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Tracking A Nonlinear System Via A Communication Network

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نتيجة الحكم على أطروحة ماجستير

بناءً على موافقة عمادة الدراسات العليا بالجامعة الإسلامية بغزة على تشكيل لجنة الحكم على أطروحة الباحث/ مهندس حسن خميس شحادة لنيل درجة الماجستير في كلية الهندسة قسم/ الهندسة الكهربائية- أنظمة التحكم وموضوعها:

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Tracking a Nonlinear System Via a Communication Network

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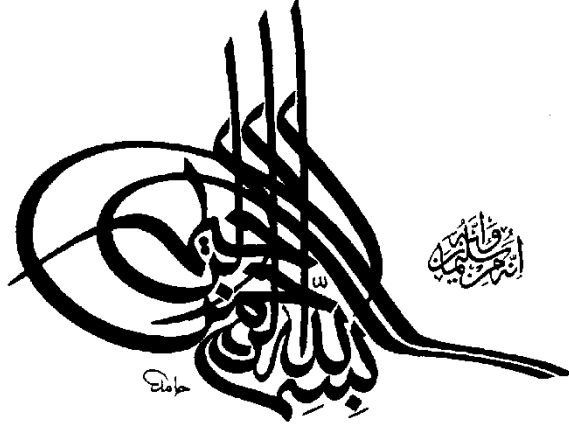
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واللجنة إذ تمنحه هذه الدرجة فإنها توصيه بتقوى الله ولزوم طاعته وأن يسخر علمه في خدمة دينه ووطنه.

والله ولي التوفيق ،،،

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﴿وَقُلْ رَبِّ زِدْنِي عِلْمًا﴾

[طه: 114]

DEDICATION

*To the teacher of the world, leader of the nation and mercy of Allah to mankind,
Prophet Muhammad peace be upon him*

To my lovely parents who are honor by this moment

To my beloved Brothers and Sisters

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I address my sincere gratitude to Allah as whenever I faced any problem I used to pray to God to help me and He always was there protecting and saving me. Then, I would like to express my deep gratitude to my advisor Prof. Dr. Mohammed Hussein, who spared much time in supporting me with all concern. I also want to thank Dr. Hatem Elaydi and Dr. Anwar Mousa for their valuable role and comments throughout my research work and for agreeing to take part in my defense.

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ABSTRACT

In a Networked Control System (NCS), sensors, controller and actuator are connected to the communication network as nodes instead of hardwiring them with point-to-point connection. This system setup has the advantages of low cost, flexibility, less wiring, and the system performance can be remotely controlled from a long distance. Besides a lot of advantages, a NCS has also shortcomings, induced by the network components, like time delays and data packet loss. Network delays degrade the NCS control performance and destabilize the system.

In this thesis, a new tracking control methodology for a special case of nonlinear control systems with time delays induced by inserting the communication network into the forward control loop is presented, which is generated from the use of gain scheduling technique. In this methodology, the design of a gain scheduled tracking controller is based on the use of output feedback control, which also includes designing an estimator for compensating the induced time delays.

This methodology is applied to an application of nonlinear systems, that is magnetic ball levitation CE152 and the results showed that the designed controller achieved the desired performance as well as treating the effect of time delays induced by the network.

ملخص الدراسة:

في نظام التحكم الشبكي، تكون عناصر نظام التحكم وهي المجسات والمتحكم والمشغل متصلة مع بعضها البعض من خلال شبكة اتصال عوضاً عن توصيلها مباشرة من خلال أسلاك. يمنح هذا الشكل لنظام التحكم مزايا عديدة منها تقليل التكلفة والمرونة وتقليل أسلاك التوصيل بالإضافة إلى أنه يضيف خاصية جديدة وهي إمكانية مراقبة أداء النظام والتحكم به عن بعد.

على الرغم من هذه المزايا الجذابة، يعاني نظام التحكم الشبكي من أوجه قصور عديدة ناتجة عن مكونات الشبكة، أهمها التأخر الزمني لوصول البيانات وفقدان العديد من حزم البيانات. علماً بأن التأخر الزمني في الشبكة يضعف أداء النظام ويقوده أحياناً إلى حالة عدم الاستقرار.

هذه الأطروحة تعرض طريقة تحكم جديدة تولدت من خلال استخدام تقنية الكسب المجدولة والتي تتضمن تتبع حالة خاصة من أنظمة التحكم اللاخطية التي تعاني من تأخير زمني ناتج عن إدراج شبكة اتصال في حلقة التحكم الأمامية لنظام التحكم.

في هذه الطريقة، عملية تصميم المتحكم التتبعي ذو الكسب المجدول تعتمد على استخدام طريقة التحكم بالتغذية الراجعة من مخرج النظام وتتضمن عملية التصميم أيضاً تصميم المقدر الذي يستخدم لتعويض التأخر الزمني في وصول البيانات.

طبقت هذه الطريقة على مثال من الأنظمة اللاخطية هو جهاز التعليق المغناطيسي وأظهرت النتائج تحقيق المتحكم المصمم للأداء المطلوب مع معالجة تأثير التأخر الزمني الناتج عن استخدام الشبكة.

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LIST OF ABBREVIATIONS

ADC	Analog to Digital Converter
CAN	Controller Area Network
DAC	Digital to Analog Converter
LAN	Local Area Network
LQG	Linear Quadratic Gaussian
MAC	Medium Accesses Control
Maglev	Magnetic Levitation
Maglev CE152	Magnetic Ball Levitation CE152
NDC	Networked Delay Compensator
NCS	Networked Control System
ZOH	Zero Order Hold

CHAPTER 1

1. INTRODUCTION

1.1 Introduction of Networked Control Systems

The point-to-point architecture is the traditional communication architecture for control systems, that is, sensors and/or actuators are connected to controllers via wires. In recent years, due to the expansion of physical setups and functionality, a traditional point-to-point architecture is no longer able to meet new requirements, such as modularity, integrated diagnostics, quick and easy maintenance, and low cost. Such requirements are particularly demanding in the control of complex control systems and remote control systems [1].

Meanwhile, data networking technologies provide several benefits on linking data points like computers. Networks enable remote data transfers and data exchanges among users, reduce the complexity in wiring connections and the costs of medias, and provide ease in maintenance[2].

The new requirements can be satisfied by replacing the traditional point-to-point architecture with those based on common-bus network which is called a Networked Control System (NCS).

In general, NCSs are a type of distributed control systems where sensors, actuators, and controllers are interconnected through a communication network as shown in fig(1.1). Sensors measure states of the plant and transmit these states over the communication network to controllers. The controllers receive these states, and calculate appropriate control actions and send them to actuators over the communication network. Actuators receive control actions and control the plant appropriately. On other meaning, in the NCSs, a feedback control loop is closed through a communication network which is the backbone of the NCS.

There are two general NCS structures can be configured as distributed control systems: Direct structure and hierarchical structure that will be described later.

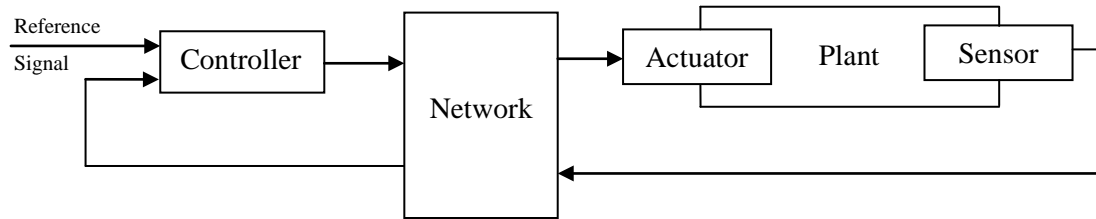


Figure 1.1: A Block Diagram of an NCS.

In the past few years, the technologies on general computer networks have also progressed very rapidly. With, increasing speed, widespread usages, numerous software and applications, and well established infrastructure, these networks have been widely applied in the control of industrial and military applications. These applications include manufacturing plants, automobiles, and aircrafts. Connecting the control system components in these applications, such as sensors, controllers, and actuators, via a network can effectively reduce the complexity of the systems with nominal economical investments. Furthermore, the applications connected through a network can be remotely controlled from a long distance. Traditionally, the networks used in the aforementioned applications are specific industrial networks, such as CAN (Controller Area Networks), and LAN (Local Area Network).

Despite the attractive benefits gained from using data networks in the control purpose, NCS suffers from many problems. One of the major problems of NCS is the delay of data transmission between the units of NCS.

The continuous-time systems with time-delays are infinite dimensional systems. The infinite dimensional continuous-time system can be reformulated to finite dimensional discrete-time system by sampling the continuous-time process.

It is clear that the infinite dimensional systems are much more difficult to deal with than the finite dimensional systems, thus , the discrete-time description of the NCS model is important in the analysis process of NCS, and can be derived by discretizing the continuous-time systems at sampling instants T .

The following Remarks should be taken into account when selecting the value of sampling time:

- Make the sampling time T small enough to reproduce the open-loop time response enough precisely ($T = t_r/10$) and to avoid aliasing effects (Nyquist frequency π/T larger than closed-loop bandwidth).
- Make the sampling time T small enough to react enough readily to disturbances affecting the system.
- Make the sampling time large enough to avoid fast and expensive control hardware.

As shown in fig.(1.2), network delays in the NCS can be categorized from the direction of data transfers as the sensor-to-controller delay (τ_k^{sc}) and the controller-to-actuator delay (τ_k^{ca}) and both network delays can be longer or shorter than the sampling time T . Network delays are modeled and analyzed in various ways. They can be modeled as a constant delay, independent random delay and delay with known probability distribution governed by *Markov chain* model.

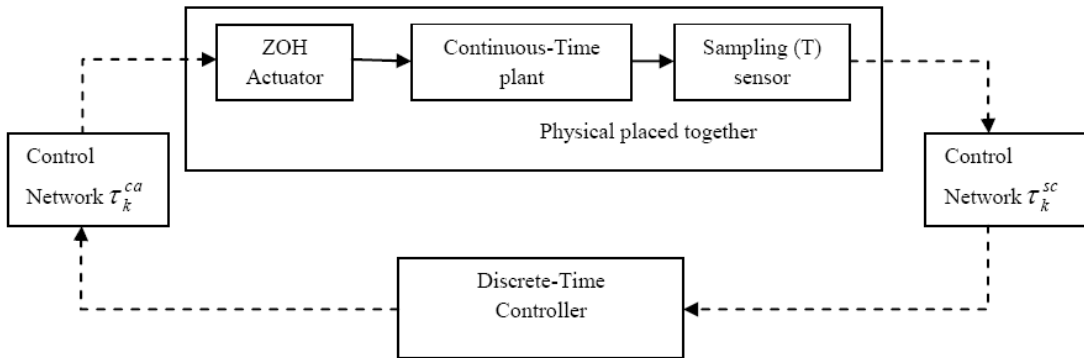


Figure 1.2: The block diagram of network-induced delay

Network delays degrade the NCS control performance and destabilize the system. Therefore, any control system constructed without considering these delays has a low performance and reliability, Thus a great emphasis are made on developing control methodologies to handle the network delay effect in NCS.

