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Semantic objects and paradox: a study of Yablo's omega-liar

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SEMANTIC OBJECTS AND PARADOX: A STUDY OF YABLO'S OMEGA-LIAR

by

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An Abstract

Of a thesis submitted in partial fulfillment of the requirements for the Doctor of Philosophy degree in Philosophy in the Graduate College of The University of Iowa

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ABSTRACT

To borrow a colorful phrase from Kant, this dissertation offers a prolegomenon to any future semantic theory. The dissertation investigates Yablo's omega-liar paradox and draws the following consequence. Any semantic theory that accepts the existence of semantic objects must face Yablo's paradox, but since restrictions on such intentional objects restrict thought, semantic objects must be abandoned.

The dissertation endeavors to position Yablo's omega-liar in a role analogous to that which Russell's paradox has for the foundations of mathematics. Russell's paradox showed that if we wed mathematics to sets, then because of the many different possible restrictions available for blocking the paradox, mathematics, intolerably, fractionates. It is similarly intolerable to have restrictions on the 'objects' of Intentionality. Hence, in the light of Yablo's omega-liar, Intentionality cannot be wed to any theory of semantic objects. We ought, therefore, to think of Yablo's paradox as a natural language paradox, and as such we must accept its implications for the semantics of natural language, namely that those entities which are 'meanings' (natural or otherwise) must not be construed as objects.

To establish our result, Yablo's paradox is examined in light of the criticisms of Priest (and his followers). Priest maintains that Yablo's original omega-liar is flawed in its employment of a Tarski-style T-schema for its truth-predicate. Priest argues that the paradox is not formulable unless it employs a "satisfaction" predicate in place of its truth-predicate. Priest is mistaken. However, it will be shown that the omega-liar paradox depends essentially on the assumption of semantic objects. No formulation of the paradox is possible without this assumption.

Given this, the dissertation looks at three different sorts of theories of propositions, and argues that two fail to specify a complete syntax for the Yablo sentences. Purely intensional propositions, however, are able to complete the syntax and thus generate the paradox. In the end, however, the restrictions normally associated with purely intensional propositions begin to look surprisingly like the hierarchies that Yablo sought to avoid with his paradox. The result is that while Yablo's paradox is syntactically formable within systems with formal hierarchies, it is not semantically so.

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CERTIFICATE OF APPROVAL

PH.D. THESIS

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INTRODUCTION

To borrow a colorful phrase from Kant, this dissertation offers a prolegomenon to any future semantic theory. The dissertation investigates Yablo's omega-liar paradox and draws the following conclusion: any semantic theory that accepts the existence of semantic objects must face Yablo's paradox. But given Brentano's principle that intentionality is a wholly unbridled capacity for thoughts to be directed toward any object whatever (intentionally inexistent or otherwise) the paradox is unsolvable for semantic object theorists. The technical notion of an "object" used here is borrowed from Frege's formal distinction between objects (that are complete) and entities that are "unsaturated" or "incomplete". A semantic object is at once an object in this technical sense and also has intrinsic intentionality. An entity has intrinsic intentionality if it is about something (represents something) intrinsically and not in virtue of its being used to mean something derivatively by an agent or otherwise.[1] The predicational component of a thought is unsaturated and not itself an object. It is in virtue of this that it accounts for the unity of thought. Our conclusion is that 'meanings', whatever they may be, are unsaturated. To impose restrictions on what can be an 'object' of thought is to reject Brentano's principle of intentionality. Yet any semantic theory embracing semantic objects must impose restrictions on what semantic objects there are to respond to Yablo's omega-liar paradox.

The dissertation positions Yablo's omega-liar in a role analogous to that which Russell's paradox has for the foundations of mathematics. Russell's paradox showed that if we understand mathematics in terms of sets, then because of the many different

[1] Any object whatsoever can have derived intentionality—it can be used to represent something.

possible restrictions available for blocking the paradox, mathematics fractionates. There would be, undesirably, different mathematics. It is similarly problematic to have restrictions on the 'objects' of intentionality. Hence, in the light of Yablo's omega-liar, intentionality cannot be wed to any theory of semantic objects. We ought, therefore, to think of Yablo's paradox as a natural language paradox, and accept its implications for the semantics of natural language.

Quine might be pleased by this result. He famously argued that 'meanings' are "creatures of darkness" and that semantic theories should avoid the "myth of the museum" according to which meanings are objects assigned to words.[2] Quine argued that only in the body of a sentence does a word have meaning and that only in the body of a theory does a sentence have meaning. He worked to build a naturalized semantics with meaning construed in terms of physical dispositions for translation. This naturalization of 'meaning' couched in behaviorism was rejected soundly by Chomsky who argued for the innateness of the transformational grammar underlying all languages. Modern externalism with respect to 'meaning' (such as found in the work of Dretske) appeals to causal regularities of stimulus-response, but it fares no better than Quine's approach. And first person introspective awareness seems wholly lost in every current naturalization project. It is not in the scope of this work to propose a new semantic theory avoiding semantic objects. The work is negative only. It does not endorse naturalism, but only demands that those entities which are 'meanings' (natural or otherwise) must not be construed as objects.

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[2] Quine, (1964)

To establish these results, Yablo's paradox is examined in light of the criticisms of Priest (and his followers). Priest maintains that Yablo's original omega-liar is flawed in its employment of a Tarski-style T-schema for its truth-predicate. Priest argues that the paradox is not formulable unless it employs a "satisfaction" predicate in place of its truth-predicate. Priest is mistaken. It will be shown that the omega-liar paradox depends essentially on the assumption of semantic objects. No formulation of the paradox is possible without this assumption. And yet, much like unrestricted classes lead to Russell's paradox, allowing unrestricted semantic objects yields paradoxes like the traditional Epimenides Liar paradox and Yablo's omega-liar paradox. Restrictions in response to these paradoxes, however promising they may seem, fail under scrutiny to deliver the desired results.

The first chapter introduces the traditional classification of paradoxes and semantic propositionalism. It sets out the traditional distinction between semantic and syntactic paradoxes, and presents the distinction between semantic entities and semantic objects. It also sets out several versions of the Liar paradox that trace the generation of such paradoxes to different forms of self-referential circularity. Yablo's intent is to show that semantic paradox can arise without self-referential circularity.

The second chapter gives a formal derivation of Yablo's omega-liar paradox. It makes a distinction between a syntactically correct string of words (a sentence^{d}) and a sentence that actually exemplifies intrinsic semantic relations (a claim). It ends with a presentation of two features that Yablo's paradox shares with the Liar paradox and four features that are unique to the Yablo paradox.

The third chapter sets out the distinction between a formal and a natural paradox as well as between questions of internal coherence and questions of external relevance. The focus of this chapter is on the relation between our written representations of our natural language and the formal interpretation we bring to bear to understand it in our philosophy of language.

The fourth chapter addresses Priest's criticism that Yablo's derivations illicitly employ a Tarski-style T-schema and that a structural self-referential circularity is revealed once the paradox is reformulated with the relation of "satisfaction". I present reasons for thinking that even if Priest is right that Yablo *must* revise the paradox in terms of satisfaction, he has failed to generate an omega-liar paradox in terms of a satisfaction predicate as Priest's version is syntactically incoherent. Priest's version characterizes the syntax of a predicate expression in terms that appeal to that very syntactic expression. It is analogous to offering a definition in which the *defeniendum* occurs in the *definiens*. Semantic self-referential circularity is bewildering, but syntactic self-referential circularity is simply incoherent. If this is the sort of self-referentiality Priest finds in the omega-liar, then there just is no omega-liar, at least nothing that might qualify as a *semantic* paradox.

Despite this, there is an omega-liar based on satisfaction that can avoid this syntactic incoherence (I present two), but only at the price of syntactic non-wellfoundedness. Obviously this is not acceptable, but it is the best one can offer if Yablo's truth-predicate is to be replaced by a satisfaction-predicate.

The fifth chapter restates Priest's concern that Yablo has not succeeded in avoiding self-referential circularity in terms of the effectiveness of Yablo's infinite sequence of sentences. A sequence is effective just in case two claims hold. First, given a position in the sequence, you can determine which element occupies that position. Second, given an element in the sequence, you can determine its position. Understood in this way, Priest's work calls attention to the fact that Yablo's paradox employed an effective consecutive series of sentences. Priest's arguments that the Yablo paradox employs structural self-reference can be captured by talking about the effectiveness of the sequence —a sort of "revenge of self-reference". This chapter presents a new version of the omega-liar that does not rely on effectiveness.

The sixth chapter begins with an overview of the conclusions drawn from our extended analysis of Yablo's paradox. It looks at three different sorts of theories of propositions, and argues that two fail to specify a complete syntax for the Yablo sentences. Purely intensional propositions, however, are able to complete the syntax and thus generate the paradox. I present a derivation and talk about the ways that a theory of propositions might be restricted to avoid certain paradoxes. In the end, the restrictions begin to look surprisingly like the hierarchies that Yablo sought to avoid with his paradox (thereby showing self-reference to be not to blame for the semantic paradoxes). And even if they did work, they only do so by violating reasonable basic assumptions about expression and thought. The supporter of semantic objects, then, must, in the face of the Yablo paradox, accept restrictions, but these restrictions are problematic in and of themselves. As such, Yablo's paradox forms the basis for this dissertation's rejection of semantic objects.

CHAPTER 1

GROUNDWORK FOR PROPOSITIONALISM AND SEMANTIC PARADOX

This dissertation investigates semantic paradox through an extended analysis of Yablo's omega-liar paradox. This chapter does three things: first, it sketches the traditional distinction between semantic paradoxes and syntactic paradoxes; second, it sketches the modern origins of propositionalism in philosophy of language followed by a rough account of the current debate; third, it presents Yablo's paradox as a descendent of the Liar.

Section 1 presents the useful traditional semantic-syntactic paradox distinction. First, it provides a framework from which to understand Yablo's paradox positively by showing which paradoxes it is more like (semantic) and negatively by showing which paradoxes is less like (logical/mathematical). Further benefit derives from the paradigms of each type of paradox. The paradigmatic semantic paradox is the Epimenides' Liar paradox. A presentation of the Liar helps flesh out the notion of semantic paradox and serves as a foundation for Yablo's paradox. The paradigmatic logical/mathematical paradox is Russell's paradox of the set of all non-self-membered sets. Not only does it make Yablo's paradox more clear, but it also caused a radical shift in set-theory around the turn of the twentieth century (right around the time Frege mounted his argument for propositionalism). The restrictions on what conditions determine sets (where different restrictions formulate different consistent set theories avoiding the paradox) serve as a model for this investigation of what restrictions might be made on semantic objects.

Section 2 presents propositionalism. Roughly, this is the view that propositions exist as mind-independent extra-linguistic abstract objects that are both truth-bearers and the meanings of declarative sentences.[3] They are mind-independent in the sense that they are external and their existence is not dependent upon minds. They are extra-linguistic in that they do not depend for their structure or existence on any linguistic practice. On the view, our linguistic practices use propositions. They are vehicles of reference insofar as a particular sentence refers to its referent via the proposition that is its meaning. In this context, when the proposition that is the meaning of a sentence accurately describes the world, we call the sentence true because the proposition is true. In short, propositionalism is the view that these entities exist and serve roughly these roles in philosophy of language.[4]

The classic statement of propositionalism comes from Frege and his discussion of contingent identity statements (specifically, the ancient Greek discovery that the Morning Star and the Evening Star were one and the same). His discussion serves a two-fold purpose. First, it introduces propositions which he calls "thoughts" (*Gedanken*) as intensional entities that are the meanings of declarative sentences. Second, it has become the paradigmatic example of the linguistic turn in philosophy, where worries about language and expression form a foundation for alterations in one's ontology. In Frege's case, this takes the form of accepting a non-mental, non-physical "third realm" populated

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[3] George Bealer, (1998) lists five qualifications of propositionalism discussed later in this chapter.

[4] The particular roles depend on individual theories, as the entities may serve as bearers of truth-values but not meanings on some view, or as mind-dependent but non-linguistic. In general, these are the parts propositions play in philosophy of language theories. They traditionally serve other roles as well, in fields as varied as value theory, logic, and philosophy of mind, among others.

by propositions. The current project takes this as a model, and proceeds from linguistic worries to a metaphysical conclusion (i.e., from the existence of Yablo's paradox to a renunciation of intensional propositions/semantic objects). First I present a sketch of modern propositionalism and the debate between intensional propositions (like Fregean thoughts) and extensional propositions (like functions, ordered sets, etc.). In this context I define the technical notion of a semantic object as an intrinsically intensional proposition.

Stephen Yablo calls his paradox Liar-like.[5] Section 3 shows its foundations in the Liar, by following the discussion of propositionalism with a presentation of several versions of the Liar paradox that build gradually toward Yablo's paradox. This makes the paradox clear while making sense of Yablo's claim that it is Liar-like.

I aim in this chapter to set forth the two key components of the dissertation (Yablo's paradox and propositionalism) and to sketch their relation to one another to give a feel for the connections that will come out through the dissertation. Along the way I make a number of distinctions and some technical definitions that will be useful for the remainder of the dissertation.

1.1 – Semantic Paradox and Syntactic Paradox

There is a traditional distinction between semantic and syntactic (sometimes called set theoretical or mathematical/logical) paradoxes.[6] As the name suggests,

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[5] Yablo, (1993), p. 252

[6] Syntax consists in the rules that govern how pieces of a language are put together to form expressions. Semantics is the interpretation of those symbols. It's about what those pieces of language (words and clauses) refer to. It is sometimes called an interpretation of the symbols of the language. Put another way, semantics is about meaning relations between words and objects.

semantic paradoxes fundamentally involve reference or meaning. They do not involve a prediction of contradictory events from the laws of physics (as in the twin paradox of special Relativity), nor the assumption of sets (as does Russell's paradox). Semantic paradoxes derive from expressions of semantic relations ("reference", "meaning", "designation", "truth" etc.). Physical and mathematical paradoxes do not. They fall out of naïve assumptions that make no explicit mention of any semantic relationship.

To make the conceptual landscape a little more clear, consider a distinction relevant to discussions of semantic paradoxes. Let "semantic entities" be a catchall phrase that is topic neutral as to whether its referent is linguistic or not, mind-dependent or not, intensional or extensional, or intrinsically or derivatively intentional. The only qualification is that all semantic entities exemplify semantic relations: they display intentionality. Let "semantic object" refer to a subset of this group of intrinsically intentional entities. They have a mind-independent existence (hence the term "object" rather than entity), and may serve as the meanings of statements. They may be understood linguistically or not, but must be independent.

The Liar is the paradigmatic example of a semantic paradox. The Liar Paradox dates back to the ancient Greeks. According to the story, Epimenides (who was a Cretan) said all Cretans were liars.[7] If he is telling the truth, then Cretans are, in fact, liars. As a Cretan himself, he must also be a liar. He thus impugns himself with his statement. So he is not telling the truth.[8] But then, if indeed all other Cretans are liars, he must be telling the truth after all.[9]

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[7] Russell, 1908, p.222

[8] On one very simplistic conception of lying, all that is involved is asserting a falsehood.

The Epimenides can be reformulated with the claim: "All propositions asserted by Cretans are false". This claim ought to give us pause. If all Cretans do indeed always assert falsehoods, then Epimenides asserts the truth if and only if he asserts a falsehood. Paradox. Observe that in this formulation, it is clear that the paradox assumes that there is an object (i.e., a proposition) that Epimenides' claim stands for. The intentional relations of this entity are the basis for the paradox. It is not just that Epimenides made noises or marked "*p*". It is that there is an object which is the meaning *p*. It is in virtue of the existence of an object that is the meaning of the sentence that we have a paradox. If we considered his sentence as a mere object (non-referentially) no paradox would arise. We would simply have a series of sounds (or shapes, if written). And as Putnam argued, following Wittgenstein's criticism of his earlier *Tractarian* theory, any theory that takes shape (iconic isomorphism) alone to generate semantic relations is occult—a magical theory of reference.[10]

To see this more clearly, consider the criticism of the Epimenides advanced by Bertrand Russell in 1906.[11]

(1) The proposition now asserted is false.

If (1) asserts a proposition, it is true if and only if it is false. But if (1) does not assert a proposition, no paradox can arise. Let Ap=df. p is a proposition now asserted. We have:

(1*) $(\exists p)(Aq \Leftrightarrow_q q{=}p\ .\&.\ {\sim}p)$

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[9] Of course, more exactly understood, to lie essentially involves intentionally misleading so as to gain advantage producing harm. The Cretan may well not have this intent, and so is not lying. What he says is false, but its not clear that it follows that what he says is true.

[10] Putnam, Reason Truth and History, Ch.1

[11] From, "On '*Insolubilia'* and their Solution by Symbolic Logic".

Now, (1*) may be regarded as true if and only if some unique proposition now asserted is false. If (1*) is not itself regarded as a proposition – i.e., if asserting (1*) is not to assert a proposition – then (1*) merely looks like other symbols that do express propositions. On this line of reasoning, as (1*) fails to actually express a proposition no paradox arises. Recall that we are using the phrase "semantic object" to refer to an object that intrinsically exemplifies an intensional relation (like a proposition). The Liar, then, is not a matter of the appearance of reference, but successful engagement of a semantic object that bears semantic relations: "meaning", "truth", and so on. The Epimenides Liar paradox arises from the fact that Epimenides' claim asserts a proposition. We can then understand the existence of a semantic object as key to the Liar paradox, and on one view, dependence on semantic objects unites semantic paradoxes.

In logical/mathematical paradoxes, like Russell's paradox, we get a contradiction, but not from using and mentioning semantic relations. The role played by semantic objects in this is akin to a breach of the distinction between use and mention. The breach of use and mention is incoherence. But semantic objects yield paradox (not incoherence). There is a sense in which a semantic object can be used (has aboutness) and yet can, at the same time, occur mentioned. This happens when we speak of Epimenides as "asserting a proposition". If this is the case, the paradox derives from certain ontological assumptions.[12] Before Russell, set theory operated using a naïve comprehension principle. In set theory, naïve comprehension principles say that every open well-formed

[12] I use ontological here to mean independent of semantic relations. It refers to objects considered without reference to their semantic relations.

formula (wff) determines a set of all and only those entities satisfying the wff.[13] No matter what wff you imagine serving as membership conditions, a theory with naïve comprehension countenances a set whose members meet those conditions.

On such a theory, many sets will be members of themselves: the set of all sets, the set of abstract objects (sets are abstract), the set of all things that are not my dog, Elenor, etc., are all members of themselves. Of course many sets will also fail to be self-members. The set of things that neither have members nor are identical to the empty set, the set of physical things (as sets are not physical), the set of my pets (since no set is itself a pet of mine). These are sets that are not members of themselves.

Russell, in accord with naïve comprehension, [14] considers the set of all non-self-membered sets – a set whose members are all and only those sets who fail to be members of themselves. Call this the "Russell set". One can then formulate a problematic question: Is the Russell set a member of itself? To see the paradox, first assume that the Russell set is a member of itself. In that case, it must meet the membership condition, (namely, non-self-membership). In which case it is *not* a member of itself, and we get the result that if the Russell set is a self-member, then it is not a self-member. It follows that the Russell set is not a member of itself. But now it meets the membership condition (namely, non-self-membership). Hence, the Russell set is a member of itself. Contradiction.

In the case of Russell's paradox, we have a situation where the existence of the set yields paradox. Of course we must use language to derive the paradox, but the paradox is there

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[13] This is also called abstraction in some cases. See, e.g., Suppes, 1972, p.5.

[14] Consider this formal definition of naïve comprehension: $(\exists y)(\forall x)(x \, \varepsilon \, y \leftrightarrow Ax)$. There is an object, y, such that for any wff A, that object contains as members all the As.

regardless of whether there are semantic relations. Liar paradoxes, in contrast, rely essentially on the assumption of semantic entities bearing semantic relations.

Russell's paradox is famous largely because of its impact on set theory as a foundation for mathematics.[15] There are many different modern set theories. Zermelo-Frankel set theory adopts axioms for sets, one among which is Zermelo's famous *A* (separation) axiom. This axiom assures that for every set there is a subset whose members meet any well-formed condition whatsoever. It follows from this axiom that there is no universal set. If there were, *Aussonderung* would assure that it has a subset of objects that are not members of themselves. This would revive Russell's paradox. The theory introduces axioms for each new kind of set, empty set, pair set, power-set, restricted complement set, etc.[16] An alternative theory, based on similar ideas is to distinguish classes from sets. A class is a set if and only if it either the empty set or is a member of some class. Some classes, like the Russell class, are too big to be members of classes or sets.[17] (All sets are classes, but not all classes are sets). There are yet other set theories. There is the simple-type theory of sets, whose grammar makes it meaningless to speak of a set being (or not being) a member of itself. On this view, there is a universal set of type (t) whose members are objects of type t. There is yet also Quine's set theory NFU (new foundations with urelements), which is the closest consistent (so far as anyone knows) theory to naïve set theory. The set theory NFU embraces a genuinely universal set and its genuine complement and its grammar is wholly type-free. It restricts the wffs

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[15] Clark, p.105

[16] See, e.g., Suppes.

[17] See, e.g., Monk.

comprehending sets to those which are simple-type stratifiable. There seems to be no neutral way to adjudicate between such rival set theories, though Zermelo-Frankel continues to be most popular. As we shall see in a later chapter, many of these rival intuitions for forming set theories can be paralleled when it comes to forming theories of semantic objects. But for the present, let us turn to a discussion of the view supporting semantic objects, normally called "propositionalism."

1.2 – Propositionalism, Old and New

In the early twentieth century Russell's paradox proved devastating to Frege's mature logical system set forth in his *Grundgesetze der Arithmetic*. Once Russell had written him to inform him of the discovery of the paradox, Frege abandoned his system. This was a major setback for Frege, but also one for Russell as they shared a common logicist goal: they wanted to prove that mathematical truths were really logical truths.[18] This was important as they held mathematical truths were mind independent, and so could not be captured by the idealism of Kant's *Transcendental Aesthetic*. Moreover, Frege and Russell agreed that if the truths of mathematics were to play their useful role, they could not be understood as happenstance psychological dispositions produced by evolution. The need for objectivity formed a cornerstone of Frege's argument that, in order to understand language and the truths of the world, we must posit an abstract third realm populated with meanings.[19] Frege called these meanings "senses" ("*Sinne*"), and called the sense of a sentence a "thought" ("*Gedanke*"). These thoughts (i.e., the

[18] Their method was one of derivation. Frege's system was designed to be the foundation from which mathematical truths would be derived.

[19] Frege, "Thought", reprinted in Frege, (1997)

meanings of sentences) are now better known by the term, “proposition”. Fregean propositions were abstract, independent bearers of truth-values and served as the meanings of sentences.

Frege had three parts to his argument for his third realm. First, he considered the informativity of astronomical discoveries such as that the Morning Star is identical to the Evening Star. He concluded that meanings (or senses) were needed to get fine-grained distinctions between different ways of thinking of a single thing (like Venus, the referent of “Morning Star” and “Evening Star”) and account for the informativity of such claims. Second, he thought meanings needed to be objective and public to serve as sharable, and thus they could not be in the head. And finally, he thought meanings needed to be non-physical, since they are not in space and time. Let’s consider what is sometimes called, “Frege’s Puzzle about Identity.”

Frege considered the case of Hesperus (the Evening Star) and Phosphorus (the Morning Star).[20] He questioned how we ought to understand contingent informative identity claims. To see this, we first need to get clear on the distinction between intensional and extensional contexts. They are ways of understanding reference relations that line up roughly with the *de re-de dicto* distinction. First let us consider the *de re-de dicto* distinction. The term “*de re*” means “of (concerning) the thing”. The term “*de dicto*” means “of the word.” We thus use these to distinguish claims and thoughts about an object itself from claims and thoughts referring to particular ways of picking out that object. We understand *de re* claims independently of how we refer to things, and *de dicto* claims in light of how we refer to them. There is no *de re* difference (i.e., no difference in

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[20] This example comes from Frege, “On *Sinn* and *Bedeutung*”, reprinted in Frege, (1997).

the object) between calling the man "Larry" and calling him "Ben's father". But there are certainly *de dicto* differences, as there are various connotations associated with one term rather than the other. There are differences in talking about him as a father and talking about him as a human named "Larry". These differences can be captured when considering *de dicto* contexts (i.e., contexts of the words). Other times we want to avoid the peculiarities of linguistic descriptions, and so we wish to talk just about the thing itself. These are *de re* contexts. This familiar distinction is useful to keep in mind when considering the difference between intensional and extensional readings of particular claims.

Consider the phrase "the tallest person in the room". When using this phrase, we may be trying to refer to a particular person or we may be trying to refer to a particular feature of a person. In the first case, if I am trying to point out Luther, and he is the tallest person in the room, then I might say, "See, the tallest person in the room is Luther." On the other hand, perhaps I want to make a claim not about a particular person (like Luther) but instead someone whose identity does not matter to my claim in the same sense as before. I might say, "I bet the tallest person in the room is good at playing basketball." In this case, I am referring as much to the height of the person as I am the person. The particular identity of the tallest individual does not matter so much as their exemplification of the property *being the tallest person in the room*. Note that in the first case, when someone taller than Luther walks in, my claim will not get the job done. Once Luther is not the tallest person in the room, the sentence is false. On the other hand, my claim about basketball skills is independent of such change. Should an even taller person walk into the room, my claim still applies, though now it applies to that person. In the first case, we have a way of talking about a particular object. In the second case, we have a way of talking about an object based on a feature of that object. We may understand the first context as extensional and the second as intensional.

Understood extensionally, then, "the tallest person in the room" simply refers to that person. To understand the phrase extensionally is to understand it as relational[21] (i.e., as only about the object referred to) or diaphanous (as if the sign gives way to the referent). These are like *de re* readings on which the object of concern is the referent (i.e., the tallest man in the room). Let the tallest man in the room be Luther, and Luther be the only person in the room wearing sunglasses. In extensional terms, there is no difference between the claims, "I'm looking for the tallest person in the room," and, "I'm looking for Luther," and, "I'm looking for anyone with sunglasses." Since the referent of each term is the same, the claims are extensionally equivalent.

Understood intensionally, however, the phrase "the tallest man in the room" refers in a particular way, in this case through Luther's exemplification of the property *being the tallest man in the room*.[22] Intensional contexts are sometimes referred to as notional (involving a particular way of thinking of a referent, a particular notion) or opaque (as the way of thinking or referring to the referent stands between an agent and the referent). These are similar to *de dicto* readings, where more than the referent matters: the mode of referring matters. In intensional terms, there is a big difference between the claims, "I'm looking for the tallest person in the room," and, "I'm looking for Luther," and, "I'm

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[21] Salmon, (1986), see, e.g., p.4

[22] Note that: For all x, refers(s, x) iff refers (s', x) does not imply s=s'. F=G is not well-formed for Frege because objects (subjects) flank the identity sign on the Fregean system, not predicates. So on his system, we need to be careful in terms of what we're putting aside the identity sign.

looking for anyone with sunglasses." It is only happenstance that these all refer to one and the same person. A taller person could walk in the room, as could a woman wearing sunglasses, and the subsequent change in referent matters to intensional contexts. In short, extensional contexts ignore differences in ways of referring and understand claims as about objects themselves. Intensional contexts focus on ways of referring, and understand claims as ways of picking out objects where the manner of reference chosen is important.

This is not to be confused with intentional contexts. Intentional contexts involve particular intentional states. Intentional contexts are even stricter about implication relations than intensional contexts. Not only do intensional contexts need to hold, but in addition an intentional implication may be blocked by the particular states of an individual.

We may formalize all this to get a little more clear on the difference between intensional and extensional contexts (which help us understand Frege's Puzzle) and intentional (with a "t") contexts, which are distinct from both other sorts. Consider a context, Γ.

a)[23] $(\forall x)(Fx \Leftrightarrow Gx) .\Rightarrow. \Gamma_F \Leftrightarrow \Gamma_{G/F}$
b)[24] $a=b .\Rightarrow. \Gamma_{Aa} \Leftrightarrow \Gamma_{Ab}$, b free for a in A.
c)[25] $A \Leftrightarrow B .\Rightarrow. \Gamma(\Theta_A) \Leftrightarrow \Gamma(\Theta_{B/A})$
d)[26] $\ulcorner A \Leftrightarrow B \urcorner$ is logically true $.\Rightarrow. \Gamma_A \Leftrightarrow \Gamma_B$

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[23] If all Fs are Gs and all Gs are Fs, then any claim about an F in context Γ holds of a G in context Γ.

[24] If a and b are identical, then any property a has in context Γ, b has also (provided b isn't illicitly bound by substitution for a in A).

[25] If the statement A holds if and only if the statement B holds, then any property of A in context Γ is a property of B in context Γ.

Where Γ is an extensional context, all four hold since each is understood in terms of claims merely about the objects of the language and not in terms of any semantic properties.

Where Γ is an intensional context, however, the first three fail, since there are different ways to refer to each thing. In other words, while there might be a material equivalence between the objects referred to, there might not be between the ways of referring. For example, even if all Fs are Gs and vice versa, it does not imply that F=F if and only if F=G. Every creature with a heart is a creature with kidneys, yet while substitutions will hold in extensional contexts (as the claims are all boiled down to the objects), they fail to hold in intensional contexts, where the object is referred to via a particular property. Since *having a heart* is different than *having kidneys*, the substitution represented in a) fails to hold. Similarly for b), just because a and b are identical does not mean that what S believes about a, S believes about b. Think of Superman and Clark Kent. While this sort of claim holds extensionally (as it is a claim about the objects) it fails to hold when considered in terms of the way that particular Kryptonian is referred to. d) still holds in intensional contexts, however, as the logical equivalence implies each will hold in the same context. Equivalent intensional entities do not differ in their properties.

Even d) fails, however, in intentional contexts. For contexts of thoughts, knowledge, belief, and the like (intentional states or propositional attitudes) one may believe one proposition but not something logically equivalent to that proposition. One

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[26] If it is logically true that A holds if and only if B holds, then A holds in Γ if and only if B holds in Γ.

can know one thing and believe something contradictory, and this captures intentional contexts.

One must keep intentional with a "t" clear from intensional with an "s". Intentionality, according to Brentano, is the peculiar aboutness that is had by certain mental states (which, on some views, then migrates to our linguistic expressions). The latter involves meaning contexts, and the former, while often understood as dependent upon the latter, is really about propositional attitudes of individuals. Intensionality is more the realm of philosophy of language, whereas talk of intentionality is more at home in philosophy of mind circles. The two are often related because many offer theories of intentional contexts that explain the failure of substitutivity by invoking intensional entities (propositions, universals, etc). In extensional contexts, only the objects matter. Intensional contexts focus more on modes of presenting the objects. Intentional contexts focus on the propositional states of particular individuals. It is clear that extensional contexts are the least restrictive and intentional are the most restrictive in terms of reasoning. One needs reason, then, to justify needing to move to intensional contexts from the more simple extensional contexts, and it is in this light that we return to Frege.

Frege's Puzzle of the Morning Star and the Evening star derives from the Greek discovery that the brightest star in the morning sky (Phosophorus) was the same celestial body as the brightest star in the evening sky (Hesperus). But if identity claims like a=b form extensional contexts, then there would be no difference between asserting, 'a = b' and 'a = a'. If a = b, then the referent of 'a' is the referent of 'b'. The two sentences in question, understood extensionally, both assert one object (namely Venus) is self-identical. Yet this reading seems to fail to capture a significant difference. Certainly this was an important discovery for the Greeks, as are many discoveries of contingent identities. "The Butler was the killer," or "The girl/boy from the party is our waiter," or "Heat is molecular motion." Frege thought these were,

> …obviously statements of differing cognitive value; a = a holds *a priori* and, according to Kant, is to be labeled analytic, while statements of the form a = b often contain very valuable extensions of our knowledge and cannot always be established *a priori*.[27]

Accordingly, the Greeks felt as if they had *discovered* something when they came to assert, "Hesperus *is* Phosphorus!" To account for this difference in cognitive value, for the difference between contingent identity claims (like, "Hesperus is Phosphorus") and identity claims which Frege through were not informative (like, "Hesperus is Hesperus"), Frege asserted the need for something to stand between the marks on the page and the celestial body. It could not be merely referential, but referential in a particular way. He wrote:

> It is natural, now, to think of there being connected with a sign (name, combination of words, written mark), besides that which the sign designates, which may be called the *Bedeutung* of the sign, also what I should like to call the sense of the sign, wherein the mode of presentation if contained...The *Bedeutung* of 'Evening Star' would be the same as that of 'Morning Star', but not the sense.[28]

He wanted an intensional entity with a mode of presentation. These entities he called senses, and took them to be abstract intensional objects (more on this later). "Hesperus" and "Phosphorus" both refer to the same celestial body (Venus, as it turns out). Understood extensionally, they are the same: they share a *Bedeutung*. They do not, however, refer to it in the same way. "Hesperus" picks out Venus because it is the

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[27] Frege, "On *Sinn* and *Bedeutung*", reprinted in Frege, (1997), p.151

[28] Frege, "On *Sinn* and *Bedeutung*", reprinted in Frege, (1997), p.152

brightest body in the evening sky. “Phosphorus”, on the other hand, picks out Venus because it is the brightest body in the morning sky. These are different modes of reference that could have picked out different things had, say, Mars been brighter in the evening. This is a difference that the semantic objects that are Fregean senses account for, and thereby, they solve Frege’s Puzzle about Identity. Fregean senses allow for what is now called fineness of grain, referring to the need for differentiating subtly different meanings in one’s epistemology or philosophy of mind.[29]

Frege’s solution to this linguistic problem was his theory of referential shift. Dualists (like Descartes) had posited the existence of two kinds of substance, one mental and one physical. Frege’s puzzle about identity moved him to endorse the existence of a third realm, neither mental nor physical. This is the realm of senses. This third realm was independent of the physical and the mental realm. This turned out to be a turning point in modern philosophy. There was a problem with how our language referred to the world, and Frege argued based on that problem that we needed to change our ontology. Concerns like this became common investigations for Frege, Russell, Wittgenstein, and many other twentieth century philosophers in what Dummett calls “the linguistic turn” in philosophy. They began thinking about the world in terms of the way we describe it, conceiving of language as a gateway to something beyond mere semantics and syntax. It is a move that motivates projects like this dissertation, which take seriously issues arising from semantic paradox.

