Durham University

Durham E-Theses

Supersymmetric model building with Dirac gauginos

BUSBRIDGE, DANIEL,WILLIAM

How to cite:

BUSBRIDGE, DANIEL,WILLIAM (2015) *Supersymmetric model building with Dirac gauginos*, Durham theses, Durham University. Available at Durham E-Theses Online: http://etheses.dur.ac.uk/11108/

Use policy

The full-text may be used and/or reproduced, and given to third parties in any format or medium, without prior permission or charge, for personal research or study, educational, or not-for-profit purposes provided that:

- a full bibliographic reference is made to the original source
- a link is made to the metadata record in Durham E-Theses
- the full-text is not changed in any way

The full-text must not be sold in any format or medium without the formal permission of the copyright holders.

Please consult the full Durham E-Theses policy for further details.

Academic Support Office, Durham University, University Office, Old Elvet, Durham DH1 3HP
e-mail: e-theses.admin@dur.ac.uk Tel: +44 0191 334 6107
http://etheses.dur.ac.uk

Supersymmetric model building with Dirac gauginos

Daniel William Busbridge

A Thesis presented for the degree of
Doctor of Philosophy

Durham University

Institute for Particle Physics Phenomenology
Department of Physics
University of Durham
England

September 2014

To my parents Ray and Diane Busbridge
for their endless support and encouragement

Supersymmetric model building with Dirac gauginos

Daniel William Busbridge

Submitted for the degree of Doctor of Philosophy
September 2014

Abstract

With the Large Hadron Collider about to start its second run, we are in an era of high–energy collider physics. The discovery of a Standard Model–like Higgs boson with a mass of 125 GeV is a fantastic achievement, but the non–observation of supersymmetry (or any other mechanism of choice that stabilises the electroweak scale) is a tantalising puzzle.

In this work, we investigate the possibility that a particular non–minimal realisation of supersymmetry — one with Dirac gauginos — can be a reasonably natural way of explaining this nonobservation, but can still can stabilise electroweak physics. We construct a simple UV completion of a model with Dirac gluinos dubbed *Constrained Dirac gluino mediation* and determine the characteristic low energy spectra, the production cross sections of key processes at the Large Hadron Collider and the degree of fine tuning for a representative range of parameters. Noting that theories with Dirac gluinos have a tendency to lose asymptotic freedom due to the presence of extra matter content, we then cast our eyes towards Seiberg Duality and its generalisation to include adjoint chiral superfields — Kutasov duality and investigate how a Dirac mass maps across this duality. We provide evidence that a Dirac gaugino mass maps between electric and magnetic Kutasov descriptions as

$$\lim_{\mu\to\infty} \frac{m_D}{g\,\kappa^{\frac{1}{k+1}}} \to \lim_{\mu\to 0} \frac{\tilde{m}_D}{\tilde{g}\,\tilde{\kappa}^{\frac{1}{k+1}}}$$

using renormalisation group arguments and harmonic superspace techniques.

Declaration

The work in this thesis is based on research carried out at the Institute for Particle Physics Phenomenology, the Department of Physics, Durham University, England. No part of this thesis has been submitted elsewhere for any other degree or qualification. The research described in this thesis, unless referenced to the contrary in the text, has been carried out in collaboration with Valya Khoze and Steven Abel.

Chapter 4 is based on the (to be) published work [1]:

> Daniel Busbridge
> ***Constrained Dirac gluino mediation***
> `arXiv:1408.4605`

Chapter 5 is based on the published work [2]:

> Steven Abel and Daniel Busbridge
> ***Mapping Dirac gaugino masses***
> Journal of High Energy Physics (June, 2013) 37
> `arXiv:1306.6323`

Copyright © 2014 by Daniel William Busbridge.
"The copyright of this thesis rests with the author. No quotations from it should be published without the author's prior written consent and information derived from it should be acknowledged".

Acknowledgements

There are a number of people who have been a great help to me during the creation of this work; for academic reasons, for maintaining my sanity or in many cases both:

- My supervisor, Valya Khoze, for his invaluable guidance and useful discussions, for incredible patience and for giving me the opportunity to investigate so many different areas of theoretical particle physics;
- My collaborator, Steve Abel, for our many enlightening discussions and for all that he taught me, both of a physics nature and of the realities of research;
- Jörg Jäckel and Céline Bœhm for their thorough (but enjoyable) examination of this thesis and the resulting amendments;
- Alberto Mariotti for helpful conversations and Mark Goodsell, Florian Lyonnet and Florian Staub for useful correspondence;
- All of my friends at the IPPP, both past and present, for generally being great. In particular I would like to thank my fellow OC118 office mates — Darren, Herr Yip, Jon, Malte, Mark, Oli, Peter, Ryan and Tom — for their shared appreciation of plush animals, enthusiasm for *question of the day*, and general surreality. I hope the OC118 (and friends) fine dining consortium continues its well established tradition, and that the latté adventure lives on;
- Mike Johnson, Ewan Steele and Oliver Smith for all things IT, and Linda Wilkinson and Trudy Forster for all things logistic;
- My housemates for the last two years, Andy and James, for their support, encouragement, and excellent cooking;

- The friends I've met through Hatfield College — particularly Alex, Hanna, Henry, Jack, Philippa, Tish and Zoe — for their indispensable help of a non–physics variety, and Anthony Bash, whose advice, support and excellent conversations over breakfast will be missed;

- The Science and Technology Facilities Council for funding my studies;

- All of my family, particularly my parents, my sister and my grandmother, for being an endless supply of encouragement over the years.

Contents

List of Figures

List of Tables

Acronyms

$\not{E}_T$ Missing Transverse Energy. 76, 80, 141, 144, 149

$\not{R}$ R Parity Violation. 76

χ**SF** Chiral Superfield. 32, 33, 35–38, 49, 50, 53, 56, 59, 69, 71, 86–89, 91, 96, 100, 125, 127, 164, 165, 167

2HDM Two Higgs–Doublet Model. 71, 120

ATP Antoniadis–Taylor–Partouche. 168, 179–183, 191, 198

BZ Banks-Zaks. 62, 169, 173

CGGM Constrained General Gauge Mediation. 125, 127, 130, 131, 133, 134, 142, 144, 145, 153, 156, 158–160, 162

CKM Cabibbo-Kobayashi-Maskawa. 13

CL Confidence Level. 75, 136

CLIC Compact Linear Collider. 162

CM Coleman–Mandula. 25, 28

CMB Cosmic Microwave Background. 11

CMSSM Constrained Minimal Supersymmetric Standard Model. 76, 125, 127, 130, 131, 133, 134, 136–138, 140, 143–147, 153–155, 157–160, 162

CP Charge Parity. 12, 13, 15, 16, 39, 73, 76, 103, 115, 149

CS Callan-Symanzik. 22, 51

d.o.f. degree of freedom. 21

DRED Dimensional Reduction. 21

DREG Dimensional Regularisation. 21

EFT Effective Field Theory. 4, 14, 17, 23, 48, 58, 118, 120, 131, 179

ESP Extended Superpartner. 80, 81, 111, 118, 120, 122, 123, 127

EW Electroweak. 4, 21, 52, 76

EWSB Electroweak Symmetry Breaking. 10, 14, 72, 103, 107, 124–126, 130–133, 137, 139, 143, 157–159

FGM Flavoured Gauge Mediation. 76

FI Fayet–Iliopoulos. 37, 39, 56, 168, 169, 171, 180, 182, 191–193, 197

FNW Fox, Nelson and Weiner. 127

FS Fayet–Sohnius. 40, 41, 44, 45, 47, 49, 179

GGM General Gauge Mediation. 130

GMSB Gauge Mediated Supersymmetry Breaking. 130

GUT Grand Unified Theory. 13, 16, 19, 28, 76, 80, 81, 111, 130, 158, 164, 165

HLS Haag–Lopuszanski–Sohnius. 25, 28

HSS Harmonic Superspace. 42–45, 47, 179, 180, 198, 232

ILC International Linear Collider. 162

IR Infrared. 20, 23, 62, 67–69, 71, 132, 137, 139, 162, 163, 165, 166, 170–172, 175, 198

IRFP Infrared Fixed Point. 63–68

LEP Large Electron Positron Collider. 76, 137, 159

LHC Large Hadron Collider. 1, 3, 4, 6, 23, 76, 81, 94, 98–100, 126, 146, 148, 155, 161

LL Leading Log. 18, 157

LO Leading Order. 4, 21, 94, 98, 146

LOSP Lightest Ordinary Supersymmetric Particle. 76, 80, 136, 140, 141, 144, 145, 161

LSP Lightest Supersymmetric Particle. 72, 144

MFV Minimal Flavour Violation. 74, 76

mGMSB Minimal Gauge Mediated Supersymmetry Breaking. 130

MSSM Minimal Supersymmetric Standard Model. 4, 28, 55, 57, 59, 71, 73–76, 79, 80, 83, 85, 86, 92, 95–97, 100, 101, 105, 107, 111, 112, 124, 126, 127, 129, 131, 136, 137, 139, 145, 148, 149, 161, 163–165

NLOSP Next to Lightest Ordinary Supersymmetric Particle. 136, 141, 145

NMSSM Next to Minimal Supersymmetric Standard Model. 109, 111

NS Nelson–Seiberg. 57, 79

NSVZ Novikov–Shifman–Vainshtein–Zakharov. 54, 62, 171

OR O'Raifeartaigh. 56, 57

OS On Shell. 21, 22, 40, 41, 46, 47

PDF Parton Distribution Function. 6

PMNS Pontecorvo-Maki-Nakagawa-Sakata. 13

pMSSM Phenomenological Mimimal Supersymmetric Standard Model. 76

PQ Peccei-Quinn. 15

QCD Quantum Chromodynamics. 6, 155

QFT Quantum Field Theory. 2, 3, 6, 11, 17, 21, 28, 71

REWSB Radiative Electroweak Symmetry Breaking. 9

RG Renormalisation Group. 5, 18, 19, 23, 52, 63, 65, 69–71, 76, 90, 92, 94, 126, 127, 131, 133, 136, 137, 139, 145, 157, 162–164, 166–170, 173, 175, 198

RGE Renormalisation Group Equation. 18, 23, 24, 51, 52, 54, 59, 75, 91, 92, 94, 123, 136, 162, 171, 172, 218

SCFT Superconformal Field Theory. 60, 63, 64, 66–68

SM Standard Model. 1–4, 6–17, 19–21, 26–28, 52, 58, 59, 71, 75, 80, 84, 85, 109, 119, 123, 124, 128, 145, 148, 165

SQCD Super Quantum Chromodynamics. 41, 47, 49, 60, 61, 63, 66, 67, 69, 168–170, 173–175, 177, 179, 182, 183, 197, 198, 233, 234

SSB Spontaneous Symmetry Breaking. 10

SUSY Supersymmetry. 3–5, 9, 16, 20, 24–28, 30–37, 39, 40, 42, 49–52, 55–61, 68, 69, 71, 74–77, 79, 85–89, 94, 100, 103, 112, 114, 125–127, 131, 132, 134, 136, 157, 158, 161, 162, 164, 168, 171, 179, 180, 184, 186, 189, 191, 198, 220, 234

SYM Super Yang–Mills. 37, 48, 52, 89, 234

UV Ultraviolet. 4, 9, 11, 20, 22, 23, 59, 68, 70, 75, 76, 80, 90, 94, 109, 115–117, 124–127, 129, 131–133, 137, 139, 143, 145, 149, 156, 158, 160, 161, 163–166, 174, 178

VEV Vacuum Expectation Value. 9, 55–57, 69, 80, 103, 107, 110, 113, 117, 118, 125, 128, 165, 176–178, 191

VSF Vector Superfield. 34–37, 50, 52, 85, 88, 89

WT Ward–Takahashi. 22

WZ Wess–Zumino. 33, 36–38, 47, 50, 85


Supersymmey
Standard
Model
Minimal
theory
mass
Dirac
TeV
gauge
GeV
gaugino
gluino
Nc SU
Renormalisation
Quantum
Chromodynamics
masses
terms
Majorana
duality
superpotential
Nf
Higgs
Group
Hu
Hd
scale
Super
one

1

Introduction

X–rays will prove to be a hoax.

– Sir William Thomson (a.k.a. Lord Kelvin)

1.1 Non–technical overview

The world's largest and most powerful particle collider, the Large Hadron Collider (LHC), is about to start its second run after being shut down for upgrades and maintenance in February 2013. It will start taking data at an energy of 13 TeV – that is, the average proton–proton collision energy will be $2.08 \times 10^{-6}\ J$, or roughly the energy of a flying mosquito. This may not seem like much, but when you consider that a mosquito contains roughly $10^{21} = 1,000,000,000,000,000,000,000$ atoms[1], then we see that 13 TeV is rather a lot of energy for two protons to have.

The LHC was primarily designed to find the *Higgs boson*: a detectable 'leftover' of the *Higgs mechanism* that gives the fundamental particles, like electrons and quarks, mass[2]. The discovery of the Higgs boson was announced in July 2012, experimentally validating the Standard Model (SM), our mathematical description of all observed particles to date and their interactions.

[1] In the original version of this thesis I used 1 gram as an estimate for the mosquito mass. This is roughly three orders of magnitude out and corresponds to a mosquito of around 10 cm in length. I thank my examiners for pointing out to me that we have been fortunate enough to not yet encounter such a species.

[2] It is important to note that the *Higgs mechanism* is *not* responsible for most of the mass of the proton — this comes from the energy required to bind the quarks together inside this proton. This energy has a mass due to Einstein's famous formula $E = m\,c^2$ that is much larger than the sum of the masses of the three quarks.

In truth, as is typical in science, the discovery of the Higgs boson raised more questions than it solved. One question that is at the forefront of the theoretical physics community's attention, and the work in this thesis is primarily concerned with, is 'Why aren't the fundamental particles much *much* heavier?', or stated another way, 'Why is the Higgs boson so light?'.

The SM is a Quantum Field Theory (QFT), and so everything in the theory undergoes *quantum corrections*. What this means is that if a measurement of anything in a QFT is performed, and yields a value n, then secretly we know this actually the sum of the normal or *classical* part $n_{\text{classical}}$ and its quantum corrections n^i_{quantum}

$$n = n_{\text{classical}} + \sum_i n^i_{\text{quantum}}. \tag{1.1.1}$$

In a QFT after doing a certain number of measurements, we can make a prediction for each n in the theory that can be tested. Indeed, all the n's of the SM match up precisely with their predicted values. There is, however, a conceptual problem with the Higgs mass. We have measured the Higgs mass (our n for the moment) to be roughly 100 (in some units), but we know that there should be some n^i_{quantum} at roughly 10^{16}. If we are to get the correct result we need something like[3]

$$100 = 10,000,000,000,000,100 - 10,000,000,000,000,000 \tag{1.1.3}$$

to happen. In any scenario it is unusual that a cancellation between very large numbers to get a small number occurs unless there is a rationale behind it. There are three popular schools of thought:

- These cancellations are well–defined within a QFT and so we shouldn't worry about them,
- We haven't properly understood the underlying theory and there are no large numbers present to be cancelled,

[3] Actually in the real calculation this is even worse since the Higgs physics is quadratically sensitive to the quantum corrections, and we should get a cancellation of

$$100^2 = (10^{16})^2 - [(10^{16})^2 - 100^2]. \tag{1.1.2}$$

- The cancellation is engineered to happen by the theory itself.

The third option is the conspiracy in theoretical physics known as *symmetry*. To be explicit, imagine for each n^i_{quantum} there is an accompanying $\tilde{n}^i_{\text{quantum}}$ where $\tilde{n}^i_{\text{quantum}} = -n^i_{\text{quantum}}$. Then we see automatically

$$n = n_{\text{classical}} + \sum_i n^i_{\text{quantum}} + \sum_i \tilde{n}^i_{\text{quantum}} = n_{\text{classical}} \tag{1.1.4}$$

irrespective of how large each quantum correction n^i_{quantum} is. The specific *symmetry* that can achieve this for the Higgs mass is known Supersymmetry (SUSY). If the SM enjoys SUSY then this answers our question.

No SUSY has been observed experimentally, so if it is there at all, is relegated to a *broken symmetry*. Part of this thesis is devoted to the exploration of various novel extensions (i.e. with new particles and new forces) to the SM with broken SUSY to see if they can be a reasonable description of reality and, if so, whether the LHC can find a remnant of such a theory.

The remainder of the thesis involves the study of the same class of extensions but within the context of *duality*. Here, *duality* is taken to mean two different descriptions of anything. A familiar example of this could be different spoken or written languages. In English, if we say 'hello,' it should have the same impact to someone who understands English as saying 'bonjour' to someone who understands French. We would technically say that the English and French languages are *dual* or there is a *duality* between them, accompanied with *dictionary* that *map* the *operators* (or here words) into each other

$$\text{hello} \qquad \longleftrightarrow \qquad \text{bonjour.} \tag{1.1.5}$$

There are of course words that cannot be one–for–one translated. A good example is the word 'boh' in Italian, which translates into English, not as a single word, but as the phrase 'I don't know,' and so we have a mapping of a *composite operator* (or here phrase) into a word and vice–versa

$$\text{I don't know} \qquad \longleftrightarrow \qquad \text{boh.} \tag{1.1.6}$$

The interesting thing about dualities in a SUSY QFT is that there are special cases calculation that cannot be performed in one language can be performed in the *dual* language. In order to to this, one must have first established the *map* between the theories. The development of the *map* that incorporates the novel SUSY SM extensions is the focus of the final part of the thesis.

1.2 Outline of thesis

We begin our journey in Chapter 2, where we quickly review the SM and take some time to look its shortcomings. We then briefly review Effective Field Theories (EFTs) since being clear about decoupling and matching in a mass independent renormalisation scheme will be important for performing a consistent calculation with the Ultraviolet (UV) models introduced in Chapter 4. We then go over some SUSY basics to clear up our conventions (of which there are a staggering number of in the literature) before introducing the more advanced topics that may be less familiar to graduate students – R symmetry, non–renormalisation, Seiberg duality and mapping soft terms. The Minimal Supersymmetric Standard Model (MSSM) and SUSY breaking are also covered.

We continue into Chapter 3 where we bring the reader up to speed on Dirac gauginos. The advantages and disadvantages of a Dirac gaugino compared to its Majorana counterpart are discussed before moving in to a discussion on generic differences between Dirac and Majorana particles. The minimum requirements for a theory with a Dirac gaugino are outlined with specific focus on symmetries and matter content. The renormalisation properties of Dirac gauginos known as *supersoftness* are then discussed, before moving on to explain *supersafeness*, the mechanism that can suppress the LHC cross sections for sparticle production. Finally, since the remainder of this thesis mainly concerns Dirac gluinos, we take some time to look at the properties of MSSM extended with gauginos for all gauge groups with particular focus on the Electroweak (EW) sector.

Chapter 4 contains the phenomenological study of a simple UV completion of the MSSM with a Dirac gluino based upon [1]. After a discussion of the various possible effective operators, the UV model is introduced and the numerical setup is outlined. The characteristic spectra, Leading Order (LO) cross sections and tuning are then discussed.

Finally, Chapter 5 contains the work based upon [2]. We identify the Renormalisation Group (RG) invariant that will give us the mapping across the Kutasov duality, but it is unfortunately not the spurion of any coupling in the theory. We show how one can get between the Kutasov duality and the $\mathcal{N} = 2$ duality via deformations in the $\mathcal{N} = 1$ picture. We then show how one can describe an $\mathcal{N} = 1$ theory as an $\mathcal{N} = 2$ theory, bypassing the *two–into–one–won't–go* theorem with a special type of SUSY breaking, before showing how the same mechanism can produce $\mathcal{N} = 0$ deformations, including Dirac and Majorana gaugino masses.

2

Foundations

The most exciting phrase to hear in science, the one that heralds new discoveries, is not "Eureka!", but "That's funny . . . "

– Isaac Asimov

2.1 The Standard Model of particle physics

2.1.1 Introduction

Almost all observed physical phenomena are accurately described by the SM of particle physics, a renormalisable QFT of quarks, leptons and gauge bosons with the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The associated properties of the gauge groups in the SM are detailed in table 2.1. The incredible success of the SM is well demonstrated by looking at the *electroweak sector* governed by $SU(2)_L \times U(1)_Y$ and the *strong sector* or Quantum Chromodynamics (QCD) governed by $SU(3)_C$ separately. In figure 2.1 one can see that there is a fantastic agreement between the theoretical predictions and experimental measurements of a range of electroweak production cross–sections at the LHC. The amount of 'physics' that goes into these calculations is huge — the calculation of the hard processes to sometimes many orders in perturbation theory, the extraction of Parton Distribution Functions (PDFs) from existing datasets of numerous past experiments at many different energy scales and the development showering algorithms just to name a few. At every stage of the calculation the SM is being tested and there is no apparent faltering yet. In figure 2.2 we can see that the theoretical predictions using lattice QCD and experimental mea-

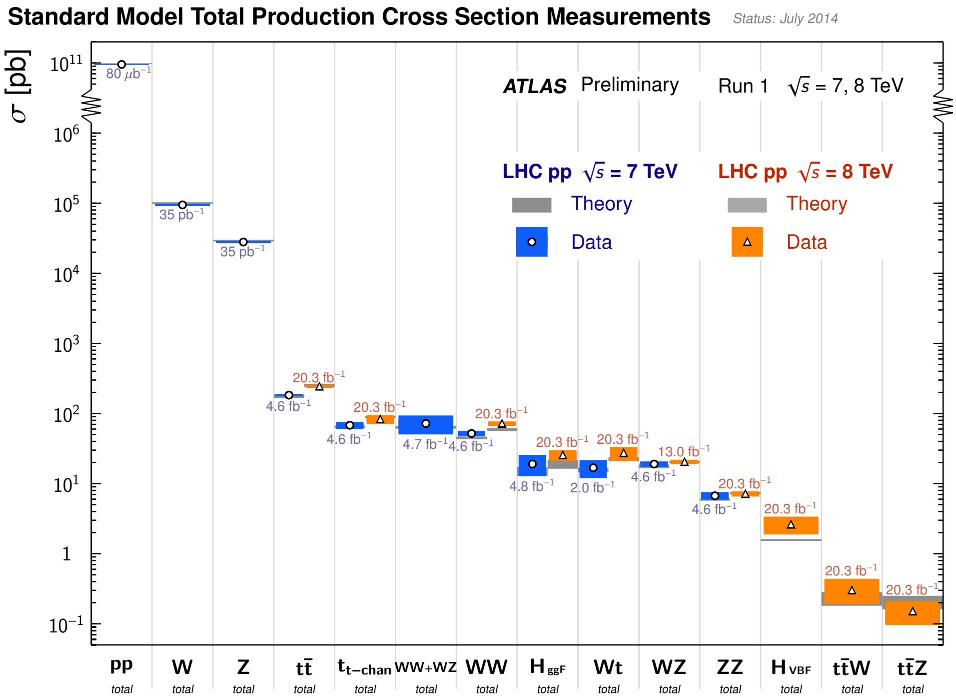


Figure 2.1: Several Standard Model total production cross section measurements compared with the corresponding theoretical expectations. All theoretical expectations were calculated at NLO or higher. The W and Z vector-boson inclusive cross sections were measured with 35 pb^{-1} integrated luminosity from the 2010 dataset. All other measurements were performed using the 2011 dataset or the 2012 dataset. The luminosity used for each measurement is indicated close to the data point. Uncertainties for the theoretical predictions are quoted from the original ATLAS papers. They were not always evaluated using the same prescriptions for PDFs and scales. Taken from [3].

surements of the baryon spectrum are also in good agreement. Together, these show the SM working in its perturbative and non-perturbative regimes within the realm of what we can calculate and should be considered a triumph for experimentalists and theorists alike.

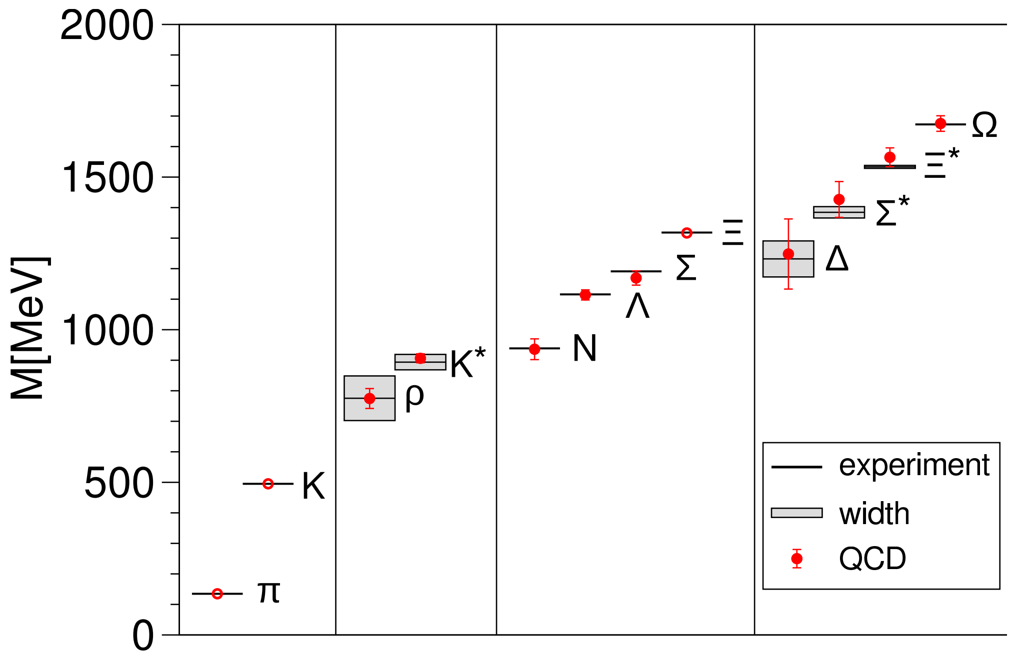


Figure 2.2: Baryonic spectrum obtained by the Budapest Marseille Wuppertal collaboration. Taken from [4].

	Gauge field	Gauge coupling	Defining generators	Structure constants	Indices
$\mathrm{U(1)_Y}$	B_μ	g'	Y	0	N/A
$\mathrm{SU(2)_L}$	W^i_μ	g	$T^i = \frac{\sigma^i}{2}$	ε^{ijk}	$i, j = 1, 2, 3$
$\mathrm{SU(3)_C}$	G^a_μ	g_3	$t^a = \frac{\lambda^a}{2}$	f^{abc}	$a, b = 1, \ldots, 8$

Table 2.1: Properties of gauge groups in the SM. Defining generators means generators in the defining or fundamental representation. Here, σ and λ are the Pauli and Gell–Mann matrices respectively.

The SM is governed by the lagrangian[1]

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}}^{\mathrm{kinetic}} + \mathcal{L}_{\mathrm{SM}}^{\mathrm{yuk}} + \mathcal{L}_{\mathrm{SM}}^{\mathrm{theta}} - V_{\mathrm{SM}}, \tag{2.1.2}$$

$$\mathcal{L}_{\mathrm{SM}}^{\mathrm{kinetic}} = |D_\mu \mathrm{H}|^2 + i \sum_{\mathrm{x}} \psi_x^\dagger \, \sigma^\mu \, D_\mu \psi_x + \frac{1}{4} \sum_{\mathrm{y}} A_y^{\mu\nu} A_{y,\mu\nu}, \tag{2.1.3}$$

$$\mathcal{L}_{\mathrm{SM}}^{\mathrm{theta}} = -\frac{\theta_{\mathrm{YM}}}{32\,\pi^2}\, G^{a,\mu\nu} \widetilde{G}^a_{\mu\nu}, \tag{2.1.4}$$

$$\mathcal{L}_{\mathrm{SM}}^{\mathrm{yuk}} = y_{\mathrm{d}}\, \mathrm{H}^\dagger \cdot q\, \overline{\mathrm{d}} + y_{\mathrm{e}}\, \mathrm{H}^\dagger \cdot \ell\, \overline{\mathrm{e}} + y_{\mathrm{u}}\, \mathrm{H} \cdot q\, \overline{\mathrm{u}} + \mathrm{h.c.}, \tag{2.1.5}$$

$$V_{\mathrm{SM}} = -\mu^2 |\mathrm{H}|^2 + \frac{1}{2}\, \lambda\, |\mathrm{H}|^4, \tag{2.1.6}$$

[1]Here we use "·" to denote the SU(2) invariant product

$$a \cdot b \equiv \varepsilon^{\alpha\beta}\, a_\alpha\, b_\beta = a_1\, b_2 - a_2\, b_1. \tag{2.1.1}$$

where we have suppressed generation indices. The sum over x and y indicates a sum over all fermions and gauge bosons,

$$\psi_x = \{q, \ell, \overline{\mathrm{u}}, \overline{\mathrm{d}}, \overline{\mathrm{e}}\}_x, \qquad A_x^{\mu\nu} = \{\mathrm{B}, \mathrm{W}, G\}_x^{\mu\nu}; \tag{2.1.7}$$

the gauge covariant derivative D_μ is determined by what it acts upon

$$D_\mu \mathcal{O} \equiv \left(\partial_\mu - i\, g_1\, \mathrm{B}^\mu\, Y_{\mathcal{O}} - i\, g_2 \mathrm{W}_\mu^i\, T^i_{\mathbf{r}\mathcal{O}} - i\, g_3\, G_\mu^a\, t^a_{\mathbf{r}\mathcal{O}}\right) \mathcal{O}; \tag{2.1.8}$$

the gauge field strengths of the SM are

$$B_{\mu\nu} \equiv \partial_\mu \mathrm{B}_\nu - \partial_\nu \mathrm{B}_\mu, \tag{2.1.9}$$

$$\mathrm{W}^i_{\mu\nu} \equiv \partial_\mu \mathrm{W}^i_\nu - \partial_\nu \mathrm{W}^i_\mu + g\, \varepsilon^{ijk} W^j_\mu W^k_\nu, \tag{2.1.10}$$

$$G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_3\, f^{abc} G^b_\mu G^c_\nu; \tag{2.1.11}$$

y_{d}, y_{e} and y_{u} are the Yukawa couplings — 3×3 complex matrices in generation space — and

$$\widetilde{G}^a_{\mu\nu} \equiv \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma} \tag{2.1.12}$$

is the Hodge dual of the gluon field strength tensor. The lagrangian in 2.1.2 is the most general renormalisable, Lorentz invariant, gauge invariant lagrangian that can be written with the SM field content given in table 2.2.

In the SM, μ is taken positive such that the scalar potential in eq. 2.1.6 is minimised for a non-zero value of H — the famous *Mexican hat* or *wine bottle* potential[2]. Consequently, the bottom component of the Higgs $\mathrm{SU}(2)_{\mathrm{L}}$ doublet acquires a Vacuum Expectation Value (VEV)

$$\mathrm{H}^0 \to \frac{1}{\sqrt{2}} \left(v + \mathrm{h} + i\, \sigma\right) \tag{2.1.13}$$

leaving the combination of generators

$$Q = T^3 + Y \tag{2.1.14}$$

[2]One issue with the SM is that there is no reason why the terms in the potential should take these relative signs. As we sill see later, one of the triumphs of SUSY is its ability to determine each term in the Higgs potential and, given some reasonable assumptions about the UV physics, predict the relative signs. This is known as Radiative Electroweak Symmetry Breaking (REWSB).

	Spin	Generations	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$H = (H^+, H^0)$	0	1	$\mathbf{1}$	$\square$	$\frac{1}{2}$
$q = (u_L, d_L)$	$\frac{1}{2}$	3	$\square$	$\square$	$\frac{1}{6}$
$\ell = (\nu, e_L)$	$\frac{1}{2}$	3	$\mathbf{1}$	$\square$	$-\frac{1}{2}$
$\bar{u} = u_R^\dagger$	$\frac{1}{2}$	3	$\bar{\square}$	$\mathbf{1}$	$-\frac{2}{3}$
$\bar{d} = d_R^\dagger$	$\frac{1}{2}$	3	$\bar{\square}$	$\mathbf{1}$	$\frac{1}{3}$
$\bar{e} = e_R^\dagger$	$\frac{1}{2}$	3	$\mathbf{1}$	$\mathbf{1}$	1

Table 2.2: SM field content.

unbroken and identified with the generators of the $U(1)_{EM}$. This phenomena — Electroweak Symmetry Breaking (EWSB) — is an explicit example of a more general phenomena called Spontaneous Symmetry Breaking (SSB), often referred to as *The Higgs Mechanism* or simply *Higgsing*

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\text{Higgsing}} SU(3)_C \times U(1)_{EM}. \tag{2.1.15}$$

The more general form of Higgsing will return in Section 5.5.3. This *Higgsing* gives masses to the B and W gauge bosons who mix to form the charged W boson $W^\pm$ and photon γ

$$\begin{pmatrix} B \\ W_1 \end{pmatrix} = Z^{\gamma,Z} \begin{pmatrix} \gamma \\ Z \end{pmatrix}, \qquad \begin{pmatrix} W_2 \\ W_3 \end{pmatrix} = Z^{W^\pm} \begin{pmatrix} W^+ \\ W^- \end{pmatrix} \tag{2.1.16}$$

where

$$Z^{\gamma,Z} = \begin{pmatrix} c_{\theta_W} & -s_{\theta_W} \\ s_{\theta_W} & c_{\theta_W} \end{pmatrix}, \qquad Z^{W^\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}. \tag{2.1.17}$$

The *Higgsing* also generates masses for all fermions in the SM except for neutrinos. There are however some limitations of the SM. Firstly, there are observed phenomena that are not explained within the setup of the SM:

- **Gravity**: The SM describes three of the four fundamental forces incredibly accurately within the realms of perturbation theory. As the couplings in the SM are dimensionless or of positive mass dimension, the most divergent a loop diagram can be is logarithmic. At each order in perturbation theory, there are only a finite number of divergent diagrams meaning that the theory can be renormalised using a finite set of counterterms. One needs to then do a finite set of measurements to fix the theory and make it predictive. Gravity is problematic in this sense as the Newton constant

$$G_{\text{Newton}} = \left(\frac{1}{8\,\pi\,M_{\text{Planck}}}\right)^2 \tag{2.1.18}$$

has a *negative* mass dimension. If we do a naïve power counting, we see that at each loop order in perturbation theory, the UV divergences become worse as we need additional powers of loop momentum to compensate the powers of Planck mass $M_{\text{Planck}} \approx 1.2209 \times 10^{10}\,\text{GeV}$ in the denominator of eq. 2.1.18. Consequently one would expect a quantum field theory involving gravity to require an infinite set of counterterms thus making it non-renormalisable and unpredictive. This is a problem with QFT and gravity in general, not just with the SM.

- **Dark matter**: There is ever growing gravitational evidence for the existence of non-luminous matter in our universe. The radial velocity profile of objects in the outer layers of galactic discs — referred to as *galactic rotation curves* — do not match those expected from Newtonian gravity if only the matter visible in the galaxy is taken into account [12]. On galactic cluster scales, gravitational lensing has provided evidence that the mass and X-rays (that trace the distribution of hot plasma) do not coincide [13]. Finally, the temperature fluctuations in the Cosmic Microwave Background (CMB) have allowed the Planck collaboration has inferred current cold dark matter and baryonic fractions ω_{c} and ω_{b} [14]:

$$\omega_{\text{c}} = 0.1199 \pm 0.0027, \qquad \omega_{\text{b}} = 0.02205 \pm 0.00028.$$

This is a problem as the SM has no dark matter candidate, though this is easily solved by any one or combination of extensions to the SM. To the present day there has been no confirmed particle dark matter observation at either direct detection or collider experiments.

- **Dark energy**: The Plank Collaboration also identified the dark energy fraction ω_Λ of our universe

$$\omega_\Lambda = 0.685^{+0.018}_{-0,016}.$$

This accounts for the majority of the mass-energy content of the universe yet is completely absent from SM. Dark energy does not behave in the same way as normal matter, so is unlikely to have a particle physics interpretation. Incorporating dark energy and the SM into the same framework is however, possible.

- **Matter-antimatter asymmetry**: It is not clear whether the SM has an answer to '*Why are we here rather than not?*' or put another way, '*Why are there unequal amounts of matter and anti–matter?*'. The necessary conditions for this are the *Sakharov conditions* [15]:

  1. Baryon number violation,
  2. C violation and Charge Parity (CP) violation,
  3. Interactions out of thermal equilibrium.

  Although $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_B$ are symmetries at the level of the SM lagrangian, they are broken anomalously

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = -\frac{N_f}{32\pi^2}\left(B_{\mu\nu}\tilde{B}^{\mu\nu} - W^i_{\mu\nu}\tilde{W}^{i,\mu\nu}\right), \tag{2.1.19}$$

  where N_f is the number of flavours. Clearly the combination $\mathrm{U}(1)_{B-L}$ is not anomalous. This allows baryon number $n_B \equiv n_b - n_{\bar{b}} = \int d^3x\, j^0_B$ and lepton number $n_L \equiv n_l - n_{\bar{l}} = \int d^3x\, j^0_L$ violation to occur non–perturbatively[3] providing that the difference in baryon and lepton number is preserved. This can happen if the theory jumps between vacua of different topological charges between two times t_1 and t_2

$$n_B(t_2) - n_B(t_1) = \int_{t_1}^{t_2} dx^0 \int d^3x\, \partial_\mu j^\mu_B = N_f\left[N_{\mathrm{CS}}(t_2) - N_{\mathrm{CS}}(t_1)\right] \tag{2.1.20}$$

[3]The nomenclature here is a bit unusual since one often thinks of the anomaly as being due to triangle diagrams in perturbation theory. The point is that since the result is *one loop exact* then the result isn't a perturbative one, and depending on how the quantity is calculated (e.g. via the Fujikawa method) one sees that the anomaly is proportional to

$$\int d^4x\, \varepsilon^{\mu\nu\rho\sigma} A_{\mu\nu} A_{\rho\sigma} \sim 8\pi^2 k \qquad k \in \mathbb{Z}$$

which is a topological term [16].

where

$$N_{\text{CS}}(t) = -\frac{1}{32\pi^2} \int d^3x \, \left(B_{\mu\nu} \, \tilde{B}^{\mu\nu} - W^i_{\mu\nu} \, \tilde{W}^{i,\mu\nu} \right) \tag{2.1.21}$$

is the Chern–Simmons number and evaluates to an integer[4]. The approximations and uncertainty in this mechanism lie in estimating and calculating the tunnelling rates between the different vacua with different N_{CS}.

Charge symmetry is clearly violated in the SM since e.g. charge reversal acting on an interacting left handed neutrino would take it into a non-interacting left handed anti-neutrino.

The SM has numerous sources of CP violation. The phase δ in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix and the equivalent phase in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing (as well as two additional phases if neutrinos are Majorana) can cause the rate of a process involving these particles to differ from the rate of its CP conjugate. Thanks to the measurement of a non-zero θ_{13} in the PMNS matrix, it should be possible to experimentally determine how large the equivalent δ is in the neutrino sector.

These interactions will become out of thermal equilibrium when the rate of reaction generating the baryon asymmetry becomes less than the rate of universal expansion [17].

Although some sources of CP violation are known, at present, the total CP violating sources in the SM are unknown, and the sphaleron processes in the early universe not yet well understood. It is currently unclear whether the SM can give rise to the current observed matter–antimatter asymmetry. It is likely that if it doesn't then additional high energy degrees of freedom are required to generate a larger $\text{U}(1)_B$ or $\text{U}(1)_L$ imbalance. This is quite normal in Grand Unified Theory (GUT) theories where the same representation of the GUT gauge group G_{GUT} may contain both quarks and leptons. The scalar and gauge degrees of freedom may then mediate interactions amongst fermions that have a different baryon number.

- **Neutrino masses**: The observation that neutrinos oscillate is almost ir-

[4] We have neglected $\int d^3x \, \partial_{\underline{\mu}}, j^{\mu}_{B}$ because fields vanish at spatial infinity.

	Spin	Generations	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\overline{\nu} = \nu_R^\dagger$	$\frac{1}{2}$	3	**1**	**1**	0

Table 2.3: Representations of right-handed neutrino.

refutable evidence that at least two of the three neutrino mass eigenstates have non–zero masses [18, 19]. Here we take (but is not always chosen in the literature) the SM to *not* include a right handed neutrino — a singlet under the SM gauge groups detailed in table 2.3. This choice is made because it is not currently known how the neutrinos acquire a mass. If we don't include a right handed neutrino, the neutrino mass arises in the form of the *Weinberg operator*, the unique gauge invariant, Lorentz invariant dimension five operator one can form using the SM degrees of freedom

$$\mathcal{L}_{\rm SM}^{\rm dim.\ 5} = \frac{(\mathrm{H}\cdot\ell)^2}{\Lambda} + \mathrm{h.c.}. \tag{2.1.22}$$

In this case we need to introduce some new physics at the scale Λ that generates this operator in the EFT.

If they aquire a Dirac mass in the same way as the rest of the SM fermions

$$\mathcal{L}_{\rm SM}^{\rm yuk} \to \mathcal{L}_{\rm SM}^{\rm yuk} + (y_\nu\,\mathrm{H}\cdot\ell\,\overline{\nu} + \mathrm{h.c.}), \tag{2.1.23}$$

where again y_ν is a 3×3 complex matrix in generation space, then the incredible smallness of the Yukawa coupling is suspicious but not impossible to believe. The commonly expected for a neutrino to acquire their small $\lesssim$ eV masses is through the *see–saw mechanism*. Here, in addition to the Dirac term in eq. 2.1.23, one allows for a large Majorana mass $M \gg \frac{y\,v}{\sqrt{2}}$ for the right handed neutrino

$$\mathcal{L}^{\rm Maj} = \frac{M}{2}\,\overline{\nu}\,\overline{\nu} + \mathrm{h.c.}. \tag{2.1.24}$$

After EWSB, the two neutrino states mix to form two Majorana states with masses

$$m_{\nu 1} \approx -\frac{(y_\nu\,v)^2}{2\,M}, \qquad m_{\nu 2} \approx M,$$

and eV masses can be achieved with $\mathcal{O}(1)$ couplings if the Majorana mass M is $\mathcal{O}(10^{13})$ GeV. In each of these three cases, the SM needs extending. It will be very interesting to see which (if any!) of these is the correct solution.

Beyond the above experimental shortcomings of the SM, there are also theoretical issues:

- **The Strong CP problem**: The upper bound on the $\theta_{\rm YM}$ coefficient in eq. 2.1.4 has been experimentally determined from Neutrino Electric Dipole measurements [20, 21]
$$|\theta_{\rm YM}| \lesssim 3 \times 10^{-10}.$$
In the SM, this does not obey the 't Hooft naturalness criterion [22] that essentially states '*A parameter in a lagrangian is allowed to be small providing that when that parameter is set to zero, the symmetry of the lagrangian is enhanced*'[5]. If $\theta_{\rm YM}$ is set to zero, there is no enhancing of the symmetry of the lagrangian — the term is gauge *and* Lorentz invariant, and the SM lagrangian already violates CP[6]. There are explanations for the smallness of this term, with the Peccei-Quinn (PQ) mechanism [23, 24] the most popular. Here, there is a $\mathrm{U}(1)_{\rm PQ}$ symmetry that is broken spontaneously at high energies and anomalously by strong interactions. The consequence is that the low energy theory has a stationary point where the total coefficient of eq. 2.1.4 is zero and is accompanied with the prediction of a new particle: the axion. The simplest models of axions have been ruled out experimentally, but there are many more general possibilities that are interesting to consider [25, 26].

- **The physical parameter problem**: The SM has 18 physical parameters.

[5]The rationale is that we have in the back of our minds that the associated term in the lagrangian has been generated by the breakdown of the symmetry we are restoring as some energy scale. In the limit of the parameter vanishing, the enhanced symmetry of the system prevent radiative corrections from inducing it, and so radiative corrections themselves must be proportional to the parameter itself.

[6]This term violates CP as can be seen by rewriting it

$$G^a_{\mu\nu}\,\widetilde{G}^{a,\mu\nu} = \sum_{\underline{\mu}} 4\, E^a_{\underline{\mu}}\, B^a_{\underline{\mu}}, \tag{2.1.25}$$

where

$$E^a_{\underline{\mu}} \equiv G^{a,\underline{\mu}\,0}, \qquad B^a_{\underline{\mu}} \equiv -\frac{1}{2}\,\varepsilon_{\underline{\mu}\,\underline{\nu}\,\underline{\rho}}\, G^{a,\underline{\nu}\,\underline{\rho}} \tag{2.1.26}$$

are the gluon analogues of the electric and magnetic fields. Under charge conjugation, $E^a_{\underline{\mu}} \to -E^a_{\underline{\mu}}, B^a_{\underline{\mu}} \to -B^a_{\underline{\mu}}$ and under parity transformations $E^a_{\underline{\mu}} \to E^a_{\underline{\mu}}, B^a_{\underline{\mu}} \to -B^a_{\underline{\mu}}$ so the term in eq. 2.1.4 violates CP maximally.

A simple way of counting the physical parameters of a system is to start with the system in the limit of its couplings taken to zero so that the system has its full symmetry. One then includes a *spurion* of that symmetry which has an associated numbers of parameters. By comparing how many generators of the symmetries with and without the *spurion* tells you how many independent rotations could be done to remove the unphysical degrees of freedom in the *spurion*, i.e.

$$\#\,\text{physical parameters} = \#\,\text{parameters} - \#\,\text{broken generators}. \qquad (2.1.27)$$

For the SM one then finds

1. $3\times$ gauge couplings,
2. $2\times$ independent Higgs parameters, e.g. $|\mu|^2$ and λ,
3. $(2\times9)-(2\times9-3) = 3\times$ independent lepton sector parameters left over from the breaking of the $\mathrm{U}(3)_\ell \times \mathrm{U}(3)_\mathrm{e} \to \mathrm{U}(1)^3$ by the presence of y_e, a convenient set are the three charged lepton masses,
4. $(2\times2\times9)-(3\times9-1) = 10\times$ independent quark sector parameters left over from the breaking of the $\mathrm{U}(3)_q \times \mathrm{U}(3)_\mathrm{u} \times \mathrm{U}(3)_\mathrm{d} \to \mathrm{U}(1)_B$ by the presence of y_u and y_d. A convenient set are the three quark lepton masses, three mixing angles and the CP violating phase.

Although it is remarkable that by fixing these 18 parameters one ends up with a theory that is incredibly predictive. One expects that at a higher energy scale that these parameters become related, for instance in a GUT, gauge couplings become related $g(M_\mathrm{GUT}) = g'(M_\mathrm{GUT}) = g_3(M_\mathrm{GUT}) = g_\mathrm{GUT}(M_\mathrm{GUT})$ in an appropriate normalisation, the concept of quarks and leptons may become unified, and the Higgs potential may become completely fixed as in the case of SUSY. In addition, the mechanism for generating the structure of the Yukawa couplings may be understood so that the parameters in the lepton and quark sector are all understood in terms of much fewer numerical inputs.

- **The gauge hierarchy problem**: The final theoretical problem I will discuss is the gauge hierarchy problem. Because of its importance in this thesis, it will be discussed in detail in Subsection 2.1.2.

In summary, the SM is an incredibly predictive theory that, for the last half a century, has not really been put into doubt experimentally (at least as the low energy EFT of something else). As discussed, there are plenty of reasons to expect new physics in experiments to come.

2.1.2 The Gauge Hierarchy Problem

The gauge hierarchy problem with the SM is the one that is of most interest to us for the remainder of this thesis. It can be phrased as '*Why is the weak scale at the scale it is, rather than another, much larger one?*'. In the lagrangian of the SM the μ parameter:

$$V_{\rm SM} \supset -\mu^2\,|{\rm H}|^2.$$

is the single dimensionful parameter, and controls the overall scale of SM physics[7]. We know experimentally that $v = \sqrt{\frac{2\,\mu^2}{\lambda}} \approx 246\,\text{GeV}$ and so we expect $\mu \approx 100\,\text{GeV}$. In isolation, the SM is a perfectly natural QFT at *any* energy scale; the issue is really whether you consider the SM to be in isolation or not. Before turning to the SM, let us consider a much simpler theory with no gauge interactions, two complex scalar fields ϕ_1 and ϕ_2 and a fermion ψ each with a single generation. Let the theory have the lagrangian

$$\mathcal{L}_{\rm simple} = \mathcal{L}^{\rm kinetic}_{\rm simple} + \mathcal{L}^{\rm yuk}_{\rm simple} + \mathcal{L}^{\rm mass}_{\rm simple} - V_{\rm simple}, \tag{2.1.28}$$

$$\mathcal{L}^{\rm kinetic}_{\rm simple} = |\partial_\mu\phi_1|^2 + |\partial_\mu\phi_2|^2 + \psi^\dagger\,\sigma^\mu\,\partial_\mu\,\psi, \tag{2.1.29}$$

$$\mathcal{L}^{\rm yuk}_{\rm simple} = y\,\phi_1\,\psi\,\psi + \text{h.c.}, \tag{2.1.30}$$

$$\mathcal{L}^{\rm mass}_{\rm simple} = M\,\psi\,\psi + \text{h.c.}, \tag{2.1.31}$$

$$V_{\rm simple} = m_1^2\,|\phi_1|^2 + m_2^2\,|\phi_2|^2 + \frac{\lambda_1}{2}\,|\phi_1|^4 + \frac{\lambda_2}{2}\,|\phi_2|^4 + \lambda_{12}|\phi_1|^2|\phi_2|^2. \tag{2.1.32}$$

[7]There are additional scales induced by dimensional transmutation.

This theory has the one loop Renormalisation Group Equations (RGEs)

$$16\,\pi^2\,\beta_y^{(1)} = 6\,y^3, \tag{2.1.33}$$

$$16\,\pi^2\,\beta_M^{(1)} = 6\,M\,y^2, \tag{2.1.34}$$

$$16\,\pi^2\,\beta_{m_1^2}^{(1)} = 4\,m_1^2\,\lambda_1 + 2\,m_2^2\,\lambda_{12} + 4\,m_1^2\,y^2 - 32\,M^2\,y^2, \tag{2.1.35}$$

$$16\,\pi^2\,\beta_{m_2^2}^{(1)} = 4\,m_2^2\,\lambda_2 + 2\,m_1^2\,\lambda_{12}, \tag{2.1.36}$$

$$16\,\pi^2\,\beta_{\lambda_1}^{(1)} = 10\,\lambda_1^2 + 2\,\lambda_{12}^2 + 8\,y^2\,\lambda_1 - 32\,y^4, \tag{2.1.37}$$

$$16\,\pi^2\,\beta_{\lambda_2}^{(1)} = 10\,\lambda_2^2 + 2\,\lambda_{12}^2, \tag{2.1.38}$$

$$16\,\pi^2\,\beta_{\lambda_{12}}^{(1)} = 4\,\lambda_{12}\left(\lambda_{12} + \lambda_1 + \lambda_2 + y^2\right). \tag{2.1.39}$$

A lot can be gained just by looking at the lagrangian and the RGEs. Firstly notice that a small fermion mass M is allowable; the RGE for M in eq. 2.1.34 is proportional to the mass itself so if we set M to zero then it will stay zero along the whole RG flow as was discussed in Section 2.1.1. This is essentially because in the limit of $M \to 0$ an additional U(1) symmetry under which the fermions rotate by a phase $\psi \to e^{i\theta}\psi$ appears — the left part of the *chiral symmetry*. The same is visibly *not* true for the parameter m_1^2 however in eq. 2.1.35. The only way m_1 can be reasonably be expected to stay at a particular scale is if the other masses in the theory m_2 and M are of that scale. If however if $M_\mathrm{h} \equiv M(\mu_\mathrm{high})$ and $m_{2,\mathrm{h}} \equiv m_2(\mu_\mathrm{high})$ are much larger than $m_{1,\mathrm{h}} \equiv m_1(\mu_\mathrm{high})$ at some renormalisation scale μ_high then when running to a lower renormalisation scale μ_low, one finds in the Leading Log (LL) approximation

$$m_{1,\mathrm{l}}^2 \approx m_{1,\mathrm{h}}^2 + \frac{1}{16\,\pi^2} \times \left(32\,M_\mathrm{h}^2\,y_\mathrm{h}^2 - 2\,m_{1,\mathrm{h}}^2\,\lambda_{12,\mathrm{h}}\right) \times \log\left(\frac{\mu_\mathrm{high}}{\mu_\mathrm{low}}\right). \tag{2.1.40}$$

Unless there is some cancellation along the RG flow, it is inevitable that one should find $m_{1,\mathrm{l}}$ to be many orders of magnitude larger than $m_{1,\mathrm{h}}$. The degree of cancellation that one might acquire to achieve a particularly small value at a point in the RG flow is called *tuning*. m_2 on the other hand can only be pushed positive in the same way by m_1, at least at one loop. This is simply because at this order in perturbation theory ϕ_2 and M_2 don't talk to each other. In order for them to interact at this level of perturbation theory, a term in the lagrangian $y'\,\phi_2\,\psi\,\psi$ would need to be generated, but we can immediately see that this will never happen because the lagrangian has a symmetry under which $\phi_2 \to e^{i\,\theta}\,\phi_2$ and all other fields held constant. Such a symmetry would be broken by the $y' \phi_2 \psi \psi$ term and so will never be generated. On top of the corrections from RG flow, there are also finite corrections to each of the particles self energies. For this we need to decompose the complex scalars into real degrees of freedom,

$$\phi_1 \to \frac{1}{\sqrt{2}} (h_1 + i\,\sigma_1) \,, \qquad \phi_2 \to \frac{1}{\sqrt{2}} (h_2 + i\,\sigma_2) \,. \tag{2.1.41}$$

Then one finds

$$\begin{aligned}
\Pi^{\mathrm{h}_1}(p^2) = &- \frac{i}{2} \left\{ \lambda_1 \left[3\,A_0(m^2_{\mathrm{h}_1}) + A_0(m^2_{\sigma_1}) \right] + \lambda_{12} \left[A_0(m^2_{\mathrm{h}_2}) + A_0(m^2_{\sigma_2}) \right] \right\} \\
&+ 2\,y^2 \left[G_0(p^2, m^2_\psi, m^2_\psi) - 2\,m^2_\psi\, B_0(p^2, m^2_\psi, m^2_\psi) \right] , && (2.1.42) \\
\Pi^{\sigma_1}(p^2) = &- \frac{i}{2} \left\{ \lambda_1 \left[A_0(m^2_{\mathrm{h}_1}) + 3\,A_0(m^2_{\sigma_1}) \right] + \lambda_{12} \left[A_0(m^2_{\mathrm{h}_2}) + A_0(m^2_{\sigma_2}) \right] \right\} \\
&+ 2\,y^2 \left[G_0(p^2, m^2_\psi, m^2_\psi) + 2\,m^2_\psi\, B_0(p^2, m^2_\psi, m^2_\psi) \right] , && (2.1.43) \\
\Pi^{\mathrm{h}_2}(p^2) = &- \frac{i}{2} \left\{ \lambda_2 \left[3\,A_0(m^2_{\mathrm{h}_2}) + A_0(m^2_{\sigma_2}) \right] + \lambda_{12} \left[A_0(m^2_{\mathrm{h}_1}) + A_0(m^2_{\sigma_1}) \right] \right\} , && (2.1.44) \\
\Pi^{\sigma_2}(p^2) = &- \frac{i}{2} \left\{ \lambda_2 \left[A_0(m^2_{\mathrm{h}_2}) + 3\,A_0(m^2_{\sigma_2}) \right] + \lambda_{12} \left[A_0(m^2_{\mathrm{h}_1}) + A_0(m^2_{\sigma_1}) \right] \right\} , && (2.1.45) \\
\Pi^{\psi}(p^2) = &- y^2 \left[B_1(p^2, m^2_\psi, m^2_{\mathrm{h}_1}) + B_1(p^2, m^2_{\psi_1}, m^2_{\sigma_1}) \right] , && (2.1.46)
\end{aligned}$$

where the A's, B's and G's are scalar integrals that are defined in Appendix A. This ensures that even if the logarithm is small in 2.1.40, then at some point in the RG flow, there will be a threshold correction where the hierarchy in scales is transferred to the smallest scalar mass.

Taking this on board, if we return to the SM, since there all observed in the SM are roughly the same mass as the μ term or might lighter, then its conceivable that the μ term is acceptable at *any* scale. On the other hand, we currently know very little of the particles that exist above say 1 TeV. As was discussed in Section 2.1.1, there are many reasons to expect new particles to come in at scales different (and much higher) than the weak scale. If the SM is to be extended to include any or all of:

- Heavy right handed neutrinos,
- Flavour physics responsible for generating Yukawa couplings,
- GUT phenomena [27],

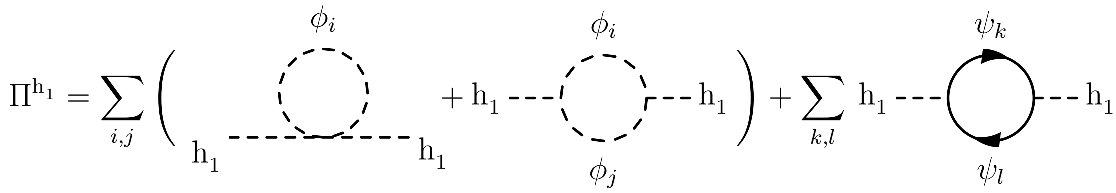


Figure 2.3: The one loop self energy for a scalar field h_1 in a theory with scalar fields ϕ_i, ϕ_i and fermion fields ψ_k, ψ_l. The sums are to be performed so that each independent diagram is only included once in the usual manner. The first diagram is quadratically sensitive to the mass of ϕ_i and the third diagram is quadratically sensitive to the mass of ψ_k and ψ_l. The second diagram is only logarithmically divergent.

- Gravitational physics,
- ... your favourite high energy idea here ...,

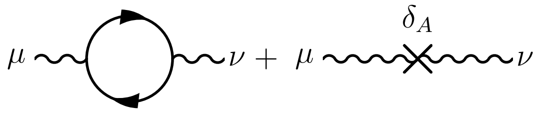
then at some order in perturbation theory, the Higgs sector *will* be coupled heavy particles associated with the above phenomena and consequently, unless engineered otherwise, the μ term will receive radiative corrections of the form

$$\delta\mu^2 \sim \left(\frac{1}{16\,\pi^2}\right)^n \times \sim \mathcal{O}(1) \text{ couplings} \times \text{high scales} \times \text{logarithms}. \tag{2.1.47}$$

Also note that if any of the phenomena involves a heavy scalar field Φ, then there is no symmetry one can impose at the lagrangian level to remove the *Higgs portal* term $\lambda\,|\mathrm{H}|^2|\Phi|^2$, and so the corrections in eq. 2.1.47 *will* happen at one loop. Understood solutions to this problem are:

- Accept an unnatural theory by tuning the UV values of parameters in the theory to match the observed value in the Infrared (IR),
- Not include any of the above phenomena and take the SM as the correct theory all the way to the Planck scale,
- Treat the scalar fields of the SM as composites of fermions (e.g. technicolour [28–30] or *fat Higgs* models [31]),
- Impose a symmetry that protects scalar masses,

amongst a plethora of other possibilities. We will see in the next chapter that SUSY is one way of implementing this last option.



$$= i\left(p^2\, g^{\mu\nu} - p^\mu\, p^\nu\right)\Pi(p^2)$$

Figure 2.4: One loop contribution to an abelian gauge boson vacuum polarisation from a charged fermion with mass m.

2.2 Effective field theories, schemes and the decoupling theorem

If one tries to go beyond LO with a theoretical prediction in a QFT, it is necessary to chose a renormalisation scheme within which to perform the calculation. If we are just calculating within the realms of the SM at the EW scale, it makes sense to choose a scheme where the parameters in the theory are connected to what has been measured experimentally. This scheme is called the On Shell (OS) scheme and is a *mass dependent* scheme, i.e. each of the counterterms are evaluated with the external particles on shell. This scheme is sometimes referred to as the *physical scheme* since the renormalised masses coincide with those measured in experiment.

An important concept for Chapter 4 is the distinction between *mass independent* renormalisation schemes. When doing calculations with a theory that contains a large separation of scales, one usually chooses a *mass independent* scheme such as Dimensional Regularisation (DREG) with modified minimal subtraction ($\overline{\text{MS}}$) or Dimensional Reduction (DRED) [32] with modified minimal subtraction ($\overline{\text{DR}}$) mainly because it is much simpler to do so! There is one caveat however. The OS scheme and other *physical schemes* automatically take into account the Applequist–Carazzone decoupling theorem [33,34] that states '*A heavy degree of freedom (d.o.f.) e.g. a particle with mass M decouples at energy scales μ much lower than its mass $\mu \ll M$ up to logarithmic contributions suppressed by powers of $\frac{\mu}{M}$*', whereas an *unphysical*, *mass independent* scheme does not. To demonstrate this and its solution in a mass–independent scheme, consider the fermion of unit charge and mass m contribution to the one loop gauge boson self energy in a U(1) gauge theory with a single fermion ψ shown in fig. 2.4. In DREG we can evaluate the diagram in fig.

2.4 using standard methods [35]

$$\Pi(p^2) = -\frac{(g/\mu^\varepsilon)^2}{(4\,\pi)^{2-\varepsilon}} \int_0^1 dx\, 8\, x\, (1-x) \frac{\Gamma(\varepsilon)}{[m^2 - x\,(1-x)\, p^2]^\varepsilon} + \delta_A(\mu) \tag{2.2.48}$$

$$= -\frac{g^2}{(4\,\pi)^2}\, \frac{4}{3}\, \Gamma(\varepsilon) + \delta_A(\mu) + \text{finite} + \mathcal{O}(\varepsilon). \tag{2.2.49}$$

In $\overline{\text{MS}}$ the counterterm is fixed to only absorb the UV divergence plus a prescribed finite piece

$$\delta_A^{\overline{\text{MS}}}(\mu) = \frac{g^2}{(4\,\pi)^{2-\varepsilon}}\, \frac{4}{3}\, \frac{\Gamma(\varepsilon)}{\mu^{2\varepsilon}}. \tag{2.2.50}$$

This does not depend on the mass of any particle and hence is a *mass independent* renormalisation scheme. In the OS scheme, we instead fix the counterterm such that $\Pi(p^2)$ vanishes for $p^2 = -\mu^2$

$$\delta_A^{\text{OS}}(\mu) = \frac{(g/\mu^\varepsilon)^2}{(4\,\pi)^{2-\varepsilon}} \int_0^1 dx\, 8\, x\, (1-x) \frac{\Gamma(\varepsilon)}{[m^2 + x\,(1-x)\, \mu^2]^\varepsilon} \tag{2.2.51}$$

which does depend on m demonstrating the mass dependence of the OS scheme. Combining eqs. 2.2.48 and 2.2.51 we find

$$\Pi^{\text{OS}}(p^2, \mu) = \frac{g^2}{(4\,\pi)^2} \int_0^1 dx\, 8\, x\, (1-x) \log\left[\frac{m^2 + x\,(1-x)\,\mu^2}{m^2 - x\,(1-x)\, p^2}\right]. \tag{2.2.52}$$

By applying the Callan-Symanzik (CS) equation it can be shown that to lowest order, the β functions are just combinations of the counterterms. δ_A contributes to the gauge beta function in the different renormalisation schemes as[8]

$$\beta_g^{\text{OS}} = \frac{g}{2} \frac{d\,\delta_A^{\text{OS}}}{dt}, \qquad \beta_g^{\overline{\text{MS}}} = \frac{g}{2} \frac{d\,\delta_A^{\overline{\text{MS}}}}{dt}. \tag{2.2.53}$$

We find

$$\beta_g^{\text{OS}} = -\frac{g^2}{(4\,\pi)^2} \int_0^1 dx\, 8\, x\, (1-x)\, \frac{x\,(1-x)\,\mu^2}{m^2 + x\,(1-x)\,\mu^2} \tag{2.2.54}$$

[8] There is no contribution from δ_ψ or δ_g due to the Ward–Takahashi (WT) identity.

which interpolates between two limits

$$\beta_g^{\text{OS}} = \frac{g^3}{12\,\pi^2} \qquad \mu \gg m, \tag{2.2.55}$$

$$\beta_g^{\text{OS}} = 0 \qquad \mu \ll m. \tag{2.2.56}$$

This is an explicit example of the decoupling theorem in action in a *mass dependent scheme* — when we are at energy scales much higher than the mass of the fermion m, its effects are included into the Greens functions and hence the RG flow of the theory. Once we past its mass in energy scale, the particle decouples on its own, both from the RGEs and other calculations in the theory. If now we turn to the $\overline{\text{MS}}$ scheme we find

$$\beta_g^{\overline{\text{MS}}} = \frac{g^3}{12\,\pi^2} + \mathcal{O}(\varepsilon). \tag{2.2.57}$$

What happens here is that the fermion is included for the entire RG flow and will similarly contribute to all theoretical calculations in the IR far below its mass. *There is no automatic decoupling when working in a mass independent scheme*. This can be viewed as a sickness of the calculation since IR physics is not screened from what is happening in the UV. In other words, by measuring something like the slope of the strong gauge coupling at the LHC, we would be able to infer the existence of strongly interacting particles near the Planck scale, should they exist. This is clearly not the case. What needs to be done is a by hand implementation of the decoupling theorem. The simplest way (that we will choose in this thesis) is to flow down to the mass of a particle and *integrate it out*, resulting in a matching of field theories, the first including the particle, and the second an EFT with the particle removed. At their boundary there will be a set of *matching conditions* or *threshold corrections* to ensure that the two theories agree in the region where they are both valid descriptions [36]. This setup is shown in figure 2.5.

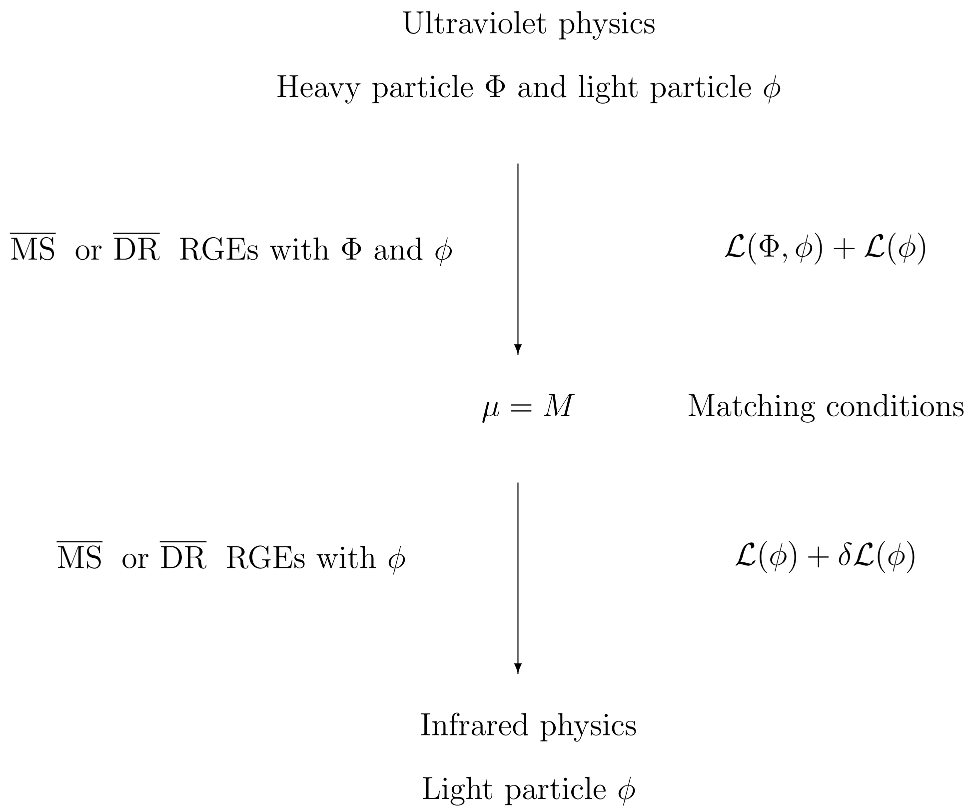


Figure 2.5: Schematic for matching and running between EFTs. At the interface between the two field theories, the terms involving Φ are removed, and then at a given order in perturbation theory, calculations are matched up between the two theories leading to an infinite tower of effective operators $\delta\mathcal{L}(\phi)$ in the LEEFT rendering the low energy theory non-renormalisable.

2.3 Supersymmetry

2.3.1 Basics

SUSY is a symmetry relating particles of different spin

$$Q^i_\alpha \, |\text{fermion}\rangle \sim |\text{boson}\rangle, \qquad Q^i_\alpha \, |\text{boson}\rangle \sim |\text{fermion}\rangle, \tag{2.3.58}$$

where Q^i_α is the generator of a SUSY. Particles related in this way will be referred to as *superpartners*. Immediately one sees striking consequences if SUSY is a symmetry realised in a theory:

- If a one particle fermionic state $|\text{fermion}\rangle$ exists, there is another one particle state $|\text{boson}\rangle$ in the theory; each one-particle state has at least one *superpartner* i.e. in a SUSY theory, one deals with supermultiplets of particle states rather than the single particle states themselves,

- The generator Q^i_α changes the spin of a particle by $\frac{1}{2}$ and hence its *space-time* properties. Q^i_α therefore also transforms in a spin $\frac{1}{2}$ representation of the Lorentz group and is a generator of a *space-time symmetry* rather than an *internal symmetry.*

The Haag–Lopuszanski–Sohnius (HLS) [37] extension of the famous *no–go* theorem of Coleman–Mandula (CM) [38] to include symmetry generators of spin $\frac{1}{2}$ greatly restricts the form that the Q^i_α are allowed to take. The N-extended SUSY algebra where $i = 1, \ldots, N$ is

$$[J_{\mu\nu}, Q^i_\alpha] = -\frac{1}{2}\,(\sigma_{\mu\nu})^{\beta}_{\alpha}\,Q^i_\beta, \qquad [J_{\mu\nu}, \bar{Q}^i_{\dot\alpha}] = \frac{1}{2}\,(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha}\,\bar{Q}^i_{\dot\beta}, \tag{2.3.59}$$

$$[P_\mu, Q^i_\alpha] = 0, \qquad [P_\mu, \bar{Q}^i_{\dot\alpha}] = 0, \tag{2.3.60}$$

$$\{Q^i_\alpha, Q^j_\beta\} = \varepsilon_{\alpha\beta}\,Z^{ij}, \qquad \{\bar{Q}_{\dot\alpha\,i}, \bar{Q}_{\dot\beta\,j}\} = \varepsilon_{\dot\alpha\dot\beta}\,\bar{Z}_{ij}, \tag{2.3.61}$$

and finally

$$\{Q^i_\alpha, \bar{Q}_{\dot\alpha\,j}\} = 2\,\delta^i_j\,(\sigma^\mu)_{\alpha\dot\alpha}\,P_\mu, \tag{2.3.62}$$

where $J_{\mu\nu}$ and P_μ are the Poincaré generators of translations and Lorentz rotations

$$i\,[J^{\mu\nu}, J^{\rho\sigma}] = \eta^{\nu\rho}J^{\mu\sigma} - \eta^{\mu\rho}J^{\nu\sigma} - \eta^{\sigma\mu}J^{\rho\nu} + \eta^{\sigma\nu}J^{\rho\mu}, \tag{2.3.63}$$

$$i\,[P^\mu, J^{\rho\sigma}] = \eta^{\mu\rho}P^\sigma - \eta^{\mu\sigma}P^\rho, \tag{2.3.64}$$

$$[P^\mu, P^\sigma] = 0, \tag{2.3.65}$$

$Z^{ij} = -Z^{ji}$ and $\bar{Z}_{ij} = (Z^{ij})^\dagger$ are *central charges* that commute with all the generators of the SUSY algebra, and $\bar{Q}_{\dot\alpha\,i} = (Q^i_\alpha)^\dagger$ are the N-extended SUSY generators. In this thesis we will only deal with the cases $N = 0, 1$ and 2 which are referred to as $\mathcal{N} = 0$ SUSY (or just a non-SUSY theory), $\mathcal{N} = 1$ SUSY (or just SUSY theory), and $\mathcal{N} = 2$ SUSY (occasionally known as *hypersymmetry*) respectively.

There are some immediate consequences for SUSY theories from the algebra above:

- All *superpartners* have the same mass because P^2 commutes with all Q^i_α,

- The energy E of a state is always zero or positive since

$$0 \leq \sum_{\alpha,\dot{\alpha}} |Q^1_\alpha|\, \phi\rangle|^2 + |\bar{Q}_{\dot{\alpha}\,1}\, |\phi\rangle|^2 = \sum_{\alpha,\dot{\alpha}} \langle\phi|\{Q^1_\alpha, \bar{Q}_{\dot{\alpha}\,1}\}|\phi\rangle = 4\, E\, \langle\phi|\phi\rangle, \quad (2.3.66)$$

- *Supermultiplets* contain equal numbers of fermionic and bosonic degrees of freedom. The operator $(-)^F \equiv (-1)^{2s}$ where s is the spin operator acts on fermions and bosons

$$(-)^F|\text{fermion}\rangle = -\,|\text{fermion}\rangle, \qquad (-)^F|\text{boson}\rangle = +\,|\text{boson}\rangle \qquad (2.3.67)$$

and anti–commutes with *all* fermionic operators — notably the Q^i_α — but commutes with all bosonic operators — notable the P^μ. One then finds

$$n_B - n_F \propto p^\mu \,\text{tr}\, \left[(-)^F\right] = \sum_i \langle i|(-)^F\, P^\mu|i\rangle = 0, \qquad (2.3.68)$$

where the $|i\rangle$ are subspace of states with common momentum $P^\mu|i\rangle = p^\mu|i\rangle$. The quantity tr $\left[(-)^F\right]$ is the *Witten index* [39].

These consequences make it clear that our world is not a SUSY one. As discussed in Section 2.1.1, the SM provides an excellent description of nature, yet no particles in the SM can possibly be *superpartners* of each other since they cannot be arranged into supermultiplets[9]. A solution to this is to complete the multiplets with particles that are yet to be discovered — their names are given in table 2.4. The problem with this is that the resulting theory is in direct conflict with what is observed experimentally. No fermionic photon, the photino, or fermionic gluon, the gluino has ever been observed experimentally. If they had been observed (which in this setup is inevitable) they would already be part of the SM.

[9] Actually this is not strictly true as very early on in the development of SUSY models it was noticed that the Higgs $SU(2)_L$ doublet *does* share quantum numbers with the neutrino and differs in spin by $\frac{1}{2}$ [40–44]. In any case, it is certainly true that not all particles in the SM can be arranged into *supermultiplets* and there is a mass difference between the neutrino and the Higgs, so SUSY will have to be broken even if they are superpartners.

Standard Model eigenstates		Superpartner eigenstates	
Gauge	Mass	Gauge	Mass
	Neutral Higgs		Neutral Higgsino
Gluon	Gluon	Gluino	Gluino
W/B boson	Z boson, photon	Wino/Bino	Zino/Photino
Quarks	Quarks	Squarks	Squarks
Charged leptons	Charged leptons	Charged sleptons	Charged sleptons
Neutrinos	Neutrinos	Sneutrinos	Sneutrinos

Table 2.4: Naming conventions for SUSY partners of SM particles in the absence of SUSY breaking.

2.3.2 Motivation

Now that we have a more concrete understanding of what SUSY *is*, since we've already commented that a fully SUSY theory cannot possibly reproduce reality, why go any further? The key is that we can recycle a technique that already exists in the SM to help us; SUSY, like any other continuous symmetry, can be *spontaneously broken*. The details of how this can happen are given in Section 2.3.7. Using SUSY breaking it is possible to reproduce observed reality. In addition to this, it maintains some properties of SUSY:

- The *gauge hierarchy problem* is, in principle, still solved. The μ term in the SM scalar potential is protected due to the *non-renormalisation* theorem as will be discussed in 2.3.6, but also the large finite pieces in the radiative corrections so scalar masses are (in the SUSY limit) are cancelled against corrections of equal magnitude by corrections from particles of the opposite spin. Another way of seeing this is that the *superpartner* ψ of the $\mathrm{SU(2)_L}$ doublet Φ containing the Higgs has a mass that is protected by chiral symmetry. Since in the SUSY limit, these masses are equal, the mass of Φ, and hence the Higgs mass, is also protected. In the case where SUSY is broken, this cancellation is approximate, and one gets corrections to scalar masses of the form

$$\delta m_\Phi^2 \sim (\text{couplings}) \times m_\psi^2 \log\left(\frac{m_\phi}{m_\psi}\right) \tag{2.3.69}$$

where the fields ψ and ϕ are related by SUSY, and their masses are equal in the SUSY limit. We now see an interesting problem. If we are to solve the hierarchy problem in a satisfactory way, then the effects of SUSY breaking should not be too large as to not make the logarithm in eq. 2.3.69 too large. The tension between the non-observation of superpartners and the size of logarithms is the *little hierarchy problem*, with the non-observation of squarks and gluinos being the main source of tension. Beyond the possible stability of the Higgs sector to radiative corrections, the presence of an approximate SUSY (that is presumably restored at some energy scale) allows us to include other scalars in the theory without worrying about them — at least from a naturalness point of view.

- A GUT theory is indicated by the apparent unification of the SM gauge couplings with the MSSM field content [45,46] (see Section 2.3.9). This is demonstrated at one loop in figure 2.6.
- The R parity conserving MSSM typically contains a viable dark matter candidate.
- SUSY is the *only* non-trivial extension of the Poincaré group allowed by the HLS [37] extension of CM [38] no-go theorem. Examples of all other types of symmetries allowed by the CM theorem are realised in nature somehow, even if they are spontaneously broken. SUSY is so far the only allowed symmetry that we haven't seen, which makes it slightly unusual. Perhaps there is a deeper reason as to why we shouldn't expect SUSY to be realised in a QFT but so far one hasn't been put forwards.

2.3.3 Writing an $\mathcal{N} = 1$ supersymmetric theory

From space–time to superspace

We will now demonstrate how to write a $\mathcal{N} = 1$ SUSY theory. The simplest way of doing this is to use the *superspace* approach developed in [47] which we take, following the conventions of [48]. Here we extend the bosonic coordinates *space-time* to include additional two complex anti-commuting (Grassman) spinors θ_α and $\bar{\theta}_{\dot{\alpha}}$. Recall that for a Grassman parameter η

$$\int d\eta = 0, \qquad \int d\eta\, \eta = \frac{d\eta}{d\eta} = 1, \tag{2.3.70}$$

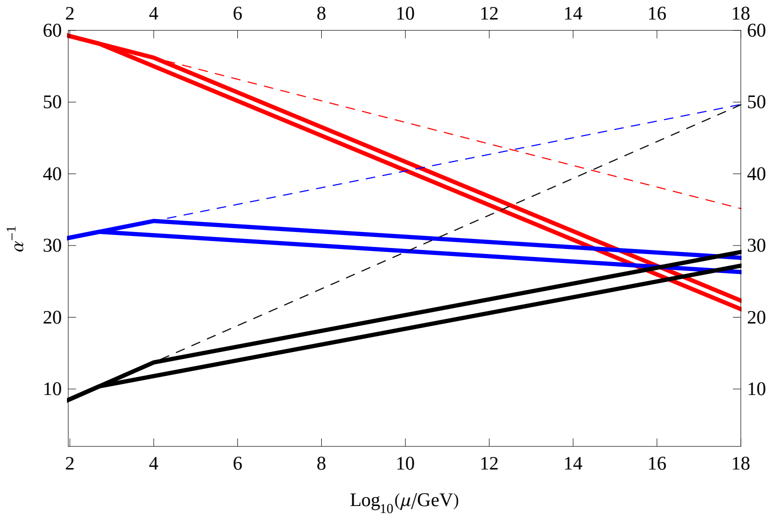


Figure 2.6: One loop RG evolution of the SM gauge couplings. The red, blue and black dashed (solid) lines show the evolution of the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge couplings in the SM (MSSM). The one loop beta coefficients are $b^{SM} = (41/10, -19/6, -7)$ and $b^{MSSM} = (33/5, 1, -3)$. We decouple the SUSY particles at a common threshold varied between 500 GeV and 10 TeV.

and that a Taylor expansion of a function of a Grassman parameter truncates in a finite number of terms

$$f(\eta) = f_0 + \eta f_1 \tag{2.3.71}$$

so that

$$\int d\eta \, f(\eta) = f_1 \tag{2.3.72}$$

and so on. Now with the fermionic coordinates we have

$$\{\theta_\alpha, \bar{\theta}_{\dot{\alpha}}\} = 0, \tag{2.3.73}$$

and it is convenient to define

$$d^2\theta \equiv -\frac{1}{4} d\theta^\alpha \, d\theta^\beta \, \varepsilon_{\alpha\beta}, \tag{2.3.74}$$

$$d^2\bar{\theta} \equiv -\frac{1}{4} d\bar{\theta}_{\dot{\alpha}} \, d\bar{\theta}_{\dot{\beta}} \, \varepsilon^{\dot{\alpha}\dot{\beta}}, \tag{2.3.75}$$

$$d^4\theta \equiv d^2\theta \, d^2\bar{\theta}, \tag{2.3.76}$$

so that

$$\int d^2\theta \, \theta^2 = \int d^2\bar{\theta} \, \bar{\theta}^2 = \int d^4\theta \, \theta^2\bar{\theta}^2 = 1. \tag{2.3.77}$$

SUSY transformations in superspace

The most general $\mathcal{N} = 1$ superfield $X(x, \theta, \bar{\theta})$ is

$$X(x,\theta,\bar{\theta}) = a + \theta\,\xi + \bar{\theta}\,\chi^{\dagger} + \theta^2\,b + \bar{\theta}^2\,c + \bar{\theta}\,\bar{\sigma}^{\mu}\,\theta\,v_{\mu} + \bar{\theta}^2\,\theta\,\eta + \theta^2\,\bar{\theta}\,\zeta^{\dagger} + \theta^2\,\bar{\theta}^2\,d \quad (2.3.78)$$

and when we perform the Grassman integration we get

$$\int d^2\theta\, X(x,\theta,\bar{\theta}) = b(x) + \bar{\theta}\,\zeta^{\dagger}(x) + \bar{\theta}^2\,d(x), \quad (2.3.79)$$

$$\int d^2\bar{\theta}\, X(x,\theta,\bar{\theta}) = c(x) + \theta\,\eta^{\dagger}(x) + \theta^2\,d(x), \quad (2.3.80)$$

$$\int d^4\theta\, X(x,\theta,\bar{\theta}) = d(x). \quad (2.3.81)$$

We want to work out how to write down a SUSY action, so we need to form differential representations of the SUSY generators Q_{α} to act on the superfields. They will be defined so that $i\,\varepsilon\,\mathcal{Q}$ generates translation in superspace $\theta \to \theta + \varepsilon$ and is accompanied by some translation in space–time $x \to x + \delta x$

$$(1 + i\,\varepsilon\,\mathcal{Q})\,X(x,\theta,\bar{\theta}) \equiv X(x + \delta x, \theta + \varepsilon, \bar{\theta}), \quad (2.3.82)$$

$$(1 + i\,\varepsilon^{\dagger}\,\bar{\mathcal{Q}})\,X(x,\theta,\bar{\theta}) \equiv X(x + \delta x, \theta, \bar{\theta} + \varepsilon^{\dagger}). \quad (2.3.83)$$

One finds[10]

$$\mathcal{Q}_{\alpha} = i\,\partial_{\alpha} - (\sigma^{\mu}\,\bar{\theta})_{\alpha}\,\partial_{\mu}, \qquad \mathcal{Q}^{\alpha} = -i\,\partial^{\alpha} + (\bar{\theta}\,\bar{\sigma}^{\mu})^{\alpha}\,\partial_{\mu}, \quad (2.3.84)$$

$$\bar{\mathcal{Q}}^{\alpha} = i\,\partial^{\dot{\alpha}} - (\bar{\sigma}^{\mu}\,\theta)^{\dot{\alpha}}\,\partial_{\mu}, \qquad \bar{\mathcal{Q}}_{\dot{\alpha}} = -i\,\partial_{\dot{\alpha}} + (\theta\,\sigma^{\mu})_{\dot{\alpha}}\,\partial_{\mu}. \quad (2.3.85)$$

It is worth noting that a consequence of these definitions is that integrating anything with respect to the whole of the superspace gives a SUSY quantity

$$\delta_{\varepsilon}\int d^4x \int d^4\theta\ (\text{anything}) \sim \int d^4x \int d^4\theta\ \Big(\sum \text{total derivatives}\Big) = 0. \quad (2.3.86)$$

[10]There is a sign difference in the ∂^{α} versus ∂_{α} terms since $\partial_{\alpha}(\theta^2) = 2\,\theta_{\alpha}$ whereas $\partial^{\alpha}(\theta^2) = -2\,\theta^{\alpha}$ from the definition of the derivative $\partial_{\alpha}\theta^{\beta} \equiv \delta_{\alpha}^{\beta}$.

A SUSY transformation acts on $X(x, \theta, \bar{\theta})$ as

$$\begin{aligned}\delta_\varepsilon X(x,\theta,\bar{\theta}) &= i\,(\varepsilon\,\mathcal{Q} + \varepsilon^\dagger\,\bar{\mathcal{Q}})\,X(x,\theta,\bar{\theta})\\ &= X[x + i(\varepsilon\,\sigma^\mu\,\bar{\theta} + \varepsilon^\dagger\,\bar{\sigma}^\mu\,\theta), \theta + \varepsilon, \bar{\theta} + \varepsilon^\dagger] - X(x,\theta,\bar{\theta}). \end{aligned} \tag{2.3.87}$$

Leading to the transformations of the component fields in eq. 2.3.78

$$\delta_\varepsilon\, a = \varepsilon\,\xi + \varepsilon^\dagger\chi^\dagger, \tag{2.3.88}$$

$$\delta_\varepsilon\,\xi_\alpha = 2\,\varepsilon_\alpha b - (\sigma^\mu\,\varepsilon^\dagger)_\alpha(v_\mu + i\,\partial_\mu\,a), \tag{2.3.89}$$

$$\delta_\varepsilon\,\chi^{\dagger\dot{\alpha}} = 2\,\varepsilon^{\dagger\dot{\alpha}}\,c + (\bar{\sigma}^\mu\,\varepsilon)^{\dot{\alpha}}\,(v_\mu - i\,\partial_\mu\,a), \tag{2.3.90}$$

$$\delta_\varepsilon\,b = \varepsilon^\dagger\,\zeta^\dagger - \frac{i}{2}\,\varepsilon^\dagger\,\bar{\sigma}^\mu\,\partial_\mu\,\xi, \tag{2.3.91}$$

$$\delta_\varepsilon\,c = \varepsilon^\dagger\,\eta^\dagger - \frac{i}{2}\,\varepsilon^\dagger\,\bar{\sigma}^\mu\partial_\mu\,\chi, \tag{2.3.92}$$

$$\delta_\varepsilon\,v^\mu = \varepsilon\,\sigma^\mu\,\zeta^\dagger - \varepsilon^\dagger\,\bar{\sigma}^\mu\,\eta - \frac{i}{2}\,\varepsilon\sigma^\nu\,\bar{\sigma}^\mu\,\partial_\nu\,\xi + \frac{i}{2}\,\varepsilon^\dagger\,\bar{\sigma}^\nu\,\sigma^\mu\,\partial_\nu\,\chi^\dagger, \tag{2.3.93}$$

$$\delta_\varepsilon\,\eta_\alpha = 2\,\varepsilon_\alpha\,d - i(\sigma^\mu\,\varepsilon^\dagger)_\alpha\,\partial_\mu\,c - \frac{i}{2}\,(\sigma^\nu\,\bar{\sigma}^\mu\,\varepsilon)_\alpha\,\partial_\mu\,v_\nu, \tag{2.3.94}$$

$$\delta_\varepsilon\,\zeta^{\dagger\dot{\alpha}} = 2\,\varepsilon^{\dagger\dot{\alpha}}\,d - i(\bar{\sigma}^\mu\,\varepsilon)^{\dot{\alpha}}\,\partial_\mu\,b + \frac{i}{2}\,(\bar{\sigma}^\nu\,\sigma^\mu\,\varepsilon^\dagger)^{\dot{\alpha}}\,\partial_\mu v_\nu, \tag{2.3.95}$$

$$\delta_\varepsilon\,d = -\frac{i}{2}\,\varepsilon^\dagger\,\bar{\sigma}^\mu\partial_\mu\,\eta - \frac{i}{2}\,\varepsilon\,\sigma^\mu\,\partial_\mu\,\zeta^\dagger. \tag{2.3.96}$$

Before we proceed it is useful to define the *chiral superspace* or *Grassman analytic* coordinate

$$y^\mu \equiv x^\mu - i\,\theta\sigma^\mu\,\bar{\theta} \tag{2.3.97}$$

and a representation for the superspace derivatives[11]

$$\mathcal{D}_\alpha = \partial_\alpha - i(\sigma^\mu\,\bar{\theta})_\alpha\,\partial_\mu = \partial_\alpha - 2\,i(\sigma^\mu\,\bar{\theta})_\alpha\,\frac{\partial}{\partial y^\mu}, \tag{2.3.98}$$

$$\mathcal{D}^\alpha = -\partial^\alpha + i(\bar{\theta}\,\bar{\sigma}^\mu)^\alpha = -\partial^\alpha + 2\,i\,(\bar{\theta}\,\bar{\sigma}^\mu)^\alpha\,\frac{\partial}{\partial y^\mu}, \tag{2.3.99}$$

$$\bar{\mathcal{D}}^{\dot{\alpha}} = \partial^{\dot{\alpha}} - i(\bar{\sigma}^\mu\,\theta)^{\dot{\alpha}}\,\partial_\mu = \partial^{\dot{\alpha}}, \tag{2.3.100}$$

$$\bar{\mathcal{D}}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} + i(\theta\,\sigma^\mu)_{\dot{\alpha}}\,\partial_\mu = -\partial_{\dot{\alpha}} \tag{2.3.101}$$

[11] The second expression for each derivative is valid only at the coordinate in eq. 2.3.97.

that anti–commute with the $\mathcal{Q}$s and $\bar{\mathcal{Q}}$s such that

$$\delta_\varepsilon\,[\mathcal{D}_\alpha\,X(x,\theta,\bar\theta)] = \mathcal{D}_\alpha[\delta_\varepsilon X(x,\theta,\bar\theta)], \tag{2.3.102}$$

$$\delta_\varepsilon\,[\bar{\mathcal{D}}_{\dot\alpha}\,X(x,\theta,\bar\theta)] = \bar{\mathcal{D}}_{\dot\alpha}[\delta_\varepsilon X(x,\theta,\bar\theta)]. \tag{2.3.103}$$

Chiral superfields

We can now start defining our *irreducible representations* of SUSY. The first one we will consider is the Chiral Superfield (χSF) (or *left handed χSF*) $\Phi(y,\theta) = \Phi(x,\theta,\bar\theta)$ that satisfies

$$\bar{\mathcal{D}}_{\dot\alpha}\,\Phi = 0. \tag{2.3.104}$$

The converse, an antiχSF (or *right handed χSF*) $\Phi^\dagger(y^\dagger,\bar\theta) = \Phi^\dagger(x,\theta,\bar\theta)$ satisfies

$$\mathcal{D}_\alpha\,\Phi^\dagger = 0. \tag{2.3.105}$$

Note that since $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_{\dot\alpha}$ satisfy the product rule, then a product of (anti)χSFs is also a(n) (anti)χSF. Also, since

$$\bar{\mathcal{D}}_{\dot\alpha}\,\bar{\mathcal{D}}^2\,(\text{anything}) = \mathcal{D}_\alpha\,\mathcal{D}^2\,(\text{anything}) = 0 \tag{2.3.106}$$

then a χSF and an anti χSF can be formed by writing

$$\Phi = \bar{\mathcal{D}}^2\,X, \qquad \Phi^\dagger = \mathcal{D}^2\,X \tag{2.3.107}$$

respectively, where $X = X(x,\theta,\bar\theta)$ is the general superfield given in eq. 2.3.78. Now if we fix the space–time location to be at y, the component form solution to the constraint in eq. 2.3.104 is solved by simply taking a function of *only* x and θ but *not* $\bar\theta$[12]

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\,\theta\,\psi(y) + \theta^2\,F(y) \tag{2.3.108}$$

$$= \phi(x) + i\,\bar\theta\,\bar\sigma^\mu\,\theta\,\partial_\mu\,\phi(x) + \frac{1}{4}\,\theta^2\,\bar\theta^2\,\partial_\mu\,\partial^\mu\,\phi(x)$$

$$+ \sqrt{2}\,\psi(x) - \frac{i}{\sqrt{2}}\,\theta^2\,\bar\sigma^\mu\,\partial_\mu\,\psi(x) + \theta^2\,F(x). \tag{2.3.109}$$

[12]The $\sqrt{2}$ is just convention.

Comparing the terms in 2.3.109 allow us to determine how the components of a χSF behave under SUSY transformations

$$\delta_\varepsilon \, \phi = \varepsilon \, \psi, \tag{2.3.110}$$

$$\delta_\varepsilon \, \psi_\alpha = -i \, (\sigma^\mu \, \varepsilon^\dagger)_\alpha \, \partial_\mu \, \phi + \varepsilon_\alpha \, F, \tag{2.3.111}$$

$$\delta_\varepsilon \, F = -i \, \varepsilon^\dagger \, \bar{\sigma}^\mu \, \partial_\mu \, \psi, \tag{2.3.112}$$

with the key thing to note that the coefficient of the θ^2 term of a χSF — known as the *F term* — transformed by only a total derivative. An action S formed from just *F terms* is therefore *supersymmetric*, i.e. is invariant under SUSY transformations

$$\delta_\varepsilon S = \delta_\varepsilon \int d^4x \, \left[\left(\int d^2\theta \, \Phi \right) + \text{h.c.} \right] = 0. \tag{2.3.113}$$

Now given a theory with a set of χSFs Φ_i, we can write a renormalisable SUSY lagrangian

$$\mathcal{L} = \int d^4\theta \, K(\Phi_i) + \left\{ \left[\int d^2\theta \, W(\Phi_i) \right] + \text{h.c.} \right\}, \tag{2.3.114}$$

where K is the *Kähler potential* with canonical form

$$K(\Phi_i) = \Phi^{\dagger i} \Phi_i \tag{2.3.115}$$

and $W(\Phi_i)$ is the *superpotential* — a χSF formed from the other Φ_i in the theory. W is also referred to as a *holomorphic* as it is a function of χSFs *only* and *not* antiχSFs. The general renormalisable form of $W(\Phi_i)$ was introduced in the Wess–Zumino (WZ) model[13]

$$W(\Phi_i) = L^i \, \Phi_i + \frac{1}{2} \, M^{ij} \, \Phi_i \, \Phi_j + \frac{1}{6} \, y^{ijk} \, \Phi_i \, \Phi_j \, \Phi_k. \tag{2.3.116}$$

where L^i is the *linear superpotential term*, $M^{ij} = M^{ji}$ is a *supersymmetic mass*, and $y^{ijk} = y^{jik} =$ (other permutations of ijk) is a *supersymmetic Yukawa coupling*. The

[13]The WZ model has $L_i = 0$.

different terms in eq. 2.3.114 in component form evaluate to

$$\mathcal{L}_K = \int d^4\theta \, K(\Phi_i) \tag{2.3.117}$$

$$= -\,\partial_\mu \phi^{\dagger i} \partial^\mu \phi_i + i\,\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + |F_i|^2 + \text{total derivatives}, \tag{2.3.118}$$

$$\mathcal{L}_W = \left[\int d^2\theta \, W(\Phi_i) \right] + \text{h.c.} \tag{2.3.119}$$

$$= \; L^i \, F_i + \frac{1}{2} \, M^{ij} \, (\phi_i \, F_j + \phi_j \, F_i - \psi_i \, \psi_j) + \frac{1}{6} \, y^{ijk} \, [\phi_i \, \phi_j \, F_k - \psi_i \, \psi_j \, \phi_k + (\text{cyclic permutations})] + \text{h.c.}. \tag{2.3.120}$$

One then solves the F *term* equations

$$\frac{\partial \mathcal{L}}{\partial \, F_i} = F^{\dagger i} + L^i + M^{ij} \, \phi_j + \frac{1}{2} \, y^{ijk} \, \phi_i \, \phi_j = 0, \tag{2.3.121}$$

$$\frac{\partial \mathcal{L}}{\partial \, F^{\dagger i}} = F_i + L_i^* + M_{ij}^* \, \phi^{\dagger j} + \frac{1}{2} \, y_{ijk}^* \, \phi^{\dagger i} \, \phi^{\dagger j} = 0 \tag{2.3.122}$$

and substitutes their solutions to find the scalar potential $V(\phi_i)$

$$V(\phi_i) = |F_i|^2. \tag{2.3.123}$$

Vector superfields

The second *irreducible representations* of SUSY we will consider is the Vector Superfield (VSF) (or *real* superfield). A VSF satisfies the constraint

$$V(x, \theta, \bar{\theta}) = V^\dagger(x, \theta, \bar{\theta}). \tag{2.3.124}$$

Upon comparing this to the general superfield $X(x, \theta, \bar{\theta})$, this just causes the identification of some coefficients and forces some coefficients to be real. After making the traditional redefinitions

$$\eta_\alpha = \lambda_\alpha - \frac{i}{2} \, (\sigma^\mu \, \partial_\mu \, \zeta^\dagger)_\alpha, \qquad d = \frac{1}{2} \, D + \partial_\mu \, \partial^\mu \, a \tag{2.3.125}$$

the component field expansion of $V(x,\theta,\bar{\theta})$ is

$$V(x,\theta,\bar{\theta}) = a + \bar{\theta}\,\bar{\sigma}^{\mu}\,\theta v_{\mu} + \theta^2\,\bar{\theta}^2\,\left(\frac{1}{2}\,D + \frac{1}{4}\,\partial_{\mu}\,\partial^{\mu}\,a\right) + \left[\theta\,\xi + \theta^2\,b + \theta^2\,\bar{\theta}\,\left(\lambda^{\dagger} - \frac{i}{2}\,\bar{\sigma}^{\mu}\,\partial_{\mu}\,\xi\right) + \text{h.c.}\right], \tag{2.3.126}$$

which when comparing the terms in 2.3.109 allow us to determine how the components of a VSF behave under SUSY transformations

$$\delta_{\varepsilon}\,a = \varepsilon\,\xi + \varepsilon^{\dagger}\,\xi^{\dagger}, \tag{2.3.127}$$

$$\delta_{\varepsilon}\,\xi_{\alpha} = 2\,\varepsilon_{\alpha}b - (\sigma^{\mu}\,\varepsilon^{\dagger})_{\alpha}\,(v_{\mu} + i\,\partial_{\mu}\,a), \tag{2.3.128}$$

$$\delta_{\varepsilon}\,b = \varepsilon^{\dagger}\,\lambda^{\dagger} - \varepsilon^{\dagger}\,\bar{\sigma}^{\mu}\,\partial_{\mu}\,\xi, \tag{2.3.129}$$

$$\delta_{\varepsilon}\,v^{\mu} = \varepsilon\,\sigma^{\mu}\,\lambda^{\dagger} - \varepsilon^{\dagger}\,\bar{\sigma}^{\mu}\,\lambda + i\,\varepsilon\,\partial^{\mu}\,\xi - i\,\varepsilon^{\dagger}\,\partial^{\mu}\,\xi^{\dagger}, \tag{2.3.130}$$

$$\delta_{\varepsilon}\,\lambda_{\alpha} = \varepsilon_{\alpha}\,D + \frac{i}{2}\,(\sigma^{\mu}\,\bar{\sigma}^{\nu}\,\varepsilon)_{\alpha}\,\left(\partial_{\mu}v_{\nu} - \partial_{\nu}\,v_{\mu}\right), \tag{2.3.131}$$

$$\delta_{\varepsilon}\,D = -i\,\varepsilon\,\sigma^{\mu}\,\partial_{\mu}\,\lambda^{\dagger} - i\,\varepsilon^{\dagger}\,\bar{\sigma}^{\mu}\,\partial_{\mu}\,\lambda. \tag{2.3.132}$$

Again, the key thing to note that the coefficient of the $\theta^2\bar{\theta}^2$ term of a VSF — known as the *D term* — transformed by only a total derivative, and so any action S formed from just *D terms* of VSFs is *supersymmetric*

$$\delta_{\varepsilon}S = \delta_{\varepsilon}\int d^4x\int d^4\theta\,V = 0. \tag{2.3.133}$$

Finally, (as we will see later) V is a VSF because it will be involved in gauge transformations of the form

$$V \to V + i\,(\Lambda - \Lambda^{\dagger}) \tag{2.3.134}$$

where Λ is a χSF. Because of this, one can make a suitable gauge transformation of the form in eq. 2.3.134 to remove unphysical degrees of freedom, bringing V into the WZ gauge[14]

$$V_{\rm WZ} = \bar\theta\,\sigma^\mu\,\theta v_\mu + \left(\bar\theta^2\,\theta\,\lambda + \text{h.c.}\right) + \frac{1}{2}\,\theta^2\,\bar\theta^2\,D. \tag{2.3.135}$$

Gauge theories

If we want to have any hope in making a connection with reality then we will need to work out how to write a supersymmetric gauge theory. First we consider an abelian gauge theory in detail before simply presenting the result for a non–abelian gauge theory.

