you are half way through. If x is 2, now you have to find y. And that equation is easiest. And there it says x+y=4, you see? And since x is two we exchange that with the number two plus y is four. What do we do to find y?	Teacher showed and told the student what to do to solve the task the easiest way *TEM*
3	Stud	Must divide?	Student’s suggestion *SQM*
4	D	No that is when it says multiply, when it is *nothing in between*. When it says plus in between we move it over. So y is four minus two. Makes two and then you have finished.	Popularising: “Nothing in between” and “move over” *TEM*

Multiply with what, add or subtract?

Both in whole class and during individual seatwork David talked about “To get rid of one of the letters” or “one letter disappears” as aims when working with equations with two unknowns. By using everyday language, “nothing in between” and “move over” David popularised the mathematical language and thus interwove what was familiar for the students with what was new. This was in line with what David said in our conversations and focus groups and also what he wrote about ideal teaching, that it is important for a teacher to be able to explain for the students in a language they understand.

In the analysis of the conversations with David I referred to an episode where the teacher demonstrated reluctance for the student to use the subtracting method. This episode is presented below (Excerpt 37). The student suggested what to multiply by to solve the exercise. However, the suggestion involved subtracting one equation from the other, so the teacher did not approve his suggestions but rather told him what to do to get minus and plus so adding would make one of the letters disappear.

The equations were:
$x-2y=3$
$4x+3y=34$

Excerpt 37, David Jan 14th, episode II-7

Nr	Who	What is said	Comments
1	Stud	Here, shall I multiply by four there?	Student questioned mathematics *SQM*
2	David	I wouldn't have done that	Refused without giving reason *TEM*
3	Stud	So you think I should rather multiply by three there and….	Teacher interrupted *SQM*
4	David	Yes, I would multiply by three there and that one with minus two because then you get minus x and plus x and then they disappear and that is the easiest.	Teacher showed and told *TEM*

The next episode, (Excerpt 38, page 286) is from the same lesson. The student's question contained some mathematics: "David, is it illegal to multiply by x?" This was more an exception than a rule. Out of the twenty-one "units of help" in this lesson, six started with a question about mathematics. The others started with either "What shall I do" or "I don't understand?" In three of the episodes which started with a mathematical question a mathematical discussion took place between the teacher and the student. One of these is presented below. The student actually argued for his way of doing the task (turn 5). He did not only listen consenting to what the teacher said which the case was in most units of help. However, he accepted the teacher's suggestion (turn 9). It is unclear what the student meant in turn 1. The teacher did not ask him to clarify but started right away telling what he had to do in this exercise apparently regardless of what the student was trying to express through his question.

David's focus was on the procedure to solve the equations. The teacher demanded the student to use the addition method "*you actually have to multiply either both or one equation with a minus number"* (turn 2) and "*Therefore you need plus and minus on the ones which shall disappear as well"* (turn 4). David asked a question (turn 2): "*how many x'es do you need if they shall disappear*?" and answered it before the student got the chance. The student's "*Do I have to*" in turn 3 could have been to question if that was the only possibility to solve the task, because the student saw the possibility to multiply the upper with 5 and the lower with 3 and thus get 15y in both (turn 5). In turn 6 the teacher opened up for the student's point of view by challenging him what to do next. But the student presented a wrong answer. I think one reason for students' difficulties with this was the focus David put on the "adding method" as the only method, the mechanical focus on needing opposite signs for those you shall “get rid of”, and not on the conceptual understanding of when to add and when to subtract. In turns 11-13 I sense a miscommunication. The student asked if he had not to multiply the other and David answered “no”. David’s answer was as if the question was: Do I not have to multiply the other by a minus-number? But the student could have meant to ask if he not could multiply the other by anything at all. However, the teacher said he should try and that he would come back to give more help if needed. There was no follow up episode with this student in this lesson, which indicates that he had managed. The equations were:

$x+3y=22$ and

$3x+5y=46$

Excerpt 38, David Jan 14th, episode II-15

NR	Who	What is said	Comments
1	Stud	David, is it illegal to multiply by x? Because then it becomes fifteen here	Mathematical question
2	David	You see, here it is not like you have plus and minus, so here there are many possibilities, but you actually have to multiply either both, or one of the equations with a minus number. You may multiply with negative numbers. If you want to get rid of, if you have 3x there, how many x’es do you need there if they shall disappear? You must then have minus three.	David explained Popularised “get rid of”
3	Stud	Do I have to…	Was interrupted
4	David	Yes, because you see, everyone you have had there has plus and minus on those that disappear. Therefore you need plus and minus on the ones which shall disappear here as well.	David explained
5	Stud	But that I don’t bother. I’d rather multiply by five and with three here.	
6	David	But then I would, - what would you do afterwards then?	David challenged
7	Stud	Add?	
8	David	No, because then you get 15y there and 15y there. If you add you’ll get 30y and they don’t disappear	David explained
9	Stud	Okay, multiply by minus 5 then	
10	David	Yes, one of them you ought to multiply by minus. Only one of them	
11	Stud	Do I not have to multiply the other?	
12	David	No, just one. Because if you get minus on both, you are all at the same	
13	Stud	But, then it is minus 5x….	Was interrupted
14	David	Yes, try that and I’ll be back if necessary	

Episode II-6, the same day (Excerpt 39, below) started with a student asking a question with mathematical content (turn 1). This episode shows that also this student demonstrated uncertainty whether to add or to subtract (3) and it also illustrates how David kept telling the student how to find x (2), (4), (6). The student just asked if she could multiply by three, she did not ask him to solve the whole exercise for her as he did (2), (4), (6). Although she had not asked for it David told her how to find y. The equations they were working with here were:

$x + y=4$
$4x-3y=2$

Excerpt 39, David Jan 14th, episode II-6

Nr	Who	What is said	Comments
1	Stud	Can I multiply here by three?	Mathematical question
2	David	Yes, you multiply the first one with three, no doubt. And then you write both equations again. And then you get 3x+3y=12, and then you write the other on just as it is and then you can add and one of the letters disappears.	David showed and told
3	Stud	Let me see if I understand it right, it gives 12x?	Student tried to understand
4	David	No, 7x. You add!	Directive
5	Stud	Oh, 7x=14 then?	Uncertainty
6	David	Yes, and then you divide by 7 and then you have x. Then you find y by putting into one of the equations up there. The first one is definitely the easiest one	David showed and told

Also one week later, Jan 21st, students demonstrated uncertainty whether to add or subtract, and also with regard to what to multiply by and why. One episode started with a boy who had big difficulties with these tasks and asked for help. The equations were:

$2x+y=7$
$x-2y=11$

The teacher explained by showing and telling that he had to multiply the upper by two. When the teacher had finished, the student asked: *How do we just find that out?* This suggests that for this boy how to find out by what to multiply was like a guess, and that he did not see the relation between what to multiply by and the purpose of multiplying. The teacher followed up by explaining again, and when the boy tried to ask a question in between the teacher's explanation the teacher just carried on.

The last episode I present as evidence for my claim that some students had not developed relational or conceptual understanding of whether to add or to subtract the equations, is from Jan 21st. A girl recognised one of the tasks as similar to one the teacher had done on the board and said it was almost the same (1). There was obvious confusion when to add and when to subtract (5-6), and this episode illustrates that the girl had some kind of conception when to add and when to subtract which she expressed to the teacher (turns 5, 13 and 15). In turn 17 I conjecture that she pointed to the signs in front of the numbers The teacher explained when to add and when to subtract (turns 8, 10, 12, 16, 18 and 22), and he showed with an example in turn 24. In turn 25, I can perceive that the student tried to express a conception, but the teacher carried on with his explanation. The equations were:

$y= x+2$

$y= 4-x$

Excerpt 40, David Jan 21st, episode II-1

Nr	Who	What is said	Comments
1	Stud	That one is nearly the same as	
2	D	That is exactly like the first one I did	
3	Stud	But that one is eight, no [ ] inaudible	
4	D	Y is three here. Because, you see, the two x'es disappear when you add	
5	Stud	Isn't it minus four?	
6	D	No, it is plus on it, isn't it?	
7	Stud	Yes	
8	D	And plus on that. Then it becomes six all together. Here you add	
9	Stud	But when did we have to subtract?	
10	D	If it had been plus x	
11	Stud	Under each other	
12	D	No, but if it is the same sign on both, if both are plus x then you have to subtract. But if it is one plus and one minus it disappears when you add	
13	Stud	But that one, then it is plus then it becomes six?	
14	D	Yes, then it is six.	
15	Stud	So, one takes minus only when it is different?	
16	D	When the signs are different I will say you take plus	
17	Stud	But the signs are similar there	
18	D	Yes, but you see, it has nothing to do with that. It is not the numbers that decide whether you shall add or subtract. The x'es and y's decide that.	
19	Stud	Oh, so if that shall be added, that one shall be added as well?	
20	D	Yes	
21	Stud	When shall you then take minus? When it is the same?	
22	D	Yes, if there had been plus there, then you could have taken minus. That hadn't worked in the equation there because then both letters had	

		disappeared	
23	Stud	But, when is it one shall take minus, is that when it is the same?	
24	D	It is obvious that if it had been like this, let us see.. it is not sure that it works with these numbers though, You see, if it stands like this, it is obvious that to get rid of the x'es, you must have to subtract. That minus that is zero.	I conjecture David wrote down an example and showed
25	Stud	Then you must take….	Was interrupted
26	D	Then you must subtract those and those as well	
27	Stud	Yes	

Geometry

Pythagoras' theorem

The three different topics David referred to in the "transition utterance" Feb 11th were: angles, angle sum in polygons, and Pythagoras' theorem. One of my claims presented above (page 283) was that the students had not gained conceptual understanding of Pythagoras' theorem. Earlier I have indicated a reason for this to be the teacher's teaching style which emphasised the procedural aspect of mathematics. Another significant feature was how David expressed making a mistake "falling into the trap" (turn 6) as if he expected them to make mistakes and that there was a trick to avoid falling into the trap. In a previous episode with Christian David said: "*Now you have made the error you just had to do*"[57], and later to the whole class he referred to this task (exercise 4.130 in the textbook, presented below) and that everybody who had asked him about it so far had fallen into the trap. This indicates that he expected the students to make this error. He said: "*When doing 4.130 you may easily fall into a trap, so I will see if anybody is smart enough not to fall into that trap.*" The error the student who asked for help in this episode had made was that he had taken $3x^2$ instead of $9x^2$, and making that mistake, was what he called "falling into the trap". I suggest turn 6 rather being an explanation or even a support rather than an example of showing and telling, and David emphasised the wrong part of the answer as a teaching strategy to make the student aware the error he had made. In turn 8 David asked a leading question, however the student was on the right track now and the indicated right answer in turn 9 was a result of David's support in turn 6, and was followed up by David's explanation how to avoid the trap (10). I conjecture he suggested using brackets. David popularised through the use of everyday language ("*half of side's the name*"). David referred to quadrilaterals and not to squares in turns 6 and 8.