1.2 Background

1.2.1 Network Types

Many different network types have been developed for the use in control systems and the delay characteristics on NCS basically depend on the type of a network used. The networks can be divided into two groups: real-time and non-real-time data.

- **Non-real-time:** This network does not have stringent time limits on their delays during data exchange, and in these systems, the concern is over whether the data arrives without error and duplication.
- **Real-time:** This network has strict time limits and the data's value is diminished as the system delay grows larger.

There are two types of real networks according to different medium access control (MAC) protocols which are used for control purpose and described as follows[1]:

1- Cyclic Service (Periodic) Network

In local area network protocols with cyclic service such as SAE token bus and time division multiple access protocol, data is transmitted in a cyclic order with deterministic behaviors. Thus, the delays are periodic and can be simply modeled as a periodic function. In practice, NCS may experience small variations on periodic delays due to several reasons, for examples, the discrepancies in clock generators on a controller and a remote system may result in delay variations.

2- Random Access Network

Carrier sense multiple access protocol is most often used in random access network whose application includes CAN, Ethernet, and Internet. The significant parts of random network delays are the waiting time delays due to queuing and frame collision on the networks. When an NCS operates across networks, several more factors can increase the randomness on network delays such as the queuing time delays at a switch or a router, and the propagation time delays from different network paths. In such network, τ_k^{sc} and τ_k^{ca} , are stochastic processes where stochastic approaches are needed to model the behaviors of both delay types.

1.2.2 NCS Configuration.

In general, there are two major types of remote control systems that utilize communication networks listed as follows[3]:

- **Direct Structure:** The NCS in the direct structure is composed of a controller and a remote system containing a physical plant, sensors and actuators. The controller and the plant are physically located at different locations and are directly linked by a data network in order to perform remote closed-loop control as shown in fig.(1.3).
- **Hierarchical Structure:** The basic hierarchical structure consists of a main controller and remote closed loop systems as depicted in fig.(1.4). Periodically, the main controller computes and sends the reference signal in a frame or a packet via a network to the remote systems. The remote system then processes the reference signal to perform local closed-loop control and returns to the sensor measurement to the main controller for networked closed-loop control.

The use of either the direct structure or the hierarchical structure is based on application requirements and designer's preferences. For example, a robotic manipulator usually requires several motors at the joints of the robot to simultaneously and smoothly rotate together. It may be more convenient and more robust to use an existing robot controller and formulate the networked control problem in the hierarchical structure. On the other hand, a designer may require a networked DC motor speed control system to have a faster control response over the network. The direct structure may be preferred in this case[3].

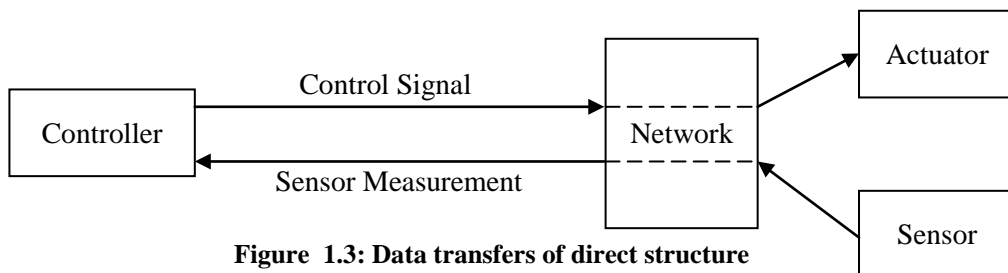


Figure 1.3: Data transfers of direct structure

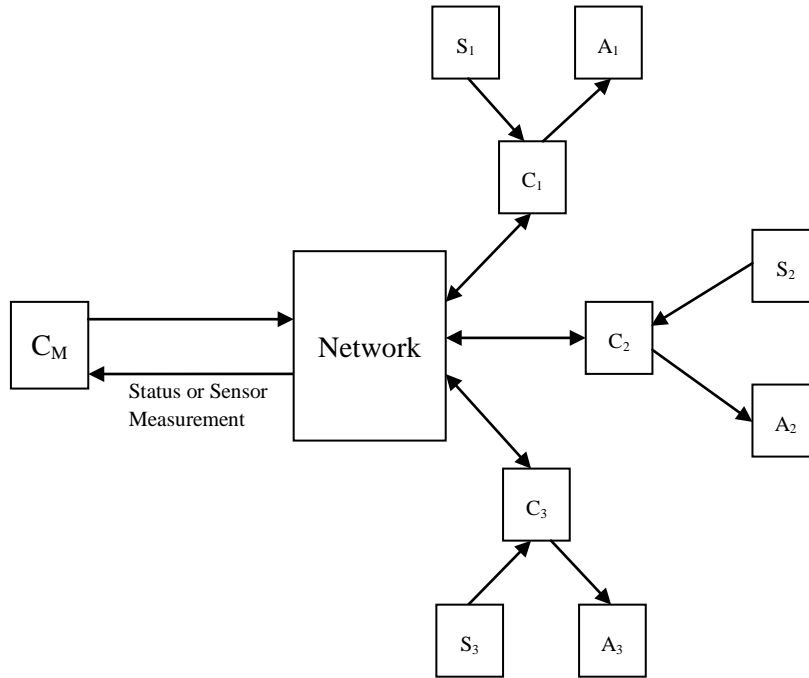


Figure 1.4: Data transfers of hierarchical structure,

Where C_M is the main controller, S_i is the i th local sensor, C_i is the i th local controller, and A_i is the i th local actuator.

1.2.3 Closed Loop Control System

In a closed-loop control system, a sensor monitors the output and feeds the data to a computer, which continuously adjusts the control output (system input) as necessary to minimize the error. Feedback allows the controller to dynamically compensate the disturbances. An ideal feedback control system cancels out all errors, effectively mitigating the effects of any forces that might or might not arise during operation and producing a response in the system that perfectly matches the user's wishes.

In reality, this cannot be achieved due to measurement of errors in the sensors, delays in the controller, and imperfections in the control input. The concept of the feedback loop: the sensed value is subtracted from the desired value to create the error signal, which is handled by the controller to produce the control signal, which can achieve the desired response [4].

1.3 Fundamental Issues with Networked Control Systems

The following two issues are the most challenging problems with NCSs that need to be properly addressed to ensure the stability and performance of the closed-loop systems[5].

1. The first issue is the network-induced delay, including sensor-to-controller delay and controller-to-actuator delay, that happens when data exchange happens among devices connected by the communication network, which will deteriorate the system performance as well as stability. This delay, depending on the network characteristics such as network load, topologies, routing schemes, etc., can be constant, time-varying, or even random.
2. The second issue is the data packet dropouts. In the NCS, data is sent through the network in packets. Due to network characteristics, such as their topologies, the used protocol, etc. therefore, any continuous time signal from the plant is first sampled to be carried over the communication network. Chances are that those packets can be lost during transmission because of uncertainty and noise in communication channels. It may also occur at the destination when out of order delivery takes place. Although most network protocols are equipped with transmission- retry mechanisms, they can only retransmit for a limited time. After this time has expired, the packets are dropped

1.4 Recent Works on Networked Control Systems

Due to network delay concerns, several methodologies have been proposed to treat time delays systems. These methodologies have been formulated based on several types of network behaviors and configurations in conjunction with different ways to treat the delay problems. The basic concepts of some control techniques that have been developed for the control of NCSs are presented here.

1. HALEVI and RAY, proposed a methodology named as the augmented deterministic discrete time model methodology to control a linear plant over a periodic delay network. This methodology is based on discrete-time state

space models. The controller uses j past measurements $z(k) = y(k - i)$, $i = \{1, \dots, j\}$ to calculate the control signal at k . The network delays are handled by augmenting the delays into the full system state model, and the stability for periodic delays is proven based on the eigenvalues of the augmented system state transition matrix[6].

2. LUCK and RAY, developed an queuing methodology which denoted as the deterministic predictor-based delay compensation methodology. This methodology uses an observer to estimate the plant states and a predictor to compute the predictive control based on past output measurements. The control and past output measurements are stored in FIFO queues and shift registers, and these are located before and after the controller in the control loop. the past measurements are used to estimate the plant state at $k - \theta + 1$, where θ is the size of the shift register between the sensor and the observer. Next, using the previous estimate, the plant state is predicted at $k + \mu$, where μ is the size of the register after the controller. The predictive control signal $u(k + \mu)$ is then calculated and stored in the shift register. Since both the observer and the predictor are model-based, the performance of the system highly depends on model accuracy[7].
3. NILSSON, developed an optimal stochastic control methodology for NCS. To control a system over a random delay network. The effects of network delays are treated as a Linear Quadratic Gaussian (LQG) problem. The controller and actuator used in this approach is event-driven while the sensor is time-driven. In this work the delay was assumed to be less than one sample time and the information of all the past delays are available. In this approach, two stochastic processes were incorporated into the system state-space equations and the goal is to minimize a cost function of the plant states and inputs. The stability of the network-based system for both independent delays and delays modeled by a *Markov chain* is discussed using stochastic stability analysis[8].

4. WALSH, BELDIMAN, YE, and BUSHNELL, developed the perturbation methodology for NCS. This methodology considered the difference between the current plant output values and the most recently transmitted plant output values as a perturbation to the system and searches for limits to this error. The stability is proven using the Lyapunov approach on the dynamics of the error. Several assumptions are made, including error free communications, fast sampling and noiseless observations, but the plant and the controller may be nonlinear and time variant[9].
5. ALMUTAIRI ET AL, proposed the fuzzy logic modulation methodology for an NCS with a linear plant and a modulated PI controller to compensate the network delay effects. In this methodology, the PI controller gains are externally updated at the controller output with respect to the system output error caused by network delays. Thus, the PI controller needs not to be redesigned, modified, or interrupted for use on a network environment. The fuzzy logic modulation methodology can be implemented in a unit called the fuzzy logic modulator, which modifies the control $u_c(t)$ by

$$u_c(t) = \beta \left(u_{pi}(t) + K_p e(t) + K_i \int_{t_0}^t e(\gamma) d\gamma \right)$$

The multiplicative factor β is used to externally adjust the controller gains at the output without interrupting the original PI controller. The value of β is selected from two fuzzy rules based on the network delay effects as follows:

$$\text{If } e(t) \text{ is SMALL, then } \beta = \beta_1,$$

$$\text{If } e(t) \text{ is LARGE, then } \beta = \beta_2,$$

Where $0 < \beta_1 < \beta_2 < 1$ [11].

6. HONG, developed the sampling time scheduling methodology to appropriately select a sampling period for an NCS. The fundamental concept of this approach is to appropriately select a long enough sampling period for a discrete-time network-based system such that communication delays do not affect the control performance, and the system remains stable. In this case, the control delay in a discrete-time control loop must be assumed to be less than

the sampling period T of the loop. The control loop consists of a time-driven sensor and controller, and an event driven actuator[12].