Abelian gauge theories: Consider a theory with some χSFs Φ_i each with charge q_i under a U(1) gauge theory with associated VSF V. The χSFs and VSF transform under the theory as

$$\Phi_i \to e^{2\,i\,g\,q_i\,\Lambda}\Phi_i, \quad \Phi^{\dagger i} \to e^{-2\,i\,g\,q_i\,\Lambda^\dagger}\Phi^\dagger_i, \quad V \to V + i(\Lambda^\dagger - \Lambda), \tag{2.3.136}$$

where Λ is a χSF. The Kähler potential in eq. 2.3.115 is in general not gauge invariant, so we need to upgrade to

$$K(\Phi_i, V) = \Phi^{\dagger i}\, e^{2\,g\,q_i\,V}\,\Phi_i. \tag{2.3.137}$$

This is a U(1) gauge invariant, and in the WZ gauge it becomes

$$\begin{aligned}\mathcal{L}_K = \int d^4\theta\, K(\Phi_i, V) = &- D_\mu\phi^{\dagger i} D^\mu\phi_i + i\,\psi^{\dagger i}\bar\sigma^\mu D_\mu\psi_i - \sqrt{2}\,g\,q_i(\phi^{\dagger i}\psi_i\lambda + \text{h.c.})\\ &+ g\,q_i\,|\phi_i|^2 D + |F_i|^2 + (\text{total derivatives}).\end{aligned} \tag{2.3.138}$$

Finally we want to include kinetic terms for the U(1) gauge bosons. To do this we define the gauge field superstrength χSF

$$\mathcal{W}_\alpha \equiv -\frac{1}{4}\,\bar{\mathcal{D}}^2\,\mathcal{D}_\alpha\,V, \qquad \overline{\mathcal{W}}_{\dot\alpha} \equiv -\frac{1}{4}\,\mathcal{D}^2\,\bar{\mathcal{D}}_{\dot\alpha}\,V. \tag{2.3.139}$$

[14]Note that the WZ gauge is broken by both SUSY and gauge transformations. If the action is SUSY however, one can always do a combination of SUSY and gauge transformations to recast V in this form.

This is a gauge invariant, and in the WZ gauge is written

$$\mathcal{W}_{\mathrm{WZ}\alpha} = \lambda_\alpha + \theta_\alpha\, D - \frac{i}{2}\,(\sigma^\mu\,\bar{\sigma}^\nu\,\theta)_\alpha v_{\mu\nu} + i\,\theta^2(\sigma^\mu\,\partial_\mu\,\lambda^\dagger)_\alpha. \qquad (2.3.140)$$

Because $\mathcal{W}$ is a χSF we can write another SUSY term in the lagrangian known as the Super Yang–Mills (SYM) term

$$\mathcal{L}_{\mathrm{SYM}} = \left(\frac{1}{4}\int d^2\theta\,\mathcal{W}^2\right) + \text{h.c.} \qquad (2.3.141)$$

that when evaluated contains kinetic terms for the gauge field and gauginos

$$\left(\frac{1}{4}\int d^2\theta\,\mathcal{W}^2\right) + \text{h.c.} = i\,\lambda^\dagger\,\bar{\sigma}^\mu\partial_\mu\,\lambda - \frac{1}{4}\,v_{\mu\nu}\,v^{\mu\nu} + \frac{1}{2}\,D^2 + (\text{total derivatives}) \quad (2.3.142)$$

where $v_{\mu\nu} \equiv \partial_\mu\,v_\nu - \partial_\nu\,v_\mu$. In the case of an abelian gauge theory, we can also write the Fayet–Iliopoulos (FI) term [49]

$$\mathcal{L}_{\mathrm{FI}} = -2\,\kappa\int d^4\theta\,V = -\kappa\,D + (\text{total derivatives}), \qquad (2.3.143)$$

and as we will see in Section 2.3.7 plays a role in spontaneous SUSY breaking. The whole lagrangian is then constructed by combining the terms

$$\mathcal{L}_{\mathrm{total}} = \mathcal{L}_{\mathrm{SYM}} + \mathcal{L}_K + \mathcal{L}_W + \mathcal{L}_{\mathrm{FI}}, \qquad (2.3.144)$$

where $\mathcal{L}_W$ is as determined in eq. 2.3.119 with the additional requirement that the superpotential $W(\Phi_i)$ is gauge invariant. After performing the relevant superspace integrals, one then solves the *F term* equations (unchanged from eqs. 2.3.121 and 2.3.122) and the *D term* equation

$$\frac{\partial\mathcal{L}_{\mathrm{total}}}{\partial D} = D + g\,q_i\,|\phi_i|^2 - \kappa = 0. \qquad (2.3.145)$$

Substituting the solutions then gives the scalar potential $V(\phi_i)$

$$V(\phi_i) = |F_i|^2 + \frac{1}{2}D^2. \qquad (2.3.146)$$

Non–abelian gauge theories: Consider a theory with χSFs Φ_i in some representation $\mathbf{r}_i$ of a gauge group G with associated VSF V^a. For convenience we define the matrices

$$V_i^{\ j} = 2\,g\,(T^a)_i^{\ j}\,V^a, \qquad \Lambda_i^{\ j} = 2\,g\,(T^a)_i^{\ j}\,V^a \tag{2.3.147}$$

so that the gauge transformation acts as[15]

$$\Phi_i \to (e^{i\,\Lambda})_i^{\ j}\,\Phi_j, \quad \Phi^{\dagger i} \to \Phi^{\dagger j}(e^{-i\,\Lambda^\dagger})_j^{\ i}, \quad e^V \to e^{i\,\Lambda^\dagger} e^V e^{-i\,\Lambda} \tag{2.3.148}$$

and

$$K(\Phi_i, V) = \Phi^{\dagger i}\,(e^V)_i^{\ j}\,\Phi_j \tag{2.3.149}$$

is gauge invariant. The gauge field strength χSF is now defined

$$\mathcal{W}_\alpha \equiv -\frac{1}{4}\,\bar{\mathcal{D}}^2\left(e^{-V}\,\mathcal{D}_\alpha\,e^V\right) \tag{2.3.150}$$

and transforms under gauge transformations as

$$\mathcal{W}_\alpha \to e^{i\,\Lambda}\,\mathcal{W}_\alpha\,e^{-i\,\Lambda}, \tag{2.3.151}$$

where $\mathcal{W}_\alpha$, Λ and T^a are matrices. In the WZ gauge, $\mathcal{W}_\alpha^a$ in the adjoint representation is written

$$\mathcal{W}^a_{\mathrm{WZ}\alpha} = \lambda^a_\alpha + \theta_\alpha\,D^a - \frac{i}{2}\,(\sigma^\mu\,\bar{\sigma}^\nu\,\theta)_\alpha\,v^a_{\mu\nu} + i\,\theta^2\,(\sigma^\mu\,D_\mu\,\lambda^\dagger)_\alpha. \tag{2.3.152}$$

where $v^a_{\mu\nu}$ is the non abelian gauge strength

$$v^a_{\mu\nu} = \partial_\mu\,v_\nu - \partial_\nu\,v_\mu + g\,f^{abc}\,v^b_\mu\,v^c_\nu. \tag{2.3.153}$$

[15]The matrix structure means that the transformation for V is now more complicated than in the abelian case.

Combining everything we get

$$
\begin{aligned}
\mathcal{L}_{\text{total}} &= \mathcal{L}_K + \mathcal{L}_{\text{SYM}} + \mathcal{L}_W, && (2.3.154)\\
\mathcal{L}_K &= \int d^4\theta\, K(\Phi_i, V), && (2.3.155)\\
\mathcal{L}_{\text{SYM}} &= \left[\left(\frac{1}{4} - \frac{i\, g^2\, \theta_{\text{YM}}}{32\,\pi^2}\right)\int d^2\theta\, \mathcal{W}^{\alpha a}\, \mathcal{W}^a_\alpha\right] + \text{h.c.}, && (2.3.156)\\
\mathcal{L}_W &= \left[\int d^2\theta\, W(\Phi_i)\right] + \text{h.c.}. && (2.3.157)
\end{aligned}
$$

and in component form

$$
\begin{aligned}
\mathcal{L}_K =& - D_\mu\phi^{\dagger i} D^\mu \phi_i + i\,\psi^{\dagger i}\bar{\sigma}^\mu D_\mu \psi_i - \sqrt{2}\, g[(\phi^\dagger\, T^a\, \psi)\lambda^a + \text{h.c.}]\\
&+ g(\phi^\dagger\, T^a\, \phi) D^a + |F_i|^2 + (\text{total derivatives}), && (2.3.158)\\
\mathcal{L}_{\text{SYM}} =& \; i\,\lambda^{a\dagger}\bar{\sigma}^\mu \partial_\mu \lambda^a - \frac{1}{4}\, v^{a\mu\nu} v^a_{\mu\nu} + \frac{g^2\,\theta_{\text{YM}}}{32\,\pi^2}\, v^{a\mu\nu}\, \tilde{v}^a_{\mu\nu}\\
&+ \frac{1}{2}\, D^a\, D^a + (\text{total derivatives}) && (2.3.159)
\end{aligned}
$$

and $\mathcal{L}_W$ is still determined in eq. 2.3.119. We have included the imaginary part of the gauge coupling in eq. 2.3.156 in order to allow the gauge invariant CP violating *theta term* in eq. 2.3.159. There is no FI term here as V is not gauge invariant. After performing the relevant superspace integrals, one then solves the F *term* equations (unchanged from eqs. 2.3.121 and 2.3.122) and the D *term* equations

$$\frac{\partial \mathcal{L}_{\text{total}}}{\partial D^a} = D^a + g\,(\phi^\dagger\, T^a\, \phi) = 0 \qquad (2.3.160)$$

Substituting the solutions then gives the scalar potential $V(\phi_i)$

$$V(\phi_i) = |F_i|^2 + \frac{1}{2} D^a\, D^a. \qquad (2.3.161)$$

2.3.4 Writing a theory with extended supersymmetry

Overview

In Chapter 5 a manifestly $\mathcal{N} = 2$ SUSY framework will be required, and so we will take some time to review the requirements here.

The inadequacy of conventional extended superspace

We follow the notation of [50]. Generalising eq. 2.3.162 for the $\mathcal{N}=1$ case, a general N–extended SUSY transformation acts on a general superfield X (that has a θ_i and $\bar{\theta}^i$ expansion rather than just a θ and $\bar{\theta}$ one)

$$\begin{aligned}\delta_\varepsilon X(x,\theta_i,\bar{\theta}^i) &= i\,(\varepsilon_i \mathcal{Q}^i + \varepsilon^{\dagger i}\bar{\mathcal{Q}}_i)\,X(x,\theta_i,\bar{\theta}^i)\\ &= X[x + i(\varepsilon^i\sigma^\mu\bar{\theta}_i - \theta^i\sigma^\mu\varepsilon_i^\dagger), \theta_i+\varepsilon_i, \bar{\theta}^i+\varepsilon^{\dagger i}] - X(x,\theta_i,\bar{\theta}^i). \quad (2.3.162)\end{aligned}$$

and we can find the associated differential operator representation for the $\mathcal{Q}$'s

$$\mathcal{Q}^i_\alpha \equiv i\,\partial^i_\alpha + (\sigma^\mu\,\bar{\theta}^i)_\alpha\,\partial_\mu, \quad (2.3.163)$$

$$\bar{\mathcal{Q}}_{\dot{\alpha}i} \equiv -i\,\partial_{\dot{\alpha}i} - (\theta_i\,\sigma^\mu)_{\dot{\alpha}}\,\partial_\mu, \quad (2.3.164)$$

and the SUSY covariant derivates

$$\mathcal{D}^i_\alpha \equiv \partial^i_\alpha + i\,(\sigma^\mu\,\bar{\theta}^i)_\alpha\,\partial_\mu = \partial^i_\alpha + 2\,i\,(\sigma^\mu\,\bar{\theta}^i)_\alpha\,\frac{\partial}{\partial\,y^\mu}, \quad (2.3.165)$$

$$\bar{\mathcal{D}}_{\dot{\alpha}i} \equiv -\partial_{i\dot{\alpha}} - i\,(\theta_i\,\sigma^\mu)_{\dot{\alpha}}\,\partial_\mu = -\partial_{i\dot{\alpha}}, \quad (2.3.166)$$

where the right hand side of eqs. 2.3.165 and 2.3.166 are only valid when acting on functions evaluated in the basis

$$y^\mu \equiv x^\mu + i\,\theta_i\,\sigma^\mu\,\bar{\theta}^i. \quad (2.3.167)$$

The $\mathcal{D}$'s and $\bar{\mathcal{D}}$'s satisfy the algebra

$$\{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}j}\} = -2\,i\,\delta^i_j\,\sigma^\mu_{\alpha\dot{\alpha}}\,\partial_\mu. \quad (2.3.168)$$

If we turn our attention to $\mathcal{N}=2$ SUSY, the irreducible representation of $\mathcal{N}=2$ needed to embed the quark superfields Q and $\tilde{Q}$ is the Fayet–Sohnius (FS) hypermultiplet $Q_{\rm FS}$ [51,52] (see figure 2.7). When put OS, the FS hypermultiplet contains four real scalar fields that form a complex $\mathrm{SU}(2)_R$ doublet $Q^i(x)$ and two $\mathrm{SU}(2)_R$ singlet fermions $\psi^\alpha_Q(x)$ and $\psi^\alpha_{\tilde{Q}}(x)$. The fields beyond this — including a spin $\frac{3}{2}$ field

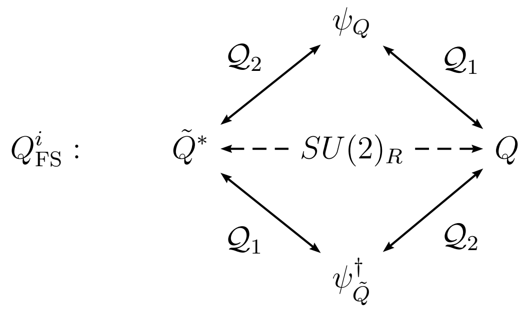


Figure 2.7: Embedding of the quarks of $\mathcal{N} = 1$ SQCD into the $\mathcal{N} = 2$ Fayet–Sohnius hypermultiplet onto $\mathcal{N} = 1$.

— are eliminated by the condition[16] [52]

$$\mathcal{D}^{(i}_\alpha \, Q^{j)}_{\rm FS} = \bar{\mathcal{D}}^{(i}_\alpha \, Q^{j)}_{\rm FS} = 0, \tag{2.3.170}$$

resulting in a Grassman expansion of the hypermultiplet

$$Q^i_{\rm FS}(x, \theta_i, \bar{\theta}^i) = Q^i(x) + \theta^i \, \psi_Q(x) + \bar{\theta}^i_{\dot{\alpha}} \, \psi^\dagger_{\tilde{Q}}(x) + (\text{derivatives}). \tag{2.3.171}$$

Unfortunately, the constraint 2.3.170 together with the algebra for covariant derivatives 2.3.168 imply that the component fields are OS

$$\partial_\mu \, \partial^\mu \, Q^i(x) = [\sigma^\mu \, \partial_\mu \, \psi_Q(x)]^{\dot{\alpha}} = [\sigma^\mu \, \partial_\mu \, \psi^\dagger_{\tilde{Q}}(x)]_\alpha = 0 \tag{2.3.172}$$

and so it is not possible to introduce interactions for the FS hypermultiplet. Our $\mathcal{N} = 2$ theory of interest will be Super Quantum Chromodynamics (SQCD) which does contain interactions. It is possible to introduce interactions in the standard $\mathcal{N} = 2$ superspace for the FS hypermultiplet by relaxing the constraint 2.3.170, however, this cannot be achieved with a finite number of auxiliary fields [53, 54]. Consequently, we will turn to the natural language of dealing with an infinite number of auxiliary fields — Harmonic superspace — for the purposes of describing $\mathcal{N} = 2$ in Chapter 5.

[16]Our conventions for symmetric indices are

$$a^{(i_1 \dots i_n)} \equiv \frac{1}{n!} \left[a^{i_1 \dots i_n} + (\text{permutations})\right]. \tag{2.3.169}$$

Harmonic superspace

Harmonic Superspace (HSS) [50,55] bypasses the problem by introducing an infinite set of auxiliary fields. These fields are essentially different harmonic modes on the 'sphere' defined by the $\mathrm{SU}(2)_R$ automorphism of the standard $\mathcal{N} = 2$ SUSY. By applying the equations of motion, we will find that the auxiliary fields will just vanish as usual, giving us the a standard physical theory with $\mathcal{N} = 2$ SUSY. In this approach, we will see the $\mathrm{SU}(2)_R$ automorphism become manifest as different modes on the sphere take on different $\mathrm{SU}(2)_R$ representations.

The standard $\mathcal{N} = 2$ superspace $\mathbb{R}^{4|8}$ is written as a coset space

$$\mathbb{R}^{4|8} = \frac{\text{Super–poincaré}}{\text{Lorentz}} = (x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i}), \tag{2.3.173}$$

i.e. a unique point in $\mathcal{N} = 2$ superspace can be reached from the origin with a super–poincaré transformation with the transformation defined up to a Lorentz transformation. Now really because of the $\mathrm{SU}(2)_R$ automorphism, we could imagine eq. 2.3.173 to be written

$$\mathbb{R}^{4|8} = \frac{\text{Super–poincaré} \times \mathrm{SU}(2)_R}{\text{Lorentz} \times \mathrm{SU}(2)_R} = (x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i}), \tag{2.3.174}$$

that is, the automorphism is contained as part of the transformation group, but each unique point in the $\mathcal{N} = 2$ superspace is unique up to a $\mathrm{SU}(2)_R$ transformation. The trick is now to no longer identify points related by an $\mathrm{SU}(2)_R$ transformation, but only the $\mathrm{U}(1) \subset \mathrm{SU}(2)_R$ subgroup under which Q^1_α and Q^2_α have opposite charges

$$\mathbb{H}^{4+2|8} = \frac{\text{Super–poincaré} \times \mathrm{SU}(2)_R}{\text{Lorentz} \times \mathrm{U}(1)} = (x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i}, u_i^\pm), \tag{2.3.175}$$

where the $u_i^\pm$ are *harmonic variables* that parameterise transformations in the coset space[17]

$$\frac{\mathrm{SU}(2)_R}{\mathrm{U}(1)} \sim S^2 \tag{2.3.176}$$

and satisfy

$$u^{+i} u_i^- = 1, \qquad (u^{+i})^\dagger = u_i^- . \tag{2.3.177}$$

[17] S^2 is the two dimensional sphere.

The $+$ and $-$ correspond to charges under the U(1), and a general function on $\mathrm{SU}(2)_R$ $f^{(q)}(u)$ with U(1) charge q has the harmonic expansion

$$f^{(q)}(u) = \sum_{n=0}^{\infty} f^{i_1 \ldots i_{n+q} j_1 \ldots j_n}\, u^+_{i_1} \ldots u^+_{i_{n+q}} u^-_{j} \ldots u^-_{j_n}. \tag{2.3.178}$$

This is the key to the success of harmonic superspace — any space time field $\phi^{(q)}(x,u)$ with a harmonic expansion

$$\phi^{(q)}(x,u) = \sum_{n=0}^{\infty} \phi^{i_1 \ldots i_{n+q} j_1 \ldots j_n}(x)\, u^+_{i_1} \ldots u^+_{i_{n+q}} u^-_{j} \ldots u^-_{j_n} \tag{2.3.179}$$

is accompanied with an infinite tower of space time fields $\phi^{i_1 \ldots i_{n+q} j_1 \ldots j_n}(x)$. Integration rules for harmonic functions are given in Section E.1. A review of HSS is beyond the scope of this thesis, however, a very good introduction to the subject can be found in [50]. Here we will just quote the necessary results for our analysis. The coset $\mathrm{SU}(2)_R/\mathrm{U}(1)$ has generators

$$T^{\pm\pm} = T^1 \pm i\, T^2 \tag{2.3.180}$$

and the U(1) factor is generated by $T^0 = 2\,T^3$. Together they form the $\mathrm{SU}(2)_R$ algebra

$$[T^{++}, T^{--}] = T^0, \qquad [T^0, T^{\pm\pm}] = \pm\, 2\, T^{\pm\pm}. \tag{2.3.181}$$

T^0 has a representation σ^3 on the $\mathcal{Q}$'s

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} Q^1_\alpha \\ Q^2_\alpha \end{pmatrix} = \begin{pmatrix} Q^1_\alpha \\ -Q^2_\alpha \end{pmatrix} \tag{2.3.182}$$

and so we use the notation

$$Q^1_\alpha \equiv Q^+_\alpha, \qquad Q^2_\alpha \equiv Q^-_\alpha \tag{2.3.183}$$

with the new algebra

$$\{Q^+_\alpha, \bar{Q}^+_{\dot\alpha}\} = \{Q^-_\alpha, \bar{Q}^-_{\dot\alpha}\} = 0, \tag{2.3.184}$$

$$\{Q^+_\alpha, \bar{Q}^-_{\dot\alpha}\} = -\{Q^-_\alpha, \bar{Q}^+_{\dot\alpha}\} = 2\,\sigma^\mu_{\alpha\dot\alpha}\, p_\mu. \tag{2.3.185}$$

One can define harmonic derivatives $\mathcal{D}^{\pm\pm}$ and the U(1) charge operator $\mathcal{D}^0$

$$\mathcal{D}^{\pm\pm} \equiv u^{\pm i}\,\frac{\partial}{\partial u^{\mp i}} \equiv \partial^{\pm\pm} \tag{2.3.186}$$

$$= \partial^{\pm\pm} - 2\,i\,\theta^{\pm}\,\sigma^{\mu}\,\bar{\theta}^{\pm}\,\frac{\partial}{\partial y^{\mu}} + \theta^{\pm\alpha}\,\frac{\partial}{\partial\theta^{\mp\alpha}} + \bar{\theta}^{\pm\dot{\alpha}}\,\frac{\partial}{\partial\bar{\theta}^{\mp\dot{\alpha}}}, \tag{2.3.187}$$

$$\mathcal{D}^0 \equiv u^{+i}\,\frac{\partial}{\partial u^{+i}} - (+\leftrightarrow -) \equiv \partial^0 \tag{2.3.188}$$

$$= \partial^0 + \left[\theta^{+\alpha}\,\frac{\partial}{\partial\theta^{+\alpha}} + \bar{\theta}^{+\alpha}\,\frac{\partial}{\partial\bar{\theta}^{+\alpha}} - (+\leftrightarrow -)\right] \tag{2.3.189}$$

that act upon the harmonic variables

$$\mathcal{D}^0\,u_i^{\pm} = \pm\,u_i^{\pm}, \qquad \mathcal{D}^{\pm\pm}\,u_i^{\mp} = u_i^{\pm}, \qquad \mathcal{D}^{\pm\pm}\,u_i^{\pm} = 0. \tag{2.3.190}$$

The expressions 2.3.187 and 2.3.189 are valid only when evaluated at

$$y^{\mu} = x^{\mu} - 2\,i\,\theta^{(i}\,\sigma^{\mu}\,\bar{\theta}^{j)}\,u_i^{+}\,u_j^{-} \tag{2.3.191}$$

and

$$\theta_{\alpha}^{\pm} \equiv u_i^{\pm}\,\theta_{\alpha}^{i}, \qquad \bar{\theta}_{\dot{\alpha}}^{\pm} \equiv u_i^{\pm}\,\bar{\theta}_{\dot{\alpha}}^{i}. \tag{2.3.192}$$

The expression 2.3.191 defines the *harmonic analytic basis*, analogous to 2.3.97.

Hypermultiplets in harmonic superspace

We are now ready to define the FS hypermultiplet in HSS. If we take the constraint 2.3.170 and contract it with the harmonics $u_i^{+}\,u_j^{+}$ we get

$$\mathcal{D}_{\alpha}^{+}\,Q_{\mathrm{FS}}^{+} = \bar{\mathcal{D}}_{\alpha}^{+}\,Q_{\mathrm{FS}}^{+} = 0, \tag{2.3.193}$$

where we have defined

$$Q_{\mathrm{FS}}^{+} \equiv Q_{\mathrm{FS}}^{i}\,u_i^{+} \tag{2.3.194}$$

and the derivatives are

$$\mathcal{D}^+_\alpha \equiv u^+_i\, \mathcal{D}^i_\alpha = \frac{\partial}{\partial\theta^{-\alpha}}, \tag{2.3.195}$$

$$\bar{\mathcal{D}}^+_{\dot\alpha} \equiv u^+_i\, \bar{\mathcal{D}}^i_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{-\dot\alpha}}, \tag{2.3.196}$$

$$\mathcal{D}^-_\alpha \equiv u^-_i\, \mathcal{D}^i_\alpha = -\frac{\partial}{\partial\theta^{+\alpha}} + 2\,i\,(\sigma^\mu\,\bar\theta^-)_\alpha\,\partial_\mu, \tag{2.3.197}$$

$$\bar{\mathcal{D}}^-_{\dot\alpha} \equiv u^-_i\, \bar{\mathcal{D}}^i_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{+\dot\alpha}} - 2\,i\,(\theta^-\sigma^\mu)_{\dot\alpha}\,\partial_\mu. \tag{2.3.198}$$

Again, the expressions on the far right side of 2.3.195 to 2.3.198 are only valid when evaluated at 2.3.191. Noticing that any function $f^{(q)}(u)$ satisfying

$$\mathcal{D}^{++}\, f^{(q)}(u) = 0 \tag{2.3.199}$$

has the solutions

$$f^{(q)}(u) = \begin{cases} 0 & \text{if } q < 0 \\ u^+_{i_1}\dots u^+_{i_q}\, f^{i_1\dots i_q} & \text{if } q \geq 0. \end{cases} \tag{2.3.200}$$

lets us rewrite the definition 2.3.194 as

$$\mathcal{D}^{++}\, Q^+_{\rm FS} = 0. \tag{2.3.201}$$

Putting all of this together, the definition of the FS hypermultiplet 2.3.170 is rewritten in HSS as

$$\mathcal{D}^{++}\, Q^+_{\rm FS} = \mathcal{D}^+_\alpha\, Q^+_{\rm FS} = \bar{\mathcal{D}}^+_{\dot\alpha}\, Q^+_{\rm FS} = 0. \tag{2.3.202}$$

We can then find an expression for $Q^+_{\rm FS}$ in the harmonic analytic basis. The derivatives $\mathcal{D}^+_\alpha$ and $\bar{\mathcal{D}}^+_{\dot\alpha}$ become short (see eqs. 2.3.195 and 2.3.196) leaving $Q^+_{\rm FS}$ with a θ

expansion in only θ^+ and $\bar{\theta}^+$:

$$\begin{aligned} Q^+_{\text{FS}} &= Q^+_{\text{FS}}(x, \theta^+, \bar{\theta}^+, u_i^\pm) && (2.3.203)\\ &= Q^+(x,u) + \theta^+\, \psi_Q(x,u) + \bar{\theta}^+\, \psi^\dagger_{\tilde{Q}}(x,u) \\ &\quad + (\theta^+)^2\, F^-(x,u) + (\bar{\theta}^+)^2\, G^-(x,u) + i\, \theta^+ \sigma^\mu\, \bar{\theta}^+\, v^-_\mu(x,u) \\ &\quad + (\theta^+)^2\, \bar{\theta}^+ \bar{\chi}^{(-2)}(x,u) + (\bar{\theta}^+)^2\, \theta^+ \lambda^{(-2)}(x,u) \\ &\quad + (\theta^+)^2\, (\bar{\theta}^+)^2\, P^{(-3)}(x,u). && (2.3.204) \end{aligned}$$

Applying the remaining constraint 2.3.201

$$\mathcal{D}^{++}\, Q^+_{\text{FS}}(x, \theta^+, \bar{\theta}^+, u_i^\pm) = \left(\partial^{++} - 2\, i\, \theta^+ \sigma^\mu\, \bar{\theta}^+\, \partial_\mu\right) Q^+_{\text{FS}}(x, \theta^+, \bar{\theta}^+, u_i^\pm) = 0, \quad (2.3.205)$$

the lowest theta components define the physical space time fields and eliminate some of the infinite tower of auxiliary fields

$$\begin{aligned} \partial^{++}\, Q^+(x,u) = 0 &\implies Q^+(x,u) = Q^i(x)\, u_i^+, && (2.3.206)\\ \partial^{++}\, \psi_Q(x,u) = 0 &\implies \psi_Q(x,u) = \psi_Q(x), && (2.3.207)\\ \partial^{++}\, \psi^\dagger_{\tilde{Q}}(x,u) = 0 &\implies \psi^\dagger_{\tilde{Q}}(x,u) = \psi^\dagger_{\tilde{Q}}(x), && (2.3.208)\\ \partial^{++}\, F^-(x,u) = 0 &\implies F^-(x,u) = 0, && (2.3.209)\\ \partial^{++}\, G^-(x,u) = 0 &\implies G^-(x,u) = 0. && (2.3.210) \end{aligned}$$

The equations

$$\begin{aligned} \partial^{++}\, v^-_\mu - 2\, \partial_\mu\, Q^+(x,u) = 0 &\implies v^-_\mu(x,u) = 2\, \partial_\mu Q^i(x)\, u_i^- && (2.3.211)\\ \partial^{++}\, P^{-3}(x,u) + \partial^\mu\, v^-_\mu(x,u) = 0 &\implies P^{-3}(x,u) = 0 && (2.3.212) \end{aligned}$$

put the $Q^i(x)$ OS

$$\partial^\mu\, v^-_\mu(x,u) = 2\, \partial_\mu\, \partial^\mu\, Q^i(x)\, u_i^- = 0 \implies \partial_\mu\, \partial^\mu\, Q^i(x) = 0, \quad (2.3.213)$$

and the equations

$$\partial^{++}\lambda_\alpha^{(-2)}(x,u) + i\,\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu\,\psi^{\dagger\dot\alpha}_{\tilde Q} = 0 \implies \lambda_\alpha^{(-2)}(x,u) = 0, \tag{2.3.214}$$

$$\partial^{++}\bar\chi_{\dot\alpha}^{(-2)}(x,u) + i\,\partial_\mu\,\psi^\alpha_Q\,\sigma^\mu_{\alpha\dot\alpha} = 0 \implies \bar\chi_{\dot\alpha}^{(-2)}(x,u) = 0 \tag{2.3.215}$$

and put ψ_Q and $\psi^\dagger_{\tilde Q}$ OS

$$[\sigma^\mu\,\partial_\mu\,\psi_Q(x)]^{\dot\alpha} = [\sigma^\mu\,\partial_\mu\,\psi^\dagger_{\tilde Q}(x)]_\alpha = 0 \tag{2.3.216}$$

as we found before in standard superspace. It would seem like we haven't achieved anything yet but actually this is not true. The FS hypermultiplet was only put OS after applying the constraint 2.3.201. In in a 'normal' free field theory, a field is put OS by solving its Euler–Lagrange equations. It is now clear how to take the hypermultiplet off–shell. If we define the FS hypermultiplet using 2.3.193 *only*, and write down the action

$$S_Q^{\text{free}} = -\int du\,d\zeta^{(-4)}\,\widetilde{Q^+_{\text{FS}}}\,\mathcal{D}^{++}\,Q^+_{\text{FS}}, \tag{2.3.217}$$

where all definition of the measures used are given in Section E.2, and $\widetilde{Q^+_{\text{FS}}}$ is the hermitian $\times$ *antipodal conjugation* of Q^+_{FS} (defined in Section E.3) and should not be confused with the right handed quarks $\tilde Q$ of SQCD. Requiring the variation of the action 2.3.217 with respect to $\tilde Q^+_{\text{FS}}$ to vanish then yields the constraint 2.3.201 and only then puts the FS hypermultiplet OS.

Gauge theories in harmonic superspace

To incorporate gauge interactions in HSS we introduce the *vector hypermultipet* V^{++}

$$V^{++} = \widetilde{V^{++}} \tag{2.3.218}$$

that is written in the WZ gauge as

$$V^{++}_{\text{WZ}} = -\,2\,i\,\theta^+\sigma^\mu\,\bar\theta^+ v_\mu(x) + 3\,(\theta^+)^2\,(\bar\theta^+)^2\,D^{ij}(x)\,u^-_i\,u^-_j + \Big[i\,\sqrt{2}\,(\bar\theta^+)^2\,X(x) + 4\,(\bar\theta^+)^2\,\theta^+\psi^i\,u^-_i + \text{c.c.}\Big], \tag{2.3.219}$$

where c.c. is the hermitian × antipodal conjugation. The action for $\mathcal{N}=2$ SYM is then written [56]

$$S^{\mathcal{N}=2}_{\rm SYM} = \sum_{n=2}^{\infty} \frac{(-i)^n}{2\,n} \mathrm{tr}\left[\int d^{12}X\, du_1 \dots du_n\, \frac{V^{++}(X,u_1)\dots V^{++}(X,u_n)}{(u_1^+\,u_2^+)\dots(u_n^+\,u_1^+)}\right], \quad (2.3.220)$$

which, amazingly, in component fields evaluates to

$$S^{\mathcal{N}=2}_{\rm SYM} = \mathrm{tr}\Bigg\{\int d^4x \Big[(\mathcal{D}_\mu X)^\dagger (\mathcal{D}^\mu X) - i\,\psi^i\,\sigma^\mu\,\mathcal{D}_\mu\,\psi_i^\dagger - \frac{1}{4}\,v_{\mu\nu}\,v^{\mu\nu} - \frac{1}{\sqrt{2}}\,\psi^i\,[X^\dagger,\psi_i] - \frac{1}{\sqrt{2}}\,\psi_i^\dagger\,[X,\psi^{\dagger i}] + \frac{1}{4}\,D^{ij}\,D_{ij}\Big]\Bigg\}. \quad (2.3.221)$$

It is also convenient to define the *gauge field hyperstrength* $\mathcal{W}$ [57, 58]

$$\overline{\mathcal{W}} \equiv -\frac{1}{4}\,(\mathcal{D}^+)^2 \sum_{n=1}^{\infty} (-i)^{n+1} \int dv_1 \dots dv_n\, \frac{V^{++}(X,v_1)\dots V^{++}(X,v_n)}{(u^+\,v_1^+)\dots(v_n^+\,u^+)} \quad (2.3.222)$$

that has a component field expansion[18] that can be found by using the expansion in eq. 2.3.219 and the harmonic superspace rules in Appendix E

$$\mathcal{W} = i\,\sqrt{2}\,X - 2\,\theta^+\lambda^i\,u_i^- + \theta^i\sigma^{\mu\nu}\theta_i\,v_{\mu\nu} + 2(\theta\theta)^A\,D^A + \dots. \quad (2.3.223)$$

A low energy EFT for $\mathcal{N}=2$ SYM can be written

$$S^{\mathcal{N}=2}_{\rm SYM} = -\frac{i}{4}\int d^4x\,(\mathcal{D})^4\mathcal{F}(\mathcal{W}) + \text{h.c.}, \quad (2.3.224)$$

where $\mathcal{F}(\mathcal{W})$ is the prepotential [59], and is a gauge invariant function of only $\mathcal{W} \equiv \mathcal{W}^a t_a$ and has the general form

$$\mathcal{F}(\mathcal{W}) = \sum_M \frac{1}{M!} \sum_{m_1\dots m_M} \frac{c_{m_1\dots m_M}}{m_1!\dots m_M!}\,\mathrm{tr}\,(\mathcal{W}^{m_1})\dots\mathrm{tr}\,(\mathcal{W}^{m_M}). \quad (2.3.225)$$

The coefficients $c_{m_1\dots m_M}$ arise from integrating out microscopic degrees of freedom, and have been exactly determined in specific cases, for example in [60, 61]. Deriva-

[18] The A index is an adjoint $\mathrm{SU}(2)_R$ index, see Section E.6 for explicit definitions.

tives of the prepotential are conveniently defined

$$\mathcal{F}_{a_1\,\dots\,a_N}(\mathcal{W}) \equiv \frac{\partial^N \mathcal{F}(\mathcal{W})}{\partial \mathcal{W}^{a_1}\,\dots\,\partial \mathcal{W}^{a_N}}, \qquad h_{ab} \equiv \mathrm{Re}\,\mathcal{F}_{ab}|, \qquad g_{ab} \equiv \mathrm{Im}\,\mathcal{F}_{ab}|, \quad (2.3.226)$$

where $\mathcal{O}| \equiv \mathcal{O}(\theta = \bar{\theta} = 0)$. The resulting theory up to four derivatives of the prepotential was derived in [58] and is presented for completeness in Appendix E.4.1.

To couple the FS hypermultiplet to the gauge theory, one simply extends the $\mathcal{D}^{++}$ derivative in the action 2.3.217 to be a gauge covariant derivative

$$S_Q^{\text{gauged}} = -\int du\, d\zeta^{(-4)}\, \widetilde{Q^+_{\text{FS}}}\, \mathscr{D}^{++}\, Q^+_{\text{FS}}, \quad (2.3.227)$$

where

$$\mathscr{D}^{++} \equiv \mathcal{D}^{++} + i\,V^{++}. \quad (2.3.228)$$

This alters its equations of motion to be those of ones coupled to a gauge theory. To describe $\mathcal{N} = 2$ SQCD, we just combine the actions 2.3.220 or 2.3.224 with 2.3.227.

2.3.5 R symmetry

In the absence of central charges the SUSY algebra in eqs. 2.3.59 to 2.3.62 has the *automorphism* group U(N)

$$Q^i_\alpha \to U_j{}^i\, Q^j_\alpha, \qquad \bar{Q}_{\dot{\alpha}j} \to \bar{Q}_{\dot{\alpha}j}\, U^{\dagger j}_i \quad (2.3.229)$$

where U is a unitary matrix. This is the *R symmetry*. Irreducible representations of SUSY will carry a representation of this automorphism group. In $\mathcal{N} = 1$ SUSY this is a U$(1)_R$ global symmetry under which the SUSY generators and superspace coordinates transform

$$Q_\alpha \to e^{-i\,\phi}\, Q_\alpha, \qquad \bar{Q}_{\dot{\alpha}} \to e^{i\,\phi}\, \bar{Q}_{\dot{\alpha}}, \quad (2.3.230)$$

$$\theta_\alpha \to e^{i\,\phi}\, \theta_\alpha, \qquad \bar{\theta}_{\dot{\alpha}} \to e^{-i\,\phi}\, \bar{\theta}_{\dot{\alpha}}. \quad (2.3.231)$$

The general χSF with charge R_X transforms as

$$X(x, \theta, \bar{\theta}) \longrightarrow e^{i\,R_X\,\phi}\, X(x, e^{i\,\phi}\theta, e^{-i\,\phi}\bar{\theta}). \quad (2.3.232)$$

	U(1)	$\mathrm{U}(1)_R$
$\hat{\Phi}$	1	1
$\hat{m}$	−2	0
$\hat{y}$	−3	−1

Table 2.5: Global charges for the holomorphic field $\hat{\Phi}$ and couplings $\hat{m}$ and $\hat{y}$.

For a χSF with $\mathrm{U}(1)_R$ charge R_Φ, the $\mathrm{U}(1)_R$ charges of its components are

$$R_\phi = R_\Phi, \qquad R_\psi = R_\Phi - 1, \qquad R_F = R_\Phi - 2. \tag{2.3.233}$$

For a VSF that is necessarily $\mathrm{U}(1)_R$ chargeless, the $\mathrm{U}(1)_R$ charges of its components are

$$R_v = 0, \qquad R_\lambda = 1, \qquad R_D = 0. \tag{2.3.234}$$

We will see in Section 2.3.7 that the $\mathrm{U}(1)_R$ symmetry has important consequences for the Majorana versus Dirac gaugino masses that are central to this thesis, and that it also has striking implications for SUSY breaking.

2.3.6 The holomorphic basis and non-renormalisation

Non-renormalisation of superpotential

The superpotential is a *holomorphic* function of χSFs. Using supergraph perturbation theory [62] and later using *holomorphy* [63, 64] it has been shown that *the superpotential is not renormalised at any order in perturbation theory.* From now on we will use hatted variables to denote holomorphic quantities and unhatted variables to denote canonical variables. Consider the WZ model that has the tree level *superpotential*

$$W^{\mathrm{tree}}(\hat{\Phi}) = \frac{\hat{m}}{2}\,\hat{\Phi}^2 + \frac{\hat{y}}{3}\,\hat{\Phi}^3. \tag{2.3.235}$$

We can think of the couplings $\hat{m}$ and $\hat{y}$ as *spurions* of the global symmetry $\mathrm{U}(1) \times \mathrm{U}(1)_R$ and as χSFs in their own right. The field $\hat{\Phi}$ and the couplings $\hat{m}$ and $\hat{y}$ are assigned the global charges in table 2.5. Now if we consider integrating out some modes to generate an *effective superpotential* down to a lower scale μ, the theory should still have the same (*spuriously* broken) global symmetries and should still have a *holomorphic superpotential.* The *effective superpotential* is then restricted to be of the form

$$W^{\text{eff}} = \hat{m}\, \hat{\Phi}^2\, f(x), \qquad \text{where } x = \frac{\hat{y}\, \hat{\Phi}}{\hat{m}}, \tag{2.3.236}$$

and $f(x)$ is a function to be determined by sensible limits of the couplings. Taking the limit $\hat{y} \to 0$ and $\hat{m} \to 0$ while holding $\hat{y}/\hat{m}$ constant needs to reproduce $W^{\text{eff}} \to W^{\text{tree}}$ since in this limit quantum corrections are turned off. In this limit we find $f(x) = 1 + x$. This is independent of x however, as $\hat{y}/\hat{m}$ can be anything, and so $f(x) = 1 + x$ *for all* x. From this one concludes that

$$W^{\text{eff}} = \frac{\hat{m}}{2}\, \hat{\Phi}^2 + \frac{\hat{y}}{3}\, \hat{\Phi}^3 = W^{\text{tree}} \tag{2.3.237}$$

i.e. *the superpotential is not renormalised.* This is the *non-renormalisation theorem*, and is true for *any superpotential* $W(\Phi)$.

Superpotential renormalisation group equations

A consequence of the *non-renormalisation theorem* is that we don't need to solve the CS equations to obtain the RGEs of a SUSY theory. Consider a theory defined at the renormalisation scale μ

$$\mathcal{L}_{\text{total}}(\mu) = \int d^4\theta\, K(\Phi, \mu) + \left\{ \left[\int d^2\theta\, W(\Phi) \right] + \text{h.c.} \right\} \tag{2.3.238}$$

where

$$K(\Phi, \mu) = Z(\mu)\, \hat{\Phi}^\dagger\, \hat{\Phi} \tag{2.3.239}$$

and with a superpotential

$$W(\hat{\Phi}) = \sum_k \hat{y}_k\, \hat{\Phi}^k \tag{2.3.240}$$

Now physical fields *do* renormalise and a physical couplings *do* run. We can make contact between the *physical* or *canonical* basis and the *holomorphic* basis by absorbing the wave function renormalisations that appear in the Kähler potential into the definition of the fields and dimensionless couplings

$$\Phi(\mu) \equiv Z(\mu)^{1/2}\, \hat{\Phi}, \qquad y_k(\mu) \equiv Z(\mu)^{-k/2}\, \mu^{-d_k}\, \hat{y}_k, \tag{2.3.241}$$

where $d_k = 3 - k$ is the *canonical* or *engineering dimension* of y_k. By *non-renormalisation* we know

$$\frac{d}{dt}\left(\hat{y}_k\,\hat{\Phi}^k\right) = 0. \tag{2.3.242}$$

for all k. Then the beta function for y_k is

$$\beta_{y_k} \equiv \frac{dy_k}{dt} = \frac{d}{dt}\left(Z^{-k/2}\,\mu^{-d_k}\right) = \left(\frac{k}{2}\,\gamma_\Phi - d_k\right) y_k \tag{2.3.243}$$

where the *anomalous dimension* γ_Φ of Φ is defined

$$\gamma_\Phi \equiv -\frac{\partial \log Z}{\partial \log \mu}. \tag{2.3.244}$$

The RGE in eq. 2.3.243 has striking implications. The most important for us is that it highlights how SUSY solves the *gauge hierarchy problem* of Section 2.1.2. In eq. 2.3.243 we see that *physical parameters in the superpotential only renormalise proportional to themselves*, i.e.

- If a SUSY parameter is becomes zero at *any* point along the RG flow then it remains zero,
- If a parameter begins small, it will remain small for a reasonably long period of running.

If the μ parameter in the SM scalar potential shown in eq. 2.1.6 then it is at least reasonable that it can be of EW size and screened from effects coming from other high mass scales by SUSY providing one can explain why it starts off at the EW scale. That, however, is a different problem entirely, and is the *SUSY μ problem.*

One loop exact renormalisation of gauge theories

The SYM expressions in Section 2.3.3 are done in the *physical* or *canonical* basis. We will now introduce the *holomorphic* basis for gauge fields where the VSFs are related

$$\hat{V} = g\,V \longleftrightarrow (\hat{v}^a_\mu, \hat{\lambda}^a, \hat{D}^a) = g\,(v^a_\mu, \lambda^a, D^a) \tag{2.3.245}$$

where *all* hatted variables are *holomorphic* quantities and *all* unhatted variables *including* the gauge coupling g are the *canonical* quantities. Then

$$\hat{\mathcal{W}}_\alpha = \mathcal{W}_\alpha(\hat{V}) \equiv -\frac{1}{4}\,\bar{\mathcal{D}}^2\left(e^{-\hat{V}}\,\mathcal{D}_\alpha\,e^{\hat{V}}\right) = g\,\mathcal{W}_\alpha(V) \equiv g\,\mathcal{W}_\alpha. \tag{2.3.246}$$

In this normalisation it is standard to collect the prefactors in eq. 2.3.156 together to define the *holomorphic gauge coupling*[19]

$$\hat{\tau} \equiv \frac{\theta_{\rm YM}}{2\,\pi} + \frac{4\,\pi\,i}{\hat{g}^2}. \tag{2.3.247}$$

Now the lagrangian is written in the *holomorphic basis*

$$\hat{\mathcal{L}}_{\rm total} = \mathcal{L}_K + \mathcal{L}_{\rm SYM} + \mathcal{L}_W, \tag{2.3.248}$$

$$\hat{\mathcal{L}}_K = \int d^4\theta\,K(\Phi_i, \hat{V}), \tag{2.3.249}$$

$$\hat{\mathcal{L}}_{\rm SYM} = \left(\frac{1}{16\,\pi\,i}\int d^2\theta\,\hat{\tau}\,\hat{\mathcal{W}}^{\alpha a}\,\hat{\mathcal{W}}^a_\alpha\right) + \text{h.c.}, \tag{2.3.250}$$

and in component form

$$\begin{aligned}\hat{\mathcal{L}}_K =&- D_\mu\phi^{\dagger i}D^\mu\phi_i + i\,\psi^{\dagger i}\bar{\sigma}^\mu D_\mu\psi_i - \sqrt{2}[(\phi^\dagger\,T^a\,\psi)\hat{\lambda}^a + \text{h.c.}]\\ &+ (\phi^\dagger\,T^a\,\phi)\hat{D}^a + |F_i|^2 + (\text{total derivatives}),\end{aligned} \tag{2.3.251}$$

$$\begin{aligned}\hat{\mathcal{L}}_{\rm SYM} =& \frac{i}{\hat{g}^2}\lambda^{a\dagger}\bar{\sigma}^\mu\partial_\mu\lambda^a - \frac{1}{4\,\hat{g}^2}\,\hat{v}^{a\mu\nu}\hat{v}^a_{\mu\nu} + \frac{\theta_{\rm YM}}{32\,\pi^2}\,\hat{v}^{a\mu\nu}\,\hat{\tilde{v}}^a_{\mu\nu}\\ &+ \frac{1}{2\,\hat{g}^2}\,\hat{D}^a\,\hat{D}^a + (\text{total derivatives})\end{aligned} \tag{2.3.252}$$

with the superpotential contribution unchanged. Since $\hat{\tau}$ is a holomorphic quantity, one might think that it does not renormalise for the same reasons as the holomorphic couplings $\hat{y}_k$ in the superpotential. In fact, the running of $\hat{\tau}$ to one loop *is* consistent with *holomorphy* [65]. Integrating out modes between μ_1 and μ_2, let us write the

[19] Again we are met some strange terminology since what is holomorphic about the gauge coupling $\hat{\tau}$? The point here is that the term $\hat{\tau}\,\hat{\mathcal{W}}^{\alpha a}\hat{\mathcal{W}}^a_\alpha$ is holomorphic in $\hat{\tau}$ with $\hat{\tau}$ promoted to a χSF. This is not true for the *canonical* gauge coupling, since it must be real sue to $V = g\,\hat{V}$ with both V and $\hat{V}$ real.

low energy $\tau(\mu_2)$ as

$$\hat{\tau}(\mu_2) = \tau(\mu_1) + f\left[\hat{\tau}(\mu_1), t\right] \qquad \text{where} \qquad t = \log\left(\frac{\mu_1}{\mu_2}\right) \tag{2.3.253}$$

with f holomorphic in $\hat{\tau}$ and continuous in t. The whole theory is unchanged under $\theta_{\mathrm{YM}} \to \theta_{\mathrm{YM}} + 2\pi$ so

$$f\left[\tau(\mu) + 1, t\right] = f\left[\tau(\mu), t\right]. \tag{2.3.254}$$

Consequently, the RGE for $\hat{\tau}$ is periodic

$$\frac{d\hat{\tau}}{dt} \equiv \beta_{\hat{\tau}}, \qquad \beta_{\hat{\tau}+1} = \beta_{\hat{\tau}} \tag{2.3.255}$$

and admits the Fourier decomposition

$$\beta_{\hat{\tau}} = \sum_{n\geq 0} c_n\, e^{i\,n\,\hat{\tau}}. \tag{2.3.256}$$

The zeroth term in this expansion can be calculated to be the one loop beta function

$$\beta_{\hat{\tau}} = \frac{2\,b\,i}{\pi} + \sum_{n\geq 1} c_n\, e^{i\,n\,\hat{\tau}} \longleftrightarrow \beta_{\hat{g}^{-2}} = \frac{b}{8\,\pi^2}. \tag{2.3.257}$$

with the remaining $n \geq 1$ terms never arising in perturbation theory. Before leaving this *holomorphic* versus *canonical* basis discussion, we comment that due to the *rescaling anomaly*, the relation between the *holomorphic* and *canonical* gauge couplings in the presence of matter is

$$\frac{1}{g^2} = \frac{1}{\hat{g}^2} - \frac{2\,T_G}{8\pi^2}\ln g - \sum_j \frac{T_j}{8\pi^2}\ln(Z_j). \tag{2.3.258}$$

Whilst we have shown that the *holomorphic* coupling $\hat{g}$ runs only to one loop, from this relation it follows that the *canonical* coupling g runs to orders in perturbation theory according to the Novikov–Shifman–Vainshtein–Zakharov (NSVZ) beta function [66]

$$\beta_g = \beta_g^{\mathrm{NSVZ}} = -\frac{g^3}{16\pi^2}\frac{3\,T_G - \sum_j T_j(1-\gamma_j)}{1 - T_G\frac{g^2}{8\pi^2}}. \tag{2.3.259}$$

2.3.7 Supersymmetry breaking

Now its time to make a connection with reality. At low energies, SUSY *must* be broken in some way, and critically, in a way that maintains all of its nice features. For us, this means that SUSY breaking felt in the MSSM must give an effective SUSY scale much more than a few TeV.

How to break supersymmetry

The order parameter for SUSY breaking is the ground state energy. If the vacuum state is not invariant under SUSY

$$Q_\alpha|0\rangle \neq 0, \qquad \bar{Q}_{\dot{\alpha}}|0\rangle \neq 0 \tag{2.3.260}$$

then the ground state energy is positive from eq. 2.3.66. If there are no fermion condensates then the vacuum energy can be written

$$\langle 0|E|0\rangle = \langle 0|V|0\rangle, \tag{2.3.261}$$

where V is the scalar potential of the theory. Since V can be written as the sums of *F terms* and *D terms*, if either an *F term* or *D term* aquires a VEV in the vacuum state then SUSY will be broken spontaneously. Writing a theory that achieves this is actually more difficult than one might imagine, as SUSY has a habit of restoring itself.

Interestingly, we can see from eq. 2.3.260 that it is not possible to begin with an N-extended SUSY theory, then, through spontaneous breaking, arrive at an M-extended SUSY theory with $N > M > 0$. This is the *two into one won't go* theorem [67, 68]. Concretely, consider the N-extended SUSY algebra

$$\{Q^A_\alpha, \bar{Q}_{\dot{\beta},B}\} = 2\,\sigma^\mu_{\alpha\dot{\beta}}\,\delta^A{}_B\,P_\mu\,, \qquad A, B = 1, \ldots, N. \tag{2.3.262}$$

The vacuum energy in these theories is

$$\langle 0|V|0\rangle = \frac{1}{4}\left(||Q^A_1|0\rangle||^2 + ||\bar{Q}_{1,A}|0\rangle||^2 + ||Q^A_2|0\rangle||^2 + ||\bar{Q}_{2,A}|0\rangle||^2\right) \tag{2.3.263}$$

and is true for every A. This vacuum energy is positive if *any* of the SUSY generators

Q_i^A or $\bar{Q}_{i,A}$ do not annihilate the vacuum. It follows if *any* of the SUSY generators are broken, then at least one of Q_i^A or $\bar{Q}_{i,A}$ is broken for *every* A in order for eq. 2.3.263 to hold for all A. (The alternative is that none of them are broken.)

D term SUSY breaking: As already mentioned, in the case of an abelian gauge theory, a term linear in D displayed in eq. 2.3.143 called the FI term can be added to the lagrangian. This leads to the FI mechanism of SUSY breaking [49]. In its presence the *D term* equations are modified as in eq. 2.3.144 and lead to a scalar potential

$$V(\phi) = \frac{1}{2}\,D^2 + (\text{F terms}) = \frac{1}{2}\left(g\,q_i\,|\phi_i|^2 - \kappa\right)^2 + (\text{F terms}). \tag{2.3.264}$$

There are three distinct situations:

- If there are no *F terms* then there is a SUSY vacuum at $g\,q_i\,|\langle\phi_i\rangle|^2 - \kappa = 0$.
- If there are no χSFs then $\langle D\rangle = -\kappa$ and SUSY is broken.
- If we include a superpotential $W(\Phi_i) = \frac{m_i}{2}\,\Phi_i^2$ this induces the *F term* scalar potential $V(\phi_i) = m_i^2|\phi_i|^2$. If $m_i^2 > g\,q_i\,\kappa$ for each i then the vacuum is at $\langle\phi_i\rangle = 0$ for all i, and again $\langle D\rangle = -\kappa$ with SUSY broken.

F term SUSY breaking: The archetype of *F term* breaking models is the O'Raifeartaigh (OR) model [69] with the superpotential

$$W^{\text{OR}}(\Phi_i) = -k\,\Phi_1 + m\,\Phi_2\,\Phi_3 + \frac{y}{2}\,\Phi_1\,\Phi_3^2. \tag{2.3.265}$$

It was initially quite difficult to find other models that broke SUSY with *F terms* until it was realised that superpotentials of the form in eq. 2.3.265 belong to a certain class of models: they have a $\text{U}(1)_R$ symmetry that is broken spontaneously [70]. Consider a theory with n fields Φ_i $i = 1, \ldots, n$ each with $\text{U}(1)_R$ charge R_i. If the nth field Φ_n acquires a VEV then we can write the superpotential with new variables X_j

$$W = \Phi_n^{2/R_n}\,f(X_j), \qquad X_j \equiv \frac{\Phi_j}{\Phi_n^{R_j/R_n}}, \qquad j = 1, \ldots, n-1, \tag{2.3.266}$$

	$U(1)_R$
Φ_1	2
Φ_3	2
Φ_3	0

Table 2.6: $U(1)_R$ charge assignment in the OR model with superpotential in eq. 2.3.265.

where the X_j have zero $U(1)_R$ charge. For a SUSY vacuum to exist, we need all of the *F terms* to vanish

$$\partial_k f(X_j) = 0, \qquad f(X_j) = 0. \tag{2.3.267}$$

There are n constraints and $n-1$ unknowns which in general has no solution. We find then that *a generic superpotential which spontaneously breaks a $U(1)_R$ symmetry also breaks SUSY*. This is the Nelson–Seiberg (NS) theorem. We can now immediately see why the OR model 2.3.265 broke SUSY. It has the $U(1)_R$ charge assignment in table 2.6. By looking at the *F terms* one notices that Φ_1 is undetermined, i.e. is a *flat direction*. Φ_1 acquires a VEV along this direction, spontaneously breaking the $U(1)_R$ symmetry and causes SUSY to break via the NS theorem.

The supertrace and its implications

The *supertrace* of a theory is [71]

$$\mathrm{STr}(m^2) \equiv \sum_j (-1)^{2s_j} (2j+1)\, \mathrm{tr}(m_j^2), \tag{2.3.268}$$

where s_j is the spin of particle j.I f one assumes that SUSY breaking is communicated through renormalisable interactions at tree level then the *supertrace* satisfies

$$\mathrm{STr}(m^2) = \mathrm{tr}(m_\phi^2) - 2\,\mathrm{tr}(m_\psi^\dagger\, m_\psi) + 3\,\mathrm{tr}(m_V^2) = -2\,g\,\mathrm{tr}(T^a)\,D^a = 0 \tag{2.3.269}$$

where m_ϕ^2, m_ψ and m_V^2 are the scalar mass squared matrix, the fermion mass matrix and the gauge boson mass squared matrix respectively, and the final equality holds for a non-anomalous U(1) gauge theory. This tells us that after SUSY breaking, for a given fermion mass, the sum of the scalar masses is a constant. This means that if SUSY breaking is only communicated in this way to the MSSM, there should be one selectron lighter than the electron and one selectron heavier, in conflict

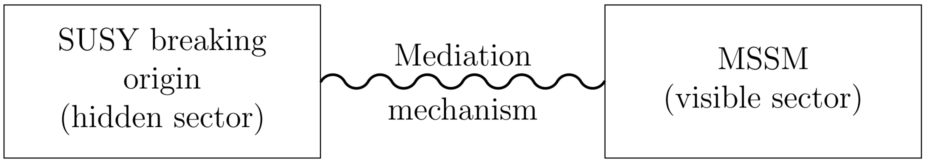


Figure 2.8: Model building setup for mediating SUSY breaking from the *hidden sector* to the *visible sector*.

with observation. One concludes that if we do live in a world with broken SUSY, then the interactions that mediate the breaking happen through non-renormalisable interactions or through loop processes.

A simple phenomenologically viable model of SUSY breaking usually imagines a *hidden sector*[20] containing one of the discussed models of SUSY breaking to a *visible sector* containing the SM. Constraints from the *supertrace* then require that the mechanism connecting the *hidden sector* is either non-renormalisable or is loop level.

Working with SUSY breaking

There are two elements to creating a model of SUSY breaking:

- The *hidden sector* that is the source of SUSY breaking,
- The mediation mechanism between the *hidden sector* and the *visible* sector.

Once these are specified, the mediation mechanism is *integrated out*, creating an EFT with a set of *soft terms*. They are called *soft terms* because they represent a *soft* breaking of the symmetry, that is, they break SUSY in a way that does not introduce quadratic divergences. For $\mathcal{N} = 1$ SUSY, the complete set of soft terms are the *standard soft terms* and *non–standard soft terms* [72]. They are

$$\mathcal{L}_{\text{soft}}^{\text{standard}} = (m^2)^j{}_i\, \phi^{\dagger i} \phi_j + \left(\frac{1}{6}\, a^{ijk}\, \phi_i\, \phi_j\, \phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\, \phi_j + \frac{1}{2}\, M\, \lambda\, \lambda + \text{h.c.} \right), \quad (2.3.270)$$

[20]Some prefer to use *dark sector* instead of *hidden sector*. I find this slightly too foreboding.

where m^2, a, b and M are *soft scalar mass squareds*, *a terms* or *soft trilinear terms*, *b terms* or *soft bilinear terms* and Majorana gaugino masses respectively, and

$$\mathcal{L}_{\text{soft}}^{\text{non-standard}} = \frac{1}{2}\, c_i^{jk}\, \phi^{\dagger i}\, \phi_j\, \phi_k + \frac{1}{2}\, M_F^{ij}\, \psi_i\, \psi_j + m_D^{ia}\, \psi_i\, \lambda_a + \text{h.c.}, \tag{2.3.271}$$

where c, M_F and m_D are the *c terms* or *non-holomorphic soft trilinear terms*, *soft fermion masses* and *Dirac gaugino masses*. The set of *soft terms* in eq. 2.3.270 are referred to as standard because they in general don't lead to quadratic divergences irrespective of field content and they are all present in the MSSM. The first two terms in eq. 2.3.271 are referred to as *non-standard* because they lead to quadratic divergences in the presence of a gauge singlet[21]. Note that the MSSM does not have a gauge singlet, so strictly these terms should be included in the analysis of the softly broken MSSM in addition to those in eq. 2.3.270. The third term in eq. 2.3.271 is *non-standard* because it requires the presence of a field that the MSSM does not have: a χSF in the **Ad** representation of the SM gauge groups such that the term $\psi_i\, \lambda_a$ is gauge invariant.

Once the set of *soft terms* is established given a specified SUSY model, the low energy physics can be calculated. This is typically done by using a *spectrum generator* such as `SoftSUSY` [73], `SPheno` [8, 74] or `SuSpect` [75]. These programs solve the RGEs of the model given the set of boundary conditions from observation and those specified in the UV by the model. Upon convergence, one can calculate the physical masses of the unobserved particles as well as other low energy properties of the model.

2.3.8 Dualities and mapping soft terms

Seiberg duality for SUSY QCD

This section will briefly cover one of the most interesting and well understood $\mathcal{N} = 1$ SUSY dualities: *Seiberg duality* [76–78]. The word *duality* has many meanings. In this case we mean that two (or more) different theories with different field content and different gauge groups produce indistinguishable long distance physics. To be

[21] There are exceptions to this — we will see later that actually in the presence of *supersoft* SUSY breaking, a particular form of the c_i^{jk} do not introduce quadratic divergences in the presence of a gauge singlet.

concrete, *Seiberg duality* for SQCD is a duality between two gauge theories:

- An $\mathcal{N} = 1$ SUSY gauge theory with gauge group $\mathrm{SU}(N_c)$, a global chiral symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_L$, a Baryon symmetry $\mathrm{U}(1)_B$ and an R symmetry $\mathrm{U}(1)_R$. There are N_f flavours of left handed quarks Q in the $\square$ representation of $\mathrm{SU}(N_c)$ and N_f flavours of right handed quarks $\tilde{Q}$ in the $\bar{\square}$ representation of $\mathrm{SU}(N_c)$. This is the *electric* theory.

- An $\mathcal{N} = 1$ SUSY gauge theory with gauge group $\mathrm{SU}(\widetilde{N_c} \equiv N_f - N_c)$, a global chiral symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_L$, a Baryon symmetry $\mathrm{U}(1)_B$ and an R symmetry $\mathrm{U}(1)_R$. There are N_f flavours of left handed quarks q in the $\square$ representation of $\mathrm{SU}(N_c)$, N_f flavours of right handed quarks $\tilde{q}$ in the $\bar{\square}$ representation of $\mathrm{SU}(N_c)$, and a gauge invariant fundamental meson φ that transforms in the $\square \times \bar{\square}$ representation of the chiral symmetry. This is the *magnetic* theory.

The above gauge theories have many different phases. The ones that interest us are the *Conformal Window* and the *Magnetic Free* where

$$\frac{3}{2}\,N_c < N_f < 3\,N_c \qquad \text{Conformal Window} \tag{2.3.272}$$

$$N_c + 1 < N_f \leq \frac{3}{2}\,N_c \qquad \text{Magnetic Free.} \tag{2.3.273}$$

We will show that in the *Conformal Window* both theories flow to an interacting Superconformal Field Theory (SCFT) and provide evidence that they are the same theory. We will then comment on what is expected to happen in the *Magnetic Free* phase.

Facts about conformal field theories: Before we begin our journey, we first need two important results from conformal field theory [16][22]:

- A chiral operator $\mathcal{O}$ of a SCFT satisfies

$$\dim(\mathcal{O}) = 1 + \frac{1}{2}\,\gamma_{\mathcal{O}} = \frac{3}{2}\,R_{\mathcal{O}}, \tag{2.3.274}$$

  where $\gamma_{\mathcal{O}}$ is the anomalous dimension of $\mathcal{O}$ and $R_{\mathcal{O}}$ is the $\mathrm{U}(1)_R$ charge of $\mathcal{O}$.

[22] These are stated without proof or explanation as they are beyond the scope of this thesis.

	$\mathrm{SU}(N_c)$	$\mathrm{SU}(N_f)_L$	$\mathrm{SU}(N_f)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	$\frac{1}{N_c}$	$\frac{N_f-N_c}{N_f}$
$\tilde{Q}$	$\bar{\square}$	$\mathbf{1}$	$\bar{\square}$	$-\frac{1}{N_c}$	$\frac{N_f-N_c}{N_f}$
M	$\mathbf{1}$	$\square$	$\bar{\square}$	0	$2\,\frac{N_f-N_c}{N_f}$
B	$\mathbf{1}$	$\binom{N_f}{N_c}$	$\mathbf{1}$	1	$N_c\,\frac{N_f-N_c}{N_f}$
$\tilde{B}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\binom{N_f}{N_c}}$	-1	$N_c\,\frac{N_f-N_c}{N_f}$

Table 2.7: Representations and charges in electric SQCD. The $\mathrm{U}(1)_R$ charges are chosen so that the $\mathrm{SU}(N_c)^2 \times \mathrm{U}(1)_R$ anomaly vanishes.

- Near conformal fixed points, a spin zero gauge invariant $\mathcal{O}$ satisfies

$$\dim(\mathcal{O}) \geq 1. \tag{2.3.275}$$

  Upon saturating the inequality in eq. 2.3.275, $\mathcal{O}$ becomes a free fields and when it violates the bound, decouples from the theory. This is called hitting the *unitarity bound*.

Electric theory: Super QCD Now that we have our superconformal tools to hand we can begin to analyse SQCD. The fundamental and composite particle representations are displayed in table 2.7 and we take an empty superpotential

$$W^{\mathrm{el}}(Q,\tilde{Q}) = 0. \tag{2.3.276}$$

The classical moduli space of a SUSY gauge theory is well described by the set of holomorphic gauge invariant polynomials [79] and in SQCD with $N_f > N_c$ the quantum moduli space is the same as classical moduli space [80]. For the range of N_f and N_c we are interested in, our quantum moduli space can be described by

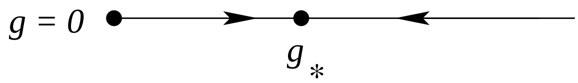


Figure 2.9: Near the loss of asymptotic freedom in SQCD, an IRFP appears at $g = g_*$ determined in eq. 2.3.282. Figure taken from [5].

mesons M, baryons B and anti-baryons $\tilde{B}$[23]

$$M^i{}_{\tilde{i}} = Q^i \tilde{Q}_{\tilde{i}} \tag{2.3.277}$$

$$B^{i_1 i_2 \cdots i_{N_c}} = \varepsilon_{a_1 a_2 \cdots a_{N_c}} Q^{i_1 a_1} Q^{i_2 a_2} \cdots Q^{i_{N_c} a_{N_c}} \tag{2.3.278}$$

$$\tilde{B}_{\tilde{i}_1 \tilde{i}_2 \cdots \tilde{i}_{N_c}} = \varepsilon^{a_1 a_2 \cdots a_{N_c}} \tilde{Q}_{\tilde{i}_1 a_1} \tilde{Q}_{\tilde{i}_2 a_2} \cdots \tilde{Q}_{\tilde{i}_{N_c} a_{N_c}}. \tag{2.3.279}$$

The all orders NSVZ beta function is

$$\beta_g^{\mathrm{NSVZ}} = -\frac{g^3}{16\pi^2} \frac{3\,N_c - N_f(1-\gamma_Q)}{1 - N_c \frac{g^2}{8\pi^2}}. \tag{2.3.280}$$

It is easy to see that the theory is asymptotically free for $b = 3\,N_c - N_f > 0$. One might expect that in the IR, we would reach a Landau pole. This turns out not to always be the case. The perturbative expansion of the quark anomalous dimension is[24]

$$\gamma_Q = -\frac{g^2}{8\,\pi^2} \frac{N_c^2 - 1}{N_c} + \mathcal{O}(g^4). \tag{2.3.281}$$

If we look close to the point where asymptotic freedom is lost $N_f = 3\,N_c - \varepsilon\,N_c$ then a perturbative fixed point appears in the IR

$$g_*^2 = \frac{8\,\pi^2}{3} \frac{N_c}{N_c^2 - 1}\,\varepsilon \tag{2.3.282}$$

which has been caused by a cancellation between the $\mathcal{O}(g^3)$ and $\mathcal{O}(g^5)$ terms in eq. 2.3.280 and is the Banks-Zaks (BZ) fixed point [81] shown in figure 2.9. Actually, something stronger can be said about the existence of such a fixed point. To all

[23] Baryons are in the $\binom{N_f}{N_c}$ representation of $\mathrm{SU}(N_f)$ as detailed in table 2.7. To see this, realise that the Baryon flavour indices are antisymmetric and there are N_c of them with each N_c index taking any possible value from 1 to N_f. The dimension of the representation is the number of independent such objects one can form. Since indices cannot repeat (as the Baryon would be identically zero), the number of independent Baryons that can be formed is then $\binom{N_f}{N_c}$.

[24] $\gamma_Q = \gamma_{\tilde{Q}}$ by symmetry.

orders, a non-trivial Infrared Fixed Point (IRFP) of the gauge coupling will exist providing there is a zero of the numerator in eq. 2.3.280 somewhere along the RG flow. If such a solution exists then

$$3\,N_c - N_f(1-\gamma_Q^{\mathrm{SC}}) = 0 \implies \gamma_Q^{\mathrm{SC}} = 1 - 3\,\frac{N_c}{N_f}. \tag{2.3.283}$$

The determination of whether a fixed point exists is reduced to simple functions of the anomalous dimensions. Using our superconformal tricks, at the fixed point

$$R_Q^{\mathrm{SC}} = \frac{2}{3} + \frac{1}{3}\,\gamma_Q^{\mathrm{SC}} = \frac{N_f - N_c}{N_c} = R_Q, \tag{2.3.284}$$

and so the non-anomalous assignment of the $\mathrm{U}(1)_R$ charges is consistent with flowing to the IRFP. We want to hit an interacting field theory, so the dimension of the spinless gauge invariants imply

$$\dim(M) = \dim(Q\,\tilde{Q}) = 2\left(1 + \frac{1}{2}\,\gamma_Q^{\mathrm{SC}}\right) > 1 \tag{2.3.285}$$

puts a constraint on the anomalous dimensions of the quarks

$$\gamma_Q^{\mathrm{SC}} > -1 \implies N_f > \frac{3}{2}\,N_c. \tag{2.3.286}$$

Finally we find that with the range of flavours and colours

$$\frac{3}{2}\,N_c < N_f < N_c$$

electric SQCD will flow to an IRFP with $g = g_*$ that is an interacting SCFT.

Magnetic theory: Super QCD plus a meson Now let us consider a similar theory with the field content displayed in table 2.8 and an empty superpotential

$$W^{\mathrm{mag}}(q, \tilde{q}, \varphi) = 0. \tag{2.3.287}$$

Note that because φ is a singlet under $\mathrm{SU}(\widetilde{N_c})$ then the $\mathrm{SU}(\widetilde{N_c})^2 \times \mathrm{U}(1)_R$ anomaly cannot be used to fix the $\mathrm{U}(1)_R$ charge of φ. Again the moduli space is parameterised

	$\mathrm{SU}(\widetilde{N}_c)$	$\mathrm{SU}(N_f)_L$	$\mathrm{SU}(N_f)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
q	$\square$	$\bar{\square}$	$\mathbf{1}$	$\frac{1}{\widetilde{N}_c}$	$\frac{N_f-\widetilde{N}_c}{N_f}$
$\tilde{q}$	$\bar{\square}$	$\mathbf{1}$	$\square$	$-\frac{1}{\widetilde{N}_c}$	$\frac{N_f-\widetilde{N}_c}{N_f}$
φ	$\mathbf{1}$	$\square$	$\bar{\square}$	0	$2\,\frac{\widetilde{N}_c}{N_f}$
m	$\mathbf{1}$	$\bar{\square}$	$\square$	0	$2\,\frac{N_f-\widetilde{N}_c}{N_f}$
b	$\mathbf{1}$	$\overline{\binom{N_f}{\widetilde{N}_c}}$	$\mathbf{1}$	1	$\widetilde{N}_c\,\frac{N_f-\widetilde{N}_c}{N_f}$
$\tilde{b}$	$\mathbf{1}$	$\mathbf{1}$	$\binom{N_f}{\widetilde{N}_c}$	-1	$\widetilde{N}_c\,\frac{N_f-\widetilde{N}_c}{N_f}$

Table 2.8: Representations and charges in magnetic SQCD. The $\mathrm{U}(1)_R$ charges are chosen so that the $\mathrm{SU}(\widetilde{N}_c)^2 \times \mathrm{U}(1)_R$ anomaly vanishes. The $\mathrm{U}(1)_R$ charge of φ isn't fixed until the non-zero superpotential in eq. 2.3.293 is added.

by gauge invariant polynomials of the fields

$$m_i^{\tilde{i}} = q_i \tilde{q}^{\tilde{i}} \tag{2.3.288}$$

$$b_{i_1 i_2 \cdots i_{\widetilde{N}_c}} = \varepsilon_{a_1 a_2 \cdots a_{\widetilde{N}_c}} Q^{a_1}_{i_1} Q^{a_2}_{i_2} \cdots Q^{a_{\widetilde{N}_c}}_{i_{\widetilde{N}_c}} \tag{2.3.289}$$

$$\tilde{b}^{\tilde{i}_1 \tilde{i}_2 \cdots \tilde{i}_{\widetilde{N}_c}} = \varepsilon^{a_1 a_2 \cdots a_{\widetilde{N}_c}} \tilde{Q}^{\tilde{i}_1}_{a_1} \tilde{Q}^{\tilde{i}_2}_{a_2} \cdots \tilde{Q}^{\tilde{i}_{\widetilde{N}_c}}_{a_{\widetilde{N}_c}} \tag{2.3.290}$$

and the theory has an all orders beta function

$$\beta^{\mathrm{NSVZ}}_{\tilde{g}} = -\frac{\tilde{g}^3}{16\pi^2} \frac{3\,\widetilde{N}_c - N_f(1-\gamma_q)}{1 - \widetilde{N}_c \frac{\tilde{g}^2}{8\pi^2}}. \tag{2.3.291}$$

Now because φ is a gauge singlet and there is no superpotential, we know that $\gamma_\varphi = 0$ always, and so will decouple in a SCFT. In the range of flavours and colours

$$\frac{3}{2}\,\widetilde{N}_c < N_f < \widetilde{N}_c$$

we know immediately that this theory flows to an IRFP with $\tilde{g} = \tilde{g}_*$ with the φ decoupled. However, notice that the term in the superpotential $q\,\varphi\,\tilde{q}$ is relevant at this fixed point

$$\dim\left(q\,\varphi\,\tilde{q}\right) = 3 + \left(1 - 3\,\frac{\widetilde{N}_c}{N_f}\right) < 3. \tag{2.3.292}$$

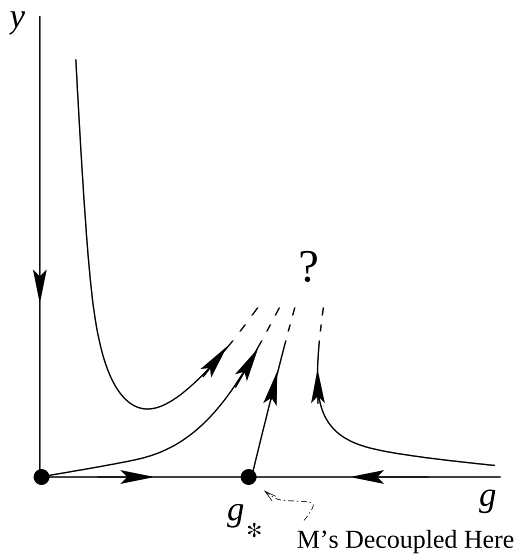


Figure 2.10: Nothing that the superpotential in eq. 2.3.293 is relevant at the IRFP with $y = 0$, $\tilde{g} = \tilde{g}_*$, we see that the flow is only onto $g = g_*$ with y exactly zero and will be away from the point $y = 0$, $\tilde{g} = \tilde{g}_*$ if y is non-zero. Figure taken from [5]. Note the M's in this diagram are our φ's.

If one then adds the superpotential

$$W^{\text{mag}}(q, \tilde{q}, \varphi) = y\, q\, \varphi\, \tilde{q} \tag{2.3.293}$$

this fixes the $\text{U}(1)_R$ charge of φ and we see the behaviour in fig. 2.10. By considering the beta function for y

$$\beta_y = \frac{y}{2}\left(2\,\gamma_q + \gamma_\varphi\right) \tag{2.3.294}$$

we notice that there is also a non-trivial fixed point $y = y_*$ where $2\,\gamma_q + \gamma_\varphi = 0$. At some point in the RG flow, it is then anticipated that we would hit another IRFP with $\tilde{g} = \tilde{g}'_*$ and $y = y_*$ as shown in figure 2.11. At this IRFP,

$$3\,\widetilde{N_c} - N_f(1 - \gamma_q^{\text{SC}}) = 0 \qquad \Longrightarrow \qquad \gamma_q^{\text{SC}} = 1 - 3\,\frac{\widetilde{N_c}}{N_f}, \tag{2.3.295}$$

$$2\,\gamma_q^{\text{SC}} + \gamma_\varphi^{\text{SC}} = 0 \qquad \Longrightarrow \qquad \gamma_\varphi^{\text{SC}} = -2\,\gamma_q^{\text{SC}}, \tag{2.3.296}$$

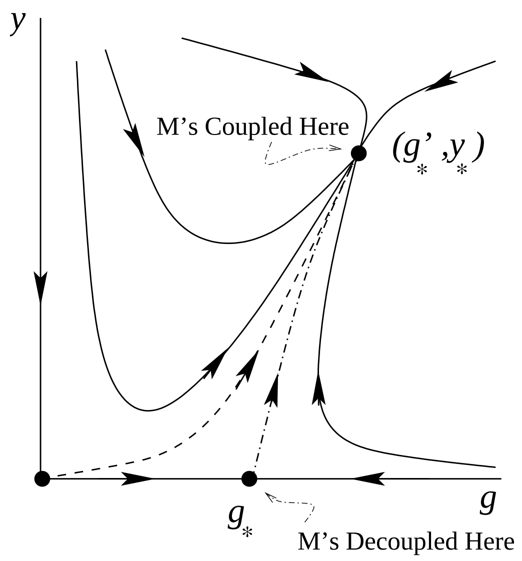


Figure 2.11: The end point of the flow for magnetic SQCD with non-zero superpotential in eq. 2.3.293 is an interacting SCFT with $y = y_*$ and $\tilde{g} = \tilde{g}'_*$. Figure taken from [5]. Note the M's in this diagram are our φ's.

giving the $\mathrm{U}(1)_R$ charges

$$R_q^{\mathrm{SC}} = \frac{2}{3} + \frac{1}{3}\,\gamma_q^{\mathrm{SC}} = \frac{N_f - \widetilde{N_c}}{N_c} = R_q, \tag{2.3.297}$$

$$R_\varphi^{\mathrm{SC}} = \frac{2}{3} + \frac{1}{3}\,\gamma_\varphi^{\mathrm{SC}} = 2\,\frac{\widetilde{N_c}}{N_c} = R_\varphi \tag{2.3.298}$$

which are precisely those determined the vanishing of the $\mathrm{SU}(\widetilde{N_c})^2 \times \mathrm{U}(1)_R$ anomaly and a $\mathrm{U}(1)_R$ invariant superpotential in eq. 2.3.293. Now to just check that we don't have any particles decoupling

$$\dim(m) = \dim(q\,\tilde{q}) = 2\left(1 + \frac{1}{2}\,\gamma_q\right) > 1 \implies \gamma_q > -1, \tag{2.3.299}$$

$$\dim(\varphi) = -2\,\gamma_q \implies \gamma_q < 0. \tag{2.3.300}$$

This gives a bound on the number of colours and flavours

$$\frac{3}{2}\,\widetilde{N_c} < N_f < 3\widetilde{N_c}, \tag{2.3.301}$$

and in this range, magnetic SQCD is an asymptotically free theory that will flow to an IRFP with $y = y_*$ and $\tilde{g} = \tilde{g}'_*$ that is an interacting SCFT.

Seiberg duality in the conformal window: If we take electric SQCD in the *conformal window*, and relate the electric and magnetic gauge groups

$$\widetilde{N_c} \equiv N_f - N_c \tag{2.3.302}$$

then both theories are asymptotically free and flow to their respective IRFPs. There is a large amount of evidence to suggest that at this fixed point, the two theories are identical. A few (not exhaustive) examples are:

- There is a one–to–one correspondence with the operators that parameterise their quantum moduli spaces[25]

$$M^i{}_{\tilde{i}} \leftrightarrow \varphi^i{}_{\tilde{i}}, \tag{2.3.303}$$

$$B^{i_1 i_2 \cdots i_{N_c}} \leftrightarrow \varepsilon^{i_1 i_2 \cdots i_{N_c} j_1 j_2 \cdots j_{\widetilde{N_c}}}\, b_{j_1 j_2 \cdots j_{\widetilde{N_c}}} \tag{2.3.304}$$

$$\tilde{B}_{\tilde{i}_1 \tilde{i}_2 \cdots \tilde{i}_{N_c}} \leftrightarrow \varepsilon_{\tilde{i}_1 \tilde{i}_2 \cdots \tilde{i}_{N_c} \tilde{j}_1 \tilde{j}_2 \cdots \tilde{j}_{\widetilde{N_c}}}\, \tilde{b}^{\tilde{j}_1 \tilde{j}_2 \cdots \tilde{j}_{\widetilde{N_c}}}, \tag{2.3.305}$$

  At the IRFP, the quantum numbers of the operators on each side match exactly.

- In the SCFTs, the global anomalies should remain unbroken and so 't Hooft anomaly matching [22, 82] should apply to both descriptions of the IR degrees of freedom. A selection of the matchings is given in table 2.9.

- The duality is preserved under deformations by quark masses:

$$\begin{array}{ccc} \text{Electric SQCD} & & \text{Magnetic SQCD+}\varphi \\ \mathrm{SU}(N_c),\ N_f & \longleftrightarrow & \mathrm{SU}(N_f - N_c),\ N_f \\ \downarrow \text{mass} & & \downarrow \text{Higgsing} \\ \mathrm{SU}(N_c),\ N_f - 1 & \longleftrightarrow & \mathrm{SU}(N_f - N_f - 1),\ N_f \end{array}$$

  Integrating out a particle has a Higgsing effect in the Seiberg dual picture.

Using symmetries and holomorphy, the holomorphic scales of the electric and magnetic theory are related by

$$\Lambda^b\, \Lambda^{\tilde{b}} = (-)^{N_f - N_c}\, \mu^{N_f} \tag{2.3.306}$$

[25]The m degrees of freedom are projected out by the F *terms* of the q's and $\tilde{q}$'s.

Global anomaly	Electric SQCD	Magnetic SQCD
$\mathrm{SU}(N_f)^3$	N_f	$\widetilde{N_c} - N_f$
$\mathrm{SU}(N_f)^2 \times \mathrm{U}(1)_R$	$\frac{1}{2} \times N_c \times \left(\frac{N_f - N_c}{N_f} - 1\right)$	$\frac{1}{2} \times \widetilde{N_c} \times \left(\frac{N_f - \widetilde{N_c}}{N_f} - 1\right)$
		$+ \frac{1}{2} \times N_f \times \left(2\frac{\widetilde{N_c}}{N_f} - 1\right)$
$\mathrm{U}(1)_B^2 \times \mathrm{U}(1)_R$	$2\, N_f\, N_c \times \left(\frac{1}{N_c}\right)^2 \times \left(\frac{N_f - N_c}{N_f} - 1\right)$	$2\, N_f\, \widetilde{N_c} \times \left(\frac{1}{\widetilde{N_c}}\right)^2 \times \left(\frac{N_f - \widetilde{N_c}}{N_f} - 1\right)$

Table 2.9: A selection of the global anomaly coefficients in Electric and Magnetic SQCD. Identifying $\widetilde{N_c} \equiv N_f - N_c$ matches all of the global anomalies.

where μ is an intrinsic scale introduced to account for the fact that φ from the point of view of the electric theory is really a dimension 2 field in the free theory $\Phi \equiv \mu\, \varphi$. The $(-)^{N_f - N_c}$ factor is determined by requiring the $(\text{dual})^2$ theory to be the same as the original theory.

Seiberg duality in the magnetic free phase: If we now take the range of flavours and colours in the magnetic free phase, we find that the electric theory is still asymptotically free, whereas the magnetic theory is IR free. In this case, the magnetic theory doesn't flow to its IRFP but instead will flow to its trivial fixed point $\tilde{g} = y = 0$. This is a free theory of massless quarks, gauge bosons, baryons, mesons and their superpartners. Because it is IR free, it is comes with a UV cut–off — its Landau pole.

All of the tests of duality in the *conformal window* are valid in the *magnetic free phase*. The critical point here is that whilst we can make the electric theory flow to a strongly coupled SCFT at its IRFP, the magnetic theory in this phase will always undergo a period of flow that is weakly coupled until approaching a free SCFT. Non–perturbative effects arising due to strong coupling in the electric theory can be done in a perturbative manner in the weakly coupled magnetic theory.

The scale matching condition in eq. 2.3.306 is now significant. As the electric theory becomes stronger, the magnetic theory becomes weaker. This is analogous of the $g \to \frac{1}{g}$ of abelian electric–magnetic duality, and is the reason why the theories in Seiberg duality acquire their names.

Mapping soft terms

Even within the context of SUSY dualities it is possible to make contact with reality [83, 84] by deforming the duality with SUSY breaking operators that can be *mapped* across the strong dynamics [85–87]. There are many ways of mapping soft terms across dualities [85–96]. The approach we consider here is to construct the dual theories in terms of couplings that are promoted to superfields and allow their auxiliary components to aquire VEVs parameterising the SUSY breaking [95–99]. One then constructs a set of RG invariants for each theory and, since the theories produce the same physics in the IR we can match the RG invariants there, and indeed anywhere. The matching of these RG invariants then gives relationships between the *soft* terms of each theory. Consider electric SQCD at a renormalisation scale μ

$$\hat{\mathcal{L}}_{\text{total}}(\mu) = \mathcal{L}_K(\mu) + \mathcal{L}_{\text{SYM}}(\mu), \tag{2.3.307}$$

$$\hat{\mathcal{L}}_K(\mu) = \int d^4\theta \left[Z(\mu) \hat{Q}^\dagger e^{\hat{V}} \hat{Q} + (Q \leftrightarrow \tilde{Q}) \right], \tag{2.3.308}$$

$$\hat{\mathcal{L}}_{\text{SYM}}(\mu) = \left[\frac{1}{2} \int d^2\theta \, S(\mu) \, \hat{\mathcal{W}}^{\alpha a} \, \hat{\mathcal{W}}^a_\alpha \right] + \text{h.c.} \tag{2.3.309}$$

where the Z and S have superfield expansions

$$\mathcal{Z}(\mu) = Z(\mu) \left\{ 1 - \left[\theta^2 \, B_Q(\mu) + \text{h.c.} \right] - \theta^2 \bar{\theta}^2 \left[m_Q^2(\mu) - |B_Q(\mu)|^2 \right] \right\}, \tag{2.3.310}$$

$$S(\mu) = \frac{1}{2\,\hat{g}^2} - i \, \frac{\theta_{\text{YM}}}{16\,\pi^2} + \theta^2 \, \frac{M_\lambda(\mu)}{\hat{g}^2(\mu)} \equiv s(\mu) + \theta^2 \, \frac{M_\lambda(\mu)}{\hat{g}^2(\mu)}, \tag{2.3.311}$$

where m_Q^2 and $m_{\tilde{Q}}^2$ are the squark scalar mass squared, B_Q and $B_{\tilde{Q}}$ are the squark B terms, and M_λ is the Majorana gaugino mass. Here, $s(\mu)$ is related to our holomorphic gauge coupling $\hat{\tau}(\mu)$ by $\hat{\tau}(\mu) = 8\,\pi\,i\,s(\mu)$, and $S(\mu)$ is related to the holomorphic RG invariant $\hat{\Lambda}$

$$\hat{\Lambda} = \mu \, e^{-16\,\pi^2 \, S(\mu)/b} \tag{2.3.312}$$

which is now also a χSF. This theory has an axial symmetry under which

$$Q \to Q \, e^X, \qquad \tilde{Q} \to \tilde{Q} \, e^X,$$
$$\mathcal{Z} \to e^{-(X+X^\dagger)} \, \mathcal{Z}, \qquad \hat{\Lambda} \to \hat{\Lambda} \, e^{2\,X\,N_f/b} \, \hat{\Lambda} \tag{2.3.313}$$

where the rotation parameter X is a χSF. Physical quantities have to be $\text{U}(1)_A$ invariant *and* RG invariant. The only such object that can be formed from parameters in the theory is

$$I \equiv \hat{\Lambda}^\dagger \, \mathcal{Z}^{2\,N_f/b} \, \hat{\Lambda}, \tag{2.3.314}$$

which has the θ^2 component

$$\int d^2\theta \, \log\left(\frac{I}{\mu}\right) = \frac{16\,\pi^2}{b} \frac{M_\lambda}{\hat{g}^2}, \tag{2.3.315}$$

The quantities above are RG invariants by construction, and so can be evaluated at any RG scale. In an asymptotically free theory, this is most conveniently the UV

$$\int d^2\theta \, \log\left(\frac{I}{\mu}\right) = \frac{16\,\pi^2}{b} \frac{M}{\hat{g}^2_\infty} \tag{2.3.316}$$

where

$$\frac{M}{\hat{g}^2_\infty} \equiv \lim_{\mu \to \infty} \frac{M_\lambda}{\hat{g}^2}. \tag{2.3.317}$$

In the dual theory there are the wave function renormalisations of the magnetic quarks $\tilde{\mathcal{Z}}$, a new holomorphic scale

$$\hat{\tilde{\Lambda}} = \mu \, e^{-16\,\pi^2 \, \tilde{S}(\mu)/\tilde{b}} \tag{2.3.318}$$

satisfying the scale matching condition 2.3.306. Because physical quantities are $\mathrm{U}(1)_A$ invariant *and* RG invariant, the magnetic invariant

$$\tilde{I} \equiv \hat{\tilde{\Lambda}}^\dagger \, \tilde{\mathcal{Z}}^{2\,N_f/\tilde{b}} \, \hat{\tilde{\Lambda}}, \tag{2.3.319}$$

must match the elecric one

$$\tilde{I} = I \implies \tilde{\mathcal{Z}}^b = \mathcal{Z}^{\tilde{b}}. \tag{2.3.320}$$

The dual holomorphic coupling $\tilde{S}(\mu)$ has the expansion

$$\tilde{S}(\mu) = \frac{1}{2\,\hat{\tilde{g}}^2} - i\,\frac{\theta_{\mathrm{YM}}}{16\,\pi^2} + \theta^2 \, \frac{\widetilde{M}_\lambda(\mu)}{\hat{\tilde{g}}^2(\mu)} \equiv \tilde{s}(\mu) + \theta^2 \, \frac{\widetilde{M}_\lambda(\mu)}{\hat{\tilde{g}}^2(\mu)}. \tag{2.3.321}$$

Consequently the θ^2 component of the RG and $\mathrm{U}(1)_A$ invariant 2.3.319 is

$$\int d^2\theta \, \log\left(\frac{\tilde{I}}{\mu}\right) = \frac{16\,\pi^2}{\tilde{b}} \frac{M_{\tilde{\lambda}}}{\hat{\tilde{g}}^2}, \tag{2.3.322}$$

	Bosons	Fermions	Generations	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
H_u	(H_u^+, H_u^0)	$(\widetilde{H}_u^+, \widetilde{H}_u^0)$	1	$\mathbf{1}$	$\square$	$\frac{1}{2}$
H_d	(H_d^0, H_d^-)	$(\widetilde{H}_d^0, \widetilde{H}_d^-)$	1	$\mathbf{1}$	$\square$	$-\frac{1}{2}$
q	$(\widetilde{u}_L, \widetilde{d}_L)$	(u_L, d_L)	3	$\square$	$\square$	$\frac{1}{6}$
ℓ	$(\widetilde{\nu}, \widetilde{e}_L)$	(ν, e_L)	3	$\mathbf{1}$	$\square$	$-\frac{1}{2}$
$\overline{u}$	$\widetilde{u}_R^\dagger$	$\overline{u} = u_R^\dagger$	3	$\bar{\square}$	$\mathbf{1}$	$-\frac{2}{3}$
$\overline{d}$	$\widetilde{d}_R^\dagger$	$\overline{d} = d_R^\dagger$	3	$\bar{\square}$	$\mathbf{1}$	$\frac{1}{3}$
$\overline{e}$	$\widetilde{e}_R^\dagger$	$\overline{e} = e_R^\dagger$	3	$\mathbf{1}$	$\mathbf{1}$	1

Table 2.10: MSSM χSF field content.

and in an IR free theory is most conveniently evaluated in the IR

$$\int d^2\theta \, \log\left(\frac{\tilde{I}}{\mu}\right) = \frac{16\,\pi^2}{\tilde{b}} \frac{\widetilde{M}}{\hat{g}_0}, \tag{2.3.323}$$

where

$$\frac{\widetilde{M}}{\hat{g}_0^2} \equiv \lim_{\mu\to 0} \frac{M_{\tilde{\lambda}}}{\hat{\tilde{g}}^2}. \tag{2.3.324}$$

Now because the RG invariants are matched, we find the mapping of the gaugino masses across Seiberg duality in the *holomorphic* basis $\widetilde{M}/(\tilde{b}\,\hat{\tilde{g}}_0^2) = M/(b\,\hat{g}_\infty^2)$ and after shifting to the *canonical* basis we have

$$\widetilde{M} = -\frac{3\,N_c - 2\,N_f}{3\,N_c - N_f}\, M. \tag{2.3.325}$$

Similarly one can find a mapping for the soft scalar masses across the duality.