[57] Norwegian: "Nå har dere gjort den feilen dere bare måtte gjøre"

The task was taken from the textbook where a right angled triangle ABC was drawn. B was 90 degrees, AB=8.5 cm and AC=3 ·BC

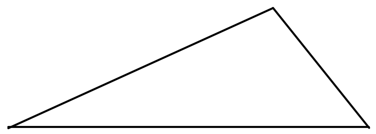
Excerpt 41, David Feb11th, episode II-19

Nr	Who	What is said	Comments
1	Stud	Is the purpose that when I do that it disappears…	Was interrupted by the teacher
2	D	You make the same mistake as Christian did	
3	Stud	When that disappears only that is left?	
4	D	No	
5	Stud	Or is it like this?	
6	D	Everything you have done is right, but you have made an error, a trap that nearly everybody falls into. And you must try to show that, you see, Pythagoras he, if that side is x, you take x squared and that is because you actually have a quadrilateral which is x that way and x that way. So x times x. And then you have a side which is eight point five, and then you take eight point five squared because it is eight point five times eight point five. If that side is 3x, you have a quadrilateral which is 3x that way and 3x that way.	
7	Stud	Okay.	
8	D	Is the area **three** x squared then?	Teaching strategy: emphasised the wrong part of the answer
9	Stud	No, nine..	
10	D	Both the digit three and the x are squared. It is not only the x. So the safest thing to do is this (?) so it becomes nine x squared. It is not only half of the side's name that is squared. And then it becomes nine there, you see, and it becomes eight there, so the numbers change a little. This is a task coming quite often, with twice as big, three times as big. We then need to take along both numbers.	I conjecture he suggested using brackets

Students' images of geometrical figures

My claim that students' images of triangles were bound to certain contexts, and to contexts in which they were most used to see them, is taken from March 10th lesson when the exercises consisted of both construction of triangles and of calculating angles in triangles. Out of 13 "units of help", 6 were about constructing a triangle where they had to start with another side than the horizontal line AB, and 4 were about an isosceles triangle where the equal sides were BC and AC. In the construction exercise they were also asked to calculate some sides. The task was :

$\angle C = 90°$, AC = 6 cm and BC = 4.5 cm



Construct the triangle ABC
What is the triangle called?
Calculate the area of the triangle
Construct the height from C to AB

The problem students demonstrated they had with this task was twofold: First, they did not manage to start constructing since they could not start with AB, as they were *used to*, according to what David said in turn 2. Second, they thought that A was 30 degrees and B was 60 degrees. The latter was probably because the 30 -60 -90 rule was focused on during the lecturing part of the whole class section of the same lesson, and they were used to practice similar exercises during seatwork as they just had been doing in the whole class section. The students were used to start the constructions with a horizontal line, usually AB, and not an oblique line AC which David told one of the students in the episode below and also aloud to the whole class. According to what David said in turn 6 they were not used to the angle C being ninety degrees. Through the way David responded in the following episode it is evident that his focus was on the procedure to carry out the construction correctly (turn 4) - he showed by making a sketch. The focus was also on the error they had to avoid (turn 2) – "you cannot start with AB". In turn 5 the student revealed that he thought that the smallest side was half of the largest which David rejected- referring to the rule but not with further elaboration.

Excerpt 42, David March 10th, episode II-3

NR	**Who**	**What is said**	**comments**
1	Stud	How..?	
2	D	You shall construct from what you know. The special thing here which you ought to see is that *you cannot start with AB as you are used to* because you don't know how long it is, you don't know anything about the angles, so you cannot start with it. You have to start with one of the two other sides	
3	Stud	How?	
4	D	Can I sketch somewhere? You see, it is nothing wrong if you make yourself a line saying that is where I want to have A and measure 6 cm	This indicates that David showed him by making a sketch in the student's workbook.

5	Stud	But that one is half of that	
6	D	No, that is only if it is 30, 60 and 90 which it says nothing about. If you start with an oblique line which is 6 cm and when you shall have 90 degrees there, then it is obvious that you can construct 90 degrees there so you get it straight out. And then it is just to measure that one four and a half cm. Then the triangle is ready. *You just start another place and the ninety degrees shall be made in C rather than where we are used to.*	Referred to the rule. Showed and told what to do
7	Stud	Start in A to C rather than..	Was interrupted
8	D	Yes you can start with A to C. You might as well start with that and construct that way.	Indicates that he pointed to BC
9	Stud	But can I start with A to B?	
10	D	No, you cannot start with AB because then you will not be able to proceed. You can start with that or that	I conjecture he pointed to BC and to AC

This episode was typical of how David answered students' request for help in doing this task. He told them what they could do and what they could not do and how to start. His focus in this episode was on the procedure: that it was not possible to start with AB.

The other task with which the students asked for help to do in this lesson was:

> In an isosceles triangle B=70 degrees, and AB=AC. Calculate $\angle A$ and $\angle C$

The problem the students demonstrated was that they thought A was 70 degrees and C was 40 degrees. I conjecture that because they had learned how an isosceles triangle looked when $\angle A=\angle B$ and the sides with the same length were AC and BC and this is how isosceles triangles often are presented in textbooks, the students' images of isosceles triangles were bound to such triangles. However, when the triangle asked for was different, AC was not equal to AB, their image of an isosceles triangle did not work and they got difficulties.

A portrait of David

> I think that generally students learn best and quickest if they get things well and clearly explained, in a manner and in a language which make them follow and understand (David during our post-lesson conversation Jan 14th).

This quotation from one of my conversations with David characterises both the teacher and the teaching I observed in his lessons. Both in our conversations, in focus groups and in his writings about ideal teaching David very clearly stated that he believed explaining being the best way for students to learn. He believed that the teacher's ability to explain was of crucial importance for the students' learning. He looked upon his role as that of conveying mathematics to the students, which also was consistent with what I found him doing in the classroom, both in whole class and during individual seat work.

The course of David’s lessons

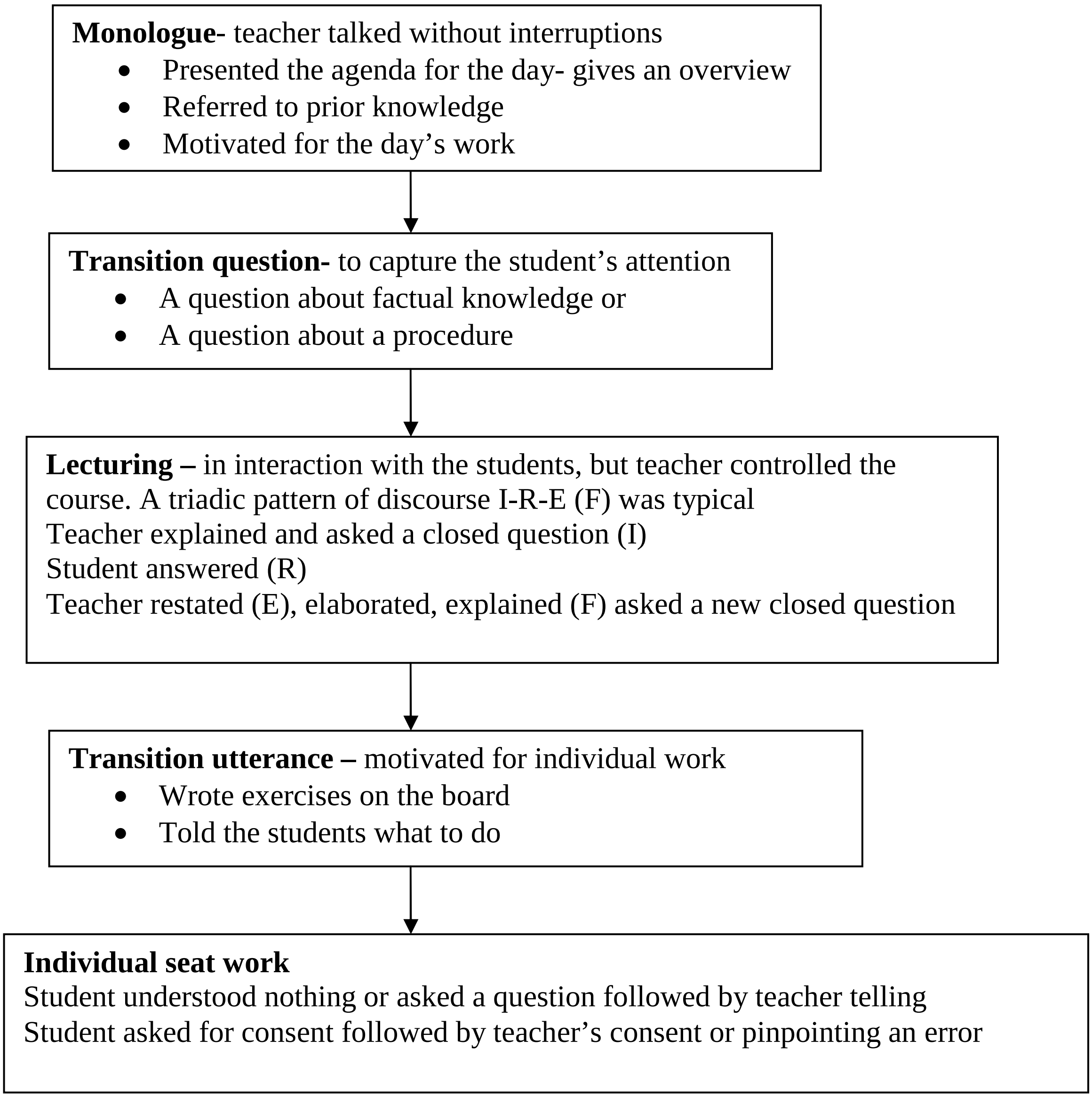


Figure 4, The course of David’s lesson

During one of the lessons in David’s class I noticed that one of the students seemed to be very bored. As an optional course of study she had chosen a course in in-depth study in mathematics once a week. In that class, they often worked with dice and tokens and did exploring activities.[58] I went over to her and asked what the matter was, and she replied

[58] In lower secondary school students have to choose compulsory additional subjects including foreign language, supplementary language study and practical project work (L97). Some schools offer in depth study in mathematics as an optional course in addition to other optional courses, which this student had chosen.

that it was so boring because every lesson was the same; the teacher presented examples on the board of which they should exercise similar tasks on their own afterwards. I asked her if she could write down some of her thoughts about mathematics and give it to me the next week which she did. She wrote:

> What I find boring within mathematics are the standard tasks in geometry, algebra, fractions and so on. What I mean is boring is that it is logical and that if you know the formula or rule, it is just to solve the task and write up the answer. What is fun within mathematics are "ponders" (tasks on which you have to ponder to solve)[59], equations and problem-solving tasks; tasks with different types of calculations where everything we have learned can be used on one task. I also like tasks where I can play with matches or dice to gain understanding.

What this girl said and what she wrote emphasised the course in the lessons which she found boring and also the focus on the use of rules and the procedural aspect of mathematics. In Figure 4 page 293 I have presented how a typical course of David's lessons was.