7. H. Elaydi and W. Sakallah, modeled the time delays using different approaches such as Pad'e approximation and Smith Predictor in continuous system and modified z-transform in discrete systems. The delays were assumed to be constant and known. The delays in the system were lumped in the plant model. They showed a design of stable and optimal controller for time-delay systems using algebraic Riccati equation solutions and PID control[13].

1.5 Thesis Assumption

In this thesis, the following assumptions are needed:

1. The controlled process should be a special case of 2nd order nonlinear system.

The linearization of the system about an equilibrium point takes the form:

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \end{pmatrix} u \\ y &= (c_1 \ 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{aligned} \tag{1.1}$$

2. The actuator is time driven.

The actuator implements zero-order hold (ZOH) reconstructing, which holds the last control signal until the next sample time arrived.

3. The sensor is time-driven.

The states of the plant are sampled periodically.

4. The controller is time-driven.

The controller calculates the new control signal at discrete time instants with a constant sampling time.

5. The proposed type for the network that used for control purpose is cyclic service network.

Data is transmitted in a cyclic order with deterministic behaviors. Thus, the delays are periodic and can be simply modeled as a periodic function.

6. Network transmissions are error-free.

7. The communication network inserted into the forward control loop of the control system.

8. The induced time delay τ_k^{ca} in the forward control loop is random, but in the same time bounded ($\tau_k^{ca} < T$).

1.6 Statement Of The Problem

A general block diagram of the networked control system proposed in this thesis is shown in fig.(1.5).The network distributed control system consists of a remote nonlinear plant, a controller designed for that nonlinear plant, and a communication network that connects between the controller and the plant.

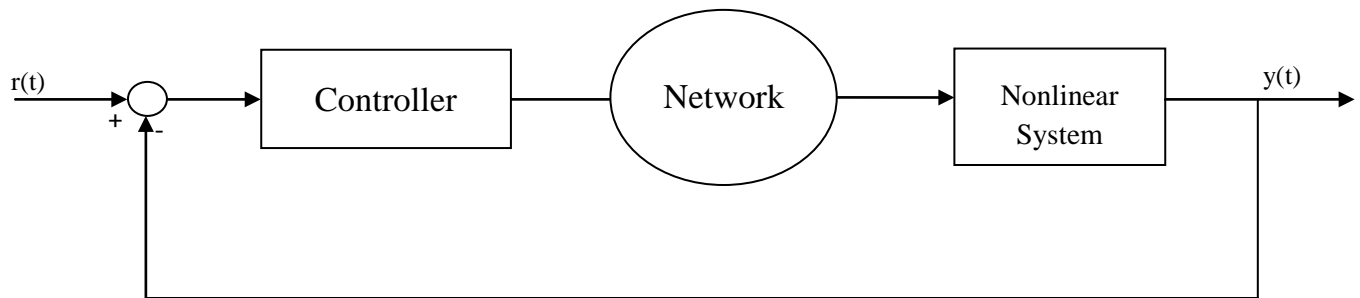


Figure 1.5: The block diagram of the networked control system

The insertion of the communication network in the feedback control system that contains a nonlinear system, makes the design of the tracking controller complex. During the designing process for the controller, two major problems will be faced and need to be taken into account: Nonlinearity of the system and the time delays induced in the forward control loop by the network.

The tracking controller for nonlinear system without considering the time delays can be built by using some tools such as: gain scheduling technique, sliding mode, feedback linearization, back stepping and adaptive control. In this thesis, the problem of nonlinearity will be solved by using gain scheduling technique.

As stated previously, the control system constructed without considering the time delays have a low performance and reliability. Therefore, it is necessary to extend the designed controller for the nonlinear system to compensate the time delays over treating nonlinearity of the controlled system.

1.7 Thesis Objectives

The main objective of this thesis is to propose and employ a new methodology to solve the tracking control problem for nonlinear control systems with time delays induced by introducing a communication network in the forward control loop for the control system. The evaluated technique will be applied on an application of a special 2nd order nonlinear systems such as magnetic ball levitation CE152.

1.8 Research Methodology

In order to achieve these objectives, the following procedure will be carried out:

1. Designing a continuous output feedback gain scheduled controller for the special case of nonlinear system without time delay consideration.
2. The continuous system is sampled, and a digital output feedback gain scheduled controller for the special nonlinear system with no time delay is also designed.
3. The communication network in the forward channel is taken into account, the discrete time model of the NCS is established, and the extended gain scheduled output feedback controllers is designed for the discrete-time networked control system.
4. Simulating each type of an output feedback gain scheduling designed controller for an application such as magnetic ball levitation CE152 and checking the performance of each controller.

1.9 Contribution Of The Thesis

This thesis presents a methodology for designing a gain scheduled controller for a special case of nonlinear systems to solve the tracking problem via a communication network introduced into the forward control loop of the control system, the tracking task is achieved by designing an extended digital output feedback gain scheduled controller which supports for compensating the induced time delays over treating nonlinearity of the system. The evaluated technique is applied on an application which is magnetic ball levitation CE152.

1.10 Thesis Outline

The contents of the thesis are as follows:

Chapter 2 provides a basic idea of the linearization and the gain scheduling techniques that used for a nonlinear control system, and presents the design procedures of an output feedback gain scheduled tracking controller for a special case of 2nd order nonlinear system.

Chapter 3 shows how to replace the continuous time model of the plant by a discrete time model, and also shows how to derive the exact discrete time model for a special case of 2nd order system, this model is used to design a digital output feedback gain scheduled controller.

Chapter 4 presents how to obtain a discrete time model for a plant with time delays at the input that induced by inserting the forward channel in the control loop of the control system, and also provides strategies for compensating the induced time delay by the forward channel of the network.

Chapter 5 shows how to obtain an approximated model for the magnetic ball levitation CE152 apparatus, and shows how to implement a simulation model of this apparatus on Matab/Simulink.

Chapter 6 presents simulation results that performed by applying the different programmed controllers to a simulation model of the magnetic ball levitation CE152 on Matalb/Simulink program using MATLAB, at the end of this chapter a discussion of these results is presented.

CHAPTER 2

2. NONLINEAR FEEDBACK CONTROL

2.1 Introduction

There are several practical tools available for nonlinear control design, including linearization, gain scheduling, feedback linearization, sliding mode control, Lyapunov redesign, backstepping, passivity-based control, and high gain observers. It is also unlikely that the whole design of a nonlinear feedback controller can be based on one particular tool. The engineer will need to employ the tool that is appropriate for the problem in hand. The applied application used in this thesis uses gain scheduling control to overcome the problem of nonlinearity because it is practical enough for the applied application.

There are many control tasks that require the use of feedback. Depending on the design goals, there are several formulations of the control problem. The tasks of regulation, tracking, and disturbance rejection or attenuation lead to a number of control problems. In each problem, we may have a state feedback version where all state variables can be measured or an output feedback version where only an output vector, whose dimension is typically less than the dimension of the state, can be measured[13].

2.2 Regulation Of Nonlinear System

We start by the regulation problems for two reasons. First, many control problems are regulation problems. Second, the tracking problems and regulation problems are related, so, the tracking problem reduces to a regulation problem. The regulation problem is the problem of remaining the system close to the origin equilibrium point.

Consider the nonlinear system,

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x, u) \end{aligned} \tag{2.1}$$

The state feedback regulation problem for system (2.1) is the problem of designing a feedback control input

$$u = \gamma(x) \quad (2.2)$$

Such that the origin $x = 0$ is a uniformly asymptotically stable equilibrium point of the closed-loop system,

$$\dot{x} = f(x, \gamma(x)) \quad (2.3)$$

The output feedback regulation problem for the system (2.1) is the problem of designing a dynamic output feedback controller,

$$\begin{aligned} u &= \gamma(y, z) \\ \dot{z} &= g(y, z) \end{aligned} \quad (2.4)$$

Such that the origin of the closed loop system ($x = 0, z = 0$) is a uniformly asymptotically stable equilibrium point.

2.3 Tracking Of A Nonlinear System

The tracking problem is the problem of regulating the system to any of its equilibrium states, not just the zero state. In a tracking problem, a reference input is defined, and the output of the system is required to be equal (or close to) the value of reference input.

Since, the regulation problem and tracking problem are related, then the later can be reduced to a regulation problem by shifting an arbitrary point x_{ss} to the origin. At the same time, the feedforward value of the input u_{ss} is needed to maintain equilibrium at x_{ss} . Since x_{ss} is an equilibrium point, we have

$$0 = f(x_{ss}, u_{ss}), \quad \forall t \geq 0 \quad (2.5)$$

To shift the desired equilibrium point to the origin, change of variables is needed and can be written as follows:

$$\begin{aligned} x_\delta &= x - x_{ss} \\ u_\delta &= u - u_{ss} \end{aligned} \quad (2.2)$$

The new coordinates of the nonlinear state equations become,

$$\begin{aligned}\dot{x}_\delta &= f_\delta(x_\delta, u_\delta) \\ y_\delta &= h_\delta(x_\delta, u_\delta)\end{aligned}\tag{2.7}$$

Where

$$\begin{aligned}\dot{x}_\delta &= f(x_{ss} + x_\delta, u_{ss} + u_\delta) = f_\delta(x_\delta, u_\delta) \\ y_\delta &= h(x_{ss} + x_\delta, u_{ss} + u_\delta) = h_\delta(x_\delta, u_\delta)\end{aligned}\tag{2.8}$$

At equilibrium point (x_{ss}, u_{ss}) , we have,

$$\begin{aligned}x &= x_{ss} \rightarrow x_\delta = 0 \\ u &= u_{ss} \rightarrow u_\delta = 0\end{aligned}\tag{2.9}$$

Substituting from (2.9) into (2.7), we get:

$$\begin{aligned}f_\delta(0,0) &\equiv 0 \text{ for all } t \geq 0. \\ h_\delta(0,0) &\equiv 0 \text{ for all } t \geq 0.\end{aligned}\tag{2.10}$$

The tracking problem is reduced to solve the regulation problem for the system (2.7) at the desired operating point, where u_δ is designed as a feedback control of x_δ or \hat{x}_δ .

The overall control $u = u_\delta + u_{ss}$ where u_δ is a feedback component and u_{ss} is a feed forward component.

2.3.1 Integral Control

The state feedback regulator for the combined integral action together with a system can then be designed to obtain a tracking system with zero steady state error. The block diagram of the state feedback with integral control is shown in fig(2.1) below:

The addition of integral control achieves asymptotic tracking under all parameter perturbation that do not destroy the stability of the closed loop system, but in the same time increases the system type, and thus, an additional state vector must be added to achieve zero steady state error as well as desired transient response is achieved.