2.3.9 The Minimal Supersymmetric Standard Model

Now we turn to the MSSM. This is a SUSY QFT constructed by taking the field content of the SM in table 2.2 and assigning them to the bosonic and fermionic components of a χSF. Due to the holomorphy of the superpotential and the cancellation of the *Witten anomaly* [100] the MSSM has two Higgs $SU(2)_L$ doublets — one for the up-type sector and one for the down-type sector. Consequently, the Higgs sector of the MSSM is a particular example of a Two Higgs–Doublet Model (2HDM). The χSF content of the MSSM is given in table 2.10. The MSSM has a superpotential

$$W^{\text{MSSM}} = y_\text{u}\,\text{H}_\text{u}\cdot q\,\overline{\text{u}} - y_\text{d}\,\text{H}_\text{d}\cdot q\,\overline{\text{d}} - y_\text{e}\,\text{H}_\text{d}\cdot \ell\,\overline{\text{e}} + \mu\,\text{H}_\text{u}\cdot\text{H}_\text{d}, \tag{2.3.326}$$

where we have assumed that R *parity*

$$P_R \equiv (-)^{3\,(B-L)+2\,s} \tag{2.3.327}$$

is preserved. R *parity* has the actions on the field content 2.10

$$P_R(\text{H}_\text{u},\text{H}_\text{d}) = +(\text{H}_\text{u},\text{H}_\text{d}), \tag{2.3.328}$$

$$P_R(q,\ell,\overline{\text{u}},\overline{\text{d}},\overline{\text{e}}) = -(q,\ell,\overline{\text{u}},\overline{\text{d}},\overline{\text{e}}) \tag{2.3.329}$$

and forbids the potentially dangerous holomorphic gauge invariants

$$W^{\text{MSSM}}_{\Delta L=1} = \frac{1}{2}\,\lambda^{ijk}\,\ell_i\,\ell_j\,\overline{\text{e}}_k + \lambda'^{ijk}\,\ell_i\,q_j\,\overline{\text{d}}_k + \mu'^i\,\ell_i\text{H}_\text{u}, \tag{2.3.330}$$

$$W^{\text{MSSM}}_{\Delta B=1} = \frac{1}{2}\,\lambda'^{ijk}\,\overline{\text{u}}_j\overline{\text{d}}_j\,\overline{\text{d}}_k \tag{2.3.331}$$

that lead to rapid proton decay through e.g. an S–channel strange anti squark. The presence of R *parity* also means that the Lightest Supersymmetric Particle (LSP) is a dark matter candidate. The superpotential 2.3.326 is supplemented by the standard soft terms

$$\begin{aligned}
-\mathcal{L}^{\text{MSSM}}_{\text{soft}} = {} & \frac{1}{2}\Big(M_3\,\widetilde{\text{g}}^a\,\widetilde{\text{g}}^a + M_2\,\widetilde{\text{W}}^i\,\widetilde{\text{W}}^i + M_1\,\widetilde{\text{B}}\,\widetilde{\text{B}} + \text{h.c.}\Big)\\
& + \Big(a_\text{u}\,\overline{\text{u}}\,q\cdot\text{H}_u - a_\text{d}\,\overline{\text{d}}\,q\cdot\text{H}_d - a_\text{e}\,\overline{\text{e}}\,\ell\cdot\text{H}_d + \text{h.c.}\Big)\\
& + m_q^2|q|^2 + m_\text{u}^2|\overline{\text{u}}|^2 + m_\text{d}^2|\overline{\text{d}}|^2 + m_\ell^2|\ell|^2 + m_\text{e}^2|\overline{\text{e}}|^2\\
& + m^2_{\text{H}_\text{u}}|\text{H}_\text{u}|^2 + m^2_{\text{H}_d}|\text{H}_d|^2 + (b_\mu\,\text{H}_u\cdot\text{H}_d + \text{h.c.})
\end{aligned} \tag{2.3.332}$$

that are anticipated to be generated by the mechanisms discussed in Section 2.3.7. After EWSB

$$\text{H}^0_\text{u} \to \frac{1}{\sqrt{2}}\big(v_\text{u} + \phi_\text{u} + i\,\sigma_\text{u}\big)\,, \qquad \text{H}^0_\text{d} \to \frac{1}{\sqrt{2}}\big(v_\text{d} + \phi_\text{d} + i\,\sigma_\text{d}\big) \tag{2.3.333}$$

where for the vacuum solution to be a minimum of the scalar potential[26]

$$|\mu|^2 + m_{\mathrm{H_u}}^2 = b_\mu \cot(\beta) + \frac{m_{\mathrm{Z}}^2}{2}\, c_{2\beta}, \tag{2.3.334}$$

$$|\mu|^2 + m_{\mathrm{H_d}}^2 = b_\mu\, t_\beta - \frac{m_{\mathrm{Z}}^2}{2}\, c_{2\beta}, \tag{2.3.335}$$

and

$$t_\beta \equiv \frac{v_{\mathrm{u}}}{v_{\mathrm{d}}}, \qquad v^2 \equiv v_{\mathrm{u}}^2 + v_{\mathrm{d}}^2 \approx (246\ \mathrm{GeV})^2 \tag{2.3.336}$$

there is mixing amongst the gauge eigenstates to form the mass eigenstates of the theory. There are the CP even neutral scalars in the basis $(\phi_{\mathrm{u}}, \phi_{\mathrm{d}})$

$$m_\phi^2 = \begin{pmatrix} |\mu|^2 + m_{\mathrm{H_u}}^2 + \frac{g_1^2+g_2^2}{8}\,(3\, v_{\mathrm{u}}^2 - v_{\mathrm{d}}^2) & -b_\mu - \frac{g_1^2+g_2^2}{4}\, v_{\mathrm{u}} v_{\mathrm{d}} \\ -b_\mu - \frac{g_1^2+g_2^2}{4}\, v_{\mathrm{u}} v_{\mathrm{d}} & |\mu|^2 + m_{\mathrm{H_d}}^2 + \frac{g_1^2+g_2^2}{8}\,(3\, v_{\mathrm{d}}^2 - v_{\mathrm{u}}^2) \end{pmatrix} \tag{2.3.337}$$

where the gauge boson masses are

$$m_{\mathrm{Z}}^2 = \frac{1}{4}(g_1^2 + g_2^2)\, v^2, \qquad m_{\mathrm{W}}^2 = \frac{1}{4}\, g_2^2\, v^2. \tag{2.3.338}$$

In the vacuum 2.3.335, the CP even neutral scalar mass matrix 2.3.337 becomes

$$m_\phi^2 = \frac{1}{2}\begin{pmatrix} m_{A_0}^2 + m_{\mathrm{Z}}^2 + (m_{A_0}^2 - m_{\mathrm{Z}}^2)\, c_{2\beta} & -2\,(m_{A_0}^2 + m_{\mathrm{Z}}^2)\, c_\beta\, s_\beta \\ -2\,(m_{A_0}^2 + m_{\mathrm{Z}}^2)\, c_\beta\, s_\beta & m_{A_0}^2 + m_{\mathrm{Z}}^2 - (m_{A_0}^2 - m_{\mathrm{Z}}^2)\, c_{2\beta} \end{pmatrix} \tag{2.3.339}$$

where $m_{A_0}^2 = \frac{2\, b_\mu}{s_{2\beta}} = 2\,|\mu|^2 + m_{\mathrm{H_u}}^2 + m_{\mathrm{H_d}}^2$ is the CP odd scalar mass. The eigenvalues of 2.3.339 are

$$m_{\mathrm{h,H}}^2 = \frac{1}{2}\left(m_{A_0}^2 + m_{\mathrm{Z}}^2 \mp \sqrt{m_{A_0}^4 + m_{\mathrm{Z}}^4 - 2\, m_{A_0}^2\, m_{\mathrm{Z}}^2\, c_{4\beta}}\right), \tag{2.3.340}$$

and the smallest mass eigenvalue is maximised in the decoupling limit $m_{A_0}^2 \to \infty$ yielding the famous tree level Higgs mass upper bound in the MSSM

$$m_{\mathrm{h}} \leq m_{\mathrm{Z}}\, c_{2\beta} = (91.2\,\mathrm{GeV})\, c_{2\beta} \tag{2.3.341}$$

[26]We use $c_\theta \equiv \cos(\theta)$, $s_\theta \equiv \sin(\theta)$ and $t_\theta \equiv \tan(\theta)$ throughout.

clearly in conflict with the mass of the Higgs boson $m_{\mathrm{h}} = 125.7 \pm 0.4\,\mathrm{GeV}$ observed by ATLAS and CMS [11, 101, 102]. As discussed in 2.3.69, scalar mass squareds are sensitive to the amount of SUSY breaking, and in the MSSM at one loop the Higgs mass is most sensitive to the non-cancellation between the top quark and squarks

$$m_{\mathrm{h}}^2 \simeq m_{\mathrm{Z}}^2\, c_{2\beta}^2 + \frac{3}{2\,\pi^2}\frac{m_{\mathrm{t}}^4}{v^2}\left[\log\left(\frac{m_{\widetilde{\mathrm{t}}_1} m_{\widetilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2}\right) + \frac{X_t^2}{m_{\widetilde{\mathrm{t}}_1} m_{\widetilde{\mathrm{t}}_2}}\left(1 - \frac{X_t^2}{12\, m_{\widetilde{\mathrm{t}}_1} m_{\widetilde{\mathrm{t}}_2}}\right)\right] \tag{2.3.342}$$

where

$$X_t \equiv (a_{\mathrm{u}})_3 - \mu\, \cot(\beta) \tag{2.3.343}$$

is the stop mixing parameter. It is therefore possible for a radiatively corrected Higgs mass to agree with the experimental observation, however, in minimal SUSY models this is typically accompanied with a degree of *fine tuning* known as the *little hierarchy problem*. The neutral gauginos and Higgsinos mix to form *neutralinos* with a mass matrix $m_{\widetilde{\chi}^0}$ in the basis $(\widetilde{\mathrm{B}}, \widetilde{\mathrm{W}}^0, \widetilde{\mathrm{H}}_{\mathrm{u}}^0, \widetilde{\mathrm{H}}_{\mathrm{d}}^0)$

$$m_{\widetilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -\frac{g_2}{2}\, v_{\mathrm{u}} & \frac{g_2}{2}\, v_{\mathrm{d}} \\ 0 & M_2 & \frac{g_1}{2}\, v_{\mathrm{u}} & -\frac{g_1}{2}\, v_{\mathrm{d}} \\ -\frac{g_2}{2}\, v_{\mathrm{u}} & \frac{g_1}{2}\, v_{\mathrm{u}} & 0 & -\mu \\ \frac{g_2}{2}\, v_{\mathrm{d}} & -\frac{g_1}{2}\, v_{\mathrm{d}} & -\mu & 0 \end{pmatrix} \tag{2.3.344}$$

and the charged gauginos bosons Higgsinos mix to form *charginos* with a mass matrix $m_{\widetilde{\chi}^\pm}$ in the basis $(\widetilde{\mathrm{W}}^-, \widetilde{\mathrm{H}}_{\mathrm{d}}^-), (\widetilde{\mathrm{W}}^+, \widetilde{\mathrm{H}}_{\mathrm{u}}^+)$

$$m_{\widetilde{\chi}^\pm} = \begin{pmatrix} M_2 & \frac{g_2}{\sqrt{2}}\, v_{\mathrm{u}} \\ \frac{g_2}{\sqrt{2}}\, v_{\mathrm{d}} & \mu \end{pmatrix}. \tag{2.3.345}$$

If Minimal Flavour Violation (MFV) is satisfied, the squark and slepton mass matrices are block diagonal within each generation. Top squarks mix to form a lighter $\widetilde{\mathrm{t}}_1$ and heavier $\widetilde{\mathrm{t}}_2$ top squark with a mass squared matrix $m_{\widetilde{\mathrm{t}}}^2$ in the basis $(\widetilde{\mathrm{t}}_L^\dagger, \widetilde{\mathrm{t}}_R^\dagger), (\widetilde{\mathrm{t}}_L, \widetilde{\mathrm{t}}_R)$

$$m_{\widetilde{\mathrm{t}}}^2 = \begin{pmatrix} (m_q^2)_3 + m_{\mathrm{t}}^2 + \left(\frac{1}{3}\, s_{\theta_W}^2 - \frac{1}{2}\right) c_{2\beta}\, m_{\mathrm{Z}}^2 & \frac{v_{\mathrm{u}}}{\sqrt{2}}\, X_t \\ \frac{v_{\mathrm{u}}}{\sqrt{2}}\, X_t & (m_{\mathrm{u}}^2)_3 + m_{\mathrm{t}}^2 + \frac{2}{3} s_{\theta_W} c_{2\beta}\, m_{\mathrm{Z}}^2 \end{pmatrix}. \tag{2.3.346}$$

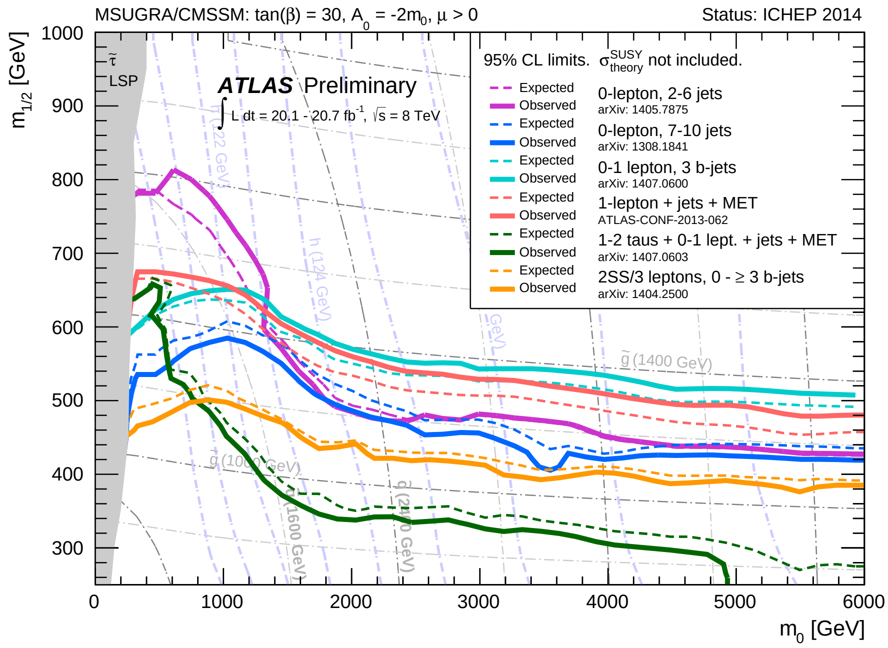


Figure 2.12: 95% CL Exclusion limits for 8 TeV analyses in the $(m_0, M_{1/2})$ plane in the CMSSM with $t_\beta = 30$, $a_0 = -2\,m_0$, sign$(\mu) > 0$. Taken from [6].

with analogous expressions for the remaining squarks and the sleptons.

2.3.10 Naturalness in trouble

Although the observation of a SM–like Higgs boson puts stringent constraints on SUSY breaking [103,104], limiting the discussion to direct searches for superpartners tells a similar story. Popular ways of interpreting limits on SUSY particles are

- **Top–down:** Construct a UV completion of the MSSM (or its extensions) that give relations at a high scale for the parameters in the soft lagrangian in terms of those of the UV model. Solving the RGEs for a given point in the parameter space of the UV model fixes the low lying spectrum and parameters, and consequently the number of events of a particular kind one expects to see. On the UV parameter space, one can then draw up an exclusion for the model at a certain Confidence Level (CL),

- **Bottom–up:** Decide that certain subset of the SUSY particles are responsible for the majority of a particular signature of interest. Decouple the remaining SUSY particles and taking the limits of relevant branching ratios to 100 %

gives a *simplified model* [105]. The number of events can then be calculated as a function of the light sparticle masses.

- **Brute–force:** Although the 105 dimensional parameter space of the MSSM is too large to reasonably study, if we assume:

    1. CP conservation,
    2. MFV at the EW scale,
    3. First and second generation sfermion masses are degenerate,
    4. Negligible Yukawa couplings or a terms for the first and second generations,

    then we end up with the 19/20 dimensional parameter space of the Phenomenological Mimimal Supersymmetric Standard Model (pMSSM) [106], where the 20 dimensional parameter space has an additional parameter for the gravitino mass. One can then analyse this parameter space without any RG evolution in the same way as the top–down approach.

During Run I of the LHC, SUSY particles were not observed directly. Using the simplified model approach, limits have been put on the stop masses and gluino masses (see figure 2.13) that, except for a few isolated strips in the plane, for a light Lightest Ordinary Supersymmetric Particle (LOSP) $m_{\widetilde{\chi}_1^0} = 0$ ($m_{\widetilde{\chi}_1^0} < 400$ GeV) imply $m_{\widetilde{\mathrm{t}}_1} \gtrsim 640$ GeV [107, 108] ($m_{\widetilde{\mathrm{g}}} \gtrsim 1.34$ TeV [109]). The situation is similar if we consider a simple UV model like the Constrained Minimal Supersymmetric Standard Model (CMSSM) in figure 2.12. We can use the fine tuning estimate [110]

$$\Delta \geq 2\,\frac{m_{\mathrm{H_u}}^2}{m_{\mathrm{h}}^2} \tag{2.3.347}$$

to see that if these squark masses are generated at the GUT scale, this results in $\Delta \gtrsim 100$ at the EW scale, i.e. fine–tuning at the 1% level (at least!).

It will be useful to also note the bounds from Large Electron Positron Collider (LEP) (see table 2.11) on the masses neutralinos, sneutrinos, charginos and sleptons for later in Chapter 4. Solutions accounting for the non-observation of SUSY are also available: compressed spectra [116–118] softens jet activity, R Parity Violation ($\not{R}$) reduces the about of Missing Transverse Energy ($\not{E}_T$) [119] and Flavoured

Sparticle	Lower Mass Limit at 95 % CL (GeV)	Reference
Neutralino (stable)	45.5	[11]
Neutralino (unstable)	96.8	[111, 112]
Sneutrino	41	[11]
Chargino	103.5	[113, 114]
Sleptons	100.2	[115]

Table 2.11: The strongest most model independent non-hadron collider limits on LOSP and NLSP masses. The lightest neutralino $\widetilde{\chi}_1^0$ is assumed to be bino–like, and allowed to decay to the gravitino $\tilde{G}$ in GMSB, emitting a photon.

Gauge Mediation (FGM) [120, 121] can break the squark mass degeneracy, weakening the reduced limits at current experiments. Combining these mechanisms with models that generate natural spectra can give a plausible explanation of SUSY non-observation *and* the Higgs mass. We will see in Section 3.5 of the next chapter that Dirac gauginos provide a different approach to this problem.

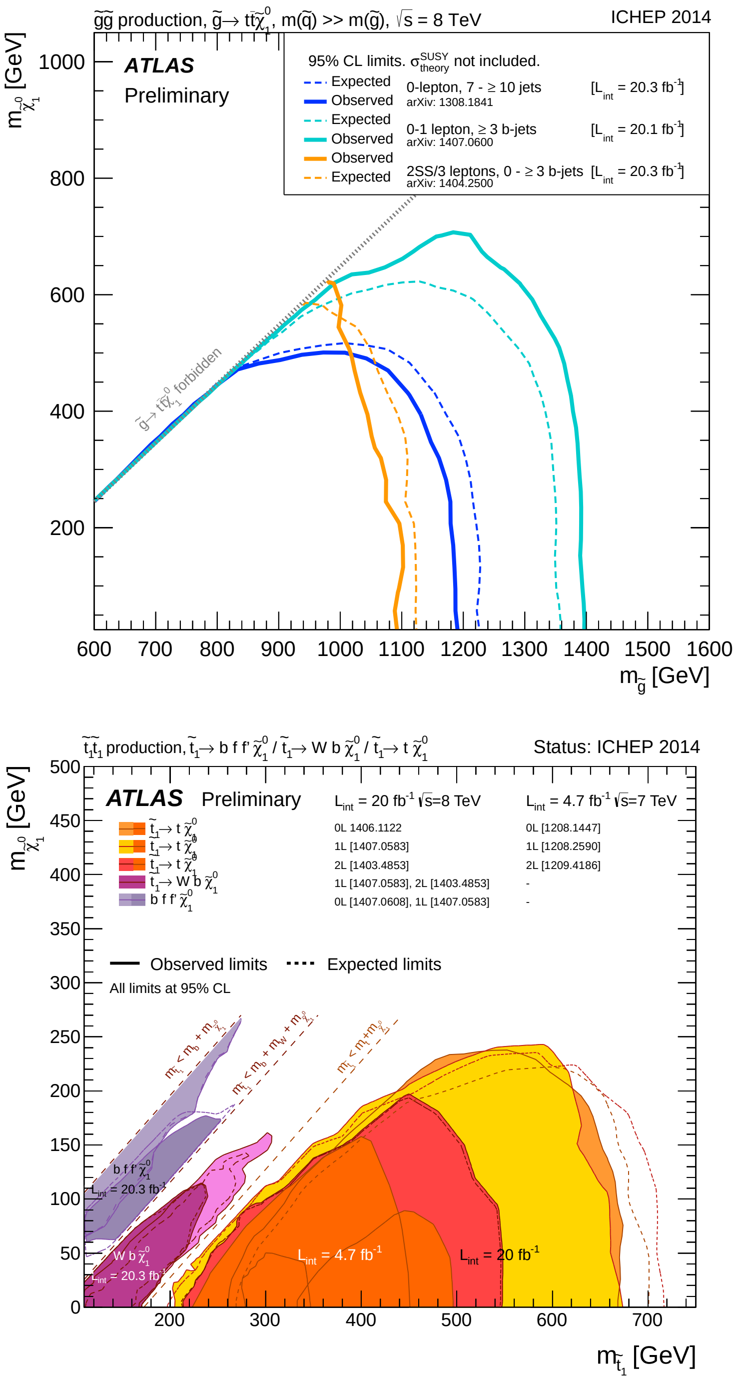


Figure 2.13: **Top:** 95% CL exclusion limits for 8 TeV analyses in the $(m_{\tilde{g}}, m_{\tilde{\chi}^0})$ plane for the simplified model *Gtt*, where a pair of gluinos decays promptly via off-shell stop to four top quarks and two neutralino LSPs. **Bottom:** Exclusion limits at 95% CL in the $\tilde{t}_1, \tilde{\chi}^0_1$ mass plane for dedicated ATLAS searches for stop pair production based on 20 fb^{-1} of proton–proton collisions at $\sqrt{s} = 8$ TeV, and 4.7 fb^{-1} of proton–proton collisions at $\sqrt{s} = 7$ TeV. Taken from [6].

3

Dirac gauginos

It does not do to leave a live dragon out of your calculations, if you live near him.

– J. R. R. Tolkien, *The Hobbit*

3.1 Introduction

So far we have only introduced the MSSM as a possible model of reality with the caveat that superpartners are decoupled by some SUSY breaking mechanism in an appropriate way. The MSSM is far from the only SUSY model that has a chance of describing our world. For the remainder of this thesis, our eyes will turn to a particular alternative: that all (or a subset) of the gauginos are (pseudo–)Dirac particles [122]. Dirac gauginos have been studied in a wide range of scenarios [1, 2, 7, 40–44, 72, 122–136, 136–184] and have numerous advantages over their Majorana counterparts:[1]

- They can preserve the $\mathrm{U}(1)_R$ symmetry, allowing for the simpler SUSY breaking models using the NS theorem discussed in Section 2.3.7,
- Key diagrams involved in sparticle production at collider experiments vanish, alleviating the bounds form direct searches at colliders [1,7,159,169,178]. This is known as *supersafeness* and is discussed in Section 3.4,

[1] It was initially thought that preserved $\mathrm{U}(1)_R$ symmetry could significantly relax flavour constraints [141], however, it has been shown that this is no longer valid beyond the mass mass insertion approximation [176].

- They have novel renormalisation properties known as *supersoft* behaviour, making them more natural [1, 72, 127, 129, 155, 167]. This is discussed in Section 3.5. Due to this mechanism it is possible for the spectrum to lie on the *wrong side* of the squark gluino plane, where squarks are lighter than gluinos. This is not typical in UV completions of the MSSM [185],

- Additional F terms can raise the SM–like Higgs mass is at tree level to its experimental value. This is investigated in in Section 3.6.4.

There are some problems with models of Dirac gauginos however:

- Tachyonic states easily arise in these theories and is discussed in Section 3.6.6,

- The VEVs of additional $\mathrm{SU}(2)_{\mathrm{L}}$ states contribute to custodial symmetry breaking, causing additional deviations of the ρ parameter from 1. This is discussed in Section 3.6.5,

- Integrating out the additional matter content sets the SM D terms to zero in the absence of a superpotential, and in the presence of a superpotential could cause an unstable vacuum. This is discussed in Section 3.6.7.

In addition to investigating the pros and cons of Dirac gauginos, the crucial differences between Majorana and Dirac particles are investigated in Section 3.2 and the minimum requirements for a model Dirac gauginos are outlined in Section 3.3. Other notable features of Dirac gauginos are:

- In order to evade limits from XENON100, Dirac neutralinos must be bino–like, and we must have either heavy squarks $m_{\widetilde{q}} \gtrsim 2$ TeV *or* $m_{\widetilde{\chi}^0} \lesssim 20$ to 380 GeV. Dirac bino–like neutralinos with masses $m_{\widetilde{\chi}^0} \sim 10$ to 380 GeV annihilate through slepton exchange to generate the correct relic abundance without requiring co–annihilation effects or near–resonant annihilation [174],

- In the case of a pseudo–Dirac bino LOSP, the process $\widetilde{\mathrm{B}}_2 \to \bar{f} f \widetilde{\mathrm{B}}_1$ has a decay length $L \propto (\Delta m)^{-5}$ where Δm is the mass splitting between the quasi–degenerate Majorana binos $\widetilde{\mathrm{B}}_1$ and $\widetilde{\mathrm{B}}_2$. A collider signal for a pseudo–Dirac bino would then be a displaced vertex $\bar{f} f$ vertex and $\not{E}_T$ [153],

- The prediction of a GUT is naïvely lost, but can be recovered by accompanying the Extended Superpartners (ESPs) with bachelor states left over from the embedding of ESPs into a complete GUT representation at the higher scale. The two main possibilities here are SU(5) (considered in [148, 180]) and $[\mathrm{SU}(3)]^3$ (considered in [129, 180]). It was found in [180] that in the SU(5) case, the gauge couplings actually diverge at two–loops before the unification scale, but the $[\mathrm{SU}(3)]^3$ is possible with $m_{\mathrm{GUT}} \approx (1.8 \pm 0.4) \times 10^{17}$ GeV,

- Sgluon production [186] can be a dominant process at the LHC. Their masses need to be $m_{\phi_{\tilde{g}}} \gtrsim 1$ TeV to avoid exclusion [145].

3.2 Dirac versus Majorana particles

3.2.1 Continuous symmetries

A neutral anti–commuting spin $\frac{1}{2}$ field $\psi_\alpha(x)$ that transforms under the $(\frac{1}{2}, 0)$ representation of the Lorentz group with mass M describes a Majorana fermion. The associated Lagrangian is

$$\mathcal{L}_{\mathrm{Majorana}} = i\,\psi^\dagger \bar{\sigma}^\mu\,\partial_\mu\,\psi - \left(\frac{M}{2}\,\psi\,\psi + \mathrm{h.c.}\right) \tag{3.2.1}$$

and on–shell, ψ satisfies

$$i\,(\bar{\sigma}^\mu\,\partial_\mu\,\psi)^{\dot{\alpha}} = M\,\psi^{\dagger\dot{\alpha}}. \tag{3.2.2}$$

Now consider a set of fermions $\hat{\psi}_i$ $i = 1, \ldots, N$ and a mass matrix M^{ij}

$$\mathcal{L} = i\,\hat{\psi}^{\dagger i} \bar{\sigma}^\mu\,\partial_\mu\,\hat{\psi}_i - \left(\frac{M^{ij}}{2}\,\hat{\psi}_i\,\hat{\psi}_j + \mathrm{h.c.}\right), \tag{3.2.3}$$

where M^{ij} is a complex symmetric matrix. In the limit $M^{ij} \to 0$ there is a U(N) flavour symmetry

$$\hat{\psi}_i \to U_i{}^j\,\hat{\psi}_j \tag{3.2.4}$$

where U is a unitary matrix. In the presence of M, the U(N) is still a symmetry providing M transforms[2]

$$M^{ij} \to U^i{}_k\,U^i{}_l\,M^{kl}, \tag{3.2.5}$$

[2] I.e. M is a *spurion* of the U(N) symmetry.

where

$$U^j{}_i \equiv (U_j{}^i)^* = (U^\dagger)_i{}^j, \qquad U_i{}^k (U^\dagger)_k{}^j = \delta_i^j. \tag{3.2.6}$$

We can now move to a new basis

$$\hat{\psi}_i = \hat{U}_i{}^j \, \psi_j \tag{3.2.7}$$

where we choose $\hat{U}$ to diagonalise M

$$M^{ij} \, \hat{U}_i{}^k \, \hat{U}_j{}^l = M_k \, \delta^{kl}, \qquad \text{(no summation over } k\text{)}. \tag{3.2.8}$$

The resulting lagrangian is

$$\mathcal{L} = i \, \psi^{\dagger i} \bar{\sigma}^\mu \, \partial_\mu \, \psi_i - \left(\frac{M_i}{2} \, \psi_i \, \psi_i + \text{h.c.} \right), \tag{3.2.9}$$

There are three cases to comment on:

- If a $M_i \neq 0$ is not degenerate with another M_j the corresponding field ψ_i describes a neutral Majorana fermion as described above,
- If a $M_i = 0$ the corresponding field ψ_i is a massless Weyl fermion,
- If two of the M_i's are degenerate, say $M_1 = M_2 \neq 0$ then the Lagrangian 3.2.9 has an O(2) flavour symmetry

$$\psi_i \to O_i{}^j, \qquad \sum_k O_i{}^k \, O_j{}^k = \delta_{ij}, \qquad i, j, k = 1, 2. \tag{3.2.10}$$

Making a change of basis

$$\chi \equiv \frac{1}{\sqrt{2}} \, (\psi_1 + i \, \psi_2) \, , \qquad \eta \equiv \frac{1}{\sqrt{2}} \, (\psi_1 - i \, \psi_2) \tag{3.2.11}$$

the Lagrangian 3.2.9 involving only $i = 1, 2$ and setting $M_1 = M_2 \equiv M$ becomes

$$\mathcal{L} = i \, \chi^\dagger \bar{\sigma}^\mu \, \partial_\mu \, \chi + i \, \eta^\dagger \bar{\sigma}^\mu \, \partial_\mu \, \eta - M \, (\chi \, \eta + \text{h.c.}) \, . \tag{3.2.12}$$

In this basis the O(2) symmetry manifests itself as a U(1) symmetry under which χ and η have opposite charges q and $-q$

$$\chi \to e^{i\,q\,\theta}\,\chi, \qquad \eta \to e^{-i\,q\,\theta}\,\eta. \tag{3.2.13}$$

On–shell the fields χ and η satisfy

$$i\,(\bar{\sigma}^{\mu}\,\partial_{\mu}\,\chi)^{\dot{\alpha}} = M\,\eta^{\dagger\dot{\alpha}}, \qquad i\,(\bar{\sigma}^{\mu}\,\partial_{\mu}\,\eta)^{\dot{\alpha}} = M\,\chi^{\dagger\dot{\alpha}} \tag{3.2.14}$$

and together they constitute a singe Dirac fermion. Given a set of fermions transforming in the $(\frac{1}{2}, 0)$ representation of the Lorentz group, the question about whether a particle is Majorana or Dirac is determined by the symmetries of the Lagrangian. If there is no continuous symmetry of the Lagrangian, all massive fermions are Majorana. If there is a continuous symmetry group G, then massive fermions in real representations of G are Majorana[3] and massive fermions in complex representations of G are Dirac. Massless fermions are simply Weyl fermions and aren't classed as Majorana or Dirac.

3.2.2 Propagators

In the $i\,\varepsilon$ prescription, there are four propagators[4] for a Majorana particle $\psi_{\alpha}(x)$ with mass M [187][5]

$$\langle 0|T\{\psi_{\alpha}(x)\,\psi^{\dagger}_{\dot{\alpha}}(y)\}|0\rangle_{\rm FT} = \frac{i\,\sigma^{\mu}_{\alpha\dot{\alpha}}\,p_{\mu}}{p^2 - M^2} = \psi^{\dagger}_{\dot{\alpha}} \;\longrightarrow\; \psi_{\alpha}\;, \tag{3.2.16}$$

$$\langle 0|T\{\psi^{\alpha}(x)\,\psi^{\dagger\dot{\alpha}}(y)\}|0\rangle_{\rm FT} = \frac{i\,\bar{\sigma}^{\mu\alpha\dot{\alpha}}\,p_{\mu}}{p^2 - M^2} = \psi^{\alpha} \;\longleftarrow\; \psi^{\dagger\dot{\alpha}}\,, \tag{3.2.17}$$

$$\langle 0|T\{\psi_{\alpha}(x)\,\psi^{\beta}(y)\}|0\rangle_{\rm FT} = \frac{i\,M\,\delta_{\alpha}{}^{\beta}}{p^2 - M^2} = \psi^{\beta} \;\leftarrow\!\!-\!\!\rightarrow\; \psi_{\alpha}\;, \tag{3.2.18}$$

$$\langle 0|T\{\psi^{\dagger\dot{\alpha}}(x)\,\psi^{\dagger}_{\dot{\beta}}(y)\}|0\rangle_{\rm FT} = \frac{i\,M\,\delta^{\dot{\alpha}}{}_{\dot{\beta}}}{p^2 - M^2} = \psi^{\dagger}_{\dot{\beta}} \;\rightarrow\!\!-\!\!\leftarrow\; \psi^{\dagger\dot{\alpha}} \tag{3.2.19}$$

---

[3] An example of this would be the gluinos of the MSSM with respect $SU(3)_C$ *only*.

[4] We do not write the $+i\,\varepsilon$ in the denominator of the propagators but it is there implicitly. We take momentum p to flow from left to right in all propagators presented.

[5] The Fourier transform $\tilde{f}(p) \equiv f(x)_{\rm FT}$ for a function $f(x)$ is defined

$$\tilde{f}(p) = \int d^4x\, f(x)\, e^{i\,p\cdot x}, \qquad f(x) = \int \frac{d^4p}{(2\,\pi)^4}\,\tilde{f}(p)\,e^{-i\,p\cdot x}. \tag{3.2.15}$$

whereas for a Dirac particle $[\chi_\alpha(x), \eta^{\dagger\dot{\alpha}}(x)]$ with mass M there are a different set of four independant propegators

$$\langle 0|T\{\chi_\alpha(x)\,\chi^\dagger_{\dot{\alpha}}(y)\}|0\rangle_{\mathrm{FT}} = \frac{i\,\sigma^\mu_{\alpha\dot{\alpha}}\,p_\mu}{p^2 - M^2} = \chi^\dagger_{\dot{\alpha}} \longrightarrow \chi_\alpha \;, \tag{3.2.20}$$

$$\langle 0|T\{\chi^\alpha(x)\,\chi^{\dagger\dot{\alpha}}(y)\}|0\rangle_{\mathrm{FT}} = \frac{i\,\bar{\sigma}^{\mu\alpha\dot{\alpha}}\,p_\mu}{p^2 - M^2} = \chi^\alpha \longleftarrow \chi^{\dagger\dot{\alpha}} \;, \tag{3.2.21}$$

$$\langle 0|T\{\chi_\alpha(x)\,\eta^\beta(y)\}|0\rangle_{\mathrm{FT}} = \frac{i\,M\,\delta_\alpha{}^\beta}{p^2 - M^2} = \eta^\beta \longleftrightarrow \chi_\alpha \;, \tag{3.2.22}$$

$$\langle 0|T\{\chi^{\dagger\dot{\alpha}}(x)\,\eta^\dagger_{\dot{\beta}}(y)\}|0\rangle_{\mathrm{FT}} = \frac{i\,M\,\delta^{\dot{\alpha}}{}_{\dot{\beta}}}{p^2 - M^2} = \eta^\dagger_{\dot{\beta}} \rightarrow\!\leftarrow \chi^{\dagger\dot{\alpha}} \tag{3.2.23}$$

where the remaining four are found by performing the swap $\chi \leftrightarrow \eta$ under which the propagators are invariant, i.e.

$$\langle 0|T\{\eta_\alpha(x)\,\eta^\dagger_{\dot{\alpha}}(y)\}|0\rangle_{\mathrm{FT}} = \langle 0|T\{\chi_\alpha(x)\,\chi^\dagger_{\dot{\alpha}}(y)\}|0\rangle_{\mathrm{FT}} \tag{3.2.24}$$

and so on. The main differences between Majorana and Dirac fermions can then be summarised as follows:

- A massive fermion is necessarily Dirac if it transforms in a complex representation of a continuous symmetry group G. Examples of this are the $\mathrm{U}(1)_{\mathrm{EM}}$ charged fermions of the SM — the quarks and charged leptons. It is currently unknown if neutrinos are Majorana or Dirac; if there is a non-anomalous $\mathrm{U}(1)_{\mathrm{L}}$ symmetry under which they are charged (which in the current version of the SM is *not* the case) then they will be Dirac. Otherwise they will be Majorana.

- Dirac fermions require $2\times$ two-component spinors: a left handed component and a right handed component, in the same representation of all continuous symmetry groups G (or equivalently, two left handed degrees of freedom in conjugate representations of the continuous symmetry groups G). Majorana fermions only require one two-component spinor.

- In the Dirac case, the chirality–flipping propagator exchanges its left and right handed components whereas the Majorana case it exchanges it only exchanges the left handed component. As will become important, this means that certain diagrams involving only couplings to the left component can become suppressed or absent in the Dirac case but present in the Majorana case. A well–known example of this is neutrino–less double beta decay [188, 189].

3.3 Can gauginos be Dirac?

3.3.1 Requirements

Considering the points made in Section 3.2, one can then ask if gauginos can be Dirac particles. In the MSSM this is clearly not possible for two reasons:

- There is no right–handed component of the gaugino[6]; each gaugino of a SM gauge group is the only fermion that is in the **Ad** representation of that gauge group in the SM,
- The left-handed gaugino in the MSSM does not transform in the complex representation of any continuous symmetry group G, so if it does acquire a mass, it will be Majorana.

The above two points need addressing if a model with Dirac gauginos is to be constructed, although the second point can be relaxed slightly if we only want Dirac-like behaviour but are happy with pseudo-Dirac particles, i.e. two Majorana states that are nearly degenerate in mass.

3.3.2 Right handed degree of freedom

For a given gaugino $\lambda_{L\alpha} \equiv \lambda_{X\alpha}$ in the **Ad** representation of a gauge group G_X (in the WZ gauge)

$$V_X = \bar{\theta}\sigma^{\mu}\theta v_{X\mu} + \left[\bar{\theta}^2\theta\lambda_X + \text{h.c.}\right] + \frac{1}{2}\,\theta^2\bar{\theta}^2 D_X. \tag{3.3.25}$$

Given the irreducible representations of SUSY discussed in Section 2.3.3, there are three possibilities at first glance:

- We can increase the gauge group $G_X \to G_X \times G_Y$ introducing a second VSF V_Y into the theory. Unfortunately, the vector fields V_X and V_Y transform differently under $G_X \times G_Y$, even if G_X and G_Y are governed by the same

[6] We conventionally identify the gauginos of the MSSM with the left–handed component of a Dirac gaugino.

group. This means in the high energy theory we cannot write down a Dirac mass is it would not be gauge invariant. One could then ask if by Higgsing the product gauge group to its diagonal subgroup $G_X \times G_Y \to G_{\text{diagonal}}$ with e.g. the superpotential

$$W_{\text{Higgsing}} = \lambda\,\varphi\,(\Phi\,\tilde{\Phi} - v^2) \tag{3.3.26}$$

where the field φ is a singlet and behaves as a Lagrange multiplier, and $(\Phi, \tilde{\Phi})$ are *link fields* that transform in the bi–fundamental and anti–bi–fundamental of $G_X \times G_Y$. $(\Phi, \tilde{\Phi})$ cause the desired Higgsing. The type of setups with G_X identified with the visible sector, (also the case with us as we identify the MSSM gaugino with the gaugino of G_X) are already studied under the name *gaugino mediation* [190–192]. One combination of the gauginos will remain massless (before SUSY breaking effects are included) and the orthogonal combination gets a mass of order v via the *super Higgs mechanism*, causing it to decouple from the spectrum. In the end we are left with effectively one left-handed degree of freedom which will either be massless or Majorana, ruling out this approach as a (at least simple) possibility. In addition, supposing we are able to keep both λ_X and λ_Y in the spectrum, and if the term $\lambda_X\,\lambda_Y$ is gauge invariant, then all the terms

$$\lambda_X\,\lambda_X, \qquad \lambda_X\,\lambda_Y, \qquad \lambda_Y\,\lambda_Y$$

would be allowed as there is no symmetry that λ_X and λ_Y can be charged under that prevents or even suppresses the Majorana masses but allows Dirac masses. Consequently, if it were possible to construct a model with the above field content, the resulting gauginos would be pseudo-Dirac at best, with two Majorana states and a mass splitting due to the Dirac mass.

- Keep the gauge group G_X and introduce a spinor χSF Ψ_α in the **Ad** representation of G_X whose $\theta = 0$ component is a spin $\frac{1}{2}$ particle [193–195]

$$\Psi^a_\alpha(y,\theta) = \psi^a_\alpha + \left[\delta_\alpha{}^\beta\,\phi^a + (\sigma^\mu\,\bar{\sigma}^\nu)_\alpha{}^\beta\,F^a_{\mu\nu}\right]\,\theta_\beta + \theta^2\,\sigma^\mu_{\alpha\dot{\alpha}}\,\partial_\mu\,\psi^{a\dagger}. \tag{3.3.27}$$

Identifying ψ^a_α with the right–handed gaugino would be particularly appealing as it is very easy to write down a theory that would lead to the Dirac mass

$$\mathcal{L}_{\text{Dirac}} = \left(m_D \int d^2\theta\, \theta^2\, \Psi^{\alpha a}\, \mathcal{W}^a_\alpha \right) + \text{h.c.} = m_D\, \psi^{a\alpha}\, \lambda^a_\alpha + \text{h.c.}. \qquad (3.3.28)$$

Unfortunately we see this isn't going to work. The SUSY lagrangian for a spinor χSF is

$$\frac{1}{8} \int d^4\theta\, (\mathcal{D}^\alpha\, \Psi_\alpha + \text{h.c.})^2 + \left[\left(\frac{m^2}{2} \int d^2\theta\, \Psi^\alpha \Psi_\alpha \right) + \text{h.c.} \right] \qquad (3.3.29)$$

is invariant under the transformation

$$\Psi_\alpha \to \Psi_\alpha + \bar{\mathcal{D}}^2\, \mathcal{D}_\alpha\, \Lambda, \qquad \Lambda = \Lambda^\dagger. \qquad (3.3.30)$$

The form of the transformation $\bar{\mathcal{D}}^2\, \mathcal{D}_\alpha\, \Lambda$ is the same as the gauge field superstrength $\mathcal{W}_\alpha$ which in its $\theta = 0$ component contains a fermion — the left handed gaugino λ. In the unbroken SUSY limit, the would be right handed gaugino can just be gauged away; ψ^a_α in eq. 3.3.27 is an unphysical degree of freedom.

- Keep the gauge group G_X and introduce a scalar χSF Φ in the **Ad** representation of G_X with the expansion

$$\Phi^a(y, \theta) = \phi^a + \sqrt{2}\, \theta\, \psi^a + \theta^2\, F^a. \qquad (3.3.31)$$

Identifying ψ^a_α with the right–handed gaugino can work, with a Dirac mass

$$\mathcal{L}_{\text{Dirac}} = \left(\sqrt{2}\, m_D \int d^2\theta\, \theta^\alpha\, \Phi^a\, \mathcal{W}^a_\alpha \right) + \text{h.c.} = m_D\, \psi^{a\alpha}\, \lambda^a_\alpha + \text{h.c.} + \ldots. \qquad (3.3.32)$$

Thankfully this does work and will be the choice we pursue in the remainder of the thesis. This operator can be generated by D term SUSY breaking

$$\mathcal{L}^D_{\text{Dirac}} = \left(\sqrt{2} \int d^2\theta\, \frac{\mathcal{W}'^\alpha\, \Phi^a\, \mathcal{W}^a_\alpha}{M} \right) + \text{h.c.}, \qquad \langle \mathcal{W}'_\alpha \rangle = \langle D' \rangle\, \theta_\alpha, \qquad (3.3.33)$$

where $\mathcal{W}'_\alpha$ is just the gauge field superstrength of a hidden $\text{U}(1)'$ gauge theory and we see $m_D = \langle D' \rangle / M$. The operator 3.3.32 can also be generated with F term SUSY breaking

$$\mathcal{L}^{F}_{\text{Dirac}} = \left(\sqrt{2} \int d^4\theta \, \frac{\mathcal{D}^\alpha (X^\dagger X) \, \Phi^a \, \mathcal{W}^a_\alpha}{M^3} \right) + \text{h.c.}, \qquad \langle X \rangle = \langle F_X \rangle \, \theta^2 \quad (3.3.34)$$

where X is a χSF in the hidden sector. We see that $m_D = \langle F_X^3 \rangle / M^5$. In both cases, M is the mediation scale for SUSY breaking to the visible sector.

3.3.3 R symmetry

Having satisfied the first criteria: the existence of a right–handed gaugino, we need to engineer the theory so that the left and right handed components transform in conjugate complex representations of a continuous symmetry group G. At first glance this would seem impossible because the left–handed gaugino λ sits inside a VSF V that is by definition real 2.3.124. Because the reality definition is only at the level of superfields, we can get around this problem by having the continuous symmetry group G be one that doesn't commute with SUSY — the R symmetry, for which in $\mathcal{N} = 1$ as discussed in Section 2.3.5 is a $\text{U}(1)_R$ symmetry under which the gaugino has charge $R_\lambda = 1$.

To satisfy this requirement, the conjugate right–handed gaugino we add must transform with a charge $R_\psi = -1$ under the $\text{U}(1)_R$ symmetry, and therefore the χSF containing the conjugate right–handed gaugino must have a charge $R_\Phi = 0$. The Dirac mass in eq. 3.3.32 is then clearly invariant

$$m_D \, \psi \, \lambda \to m_D \, (e^{-i\,\theta} \, \psi) \, (e^{i\,\theta} \, \lambda) = m_D \, \psi \, \lambda \qquad (3.3.35)$$

whereas a Majorana gaugino mass M is forbidden

$$\frac{M}{2} \, \lambda \, \lambda \to \frac{M}{2} \, (e^{i\,\theta} \, \lambda) \, (e^{i\,\theta} \, \lambda) \neq \frac{M}{2} \, \lambda \, \lambda. \qquad (3.3.36)$$

3.3.4 Origins from extended supersymmetry

We have now decided that the only reasonable way of introducing additional right–handed gaugino in order to construct a Dirac gaugino is to introduce a χSF with $\text{U}(1)_R$ charge $R_\Phi = 0$. A reasonable question to then ask is '*Where does it come from?*'. One of the most natural realisations of Dirac gauginos occurs in $\mathcal{N} = 2$

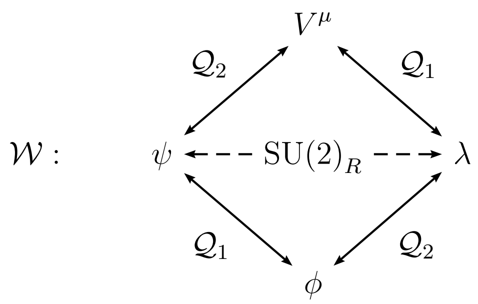


Figure 3.1: Diamond representation of an $\mathcal{N} = 2$ vector multiplet. Together, V^μ and λ form an $\mathcal{N} = 1$ VSF, and ψ and ϕ form an $\mathcal{N} = 1$ χSF. The basis is chosen such that the SUSY generators $\mathcal{Q}_1^\alpha$ are the $\mathcal{N} = 1$ generators that relate the different components of the $\mathcal{N} = 1$ superfields and $\mathcal{N} = 2$ is the orthogonal combination.

theories [134,135,159] and hybrid $\mathcal{N} = 2/\mathcal{N} = 1$ models [129,151]. In both of these cases, the gauge sector is extended from enjoying $\mathcal{N} = 1$ SUSY to $\mathcal{N} = 2$ SUSY. The $\mathcal{N} = 2$ vector multiplet has a decomposition in terms of an $\mathcal{N} = 1$ VSF and an $\mathcal{N} = 1$ χSF in the **Ad** representation. This is most easily seen diagrammatically in component superfield 'diamonds' [196] and is displayed in figure 3.1. Using a naïve $\mathcal{N} = 2$ superspace [52,197,198] parameterised in terms of θ, $\tilde{\theta}$ and their conjugates, we can write the $\mathcal{N} = 2$ vector multiplet Ψ as [199–201]

$$\Psi^a(y', \theta, \tilde{\theta}) = \Phi^a(y', \theta) + \tilde{\theta}^\alpha \, \mathcal{W}^a_\alpha(y', \theta) + (\text{auxiliary fields}) \tag{3.3.37}$$

where

$$y'^\mu = y^\mu + i\,\tilde{\theta}\,\sigma^\mu\,\bar{\tilde{\theta}} = x^\mu + i\,\theta\,\sigma^\mu\,\bar{\theta} + (\theta \leftrightarrow \tilde{\theta}) \tag{3.3.38}$$

and $\Phi(y, \theta)$, and $\mathcal{W}_\alpha(y, \theta)$ are $\mathcal{N} = 1$ χSFs. The $\mathcal{N} = 2$ SYM lagrangian is then schematically

$$\mathcal{L}^{\mathcal{N}=2}_{\text{SYM}} = \left(\int d^2\theta\, d^2\tilde{\theta}\, \frac{1}{2\,\hat{g}^2}\, \Psi^a\, \Psi^a \right) + \text{h.c.} \tag{3.3.39}$$

and a spurious definition of the gauge coupling [86]

$$\frac{1}{2\,g^2} \to \frac{1}{2\,\hat{g}^2} \left(1 + 2\sqrt{2}\,\theta\,\tilde{\theta}\,m_D \right) \tag{3.3.40}$$

introduces the Dirac mass m_D for the gauginos in eq. 3.3.32. Majorana masses as well as other soft terms can also be introduced with a spurious redefinition of the gauge coupling.

3.4 Supersoftness

Theories with Dirac gaugino masses are often referred to as *supersoft* theories. These theories are much less UV sensitive than theories with Majorana gauginos as we will now show. Consider the Dirac mass operator written in the holomorphic basis

$$\sqrt{2}\,\frac{\hat{m}_D}{\hat{g}}\int d^2\theta\,\theta^\alpha\,\hat{\mathcal{W}}_\alpha\,\hat{\Phi} \tag{3.4.41}$$

If this term is holomorphic, and therefore protected from renormalisation, then we must have

$$\beta_{\hat{m}_D} = \frac{\hat{m}_D}{\hat{g}}\,\beta_{\hat{g}}. \tag{3.4.42}$$

If we now switch to the canonical basis for gauge fields and chiral fields

$$\sqrt{2}\,\frac{\hat{m}_D}{g}\int d^2\theta\,\theta^\alpha\,g\,\mathcal{W}_\alpha\,\Phi\,Z_\Phi^{-1/2} \equiv \sqrt{2}\,m_D\int d^2\theta\,\theta^\alpha\,\mathcal{W}_\alpha\,\Phi \tag{3.4.43}$$

and so the physical Dirac mass is

$$m_D = Z_\Phi^{-1/2}\,\hat{m}_D \tag{3.4.44}$$

and along the RG flow [127, 129, 167]

$$\begin{aligned}\beta_{m_D} \equiv \frac{dm_D}{dt} &= \hat{m}_D\,\frac{\partial Z_\Phi^{-1/2}}{\partial t} + Z_\Phi^{-1/2}\,\beta_{\hat{m}_D}\\ &= \frac{Z_\Phi^{-1/2}\,\hat{m}_D}{2}\left(-\frac{1}{Z_\Phi}\frac{\partial Z_\Phi}{\partial t}\right) + Z_\Phi^{-1/2}\,\frac{\hat{m}_D}{\hat{g}}\,\beta_{\hat{g}}\\ &= m_D\left(\frac{\gamma_\Phi}{2} + \frac{\beta_g}{g}\right),\end{aligned} \tag{3.4.45}$$

where γ is the anomalous dimension of Φ [202]

$$\gamma_\Phi \equiv -\frac{\partial \log Z_\Phi}{\partial t} = -\frac{1}{Z_\Phi}\frac{\partial Z_\Phi}{\partial t}. \tag{3.4.46}$$

Note that this result differs by the one found in [127, 167]

$$\beta_{m_D} = m_D\left(\gamma_\Phi + \frac{\beta_g}{g}\right),$$

because our definitions of the anomalous dimension differ by a factor $\frac{1}{2}$ such that our quantum dimensions of chiral operators satisfy

$$\dim(\mathcal{O}) = 1 + \frac{1}{2}\gamma_{\mathcal{O}}, \qquad \gamma_{\mathcal{O}} \equiv -\frac{\partial \log Z_{\mathcal{O}}}{\partial t}. \tag{3.4.47}$$

The result 3.4.45 was checked explicitly to two loops using RGEs derived in [72,127] strongly indicating that the Dirac mass operator 3.4.41 is indeed holomorphic since it is only receives wave function renormalisations. This is not a proof however, and an argument using either supergraph techniques [62] or holomorphic arguments as in [63] would be needed in order to verify this at all orders in perturbation theory.

In any case, for phenomenological purposes, the observation in [72, 127] that eq. 3.4.45 is obeyed up to two loops is strong enough to have striking implications. Consider a theory with fields ϕ_i and Φ^a charged under a gauge group G supplemented with the supersoft operator for Φ^a, which in full is

$$\left(\sqrt{2}\, m_D \int d^2\theta\, \theta^a\, \Phi^a\, \mathcal{W}^a_\alpha\right) + \text{h.c.} = m_D \left(\psi^{a\alpha}\, \lambda^a_\alpha + \sqrt{2}\, \Phi^a\, D^a + \text{h.c.}\right) \tag{3.4.48}$$

Upon solving the *D term* equations we find that a further non–standard soft term is generated beyond the Dirac gaugino mass

$$c_i^{aj} \equiv c_{\phi^{\dagger i}}^{\Phi^a\, \phi_j} = \sqrt{2}\, g\, m_D\, \left[R^a(\phi)\right]_i{}^j \tag{3.4.49}$$

where $R^a(\phi)$ is the a^{th} generator of the gauge group G in the representation of the field ϕ. In a U(1) theory this is just the charge of the χSF under the U(1). Here the ϕ's are the scalar components of other χSFs charged under G and are coupled to the D term through the standard Kähler potential.

$$-\mathcal{L}_{\text{soft}}^{\text{non–standard}} = c_i^{aj}\, \phi^{\dagger i}\, \Phi^a\, \phi_j + m_D\, \psi^{a\alpha}\, \lambda^a_\alpha + \text{h.c.}. \tag{3.4.50}$$

In an anomaly free U(1) gauge theory, the sum of the c term coefficients is proportional to the charges of all χSFs in the theory

$$\sum_i c_i^{ai} = \sqrt{2}\, g\, m_D \sum_i Q(\phi_i) = 0. \tag{3.4.51}$$

The quadratic divergences due to the presence of a singlet and the c terms are proportional to this at one loop and therefore vanish [203]. The supersoft operator also creates a shift in the scalar mass squared for Φ and its b term There is an RG invariant relationship between soft terms with and without non–standard soft term of the form in eq. 3.4.50

$$m_\Phi^2 = m_{\Phi,\mathrm{s}}^2 + 2\, m_D^2, \qquad b_\Phi = b_{\Phi,\mathrm{s}} + 2\, m_D^2, \tag{3.4.52}$$

$$(m^2)_i{}^j = (m_\mathrm{s}^2)_i{}^j, \qquad b_\phi = b_{\phi,\mathrm{s}}, \tag{3.4.53}$$

where $m_{\Phi,\mathrm{s}}^2$, $b_{\Phi,\mathrm{s}}$, $(m_\mathrm{s}^2)_i{}^j$ and b_s^{ij} are the standard soft terms that solve the RGEs of [204] in the limit $\mathcal{L}_\mathrm{soft}^\text{non–standard} \to 0$

$$-\mathcal{L}_\mathrm{soft}^\mathrm{standard} = m_{\Phi,\mathrm{s}}^2\, |\Phi|^2 + (m_\mathrm{s}^2)^j{}_i\, \phi^{\dagger i}\, \phi_j + \frac{1}{2}\left(b_{\Phi,\mathrm{s}}\, \Phi^2 + b_\mathrm{s}^{ij}\, \phi_i\, \phi_j + \mathrm{h.c.}\right). \tag{3.4.54}$$

Again, it is important to note that the relationships 3.4.52 and 3.4.53 have only been verified to two loops although it is *anticipated* that if the *supersoft* behaviour is due to the holomorphy of the operator 3.4.41 then the relationships will hold to all orders.

The RG invariant relationships 3.4.52 and 3.4.53 are precisely supersoftness. Together they mean that the RGEs of a theory with Dirac gaugino masses induced by the supersoft operator 3.4.41 can be evolved *ignoring* the Dirac mass[7] and then we can perform the shifts 3.4.52 and 3.4.53 at any renormalisation scale to get to the theory with a Dirac gaugino mass. The Dirac gaugino mass must also be evolved to the scale in question but as already shown, this only receives wave function renormalisation 3.4.45. The scalar sector sensitivity to a Dirac gaugino mass is therefore only through the shifts 3.4.52 and 3.4.53 and any finite corrections. This is why theories with Dirac gauginos can have very large gaugino masses without worrying (as much) about inducing a large amount of fine tuning.

To get a feeling for why this happens perturbatively, consider the MSSM with a Dirac gluino (that will be the subject of our discussion in Chapter 4). For this, it is convenient to decomposing the scalar adjoint $\Phi_{\tilde{\mathrm{g}}}$ into a scalar $\phi_{\tilde{\mathrm{g}}}$ and pseudo–scalar

[7] Of course we can't ignore the presence of the additional field content which, to one loop affects the running of gauge couplings and to two loops contributes in many places through its correction to the gauge boson propagator.

$\sigma_{\widetilde{g}}$

$$\Phi_{\widetilde{g}} = \frac{1}{\sqrt{2}} \left(\phi_{\widetilde{g}} + i\, \sigma_{\widetilde{g}} \right) \tag{3.4.55}$$

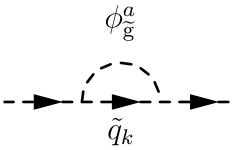
then the supersoft operator induces the scalar and pseudo–scalar masses

$$m^2_{\phi_{\widetilde{g}}} = 4\, m^2_{D\widetilde{g}}, \qquad m^2_{\sigma_{\widetilde{g}}} = 0. \tag{3.4.56}$$

and only $\phi_{\widetilde{g}}$ couples to squarks through the c term

$$c_i^{aj}\, \phi^{\dagger i}\, \Phi^a\, \phi_j + \text{h.c.} = \sqrt{2}\, c_i^{aj}\, \phi^{\dagger i}\, \phi^a_{\widetilde{g}}\, \phi_j \tag{3.4.57}$$

The c term allows the scalar diagram

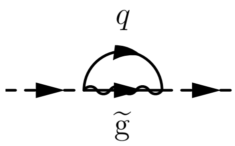


$$\begin{aligned} &= (\sqrt{2}\, c_i^{ak})(\sqrt{2}\, c_k^{aj}) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2\,(k^2 - m^2_{\phi_{\widetilde{g}}})} \\ &= 4\, i\, g_3^2\, m^2_{D\widetilde{g}}\, [C_2^{\tilde{q}}]_i^j \int_0^1 dx \int \frac{d^4k_E}{(2\pi)^4} \frac{1}{(k_E^2 + x\, m^2_{\phi_{\widetilde{g}}})^2} \\ &= \frac{4\, i\, g_3^2\, m^2_{D\widetilde{g}}\, [C_2^{\tilde{q}}]_i^j}{16\, \pi^2} \int_0^1 dx \left[\frac{2}{\varepsilon} - \gamma - \log\left(\frac{x\, m^2_{\phi_{\widetilde{g}}}}{4\, \pi\, \mu^2} \right) + \mathcal{O}(\varepsilon) \right] \\ &= \frac{4\, i\, g_3^2\, m^2_{D\widetilde{g}}\, [C_2^{\tilde{q}}]_i^j}{16\, \pi^2} \left[\frac{2}{\varepsilon} - \gamma + 1 + \log(4\pi) - \log\left(\frac{m^2_{\phi_{\widetilde{g}}}}{\mu^2} \right) + \mathcal{O}(\varepsilon) \right], \end{aligned}$$

where we have used the shorthand $[C_2^{\tilde{q}}]_i^j \equiv C_2(\mathbf{r}_{\tilde{q}})\, \delta_i^j$ with $C_2(\mathbf{r})$ the quadratic casimir in the representation $\mathbf{r}$ under the gauge group G

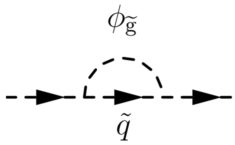
$$(T^a_{\mathbf{r}})^i_k\, (T^a_{\mathbf{r}})^k_j \equiv C_2(\mathbf{r})\, \delta^i_j \equiv [C_2^{\mathbf{r}}]^i_j \tag{3.4.58}$$

and μ is the renormalisation scale. The diagram involving the Dirac gaugino is

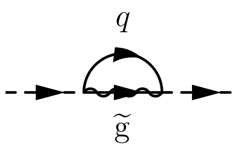


$$
\begin{aligned}
&= -4\, g_3^2\, [C_2^{\tilde{q}}]_i^j \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{k^2\,(k^2 - m_{D\tilde{g}}^2)} \\
&= 4\, i\, g_3^2\, [C_2^{\tilde{q}}]_i^j \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2 + m_{D\tilde{g}}^2} \\
&= 4\, i\, g_3^2\, [C_2^{\tilde{q}}]_i^j \lim_{\varepsilon \to 0} \left[\frac{1}{(4\pi)^{2-\varepsilon/2}} \Gamma(-1+\varepsilon/2) \left(m_{D\tilde{g}} \right)^{2-\varepsilon} \right] \\
&= \frac{4 i g_3^2 m_{D\tilde{g}}^2\, [C_2^{\tilde{q}}]_i^j}{16\,\pi^2} \left[-\frac{2}{\varepsilon} + \gamma - 1 - \log(4\pi) + \log\left(\frac{m_{D\tilde{g}}^2}{\mu^2} \right) + \mathcal{O}(\varepsilon) \right].
\end{aligned}
$$

Summing the real scalar and the Dirac gluino contribution at a common renormalisation scale



$+$



$$
= \frac{i\, g_3^2\, m_{D\tilde{g}}^2\, [C_2^{\tilde{q}}]_i^j}{4\,\pi^2} \log\left(\frac{m_{D\tilde{g}}^2}{m_{\phi_{\tilde{g}}}^2} \right) \tag{3.4.59}
$$

in accord with the expression found in [129]. The supersoft effect can be seen here where the terms in $1/\varepsilon$ cancelled between the diagrams involving the real scalar gluon (or sgluon) and the Dirac gluino — the $1/\varepsilon$ terms in a theory regularised using dimensional regularisation signify a UV divergence. Because they cancel here at one loop, no terms that depend on the $m_{D\tilde{g}}$ will enter the RGEs of the squark mass squared at one loop because there is no UV divergence that depends on $m_{D\tilde{g}}$. This is an explicit example of ther supersoft effect that we saw in the RG invariant relations 3.4.52 and 3.4.53 but now we understand physically what is happening. The usual UV sensitivity induced by a gaugino mass is accompanied by a UV sensitivity of opposite sign and magnitude through the corresponding scalar degrees of freedom.

3.5 Supersafeness

Theories with Dirac gaugino masses (and specifically Dirac gluino masses) have been referred to as *supersafe* [7, 169, 178]. In a proton–proton collider such as the LHC, strong interactions dominate hard processes including the production of SUSY particles. In the limit of the gluino becoming Dirac, many of the dominant LO diagrams for squark squark production vanish, as we will now show.

In the basis $(\widetilde{g}, \psi_{\widetilde{g}})$ the most general gluino mass matrix $M_{\widetilde{g}}$ is

$$M_{\widetilde{g}} = \begin{pmatrix} M_3 & m_{D\widetilde{g}} \\ m_{D\widetilde{g}} & M_{\psi_{\widetilde{g}}} \end{pmatrix} \tag{3.5.60}$$

where M_3 is a Majorana gluino mass, $m_{D\widetilde{g}}$ is a Dirac gluino mass, and $M_{\psi_{\widetilde{g}}}$ is a Majorana mass for $\psi_{\widetilde{g}}$. $\widetilde{g}$ and $\psi_{\widetilde{g}}$ mix to form mass eigenstates $\widetilde{g}_1$ and $\widetilde{g}_2$

$$\begin{pmatrix} \widetilde{g}_2 \\ \widetilde{g}_1 \end{pmatrix} = \begin{pmatrix} c_{\theta_{\widetilde{g}}} & s_{\theta_{\widetilde{g}}} \\ -s_{\theta_{\widetilde{g}}} & c_{\theta_{\widetilde{g}}} \end{pmatrix} \begin{pmatrix} \widetilde{g} \\ \psi_{\widetilde{g}} \end{pmatrix}, \tag{3.5.61}$$

where

$$4\, c^2_{\theta_{\widetilde{g}}} = 1 + \frac{\Delta_M}{\sqrt{\Delta_M^2 + 4\, m^2_{D\widetilde{g}}}}, \qquad \Delta_M \equiv M_3 - M_{\psi_{\widetilde{g}}} \tag{3.5.62}$$

and the physical eigenstates $\widetilde{g}_1$ and $\widetilde{g}_2$ have masses

$$m_{\widetilde{g}_1, \widetilde{g}_2} = \frac{1}{2} \left(M_3 + M_{\psi_{\widetilde{g}}} \mp \sqrt{\Delta_M^2 + 4\, m^2_{D\widetilde{g}}} \right). \tag{3.5.63}$$

In the pure Dirac gluino case $M_3 = M_{\psi_{\widetilde{g}}} = 0$, $m_{D\widetilde{g}} \neq 0$

$$c_{\theta_{\widetilde{g}}} = s_{\theta_{\widetilde{g}}} = \frac{1}{\sqrt{2}}, \tag{3.5.64}$$

$$m_{\widetilde{g}_1, \widetilde{g}_2} = \mp\, m_{D\widetilde{g}}, \tag{3.5.65}$$

$$\widetilde{g}_1, \widetilde{g}_2 = \frac{1}{\sqrt{2}} \left(\psi_{\widetilde{g}} \mp \widetilde{g} \right) \tag{3.5.66}$$

and in the pure Majorana gluino case $M_{\psi_{\widetilde{g}}} = m_{D\widetilde{g}} = 0$, $M_3 \neq 0$

$$c_{\theta_{\widetilde{g}}} = 1, \qquad s_{\theta_{\widetilde{g}}} = 0, \tag{3.5.67}$$

$$m_{\widetilde{g}_1} = 0, \qquad m_{\widetilde{g}_2} = M_3, \tag{3.5.68}$$

$$\widetilde{g}_1 = \psi_{\widetilde{g}}, \qquad \widetilde{g}_2 = \widetilde{g}. \tag{3.5.69}$$

The kinetic term in the MSSM only couples the gluino $\widetilde{g}$ but not $\psi_{\widetilde{g}}$ to the strongly interacting χSFs through the Kähler term

$$\int d^4\theta\, \Phi^{\dagger i}\, (e^V)_i{}^j\, \Phi_j \supset -\sqrt{2}\, g_3\, (\phi^\dagger\, T^a\, \phi)\, \widetilde{\mathrm{g}}^a + \text{h.c.} \tag{3.5.70}$$

$$= -\sqrt{2}\, g_3 \Big(\widetilde{\mathrm{u}}_L^\dagger\, T^a\, \mathrm{u}_L + \widetilde{\mathrm{d}}_L^\dagger\, T^a\, \mathrm{d}_L - \widetilde{\mathrm{u}}_R\, T^a\, \overline{\mathrm{u}} - \widetilde{\mathrm{d}}\, T^a\, \overline{\mathrm{d}} \Big)\, \widetilde{\mathrm{g}}^a + \text{h.c.}, \tag{3.5.71}$$

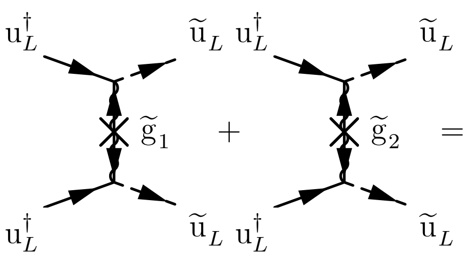
where there is an implicit sum over (s)quark generations. We can write the MSSM gluino in terms of the mass eigenstates

$$\widetilde{\mathrm{g}}^a = -s_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{g}}_1^a + c_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{g}}_2^a \tag{3.5.72}$$

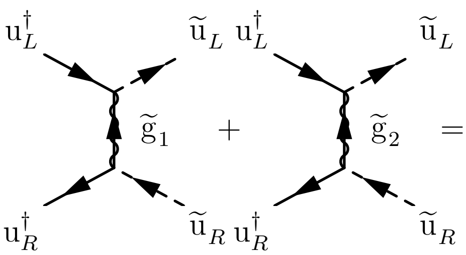
then one finds the quark–squark–gluino interactions between the gluino mass eigenstates with mixing

$$\begin{aligned} \frac{\mathcal{L}_{\widetilde{q}\, q\, \widetilde{\mathrm{g}}}}{\sqrt{2}\, g_3} =\; & s_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{u}}_L^\dagger\, T^a\, \mathrm{u}_L\, \widetilde{\mathrm{g}}_1^a - c_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{u}}_L^\dagger\, T^a\, \mathrm{u}_L\, \widetilde{\mathrm{g}}_2^a + s_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{d}}_L^\dagger\, T^a\, \mathrm{d}_L\, \widetilde{\mathrm{g}}_1^a - c_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{d}}_L^\dagger\, T^a\, \mathrm{d}_L\, \widetilde{\mathrm{g}}_2^a \\ & - s_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{u}}_R\, T^a\, \overline{\mathrm{u}}\, \widetilde{\mathrm{g}}_1^a + c_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{u}}_R\, T^a\, \overline{\mathrm{u}}\, \widetilde{\mathrm{g}}_2^a - s_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{d}}_R\, T^a\, \overline{\mathrm{d}}\, \widetilde{\mathrm{g}}_1^a + c_{\theta_{\widetilde{\mathrm{g}}}}\, \widetilde{\mathrm{d}}_R\, T^a\, \overline{\mathrm{d}}\, \widetilde{\mathrm{g}}_2^a \\ & + \text{h.c.}. \end{aligned} \tag{3.5.73}$$

If we consider then the possible diagrams for squark–squark production with an intermediate gluino we have

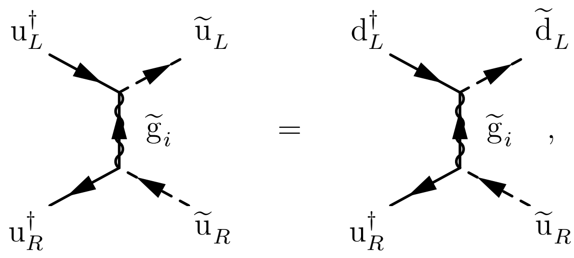


$$= g_3^2\, C_2\, \mathrm{u}_{L\dot{\alpha}}^\dagger \left(\frac{c_{\theta^2_{\widetilde{\mathrm{g}}}}\, i\, m_{\widetilde{\mathrm{g}}_1}}{p^2 - m^2_{\widetilde{\mathrm{g}}_1}} + \frac{s_{\theta^2_{\widetilde{\mathrm{g}}}}\, i\, m_{\widetilde{\mathrm{g}}_2}}{p^2 - m^2_{\widetilde{\mathrm{g}}_2}} \right) \mathrm{u}_L^{\dot{\alpha}}, \tag{3.5.74}$$



$$= g_3^2\, C_2\, \mathrm{u}_R^{\dagger\alpha} \left(\frac{c_{\theta^2_{\widetilde{\mathrm{g}}}}\, i\, \sigma^\mu_{\alpha\dot{\alpha}} p_\mu}{p^2 - m^2_{\widetilde{\mathrm{g}}_1}} + \frac{s_{\theta^2_{\widetilde{\mathrm{g}}}}\, i\, \sigma^\mu_{\alpha\dot{\alpha}} p_\mu}{p^2 - m^2_{\widetilde{\mathrm{g}}_2}} \right) \mathrm{u}_L^{\dagger\dot{\alpha}}, \tag{3.5.75}$$

where at each vertex, (s)quark flavour is presered. The *kernels* of the transition amplitudes remain invariant under switching in–out pairs of up–type with down–type, i.e.



$$, \quad i = 1, 2 \tag{3.5.76}$$

ignoring the external fermionic states. Now we can see how Dirac gauginos are *supersafe*. In the pure Dirac limit, the *kernel* for $\widetilde{u}_L \widetilde{u}_L$, $\widetilde{u}_L \widetilde{u}_R^\dagger$ and the remaining combinations with any of the $u \to d$ vanishes

$$\frac{c_{\theta^2_{\widetilde{g}}}\, i\, m_{\widetilde{g}_1}}{p^2 - m^2_{\widetilde{g}_1}} + \frac{s_{\theta^2_{\widetilde{g}}}\, i\, m_{\widetilde{g}_2}}{p^2 - m^2_{\widetilde{g}_2}} = \frac{(1/\sqrt{2})^2\, i\, (-m_{D\widetilde{g}})}{p^2 - (-m_{D\widetilde{g}})^2} + \frac{(1/\sqrt{2})^2\, i\, m_{D\widetilde{g}}}{p^2 - m^2_{D\widetilde{g}}} = 0; \tag{3.5.77}$$

the contributions from $\widetilde{g}_1$ are cancelled by contributions from $\widetilde{g}_2$. On the other hand, the *kernel* for $\widetilde{u}_L \widetilde{u}_L^\dagger$, $\widetilde{u}_L \widetilde{u}_R$ and the remaining combinations with any of the $u \to d$ are essentially unaffected by the gluino's Dirac or Majorana nature. Another way of understanding this is to consider the theory with a genuinely Dirac particle (rather than a Dirac particle that is in disguise as two Majorana particles). The chirality flipping propagator in the case of a Dirac particle given in eq. 3.2.22 exchanges the left hand degree of freedom for the conjugated right handed degree of freedom. With a Dirac gluino, this would swap $\widetilde{g}$ and $\psi_{\widetilde{g}}$. Since $\psi_{\widetilde{g}}$ does interact with (s)quarks then the diagram vanishes. This is the same effect as the absence of neutrino–less double beta decay in the limit of a Dirac neutrino [188, 189].

A final way of understanding this is to consider the MSSM superpotential in eq. 2.3.326. There is an unbroken $U(1)_R$ symmetry (if we treat the μ term as a spurion of the $U(1)_R$ symmetry) under which the fields have $U(1)_R$ charge assignments given in table 3.1. In the limit of a pure Dirac mass, there is a full $U(1)_R$ symmetry which forbids the effective operators corresponding to $\widetilde{u}_L \widetilde{u}_L$, $\widetilde{u}_L \widetilde{u}_R^\dagger$ and the remaining combinations with any of the $u \to d$

$$\frac{u_L^\dagger\, u_L^\dagger\, \widetilde{u}_L\, \widetilde{u}_L}{\Lambda} \to \frac{u_L^\dagger\, u_L^\dagger\, (e^{i\theta}\, \widetilde{u}_L)\, (e^{i\theta}\, \widetilde{u}_L)}{\Lambda} \neq \frac{u_L^\dagger\, u_L^\dagger\, \widetilde{u}_L\, \widetilde{u}_L}{\Lambda} \tag{3.5.78}$$

	$\mathrm{U}(1)_R$
$\mathrm{H_u}$, $\mathrm{H_d}$, $\Phi_{\widetilde{\mathrm{g}}}$, $\Phi_{\widetilde{\mathrm{W}}}$, $\Phi_{\widetilde{\mathrm{B}}}$	0
q, ℓ, $\overline{\mathrm{u}}$, $\overline{\mathrm{d}}$, $\overline{\mathrm{e}}$	1
R_u, R_d	2

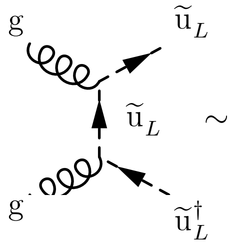
Table 3.1: MRSSM $\mathrm{U}(1)_R$ charge assignments.

but allows the remaining effective operators

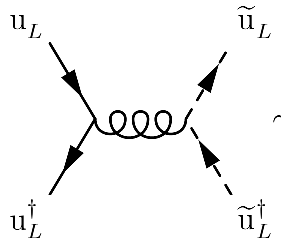
$$\frac{\mathrm{u}_L^\dagger\,\mathrm{u}_R^\dagger\,\widetilde{\mathrm{u}}_L\,\widetilde{\mathrm{u}}_R}{\Lambda} \to \frac{\mathrm{u}_L^\dagger\,\mathrm{u}_R^\dagger\,(e^{i\theta}\,\widetilde{\mathrm{u}}_L)\,(e^{-i\theta}\,\widetilde{\mathrm{u}}_R)}{\Lambda} = \frac{\mathrm{u}_L^\dagger\,\mathrm{u}_R^\dagger\,\widetilde{\mathrm{u}}_L\,\widetilde{\mathrm{u}}_R}{\Lambda}, \tag{3.5.79}$$

where Λ is the high scale corresponding to integrating out the gluino(s).

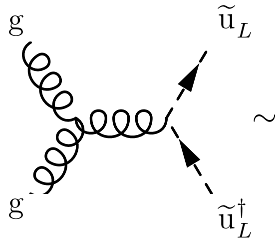
The remaining tree–level diagrams contributing to the production of squarks via strong interactions are



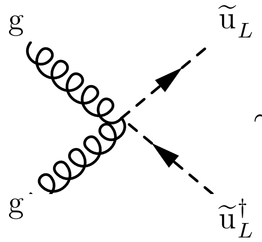
$\sim\ g_3^2\,(T^a\,T^b)_i^{\ j}\,\dfrac{p_1\,p_2}{p^2 - m^2_{\widetilde{\mathrm{u}}_L}}$,



$\sim\ g_3^2\,C_2\,\dfrac{\sigma\cdot p}{p^2}$, (3.5.80)



$\sim\ g_3^2\,f^{abc}\,(T^c)_i^{\ j}\,\dfrac{p_1\,p_2}{p^2}$,



$\sim\ g^2\,(T^a\,T^b)_i^{\ j}$. (3.5.81)

Together with the contributions in eq. 3.5.75, they form the strong LO production mechanisms for squarks at the LHC. For a reasonable squark mass $m_{\widetilde{q}} \gtrsim 1.5\,\mathrm{TeV}$ in the pure Majorana case these contributions are sub–dominant to those involving the t channel gluino contribution to squark–squark production (see fig. 3.2). The extent of the suppression of production cross–section is well characterised by the three simplified modesl [105] of [7, 169] detailed in table 3.2. Although a Dirac gluino

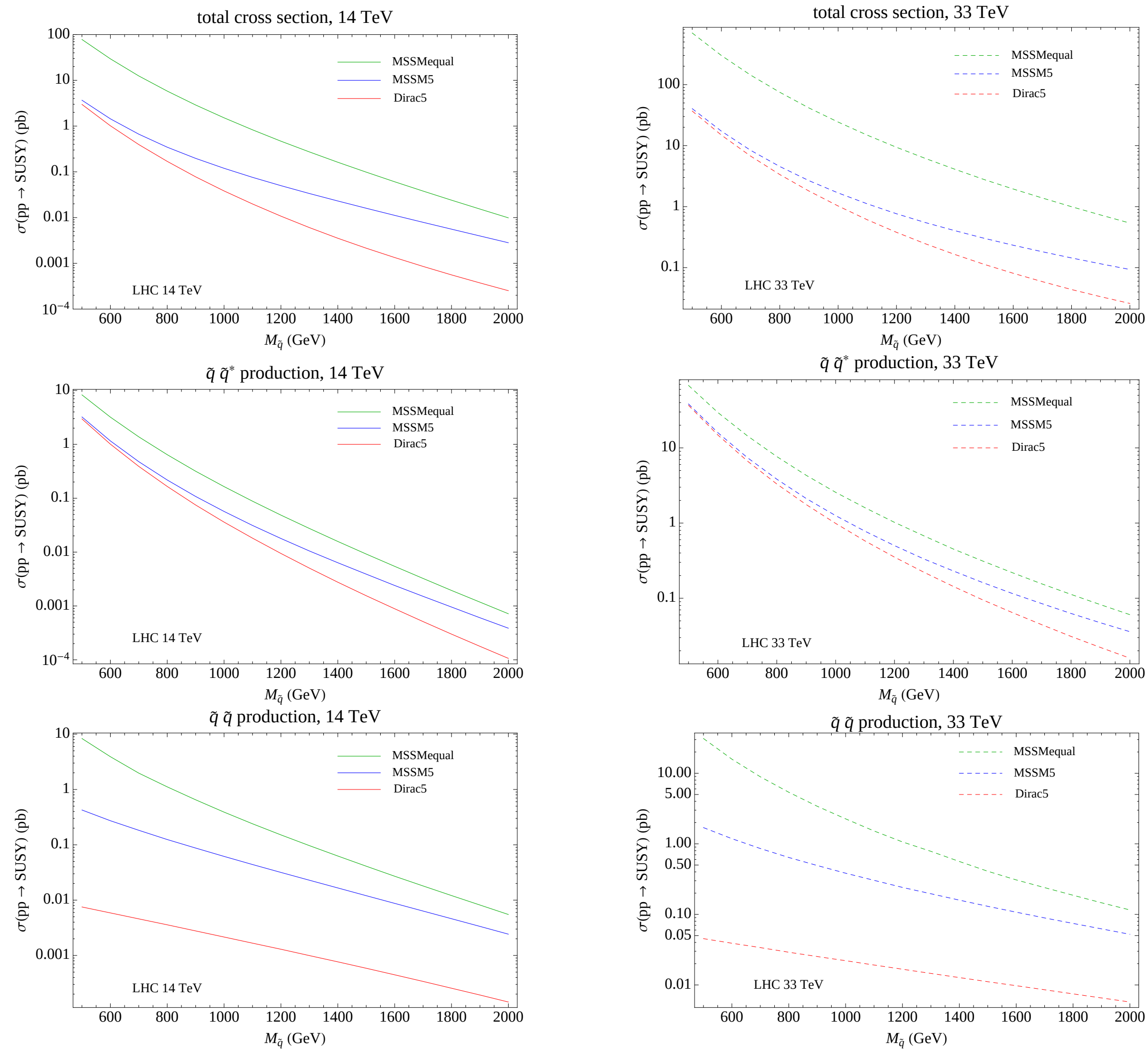


Figure 3.2: LO production cross sections for the simplified models detailed in table 3.2 for LHC at $\sqrt{s} = 14$ and 33 TeV. Squark production refers to the sum of allowed combinations of the first two generations. Total cross section is the sum of the squark production, gluino pair production and gluino–squark production. Taken from [7].

can reasonably be significantly heavier than its Majorana counterpart due to the supersoftness discussed in Section 3.4, the focus of these studies was to demonstrate how Majorana and Dirac gluinos of similar mass have sparticle different production cross–sections. The results of their study for the LHC at 14 and 33 TeV are shown in figure 3.2. There are two things to note:

- The squark–squark production is suppressed much more than squark–anti–squark production in the Dirac5 simplified model as we expected. At both 14 and 33 TeV this suppression is roughly two orders of magnitude. The squark–squark production is not exactly zero however. The reason for this is

	Dirac5	MSSM5	MSSMequal
$M_{\tilde{g}}$	5	5	$= M_{\tilde{q}}$
$M_{\tilde{q}}$	varies	varies	$= M_{\tilde{g}}$
Light squarks	First two gens.	First two gens.	First two gens.
BR($\tilde{q} \to q + \text{LSP}$)	100%	100%	100%
LSP mass	0	0	0

Table 3.2: The simplified models considered in [7]. All masses are in TeV. All other sparticles are decoupled.

only notational — the papers [7, 169] are including processes $\mathrm{p\,p} \to \widetilde{\mathrm{u}}_L\, \widetilde{\mathrm{u}}_R$ in the squark–squark production processes, whereas we are considering it to be a part of squark–anti–squark production (since $\widetilde{\mathrm{u}}_R$ is the scalar component of an anti χSF).

- The cross sections for Dirac5 at the LHC are too small to be seen at the planned integrated luminosity, but at 33 TeV it is reasonable that squarks and gluinos in Dirac5 can be studied.

Interestingly, if one considers pseudo-Dirac particles, the production cross section for coloured sparticles at the LHC is even lower than in the Dirac case [178]. Although reintroducing a Majorana gluino mass has been introduced to allow the chirality flipping squark–squark production that was forbidden in the pure Dirac case, the strong squark–anti–squark production drops at a faster rate than squark–squark production increases. The squark–anti–squark production drops because the gluino that couples the most to the squarks is the one that is mostly the gluino from the MSSM. Upon diagonalising, this has a mass that is increased by the Majorana mass, whereas the eigenstate that couples less to the squarks is the lighter one.

3.6 The MSSM with Dirac gauginos

3.6.1 General superpotential and soft terms

We have now discussed what the minimal model building requirements for introducing Dirac gauginos in a SUSY setup are, and some of their immediate consequences: *supersoftness* and *supersafeness*. Now let us consider extending the MSSM of Section 2.3.9 with a Dirac gaugino for each gauge group. We need to add a χSF in

	Bosons	Fermions	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\Phi_{\widetilde{g}}$	$\Phi_{\widetilde{g}}$	$\psi_{\widetilde{g}}$	**Ad**	**1**	0
$\Phi_{\widetilde{W}}$	$\frac{1}{2}\begin{pmatrix} \Phi^0_{\widetilde{W}} & \sqrt{2}\,\Phi^+_{\widetilde{W}} \\ \sqrt{2}\,\Phi^-_{\widetilde{W}} & -\Phi^0_{\widetilde{W}} \end{pmatrix}$	$\frac{1}{2}\begin{pmatrix} \psi^0_{\widetilde{W}} & \sqrt{2}\,\psi^+_{\widetilde{W}} \\ \sqrt{2}\,\psi^-_{\widetilde{W}} & -\psi^0_{\widetilde{W}} \end{pmatrix}$	**1**	**Ad**	0
$\Phi_{\widetilde{B}}$	$\Phi_{\widetilde{B}}$	$\psi_{\widetilde{B}}$	**1**	**1**	0

Table 3.3: Additional χSF field content required for introducing a Dirac gaugino for each SM gauge group. The generation number for each of the Φ's is 1.

the **Ad** representation of each of the gauge groups $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$[8] that is also a singlet under the other two gauge groups. This field content is detailed in table 3.3, and supplements the MSSM field content in table 2.10. $\Phi_{\widetilde{W}}$ is defined such that

$$\Phi_{\widetilde{W}} \equiv T^i\,\Phi^i_{\widetilde{W}} = \frac{1}{2}\begin{pmatrix} \Phi^0_{\widetilde{W}} & \sqrt{2}\,\Phi^+_{\widetilde{W}} \\ \sqrt{2}\,\Phi^-_{\widetilde{W}} & -\Phi^0_{\widetilde{W}} \end{pmatrix}, \quad \Phi^i_{\widetilde{W}} = 2\,\mathrm{tr}(T^i\,\Phi_{\widetilde{W}}), \tag{3.6.82}$$

so that

$$\Phi^\pm_{\widetilde{W}} = \frac{\Phi^1_{\widetilde{W}} \mp i\,\Phi^2_{\widetilde{W}}}{\sqrt{2}}, \qquad \Phi^0_{\widetilde{W}} = \Phi^3_{\widetilde{W}}. \tag{3.6.83}$$

The gauge invariant superpotential that we can write for this theory is conveniently divided into a part that preserves the $U(1)_R$ symmetry in table 3.1 — the Yukawa terms $W_{\rm yuk}$ and a part which breaks the $U(1)_R$ symmetry $W_{\not{R}}$

$$W = W_{\rm yuk} + W_{\not{R}} \tag{3.6.84}$$

where

$$W_{\rm yuk} = y_{\rm u}\,{\rm H_u}\cdot q\,\overline{\rm u} - y_{\rm d}\,{\rm H_d}\cdot q\,\overline{\rm d} - y_{\rm e}\,{\rm H_d}\cdot \ell\,\overline{\rm e}, \tag{3.6.85}$$

$$\begin{aligned} W_{\not{R}} = {} & \mu\,{\rm H_u}\cdot{\rm H_d} + \lambda_{\widetilde{B}}\,\Phi_{\widetilde{B}}\,{\rm H_u}\cdot{\rm H_d} + 2\,\lambda_{\widetilde{W}}\,{\rm H_d}\,\Phi_{\widetilde{W}}\cdot{\rm H_u} \\ & + L_{\widetilde{B}}\,\Phi_{\widetilde{B}} + \frac{M_{\widetilde{B}}}{2}\,\Phi^2_{\widetilde{B}} + \frac{\kappa_{\widetilde{B}}}{3}\,\Phi^3_{\widetilde{B}} + M_{\widetilde{W}}\,\Phi^2_{\widetilde{W}} + M_{\widetilde{g}}\,\Phi^2_{\widetilde{g}} \\ & + \lambda_{\widetilde{B}\widetilde{W}}\,\Phi_{\widetilde{B}}\,\Phi^2_{\widetilde{W}} + \lambda_{\widetilde{B}\widetilde{g}}\,\Phi_{\widetilde{B}}\,\Phi^2_{\widetilde{g}} + \frac{\kappa_{\widetilde{g}}}{3}\,\Phi^3_{\widetilde{g}}. \end{aligned} \tag{3.6.86}$$

[8] The adjoint of a U(1) gauge group just means that the field is a singlet of that gauge group.