David's teaching strategies

David demonstrated a wide range of teaching strategies in whole class:

- He popularised the mathematical language to make it closer to everyday language and thus more understandable for the students;
- By repeating a student's comment or question, he was sharing so everybody in the class got the possibility to take part in the issue discussed;
- He pointed to possible errors and how to avoid them;
- When a student gave a wrong answer, he repeated the wrong answer emphasising what was wrong (which he also did during individual work);
- He used "we" both as a collective term for the whole class and also as an emphasis of the conventional aspect of mathematics;
- To activate the students and to make the lecturing part of the lesson different from that of the monologue he asked closed questions for the students to answer. He thus ensured their attention, and at the same time kept control of the course of the lesson;
- When having to reprimand a student he did it in a subordinate aside and thus he did not make a big deal out of it.

During individual seatwork, David was mainly showing and telling when the students asked for help. Since I have presented several episodes indicating that David interrupted his students, I will point to David's eagerness to tell the students how to do mathematics correctly. Again I will emphasise David's long experience as a mathematics teacher and hence the knowledge he had gained about students' possible errors and mis-

[59] I am aware that "ponders" is not an appropriate English word. Neither is the Norwegian word "grublis" which I have translated into "ponders". To ponder in English is "gruble" in Norwegian.

conceptions which he wanted the students to avoid by telling them how to do the mathematics. Based on his experience he had developed awareness and a sense of what the students were going to say and he used his experience and knowledge to save time and to save the students for extra work by telling them the right way to work out mathematics rather than they should find it out on their own. This is in accordance with what I presented in the section about David’s beliefs about teaching and learning mathematics earlier in this chapter.

Characteristics of David’s teaching

In the table below I present characteristics of David’s teaching that I found most prominent in my work with him. The characteristics are presented in the left hand column and examples from conversations and classroom observations are presented in the other columns as evidence for these characteristics. I also quote relevant statements from L97 with regard to the characteristics I present.

David

<table>
<tr><th>Characteristics</th><th>Conversations</th><th>Classroom observations</th></tr>
<tr><td rowspan="2">David had professional pride and demonstrated a high level of professional confidence and he addressed his students in a friendly way.</td><td>Based on what he said he liked teaching and he looked upon himself as doing a good job and he enjoyed the company with his students (Jan 14th, post).</td><td rowspan="2">When he reprimanded students he did it in a subordinate aside. In all lessons there was a spirit of good fellowship and the teacher demonstrated that he was an experienced teacher with good subject content knowledge.</td></tr>
<tr><td>He said he managed to push and prompt his students (March 3rd, post)</td></tr>
<tr><td colspan="3">L97: The most important tool the teachers have is themselves. For this reason they must dare to acknowledge their own personality and character, and to stand forth as robust and mature adults in relation to young people (p. 38)</td></tr>
<tr><td rowspan="2">David’s teaching style was transmitting. He presented detailed explanations of how to solve mathematical exercises</td><td>Based on what David said he liked to be a conveying teacher although he acknowledged it not being politically correct (March 3rd, post)</td><td rowspan="2">In all lessons I observed David was explaining or showing and telling. I found a typical pattern of discourse in his lessons where he was in charge of the course of the lesson</td></tr>
<tr><td>He said that students learn best if the teacher explains well (Jan 14th, post)</td></tr>
<tr><td colspan="3">L97: Good teachers have a sure grasp of their material, and know how it should be conveyed to kindle curiosity, ignite interest and win respect for the subject (p. 37)</td></tr>
<tr><td>When presenting a new mathematical topic, David linked to students’ prior knowledge and thus inter-</td><td>He outlined how he would connect what was new when solving equations with two unknowns to what they had already done(Jan 14th, pre)</td><td>In the monologues he presented an overview of the lesson and related what they should work with to what they had done so far.</td></tr>
</table>

<table>
<tr><td>wove familiar and new knowledge.</td><td>He said he would link graphical solutions of equations to algebraic solutions and to functions on which they had worked earlier (Jan 21st, pre)</td><td>In the lecturing parts of all lessons he asked closed questions to link what he was presenting to what students were supposed to have learned</td></tr>
<tr><td colspan="3">L97: Mathematics has a variety of aspects, and learning takes place in a variety of ways. Pupils’ experience and previous knowledge, and the assignments they are given are important elements in the learning process (p. 167). [ ] Pupils must be challenged to build up chains of reasoning and combine knowledge from various areas of mathematics (p.167)</td></tr>
<tr><td rowspan="3">David focused on mastering methods, exercising skills and procedures.</td><td>The goal for the lesson was to master the method of solving equations with two unknowns (Jan 14th pre).</td><td>He told the students that it was important to master the methods when solving equations with two unknowns (Jan 14th and Jan 21st)</td></tr>
<tr><td>He said it did not matter if some did it (Pythagoras’ theorem) mechanically (Feb 11th post)</td><td>He focused on the technical use of Pythagoras’ theorem (Feb 11th)</td></tr>
<tr><td>He did not look upon it as wrong telling the students how to solve a task (March 3rd, post)</td><td>He focused on the 30-60-90 rule (March 10th)
In all lessons he gave students tasks to exercise skills and procedures</td></tr>
<tr><td colspan="3">L97: Pupils’ own activities are of the greatest importance in the study of mathematics. The mathematics teaching must at all levels provide pupils with opportunities to ….[ ] - exercise skills, knowledge and procedures (p. 168)</td></tr>
<tr><td rowspan="4">David had concern for students’ different abilities in mathematics.</td><td>Based on what he said he talked mostly to the clever students when reviewing tests (Jan 21st, pre)</td><td rowspan="2">He gave some of the weaker students other tasks to work with so they could get a “three” in mathematics. They did not have to practice solving equations with two unknowns (Jan 14th)</td></tr>
<tr><td>He prepared exercises with different level of difficulties for different students (Jan 14th pre)</td></tr>
<tr><td>He claimed that the clever students often become losers in our school system.</td><td rowspan="2">Clever students do not have to do trivial exercises in geometry (March 10th)</td></tr>
<tr><td>He said he would leave for the students to choose what exercises they should do (March 10th, pre)</td></tr>
<tr><td colspan="3">L97: The teaching of mathematics must be attuned to the abilities of individual pupils, who must be given tasks which they find meaningful and capable of carrying out (p. 166)</td></tr>
</table>

Table 16, Characteristics of David’s teaching

What David said he believed in and about students' learning were consistent with what I observed in his lessons in the classroom. Before the lessons he said he was going to show and tell; that they were going to exercise methods and procedures; that he was going to give weaker students different exercises rather than solving equations with two unknowns; how he was teaching in interaction with the students (asking easy manageable closed questions) and even some of the questions he was going to ask. In the conversations he also told me about the spirit of fellowship which I sensed in the lessons. I thus observed a great extent of coherence between David's beliefs about the teaching of mathematics – what he said he was doing and what he said he believed in as good teaching practice - and his classroom practice with regard to the characteristics presented above. This gives me part of an answer to my third research question: How are teachers' practices in the classroom related to their beliefs about teaching and learning mathematics?

In the table above I quoted relevant passages from L97 with regard to each characteristic. With regard to the characteristic of David as a teacher who conveyed and transmitted mathematics, the quotation from L97 that I have linked to this characteristic, "good teachers have a sure grasp of their material, and know how it should be conveyed to kindle curiosity, ignite interest and win respect for the subject" (L97 p.37), is far more comprehensive and incorporates a lot more than showing and telling which I found most prominent in David's lessons.

According to the general aims for mathematics in the mathematical part of L97 (p. 170), students are supposed to develop a positive attitude to mathematics; mathematics shall become a tool which they shall find useful in school, in leisure activities, in work and social life; students shall be stimulated to use their imaginations, personal resources and knowledge to find methods of solutions and alternatives through exploratory and problem-solving activities; they shall develop skills in reading, formulating and communicating issues and ideas in which it is natural to use the language and symbols of mathematics; they shall develop insight into fundamental mathematical concepts and methods and develop ability to see relations and structures; and they shall develop insight into the history in mathematics and into its role in culture and science. These aims for mathematics as a school subject were, according to the leader of the committee developing the written mathematical part of L97, taken from the Cockcroft Report (1982). However, there was one point from the Cockcroft Report that was not taken into L97: the point about *teacher's exposition*. This emphasises that the teacher's exposition is consciously toned down in L97, and the weight I find that David put on the teacher's exposition is not in line with L97's intentions.

The other characteristic presented which I claim is not in line with L97's intention is the focus on methods and exercising skills and procedures. Although L97 says that students shall have the opportunity to exercising skills and procedures, this is only one of many intended approaches to the study of mathematics presented in the curriculum. The six activities which L97 explicitly says that mathematics teaching must provide opportunities for the students to do are to:

> carry out practical work and gain concrete experience; investigate and explore connections, discover patterns and solve problems; talk about mathematics, write about their work, and formulate results and solutions; *exercise skills, knowledge and procedures*; reason, give reason, and draw conclusions; work co-operatively on assignments and problems (L97 p. 168, my emphasis).

Focusing nearly exclusively on methods and exercising skills and procedures as I found in David's lessons indicates that L97 was not implemented with regard to the other aspects of mathematics. However, David was teaching according to what he said he did, and also based on what he said, according to what he believed in with regard to how students learn mathematics.

When presenting the two other teachers, I have pointed out constraints influencing their teaching practice seen in relation to their beliefs about the teaching and learning of mathematics. For Bent I claimed that the constraints were lying between the teacher's beliefs and his classroom practice and for Cecilie I claimed that the constraints preventing the teacher to teach according to her beliefs were found in the classroom practice, in the enacted curriculum. With regard to David, he mentioned the final exam as a constraint preventing him for not having time for play and exploring activities, but I see David as a teacher who was teaching according to his beliefs. However, studying the teaching through the lenses of L97, the constraints preventing the teacher to teach according to L97's recommendations were lying in the teacher's beliefs. I see a teacher who did not believe in many of the L97's recommendations, and therefore he did not teach according to those recommendations.

9. Synthesis and conclusions

In this final chapter I will

- Draw together the findings from Chapters six, seven and eight and provide further evidence to support the individual findings from each teacher;
- Under the heading “One curriculum intended, three enacted” provide a cross-case analysis of data from the three teachers and some findings related to the research questions;
- Offer a reflection on the analytical tool developed, on strengths and limitations in my research and on curricula coming and going, before I suggest some consequences for teacher education and further research.

Further evidence to support my findings

In the chapters on methodology and methods I wrote that I also used a questionnaire (appendix 1) as a research method to find out what the teachers were thinking about teaching and learning mathematics and about their view on L97. I did not refer to how they responded to this in the analysis of each teacher, preferring rather to refer their responses in this synthesis. The questionnaire served two purposes; both as a validation of my findings from the analysis of each teacher and also as a means to compare the three teachers across the same questions. The latter I found especially important with regard to questions involving L97 and also for questions which involved the teachers’ views on mathematics and on teaching and learning mathematics.