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[29] Think of the difference between the claims, “Lois Lane believes that Superman can fly,” and, “Lois Lane believes Clark Kent can fly.”

Now, recall that Frege called the sense of a sentence a thought[30]. To see just what he had in mind, then, I turn an eye to Frege's third realm. He writes that, "thoughts are neither things in the external world nor ideas."[31] With this, he rejects the view that they are physical (they cannot be perceived) as well as the psychologism that he opposed (as ideas are not sharable, whereas expression requires sharable content). Frege concludes,

> A third realm must be recognized. Anything belonging to this realm has it in common with ideas that it cannot be perceived by the senses, but has it in common with things that it does not need an owner so as to belong to the contents of his consciousness.[32]

This is a Platonic realm of meanings graspable through ideas, sharable, and yet independent. It shares features of both the mental and the physical realm. Frege gives us an example:

> The thought we have expressed in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true. It needs no owner. It is not true only from the time when it is discovered; just as a planet, even before anyone saw it, was in interactions with the other planets.[33]

Notice the realism in Frege's remarks. Thoughts have ontological status no less shadowy than physical objects like planets. They are just as independent of minds. In addition, if the thought is true, and it is not just true independently of anyone, but atemporally, and

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[30] While the term 'thought' immediately suggests itself to the mental realm, he has something different in mind, a non-mental, yet abstract object.

[31] Frege, "Thought", reprinted in Frege, (1997), p. 336

[32] Frege, "Thought", reprinted in Frege, (1997), p. 337

[33] Frege, "Thought", reprinted in Frege, (1997), p.337

can, in addition, be shared across time. Fregean thoughts refer to truth-values.[34] They exist eternally and independently. Frege's third realm is populated by senses, and included are senses that are objects such as the sense of a proper name, the sense of a definite description, and the sense (proposition) indicated by a nominalized sentence.[35]

Kaplan famously distinguished[36] between Fregean propositions and their later counterparts: Russellian propositions. Frege believed that thoughts were always composed of senses. They were purely intensional entities. Russell rejects a purely intensional understanding and instead endorsed what Kaplan called singular propositions. Singular propositions have aboutness (exemplify intrinsically semantic relations) and yet can have individuals that are not senses as constituents.[37] This Fregean-Russellian divide foreshadows the modern debate between intensional propositions and extensional propositions.

While much of the twentieth century focused on meanings as assertability conditions (like Wittgenstein), methods of verification (like Ayer and the logical positivists), and external relations (like Burge and Putnam), propositionalism has come back into vogue. George Bealer has a nice discussion of modern propositionalism, and he defines it by five theses:

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[34] This can be understood as a form of being the bearers of truth-values. While Frege had a questionable ontology of Truth and Falsehood, Fregean thoughts referred to these through the senses that formed their content, and it is in this that they have a particular meaning and "bear" truth or falsehood.

[35] This realm is also populated by senses that are not objects, for example the sense of a predicate expression.

[36] See, e.g., Kaplan, (1989)

[37] Fitch and Nelson, (2009)

1. Propositions are the primary bearers of such properties as necessity, possibility, impossibility, truth, and falsity.

2. Propositions are mind-independent extra-linguistic abstract objects.

3. A belief state consists in a subject standing in the relation of believing to a proposition, and that a proposition is the content of the belief (likewise for other intentional states-desire, decision, memory, etc.).

4. Propositions are typically public: people commonly believe one and the same proposition and doing so is a prerequisite for successful communication.

5. Propositions are what (literal utterance of declarative) sentences express or mean.

Notice the breadth of application here: modality, logic, philosophy of mind, philosophy of language, and so on. He then sums up the modern landscape succinctly:

> Of course, some philosophers have been skeptical about abstract objects in general and for that reason alone have been skeptical about the traditional theory of propositions. But with the rise of modal logic, the resurgence of modal metaphysics, and the revolution in cognitive psychology and its realism about intentional states, this general skepticism strikes most philosophers as idle. Today, the traditional theory of propositions is the dominant view.[38]

To see the importance of propositionalism, consider the following sketches of their relevance to the varied fields Bealer mentions. Modal Metaphysics looks at the nature of possible worlds. Propositionalism allows viewing possible worlds as composed of propositions (Kripke and Plantinga) rather than actual causally isolated universes (Lewis).[39] Propositions can compose the building blocks of possible worlds that are often used as the underpinning of understanding how things might have been and how they must be.

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[38] Bealer, (1998), p.1-2

[39] For a discussion, see Van Inwagen, (2001), Chapter 12

As modal logic is the logic of possibility and necessity, it is key to thinking about this sort of logic that something bear the relevant modal properties. Abstract propositions are ideally suited to the task and can help modal logic to work as they can help non-modal logic. We need something to exemplify entailment relations. Saying that one state of affairs entails another strikes some as a queer view, as states of affairs are not normally seen as intentional entities and it is often something about the meaning of one thing that allows it to entail another. And in modal cases, talk of possibility and necessity puts one in the realm of what is not. For a possible truth, what better to serve as a truth bearer than an abstract entity? Its non-concrete nature sets it outside, in some sense, of the actual, and propositions' intensional nature can capture possible states of affairs. Propositions can thus serve as foundations for both logic and modal logic.

In terms of cognitive psychology, Bealer references realism about intentional states. First think about a Spaniard thinking that snow is white and a Brit thinking snow is white. The Spaniard would talk about it saying, "Nieve es blanco," whereas the Brit would say, "Snow is white." Yet it seems like their mental states have something in common, and this intuition is sometimes captured at the level of content: they both stand in the believing relation to the same proposition. It is non-linguistic (so independent of both Spanish and English). The fact that propositions (as non-linguistic) are sharable allows them to fulfill just this sort of role, and form natural ways of arguing for the existence of propositions.

Intentional states take something as their object (something that is believed, desired, perceived, etc.). The object needs to be something that looks a lot like a proposition to make sense of intentional notions like true belief, false belief, seeming, and more. Consider my hope that Pegasus exists. This is most easily understood when there is an object of my hoping, a thing that I hope, something called the Indispensibility argument,[40] and propositions can fulfill this role easily.

Further, we are afforded a simplifying tie when propositions function in one of their key roles: truth-bearers. The waters are muddied if, for example, the object of my belief is something other than the bearer of a truth-value. In that case, we would need to explain the relation between the two and talk about how they are connected. When they are the same, however, we can easily explain what it means to say that my belief is true: it is just to say that the proposition to which I stand in the believing relation is a true proposition. And no further explanation is possible. We can talk about the truth conditions laid out by the propositions, but this is to talk about what it means to say that a proposition is true, not to explain the relation of a true belief to the bearers of truth values.

As is clear, there are many circumstances in which the existence of propositions is remarkably handy, and a simplifying assumption. All this goes to motivate the importance of propositions to modern philosophical theories. As Bealer says, “Today, the traditional theory of propositions is the dominant view.”[41] To see the modern landscape a little more clearly, consider some of the different propositionalist variants. Bealer himself holds an anti-reductionist view, conceiving of propositions as intensional entities. Reductionists, on the other hand, agree there are propositions, and that those propositions are the bearers of truth-values while disagreeing about the nature of those propositions.

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[40] Landini, (201x)

[41] Bealer, (1998), p.2

There are those, like Bealer, Frege, and the early Russell, who hold propositions are abstract *objects* in a metaphysically significant sense of the word (i.e., Frege's technical sense). Bealer puts the debate as follows:

> …the primary question with which we are concerned is what propositions *are*. Are they identical to extensional functions, ordered sets, sequences, etc.; or are they *sui generis* entities, belonging to an altogether new category?[42]

Bealer, here, describes the divide as one between those who accept extensionality in their propositions (the reductionists) and those who do not (Bealer and the anti-reductionists). This is the sense in which the Russell-Frege debate foreshadows the modern one. Frege's purely general propositions are *sui generis* forerunners of Bealer's propositions. They are composed solely of intensions that bear intrinsic semantic relations to things in the world. Russellian propositions, on the other hand, have things in the world as constituents, and so countenance extensional parts of propositions. In this sense, they are trailblazers for the purely extensional reductionist routes popular today (which see propositions as, for example, functions). They are diaphanous in that they have no mode of presentation: they simply relate our signs to their referents. Robust propositionalists, on the other hand, hold that intensional objects are the meanings of sentences, and refer via a particular mode of presentation. As is clear, propositionalism comes in varying degrees of intensional and extensional.

First recall the notion of a semantic object: an independent, intrinsically intensional entity. Next consider a more general definition of proposition: Propositions are the meanings of sentences, and good candidates for possible worlds semantics and

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[42] Bealer, (1998), p.4

other such things. They are normally understood as mind-independent.[43] There are both intensional and extensional propositionalists. Intensional propositions include Fregean thoughts and are what I call semantic objects: they have intrinsic intentionality. Extensional versions can be ordered pairs (captured by functions). No propositions are linguistic. They are either non-linguistic semantic objects (which exhibit natural intentionality) or they are non-linguistic extensional entities (which, perhaps, exhibit derived intentionality).

Semantic objects obey strong conditions of identity.[44] They have a mode of reference, and populate, at least on the Fregean view, a third metaphysical realm distinct from the physical and the mental. We can define the Semantic Object Thesis: Propositionalism in accord with Bealer's definition + the thesis that propositions=intensional objects. This distinction and battle is important because one main goal of the dissertation is to argue that Yablo's paradox is not formable without semantic objects.

I will argue that anyone that holds the Semantic Object Thesis will fall prey to Yablo's paradox, and the Thesis is, as such, to be avoided. Recall that, while Russell's paradox invalidated Frege's logical system, it also undermined naïve set theory. Naïve comprehension of sets was thrown out and replaced by intuitions of an iterative conception of a set. Thus it might be considered that some extensional version of propositionalism is the way to go. There are reasons, however, to avoid them as well, and so the options open to set theory upon the rejection of naïve comprehension are not so

[43] There is conceptual space for mind-dependent versions.

[44] More formally three conditions: 1. $\{p \supset q\}=\{r \supset s\}$.⊃. p=r & q=s
2. $\{\sim p\}=\{\sim q\}$. ⊃. p=q 3. $\{(x)Ax\} = \{(x)Bx\}$. ⊃. $(x)(\{Ax\}=\{Bx\})$

promising for philosophers of language. Propositionalism and paradox have roots in the dialogues between Frege and Russell, and their intensional and extensional versions remain relevant today, forming the backdrop for the current discussion of paradox.

1.3 – From Liar toward Omega-Liar

Consider: Yablo calls his paradox liar-like. This must mean that it fundamentally involves the notion of reference. As such, I propose to investigate Yablo's paradox with an eye to current theories of reference, specifically propositionalism. This third and final section lays out Yablo's paradox as a variant of the Liar, setting the stage for the formal presentation of Chapter 2 while ensuring the current investigation remains true to its roots in semantic paradox.

I begin with the Liar paradox, presenting seven different versions building toward an informal presentation of Yablo's paradox.

1.3.1 The Simple Liar[45]

Imagine a friend, Ash, walks up to you on the street and says sentence L: "I am lying." You are faced with a choice between believing Ash and not believing Ash. On one hand, if you believe him, you believe that L is true, namely that he is lying. If you believe he is lying, you do not believe what he says. In this case, believing him implies that you do not believe him. On the other hand, if you do not believe him, you think that

[45] Clark, (2007), p. 99

he is not telling the truth. But that is precisely what he is asserting! As such, if you do not believe him, you believe him after all! Paradox.[46]

This natural language version of the Liar paradox has been strengthened in many ways. People often talk of the Strengthened Liar sentence[47] as one that refers to itself. This can be accomplished in at least three ways: via description, via naming, and via indexicals. I present the first two as Non-indexical Liars, and the latter as Indexical Liars. Note that here these paradoxes comes not as belief paradoxes (i.e., whether or not to believe Ash) but instead as about the truth or falsehood of a particular sentence. We cannot avoid the issue by withholding judgment. Whether or not we actively engage, their truth-values are a problem. The same holds for the final version I will present: the Looping Liar.

1.3.2 The Strengthened Non-indexical Liar, Descriptive

Consider the following descriptive sentence, Ld:

> The first indented sentence in Section 3.b of Ch.I of Hassman's dissertation is false.

If Ld is true, then the sentence that fits the description is false. But Ld fits the description and so must be false. On the other hand, if Ld is false, then the first sentence in Section

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[46] This natural version of the paradox is distilled from the ancient Cretan, Epimenides, and makes an appearance in the Bible. Its longevity as a point of interest shows this simple situation hides a gnarly problem.

[47] Sainsbury, (1995), p. 111

3.b of Ch.1 of Hassman's Disseration is false. As this is the claim made by Ld, Ld must be true. Paradox.[48]

1.3.3 The Strengthened Non-indexical Liar, Named

The same paradox can be generated through naming. Consider the following sentence, which I call "Ln":

Ln is false.

This generates paradox as does the descriptive sentence above. If Ln is true, then the sentence named, "Ln" is false. But Ln is that sentence, and so must be false if true. If it false, however, then, "Ln is false" is true. As this is the claim Ln makes, Ln is true if false. Paradox.

This Named Liar and the previous Descriptive Liar generate the paradox using names and descriptions rather than indexicals used by the next three versions. It is their use of indexicals that makes them more complicated.

1.3.4 The Simple Strengthened Indexical Liar

Consider the following sentence, call it Li:

This sentence is false.

If Li is true, then what it says is true, namely that "this sentence" (which refers to Li) is false. So we may assert the following conditional: If Li is true, then Li is false. Hence it is

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[48] Tarski focused on a similar premise for his version of the Liar. He imagined a sentence written on the chalkboard of room 301: "The sentence on the chalkboard in room 301 is false." As I'll argue in Chapter 3, Yablo thinks of his sentences this way: as resultant from natural language rather than concerns only for formal systems. See Yablo, (2006) for a discussion.

false. If it is false, however, then what it asserts is not the case, namely, it is not the case that “this sentence” (which refers to Li) is false, i.e., Li is true.[49] And so we may assert the conditional: If Li is false, then Li is true. Hence it is true. Paradox![50]

Indexical Liars are special cases because indexicals are shifty creatures. To generate the paradox, Li needs the “this sentence” to refer to Li. But consider the following plausible claim:

(NTS): Sentences with indexicals have no truth-conditions *simpliciter*; only on an occasion of utterance does the indexical acquire a reference.

NTS acknowledges that the referential abilities of indexicals are fluid, and depend on context to be determined. If I have a sheet of paper on which is written Li, there is nothing to determine that Li refers to itself rather than some other sentence. If it did refer to some sentence other than Li, then we would have no means of determining whether Li was or was not paradoxical.

Of course, there are many instances of Li that are non-paradoxical and that successfully refer. When I am in class, and I write on the board, “My name is Frank,” and, pointing at the sentence, declare, “This sentence is false,” surely no paradox ensues. Why? The context of utterance does not fix Li as the referent of my teaching statement, so we have no basis for asserting Li’s falsehood. This dependence on context for reference is precisely why NTS makes sense. If a sentence is dependent on context to determine one among many as the referent of the sentence, then without that context one

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[49] I avoid considerations of bivalence here, as this is a sketch intended to give the reader a feel for the origins of Yablo’s paradox. I admit this presentation is cursory.

[50] This employs two derivation rules that hold in classical logic: $(p \Rightarrow \sim p) \Rightarrow \sim p$, and $(\sim p \Rightarrow p) \Rightarrow p$.

might hold that there is no reference. Without a referent no claim is made and so there are no truth-conditions for the sentence. Thus, NTS suggests that without context there are no truth-conditions simpliciter for indexical sentences.

It would not be possible to avoid NTS and form an indexical liar by saying “By ‘this sentence’ I just mean Li.” This is simply a disguised version of a non-indexical Liar. To truly generate an indexical liar, you need to reject NTS altogether and maintain that there are singular indexical intrinsically semantic objects since these get their meaning independently of any interpretation or other contingent feature of the world.

Perhaps, however, this is too hasty. Let’s consider two other ways of attempting to generate indexical liars without resorting to semantic objects and rejecting NTS.

1.3.5 The Strengthened Indexical Gappy Liar

One line of response to Liar paradoxes uses the possibility of truth gaps to account for the difficulties of Liar sentences. If a Liar sentence cannot be true and it cannot be false, perhaps there is some other value that it holds. The simplest of these are three-valued logics that admit of Truth, Falsehood, and a third value that is neither true nor false. On this view, Liar sentences motivate our acceptance of more complicated logics that allow Liar sentences to be neither true nor false. These sorts of responses, however, are often subject to Revenge claims: Liar-style sentences that use the added truth-values to generate paradox. With this in mind, consider the following sentence:

LiTF: This sentence is neither true nor false.

NTS holds that LiTF, as an indexical sentence, has no truth conditions *simpliciter*. In other words, it is neither true (*simpliciter*) nor false (*simpliciter*) independent of context.

This seems to be just what LiTF claims. But it is not what it claims. Recall that we are using the definition:

false =df not true

Thus, LiTF claims that it is not true (*simpliciter*) and that it is not false (*simpliciter*). By our definition, it claims that it is false (*simpliciter*) and that it is true (*simpliciter*). But by NTF it has no truth conditions *simpliciter*. In a context of utterance, LiTF does get truth-conditions. But in every context of utterance it is false.

1.3.6 The Strengthened Indexical Contextual Liar

Consider a final attempt to generate an indexical Liar without embracing semantic objects and rejecting NTF. Let C be some context of utterance:

LiC: This sentence is false-in-C.

Here it seems like LiC will generate a paradox without rejecting NTS because it builds the context of utterance into the sentence. But for this sentence to work, even uttered in C, we need to ensure the "C" in the sentence refers to the context, C, of utterance to generate the paradox. Nothing assures this if the semantic relations that fix the referent of "C" are extrinsic to LiC itself. If we embrace LiC as a semantic object, however, "C" in "This sentence is false-in-C" refers to C because the semantic relation between "C" and C is intrinsic to the object LiC. We might still try to avoid this by considering

LiiC: This sentence is false-in-this-context.

Now when "This sentence is false-in-this-context" is uttered in a context C, the first occurrence of "this" has no referent until the second occurrence of "this" gets a referent. (Just compare it to the analogous case of "This man killed this most famous of child pharaohs".) But the second indexical ("this context") gets a referent only in virtue of the first occurrence already having a referent to some particular sentence. All we know about the context is that it is the one in which this sentence is uttered, and so we are not in any position to know what context is being referred to until we know what sentence "this" refers to. In short, there seems no reason to think that the sentence "This sentence is false-in-this-context" is the sentence being referred to when it is asserted in C unless we are working on a semantic object view. Then the intension can pick out the context or sentence independently of the other referential features of the sentence, and so while the sentence is not clearly a paradox without semantic objects, the issues with proper semantic relations can be handled by accepting intrinsically intentional objects. Our discussion of these paradigmatic cases of indexical Liars make clear that the existence of semantic objects (and the accompanying rejection of NTF) are necessary for generating semantic paradoxes of this sort.

1.3.7 The Looping Liar[51]

In this case, we have multiple sentences that form a referential loop.

(LL1): LL2 is false.

(LL2): LL1 is true.

If LL1 is true, then LL2 is false, namely, it is not the case that, "LL1 is true". So: if LL1 is true, then LL1 is false. Hence it is false. On the other hand, if LL1 is false, then what it says is not the case, namely LL2 is true, and so what it asserts is true: that LL1 is true. So: if LL1 is false, then LL1 is true. Hence, LL1 is true. Paradox! Here we move away

[51] Clark, (2007), p.100

from self-reference to the more general category of circularity (where self-reference is a kind of circularity). Rather than LL1 or LL2 referring to themselves directly, they do so via a referential chain. LL1 refers to itself because its actual referent, LL2, refers to it. Similar remarks apply to LL2. These two, then, are jointly paradoxical.

Note that while this is a short loop, but the loop could be any length: imagine a corporation filled with middle management. A loyal worker, John, claims, “What my manager says is true.” The manager, with someone to report to, says, “What my manager says is true,” and so on up to the president of the company, who wishes to appear more knowledgeable than the staff, saying, “I’m sure what John said was false.” All the company has managed to do is contradict itself. Looping liars, as such, can be lengthy, and one might imagine empirical examples are more likely to arise in this sort of complex scenario.

The important question for looping Liars will arise for the Yablo as well. Does anyone in the above corporation succeed in making a claim? If not, then their utterances are not the sorts of things that have truth-values. On the other hand, if there are semantic objects, then it seems that each of their utterances pairs with an intensional entity that intends something in the world, and has a meaning independently of the nature of that referent due to its intrinsic intensionality. The question of whether Looping Liars successfully refer is answered for the propositionalist because the referring is built into the ontological machinery of the semantic object.

1.3.8 Yablo's Omega-Liar Paradox

Yablo asks his reader to imagine an infinite sequence of sentences each of which intuitively claims all the rest in the sequence are untrue.

(S_1): For all $k > 1$, S_k is not true.

(S_2): For all $k > 2$, S_k is not true.

And so on.[52] The intuitive paradox can be seen by considering these first two members of the infinite sequence. Assuming S_1, we know that S_2 is untrue and all the rest are untrue. S_2 asserts that all those after it are untrue. For S_2 to be untrue, then, requires some sentence later in the sequence not being untrue (i.e., some later sentence being true). But if there is some sentence later than the second that is true, then all those after the first fail to be untrue (as S_1 claims). Hence our assumption, S_1, must be untrue. But notice that our reasoning will hold no matter which sentence in the sequence we use as our *reductio* assumption. Given this, we can see how the sketch showing S_1 is untrue shows *every* sentence must be untrue. Here's the crux, however: if every sentence is untrue, then every sentence after some particular sentence, say the second, is untrue. And that is precisely what the second sentence claims. This means that the truth conditions for the second sentence (as well as all the rest of the sentences) holds. In other words, given that each of the sentences S_1, S_2, S_3, and so on are untrue, then all the sentences S_1, S_2, S_3, and so on are true.

Yablo makes clear the roots of his paradox lie in versions of the Liar, and this presentation brings out how one might make a case for building a Yablo out of the Liar. Some versions of the Liar need semantic objects to get going, and while it may seem easy

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[52] I match the informal presentation of the other Liar paradoxes in this section. A formal derivation of the paradox begins Chapter 2.

to escape, the different versions presented here as outlets around that need make clear that there is no easy route around this need, and that, along with a lineage that connects the Yablo to the Liar, sets the stage for the current investigation.

1.4 – Conclusion

Each of the Liar variants depends on a meaning relation holding between a part of the sentence and the sentence itself (either via name, description, indexical, or loop) and it is through these intentional relations that they generate paradox. This reliance on intentional (i.e., meaning) relations makes these semantic paradoxes. Similarly, the Yablo paradox relies on meaning relations between individual sentences, specifically that earlier sentences in the Yablo sequence make claims about later sentences in the Yablo sequence. Yablo's paradox is, as such, a semantic paradox, and may be generated from modern propositionalism's semantic objects as are other versions. Yablo's paradox may change how philosophers view semantic objects such as propositions. As such, I've attempted to lay out the modern notion of proposition, setting it in place (along with the methodological grounding of the linguistic turn) in Frege and Russell and their views on paradox and propositions. Yablo's paradox is a semantic paradox and as such we must look at it in the light of theories of meaning to see how much and to what extent it impugns our ideas about the metaphysical underpinnings of the semantics of natural language.

CHAPTER 2

YABLO'S OMEGA-LIAR PARADOX

This chapter is an analysis of Yablo's paradox in light of the Liar paradox. It begins with a formal derivation of Yablo's omega-liar paradox (most commonly referred to simply as "Yablo's paradox"). The first section includes, in addition, several definitions for navigating discussions of Yablo's paradox. Section 2 distinguishes a technical notion of a claim from that of a declarative sentence (a "sentence^{d}"). Finally, Section 3 analyzes Yablo's paradox into its necessary pieces and notes which are unique to the Yablo and which it shares with Liar paradoxes. I argue that there are two features shared by Yablo and the Liar. First, that the relevant Yablo sentences and Liar sentence(s) need to be claims in the technical sense laid out in Section 2. Second, each set of sentences must be assertions involving untruth or falsehood. In addition to these two requirements, there are four requirements unique to Yablo's paradox. First, there can be no final claim in the sequence. If there is, no paradox will result. Second, the untruth/falsehood claims of the Yablo sentences must refer to an infinite subset of the list. If they assert that only a finite subset is untrue, no paradox arises. Third, there must be some reference ordering that determines the positions of the Yablo sentences in the sequence. Call this the "Ordering Problem". Finally, there must be some way to ensure the existence of the infinite number of sentences in the sequence, a problem that does not arise for the finite Liar variants. Call this the "Generation Problem." These are the six necessary features for generating the Yablo paradox.

2.1 – Formal Derivation of Yablo's Paradox

In this section I present a formal derivation of Yablo's paradox. Preceding it, I define a series of terms that help navigate discussions of Yablo's paradox and the infinite sequence of sentences from which it derives. First recall that Yablo asks us[53] to imagine an infinite set sentences:

(Y_1): For all $k > 1$, Y_k is not true.

(Y_2): For all $k > 2$, Y_k is not true.

And so on. Call this set of sentences a "Yablo sequence". Each sentence is an instance of the schematic expression, "For all $k > \Delta$, Y_k is untrue". Call these sentences "Yablo sentences". Each sentence differs from others only in the occurrence of a different numeral in place of Δ. Call the numeral in the Δ-slot the "content numeral" of that sentence. When a numeral is inserted for Δ, then, the schema yields a complete sentence. Note two things about the name of the sentence: first, it includes a numeral that marks its position in the sequence (i.e., the forty-third sentence is Y_{43}), second the numeral that marks its position is identical to the content numeral of that sentence. Call the numeral "s" in the name "Y_S" that marks its position in the sequence the "position numeral". Intuitively, each sentence in the ordering claims that all the sentences that follow it are not true (so the fifth sentence says the sixth, seventh, etc. are not true). This set of sentences is also a consecutively ordered set. Because the ordering determines the reference of each claim (via each sentence claiming, "All the rest are false,"), call this the "reference-ordering". The situation then, consists of an infinite set (the Yablo sequence) of Yablo sentences in a particular reference-ordering.

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[53] Yablo, (1993), p.252

Attempts to assign truth-values to the Yablo sentences falter, however. Consider this formal derivation. We run this derivation in a formal language with a truth-predicate T that obeys relevant rules from the literature[54]:

Capture: $\alpha \Rightarrow T`\alpha'$

Release: $T`\alpha' \Rightarrow \alpha$

Capture justifies inferring the truth of a statement from the assertion of its truth conditions. In this case, the truth conditions are captured by α and the truth of the statement characterizing those conditions is given by the consequent, T'α', where 'α' is a name for α. It is the sign or symbol that relates to or captures those particular truth conditions. Famously: If snow is white, then it is true that, "Snow is white." Release works the other way around. It takes the assertion of the truth of the statement and asserts the truth conditions (or content) of that statement. 'α' is still a name for α, and Release allows asserting the content based on the truth (intuitively) of 'α'. Given these, the proof goes as follows:

1. Assume: $T(Y_n)$ — *reductio* assumption
2. $\forall k>n, \neg T(Y_k)$ — 1, release
3. $\neg T(Y_{n+1})$ — 2, $(n+1)>n$
4. $\forall k>n+1, \neg T(Y_k)$ — 2, logic
5. $T(Y_{n+1})$ — 4, Capture
6. $T(Y_{n+1})$.&. $\neg T(Y_{n+1})$ — 3, 5, Conjunction
7. $\neg T(Y_n)$ — 1-6, *reductio*
8. $\forall n, \neg T(Y_n)$ — 7, UG, schematic *reductio*

[54] See, e.g., Beall (2007), p.1

9. $\forall n, \forall k>n, \neg T(Y_k)$ 8, logic

10. $\forall n, T(Y_n)$ 9, Capture

11. $\forall n, T(Y_n)$.&. $\forall n, \neg T(Y_n)$ 8, 10, Conjunction

Assume sentence Y_n in the sequence. From this assumption we derive the untruth of a subsequent sentence, i.e., $\neg T(Y_{n+1})$, in step three. But we may also derive from the assumption that every sentence following n+1 is untrue, and the untruth of each of those sentences are the truth conditions of Y_{n+1}. As such, Y_{n+1} must be both true and untrue. This is impossible, and so we deny the assumption in line 7: $\neg T(Y_n)$. But the proof for line 7 was schematic: it works no matter which n you choose. This legitimizes a universal generalization on line 7, yielding the universal claim on line 8. And if line 8 holds, then every sentence following any particular sentence is untrue (i.e., line 9, that for any n, all subsequent sentences are untrue). As these are the truth conditions for all the Yablo sentences in the sequence, each of the sentences in the sequence must be true. Contradiction.

As such, there is no stable assignment of truth-values for the sentences in the Yablo sequence: it is paradoxical.

2.2 – Claims in the Technical Sense

Before I turn to a discussion of the necessary features of Yablo's paradox, I turn to a discussion of sentences and successful expressions. First, note that not every declarative sentence (hereafter sentence^d) expresses a proposition. There is a distinction between successfully referential sentences^d and unsuccessfully referential sentences^d.[55]

[55] This distinction shows up in many places, including Kripke, (1975), and Ayer, (1946).

Marks on paper and sounds are how we converse, one to another. It is by means of these shapes and sounds that I can order a cup of coffee or assure the waiter I want my eggs overeasy. There are many rules governing these sorts of interactions, and these rules are separated into two sorts. The first, syntactic rules, are traditionally the domain of logicians and linguists. They deal with the ways different parts of speech attach to one another to make the sort of thing we think of as referential. The simplest and often first rule English speakers learn is that each sentence[d] must contain a verb and a noun: an action and something acting, respectively. This is a basic syntactic rule. It doesn't speak of the utterance's meaning, but instead describes rules for its *structure*, in this case, the necessity of both a verb and a noun. The second kind of rule, semantic, governs how we put together sentences[d] as well, but these rules are not at the level of parts of language, but rather the much more complicated engagement with individual terms. These rules fall out of the meanings of specific terms rather than the types of terms involved in meaningful expression. This latter set of rules, as it is more particular, is more discerning: it is stricter in terms of its rules than are those of syntax. If a set of shapes or sounds is semantically correct, then it is syntactically correct (note that I am not saying meaningful, as it is clear you could have a meaningful sentence that wasn't syntactically correct but got the job done anyway).

The converse, however, is not true. A set of shapes or sounds can be syntactically correct without being semantically correct. Take an example: "Blue ideas run angularly through the vacuum." This sentence[d] is syntactically immaculate: it has a verb and a noun, and adverbs adjust verbs, and adjectives adjust nouns. Yet there is something wrong with the sentence[d]. Ideas, as they are non-physical, cannot be colored, and so cannot be blue. I distinguish here between an idea *of* blue and an idea's being blue, which we may do even if we choose to speak in shorthand of a 'blue idea' caused by looking over a stormy ocean. The immaterial nature of ideas also makes it meaningless to discuss their running through three-dimensional space, and “angularly” is a topological description, and so isn't the sort of thing that would apply to running (though it might apply to what we call a 'run', referring to the route ran, which has a topology, though to describe it as angular might be a stretch). But despite this each of the terms is meaningful. It can get lost of the discussion of fit between adverbs and verbs, adjectives and nouns, but each of these has a clear meaning in English. Yet, while each of the terms are meaningful, and their location is meaningful, the whole is not. It simply does not mean anything to say those words. Consider one more enjoyable example from the literature: “Monkeys multiplied by grass snakes equal tuxedos.”[56] Syntactically the sentence^{d} is fine, but it is a semantic wash, a conversational failure.