In addition to the superpotential 3.6.84 there are a staggering number of soft terms [148]

$$\mathcal{L}_{\rm soft} = \mathcal{L}_{\rm soft}^{\rm standard} + \mathcal{L}_{\rm soft}^{\rm non-standard} + \mathcal{L}_{\rm soft}^{\rm tadpoles} \tag{3.6.87}$$

where

$$\mathcal{L}_{\rm soft}^{\rm standard} = \mathcal{L}_{\rm soft}^{\rm MSSM} + \mathcal{L}_{\rm soft}^{\rm adjoint\ quadtratic} + \mathcal{L}_{\rm soft}^{\rm adjoint\ a\ terms}, \tag{3.6.88}$$

$$\mathcal{L}_{\rm soft}^{\rm non\text{-}standard} = \mathcal{L}_{\rm soft}^{\rm Dirac\ gaugino} + \mathcal{L}_{\rm soft}^{\rm c\ terms} \tag{3.6.89}$$

and finally

$$\begin{aligned}
-\mathcal{L}_{\rm soft}^{\rm MSSM} &= \frac{1}{2}\Big(M_3\,\widetilde{\rm g}^a\,\widetilde{\rm g}^a + M_2\,\widetilde{\rm W}^i\,\widetilde{\rm W}^i + M_1\,\widetilde{\rm B}\,\widetilde{\rm B} + \text{h.c.}\Big) \\
&+ \big(a_{\rm u}\,\overline{\rm u}\,q\cdot{\rm H}_u - a_{\rm d}\,\overline{\rm d}\,q\cdot{\rm H}_d - a_{\rm e}\,\overline{\rm e}\,\ell\cdot{\rm H}_d + \text{h.c.}\big) \\
&+ m_q^2\,|q|^2 + m_{\rm u}^2\,|\overline{\rm u}|^2 + m_{\rm d}^2\,|\overline{\rm d}|^2 + m_\ell^2\,|\ell|^2 + m_{\rm e}^2\,|\overline{\rm e}|^2 \\
&+ m_{{\rm H}_{\rm u}}^2\,|{\rm H}_{\rm u}|^2 + m_{{\rm H}_d}^2\,|{\rm H}_d|^2 + (b_\mu\,{\rm H}_u\cdot{\rm H}_d + \text{h.c.}),
\end{aligned} \tag{3.6.90}$$

$$\begin{aligned}
-\mathcal{L}_{\rm soft}^{\rm adjoint\ quadratic} &= m_{\widetilde{\rm B}}^2\,|\Phi_{\widetilde{\rm B}}|^2 + \frac{1}{2}\,b_{\widetilde{\rm B}}\,(\Phi_{\widetilde{\rm B}}^2 + \text{h.c.}) + 2\,m_{\widetilde{\rm W}}^2\,|\Phi_{\widetilde{\rm W}}^2| \\
&+ b_{\widetilde{\rm W}}\,(\Phi_{\widetilde{\rm W}}^2 + \text{h.c.}) + 2\,m_{\widetilde{\rm g}}^2\,|\Phi_{\widetilde{\rm g}}|^2 + b_{\widetilde{\rm g}}\,(\Phi_{\widetilde{\rm g}}^2 + \text{h.c.}),
\end{aligned} \tag{3.6.91}$$

$$\begin{aligned}
-\mathcal{L}_{\rm soft}^{\rm adjoint\ a\ terms} &= a_{\widetilde{\rm B}}\,\Phi_{\widetilde{\rm B}}\,{\rm H}_{\rm d}\cdot{\rm H}_{\rm u} + 2\,a_{\widetilde{\rm W}}\,{\rm H}_{\rm d}\cdot\Phi_{\widetilde{\rm W}}\,{\rm H}_{\rm u} + \frac{1}{3}\,a_{\kappa_{\widetilde{\rm B}}}\,\Phi_{\widetilde{\rm B}}^3 \\
&+ a_{\widetilde{\rm B}\widetilde{\rm W}}\,\Phi_{\widetilde{\rm B}}\,\Phi_{\widetilde{\rm W}}^2 + a_{\widetilde{\rm B}\widetilde{\rm g}}\,\Phi_{\widetilde{\rm B}}\,\Phi_{\widetilde{\rm g}}^2 + \frac{1}{3}\,a_{\kappa_{\widetilde{\rm g}}}\,\Phi_{\widetilde{\rm g}}^3 + \text{h.c.},
\end{aligned} \tag{3.6.92}$$

$$-\mathcal{L}_{\rm soft}^{\rm Dirac\ gaugino} = m_{D\widetilde{\rm g}}\,\widetilde{\rm g}^a\,\psi_{\widetilde{\rm g}}^a + m_{D\widetilde{\rm W}}\,\widetilde{\rm W}^i\,\psi_{\widetilde{\rm W}}^i + m_{D\widetilde{\rm B}}\,\widetilde{\rm B}\,\psi_{\widetilde{\rm B}} + \text{h.c.}, \tag{3.6.93}$$

$$\begin{aligned}
-\mathcal{L}_{\rm soft}^{\rm c\ terms} &= \Phi_{\widetilde{\rm g}}^a\Big[q^\dagger\,(c_{\widetilde{\rm g}q}^a)\,q + \overline{\rm u}^\dagger\,(c_{\widetilde{\rm g}{\rm u}}^a)\,\overline{\rm u} + \overline{\rm d}^\dagger\,(c_{\widetilde{\rm g}{\rm d}}^a)\,\overline{\rm d}\Big] \\
&+ \Phi_{\widetilde{\rm W}}^i\Big[q^\dagger\,(c_{\widetilde{\rm W}q}^i)\,q + \ell^\dagger\,(c_{\widetilde{\rm W}\ell}^i)\,\ell + {\rm H}_{\rm u}^\dagger\,(c_{\widetilde{\rm W}{\rm H}_{\rm u}}^i)\,{\rm H}_{\rm u} + {\rm H}_{\rm d}^\dagger\,(c_{\widetilde{\rm W}{\rm H}_{\rm d}}^i)\,{\rm H}_{\rm d}\Big] \\
&+ \Phi_{\widetilde{\rm B}}\Big[c_{\widetilde{\rm B}q}\,|q|^2 + c_{\widetilde{\rm B}{\rm u}}\,|\overline{\rm u}|^2 + c_{\widetilde{\rm B}{\rm d}}\,|\overline{\rm d}|^2 + c_{\widetilde{\rm B}\ell}\,|\ell|^2 \\
&+ c_{\widetilde{\rm B}{\rm e}}\,|\overline{\rm e}|^2 + c_{\widetilde{\rm B}{\rm H}_{\rm u}}\,|{\rm H}_{\rm u}|^2 + c_{\widetilde{\rm B}{\rm H}_{\rm d}}\,|{\rm H}_{\rm d}|^2\Big],
\end{aligned} \tag{3.6.94}$$

$$-\mathcal{L}_{\rm soft}^{\rm tadpoles} = t_{\widetilde{\rm B}}\,\Phi_{\widetilde{\rm B}}, \tag{3.6.95}$$

where we have only included the c terms that would be induced by the *supersoft operator* 3.4.41. Where indices haven not been made explicit, there is an implicit trace

$$\Phi_{\widetilde{\rm W}}^2 \equiv \text{tr}\left(\Phi_{\widetilde{\rm W}}^2\right) = \frac{1}{2}\,\Phi_{\widetilde{\rm W}}^i\,\Phi_{\widetilde{\rm W}}^i. \tag{3.6.96}$$

3.6.2 Electroweakino masses

Neutralinos

In the basis $(\widetilde{\mathrm{B}}, \psi_{\widetilde{\mathrm{B}}}, \widetilde{\mathrm{W}}, \psi_{\widetilde{\mathrm{W}}}, \widetilde{\mathrm{H}}^0_{\mathrm{u}}, \widetilde{\mathrm{H}}^0_{\mathrm{d}})$ the neutralino mass matrix is

$$m_{\widetilde{\chi}^0} = \begin{pmatrix} M_1 & m_{D\widetilde{\mathrm{B}}} & 0 & 0 & \frac{g_1\, v_u}{2} & -\frac{g_1\, v_d}{2} \\ m_{D\widetilde{\mathrm{B}}} & M_{\widetilde{\mathrm{B}}} + \sqrt{2}\kappa_{\widetilde{\mathrm{B}}} v_{\widetilde{\mathrm{B}}} & 0 & 0 & -\frac{\lambda_{\widetilde{\mathrm{B}}}\, v_d}{\sqrt{2}} & -\frac{\lambda_{\widetilde{\mathrm{B}}}\, v_u}{\sqrt{2}} \\ 0 & 0 & M_2 & m_{D\widetilde{\mathrm{W}}} & -\frac{g_2\, v_u}{2} & \frac{g_2\, v_d}{2} \\ 0 & 0 & m_{D\widetilde{\mathrm{W}}} & M_{\widetilde{\mathrm{W}}} & -\frac{\lambda_{\widetilde{\mathrm{W}}}\, v_d}{\sqrt{2}} & -\frac{\lambda_{\widetilde{\mathrm{W}}}\, v_u}{\sqrt{2}} \\ \frac{g_1\, v_u}{2} & -\frac{\lambda_{\widetilde{\mathrm{B}}}\, v_d}{\sqrt{2}} & -\frac{g_2\, v_u}{2} & -\frac{\lambda_{\widetilde{\mathrm{W}}}\, v_d}{\sqrt{2}} & 0 & -\mu^{\mathrm{eff}} \\ -\frac{g_1\, v_d}{2} & -\frac{\lambda_{\widetilde{\mathrm{B}}}\, v_u}{\sqrt{2}} & \frac{g_2\, v_d}{2} & -\frac{\lambda_{\widetilde{\mathrm{W}}}\, v_u}{\sqrt{2}} & -\mu^{\mathrm{eff}} & 0 \end{pmatrix} \tag{3.6.97}$$

Charginos

In the basis $(\widetilde{\mathrm{W}}^+, \psi^+_{\widetilde{\mathrm{W}}}, \widetilde{\mathrm{H}}^+_{\mathrm{u}})/(\widetilde{\mathrm{W}}^-, \psi^-_{\widetilde{\mathrm{W}}}, \widetilde{\mathrm{H}}^-_{\mathrm{d}})$ the chargino mass matrix is

$$m_{\widetilde{\chi}^\pm} = \begin{pmatrix} M_2 & m_{D\widetilde{\mathrm{W}}} - \frac{g_2\, v_{\widetilde{\mathrm{W}}}}{2} & \frac{g2\, v_u}{\sqrt{2}} \\ \frac{g2\, v_u}{\sqrt{2}} & M_{\widetilde{\mathrm{W}}} & \lambda_{\widetilde{\mathrm{W}}}\, v_d \\ \frac{g2\, v_d}{\sqrt{2}} & -\lambda_{\widetilde{\mathrm{W}}}\, v_u & \mu + \frac{1}{\sqrt{2}}(\lambda_{\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}} - \lambda_{\widetilde{\mathrm{W}}}\, v_{\widetilde{\mathrm{W}}}) \end{pmatrix} \tag{3.6.98}$$

3.6.3 Higgs sector Electroweak symmetry breaking

In this model after EWSB, there are four CP even scalar fields that can acquire VEVs

$$\mathrm{H}^0_{\mathrm{u}} \to \frac{1}{\sqrt{2}}\left(v_{\mathrm{u}} + \phi_{\mathrm{u}} + i\,\sigma_{\mathrm{u}}\right), \qquad \mathrm{H}^0_{\mathrm{d}} \to \frac{1}{\sqrt{2}}\left(v_{\mathrm{d}} + \phi_{\mathrm{d}} + i\,\sigma_{\mathrm{d}}\right), \tag{3.6.99}$$

$$\Phi^0_{\widetilde{\mathrm{B}}} \to \frac{1}{\sqrt{2}}\left(v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}} + i\,\sigma_{\widetilde{\mathrm{B}}}\right), \qquad \Phi^0_{\widetilde{\mathrm{W}}} \to \frac{1}{\sqrt{2}}\left(v_{\widetilde{\mathrm{W}}} + \phi_{\widetilde{\mathrm{W}}} + i\,\sigma_{\widetilde{\mathrm{W}}}\right), \tag{3.6.100}$$

where $\phi_{\widetilde{\mathrm{B}}}$ and $\phi^0_{\widetilde{\mathrm{W}}}$ are the sbino and swino, and $\sigma_{\widetilde{\mathrm{B}}}$ and $\sigma^0_{\widetilde{\mathrm{W}}}$ as the pseudo–sbino and pseudo–swino. The particles ϕ_{u}, ϕ_{d}, $\phi_{\widetilde{\mathrm{B}}}$ and $\phi_{\widetilde{\mathrm{W}}}$ then mix to form the CP even neutral Higgs bosons of the theory. The SUSY scalar potential is written as in eq. 2.3.161

$$V = \sum_i |F_{\Phi_i}|^2 + \frac{1}{2}\sum_j D_j^2, \tag{3.6.101}$$

where the sum over j sums over the gauge groups $\mathrm{U}(1)_\mathrm{Y}$ and $\mathrm{SU}(2)_\mathrm{L}$ and

$$\Phi_i = \left(\mathrm{H}_\mathrm{u}^+,\, \mathrm{H}_\mathrm{u}^0,\, \mathrm{H}_\mathrm{d}^0,\, \mathrm{H}_\mathrm{d}^-,\, \Phi_{\widetilde{\mathrm{W}}}^+,\, \Phi_{\widetilde{\mathrm{W}}}^-,\, \Phi_{\widetilde{\mathrm{W}}}^0,\, \Phi_{\widetilde{\mathrm{B}}}\right)_i. \tag{3.6.102}$$

For this purpose it is useful to explicitly write the $\mathrm{SU}(2)_\mathrm{L}$ decompositions

$$\begin{aligned}
\mathrm{H}_\mathrm{u}\cdot\mathrm{H}_\mathrm{d} &= \mathrm{H}_\mathrm{u}^+\,\mathrm{H}_\mathrm{d}^- - \mathrm{H}_\mathrm{u}^0\,\mathrm{H}_\mathrm{d}^0, && (3.6.103)\\
\Phi_{\widetilde{\mathrm{W}}}\,\mathrm{H}_\mathrm{u} &= \frac{1}{2}\begin{pmatrix}\Phi_{\widetilde{\mathrm{W}}}^0\,\mathrm{H}_\mathrm{u}^+ + \sqrt{2}\,\Phi_{\widetilde{\mathrm{W}}}^+\,\mathrm{H}_\mathrm{u}^0\\ \sqrt{2}\,\Phi_{\widetilde{\mathrm{W}}}^-\,\mathrm{H}_\mathrm{u}^+ - \Phi_{\widetilde{\mathrm{W}}}^0\,\mathrm{H}_\mathrm{u}^0\end{pmatrix}, && (3.6.104)\\
\mathrm{H}_\mathrm{d}\cdot\Phi_{\widetilde{\mathrm{W}}}\,\mathrm{H}_\mathrm{u} &= \frac{1}{2}\Big(\sqrt{2}\,\mathrm{H}_\mathrm{d}^0\,\Phi_{\widetilde{\mathrm{W}}}^-\,\Phi_{\widetilde{\mathrm{W}}}^+ - \mathrm{H}_\mathrm{d}^0\,\Phi_{\widetilde{\mathrm{W}}}^0\,\mathrm{H}_\mathrm{u}^0\\
&\qquad - \mathrm{H}_\mathrm{d}^-\,\Phi_{\widetilde{\mathrm{W}}}^0\,\mathrm{H}_\mathrm{u}^+ - \sqrt{2}\,\mathrm{H}_\mathrm{d}^-\,\Phi_{\widetilde{\mathrm{W}}}^+\,\mathrm{H}_\mathrm{u}^0\Big), && (3.6.105)\\
\Phi_{\widetilde{\mathrm{W}}}^2 &= \frac{1}{4}\begin{pmatrix}(\Phi_{\widetilde{\mathrm{W}}}^0)^2 + 2\,\Phi_{\widetilde{\mathrm{W}}}^+\,\Phi_{\widetilde{\mathrm{W}}}^- & \sqrt{2}\,\Phi_{\widetilde{\mathrm{W}}}^0\left(\Phi_{\widetilde{\mathrm{W}}}^+ - \Phi_{\widetilde{\mathrm{W}}}^-\right)\\ -\sqrt{2}\,\Phi_{\widetilde{\mathrm{W}}}^0\left(\Phi_{\widetilde{\mathrm{W}}}^+ - \Phi_{\widetilde{\mathrm{W}}}^-\right) & (\Phi_{\widetilde{\mathrm{W}}}^0)^2 + 2\,\Phi_{\widetilde{\mathrm{W}}}^+\,\Phi_{\widetilde{\mathrm{W}}}^-\end{pmatrix}, && (3.6.106)\\
\mathrm{tr}(\Phi_{\widetilde{\mathrm{W}}}^2) &= \frac{1}{2}\,(\Phi_{\widetilde{\mathrm{W}}}^0)^2 + \Phi_{\widetilde{\mathrm{W}}}^+\,\Phi_{\widetilde{\mathrm{W}}}^-. && (3.6.107)
\end{aligned}$$

For looking at the soft terms, the decompositions

$$|\Phi_{\widetilde{\mathrm{W}}}|^2 = \frac{1}{4}\begin{pmatrix}|\Phi_{\widetilde{\mathrm{W}}}^0|^2 + 2\,|\Phi_{\widetilde{\mathrm{W}}}^+|^2 & \sqrt{2}\,\Big[\Phi_{\widetilde{\mathrm{W}}}^0\,(\Phi_{\widetilde{\mathrm{W}}}^-)^\dagger - \Phi_{\widetilde{\mathrm{W}}}^+\,(\Phi_{\widetilde{\mathrm{W}}}^0)^\dagger\Big]\\ \sqrt{2}\,\Big[(\Phi_{\widetilde{\mathrm{W}}}^0)^\dagger\Phi_{\widetilde{\mathrm{W}}}^- - (\Phi_{\widetilde{\mathrm{W}}}^+)^\dagger\Phi_{\widetilde{\mathrm{W}}}^0\Big] & |\Phi_{\widetilde{\mathrm{W}}}^0|^2 + 2\,|\Phi_{\widetilde{\mathrm{W}}}^-|^2\end{pmatrix},$$

$$\mathrm{tr}(|\Phi_{\widetilde{\mathrm{W}}}^2|) = \frac{1}{2}\left(|\Phi_{\widetilde{\mathrm{W}}}^0|^2 + |\Phi_{\widetilde{\mathrm{W}}}^+|^2 + |\Phi_{\widetilde{\mathrm{W}}}^-|^2\right) \tag{3.6.108}$$

are useful. The Higgs part of the superpotential is then decomposed as

$$\begin{aligned}
W^{\mathrm{Higgs}} = &-\left(\mu + \lambda_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} + \lambda_{\widetilde{\mathrm{W}}}\,\Phi_{\widetilde{\mathrm{W}}}^0\right)\mathrm{H}_\mathrm{u}^0\,\mathrm{H}_\mathrm{d}^0 + \left(\mu + \lambda_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} - \lambda_{\widetilde{\mathrm{W}}}\,\Phi_{\widetilde{\mathrm{W}}}^0\right)\mathrm{H}_\mathrm{u}^+\,\mathrm{H}_\mathrm{d}^-\\
&+ \sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\left(\mathrm{H}_\mathrm{d}^0\,\Phi_{\widetilde{\mathrm{W}}}^-\,\mathrm{H}_\mathrm{u}^+ - \mathrm{H}_\mathrm{d}^-\,\Phi_{\widetilde{\mathrm{W}}}^+\,\mathrm{H}_\mathrm{u}^0\right) + L_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} + \frac{M_{\widetilde{\mathrm{B}}}}{2}\,\Phi_{\widetilde{\mathrm{B}}}^2 + \frac{\kappa_{\widetilde{\mathrm{B}}}}{3}\,\Phi_{\widetilde{\mathrm{B}}}^3\\
&+ \left(M_{\widetilde{\mathrm{W}}} + \lambda_{\widetilde{\mathrm{B}}\widetilde{\mathrm{W}}}\,\Phi_{\widetilde{\mathrm{B}}}\right)\left[\frac{1}{2}\,(\Phi_{\widetilde{\mathrm{W}}}^0)^2 + \Phi_{\widetilde{\mathrm{W}}}^+\,\Phi_{\widetilde{\mathrm{W}}}^-\right].
\end{aligned} \tag{3.6.109}$$

with the corresponding F terms

$$-F^\dagger_{\mathrm{H_u^+}} = \left(\mu + \lambda_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} - \lambda_{\widetilde{\mathrm{W}}}\,\Phi^0_{\widetilde{\mathrm{W}}}\right)\mathrm{H_d^-} + \sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\,\mathrm{H_d^0}\,\Phi^-_{\widetilde{\mathrm{W}}}, \tag{3.6.110}$$

$$-F^\dagger_{\mathrm{H_u^0}} = -\left(\mu + \lambda_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} + \lambda_{\widetilde{\mathrm{W}}}\,\Phi^0_{\widetilde{\mathrm{W}}}\right)\mathrm{H_d^0} - \sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\,\mathrm{H_d^-}\,\Phi^+_{\widetilde{\mathrm{W}}}, \tag{3.6.111}$$

$$-F^\dagger_{\mathrm{H_d^0}} = -\left(\mu + \lambda_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} + \lambda_{\widetilde{\mathrm{W}}}\,\Phi^0_{\widetilde{\mathrm{W}}}\right)\mathrm{H_u^0} + \sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\,\Phi^-_{\widetilde{\mathrm{W}}}\,\mathrm{H_u^+}, \tag{3.6.112}$$

$$-F^\dagger_{\mathrm{H_d^-}} = \left(\mu + \lambda_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} - \lambda_{\widetilde{\mathrm{W}}}\,\Phi^0_{\widetilde{\mathrm{W}}}\right)\mathrm{H_u^+} - \sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\,\Phi^+_{\widetilde{\mathrm{W}}}\,\mathrm{H_u^0}, \tag{3.6.113}$$

$$-F^\dagger_{\Phi^+_{\widetilde{\mathrm{W}}}} = -\sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\,\mathrm{H_d^-}\,\mathrm{H_u^0} + \left(M_{\widetilde{\mathrm{W}}} + \lambda_{\widetilde{\mathrm{B}}\widetilde{\mathrm{W}}}\,\Phi_{\widetilde{\mathrm{B}}}\right)\Phi^-_{\widetilde{\mathrm{W}}}, \tag{3.6.114}$$

$$-F^\dagger_{\Phi^0_{\widetilde{\mathrm{W}}}} = -\lambda_{\widetilde{\mathrm{W}}}\left(\mathrm{H_u^+}\,\mathrm{H_d^-} + \mathrm{H_u^0}\,\mathrm{H_d^0}\right) + \left(M_{\widetilde{\mathrm{W}}} + \lambda_{\widetilde{\mathrm{B}}\widetilde{\mathrm{W}}}\,\Phi_{\widetilde{\mathrm{B}}}\right)\Phi^0_{\widetilde{\mathrm{W}}}, \tag{3.6.115}$$

$$-F^\dagger_{\Phi^-_{\widetilde{\mathrm{W}}}} = \sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\,\mathrm{H_d^0}\,\mathrm{H_u^+} + \left(M_{\widetilde{\mathrm{W}}} + \lambda_{\widetilde{\mathrm{B}}\widetilde{\mathrm{W}}}\,\Phi_{\widetilde{\mathrm{B}}}\right)\Phi^+_{\widetilde{\mathrm{W}}}, \tag{3.6.116}$$

$$\begin{aligned}-F^\dagger_{\Phi_{\widetilde{\mathrm{B}}}} &= \lambda_{\widetilde{\mathrm{B}}}\left(\mathrm{H_u^+}\,\mathrm{H_d^-} - \mathrm{H_u^0}\,\mathrm{H_d^0}\right) + L_{\widetilde{\mathrm{B}}} + M_{\widetilde{\mathrm{B}}}\,\Phi_{\widetilde{\mathrm{B}}} + \kappa_{\widetilde{\mathrm{B}}}\,\Phi^2_{\widetilde{\mathrm{B}}} \\ &\quad + \lambda_{\widetilde{\mathrm{B}}\widetilde{\mathrm{W}}}\left[\frac{1}{2}\,(\Phi^0_{\widetilde{\mathrm{W}}})^2 + \Phi^+_{\widetilde{\mathrm{W}}}\,\Phi^-_{\widetilde{\mathrm{W}}}\right].\end{aligned} \tag{3.6.117}$$

Assuming the Dirac gaugino masses are generated by the supersoft operator, the D term for $\mathrm{U(1)_Y}$ appears in the lagrangian

$$\begin{aligned}\mathcal{L}^{\text{D term}}_{\mathrm{U(1)_Y}} &= \frac{1}{2}\,D_Y^2 + \frac{1}{2}\,g_1\,D_Y\,\left(|\mathrm{H_u^+}|^2 + |\mathrm{H_u^0}|^2 - |\mathrm{H_d^0}|^2 - |\mathrm{H_d^-}|^2\right) \\ &\quad + \sqrt{2}\,m_{D\widetilde{\mathrm{B}}}\,D_Y\,\left(\Phi_{\widetilde{\mathrm{B}}} + \text{h.c.}\right),\end{aligned} \tag{3.6.118}$$

where the first line is the same as in the MSSM and the second line is from the supersoft operator. Similarly for $\mathrm{SU(2)_L}$

$$\begin{aligned}\mathcal{L}^{\text{D term}}_{\mathrm{SU(2)_L}} &= \frac{1}{2}\,D_L^i\,D_L^i + \frac{1}{2}\,g_2\,D_L^i\,\left(\mathrm{H_u^\dagger}\,T^i\,\mathrm{H_u} + \mathrm{H_d^\dagger}\,T^i\,\mathrm{H_d} + \Phi^\dagger_{\widetilde{\mathrm{W}}}\,T^i\,\Phi_{\widetilde{\mathrm{W}}}\right) \\ &\quad + \sqrt{2}\,m_{D\widetilde{\mathrm{W}}}\,D_L^i\,\left(\Phi^i_{\widetilde{\mathrm{W}}} + \text{h.c.}\right).\end{aligned} \tag{3.6.119}$$

The D terms for $\mathrm{U(1)_Y}$ and $\mathrm{SU(2)_L}$ are then

$$-D_Y = \frac{1}{2}\,g_1\,\left(|\mathrm{H_u^+}|^2 + |\mathrm{H_u^0}|^2 - |\mathrm{H_d^0}|^2 - |\mathrm{H_d^-}|^2\right) + \sqrt{2}\,m_{D\widetilde{\mathrm{B}}}\,\left(\Phi_{\widetilde{\mathrm{B}}} + \text{h.c.}\right), \tag{3.6.120}$$

$$-D_L^i = \frac{1}{2}g_2\left(\mathrm{H_u^\dagger}T^i\mathrm{H_u} + \mathrm{H_d^\dagger}T^i\mathrm{H_d} + \Phi^\dagger_{\widetilde{\mathrm{W}}}T^i\Phi_{\widetilde{\mathrm{W}}}\right) + \sqrt{2}m_{D\widetilde{\mathrm{W}}}\left(\Phi^i_{\widetilde{\mathrm{W}}} + \text{h.c.}\right). \tag{3.6.121}$$

Now we finally have enough to write down the Higgs potential. The F term part of the scalar potential is easily found by summing the modulus squared of the terms 3.6.110 to 3.6.117. The $\mathrm{U(1)_Y}$ D term contribution to the scalar potential is

$$V_{\mathrm{U(1)_Y}} = \frac{1}{2}\,D_Y^2 = \frac{1}{2}\Big[\frac{1}{2}\,g_1\left(|\mathrm{H_u^+}|^2 + |\mathrm{H_u^0}|^2 - |\mathrm{H_d^0}|^2 - |\mathrm{H_d^-}|^2\right) + \sqrt{2}\,m_{D\tilde{\mathrm{B}}}\,\left(\Phi_{\tilde{\mathrm{B}}} + \mathrm{h.c.}\right)\Big]^2, \qquad (3.6.122)$$

and the $\mathrm{SU(2)_L}$ D term contribution to the scalar potential can be calculated by $\mathrm{SU(2)_L}$ decomposing the D term

$$\mathrm{tr}\,(D_L^2) = \frac{1}{2}\,D_L^i\,D_L^i = \frac{1}{2}\,(D_L^0)^2 + D_L^+\,D_L^- \qquad (3.6.123)$$

where

$$D_L \equiv D_L^i\,T^i = \frac{1}{2}\begin{pmatrix} D_L^0 & \sqrt{2}\,D_L^+ \\ \sqrt{2}\,D_L^- & -D_L^0 \end{pmatrix} \qquad (3.6.124)$$

as was done for $\Phi_{\widetilde{\mathrm{W}}}$ and $\psi_{\widetilde{\mathrm{W}}}$. The solutions for the D terms are then

$$-D_L^+ = g_2\left\{(\mathrm{H_u^0})^\dagger\,\mathrm{H_u^+} + (\mathrm{H_d^-})^\dagger\,\mathrm{H_d^0} + \sqrt{2}\,\left[(\Phi_{\widetilde{\mathrm{W}}}^-)^\dagger\,\Phi_{\widetilde{\mathrm{W}}}^0 + (\Phi_{\widetilde{\mathrm{W}}}^0)^\dagger\,\Phi_{\widetilde{\mathrm{W}}}^+\right]\right\} + 2\,m_{D\widetilde{\mathrm{W}}}\,\left[\Phi_{\widetilde{\mathrm{W}}}^+ + (\Phi_{\widetilde{\mathrm{W}}}^-)^\dagger\right], \qquad (3.6.125)$$

$$-D_L^0 = \frac{1}{2}\,g_2\left[|\mathrm{H_u^+}|^2 - |\mathrm{H_u^0}|^2 + |\mathrm{H_d^0}|^2 - |\mathrm{H_d^-}|^2 + 2\,\left(|\Phi_{\widetilde{\mathrm{W}}}^+|^2 - |\Phi_{\widetilde{\mathrm{W}}}^-|^2\right)\right] + \sqrt{2}\,m_{D\widetilde{\mathrm{W}}}\,\left[\Phi_{\widetilde{\mathrm{W}}}^0 + \mathrm{h.c.}\right], \qquad (3.6.126)$$

$$-D_L^- = g_2\left\{(\mathrm{H_u^+})^\dagger\,\mathrm{H_u^0} + (\mathrm{H_d^0})^\dagger\,\mathrm{H_d^-} + \sqrt{2}\,\left[(\Phi_{\widetilde{\mathrm{W}}}^0)^\dagger\,\Phi_{\widetilde{\mathrm{W}}}^- + (\Phi_{\widetilde{\mathrm{W}}}^+)^\dagger\,\Phi_{\widetilde{\mathrm{W}}}^0\right]\right\} + 2\,m_{D\widetilde{\mathrm{W}}}\,\left[\Phi_{\widetilde{\mathrm{W}}}^- + (\Phi_{\widetilde{\mathrm{W}}}^+)^\dagger\right]. \qquad (3.6.127)$$

The $\mathrm{SU(2)_L}$ D term contribution to the scalar potential then follows from substituting eqs. 3.6.125, 3.6.126 and 3.6.127 into eq. 3.6.123. The decomposed Higgs soft terms are

$$\begin{aligned} V_{\rm soft}^{\rm Higgs} =& \, m_{\rm H_u}^2 \left(|{\rm H_u^+}|^2 + |{\rm H_u^0}|^2 \right) + m_{\rm H_d}^2 \left(|{\rm H_d^0}|^2 + |{\rm H_d^-}|^2 \right) + m_{\widetilde{\rm B}}^2 \, |\Phi_{\widetilde{\rm B}}|^2 \\ &+ m_{\widetilde{\rm W}}^2 \left(|\Phi_{\widetilde{\rm W}}^0|^2 + |\Phi_{\widetilde{\rm W}}^+|^2 + |\Phi_{\widetilde{\rm W}}^-|^2 \right) + \Bigg\{ t_{\widetilde{\rm B}} \, \Phi_{\widetilde{\rm B}} + \frac{b_{\widetilde{\rm B}}}{2} \, \Phi_{\widetilde{\rm B}}^2 + \frac{a_{\kappa_{\widetilde{\rm B}}}}{3} \, \Phi_{\widetilde{\rm B}}^3 \\ &- \left(b_\mu + a_{\widetilde{\rm B}} \, \Phi_{\widetilde{\rm B}} + a_{\widetilde{\rm W}} \, \Phi_{\widetilde{\rm W}}^0 \right) {\rm H_u^0} \, {\rm H_d^0} + \left(b_\mu + a_{\widetilde{\rm B}} \, \Phi_{\widetilde{\rm B}} - a_{\widetilde{\rm W}} \, \Phi_{\widetilde{\rm W}}^0 \right) {\rm H_u^+} \, {\rm H_d^-} \\ &+ \sqrt{2} \, a_{\widetilde{\rm W}} \left({\rm H_d^0} \, \Phi_{\widetilde{\rm W}}^- \, {\rm H_u^+} - {\rm H_d^-} \, \Phi_{\widetilde{\rm W}}^+ \, {\rm H_u^0} \right) \\ &+ \left(b_{\widetilde{\rm W}} + a_{\widetilde{\rm B}\widetilde{\rm W}} \, \Phi_{\widetilde{\rm B}} \right) \left[\frac{1}{2} \, (\Phi_{\widetilde{\rm W}}^0)^2 + \Phi_{\widetilde{\rm W}}^+ \, \Phi_{\widetilde{\rm W}}^- \right] + {\rm h.c.} \Bigg\}. \end{aligned} \tag{3.6.128}$$

Summing all of these terms gives the total Higgs scalar potential

$$V = \sum_i |F_{\Phi_i}|^2 + V_{\rm U(1)_Y} + V_{\rm SU(2)_L} + V_{\rm soft}^{\rm Higgs}. \tag{3.6.129}$$

We can check the existence of a vacuum that preserves $\rm U(1)_{em}$. As is done in the MSSM, we can use the $\rm SU(2)_L$ symmetry to set $\langle {\rm H_u^+} \rangle = 0$. In this vacuum we should have

$$\frac{\partial V}{\partial {\rm H_u^+}} = \frac{\partial V}{\partial {\rm H_d^-}} = \frac{\partial V}{\partial \Phi_{\widetilde{\rm W}}^+} = \frac{\partial V}{\partial \Phi_{\widetilde{\rm W}}^-} = 0 \tag{3.6.130}$$

which has a solution

$$\langle {\rm H_d^-} \rangle = \langle \Phi_{\widetilde{\rm W}}^+ \rangle = \langle \Phi_{\widetilde{\rm W}}^- \rangle = 0. \tag{3.6.131}$$

For analysing EWSB it is therefore at least consistent to set the VEVs of the charged scalar fields to zero. Now we can move to the vacuum with broken electroweak symmetry by applying the shifts 3.6.99 and 3.6.100, and sitting in the electroweak preserving vacuum we have (setting $L_{\widetilde{\rm B}} = 0$ for brevity and taking all couplings real for simplicity)

$$
\begin{aligned}
V = & \frac{m_{\mathrm{H_u}}^2}{2} \left[(v_u + \phi_u)^2 + \sigma_u^2 \right] + \frac{m_{\mathrm{H_d}}^2}{2} \left[(v_d + \phi_d)^2 + \sigma_d^2 \right] \\
& + \frac{\lambda_{\widetilde{\mathrm{B}}}^2 + \lambda_{\widetilde{\mathrm{W}}}^2}{4} \left[(v_u + \phi_u)^2 + \sigma_u^2 \right] \left[(v_d + \phi_d)^2 + \sigma_d^2 \right] \\
& + \frac{m_{\phi_{\widetilde{\mathrm{B}}}}^2}{2} (v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}})^2 + \frac{m_{\sigma_{\widetilde{\mathrm{B}}}}^2}{2} \sigma_{\widetilde{\mathrm{B}}}^2 + \frac{m_{\phi_{\widetilde{\mathrm{W}}}}^2}{2} (v_{\widetilde{\mathrm{W}}} + \phi_{\widetilde{\mathrm{W}}})^2 + \frac{m_{\sigma_{\widetilde{\mathrm{W}}}}^2}{2} \sigma_{\widetilde{\mathrm{W}}}^2 \\
& + \frac{1}{2} \left[(v_u + \phi_u)^2 + \sigma_u^2 + (v_d + \phi_d)^2 + \sigma_d^2 \right] \Bigg\{ \mu^2 + \frac{\lambda_{\widetilde{\mathrm{B}}}^2}{\sqrt{2}} \left[(v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}})^2 + \sigma_{\widetilde{\mathrm{B}}}^2 \right] \\
& + \frac{\lambda_{\widetilde{\mathrm{W}}}^2}{\sqrt{2}} \left[(v_{\widetilde{\mathrm{W}}} + \phi_{\widetilde{\mathrm{W}}})^2 + \sigma_{\widetilde{\mathrm{W}}}^2 \right] + \lambda_{\widetilde{\mathrm{B}}} \lambda_{\widetilde{\mathrm{W}}} \left[(v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}})(v_{\widetilde{\mathrm{W}}} + \phi_{\widetilde{\mathrm{W}}}) + \sigma_{\widetilde{\mathrm{B}}} \sigma_{\widetilde{\mathrm{W}}} \right] \\
& + \sqrt{2} \mu \left[\lambda_{\widetilde{\mathrm{B}}} (v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}}) + \lambda_{\widetilde{\mathrm{W}}} (v_{\widetilde{\mathrm{W}}} + \phi_{\widetilde{\mathrm{W}}}) \right] \Bigg\} - \left[(v_u + \phi_u)(v_d + \phi_d) - \sigma_u \sigma_d \right] \\
& \times \Bigg\{ b_\mu + \frac{\lambda_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}}}{\sqrt{2}} (v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}}) + \frac{\lambda_{\widetilde{\mathrm{W}}} M_{\widetilde{\mathrm{W}}} + a_{\widetilde{\mathrm{W}}}}{\sqrt{2}} (v_{\widetilde{\mathrm{W}}} + \phi_{\widetilde{\mathrm{W}}}) \\
& + \frac{1}{2} \lambda_{\widetilde{\mathrm{B}}} \kappa_{\widetilde{\mathrm{B}}} \left[(v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}})^2 - \sigma_{\widetilde{\mathrm{B}}}^2 \right] \Bigg\} - \left[(v_u + \phi_u)\, \sigma_d + (v_d + \phi_d)\, \sigma_u \right] \\
& \times \Bigg\{ \frac{\lambda_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}} - a_{\widetilde{\mathrm{B}}}}{\sqrt{2}} \sigma_{\widetilde{\mathrm{B}}} + \frac{\lambda_{\widetilde{\mathrm{W}}} M_{\widetilde{\mathrm{W}}} - a_{\widetilde{\mathrm{W}}}}{\sqrt{2}} \sigma_{\widetilde{\mathrm{W}}} + \lambda_{\widetilde{\mathrm{B}}} \kappa_{\widetilde{\mathrm{B}}} (v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}}) \sigma_{\widetilde{\mathrm{B}}} \Bigg\} \\
& + \frac{\kappa_{\widetilde{\mathrm{B}}}^2}{4} \left[(v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}})^2 + \sigma_{\widetilde{\mathrm{B}}}^2 \right]^2 + \frac{\kappa_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}}}{\sqrt{2}} \left[(v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}})^2 + \sigma_{\widetilde{\mathrm{B}}}^2 \right] (v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}}) \\
& + \frac{a_{\kappa_{\widetilde{\mathrm{B}}}}}{3\sqrt{2}} \left[(v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}})^2 - 3\, \sigma_{\widetilde{\mathrm{B}}}^2 \right] (v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}}) \\
& + \left[(v_u + \phi_u)^2 + \sigma_u^2 - (v_d + \phi_d)^2 - \sigma_d^2 \right] \left[g_1^2\, m_{D\widetilde{\mathrm{B}}} (v_{\widetilde{\mathrm{B}}} + \phi_{\widetilde{\mathrm{B}}}) - g_2^2\, m_{D\widetilde{\mathrm{W}}} (v_{\widetilde{\mathrm{W}}} + \phi_{\widetilde{\mathrm{W}}}) \right] \\
& + \frac{g_1^2 + g_2^2}{32} \left[(v_u + \phi_u)^2 + \sigma_u^2 - (v_d + \phi_d)^2 - \sigma_d^2 \right]^2 , \qquad (3.6.132)
\end{aligned}
$$

where we have defined the mass squareds of the sbino and swino

$$ m_{\phi_{\widetilde{\mathrm{B}}}}^2 = m_{\widetilde{\mathrm{B}}}^2 + M_{\widetilde{\mathrm{B}}}^2 + 4\, m_{D\widetilde{\mathrm{B}}}^2 + b_{\widetilde{\mathrm{B}}}, \qquad (3.6.133) $$

$$ m_{\phi_{\widetilde{\mathrm{W}}}}^2 = m_{\widetilde{\mathrm{W}}}^2 + M_{\widetilde{\mathrm{W}}}^2 + 4\, m_{D\widetilde{\mathrm{W}}}^2 + b_{\widetilde{\mathrm{W}}} \qquad (3.6.134) $$

and the mass squareds of the pseudo–sbino and pseudo–swino

$$m^2_{\sigma_{\widetilde{\mathrm{B}}}} = m^2_{\widetilde{\mathrm{B}}} + M^2_{\widetilde{\mathrm{B}}} - b_{\widetilde{\mathrm{B}}}, \tag{3.6.135}$$

$$m^2_{\sigma_{\widetilde{\mathrm{W}}}} = m^2_{\widetilde{\mathrm{W}}} + M^2_{\widetilde{\mathrm{W}}} - b_{\widetilde{\mathrm{W}}}. \tag{3.6.136}$$

It's worth keeping an eye on the terms in the second line of 3.6.132 as they are going to life the SM–like Higgs mass at tree level like in the Next to Minimal Supersymmetric Standard Model (NMSSM). The equations for minimising the scalar potential are

$$\begin{aligned}
0 = \frac{1}{v_u}\frac{\partial V}{\partial v_u} &= m^2_{\mathrm{H_u}} + (\mu^{\mathrm{eff}})^2 + g_1\, m_{D\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}} - g_2\, m_{D\widetilde{\mathrm{W}}}\, v_{\widetilde{\mathrm{W}}} \\
&\quad - b^{\mathrm{eff}}_\mu \cot(\beta) - \frac{m^2_{\mathrm{Z}}}{2} c_{2\beta} + \frac{\lambda^2_{\widetilde{\mathrm{B}}} + \lambda^2_{\widetilde{\mathrm{B}}}}{2} v^2\, c^2_\beta, &(3.6.137)\\
0 = \frac{1}{v_d}\frac{\partial V}{\partial v_d} &= m^2_{\mathrm{H_d}} + (\mu^{\mathrm{eff}})^2 - g_1\, m_{D\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}} + g_2\, m_{D\widetilde{\mathrm{W}}}\, v_{\widetilde{\mathrm{W}}} \\
&\quad - b^{\mathrm{eff}}_\mu\, t_\beta - \frac{m^2_{\mathrm{Z}}}{2} c_{2\beta} + \frac{\lambda^2_{\widetilde{\mathrm{B}}} + \lambda^2_{\widetilde{\mathrm{B}}}}{2} v^2\, s^2_\beta, &(3.6.138)\\
0 = \frac{1}{v_{\widetilde{\mathrm{B}}}}\frac{\partial V}{\partial v_{\widetilde{\mathrm{B}}}} &= m^2_{\phi_{\widetilde{\mathrm{B}}}} + \frac{\lambda_{\widetilde{\mathrm{B}}}}{2} v^2 \left(\lambda_{\widetilde{\mathrm{B}}} + \kappa\, s_{2\beta}\right) + \frac{\kappa_{\widetilde{\mathrm{B}}}}{\sqrt{2}} (3\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}})\, v_{\widetilde{\mathrm{B}}} + \kappa^2_{\widetilde{\mathrm{B}}}\, v^2_{\widetilde{\mathrm{B}}} \\
&\quad + \frac{v^2}{v_{\widetilde{\mathrm{B}}}}\left[\frac{\mu\lambda_{\widetilde{\mathrm{B}}}}{\sqrt{2}} - \frac{\lambda_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}}}{2\sqrt{2}} s_{2\beta} + \frac{\lambda_{\widetilde{\mathrm{B}}}\lambda_{\widetilde{\mathrm{W}}} v_{\widetilde{\mathrm{W}}}}{2} - \frac{g_1 m_{D\widetilde{\mathrm{B}}} c_{2\beta}}{2}\right], &(3.6.139)\\
0 = \frac{1}{v_{\widetilde{\mathrm{W}}}}\frac{\partial V}{\partial v_{\widetilde{\mathrm{W}}}} &= m^2_{\phi_{\widetilde{\mathrm{W}}}} + \frac{\lambda^2_{\widetilde{\mathrm{W}}}}{2} v^2 + \frac{v^2}{v_{\widetilde{\mathrm{W}}}}\left[\frac{\mu\, \lambda_{\widetilde{\mathrm{W}}}}{\sqrt{2}} - \frac{\lambda_{\widetilde{\mathrm{W}}}\, M_{\widetilde{\mathrm{W}}} + a_{\widetilde{\mathrm{W}}}}{2\sqrt{2}} s_{2\beta}\right. \\
&\quad \left. + \frac{\lambda_{\widetilde{\mathrm{B}}}\lambda_{\widetilde{\mathrm{W}}} v_{\widetilde{\mathrm{B}}}}{2} + \frac{g_2\, m_{D\widetilde{\mathrm{W}}}}{2} c_{2\beta}\right], &(3.6.140)
\end{aligned}$$

where we have defined the effective Higgsino mass μ^{eff} and the effective Higgs b term b^{eff}_μ

$$\mu^{\mathrm{eff}} \equiv \mu + \frac{1}{\sqrt{2}} \left(\lambda_{\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}} + \lambda_{\widetilde{\mathrm{W}}}\, v_{\widetilde{\mathrm{W}}}\right), \tag{3.6.141}$$

$$b^{\mathrm{eff}}_\mu \equiv b_\mu + \frac{1}{\sqrt{2}} \left[(\lambda_{\widetilde{\mathrm{B}}}\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}})\, v_{\widetilde{\mathrm{B}}} + (\lambda_{\widetilde{\mathrm{W}}}\, M_{\widetilde{\mathrm{W}}} + a_{\widetilde{\mathrm{W}}})\, v_{\widetilde{\mathrm{W}}}\right]. \tag{3.6.142}$$

Solving the vacuum minimisation equations is typically done for $m^2_{\mathrm{H_u}}$, $m^2_{\mathrm{H_d}}$, $m^2_{\phi_{\widetilde{\mathrm{B}}}}$ and $m^2_{\phi_{\widetilde{\mathrm{W}}}}$ as the equations are linear in these. Unfortunately doing this moves away from UV models that can generate soft terms at some high scale, although numerically solving the vacuum equations in terms of the non–linear variables such as the VEVs is possible (though currently less stable).

3.6.4 The (T)NMSSM effect

From the scalar potential 3.6.132 we find that the Higgs mass matrix in the basis $(\phi_u,\, \phi_d,\, \phi_{\widetilde{\mathrm{B}}},\, \phi_{\widetilde{\mathrm{W}}})$ is

$$m_\phi^2 = \begin{pmatrix} m_{uu}^2 & m_{du}^2 & m_{\widetilde{\mathrm{B}}u}^2 & m_{\widetilde{\mathrm{W}}u}^2 \\ m_{du}^2 & m_{dd}^2 & m_{\widetilde{\mathrm{B}}d}^2 & m_{\widetilde{\mathrm{W}}d}^2 \\ m_{\widetilde{\mathrm{B}}u}^2 & m_{\widetilde{\mathrm{B}}d}^2 & m_{\widetilde{\mathrm{B}}\widetilde{\mathrm{B}}}^2 & m_{\widetilde{\mathrm{W}}\widetilde{\mathrm{B}}}^2 \\ m_{\widetilde{\mathrm{W}}u}^2 & m_{\widetilde{\mathrm{W}}d}^2 & m_{\widetilde{\mathrm{W}}\widetilde{\mathrm{B}}}^2 & m_{\widetilde{\mathrm{W}}\widetilde{\mathrm{W}}}^2 \end{pmatrix} \tag{3.6.143}$$

where

$$m_{uu}^2 = m_{\mathrm{H_u}}^2 + (\mu^{\mathrm{eff}})^2 + \frac{3}{2} m_{\mathrm{Z}}^2\, s_\beta^2 + \frac{\Delta}{2}\, c_\beta^2 + g_1\, m_{D\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}} - g_2\, m_{D\widetilde{\mathrm{W}}}\, v_{\widetilde{\mathrm{W}}}, \tag{3.6.144}$$

$$m_{du}^2 = s_\beta\, c_\beta\, (\Delta - m_{A^0}^2), \tag{3.6.145}$$

$$m_{\widetilde{\mathrm{B}}u}^2 = v\left[s_\beta\left(\sqrt{2}\,\lambda_{\widetilde{\mathrm{B}}}\mu^{\mathrm{eff}} + g_1\, m_{D\widetilde{\mathrm{B}}}\right) - c_\beta\left(\frac{\lambda_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}}}{\sqrt{2}} + \lambda_{\widetilde{\mathrm{B}}}\, \kappa_{\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}}\right)\right], \tag{3.6.146}$$

$$m_{\widetilde{\mathrm{W}}u}^2 = v\left[s_\beta\left(\sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\, \mu^{\mathrm{eff}} - g_2\, m_{D\widetilde{\mathrm{W}}}\right) - \frac{c_\beta}{\sqrt{2}}\left(\lambda_{\widetilde{\mathrm{W}}} M_{\widetilde{\mathrm{W}}} + a_{\widetilde{\mathrm{W}}}\right)\right], \tag{3.6.147}$$

$$m_{dd}^2 = m_{\mathrm{H_d}}^2 + (\mu^{\mathrm{eff}})^2 + \frac{3}{2} m_{\mathrm{Z}}^2\, c_\beta^2 + \frac{\Delta}{2}\, s_\beta^2 - g_1\, m_{D\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}} + g_2\, m_{D\widetilde{\mathrm{W}}}\, v_{\widetilde{\mathrm{W}}}, \tag{3.6.148}$$

$$m_{\widetilde{\mathrm{B}}d}^2 = v\left[-s_\beta\left(\frac{\lambda_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}}}{\sqrt{2}} + \lambda_{\widetilde{\mathrm{B}}}\kappa_{\widetilde{\mathrm{B}}} v_{\widetilde{\mathrm{B}}}\right) + c_\beta\left(\sqrt{2}\lambda_{\widetilde{\mathrm{B}}}\mu^{\mathrm{eff}} - g_1 m_{D\widetilde{\mathrm{B}}}\right)\right], \tag{3.6.149}$$

$$m_{\widetilde{\mathrm{W}}d}^2 = v\left[-\frac{s_\beta}{\sqrt{2}}\left(\lambda_{\widetilde{\mathrm{W}}}\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{W}}}\right) + \frac{c_\beta}{\sqrt{2}}\left(\sqrt{2}\,\lambda_{\widetilde{\mathrm{W}}}\, \mu^{\mathrm{eff}} + g_2\, m_{D\widetilde{\mathrm{W}}}\right)\right], \tag{3.6.150}$$

$$m_{\widetilde{\mathrm{B}}\widetilde{\mathrm{B}}}^2 = m_{\phi_{\widetilde{\mathrm{B}}}}^2 + \sqrt{2}\left(3\kappa_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}} + a_{\kappa_{\widetilde{\mathrm{B}}}}\right) v_{\widetilde{\mathrm{B}}} + 3\kappa_{\widetilde{\mathrm{B}}}^2 v_{\widetilde{\mathrm{B}}}^2 - \lambda_{\widetilde{\mathrm{B}}}\kappa_{\widetilde{\mathrm{B}}} s_\beta c_\beta + \frac{v^2 \lambda_{\widetilde{\mathrm{B}}}}{2}, \tag{3.6.151}$$

$$m_{\widetilde{\mathrm{W}}\widetilde{\mathrm{B}}}^2 = \lambda_{\widetilde{\mathrm{B}}}\, \lambda_{\widetilde{\mathrm{W}}}\, \frac{v^2}{2}, \tag{3.6.152}$$

$$m_{\widetilde{\mathrm{W}}\widetilde{\mathrm{W}}}^2 = m_{\phi_{\widetilde{\mathrm{B}}}}^2 + \frac{v^2 \lambda_{\widetilde{\mathrm{B}}}}{2}, \tag{3.6.153}$$

and

$$\Delta \equiv (\lambda_{\widetilde{\mathrm{B}}}^2 + \lambda_{\widetilde{\mathrm{W}}}^2)\, v^2 - m_{\mathrm{Z}}^2, \qquad m_{A^0}^2 = \frac{2\, b_\mu^{\mathrm{eff}}}{s_{2\beta}}. \tag{3.6.154}$$

Substituting the solutions for the vacuum equations 3.6.137 to 3.6.140 into the mass matrix 3.6.143 we find the 2×2 upper left sub–matrix becomes

$$m_\phi^2 = \begin{pmatrix} m_Z^2\, s_\beta^2 + m_{A_0}^2\, c_\beta^2 & (\Delta - m_{A_0}^2)\, s_\beta\, c_\beta & \cdot & \cdot \\ (\Delta - m_{A_0}^2)\, s_\beta\, c_\beta & m_Z^2\, c_\beta^2 + m_{A_0}^2\, s_\beta^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \tag{3.6.155}$$

with the eigenvalues

$$m_{\mathrm{h,H}}^2 = \frac{1}{2}\left[m_Z^2 + m_{A_0}^2 \mp \sqrt{(\Delta - m_{A_0}^2)^2\, s_{2\beta}^2 + (m_Z^2 - m_{A_0}^2)^2\, c_{2\beta}^2}\right] \tag{3.6.156}$$

and in the decoupling limit $m_{A_0}^2 \to \infty$

$$m_{\mathrm{h}}^2 = m_Z^2\, c_{2\beta}^2 + \frac{\lambda_{\widetilde{B}}^2 + \lambda_{\widetilde{W}}^2}{2}\, v^2\, s_{2\beta}^2. \tag{3.6.157}$$

In the presence of a $\mathrm{U}(1)_Y$ singlet $\Phi_{\widetilde{B}}$ and a triplet of SU(2) $\Phi_{\widetilde{W}}$ we see that the MSSM tree level upper bound on the Higgs boson mass is lifted [205]

$$m_{\mathrm{h,tree}}^2 \leq m_Z^2\, c_{2\beta}^2 + \frac{\lambda_{\widetilde{B}}^2 + \lambda_{\widetilde{W}}^2}{2}\, v^2\, s_{2\beta}^2 \tag{3.6.158}$$

by new F terms contributions coming from

$$W \supset \lambda_{\widetilde{B}}\, \Phi_{\widetilde{B}}\, \mathrm{H_u} \cdot \mathrm{H_d} + 2\, \lambda_{\widetilde{W}}\, \mathrm{H_d}\, \Phi_{\widetilde{W}} \cdot \mathrm{H_u}, \tag{3.6.159}$$

with the inequality in eq. 3.6.158 saturated in the decoupling limit $m_{A_0}^2 \to \infty$ and zeroing the terms that mix the MSSM Higgs fields with the ESPs

$$m_{\widetilde{B}u}^2 = m_{\widetilde{W}u}^2 = m_{\widetilde{B}d}^2 = m_{\widetilde{W}d}^2 = 0. \tag{3.6.160}$$

This effect with $\lambda_{\widetilde{W}} = 0$ is well–known in the NMSSM [206, 207]. Unfortunately, the new contributions are suppressed at large t_β, where the D-term contributions to the tree level Higgs mass are maximised. Typically $m_{\tilde{t}} \sim 1$ TeV is required if perturbativity holds all the way to the GUT scale, leaving the NMSSM with a *little hierarchy problem*. Now allowing for $\lambda_{\widetilde{W}} \neq 0$, we have a Triplet Extended NMSSM.

In this case, the additional Higgs interactions induced that allow the correct Higgs mass to be achieved at tree level within the realms of perturbativity [162, 173].

3.6.5 The rho parameter and custodial symmetry breaking

In the limit $g_1 \to 0$, the MSSM has the custodial symmetry $\mathrm{SU(2)_L \times SU(2)_R}$ [208]. Consider writing the Higgs sector as a *bi–doublet*

$$\Phi_\mathrm{H} \equiv \begin{pmatrix} \mathrm{H_d} & \mathrm{H_u} \end{pmatrix} = \begin{pmatrix} \mathrm{H_d^0} & \mathrm{H_u^+} \\ \mathrm{H_d^-} & \mathrm{H_u^0} \end{pmatrix}. \tag{3.6.161}$$

such that[9]

$$\mathrm{tr}\left[(D_\mu \Phi_\mathrm{H})^\dagger (D^\mu \Phi_\mathrm{H})\right] = (D_\mu \mathrm{H_u})^\dagger (D^\mu \mathrm{H_u}) + (D_\mu \mathrm{H_d})^\dagger (D^\mu \mathrm{H_d}). \tag{3.6.162}$$

The lagrangian has the standard $\mathrm{SU(2)_L \times U(1)_Y}$ symmetry that acts on Φ_H and $\Phi_{\widetilde{\mathrm{W}}}$ as

$$\mathrm{SU(2)_L}: \qquad \Phi_\mathrm{H} \to L\,\Phi_\mathrm{H}, \qquad \Phi_{\widetilde{\mathrm{W}}} \to L\,\Phi_\mathrm{H}\,L^\dagger \tag{3.6.163}$$

$$\mathrm{U(1)_Y}: \qquad \Phi_\mathrm{H} \to e^{-i\,\frac{1}{2}\,\sigma_3\,\theta}\,\Phi_\mathrm{H}, \qquad \Phi_{\widetilde{\mathrm{W}}} \to \Phi_{\widetilde{\mathrm{W}}}, \tag{3.6.164}$$

where L is a matrix that acts on the standard $\mathrm{SU(2)_L}$ indices to rotate the components within each Higgs doublet into each other. If we turn off the $\mathrm{SU(2)_L \times U(1)_Y}$ interaction by setting $g_1 \to 0$ then there is an additional global $\mathrm{SU(2)_R}$ symmetry that can mix components between different Higgs doublets

$$\mathrm{SU(2)_R}: \qquad \Phi_\mathrm{H} \to \Phi_\mathrm{H}\,R^\dagger. \tag{3.6.165}$$

This is violated by the hypercharge interaction because $\mathrm{H_u}$ and $\mathrm{H_d}$ have different charges under $\mathrm{SU(2)_L \times U(1)_Y}$. In the absence of hypercharge, the lagrangian then has the *custodial symmetry* $\mathrm{SU(2)_L \times SU(2)_R}$

$$\Phi_\mathrm{H} \to L\,\Phi_\mathrm{H}\,R^\dagger. \tag{3.6.166}$$

[9]Note that only in the limit $t_\beta \to 1$ can the SUSY lagrangian be written in terms of the *bi–doublet* Φ_H, but this isn't important for the present discussion.

VEVs for H_u^0, H_d^0 and $\Phi_{\widetilde{W}}$ break this symmetry

$$\langle \Phi_H^\dagger \, \Phi_H \rangle = \frac{1}{2} \begin{pmatrix} v_d^2 & 0 \\ 0 & v_u^2 \end{pmatrix}, \qquad \langle \Phi_{\widetilde{W}}^\dagger \, \Phi_{\widetilde{W}} \rangle = \frac{1}{4} \begin{pmatrix} v_{\widetilde{W}}^2 & 0 \\ 0 & -v_{\widetilde{W}}^2 \end{pmatrix}, \tag{3.6.167}$$

which in the limit $t_\beta \to 1$ and $v_{\widetilde{W}} \to 0$ has the symmetry under which $L = R$

$$L \, \langle \Phi_H \rangle \, L^\dagger = \langle \Phi_H \rangle. \tag{3.6.168}$$

The number of generators of the custodial symmetry broken by the Higgs VEVs is $2 \times 3^2 - 3 = 3$, corresponding to three Goldstone modes that get eaten by the $W^\pm$ and Z bosons

$$(D_\mu \langle H_u \rangle)^\dagger \, (D^\mu \langle H_u \rangle) = \frac{v_u^2}{4} \left(\frac{g_1^2 + g_2^2}{2} \, Z^2 + g_2^2 \, W^+ \, W^- \right), \tag{3.6.169}$$

$$(D_\mu \langle H_d \rangle)^\dagger \, (D^\mu \langle H_d \rangle) = \frac{v_d^2}{4} \left(\frac{g_1^2 + g_2^2}{2} \, Z^2 + g_2^2 \, W^+ \, W^- \right), \tag{3.6.170}$$

$$(D_\mu \langle \Phi_{\widetilde{W}} \rangle)^\dagger \, (D^\mu \langle \Phi_{\widetilde{W}} \rangle) = \frac{g_2^2 \, v_{\widetilde{W}}^2}{2} \, W^+ \, W^-, \tag{3.6.171}$$

who then acquire the masses

$$m_Z^2 = \frac{g_1^2 + g_2^2}{4} \, v^2, \qquad m_W^2 = \frac{g_2^2}{4} \, v^2 \left(1 + \frac{2 \, v_{\widetilde{W}}^2}{v^2} \right) \tag{3.6.172}$$

and in the custodial limit

$$m_Z^2 = \frac{g_1^2 + g_2^2}{4} \, v^2, \qquad m_W^2 = \frac{g_2^2}{4} \, v^2. \tag{3.6.173}$$

The tree level relations hold for all values of t_β. We can then define the electroweak precision rho parameter ρ [209]

$$\rho \equiv \frac{m_W^2}{m_Z^2 \, c_{\theta_W}^2} \equiv 1 + \Delta\rho \tag{3.6.174}$$

where $\Delta\rho$ is essentially a measure of *custodial symmetry breaking*. In general $\Delta\rho$ gets radiative corrections from parameters that break the custodial symmetry such as deviations of t_β from one and Yukawa couplings. At tree level, any VEV for $\Phi_{\widetilde{W}}$

breaks the custodial symmetry as

$$L\,\langle\Phi_{\widetilde{\mathrm{W}}}\rangle\,L^\dagger \neq \langle\Phi_{\widetilde{\mathrm{W}}}\rangle, \tag{3.6.175}$$

and we find a tree level contribution to $\Delta\rho$

$$\Delta\rho = 2\,\frac{v_{\widetilde{\mathrm{W}}}^2}{v^2}. \tag{3.6.176}$$

The experimental limit on $\Delta\rho$ is [210–212]

$$\Delta\rho = (4.2 \pm 2.7) \times 10^{-4} \tag{3.6.177}$$

which puts the stringent upper bound on $v_{\widetilde{\mathrm{W}}}$

$$v_{\widetilde{\mathrm{W}}} \lesssim 3.7\,\mathrm{GeV} \tag{3.6.178}$$

which is considerably a few orders of magnitude smaller than the electroweak scale, indicating a potential source of fine tuning since the other massive parameters involved with the triplet are typically of order the SUSY scale; multi–TeV SUSY particles would give an estimated tuning Δ^{-1} of

$$\Delta \approx 2\,\frac{m_{\mathrm{SUSY}}^2}{v_{\widetilde{\mathrm{W}}}^2} \tag{3.6.179}$$

i.e. tuning at the sub–percent level.

3.6.6 Tachyons

Pseudoscalars

As was first noticed in [122], Dirac gauginos are often companied with tachyonic states. The first place these arise can arise is in the pseudo–scalar sector. For strong interactions this is transparent as the pseudo–sgluon has a mass given by

$$m_{\sigma_{\widetilde{\mathrm{g}}}}^2 = m_{\widetilde{\mathrm{g}}}^2 + M_{\widetilde{\mathrm{g}}}^2 - b_{\widetilde{\mathrm{g}}}. \tag{3.6.180}$$

Clearly one needs to arrange $m^2_{\widetilde{g}} + M^2_{\widetilde{g}} - b_{\widetilde{g}} > 0$ in order to not break CP and give gluons mass. From a phenomenological point of view this is just a simple algebraic constraint, but from a UV perspective this is surprisingly difficult to achieve:

- Taking $M^2_{\widetilde{g}}$ large introduces $\mathrm{U}(1)_R$ symmetry breaking into the picture and generically generates Majorana gaugino masses, introducing phenomenological consequences that we are trying to avoided by making the gluino Dirac in the first place,

- Taking $b_{\widetilde{g}}$ negative just makes the sgluon tachyonic

$$m^2_{\phi_{\widetilde{g}}} = m^2_{\widetilde{g}} + 4\, m^2_{D\widetilde{g}} + M^2_{\widetilde{g}} + b_{\widetilde{g}}. \tag{3.6.181}$$

  From a model building perspective it is also difficult to argue why $b_{\widetilde{g}}$ could be small whilst keeping $m^2_{D\widetilde{g}}$ large because although $m_{D\widetilde{g}}$ and $b_{\widetilde{g}}$ are typically generated at the same loop order, $m_{D\widetilde{g}}$ has mass dimension one, $b_{\widetilde{g}}$ has mass dimension two and one therefore finds $b_{\widetilde{g}} \sim (16\,\pi)^2\, m^2_{D\widetilde{g}}$. This is known as the $m_D - b_M$ problem [175] and is discussed further in Chapter 4.

Slightly more obscured is the situation with electroweak pseudo–scalars. The corresponding mass matrix is

$$m^2_\sigma = \begin{pmatrix} m^2_{uu} & m^2_{du} & m^2_{\widetilde{B}u} & m^2_{\widetilde{W}u} \\ m^2_{du} & m^2_{dd} & m^2_{\widetilde{B}d} & m^2_{\widetilde{W}d} \\ m^2_{\widetilde{B}u} & m^2_{\widetilde{B}d} & m^2_{\widetilde{B}\widetilde{B}} & m^2_{\widetilde{W}\widetilde{B}} \\ m^2_{\widetilde{W}u} & m^2_{\widetilde{W}d} & m^2_{\widetilde{W}\widetilde{B}} & m^2_{\widetilde{W}\widetilde{W}} \end{pmatrix} \tag{3.6.182}$$

where

$$m^2_{uu} = m^2_{\mathrm{H}_u} + (\mu^{\mathrm{eff}})^2 + \frac{m^2_{\mathrm{Z}}}{2}\, s^2_\beta + \frac{\Delta}{2}\, c^2_\beta + g_1\, m_{D\widetilde{B}}\, v_{\widetilde{B}} - g_2\, m_{D\widetilde{W}}\, v_{\widetilde{W}}, \tag{3.6.183}$$

$$m^2_{du} = m^2_{A_0}\, s_\beta\, c_\beta, \tag{3.6.184}$$

$$m^2_{\widetilde{B}u} = -v\, c_\beta \left(\frac{M_{\widetilde{B}}\, \lambda_{\widetilde{B}} - a_{\widetilde{B}}}{\sqrt{2}} + \kappa_{\widetilde{B}}\, \lambda_{\widetilde{B}}\, v_{\widetilde{B}} \right), \tag{3.6.185}$$

$$m^2_{\widetilde{W}u} = -\frac{v\, c_\beta}{\sqrt{2}} \left(\lambda_{\widetilde{W}}\, M_{\widetilde{W}} - a_{\widetilde{W}} \right), \tag{3.6.186}$$

$$m^2_{dd} = m^2_{\mathrm{H}_u} + (\mu^{\mathrm{eff}})^2 + \frac{m^2_{\mathrm{Z}}}{2}\, c^2_\beta + \frac{\Delta}{2}\, s^2_\beta - g_1\, m_{D\widetilde{B}}\, v_{\widetilde{B}} + g_2\, m_{D\widetilde{W}}\, v_{\widetilde{W}}, \tag{3.6.187}$$

$$m^2_{\widetilde{\mathrm{B}}d} = -v\, s_\beta \left(\frac{M_{\widetilde{\mathrm{B}}}\, \lambda_{\widetilde{\mathrm{B}}} - a_{\widetilde{\mathrm{B}}}}{\sqrt{2}} + \kappa_{\widetilde{\mathrm{B}}}\, \lambda_{\widetilde{\mathrm{B}}}\, v_{\widetilde{\mathrm{B}}} \right), \tag{3.6.188}$$

$$m^2_{\widetilde{\mathrm{W}}d} = -\frac{v\, s_\beta}{\sqrt{2}} \left(\lambda_{\widetilde{\mathrm{W}}}\, M_{\widetilde{\mathrm{W}}} - a_{\widetilde{\mathrm{W}}} \right), \tag{3.6.189}$$

$$m^2_{\widetilde{\mathrm{B}}\widetilde{\mathrm{B}}} = m^2_{\phi_{\widetilde{\mathrm{B}}}} + \lambda_{\widetilde{\mathrm{B}}} v^2 \left(\kappa_{\widetilde{\mathrm{B}}} s_\beta c_\beta + \frac{\lambda_{\widetilde{\mathrm{B}}}}{2} \right) + \sqrt{2} v_{\widetilde{\mathrm{W}}} \left(\kappa_{\widetilde{\mathrm{B}}} M_{\widetilde{\mathrm{B}}} - a_{\kappa_{\widetilde{\mathrm{B}}}} \right) + \kappa^2_{\widetilde{\mathrm{B}}} v^2_{\widetilde{\mathrm{B}}}, \tag{3.6.190}$$

$$m^2_{\widetilde{\mathrm{W}}\widetilde{\mathrm{B}}} = \frac{\lambda_{\widetilde{\mathrm{B}}}\, \lambda_{\widetilde{\mathrm{W}}}}{2}\, v^2, \tag{3.6.191}$$

$$m^2_{\widetilde{\mathrm{W}}\widetilde{\mathrm{W}}} = m^2_{\phi_{\widetilde{\mathrm{W}}}} + \frac{\lambda^2_{\widetilde{\mathrm{W}}}}{2}\, v^2. \tag{3.6.192}$$

From a UV perspective, the problems with the pseudo–sbino and pseudo–swino are as in the pseudos–gluon case. In addition to this, if we substitute the vacuum solutions 3.6.137 to 3.6.140 into the mass matrix 3.6.182 we find

$$m^2_{uu} = m^2_{A_0}\, c^2_\beta, \tag{3.6.193}$$

$$m^2_{dd} = m^2_{A_0}\, s^2_\beta, \tag{3.6.194}$$

$$\begin{aligned} m^2_{\widetilde{\mathrm{B}}\widetilde{\mathrm{B}}} = {} & -4\, m^2_{D\widetilde{\mathrm{B}}} - 2\, b_{\widetilde{\mathrm{B}}} + v^2 \Bigg\{ \lambda_{\widetilde{\mathrm{B}}} \left(\frac{\lambda_{\widetilde{\mathrm{B}}}}{2} + \kappa_{\widetilde{\mathrm{B}}}\, s_{2\beta} \right) + \frac{1}{v_{\widetilde{\mathrm{B}}}} \Bigg[\frac{s_{2\beta}}{2\sqrt{2}} \Big(\lambda_{\widetilde{\mathrm{B}}}\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}} \Big) \\ & + \frac{g_1}{2}\, m_{D\widetilde{\mathrm{B}}}\, c_{2\beta} - \frac{\lambda_{\widetilde{\mathrm{B}}}}{\sqrt{2}}\, \mu^{\mathrm{eff}} \Bigg] \Bigg\} - \frac{v_{\widetilde{\mathrm{B}}}}{\sqrt{2}} \left(\kappa_{\widetilde{\mathrm{B}}}\, M_{\widetilde{\mathrm{B}}} + 3\, a_{\widetilde{\mathrm{B}}} \right), \end{aligned} \tag{3.6.195}$$

$$\begin{aligned} m^2_{\widetilde{\mathrm{W}}\widetilde{\mathrm{W}}} = {} & -4\, m^2_{D\widetilde{\mathrm{W}}} - 2\, b_{\widetilde{\mathrm{W}}} + v^2 \Bigg\{ \frac{\lambda^2_{\widetilde{\mathrm{W}}}}{2} + \frac{1}{v_{\widetilde{\mathrm{W}}}} \Bigg[\frac{s_{2\beta}}{2\sqrt{2}} \Big(\lambda_{\widetilde{\mathrm{W}}}\, M_{\widetilde{\mathrm{W}}} + a_{\widetilde{\mathrm{W}}} \Big) \\ & - \frac{g_2}{2}\, m_{D\widetilde{\mathrm{W}}}\, c_{2\beta} - \frac{\lambda_{\widetilde{\mathrm{W}}}}{\sqrt{2}}\, \mu^{\mathrm{eff}} \Bigg] \Bigg\}, \end{aligned} \tag{3.6.196}$$

where elements that haven't been indicated are unchanged from their out–of-vacuum solutions. The difficulty even from a phenomenological point of view is now highlighted by the on–diagonal elements 3.6.195 and 3.6.196. In models with large Dirac gaugino masses, the terms $-2\, b^2_{\widetilde{\mathrm{B}}}$, $-2\, b^2_{\widetilde{\mathrm{W}}}$, $-4\, m^2_{D\widetilde{\mathrm{B}}}$ and $-4\, m^2_{D\widetilde{\mathrm{W}}}$ dominate the expressions with the other dimensionful quantities tied to the electroweak scale. Also noting the limit on $v_{\widetilde{\mathrm{W}}}$ from $\Delta\rho$ (see eq. 3.6.178) the term

$$-\frac{g_2}{2}\, c_{2\beta}\, m_{D\widetilde{\mathrm{W}}}\, \frac{v^2}{v_{\widetilde{\mathrm{W}}}}$$

can also be large and negative, depending on the size of t_β and $\text{sign}(m_{D\widetilde{\mathrm{W}}})$. Although it can be arranged for the eigenvalues of the matrix 3.6.182 to be positive[10], it is certainly not automatic and can easily lead to tachyons without even worrying about the problems of building a UV model that realistically achieves such a set of parameters.

Squarks and sleptons

The potential tachyonic effects aren't confined to the pseudo–scalar sector. If we consider the effect of the *supersoft* operator on the D terms *after* VEVs have been acquired for $\Phi_{\widetilde{\mathrm{B}}}$ and $\Phi_{\widetilde{\mathrm{W}}}$ then we find

$$-D_Y = g_1 \left[\frac{1}{6}\left(|\widetilde{\mathrm{u}}_L|^2 + |\widetilde{\mathrm{d}}_L|^2\right) - \frac{2}{3}|\widetilde{\mathrm{u}}_R|^2 + \frac{1}{3}|\widetilde{\mathrm{d}}_R|^2 + \cdots\right] + 2\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}}, \quad (3.6.197)$$

$$-D_L^3 = \frac{1}{2}\, g_2 \left(q^\dagger\, T^3\, q + \ell^\dagger\, T^3\, \ell + \cdots\right) + 2\, v_{\widetilde{\mathrm{W}}}\, m_{D\widetilde{\mathrm{W}}}. \quad (3.6.198)$$

The cross terms in the potential then give positive and negative contributions to the squark and slepton mass matrices.

Up squarks: In the basis $(\widetilde{\mathrm{u}}_L, \widetilde{\mathrm{u}}_R)$ the mass matrix matrix $m^2_{\widetilde{\mathrm{u}}}$ receives a shift in block–diagonal form

$$m^2_{\widetilde{\mathrm{u}}} \to m^2_{\widetilde{\mathrm{u}}} + \begin{pmatrix} \frac{g_1}{3}\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} + g_2\, v_{\widetilde{\mathrm{W}}}\, m_{D\widetilde{\mathrm{W}}} & 0_{3\times 3} \\ 0_{3\times 3} & -\frac{4\, g_1}{3}\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} \end{pmatrix}. \quad (3.6.199)$$

Down squarks: In the basis $(\widetilde{\mathrm{d}}_L, \widetilde{\mathrm{d}}_R)$ the mass matrix matrix $m^2_{\widetilde{\mathrm{d}}}$ recieves a shift in block–diagonal form

$$m^2_{\widetilde{\mathrm{d}}} \to m^2_{\widetilde{\mathrm{d}}} + \begin{pmatrix} \frac{g_1}{3}\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} - g_2\, v_{\widetilde{\mathrm{W}}}\, m_{D\widetilde{\mathrm{W}}} & 0_{3\times 3} \\ 0_{3\times 3} & \frac{2\, g_1}{3}\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} \end{pmatrix}. \quad (3.6.200)$$

[10]Except of course for the massless Goldstone mode that gets eaten by the Z boson.

Sleptons: In the basis $(\widetilde{\mathrm{e}}_L, \widetilde{\mathrm{e}}_R)$ the mass matrix matrix $m^2_{\widetilde{\mathrm{e}}}$ recieves a shift in block–diagonal form

$$m^2_{\widetilde{\mathrm{e}}} \to m^2_{\widetilde{\mathrm{e}}} + \begin{pmatrix} -g_1\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} - g_2\, v_{\widetilde{\mathrm{W}}}\, m_{D\widetilde{\mathrm{W}}} & 0_{3\times 3} \\ 0_{3\times 3} & 2\, g_1\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} \end{pmatrix}. \tag{3.6.201}$$

Sneutrinos: The mass matrix matrix $m^2_{\widetilde{\nu}}$ recieves a shift in block–diagonal form

$$m^2_{\widetilde{\nu}} \to m^2_{\widetilde{\nu}} - g_1\, v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} + g_2\, v_{\widetilde{\mathrm{W}}}\, m_{D\widetilde{\mathrm{W}}}. \tag{3.6.202}$$

Consequently, when building models where the ESPs can acquire VEVs, one must be wary in these models then if the combination of the Dirac bino and wino masses with their corresponding VEVs is sufficiently large

$$v_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}} \qquad \text{or} \qquad v_{\widetilde{\mathrm{W}}}\, m_{D\widetilde{\mathrm{W}}} \gtrsim m^2_{\mathrm{SUSY}} \tag{3.6.203}$$

where m_{SUSY} is the characteristic scale of superpartner masses, then certain particles can be driven tachyonic.

3.6.7 Higgs quartic coupling suppression

Tree level thresholds via equations of motion

There is another interesting (and potentially dangerous) effect induced by the supersoft operator. If we just consider an empty superpotential and adding the supersoft operator for a general non–abelian gauge theory with the ESP $\Phi = (\phi + i\,\sigma)/\sqrt{2}$. At low energies, the only term in the lagrangian involving Φ before integrating out the D term is

$$\mathcal{L}_\phi = 2\, m_D\, \phi^a\, D^a. \tag{3.6.204}$$

The field ϕ^a then acts as a *Lagrange multiplier* for D^a such that when we integrate out ϕ^a due to its large mass m_D

$$\frac{\partial \mathcal{L}_\phi}{\partial \phi^a} = 2\, m_D\, D^a = 0. \tag{3.6.205}$$

Below m_D of ϕ^a we should then switch to an EFT where the corresponding D^a vanish. Of course, D^a has its own solution that is determined *before* ϕ^a because D^a

is an *auxiliary field* and therefore *infinitely massive*. Whatever solution is found for for D^a will then be set to zero upon integrating out ϕ^a.

This is a problem because if we add in Dirac masses for the wino and bino (but no further interactions for them) then below their corresponding masses, the D terms for $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$ will vanish. In particular, this causes the quartic terms that are responsible for giving the SM–like Higgs boson its mass to vanish

$$V_Y \supset \frac{1}{8}\, g_1^2 \left(|\mathrm{H_u^0}|^2 - |\mathrm{H_d^0}|^2\right)^2 \to 0, \quad V_L \supset \frac{1}{4}\, g_2^2 \left(\mathrm{H_u^\dagger} T^i \mathrm{H_u} + \mathrm{H_d^\dagger} T^i \mathrm{H_d}\right) \to 0. \quad (3.6.206)$$

Let's consider what happens if we allow a mass squared for ϕ

$$\mathcal{L}_\phi = 2\, m_D\, \phi^a\, D^a - \tilde{V}(\phi), \qquad \tilde{V}(\phi) = \frac{1}{2}\, m_\phi^2\, \phi^a\, \phi^a. \qquad (3.6.207)$$

The equations of motion set

$$\frac{\partial \mathcal{L}_\phi}{\partial \phi^a} = 2\, m_D\, D^a - m_\phi^2\, \phi^a = 0, \qquad (3.6.208)$$

and so for the D terms we find

$$\mathcal{L}_D = \frac{1}{2}\, \frac{m_\phi^2 + 4\, m_D^2}{m_\phi^2}\, D^a\, D^a + g\, D^a\, \phi_l^\dagger\, T^a_{\mathbf{r}_{\phi_l}}\, \phi_l, \qquad (3.6.209)$$

where ϕ_l are the light fields with mass $m_{\phi_l}^2 \ll m_\phi^2 + 4\, m_D^2$. The scalar potential after solving for the D terms is

$$V = \frac{1}{2}\, D^a D_i^a = \frac{1}{2}\, \frac{m_\phi^2}{m_\phi^2 + 4\, m_D^2}\, g^2 \left(\phi_l^\dagger\, T^a_{\mathbf{r}_{\phi_l}}\, \phi_l\right)^2 \qquad (3.6.210)$$

and we see the rescaling behaviour

$$(\text{D terms}) \to \frac{m_\phi^2}{m_\phi^2 + 4\, m_D^2}\, (\text{D terms}) \qquad (3.6.211)$$

with the correct limiting behaviour:

- Taking $m_D \to 0$ removes the supersoft operator causing the problem in the first place and the suppression vanishes,
- Taking $m_\phi \to 0$ removes the additional interactions for ϕ and so the quartic

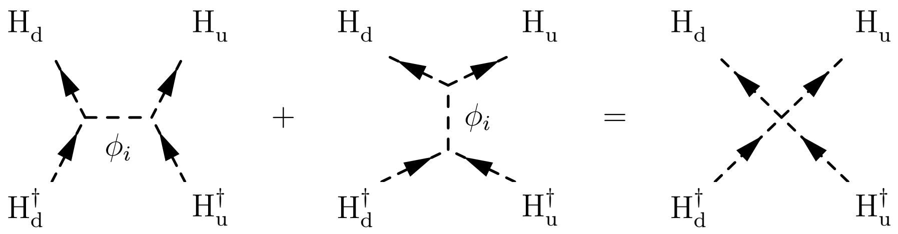


Figure 3.3: Tree level matching conditions for λ_3 upon integrating out $\phi_1 \equiv \phi_{\widetilde{\mathrm{B}}}$ and $\phi_2 \equiv \phi_{\widetilde{\mathrm{W}}}$ at $\mu^2 = m^2_{\phi_{\widetilde{\mathrm{B}}}}$ and $\mu^2 = m^2_{\phi_{\widetilde{\mathrm{W}}}}$ respectively.

couplings vanish as before.

Tree level threshold matching

An alternative way to see what happens (that is also easier to do for a more general choice of $\tilde{V}(\phi)$) is to switch to the EFT by matching coefficients order by order in perturbation theory as was discussed in Section 2.2. Our main interest is the effect on the Higgs potential. The most general gauge invariant potential that we can write is that of the 2HDM

$$\begin{aligned} V_{\text{eff}} = &(m^2_{\mathrm{H_u}} + \mu^2)|\mathrm{H_u}|^2 + (m^2_{\mathrm{H_d}} + \mu^2)|\mathrm{H_d}|^2 - \big(m^2_{12}\, H_u \cdot H_d + \text{h.c.}\big) \\ &+ \frac{\lambda_1}{2}\,|H_d|^4 + \frac{\lambda_2}{2}\,|H_u|^4 + \lambda_3\,|H_d|^2\,|H_u|^2 + \lambda_4\,|H_d \cdot H_u|^2 \\ &+ \left[\frac{\lambda_5}{2}\,(H_d \cdot H_u)^2 + \big(\lambda_6\,|H_d|^2 + \lambda_7\,|H_u|^2\big)(H_d \cdot H_u) + \text{h.c.}\right]. \end{aligned} \tag{3.6.212}$$

At the point where we integrate out the real ESPs, the λ_i have boundary conditions at the ESP masses given

$$\lambda_i = \lambda_i^{(\mathrm{MSSM})} + \lambda_i^{(\mathrm{DG})} + \lambda_i^{(1,\,\mathrm{tree})} + \lambda_i^{(2,\,\mathrm{tree})} + \dots, \tag{3.6.213}$$

where the $\lambda_i^{(\mathrm{MSSM})}$ are the standard quartic D terms, the $\lambda_i^{(\mathrm{DG})}$ are contributions from the new Lagrangian couplings in the superpotential from the ESPs

$$\lambda_1^{(\mathrm{MSSM})} = \frac{g_1^2 + g_2^2}{4} \qquad \lambda_1^{(\mathrm{DG})} = 0 \tag{3.6.214}$$

$$\lambda_2^{(\mathrm{MSSM})} = \frac{g_1^2 + g_2^2}{4} \qquad \lambda_2^{(\mathrm{DG})} = 0 \tag{3.6.215}$$

$$\lambda_3^{(\mathrm{MSSM})} = \frac{-g_1^2 + g_2^2}{4} \qquad \lambda_3^{(\mathrm{DG})} = 2\,\lambda_{\widetilde{\mathrm{W}}}^2 \tag{3.6.216}$$

$$\lambda_4^{(\mathrm{MSSM})} = -\frac{g_2^2}{2} \qquad \lambda_4^{(\mathrm{DG})} = \lambda_{\widetilde{\mathrm{B}}}^2 - \lambda_{\widetilde{\mathrm{W}}}^2 \tag{3.6.217}$$

and

$$\lambda_5^{(\mathrm{MSSM})} = \lambda_6^{(\mathrm{MSSM})} = \lambda_7^{(\mathrm{MSSM})} = \lambda_5^{(\mathrm{DG})} = \lambda_6^{(\mathrm{DG})} = \lambda_7^{(\mathrm{DG})} = 0. \tag{3.6.218}$$

The $\lambda_i^{(1,\,\mathrm{tree})}$ and $\lambda_i^{(2,\,\mathrm{tree})}$ arise due to matching conditions from integrating out $\phi_{\widetilde{\mathrm{B}}}$ and $\phi_{\widetilde{\mathrm{W}}}$ respectively. A diagrammatic representation of the matching conditions for $\lambda_3^{(i,\mathrm{tree})}$ is shown in fig. 3.3. Using `FeynArts` and `FormCalc` [213], we find that the $\lambda_i^{(1,\,\mathrm{tree})}$ are:

$$m_{\phi_{\widetilde{\mathrm{B}}}}^2\,\lambda_1^{(1)} = -\left[g_1^2\, m_{D\widetilde{\mathrm{B}}}^2 + 2\sqrt{2}\, g_1\, \lambda_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}}\, \mu + 2\,(\lambda_{\widetilde{\mathrm{B}}}\mu)^2\right], \tag{3.6.219}$$

$$m_{\phi_{\widetilde{\mathrm{B}}}}^2\,\lambda_2^{(1)} = -\left[g_1^2 m_{D\widetilde{\mathrm{B}}}^2 - 2\sqrt{2}\, g_1\, \lambda_{\widetilde{\mathrm{B}}}\, m_{D\widetilde{\mathrm{B}}}\, \mu + 2\,(\lambda_{\widetilde{\mathrm{B}}}\mu)^2\right], \tag{3.6.220}$$

$$m_{\phi_{\widetilde{\mathrm{B}}}}^2\,\lambda_3^{(1)} = g_1^2 m_{D\widetilde{\mathrm{B}}}^2 - 2\,(\lambda_{\widetilde{\mathrm{B}}}\mu)^2, \tag{3.6.221}$$

$$m_{\phi_{\widetilde{\mathrm{B}}}}^2\,\lambda_4^{(1)} = -(\lambda_{\widetilde{\mathrm{B}}}\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}})^2, \tag{3.6.222}$$

$$m_{\phi_{\widetilde{\mathrm{B}}}}^2\,\lambda_5^{(1)} = (\lambda_{\widetilde{\mathrm{B}}}\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}})^2, \tag{3.6.223}$$

$$m_{\phi_{\widetilde{\mathrm{B}}}}^2\,\lambda_6^{(1)} = (\lambda_{\widetilde{\mathrm{B}}}\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}})(2\,\lambda_{\widetilde{\mathrm{B}}}\mu + \sqrt{2}\, g_1\, m_{D\widetilde{\mathrm{B}}}), \tag{3.6.224}$$

$$m_{\phi_{\widetilde{\mathrm{B}}}}^2\,\lambda_7^{(1)} = (\lambda_{\widetilde{\mathrm{B}}}\, M_{\widetilde{\mathrm{B}}} + a_{\widetilde{\mathrm{B}}})(2\,\lambda_{\widetilde{\mathrm{B}}}\mu - \sqrt{2}\, g_1\, m_{D\widetilde{\mathrm{B}}}), \tag{3.6.225}$$

and the $\lambda_i^{(2,\,\mathrm{tree})}$ are:

$$m^2_{\phi_{\widetilde{W}}}\,\lambda_1^{(2)} = -\left[g_2^2\, m^2_{D\widetilde{W}} + 2\,(\lambda_{\widetilde{W}}\mu)^2\right], \tag{3.6.226}$$

$$m^2_{\phi_{\widetilde{W}}}\,\lambda_2^{(2)} = -\left[g_2^2\, m^2_{D\widetilde{W}} + 2\,(\lambda_{\widetilde{W}}\mu)^2\right], \tag{3.6.227}$$

$$m^2_{\phi_{\widetilde{W}}}\,\lambda_3^{(2)} = -\left[(\lambda_{\widetilde{W}}\, M_{\widetilde{W}} + a_{\widetilde{W}})^2 + g_2^2\, m^2_{D\widetilde{W}} - 2\,(\lambda_{\widetilde{W}}\,\mu)^2\right], \tag{3.6.228}$$

$$m^2_{\phi_{\widetilde{W}}}\,\lambda_4^{(2)} = (\lambda_{\widetilde{W}}\, M_{\widetilde{W}} + a_{\widetilde{W}})^2 + 4\,(g_2^2\, m^2_{D\widetilde{W}} - \lambda^2_{\widetilde{W}}\,\mu^2), \tag{3.6.229}$$

$$m^2_{\phi_{\widetilde{W}}}\,\lambda_5^{(2)} = (\lambda_{\widetilde{W}}\, M_{\widetilde{W}} + a_{\widetilde{W}})^2, \tag{3.6.230}$$

$$m^2_{\phi_{\widetilde{W}}}\,\lambda_6^{(2)} = 0, \tag{3.6.231}$$

$$m^2_{\phi_{\widetilde{W}}}\,\lambda_7^{(2)} = 2\,(a_{\widetilde{W}} + \lambda_{\widetilde{W}} M_{\widetilde{W}})\,\lambda_{\widetilde{W}}\,\mu. \tag{3.6.232}$$

The remaining "..." in eq. 3.6.213 are for higher order matchings that we aren't important to consider here. Now we can see more generally that each quartic coupling can be saved from being set to zero by taking a non–zero superpotential. If we just take λ_1 as an example (ignoring higher order corrections for the remainder of this discussion)

$$\lambda_1 = \lambda_1^{(\mathrm{MSSM})} + \lambda_1^{(\mathrm{DG})} + \lambda_1^{(1,\,\mathrm{tree})} + \lambda_1^{(2,\,\mathrm{tree})} \tag{3.6.233}$$

$$= g_1^2\left(\frac{1}{4} - \frac{m^2_{D\widetilde{B}}}{m^2_{\phi_{\widetilde{B}}}}\right) + g_2^2\left(\frac{1}{4} - \frac{m^2_{D\widetilde{W}}}{m^2_{\phi_{\widetilde{W}}}}\right) - 2\left[\frac{\sqrt{2}\,g_1\,\lambda_{\widetilde{B}}\,m_{D\widetilde{B}}\,\mu + (\lambda_{\widetilde{B}}\mu)^2}{m^2_{\phi_{\widetilde{B}}}} + \frac{(\lambda_{\widetilde{W}}\mu)^2}{m^2_{\phi_{\widetilde{W}}}}\right]. \tag{3.6.234}$$

The first line of eq. 3.6.234 vanishes if we only have the supersoft operator and in the absence of a superpotential we would then see $\lambda_1 \to 0$ as before. Now we see that either giving the ESPs sources of mass not from the supersoft operator *or* having a non–trivial superpotential for the ESPs will result in non–zero λ_1 upon integrating out $\phi_{\widetilde{B}}$ and $\phi_{\widetilde{W}}$. The same behaviour can be observed in the remaining λ_i. One needs to be careful though to make sure that the additional contributions to the superpotential don't destabilise the vacuum upon integrating out, i.e. we should have

$$g_1^2\left(\frac{1}{4} - \frac{m^2_{D\widetilde{B}}}{m^2_{\phi_{\widetilde{B}}}}\right) + g_2^2\left(\frac{1}{4} - \frac{m^2_{D\widetilde{W}}}{m^2_{\phi_{\widetilde{W}}}}\right) - 2\left[\frac{\sqrt{2}\,g_1\,\lambda_{\widetilde{B}}\,m_{D\widetilde{B}}\,\mu + (\lambda_{\widetilde{B}}\mu)^2}{m^2_{\phi_{\widetilde{B}}}} + \frac{(\lambda_{\widetilde{W}}\mu)^2}{m^2_{\phi_{\widetilde{W}}}}\right] > 0$$

and similarly for the other λ_i.

A curious use of this zeroing quartic terms could be to explain the existence of a zero in the SM quartic coupling λ at a very high scale [214–220]. If the ESPs are given an extremely large mass, then at this scale, what would become the SM quartic coupling is set to zero and can then start running according to its RGEs. This possibility was explored in the *split Dirac gaugino* scenario [182], where Dirac gauginos are given masses $m_D \sim 10^8$ to 10^{11} GeV, the corresponding scalars have their usual loop–suppressed masses, and the Higgsino is much lighter with a mass dictated by μ.

4

Constrained Dirac gluino mediation

The Guide is definitive. Reality is frequently inaccurate.

– Douglas Adams, *The Restaurant At End Of The Universe*

This chapter is based on my single–authored work [1]. The text here follows it closely.

4.1 Overview

As was discussed in Section 2.3.10, UV completions of the MSSM are in the middle of a naturalness crisis, with the two driving factors being a rather heavy for the MSSM SM–like Higgs boson mass *and* direct limits on sparticle masses. We saw in Chapter 3 how adding Dirac gaugino masses can aleviate many of the problems the MSSM has. This can be summarised as follows:

- *Supersoftness* (see Section 3.4) reduces the logarithmic dependence of EWSB upon the UV parameters,
- *Supersafeness* (see Section 3.5) decreases the direct search constrains on sparticle masses,
- Additional Higgs F terms (see Section 3.6.4) can give the correct Higgs mass at tree level, removing the need to rely on heavy sparticles.

We also saw in Chapter 3 that introducing electroweak Dirac gauginos came with a set of problems, including:

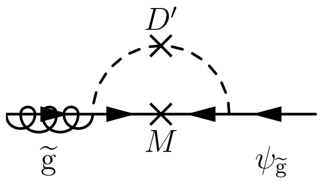
- Contributions to $\Delta\rho$ at tree level,
- A huge number of free parameters,
- Very complicated EWSB breaking,
- Tachyons.

Our aim in this chapter is to construct a simple UV model along the lines of the CMSSM or Constrained General Gauge Mediation (CGGM), such that a reasonable phenomenological study can be done of a theory that exhibits the signature properties of Dirac gauginos. The choice we make is to give a Dirac mass to the gluino *only* (by providing the appropriate χSF content), whilst not introducing the field content for electroweak Dirac gauginos. Although this drops the extra tree level contributions the Higgs mass, this sidesteps most of the issues we saw in Chapter 3. At the same time, we maintain the *supersoftness* and *supersafness* where they really count — the $\mathrm{SU}(3)_\mathrm{C}$ sector.

4.2 Generating a gluino mass

The simplest known way of generating a Dirac gluino mass $m_{D\widetilde{g}}$ is to generate it at the messenger scale M by integrating out the messenger sector coupled to a source of D term breaking (see Section 2.3.7)

$$\delta m_{D\widetilde{g}} =$$



$$= \frac{y\, g_3}{16\,\pi^2}\frac{D'}{M}, \tag{4.2.1}$$

where D' is the SUSY breaking D-term VEV of a $\mathrm{U}(1)'$ gauge group in the hidden sector: $\langle \mathcal{W}'_\alpha \rangle = \theta_\alpha\, D'$ and M is the messenger scale and y is couples vector-like messengers $(\Phi, \bar{\Phi})$ to the chiral field $\Phi_{\widetilde{g}}$ containing the right handed component of the Dirac gluino $\widetilde{g}_R = (\psi_{\widetilde{g}})^\dagger$

$$W_{\text{Mess}} = \sqrt{2}\, y\, \bar{\Phi}\, \Phi_{\widetilde{g}}\, \Phi + M\, \bar{\Phi}\, \Phi. \tag{4.2.2}$$

This theory is RG evolved to the physical Dirac gluino mass where we must switch to an effective theory with the gluino and the sgluons integrated out. This generates one loop threshold corrections for the squarks given in eq. 3.4.59. The theory is then RG evolved to the SUSY scale m_{SUSY} which we take to be the geometric stop mass

$$m_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \tag{4.2.3}$$

where the renormalisation scale dependence for the calculation of the spectrum is minimised [221–223].

If the majority of the squark mass is generated through integrating out the gluino and its corresponding scalar degrees of freedom, the sensitivity of electroweak parameters to the parameters defined at M is reduced as the most sensitive period of running is now effectively from $m_{D\tilde{g}}$ rather than M to m_{SUSY}. It is straightforward to give Dirac masses to all of the gauginos in the MSSM in this way, each accompanied by analogous threshold corrections to the scalar spectrum, though this can introduces further complications such as tachyons and electroweak precision measurements.

We will first construct two simple models that that have the following properties:

- Natural from the point of view of EWSB — electroweak sparticles all at electroweak scale.
- A minimal set of free parameters in the UV.
- *Supersoftess* to reduce fine tuning.
- *Supersafeness* to alleviate collider bounds.

We will then implement these models and the supersoft mechanism into a spectrum generator and perform a study, discussing the consequences for hadron collider phenomenology and fine tuning.

4.3 Constrained Dirac gluino mediation

4.3.1 Overview

As the LHC is a proton-proton collider, the non-observation of SUSY, and particularly of gluinos, indicates that the strongly interacting SUSY particles should be

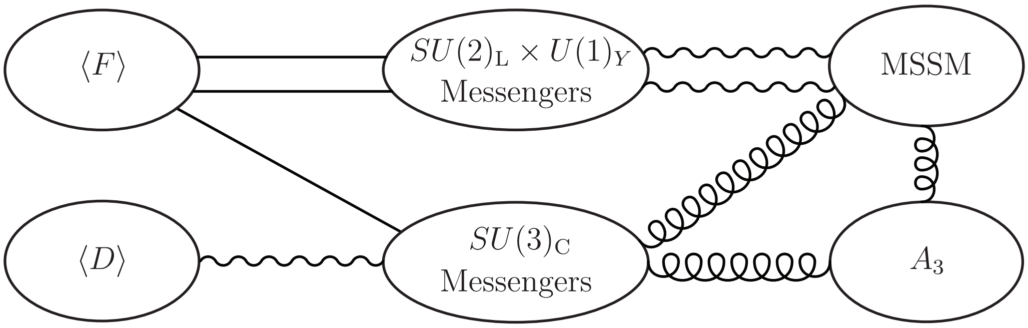


Figure 4.1: The different sectors used in our setup.

moderately heavy to evade exclusion. To achieve this, we supplement the CMSSM and CGGM with a Dirac gluino. We will refer to these scenarios as *Constrained Dirac gluino mediation*. Due to the one loop supersoft nature of the Dirac gauginos, the higher scale of the strong sector is not transferred to the electroweak sector through RG running, and so electroweak sparticles can remain light (depending on the region of parameter space). Specifically, we couple $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ to either the CMSSM or the CGGM, and couple only $\mathrm{SU}(3)_{\mathrm{C}}$ to a sector of D term breaking to the mechanism of [129] (see fig. 4.1 for the CGGM setup). The field content is the same as the MSSM plus the ESP χSF $\Phi_{\tilde{\mathrm{g}}}$ detailed in table 4.1.

We will now recap the effects of integrating out a messenger sector in terms of the presence of D term SUSY breaking before moving on to discuss the full UV boundary conditions of the model.

4.3.2 Boundary conditions at the Messenger scale

D-term breaking effective operators

Fox, Nelson and Weiner (FNW) [129] identified two operators generated by D-term breaking in the presence of ESPs. The first is our *supersoft* operator 3.4.41 that we rewrite for convenience, and the other generates the sgluon b term,

$$\mathcal{L}^{(1)}_{\text{Supersoft}} = \sqrt{2}\int d^2\theta\, \frac{\mathcal{W}'\cdot\mathcal{W}^a_3\, A^a_3}{M} = \frac{D'}{M}\left(\widetilde{\mathrm{g}}^a\cdot\tilde{A}^a_3+\sqrt{2}\,A^a_3\,D^a_3\right)+\cdots$$
$$= m_{D\widetilde{\mathrm{g}}}\left(\widetilde{\mathrm{g}}^a\cdot\tilde{A}^a_3+\sqrt{2}\,A^a_3\,D^a_3\right)+\cdots, \quad (4.3.4)$$

$$\mathcal{L}^{(2)}_{\text{Supersoft}} = \int d^2\theta\, \frac{\mathcal{W}'\cdot\mathcal{W}'\,A^a_3\,A^a_3}{M^2} = \left(\frac{D'}{M}\right)^2\Phi^a_{\widetilde{\mathrm{g}}}\,\Phi^a_{\widetilde{\mathrm{g}}}$$
$$= b_{\widetilde{\mathrm{g}}}\,\Phi^a_{\widetilde{\mathrm{g}}}\,\Phi^a_{\widetilde{\mathrm{g}}}, \quad (4.3.5)$$

where M is the scale of physics integrated out to generate the operators in eqs. 4.3.4, and 4.3.4, and D' is the VEV of a hidden sector $U(1)'$: $\langle\mathcal{W}'_\alpha\rangle = \theta_\alpha\, D'$. The "$\cdots$" in eq. 4.3.4 correspond to operators that vanish upon including their hermitian conjugates. In a messenger setup, both of these operators are generated at one loop, leading to a tachyon in the spectrum. Indeed, this is the original reason for abandoning these models [122]. There is one further operator generated at two loops by D-term breaking identified by Csáki et al. [175]

$$\mathcal{L}^{(1)}_{\text{Not supersoft}} = \int d^4\theta\, \frac{S^\dagger e^V S+\tilde{S}^\dagger e^{-V}\tilde{S}}{M^2}\,\Phi^\dagger_{\widetilde{\mathrm{g}}}\,\Phi_{\widetilde{\mathrm{g}}} = \left(\frac{D'}{M}\right)^2\Phi^\dagger_{\widetilde{\mathrm{g}}}\Phi_{\widetilde{\mathrm{g}}}$$
$$= m^2_{\Phi_{\widetilde{\mathrm{g}}}}\,\Phi^\dagger_{\widetilde{\mathrm{g}}}\Phi_{\widetilde{\mathrm{g}}} \quad (4.3.6)$$

where S and $\tilde{S}$ are singlets under the SM but charged under the $U(1)'$. These give rise to the non-vanishing $D' \propto |S|^2 - |\tilde{S}|^2$ and break the $U(1)'$ gauge symmetry. Note that the operator in eq. 4.3.6 is still picks out a coefficient $\sim (D'/M)^2$. Upon introducing messenger mixing, 4.3.6 is generated at one loop instead of two, and then the mixing freedom can be used to tune[1] 4.3.5 to be two loop size [137, 148, 175]. We then find the phenomenologically acceptable boundary conditions

$$m_{D\widetilde{\mathrm{g}}} \sim \frac{1}{16\pi^2}\frac{D'}{M}, \qquad m^2_{\Phi_{\widetilde{\mathrm{g}}}} \sim \frac{1}{16\pi^2}\left(\frac{D'}{M}\right)^2, \qquad b_{\widetilde{\mathrm{g}}} \sim \frac{\varepsilon}{16\pi^2}\left(\frac{D'}{M}\right)^2, \quad (4.3.7)$$

where $\varepsilon \sim 1/(16\pi^2)$ is a parameter that arises due to a cancellation between different contributions to $b_{\widetilde{\mathrm{g}}}$[2]. Note that the operator 4.3.6 is not supersoft at two loops,

[1]The tuning is typically $\mathcal{O}\left(\frac{1}{16\pi^2}\right)$.

[2]Strictly this is a cancellation between terms linear and quadratic in D', though this is not so important for our discussion.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\Phi_{\widetilde{g}}$	**Ad**	**1**	0

Table 4.1: Additional field content required to give a Dirac mass to the gluino.

however, and will generate

$$K_{\text{Sfermion}} = \int d^4\theta \, \frac{S^\dagger e^V S + \tilde{S}^\dagger e^{-V} \tilde{S}}{M^2} \, q^\dagger \, q \tag{4.3.8}$$

as can be observed from the squark two loop beta function

$$(16\pi^2)^2 \beta^{(2)}_{m^2_{q}} = 32 \, g_3^2 \, m^2_{\Phi_{\widetilde{g}}} + \cdots . \tag{4.3.9}$$

Supersoftness is then broken at two loops, rendering a UV sensitivity to the scale at which the Dirac gluino mass is generated [171].

Combined D and F term

Upon integrating out the messenger sector, we still have the MSSM superpotential 2.3.326 and a soft lagrangian conveniently decomposed into

$$\mathcal{L}_{\text{Soft}} = \mathcal{L}^F_{\text{Soft}} + \mathcal{L}^D_{\text{Soft}}. \tag{4.3.10}$$

$\mathcal{L}^F_{\text{Soft}}$ is the standard soft lagrangian of the MSSM 2.3.332 supplemented with $\Phi_{\widetilde{g}}$

$$\begin{aligned} -\mathcal{L}^F_{\text{Soft}} = & \frac{1}{2}\left(M_3 \, \widetilde{g} \cdot \widetilde{g} + M_2 \, \widetilde{W} \cdot \widetilde{W} + M_1 \, \widetilde{B} \cdot \widetilde{B} + \text{h.c.} \right) \\ & + \left(a_u \, \overline{u} \, q \cdot H_u - a_d \, \overline{d} \, q \cdot H_d - a_e \, \overline{e} \, \ell \cdot H_d + \text{h.c.} \right) \\ & + m^2_q |q|^2 + m^2_u |\overline{u}|^2 + m^2_d |\overline{d}|^2 + m^2_\ell |\ell|^2 + m^2_e |\overline{e}|^2 + m^2_{A^F_3} |\Phi_{\widetilde{g}}|^2 \\ & + \left(b_{\widetilde{g}} \, \Phi_{\widetilde{g}} \, \Phi_{\widetilde{g}} + \text{h.c.} \right) \\ & + m^2_{H_u} |H_u|^2 + m^2_{H_d} |H_d|^2 + (b_\mu \, H_u \cdot H_d + \text{h.c.}). \end{aligned} \tag{4.3.11}$$

The boundary conditions for these terms at m_{GUT} in the CMSSM are [48]

$$m^2_{\tilde{f}} = m_0^2, \qquad \tilde{f} = q, \overline{\mathrm{u}}, \overline{\mathrm{d}}, \ell, \mathrm{e}, \mathrm{H}_u, \mathrm{H}_d, A_3^F, \tag{4.3.12}$$

$$M_i = M_{1/2}, \qquad i = 1, 2, 3, \tag{4.3.13}$$

$$a_i\, y_i^{-1} = A_0, \qquad i = \mathrm{u}, \mathrm{d}, \mathrm{e}, \tag{4.3.14}$$

with b_μ and μ determined from EWSB at the low scale

$$m_{\mathrm{Z}}^2 = \frac{m^2_{\mathrm{H}_d} - m^2_{\mathrm{H}_u}}{\sqrt{1 - s^2_{2\beta}}} - m^2_{\mathrm{H}_u} - m^2_{\mathrm{H}_d} - 2\,|\mu|^2, \tag{4.3.15}$$

$$s_{2\beta} = \frac{2\, b_\mu}{m^2_{\mathrm{H}_u} + m^2_{\mathrm{H}_d} + 2\,|\mu|^2} \tag{4.3.16}$$

and $b_{\tilde{\mathrm{g}}} = 0$ for simplicity. The boundary conditions at m_{Mess} for General Gauge Mediation (GGM) are [224]

$$M_i = \frac{g_i^2}{16\,\pi^2}\,\Lambda_{G_i}, \qquad i = 1, 2, 3, \tag{4.3.17}$$

$$a_i = 0, \qquad i = \mathrm{u}, \mathrm{d}, \mathrm{e}, \tag{4.3.18}$$

$$m^2_{\tilde{f}} = 2\sum_{i=1}^{3} C_2(\mathbf{r}^i_{\tilde{f}}, i)\, k_i\, \frac{g_i^4}{(16\,\pi^2)^2}\,\Lambda^2_{S_i}, \qquad \tilde{f} = q, \overline{\mathrm{u}}, \overline{\mathrm{d}}, \ell, \mathrm{e}, \mathrm{H}_u, \mathrm{H}_d, A_3^F, \tag{4.3.19}$$

with b_μ and μ again determined from EWSB at the low scale as in eqs. 4.3.15 and 4.3.16. $C_2(\mathbf{r}^i_{\tilde{f}}, i)$ is the quadratic Casimir of the representation $\mathbf{r}^i_{\tilde{f}}$ under the i^{th} gauge group and $k_i = (3/5, 1, 1)$ is the standard GUT normalisation. To compare like with like, we will take the CGGM parameter space

$$\Lambda_{G_i} = \Lambda_G, \qquad \Lambda_{S_i} = \Lambda_S, \qquad i = 1, 2, 3, \tag{4.3.20}$$

and looking along the line $\Lambda_S = \Lambda_G$ gives the boundary conditions of the Minimal Gauge Mediated Supersymmetry Breaking (mGMSB) [225–227] subspace of models originally developed in [228–233]. We concede that we have not solved the b_μ problem of Gauge Mediated Supersymmetry Breaking (GMSB). With a future study one could take supplement GMSB with a Dirac gluino. Then as was studied in [103, 234–236] t_β would be taken as an output rather than input, and a small value of b_μ would be specified at the high scale. $\mathcal{L}^D_{\mathrm{Soft}}$ contains the operators, including the non-standard soft terms [72] generated by the D-term SUSY breaking discussed in 4.3.2

$$\begin{aligned}-\mathcal{L}^{D}_{\text{Soft}} = & \left(i\, m_{D\widetilde{\text{g}}}\, \widetilde{\text{g}}^{a} \cdot \psi^{a}_{\widetilde{\text{g}}} + \text{h.c.}\right) + \frac{m^2_{\phi 3}}{2}\, \phi^2_{\widetilde{\text{g}}} + \frac{m^2_{\sigma 3}}{2}\, \sigma^2_{\widetilde{\text{g}}} \\ & + 2\, g_3\, m_{D\widetilde{\text{g}}}\, \phi^{a}_{\widetilde{\text{g}}} \left(q^\dagger\, T^a\, q + \overline{\text{u}}^\dagger\, T^a\, \overline{\text{u}} + \overline{\text{d}}^\dagger\, T^a\, \overline{\text{d}}\right),\end{aligned} \tag{4.3.21}$$

where the T^a are the generators of $\text{SU}(3)_\text{C}$ in the fundamental representation. The second line in 4.3.21 is the origin of the supersoftness of these models, and provides the additional interaction required for the diagram on the second line of 3.4.59, cutting off the sensitivity to the UV scale where $m_{D\widetilde{\text{g}}}$ is generated. Finally, for both the CMSSM and CGGM we take

$$m_{D\widetilde{\text{g}}} = \frac{1}{16\pi^2}\Lambda_D, \qquad m^2_{\Phi^D_{\widetilde{\text{g}}}} = \frac{c_1^2}{16\pi^2}\,\Lambda_D^2, \qquad b_{\widetilde{\text{g}}} = 0, \tag{4.3.22}$$

where c_1 represents $\mathcal{O}(1)$ mixings in the messenger sector that have been tuned to make $b_{\widetilde{\text{g}}}$ phenomenologically negligible as already discussed.

4.3.3 One loop thresholds at the Dirac gluino mass

Significance

The Dirac gluinos and the sgluons play the role of messengers of D–term SUSY breaking for the strongly interacting sparticles. As discussed in Section 2.2, as we are calculating in the mass–independent scheme $\overline{\text{DR}}$, in order to treat the large hierarchy between the gluino mass and the rest of the SUSY spectrum correctly, we need to integrate out the gluino and the sgluons at their mass and switch to an EFT. This leads to a different behaviour of the RG compared to the MSSM. The most important contributions to take into account are the corrections to squark masses and to the strong gauge coupling g_3. We will see that this alters where EWSB occurs and can increase the naturalness of these models.

Threshold corrections

Squark masses: The gluino in these models is not pure Dirac, although in some regions of parameter space this may be approximately true. Consequently, instead of using the analytic formulae in eq. 3.4.59, we will numerically compute the full one loop threshold correction to squark masses[3]

$$m^2_{\widetilde{q}} \to m^2_{\widetilde{q}} - \Pi^{\widetilde{g}}_{\widetilde{q}}(m_{\widetilde{q}}) - \Pi^{\phi_{\widetilde{g}}}_{\widetilde{q}}(m_{\widetilde{q}}) \tag{4.3.23}$$

where

$$\Pi^{\widetilde{g}}_{\widetilde{q}}(p) = \frac{g_3^2}{6\,\pi^2}\,|(Z_g)_{i,1}|^2\,G_0(p, m_{\widetilde{g}_i}, 0), \quad \Pi^{\phi_{\widetilde{g}}}_{\widetilde{q}}(p) = \frac{g_3^2}{3\,\pi^2}\,m^2_{D\widetilde{g}}\,B_0(p, m_{\widetilde{q}}, m_{\phi_{\widetilde{g}}}) \tag{4.3.24}$$

and Z_g is the matrix that diagonalises the gluino mass matrix $m_{\widetilde{g}}$

$$m_{\widetilde{g}} = \begin{pmatrix} M_3 & m_{D\widetilde{g}} \\ m_{D\widetilde{g}} & 0 \end{pmatrix}, \qquad Z_g\,m_{\widetilde{g}}\,Z_g^\dagger = \mathrm{diag}(m_{\widetilde{g}_1}, m_{\widetilde{g}_2}) \tag{4.3.25}$$

where $m_{\widetilde{g}}$ is in the $(\widetilde{g}, \psi_{\widetilde{g}})$ basis. B_0 and G_0 are scalar integrals given in appendix A.