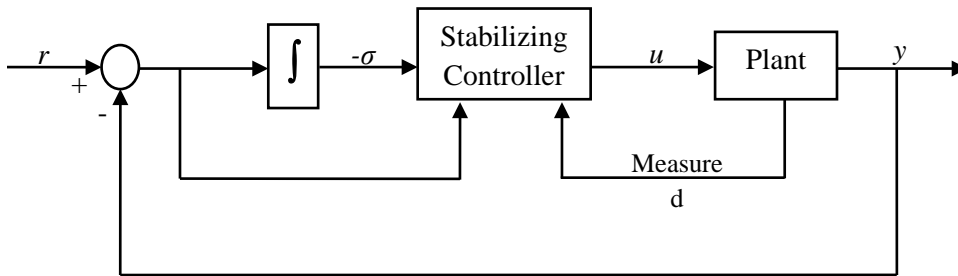


Figure 2.1: Block diagram of integral control.

Consider the nonlinear system,

$$\begin{aligned}\dot{x} &= f(x, u, \omega) \\ y &= h(x, \omega)\end{aligned}\tag{2.11}$$

Where ω is a vector of the unknown constant parameters and disturbance. The tracking task here will be achieved by regulating the system at the desired equilibrium point (x_{ss}, u_{ss}) such that at this point $y = r$.

At equilibrium, the following equations must be satisfied

$$\begin{aligned}0 &= f(x_{ss}, u_{ss}, \omega) \\ r &= h(x_{ss}, \omega)\end{aligned}\tag{2.12}$$

The new state σ generated from integrating the regulation error $e = y - r$ where:

$$\dot{\sigma} = e\tag{2.13}$$

Then the augmented state space model can be written as :

$$\begin{aligned}\dot{x} &= f(x, u, \omega) \\ \dot{\sigma} &= h(x, \omega) - r\end{aligned}\tag{2.14}$$

The tracking task here is reduced to design a regulated feedback controller that regulates the augmented state space model (2.14) at an equilibrium point (x_{ss}, σ_{ss}) where σ_{ss} produces the desired u_{ss} .

The nonlinear feedback controller that regulates the augmented model takes the form,

$$u = \gamma(x, \sigma, e)\tag{2.15}$$

Such that at the equilibrium point (x_{ss}, σ_{ss}) , the controlled output is tracking the reference signal with zero steady state error.

2.4 Linearization Tool

Linearization is a linear approximation of a nonlinear system that is valid in a small region around the operating point. The linearization tool is useful in model analysis and control design tasks, some of these tasks are regulation and tracking.

2.4.1 State Feedback Regulation

Consider the nonlinear system,

$$\dot{x} = f(x, u) \quad (2.16)$$

By using Taylor series, the linearization of a system about $(x = 0, u = 0)$ can be written as:

$$\dot{x} = Ax + Bu \quad (2.17)$$

Where:

$$A = \left. \frac{\partial f}{\partial x}(x, u) \right|_{x=0, u=0}, \quad B = \left. \frac{\partial f}{\partial u}(x, u) \right|_{x=0, u=0} \quad (2.18)$$

After applying the linear state feedback control $u = -Kx$ to the nonlinear system (2.16), the closed loop system becomes:

$$\dot{x} = f(x, -Kx) \quad (2.19)$$

The linearization of the closed loop system (2.19) about the origin $(x = 0)$ is given by:

$$\dot{x} = \left. \frac{\partial f}{\partial x}(x, -Kx) \right|_{x=0} + \left. \frac{\partial f}{\partial u}(x, -Kx) \cdot \frac{\partial u}{\partial x} \right|_{x=0} = (A - BK)x \quad (2.20)$$

The origin of the closed loop system is asymptotically stable equilibrium point if and only if the closed loop matrix $(A - BK)$ is Hurwitz, thus, the state feedback regulation problem of the nonlinear system is reduced to a problem of designing a linear state feedback controller $u = -Kx$ to yield the required closed loop poles values that depend on desired transient response.

2.4.2 Output Feedback Regulation

A problem in using state feedback controller comes about when it is difficult to measure all of the state variables of the plant. Furthermore, it is too expensive to measure all the states of many applications and send them to controller such as send them through a network. If the state variables are not available because of the system configuration or cost, it is possible to determine the estimated states of a system from available measurements and a model by using an observer. For an output feedback controller, a state feedback applied from estimated state, rather than actual states.

For the output feedback regulation problem of the nonlinear system (2.1), the linearization of the system about of the origin results in the standard linear form,

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}\tag{2.21}$$

Where A and B are defined previously, and

$$C = \left. \frac{\partial h}{\partial x}(x) \right|_{x=0}\tag{2.22}$$

The output feedback regulation of a nonlinear system reduces to design a linear output feedback controller. This controller consists of designing state feedback controller and an observer to estimate the states of the plant that used for the purpose of state feedback. The linear output feedback controller called observer-based regulator and it can be written as:

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + bu - H(y - \hat{y}) \\ u &= -K\hat{x}\end{aligned}\tag{2.23}$$

To regulate the nonlinear system (2.1), the state feedback gains vector K is designed such that the closed loop matrix $(A - BK)$ is Hurwitz, while the observer gains vector H is also designed separately such that the closed loop matrix $(A - HC)$ is Hurwitz.

2.4.3 State Feedback Tracking

The linear form of the state feedback control input (2.15) is given by

$$u = -K_x x - K_\sigma \sigma - K_e e\tag{2.24}$$

When the linear control input (2.24) is applied to system (2.1), we get the closed loop system as,

$$\begin{aligned}\dot{x} &= f\left(x, -K_x x - K_\sigma \sigma - K_e (h(x, \omega) - r), \omega\right) \\ y &= h(x, \omega)\end{aligned}\tag{2.25}$$

Linearization of system (2.25) about the equilibrium point (x_{ss}, σ_{ss}) yields the linear augmented state space model by,

$$\dot{\xi}_\delta = (\mathcal{A} - \mathcal{BK})\xi_\delta\tag{2.26}$$

Where:

$$\xi_\delta = \begin{pmatrix} x - x_{ss} \\ \sigma - \sigma_{ss} \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \quad \mathcal{C} = (C \quad 0), \quad \mathcal{K} = (K_x \quad K_\sigma) \quad (2.27)$$

$$A = \left. \frac{\partial f}{\partial x}(x, u, \omega) \right|_{x=x_{ss}, u=u_{ss}}, \quad B = \left. \frac{\partial f}{\partial u}(x, u, \omega) \right|_{x=x_{ss}, u=u_{ss}}, \quad C = \left. \frac{\partial h}{\partial u}(x, u, \omega) \right|_{x=x_{ss}, u=u_{ss}}$$

From (2.26), the tracking problem is reduced to design the state feedback gains vector \mathcal{K} such that the closed loop matrix $(A - B\mathcal{K})$ is Hurwitz.

2.4.4 Output Feedback Tracking

The linear form of the output feedback tracking controller to be designed is given by:

$$\begin{aligned} \hat{x} &= A\hat{x} + Bu - HC(x - \hat{x}) \\ u &= -K_x \hat{x} - K_\sigma \sigma \end{aligned} \quad (2.28)$$

When the controller (2.28) is applied to the augmented model (2.14), the result of the closed loop system is:

$$\begin{aligned} \dot{x} &= f(x, -K_x \hat{x} - K_\sigma \sigma) \\ \hat{x} &= A\hat{x} + Bu - HC(x - \hat{x}) \end{aligned} \quad (2.29)$$

Let $e_x = x - \hat{x}$,

After linearizing the closed loop system (2.29) about the equilibrium point, the result becomes in a linear form as,

$$\begin{aligned} \begin{pmatrix} \dot{\xi} \\ \dot{e}_x \end{pmatrix} &= \begin{pmatrix} \mathcal{A} - \mathcal{B}\mathcal{K} & \mathcal{B}\mathcal{K} \\ 0 & A - HC \end{pmatrix} \begin{pmatrix} \xi \\ e_x \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} r \\ y &= (C \quad 0) \begin{pmatrix} \xi \\ e_x \end{pmatrix} \end{aligned} \quad (2.30)$$

Because the closed loop matrix is an upper-triangular, the eigenvalues of the closed loop system are the eigenvalues of the diagonal blocks.

$$eig(A_{CL}) = eig(A - B\mathcal{K}) \cup eig(A - H\mathcal{K}) \quad (2.31)$$

Where:

$$A_{CL} = \begin{pmatrix} A - B\mathcal{K} & \mathcal{B}\mathcal{K} \\ 0 & A - HC \end{pmatrix} \quad (2.32)$$

Eq(2.32) is called the separation principle, which refers to the fact that the designs of state feedback gains and observer gains can be performed separately.

From (2.30), the tracking problem is reduced to design the state feedback gains vector \mathcal{K} such that the closed loop matrix $(A - B\mathcal{K})$ is Hurwitz and also designing the observer gains vector H such that the matrix $(A - HC)$ is Hurwitz in which the estimated vector e_x will decay to zero.

2.5 Gain Scheduling

In control theory, gain scheduling is an approach to control of non-linear systems that uses a family of linear controllers, each of which provides satisfactory control for a different operating point of the system[14]. The limitation of the design via linearization technique is that the linearized model is valid only for a limited range in a neighborhood of an operating point at which the linearization is carried out. To extend the approximate modeling by linearization over a wide range of operating points, gain scheduling technique should be used. For the gain scheduling technique, the nonlinear system is linearized about a set of operating points that parameterized by one or more variables, which called scheduling variables. To achieve the performance requirements for the nonlinear system, we need to design a single linear controller whose parameters are functions of the scheduling variables, such a controller is called a gain scheduled controller.

The procedure for designing gain scheduling controller of the nonlinear systems is developed by the following steps[13]:

1. Linearize the nonlinear system about the family of operating points, parameterized by the scheduling variables.
2. Design a parameterized family of linear controllers to achieve the specified performance for the parameterized family of linear systems at each operating point.
3. Construct a gain scheduled controller such that, at each constant operating point, the controller provides a constant control value yielding zero error, the linearization of the closed-loop nonlinear system at each operating point is the

same as the feedback connection of the parameterized linear system and the corresponding linear controller.

4. Check the performance of the gain scheduled controller for the nonlinear model by simulation.

Now we want to apply the last procedures for designing a gain scheduled tracking controller for a special case of 2nd order nonlinear system which describes many physical systems, especially systems in which the position and velocity are the state variables, for example, the magnetic ball levitation CE152 and the inverted pendulum.