Let the term “sentence^{d}” be an uninterpreted well-formed expression: let it refer to a syntactically proper set of sounds or shapes. These are the sort of entity that we might describe as syntactically proper. They follow our syntactic rules. Put another way, there are no missing pieces based on the syntax of the sentence. Different philosophers discuss this differently. As an example, take Kripke on Strawson:

> …we can regard a sentence as an attempt to make a statement, express a proposition, or the like. The meaningfulness or well-formedness of the sentence lies in the fact that there are specifiable circumstances under which it has determinate truth conditions (expresses a proposition), not that it always does express a proposition.[57]

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[56] Woozley, (1968), p.27

[57] Kripke, (1975), p.699

The idea is that the terms in the sentence^{d} and their syntactic configuration provide us with an account of what it *would* mean if it did. It may fail in such a case to express a proposition and thereby have a determinate truth-value. (Kripke calls such failure as a truth-gap). So for our sentence above, we know, based on the meaning of the individual terms and their position in the structure of the sentence what circumstances would yield a truth-value. Abstract objects would need to be able to be colored, and to run, and it would need to mean something to run angularly. As it is, none of that is the case. It does not make sense. There are other sentences that obviously do. "Because of an old case of bronchitis, the cat's purring sounded like a coffee percolator." And then there are some of which we are unsure. Take, for example, a particular verbizing of a noun: "So I motorbiked it down to the store and picked up two thick steaks." We have an intuitive sense of what this means, but might, dependent on our philosophy of language, think that it fails to follow the proper semantic rules. If, for example, we identify meaningfulness with acceptance into the linguistic community, then if this is the first usage, it would be illicit. Each of these three examples, however, has no syntactic problem. Our term "sentence^{d}" will refer to any of these three that *may* express propositions, and *may* have a determinate truth-value.

This notion of sentence^{d} allows for a corresponding notion of a *successful* sentence^{d}. This second successful set is a subset of the first. Call such successful sentences^{d} (that have truth values, and are, I like to say, semantically successful), 'claims'. Claims are all syntactically proper. Claims are *in addition* meaningful expressions that have intentionality, and express propositions which are the bearers of truth-values, whether directly (as with Frege's abstract thoughts) or indirectly (as with shapes, on a view where the marks on the page are true or false, yet only via an interpretation function from sentences^{d} to the actual world, as they do in modal logic[58]). All claims are sentences^{d}, but not all sentences^{d} are claims. Claims do not merely have the appearance of reference (through using referential parts), but instead bear genuine semantic relations to objects in the world. It may be that the relations they bear are such that the world falsifies the claim. The point is that to be a claim is to be related through meaning to the world such that you either accurately describe the world (and are true) or do not (and are false).[59]

Recall that the definitions from the previous chapter of semantic entities (which are anything that exemplifies meaning relations), semantic objects (which exemplify intrinsic intensionality), and propositions (which are non-linguistic meanings of sentences and often understood as the mind-independent bearers of truth-values). Given this, let's look at a few claims about their interrelations. 1. All claims are semantic entities. 2. All claims are sentences^{d}, but some sentences^{d} are not claims. 3. Semantic objects may be claims (and, by definition, linguistic entities and sentences^{d}). 4. No non-claim sentences^{d} are semantic objects.

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[58] See, e.g., Bealer, (1998)

[59] This description of claims assumes bivalence, but note: in normal three-valued logics (that utilize an undecided value or a neither-true-nor-false value) the third value is used for sentences that don't properly refer. In other words, undecided is an appropriate value for sentences that seem like they refer (perhaps through syntactic completeness) yet do not, as per Kripke. It is the perfect value for things that aren't claims, and we might include tables and chairs and feelings in that category. Similarly, infinitary logics see truth as coming in degrees, and the notion of a claim is consistent with that. In that case, claims are those things that fall along the spectrum of truth-values. In either case, the notion of a claim should be clear as that which exemplifies what we might call completed semantic relations. The relations aren't random. They are not a simple collection of referring terms, but terms with a particular structure that coincides with how the terms are appropriately used.)

1 and 2 are pretty straightforward. Claims are, by definition, semantically successful, and as such exemplify semantic relations. That, however, is the very notion of a semantic entity. And claims are sentences[d] that exemplify semantic relations. 3 means that, while propositions are understood non-linguistically, all we mean when we refer to semantic objects is that they exemplify semantic relations intrinsically. Insofar as they do, they might have linguistic properties (like syntactic structure) or they might not (more like propositions). As defined (or at least this is the intention) they may be either. 4 holds because non-claim linguistic entities fail to exemplify semantic relations, they cannot be semantic objects as semantic objects by definition exemplify semantic relations intrinsically.

The real use in talking about claims rather than simply propositions is that propositions are normally not understood as linguistic. Claims are linguistic, and Yablo's sequence is composed of linguistic entities. The Yablo sequence is a list of sentences[d] and it is important to talk about the distinction between a linguistic symbol that exemplifies semantic relations and the non-linguistic objects through which it means things in the world. We might say that claims are sentences[d] that express propositions. In this light, "semantic entities" is a catchall for anything that exemplifies semantic relations, and "semantic objects" is a term for those non-reductive views of propositions that take intrinsically intensional objects seriously.

2.3 – The Parts of Yablo's Paradox and the Liar Paradox

There are six features that are integral to this paradox, some of which are necessary parts of the Liar, and some of which are unique to the Yablo. I begin with two shared features, and continue to those unique the Yablo, and follow each list with an explanation of each feature.

There are two features shared between Yablo's paradox and the Liar paradoxes:

1. The sentences^{d} must be claims (in my technical sense). Unless they are successful in their attempted reference, they will not generate a paradox (Goldstein has a nice example[60] relevant in this sense to both the Liar and the Yablo paradox).

2. Each sentence^{d} must make an assertion about the untruth/falsehood of other sentences^{d} (that are claims on the list for Yablo, or itself for the Liar). These claims interrelate the Yablo sentences in terms of their truth and falsehood, and are necessary parts of the proof of contradiction from the Yablo sequence, much as the Ln relates the Liar sentence to itself.

There are four features unique to Yablo's paradox:

1. There must be a Yablo sequence such that there isn't a last claim. With only finitely many (or if there is a last claim in the sequence), no contradiction is derivable from the Yablo sequence.

2. Each claim must assert the untruth/falsehood of an infinite subset of the list. If a claim were to make its assertion about a finite subset, then, as per the reference-ordering, that subset would have a last member, and as is suggested in the first qualification, finitude of reference fails to generate Yablo's paradox because a finite number of sentences implies a last member. As was noted above this is problematic even if the claims are dense (i.e.,

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[60] Goldstein, (1996), where he talks about a building project asking for bids where each contractor sends in a letter claiming a bid of $1000 below the lowest bid. It would seem, no matter how many letters were received, that no one had succeeded in bidding anything.

between any two, there is a third, like the rational numbers). If there is a last, there is a consistent assignment of truth-values to the claims of the sequence.

3. There must be a reference-ordering to gather the infinite number of claims into a sequence. The sentences of the Yablo sequence need to be consecutively ordered into that sequence. The consecutive ordering, however, need not be the natural ordering of the natural numbers[61].

4. There must be some way of ensuring the existence of the infinite number of sentences in the Yablo sequence. The need for an *infinite* number of sentences requires this discussion in a way it is unnecessary for the finite Liars discussed in Chapter 1.

Let us now discuss each of these six requirements in detail.

2.3.1 Shared features of Yablo and the Liar

1. That The Sentences Comprehend Claims (in my technical sense)

I'm using the term “claim” in a technical sense. Claims are distinct from mere sentences^{d}. Claims are semantically successful, and form a subset of the latter, which only requires that a sentence^{d} be syntactically proper. In cases of paradox, we must always ask ourselves whether the sentences^{d} we're dealing with qualify as genuine claims that have intentionality. This is especially important when it comes to instances like Liar sentences^{d} and Yablo's sentences^{d} which include mention of truth and falsehood. After all, in some sense, to say that something is a claim is to say that it has a truth-value (insofar as it makes an assertion, i.e., succeeds in describing the world). Given this notion

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[61] See Chapter 5.

of claim, we can state that one of the requirements of the Yablo's paradox: the Liar and Yablo sentences[d] that compose his paradoxical sequence must comprehend claims.

In set theory, comprehension principles forge a relation between well-formed formulas (hereafter, wffs) and the sets countenanced by the theory. Naïve comprehension principles assert every wff comprehends a set. In other words, for any wff whatsoever you create, there exists a set of all and only those entities satisfying the wff. Our metaphysics can take a card from this deck: we can speak of wffs comprehending properties. Naïve comprehension of properties asserts the existence of a property or relation for any wff (of *n*-places) whatsoever of the formal language. Different views of properties will restrict this relation in various ways.

In terms of the Liar and Yablo's paradox, comprehension becomes a question of whether the sentences[d] succeed in being claims. This is a worry as the Liar must succeed in referring to itself to generate a paradox. Similarly, sentences[d] in the Yablo sequence must successfully refer to other sentences[d] that are claims in order to generate a paradox. The Liar and Yablo sentences must comprehend claims to generate paradox rather than merely constructing sentences[d].

To see this, consider the sorts of proofs that derive a contradiction from the Liar and the Yablo sequence. They are not, like Russell's paradox, sheer manipulations of symbols. Rather, they require premises that discharge names for sentences[d] thereby yielding the semantic content of those sentences[d]. These semantic paradoxes require Capture and Release (as defined in Section 1) to derive their contradiction. Let’s take a look back to the Liar and to Yablo’s paradox to see just how semantic relations (and not merely notions of set and set-membership) are involved.

In the Liar, we assume Ln, and infer its content (recall that Ln: Ln is false). We assume: T(Ln), and then infer that ¬T(Ln). For this to work properly requires use of the Release rule: T'α'⇒α. This rule requires 'α' be a name or a symbol for α, where α is some set of conditions in the world. If 'α' did not, through exemplifying semantic relations, refer to α, then use of Release would not be legitimate and the argument to contradiction from the Liar sentence would not go through. Specifically, in the case of the Liar, we need to know that Ln is in fact a name for the claim 'Ln is false.' Without this semantic relation, Release would not justify the move to assert the falsehood of Ln based on assuming it. Similarly, we could not get its truth through Capture if it were not the case that Ln's being false was named by Ln.

For the Yablo, we assume $T(Y_n)$, and derive, $\forall k>n$, $\neg T(Y_k)$. This requires semantic relations and Release just as did the Liar. It is possible because Y_n *means* $\forall k>n$, $\neg T(Y_k)$. Further, we instantiate the meaning of Y_n, yielding truths about other sentences[d]. This instantiation, again, is only possible given semantic relations intrinsic to sentences[d] like Y_n and the other sentences in the sequence.

In that light, Yablo's sequence could not be shown to be paradoxical. As such, it is necessary for the sentences[d] in the Yablo sequence and the Liar to be claims.

2. That each sentence must make an assertion about the untruth/falsehood of other sentences[d] (on the list for Yablo, of itself for the Liar).

Fundamental to Yablo's paradox (as it is to versions of the Liar) is that its sentences involve assertions about the truth or falsity of other sentences[d]. They are essentially of the form, p is true/false/untrue, for some sentence[d], p. This reference to notions of truth (a semantic relation) is part of what makes these semantic paradoxes. In the case of the Liar, we have a sentence^{d}, Ln, that asserts: Ln is false. In the case of Yablo we have universal generalizations, but it is integral to running a proof that one can deny later sentences^{d} in the sequence based on the truth (or assumption) of earlier Yablo sentences^{d}. Without claims of truth or falsity, these situations wouldn't bother anyone.

To see why this is worth mentioning, consider some theories of what it means to say that something is true. Deflationists hold that there is no difference between a sentence^{d}, 'p', and the equivalent sentence^{d} 'p*': 'p is true'. Ayer, for example, calls "is true" logically superfluous.[62] The idea is that we must deflate the phrase "is true" out of the language. Their theory holds that it is vestigial, and plays no role. On an account like this, there is no Liar paradox because it cannot be stated in a properly formed language (i.e., one that excludes any references to truth or falsehood). An attempt to translate Ln into a sentence^{d} without an ascription of falsehood yields nothing save an incomplete sentence^{d}. Taking out "is false" leaves us with a name that names only the name itself, not a complete sentence^{d}. Similar remarks apply to the Yablo sentences^{d}.

Deflationary views are in vogue, but classical correspondence and coherentist theories are still viable. I will not rehearse them here. There are reasons to think that "is true" is more than a superfluous add-on. If they are correct, they show what many are naturally inclined to believe: that truth is a genuine property corresponding to our predicate "is true". In the end, the key is that claims about truth and falsehood are integral to Yablo's paradox, as they are to the Strengthened Liar. Without them, a contradiction is underivable from the Liar and Yablo. That the relevant sentences^{d} use notions of truth and falsehood is integral to these paradoxes, and that makes them semantic paradoxes.

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[62] Ayer, (1946), p.88

2.3.2 Four Features Unique to the Yablo

1. No Last Sentence

Yablo's paradox derives part of its force in one sense from the intuition that we can assign consistent truth-values to any set of sentences. Take, for example, {p, ¬p}. In this case, we have two consistent options. First, p is true and ¬p is false. Second, p is false, and ¬p is true. Certainly 'p' is inconsistent with '¬p', but we can assign truth-values consistently. This is why Liar sentences[d] are so troubling. We feel like, provided we understand the sentence, we can assign it a truth-value. There are cases where we understand and yet remain agnostic: "A meteor strike just off the Yucatan peninsula was the primary cause of the mass-extinction of the dinosaurs." But this is very different from the Liar case. We lack evidence in the dinosaur case. And while we might be unwilling to assign it a particular value, certainly many have asserted it true or false, and there is a fact of the matter (i.e., the claim is either true or false). In the Liar case, on the other hand, it seems we understand the sentence[d] and have all the relevant information. But any attempt to assign a truth-value is thwarted. This challenges our normal comfort with the notions of truth and falsehood. In this light, the problem posed by Yablo's paradox is how to assign truth-values to the sentences[d] in a consistent manner. Put in a different way, to generate the paradox requires successfully capturing a situation in which no consistent assignment of truth-values can be given.

To see why Yablo cannot have a last member (which he overcomes by positing an infinite number of consecutively ordered sentences[63]), consider a situation with a finite number of Yablo sentences, say, three.

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[63] Yablo, (1993)

(S1) ∀k > 1, Sk is untrue.
(S2) ∀k > 2, Sk is untrue.
(S3) ∀k > 3, Sk is untrue.

In this case there is, in fact, a consistent assignment of truth-values. S1 and S2 are false, and S3 is true. The truth of S3 makes S1 and S2 false, and there is no problematic consequence to S3's truth as there is in the case of an infinite number of sentences^{d}. Further, it is clear that this result generalizes to any finite number of consecutively ordered sentences^{d} in a Yablo sequence. As long as there is a last sentence, that sentence can be true and thereby falsify all the rest (even an infinite number of sentences in the case of a dense ordering), giving us our consistent assignment of truth-values. In the case of an infinite number of consecutively ordered Yablo sentences, however, there will always be more beyond any sentence we attempt to assign as true. When there are further sentences^{d}, the formal reasoning above that led to the contradiction is applicable, and the attempt at a consistent assignment of truth-values fails. As such, an infinite number of Yablo sentences^{d} is integral to the Yablo paradox.

This analysis needs one note. It requires a particular disambiguation of the Yablo claim. There are, as has not been noted in the literature, two ways we might understand a Yablo sentence, ∀k> 1, Sk is untrue. We might think this is a conditional: $(\forall k)([(k > 1)$ & $(\mathrm{S}k$ is in the Yablo sequence$)] .\Rightarrow. \neg \mathrm{T}(\mathrm{S}k))$. But there is another way to read this line in terms of a conjunction: $(\forall k)(k > 1 .\Rightarrow. (\exists k)(\mathrm{S}k$ is in the Yablo sequence & $\neg \mathrm{T}(\mathrm{S}k))$. The former is more intuitive, but the latter is a possible reading. The above finite Yablo sequence depends on the analysis of the sentence in terms of the conditional. If the last line were a conjunction, then it would not be vacuously true, and contradiction might ensue. This, if nothing else, ought to suggest that the conditional version is the correct analysis.

Another way to put the main point: it seems that if we can consistently assign truth-values to every finite subset of a set, then we can consistently assign truth-values to the set (even if it is infinite). As noted above, this can be done for the Yablo sequence: let all be false save the last in the subset. This relates to compactness: if every finite subset of a set of sentences has a model, then the whole set (be it infinite or finite) has a model. This is trivial in the finite case, but *not trivial* in the infinite case. Not every set is compact in the above sense. But sets of 1st order languages are compact. This must mean that the Yablo sentences^{d} are not first order. And this makes sense. Our intuitions get less sure footing as we increase complexity of claims. Higher-order languages are more complex, and so seem better candidates for the sort of intuition-troubling situation captured by Yablo's paradox. Yablo's sequence must not be compact, and must not have a last element.

2. That Each Sentence^{d} Asserts the Untruth/Falsehood of an Infinite Subset of the List/in the Sequence

If it were some finite subset, then the Yablo sequence would not be paradoxical. The reasoning here is similar to that discussion of having an infinite number of Yablo sentences^{d}. If there is a finite subset each of which is claimed untrue, then two things are possible. First note that for any finite subset, there are an infinite number of statements left to which that particular sentence^{d} fails to refer. This means there are plenty of statements outside the realm of reference to falsify each sentence referred to. To see this, imagine that our sentence^{d}, S9, refers only to sentences S10 and S11. It claims that they are untrue. Each of them makes claims all their own. Let S10 claim S13 and S14 are untrue, and let S11 claims S15 and S16 are untrue. Then, if, say S13 is true, and S15 is true, S10 and S11 are untrue. Further, this assignment of truth-values is consistent, as our original sentence[d], S9, doesn't claim that S13 or S15 are untrue. There is no contradiction. And since we have an infinite number of claims, we can assign truth-values consistently provided the referent of each Yablo sentence[d] is a finite subset. As such, I conclude that Yablo's paradox requires that each Yablo sentence asserts the untruth/falsehood of an infinite subset of the consecutive sequence. This overlap means that any sentence that could falsify a particular Yablo sentence by being true is also claimed untrue by the preceding sentence. Since the overlap is required, and this is achieved through reference to an infinite subset of Yablo sentences, the paradox requires that each sentence make its claim of untruth or falsehood about an infinite subset of the sequence.

3. That There is a Consecutive Reference-Ordering of Sentences[d] into a Sequence

The sentences in the Yablo sequence need to be ordered consecutively. Each Yablo sentence[d] is a generalization. Yet the assertion of each claim is dependent on something other than the content of the claim. It depends on the order of the claims in the list. Recall that the form of the Yablo sentences:

(Sn): $\forall k > n$, Sk is untrue.

It asserts that all claims that share a certain feature are untrue. Which feature? The feature is that of occupying a position in the Yablo sequence antecedent to Sn. So if we are looking at sentence S8, it claims that S9 is false, and S10, and so on. It uses positions to refer to claims. Which claims does it refer to? Well that depends on the consecutive ordering. The consecutive ordering, in that sense, determines which claims a particular

Yablo sentence refers to. If after S1 Yablo had specified that the remaining claims were of the form, Sn: "n=n+1", then those would be the claims referred to by S1. The claims do pick out a certain property, but the ordering of the claims is the origin of that property.

The ordering need not be the natural ordering of the natural numbers. One could imagine a sequence populated by Yablo sentences in some other order. Perhaps Yablo's original S8 comes first, and all the rest are the same. It is not their lining up with the natural numbers that matters, but rather *that they are lined up*. In this case, the ordering needs to be consecutive (i.e., have a next element) because the sequence will not generate a paradox if densely ordered. Consecutive orderings have a distinct next element. This is opposed to a dense ordering in which there is always another element between any two. As an example, consider the natural numbers. They are consecutive. After 2 comes 3. The rationals, however, are dense. Between 2 and 3, there is 5/2, and between 5/2 and 6/2, there is 11/4, and so on. Generally, $a/b < c/d$, then $a/b < [(a+c)/(b+d)] < c/d$. The proof[64] of contradiction from a sequence not ordered as the natural numbers fails to go through if there are not just infinitely many on the list, but infinitely many between any two claims, and so Yablo needs a consecutive ordering.

One can imagine the Yablo sequence generated by a function, but it also could be done by naming. Imagine that there is no end to the world: it simply proceeds *ad infinitum*. Also imagine in each generation a person names and utters a sentence: "I'm calling this one S4, and it goes like this: 'All sentences S5 or higher are untrue.'" Imagine no one else ever names anything with an S-name. The naming itself, in this case, determines the reference. This referential structure provides order to the sentences. One

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[64] See Chapter 4.

can think of the sentences as ordered, and the ordering thereby determining their referents, but one can just as easily think of the referents being stipulated (e.g., via naming) and that stipulation determining an order. This is not to say that the infinity is ever *completed*, as that would be a misunderstanding of the infinite. But if the process will go on forever, then one might worry about the truth-values of assertions about those future claims in much the same way.

The key is not the way in which the Yablo sentences get consecutively ordered, but that they do, in fact have a consecutive reference-ordering. There needs to be some way of determining the referent or referents of each sentence. This is necessary because otherwise the proof of contradiction cannot proceed. During the proof, one needs to be able to conclude, given Y_n, it is not the case that $\forall k > n+2$, Y_k is untrue. This move is dependent on a reference ordering to determine the sentences Y_n refers to. And so the paradox needs a reference ordering, which I have called the, “Ordering Problem”.

4. That there must be some way of ensuring the existence of the infinite number of sentences in the Yablo sequence.

The Yablo paradox requires an infinite number of sentences for many of the reasons listed in this section. Sentences need to assert the untruth of an infinite number of other sentences. There can be no last sentence in the sequence, and an infinite number of sentences is one way to accomplish this (as would be the circularity of the Looping Liar from Chapter 1). All these are important to the Yablo paradox, and the infinite number of sentences helps accomplish this goal.

But while it is useful, it also comes at a price. When we consider an infinite number of sentences, we must ask ourselves whether that set of sentences exists. In finite cases, we can be assured that they exist by uttering the sentences ourselves, or writing them down. In longer cases, we have computers display them or print them out. But even the most complex computer is finite, and Yablo's paradox requires an infinite number of sentences. There is no way to write down or utter an infinite number of sentences. In other cases, we feel assured by the nature of the subject. It seems reasonable that there are an infinite number of numbers, or an infinite number of ratios between 1 and 2. That seems to be a claim about structures in one way or another independent of us. But when we are talking about sentences that refer to language (that refer to other sentences), then the situation becomes more murky. Some deny the existence of the sequence, others assure themselves of it via reference to mathematical logic. But we know we cannot utter an infinite number of sentences, and it is a real question whether we can think of or represent any kind of infinity in thought, and this implies that generating the requisite infinite number of Yablo sentences might be difficult. It seems most tacitly assume a function can generate them in the same way mathematicians talk about generating an infinite number of truths with a function like f(x)=x+1 with a domain of the natural numbers.

But it is less clear with the sentences in the Yablo sequence that any function of this sort will be legitimate. Indeed, there is a worry buried in this sort of construction that mirrors some of the issues we saw arise for Indexical Liars in the first chapter. The Indexical Liars needed some way of ensuring their indexical components referred to the very Liar sentence in question. Without that assurance, there was no way to make the necessary moves in an argument to contradiction. Similarly, we need some way of ensuring the sentences in the Yablo sequence refer to later sentences in the sequence.

This can be done by reference to an ordering function that assigns a position to each of the Yablo sentences. On this reading, each Yablo sentence asserts other sentences are false based on what position the ordering function assigns them. It is because Y_1 is first that it refers to all the rest. Which rest? Well, the others *assigned by the function*. We might thus rewrite the sequence above with an augmented version that acknowledges the way that Y_1 refers to the rest is that the rest are assigned later positions by an ordering function, O. $\forall k>1, \neg T(O(Y_k))$ For any number greater than one, the sentence that O assigns to that number is untrue. This construction seems to make explicit the need for a function to order the sequence.

But the story does not stop here. It cannot simply be a matter of putting the symbol "O" into the sentence. As we saw with the Indexical Liars, the question that needs answering is whether or not "O" refers to the function that actually orders the sequence, and this is where it becomes an issue to see O as not simply ordering but also *generating* the Yablo sentences (hence the name "Generation Problem"). We can certainly refer in a Yablo sentence to a function, but there is an issue with referencing the function doing the ordering in the very sentences being ordered. Defining a function requires that the elements of the range are well-defined. For a range of sentences, this means that we need concrete pieces of expression. But as we saw in the Indexical Liar cases, the real issue was coming up with an interpretation that would link the indexical terms to the sentence itself. In the case of the Yablo sentences, we need some way to ensure "O" is a name for the ordering/generating function. But the function only exists after it is defined over the sentences in the range, so "O" can only refer to the function *after* that. In other words, the sentences cannot be interpreted as referring to the function before ordering since there is no function to refer to![65]

Here the need for generating claims becomes clear. If the Yablo sentences are claims, then the "O" refers as claims successfully exemplify semantic relations. If one has to appeal to an interpretation to get the relevant semantic relations going, then one can simply argue that the Yablo sequence does not exist, and no proof of contradiction could be given, and indeed, one might argue that no such function could generate the necessary Yablo sentences. There exist options for ensuring the existence of the sequence, but some sort of assurance is necessary for the Yablo sequence to generate paradox.

2.4 – Conclusion

Yablo's paradox is a semantic paradox. It relies on its constituent sentences^{d} being claims. As I will consider later in the dissertation, modern propositionalism may argue that claims are most easily understood in terms of the semantic objects defined in Chapter 1. Whether they are identical or claims express semantic objects, the propositionalist has an answer to how certain sentences (the claims) succeed in their reference as opposed to other sentences^{d} which are merely syntactically correct. It is in this light that Yablo's paradox requires us to think about the varied notions in philosophy of language set out thus far, and may change how philosophers view issues like comprehension of semantic objects. To show this connection, I have argued that there are six key assumptions to Yablo's paradox. It shares two with Liar paradoxes: its sentences

[65] Landini, (2008)

must be claims and they must assert the untruth/falsehood of some sentence[d]. Unique to the Yablo are four theses: that it have no last sentence; that its sentences assert an *infinite* set of sentences are untrue/false; that there be a reference ordering; and that there be some assurance that the infinite set of sentences exists.

CHAPTER 3

THE AIM OF YABLO'S PARADOX

Yablo describes his paradox as a Liar-like paradox that avoids any kind of circularity[66]. In considering this paradox, there is reason to set it in the context of what Yablo wanted to accomplish. While the literature focuses on what I'll call internal coherence questions (e.g., in what sense is it Liar-like?), the real import lies in questions of what I'll call its external relevance (e.g., does the paradox give us reason to revise our concept of truth?). After all, paradoxes only come alive in their (sometimes dramatic) consequences for our conceptions of the world. I hold that the import of Yablo's paradox lies in its external relevance to semantic theory and natural language. In this chapter, I begin with a series of distinctions useful in discussion of Yablo's paradox and proceed to an account of the omega-liar paradox based on an analysis of the differences in Yablo's two presentations of it. The first is "Paradox Without Self-reference" (1993) and the second is "Circularity and Paradox" (2006). [67] I seek to get clear about Yablo's goal by interpreting the differences between these presentations in light of the intervening literature. This effort reveals how Yablo sees his paradoxical sequence interacting with the fields of study in question and helps focus the subsequent discussion. I'll first draw two distinctions, one mentioned above between questions of internal coherence and questions of external relevance, the other between natural and formal paradoxes. The first

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[66] Yablo (1993), p. 252

[67] The presentation in Yablo, (1985), is not significantly different enough to be regarded as a separate version of the paradox.

separates two different sorts of questions for investigating philosophical phenomena. The second is a rough distinction between two kinds of paradox broadly understood. The first will clarify the subject matter of our discussion of Yablo's goals. The second will help clarify what those goals are, and make clear and resolve an apparent tension. In the end, I argue that Yablo's paradox is best understood as aimed at how we interpret our natural language.

3.1 – The Setup

In this section, I first sketch Yablo's paradox, followed by two distinctions relevant to the remainder of the chapter. The first distinguishes questions of internal coherence of a phenomenon from questions of its external relevance. The second works to sketch a distinction between natural paradoxes (which can arise from natural language) and formal paradoxes (which result from our formal attempts to understand our language).

3.1.1 Yablo's paradox

Recall that Yablo's paradox involves an infinite set of sentences[d].

(Y_1): For all $k > 1$, Yk is untrue.

(Y_2): For all $k > 2$, Yk is untrue.

And so on. This Yablo sequence is paradoxical because assuming any Yablo sentence entails all subsequent sentences as well as entailing their negations. As such, no Yablo sentence is true. But this satisfies the truth conditions of every Yablo sentence! As such, the sequence is paradoxical. I call the Y subscript the position numeral as it represents the sentence's position in the sequence. The numeral in the sentence (the n in "For all $k>n$, Yk is untrue") I call the content numeral as it forms the difference in content from sentence to sentence. This set of sentences^{d} is a consecutive set with a particular reference-ordering, so-called because the ordering determines the reference of each claim (via each sentence^{d} claiming, "all the rest are false"). The situation then, consists of an infinite set (the Yablo sequence) of Yablo sentences^{d} in a particular reference-ordering. There is no stable assignment of truth-values for this Yablo sequence.

As I suggested in Chapter 1, one way to think of Yablo's paradox is resultant from versions of the Liar paradox. It is similar to a Looping Liar, but replaces the circularity of the loop with an infinite number of sentences^{d}. Classically (think Russell[68]) the Liar paradoxes were considered symptoms of self-referential circularity. Much as the Looping Liar avoids self-reference (while still exhibiting circularity) the Yablo paradox pushes the envelope one step further in an effort to produce a liar-like paradox that exhibits no circularity whatsoever.[69]

3.1.2 Internal Coherence vs. External Relevance

Paradoxes are grist for the philosophical mill. We characterize these sorts of circumstances, then draw our conclusions about the world around us. In this setup is a clear demarcation between two sorts of inquiry gestured at in the introduction. We can distinguish between questions of internal coherence and questions of external relevance. The former investigate the particulars of a situation (argument, paradox, etc.). They ask

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[68] Russell (1908)

[69] Yablo (1993), p. 252

about the pieces necessary to make a situation make sense. The latter investigates the broader implications within the conceptual landscape. They ask about the sorts of conclusions we ought to draw about the world around us based on the situation in consideration.

As an example, consider an experiment of Hume's. When you push on one eye with a finger, you see double. There are two of all the objects of visual perception.[70] A question of internal coherence arises. Are there differences in character between the two images that result and the single original image? In other words, do the images, so to speak, lose nothing or are there relevant differences (warping of lines, lack of depth, etc.)? This question asks about the very premises from which we are to reason. On the other hand, a question of external relevance asks whether this experiment ought to push us towards some version of indirect realism. If we are seeing two when there is really just one, ought we to subscribe to the thesis that the objects of awareness are not independent and external? Notice this is a conditional question where the antecedent seeks to fix the internal coherence of the situation in order to draw some external conclusion. This sort of question relates the situation to broader inquiry.

In terms of Yablo's paradox, then, internal coherence questions include, "Is it circular?" "How must the sentences be worded?" "Is there a real contradiction here?" and questions about the necessary features of the Liar and Yablo paradoxes discussed in Chapter 2. These are the sorts of question the focused on by the literature. Examples of external relevance questions of Yablo's paradox ask about its implications for theories of

[70] Hume (1978), p.210. "When we press one eye with a finger, we immediately perceive all the objects to become double, and one half of them to be remov'd from their common and natural position."

the causes of paradox, and for notions of truth and reference and intentionality. As I suggested above, paradoxes come alive in their consequences. As such, these questions of external relevance form the main focus for the dissertation. I'll address questions of internal coherence in Chapters 4 and 5, and do so in light of their usefulness to questions of external relevance (as will be clear, the focus will be on determining a completed syntax for the Yablo sentences, thereby ensuring they qualify as sentences[d]). This chapter, then, functions as a setup for questions of external relevance for Yablo's paradox by focusing on Yablo's intent in presenting the paradox.