Strong gauge coupling: The 1-loop threshold corrections to g_3 at $m_{D\widetilde{g}}$ are [36]

$$g_3 \to g_3 \left\{ 1 \pm \frac{g_3^2}{16\,\pi^2} \left[\sum_i \log\left(\frac{m^2_{\widetilde{g}_i}}{m^2_{D\widetilde{g}}}\right) + \frac{1}{4}\log\left(\frac{m^2_{\phi_{\widetilde{g}}}}{m^2_{D\widetilde{g}}}\right) \right] \right\} \tag{4.3.26}$$

where the positive (negative) contribution occurs when running from the UV (IR) to the IR (UV) and all parameters are evaluated at the renormalisation scale $\mu(m_{D\widetilde{g}}) = m_{D\widetilde{g}}$.

Quark masses: We do not implement the quark mass threshold corrections from the gluinos and sgluons. To correctly do this would be quite technical and we anticipate that the overall impact on the areas we are interested in (such as the SUSY spectrum, EWSB and tuning) should be minimal; corrections of this kind must be proportional to chiral symmetry breaking and since the quarks are essentially massless at $m_{D\widetilde{g}}$ the remaining correction is proportional to the Majorana gluino

[3] There is no contribution from $\Pi^{\sigma_{\widetilde{g}}}_{\widetilde{q}}(m_{\widetilde{q}})$ as the $\sigma_{\widetilde{g}}$ coupling to squarks is zero.

mass. For the top quark [237]

$$\delta m_{\rm t} = -\frac{g_3^2}{12\,\pi^2}\sin(\theta_{\tilde{\rm t}})\,M_3\left[B_0(0,M_3,m_{\tilde{\rm t}_1}) - B_0(0,M_3,m_{\tilde{\rm t}_2})\right], \tag{4.3.27}$$

where $\theta_{\tilde{\rm t}}$ is the stop mixing angle. This will alter the Yukawa couplings in the UV and hence only affect the running of UV parameters that depend on the Yukawa couplings. We expect the low energy physics to be largely unaffected however, and instead we include the loop contributions to the quark masses from gluinos and sgluons at $m_{\rm Z}$ and $m_{\rm SUSY}$. By doing this we make a systematic error proportional to $(16\pi^2)^{-2} \times \log(m_{D\tilde{\rm g}}/m_{\rm SUSY}) \times \log(m_{GUT}/m_{\rm SUSY}) \lesssim 0.1\%$.

4.4 Numerical setup

We use the standard top-down approach where we fix a set of UV boundary conditions at either $m_{\rm GUT}$ in the CMSSM or $m_{\rm Mess}$ in CGGM. The low energy spectrum is found through RG evolution, and then the corresponding flavour observables and fine tuning are calculated.

To achieve this, we have used the Mathematica package `SARAH` 4.3.0 [238–243] to generate source code for the spectrum generator `SPheno` 3.3.2 [8, 74]. `SPheno` solves the RG equations taking into account the presence of the Dirac gluino at one and two loops. This program then calculates the one loop masses for all particles in the model, the branching ratios for all kinematically allowed two body decays and the branching ratios for three body decays involving intermediate W and Z bosons.

The UV boundary conditions discussed in Section 4.3.2 are implemented we can then solve the EWSB minimisation conditions for μ and b_μ. We only study the $\mu > 0$ case in order to maximise the effect from stop mixing upon the Higgs sector. The `SPheno` code has been modified to include an intermediate step in RG running where the gluino and its corresponding scalar degrees of freedom are integrated out at the gluino mass. This implements the EWSB mechanism of supersoft models outlined in [129]. The masses for the gluino and the real sgluon are calculated at the this intermediate scale instead of $m_{\rm SUSY}$. A schematic of this algorithm is shown in fig. 4.2.

4.5 Spectra

On each of the parameter space plots we include the relevant limits on SUSY particle masses. As the production cross section is suppressed for all SUSY particles in models with Dirac gluinos (shown in Section 4.6), we take only the strongest most model independent limits available set by lepton colliders, outlined in table 2.11. For the CMSSM, the stable neutralino limit is applied, whereas for CGGM the unstable limit is used instead. The red, purple and green solid lines indicate the limit on the slepton, neutralino and sneutrino masses, and the blue dashed line indicates the limit on the chargino masses.

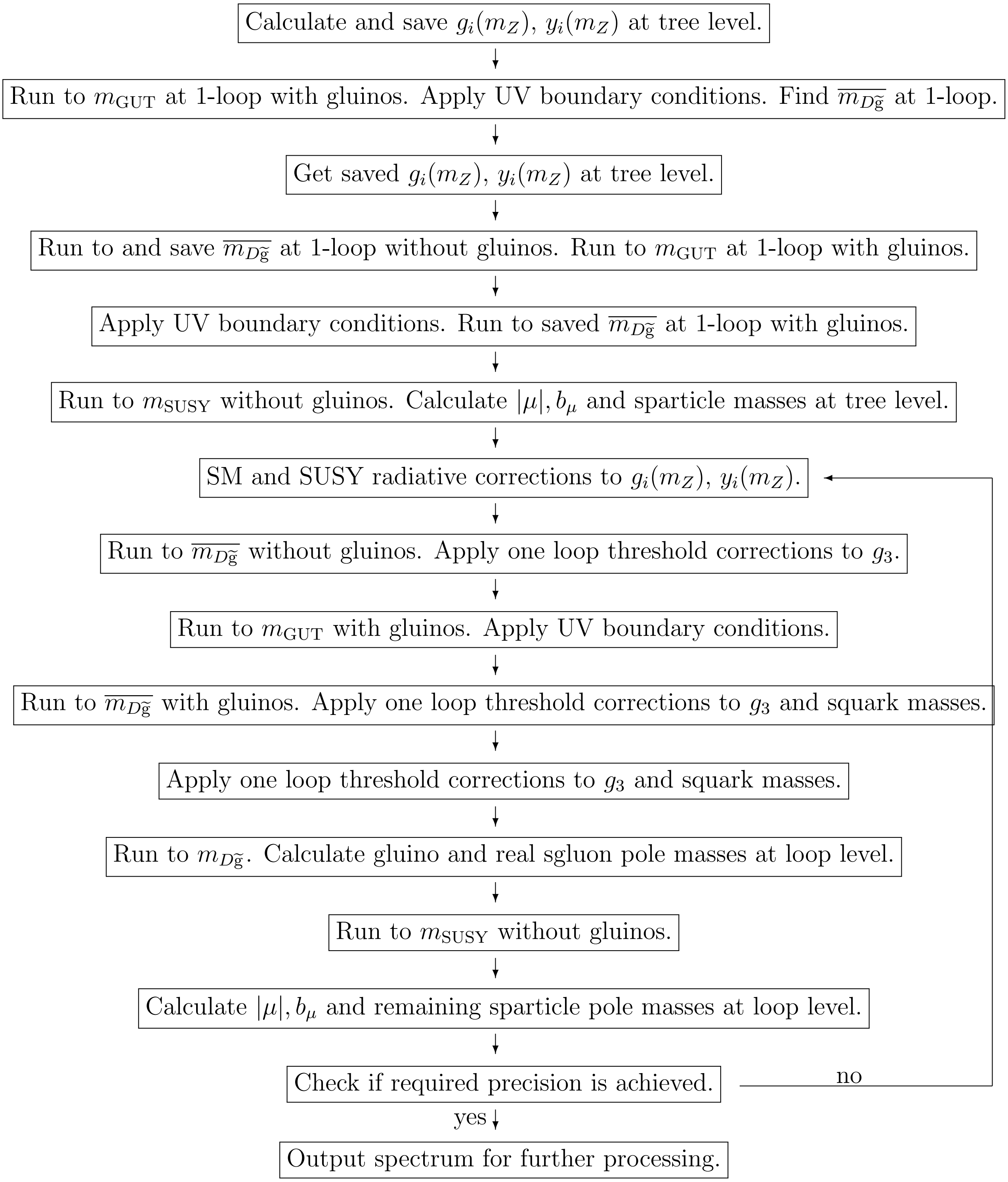


Figure 4.2: Algorithm used to calculate the spectrum. Adapted from fig. 1 in [8]. Note that apart from where it explicitly states running to a saved value of $\overline{m_{D\tilde{g}}}$, the scale is found by requiring a solution to $\mu(m_{D\tilde{g}}) = m_{D\tilde{g}}$. This typically updates with each iteration since it depends on the behaviour of g_3 whose running is determined by the location of $\overline{m_{D\tilde{g}}}$.

Points below and to the left of these lines are excluded at the 95% CL. We will present three types of graphs in the $(m_0, M_{1/2})$ and (Λ_G, Λ_S) planes to illustrate the similarities and differences between spectra:

- Gradients of Higgs boson masses with contours of the parameters entering into the one loop Higgs mass approximation in eq. 2.3.342.
- LOSP species with mass contours of the typical candidates.
- Next to Lightest Ordinary Supersymmetric Particle (NLOSP) species.

In the MSSM the two loop contribution from gluinos gives quite a significant contribution. Because the two loop Higgs mass has not yet been computed in the presence of a Dirac gluino, we will not impose achieving the correct value as a strict requirement, as we would be incorrectly ruling out viable regions of parameter space. Although the full calculation will be completed in the future [244], the effective field theory framework used here requires a different approach. At the gluino scale, one would need to match the theory onto a theory with broken SUSY with RGEs. This requires removing the approximation that e.g. the stop-Higgs quartic coupling and the Higgs–top Yukawa terms remain equal along the RG flow

$$y_{\rm t}\, \bar{\rm t}\, q \cdot {\rm H}_u \longleftrightarrow |y_{\rm t}|^2 |\widetilde{\rm t}|^2 |{\rm H}_u|^2.$$

Instead, the coefficients of the operators $\bar{\rm t}\, q \cdot {\rm H}_u$ and $|\widetilde{\rm t}|^2 |{\rm H}_u|^2$ should have different RGEs below the Dirac gluino mass. After applying threshold corrections to each coupling, flowing down from the gluino mass to the SUSY scale would then correctly include the two loop contributions to the Higgs mass with gluino integrated out. With the new non-SUSY RGE calculators becoming available [242,245], the possibility to correctly incorporate these kinds of particle threshold effects into spectrum generators in the future is a very interesting possibility

Only a subset of the scans are presented in the body of the text. The remaining parameter configurations can be found in appendix B. The generic dependence of the spectrum and low energy parameters on the UV boundary conditions can be inferred by analysing the cases we present.

We first present the comparison of the CMSSM with and without a Dirac gluino.

We scan

$$0 \text{ TeV } \leq m_0 \leq 6 \text{ TeV} \qquad\qquad 0 \text{ TeV } \leq M_{1/2} \leq 4 \text{ TeV} \tag{4.5.28}$$

and take a moderate and large $t_\beta = 10, 25$. In the presence of a Dirac gluino, we set $m_{D\tilde{g}}(m_{\text{GUT}}) = 5, 7.5, 10$ TeV which, due to RG running, lead to a significant spread of physical Dirac gluino masses that can be estimated using

$$\overline{m_{D\tilde{g}}}|_{\text{approx}} = \left[m_{D\tilde{g}}(\Lambda)\, \Lambda^{\frac{3\, g_3^2(\Lambda)}{8\,\pi^2}} \right]^{\frac{1}{1+\frac{3\, g_3^2(\Lambda)}{8\,\pi^2}}} \tag{4.5.29}$$

where Λ can be any scale, but is most conveniently taken as the UV scale.

The first thing to note is that there is a new region of parameter space in the $(m_0, M_{1/2})$ plane opening up for very low $M_{1/2}$ but non-zero m_0 in the presence of a Dirac gluino. This region isn't populated in the MSSM due to an absence of EWSB when $m^2_{\text{H}_u}$ isn't pushed negative enough for a positive $|\mu|^2$ solution; at this point in parameter space in the CMSSM one needs extra logs from M_3 to push the squark mass up along the RG trajectory. In the case of a Dirac gluino, one can essentially ignore the need for a Majorana gluino mass, as the threshold correction on its own is enough to lift the squark mass in the IR, triggering EWSB for even zero $M_{1/2}$. Here however, the LEP bound on the chargino mass becomes important, putting an experimental lower limit on $M_{1/2}$ of $\mathcal{O}(100)$ GeV.

Higgs: In figures 4.3 and B.1 we show the Higgs mass and the parameters entering the one loop Higgs mass formula in eq. 2.3.342. Even though we are taking $A_0(m_{\text{GUT}}) = 0$, a non-zero value is generated by running. In the large y_t limit (see eq. C.28 for the complete expression)

$$16\pi^2 \beta^{(1)}_{a_{\text{t}}} \supset a_{\text{t}} \left[18\, |y_{\text{t}}|^2 + \frac{16}{3} \left(\theta_{\tilde{g}} - 2 \right) g_3^2 \right] + \frac{32}{3}\, y_{\text{t}}\, g_3^2\, M_3\, \theta_{\tilde{g}}, \tag{4.5.30}$$

where

$$\theta_{\tilde{g}} = 1 \quad \text{if} \quad \mu \geq \overline{m_{D\tilde{g}}}, \qquad\qquad \theta_{\tilde{g}} = 0 \quad \text{if} \quad \mu < \overline{m_{D\tilde{g}}}. \tag{4.5.31}$$

with the precise definitions given in appendix C.2. In the CMSSM without a gluino, $\theta_{\tilde{g}} = 1$ always in eq. 4.5.30. Note that we do not observe the more negative values

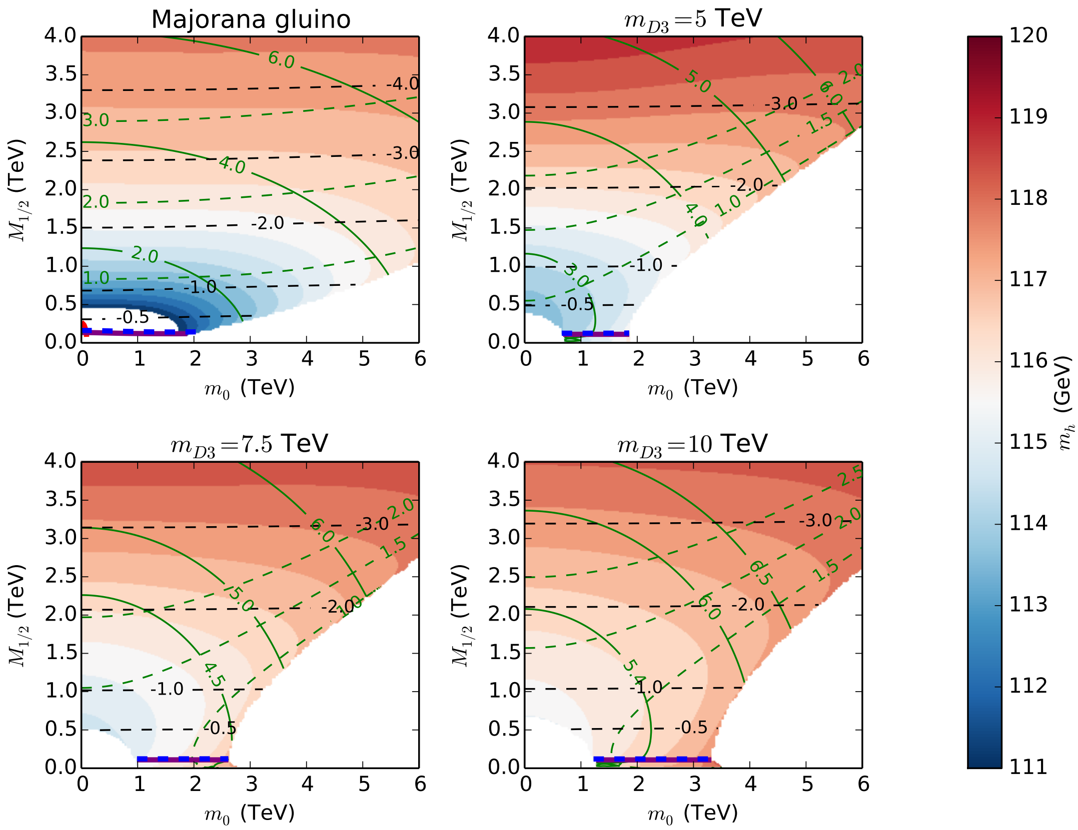


Figure 4.3: Higgs sector parameters in the CMSSM with $t_\beta = 10$ and $m_{D\tilde{g}}$ fixed as indicated. The gradient indicates the Higgs mass. The black dashed, green dashed and green solid lines are contours of $a_t(m_{\rm SUSY})$, $\mu(m_{\rm SUSY})$, and $m_{\rm SUSY}$ respectively. All contours unless otherwise specified are in TeV.

of a_t in the presence of the gluino that were found in [246]. This can be understood by considering the running of the Majorana gluino mass in the presence of a Dirac gluino

$$16\pi^2\beta^{(1)}_{M_3} = -6\,g_3^2\,M_3 \qquad \text{MSSM} \qquad (4.5.32)$$

$$16\pi^2\beta^{(1)}_{M_3} = 0 \qquad \text{MSSM with Dirac Gluino.} \qquad (4.5.33)$$

Because we are taking $a_t(m_{\rm GUT}) = 0$ then the gluino term dominates for most of the flow, and in the CMSSM, this term becomes larger that in the CMSSM with a Dirac gluino as demonstrated in fig. 4.4. The contours of $m_{\rm SUSY}$ in the presence of a Dirac gluino are increased to the minimum squark mass possible in the model (i.e. determined by eq. 3.4.59). For large values of m_0 and $M_{1/2}$ contours of $m_{\rm SUSY}$

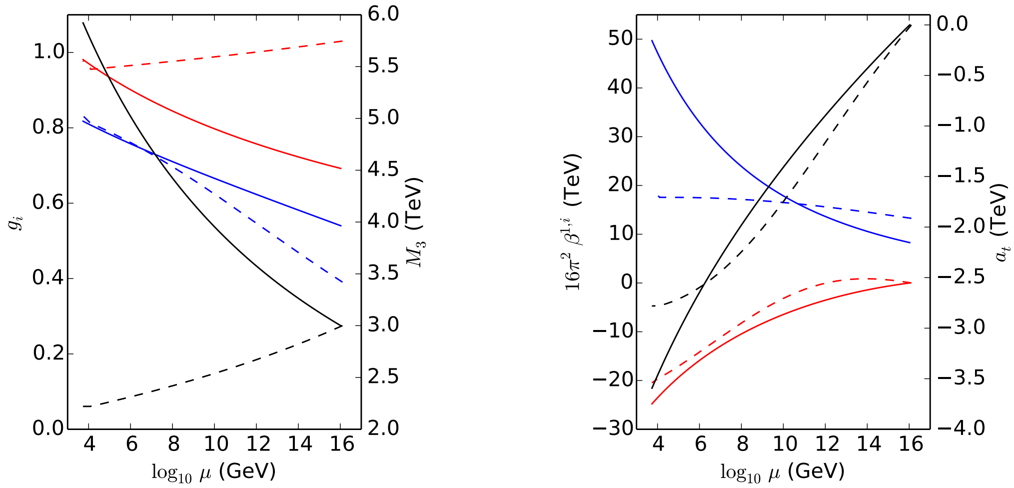


Figure 4.4: RG evolution of dominant parameters contributing to the running of $a_{\rm t}$ in the CMSSM with $m_0 = 4.5$ TeV, $M_{1/2} = 4$ TeV, $m_{D\tilde{\rm g}} = 5$ TeV and $t_\beta = 25$. Solid lines correspond to the CMSSM and dashed lines correspond to the CMSSM supplemented with a Dirac gluino. **Left**: The blue, red and black lines show the evolution of $y_{\rm t}$, g_3 and M_3 respectively. **Right**: The blue, red and black lines show the evolution of $\frac{32}{3}\, y_{\rm t}\, g_3^2\, M_3$, $a_{\rm t}\left(18\,|y_{\rm t}|^2 - \frac{16}{3}\, g_3^2\right)$ and $a_{\rm t}$ respectively.

across the different models approach each other.

The μ parameter is seen to increase with increasing Dirac gluino mass. This can be understood by considering the EWSB conditions in the large t_β limit

$$|\mu|^2 = -m_{{\rm H}_u}^2 - \frac{m_{\rm Z}^2}{2} + \mathcal{O}(t_\beta^{-2}). \tag{4.5.34}$$

$m_{{\rm H}_u}^2$ is driven negative by the squark soft scalar masses

$$16\pi^2 \beta^{(1)}_{m^2_{{\rm H}_u}} \supset 6\,|y_{\rm t}|^2\left(m_q^2 + m_{\rm t}^2\right) \tag{4.5.35}$$

which are in turn determined by the Dirac gluino mass through eq. 3.4.59. The values of μ in the MSSM for moderate $(m_0, M_{1/2})$ are actually lower with a Dirac gluino than without. Considering the RG equation for $y_{\rm t}$

$$16\pi^2 \beta^{(1)}_{y_{\rm t}} \supset \frac{8}{3}(\theta_{\tilde{\rm g}} - 3)\, g_3^2. \tag{4.5.36}$$

This term causes $y_{\rm t}$ to decrease in the flow from the IR to the UV. In the MSSM, the strong interactions retain asymptotic freedom, whereas with a Dirac gluino present, g_3 remains roughly constant along the entire flow. In the Dirac gluino case, this

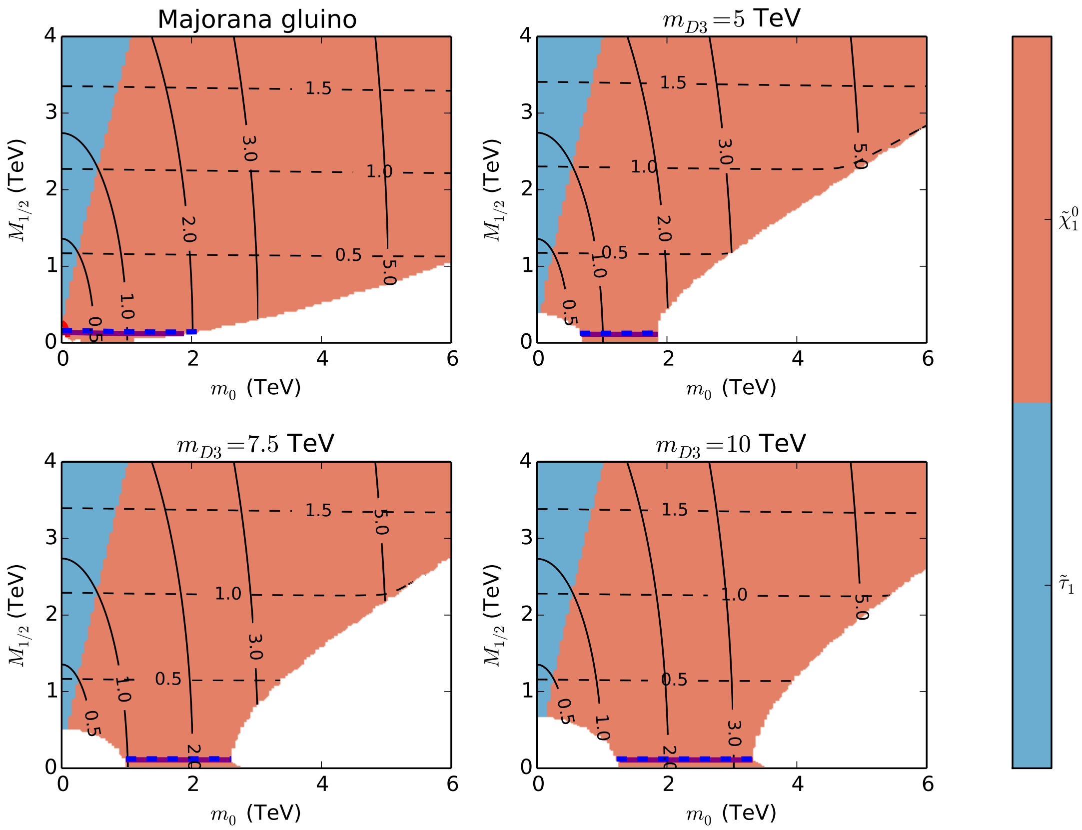


Figure 4.5: LOSP species in the CMSSM with $t_\beta = 10$ and $m_{D\tilde{g}}$ fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

causes y_t to decrease much more rapidly, and so the the integrated term of eq. 4.5.35 with a Dirac gluino than without.

The lower limit on squark masses translates into a lower limit on the Higgs mass. Apart from at low $(m_0, M_{1/2})$ where we get a separation between the strong and electroweak sectors it is difficult to distinguish the CMSSM with and without a gluino. The presence of a Dirac gluino allows us, for a given Higgs mass, to realise a lighter electroweak scalar spectrum for low $(m_0, M_{1/2})$.

LOSP: The LOSP candidate in the presence of a Dirac gluino is essentially unchanged in the CMSSM. The blue regions in figs. 4.5 and B.2 have a charged stau $\tilde{\tau}_1$ as the LOSP and so are excluded. The remainder of the parameter space is entirely bino–like neutralino $\tilde{\chi}_1^0$ LOSP, a good dark matter candidate.

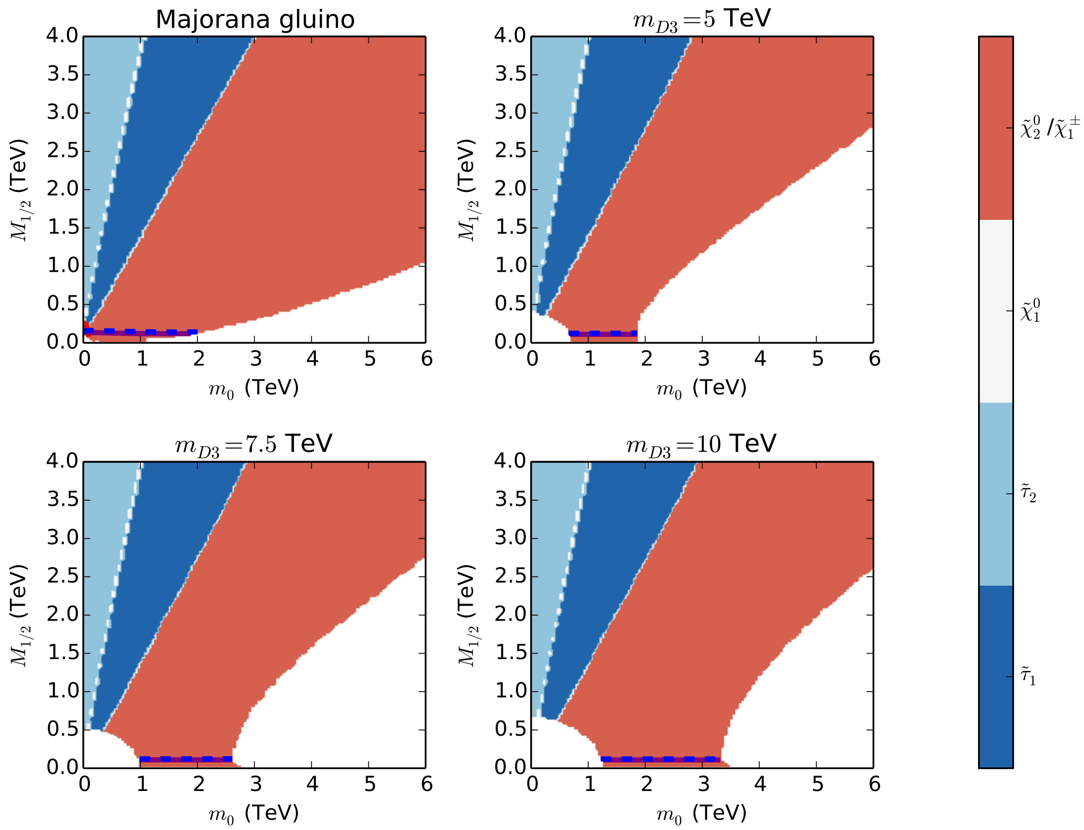


Figure 4.6: NLOSP species in the CMSSM with $t_\beta = 10$ and $m_{D\tilde{g}}$ fixed as indicated.

NLOSP: The NLOSP candidate in the presence of a Dirac gluino is similarly relatively unchanged essentially when compared to the Majorana case. The light blue regions in figs. 4.6 and B.3 have the second lightest stau $\widetilde{\tau}_2$ as the NLOSP but are excluded as the corresponding region has a lightest stau $\widetilde{\tau}_1$ LOSP. The dark blue region has lightest stau $\widetilde{\tau}_1$ LOSP and leads to one lepton and $\not{E}_T$ or jets and $\not{E}_T$ in the final state, as does the red region with wino-like chargino $\widetilde{\chi}^{\pm}$ NLOSP. This chargino $\widetilde{\chi}^{\pm}$ is also coincident with the wino-like neutralino $\widetilde{\chi}_2^0$ which instead leads to either entirely $\not{E}_T$ in the final state or $\not{E}_T$ with either two leptons of opposite sign or a jet.

It is clear that nature of the light spectrum is largely unaffected by the presence of a Dirac gluino, except that it is now possible to raise the strongly interacting sector almost[4] independently of the electroweak sector, giving some freedom to aleviate

[4]There will always arise terms proportional to $(16\,\pi^2)^{-1}\log(m_{D\widetilde{g}}/m_{\rm SUSY})$, and there two loop sensitivity to the sgluon soft mass in eq. 4.3.9 present.

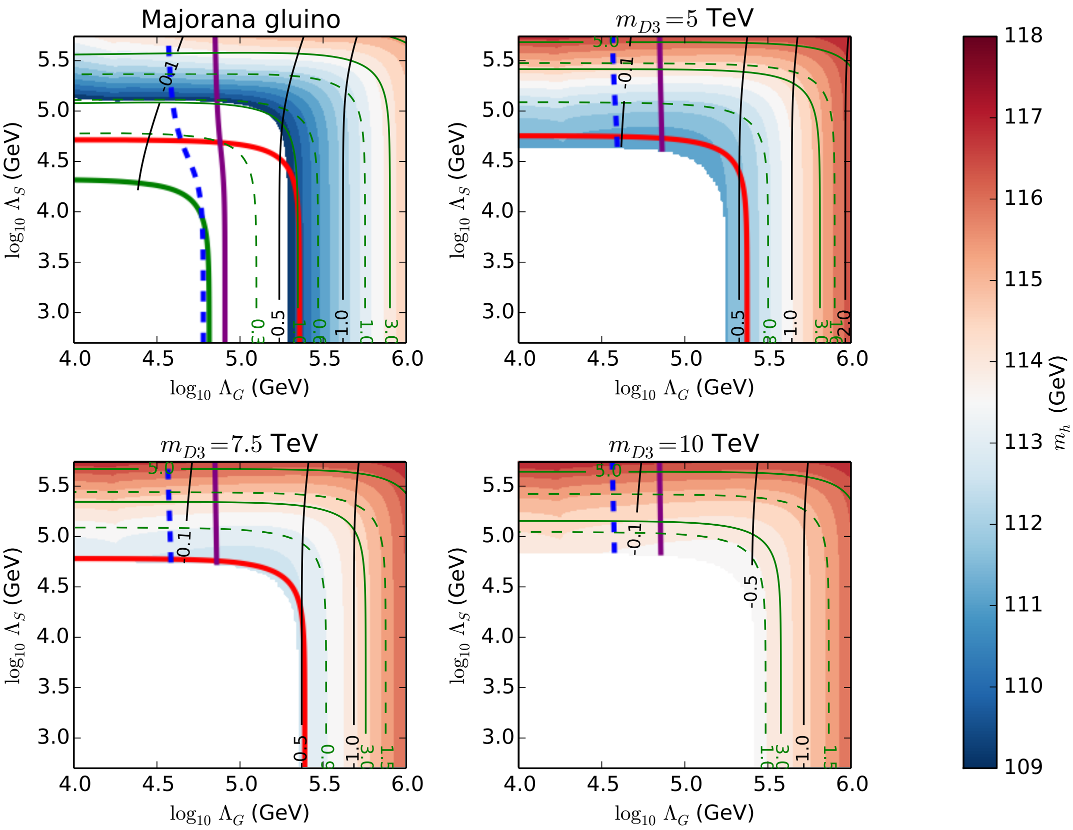


Figure 4.7: Higgs sector parameters in CGGM with $t_\beta = 10$, $m_{\text{Mess}} = 10^7$ GeV and $m_{D\tilde{g}}$ fixed as indicated. The gradient indicates the Higgs mass. The black dashed, green dashed and green solid lines are contours of $a_t(m_{\text{SUSY}})$, $\mu(m_{\text{SUSY}})$, and m_{SUSY} respectively. All contours unless otherwise specified are in TeV.

the tension with results at hadron collider experiments to date.

4.5.1 Constrained General Gauge Mediation

We now present the comparison of CGGM with and without Dirac gluino. A recent comprehensive study of the parameter space of CGGM was done in [247]. We scan

$$10^3 \text{ GeV } \leq \Lambda_G \leq 10^7 \text{ GeV} \qquad 10^3 \text{ GeV } \leq \Lambda_S \leq 10^7 \text{ GeV} \qquad (4.5.37)$$

whilst taking $t_\beta = 10, 25$ and again we again take $m_{D\tilde{g}}(m_{\text{GUT}}) = 5, 7.5, 10$ TeV in the presence of a Dirac gluino. We take two messenger scales $m_{\text{Mess}} = 10^7$ GeV and 10^{12} GeV to represent short and long periods of running.

The theoretically allowed parameter space is reduced by the presence of a Dirac

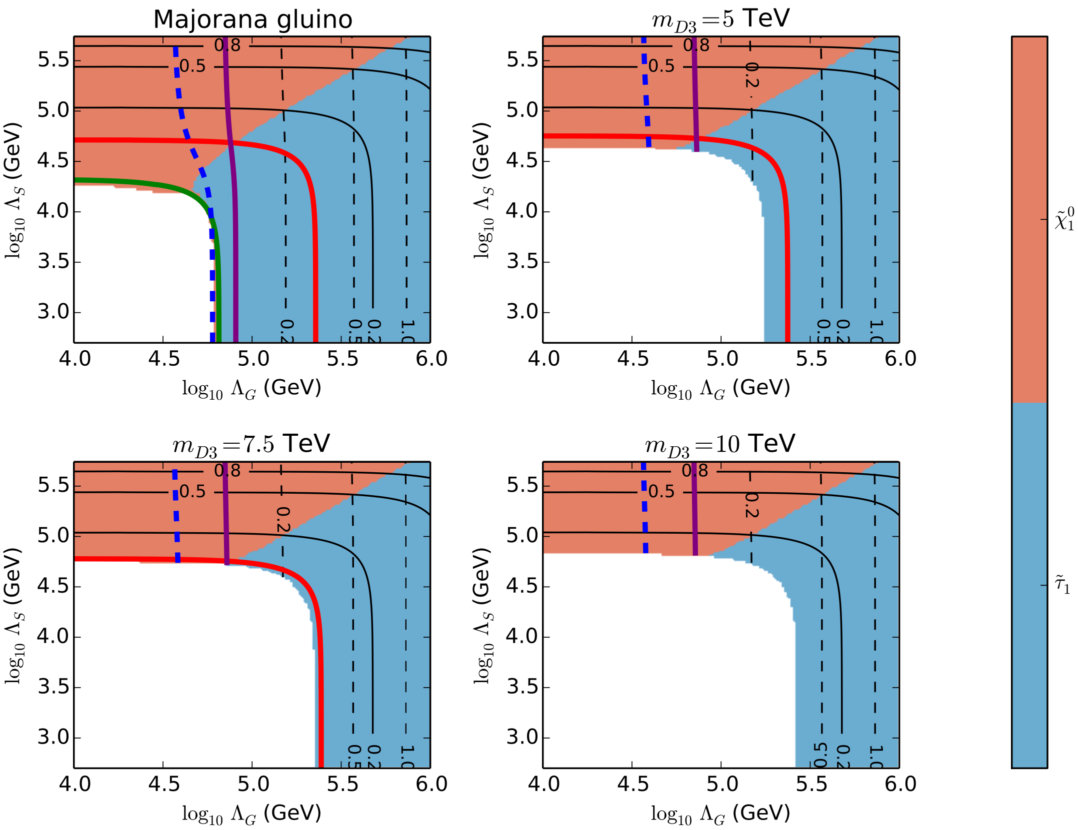


Figure 4.8: LOSP species in CGGM with $t_\beta = 10$, $m_{\text{Mess}} = 10^7$ GeV and $m_{D\tilde{g}}$ fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

gluino as is seen in fig. 4.8. Although viable EWSB is occurring, the the lightest stau $\tilde{\tau}_1$ is being driven tachyonic for a larger portion of the parameter space. This is induced by the Dirac gluino much for much higher UV stau mass set by eq. 4.3.19. This is caused by larger values of $|\mu|^2$ for a given (Λ_G, Λ_S) by the threshold corrections at the Dirac gluino scale, driving the smallest eigenvalue of the stau mass matrix

$$m^2_{\tau,\text{mat}} = \begin{pmatrix} m^2_{\ell_{3,3}} + D\,\text{terms} & v\,(a^*_\tau\, c_\beta - \mu\, y_\tau\, s_\beta) \\ v\,(a_\tau c_\beta - \mu^*\, y_\tau\, s_\beta) & m^2_{e_{3,3}} + D\,\text{terms} \end{pmatrix} \tag{4.5.38}$$

negative.

Higgs: In figures 4.7, B.5, B.6 and B.7 we show the Higgs mass and the parameters entering the one loop Higgs mass formula in eq. 2.3.342. The characteristic properties here are essentially unchanged from the CMSSM counterpart as we have

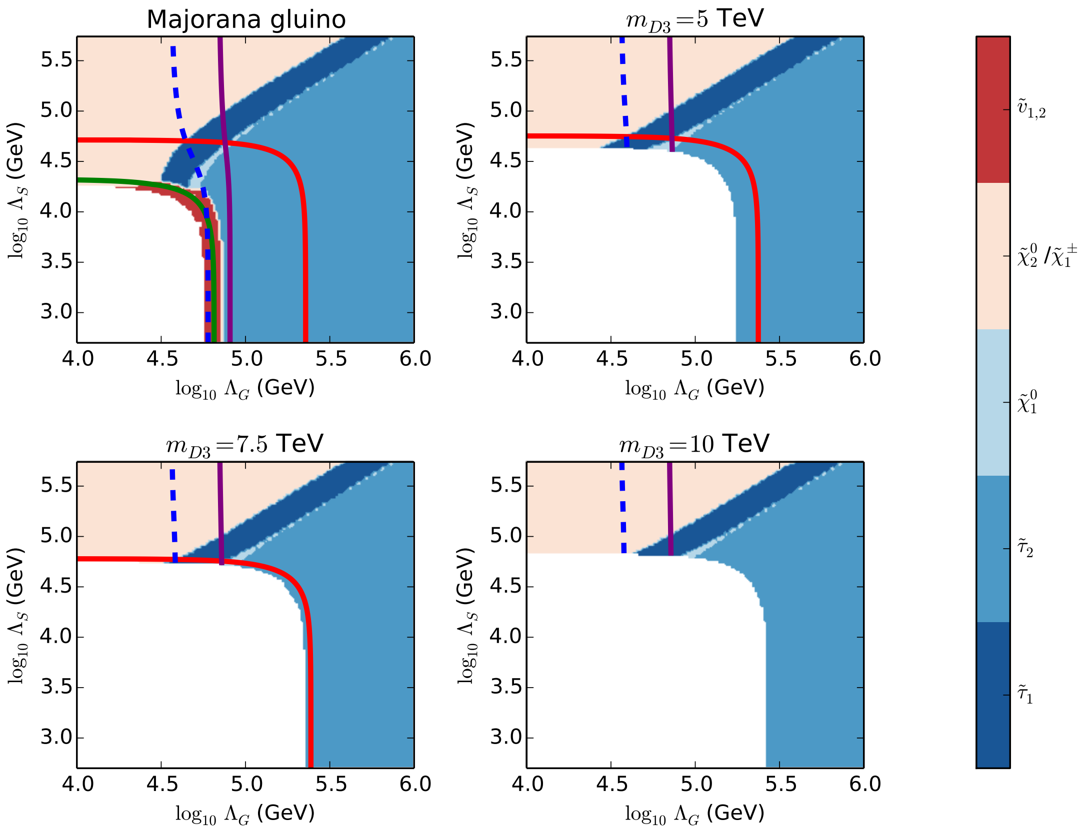


Figure 4.9: NLOSP species in CGGM with $t_\beta = 10$, $m_{\text{Mess}} = 10^7$ GeV and $m_{D\tilde{g}}$ fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

only considered the CMSSM case $A_0 = 0$.

LOSP: The LOSP candidates in CGGM with and without a Dirac gluino are similar to those of the CMSSM as can be seen in figs. 4.8, B.8, B.9 and B.10. The difference here is that the blue regions that correspond to stau $\tilde{\tau}_1$ LOSP are now viable as the LSP in these models is the gravitino $\tilde{G}$. The stau can either be long lives produce a missing energy signature or it can undergo the decay $\tilde{\tau} \to \tilde{G}\,\tau$ inside the detector depending on its mass. If it does decay it will lead to one lepton and $\not{E}_T$ or jets and $\not{E}_T$. The remainder of the parameter space is has entirely bino-like neutralino $\tilde{\chi}_1^0$ LOSP, whose decay rate can be calculated from the standard formula [48]

$$\Gamma(\tilde{X} \to X\,\tilde{G}) = \frac{m_{\tilde{X}}^5}{16\,\pi\,\langle F \rangle^2} \left(1 - \frac{m_X^2}{m_{\tilde{X}}^2} \right)^4 \tag{4.5.39}$$

and typically undergos the decay $\widetilde{\chi}_1^0 \to \tilde{G}\,\gamma$ well inside the detector. This decay is responsible for the stronger lower bounds on the neutralino mass $m_{\widetilde{\chi}_1^0}$ in CGGM.

NLOSP: In CGGM we have a sneutrino NLOSP candiate in addition to those found in the CMSSM. These are shown in figs. 4.9, B.11, B.12 and B.13. This only happens without a Dirac gluino however, as in the region where a sneutrino $\widetilde{\nu}$ NLOSP would be achieved, the lightest stau $\widetilde{\tau}_1$ has already been pushed tachyonic. The region with sneutrino $\widetilde{\nu}$ NLOSP is ruled out by collider searches. The remaining NLOSP candidates have the same decays as seen in the CMSSM except that they may be accompanied by an additional photon in the final state.

4.5.2 Overview

Overall, one sees that when each the CMSSM and CGGM are supplemented with a Dirac gluino, very little changes in the electroweak spectrum. This is of course by construction since the effective theory is essentially the MSSM without a gluino. The Higgs mass however, is raised across the whole parameter space and can be made largely independent of $(m_0, M_{1/2})$ or (Λ_G, Λ_S) at sufficiently low values of these parameters. Note that this is different to having non-universal scalar masses and gaugino masses, since giving a large mass to squarks and or gluinos in the UV will lead to a very large value for μ, giving very heavy Higgsinos and non-SM-like Higgses as well as being accompanied by considerable fine tuning. The Wino mass will also be lifted along the RG flow since

$$(16\,\pi^2)^2\beta_{M_2} \supset 48\,(g_2\,g_3)^3\,M_3 \tag{4.5.40}$$

causes M_2 to increase by ~ 500 GeV for a 10 TeV Majorana gluino. A characteristic plot of the spectra in the CMSSM with and without a Dirac gluino is shown in fig. 4.10. Since the overall result is a light set of electroweak particles with the neutralino as the LOSP, the detailed phenomenology is expected to be very similar to that of the *well-tempered neutralino* [248, 249]. One could also take all of the orderings of our electroweak states and map them on to the analysis in [250].

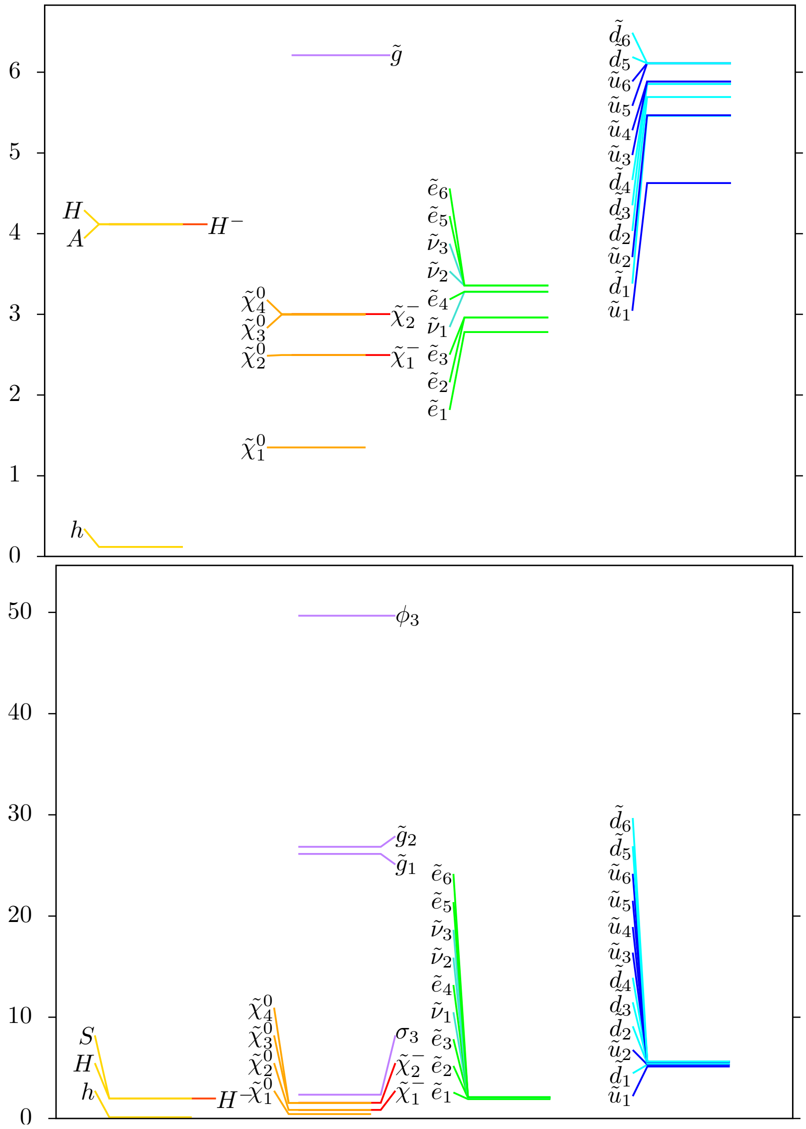


Figure 4.10: Sparticle spectra the CMSSM (top) and the CMSSM with a Dirac gluino (bottom) for the benchmark points in table 4.7. The y–axis is in TeV.

4.6 Cross sections

Here we present the LO cross sections at 8 and 13 TeV LHC with and without a Dirac gluino in the CMSSM. We fixed $t_\beta = 10$, $m_0 = 200\,\text{GeV}$ and scanned over

$$M_{1/2} \in [200, 1600]\,\text{GeV} \qquad \text{CMSSM}$$

$$M_{1/2} = 400,\ m_{D\tilde{g}} \in [500, 5000]\,\text{GeV} \qquad \text{CMSSM with Dirac gluino}$$

leading to the spread of squark masses shown in fig. 4.11. For di–squark production, we can see that there is suppression in the Dirac gluino case of approximately two

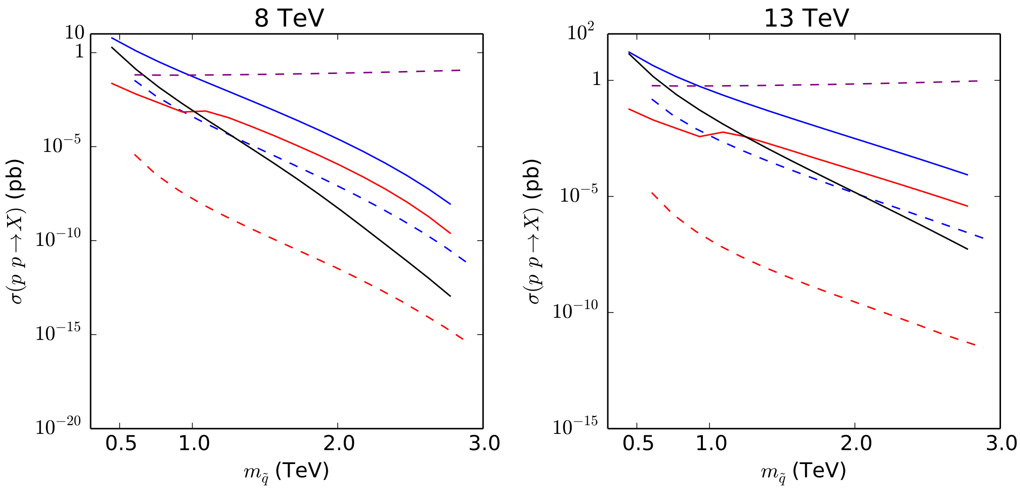


Figure 4.11: LO cross sections for various processes at 8 TeV (left) and 13 TeV (right) LHC. The solid and dashed lines indicate cross sections in the CMSSM with and without a Dirac gluino respectively. The blue, black and purple lines indicate total di–squark ($\widetilde{q}_i\,\widetilde{q}_j$), di–gluino ($\widetilde{g}_i\,\widetilde{g}_j$) di–pseudo–sgluon production ($\sigma_{\widetilde{g}}\,\sigma_{\widetilde{g}}$). The red lines indicate cross section × branching ratio for processes beginning with di–squark production yielding two jets, two same sign leptons and $\not{E}_T$ in the final state. The cross–sections were calculated using `Madgraph5_aMC@NLO` with the MSTW2008lo68cl PDF set. All two and three body branching ratios were calculated using `SPheno`. Although the x-axis shows squark mass, we are indeed scanning over the Dirac gluino mass. The Dirac gluino mass essentially determines the squark mass through eq. 3.4.59.

orders of magnitude due to the *supersafe* mechanism discussed in Section 3.5. Note that this is only true for di–squark production, but is not true for squark–anti–squark production as the dominant diagrams required for these processes do not involve the Majorana nature of gluinos as was discussed in [7, 169].

Di–gluino production is only displayed for the CMSSM without a Dirac gluino, since for the parameter space displayed, di–gluino production is kinematically forbidden in the Dirac gluino case. Similarly, di–sgluon production is kinematically forbidden. The di–pseudo–sgluon production rate, however, is relatively high due to its light mass and its large $SU(3)_C$ charge.

Finally we display the product of branching ratios approximation for the cross

Decaying particle	Decay products	Branching fraction
Z	invisible	0.2000 ± 0.0006
W^+	$\mathrm{e}^+ \nu_\mathrm{e}$	0.1075 ± 0.0013
	$\mu^+ \nu_\mu$	0.1057 ± 0.0015
	$\tau^+ \nu_\tau$	0.1125 ± 0.0020
τ^+	$\overline{\nu}_\tau \mathrm{e}^+ \nu_\mathrm{e}$	0.1783 ± 0.0004
	$\overline{\nu}_\tau \mu^+ \nu_\mu$	0.1741 ± 0.0004
t^-	$\mathrm{W}^+ \mathrm{b}$	0.91 ± 0.04

Table 4.2: SM branching ratios used in calculation of branching ratios × cross sections. All are the world averages taken from [11].

section for two jets, two same sign leptons and missing energy

$$2 \times \Big[\sum_{i \leq j} \sigma(\mathrm{p\,p} \to \widetilde{\mathrm{u}}_i \, \widetilde{\mathrm{u}}_j) \times \mathrm{Br}(\widetilde{\mathrm{u}}_i \to \mathrm{jet} + \ell^+ + \not{E}) \times \mathrm{Br}(\widetilde{\mathrm{u}}_j \to \mathrm{jet} + \ell^+ + \not{E}) + \\ \sum_{i \leq j} \sigma(\mathrm{p\,p} \to \widetilde{\mathrm{d}}_i \, \widetilde{\mathrm{d}}_j) \times \mathrm{Br}(\widetilde{\mathrm{d}}_i \to \mathrm{jet} + \ell^- + \not{E}) \times \mathrm{Br}(\widetilde{\mathrm{d}}_j \to \mathrm{jet} + \ell^- + \not{E}) \Big],$$

where the squark branching ratios are given by all possible combinations of kinematically allowed decays leading to one jet, one lepton and missing energy

$$\mathrm{Br}(\widetilde{\mathrm{u}}_i \to \mathrm{jet} + \ell^+ + \not{E}) \sim \mathrm{Br}(\widetilde{\mathrm{u}}_i \to \mathrm{d}\ \widetilde{\chi}_1^+) \times \mathrm{Br}(\widetilde{\chi}_1^+ \to \ell^+ \, \nu \, \widetilde{\chi}_1^0) + \cdots . \quad (4.6.41)$$

Although this approximation misses effects coming from off-shell intermediate sparticles in the decay chain that increase the cross section × branching ratio, it can still serve as an indicator of what to expect if one simulated the high multiplicity final states fully. All branching ratios are calculated as a function of the parameter space scanned by `SPheno`. All other branching ratios are SM branching ratios which can are given in table 4.2. All decay products in the chain considered are displayed in table 4.3. Whilst the Majorana case still allows a number of events visible at the LHC given an integrated luminosity of $23.26\,\mathrm{fb}^{-1}$ such that the same sign lepton analyses [251] are sensitive in the direct squark (via sleptons) models, the case with a Dirac gluino is far beyond producing any same sign di–leptons plus two jet events at the LHC with the current integrated luminosity. In addition, the Majorana di–gluino production is the dominant process leading to two same-sign di–leptons with $\widetilde{\mathrm{g}} \to 2\,\mathrm{jets} + l^\pm + \not{E}$. This decay is simply absent with a heavy Dirac gluino.

One feature to note is that in the MSSM, there is a rise in the branching ratio

Particle	Relevant Decay Products
$\widetilde{\mathrm{u}}_{1,\ldots,6}$	$\mathrm{d}_{1,2,3}\,\widetilde{\chi}^{+}_{1,2}$
$\widetilde{\mathrm{d}}_{1,\ldots,6}$	$\mathrm{u}_{1,2}\,\widetilde{\chi}^{-}_{1,2}$
$\widetilde{\chi}^{+}_{2}$	$\mathrm{e}^{+}_{1,\ldots,3}\,\widetilde{\nu}_{1,2,3};\ \nu_{1,2,3}\,\widetilde{\mathrm{e}}^{+}_{1,\ldots,6};\ \mathrm{W}^{+}\,\widetilde{\chi}^{0}_{1,2};\ \mathrm{Z}\,\widetilde{\chi}^{+}_{1}$
$\widetilde{\chi}^{+}_{1}$	$\mathrm{e}^{+}_{1,\ldots,3}\,\widetilde{\nu}_{1,2,3};\ \nu_{1,2,3}\,\widetilde{\mathrm{e}}^{+}_{1,\ldots,6};\ \mathrm{W}^{+}\,\widetilde{\chi}^{0}_{1,2}$
$\widetilde{\chi}^{0}_{2}$	$\mathrm{Z}\,\widetilde{\chi}^{0}_{1}$
$\widetilde{\mathrm{e}}^{-}_{1,\ldots,6}$	$\mathrm{e}^{-}_{1,\ldots,3}\,\widetilde{\chi}^{0}_{1,2};\ \nu_{1,\ldots,3}\,\widetilde{\chi}^{+}_{1}$
$\widetilde{\nu}_{1,2,3}$	$\widetilde{\chi}^{+}_{1,2}\,\mathrm{e}^{-}_{1,2,3};\ \widetilde{\chi}^{0}_{1,2}\,\nu_{1,2,3}$

Table 4.3: Decays considered for the squark to one jet, one lepton and $\not{E}_T$.

$\times$ cross section for 1 TeV squarks in both the 8 and 13 TeV cases. This doesn't occur in with a Dirac gluino in the parameter space studied. In the MSSM, we are raising $M_{1/2}$ in order to raise the squark masses. As this happens, a gap between the lightest chargino and the sneutrino masses opens up. The chains that involving $\widetilde{\chi}^{+}_{1} \rightarrow \widetilde{\nu}\,\ell^{+}$ account for 10 % of the overall branching ratio of a squark into one lepton, one jet and $\not{E}_T$ and only turn on once $M_{1/2}$ becomes large enough. In the Dirac gluino case this channel never opens up as we raise $m_{D\widetilde{\mathrm{g}}}$ to raise the squark masses instead of $M_{1/2}$.

4.7 Decays of the pseudosgluon

Since the pseudosgluon is the lightest strongly interacting sparticle in our spectrum and is CP-odd, at first glance it would seem that it may be a dark matter candidate as there is no relevant tree–level decay present in the UV lagrangian[5]. In fact, this turns out not to be the case as the pseudo–sgluon undergoes a loop level decay to quarks via gluinos and squarks shown in figure 4.12. Upon integrating out the gluino at $\mu^2 = m^2_{\widetilde{\mathrm{g}}}$, this generates a new three–point interaction in the effective Lagrangian

$$-\mathcal{L}_{\text{eff}} \supset -c_q\, i\, \sigma_{\widetilde{\mathrm{g}}}\, \psi_q\, \psi_q + \text{h.c.} \tag{4.7.42}$$

[5]Stability for this particle would be somewhat disastrous as its large $SU(3)_C$ charge would imply that some non-perturbative interaction with the nucleus would have showed up in dark matter direct detection experiments.

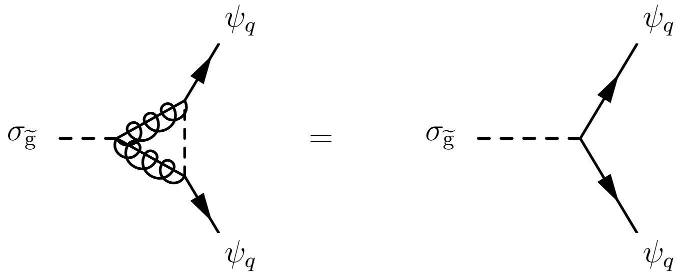


Figure 4.12: The generation of the $\sigma_{\widetilde{g}}\,\psi_q\,\psi_q$ interaction in the effective theory upon integrating out $\widetilde{g}$ at $\mu^2 = m_{\widetilde{g}}^2$.

where the coefficient c is determined by the matching in figure 4.12 to be [145, 183, 186]

$$c_q = \frac{3}{\sqrt{2}}\frac{g_3}{16\,\pi^2}\,m_{D\widetilde{g}}\,m_q\,I_q, \tag{4.7.43}$$

where

$$I_q = V_{qi}^L\,C_{0q}^L\,V_{qi}^{L\dagger} - (L \leftrightarrow R), \tag{4.7.44}$$

the $V_{qi}^{L,R\,\dagger} = U_{qj}^{L,R}\,Z_{ji}^{L,R}$ is the product of the appropriate squark and quark mixing matrices, and

$$C_{0q}^{L/R} = C_0(m_{\sigma_{\widetilde{g}}}, m_q, m_{D\widetilde{g}}, m_{D\widetilde{g}}, m_{\widetilde{q}_{L/R}}) \tag{4.7.45}$$

where C_0 is a standard scalar integral given in appendix A. In the limit of negligible flavour mixing (which our model has by construction), I_q is approximated by [183]

$$I_q = \frac{m_{D\widetilde{g}}^2 - m_{\widetilde{q}_R}^2(1 - \log m_{D\widetilde{g}}^2/m_{\widetilde{q}_R}^2)}{(m_{D\widetilde{g}}^2 - m_{\widetilde{q}_R}^2)^2} - (L \leftrightarrow R) \tag{4.7.46}$$

$$= \frac{\delta m^2}{m_{\widetilde{q}}^4}\frac{(1+x)[x-1-\log(x)]}{(x-1)^3} + \mathcal{O}(\delta m^4/m_{\widetilde{q}}^4) \tag{4.7.47}$$

where $m_{\widetilde{q}_{L/R}} \neq m_{\widetilde{g}}$, $x \equiv m_{D\widetilde{g}}^2/m_{\widetilde{q}}^2$ and we have taken

$$\delta m^2 \equiv m_{\widetilde{q}_R}^2 - m_{\widetilde{q}_L}^2 \equiv m_{\widetilde{q}_R}^2 - m_{\widetilde{q}}^2. \tag{4.7.48}$$

Now we are in a position to calculate the decay rate of $\sigma_{\widetilde{g}}$. The general decay rate formula for a particle of mass m_X decaying into n particles with a set of momenta

$\{p_f\}$ and corresponding matrix element $\mathcal{M}$ is

$$d\Gamma = \frac{1}{2\,m_X}\left[\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right]|\mathcal{M}(m_X \to \{p_f\})|^2\,(2\pi)^4\delta^{(4)}\left(p_X - \sum p_f\right). \quad (4.7.49)$$

For a two–particle decay, the corresponding total width is then given by integrating over the phase space. Since the two final states we are interested in have equal masses $m \equiv m_1 = m_2$, their momenta will be definite and hence

$$\mathcal{M}(p_i) = \mathcal{M}. \quad (4.7.50)$$

Since we are working with manifestly Lorentz invariant expressions, we can choose the center–of–momentum frame

$$E_1 = E_2 \equiv E = \frac{E_{\sigma_{\tilde{g}}}}{2} = \frac{m_{\sigma_{\tilde{g}}}}{2}, \qquad \mathbf{p}_1 + \mathbf{p_2} = 0, \quad \mathbf{p}_{\sigma_{\tilde{g}}} = 0 \quad (4.7.51)$$

for the calculation. The total decay rate is then

$$\Gamma = \frac{1}{2m_{\sigma_{\tilde{g}}}}|\mathcal{M}|^2\int\frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}(2\pi)^4\delta^{(4)}\left[p_{\sigma_{\tilde{g}}} - (p_1+p_2)\right] \quad (4.7.52)$$

$$= \frac{1}{32\pi^2 m_{\sigma_{\tilde{g}}}}|\mathcal{M}|^2\int d^3p_1 d^3p_2\frac{1}{E^2}\delta^{(4)}\left[p_{\sigma_{\tilde{g}}} - (p_1+p_2)\right]. \quad (4.7.53)$$

We can decompose the delta function as

$$\delta^{(4)}\left[p_{\sigma_{\tilde{g}}} - (p_1+p_2)\right] = \delta\left[m_{\sigma_{\tilde{g}}} - 2E\right]\delta^{(3)}\left[\mathbf{p}_1 + \mathbf{p}_2\right] \quad (4.7.54)$$

where

$$E = m^2 + \mathbf{p}^2, \qquad \mathbf{p}^2 \equiv \mathbf{p}_1^2 = \mathbf{p}_2^2. \quad (4.7.55)$$

Performing the p_2 integral yields some of the constraints in eq. 4.7.51 and we then have

$$\Gamma = \frac{1}{32\pi^2 m_{\sigma_{\tilde{g}}}}|\mathcal{M}|^2\int d^3p_1\frac{1}{E^2}\delta\left[m_X - 2E\right] \quad (4.7.56)$$

$$= \frac{1}{32\pi^2 m_{\sigma_{\tilde{g}}}}|\mathcal{M}|^2\int dp\,p^2\,d\Omega\,\frac{1}{E^2}\delta\left[m_X - 2E\right], \quad (4.7.57)$$

where $d\Omega = \sin(\theta)\, d\theta\, d\phi$ is the usual differential solid angle, and $p^2 \equiv \mathbf{p}^2 = |\mathbf{p}^2|$. We can express eq. 4.7.57 as

$$\Gamma = \frac{1}{32\pi^2 m_{\sigma_{\tilde{g}}}} |\mathcal{M}|^2 \int dp\, d\Omega\, g(p)\, \delta[f(p)] \tag{4.7.58}$$

where

$$g(p) = \frac{p^2}{E^2} = \frac{p^2}{m^2 + p^2}, \qquad f(p) = m_{\sigma_{\tilde{g}}} - 2\sqrt{m^2 + p^2}. \tag{4.7.59}$$

Using the property of delta functions

$$\delta[f(x)] = \left| \frac{df(x)}{dx} \right|^{-1}_{x^*} \delta(x - x^*) \tag{4.7.60}$$

It follows that

$$\int dp\, g(p) \delta[f(p)] = g(p^*) \left| \frac{df(p)}{dp} \right|^{-1}_{p^*} \tag{4.7.61}$$

where

$$p^* = \frac{m_{\sigma_{\tilde{g}}}}{2} \sqrt{1 - \frac{4\, m^2}{m^2_{\sigma_{\tilde{g}}}}} \tag{4.7.62}$$

satisfies the original delta function. We know

$$\frac{df(p)}{dp} = -\frac{2\, p}{\sqrt{m^2 + p^2}} = -2\, \frac{p}{E} \tag{4.7.63}$$

and so we find the phase space part of the total decay rate

$$\Gamma = \frac{1}{32\pi^2 m_{\sigma_{\tilde{g}}}} |\mathcal{M}|^2 \frac{(p^*)^2}{E^2} \frac{E}{2\, p^*} \int d\Omega = \frac{1}{16\pi m_{\sigma_{\tilde{g}}}} |\mathcal{M}|^2 \sqrt{1 - \frac{4\, m^2}{m^2_{\sigma_{\tilde{g}}}}}. \tag{4.7.64}$$

We now need to compute $|\mathcal{M}|^2$ in our effective theory. The pseudosgluon can decay to either the combination $\psi_f\, \psi_f$ or $\psi_f^\dagger\, \psi_f^\dagger$ and so the matrix element is given by

$$i\mathcal{M} = -i\, c_q\, y(\mathbf{p_1}, s_1)\, y(\mathbf{p_2}, s_2) + i\, c_q\, x^\dagger(\mathbf{p_1}, s_1)\, x^\dagger(\mathbf{p_2}, s_2). \tag{4.7.65}$$

The squared amplitude is then

$$|\mathcal{M}|^2 = |c_q|^2 \left(y_1\, y_2\, y_2^\dagger\, y_1^\dagger + x_1^\dagger\, x_2^\dagger\, x_2\, x_1 - x_1^\dagger\, x_2^\dagger\, y_2^\dagger\, y_1^\dagger - y_1\, y_2\, x_2\, x_1 \right). \tag{4.7.66}$$

u	d	c	s	t	b
2.3 MeV	4.8 MeV	1.275 GeV	95 MeV	171.21 GeV	4.18 GeV

Table 4.4: Central value world average quark masses taken from [11].

m_0 (TeV)	$m_{\tilde{g}}$ (TeV)	τ (s)	L (m)
0.25	5	1.45×10^{-11}	4.33×10^{-3}
0.25	15	1.05×10^{-8}	3.16
0.5	5	6.45×10^{-15}	1.94×10^{-6}
0.5	15	4.71×10^{-12}	1.41×10^{-3}
1	5	2.50×10^{-15}	7.51×10^{-7}
1	15	1.83×10^{-12}	5.47×10^{-4}

Table 4.5: Pseudosgluon decays in the CMSSM.

We need to sum over the final state helicities using standard spin projection techniques. Performing the sum over final state antifermions

$$\sum_{\lambda_2} |\mathcal{M}|^2 = |c_q|^2 \left(y_1\, p_2 \cdot \sigma\, y_1^\dagger + x_1^\dagger\, p_2 \cdot \sigma\, x_1 + x_1^\dagger\, m\, y_1^\dagger + y_1\, m\, x_1 \right), \tag{4.7.67}$$

and then summing over fermion spins

$$\sum_{\lambda_1 \lambda_2} |\mathcal{M}|^2 = 4\, |c_q|^2 \left(p_1 \cdot p_2 + m^2 \right) = 2\, |c_q|^2\, m_{\sigma_{\tilde{g}}}^2. \tag{4.7.68}$$

Finally, we find the decay rate of the pseduosgluon into a quark–antiquark pair is given by

$$\Gamma_q = \frac{|c_q|^2\, m_{\sigma_{\tilde{g}}}}{8\pi} \sqrt{1 - \frac{4\, m_q^2}{m_{\sigma_{\tilde{g}}}^2}}. \tag{4.7.69}$$

The total decay rate rate for the pseduosgluon is then given by

$$\Gamma_{\text{Total}} = \sum_q \Gamma_q. \tag{4.7.70}$$

Note that due to the nature of the coefficient c_q this is heavily dependent on the spectrum. One can immediately see that the pseudosgluon will decay more rapidly in CGGM than in the CMSSM, though in both cases it is unstable.

CMSSM: To a good approximation, the splitting between the mass of the left and right handed squarks can be taken to be the order of the Z boson mass[6]. For simplicity, in the CMSSM we therefore take[7]

$$\delta m^2 = m_{\rm Z}^2. \tag{4.7.73}$$

The sign is irrelevant due to the modulus squared in eq. 4.7.69. The remaining masses are taken to be either the measured physical quark masses given in table 4.4 or determined by the choice of m_0 and $m_{D\tilde{\rm g}}$ through the approximations

$$m_{\sigma_{\tilde{\rm g}}}^2 = m_0^2, \qquad\qquad m_{\tilde{q}}^2 = \frac{g_3^2\, m_{D\tilde{\rm g}}^2}{3\,\pi^2}\log(4). \tag{4.7.74}$$

The lifetime τ is then calculated by[8]

$$\tau = \frac{1}{\Gamma_{\rm Total}}. \tag{4.7.76}$$

Taking $g_3 = 1.22$, we find the lifetimes and decay lengths in table 4.5.

CGGM: The dominant splitting between the mass of the left and right handed squarks is generated by an additional gauge mediated contribution from the $\rm SU(2)_L$ messengers

$$\delta m^2 = \frac{g_2^4}{(16\pi^2)^2}\Lambda_S^2. \tag{4.7.77}$$

---

[6] In the presence of large stop mixing, the difference in the stop masses will be larger, but we do not consider this case here as we are more interested in an upper limit on the lifetime.

[7] Note that from section 3.6.6 we know that the additional supersoft operator causes a splitting between the left and right squark masses at tree level, giving contributions to the relevant δm^2 of the form

$$\delta m_{\rm u}^2 \to \delta m_{\rm u}^2 - \frac{5\,g_1}{3}\, v_{\tilde{\rm B}}\, m_{D\tilde{\rm B}} - g_2\, v_{\widetilde{\rm W}}\, m_{D\widetilde{\rm W}} \tag{4.7.71}$$

$$\delta m_{\rm d}^2 \to \delta m_{\rm d}^2 + \frac{g_1}{3}\, v_{\tilde{\rm B}}\, m_{D\tilde{\rm B}} + g_2\, v_{\widetilde{\rm W}}\, m_{D\widetilde{\rm W}}. \tag{4.7.72}$$

Decays of the a pseduosgluon in a more general CMSSM with Dirac Binos and Winos would be more prompt.

[8] Recall that

$$1\ {\rm GeV}^{-1} = 6.58\times 10^{-25}\, s. \tag{4.7.75}$$

Λ_S (GeV)	$m_{\tilde{g}}$ (TeV)	τ (s)	L (m)
10^3	5	3.63×10^{-4}	1.09×10^5
10^3	15	2.64×10^{-1}	7.93×10^7
10^4	5	3.09×10^{-9}	9.28×10^{-1}
10^4	15	2.26×10^{-6}	6.76×10^2
10^5	5	2.06×10^{-17}	6.19×10^{-9}
10^5	15	1.51×10^{-14}	4.51×10^{-6}

Table 4.6: Pseudosgluon decays in CGGM.

The remaining elements of the integral are approximated

$$m^2_{\sigma_{\tilde{g}}} = \frac{8}{3}\frac{g_3^4}{(16\pi^2)^2}\Lambda_S^2, \qquad m^2_{\tilde{q}} = \frac{g_3^2\, m^2_{D\tilde{g}}}{3\,\pi^2}\log(4). \tag{4.7.78}$$

Taking $g_2 = 0.652$ and $g_3 = 1.22$, we find the lifetimes and decay lengths in table 4.6.

Consequences for LHC searches: For a relatively light pseudosgluon $m_{\sigma_{\tilde{g}}} < 2\,m_{\rm t}$, then it will decay via the loop interaction into light quarks that hadronise and are hidden in the low energy QCD background. As soon as the pseudosgluon can undergo decay to two top quarks, i.e. $m_{\sigma_{\tilde{g}}} > 2\,m_{\rm t}$, then there is the possibility to constrain the pseudosgluon via the one or two lepton decay topology as was done in [9, 138, 145, 186, 252]. The most useful for our purposes is the study in [9] where a simplified model approach is taken, and a limit on the pseudosgluon mass was derived as a function of its effective a_t coupling to two top quarks (see figure 4.13).

$$-i\sqrt{2}\,\mathcal{L}_{\rm eff} \supset a_{\rm t}\,\sigma_{\tilde{g}}\,\psi_{\rm t}\,\psi_{\rm t}. \tag{4.7.79}$$

For us, $a_{\rm t}$ is simply determined through

$$a_{\rm t} = \sqrt{2}\,c_{\rm t}, \tag{4.7.80}$$

where $c_{\rm t}$ is just a function of the gluino and squark mass spectrum. If the gluinos and squarks are sufficiently heavy then we can see there is no limit LHC searches on the multitop decay of the pseudosgluon. In the CMSSM if we take again the

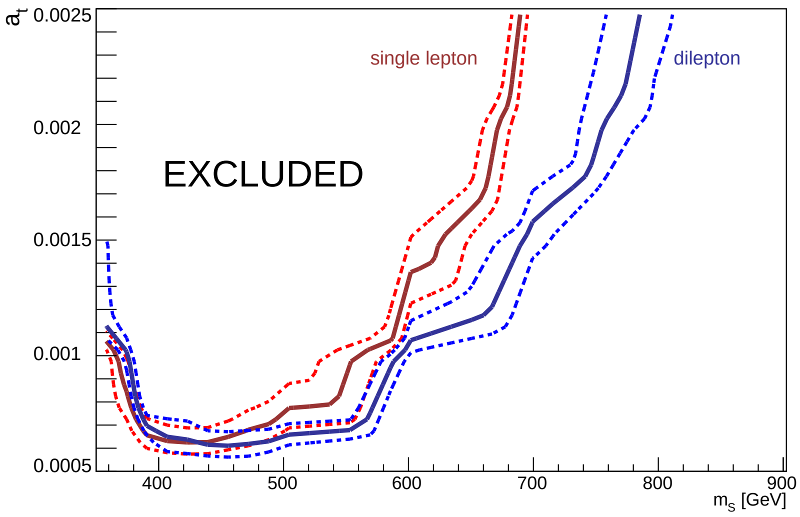


Figure 4.13: The excluded regions in the $(m_S, a_{\rm t})$ space derived from the single lepton analysis (red solid line), and lepton analysis (blue solid line). In both cases, the dashed lines correspond to the exclusion regions obtained when a_g is varied by $\pm 10\%$. Taken from [9]. a_g parameterises the coupling strength of a single sgluon to two gluons. a_g/Λ is taken to be the reference value 1.5×10^{-6} (GeV)$^{-1}$.

approximations in eqs. 4.7.72 and 4.7.73 then we find

$$m_{D\tilde{g}} \gtrsim 980 \text{ GeV}. \tag{4.7.81}$$

In CGGM if we take again the approximations in eqs. 4.7.77 and 4.7.78 then we find

$$\frac{m^3_{D\tilde{g}}}{\Lambda^2} \gtrsim 0.82 \text{ GeV}. \tag{4.7.82}$$

The limits in eqs. 4.7.81 and 4.7.82 are easy to achieve in a UV complete model as has been demonstrated in this chapter, and are indeed typical in order to achieve a correct Higgs mass. It may be possible to exploit the considerable decay lengths of the pseudosgluon observed in tables 4.5 and 4.6 to identify misplaced vertices for heavy gluinos. This would still be a challenge however, due to the low production cross sections show in fig. 4.11 and requires a separate, detailed investigation.

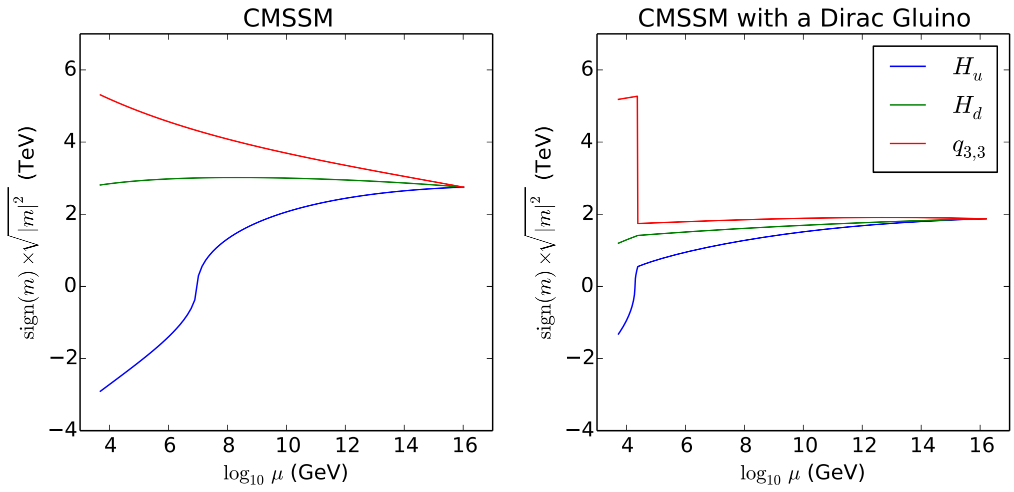


Figure 4.14: RGE of $m^2_{\mathrm{H}_u}$ (blue), $m^2_{\mathrm{H}_d}$ (green) and $m^2_{\tilde{q}_{3,3}}$ (red) from the GUT scale to the SUSY scale in the CMSSM (left) and the CMSSM with a Dirac gluino (right) for the benchmark points given in table 4.7

Model	m_0 (TeV)	$M_{1/2}$ (TeV)	$m_{D\tilde{g}}$ (TeV)	$m^{(1)}_{\mathrm{h}}$ (GeV)	$m^{(2)}_{\mathrm{h}}$ (GeV)
CMSSM	2.750	3.000	N/A	118.1	127.4
CMSSM + DG	1.875	1.000	10.00	117.3	unknown

Table 4.7: Benchmark points for the RG evolution of parameters in the CMSSM with and without a Dirac gluino shown in figure 4.14.

4.8 EWSB and fine tuning

As has already been indicated, EWSB in a model with a Dirac gluino is triggered much closer to the electroweak scale. As is well understood in most SUSY models, it is the stop mass (and at two loops a Majorana gluino mass) that causes this to happen. The same is true with a Dirac gluino. The difference here is that the stop mass can be negligible along the whole RG flow until the Dirac gluino mass is reached. The supersoft contribution from integrating out the gluino is applied to the squark masses, and they drive $m^2_{\mathrm{H}_u}$ negative for the remainder of the flow through its RG equation given in eq. 4.5.35. This effect is demonstrated in fig. 4.14. The upshot is that for a particularly large final squark mass, there is some control over how large $m^2_{\mathrm{H}_u}$ (and consequently $|\mu|^2$) is. In the LL approximation at one loop we find

$$m_{\mathrm{H}_u}^2(m_{\mathrm{SUSY}}) = m_{\mathrm{H}_u}^2(m_{\mathrm{GUT}}) - \beta^{(1)}_{m^2_{\mathrm{H}_u}} \times \log\left(\frac{m_{\mathrm{GUT}}}{m_{\mathrm{SUSY}}}\right)$$
$$\approx m_0^2\left[1 - \frac{3\,|y_{\mathrm{t}}|^2}{4\,\pi^2}\right] \times \log\left(\frac{m_{\mathrm{GUT}}}{m_0}\right) \tag{4.8.83}$$

in the CMSSM and

$$m_{\mathrm{H}_u}^2(m_{\mathrm{SUSY}}) \approx m_0^2 - (m_0^2 + m_q^2) \times \frac{3\,|y_{\mathrm{t}}|^2}{4\,\pi^2} \times \log\left(\frac{m_{D\tilde{g}}}{m_0 + m_q}\right) \tag{4.8.84}$$

in the CMSSM with a Dirac gluino where m_q^2 is given by eq. 3.4.59. Since $m^2_{\mathrm{H}_d}$ is so linked to the electroweak UV sensitivity, it is reasonable to expect that Dirac gluinos have the ability to reduce the amount of fine tuning in the presence of larger squark masses.

To quantify the impact this difference in triggering EWSB has on fine tuning, we take the measure Δ from [253]

$$\Delta \equiv \max\left[\mathrm{Abs}(\Delta_{\mathcal{O}})\right], \qquad\qquad \Delta_{\mathcal{O}} \equiv \frac{\partial \log v^2}{\partial \log \mathcal{O}} \tag{4.8.85}$$

such that Δ^{-1} gives a measure of how tuned the parameters $\mathcal{O}$ need to be tuned to achieve the observed EWSB scale v. This measure was compared to the naïve one we used in eq. 2.3.347 by [254], and found although they were comparable, the one in eq. 2.3.347 tends to overestimate the tuning since it cannot account for correlantions between the parameters. The Δ for the analysis at hand was calculated at the SUSY scale using the routines generated by `SARAH` modified to include the thresholds discussed in Section 4.3.3 where appropriate. Since we are interested in UV sensitivity, we take the $\mathcal{O}$s as the set of parameters that would be fixed by the UV model at either the GUT scale in CMSSM or the messenger scale in CGGM. These are

$$\mathcal{O}|_{\mathrm{CMSSM}} \in \{m_0, M_{1/2}, \mu, b_\mu, m_{D\tilde{g}}\}, \quad \mathcal{O}|_{\mathrm{CGGM}} \in \{\Lambda_G, \Lambda_S, m_{\mathrm{Mess}}, \mu, b_\mu, m_{D\tilde{g}}\}. \tag{4.8.86}$$

The tuning in the CMSSM for the parameter space investigated in Section 4.5 is shown in figs. 4.15 and B.4, and the tuning in CGGM is shown in figs. 4.16, B.14, B.15 and B.16.

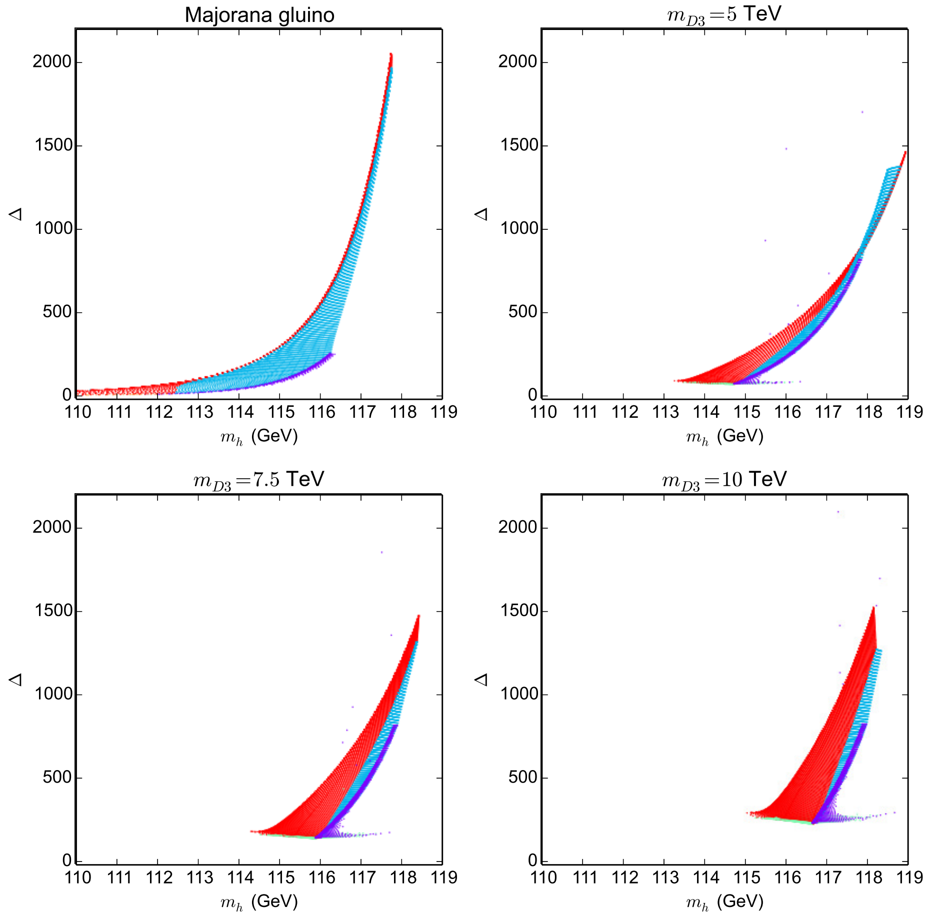


Figure 4.15: Fine tuning in the CMSSM with $t_\beta = 10$ and $m_{D\tilde{g}}$ fixed as indicated. The red, purple, blue, and green regions correspond to μ, m_0, $M_{1/2}$ and $m_{D\tilde{g}}$ as the dominant source of tuning.

In the CMSSM and in CGGM it is observed that, for a given Higgs mass, new points exist with a reduction in fine tuning of typically up to a factor of two or three. In the CMSSM also a line of points opening up with moderately large Higgs mass mass but low ($\Delta \sim 200$) fine tuning. These points occur where the two terms in eq. 4.8.84 approximately cancel, giving low — $\mathcal{O}(0.5 - 1\,\text{TeV})$ — values of m_{H_u} and μ. The strip is very thin, since an increase in either m_0 or $M_{1/2}$ makes the right hand side become more positive in eq. 4.8.84, leaving no EWSB and decreasing m_0 or $M_{1/2}$ leads to a reduction in the Higgs mass. Unfortunately since these points are at very low values of $M_{1/2}$ that give rise to neutralino and chargino masses that are excluded by LEP.

The reduction in tuning in CGGM is less drastic than that seen in the CMSSM. This is because the mechanism reduces tuning through making logarithms smaller.

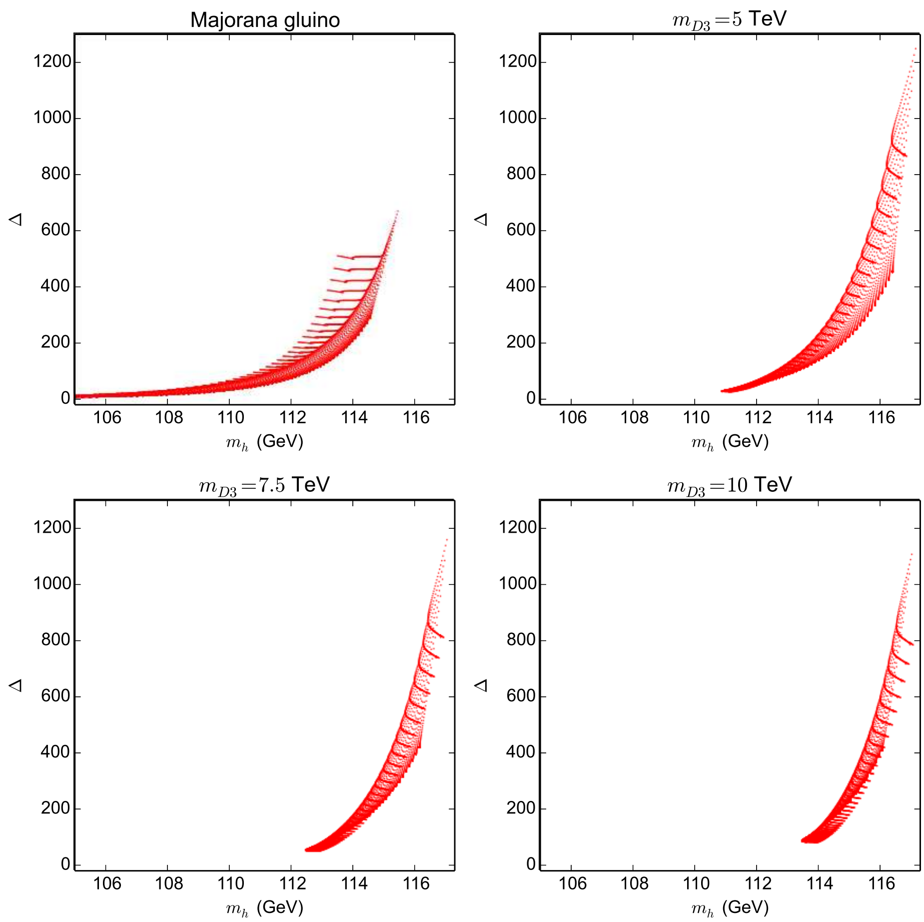


Figure 4.16: Fine tuning in the CGGM with $t_\beta = 10$, $m_{\text{Mess}} = 10^7$ GeV and $m_{D\tilde{g}}$ fixed as indicated. The dominant source of tuning is entirely from the μ parameter.

In the CMSSM we have the log reduced $\log(m_{\text{GUT}}/m_{\text{SUSY}}) \to \log(m_{D\tilde{g}}/m_{\text{SUSY}})$, whereas in CGGM this is only the factor $\log(m_{\text{Mess}}/m_{\text{SUSY}}) \to \log(m_{D\tilde{g}}/m_{\text{SUSY}})$. Similarly, the reduction in fine tuning in CGGM is less drastic in the case of the lower messenger scale than the higher messenger scale. In the CMSSM one can see the full range of UV parameters becoming the dominant source of tuning whereas in CGGM it is mainly the μ parameter across the entire space. However, in both the CMSSM and CGGM, all the underlying UV parameters considered do have associated tunings across the respective parameter spaces.

4.9 Chapter summary

In this chapter we constructed a set of simple UV models with the supersoft mechanism outlined in Section 3.4 by extending the MSSM field content by only what was required to give the gluino a Dirac mass. We then performed the first implementation of the supersoft mechanism into a state of the art spectrum generator and carried out an analysis of the spectra, the production rates at LHC8 and LHC13, and fine tuning.

In the presence of a Dirac gluino, we find that it is possible to essentially decouple the strong sparticles without affecting the electroweak spectrum except that one finds that the pseudo–sgluon usually remains light and may even be a novel dark matter candidate by forming neutral bound states with other strongly interacting particles.

The decoupling of the strongly interacting sparticles from the electroweak sparticles has been shown to give a handle on the production cross sections at the LHC. Using a product of branching ratios approximation, we have shown that the Dirac gluino completely removes the same sign di–lepton as a visible signature in current LHC data. A full simulation of the decay chain needs to be done to confirm this and it should also include the usually sub–dominant purely electroweak contributions to these events as these may now be important. It would also be interesting to investigate how many charginos and neutralinos are still produced in these cases with t-channel squarks.

Taking account the spectra and cross section suppression, we find that the final states of these models at the LHC are therefore altered in the following way:

- The number of events involving the Majorana gluino propagator are suppressed by roughly two orders of magnitude. This includes the same sign di–lepton events.

- Events involving the pair production of gluinos are absent due to kinematic inaccessibility.

- The mass hierarchy between the strong and electroweak sectors causes hard jets in a SUSY cascade to be harder than usual.

- LOSP candidates are typical, yielding a number of leptons and missing energy in the final stages of a cascade. In the case of CGGM this may also include the emission of a photon.

- The number of events with jets and missing energy will increase in the case of a stable pseudo–sgluon.

Unfortunately there are no smoking gun signatures for these models. Their main distinguishing characteristic is that there are different numbers of each type of visible event compared to models without a Dirac gluino — generally fewer. Note that for models of this type, a new lepton collider such as the International Linear Collider (ILC) or Compact Linear Collider (CLIC) would be able to simply bypass the strong sparticle sector and directly probe the much lighter accessible electroweak states.

Finally, the allowed tuning in these models is found to be reduced. In allowed regions of parameter space, the reduction for a given Higgs mass is generally by a factor of two or three, although one has to keep in consideration that a reduction in fine tuning is being achieved whilst the gluino mass is being taken up to ten times greater that which is usually considered for precisely reasons of tuning.

There are two obvious extensions of this study:

- The accuracy of the Higgs mass calculation needs improving in order to say something more concrete and more tightly constrain the model. In order to achieve this, the full set of general broken SUSY two loop RGEs should be used below the Dirac gluino mass, allowing a two loop accurate Higgs mass prediction. This should be possible with the general two loop RGE calculators on the market [242,245]. Since these calculations are in $\overline{\mathrm{MS}}$ scheme, one would need to take care to convert to the $\overline{\mathrm{DR}}$ scheme before implementing them into a SUSY spectrum generator [255].

- In the case of the CMSSM, we kept the A terms zero for simplicity. As was noted in [246], the presence of additional scalar octets allows g_3 to remain much larger over the RG flow, and can consequently generate large negative A terms in the IR providing one starts with a negative A term. This model has the potential to reduce tuning much further by allowing a reduction in the squark masses and at the same time the length of flowing between the Dirac gluino mass and the SUSY scale.

5

Mapping Dirac gaugino masses

But please remember: this is only a work of fiction. The truth, as always, will be far stranger.

– Arthur C. Clarke, *2001: A Space Odyssey*

This chapter is based on my work done in collaboration with Steven Abel [2]. The text has been partially rewritten.

5.1 Background and purpose

There have been attempts in the literature to create a phenomenological model of the MSSM as an (at least partial) Seiberg dual of some UV complete theory [83,84]. In these models, the mapping of the soft terms discussed in Section 2.3.8 generates a set of boundary conditions for the IR theory, determining its spectrum in terms of the spectrum of the UV theory

$$\tilde{M} = -\frac{3\,N_c - 2\,N_f}{3\,N_c - N_f}\,\tilde{M}. \tag{5.1.1}$$

$$m_\varphi^2 = 2\,\frac{3\,N_c - 2\,N_f}{b}\,m_{\mathrm{UV}}, \qquad m_q^2 = -\frac{3\,N_c - 2\,N_f}{b}\,m_{\mathrm{UV}}, \tag{5.1.2}$$

The relation 5.1.1 was derived in Section 2.3.8 from the RG invariance of the dimensional transmutation scale, and the relations in eq. 5.1.2 just follow from a more general RG invariant function of the dimensional transmutation scale and powers of the superfield wavefunction renormalisation. These results are interesting because it allows us to make sense of theories that become strongly coupled due to their matter content on a microscopic level.

Theories with a Dirac gluino and a simple GUT structure[1] lose their asymptotic freedom before the GUT scale [180], so it is worth investigating if at least the $\mathrm{SU}(3)_\mathrm{C}$ sector of the MSSM with a Dirac gluino could be a Seiberg–like dual of a UV free theory. That is the purpose of this chapter. The additional $\mathrm{SU}(3)_\mathrm{C}$ superfield content requires the generalisation of Seiberg duality to Kutasov duality [10, 256, 257]. The aim is to arrive at a relation similar to eqs. 5.1.1 and 5.1.2, except for a Dirac gaugino mass. The difficulty to overcome lies in identifying the RG invariant relationship that can be compared across the duality, since there is no obvious superfield spurion in the $\mathcal{N} = 1$ language that can be used to derive such a result as was done for the Majorana case. What is identified is an $\mathcal{N} = 2$ spurion that could achieve this, and independently, an RG invariant relationship originating from the supersoftness of theories with Dirac gauginos

$$\frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} = \text{RG invariant.} \tag{5.1.3}$$

In the remaining part of the chapter it is demonstrated how an $\mathcal{N} = 1$ Kutasov theory could be written as an $\mathcal{N} = 2$ theory in the presence of a special kind of SUSY breaking, and demonstrate how the same kind of breaking can induce the Dirac gaugino mass. It is shown that the Dirac mass maps on to a dual Dirac mass, and a sequence of RG flows and higgsings exists that connects the dual Kutasov theories together (shown in fig. 5.1). Combining all of this means that we can follow the RG invariant relationship along the flow, via the $\mathcal{N} = 2$ pair, and to the other side of the Kutasov theory, yielding the result

$$\lim_{\mu\to\infty} \frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} = \lim_{\mu\to 0} \frac{\tilde{m}_D}{\hat{\tilde{g}}\,\tilde{\kappa}^{\frac{1}{k+1}}}. \tag{5.1.4}$$

5.2 Introduction

In the previous chapter, we saw that the introduction of a χSF in the **Ad** of $\mathrm{SU}(3)_\mathrm{C}$ brought the theory close to losing asymptotic freedom (see fig. 4.4). Indeed, in [180],

[1] Note: $(\mathrm{SU}(3))^3$ is fine in this respect, but serious issues are encountered when one considers an SU(5) GUT.

gauge unification in the SU(5) case was seen to be impossible because the theory hit a Landau pole before the GUT scale. It was noticed in [258, 259] that achieving a GUT beyond the Landau pole is possible in *dualification* if the low energy theory is actually the dual magnetic description of an asymptotically free electric theory (as discussed in Section 2.3.8). In this scenario, Seiberg duality acts upon the strongly coupled gauge group then unification happens in the dual picture at physical values of all the gauge couplings. There have also been attempts to construct a Seiberg dual for the MSSM [83,84], involving the mapping of soft terms discussed in Section 2.3.8, though in these cases, the SM gauge groups is a spectator gauge group to the Seiberg duality.

Almost all of the literature dealing with Dirac gaugino masses considers them in a perturbative setting. An exception is [154], where the adjoint fermions that become the right–handed gauginos are the mesinos of a strongly coupled $\mathcal{N} = 1$ gauge theory. In this case, as with [83, 84], the gauge symmetry of interest is just a spectator flavour symmetry of the duality. Dirac gaugino mass terms can originate from operators in the UV

$$W_{\text{eff}}^{\text{el Dirac}} = \frac{1}{M^2}\,\text{tr}\,(\tilde{Q}\,Q)\,\mathcal{W}'^{\alpha}\,\mathcal{W}_{F\alpha}, \tag{5.2.5}$$

where $\mathcal{W}_{F\alpha}$ is the gauge field superstrength of the flavour symmetry and $\mathcal{W}'^{\alpha}$ is the gauge field superstrength of a hidden $\text{U}(1)'$ gauge symmetry that aquires a D term VEV $\langle \mathcal{W}'^{\alpha} \rangle = \theta_\alpha \langle D' \rangle$. In this case, however, then the whole operator is blind to the duality and is therefore trivial to map to the IR

$$W_{\text{eff}}^{\text{mag Dirac}} = \frac{1}{M}\,\varphi\,\mathcal{W}'^{\alpha}\,\mathcal{W}_{F\alpha}, \tag{5.2.6}$$

where the mesino $\tilde{\varphi} \sim \Lambda^{-1}\,\text{tr}(\tilde{Q}\,\psi_Q)$ aquires a Dirac mass $m_D \sim \Lambda\, D/M^2$ with the flavour gaugino λ_F.