Consider the special case 2nd order nonlinear system described by:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2, u) \\ y &= h(x_1)\end{aligned}\tag{2.33}$$

For the tracking task, the system is regulated to a set of operating points parameterized by a scheduling variables. Define α as a scheduling variable so that when $r = \alpha = \text{constant value}$, the following equations are satisfied,

$$\begin{aligned}0 &= x_{2SS}(\alpha) \\ 0 &= f(x_{1SS}(\alpha), 0, u_{SS}(\alpha)) \\ \alpha &= h(x_{1SS}(\alpha))\end{aligned}\tag{2.34}$$

The linearization of the nonlinear system (2.33) about a family of parameterized operating points $(x_{ss}(\alpha), u_{ss}(\alpha))$ can be written as :

$$\begin{aligned}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -a_2(\alpha) & -a_1(\alpha) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2(\alpha) \end{pmatrix} u \\ y &= (c_1(\alpha) \quad 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\end{aligned}\tag{2.35}$$

After adding the integrator, the parameterized augmented model can be expressed as:

$$\begin{aligned}\dot{\xi} &= \mathcal{A}(\alpha)\xi + \mathcal{B}(\alpha)u \\ y &= \mathcal{C}(\alpha)\xi\end{aligned}\quad (2.36)$$

Where

$$\begin{aligned}\xi &= \begin{pmatrix} x_1 \\ x_2 \\ \sigma \end{pmatrix}, \mathcal{A} = \begin{pmatrix} 0 & 1 & 0 \\ -a_2(\alpha) & -a_1(\alpha) & 0 \\ c_1(\alpha) & 0 & 0 \end{pmatrix} = \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix}, \mathcal{B} = \begin{pmatrix} 0 \\ b_2(\alpha) \\ 0 \end{pmatrix} = \begin{pmatrix} B \\ 0 \end{pmatrix} \\ \mathcal{C} &= (c_1(\alpha) \ 0 \ 0) = (C \ 0)\end{aligned}\quad (2.37)$$

The gain scheduled observer-based integral controller is given by:

$$\begin{aligned}\hat{x} &= A(\alpha)\hat{x} + B(\alpha)u - H_z(\alpha)C(x - \hat{x}) \\ u &= -K_z(\alpha)\hat{x} - K_\sigma(\alpha)\sigma\end{aligned}\quad (2.38)$$

Where

$$K_z = (K_{z1}(\alpha) \ K_{z2}(\alpha)) \text{ and } H_z = (h_{z1}(\alpha) \ h_{z2}(\alpha))^T \quad (2.39)$$

After Substituting from (3.38) for u in (3.36), the closed loop gain scheduling Observer-based integral controller becomes,

$$\begin{aligned}\begin{pmatrix} \dot{\xi} \\ \dot{e}_x \end{pmatrix} &= \begin{pmatrix} \mathcal{A}(\alpha) - \mathcal{B}(\alpha)\mathcal{K}(\alpha) & \mathcal{B}(\alpha)\mathcal{K}(\alpha) \\ 0 & A(\alpha) - H(\alpha)C(\alpha) \end{pmatrix} \begin{pmatrix} \xi \\ e_x \end{pmatrix} + \begin{pmatrix} \mathcal{D} \\ 0 \end{pmatrix} r \\ y &= (\mathcal{C}(\alpha) \ 0) \begin{pmatrix} \xi \\ e_x \end{pmatrix}\end{aligned}\quad (2.40)$$

Where

$$\mathcal{K} = (K_{z1}(\alpha) \ K_{z2}(\alpha) \ K_\sigma(\alpha)) \text{ and } \mathcal{D}^T = (0 \ 0 \ -1) \quad (2.41)$$

Because the closed loop matrix is in upper-triangular form, the separation principle can be applied as stated previously, which means that the state feedback control with integral action and the observer can be designed separately.

For designing the state feedback gain regulators for systems that are not represented in CCF, four steps can be used as follow:

- 1- Calculate the desired closed-loop denominator polynomial of the system $P(s)$.
- 2- Transform the system into controllable canonical form by using linear transformation.
- 3- Calculate the feedback vector of gains K_x for the transformed design model that assigns the poles of the closed-loop system at the roots of $P(s)$.
- 4- Finally, find the original state feedback vector of gains K_z in terms of K_x .

For specifying the desired locations of the closed loop system poles, the method that based on the classical control concept of the dominant second-order poles is used, so the desired polynomial for a second-order prototype system is written as follows:

$$P(S) = s^2 + 2\zeta\omega_n s + \omega_n^2 \quad (2.3)$$

The desired poles are : $s_{1,2} = -\zeta\omega_n \pm \sqrt{1-\zeta^2}$.

Because the system becomes third order system due to adding an integral action, we must place the remaining pole so that their real parts are less than $-4\zeta\omega_n$ [15]. The desired polynomial can be computed as:

$$\begin{aligned} P(s) &= (s + \beta_1\zeta\omega_n)(s^2 + 2\zeta\omega_n s + \omega_n^2) \\ &= s^3 + (\beta_1 + 2)\zeta\omega_n s^2 + (1 + 2\beta_1\zeta^2)\omega_n^2 s + \beta_1\zeta\omega_n^3 \end{aligned} \quad (2.43)$$

Where $\beta_1 > 4$

Let :

$$\begin{aligned} d_1 &= (\beta_1 + 2) \\ d_2 &= (1 + 2\beta_1\zeta^2) \\ d_3 &= \beta_1\zeta\omega_n^3 \end{aligned} \quad (2.44)$$

The desired polynomial becomes,

$$P(S) = s^3 + d_1 s^2 + d_2 s + d_3 \quad (2.45)$$

The controllability matrix is:

$$C_{mz}(\alpha) = \begin{pmatrix} B(\alpha) & A(\alpha)B(\alpha) & A^2(\alpha)B(\alpha) \end{pmatrix} = \begin{pmatrix} 0 & b_2(\alpha) & -a_1(\alpha)b_2(\alpha) \\ (\alpha)b_2(\alpha) & -a_1(\alpha)b_2(\alpha) & (a_1^2(\alpha) - a_2(\alpha))b_2(\alpha) \\ 0 & 0 & c_1(\alpha)b_2(\alpha) \end{pmatrix} \quad (2.4)$$

From the controllability matrix (2.46), the system is controllable. The transfer function of the system (2.36) is:

$$T(S) = \frac{b_2(\alpha)c_1(\alpha)}{s(s^2 + a_1(\alpha)s + a_2(\alpha))} = \frac{b_2(\alpha)c_1(\alpha)}{s^3 + a_1(\alpha)s^2 + a_2(\alpha)s} \quad (2.47)$$

The CCF can be derived from the transfer function above as :

$$\begin{pmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \end{pmatrix} = \begin{pmatrix} -a_1(\alpha) & -a_2(\alpha) & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u \quad (2.5)$$

$$y = \begin{pmatrix} b_2(\alpha)c_1(\alpha) & 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix}$$

The controllability matrix of the transformed model is:

$$C_{mx}(\alpha) = \begin{pmatrix} \bar{B}(\alpha) & \bar{A}(\alpha)\bar{B}(\alpha) & \bar{A}^2(\alpha)\bar{B}(\alpha) \end{pmatrix} = \begin{pmatrix} 1 & -a_1(\alpha) & a_1^2(\alpha) - a_2(\alpha) \\ 0 & 1 & -a_1(\alpha) \\ 0 & 0 & 1 \end{pmatrix} \quad (2.6)$$

Where

$$\bar{A}(\alpha) = \begin{pmatrix} -a_1(\alpha) & -a_2(\alpha) & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } \bar{B}(\alpha) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (2.7)$$

The transformation matrix $T_c(\alpha)$ between the two forms can be evaluated as:

$$T_c(\alpha) = C_{mz}(\alpha)(C_{mx}(\alpha))^{-1} = \begin{pmatrix} 0 & b_2(\alpha) & 0 \\ b_2(\alpha) & 0 & 0 \\ 0 & 0 & c_1(\alpha)b_2(\alpha) \end{pmatrix} \quad (2.51)$$

The control signal of this form can be written as:

$$u = -k_{x_1}(\alpha)\bar{x}_1 - k_{x_2}(\alpha)\bar{x}_2 - k_{x_3}(\alpha)\bar{x}_3 \quad (2.52)$$

Substituting for u in (2.48), The closed loop system becomes,

$$\begin{pmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \end{pmatrix} = \begin{pmatrix} -(a_1(\alpha) + k_{x_1}(\alpha)) & -(a_2(\alpha) + k_{x_2}(\alpha)) & -k_{x_3}(\alpha) \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} \quad (2.53)$$

The characteristic equation of the closed loop system (2.53) is:

$$\det(sI - (\bar{A}(\alpha) - \bar{B}(\alpha)k_{x(\alpha)})) = s^3 + (a_1(\alpha) + k_{x_1}(\alpha))s^2 + (a_2(\alpha) + k_{x_2}(\alpha))s + k_{x_3}(\alpha) \quad (2.54)$$

Then, by matching the coefficients of (2.54) with the desired polynomial (2.43), yields:

$$\begin{aligned} d_1 &= a_1(\alpha) + k_{x_1}(\alpha) \rightarrow k_{x_1}(\alpha) = d_1 - a_1(\alpha) \\ d_2 &= a_2(\alpha) + k_{x_2}(\alpha) \rightarrow k_{x_2}(\alpha) = d_2 - a_2(\alpha) \\ k_{x_3}(\alpha) &= d_3 \end{aligned} \quad (2.55)$$

Substituting for the values of d_1 , d_2 , and d_3 from eq(2.44) into eq(2.55), yield

$$\begin{aligned} k_{x_1}(\alpha) &= (\beta_1 + 2)\zeta\omega_n - a_1(\alpha) \\ k_{x_2}(\alpha) &= (1 + 2\beta_1\zeta^2)\omega_n^2 - a_2(\alpha) \\ k_{x_3}(\alpha) &= \beta_1\zeta\omega_n^3 \end{aligned} \quad (2.56)$$

By using similtary transformation, the feedback gain vector of the original model is:

$$K_z(\alpha) = K_x(\alpha)T_c^{-1}(\alpha)$$

$$K_z(\alpha) = \begin{pmatrix} k_{x1}(\alpha) & k_{x2}(\alpha) & k_{x3}(\alpha) \end{pmatrix} \begin{pmatrix} 0 & b_2(\alpha) & 0 \\ b_2(\alpha) & 0 & 0 \\ 0 & 0 & c_1(\alpha)b_2(\alpha) \end{pmatrix}^{-1} = \begin{pmatrix} \frac{k_{x2}(\alpha)}{b_2(\alpha)} & \frac{k_{x1}(\alpha)}{b_2(\alpha)} & \frac{k_{x3}(\alpha)}{b_2(\alpha)c_1(\alpha)} \end{pmatrix} \quad (2.8)$$

Where

$$K_{z(\alpha)} = \begin{pmatrix} k_{z1}(\alpha) & k_{z2}(\alpha) & k_{\sigma}(\alpha) \end{pmatrix}$$

Substituting for the values of k_{x1} , k_{x2} , and k_{x3} from eq(2.56) into eq(2.57), the scheduled state feedback controller gains are:

$$\begin{aligned} k_{z1}(\alpha) &= \frac{(1+2\beta_1\zeta^2)\omega_n^2 - a_2(\alpha)}{b_2(\alpha)} \\ k_{z2}(\alpha) &= \frac{(\beta_1+2)\zeta\omega_n - a_1(\alpha)}{b_2(\alpha)} \\ k_{\sigma}(\alpha) &= \frac{\beta_1\zeta\omega_n^3}{b_2(\alpha)c_1(\alpha)} \end{aligned} \quad (2.58)$$

For designing an observer for any second order system that is not represented in OCF four steps can be used as follows:

1. Selecting the desired characteristic equation for an observer $P_o(s)$.
2. Transform the system into observable canonical form by using linear transformation.
3. Calculate the observer gains vector H_x for the transformed design model that assigns the poles of the observer at the roots of $P_o(s)$.
4. Finally, Find the original observer vector of gains H_z in terms of H_x .