3.1.3 Natural Paradoxes vs. Formal paradoxes

Now, I understand that it is a daunting (if even possible) task to give a rigorous taxonomy of the different 'types' of paradox and I do not intend for the question to ride on a taxonomy that we can all shake hands to and agree upon. That said, I think this is one place where confusion can arise and where that confusion can be helped by realizing that trying to type the paradox is a question of external relevance. Let those paradoxes that pose a problem for meaning in natural language be called "natural paradoxes." For example, the Simple Indexical Liar paradox (sketched in Chapter 1) arises in natural language with an utterance of "This sentence is false." One might wonder whether this is simply the semantic-syntactic distinction recast. It is similar (as can be seen by the examples I've used to carve out the distinction) but the key difference is that semantic paradoxes can have both natural and formal versions. All syntactic/mathematical/logical paradoxes are formal paradoxes, but only some semantic paradoxes are natural. It is clear that Yablo's paradox is at least intended as a semantic paradox.[71] The question then becomes whether it is a formal or a natural (semantic) paradox. The paradox is not merely academic. It poses a serious problem for the consistency of the natural language rules governing the use of indexicals such as "this." Let those paradoxes that pose a problem for formal ontological theories be called "formal paradoxes." For example, there is Russell's paradox, which arises for any naïve theory of sets or classes. With this distinction in place, we can articulate a key question in thinking about Yablo's paradox:

Is Yablo's paradox a formal paradox or a natural paradox?

How we answer this question is of central importance to what we take Yablo to have intended to show by means of his omega-liar paradox, and whether our inquiry should skew towards a discussion of formal languages or of semantics.

One way to answer the question: Yablo was attempting to criticize a certain diagnosis of Liar paradoxes. On this view, Yablo offered his paradox to show that a Tarski-style restriction on the "truth" predicate is an unjustified over-reaction to a fear that self-reference generates paradox.[72] In dealing with paradoxes involving a "truth" or "falsehood" predicate, Tarksi famously proved that the only intelligible (non-contradictory) "truth" and "falsehood" predicates for a formal language L are those indexed to L. For a formal language L, "truth" is an indefinable. In contrast, "Truth-in-L" can be defined, but it can only be deployed at the metalinguistic level. The classical Tarski semantics is built around this notion.

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[71] I take this much from his naming it a Liar and calling it Liar-like (1993), as from his acknowledging that there are corresponding syntactic versions, which strongly implies that his is semantic.

[72] Using a cannonball to kill a fly, as he puts it.

Take the Curry paradox as a motivating example. Consider the naïve T-schema: ‘p’ is true⇔p. The Curry paradox, on one reading, results from this naïve version. Take the following sentence:

a) a is true .⇒. q & ¬q

By the naïve schema:

a is true ⇔ (a is true .⇒. q & ¬q)

But then we get:

a is true ⇒ (a is true .⇒. q & ¬q)

And by exportation:

a is true & a is true .⇒. q & ¬q

By tautology:

a is true .⇒. q & ¬q

And via the naïve schema:

a is true

And by Modus Ponens:

q & ¬q

Tarski’s schema uses the notion of truth-in-L to restrict truth attributions:

Tarski T-Schema: ‘p’ is true_L⇔p

In this case, the Curry sentence becomes:

a) a is true_L⇒ q & ¬q

And the instance of the Tarski schema:

a is true_{L*} ⇔a is true_L⇒ q & ¬q

for L*, a metalanguage of L that includes “true-in-L”. We can get:

(a is $true_{L*}$ &a is $true_{L}$)$\Rightarrow$ q & ¬q

But this cannot simplify via a tautology rule since a's truth in different languages (L and L*) is not tautologous. Such is the Tarski-style semantics as a response to problems like the Curry and the Liar.

The crux of these formal semantics is that we have a particular language with constants and variables and predicates, and in creating an interpretation for that language, we assign properties (or sets) to predicates, objects to constants, and by appeal to denumerable sequences of objects in a given domain, we assign satisfaction-in-L to open wffs of the formal language and truth-in-L or falsehood-in-L to closed statements (sentences).[73] In this light, Yablo's paradox becomes one of *assigning truth or falsehood* to the sentences (strings of symbols, as it were) in the Yablo sequence. This interpretation sees Yablo's paradox as a formal paradox, and one that endeavors to criticize those who, employing Tarski's formal theory, claim that vicious circularity warrants the conclusion that there are no univocal properties of "truth" and "falsehood."

A second way to think about Yablo's intent, is that the omega-liar paradox endeavors to show that within natural language, contradictions can arise without self-reference. Our natural language seems to allow Yablo's sequence, and the sequence is contradictory. It seems easy, after all, to imagine an infinite set of claims. Take this sequence:

1) All the rest of the claims are false.

2) 2=2+1

3) 3=3+1

[73] Hunter, (1996), p.148

And so on. The first claim is true and akin to each entry in the Yablo sequence insofar as each sentence claims that an infinite subset of sentences is false. With this sort of empirical example, we might see the issue as not one of formal languages but of the language we engage and deploy in ordinary circumstances. Anyone (i.e., not just a logician supplied with a formal interpretation, I_L) can see what each of the Yablo sentences^d means, and can evaluate it accordingly. *Anyone* considering the Yablo sentences^d runs into problems in evaluating their truth or falsehood. It is not, as the previous interpretation held, a problem with a formal interpretation (or the process of laying down a formal interpretation), but instead their evaluation trouble is a problem with the *language itself*, arising not out of a formal construction, but a naturally occurring language. This interpretation suggests the issues with Yablo's paradox indict the notions that we deploy in an attempt to understand our natural language.

These paradox types are not mutually exclusive (one phenomena could entwine both sorts), nor are they exhaustive, yet they will be useful to keep them in mind as it will help keep certain questions of external relevance in place, making the big picture come through with more clarity than if we ignored the distinction.

Thirteen years pass between Yablo's two presentations of what he calls the omega-liar paradox, a wrinkle that has become known as Yablo's paradox. A look at the changes in presentation, informed by the intervening literature, sheds light on Yablo's interpretation of his paradox, and provides a starting point for further investigation.

3.2 – Yablo (1993) "*Paradox without Self-Reference*"

Yablo's first presentation imagines an infinite sequence of sentences S_1, S_2, S_3,…, S*n*,… each to the effect that every subsequent claim is untrue:

(S_1) For all $k > 1$, S_k is untrue
(S_2) For all $k > 2$, S_k is untrue
(S_3) For all $k > 3$, S_k is untrue…[74]

Each Yablo statement is labeled with a numerically subscripted S whose position numeral corresponds to the content numeral of the sentence it names. Yablo gestures to logical vernacular, using a natural language version of the material conditional and a universal quantifier while including neither. In fact, the only symbol involved here is a greater-than symbol (>) that well-orders the natural numbers. Yablo avoids the locution, "is not true," in his quasi-consequent (i.e., 'S_k is untrue'), instead opting for the more neutral term, 'untrue'. Each statement, then, in this Yablo sequence has 1) a position numeral that corresponds to its content numeral, 2) an avoidance of logical symbols in lieu of natural language, and 3) an ambiguous claim of untruth as its result. It is of note that the correspondence between position- and content numerals ensures we know i) the sentence in any position (insert the position numeral into the Yablo sequence schema) and ii) each sentence's position (the schema instance with the *n* in the slot is the *n*th sentence). These two features of a sequence are called its effectiveness. Put more simply, effective orderings imply two things: first, one can know which element in the ordering occupies a particular position. Second, one can know the position of any element.[75]

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[74] Yablo (1993), p. 251

[75] Sequences exhibiting these two features are called, "effective enumerations" and that Yablo's sequence is effective will become especially useful in Chapter 5.

Yablo continues on from his presentation of the sequence toward derivation of a contradiction:

> Suppose for contradiction that some S_n is true. Given what S_n says, for all $k > n$, S_k is untrue. Therefore (a) S_{n+1} is untrue, and (b) for all $k >$ n+1, S_k is untrue. By (b), what S_{n+1} says is in fact the case, whence contrary to (a) S_{n+1} is true! So *every* sentence S_n in the sequence is untrue. But then the sentences *subsequent* to any given S_n are all untrue, whence S_n is true after all! I conclude that self-reference is neither necessary nor sufficient for Liar-like paradox.[76]

First note that the form of the argument begins with a *reductio ad absurdum*. Accordingly, let's look first at the *reductio*. First we are given the following assumption: Some S_n is true. Notice that this is not a particular claim, but a schema for a claim: he has used a schema with variable *n* in the content numeral slot. He reasons schematically so his results will be applicable to any sentence in the sequence. More formally, he assumes a sentence with a free variable *n* so his results are generalizable to all the claims in the sequence.

Within the scope of the *reductio* assumption, Yablo draws two inferences based on S_n (i.e., For all $k > n$, S_k is untrue): (a) S_{n+1} is untrue, and (b) for all $k > n+1$, S_k is untrue. Yablo gets (a) by discharging S_n's antecedent with the arithmetic truth: $n+1>n$. For (b), note that if $k > n+1$, then $k > n$. Yablo reasons from "All As are Bs," to "All Cs are Bs," on the grounds that "All Cs are As." Here Yablo moves from his *reductio* assumption (which assures that all the rest in the sequence are untrue) to the same claim about a restricted domain. In short, (a) results from our *reductio* assumption when combined with an arithmetic truth, and (b) follows by a truth of logic and arithmetic (by the "All Cs are As" claim and $(\forall mn)(k>n+m \;.\!\Rightarrow\!.\; k>n)$).

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[76] Yablo (1993), p. 252

Yablo proceeds to note, "By (b), what S_{n+1} says is in fact the case, whence contrary to (a) S_{n+1} is true!"[77] Recall that we know what claim is assigned to each number, so we know that S_{n+1} claims that for all $k > n+1$, S_k is untrue. But this just is (b), and since what it asserts is in fact the case, Yablo concludes (under the *reductio* assumption) that S_{n+1} is true, which contradicts (a)'s assertion that S_{n+1} is untrue. If S_n is true, then S_n is untrue. Hence, S_n is untrue.

Closing off the *reductio*, Yablo concludes that, "So *every* sentence S_n in the sequence is untrue."[78] A normal *reductio* would only assert the falsehood (or in this case, untruth) of its specific assumption. As was noted earlier, however, Yablo's assumption was schematic: this proof assumed a generic sentence of the sequence, and as such, Yablo's *reductio* reasoning remains applicable no matter which particular sequence-statement functions as assumption.[79] In other words, the proof uses no particular information that ties it to a particular (absolute) location on the list. The generic *n* of his schematic proof allows Yablo to assert the untruth of every sentence[d] in the sequence.

Now recall that we know what each S_n claims: specifically that every sentence[d] coming after it is untrue. Yablo notes that if his result that every sentence[d] in the sequence is untrue, "…then the sentences *subsequent* to any given S_n are all untrue, whence S_n is true after all!"[80] Yablo's *reductio* proof works for any old S_n, so all S_n must

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[77] Yablo, 1993, p.252

[78] Yablo, (1993), his emphasis.

[79] Specifically, proof of F*n* for variable *n* allows for universal generalization to $(\forall n)Fn$.

[80] Stephen Yablo, (1993)

be untrue. But each S_n says that all subsequent sentences[d] are untrue. As such, each S_n is false and that proves that each S_n is true. Paradox!

In the preceding discussion, three keys to this proof have revealed themselves: first, the schematic *reductio* itself; second, the fact that the schematic *reductio* was performed using a schematic expression "S_n" assuring that Yablo's results are applicable to every sentence[d] in the sequence; third, the ability to effectively produce the *n*th member in the series. Effectiveness is needed for it is only through knowing the content of statement S_n that we can show its assertion to hold through the universal applicability of the *reductio* proof. These are the three key features of Yablo's 1993 presentation of the omega-liar paradox.

3.3 – Yablo (2006): "*Circularity and Paradox*"

Thirteen years later, Yablo presented his paradox in a new way, changing the description of the sentences in his Yablo sequence as well as shifting the technique of his derivation of the contradiction. He states:

> An example is what we may call the Ω-liar. This involves an infinite series of sentences S_i, each describing as false all the S_js occurring later in the sequence:
>
> $S_0 = (\forall n \geq 1) \sim T\,[S_n]$
> $S_1 = (\forall n \geq 2) \sim T\,[S_n]$
> .
> .
> .
> $S_i = (\forall n \geq i+1) \sim T\,[S_n]$
> .
> .
> .

In this version,[81] Yablo still names each sentence with a numerically subscripted S. Rather than the earlier version where the content numeral is identical with the position numeral, we have a presentation where the content numeral is one higher than the position numeral.[82] In these Yablo sentences, Yablo has replaced "S_k is untrue," by "~T[S_k]". This version uses the more formal language of the Tarski-style truth definition. Here we get a universal quantifier with the variable *n*[83] restricted by the greater-than-or-equal-to sign, '≥'. As such, the domain of reference for the universal claim is the set of all natural numbers larger than or equal to that named in the sentence. What does the sentence say about that domain? It says, ~T [S_n]. Yablo inserts a negated truth operator over S_n, where *n* is bound by the quantifier.

As in his first presentation, each sentence[d] denies all those that come after it. This time, however, Yablo uses a truth-operator within a universally quantified statement rather than natural language analogues of these formal logical symbols. As such we have: 1) a position numeral that corresponds to [its content numeral – 1], 2) an embrace of logical symbols, particularly a first-order quantifier and a truth-predicate, and 3) a formal negation sign (~) of the truth of referenced sentences. It is of note that the correspondence still ensures the effectiveness of the sequence.

After describing the sequence, Yablo continues, deriving his contradiction as follows:

Earlier S_is entail later ones, so if any S_i is true so are all the ones after it.

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[81] Yablo, (2006), p.140

[82] (Position numeral) + 1 = Content numeral

[83] Clearly *n* is shorthand, in this case, for the natural numbers, if only because numbers are the only entities for which '>' can be meaningfully flanked.

> At the same time S_i is true only if the sentences after it are false. Therefore no S_i can be true; the only consistent assignment makes them all false. However that assignment is not consistent either, since now the truth conditions of each S_i are fulfilled. So we have an intuitive contradiction.[84]

Yablo begins by noting that S_i entails S_{i+m}. This is because the claims made by S_{i+m} are a subset of those by S_i for any m. To see this, consider the claim made by S_{i+m}:

$$(\forall n \geq i+m+1), \sim T\ [S_n]$$

Recall that S_i:

$$(\forall n \geq i+1), \sim T\ [S_n]$$

And:

$$(\forall k)(k > [i+1]) \Rightarrow (k > [i+1+m])$$

From this and S_i we can get S_{i+m}. Hence, the truth-conditions of any lower subscripted S-sentence contain (refer to, assert…) the truth-conditions of any higher-subscripted Yablo sentence. [85] As such, if lower numbered subscripts (i.e., earlier sentences) are true, then higher numbered statements are true. Given a Yablo sequence, one can assert an infinity of conditional statements, each a discharged instance of:

$$(\forall n>0)(\forall m)(n > m\ .\supset.\ T[S_m] \supset [TS_n])$$ [86]

Or, in other words: For any n greater than m, $T[S_m] \supset [TS_n]$. This is an entailment

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[84] Yablo, (2006), p.140

[85] Take a natural example: 1. Dave likes coffee and Dave likes cookies. 2. Dave likes cookies. Now note that if 1 is true, so is 2, as 2 is contained in 1 (or claimed by 1, or asserted by 1, pick the metaphor that best suits).

[86] The convention here sees dots flank connectives as punctuation, used symmetrically. The most dots flank the main connective, 2nd most flank the secondary connectives, etc.

relation between the semantic contents of the sentences of this Yablo sequence.

Yablo follows this by noting the claim each makes about sentence^{d} names: "At the same time S_i is true only if the sentences after it are false."[87] Every sentence makes a claim about those larger numbers that subscript certain S-sentences^{d}, namely that those sentences^{d} are not true (false). Take a particular statement, S_8, that claims: $(\forall n \geq 9)$, $\sim T\ [S_n]$. Since $13 \geq 9$, $\sim T(S_{13})$, and likewise for all other numbers (and their corresponding S- sentences^{d}) greater than 9. Yablo is, in essence, noting that each sentence^{d} makes a claim about other sentences named in a particular way: the "after it" just means all those statements with larger position numerals.

Putting the preceding two claims together, Yablo concludes: "Therefore no S_i can be true; the only consistent assignment makes them all false." In this, he generalizes to all the sentences^{d} in the sequence much like he did with the *reductio* proof in 1993. Recall that the claim I produced above that asserts the entailment between any $T[S_i]$ of any sentence^{d} in a Yablo sequence and $T[S_{i+m}]$ of any later statement S_{i+m} in that same sequence. That is, all sentences^{d} higher in the series are false. Yablo concludes that each $T[S_{i+m}]$ yields a contradiction with $T[S_i]$, and summarily concludes the only consistent assignment of truth values remaining makes all the sentences^{d} in the sequence false.

This is not the end of the story: "However that assignment is not consistent either, since now the truth conditions of each S_i are fulfilled."[88] In other

[87] Yablo, (2006), p.140

[88] Yablo, (2006), p.140

words, since we know the content of each sentence^d (i.e., that all later sentences^d are false) we know its truth conditions. If we assign *all* of them false, then whichever subset a particular S_i refers to is false, thus fulfilling its truth conditions, whence it should be true. Yablo finishes: "So we have an intuitive contradiction."[89]

The keys in this second presentation are: first, the entailment relation between earlier sentences^d and later sentences^d, $T[S_i] \Rightarrow T[S_{i+m}]$, for $m>0$, (which stems from the truth-conditions of later statements being subsets of the truth-conditions of earlier statements); second, the claim every sentence^d makes involved in this guaranteeing any sentence with a higher position number is not-true; third, the effectiveness of the sequence that allows our reasoning about their contents, and fourth, the fact that every sentence^d makes the same claim about the statements following it (it is this uniformity that allows Yablo to generalize).

3.4 – The Difference between 1993 and 2006

In the two papers, Yablo changes the way he words the sentences designed to capture the Yablo sequence. He changes the notation he uses from an S_n to an S_i.[90] More importantly, he changes around his proof structure, moving away from the *reductio*-dependent 1993 version and toward the entailment version from *Circularity and Paradox*.

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[89] Yablo, (2006), p.140

[90] This is an interesting change, as it does nothing substantive, yet does change the subscript variable Priest (1997) mentions in his analysis of Yablo's original article. Nothing rides on this, but it does seem convenient.

He also changes his proof of contradiction from a *reductio* to a more metalinguistic version focusing on entailment relations. One key piece remains, and it forms the commonality between the following two quotes:

1. "By (b), what S_{n+1} says is in fact the case, whence contrary to (a) S_{n+1} is true! So *every* sentence S_n in the sequence is untrue."[91]
2. "At the same time S_i is true only if the sentences after it are *false*. Therefore no S_i can be true."[92]

You can see the key move is from a randomly chosen sentence to the untruth of *all* the sentences in the sequence made possible *because the sentence chosen is random* (i.e., he is reasoning schematically). Any generalization requires a free variable. In this case, the only option is the *n*, as all other potential variables are bound. Indeed, the *n* is free. A lot rides on this. Two things are true because his proof will work for any *n*. First the generalization is legitimate (i.e., the premise that each Yablo sentence is false is justified). Second, the reassertion of each Yablo sentence is legitimate. The second claim derives from the first. Because we can argue that each Yablo sentence^d is false, the truth conditions of each obtain, specifically that each member of the relevant subset of sentences^d is false (namely that subset containing all sentences^d subsequent to a given claim). Despite the intervening literature including Graham Priest's claim that this move leads to circularity, discussed at length in Chapter 4, and the existence of alternate non-generalizing arguments like Bueno and Colyvan, (2003), Yablo leaves this generalization in. His continued use of this particular move seems to threaten his ability to derive a

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[91] Yablo, (1993), p.252, his emphasis.

[92] Yablo, (2006), p.140, his emphasis.

contradiction from the Yablo sequence, at least without specifying some particular explicit syntactic form capable of responding to accusations of circularity.

In addition to the notational and proof differences and retained generalization, Yablo also adds in defense of the paradox from certain concerns. Specifically, he argues for i) the existence of the infinite set of sentences and ii) gives conditions for avoiding structural self-reference in the sentences' claims.

In the first case, he argues we might characterize the sentences empirically or formally and then empirically investigate the consequences of such a formal characterization. He is explicitly at a loss for a non-*ad hoc* way to criticize the sentences of the sequence: "How an empirical description could fail to apply because of an object's semantical properties it is not easy to see."[93] His sentences seem to describe later sentences in normal ways. I might refer to the other sentences in Chapter 3 of my dissertation, or hope that all the claims in Chapter 4 are true. These are empirical descriptions much as are Yablo's, and as we have no problem with them, we ought to not have a problem with Yablo's sentences[d]. The sentences are in plain English, and we can construct perfectly clear claims that seem to capture the Yablo sentences[d]. First we need claims within a sequence about the other claims in that sequence. Second we need claims about the truth or falsehood of an infinite set of sentences[d]. These two give us the conditions of the Yablo, and both are normal sorts of claims. As such, Yablo claims, denying existence to Yablo sentences[d] on this basis seems *ad hoc*. In the second case, he addresses the any worry that a particular statement might self-refer (by referring to sentences[d] of a particular form, one which it shares with them, i.e., exhibit structural self-

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[93] Yablo, (2006), p.142

reference). Think here of a claim like, "All generalizations are false." This attributes a particular property (*being false*) to sentences of a particular form (universal sentences). That form, however, is one that it shares, and so it refers to itself. One might worry that the Yablo sentences, insofar as they seem to pick out other sentences[d] by their form (which is shared between all Yablo sentences[d]) it might in that fashion self-refer. These cases can be assuaged, Yablo claims, by subtly changing the structure of each sentence to make it unique. Each addresses a concern of Priest's. He worries briefly about the existence of the sequence, and Yablo reads Priest's circularity concern as a worry about structural self-reference. I'll present these concerns in the next section.

In summation, Yablo must have seen some importance in increasing the notation, avoiding the *reductio*, changing the proof structure, keeping the universal generalization, addressing the existence concern, and combating the structural self-reference claim.

3.5 – Priest's Objection and The Intervening Literature

In the literature following Yablo's 1996 "*Paradox without self-reference*" there are two distinct periods. In the first couple years, Goldstein, Hardy, and Tennant use their articles largely to clarify the paradox. The second period, ushered in by Priest's "*Yablo's Paradox*", focuses on criticism (and defense) of the paradox. Priest sets off a series of clashing entries investigating (among other things) the relation between the sentences used to describe the Yablo sequence and the sequence itself. I leave the clarificatory period for now and focus on the critical period as represented mainly by Priest's article.

Priest's analysis boils down to two key concerns: the existence of the sequence (a concern which he dismisses relatively summarily) and the circularity of the paradox (on which he spends most of his time). Priest's brief aside about the existence of the sequence turns on his discussion of circularity. As such, I begin there, and proceed forward.

Priest's argument for circularity turns on Yablo using a free variable during his schematic argument. Priest writes one line of proof: $Ts_n \Rightarrow (\forall k > n), \neg Ts_k$[94]. He asks his reader to consider the justification for this line (along with other similar lines): "It is natural to suppose that this is the *T*-schema, but it is not."[95] He refers to a Tarski style *T*-schema: *T*'*p*' iff *p*, for the truth predicate where 'p' is a structural representation of p. Priest proceeds, focusing on the '*n*' from the above line of proof. He writes, "The *n* involved in each step of the *reductio* argument is a free variable, since we apply universal generalization to it a little later; and the *T*-schema applies only to sentences, not to things with free variables in." The *T*-schema above, in other words, requires closed wffs (sentences) to appear on the left and right of the "iff". For example:

S_n is true iff $(\forall k > n), \neg T[S_k]$,

since S_n names "$(\forall k > n), \neg T[S_k]$." Yablo can work with this example, but cannot then go on to regard *n* as a free variable, ready for universal generalization. As such, Priest argues the move from Yablo's proof (i.e., $Ts_n \Rightarrow (\forall k > n), \neg Ts_k$) cannot be justified by a Tarski-schema[96] $T[S_n]$ iff $(\forall k > n), \neg Ts_k$. If S_n has a free variable, then the move is illegitimate.

As such, a lot turns on how Priest understands the *n*. Priest reads the *n* in terms of a particular move in Yablo's argument. Yablo shows how to argue for untruth for a single schematic Yablo sentence and then concludes that they are *all* untrue. This is the

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94 Priest, (1997), p. 237

95 Priest, (1997), p. 237

96 In the literature and earlier discussions this is sometimes called Release.

universal generalization (i.e., UG) Priest refers to in his criticism. As the UG is centrally involved for Yablo's derivation, and a UG is only legitimate on a free variable[97], Priest concludes the Tarski *T*-schema cannot apply.[98]

Priest's article went on to spawn a long line of work[99] based largely on this particular turn and the circularity he bases on it. Some defend Yablo and others argue that the Yablo sentences are circular. Something of note here is that while Priest focuses on the satisfaction requirements, the later literature moves towards a discussion of circularity of description (i.e., sentence) and circularity of object (i.e., that which exemplifies semantic relations). This is the focus of most of the discussion in the literature.

Priest attempts a revision of the set of sentences in terms of satisfaction, which he claims will avoid the restrictions of Tarski-biconditionals (i.e., it can form something akin to Tarski-biconditionals[100] that allows for sentences including free variables). Priest claims his resultant sequence still paradoxical, but also finds self-referential circularity.[101] Rather than sentences, satisfaction is a relation between open sentences (Priest calls his

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[97] For one instance of an explanation, consider Copi, (1979), p.72: "…what is true of *any arbitrarily selected individual* must be true of *all* individuals." He follows with an example of a fallacy on p. 73, attempting to generalize from a constant rather than a free variable. In Yablo's case, he uses an arbitrarily chosen member, NOT a particular sentence. As such, it must have a free variable in it, and the only thing unbound is *n*.

[98] Priest (1997), p. 238

[99] See, e.g., Sorensen, (1998), Beall, (2001), Bueno and Colyvan (2003, 2003, 2005), Ketland (2004), Bringsjord and Van Heuveln (2003).

[100] It will be akin to Tarski bi-conditionals insofar we have:

$p \Rightarrow T`p'$ (Capture)

and

$T`p' \Rightarrow p$ (Release)

[101] Priest (1997), p. 238. I leave a detailed exposition of Priest to Chapter 4.

particular ones predicates) and elements (in this case, numbers). The *x*th satisfaction predicate in the sequence, according to Priest, "…is the predicate 'no number greater than *x* satisfies this predicate'. The circularity is now manifest."[102] In other words, the sort of open sentence required by Yablo's generalization is a sentence about satisfaction. To derive the contradiction from a sequence filled with satisfaction sentences requires self-referential open sentences (Priest's predicates). As such, Yablo's argument (which needs the generalization to derive the contradiction) is inherently circular, according to Priest. Priest's argument, in turn, rests solely on the generalization, and the fact that the Yablo sentences need be open formulas for the generalization to work.

Priest also worries about the existence of the Yablo sequence. "How can one be sure that there is such a sequence? (We can imagine all sorts of things that do not exist.)"[103] He goes on to say we can be sure because the sequence can be defined in terms of his satisfaction predicates (i.e., the open sentences that compose Priest's version of the sequence). The thing to notice here is that his resolution to this worry is dependent on his version of the Yablo sequence. He is assured the sequence exists because he is sure the relevant predicates exist.

One final piece of the literature relevant to the current inquiry is an argument by Bueno and Colyvan from their *Paradox without satisfaction*.[104] They present a different argument to contradiction from a Yablo sequence. Their argument, avoids the generalization Priest criticizes inhis analysis of Yablo. They note that they argue for a

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[102] Priest (1997), p.238

[103] Priest (1997), p.238

[104] Bueno and Colyvan, (2003)

weaker contradiction, yet assert about the sentence, s_1, that they show contradictory: "Even if 's_1' were the only paradoxical sentence in the Yablo list, this would be sufficient to conclude that Yablo's paradox (i) is a paradox, and (ii) is not circular – or, at least, it's not circular in the sense [of Priest]."[105] I think what they have in mind is largely correct. In their proof, the *arbitrariness* of the sentence in question isn't used to derive the contradiction as it is in Yablo. They avoid the UG. Proofs like this show Priest's analysis is questionable. Yablo's particular derivation of a contradiction is not essential for for Yablo's paradox. As such, even if one grants the entirety of Priest's analysis, the paradox is not thereby impugned.

3.6 – An Interpretation of Yablo's Paradox

By way of summary, let's look back at the pieces that we need to build this interpretation. First, we have the commonalities of Yablo's two proofs:

1) The infinite sequence of sentences[d];

2) The use of UG in the proof;

3) Using the generalized sentence[d] (that asserts that each later sentence[d] in the sequence is false) to reassert all the sentences[d] (as their truth conditions are contained in the claim made by the generalized sentence[d]);

4) The effectiveness of the sequence;

Second, we have the changes:

1) Move from a *reductio* structure to metalinguistic claims about entailment;

2) The move from avoiding logical symbols to embracing them;

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[105] Bueno and Colyvan, (2003), p.155

3) A notational change from the particular variable (that shows up in Priest's analysis);

Third, we have the concerns Yablo chooses to address in his second presentation:

1) The existence of the sequence;

2) The structural self-reference worry;

Fourth, we have the intervening literature:

1) Priest's argument that Yablo's sequence is circular based on *n* used as a variable, and his question of whether a Yablo sequence exists;

2) The subsequent discussion of circularity of description and object;

3) Bueno and Colyvan's "*Paradox without Satisfaction*";

These are the factors needed to answer whether or to what extent the aim of Yablo's paradox makes it more of a natural paradox or a formal paradox based on how these factors points toward its external relevance.

There is a tension here: In adding symbolic notation, Yablo seems to embrace the paradox's relation to formal language, giving us reason to think of the paradox as a formal one. Yet while his most clearly stated goal is to avoid self-reference and circularity, he does not work to omit the generalization that Priest uses to argue the paradox *is* circular. He does this in the face of Bueno and Colyvan's argument to contradiction for a Yablo sequence that avoids satisfaction by utilizing particular *n* (rather than arbitrary *n*), an argument that avoids the universal generalization. Further, while Yablo seeks to address the structural self-reference worry (which he sees as Priest's), he does nothing to address the longer and broader questions of self-reference, nor does he address satisfaction worries, all of which seem to spring from the generalization (via

Priest). So why does he leave it in? Why not take Bueno and Colyvan's derivation? He ends his latter presentation of the paradox, saying, "So we have an intuitive contradiction."[106] It would seem then that he is only interested in the 'intuitive contradiction', not the formal contradiction. Yet he moves to a more symbol-heavy presentation. These two pieces of his presentation seem at odds.

There is a way to understand the two seemingly opposed motives together. First consider: Tarski semantics are for formal languages, but they are dependent upon an interpretation. I suggest that Yablo sees us as using Tarski's theory to understand our own natural language. Tarski, one might think, should give us insight into how our language-use deals with the world. And while, in some sense, the problem with the Yablo sequence lies in not being able to assign truth-values full-stop, the better answer lies in a footnote Yablo appends to his second presentation. In it, he seems unconcerned with the logical apparatus, yet describes the problem as the inability to properly assess the Yablo sequence under its *intended interpretation*.[107] One might think of this not as a formal interpretation (set out for all the symbols of the language) but rather as a *natural* interpretation, the one that flows out of our linguistic practices (or perhaps the one determined by our language). In terms of the question at hand, the way to dissolve the tension between increased notation and lack of concern for logic is to see Yablo as using a formal language (insofar as he thinks Tarski has something to tell us) in an attempt to capture our natural language. As Tarski requires an interpretation, we as language users supply one: the intended interpretation, the one we are attempting to deploy in our

[106] Yablo (2006), p.166

[107] Yablo (2006), p.166

everyday conversations. Returning to the language we used to start the chapter, Yablo's paradox is a natural paradox. It uses formal language constructs to critique the way we interpret our language. If successful, then, Yablo's paradox puts restraints on how we understand our language, as we would expect from a natural paradox.

3.7 – Conclusion

I have argued that Yablo hopes his paradox reveals important features of how we understand our language. It is about the natural interpretation (the non-artificial interpretation) we bring to the table. Paradoxes, on one view, are glitches in our understanding of the world. For example, the barber paradox[108] is a simple miscalculation on our part: we simply don't realize such a man cannot exist until we think about it. With other paradoxes, the conclusion (or resolution or dissolution) is much more complex, and, accordingly, much more interesting. Yablo's paradox is one such paradox that, understood in this way, is a natural paradox, and one that calls into question the way we understand our natural language. What are the meanings involved? Is there reason to think all our non-first level claims need to be understood as first level claims *or not at all*? What is the metaphysical grounding of the marks, "The dog is on the rug," to dogs and rugs? These are some of the questions of external relevance upon which Yablo might bear. They are not formal questions of the relations of symbols (as might be questions of internal coherence), but of that more complex and intricate realm of semantics. The paradox assumes that there are claims that are intentional entities (i.e., they have aboutness) and that an important part of moving forward in understanding our language is

[108] There lives a man in a small village who shaves all and only men of the village who do not shave themselves. He both shaves and does not shave himself, a difficult task.

thinking about how linguistic objects (like claims and sentences[d]) relate to the metaphysics of meaning (i.e., those things, whatever they are, that relate our symbols to our world).