The questionnaire

Two questions in the questionnaire involved L97 explicitly. Below I present the question, the response options (in brackets) and each of the teachers’ responses.

When you plan a mathematics lesson, to what extent do you take the following into account: (great, some, little, no extent)	**Bent**	**Cecilie**	**David**
A lesson you have just had	some	some	some
The textbook	great	no	great
A joint developed teaching plan at school	little	little	little
Other teachers teaching mathematics	some	no	little
The textbook’s guidance	little	no	little
The curriculum, L97	little	great	little

Table 17, Teachers’ responses to question 1 on the questionnaire

The teachers’ responses to the first question show that Cecilie was the only teacher who answered that she to a great extent was taking L97 into account, and who did not use the textbook in her planning of a mathematics lesson. David's and Bent's responses to this question were almost the opposite of that of Cecilie which emphasises my presented findings based on the analysis of conversations and classroom observations. Both David and Bent used the textbook in every lesson, and they also said that they did not use L97 in their daily planning.

Where do you search for information when you shall (L97, local syllabus, textbook, students' wishes, other)	**Bent**	**Cecilie**	**David**
decide topics and goals for the teaching	L97	L97	L97 and textbook
choose how to present the topic	textbook	other	Textbook, other
choose problems and exercises for work in class and homework	textbook	other	Textbook, other
choose problems and tasks for assessing the students	other	L97	other
let students do theme- and project work	other	other	Local syllabus

Table 18, Teachers' responses to question 2 on the questionnaire

Also the teachers' responses to the second question, concerning where they searched for information when deciding topics and tasks, show that Cecilie did not rely on the textbook but rather on other resources and L97. This is consistent with what she said in conversations and also with what I saw in her classroom. They all responded that they searched for information in L97 to decide topics and goals. Although in a conversation David said that he never leafed through that book (L97), I suggest that he had some ideas as to what was presented in L97 as mathematical topics, and the textbook he used was approved according to L97.

In addition to the questions above, the teachers were asked to what extent they agreed; *strongly agree*, *agree a little*, *disagree a little* and *strongly disagree* with some statements. There was no option to be "neutral"; hence they had to take a stand. Below I present how they responded to the different statements, and how their responses relate to earlier presented findings from the analysis of the teachers and also to some of L97's recommendations.

- When being asked about their view on mathematics, they all strongly agreed that "*Mathematics is an interesting and challenging subject*". Both in focus groups, in conversations and in classroom observations I experienced that they were interested in the subject, and I perceived enthusiasm for the subject in their classrooms.
- David strongly agreed that "*rules and routines are essential parts of the subject*", to which Bent agreed a little and Cecilie disagreed a little. This is also consistent with my other findings, what they

said and what I observed that they emphasised in their classes. David focused more on exercising drill and procedures than the other teachers.

- They all agreed a little that "*mathematics is to find the right answer to a problem*" which emphasises what has been traditionally focused upon in schools and exams, however not emphasised so much in L97. Both Cecilie and David agreed a little that *"the solution of a mathematical task is either right or wrong"* on which Bent strongly disagreed. This demonstrates some of the open-mindedness to a wider perspective of mathematics I perceived Bent had, and emphasises David's more traditional view. With regard to Cecilie this reflects a more absolutist view on mathematics than what I experienced she had when preparing the lessons, and different from what she expressed as her view upon the teaching and learning of mathematics.
- Cecilie strongly agreed with the two statements "*mathematics is mainly an abstract subject"* and *"to master mathematics you need innate gifts for the subject"* which reflects what I experienced in her class. Sometimes she had long dialogues with one single clever student while other students were not paying attention because the topic discussed was beyond their interest and/or understanding. Bent's responses were the opposite. He strongly disagreed that mathematics is mainly an abstract subject and that to master mathematics you need innate gifts. He rather expressed a view that mathematics is for everybody, which I also experienced in his classroom by the way he supported the students in their building of mathematical structures. In the analysis of Bent's lessons I indicated that the experiences he gave the students in their work supported the students in developing their abilities to conceptualise the mathematics they were struggling with. David's view was closer to that of Cecilie, but he agreed only a little. This seemed to be reflected in how he worked with his students. He prepared separate tasks for the "weaker" students, so they did not need to struggle through complicated algebra tasks but rather to practice mathematics they could master on the exam.
- All three teachers strongly agreed that *"Mathematics is a practical and systematic way to solve real problems"*. This is in line with one of the general aims for mathematics in L97: "for mathematics to become a tool which pupils will find useful at school, in their leisure activities, and in their working and social lives" (p.170).
- They all strongly agreed that "*Mathematics is processes, generalisations, and understandings*". These aspects are also reflected in L97. With regard to David, this is consistent with how he in the

estimation form valued the process aspect of ideal teaching (with 15 points). He also estimated the process aspect in his own teaching with 15 points. However, as I pointed out in the chapter on David, this aspect was not the one I found prominent in his classroom. I found rather the procedural aspect being more focused there. Bent and Cecilie also valued the process aspect highly in ideal teaching (13 and 15 respectively). They estimated their own teaching with 7 and 12 points respectively, which indicates that they both realised that they have, as Bent expressed it "a way to go" with regard to the process aspect, and according to their estimation Bent acknowledged a longer way than Cecilie.

In responding to the statement "*This is important when the students shall learn mathematics*" all three teachers strongly agreed or agreed a little in the following statements:

1. *That they [the students] must explain a reasoning*
2. *That the teacher presents new material on the board* (David strongly agreed, Bent and Cecilie agreed a little)
3. *That the teacher explains errors they have made* (David strongly agreed, Bent and Cecilie agreed a little)
4. *That they shall discuss mathematical tasks with each other*
5. *That they shall solve mathematical tasks in groups*
6. *That they get individual supervision from the teacher*
7. *That the student works on his/her own*
8. *That they reflect on what to be learned*
9. *That they take what they already know as a starting point*

I experienced all these activities with all three teachers and especially those presented in 2, 6, 7 and 9. To emphasise the weight David put on presenting new materials on the board and explaining errors, I have indicated his strong agreements in brackets with statements 2 and 3 above.

The 10th statement in the questionnaire was "*that the student shall explain an error s/he has made*". David strongly agreed whereas Cecilie agreed a little and Bent disagreed a little. This together with David's response on the third statement above emphasises the focus I experienced David had on students' errors and how to "avoid traps". None of the teachers agreed on the statement that "*learning material should be organised thematically across subjects*", which is encouraged in L97, but which I did not see with any of the teachers.

Focus Group 4

The last focus group I had with the teachers who had been part of my study took place towards the end of my work with them. I have chosen to comment briefly on my findings from Focus group 4 in this final chapter for the purpose of cross case-analysis and also to illuminate and validate my findings from the rest of my study with the teachers.

I had asked the teachers to prepare two issues to share with the group; first, one issue they felt they had succeeded in carrying out as a mathematics teacher and one issue they felt they not yet had accomplished. They found the task difficult. However, after a few minutes discussing and reflecting on the difficulty of the task, Cecilie volunteered to start with hers (to which I referred in Chapter 7). She felt she had succeeded in challenging and motivating the clever students, which is in accordance with what she had expressed in our conversations. The task she felt she had not yet accomplished was enabling the students to copy out their written work in mathematics clearly. Bent responded by expressing that more important for the students than the written presentation of mathematics is for them to understand when to multiply and when to divide in working it out. This emphasises Bent's focus on students' conceptual understanding which I also found through my work with him in the classroom and in our conversations.

Bent chose to present issues from two of the lessons I had been observing with regard to what he felt he had succeeded in and what he not yet had accomplished. His presentation of the issues revealed that he had been reflecting on these lessons. About the fraction lesson he said that he felt he had succeeded to a certain extent. However, he could have done more with it. With regard to the use of concrete materials, he expressed a disappointment that the effect had not been as intended. It had however been better in the other 9th grade class he was teaching. He thus expressed a feeling of having succeeded with the use of concrete materials in that class (in which I did not observe). This suggests that the complexity of the classroom and the classroom discourse often influence the outcome of an activity, and thus the enacted curriculum which is jointly constructed by the teacher and the students and the materials used.

Presenting what he felt he had been successful with, David said: "I have managed to make them cleverer in doing percentage calculations". This emphasises how he looked upon himself as conveying mathematics to the students and that students' learning is dependent on the teacher's ability to explain. When he was asked by the others in the group how he had done it he said: "It is just to explain as well as possible". This emphasises further how he looked upon explaining as the most "efficient" teaching strategy. However, he also offered an elaboration of how he had done it which revealed that he as a teacher was consciously systematic when presenting mathematics for his students. He said:

> I have been very systematic with percentage types 1, 2, 3, 4, 5. Therefore, when one of the types turns up, I refer to the type. Number 1 is like "3 students absent how many percent?" Then it is in connections with changes, then having to calculate backwards, and then comparing two numbers.

David's systematic way of preparing the mathematics to be taught was a feature in his teaching.

With regard to what he had not yet accomplished, David focused on kinds of errors students made, especially how they used the equal sign wrongly, and he also supported Cecilie in her suggestion: how to enable students to copy out mathematics in a lucid written way which clearly showed how they had solved the task.

What was said in this last focus group emphasises my findings from the analysis of the individual teachers: Cecilie felt she was successful in her work with the clever students, but had difficulties enabling students to present written mathematics with a clear overview; Bent reflected upon both success and not-yet-accomplished aspects of the issues presented; and David felt success in explaining and had not yet found out how students could avoid making errors.

However, this last meeting provided me with information beyond what I had observed in the classroom, and what I had talked with the teachers about in the conversations. Bent offered his reflections around his work with fractions and use of concrete materials. Cecilie shared her difficulties with enabling students copying out their written work clearly, in which David supported her. By challenging David about *what* he had done to make students become good in percentage calculations we were initiated into a systematic way of preparing his teaching. This demonstrates that the use of focus groups provide researchers with information beyond what can be obtained otherwise.

One Curriculum intended, three enacted

In the literature review I referred to research which revealed that there were great differences in teaching practices, both within the same nation (Stigler & Hiebert, 1999) and also within the same school (Kilpatrick, 2003a; Stigler & Hiebert, 2004; Tarr, Chávez, Reys, & Reys, 2006). Several explanations have been given why teaching style both across schools and within the same school vary (Collopy, 2003; Rowan, Harrison, & Hayes, 2004). Hence the enacted curriculum has been subject of research when investigating how teachers respond to curriculum reforms.

In my research I have studied one curriculum, L97, and three teachers. In this part of the synthesis I will discuss the differences in teaching styles among the teachers I found, and hence three different enacted curricula.

In presenting the three teachers, I chose to write one chapter on each. First I presented the analysis of the conversations and then the analysis of the classroom observation with each teacher. I concluded with a portrait of each teacher including some characteristics of each teacher's teaching with relation to L97. This portrait is partly verified by the teachers' responses to the questionnaire.

