Duality and Models of Supersymmetry Breaking

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Abstract

Duality is often most clearly manifest in supersymmetric theories, where the rigid mathematical structure affords good control over the behaviour of the system. In many real-world applications, and particularly in particle physics at the TeV scale, supersymmetry can only be present as a broken symmetry. In this thesis we explore various situations in which duality can continue be important when supersymmetry is broken spontaneously, or even explicitly.

We first focus on the AdS/CFT correspondence, and consider the effect of instantons in a non-supersymmetric gauge theory obtained via a marginal deformation of $\mathcal{N} = 4$ super Yang-Mills. This gauge theory is expected to be dual to type IIB string theory on a background that is the product of five-dimensional anti-de Sitter spacetime and a *deformed* five-sphere. By performing an instanton calculation in the deformed gauge theory we extract a prediction for the dilaton-axion field τ in dual string theory. In the limit of small deformations where the supergravity regime is valid, our instanton result reproduces the expression for τ of the supergravity solution originally found by Frolov, thus supporting the validity of the correspondence.

We then go on to look at how supersymmetry breaking in a metastable vacuum allows one to build simple and concrete models of gauge mediation. In the prototypical model of Intriligator, Seiberg and Shih (ISS), Seiberg duality plays an important rôle in ensuring the longevity of the metastable vacuum. In a move to construct more realistic models we deform the ISS model by adding a baryon term to the superpotential. This simple deformation causes spontaneous breaking of the approximate R-symmetry of the metastable vacuum. We then gauge an $\mathrm{SU}(5)_f$ flavour group and identify it with the parent gauge symmetry of the supersymmetric Standard Model.

This implements direct mediation of supersymmetry breaking without the need for an additional messenger sector. A reasonable choice of parameters leads to gaugino masses of the right order.

To further explore the phenomenology of metastable SUSY breaking we distinguish different types of models by the manner in which R-symmetry is broken in the metastable vacuum. In general, there are two possible ways to break R-symmetry: explicitly or spontaneously. We find that the MSSM phenomenology can be greatly affected how this breaking occurs in the Hidden Sector. Explicit R-symmetry breaking models lead to fairly standard gauge mediation patterns, but we argue that in the context of ISS-type models this only makes sense if $B_\mu = 0$ at the mediation scale. This leads to high values of $\tan\beta$ as a generic prediction. If on the other hand R-symmetry is broken spontaneously, then R-violating soft terms tend to be suppressed with respect to the R-symmetry preserving ones, and one is led to a scenario with large scalar masses. These models interpolate between standard gauge mediation and split SUSY models. We provide benchmark points for the two scenarios, which serve to demonstrate that the specific dynamics of the Hidden Sector — the underlying nature of supersymmetry and R-symmetry breaking — can considerably affect the mass spectrum of the MSSM.

Declaration

This thesis is the result of my own work, except where explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university.

Chapter 3 is based on the published work [1]:

"Instanton test of non-supersymmetric deformations of the $AdS_5 \times S^5$"
C. Durnford, G. Georgiou and V. V. Khoze
JHEP **0609**, 005 (2006) [arXiv:hep-th/0606111]

Chapters 4, 5 and 6 are based on the published works [2, 3]:

"Dynamical breaking of $U(1)_R$ and supersymmetry in a metastable vacuum"
S. Abel, C. Durnford, J. Jaeckel and V. V. Khoze
Phys. Lett. B **661**, 201 (2008) [arXiv:0707.2958 [hep-ph]]

"Patterns of Gauge Mediation in Metastable SUSY Breaking"
S. A. Abel, C. Durnford, J. Jaeckel and V. V. Khoze
JHEP **0802**, 074 (2008) [arXiv:0712.1812 [hep-ph]]

This thesis does not exceed the word limit for the respective Degree Committee.

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Chapter 1.

Introduction

"The eternal mystery of the world
is in its comprehensibility."
— Albert Einstein

1.1. Non-technical Overview

With the immanent start-up of the Large Hadron Collider, this is an exciting time to be a theoretical particle physicist. Whilst working to develop elegant models with robust predictions, we can be happy in the knowledge that our ideas will soon be tested against experimental data. Frequently when building such models, one runs into problems of strong coupling, where the standard mathematical approach used to make predictions — perturbation theory — stops working. The core of my PhD research has been to shed light on such problems with the help of duality; the fascinating idea that one can calculate interesting physical quantities by altering how one mathematically frames the problem. This change of perspective can also require us to re-examine what we perceive as the fundamental degrees of freedom of a system. Should we think in terms of interacting quarks and electrons, bits of strings vibrating in higher dimensions, or something else? Providing answers to these questions, in turn, modifies our understanding of the world around us.

Broadly speaking, research into duality falls into two distinct areas. Firstly, the existence of any such correspondence between different theories must be established. This involves building up a bank of non-trivial cross-checks (when direct proof is too difficult), and also exploring the domain of validity of the duality to see how far the phenomenon persists. Our work described in Chapter 3 is a good example of this: by carefully modifying the well-understood correspondence between a particular string theory and a particular gauge theory,[1] a new class of dual solutions was proposed in the papers [4] and [5]; we find that a multi-instanton calculation in the gauge theory correctly reproduces the behaviour of the proposed gravity solution, thus affirming the correspondence in this new regime.

Secondly, one can construct models that make use of duality to calculate quantities that are beyond the scope of standard quantum field theory. Duality is often most clearly manifest in supersymmetric theories, where the rigid mathematical structure ensures that the high energy behaviour is well under control. However, to connect with particle physics experiment, the rigid structure of supersymmetry must be broken. In Chapter 4 we see why this has to happen, and explore how the breaking may be achieved, thus connecting supersymmetric theories — which we understand — to the real world — which we would like to understand!

One intriguing phenomenon that features in recent studies of supersymmetry breaking [6], and that seems to be evident in a wide class of models (cf. Section 4.2.2), is that our Universe is only *metastable*. In this case the low energy breaking of supersymmetry is only temporary: quantum effects can potentially restore supersymmetry and thus fundamentally change the nature of matter. Discovery of such an instability would profoundly change our understanding of the Universe; for a non-technical review of these ideas see reference [7]. The supersymmetry breaking model we construct in Chapter 5 makes vital use of Seiberg duality — a correspondence between two different gauge theories. This duality makes it possible to calculate the low energy behaviour of the model whilst ensuring that the Universe is sufficiently stable.

When discussing possible models of supersymmetry breaking, it is important to calculate how their consequences impact on experimental data — both the array of existing results, and the eagerly anticipated output of the Large Hadron Collider. As the required

[1] String theory is a quantum theory of gravity. Gauge theories are used to describe all the forces of Nature *except* gravity. A special duality can sometimes be found between the two of them, and is known as the AdS/CFT correspondence.

breaking of supersymmetry can potentially show up in our detectors in a variety of different ways,[2] it is going to take a lot of work to establish which scenarios are preferred by the data (and in fact whether supersymmetry is relevant to low scale physics at all). In Chapter 6 we investigate the phenomenology of different classes of metastable supersymmetry breaking models, and discuss interesting features that distinguish them amongst the pantheon of other models. Such studies will be vital in exploring the connection between theory and experiment when data from the LHC begins to reshape the landscape of particle physics models.

1.2. Outline of Thesis

After this short overview, we begin in earnest in Chapter 2 by presenting the four main areas of theoretical physics that our subsequent work will be drawn from. These are: Renormalisation, Supersymmetry, String Theory and the AdS/CFT Correspondence. The main purpose of reviewing this material is really twofold. We will establish the language and notation that will be of use to us later on, and we will also be able to focus attention on aspects of the theory that later become central to our purpose. All the physics described in this chapter can be found in the original research papers, and also in a variety of books and review articles as indicated in the references. The discussion of instantons in the AdS/CFT correspondence (Section 2.4.3) is well established in the literature, but perhaps less well known to the average graduate student.

In Chapter 3 we further explore the duality, described in Section 2.4, between gravity and gauge theories. Following our published work [1] we consider instanton effects in a non-supersymmetric gauge theory that is constructed by making a marginal deformation of $\mathcal{N} = 4$ super Yang-Mills. Under the AdS/CFT correspondence, this conformal gauge theory is expected to be dual to Type IIB string theory with background geometry that is a product of five-dimensional Anti-de Sitter space and a deformed five-sphere. From an instanton calculation in the deformed gauge theory, in Section 3.3 we extract a prediction for the dilaton-axion field τ in its dual string theory. In the limit of small deformations where the supergravity regime is valid, our instanton result reproduces the expression for τ in the supergravity solution found by Frolov [5]. This provides further support in favour the conjectured correspondence.

[2] This is the truism that you don't quite know what you're looking for until you find it.

We then turn our attention in Chapter 4 to the subject of supersymmetry breaking. Section 4.1 explains some of the mathematical and physical contraints that one must be aware of when constructing realistic SUSY breaking models. This is followed by a discussion of metastablility: recent interest in this idea was sparked by reference [6], where a simple model was presented in which SUSY is dynamically broken in a metastable vacuum. This has become known as the ISS model; we will see how it works in Section 4.2.1. It was further argued in reference [6] that metastable SUSY breaking vacua are quite commonplace, and so should allow one to build simple, physically viable models of particle physics. Indeed, subsequent work [3, 8] has shown that, given some rather general assumptions that we outline in Section 4.2.2, metastablility is actually *unavoidable*; R-symmetry plays a key rôle in establishing this conclusion. To add further credence to the metastable SUSY breaking paradigm, there also turn out to be compelling cosmological reasons for why we should find ourselves trapped in a metastable vacuum [9–11].

Picking up the gauntlet laid down by ISS [6], in Chapter 5 we investigate the feasability of building an simple model in which SUSY is broken in a metastable vacuum with the effects communicated *directly* to the MSSM (i.e. without invoking an additional set of messenger fields). Such models have the benefit of being both compact and predictive. We argue in Section 5.1 that an important aspect of any such construction is the manner in which R-symmetry is broken. The model we introduce in Section 5.2 is constructed by deforming the ISS model with the addition of a baryon-type operator. This initially leads to a runaway potential, which we show is stabilised by Coleman-Weinberg corrections at one loop. As a result, we end up with a metastable SUSY breaking vacuum in which R-symmetry is broken *spontaneously*. We can then go ahead and gauge an $SU(3) \times SU(2) \times U(1)$ subgroup of the ISS flavour symmetry to directly communicate SUSY breaking to the MSSM. In Section 5.3.1 we make a first pass at understanding the phenomenology of this model, with a more comprehensive survey postponed until Chapter 6. One preliminary observation is that gaugino masses are suppressed relative to expectations based on standard gauge mediation models. We also discuss the R-axion that arises as the Goldstone boson of spontaneously broken R-symmetry, and show that non-perturbative effects give it sufficient mass to evade potentially dangerous astrophysical bounds.

Chapter 6 is largely drawn from our work [3] and is organised as follows. After setting the scene in Section 6.1, we go on to study a particular scenario for metastable gauge mediation that was formulated by Murayama & Nomura and Aharony & Seiberg in references [12, 13]. This class of models has a dedicated messenger sector in which the R-symmetry of the ISS sector is broken explicitly. In this case the effective R-symmetry breaking is weak is because the messengers are coupled to the Hidden Sector fields only via $1/M_{\text{Pl}}$-suppressed operators, cf. equation (6.1). In the limit where $M_{\text{Pl}} \to \infty$, both R-symmetry and the supersymmetry of the MSSM are exact, since the ISS Hidden Sector decouples from the messengers. As a result, in these models the effective R-symmetry breaking and the effective SUSY breaking scales in the Visible Sector are essentially the same. The generated gaugino and scalar soft mass terms are of the same order, so the resulting phenomenology of these models [12, 13] is largely of the usual form [14].

In Section 6.3 we study gauge mediation with spontaneous R-symmetry breaking. Specifically, we concentrate on the *direct* gauge mediation model of Chapter 5 where the entire Hidden-plus-Messenger sector consists of only the baryon-deformed ISS theory with $N_f = 7$ flavours and $N_c = 5$ colours. The resulting gaugino and sfermion soft masses are discussed in Section 6.3.2. In Section 6.3.3 we analyse the phenomenology of this class of models, which turns out to be quite different from the usual gauge mediation scenarios [14]. A significant reason for this difference is the fact that R-symmetry is broken spontaneously by one-loop corrections, and as such the scale of R-breaking is naturally smaller than the scale of SUSY breaking, leading to the gaugino masses being smaller than the scalar masses. This is different from the usual gauge-mediation assumption that the R-symmetry breaking is larger than the SUSY breaking. Thus one generally expects a Hidden sector with spontaneous R-symmetry violation to interpolate between standard gauge mediation and split SUSY models [15–17].

Throughout Chapter 6 we will be treating the μ_{MSSM} parameter of the MSSM as a free parameter. As it is SUSY preserving it does not have to be determined by the ISS Hidden Sector, and we will, for this discussion, have little to say about it: we will not address the question of why it should be of order the weak scale, the so-called μ problem (cf. Section 4.1.1). However the corresponding SUSY breaking parameter B_μ cannot consistently be taken to be a free parameter. It is determined by the models at the messenger scale, and in both cases it is approximately zero, as will be explained in detail. In Section 6.4 we display a couple of benchmark points to illustrate the general phenomenological features expected in metastable SUSY breaking models, and critically apraise the various assumptions we made in deriving these results.

We conclude in Section 6.5 by summarising what has been learnt about the phenomenology of metastable gauge mediation, and by indicating some interesting directions for future research. These include ambitious ideas for dealing with Landau poles, and suggestions for what we might try if the LHC doesn't work!

Appendix A is a digest of the different notation used throughout this work, including the metric and spinor conventions that make supersymmetric calculations so entertaining. We use Appendix B to clarify a subtle point relating to the precise R-symmetry that is used in the construction of our baryon-deformed model of Chapter 5.

Chapter 2.

Fundamentals

"Everything is vague to a degree you do not realise
till you have tried to make it precise."
— Bertrand Russell

This chapter is an exposition the key ideas that will be combined and explored in the remainder of the thesis. The existing literature on these subjects is sufficient to fill a small library, so here we only aim to set the scene and establish notation, leaving any form of comprehensive review to the indicated works.

2.1. Renormalisation

Quantum field theory is truly a triumph of 20th century physics. It is a conceptual framework that allows us to predict a wide variety of observable phenomena with unprecedented accuracy. Unsurprisingly, the calculations required to make such predictions are not without their complications. For example, it is quite common to arrive at answers that are formally infinite. The prescription for dealing with these divergences is called RENORMALISATION, and although when one first meets it, it feels a bit like "brushing things under a rug", careful study of the origin of the divergences leads to an improved understanding of the physical situation and to the very definition of a quantum field theory.

Every quantum field theory textbook ever written contains a discussion of renormalisation. My favourites are Peskin & Schroeder [18] and Zee [19]. A more mathematical perspective is given in Banks' recent book [20], or Gross' lectures in [21]. There is also a set of lecture notes available from Tim Hollowood's website [22] that tie everything together nicely.

2.1.1. The Renormalisation Group

The important idea here is that the physics of a system is strongly dependent on the energy scale μ at which one is probing the system. This allows us to model the system in terms of MACROSCOPIC variables which capture the behaviour at this energy scale without requiring precise knowledge of the physics at higher energies. Of course, the macroscopic description of a system will depend on this MICROSCOPIC physics, but one does not require the full resolving power of very high energy degrees of freedom to be able to describe long distance physics with good accuracy. This is the reason physics works! For instance, we would be in real trouble if we had to take into account the behaviour of all the quarks and electrons involved when calculating the trajectory of a tennis ball.[1]

Sometimes the macroscopic description of physics can be given in terms of variables that are different to those of the microscopic theory. The low energy theory is then said to provide an EFFECTIVE FIELD THEORY description of the underlying physics. There is no need for the effective variables to even make sense at much higher energies, indeed the breakdown of an effective description at high energy is precisely what signals the need to introduce new physics. Despite their limited domain of validity, effective theories are incredibly useful: they can make calculations more tractable by boiling everything down to the most important physical effects.

Taking this notion one step further, we are lead to the idea of DUALITY. This is where two different mathematical models describe the same physical system at all energy scales. This fascinating phenomenon can be very useful because each mathematical 'picture' generally reveals different aspects of the physics. Quantities that are hard to calculate in one framework may be much easier to understand from the alternative point of view. We'll see examples of this at many points in the coming chapters. The

[1] By which we mean non-relativistic tennis — nothing fancy.

distinction between effective field theories and dualities is often blurred in practice, as will be evident in our discussion of Seiberg duality in Section 2.2.2.

We should now take a more careful look at the issue of scale dependence. Consider a scalar field theory defined by the Lagrangian[2]

$$\mathcal{L}_0 = \frac{1}{2}\partial^\mu \phi_0^\dagger \, \partial_\mu \phi_0 + \sum_i g_{0,\,i}\, \mathcal{O}_i(\phi_0, \partial_\mu) \; . \tag{2.1}$$

The operators $\mathcal{O}_i$ are built from products of the scalar field ϕ_0 and possibly derivatives thereof. For the time being, we will assume that all the operators have a scaling dimension that is equal to the number of ambient spacetime dimensions, so each comes with its own *dimensionless* coupling $g_{0,\,i}$. The resulting physics is encoded in the correlation functions

$$\big\langle \phi_0(x_1) \cdots \phi_0(x_n) \big\rangle = \big\langle 0 \big| \, T\, \phi_0(x_1) \cdots \phi_0(x_n) \big| 0 \big\rangle \tag{2.2a}$$

$$\equiv G_n^{(0)}(x_i, \{g_0\}) \; , \tag{2.2b}$$

which can be used to calculate scattering amplitudes, for example. Now suppose we're looking for an effective theory with field ϕ and Lagrangian $\mathcal{L}_\mu$ that describes the same physics as $\mathcal{L}_0$, but at a characteristic scale μ. For the time-being we will make the reasonable assumption that the Lagrangian $\mathcal{L}_\mu$ is structurally the same as $\mathcal{L}_0$, i.e. it contains the same operators, but perhaps with different *renormalised* values for the coupling coefficients

$$\mathcal{L}_\mu = \frac{1}{2} Z \, \partial^\mu \phi^\dagger \, \partial_\mu \phi + \sum_i g_i \, Z^{d_i/2} \, \mathcal{O}_i(\phi, \partial_\mu) \; . \tag{2.3}$$

Note that here we have also allowed for the possibility of WAVEFUNCTION RENORMALISATION by including the factors of Z. The integers d_i tell us how many powers of the field there are in $\mathcal{O}_i$.

To understand what is meant by 'physics at the characteristic scale μ' one has to specify RENORMALISATION CONDITIONS. These are a set of equations that connect the parameters $\{g\}$ in the effective Lagrangian $\mathcal{L}_\mu$ to physical quantities, such as correlation functions, measured at the scale μ. The statement that both theories $\mathcal{L}_0$ and $\mathcal{L}_\mu$ describe

[2] For simplicity we will assume a Lagrangian description of physics exists, although the discussion will actually be in terms of correlation functions, which is more generally applicable.

the same physics, is the same as saying they lead to the same correlation functions:

$$G_n^{(0)}(x_i, \{g_0\}) \;\equiv\; Z^{n/2} G_n(x_i, \{g\}, \mu) \;. \tag{2.4}$$

Here we have used the (hopefully obvious) notation that $G_n^{(0)}$ follows from $\mathcal{L}_0$. Now observe that the left hand side of equation (2.4) does not depend on the renormalisation scale μ, i.e.

$$\mu \frac{d}{d\mu} G_n^{(0)}(x_i, \{g_0\}) \;=\; 0 \;. \tag{2.5}$$

It therefore follows that any explicit μ dependence in $G_n(x_i, \{g\}, \mu)$ must be accounted for by a concomitant change in the couplings $\{g\}$ and Z. Equation (2.5) then leads to the CALLAN-SYMANZIK EQUATIONS

$$\left(\mu \frac{\partial}{\partial \mu} \;+\; \beta_i(g) \frac{\partial}{\partial g_i} \;+\; \frac{n}{2}\gamma(g)\right) G_n(x_i, \{g\}, \mu) \;=\; 0 \;. \tag{2.6a}$$

where we define the BETA FUNCTIONS

$$\beta_i(g) \;=\; \mu \frac{d}{d\mu} g_i \;, \tag{2.6b}$$

and ANOMALOUS DIMENSION of the field

$$\gamma(g) \;=\; \mu \frac{d}{d\mu} \log{(Z)} \;\;. \tag{2.6c}$$

Equations (2.6b) and (2.6c) are known as the RENORMALISATION GROUP EQUATIONS. They describe how the renormalised parameters change, or RUN, as one varies the renormalisation scale μ. It is hard to overstate the importance of these equations. They are widely applied across much of modern physics (and mathematics), and will be recurring in many guises throughout this thesis.

Dealing with Divergences

Renormalisation is vital for controlling the plethora of divergences that arise when calculating physical quantities in quantum field theory. For example, quantum corrections to scattering amplitudes, which are often calculated as a (perturbative) expansion in the coupling constants, usually contain integrals over loop momenta that are formally divergent. To make sense of the mathematics, one must first REGULARISE these integrals — make them finite in some way. This can be done, for example, by working in a higher number of dimensions, or more simple-mindedly by truncating the range of integration for loop momenta. Given the arbitrary nature of this regularisation procedure, it would be a disaster if physical quantities were found to depend on the regularisation parameter. This is where renormalisation comes to the rescue: one can carefully adjust the renormalised couplings $\{g\}$ and Z such that when the regulator is removed, the renormalisation conditions are preserved, and all correlators of physical fields remain finite.

There is an important caveat here, pertaining to the 'reasonable' assumption we made above: that the effective Lagrangian $\mathcal{L}_\mu$ contains the same operators as the bare Lagrangian $\mathcal{L}_0$. This is only valid for a certain class of models, which are termed RENORMALISABLE. By definition, these are theories in which only a finite number of terms in the Lagrangian need to be adjusted to remove UV divergences. Otherwise, a theory is deemed NON-RENORMALISABLE — in this case, cancellation of UV divergences requires one to introduce an infinite number of new operators to the renormalised action (one is eventually forced to introduce every operator consistent with the symmetries at some order in perturbation theory). The proliferation of operators required to fix the divergences in non-renormalisable models indicates that there is a real problem defining the limit in which the (artificial) UV regulator is removed. For this reason it only makes sense to think of such models as effective theories, valid up to a certain energy scale.

However, this is not the end of the story; if one can show that the effective theory emerges as the low energy behaviour of a theory that *does* have a well defined UV limit, then all is well. The microscopic model is known as a UV COMPLETION of the effective theory. Nothing we have said here precludes the existence of two *different* UV completions for a single macroscopic model. This reflects our earlier comment about why effective theories are so effective for studying physics: they provide a useful calculational tool without requiring us to know the precise details of ultra-high energy physics.

2.1.2. The Relevance of Fixed Points

It is often useful to think about the renormalisation group (RG) equations (2.6) by how they act on the space of Lagrangians of a theory. Different directions in this space correspond to the different operators that may appear in the Lagrangian, with the couplings providing local coordinates on the space. When continuously changing the renormalisation scale from μ to another, μ', with $\mu' < \mu$, the RG equations define a

FLOW, which describes how the couplings must change in order to keep the underlying physics invariant. Following the flow from one scale to another induces a transformation on the couplings, with the set of all such transformations forming what is known as the RENORMALISATION GROUP.

Special significance is attached to fixed points of the RG flow. By definition the physics associated to such points is SCALE INVARIANT, and clearly the beta functions defined in equation (2.6b), which describe how the couplings vary with the renormalisation scale, must also vanish. It is common lore that Lorentz invariant quantum field theories that are scale invariant have their spacetime symmetries enhanced from the Poincaré group to the CONFORMAL group. There are a few known exceptions to this rule, but they are rather pathological, so it is common to be lax and take *scale invariant* and *conformal* as synonymous. One of the reasons conformal fixed points are interesting is because the enhanced symmetry provides extra constraints on physics that makes the whole system mathematically more tractable. This is particularly evident in two dimensions, where the conformal group is infinite dimensional, but as we will see in Chapter 3, conformality also provides a useful tool for analysing field theories in higher numbers of dimensions.

Working at a conformal fixed point allows us to make the above definition of the field's anomalous dimension particularly transparent. One way of thinking about the canonical 'engineering' dimensions of a quantity is to ask how it transforms under a rescaling of units $x^\mu \to s^{-1}x^\mu$. For example, a scalar field $\phi(x)$ in D dimensions will pick up a factor of s^{d_ϕ} where $d_\phi = \frac{D}{2} - 1$. Translating this in terms of correlation functions gives us

$$\left(s\frac{\partial}{\partial s} + \mu\frac{\partial}{\partial\mu}\right) G_n(s^{-1}x_i, \{g\}, \mu) \;=\; n\, d_\phi\, G_n(s^{-1}x_i, \{g\}, \mu)\ . \tag{2.7}$$

Subtracting the Callan-Symanzik equation (2.6a) evaluated at a fixed point with couplings $g_i = g_i^*$ leads to

$$\left(s\frac{\partial}{\partial s} - n\left[d_\phi + \frac{1}{2}\gamma(g^*)\right]\right) G_n(s^{-1}x_i, \{g\}, \mu) \;=\; 0\ . \tag{2.8}$$

Notice that the contribution $\frac{1}{2}\gamma(g^*)$ from wavefunction renormalisation enters this equation in the same way as the canonical dimension of the field, thus accounting for the name: the combination $d_\phi + \frac{1}{2}\gamma(g^*)$ is known as the anomalous scaling dimension of the field.

Once the physics of a fixed point is understood, one can investigate the effects of perturbing it by adding all sorts of new operators into the Lagrangian. Consider adding operators of canonical dimension k_i to the renormalised Lagrangian (2.3) in 4 dimensions:

$$\widehat{\mathcal{L}} = \mathcal{L}_\mu + \sum_i \hat{g}_i \, \mu^{4-k_i} \, \mathcal{O}_i(\phi, \partial_\mu) \; . \tag{2.9}$$

Here we have included explicit factors of the renormalisation scale to soak up the engineering dimensions of the operators and thus render the couplings $\hat{g}_i$ dimensionless. Proceeding as above, one can go on to derive RG equations

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_i(g) \frac{\partial}{\partial g_i} + \frac{n}{2} \gamma(g) + \sum_i \Big[\gamma_i(g) - 4 + k_i \Big] \, \hat{g}_i \, \frac{\partial}{\partial \hat{g}_i} \right) G_n(x_i, \{g\}, \{\hat{g}\}, \mu) = 0 \; . \tag{2.10}$$

In this equation, the explicit μ dependence of the perturbation in equation (2.9) is offset by a rescaling of $\hat{g}_i$, and we have also introduced the anomalous dimension of the *operator* $\mathcal{O}_i$ in much the same way as it is defined for the scalar field itself:

$$\gamma_i(g) = \mu \frac{d}{d\mu} \log\left(\hat{g}_i\right) \; . \tag{2.11}$$

Observe that it is also possible to think of the coefficient of $\frac{\partial}{\partial \hat{g}_i}$ as another beta function that indicates how the dimensionless coupling $\hat{g}_i$ runs:

$$\beta_i(g) = \Big[\gamma_i(g) - 4 + k_i \Big] \, \hat{g}_i \; . \tag{2.12}$$

Now consider the flow of couplings $\hat{g}_i$ in the vicinity of a fixed point g_i^*. This can be understood by integrating the beta function (2.12) to give

$$\hat{g}_i(\mu) = \left(\frac{\mu}{\mu_0} \right)^{\gamma_i(g^*) - 4 + k_i} \hat{g}_i(\mu_0) \; . \tag{2.13}$$

It is now possible to discern three different types of behaviour for the perturbation. If the exponent $\gamma_i(g^*) - 4 + k_i$ is positive, then the coupling $\hat{g}_i(\mu)$ shrinks as μ flows to lower values, and we say the corresponding operator $\mathcal{O}_i$ is infra-red IRRELEVANT — for sufficiently small values of μ, the coefficient of the operator is vanishingly small and so it will have a negligible effect on the low energy physics *described by this fixed point*. Conversely, if the exponent in equation (2.13) is negative, then the operator corresponding to $\hat{g}_i$ will come to dominate infra-red physics. The RG flow will be driven away from this particular fixed point, and so the operator is clearly infra-red RELEVANT.[3]

The intermediate situation, where the exponent in equation (2.13) vanishes, defines a MARGINAL operator. In this case, further analysis is required to establish the infra-red behaviour of the deformation. For example, if corrections to the beta function (2.12) that are of higher order in $\hat{g}_i$ come with a positive coefficient, we say the operator is MARGINALLY IRRELEVANT. In some situations, such as we'll meet in Section 2.2.2 and Chapter 3, the beta functions can be shown to vanish to all orders, and so the deformation is EXACTLY MARGINAL, with the associated couplings parameterising a manifold of conformal fixed points.

To make contact with the perhaps more familiar terminology of perturbative renormalisation, consider the case where $\widehat{\mathcal{L}}$ in (2.9) is an expansion about a free field theory. We can then drop the anomalous dimension contribution to equation (2.13), and conclude that irrelevant perturbations arise from operators with mass dimension greater than 4. Such operators are usually referred to as non-renormalisable interactions. Similarly, if all operators have mass dimension less than or equal to 4, then the perturbed Lagrangian is renormalisable in the traditional sense: UV divergences can be dealt with by adjusting a finite number of parameters at each order in perturbation theory.

Shedding Degrees of Freedom

Another useful point of view on the renormalisation group, developed by Wilson [23], is found by thinking more directly in terms of the path integral definition of correlation functions:

$$\big\langle \phi(x_1)\cdots\phi(x_n)\big\rangle_\Lambda \sim \int \mathcal{D}\phi_\Lambda\, \phi(x_1)\cdots\phi(x_n)\; e^{-S_\Lambda}\ . \tag{2.14}$$

We suppose that the theory has already been regularised with the introduction of a momentum scale Λ, and that the effective action S_Λ contains all possible operators consistent with the symmetries of the theory:

$$S_\Lambda = \int \mathrm{d}^D x \left\{ \frac{1}{2}\, Z\, \partial^\mu\phi^\dagger\, \partial_\mu\phi + \sum_i Z^{\,d_i/2}\, g_i\, \Lambda^{D-k_i}\, \mathcal{O}_i(\phi,\partial_\mu) \right\}\ . \tag{2.15}$$

[3]Obviously, an IR relevant operator is UV irrelevant. For consicion, from now on we will describe operators (and their associated couplings) as simply *relevant* or *irrelevant*, with the implicit understanding that this relates to infra-red physics.

Here we have again used the cutoff scale to counter-balance the canonical dimension of each operator, thus rendering the couplings g_i dimensionless. Also, the operator $\mathcal{O}_i$ has canonical dimension k_i and contains d_i powers of the field.

In this context, doing a renormalisation group transformation from the scale Λ to Λ' is equivalent to partially performing the path integral. By integrating over the high frequency field modes with support in the range $[\Lambda', \Lambda]$, one is left with a theory described by the (Wilsonian) effective action

$$S_{\Lambda'} = \int \mathrm{d}^D x \left\{ \frac{1}{2} Z' \partial^\mu \phi^\dagger \partial_\mu \phi + \sum_i Z'^{\,d_i/2} g_i' \Lambda'^{\,D-k_i} \mathcal{O}_i(\phi, \partial_\mu) \right\} , \tag{2.16}$$

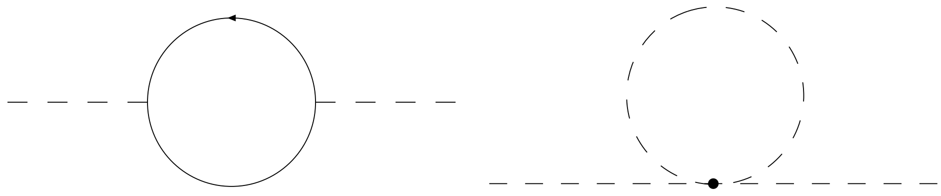
which is of the same form as equation (2.15) but with couplings adjusted as per the RG flow (2.6). Deriving an effective action in this way is known as INTEGRATING OUT the high energy degrees of freedom.

2.2. Supersymmetry

Everyone has their own reasons for liking supersymmetry.[4] It can be motivated as the unique way of extending the Poincaré group to give a consistent $(1 + 3)$-dimensional quantum field theory that still allows for non-trivial scattering. This result is an extension of the Coleman-Mandula No-Go theorem [24], given by Haag, Łopuszański and Sohnius to include spinorial generators [25]. Mathematically, the extra structure imposed by supersymmetry is also very useful for simplifying the behaviour of field theory. This leads to some powerful and far-reaching results, as we shall see shortly.

Another reason for liking supersymmetry (SUSY) is that it provides a nice mechanism for stabilising the electro-weak scale. In the Standard Model, most fields obtain a mass when electro-weak symmetry is broken by a fundamental scalar field — the Higgs field — which acquires a vacuum expectation value at energies below about 246 GeV. In this process, part of the Higgs field gets eaten by the gauge fields that correspond to broken symmetry generators, resulting in the massive $W^\pm$ and Z bosons, and the remaining fluctuation of the Higgs field ends up with a mass of order 100 GeV. For the record, direct searches at the Large Electron-Positron collider (LEP) at CERN suggest the Standard Model Higgs mass is greater than 114.4 GeV to a 95% confidence level [26]. There is

[4] Or not, as the case may be.



(a) Fermion Loop $\sim\ -y^2\Lambda^2\phi^\dagger\phi$ **(b)** Scalar Loop $\sim\ \lambda\Lambda^2\phi^\dagger\phi$

Figure 2.1.: Feynman diagrams that contribute to the Higgs mass. Both these corrections are quadratically dependent on the UV cut-off Λ. Supersymmetry aligns the couplings $y^2 = \lambda$ so the most severe divergences cancel.

known to be a slight tension between this result and the Standard Model prediction, perhaps indicative of supersymmetry...

The instability problem occurs when one computes quantum corrections to the Higgs mass. The integral of interest, given schematically by diagram (a) in Figure 2.1 is found to be quadratically dependent on the cut-off. Simply put, this tells us that the Higgs mass would naturally like to be as big as the next highest mass scale, which for the sake of argument we will take to be the Planck scale, $M_{\rm Pl} \sim 10^{19}$ GeV. We then see that when renormalising, the bare mass parameter has to be very finely tuned to allow for the following delicate cancellation:

$$-(100\ \text{GeV})^2\ =\ m^2_{\text{bare}}\ +\ (10^{19}\,\text{GeV})^2 \tag{2.17}$$

This situation, which to some is little more than an aesthetic deficiency, is known as the Hierarchy Problem. It is certainly a little odd, and may indicate a way beyond the Standard Model.

Supersymmetry addresses the Hierarchy Problem by necessarily introducing extra matter fields, and arranging a conspiracy of couplings such that the leading divergences in the diagrams of Figure 2.1 cancel. The Higgs mass is then only logarithmically divergent, which is altogether more satisfactory. This good behaviour at high energies is typical of supersymmetric models, and lends further support to the belief that SUSY will be an important component in a unified theory of physics. We shall see further niceties of SUSY when we supersymmetrise the Standard Model in Section 2.2.3.

Of the many good texts on supersymmetry, a great all-round reference with a suitably phenomenological bent is Martin's Primer [27]. For a more mathematical approach that covers the material of Section 2.2.2 particularly well, one could consult Terning's book [28] or Argyres' lecture notes [29]. Of course, Weinberg (Volume III) [30] also makes good reading.

2.2.1. The Language of Supersymmetry

What is Supersymmetry?

The essential idea of supersymmetry is that there exists a symmetry, with generator Q say, which relates bosonic states $|B\rangle$ to fermionic states $|F\rangle$:

$$Q\,|B\rangle = |F\rangle\;, \qquad Q\,|F\rangle = |B\rangle\;.$$

From here we immediately see the generators themselves must be fermionic in nature, i.e. they must transform as spinors under the Poincaré group. In more detail, supersymmetry extends the Poincaré group with its usual set of generators P_μ (translations) and $M^{\mu\nu}$ (Lorentz boosts/rotations) by including spinorial generators $Q_\alpha, \overline{Q}_{\dot\beta}$ with the following (anti-)commutation relations:

$$\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} = 2\,\sigma^\mu_{\alpha\dot\beta}\,P_\mu \tag{2.18a}$$

$$\begin{aligned} \left\{Q_\alpha, Q_\beta\right\} &= 0 & \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} &= 0 \\ \left[Q_\beta, P_\mu\right] &= 0 & \left[\overline{Q}_{\dot\alpha}, P_\mu\right] &= 0 \\ \left[M^{\mu\nu}, Q_\alpha\right] &= (\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta & \left[M^{\mu\nu}, \overline{Q}^{\dot\alpha}\right] &= (\overline{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta}\,\overline{Q}^{\dot\beta} \end{aligned} \tag{2.18b}$$

Here the indices α, $\dot\alpha$ can each take the values 1 or 2, with the undotted/dotted cases corresponding to the left- and right-handed Weyl spinor representation of the Lorentz group $\mathrm{SO}(1,3) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ respectively. Explicit expressions for the matrices $\sigma^\mu_{\alpha\dot\beta}$ and $(\sigma^{\mu\nu})_\alpha{}^\beta$ that intertwine the spinor and Lorentz vector indices can be found in Appendix A along with a more detailed discussion of the spinor notation used throughout this work.

As all single particle states must necessarily fall into representations — known as SUPERMULTIPLETS — of the above SUSY algebra (2.18), we can derive many important properties of supersymmetric field theory by studying these commutation relations. For example, because

$$[Q_\beta\,, P_\mu] \;=\; 0 \qquad \text{implies} \qquad [Q_\beta\,, P^\mu P_\mu] \;=\; 0\;,$$

we see that all the particles in a supermultiplet must have the same mass.

Of all the commutation relations (2.18), the most interesting is probably equation (2.18a). For instance it can be used to show there are an equal number of bosonic and fermionic degrees of freedom in each supermultiplet: Consider the operator $(-1)^F$ which reads $+1$ on bosonic states and -1 on fermionic states. It must anticommute with the supercharges Q_α. By using this fact, and the cyclicity of the trace, we see the following quantity vanishes:

$$\begin{aligned}
\mathrm{Tr}\left[(-1)^F\;\{Q_\alpha,\overline{Q}_{\dot\beta}\}\right] \;&=\; \mathrm{Tr}\left[(-1)^F\,Q_\alpha\overline{Q}_{\dot\beta} + (-1)^F\,\overline{Q}_{\dot\beta}Q_\alpha\right] \\
&=\; \mathrm{Tr}\left[\overline{Q}_{\dot\beta}(-1)^F\,Q_\alpha - \overline{Q}_{\dot\beta}(-1)^F\,Q_\alpha\right] \\
&=\; 0\;,
\end{aligned} \tag{2.19}$$

where the trace runs over all the states of a supermultiplet. On the other hand, by combining equation (2.19) with the relation (2.18a) one finds

$$0 \;=\; 2\,\sigma^\mu_{\alpha\dot\beta}\,\mathrm{Tr}\left[(-1)^F\,P_\mu\right]\;.$$

For a supermultiplet with arbitrary momentum, we must therefore have $\mathrm{Tr}\left[(-1)^F\right] = 0$, from which it is easy to see there are equal numbers of bosonic and fermionic degrees of freedom:

$$\begin{aligned}
0 \;=\; \mathrm{Tr}\left[(-1)^F\right] \;&\equiv\; \sum_B \langle B|(-1)^F|B\rangle \;+\; \sum_F \langle F|(-1)^F|F\rangle \\
&=\; \sum_B \langle B|B\rangle \;-\; \sum_F \langle F|F\rangle \\
\Longrightarrow \qquad n_b \;&=\; n_f\;.
\end{aligned} \tag{2.20}$$

Another relation, which will be essential in Section 2.2.3 and beyond, can be found by contracting equation (2.18a) with $g_{0\mu}\,\overline{\sigma}^{\mu\,\dot\beta\alpha}$. The result is a simple expression for the

Hamiltonian in terms of the supercharges

$$H \equiv P_0 = \frac{1}{4}\left(Q_1\overline{Q}_1 + \overline{Q}_1 Q_1 + Q_2\overline{Q}_2 + \overline{Q}_2 Q_2\right) \ . \tag{2.21}$$

The right hand side of this is a sum of positive definite operators, so when computing the vacuum energy, we crucially find it must be non-negative

$$E \;=\; \langle\, 0\,|H|\,0\,\rangle \;\geqslant 0\ .$$

Moreover, the vacuum energy will be non-zero *if and only if* one of the supercharges fails to annihilate the vacuum state, i.e. when the vacuum spontaneously breaks supersymmetry

$$E > 0 \qquad \Longleftrightarrow \qquad Q_\alpha|\,0\,\rangle \;\neq\; 0\ .$$

This is so important it is worth saying the other way round: a theory has exact supersymmetry *if and only if* the vacuum energy vanishes.

Extending the Poincaré group of $(1+3)$-dimensional spacetime by one pair of supercharges $Q_\alpha, \overline{Q}_{\dot\beta}$ defines what is called an $\mathcal{N}=1$ supersymmetric theory. Although it is this kind of model that is of most direct relevance to particle physics, we will meet other theories with higher amounts of supersymmetry at the end of this section.

Super-Everything

Having extended the symmetry group of spacetime by fermionic generators, it turns out [35] that a convenient notation for keeping track of the representation theory of the resulting graded Lie algebra is to formally enlarge spacetime to a SUPERSPACE by introducing Grassmannian coodinates θ^α, $\overline{\theta}_{\dot\alpha}$. Various properties of these coordinates can be found in Appendix A. The different fields of a supermultiplet can then be represented as one SUPERFIELD living on this superspace $(x^\mu, \theta^\alpha, \overline{\theta}_{\dot\alpha})$. Under a supersymmetry transformation, the superfield Φ transforms as $\delta\Phi = (\epsilon^\alpha Q_\alpha + \overline{\epsilon}_{\dot\alpha}\overline{Q}^{\dot\alpha})\Phi$ with the operators Q and $\overline{Q}$ represented as differential operators on superspace:

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\,\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\frac{\partial}{\partial x^\mu}\ , \qquad \overline{Q}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i\,\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial x^\mu}\ . \tag{2.22}$$

Although the most general superfield has too many components to be of much use, it provides a reducible representation of the SUSY algebra and so can be cut down to a more manageble size by imposing various constraints. To do this, and also to be able to write down supersymmetric actions in this notation, it is also useful to have superderivatives:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\,\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\frac{\partial}{\partial x^\mu}\ , \qquad \overline{D}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} - i\,\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial x^\mu}\ . \tag{2.23}$$

As these anticommute with the differential operators (killing vectors) in equation (2.22) they allow us to impose SUSY invariant conditions on the most general of superfields in order to define various irreducible supermultiplets.

An important one is the CHIRAL SUPERFIELD Φ, defined to satisfy $\overline{D}_{\dot\alpha}\Phi = 0$. We can solve this constraint nicely by introducing the coordinate $y^\mu = x^\mu + i\,\theta\sigma^\mu\overline{\theta}$ that satisfies $\overline{D}_{\dot\alpha}y^\mu = 0$. The chiral superfield is then $\Phi\,(y^\mu, \theta_\alpha)$. Expanding in the Grassmannian coordinates

$$\Phi(x^\mu, \theta_\alpha, \overline{\theta}_{\dot\alpha}) \;=\; \phi(y^\mu) \;+\; \sqrt{2}\,\theta\,\psi(y^\mu) \;+\; \theta^2 F(y^\mu) \tag{2.24}$$

reveals the field content of the chiral multiplet to be a complex scalar field ϕ, a left-handed Weyl spinor ψ_α, and an auxiliary complex scalar field F, which we will come to shortly.

Under an infinitesimal SUSY transformation with parameters ϵ_α and $\overline{\epsilon}_{\dot\alpha}$, the components of a chiral superfield transform as:

$$\delta\phi \;=\; \sqrt{2}\,\epsilon\,\psi \tag{2.25a}$$

$$\delta\psi \;=\; \sqrt{2}\,\epsilon\,F \;+\; i\sqrt{2}\,\sigma^\mu\overline{\epsilon}\,\partial_\mu\phi \tag{2.25b}$$

$$\delta F \;=\; -\,i\sqrt{2}\,\partial_\mu\psi\,\sigma^\mu\,\overline{\epsilon} \tag{2.25c}$$

Another representation of the SUSY algebra that will be of interest to us is the VECTOR SUPERMULTIPLET. This is defined as a superfield V satisfying $V^\dagger = V$, and can be used to write down gauge transformations in a supersymmetry-invariant way. The generators of the gauge group are made evident in the usual way by writing $V = V^a T^a$. A generalised gauge transformation can be defined, whereby V transforms as

$$V \longrightarrow e^{-i\Lambda^\dagger} V e^{i\Lambda}\ . \tag{2.26}$$

The parameter here, Λ, is actually a chiral superfield; the transformation can be thought of as a gauge transformation with *complex* parameter. We can reduce this to the usual (real) gauge freedom by choosing WESS-ZUMINO GAUGE, in which the vector superfield can be expanded to give[5]

$$V^a = \theta\sigma^\mu\overline{\theta}\, A^a_\mu + i\theta^2\overline{\theta\lambda}^a - i\overline{\theta}^2\theta\lambda^a + \frac{1}{2}\overline{\theta}^2\theta^2 D^a \ . \tag{2.27}$$

The field content is then seen to be a vector field A^a_μ, its superpartner λ^a_α, which is a Majorana spinor known as the gaugino, and an auxiliary real scalar field D^a. The field strength $F_{\mu\nu}$ for this gauge field is contained in the following superfield

$$W_\alpha = -\frac{1}{4}\overline{DD}\, e^{-V} D_\alpha\, e^V \tag{2.28a}$$
$$= -i\lambda_\alpha - (\sigma^{\mu\nu}\theta)_\alpha\, F_{\mu\nu} + \cdots \ , \tag{2.28b}$$

which is chiral and gauge covariant (invariant for Abelian gauge groups).

Supersymmetry invariant Lagrangian densities can now be written down by taking products and SUSY derivatives of the above superfields, and integrating them over the Grassmannian coordinates of superspace. It can be shown that the resulting Lagrangian only changes by a total derivative under a supersymmetry transformation. The most general gauge invariant and SUSY invariant Lagrangian has quite a restricted form: kinetic terms for chiral superfields, and also derivative interactions, follow from a real-valued function – the Kähler potential, $K(\cdot\,,\cdot)$:

$$\mathcal{L} \supset \int d^2\theta d^2\overline{\theta}\, K\big(\Phi^\dagger, e^{gT^aV^a}\Phi\big) \ . \tag{2.29}$$

If, as is often the case, we want to restrict attention to renormalisable interactions, it is sufficient to consider the *canonical* Kähler potential:

$$\mathcal{L} \supset \int d^2\theta d^2\overline{\theta}\ \Phi^\dagger e^{gV}\Phi \ . \tag{2.30}$$

Kinetic terms for gauge superfields come from

$$\mathcal{L} \supset \frac{1}{16\pi i}\int d^2\theta\, \tau\, W^{a\alpha}W^a_\alpha + \text{Complex Conjugate} \ , \tag{2.31}$$

[5]The choice of Wess-Zumino gauge is not SUSY invariant. After performing a SUSY transformation, a further generalised gauge transformation is usually required to return to WZ gauge.

where the gauge coupling g and theta angle θ have been packaged up into the COMPLEXIFIED GAUGE COUPLING

$$\tau \equiv \frac{4\pi i}{g^2} + \frac{\theta}{2\pi} \ . \tag{2.32}$$

There is another gauge invariant term one can add for Abelian gauge fields

$$\mathcal{L} \supset \int d^2\theta d^2\overline{\theta}\, \xi V = \xi D \ , \tag{2.33}$$

with constant coefficient ξ. This is known as a FAYET-ILIOPOULOS term.

Last, but not least, non-derivative interactions follow from a function of chiral superfields called the Superpotential, $W(\,\cdot\,)$:

$$\mathcal{L} \supset \int d^2\theta\, W(\Phi) \ + \ \text{Complex Conjugate} \ . \tag{2.34}$$

The fact that the superpotential is forced by the SUSY algebra to be a *holomorphic* function is highly significant and leads to many of the interesting effects, which we will study in Section 2.2.2.

The auxiliary fields F_i and D^a deserve more attention.[6] They are special in that they have no kinetic terms, only appearing in the Lagrangian as

$$V(\phi_i) \ = \ F_i^\dagger F_i \ + \ \frac{\partial W}{\partial \phi_i} F_i \ + \ \frac{1}{2} D^a D^a \ - \ g\, \phi_i^\dagger T^a \phi_i D^a \ , \tag{2.35}$$

so they can be eliminated via their equations of motion to give the SCALAR POTENTIAL

$$V(\phi_i) \ = \ \left| \frac{\partial W}{\partial \phi_i} \right|^2 \ + \ \frac{1}{2}\, g^2 \Big(\phi_i^\dagger T^a \phi_i \Big) \Big(\phi_k^\dagger T^a \phi_k \Big) \ . \tag{2.36}$$

The first terms in this equation are known as F-terms, and the second are known as D-terms, for hopefully obvious reasons.

[6]Here we consider a set of chiral superfields indexed by i, and one non-Abelian vector superfield with coupling g and gauge group generators indexed by a.