CHAPTER 4

PRIEST'S (DIS)SATISFACTION YABLO

Graham Priest argues that Yablo's paradox will not work in its original form. He offers a reconstruction in terms of the satisfaction of an open wff, which, he thinks, recovers a semantic Liar-like paradox but cannot realize Yablo's original goal of forming such a paradox without self-referential circularity. Priest maintains that once the paradox is characterized in terms of satisfaction, we see that it essentially involves a self-referential fixed point[109]. In this chapter, I present Priest's critique of Yablo's paradox and a detailed attempt to work out the particulars of his account.

Whereas each of Yablo's sentences claim all those subsequent to it in the ordering are untrue, Priest's version uses satisfaction of an open well-formed formula in place of truth. After laying out Priest's critique, I define satisfaction and discuss its usefulness (on Priest's view) to Yablo's original derivations of the paradox. This allows a reconstruction of how Priest understood Yablo's use of universal generalization in the original demonstration of the paradox. With this in place, I explain Priest's reconstruction of Yablo's paradox and call it a "satisfaction Yablo paradox". As noted above, Priest holds that there is self-referential circularity manifest in the satisfaction Yablo. I shall show, however, that Priest's formulation of the satisfaction Yablo is ill formed. It involves a syntactic characterization of an expression that is incoherent because it uses the very expression in its own characterization. This is vicious circularity with a vengeance and

[109] People talk of fixed points of a function where f(x)=x such that talk of f(x) just is talking of x, a sort of mathematical "self-referential circularity" found in Gödel and others.

surely cannot be a viable reconstruction of Yablo's paradox. I cite a sketch of Gödel's result (paradigmatic of a legitimately formed self-referential fixed point) to help clarify Priest's error. In an attempt to salvage Priest's argument, I present two alternate satisfaction Yablo paradox versions. In the end, I conclude that they are viable versions of Yablo's Paradox that are not self-referential (at least not in Priest's sense). I conclude that even if Priest is correct that the original Yablo paradox must be reconstructed in terms of satisfaction, he fails to reveal that circular self-reference is involved.

4.1 – Priest's Yablo Critique

Yablo's paradox derives from an infinite sequence of sentences each of which intuitively asserts all the rest are false:

$s_1 = \forall k > 1, \neg Ts_k$ [110]

Yablo's argument to contradiction involves moving from names of sentences (e.g., s_1, s_2, s_3, etc.) to the assertion of sentences themselves (e.g., $\forall k > 1, \neg Ts_k$, $\forall k > 2, \neg Ts_k$, $\forall k > 3, \neg Ts_k$, etc.) both in terms of assertions like Ts_2 and denials like $\neg Ts_1$. As such, he employs conditionals of the form:

$Ts_n \Rightarrow \forall k > n, \neg Ts_k$ [111]

$\neg Ts_n \Rightarrow \neg \forall k > n, \neg Ts_k$

These are the focus of Priest's criticisms.

Now focus on such inferences in Yablo's paradox naturally turns our attention to Tarski's formal recursive definition of "truth" in terms of satisfaction (a semantic notion

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[110] This is Priest's construction of the original Yablo sentences with a formal truth predicate, T.

[111] Priest, (1997), p. 237

defined over both closed and open wffs, properly defined in Section 2). Tarski adopted the following now-famous schema:

(Tarski T-schema) ‘*p*’ is true-in-L ⇔ *p*,

where ‘*p*’ is a name or structural description of the well-formed formula (wff) *p*. The T-schema has come to be called a “disquotational schema” because the quote marks (or other apparatus of naming) on the left side of the biconditional is removed (disquoted) on the right. Now Tarski discovered that a formal “truth” predicate is not *arithmetic*—it cannot be consistently expressed in the object-language of any formal recursively axiomatizable theory adequate to elementary arithmetic. A truth-predicate can be made formally acceptable, but Tarski maintained that to avoid contradictions one has to adopt a predicate “true-in-L” for a specific recursively specified formal language L. Indeed, Curry showed how easy it is to formulate a contradiction with a naïve T-schema such as this:

T‘*p*’ ⇔ *p*

where ‘*p*’ is a name or structural description of the well-formed formula (*wff*) *p*. Curry imagines using Gödel numbering or some such apparatus to arrive at

a: ‘a’ is true ⇒ (*q* & ~*q*).

Then applying the naïve T-schema above, we have the following derivation of a contradiction:

1. ‘a’ is true ⇔ (‘a’ is true ⇒ (*q* & ~*q*))	Naïve Tarski Instance
2. ‘a’ is true & ‘a’ is true .⇒. (*q* & ~*q*))	1, Export
3. ‘a’ is true ⇒ (*q* & ~*q*))	2, Tautology
4. ‘a’ is true	1, 3 MP
5. q&~q	3, 4, MP

Thus the naïve T-schema, taken together with an apparatus of self-reference, yields contradiction.[112]

Now Yablo's derivation cannot use Tarski's T-schema rather than the naïve T-schema. If each Yablo sentence is really an abbreviation of

$s_n : \forall k > n, \neg T^L s_k$

where $T^L s_k$ abbreviates "s_k is true-in-L". Then, in accordance with Tarski's T-schema one would have:

$T^{L^*} s_n \Rightarrow \forall k > n, \neg T^L s_k$

This would undermine Yablo's demonstration altogether. Yablo could mount a reply that the naïve T-schema (with its univocal "truth" predicate) is not, in itself, known to be self-referentially circular. The Curry paradox indeed involves self-referential circularity, but that is due to the construction of

a: 'a' is true ⇒ (q & ~q).

This in no way shows that a univocal "truth-predicate" is self-referentially circular. As such Yablo might be fine with the naïve schema rather than the more restrictive one in which his paradox is not formable. With this clarification, then, Yablo's use of the naïve T-schema seems acceptable.

Priest is worried, however, about a different feature of Yablo's use of a naïve T-schema n his derivation. As mentioned above, he considers the justification for lines like $Ts_n \Rightarrow \forall k > n, \neg Ts_k$, saying: "It is natural to suppose that this is the *T*-schema, but it is not."[113] Priest focuses on the '*n*' from the above line of proof. He explains that, "The *n*

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[112] For a more thorough treatment, see Chapter 3.

[113] Priest, (1997), p. 237

involved in each step of the *reductio* argument is a free variable, since we apply universal generalization to it a little later; and the *T*-schema applies only to sentences, not to things with free variables in." In classic predicate logic, a formula φx is generalizable to $(\forall x)\varphi x$ only if x is free in φx. Priest's claim is that since Yablo's proof involves a generalization on the variable n in a Yablo sentence s_n, that sentence must involve a free variable. But if s_n involves a free variable n, the T-schema cannot be employed in the derivations. For example, the needed inference (i.e., $Ts_n \Rightarrow \forall k > n, \neg Ts_k$) is illicit. The *T*-schema requires closed well-formed formulas to appear on the left and right of the "⇔". For example:

S_1 is True ⇔ $\forall k > 1$, S_k is untrue,

Since S_1 names "$\forall k > 1$, S_k is untrue." Yablo can work with this instance of the schema, i.e., "$Ts_n \Rightarrow \forall k > n, \neg Ts_k$", only if n is a constant (not a variable). But if it *is* a constant, he cannot proceed to regard n as a free variable, ready for universal generalization. As such, Priest concludes, the line from Yablo's proof (i.e., $Ts_n \Rightarrow \forall k > n, \neg Ts_k$) cannot be justified by a Tarski-schema.

Priest entirely reconstructs the Yablo paradox so as to avoid this problem. Since Yablo uses universal generalization, Priest revises the Yablo sequence in terms of satisfaction and open formulas which he calls "predicates." Priest then rewrites $T\mathrm{s}_n \Rightarrow \forall k > n, \neg T\mathrm{s}_k$:

$S(n, s\hat{}) \Rightarrow \forall k > n, \neg T\mathrm{s}_k$[114]

As the n here is free, it is variable, and so is a legitimate candidate for universal generalization.

This revision, Priest argues, is not enough for the argument to go through:

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[114] Priest, (1997), p.237. He later suggests we need to revise the truth predicate out of every sentence.

> But then every other line of the argument needs to be rewritten to make it work, truth being replace by satisfaction. In particular, *s^* has to be taken as the predicate $\forall k > x$, $\neg$S(k, *s^*). Rewriting this way, the argument goes through straightforwardly, as may be checked. The final contradiction is $\forall k > 0$, $\neg$S(k, *s^*) and its negation.[115]

If the argument to contradiction requires reasoning about something with a free variable and predicates involving semantic relations (like being true), then satisfaction is the best bet. Satisfaction is a semantic relation defined for a formal language and a particular interpretation. It is truth-like (in the sense that it is also a semantic relation) and yet works with open sentences as is required by Yablo's use of universal generalization. Priest's claim, then, is that satisfaction is the sort of relation that works for the moves Yablo needs and remains semantic in nature as Yablo desires. But rewriting the necessary lines, Priest argues, requires rewriting the entire proof. When he does, he finds the argument to contradiction goes through, but involves self-referential circularity. Specifically, it involves the (open) predicate s^: $\forall k > x$, $\neg$S(k, *s^*).[116] Note that s^ specifically refers to itself, and therein lies circularity.

In the next section, Priest talks about infinitary reasoning. Specifically he looks to refute the possibility that satisfaction may be avoided by an application the ω-rule:

$$\frac{\alpha(0), \alpha(1), \ldots}{\forall x \alpha(x)}$$ [117]

The ω-rule is for infinite reasoning, where $\{\alpha(0), \alpha(1), \ldots, \alpha(n), \ldots\}$ is an infinite set. In the case where we have every $\alpha(n)$ (an infinite number of them), the ω-rule justifies the universal claim $\forall x \alpha(x)$. Priest notes that using the ω-rule would justify the universal

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[115] Priest, (1997), p.237-8

[116] Priest, (1997), p.238

[117] Priest, (1997), p.239

generalization without the need to involve satisfaction. After all, $\alpha(0)$ is a closed well-formed formula. The relevant Yablo analogue is $\neg\forall k > 0, \neg T\mathrm{s}_k$. After proving that "s0" named an untrue sentence, we would move on to prove that "s1" named an untrue sentence, and so on until proofs were completed for every natural number. After completion of each of these instances, the ω-rule allows assertion of the universal claim, $\forall n(\neg T\mathrm{s}_n)$. Since the proof involving truth is all that is necessary to prove any particular instance, truth (and *not* satisfaction) is all that is need to prove *every* instance. The ω-rule would thus allow the argument to go through without reference to satisfaction.

But while this would work, Priest argues that it is not a viable option: "As a matter of fact, we did not apply the ω-rule, and could not have. The reason we know that $\neg Ts_n$ is provable for all n is that we have a uniform proof, i.e., a proof for *variable n*."[118] The way that we know the argument goes through for each instance, in other words, is that it goes through for variable n: it goes through schematically. Yablo runs the proof of $\neg S_n$ in a way that avoids reference to any particular number. The structural properties, then, that are necessary to run such a proof are the same anywhere in the sequence, and that is how we know that we can prove $\neg T\mathrm{s}_n$ for any s_n: because we reasoned about schematic n. And recall that it was precisely the fact that n was a variable that Priest argued required satisfaction in the first place. If the retreat to the ω-rule requires a variable n, then the jig is up and satisfaction is required here as well.

This turns, in some sense, on Priest's next claim:

> Moreover, no finite reasoner ever really applies the ω-rule. The only way that they can know that there is a proof of each $\alpha(i)$ is because they have a

[118] Priest, (1997), p.239, his emphasis.

> uniform method of constructing such proofs. And it is this finite information that grounds the conclusion $\forall x\alpha(x)$.[119]

Only an infinite mind could comprehend the infinity of proofs necessary to deploy the ω-rule. The way we know the necessary premises is through exactly the sort of proof Priest's worry aims at. The uniform method (requiring variable *n*) is the access point for finite human minds: it is through the variable (and its finite information) that we can legitimately assert the necessary claim $\forall x\alpha(x)$, or, in Yablo's case, $(\forall n)\neg Ts_n$. Priest concludes that we can get the necessary claim, but not without satisfaction, and that requires a self-referential predicate. Since we need the universal claim, and cannot get it except through the free-variable-requiring universal generalization, Priest argues that Yablo has failed to produce a paradox that avoids self-referential circularity and has instead simply given an alternate version of a circular Liar paradox.

4.2 – (dis)Satisfaction

Despite its popularity in the literature, exactly how Priest's argument is supposed to go is not clear. Given his emphasis on the generalization Yablo utilizes in his proof, there must be a free variable involved somehow. Recall that Priest claimed the final contradiction was: $\forall k > 0$, $\neg S(k, s\^{})$ and its negation. This uses satisfaction, and so in what follows we begin by defining "satisfaction" for an interpretation and proceed toward the definition of "truth" in an attempt to make clear Priest's reconstruction of Yablo. Seeing the relation between satisfaction and truth will help make clear what Priest's claims are, and why they require self-reference, as well as why satisfaction is indeed a legitimate alternative semantic relation to truth. I will lay out the definition of satisfaction

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[119] Priest, (1997), p.239

for different types of formulas building toward a definition of truth in terms of satisfaction. The relevant concept is truth for an interpretation, i.e., truth given a particular assignment of predicates to properties and relations and constants to elements.

As Priest suggests, satisfaction allows us to make sense out of a truth-like relation that is, unlike truth, well-defined for open well-formed formulas of a formal language. Where "R^2" is a two-placed predicate letter of a language L whose *wffs* are recursively definable, and "x_i" and "xj" are variables (of an effective enumeration of the variables of L), one might conveniently describe satisfaction in a interpretation *I* (satisfaction_I) like this: The atomic wff "$R^2(x_i, x\text{j})$" is satisfied_I by a denumerable sequence of objects in a given domain when the i^{th} member of the sequence and the j^{th} member of the sequence are related by the relation that the interpretation I assigns to "R^2." The base of the recursion is the satisfaction_I definition for the atomic formulas. This must include formulas that involve several free variables. But in any case, because the complex *wffs* of L are built up recursively from the atomic *wffs*, a recursive definition of satisfaction_I can be given for complex formulas including universal and existential quantifications. But the recursive definition is not necessary here.

Now it is convenient in cases where there is just one free-variable "x_i" in a *wff* Ax_i to speak of a member of the domain, say *k*, satisfying Ax_i. This is just shorthand, meaning that *k* occupies the *i*-th position in some denumerable sequence of members of the domain which satisfies Ax_i . In fact, we may conveniently say that

k satisfies $Ax_i \Leftrightarrow Ak$

as long as we recall that "k satisfies Ax_i" means that k occurs at the *i*-th position in some sequence of members of the domain of interpretation I.

Less formally, satisfaction is about the relation between the formulas and terms of a formal system relative to the domain of objects assigned as meanings by a particular interpretation. More formally, an interpretation, I, of a formal language, L, is an assignment of meanings to the symbols and formulas of L.[120] I assigns truth or falsehood (normally represented by 1 and 0) to propositional symbols, assigns entities to constants, assigns functions (with arguments and values in the domain) to functions symbols, and assigns properties or relations to predicates.[121]

We talk about sequences of elements of the domain satisfying a well-formed formula of the language[122] dependent upon the particular semantic assignments of I.[123] A sequence is an ordered denumerable series of elements of the domain of our interpretation, I. Elements of the domain may repeat in the sequence (in fact, some denumerable sequences are mere repetitions of a single entity) and may include any and all elements in the domain in any order whatsoever. It needs to be ordered to accommodate formulas with more than one free variable. In those cases, free variables will be subscripted (e.g., x_3) telling us to take the corresponding element in the denumerable sequence (in this case, the third). That is why part of accommodating these formulas is speaking of such a sequence rather than just speaking of elements in the domain. Talk of a sequence satisfying a formula depends on particular elements in the

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[120] Hunter, (1996), p.6

[121] Hunter, (1996), p. 141

[122] Hunter, (1996), p.142

[123] It is the terms in the sequence that are relevant to the satisfaction of a formula, and so some speak informally of terms (or the elements they stand for) satisfying formulas. This is simply shorthand for the more intricate relation between denumerable sequences and well-formed formulas of the language.

sequence satisfying that formula. A sequence can satisfy all formula types from propositional symbols (which have no variables of any sort) to open, universally quantified formulas. As such, speaking of sequences allows a unified approach to satisfaction, one that works for any well-formed formula. Given *I*, elements in a sequence satisfy some formulas in the formal system depending on the particular semantic assignments of *I*. As Priest hints, the notion of satisfaction is part of the complicated interpretation necessary to cope with open formulas (i.e., formulas with free variables) when Capture and Release[124] are important.

Satisfaction is defined recursively first for propositional symbols, then for molecular *wffs* involving the logical particles (~, ⇒, etc.), then for open atomic well-formed formulas, and finally for *wffs* involving universal and existential quantifiers (if both are taken as primitive). We define satisfaction[125] for an interpretation *I* over domain *D*, where *t* is an arbitrary term, and *s* is an arbitrary denumerable sequence of members of *D*. Defining satisfaction depends on defining a function, *, with the terms of our language, *L*, as arguments, and values in *D* by the following rules: If *t* is a constant, then t**s* is the member of D in the sequence *s* assigned by *I* to the constant *t*. If *t* is the k^{th} variable in our enumeration, then t**s* is the *k*th term in *s*. Finally, if *t* is of the form $f(t_1 \ldots t_n)$, where *f* is an *n*-place function symbol, and $t_1 \ldots t_n$ are terms, and *f* is the function assigned by *I* to g, then $t{*}s = f(t_1{*}s, \ldots, t_n{*}s)$. These three cases characterize the relation between the elements of *D* (enumerated in the sequence) and formal language *L* under interpretation *I.* Given these claims, satisfaction may be defined in five claims:

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[124] See Chapter 2 and Chapter 3.

[125] This construction comes from Hunter, (1996), p.147-8

1. If A is a propositional symbol, then a sequence, *s*, satisfies A iff *I* assigns the truth value truth to A.

2. If A is an atomic well-formed formula of the form $F(t_1, \ldots t_n)$, where F is an *n*-place predicate symbol and $t_1,\ldots, t_n$ are terms, then *s* satisfies A iff $\{t_1{*}s,\ldots, t_n{*}s\}$ is a member of the set of ordered *n*-tuples assigned by *I* to F.

3. If A is of the form ~B, then *s* satisfies A *iff s* does not satisfy B.

4. If A is of the form (B⊃C), then *s* satisfies A *iff* either *s* does not satisfy B or *s* does satisfy C.

5. If A is of the form $\forall t_i B$, where t_i is the *i*th variable in our enumeration, then *s* satisfies A *iff* every denumerable sequence of members of D that differs from *s* in at most the i^{th} term satisfies B.

Above I suggested that talk of sequences satisfying formulas allows a unified approach to satisfaction, and these five definitions bring that out. Rather than heterogeneous talk of elements satisfying one type of formula and sequences another, we define satisfaction in a way that simplifies application: only one thing (a sequence) can satisfy a formula. In each of the five cases defined above, the satisfaction relation holds between elements in the sequence and formulas of *L*. Note two things: first, we need to talk of the sequence to accommodate open and quantified formulas (i.e., in order to capture the notion of the variable), and second, the satisfaction relation has broader applicability than the truth relation (i.e., it applies to more formulas than does truth). I discuss each of these two claims briefly.

In the first case, one might think we can just speak of elements satisfying rather than sequences, and this would work for closed atomic sentences. The nice part about using sequences in discussions of satisfaction, however, is that a sequence can serve as an appropriate *relata* for quantified and open claims as well as polyadic predicates (especially asymmetric ones). We talk of a denumerable sequence of elements to capture the notion that for a quantified statement, we are talking about some element among many. The definition of satisfying a quantified formula uses a position in the sequence to pick out a particular term, and then the notion that the sequence could have had any other term in the domain there to capture the idea that it does not matter which term you use (hence, the universality of the claim). Similar remarks apply to the variability of the variable. Subscripted positions accommodate formulas with multiple free variables (e.g., $\varphi(x_1, x_{12})$, which says x_1 relates φ-ly to x_{12}, or, more formally, the first element in the denumerable sequence relates φ-ly to the twelfth element in the sequence).

Talk of the sequence, in short, allows a uniform approach to satisfaction of formulas no matter their type.

In the second case, note that propositional symbols are assigned truth-values under *I*. Each propositional symbol is either true-in-*I* or false-in-*I*. Truth for the logical connectives can built from these in the usual fashion (i.e., $\neg$A is true-in-*I* *iff* A is not true-in-*I,* and A⇒B is true-in-*I* *iff* either A is not true-in-*I*, or B is true-in-*I*). Similar remarks apply to quantified formulas provided they are closed. But truth is not defined for open formulas. They are not assigned t-values by *I* because it would be an illicit assignment. What would it mean to say "*x* is tall" is true? Or false? For a predicate, φ, and variable, x,'φx is true' is not well-formed. Priest puts this point in terms of the naïve T-schema (for a truth predicate *T*): "…the T-schema applies only to sentences, not to things with free variables in. It is nonsense to say, for example, *T*'*x* is white' *iff* x is white."[126] Satisfaction, however, applies to just this sort of formula AND the closed formulas that serve as a *relata* for the truth relation. The ability to engage with open formulas is part of what makes satisfaction useful, and exactly why Priest turns to satisfaction after he notices that Yablo employs a universal generalization and so reasons about a formula with a free variable.

We can now define truth-in-*I*:

A wff A of *L* is true for a given interpretation *I* of *L iff* every denumerable sequence of members of the domain of *I* satisfies A.

A wff *of* L *is* false for a given interpretation *I* of *L, iff no* denumerable sequence of members of the domain of *I* satisfies A.[127]

Now we can see the relation between satisfaction and truth in a system and under an interpretation. For propositional symbols, every sequence satisfies them just is case they are true. For closed formula, φa, they are satisfied if the element *I* assigns to a has the property *I* assigns to φ. In both these cases,

*T*A⇔ (∀a)(a is a den. seq.⇒ a sat A).

For a statement more complex than a propositional symbol to be true, it needs be satisfied for *any* sequence of elements in the domain. Accordingly, truth standards are much higher than satisfaction standards. Satisfaction simply means that *some* way of thinking about the statement works. So for our open formula, φ*x*, as long as there is one thing in our domain that has the property assigned to φ, φ*x* is satisfiable, but it is true only if

[126] Priest, (1997), p.237

[127] Hunter, (1996), p.148

everything in our domain is φ. Also note that this leaves room for truth gaps: well-formed formulas can be neither true nor false. To say that a well-formed formula A is *not true* (for interpretation I) is simply to say that not every sequence of that interpretation satisfies it (i.e. there is some sequence of that interpretation that fails to satisfy A). Likewise, A is *not false* (for interpretation I) if it is not the case that every sequence fails to satisfy A (i.e. there is some sequence of the interpretation that satisfies A). A's being *not true* for in interpretation does not imply A's being false for that interpretation; and A's being *not false* for an interpretation does not imply that it is true for that interpretation. Satisfaction allows us to formalize these notions of truth-gaps for open formulas. A closed formula B, on the other hand, is such that B is not true for an interpretation if and only if B is false for that interpretation.

To see how this works a little more clearly, take an example. Assign "H" to the property '*is human*,' and let "M" be assigned to the property '*is a mountain*,' and consider the domain of the interpretation to be the set {Katie, Katie's dog, Katie's computer}. Let "k" be a constant that the interpretation assigns to Katie, let "d" stands for Katie's dog, and "c" stand for Katie's computer. In this case, "Fk," "¬Fd," and "¬Fc" are true as they are satisfied by every sequence (since these are determined by the assignment made by the interpretation to the constants). In contrast, "Fx_i" is satisfiable (since there is a sequence in which Katie appears as the i^{th} member. Such a sequence satisfies "Fx_i" in virtue of Katie's having the property of *being human*. However, "Fx_i" is not true, since some sequences of member of the domain have Katie's dog and some have Katie's computer as their i^{th} members and neither of these has the property *being human*. These sequences do not satisfy "Fx_i". Thus, "Fx_i" is open well-formed formula that is not true for the interpretation and not false for the interpretation. In contrast, consider "Mx_i". No sequence of elements in the domain of our interpretation satisfies it (neither Katie, nor her dog, nor her computer are mountains). This is highly simplified, but it offers us the necessary background to follow the details of Priest's attempt to reformulate Yablo's paradox in terms of satisfaction.

Talking of satisfaction gives us something to say about wffs that are not true and are not false (or that might be true or might be false, but we do not know or cannot tell). In so doing, it allows us to talk about statements with free variables. Priest's diagnosis suggests wffs with free variables are required for a proper formulation of Yablo's paradox. The free variables allow for the use of universal generalization, a use that would be excluded if the paradox relies on the T-schema. Priest claims that his reformulation of Yablo's paradox in terms of satisfaction yields a Liar-style paradox. But it is a paradox that Priest exposes as involving circular self-reference. Thus, if Priest's reformulation is required, Yablo has failed in his effort to generate a Liar-style paradox without self-reference.

4.3 – Reconstructing Priest

The Yablo sequence consists of an infinite number of sentences[d] each of which intuitively claims that all the entries on the list later in the ordering are untrue. The Priest list, then, seems to be based on infinite number of sentences each of which claims that no number greater than *n* satisfies the open wff "A*x*". This at once generates a puzzle. What precisely is the formula A*x* with free variable "*x*"?

The Priest list (which is to replace the Yablo list) is defined by the *n*th member as, "exactly the predicate s^ with '*x*' replaced by '<u>*n*</u>'."[128] In an effort to see what Priest meant, I will first present a version of Yablo's original paradox based on an explicit use of the T-schema and the lines Priest does note, then translate it as he suggests into an infinite paradoxical sequence about satisfaction.

4.3.1 Priest's account of Yablo (enhanced by annotations)

1. Assume: $T(s_n)$	*reductio* assumption
2. $T(s_n) \Leftrightarrow \forall k>n, \neg T(s_k)$	disquotation instance[129]
3. $\neg T(s_{n+1})$	$n+1>n$ and $\forall k>n, \neg T(s_k)$
4. $T(s_n) \Leftrightarrow \forall k>n, \neg T(s_k)$	repeat of 2, by disquotation[130]
5. $T(s_n) \Rightarrow \forall k>n+1, \neg T(s_k)$	From 4, logic
6. $T(s_{n+1}) \Leftrightarrow \forall k>n+1, \neg T(s_k)$	disquotation instance
7. $T(s_n) \Rightarrow T(s_{n+1})$	5, 6, syllogism
<u>8. $T(s_{n+1})$ & $\neg T(s_{n+1})$</u>	3; 1,7, MP
9. $\neg T(s_n)$	1-9, *Reductio*
10. $\forall n \neg T(s_n)$	9, UG
11. $\forall k>0, \neg T(s_k)$	10, Logic
12. $T(s_0) \Leftrightarrow (\forall k>0, \neg T(s_k))$	Disquotation Instance
13. $\neg T(s_0)$	10, UI

[128] Priest, (1997), p.238

[129] See Section 1 of this chapter.

[130] This is unnecessary, but among the few lines Priest presents, he cites this twice.

14. $\neg(\forall k>0, \neg T(s_k))$ 12, 13 MT

15. $[\forall k>0, \neg T(s_k)]$ & $\neg[\forall k>0, \neg T(s_k)]$ 11, 14, Conj

In this version of the proof, we assume a random sentence (sentence s_n) is true. Since it is true, what it says must be the case.[131] The second line asserts the content of s_n and leads to a contradiction in line 8. Then we discharge our *reductio* assumption and generalize. But if the universal claim (line 10) is the case, then each of the Yablo sentences is untrue, which we can write as line 11. Since line 11 asserts the content of s_0, and line 10 asserts it untrue, we get our contradiction: the assertion and the denial of the content of s_0 (line 15).

Priest's problem here is with lines like 2 and 4. As suggested above, they are fine if *n* is a constant, but are not well-formed if *n* is to be a variable. Specifically, Ts_n for free x_i is ill-formed since s_n needs a free variable if we are generalize its denial in line 10. As is clear from the discussion of satisfaction, claims about open formulas being satisfied can be well-formed (unlike truth claims about them). As such, Priest demands that the paradox be reformulated in terms of satisfaction.

4.3.2 The Satisfaction Version

Given Priest's explicit claims about individual lines of Yablo's original paradox, I worked up the following reconstruction:

1. Assume: $S(n, s\^{})$ *reductio* assumption
2. $S(n, s\^{}) \Leftrightarrow \forall k>n, \neg S(k, s\^{})$ Disquotation Instance
3. $\neg S(n+1, s')$ $n+1>n$ and $\forall k>n, \neg S(k, s\^{})$
4. $S(n, s\^{}) \Leftrightarrow \forall k>n, \neg S(k, s\^{})$ Disquotation Instance (repeat)

[131] I put it this way to emphasize the Tarski-connection highlighted by Priest.

5. $S(n, s^\wedge) \Leftrightarrow \forall k > n+1, \neg S(k, s^\wedge)$ From 4, Logic

6. $S(n+1, s^\wedge) \Leftrightarrow \forall k > n+1, \neg S(k, s^\wedge)$ Disquotation Instance

7. $S(n, s^\wedge) \Leftrightarrow S(n+1, s^\wedge)$ 5, 6 , syllogism

8. $\underline{S(n+1, s^\wedge) \ \& \ \neg S(n+1, s^\wedge)}$ 3; 1,7 MP

9. $\neg S(n, s^\wedge)$ 1-9 *Reductio*

10. $\forall n \ \neg S(n, s^\wedge)$ 9, UG

11. $\forall k > 0, \neg S(k, s^\wedge)$ 10, Instance

12. $S(0, s^\wedge) \Leftrightarrow \forall k > 0, \neg S(k, s^\wedge)$ Disquotation Instance

13. $\neg S(0, s^\wedge)$ 10, UI

14. $\neg \forall k > 0, \neg S(k, s^\wedge)$ 12, 13 MT

15. $[\forall k \ \neg S(k, s^\wedge)] \ \& \ \neg[\forall k > 0, \neg S(k, s^\wedge)]$ 11, 14 Conj

In this version, we assume that some number, *n*, in fact satisfies the predicate s^. For that to be the case, all numbers greater than *n* must fail to satisfy s^. Recall that claims about open formulas being *satisfied* can be well-formed (unlike the sides of the T-schema). Priest seems to see the structure of this proof as the same. In this case, the Yablo sentences are each instances of the s^ predicate (i.e., $\forall k > n, \neg S(k, s^\wedge)$) with numerals in the *n* slot.[132]

s_0: $\forall k > 0, \neg S(k, s^\wedge)$

s_1: $\forall k > 1, \neg S(k, s^\wedge)$

and so on, each sharing the form of the predicate, s^.

[132] Priest, (1997), p.238

4.4 – Circularity and Vicious Circularity

Yablo's paradox is an attempt to generate a Liar-like paradox without circularity. Certainly the Priest satisfaction version is Liar-like, and yet involves circularity. If Yablo's paradox reduces through analysis to Priest's version, then Yablo's paradox fails. In this section I will argue that things are not so simple as Priest suggest. His predicate s^ is syntactically ill-formed and as such needs revision.

Priest describes the self-referential circularity he finds as a "fixed point." Now, fixed point constructions are normally arguments and functions such that f(x)=x. For example, the square function (i.e., f(x)=x^2), has fixed points when x=1 and also when x=0, for we have f(1)=1 and f(0)=0). But for any other natural number, the function yields different results.[133] The fixed point is a particular element (or set of elements) in the domain. But notice what has happened in Priest's construction of s^. Rather than finding a fixed point, Priest has defined a fixed point into the very nature of the sequence. Each Yablo sentence in Priest's version is a fixed point because s^ is defined in terms of itself. As such, it is not surprising that Priest's reformulation of the Yablo in terms of satisfaction is a paradox involving vicious circularity. This kind of syntactic incoherence is not found in the self-referentially circular Liar-like paradoxes. Indexical and Simple Strengthened Liars use linguistic tools to give a complete syntax while self-referring. In contrast, consider:

s_0: $\forall k>0, \neg S(k, s\text{^})$

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[133] For example, f(f(2))=16, i.e., $(2^2)^2=4^2=16$; f(f(f(3)))=$((3^2)^2)^2=(9^2)^2=81^2=6561$.

What does this come to? We are trying to understand precisely what expression s^ is supposed to be. What is the free variable "*z*" in it? The only obvious option is as Priest puts it:

$\forall k{>}z, \neg S(k, s\hat{})$

This expression uses s^ and so we are back to our original problem.

We are no closer to specifying the sentence s_0, and repeated attempts to flesh out the actual claim will be similarly thwarted. This isn't mere circularity, but vicious incoherence.

Priest provides us with sentences that look similar to Yablo's when written on the page, but that are based on an ill-defined predicate s^. His fixed point is defined into each line of his version of the Yablo paradox. One might think that there must be another way of doing it where we use the satisfaction required by the generalization while avoiding such incoherence.