For the purpose of contrasting and comparing the three teachers I have made the overview in the table below. There is one column for each teacher. In the first row I quote an expression from each teacher about L97; in the next I have chosen a quotation which according to the analysis of each teacher characterises his/her beliefs about teaching and learning mathematics; in the third row I present the most significant teaching strategies based on the analysis of classroom observations. This overview allows me to compare the teachers across what they said about L97, across their beliefs and across their teaching strategies. The last row deals with "constraints" and issues which I discuss later in this section.

	Bent	**Cecilie**	**David**
Quote about L97	"If the students shall be more exploring like L97 encourages, I think it will take more time. However, I believe they learn better that way" (April 1st post)	She said that she liked L97 and "when I start a new topic, I read L97" (Jan 21st post)	"I never leaf through that book" (David about L97)
Expressed thoughts about teaching and learning mathematics	"I think students learn best by exploring things themselves" (from Bent's writings)	"students are supposed to explore things themselves,[ ] it becomes more exciting that way and I believe they will learn some mathematics they won't learn by learning formulae by heart" (Jan 21st post)	"I think students learn best and quickest if they get it clearly and well explained" (David in FG3)
Teaching strategies observed in the classroom	He probed students' thinking through the way he asked questions and responded to their questions	She prepared for exploring activities and linked school mathematics to mathematics history	He explained by showing and telling and exercising skills and procedures
Constraints and issues	There were constraints between his beliefs about good teaching which are in line with L97, and his classroom practice	Her visions about good teaching and L97's recommendations were not so easily transformed into the classroom. Mismatch between mathematical focus and students' abilities	There was lack of agreement between his beliefs about good teaching and L97's recommendations

Table 19, Overview of the three teachers

The teachers' responses to the questionnaire as well as the overview which is based on the analysis of each of the teachers, show that they responded differently to the curriculum L97, both in terms of what they expressed and also in terms of what they did in the classroom. Hence from one intended curriculum, I saw three different curricula enacted.

David said very explicitly that he did not relate to L97, which also characterised his teaching. However, he thought very carefully about his way of teaching which had developed throughout many years of teaching experience and work with mathematics. He expressed a greater belief and faith in his own judgement of good teaching than what was recommended in L97. His beliefs about teaching and learning mathematics were thus very socio-culturally rooted both in his own educational background, in his own experience as a teacher and in the school context. He had experienced that his way of teaching mathematics had worked well; he had experienced success as a teacher; his students performed well on exams and according to what he said, students and parents liked his way of teaching. David demonstrated a sure grasp of mathematics as a subject, and he always had an answer ready to present when a student asked for help. He also strongly advised the students what to have in their rule books and he handed out photocopies he had made for them to paste in. Their use of the rule books on the exam had shown to work well. Based on this there was no reason for David to consider changing his way of teaching. Why should he?

David and Cecilie were teachers at the same school, Dalen. They did not collaborate in their daily work when preparing lessons. This is emphasised by how they answered the question: "When you plan a mathematics lesson, to what extent do you take other teachers' teaching of mathematics into account?" in the questionnaire. David ticked off "little extent" and Cecilie ticked off "no extent". David and Cecilie had two 10^{th} grade classes each. There were six 10^{th} grade classes in the school. A third teacher had the other two. He was not involved in my project, but David was a second teacher in his two classes once a week. I refer to this towards the end of the chapter in seeing a collaboration between teachers as a possibility for professional growth.

Unlike David, Cecilie expressed that she liked L97 and that she used it in planning her lessons. In the conversation I had with David and Cecilie together, when David said he never leafed through L97, but that he rather wanted to focus on "misuse of statistics" because it was fun, Cecilie retorted: "Then [if you leaf through it] you will probably find out that it is written in L97" (Jan 28^{th} pre). This was the only time I could perceive a small tension between the two teachers with regard to their views on L97 and its recommendations. To me they seemed to respect each other's view, and also each other's way of teaching and planning lessons. They also had common “whole-day tests” which 10^{th} grade classes in the same school usually have; it is part of the socio-cultural practice in schools, not a central given law but a common practice.

Cecilie was the only teacher who said that she actively used L97 in her teaching and that she rarely used textbooks. Just like David rather “used his own head” (David’s own expression) rather than L97 in his teaching of mathematics, Cecilie used her own ideas and ideas picked from other literature rather than the textbook. “I liked it [i.e. L97], but I did not like the textbooks following it”, she said. Thus both Cecilie and David can be seen as teachers who had faith in what they were doing, and who had made their own judgements how to teach and on what aspect of mathematics to focus. They had constructed their own conceptions of good mathematics teaching based on their own ideas and experience. Cecilie believed that students learn best from exploring things themselves. They also learn some mathematics they would not learn by only using “ready made” formulae. She therefore prepared for “exploring activities”. However, the way it turned out in the classroom, the enacted lessons was that she being *the teacher* did the exploring and the students were channelled through the activity by answering easy manageable closed questions. Another significant aspect in the course of her lessons was that many students lost track throughout her exploration and stopped paying attention. A few clever students followed her and contributed with comments and suggestions. This shows that factors such as having students with different mathematical abilities and different interest for the subject in the same classroom and the complexity of the classroom, in which there were contradictory demands on individual students (some students were very interested and captured the teacher’s attention while others talked to their class mates), influenced the enactment of the lesson. This suggests how Cecilie’s visions about doing exploring activities were not so easily translated into her classroom practice.

The third teacher, Bent, expressed that he both wished and thought he ought to do more exploring activities, as recommended in L97, than he currently did. He thus expressed more uncertainty about his own teaching than the other two teachers. Contrary to Cecilie, who was able to prepare exploring activities, Bent indicated that he was not sure how to do that. He said “there I have a way to go myself”. Bent reflected more than the other two teachers on how he saw himself as not yet sufficiently accomplished as teacher. Furthermore he demonstrated a more inquiring attitude towards his practice than the other teachers.[60]

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[60] In that connection I as a teacher educator at a teacher training college, am tempted to reflect upon the educational backgrounds of the three teachers. Bent was the only teacher having his education from a teacher training college where mathematics and didactics are taught as one interrelated subject, whereas the other teachers had studied mathematics at a University first and then taken “Pedagogical

In addition to admitting that he did not know how to do exploring activities, Bent suggested other reasons for not responding adequately (as he saw it) to L97. I have referred to these as “constraints” in the chapter on Bent. Time pressure and parents’ and students’ expectations were the most evident ones. In the analysis of the observed lessons with Bent, I point out how he was dealing with a highly complex classroom with many disciplinary issues and with demanding students. However, I also point out that he took the often demanding students’ contributions into account in whole class and that he challenged and structured their thinking during individual seatwork. These were also elements of teaching mathematics reflected in L97, a challenge he thus seemed to have accomplished.

In the analysis of Bent in Chapter 6 I presented his reflections on how much time to spend on conceptual understanding as opposed to the method of mastering a procedure and that some students are happy just knowing the rule and using it. David also expressed the same kind of awareness with regard to relation between students’ abilities and working methods, which I discussed in Chapter 8. There was a difference, however, in how the awareness was presented. Whereas Bent offered a reflection on and expressed an uncertainty how much weight to put on computational methods as opposed to relational understanding, David expressed a certainty that the weak students would manage in the classroom while exercising procedures, but would forget later because, as he said, “they won’t digest it”. He demonstrated an acceptance of that.

Both Bent and David expressed a view that for the weaker students it is better to focus on the method than to spend a lot of time to explain the *why*. Bent expressed an uncertainty about how much time to spend on the *why* to make a few more students understand, and David said that there is a “balance” how much time to spend, and therefore some students can rather “do it mechanically”. In the analysis of Cecilie, I presented an overview of the relation between the working methods and students’ abilities in her teaching. I suggested that she focused more on methods and the procedural aspect of mathematics for the weaker students than for the clever ones. Hence, an indicated relation between focus on the procedural aspect of mathematics and students’ abilities was common for all three teachers.

Three types of teaching

In the Literature Review I referred to findings in research more widely how teachers, even when teaching in the same school, responded differently to a reform, Based on the overview (on page 305), which is a con-

Seminar” or “Practical Pedagogical Education” as a separate course. However, having studied only one teacher with educational background from teacher training college, I will not make any generalisation of that as an issue.

densed version of some of the findings from the analysis of the three teachers, I see three types of teaching:

- Bent focused on students' conceptual understanding. In his teaching he challenged students' thinking and encouraged them to see connections between different mathematical entities. He was thus "bridging" between previous and new knowledge.
- Cecilie prepared exploring activities. She expressed a belief that students learn best by exploring things themselves, and that they then will discover mathematics which cannot be learned from only using ready made formulae. From my perspective, the lessons turned out differently from what the teacher (according to what she said) had intended. The teacher carried out the exploring activities through which the students were channelled by easy manageable questions.
- According to David the best way for students to learn mathematics is to have it well explained. The mathematical focus in his lessons was procedural and the discourse in the lessons was characterised by him showing and telling as if mathematics could be transmitted from the teacher to the students.

Although I have only analysed three teachers in detail and that led to three different types or styles of teaching, my feeling is that if I had analysed the fourth teacher in my study, Alfred, as thoroughly as the other three teachers, he would have fit very close to David. These three styles of teaching can be compared with the three models which Askew et al. (1997b) and Askew et al. (2000) characterised in their study and which I referred to in the Chapter 2: A Connectionist who emphasises the connections within mathematics putting weight on sharing ideas (as Bent was aiming at); a Discovery orientation whose view is that mathematics ought to be discovered by the students (which Cecilie expressed as her intention); and a Transmission orientation whose view is that mathematics consist of a set of routines and procedures which can be transmitted from one person to another (which was most prominent in David's lessons).

Three types of constraints

In Chapter 2 I discussed obstacles, constraints and issues in teachers' decision making which are pointed out in mathematics educational research. As an outcome of the analysis of the three teachers in my study, I see three types or levels of constraints influencing the different stages in teachers' implementation of a curriculum. To answer my first research question, *how are teachers in their mathematics teaching practice responding to the L97's recommendations*, I have had conversations with the teachers (both in focus groups and individual conversations), estimation form, questionnaire, teachers' writing about ideal teaching and classroom observations. Based on what the teachers said about L97 and about their own teaching related to L97, I have got ideas of what beliefs each teacher had about L97. I see these expressed beliefs which are highly influenced by socio-cultural factors as one level of possible constraints preventing the teacher from implementing a reform curriculum. If a teacher does not believe in the reform, if s/he does not want to teach according to it, if s/he believes that the way of teaching mathematics s/he has always done is the best way, then one cannot expect that s/he implements the curriculum. I look upon as this as one type of constraints. These constraints which are preventing the teachers from implementing the reform are lying in the teacher's beliefs. This is the level of constraints that I found most visible in David's teaching.

The second type of possible constraints influencing the teacher in another stage is seen when the teacher expresses a wish to implement the reform. A teacher believes in the reform, s/he believes that L97's recommendations enhance students' possibilities for learning mathematics, but does not teach according to this to the extent s/he wishes because factors like parents' expectations, students' demands, the work plan and lack of time are constraints that prevent him/her from doing it. These constraints are lying between the teacher's beliefs and his/her teaching practice in the classroom, and they influence the extent to which the teacher teaches according to his/her beliefs. This was where I found the constraints in Bent's teaching most visible.