A more interesting question is 'what happens to Dirac mass terms involving the gauginos of the colour gauge symmetry that becomes strongly coupled?'. To make the question precise, we will focus on the $\mathcal{N} = 1$ generalisation of Seiberg duality to include a χSF X in the **Ad** of the colour symmetry $\text{SU}(N_c)$ known as Kutasov duality [10, 256, 257, 260, 261]. In the free magnetic phase, an asymptotically free electric $\text{SU}(N_c)$ theory with N_f flavours of left–handed quarks Q and right–handed quarks $\tilde{Q}$, and an adjoint X with a superpotential

$$W^{\rm el} = \frac{\kappa}{k+1}\,{\rm tr}(X^{k+1}), \tag{5.2.7}$$

flows to an IR free SU($\widetilde{N_c}$) theory with N_f flavours of magnetic left–handed quarks q and right–handed quarks $\tilde{q}$, and a chiral adjoint x, and a set of mesons φ_i with a superpotential

$$W^{\rm mag} = \frac{\tilde{\kappa}}{k+1}{\rm tr}(x^{k+1}) + \sum_{j=1}^{k} \varphi_j\,\tilde{q}\,x^{k-j}\,q. \tag{5.2.8}$$

In this chapter we provide evidence that the Dirac gaugino mass terms

$$W^{\rm el\ Dirac} = \sqrt{2}\,m_D \int d^2\theta\,\theta^\alpha\,X^a\,\mathcal{W}^a_\alpha + {\rm h.c.}, \tag{5.2.9}$$

$$W^{\rm mag\ Dirac} = \sqrt{2}\,\tilde{m}_D \int d^2\theta\,\theta^\alpha\,x^a\,\tilde{\mathcal{W}}^a_\alpha + {\rm h.c.} \tag{5.2.10}$$

map from the UV to the IR as[2]

$$\lim_{\mu\to\infty} \frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} = \lim_{\mu\to 0} \frac{\tilde{m}_D}{\hat{\tilde{g}}\,\tilde{\kappa}^{\frac{1}{k+1}}} \tag{5.2.14}$$

across the Kutasov duality, analogous to eq. 2.3.324. Here the coupling κ is a canonically normalised electric superpotential coupling $\kappa\,X^{k+1}$ appearing in eq. 5.2.7, $\hat{g}$ is the holomorphic electric gauge coupling, and $(\tilde{\kappa}, \hat{\tilde{g}})$ are the corresponding dual

[2]Actually, as we will see later, the relationship we discover is much more like

$$c_1 \lim_{\mu\to\infty} \frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} = c_2 \lim_{\mu\to 0} \frac{\tilde{m}_D}{\hat{\tilde{g}}\,\tilde{\kappa}^{\frac{1}{k+1}}}, \tag{5.2.11}$$

i.e. we are able to determine the form of the mapping up to a prefactor. This is not the same scenario as with e.g. the Majorana gaugino mass where we are able to determine precisely

$$\tilde{M} = -\frac{3\,N_c - 2\,N_f}{3\,N_c - N_f}\,\tilde{M}. \tag{5.2.12}$$

The reason for this difference is that eq. 5.2.12 steps from the relationship between two RG invariants Λ and $\tilde{\Lambda}$ that are *known* to be equal

$$\Lambda = \tilde{\Lambda} \tag{5.2.13}$$

since the physics matches across the duality. In the case if the Dirac gluino mass, we only have the relation 5.1.3, as well (as we will show in this Chapter) an RG flow that connects this RG invariant combination to its dual combination. Unfortunately we are not in a position to say which RG invariants need to be equal, but are in a position to say that there is good evidence one exists, leaving the mapping of the Dirac gaugino mass ambiguous up to a prefactor.

magnetic variables.

5.3 A key observation

We have already encountered an all-orders RG invariant relationship involving m_D in eq. 3.4.45 due to the (potentially) holomorphic nature of the *supersoft* operator. For convenience, we rewrite the expression here (where to match the notation of Kutasov, our χSF is X rather than Φ)

$$\beta_{m_D} = m_D \left(\frac{\gamma_X}{2} + \frac{\beta_{\hat{g}}}{\hat{g}} \right). \tag{5.3.15}$$

By the non–renormalisation theorem, we can rewrite γ_X as

$$\beta_\kappa = \frac{k+1}{2}\, \kappa\, \gamma_X \tag{5.3.16}$$

and so the expression 5.3.17 becomes

$$\beta_{m_D} = m_D \left[\frac{\beta_\kappa}{\kappa\,(k+1)} + \frac{\beta_{\hat{g}}}{\hat{g}} \right]. \tag{5.3.17}$$

One then sees that the combination $m_D/(\hat{g}\,\kappa^{\frac{1}{k+1}})$ is an all–orders RG invariant

$$\frac{d}{dt}\, \frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} = -\frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} \left[\frac{\beta_\kappa}{\kappa\,(k+1)} + \frac{\beta_{\hat{g}}}{\hat{g}} \right] + \frac{\beta_{m_D}}{\hat{g}\,\kappa^{\frac{1}{k+1}}} = 0. \tag{5.3.18}$$

Our task would be to then identify which holomorphic RG invariant Λ contains the combination $m_D/(\hat{g}\,\kappa^{\frac{1}{k+1}})$

$$\Lambda = \ldots + \theta^i\, \theta^{\dagger j}\, \frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} + \ldots \tag{5.3.19}$$

for some i, j, and in the dual picture

$$\tilde{\Lambda} = \ldots + \theta^i\, \theta^{\dagger j}\, \frac{\tilde{m}_D}{\tilde{\hat{g}}\,\tilde{\kappa}^{\frac{1}{k+1}}} + \ldots. \tag{5.3.20}$$

Arguing $\Lambda = \tilde{\Lambda}$ as was done in Section 2.3.8 would then give the map 5.2.14. Establishing the map 5.2.14 is not as straightforward as it was for Majorana gauginos however, as there is no coupling in a renormalisable $\mathcal{N} = 1$ theory that can be promoted to a spurious superfield that contains the Dirac mass. Consequently, no all–orders RG invariant Λ can be immediately constructed and matched. It was however shown in Section 3.3.4 that a spurious redefinition of the $\mathcal{N}=2$ gauge coupling *can* introduce a Dirac gaugino mass (see eq. 3.3.40) suggesting a way forward.

5.4 Overview of method

As mentioned, in Kutasov theory (or indeed *any* $\mathcal{N}=1$ SUSY gauge theory) there is no RG invariant that can be built from the couplings of the $\mathcal{N}=1$ theory which can incorporate a Dirac mass from promoting the coupling constants of the theory to superfields. It is possible to achieve this in an $\mathcal{N}=2$ theory where the X becomes part of the $\mathcal{N}=2$ gauge supermultiplet (see figure 3.1). One way of achieving this is to write the Kutasov theory as the spurion of an $\mathcal{N}=2$ theory and then introduce a spurion for the gauge coupling to generate a Dirac gaugino mass. The *2 into 1 won't go* theorem [67, 68] (see Section 2.3.7) greatly restricts how $\mathcal{N}=2 \to \mathcal{N}=1$ breaking can occur. We will therefore break SUSY in a way that evades the assumptions of the theorem by using a combination of electric and magnetic FI terms [262–264] inspired by [60, 61] and will be referred to as the Antoniadis–Taylor–Partouche (ATP) mechanism.

The remainder of this Chapter proceeds as follows:

- We consider the $N_f = 2\,N_c$ version of the $\mathcal{N}=1$, SU(N_c) Kutasov theory with a superpotential deformation $h\,\tilde{Q}\,X\,Q$ – where $h \ll g$ is parametrically small. We show perturbatively that for $k=2$ this theory can flow to the $\mathcal{N}=2$ fixed line in the IR, where $h \to g$ and $\kappa \to 0$, i.e. $\mathcal{N}=2$ SQCD with a superpotential deformation $\kappa\,X^{k+1}$ where now $\kappa \ll g$ is parametrically small,

- We show that the h coupling in the magnetic description of the deformed Kutasov theory induces the correct Higgsing for any k, causing the magnetic description to also flow to an $\mathcal{N}=2$ SQCD theory,

- We establish that the above deformations can be generated by electric and magnetic FI terms in an $\mathcal{N}=2$ theory with an appropriate prepotential.

This completes a route that goes from an electric $\mathcal{N}=1$, SU(N_c) Kutasov theory to its magnetic dual via an intermediate pair of $\mathcal{N}=2$ duals. The Dirac masses can

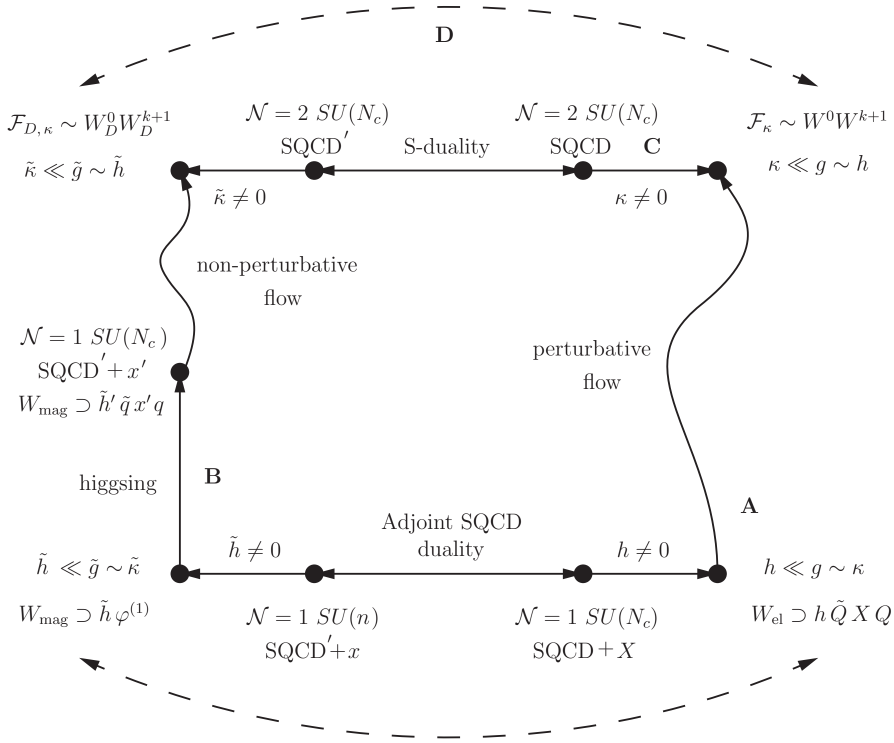


Figure 5.1: The flow between $\mathcal{N}=2$ S–duality and $\mathcal{N}=1$ Kutasov duality. **A**: The duality of [10] is deformed with a parametrically small $\mathcal{N}=2$ gauge interactions for the quarks. The resulting perturbative flow to $\mathcal{N}=2$ SQCD is analysed in Section 5.5.2. **B**: The the magnetic dual of the $\mathcal{N}=2$ quark gauge interactions are observed to Higgs the magnetic theory down to a gauge group of the same rank as the electric theory. This theory then flows to $\mathcal{N}=2$ SQCD′, as discussed in Section 5.5.3. **C**: The electric theory of [10] is now written as an $\mathcal{N}=2$ theory broken to $\mathcal{N}=1$ at low energies by electric and magnetic FI terms, as discussed in Section 5.6.3. **D**: The existence of a small dual x^{k+1} deformation is shown to be required in the presence of a small electric X^{k+1} deformation.

then be added by additional FI terms and tracked down the dual RG trajectories to the dual Kutasov theories using eq. 5.3.15. A schematic of the overall picture (before adding the soft terms) is shown in figure 5.1.

5.5 From Kutasov duality to $\mathcal{N}=2$ duality

In this section we will try and understand the RG flow from a pair of dual Kutasov theories to a pair of dual $\mathcal{N}=2$ theories. Ideally we would like to be able to study the flow from the electric Kutasov theory at a fixed point to the fixed line of $\mathcal{N}=2$ SQCD since then we would know the anomalous dimensions precisely (see Section 2.3.8). In particular one might imagine that there would be a BZ–like fixed point for the Kutasov theory with a parametrically small superpotential deformation

	$\mathrm{SU}(N_c)$	$\mathrm{SU}(N_f)_L$	$\mathrm{SU}(N_f)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	$\frac{1}{N_c}$	$1-R_X\frac{N_c}{N_f}$
$\tilde{Q}$	$\bar{\square}$	$\mathbf{1}$	$\bar{\square}$	$-\frac{1}{N_c}$	$1-R_X\frac{N_c}{N_f}$
X	**Ad**	$\mathbf{1}$	$\mathbf{1}$	0	R_X

Table 5.1: The matter content of the electric Kutasov theory with the superpotential deformation $\tilde{Q}\,X\,Q$. All the flavour charges are anomaly-free with respect to the gauge symmetry. R_X is fixed by the superpotential.

$h\,\tilde{Q}\,X\,Q$ and with $N_f = 2\,N_c$. Such a theory could flow to the $\mathcal{N}=2$ fixed line. Unfortunately this turns out to be impossible due to the a theorem [265–267] as we shall show; either the $\mathcal{N}=1$ theory or the $\mathcal{N}=2$ theory cannot be at a fixed point (line) if a RG flow is to connect them.

The next best thing — the RG flow from one theory *not* at a fixed point (line) to the other at a fixed line (point) — does occur perturbatively in the $k=2$, $N_f=2N_c$ case, as we show in subSection 5.5.2. We then identify the Higgsing mechanism whereby the strongly coupled dual $\mathcal{N}=1$ theory flows to the dual $\mathcal{N}=2$ theory for any k.

Finally, we propose a way to extend the study to regions of parameter space where neither dual is perturbative.

5.5.1 No flowing between fixed points and fixed lines

The theory of interest

Consider $\mathcal{N}=1$ Kutasov theory with a superpotential deformation $h\,\tilde{Q}\,X\,Q$ coupling. The full electric superpotential is

$$W^{\text{el}} = h\,\tilde{Q}\,X\,Q + \frac{\kappa}{k+1}\,\mathrm{tr}(X^{k+1}). \tag{5.5.21}$$

The field content and representations are detailed in table 5.1. The $h=g$ and $\kappa=0$ limit corresponds to $\mathcal{N}=2$ SQCD and the $h=0$ limit corresponds to the electric Kutasov theory. Providing $\frac{N_f}{N_c} \le 3+\sqrt{7}$ and $k \le 15$, the X^{k+1} term is relevant at the IR fixed point in the limit $h \to 0$ if [268]

$$\frac{N_f}{N_c} > x_k, \qquad x_k = \sqrt{\frac{1}{20}\left[\frac{(5\,k-4)^2}{9}+1\right]}. \tag{5.5.22}$$

It is important that the X^{k+1} term is relevant as it was shown in [256] that if it is marginal or irrelevant in the IR, the theory has no stable vacuum. The Kutasov theory has a conformal window for

$$\frac{1}{k-\frac{1}{2}}N_c < N_f < 2N_c, \tag{5.5.23}$$

and is in the free magnetic phase for

$$\frac{1}{k}(N_c+1) < N_f \leq \frac{1}{k-\frac{1}{2}}N_c. \tag{5.5.24}$$

In this chapter we are intending to flow from this theory to the $\mathcal{N}=2$ theory with small κ induced by a FI term. We mainly interested in the influence of the operator $\tilde{Q}\,X\,Q$, and anticipate that the RG flow will be dominated by either h or κ in different regions. Therefore these bounds cannot be immediately used to draw conclusions for our investigation.

RG flow constraints from the a theorem

Defining the dimensionless coupling $\eta_\kappa \equiv \kappa\,\mu^{k-2}$, the SUSY RGEs are to all orders

$$\frac{dg^2}{dt} = 2\,g\,\beta_g, \qquad \frac{dh^2}{dt} = h^2(\gamma_X + 2\,\gamma_Q), \qquad \frac{d\eta_\kappa^2}{dt} = \eta_\kappa^2\left[(k+1)(\gamma_X+2)-6\right],$$

$$\beta_g = -\frac{g^3}{16\pi^2}\,\frac{3\,C_{2\,\mathbf{Ad}} - 2\,N_f\,T_\square(1-\gamma_Q) - T_{\mathbf{Ad}}(1-\gamma_X)}{1 - T_{\mathbf{Ad}}\,\frac{g^2}{8\pi^2}},$$

$$T_\square = \frac{1}{2}, \qquad C_{2\,\square} = \frac{N_c^2-1}{2N_c}, \qquad C_{2\,\mathbf{Ad}} = T_{\mathbf{Ad}} = N_c,$$

where the first line is by definition, and where β_g is the all orders NSVZ beta function for the *canonical* gauge coupling (see eq. 2.3.259). If we assume that both theories *can* reach a fixed point with the same values of N_c and N_f then the vanishing of the NSVZ β-function set

$$0 = \gamma_X + 2\,\gamma_Q, \tag{5.5.25}$$

$$0 = (k+1)(\gamma_X+2) - 6, \tag{5.5.26}$$

$$0 = 3\,N_c - N_f\,(1-\gamma_Q) - N_c\,(1-\gamma_X), \tag{5.5.27}$$

or equivalently, using eq. 2.3.274

$$0 = R_X + 2\,R_Q - 2, \tag{5.5.28}$$

$$0 = (k+1)\,R_X - 2, \tag{5.5.29}$$

$$0 = N_f\,R_Q + N_c\,R_X - N_f. \tag{5.5.30}$$

The equations 5.5.29 and 5.5.30 set the $\mathrm{U}(1)_R$ charges to be the ones of the standard anomaly free electric Kutasov theory

$$R_X = \frac{2}{k+1}, \qquad R_Q = 1 - R_X\,\frac{N_c}{N_f}. \tag{5.5.31}$$

We see however that eq. 5.5.28 then becomes

$$0 = R_X + 2\left(1 - R_X\,\frac{N_c}{N_f}\right) - 2 = R_X\left(1 - 2\,\frac{N_c}{N_f}\right), \tag{5.5.32}$$

and so in the anomaly free theory, to maintain a $\mathrm{U}(1)_R$ symmetry we can have:

- $N_f = 2\,N_c$ with $R_X = 2/(k+1)$ and *both* the X^{k+1} and $\tilde{Q}\,X\,Q$ operator,
- $N_f \neq 2\,N_c$ with $R_X = 2/(k+1)$ and *only* the X^{k+1} operator,
- $N_f \neq 2\,N_c$ with $R_X = 0$ and *only* the $\tilde{Q}\,X\,Q$ operator,
- $N_f \neq 2\,N_c$ with $R_X \neq 2/(k+1)$ or 0 and an empty superpotential. This is of course not possible as already mentioned the theory has an unstable vacuum.

If $N_f \neq 2N_c$ there can be no fixed point behaviour unless either h or η_κ are zero. If and only if $N_f = 2\,N_c$, can one find fixed point solutions of the RGEs with non–zero h and η_κ. The corresponding $\mathrm{U}(1)_R$ charges are

$$R_X = \frac{2}{k+1}, \qquad R_Q = 1 - \frac{R_X}{2} = 1 - \frac{1}{k+1} \tag{5.5.33}$$

and the anomalous dimensions at the IR fixed point are

$$\gamma_X = 2\,\frac{k-2}{k+1}. \qquad \gamma_Q = \frac{k-2}{k+1}. \tag{5.5.34}$$

We now see the problem: h and κ preserve the same $\mathrm{U}(1)_R$ symmetry and the corresponding charges are completely constrained. The a theorem [265–267] says that $a_{\rm UV} > a_{\rm IR}$ where at a fixed point a can be determined in terms of the R charges [269, 270]

$$a = \frac{3}{32}\left[3\,{\rm tr}(R^3) - {\rm tr}(R)\right] \equiv \frac{3}{32}\,\tilde{a}, \tag{5.5.35}$$

where the trace is over the fermions in the theory. In the electric Kutasov theory

$$\begin{aligned}\tilde{a} = {} & 2\,(N_c^2-1)\\ & - 2\,N_c\,N_f\,(R_Q-1) + 6\,N_c\,N_f\,(R_Q-1)^3\\ & - (N_c^2-1)\,(R_X-1) + 3\,(N_c^2-1)\,(R_X-1)^3,\end{aligned} \tag{5.5.36}$$

where the first, second and third lines are the gaugino, quark and adjoint fermion contributions respectively. If some RG flow were to occur in a theory that allowed non–trivial fixed points for $h \neq 0$ and $\kappa \neq 0$, we see that a flow from a non–trivial BZ–like fixed point in the Kutasov picture and the $\mathcal{N}=2$ fixed like cannot occur because they have the same value of a.

5.5.2 Perturbative flow to $\mathcal{N}=2$ SQCD via Kutasov theory

Instead we will look at the perturbative flow from a different fixed point in the theory — one without a superpotential (but including the superpotential terms as deformations with $h, \eta_\kappa \ll g$) — to the $\mathcal{N}=2$ SQCD fixed line via the $\mathcal{N}=1$ Kutasov theory that is not at a fixed point. To be concrete, we will set $k=2$ and will take $N_f = 2\,N_c$ in order to allow the presence of both the Kutasov and $\mathcal{N}=2$ gauge interactions

$$W^{\rm el} = h\,\tilde{Q}\,X\,Q + \frac{\kappa}{3}\,{\rm tr}(X^3). \tag{5.5.37}$$

The R charges in the theory are

$$R_X = R_Q = \frac{2}{3} \tag{5.5.38}$$

and the perturbative anomalous dimensions are

$$\gamma_Q = \frac{1}{4\pi^2}\,C_{2\,\Box}\,\left(h^2 - g^2\right), \tag{5.5.39}$$

$$\gamma_X = \frac{1}{4\pi^2}\left[N_f\,T_\Box\,h^2 + \delta_{k,2}\left(4\,C_{2\,\Box} - \frac{3}{2}\,T_{\mathbf{Ad}}\right)\eta_\kappa^2 - C_{2\,\mathbf{Ad}}\,g^2\right]. \tag{5.5.40}$$

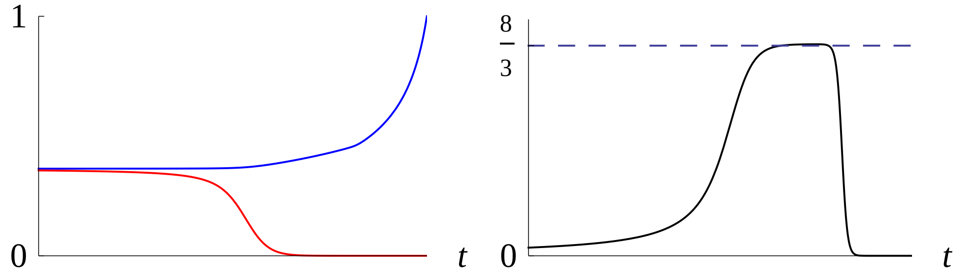


Figure 5.2: RG flow of g^2 (blue), h^2 (red) and η_κ^2/g^2 (black) from the UV (right) to the (IR) left. The horizontal axis is $t = \log\mu$, and we take $N_c = 4$, $N_f = 8$. $8/3$ is the η_κ^2/g^2 quasi–fixed point value for this N_c and N_f given by eq. 5.5.41. We see the couplings h^2 and η_κ^2 grow form the UV to the IR. Initially η_κ dominates and we are in the electric Kutasov theory at a quasi–fixed point, then h takes over, pushing $\eta_\kappa \to 0$ and $h \to g$. Eventually $\beta_h = \beta_g = 0$ and we arrive at the fixed line of $\mathcal{N}=2$ SQCD in the IR.

Note that since the theory will be weakly coupled in this analysis, the anomalous dimensions will be small and so the problem of particles hitting the unitarity bound can be ignored.

Starting from the theory without a superpotential, one might think that imagine that the κ operator would be marginally irrelevant. However the theory exhibits quasi–fixed point behaviour and η_κ/g runs to a fixed value

$$\left(\frac{\eta_\kappa}{g}\right)^2\Bigg|_{\text{quasi–fixed}} = \frac{2\,C_{2\,\mathbf{Ad}}}{8\,C_{2\,\square} - 3\,T_{\mathbf{Ad}}}. \tag{5.5.41}$$

A numerically solved example is shown in figure 5.2 with further examples with different N_c shown in appendix D. In all of these examples, the UV boundary conditions are $g = 1$ and $h = \eta_\kappa = 10^{-15}$ and the period of running for t is $[-10^3,\ 0\,]$, where $t = 0$ corresponds to the UV.

5.5.3 Higgsing in the dual theory and flow to $\mathcal{N}=2$ SQCD′

In Section 5.5.2 we showed that electric Kutasov theory with $k=2$, $N_f = 2\,N_c$ and (almost) empty superpotential flows first to a quasi–fixed electric Kutasov theory with $h \ll \eta_\kappa \sim g$ and then onwards to the $\mathcal{N}=2$ SQCD fixed line. Let us now consider the dual theory.

We know that the dual of the $\mathcal{N}=2$ theory is also an SU(N_c) gauge theory, the dual of Kutasov theory is an SU$(\widetilde{N_c})$ = SU$(k\,N_f - N_c)$. For $k=2$, this is a

	SU(()N_c)	SU(()$N_f)_L$	SU(()$N_f)_R$	U(1)$_B$	U(1)$_R$
q	$\square$	$\bar{\square}$	**1**	$\frac{1}{\widetilde{N_c}}$	$1 - R_x \frac{\widetilde{N_c}}{N_f}$
$\tilde{q}$	$\bar{\square}$	**1**	$\square$	$-\frac{1}{\widetilde{N_c}}$	$1 - R_x \frac{\widetilde{N_c}}{N_f}$
x	**Ad**	**1**	**1**	0	R_x
$\varphi^{(j)}$	**1**	$\square$	$\bar{\square}$	0	$2 - 2\,R_x \frac{N_c}{N_f} - R_x\,(j-1)$

Table 5.2: The matter content of the magnetic Kutasov theory where $\widetilde{N_c} = k\,N_f - N_c$.

SU($3\,N_c$) theory. Clearly $N_c \neq 3\,N_c$ and so something has to happen in the dual theory to make the ranks of the gauge groups match up. Interestingly, the growing h coupling that becomes the $\mathcal{N}=2$ SQCD quark gauge interaction induces the necessary Higgsing for *any* value of k:

$$\begin{array}{ccc} \text{SU}(N_c)\text{ Electric Kutasov} & \longleftrightarrow & \text{SU}(k\,N_f - N_c)\text{ Magnetic Kutasov} \\ \downarrow h\,\tilde{Q}\,X\,Q & & \downarrow \text{Higgsing} \\ \mathcal{N}=2\text{ SU}(N_c)\text{ SQCD} & & \text{SU}(N_c)\text{ ``Electric'' Kutasov} + h\,\tilde{q}'\,x'\,q' \\ & & \downarrow ? \end{array}$$

The dual theory in our region of interest Higgses down to a theory with the same field content and superpotential (up to relabelling) as the original electric Kutasov theory, hence "electric" theory. In this region the theory is strongly coupled however, and so we cannot claim for certain that it will end up hitting the $\mathcal{N}=2$ SU(N_c) SQCD fixed line in the IR as the true electric theory does. The anomaly free R charges do allow this to happen however, and in any case, if the theory becomes weakly coupled at any point then it will be in a perturbative regime that inevitably flows to the $\mathcal{N}=2$ fixed line. Since our method uses RG invariants, if this occurs at any point in the RG flow (not just in the IR) then this is sufficient to establish the map.

Let us now show this Higgsing. The field content in the magnetic SU($\widetilde{N_c}$) = SU($kN_f - N_c$) Kutasov this theory is N_f flavours of left–handed magnetic quarks q and right–handed quarks $\tilde{q}$, an ajoint x, and k mesons $\varphi^{(j)}$ that are identified in the electric theory as

$$m^{(j)} \leftrightarrow \tilde{Q}\,X^{j-1}\,Q, \qquad j = 1, \ldots, k, \tag{5.5.42}$$

with canonically normalized fields $\varphi^{(j)} \sim \Lambda^{-j} m^{(j)}$. The field representations are detailed in table 5.2. In the magnetic superpotential is

$$W^{\text{mag}} = h\,\varphi^{(2)}_{nm}\,\delta_{nm} + \frac{\tilde{\kappa}}{k+1}\,\text{tr}(x^{k+1}) + \sum_{j=1}^{k} \tilde{c}_j\,\varphi^{(j)}_{nm}\,\tilde{q}_m x^{k-j}\,q_n \tag{5.5.43}$$

where n, m are flavour indices.

Higgsing for $k = 2$

If we set $k = 2$, the superpotential becomes (dropping the indices)

$$W_{\text{mag}} = h\,\varphi^{(2)} + \frac{\tilde{\kappa}}{3}\,x^3 + \tilde{c}_1\,\varphi^{(1)}\,\tilde{q}\,x\,q + \tilde{c}_2\,\varphi^{(2)}\,\tilde{q}\,q. \tag{5.5.44}$$

The $\varphi^{(2)}$ F term sets

$$\tilde{c}^{(2)}\,\tilde{q}\,q = -h. \tag{5.5.45}$$

These equations have rank $N_f = 2\,N_c$ and thus, once it turns on, the coupling h induces the Higgsing $\text{SU}(3\,N_c) \to \text{SU}(N_c)$ as required. Using a combination of flavour and colour rotations, we can arrange the VEVs for q and $\tilde{q}$ to be

$$\langle q \rangle = \langle \tilde{q} \rangle \sim \begin{pmatrix} \mathbb{I}_{N_c \times N_c} & \cdot \\ \cdot & \mathbb{I}_{N_c \times N_c} \\ \cdot & \cdot \end{pmatrix}. \tag{5.5.46}$$

Writing the $\text{SU}(3N_c)$ adjoints as

$$x = \begin{pmatrix} z & y \\ \tilde{y} & \hat{x} \end{pmatrix} \tag{5.5.47}$$

where z is $2\,N_c \times 2\,N_c$ and $\hat{x}$ is $N_c \times N_c$, the $\tilde{c}_1$ coupling then becomes an effective mass term for the adjoint z and the traceless mesons

$$\bar{\varphi}^{(1)} \equiv \varphi^{(1)} - \frac{1}{2\,N_c}\,\text{tr}(\varphi^{(1)}), \tag{5.5.48}$$

where the mass is of the form

$$-\frac{h\,\tilde{c}_1}{\tilde{c}_2}\,\bar{\varphi}^{(1)}\,z. \tag{5.5.49}$$

The flavour is also broken to the diagonal $\text{SU}(N_f)_L \times \text{SU}(N_f)_L \to \text{SU}(N_f)_D$. In addition $\varphi^{(2)}$ gets a mass together with the Higgsing $2\,N_c$ block of q. Explicitly, we can parameterise the VEVs of q and $\tilde{q}$ by

$$q = \begin{pmatrix} v+\eta \\ \rho \end{pmatrix}, \qquad \tilde{q} = \begin{pmatrix} v+\tilde{\eta} \\ \tilde{\rho} \end{pmatrix}. \tag{5.5.50}$$

and se a a mass term term $\tilde{c}_2\,(\eta+\tilde{\eta})\,\varphi^{(2)}\,v$ is generated in the superpotential with the $8\,N_c^2$ massless combinations $\eta-\tilde{\eta}$ corresponding to the Goldstone modes that are eaten by the $8N_c^2$ heavy gauge bosons of the broken $\mathrm{SU}(3\,N_c)$. The left handed quarks ρ and right handed quarks $\tilde{\rho}$ are left massless and are the light quarks of the unbroken $\mathrm{SU}(N_c)$. The superpotential after the Higgsing for the effective theory is then $\mathrm{SU}(N_c)$ theory is

$$W^{\mathrm{mag}} = \frac{\tilde{\kappa}}{3}\,x^3 + \tilde{h}\,\tilde{\rho}\,x\,\rho \tag{5.5.51}$$

We see that this theory has the same light field content, superpotential and anomalies as the original electric theory. We therefore anticipate that this theory now flows to the $\mathcal{N}=2$ SQCD fixed line.

Higgsing for general k

We will now show that that the h coupling induces the required Higgsing from $\mathrm{SU}[(2k-1)N_c] \to \mathrm{SU}(N_c)$. From eq. 5.5.44 the x and φ equations of motion are

$$\varphi^{(j)}: \quad 0 = h\,\delta_{nm}\,\delta_{2j} + \tilde{c}_j\,\tilde{q}_m\,x^{k-j}\,q_n, \tag{5.5.52}$$

$$x: \quad 0 = \tilde{\kappa}\,x^k + \sum_{j=1}^{k} \tilde{c}_j\,\varphi^{(j)}_{nm} \sum_{r=0}^{k-j-1} x^{k-j-1-r}\,q_n\,\tilde{q}^T_m\,(x^r)^T. \tag{5.5.53}$$

From the first condition we see for $k \geq 3$ and non-zero $\tilde{c}_j$

$$\langle \tilde{q}_m\,x^{k-1}\,q_n\rangle = \langle \tilde{q}_m\,x^{k-3}\,q_n\rangle = \ldots = \langle \tilde{q}_m\,x\,q_n\rangle = \langle \tilde{q}_m\,q_n\rangle = 0 \tag{5.5.54}$$

$$\langle \tilde{q}_m\,x^{k-2}\,q_n\rangle \neq 0. \tag{5.5.55}$$

Let us write x, q and $\tilde{q}$ as

$$x = \begin{pmatrix} z & y \\ \tilde{y} & \hat{x} \end{pmatrix}, \quad q = \begin{pmatrix} v+\eta \\ \rho_1 \\ \rho_2 \end{pmatrix}, \quad \tilde{q}^T = \begin{pmatrix} \tilde{\rho}_1 \\ v+\tilde{\eta} \\ \tilde{\rho_2} \end{pmatrix}, \tag{5.5.56}$$

where z is an $(k-1)\,N_f \times (k-1)\,N_f$ matrix, v, η and $\tilde{\eta}$ are $N_f \times N_f$ matrices, ρ_1 and $\tilde{\rho}_1$ are $(k-2)\,N_f \times N_f$ matrices, and ρ_2 and $\tilde{\rho}_2$ are $N_c \times N_f$ matrices. We can solve equations 5.5.54 and 5.5.55 by taking z as

$$\langle z \rangle \sim \begin{pmatrix} 0_{N_f\times N_f} & \mathbb{I}_{N_f\times N_f} & \cdot & \cdot \\ \cdot & \cdot & \ddots & \cdot \\ \cdot & \cdot & \cdot & \mathbb{I}_{N_f\times N_f} \\ \cdot & \cdot & \cdot & 0_{N_f\times N_f} \end{pmatrix} \tag{5.5.57}$$

such that

$$\langle z^{k-2} \rangle \sim \begin{pmatrix} \cdot & \mathbb{I}_{N_f\times N_f} \\ \cdot & \cdot \end{pmatrix} \tag{5.5.58}$$

and then separating the VEVs of q and $\tilde{q}$ by $k-2$ permutations,

$$\langle \tilde{q} \rangle \sim \begin{pmatrix} \mathbb{I}_{N_f\times N_f} \\ \cdot \end{pmatrix}, \qquad \langle q \rangle \sim \begin{pmatrix} 0_{(k-2)N_f\times N_f} \\ \mathbb{I}_{N_f\times N_f} \\ \cdot \end{pmatrix}, \tag{5.5.59}$$

so that clearly

$$\langle x^{k-2} q \rangle \sim \begin{pmatrix} \mathbb{I}_{N_f\times N_f} \\ \cdot \end{pmatrix} \sim \langle \tilde{q} \rangle, \tag{5.5.60}$$

as required. Then $\langle z \rangle$ which is rank $(k-2)N_f$, together with $\langle q \rangle$, leave the bottom ρ_2, $\tilde{\rho}_2$ block and hence $\mathrm{SU}(N_c)$ unbroken.

5.5.4 Flow away from $N_f = 2\,N_c$

Now one can see a possible way to extend the analysis away from the constrained $N_f = 2\,N_c$ regime — although we leave this for future study. From our $N_f = 2\,N_c$ electric theory we can add n additional heavy quarks Q' and $\tilde{Q}'$ with a superpotential mass term $m\,\tilde{Q}'\,Q'$ and m chosen such that it is in the period where $\eta_\kappa \gg h$. In the UV, these quarks give a new contribution to the beta function that pushes the theory to a Landau pole rather than being asymptotically free. In the dual picture, the mass term is a linear term for a new meson $\varphi' \sim \Lambda^{-1}\,\tilde{Q}'\,Q'$ which causes a Higgsing for the new magnetic quarks q' and $\tilde{q}'$. This theory is asymptotically free. It will be useful in our setup to note that mass deformations can be introduced in a manifestly $\mathcal{N}=2$ SUSY way [52, 271].

5.6 $\mathcal{N}=2 \to \mathcal{N}=1$ with a superpotential for X

5.6.1 Overview

We will now show that the Kutasov theory can be written in a manifestly $\mathcal{N}=2$ SUSY way and induced by the ATP mechanism in HSS. The ATP mechanism was formulated in HSS in [272], and has been coupled to a number of interesting theories [58, 273–278], of which the most relevant for this study is [58] where the the theory in question is $\mathcal{N}=2$ SQCD. We will proceed as follows:

- We write $\mathcal{N}=2$ SQCD in the HSS formalism described in Section 2.3.4. For comparison, we also write this theory in the standard $\mathcal{N}=1$ superspace in Section E.4.2,
- Noting the restriction from the *2 into 1 won't go* theorem [67, 68], we collect the necessary ingredients to achieve $\mathcal{N}=2 \to \mathcal{N}=1$ breaking, and check that it successfully reproduces the ATP mechanism,
- We show that a specific choice of the prepotential $\mathcal{F}(\mathcal{W})$ generates the required scalar potential and fermion interactions, matching the known result from $\mathcal{N}=1$ superspace which is presented in Appendix E.5.1.

5.6.2 $\mathcal{N}=2$ SU(N_c) SQCD

The low energy EFT for $\mathcal{N}=2$ $SU(N_c)$ SQCD is [279]

$$S_{\text{QCD}}^{\mathcal{N}=2} = S_{\text{SYM}}^{\mathcal{N}=2} + S_{Q}^{\mathcal{N}=2}, \tag{5.6.61}$$

$$S_{\text{SYM}}^{\mathcal{N}=2} = \frac{1}{16\,\pi\, i}\int d^4x\,(\mathcal{D})^4\,\mathcal{F}(\mathcal{W}) + \text{h.c.}, \tag{5.6.62}$$

$$S_{Q}^{\mathcal{N}=2} = -\int du\, d\zeta^{-4}\,\widetilde{Q^+}\,\mathscr{D}^{++}Q^+, \tag{5.6.63}$$

where Q^+ is a FS hypermultiplet with gauge and global representations given in table 5.3, V^{++} is a $\mathcal{N}=2$ vector multiplet, and $\mathcal{W}$ is gauge field hyperstrength with component field expansions 2.3.204, 2.3.219 and 2.3.223 respectively. The notation and derivatives are all detailed in Section 2.3.4 with the measures and normalisations

	$\mathrm{SU}(N_c)$	$\mathrm{SU}(N_f)$
Q^+	$\square$	$\square$

Table 5.3: $\mathcal{N}=2$ superfield representations in $\mathcal{N}=2$ SQCD

given in Section E.2. The difference in the prefactor of the $S^{\mathcal{N}=2}_{\mathrm{SYM}}$ piece is purely conventional.

5.6.3 $\mathcal{N}=2 \to \mathcal{N}=1$ SU(N_c) SQCD

Evading the *2 into 1 won't go theorem*

Before embarking into HSS for a second time, it is worth briefly commenting on how the mechanism we are about to use — the ATP mechanism — circumvents the *2 into 1 won't go theorem* discussed in Section 2.3.7. this argument requires noting that eq. 2.3.262 isn't always valid in the case of spontaneously broken SUSY. The SUSY algebra in eq. 2.3.262 follows from the supercurrent algebra

$$\int d^3y\, \{J^A_{\nu,\alpha}(x), J_{B,0,\dot\beta}(y)\} = 2\, \sigma^\mu_{\alpha\dot\beta}\, \delta^A{}_B\, T_{\mu\nu}(x). \tag{5.6.64}$$

This is *not* the most general current algebra consistent with SUSY [280], as the Jacobi identities of SUSY [37] allow an additional field-independent constant piece

$$\Delta = \sigma_{\mu,\alpha\dot\alpha}\, C^A{}_B \tag{5.6.65}$$

to be added. Δ commutes with all quantities in the theory so the SUSY algebra on the fields is not modified [281]. If $C^A{}_B = 0$, then we can integrate eq. 5.6.64 over the x 3-space to reproduce eq. 2.3.262 as is usually understood and the no-go theorem holds. When $C^A{}_B \neq 0$ there is an infinite contribution to the right hand side of eq. 2.3.262 from $\Delta \int d^3x$ making the SUSY algebra derived in this manner ill-defined, and allowing evasion of the no–go theorem. The ATP mechanism is precisely a realization of a physical model inducing a non–zero $C^A{}_B$ [282], where the vacuum energy in the partially broken SUSY vacuum is now related to the FI terms [273].

	$\mathrm{SU}(N_c)$	$\mathrm{U}(1)_\circ$	$\mathrm{SU}(N_f)$
Q^+	$\square$	1	$\square$

Table 5.4: $\mathcal{N}=2$ superfield representations in $\mathcal{N}=2$ SQCD coupled to $\mathrm{U}(1)_\circ$

Formulation in harmonic superspace: the ATP mechanism

To achieve spontaneous breaking of $\mathcal{N}=2 \to \mathcal{N}=1$ via the ATP mechanism, we first extend the gauge theory $\mathrm{SU}(N_c) \to \mathrm{SU}(N_c) \times \mathrm{U}(1)_\circ$, where Q^+ is charged under the $\mathrm{U}(1)_\circ$ factor as shown in table 5.4. The resulting action is the same as in 5.6.61 and 5.6.63 with prepotential $\mathcal{F}(\mathcal{W}, \mathcal{W}^\circ)$ written as a general expansion in $\mathcal{W}$'s, and the covariant derivative

$$\mathscr{D}^{++} = D^{++} + i\,(V^{++} + V_\circ^{++}). \tag{5.6.66}$$

The $\circ$-index on $V_\circ^{++}$ or $\mathcal{W}^\circ$ is equivalent to the trace $\mathrm{U}(1)$ element of the $\mathrm{U}(N_c)$ gauge group in [58], in the sense that we can define a Kähler metric for the whole gauge theory through the prepotential $\mathcal{F}_{a_1\ldots a_N}(\mathcal{W}, \mathcal{W}^\circ)$. From now on we use the following notation to distinguish $\mathrm{SU}(N_c)$ and $\mathrm{U}(1)_\circ$ indices

$$\tilde{a} = 1, \ldots, N_c^2 - 1, \qquad a = \circ, 1, \ldots, N_c^2 - 1,$$

and we normalize the $\mathrm{U}(1)_\circ$ generator as $t_\circ = \frac{1}{\sqrt{2\,N_c}}\,\mathbb{I}_{N_c\times N_c}$. $\mathcal{N}=2$ SUSY can be broken spontaneously by giving the electric or dual magnetic[3] D terms of the $\mathrm{U}(1)_\circ$ gauge a constant shift. The dual magnetic D term $D^A_{D,\circ}$ is shifted by the electric FI term

$$4\pi\, S^{\mathcal{N}=2}_{\mathrm{El\ FI},\circ} = \int du\, d\zeta^{(-4)}\, \xi^{++} (V^\circ)^{++} + \mathrm{h.c.} = 2 \int d^4x\, \xi^A\, D^{\circ,A} + \mathrm{h.c.} \tag{5.6.67}$$

where $\xi^{++} \equiv \xi^{ij}\, u_i^+\, u_j^+ = 2\,\xi^A (u^+ u^+)^A$. This shift can be seen by writing the whole action as an integral over the analytic subspace and varying it with respect to $V_\circ^{++}$ yielding the equation of motion [56, 272]

$$(D^+)^2\, \mathcal{F}_\circ - \mathrm{h.c.} = 4\, i\, \xi^{++}. \tag{5.6.68}$$

[3] see Section 5.8.

Because $\mathcal{F}_\circ \equiv \mathcal{W}_{D,\circ} \supset 2(\theta\theta)^A D^A_\circ$, the equation of motion 5.6.68 shifts the magnetic dual D term $D^A_{D,\circ}$ by an imaginary part on-shell [58]:

$$\mathbf{D}^A_{D,\circ} = D^A_{D,\circ} + 4\,i\,\xi^A, \qquad \bar{\mathbf{D}}^A_{D,\circ} = \bar{D}^A_{D,\circ} - 4\,i\,\bar{\xi}^A. \tag{5.6.69}$$

Similarly, the electric D term is shifted by a FI term for the dual magnetic gauge field of the form

$$4\pi\, S^{\mathcal{N}=2}_{\text{Mag FI},\circ} = 2\int d^4x\, \xi^A_D \left\{(D)^4(\theta\theta)^A \left[\mathcal{F}_\circ + \mathcal{F}_{\circ\circ}\, 4i\xi^B_D(\theta\theta)^B\right] - 2\,\mathcal{Q}^A_\circ\right\} + \text{h.c.} \tag{5.6.70}$$

where

$$\mathcal{Q}^{ij}_a \equiv 4\,\pi\,\bar{Q}^{(i}\,t_a\,Q^{j)} = -\bar{\mathcal{Q}}^{ij}_a. \tag{5.6.71}$$

The $\mathcal{Q}$'s have an explicit $\mathrm{SU}(2)_R$ decomposition that will be useful later

$$\frac{\mathcal{Q}^1_a}{2\,\pi\,i} = -(\overline{Q^2}\,t_a\,Q^1 + \overline{Q^1}\,t_a\,Q^2), \tag{5.6.72}$$

$$\frac{\mathcal{Q}^2_a}{2\pi} = \overline{Q^2}\,t_a\,Q^1 - \overline{Q^1}\,t_a\,Q^2, \tag{5.6.73}$$

$$\frac{\mathcal{Q}^3_a}{2\pi i} = \overline{Q^2}\,t_a\,Q^2 - \overline{Q^1}\,t_a\,Q^1. \tag{5.6.74}$$

It has been shown that the presence of $S^{\mathcal{N}=2}_{\text{Mag FI},\circ}$ shifts the electric D term $D^A_\circ$ by an imaginary constant off-shell, allowing us to write $S^{\mathcal{N}=2}_{\text{SQCD}} + S^{\mathcal{N}=2}_{\text{Mag FI},\circ}$ as

$$\left[\frac{1}{16\,\pi\,i}\int d^4x\,(\mathcal{D})^4\mathcal{F}(\mathcal{W},\mathcal{W}^\circ) - \frac{1}{2}\int du\,d\zeta^{-4}\,\widetilde{Q^+}\,\mathscr{D}^{++}\,Q^+\right]\Bigg|_{D^A_\circ \to \mathbf{D}^A_\circ} + \text{h.c.}, \tag{5.6.75}$$

where

$$\mathbf{D}^A_\circ = \mathcal{D}^A_\circ + 4\,i\,\xi^A_D, \qquad \bar{\mathbf{D}}^A_\circ = \mathcal{D}^A_\circ - 4\,i\,\bar{\xi}^A_D. \tag{5.6.76}$$

Taking the full off-shell action as

$$S_{\text{off-shell}} = S^{\mathcal{N}=2}_{\text{SQCD}} + S^{\mathcal{N}=2}_{\text{El FI},\circ} + S^{\mathcal{N}=2}_{\text{Mag FI},\circ} \tag{5.6.77}$$

and solving the D terms up to third derivatives in the prepotential, we finally arrive at the desired on-shell action for $\mathcal{N}=2$ SQCD coupled to the ATP mechanism:

$$S_{\text{on-shell}} = \int d^4x\,\left(\mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} + \mathcal{L}_{\text{Pauli}} + \mathcal{L}'_{4\text{ Fermi}} + \mathcal{L}'_{\text{D Fermi}} - V'\right), \tag{5.6.78}$$

where $\mathcal{L}_{\text{kin}}$, $\mathcal{L}_{\text{yuk}}$, and $\mathcal{L}_{\text{Pauli}}$ are unchanged from their respective forms in Appendix E.4.1, and

$$-4\,\pi\,\mathcal{L}'_{4\text{ Fermi}} = \left(\frac{2\pi i}{3}\,\mathcal{F}_{abcd}|\,\lambda^a\lambda^b - \frac{1}{8}\,g^{ab}(\mathcal{F}_{aef}|\,\lambda^e\lambda^f - \bar{\mathcal{F}}_{afe}|\,\bar{\lambda}^e\bar{\lambda}^f)\mathcal{F}_{bcd}|\right)^A \times(\lambda^c\lambda^d)^A + \text{h.c.}, \tag{5.6.79}$$

$$4\,\pi\,\mathcal{L}'_{\text{D Fermi}} = \frac{i}{2}\,\mathbf{D}^{a,\,A}|\,\mathcal{F}_{abc}|\,(\lambda^b\lambda^c)^A + \text{h.c.}, \tag{5.6.80}$$

$$4\,\pi\,V' = \frac{1}{2}\,g_{ab}\,\mathbf{D}^{a,\,A}_\phi|\,\bar{\mathbf{D}}^{b,\,A}_\phi| + 4\,\pi\,\overline{Q^i}\,\{\bar{X},X\}\,Q^i - \frac{1}{2}\,g_{ab}\,f^a_{cd}\,f^b_{ef}\,\bar{X}^cX^d\,\bar{X}^e\,X^f + 4\,i\,(\xi^A+\bar{\xi}^A)(\xi^A_D-\bar{\xi}^A_D), \tag{5.6.81}$$

where the solutions of the D terms have the convenient decomposition

$$\mathbf{D}^{a,\,A} = \mathbf{D}^{a,\,A}_X + D^{a,\,A}_Q + D^{a,\,A}_\lambda, \tag{5.6.82}$$

$$\mathbf{D}^{a,\,A}_\phi = \mathbf{D}^{a,\,A}_X + D^{a,\,A}_Q, \tag{5.6.83}$$

$$\boldsymbol{\xi}^A_a \equiv (\xi^A+\bar{\xi}^A)\,\delta^\circ_a + (\xi^A_D+\bar{\xi}^A_D)\,\bar{\mathcal{F}}_{\circ a}|, \tag{5.6.84}$$

$$\mathbf{D}^{a,\,A}_X = -2\,g^{ab}\,\boldsymbol{\xi}^A_b, \tag{5.6.85}$$

$$D^{a,\,A}_Q = -2\,i\,g^{ab}\,\mathcal{Q}^A_b, \tag{5.6.86}$$

$$D^{a,\,A}_\lambda = -\frac{i}{2}\,g^{ab}\,\mathcal{F}_{bcd}|\,(\lambda^c\,\lambda^d)^A + \text{h.c.}. \tag{5.6.87}$$

We shall refer back to these equations frequently below.

$\mathcal{N}=1$ conditions: scalar potential

We will now ensure that the properties of $\mathcal{N}=2$ SQCD coupled to the ATP mechanism as described in Section 5.6.3 are those of the $\mathcal{N}=1$ theory presented in appendix E.5.1. There are three conditions that one could consider for the vacuum to respect $\mathcal{N}=1$:

- Vacuum stability,
- Zero vacuum energy.
- A scalar potential corresponding to the $\mathcal{N}=1$ superpotential in eq. E.32.

As we shall see the first two of these provide a constraint on the FI terms while the third is observed to be generally true, and relates the prepotential to the desired

$\mathcal{N}=1$ deformations. In addition, although it is possible to set the vacuum energy to zero, it is not obligatory for preserving $\mathcal{N}=1$ SUSY [264], but it is natural to apply it. Results for the first two are available in the literature but somewhat scattered, so it is worth collating all three elements here.

Vacuum stability: Stable SUSY breaking vacua exist on the Coulomb branch (i.e. with $\langle Q\rangle=0$) which can be achieved by assuming $X^\circ\neq 0$ [58, 273–276, 278] or on the Higgs branch when $X^\circ=0$. In order to study the latter without breaking $SU(N_c)$ one could introduce hypermultiplets charged only under $\mathrm{U}(1)_\circ$, but this case is more complicated to analyse as the Goldstino comes from a linear combination of the new quarks and the λ's, so we will restrict the discussion to the former case[4].

Noting that the scalar potential 5.6.81 contains[5]

$$-4\,\pi\,V\supset\frac{1}{2}\,g_{ab}\,f^a_{cd}\,f^b_{ef}\,\bar{X}^c\,X^d\,\bar{X}^e\,X^f,$$

it follows that $\langle X^{\hat{a}}\rangle=0$ where $t_{\hat{a}}$ are non–Cartan generators. Therefore only $\langle X^{\underline{a}}\rangle\neq 0$ is possible, where $t_{\underline{a}}$ are Cartan generators. The vacuum condition is [275]

$$4\,\pi\left\langle\frac{\partial V}{\partial(\mathcal{W}^a|)}\right\rangle=\frac{i}{4}\,\langle\mathcal{F}_{abc}\,\mathbf{D}^{b,\,A}\,\mathbf{D}^{c,\,A}\rangle=0. \tag{5.6.88}$$

The only non-vanishing $\langle\mathcal{F}_{ab}\rangle$ are the diagonal elements $\langle\mathcal{F}_{\hat{a}\,\hat{a}}\rangle$ and $\langle\mathcal{F}_{\underline{a}\,\underline{a}}\rangle$, whilst the only non-vanishing $\langle\mathcal{F}_{abc}\rangle$ are $\langle\mathcal{F}_{\underline{a}\,\underline{a}\,\underline{a}}\rangle$ and $\langle\mathcal{F}_{\underline{a}\,\hat{b}\,\hat{b}}\rangle$. It follows that $\langle\mathbf{D}^{\hat{a}}\rangle=0$ and so condition 5.6.88 becomes

$$\langle\mathcal{F}_{\underline{a}\,\underline{a}\,\underline{a}}\,\mathbf{D}^{\underline{a},\,A}\,\mathbf{D}^{\underline{a},\,A}\rangle=0. \tag{5.6.89}$$

The choice $\langle\mathcal{F}_{\underline{a}\,\underline{a}\,\underline{a}}\rangle=0$ corresponds to unstable saddle points, and so a stable vacuum must satisfy

$$\langle\mathbf{D}^{\underline{a},\,A}\,\mathbf{D}^{\underline{a},\,A}\rangle=0 \tag{5.6.90}$$

[4] By *Coulomb branch* we are referring to $X^\circ\neq 0$. In this vacuum the hypermultiplets acquire mass from X° but the $X^{\tilde{a}}$ are unconstrained by the equations of motion because of the extra degree of freedom provided by X°. In the presence of the superpotental term $W\supset X^3$ (assuming that we can eventually make it), setting $X^\circ=0$ would force some $X^{\tilde{a}}\neq 0$, with the theory sitting at an Argyres–Douglas point [283]. This would break the gauge symmetry, and may be interesting for phenomenology; we leave this possibility for future study.

[5] We will refer to the scalar potential after D term shifts and substitution as V instead of V' as was used in 5.6.81 in order to avoid confusion with derivatives and to reduce clutter.

for every $\underline{a}$. By fixing the $SU(2)_R$ direction appropriately, this condition is solved by

$$\langle \mathcal{F}_{\circ\circ} \rangle = -\frac{1}{m}\,(e + i\,\xi)\,, \qquad \xi^A + \bar{\xi}^A = (0,e,\xi)^A, \qquad \xi_D^A + \bar{\xi}_D^A = (0,m,0)^A, \quad (5.6.91)$$

where e, m and ξ are real constants. Without loss of generality, taking $\frac{\xi}{m} < 0$ fixes the sign of the solution as we demand a positive metric, $\langle g_{\circ\,\circ} \rangle = -\frac{\xi}{m} \geq 0$.

Zero vacuum energy: The vacuum energy is given by

$$\langle 4\,\pi\,V \rangle = -4\,\xi\,m - 4\,i\,(\xi^A + \bar{\xi}^A)(\xi_D^A - \bar{\xi}_D^A), \quad (5.6.92)$$

so that the choice

$$\xi_D^A - \bar{\xi}_D^A = (0,0,i\,m)^A \quad (5.6.93)$$

makes it vanish [262, 278]. The form of ξ_D^A is then completely fixed, whereas the imaginary part of ξ^A is still undetermined,

$$\mathrm{Re}\,\xi^A = \frac{1}{2}\,(0,e,\xi)^A, \qquad \xi_D^A = \frac{m}{2}\,(0,1,i)^A. \quad (5.6.94)$$

A scalar potential corresponding to $W_{\mathbf{def}}$ in E.32: Our final requirement is that we can describe W_{def} correctly in this setup. The first term in 5.6.81 is

$$4\,\pi\,V \supset 2\,g^{ab}\,[\boldsymbol{\xi}_a - i\,\mathcal{Q}_a]^A\,[\boldsymbol{\xi}_b - i\,\mathcal{Q}_b]^{A\,\dagger}\,. \quad (5.6.95)$$

From the above, 5.6.74 and 5.6.81, the $\mathrm{U}(1)_\circ$ part of the potential takes the form

$$V = |X^\circ|^2\,|Q^i|^2 + \frac{g^2}{2}\,\left|\overline{Q^2}\,Q^1 - \overline{Q^1}\,Q^2\right|^2 + \frac{g^2}{2}\xi^2 + \frac{g^2}{2}\left|\xi - |Q^1|^2 + |Q^2|^2\right|^2, \quad (5.6.96)$$

confirming that it is stable if $X^\circ > g\,\xi$. Note for later reference that along the Coulomb branch the quarks all gain masses and decouple.

Now consider the $\mathrm{SU}(N_c)$ part. The kinetic terms already identify $g_{ab} = \tau_2\,K_{ab}$, so in order to reproduce the scalar potential E.32, the above together with eq. 5.6.71

suggest the identification

$$|\boldsymbol{\xi}_a^{(2)}| \leftrightarrow \frac{4\,\pi}{\sqrt{2}} \left| \frac{\partial W_{\rm eff}}{\partial X^a} \right|. \tag{5.6.97}$$

Defining a rescaled superpotential $\hat{W}_{\rm eff} = 4\,\pi\, W_{\rm eff}$ (noting that $\mathcal{W}^a| = i\,\sqrt{2}\,X^a$), this implies

$$\hat{W}_{\rm eff} \supset (e\,\mathcal{W}^\circ + m\,\mathcal{F}_\circ)| + \dots. \tag{5.6.98}$$

Hence a reasonable guess is that in order to preserve an $\mathcal{N}=1$ SUSY gauge theory with an effective rescaled superpotential $\hat{W}_{\rm def}$ for the traceless SU(N_c) matter in the **Ad** rep (which we will henceforth denote $\tilde{X}$), one should take

$$\mathcal{F}(\mathcal{W}) = \frac{\tau}{2}\,\mathcal{W}^a\,\mathcal{W}^a + \frac{\mathcal{W}^\circ}{\Lambda^2}\,\hat{W}_{\rm def}, \tag{5.6.99}$$

where $\Lambda^2 = m$ is the scale of new physics integrated out to form the effective prepotential, and the conditions above give $\mathrm{Im}(\tau) = -\frac{\xi}{m}$. For example deformations of the Kutasov type can be embedded by choosing

$$\hat{W}_{\rm def} = 4\,\pi\,\frac{\kappa}{k+1}\,\mathrm{tr}(\tilde{X}^{k+1}). \tag{5.6.100}$$

Note that in order to reduce clutter, until further notice the κ we refer to will be the holomorphic coupling, not the running coupling of the canonically normalised theory. Let us check that the $\mathcal{N}=1$ scalar lagrangian is recovered in the decoupling limit with this prepotential. We have

$$g_{\circ\circ} = \tau_2, \tag{5.6.101}$$

$$g_{\tilde{a}\circ} = \frac{1}{m}\,\partial_{\tilde{a}}\,\hat{W}_{\rm def}, \tag{5.6.102}$$

$$g_{\tilde{a}\tilde{b}} = \tau_2\,\delta_{\tilde{a}\tilde{b}} + \frac{\mathcal{W}^\circ}{m}\,\partial_{\tilde{a}}\,\partial_{\tilde{b}}\,\hat{W}_{\rm def}, \tag{5.6.103}$$

where $\tau_2 \equiv \mathrm{Im}(\tau)$. This metric is, in matrix form

$$g = \frac{1}{m}\begin{pmatrix} m\,\tau_2 & \mathrm{Im}(\hat{W}_{{\rm def},\,\tilde{a}}) \\ \mathrm{Im}(\hat{W}_{{\rm def},\,\tilde{b}}) & m\,\tau_2\,\delta_{\tilde{a}\tilde{b}} + \mathcal{W}^\circ\,\mathrm{Im}(\hat{W}_{{\rm def},\,\tilde{a}\,\tilde{b}}) \end{pmatrix}, \tag{5.6.104}$$

where we have introduced the notation

$$\hat{W}_{\mathrm{def},\,\tilde{a}} \equiv \partial_{\tilde{a}}\,\hat{W}_{\mathrm{def}} \tag{5.6.105}$$

and similarly for the other subscripts of $\hat{W}_{\mathrm{def}}$. The inverse metric g^{-1} in the decoupling limit $(m,\,\xi) \to \infty$, $\tau =$ constant is

$$g^{-1} = \begin{pmatrix} g^{\circ\circ} & g^{\tilde{a}\circ} \\ g^{\circ\tilde{b}} & g^{\tilde{a}\tilde{b}} \end{pmatrix} \tag{5.6.106}$$

where

$$\tau_2^3\, m^2\, g^{\circ\circ} = \hat{W}_{\mathrm{def},\,\tilde{a}}\,\hat{W}_{\mathrm{def},\,\tilde{b}} + m^2\,\tau_2^2 + \mathcal{O}(m^{-1}), \tag{5.6.107}$$

$$\tau_2^3\, m^2\, g^{\tilde{a}\circ} = \hat{W}_{\mathrm{def},\,\tilde{a}}\,\left(\mathcal{W}^\circ\,\hat{W}_{\mathrm{def},\,\tilde{a}\tilde{b}} - m\,\tau_2\right) + \mathcal{O}(m^{-1}), \tag{5.6.108}$$

$$\tau_2^3\, m^2\, g^{\circ\tilde{b}} = \hat{W}_{\mathrm{def},\,\tilde{b}}\,\left(\mathcal{W}^\circ\,\hat{W}_{\mathrm{def},\,\tilde{a}\tilde{b}} - m\,\tau_2\right) + \mathcal{O}(m^{-1}), \tag{5.6.109}$$

$$\tau_2^3\, m^2\, g^{\tilde{a}\tilde{b}} = \hat{W}_{\mathrm{def},\,\tilde{a}}\hat{W}_{\mathrm{def},\,\tilde{b}} + \hat{W}_{\mathrm{def},\,\tilde{a}\tilde{b}}\mathcal{W}^\circ\,(\mathcal{W}^\circ - m\,\tau_2) + m^2\,\tau_2^2 + \mathcal{O}(m^{-1}), \tag{5.6.110}$$

where we are currently taking $\hat{W}_{\mathrm{def}}$ to mean $\mathrm{Im}(\hat{W}_{\mathrm{def}})$ for brevity until stated otherwise. The F term part of the scalar potential is contained in

$$\begin{aligned} -4\,\pi\,V \supset{}& 2\,g^{ab}\,\left[\boldsymbol{\xi}_a - i\,\mathcal{Q}_a\right]^A\left[\boldsymbol{\xi}_b - i\,\mathcal{Q}_b\right]^{A\,\dagger} \\ ={}& 2\,g^{ab}\,V_{ab}, \end{aligned} \tag{5.6.111}$$

where

$$V_{ab} = V_a^A\,V_b^{A\dagger}, \tag{5.6.112}$$

$$V_a^A = \left[(0,e,\xi)\,\delta_a^\circ + (0,\hat{W}_{\mathrm{def},\,a},0) - i\,\mathcal{Q}_a\right]^A \tag{5.6.113}$$

and decomposed in indices

$$V_\circ^A = (-i\,\mathcal{Q}_\circ^1, e - i\,\mathcal{Q}_\circ^2, \xi - i\,\mathcal{Q}_\circ^3)^A, \tag{5.6.114}$$

$$V_{\tilde{a}}^A = (-i\,\mathcal{Q}_{\tilde{a}}^1, \hat{W}_{\mathrm{def},\,\tilde{a}} - i\,\mathcal{Q}_{\tilde{a}}^2, -i\,\mathcal{Q}_{\tilde{a}}^3)^A. \tag{5.6.115}$$

Terms arising in the scalar potential are then

$$V_{\circ\circ} = -|\mathcal{Q}^1_\circ|^2 + |e - i\,\mathcal{Q}^2_\circ|^2 + |\xi - i\,\mathcal{Q}^3_\circ|^2, \tag{5.6.116}$$

$$V_{\tilde{a}\circ} = -\mathcal{Q}^1_{\tilde{a}}\,\mathcal{Q}^{1\dagger}_\circ + (\hat{W}_{\text{def},\tilde{a}} - i\,\mathcal{Q}^2_{\tilde{a}})(e - i\,\mathcal{Q}^2_\circ)^\dagger - i\,\mathcal{Q}^3_{\tilde{a}}\,(\xi - i\,\mathcal{Q}^3_\circ)^\dagger \tag{5.6.117}$$

$$V_{\tilde{a}\tilde{b}} = -\mathcal{Q}^1_{\tilde{a}}\,\mathcal{Q}^{1\dagger}_{\tilde{b}} + (\hat{W}_{\text{def},\tilde{a}} - i\,\mathcal{Q}^2_{\tilde{a}})(\hat{W}_{\text{def},\tilde{b}} - i\,\mathcal{Q}^2_{\tilde{b}})^\dagger - \mathcal{Q}^3_{\tilde{a}}\,\mathcal{Q}^{3\dagger}_{\tilde{b}}. \tag{5.6.118}$$

If we now focus on just the $\mathrm{SU}(N_c)$ sector of interest, keeping only the highest powers of ξ and m, we see that, reintroducing the $\mathrm{Im}(\hat{W})$ notation

$$\tau_2\, g^{\tilde{a}\circ}\, V_{\tilde{a}\circ} \supset i\,\mathrm{Im}(\hat{W}_{\text{def},\tilde{a}})\,\mathcal{Q}^3_{\tilde{a}}, \tag{5.6.119}$$

$$\tau_2\, g^{\tilde{a}\tilde{b}}\, V_{\tilde{a}\tilde{b}} \supset -\mathcal{Q}^1_{\tilde{a}}\,\mathcal{Q}^{1\dagger}_{\tilde{b}} + (\hat{W}_{\text{def},\tilde{a}} - i\,\mathcal{Q}^2_{\tilde{a}})\,(\hat{W}_{\text{def},\tilde{b}} - i\,\mathcal{Q}^2_{\tilde{b}})^\dagger - \mathcal{Q}^3_{\tilde{a}}\,\mathcal{Q}^{3\dagger}_{\tilde{b}}. \tag{5.6.120}$$

We then see that

$$\begin{aligned} -2\,\tau_2\,\pi\,V \supset &- \mathcal{Q}^1_{\tilde{a}}\,\mathcal{Q}^{1\dagger}_{\tilde{a}} - \mathcal{Q}^2_{\tilde{a}}\,\mathcal{Q}^{2\dagger}_{\tilde{a}} - \mathcal{Q}^3_{\tilde{a}}\,\mathcal{Q}^{3\dagger}_{\tilde{a}} \\ &+ i\,\mathrm{Im}(\hat{W}_{\text{def},\tilde{a}})\,\mathcal{Q}^3_{\tilde{a}} + i\,\mathrm{Im}(\hat{W}_{\text{def},\tilde{a}})\,\mathcal{Q}^3_{\tilde{a}} \\ &+ (\hat{W}_{\text{def},\tilde{a}} - i\,\mathcal{Q}^2_{\tilde{a}})\,(\hat{W}_{\text{def},\tilde{a}} - i\,\mathcal{Q}^2_{\tilde{a}})^\dagger, \end{aligned} \tag{5.6.121}$$

eventually yielding

$$4\,\pi\,V \supset \frac{2}{\tau_2}\left|\frac{1}{i\,\sqrt{2}}\frac{\partial\hat{W}_{\text{def}}}{\partial X^a} + \mathcal{Q}^3_a - i\,\mathcal{Q}^2_a\right|^2. \tag{5.6.122}$$

Comparing with eqs. 5.6.74 we see that we must have

$$\mathcal{Q}^3_a - i\,\mathcal{Q}^2_a = 2\,\pi\,i\,(Q^1 - Q^2)\,(\overline{Q^1} + \overline{Q^2}). \tag{5.6.123}$$

Therefore the quarks of the $\mathcal{N}=1$ theory are identified as can be identified as

$$Q \equiv \frac{1}{\sqrt{2}}\,(Q^1 - Q^2), \qquad \tilde{Q} \equiv \frac{1}{\sqrt{2}}\,(\overline{Q^1} + \overline{Q^2})\,, \tag{5.6.124}$$

and we find

$$V \supset \frac{4\pi}{\tau_2}\left|\partial_a\,W_{\text{def}} + \sqrt{2}\,Q\,t_a\,\tilde{Q}\right|^2. \tag{5.6.125}$$

This matches the $\mathcal{N}=1$ expression in eq. E.32. The $\mathrm{U}(1)_R$ symmetry of the $\mathcal{N}=1$ theory is then identified with the σ^1 generator of $\mathrm{SU}(2)_R$, under which Q and $\tilde{Q}$

have equal charges. As discussed above, on the Coulomb branch we have $X^\circ > g\,\xi$ for stability, so the quarks will decouple as well, although one can arrange to keep them in the spectrum by choosing $g_\circ \ll g_{\mathrm{SU}(N_c)}$.

Gaugino–fermion lagrangian

Finally we show that the correct $\mathcal{N}=1$ fermion lagrangian is also induced by eq. 5.6.99, and check the existence of a massless gaugino that will be the goldstino corresponding to the broken SUSY generators. The term providing the fermion contributions coming from the partial SUSY breaking 5.6.80 is

$$4\,\pi\,\mathcal{L}_{\mathrm{D\ Fermi}} = \frac{i}{2}\,\mathbf{D}^{a,\,A}|\,\mathcal{F}_{abc}|\,(\lambda^b\,\lambda^c)^A + \mathrm{h.c.}. \qquad (5.6.126)$$

This, together with the Yukawa interaction

$$4\,\pi\,\mathcal{L}_{\mathrm{yuk}} \supset \frac{i}{\sqrt{2}}\,g_{ab}\,f^b_{cd}\,\lambda^{a,i}\,\bar{X}^c\,\lambda^d_i + \mathrm{h.c.}$$

gives rise to the adjoint fermion masses. Since we are only interested in the phase where $\langle X^{\hat{a}}\rangle = 0$, we can ignore the Yukawa term for a spectrum analysis for the $\mathrm{SU}(N_c)$ part. For the $\mathrm{U}(1)_\circ$ theory this coupling does not exist because there are no abelian self interactions. Noting that $\langle \mathcal{F}_{\hat{a}\circ\circ}\rangle = 0$, we can decompose 5.6.126 into the $\mathrm{U}(1)_\circ$ and $\mathrm{SU}(N_c)$ parts as

$$-\mathcal{L}_{\mathrm{D\ Fermi}} = \frac{1}{2}\,M^{ij}_\circ\,\lambda^\circ_i\,\lambda^\circ_j + \frac{1}{2}\,M^{ij}\,\lambda^{\tilde{a}}_i\,\lambda^{\tilde{a}}_j + \mathrm{h.c.} \qquad (5.6.127)$$

where the fermion mass matrices are

$$M^{ij}_\circ = \frac{i\,g^{\circ\circ}}{4\pi}\begin{pmatrix} e + m\,\bar{\mathcal{F}}_{\circ\circ}| & -i\,\xi \\ -i\,\xi & e + m\,\bar{\mathcal{F}}_{\circ\circ} \end{pmatrix}^{ij}\mathcal{F}_{\circ\circ\circ} \qquad (5.6.128)$$

$$M^{ij} = \frac{i\,g^{\circ\circ}}{4\pi}\begin{pmatrix} e + m\,\bar{\mathcal{F}}_{\circ\circ} & -i\,\xi \\ -i\,\xi & e + m\,\bar{\mathcal{F}}_{\circ\circ} \end{pmatrix}^{ij}\mathcal{F}_{\circ\tilde{a}\tilde{a}}|. \qquad (5.6.129)$$

In the vacuum determined above 5.6.3 these become

$$M_{\circ}^{ij} = -\frac{m}{4\,\pi}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{ij} \langle \mathcal{F}_{\circ\circ\circ}\rangle, \tag{5.6.130}$$

$$M^{ij} = -\frac{m}{4\,\pi}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{ij} \langle \mathcal{F}_{\circ\tilde{a}\tilde{a}}\rangle. \tag{5.6.131}$$

Note that the latter term can be rewritten as

$$M^{ij} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{ij} \frac{\partial^2 W_{\rm def}}{\partial X^{\tilde{a}}\,\partial X^{\tilde{a}}}. \tag{5.6.132}$$

This correctly matches eq. E.33 as required. Since for $m, \langle \mathcal{F}_{\circ\circ\circ}\rangle$, and $\langle \mathcal{F}_{\circ\tilde{a}\tilde{a}}\rangle$ all non-zero we have

$$\det\, M_{\circ} \;=\; \det\, M \;=\; 0, \qquad {\rm tr}\, M_{\circ} \neq 0, \qquad {\rm tr}\, M \neq 0, \tag{5.6.133}$$

the $U(1)_{\circ}$ fermions and the $SU(N_c)$ fermions each have one linear combination that corresponds to a massless eigenstate, and one linear combination that corresponds to an eigenstate of mass

$$\frac{m\,\langle \mathcal{F}_{\circ\circ\circ}\rangle}{2\,\pi} \qquad \text{and} \qquad \frac{m\,\langle \mathcal{F}_{\circ\tilde{a}\tilde{a}}\rangle}{2\pi} = \partial_{X^{\tilde{a}}}\partial_{X^{\tilde{b}}} W_{\rm def}$$

respectively. The massless $U(1)_{\circ}$ combination is the Nambu–Goldstone fermion of partial SUSY breaking, and the massless $SU(N_c)$ combination is the gaugino of the unbroken gauge symmetry as required[6]. In the $\mathcal{N}=1$ preserving vacuum, note that the massless $SU(N_c)$ gaugino does not enter the superpotential, only the (potentially) massive $SU(N_c)$ combination will.

[6]This can be seen by calculating the SUSY transformations where one finds [273]

$$\langle \delta_Q\, \lambda^{\circ}_{\rm massless}\rangle \sim \langle \mathbf{D}^{\circ}_{\rm massless}\rangle \neq 0, \qquad \langle \delta_Q\, \lambda^{\tilde{a}}_{\rm massless}\rangle \sim \langle \mathbf{D}^{\tilde{a}}_{\rm massless}\rangle = 0. \tag{5.6.134}$$

5.7 $\mathcal{N}=2 \to \mathcal{N}=0$ with gaugino masses

5.7.1 Overview

We have shown that one can write an $\mathcal{N}=1$ theory as a spontaneously broken $\mathcal{N}=2$ theory using the ATP mechanism. Now this can be extended by enhancing the gauge symmetry to $SU(N_c)\times \mathrm{U}(1)^3$; we can then assign a combination of FI terms to pick out an $\mathcal{N}=1$ preserving direction, and as a perturbation, assign a different combination of FI terms to fully break SUSY. This provides us with a description of an $\mathrm{SU}(N_c)$ $\mathcal{N}=2$ theory augmented by both $\mathcal{N}=1$ deformations and soft terms that can all be mapped under electric–magnetic duality.

5.7.2 How to add a Dirac gaugino mass

We will be thinking of the additional U(1)'s as a perturbation on the $\mathcal{N}=1$ theory (in the sense that $m_D \ll \Lambda$) and will take the FI terms for $\mathrm{U}(1)_\circ$ to be as described above. Although Dirac mass-terms can famously preserve an R-symmetry, in the context of Kutasov duality they will break it (since the $\mathcal{N}=1$ gauginos have R-charge 1 and therefore the Dirac mass requires $\tilde{X}$ to have R-charge zero, in conflict with $W_{\mathrm{def}} \supset \kappa X^{k+1}$). Therefore the FI terms for the new U(1)'s must have some component along the σ^1 direction of $\mathrm{SU}(2)_R$ which as we saw in Section 5.6.3 is the $\mathrm{U}(1)_R$ direction of the $\mathcal{N}=1$ theory. Furthermore the contribution from FI terms to the fermion mass matrix M^{ij} are $M^{ij} \sim \boldsymbol{\xi}^A(\sigma^A\varepsilon)^{ij}$ where ε is the $\mathrm{SU}(2)_R$ metric. But the stability condition essentially fixes $\boldsymbol{\xi}$ to be null. We can parameterise a general null $\boldsymbol{\xi}^A$ by

$$\boldsymbol{\xi}^A = (\alpha, i\sqrt{\alpha^2+\beta^2}, \beta) \tag{5.7.135}$$

irrespective of the origin of α and β. The stability conditions for $\boldsymbol{\xi}$ then simply fix the VEVs of the $\mathcal{F}_{abc}$ to satisfy this condition (the specific case above has $\alpha=0$, $\beta=\xi$). Shifting to the basis in which the $\mathcal{N}=1$ created by $\mathrm{U}(1)_\circ$ is diagonal, we find that additional terms from a single extra U(1) are of the form

$$\delta M^{ij} \sim \begin{pmatrix} -\beta+\sqrt{\alpha^2+\beta^2} & -\alpha \\ -\alpha & \beta+\sqrt{\alpha^2+\beta^2} \end{pmatrix}. \tag{5.7.136}$$

	$\mathrm{SU}(N_c)$	$\mathrm{U}(1)_{\circ}$	$\mathrm{U}(1)_{𝅗𝅥}$	$\mathrm{U}(1)_{♩}$	$\mathrm{SU}(N_f)$
Q^+	$\square$	1	0	0	$\square$

Table 5.5: $\mathcal{N}=2$ representations for $\mathcal{N}=2$ SQCD coupled to $\mathrm{U}(1)_{\circ} \times \mathrm{U}(1)_{𝅗𝅥} \times \mathrm{U}(1)_{♩}$.

Clearly for any choice of α and β one can never set the δM^{11} and δM^{22} components to zero unless α is zero as well, and it is therefore impossible to introduce a pure Dirac mass with a single extra U(1). On the other hand it is always possible (by tuning parameters) to do this with two additional U(1)'s.

In order to add a Dirac mass, the theory we need to consider is therefore an $\mathrm{SU}(N_c) \times \mathrm{U}(1)_{\circ} \times \mathrm{U}(1)_{𝅗𝅥} \times \mathrm{U}(1)_{♩}$ theory, where the Q^+ is charged under only the $\mathrm{U}(1)_{\circ}$ as displayed in table 5.5. This theory is in the same form as in 5.6.61 and 5.6.63 with the prepotential $\mathcal{F}(\mathcal{W}, \mathcal{W}^{\circ}, \mathcal{W}^{𝅗𝅥}, \mathcal{W}^{♩})$ again being a generic function of $\mathcal{N}=2$ gauge hyperstrengths, and the gauge covariant derivative acting on the hypermultiplets remaining unchanged. The corresponding additional FI pieces in the action take the same form as in equations 5.6.67 and 5.6.70 with the obvious replacement of gauge group. The vacuum stability conditions in the $\mathcal{N}=0$ theory still set

$$\langle \mathbf{D}^{\underline{a},\,A}\mathbf{D}^{\underline{a},\,A}\rangle = 0 \tag{5.7.137}$$

for $\underline{a}$'s corresponding to each of the U(1) factors, where as before there is summation over A but not over $\underline{a}$.

There are many combinations that one could consider for the prepotential and the new FI-terms. A simple solution is to allow only $\mathcal{F}_{\circ 𝅗𝅥}$ and $\mathcal{F}_{\circ ♩}$ mixing, and just electric FI terms for the $\mathrm{U}(1)_{𝅗𝅥}$ and $\mathrm{U}(1)_{♩}$ factors in the σ^1 and σ^2 directions (i.e. we are going to add two $\beta = 0$ type solutions and make the Majorana masses cancel). The three vacuum stability equations then translate into the conditions

$$g_{\circ\circ}\,\mathrm{Re}(\xi^{(2)}_{D,\circ}) = \mathrm{Re}(\xi^{(3)}_{\circ}), \tag{5.7.138}$$

$$g_{\circ 𝅗𝅥}\,\mathrm{Re}(\xi^{(2)}_{D,\circ}) = \mathrm{Re}(\xi^{(1)}_{𝅗𝅥}), \tag{5.7.139}$$

$$g_{\circ ♩}\,\mathrm{Re}(\xi^{(2)}_{D,\circ}) = -\,\mathrm{Re}(\xi^{(1)}_{♩}). \tag{5.7.140}$$

The first of these is essentially the same condition as in eq. 5.6.91. The imaginary parts can be set to satisfy the zero vacuum energy conditions if desired. In order to get non–zero gaugino masses the prepotential is of the form

$$\mathcal{F}(\mathcal{W}) = \frac{\tau_{ab}}{2}\,\mathcal{W}^a\,\mathcal{W}^b + \frac{\mathcal{W}^\circ}{\Lambda^2}\,\hat{W}_{\rm def} + \frac{1}{2\,\Lambda}(\mathcal{W}^{\text{𝅗𝅥}} - \mathcal{W}^{\text{♩}})\mathcal{W}^{\tilde a}\,\mathcal{W}^{\tilde a}, \tag{5.7.141}$$

where $\tau_{ab} = \mathcal{F}_{ab}|$, and we neglect higher order terms in the leading part. Note that the mass-inducing third term only involves the two additional U(1)'s. The contribution to the gaugino masses is of the form

$$\delta M^{ij} = -\frac{(\sigma^A\,\varepsilon)^{ij}}{4\,\pi\,\Lambda}\left[\boldsymbol{\xi}^A_\circ\,(g^{\circ\text{𝅗𝅥}} - g^{\circ\text{♩}}) + (g^{\text{𝅗𝅥}\text{𝅗𝅥}}\,\boldsymbol{\xi}^A_{\text{𝅗𝅥}} - g^{\text{♩}\text{♩}}\,\boldsymbol{\xi}^A_{\text{♩}})\right]. \tag{5.7.142}$$

In order to forbid additional $\mathcal{N}=1$ mass terms for the adjoints $X^{\tilde a}$, we must choose $g^{\circ\text{𝅗𝅥}} = g^{\circ\text{♩}}$ to make the first term vanish. By eq. 5.7.138 we then have $\boldsymbol{\xi}^{(1)}_{\text{𝅗𝅥}} = -\boldsymbol{\xi}^{(1)}_{\text{♩}}$. Choosing for simplicity $g_{\circ\text{𝅗𝅥}} = g_{\circ\text{♩}} \ll g_{\circ\circ}$, $g_{\text{𝅗𝅥}\text{𝅗𝅥}} = g_{\text{♩}\text{♩}}$ together with $g_{\text{𝅗𝅥}\text{♩}} = 0$, we then have $g_{\circ\text{𝅗𝅥}} = g_{\circ\text{♩}} \equiv -\alpha/m$. Hence $\boldsymbol{\xi}_{\text{𝅗𝅥}} = (\alpha, i\,\alpha, 0)$ and $\boldsymbol{\xi}_{\text{♩}} = (-\alpha, i\,\alpha, 0)$, giving a gaugino mass matrix of the form

$$\delta M^{ij} = -\frac{\alpha}{2\,\pi\,\Lambda}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \tag{5.7.143}$$

as required. Along with these terms we see the supersoft operator of eq. 3.4.41 is also induced in the scalar potential 5.6.95, arising in the cross terms of

$$g^{\circ\text{𝅗𝅥}}\,\mathcal{Q}^\dagger_{\text{𝅗𝅥}}\,\boldsymbol{\xi}_\circ + g^{\circ\text{♩}}\,\mathcal{Q}^\dagger_{\text{♩}}\,\boldsymbol{\xi}_\circ + \text{h.c.}.$$

It is much easier to generate pure Majorana mass as this only requires a single additional $\mathrm{U}(1)_{\text{𝅗𝅥}}$, and a prepotential of the form

$$\mathcal{F}(\mathcal{W}) = \frac{\tau_{ab}}{2}\,\mathcal{W}^a\,\mathcal{W}^b + \frac{\mathcal{W}^\circ}{\Lambda^2}\,\hat{W}_{\rm def} + \frac{1}{2\,\Lambda}\,\mathcal{W}^{\text{𝅗𝅥}}\,\mathcal{W}^{\tilde a}\,\mathcal{W}^{\tilde a}, \tag{5.7.144}$$

choosing FI terms such that $\alpha = 0$ in eq. 5.7.136. Furthermore, to avoid this becoming just another $\mathcal{N}=1$ mass-term for the adjoint fields, the sign of β is chosen so that the non–zero eigenvalue falls in the block that has just been identified by the $\mathrm{U}(1)_\circ$ FI terms as belonging to the $\mathcal{N}=1$ gauginos. That is with $\boldsymbol{\xi}^A_\circ = (0, i\,\xi, \xi)$ we choose $\boldsymbol{\xi}^A_{\text{𝅗𝅥}} = (0, i\,\beta, -\beta)$, with both ξ and $\beta > 0$.

5.8 Duality relations for the broken theory

5.8.1 $\mathcal{N} = 1$ couplings and gaugino masses

Let finally return to our objective, which (recall) is to determine how couplings as well as Dirac gaugino masses map under $\mathcal{N} = 2$ duality, and that the prepotential maps consistently under $\mathcal{N} = 2$ duality. We should at this point be clear that we are not about to solve the $\mathcal{N} = 2$ system for arbitrary N_c and N_f. Nevertheless it is possible to make general statements about the constraints such a duality should give on the prepotential. This is enough to establish that it contains all the same operators as the weakly coupled electric superpotential. After this use the spurion technique of [86] determines the precise coefficients.

The theory can be written in either electric variables

$$\mathcal{W}(X, \lambda, D, v), \quad \mathcal{F} \tag{5.8.145}$$

or dual magnetic ones,

$$\mathcal{W}_D(X_D, \lambda_D, D_D, v_D), \quad \mathcal{F}_D, \tag{5.8.146}$$

with the relations [272]

$$\mathcal{W}_D^a = \frac{\partial \mathcal{F}}{\partial \mathcal{W}_a}, \qquad \mathcal{W}^a = -\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,a}}. \tag{5.8.147}$$

From eq. 5.8.147, we can infer

$$\mathcal{F}_D(\mathcal{W}_D) = \mathcal{F}[\mathcal{W}(\mathcal{W}_D)] - \mathcal{W}_D\, \mathcal{W}(\mathcal{W}_D). \tag{5.8.148}$$

This means the magnetic prepotential is given by taking the electric one and replacing $\mathcal{W}$ with $\mathcal{W}(\mathcal{W}_D)$ determined as a function of $\mathcal{W}_D$. In general this is extremely complicated, however we can demonstrate the perturbative behaviour with our deformation of interest. Suppose one knows the dual prepotential $\mathcal{F}_D^{(0)}(\mathcal{W}_D)$ of an undeformed $\mathcal{N} = 2$ theory, with prepotential $\mathcal{F}^{(0)}(\mathcal{W})$. If the theory is then deformed to $\mathcal{F}(W) = \mathcal{F}^{(0)} + \kappa\, \mathcal{F}_\kappa$, where κ is parametrically small, then in a κ expansion, a dual prepotential of the form

$$\mathcal{F}_D(\mathcal{W}_D) = \mathcal{F}^{(0)}[\mathcal{W}^{(0)}(\mathcal{W}_D)] + \kappa\, \mathcal{F}_\kappa[\mathcal{W}^{(0)}(\mathcal{W}_D)], \tag{5.8.149}$$

where $W^{(0)}(W_D)$ is the function determined from $W_D = \partial \mathcal{F}^{(0)}/\partial W$, is seen to correctly solve equations 5.8.147 and 5.8.148 to $\mathcal{O}(\kappa^2)$. Let us show this explicitly. The electric prepotential of interest is

$$\mathcal{F}(\mathcal{W}) = \frac{\tau}{2}\,\mathcal{W}^a\,\mathcal{W}^a + \frac{\mathcal{W}^\circ}{\Lambda^2}\,\hat{W}_{\mathrm{def}}, \tag{5.8.150}$$

$$\hat{W}_{\mathrm{def}} = 4\,\pi\,\frac{\kappa}{k+1}\,\mathrm{tr}\,\tilde{X}^{k+1} = \frac{c\,\kappa}{k+1}\,\mathrm{tr}(\tilde{\mathcal{W}}^{k+1}) \tag{5.8.151}$$

where

$$c = 4\,\pi\,\left(\frac{1}{i\,\sqrt{2}}\right)^{k+1}, \tag{5.8.152}$$

$$\mathrm{tr}\,(\tilde{W}^n) = W^{\tilde{a}_1}\ldots W^{\tilde{a}_n}\,\mathrm{tr}(T^{\tilde{a}_1}\ldots T^{\tilde{a}_n}). \tag{5.8.153}$$

The equations we need to solve to find the dual prepotential are

$$\mathcal{W}_D^0 = -\,\tau\,\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,0}} + \frac{(-1)^{k+1}}{\Lambda^2}\frac{c\,\kappa}{k+1}\mathrm{tr}\left(\frac{\partial \mathcal{F}_D}{\partial \tilde{W}_D}\right)^{k+1}, \tag{5.8.154}$$

$$\begin{aligned}\mathcal{W}_D^{\tilde{a}} &= -\,\tau\,\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,\tilde{a}}} \\ &\quad + c\,\kappa\,\frac{(-1)^{k+1}}{\Lambda^2}\,\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,0}}\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,\tilde{b}_1}}\cdots\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,\tilde{b}_k}}\mathrm{tr}\left(T^{\tilde{a}}T^{\tilde{b}_1}\cdots T^{\tilde{b}_k}\right).\end{aligned} \tag{5.8.155}$$

We see that a dual prepotential of the form

$$\mathcal{F}_D = -\frac{1}{2\,\tau}\,\mathcal{W}_D^a\,\mathcal{W}_D^a + \frac{\mathcal{W}_D^0}{\Lambda^2}\,\frac{c\,\kappa}{(k+1)\,\tau^{k+2}}\,\mathrm{tr}(\tilde{\mathcal{W}}_D^{k+1}) + \mathcal{O}(\kappa^2) \tag{5.8.156}$$

solves this up to order κ^2. To see this, the relevant derivatives are

$$\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,0}} = -\frac{1}{\tau}\,\mathcal{W}_D^0 + \frac{1}{\Lambda^2}\,\frac{c\,\kappa}{(k+1)\,\tau^{k+2}}\,\mathrm{tr}(\tilde{\mathcal{W}}_D^{k+1}), \tag{5.8.157}$$

$$\frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\tilde{a}}} = -\frac{1}{\tau}\,\mathcal{W}_D^{\tilde{a}} + \frac{\mathcal{W}_D^0}{\Lambda^2}\,\frac{c\,\kappa}{\tau^{k+2}}\mathcal{W}_D^{\tilde{b}_1}\ldots\mathcal{W}_D^{\tilde{b}_k}\,\mathrm{tr}\left(T^{\tilde{a}}T^{\tilde{b}_1}\cdots T^{\tilde{b}_k}\right). \tag{5.8.158}$$

This implies that eqs. (5.8.154) and (5.8.155) become

$$\begin{aligned}
\mathcal{W}_D^0 &= -\tau \frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,0}} + \frac{(-1)^{k+1}}{\Lambda^2} \frac{c\,\kappa}{k+1} \,\mathrm{tr} \left(\frac{\partial \mathcal{F}_D}{\partial \tilde{\mathcal{W}}_D} \right)^{k+1} \\
&= \mathcal{W}_D^0 - \frac{\tau}{\Lambda^2} \frac{\kappa}{(k+1)\,\tau^{k+2}} \,\mathrm{tr}(\tilde{\mathcal{W}}_D^{k+1}) + \frac{(-1)^{k+1}}{\Lambda^2} \frac{\kappa}{k+1} \mathrm{tr} \left(-\frac{\tilde{\mathcal{W}}_D}{g} \right)^{k+1} + \mathcal{O}(\kappa)^2 \\
&= \mathcal{W}_D^0 + \mathcal{O}(\kappa)^2 \qquad (5.8.159)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{W}_D^{\tilde{a}} &= -\tau \frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,\tilde{a}}} + c\,\kappa \frac{(-1)^{k+1}}{\Lambda^2} \frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,0}} \frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,\tilde{b}_1}} \cdots \frac{\partial \mathcal{F}_D}{\partial \mathcal{W}_{D,\,\tilde{b}_k}} \,\mathrm{tr} \left(T^{\tilde{a}} T^{\tilde{b}_1} \cdots T^{\tilde{b}_k} \right) \\
&= \mathcal{W}_D^{\tilde{a}} - \tau \frac{\mathcal{W}_D^0}{\Lambda^2} \frac{c\,\kappa}{\tau^{k+2}} \mathcal{W}_D^{\tilde{b}_1} \ldots \mathcal{W}_D^{\tilde{b}_k} \mathrm{tr} \left(T^{\tilde{a}} T^{\tilde{b}_1} \cdots T^{\tilde{b}_k} \right) \\
&\quad + c\,\kappa \frac{(-1)^{k+1}}{\Lambda^2} \left(-\frac{\mathcal{W}_D^0}{\tau} \right) \left(-\frac{\mathcal{W}_D^{\tilde{b}_1}}{\tau} \right) \cdots \left(-\frac{\mathcal{W}_D^{\tilde{b}_k}}{\tau} \right) \mathrm{tr} \left(T^{\tilde{a}} T^{\tilde{b}_1} \cdots T^{\tilde{b}_k} \right) + \mathcal{O}(\kappa)^2 \\
&= \mathcal{W}_D^{\tilde{a}} + \mathcal{O}(\kappa)^2 \qquad (5.8.160)
\end{aligned}$$

respectively. This implies that under the change to dual magnetic variables (5.8.146), a prepotential

$$\mathcal{F} = \frac{\tau}{2} \mathcal{W}^a \mathcal{W}^a + \frac{\mathcal{W}^0}{\Lambda^2} \hat{W}_{\mathrm{def}}, \qquad \hat{W}_{\mathrm{def}} = 4\,\pi \frac{\kappa}{k+1} \,\mathrm{tr}(X^{k+1}) \qquad (5.8.161)$$

implies a dual

$$\mathcal{F}_D = -\frac{1}{2\,\tau} \mathcal{W}_D^a \mathcal{W}_D^a + \frac{\mathcal{W}_D^0}{\Lambda^2\,\tau^{k+2}} \hat{W}_{D,\mathrm{def}}, \qquad \hat{W}_{D,\mathrm{def}} = \frac{\kappa}{(k+1)} \,\mathrm{tr}(x^{k+1}), \qquad (5.8.162)$$

We can now use the spurion technique of [86] to fix the coefficients of the terms in the κ deformation of the magnetic prepotential even in the presence of Majorana and gaugino masses. In particular, the electric prepotential

$$\mathcal{F}(\mathcal{W}) = \frac{\tau}{2} \mathcal{W}^a \mathcal{W}^b + \frac{\mathcal{W}^{\circ}}{\Lambda^2} \hat{W}_{\mathrm{def}} + \frac{1}{2\,\Lambda} (\mathcal{W}^{\text{𝅗𝅥}} - \mathcal{W}^{\text{♩}}) \mathcal{W}^{\tilde{a}} \mathcal{W}^{\tilde{a}}, \qquad (5.8.163)$$

where recall the electric scale is $\Lambda^2 = m$, needs a dual prepotential of the form

$$\begin{aligned}\mathcal{F}_D(W_D) = &- \frac{1}{2\,\tau}\,\mathcal{W}^a_D\,\mathcal{W}^a_D + \frac{\mathcal{W}^{\circ}_D}{\Lambda^2_D}\,\hat{W}_{D,\mathrm{def}} \\ &+ \frac{1}{2\,\Lambda_D}(\mathcal{W}^{𝅗𝅥}_D - \mathcal{W}^{♩}_D)\,\mathcal{W}^{\tilde{a}}_D\,\mathcal{W}^{\tilde{a}}_D + \mathcal{O}(\kappa^2),\end{aligned} \tag{5.8.164}$$

for the mapping to be correct, where the magnetic scale $\Lambda^2_D = -(e + i\,\xi)$, and the scales Λ and Λ_D satisfy

$$\Lambda^2_D = \tau\,\Lambda. \tag{5.8.165}$$

To see how the Dirac gaugino mass is treated by the mapping of the prepotential and the swapping of electric and magnetic FI terms we just need to follow the prefactor in $\mathcal{L}_{\mathrm{fermion}}$ of eq. 5.6.126. In the electric theory this is

$$g^{\circ\circ}\,\boldsymbol{\xi}^A_{\circ}\,\mathcal{F}_{\circ\tilde{a}\tilde{a}} = -m\,(0, i, 1)\,\mathcal{F}_{\circ\tilde{a}\tilde{a}}. \tag{5.8.166}$$

and in the magnetic theory the stability conditions for $\boldsymbol{\xi}^A_{\circ} = (0, -m, 0) + (0, e, \xi)\bar{\mathcal{F}}_{D,\circ\circ}$ give $\mathcal{F}_{D,\circ\circ} = m/(e + i\xi) = -1/\mathcal{F}_{\circ\circ}$ and so the prefactor is

$$\tilde{g}^{\circ\circ}\,\tilde{\boldsymbol{\xi}}^A\mathcal{F}_{D,\circ\tilde{a}\tilde{a}} = (e + i\,\xi)\,(0, i, 1)\,\mathcal{F}_{D,\circ\tilde{a}\tilde{a}}. \tag{5.8.167}$$

We see that the Dirac gaugino mass given by the prepotential deformation is mapped into another Dirac gaugino mass with the dual prepotential deformation. The same behaviour is observed for Majorana masses.

5.8.2 A note on quarks under electric–magnetic duality

Let us briefly comment on the mapping of the quark hypermultiplet Q^+ under the $\mathcal{N} = 2$ S–duality. By considering finiteness, the mapping of gauge invariants, and requiring that known non–self dual points are not mapped onto each other, refs. [284, 285] argue that a natural map for $\mathrm{SU}(N_c)$ $\mathcal{N} = 2$ SQCD deformed by a mass for the chiral adjoint in the unbroken phase is into a similar theory $\mathrm{SU}(N_c)$ $\mathcal{N} = 2$ SQCD$'$ with the charge conjugation acting on the flavour structure. The new hypermultiplets q^+ are interpreted as the general N_f case of the semi–classical monopoles of [60, 61], and the mass for the chiral adjoint is mapped to itself. For our purposes, we have already shown that a mass for the chiral adjoint is mapped to itself in Section 5.8.1, and so we expect the conclusions of [284,285] to apply here as well.

5.9 Chapter summary

We have presented evidence that a Dirac gaugino maps across Kutasov duality as

$$\lim_{\mu\to\infty} \frac{m_D}{\hat{g}\,\kappa^{\frac{1}{k+1}}} = \lim_{\mu\to 0} \frac{\tilde{m}_D}{\hat{\tilde{g}}\,\tilde{\kappa}^{\frac{1}{k+1}}}. \qquad (5.9.168)$$

The reasoning is as follows. We have shown that both the left hand side and the right hand side of this expression are all orders RG invariants, thus it doesn't matter when they are made equal, providing they are found to be equal at any point along the RG flow. The standard way of determining them to be equal (since they are SUSY breaking operators) is to embed them into the superfield expansion of a physical SUSY RG invariant of the theory that can be matched between both theories. This was not possible to achieve in the $\mathcal{N}=1$ language but it is possible in the language of $\mathcal{N}=2$ HSS, since there, a spurious redefinition of the gauge coupling can induce a Dirac gaugino mass. We showed that the electric Kutasov theory was connected by RG flow to an $\mathcal{N}=2$ SQCD theory in the IR, and, after showing that the magnetic Kutasov theory undergoes the correct Higgsing, argued that it can flow to the dual $\mathcal{N}=2$ SQCD at some point. Finally we showed that it was possible via the ATP mechanism to embed the $\mathcal{N}=1$ Kutasov theory into $\mathcal{N}=2$ HSS, and that one can, as a perturbation, add Dirac gaugino masses by breaking the orthogonal SUSY direction. Under the mapping the prepotential, the Kutasov deformation becomes the dual Kutasov deformation, and a Dirac gaugino mass becomes a dual Dirac gaugino mass. This is suggestive that there exists an orders RG invariant in the $\mathcal{N}=2$ theory that encodes the $\mathcal{N}=1$ Kutasov deformation *and* the Dirac gaugino mass, that is mapped to itself under the $\mathcal{N}=2$ S–duality, leading to 5.9.168 as one of the terms in its Grassman expansion. Focusing on the Dirac mass, we see that m_D/g^2 is an RG invariant but of course only in the $\mathcal{N}=2$ theory (as in [86]); away from $\mathcal{N}=2$, the h and g couplings go their separate ways and m_D/g^2 will begin to pick up corrections of order κ^2, but as we know the combination $m_D/g\kappa^{\frac{1}{k+1}}$ remains an RG invariant even as we flow back to $\mathcal{N}=1$.