Recognising Broken Supersymmetry

The scalar potential plays a central rôle in understanding when global SUSY is broken. To see this, first recall the discussion around equation (2.21) in which we showed (from the SUSY algebra) that a vacuum is supersymmetric precisely when the vacuum energy vanishes:

$$E = \langle 0 | H | 0 \rangle \geqslant 0 \qquad \text{with} \qquad E = 0 \iff Q_\alpha | 0 \rangle = 0 \, . \tag{2.37}$$

The crucial observation now is that the scalar potential is a positive definite function of the auxiliary fields

$$V(\phi_i) = F_i^\dagger F_i + \frac{1}{2} D^a D^a \geqslant 0 \, , \tag{2.38}$$

which means it only vanishes when both the F- and D-terms vanish:

$$V(\phi_i) = 0 \qquad \iff \qquad F_i = 0 \quad \& \quad D^a = 0 \, . \tag{2.39}$$

This allows us to rephrase the condition (2.37) for a vacuum to be supersymmetric in terms of the vanishing of the F- and D-terms:

$$Q_\alpha | 0 \rangle = 0 \qquad \iff \qquad F_i = 0 \quad \& \quad D^a = 0 \, . \tag{2.40}$$

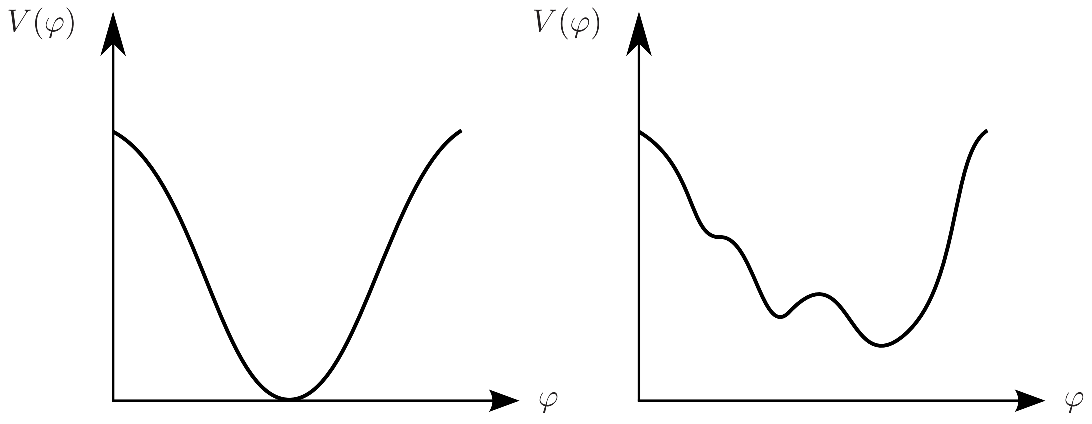
Thus SUSY preserving vacua can be seen to correspond to minima of the scalar potential with vanishing vacuum energy, as illustrated in Figure 2.2. Note that the vacuum energy serves as an order parameter for the breaking of global supersymmetry.

R-symmetry

Most internal symmetry transformations, such as colour or flavour rotations, commute with the action of supersymmetry. The exception to this is what is called an R-SYMMETRY. The generators of an R-symmetry satisfy the following relations

$$[Q, R] = Q \, , \qquad [\overline{Q}, R] = -\overline{Q} \, . \tag{2.41}$$

In more technical language, an R-symmetry is an automorphism of the SUSY algebra, meaning that it reflects some arbitrariness in our choice of supercharges. Not all super-



(a) A SUSY preserving vacuum. **(b)** SUSY breaking vacuua.

Figure 2.2.: A sketch of how supersymmetry breaking vacua can be recognised from the scalar potential.

symmetric theories have an R-symmetry, though in some cases it is possible to define a continuous family of such symmetries. R-symmetry can also be manifest as an anomalous and/or spontaneously broken symmetry, as we will see in later chapters, and can provide valuable guidance when building SUSY breaking models.

To understand why R-symmetry is so useful, consider equation (2.41). It is easy to see that the different components of a superfield (which are shuffled amongst themselves under a SUSY transformation) must carry different R-charge. For example, for the scalar and fermionic components of a chiral superfield (2.24):

$$R[\phi] \;=\; s\;, \quad \psi \;=\; Q\,(\phi) \qquad \Longrightarrow \qquad R[\psi] \;=\; s-1\;.$$

We can therefore consistently assign non-zero R-charge to the Grassmannian coordinates:

$$R[\theta] \;=\; 1\;, \quad R[d\theta] \;=\; -1\;.$$

As all terms in the Lagrangian must of course be R-charge neutral, we find the superpotential must carry an R-charge of two:

$$R\left[\int d^2\theta\, W(\Phi)\right] = 0 \qquad \Longrightarrow \qquad R[W] = 2\;. \tag{2.42}$$

This has important consequences for SUSY breaking, which we will come back to in Chapter 4.

Extended Supersymmetry and Beyond

Up until now we've been discussing so-called $\mathcal{N} = 1$ global supersymmetry, generated by one pair of supercharges $Q_\alpha, \overline{Q}_{\dot{\beta}}$. It is reasonable to wonder what happens if more supercharges are introduced; if we suppose they carry an index K running from 1 to $\mathcal{N}$, then $Q^K_\alpha, \overline{Q}_{K\dot{\beta}}$ form a SUSY algebra that is a slight extension of (2.18), with the most important relations being

$$\{Q^M_\alpha, \overline{Q}_{N\dot{\beta}}\} = 2\, \delta^M_N\; \sigma^\mu_{\alpha\dot{\beta}}\; P_\mu \;, \tag{2.43a}$$

$$\{Q^M_\alpha, Q^N_\beta\} = 2\sqrt{2}\,\varepsilon_{\alpha\beta} Z^{MN} \;, \qquad \{\overline{Q}_{M\dot{\alpha}}, \overline{Q}_{N\dot{\beta}}\} = 2\sqrt{2}\,\varepsilon_{\dot{\alpha}\dot{\beta}} Z^*_{MN} \;. \tag{2.43b}$$

The new components Z^{MN}, which are anti-symmetric in their indices, are bosonic symmetry generators known as CENTRAL CHARGES, meaning that they commute with all elements of the Poincaré/SUSY algebra, including amongst themselves. They take different values on different representations of the algebra, and allow one to define special 'short' multiplets — smaller than the usual 'long' representations — whose mass is related to (rather than just bounded by) the central charges. States in a shortened multiplet turn out to be annihilated by some fraction of the supercharges, and have particularly nice behaviour under quantum corrections. We won't need to know much about these representations of extended SUSY algebras, but we just mention that they play an important rôle in the AdS/CFT correspondence of Section 2.4.

With more supersymmetry comes a more constrained theory. For example, when $\mathcal{N} = 2$, the superpotential and the most relevant terms of the Kähler potential can both be derived from one function, the prepotential $\mathcal{F}$. This rigid structure allowed Seiberg and Witten to give complete expressions for the low energy behaviour of such theories: exact results for the coupling and BPS spectrum, and a metric on the quantum moduli space [36–38].

Life's not all a bed of roses though. The extra supersymmetry may simplify field theory, but it proves to be too stringent a constraint for real-world physics. The main problem is that fields in extended SUSY models always come in pairs such that for every left-handed field there is a right-handed field with the opposite quantum numbers. This leaves little room for constructing the Standard Model, which is a chiral theory, i.e. one in which such a matching is not possible.

Even more remarkable things happen if we proceed to $\mathcal{N} = 4$. If we require that there be no fields with spin greater than 1, then there is so little room for manoeuvre that a $(1+3)$-dimensional field theory with $\mathcal{N} = 4$ supersymmetry is essentially unique. The only supermultiplet one can construct necessarily contains spin 1 fields, so the only freedom left is to choose what gauge group these bosons transform under. The R-symmetry group turns out to be SU(4), with the supercharges Q_α^K transforming in the fundamental representation **4**. The supermultiplet consists of six real scalar fields ϕ_i transforming in the **6** of the R-symmetry group, four Majorana fermions $\lambda_{\alpha K}$ in the $\overline{\mathbf{4}}$ of SU(4), and a vector field A_μ. As all fields in a multiplet carry the same representation under internal (non-R-) symmetries, and the multiplet contains a gauge field, all these fields transform in the adjoint of the gauge group.

By combining the 6 real scalars into 3 complex scalars, as explained in Appendix A, this field content can be conveniently written in $\mathcal{N} = 1$ superspace language as three chiral superfields Φ_i and a vector superfield V. The superpotential is then fixed to be

$$W = ig \, \mathrm{Tr} \left[\Phi_1 \Phi_2 \Phi_3 - \Phi_1 \Phi_3 \Phi_2 \right] \ . \tag{2.44}$$

One crucial aspect of $\mathcal{N} = 4$ SUPER YANG-MILLS, as this theory is known, is that it is exactly conformal, in the sense of Section 2.1. The beta function for the theory is zero at both the classical and quantum level. If you don't think the shear existence of an interacting, exactly conformal field theory in four dimensions is interesting enough, in Section 2.4 we will see at least one more good reason why $\mathcal{N} = 4$ super Yang-Mills (sYM) is interesting.

Increasing the amount supersymmetry still further takes us to $\mathcal{N} = 8$. The smallest supermultiplet we construct now contains fields of spin two — we have entered the strange world of 4-dimensional Supergravity. As this is a quantum field theory of gravity, the conventional wisdom says it is non-renormalisable. Indeed, it can currently only be understood as an effective field theory. For example, it can be viewed as the dimensional reduction to 4 dimesions of the 10-dimensional Type II superstring theory — see Section 2.3. However, the remarkable properties of this theory are still a hot topic of research, particularly the question of whether the theory is in fact UV finite (perturbatively).

2.2.2. The Magic of Supersymmetry

Holomorphy and Exact Results

Many remarkable properties of supersymmetric theories follow from the observation that the superpotential must be a *holomorphic* function of chiral superfields. One famous example is the perturbative non-renormalisation of the superpotential. To illustrate this we follow the example from reference [39] and consider a simple Wess-Zumino model describing the behaviour of a single chiral superfield Φ. The theory is defined at high energies by the superpotential:

$$W_{\text{tree}} = \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3 . \tag{2.45}$$

Thinking of the couplings m and g as the lowest components of SPURION superfields, this superpotential has two U(1) symmetries, with the charge assignments indicated in Table 2.1. When the spurions acquire VEVs, these symmetries are spontaneously broken, but they can still be used to constrain the structure of the Wilsonian effective action, which must be a holomorphic function of Φ, m and g, with the charge assignment indicated in the bottom line of Table 2.1. Thus, the most general effective action must take the form:

$$W_{\text{eff}} = \frac{1}{2} m \Phi^2 f\left(\frac{g \Phi}{m}\right) , \tag{2.46}$$

for some function $f(z) = \sum_{-\infty}^{\infty} f_n z^n$. Now consider taking a few judicious limits:

1. When $g \to 0$ for fixed m, the theory is free, so there can be no negative powers of z in $f(z)$.

2. Requiring a smooth massless limit $m \to 0$ shows quadratic and higher powers of z must also be absent.

3. To match with the microscopic superpotential at weak coupling, we must therefore have $f(z) = 1 + \frac{2}{3} z$.

Putting all of this together we see

$$W_{\text{eff}} = W_{\text{tree}} . \tag{2.47}$$

	U(1)	$U(1)_R$
Φ	1	1
m	−2	0
g	−3	−1
W_{eff}	0	2

Table 2.1.: Spurious symmetries of a simple Wess-Zumino model.

In other words, the superpotential is not renormalised at all, even by non-perturbative effects. In more general theories, the symmetries used to constrain the superpotential can be anomalous, in which case the above argument holds to all orders in perturbation theory, but there is some scope for the superpotential to receive non-perturbative corrections, as we will see shortly.

Similar reasoning also shows that Fayet-Iliopoulos terms don't change under renormalisation and that the complexified gauge coupling (or more generally, gauge kinetic function) only receives perturbative contributions at one loop order. Disappointingly, the Kähler potential enjoys no such protection from quantum corrections.

As we will soon be interested in theories with spontaneously broken supersymmetry, it is worth mentioning one important corollary of the non-renormalisation theorems. In the absence of any Fayet-Iliopoulos terms (so there is no D-term SUSY breaking), if the tree-level F-term equations can be solved to find a supersymmetric vacuum state, then this SUSY preserving vacuum will persist to all orders in perturbation theory [40].

Instantons

Instantons began life as self-dual solutions to the Euclidean Yang-Mills equations with finite action:

$$S_{\text{inst}} = \frac{1}{2\,g^2}\int \mathrm{d}^4x\,\mathrm{Tr}\,F_{mn}F^{mn} \tag{2.48a}$$

$$= \frac{1}{4\,g^2}\int \mathrm{d}^4x\,\mathrm{Tr}\left[\left(F_{mn} - *F^{mn}\right)^2 + 2\,*F_{mn}F^{mn}\right] \tag{2.48b}$$

$$= \frac{8\pi^2}{g^2}\;. \tag{2.48c}$$

In Minkowski space they can be thought of as tunnelling events between different vacuum configurations of a gauge theory, and provide the leading semi-classical correction to path integrals, and hence correlation functions of a theory. For an introduction to instantons, their applications, and their relation to other non-perturbative effects see references [41–43]. A comprehensive review of the multi-instanton calculus with and without supersymmetry, is found in reference [44].

Another very attractive feature of supersymmetric theories is that computing the effects of one or more instanton is often a manageable exercise. Why do the instanton calculations work so well? The general idea is to calculate the contribution to path integrals from instanton configurations. As can be seen from (2.48b), they are local minima of the action, so we can expand around these configurations to quadratic order and try to perform the resulting Gaussian integration. For massive fluctuations this causes no problems, and one gets the usual bosonic ($\frac{1}{\det^{1/2}\Delta_b}$) and fermionic ($\det\Delta_f$) determinants. Massless fluctuations (zero modes of the field equations) cause more of a problem; they correspond to changes in the field configuration that don't alter the action. Such directions can be parameterised by COLLECTIVE COORDINATES in the instanton solution. The zero mode integrations are dealt with by first converting them to integrals over the collective coordinates (both bosonic X and fermionic ξ) of the background. This change of variables results in Jacobian factors in the instanton measure, which heuristically then takes the form

$$d\mu_{\text{inst}} = dX\, d\xi\, J_b\, J_f\, \frac{(\det\Delta_f)^{n_f}}{(\det\Delta_b)^{n_b}} e^{-S_{\text{inst}}} \,, \tag{2.49}$$

where n_b and n_f are the number of bosonic and fermionic degrees of freedom. We are now in a position to see why the SUSY instanton calculations work well: the balance of bosonic and fermionic degrees of freedom in each supermultiplet (see Section 2.2.1) leads to a cancellation of determinants in the instanton measure. Despite this simplification, there are still many subtleties, such as the calculation of the Jacobian factors which we will have to address when we actually perform an instanton calculation in Chapter 3.

Another thing we can learn from equation (2.49) is that instantons are only able to contribute to certain correlation functions. The point is that unless the correlator itself contains the right number of fermionic zero modes, the $d\xi$ integrals will cause the contribution to vanish.

	$\mathrm{SU}(N_c)$	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_A$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	1	1	$1-\frac{N_c}{N_f}$
$\widetilde{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	1	-1	$1-\frac{N_c}{N_f}$

Table 2.2.: The local and global symmetries of massless SQCD with N_c colours and N_f flavours.

SQCD

The archetype of a supersymmetric gauge theory, and one we will return to many times in this thesis, is Supersymmetric Quantum Chromodynamics (SQCD). Due to the magic of supersymmetry, there is a lot one can say about this theory. It has been well studied in the literature, with the must-read review being the lectures of Intriligator and Seiberg [45]. We will now document some of the key results that will be of use to us in future chapters.

SQCD is defined to be an $\mathrm{SU}(N_c)$ gauge theory with $\mathcal{N}=1$ supersymmetry, coupled to N_f pairs of chiral superfields $\left(Q,\widetilde{Q}\right)$ that transform in the the (anti-)fundamental representation of the gauge group $\left(\square,\overline{\square}\right)$, just like the usual quarks of QCD. In the case where there is no superpotential, so in particular the quarks are massless, there are also various global symmetries whose charges are found in Table 2.2. The $\mathrm{U}(1)_R$ charges have been chosen such that the symmetry is anomaly free.

It is interesting to ask what the moduli space of vacuum solutions is for SQDC. The answer, unsurprisingly, depends on the values of N_c and N_f, and involves an interplay between many of the ideas we have already discussed. A useful place to start is to find the Wilsonian effective superpotential that describes low energy physics. As we have seen above, this should be determinable from holomorphy, the symmetries of the microscopic theory, and by requiring smooth behaviour in various limits. Unfortunately there's a twist: the $\mathrm{U}(1)_A$ symmetry is anomalous, i.e. it is a classical symmetry that is violated in the quantum theory. Recall that for such a symmetry, the anomaly is manifest as the non-conservation of the associated Noether current,

$$\partial_m\, j_A^m \;=\; \sum_r \frac{T(r)}{8\,\pi^2}\,\mathrm{Tr}\; {*F_{mn}}F^{mn}\;, \tag{2.50}$$

where $T(r)$ is (one half of) the Dynkin index of the representation r, and the sum runs over all fermions charged under the symmetry. By comparing with the instanton action, equation (2.48), it is clear that in the presence of an instanton, $\mathrm{U}(1)_A$ charge conservation is violated by $\sum 2\,T(r) = 2N_f$. Another thing to note is that in an instanton background, Euclidean correlation functions will be weighted by the factor

$$e^{-S_{\text{inst}}} = \exp\left(-\frac{1}{2\,g^2}\int \mathrm{d}^4x\,\mathrm{Tr}\,F_{mn}F^{mn} + i\frac{\theta}{16\,\pi^2}\int \mathrm{d}^4x\,\mathrm{Tr}\,{*F_{mn}}F^{mn}\right) \tag{2.51a}$$

$$= \exp\left(-\frac{8\pi^2}{g^2(\mu)} + i\theta\right) \tag{2.51b}$$

$$= \left(\frac{\Lambda}{\mu}\right)^{3N_c-N_f} \tag{2.51c}$$

where the last line follows from integrating the one loop beta function $\beta(g) = \frac{-g^3\,b_0}{16\pi^2}$ with $b_0 = 3N_c - N_f$.[7] This then suggests a cunning way to deal with the $\mathrm{U}(1)_A$ anomaly: if we think of the instanton factor Λ^{b_0} as a spurious superfield, we can assign it a charge of $2N_f$ under $\mathrm{U}(1)_A$ to account for the anomalous shift.

One can then argue by holomorphy, symmetry and smoothness that for $N_f < N_c$, the only possible effective superpotential must take the form:

$$W_{\text{eff}} = C_{N_c,N_f}\left[\frac{\Lambda^{3N_c-N_f}}{\det\left(Q\cdot\widetilde{Q}\right)}\right]^{\frac{1}{N_c-N_f}} . \tag{2.52}$$

This is the Affleck-Dine-Seiberg (ADS) superpotential [46]. The coefficients C_{N_c,N_f} can be interrelated by turning on quark VEVs to Higgs the theory, or by turning on masses and integrating out the corresponding quarks. The overall normalisation can then be fixed by an instanton calculation with $N_c = 2$, $N_f = 1$ [47], leading to the globally consistent formula $C_{N_c,N_f} = N_c - N_f$.

From the ADS superpotential one can derive a variety of interesting things. For example, the classical moduli space of SQCD with $0 < N_f < N_c$, which is parameterised by the scalar VEV of the gauge invariant quantity $Q\cdot\widetilde{Q}$, acquires a potential due to equation (2.52). The profile of this scalar potential runs away to infinity, so there is in fact no quantum vacuum for massless SQCD with fewer flavours than colours.

[7] We will often refer to Λ as the dimensional transmutation scale, whereas strictly this is just the modulus of Λ, with the phase being θ/b_0.

Another interesting phenomenon is observed when there are no quark superfields at all (pure supersymmetric Yang-Mills theory). In this case one cannot construct a non-anomalous R-symmetry, so $\mathrm{U}(1)_R$ charge conservation is violated by $\sum 2\,T(r) = 2N_c$. As the theta angle in equation (2.51b) is only defined modulo 2π, the charge violation will have no observable consequences if $2N_c$ is a multiple of 2π. In other words, the anomaly breaks $\mathrm{U}(1)_R$ down to the discrete subgroup $\mathbb{Z}_{2N_c}$. Furthermore, as we have just argued, non-perturbative dynamics generate an effective superpotential

$$W_{\text{eff}} \;=\; N_c\,\Lambda^3\;. \tag{2.53}$$

As gauginos form the lowest component of the field strength superfield (2.28), and the field strength squared is sourced in the effective action by the running gauge coupling τ (cf. equation (2.31)), it follows that there is a GAUGINO CONDENSATE

$$\begin{aligned}\left\langle\lambda^a\lambda^a\right\rangle \;&=\; 16\pi\,i\frac{\partial}{\partial\tau}W_{\text{eff}} &(2.54\text{a})\\ &=\; -32\pi^2\,\Lambda^3\;, &(2.54\text{b})\end{aligned}$$

which spontaneously breaks the discrete R-symmetry (under which gauginos rotate) down to $\mathbb{Z}_2$. In fact, the different phases of this condensate lead to N_c physically distinct supersymmetric vacua.

For $N_f \geq N_c$, the set of gauge invariants that can potentially parameterise the classical moduli space expands to

$$\begin{aligned}&\text{MESONS} && M_i^j \;=\; \widetilde{Q}_i^a Q_a^j\;,\\ &\text{BARYONS} && B^{i\ldots k} \;=\; \varepsilon^{a\ldots c}\,Q_a^i\cdots Q_c^k\;,\\ &\text{ANTI-BARYONS} && \widetilde{B}_{i\ldots k} \;=\; \varepsilon_{a\ldots c}\,\widetilde{Q}_i^a\cdots\widetilde{Q}_k^c\;,\end{aligned}$$

where $1 \leq a, c \leq N_c$ are gauge indices, and $1 \leq i, j \leq N_f$ are flavour indices. In general there are more mesons and baryons than there are quarks, so this over-complete set of parameters will be constrained by a set of algebraic relations.

In the case where $N_f = N_c$ the superpotential (2.52) doesn't make sense, so one might expect the classical moduli space, parameterised by M_i^j, B and $\widetilde{B}$ (where $\varepsilon^{i\ldots k}B = B^{i\ldots k}$), to remain in the quantum theory. This is *almost* what happens. Classically, the moduli satisfy the constraint $\det M - B\widetilde{B} = 0$, but this gets modified by quantum effects to $\det M - B\widetilde{B} = \Lambda^{2N_c}$.

When $N_f = N_c + 1$ the classical parameters M_i^j, B_i and $\widetilde{B}^i$ (where $\varepsilon^{ij\ldots k} B_i = B^{j\ldots k}$) are constrained by

$$B_i M^i_j = 0 \ , \qquad M^i_j \widetilde{B}^j = 0 \ , \qquad B_i \widetilde{B}^j - (M^{-1})^i_j \det M = 0 \ . \tag{2.55}$$

Quantum mechanically, an effective superpotential is generated, which is restricted by holomorphy and symmetries to be

$$W_{\text{eff}} = \frac{1}{\Lambda^{2N_c - 1}} \left[B_i M^i_j \widetilde{B}^j - \det M \right] \ . \tag{2.56}$$

The equations of motion for the mesons and baryons enforce the classical constraints, so we see that the classical moduli space is not modified by quantum corrections.

There is a subtle but important difference between the classical and quantum descriptions of the moduli space, regarding the interpretation of the singularity at $M = B = \widetilde{B} = 0$. Classically, at this point none of the gauge symmetry is Higgsed, so one can attribute the singularity to the fact we have not accounted for massless gluons in our effective theory of mesons and baryons. Quantum mechanically, however, the theory confines at a scale Λ below which there should only be mesons and baryons. It is *these* composite degrees of freedom that become massless at the origin of the quantum moduli space.

How confident can we be that all the relevant degrees of freedom have been accounted for in our low energy effective field theory description? Fortunately there is a stringent test, proposed by 't Hooft [48], that addresses this concern: anomalies in the global symmetries must match in both the microscopic and macroscopic pictures. For the $N_f = N_c + 1$ effective potential (2.56) the anomalies do indeed match, thus supporting the effective description we outlined. For $N_f \geq N_c + 2$, the 't Hooft anomalies generated by high energy SQDC[8] are not the same as those of a naïve effective description in terms of mesons and baryons; there must be more to the low energy description...

Seiberg Duality

When SQCD has more than $N_c + 1$ quark flavours and is still asymptotically free (so $N_f < 3N_c$) Seiberg found that the low energy physics could be described by a similar theory, with gauge group $SU(N_f - N_c)$, N_f quark fields φ, $\widetilde{\varphi}$ and a set of gauge singlets

[8] This is asymptotically free, and so has a well defined UV limit, provided $N_f < 3N_c$.

Φ. The global symmetries are the same as in the original theory (henceforth known as the ELECTRIC theory), with charges assigned to the MAGNETIC picture fields as indicated in Table 2.3. The magnetic theory also has a tree-level superpotential

$$W_{\text{tree}} = h \text{ Tr } \varphi_i \Phi^i_j \widetilde{\varphi}^j , \tag{2.57}$$

which is crucial for reproducing the moduli space of the electric theory. The dictionary between electric and magnetic fields is most easily expressed in terms of gauge invariant quantities. The matching of global symmetries then essentially dictates that electric baryons map to magnetic baryons

$$B^{i\ldots k} = C\, \varepsilon^{i\ldots k\, m\ldots p}\, b_{m\ldots p} \tag{2.58}$$

where $C = \sqrt{-(-\mu)^{N_c - N_f} \Lambda_e^{3N_c - N_f}}$ and $b_{m\ldots p} = \varepsilon^{a\ldots c}\, \varphi_{a\,m} \cdots \varphi_{c\,p}$, with a similar expression for the anti-baryons, and that electric mesons $\left(M^i_j = \widetilde{Q}^a_j Q^{\,i}_a \right)$ map to the magnetic singlet

$$M^i_j = \mu\, \Phi^i_j . \tag{2.59}$$

A new scale, μ, appears in the above matching relations. Although for fixed N_c and N_f one might expect to be able to dispense with it by redefining the strong coupling scale Λ_e of the electric theory, its presence is required to make this whole picture of SQCD consistent under deformations that alter N_c and N_f, as shown in reference [45]. This requirement also ties together the strong coupling scales of the electric and magnetic theories via[9]

$$\Lambda_e^{3N_c - N_f}\, \Lambda_m^{3(N_f - N_f) - N_f} = (-1)^{N_f - N_c}\, \mu^{N_f} . \tag{2.60}$$

The behaviour of the magnetic theory is dictated by its beta function, which is proportional to $3(N_f - N_c) - N_f$. The fact this changes sign at $N_f = \frac{3}{2} N_c$ has repercussions for the low energy dynamics of SQCD, and also on how we view the duality. For $N_c + 1 < N_f < \frac{3}{2} N_c$, a regime known as the FREE MAGNETIC WINDOW, the magnetic gauge coupling is infrared free, the operator (2.57) is irrelevant, and so as the name suggests, the theory is entirely non-interacting at low energy. From the perspective of the magnetic theory, there appears to be a Landau pole problem, with the coupling

[9] The sign here is fixed by demanding that the dual of the dual theory is the original theory.

	$\mathrm{SU}(N_f - N_c)$	$\mathrm{SU}(N_f)$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_R$
φ	$\square$	$\overline{\square}$	$\mathbf{1}$	$\frac{N_c}{N_f - N_c}$	$\frac{N_c}{N_f}$
$\widetilde{\varphi}$	$\overline{\square}$	$\mathbf{1}$	$\square$	$-\frac{N_c}{N_f - N_c}$	$\frac{N_c}{N_f}$
Φ	$\mathbf{1}$	$\square$	$\overline{\square}$	0	$2\left(1 - \frac{N_c}{N_f}\right)$

Table 2.3.: The local and global symmetries of SQCD+Φ, the magnetic Seiberg dual of SQCD.

blowing up at high energy. With our understanding of duality, this problem evaporates — the electric dual theory provides a well-behaved UV completion, as discussed in Section 2.1.1.

When $\frac{3}{2}N_c < N_f < 3N_c$, both electric and magnetic theories are asymptotically free and consequently strongly coupled in the IR. Duality tells us that they share the same long distance physics. The interesting feature is that this physics is controlled by an *interacting* conformal fixed point; many quantities of interest, such as the anomalous dimensions of chiral operators, can be calculated using the superconformal algebra. This range of N_f is accordingly known as the CONFORMAL WINDOW.

As the electric and magnetic theories are only indistinguishable at sufficiently low energies one might ask why this is known as a *duality*, and not just an effective field theory description of SQCD. One reason is because Seiberg duality has many similarities to Olive-Montonen duality [49, 50]. This is a generalisation of the electromagnetic duality[10] of Maxwell's equations, which relates strongly coupled $\mathcal{N} = 4$ sYM to essentially the same theory at weak coupling, and is an exact quantum equivalence *at all scales*. In the $\mathcal{N} = 4$ case, fundamental objects that are electrically charged are exchanged for composite objects that carry magnetic charge (and vice-versa) much like the relation (2.59) of Seiberg duality. From equation (2.60) one can also see that if one picture is strongly coupled, the Seiberg dual will be weakly coupled. Another reason for the use of the term duality is that in some situations the transformation really is exact at all scales [51, 52].

All of the interesting phenomena we have outlined in this section, from the quantum deformed moduli space to Seiberg duality, can be employed to model physical systems. They are particularly useful when investigating the breaking of supersymmetry, as we will see in Chapter 4.

[10]This connection accounts for much of the terminology used in Seiberg duality.

2.2.3. The Physics of Supersymmetry

By now the reader is hopefully convinced that SUSY field theories provide a rich and highly calculable framework. We now turn to the task of using these tools to build models that address various short-comings of the Standard Model. The place to start is. . .

The Minimal Supersymmetric Standard Model

To construct the Minimal Supersymmetric Standard Model (MSSM), just take the Standard Model and promote each matter field to a left-handed chiral superfield:

$$\begin{gathered} L_i \sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right) \qquad \overline{e}_i \sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right) \\ Q_i \sim \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right) \qquad \overline{u}_i \sim \left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right) \qquad \overline{d}_i \sim \left(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right) \end{gathered} \tag{2.61}$$

There is also a vector superfield for each gauge group factor $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. The numbers in (2.61) indicate the charge of each superfield with respect to these groups.

As non-derivative interactions have to come from the superpotential, which is necessarily holomorphic, we are forced to introduce two Higgs doublets

$$H_u \sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right) \qquad H_d \sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right) \tag{2.62}$$

so that both the u and d quarks can have Yukawa couplings

$$W = \overline{u}\, Y_u\, Q\, H_u - \overline{d}\, Y_d\, Q\, H_d - \overline{e}\, Y_e\, L\, H_d + \mu\, H_u H_d \,. \tag{2.63}$$

Notice that although we have more than twice the field content, we have only introduced one extra parameter over and above those of the Standard Model, μ, which couples the Higgs superfields together and has dimensions of mass.

The following terms appear to be allowed by all the symmetries too:

$$W_{RPV} = \alpha^{kmj} Q_k\, L_m\, \overline{d}_j + \beta^{kmj} L_k\, L_m\, \overline{e}_j + \gamma^m L_m H_u + \delta^{kmj} \overline{d}_k \overline{d}_m \overline{u}_j \,. \tag{2.64}$$

These can lead to too-fast proton decay and so need to be suppressed/forbidden in some way. A widely used resolution to this problem is to impose R-PARITY, a $\mathbb{Z}_2$

(discrete) subgroup of the R-symmetry group. This is a combination of matter parity and the fermion number operator introduced in Section 2.2.1 which essentially distinguishes between the usual Standard Model fields and their newly-introduced superpartners:

$$R_P = (-1)^{3(B-L)+F} \quad \Rightarrow \quad \begin{aligned} R_P[\text{Standard Model}] &= +1 \\ R_P[\text{Non-Standard Model}] &= -1 \end{aligned} \tag{2.65}$$

One can check that requiring R-parity forbids the operators in (2.64) whilst still allowing the desired Yukawas. For the rest of this thesis we will work in the R-parity preserving scenario.

A More Serious Problem

Although we have been able to dispense with some unwanted couplings, there is a more immediate problem. Recall from Section 2.2.1, a simple corollary of the SUSY algebra is that particles in the same supermultiplet will have the same mass. This then raises the question: if a supersymmetric model is supposed to be used to describe our Universe, why have we not seen a *single* superpartner? We have yet to see a scalar electron, for instance, or a Fermionic photon. In fact, none of the currently known particles can be partnered with another with the same quantum numbers but opposite statistics. If supersymmetry were realised exactly in nature, such particles would be unavoidable and we must surely have found them by now. The only possibility that allows us to keep SUSY as a useful organising principle for the high energy degrees of freedom, is for it to be manifest as a symmetry which is broken at low energies. We will learn how to approach this in Chapter 4. With our new-found understanding we can then carefully construct realistic models, a task that will occupy us for Chapters 5 and 6.

2.3. String Theory

Constructing a quantum theory of Gravity is a tricky business. Classical gravity is very well described by Einstein's General Theory of Relativity, but attempts to quantise it as one might a classical field theory seem destined to failure. The problem is seen when one tries to remove ultraviolet divergences, which are associated to the locality of interactions, by the usual renormalisation procedures. Newton's constant G, the coupling that controls the strength of gravitational interactions, has mass dimension -2

and so in attempting to remove the divergences we are forced to introduce an infinite number of counter-terms. Hence the theory is deemed non-renormalisable — not a desirable feature for a putative theory of everything.

String theory ameliorates this problem by replacing the worldline of pointlike particles with the WORLDSHEET of a one-dimensionally extended object — a STRING — with characteristic length ℓ_s. At low energies the worldsheets still look like worldlines, but as string interactions cannot be localised below the scale ℓ_s this provides a natural regularisation of the UV divergences.[11] Upon quantisation, one can easily find a massless spin two resonance in the spectrum of the closed string, which is identified as the graviton to provide a theory of quantum gravity.

So fluctuations in the worldsheet of a string can change the shape of spacetime; how can we picture this? A useful perspective one can adopt is to imagine the map that embeds the two-dimensional worldsheet, parameterised by τ and σ, into spacetime. Then from this point of view the coordinates of spacetime become fields living on the worldsheet: $X^M(\tau,\sigma)$. In fact, string theory is a *conformal* theory on the worldsheet.

There are many other famous consequences of quantising a theory of strings. Consistency at the quantum level requires the theory to live in a certain number of spacetime dimensions — from the above perspective this is the requirement that the conformal anomaly vanishes. There are no free dimensionless parameters in the theory; quantities such as the string coupling strength are determined dynamically. Above the lowest excitations of the string (which on the superstring are massless) there sits an infinite tower of higher harmonics that gives a sequence of fields with increasing mass, spaced in units of ℓ_s^{-1}. If strings really do provide the Theory of Everything, the weakness of gravity in the real world leads us to expect that $\ell_s \sim 10^{-13}\, cm$ and so even the first harmonic above ordinary matter would reside up near the Planck scale.

For a cross-section of vast literature on string theory, one could do a lot worse than consult references [53–57].

[11] Other dimensionful parameters are often used instead of ℓ_s:

$$\text{Regge Slope} \quad \alpha' = \ell_s^2 \ , \qquad \text{String Tension} \quad T = \frac{1}{2\pi\ell_s^2} \ .$$

2.3.1. The Anatomy of a Superstring

To be able to describe spacetime fermions, it turns out that string theory must turn to our old friend supersymmetry, although this is often initially imposed as a symmetry of the *worldsheet* rather than of spacetime. The story of how fermions and SUSY on the worldsheet can then conspire to deliver fermions (and SUSY) in spacetime is quite convoluted, and is found in any of the above references. For our purposes it will be sufficient to be aware of a few of the key facts about superstring theories, including their spectrum, which we now recount.

First off, a perturbative superstring theory can be constructed in essentially five different ways (to fully appreciate the careful choice of words here, see the discussion of Section 2.3.3). Each of these theories requires $(1 + 9)$ spacetime dimensions to be be fully consistent, and all five also have some degree of spacetime supersymmetry. All theories contain closed strings, whose quantisation yields the spacetime metric G_{MN}, and also the scalar dilaton field ϕ whose vacuum expectation value sets the strength of closed string interactions

$$g_s = \left\langle e^{\phi} \right\rangle .$$

There are four consistent theories that only describe the propagation of closed, oriented strings. Their main distinguishing features are displayed in Table 2.4. The amount of 10d (local) supersymmetry they preserve is counted by the supercharge index $\mathcal{N}$. Along with the dilaton and metric, each has various bosonic matter fields that are the components of differential forms on spacetime (and hence totally antisymmetric on their spacetime indices M, $N \ldots$). These are akin to gauge fields (which can themselves be thought of as 1-forms), and so have corresponding field strengths given by $F = d_R C$ where d_R is a generalisation of the exterior derivative. There are also various fermionic fields, as dictated by the supersymmetry.

The only other option, known as Type I, describes unoriented[12] open and closed strings. It is also chiral (indicating that work is required to show the theory is anomaly free), and carries $\mathcal{N} = 1$ supersymmetry on its spacetime. In the massless bosonic spectrum one will find the usual dilaton, graviton and Kalb-Ramond B_{MN} fields, and also a gauge field A^a_M in the adjoint representation of SO(32). The gauge degrees of

[12] This means it is not possible to define a consistent orientation on the worldsheet.

Type	$\mathcal{N}$	Chiral?	Bosonic Massless Spectrum
IIA	2	No	ϕ, G_{MN}, B_{MN} C_M, C_{MNP}
IIB	2	Yes	ϕ, G_{MN}, B_{MN} C_0, C_{MN}, C_{MNPQ}
Heterotic $E_8 \times E_8$	1	Yes	ϕ, G_{MN}, B_{MN} A^a_M in Adjoint of $E_8 \times E_8$
Heterotic SO(32)	1	Yes	ϕ, G_{MN}, B_{MN} A^a_M in Adjoint of SO(32)

Table 2.4.: Assorted properties of the closed superstring theories.

freedom, known as Chan-Paton factors, are associated to the ends of open strings, which leads us nicely into a short discussion of...

2.3.2. *D*-branes

The realisation that string theory isn't just all about strings revolutionised the field. At the end of the 1990's, objects known as p-branes were found within the theory. They are solitonic states which, when thought of as objects in their own right, provide a new point of view on the degrees of freedom in string theory. Their non-perturbative nature extended the above understanding of (perturbative) strings to such a point that people realised each incarnation of the superstring was actually just a different corner of a larger, all-encompassing theory. We will pick up this story again in Section 2.3.3, but first we need some familiarity with branes.

In general, a p-brane is object with p space-like dimensions (and so it traces out a $(p+1)$-dimensional WORLDVOLUME). We have already met one such beast: the fundamental string is a 1-brane. Another particularly useful class are D-branes. In Type I or II superstring theory, Dp-branes can be thought of as $(p+1)$-dimensional spacetime hypersurfaces on which fundamental open strings can end [58]. They get their name from the fact the string endpoints obey Dirichlet boundary conditions (fixed ends) in the directions transverse to the brane worldvolume. On the brane itself the endpoints of the string are free to more around (Neumann boundary conditions); as open string endpoints carry Chan-Paton factors, they will define a U(1) gauge theory on the worldvolume of the brane. Also, the location of the brane in $9-p$ transverse coordinates can be interpreted as $9-p$ real scalar fields on the worldvolume.

If we consider a stack of N coincident Dp-branes, orientable open strings of vanishing length can attach to any one of the N branes, and so we find the gauge symmetry is enhanced to $\mathrm{U}(N)$. Unoriented open strings, which arise in the presence of an orientifold, can similarly lead to orthogonal $\mathrm{SO}(N)$ or symplectic $\mathrm{USp}(N)$ groups. Thus branes gives us a nice geometric way of constructing gauge theories out of strings. This approach has all manner of uses in string phenomenology, and will be of great importance in Section 2.4.

When the two ends of an open string attached to a D-brane come together, a closed string is formed. This is now no longer bound to the brane and so can also move in directions transverse to the brane. The fact that D-branes can emit closed strings tells us they gravitate, i.e. they have mass. The mass per unit volume of the brane is known as the TENSION, and for a D-brane is given by[13]

$$T_p = \frac{\sqrt{\pi}}{g_s \kappa_{10}} (4\pi^2\alpha')^{\frac{3-p}{2}} . \tag{2.66}$$

Notice the dependence on g_s goes as $\frac{1}{g_s}$. This is why you would never see branes in a small g_s perturbation expansion. An important fact about D-branes is that as well as mass, they also carry RR-charge. This means that in the low energy effective action, they couple to the n-form fields $C_{M\dots N}$ of Table 2.4, either ELECTRICALLY via

$$\mu_p \int_{V_{p+1}} d\sigma^a \dots d\sigma^b \, C_{M\dots N} \, \partial_a X^M \dots \partial_b X^N \tag{2.67}$$

or MAGNETICALLY via

$$\mu_p \int_{V_{p+1}} d\sigma^a \dots d\sigma^b \, \widetilde{C}_{M\dots N} \, \partial_a X^M \dots \partial_b X^N \tag{2.68}$$

where $\widetilde{C}_{M\dots N}$ are related to $C_{M\dots N}$ by 10d Hodge duality of their field strengths:

$$d\widetilde{C} = *dC .$$

The map $X^M(\sigma^a)$ describes the embedding into spacetime of the brane worldvolume, parameterised by σ^a with $a = 0, 1, \dots, p$. In both cases μ_p is the RR-charge. We see that in Type IIA there exists stable Dp-branes for $p = 0, 2, 4, 6, 8$, whereas in Type IIB, $p = -1, 1, 3, 5, 7$.

[13] κ_{10} is the 10d gravitational coupling and $\alpha' = \ell_s^2$.

What is remarkable is that the RR-charge of a Dp-brane is directly related to its tension: $\mu_p^2 = g_s^2 T_p^2$. This singles D-branes out as BPS STATES, much like the short multiplets we encountered in extended SUSY models, and similarly leads to them having nice (essentially invariant) properties under renormalisation. Another useful fact about D-branes that follows from their being BPS states: they break only half of the ambient supersymmetry. All this gives us reason to trust our formulae for the brane charge and mass, despite their having been calculated in a specific (supergravity) limit. In the full theory, g_s and α' corrections are expected for the states, but these formulae (which essentially follow from the 10d SUSY algebra) shouldn't change.

For future reference it will be useful to know a thing or two about the worldvolume theory of a stack of D-branes. At low energies, for the case of one brane this is described by the Dirac-Born-Infeld action [59]

$$S_{\rm DBI} = -T_p \int_{V_{p+1}} d^{p+1}\sigma \, e^{-\phi} \sqrt{\det \left(\mathcal{P}[G+B]_{ab} + 2\pi\alpha' F_{ab}\right)} \ , \tag{2.69}$$

where $\mathcal{P}$ indicates the pull-back of the spacetime metric and 2-form to the worldvolume of the brane, and F_{ab} is the field strength of the gauge field on the brane. The expression (2.69) is exact in α' for slowly varying fields, meaning that it will receive corrections $\mathcal{O}(\alpha'^2 F^4)$. The action also has additional Wess-Zumino terms

$$S_{\rm WZ} = \mu_p \int_{V_{p+1}} \mathcal{P}\left[\sum_n C^{(n)} e^B\right] e^{2\pi\alpha' F} \ . \tag{2.70}$$

Note the integration only picks out (from the formal sum and exponentials) those combinations of n-form whose total degree is $p+1$. The effective action in the case of a stack of multiple branes is similar, but unfortunately complicated by various subtleties, which can be read about in [60].

One interesting consequence of equation (2.70) can be seen when considering an instanton (field configuration with $\mathrm{Tr}\int F\wedge F = \frac{8\pi^2}{g^2}$) on a Dp-brane. This necessarily has a Wess-Zumino term

$$\mu_p \ \mathrm{Tr} \int_{V_{p+1}} C^{(p-3)} \, 2\pi^2 \alpha'^2 \, F\wedge F \ ,$$

which by using $\mu_p = (4\pi^2\alpha')^{-2}\mu_{p-4}$ can be rearranged to give

$$\mu_{p-4}\ \mathrm{Tr}\int_{V_{p-3}} C^{(p-3)}\ . \qquad (2.71)$$

This can be recognised (via (2.67)) as how a $D(p-4)$-brane electrically sources the $(p-1)$-form $C^{(p-3)}$. The upshot is that an instanton on a Dp-brane is, for all intents and purposes, a $D(p-4)$-brane living inside the Dp-brane. Taking the specific case $p = 3$, we find $D(-1)$-branes (otherwise known as D-instantons) living inside a $D3$-brane are none other than (super) Yang-Mills instantons in the $D3$-brane's worldvolume gauge theory.

This correspondence has deep implications. For starters one can provide a clear geometrical interpretation of the ADHM construction of self-dual solutions to the Yang-Mills equations [61]. The whole construction can be viewed from the perspective of the worldvolume of the lower dimensional brane, in which case the Higgs branch[14] of the moduli space of vacua is found to be identical to the ADHM instanton moduli space (for a detailed account, see reference [44]). In the case of one D-instanton, as it can be placed anywhere inside the $D3$-brane, we would expect the 1-instanton moduli space to be the full worldvolume of the $D3$-brane. In this way the instanton is seen to be a probe of the background geometry, an idea that will be important to us in Section 2.4.3.

2.3.3. Dualities

The 10d supergravity theory obeyed by the massless modes of Type IIB strings at low energy has a special feature: it is invariant under $\mathrm{SL}(2,\mathbb{R})$ transformations that simultaneously act on the two-forms, and a combination τ of the dilaton and axion (zero-form)

$$\begin{pmatrix} B_{MN} \\ C_{MN} \end{pmatrix} \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} B_{MN} \\ C_{MN} \end{pmatrix}\ , \qquad \tau \longrightarrow \frac{a\tau+b}{c\tau+d}\ , \qquad \tau \equiv \frac{C_0}{2\pi} + ie^{-\phi}\ . \qquad (2.72)$$

Here, $ad - bc = 1$ with $a, b, c, d \in \mathbb{R}$. In the full theory, the two-form fluxes are quantised (obeying a generalisation of the Dirac quantisation condition) and so only an $\mathrm{SL}(2,\mathbb{Z})$ subgroup of this survives. This is known as an S-duality, and is still highly significant:

[14] On the Higgs branch, hypermultiplets charged under the gauge group acquire VEVs, thus Higgsing (and in general, completely breaking) the gauge symmetry.

considering the action of the group element $\left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ on the dilaton, this is recognised as a duality that maps a strongly coupled theory $g_s = \langle e^{\phi} \rangle \gg 1$ to a weakly coupled one $g_s = \langle e^{\phi} \rangle \ll 1$.

Weak-strong dualities like this are notoriously difficult to prove because one tends to only have good control over one side of the duality. This is where BPS states such as the D-branes, discussed above, can lend vital understanding. These states have well established properties and are expected to survive the transition from weak to strong coupling. One can therefore look for them on either side of the duality. In the case of Type IIB S-duality, the correspondence is

$$\begin{array}{ccc} \text{Weak Coupling} & \stackrel{S}{\longleftrightarrow} & \text{Strong Coupling} \\ \text{Fundamental String} & \longleftrightarrow & D1\text{-brane} \\ D1\text{-brane} & \longleftrightarrow & \text{Fundamental String} \\ D5\text{-brane} & \longleftrightarrow & \text{NS5-brane} \\ \text{NS5-brane} & \longleftrightarrow & D5\text{-brane} \\ D3\text{-brane} & \longleftrightarrow & D3\text{-brane} \end{array}$$

For the record, an NS5-brane is a solitonic object with tension $\sim \frac{1}{g_s^2}$, which couples magnetically to the Kalb-Ramond two-form B_{MN}. The S-duality of Type IIB strings places tight restrictions on the form of stringy corrections to correlation functions, as they too must respect the duality. We return to this in Section 2.4.3 and Chapter 3.

Similar reasoning to the above has uncovered many other dualities between the different superstring theories. For example, Type I turns out to be S-dual to the Heterotic SO(32) theory. There are also dualities that act in other ways. One that we will come across in Chapter 3 is T-duality, which essentially relates a theory with one direction compactified on a circle of radius R to a different theory compactified on a circle of radius $1/R$. Under such a transformation:

$$\begin{array}{ccc} \text{Type IIA} & \stackrel{T}{\longleftrightarrow} & \text{Type IIB} \\ \text{Heterotic SO(32)} & \stackrel{T}{\longleftrightarrow} & \text{Heterotic } \mathrm{E}_8 \times \mathrm{E}_8 \end{array}$$

Another famous result: it had been known for a long time that $\mathcal{N} = 1$ supergravity in 11 dimensions, when compactified on a circle gives the low energy spectrum of Type IIA supergravity. Through brane-based reasoning, Witten conjectured that the strong

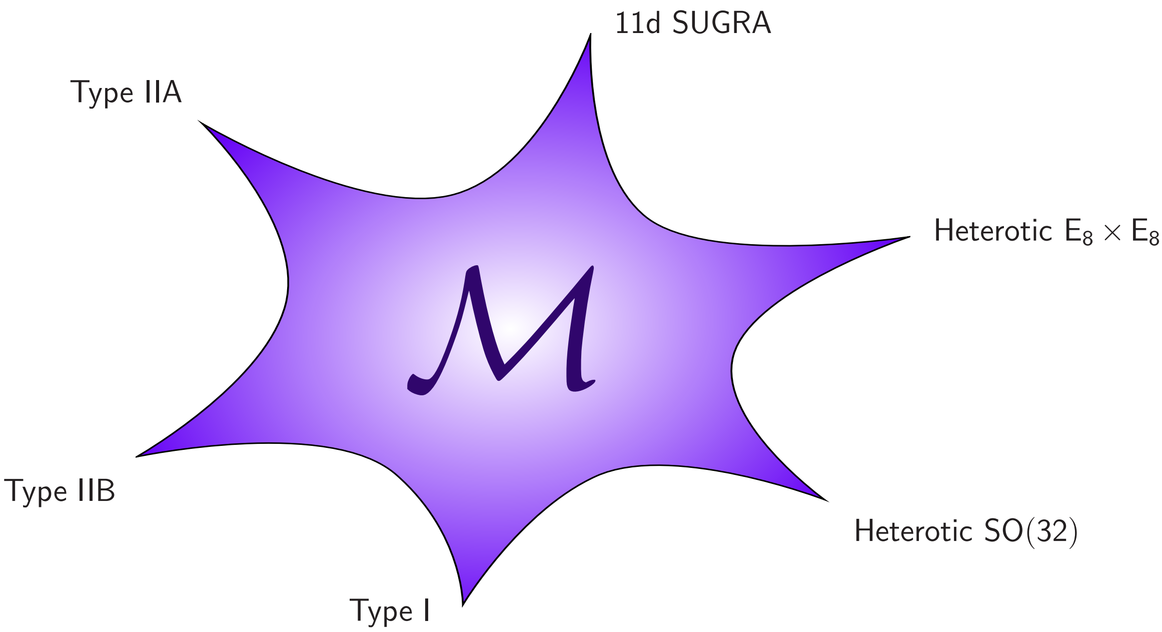


Figure 2.3.: A non-artist's impression of the M-Theory moduli space.

coupling limit of Type IIA strings is actually an 11-dimensional theory of membranes, known as M-Theory. This same theory can be compactified on a line interval to give the 10d Heterotic $E_8 \times E_8$ theory.