The transient response of the observer is designed so that it is much quicker than the transient response of the controlled closed loop system in order to yield rapidly state estimate of the system[16]. To make the transient response for the observer faster than for the closed loop system, the natural frequency for the observer is chosen to be

equal to the natural frequency for the controlled closed loop system multiplied by a factor β_0 , The desired characteristic equation for the closed loop observer becomes,

$$P_o(S) = s^2 + 2\beta_0\zeta\omega_n s + \beta_0^2\omega_n^2 \quad (2.59)$$

Let

$$\begin{aligned} d_1 &= 2\beta_0\zeta\omega_n \\ d_2 &= \beta_0^2\omega_n^2 \end{aligned} \quad (2.60)$$

The desired observer characteristic equation becomes,

$$P_o(s) = s^2 + d_1s + d_2 \quad (2.61)$$

The observability matrix for the special case of 2nd order system described in (2.35) is:

$$O_{mz}(\alpha) = \begin{pmatrix} C(\alpha) \\ C(\alpha)A(\alpha) \end{pmatrix} = \begin{pmatrix} c_1(\alpha) & 0 \\ 0 & c_1(\alpha) \end{pmatrix} \quad (2.62)$$

The system is said to be completely observable because the matrix $O_{mz}(\alpha)$ has a full rank. From the transfer function of the system (2.35), the observable canonical form can be written as:

$$\begin{aligned} \begin{pmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{pmatrix} &= \begin{pmatrix} -a_1(\alpha) & 1 \\ -a_2(\alpha) & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2(\alpha)c_1(\alpha) \end{pmatrix} u \\ y &= (1 \ 0) \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} \end{aligned} \quad (2.63)$$

The observability matrix of the transformed model is:

$$O_{mx}(\alpha) = \begin{pmatrix} C(\alpha) \\ C(\alpha)A(\alpha) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -a_1(\alpha) & 1 \end{pmatrix} \quad (2.64)$$

The transformation matrix $T_o(\alpha)$ can be evaluated by:

$$\begin{aligned} T_o(\alpha) &= O_{mz}(\alpha)^{-1} O_{mx}(\alpha) \\ T(\alpha)_o &= \begin{pmatrix} c_1(\alpha) & 0 \\ 0 & c_1(\alpha) \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ -a_1(\alpha) & 1 \end{pmatrix} = \begin{pmatrix} 1/c_1(\alpha) & 0 \\ -a_1(\alpha)/c_1(\alpha) & 1/c_1(\alpha) \end{pmatrix} \end{aligned} \quad (2.65)$$

The observer scheduled gains for the system found in OCF is given by:

$$H_x^T(\alpha) = (h_{x1}(\alpha) \quad h_{x2}(\alpha))$$

The closed loop matrix $(\bar{A}(\alpha) - H_x(\alpha)\bar{C}(\alpha))$ is computed as :

$$(\bar{A}(\alpha) - H_x(\alpha)\bar{C}(\alpha)) = \begin{pmatrix} -(a_1(\alpha) + h_{x1}(\alpha)) & 1 \\ -(a_2(\alpha) + h_{x2}(\alpha)) & 0 \end{pmatrix} \quad (2.66)$$

Then, the characteristic equation of the closed loop observer (2.66) is evaluated by :

$$\begin{aligned} \det(sI - (\bar{A}(\alpha) - H_x(\alpha)\bar{C}(\alpha))) \\ = s^2 + (a_1(\alpha) + h_{x1}(\alpha))s + (a_2(\alpha) + h_{x2}(\alpha)) \end{aligned} \quad (2.67)$$

By matching the coefficients of the characteristic polynomial (2.67) to the desired observer polynomial (2.61), we have :

$$\begin{aligned} d_1(\alpha) &= a_1(\alpha) + h_{x1}(\alpha) \rightarrow h_{x1}(\alpha) = d_1(\alpha) - a_1(\alpha) \\ d_2(\alpha) &= a_2(\alpha) + h_{x2}(\alpha) \rightarrow h_{x2}(\alpha) = d_2(\alpha) - a_2(\alpha) \end{aligned} \quad (2.68)$$

By Substituting the values of d_1 and d_2 from eq(2.60) into (2.68), we get:

$$\begin{aligned} h_{x1}(\alpha) &= 2\beta_0\zeta\omega_n - a_1(\alpha) \\ h_{x2}(\alpha) &= \beta_0^2\omega_n^2 - a_2(\alpha) \end{aligned} \quad (2.69)$$

The observer gains vector of original system is:

$$\begin{aligned} H_z(\alpha) &= T_o(\alpha)H_x(\alpha) = (1/c_1(\alpha)) \\ &\left(\begin{array}{c} 2\beta_0\zeta\omega_n - a_1(\alpha) \\ \beta_0^2\omega_n^2 - a_2(\alpha) - 2a_1(\alpha)\beta_0\zeta\omega_n + a_1^2(\alpha) \end{array} \right) \end{aligned} \quad (2.70)$$

The observer scheduled gains (2.70) can be rewritten as :

$$\begin{aligned} h_{z1}(\alpha) &= \frac{2\beta_0\zeta\omega_n - a_1(\alpha)}{c_1(\alpha)} \\ h_{z2}(\alpha) &= \frac{\beta_0^2\omega_n^2 - a_2(\alpha) - 2a_1(\alpha)\beta_0\zeta\omega_n + a_1^2(\alpha)}{c_1(\alpha)} \end{aligned} \quad (2.71)$$

The procedure for designing a gain scheduled observer-based integral controller can be summarized as follows :

- 1- Attempt the values of damping ratio ζ and the natural frequency ω_n of the desired pole locations for a prototype second order system, by choosing the values of settling time and overshoot for the transient response specification.
- 2- Choose the value of factor β_1 which guarantees that the third pole is less than $-4\zeta\omega_n$ so that the third pole does not affect the behavior of the two dominant poles.
- 3- Choose the value of factor β_0 , so that the desired transient response of the observer is much faster than that of the plant.
- 4- Calculate the gain scheduled state feedback controller gains $(k_{z_1}(\alpha), k_{z_2}(\alpha), k_{z_\sigma}(\alpha))$, as described in (2.58).
- 5- Calculate the gain scheduled observer gains $(h_{z_1}(\alpha), h_{z_2}(\alpha))$ as described in (2.71).

3. DIGITAL CONTROL

3.1 Introduction

A digital control system operates on discrete-time rather than continuous-time signals. A digital computer is used as the controller in such a system. Using computers or microcontrollers to implement digital controllers has substantial advantages. Many of the difficulties with analog implementation can be avoided. For example, there are no problems with accuracy or drift of the components. It is very easy to have sophisticated calculations in the control law, and it is easy to include logic and nonlinear functions. Tables can be used to store data in order to accumulate knowledge about the properties of the system. It is also possible have effective user interfaces[17].

The block diagram of digital control system is shown in fig.(3.1). The digital computer perform the controller function within the system by running an algorithm. The A/D converter converts the measured signal by a sensor, which is a continuous signal, into sampled data form so that it can be processed by the controller . At the controller output, the D/A converter converts the sampled data output of the controller into a form which can be used to drive the plant.

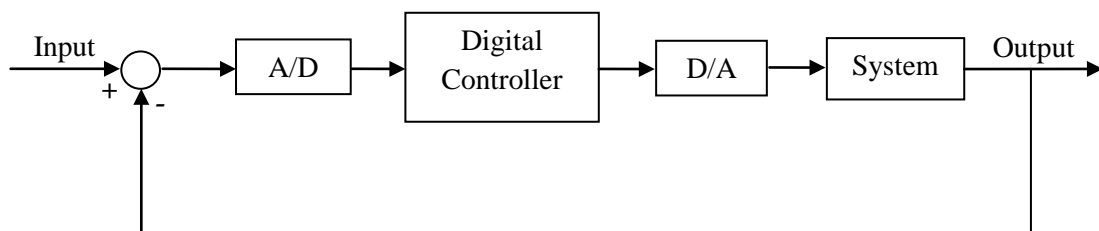


Figure 3.1: Digital control system.

3.2 Sampling and Reconstruction Processes

The operation that transforms continuous time signal into discrete time data is called sampling or discretization. The reverse operation, that transforms the discrete time data into continuous time signal, is called data reconstruction[18].

3.2.1 Sampling Process

A sampling process replaces the continuous time signal by a sampled data signal which is a sequence of values at discrete time points. A sampled data signal can be generated by using A/D converter which can be approximated by an ideal sampler in the sampled data form. As shown in fig.(3.2), an ideal sampler is basically a switch that closes instantaneously at every time interval T . When a continuous signal $y(t)$ is sampled by an ideal sampler at periodic intervals T , the resulting sampled data signal is shown in fig.(3.3).

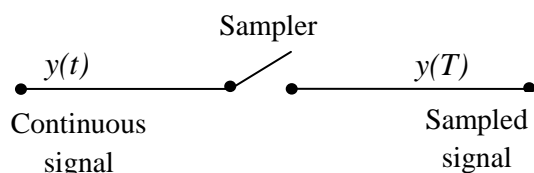


Figure 3.2: Ideal sampler

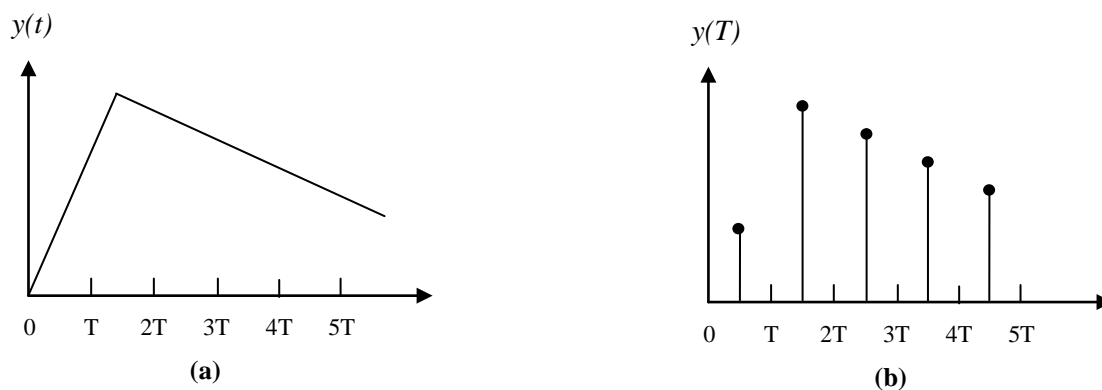


Figure 3.3: (a) Analog signal, (b) Sampled data signal

3.2.2 Reconstruction Process

Many control systems have controlled processes that contain analog devices. These devices are driven by an analog signal input. Thus, when the sampled data appear in a control system, they should be first converted into analog signal before being applied to the controlled process[19].