The third type of possible constraints is seen when the teacher believes in the reform, prepares the lessons according to it by choosing exploring activities and thus an investigative approach to teaching as L97 recommends. However, the way it turns out in the classroom becomes quite traditional. The constraints are in the classroom. The teacher's classroom practices together with the complexity of the classroom are the constraints; they are lying in the activities jointly constructed by the teacher, the students and the teaching material used, in the enacted curriculum. This was most visible within Cecilie's teaching.

Research questions and related findings

How are teachers in their mathematics teaching practice responding to the L97 curriculum?

I have studied three teachers who responded differently to L97. One teacher did not relate to the curriculum. His teaching seemed sometimes to be contradictory to the curriculum. However, that does not mean that it was not possible to see a lot being valuable in his teaching. The other two teachers related to the curriculum, and they saw a lot of what L97 suggests as valuable and they responded to that. However, there were constraints preventing them from implementing L97's suggestions. These constraints were explicitly expressed by one of them, and I have suggested the complexity of the classroom as a constraint resulting in a lesson that turns out differently from what was teacher's intention. Identifying and highlighting the complexity of the classroom as constraints illuminated the difficulty of the transition of a teacher's vision about teaching into the teaching practice.

What kinds of teaching practices are observable in the mathematics classroom?

In the portraits of each teacher I have identified a wide range of teaching strategies they used. Here I point out a few strategies which I found were used by two or all three of them:

- They invited the students to participate;
- They reminded the students about their previous knowledge;
- They facilitated tasks by simplifying numbers;
- They used concrete materials;
- They challenged and structured students' thinking through their questions;
- They took students' abilities into account when giving support;
- They shared a student's contribution with the rest of the class so all students had the possibility to participate in the process.

How are teachers' practices related to their beliefs about teaching and learning mathematics?

As discussed in the Literature Review (Chapter 2) there have been reported varying degrees of consistency between teachers' beliefs and their practice. There are many factors influencing a teacher's decision making when teaching in the classroom, and I referred to Skott (2001b) who claimed that teachers cannot be inconsistent, and that if inconsistency is observed, that is from the observer's perspective. In my research I have emphasised that the findings presented are findings from my perspective through the conceptual lenses I have used, hence my agreement with Skott. Therefore, I have tried to account for the relation between what I interpreted as a teacher's beliefs and his/her teaching practice and to identify some constraints being in the way. Leatham (2006) suggests that some beliefs are more central than others, for example the wish to keep control of the class is more central than believing in group work. The constraints I identified can be looked upon as beliefs being more central than other beliefs. A belief that it is important to comply with demands from parents and thus to teach from the board, seemed to be more central to Bent than his belief that students ought to engage in exploring activities. For Cecilie a belief that doing (showing on the board) exploring activities was important seemed to be more central than a belief that all students in class ought to participate in the activity. With regard to David, he did what he said he did and what he believed was the best way to teach and thus for students to learn. Thus in the case of David, consistency was observed. In the analysis of David I characterised his teaching as being traditional in style. Thompson (1992) reports findings in research about seemingly higher degree of consistency between teachers' beliefs and their teaching practice when they express traditional conceptions about mathematics teaching. Thus in the case of David, the strong relation I found between what he said and what he did is also recognisable in the literature. However, in the estimation form, David valued the process aspect highly with regard to his real teaching. That was inconsistent from my perspective as an observer. In retrospect I wish I had discussed this with him in one of our conversations. My conjecture is that he looked upon the students' learning as a process for which he through his teaching created conditions.

Concluding remarks and the way ahead

To conclude this thesis I will first offer some reflections on the analytical tool developed in this research and on strengths and limitations in my work. Then, under the heading "Curricula come curricula go – the classroom practices endure" I discuss the role of curricula and how frequent changes in curricula may influence their effects. Finally I present possible consequences my research can have for teacher education and further research.

The analytical tool developed

I see generality in my research lying in the methodology used; both in the use of the different research methods and in applying the three categories Conditions for possibilities of learning, CPL, Mathematical focus, MF, and Students abilities, SA, in the analytical process. These categories which became an analytical tool arose through the analysis of one of the teachers and showed its applicability in the analysis of the other two teachers. The use of these categories in the analysis has revealed issues which are important in the developmental process of teachers.

Having developed these three categories and applied them in the analysis of the teachers, I became aware of a potential correspondence between this tool and a construct in the literature, called the "Teaching triad" (Jaworski, 1994; Potari & Jaworski, 2002); the categories are sufficiently similar for me to make a comparison. The teaching triad emerged from a grounded theory of analysis of teachers who wanted to teach mathematics investigatively (Jaworski, 1994). The applicability of the teaching triad was shown in later studies (Potari & Jaworski, 2002; Zaslavsky & Leikin, 2004). The three categories in the triad are: "Management of learning" (ML) which describes the teachers' role in the constitution of the classroom learning environment by the teacher and students; "Sensitivity to students" (SS) which describes the teacher's knowledge of and interactions with students and attention to their needs, the ways in which the teacher interacts with individuals and guides group interactions; and "Mathematical challenge" (MC) which describes the challenges offered to students to engender mathematical thinking. Unlike the categories developed in my research, the teaching triad was developed through a study of teachers who wanted to teach investigatively and the teachers were involved in the researcher's development of the construct (Jaworski, 1994). In the later study (Potari & Jaworski, 2002) the teaching triad was used by the researchers and also by the teachers in doing research on their own practice for the purpose of professional development. The concept of "Harmony" between the three elements in the teaching triad had then emerged as "a key factor in explaining the apparent success of a teaching/learning episode" (Potari & Jaworski, 2002, p. 357).

Linking to my study I see similarities between Conditions for possibilities of learning, CPL, which includes teachers' teaching strategies, aspects of classroom culture and discipline and Management of learning, ML in the teaching triad; between Mathematical focus, MF, including conceptual, procedural and conventional aspects and the mathematical topic studied in my study and Mathematical challenge in the teaching triad, and Students' abilities, SA, including aspects of differentiating, how different students learn and issues about different students' mathematical knowledge based on their achievements and expressed by the teacher and Sensitivity to students in the teaching triad.

I have indicated a mismatch between mathematical focus and students' abilities which was especially visible in some of Cecilie's teaching. Using the constructs from the teaching triad, this mismatch could be expressed as lack of harmony between mathematical challenge and sensitivity to students (Potari & Jaworski, 2002).

This indicated relation to a theoretical construct which was already in the literature, suggests that the analytical tool I developed can be recognised more widely. Although I knew about the teaching triad before I started my research, that was not influential in the development of the categories in my study. The categories Conditions for possibilities of Learning, Mathematical focus and Students' abilities emerged from the multiple codes I used in the initial analysis of my data and as I described in Chapter 6 about Bent.

Strengths and limitation in my study

Following scientific criteria suggested in the literature, (Bassey, 1999; Bryman, 2001; Jaworski, 1994; Kilpatrick, 1993; Sierpinska, 1993) I will offer a reflection on some strengths and limitations in my research. The first issue to address is the *Relevance* of my research and one question to ask is: Was the research worth doing, and for whom? This question was addressed partly in my discussion of ethical aspects in the research project in Chapter 4, where I indicated that research not worth doing is waste of time for people involved.

According to Sierpinska (1993) relevance of a research study is related both to the research questions and the outcome of the research study. Although an ultimate goal of mathematics educational research might be that it could lead to improvement of teachers' teaching practice and a study's "relevance to teachers typically comes when a line of investigation, a set of studies has been synthesized to yield some clear implications for practice" (Kilpatrick, 1993, p. 19), "every research study in mathematics education contributes to the shared knowledge needed for a profession" (Kilpatrick, 1993, p. 20). In mathematics education the profession involves teachers, teacher educators and researchers, thus the outcome of a study like the one I have undertaken, will contribute to knowledge needed for teachers, teacher educators and researchers.

Other criteria to address in research are *validity, rigour and trustworthiness.* I addressed the latter in Chapter 4 (page 92) when discussing the different research methods used in my study. A research study is not valid in itself, but the validity refers to the conclusion drawn from the study. A criterion for internal validity is that there is a good match between researchers' observations and theoretical ideas developed (Bryman, 2001). The related criterion, rigour, in a project is to show how conclusions drawn are based on evidence. To provide rigour it is important for the researcher to prevent doubt surrounding a phenomenon or conclusion presented, by presenting it as completely and understandably as possible (Kilpatrick, 1993). To meet this criterion I have tried to account for the subjective decisions I have made and to show how conclusions drawn from my findings were based on evidence in my data. When drawing conclusions I have constantly gone back to the data which I reread and analysed again. I also went back to transcribe more of the audio-recorded data for the purpose of providing more extensive support to my findings. Thus the conclusions drawn are based on reflexivity between the data gathered and the analytical process in striving to convince the reader that there is a fit between the empirical data and the presented analysis.

According to Kilpatrick (1993) *objectivity* is a criterion a researcher should strive to meet and therefore it is important to identify obvious biases brought to the research by the researcher. In the Methodology chapter, under the heading "My role as the researcher" I pointed out my experience within the educational field as a possible bias in the research and I have tried to show how I have come to the conclusions I have drawn by justifying the subjective decisions I made. By using "I" in my account throughout the whole thesis, I have emphasised that I am the one who has made the subjective decisions.

However, it might be seen as a limitation in my research that I did not let the teachers read anything of my analysis during the research process. In retrospect I see that as a weakness of my study and that the teachers' comments to initial analysis, as respondent validation, could possibly have provided the research with a greater degree of objectivity.

Greater objectivity could also have been achieved using other methods such as "secondary observation from colleagues" or "a stimulus-recall work" using classroom video (Jaworski, 1994, p. 76). However, such methods were not part of my study.

A further limitation in my study is that the teachers, who participated, did not participate as researchers. The purpose of my study was not for the development of teachers' teaching practice. However, if they had been more involved in the research, it could have added more information to the outcome of the study and it could also have offered the teachers the possibility to reflect on and develop their own teaching practice.

As a research study I see this study as having its strengths in

- *The multiple of methods used*. According to Kilpatrick (1993) one research method can never tell the whole story; "multiple methods will yield a body of research that collectively can be of high quality even when the individual studies are deficient" (p.18). The multiple of methods I used allowed me to investigate what I had intended and allowed me to draw conclusions about how teachers respond to a curriculum reform.
- *The richness in the data I have gathered*. Going back to the recorded data of the lessons of which I did not present a detailed analysis, I found that those lessons did not add anything new with regard to the teachers' teaching practice. Hence the data gathered from each of the teachers present the richness of each of the teachers' teaching practice. Throughout the analytical process I experienced that I had a great deal of data from each teacher to provide evidence for each of the features presented. Hence one issue which has been important for me to consider in the presentation of my study has been the balance between sufficient evidence and being repetitive.
- *The good match between what I observed and the theoretical ideas presented*. The long period of time I participated in the teachers' life allowed me to ensure a "high level of congruence between concepts and observations" (Bryman, 2001, p. 272).
- *I have pointed out that the implementation of a curriculum reform is not straightforward*

- by highlighting different constraints influencing teachers in their implementation of it;
- by illustrating teachers' different beliefs related to teaching and learning of mathematics;
- by illustrating teachers' different readings of a curriculum and their different teaching practices.