In this way a picture is building up in which all the apparently different superstring theories are just different limits of a greater, all-encompassing model, also now known as M-Theory. Evidence for all the dualities conjectured above is still accruing, and new mathematical techniques that let us explore the M-Theory landscape (illustrated in Figure 2.3) are still being developed. It will be interesting to see how this picture evolves, and whether we can one day gain new physical insight from this remarkable and (from a mathematical point of view) rather unique-looking model.

The ostensible uniqueness of the superstring framework is spoilt by one of its greatest virtues: the prediction of extra spacial dimensions. As every experiment to date has been consistent with there being three space dimensions, string phenomenologists have to compactify the extra dimensions down to very small scales to account for their non-detection. There are many different ways of compactifying,[15] parameterised by MODULI, each choice of which gives slightly different 4d physics, so our Theory of Everything now appears to be a Theory of Anything. The correct way to approach this LANDSCAPE of possibilities varies depending on who you talk to. Some people think we should search for dynamical mechanisms capable of selecting one out of the multitude of vacua, whereas others prefer to invoke semi-anthropic arguments to justify the observed laws of physics.

[15]Recent estimates suggest there may be as many as 10^{500} distinct ways.

Despite this apparently vast freedom, nobody has yet produced a model in which all the moduli are fixed, and that has only the Standard Model spectrum at low energies.

2.4. AdS/CFT Correspondence

2.4.1. Large N Gauge Theories

The idea that stringy behaviour is of importance in understanding the physics of hadrons pre-dated both the development of Quantum Chromodynamics (QCD) — our current understanding of the hadronic world — and the realisation that oscillating strings can be used to build a consistent theory of quantum gravity. When plotting the spin versus mass-squared of various mesons, the results were found to lie on lines of constant slope α' (known as Regge trajectories). Such behaviour can be explained by modelling the meson as a quark-antiquark pair bound together by a string with constant tension $1/\alpha'$. Unfortunately, such Dual Resonance Models were found to be phenomenologically unviable.

In time, experiment established that hadrons should be thought of as a collection of quarks, bound together by a strongly interacting non-Abelian gauge theory. At high energies the strength of this interaction drops off (asymptotic freedom), so the quarks' behaviour is well modelled by a perturbation series in the gauge coupling, expanded around the free theory. At low energies though, the story is very different: the interaction strength grows, and the quarks become confined into meson and baryon bound-states. The large size of the coupling means perturbation theory is no longer valid, so understanding how this confinement takes place becomes a very difficult question.[$$$] Unfortunately for us this hadronisation process is crucial for linking theoretical predictions (which are largely based on perturbative techniques) to the results of experiment.

In an effort to better understand the mysterious strong coupling behaviour of non-Abelian gauge theories, 't Hooft considered a limit of QCD where the number of colours, instead of being three, was a large integer N [63]. He found that in order to keep the strong coupling scale Λ_{QCD} finite and non-zero, he had to take the limit $N \to \infty$ whilst keeping the combination $\lambda \equiv g_{YM}^2 N$ fixed — this is now known as the 't Hooft limit.

[$$$]In fact, it's a Million Dollar question [62].

The g_{YM} perturbation expansion of any process can trivially be rearranged into a double expansion in terms of N and λ, but 't Hooft made a further observation: the order of N in which a given Feynman diagram contributes to a process is completely determined by its topology. More precisely, a two dimensional surface can be assigned to each diagram (essentially, the simplest surface on which the diagram can be drawn with no self-intersections). The number of factors of N that comes with each diagram is found to be the Euler characteristic χ of the associated surface, which for a orientable closed surface is given in terms of the genus g (number of holes) and number of boundaries b by $\chi = 2 - 2g - b$. Now, with all diagrams weighted by factors of N^{2-2g-b} we see that as $N \to \infty$ the theory simplifies, because all processes are dominated by diagrams with the topology of a sphere (genus 0 and no boundaries). Note also that in this limit, although $g_{YM} \to \infty$, one is still left with a perturbation expansion in λ. This therefore becomes the effective coupling (known as the 't Hooft coupling).

The $1/N$ expansion in gauge theory is highly reminiscent of the genus expansion of string theory — that only one topology of string diagram contributes at each order in a g_s perturbation expansion. This is yet another indication that string-like behaviour may be of relevance to the physics of hadrons. Taking this idea any further, for instance to find a string theory which describes the strong coupling behaviour of QCD, is a monumentally tricky task. However, progress was made after the discovery of D-branes brought a new perspective on gauge theory.

2.4.2. Establishing a Correspondence

In the remarkable paper [64], Maldacena proposed a correspondence between the maximally supersymmetric $(1+3)$-dimensional conformal field theory ($\mathcal{N} = 4$ super Yang-Mills) and Type IIB superstring theory on the background $\mathrm{AdS}_5 \times \mathrm{S}^5$. It was the first concrete realisation of a duality between large N gauge theory and strings, opened up a plethora of new avenues for research, and has lead to a far deeper understanding of the dynamics of both gauge theories and gravity. By way of introduction we can recommend the review articles [65, 66].

The correspondence can be revealed by considering a stringy brane construction from two different points of view. We begin with Type IIB superstrings in $(1+9)$-dimensional flat space and place a stack of N coincident $D3$-branes at the origin. At low energy, in the bulk far from the branes we would expect to find the usual spectrum of Type IIB in flat space, but on their worldvolume we know from Section 2.3.2 that the branes support a U(N) gauge theory. Also, as the branes break half of the ambient 32 supersymmetries, we know precisely what this $(1+3)$-dimensional gauge theory with 16 supercharges is — it's the $\mathcal{N}=4$ super Yang-Mills theory we met in Section 2.2.1. Schematically the action for this open string view of the brane construction splits into three parts

$$S_{\text{open}} \;=\; S_{\text{bulk}} \;+\; S_{\mathcal{N}=4} \;+\; S_{\text{int}} \tag{2.73}$$

describing the bulk excitations, the gauge theory on the branes, and the interaction between the two, respectively. These last two terms are described more correctly by the Dirac-Born-Infeld action [59]. To leading order in α' this reduces to the $\mathcal{N}=4$ action, so to turn off the bulk-brane interaction, one should take the limit $\alpha' \to 0$.

Alternatively, as the branes are massive objects they must to some extent warp the geometry in which they sit. Taking this BACKREACTION into account one can replace the branes with the geometry they create to find a space with metric

$$ds^2 \;=\; \frac{1}{\sqrt{H(r)}}\left(-dt^2 \;+\; \sum_{i=1}^{3} dx^i dx^i\right) \;+\; \sqrt{H(r)}\left(dr^2 + r^2 d\Omega_5^2\right) \;, \tag{2.74}$$

where

$$H(r) = 1 + \left(\frac{R}{r}\right)^4 \qquad \text{with} \qquad R^4 = 4\pi g_s N \alpha'^2 \;. \tag{2.75}$$

This metric, along with a constant dilaton, and five-form flux N over the five-sphere (Ω_5), is a solution of the classical supergravity equations of motion that is valid provided $g_s < 1$ and that the curvature (of characteristic scale R) is much larger than the string scale $\sqrt{\alpha'}$, i.e. $4\pi g_s N \gg 1$.

Note that far from the branes, $r \gg R$, so $H(r) \approx 1$ and the metric (2.74) returns to that of 10d flat space. Near the branes, $H(r) \approx \left(\frac{R}{r}\right)^4$ and we find the metric

$$ds^2 \;=\; \frac{r^2}{R^2}\left(-dt^2 \;+\; \sum_{i=1}^{3} dx^i dx^i\right) \;+\; R^2\frac{dr^2}{r^2} \;+\; R^2 d\Omega_5^2 \tag{2.76}$$

which is the geometry of $\text{AdS}_5 \times \text{S}^5$ with each factor having radius R. Again, we can write the action of this closed string point of view in a suggestive form:

$$S_{\text{closed}} \;=\; S_{\text{bulk}} \;+\; S_{\text{AdS}_5\,\times\,\text{S}^5} \;+\; S_{\text{int}} \;. \tag{2.77}$$

The idea that arises from comparing (2.73) and (2.77) is to define a DECOUPLING LIMIT in which the interaction terms vanish and each action neatly splits into two parts. Given that the bulk excitations are the same in both pictures, we are lead to conclude that $(1+3)$-dimensional $\mathcal{N}=4$ super Yang-Mills is dual to Type IIB superstrings on the background $\mathrm{AdS}_5 \times \mathrm{S}^5$. We now look at this proposition more carefully.

The Decoupling Limit

We need to consider the limit $\alpha' \to 0$ so the low energy dynamics of the brane are modelled by $\mathcal{N}=4$ sYM, but at the same time we should keep fixed the energy E_∞ of excitations in the bulk, as measured by an observer at infinity. Moreover, we would also like to retain the tower of stringy excitations, which has a spacing between states of $\sqrt{\alpha'}E_r$, where E_r is the energy measured by an observer at radial position r. From the metric (2.74) we can read off the redshift factor that relates this to to the asymptotic energy: $E_\infty = E_r\,[H(r)]^{-1/4}$. Putting this all together

$$\underbrace{E_\infty}_{\text{Fixed}} = E_r\left[1+\left(\frac{R}{r}\right)^4\right]^{-1/4} \approx \frac{r}{\alpha'}\underbrace{\frac{E_r\sqrt{\alpha'}}{\sqrt[4]{4\pi g_s N}}}_{\text{Fixed}} . \tag{2.78}$$

So if all other parameters are to remain fixed, the required decoupling limit is

$$\alpha' \to 0 \qquad \text{with} \qquad \frac{r}{\alpha'} \qquad \text{fixed} . \tag{2.79}$$

One can introduce coordinates that are better suited to taking this limit by defining $U = r/\alpha'$. Then metric (2.74) in the decoupling limit becomes

$$ds^2 = \alpha'\left[\frac{U^2}{\sqrt{4\pi g_s N}}\left(-dt^2 + \sum_{i=1}^{3} dx^i dx^i\right) + \sqrt{4\pi g_s N}\,\frac{dU^2}{U^2} + \sqrt{4\pi g_s N}\,d\Omega_5^2\right] , \tag{2.80}$$

which is the metric on $\mathrm{AdS}_5 \times \mathrm{S}^5$ with both factors having radius $R = (4\pi g_s N \alpha'^2)^{1/4}$ as advertised.

To connect with the description of $\mathcal{N}=4$ sYM, we need to know how to associate parameters in the string theory with those of the gauge theory. The following combination of dilaton and axion fields often occurs in string theory

$$\tau = \frac{i}{g_s} + \frac{C_0}{2\pi} \qquad \text{where} \qquad g_s = e^{\phi} \ . \tag{2.81}$$

Similarly, a combination of the gauge coupling g_{YM} and vacuum angle θ which arises naturally in many contexts is

$$\tau_0 = \frac{4\pi i}{g_{YM}^2} + \frac{\theta}{2\pi} \ . \tag{2.82}$$

From expanding the DBI action in search of the coefficient of $\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$, one finds

$$4\pi\, g_s = g_{YM}^2 \ , \qquad \text{or more generally} \qquad \tau = \tau_0 \ . \tag{2.83}$$

We explore the significance of this relation in Section 2.4.3, and in Chapter 3 we'll also see how it may be modified under deformations the correspondence.

Our stack of N $D3$-branes supports a $\mathrm{U}(N)$ gauge theory, but each brane is also charged under the 4-form C_{MNPQ}, which has a 5-form field strength F_5. Therefore the rank of the gauge group must be proportional to the flux of this 5-form through a surface surrounding the branes:

$$N = \int_{S^5} F_5 \ .$$

Validity. . . Verification. . . Proof?

As a pleasing sanity-check, one can observe the same symmetry groups on either side of the duality. The maximal bosonic symmetry is the group $\mathrm{SO}(2,4) \times \mathrm{SO}(6)$ which on the gravity side is just the isometry group of $\mathrm{AdS}_5 \times \mathrm{S}^5$. In the gauge theory, $\mathrm{SO}(2,4)$ is the conformal group of $(1+3)$-dimensional spacetime whereas $\mathrm{SO}(6)$, which is locally isomorphic to $\mathrm{SU}(4)$, arises as the R-symmetry group discussed in Section 2.2.1.

The above supergravity perspective is good so long as $g_s < 1$ and $g_s N \gg 1$, which implies $N \gg 1$. Also, the effective coupling of the gauge theory $\lambda = g_{YM}^2 N \gg 1$, so we see that AdS/CFT duality realises 't Hooft's dream to understand the strong coupling behaviour of a large N gauge theory in terms of strings. The duality is conjectured to hold at large N but with finite $g_s N$. This allows for the possibility of a gauge theory perturbation expansion in λ, but now there is a problem on the string side; string theory on a curved background, especially one with non-zero RR fluxes, is poorly understood, even in the classical limit ($g_s < 1$).

Despite the lack of perturbative control, the strongest form of the AdS/CFT duality asserts that for any value of the couplings there should be a complete quantum equivalence between $\mathcal{N} = 4$ sYM and Type IIB strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$. Given the fact that — at best — we can only have control over one side of the correspondence, proving this statement is currently an impossible task, but the circumstantial evidence in its favour is mounting, as we shall see below. Turning things around, by accepting the existence of such a correspondence one inherits an arsenal of new tools for exploring gauge theory and gravity in regimes beyond the scope of standard techniques.

Another great success of the AdS/CFT correspondence is that it provides a concrete realisation of the Holographic Principle. This is the very general (and consequently somewhat vague) notion that in any theory of quantum gravity in a volume V, the number of degrees of freedom of the system should scale as the size of the *boundary* of V (rather than with V itself, as in a standard quantum field theory). The principle can be motivated by considering the Bekenstein-Hawking entropy of a black hole, which is proportional to the area of the event horizon. In the context of AdS/CFT, the concept of holography arises when trying to make the correspondence more precise.

The hint of where to start comes from considering the string coupling g_s. The magnitude of this is controlled by the VEV of the dilaton field, which in turn is dictated by its boundary conditions. The boundary of $\mathrm{AdS}_5 \times \mathrm{S}^5$ is perhaps most clearly seen by substituting $y = \frac{\sqrt{4\pi g_s N}}{U}$ into the metric (2.80) and setting $R = 1$ for convenience:

$$ds^2 \;=\; \frac{1}{y^2}\left(-dt^2 + d\vec{x}^2 + dy^2\right) \;+\; d\Omega_5^2 \;. \tag{2.84}$$

The boundary lies at $y = 0$ and is clearly conformally equivalent to $(1+3)$-dimensional Minkowski space. So as the boundary conditions of the dilaton, and its dual, the gauge coupling, are both determined by their values on a 4d flat space, it is natural to think of the gauge theory as 'living on the boundary' of $\mathrm{AdS}_5 \times \mathrm{S}^5$ — the gauge theory on the boundary holographically encodes the behaviour of gravity on the contained volume. Recalling that the gauge coupling effectively acts as a source for the operator $\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$ allows this holographic correspondence to be generalised [67, 68]. For a general operator $\mathcal{O}(\vec{x})$ in the gauge theory, its source term $\phi_0(\vec{x})$ acts as the boundary condition for a corresponding field $\phi(\vec{x}, y)$ in the string theory. This is usually stated as a relationship between the string partition function and the CFT's generating functional:

$$\left\langle e^{\int d^4x\, \phi_0(\vec{x})\mathcal{O}(\vec{x})}\right\rangle_{\rm CFT} = \mathcal{Z}_{\rm string}\left[\phi(\vec{x},y)\Big|_{y=0} = \phi_0(\vec{x})\right] . \tag{2.85}$$

Taking the more prosaic supergravity limit $g_s < 1$ and $g_s N \gg 1$, we know the string partition function is dominated by the supergravity action (assuming there's only one saddle point for simplicity), and so we find the CFT's *connected* generating functional for large N and at strong 't Hooft coupling is given by

$$W_{\rm CFT}[\phi_0] \equiv -\log\left\langle e^{\int d^4x\, \phi_0(\vec{x})\mathcal{O}(\vec{x})}\right\rangle_{\rm CFT} = \underset{\phi|_{y=0}=\phi_0}{\text{infimum}}\; S_{\rm SUGRA}[\phi] . \tag{2.86}$$

A more sophisticated test of the correspondence can be made by recalling from Section 2.2.1 that in $\mathcal{N}=4$ sYM, certain operators transform in short representations of the superalgebra, which implies they are immune to quantum corrections. We would therefore expect to find states with the appropriate quantum numbers in the spectrum of supergravity on $\mathrm{AdS}_5 \times \mathrm{S}^5$. Such operators have been completely classified [69], and the corresponding supergravity states can be precisely identified [70], thus affirming the conjectured duality.

2.4.3. The Story of Instanton Matching

In Section 2.3.3 we touched upon a curious property of Type IIB string theory: it is self-dual under an S-duality which acts on the dilaton-axion parameter τ as

$$\tau \longrightarrow \frac{a\tau+b}{c\tau+d} , \qquad \text{where} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) . \tag{2.87}$$

If there is to be an exact duality between Type IIB strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathcal{N}=4$ sYM under which τ is identified with τ_0, then the gauge theory had better also be invariant under an $\mathrm{SL}(2,\mathbb{Z})$ action with τ_0 transforming analogously to (2.87). Indeed it does; there is generalisation of the ELECTROMAGNETIC DUALITY of Maxwell's equations to the spectrum of spontaneously broken gauge theories [49] in which elementary field excitations are interchanged with BPS dyons[16] when the complexified coupling τ_0 undergoes an $\mathrm{SL}(2,\mathbb{Z})$ transformation. The duality can be further extended to the realms of $\mathcal{N}=4$

[16] Dyons are similar to monopoles, but carry both electric and magnetic charge.

sYM where quantum corrections are more safely under control thanks to the high degree of supersymmetry [50].

The AdS/CFT correspondence has clearly dodged another bullet, but as S-duality is expected to be exact beyond the supergravity approximation, it is reasonable to hope that it may be used as a more incisive probe of the correspondence, particularly if formulae like equation (2.85) are to be believed. We will now review some of the fruits of this observation, which are comprehensively chronicled in [71].

S-duality can be used to construct the leading order stringy corrections to the Type IIB supergravity action on a flat background [72]. They correspond to accounting for the effect of D-instantons (aka $D(-1)$-branes) and include terms (in string frame) of the form

$$(\alpha')^{-1}\int d^{10}x\,\sqrt{-G_{10}}\,e^{-\phi/2}\,f_4(\tau,\overline{\tau})\,\mathcal{R}^4\ , \tag{2.88}$$

and

$$(\alpha')^{-1}\int d^{10}x\,\sqrt{-G_{10}}\,e^{-\phi/2}\,f_{16}(\tau,\overline{\tau})\,\Lambda^{16}\quad+\quad\text{c.c.}\ , \tag{2.89}$$

where $\mathcal{R}$ is a specific contraction of 10d Riemann tensors, given in [72], and Λ is the 10d dilatino. The functions $f_n(\tau,\overline{\tau})$ are MODULAR FORMS, which transform under $SL(2,\mathbb{Z})$ in precisely the right way to leave the action S-duality invariant.

In [73] Banks and Green observed that the same effective action also provides the leading corrections to the background $AdS_5\times S^5$, so via the gauge-string correspondence this gravity result can be used to make predictions for the behaviour of the gauge theory. From the discussion of Section 2.3.2 we would expect the D-instanton corrections to correspond to a multi-instanton effect in the gauge theory. This possibility is made more evident by performing a weak coupling expansion of the above modular forms to extract terms like

$$e^{-\phi/2}f_n(\tau,\overline{\tau})\ \ni\ \sum_{k=1}^{\infty}\text{const}\times\left(\frac{k}{g_s^2}\right)^{n-7/2}\times e^{2\pi ik\tau}\times\sum_{d|k}\frac{1}{d^2}\ , \tag{2.90}$$

where the sum on d runs over positive integral divisors of k. With the AdS/CFT correspondence making the association $\tau = \tau_0$ (cf. equation (2.83)) this very much looks like a gauge theory instanton expansion.[17]

The actual test of the correspondence comes from comparing correlation functions (with the relevant instanton corrections) on both the gravity and gauge theory side. As mentionned in Section 2.2.2, instantons can only modify those correlation functions that are able to saturate the fermionic zero modes of the new terms. In this case, as the D-instanton breaks half of the 16 SUSY and 16 superconformal generators of the background, there are 16 exact fermionic zero modes. An interesting class of correlators to consider are therefore

$$\big\langle \Lambda_1(\vec{x}_1)\ldots\Lambda_{16}(\vec{x}_{16})\big\rangle \;, \tag{2.91}$$

with each dilatino absorbing one zero mode. Note that here the dilatinos originate on the boundary of $AdS_5 \times S^5$. Further correlation functions that can be considered are found in [74, 75].

The superstring prediction for the correlation function (2.91) can be found by propagating the dilatinos into the bulk to meet at an effective vertex at y_0 (derived from equation (2.89)). One then integrates this point $y_0 \equiv (\vec{X}, \rho, \hat{\Omega}_i)$ over all of $AdS_5 \times S^5$ and performs the Grassmann integrals of the fermi zero modes. The result is

$$\big\langle \Lambda_1(\vec{x}_1)\ldots\Lambda_{16}(\vec{x}_{16})\big\rangle \sim (\alpha')^{-1} e^{-\phi/2} f_{16}(\tau,\overline{\tau})\, t_{16} \int d^5\hat{\Omega}_5 \int \frac{d^4X\,d\rho}{\rho^5} \prod_{i=1}^{16} K^F_{7/2}(\vec{X},\rho;\vec{x}_i,0) \;, \tag{2.92}$$

where the dilatino bulk-to-boundary propagator is

$$K^F_{7/2}(\vec{X},\rho;\vec{x},0) \;=\; K_4(\vec{X},\rho;\vec{x},0)\left(\rho^{1/2}\gamma_5 - \rho^{-1/2}(x-X)_n\gamma^n\right) \;, \tag{2.93a}$$

with

$$K_4(\vec{X},\rho;\vec{x},0) \;=\; \frac{\rho^4}{(\rho^2+(x-X)^2)^4} \;. \tag{2.93b}$$

The 16-index antisymmetric tensor t_{16} comes from performing the Grassmann integration, and encodes the dilatino's fermi statistics. It also ensures that eight copies each of

[17] For fixed k, each of these terms receives g_s^2 corrections. Also, the contribution from anti-instantons is suppressed by a factor g_s^{4n-16} — cf. equation (1.12) of [71].

the terms $\rho^{1/2}\gamma_5$ and $\rho^{-1/2}\gamma^n$ are picked out of the product of propagators. To facilitate comparison with the related object in gauge theory, observe from (2.90) the form of the correction to leading order in α' and g_s:

$$\big\langle \Lambda_1(\vec{x}_1)\dots\Lambda_{16}(\vec{x}_{16})\big\rangle \sim (\alpha')^{-1} g_s^{-\frac{25}{2}} \ . \tag{2.94}$$

Under the AdS/CFT correspondence, the dilatino sources a particular superconformal Noether current in the gauge theory, which is

$$\Lambda^A_\alpha \;=\; \frac{1}{g^2_{YM}}\,\sigma^{mn}{}_\alpha{}^\beta\,\mathrm{tr}_N\,F_{mn}\lambda^A_\beta \ . \tag{2.95}$$

Here we recognise the $\mathcal{N}=4$ sYM field strength F_{mn} and gaugino λ^A_β. The index $A = 1,\dots,4$ counts the supersymmetries, and to be consistent with the instanton literature, from now until the end of Chapter 3, spacetime indices are denoted by lower-case roman letters $m,n = 0,\dots,3$. Details of the σ^{mn} matrices can be found in Appendix A.

The one instanton correction to correlation function (2.91) was first computed in reference [74] for the case of a single SU(2) Yang-Mills instanton. This was extended to general N in reference [76] which allowed for the large N limit to be taken. Let's look at this a bit more closely.

We begin with the 1-instanton partition function appropriate for the calculation of gauge invariant correlation functions [71]

$$\int d\mu^1_{\mathrm{phys}}\, e^{-S^1_{\mathrm{inst}}} \;=\; \frac{g^8_{YM}e^{2\pi i\tau_0}}{2^{31}\pi^{13}(N-1)!(N-2)!}\int d^4X\, d\rho\, d^5\hat{\Omega}_5\, \rho^{4N-7}\, I_N \prod_{A=1}^{4} d^2\xi^A\, d^2\bar{\eta}^A \ , \tag{2.96}$$

where we have separated out an an integral over the collective coordinate r, which takes the form:

$$I_N \;=\; \int_0^\infty dr\, r^{4N-3}\, e^{-2\rho^2 r^2} \ . \tag{2.97}$$

In equation (2.96), X and ρ are the bosonic collective coordinates corresponding to instanton position and scale size respectively. The variables $\chi_a \equiv \{r, \hat{\Omega}_5\}$ are a 6-vector of bosonic collective coordinates expressed in 6d polar coordinates, about which we will have more to say in Section 3.3.1. The fermionic zero modes are ξ and $\bar{\eta}$, and correspond to the supersymmetric and superconformal symmetries broken by the instanton.

Due to a quirk of the one instanton sector, the integral (2.97) can be done exactly,[18] but as this trick doesn't generalise to the multi-instanton case it is more enlightening to solve in the large N limit with a saddle-point approximation:

$$I_N \;=\; N^{2N-1}\int_0^\infty dr\, r^{-3}e^{2N(\log r^2-\rho^2r^2)} \;=\; N^{2N-1}\left(\rho^{2-4N}e^{-2N}\sqrt{\frac{\pi}{4N}}\;+\;\mathcal{O}\left(N^{-3/2}\right)\right)\;. \tag{2.98}$$

The saddle-point lies at $r=\rho^{-1}$. Collecting powers of ρ we find the bosonic part of the partition function is

$$\int\frac{d^4X\,d\rho}{\rho^5}\int d^5\Omega_5\;, \tag{2.99}$$

which is precisely the measure on $\mathrm{AdS}_5\times\mathrm{S}^5$. We appear to be on the right track: the moduli space of the Yang-Mills instanton has the same geometry as the supergravity background probed by the D-instanton. It is also encouraging to see that only in the large N limit (where the saddle point equation applies) is the size of the five-sphere r related to the radial coordinate ρ of AdS_5, as required by the AdS/CFT correspondence.

The other ingredient we require for computing the correlation function is an expression for the gaugino λ^A_β of equation (2.95) in a superinstanton background. This takes the form (cf. equations (4.3a) and (A.5) of reference [77])

$$\lambda^{A\,\beta}(x)\;=\;-\left(\xi^{A\gamma}-\overline{\eta}^A_{\dot\gamma}\,\overline{\sigma}^{\dot\gamma\gamma}_m\cdot(x^m-x^m_0)\right)\sigma^{kl}{}_\gamma{}^\beta\,F_{kl}(x-x_0)\;+\;\ldots \tag{2.100}$$

where the ellipsis denotes terms containing fermi zero modes that are lifted by the instanton action, and so don't contribute to the correlation function of interest. We can now construct the Noether current (2.95), which can be expressed in the following form:

$$\begin{aligned}
\Lambda^A_\alpha \;&=\; -\frac{1}{g^2_{YM}}\left(\xi^{A\gamma}-\overline{\eta}^A_{\dot\gamma}\,\overline{\sigma}^{\dot\gamma\gamma}_s\cdot(x^s-x^s_0)\right)\sigma^{mn}{}_\alpha{}^\beta\,\sigma^{kl}_{\gamma\beta}\;\mathrm{tr}_N\left[F_{mn}\,F_{kl}(x-x_0)\right] && \text{(2.101a)}\\
&=\; -\frac{1}{3\,g^2_{YM}}\left(\xi^{A\gamma}-\overline{\eta}^A_{\dot\gamma}\,\overline{\sigma}^{\dot\gamma\gamma}_s\cdot(x^s-x^s_0)\right)\sigma^{mn}{}_\alpha{}^\beta\,\sigma^{kl}_{\gamma\beta}\,\mathcal{P}_{mn,\,kl}\;\mathrm{tr}_N\left[F^2_{pq}(x-x_0)\right] && \text{(2.101b)}\\
&=\; -\frac{1}{g^2_{YM}}\,\varepsilon_{\alpha\gamma}\left(\xi^{A\gamma}-\overline{\eta}^A_{\dot\gamma}\,\overline{\sigma}^{\dot\gamma\gamma}_s\cdot(x^s-x^s_0)\right)\;\mathrm{tr}_N\left[F^2_{pq}(x-x_0)\right] && \text{(2.101c)}
\end{aligned}$$

The second line follows from the identity $\mathrm{tr}_N\,F_{mn}\,F_{kl}\;=\;\frac{1}{3}\,\mathcal{P}^{\mathrm{SD}}_{mn,kl}\;\mathrm{tr}_N(F_{pq})^2$, where $\mathcal{P}^{\mathrm{SD}}_{mn,kl}$ projects onto self-dual Lorentz structure. Fortunately we don't need to know the precise

[18] $I_N=\frac{1}{2}(2\rho^2)^{1-2N}\int_0^\infty dx\,x^{2N-2}\,e^{-x}=\frac{1}{2}(2\rho^2)^{1-2N}(2N-2)!$

structure of this projector because, as reviewed in Appendix A, the sigma matrices are already self-dual, and so the whole thing reduces to equation (2.101c). One can then insert the standard charge-1 instanton field strength,[19]

$$F^a_{mn} = -4\,\eta^a_{mn}\frac{\rho^2}{\left(\rho^2+(x-x_0)^2\right)^2} ,$$

with η^a_{mn} being the 't Hooft matrices listed in Appendix A, to find

$$\begin{aligned}\Lambda^A_\alpha &= -\frac{96}{g^2_{YM}}\left(\xi^A_\alpha - \sigma^s_{\alpha\dot\alpha}\,\overline{\eta}^{\dot\alpha A}\cdot(x-x_0)_s\right)\frac{\rho^4}{\left(\rho^2+(x-x_0)^2\right)^4} &\text{(2.102a)}\\ &= -\frac{96}{g^2_{YM}}\left(\xi^A_\alpha - \sigma^s_{\alpha\dot\alpha}\,\overline{\eta}^{\dot\alpha A}\cdot(x-x_0)_s\right)K_4(\vec{x}_0,\rho;\vec{x},0) . &\text{(2.102b)}\end{aligned}$$

In the second line we have identified the factor K_4 which we saw before in the dilatino bulk-to-boundary propagator (2.93).

We are finally in a position to combine our results for the partition function and operators to compute the correlation function:

$$\begin{aligned}\left\langle\Lambda_1(\vec{x}_1)\ldots\Lambda_{16}(\vec{x}_{16})\right\rangle &= C_N\int\frac{d^4X d\rho}{\rho^5}\int\prod_{A=1}^{4}d^2\xi^A\,d^2\overline{\eta}^A &\text{(2.103)}\\ &\quad\times\left(\xi^1_{\alpha_1}-\overline{\eta}^1_{\dot\gamma}\overline{\sigma}^{\dot\gamma}_{m\alpha_1}\cdot(x_1-X)^m\right)K_4(\vec{X},\rho;\vec{x}_1,0)\\ &\quad\;\vdots\\ &\quad\times\left(\xi^4_{\alpha_{16}}-\overline{\eta}^4_{\dot\gamma}\overline{\sigma}^{\dot\gamma}_{m\alpha_{16}}\cdot(x_{16}-X)^m\right)K_4(\vec{X},\rho;\vec{x}_{16},0) ,\end{aligned}$$

in which the prefactor is

$$C_N = g^{-24}_{YM}\frac{(2N-2)!}{(N-1)!(N-2)!}\,2^{-2N+49}\,3^{16}\,\pi^{-10} . \qquad (2.104)$$

We can take the $N\to\infty$ limit using Stirling's approximation $n!\approx\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ to find

$$C_N \longrightarrow g^{-24}_{YM}\sqrt{N}\,2^{47}\,3^{16}\,\pi^{-21/2} . \qquad (2.105)$$

[19] We are considering the SU(N) instanton as an SU(2) instanton embedded in a diagonal 2×2 subgroup of SU(N). Other embeddings have higher topological charge.

Using the AdS/CFT dictionary (2.75) and (2.83) to translate equation (2.94) into gauge language,

$$\left\langle \Lambda_1(\vec{x}_1)\ldots\Lambda_{16}(\vec{x}_{16})\right\rangle \sim (\alpha')^{-1} g_s^{-25/2} \tag{2.106}$$

$$\sim \sqrt{N} g_{YM} \left(\frac{g_{YM}^2}{4\pi}\right)^{-25/2} = \sqrt{N} g_{YM}^{-24} \, , \tag{2.107}$$

we see the Yang-Mills instanton calculation reproduces the gravity result in the large N limit.

It is important to note that the domains of validity of the above two results do not overlap. To have perturbative control over the large N gauge theory, the 't Hooft coupling must be small, and so $g_{YM} < 1$, whereas for the supergravity calculation to be trusted, we must be in a regime where the curvature radii of the background geometry are significantly greater than the string scale, i.e. $R^2 \gg \alpha'$, and also $g_s < 1$. The AdS/CFT correspondence then implies $g_{YM} \gg 1$ for the SUGRA calculation to be valid. It is therefore remarkable that we see such close agreement for this correlation function in each framework. Indeed, a similar matching can be obeserved for a wide variety corrrelators to which instantons contribute, including Kaluza-Klein excitations which truly probe the geometry of the five-sphere.[20] This has lead to the conjectured existence of a non-renormalisation theorem for these non-perturbative phenomena. We will explore this idea more in Chapter 3 once we have accumulated a bit more evidence.

By making careful study of a supersymmetric version of the ADHM construction [61], the above Yang-Mills result was extended to a full multi-instanton background in reference [78] with complete details of the calculation reported in reference [71]. This long calculation contains yet more suprises. The first observation is that our manipulation of equation (2.102) used the fact that the squared field strength of a one instanton solution essentially takes the form of a SUGRA bulk-to-boundary propagator (2.93). This isn't true at the multi-instanton level, except perhaps in the DILUTE GAS APPROXIMATION, which is only valid in those regions of the k-instanton moduli space that can be understood as being built from k widely separated single instanton solutions. Even in this special case something strange seems to be going on, because one would expect the moduli space to resemble k copies of $\mathrm{AdS}_5 \times \mathrm{S}^5$ (one for each instanton) rather than the single copy required by the AdS/CFT correspondence.

[20]Note: the correlator considered in equation (2.91) has no dependence on the coordinates of S^5.

This issue was shown to have a fascinating resolution in references [71, 78]. Careful consideration of the small fluctuations about a multi-instanton background reveals that in the large N limit, a potential is generated on moduli space that attracts all the instantons to a point, whilst leaving them in mutually commuting SU(2) subgroups of the full SU(N) gauge group. This substantially reduces the k-instanton moduli space, and causes the effective measure to factorise into one copy of the measure on $\mathrm{AdS}_5 \times \mathrm{S}^5$ (as per the conjectured duality), times the partition function $\hat{\mathcal{Z}}_k$ of a 10-dimensional $\mathcal{N} = 1$ supersymmetric SU(k) gauge theory, dimensionally reduced to zero dimensions. The resulting configuration is interpreted on the string side as a bound state of D-instantons.

One can in fact go further, and solve the SU(k) matrix model [79] to extract the full k-dependence,

$$\hat{\mathcal{Z}}_k = 2^{17k^2/2-k/2-8} \, \pi^{9k^2/2-9/2} \, k^{-1/2} \sum_{d|k} \frac{1}{d^2} \,, \tag{2.108}$$

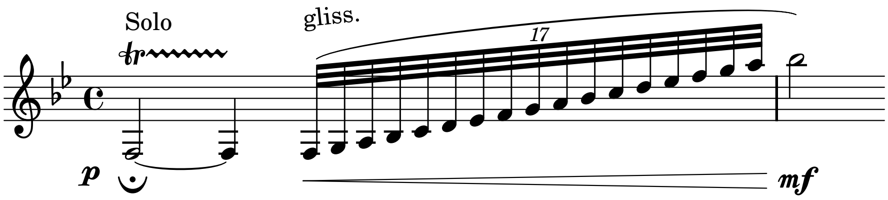
where the sum runs over the positive integral divisors d of k. This then provides all relevant correlation functions with the correct k dependence to precisely match the weak (string) coupling expansion (2.90) of the modular forms residing in the string action. The factors of k actually arise from various places. For example, in computing the sixteen dilatino correlator (2.91) the currents[21] themselves contribute k^{16} and equation (2.108) gives a further $k^{-1/2}$. In factoring the $\mathrm{AdS}_5 \times \mathrm{S}^5$ factor out of the measure, we incur a Jacobian $\sim k^{\pm 1/2}$ for each of the collective coordinates ($+ \leftrightarrow$ bosonic; $- \leftrightarrow$ fermionic) which provides an extra k^{-3}. The overall factor is thus $k^{16-1/2-3} = k^{25/2}$, as expected. We will examine the multi-instanton calculation further in Chapter 3 when we set about testing deformations of the correspondence.

[21]The dilatino current on the k instanton saddle-point background is k times the 1-instanton current:

$$\Lambda^A_\alpha\big|_{k\text{-inst}} = k \cdot \Lambda^A_\alpha\big|_{1\text{-inst}} \,.$$

Chapter 3.

Instanton Test of Deformed AdS/CFT Correspondence



— George Gershwin

3.1. Deforming the Correspondence

Having well established the existence of a gauge theory–string theory duality in one specific case, it is then interesting to see how far the phenomenon persists when this theory is carefully modified. Many possible avenues have been explored, for example one can try taking the decoupling limit of Section 2.4.2 on different background geometries (orbifold, conifold, . . .) or with other kinds of brane, rather than just a stack of $D3$-branes. One can add fundamental quarks into the mix, and can even investigate putting the field theory at finite temperature.

Another option is to introduce new operators that break some of the symmetries, and then reassess the behaviour of both the gauge and string theories to see if the correspondence still holds. In this chapter we will be concerned with adding *exactly marginal* operators, which break various symmetries, including supersymmetry, but that leave the theory with conformal invariance at the quantum level. The importance of this class of operators, and their connection with renormalisation group flow was outlined in Section 2.1.

3.1.1. Marginal Deformations

Much has been learnt recently about gauge theory–string theory duality by investigating how the AdS/CFT correspondence [64] is realised when the $\mathcal{N} = 4$ supersymmetric gauge theory is deformed by exactly marginal operators [4, 5, 51, 80–92]. Since the gauge theory stays conformal it is expected to be dual (in the appropriate limit) to a supergravity solution with Anti-de Sitter geometry. There are two points that make these marginal deformations particularly interesting. First, is that these deformations give a continuous family of theories parameterised by the deformation parameters β_i. The AdS/CFT duality provides a mapping between a gauge theory and a string theory for each value of β_i. By studying the β-dependence in gauge theory and in the dual supergravity (or string theory) one thus acquires a more detailed understanding of the AdS/CFT correspondence. The second feature of marginal β-deformations is that they break (partially or completely) the supersymmetry of the original $\mathcal{N} = 4$ theory.

Lunin and Maldacena [4] have constructed a supergravity dual of the β-deformed $\mathcal{N} = 4$ super Yang-Mills theory (β-SYM) which preserves $\mathcal{N} = 1$ supersymmetry. One notable feature of this background is that the dilaton picks up a non-trivial dependence on the coordinates of the deformed 5-sphere. This leads to a bit of a puzzle in relation to the matching of instantons discussed in Section 2.4.3. On the face of it, one would expect the Yang-Mills instantons to contribute factors of $e^{2\pi i k\tau_0}$ where τ_0 is the complexified gauge coupling, whereas the analogous D-instanton calculation should result in factors of $e^{2\pi i k\tau}$ with $\tau = \hat{\tau} + \delta\tau$ being the *deformed* axio-dilaton. Recall that from the undeformed correspondence we expect $\hat{\tau} \equiv \tau_0$ (cf. equation (2.83)). To shed light on this issue, in reference [90] the supergravity solution of [4], and the resulting string theory effective action, was tested against an instanton calculation on the gauge theory side, in an analysis similar to Section 2.4.3. It was found that the correct expression for the dilaton-axion supergravity field τ was indeed reproduced by instanton effects in gauge theory, and that the higher-derivative terms in the string theory effective action included the appropriate modular forms $f_n(\tau, \bar{\tau})$ of this τ as one would expect from the $SL(2, \mathbb{Z})$ duality of IIB string theory.

One way of realising the solution generating method of [4] is by operating with a combined T-duality-shift-T-duality (TsT) transformation on the supergravity $AdS_5 \times S^5$ geometry. This approach enabled Frolov [5] to extend the method and to find a three-parameter family of non-supersymmetric supergravity solutions. This background has to be AdS/CFT dual to a non-supersymmetric conformal gauge theory obtained by a certain three-parameter deformation of the $\mathcal{N} = 4$ sYM.

In this chapter we apply the instanton approach of references [71, 90] to investigate this non-supersymmetric gauge theory and to test the supergravity solution of reference [5]. We start in Section 3.2 by writing down the supergravity solution of [5], which is parameterised by three real deformations γ_i, and then specifying the corresponding γ_i-deformed gauge theory. In Section 3.3 we then carry out an instanton calculation in this γ_i-deformed gauge theory with a view to reconstructing the dilaton-axion supergravity field τ from gauge theory. Taking the appropriate double-scaling limit, $\gamma_i \ll 1$, our result

$$\tau \;=\; \tau_0 + 2N\pi i \left(\gamma_3^2\, \mu_1^2\mu_2^2 \;+\; \gamma_1^2\, \mu_2^2\mu_3^2 \;+\; \gamma_2^2\, \mu_3^2\mu_1^2\right) \tag{3.1}$$

reproduces the τ-field of Frolov's supergravity dual. Here τ_0 is the usual complexified coupling constant in gauge theory (cf. equation (2.82)), γ_i are the three deformation parameters, and μ_i are coordinates on the deformed S^5 sphere in supergravity that emerge in the gauge theory as important collective coordinates of the non-perturbative sector in the large N limit. We generalise our set-up in Section 3.5 to include complex-valued deformations $\beta_i = \gamma_i + i\sigma_i$, and discuss the interpretation of all our results in Section 3.6. We will round this chapter off by indicating some of the interesting fields of research that are related to this work.

3.2. Three-parameter Deformation of $\mathrm{AdS}_5 \times \mathrm{S}^5$

We begin by reviewing the theories on each side of the gauge/string duality we wish to study. The solution generating tool on the supergravity side is the combination of T-dualities and coordinate shifts known as a TsT transformation. These allow one to start with the known duality between IIB supergravity on a flat background and $\mathcal{N} = 4$ sYM, and generate new supergravity backgrounds [4, 5]. The deformation on the gauge theory side will be incorporated by introducing an appropriate star-product between fundamental fields. For the most part we will concern ourselves with real valued deformations of the theory. The issues that arise for complex deformations will be discussed in Section 3.5.

3.2.1. Supergravity Dual

In order to perform supergravity TsT transformations one must first identify suitable tori in the initial geometry. In the case of [4] this torus was chosen to be the one dual to the $\mathrm{U}(1) \times \mathrm{U}(1)$ global symmetry of β-SYM. If we parameterise this torus with angular variables (φ_1, φ_2), then a TsT transformation with parameter $\hat{\gamma}$ is the following:

T — T-dualise in the φ_1 direction

s — Perform the shift $\varphi_2 \to \varphi_2 + \hat{\gamma}\varphi_1$

T — T-dualise again along φ_1

The resulting supergravity solution was shown in [4] to be dual to β-SYM for small, real β under the association $\hat{\gamma} = R^2\beta$ where R is the radius of S^5.

The S^5 factor of $\mathrm{AdS}_5 \times \mathrm{S}^5$ can be parameterised with the coordinates μ_1, μ_2, μ_3 with $0 \le \mu_i \le 1$ subject to $\mu_1^2 + \mu_2^2 + \mu_3^2 = 1$ and the angular coordinates ϕ_1, ϕ_2, ϕ_3. There are clearly three independent choices of torus corresponding to the pairs (ϕ_1, ϕ_2), (ϕ_2, ϕ_3) and (ϕ_3, ϕ_1). The three parameter deformation constructed in reference [5] follows by performing a separate TsT transformation on each of these, with shift parameters $\hat{\gamma}_3$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$ respectively. The resulting Type IIB supergravity background of Frolov, written in string frame with $\alpha' = 1$, takes the form:

$$
\begin{aligned}
ds^2_{\mathrm{str}} &= R^2 \left[ds^2_{\mathrm{AdS}} + \sum_i \left(d\mu_i^2 + G\, \mu_i^2\, d\phi_i^2 \right) + G\, \mu_1^2\, \mu_2^2\, \mu_3^2 \left(\sum_i \hat{\gamma}_i\, d\phi_i \right)^2 \right] , \qquad (3.2) \\
G^{-1} &= 1 + \hat{\gamma}_3^2\, \mu_1^2\, \mu_2^2 + \hat{\gamma}_1^2\, \mu_2^2\, \mu_3^2 + \hat{\gamma}_2^2\, \mu_3^2\, \mu_1^2 , \qquad e^{2\phi} = e^{2\phi_0}\, G , \\
B^{NS} &= R^2 G \left(\hat{\gamma}_3\, \mu_1^2\, \mu_2^2\, d\phi_1 \wedge d\phi_2 + \hat{\gamma}_1\, \mu_2^2\, \mu_3^2\, d\phi_2 \wedge d\phi_3 + \hat{\gamma}_2\, \mu_3^2\, \mu_1^2\, d\phi_3 \wedge d\phi_1 \right) .
\end{aligned}
$$

We present here only the fields that will be relevant for our purposes. The full complement, including the RR forms C_2 and C_4 and self-dual five-form fields is given in reference [5]. To make contact with the dual gauge theory we have the usual AdS/CFT relation $R^4 = 4\pi e^{\phi_0} N \equiv \lambda$. The real deformation parameters $\hat{\gamma}_i$ appearing in (3.2) are related to the γ_i deformations on the gauge theory side via a simple rescaling, $\hat{\gamma}_i = R^2 \gamma_i$. We note that the dilaton field ϕ in (3.2) is not simply a constant, but depends on the coordinates of the deformed sphere $\widetilde{\mathrm{S}}^5$. The axion field $C = C^0$ is a constant for real-valued deformations γ_i, but will also acquire a non-trivial parameter dependence when we consider complex deformations in Section 3.5.

A few further comments: For the supergravity regime to be valid, we need both $R \gg 1$ and $R\gamma_i = \hat{\gamma}_i/R \ll 1$. This last inequality ensures that the sizes of the tori on which we TsT transform are not smaller than the string scale. Also, when all three deformation parameters are equal, $\hat{\gamma}_1 = \hat{\gamma}_2 = \hat{\gamma}_3 \equiv \hat{\gamma}$, this solution reverts to that of Lunin and Maldacena [4], and the dual gauge theory is β-SYM.

3.2.2. Gauge Theory Formulation

Frolov's supergravity solution (3.2) with three real deformations γ_i contains an AdS_5 factor. It is thus expected to be dual to a conformal gauge theory obtained by exactly marginal but non-supersymmetric deformations of the $\mathcal{N} = 4$ sYM. More precisely, the gauge theory should be conformal in the large number of colours limit (which we always assume in this chapter) where the supergravity approximation to string theory can be trusted.

We will be considering non-supersymmetric deformations of the $\mathcal{N} = 4$ gauge theory, parameterised by three phases, $e^{i\pi\gamma_1}$, $e^{i\pi\gamma_2}$ and $e^{i\pi\gamma_3}$, with the parameters γ_i taken to be real for the time being. It is convenient to account for these phase-deformations by introducing a STAR-PRODUCT — this helps ensure conformal invariance of the theory in the large N limit, as we will see shortly. Our first task will be to take the component Lagrangian of $\mathcal{N} = 4$ supersymmetric Yang-Mills and modify all products of fields into star-products. For any pair of fields f and g, the star-product that gives rise to our deformation is [87]:

$$f * g \equiv e^{-i\pi\, Q_i^f Q_j^g\, \epsilon_{ijk}\gamma_k}\, fg \; . \tag{3.3}$$

Here Q_i^f and Q_i^g are the charges of the fields f and g under the $i = 1, 2, 3$ Cartan generators of the $\mathrm{SU}(4)_R$ R-symmetry belonging to the original $\mathcal{N} = 4$ sYM. The value of these charges for all component fields is the same as in reference [84] and is given in Table 3.1.

These values are easy to derive from the fact that the integral of the superpotential of $\mathcal{N} = 4$ sYM

$$\int d^2\theta\, \mathcal{W}_{\mathcal{N}=4} = \int d^2\theta\, i\, g\, \mathrm{Tr}(\Phi_1\Phi_2\Phi_3 - \Phi_1\Phi_3\Phi_2) \;, \tag{3.4}$$

	Φ_1	Φ_2	Φ_3	A_μ	λ_1	λ_2	λ_3	λ_4
Q_1	1	0	0	0	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
Q_2	0	1	0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
Q_3	0	0	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$

Table 3.1.: Charges Q_i of the component fields in the theory under the Cartan subgroup of the $\mathrm{SU}(4)_R$.

is invariant under the action of each of these Cartan generators on the superfields Φ_i:

$$\Phi_1 \to e^{i\phi_1}\,\Phi_1\ , \qquad \Phi_2 \to e^{i\phi_2}\,\Phi_2\ , \qquad \Phi_3 \to e^{i\phi_3}\,\Phi_3\ . \tag{3.5}$$

This implies that the Grassmann $\mathcal{N}=1$ superspace coordinate θ_α is charged under these transformations with $Q^\theta = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. The charges of the scalar fields Φ_i are precisely the same as of their parent superfields Φ_i in (3.5) and the charges of the fermions λ_A in Table 3.1 can mostly be read off from equation (3.5) keeping in mind

$$\Phi_i(x,\theta) \;=\; \Phi_i(x) + \theta\cdot\lambda_i(x) + \cdots\ .$$

The fourth fermion λ_4 is the $\mathcal{N}=1$ superpartner of A_μ. It's charge assignment follows from the invariance of the gauge kinetic term, $\int d^2\theta\, WW$, where

$$W_\alpha \;=\; -\,i\,\lambda_{4\,\alpha} \;-\; (\sigma^{\mu\nu}\theta)_\alpha\, F_{\mu\nu} \;+\; \cdots$$

is the usual field-strength chiral superfield. We can also see from here that the gauge field A_μ is neutral.