The process of converting sampled data signal $u(kT)$ to an analog signal $u(t)$ is called reconstruction. A device that performs this reconstruction process is called a digital to analog (D/A) converter. The most common form of reconstruction used in practice is zero order hold (ZOH) reconstruction which also called the actuators. As shown in fig.(3.4), a ZOH circuit reconstructs a continuous signal from a discrete signal by producing a constant output value that is proportional to a given input sample for a fixed amount of time. Then, the output changes to a new constant value that is proportional to the value of the next sample.

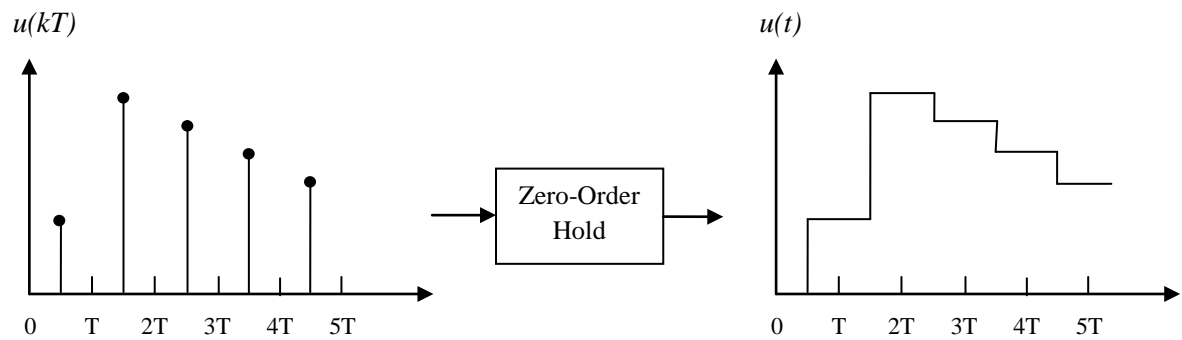


Figure 3.4: Data reconstruction by using ZOH device.

The transfer function of the ZOH device is given by[20]:

$$G_{ho}(s) = \frac{1 - e^{-Ts}}{s}$$

After approximating the A/D and D/A devices by an ideal sampler and ZOH device, the approximated hybrid data control system is shown in fig.(3.5).

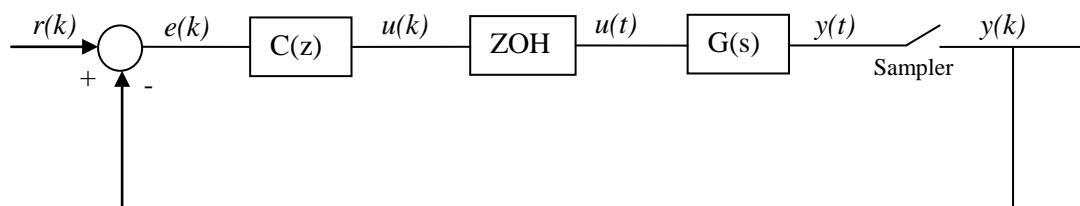


Figure 3.5: Approximated hybrid data control system.

3.3 Discrete Time State Space Model

The analysis and design of a digital control system is complicated by the fact that the plant is a continuous-time system while the compensator is a discrete time system. One way of dealing with this problem is to replace the plant with a discrete-time model that specifies the behavior at the sampling instants. In this way, the entire control system becomes discrete time, and the standard discrete time theory can be used to design the compensator[15].

3.3.1 ZOH Equivalent Model For An Analog System

The discrete data model for an analog plant driven by a piecewise constant input is called the ZOH equivalent model, which is an exact model for the plant at sampling instants. The block diagram of digital control system in which the analog plant has been replaced by its ZOH equivalent is shown in fig.(3.6).

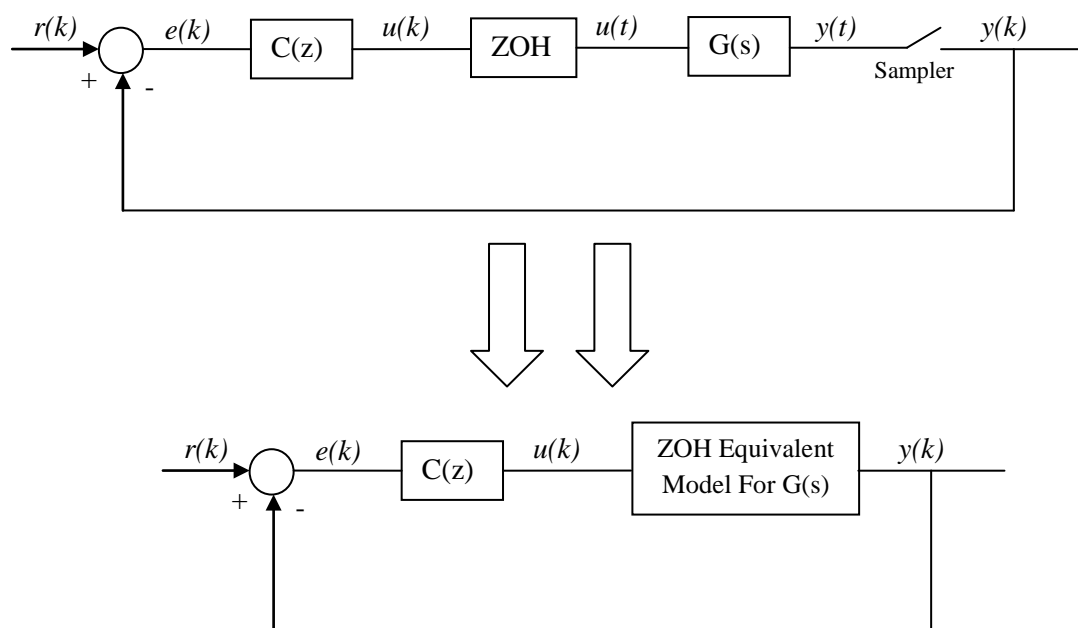


Figure 3.6: Digital control system with ZOH equivalent model for an analog plant.

The resulted discrete state equation when an analog plant is replaced by a discrete data model (ZOH model) is described by the following equations[15]:

$$\begin{aligned} x(k+1) &= \Phi(T)x(k) + \Gamma(T)u(k) \\ y(k) &= Cx(k) \end{aligned} \quad (3.1)$$

Where:

$$\Phi(T) = e^{AT}, \quad \Gamma(T) = \int_0^T e^{As} ds B \quad (3.2)$$

3.3.2 ZOH Equivalent Model For A Special Case 2nd Order System

Consider a special case of 2nd order system is given by:

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \end{pmatrix} u \\ y &= (c_1 \ 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{aligned} \quad (3.3)$$

The characteristic matrix $(sI - A)$ is computed as:

$$(sI - A) = \begin{pmatrix} s & -1 \\ a_2 & s + a_1 \end{pmatrix} \quad (3.4)$$

The transition matrix is evaluated as:

$$\begin{aligned} \Phi(t) &= L^{-1} \left((sI - A)^{-1} \right) = \\ &= \begin{pmatrix} e^{-\lambda t} (\cosh(\psi t) + (\lambda/\psi) \sinh(\psi t)) & \frac{e^{-\lambda t} \sinh(\psi t)}{\psi} \\ \frac{-a_2 e^{-\lambda t} \sinh(\psi t)}{\psi} & e^{-\lambda t} (\cosh(\psi t) - (\lambda/\psi) \sinh(\psi t)) \end{pmatrix} \end{aligned} \quad (3.5)$$

Where

$$\lambda = \frac{a_1}{2}, \quad \psi = \frac{1}{2} \sqrt{a_1^2 - 4a_2}$$

The vector $\Gamma(t)$ can be evaluated as:

$$\begin{aligned} \Gamma(t) &= \int_0^T e^{A\gamma} B \, d\gamma = \int_0^T \begin{pmatrix} b_2 \frac{e^{-\lambda\gamma} \sinh(\psi\gamma)}{\psi} \\ b_2 e^{-\lambda\gamma} (\cosh(\psi\gamma) - (\lambda/\psi) \sinh(\psi\gamma)) \end{pmatrix} d\gamma \\ &= \begin{pmatrix} \frac{b_2}{a_2} [1 - e^{-\lambda t} (\cosh(\psi t) + (\lambda/\psi) \sinh(\psi t))] \\ \frac{b_2}{\psi} [e^{-\lambda t} \sinh(\psi t)] \end{pmatrix} \end{aligned} \quad (3.6)$$

For discretization, by setting $t=kT$, the discrete state space model can be written as:

$$\begin{aligned} \begin{pmatrix} x_1(k+1) \\ x_1(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_1(k) \end{pmatrix} + \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} u(k) \\ y(k) &= (c_1 \ 0) \begin{pmatrix} x_1(k) \\ x_1(k) \end{pmatrix} \end{aligned} \quad (3.7)$$

Where

$$\begin{aligned} \Phi_{11} &= e^{-\lambda T} (\cosh(\psi T) + (\lambda/\psi) \sinh(\psi T)) \\ \Phi_{12} &= \frac{e^{-\lambda T} \sinh(\psi T)}{\psi} \\ \Phi_{21} &= \frac{-a_2 e^{-\lambda T} \sinh(\psi T)}{\psi} \\ \Phi_{22} &= e^{-\lambda T} (\cosh(\psi T) - (\lambda/\psi) \sinh(\psi T)) \end{aligned} \quad (3.8)$$

And

$$\begin{aligned} \Gamma_1 &= \frac{b_2}{a_2} [1 - e^{-\lambda T} (\cosh(\psi) + (\lambda/\psi) \sinh(\psi))] = \frac{b_2}{a_2} [1 - \Phi_{11}] \\ \Gamma_2 &= \frac{b_2}{\psi} [e^{-\lambda T} \sinh(\psi T)] = b_2 \Phi_{12} \end{aligned} \quad (3.9)$$

The discrete transfer function can be calculated as:

$$\begin{aligned} TF &= C (zI - \Phi)^{-1} \Gamma = \\ & \left(\frac{c_1 b_2}{a_2} \right) \frac{[(1 - \cosh(\psi) - (\lambda/\psi) \sinh(\psi))z + (e^{-2\lambda T} - \cosh(\psi) + (\lambda/\psi) \sinh(\psi))]}{z^2 - 2e^{-\lambda T} \cosh(\psi T)z + e^{-2\lambda T}} \end{aligned} \quad (3.10)$$