A

One loop scalar integrals

Here we present the scalar integrals used in this thesis. They are calculated the in the $\overline{\text{DR}}$ scheme and regularised in $d = 4 - 2\,\varepsilon$ dimensions with renormalisation scale μ [237, 286]. We denote

$$\frac{1}{\widehat{\varepsilon}} \equiv \frac{1}{\varepsilon} - \gamma + \log(4\pi), \tag{A.1}$$

where

$$\gamma \approx 0.5772 \tag{A.2}$$

is the Euler–Mascheroni gamma constant. The scalar integrals A_0 and B_0 are

$$\begin{aligned} A_0(m) &= \frac{1}{i\,\pi^2} \int d^d q\, \frac{1}{q^2 + m^2} \\ &= m^2 \left[\frac{1}{\widehat{\varepsilon}} + 1 - \log\left(\frac{m^2}{\mu^2}\right) \right], \end{aligned} \tag{A.3}$$

$$\begin{aligned} B_0(p, m_1, m_2) &= \frac{1}{i\,\pi^2} \int d^d q\, \frac{1}{(q^2 + m_1^2)[(q+p)^2 + m_2^2]} \\ &= \frac{1}{\widehat{\varepsilon}} - \log\left(\frac{p^2}{\mu^2}\right) - f_B(x_+) - f_B(x_-), , \end{aligned} \tag{A.4}$$

where

$$x_\pm \equiv \frac{s \pm \sqrt{s^2 - 4\,p^2(m_1^2 - i\,\epsilon)}}{2p^2}, \tag{A.5}$$

$$f_B(x) \equiv \log(1-x) - x\,\log(1 - x^{-1}) - 1, \tag{A.6}$$

$$s = p^2 + m_1^2 - m_2^2. \tag{A.7}$$

A useful limit of B_0 is its zero momentum limit

$$B_0(0, m_1, m_2) = \frac{1}{\widehat{\varepsilon}} + 1 + \log\left(\frac{\mu^2}{m_2^2}\right) + \frac{m_1^2}{m_1^2 - m_2^2}\,\log\left(\frac{m_2^2}{m_2^2}\right). \tag{A.8}$$

The remaining scalar integral used in this theses is just a combination of eqs. A.3 and A.4

$$G_0(p, m_1, m_2) = (p^2 - m_1^2 - m_2^2)\,B_0(p, m_1, m_2) - A_0(m_1) - A_0(m_2), \tag{A.9}$$

$$B_1(p, m_1, m_2) = \frac{1}{2\,p^2}\left[(p^2 + m_1^2 - m_2^2)\,B_0(p, m_1, m_2) + A_0(m_2) - A_0(m_1)\right]. \tag{A.10}$$

The triangle integral C_0 is

$$C_0[p_1, p_2, m_1, m_2, m_3] \equiv \frac{1}{i\pi^2}\int \frac{d^dq}{(q^2 + m_1^2)[(q + p_1)^2 + m_2^2][(q + p_2)^2 + m_3^2]} \tag{A.11}$$

and has the zero momentum limit

$$C_0(m_1, m_2, m_3) = \frac{1}{m_2^2 - m_3^2}\left[\frac{m_2^2}{m_1^2 - m_2^2}\log\left(\frac{m_2^2}{m_1^2}\right) - \frac{m_3^2}{m_1^2 - m_3^2}\log\left(\frac{m_3^2}{m_1^2}\right)\right]. \tag{A.12}$$

B

Additional plots for CDGM

B.1 The Constrained MSSM

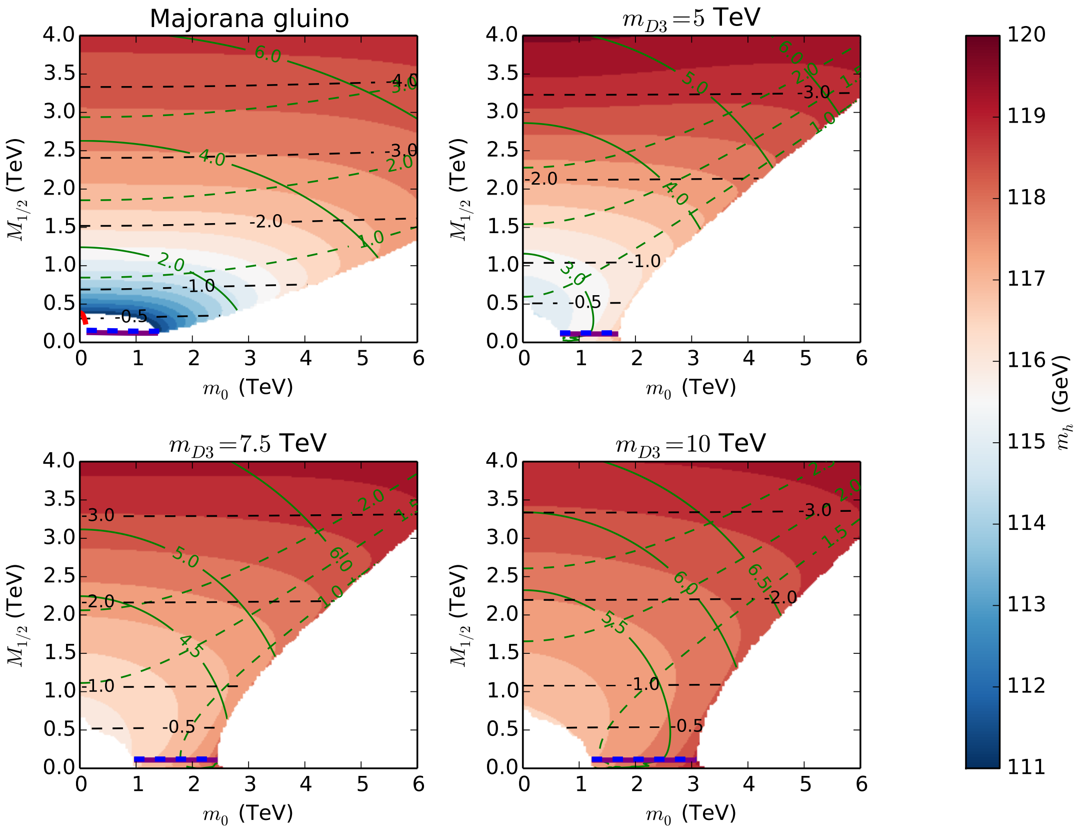


Figure B.1: Higgs sector parameters in the CMSSM with $t_\beta = 25$ and m_{D3} fixed as indicated. The gradient indicates the Higgs mass. The black dashed, green dashed and green solid lines are contours of $a_t(m_{\text{SUSY}})$, $\mu(m_{\text{SUSY}})$, and m_{SUSY} respectively. All contours unless otherwise specified are in TeV.

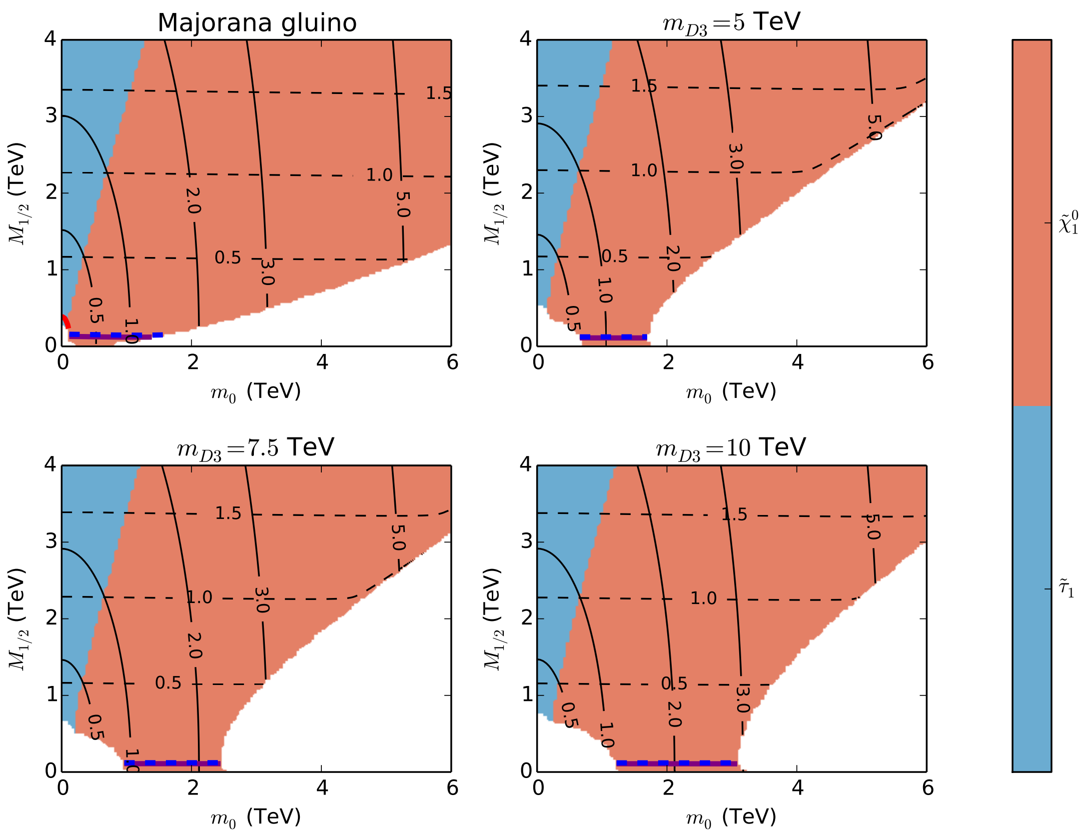


Figure B.2: LOSP species in the CMSSM with $t_\beta = 25$ and m_{D3} fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

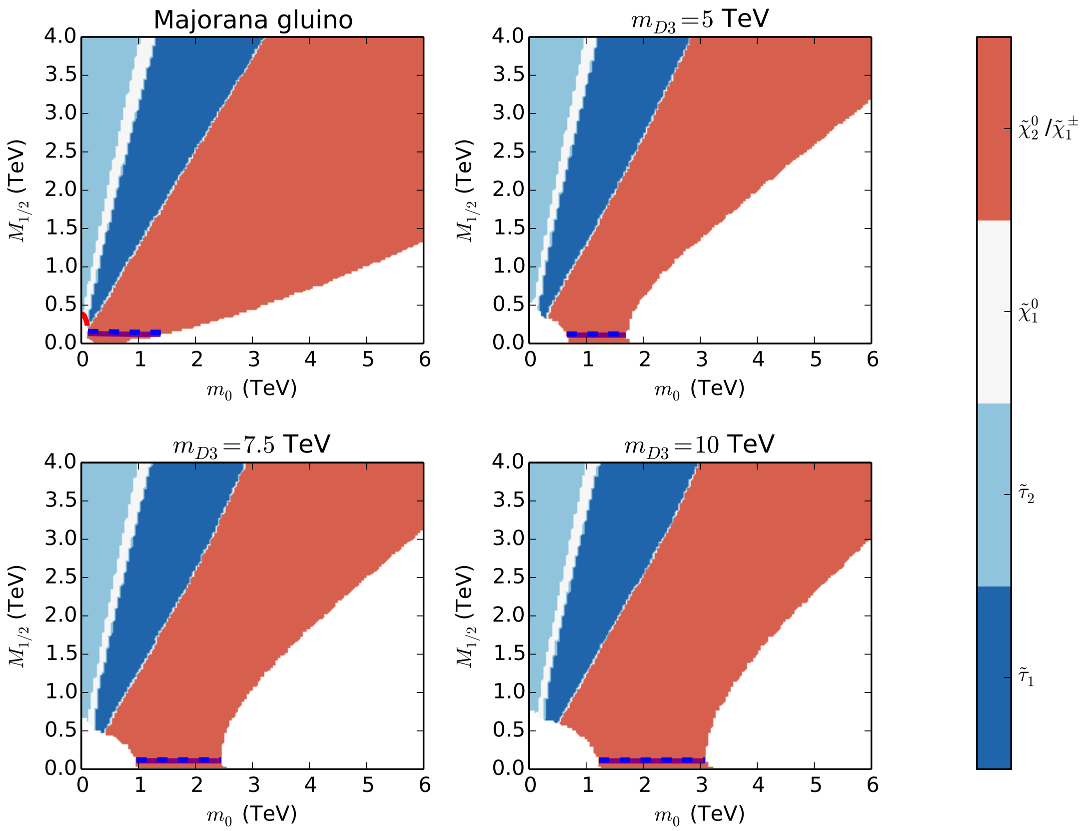


Figure B.3: NLOSP species in the CMSSM with $t_\beta = 25$ and m_{D3} fixed as indicated

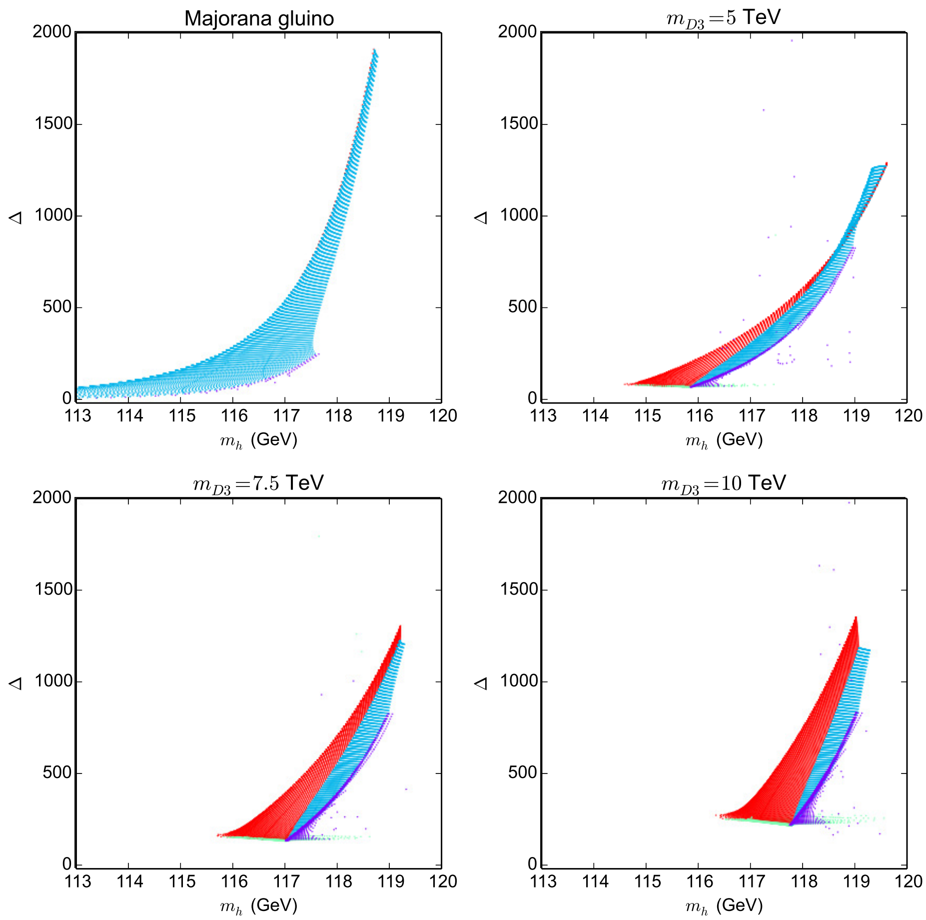


Figure B.4: Fine tuning in the CMSSM with $t_\beta = 25$ and m_{D3} fixed as indicated. The red, purple, blue, and green regions correspond to μ, m_0, $M_{1/2}$ and m_{D3} as the dominant source of tuning.

B.2 Constrained General Gauge Mediation

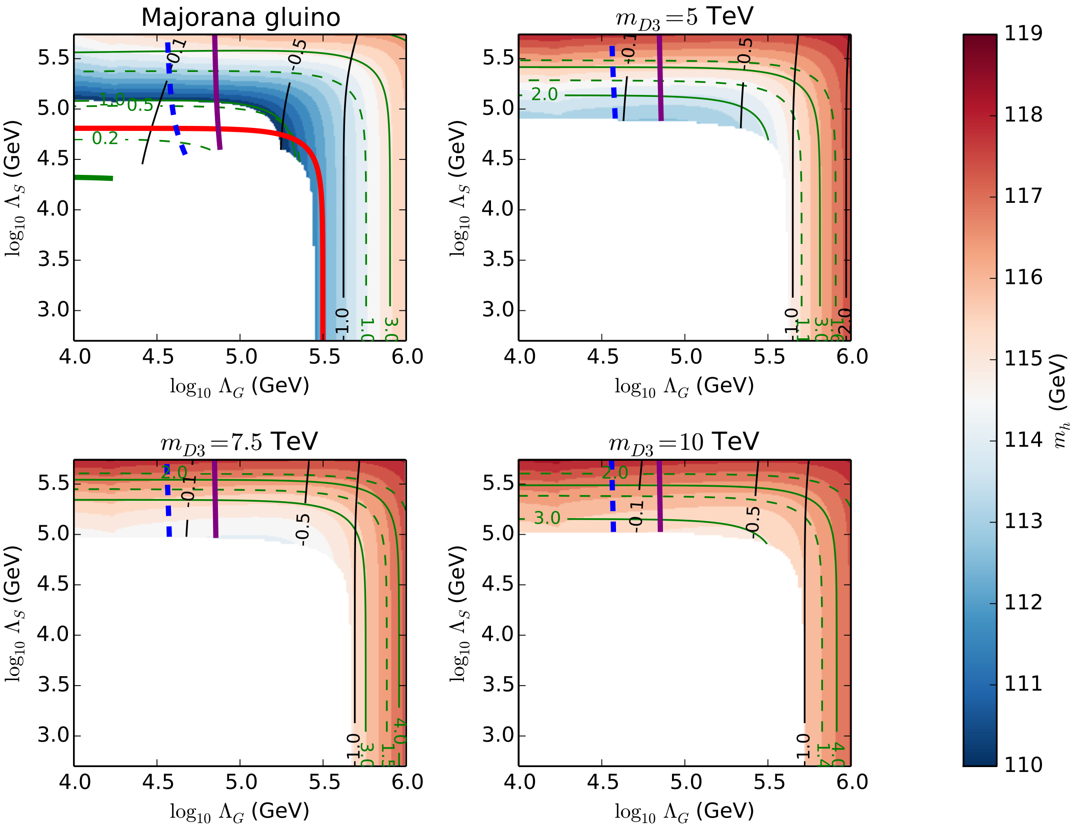


Figure B.5: Higgs sector parameters in CGGM with $t_\beta = 25$, $m_{\text{Mess}} = 10^7$ GeV and m_{D3} fixed as indicated. The gradient indicates the Higgs mass. The black dashed, green dashed and green solid lines are contours of $a_t(m_{\text{SUSY}})$, $\mu(m_{\text{SUSY}})$, and m_{SUSY} respectively. All contours unless otherwise specified are in TeV.

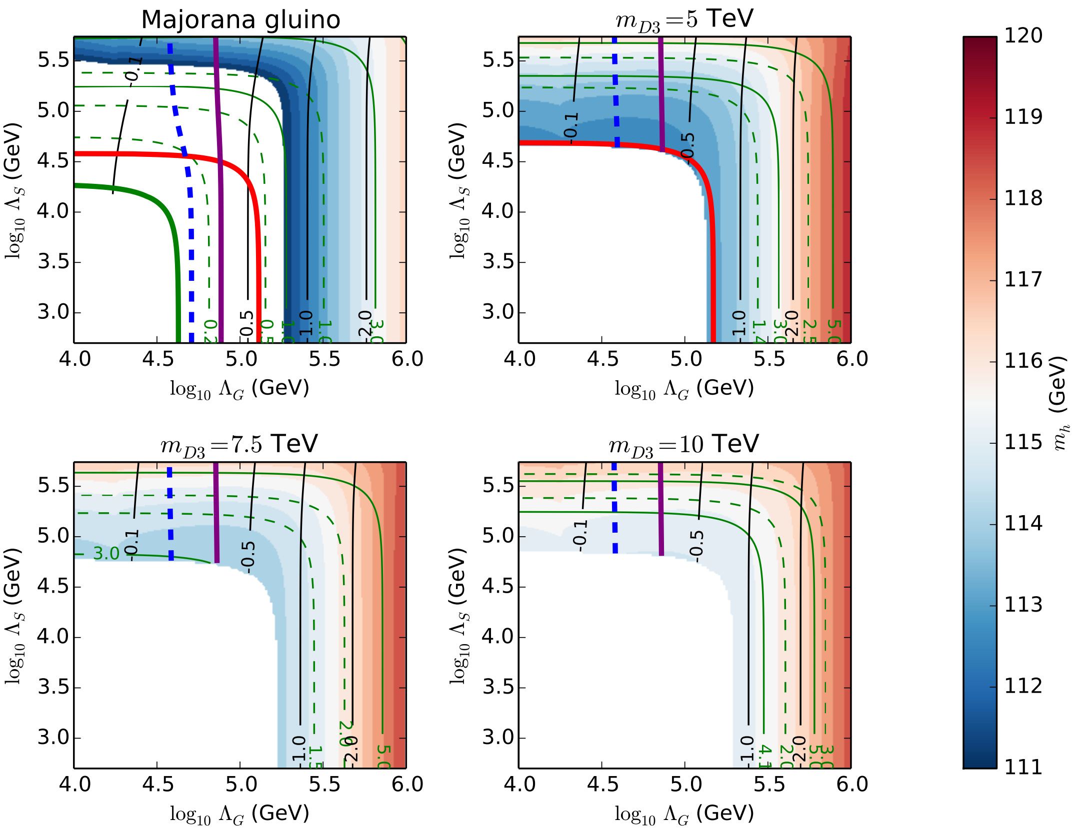


Figure B.6: Higgs sector parameters in CGGM with $t_\beta = 10$, $m_{\text{Mess}} = 10^{12}$ GeV and m_{D3} fixed as indicated. The gradient indicates the Higgs mass. The black dashed, green dashed and green solid lines are contours of $a_t(m_{\text{SUSY}})$, $\mu(m_{\text{SUSY}})$, and m_{SUSY} respectively. All contours unless otherwise specified are in TeV.

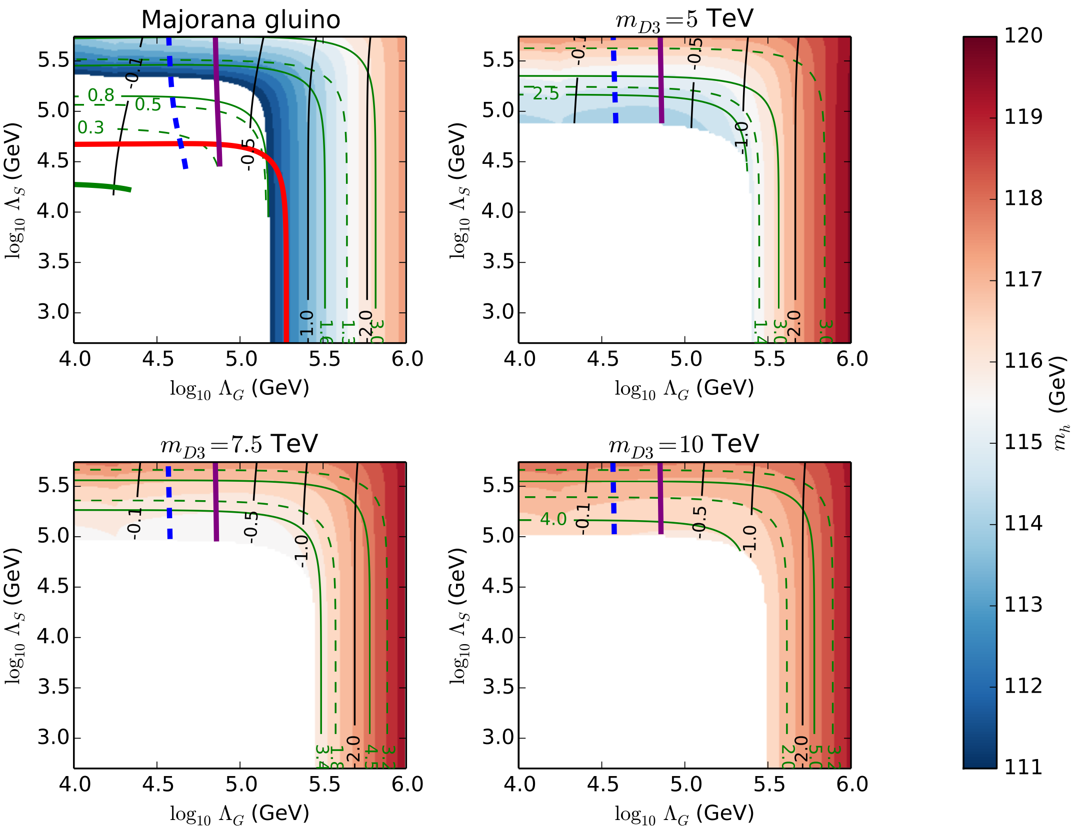


Figure B.7: Higgs sector parameters in CGGM with $t_\beta = 25$, $m_{\text{Mess}} = 10^{12}$ GeV and m_{D3} fixed as indicated. The gradient indicates the Higgs mass. The black dashed, green dashed and green solid lines are contours of $a_t(m_{\text{SUSY}})$, $\mu(m_{\text{SUSY}})$, and m_{SUSY} respectively. All contours unless otherwise specified are in TeV.

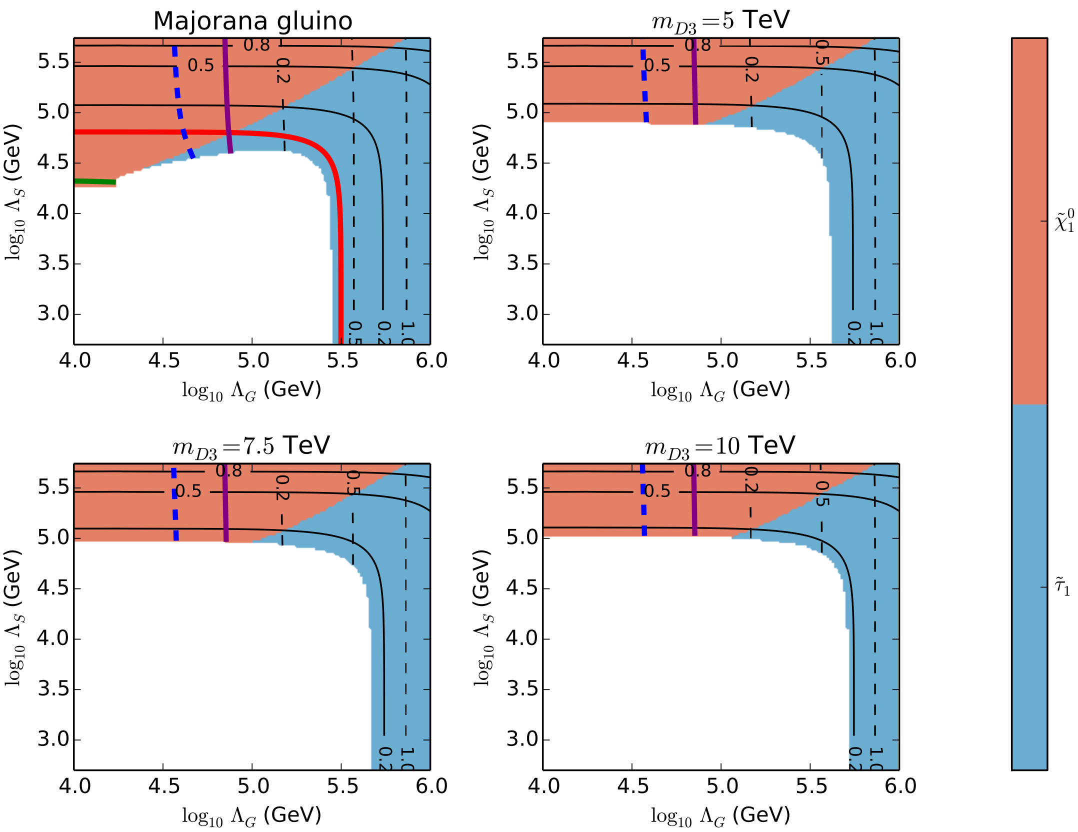


Figure B.8: LOSP species in CGGM with $t_\beta = 25$, $m_{\rm Mess} = 10^7$ GeV and m_{D3} fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

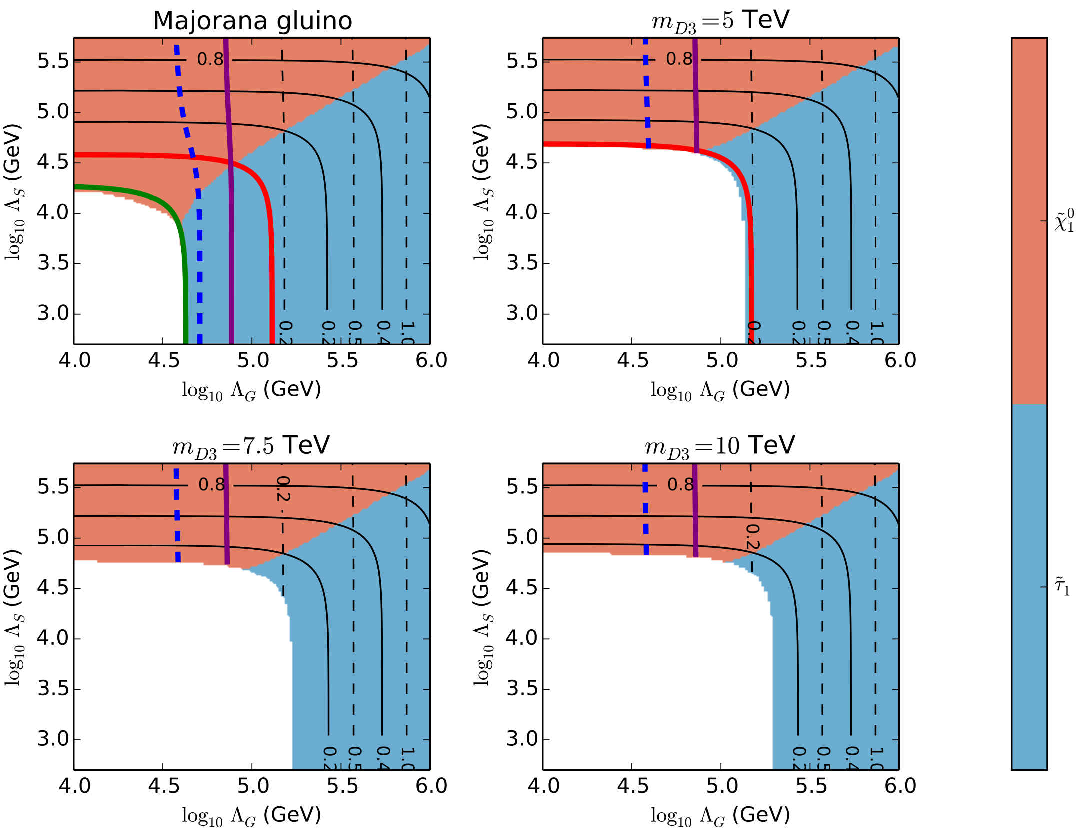


Figure B.9: LOSP species in CGGM with $t_\beta = 10$, $m_{\text{Mess}} = 10^{12}$ GeV and m_{D3} fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

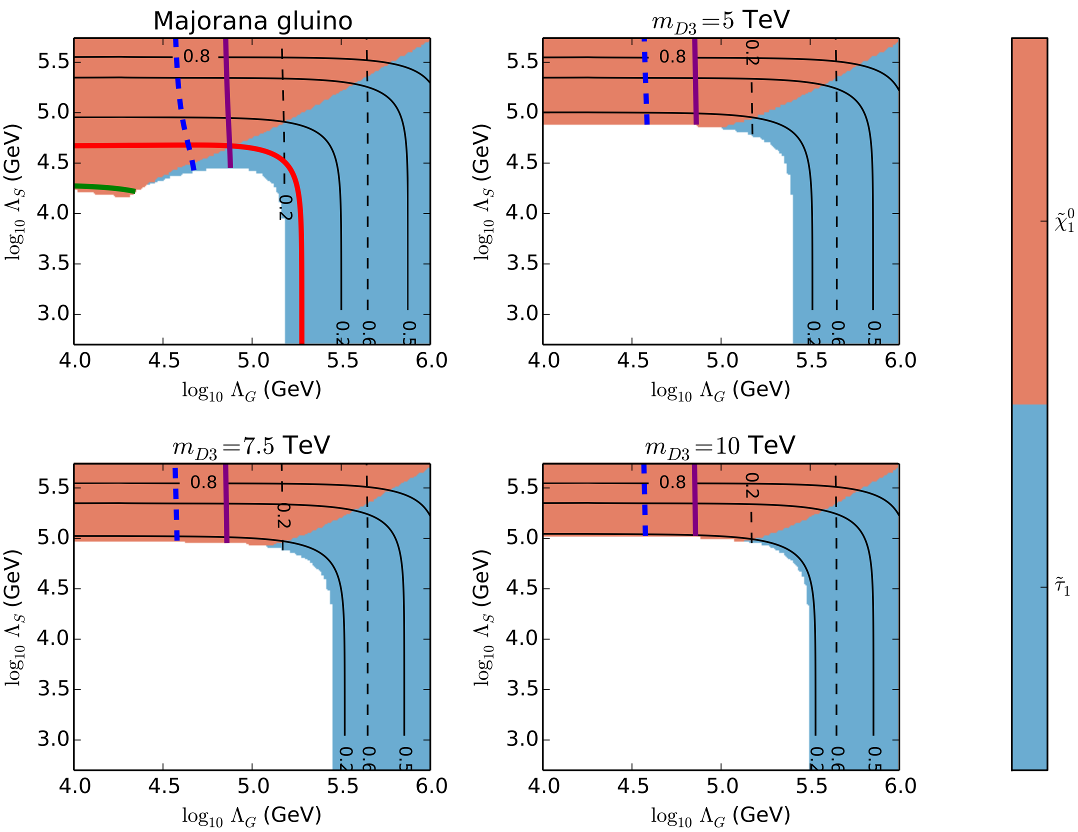


Figure B.10: LOSP species in CGGM with $t_\beta = 25$, $m_{\text{Mess}} = 10^{12}$ GeV and m_{D3} fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

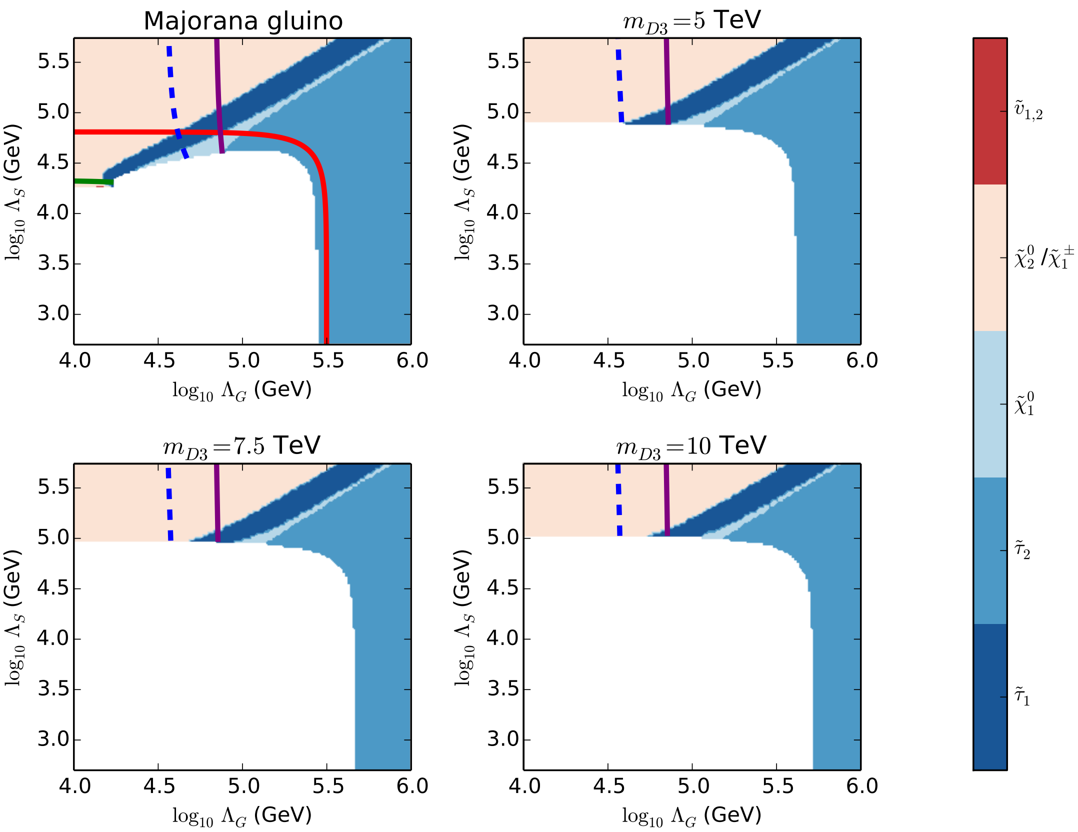


Figure B.11: NLOSP species in CGGM with $t_\beta = 25$, $m_{\rm Mess} = 10^7$ GeV and m_{D3} fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

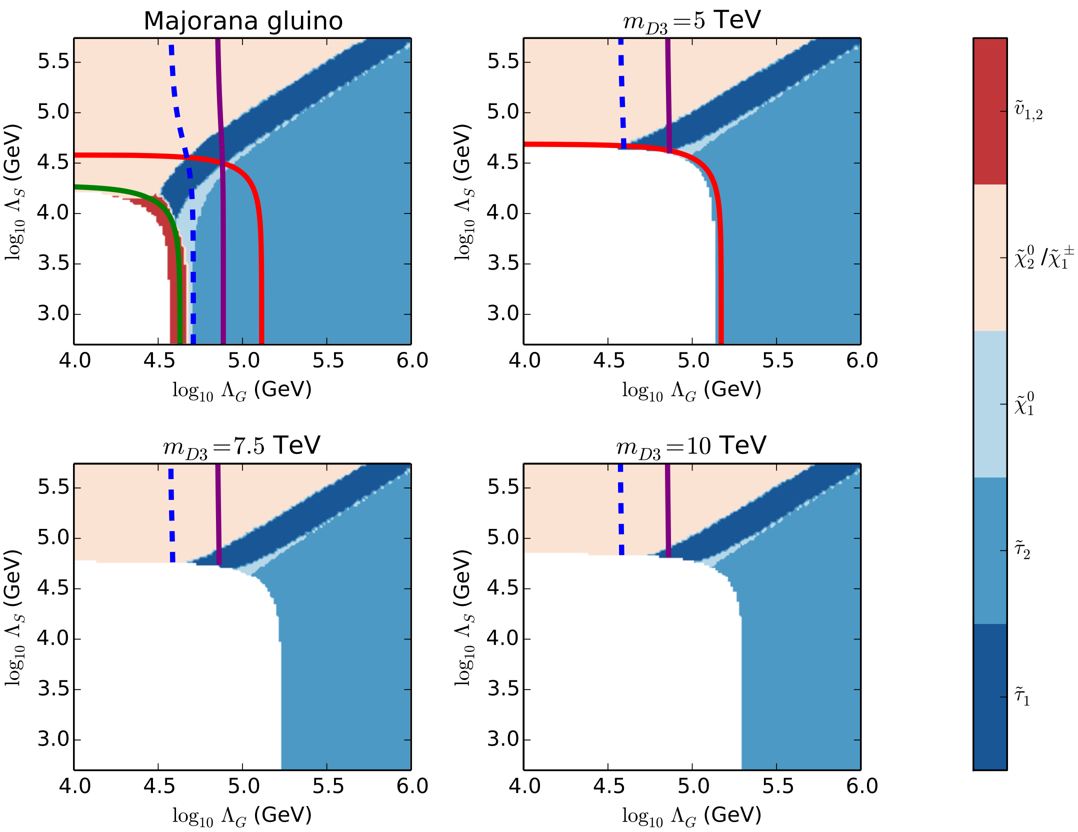


Figure B.12: NLOSP species in CGGM with $t_\beta = 10$, $m_{\rm Mess} = 10^{12}$ GeV and m_{D3} fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}^0_1}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

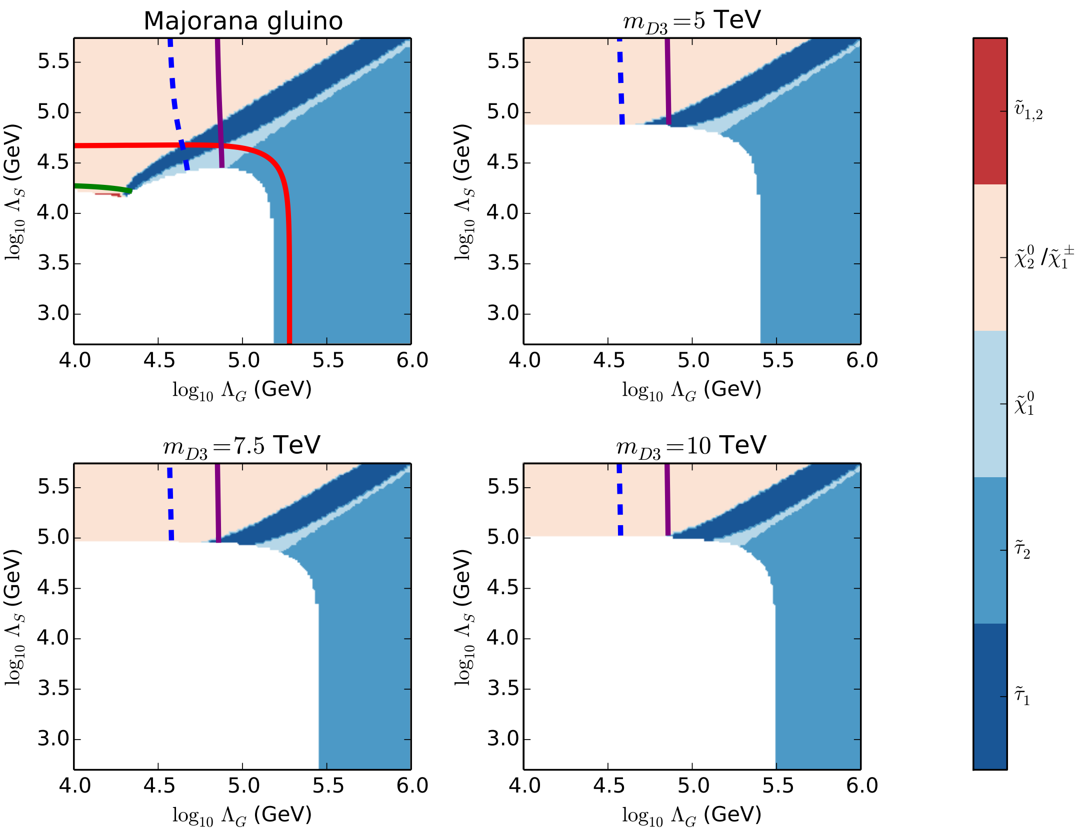


Figure B.13: NLOSP species in CGGM with $t_\beta = 25$, $m_{\text{Mess}} = 10^{12}$ GeV and m_{D3} fixed as indicated. The black dashed and black solid lines are contours of lightest neutralino mass $m_{\tilde{\chi}_1^0}$ and stau mass $m_{\tilde{\tau}}$ in TeV.

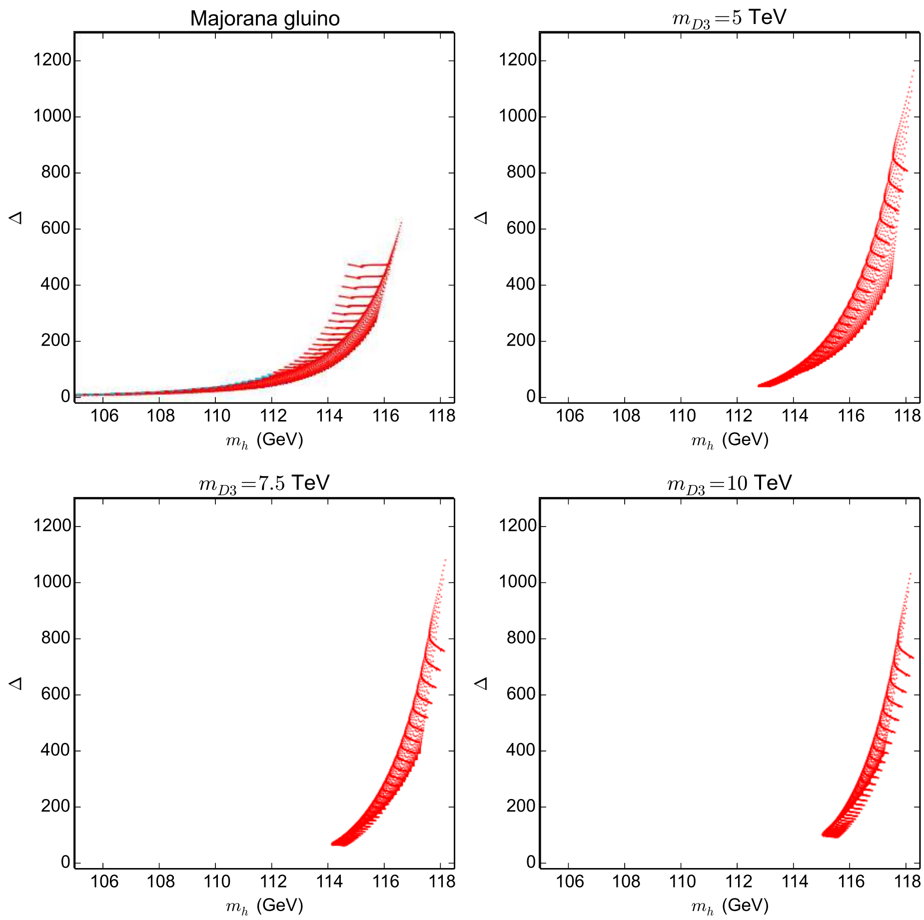


Figure B.14: Fine tuning in CGGM with $t_\beta = 25$, $m_{\rm Mess} = 10^7$ GeV and m_{D3} fixed as indicated. The red and blue regions correspond to μ and Λ_S as the dominant source of tuning.

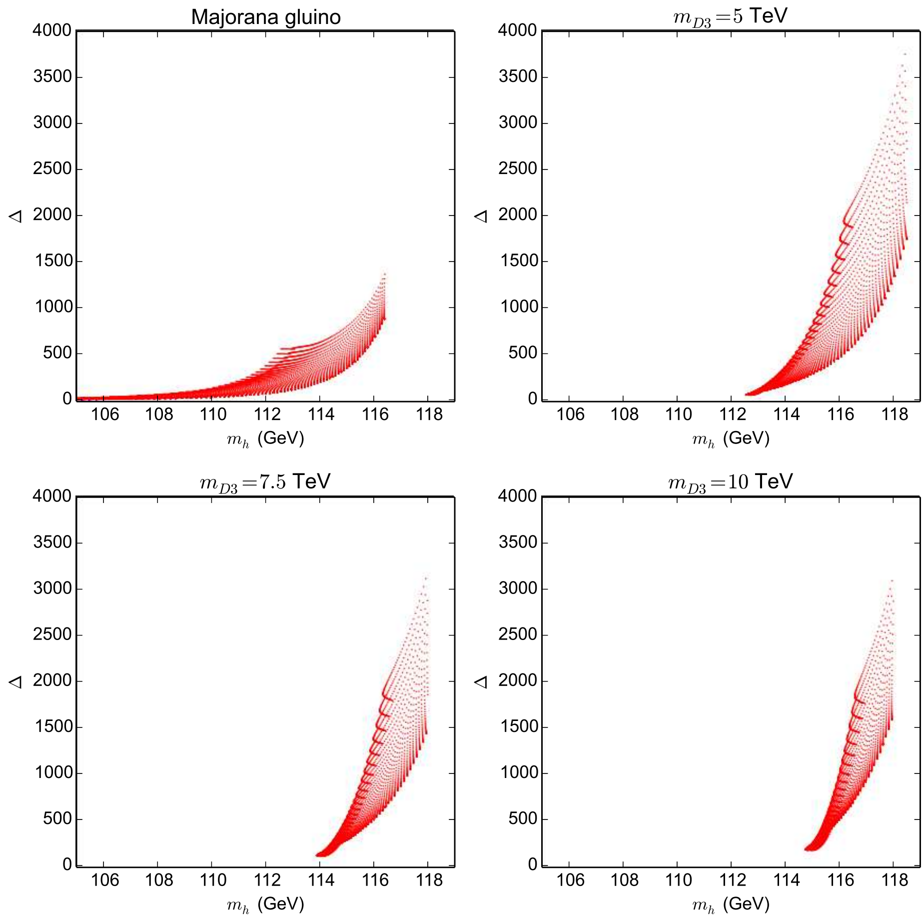


Figure B.15: Fine tuning in CGGM with $t_\beta = 10$, $m_{\rm Mess} = 10^{12}$ GeV and m_{D3} fixed as indicated. The dominant source of tuning is entirely from the μ parameter.

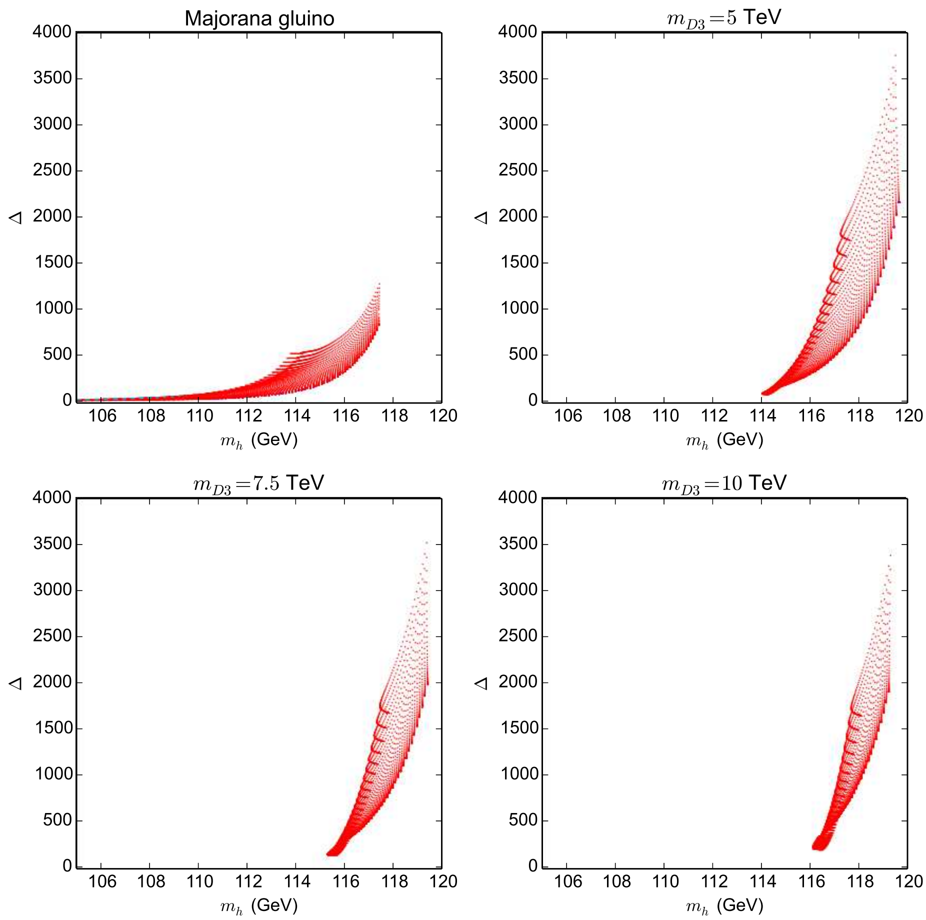


Figure B.16: Fine tuning in CGGM with $t_\beta = 25$, $m_{\text{Mess}} = 10^{12}$ GeV and m_{D3} fixed as indicated. The red and blue regions correspond to μ and Λ_S as the dominant source of tuning.

C

RGEs with Dirac gluino decoupling

C.1 Method and notation

These RGEs were calculated using a combination of `SARAH`, `PyR@TE` [245] and results from [287, 288]. We decouple the gluino and the sgluons at renormalisation scales μ below $\mu(m_{D\tilde{g}}) = m_{D\tilde{g}} \equiv \overline{m_{D\tilde{g}}}$. We therefore define

$$\theta_{\tilde{g}} = 1 \quad \text{if} \quad \mu \geq \overline{m_{D\tilde{g}}}, \qquad\qquad \theta_{\tilde{g}} = 0 \quad \text{if} \quad \mu < \overline{m_{D\tilde{g}}}. \tag{C.1}$$

Decoupling is achieved at two loop accuracy for the gauge coupling for all particles, whereas the decoupling for the remaining terms is correct to one loop for all particles and correct to two loop for the sgluons and right handed gluino.

C.2 Renormalisation group equations

C.2.1 SUSY parameters

Gauge couplings

$$\beta_{g_1}^{(1)} = \frac{33}{5} g_1^3, \tag{C.2}$$

$$\beta_{g_1}^{(2)} = \frac{1}{25} g_1^3 \Big[-130 \,\mathrm{tr}\left(y_\mathrm{u} y_\mathrm{u}^\dagger\right) + 135 g_2^2 + 199 g_1^2 + 220(3-\theta_{\tilde{\mathrm{g}}}) g_3^2 - 70 \,\mathrm{tr}\left(y_\mathrm{d} y_\mathrm{d}^\dagger\right) - 90 \,\mathrm{tr}\left(y_\mathrm{e} y_\mathrm{e}^\dagger\right)\Big], \tag{C.3}$$

$$\beta_{g_2}^{(1)} = g_2^3, \tag{C.4}$$

$$\beta_{g_2}^{(2)} = \frac{1}{5} g_2^3 \Big[-10 \,\mathrm{tr}\left(y_\mathrm{e} y_\mathrm{e}^\dagger\right) + 60(3-\theta_{\tilde{\mathrm{g}}}) g_3^2 + 125 g_2^2 - 30 \,\mathrm{tr}\left(y_\mathrm{d} y_\mathrm{d}^\dagger\right) - 30 \,\mathrm{tr}\left(y_\mathrm{u} y_\mathrm{u}^\dagger\right) + 9 g_1^2\Big], \tag{C.5}$$

$$\beta_{g_3}^{(1)} = -\frac{9}{2}(1-\theta_{\tilde{\mathrm{g}}}) g_3^3, \tag{C.6}$$

$$\beta_{g_3}^{(2)} = \frac{1}{5} g_3^3 \Big[11 g_1^2 - 20 \,\mathrm{tr}\left(y_\mathrm{d} y_\mathrm{d}^\dagger\right) - 20 \,\mathrm{tr}\left(y_\mathrm{u} y_\mathrm{u}^\dagger\right) + 5(39 + 29\,\theta_{\tilde{\mathrm{g}}}) g_3^2 + 45 g_2^2\Big]. \tag{C.7}$$

Yukawa couplings

$$\beta_{y_\mathrm{d}}^{(1)} = 3 y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} + y_\mathrm{d}\Big[-3 g_2^2 + 3\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) + \frac{8}{3}\Big(\theta_{\tilde{\mathrm{g}}} - 3\Big) g_3^2 - \frac{7}{15} g_1^2 + \mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big)\Big] + y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u}, \tag{C.8}$$

$$\begin{aligned}\beta_{y_\mathrm{d}}^{(2)} = {} & \frac{4}{5} g_1^2 y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} - 4 y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} - 2 y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} - 2 y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} \\ & + y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d}\Big[6 g_2^2 - 3\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 9\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) + \frac{4}{5} g_1^2\Big] - 3 y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) \\ & + y_\mathrm{d}\Big[\frac{287}{90} g_1^4 + g_1^2 g_2^2 + \frac{15}{2} g_2^4 + \frac{8}{9} g_1^2 g_3^2 + 8 g_2^2 g_3^2 + \frac{128}{9} g_3^4 - \frac{2}{5}\Big(g_1^2 - 40 g_3^2\Big)\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) \\ & + \frac{6}{5} g_1^2\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 9\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 3\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger\Big) - 3\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} y_\mathrm{e}^\dagger\Big)\Big],\end{aligned} \tag{C.9}$$

$$\beta_{y_\mathrm{e}}^{(1)} = 3 y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} + y_\mathrm{e}\Big[-3 g_2^2 + 3\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - \frac{9}{5} g_1^2 + \mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big)\Big], \tag{C.10}$$

$$\begin{aligned}\beta_{y_\mathrm{e}}^{(2)} = {} & -4 y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} + y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e}\Big[-3\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) + 6 g_2^2 - 9\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big)\Big] \\ & + \frac{1}{10} y_\mathrm{e}\Big\{ 3\Big[45 g_1^4 + 6 g_1^2 g_2^2 + 25 g_2^4 + 4 g_1^2\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 30\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger\Big) \\ & - 10\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger\Big) - 10\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} y_\mathrm{e}^\dagger\Big)\Big] - 4\Big(-40 g_3^2 + g_1^2\Big)\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big)\Big\},\end{aligned} \tag{C.11}$$

$$\beta_{y_\mathrm{u}}^{(1)} = 3 y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} - \frac{1}{15} y_\mathrm{u}\Big[13 g_1^2 + 45 g_2^2 - 45\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) + 40\Big(3 - \theta_{\tilde{\mathrm{g}}}\Big) g_3^2\Big] + y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d}, \tag{C.12}$$

$$\beta_{y_u}^{(2)} = \frac{2}{5}g_1^2 y_u y_u^\dagger y_u + 6g_2^2 y_u y_u^\dagger y_u - 2y_u y_d^\dagger y_d y_d^\dagger y_d - 2y_u y_d^\dagger y_d y_u^\dagger y_u$$
$$- 4y_u y_u^\dagger y_u y_u^\dagger y_u + y_u y_d^\dagger y_d \Big[- 3\,\mathrm{tr}\Big(y_d y_d^\dagger\Big) + \frac{2}{5}g_1^2 - \mathrm{tr}\Big(y_e y_e^\dagger\Big)\Big] - 9y_u y_u^\dagger y_u\,\mathrm{tr}\Big(y_u y_u^\dagger\Big)$$
$$+ y_u\Big[\frac{2743}{450}g_1^4 + g_1^2 g_2^2 + \frac{15}{2}g_2^4 + \frac{136}{45}g_1^2 g_3^2 + 8g_2^2 g_3^2 + \frac{128}{9}g_3^4$$
$$+ \frac{4}{5}\Big(20g_3^2 + g_1^2\Big)\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) - 3\,\mathrm{tr}\Big(y_d y_u^\dagger y_u y_d^\dagger\Big) - 9\,\mathrm{tr}\Big(y_u y_u^\dagger y_u y_u^\dagger\Big)\Big]. \tag{C.13}$$

SUSY masses

$$\beta_\mu^{(1)} = 3\mu\,\mathrm{tr}\Big(y_d y_d^\dagger\Big) - \frac{3}{5}\mu\Big(5g_2^2 - 5\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) + g_1^2\Big) + \mu\,\mathrm{tr}\Big(y_e y_e^\dagger\Big), \tag{C.14}$$
$$\beta_\mu^{(2)} = \frac{1}{50}\mu\Big[207g_1^4 + 90g_1^2 g_2^2 + 375g_2^4 - 20\Big(- 40g_3^2 + g_1^2\Big)\,\mathrm{tr}\Big(y_d y_d^\dagger\Big) + 60g_1^2\,\mathrm{tr}\Big(y_e y_e^\dagger\Big)$$
$$+ 800g_3^2\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) - 450\,\mathrm{tr}\Big(y_d y_d^\dagger y_d y_d^\dagger\Big) - 300\,\mathrm{tr}\Big(y_d y_u^\dagger y_u y_d^\dagger\Big) - 150\,\mathrm{tr}\Big(y_e y_e^\dagger y_e y_e^\dagger\Big)$$
$$+ 40g_1^2\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) - 450\,\mathrm{tr}\Big(y_u y_u^\dagger y_u y_u^\dagger\Big)\Big]. \tag{C.15}$$

C.2.2 SUSY breaking parameters

Majorana gaugino masses

$$\beta_{M_1}^{(1)} = \frac{66}{5}g_1^2 M_1, \tag{C.16}$$
$$\beta_{M_1}^{(2)} = \frac{2}{25}g_1^2\Big[398g_1^2 M_1 + 135g_2^2 M_1 + 440g_3^2 M_1 + 440g_3^2 M_3\,\theta_{\widetilde{g}} + 135g_2^2 M_2$$
$$- 70M_1\,\mathrm{tr}\Big(y_d y_d^\dagger\Big)$$
$$- 90M_1\,\mathrm{tr}\Big(y_e y_e^\dagger\Big) - 130M_1\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) + 70\,\mathrm{tr}\Big(y_d^\dagger a_d\Big) + 90\,\mathrm{tr}\Big(y_e^\dagger a_e\Big)$$
$$+ 130\,\mathrm{tr}\Big(y_u^\dagger a_u\Big)\Big], \tag{C.17}$$
$$\beta_{M_2}^{(1)} = 2g_2^2 M_2, \tag{C.18}$$
$$\beta_{M_2}^{(2)} = \frac{2}{5}g_2^2\Big[9g_1^2 M_1 + 120g_3^2 M_3\,\theta_{\widetilde{g}} + 9g_1^2 M_2 + 250g_2^2 M_2 + 120g_3^2 M_2 - 30M_2\,\mathrm{tr}\Big(y_d y_d^\dagger\Big)$$
$$- 10M_2\,\mathrm{tr}\Big(y_e y_e^\dagger\Big) - 30M_2\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) + 30\,\mathrm{tr}\Big(y_d^\dagger a_d\Big) + 10\,\mathrm{tr}\Big(y_e^\dagger a_e\Big)$$
$$+ 30\,\mathrm{tr}\Big(y_u^\dagger a_u\Big)\Big], \tag{C.19}$$
$$\beta_{M_3}^{(1)} = 0, \tag{C.20}$$
$$\beta_{M_3}^{(2)} = \frac{2}{5}g_3^2\Big[11g_1^2 M_1 + 11g_1^2 M_3 + 45g_2^2 M_3 + 680g_3^2 M_3 + 45g_2^2 M_2 - 20M_3\,\mathrm{tr}\Big(y_d y_d^\dagger\Big)$$

$$- 20M_3 \operatorname{tr}\Big(y_u y_u^\dagger\Big) + 20 \operatorname{tr}\Big(y_d^\dagger a_d\Big) + 20 \operatorname{tr}\Big(y_u^\dagger a_u\Big)\Big]\theta_{\widetilde{g}}. \tag{C.21}$$

Dirac gluino mass

$$\beta^{(1)}_{m_{D\widetilde{g}}} = -6\, g_3^2 m_{D\widetilde{g}}\, \theta_{\widetilde{g}}, \tag{C.22}$$

$$\beta^{(2)}_{m_{D\widetilde{g}}} = \frac{1}{5}\, g_3^2 m_{D\widetilde{g}} \Big[11g_1^2 + 45g_2^2 + 520g_3^2 - 20 \operatorname{tr}\Big(y_d y_d^\dagger\Big) - 20 \operatorname{tr}\Big(y_u y_u^\dagger\Big)\Big]\theta_{\widetilde{g}}. \tag{C.23}$$

Trilinear Soft-Breaking Parameters

$$\begin{aligned}
\beta^{(1)}_{a_d} = {}& 4y_d y_d^\dagger a_d + 2y_d y_u^\dagger a_u + 5a_d y_d^\dagger y_d + a_d y_u^\dagger y_u - \frac{7}{15}g_1^2 a_d - 3g_2^2 a_d + \frac{16}{3}\Big(\theta_{\widetilde{g}} - 2\Big)g_3^2 a_d \\
&+ 3a_d \operatorname{tr}\Big(y_d y_d^\dagger\Big) + a_d \operatorname{tr}\Big(y_e y_e^\dagger\Big) + y_d \Big[2 \operatorname{tr}\Big(y_e^\dagger a_e\Big) + 6g_2^2 M_2 + 6 \operatorname{tr}\Big(y_d^\dagger a_d\Big) \\
&+ \frac{14}{15}g_1^2 M_1 + \frac{32}{3}g_3^2 M_3\, \theta_{\widetilde{g}}\Big],
\end{aligned} \tag{C.24}$$

$$\begin{aligned}
\beta^{(2)}_{a_d} = {}& \frac{6}{5}g_1^2 y_d y_d^\dagger a_d + 6g_2^2 y_d y_d^\dagger a_d - \frac{8}{5}g_1^2 M_1 y_d y_u^\dagger y_u + \frac{8}{5}g_1^2 y_d y_u^\dagger a_u \\
&+ \frac{6}{5}g_1^2 a_d y_d^\dagger y_d + 12g_2^2 a_d y_d^\dagger y_d + \frac{4}{5}g_1^2 a_d y_u^\dagger y_u - 6y_d y_d^\dagger y_d y_d^\dagger a_d \\
&- 8y_d y_d^\dagger a_d y_d^\dagger y_d - 2y_d y_u^\dagger y_u y_d^\dagger a_d - 4y_d y_u^\dagger y_u y_u^\dagger a_u - 4y_d y_u^\dagger a_u y_d^\dagger y_d \\
&- 4y_d y_u^\dagger a_u y_u^\dagger y_u - 6a_d y_d^\dagger y_d y_d^\dagger y_d - 4a_d y_u^\dagger y_u y_d^\dagger y_d - 2a_d y_u^\dagger y_u y_u^\dagger y_u \\
&+ \frac{287}{90}g_1^4 a_d + g_1^2 g_2^2 a_d + \frac{15}{2}g_2^4 a_d + \frac{8}{9}g_1^2 g_3^2 a_d + 8g_2^2 g_3^2 a_d + \frac{128}{9}g_3^4 a_d \\
&- 12y_d y_d^\dagger a_d \operatorname{tr}\Big(y_d y_d^\dagger\Big) - 15a_d y_d^\dagger y_d \operatorname{tr}\Big(y_d y_d^\dagger\Big) - \frac{2}{5}g_1^2 a_d \operatorname{tr}\Big(y_d y_d^\dagger\Big) \\
&+ 16g_3^2 a_d \operatorname{tr}\Big(y_d y_d^\dagger\Big) - 4y_d y_d^\dagger a_d \operatorname{tr}\Big(y_e y_e^\dagger\Big) - 5a_d y_d^\dagger y_d \operatorname{tr}\Big(y_e y_e^\dagger\Big) \\
&+ \frac{6}{5}g_1^2 a_d \operatorname{tr}\Big(y_e y_e^\dagger\Big) - 6y_d y_u^\dagger a_u \operatorname{tr}\Big(y_u y_u^\dagger\Big) - 3a_d y_u^\dagger y_u \operatorname{tr}\Big(y_u y_u^\dagger\Big) \\
&- \frac{2}{5}y_d y_d^\dagger y_d \Big[15 \operatorname{tr}\Big(y_e^\dagger a_e\Big) + 30g_2^2 M_2 + 45 \operatorname{tr}\Big(y_d^\dagger a_d\Big) + 4g_1^2 M_1\Big] - 6y_d y_u^\dagger y_u \operatorname{tr}\Big(y_u^\dagger a_u\Big) \\
&- 9a_d \operatorname{tr}\Big(y_d y_d^\dagger y_d y_d^\dagger\Big) - 3a_d \operatorname{tr}\Big(y_d y_u^\dagger y_u y_d^\dagger\Big) - 3a_d \operatorname{tr}\Big(y_e y_e^\dagger y_e y_e^\dagger\Big) \\
&- \frac{2}{45}y_d \Big[287g_1^4 M_1 + 45g_1^2 g_2^2 M_1 + 40g_1^2 g_3^2 M_1 + 40g_1^2 g_3^2 M_3\, \theta_{\widetilde{g}} + 360g_2^2 g_3^2 M_3\, \theta_{\widetilde{g}} \\
&+ 1280g_3^4 M_3\, \theta_{\widetilde{g}} + 45g_1^2 g_2^2 M_2 + 675g_2^4 M_2 + 360g_2^2 g_3^2 M_2 \\
&+ 18\Big(40g_3^2 M_3\, \theta_{\widetilde{g}} - g_1^2 M_1\Big) \operatorname{tr}\Big(y_d y_d^\dagger\Big) + 54g_1^2 M_1 \operatorname{tr}\Big(y_e y_e^\dagger\Big) + 18g_1^2 \operatorname{tr}\Big(y_d^\dagger a_d\Big) \\
&- 720g_3^2 \operatorname{tr}\Big(y_d^\dagger a_d\Big) - 54g_1^2 \operatorname{tr}\Big(y_e^\dagger a_e\Big) + 810 \operatorname{tr}\Big(y_d y_d^\dagger a_d y_d^\dagger\Big)
\end{aligned}$$

$$+ 135\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{u}^\dagger a_\mathrm{u} y_\mathrm{d}^\dagger\Big) + 270\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e} y_\mathrm{e}^\dagger\Big) + 135\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d} y_\mathrm{u}^\dagger\Big)\Big], \tag{C.25}$$

$$\beta_{a_\mathrm{e}}^{(1)} = 4 y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e} + 5 a_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} - \frac{9}{5} g_1^2 a_\mathrm{e} - 3 g_2^2 a_\mathrm{e} + 3 a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) + a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$+ y_\mathrm{e}\Big[2\,\mathrm{tr}\Big(y_\mathrm{e}^\dagger a_\mathrm{e}\Big) + 6 g_2^2 M_2 + 6\,\mathrm{tr}\Big(y_\mathrm{d}^\dagger a_\mathrm{d}\Big) + \frac{18}{5} g_1^2 M_1\Big], \tag{C.26}$$

$$\beta_{a_\mathrm{e}}^{(2)} = +\frac{6}{5} g_1^2 y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e} + 6 g_2^2 y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e} - \frac{6}{5} g_1^2 a_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} + 12 g_2^2 a_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e}$$
$$- 6 y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e} - 8 y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} - 6 a_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} + \frac{27}{2} g_1^4 a_\mathrm{e} + \frac{9}{5} g_1^2 g_2^2 a_\mathrm{e} + \frac{15}{2} g_2^4 a_\mathrm{e}$$
$$- 12 y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 15 a_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - \frac{2}{5} g_1^2 a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big)$$
$$+ 16 g_3^2 a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 4 y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 5 a_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$+ \frac{6}{5} g_1^2 a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 6 y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e}\Big[2 g_2^2 M_2 + 3\,\mathrm{tr}\Big(y_\mathrm{d}^\dagger a_\mathrm{d}\Big) + \mathrm{tr}\Big(y_\mathrm{e}^\dagger a_\mathrm{e}\Big)\Big]$$
$$- 9 a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 3 a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger\Big) - 3 a_\mathrm{e}\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger y_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$- \frac{2}{5} y_\mathrm{e}\Big[135 g_1^4 M_1 + 9 g_1^2 g_2^2 M_1 + 9 g_1^2 g_2^2 M_2 - \Big(2 g_1^2 M_1 - 80 g_3^2 M_3\,\theta_{\widetilde{\mathrm{g}}}\Big)\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big)$$
$$+ 75 g_2^4 M_2 + 6 g_1^2 M_1\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) + 2 g_1^2\,\mathrm{tr}\Big(y_\mathrm{d}^\dagger a_\mathrm{d}\Big) - 80 g_3^2\,\mathrm{tr}\Big(y_\mathrm{d}^\dagger a_\mathrm{d}\Big) - 6 g_1^2\,\mathrm{tr}\Big(y_\mathrm{e}^\dagger a_\mathrm{e}\Big)$$
$$+ 90\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger a_\mathrm{d} y_\mathrm{d}^\dagger\Big) + 15\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{u}^\dagger a_\mathrm{u} y_\mathrm{d}^\dagger\Big) + 30\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger a_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$+ 15\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d} y_\mathrm{u}^\dagger\Big)\Big], \tag{C.27}$$

$$\beta_{a_\mathrm{u}}^{(1)} = 2 y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d} + 4 y_\mathrm{u} y_\mathrm{u}^\dagger a_\mathrm{u} + a_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} + 5 a_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} - \frac{13}{15} g_1^2 a_\mathrm{u} - 3 g_2^2 a_\mathrm{u} + \frac{16}{3}\Big(\theta_{\widetilde{\mathrm{g}}} - 2\Big) g_3^2 a_\mathrm{u}$$
$$+ 3 a_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) + y_\mathrm{u}\Big[6 g_2^2 M_2 + 6\,\mathrm{tr}\Big(y_\mathrm{u}^\dagger a_\mathrm{u}\Big) + \frac{26}{15} g_1^2 M_1 + \frac{32}{3} g_3^2 M_3\,\theta_{\widetilde{\mathrm{g}}}\Big], \tag{C.28}$$

$$\beta_{a_\mathrm{u}}^{(2)} = \frac{4}{5} g_1^2 y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d} - \frac{4}{5} g_1^2 M_1 y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} - 12 g_2^2 M_2 y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} + \frac{6}{5} g_1^2 y_\mathrm{u} y_\mathrm{u}^\dagger a_\mathrm{u}$$
$$+ 6 g_2^2 y_\mathrm{u} y_\mathrm{u}^\dagger a_\mathrm{u} + \frac{2}{5} g_1^2 a_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} + 12 g_2^2 a_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} - 4 y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger a_\mathrm{d}$$
$$- 2 y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger a_\mathrm{u} - 4 y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} - 4 y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} - 6 y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger a_\mathrm{u}$$
$$- 8 y_\mathrm{u} y_\mathrm{u}^\dagger a_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} - 2 a_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} - 4 a_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} - 6 a_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} + \frac{2743}{450} g_1^4 a_\mathrm{u}$$
$$+ g_1^2 g_2^2 a_\mathrm{u} + \frac{15}{2} g_2^4 a_\mathrm{u} + \frac{136}{45} g_1^2 g_3^2 a_\mathrm{u} + 8 g_2^2 g_3^2 a_\mathrm{u} + \frac{128}{9} g_3^4 a_\mathrm{u} - 6 y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big)$$
$$- 3 a_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 2 y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d}\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - a_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d}\,\mathrm{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$- 12 y_\mathrm{u} y_\mathrm{u}^\dagger a_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) - 15 a_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) + \frac{4}{5} g_1^2 a_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big)$$
$$+ 16 g_3^2 a_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) - \frac{2}{5} y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d}\Big[15\,\mathrm{tr}\Big(y_\mathrm{d}^\dagger a_\mathrm{d}\Big) + 2 g_1^2 M_1 + 5\,\mathrm{tr}\Big(y_\mathrm{e}^\dagger a_\mathrm{e}\Big)\Big]$$
$$- 18 y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u}^\dagger a_\mathrm{u}\Big) - 3 a_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger\Big) - 9 a_\mathrm{u}\,\mathrm{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger\Big)$$
$$- \frac{2}{225} y_\mathrm{u}\Big\{2743 g_1^4 M_1 + 225 g_1^2 g_2^2 M_1 + 680 g_1^2 g_3^2 M_1 + 680 g_1^2 g_3^2 M_3\,\theta_{\widetilde{\mathrm{g}}}$$

$$+ 1800 g_2^2 g_3^2 M_3\,\theta_{\widetilde{g}} + 6400 g_3^4 M_3\,\theta_{\widetilde{g}} + 225 g_1^2 g_2^2 M_2 + 3375 g_2^4 M_2 + 1800 g_2^2 g_3^2 M_2$$
$$- 180\Big(20 g_3^2 + g_1^2\Big)\,\mathrm{tr}\Big(y_u^\dagger a_u\Big) + 675\,\mathrm{tr}\Big(y_d y_u^\dagger a_u y_d^\dagger\Big) + 675\,\mathrm{tr}\Big(y_u y_d^\dagger a_d y_u^\dagger\Big)$$
$$+ 4050\,\mathrm{tr}\Big[y_u y_u^\dagger a_u y_u^\dagger + 180\Big(20 g_3^2 M_3\,\theta_{\widetilde{g}} + g_1^2 M_1\Big)\,\mathrm{tr}\Big(y_u y_u^\dagger\Big)\Big]\Big\}. \tag{C.29}$$

Bilinear Soft-Breaking Parameters

$$\beta_{b_\mu}^{(1)} = \frac{6}{5} g_1^2 M_1 \mu + 6 g_2^2 M_2 \mu + b_\mu\Big[3\,\mathrm{tr}\Big(y_d y_d^\dagger\Big) - 3 g_2^2 + 3\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) - \frac{3}{5} g_1^2 + \mathrm{tr}\Big(y_e y_e^\dagger\Big)\Big]$$
$$+ 6\mu\,\mathrm{tr}\Big(y_d^\dagger a_d\Big) + 2\mu\,\mathrm{tr}\Big(y_e^\dagger a_e\Big) + 6\mu\,\mathrm{tr}\Big(y_u^\dagger a_u\Big), \tag{C.30}$$

$$\beta_{b_\mu}^{(2)} = b_\mu\Big[\frac{207}{50} g_1^4 + \frac{9}{5} g_1^2 g_2^2 + \frac{15}{2} g_2^4 + \frac{2}{5}\Big(g_1^2 - 40 g_3^2\Big)\,\mathrm{tr}\Big(y_d y_d^\dagger\Big) + \frac{6}{5} g_1^2\,\mathrm{tr}\Big(y_e y_e^\dagger\Big)$$
$$+ \frac{4}{5} g_1^2\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) + 16 g_3^2\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) - 9\,\mathrm{tr}\Big(y_d y_d^\dagger y_d y_d^\dagger\Big) - 6\,\mathrm{tr}\Big(y_d y_u^\dagger y_u y_d^\dagger\Big)$$
$$- 3\,\mathrm{tr}\Big(y_e y_e^\dagger y_e y_e^\dagger\Big) - 9\,\mathrm{tr}\Big(y_u y_u^\dagger y_u y_u^\dagger\Big)\Big] - \frac{2}{25}\mu\Big[207 g_1^4 M_1$$
$$+ 45 g_1^2 g_2^2 M_1 + 45 g_1^2 g_2^2 M_2 + 375 g_2^4 M_2 - 20 g_1^2\,\mathrm{tr}\Big(y_u^\dagger a_u\Big)$$
$$+ 30 g_1^2 M_1\,\mathrm{tr}\Big(y_e y_e^\dagger\Big) + 10\Big(g_1^2 M_1 - 40 g_3^2 M_3\,\theta_{\widetilde{g}}\Big)\,\mathrm{tr}\Big(y_d y_d^\dagger\Big) + 20 g_1^2 M_1\,\mathrm{tr}\Big(y_u y_u^\dagger\Big)$$
$$+ 400 g_3^2 M_3\,\theta_{\widetilde{g}}\,\mathrm{tr}\Big(y_u y_u^\dagger\Big) + 10 g_1^2\,\mathrm{tr}\Big(y_d^\dagger a_d\Big) - 400 g_3^2\,\mathrm{tr}\Big(y_d^\dagger a_d\Big) - 30 g_1^2\,\mathrm{tr}\Big(y_e^\dagger a_e\Big)$$
$$+ 450\,\mathrm{tr}\Big(y_d y_d^\dagger a_d y_d^\dagger\Big) + 150\,\mathrm{tr}\Big(y_d y_u^\dagger a_u y_d^\dagger\Big) + 150\,\mathrm{tr}\Big(y_e y_e^\dagger a_e y_e^\dagger\Big) + 150\,\mathrm{tr}\Big(y_u y_d^\dagger a_d y_u^\dagger\Big)$$
$$+ 450\,\mathrm{tr}\Big(y_u y_u^\dagger a_u y_u^\dagger\Big) - 400 g_3^2\,\mathrm{tr}\Big(y_u^\dagger a_u\Big)\Big], \tag{C.31}$$

$$\beta_{b_{\widetilde{g}}}^{(1)} = -12 g_3^2 b_{\widetilde{g}}, \tag{C.32}$$

$$\beta_{b_{\widetilde{g}}}^{(2)} = 72 g_3^4 b_{\widetilde{g}}. \tag{C.33}$$

Soft-Breaking Scalar Masses

$$\beta_{m_q^2}^{(1)} = -\frac{2}{15} g_1^2 |M_1|^2 - \frac{32}{3} g_3^2 |M_3|^2\,\theta_{\widetilde{g}} - 6 g_2^2 |M_2|^2 + 2 m_{H_d}^2 y_d^\dagger y_d + 2 m_{H_u}^2 y_u^\dagger y_u + 2 a_d^\dagger a_d$$
$$+ 2 a_u^\dagger a_u + m_q^2 y_d^\dagger y_d + m_q^2 y_u^\dagger y_u + 2 y_d^\dagger m_d^2 y_d + y_d^\dagger y_d m_q^2 + 2 y_u^\dagger m_u^2 y_u$$
$$+ y_u^\dagger y_u m_q^2 + \frac{1}{\sqrt{15}} g_1 \sigma_{1,1}, \tag{C.34}$$

$$\beta_{m_q^2}^{(2)} = \frac{2}{5} g_1^2 g_2^2 |M_2|^2 + 33 g_2^4 |M_2|^2 + 32 g_2^2 g_3^2 |M_2|^2 + \frac{1}{5} g_1^2 g_2^2 M_1 M_2^*$$

$$+ \frac{16}{45} g_3^2 \Big\{ 15 \Big[10 g_3^2 M_3\, \theta_{\tilde{\mathrm{g}}} + 3 g_2^2 \Big(2 M_3\, \theta_{\tilde{\mathrm{g}}} + M_2 \Big) \Big] + g_1^2 \Big[2 M_3\, \theta_{\tilde{\mathrm{g}}} + M_1 \Big] \Big\} M_3^*\, \theta_{\tilde{\mathrm{g}}}$$
$$+ 16 g_2^2 g_3^2 M_3 M_2^*\, \theta_{\tilde{\mathrm{g}}} + \frac{4}{5} g_1^2 m_{\mathrm{H}_d}^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} + \frac{8}{5} g_1^2 m_{\mathrm{H}_u}^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}}$$
$$+ \frac{1}{225} g_1^2 M_1^* \Big[\Big\{ 5 \Big[16 g_3^2 \Big(2M_1 + M_3\, \theta_{\tilde{\mathrm{g}}} \Big) + 9 g_2^2 \Big(2M_1 + M_2 \Big) \Big] + 597 g_1^2 M_1 \Big\}$$
$$+ 180 \Big\{ 2 M_1 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} - 2 y_{\mathrm{u}}^\dagger a_{\mathrm{u}} + 4 M_1 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} - y_{\mathrm{d}}^\dagger a_{\mathrm{d}} \Big\} \Big]$$
$$- \frac{4}{5} g_1^2 M_1 a_{\mathrm{d}}^\dagger y_{\mathrm{d}} + \frac{4}{5} g_1^2 a_{\mathrm{d}}^\dagger a_{\mathrm{d}} - \frac{8}{5} g_1^2 M_1 a_{\mathrm{u}}^\dagger y_{\mathrm{u}} + \frac{8}{5} g_1^2 a_{\mathrm{u}}^\dagger a_{\mathrm{u}}$$
$$+ \frac{2}{5} g_1^2 m_q^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} + \frac{4}{5} g_1^2 m_q^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} + \frac{4}{5} g_1^2 y_{\mathrm{d}}^\dagger m_{\mathrm{d}}^2 y_{\mathrm{d}} + \frac{2}{5} g_1^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} m_q^2$$
$$+ \frac{8}{5} g_1^2 y_{\mathrm{u}}^\dagger m_{\mathrm{u}}^2 y_{\mathrm{u}} + \frac{4}{5} g_1^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} m_q^2 - 8 m_{\mathrm{H}_d}^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} y_{\mathrm{d}}^\dagger y_{\mathrm{d}} - 4 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} a_{\mathrm{d}}^\dagger a_{\mathrm{d}}$$
$$- 4 y_{\mathrm{d}}^\dagger a_{\mathrm{d}} a_{\mathrm{d}}^\dagger y_{\mathrm{d}} - 8 m_{\mathrm{H}_u}^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} y_{\mathrm{u}}^\dagger y_{\mathrm{u}} - 4 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} a_{\mathrm{u}}^\dagger a_{\mathrm{u}} - 4 y_{\mathrm{u}}^\dagger a_{\mathrm{u}} a_{\mathrm{u}}^\dagger y_{\mathrm{u}}$$
$$- 4 a_{\mathrm{d}}^\dagger y_{\mathrm{d}} y_{\mathrm{d}}^\dagger a_{\mathrm{d}} - 4 a_{\mathrm{d}}^\dagger a_{\mathrm{d}} y_{\mathrm{d}}^\dagger y_{\mathrm{d}} - 4 a_{\mathrm{u}}^\dagger y_{\mathrm{u}} y_{\mathrm{u}}^\dagger a_{\mathrm{u}} - 4 a_{\mathrm{u}}^\dagger a_{\mathrm{u}} y_{\mathrm{u}}^\dagger y_{\mathrm{u}} + \frac{4}{\sqrt{15}} g_1 \sigma_{3,1}$$
$$- 2 m_q^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} y_{\mathrm{d}}^\dagger y_{\mathrm{d}} - 2 m_q^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} y_{\mathrm{u}}^\dagger y_{\mathrm{u}} - 4 y_{\mathrm{d}}^\dagger m_{\mathrm{d}}^2 y_{\mathrm{d}} y_{\mathrm{d}}^\dagger y_{\mathrm{d}} - 4 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} m_q^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}}$$
$$- 4 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} y_{\mathrm{d}}^\dagger m_{\mathrm{d}}^2 y_{\mathrm{d}} - 2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} y_{\mathrm{d}}^\dagger y_{\mathrm{d}} m_q^2 - 4 y_{\mathrm{u}}^\dagger m_{\mathrm{u}}^2 y_{\mathrm{u}} y_{\mathrm{u}}^\dagger y_{\mathrm{u}} - 4 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} m_q^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}}$$
$$- 4 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} y_{\mathrm{u}}^\dagger m_{\mathrm{u}}^2 y_{\mathrm{u}} - 2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} y_{\mathrm{u}}^\dagger y_{\mathrm{u}} m_q^2 + 6 g_2^4 \sigma_{2,2} + \frac{32}{3} g_3^4 \sigma_{2,3} + \frac{2}{15} g_1^2 \sigma_{2,11}$$
$$- 12 m_{\mathrm{H}_d}^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{d}} y_{\mathrm{d}}^\dagger \Big) - 6 a_{\mathrm{d}}^\dagger a_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{d}} y_{\mathrm{d}}^\dagger \Big) - 3 m_q^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{d}} y_{\mathrm{d}}^\dagger \Big)$$
$$- 6 y_{\mathrm{d}}^\dagger m_{\mathrm{d}}^2 y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{d}} y_{\mathrm{d}}^\dagger \Big) - 3 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} m_q^2 \operatorname{tr}\Big(y_{\mathrm{d}} y_{\mathrm{d}}^\dagger \Big) - 4 m_{\mathrm{H}_d}^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{e}} y_{\mathrm{e}}^\dagger \Big)$$
$$- 2 a_{\mathrm{d}}^\dagger a_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{e}} y_{\mathrm{e}}^\dagger \Big) - m_q^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{e}} y_{\mathrm{e}}^\dagger \Big) - 2 y_{\mathrm{d}}^\dagger m_{\mathrm{d}}^2 y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{e}} y_{\mathrm{e}}^\dagger \Big)$$
$$- y_{\mathrm{d}}^\dagger y_{\mathrm{d}} m_q^2 \operatorname{tr}\Big(y_{\mathrm{e}} y_{\mathrm{e}}^\dagger \Big) - 12 m_{\mathrm{H}_u}^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} \operatorname{tr}\Big(y_{\mathrm{u}} y_{\mathrm{u}}^\dagger \Big) - 6 a_{\mathrm{u}}^\dagger a_{\mathrm{u}} \operatorname{tr}\Big(y_{\mathrm{u}} y_{\mathrm{u}}^\dagger \Big)$$
$$- 3 m_q^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} \operatorname{tr}\Big(y_{\mathrm{u}} y_{\mathrm{u}}^\dagger \Big) - 6 y_{\mathrm{u}}^\dagger m_{\mathrm{u}}^2 y_{\mathrm{u}} \operatorname{tr}\Big(y_{\mathrm{u}} y_{\mathrm{u}}^\dagger \Big) - 3 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} m_q^2 \operatorname{tr}\Big(y_{\mathrm{u}} y_{\mathrm{u}}^\dagger \Big)$$
$$- 6 a_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{d}}^\dagger a_{\mathrm{d}} \Big) - 2 a_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(y_{\mathrm{e}}^\dagger a_{\mathrm{e}} \Big) - 6 a_{\mathrm{u}}^\dagger y_{\mathrm{u}} \operatorname{tr}\Big(y_{\mathrm{u}}^\dagger a_{\mathrm{u}} \Big)$$
$$- 6 y_{\mathrm{d}}^\dagger a_{\mathrm{d}} \operatorname{tr}\Big(a_{\mathrm{d}}^* y_{\mathrm{d}}^T \Big) - 6 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(a_{\mathrm{d}}^* a_{\mathrm{d}}^T \Big) - 2 y_{\mathrm{d}}^\dagger a_{\mathrm{d}} \operatorname{tr}\Big(a_{\mathrm{e}}^* y_{\mathrm{e}}^T \Big)$$
$$- 2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(a_{\mathrm{e}}^* a_{\mathrm{e}}^T \Big) - 6 y_{\mathrm{u}}^\dagger a_{\mathrm{u}} \operatorname{tr}\Big(a_{\mathrm{u}}^* y_{\mathrm{u}}^T \Big) - 6 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} \operatorname{tr}\Big(a_{\mathrm{u}}^* a_{\mathrm{u}}^T \Big)$$
$$- 6 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(m_{\mathrm{d}}^2 y_{\mathrm{d}} y_{\mathrm{d}}^\dagger \Big) - 2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(m_{\mathrm{e}}^2 y_{\mathrm{e}} y_{\mathrm{e}}^\dagger \Big) - 2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(m_\ell^2 y_{\mathrm{e}}^\dagger y_{\mathrm{e}} \Big)$$
$$- 6 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \operatorname{tr}\Big(m_q^2 y_{\mathrm{d}}^\dagger y_{\mathrm{d}} \Big) - 6 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} \operatorname{tr}\Big(m_q^2 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} \Big) - 6 y_{\mathrm{u}}^\dagger y_{\mathrm{u}} \operatorname{tr}\Big(m_{\mathrm{u}}^2 y_{\mathrm{u}} y_{\mathrm{u}}^\dagger \Big), \qquad \text{(C.35)}$$

$$\beta_{m_\ell^2}^{(1)} = -\frac{6}{5} g_1^2 |M_1|^2 - 6 g_2^2 |M_2|^2 + 2 m_{\mathrm{H}_d}^2 y_{\mathrm{e}}^\dagger y_{\mathrm{e}} + 2 a_{\mathrm{e}}^\dagger a_{\mathrm{e}} + m_\ell^2 y_{\mathrm{e}}^\dagger y_{\mathrm{e}} + 2 y_{\mathrm{e}}^\dagger m_{\mathrm{e}}^2 y_{\mathrm{e}}$$
$$+ y_{\mathrm{e}}^\dagger y_{\mathrm{e}} m_\ell^2 - \sqrt{\frac{3}{5}} g_1 \sigma_{1,1}, \qquad \text{(C.36)}$$

$$\beta_{m_\ell^2}^{(2)} = \frac{3}{5} g_2^2 \Big[3 g_1^2 \Big(2 M_2 + M_1 \Big) + 55 g_2^2 M_2 \Big] M_2^* + \frac{12}{5} g_1^2 m_{\mathrm{H}_d}^2 y_{\mathrm{e}}^\dagger y_{\mathrm{e}} - \frac{12}{5} g_1^2 M_1 a_{\mathrm{e}}^\dagger y_{\mathrm{e}}$$

$$
\begin{aligned}
&+ \frac{3}{25} g_1^2 M_1^* \Big\{ - 20 y_e^\dagger a_e + 3\Big[5 g_2^2 \Big(2M_1 + M_2\Big) + 69 g_1^2 M_1\Big] + 40 M_1 y_e^\dagger y_e \Big\} \\
&+ \frac{12}{5} g_1^2 a_e^\dagger a_e + \frac{6}{5} g_1^2 m_\ell^2 y_e^\dagger y_e + \frac{12}{5} g_1^2 y_e^\dagger m_e^2 y_e + \frac{6}{5} g_1^2 y_e^\dagger y_e m_\ell^2 \\
&- 8 m_{H_d}^2 y_e^\dagger y_e y_e^\dagger y_e - 4 y_e^\dagger y_e a_e^\dagger a_e - 4 y_e^\dagger a_e a_e^\dagger y_e - 4 a_e^\dagger y_e y_e^\dagger a_e \\
&- 4 a_e^\dagger a_e y_e^\dagger y_e - 2 m_\ell^2 y_e^\dagger y_e y_e^\dagger y_e - 4 y_e^\dagger m_e^2 y_e y_e^\dagger y_e - 4 y_e^\dagger y_e m_\ell^2 y_e^\dagger y_e \\
&- 4 y_e^\dagger y_e y_e^\dagger m_e^2 y_e - 2 y_e^\dagger y_e y_e^\dagger y_e m_\ell^2 + 6 g_2^4 \sigma_{2,2} + \frac{6}{5} g_1^2 \sigma_{2,11} - 4\sqrt{\frac{3}{5}} g_1 \sigma_{3,1} \\
&- 12 m_{H_d}^2 y_e^\dagger y_e \operatorname{tr}\Big(y_d y_d^\dagger\Big) - 6 a_e^\dagger a_e \operatorname{tr}\Big(y_d y_d^\dagger\Big) - 3 m_\ell^2 y_e^\dagger y_e \operatorname{tr}\Big(y_d y_d^\dagger\Big) \\
&- 6 y_e^\dagger m_e^2 y_e \operatorname{tr}\Big(y_d y_d^\dagger\Big) - 3 y_e^\dagger y_e m_\ell^2 \operatorname{tr}\Big(y_d y_d^\dagger\Big) - 4 m_{H_d}^2 y_e^\dagger y_e \operatorname{tr}\Big(y_e y_e^\dagger\Big) \\
&- 2 a_e^\dagger a_e \operatorname{tr}\Big(y_e y_e^\dagger\Big) - m_\ell^2 y_e^\dagger y_e \operatorname{tr}\Big(y_e y_e^\dagger\Big) - 2 y_e^\dagger m_e^2 y_e \operatorname{tr}\Big(y_e y_e^\dagger\Big) \\
&- y_e^\dagger y_e m_\ell^2 \operatorname{tr}\Big(y_e y_e^\dagger\Big) - 6 a_e^\dagger y_e \operatorname{tr}\Big(y_d^\dagger a_d\Big) - 2 a_e^\dagger y_e \operatorname{tr}\Big(y_e^\dagger a_e\Big) \\
&- 6 y_e^\dagger a_e \operatorname{tr}\Big(a_d^* y_d^T\Big) - 6 y_e^\dagger y_e \operatorname{tr}\Big(a_d^* a_d^T\Big) - 2 y_e^\dagger a_e \operatorname{tr}\Big(a_e^* y_e^T\Big) \\
&- 2 y_e^\dagger y_e \operatorname{tr}\Big(a_e^* a_e^T\Big) - 6 y_e^\dagger y_e \operatorname{tr}\Big(m_d^2 y_d y_d^\dagger\Big) - 2 y_e^\dagger y_e \operatorname{tr}\Big(m_e^2 y_e y_e^\dagger\Big) \\
&- 2 y_e^\dagger y_e \operatorname{tr}\Big(m_\ell^2 y_e^\dagger y_e\Big) - 6 y_e^\dagger y_e \operatorname{tr}\Big(m_q^2 y_d^\dagger y_d\Big),
\end{aligned}
\tag{C.37}
$$

$$
\begin{aligned}
\beta_{m_{H_d}^2}^{(1)} &= -\frac{6}{5} g_1^2 |M_1|^2 - 6 g_2^2 |M_2|^2 - \sqrt{\frac{3}{5}} g_1 \sigma_{1,1} + 6 m_{H_d}^2 \operatorname{tr}\Big(y_d y_d^\dagger\Big) + 2 m_{H_d}^2 \operatorname{tr}\Big(y_e y_e^\dagger\Big) \\
&+ 2 \operatorname{tr}\Big(a_e^* a_e^T\Big) + 6 \operatorname{tr}\Big(m_d^2 y_d y_d^\dagger\Big) + 2 \operatorname{tr}\Big(m_e^2 y_e y_e^\dagger\Big) + 2 \operatorname{tr}\Big(m_\ell^2 y_e^\dagger y_e\Big) \\
&+ 6 \operatorname{tr}\Big(a_d^* a_d^T\Big) + 6 \operatorname{tr}\Big(m_q^2 y_d^\dagger y_d\Big),
\end{aligned}
\tag{C.38}
$$

$$
\begin{aligned}
\beta_{m_{H_d}^2}^{(2)} &= \frac{1}{25}\Big\{15 g_2^2 \Big[3 g_1^2 \Big(2M_2 + M_1\Big) + 55 g_2^2 M_2\Big] M_2^* + g_1^2 M_1^* \Big[621 g_1^2 M_1 + 90 g_2^2 M_1 \\
&+ 45 g_2^2 M_2 - 40 M_1 \operatorname{tr}\Big(y_d y_d^\dagger\Big) + 120 M_1 \operatorname{tr}\Big(y_e y_e^\dagger\Big) + 20 \operatorname{tr}\Big(y_d^\dagger a_d\Big) - 60 \operatorname{tr}\Big(y_e^\dagger a_e\Big)\Big] \\
&+ 10\Big[15 g_2^4 \sigma_{2,2} + 3 g_1^2 \sigma_{2,11} + \Big(160 g_3^2 |M_3|^2 \theta_{\tilde{g}} - 2 g_1^2 m_{H_d}^2 + 80 g_3^2 m_{H_d}^2\Big) \operatorname{tr}\Big(y_d y_d^\dagger\Big) \\
&+ 6 g_1^2 m_{H_d}^2 \operatorname{tr}\Big(y_e y_e^\dagger\Big) - 80 g_3^2 M_3^* \operatorname{tr}\Big(y_d^\dagger a_d\Big) \theta_{\tilde{g}} + 2 g_1^2 M_1 \operatorname{tr}\Big(a_d^* y_d^T\Big) \\
&- 2 g_1^2 \operatorname{tr}\Big(a_d^* a_d^T\Big) + 80 g_3^2 \operatorname{tr}\Big(a_d^* a_d^T\Big) - 6 g_1^2 M_1 \operatorname{tr}\Big(a_e^* y_e^T\Big) + 6 g_1^2 \operatorname{tr}\Big(a_e^* a_e^T\Big) \\
&- 2 g_1^2 \operatorname{tr}\Big(m_d^2 y_d y_d^\dagger\Big) + 80 g_3^2 \operatorname{tr}\Big(m_d^2 y_d y_d^\dagger\Big) + 6 g_1^2 \operatorname{tr}\Big(m_e^2 y_e y_e^\dagger\Big) + 6 g_1^2 \operatorname{tr}\Big(m_\ell^2 y_e^\dagger y_e\Big) \\
&- 2 g_1^2 \operatorname{tr}\Big(m_q^2 y_d^\dagger y_d\Big) + 80 g_3^2 \operatorname{tr}\Big(m_q^2 y_d^\dagger y_d\Big) - 90 m_{H_d}^2 \operatorname{tr}\Big(y_d y_d^\dagger y_d y_d^\dagger\Big) \\
&- 90 \operatorname{tr}\Big(y_d y_d^\dagger a_d a_d^\dagger\Big) - 30 \operatorname{tr}\Big(y_e y_e^\dagger a_e a_e^\dagger\Big) - 30 \operatorname{tr}\Big(y_e a_e^\dagger a_e y_e^\dagger\Big) \\
&- 15 m_{H_d}^2 \operatorname{tr}\Big(y_d y_u^\dagger y_u y_d^\dagger\Big) - 15 m_{H_u}^2 \operatorname{tr}\Big(y_d y_u^\dagger y_u y_d^\dagger\Big) - 15 \operatorname{tr}\Big(y_d y_u^\dagger a_u a_d^\dagger\Big) \\
&- 90 \operatorname{tr}\Big(y_d a_d^\dagger a_d y_d^\dagger\Big) - 15 \operatorname{tr}\Big(y_d a_u^\dagger a_u y_d^\dagger\Big) - 30 m_{H_d}^2 \operatorname{tr}\Big(y_e y_e^\dagger y_e y_e^\dagger\Big)
\end{aligned}
$$