The Lagrangian of the deformed theory follows from the component Lagrangian of $\mathcal{N}=4$ sYM and the definition of the star-product (3.3). We have[1]

$$\begin{aligned}\mathcal{L} \;=\; \frac{1}{g^2}\,\mathrm{Tr}\Bigg(&\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\bar{\Phi}^i)(D_\mu\Phi_i) - \frac{1}{2}[\Phi_i,\Phi_j]_{C_{ij}}[\bar{\Phi}^i,\bar{\Phi}^j]_{C_{ij}} + \frac{1}{4}[\Phi_i,\bar{\Phi}^i][\Phi_j,\bar{\Phi}^j]\\ &+ \lambda_A\sigma^\mu D_\mu\bar{\lambda}^A - i\,[\lambda_4,\lambda_i]_{B_{4i}}\bar{\Phi}^i + i\,[\bar{\lambda}^4,\bar{\lambda}^i]_{B_{4i}}\Phi_i\\ &+ \frac{i}{2}\epsilon^{ijk}[\lambda_i,\lambda_j]_{B_{ij}}\Phi_k + \frac{i}{2}\epsilon_{ijk}[\bar{\lambda}^i,\bar{\lambda}^j]_{B_{ij}}\bar{\Phi}^k\Bigg)\ .\end{aligned} \tag{3.6}$$

[1]For convenience, all fields have been rescaled $\Phi \to \frac{1}{g}\Phi$ to pull out an overall factor of $\frac{1}{g^2}$.

This Lagrangian contains only ordinary products between the fields; all modifications due to the star-product (3.3) are assembled in (3.6) into the deformed commutators of scalars Φ_i, $\bar{\Phi}^i$ and fermions λ_A, $\bar{\lambda}^A$. These deformed commutators are

$$[\Phi_i, \Phi_j]_{C_{ij}} := e^{iC_{ij}}\, \Phi_i \Phi_j - e^{-iC_{ij}}\, \Phi_j \Phi_i\ , \qquad i,j = 1,2,3\ , \tag{3.7a}$$

$$[\lambda_A, \lambda_B]_{B_{AB}} := e^{iB_{AB}}\, \lambda_A \lambda_B - e^{-iB_{AB}}\, \lambda_B \lambda_A\ , \qquad A,B = 1,\ldots,4\ . \tag{3.7b}$$

Deformed commutators for $\bar{\Phi}$ and $\bar{\lambda}$ fields are defined in the same way as in (3.7), and we note in particular that the commutator $[\Phi_i, \bar{\Phi}^i]$ in (3.6) is undeformed. The matrices C and B are the same as in reference [84], and read

$$C = \pi \begin{pmatrix} 0 & -\gamma_3 & \gamma_2 \\ \gamma_3 & 0 & -\gamma_1 \\ -\gamma_2 & \gamma_1 & 0 \end{pmatrix}, \tag{3.8a}$$

$$B = \pi \begin{pmatrix} 0 & -\frac{1}{2}(\gamma_1+\gamma_2) & \frac{1}{2}(\gamma_3+\gamma_1) & \frac{1}{2}(\gamma_2-\gamma_3) \\ \frac{1}{2}(\gamma_1+\gamma_2) & 0 & -\frac{1}{2}(\gamma_2+\gamma_3) & \frac{1}{2}(\gamma_3-\gamma_1) \\ -\frac{1}{2}(\gamma_3+\gamma_1) & \frac{1}{2}(\gamma_2+\gamma_3) & 0 & \frac{1}{2}(\gamma_1-\gamma_2) \\ -\frac{1}{2}(\gamma_2-\gamma_3) & -\frac{1}{2}(\gamma_3-\gamma_1) & -\frac{1}{2}(\gamma_1-\gamma_2) & 0 \end{pmatrix}. \tag{3.8b}$$

We see that the whole effect of the 3-parameter deformation is encoded in these matrices, which introduce the appropriate phases into the 4-scalar and Yukawa interactions of the deformed theory (3.6). It is important to note that the induced phases of the fermions (determined by the matrix B) are different from those of the scalars (in C). Also, the ranks of B and C are different because the matrix B introduces phases to the Yukawa interactions involving all fermions, including the gaugino λ_4. The Lagrangian (3.6) correctly incorporates the four-scalar interactions written down in [5, 87]. In addition to these, equations (3.6) and (3.8) give the precise form of the interactions with fermions, which we will require for the instanton calculations performed later in this chapter.

For the special case of all γ_i being equal, the matrices B and C essentially coincide, giving the same phase factors to scalars and fermions. In this case, the gauge theory is $\mathcal{N} = 1$ supersymmetric and is dual to the supergravity solution of Lunin and Maldacena [4]. In the general case of unequal deformations γ_i, the fermion and scalar phases differ and the gauge theory is non-supersymmetric.

Finally, we need to make sure that the γ-deformed gauge theory defined by equations (3.6) and (3.8) is exactly marginal in the large N limit. In general, this would be a non-trivial task since the theory is not supersymmetric and one cannot use the approach of Leigh and Strassler [51] to establish the required conformal invariance. Instead, the marginality of the theory follows from the use of the star-product. It is known [93] that the Moyal star-product, often used in the formulation of noncommutative field theory, does not affect large N perturbation theory. More precisely, the planar diagrams of the theories with and without the star-products can differ only by an overall phase-factor that depends on the external lines. This argument essentially uses only the associativity property of the star-product, and it also applies to our choice (3.3), see section **3.2** of reference [94] for more details. This implies that all planar perturbative contributions to the beta functions and anomalous dimensions of our deformed theory are proportional to those in the conformal $\mathcal{N} = 4$ theory, and hence vanish. Thus, the γ-deformed theory (3.6), (3.8) is conformal in large N perturbation theory.

The γ-deformed theory is an interesting field theory on its own right. It is a non-supersymmetric theory which fully inherits the remarkable perturbative structure of large N superconformal $\mathcal{N} = 4$ sYM. In reference [95] it was argued that the Maximally-Helicity-Violating (MHV) n-point amplitudes of $\mathcal{N} = 4$ sYM have an iterative structure, such that the kinematic dependence of all higher-loop MHV amplitudes can be determined from the known one-loop results. It then follows [94] that the same must be true for the planar MHV amplitudes of the deformed theory. This is yet-another consequence of the fact that the deformations were introduced via a star-product of the type (3.3). It is remarkable that such an intricate, iterative structure can exist for the multi-loop amplitudes of a non-supersymmetric theory.

3.3. Instanton Effects

The effect of instantons in β-deformed $\mathcal{N} = 4$ gauge theory was investigated in detail by Georgiou and Khoze in reference [90]. We will now précis their discussion, generalising where necessary to make it applicable to our non-supersymmetric γ-deformed theory.

The first point to note is that the deformation only enters through interaction terms, so the field content of the non-supersymmetric theory will be the same as for $\mathcal{N} = 4$ sYM. This leads to the same cancellation of determinant factors in the instanton measure that we noted in Section 2.2.2 simplifies supersymmetric calculations, and also suggests that we use a notation that makes the connection to the undeformed $\mathcal{N} = 4$ sYM case transparent.

Our eventual aim is to calculate the contribution to correlation functions coming from perturbing around a multi-instanton background. Unfortunately, even in the maximally supersymmetric case, constructing an instanton solution is a formidable task. Solving the coupled Euler-Lagrange equations that follow from the $\mathcal{N} = 4$ Lagrangian, although possible in principle, is too difficult to be practical. Not having an explicit expression for the saddle point unsurprisingly poses a real obstacle to computing its effect on correlators. Fortunately, if we're only interested in the corrections that arise at leading semi-classical order, it suffices to solve the Euler-Lagrange equations order-by-order in the gauge coupling [71]. In this case, the approximate instanton configuration in our deformed theory is defined (to leading order in g) to satisfy the following equations for the gauge field,

$$F_{mn} = *F_{mn} \ , \tag{3.9}$$

fermions,

$$\bar{\not{\mathcal{D}}}^{\dot{\alpha}\alpha} \lambda^A_\alpha = 0 \ , \tag{3.10}$$

and scalars,

$$\mathcal{D}^2 \, \Phi_{AB} = \sqrt{2}\, i \left(e^{iB_{AB}} \lambda_A \lambda_B - e^{-iB_{AB}} \lambda_B \lambda_A \right) . \tag{3.11}$$

Here $\bar{\not{\mathcal{D}}}^{\dot{\alpha}\alpha} = D^\mu \bar{\sigma}^{\dot{\alpha}\alpha}_\mu$ and $\mathcal{D}^2 = D^\mu D_\mu$ where D_μ is the covariant derivative in the instanton background A_μ. The matrix B is given in (3.8).

There are $8kN$ fermionic zero modes that correspond to solutions of (3.10) in the approximate k-instanton background. Only those which are protected by some form of symmetry are expected to survive to all orders and solve the full saddle point equations. The failure of the other fermi zero modes to be exact is felt as additional contributions to the instanton action, which LIFT them. In $\mathcal{N} = 4$ sYM 16 of these fermion zero modes are exact. These solutions correspond to $2\mathcal{N} = 8$ supersymmetric and $2\mathcal{N} = 8$ superconformal fermion zero modes of the original $\mathcal{N} = 4$ gauge theory. In our deformed theory, supersymmetry is lost and all of the fermion zero modes are lifted in the instanton action as will be seen shortly.

The scalar field equation (3.11) follows from the Yukawa interactions of the Lagrangian (3.6); the four-scalar interactions do not enter the instanton construction to leading order in g. The scalar fields are written in a basis $\Phi_{AB} = -\Phi_{BA}$ that is related to the usual basis Φ_i used in (3.6) as follows:

$$\begin{aligned}
\Phi_1 = \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2) \quad &= \quad 2\,\bar{\Phi}_{32} = 2\,\Phi_{41} \\
\Phi_2 = \frac{1}{\sqrt{2}}(\phi^3 + i\phi^4) \quad &= \quad 2\,\bar{\Phi}_{13} = 2\,\Phi_{42} \\
\Phi_3 = \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6) \quad &= \quad 2\,\bar{\Phi}_{21} = 2\,\Phi_{43}
\end{aligned} \tag{3.12}$$

This representation is discussed further in Appendix A. The instanton configuration defined as the solution of equations (3.9)-(3.11), is used to construct a semi-classical instanton integration measure, to which we now turn.

3.3.1. The γ-deformed Instanton Measure

To calculate correlation functions in an instanton background, we need a measure that determines how each configuration is weighted in the path integral. This is usually provided as an integral over the collective coordinates of the instanton solution. The general multi-instanton measure was constructed in reference [71] for $\mathcal{N} = 4$ sYM and generalised in [90] to account for the supersymmetry preserving β-deformations. The result of reference [90] can now be straightforwardly adapted to the case of non-supersymmetric γ_i deformations. We will concentrate here on the simplest case of the single-instanton measure; the multi-instanton measure can be similarly built up by modifying the construction of [71, 90].

The 1-instanton measure of the γ_i-deformed theory, valid for the calculation of correlation functions of gauge invariant operators, reads:[2]

$$\int d\mu\, e^{-S_{1-\text{inst}}} = \frac{2^{-31}\pi^{-4N-5}g^{4N}}{(N-1)!(N-2)!}\int d^4x_0\, d\rho\, d^6\chi \prod_{A=1}^{4} d^2\xi^A\, d^2\bar{\eta}^A\, d^{(N-2)}\nu^A\, d^{(N-2)}\bar{\nu}^A$$
$$\rho^{4N-7}\exp\left[-\frac{8\pi^2}{g^2} + i\theta - 2\rho^2\chi^a\chi^a + \frac{4\pi i}{g}\chi_{AB}\Lambda^{AB}\right]. \tag{3.13}$$

[2] For the derivation of this result, see Section 4 of reference [90].

The integral above is over the bosonic and fermionic (Grassmann) collective coordinates of the instanton. The fermionic ones comprise $4(N-2)$ parameters ν_i^A (where $i = 1, \ldots N-2$) that can be thought of as the $\mathcal{N}=4$ superpartners of the parameters that specify the embedding of the instanton's SU(2) in the full SU(N) gauge group, 8 supersymmetric coordinates ξ_α^A and 8 superconformal modes $\bar{\eta}_{\dot{\alpha}}^A$ (where $\alpha = 1,2$ and $\dot{\alpha} = 1,2$). The bosonic collective coordinates include the instanton position x_0^μ, the scale-size ρ and the 6 additional variables χ^a that are coupled to fermion modes in the instanton action in the exponent in (3.13).

The variables χ^a or χ_{AB} deserve further comment. They transform in the vector representation of the SO(6) $\cong$ SU(4) R-symmetry and are subject to the reality condition $\overline{\chi}_{AB} = \frac{1}{2}\epsilon_{ABCD}\chi_{CD}$, as explained in Appendix A. They are introduced in the derivation of equation (3.13) in order to bilinearise the fermionic quadrilinear term $\sim\epsilon_{ABCD}\Lambda^{AB}\Lambda^{CD}$ that appears in the instanton action. It is a curious and interesting fact that although the χ_{AB} variables initially only appear to be required as a mathematical crutch, they serve a very important rôle in the mechanism of the AdS/CFT correspondence; in the large N limit they metamorphose into collective coordinates that are dual to the non-AdS$_5$ part of the geometry (in the case of $\mathcal{N}=4$ sYM, this is just S^5).

Finally we come to the term Λ^{AB} in the instanton action (the exponent of equation (3.13)). This is a fermionic bilinear defined as

$$\begin{aligned}\Lambda^{AB} = \frac{1}{2\sqrt{2}} \sum_{i=1}^{N-2} \Big(e^{iB_{AB}}\, \bar{\nu}_i^A \nu_i^B - e^{-iB^{AB}}\, \bar{\nu}_i^B \nu_i^A \Big) \\ + i\, 8\sqrt{2}\, \sin(B_{AB})\, \big(\rho^2 \bar{\eta}^A \cdot \bar{\eta}^B + \xi^A \cdot \xi^B \big) \; .\end{aligned} \tag{3.14}$$

The 4×4 antisymmetric matrix B_{AB} was defined in (3.8). The fact that the instanton action in (3.13) depends on all of the fermionic collective coordinates (through Λ_{AB}) implies that they are all lifted. This is to be expected in the non-supersymmetric theory.

When dealing with a path integral measure such as equation (3.13) one must always bear in mind the kind of correlation functions it can be used to compute. For example, we have already seen how the fermionic zero mode structure dictates which observables are sensitive to the instanton background. For the purposes of this chapter — understanding the matching of exponents discussed in Section 3.1.1 — we will only need to look at the sixteen dilatino current correlator (2.91) that featured in Section 2.4.3. This enables us to exploit a few simplifications in the measure.

Recall that in the original $\mathcal{N} = 4$ theory we chose to look at the correlator (2.91) because it saturated all 16 exact fermi zero modes of the instanton. In the non-supersymmetric case at hand, every one of these zero modes is lifted in the effective action by the second line of equation (3.14), but this is of little consequence. The fact that the Grassmann integration of $\bar{\eta}$ and ξ modes is always saturated by the operator insertion means we can legitimately neglect the second term in equation (3.14) (it can never contribute). This relies crucially on the absence of ν and $\bar{\nu}$ modes in the sixteen dilatino operator. Other current correlators,[3] such as those corresponding to higher Kaluza-Klein excitations on the (deformed) five-sphere, have explicit dependence on the ν and $\bar{\nu}$. In this situation, saturating all fermionic integrals would, for some terms, require pulling down appropriate factors of $\bar{\eta}$ and ξ from the effective action, thus requiring us to keep the full expression (3.14) and leading to a more involved calculation. In summary, by restricting our attention to the sixteen dilatino current correlator, we can always drop the second term on the right hand side of equation (3.14), which is what we will do from now on.

3.4. Large N Saddle-point Integration

Following the approach of [90] we proceed by integrating out fermionic collective coordinates ν_i^A and $\bar{\nu}_i^A$ from the instanton partition function (3.13). For each value of $i = 1, \ldots, N-2$ this integration gives a factor of

$$\left(\frac{4\pi}{g}\frac{1}{\sqrt{2}}\right)^4 \det{}_4\left(e^{iB_{AB}}\,\chi_{AB}\right) \ . \tag{3.15}$$

The determinant above can be calculated directly. It will be useful to express the result in terms of the three complex variables X_i that are defined in terms of χ^{AB} in a way analogous to equations (3.12):

$$\begin{aligned} X_1 = \chi^1 + i\chi^2 &= 2\sqrt{2}\,\overline{\chi}_{32} = 2\sqrt{2}\,\chi_{41} \\ X_2 = \chi^3 + i\chi^4 &= 2\sqrt{2}\,\overline{\chi}_{13} = 2\sqrt{2}\,\chi_{42} \\ X_3 = \chi^5 + i\chi^6 &= 2\sqrt{2}\,\overline{\chi}_{21} = 2\sqrt{2}\,\chi_{43} \end{aligned} \tag{3.16}$$

[3] For more detail on the zero mode structure of current correlators, we refer the reader to reference [75].

In terms of these degrees of freedom, the determinant takes the form

$$\det_4\left(e^{iB_{AB}}\,\chi_{AB}\right) = \begin{vmatrix} 0 & X_3^\dagger\,e^{-\frac{i\pi}{2}(\gamma_1+\gamma_2)} & -X_2^\dagger\,e^{\frac{i\pi}{2}(\gamma_3+\gamma_1)} & X_1\,e^{\frac{i\pi}{2}(\gamma_2-\gamma_3)} \\ -X_3^\dagger\,e^{\frac{i\pi}{2}(\gamma_1+\gamma_2)} & 0 & X_1^\dagger\,e^{-\frac{i\pi}{2}(\gamma_2+\gamma_3)} & X_2\,e^{\frac{i\pi}{2}(\gamma_3-\gamma_1)} \\ X_2^\dagger\,e^{-\frac{i\pi}{2}(\gamma_3+\gamma_1)} & -X_1^\dagger\,e^{\frac{i\pi}{2}(\gamma_2+\gamma_3)} & 0 & X_3\,e^{\frac{i\pi}{2}(\gamma_1-\gamma_2)} \\ -X_1\,e^{-\frac{i\pi}{2}(\gamma_2-\gamma_3)} & -X_2\,e^{-\frac{i\pi}{2}(\gamma_3-\gamma_1)} & -X_3\,e^{-\frac{i\pi}{2}(\gamma_1-\gamma_2)} & 0 \end{vmatrix} . \tag{3.17}$$

Multiplying everything out, this evaluates to

$$\begin{aligned} \det_4\left(e^{iB_{AB}}\,\chi_{AB}\right) \;=\; & \frac{1}{64}\,(|X_1|^2+|X_2|^2+|X_3|^2)^2 \;-\; \frac{1}{16}\,\sin^2(\pi\gamma_3)\,|X_1|^2|X_2|^2 \\ & -\frac{1}{16}\,\sin^2(\pi\gamma_1)\,|X_2|^2|X_3|^2 \;-\; \frac{1}{16}\,\sin^2(\pi\gamma_2)\,|X_3|^2|X_1|^2\;. \end{aligned} \tag{3.18}$$

We note that the expression above depends only on the three absolute values of X_i and is independent of the three angles. We can further change variables as follows:

$$|X_i| = r\,\mu_i\;, \qquad \sum_{i=1}^{3}\mu_i^2 \;=\; 1\;, \tag{3.19}$$

and write

$$\begin{aligned} \left(\frac{4\pi}{g}\frac{1}{\sqrt{2}}\right)^4 \det_4\left(e^{iB_{AB}}\,\chi_{AB}\right) \;=\; & \left(\frac{\pi}{g}\right)^4 r^4\,\Big(1-4\,\sin^2(\pi\gamma_3)\,\mu_1^2\mu_2^2 \\ & -\;4\,\sin^2(\pi\gamma_1)\,\mu_2^2\mu_3^2\;-\;4\,\sin^2(\pi\gamma_2)\,\mu_3^2\mu_1^2\Big)\;. \end{aligned} \tag{3.20}$$

In summary after integrating out all of the ν and $\bar\nu$ fermionic collective coordinates we find the following generic instanton factor in the measure:

$$\begin{aligned} \mathcal{F}_{\rm inst} \;:=\; & e^{-\frac{8\pi^2}{g^2}+i\theta}\,\left(1\;-\;4\,\sin^2(\pi\gamma_3)\,\mu_1^2\mu_2^2\;-\;4\,\sin^2(\pi\gamma_1)\,\mu_2^2\mu_3^2\;-\;4\,\sin^2(\pi\gamma_2)\,\mu_3^2\mu_1^2\right)^{N-2} \\ \equiv\; & e^{2\pi i\tau_0}\,(1\;-\;\mathcal{Q}(\mu_i,\gamma_i))^{N-2}\;. \end{aligned} \tag{3.21}$$

This factor is integrated over the $\mathrm{AdS}_5\times \mathrm{S}^5$ space spanned by x_0^μ, ρ and the five angles of χ^a

$$\int d^4x_0\,\frac{d\rho}{\rho^5}\,d^5\hat\chi \;=\; (2\pi)^3\int d^4x_0\,\frac{d\rho}{\rho^5}\,d\mu_1\,d\mu_2\,d\mu_3\,\delta(\mu_1^2+\mu_2^2+\mu_3^2-1)\;, \tag{3.22}$$

exactly as in [71, 90]. As we are interested in the limit $N \to \infty$ we can rewrite equation (3.21) as a total exponent and evaluate the integrals over μ_i via a saddle-point approximation,

$$\begin{aligned} \int_{\mu_i} e^{2\pi i\tau_0} \left(1 - \mathcal{Q}(\mu_i)\right)^{N-2} &= \int_{\mu_i} \exp\Big(2\pi i\tau_0 + (N-2)\log\big(1 - \mathcal{Q}(\mu_i)\big)\Big) \\ &\approx \exp\big(2\pi i\tau_0 - N\,\mathcal{Q}(\mu_i|_{\text{saddle}})\big) \,. \end{aligned} \tag{3.23}$$

This method selects the dominant value of the function $\mathcal{Q}(\mu_i)$ to be $\mathcal{Q}(\mu_i|_{\text{saddle}}) \sim \frac{1}{N}$ and has therefore allowed us to expand the log to leading power in $\mathcal{Q}$ in the last line.

What we have calculated so far is a large N expression for the characteristic instanton factor

$$\mathcal{F}_{\text{inst}} = \exp\big(2\pi i\tau_0 - N\,\mathcal{Q}(\mu_i|_{\text{saddle}}, \gamma_i)\big) \,. \tag{3.24}$$

This factor arises in instanton calculations of generic correlation functions in gauge theory. As we explored in Section 2.4.3, when applied to the calculation of Yang-Mills correlators involving operators that are dual to supergravity fields, the instanton result in gauge theory must match with the corresponding D-instanton contribution in string theory. This means that the characteristic factor (3.24) due to the Yang-Mills instanton must correspond to $\exp\big(2\pi i\tau\big)$, where τ is the dilaton-axion field in dual string theory.[4] By matching exponents we read off the instanton prediction for the dilaton-axion field:

$$\tau = \tau_0 - \frac{N}{2\pi i}\left(4\,\sin^2(\pi\gamma_3)\,\mu_1^2\mu_2^2 + 4\,\sin^2(\pi\gamma_1)\,\mu_2^2\mu_3^2 + 4\,\sin^2(\pi\gamma_2)\,\mu_3^2\mu_1^2\right) \,. \tag{3.25}$$

This semi-classical field theory result is valid for any value of the parameters γ_i and, as such, can be interpreted [91] as a (weak-coupling) prediction for the τ field in the exact string theory background.

The supergravity regime is reached in the limit of $\gamma_i \ll 1$ which gives:

$$\tau \to \tau_0 + 2N\pi i\left(\gamma_3^2\,\mu_1^2\mu_2^2 + \gamma_1^2\,\mu_2^2\mu_3^2 + \gamma_2^2\,\mu_3^2\mu_1^2\right) \,. \tag{3.26}$$

[4] More detail about instanton and D-instanton contributions to the string effective action can be found in [71, 73, 74, 90].

This precisely matches with Frolov's three parameter supergravity solution (3.2) for the dilaton-axion field:

$$\begin{aligned}
\tau &= ie^{-\phi} + C && \text{(3.27a)}\\
&= ie^{-\phi_0}\Big(1 + \hat{\gamma}_3^2\,\mu_1^2\mu_2^2 + \hat{\gamma}_1^2\,\mu_2^2\mu_3^2 + \hat{\gamma}_2^2\,\mu_3^2\mu_1^2\Big)^{1/2} + C^0 && \text{(3.27b)}\\
&= \tau_0 + \frac{ie^{-\phi_0}}{2}\left(\hat{\gamma}_3^2\,\mu_1^2\mu_2^2 + \hat{\gamma}_1^2\,\mu_2^2\mu_3^2 + \hat{\gamma}_2^2\,\mu_3^2\mu_1^2\right)\ , && \text{(3.27c)}
\end{aligned}$$

where the deformation parameters are $\hat{\gamma}_i^2 = N\,g^2\,\gamma_i^2$ and one identifies the coordinates on the deformed supergravity $\widetilde{\mathrm{S}}^5$ sphere with the χ-collective coordinates of the instanton.

One might be concerned that in the instanton measure we integrate over the angular components of the χ^a variables, which form a proper S^5, whereas in the supergravity picture this integral should correspond to the volume of the *deformed* five-sphere $\widetilde{S}^5$. This incongruence can be reconciled be considering the order in which limits are taken. As we use an instanton multiplet that is only defined to leading order in g, we are forced to work in the small g regime, but γ_i can be arbitrary. This allows us to make classical string predictions such as equation (3.25). Only then does the supergravity limit $\gamma_i \ll 1$ need to be taken. The volume of the deformed $\widetilde{S}^5$ is found in the supergravity solution (3.2) to be

$$\int \widetilde{\omega}_5 = \int d^5\hat{\Omega}\,G\ , \tag{3.28}$$

where the integral on the right-hand side is over the angles of an S^5, and

$$\begin{aligned}
G &= \Big(1 + N\,g^2\left(\gamma_3^2\,\mu_1^2\,\mu_2^2 + \gamma_1^2\,\mu_2^2\,\mu_3^2 + \gamma_2^2\,\mu_3^2\,\mu_1^2\right)\Big)^{-1}\\
&= 1 - N\,g^2\left(\gamma_3^2\,\mu_1^2\,\mu_2^2 + \gamma_1^2\,\mu_2^2\,\mu_3^2 + \gamma_2^2\,\mu_3^2\,\mu_1^2\right) + \ldots && \text{(3.29)}\\
&\approx 1 \quad \text{for} \quad g \ll 1\ .
\end{aligned}$$

We see from the last line that terms containing the deformation can (and should) be neglected in the small g limit in which we work. So the instanton measure can not be expected to be sensitive to the deformation at leading order in g. Nevertheless, as we saw in equation (3.27), the deformation is still evident in the exponent.

It is clear in the above that an analogous calculation for the case of one *anti*-instanton would yield the same type of gauge/supergravity matching for the conjugate parameter $\bar{\tau}$.

One can also extend this calculation to include the multi-instanton sectors, as in [71, 90]. In the large N limit the partition function in the k-instanton sector is:

$$\int d\mu^k_{\rm inst}\, e^{-S^k_{\rm inst}} = \frac{\sqrt{Ng^2}}{2^{33}\pi^{27/2}} \frac{k}{g^2}^{-7/2} \sum_{d|\,k} \frac{1}{d^2} \int \frac{d^4x\, d\rho}{\rho^5}\, d^5\hat{\Omega} \prod_{A=1,2,3,4} d^2\xi^A d^2\bar{\eta}^A e^{2\pi i k\tau} \;, \quad (3.30)$$

where the value of τ here is given by the same γ-deformed complex coupling shown in equation (3.25). The way this measure facilitates the matching of gauge theory correlators with terms in the dual supergravity effective action, including the intriguing sum over integral divisors of the instanton number, was described at the end of Section 2.4.3.

3.5. Complex β Deformations

In this section we consider the more general case of marginal deformations with complex values of the deformation parameters $\beta_i \in \mathbb{C}$

$$\beta_1 = \gamma_1 + i\,\sigma_1 \;, \quad \beta_2 = \gamma_2 + i\,\sigma_2 \;, \quad \beta_3 = \gamma_3 + i\,\sigma_3 \;. \quad (3.31)$$

The supergravity solution corresponding to this case was obtained in reference [5] by performing three consecutive $STsTS^{-1}$ transformations (where S is an S-duality) acting on the three natural tori of S^5. This family of solutions is expected to be dual to a deformed Yang-Mills theory with three complex deformation parameters.

We will first explain how to extend the instanton calculation on the gauge theory side from real to complex β_i-deformations. We will again carry out this calculation for arbitrary (not necessarily small) values of the deformation parameter $\beta_i \in \mathbb{C}$. The main result of this section is the instanton prediction for the dilaton-axion field τ. We will show that in the limit of small $|\beta_i|$ it will match precisely with the τ field of Frolov's supergravity dual [5]. As before, the small-β_i limit is required to ensure the validity of the supergravity approximation to full string theory.

We now need to specify the deformed gauge theory. The absence of supersymmetry and the complex-valuedness of the deformation parameters β_i make this more difficult than then previous (real-valued) case. It is not entirely clear how to uniquely define this theory and, more importantly, whether one can guarantee its marginality in the large N limit. The absence of supersymmetry prevents one from using the Leigh-Strassler approach [51] in terms of conformal constraints, while the complex-valuedness of the deformation parameters makes it difficult to use the star-product formulation.[5] Fortunately, the instanton calculation that we are about to present does not require full knowledge of the gauge-theory Lagrangian, only its gauge and Yukawa interactions specified below.

The instanton configuration at leading order in a weak gauge coupling expansion is defined as in equations (3.10)-(3.11) with the scalar field equation (3.11) taking the form:

$$\mathcal{D}^2\Phi_{AB} = \frac{h}{g}\sqrt{2}\,i\,(\,e^{i\pi B_{AB}}\,\lambda_A\lambda_B - e^{-i\pi B_{AB}}\,\lambda_B\lambda_A\,) \qquad \text{for } A,\, B \neq 4\;, \tag{3.32a}$$

$$\mathcal{D}^2\Phi_{AB} = \sqrt{2}\,i\,(\,e^{i\pi B_{AB}}\,\lambda_A\lambda_B - e^{-i\pi B_{AB}}\,\lambda_B\lambda_A\,) \qquad \text{for } A \text{ or } B = 4\;. \tag{3.32b}$$

Here B_{AB} is a complex-valued matrix obtained from the one in (3.8) by the substitution $\gamma_i \to \beta_i$. The factor of h/g on the right hand side of (3.32a) accounts for the change of the coupling constant from g to h in the Yukawa couplings, where h is an new complex parameter. This coupling was also secretly present in the real-deformation case, but a constraint that follows from the exact marginality of the theory sets $g = h$ to leading order. A similar constraint could, in principle, be imposed here, but this will turn out to be unnecessary. For our calculation we will not need to use an explicit resolution of this constraint.

We note that the resulting instanton configuration depends holomorphically on h: at leading order in g the dependence on h^* can come only through the equation conjugate to (3.32), which involves anti-fermion zero modes $\bar{\lambda}$ on the right hand side. These are vanishing in the instanton background. It is then also clear that the anti-instanton configuration, will depend on h^* and not on h.

Just as we did in Section 3.3 we integrate out the fermionic collective coordinates ν_i^A and $\bar{\nu}_i^A$. For each value of $i = 1, \ldots, N-2$ this integration gives a factor of the determinant (3.15) times an appropriate rescaling by h/g. We find

$$\left(\frac{1}{g}\right)^4 \det{}_4\left(e^{i\pi \mathrm{B}_{AB}}\,\chi_{AB}\right) \longrightarrow \left(\frac{1}{g}\right)^4 \left(\frac{h}{g}\right)^2 \det{}_4\left(e^{i\pi \mathrm{B}_{AB}}\,\chi_{AB}\right)\;. \tag{3.33}$$

[5] For related discussion on this point, see reference [96].

After evaluating this determinant, the result for the characteristic instanton factor in the large N limit is:

$$\mathcal{F}_{\text{inst}} \;=\; \exp\left[2\pi i\tau_0 \;+\; 2N\log\left(\frac{h}{g}\right) \;+\; N\log\big(1 \;-\; \mathcal{Q}(\mu_i,\beta_i)\big)\right], \tag{3.34}$$

where $\mathcal{Q}(\mu_i,\beta_i)$ is the same function as before, but with the complex β_i parameters in place of real γ_i,

$$\mathcal{Q}(\mu_i,\beta_i) \;=\; 4\,\left(\sin^2(\pi\beta_3)\,\mu_1^2\mu_2^2 \;+\; \sin^2(\pi\beta_1)\,\mu_2^2\mu_3^2 \;+\; \sin^2(\pi\beta_2)\,\mu_3^2\mu_1^2\right)\;. \tag{3.35}$$

By taking the small deformation limit, $|\beta_i|^2 \ll 1$, appropriate for comparison with the supergravity solution, we find

$$\begin{aligned}\mathcal{F}_{\text{inst}} \;=\; \exp\Big[2\pi i\tau_r - 4\pi^2\,N\,\big((\gamma_1^2-\sigma_1^2+2i\gamma_1\sigma_1)\mu_2^2\mu_3^2 \\ +\;(\gamma_2^2-\sigma_2^2+2i\gamma_2\sigma_2)\mu_1^2\mu_3^2 \;+\; (\gamma_3^2-\sigma_3^2+2i\gamma_3\sigma_3)\mu_2^2\mu_1^2\big)\Big]\;.\end{aligned} \tag{3.36}$$

The constant τ_r appearing in this expression is defined as

$$\tau_r \;:=\; \tau_0 \;-\; \frac{iN}{\pi}\,\log\frac{h}{g} \tag{3.37}$$

and can be interpreted as a 'renormalised' Yang-Mills coupling. The importance of this shift was first identified for the case of complex parameters $\beta_1=\beta_2=\beta_3=\beta$ in reference [82], where the authors argued τ_r acts as the modular parameter of an $\mathrm{SL}(2,\mathbb{Z})$ transformation that permutes the vacuum solutions, with various combinations of the parameters behaving as modular forms:

$$\tau_r \;\longrightarrow\; \frac{a\tau_r+b}{c\tau_r+d}\,, \qquad \text{where} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})\;,$$

$$\beta \;\longrightarrow\; \frac{\beta}{c\tau_r+d}\,, \qquad (h/g)^2\sin(\pi\beta) \;\longrightarrow\; \frac{(h/g)^2\sin(\pi\beta)}{c\tau_r+d}\;.$$

This can be thought of as a vestige of the electromagnetic duality of $\mathcal{N}=4$ sYM. From this perspective the appearance of τ_r in equation (3.36) is not all that surprising, because as was explained in Section 2.4.3, the $\mathrm{SL}(2,\mathbb{Z})$-action lies at the root of our instanton test of the gauge–string duality.

The dilaton and axion field components of Frolov's supergravity dual with three complex deformations are given by:

$$e^{\phi} = e^{\phi_0} G^{1/2} H , \qquad C = C^0 + e^{-\phi_0} H^{-1} Q , \tag{3.38}$$

where expressions for the functions G, H and Q can be found in Appendix B of [5]. By utilising these expressions and (3.38) one can easily calculate the axion-dilaton field for the case of complex deformations. The result thus obtained reads

$$\begin{aligned} e^{2\pi i \tau} &= e^{2\pi i\,(ie^{-\phi}+C)} \\ &= \exp\Big[- 2\pi e^{-\phi_0} \big[1 + \tfrac{1}{2}(\hat{\gamma}_1^2 - \hat{\sigma}_1^2)\mu_2^2\mu_3^2 + \tfrac{1}{2}(\hat{\gamma}_2^2 - \hat{\sigma}_2^2)\mu_1^2\mu_3^2 + \tfrac{1}{2}(\hat{\gamma}_3^2 - \hat{\sigma}_3^2)\mu_2^2\mu_1^2\big] \\ &\qquad + 2\pi i \left(C^0 + e^{-\phi_0}(\hat{\gamma}_1\hat{\sigma}_1\mu_2^2\mu_3^2 + \hat{\gamma}_2\hat{\sigma}_2\mu_1^2\mu_3^2 + \hat{\gamma}_3\hat{\sigma}_3\mu_2^2\mu_1^2)\right)\Big] \end{aligned} \tag{3.39}$$

By making the identification

$$\hat{\gamma}_i = g_r\sqrt{N}\gamma_i , \qquad \hat{\sigma}_i = -g_r\sqrt{N}\sigma_i , \qquad \tau_r = ie^{-\phi_0} + C^0 , \tag{3.40}$$

one can immediately see this supergravity result is in perfect agreement with our field theory prediction (3.36).

3.6. Discussion

The main result of this chapter is the fact that instanton contributions in gauge theory confirm the non-supersymmetric supergravity solution of reference [5]. Both expressions, in gauge theory and in supergravity, are continuous functions of the three complex deformation parameters. What is interesting about this matching is not merely the fact that there is a non-trivial agreement between gauge theory and supergravity, but also that the Yang-Mills instanton calculation which is intrinsically valid only at weak coupling, $g^2N \ll 1$, with $N \to \infty$, appears to give the correct result in the strong coupling limit, $g^2N \gg 1$, relevant for comparison with the supergravity. This agreement between the strong and the weak coupling limits is completely analogous to the previously known instanton tests of the AdS/CFT correspondence that we discussed in Section 2.4.3 in the $\mathcal{N} = 4$ sYM context and that was more recently observed to persist under supersymmetry-preserving β-deformations in reference [90]. In all known cases, leading order contributions of Yang-Mills instantons calculated at $g^2N \ll 1$, match with the contribution of D-instantons in supergravity in the opposite limit $g^2 N \gg 1$. The agreement only holds for the instanton part of the answer; it is known that perturbative contributions in gauge theory and in string theory do not match [74]. This suggests that there should exist a non-renormalisation theorem that applies to the instanton effects and explains the agreement. For a more detailed discussion on this point, we refer the reader to references [44, 71, 90] and [97].

In this chapter we saw that the matching in the instanton sector persists for non-supersymmetric deformations of $\mathcal{N} = 4$ sYM. This implies that the non-renormalisation theorem is not dictated by supersymmetry. There is a slightly subtle point here: the γ deformed theory still posesses supersymmetric *field content*, which ensures the leading order cancellation of determinants discussed in Section 2.2.2. Supersymmetry is broken due to the form of the interaction between these fields. We therefore expect that the origin of this agreement lies in identification of Yang-Mills instantons with D-instantons, as they each arise as 'extended' objects or defects in their respective theories.

3.7. Future Directions

At the level of mathematical rigour, the AdS/CFT correspondence is still a conjecture, but over the last ten years its predictions have repeatedly stood firm against intense levels of scrutiny. The framework has been extended in many different directions, and every time the results are shown to be astonishingly self-consistant. So with the veracity of the correspondence established to such a high degree, some researchers are now looking at its practical applications. As a tool in the theorist's arsenal, it has the potential to let us attack all sorts of strongly coupled field theory problems by describing them in terms of a weakly coupled holographic gravity dual. These techniques have already been applied to calculate various properties of strongly coupled Quark-Gluon Plasmas [98]. Despite the fact these calculations are performed in the gravity dual of $\mathcal{N} = 4$ sYM, not QCD (the dual of which is not known), the qualitative agreement (of certain observables) with experimental data is apparently very good. The experiments in question involve colliding heavy ions such as Gold or Lead nuclei, and are currently taking place at RHIC (Brookhaven National Laboratory) and hopefully soon also at the LHC (CERN).

Another particularly pertinent application of the AdS/CFT correspondence is in the study of gravity duals of non-relativistic systems, initiated in reference [99]. The non-relativistic analog of the conformal group is known as the Schrödinger group. Field theories with this symmetry are used to describe the critical point behaviour of various condensed matter systems. In recent work by various authors [100–102] it was shown that gravity duals of QFTs with Schrödinger symmetry can be obtained from known pairs with $\mathcal{N} = 1$ superconformal symmetry by performing a TsT transformation, similar to the method described in Section 3.2 for generating the γ-deformed field theory. The main difference is, for the γ-deformation, both the T-duality and shift are performed along spacelike directions on the S^5, and so leave the conformal structure (associated with AdS_5) well alone, whereas in [101] the shift is carefully taken in a null direction (i.e. along the lightcone) to generate theories with Schrödinger symmetry. A nice introduction to this emerging field — applying the gauge–gravity correspondence to problems in fluid dynamics — is given in reference [103].

Chapter 4.

Supersymmetry Breaking

"Life is really simple, but we insist
on making it complicated."
— Confucius

Supersymmetry certainly helps to tame quantum field theory, but unfortunately it makes things too simple. As we saw in Section 2.2.3, if the Universe were exactly supersymmetric, for every type of particle there would exist a corresponding superpartner with the same quantum numbers but opposite statistics. As there is no such pairing amongst any of the known particles, we are going to have to work harder to keep any of the benefits of SUSY.

At high energies, supersymmetry is very good at ensuring the ubiquitous divergences of quantum field theory are not too severe, and in fact all phenomenologically viable incarnations of string theory are currently reliant on SUSY too. At low energies, as we have seen SUSY must be broken in some way, but we must be careful: does the breaking of supersymmetry re-introduce a hierarchy of the sort we initially set out to avoid? This doesn't necessarily have to be the case, provided the breaking spontaneously occurs due to some dynamical mechanism. For example, non-perturbative effects can generate a natural hierarchy of scales [104]. The situation is akin to QCD; nobody gets too worried that the dimensional transmutation scale Λ_{QCD} is much less than the Planck scale because the relationship is approximately logarithmic:

$$\left(\frac{\Lambda_{QCD}}{\mu}\right)^b = \exp\left(\frac{-8\pi^2}{g^2(\mu)}\right) \qquad \Longrightarrow \qquad \Lambda_{QCD} = M_{\rm Pl}\exp\left(\frac{-8\pi^2}{b\, g^2(M_{\rm Pl})}\right) .$$

Similarly, if SUSY could be dynamically broken by the effects of gaugino condensation for example (cf. Section 2.2.2), then worrisome quadratic divergences would — at worst — be exchanged for an altogether more palatable logarithmic fine-tuning.

4.1. Why It's Hard

Compared to the relative order found in exactly supersymmetric theories, SUSY breaking as an enterprise is beset with complications. Broadly speaking, it is possible to identify two main themes:

Too much freedom — there is certainly more than one way to break supersymmetry, and depending on how this is achieved, a wide variety of physics can result. From a phenomenological point of view, the vast parameter space arising in even the most minimal of models makes distinguishing the different SUSY breaking mechanisms very challenging. There is, of course, also the perennial problem of there being very little experimental input for such high energy physics. With any luck, at least this will aspect will soon be changing.

SUSY is resilient — in short, SUSY doesn't like being broken. One can work hard to engineer a model with broken supersymmetry, but modifying or extending the model often leads to a restoration of supersymmetry. This phenomenon has its roots in the rigid mathematics of supersymmetry; there are various general theorems one can explore to understand why it happens, and to also find useful ways of proceeding.

We will now investigate these two issues in more detail.

4.1.1. Physical Perspective

The Soft Option

For want of a more fundamental understanding of SUSY breaking, and to allow for some dialogue between SUSY model builders and experiment, it is useful to "parameterise our ignorance" and classify all the possible terms that could arise in a low energy effective Lagrangian that *softly* breaks supersymmetry. Within the framework of the MSSM, detailed in Section 2.2.3, the following SOFT TERMS break SUSY whilst preserving the desired cancellation of quadratic divergences:

$$\begin{aligned}\mathcal{L}_{\text{soft}} = &- \frac{1}{2}\left(M_3\,\widetilde{G}\widetilde{G} + M_2\,\widetilde{W}\widetilde{W} + M_1\,\widetilde{B}\widetilde{B}\right) + h.c. &\quad (4.1a)\\
&- \left(\widetilde{\overline{u}}\,\mathbf{A}_u\,\widetilde{Q}\,H_u - \widetilde{\overline{d}}\,\mathbf{A}_d\,\widetilde{Q}\,H_d - \widetilde{\overline{e}}\,\mathbf{A}_e\,\widetilde{L}\,H_d\right) + h.c. &\quad (4.1b)\\
&- \widetilde{Q}^*\mathbf{m}^2_Q\,\widetilde{Q} - \widetilde{L}^*\mathbf{m}^2_L\,\widetilde{L} - \widetilde{\overline{u}}^*\,\mathbf{m}^2_{\overline{u}}\,\widetilde{\overline{u}} - \widetilde{\overline{d}}^*\,\mathbf{m}^2_{\overline{d}}\,\widetilde{\overline{d}} - \widetilde{\overline{e}}^*\,\mathbf{m}^2_{\overline{e}}\,\widetilde{\overline{e}} &\quad (4.1c)\\
&- m^2_{H_u}\,H_u^*H_u - m^2_{H_d}\,H_d^*H_d &\quad (4.1d)\\
&- \left(B_\mu\,H_uH_d + h.c.\right) &\quad (4.1e)\end{aligned}$$

The first line of (4.1) represents mass terms for gauginos.[1] The subsequent lines all involve just the scalar components of the matter/Higgs chiral superfields: (4.1b) are the TRILINEAR A TERMS, (4.1c) are squark and slepton masses, (4.1d) give the Higgs fields mass, and (4.1e) is the so-called B_μ TERM, which mixes the up- and down-type Higgses.

The MSSM only has one parameter (μ) more than the Standard Model, but when we also take into consideration the soft couplings M_i, $\mathbf{A}_i$, $\mathbf{m}_i^2$, $m^2_{H_i}$ and B_μ, this extra freedom jumps to a horrifying 105 variables. This is the source of many complications associated to SUSY breaking; a wide variety of different physics can, in principle, result depending on the breaking scenario. Although there is a lot of leeway in the precise values of soft breaking parameters, general phenomenological considerations indicate they should have a characteristic energy scale of the order of 1 TeV. This is the case, for example, if supersymmetry is to provide a satisfactory resolution to the hierarchy problem discussed in Section 2.2.

Mediation Mechanisms

Clearly, measuring over one hundred independent parameters is not viable, so to proceed we have to make assumptions about how supersymmetry is broken and how the effects are communicated to the MSSM fields. This interrelates many of the soft parameters and hence cuts down the parameter space to something more manageable. One can then attempt to identify general patterns in the masses and couplings, and place bounds on the soft parameter values allowed by each mechanism.

[1] We adhere to the convention that a tilde ($\widetilde{\ }$) indicates the superpartner of a Standard Model field.

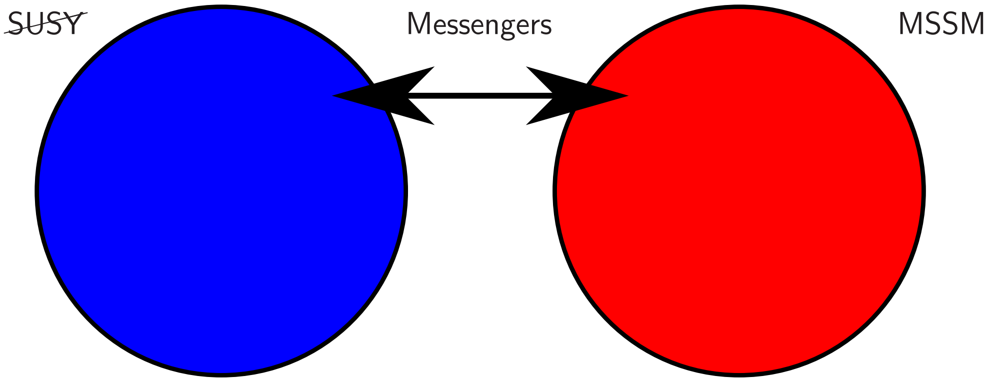


Figure 4.1.: Illustration of disjoint SUSY breaking (blue) and MSSM (red) sectors interacting via messenger fields.

The first thing one might think to do is engineer tree-level spontaneous breaking, much like the electroweak sector of the Standard Model. For this form of DIRECT MEDIATION, the SUSY breaking fields are directly coupled to MSSM fields. Unfortunately, this idea falls at the first hurdle for the following reason. Even when SUSY is broken, for the quadratic UV divergences to cancel, one can show that each supermultiplet still obeys SUM RULES. For the down-type (s)quarks this forces the equality

$$m_{\tilde{d}}^2 + m_{\tilde{s}}^2 + m_{\tilde{b}}^2 \;=\; 2 \cdot \left(m_d^2 + m_s^2 + m_b^2\right)$$

We know the right-hand side is $\approx 2 \cdot (5\text{ GeV})^2$, which implies all squarks must be lighter than 7 GeV. The existence of such particles was ruled out long ago whilst searching for the top quark.

A more flexible possibility, which has very much become the norm for studies of SUSY signatures in particle physics, is to separate out the supersymmetry breaking sector from the MSSM sector, and couple the two with messenger fields, as illustrated in Figure 4.1. This has the advantage of decoupling the actual mechanism of SUSY breaking from constraints of the real world; the phenomenology, i.e. the pattern of soft terms, depends primarily on the nature of the messengers.