Before concluding this section I want to comment on two issues in my study.

- First: the two schools in which I did the classroom observations were in the same (rich) community recruiting students from the same kind of social level in society (well educated parents with high income). There were no students with multilingual background in the classes I observed. The nature of schools in Norway generally is more multifaceted than in the schools I observed.
- Second: all four teachers were well educated, with at least one year of mathematics study beyond upper secondary school, and had several years of experience from teaching mathematics. Thus my study was not to investigate how teachers' education, their mathematical knowledge or knowledge about teaching influenced their classroom practice.

Curricula come curricula go – classroom practices endure

In Norwegian there is a proverb which applied to curricula says: "*Læreplaner kommer, læreplaner går, klasserommet består*" which means that classroom practices remain the same despite changes in curriculum. As I indicated in Chapter 1, frequent shifts in governments have led to frequent changes in curricula. L97 was operative for nine years only (ten for one class). Looking back on history, that is the shortest time for any curriculum in Norway. Stigler and Hiebert (1999) suggested that frequent reforms can make the teachers "grow weary". Teachers are asked to change over and over again, and nothing happens.

L97 goes

In my study I have pointed to research which suggests that the mathematical part of L97 has not been implemented as intended (Alseth, Brekke, & Breiteig, 2003). Similar findings, that features from the NCTM standards were implemented only at the margin of the teaching rather than its core, were presented by Jacobs et al. (2006). I have also pointed out that students' performances on similar tasks (the attained curriculum) were lower after the implementation of L97 than before. The latter can be explained by Stigler and Hiebert's argument that only individual features of the curriculum have been changed and that has been "downright risky".

In my study I have pointed out possible reasons, or constraints, for why the curriculum, L97, not has been implemented as intended:

- Teachers do not want to do what the curriculum says;
- there are socio-cultural issues preventing them from implementing it;
- teachers' visions are not easily translated into classroom practice.

LK06 comes

In the introduction I referred to interviews with two of the members of the committee who developed the mathematical part of L97. One of the issues I wanted to discuss with them was what kinds of guidelines they received together with the task to write the curriculum. I did the same to find out about the new curriculum, LK06 "Kunnskapsløftet" (Kunnskapsdepartementet, 2006), Knowledge Promotion[61]. I conducted an interview with one member of the committee developing the mathematical part of LK06 and asked how LK06 differs from L97 and what was suggested to remain the same.

The committee writing LK06 did not get any guidelines regarding theories about teaching and learning mathematics, but according to what the one interviewed said, they perceived an emphasis on the social practice in the classroom. Furthermore, they looked upon the description in LK06 of competence in mathematics as rather reflecting a more structural view on mathematics (weight is put on connections between different mathematical entities and concepts), than a view on teaching and learning. Linguistic and problem-solving aspects are reflected in the description of basic skills in mathematics.

LK06 is built on the reform R97 which was the wide ranging school reform where L97 was the curriculum. The core curriculum of L97 is retained in LK06. Principles and Guidelines in L97 which was supposed to be the "bridge" between the core curriculum and the subject syllabus is exchanged with a "Learning programme" (Læringsplakaten) which comprises "important principles for the school's activity and must be seen in relation with law and regulation and the core curriculum" (Utdannings- og forskningsdepartementet, 2005, p. 7, my translation). The committee which wrote the mathematical part of LK06 was told that the new curriculum should build on R97, but the description of competence in the subject should not be as detailed as in L97. There should be stated clear aims for competence, not for each class but for each stage (1-4, 5-7, 8-10, and one for each year in upper secondary school). Neither

61 http://www.dep.no/filarkiv/291143/Kunnskapsloftet2006_engelsk_II.pdf
http://www.dep.no/kd/english/topics/knowledgepromotion/bn.html
http://www.dep.no/kd/norsk/tema/kunnskapsloeftet/bn.html

working methods nor ways of organising the teaching activities should be specified in LK06. In L97 what to be learned, and when, was specified. Following LK06, the teacher or school can decide for themselves about organisation and working methods. The most important changes in LK06 are that “Basic skills are to be strengthened”. Basic skills in mathematics are explained as “abilities to do arithmetic”. Another change is “Freedom at the local level with respect to work methods, teaching materials and the organization of classroom instruction” which involves an option for “mixed age schooling”, meaning that for example children of different ages can be taught in the same class. [62]

Who is to blame?

Under the heading “Who is to blame?”[63] in a national newspaper, Dagbladet (2004), Gudmund Hernes who was the Minister of Education when L97 was developed and the man behind the reform, R97; discussed several explanations, presented by the public in the media, conveying why Norwegian students performed lower after the implementation of the reform than before. Many of these explanations put the responsibility on the reform. Hernes suggests that these explanations either put the responsibilities on groups of people (students, parents, teachers, politicians) or on systems which emphasise structural changes (lack of discipline, lack of pedagogical demands, many immigrants in school, teacher’s low status, curriculum is too weak, students are to a great extent influenced by media, bureaucracy, allocation of subjects, lack of resources). Instead of emphasising what does not work, Hernes suggested focusing on what works, giving an example from Finland. In Finland, the effect of parents’ status is toned down. The teachers are well trained and gain status as an effect of their knowledge, and thus autonomy. The core of his message in the article is: It is the class and the teacher who play the decisive role in the students’ development. Claiming that “teacher education in Norway is a black box – what is going on or not going on there, we know little about”, he emphasises the responsibilities of those who work in teacher education and the importance of teacher education and in-service training of teachers.

Consequences for teacher education and further research

In the Methodology chapter I wrote that my intention was from the outcome of the study to make “fuzzy generalisations” (Bassey, 1999). One such generalisation is to treat the findings from the three teachers as possible characteristics of teachers more widely, and to consider consequences of this. I therefore suggest that the mathematical part of the cur-

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[62] http://www.dep.no/filarkiv/291143/Kunnskapsloftet2006_engelsk_II.pdf
http://www.dep.no/kd/english/topics/knowledgepromotion/bn.html

[63] The Norwegian heading was ”Hvem har skylda?”

riculum L97 has not been implemented as intended, although plans for in-service training courses and guidelines for teachers were developed and carried out.

In this study I have learned that teachers respond differently to the curriculum L97. I found both similarities and differences in their teaching practice, they expressed different beliefs about teaching and learning mathematics and I have identified constraints and issues influencing and as I see it, partly preventing the teachers from implementing the curriculum. Hence, constraints, *cultural factors influencing teachers' beliefs and teaching practice which may conflict with the recommendations of the curriculum and internally with each other*, are important aspects of how teachers respond to a curriculum.

Based on my findings I see a need for further educational development in mathematics. I have long experience as a mathematics teacher and as a mathematics teacher educator. I have been largely involved in in-service training courses for teachers, and I was much involved in these kinds of courses when L97 should be implemented. After having carried out this research and reflecting upon my findings which have provided me with insights into teachers' thinking about teaching and also into their practice, I am asking: what does all this mean for me and my colleagues as teacher educators and what is valuable to know as a result of my study for the purpose of enhancing mathematics teacher education and mathematics teachers' teaching practice? As concluding remarks in this thesis I will try briefly to indicate how my findings can encourage me and I hope other teacher educators in their work with mathematics teachers.

All suggestions in the literature I referred in Chapter 2 about how reforms in mathematics education could serve the purpose of enhancing mathematics teachers' teaching practice and thus students' learning outcome, took *the teacher* as a starting point. Also Hernes in his article "Who is to blame?" suggested the teacher and class as crucial factors in students' learning. Thus the teacher and not the curriculum is the crucial issue regarding how conditions for students' possibilities of learning are created.

For a teacher educator with the purpose of in-service training, knowledge about constraints is valuable. I suggest that enhancement in a teacher's practice has a greater possibility to take place when factors constraining the practice are identified. I reported constraints as lying in the complexity of the classroom, in the difficulties of transition of visions about good mathematics teaching into practice; between the teacher's beliefs and practice, in the socio-cultural environment as society's and parents' expectations and the school context. Being conscious of such factors, which to a certain extent can be dealt with, can thus open up possibilities for professional development of mathematics teacher educators' and teachers' teaching practice. However, constraints lying in the teacher's beliefs are more difficult to deal with, because only the teacher him/herself can change his/her own beliefs. Mason (2002) writes: "I cannot change others, but I can work at changing myself" (p. xii). I suggest that collaboration between teachers and with teacher educators can influence beliefs so the teachers and teacher educators can work at changing themselves.

I experienced the use of focus group in working with teachers as valuable: In these groups the teachers eagerly discussed issues and challenges in mathematics teaching in general and issues from their own experiences in the classroom, and they all expressed that the discussions which had taken place were interesting and instructive. Such groups can serve as a collaborative forum for teachers to discuss issues, exchange ideas and plan lessons. Hence, through the collaboration in focus groups, teachers can be curriculum *developers* rather than curriculum *implementers*.

Two of the teachers in my research were teaching in the same school and at the same level. My study suggests that one did not want to change his practice towards an L97 orientation because he had experienced success teaching the way he did and he got positive feedback on his teaching from both students and parents. I asked the question "why should he?" I could also have asked: "How is it at all possible not to change the way of teaching when a new curriculum is implemented?" As long as teachers can conduct their own "private practice" behind a closed door into the classroom, this is possible. Having studied two teachers closely who were teaching parallel classes in the same school, both having strong but very different visions about good teaching practice, both having strengths and weaknesses in their practice, made me reflect upon how these two teachers could both have benefited from collaboration with each other. In all educational research for the purpose of enhancing teachers' teaching practice it is important to focus on each teacher's strengths and take that as starting points for further development. Teachers can also learn from each other's strengths.

As a generality in my study I have suggested the analytical tool developed showed its applicability in the analysis of all three teachers. Knowledge about the analytical tool is not only valuable to serve the purpose for educational researchers to analyse teachers' teaching practice, but also for teachers in carrying out research on their own practice.

My hope is that the doctoral work I have carried out will contribute to knowledge about how teachers respond to a curriculum, how they think about the curriculum and about their own practice and how their mathematics teaching practices are constrained by socio-cultural factors.

I also hope I can contribute in further mathematics educational research with the experience I have gained about research methods, methods of analysis data and the research process more generally.