$$- 15\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{d}^\dagger a_\mathrm{d}a_\mathrm{u}^\dagger\Big) - 15\,\mathrm{tr}\Big(y_\mathrm{u}a_\mathrm{d}^\dagger a_\mathrm{d}y_\mathrm{u}^\dagger\Big) - 90\,\mathrm{tr}\Big(m_\mathrm{d}^2 y_\mathrm{d}y_\mathrm{d}^\dagger y_\mathrm{d}y_\mathrm{d}^\dagger\Big)$$
$$- 15\,\mathrm{tr}\Big(m_\mathrm{d}^2 y_\mathrm{d}y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{d}^\dagger\Big) - 30\,\mathrm{tr}\Big(m_\mathrm{e}^2 y_\mathrm{e}y_\mathrm{e}^\dagger y_\mathrm{e}y_\mathrm{e}^\dagger\Big) - 30\,\mathrm{tr}\Big(m_\ell^2 y_\mathrm{e}^\dagger y_\mathrm{e}y_\mathrm{e}^\dagger y_\mathrm{e}\Big)$$
$$- 90\,\mathrm{tr}\Big(m_q^2 y_\mathrm{d}^\dagger y_\mathrm{d}y_\mathrm{d}^\dagger y_\mathrm{d}\Big) - 15\,\mathrm{tr}\Big(m_q^2 y_\mathrm{d}^\dagger y_\mathrm{d}y_\mathrm{u}^\dagger y_\mathrm{u}\Big) - 15\,\mathrm{tr}\Big(m_q^2 y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{d}^\dagger y_\mathrm{d}\Big)$$
$$- 2\sqrt{15}g_1\sigma_{3,1} - 80g_3^2M_3\,\mathrm{tr}\Big(a_\mathrm{d}^*y_\mathrm{d}^T\Big)\,\theta_{\widetilde{\mathrm{g}}} - 15\,\mathrm{tr}\Big(m_\mathrm{u}^2 y_\mathrm{u}y_\mathrm{d}^\dagger y_\mathrm{d}y_\mathrm{u}^\dagger\Big)\Big]\Big\}, \tag{C.39}$$

$$\beta_{m_{\mathrm{H}_u}^2}^{(1)} = -\frac{6}{5}g_1^2|M_1|^2 - 6g_2^2|M_2|^2 + \sqrt{\frac{3}{5}}g_1\sigma_{1,1} + 6m_{\mathrm{H}_u}^2\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{u}^\dagger\Big) + 6\,\mathrm{tr}\Big(a_\mathrm{u}^*a_\mathrm{u}^T\Big)$$
$$+ 6\,\mathrm{tr}\Big(m_q^2 y_\mathrm{u}^\dagger y_\mathrm{u}\Big) + 6\,\mathrm{tr}\Big(m_\mathrm{u}^2 y_\mathrm{u}y_\mathrm{u}^\dagger\Big), \tag{C.40}$$

$$\beta_{m_{\mathrm{H}_u}^2}^{(2)} = \frac{3}{5}g_2^2\Big[3g_1^2\Big(2M_2 + M_1\Big) + 55g_2^2M_2\Big]M_2^* + 6g_2^4\sigma_{2,2} + \frac{6}{5}g_1^2\sigma_{2,11} + 4\sqrt{\frac{3}{5}}g_1\sigma_{3,1}$$
$$+ \frac{8}{5}g_1^2m_{\mathrm{H}_u}^2\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{u}^\dagger\Big) + 32g_3^2m_{\mathrm{H}_u}^2\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{u}^\dagger\Big) + 64g_3^2|M_3|^2\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{u}^\dagger\Big)\,\theta_{\widetilde{\mathrm{g}}}$$
$$+ \frac{1}{25}g_1^2M_1^*\Big[- 40\,\mathrm{tr}\Big(y_\mathrm{u}^\dagger a_\mathrm{u}\Big) + 45g_2^2M_2 + 621g_1^2M_1 + 80M_1\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{u}^\dagger\Big) + 90g_2^2M_1\Big]$$
$$- 32g_3^2M_3^*\,\mathrm{tr}\Big(y_\mathrm{u}^\dagger a_\mathrm{u}\Big)\,\theta_{\widetilde{\mathrm{g}}} - \frac{8}{5}g_1^2M_1\,\mathrm{tr}\Big(a_\mathrm{u}^*y_\mathrm{u}^T\Big) - 32g_3^2M_3\,\mathrm{tr}\Big(a_\mathrm{u}^*y_\mathrm{u}^T\Big)\,\theta_{\widetilde{\mathrm{g}}}$$
$$+ 32g_3^2\,\mathrm{tr}\Big(a_\mathrm{u}^*a_\mathrm{u}^T\Big) + \frac{8}{5}g_1^2\,\mathrm{tr}\Big(m_q^2y_\mathrm{u}^\dagger y_\mathrm{u}\Big) + 32g_3^2\,\mathrm{tr}\Big(m_q^2y_\mathrm{u}^\dagger y_\mathrm{u}\Big) + \frac{8}{5}g_1^2\,\mathrm{tr}\Big(m_\mathrm{u}^2y_\mathrm{u}y_\mathrm{u}^\dagger\Big)$$
$$+ 32g_3^2\,\mathrm{tr}\Big(m_\mathrm{u}^2y_\mathrm{u}y_\mathrm{u}^\dagger\Big) - 6m_{\mathrm{H}_d}^2\,\mathrm{tr}\Big(y_\mathrm{d}y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{d}^\dagger\Big) - 6m_{\mathrm{H}_u}^2\,\mathrm{tr}\Big(y_\mathrm{d}y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{d}^\dagger\Big)$$
$$- 6\,\mathrm{tr}\Big(y_\mathrm{d}y_\mathrm{u}^\dagger a_\mathrm{u}a_\mathrm{d}^\dagger\Big) - 6\,\mathrm{tr}\Big(y_\mathrm{d}a_\mathrm{u}^\dagger a_\mathrm{u}y_\mathrm{d}^\dagger\Big) - 6\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{d}^\dagger a_\mathrm{d}a_\mathrm{u}^\dagger\Big) - 36m_{\mathrm{H}_u}^2\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{u}^\dagger\Big)$$
$$- 36\,\mathrm{tr}\Big(y_\mathrm{u}y_\mathrm{u}^\dagger a_\mathrm{u}a_\mathrm{u}^\dagger\Big) - 6\,\mathrm{tr}\Big(y_\mathrm{u}a_\mathrm{d}^\dagger a_\mathrm{d}y_\mathrm{u}^\dagger\Big) - 36\,\mathrm{tr}\Big(y_\mathrm{u}a_\mathrm{u}^\dagger a_\mathrm{u}y_\mathrm{u}^\dagger\Big) + \frac{8}{5}g_1^2\,\mathrm{tr}\Big(a_\mathrm{u}^*a_\mathrm{u}^T\Big)$$
$$- 6\,\mathrm{tr}\Big(m_\mathrm{d}^2y_\mathrm{d}y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{d}^\dagger\Big) - 6\,\mathrm{tr}\Big(m_q^2y_\mathrm{d}^\dagger y_\mathrm{d}y_\mathrm{u}^\dagger y_\mathrm{u}\Big) - 6\,\mathrm{tr}\Big(m_q^2y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{d}^\dagger y_\mathrm{d}\Big)$$
$$- 36\,\mathrm{tr}\Big(m_q^2y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{u}^\dagger y_\mathrm{u}\Big) - 6\,\mathrm{tr}\Big(m_\mathrm{u}^2y_\mathrm{u}y_\mathrm{d}^\dagger y_\mathrm{d}y_\mathrm{u}^\dagger\Big) - 36\,\mathrm{tr}\Big(m_\mathrm{u}^2y_\mathrm{u}y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{u}^\dagger\Big), \tag{C.41}$$

$$\beta_{m_\mathrm{d}^2}^{(1)} = -\frac{8}{15}g_1^2|M_1|^2 - \frac{32}{3}g_3^2|M_3|^2\,\theta_{\widetilde{\mathrm{g}}} + 4m_{\mathrm{H}_d}^2y_\mathrm{d}y_\mathrm{d}^\dagger + 4a_\mathrm{d}a_\mathrm{d}^\dagger + 2m_\mathrm{d}^2y_\mathrm{d}y_\mathrm{d}^\dagger + 4y_\mathrm{d}m_q^2y_\mathrm{d}^\dagger$$
$$+ 2y_\mathrm{d}y_\mathrm{d}^\dagger m_\mathrm{d}^2 + 2\frac{1}{\sqrt{15}}g_1\sigma_{1,1}, \tag{C.42}$$

$$\beta_{m_\mathrm{d}^2}^{(2)} = \frac{32}{45}g_3^2\Big[2g_1^2\Big(2M_3 + M_1\Big) + 75g_3^2M_3\Big]M_3^*\,\theta_{\widetilde{\mathrm{g}}} + \frac{4}{5}g_1^2m_{\mathrm{H}_d}^2y_\mathrm{d}y_\mathrm{d}^\dagger + 12g_2^2m_{\mathrm{H}_d}^2y_\mathrm{d}y_\mathrm{d}^\dagger$$
$$+ 24g_2^2|M_2|^2y_\mathrm{d}y_\mathrm{d}^\dagger - \frac{4}{5}g_1^2M_1y_\mathrm{d}a_\mathrm{d}^\dagger - 12g_2^2M_2y_\mathrm{d}a_\mathrm{d}^\dagger$$
$$+ \frac{4}{225}g_1^2M_1^*\Big\{2\Big[303g_1^2M_1 + 40g_3^2\Big(2M_1 + M_3\,\theta_{\widetilde{\mathrm{g}}}\Big)\Big] - 45a_\mathrm{d}y_\mathrm{d}^\dagger + 90M_1y_\mathrm{d}y_\mathrm{d}^\dagger\Big\}$$
$$+ \frac{4}{5}g_1^2a_\mathrm{d}a_\mathrm{d}^\dagger + 12g_2^2a_\mathrm{d}a_\mathrm{d}^\dagger + \frac{2}{5}g_1^2m_\mathrm{d}^2y_\mathrm{d}y_\mathrm{d}^\dagger + 6g_2^2m_\mathrm{d}^2y_\mathrm{d}y_\mathrm{d}^\dagger - 12g_2^2M_2^*a_\mathrm{d}y_\mathrm{d}^\dagger$$
$$+ \frac{4}{5}g_1^2y_\mathrm{d}m_q^2y_\mathrm{d}^\dagger + 12g_2^2y_\mathrm{d}m_q^2y_\mathrm{d}^\dagger + \frac{2}{5}g_1^2y_\mathrm{d}y_\mathrm{d}^\dagger m_\mathrm{d}^2 + 6g_2^2y_\mathrm{d}y_\mathrm{d}^\dagger m_\mathrm{d}^2$$
$$- 8m_{\mathrm{H}_d}^2y_\mathrm{d}y_\mathrm{d}^\dagger y_\mathrm{d}y_\mathrm{d}^\dagger - 4y_\mathrm{d}y_\mathrm{d}^\dagger a_\mathrm{d}a_\mathrm{d}^\dagger - 4m_{\mathrm{H}_d}^2y_\mathrm{d}y_\mathrm{u}^\dagger y_\mathrm{u}y_\mathrm{d}^\dagger$$

$$- 4m_{\mathrm{H}_u}^2 y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger - 4y_\mathrm{d} y_\mathrm{u}^\dagger a_\mathrm{u} a_\mathrm{d}^\dagger - 4y_\mathrm{d} a_\mathrm{d}^\dagger a_\mathrm{d} y_\mathrm{d}^\dagger - 4y_\mathrm{d} a_\mathrm{u}^\dagger a_\mathrm{u} y_\mathrm{d}^\dagger$$
$$- 4a_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} a_\mathrm{d}^\dagger - 4a_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} a_\mathrm{d}^\dagger - 4a_\mathrm{d} a_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger - 4a_\mathrm{d} a_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger$$
$$- 2m_\mathrm{d}^2 y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger - 2m_\mathrm{d}^2 y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger - 4y_\mathrm{d} m_q^2 y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger - 4y_\mathrm{d} m_q^2 y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger$$
$$- 4y_\mathrm{d} y_\mathrm{d}^\dagger m_\mathrm{d}^2 y_\mathrm{d} y_\mathrm{d}^\dagger - 4y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} m_q^2 y_\mathrm{d}^\dagger - 2y_\mathrm{d} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{d}^\dagger m_\mathrm{d}^2 - 4y_\mathrm{d} y_\mathrm{u}^\dagger m_\mathrm{u}^2 y_\mathrm{u} y_\mathrm{d}^\dagger$$
$$- 4y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} m_q^2 y_\mathrm{d}^\dagger - 2y_\mathrm{d} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{d}^\dagger m_\mathrm{d}^2 + \frac{32}{3} g_3^4 \sigma_{2,3} + \frac{8}{15} g_1^2 \sigma_{2,11} + 8\frac{1}{\sqrt{15}} g_1 \sigma_{3,1}$$
$$- 24m_{\mathrm{H}_d}^2 y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 12a_\mathrm{d} a_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 6m_\mathrm{d}^2 y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big)$$
$$- 12y_\mathrm{d} m_q^2 y_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 6y_\mathrm{d} y_\mathrm{d}^\dagger m_\mathrm{d}^2 \operatorname{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 8m_{\mathrm{H}_d}^2 y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$- 4a_\mathrm{d} a_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 2m_\mathrm{d}^2 y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 4y_\mathrm{d} m_q^2 y_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$- 2y_\mathrm{d} y_\mathrm{d}^\dagger m_\mathrm{d}^2 \operatorname{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 12y_\mathrm{d} a_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{d}^\dagger a_\mathrm{d}\Big) - 4y_\mathrm{d} a_\mathrm{d}^\dagger \operatorname{tr}\Big(y_\mathrm{e}^\dagger a_\mathrm{e}\Big)$$
$$- 12a_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(a_\mathrm{d}^* y_\mathrm{d}^T\Big) - 12y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(a_\mathrm{d}^* a_\mathrm{d}^T\Big) - 4a_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(a_\mathrm{e}^* y_\mathrm{e}^T\Big)$$
$$- 4y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(a_\mathrm{e}^* a_\mathrm{e}^T\Big) - 12y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(m_\mathrm{d}^2 y_\mathrm{d} y_\mathrm{d}^\dagger\Big) - 4y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(m_\mathrm{e}^2 y_\mathrm{e} y_\mathrm{e}^\dagger\Big)$$
$$- 4y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(m_\ell^2 y_\mathrm{e}^\dagger y_\mathrm{e}\Big) - 12y_\mathrm{d} y_\mathrm{d}^\dagger \operatorname{tr}\Big(m_q^2 y_\mathrm{d}^\dagger y_\mathrm{d}\Big), \tag{C.43}$$

$$\beta_{m_\mathrm{u}^2}^{(1)} = -\frac{32}{15} g_1^2 |M_1|^2 - \frac{32}{3} g_3^2 |M_3|^2\, \theta_{\tilde{\mathrm{g}}} + 4m_{\mathrm{H}_u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger + 4a_\mathrm{u} a_\mathrm{u}^\dagger + 2m_q^2 y_\mathrm{u} y_\mathrm{u}^\dagger + 4y_\mathrm{u} m_q^2 y_\mathrm{u}^\dagger$$
$$+ 2y_\mathrm{u} y_\mathrm{u}^\dagger m_\mathrm{u}^2 - 4\frac{1}{\sqrt{15}} g_1 \sigma_{1,1}, \tag{C.44}$$

$$\beta_{m_\mathrm{u}^2}^{(2)} = \frac{32}{45} g_3^2 \Big[75 g_3^2 M_3\, \theta_{\tilde{\mathrm{g}}} + 8g_1^2 \Big(2M_3 + M_1\Big)\Big] M_3^*\, \theta_{\tilde{\mathrm{g}}} - \frac{4}{5} g_1^2 m_{\mathrm{H}_u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger + 12g_2^2 m_{\mathrm{H}_u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger$$
$$+ 24g_2^2 |M_2|^2 y_\mathrm{u} y_\mathrm{u}^\dagger + \frac{4}{5} g_1^2 M_1 y_\mathrm{u} a_\mathrm{u}^\dagger - 12g_2^2 M_2 y_\mathrm{u} a_\mathrm{u}^\dagger - 12g_2^2 M_2^* a_\mathrm{u} y_\mathrm{u}^\dagger$$
$$+ \frac{4}{225} g_1^2 M_1^* \Big\{45\Big[-2M_1 y_\mathrm{u} y_\mathrm{u}^\dagger + a_\mathrm{u} y_\mathrm{u}^\dagger\Big] + 8\Big[321 g_1^2 M_1 + 40 g_3^2 \Big(2M_1 + M_3\, \theta_{\tilde{\mathrm{g}}}\Big)\Big]\Big\}$$
$$+ 12g_2^2 a_\mathrm{u} a_\mathrm{u}^\dagger - \frac{2}{5} g_1^2 m_\mathrm{u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger + 6g_2^2 m_\mathrm{u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger - \frac{4}{5} g_1^2 y_\mathrm{u} m_q^2 y_\mathrm{u}^\dagger - \frac{4}{5} g_1^2 a_\mathrm{u} a_\mathrm{u}^\dagger$$
$$+ 12g_2^2 y_\mathrm{u} m_q^2 y_\mathrm{u}^\dagger - \frac{2}{5} g_1^2 y_\mathrm{u} y_\mathrm{u}^\dagger m_\mathrm{u}^2 + 6g_2^2 y_\mathrm{u} y_\mathrm{u}^\dagger m_\mathrm{u}^2 - 4m_{\mathrm{H}_d}^2 y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger$$
$$- 4m_{\mathrm{H}_u}^2 y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger - 4y_\mathrm{u} y_\mathrm{d}^\dagger a_\mathrm{d} a_\mathrm{u}^\dagger - 8m_{\mathrm{H}_u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger - 4y_\mathrm{u} y_\mathrm{u}^\dagger a_\mathrm{u} a_\mathrm{u}^\dagger$$
$$- 4y_\mathrm{u} a_\mathrm{d}^\dagger a_\mathrm{d} y_\mathrm{u}^\dagger - 4y_\mathrm{u} a_\mathrm{u}^\dagger a_\mathrm{u} y_\mathrm{u}^\dagger - 4a_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} a_\mathrm{u}^\dagger - 4a_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} a_\mathrm{u}^\dagger$$
$$- 4a_\mathrm{u} a_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger - 4a_\mathrm{u} a_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger - 2m_\mathrm{u}^2 y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger - 2m_\mathrm{u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger$$
$$- 4y_\mathrm{u} m_q^2 y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger - 4y_\mathrm{u} m_q^2 y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger - 4y_\mathrm{u} y_\mathrm{d}^\dagger m_\mathrm{d}^2 y_\mathrm{d} y_\mathrm{u}^\dagger$$
$$- 4y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} m_q^2 y_\mathrm{u}^\dagger - 2y_\mathrm{u} y_\mathrm{d}^\dagger y_\mathrm{d} y_\mathrm{u}^\dagger m_\mathrm{u}^2 - 4y_\mathrm{u} y_\mathrm{u}^\dagger m_\mathrm{u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger - 4y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} m_q^2 y_\mathrm{u}^\dagger$$
$$- 2y_\mathrm{u} y_\mathrm{u}^\dagger y_\mathrm{u} y_\mathrm{u}^\dagger m_\mathrm{u}^2 + \frac{32}{3} g_3^4 \sigma_{2,3} + \frac{32}{15} g_1^2 \sigma_{2,11} - 16\frac{1}{\sqrt{15}} g_1 \sigma_{3,1} - 24m_{\mathrm{H}_u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger \operatorname{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big)$$
$$- 12a_\mathrm{u} a_\mathrm{u}^\dagger \operatorname{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) - 6m_\mathrm{u}^2 y_\mathrm{u} y_\mathrm{u}^\dagger \operatorname{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big) - 12y_\mathrm{u} m_q^2 y_\mathrm{u}^\dagger \operatorname{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big)$$

$$
\begin{aligned}
&- 6y_u y_u^\dagger m_u^2 \operatorname{tr}\Big(y_u y_u^\dagger\Big) - 12 y_u a_u^\dagger \operatorname{tr}\Big(y_u^\dagger a_u\Big) - 12 a_u y_u^\dagger \operatorname{tr}\Big(a_u^* y_u^T\Big) \\
&- 12 y_u y_u^\dagger \operatorname{tr}\Big(a_u^* a_u^T\Big) - 12 y_u y_u^\dagger \operatorname{tr}\Big(m_q^2 y_u^\dagger y_u\Big) - 12 y_u y_u^\dagger \operatorname{tr}\Big(m_u^2 y_u y_u^\dagger\Big), \qquad \text{(C.45)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_e^2}^{(1)} &= -\frac{24}{5} g_1^2 |M_1|^2 + 2\Big(2m_{H_d}^2 y_e y_e^\dagger + 2a_e a_e^\dagger + 2y_e m_\ell^2 y_e^\dagger + m_e^2 y_e y_e^\dagger + y_e y_e^\dagger m_e^2\Big) \\
&+ 2\sqrt{\frac{3}{5}} g_1 \sigma_{1,1}, \qquad \text{(C.46)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_e^2}^{(2)} &= \frac{2}{25}\Big[6g_1^2 M_1^* \Big\{234 g_1^2 M_1 + 5\Big[-2M_1 y_e y_e^\dagger + a_e y_e^\dagger\Big]\Big\} + 20 g_1\Big(3g_1\sigma_{2,11} + \sqrt{15}\sigma_{3,1}\Big) \\
&- 5\Big\{30 g_2^2 M_2^* a_e y_e^\dagger + 6g_1^2 a_e a_e^\dagger - 30 g_2^2 a_e a_e^\dagger + 3g_1^2 m_e^2 y_e y_e^\dagger \\
&- 15 g_2^2 m_e^2 y_e y_e^\dagger + 6 g_1^2 y_e m_\ell^2 y_e^\dagger - 30 g_2^2 y_e m_\ell^2 y_e^\dagger + 3 g_1^2 y_e y_e^\dagger m_e^2 \\
&- 15 g_2^2 y_e y_e^\dagger m_e^2 + 20 m_{H_d}^2 y_e y_e^\dagger y_e y_e^\dagger + 10 y_e y_e^\dagger a_e a_e^\dagger + 10 y_e a_e^\dagger a_e y_e^\dagger \\
&+ 10 a_e y_e^\dagger y_e a_e^\dagger + 10 a_e a_e^\dagger y_e y_e^\dagger + 5 m_e^2 y_e y_e^\dagger y_e y_e^\dagger + 10 y_e m_\ell^2 y_e^\dagger y_e y_e^\dagger \\
&+ 10 y_e y_e^\dagger m_e^2 y_e y_e^\dagger + 10 y_e y_e^\dagger y_e m_\ell^2 y_e^\dagger + 5 y_e y_e^\dagger y_e y_e^\dagger m_e^2 + 30 a_e a_e^\dagger \operatorname{tr}\Big(y_d y_d^\dagger\Big) \\
&+ 15 m_e^2 y_e y_e^\dagger \operatorname{tr}\Big(y_d y_d^\dagger\Big) + 30 y_e m_\ell^2 y_e^\dagger \operatorname{tr}\Big(y_d y_d^\dagger\Big) + 15 y_e y_e^\dagger m_e^2 \operatorname{tr}\Big(y_d y_d^\dagger\Big) \\
&+ 10 a_e a_e^\dagger \operatorname{tr}\Big(y_e y_e^\dagger\Big) + 5 m_e^2 y_e y_e^\dagger \operatorname{tr}\Big(y_e y_e^\dagger\Big) + 10 y_e m_\ell^2 y_e^\dagger \operatorname{tr}\Big(y_e y_e^\dagger\Big) \\
&+ 5 y_e y_e^\dagger m_e^2 \operatorname{tr}\Big(y_e y_e^\dagger\Big) + y_e a_e^\dagger\Big[10\operatorname{tr}\Big(y_e^\dagger a_e\Big) + 30 g_2^2 M_2 + 30\operatorname{tr}\Big(y_d^\dagger a_d\Big) - 6 g_1^2 M_1\Big] \\
&+ 30 a_e y_e^\dagger \operatorname{tr}\Big(a_d^* y_d^T\Big) + 10 a_e y_e^\dagger \operatorname{tr}\Big(a_e^* y_e^T\Big) + 2 y_e y_e^\dagger\Big[3 g_1^2 m_{H_d}^2 - 15 g_2^2 m_{H_d}^2 \\
&+ 30 m_{H_d}^2 \operatorname{tr}\Big(y_d y_d^\dagger\Big) + 10 m_{H_d}^2 \operatorname{tr}\Big(y_e y_e^\dagger\Big) + 15\operatorname{tr}\Big(a_d^* a_d^T\Big) + 5\operatorname{tr}\Big(a_e^* a_e^T\Big) \\
&- 30 g_2^2 |M_2|^2 + 15\operatorname{tr}\Big(m_d^2 y_d y_d^\dagger\Big) + 5\operatorname{tr}\Big(m_e^2 y_e y_e^\dagger\Big) + 5\operatorname{tr}\Big(m_\ell^2 y_e^\dagger y_e\Big) \\
&+ 15\operatorname{tr}\Big(m_q^2 y_d^\dagger y_d\Big)\Big]\Big\}\Big], \qquad \text{(C.47)}
\end{aligned}
$$

$$
\beta_{m_{\tilde{g}}^2}^{(1)} = -24 g_3^2 |M_3|^2\, \theta_{\tilde{g}}, \qquad \text{(C.48)}
$$

$$
\beta_{m_{\tilde{g}}^2}^{(2)} = 24 g_3^4\Big(15|M_3|^2\,\theta_{\tilde{g}} + \sigma_{2,3}\Big). \qquad \text{(C.49)}
$$

where

$$
\begin{aligned}
\sigma_{1,1} &= \sqrt{\frac{3}{5}} g_1\Big[m_{H_u}^2 - 2\operatorname{tr}\Big(m_u^2\Big) - \operatorname{tr}\Big(m_\ell^2\Big) - m_{H_d}^2 + \operatorname{tr}\Big(m_d^2\Big) \\
&+ \operatorname{tr}\Big(m_e^2\Big) + \operatorname{tr}\Big(m_q^2\Big)\Big], \qquad \text{(C.50)}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{2,11} &= \frac{1}{10} g_1^2\Big[2\operatorname{tr}\Big(m_d^2\Big) + 3\operatorname{tr}\Big(m_\ell^2\Big) + 3m_{H_d}^2 + 3m_{H_u}^2 + 6\operatorname{tr}\Big(m_e^2\Big) \\
&+ 8\operatorname{tr}\Big(m_u^2\Big) + \operatorname{tr}\Big(m_q^2\Big)\Big], \qquad \text{(C.51)}
\end{aligned}
$$

$$
\sigma_{3,1} = \frac{1}{20\sqrt{15}} g_1\Big[9 g_1^2 m_{H_u}^2 - 9 g_1^2 m_{H_d}^2 - 45 g_2^2 m_{H_d}^2 + 45 g_2^2 m_{H_u}^2 + 4\Big(20 g_3^2 + g_1^2\Big)\operatorname{tr}\Big(m_d^2\Big)
$$

$$
\begin{aligned}
&- 9g_1^2 \operatorname{tr}\Big(m_\ell^2\Big) - 45g_2^2 \operatorname{tr}\Big(m_\ell^2\Big) + g_1^2 \operatorname{tr}\Big(m_q^2\Big) + 45g_2^2 \operatorname{tr}\Big(m_q^2\Big) + 80g_3^2 \operatorname{tr}\Big(m_q^2\Big)\\
&- 160g_3^2 \operatorname{tr}\Big(m_q^2\Big) + 90m_{\mathrm{H}_d}^2 \operatorname{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger\Big) + 30m_{\mathrm{H}_d}^2 \operatorname{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger\Big) - 90m_{\mathrm{H}_u}^2 \operatorname{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger\Big)\\
&- 30 \operatorname{tr}\Big(y_\mathrm{d} m_q^{2*} y_\mathrm{d}^\dagger\Big) - 60 \operatorname{tr}\Big(y_\mathrm{e} y_\mathrm{e}^\dagger m_\mathrm{e}^{2*}\Big) + 30 \operatorname{tr}\Big(y_\mathrm{e} m_\ell^{2*} y_\mathrm{e}^\dagger\Big) + 120 \operatorname{tr}\Big(y_\mathrm{u} y_\mathrm{u}^\dagger m_\mathrm{u}^{2*}\Big)\\
&- 32g_1^2 \operatorname{tr}\Big(m_\mathrm{u}^2\Big) - 30 \operatorname{tr}\Big(y_\mathrm{u} m_q^{2*} y_\mathrm{u}^\dagger\Big) + 36g_1^2 \operatorname{tr}\Big(m_\mathrm{e}^2\Big) - 60 \operatorname{tr}\Big(y_\mathrm{d} y_\mathrm{d}^\dagger m_\mathrm{d}^{2*}\Big)\Big], \qquad \text{(C.52)}
\end{aligned}
$$

$$
\sigma_{2,2} = \frac{1}{2}\Big[3 \operatorname{tr}\Big(m_q^2\Big) + m_{\mathrm{H}_d}^2 + m_{\mathrm{H}_u}^2 + \operatorname{tr}\Big(m_\ell^2\Big)\Big], \qquad \text{(C.53)}
$$

$$
\sigma_{2,3} = \frac{1}{2}\Big[2 \operatorname{tr}\Big(m_q^2\Big) + 3(1+\theta_{\widetilde{\mathrm{g}}}) m_{\widetilde{\mathrm{g}}}^2 + \operatorname{tr}\Big(m_\mathrm{d}^2\Big) + \operatorname{tr}\Big(m_\mathrm{u}^2\Big)\Big]. \qquad \text{(C.54)}
$$

D

Numerically solved perturbative flows to $\mathcal{N} = 2$ SQCD

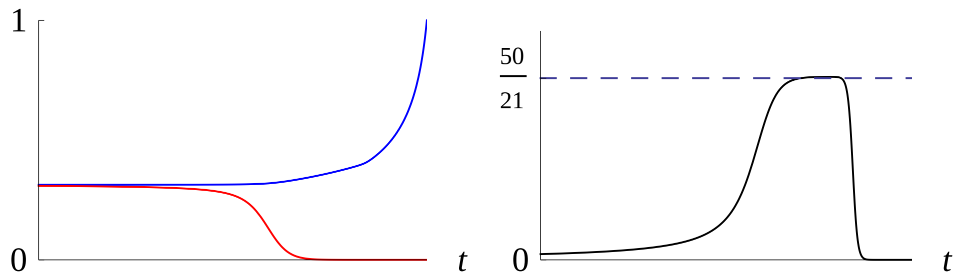


Figure D.1: RG flow of g^2 (blue), h^2 (red) and η_κ^2/g^2 (black) from the UV (right) to the (IR) left. The horizontal axis is $t = \log\mu$, and we take $N_c = 5$, $N_f = 10$. 50/21 is the η_κ^2/g^2 quasi–fixed point value for this N_c and N_f given by eq. 5.5.41.

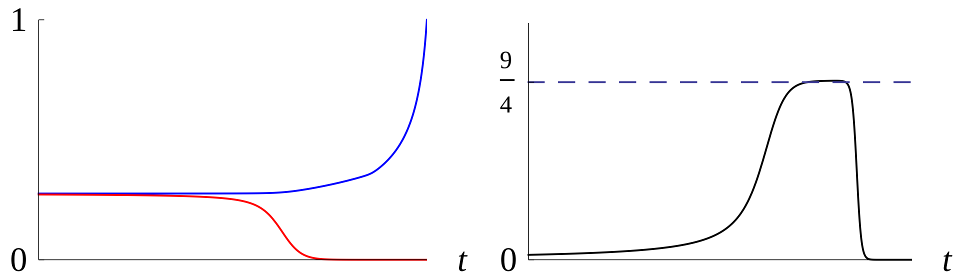


Figure D.2: RG flow of g^2 (blue), h^2 (red) and η_κ^2/g^2 (black) from the UV (right) to the (IR) left. The horizontal axis is $t = \log\mu$, and we take $N_c = 6$, $N_f = 12$. 9/4 is the η_κ^2/g^2 quasi–fixed point value for this N_c and N_f given by eq. 5.5.41.

E

Harmonic superspace and $\mathcal{N} = 2$ SQCD

E.1 Integration rules for harmonic functions

Here we collect the rules for integrating harmonic functions over the sphere [50].

$$\int du\, f^{(q)}(u) = 0 \quad \text{if } q \neq 0, \tag{E.1}$$

$$\int du\, 1 = 1, \tag{E.2}$$

$$\int du\, u^+_{(i_1} \dots u^+_{i_n} u^-_{j_1} \dots u^-_{j_n)} = 0 \quad \text{if } n \geq 1, \tag{E.3}$$

$$\int du\, (u^+)^{(m} (u^-)^{n)} (u^+)_{(k} (u^-)_{l)} = \frac{(-)^n\, m!\, n!}{(m+n+1)!}\, \delta^{(i_1}_{(j_1} \dots \delta^{i_{m+n})}_{(j_{m+n})}\, \delta_{ml}\, \delta_{nk}.. \tag{E.4}$$

Equation E.4 can be inverted to find the coefficients of a harmonic function $f^{(q)}(u)$

$$f^{(i_1 \dots i_{n+q} j_1 \dots j_n)} = \frac{(-)^{n+q}\, (2\ + q + 1)!}{(n+q)!\, n!} \int du\, (u^+)^{(n} (u^-)^{n+q)} f^{(q)}(u). \tag{E.5}$$

E.2 Measures for harmonic superspace

Here we collect the relevant measures and normalisations for integration over HSS [50]. The measures are

$$\int du\, d^{12}X \equiv \int du\, d^4x\, d^8\theta = \int du\, d^4x_A\, d^4\theta^+ d^4\theta^- \\ = \frac{1}{256}\int du\, d^4x_A\, (\mathcal{D}^-)^2(\bar{\mathcal{D}}^-)^2(\mathcal{D}^+)^2(\bar{\mathcal{D}}^+)^2, \tag{E.6}$$

$$\int du\, d\zeta^{(-4)} \equiv \int du\, d^4x_A\, d^4\theta^+ = \frac{1}{16}\int du\, d^4x_A\, (\mathcal{D}^-)^2(\bar{\mathcal{D}}^-)^2, \tag{E.7}$$

with normalisations

$$\int d^8\theta\, \theta^8 = \int d^4\theta^+\, (\theta^+)^4 = \int d^4\theta\, (\theta)^4 = \int d^4\bar\theta\, (\bar\theta)^4 = 1. \tag{E.8}$$

where

$$\theta^8 = (\theta^+)^4(\theta^-)^4 = (\theta)^4(\bar\theta)^4, \qquad (\theta^\pm)^4 = (\theta^\pm)^2(\bar\theta^\pm)^2, \tag{E.9}$$

$$(\theta)^4 = (\theta^+)^2(\theta^-)^2, \qquad (\bar\theta)^4 = (\bar\theta^+)^2(\bar\theta^-)^2. \tag{E.10}$$

E.3 Conjugation rules for harmonic superspace

E.3.1 Conjugation rules

Complex conjugation $\bar{\mathcal{O}}$ is defined as

$$\overline{\theta_{\alpha i}} = \bar\theta^i_{\dot\alpha}, \qquad \overline{\theta^i_\alpha} = -\bar\theta_{\dot\alpha i}; \tag{E.11}$$

$$\overline{u^{+i}} = u^-_i, \qquad \overline{u^+_i} = -u^{-i}; \tag{E.12}$$

$$\overline{f^{i_1\ldots i_n}} \equiv \bar f_{i_1\ldots i_n}, \qquad \overline{f_{i_1\ldots i_n}} = (-1)^n \bar f^{i_1\ldots i_n}. \tag{E.13}$$

Antipodal conjugation $\mathcal{O}^\star$

$$(u^{+i})^\star = u^{-i}, \qquad (u^+_i)^\star = u^-_i, \tag{E.14}$$

$$(u^{-i})^\star = -u^{+i}, \qquad (u^-_i)^\star = -u^+_i. \tag{E.15}$$

Combined complex and antipodal conjugation $(\bar{\mathcal{O}})^\star = \overline{(\mathcal{O})^\star} \equiv \widetilde{\mathcal{O}}$

$$\widetilde{(u_i^\pm)} = u^{\pm i}, \qquad \widetilde{(u^{\pm i})} = -u_i^\pm. \tag{E.16}$$

It is convenient to note that

$$\overline{Q^1} = \bar{Q}_1 = \varepsilon_{12}\,\bar{Q}^2 = -\bar{Q}^2 = \overline{Q_2}, \qquad \overline{Q^2} = \bar{Q}_2 = \varepsilon_{21}\,\bar{Q}^1 = \bar{Q}^1 = -\overline{Q_1}. \tag{E.17}$$

E.4 $\mathcal{N}=2$ SQCD

E.4.1 Formulation in harmonic superspace

The lagrangian $\mathcal{L}_{\text{total}}$ for $\mathcal{N}=2$ SQCD arising from eqs. 5.6.61, 5.6.63, and 2.3.225 up to four derivatives in the prepotential $\mathcal{F}(\mathcal{W})$ is

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} + \mathcal{L}_{\text{Pauli}} + \mathcal{L}_{\text{D Fermi}} + \mathcal{L}_{\text{4 Fermi}} - V \tag{E.18}$$

where

$$\begin{aligned}
-\mathcal{L}_{\text{kin}} &= \frac{g_{ab}}{4\pi}\left(\mathcal{D}^\mu X^a\,\mathcal{D}_\mu \bar{X}^b + i\,\lambda^{i,a}\,\sigma^\mu\,\mathcal{D}_\mu\,\bar{\lambda}_i^b - \frac{1}{4}\,F^a_{\mu\nu}\,F^{b,\mu\nu}\right) + \frac{h_{ab}}{16\pi}\,F^a_{\mu\nu}\,\tilde{F}^{b,\mu\nu} \\
&\quad + \bar{Q}^i\,\mathcal{D}^\mu\mathcal{D}_\mu\,Q_i + \frac{i}{2}\left(\bar{\psi}_Q\bar{\sigma}^\mu\mathcal{D}_\mu\psi_Q + \psi_{\tilde{Q}}\sigma^\mu\mathcal{D}_\mu\bar{\psi}_{\tilde{Q}}\right), &\text{(E.19)}\\
-\mathcal{L}_{\text{yuk}} &= \frac{i g_{ab}}{4\pi\sqrt{2}}\,f^b_{cd}\,\lambda^{a,i}\,\bar{X}^c\,\lambda^d_i + i\left(\bar{Q}^i\,\lambda_i\,\psi_Q - \psi_{\tilde{Q}}\,\lambda^i\,Q_i\right) \\
&\quad - \frac{1}{\sqrt{2}}\,\psi_{\tilde{Q}}\,X\,\psi_Q + \text{h.c.}, &\text{(E.20)}\\
\mathcal{L}_{\text{D Fermi}} &= \frac{i}{2}\,\mathcal{F}_{abc}|(\lambda^a\lambda^b)^A D^{c,A} + \text{h.c.}, &\text{(E.21)}\\
-\mathcal{L}_{\text{Pauli}} &= \frac{i}{4}\,\mathcal{F}_{abc}|\,\lambda^{a,i}\sigma^{\mu\nu}\lambda^b_i\,F^c_{\mu\nu} + \text{h.c.}, &\text{(E.22)}\\
-\mathcal{L}_{\text{4 Fermi}} &= \frac{i}{6}\,\mathcal{F}_{abcd}|\,(\lambda^a\lambda^b)^A(\lambda^c\lambda^d)^A + \text{h.c.}, &\text{(E.23)}\\
V &= \bar{Q}^i\,\{\bar{X}, X\}\,Q_i - \frac{g_{ab}}{4\pi}\left(\frac{1}{2}\,f^a_{cd}\,f^b_{ef}\bar{X}^c X^d \bar{X}^e X^f + \frac{1}{2}\,D^{a,A}|\,D^{b,A}|\right), &\text{(E.24)}
\end{aligned}$$

and the traced $\mathrm{SU}(2)_R$ tensor products are written as three vector dot products

$$a^i{}_j \equiv i\,a^A\left(\sigma^A\right)^i{}_j, \qquad a^{ij}\,b_{ij} = -a^i{}_j\,b^j{}_i = a^A\,b^B\,\mathrm{tr}_R\left(\sigma^A\cdot\sigma^B\right) = 2\,a^A\,b^A, \tag{E.25}$$

	SU(N_c)	SU(N_f)	U(1)$_R$
Q	$\square$	$\square$	$1-R_X\,\frac{N_c}{N_f}$
$\tilde{Q}$	$\bar{\square}$	$\bar{\square}$	$1-R_X\,\frac{N_c}{N_f}$
X	**Ad**	**1**	R_X

Table E.1: $\mathcal{N}=1$ superfield representations in $\mathcal{N}=2$ SQCD.

where tr_R is a trace over the $\mathrm{SU}(2)_R$ indices and we use the conventions of Appendix E.6. The standard renormalisable $\mathcal{N}=2$ SQCD lagrangian can be obtained by integrating out the $D^{a,A}$ and taking the canonical prepotential

$$\mathcal{F}(\mathcal{W})=\tau\frac{(\mathcal{W}^a)^2}{2},\qquad \tau\equiv\frac{\theta_{\rm YM}}{2\,\pi}+\frac{4\,\pi\,i}{g^2}\equiv\tau_1+i\,\tau_2,\qquad \tau_1,\,\tau_2\in\mathbb{R}. \tag{E.26}$$

One then finds the kinetic terms in the holomorphic basis

$$-\mathcal{L}_{\rm kin}=\frac{1}{g^2}\left(\mathcal{D}^\mu X^a\,\mathcal{D}_\mu\bar{X}_a+i\,\lambda^{i,\,a}\,\sigma^\mu\,\mathcal{D}_\mu\,\bar{\lambda}_{i,\,a}+\frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a\right)+\frac{\theta_{\rm YM}}{32\pi^2}\,F^a_{\mu\nu}\,\tilde{F}^{\mu\nu}_a, \tag{E.27}$$

as well as the familiar yukawa interactions and scalar potential.

E.4.2 Formulation in $\mathcal{N}=1$ superspace

Because we are ultimately interested in the $\mathcal{N}=1$ electric and magnetic Kutasov theories, in this appendix we recast $\mathcal{N}=2$ SQCD in $\mathcal{N}=1$ superspace [47]. The appropriate $\mathcal{N}=1$ superfield content [60, 289] is given in table E.1, and the $\mathcal{N}=2$ SQCD action composed of two parts as in 5.6.61. From the full $\mathcal{N}=2$ superspace point of view, after fixing an $\mathrm{SU}(2)_R$ direction so that a particular $\mathcal{Q}_i$ is the canonical $\mathcal{N}=1$ SUSY

The SYM part is written in terms of an analytic prepotential $\mathcal{F}(i\sqrt{2}X)=\mathcal{F}(\mathcal{A})$ [59],

$$S^{\mathcal{N}=2}_{\rm SYM}=\frac{1}{16\pi i}\int d^4x\,d^2\theta\left(\mathcal{F}_{ab}\mathcal{W}^a\mathcal{W}^b-\int d^2\bar{\theta}\,\frac{i}{\sqrt{2}}\mathcal{F}_a(e^V)^a{}_b\bar{X}^b\right)+\text{h.c.} \tag{E.28}$$

whereas the QCD part is

$$S^{\mathcal{N}=2}_{Q}=\int d^4x\,d^2\theta\left(\sqrt{2}\,\tilde{Q}\,X\,Q+\frac{1}{2}\int d^2\bar{\theta}\,\left[K_Q+K_{\tilde{Q}}\right]\right)+\text{h.c.} \tag{E.29}$$

and K_ϕ is the Kähler potentials for the superfield ϕ. The Kähler potential for X and effective gauge coupling for the standard renormalisable $\mathcal{N}=2$ theory can be recovered by taking E.26,

$$\mathcal{F}(\mathcal{A}) = \tau\frac{(\mathcal{A}^a)^2}{2} \implies S^{\mathcal{N}=2}_{\rm SYM} = \frac{\tau}{4\pi i}\int d^4x\, d^2\theta\left(\frac{1}{4}\mathcal{W}^2 + \frac{1}{2}\int d^2\bar\theta\, K_X\right) + \text{h.c.} \quad (E.30)$$

E.5 $\mathcal{N}=2$ SQCD in the presence of $W(X)$

E.5.1 Formulation in $\mathcal{N}=1$ superspace

An $\mathcal{N}=2$ breaking X deformation $W_{\rm def}(X)$ causes the shift in in the action

$$S^{\mathcal{N}=2}_{\rm SQCD} \to S^{\mathcal{N}=2}_{\rm SQCD} + S^{\mathcal{N}=1}_{\rm def}, \qquad S^{\mathcal{N}=1}_{\rm def} = \int d^4x\, d^2\theta\, W_{\rm def}(X) + \text{h.c.} \quad (E.31)$$

and yields the additional terms in the lagrangian

$$V_{\rm def} = \frac{4\pi}{\tau_2}K_X^{ab}\left[\frac{\partial W_{\rm def}}{\partial X^a} + \sqrt{2}\,\tilde Q\, t_a\, Q\right]\left[\frac{\partial W_{\rm def}}{\partial X^b} + \sqrt{2}\,\tilde Q\, t_b\, Q\right]^\dagger \quad (E.32)$$

$$\mathcal{L}^{\rm fermion}_{\rm def} = -\frac{1}{2}\frac{\partial^2 W_{\rm def}}{\partial X^a \partial X^b}\,\psi^a_X\,\psi^b_X + \text{h.c.}, \quad (E.33)$$

where K_X^{ab} is the inverse of the Kähler metric for the physically normalised X

$$(K_X)_{ab} \equiv \frac{\partial^2 K_X}{\partial X^a \partial \bar X^b}, \quad (E.34)$$

and $\tau_2 = \frac{4\pi}{g^2}$ is the imaginary part of the holomorphic gauge coupling defined in eq. E.26.

E.6 $\mathrm{SU}(2)_R$ and index conventions

The index conventions used can be found in table E.2. Our $\mathrm{SU}(2)_R$ conventions are $\varepsilon^{12} = +1$, and that if $a^i{}_j \equiv i\,a^A(\sigma^A)^i{}_j$ then clearly $a^A = \frac{1}{2i}\mathrm{tr}(\sigma^A\, a)$, and in components

$$a^i{}_j = \begin{pmatrix} i\,a^3 & i\,a^1 + a^2 \\ i\,a^1 - a^2 & -i\,a^3 \end{pmatrix}, \quad a^{ij} = \begin{pmatrix} i\,a^1 + a^2 & -i\,a^3 \\ -i\,a^3 & -i\,a^1 + a^2 \end{pmatrix},$$

Label	Type	Range
μ, ν, ρ, σ	space-time	0 to 3
$\alpha, \dot{\alpha}, \beta, \dot{\beta}$	spinor	$1, 2$
i, j, k, l	$\mathrm{SU}(2)_R$	$1, 2$
$\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$	$\mathrm{SU}(N_c)$ adjoint	1 to $(N_c^2 - 1)$
a, b, c, d	all adjoints	𝅝, 𝅗𝅥, 𝅘𝅥, 1 to $(N_c^2 - 1)$

Table E.2: Conventions used throughout Chapter 5.

$$a_{ij} = \begin{pmatrix} -i\, a^1 + a^2 & i\, a^3 \\ i\, a^3 & i\, a^1 + a^2 \end{pmatrix}. \tag{E.35}$$

Bibliography

[1] D. Busbridge, *Constrained Dirac gluino mediation*, arXiv:1408.4605.

[2] S. A. Abel and D. Busbridge, *Mapping Dirac gaugino masses*, *Journal of High Energy Physics* (June, 2013) 37, [arXiv:1306.6323].

[3] The ATLAS Collaboration, *Summary plots from the ATLAS Standard Model physics group*, 2014.

[4] K. Jansen, *Lattice QCD: a critical status report*, arXiv:0810.5634.

[5] M. J. Strassler, *The Duality Cascade*, 0505153.

[6] The ATLAS Collaboration, *Summary plots from the ATLAS SUSY physics group*, 2014.

[7] G. D. Kribs and A. Martin, *Dirac Gauginos in Supersymmetry – Suppressed Jets + MET Signals: A Snowmass Whitepaper*, arXiv:1308.3468.

[8] W. Porod, *SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at e+e colliders*, *Computer Physics Communications* **153** (June, 2003) 275–315.

[9] L. Beck, F. Blekman, D. Dobur, B. Fuks, and K. Mawatari, *Probing top-philic sgluons with LHC Run I data*, arXiv:1501.0758.

[10] D. Kutasov, *A comment on duality in N = 1 supersymmetric non-abelian gauge theories*, *Physics Letters B* **351** (May, 1995) 230–234, [9503086].

[11] J. Beringer et al. (Particle Data Group), *Review of Particle Physics*, *Physical Review D* **86** (July, 2012) 010001.

[12] A. Klypin, H. Zhao, and R. S. Somerville, *ΛCDM based Models for the Milky Way and M31. I. Dynamical Models*, *The Astrophysical Journal* **573** (July, 2002) 597–613, [0110390].

[13] D. Paraficz, J. P. Kneib, J. Richard, A. Morandi, M. Limousin, and E. Jullo, *The Bullet Cluster revisited: New results from new constraints and improved strong lensing modeling technique*, arXiv:1209.0384.

[14] Planck Collaboration, *Planck 2013 results. XVI. Cosmological parameters*, arXiv:1303.5076.

[15] A. Sakharov, *Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe*, *JETP Lett.* **34** (1967), no. May 392–393.

[16] J. Terning, *Modern supersymmetry: Dynamics and duality*, vol. 132. Oxford University Press, USA, 2006.

[17] A. Riotto and M. Trodden, *Recent progress in baryogenesis*, *Annual Review of Nuclear and Particle Science* **49** (Dec., 1999) 35–75, [9901362].

[18] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and a. M. Rotunno, *Global analysis of neutrino masses, mixings, and phases: Entering the era of leptonic CP violation searches*, *Physical Review D* **86** (July, 2012) 013012, [arXiv:1205.5254].

[19] D. V. Forero, M. Tórtola, and J. W. F. Valle, *Global status of neutrino oscillation parameters after Neutrino-2012*, *Physical Review D* **86** (Oct., 2012) 073012, [arXiv:1205.4018].

[20] M. Pospelov and A. Ritz, *Theta vacua, QCD sum rules, and the neutron electric dipole moment*, *Nuclear Physics B* **573** (May, 2000) 177–200, [9908508].

[21] M. Dine, *TASI Lectures on The Strong CP Problem*, 0011376.

[22] G. 't Hooft, C. Itzykson, A. Jaffe, H. Lehmann, P. Mitter, I. Singer, and R. Stora, *Recent Developments in Gauge Theories.* Springer US, Boston, MA, 1980.

[23] R. Peccei and H. Quinn, *Constraints imposed by CP conservation in the presence of pseudoparticles*, *Physical Review D* (1977).

[24] R. Peccei and H. Quinn, *CP conservation in the presence of pseudoparticles*, *Physical Review Letters* **387** (1977), no. 25.

[25] J. Jaeckel and A. Ringwald, *The Low-Energy Frontier of Particle Physics*, `arXiv:1002.0329`.

[26] K. Baker, G. Cantatore, S. a. Cetin, M. Davenport, K. Desch, B. Döbrich, H. Gies, I. G. Irastorza, J. Jaeckel, A. Lindner, T. Papaevangelou, M. Pivovaroff, G. Raffelt, J. Redondo, A. Ringwald, Y. Semertzidis, A. Siemko, M. Sulc, A. Upadhye, and K. Zioutas, *The quest for axions and other new light particles*, *Annalen der Physik* **525** (June, 2013) A93–A99, [`arXiv:1306.2841`].

[27] H. Georgi and S. Glashow, *Unity of all elementary-particle forces*, *Physical Review Letters* **32** (1974), no. 8 438–441.

[28] K. Lane, *Two Lectures on Technicolor*, `0202255`.

[29] C. T. Hill and E. H. Simmons, *Strong dynamics and electroweak symmetry breaking*, *Physics Reports* **381** (July, 2003) 235–402, [`0203079`].

[30] R. Shrock, *Some Recent Results on Models of Dynamical Electroweak Symmetry Breaking*, `0703050`.

[31] N. Craig, D. Stolarski, and J. Thaler, *A Fat Higgs with a Magnetic Personality*, *Spectrum* (June, 2011) 33, [`arXiv:1106.2164`].

[32] W. Siegel, *Supersymmetric dimensional regularization via dimensional reduction 1*, *Physics Letters B* **84** (1979), no. 2 193–196.

[33] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields*, *Physical Review D* **11** (1975), no. 10.

[34] S. Weinberg, *Effective gauge theories*, *Physics Letters B* **91** (1980), no. 1.

[35] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*. Westview Press, 1995.

[36] L. J. Hall, *Grand unification of effective gauge theories*, *Nuclear Physics B* **178** (1981).

[37] R. Haag, J. T. Lopuszanski, and M. F. Sohnius, *All possible generators of supersymmetries of the S-matrix*, *Nuclear Physics B* **88** (Mar., 1975) 257–274.

[38] S. Coleman and J. Mandula, *All possible symmetries of the S matrix*, *Physical Review* **159** (1967), no. 5 1251.

[39] E. Witten, *Constraints on supersymmetry breaking*, *Nuclear Physics B* **202** (1982), no. 2 253–316.

[40] C. Frugiuele, T. Gregoire, P. Kumar, and E. Ponton, *"L=R" - $U(1)_R$ as the Origin of Leptonic 'RPV'*, arXiv:1210.0541.

[41] C. Frugiuele and T. Grégoire, *Making the sneutrino a Higgs particle with a $U(1)_R$ lepton number*, *Physical Review D* **85** (Jan., 2012) 015016, [arXiv:1107.4634].

[42] F. Riva, C. Biggio, and A. Pomarol, *Is the 125 GeV Higgs the superpartner of a neutrino?*, arXiv:1211.4526.

[43] C. Frugiuele, T. Gregoire, P. Kumar, and E. Ponton, *"L=R" – $U(1)_R$ Lepton Number at the LHC*, arXiv:1210.5257.

[44] E. Bertuzzo and C. Frugiuele, *Fitting neutrino physics with a U(1) R lepton number*, *Journal of High Energy Physics* **2012** (May, 2012) 100, [arXiv:1203.5340].

[45] S. Dimopoulos and H. Georgi, *Softly broken supersymmetry and SU (5)*, *Nuclear Physics B* **193** (1981), no. 1 150–162.

[46] J. Ellis and D. V. Nanopoulos, *Grand unification in simple supergravity*, *Physics Letters B* **121** (1983), no. 2 123–129.

[47] A. Salam and J. Strathdee, *Superfields and fermi-bose symmetry*, *Physical Review D* **11** (1975), no. 6 1521.

[48] S. P. Martin, *A Supersymmetry Primer*, *Nature* (Sept., 1997) 152, [9709356].

[49] P. Fayet and J. Iliopoulos, *Spontaneously broken supergauge symmetries and Goldstone spinors*, *Physics Letters B* **51** (1974), no. 5 461–464.

[50] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky, and E. S. Sokatchev, *Harmonic Superspace*. Cambridge University Press, Cambridge, 1st ed., 2001.

[51] P. Fayet, *Fermi-Bose hypersymmetry*, *Nuclear Physics B* **13** (1976), no. 1 135–155.

[52] M. F. Sohnius, *Supersymmetry and central charges*, *Nuclear Physics B* **138** (1978) 109–121.

[53] P. Howe, K. Stelle, and P. West, *N= 1, d= 6 harmonic superspace*, *Classical and Quantum Gravity* (1985).

[54] K. Stelle, *Manifest realizations of extended supersymmetry*, *Proceedings of the Santa Fe meeting of the Division of ...* (1985).

[55] A. S. Galperin, E. A. Ivanov, S. Kalitzin, V. I. Ogievetsky, and E. S. Sokatchev, *Unconstrained N = 2 matter, Yang-Mills and supergravity theories in harmonic superspace*, *Classical and Quantum Gravity* **2** (Jan., 1985) 127–127.

[56] B. M. Zupnik, *The action of the supersymmetric N= 2 gauge theory in harmonic superspace*, *Physics Letters B* **183** (1987), no. 2 175–176.

[57] A. S. Galperin, E. Ivanov, V. I. Ogievetsky, and E. S. Sokatchev, *Harmonic supergraphs: Green functions*, *Classical and Quantum Gravity* **2** (1985) 601.

[58] K. Fujiwara, H. Itoyama, and M. Sakaguchi, *Partial supersymmetry breaking and $\mathcal{N} = 2$ $U(N_c)$ gauge model with hypermultiplets in harmonic superspace*, *Nuclear Physics B* **740** (Apr., 2006) 58–78, [0510255].

[59] N. Seiberg, *Supersymmetry and non-perturbative beta functions*, *Physics Letters B* **206** (1988), no. 1 75–80.

[60] N. Seiberg and E. Witten, *Monopoles, duality and chiral symmetry breaking in N = 2 supersymmetric QCD*, *Nuclear Physics B* **431** (Dec., 1994) 484–550, [9408099].

[61] N. Seiberg and E. Witten, *Electric-magnetic duality, monopole condensation, and confinement in N=2 supersymmetric Yang-Mills theory*, *Nuclear Physics B* **426** (Sept., 1994) 19–52, [9407087].

[62] M. T. Grisaru and W. Siegel, *Improved methods for supergraphs*, *Nuclear Physics B* **159** (1979), no. 3 429–450.

[63] N. Seiberg, *Naturalness versus supersymmetric non-renormalization theorems*, *Physics Letters B* **318** (1993), no. 3 469–475.

[64] N. Seiberg, *The Power of Holomorphy – Exact Results in 4D SUSY Field Theories*, *Physics* (Aug., 1994) 13, [9408013].

[65] N. Arkani-Hamed and H. Murayama, *Holomorphy, Rescaling Anomalies and Exact beta Functions in Supersymmetric Gauge Theories*, 9707133.

[66] V. Novikov, M. Shifman, A. Vainshtein, and V. Zakharov, *Exact Gell-Mann-Low function of supersymmetric Yang-Mills theories from instanton calculus*, *Nuclear Physics B* **229** (Dec., 1983) 381–393.

[67] S. Cecotti, L. Girardello, and M. Porrati, *Constraints on partial super-Higgs*, *Nuclear Physics B* (1986).

[68] S. Cecotti, L. Girardello, and M. Porrati, *Two into one won't go*, *Physics Letters B* (1984).

[69] L. O'Raifeartaigh, *Spontaneous symmetry breaking for chirals scalar superfields*, *Nuclear Physics B* **96** (1975), no. 2 331–352.

[70] A. E. Nelson and N. Seiberg, *R-symmetry breaking versus supersymmetry breaking*, *Nuclear Physics B* **416** (Mar., 1994) 46–62, [9309299].

[71] S. Ferrara, L. Girardello, and F. Palumbo, *General mass formula in broken supersymmetry*, *Physical Review D* **20** (1979), no. 2 403.

[72] I. Jack and D. R. T. Jones, *Non-standard soft supersymmetry breaking*, *Physics Letters B* **457** (Mar., 1999) 101, [9903365].

[73] B. Allanach, *SOFTSUSY: a program for calculating supersymmetric spectra*, *Comput. Phys. Commun.* **143** (Apr., 2001) 305–331, [0104145].

[74] W. Porod and F. Staub, *SPheno 3.1: extensions including flavour, CP-phases and models beyond the MSSM*, *Computer Physics Communications* **183** (Nov., 2012) 2458–2469.

[75] A. Djouadi, J.-L. Kneur, and G. Moultaka, *SuSpect: A Fortran code for the Supersymmetric and Higgs particle spectrum in the MSSM*, *Computer Physics Communications* **176** (Mar., 2007) 426–455, [0211331].

[76] N. Seiberg, *Electric-magnetic duality in supersymmetric non-Abelian gauge theories*, *Nuclear Physics B* **435** (Feb., 1995) 129–146, [9411149].

[77] N. Seiberg, *The Power of Duality - Exact Results in 4D SUSY Field Theory*, *International Journal of Modern Physics A* **16** (Oct., 2001) 4365–4376, [9506077].

[78] K. Intriligator and N. Seiberg, *Lectures on supersymmetric gauge theories and electric-magnetic duality*, *Nuclear Physics B - Proceedings Supplements* **45** (Feb., 1996) 1–28, [9509066].

[79] M. Luty and W. Taylor, *Varieties of vacua in classical supersymmetric gauge theories*, *Technology* (June, 1995) 14, [9506098].

[80] N. Seiberg, *Exact results on the space of vacua of four-dimensional SUSY gauge theories*, *Physical Review D* **49** (June, 1994) 6857–6863, [9402044].

[81] T. Banks and A. Zaks, *On the phase structure of vector-like gauge theories wtih massless fermions*, *Nuclear Physics* **196** (1982) 189–204.

[82] Y. Frishman, A. Schwimmer, T. Banks, and S. Yankielowicz, *The axial anomaly and the bound-state spectrum in confining theories*, *Nuclear Physics B* **177** (1981) 157–171.

[83] C. Csáki, Y. Shirman, and J. Terning, *A Seiberg Dual for the MSSM: Partially Composite W and Z*, `arXiv:1106.3074`.

[84] C. Csaki, L. Randall, and J. Terning, *Light Stops from Seiberg Duality*, `arXiv:1201.1293`.

[85] N. Arkani-Hamed and R. Rattazzi, *Exact results for non-holomorphic masses in softly broken supersymmetric gauge theories*, *Physics Letters B* **454** (May, 1999) 290–296, [9804068].

[86] M. Luty and R. Rattazzi, *Soft supersymmetry breaking in deformed moduli spaces, conformal theories, and Script N = 2 Yang-Mills theory*, *Journal of High Energy Physics* **1999** (Nov., 1999) 1–37, [9908085].

[87] S. A. Abel, M. Buican, and Z. Komargodski, *Mapping Anomalous Currents in Supersymmetric Dualities*, *Physics* (May, 2011) 23, [arXiv:1105.2885].

[88] N. Evans, S. D. HSU, M. Schwetz, and S. B. Selipsky, *Exact results and soft breaking masses in supersymmetric gauge theory*, *Nuclear Physics B* **456** (Dec., 1995) 205–218, [9508002].

[89] O. Aharony, J. Sonnenschein, M. E. Peskin, and S. Yankielowicz, *Exotic nonsupersymmetric gauge dynamics from supersymmetric QCD*, *Physical Review D* **52** (Nov., 1995) 6157–6174, [9507013].

[90] N. Evans, S. D. Hsu, and M. Schwetz, *Exact results in softly broken supersymmetric models*, *Physics Letters B* **355** (Aug., 1995) 475–480, [9503186].

[91] A. Karch, T. Kobayashi, J. Kubo, and G. Zoupanos, *Infrared behaviour of softly broken SQCD and its dual*, *Physics Letters B* **441** (Nov., 1998) 235–242, [9808178].

[92] G. F. Giudice and R. Rattazzi, *Extracting supersymmetry-breaking effects from wave-function renormalization*, *Nuclear Physics B* **511** (1998), no. 1-2 25–44.

[93] N. Arkani-Hamed, G. F. Giudice, M. Luty, and R. Rattazzi, *Supersymmetry-breaking loops from analytic continuation into superspace*, *Physical Review D* **58** (Oct., 1998) 115005, [9803290].

[94] H.-c. Cheng and Y. Shadmi, *Duality in the Presence of Supersymmetry Breaking*, *Science* (Jan., 1998) 27, [9801146].

[95] T. Kobayashi and K. Yoshioka, *New RG-invariants of soft supersymmetry breaking parameters*, *Physics Letters B* **486** (July, 2000) 223–227, [0004175].

[96] A. E. Nelson and M. J. Strassler, *Exact Results for Supersymmetric Renormalization and the Supersymmetric Flavor Problem*, *Journal of High Energy Physics* **2002** (July, 2002) 021–021, [0104051].

[97] L. V. Avdeev, D. I. Kazakov, and I. N. Kondrashuk, *Renormalizations in softly broken SUSY gauge theories*, *Nuclear Physics B* **510** (1998) 289–312.

[98] D. I. Kazakov, *Exploring softly broken SUSY theories via Grassmannian Taylor expansion*, *Physics Letters B* **449** (Mar., 1999) 201–206, [9812513].

[99] D. I. Kazakov, *Renormalization Properties of Softly Broken SUSY Gauge Theories*, *Advances* (Aug., 2002) [0208200].

[100] L. Alvarez-Gaume and E. Witten, *Gravitational anomalies*, *Nuclear Physics B* **234** (1984).

[101] The ATLAS Collaboration, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, arXiv:1207.7214.

[102] The CMS Collaboration, *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, arXiv:1207.7235.

[103] D. Grellscheid, J. Jaeckel, V. V. Khoze, P. Richardson, and C. Wymant, *Direct SUSY Searches at the LHC in the light of LEP Higgs Bounds*, *Physics* (Nov., 2011) 13, [arXiv:1111.3365].

[104] P. Draper, P. Meade, M. Reece, and D. Shih, *Implications of a 125 GeV Higgs for the MSSM and Low-Scale SUSY Breaking*, arXiv:1112.3068.

[105] J. Alwall, P. Schuster, and N. Toro, *Simplified Models for a First Characterization of New Physics at the LHC*, arXiv:0810.3921.

[106] A. Djouadi et al. [MSSM Working Group Collaboration], *The Minimal Supersymmetric Standard Model: Group Summary Report*, 9901246.

[107] The ATLAS Collaboration, *Search for direct pair production of the top squark in all-hadronic final states in proton-proton collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, Journal of High Energy Physics **2014** (Sept., 2014) 15, [arXiv:1406.1122].

[108] The ATLAS Collaboration, *Search for top squark pair production in final states with one isolated lepton, jets, and missing transverse momentum in $\sqrt{s} = 8$ TeV pp collisions with the ATLAS detector*, arXiv:1407.0583.

[109] The ATLAS Collaboration, *Search for strong production of supersymmetric particles in final states with missing transverse momentum and at least three b-jets at $\sqrt{s} = 8$ TeV proton-proton collisions with the ATLAS detector*, arXiv:1407.0600.

[110] X. Lu, H. Murayama, J. T. Ruderman, and K. Tobioka, *Natural Higgs Mass in Supersymmetry from Nondecoupling Effects*, Physical Review Letters **112** (May, 2014) 191803, [arXiv:1308.0792].

[111] The DELPHI Collaboration and J. Abdallah, *Photon events with missing energy in e + e- collisions at $\sqrt{s} =$ 130 to 209 GeV*, The European Physical Journal C **38** (Jan., 2005) 395–411, [0406019].

[112] The OPAL Collaboration and G. Abbiendi, *Searches for Gauge-Mediated Supersymmetry Breaking topologies in e^+e^- collisions at centre-of-mass energies up to $\sqrt{s} = 209\mathrm{GeV}$*, The European Physical Journal C **46** (Mar., 2006) 307–341, [0507048].

[113] LEPSUSYWG, ALEPH, DELPHI, L3, and OPAL Experiments, *Combined LEP Chargino Results, up to 208 GeV for large m0, note LEPSUSYWG/01-03*, 2001.

[114] LEPSUSYWG, ALEPH, DELPHI, L3, and OPAL Experiments, *Combined LEP Chargino Results, up to 208 GeV for low DM, note LEPSUSYWG/02-04*, 2002.

[115] LEPSUSYWG, ALEPH, DELPHI, L3, and OPAL Experiments, *Combined LEP Selectron/Smuon/Stau Results, 183-208 GeV, note LEPSUSYWG/04-01*, 2004.

[116] T. J. LeCompte and S. P. Martin, *Large Hadron Collider reach for supersymmetric models with compressed mass spectra*, *Physical Review D* **84** (July, 2011) 015004, [arXiv:1105.4304].

[117] H. K. Dreiner, M. Krämer, and J. Tattersall, *How low can SUSY go? Matching, monojets and compressed spectra*, arXiv:1207.1613.

[118] B. Bhattacherjee, A. Choudhury, K. Ghosh, and S. Poddar, *Compressed supersymmetry at 14 TeV LHC*, *Physical Review D* **89** (Feb., 2014) 037702, [arXiv:1308.1526].

[119] R. Barbier, C. Bérat, M. Besançon, M. Chemtob, A. Deandrea, E. Dudas, P. Fayet, S. Lavignac, G. Moreau, E. Perez, and Y. Sirois, *R-Parity-violating supersymmetry*, *Physics Reports* **420** (Nov., 2005) 1–195, [0406039].

[120] M. Abdullah, I. Galon, Y. Shadmi, and Y. Shirman, *Flavored Gauge Mediation, A Heavy Higgs, and Supersymmetric Alignment*, arXiv:1209.4904.

[121] I. Galon, G. Perez, and Y. Shadmi, *Non-Degenerate Squarks from Flavored Gauge Mediation*, arXiv:1306.6631.

[122] P. Fayet, *Massive gluinos*, *Physics Letters B* **78** (1978), no. 4 417–420.

[123] J. Polchinski and L. Susskind, *Breaking of supersymmetry at intermediate energy*, *Physical Review D* **26** (1982), no. 12 3661–3673.

[124] L. J. Hall and L. Randall, $U(1)_R$ *symmetric supersymmetry*, *Nuclear Physics B* **352** (1991) 289–308.

[125] I. Antoniadis and K. Benakli, *Fractionally charged particles and supersymmetry breaking in 4D strings*, *Physics Letters B* **295** (Dec., 1992) 219–224, [9209020].

[126] M. Dine and D. a. MacIntire, *Supersymmetry, naturalness, and dynamical supersymmetry breaking*, *Physical Review D* **46** (Sept., 1992) 2594–2601, [9205227].

[127] I. Jack and D. R. T. Jones, *Quasi-infrared fixed points and renormalization group invariant trajectories for nonholomorphic soft supersymmetry breaking*, *Physical Review D* **61** (Mar., 2000) 095002, [9909570].

[128] D. Martí and A. Pomarol, *Supersymmetric theories with compact extra dimensions in N=1 superfields*, *Physical Review D* **64** (Oct., 2001) 105025, [0106256].

[129] P. J. Fox, A. E. Nelson, and N. Weiner, *Dirac Gaugino Masses and Supersoft Supersymmetry Breaking*, *Journal of High Energy Physics* **2002** (Aug., 2002) 035–035, [0206096].

[130] A. E. Nelson, N. Rius, V. Sanz, and M. Unsal, *The Minimal Supersymmetric Model without a μ term*, *Journal of High Energy Physics* **2002** (Aug., 2002) 039–039, [0206102].

[131] A. E. Nelson, N. Rius, V. Sanz, and M. Unsal, *The MSSM without mu term*, 0211102.

[132] I. Antoniadis, A. Delgado, K. Benakli, M. Quirós, and M. Tuckmantel, *Splitting extended supersymmetry*, *Physics Letters B* **634** (Mar., 2006) 302–306, [0507192].

[133] L. M. Carpenter, P. J. Fox, and D. E. Kaplan, *The NMSSM, Anomaly Mediation and a Dirac Bino*, *arXiv preprint hep-ph/0503093* (Mar., 2005) [0503093].

[134] I. Antoniadis, K. Benakli, A. Delgado, and M. Quirós, *A New Gauge Mediation Theory*, 0610265.

[135] I. Antoniadis, K. Benakli, A. Delgado, M. Quirós, and M. Tuckmantel, *Split extended supersymmetry from intersecting branes*, *Nuclear Physics B* **744** (June, 2006) 156–179, [0601003].

[136] S. D. L. Amigo, A. E. Blechman, P. J. Fox, and E. Poppitz, *R -symmetric gauge mediation*, *Journal of High Energy Physics* **2009** (Jan., 2009) 018–018, [arXiv:0809.1112].

[137] K. Benakli and M. D. Goodsell, *Dirac gauginos in general gauge mediation*, *Nuclear Physics B* **816** (July, 2009) 185–203, [arXiv:0811.4409].

[138] S. Y. Choi, M. Drees, J. Kalinowski, J. Kim, E. Popenda, and P. Zerwas, *Color-octet scalars of supersymmetry at the LHC*, *Physics Letters B* **672** (Feb., 2009) 246–252, [arXiv:0812.3586].

[139] S. Y. Choi, M. Drees, A. Freitas, and P. Zerwas, *Testing the Majorana nature of gluinos and neutralinos*, *Physical Review D* **78** (Nov., 2008) 095007, [arXiv:0808.2410].

[140] K. Hsieh, *Pseudo-Dirac bino dark matter*, *Physical Review D* **77** (Jan., 2008) 015004, [arXiv:0708.3970].

[141] G. D. Kribs, E. Poppitz, and N. Weiner, *Flavor in supersymmetry with an extended R symmetry*, *Physical Review D* **78** (Sept., 2008) 055010.

[142] G. D. Kribs, A. Martin, and T. S. Roy, *Supersymmetry with a chargino NLSP and gravitino LSP*, *Journal of High Energy Physics* **2009** (Jan., 2009) 023–023, [arXiv:0807.4936].

[143] G. Belanger, K. Benakli, M. D. Goodsell, C. Moura, and A. Pukhov, *Dark matter with Dirac and Majorana gaugino masses*, *Journal of Cosmology and Astroparticle Physics* **2009** (Aug., 2009) 027–027, [arXiv:0905.1043].

[144] A. E. Blechman, *R-symmetric Gauge Mediation and the MRSSM*, *Modern Physics Letters A* **24** (Mar., 2009) 14, [arXiv:0903.2822].

[145] S. Y. Choi, J. Kalinowski, J. M. Kim, and E. Popenda, *Scalar gluons and Dirac gluinos at the LHC*, arXiv:0911.1951.

[146] A. Freitas, *Distinguishing Majorana and Dirac Gluinos and Neutralinos*, *Arxiv preprint arXiv:0909.5308* (Sept., 2009) 4, [arXiv:0909.5308].

[147] G. D. Kribs, T. Okui, and T. Roy, *Viable gravity-mediated supersymmetry breaking*, *Physical Review D* (2010).

[148] K. Benakli and M. D. Goodsell, *Dirac Gauginos, Gauge Mediation and Unification*, arXiv:1003.4957.

[149] K. Benakli and M. D. Goodsell, *Dirac gauginos and kinetic mixing*, *Nuclear Physics B* **830** (Mar., 2010) 315–329, [arXiv:1003.4957].

[150] L. M. Carpenter, *Dirac Gauginos, Negative Supertraces and Gauge Mediation*, arXiv:1007.0017.

[151] S. Y. Choi, D. Choudhury, A. Freitas, J. Kalinowski, J. Kim, and P. M. Zerwas, *Dirac neutralinos and electroweak scalar bosons of N = 1/N = 2 hybrid supersymmetry at colliders*, *Journal of High Energy Physics* **2010** (Aug., 2010) 25, [arXiv:1005.0818].

[152] E. J. Chun, J.-C. Park, and S. Scopel, *Dirac gaugino as leptophilic dark matter*, *Journal of Cosmology and Astroparticle Physics* **2010** (Feb., 2010) 015–015, [arXiv:0911.5273].

[153] A. De Simone, V. Sanz, and H. P. Sato, *Pseudo-Dirac Dark Matter Leaves a Trace*, *Structure* (Apr., 2010) 16, [arXiv:1004.1567].

[154] S. A. Abel and M. D. Goodsell, *Easy Dirac gauginos*, *Journal of High Energy Physics* **2011** (June, 2011) 64, [arXiv:1102.0014].

[155] K. Benakli, M. D. Goodsell, and A.-K. Maier, *Generating μ and $B\mu$ in models with Dirac gauginos*, *Nuclear Physics B* **851** (Oct., 2011) 445–461.

[156] K. Benakli, *Dirac Gauginos: A User Manual*, *ArXiv* (June, 2011) 4, [arXiv:1106.1649].

[157] R. Davies and M. McCullough, *Small m_ν due to R-symmetry breaking for small Λ*, *Physical Review D* **86** (July, 2012) 025014, [arXiv:1111.2361].

[158] R. Davies, J. March-Russell, and M. McCullough, *A supersymmetric one Higgs doublet model*, *Journal of High Energy Physics* **2011** (Apr., 2011) 108, [arXiv:1103.1647].

[159] M. Heikinheimo, M. Kellerstein, and V. Sanz, *How Many Supersymmetries?*, *Simulation* (Nov., 2011) 7, [arXiv:1111.4322].

[160] H. Itoyama and N. Maru, *D-term Dynamical Supersymmetry Breaking Generating Split N=2 Gaugino Masses of Mixed Majorana-Dirac Type*, arXiv:1109.2276.

[161] J. Kalinowski, *Phenomenology of R-symmetric Supersymmetry*, *Acta Physica Polonica B* **42** (2011), no. 11 2425–2432.

[162] T. Basak and S. Mohanty, *Triplet-singlet extension of the MSSM with a 125 GeV Higgs boson and dark matter*, *Physical Review D* **86** (Oct., 2012) 075031, [arXiv:1204.6592].

[163] K. Benakli, M. D. Goodsell, and F. Staub, *Dirac Gauginos and the 125 GeV Higgs*, arXiv:1211.0552.

[164] P. Byakti, *Supersymmetry Breaking: constraint on U(1) R-charge*, *Nuclear Physics* (Jan., 2012) 5, [arXiv:1201.1180].

[165] R. Davies, *Dirac gauginos and unification in F-theory*, *Journal of High Energy Physics* **2012** (Oct., 2012) 10, [arXiv:1205.1942].

[166] R. Fok, G. D. Kribs, A. Martin, and Y. Tsai, *Electroweak baryogenesis in R-symmetric supersymmetry*, *Physical Review D* **87** (Mar., 2013) 055018, [arXiv:1208.2784].

[167] M. D. Goodsell, *Two-loop RGEs with Dirac gaugino masses*, arXiv:1206.6697.

[168] H. Itoyama and N. Maru, *D-term Dynamical SUSY Breaking*, arXiv:1207.7152.

[169] G. D. Kribs and A. Martin, *Supersoft supersymmetry is super-safe*, *Physical Review D* **85** (June, 2012) 115014, [arXiv:1203.4821].

[170] Y. Morita, H. Nakano, and T. Shimomura, *Neutrino Mass and Proton Decay in a $U(1)_R$ Symmetric Model*, arXiv:1212.4304.

[171] A. Arvanitaki, M. Baryakhtar, X. Huang, K. Van Tilburg, and G. Villadoro, *The Last Vestiges of Naturalness*, *arXiv preprint arXiv: ...* (Sept., 2013) 22, [arXiv:1309.3568].

[172] T. Banks, *Dirac Gluinos in the Pyramid Scheme*, *arXiv preprint arXiv:1311.4410* **30** (2013) 1–11, [arXiv:1311.4410].

[173] T. Basak and S. Mohanty, *130 GeV gamma ray line and enhanced Higgs di-photon rate from Triplet-Singlet extended MSSM*, arXiv:1304.6856.

[174] M. R. Buckley, D. Hooper, and J. Kumar, *Phenomenology of Dirac Neutralino Dark Matter*, arXiv:1307.3561.

[175] C. Csáki, J. Goodman, R. Pavesi, and Y. Shirman, *The mDbM problem of Dirac gauginos and its solutions*, *Physical Review D* **89** (Mar., 2014) 055005, [arXiv:1310.4504].

[176] E. Dudas, M. D. Goodsell, L. Heurtier, and P. Tziveloglou, *Flavour models with Dirac and fake gluinos*, *Nuclear Physics B* **884** (July, 2014) 632–671, [arXiv:1312.2011].

[177] H. Itoyama and N. Maru, *D-term triggered dynamical supersymmetry breaking*, *Physical Review D* **88** (July, 2013) 025012, [arXiv:1301.7548].

[178] G. D. Kribs and N. Raj, *Mixed gauginos sending mixed messages to the LHC*, *Physical Review D* **89** (Mar., 2014) 055011, [arXiv:1307.7197].

[179] H. Beauchesne and T. Gregoire, *Electroweak precision measurements in supersymmetric models with a U(1)R lepton number*, arXiv:1402.5403.

[180] K. Benakli, M. D. Goodsell, W. Porod, and F. Staub, *Constrained minimal Dirac gaugino supersymmetric standard model*, *Physical Review D* **90** (Aug., 2014) 045017, [arXiv:1403.5122].

[181] E. Bertuzzo, C. Frugiuele, T. Gregoire, and E. Ponton, *Dirac gauginos, R symmetry and the 125 GeV Higgs*, arXiv:1402.5432.

[182] P. J. Fox, G. D. Kribs, and A. Martin, *Split Dirac Supersymmetry: An Ultraviolet Completion of Higgsino Dark Matter*, arXiv:1405.3692.

[183] M. D. Goodsell and P. Tziveloglou, *Dirac Gauginos in Low Scale Supersymmetry Breaking*, arXiv:1407.5076.

[184] S. Ipek, D. McKeen, and A. E. Nelson, *CP Violation in Pseudo-Dirac Fermion Oscillations*, arXiv:1407.8193.

[185] J. Jaeckel, V. V. Khoze, T. Plehn, and P. Richardson, *Travels on the squark-gluino mass plane*, *Physical Review D* **85** (Jan., 2012) 015015, [arXiv:1109.2072].

[186] T. Plehn and T. M. P. Tait, *Seeking sgluons*, *Journal of Physics G: Nuclear and Particle Physics* **36** (July, 2009) 075001.

[187] H. K. Dreiner, H. E. Haber, and S. P. Martin, *Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry*, *Physics* (Dec., 2008) 311, [arXiv:0812.1594].

[188] M. Goeppert-Mayer, *Double beta-disintegration*, *Physical Review* **161** (1935), no. V 512–516.

[189] a. S. Barabash, *75 years of double beta decay: yesterday, today and tomorrow*, arXiv:1101.4502.

[190] M. Schmaltz and W. Skiba, *Minimal gaugino mediation*, *Physical Review D* **62** (Oct., 2000) 095005, [0001172].

[191] H. Cheng, D. E. Kaplan, M. Schmaltz, and W. Skiba, *Deconstructing gaugino mediation*, *Physics Letters B* **515** (2001), no. August 395–399.

[192] D. Green, Z. Komargodski, and A. Katz, *Direct Gaugino Mediation*, *Physical Review Letters* **106** (Feb., 2011) 061801, [arXiv:1008.2215].

[193] W. Siegel, *Gauge spinor superfield as scalar multiplet*, *Physics Letters B* **85** (1979), no. 4 333–334.

[194] J. Louis and J. Swiebodzinski, *Couplings of N=1 chiral spinor multiplets*, *The European Physical Journal C* **51** (May, 2007) 731–739.

[195] J. a. Dixon, *Some Properties of Chiral Dotted Spinor Superfields*, arXiv:0911.0199.

[196] P. Fayet, *Six-dimensional supersymmetric QED, R-invariance and N = 2 supersymmetry breaking by dimensional reduction*, *Nuclear Physics B* **263** (Jan., 1986) 649–686.

[197] M. F. Sohnius, *Bianchi identities for supersymmetric gauge theories*, *Nuclear Physics B* **136** (1978) 461–474.

[198] R. Grimm, M. F. Sohnius, and J. Wess, *Extended supersymmetry and gauge theories*, *Nuclear Physics B* (1978).

[199] P. C. Argyres, *An Introduction to Global Supersymmetry*, *New York* no. 3.

[200] J. D. Lykken, *Introduction to Supersymmetry*, 9612114.

[201] M. Bertolini, *Lectures on Supersymmetry*, .

[202] S. Weinberg, *Nonrenormalization Theorems in Nonrenormalizable Theories*, *Physical Review Letters* **80** (Apr., 1998) 3702–3705, [9803099].

[203] L. Girardello and M. T. Grisaru, *Soft breaking of supersymmetry*, *Nuclear Physics B* **194** (1982), no. 1 65–76.

[204] S. P. Martin and M. T. Vaughn, *Two-Loop Renormalization Group Equations for Soft Supersymmetry-Breaking Couplings*, 9311340.

[205] J. Espinosa and M. Quirós, *On Higgs boson masses in non-minimal supersymmetric standard models*, *Physics Letters B* **279** (1992) 92–97.

[206] U. Ellwanger, C. Hugonie, and A. M. Teixeira, *The Next-to-Minimal Supersymmetric Standard Model*, *Physics Reports* **496** (Nov., 2010) 1–77, [arXiv:0910.1785].

[207] U. Ellwanger, *Higgs Bosons in the Next-to-Minimal Supersymmetric Standard Model at the LHC*, arXiv:1108.0157.

[208] S. Willenbrock, *Symmetries of the Standard Model*, 0410370.

[209] P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, *Isospin breaking in technicolor models*, *Nuclear Physics B* **173** (1980) 189–207.

[210] The D0 Collaboration, *Measurement of the W Boson Mass with the D0 Detector*, *Physical Review Letters* **108** (Apr., 2012) 151804, [arXiv:1203.0293].

[211] Tevatron Electroweak Working Group, for the CDF Collaboration, and D0 Collaboration, *2012 Update of the Combination of CDF and D0 Results for the Mass of the W Boson*, `arXiv:1204.0042`.

[212] V. Barger, P. Huang, M. Ishida, and W.-Y. Keung, *Scalar-Top Masses from SUSY Loops with 125 GeV mh and precise Mw, mt*, `arXiv:1206.1777`.

[213] T. Hahn and M. Perez-Victoria, *Automatized One-Loop Calculations in 4 and D dimensions*, *Computer* (July, 1998) 16, [9807565].

[214] J. Casas, J. Espinosa, and M. Quirós, *Improved Higgs mass stability bound in the standard model and implications for supersymmetry*, *Physics Letters B* **342** (Jan., 1995) 171–179, [9409458].

[215] J. Casas, J. Espinosa, and M. Quirós, *Standard model stability bounds for new physics within LHC reach*, *Physics Letters B* **382** (Aug., 1996) 374–382, [9603227].

[216] G. Isidori, G. Ridolfi, and A. Strumia, *On the metastability of the Standard Model vacuum*, *Nuclear Physics B* **609** (Aug., 2001) 387–409, [0104016].

[217] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto, and A. Strumia, *Higgs mass implications on the stability of the electroweak vacuum*, `arXiv:1112.3022`.

[218] G. Degrassi, S. Di Vita, J. Elias-Miró, J. R. Espinosa, G. F. Giudice, G. Isidori, and A. Strumia, *Higgs mass and vacuum stability in the Standard Model at NNLO*, `arXiv:1205.6497`.

[219] F. Bezrukov, M. Y. Kalmykov, B. a. Kniehl, and M. Shaposhnikov, *Higgs boson mass and new physics*, `arXiv:1205.2893`.

[220] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, *Investigating the near-criticality of the Higgs boson*, `arXiv:1307.3536`.

[221] G. Gamberini, G. Ridolfi, and F. Zwirner, *On radiative gauge symmetry breaking in the minimal supersymmetric model*, *Nuclear Physics B* **331** (1990) 331–349.

[222] P. Nath and R. Arnowitt, *Loop corrections to radiative breaking of electroweak symmetry in supersymmetry*, *Physical Review D* **46** (1992), no. 9 3981.

[223] B. de Carlos and J. A. Casas, *One-loop analysis of the electroweak breaking in supersymmetric models and the fine-tuning problem*, *Physics Letters B* **309** (1993), no. 3 320–328.

[224] S. P. Martin, *Generalized messengers of supersymmetry breaking and the sparticle mass spectrum*, *Physical Review D* **55** (Mar., 1997) 3177–3187, [9608224].

[225] M. Dine and A. E. Nelson, *Dynamical supersymmetry breaking at low energies*, *Physical Review D* (1993).

[226] M. Dine, A. Nelson, and Y. Shirman, *Low energy dynamical supersymmetry breaking simplified*, *Physical Review D* **51** (1995), no. 3 1362–1370.

[227] M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, *New tools for low energy dynamical supersymmetry breaking.*, *Physical review D: Particles and fields* **53** (Mar., 1996) 2658–2669.

[228] S. Dimopoulos and S. Raby, *Supercolor*, *Nuclear Physics B* **192** (1981), no. 1981 353–368.

[229] M. Dine, W. Fischler, and M. Srednicki, *Supersymmetric technicolor*, *Nuclear Physics B* **189** (1981) 575–593.

[230] M. Dine and W. Fischler, *A phenomenological model of particle physics based on supersymmetry*, *Physics Letters B* **110** (1982), no. 3 227–231.

[231] M. Dine and W. Fischler, *A supersymmetric gut*, *Nuclear Physics B* **204** (1982) 346–364.

[232] C. R. Nappi and B. A. Ovrut, *Supersymmetric extension of the SU(3)SU(2)U(1) model*, *Physics Letters B* **113** (June, 1982) 175–179.

[233] L. Alvarez-Gaume, M. Claudson, and M. Wise, *Low-energy supersymmetry*, *Nuclear Physics B* **207** (1982) 96–110.

[234] S. A. Abel, M. J. Dolan, J. Jaeckel, and V. V. Khoze, *Phenomenology of Pure General Gauge Mediation*, *Physics* (Oct., 2009) 30, [`arXiv:0910.2674`].

[235] S. A. Abel, M. J. Dolan, J. Jaeckel, and V. V. Khoze, *Pure General Gauge Mediation for Early LHC Searches*, *Physics* (Sept., 2010) 22, [`arXiv:1009.1164`].

[236] M. J. Dolan, D. Grellscheid, J. Jaeckel, V. V. Khoze, and P. Richardson, *New Constraints on Gauge Mediation and Beyond from LHC SUSY Searches at 7 TeV*, *Constraints* (Apr., 2011) 20, [`arXiv:1104.0585`].

[237] D. M. Pierce, J. A. Bagger, K. T. Matchev, and R.-J. Zhang, *Precision Corrections in the Minimal Supersymmetric Standard Model*, *Energy* (June, 1996) 57, [`9606211`].

[238] F. Staub, *Sarah*, `arXiv:0806.0538`.

[239] F. Staub, *From superpotential to model files for FeynArts and CalcHep/CompHEP*, *Computer Physics Communications* (2010) [`0909.2863`].

[240] F. Staub, *Automatic calculation of supersymmetric renormalization group equations and loop corrections*, *Computer Physics Communications* (2011) 1–32, [`1002.0840`].

[241] F. Staub, *SARAH 3.2: Dirac gauginos, UFO output, and more*, *Computer Physics Communications* (July, 2013) 1–24, [`arXiv:1207.0906`].

[242] F. Staub, *SARAH 4: A tool for (not only SUSY) model builders*, *arXiv preprint arXiv:1309.7223* (Sept., 2013) 24, [`arXiv:1309.7223`].

[243] W. Porod, F. Staub, and A. Vicente, *A Flavor Kit for BSM models*, `arXiv:1405.1434`.

[244] M. D. Goodsell and P. Slavich, *No title*, *in preparatation* (2014).

[245] F. Lyonnet, I. Schienbein, F. Staub, and A. Wingerter, *PyR@TE: Renormalization Group Equations for General Gauge Theories*, `arXiv:1309.7030`.

[246] G. Bhattacharyya and T. Ray, *Pushing the SUSY Higgs mass towards 125 GeV with a color adjoint*, *Physical Review D* **87** (Jan., 2013) 015017.

[247] P. Grajek, A. Mariotti, and D. Redigolo, *Phenomenology of General Gauge Mediation in light of a 125 GeV Higgs*, arXiv:1303.0870.

[248] N. Arkani-Hamed, A. Delgado, and G. F. Giudice, *The well-tempered neutralino*, *Nuclear Physics B* **741** (May, 2006) 108–130, [0601041].

[249] R. Brock, M. E. Peskin, K. Agashe, M. Artuso, J. Campbell, S. Dawson, R. Erbacher, C. Gerber, Y. Gershtein, A. Gritsan, K. Hatakeyama, J. Huston, A. Kotwal, H. Logan, M. Luty, K. Melnikov, M. Narain, M. Papucci, F. Petriello, S. Prell, J. Qian, R. Schwienhorst, C. Tully, R. Van Kooten, D. Wackeroth, L. Wang, and D. Whiteson, *Planning the Future of U.S. Particle Physics (Snowmass 2013): Chapter 3: Energy Frontier*, arXiv:1401.6081.

[250] M. Berggren, T. Han, J. List, S. Padhi, S. Su, and T. Tanabe, *Electroweakino Searches: A Comparative Study for LHC and ILC (A Snowmass White Paper)*, arXiv:1309.7342.

[251] The ATLAS Collaboration, *Search for strongly produced supersymmetric particles in final states with two same-sign leptons and jets with the ATLAS detector using 21 fb1 of proton-proton collisions at* $\sqrt{s} = 8$ *TeV*, in *ATLAS NOTE, ATLAS-CONF-2013-007*, pp. 0–26, 2013.

[252] S. Calvet, B. Fuks, P. Gris, and L. Valéry, *Searching for sgluons in multitop events at a center-of-mass energy of 8 TeV*, *Journal of High Energy Physics* **2013** (2013) [1212.3360].

[253] R. Barbieri and G. F. Giudice, *Upper bounds on supersymmetric particle masses*, *Nuclear Physics B* **306** (1988) 63–76.

[254] A. Kaminska, G. G. Ross, K. Schmidt-Hoberg, and F. Staub, *A precision study of the fine tuning in the DiracNMSSM*, *Journal of High Energy Physics* **2014** (June, 2014) 153, [arXiv:1401.1816].

[255] S. P. Martin and M. T. Vaughn, *Regularization Dependence of Running Couplings in Softly Broken Supersymmetry*, 9308222.

[256] D. Kutasov and A. Schwimmer, *On duality in supersymmetric Yang-Mills theory*, *Physics Letters B* **354** (July, 1995) 315–321, [9505004].

[257] D. Kutasov, A. Schwimmer, and N. Seiberg, *Chiral rings, singularity theory and electric-magnetic duality*, *Nuclear Physics B* **459** (1996), no. 3 455–496.

[258] S. A. Abel and V. V. Khoze, *Direct mediation, duality and unification*, *Journal of High Energy Physics* **2008** (Nov., 2008) 024–024, [arXiv:0809.5262].

[259] S. A. Abel and V. V. Khoze, *Dual unified SU(5)*, *Journal of High Energy Physics* **2010** (Jan., 2010) 6, [arXiv:0909.4105].

[260] K. Intriligator, R. G. Leigh, and M. J. Strassler, *New examples of duality in chiral and non-chiral supersymmetric gauge theories*, *Nuclear Physics B* **456** (Dec., 1995) 567–621, [9506148].

[261] J. H. Brodie and M. J. Atrassler, *Patterns of duality in N = 1 SUSY gauge theories*, *Nuclear Physics B* **524** (July, 1998) 224–250, [9611197].

[262] I. Antoniadis, H. Partouche, and T. R. Taylor, *Spontaneous breaking of N= 2 global supersymmetry*, *Physics Letters B* **372** (1996), no. April 83–87.

[263] I. Antoniadis and T. R. Taylor, *Dual N = 2 SUSY Breaking*, *Fortschritte der Physik/Progress of Physics* **44** (Apr., 1996) 487–492, [9604062].

[264] H. Partouche and B. Pioline, *Partial spontaneous breaking of global supersymmetry*, *Nuclear Physics B - Proceedings Supplements* **56** (July, 1997) 322–327, [9702115].

[265] K. Intriligator and B. Wecht, *The exact superconformal R-symmetry maximizes a*, *Nuclear Physics B* **667** (Sept., 2003) 183–200, [0304128].

[266] K. Intriligator and B. Wecht, *RG fixed points and flows in SQCD with adjoints*, *Nuclear Physics B* **677** (2004), no. 1-2 59.

[267] Z. Komargodski and A. Schwimmer, *On Renormalization Group Flows in Four Dimensions*, arXiv:1107.3987.

[268] D. Kutasov, A. Parnachev, and D. A. Sahakyan, *Central charges and $U(1)_R$ symmetries in $\mathcal{N} = 1$ super Yang-Mills*, *Journal of High Energy Physics* **2003** (Nov., 2003) 013–013, [0308071].

[269] D. Anselmi, D. Freedman, M. Grisaru, and A. Johansen, *Non-perturbative formulas for central functions of supersymmetric gauge theories*, *Nuclear Physics B* **526** (Aug., 1998) 543–571, [9708042].

[270] D. Anselmi, J. Erlich, D. Freedman, and A. Johansen, *Positivity constraints on anomalies in supersymmetric gauge theories*, *Physical Review D* **57** (June, 1998) 7570–7588, [9711035].

[271] N. Ohta, H. Sugata, and H. Yamaguchi, *N = 2 harmonic superspace with central charges and its application to self-interacting massive hypermultiplets*, *Annals of Physics* **172** (Nov., 1986) 26–39.

[272] E. A. Ivanov and B. M. Zupnik, *Modified N=2 supersymmetry and Fayet-Iliopoulos terms*, 9710236.

[273] K. Fujiwara, H. Itoyama, and M. Sakaguchi, *Supersymmetric U (N) Gauge Model and Partial Breaking of N =2 Supersymmetry*, *Progress of Theoretical Physics* **113** (Feb., 2005) 429–455, [0409060].

[274] K. Fujiwara, H. Itoyama, and M. Sakaguchi, *Partial breaking of supersymmetry and of gauge symmetry in the gauge model*, *Nuclear Physics B* **723** (Sept., 2005) 33–52, [0503113].

[275] K. Fujiwara, H. Itoyama, and M. Sakaguchi, *U(N) Gauge Model and Partial Breaking of N=2 Supersymmetry*, 0410132.

[276] K. Fujiwara, H. Itoyama, and M. Sakaguchi, *Spontaneous Partial Breaking of N = 2 Supersymmetry and the U(N) Gauge Model*, in *AIP Conference Proceedings*, vol. 903, pp. 521–524, AIP, Nov., 2007. 0611284.

[277] K. Fujiwara, *Partial breaking of supersymmetry and decoupling limit of NambuGoldstone fermion in gauge model*, *Nuclear Physics B* **770** (May, 2007) 145–153, [0609039].

[278] H. Itoyama, K. Maruyoshi, and M. Sakaguchi, *N=2 quiver gauge model and partial supersymmetry breaking*, *Nuclear Physics B* **794** (May, 2008) 216–230, [arXiv:0709.3166].

[279] S. J. Gates, *Superspace formulation of new non-linear sigma models*, *Nuclear Physics B* **238** (1984) 349–366.

[280] J. Hughes, J. Liu, and J. Polchinski, *Supermembranes*, *Physics Letters B* **180** (1986), no. 4 370–374.

[281] J. T. Lopuszanski, *The spontaneously broken supersymmetry in quantum field theory*, *Reports on Mathematical Physics* **13** (1978), no. 1.

[282] S. Ferrara, L. Girardello, and M. Porrati, *Spontaneous breaking of N= 2 to N= 1 in rigid and local supersymmetric theories*, *Physics Letters B* (1996) [9512180v1].

[283] P. C. Argyres and M. Douglas, *New phenomena in SU(3) supersymmetric gauge theory*, *Nuclear Physics B* (May, 1995) [9505062v1].

[284] R. G. Leigh and M. J. Strassler, *Exactly marginal operators and duality in four dimensional $N = 1$ supersymmetric gauge theory*, *Nuclear Physics B* **447** (July, 1995) 95–133, [9503121].

[285] M. J. Strassler, *Manifolds of Fixed Points and Duality in Supersymmetric Gauge Theories*, *Progress of Theoretical Physics Supplement* **123** (Feb., 1996) 373–380, [9602021].

[286] G. Passarino and M. Veltman, *One-loop corrections for e+e annihilation into μ+μ in the Weinberg model*, *Nuclear Physics B* **160** (1979) 151–207.

[287] A. D. Box and X. Tata, *Threshold and flavor effects in the renormalization group equations of the MSSM. II*, *Physical Review D* **82** (Dec., 2010) 119905, [arXiv:0810.5765].

[288] A. D. Box and X. Tata, *Threshold and flavor effects in the renormalization group equations of the MSSM: Dimensionless couplings*, *Physical Review D* **82** (Dec., 2010) 119904, [arXiv:0712.2858].

[289] P. C. Argyres, M. Ronen Plesser, and N. Seiberg, *The moduli space of vacua of $N = 2$ SUSY QCD and duality in $N = 1$ SUSY QCD*, Nuclear Physics B **471** (July, 1996) 159–194, [9603042].