One of the most studied messenger mechanisms is GRAVITY MEDIATION. This is perhaps the most natural way of communicating SUSY breaking from a Hidden Sector to the supersymmetric Standard Model, in the sense that all objects that have energy are expected to interact gravitationally, so some degree of gravity mediation is unavoidable. The weakness of the gravitational force (couplings are suppressed by factors of the Planck scale $M_{\text{Pl}} \sim 1.2 \times 10^{19}$GeV) means the SUSY breaking scale has to be relatively high $\sim 10^{11}$ GeV to account for TeV size soft masses. The fact that gravity couples everything to everything else, indiscriminately, causes difficulties for gravity mediated models; they generally suffer from an excess of flavour-changing neutral current type processes, which are tightly constrained by experimental data. Under additional assumptions about the structure of gravitational interactions at high energy, a model of gravity mediation can be derived in which all the soft terms depend on just four parameters. This is the minimal supergravity inspired SUSY breaking scenario (mSUGRA) that is often used as the basis for phenomenological studies of supersymmetric particle physics. By altering the assumptions made in connecting the Standard Model with a theory of gravity, one arrives at different mechanisms such as ANOMALY MEDIATION and GRAVITINO MEDIATION. Each of these can lead to distinctive signatures in the spectrum of particles found at low energy.

Given the success of gauge theories in modelling particle physics, and also the uncertainties inherent in our current understanding of quantum theories of gravity, in this thesis we will mostly be interested in constructing SUSY breaking models in which the effects of gravity play a sub-dominant rôle; the Messenger Sector (and the Hidden Sector itself) will be described entirely in terms of supersymmetric gauge field theory, with the fields that communicate supersymmetry breaking to the Visible Sector carrying charge under the Standard Model gauge groups. This GAUGE MEDIATION paradigm for supersymmetry breaking (GMSB) was introduced in the early days of SUSY model building in references [105–110] and was subsequently revived in [111–113] (see [14] for a comprehensive review of GMSB patterns and phenomenology). One immediate consequence of mediating SUSY breaking in this way is that there is no serious problem with flavour changing processes. This is because the effects of SUSY breaking enter through the standard gauge interactions and so are FLAVOUR BLIND, introducing no additional flavour structure (at tree level) beyond the usual Yukawa matrices.

We can make a rough estimate of the SUSY breaking scale required for gauge mediated scenarios to deliver TeV scale soft masses. Taking SUSY to be broken in a Hidden Sector at a scale $\sqrt{F}$, the soft masses induced by loops of messenger superfields with mass M_{mess} are typically

$$m_{\text{soft}} \sim \frac{\alpha_s}{4\pi}\frac{F}{M_{\text{mess}}} \ . \tag{4.2}$$

If $\sqrt{F}$ and M_{mess} are of the same order of magnitude, requiring $m_{\text{soft}} \sim 1$ TeV then allows supersymmetry to be broken at a scales as low as $\sqrt{F} \sim 10^5$ GeV. In contradistinction to gravity mediated scenarios, gauge mediation is thus known as a LOW-SCALE MEDIATION scenario, and is largely independent of the intricacies of gravity. An obvious exception to this rule is the gravitino (superpartner of the graviton), which is expected to acquire a mass $m_{3/2} \sim \frac{F}{M_{\rm Pl}}$ when SUSY is spontaneously broken. For low-scale scenarios, the gravitino is very often the lightest supersymmetric particle, and as such plays an important rôle in both particle phenomenology and cosmology.

Renormalisation Group Running

For a given model of SUSY breaking, the effective soft terms (4.1) are usually derived by integrating out the messengers fields. This gives the soft terms at the mass scale of the messengers $Q_{\rm Mess}$. Typically the messenger mass scale is quite a few orders of magnitude higher than the electroweak scale, so for comparison with experiment the derived soft terms then need be RG evolved (cf. Section 2.1) down to lower scales. The appropriate RG equations are usefully collected in the appendix of reference [114].

One attractive feature of the MSSM soft terms is they can provide a dynamical explanation of electroweak symmetry breaking. Recall how in the Standard Model one must simply posit a 'mexican hat' profile for the Higgs potential. In the MSSM, even if at a high scale both the Higgs soft masses are positive, under renormalisation group evolution it can happen that the $m^2_{H_u}$ term is forced to a negative value at lower energies. Thus the Higgs potential develops a minimum away from the origin which gives H_u a non-zero vacuum expectation value and hence precipitates the breaking of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ to $\mathrm{U}(1)_\gamma$. This phenomenon is known as RADIATIVE ELECTROWEAK SYMMETRY BREAKING (REWSB), and is such a desirable feature that its successful implementation is often used as a constraint on SUSY breaking models (at the expense of some freedom in other parameters) as we shall now explain.

From equations (2.63) and (4.1) one finds the electrically neutral components of the Higgs fields feel a potential

$$\begin{aligned} V(H_u^0, H_d^0) \;=&\; \left(|\mu|^2 + m^2_{H_u}\right) \left|H_u^0\right|^2 + \left(|\mu|^2 + m^2_{H_d}\right) \left|H_d^0\right|^2 \\ &- \left(B_\mu\, H_u^0 H_d^0 + \text{h.c.}\right) + \frac{1}{8}(g^2 + g'^2) \left(\left|H_u^0\right|^2 - \left|H_d^0\right|^2\right)^2 \;, \end{aligned} \tag{4.3}$$

where g and g' correspond to the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ couplings respectively. At the minimum of this potential, the Higgs fields obtain VEVs

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}} \qquad \langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}} \,, \tag{4.4}$$

which lead to masses for the W and Z bosons

$$M_W^2 = \frac{1}{4} g^2 v^2 \qquad M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2 \qquad \text{with} \qquad v^2 = v_u^2 + v_d^2 \,. \tag{4.5}$$

Reproducing the observed values of these masses requires $v \approx 246$ GeV. If we parameterise the ratio of Higgs VEVs by

$$\tan\beta = \frac{v_u}{v_d} \,, \tag{4.6}$$

it is easy to show the minimisation conditions for potential (4.3) become

$$|\mu|^2 + m_{H_u}^2 = B_\mu \cot\beta + \frac{M_Z^2}{2} \cos 2\beta \tag{4.7a}$$

$$|\mu|^2 + m_{H_d}^2 = B_\mu \tan\beta - \frac{M_Z^2}{2} \cos 2\beta \,. \tag{4.7b}$$

To ensure REWSB takes place, it is common practice in phenomenological studies of SUSY breaking to take the known value of M_Z and an arbitrary value of $\tan\beta$ ($4 \lesssim \tan\beta < 60$) at the electroweak scale, and use equations (4.7) to determine suitable values of μ and B_μ. It may seem odd to marginalise these two parameters, at least one of which (B_μ) should be derivable from any respectable SUSY breaking mechanism, but treating them this way is often expedient owing to unresolved issues involving μ and B_μ, which we will come to shortly.

The first real test of the viability of a SUSY breaking scenario comes from obtaining its spectrum of MSSM particle masses at collider energy scales. To do this one first RG evolves the known Standard Model Yukawa couplings from the electroweak scale up to a high energy scale, which for sake of argument[2] we take to be the messenger mass scale Q_{Mess}. Note, the scale evolution in this step involves making an arbitrary choice for the initial values of the soft parameters. The values of soft parameters (except μ and B_μ) derived from the SUSY breaking model can then be imposed (they comprise the high-scale boundary conditions of the system). All couplings are then evolved down to

[2] As an alternative, one might consider using the scale of gauge coupling unification.

a low scale Q_{SUSY} at which point the electroweak symmetry breaking conditions (4.7) are imposed to determine the μ and B_μ values required by REWSB.[3]

As the RG equations that allow one to compare parameters at the high and low scales themselves depend on the sparticle mass thresholds and soft parameters, the above procedure must be iterated to wash out the guess we had to make for their initial values. With any luck the routine will converge to a stable set of values and the spectrum of SUSY particle (pole) masses can then be calculated. Fortunately there are a variety of freely available programs that implement the algorithm just described to multi-loop accuracy, for example `SoftSusy` [116], `SuSpect` [117] and `Spheno` [118]. In Chapter 6 we will use a modified version of `SoftSusy` to compare the predicted spectra of various models of gauge mediated SUSY breaking.

μ Problem

One curious issue with the softly broken MSSM can be seen from equations (4.7a) and (4.7b). Rearranging these gives

$$|\mu|^2 = -\frac{M_Z^2}{2} - \frac{\tan^2\beta\, m_{H_u}^2 - m_{H_d}^2}{\tan^2\beta - 1} , \tag{4.8a}$$

$$B_\mu = \frac{\sin 2\beta}{2}\left(m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2\right) . \tag{4.8b}$$

This requires there to be a link between the *supersymmetric* parameter μ and the SUSY *breaking* terms m_{H_i}. Furthermore, if there is to be no careful tuning the parameters then we would expect μ to be of order the electroweak scale $|\mu| \sim \mathcal{O}(M_Z)$. Depending on the mediation mechanism, it can be quite a challenge to arrange for such a relationship to arise naturally — this is the μ PROBLEM. One way to approach it is to introduce a discrete symmetry, or a Peccei-Quinn symmetry[4] that forbids the superpotential term $\mu\, H_u H_d$ at tree level. Radiative breaking of this symmetry can then generate a μ term via loop effects in the same manner as the soft terms. This often works well for gravity mediated models [119], but is more of a headache for gauge mediation [120]. In this case, μ and B_μ typically both get generated at the same loop order, so μ^2 has an extra

[3]The scale Q_{SUSY} is often taken to be $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, the geometric mean of the stop masses, which lies slightly above the electroweak scale. This is found to minimize the scale dependence of the result, as discussed in reference [115] and Section 6.3.3.

[4]Under a U(1) PQ symmetry, Higgs fields are rotated in the opposite direction to other fields.

suppression factor relative to B_μ, i.e.

$$B_\mu \sim 16\pi^2\mu^2 \gg \mu^2 \ .$$

To satisfy equation (4.8b) we must then have $B_\mu \sim \left|m^2_{H_u}\right|, \left|m^2_{H_d}\right|$, and so from equation (4.8a) it follows that $|\mu| \gg M_Z$. Such a hierarchy can be achieved by fine tuning the model parameters; providing a more convincing explanation of this difference of scales is the content of the B_μ Problem.

No clear solution to the μ/B_μ problems has yet emerged. One possibility is to extend the MSSM by a singlet chiral superfield N and take the superpotential

$$W_{\text{NMSSM}} = W_{\text{Yukawa}} + \lambda\, N\, H_u H_d + \frac{1}{3}\kappa\, N^3 \ . \tag{4.9}$$

The μ/B_μ terms can then be dynamically generated if N acquires a non-zero VEV. This Next-to-Minimal Supersymmetric Standard Model (NMSSM) still has various phenomenological issues, and doesn't shed any extra light on the nature of supersymmetry breaking, so we shall not consider it or similar extensions in the remainder of this thesis. Instead we adopt a more prosaic attitude towards the μ problem: we will be content to find *some* value of μ which allows for REWSB in our models. Such an approach is permissible because μ is a SUSY preserving parameter, and so is at least logically distinct from the SUSY breaking sector we seek to understand. In this way we can at least *accommodate* the μ problem, albeit without specifying or understanding its resolution.

4.1.2. Mathematical Perspective

It is somewhat ironic that the very same mathematical framework that underpins the power and beauty of supersymmetric field theories also contrives to undermine the construction of supersymmetric models that are physically realistic. The following sections unearth some far-reaching consequences of the SUSY algebra that teach us what can and can't be achieved in such models.

The Witten Index

Knowing whether a given model admits supersymmetric vacua is sometimes relatively easy. A useful place to start is to consider the trace of the operator $(-1)^F$, introduced in Section 2.2.1, which distinguishes between bosonic $|B\rangle$ and fermionic states $|F\rangle$

$$\mathcal{W} = \mathrm{Tr}\,(-1)^F = \sum_B \langle B|B\rangle - \sum_F \langle F|F\rangle \, .$$

This is known as the Witten Index [121]. To understand why it is useful, recall from equation (2.21) that the Hamiltonian H of any theory with global (not local) supersymmetry is a positive definite function of the supercharges. In particular

$$H|B\rangle = 0 \qquad \Longleftrightarrow \qquad Q|B\rangle = 0 \, ,$$

and similarly for fermionic zero energy states, whereas for $H|B\rangle \neq 0$ we are guaranteed a state $|F\rangle = Q|B\rangle$ with the same energy but opposite statistics. We see that in the Witten index, states with non-zero energy cancel pairwise, so it actually counts the difference in number of zero energy bosonic and fermionic states. The important point is that if the Witten index is non-vanishing, there must exist zero energy states, so a supersymmetric vacuum must exist. The converse doesn't necessarily hold though: if the index vanishes, susy may or may not be broken — more work is required to decide either way.

The power of the Witten index comes from its topological nature: the argument works irrespective of the value of the couplings, so if one can compute the index in some weak coupling regime, the conclusion can generally be extrapolated to other values of the coupling that are less under control. Naturally, there are some caveats which complicate the matter — operators must be properly defined, one must worry about the distinction between finite/infinite volume, and one must be aware of the asymptotic values of the potential changing, which can allow new zero-energy states in from infinity — but the punchline is that susy vacua are relatively easy to find.

There is a simple application of this theorem that frustrates attempts at model building. It can be shown that the Witten index for a simple Yang-Mills gauge theory is not zero (for the common case of an $\mathrm{SU}(N_c)$ gauge group there are N_c distinct vacua). As a corollary, for any gauge theory in which all matter fields can be given a mass — "vector-like" theories — the matter can be integrated out, so at low energies one is left with a pure Yang-Mills theory. Thus all vector-like theories (modulo a few caveats) have a non-vanishing Witten index, and so admit supersymmetric vacua. Having apparently ruled out a large class of models undoubtably complicates our effort to find SUSY breaking models, but as we will see in Section 4.2 there is more one can do with vector-like theories: they shouldn't be discarded so soon.

The Nelson-Seiberg Theorem

This provides a connection between R-symmetry and SUSY breaking that, again, apparently scuppers attempts to construct simple physical models. The theorem was first published in reference [122] and states that

Dynamical supersymmetry breaking in a generic, calculable model requires a spontaneously broken R-symmetry.

A GENERIC model is one in which any operator that is allowed by the symmetries necessarily appears in the Lagrangian. In CALCULABLE models the low energy behaviour, particularly that arising from non-perturbative effects, is understood/under control. For completeness we now sketch a proof of the theorem.

To find a SUSY vacuum we should look for stationary points of the low energy effective action. In the type of models considered, this boils down to solving the following simultaneous equations that are derived from the effective superpotential:

$$\frac{\partial}{\partial \phi_i} W_{\text{eff}}(\phi_j) = 0 , \qquad i, j = 1 \ldots n . \qquad \spadesuit$$

It is sometimes possible to use symmetries of the model to further constrain this system of equations. Consider the following cases:

No symmetries: With no extra restrictions, ♠ represents n equations in n unknowns. In general, such a system is soluble, i.e. it is possible to find a supersymmetric vacuum. Note that here, as well as the genericity assumption, we have to make use of the holomorphy of the superpotential (Section 2.2.2). This guarantees ♠ are simply a bunch of polynomials, and hence soluble.

Global U(1) non-R-symmetry: Under the symmetry, W_{eff} carries no charge so must be a holomorphic function of the $n-1$ variables:

$$X_a = \frac{\phi_a}{\langle\phi_n\rangle^{q_a/q_n}} , \qquad a = 1, \ldots, n-1 \qquad (\text{if } \langle\phi_n\rangle \neq 0)$$

where q_a is the R-charge of ϕ_a. One vacuum equation is trivially satisfied so there ends up being $n-1$ equations in $n-1$ unknowns. Again, this is soluble in general.

Global U(1) R-symmetry: We highlighted in Section 2.2.1 that one special feature of R-symmetry is that the superpotential carries R-charge 2. It must therefore be possible to write W_{eff} in terms of a new holomorphic function:

$$W_{\text{eff}} = \phi_n^{2/q_n} f(X_a) .$$

In this notation the vacuum equations become

$$\partial_a f(X_b) = 0 \qquad \text{and} \qquad f(X_b) = 0 \qquad (\text{provided } \langle\phi_n\rangle \neq 0) .$$

This time we have n equations in $n-1$ unknowns, which is generically insoluble. In this case one cannot find a supersymmetric vacuum, and so SUSY must be spontaneously broken.

Generic and calculable models of dynamical SUSY breaking are highly desirable; in theory they are attractive models with no fine-tuning or hierarchy issues, which provide concrete predictions for low energy physics. Unfortunately, the Nelson-Seiberg theorem tells us that the Lagrangian of any such a theory must also posess an R-symmetry. This in turn forbids the gaugino[5] Majorana mass operator

$$m_\lambda \left(\lambda\lambda + \overline{\lambda\lambda} \right)$$

Massless gauginos are problematic because they have already been experimentally ruled out. Spontaneously breaking the R-symmetry that protects gaugino mass may sound like a good idea, but the resulting Goldstone boson, the R-AXION, is also relatively constrained by (non-)observation. This is clearly a dangerous game; we will see what more can be learnt from these ideas in Section 4.2.2.

[5] The fact that gauginos have R-charge equal to one can be seen from how they enter the vector supermultiplet: $V = i(\theta\theta)(\overline{\theta\lambda}) - i(\overline{\theta\theta})(\theta\lambda) + \ldots$

4.2. Story of Metastability

The issue of supersymmetry breaking has recently been reinvigorated by Intriligator, Seiberg and Shih [6] (ISS). They made the observation that METASTABLE SUSY breaking vacua can arise naturally and dynamically in the low-energy limit of supersymmetric SU(N) gauge theories. This has important implications for our understanding of how SUSY is broken in nature. Following the work of ISS, there has been exploration of both the cosmological consequences [9–11, 123, 124], and the possibilities for gauge or direct mediation of the SUSY breaking to the Visible Sector [6, 8, 12, 13, 125–143]. On the cosmological side, an attractive feature of these models is that the metastable vacua are naturally long lived due to the flatness of the potential. Moreover, at high temperatures the SUSY breaking vacua are dynamically favoured over the SUSY preserving ones, for reasons to be explained in Section 4.2.3, so the early Universe would naturally have been driven into them.

On the phenomenological side, attention has focussed on a striking aspect of metastability, namely that the models do not have an exact $\mathrm{U}(1)_R$ symmetry, and indeed that the $\mathrm{U}(1)_R$ symmetry is anomalous under the same gauge group that dynamically restores supersymmetry in the supersymmetric global minima. *In principle* this allows one to evade the Nelson-Seiberg theorem of Section 4.1.2, that SUSY breaking requires R-symmetry in a generic model (i.e. one that includes all couplings compatible with the symmetries) [122]. R-symmetry is unwelcome because it implies that gauginos are massless, so the fact that it can be broken by metastability is an encouraging sign. The remainder of this thesis is aimed at exploring how shifting ones point of view to the metastable paradigm plays out *in practice*.

One hope that was expressed in reference [6] is that allowing the vacuum to be metastable might permit the construction of simpler, more robust models of SUSY breaking. This expectation was illustrated with an example that has become known as the ISS model of metastable SUSY breaking. We will now review its salient features before turning to some more general arguments in favour of metastability in Section 4.2.2.

4.2.1. The ISS Model

To demonstrate the ubiquity of metastable SUSY breaking models, Intriligator, Seiberg and Shih [6] invite us to consider perhaps the simplest supersymmetric gauge theory with massive matter: the SQCD theory introduced in Section 2.2.2. In more detail, this is an $\mathcal{N}=1$ super Yang-Mills theory with the gauge group $\mathrm{SU}(N_c)$ coupled to N_f pairs of (anti-)fundamental quark supermultiplets Q, $\widetilde{Q}$. The Kähler potential is taken to be canonical, and the superpotential is

$$W_{\text{tree}} \,=\, m_{ij}\, Q_i \cdot \widetilde{Q}_j \ . \tag{4.10}$$

Metastable vacua $|\text{vac}\rangle_+$ can be shown to occur in this model when N_f is in the 'free magnetic' range:

$$N_c + 1 \,\leq\, N_f \,\leq\, \frac{3}{2} N_c \ .$$

These vacua are apparent in the Seiberg dual formulation of the theory, which has the advantage of being weakly coupled in the vicinity of $|\text{vac}\rangle_+$. The magnetic Seiberg dual of the ISS theory is given [144, 145] by the $\mathrm{SU}(N)_{\text{mg}}$ gauge theory, with $N = N_f - N_c$, coupled to N_f magnetic quark/anti-quark pairs φ, $\widetilde{\varphi}$. The tree-level superpotential of the magnetic theory is of the form

$$W_{\text{cl}} \,=\, \Phi_{ij}\, \varphi_i \cdot \widetilde{\varphi}_j \,-\, \mu^2_{ij} \Phi_{ji} \ , \tag{4.11}$$

where $i, j = 1, \ldots, N_f$ are flavour indices. Φ_{ij} is the gauge-singlet 'meson' superfield, which is related to the original electric quarks via $\Phi_{ij} \propto \Lambda^{-1} Q_i \cdot \widetilde{Q}_j$ and Λ is the dynamical scale of the ISS theory [6]. The matrix μ^2_{ij} (which can be diagonalised without loss of generality) arises from the masses of electric quarks, $\mu^2_{ii} = -\Lambda\, m_{Q_i}$. All of its eigenvalues μ_i are taken to be much smaller than the UV cutoff of the magnetic theory, $\mu_i \ll \Lambda$. This magnetic theory is free and calculable in the IR and becomes strongly coupled in the UV where one should instead use the electric Seiberg dual, i.e. the original $\mathrm{SU}(N_c)$ SQCD, which is asymptotically free.

The usual holomorphicity arguments imply that the superpotential (4.11) receives no corrections in perturbation theory. However, there is a non-perturbative contribution to the full superpotential of the theory, $W = W_{\text{cl}} + W_{\text{dyn}}$, which is generated dynamically [6] and is given by

$$W_{\text{dyn}} \,=\, N \left(\frac{\det_{N_f} \Phi}{\Lambda^{N_f - 3N}} \right)^{\frac{1}{N}} \ . \tag{4.12}$$

The authors of [6] have studied the vacuum structure of the theory and established the existence of the metastable vacuum $|\mathrm{vac}\rangle_+$ with non-vanishing vacuum energy V_+ as well as the SUSY preserving stable vacua $|\mathrm{vac}\rangle_0$.

The supersymmetry breaking vacuum $|\mathrm{vac}\rangle_+$ originates from the so-called rank condition, which implies that there are no solutions to the F-flatness equation

$$F_{\Phi_{ij}} = (\varphi_i \cdot \widetilde{\varphi}_j - \mu_{ij}^2) = 0 \tag{4.13}$$

for the classical superpotential W_{cl} (equation (4.13) can only be satisfied for a rank-N submatrix of the $N_f \times N_f$ matrix F_Φ). The SUSY preserving vacuua appear by allowing the meson Φ to develop a VEV that is stabilised by the non-perturbative superpotential (4.12) and that is interpreted in the ISS model as a non-perturbative or dynamical *restoration* of supersymmetry [6]. The lowest lying SUSY breaking vacuum $|\mathrm{vac}\rangle_+$ is characterised by

$$\langle\varphi\rangle = \langle\tilde{\varphi}^T\rangle = \begin{pmatrix} \mathrm{diag}(\mu_1, \ldots, \mu_N) \\ 0_{N_f-N} \end{pmatrix}, \quad \langle\Phi\rangle = 0\,, \qquad V_+ = \sum_{i=N+1}^{N_f} |\mu_i^4|\,. \tag{4.14}$$

Here μ_i are the ordered eigenvalues μ matrix, such that $|\mu_1| \geq |\mu_2| \geq \ldots \geq |\mu_{N_f}|$. In this way, the vacuum energy V_+ above receives contributions from $(N_f - N)$ of the smallest μ's while the VEV $\langle\varphi\rangle$ is determined by the largest μ's.

The SUSY-preserving vacuum $|\mathrm{vac}\rangle_0$ can also be found in terms of the magnetic variables. It is described by

$$\langle\varphi\rangle = \langle\tilde{\varphi}^T\rangle = 0\,, \qquad \langle\Phi\rangle = \left(\frac{\Lambda}{\mu}\right)^{\frac{N_f-3N}{N_f-N}} \mu\,\mathbb{1}_{N_f}\,, \qquad V_0 = 0\,, \tag{4.15}$$

where for simplicity we have specialised to the degenerate case, $\mu_{ij} = \mu\delta_{ij}$. There are precisely $N_f - N = N_c$ of such vacua differing by the phase $e^{2\pi i/(N_f-N)}$, as there must be to match the Witten index of the electric ISS theory (cf. Section 4.1.2). For $\mu/\Lambda \ll 1$ the metastable vacuum is exponentially long-lived and the lifetime of $|\mathrm{vac}\rangle_+$ can easily be made much longer than the age of the Universe, as we will see shortly.

The issue of R-symmetry is quite subtle in ISS SQCD, but given its importance in building realistic models, it pays to understand it a little better. The tree-level superpotential in the magnetic picture (4.11) admits an R-symmetry under which Φ has R-charge 2 (φ and $\widetilde{\varphi}$ can have any charges, as long as they are opposite). This symmetry is anomalous with respect to the magnetic gauge group, a fact which is reflected in the symmetry being explicitly broken by the non-perturbative contribution (4.12) to the effective superpotential. From the perspective of the electric theory, the magnetic R-symmetry is broken explicitly by the mass terms of electric quarks m_Q. Thinking of the electric theory as *more fundamental* (because it has a well-defined high energy limit), we see the magnetic R-symmetry emerges as an ACCIDENTAL SYMMETRY of the low energy theory around the metastable vacuum (where the effects of (4.12) are negligible). A more detailed discussion of this point, pertinent to our model building exploits in Chapter 5, can be found in Appendix B.

Estimating the Lifetime of the False Vacuum

The essential premise of ISS-type models is that our Universe is only metastable. This means the low energy breaking of supersymmetry is only a temporary phenomenon, and that quantum effects can — at any point in time — restore supersymmetry and thus fundamentally change the nature of matter. To make these models physically viable one must ensure the predicted timescale for such a cataclysmic event to occur is longer than the current age of the Universe. This proves to be quite a weak constraint on ISS models, as we now review from reference [6].

The rate of decay (per unit volume) for starting in the metastable vacuum $|\text{vac}\rangle_+$ and ending up in a SUSY preserving vacuum $|\text{vac}\rangle_0$ is predominantly set by the Euclidean BOUNCE action

$$\frac{\Gamma_4}{V_4} \sim e^{-S_{\text{bounce}}} \,, \tag{4.16}$$

where S_{bounce} is the difference between the Euclidean action describing nucleation of an O(4)-symmetric bubble of SUSY vacuum in the metastable vacuum, and the action associated with remaining in the metastable vacuum. The dominant tunnelling configuration corresponds to the path in field space with the smallest potential barrier between $|\text{vac}\rangle_+$ and $|\text{vac}\rangle_0$. To get a rough idea of the shape of this barrier, consider the scalar potential near $|\text{vac}\rangle_+$ (where the dynamical superpotential (4.12) can be neglected)

$$V_{\text{tree}} = \left|\varphi_i \cdot \widetilde{\varphi}_j - \mu^2 \, \mathbb{1}_{N_f}\right|^2 + |\Phi\,\widetilde{\varphi}|^2 + |\varphi\,\Phi|^2 \,. \tag{4.17}$$

Clearly, if we start out in the metastable vacuum in which the quark fields $(\varphi, \widetilde{\varphi})$ are adjusted to minimise the first term of (4.17), then turning on both mesons (Φ) and

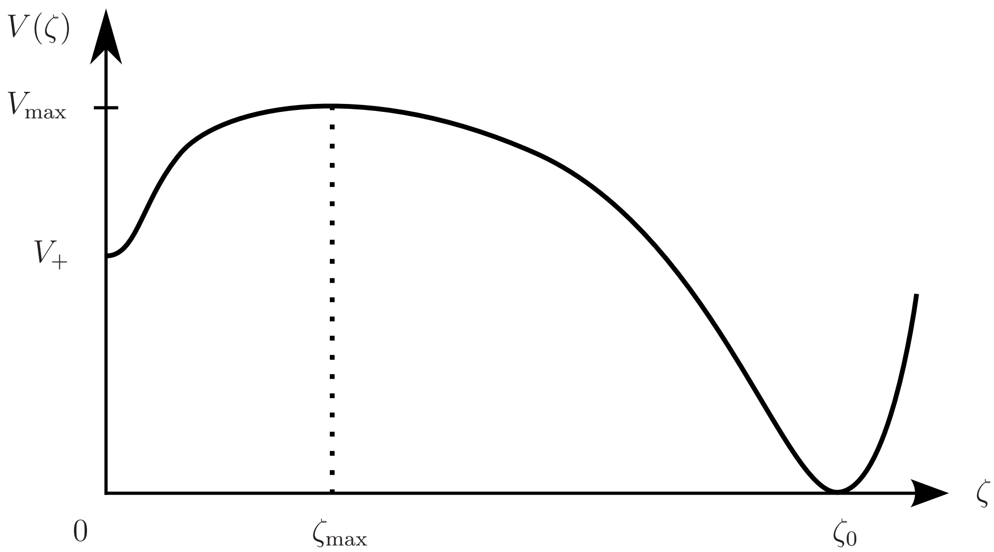


Figure 4.2.: Sketch of the scalar potential of an ISS model along a path that minimises the barrier height V_{max} at ζ_{max}. The metastable vacuum $|\text{vac}\rangle_+$ is situated at the origin, with the SUSY preserving vacuum $|\text{vac}\rangle_0$ at $\zeta_0 \gg \zeta_{\text{max}}$.

quark fields at the same time leads to an increase in the potential energy (and so also to the *Euclidean* action). Therefore, parameterising the path of minimal action by ζ, we see it will prefer to run from $|\text{vac}\rangle_+$ at $\zeta = 0$ with potential energy density $V_+ \approx \mu^4$ up to a local maximum near $\langle\varphi\rangle \sim \langle\widetilde{\varphi}\rangle \sim \langle\Phi\rangle \approx 0$ (at which point the energy V_{max} is still only of order μ^4). The meson direction can then switch on in the long run down to the supersymmetric vacuum $|\text{vac}\rangle_0$ at $\zeta_0 \gg \zeta_{\text{max}}$. We show a rough sketch of this tunneling profile in Figure 4.2. Modelling the barrier as a triangular potential, we can borrow the bounce action as calculated in reference [146] to give

$$S_{\text{bounce}} \sim \frac{(\Delta\Phi_0)^4}{V_+} \sim \left(\frac{\Lambda}{\mu}\right)^{4\cdot\left(\frac{N_f-3N}{N_f-N}\right)} , \tag{4.18}$$

where $\Delta\Phi_0 \approx \langle\Phi\rangle$ is the distance traversed in field space. Note that although the potential barrier is not very high, it is really the ratio of height-to-width of the potential that determines the size of the bounce action. As the SUSY preserving vacuum is generated non-perturbatively (in the magnetic picture in which we work) it is naturally distant in field space from the metastable vacuum; this essentially ensures the longevity of $|\text{vac}\rangle_+$, which can be expressed as $S_{\text{bounce}} \gtrsim 400$ (see for example reference [147]). Indeed, for the minimal incarnation of an ISS model with non-trivial gauge dynamics at low energy[6]

[6] We will employ this particular scenario later when building realistic models.

(corresponding to $N_c = 5$, $N_f = 7$), the separation of scales $\epsilon = \mu/\Lambda$ is only required to be $\epsilon \lesssim 5 \times 10^{-4}$ to accommodate the observation that Armageddon has not yet arrived.

4.2.2. The Unavoidablity of Metastability

If SUSY is discovered at the LHC and is of the gauge mediation type, then metastability of the vacuum is likely to be unavoidable. This conclusion, drawn in [8] and discussed in [3], is largely based on two pieces of evidence: gauginos are massive, and so too are R-axions. To substantiate the claim, we will briefly explain the logic in full generality and independently of the models of ISS. To see that metastability follows from these two pieces of evidence, the first important observation comes in the form of the Nelson-Seiberg theorem [122], discussed in Section 4.1.2, that an exact R-symmetry is *necessary and sufficient* to break SUSY in a generic, calculable theory (of the Hidden Sector). At the same time, Majorana mass terms for gauginos have non-vanishing R-charge. Thus we have a phenomenological problem that could be called the GAUGINO MASS PROBLEM: gaugino masses require both supersymmetry and R-symmetry breaking, but reference [122] tells us that these two requirements are mutually exclusive. How can we get around this?

One approach [8] is to assume that the Lagrangian is of the form

$$\mathcal{L} = \mathcal{L}_R + \varepsilon \mathcal{L}_{R-\text{breaking}}, \tag{4.19}$$

where $\mathcal{L}_R$ preserves R-symmetry, the second term, $\mathcal{L}_{R-\text{breaking}}$, is higher order in fields and breaks R-symmetry, and ε is parametrically small (we discuss why this should be shortly). Because R-symmetry is broken explicitly by the second term, the Nelson-Seiberg theorem requires that a global supersymmetry-preserving minimum must appear at order $1/\varepsilon$ away from the SUSY breaking one, which now becomes metastable. Note that this statement is completely general. Any attempt to mediate SUSY breaking to gauginos even from models that initially have no SUSY-preserving vacuum results in the appearance of a global SUSY minimum. Also the gaugino masses depend, as one would expect, on both the scale of SUSY breaking *and* the scale of R-symmetry breaking, whereas the scalar masses depend only on the former. (This point was used previously in [17] in support of split SUSY [15, 16]). The gaugino masses are directly related to ε and hence to the stability of the metastable vacuum.

The second possibility is to break the tree-level R-symmetry spontaneously. Spontaneous (rather than explicit) breaking of R-symmetry does not introduce new global SUSY preserving minima. As such it does not destabilise the SUSY breaking vacuum and does not require any fine-tuning of coefficients in the Lagrangian. At the same time, gauginos do acquire masses. This scenario, however, leads to a massless Goldstone mode of the spontaneously broken $\mathrm{U}(1)_R$ symmetry, so there is an R-AXION PROBLEM. In order to avoid astrophysical (and experimental) bounds, the R-axion should also acquire a mass. This means that R-symmetry must also be explicitly broken and by the earlier arguments this again means that the vacuum is metastable. However in this case [2] the gaugino mass is divorced from the size of *explicit* R-breaking ε which now determines the R-axion mass instead. This exhausts the logical possibilities and shows that, for a theory with a generic superpotential where the Nelson-Seiberg theorem applies, massive gauginos and massive R-axions imply metastability.

At this point the question arises as to how one might generate a Lagrangian of the form (4.19). Unless there is a compelling reason for the smallness of ε, the Lagrangian $\mathcal{L}_R$ is by definition non-generic, and $\mathcal{L}_{R-\text{breaking}}$ may allow many couplings which are compatible with the symmetries that one has to set to be small in order to avoid too rapid decay of the metastable vacuum. One requires an *almost* non-generic model, broken by small operators, which in general seems unlikely. However, realistic and natural gauge mediation models of this type were constructed in [12, 13]. The main idea of these models is to break R-symmetry by operators which are suppressed by powers of M_{Pl}. We will consider these models and their phenomenology in Section 6.2.

In Chapter 5 we will suggest an alternative approach where ε is not induced by external $1/M_{\text{Pl}}$ corrections and where R-symmetry is broken spontaneously. In the original ISS model [6], the Nelson-Seiberg theorem manifests itself in a simple way: the theory has an exact R-symmetry at tree-level. However the R-symmetry is anomalous and terms of the type $\varepsilon\, \mathcal{L}_{R-\text{breaking}}$ are generated dynamically [6] without having to appeal to Planck suppressed operators. Here ε is a naturally small parameter since it is generated non-perturbatively via instanton-like configurations, which are naturally suppressed by the usual instanton factor $e^{-8\pi^2/g^2} \ll 1$. Hence, the non-genericity in these models is fully calculable and under control. When, in addition to these non-perturbative effects, the R-symmetry is also broken spontaneously by perturbative contributions, gauginos receive sufficiently large masses $m_{\text{gaugino}} > 100\,\text{GeV}$ as required by their non-observation by current experiments. At the same time the R-axion receives a mass from the anomalously induced R-breaking terms. (Note that a possible additional contribution to the

R-axion mass may arise when the theory is embedded in supergravity [148]. However such noncalculable effects are suppressed.)

The spontaneous breaking of R-symmetry by radiative perturbative corrections is easy to achieve [128, 136]. For example, this happens [2] when the basic ISS model is deformed by adding a baryon-like term to the superpotential. This is the simplest deformation of the ISS model which preserves R-symmetry at tree-level. At one-loop level this deformation causes the R-symmetry to break spontaneously, while the R-axion gets a sufficiently large mass $m_{\text{axion}} > 100\,\text{MeV}$ to avoid astrophysical constraints from the non-perturbative anomalous R-symmetry breaking [2]. No new global minima appear other than those of the original ISS model, so the SUSY breaking scale can be sufficiently low to be addressed at the LHC. We will derive the phenomenological consequences of these models in Section 6.3.

4.2.3. Thermal History of the Universe

Many of the arguments in favour of supersymmetry and its realisation as a broken symmetry depend on the idea of NATURALNESS: large hierarchies in dimensional parameters are explained dynamically, models should be generic, etc. As we have just seen, such reasoning leads to a relatively general prediction that the SUSY breaking vacuum is metastable. However, one may now raise the legitimate concern that presupposing we reside in a metastable vacuum is, in itself, *unnatural*. Why should we find ourselves in this vacuum in the first place, rather than the more symmetric minimum energy configuration? It is a very appealing aspect of ISS-type metastable models that, instead of having to invoke anthropic arguments, cosmology provides us with a nice dynamical answer to this question: metastable ISS vacua appear to be *favoured* by the evolution of the early universe [9–11, 123, 124].

The reasoning goes as follows [9]. A long time ago, the Universe was smaller and hotter.[7] When sufficiently small and hot, finite temperature effects lead to a modification of the effective potential [149]

$$V_T(\Psi) = V_{T=0}(\Psi) + \frac{T^4}{2\pi^2}\sum_i \pm n_i \int_0^\infty \mathrm{d}q\, q^2 \ln\left(1 \mp \exp(-\sqrt{q^2 + m_i^2(\Psi)/T^2})\right) \quad (4.20)$$

[7] The temperature in deep space is currently about 2.7 K, equivalent to an energy of 2.3×10^{-4} eV.

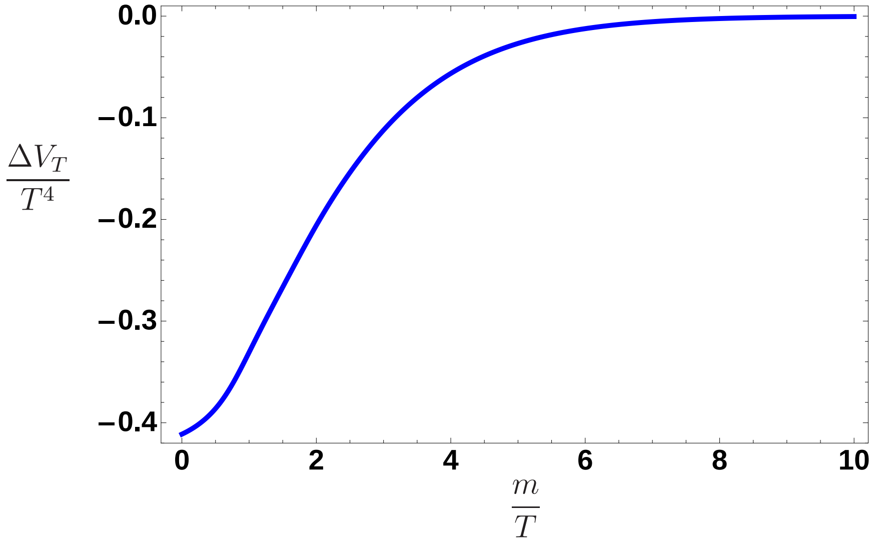


Figure 4.3.: Mass dependence of the thermal contribution of a superfield to the effective potential.

The first term on the right-hand side is the standard zero-temperature potential that follows from superpotential contributions (4.11) and (4.12). The second term is the one-loop correction due to temperature, which depends on the number of real degrees of freedom n_i and their corresponding masses m_i. As indicated, these are induced by the background field VEVs, collectively denoted Ψ. The upper signs in (4.20) are to be taken for bosonic fluctuations, and the lower for fermionic. To get a better feel for this equation, we plot the thermal contribution to the potential due to one chiral superfield of mass m in Figure 4.3. This makes it clear that (in the approximately supersymmetric case) the dominant thermal effects arise from light fields. To establish the early Universe behaviour of our zero-temperature vacua we therefore need to discuss which region of field space has the most light states. The big difference lies in the quark $(\varphi, \widetilde{\varphi})$ sector: around the metastable vacuum $|\text{vac}\rangle_+$ the quarks are relatively light, with masses of order μ at most, whereas in the supersymmetric vacuum $|\text{vac}\rangle_0$ one can see from equation (4.17) that the large VEV for Φ makes the quarks relatively heavy. The gauge degrees of freedom were also taken into account in reference [123], but they prove to give a sub-leading correction that does not qualitatively change our discussion.

The upshot is that as temperature increases, the extra light states near the origin adjust the potential so that the previously metastable vacuum has *lower* vacuum energy than the erstwhile supersymmetric one. Moreover, for sufficiently high temperatures, $|\text{vac}\rangle_0$ is washed out completely. This effect is most clearly illustrated by Figure 4.4,

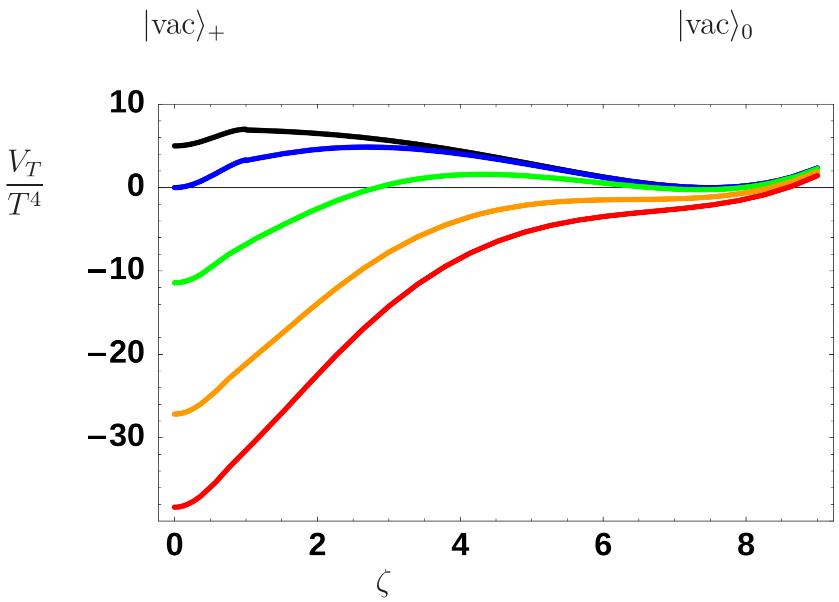


Figure 4.4.: The result of including finite temperature effects in the ISS effective potential (4.20). At high temperatures, shown as the bottom curve (red), only the vacuum $|\text{vac}\rangle_+$ is evident. As the temperature decreases (intermediate curves) $|\text{vac}\rangle_0$ also emerges, with the standard metastable profile (top, black curve) recovered at $T = 0$. For $T \neq 0$ a constant shift of the potential $\sim T^4$ has been suppressed. This graph is reproduced from reference [9] with the authors' consent.

which plots the thermal effective potential[8] for different values of the temperature. Above a critical temperature T_{crit}, tunnelling *into* $|\text{vac}\rangle_+$ can occur. Even if the Universe is sitting in $|\text{vac}\rangle_0$ at the end of inflation (where $T \sim 0$), as long as reheating takes the temperature above T_{crit}, the Universe is rapidly driven towards $|\text{vac}\rangle_+$. As the Universe again cools towards the present day, and the potential relaxes back to the metastable profile of Figure 4.2, we find ourselves naturally trapped in the metastable vacuum.

[8]There is also a constant shift in the potential proportional to T^4, coming from states that remain light for all ζ, which we suppress as it will not feature in the following.

Chapter 5.

A Simple Model of Direct Mediation

"Make everything as simple as possible,
but not simpler."
— Albert Einstein

5.1. Introduction

Just how simple a model of particle physics can one construct with metastable SUSY breaking vacua? The key to answering this question lies, once again, in how R-symmetry is broken. As we emphasised in Section 4.2.2, the relation between SUSY breaking and R-symmetry is a continuous one, in the sense that the lifetime of a metastable vacuum decreases in proportion to the size of any explicit R-symmetry breaking terms that one adds to the theory. This allows one to play the "approximate R-symmetry" game: add to the superpotential of the effective theory explicit R-symmetry breaking terms of your choosing, whilst trying to keep the metastable minimum as stable as possible.

Clearly there is some tension in this procedure. For example the gauge mediation scenario explored in references [12, 133] invokes a messenger sector (denoted by f). The field f has to have an explicit R-breaking mass-term to give gauginos a mass, and consequently a new SUSY restoring direction opens up along which f gets a VEV. One is then performing a rather delicate balancing act: in order to avoid disastrously fast decay of the metastable vacuum, large SUSY breaking scales must be invoked so that the R-breaking mass can be sufficiently small.

It should also be noted that here the R-symmetry breaking responsible for the globally supersymmetric minima of ISS models plays no direct role in the generation of gaugino masses, and consequently this is expected to be a generic problem for gauge mediation of metastable SUSY breaking. This is also a problem for the models that were constructed to implement direct mediation [130], and again, in those cases certain operators had to be forbidden by hand, making the superpotential non-generic.

To avoid these problems, the next option for generating non-zero gaugino masses would be to use the explicit R-breaking of the ISS model itself, associated with the metastability and the existence of a global supersymmetric groundstate. This is in fact a more difficult proposition than one might suppose for the following reason. At the metastable minimum there is an unbroken approximate R-symmetry (which is of course why it is metastable in the first place). The R-symmetry is explicitly (more precisely anomalously) broken only by the non-perturbative term,

$$W_{np} \propto (\det{}_{N_f}\Phi)^{\frac{1}{N}} \sim \Phi^{\frac{N_f}{N}} \ , \tag{5.1}$$

where Φ is the meson field, $\mathrm{SU}(N)_{\mathrm{mg}}$ is the gauge group of the magnetic theory, and $N = N_f - N_c$ with $\mathrm{SU}(N_c)$ being the gauge group of the electric theory [6]. One might hope that this would induce (for example) R-symmetry breaking mass-terms that contribute to gaugino masses in perturbation theory. However such mass-terms will be typically of order $\frac{\partial^2 W}{\partial \Phi^2} \sim \Phi^{\frac{2N_c - N_f}{N}}$. Thus since ISS models are valid in the interval $N_c + 1 \leq N_f < \frac{3}{2}N_c$, they are exactly zero in the metastable minimum where $\langle \Phi \rangle = 0$.

We are led to an alternative — the focus of this chapter — which is to *spontaneously* break the approximate R-symmetry of the ISS model to generate gaugino masses. The explicit breaking of the model then ensures that any R-axions get a mass and are made safe. The natural avenue to explore is to gauge (part of) the $\mathrm{SU}(N_f)$ flavour symmetry of the ISS model, identifying it with the Standard Model gauge groups. This would allow the quarks and mesons in the theory to mediate SUSY breaking directly to the Standard Model, thereby avoiding the need for any messenger sectors, which as we have seen are liable to destabilise the metastable vacuum. Once spontaneous R-breaking has been achieved, there is in principle nothing to prevent it being mediated via these fields to the rest of the model, allowing the generation of gaugino masses.[1] (Note that the

[1]Indeed the very same point was made in reference [128] which was presented in the language of retrofitting [127]. There however, successful mediation required a messenger sector which, in general, may lead to new and unstable directions.

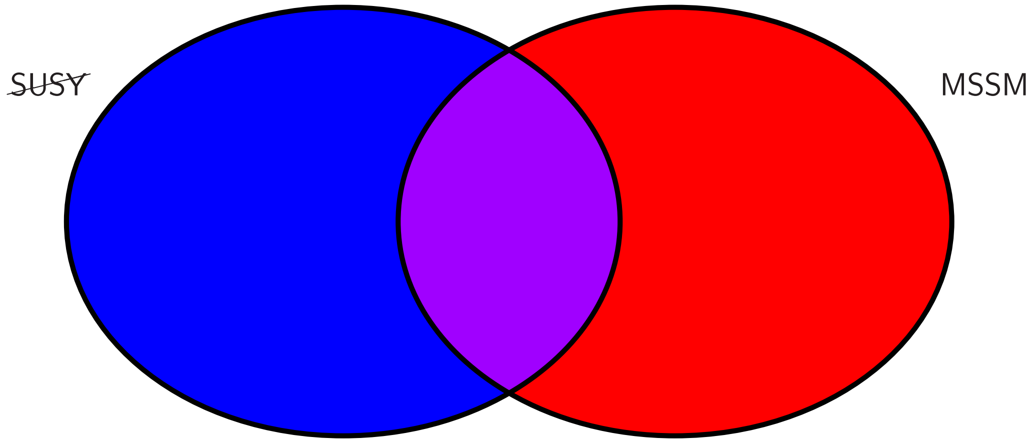


Figure 5.1.: Illustration of directly mediated supersymmetry breaking, indicating the overlap between SUSY breaking (blue) and MSSM (red) sectors.

non-perturbative explicit R-breaking can also now contribute to gaugino masses since Φ will get a VEV.)

5.1.1. A Return to Direct Mediation

The purported simplicity of metastable SUSY breaking models now compels us to reconsider the possibility of *direct* gauge mediation, whereby matter fields of the SUSY breaking sector carry charges under the gauge groups of the Standard Model and there is no need for a separate messenger sector. In ordinary gauge mediation, the details of SUSY breaking are generally 'hidden' from the Matter Sector, with the most important phenomenological features arising from the messenger particle content. To clarify the distinction, it may be helpful to contrast the representation of direct mediation models found in Figure 5.1 with the picture of standard mediation via messenger fields given in Figure 4.1. The elegance of direct gauge mediation models lies in their compactness and predictivity. Previously, direct mediation of metastable SUSY breaking was considered in this context in references [129] and [130].