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Appendices

Appendix 1	Questionnaire
Appendix 2	Estimation Form
Appendix 3 (In Norwegian)	Informed consent
Appendix 4 (In Norwegian)	
Appendix 5 (In Norwegian)	

Appendix 1: Questionnaire (2 pages)

Thoughts about mathematics teaching

1. Planning lessons

When you plan a mathematics lesson, to what extent do you take the following into account? (Great extent, certain extent, little extent, no exgtent)

- A lesson you have just had
- The textbook
- A joint developed teaching plan at school
- Other teachers teaching mathematics
- The textbook's guidance
- The curriculum, L97

2. Information sources

Where do you search for information when you shall (L97, Local syllabus, Text-book, Students' wishes, Other)

- Decide topics and goals for the teaching
- Choose how to present the topic
- Choose problems and exercises for work in class and homework
- Choose problems and tasks for assessing the students
- Let students do theme- and project work

3. Your view on mathematics

What is your meaning about the claims below? (Strongly agree, Agree a little, Disagree a little, Strongly disagree)

- Mathematics is an interesting and challenging subject
- Rules and routines were essential parts of the subject
- Mathematics is to find the right answer to a problem
- Mathematics is mainly an abstract subject
- Mathematics is a formal way to describe the real world
- Mathematics is a practical and systematic way to solve real problems
- The solution of a mathematical task is either right or wrong
- To master mathematics you need innate gifts for the subject

4. Learning mathematics

This is important when the students shall learn mathematics (strongly agree, agree a little, disagree a little, strongly disagree)

- That they must explain a reasoning
- That the teacher presents new material on the board

- That the teacher explains errors they have made
- That they shall discuss mathematical tasks with each other
- That they shall solve mathematical tasks in groups
- That they get individual supervision from the teacher
- That the student works on his/her own
- That they reflect on what to be learned
- That they can use fantasy and creativity in their work
- That they take what they already know as a starting point
- That the student explains the error s/he has made
- That learning material should be organised thematically across subjects

5. Teaching mathematics

When teaching mathematics it is important to (very important, important, a little important, not important)

- Use more than one illustration (drawings, pictures, tokens, concrete materials, etc.)
- Encourage students to find their own solutions and discuss different solutions with each other
- Emphasise that the students shall learn rules and routines by heart
- Concentrate the work around the textbook
- Let students use play and games
- Let students solve tasks individually
- Let mathematics be part of a project across several subjects
- Integrate mathematics in other subjects
- Take students' own experiences as starting point
- Encourage students' creativity and ability to think in new ways

6. Students' relations to mathematics

To what extent do you experience that your students in the mathematics lessons are: (great extent, certain extent, little extent, no extent)

- Hardworking
- Interested and motivated
- Independent
- Noisy and un-concentrated
- Clever in solving subject matter problems
- Creative and inventive
- Willing to collaborate with/help fellow students
- Express comfort and joy when working

Appendix 2: Estimation form

The following three perspectives can be used as a rough classification of mathematical view and the view of teaching mathematics

T: Mathematics is a large toolbox: Doing mathematics means working with figures, applying rules and procedures and using formulae

S: Mathematics is a formal, rigorous system: Doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts

P: Mathematics is a constructive process. Doing mathematics means learning to think, deriving formulae, applying reality to mathematics and working with concrete problems.

I

Distribute a total of 30 points corresponding to your estimation of the factors T, S and P in which you value

- Your own teaching of mathematics
- An ideal teaching of mathematics
- L97's view on teaching mathematics

	T	S	P
Your teaching			
Ideal teaching			
L97's view on teaching			

II

On the back of this sheet try to express thoroughly what you mean ideal teaching of mathematics is

Appendix 3: Informed Consent

Samtykkeerklæring ved innsamling og bruk av personopplysninger til forskningsformål

Prosjektleder: Bodil Kleve, ALU, Høgskolen i Oslo
Prosjekttittel: Curriculum Transition in Mathematics Education
Formål: Å undersøke hvordan lærere implementerer L97's matematikkdel. Gjennom fokusgrupper, klasseromsobservasjon, intervju (og spørreskjema) er målet å undersøke sammenhenger mellom læreres tanker om det å undervise i matematikk, deres tanker om elevers læring av matematikk og det som skjer i klasserommet, analysert i forhold til L97.

Jeg samtykker i at opplysninger om meg samlet inn på følgende måter kan brukes i dette prosjektet:

1. Gruppesamtaler og lærermøter
2. Klasseromsobservasjon
3. Intervju
4. Spørreskjema

Jeg samtykker videre i at de innsamlede opplysninger innhentet fra ovennevnte situasjoner, kan oppbevares etter prosjektavslutning ved en institusjon godkjent av Datatilsynet, for slik lagring.

Jeg samtykker videre i at de innsamlede opplysninger kan brukes i en eventuell oppfølgingsundersøkelse av samme forsker som er ansvarlig for dette prosjektet.

Hvis det skulle være aktuelt med bruk av opplysningene i en annen undersøkelse, vil dette ikke kunne skje uten samtykke fra Datatilsynet.

Jeg er også kjent med at deltagelse i prosjektet er frivillig, og at jeg når som helst kan be om å få slettet de opplysninger som er registrert om meg. Dette gjelder også etter prosjektet er avsluttet.

Sted Dato Underskrift av informanten

Har du spørsmål angående lagring av opplysningene, kan du kontakte datatilsynet.

Appendix 4: Declaration of secrecy

Taushetserklæring

Prosjektleder: Bodil Kleve, ALU, Høgskolen i Oslo

Prosjekttittel: Curriculum Transition in Mathematics Education

Formål: Å undersøke hvordan lærere implementerer L97's matematikkdel. Gjennom fokusgrupper, klasseromsobservasjon, intervju (og spørreskjema) er målet å undersøke sammenhenger mellom læreres tanker om det å undervise i matematikk, deres tanker om elevers læring av matematikk og det som skjer i klasserommet, analysert i forhold til L97.

Personopplysninger som samles inn gjennom dette prosjektet skal behandles etter bestemmelser gitt i konsesjon fra Datatilsynet, jfr. § 9 i Lov om personregistre m.m.

Undertegnede prosjektansvarlig vil være den eneste som vil ha tilgang til de opplysninger som samles inn i forbindelse med prosjektet, og som kan tilbakeføres til enkeltpersoner.

Jeg erklærer med dette at ingen personopplysninger som kommer meg i hende i forbindelse med prosjektarbeidet vil være tilgjengelig for andre. I forbindelse med utgivelse av en publikasjon eller lignende vil kun anonymiserte opplysninger bli gitt ut.

Sted | Dato | Underskrift av prosjektansvarlig

Appendix 5: Letter of accept from NSD (2 pages)

Norsk samfunnsvitenskapelig datatjeneste AS
NORWEGIAN SOCIAL SCIENCE DATA SERVICES

NSD

Hans Holmboes gate
N-5007 Bergen
Norway
Tel: +47/ 55 58 21
Fax: +47/ 55 58 96
nsd@nsd.uib.no
www.nsd.uib.no
Org.nr. 985 321 88

Bodil Kleve
Avdeling for lærerutdanning
Høgskolen i Oslo
Postboks 4 St. Olavs plass
0130 OSLO

Vår dato: 27.08.2004 Vår ref: 200400908 PB /RH Deres dato: Deres ref:

MELDING OM BEHANDLING AV PERSONOPPLYSNINGER

Vi viser til melding om behandling av personopplysninger, mottatt 05.08.2004. Meldingen gjelder prosjektet:

11334 Curriculum Transition in Mathematics Education. Interpretation and implementation of L97's mathematics curriculum

Meldingen er behandlet av Norsk samfunnsvitenskapelig datatjeneste AS (NSD). Etter gjennomgang av opplysninger gitt i meldeskjemaet og dokumentasjon, finner vi at prosjektet ikke medfører behandling av personopplysninger i henhold til personopplysningsloven §§ 1 til 3, og følgelig ikke utløser meldeplikt eller konsesjonsplikt etter personopplysningslovens §§ 31 og 33.

Vedlagt følger vår vurdering. Prosjektet kan settes igang.

Dersom prosjektopplegget endres i forhold til de punktene som ligger til grunn for vår vurdering, skal prosjektet meldes på nytt.

Vennlig hilsen

Bjørn Henrichsen

Pernilla Bollman

Kontaktperson: Pernilla Bollman tlf: 55583348
Vedlegg: Prosjektbeskrivelse

Avdelingskontorer / *District Offices*
OSLO: NSD. Universitetet i Oslo, Postboks 1055 Blindern, 0316 Oslo. Tel. +47/ 22 85 52 11. nsd@uio.no
TRONDHEIM: NSD. Norges teknisk-naturvitenskapelige universitet, 7491 Trondheim. Tel. +47/ 73 59 19 07. kyrre.svarva@svt.ntnu.no

Prosjektbeskrivelse

Behandlingsansvarlig:
Bodil Kleve

Avdeling for lærerutdanning
Høgskolen i Oslo

Postboks 4 St. Olavs plass

0130 OSLO

Daglig ansvar/prosjektleder:
Bodil Kleve

Avdeling for lærerutdanning
Høgskolen i Oslo

Postboks 4 St. Olavs plass

0130 OSLO

11334 Curriculum Transition in Mathematics Education. Interpretation and implementation of L97's mathematics curriculum

Prosjektet ble startet allerede 01.01.2003, men meldt til personvernombudet først 04.08.2004.

Formålet med prosjektet er å analysere matematikkdelen i L97 i et teoretisk perspektiv samt å undersøke hvordan lærere implementerer matematikkplanen i L97. Meningen er å undersøke sammenhenger mellom lærernes tanker om det å undervise i matematikk, deres tanker om elevenes læring og det som faktisk skjer i klasserommet sett i lys av L97.

Utvalget består av fire lærere og deres matematikkelever. Førstegangskontakt ble opprettet via skolens ledelse. Datainnsamlingen foregår/har foregått ved hjelp av gruppeintervjuer, personlige intervjuer, spørreskjemaer og klasseromsobservasjon samt lydopptak av klasseromsaktiviteter. Lærerne har samtykket til deltagelse i prosjektet og til oppbevaring av analoge lydopptak etter prosjektslutt. Vedrørende lydopptak fra klasserommene, som inkluderer elever, forutsettes det at elevene selv og deres foreldre er informert om og har samtykket til innspilling og oppbevaring av disse. Det vises i denne sammenheng til Forskningsetiske retningslinjer for samfunnsvitenskap, jus og humaniora: http://www.etikkom.no/retningslinjer/NESHretningslinjer

Datamaterialet oppbevares og behandles på analogt lydopptak (kassett) og på pc i nettverkssystem. Prosjektleder opplyser at ingen direkte personidentifiserbare opplysninger (for eksempel navn) vil bli registrert. På analogt lydopptak vil det registreres indirekte personidentifiserbare opplysninger. Personvernombudets vurdering av prosjektet som ikke meldepliktig baserer seg på prosjektleders opplysning at det ikke innhentes sensitive personopplysninger i tillegg til at ingen personopplysninger oppbevares eller behandles elektronisk. Det vil si at ved overføring av opplysninger til pc (transkribering) vil ingen indirekte personidentifiserbare opplysninger registreres.

Prosjektslutt er angitt til 31.12.2005, men lydopptak vil bli lagret også etter dette.