4.5 – Revising Priest

A proper formulation of Priest's sequence needs a Yablo style sequence of open wffs. Each wff will contain one free variable and each wff will say[134] that an infinite number of other open wffs on the list are not satisfied.[135] What we need is to look at different ways of constructing such a list of such open wffs. If there is self-reference in such a sequence, then Priest has a case that a reformulation of Yablo's paradox in terms of satisfaction does not realize Yablo's goal for forming a Liar-like paradox without self-

[134] "Say" relative to a given assignment of that free variable by an interpretation.

[135] As per the requirements laid out in Chapter 2.

reference. I will look at two attempts to form a legitimate satisfaction sequence. I draw the conclusion that while a satisfaction Yablo paradox is formable that can respond to Priest's worries, the sequence is doable only with a feature that, while less unacceptable than Priest's circularity via definition, still leaves us wanting (though on par with the original Yablo sequence).

4.5.1 The Universal Subscripted Variable Sequence

In this list of distinct open wffs*,* we deviate from Yablo's list of closed wffs. I use a(*n/i*) to mean a denumerable sequence of members of the domain of an interpretation that has the natural number *n* in the i^{th} position. I use S[*a*, *p*] to mean that the sequence *a* satisfies the wff *p*. Now, imagine an infinite series of open wffs, starting with:

$Y(x_1)$: $\forall k>x_1$, $\forall$a,¬S[a(*k*/*k*), $Y(x_k)$]

$Y(x_2)$: $\forall k>x_2$, $\forall$a,¬S[a(*k*/*k*), $Y(x_k)$]

$Y(x_3)$: $\forall k>x_3$, $\forall$a,¬S[a(*k*/*k*), $Y(x_k)$]

And so on, all of the form:

$Y(x_i)$: $\forall k>x_i$, $\forall$a,¬S[a(*k*/*k*), $Y(x_k)$]

The way to think of these claims is to imagine denumerable sequence.[136] The *i*th member of the sequence is $Y(x_i)$. It says that any number, *k*, larger than the number in the *i*th

[136] It need not be all numbers, but denumerable sequences of numbers are the only interesting cases, as others are vacuously true as no non-number are greater than or less than anything, and so the antecedent fails in all non-number cases. As such, I leave those to one side.

position of the denumerable sequence is such that *any* sequence, a, that has k in the kth position does not satisfy $Y(x_k)$.[137]

Each of these claims is telling us to look to the positions in the sequence after the one in the ith slot. So if the number is 47, we look at all those after the 47th wff in the sequence. In that case, the formula in question would be satisfied if all the numbers after 47 were such that they did not satisfy the formula in their position with them in their spot in the sequence. This is simply another way of saying that any number, k, larger than the number in the ith position of the denumerable sequence is such that *any* sequence, a, that has k in the kth position does not satisfy $Y(x_k)$.

The derivation of the paradox runs as follows:

1. $S[a(x_i/x_i), Y(x_i)]$ — *reductio* assumption

2. $S[a(x_i/x_i), Y(x_i)]$.⇔.

$\forall k>x_i, \forall a, \neg S[a(k/k), Y(x_k)]$ — Disquotation instance

3. $\forall k>x_i, \forall a, \neg S[a(k/k), Y(x_k)]$ — 1, 2, MP

For sake of this proof, let $x_i=n$, yielding:

3*. $\forall k>n, \forall a, \neg S[a(k/k), Y(x_k)]$

4. $\forall a, \neg S[a(n+1/n+1), Y(x_{n+1})]$ — 3*, UI, $n+1>n$

5. $\exists a, S[a(n+1+m/n+1+m), Y(x_{n+1+m})]$ — 4, df $Y(x_{n+1})$

6. $\forall a, \neg S[a(n+1+m/n+1+m), Y(x_{n+1+m})]$ — 4, UI, $n+1+n>n$

7. $\neg\exists a, S[a(n+1+m/n+1+m), Y(x_{n+1+m})]$ — 6, quantifier =df.

8. $\exists a, S[a(n+1+m/n+1+m), Y(x_{n+1+m})]$.&.
$\neg\exists a, S[a(n+1+m/n+1+m), Y(x_{n+1+m})]$ — 5, 7, Conj

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[137] It could be that the *xi*th variable in the sequence was *i* itself. This adds in some form of self-reference insofar as it picks out it's own location as the relevant one.

9. $\neg S[a(x_i/x_i), Y(x_i)]$ 1-8, *reductio*

10. $\forall x_i \neg S[a(x_i/x_i), Y(x_i)]$ 9, UG

11. $S[a(x_i/x_i), Y(x_i)]$.⇔.
$\forall k>x_i, \forall a, \neg S[a(k/k), Y(x_k)]$ Disquotation Instance

12. $\neg\forall k>x_i, \forall a, \neg S[a(k/k), Y(x_k)]$ 9, 11 MT

13. $\exists k>x_i, \exists a, S[a(k/k), Y(x_k)]$ 12, quantifier =df.

14. $S[a(x_i+n/x_i+n), Y(x_i+n)]$ 13, EI, EI

15. $\neg S[a(x_i+n/x_i+n), Y(x_i+n)]$ 10, UI

16. $S[a(x_i+n/x_i+n), Y(x_i+n)]$.&.
$\neg S[a(x_i+n/x_i+n), Y(x_i+n)]$ 14, 15 Conj

Contradiction.

4.5.2 The Existential Subscripted Variable Sequence

Imagine an infinite series of open *wffs*, starting with:

$Y(x_1)$: $\forall k>x_1, \exists a, \neg S[a(k/k), Y(x_k)]$

$Y(x_2)$: $\forall k>x_2, \exists a, \neg S[a(k/k), Y(x_k)]$

$Y(x_3)$: $\forall k>x_3, \exists a, \neg S[a(k/k), Y(x_k)]$

And so on, all of the form:

$Y(x_i)$: $\forall k>x_i, \exists a, \neg S[a(k/k), Y(x_k)]$

This sequence trades the universal claim of the earlier sequence for an existential claim. Rather than saying that *every* sequence is such that for k greater than x_i, that sequence with k in the kth position will not satisfy $S(x_k)$, this sequence says that *some* sequence does not so satisfy. The concepts involved are the same, but in this is an alternate way of understanding the sort of sequence of claims necessary for generating a Yablo-style paradox of satisfaction. The paradox goes through as follows:

1. $S[a(x_i/x_i), Y(x_i)]$ — *reductio* assumption

2. $S[a(x_i/x_i), Y(x_i)]$.⇔.
 $\forall k>x_i, \exists a, \neg S[a(k/k), Y(x_k)]$ — Disquotation instance

3. $\forall k>x_i, \exists a, \neg S[a(k/k), Y(x_k)]$ — 1, 2, MP

For sake of this proof, let $x_i=n$, yielding:

3*. $\forall k>n, \exists a, \neg S[a(k/k), Y(x_k)]$

4. $\exists a, \neg S[a(n+1/n+1), Y(x_{n+1})]$ — 3*, UI, $n+1>n$

5. $\neg S[a(n+1/n+1), Y(x_{n+1})]$ — 4, EI

6. $S[a(n+1/n+1), Y(x_{n+1})]$.⇔.
 $\forall k>n+1, \exists a, \neg S[a(k/k), Y(x_k)]$ — Disquotation Instance

7. $\neg \forall k>n+1, \exists a, \neg S[a(k/k), Y(x_k)]$ — 5, 6 MT

8. $\forall k>n+1, \exists a, \neg S[a(k/k), Y(x_k)]$ — 3, logic

9. $\forall k>n+1, \exists a, \neg S[a(k/k), Y(x_k)]$.&.
 $\neg \forall k>n+1, \exists a, \neg S[a(k/k), Y(x_k)]$ — 7, 8 Conj

10. $\neg S[a(x_i/x_i), Y(x_i)]$ — 1-9 *reductio*

11. $\forall x_i \neg S[a(x_i/x_i), Y(x_i)]$ — 10, UG

12. $S[a(x_i/x_i), Y(x_i)]$.⇔.
 $\forall k>x_i, \exists a, \neg S[a(k/k), Y(x_k)]$ — Disquotation Instance

13. $\neg \forall k>x_i, \exists a, \neg S[a(k/k), Y(x_k)]$ — 10, 12 MT

14. $\exists k>x_i, \forall a, S[a(k/k), Y(x_k)]$ — 13, quantifier =df.

15. $\forall a, S[a(x_i+n/x_i+n), Y(x_i+n)]$ — 14, EI

16. $S[a(x_i+n/x_i+n), Y(x_i+n)]$ — 15, UI

17. ¬S[a(x_i+n/x_i+n), Y(x_i+n)] 11, UI

18. S[a(x_i+n/x_i+n), Y(x_i+n)] .&.
¬S[a(x_i+n/x_i+n), Y(x_i+n)] 16, 17 Conj

Contradiction.

Here we have two alternate versions that use satisfaction and thus enable the universal generalization Priest thinks is essential to the generation of the contradiction. And these seem to use satisfaction more in the way Priest had in mind. After all, these sequences deviate from Yablo's initial sequence. Rather than making claims about the truth or untruth of particular sentences (as did the original Yablo) this one is fundamentally about numbers, just as is Priest's version. The difference between these and Priest is that the *relata* of the satisfaction relation are the open wffs on the list rather Priest's incoherent structural predicate s^ (which is defined in terms of itself). In this way, we avoid the illicit definition involved in Priest's version, and yet have a Yablo style paradox.

That our new constructions work makes sense in the face of Priest's questionable construction in comparison to the paradigmatic fixed-point result from Gödel. Gödel's incompleteness theorem showed that powerful logical systems like *Principia Mathematica* were not complete, as there were statements that are true that are representable but not provable within the system.[138] He accomplished this through a process called Gödel numbering. Gödel numbering is a way of using strings of natural numbers to represent the logical symbols including statements and strings of statements (i.e., proofs) of a system like *Principia*, and thereby prove things about the system by

[138] For an excellent discussion, see Nagel and Newman, (2008).

proving things about the Gödel numbers that represent statements and proofs within the system.

Gn "(∀y) (U(x, y)⇒(∀z)~proof(z,y))" = *r*

Let the Gödel number of the above claim be *r*. The claim is that a number, *x*, is undemonstrable when there is no number *y* that is the Gödel number for a proof in *Principia*. The predicate, U, is undemonstrable, and proof (z, y) says that y is the Gödel number of a proof for z. We said the statement above has a Gödel number we call *r*. And so we can ask: is *r* undemonstrable? In other words, is it that case that:

(∀y) (U($\underline{r}$, y)⇒(∀z)~proof(z,y))

Let the Gödel number of this sentence be g. In other words, Gn "(∀y) (U($\underline{r}$, y)⇒(∀z)~proof(z,y)"=g. The fixed point comes from the fact that given these non-circular definitions, it is provable in the system that:

G⇔(∀z)~proof($\underline{g}$, z)

And this is the fixed point since g occurs on both sides of the bi-conditional.

But notice that our (rough) definitions of U and proof (as well as the notion of Gödel numbering) are not self-referential. They are not self-referential by definition, but become a fixed point only when deployed in a particular way acceptable in systems like *Principia*. This is part of why Gödel had to work so hard to show his fixed-point result, and why it is not surprising that Priest's was illicit since he "found" it quite so easily.

4.6 – Two Kinds of Incoherence

For all its benefits in recovering Priest's intent, the paradoxes formed with subscripted variables fail to work. The problem is that no open wff on the list has been well-defined. The issue is similar to the problem with Priest's revision, but subtly different (and subtly better).

To see this, first consider two possibilities for constructing the Yablo sentences. Consider the schematic Yablo sentence: $\forall k>n, \neg T(s_k)$. The question is the form of the latter part of this sentence. How are we to understand the s_k? There are two options. First, s_k could be a name, a set of marks that refers to a particular object. Second, s_k could be a function that picks out a particular sentence[d] via a particular syntactic description determined by the function.

Understanding it as a name seems desirable. It aligns with how the Simple Indexical Strengthened Liar achieves the necessary self-reference.[139] As a name it completes the syntax of the Yablo sentence in question. It answers the question of what the sentence asserts is untrue: the thing associated with that name. But while this has potential, it occurs with a free variable in the sentence. We need to be able to bind up the k in s_k with the k at the beginning (i.e., the $k>n$). If it has a free variable, then s_k cannot be a name because a name must be a completed sign. It would be best, then, to rewrite the Yablo sentences:

(s_n): $\forall k>n, \neg T(\text{“}s_k\text{”})$

This emphasizes that the s_k is merely a mark standing in for something else.
It is illicit to quantify into quotation marks, and this version makes clear that that is precisely what would be necessary if it were a name for a sentence.

Perhaps, then, it is a function. Functions pick out their argument via some structural description. In the case of sentences, this is a syntactic description. For the

[139] See Chapter 1.

function that orders the Yablo sentences, it picks them out via their syntax. We know that $f(n)= \forall k>n, \neg\ T(s_k)$, where there is an open slot where the "*n*" is, and the rest of the sentence gives the syntax of the resultant entity. In the case of the Yablo, then, the s_k can stand for a function. In this case, we might rewrite the sentence:

$\forall k>n, \neg\ T(s(k))$

This emphasizes that the syntactic structure is not complete. The structure of the latter part of the sentence is hidden behind the functional notation. This seems like the best option since the Yablo requires quantifying in, and functions are designed, in some sense, for that. The functional reading works best for the traditional Yablo.

With Liar paradoxes we have come to expect self-referential circularity. But syntax of sentences^d (and possible claims) of any ordinary language *must* be well-defined at a given time. That is just what it is to be a language in the ordinary sense. This applies to natural languages as much as it does for formal languages. For a sentence^d to be meaningful, it must have a determinate structure.

Priest's reformulation of Yablo's paradox in terms of s^ violates this requirement. Each entry on his list is syntactically uncharacterizable: each is characterized in terms of itself. In the revised form with subscripted variables that I have offered on behalf of Priest, no Yablo list has been formulated. The flaw this time, however, is not that individual sentences^d are self-referential (leading, in Priest's case, to syntactic incoherence). In this case the flaw is rather syntactic indeterminacy. Through the function, the syntax of any Yablo sentence gets pushed off to another sentence. Having the function sign imbedded means that the syntax of the resultant sentence is meant to be inserted into that slot in the original sentence. For

$\forall k > n, \neg T(s_k)$

we are to think of it in terms of the structure of s_k. inserted into the sentence where "s_k" occurs. This yields:

$\forall k > n, \neg T(\forall m > k, \neg T(s_m))$

This, however, is not a completed syntax either, as we now have a different functional description imbedded. Repeated iterations will not, it is clear, yield a completed syntax. If our choices are between a name and a function, then a function works better, and gets us closer than Priest's revision, but since the syntax of each entry on the list is described in terms of the syntax of other entries on the list, we have given no completed syntactic characterization of any open formula on the list.

In other words, there is a different sort of incoherence here than in the Priest. This one is less circular, so to speak, but still damaging to an attempt at a satisfaction Yablo paradox since it involves syntactic incoherence. To generate a liar-like paradox we would need problems derivative from *semantic* relations of some sort, and these constructions just fail to get there.

Priest may respond here that there is a collapse worry for both the Existential and Universal paradoxes presented above. The universal sequence may simply collapse into Yablo. The existential sequence may collapse into Priest. I look at each worry in turn.

Recall that the Universal Subscripted Variable Sequence had sentences that look like: $Y(x_i)$: $\forall k > x_i$, $\forall a, \neg S[a(k/k), Y(x_k)]$. But then think back to the definition of truth in terms of satisfaction: A wff A of *L* is true for a given interpretation *I* of *L iff* every denumerable sequence of members of the domain of *I* satisfies A. The key here is that the notion of truth is tied to the notion of *every* denumerable sequence. In the Universal

Subscripted Variable Sequence, the universal claim is made about a totality of sequences. Priest might, in this case, think that this sequence is just another way of putting Yablo's sequence: each formula is really just about calling later sentences untrue (i.e., saying that every sequence fails to satisfy them). As it is simply a disguised version of Yablo, the response might go, it is equally susceptible to Priest's claims and *it* needs to be translated into Priest's satisfaction sequence just as did Yablo's sequence.

But while this is an avenue open to Priest, it does not strike me as a fruitful one for a couple reasons. First, the universal claim is restricted to certain kinds of sequences (i.e., sequences that have k in the kth position for $k>x_i$). Other sequences (like those with x_i in the x_ith position) may perfectly well satisfy or not independent of the claims in this satisfaction version of a Yablo sequence. Second, the form of these claims differs dramatically from that of Yablo. Yablo's sequence avoids self-reference by reference to an infinite number of sentences each of which only refers to later sentences in the list. In this case, however, the claim is really about numbers that occupy certain positions in a denumerable sequence. We use the ordering of those variables into a denumerable sequence to make our claims, and even if we grant that every sequence fails to satisfy, this is enough difference to ensure that this Universal Subscripted Variable Sequence does not collapse into Yablo's sequence.

Priest may argue in some fashion that the proper version of this satisfaction sequence is the Existential Subcripted Variable sequence. Recall that it looks like: $Y(x_i)$: $\forall k>x_i, \exists a, \neg S[a(k/k), Y(x_k)]$. If he can argue that this is the proper version, then he can mount an argument that this version simply collapses into his. He may think that the relevant predicates involved in the augmented version presented here are equally self-referential to his s^ predicate, and since they come to the same thing, they do no work to save Yablo from either the UG worry or the self-referential satisfaction worry.

This, however, seems a red herring to me. In Priest's reconstruction, the ordering is determined in a particular fashion that mimics the Yablo construction. Specifically, the other reference necessary for generating the paradox comes from ordering the *sentences*^{d} themselves. That is what makes them all have the same structure, and all depend upon the same predicate, s^. In my revision of Priest, however, the ordering comes from the ordering of variables in the satisfying sequence of elements of the domain. It, in some sense, captures the move to satisfaction more completely because it does not simply try to mimic truth claims with satisfaction claims, but instead uses the basic pieces of the notion of satisfaction to generate the other-reference needed to derive a contradiction. It is certainly different enough from Priest's version to escape collapsing into it.[140]

4.7 – Conclusion

Priest presented an argument that Yablo's paradox could not prove the contradiction it needed from a Yablo sequence involving truth. Specifically, he claimed that since the proof requires a generalization, Yablo needs to reason from sentences involving a free variable. The free variable is required for generalization, but not allowed for claims involving truth since truth only applies to "closed formulas, not things with

[140] Other versions might have collapsed into Priest, or back into Yablo's sequence. For example, I explored at one point a version of the sequence that write truth claims in satisfaction terms, and since they are interdefinable, would likely have simply collapsed back into Yablo.

free variables in."[141] Priest suggests we need to move to a Yablo sequence involving satisfaction, a truth-like relation that can work with open formulas. He says the contradiction is provable with his revised satisfaction sequence, but does not give us the proof. As such, I have worked here to capture as much of Priest's Yablo proof as possible, and use it (along with Priest's explicit revisions) to give the version Priest had in mind. This version, however, is syntactically incoherent by being explicitly defined by appeal to itself. It does not find self-reference so much as define it into the sequence by making the claims about a predicate that falls within its own claim. To combat this, I have presented on behalf of Priest two possible revisions of Yablo involving satisfaction. Both fail to recover a genuine paradox as they cannot complete the necessary syntax, and thus fail to present claims. It is time to retrench. Let us return to the use of the T-schema and Yablo's original paradox. As we shall see, Priest is correct that Yablo's original derivations are defective because they universally generalize a variable and this makes appeal to the T-schema (disquotation) illicit. But there is a way to formulate the paradox without appeal to such a universal generalization that gives a completed syntax for the Yablo sentences[d] using semantic objects.

[141] Priest, (1997) p.237

CHAPTER 5

REIMAGINING PRIEST AND THE NON-EFFECTIVE YABLO PARADOX

In the previous chapter, I argued that Priest's formal augmentation of Yablo's paradox was problematic and attempted a formal revision. But some in the literature have suggested that the real Priest worry is less a specific formal worry and instead is more about what we might call structural self-reference. In this chapter, I begin with a sketch of the Yablo literature resulting from Priest's analysis. Their focus on structural self-reference can be recast in a way that captures Priest's intuition. It involves the fact that Yablo's sequence is an effective enumeration of the Yablo sentences. In this chapter, I present this worry as an interpretation of Priest, address it, and respond to possible objections.

An enumeration is effective just in case two claims hold. First, for any position, you can deduce which element occupies that position. Second, for any element, you can deduce which position it occupies. To call Yablo's sequence an effective enumeration of the Yablo sentences is just to say that it is an enumeration where there is a recipe for figuring out any sentence's position in the sequence AND the sentence that occupies a particular position in the sequence.

In this chapter, I argue we need not be worried about this revision of Priest's structural self-reference worry. I first explain why the effectiveness of the sequence might give us pause as to whether the sequence, independent of the debate over truth and satisfaction, is self-referential. Then I will argue that we need not worry about effectiveness since we can formulate Yablo's paradox from a non-effective sequence. As has been noted with many attempts to excise self-reference, one might worry that the non-effective sequence falls subject to revenge phenomena: it may involve self-reference. Fortunately for the non-effective Yablo, the self-reference, if it is involved, is non-essential. There is a specific set of circumstances in which the self-reference is causally responsible for the contradiction, and this set of circumstances does not arise for the infinite non-effective Yablo sequence. The chapter closes with a discussion of three benefits of the non-effective Yablo paradox. First, it shows that we need not be worried that effectiveness plays a central role in the Yablo paradox. Second, it focuses us on the need to discuss the existence of the Yablo sentences, as the non-effective proof will not work without them. Finally, this proof bypasses the features of Yablo's original derivation that was the source of Priest's worry in the first place.

5.1 – Priest and the Revised Worry

Roy Sorensen praises Yablo for substituting, "the cramped circularity of self-reference with the luxuriant linearity of an infinite series."[142] Rather than generate paradox from self-reference, Yablo does it through an infinity of other-reference, specifically all those after a particular sentence in the sequence, a move Sorensen finds laudable.

Sorensen was the first to respond to Priest's article in defense of Yablo. As Beall notes, "Evidently, Sorensen's defence has been thought by many to be successful; there has been little discussion, and no challenges, thereafter."[143] In Priest's initial article, the

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[142] Sorensen, (1998), p.139

[143] Beall, (2001), p.176

worry is put in terms of the form of Yablo's sentences[d] necessary to accommodate Yablo's derivation of a contradiction from his sequence. To see the current move to a focus on structural self-reference, Sorensen's Infinite Queue Paradox is a good place to start.

> An infinite queue of students receives a lecture on human fallibility. Each student thinks
>
> (Q) Some of the students behind me are now thinking an untruth.
>
> As it happens, each student is thinking just one thought: (Q).[144]

Soresen was inspired by, in part, Yablo. If student *n* is thinking an untruth, then all those students behind are thinking true thoughts. But each of them can only be thinking true thoughts if some student behind them is thinking an untruth. Student *n* must be thinking something true, then, implying that some successor is thinking an untruth. But since all these successors are thinking (Q) as well, and it is clear that thinking (Q) cannot be untrue, we have a contradiction.

Yablo diagnoses this worry structurally.

> The demonstrative form of the paradox–the beliefs of *those* people are false–gives rise to a different worry. If everyone is in structurally speaking the same situation–each stands are the front of an infinite string of people each thinking 'the beliefs of the people back there are false'–how are the various thoughts to be distinguished? One might even worry that everyone in the line is thinking in some sense *the very same thing*.[145]

He sees this situation as a version of Priest, and sees the worry as whether the sentence (or thought) manages to refer to its own structure. The above quote suggests that when Yablo references the same situation "structurally speaking" that it is the syntax of a

[144] Sorensen, (1998), p.137

[145] Yablo, (2006), p.168

particular claim or thought that individuates some state of affairs rather than another through structural properties. It is the fact that such and such people stand in such and such a line behind someone thinking such and such that makes them suitable referents for the claim. But there is no unique exemplar of these properties, and in fact *all* the students (or, one might think, the Yablo sentences) stand in the appropriate place. As such, a claim about one is a claim about all, and the self-reference is, as Priest puts it, manifest.

Sorensen has an analysis of Priest's criticism of self-reference. He sees it as an equivocation between attributes of a description and attributes of the sequence described. "If there were only finitely many statements, we could exhaustively enumerate each statement and so would have no need of a *description* of the sequence." He goes on to say, "Sequences can be specified demonstratively instead of descriptively," and,

> There are infinitely many Yablo paradoxes that cannot be specified by any recursive formula. Since no finite being can specify such a sequence, no finite being can verify the paradoxicality of such a sequence. But we can know an existential generalization (namely, that there are random Yablo sequences) without applying the inference rule of existential generalization to a known specimen.[146]

Here he speaks of other versions inaccessible to finite beings. The debate surrounds the ability of finite human beings to specify an infinite sequence without circularity. He suggests that it is merely a contingent happenstance of our finite natures that we cannot do so: "Our use of a self-referential specification is merely a useful heuristic."[147] This draws a distinction between a self-referential (or circular) description and a self-referential (or circular) referent.

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[146] Sorensen, (1998), p.146

[147] Sorensen, (1998), p.145

Beall argues against Sorensen in support of Priest. He suggests that while much of what Sorensen says is true, it is also independent of whether Yablo's paradox is circular. He says,

> …the problem with Sorensen's replies is not so much that they are incorrect; rather , they are simply off the mark. What needs to be shown is that, contrary to Priest's point…we have reason to believe that when we use 'Yablo's paradox' (or the like) we are talking about a non-circular paradox. Nothing Sorensen has said begins to provide such a reason, and that is the main problem with his defence against Priest's point.[148]

Beall mentions several of Sorensen's arguments in his article and dismisses them all. Given what he says above, I take his point to be that there is a distinction between a circular description and a circular referent. But that they are conceptually distinct, he argues, does not give us reason to think they are actually distinct in *this* case. All Sorensen has shown is that the sequence *could* be non-circular despite the fact that we must use a circular (in this case, recursive) description to pick it out.

Beall and Sorensen are followed by Bueno and Colyvan who argue in support of Sorensen and Yablo that there are many ways to refer to infinite sets, and that we might be best thinking not about *either* demonstration or description, but about demonstrating the first few and *then* describing.[149] Similarly, Bringsjord and Van Heuveln pick up on a note of Beall's and present a defense of our ability to think of infinite objects (called the "Mental Eye Defence").[150] These questions are about how our finite abilities of description relate to the objects we describe with the circularity of Yablo's paradox at stake.

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[148] Beall, (2001), p.184

[149] See Bueno and Colyvan, (2003)

[150] See Bringsjord and Van Heuveln, (2003)

The way to check whether the sequence is circular, then, is to put it to the test as to whether or not the circularity of description (of the recursive description, as Sorensen puts it) makes its way into sequence denoted. The way to check it is to look at a feature of a sequence of sentences[d] (i.e., a sequence of descriptions) and think about how this relation might be represented. The easiest way to see the connection between the current discussion and how this circular description might be represented comes from Sorensen. He puts the structural worry in a way that points toward the worry considered in this chapter. He says,

> As a finite thinker, Yablo can only generate his infinite sequence with a quantified expression of the form
>
> (Y*n*) For all *k* greater than *n*, Y*k* is not true.
>
> The need for this proposition is disguised by casual presentations that merely list the first few members and then recourse to a vague "etc.", "and so on", or "…". This explicit (Y*n*) formulation is self-referential in the sense that (Y*n*) uses its own location in the sequence as a reference point to specify which statements are not true i.e. the statements after (Y*n*). Priest stokes the suspicion that this is a relevant sense of "self-reference" by casting the point as a fixed-point theorem.[151]

Here Sorensen acknowledges the need for a sort of self-reference in reference to the Yablo sequence. Attributing this point to Priest, he suggests that its self-reference lies in referring to other sentences[d] via its own position in the sequence. Recall that the intuitive meaning of the formal statements of the Yablo is, "All later claims are false." While the numeric presentation shadows it, Sorensen suggests, "later" must be "after this sentence in the sequence".

I recast this tight connection between content and position in terms of effectiveness of the sequence. Below I lay out how we might think of the sequence as

[151] Sorensen, (1998), p.144-5

effective and test (through an attempt to generate a non-effective yet paradoxical Yablo sequence) whether the circularity of the description shows up (at least in Priest's sense) regardless.

As Yablo originally presented his infinite list of sentences of the form $\forall k>n$, $\neg T s_k$, the list is effectively ordered. Yablo's list simultaneously renders a presentation of the sentences together with their reference determining position in the sequence. The contents are used to order sentences. $\forall k>1$, $\neg T s_k$ is first, and for all sentences, their content numeral just is the number designated by their position numeral[152] in the sequence. But what Yablo wants is a list of sentences each of which intuitively claims all the rest are untrue. In this way, the contents are dependent on their position in the sequence. For example, it is only once we know a particular sentence is fourth that we can assign it the appropriate content numeral for it to claim all those coming after it are untrue. Order⇒content⇒order. Or, content⇒order⇒content. In either case, Yablo's simultaneous presentation of order and content seems to define the Yablo sequence with structural self-reference.

We might tie the worry to Priest with the following quote from Priest's article where he describes the self-reference of the paradox:

> The paradox concerns a sequence of sentences, s_n, or $s(n)$, to remind the reader that the subscript notation is just a notational variant of a function applied to its argument. The function *s* is defined by specifying each of its values, but each of these is defined with reference to *s*. (As a glance at Yablo's original formulation suffices to demonstrate.) It is now the function *s* that is a fixed point. *s* is the function which, applied to any

[152] Recall for $s_n=\forall k>n$, $\neg T s_k$, the subscript on the s is the position numeral (marking the sentence's position in the sequence) and *n* following the quantifier is the content numeral.

> number, gives the claim that all claims obtained by applying *s itself* to subsequent number are not true.[153]

Here we get something like a statement of two sorts of problems called (in Chapter 2) the Generation Problem and the Ordering Problem. The Generation Problem is that of generating the infinite number of Yablo sentences needed to run the paradox. The Ordering Problem is the need to order those infinite sentences in a particular reference-fixing fashion.

Many have questioned whether we have reason to believe Yablo's sequence exists.[154] The Generation Problem is that Yablo's paradox derives from an infinite sequence of sentences. It is unclear, one might think, just how these sentences are generated. Priest's suggestion above is that their content is generated by a function, *s*. The Yablo sentences are, on one reading, generated by defining successfully the function *s* from natural numbers onto sentences with Yablo's form. Certainly no finite being can utter (or write down) the infinite sequence. And formal issues (like that of Priest) ought to give us pause about the existence of the necessarily infinite number of sentences. Such is the Generation problem of getting the claims necessary for generating Yablo's paradox.

Since the Yablo sentences work to capture the intuitive claim, "All the rest are untrue," the ordering is important. If a statement B comes after a statement, A, we want A to refer to B. Specifically we want A to imply B is untrue. In Priest's quote above it seems this ordering is done by the function *s* as well,[155] where *s*(1) is first, *s*(2) is second,

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[153] Priest, (1997), p. 239

[154] E.g., Priest, (1997), and Landini, (2009)

[155] Where Priest's s(x) function then provides both content and ordering ot the Yablo sentences.

etc. On this view, the luxuriant other-reference derives from the function, s. Call how Yablo gets this needed ordering the Ordering Problem.

To see how the Generation Problem and the Ordering Problem might open the door for self-referential circularity, think of how the sentences are ordered in Priest's quote above. One function *s* is both generating and ordering the sentences. But to successfully define a function, it must have a definite domain of elements. The function, s, is supposed to take numbers as arguments and return closed formulas (i.e., the Yablo sentences). But since you need to know a sentence's position to know which closed formula it is and the only way to determine its position is to know the content of the sentence, it seems as if there is no way to define the function s non-circularly. In this case the simultaneous presentation worry aligns with Priest's structural self-reference worry because both derive from Yablo running together generation and ordering. It seems, then, that without self-referential circularity we cannot have one function both generate and order the Yablo sentences.

5.2 – Effectiveness and Three Questions

As I suggested briefly in the previous section, one way to talk about this close tie between the ordering and the content of the Yablo sentences is to talk about the effectiveness of the sequence. The simultaneous presentation of position and content guarantees the effectiveness of the sequence. It is the simultaneous presentation that ensures we can know any element's position and any position's element. Now, any ordering of elements is effective when two claims hold of that ordering. First, for any position, you can determine which element occupies that position. Second, for any element, you can determine the position it occupies. [156] Take the list of natural numbers, 1, 2, and so on. If you want to know what the 8th member is, or the member after a certain element, you can find out. In the case of the natural numbers, the 8th member is the number 8. Put another way, there is a recipe (the *n*th member=*n*) for figuring out which element is in a particular position and which position an element occupies.[157] Plug in either your element or your position, and you get an answer.