In this chapter we demonstrate that perfectly viable direct mediation of SUSY breaking can indeed be implemented within a metastable framework. We show this by making the simplest deformation to the ISS model that one can imagine, namely the addition of a baryon term to the superpotential. This "baryon-deformed" QCD model has a runaway direction to a non-supersymmetric metastable minimum of the ISS type, along a particular direction of field space that is lifted by the Coleman-Weinberg potential and therefore stabilised. Along this direction the meson modes Φ_{ij} acquire a VEV, and the approximate R-symmetry is spontaneously broken. Importantly the diagonal (U(1)-trace) component of the pseudo-Goldstone modes (i.e. those modes of Φ_{ij} whose flavour indices correspond to a SM gauge group) acquires a VEV at this point as well; the latter gives R-breaking masses to the magnetic quarks that are charged under the SM gauge groups. This enables them to act as messenger fields giving the gauginos masses at one-loop. We stress that all of this happens automatically upon adding a baryon. There is no need for any messenger sector outside the ISS model, and therefore no additional instability is induced. Moreover, we will show that the resulting gaugino masses can be naturally of the right order.

5.2. The Baryon Deformed ISS Model

Let us begin by introducing our model, which is based on the ISS SUSY breaking model with $\mathrm{SU}(N_c)$ gauge symmetry and N_f flavours of quark/anti-quark pairs in the electric theory. As we saw in Section 4.2.1, the low energy dynamics can be understood in terms of a magnetic dual theory that has $\mathrm{SU}(N)_{\mathrm{mg}}$ gauge symmetry, where $N = N_f - N_c$, N_f flavours of fundamental quark/anti-quark pairs, φ_i, $\widetilde{\varphi}$, and a 'meson' field Φ that is a singlet under the gauge group. This theory is IR free if $N_c + 1 \leq N_f < \frac{3}{2}N_c$. The minimal values consistent with this equation and leading to a non-trivial magnetic gauge group are $N_f = 7$ and $N_c = 5$ giving $\mathrm{SU}(2)_{\mathrm{mg}}$ in the magnetic dual theory. Now consider the following superpotential:

$$W = \Phi_{ij}\,\varphi_i \cdot \widetilde{\varphi}_j - \mu^2_{ij}\Phi_{ji} + m\,\varepsilon_{ab}\varepsilon_{rs}\varphi^a_r\varphi^b_s\,, \tag{5.2}$$

where $i, j = 1, \dots, 7$ are flavour indices, $r, s = 1, 2$ run over the first two flavours only, and a, b are $\mathrm{SU}(2)_{\mathrm{mg}}$ indices (we set the coupling $h = 1$ for simplicity). This is the superpotential of ISS with the exception of the last term which is a baryon of $\mathrm{SU}(2)_{\mathrm{mg}}$. Note that the $1, 2$ flavour indices and the $3, \dots, 7$ indices have a different status. Consequently, the flavour symmetry is broken explicitly to $\mathrm{SU}(2)_f \times \mathrm{SU}(5)_f$. The $\mathrm{SU}(5)_f$ factor will be later be gauged and identified with the parent SU(5) of the Standard Model.[2]

The baryon deformation is the leading order deformation of the ISS model that is allowed by R-symmetry (as well as the gauge and flavor symmetries discussed above).

[2]Note that the breaking of SU(5) is assumed to take place or be included explicitly in the SM sector.

Terms quadratic in the mesons that could arise from lower dimensional irrelevant operators in the electric theory are forbidden by R-symmetry.

Using the $\mathrm{SU}(2)_f \times \mathrm{SU}(5)_f$ symmetry, the matrix μ^2_{ij} can be brought to a diagonal form

$$\mu^2_{ij} = \begin{pmatrix} \mu^2 \mathbb{1}_2 & 0 \\ 0 & \hat{\mu}^2 \mathbb{1}_5 \end{pmatrix} . \tag{5.3}$$

We will assume that $\mu^2 > \hat{\mu}^2$. The parameters μ^2, $\hat{\mu}^2$ and m have an interpretation in terms of the electric theory: $\mu^2 \sim \Lambda m_Q$ and $\hat{\mu}^2 \sim \Lambda \hat{m}_Q$ come from the electric quark masses m_Q, $\hat{m}_Q$, where Λ is the Landau pole of the theory.[3] The baryon operator can be identified with a corresponding operator in the electric theory. Indeed the mapping from baryons B_E in the electric theory to baryons b_M of the magnetic theory (neglecting factors of order one) is

$$b_M \, \Lambda^{-N} \quad \longleftrightarrow \quad B_E \, \Lambda^{-N_c} \; . \tag{5.4}$$

Thus one expects

$$m \sim M \left(\frac{\Lambda}{M} \right)^{2N_c - N_f} = \frac{\Lambda^3}{M^2} \; . \tag{5.5}$$

Here M represents the scale of new physics in the electric theory at which the irrelevant operator B_E is generated. We will think of it as being M_P or M_{GUT} although as we shall see a large range of values can be accommodated.

It is encouraging that this rather minimal choice of parameters allows us to identify an $\mathrm{SU}(5)_f$ flavour symmetry with the Standard Model gauge groups.[4] Thus the magnetic quarks φ, $\widetilde{\varphi}$ decompose into 4 singlets (which we will call ϕ, $\widetilde{\phi}$) plus 2 fundamentals of $\mathrm{SU}(5)_f$ (which we call ρ, $\widetilde{\rho}$), while the magnetic mesons Φ_{ij} decompose into 4 fundamentals of $\mathrm{SU}(5)_f$ (Z and $\widetilde{Z}$), an adjoint+trace singlet of $\mathrm{SU}(5)_f$ (X), plus 4 more singlets (Y). The complete breakdown of charges can be found in Table 5.1.

As we discussed in Section 4.2.1, it is known that the R-symmetry of ISS SQCD manifests itself only as an approximate symmetry of the magnetic formulation, which is

[3] We take the strong coupling scales in equation (2.60) equal $\Lambda_e = \Lambda_m \equiv \Lambda$ for simplicity.

[4] It is also an amusing coincidence that the electric theory has the same gauge groups for colour and flavour, $\mathrm{SU}(5)_f \times \mathrm{SU}(5)_c$.

	SU(2)	$\mathrm{SU}(2)_f$	$\mathrm{SU}(5)_f$	$\mathrm{U}(1)_R$
$\Phi_{ij} \equiv \begin{pmatrix} Y & Z \\ \widetilde{Z} & X \end{pmatrix}$	$\mathbf{1}$	$\begin{pmatrix} Adj+\mathbf{1} & \overline{\square} \\ \square & \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} & \square \\ \overline{\square} & Adj+\mathbf{1} \end{pmatrix}$	2
$\varphi \equiv \begin{pmatrix} \phi \\ \rho \end{pmatrix}$	$\square$	$\begin{pmatrix} \overline{\square} \\ \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} \\ \overline{\square} \end{pmatrix}$	1
$\widetilde{\varphi} \equiv \begin{pmatrix} \widetilde{\phi} \\ \widetilde{\rho} \end{pmatrix}$	$\overline{\square}$	$\begin{pmatrix} \square \\ \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} \\ \square \end{pmatrix}$	-1

Table 5.1.: We list matter fields and their decomposition under the gauge SU(2), the flavour $\mathrm{SU}(2)_f \times \mathrm{SU}(5)_f$ symmetry, and their charges under the R-symmetry of the model in (5.2).

broken explicitly in the electric theory by the mass terms of electric quarks m_Q. It is also broken anomalously, but this is already accounted for by the dynamical superpotential (4.12). In Appendix B we point out that the R-symmetry is broken in the electric theory in a controlled way by the small parameter, $m_Q/\Lambda = \mu^2/\Lambda^2 \ll 1$. As such the R-symmetry is preserved to that order in the superpotential.

Thanks to the baryon deformation, this model has R-charges that are not 0 or 2. As discussed in reference [136] this condition is necessary for Wess-Zumino models to spontaneously break R-symmetry. Therefore, our model allows for spontaneous R-symmetry breaking; we will see in the following that this does indeed happen. We also stress that our baryon deformation is the leading order deformation of the ISS model that is allowed by the R-symmetry of the full theory imposed at the Lagrangian level. As explained in Appendix B, this is a self-consistent approach since R-symmetry breaking in the electric theory is controlled by a small parameter. Terms quadratic in the meson Φ that could arise from lower dimensional irrelevant operators in the electric theory are forbidden by R-symmetry. Thus, our deformation is described by a *generic* superpotential and (5.2) gives its leading-order terms.

5.2.1. Locating the Metastable Vacuum

Let us consider the potential at tree-level. The F-term contribution to the potential at tree-level is

$$\begin{aligned} V_F \;=\; & \sum_{ar} \left| Y_{rs}\, \widetilde{\phi}^a_s + Z_{r\hat{\imath}}\, \widetilde{\rho}^{\,a}_{\hat{\imath}} + 2m\, \varepsilon_{ab}\varepsilon_{rs}\phi^b_s \right|^2 \\ & + \sum_{a\hat{\imath}} \left| \widetilde{Z}_{\hat{\imath}r}\, \widetilde{\phi}^a_r + X_{\hat{\imath}\hat{\jmath}}\, \widetilde{\rho}^{\,a}_{\hat{\jmath}} \right|^2 + \sum_{as} \left| \phi^a_r\, Y_{rs} + \rho^{\,a}_{\hat{\imath}}\, \widetilde{Z}_{\hat{\imath}s} \right|^2 + \sum_{a\hat{\jmath}} \left| \phi^a_r Z_{r\hat{\jmath}} + \rho^{\,a}_{\hat{\imath}} X_{\hat{\imath}\hat{\jmath}} \right|^2 \\ & + \sum_{rs} \left| \phi_r \cdot \widetilde{\phi}_s - \mu^2 \delta_{rs} \right|^2 + \sum_{r\hat{\imath}} \left| \phi_r \cdot \widetilde{\rho}_{\hat{\imath}} \right|^2 + \sum_{r\hat{\imath}} \left| \rho_{\hat{\imath}} \cdot \widetilde{\phi}_s \right|^2 + \sum_{\hat{\imath}\hat{\jmath}} \left| \rho_{\hat{\imath}} \cdot \widetilde{\rho}_{\hat{\jmath}} - \hat{\mu}^2 \delta_{\hat{\imath}\hat{\jmath}} \right|^2 , \end{aligned} \tag{5.6}$$

where a, b are $\mathrm{SU}(2)_{\mathrm{mg}}$ indices. The flavor indices r, s and $\hat{\imath}, \hat{\jmath}$ correspond to the $\mathrm{SU}(2)_f$ and $\mathrm{SU}(5)_f$, respectively. It is straightforward to see that the rank condition works as in ISS; that is the minimum for a given value of X, Y, Z and $\widetilde{Z}$ is along $\rho = \widetilde{\rho} = 0$ and

$$\langle \phi \rangle \;=\; \frac{\mu^2}{\xi}\, \mathbb{1}_2 \;, \tag{5.7a}$$

$$\langle \widetilde{\phi} \rangle \;=\; \xi\, \mathbb{1}_2 \;, \tag{5.7b}$$

where ξ parameterises a runaway direction that will eventually be stabilised by the Coleman-Weinberg contribution to the potential. This then gives

$$Z \;=\; \widetilde{Z} \;=\; 0 \;, \tag{5.8}$$

but the pseudo-Goldstone modes $X = \chi\, \mathbb{1}_5$ are undetermined. (Note that all the D-terms are zero along this direction and the $\mathrm{SU}(2)_{\mathrm{mg}}$ is Higgsed but $\mathrm{SU}(5)_f$ is unbroken.) In addition Y becomes diagonal and real (assuming m is real). Defining $\langle Y_{rs} \rangle = \eta\, \mathbb{1}_2$, the full potential is

$$V \;=\; 2\left| \eta\, \xi + 2m \frac{\mu^2}{\xi} \right|^2 + 2\left| \eta \frac{\mu^2}{\xi} \right|^2 + 5\hat{\mu}^4 \;. \tag{5.9}$$

Using R-symmetry we can choose ξ to be real.[5] Minimizing in η we find

$$\eta \;=\; -2m \left(\frac{\xi^2}{\mu^2} + \frac{\mu^2}{\xi^2} \right)^{-1} \;. \tag{5.10}$$

[5] The phase of ξ corresponds to the R-axion which will be dealt with later.

Substituting $\eta(\xi)$ into equation (5.9) we see that $\xi \to \infty$ is a runaway direction along which

$$V(\xi) \;=\; 8m^2\mu^2\left(\frac{\xi^6}{\mu^6}+\frac{\xi^2}{\mu^2}\right)^{-1} \;+\; 5\hat{\mu}^4\;. \tag{5.11}$$

It is worth emphasising that even in the limit $\xi \to \infty$, the scalar potential V is non-zero, so we have a runaway to *broken* SUSY (a 'pseudo-runaway' in the language of reference [138]). Proceeding to one loop, the Coleman-Weinberg contribution to the potential is therefore expected to lift and stabilise this direction at the same time as lifting the pseudo-Goldstone modes χ.

Let's see how this works. Firstly, recall that the Coleman-Weinberg effective potential [150] sums up all one-loop quantum corrections into the following form:

$$\begin{aligned} V^{(1)}_{\text{eff}} \;&=\; \frac{1}{64\pi^2}\,\text{STr}\;\mathcal{M}^4\log\frac{\mathcal{M}^2}{\Lambda^2_{UV}} \qquad (5.12)\\ &\equiv\; \frac{1}{64\pi^2}\left(\text{Tr}\;m_0^4\log\frac{m_0^2}{\Lambda^2_{UV}} - 2\;\text{Tr}\;m_{1/2}^4\log\frac{m_{1/2}^2}{\Lambda^2_{UV}} + 3\;\text{Tr}\;m_1^4\log\frac{m_1^2}{\Lambda^2_{UV}}\right)\;, \end{aligned}$$

where Λ_{UV} is the UV cutoff,[6] and the mass matrices are given by [151]:

$$m_0^2 \;=\; \begin{pmatrix} W^{ab}W_{bc} + D^{\alpha a}D^{\alpha}{}_{c} + D^{\alpha a}_{\;\;c}D^{\alpha} & W^{abc}W_b + D^{\alpha a}D^{\alpha c} \\ W_{abc}W^b + D^{\alpha}{}_{a}D^{\alpha}{}_{c} & W_{ab}W^{bc} + D^{\alpha}{}_{a}D^{\alpha c} + D^{\alpha c}_{\;\;a}D^{\alpha} \end{pmatrix} \tag{5.13}$$

$$m_{1/2}^2 \;=\; \begin{pmatrix} W^{ab}W_{bc} + 2D^{\alpha a}D^{\alpha}{}_{c} & -\sqrt{2}W^{ab}D^{\beta}{}_{b} \\ -\sqrt{2}D^{\alpha b}W_{bc} & 2D^{\alpha c}D^{\beta}{}_{c} \end{pmatrix} \qquad m_1^2 \;=\; D^{\alpha}{}_{a}D^{\beta a} + D^{\alpha a}D^{\beta}{}_{a}. \tag{5.14}$$

As usual, $W_c \equiv \partial W/\partial\Phi^c$ denotes a derivative of the superpotential with respect to the scalar component of the superfield Φ^c, and D^α are the appropriate D-terms, $D^\alpha = gz_aT^{\alpha a}_{\;\;b}z^b$. Of course, D-terms can be switched off by setting the gauge coupling g to zero, which we will do until further notice. All the above mass matrices will generally depend on field expectation values. The effective potential $V_{\text{eff}} = V_F + V^{(1)}_{\text{eff}}$ is the sum of the F-term (tree-level) and the Coleman-Weinberg contributions. To find the vacua

[6] As usual we can "eliminate" Λ_{UV} by trading it for a renormalisation scale at which the couplings are defined.

of the theory we now have to minimize V_{eff}. The true, stable vacuum will be the global minimum, with other minima being only meta-stable.

It is interesting to note that as the 1-loop corrections are of a supertrace form, they vanish around supersymmetric vacua. If the runaway was to a supersymmetric vacuum at infinity, the Coleman-Weinberg corrections wouldn't lift it. In our case, we have a runaway to a non-supersymmetric vacuum at infinity, so it is reasonable to expect that these loop corrections will modify the asymptotic behaviour.

5.2.2. Catching the Runaway Field

Now let's see how the classical runaway direction is lifted by quantum effects. We parameterise the pseudo-Goldstone and runaway field vacuum expectation values by

$$\langle\widetilde{\phi}\rangle = \xi\,\mathbb{1}_2 \qquad \langle\phi\rangle = \kappa\,\mathbb{1}_2\,, \tag{5.15a}$$

$$\langle Y\rangle = \eta\,\mathbb{1}_2 \qquad \langle X\rangle = \chi\,\mathbb{1}_5\,. \tag{5.15b}$$

These are the most general VEVs consistent with the tree level minimization. It can be checked that at one loop order all other field VEVs are zero in the lowest perturbative vacuum. By computing the masses of all fluctuations about this valley we can go about constructing the one-loop effective potential from equation (5.12). We have done this numerically using `Mathematica` as well as `Vscape`, a program specifically written to explore the properties of metastable vacua [152].

Table 5.2 shows the result of minimizing the VEVs in the one-loop effective potential for some sample values of the parameters. As expected, the VEVs in equations (5.15a) and (5.15b) are seen to approximately, i.e. up to small Coleman-Weinberg corrections, satisfy the analytic tree-level relations (5.7), (5.10)

$$\kappa = \frac{\mu^2}{\xi} \qquad \eta = -2m\left(\frac{\xi^2}{\mu^2}+\frac{\mu^2}{\xi^2}\right)^{-1}. \tag{5.16}$$

We have checked that this is indeed the case for a wide range of input parameters. Hence, in what follows, we impose the conditions above and only consider the two independent VEVs ξ and χ. After studying the phenomenology of this model in the next chapter, we will return to question the imposition of these tree-level constraints in an effort to refine the model in Section 6.4.1.

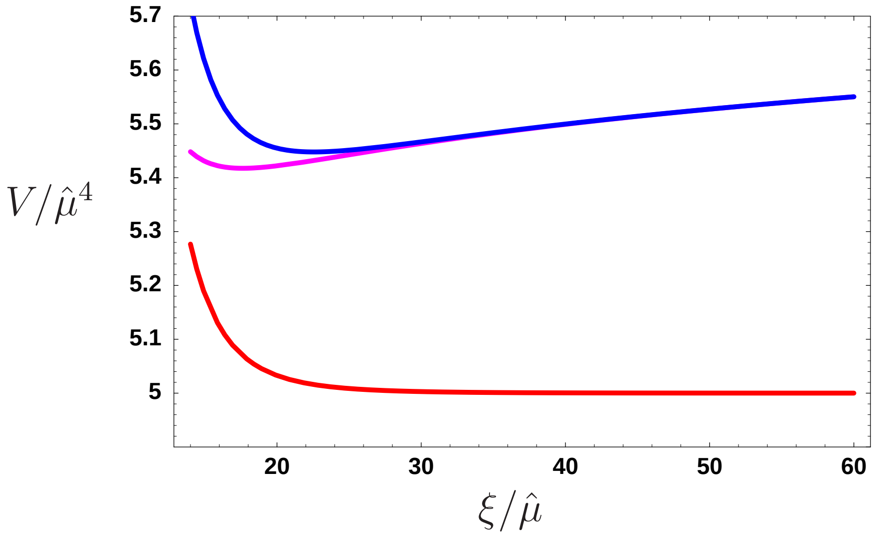


Figure 5.2.: This plot demonstrates the stabilisation of the runaway ξ direction. The red curve (bottom) is the tree-level runaway potential. The purple is the Coleman-Weinberg contribution (we have added a constant shift of 5 to it). The blue line (top) depicts the full stabilised potential. (We use $\mu = 4\hat{\mu}$, $m = 2\hat{\mu}$.)

Model	$\xi/\hat{\mu}$	$\kappa/\hat{\mu}$	$\eta/\hat{\mu}$	$\chi/\hat{\mu}$
Vscape Unconstrained	22.55451	0.709338	−0.125660	−1.00041
Vscape Constrained	22.55581	0.709352†	−0.125671†	−1.00132
Mathematica	22.5559	0.70935†	−0.12567†	−1.0014
Gauged $SU(2)_{mg}$, $g = 0.4$	22.4385	0.71306†	−0.12699†	−1.0115

Table 5.2.: Stabilised VEVs for different minimization models: $\mu = 4\hat{\mu}$, $m = 2\hat{\mu}$. The values† are obtained from the tree-level constraint equations (5.7) and (5.10).

A plot of the potential in the ξ direction, Figure 5.2, shows the Coleman-Weinberg terms do indeed stabilise the $\xi \to \infty$ runaway at finite, non-zero values of the fields. A contour plot in the $\xi - \chi$ plane, Figure 5.3, reveals that the pseudomodulus χ is also stabilised at a non-zero value $\mathcal{O}(\hat{\mu})$.

Thus, for a natural choice of parameters, all the VEVs ξ, κ, η and χ obtain stable, finite $\mathcal{O}(\hat{\mu})$ values. Notice that Φ, φ and $\widetilde{\varphi}$ all carry R-charge, so the R-symmetry of the model is spontaneously broken in this minimum.

Until now we have neglected the D-terms from $SU(2)_{mg}$ but, as we can see from Table 5.2, including them does not significantly alter the VEV-structure of the vacuum.

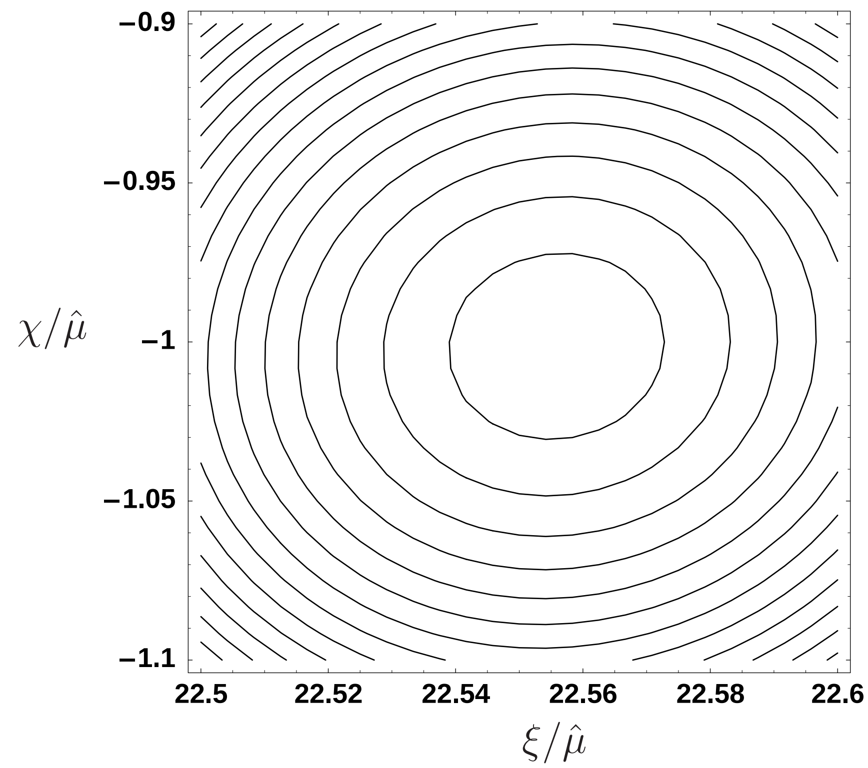


Figure 5.3.: This contour plot of the effective potential V_{eff} shows that the pseudo-modulus χ is also stabilised at a non-vanishing VEV. (We use $\mu = 4\hat{\mu}$, $m = 2\hat{\mu}$.)

What about the stability of this vacuum? When the gauge fields are turned on, this model has non-zero Witten index, so the global minimum will be supersymmetric. As in the ISS model, this minimum is induced by the non-perturbative contribution to the superpotential

$$W_{\text{np}} = 2\Lambda^3 \left[\det\left(\frac{\Phi}{\Lambda}\right) \right]^{\frac{1}{2}} . \tag{5.17}$$

Adapting the supersymmetric vacuum solution from the ISS model to our case with $\mu > \hat{\mu}$ we find

$$\varphi = 0 \, , \qquad \widetilde{\varphi} = 0 \, , \qquad \eta = \hat{\mu}^2 \mu^{-\frac{6}{5}} \Lambda^{\frac{1}{5}} \, , \qquad \chi = \mu^{\frac{4}{5}} \Lambda^{\frac{1}{5}} \, . \tag{5.18}$$

Note that the supersymmetric minimum lies at $\varphi = \widetilde{\varphi} = 0$ and is completely unaffected by the baryon deformation.

5.3. Preliminary Phenomenology

5.3.1. Communicating Breaking to the MSSM

So far we have established that supersymmetry is broken dynamically and R-symmetry can be broken spontaneously in the metastable vacuum of the ISS sector. We now need to transmit both these effects to the Standard Model. The most concise way to do this is to gauge the $\mathrm{SU}(5)_f$ flavour group and identify it with the parent gauge group the Standard Model. Since both supersymmetry and R-symmetry are broken,[7] gauginos do acquire a mass.

To discuss the general characteristics of our model it is useful to be *au fait* with the standard behaviour of gauge mediated models. It is usually assumed that the effects of Hidden Sector SUSY breaking can, to a first approximation, be accounted for by a SPURION chiral superfield X that acquires a VEV $\langle X \rangle = M + \theta^2 F_X$. This is taken to have a tree level superpotential coupling $W = f\widetilde{f}X$ to the messenger superfields f and $\widetilde{f}$ that carry charge under the Standard Model gauge groups (they transform in representations such that their product $f\widetilde{f}$ is invariant). The spurion VEV induces masses for the fermionic (ψ_f) and scalar (ϕ_f) components of the messengers:

$$m_{\psi_f} = M \, , \qquad\qquad m^2_{\phi_f} = M^2 \pm F \, . \tag{5.19}$$

Note that to avoid a tachyonic scalar in the messenger sector we must have $F/M^2 \leq 1$. Integrating out the messengers then induces soft gaugino (λ_A) and sfermion (scalar) masses in the MSSM, which *very roughly* take the form

$$m_{\lambda_A} \sim \frac{g_A^2}{16\pi^2}\frac{F}{M} \, , \tag{5.20a}$$

$$m^2_{\rm sc} \sim \sum_A \left(\frac{g_A^2}{16\pi^2}\right)^2 \frac{FF^\dagger}{MM^\dagger} \, . \tag{5.20b}$$

More precise one-loop formulae for the MSSM soft terms can of course be obtained, either via a slick renormalisation group argument [153] (valid for $\frac{F}{M^2} \ll 1$) or by directly calculating the Feyman diagrams indicated in figures 5.4 and 6.3. It is interesting to observe that the full calculation of references [154, 155], even in the most extreme regime

[7] In contrast to the ISS model, which only has small anomalous R-symmetry breaking, our model has in addition a rather large spontaneous R-symmetry breaking by the vacuum expectation value $\langle \chi \rangle$.

(where $F = M^2$) differs from the predictions of the simpler RG calculation by less than an order of magnitude.

In our model, gaugino masses are generated at one loop order, but there are various subtleties involved in ascertaining their correct size. To establish a ballpark figure, one might begin with an estimate along the lines of (5.20a), with the rôle of the spurion being played by the X components of the ISS meson. Assuming that the dominant effect comes from magnetic quarks, ρ and $\widetilde{\rho}$, propagating in the loop, as shown in Figure 5.4 and working to the leading order in SUSY breaking, i.e. to order F_χ, gaugino mass goes as

$$m_{\lambda_A}^{\text{naïve}} \sim \frac{g_A^2}{16\pi^2}\frac{\langle F_\chi\rangle}{\langle\chi\rangle} \sim \frac{g_A^2}{16\pi^2}\frac{\hat{\mu}^2}{\langle\chi\rangle} \sim \frac{g_A^2}{16\pi^2}\hat{\mu}\ . \tag{5.21}$$

For the last part of (5.21) we have assumed that all VEVs and mass parameters are of the same order $\mathcal{O}(\hat{\mu})$, so as not to introduce any large hierarchies by hand. One obvious regard in which this estimate is deficient can be seen by considering the scalar potential (5.6): we have not accounted for the mixing $\rho \leftrightarrow Z$ and $\widetilde{\rho} \leftrightarrow \widetilde{Z}$ between all fields that can propagate in the loop.

A more detailed calculation of the gaugino (and sfermion) masses will be given in Chapter 6 where, due to the non-diagonal form of the messenger mass matrices (6.25), (6.27), it will be most expedient to evaluate the appropriate expressions numerically. Borrowing the calculational method of Section 6.3.2, for the purposes of this chapter we will focus on generating the largest possible values for gaugino masses relative to the SUSY breaking scale $\hat{\mu}$.[8] We find that this occurs when $\mu \simeq \hat{\mu}$ (the reason for this will be elucidated in Section 6.3.2). For example, for $\mu = 1.1\,\hat{\mu}$ and $m = 0.3\,\hat{\mu}$ we have

$$m_{\lambda_A} \simeq \frac{g_A^2}{16\pi^2}\,0.0089\,\hat{\mu}\ , \tag{5.22}$$

where $A = 1, 2, 3$ labels the three gauge groups of the Standard Model. Requiring that all the gaugino masses are

$$m_{\lambda_A} \sim (0.1 - 1)\ \text{TeV}\ , \tag{5.23}$$

[8] We will keep the SUSY breaking scale $\hat{\mu}$ fixed and measure all other dimensionful parameters in units of $\hat{\mu}$. Then for $\hat{\mu} = 1$ there are only two independent input parameters, μ and m: the VEVs ξ, κ, η and χ that enter the messenger mass matrices are generated by minimizing the effective potential, as in Section 5.2.2.

we conclude that

$$\hat{\mu} \sim (10^4 - 10^5)\,\text{TeV} \tag{5.24}$$

at this point in the parameter space of our model.

To gain a qualitative understanding of the suppression (relative to our naïve estimate (5.21)) of gaugino masses found in equation (5.22), recall that to generate a soft gaugino mass both supersymmetry *and* R-symmetry must be broken. In our model, the order parameter for SUSY breaking is $\hat{\mu}$, whereas the degree of R-symmetry breaking is set by the scale m, and for the above parameter point in particular, $m < \hat{\mu}$. This suggests that the comparative smallness of gaugino masses can, in part, be attributed to the degree R-symmetry breaking. To test this hypothesis, one should calculate the sfermion soft masses: since scalars are not protected by R-symmetry, the generation of their masses is less constrained. Indeed, as long as supersymmetry is broken, we can have scalar masses even when R-symmetry is unbroken. Hence, we expect the appropriate two-loop diagrams (shown in Figure 6.3 and discussed, for example, in references [155, 156]) to give something closely approximating a naïve estimate for scalar masses derived along the lines of equation (5.20b). These heuristic expectations will be born out by the detailed calculations of Chapter 6, making it clear that R-symmetry breaking (together with the structure of the messenger mass matrices) plays a crucial role in suppressing gaugino masses.

It is interesting to contrast the behaviour of our model with the usual expectations of gauge mediation, typified in equation (5.20). In that case, clearly the scalar masses should be roughly similar to the gaugino masses $m_{\lambda_A} \sim m_{\text{sc}}$. Relating this to the argument of the previous paragraph, note that the scale of SUSY breaking is set by the F-term of the spurion $\sqrt{F_X}$, whereas R-symmetry need only be broken when the spurion's scalar component M gets a VEV. The crucial point then is that to avoid tachyonic scalars, the standard scenario comes with the requirement $F \leq M^2$. In words, this is telling us that the scale of R-symmetry breaking is always greater than the SUSY breaking scale, and so both the gaugino and sfermion soft masses are essentially controlled by the same scale ($\sqrt{F_X}$).

In our model therefore the scalars are always heavier than the gauginos. The phenomenology for this particular type of model is expected to be of the "heavy-scalar" type as reviewed in reference [157]. In the region $\hat{\mu} \simeq \mu \sim m$ their masses are only about two orders of magnitude larger than the gaugino masses, and a focus-point type of phe-

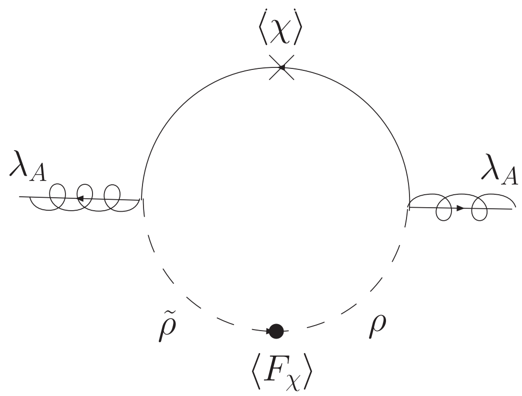


Figure 5.4.: One-loop contribution to the gaugino masses. The blob on the scalar line indicates an appropriate number of insertions of $\langle F_\chi \rangle$ to make the diagram non-vanishing.

nomenology [158] may be possible. Increasing μ and decreasing m takes us continuously to the split SUSY scenario [15, 16]. A more detailed phenomenological investigation will be carried out in Chapter 6.

Non-perturbative effects due to $W_{\rm np}$ are suppressed by the scale Λ of the Landau pole of the ISS sector, which we have not yet constrained. Choosing $\Lambda \gg \hat{\mu}$ so that the magnetic theory is weakly coupled and the metastable vacuum is long lived, the non-perturbative corrections to our discussion are small.

5.3.2. R-axions

Our model has a spontaneously broken R-symmetry that is explicitly broken only by the non-perturbative contribution $W_{\rm np}$ to the superpotential. In such a situation we generally expect a pseudo-Goldstone boson — the R-axion a_R. For more on this phenomenon, see the discussion in references [122, 148] and [159]. If such a particle is light it can have dangerous phenomenological consequences [160–162]. Since the R-symmetry is an axial symmetry, triangle diagrams typically couple the R-axion to gauge fields via a term (see, for example, reference [160])

$$\frac{\alpha}{2\pi f_R} a_R F^{\mu\nu} \widetilde{F}_{\mu\nu} \ , \qquad (5.25)$$

where $F^{\mu\nu}$ is a gauge field and f_R is the scale of spontaneous R-symmetry breaking. Particularly dangerous are the couplings of this type to gluons and photons. Moreover, there can exist couplings of the R-axion to matter fields. For small masses $m_{a_R} \lesssim 100\,\text{MeV}$ astrophysical considerations [161, 162] constrain the scale of spontaneous R-symmetry breaking to be

$$f_R \gtrsim \text{few} \times 10^7\,\text{GeV} \qquad \text{for} \qquad m_{a_R} \lesssim 100\,\text{MeV}. \tag{5.26}$$

Let us now estimate the mass of the R-axion in our model to check whether it is harmless. The R-axion is the phase of the fields that spontaneously break R-symmetry,

$$\eta = |\eta| \exp\left(2i\frac{a_R}{f_R}\right) , \qquad \chi = |\chi| \exp\left(2i\frac{a_R}{f_R}\right) , \tag{5.27}$$

where the 2 arises from the R-charge 2 of the Φ-field. The dominant contribution to spontaneous R-symmetry breaking comes from $\langle\eta\rangle$. This sets the scale

$$f_R \sim \langle\eta\rangle . \tag{5.28}$$

The R-axion mass arises[9] from explicit R-symmetry breaking due to the non-perturbative superpotential term W_{np}. More precisely, taking into account the contribution of W_{np} to the F_X-terms,

$$\begin{aligned} V_F \ni |F_X|^2 &\sim \left| \langle\eta\rangle\langle\chi\rangle^{\frac{3}{2}} \exp\left(5i\frac{a_R}{\langle\eta\rangle}\right)\Lambda^{-\frac{1}{2}} - \hat{\mu}^2 \right|^2 \\ &= \left[\langle\eta\rangle^2\langle\chi\rangle^3\Lambda^{-1} + \hat{\mu}^4 - 2\hat{\mu}^2\langle\eta\rangle\langle\chi\rangle^{\frac{3}{2}}\Lambda^{-\frac{1}{2}}\cos\left(5\frac{a_R}{\langle\eta\rangle}\right)\right] , \end{aligned} \tag{5.29}$$

the R-axion mass arises from the last term on the right hand side. For simplicity, we have chosen $\hat{\mu}$ and all the VEVs to be real. Expanding to second order in a_R we find the R-axion mass to be

$$m_{a_R}^2 \sim \hat{\mu}^2\langle\eta\rangle^{-1}\langle\chi\rangle^{\frac{3}{2}}\Lambda^{-\frac{1}{2}} . \tag{5.30}$$

For our values this turns out to be sufficiently heavy to easily avoid the astrophysical constraints for any $\Lambda < M_{\text{Pl}}$.

[9] Another contribution to the R-axion mass may come from supergravity. A constant term in the superpotential that cancels the cosmological constant also breaks R-symmetry explicitly [148].

5.4. Summary

The take-home message of this chapter is that direct mediation (i.e. mediation in which there is no separate messenger sector) is relatively simple to implement in ISS-like models. It can be achieved by inducing spontaneous breakdown of the approximate R-symmetry associated with the metastable minimum. This in turn allows us to generate gaugino masses alongside other soft SUSY breaking terms.

We presented a baryon-deformed ISS model in which spontaneous R-symmetry breaking occurs automatically due to the Coleman-Weinberg potential. Once the R-symmetry is broken, the magnetic quarks of the ISS sector are able to play the role of messengers by identifing an $\mathrm{SU}(5)_f$ subset of the flavour symmetry with parent SU(5) of the Standard Model gauge groups. The reward for constructing things in this way is a compact, calculable model with interesting low-energy physics. We will investigate the associated phenomenology in more detail in Chapter 6.

Landau Poles

We would like to end this chapter by commenting on a particular feature of our model, and indeed all direct mediation models based on embedding the Standard Model gauge groups into a flavor subgroup of the ISS sector. As already mentioned in references [6] and [130] this embedding adds a significant number of matter multiplets charged under the SM gauge groups. Above the mass thresholds of these fields this leads to all Standard Model gauge groups being not asymptotically free, and therefore to Landau poles in the SM sector. Since the additional fields are in SU(5) multiplets, the beta functions of the SM gauge couplings are modified universally. For example, in our model there is a shift

$$b_A = b_A^{(\mathrm{MSSM})} - 9 , \tag{5.31}$$

where the additional contributions are 2 from φ and $\widetilde{\varphi}$, and 7 from Φ. The SM gauge couplings at a scale $Q > \mu$ in our model are therefore related to the traditional MSSM ones as

$$\alpha_A^{-1} = (\alpha_A^{-1})^{(\mathrm{MSSM})} - \frac{9}{2\pi} \log(Q/\mu) , \tag{5.32}$$

where the fields φ, $\widetilde{\varphi}$ and Φ contribute to the running above the scale μ. The Landau pole $Q \equiv \Lambda^{(\mathrm{MSSM})}$ we will take to be situated where $g_A \sim 4\pi$ which corresponds roughly to

$$\frac{\Lambda^{(\mathrm{MSSM})}}{\mu} \sim 10^7 \; . \tag{5.33}$$

Values of $\mu \gtrsim 10^6\,\mathrm{TeV}$ would appear to be required in order to reach the conventional GUT scale in the MSSM sector before the Landau pole. However, it was recently pointed out in reference [163] that this estimate misses an important feature of Seiberg duality. Above the duality scale Λ_{ISS}, the magnetic quarks are composite degrees of freedom, and so no longer contribute to the running of the Standard Model gauge couplings. This reduction of the effective number of messengers modifies equation (5.32) to

$$\alpha_A^{-1} \;=\; (\alpha_A^{-1})^{(\mathrm{MSSM})} \;-\; \frac{9}{2\pi}\log(\Lambda_{\mathrm{ISS}}/\mu) \;-\; \frac{5}{2\pi}\log(Q/\Lambda_{\mathrm{ISS}}) \; , \tag{5.34}$$

which weakens the restriction on allowed values of μ, and makes it far easier for us to live with Landau poles. Even though preserving the longevity of the metastable vacuum requires $\Lambda_{\mathrm{ISS}} > 10^4\,\mu$, a rough estimate indicates the Standard Model can still be perturbative up to the GUT scale for $\mu \gtrsim 10^4\,\mathrm{TeV}$. This can easily be accommodated within our direct mediation model (cf. equation (5.24)). To describe this phenomenon the authors of [163] coined the phrase DEFLECTED UNIFICATION.

We would now like to suggest that the change of sign in the slopes of the Standard Model gauge couplings and the very existence of Landau poles is an interesting feature rather than an distasteful problem. The presence of Landau poles in all sectors of theory indicates that we should interpret not only the ISS sector as a magnetic dual of an asymptotically free theory, but also apply the same reasoning to the Standard Model itself. In other words, at energy scales above $\hat{\mu}$ the Standard Model sector and the ISS sector are not decoupled from each other and, in general, should be treated as part of the same theory. We already know that the UV completion of the ISS sector is its electric Seiberg dual and we propose the whole theory has such a UV completion. This seems to be a rather symmetric construction. One consequence of this interpretation is, of course, that gauge unification is lost, or at least buried in the unknown details of the dual theory. We will expand on the possibilities of this scenario in Section 6.5.2.

Chapter 6.

Patterns of Gauge Mediation

"To understand is to perceive patterns."
— Isaiah Berlin

In Chapter 5 we saw that direct mediation of SUSY breaking (i.e. mediation in which there is no separate messenger sector) is relatively simple to implement in ISS-like models. One can induce spontaneous breakdown of the approximate R-symmetry associated to the metastable minimum, which in turn allows the generation of gaugino masses alongside other soft SUSY breaking terms. With such a model to hand there are numerous interesting avenues available to explore.

One important venture, upon which we embark in this chapter, is to calculate how the consequences of such direct mediation models impact on experimental data — both the array of existing results, and the eagerly anticipated output of the Large Hadron Collider. We will solve the renormalisation group evolution of various metastable SUSY breaking models (coupled to the MSSM) to derive their sparticle mass spectrum at modern collider energies. By identifying distinguishing features of each spectrum we can compare the behaviour of metastable models to the phenomenology of other SUSY breaking mechanisms. In this way we should be in a position to say something about the nature of supersymmetry breaking in the Universe when data from the LHC begins to reshape the landscape of particle physics models.

6.1. The Phenomenology of Metastable Gauge Mediation

In the current paradigm of SUSY breaking, as presented in Section 4.1.1, supersymmetry is dynamically broken in a Hidden Sector of the full theory and the effects of this are mediated to the Visible Sector (MSSM) by so-called messenger fields. In the usual formulation, one essentially ignores the Hidden Sector theory and subsumes its details into a few parameters; the scale $M_{\rm SUSY}$ at which SUSY is broken in the Hidden Sector, the nature of mediation (gravity, gauge, extra dimensions, etc.) and the types of messenger fields. Thus it is tempting to assume that the details of the Hidden Sector are largely irrelevant to Visible Sector phenomenology, and that the entire pattern of the SUSY breaking soft terms in the MSSM is generated and determined by the messengers. The recent breakthrough made by Intriligator, Seiberg and Shih [6] in realising the dynamical SUSY breaking (DSB) via metastable vacua, provides a very minimal and simple class of candidates for the Hidden Sector, and makes it natural to reexamine this assumption. In particular one might ask: is it possible to distinguish different types of Hidden Sector physics for a given type of mediation and messenger?

We shall address this question in the context of models with low scale SUSY breaking, i.e. gauge mediation (GMSB). In this case, the usual insensitivity of Visible Sector physics to the behaviour of the Hidden Sector gives us pause for thought. We saw in Section 4.2.2 that metastability was all-but-unavoidable in low scale SUSY breaking scenarios. It is therefore reasonable to wonder whether this generic prediction of a metastable vacuum is reflected in the Visible Sector physics we observe. As the very definition of a *Hidden Sector* may suggest, any such correlation within the standard framework will be quite subtle. However, as we will see, the new model building possibilities afforded by embracing metastable vacua can deliver more distinctive phenomenology.

The main advantage of gauge mediation from a phenomenological point of view is the automatic disposal of the flavour problem that plagues gravity mediation. In GMSB the messenger fields interact only with the gauge field supermultiplets in the MSSM and the gauge interactions do not generate unwanted flavour changing soft terms in the MSSM. The sfermion soft masses are universal in flavour space and the only source of flavour violation is through the Yukawa matrices, which is already incorporated correctly into the Standard Model. Furthermore, the SUSY scale in GMSB is relatively low, $M_{\rm SUSY} \ll \sqrt{m_W M_{\rm Pl}}$, and one can determine the field theory in its entirety without appealing to the uncalculable details of an underlying supergravity theory, as one must in gravity mediation. Indeed, the recent realisation [6] that the dynamical breaking of supersymmetry can be achieved easily in ordinary SQCD-like gauge theories implies that now one can formulate complete and calculable models of gauge mediated SUSY breaking including the Hidden (and Visible) sectors. The goal of this chapter is to study and classify these models, and to show how the generic patterns of SUSY breaking generated in the MSSM depend on the details of the Hidden Sector.

To anticipate our findings, Visible Sector phenomenology depends essentially on how R-symmetry is broken in the Hidden Sector. Explicit R-symmetry breaking models such as can be found in references [12, 13] lead to fairly standard gauge mediation, but we argue that in the context of ISS-type models this only makes sense if $B_\mu = 0$ at the mediation scale, which leads to high $\tan\beta$. If, on the other hand, R-symmetry is broken spontaneously, as in the model of Chapter 5, then R-symmetry violating operators in the MSSM sector (e.g. gaugino masses) tend to be suppressed with respect to R-symmetry preserving ones (e.g. scalar masses), and one is led to a scenario with large scalar masses (and of course more fine-tuning). In the limit of small R-symmetry breaking we recover so-called split SUSY models [15–17]. We will also produce benchmark points (mass spectra) for both scenarios in Section 6.4.

Other recent investigations of metastable SUSY breaking applied to model building include references [129, 130, 135, 138, 142, 164–166].

6.2. Gauge Mediation with Explicit R-breaking

Let us start by considering the gauge mediation models of references [12, 13]. These are working models of metastable SUSY breaking with a messenger sector that explicitly breaks the R-symmetry of the ISS sector. The general philosophy is to appeal to details of the messengers' couplings to the ISS electric theory to explain why the explicit R-symmetry breaking is so weak in the effective theory. The net result is that one only breaks the R-symmetry by operators suppressed by powers of M_{Pl}. Various different constructions of such models are surveyed in reference [133].

Although the phenomenology is expected to broadly follow that of the gauge mediation paradigm [14], there is a difference. We will argue that, in the present context the Higgs bilinear B_μ parameter of the MSSM (the SUSY breaking counterpart of

$\mu_{\text{MSSM}} H_u H_d$) naturally vanishes at the mediation scale.[1] This is because R-symmetry breaking operators are (by assertion) suppressed by powers of M_{Pl} and this restricts the possibilities for generating the B_μ parameter: it is either many orders of magnitude too large or forbidden by symmetries to be zero.

We begin by recapping reference [12] and considering this issue in detail, before presenting the SUSY breaking phenomenology. An example benchmark point exhibiting typical sparticle masses follows in Section 6.4. The model augments the original ISS model with a pair of messengers "quarks" charged under the SM gauge group denoted f and $\widetilde{f}$ of mass M_f. For simplicity we shall assume that they transform as a fundamental and antifundamental respectively of the parent SU(5) of the SM. It was proposed that these couple maximally to the electric theory via a piece of the form

$$W_R = \frac{\lambda}{M_{\text{Pl}}}(\widetilde{Q}Q)(\widetilde{f}f) + M_f \widetilde{f}f \,, \tag{6.1}$$

where M_{Pl} is the scale of new physics at which the operator is generated, hereafter assumed to be the Planck scale. For simplicity, in this discussion we shall consider both μ^2 and λ to be flavour independent couplings. The essential observation of reference [12] is that, in the magnetic theory, this appears as an extremely weak violation of R-symmetry due to the large energy scale at which the operator is generated

$$W_R = \lambda' \Phi \widetilde{f}f + M_f \widetilde{f}f \equiv S_{\text{mess}} \widetilde{f}f \,, \tag{6.2}$$

where we introduced spurion superfield S_{mess} as in the standard gauge mediation set-up (cf. Section 4.1.1). By assumption the high energy scale M_{Pl} is much larger than Λ so that

$$\lambda' = \frac{\lambda\Lambda}{M_{\text{Pl}}} \ll 1 \,. \tag{6.3}$$

Since the R-symmetry is not respected by W_R the Nelson-Seiberg theorem [122] necessarily leads to the appearance of a new SUSY-preserving vacuum, but as long as λ' is small enough, the transition rate from $|\text{vac}\rangle_+$ to this new vacuum is suppressed and the original ISS picture is unchanged. Indeed the meson Φ field can remain trapped in $|\text{vac}\rangle_+$ near the origin, with the effective messenger F-term and scalar VEVs of the

[1]From now on we denote the SUSY preserving biliner term of the MSSM by μ_{MSSM}, reserving μ and $\hat{\mu}$ for parameters of ISS model.

spurion superfield given by

$$\langle F_{\text{mess}} \rangle \equiv \lambda' \langle F_\Phi \rangle = \lambda' \mu^2 , \tag{6.4a}$$

$$\langle S_{\text{mess}} \rangle \equiv \lambda' \langle \Phi \rangle + M_f \approx M_f . \tag{6.4b}$$

As in usual gauge mediation, a gaugino mass is induced at one loop, and is of order

$$m_\lambda \sim \frac{g^2}{16\pi^2} \frac{\langle F_{\text{mess}} \rangle}{\langle S_{\text{mess}} \rangle} \sim \frac{g^2}{16\pi^2} \frac{\lambda' \mu^2}{M_f} , \tag{6.5}$$

whereas a scalar mass-squared of the same order is induced at two loops

$$m_{\tilde{q}}^2 \sim m_\lambda^2 . \tag{6.6}$$

As we discussed at the end of Section 5.3.1, this last equation is a consequence of the fact that R-symmetry breaking, which controls gaugino masses, is linked to (i.e. not much smaller than) the SUSY breaking scale of the Visible Sector.