With a non-effective enumeration, however, there is no way to figure out which is the *n*th element, nor know the position in the sequence of a particular element. Take the list of natural numbers I would randomly shout if given an infinite amount of time. There is no way of knowing the, say, 73rd member I would spout off. Nor is there any way of figuring out when I would say, "42!" There is an ordering, just a *non-effective* ordering: an ordering where we cannot tell what the *n*th element is, nor can we tell the position of any given element.

On Yablo's rendition of the paradox, each sentence gets its name (and thereby position in the reference-ordering) based on its content. Consider: the fifth member, S_5 contains the numeral "5" referring to the number 5. Now consider the schematic expression, "$\forall k > \Delta$, S_k is untrue". Each Yablo sentence is an instance of this schema, differing from others only in the occurrence of a different numeral in place of Δ. When a numeral is inserted for Δ then, the schema yields a complete sentence. For example, inserting "5" yields, "$\forall k > 5$, S_k is untrue." Via their content numerals, these sentences

[156] Hunter, (1996), p.26

[157] Hunter, (1996), p.15

(i.e., these schema instances) are ordered as are the natural numbers (i.e., the sentence, "$\forall k > 1$, S_k is untrue," is first, "$\forall k > 2$, S_k is untrue," is second, and so on).

This sequence is effective. Finding the unique sentence in a particular position is easy. It is merely a matter of inserting the position numeral into the sentence schema. Similarly, figuring out which position a particular sentence occupies is simply pulling the content numeral from the sentence. The recipe, in this case, is: the *n*th sentence= "$\forall k > n$, S_k is untrue." Yablo's original sequence is effective.

Commenters in the literature follow suit. For example, Priest's satisfaction Yablo sequence is still an effective enumeration. The key tie remains in his sequence between the content numeral (the "*n*") and the position (the *n*th). In Priest's sequence, we can cite the sentence for any position, and give the position for any Yablo sentence. His sequence maintains the effectiveness of Yablo's original.

With effectiveness clearly defined, and the effectiveness of Yablo's sequence established, we can ask this as an analogue of Priest's structural self-reference worry: Is the effectiveness a key piece of the sequence's being paradoxical? This matter is entirely missed in the literature. Yet the effectiveness is a key feature of Yablo's original ordering (which determines the referent of each sentence) as well as all those that follow in the literature. Without *some* ordering, the sentences^{d} fail to refer to the necessary sentences^{d},[158] and paradox cannot ensue. Further, the ordering is key in the attempt to avoid self-reference. People worry about Truth and circularity and non-wellfoundedness as subjects for the lessons of the paradox. But is the effectiveness of the ordering playing a causal role in generating the paradox? If so, there would be no paradox for a non-

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[158] See Chapter 2.

effective Yablo sequence, and we would need to add effectiveness to that list of subjects for the lessons of Yablo's paradox.

As such, Yablo's simultaneous presentation of Yablo sentences and ordering names guarantees effectiveness, which represents an interpretation of Priest's structural self-reference worry. Further, effectiveness justifies certain moves in Yablo's argument to contradiction. Put another way, Yablo uses effectiveness to argue that his sequence is, in fact, paradoxical. To see Yablo's use of effectiveness, consider the following moves from his two arguments to contradiction (one pre-Priest (1993), the other post-Priest (2006)).

In the first, proof, Yablo needs to know that the $n+1^{th}$ sentence says "For all $k > n+1$, S_k is untrue". Knowledge of the content allows him to assert, "By [for all $k > n+1$, S_k is untrue], what S_{n+1} says is in fact the case…"[159] Without knowledge of the content, he could not know, "what S_{n+1} says", and as such, could not know that what it said was in fact the case. Knowledge of the content of S_{n+1} derives from the ability to know its content *based on the fact that it is* the $n+1^{th}$ statement (i.e., based on its position). In other words, knowledge of its content derives from the effectiveness of the sequence of Yablo sentences: it derives from the ability to reproduce the n^{th} sentence in the Yablo sequence. Effectiveness, as such, justifies the moves of the argument to contradiction in Yablo (1993).

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[159] Yablo, (1993), p. 252

Yablo's later proof also turns upon the effectiveness of the enumeration of Yablo sentences. In it, he claims, "…S_i is true only if the sentences after it are *false*."[160] The only way to know this is to know two things: (1) In his second proof, S_i is the i+1[th] statement, and (2) to know the content of the i+1[th] statement. This is straightforward use of the effectiveness of the sequence: deriving the object (the specific sentence) solely from its position in the sequence. The effectiveness of his sequence serves as justification for this move as well. I conclude that both his arguments cannot work without using the effectiveness of the sequence as part of the proof.

With the situation prepared, I ask three questions which should, through their answers, clarify whether effectiveness is an integral part of Yablo's paradox. First, is it possible to characterize a Yablo sequence without effectiveness? Second, is it possible to derive a contradiction from such a non-effective sequence without appeal to circular self-reference? And third, what benefits come from such a non-effective derivation. I answer yes, yes, and three: effectiveness is exonerated (and thus this reimagining of Priest is not a worry for Yablo), the existence of the sentences is highlighted, and our proof sidesteps the very universal generalization that formed the basis of Priest's critique (as does the argument to contradiction by Bueno and Colyvan,[161] which is similar in sentiment).

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[160] Yablo, (2006), p.166. Note he moves in his second proof to talk of "false" rather than "untrue," though nothing turns on it in this context.

[161] Bueno and Colyvan, (2003)

5.3 – The non-effective Yablo paradox

As we noted in the previous section, Yablo guarantees the effectiveness of his sequence by tying sentence contents to their position in the sequence (specifically, the second statement has the numeral "2" in it referring to 2, and so on). In Yablo's presentations the ordering of sentences is determined by their content numerals. This content numeral-position numeral connection guarantees effectiveness.

The clearest way, then to characterize a Yablo sequence non-effectively is to divorce the position from its content numeral. Yablo's formulation of the paradox relies on the existence of a one-to-one function *f* from natural numbers *n* (where *n* is the position numeral) onto sentences of the form "$\forall k > n$, S_k is untrue." But clearly there is a function *g* from natural numbers *n* (again, position numerals) onto sentences of the form "$\forall k > m$, S_k is untrue," where "*m*" may well *not* be the numeral for the number *n* (i.e., the content numeral can differ from the position numeral). This function would generate a non-effective enumeration of Yablo sentences.[162] This function takes the same sentences and orders them via their content numerals, but the actual order remains unknown. We know that, for some number *m*, "$\forall k > m$, S_k is untrue," is somewhere on the list, but we do not know where. Nor can we know which sentence is sixth because we don't know the particular permutation of the natural numbers by which *g* orders the sentences.[163] We have no recipe. Thus our first question has been answered. There is a non-effective

[162] One way to see this is to simply note that by permutation we can readily convert Yablo's effective ordering (which relies on the function *f*) to a non-effective sequence relying on a function *g* (which simply maps it to some other ordering of the natural numbers).

[163] It could be all the evens then all the odds, or start at 1,000 and proceed normally. It could be any permutation of the natural numbers whatsoever.

ordering of the Yablo list: it has a non-effective order via a function *g* that orders sentences by their content numerals according to some permutation of the natural numbers. I next offer a proof of a contradiction from this non-effective Yablo sequence.

Let us now show how a proof of contradiction can be given without relying of effectiveness. I use italicized letters for number variables and subscripted S for sentence names where S_n is the *n*th sentence in the non-effective enumeration. Assume S_n. As per our non-effective ordering, sentence S_n is "$\forall k > m$, S_k is untrue," where "m" is a numeral for some natural number. So our assumption is this: $\forall k > m$, S_k is untrue.

Now either n > *m* or $n = m$ or $n < m$. As such, we have the following: If $n > m$: then S_n is untrue, contradicting our assumption, so not-($n > m$). Hence, given our assumption, we have $n \leq m$. This means that for some possibly non-consecutive infinite series of natural numbers *e, e*, e***, etc., we have the following sequence:

$S_n = \forall k > m$, S_k is untrue.

$S_m = \forall k > e$, S_k is untrue

$S_{m+1} = \forall k > e^*$, S_k is untrue

S_{m+2} = "$\forall k > e^{**}$, S_k is untrue"

Now S_{m+1}, $S_{m+2,}$ and so on are all untrue by our assumption of S_n. Hence, in particular S_{m+1} is untrue, i.e., not-($\forall k > e^*$, S_k is untrue). Thus, $(\exists k)(k > e^*$ & S_k is true). Similarly $(\exists k)(k > e^{**}$ & S_k is true), and so on. (Recall, however, that we don't know the relation between *e** and *e*** and so on). Existentially instantiating, we have:

$p > e^*$ & S_p is true,

$p^* > e^{**}$ & S_{p^*} is true,

and so on for infinitely many in the e-series (where we also don't know the ordering relation between *p* and *p** and so on). Each *p*, *p**, and so on is a falsifier for one of S_m, S_{m+1}, and so on. But by our assumption S_n, all of these *p*, *p**, and so on, all come before *m* since our assumption stated all those after *m* are false. Thus every one of the infinitely many in the *e*-series must come before *m* since each *e* comes before its corresponding *p*. That is impossible, since *m* is a natural number, and can only be preceded by a finite number of numbers. Thus, at last we see that our assumption S_n is untrue. But now we know that S_n is untrue. So for some *m*, $(\exists k)(k > m$ & S_k is true). By existential instantiation, we have $k > m$ & S_k is true. We can then return to the reasoning above that showed that S_n is untrue to show S_k is untrue. Thus we have a contradiction.

The new Yablo list offers a non-effective sequence, and yet the contradiction is provable. Since a non-effective Yablo paradox is formulable (if the original Yablo is formulable), I conclude that effectiveness is not an essential part of the construction. And note that in the non-effective Yablo there is no point where all sentences are asserted false. This proof avoids the generalization that forms the basis for Priest's criticism of Yablo's argument (addressed in Chapter 4) that Yablo's paradox involves self-referential circularity.

5.4 – The Baby and the Bathwater

One might worry, however, that the non-effective Yablo is either uninteresting (because it collapses into the Yablo sequence), or that it is self-referential and thus antithetical to the spirit of the original Yablo paradox.[164] Let's call this the "bathwater" worry. I'll first lay out the bathwater worry and its corresponding concerns in this section. Then in Section 5 I will lay out my response that, while *prima facie* worrisome, neither of the disjuncts gives us a true reason for concern regarding the non-effective Yablo sequence and the paradox it captures.

Recall that the non-effective Yablo sequence sought to divorce content from position by letting some permutation of the natural numbers order the Yablo sentences via their content such that content numerals could diverge from position numerals. It could be that they are ordered with 1 at the end, or in some purely random fashion, or with the primes first, followed by the rest.[165] But be sure to recall that the sequence is composed of the same sentences as the original with a possibly different ordering. We simply don't know which ordering (this, I take it, is part of the benefit of the non-effective Yablo proof: we don't need to know the ordering to show the sequence yields a contradiction, and needing to know was the crutch that required effectiveness and corresponding worries about self-reference). It is against this ordering, whatever it may be, that the bathwater worry is focused.

Here is the dilemma: either this non-effective sequence will have an identical ordering to Yablo's sequence or it will not. On might find problems with both. First

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[164] This response derives from comments Kevin Klement made on "How a non-effective Yablo Paradox Works," Hassman (2010) at the 2011 Central APA.

[165] I understand that you never "run out" of prime numbers, and similar remarks apply to thinking about the odds and the evens, but this is a perfectly legitimate ordering of the natural numbers, and as such, will function for our purposes. It is not necessary that we come to the end of anything to think about the natural numbers being ordered in this fashion.

consider the case in which it turns out that the non-effective sequence is ordered just as is Yablo's original. What, then, is the benefit of the non-effective proof? It seems as if we have gone to a lot of trouble to show something that has already been shown and is self-referential just in case Yablo's original sequence is self-referential. It seems as if no genuine work has been done if all we are doing is showing that Yablo's paradoxical sequence is… paradoxical. As such, this disjunct is undesirable.

The latter disjunct is even worse. As noted in the previous chapter, Priest was concerned about covert self-reference. But there seems to be overt self-reference in the non-effective sequence. If the non-effective sequence does not collapse into Yablo's sequence, then some sentence in the non-effective sequence must quantify over itself, thereby claiming itself untrue. This sentence, as such, would be self-referential. Let the first sentence be S_1: "$\forall k > 20$, S_k is untrue." Since I defined the ordering function, *g*, as 1-1 and onto[166] the natural numbers, $\forall k > 1$, S_k is untrue, must be on the list somewhere. Let it be S_n. In this case, S_n is itself correlates with a number in the range of its quantifier and so includes itself among the many it claims to be untrue. In this case, the bathwater worry is that we have only gotten rid of the Priest style structural self-reference (captured by effectiveness) by smuggling in self-reference to the sentences in the sequence. We have thrown out the baby with the bathwater. That is, either the non-effective sequence is uninteresting (because it collapse into the Yablo paradox) or it is inherently self-referential.

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[166] 1-1 and onto means there is a unique argument for each value, only one value for each argument, and that no natural number is not defined by the function.

5.5 – Non-essential Self-reference

First and foremost, a non-effective Yablo sequence does not require that for every S*n*, there is a sentence

$\forall k > n$, S_n is untrue

whose position numeral is *n* that is on the list. The bathwater worry assumes that there is. That is, the non-effective Yablo includes cases where infinitely many sentences of the form

$\forall k > n$, S_n is untrue

are left off of the list. In other words, the Yablo sentence with any particular natural number as its content numeral may be left out of a particular non-effective sequence. The derivation of contradiction still goes through. This is important because the bathwater worry is that the non-effective Yablo essentially relies on forcing the case where there is some S*n* on the list whose quantifier $\forall k$ ranges over numbers including *n*.

Note also that the Yablo sentences quantify over the natural numbers, not sentences. In the original Yablo, we have

S*n*: "$\forall k > n$, S_n is untrue."

Yablo does not take S*n* to be self-referential even though it contains the numeral "*n*" which, by the list generator, is correlated uniquely with the sentence S*n*. Indeed, Yablo wouldn't describe the content of S*n* as saying: "Every sentence in this list numbered greater than me is untrue." That would be explicitly self-referential. But that is not its content. In fact, the situation with the non-effective Yablo is better in this regard than even the original Yablo list. No Yablo sentence in the non-effective list can be regarded as asserting explicitly that all sentences "greater than me" are is untrue. We have instead

S*n*: "$\forall k > m$, S_n is untrue."

The content of S*n* is the *same* no matter what the ordering may be. This is part of the point of forming a non-effective Yablo. The content of S*n* is completely independent of the ordering of the sentences on the list. If it were dependent, then effectiveness would be essential to the paradox as self-reference would have shown back up in the attempted non-effective sequence.

The issue of whether there is essential circular self-reference of content is tied to the issue of what numbers are in the range of the quantifier. To avoid self-reference, Yablo's effective ordering assures that the range of the quantifier in S*n* never includes a number which the list generator correlates with *n*. But this is not required. All that is required is that the existence of such a number in the range of the quantifier not be a cause that produces (or helps produce) the contradiction.

In this section, I argue two claims. First, that the non-effective Yablo sequence is useful no matter what the sequence looks like. Second, and more importantly, the possible self-reference the bathwater worry finds in the non-effective Yablo sequence is non-essential. Specifically, while you can show that self-referential Yablo sentences are paradoxical in certain finite Yablo sequences, the particular feature that unites them all is not a feature of any infinite Yablo sequence. To see this, I first consider a couple finite Yablo sequences, after which I will show the feature that unites them, and explain why it cannot be a feature of any infinite Yablo sequence. I begin with a responding to the possible collapse into Yablo's sequence.

One way to think of the import of the non-effective proof is that it is at a greater level of generality than any that exist in the literature. So even if (on some orderings) the non-effective sequence collapses into the Yablo sequence, we get the result that the original Yablo sequence is paradoxical even if we do not know which sentences occupy which positions in the sequence. And this more general proof, then, provides opportunities to avoid the worries of Priest. It does this by avoiding the effectiveness that, I have suggested, might code the self-reference, and allows a proof of contradiction that avoids the universal generalization that concerns Priest.

The self-reference involved in the non-effective Yablo is non-essential. In other words, it is not causally efficacious in producing the paradox of the non-effective Yablo sequence.

To see the role of the quantifiers in the Yablo paradox, consider some examples of finite Yablo paradoxes where content numbers correlate with positions in some, but not all cases. For simplicity, let us consider only those cases where for every S*n* on the finite list, there is a sentence with position numeral *n* which is on the list. There are two different sets of finite strings of Yablo sentences (call them "finite Yablo sequences") relevant to this response to the bathwater worry. There are sequences in which the last sentence's content numeral corresponds to its position numeral (i.e., $S_l = \forall k > l$, S_k is untrue, for S_l the last sentence in the finite sequence), and there are sequences in which the last sentence's content numeral doesn't correspond to its position numeral. To see this, consider the set of the first three sentences in Yablo's original sequence.

$\forall k > 1$, S_k is untrue
$\forall k > 2$, S_k is untrue
$\forall k > 3$, S_k is untrue

There are exactly six distinct ways to order these sentences: 123, 132, 213, 231, 312, and 321. First recall that finite Yablo sequences are not paradoxical. In the above case, there is a consistent assignment of truth-values. S1 and S2 are false and S3 is true. And not only is this a consistent assignment of truth-values, but S3 is vacuously true, providing the basis for an argument that these are the truth-values of the members of this sequence independent of consistency concerns. This is a non-self-referential and non-paradoxical sequence. There is a consistent assignment of truth-values to the first three sentences [167]

Now take an example where content numerals do not line up with position numerals. Suppose we have the following ordering for a finite Yablo:

132 Yablo

S1: $\forall k > 1$, S_k is untrue
S2: $\forall k > 3$, S_k is untrue
S3: $\forall k > 2$, S_k is untrue

Note that the last member in this list (i.e., S3) has a position numeral that fails to correspond to its position numeral. S3 self-refers as 3>2. This means that S3 cannot be true. But this means that S3 is also true, since what it says is the case, namely that all sentences >2 are untrue. As such, we have a paradox.

Compare the following finitary Yablo paradox with a different ordering:

213 Yablo

S1: $\forall k > 2$, S_k is untrue.
S2: $\forall k > 1$, S_k is untrue
S3: $\forall k > 3$, S_k is untrue

Note that the last member in this sequence (i.e., S3) has a position numeral that corresponds to its content numeral. As noted in the *123 Yablo* above, S3 is vacuously true

[167] This analysis assumes the best disambiguation is as a conditional rather than as a conjunction: $(\forall k)([(k > 1)$ & (Sk is in the Yablo sequence)] $.\Rightarrow.$ $\neg$T(Sk)). For discussion see Chapter 2.

since there is no later statement.[168] S1 is false, since there is a statement numbered greater than 2 that is not untrue (i.e., S3). But what about S2? It is the self-referential sentence here. Since it asserts that any claim numbered higher than 1 is untrue and it is higher than one, then it cannot be true. Here is where a crucial step in deriving the contradiction fails. We cannot assert, on this basis, that S2 is true, as there is also a claim numbered higher than 1 that is true, namely S3. As such, there is a consistent assignment of truth values: S1 is false, S2 is false, and S3 is true, and this sequence, despite being self-referential, is not paradoxical.

The difference in the two finite Yablo sequences lies in the correspondence (or lack thereof) between the content numeral and position numeral of the ultimate sentence in the sequence. In cases where the final sentence has a position that corresponds to its content, a finite Yablo sequence is not paradoxical, even for sequences that include self-referential sentences earlier in the sequence. In all cases where the final sentence has a position larger than its number (i.e., such that the position numeral is greater than the content numeral, making it self-referential) there is a paradox, and one that can be generated from that sentence alone.

The relevant difference between the two sorts of sequence is that in the cases of non-paradoxical self-reference, the self-referential sentence is followed by other sentences. Since these subsequent sentences may be true or untrue independently of the self-referential sentence, there is a move in the argument to contradiction from such sequences that gets blocked. To prove a contradiction from the sequence it is necessary to be able to re-assert a particular sentence based on the falsehood of all remaining

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[168] See Chapter 2 for a discussion of a conditional disambiguation.

sentences.

The cases are generalizable. A paradox with a finite Yablo requires essential appeal an ordering that does not end in a sentence whose content numeral matches its position numeral. Now the worry about the non-effective Yablo is that it cannot generate a contradiction without *essentially* appealing to an ordering that forces at least one of the sentences on the list to include its own number in the range of its quantifier. This, however, is not so. There is no essential appeal in the non-effective Yablo. The derivation in the non-effective Yablo assumes S_n to be true, where (given a non-effective ordering) sentence S_n is "$\forall k > m$, S_k is untrue," where "m" is a numeral for some natural number. Now we do not know what ordering in involved, we only know that either n > m or $n = m$ or $n < m$. There is therefore no essential appeal to least one of the sentences on the list including its own number in the range of its quantifier. To be sure in the case of n>m, this happens, but it is not causally responsible for the contradiction unless there it can be proven that the last sentence has a content numeral that fails to correspond to its position numeral. This means that of the bathwater worry is not really problematic.

5.6 – Three Answers

I would like to end noting three claims. First, that the effectiveness of Yablo's ordering is exonerated (i.e., it is not essential to Yablo's paradox). Second, that the non-effective version of the paradox makes clear the burden placed upon the existence of the Yablo sentences. Third, that my argument to contradiction for a non-effective Yablo sequence avoids (as does another in the literature) the basis for Graham Priest's criticism that Yablo's paradox is circular after all.

As mentioned earlier, the tight content-ordering connection has seemed suspect to me. This seems worse given its integral role in Yablo's (and others') arguments to contradiction. The current proof from a non-effective Yablo sequence shows this worry illusory. By Yablo's own standards, effectiveness is not a partial cause of Yablo's paradox.[169] We can also respond to the bathwater worry. Effectiveness is thus exonerated.

Second, the function *g* sheds light on a piece of the paradox that does not get much attention: the *generation problem*. The function cannot do the ordering without sentences already existing to order. It is true that a function could *re*-order Yablo's initial sequence in accord with some permutation of the natural numbers, but this just passes the buck to Yablo.[170] Moving to a non-effective ordering breaks the content numeral-position numeral identity, and as such divorces *generating* the sentences from *ordering* them. It treats the generation problem as separate from the ordering problem, allowing these to be separate question as they should be despite Yablo's original presentation.

Recall that Yablo presents the names (i.e., the reference-ordering) at the same time he presents the sentences and their contents. It is as if we are supposed to think of the sequence as having the same attribute as the presentation: simultaneousness of sentence-generation and sentence-ordering. Yablo does defends the existence of the sequence in 2006, appealing to the plausibility of successful reference for empirical claims like, "The sentence written on the board in room 301 is false." And these sorts of claims seem reasonable and meaningful. Yablo says, "How an empirical description

[169] Yablo, (2006), p.166, see definition of 'circularity-based' paradox.

[170] Yablo has argued the necessary sentences exist, but all he's really shown is that they are all *possible*: nothing blocks any particular assertion. He still needs the infinite number of sentences.

could fail to apply because of an object's semantical properties it is not easy to see."[171] We might grant that this is hard to see. But granting that there is no *prohibition* on Yablo sentences is not the same as admitting that the infinity of Yablo sentences *exist*.

Put another way, Yablo argues persuasively for the claim that there is no Yablo sentence such that *it* cannot exist. This is a far cry, however, from arguing that every Yablo sentence exists. For this reason, it is important to talk about generation of Yablo sentences as a separate problem, one that gets highlighted by the non-effective proof. The non-effective sequence is possible *relative to* the existence of the Yablo sentences. They must already be generated since we cannot attempt to rely on the position numeral to supply the content for the sentence, as does Yablo. For the non-effective proof (and, I believe, for Yablo's original version), the generation problem needs be discussed.

Third, this kind of proof avoids the basis for Graham Priest's argument that the paradox is circular. Recall from Chapter 4 that Priest's argument turns on Yablo using universal generalization during his argument. Priest argues that the natural justification for lines like, $Ts_n \Rightarrow \forall k > n, \neg Ts_k$ is an instance a naïve T-schema. [172] In this case, since "s_n" is the name for $Ts_n \Rightarrow \forall k > n, \neg Ts_k$, the relevant instance of the T-schema is: $Ts_n \Rightarrow (\forall k > n, \neg Ts_k)$. Intuitively, this sentence claims that if the sentence named by "s_n" is true, then the content of s_n, namely $Ts_n \Rightarrow \forall k > n, \neg Ts_k$, holds. Similar remarks apply to all cases where Yablo needs to swap the name of a sentence for the content of that sentence.

It is in this light that it becomes most clear why talk of effectiveness relates to Priest's concerns. The names of Yablo sentences mark out their position in the sequence,

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[171] Yablo, (2006), p.168

[172] Priest, (1997), p. 237

and their content just is the sentence that occupies that position. In this case, instances of the *T*-schema can be read as claims dependent upon the effectiveness of the sequence. The effectiveness is what lets us know the relationship between any given Yablo sentence name (which marks a position in the sequence) and the sentence named (which marks an element in the sequence).

In this chapter, I have argued that we need not be worried about effectiveness, as the sequence is still paradoxical (though more difficult to work with) without an effective enumeration of the Yablo sentences. But Priest takes a different route. Seeing the need for a naïve *T*-schema instances, he notes that Yablo uses a universal generalization in his argument to contradiction. This is only legitimate when there is a free variable as part of the sentence for generalizing. Priest puts this point, "The *n* involved in each step of the *reductio* argument is a free variable, since we apply universal generalization to it a little later; and the *T*-schema applies only to sentences, not to things with free variables in."[173] Given this, the generalization requires a move to a formal satisfaction predicate, which does apply to things with free variables in, and it is in this more to satisfaction that Priest finds self-referential circularity.[174]

In this case, then, a lot turns on how we are to understand the *n*. Is it a constant? This would make the *T*-schema applicable, yet disallow the generalization. Is it a free variable? This would outlaw use of the *T*-schema, yet allow the generalization. Further note the generalization is part of both the 1993 and 2006 arguments to contradiction presented by Yablo. He argues that some particular Yablo sentence is untrue, and from

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[173] Priest, (1997), p.237

[174] His argument is given a thorough treatment in Chapter 4.

that concludes that *all* the Yablo sentences are untrue. This is the universal generalization Priest refers to in his criticism. As the UG is necessary for Yablo's version of the argument, and a UG is only legitimate on a free variable[175], Priest concludes Tarski *T*-schema is irrelevant, leading him to circularity.[176] His article went on to spawn a long line of work[177] based largely on this particular turn and the circularity he bases on it.

Fortunately for Yablo, there are other options for arguing to contradiction from a Yablo sequence. One such argument shows up the literature by Bueno and Colyvan.[178] Another is this chapter's proof from a non-effective Yablo sequence. Both use a similar structure, deriving a contradiction from an assumed statement, then asserting its denial and deriving a contradiction from that denial.[179] Bueno and Colyvan note that this structure argues for a weaker contradiction, yet assert, "Even if 's_1' were the only paradoxical sentence in the Yablo list, this would be sufficient to conclude that Yablo's paradox (i) is a paradox, and (ii) is not circular – or, at least, it's not circular in sense [of

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[175] For one instance of an explanation, consider Copi, (1979), p.72: "…what is true of *any arbitrarily selected individual* must be true of *all* individuals." He follows with an example of a fallacy on p. 73, attempting to generalize from a constant rather than a free variable. In Yablo's case, he uses an arbitrarily chosen member, NOT a particular sentence. As such, it must have a free variable in it, and the only thing unbound is *n*.

[176] Priest, (1997), p. 238

[177] See, e.g., Sorensen, (1998), Beall, (2001), Bueno and Colyvan (2003, 2003, 2005), Ketland (2004), Bringsjord and Van Heuveln (2003).

[178] Bueno and Colyvan, (2003)

[179] The substance is the same, though Bueno and Colyvan end their proof noting that, "…a contradiction can be derived from the truth or untruth of a particular sentence, 's_1', in the Yablo list," p.155.

Priest].”[180] The weaker conclusion of their argument and the non-effective sequence is enough.

While I would not want to say s_1 being paradoxical is sufficient to conclude that Yablo’s paradox is not circular, what they have in mind is, I think, largely correct. In the non-effective proof above and in Bueno and Colyvan, the *arbitrariness* of the sentence in question is not used to derive the contradiction as it is in Yablo. They avoid the universal generalization. Proofs like the non-effective one above and Bueno and Colyvan’s show Priest’s analysis is a worry merely for Yablo’s particular *argument* to contradiction, not one for Yablo’s paradox itself. As such, even if we grant the entirety of Priest’s analysis, the paradox is not thereby impugned.

5.7 – Conclusion

In sum, this Yablo sequence remains paradoxical even for a non-effective enumeration. As such, the effectiveness of the sequence of sentences is not even a partial cause of Yablo’s paradox. It does shed light on the need to address the generation problem and the ordering problem separately. This non-effective paradox also avoids the key worry in Priest’s seminal analysis while making clear the burden placed upon the existence of the sentences.

Priest was worried about Yablo’s particular argument to contradiction, and revised it in accord with the justifications necessary for Yablo’s moves within that argument. Specifically, Yablo generalizes a result on a sentence that has no free variable (if understood as an apt argument for the truth-predicate). This generalization is

[180] Bueno and Colyvan, (2003), p.155.

illegitimate unless Yablo wants the constant variable to instead represent a free variable. But in that case it will not be the sort of thing for which a Tarski-style T-schema works. As such, Priest argues Yablo needs to revise his Yablo sentences to claims about satisfaction rather than truth. In Chapter 4 I argued this does not work as he presents it, and what seems like an appropriate revision does not involve self-reference.

This chapter discusses the role of effectiveness in the original Yablo paradox. Yablo builds his sequence in a way that guarantees effectiveness. This makes the ordering dependent on the content and the content dependent on the order. His sequence is effective because of this simultaneous presentation of the sentences and their order in the sequence. This may be just the sort of circularity Priest was worried about.

Despite this worry, I hold that we can characterize the sequence without effectiveness, and that the resultant non-effective sequence is still paradoxical. And while the bathwater worry is that the non-effective sequence, if interesting (and not just Yablo's original paradox), will involve the range of the quantifier in S*n* including a number assigned to S*n*, such self-reference, if it appears, is non-essential. As such, effectiveness is merely a useful shorthand in discussions of Yablo's paradox. In subsequent chapters, we need to discuss the generation of Yablo's sentences. And we need to discuss how Yablo can avoid Priest's objection even if he decides to grant Priest's position *in toto*.

CHAPTER 6

YABLO AND PROPOSITIONALISM

This final chapter argues that Yablo's paradox is formable for propositionalisms that embrace purely intensional propositions (i.e., semantic objects). I look to establish two theses based on this. First, that Yablo's aim is in trouble, as the need to resort to semantic objects puts Yablo's paradox on par with traditional Liar paradoxes given normal restrictions to propositions. Second, those seeking semantic objects for use in their formal semantics are left with a difficult dilemma. They must either accept semantic objects unrestricted (and thus face Yablo's paradox another way) or they must accept restrictions on the number of semantic objects. Neither of these is acceptable, as there is no clear way to avoid the paradox on an intensional propositionalist view without restrictions, and because any such restrictions would be unintuitive given reasonable theses about the lack of restrictions on thinking. In this light, I turn to Brentano and Meinong, looking to their ideas about intentionality and the mental as a sounding board for what we really want out of our intentional entities. In the end, I hold that Yablo's paradox is a good foundation for rejecting semantic objects, and that we will have to find another way to do our formal semantics.

I first lay out an overview of the project so far including a description of Yablo's paradox in terms of the definitions and results established thus far. Then I present three ways of understanding propositions: purely intensional (Fregean style-propostions), partially intensional-partially extensional (Russellian-style propositions), and purely extensional (e.g., sets or ordered pairs). The latter two are extensional in ways that prevent the completion of the syntax of the Yablo sentences^{d}. The first, however, leads to Yablo's paradox. To see this, I discuss the difference between Russellian and Fregean propositions, showing why a discussion of their different accounts of proper names leads Russellian propositions into syntactic regress for Yablo sentences^{d} and Fregean propositions into Yablo's paradox.[181] As Fregean style propositions are semantic objects, I hold that we ought to reject semantic objects. Then, I look to Kevin Klement's ideas about restricting the number and kinds of propositions as a way of responding to the Yablo. This response, I argue, is the very response Yablo wanted to attack, but because his paradox is only formable given purely intensional semantic objects, his paradox is just as vulnerable to the response as the supposedly weaker (through their self-reference) traditional Liar variants. Finally, I turn to a brief note about Brentano and his student, Meinong, for a discussion of why we might want to avoid restrictions on semantic objects if we hold them to be the meanings of sentences and the objects of thought.