There is a new global minimum where the rank condition (4.13) is satisfied and the μ^2-ISS term is cancelled in the ISS potential

$$\begin{aligned} \langle \tilde{f} f \rangle &= \frac{\mu^2}{\lambda'} , \\ \langle \Phi \rangle &= \frac{M_f}{\lambda'} , \end{aligned} \tag{6.7}$$

however for small enough λ' these minima can be much further from the origin than Λ, beyond which all that one can say is there will be a global minimum of order $\langle \Phi \rangle \sim \Lambda$. Such far-flung minima do not change the ISS picture of metastability, and this is why the weakness of λ' is welcome. The resulting bound is $M_f \gtrsim \lambda' \mu$ [12]. Coupled with the gaugino mass being of order m_W, we find only very weak bounds:

$$\mu \gtrsim 16\pi^2 m_W . \tag{6.8}$$

There are a number of additional constraints, two of the most important being that the messengers f, $\tilde{f}$ are non-tachyonic, which gives

$$M_f^2 > \lambda' \mu^2 ,$$

and that gravity mediation effects are subdominant to the gauge mediation contribution, leading to

$$\frac{M_f}{M_{\rm Pl}} \lesssim 10^{-4}\lambda' \ .$$

Further constraints come from the possibility of additional operators such as

$$\delta W_{\rm mag} = \frac{1}{2}\frac{\Phi^2}{M_{\rm Pl}} \ ,$$

which are now allowed in the superpotential, however all of these can be easily satisfied for high values of Λ.

6.2.1. Forbidden Operators and $B_\mu = 0$

If one considers the MSSM sector as well, then there are further Planck-suppressed R-symmetry breaking operators that somehow had to be forbidden in references [12, 13]. Normally in gauge mediation one is justified in neglecting gravitationally induced operators altogether, however as we have seen, in these models the leading Planck-suppressed operator plays a pivotal rôle. Hence it is important to determine what effect other Planck-suppressed operators may have. The most important conclusion of this discussion will be that phenomenological consistency requires $B_\mu \approx 0$ at the mediation scale.

Before considering the operators in question, it is worth recalling the problem with B_μ in usual gauge mediation, in which supersymmetry breaking is described by a Hidden sector spurion superfield $S_{\rm mess}$. As we discussed in Section 4.1.1, the problem arises when one tries to generate the $\mu_{\rm MSSM} H_u H_d$ term of the MSSM (see, also, reference [167] for a recent review). Consider generating $\mu_{\rm MSSM}$ directly in the superpotential. There are two possibilities, either the parameter $\mu_{\rm MSSM}$ depends on $\langle S_{\rm mess}\rangle$ in which case a B_μ-term is generated, or it does not, in which case $B_\mu = 0$. Let us suppose that it does, and that the superpotential contains $W \supset \mu_{\rm MSSM}\left(S_{\rm mess}\right) H_u H_d$. The B_μ term is given by

$$\frac{B_\mu}{\mu_{\rm MSSM}} = \frac{\mu_{\rm MSSM}{}'}{\mu_{\rm MSSM}} F_{\rm mess} \sim \frac{F_{\rm mess}}{S_{\rm mess}}, \tag{6.9}$$

where $\mu_{\rm MSSM}{}' = \frac{d\mu_{\rm MSSM}}{dS_{\rm mess}}$ and the final relation follows from a dimensional analysis. This should be compared with the SUSY breaking contribution to the gaugino masses, which appears at one loop

$$m_\lambda \sim \frac{g^2}{16\pi^2}\frac{F_{\rm mess}}{S_{\rm mess}} ,$$

so that

$$\frac{B_\mu}{\mu_{\rm MSSM}} \sim \frac{(16\pi^2)}{g^2} m_\lambda . \tag{6.10}$$

Hence one finds that B_μ is two orders of magnitude too large. More generally because the $\mu_{\rm MSSM}$ and B_μ terms are both forbidden by a Peccei-Quinn symmetry, they tend to be generated at the same order, whereas B_μ should have an additional loop suppression (in order to be comparable to the scalar mass-squareds). One can then assume that $\mu_{\rm MSSM}$ is independent of $S_{\rm mess}$ in which case $B_\mu = 0$, or try to find a more sophisticated dynamical reason that the B_μ term receives loop suppression factors.

Now let us turn to the models of references [12, 13]. Here the situation is rather more pronounced for the very same reason that the R-symmetry breaking is under control, namely that the spurion is related to a meson of the electric theory. The $\mu_{\rm MSSM}$ term will be a function of

$$\frac{Q\widetilde{Q}}{M_{\rm Pl}} = \frac{\Lambda\Phi}{M_{\rm Pl}}, \tag{6.11}$$

and will be dominated by the leading terms in the $\frac{1}{M_{\rm Pl}}$ expansion. The leading operators involving $H_u H_d$ we can consider are

$$W \supset \mu_0\, H_u H_d + \frac{\lambda_2}{M_{\rm Pl}} H_u H_d\, \widetilde{f} f + \frac{\lambda_3}{M_{\rm Pl}} H_u H_d\, Q\widetilde{Q} , \tag{6.12a}$$

$$K \supset \lambda_4 \frac{(Q\widetilde{Q})^\dagger H_u H_d}{M_{\rm Pl}^2} + \text{h.c.} . \tag{6.12b}$$

where $\lambda_{2,3,4} \sim 1$. For generality we will allow a μ_0 term, which is consistent with R-symmetry in the renormalizable theory; this represents supersymmetric contributions to the $\mu_{\rm MSSM}$-term that do not involve the ISS sector. (It would of course be inconsistent to allow further SUSY breaking in the non-ISS sector.) The remaining R-violating operators we will take to be Planck suppressed as prescribed in references [12, 13].

Unfortunately it is clear that the Kähler potential term cannot be responsible for the $\mu_{\rm MSSM}$-term (as it could in references [119, 168]). Its contribution is of order

$$\mu_{\rm MSSM} \sim \frac{\Lambda}{M_{\rm Pl}^2}\,\mu^2 \ , \tag{6.13}$$

but we require $\mu^2 \ll M_{\rm Pl}\, m_W$ for gauge-mediation to be dominant, which would imply $\mu_{\rm MSSM} \ll \frac{\Lambda}{M_{\rm Pl}}\, m_W$.

Similar considerations apply to operators in the Kähler potential with factors of $D^2[\Phi^\dagger\Phi]$ as in reference [169]. The D_α appearing here is the superderivative introduced in Section 2.2.1. D^2 acting on anything is automatically antichiral, so terms of the form $\int d^4\theta\, H_u H_d\, D^2[\Phi^\dagger\Phi]$ can only generate a $\mu_{\rm MSSM}$ term, and not B_μ at leading order.

Turning instead to the leading superpotential terms, and assuming the messengers remain VEVless, one has

$$\mu_{\rm MSSM} \;=\; \mu_0 \,+\, \lambda_3\, \frac{\Lambda}{M_{\rm Pl}} \langle\Phi\rangle \sim \mu_0 \,+\, \lambda_3\, 16\pi^2 \frac{\Lambda^3}{M_{\rm Pl}^2} \ , \tag{6.14a}$$

$$B_\mu \;=\; \lambda_3\, \frac{\Lambda}{M_{\rm Pl}}\, \mu^2 \ , \tag{6.14b}$$

$$m_{\rm Higgs}^2 \sim \frac{g^4}{(16\pi^2)^2}\, \frac{\Lambda^2}{M_{\rm Pl}^2 M_f^2}\, \mu^4 \ , \tag{6.14c}$$

$$m_\lambda \sim \frac{g^2}{16\pi^2}\, \frac{\Lambda}{M_{\rm Pl}}\, \frac{\mu^2}{M_f} \ , \tag{6.14d}$$

where we used the fact that the Φ field is only expected to get a small VEV due to the presence of R-symmetry breaking operators. This was estimated in reference [12] to be

$$\langle\Phi\rangle \sim 16\pi^2 \frac{\Lambda^2}{M_{\rm Pl}} \ .$$

Combining the estimates in (6.14) one has

$$B_\mu \sim \frac{16\pi^2}{g^2} \lambda_3\, m_\lambda M_f \ .$$

Typically, the messenger mass M_f has to be orders of magnitude above m_W, so the situation is considerably worse than in usual gauge mediation unless a symmetry forbids the λ_3 coupling. A global R-symmetry would not be respected by gravitationally suppressed operators, however it *is* possible that particular operators can be suppressed.

If, for example, $\mu_{\rm MSSM}$ is charged under an additional gauge symmetry then one might expect

$$\lambda_2 \sim \lambda_3 \sim \frac{\mu_{\rm MSSM}}{M_{\rm Pl}} \ ,$$

in which case the effect of these operators is utterly negligible and we effectively have

$$\mu_{\rm MSSM} \approx \mu_0 \ , \qquad (6.15a)$$

$$B_\mu \approx 0 \ . \qquad (6.15b)$$

Note the importance of the interpretation of the effective ISS theory as a magnetic dual in this discussion. For example one could also have considered the effective operator

$$W_{R/\rm MSSM} = \frac{\lambda_4}{M_{\rm Pl}} H_u H_d \,{\rm Tr}\left(\widetilde{\varphi}\cdot\varphi\right) \ . \qquad (6.16)$$

This would have given $\mu_{\rm MSSM} \sim \frac{\mu^2}{M_{\rm Pl}}$ similar to the Giudice-Masiero mechanism [119], above. However, because the magnetic quarks φ and $\widetilde{\varphi}$ are composite objects, the coupling λ_4 will be suppressed by many powers of $\Lambda/M_{\rm Pl}$, so this contribution to $\mu_{\rm MSSM}$ would always be negligible.

In conclusion, by surveying the options available within the framework of references [12, 13] we see that the only phenomenologically viable possibility is to have $B_\mu = 0$ at the messenger mass scale.

6.3. Gauge Mediation with Spontaneous R-symmetry Breaking

In search of new, simple implementations of supersymmetry breaking, Chapter 5 saw us reconsider *direct* gauge mediation in the light of metastable model building. The essential difference between the direct gauge mediation of SUSY breaking and models with explicit messengers, such as references [12, 13] and Section 6.2 above, is that the 'direct messengers' form an integral part of the Hidden ISS sector. As such, their interactions with the SUSY breaking VEVs are not suppressed by inverse powers of $M_{\rm Pl}$. This means that the R-symmetry of the SUSY breaking sector (required by the existence of the SUSY breaking vacuum) cannot be an accidental symmetry that is only violated in the full theory by $1/M_{\rm Pl}$ corrections, as in [12, 13]. On the other hand, any large explicit violations of R-symmetry in the full theory will necessarily destabilise the SUSY breaking metastable vacuum.

Thus, it was proposed in Section 5.1 that the R-symmetry must be *spontaneously* broken by radiative corrections arising from the Coleman-Weinberg potential. In this case the Nelson-Seiberg theorem does not force upon us a nearby supersymmetric vacuum and at the same time non-zero gaugino masses can be generated since the R-symmetry is broken.

We will show below that in this approach the direct gauge mediation scenarios give phenomenology quite distinct from the usual gauge mediation scenarios [14].

6.3.1. The Baryon-deformed ISS Model

We will study a particular instance of the SUSY breaking model developed in Section 5.2. In particular, we take an ISS model with $N_c = 5$ colours and $N_f = 7$ flavours, which has a magnetic dual description as an SU(2) theory, also with $N_f = 7$ flavours. These are the minimal allowed values of N_c and N_f that still lead to a non-trivial magnetic gauge group — in this case $\mathrm{SU}(2)_{\rm mg}$. As we saw above, interesting things happen when we deform this theory by the addition of a baryonic operator.

For the reader's convenience, and to fix notation, we now outline some salient features of the model. As it is our goal to find the low energy spectrum, we will almost exclusively be working in the magnetic picture. In terms of these variables the superpotential of the theory is given by

$$W \;=\; \Phi_{ij}\,\varphi_i\cdot\widetilde{\varphi}_j \;-\; \mu_{ij}^2\Phi_{ji} \;+\; m\,\varepsilon_{ab}\varepsilon_{rs}\varphi_r^a\varphi_s^b\;, \tag{6.17}$$

where $i, j = 1, \ldots, 7$ are flavour indices, $r, s = 1, 2$ run over the first two flavours only, and a, b are $\mathrm{SU}(2)_{\rm mg}$ indices. As the baryon deformation (controlled by parameter m) singles out the $1, 2$ flavour indices to be treated differently from the $3, \ldots, 7$ indices, the flavour symmetry is explicitly broken concomitantly to $\mathrm{SU}(2)_f \times \mathrm{SU}(5)_f$. The $\mathrm{SU}(5)_f$ factor is gauged separately and will now be identified with the parent SU(5) of the Standard Model.The decomposition of matter fields under the magnetic $\mathrm{SU}(2)_{\rm mg} \times \mathrm{SU}(5)_f \times \mathrm{SU}(2)_f$ is given in Table 6.1, along with the associated $\mathrm{U}(1)_R$ charges.

	$SU(2)_{mg}$	$SU(2)_f$	$SU(5)_f$	$U(1)_R$
$\Phi_{ij} \equiv \begin{pmatrix} Y & Z \\ \widetilde{Z} & X \end{pmatrix}$	$\mathbf{1}$	$\begin{pmatrix} Adj+\mathbf{1} & \overline{\square} \\ \square & \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} & \square \\ \overline{\square} & Adj+\mathbf{1} \end{pmatrix}$	2
$\varphi \equiv \begin{pmatrix} \phi \\ \rho \end{pmatrix}$	$\square$	$\begin{pmatrix} \overline{\square} \\ \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} \\ \overline{\square} \end{pmatrix}$	1
$\widetilde{\varphi} \equiv \begin{pmatrix} \widetilde{\phi} \\ \widetilde{\rho} \end{pmatrix}$	$\overline{\square}$	$\begin{pmatrix} \square \\ \mathbf{1} \end{pmatrix}$	$\begin{pmatrix} \mathbf{1} \\ \square \end{pmatrix}$	-1

Table 6.1.: We list matter fields of the model (6.17), their decomposition under the gauge SU(2) and flavour $SU(2)_f \times SU(5)_f$ symmetries, and their charges under the R-symmetry.

Using the $SU(2)_f \times SU(5)_f$ symmetry, the matrix μ^2_{ij} can be brought to a diagonal form

$$\mu^2_{ij} = \begin{pmatrix} \mu^2 \mathbb{1}_2 & 0 \\ 0 & \hat{\mu}^2 \mathbb{1}_5 \end{pmatrix} . \tag{6.18}$$

We will assume that $\mu^2 > \hat{\mu}^2$. The parameters μ^2, $\hat{\mu}^2$ and m have an interpretation in terms of the electric theory: $\mu^2 \sim \Lambda m_Q$ and $\hat{\mu}^2 \sim \Lambda \hat{m}_Q$ come from the electric quark masses m_Q, $\hat{m}_Q$, where Λ is the Landau pole of the theory. The baryon operator can be identified with a corresponding operator in the electric theory. Indeed the mapping from baryons B_E in the electric theory to baryons b_M of the magnetic theory (neglecting factors of order one) is

$$b_M \, \Lambda^{-N} \quad \longleftrightarrow \quad B_E \, \Lambda^{-N_c} \, . \tag{6.19}$$

Thus one expects

$$m \sim M \left(\frac{\Lambda}{M} \right)^{2N_c - N_f} = \frac{\Lambda^3}{M^2} \, . \tag{6.20}$$

Here M represents the scale of new physics in the electric theory at which the irrelevant operator B_E is generated. We will think of it as being M_P or M_{GUT} although as we shall see a large range of values can be accommodated.

As explained in Section 5.2.1, this theory has a classical runaway direction $\langle\tilde{\varphi}\rangle \to \infty$ (with $\langle\tilde{\varphi}\rangle\langle\varphi\rangle$ fixed) to a non-supersymmetric vacuum. The quantum dynamics, namely the one-loop Coleman-Weinberg potential [150], stabilises the runaway at a point which breaks both supersymmetry and R-symmetry, thus creating a meta-stable vacuum state. We parameterise the pseudo-Goldstone and runaway VEVs by

$$\langle\tilde{\varphi}\rangle = \xi\,\mathbb{1}_2 \qquad \langle\phi\rangle = \kappa\,\mathbb{1}_2 \tag{6.21a}$$

$$\langle Y\rangle = \eta\,\mathbb{1}_2 \qquad \langle X\rangle = \chi\,\mathbb{1}_5. \tag{6.21b}$$

These are the most general VEVs consistent with the tree level minimization. It can be checked that at one loop order all other field VEVs are zero in the lowest perturbative vacuum. By computing the masses of all fluctuations about this valley we can go about constructing the one-loop effective potential. We have done this numerically using the `Vscape` program of reference [152]. Table 6.2 shows the VEVs stabilised by the one loop effective potential at a selection of points relevant to this chapter.

μ	m	ξ	κ	η	χ
10	0.3	41.0523	2.43592	−0.035477	−1.761261
1.1	0.3	2.1370	0.566214	−0.148546	−0.083296
1.01	0.3	1.8995	0.537043	−0.155796	−0.073474
1.003	0.3	1.8809	0.534848	−0.157752	−0.072738

Table 6.2.: Stabilised VEVs from `Vscape` [152] for various parameter points. All values are given in units of $\hat{\mu}$.

To summarise, we have identified a SUSY breaking vacuum of the deformed ISS model, which also breaks R-symmetry spontaneously via radiative corrections. This is a long-lived metastable vacuum. The only SUSY-preserving vacua of this model are those generated by the non-perturbative suprepotential

$$W_{\text{np}} = 2\Lambda^3 \left[\det\left(\frac{\Phi}{\Lambda}\right)\right]^{\frac{1}{2}} . \tag{6.22}$$

Adapting the supersymmetric vacuum solution from the ISS model to our case with $\mu > \hat{\mu}$ we find

$$\varphi = 0\,, \qquad \tilde{\varphi} = 0\,, \qquad \eta = \hat{\mu}^2\mu^{-\frac{6}{5}}\Lambda^{\frac{1}{5}}\,, \qquad \chi = \mu^{\frac{4}{5}}\Lambda^{\frac{1}{5}}\,. \tag{6.23}$$

Note that the supersymmetric minimum lies at $\varphi = \widetilde{\varphi} = 0$ and is completely unaffected by the baryon deformation. As we are not breaking R-symmetry explicitly, no other supersymmetric vacua are generated. As a result, the decay rate of our metastable vacuum is exponentially small, just as in the original ISS model.

6.3.2. Direct Gauge Mediation and Generation of Soft Masses

As mentioned above, the $\mathrm{SU}(5)_f$ symmetry of the superpotential (6.17) is gauged and identified with the parent SU(5) of the MSSM sector. This induces direct gauge mediation of SUSY breaking from the metastable vacuum of the Hidden ISS sector to the MSSM. The Hidden sector matter fields ρ, $\widetilde{\rho}$, Z, $\widetilde{Z}$ and X are all charged under the SU(5) and serve as direct messengers. We will now see how these fields induce all the soft SUSY breaking terms of the MSSM sector, including gaugino and sfermion masses.

Gaugino Masses

Gaugino masses are generated at one loop order (cf. Figure 6.1). The fields propagating in the loop are fermion and scalar components of the direct mediation 'messengers' ρ, $\widetilde{\rho}$ and Z, $\widetilde{Z}$. The adjoint part of X is also charged under the Standard Model gauge groups and therefore, in principle, can also mediate SUSY breaking. However, at tree-level X does not couple to the supersymmetry breaking F-term, and its fermionic and bosonic components have identical (zero) mass. This degeneracy is only lifted at the one-loop level by the Coleman-Weinberg potential. For the time-being we therefore neglect the contribution from X which we expect to be subdominant. Since gaugino masses are forbidden by R-symmetry, one crucial ingredient in their generation is the presence of a non-vanishing R-symmetry breaking VEV — in our case $\langle\chi\rangle$, generated by the non-vanishing baryon deformation m.

In contrast to the gaugino masses m_λ, sfermion masses $m_{\widetilde{f}}$ are not protected by R-symmetry. Hence, as long as supersymmetry remains broken, we can have non-vanishing sfermion masses even in the absence of an R-symmetry breaking VEV. In our model this means that the sfermion masses are non-vanishing even in the case of vanishing baryon deformation. This shows that in a general (gauge) mediation scenario sfermion and gaugino masses are generated by quite different mechanisms. Accordingly, the simple relation $m_\lambda \sim m_{\widetilde{f}}$ does not necessarily hold in general gauge mediation scenarios. Indeed, our model is an explicit example where it fails.

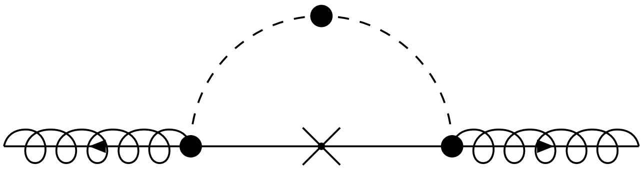


Figure 6.1.: One-loop contribution to the gaugino masses. The dashed (solid) line is a bosonic (fermionic) messenger. The blob on the scalar line indicates an insertion of $\langle F_\chi \rangle$ into the propagator of the scalar messengers and the cross denotes an insertion of the R-symmetry breaking VEV into the propagator of the fermionic messengers.

Let us now turn to the practical evaluation of the gaugino masses. For fermion components of the messengers

$$\psi \;=\; \Big(\rho_{ia}\,,Z_{ir}\Big)_{ferm}\;, \qquad \widetilde{\psi} \;=\; \Big(\widetilde{\rho}_{\,ia}\,,\widetilde{Z}_{ir}\Big)_{ferm}\;, \tag{6.24}$$

the mass matrix is given by

$$m_f \;=\; \mathbb{1}_5 \otimes \mathbb{1}_2 \otimes \begin{pmatrix} \chi & \xi \\ \frac{\mu^2}{\xi} & 0 \end{pmatrix}\;. \tag{6.25}$$

We can also assemble the relevant scalars into

$$\Big(\rho_{ia}, Z_{ir}, \widetilde{\rho}^{\,*}_{\,ia}, \widetilde{Z}^*_{ir}\Big)_{\rm sc}\;, \tag{6.26}$$

and for the corresponding scalar mass-squared matrix we have

$$m^2_{\rm sc} \;=\; \mathbb{1}_5 \otimes \mathbb{1}_2 \otimes \begin{pmatrix} |\xi|^2 + |\chi|^2 & \chi^* \kappa & -\hat{\mu}^2 & \eta\,\kappa \\ \chi\,\kappa^* & |\kappa|^2 & \xi\,\eta + 2m\kappa & 0 \\ -\hat{\mu}^2 & (\xi\eta)^* + 2m\kappa^* & |\kappa|^2 + |\chi|^2 & \chi\,\xi^* \\ \eta^*\kappa^* & 0 & \chi^*\xi & |\xi|^2 \end{pmatrix}\;. \tag{6.27}$$

Gaugino masses arise from the one-loop diagram in Figure 6.1. To evaluate the diagram it is convenient to diagonalise the non-diagonal mass matrices (6.25) and (6.27) using unitary matrices

$$\hat{m}^2_{\rm sc} \;=\; Q^\dagger\, m^2_{\rm sc}\, Q \;, \tag{6.28a}$$
$$\hat{m}_f \;=\; U^\dagger\, m_f\, V \;. \tag{6.28b}$$

The fields in the new basis are given by

$$\hat{S} \;=\; S \cdot Q \;, \tag{6.29a}$$
$$\hat{\psi}_+ \;=\; \psi \cdot U \;, \tag{6.29b}$$
$$\hat{\psi}_- \;=\; \widetilde{\psi} \cdot V^*. \tag{6.29c}$$

In order to calculate the gaugino mass, we need the gauge interaction terms given by

$$\mathcal{L} \supset i\sqrt{2}\, g_A \lambda_A \left(\psi_1 T^A S_1^* + \psi_2 T^A S_2^* + \widetilde{\psi}_1 T^{*A} S_3 + \widetilde{\psi}_2 T^{*A} S_4\right) \;+\; \text{h.c.} \tag{6.30a}$$
$$= i\sqrt{2}\, g_A \lambda_A \left[\hat{\psi}_{+i} \hat{S}_k^* \left(U^\dagger_{i1} Q_{1k} + U^\dagger_{i2} Q_{2k}\right) + \hat{\psi}_{-i} \hat{S}_k \left(Q^\dagger_{k3} V_{1i} + Q^\dagger_{k4} V_{2i}\right)\right] \;+\; \text{h.c.} \;, \tag{6.30b}$$

where we have expressed everything in terms of mass eigenstates in the second line.

Using the gauge interactions equation (6.30b), the diagram in Figure 6.1 contributes to gaugino masses as follows[2]

$$m_{\lambda_A} \;=\; 2\, g_A^2\, \text{Tr}\big(T^A T^B\big) \sum_{ik} \left(U^\dagger_{i1} Q_{1k} + U^\dagger_{i2} Q_{2k}\right)\left(Q^\dagger_{k3} V_{1i} + Q^\dagger_{k4} V_{2i}\right) I(\hat{m}_{f,i}, \hat{m}_{{\rm sc},k}) \tag{6.31}$$

where the 1-loop integral I evaluates to

$$I(a,b) \;=\; \frac{-a(\eta+1)}{16\pi^2} \;+\; \frac{1}{16\pi^2}\frac{a}{(a^2-b^2)}\left[a^2 \log\left(\frac{a^2}{\Lambda^2}\right) - b^2 \log\left(\frac{b^2}{\Lambda^2}\right)\right] \;, \tag{6.32}$$

with

$$\eta \;=\; \frac{2}{4-D} \;+\; \log(4\pi) \;-\; \gamma_E \;. \tag{6.33}$$

[2] More precisely, in evaluating (6.31), we use the diagram in Figure 6.1 without explicit insertions of $\langle F_\chi \rangle$ and $\langle \chi \rangle$ in the messenger propagators. In the loop we use mass-eigenstate propagators and insert the diagonalisation matrices at the vertices. Appropriate dependence on $\langle F_\chi \rangle$ and $\langle \chi \rangle$ is automatically introduced by the diagonalisation matrices.

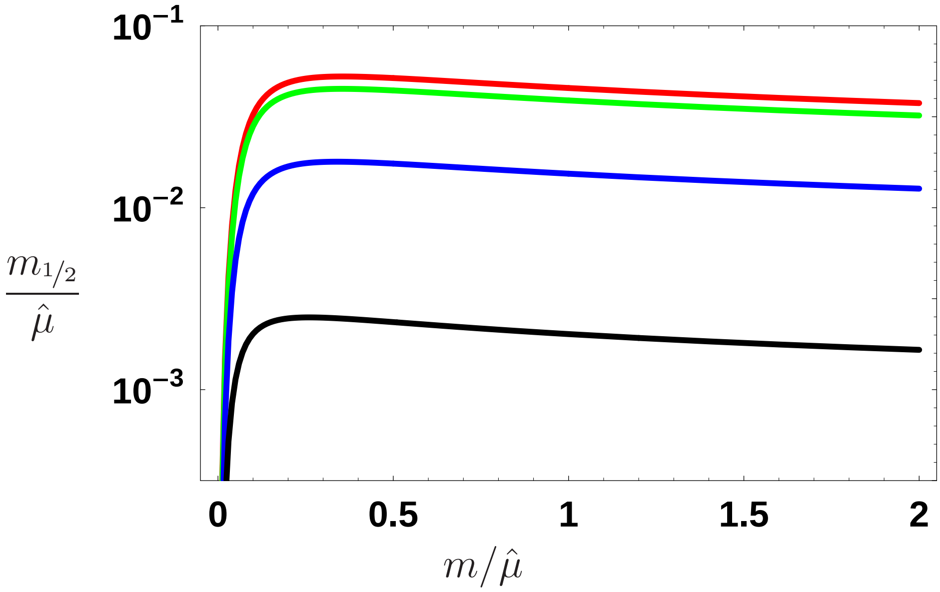


Figure 6.2.: Gaugino mass scale $m_{1/2}$ as a function of the baryon deformation m, for various values of μ: red ($\mu = 1.003\,\hat{\mu}$), green ($\mu = 1.01\,\hat{\mu}$), blue ($\mu = 1.1\,\hat{\mu}$) and black ($\mu = 1.5\,\hat{\mu}$). The mass scale $m_{1/2}$ is defined in equation (6.43a).

The integral $I(a, b)$ is UV-divergent, but the divergences cancel in the sum over eigenstates as they should.

Keeping the SUSY breaking scale $\hat{\mu}$ fixed we can now study the dependence of the gaugino mass on the two remaining parameters μ and m. The VEVs ξ, κ, η and χ are generated from minimizing the effective potential, as above. The results are shown in Figure 6.2. The first thing to note is that the gaugino masses do indeed vanish as expected when the deformation that controls spontaneous R-breaking disappears ($m \to 0$). Another interesting observation is the marked suppression of the masses relative to the rough estimate (5.21) we made in Section 5.3.1, which has $m_{1/2} \sim \hat{\mu}$. Indeed, the way gaugino mass levels off into a plateau for values of m somewhat below the SUSY breaking scale $\hat{\mu}$ indicates that there must be further effects inhibiting the generation of mass, over and above the degree of R-symmetry breaking we discussed in Section 5.3.1.

To get a better handle on this behaviour, it pays to be a bit more careful in deriving an analytic expression for gaugino mass from Figure 6.1. In a limit where the F-terms are somewhat less than μ^2, it can be shown that the gaugino masses vanish to leading order in F_χ, so one must go to order F_χ^3 to find a non-vanishing contribution. This effect, which is largely due to the structure of the fermion mass matrix (6.25), was first pointed out in reference [170], and has recently been observed as a general feature of directly mediated metastable SUSY breaking models [171]. As explained in the appendix of [171], the leading order gaugino mass should be

$$m_{\lambda_A} \sim \frac{g_A^2}{8\pi^2} 5 \, \mathrm{tr}\left(T^A T^B\right) \mathrm{Tr}\left(\mathcal{F} \cdot m_f^{-1}\right) + \mathcal{O}\left(\mathcal{F}^3\right) \;, \tag{6.34}$$

where

$$\mathcal{F}^{ab} = W^{abc} W_c = \begin{pmatrix} F_\chi & F_{\tilde{\phi}} \\ F_\phi & 0 \end{pmatrix} \qquad \text{and} \qquad m_f^{-1} = \begin{pmatrix} 0 & \frac{1}{\kappa} \\ \frac{1}{\xi} & \frac{-\chi}{\kappa\xi} \end{pmatrix} \;. \tag{6.35}$$

The last equation is found by inverting equation (6.25). On expanding out the trace over fields, one finds the zero element of m_f^{-1} prevents F_χ from contributing to the leading order result, which consequently reads

$$m_{\lambda_A} \sim \frac{g_A^2}{8\pi^2} 5 \, \mathrm{tr}\left(T^A T^B\right) \left[\frac{F_\phi}{\kappa} + \frac{F_{\tilde{\phi}}}{\xi}\right] + \mathcal{O}\left(\mathcal{F}^3\right) \;. \tag{6.36}$$

Furthermore, minimising the tree-level scalar potential (5.6) with respect to Y^* imposes the constraint

$$\frac{\partial V}{\partial Y^*} = 2\left(\xi \, F_\phi + \kappa \, F_{\tilde{\phi}}\right) = 0 \;, \tag{6.37}$$

which forces the term in square brackets in equation (6.36) to vanish, so

$$m_{\lambda_A} \sim 0 + \mathcal{O}\left(\mathcal{F}^3\right) \;. \tag{6.38}$$

as claimed.

This result gives some insight into why, in Section 5.3.1, we had to take $\hat{\mu} \sim \mu$ when looking for a realistic parameter point that also had a reasonably small splitting between gaugino and sfermion masses. Our discussion of R-symmetry breaking suggests we should only need to take $\hat{\mu} < m \ll \mu$ to return to the standard GMSB picture, but the above argument shows gaugino masses vanish to leading order in this corner of parameter space. To evade this troublesome conclusion, we therefore took $\hat{\mu} \sim \mu$ when computing the gaugino mass estimate in equation (5.22). We will further reflect upon the suppression of gaugino masses when looking to refine our calculations in Section 6.4.1.

For a recent discussion of the leading order prohibition of gaugino mass in directly mediated O'Raifearteagh models of SUSY breaking see reference [172]. There it is shown that the suppression will always be present for generic models with a *stable* tree-level pseudo-moduli space. The baryon deformation in our model induces runaway behaviour at tree-level, thus circumventing this conclusion and slightly aleviating the mass suppression, but nevertheless it is clear that the sheer *directness* of the mediation mechanism imposes structural constraints on the model that inhibit the generation of gaugino mass [171].

Sfermion Masses

Having determined the gaugino masses in equation (6.31) and Figure 6.2, we now turn to the generation of masses for the sfermions of the supersymmetric Standard Model. Here we will closely follow the calculation in reference [155] adapting it for our more general set of messenger particles. As already mentioned at the beginning of this section, sfermion masses are generated by a different mechanism to that of the gaugino masses. Indeed, they are generated by the two-loop diagrams shown in Figure 6.3. In reference [155] the contribution of these diagrams to the sfermion masses was determined to be

$$m_{\tilde{f}}^2 = \sum_{\text{mess.}} \sum_a g_a^4 \, C_a \, S_a(\text{mess.}) \Big[\text{sum of graphs in Figure 6.3} \Big] \,, \tag{6.39}$$

where we sum over all gauge groups under which the sfermion is charged, g_a is the corresponding gauge coupling, $C_a = (N_a^2 - 1)/(2N_a)$ is the quadratic Casimir and $S_a(\text{mess.})$ is (one half of) the Dynkin index of the messenger fields (normalised to $1/2$ for fundamentals).

In the following we will only describe the new features specific to the messenger fields of our direct mediation model. The explicit expressions for the loop integrals and the algebraic prefactors resulting from γ-matrix algebra etc. can be found in the appendix of [155]. To simplify the calculation we also neglect the masses of the MSSM fields relative to the messenger masses.

As in the calculation of gaugino masses we use propagators in a diagonal form and insert the diagonalisation matrices directly at the vertices. For the diagrams 6.3a to 6.3f we have closed loops of purely bosonic or purely fermionic mass eigenstates of our messenger fields. It is straightforward to check that in this case the unitary matrices from the diagonalisation drop out. We then simply have to sum over all mass eigenstates the results for these diagrams computed in reference [155].

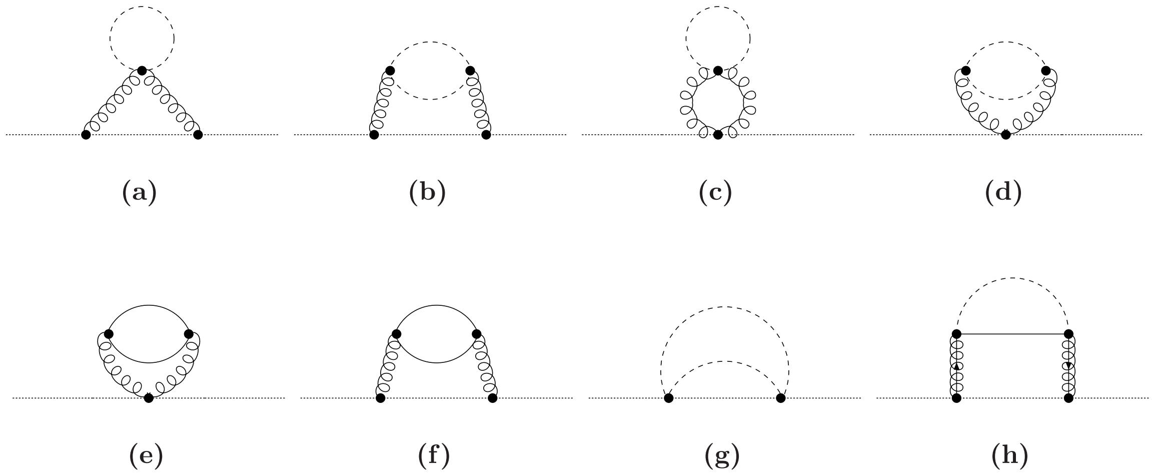


Figure 6.3.: Two-loop diagrams contributing to the sfermion masses. The long dashed (solid) line is a bosonic (fermionic) messenger. Standard Model sfermions are depicted by short dashed lines.

The next diagram 6.3g is slightly more involved. This diagram arises from the D-term interactions. D-terms distinguish between chiral and antichiral fields, in our case ρ, Z and $\widetilde{\rho}^*$, $\widetilde{Z}^*$ respectively. We have defined our scalar field S in (6.26) such that all component fields have equal charges. Accordingly, the ordinary gauge vertex is proportional to a unit matrix in the component space (cf. equation (6.30a)). This vertex is then 'dressed' with our diagonalisation matrices when we switch to the $\hat{S}$ basis, (6.30b). This is different for diagram 6.3g. Here we have an additional minus-sign between chiral and antichiral fields. In field space this corresponds to a vertex that is proportional to a matrix $V_D = \text{diag}(1, 1, -1, -1)$. We therefore obtain

$$\text{Figure 6.3g} = \sum_{i,m} \left(Q^T V_D Q\right)_{i,m} J\left(\hat{m}_{\text{sc},m}, \hat{m}_{\text{sc},i}\right) \left(Q^T V_D Q\right)_{m,i} , \tag{6.40}$$

where J is the appropriate two-loop integral for Figure 6.3g, which can be found in reference [155]. As a function it is evaluated on the eigenvalues of matrices (6.28).

Finally, in 6.3h we have a mixed boson/fermion loop. The subdiagram containing the messengers is similar to the diagram for the gaugino mass. The only difference is the direction of the arrows on the gaugino lines. Indeed, the one-loop sub-diagram corresponds to a contribution to the kinetic term rather than a mass term for the gauginos. (Of course, the mass term will also contribute, but will be suppressed by the smallness

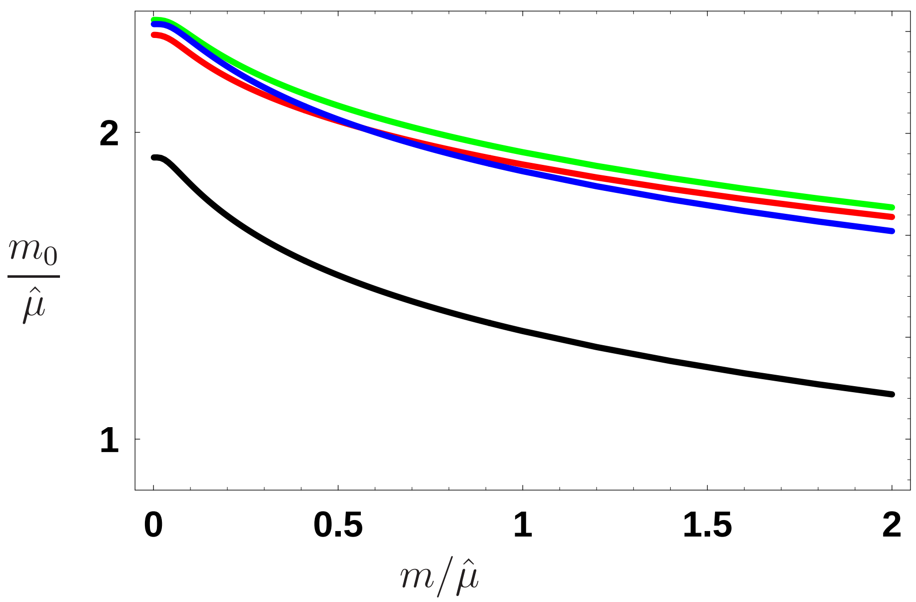


Figure 6.4.: Sfermion mass scale m_0 as a function of the baryon deformation m, for various values of μ: red ($\mu = 1.003\,\hat{\mu}$), green ($\mu = 1.01\,\hat{\mu}$), blue ($\mu = 1.1\,\hat{\mu}$) and black ($\mu = 1.5\,\hat{\mu}$). The mass scale m_0 is defined in equation (6.43b).

of the quark masses.) Using equation (6.30b) we find

$$\text{Figure 6.3h} = \sum_{ik} \left(\left| U^\dagger_{i1} Q_{1k} + U^\dagger_{i2} Q_{2k} \right|^2 + \left| Q^\dagger_{k3} V_{1i} + Q^\dagger_{k4} V_{2i} \right|^2 \right) L\big(\hat{m}_{f,i}, \hat{m}^2_{\text{sc},k}\big) \ , \quad (6.41)$$

where L is again the appropriate loop integral from [155].

Summing over all diagrams we find the sfermion masses depicted in Figure 6.4. Comparing to the gaugino masses in Figure 6.2 we find the sfermion masses to be significantly larger. Indeed, the scalar masses roughly follow the naïve estimate

$$m^2_{\tilde{f},\,\text{naïve}} \sim \frac{g^4}{(16\pi^2)^2} \hat{\mu}^2 \ . \quad (6.42)$$

This demonstrates again the fundamental difference between the generation of gaugino masses and the generation of sfermion masses.

The main results of this section, equations (6.31) and (6.39), give the gaugino and scalar masses generated at the messenger mass scale μ. It is useful to factor out the particle-type-dependent overall constants and define the *universal* fermion and scalar mass contributions $m_{1/2}$ and m_0 via

$$m_{\lambda_A}(\mu) := \frac{g_A^2}{16\pi^2}\, m_{1/2} \tag{6.43a}$$

$$m_{\tilde{f}}^2(\mu) := \sum_A \frac{g_A^4}{(16\pi^2)^2}\, C_A S_A\, m_0^2 \tag{6.43b}$$

We can then re-express equations (6.31) and (6.39) in terms of $m_{1/2}$ and m_0 which we calculate numerically using the VEVs generated by `Vscape`. As an example, in Table 6.3 we show the values for $m_{1/2}$ and m_0^2 obtained for the same parameters as in Table 6.2.

A more thorough treatment of gaugino and scalar masses in the context of direct mediation models has recently been given in reference [171]. We will highlight some of the progress made in this paper when discussing possible ways to improve the above calculations in Section 6.4.1.

μ	m	$m_{1/2}$	m_0^2
10	0.3	1.03984×10^{-7}	0.026787
1.1	0.3	0.017843	4.89783
1.01	0.3	0.044771	5.12698
1.003	0.3	0.052320	4.74031

Table 6.3.: Gaugino and sfermion mass coefficients for various parameter points. All values are in units of $\hat{\mu}$.

6.3.3. Renormalisation Group Running, Mass Spectrum and Electroweak Symmetry Breaking

In the previous section we calculated the soft SUSY breaking masses for gauginos and sfermions at the messenger scale μ. The Higgs masses $m_{H_1}^2$ and $m_{H_2}^2$ are calculated in the same way as the sfermion masses above[3]

$$m_{H_1}^2(\mu) = m_{H_2}^2(\mu) = \left[\frac{3}{4}\frac{g_2^4}{(16\pi^2)^2} + \frac{3}{20}\frac{g_1^4}{(16\pi^2)^2}\right] m_0^2 \tag{6.44}$$

The other soft SUSY breaking terms in the MSSM, such as the A-terms and the B_μ-term are generated at two-loop level. Indeed the diagrams giving rise to the B_μ-term require

[3] We use the GUT normalisation convention for the g_1 gauge couplings.

an insertion of the Peccei-Quinn violating parameter μ_{MSSM} *and* a SUSY breaking gaugino "mass loop". Thus its magnitude at the messenger scale μ is of order [173]

$$B_\mu \sim \frac{g^2}{16\pi^2} m_\lambda \, \mu_{\text{MSSM}} \sim \frac{g^4}{(16\pi^2)^2} m_{1/2} \, \mu_{\text{MSSM}} \,, \tag{6.45}$$

and is loop suppressed with respect to gaugino masses. For the accuracy required here, it will be sufficient to take $B_\mu = 0$ at the messenger scale.

Having successfully communicated the effects of broken supersymmetry to the Visible Sector, we now turn to the phenomenology in full. The next step is to use renormalisation group running to determine the soft SUSY breaking parameters at the Weak Scale. Using these, one can then solve the electroweak symmetry breaking conditions (4.7), and derive the mass spectrum of the MSSM. Throughout the following we will be using the conventions of references [116, 174] with the obvious replacement $\mu \to \mu_{\text{MSSM}}$. The pattern of SUSY breaking here is expected to be different from the standard gauge mediation form for two reasons. Firstly our model naturally predicts significantly larger values of m_0 relative to $m_{1/2}$. Secondly, for reasons explained above, we take $B_\mu = 0$ at the messenger scale. The phenomenology of the $B_\mu = 0$ case has been discussed in references [173, 175–177]. The main prediction is that high $\tan\beta$ is required to achieve successful electroweak symmetry breaking.

In order to see why, consider the tree level minimization conditions (4.8), which can be rearranged in terms of H_u and H_d to give

$$|\mu_{\text{MSSM}}|^2 = -\frac{M_Z^2}{2} - \frac{\tan^2\beta \, m_{H_u}^2 - m_{H_d}^2}{\tan^2\beta - 1} \,, \tag{6.46a}$$

$$B_\mu = \frac{\sin 2\beta}{2} \left(m_{H_u}^2 + m_{H_d}^2 + 2|\mu_{\text{MSSM}}|^2 \right) \,. \tag{6.46b}$$

Since B_μ is only generated radiatively, the right hand side of the second equation has to be suppressed by small $\sin(2\beta)$ with β approaching $\pi/2$. One additional feature of the B_μ-parameter that complicates the analysis somewhat, is that as noted in reference [173] there is an accidental cancellation of renormalization group contributions to its running close to the weak scale. Of course, this model becomes more fine-tuned as $m_0 \gg m_{1/2}$ since we are decoupling the superpartners in that limit. It is worth understanding what has to be fine-tuned. Since $\tan\beta \gg 1$ when $m_0 \gg m_{1/2}$, the first equation tells us that we must have $|\mu_{\text{MSSM}}|^2 \approx -m_{H_u}^2$. In order to have a hope of satisfying the second equation there then has to be a cancellation of the terms inside the bracket,

$m_{H_u}^2 + m_{H_d}^2 + 2|\mu_{\rm MSSM}|^2 \approx 0$ and therefore $m_{H_d}^2 \approx m_{H_u}^2$ near the minimization scale. This is consistent with large $\tan\beta$, where the top and bottom Yukawa couplings become approximately degenerate.[4]

To calculate the spectrum of these models we have modified the `Softsusy2.0` program of reference [116]. In its unmodified form this program finds Yukawa couplings consistent with soft SUSY breaking terms (specified at the messenger scale $Q_{\rm Mess} = \mu$) and electroweak symmetry breaking conditions (imposed at a scale $Q_{\rm SUSY}$ to be discussed later). It is usual to take the ratio of Higgs VEVs $\frac{v_u}{v_d} \equiv \tan\beta$ at $Q_{\rm SUSY}$ as an input parameter instead of the soft SUSY breaking term B_μ at $Q_{\rm Mess}$. This term, and the SUSY preserving $\mu_{\rm MSSM}$ are subsequently determined through the EWSB conditions (6.46a) and (6.46b).

As the models we are considering have $B_\mu = 0$ at $Q_{\rm Mess}$ to two loops, $\tan\beta$ is not a free parameter, and must (as noted above) be adjusted in `Softsusy2.0`, so that this boundary condition is met. In detail the iteration procedure works as follows: initially, a high value of $\tan\beta$ is chosen and all the gauge and Yukawa couplings are evolved to $Q_{\rm Mess}$. The soft parameters are then set, as per the SUSY breaking model, *including* the condition $B_\mu = 0$. The whole system is then evolved down to $Q_{\rm SUSY}$, where $\tan\beta$ is adjusted to bring the program closer to a solution of the EWSB condition in equation (6.46b) (including the 1-loop corrections to the soft masses $m_{H_d}^2$ and $m_{H_u}^2$ and the self-energy contributions to the $\overline{DR}$ mass-squared of the axial Higgs m_A^2). We then run back up to $Q_{\rm Mess}$ where we reimpose the soft breaking boundary conditions, and the whole process is repeated until the value of $\tan\beta$ converges.

The scale $Q_{\rm SUSY}$ at which the tree-level minimisation conditions (6.46a) and (6.46b) are imposed is chosen so as to minimise the radiative corrections to the results. It is usually taken to be $Q_{\rm SUSY} \equiv x\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ where x (`QEWSB` in the language of reference [116]) is a number of order unity. As we see from Table 6.4, the lightest Higgs mass (in the model with spontaneously broken R-symmetry) depends less on scale for lower values of $Q_{\rm SUSY}$, and so in this model we will therefore be using

$$Q_{\rm SUSY} = 0.8 \times \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \ . \tag{6.47}$$

Note that only the Higgs masses are sensitive to this choice and the other parameters are largely unaffected.

[4] Using the conventional definition [178, 179], the fine-tuning is then of order $\mu_{\rm MSSM}/m_Z$.

Q_{SUSY}	$\times$ 0.75	$\times$ 0.80	$\times$ 0.85	$\times$ 0.90	$\times$ 0.95	$\times$ 0.99	$\times$ 1.00	$\times$ 1.01
h_0	124.5	124.5	124.2	124.1	123.8	101.5	93.3	78.6

Table 6.4.: Checking the scale dependence of the lightest Higgs mass (in GeV).

What the construction we are discussing predicts in most of its parameter space (i.e. generic $\mu > \hat{\mu}$) is clearly split-SUSY like because of the suppression of R-symmetry violating operators (i.e. $m_{1/2} \ll m_0$ in Table 6.3). It provides a first-principles model that can implement split-SUSY [15–17]. For other realisations of split-SUSY scenarios see, for example, [180]. Our purpose here however is to examine how close the models with radiative R-symmetry breaking can get to the usual gauge mediation scenarios [14]. For this reason we want to reduce the m_0 to $m_{1/2}$ ratio as far as possible and to take μ approaching $\hat{\mu}$, i.e. the last two rows in Table 6.2.

6.4. Benchmarks

With supersymmetry and R-symmetry broken, and the renormalisation group running of soft parameters accounted for, we are now in a position to present complete low energy MSSM spectra of the metastable SUSY breaking models discussed in this chapter. All masses given have been extracted from `SoftSusy2.0` in a format compatible with the SUSY les Houches Accord [181] — essentially $\overline{DR}$ masses at the scale Q_{SUSY} (6.47). The exact numbers quoted are subject to various ambiguities discussed in Section 6.4.1, and are really only intended to illustrate general patterns in the spectra.

This exercise plainly demonstrates one strength of the metastable gauge mediation paradigm: from well-motivated assumptions and relatively few input parameters, the whole spectrum of observable low energy physics can be systematically calculated. A more in-depth phenomenological study of the parameter space of these models would clearly be desirable, with subsequent Monte Carlo simulations employed to search for potential ways to distinguish this particular mechanism of SUSY breaking at future particle colliders.