6.1 – Survey

Chapter 1 and Chapter 2 set up and argue that Yablo's paradox may be understood as a semantic paradox: one that derives from notions of meaning, reference, truth, and the like (including satisfaction!). Chapter 1 focuses on its history in Frege and Russell in an attempt to show its roots in the traditional Liar paradox (roots analyzed more thoroughly in Chapter 2). It also sets up the discussion of modern propositionalism. Chapter 3 argues that the best way to understand the external relevance of Yablo's paradox is in terms of natural language (or, more specifically, how our formal language

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[181] As per the Chapter 4 worry about Priest's revision.

constructions help us understand natural language). It argues this through a close reading of Yablo's two presentations and attention to the distinction between formal and natural paradoxes. Chapter 4 and 5 respond to the most influential criticism in the literature, arguing that Priest's analysis is a non-starter as his satisfaction revision is syntactically incoherent. Chapter 4 presents alternative Satisfaction Yablo Paradoxes that are merely syntactically non-wellfounded (rather than syntactically incoherent). Chapter 5 catalogues the subsequent literature, which understood the spirit of Priest's analysis as a focus on structural self-reference in the construction of the sequence. I argued this is best understood in terms of the effectiveness of Yablo's sequence, and tested the need for such a self-referential description through an attempt to create a non-effective Yablo paradox. The attempt succeeded, and possible criticisms regarding self-referential revenge are addressable.

Yablo's paradox consists of an infinite number of sentences^{d}. For the paradox to work, these need not just be syntactically well-formed, but also meaningful. Part of this is tied to the Generation Problem: the need to ensure the relevant Yablo sentences exist. Based on the description we have from Yablo, we know roughly the signs involved in the sentences^{d}. But given the worries about syntactic features of the satisfaction revisions[182] the paradox is formable given a way to complete the syntax of the sentences, thereby ensuring they are claims.

Recall that the distinction between sentences^{d} and claims is at the linguistic level: it is about our marks on the page or the sounds from our mouths. The thing we need to discuss is how the claims are successful in their reference: in virtue of what are they

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[182] See Chapter 4.

intentional? In accord with definitions laid out in Chapter 1, all claims are semantic entities: they count as the sorts of things that exhibit intentionality. Semantic objects are intrinsically intentional. They do not have derived intentionality, but have it independently. They are non-physical and non-linguisitic, and exist independently of humans. One way to describe propositionalism, then, is the view that semantic objects exist and are the meanings of our linguistic entities: the meanings of our claims (in the technical sense).

The analysis of Yablo's paradox in this dissertation suggests two requirements must be met in order to formulate the paradox. First, to ensure we are actually working with sentences^{d}, we need the completed syntax of the Yablo sentence. Second, we need those sentences^{d} to be claims, i.e., to exemplify intentional relations. I address each in turn.

In the first case, recall the issue from Chapter 4. For a Yablo sentence

$$\forall k>n, \neg \mathrm{T}(\mathrm{Y}_k)$$

how are we to read the "Y_k"? In Chapter 4 I suggested it could be a name for an entity, or it could be a function (i.e., a description of the structure of something else, in this case, another sentence^{d}). The name of an entity is problematic because it does not allow for quantifying in, and so could not work. The function works, but fails to specify a completed syntactic form, as the functional notation allows the function to stand in place of the syntax of the value of the function for a particular argument. In the case of Yablo, this "standing in" never ends, as the sentence^{d} the functional notation stands in for itself contains functional notation. Thus both the name and the function reading are problematic.

A semantic object view offers a new reading. While it does describe a syntax in some sense, with a semantic object we no longer get the regress of non-wellfounded syntactic form. The semantic object is able to refer to the function in question (through its intrinsic intentionality) and the sentence can be represented:

$$(\forall k{>}n)[(\exists \mathrm{p})(f(k)=\mathrm{p}) \Rightarrow \neg \mathrm{T}(\mathrm{p})]$$

In other words, for any number *k* greater than a particular number *n* (representing some position in the sequence) then if the ordering function (*f*) assigns *k* to an object *p*, then that object is not true. No object assigned by *f* to a number higher than *n* is true. Our reference to the semantic object p no longer need be only a syntactic put-off: it completes the syntax of the Yablo claim.

To see this, recall the distinction between extensional contexts and intensional contexts. In the former, all that matters is the referent of a particular phrase. It does not matter how that object is picked out. All there is to an extensional context is the object referred to. In the latter case, however, the method of reference is an important part of the puzzle. For extensional readings, functions are reliant on syntactic form. If they can be given an *intensional* reading, however, then we have a situation much like Frege's Puzzle of the Morning Star and the Evening Star. In Frege's famous example, he decided a sense (*Sinn*) stood between the noun phrases and Venus such that, "The Morning Star is the Evening Star," could be meaningful, even though all cases of "x=x," are trivial. A semantic object, similarly, stands between the further syntactic structure and the completed original claim. Much as senses, as intensional objects, stood between, semantic objects can, through their intension, complete the syntax of the Yablo sentences.

In the Priest case, we did not consider whether or how semantic objects worked within the structure. We understood functions as they are normally understood: extensional entities. f(x) is a description of something rather than the name of something. f(x) does stand in for what f assigns to x, but extensionally. There is no thing that is "f(x)" in virtue of which the function refers to the f(x). All the function *does* is refer.

This need not be the case. An intensionally understood function has a mode of presentation: something between the description and the described. Intensional things act like a name for the purpose of completing syntax because they, like the name, stand between rather than pushing off the syntax onto another entity.

Think of the Simple Strengthened Indexical Liar sentence: This sentence is false. Its syntax is a simple subject-predicate form, much like the Named Non-Indexical Liar, Ln: Ln is false. In each case we have a determinate syntax. If one wants to challenge these sorts of sentences, it is often in terms of semantics. For example, it is not clear that indexical terms have reference independent of a context of utterance. When we write an Indexical Liar sentence, then, we need to specify a context of utterance to ensure that the "this sentence" self-refers. But this is, as I suggested before, a matter of semantics, not a question about the syntax of the sentence. Similarly, when the function symbol at the end of the Yablo sentence refers to an intensional object, then it, too, has a definite syntactic structure and qualifies as a sentence[d] and we can turn to the question of whether or to what extent it is a claim.

I have suggested that there is a problem with a single function simultaneously generating and ordering the Yablo sentences. This comes from the need to reference the function in the sentence in order to generate the necessary reference. A particular Yablo sentence (with its particular position in the sequence) picks out all those after via reference to the function that orders the sequence. There seem to be just two options here. Either we can reference *some* function (where there is no guarantee that it will be the ordering function since the sentences need to be fixed before they can be assigned to arguments for the function, and if they are fixed previous to defining the function, then the function does not exist and there is no specifying that function as the one referred to) or we cannot refer to the function because it cannot exist until the sentences are fully formed. In both cases, we cannot get what we need.[183]

With semantic objects there exists a third option. Their intensionality allows us to think of the generation problem as distinct from the ordering problem. Because semantic objects exist independently from the function, these need not be generated by the function. They only need to be ordered by it. And if there are semantic objects that refer to functions via their intrinsic intension, the functions referenced will include an appropriate ordering function for the Yablo paradox. In this case, the Yablo sentences pre-exist the function, and so the function can be defined over a range filled with these Yablo sentences. The sentences can refer to the function because that reference is not assigned in an interpretation (which would not work for the reasons cited above) but instead derives from the intension the semantic objects exhibit. Pre-existing semantic objects, as such, solve the generation problem, and allow the ordering problem to dissolve as it makes it easy to define a function that takes numbers as arguments and produces Yablo sentences as values. This means the issues that need addressing (proper, specified syntactic structure and semantic reference) are addressed on a theory that

[183] Landini, (2008)

countenances semantic objects. Semantic objects do not need generating, as they exist mind-independently, thereby solving the generation problem. This allows them to be ordered by a function defined over them rather than ordered *and* generated. Finally, they can have a complete and specific syntax, which escapes the issues made clear by investigating Priest and satisfaction Yablo paradoxes.

6.2 – The Kinds of Propositions

In the previous section I argued that Yablo's paradox is formable for semantic theories that countenance semantic objects. Many have argued in support of propositions for independent reasons in modal logic, modal metaphysics, logic, epistemology, philosophy of mind, and more.[184] For our purposes, it is easiest to think of propositions as split into three different kinds. There are purely intensional, partly intensional-partly extensional, and purely extensional. Kaplan famously distinguishes between Fregean propositions (which are examples of the purely intensional) and Russellian "singular propositions"[185] (which are examples of the partly intensional-partly extensional).

Fregean propositions see singular terms as referring via, "a concept, something like a description in purely qualitative language".[186] Frege solved the puzzle of the Morning Star and the Evening Star by positing a third realm of senses that include modes of presentation and account for the difference between contingent identity statements (i.e., the difference between "'a'='a'" and "'a'='b'") by viewing our linguistic terms as

[184] See Chapter 1 for a more detailed discussion of the uses of propositions.

[185] Kaplan, (1989), p.483

[186] Kaplan (1989), p.485

connecting to senses that themselves refer to objects in the world. Senses allow us to understand statements like, "The Morning Star is the Evening Star," as meaningful rather than trivial. The concepts posited are intensional entities, and on this purely intensional view, the concept stands between signs and signified.

By contrast, singular propositions are, "…theories of meaning according to which certain singular terms refer directly without the mediation of a Fregean *Sinn* as meaning."[187] On this view (a Russellian view) singular terms are understood as constituents of the proposition because the singular directly refer. There is no concept nor *Sinn* nor mode of presentation that stands between the sign and the signified for singular terms. There is simply the object, and this is the sense in which these are partly-extensional: singular terms involve no intension, but instead the signified as consituent. Non-singular terms refer via something like a concept, much as they do in Fregean purely intensional propositions. This is the sense in which Russellian propositions are partly-intensional.

The third sort is referenced by Bealer when he speaks of "reductionism"[188]. Propositions in this sense "are really extensional functions from possible worlds to truth values…are nothing but ordered sets (sequences, abstract trees, etc.) consisting of properties, relations, and perhaps particulars;"[189] This final account (which I call "purely extensional") has propositions but no semantic objects. Functions, ordered pairs, and the like have no intrinsic intentionality as they are defined (or interpreted), and it is through

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[187] Kaplan, (1989), p.483

[188] Bealer, (1998), p.2

[189] Bealer, (1998), p.2

this definition or interpretation that they come to mean what they do. Functions do describe a certain structure, but the structure belongs to the values that result. We talk about functions in mathematics, but at its heart, the notion of a function is a description of a commonality of structure between all the values of the function. To nail down the terminology, a function is a process that takes arguments (some kind of entity) and plugs it into a particular gappy structure (to borrow a phrase from Frege). The gappy structure captures, in some sense, the structural properties that all the results have in common. We call these results "values". Thus an argument+the function=a value. So for the square function (i.e., $f(x)=x^2$) we can say that for the argument, 4, we have a value of 16. Written out, it looks like this: $4^2=16$. Recall that, importantly, what the identity sign means in mathematical contexts. It is an identity sign in every sense of the term. So when we write f(4)=16, that means that these two objects are identical. The method of reference "f(4)" denotes precisely the same object. It is not contingent, and there is no extra mode of presentation here. The value of the function is its only contribution.

Propositions as functions, then, are purely extensional because they do nothing but push on to the states of affairs they reference. The structure we're looking at involves signs (marks on a page, sounds, etc.), something standing in between (in this case, a function) and some particular referent (call it a state of affairs for clarity's sake). Since f(4) is no different than 16, the function standing between is merely a placeholder, a way of understanding the real relation, which is between the sign and the signified. This relation is akin to what Kaplan calls "directly referential terms,"[190] a name that pays

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[190] Kaplan, (1989), p.484

tribute to the extensionality of the term. Functions use no concept nor Fregean sense to pick out their term, but instead refer directly.

Propositionalists differ in terms of how they understand their propositions, and we can now consider the sorts of views that will need to deal with Yablo's paradox.

6.3 – Propositions and Yablo

Yablo's paradox, as is suggested in the survey above, needs to be able to have a structure that looks something like this for some ordering function, *f*:

$$(\forall k{>}n)[(\exists \mathrm{p})(f(k)=\mathrm{p}) \Rightarrow \neg \mathrm{T}(\mathrm{p})]$$

Now, it is clear that there needs to be here some way of getting the "*f*" to refer to the ordering function. I have suggested that this can be done by semantic objects: objects with intrinsic intentionality.[191] But recognizing that there are multiple conceptions of what propositions look like (including those that are not intensional), more needs to be said about the sort of view susceptible to above formulation.

Traditionally, functions are understood extensionally. They are seen as merely describing a structure. As we saw previously[192] this leads to syntactic regress in the case of the Yablo sentence as each sentence makes reference to the function. Any attempts to cache out the syntax in terms of that described by the function will lead to a longer claim that still includes the functional term. Such is the regress, and why no purely extensional account of propositions can capture Yablo's paradox. Specifically, for a view on which propositions are merely functions themselves, there are no intensions involved. As such,

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[191] Like Fregean senses.

[192] See Chapter 4.

the "*f*(*k*)" in the above sentence would not refer intensionally to the function, but would instead only refer to what the function *f* assigned to the argument *k*. This is precisely the sort of extensionalism that led Chapter 4 satisfaction Yablo paradoxes to a syntactically non-wellfounded regress.

The question, then, becomes whether or not Russellian or Fregean propositions can sustain the Yablo as they are the two kinds of proposition that are at least partially intensional. Recall that the main difference between them is in how they deal with singular terms. Russellian propositions have the referents of singular terms as constituents (and are thus partly extensional). Fregean propositions are purely intensional, and so pick out even singular terms using senses, concepts, or the like.

The answer is that reference to the ordering function (at least when it is explicit in the above formulation and other e.g., Sorensen (1998)) is clearly a singular term. It is not talking about a kind or a sort but a particular function: the one that orders Yablo sentences[d] in the relevant fashion. As such, Russellian propositions cannot complete the syntax of the Yablo sentences. Since Russellian singular terms only add their denotation to the proposition, even though it is a semantic object, does not generate the paradox as the syntax needed is not specified. No entity that is either purely extensional (the function view) or extensional for singular terms (the Russellian view) can.

Purely intensional propositions, on the other hand, have an intension in the spot where the functional term occurs. Rather than pushing on to the object, the intension fills the syntactic slot. This means that the syntax necessary for a Yablo sentences is capturable for an intensional proposition. As such, purely intensional propositions are those that solve all the issues for Yablo's paradox. With an infinity of purely intensional propositions of the form:

$$(\forall k>n)[(\exists p)(f(k)=p)] .\Rightarrow.\neg T(p)$$

we can generate Yablo's paradox with the following sequence

SO1) $(\forall k>1)[(\exists p)(f(k)=p) .\Rightarrow.\neg T(p)]$

SO2) $(\forall k>2)[(\exists p)(f(k)=p) .\Rightarrow.\neg T(p)]$

SO3) $(\forall k>3)[(\exists p)(f(k)=p) .\Rightarrow.\neg T(p)]$

and so on. I do not assume, as was mentioned in Chapter 5 regarding non-effective Yablo sequences, that there is an entry for each natural number. I only assume that there is an infinite number of Yablo sentences[d] and that they are well-ordered under the >-relation.

1. $(\forall k>1)[(\exists p)(f(k)=p) .\Rightarrow.\neg T(p)]$	assume for *reductio*
2. $\neg((\forall m>1)[(\exists p)(f(m)=p) .\Rightarrow. T(p)])$	1, =df of the list
3. $(\exists m>1)\neg[(\exists p)(f(m)=p) .\Rightarrow. T(p)])$	2, Quantifier =df.
4. $(\exists m>1)[(\exists p)(f(m)=p) .\&. T(p)])$	3, DM, Double Negation
5. $T(p)$	4, EI, simp
6. $(\exists m>1)(\exists p)(f(m)=p)$	4, EI, simp
7. $\neg T(p)$	1, 6, MP
8. $T(p) \& \neg T(p)$	5, 7, conj
9. $\neg(\forall k>1)[(\exists p)(f(k)=p) .\Rightarrow.\neg T(p)]$	1-8, *reductio*
10. $(\exists m>1)\neg[(\exists p)(f(m)=p) .\Rightarrow. T(p))]$	9, Quantifier =df.
11. $(\exists m>1)[(\exists p)(f(m)=p) .\&. T(p))]$	10, DM, Double Negation
12. $T(p)$	11, EI, Simp
13. $(\forall k>m)[(\exists r)(f(k)=r) .\Rightarrow.\neg T(r)]$	12, semantic object release

And at this point, repeat the structure from the *reductio* to derive a contradiction. Paradox.

6.4 – How Many Propositions are there?

While the paradox is, in fact, formable within a theory of purely intensional semantic objects, this does not imply that Yablo's paradox arises for any theory of intensional entities, but it does for what might be called naïve comprehension of semantic objects (after set theoretical comprehension principles) and other similar theories. On a naïve version, any *wff* whatsoever comprehends a proposition. Given what we have said about semantic objects finalizing the syntax of the Yablo sentences^{d}, those sentences^{d} satisfy as *wffs*, and so naïve comprehension of semantic objects holds that there is a proposition corresponding to the assertion of each Yablo sentence^{d}. Given this comprehension principle, then, we can generate Yablo's paradox from any such theory of semantic objects.

Naïve comprehension has gotten us into trouble before. Many, for example, reject naïve comprehension of sets because it countenances the Russell set.[193] We might, when thinking of Yablo's paradox, accept some restriction on expression and thought similar to Prior as he considered accidental empirical self-reference:

> …we must just accept the fact that thinking, fearing, etc., because they are attitudes in which we put ourselves in relation to the real world, must from time to time be oddly blocked by factors in the world, and we must just let Logic teach us where these blockages will be encountered.[194]

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[193] The class of all classes that are non-self-members.

[194] Prior, (1961), p.32. The paradox is that of Mr. X who hates Mr. Y, and upon retiring to their hotel rooms, Mr. X believes Mr. Y has gone into room 7 and thinks, "Nothing currently thought in room 7 is true." But Mr. X has made a mistake. *He* is in room 7, and his thought actually picks out Mr. X himself. Mr. Y resides safely in room 8.

Prior here simply bites the bullet here and accepts limits on thought revealed by paradoxical results in logic. But the exact form that these limits take might not be numeric (i.e., it might not be so much a restriction in the *number* of semantic objects as it is in the type or structure of those objects). After all, for any propositionalism interested in the truths of mathematics being expressible, an infinite number of propositions will be desirable.

In a discussion of modern Fregeanism, Kevin Klement suggests that restricting the number of semantic objects[195] will be difficult. His article specifically addresses Fregean senses (which he takes to be senses of particular words and phrases) but as his examples make clear, his thoughts apply equally to propositions for our purposes.[196] I will outline the principles he suggests will make trouble for those looking at restrictions, principles that are analogues of naïve comprehension of propositions. Then I present his response to proliferation of these intensions, and talk about how it, surprisingly, might make Yablo's paradox seem much more Liar-like than before, and in fact, no different in application. His response looks to restrict the comprehension of propositions much as Russell's type-theory restricted classes.

Klement outlines two relevant principles[197] that make for trouble.

1. Principle of Classes as Entities (PCE): classes are entities.

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[195] He puts it in terms of "senses".

[196] I am aware that Frege spoke of thoughts as the sense of a sentence, but Klement seems to make a distinction, and so I follow him here.

[197] Klement (2003), p.305

This is, in some sense, just to say that we can refer to classes. If I want to refer to the class of all my tools, then there is a sense that picks out the collection of those tools.

2. Principle of Propositions (PP): for every entity, it is possible to generate a different proposition.

These two together generate an infinity of senses, and in fact, violate Cantor's power class theorem[198], as any diagonalization will yield a new class, but PP will simply generate a new proposition for that class.

Of course, on a theory of propositions where they are not made but are in fact abstract and eternal (like Frege's) diagonalization will never "produce" any new propositions: the intensions have always been out there. And both Klement's principles are plausible, especially if put independently of a commitment to a particular theory of meaning. The first might well say that we can talk about collections of things. The second says we can talk about anything. And no matter your theory of meaning, these are highly intuitive. All the ideas we have and things we experience are such that we can refer to them. If your theory of meaning does not include that, then it is inadequate.

Klement presents several paradoxes, including one of note for the current discussion: the class/proposition paradox:

> Consider the (false) propositions expressed by such sentences as 'every entity is in the class of humans', 'every entity is in the null class', 'every entity is in the class of propositions'. Some of these propositions are themselves in the classes they are about, such as the final example. Others, like the first two examples, are not in the classes they are about. Define W as the class containing every propositions of this form that is not in the

[198] Klement (2003), p.304, puts it: "…there must be more classes of entities in a certain domain (i.e., subclasses of that domain) than there are entities in that domain."

> class it is about. Then consider R, the proposition that every entity is in W. Is R in W? It is just in case it is not.[199]

If propositions generated by highly intuitive principles lead to paradox so quickly, it seems to me that there must be *some* kind of restriction on them. Klement agrees, noting that Frege already had restrictions that could respond to other paradoxes Klement presents.[200] The answer, Klement suggests, lies in some kind of ramified type-theory:

> Specifically, we must divide propositions into various orders. Propositions that are about other propositions or involve quantification over other propositions would necessarily be of a higher order than that of which they are about or over which they quantify.[201]

In other words, the propositions themselves would be typed in one way or another that would prevent paradoxes like Klement's class/proposition paradox from getting off the ground.

Even if one were sympathetic to abstract independent eternal meanings, it would be difficult to be *so* sympathetic as to overlook obvious paradoxes like Klement's. As such, some sort of structuring of propositions might seem prudent for propositionalists. In other words, they ought to have some restrictions on the semantic objects they countenance.

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[199] Klement, (2003), p.305-6

[200] Specifically, a paradox of class/sense would be unformable since Fregean senses were unsaturated things and so could not be talked about independently of saturation by a particular. As such, no senses would have them as referents. Think here of the paradox of the concept 'horse' is not a concept.

[201] Klement, (2003), p.318. While Klement suggests this, it is unclear whether this is his considered view or not. The exegetical point is not important to the current project, but deserves mention.

6.5 – Avoiding Circularity

Yablo's paradox is designed to avoid circularity. He famously says in a footnote describing the article that introduces Yablo's paradox, "This note gives an example of a Liar-like paradox that is not in *any* way circular."[202] I take that to mean that he has produced a semantic paradox (and, based on other factors, a natural paradox) that seeks to vindicate self-referential circularity famously blamed for the existence of semantic paradox. Responses to paradoxes that seek to avoid self-reference say, "This may be using a cannon against a fly…but at least it stops the fly."[203] Yablo responds: "Except that it does *not* stop the fly: paradoxes like the Liar are possible the complete absence of self-reference."[204] And Priest attacks Yablo based on finding circularity in his construction, and Sorensen defends Yablo's paradox (and presents other non-circular Yablo-esque paradoxes), and others take up either banner. Yablo makes explicit the sorts of response to the Liar that are inadequate for his paradox in 2006:

> It shows up in the frequently heard claims that one sure way to avoid the semantic paradoxes is to insist with Tarski on a rigid separation of object language from meta-language, and one sure way to avoid the set paradoxes is to insist with Russell on a rigid hierarchy of types.[205]

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[202] Yablo, (1993), p.251, his emphasis

[203] Yablo, (1993), p.251

[204] Yablo, (1993), p.251

[205] Yablo, (2006), p.166

Yablo's paradox is designed to show the inadequacies of hierarchical approaches like that of Russell and Tarski. Yablo's 2006 paper gets explicit, formulating a Yablo-style paradox within set theory.[206]

In short, Yablo sees these responses as inadequate *because of problems like Yablo's Omega-Liar paradox*.

And that is where a strange thing happens once we put together Klement's comments with the arguments presented in this dissertation. Based on issues of syntax and the need to solve the ordering problem, I have argued that the only way to formulate the paradox is with unrestricted comprehension of semantic objects. In other words, the paradox is formable on a view on which any successful expression corresponds to a semantic object that is the meaning of that expression. Much as Russell attempted to use type theory to respond to his paradox, Klement suggests we can use a ramified theory to respond to paradoxes of propositions.

If semantic objects are ramified, then we cannot form Yablo's paradox. Recall that, "Propositions that are about themselves or about other propositions would be of a higher order."[207] Now think about the Yablo sentences involved in the derivation above: $(\forall k>1)[(\exists p)(f(k)=p) .\Rightarrow. \neg T(p)]$. This sentence, SO1, is about other propositions, namely all those that the ordering function *f* assigns to natural numbers. While Klement's note is not a formal definition of how the ramification would go, we do know that for a proposition q and a proposition r, if q is about r, then q needs to be of a higher order than r. But recall that one of the necessary pieces of the Yablo paradox from Chapter 2: each

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[206] Yablo, (2006), p.173. His account is patterned after Goldstein, (1994).

[207] Klement, (2003), p.318

Yablo sentence^d must claim an infinite number of other propositions false. In this case, a particular Yablo sentence^d needs to be of a higher order than *each* of the infinite number of sentences. This in itself is not a problem: we might have, for example, a sentence that asserts all claims of the form: $x^2=x*x$ are true. This would range over an infinite number of propositions, yet they all (depending on your philosophy of mathematics) are of the same level. For the Yablo sentences^d, the case is quite the contrary. Not only does the sentence need to be of a higher order than all it refers to, but each of those referents refer to an infinite number of sentences. And each of *those* refers to an infinite number such that they need, on this view, a higher order. This continues *ad infinitum*. This means that each Yablo sentence, on this theory, would need to be of an infinitely large order. Would this mean that some Yablo sentences need to have to higher orders of infinity as their order? Maybe, but it easy to see how this flies in the face of a natural paradox and would require orders that never stopped increasing, orders that were non-wellfounded. The ordering attempt is a process which would not stop, and as such, the Yablo sentences are not expressible within such a ramified theory of semantic objects.

The rub of this is that if Yablo wished to show how his paradox shows these sorts of responses inadequate, then he needs to show, against what I have argued here, that it is formable (above merely showing its true syntactic form) outside of the realm of semantic objects. As it is, those objects generate his paradox, but also all the other common paradoxes. Common responses to them also, given that the Yablo is only capturable by semantic objects, trump Yablo's omega-liar. If the semantic argument laid out here put his paradox in the same camp as other versions of the Liar, then he has failed to vindicate self-reference. It seems the cannonball *does* stop the fly. Does this mean that Priest and others are right that there is self-reference buried somewhere in the Yablo sequence? I do not think this is proof of anything like that, though there may be. But what it does show is that whether there is self-reference or not, Yablo has not vindicated self-reference in presenting his paradox.

As the Omega-Liar results from naïve acceptance of semantic objects, it does, at least, present an interesting case against such a propositionalism. His paradox is formable under an unrestricted theory of semantic objects, yet because that is the *only* way it is formable, his larger project seems put off balance as normal responses to propositions include just the sort of hierarchies Yablo wished his theory would skirt. Needing semantic objects (with their corresponding restrictions) turns Yablo's paradox into merely another variant of the Liar, and as such, no conclusion can be drawn from the Yablo distinct from the Liar. But these are restrictions on our meanings, and on many views our very thoughts. Let's briefly consider whether these sorts of restrictions might not be undesirable.

6.6 – The Borders of Thought

It is worth noting here at the end that restrictions on propositions are not accepted without reservation. Semantic objects are intensional, and those driven to accept them in philosophy of language are likely to accept them as the source of mental intentionality.[208] Chisolm, for example, worries that any restrictions on intentional objects will lead to fineness of grain problems that are hard to swallow in philosophy of language and

[208] This need not be the case. One could have a view on which each got intentionality from a different source, but this strikes me as more cumbersome rather than less.

philosophy of mind.[209] He talks about partially extensional entities as restrictions since for Russellian propositions, there is no way to account for different ways of thinking about the referents of singular terms. Those referents are already in the propositions as constituents.

But worries of fineness of grain aren't the only issue with restrictions. Indeed worries go back as far as Brentano's thesis that intentionality is the mark of the mental. He was the first to suggest that problematic distinction between mental phenomena and physical phenomena could be made by reference to aboutness. This intentionality of the mental was a characteristic unique to it. My idea of the cup of coffee somehow *refers to* or *means* or *represents* the cup of coffee. When I hope that the sun will come out, my hoping is directed at the sun. No physical phenomenon is like this. My cup of coffee is just that: a cup of coffee. It does not represent anything further, at least not in itself. As Putnam's famous example goes, the ant who, crawling in the sand, manages to trace the outline of Winston Churchill does not thereby make a *representation* of Churchill. There is nothing inherent in the shape of the ant tracks or the cup of coffee in virtue of which they represent other things in the world. Now, certainly I can *take* them to represent other things. My cup of coffee may represent morning to me (though, alas, it is more likely to represent the evening), or it may represent a new beginning, or the friend who gave it to me. But all of these are *derivative* on the associations I have with the cup of coffee, and similarly with our interpretation of the ant tracks in the sand. With these genuine intentional things (with thought), there can be no restriction.

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[209] Chisolm, (1989)

Meinong, Brentano's student, was infamous for the position that "there are objects of which it is true to say that they are not" and for his principle of the independence of *Sosein* (being so) from *Sein* (being). Meinong's point was that phenomenology must be independent (and prior) to ontology. When we search our minds for places where our thought is blocked, we find none. There ought to be no restrictions on thoughts in our theories of intentionality. If those theories are captured by semantic objects, then Brentano and Meinong would hold that those semantic objects could not be restricted.

And this is an intuitive position. Certainly it seems strange to say there are certain things we cannot think. After all, part of the reason we do not turn to Tarski in an attempt to understand language and thought is that it seems clear that our thoughts and utterances are not typed. There is no static object language that we normally operate in, nor such a meta-language that we move to once we decide to make claims about truth and falsehood.[210] It may be that some features of our natural language are captured by these sorts of structures, but certainly they are representative of our actual thoughts and expressions. After all, these restrictions are formal constructions. They seek to respond to issues like Russell's paradox and Curry and Epimenides' Liar. While this may be true of the ontology of language, it cannot be used to restrict our thoughts and expression in such dramatic fashion. To borrow a phrase from Yablo, when it comes to thought and utterances, we can't be using cannonballs.

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[210] Recall that Tarski's point was that we cannot make good sense of an univocal notion of truth, but only truth-in-a-language.

Yablo himself makes such a point when he is responding to Priest in 2006. Yablo sees his sentences as each perfectly normal. The only reason, he suggests, that one might oppose their reference is because of their semantic properties. "How an empirical description could fail to apply because of an object's semantical properties it is not easy to see."[211] This suggests that while there are going to be restrictions at some level, restricting the *descriptions* (i.e., the content by which an intentional entity refers) is not a reasonable option. There ought to be no restrictions on what we can think.

And there are other options as well. Russell's famous theory of definite descriptions suggested there were ways to understand how we can think about non-existent things (which is required by the lack of restrictions Brentano suggests and the phenomenology of Meinong as including thought of non-existent things). Russell's theory, in essence, suggests that we think quantificationally, and thereby capture, at least in many cases, the seeming thought that, "The present King of France is bald." In these cases of non-existent objects we think of the relevant *properties*. The one and only thing that is a man and is King of France is bald. This claim is false because there is no such thing.[212] He thereby blocked the problem of thinking of non-existent objects without restricting the thinking part. He simply did away with the object, and this is where, presumably, one wants to go with philosophy of language and philosophy of mind. After all, Meinong is infamous partially because people assume he was doing ontology rather than phenomenology, and they think that non-existent things having some existence is ridiculous.

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[211] Yablo, (2006), p.142

[212] See Landini, (201x) for an extended discussion.

One might think the same way about Yablo's paradox, endangering the semantic object propositionalists. If it is to be blocked, it cannot be by some *ad hoc* restriction on our meanings. We need to keep the unrestricted ability to think, and restrictions on semantic objects (if they are to be the contents of thought) are restrictions on thinking, and this is unintuitive. But if those who countenance semantic objects wish to avoid Yablo's paradox, they need to do just that.

And recall that the Yablo sentences have at least an intuitive appeal. We don't cry foul, for example, when the math teacher explains to their students that every one of an infinite number of statements asserting that two even numbers can add up to an odd number are false. The math teacher's claim strikes us as perfectly legitimate. Certainly this is part of what Brentano sought to capture with his thesis of unrestricted thought. If Brentano is right and we cannot restrict semantic objects, then the paradox is unsolvable for any such theory, and those who countenance semantic objects are left with no recourse. The proper conclusion is to reject the existence of semantic objects.

6.7 – Conclusion

In this chapter, I have argued that Yablo's paradox does not get where it wants to go. Building upon earlier work, I argued that an unrestricted theory of semantic objects was the only way to generate Yablo's paradox given the issues with ordering and syntax turned up by our investigations into the paradox. Given common ways of restricting these semantic objects, however, it seems as if Yablo cannot escape the hierarchies he set out to avoid. And perhaps the language hierarchies of Tarski or the context hierarchies of Burge will do the trick against all Liar-like paradoxes (even Yablo's Omega-Liar) given the arguments here tying Yablo's paradox to semantic objects. But since such restrictions are problematic as they restrict thought, the semantic object theorist is left with little option, and ought to abandon acceptance of such semantic objects.

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