In Table 6.5 (page 147) we present a benchmark point (Benchmark Point A) with the full spectrum of the direct mediation model with spontaneous R-symmetry breaking found in Section 6.3. This point, with $\mu = 1.003\,\hat{\mu}$ and $m = 0.3\,\hat{\mu}$, corresponds to a phenomenologically viable region of parameter space near the boundary. The important features to note: it has heavy scalars, light charginos and neutralinos, and exhibits radiative electroweak symmetry breaking. This point is still quite distinct from the usual gauge mediation scenarios, and as we saw in Section 6.2, from predictions of gauge mediation models with *explicit* R-symmetry breaking [12, 13]. To make this comparison more transparent, in Table 6.5 we also present another benchmark point (Benchmark Point B) typifying the spectrum of a model with explicit R-breaking [12, 13] such we discussed in Section 6.2. As expected, it conforms to the standard gauge mediation form, with the requirement that $B_\mu = 0$ at the mediation scale leading to large $\tan\beta$.

6.4.1. Refinements

In this chapter we have seen that the details of the dynamics of a metastable Hidden Sector — the nature of R-symmetry and SUSY breaking — leave a clear imprint on the phenomenology of the MSSM. Although both the scenarios investigated here can be seen to arise in particular corners of the phenomenological parametrisation of gauge mediated models known as General Gauge Mediation [182], it is clearly of interest to have an understanding of the physical mechanisms that give rise to this structure. The different ways in which R-symmetry may be manifest in the SUSY breaking sector appear to make sufficient difference to the spectrum of masses as to allow us the hope of one day distinguishing them by experiment. It would be interesting to broaden the scope of our study to include other models with either spontaneous or explicit R-symmetry breaking, and to see if the general pattern outlined here persists.

The first steps in this direction were taken in reference [171], where the authors investigated generalisations of our minimal deformation of the ISS model from Chapter 5. The models they considered still break R-symmetry spontaneously, and communicate SUSY breaking directly to the MSSM by the appropriate gauging of a flavour symmetry. Just as we saw in Section 6.3.2, all the direct mediation models were found to suffer from suppressed gaugino masses at leading order. A number of useful observations were made in [171] relevant to the calculation of these masses.

Firstly, as we explained around equation (6.36), in the limit $\hat{\mu} \ll \mu$ the leading order disappearance of gaugino mass can be attributed to the imposition of a *tree-level* relation between various VEVs and F-terms. This tree-level relation is not strictly respected by the Coleman-Weinberg corrections that are used to determine the parameters that feed

	Model A	Model B
$Q_{\rm Mess}$	8.32×10^5	1×10^7
$\tan\beta$	58.7	38.9
${\rm sgn}\mu_{\rm MSSM}$	$+$	$+$
$\mu_{\rm MSSM}(Q_{\rm SUSY})$	2891	939
$\widetilde{e}_L, \widetilde{\mu}_L$	4165	747.9
$\widetilde{e}_R, \widetilde{\mu}_R$	2133	399.8
$\widetilde{\tau}_L$	1818	319.4
$\widetilde{\tau}_R$	4093	737.5
$\widetilde{u}_1, \widetilde{c}_1$	11757	1963
$\widetilde{u}_2, \widetilde{c}_2$	11205	1867
$\widetilde{t}_1$	10345	1593
$\widetilde{t}_2$	11061	1825
$\widetilde{d}_1, \widetilde{s}_1$	11784	1973
$\widetilde{d}_2, \widetilde{s}_2$	11144	1851
$\widetilde{b}_1$	10298	1754
$\widetilde{b}_2$	11060	1822
χ_1^0	60.8	270.3
χ_2^0	125.0	524.8
χ_3^0	2906	949.0
χ_4^0	2929	950.3
$\chi_1^\pm$	100.7	526.5
$\chi_2^\pm$	2894	945.6
h_0	124.8	137.6
$A_0,\ H_0$	184.5	975.1
$H^\pm$	207.4	978.6
$\widetilde{g}$	414.2	1500
$\widetilde{\nu}_{1,2}$	4175	740.2
$\widetilde{\nu}_3$	4095	724.4

Table 6.5.: Sparticle spectra for SUSY breaking models with spontaneously broken (Model A) and explicitly broken (Model B) R-symmetry. All masses are in GeV.

into the gaugino mass matrix. So even though we checked in Table 5.2 that the imposition of this tree-level constraint has a small effect on the stabilised VEVs, implementing it as part of the Coleman-Weinberg minimisation is not entirely legitimate. Due to the accidental suppression — which is partly *enforced* by the constraint — this small effect can provide an appreciable contribution to the gaugino masses.

Secondly, reference [171] takes into account the effect that adjoint matter has in mediating SUSY breaking. We chose to neglect the contribution of these messenger fields to our leading order calculation of soft terms because the adjoint bosonic and fermionic masses are only split by a one-loop effect. Technically their effect should only be included in a higher-order calculation, but due to the accidental smallness of leading order gaugino masses, this sub-dominant contribution can also be important.

In summary, because the gaugino masses we calculated in Section 6.3.2 turned out to be uncharacteristically small, there are various approximations we made along the way that are less well justified than they may at first appear. Taking the above refinements into account lifts the universal gaugino mass (6.43a) by about two orders of magnitude. This alleviates the fine tuning of $\hat{\mu} \sim \mu$ that we found was necessary to minimise the split aspects of the phenomenology of our direct mediation model.

Another important phenomenological point raised in reference [183] and also discussed in [171] again relates to the adjoint superfields X of the SUSY breaking sector. They are pseudo-Goldstone modes that acquire masses proportional to the SUSY breaking scale by the Coleman-Weinberg mechanism. A naïve estimate might put their masses about a loop factor below the SUSY breaking scale, but in fact some of the fermionic components of X can have masses of the same order as the (suppressed) gauginos. As X is charged under the Standard Model, these exotic light states with masses around the TeV scale would therefore be expected to show up alongside the usual MSSM spectrum at the LHC in the coming years. Although its precise nature is relatively model dependent, the presence of new TeV scale matter invading observable physics from the SUSY breaking sector is a fairly generic expectation of direct gauge mediated models; one shouldn't be surprised to catch a glimpse of the SUSY breaking sector if it fits so closely together with the Standard Model.

In light of the above comments, we feel it necessary to reiterate that the sample MSSM spectrum for our direct mediation model, Benchmark Point A in Table 6.5, will be noticeably altered by a more careful calculation of the soft terms. The renormalisation group running must also be modified to account for the effect of light matter from the adjoint field X. Despite the approximations made, the general features of this benchmark point, such as the splitting between gaugino and sfermion masses and the large value of $\tan\beta$, are expected to still be evident in a more comprehensive calculation of the low energy spectrum; we hope to return to this point in future work.

6.5. Conclusions

6.5.1. Summary

As we argued in Chapter 4, in generic models of low scale supersymmetry breaking (where gravity effects can be neglected) metastability is inevitable.

In this chapter we compared SUSY breaking patterns generated in two distinct and complementary scenarios of gauge mediated supersymmetry breaking. Both scenarios employ an explicit formulation of the Hidden Sector in terms of an ISS-like gauge theory with a long-lived metastable vacuum. This, in both cases, provides a simple and calculable model to implement metastable DSB.

One important difference between the two approaches lies in the mechanism of R-symmetry breaking. The first approach, outlined in Section 6.2, is based on the gauge mediation models of references [12, 13] with a messenger sector that explicitly breaks the R-symmetry of the ISS sector by operators suppressed by powers of $M_{\rm Pl}$. We argue that these models lead to phenomenology broadly similar to standard gauge mediation, but with an additional constraint that $B_\mu = 0$ at the mediation scale.

The second strategy, described in Section 6.3, employs spontaneous R-symmetry breaking induced by radiative corrections. It is based on the direct gauge mediation model introduced in Chapter 5. We find that R-symmetry violating soft terms (such as gaugino masses) tend to be suppressed with respect to R-symmetry preserving ones, leading to a scenario with large scalar masses. These models effectively interpolate between split SUSY models and standard gauge mediation, although subsequent work [171, 172] has shown that the *directness* of mediation leads to an irreducible degree of gaugino-sfermion mass splitting.

Determining the complete spectrum of superpartner masses at benchmark points (see Table 6.5) we find that apart from high values of $\tan\beta$ (arising from the condition that $B_\mu \approx 0$ at the messenger scale in both models) the phenomenology of these models is quite different. For the model with explicit R-symmetry breaking (Benchmark Point B) we find that it closely follows the usual gauge mediation scenario where gauginos and sfermions have roughly equal masses. In contrast, the direct mediation model with spontaneous R-symmetry breaking typically has sfermions that are considerably heavier than the gauginos — resembling a scenario of split SUSY. Benchmark Point A represents such a model at a region in parameter space where the 'split aspects' of supersymmetry are minimal. At the same time it is quite distinct from the usual gauge mediation scenarios, having relatively heavy scalars and light charginos and neutralinos.

6.5.2. Landau Poles

The ISS approach to supersymmetry breaking has important implications for how the theory behaves at high energies. In gauge mediated scenarios, and certainly for those with low scale direct gauge mediation (i.e. in which the global flavour symmetries of the ISS model are identified with the gauge symmetries of the MSSM), there are many new degrees of freedom, corresponding to the magnetic quarks and mesons of the ISS sector. As we saw at the end of Chapter 5, this induces a Landau pole within the Visible Sector that leads, unless one is careful, to the theory becoming non-perturbative and incalculable at energies below the Planck (or GUT) scale — a long standing problem with direct gauge mediation. Rather than discard such theories as sick, in Section 5.4 we speculated that a more interesting resolution may found by mimicking the ISS sector: when the couplings become strong, *perform a Seiberg duality*.

What would be the expectation for an electric dual of the MSSM? The precise details depend of course on the exact form of the Seiberg duality, but generically, as a result of dualising all or part of the MSSM at its Landau pole, one would expect to find larger Visible Sector gauge groups, or equivalently more flavours for the ISS sector. The latter would then hit another Landau pole, probably in even shorter order than it did the first one, and so on. At each Landau pole the rank of a gauge group increases, and so the cycle repeats. Eventually perturbativity is lost when the 't Hooft coupling becomes greater than unity. The phenomenon we are describing is known as a DUALITY CASCADE.

One can think of the cascade as a renormalisation group flow where different energies are best described by different gauge theories. These theories are related by Seiberg duality: as one description becomes strongly coupled one can perform a duality transformation to new variables in terms of which the theory is better behaved. Following the renormalisation group flow towards the UV, after many dualities the rank of each dualising gauge group will be very large, so it is reasonable to expect that the UV end of a duality cascade may have a holographic supergravity/string theory interpretation along the lines of the AdS/CFT correspondence described in Section 2.4.

The idea of a cascade was first proposed in the remarkable work of Klebanov and Strassler [52, 184], where they gave a complete description of the cascading mechanism of a particular theory from both the gauge and string theory points of view (the geometry in this case is a warped, deformed conifold). Since then, much work has been done in generalising the cascading phenomenon, and also in developing tools that facilitate the construction of physically interesting cascading models. The possibility that the MSSM may lie at the bottom of a cascade was suggested in the early papers [184, 185]. Recent proposals for how this idea may be realised have been presented in references [186–188].

In many respects, having the MSSM lie at the bottom of a cascade is an attractive possibility because, although gauge unification is evidently lost,[5] one gains an explanation for why the gauge groups of the MSSM have low ranks despite there being a virtually limitless number of high-rank candidates available. In the UV, the theory has a large number of D-branes and can be well described by a gravitational dual in which the branes have "melted" into the geometry. Towards the IR the theory sheds D-branes down the cascade upon successive applications of Seiberg dualities, and ends up in a regime with low-rank gauge groups, described by the world-volume theory of a few fractional branes trapped on a singularity at the "tip" of the geometry.

As our understanding of the duality cascade is heavily reliant on the magic of $\mathcal{N} = 1$ supersymmetry, it is clear that SUSY breaking must occur somewhere below the lowest energy Seiberg duality scale, Λ_1. There has already been considerable interest in incorporating metastable supersymmetry breaking at the bottom of a duality cascade [164, 189–194]. Indeed, with Seiberg duality featuring so strongly in both the cascade and the ISS model, uniting the two seems rather natural. Attention has largely focused on QUIVER gauge theories — these have the correct field content and interactions to be realised as the low energy dynamics of a stack of D-branes. One interesting element of this framework is that the strong coupling phenomena responsible for dynamical SUSY breaking in the gauge theory, correspond to the effects of STRINGY INSTANTONS[6] from the dual gravity point of view. For example, reference [193] found simple representatives

[5] A possible way to dualise the MSSM whilst maintaining gauge coupling unification was explored in reference [163].

[6] We saw in Section 2.3.2 that an instanton in the low energy gauge theory of a stack of Dp-branes branes can be interpreted as a $D(p-4)$-brane sitting inside the worldvolume of the stack. A stringy instanton is, roughly speaking, what you get when the $D(p-4)$-brane wraps a cycle in the geometry

of Polonyi, O'Raifeartaigh and Fayet models of SUSY breaking on singularities derived from the conifold, that are induced by exponentially small D-instanton effects. This is nice because it provides a geometric understanding of the scales involved in SUSY breaking,[7] and the resulting Hidden Sector can naturally be quite compact.

The next challenge is to construct realistic models that cascade to the MSSM (or some variant thereof) in which dynamical supersymmetry breaking is directly mediated naturally at the bottom of the cascade. Such models would be very economical, and offer an interesting new perspective on the interface between string theory and particle physics. One potential construction has been suggested in reference [183], whereby SUSY breaking is mediated by via a mechanism with explicit R-symmetry breaking (such as we discussed in Section 6.2) although the *non-genericity* of this model obfuscates the cascading procedure. It would be interesting to find a realisation of our metastable SUSY breaking model with spontaneously broken R-symmetry at the bottom of a duality cascade. One expectation is that because baryons are essentially invariant under the action of Seiberg duality, the baryon deformation (5.2) should be visible all the way up the cascade, with a clear interpretation in the geometry of the gravitational dual that describes UV physics. This is a work in progress.

Even with an infinite number of Seiberg dualities, the aforementioned Landau pole problem isn't necessarily absent from the field theory perspective. One often finds the scales at which one must dualise reach an accumulation point below M_{Pl} — a DUALITY WALL [185]. This is where the holographic description becomes vitally important: It is hoped [188] that a gauge theory with such poor UV behaviour may be repaired by switching at the appropriate energy scale to a description of the system in terms of strings moving in a singular background. The challenge here, of course, is identifying the singular geometry relevant to a given gauge theory.

that is *not* contained in the worldvolume of the higher dimensional stack. See reference [195] for a recent reveiw.

[7] Small mass scales in supersymmetry breaking models may be explained dynamically, purely in field theoretic terms, by RETROFITTING along the lines of reference [127].

6.5.3. Future Directions

Up until now we have largely focused on what can be learnt about supersymmetry from smashing particles together in a controlled environment, such as the LHC. In this case, to probe the SUSY breaking sector more deeply one just has to build bigger and bigger machines that collide particles at increasingly high energies. In a world with infinite resources, this would eventually allow us to catalogue the properties of all matter, but in reality we are limited by the constraints of space, time and money. The development of the LHC alone has taken over 15 years and has necessarily been funded by a large collaboration of nations. It is therefore wise to devise alternative strategies for probing this unknown physics.

Axion-like Particles

One interesting approach is to look for low mass, weakly interacting Hidden Sector particles in low energy experiments. The general set-up of such studies usually involves shining laser light through a strong magnetic field. Although the energies attained are insufficient to directly produce the majority of Hidden Sector particles, the extreme sensitivity and terrific statistics achievable in photon experiments opens the door to a wide selection of effects from the Hidden Sector; these experiments are already putting competitive bounds on the properties of axion-like particles (ALPs). The modest scale of low energy experiments, and rapid development of the relevant technology, means that significant advances are expected to be made in ALP detection over the coming years. With this in mind, it is a good idea to have a comprehensive understanding of the theoretical possibilities if we are to be in a position to assimilate and exploit the new data when it comes in.

As we discussed in Section 4.2.2, models of low scale supersymmetry breaking (essentially, where gravity effects are subdominant), must necessarily possess an R-symmetry. Such a symmetry is phenomenologically unacceptable, and any attempt to remove it, either by explicit or spontaneous symmetry breaking, results in the vacuum becoming metastable. As we have seen in this chapter, the generic prediction of metastability arising in a wide class of SUSY breaking models has interesting consequences for what might be observed at the LHC. It would therefore be interesting to extend the scope of this phenomenological investigation to take account of data from low energy experiments.

For example, if R-symmetry is spontaneously broken, as in as in our model from Chapter 5, then Goldstone's theorem tells us the SUSY breaking sector must contain a light particle, known as an R-axion. As its name suggests, the R-axion has similar properties to the standard axion of QCD, and so the allowed values of its mass and decay constant are constrained by data from the on-going search for axion-like particles. We made cursory use of this information in Section 5.3.2 when we showed that the R-axion mass predicted by one specific model was compatible with the latest astrophysical bounds, but we believe much more can be learnt about the physics of SUSY breaking by studying low energy signatures.

It is important to establish the existence and nature of the R-axion, because the mechanism that determines its couplings and mass scale lies at the very heart of the SUSY breaking sector. A good avenue for future investigations would be to study the emergence of R-axions in metastable SUSY breaking models, and more specifically to examine the model dependence of their properties, along the lines of reference [148]. This would be useful for two reasons: In the case of the discovery of an axion-like particle, one would be able to test the hypothesis that it is an R-axion, i.e. such research should provide a way to distinguish R-axions from other axion-like particles. Also, by combining the properties of R-axions with knowledge of other aspects of SUSY breaking (such as the sparticle masses that may shortly be obtained from the LHC) it ought to be possible to refine the current methods used to search for R-axions.

Low scale SUSY breaking models have the benefit of being *predictive*, essentially because they are based on field theory models of the Hidden Sector, and so assume very little about the nature of quantum gravity (the details of which should only be relevant at very high energies). In these scenarios one will often find pseudo-Nambu-Goldstone bosons that are associated with the existence of spontaneously broken approximate symmetries, just as the R-axion follows from spontaneous R-symmetry breaking. The mass and coupling of these pNG bosons is tied to the scale at which the corresponding symmetry is broken, so as a further extension of the above study one could also explore how the (non-)detection of light pNG bosons can guide model building. In this way, searching for light particles in low energy experiments can enable us to uncover the structure of Hidden Sector physics, providing a wealth of information about the symmetries and energy scales involved.

Cosmology

A complementary approach that can potentially shed light on the nature of SUSY breaking is to consider cosmological information. In principle this allows us to look back to times when Hidden Sector fields were last in thermal equilibrium with the Standard Model; in practice, requiring that Hidden Sector physics fits well with our current understanding of the evolution of the Universe can be used to place bounds on the behaviour of these models. The soundness of this approach has already been tried and tested: as we described in Section 4.2.3 the thermal history of the Universe was examined in [9–11, 123, 124] to explain how we could have ended up trapped in a metastable vacuum. Here, the temperature after reheating can be used to constrain the scale of supersymmetry breaking.

A more sophisticated test of metastable ideas could come from incorporating inflationary dynamics into the Hidden Sector. One would then calculate how the signature of these models is imprinted as subtle correlations in the cosmic microwave background, and, through comparison with precision cosmology, extract bounds on the SUSY breaking sector. Work is already being undertaken in this area: in reference [196] the authors augment our model of SUSY breaking from Chapter 5 by including an inflationary sector. They then consider how gravitational effects in the combined Hidden Sector interrelate the scales in SUSY breaking with inflationary observables.

One intriguing aspect of the setup of [196] is how, during reheating, the inflaton decays into Standard Model particles through fields in the supersymmetry breaking sector. Thus, one expects that details of how broken SUSY is mediated to the MSSM should be encoded in inflationary observables. In short, we expect such scenarios to be highly predictive, with potentially very interesting consequences, making them a worthwhile focus for future studies.

Appendix A.

Notation

Spacetime Metric

In this thesis we work with the 'mostly minus' metric convention for Minkowski spacetime, in which the metric is $g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. Unless otherwise stated, indices from the latter part of the Greek alphabet (μ, $\nu, \ldots$) assume the values 0, 1, 2 and 3.

Spinor Conventions

We will work with two-component spinors, for the most part adhering to the conventions for supersymmetric field theory laid out in Wess and Bagger [31]. One slight quibble is that [31] uses the opposite metric convention to us. This mismatch is accounted for in [32]. For a very careful treatment of two-component spinors that discusses how the notation depends on metric conventions (and that also includes all the spinor identities you could ever wish for) we defer to reference [33].

When classifying representations of the Lorentz group SO(1, 3), it is generally easier to consider its universal cover, the group SL(2, ℂ) (this way we avoid having to deal with projective representations). The fundamental representation in this case is a two-component LEFT-HANDED WEYL SPINOR ψ_α that transforms as

$$\psi_\alpha \to \psi'_\alpha = M_\alpha{}^\beta \psi_\beta \ , \qquad M \in \mathrm{SL}(2, \mathbb{C}) \ . \tag{A.1}$$

Similarly, RIGHT-HANDED WEYL SPINORS $\overline{\chi}^{\dot\alpha}$ are defined to transform as:

$$\overline{\chi}^{\dot\alpha} \to \overline{\chi}'^{\dot\alpha} = \left[\left(M^{-1}\right)^\dagger \right]^{\dot\alpha}{}_{\dot\beta} \overline{\chi}^{\dot\beta} \ , \qquad M \in \mathrm{SL}(2, \mathbb{C}) \ . \tag{A.2}$$

Here the indices α, $\dot{\alpha}$ can each take the values 1 or 2, with the undotted/dotted cases indicating that each spinor transforms under a different subgroup of the real section $SU(2)_L \times SU(2)_R$ of $SL(2, \mathbb{C})$: left-handed Weyl spinors carry charge $(1/2, 0)$ whereas the right-handed ones have charge $(0, 1/2)$.

The raising and lowering of these indices is achieved with the $SL(2, \mathbb{C})$-invariant antisymmetric matrices

$$\varepsilon^{\alpha\beta} = \varepsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \qquad \varepsilon_{\alpha\beta} = \varepsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} , \tag{A.3}$$

which always introduce a dummy variable on their right-most index:

$$\psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta , \qquad \psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta , \qquad \overline{\chi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\overline{\chi}_{\dot{\beta}} , \qquad \overline{\chi}^{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\overline{\chi}^{\dot{\beta}} . \tag{A.4}$$

This allows us to define invariant products of left- and right-handed spinors:

$$\big(\psi\phi\big) = \psi^\alpha\phi_\alpha , \qquad \big(\overline{\chi\eta}\big) = \overline{\chi}_{\dot{\alpha}}\overline{\eta}^{\dot{\alpha}} . \tag{A.5}$$

Note, the direction of contraction ($\searrow$) or ($\nearrow$) is important here. The epsilon tensors satisfy

$$\varepsilon_{\alpha\beta}\,\varepsilon^{\gamma\delta} = -\,\delta_\alpha^\gamma\,\delta_\beta^\delta + \delta_\alpha^\delta\,\delta_\beta^\gamma , \tag{A.6}$$

which upon contracting with a bunch of left-handed spinors ${\psi_1}^\alpha\,{\psi_2}^\beta\,\psi_{3\,\gamma}\,\psi_{4\,\delta}$ leads to cyclic relations between the spinor products (A.5)

$$(\psi_1\psi_2)\,(\psi_3\psi_4) = -\,(\psi_1\psi_3)\,(\psi_4\psi_2) - (\psi_1\psi_4)\,(\psi_2\psi_4) , \tag{A.7}$$

with similar results for right-handed spinors. These are know as Fierz identities, and often come in handy when manipulating spinor expressions.

To clarify a point that often confuses: the bar in this notation means Hermitian conjugation $(\psi_\alpha)^\dagger \equiv \overline{\psi}_{\dot{\alpha}}$, as can be seen, for example, if we calculate the transformation properties of the right-handed spinor

$$\overline{\psi}'^{\,\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\,\overline{\psi}'_{\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\left(\psi'_{\beta}\right)^{\dagger} = \varepsilon^{\dot{\alpha}\dot{\beta}}\left({M_{\beta}}^{\gamma}\psi_{\gamma}\right)^{\dagger} \tag{A.8a}$$

$$= \varepsilon^{\dot{\alpha}\dot{\beta}}\,\overline{\psi}_{\dot{\gamma}}\left(M^{\dagger}\right)^{\dot{\gamma}}{}_{\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\,{M^{*}}_{\dot{\beta}}{}^{\dot{\gamma}}\,\overline{\psi}_{\dot{\gamma}} \tag{A.8b}$$

$$= \left(M^{-1}\right)^{*}{}_{\dot{\beta}}{}^{\dot{\alpha}}\,\varepsilon^{\dot{\beta}\dot{\gamma}}\,\overline{\psi}_{\dot{\gamma}} = \left[\left(M^{-1}\right)^{\dagger}\right]^{\dot{\alpha}}{}_{\dot{\beta}}\,\overline{\psi}^{\dot{\beta}}\ . \tag{A.8c}$$

The last line here follows from considering the fact that $\det(M) = 1$ for $M \in \mathrm{SL}(2,\mathbb{C})$, which in our notation takes the guise

$$-\frac{1}{2}\,\varepsilon_{\alpha\beta}\,\varepsilon^{\gamma\delta}{M_{\gamma}}^{\alpha}{M_{\delta}}^{\beta} = 1 \qquad \Rightarrow \qquad \varepsilon^{\gamma\delta}{M_{\gamma}}^{\alpha}{M_{\delta}}^{\beta} = \varepsilon^{\alpha\beta}$$

$$\Rightarrow \qquad \varepsilon^{\gamma\delta}{M_{\delta}}^{\beta} = \varepsilon^{\alpha\beta}\left(M^{-1}\right)_{\alpha}{}^{\gamma}\ .$$

The epsilon tensors (A.3) also crop up in the similarity transformation that shows right-handed spinors transform in a representation that is equivalent to the dual representation of the left-handed case:

$$\left[\left(M^{-1}\right)^{\dagger}\right]^{\dot{\alpha}}{}_{\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\gamma}}\left[{M^{*}}_{\dot{\gamma}}{}^{\dot{\delta}}\right]\varepsilon_{\dot{\delta}\dot{\beta}}\ .$$

In Section 2.2.1 we extend Minkowski space by Grassmannian coordinates θ^{α}, $\overline{\theta}_{\dot{\alpha}}$ with $\alpha\,,\dot{\alpha} = 1,2$, to form superspace $(x^{\mu},\theta^{\alpha},\overline{\theta}_{\dot{\alpha}})$. This provides the basis of a manifestly supersymmetric notation that is widely used throughout this thesis and beyond. It is useful to establish conventions that will allow us to do calculus on superspace. Differentiating the Grassmann variables works as follows:

$$\partial_{\alpha} \equiv \frac{\partial}{\partial\theta^{\alpha}} \qquad\qquad \overline{\partial}^{\dot{\alpha}} \equiv \frac{\partial}{\partial\overline{\theta}_{\dot{\alpha}}} \tag{A.9a}$$

$$\Longrightarrow \qquad \partial_{\alpha}\,\theta^{\beta} = \delta^{\beta}_{\alpha} \qquad\qquad \overline{\partial}^{\dot{\alpha}}\,\theta_{\dot{\beta}} = \delta^{\dot{\alpha}}_{\dot{\beta}}$$

$$\partial^{\alpha} \equiv -\,\varepsilon^{\alpha\beta}\partial_{\beta} \qquad\qquad \overline{\partial}_{\dot{\alpha}} \equiv -\,\varepsilon_{\dot{\alpha}\dot{\beta}}\,\overline{\partial}^{\dot{\beta}}\ . \tag{A.9b}$$

Due to the signs in the last line here, one must be careful when raising/lowering derivative indices with the epsilon tensor. Grassmann integration is largely defined in the usual way, but it is useful to define the measures

$$\mathrm{d}^2\theta = -\frac{1}{4}\,\varepsilon_{\alpha\beta}\,\mathrm{d}\theta^{\alpha}\,\mathrm{d}\theta^{\beta}\ , \qquad \mathrm{d}^2\overline{\theta} = -\frac{1}{4}\,\varepsilon^{\dot{\alpha}\dot{\beta}}\,\mathrm{d}\overline{\theta}_{\dot{\alpha}}\,\mathrm{d}\overline{\theta}_{\dot{\beta}}\ , \qquad \mathrm{d}^2\theta = \mathrm{d}^2\theta\,\mathrm{d}^2\overline{\theta}\ , \tag{A.10}$$

to simplify integration over the invariant products in equation (A.5)

$$\int d^2\theta\,(\theta\theta) = 1\,, \qquad \int d^2\overline{\theta}\,(\overline{\theta\theta}) = 1\,. \tag{A.11}$$

To make contact with the usual vector notation we introduce

$$\sigma^\mu_{\alpha\dot{\alpha}} = \left\{\mathbb{1}_{2\times 2}, \tau^1, \tau^2, \tau^3\right\}_{\alpha\dot{\alpha}}\,, \tag{A.12}$$

where the last three matrices on the right hand side are the usual Pauli matrices. It is useful to similarly define

$$\overline{\sigma}^{\mu\,\dot{\alpha}\alpha} = \left\{\mathbb{1}_{2\times 2}, -\tau^1, -\tau^2, -\tau^3\right\}^{\dot{\alpha}\alpha}\,, \tag{A.13}$$

which obey the relation $\overline{\sigma}^{\mu\,\dot{\alpha}\alpha} = \varepsilon^{\alpha\beta}\,\varepsilon^{\dot{\alpha}\dot{\beta}}\,\sigma^\mu_{\beta\dot{\beta}}$. These matrices can then be used to define a BISPINOR

$$B_{\alpha\dot{\alpha}} = \sigma^\mu_{\alpha\dot{\alpha}}\,B^\nu\,g_{\mu\nu} \tag{A.14}$$

in terms of a standard Lorentz vector B^ν. This relationship can also be inverted with aid of the identity $\mathrm{Tr}\left[\sigma^\mu\overline{\sigma}^\nu\right] = 2\,g^{\mu\nu}$.

We can now define yet more matrices, which turn out to be rather useful:

$$(\sigma^{\mu\nu})_\alpha^{\ \ \beta} = \frac{i}{4}\Big(\sigma^\mu_{\alpha\dot{\alpha}}\,\overline{\sigma}^{\nu\,\dot{\alpha}\beta} - \sigma^\nu_{\alpha\dot{\alpha}}\,\overline{\sigma}^{\mu\,\dot{\alpha}\beta}\Big) \tag{A.15a}$$

$$(\overline{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \ \dot{\beta}} = \frac{i}{4}\Big(\overline{\sigma}^{\mu\,\dot{\alpha}\alpha}\,\sigma^\nu_{\alpha\dot{\beta}} - \overline{\sigma}^{\nu\,\dot{\alpha}\alpha}\,\sigma^\mu_{\alpha\dot{\beta}}\Big)\,. \tag{A.15b}$$

The matrices $\sigma^{\mu\nu}$ and $\overline{\sigma}^{\mu\nu}$ are the generators of Lorentz transformations acting on left- and right-handed Weyl spinors respectively:

$$\psi_\alpha \to \psi'_\alpha = (e^{-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}})_\alpha^{\ \ \beta}\,\psi_\beta\,, \qquad \overline{\chi}^{\dot{\alpha}} \to \overline{\chi}'^{\dot{\alpha}} = (e^{-\frac{i}{2}\omega_{\mu\nu}\overline{\sigma}^{\mu\nu}})^{\dot{\alpha}}_{\ \ \dot{\beta}}\,\overline{\chi}^{\dot{\beta}}\,.$$

They also have special properties under hodge duality:

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{1}{2i}\,\varepsilon^{\mu\nu\rho\tau}\,\sigma_{\rho\tau} && \text{selfdual} \\ \overline{\sigma}^{\mu\nu} &= -\frac{1}{2i}\,\varepsilon^{\mu\nu\rho\tau}\,\overline{\sigma}_{\rho\tau} && \text{anti-selfdual}\,. \end{aligned} \tag{A.16}$$

In Section 2.4.3 and Chapter 3 we perform instanton calculations which require us to work with a Euclidean spacetime metric $g_{\mu\nu} = \mathrm{diag}(+1,+1,+1,+1)$. We will make this difference clear by using Roman letters from the middle of the alphabet $(m, n, \ldots)$ to index Euclidean spacetime with the values 1, 2, 3 and 4. This change also alters some of the above definitions. Particularly, we now have

$$\sigma_{m\,\alpha\dot{\alpha}} = \left\{i\tau^1, i\tau^2, i\tau^3, \mathbb{1}_{2\times 2}\right\}_{\alpha\dot{\alpha}} , \qquad \overline{\sigma}_{m\,\dot{\alpha}\alpha} = \left\{-i\tau^1, -i\tau^2, -i\tau^3, \mathbb{1}_{2\times 2}\right\}^{\dot{\alpha}\alpha} , \tag{A.17}$$

and

$$\sigma_{mn} = \frac{1}{4}\left(\sigma_m \overline{\sigma}_n - \sigma_n \overline{\sigma}_m\right) , \qquad \overline{\sigma}_{mn} = \frac{1}{4}\left(\overline{\sigma}_m \sigma_n - \overline{\sigma}_n \sigma_m\right) , \tag{A.18}$$

with the (anti-)selfduality identities becoming:

$$\sigma_{mn} = \frac{1}{2}\varepsilon_{mnrt}\sigma_{rt} \qquad \text{selfdual} \tag{A.19}$$

$$\overline{\sigma}_{mn} = -\frac{1}{2}\varepsilon_{mnrt}\overline{\sigma}_{rt} \qquad \text{anti-selfdual} . \tag{A.20}$$

The sigma matrices (both σ^m and σ^{mn}) satisfy a plethora of identities that are invaluable when manipulating spinors. We list a few of the most useful ones here; see references [31] or [33] for more a comprehensive list of such identities (and their derivation).

$$\mathrm{Tr}\left[\sigma^m \overline{\sigma}^n\right] = \mathrm{Tr}\left[\overline{\sigma}^m \sigma^n\right] = 2\,g^{mn} \tag{A.21}$$

$$\left[\sigma^m \overline{\sigma}^n + \sigma^n \overline{\sigma}^m\right]_{\alpha}^{\ \beta} = 2\,g^{mn}\delta_{\alpha}^{\beta} \tag{A.22}$$

$$\sigma_{mn\,\alpha}^{\ \ \ \ \beta}\,\sigma_{mn\,\gamma}^{\ \ \ \ \delta} = \delta_{\alpha}^{\beta}\,\delta_{\gamma}^{\delta} - 2\,\delta_{\alpha}^{\delta}\,\delta_{\gamma}^{\beta} \tag{A.23}$$

$$\sigma^m_{\alpha\dot{\alpha}}\,\overline{\sigma}^{\dot{\beta}\beta}_m = 2\,\delta_{\alpha}^{\beta}\,\delta_{\dot{\beta}}^{\dot{\alpha}} \tag{A.24}$$

The last equation here can be usefully combined with equation (A.6) to provide further Fierz identities that aid the manipulation of spinor quantities containing σ matrices.

’t Hooft Matrices

The following matrices, which intertwine (Euclidean) spacetime indices $(m, n, \ldots)$ and SU(2) gauge group $(a, b, \ldots)$ indices were introduced by ’t Hooft [34] to facilitate the study of instantons.

$$\eta^a_{mn} \equiv \varepsilon_{amn} + \delta_{am}\delta_{n4} - \delta_{an}\delta_{4m} \ , \tag{A.25}$$

$$\overline{\eta}^a_{mn} \equiv \varepsilon_{amn} - \delta_{am}\delta_{n4} + \delta_{an}\delta_{4m} \ . \tag{A.26}$$

They are also (anti-)selfdual,

$$\eta^a_{mn} = \frac{1}{2}\varepsilon_{mnrt}\eta^a_{rt} \ , \qquad \overline{\eta}^a_{mn} = -\frac{1}{2}\varepsilon_{mnrt}\overline{\eta}^a_{rt} \ , \tag{A.27}$$

and can be related to the above sigma matrices:

$$\sigma_{mn} = \frac{i}{2}\eta^a_{mn}\tau^a \ , \qquad \overline{\sigma}_{mn} = \frac{i}{2}\overline{\eta}^a_{mn}\tau^a \ , \tag{A.28}$$

with τ^a being the usual Pauli matrices. One final relation that is useful when computing the instanton action is:

$$\eta^a_{mn}\,\eta^a_{pq} = \delta_{mp}\delta_{nq} - \delta_{mq}\delta_{np} + \varepsilon_{mnpq} \qquad \Rightarrow \qquad \eta^a_{mn}\,\eta^a_{mn} = 12 \ . \tag{A.29}$$

Scalar Conventions

It is often convenient to package the six real scalars fields ϕ^a of $\mathcal{N}=4$ sYM into three complex-valued fields:

$$\begin{aligned}
\Phi_1 &= \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2) \\
\Phi_2 &= \frac{1}{\sqrt{2}}(\phi^3 + i\phi^4) \\
\Phi_3 &= \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6)
\end{aligned} \tag{A.30}$$

This allows us to use the $\mathcal{N}=1$ superspace formalism to decompose the unique supermultiplet of $\mathcal{N}=4$ sYM into $\mathcal{N}=1$ multiplets. We end up with one vector superfield V and three chiral superfields $\Phi_{1,2,3}$ (here we use the same character to denote both the chiral superfield and its scalar component). This appears to break the R-symmetry $\mathrm{SU}(4)\to\mathrm{SU}(3)$ because to employ this language we have singled out a particular $\mathcal{N}=1$ sub-algebra, but this is just an artefact of the notation.

R-symmetry acts on the six real scalars ϕ^i of $\mathcal{N}=4$ sYM as an SO(6) rotation — the scalars transform in the vector (**6**) representation. In Chapter 3, when we carry out multi-instanton calculations in the formalism of references [44] and [71], we will find it more convenient to switch to a different basis for the scalar fields — one that makes use of the local isomorphism SO(6) $\cong$ SU(4). We take an antisymmetric tensor field $\Phi_{AB}(x)$ with $A, B = 1, \ldots, 4$, that transforms in the adjoint of the SU(4) R-symmetry group. To ensure this is fully equivalent to the vector (**6**) representation of SO(6), we must also impose a specific reality condition:

$$\frac{1}{2}\epsilon_{ABCD}\,\Phi_{CD} \;=\; \bar{\Phi}_{AB}\,. \tag{A.31}$$

In terms of the six real scalars ϕ^a this new representation can be written as [71]

$$\Phi_{AB} = \frac{1}{\sqrt{8}}\bar{\Sigma}^a_{AB}\phi^a\;, \qquad \bar{\Phi}_{AB} = -\frac{1}{\sqrt{8}}\Sigma^a_{AB}\phi^a\;, \qquad a = 1,\ldots,6\;, \tag{A.32}$$

where the coefficients Σ^a_{AB} and $\bar{\Sigma}^a_{AB}$ are expressed in terms of the 't Hooft η-symbols:

$$\Sigma^a_{AB} = \left(\eta^1_{AB}, i\bar{\eta}^1_{AB}, \eta^2_{AB}, i\bar{\eta}^2_{AB}, \eta^3_{AB}, i\bar{\eta}^3_{AB}\right)\;, \tag{A.33}$$

$$\bar{\Sigma}^a_{AB} = \left(-\eta^1_{AB}, i\bar{\eta}^1_{AB}, -\eta^2_{AB}, i\bar{\eta}^2_{AB}, -\eta^3_{AB}, i\bar{\eta}^3_{AB}\right)\;. \tag{A.34}$$

The Σ^a_{AB} matrices provide an isomorphism between the Lie algebras of SO(6) and SU(4) in much the same way the $\sigma^\mu_{\alpha\dot{\alpha}}$ matrices specify the local isomorphism SO(1,3) $\cong$ SU(2) $\times$ SU(2) in equation (A.14).

For completeness we show the explicit relationship between scalars in each basis, including the complex scalars of equation (A.30):

$$\begin{aligned}
\Phi_1 &= \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2) = 2\,\bar{\Phi}_{32} = 2\,\Phi_{41}\;, \\
\Phi_2 &= \frac{1}{\sqrt{2}}(\phi^3 + i\phi^4) = 2\,\bar{\Phi}_{13} = 2\,\Phi_{42}\;, \\
\Phi_3 &= \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6) = 2\,\bar{\Phi}_{21} = 2\,\Phi_{43}\;.
\end{aligned} \tag{A.35}$$

Appendix B.

The R-symmetry of the baryon-deformed ISS model

It is known that the R-symmetry of ISS SQCD manifests itself only as an approximate symmetry of the magnetic formulation that is broken explicitly in the electric theory by the mass terms of electric quarks m_Q. Here we want to quantify this statement and show that the R-symmetry breaking in the microscopic theory is controlled by a small parameter, $m_Q/\Lambda = \mu^2/\Lambda^2 \ll 1$. As such, the intrinsic R-breaking effects and deformations can be neglected. This justifies the approach we follow in Chapter 5 where the R-symmetry of the magnetic theory is used to constrain the allowed deformations. Consequently, the R-symmetry-preserving baryon deformation in equation (5.2) constitutes a generic superpotential.

We first consider the massless undeformed SQCD theory. As we saw in Chapter 2 the global symmetry is $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_B \times \mathrm{U}(1)_A \times \overline{\mathrm{U}(1)}_R$. Table B.1 lists the charges of matter fields of the electric and the magnetic formulations. Following the well-established conventions of reference [45] the $\overline{\mathrm{U}(1)}_R$ symmetry is taken to be anomaly-free, and the axial symmetry $\mathrm{U}(1)_A$ is anomalous. The $\mathrm{U}(1)_R$ symmetry featuring in Chapter 5 will be constructed below as an anomalous linear combination of the $\overline{\mathrm{U}(1)}_R$, $\mathrm{U}(1)_A$ and $\mathrm{U}(1)_B$.

The scale Λ is charged only under the $\mathrm{U}(1)_A$, which identifies it as the anomalous $\mathrm{U}(1)$. In the usual fashion, the $\mathrm{U}(1)_A$-charge of Λ in Table B.1 is determined from the non-perturbative superpotential, cf. equation (4.12),

$$W_{\rm dyn} = (N_f - N_c)\left(\frac{\det_{N_f}\widetilde{Q}Q}{\Lambda^{3N_c-N_f}}\right)^{\frac{1}{N_f-N_c}} \tag{B.1}$$

Table B.1 also shows that the superpotential W is only charged under the $\overline{\rm U(1)}_R$, making it clear that this is an R-symmetry (such that $\int d^2\theta\, W$ is neutral).

Finally, the charges of magnetic quarks φ, $\widetilde{\varphi}$ are derived from the matching between electric and magnetic baryons, $B_E/\Lambda^{N_c} = b_M/\Lambda^{N_f-N_c}$, $\widetilde{B}_E/\Lambda^{N_c} = \widetilde{b}_M/\Lambda^{N_f-N_c}$ which implies (schematically)

$$\left(\frac{\varphi}{\Lambda}\right)^{N_f-N_c} = \left(\frac{Q}{\Lambda}\right)^{N_c}, \qquad \left(\frac{\widetilde{\varphi}}{\Lambda}\right)^{N_f-N_c} = \left(\frac{\widetilde{Q}}{\Lambda}\right)^{N_c}. \tag{B.2}$$

The charges of Φ are read off from its definition, $\Phi = \frac{Q\widetilde{Q}}{\Lambda}$. As a consistency test on these charges, one can easily verify that the magnetic superpotential $W = \varphi\Phi\widetilde{\varphi}$ is automatically neutral under $\mathrm{U}(1)_A$, $\mathrm{U}(1)_B$ and has the required charge 2 under the R-symmetry.

We now introduce mass terms $m_Q\widetilde{Q}Q$ in the superpotential of the electric theory. We want to continue describing the symmetry structure in terms of the parameters of the IR magnetic theory. For this purpose we write the quark masses as $m_Q = \frac{\mu^2}{\Lambda}$. This mass-deformation breaks the flavour group $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ to the diagonal $\mathrm{SU}(N_f)$ (if, for example, all quark masses were the same). It also breaks $\mathrm{U}(1)_A \times \overline{\mathrm{U}(1)}_R$, to a

	$\mathrm{SU}(N_f)_L$	$\mathrm{SU}(N_f)_R$	$\mathrm{U}(1)_B$	$\mathrm{U}(1)_A$	$\overline{\mathrm{U}(1)}_R$
Q	$\square$	$\mathbf{1}$	1	1	$\frac{N_f-N_c}{N_f}$
$\widetilde{Q}$	$\mathbf{1}$	$\overline{\square}$	-1	1	$\frac{N_f-N_c}{N_f}$
Λ	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2N_f}{3N_c-N_f}$	0
W	$\mathbf{1}$	$\mathbf{1}$	0	0	2
φ	$\overline{\square}$	1	$\frac{N_c}{N_f-N_c}$	$\frac{2N_f-3N_c}{3N_c-N_f}$	$\frac{N_c}{N_f}$
$\widetilde{\varphi}$	1	$\square$	$-\frac{N_c}{N_f-N_c}$	$\frac{2N_f-3N_c}{3N_c-N_f}$	$\frac{N_c}{N_f}$
$\Phi = \frac{Q\widetilde{Q}}{\Lambda}$	$\square$	$\overline{\square}$	0	$2 - \frac{2N_f}{3N_c-N_f}$	$2\frac{N_f-N_c}{N_f}$

Table B.1.: Charges of electric and magnetic picture fields under the global symmetries of massless SQCD.

	$U(1)_B$	$U(1)_A$	$\overline{U(1)}_R$
Q	1	1	$\frac{2}{7}$
$\widetilde{Q}$	-1	1	$\frac{2}{7}$
Λ	0	$\frac{7}{4}$	0
W	0	0	2
φ	$\frac{5}{2}$	$-\frac{1}{8}$	$\frac{5}{7}$
$\widetilde{\varphi}$	$-\frac{5}{2}$	$-\frac{1}{8}$	$\frac{5}{7}$
Φ	0	$\frac{1}{4}$	$\frac{4}{7}$

Table B.2.: Charges under $U(1)_B \times U(1)_A \times \overline{U(1)}_R$ for $N_c = 5$ and $N_f = 7$

linear combination U(1) subgroup. If in addition, we introduce the baryon deformation, as in Section 5.2, it breaks the third $U(1)_B$ factor. In total, the combined effect of the two deformations breaks $U(1)_B \times U(1)_A \times \overline{U(1)}_R$ to a single $U(1)_R$. This is the R-symmetry used in Chapter 5 and it is anomalous since Λ is charged under it.[1]

To explicitly construct this surviving $U(1)_R$ for the model of Section 5.2, we set $N_c = 5$ and $N_f = 7$ and list the three U(1) charges in Table B.2. It is now clear that the $U(1)_R$ symmetry of Section 5.2 is the linear combination of the three U(1)'s with charge

$$R = \overline{R} + \frac{40}{7} A + \frac{2}{5} B \ . \tag{B.3}$$

This is the unique unbroken linear combination surviving both the mass- and baryon-deformation, $\delta W = -\mu^2 \Phi + m\varphi^2$, of the magnetic theory with the charges listed in Table B.3. In the magnetic Seiberg-dual formulation, the $U(1)_R$ symmetry is manifest. It is the symmetry of the perturbative superpotential (5.2) which is only broken anomalously.

In the electric picture, the $U(1)_R$ symmetry is broken by the mass terms $m_Q \widetilde{Q} Q$ on account of the explicit Λ-dependence of the masses $m_Q = \frac{\mu^2}{\Lambda}$. It is also broken by the baryon deformation (again in the electric theory language) $\frac{1}{M_{Pl}^2} Q^5$ because the magnetic baryon deformation parameter m in equation (6.20) explicitly depends on Λ. Thus the apparent $U(1)_R$ symmetry of the IR theory is only approximate, and is lifted in the UV theory. However, the R-symmetry is broken in a controlled way, by a parameter of the order of m_Q/Λ. To verify this, note that in the limit $m_Q \to 0$, the

[1] Note that the two deformations are associated with orthogonal U(1)'s and are therefore independent.

	$U(1)_R$
φ	1
$\widetilde{\varphi}$	−1
Φ	2
Λ	10
μ	0
W	2
Q	$6+\frac{2}{5}$
$\widetilde{Q}$	$6-\frac{2}{5}$
$m_Q=\frac{\mu^2}{\Lambda}$	−10

Table B.3.: Charges under $U(1)_R$ for $N_c = 5$ and $N_f = 7$

electric quark masses disappear while the baryon deformation $\frac{1}{M_{Pl}^2}Q^5$ is invariant under the R-symmetry $U(1)_{R'}$ generated by:

$$R' = \overline{R} + \frac{5}{7}A - \frac{3}{5}B\ . \tag{B.4}$$

This linear combination is different from the one in equation (B.3), but in the massless limit we are considering it is a perfectly valid, classically conserved R-symmetry that protects the baryon deformation in the electric theory and forbids e.g. anti-baryon deformations of the form $\frac{1}{M_{Pl}^2}\widetilde{Q}^5$. Thus in the massless limit there is always an R-symmetry that protects baryon deformations either in the electric or in the magnetic formulation. When quark masses are non-vanishing, this R-symmetry is broken by m_Q/Λ. Indeed, if one formally sends $\Lambda \to \infty$ holding μ and m fixed, the dynamical non-perturbative superpotential disappears and the exact $U(1)_R$ is recovered.

In general, anomalous global symmetries do not match in the magnetic and the electric descriptions. The $U(1)_R$ of Chapter 5 is an approximate symmetry so in principle one should allow generic $U(1)_R$-violating deformations. For example, one can add an antibaryon $\widetilde{b}$ deformation to the superpotential (5.2). However, these deformations are suppressed relative to the $U(1)_R$-preserving ones by the small parameter, $m_Q/\Lambda = \mu^2/\Lambda^2 \ll 1$, and therefore can be neglected.

Colophon

This thesis was made in $\LaTeX\,2_\varepsilon$ using the "hepthesis" class. The Wordle on page 1 was generated by the fabulous applet at `http://www.wordle.net/`.

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