3.3.3 The ZOH Pole - Mapping Formula

The performance specifications for the continuous time system are achieved by assigning the closed loop poles at the desired values in the s-plane. When the plant is replaced by its ZOH equivalent, the design model becomes discrete time system, and the desired closed loop s-plane poles must be mapped into an equivalent set of the desired z-plane poles. Using ZOH mapping formula, the desired s-plane pole maps into the z-plane by the following equation.

$$z_i = e^{s_i T} \quad (3.5)$$

For designing the state feedback controller, the desired continuous polynomial (2.43) that derived in chapter 2 will be mapped into the discrete equivalent by using eq(3.11) as:

$$\begin{aligned}
P(z) &= (z - z_1)(z - z_2)(z - z_3) \\
&= (z^2 - 2e^{-\zeta\omega_n T} \cos(\omega_d T)z + e^{-2\zeta\omega_n T})(z - e^{-\beta_1\zeta\omega_n T}) \\
&= z^3 + d_1z^2 + d_2z + d_3
\end{aligned} \tag{3.6}$$

Where:

$$\begin{aligned}
d_1 &= -e^{-\zeta\omega_n T} \left(e^{-(m-1)\zeta\omega_n T} + 2 \cos(\omega_d T) \right) \\
d_2 &= e^{-2\zeta\omega_n T} \left(1 + 2e^{-(m-1)\zeta\omega_n T} \cos(\omega_d T) \right) \\
d_3 &= -e^{-(m+2)\zeta\omega_n T}
\end{aligned} \tag{3.13}$$

Also for designing an observer the continuous desired observer polynomial (2.59) will be mapped into discrete equivalent as:

$$P(z) = z^2 + P_1z + P_2 \tag{3.7}$$

Where ,

$$P_1 = -2e^{-\beta_0\zeta\omega_n T} \cos(\beta_0\omega_d T) \text{ and } P_2 = e^{-2\beta_0\zeta\omega_n T} \tag{3.15}$$

3.4 Digital State Feedback Control

The design of discrete control system by state feedback is similar in principle to that of continuous data control system. In chapter 2, we showed how to design a state feedback regulator for a controllable continuous system to keep the state variables of the system at zero values. And also we showed how the regulation problem was reduced to a problem of designing a vector K of feedback gains so that the poles of the closed loop system $(A - BK)$ are assigned at the desired locations in the left half s-plane.

For discrete time system, the design of digital controller based on state feedback will be reduced to a problem of calculating the feedback vector L so that the poles of the closed loop discrete system $(\Phi - \Gamma L)$ are assigned at the desired location in the interior of the unit circle in the z-plane.

3.4.1 Digital State Feedback Tracking Controller

As discussed in chapter 2, the tracking task for the continuous time system is achieved with zero steady state error by introducing an integral action together with a state feedback controller. The same task will be achieved by zero steady state error for the

discrete data control system in which the controlled plant is replaced by its ZOH equivalent by introducing an additional dynamics system as a part of digital compensator. The cascaded combination of the additional dynamics and the plant is shown in fig.(3.7).

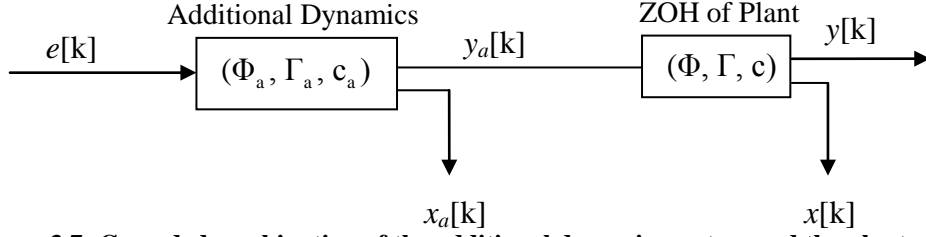


Figure 3.7: Cascaded combination of the additional dynamics system and the plant

The transfer function of the additional dynamics system which must be used to have a tracking system with zero steady state error can be defined as[15]:

$$\frac{z^m}{\delta(z)} \quad (3.16)$$

Where m represents the number of roots of reference input that are not eigenvalues of Φ and

$$\delta(z) = (z - z_1)(z - z_2) \cdots (z - z_m)$$

Where (z_1, z_2, \dots, z_m) are the poles of $d(z)$ that are not eigenvalues of Φ .

The state space model of the additional dynamics is given by[15]:

$$\begin{aligned} x_a(k+1) &= \Phi_a x_a(k) + \Gamma_a u(k) \\ y_a(k) &= (1 \ 0 \ \cdots \ 0)x_a(k) + u(k) \end{aligned} \quad (3.17)$$

Where

$$\Phi_a = \begin{pmatrix} -\delta_1 & 1 & 0 & \cdots & 0 \\ -\delta_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -\delta_{m-1} & 0 & 0 & \cdots & 1 \\ -\delta_m & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_a = \begin{pmatrix} -\delta_1 \\ -\delta_2 \\ \vdots \\ -\delta_{m-1} \\ -\delta_m \end{pmatrix}$$

And $x_a(k)$ is the new state variable generated from introducing the additional dynamics system. Then, the augmented state variables becomes:

$$x_d(k) = \begin{pmatrix} x(k) \\ x_a(k) \end{pmatrix} \quad (3.18)$$

And, the augmented state space model for the cascaded combination of the additional dynamics and the plant can be viewed as:

$$\begin{aligned}x_d(k+1) &= \Phi_d x_d(k) + \Gamma_d u(k) \\ y(k) &= C_d x_d(k)\end{aligned}\quad (3.19)$$

Where:

$$\begin{aligned}\Phi_d &= \begin{pmatrix} \Phi & 0 \\ \Gamma_a C & \Phi_a \end{pmatrix}, \quad \Gamma_d = \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} \\ C_d &= (C \quad 0)\end{aligned}\quad (3.20)$$

For example, the transfer function of the additional dynamics system which must be implemented to track a unit step or ramp input with zero steady state error can be derived from eq(3.16) as:

$$R(z) = \frac{z}{z-1} \quad (3.21)$$

The state space matrices Φ_a and Γ_a for dynamics system (3.17) can be found as:

$$\Phi_a = 1 \text{ and } \Gamma_a = 1 \quad (3.22)$$

Substituting for Φ_a and Γ_a from eq(3.22) into eq(3.19), The state space representation for the augmented model becomes:

$$\begin{aligned}\begin{pmatrix} x(k+1) \\ x_a(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi & 0 \\ C & 1 \end{pmatrix} \begin{pmatrix} x(k) \\ x_a(k) \end{pmatrix} + \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} u(k) \\ y &= (C \quad 0) \begin{pmatrix} x(k) \\ x_a(k) \end{pmatrix}\end{aligned}\quad (3.23)$$

The tracking property for the system (3.7) will be satisfied by designing a state feedback regulator to the augmented model (3.23) as shown in fig.(3.8). The gains vector L for the state feedback regulator has a dimension of $(n+1)$ and it can be partitioned into two elements as follows:

$$L = (L_z \quad L_a) \quad (3.24)$$

Where:

L_z is the feedback gains vector for the controlled plant and L_a is the feedback gain for the additional dynamics system.

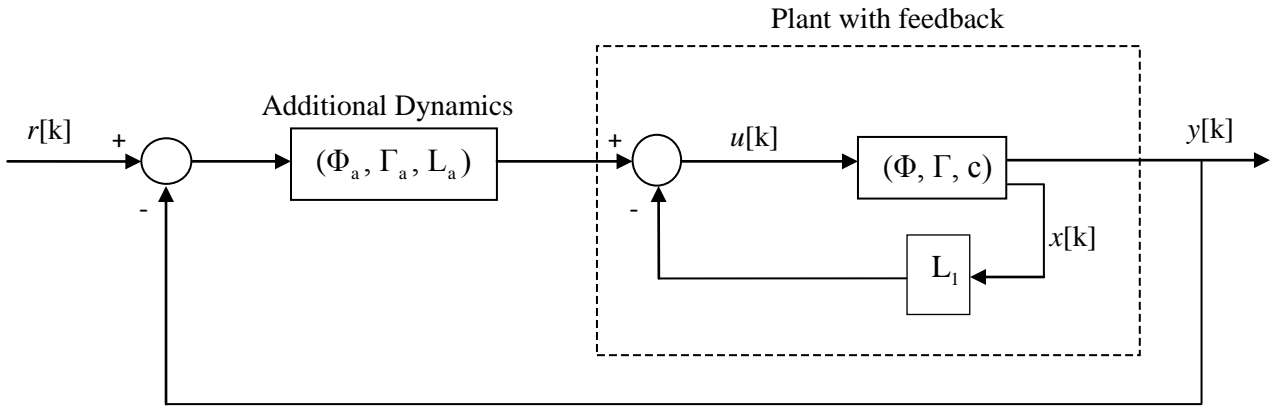


Figure 3.8: State feedback tracking controller

3.4.2 Designing A State Feedback Tracking Controller For A Special Case

System

The a special case of 2nd order nonlinear system is given by:

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \end{pmatrix} u \\ y &= (c_1 \ 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{aligned} \quad (3.25)$$

From eq(3.7), the ZOH equivalent model can be viewed as:

$$\begin{aligned} \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix} u(k) \\ y(k) &= (c_1 \ 0) \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} \end{aligned} \quad (3.26)$$

Where the values of Φ_{11} , Φ_{12} , Φ_{21} , Φ_{22} , Γ_1 and Γ_2 are described in eq(3.8).

After introducing an additional dynamics to track a step or ramp input with zero steady state error, the augmented state space model for the system (3.26) can be derived from eq (3.23) as:

$$\begin{aligned}
\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_a(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi_{11} & \Phi_{12} & 0 \\ \Phi_{21} & \Phi_{22} & 0 \\ c_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_a(k) \end{pmatrix} + \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ 0 \end{pmatrix} u \\
y &= (c_1 \quad 0 \quad 0) \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_a(k) \end{pmatrix}
\end{aligned} \tag{3.27}$$

The controllability matrix for the augmented model (3.27) is:

$$\begin{aligned}
C_{mz} &= (\Gamma \quad \Phi\Gamma \quad \Phi^2\Gamma) \\
&= \begin{pmatrix} \Gamma_1 & \Phi_{11}\Gamma_1 + \Phi_{12}\Gamma_2 & (\Phi_{11}^2 + \Phi_{12}\Phi_{21})\Gamma_1 + \Phi_{12}(\Phi_{11} + \Phi_{22})\Gamma_2 \\ \Gamma_2 & \Phi_{21}\Gamma_1 + \Phi_{22}\Gamma_2 & \Phi_{21}(\Phi_{11} + \Phi_{22})\Gamma_1 + (\Phi_{22}^2 + \Phi_{12}\Phi_{21})\Gamma_2 \\ 0 & c_1\Gamma_1 & c_1(1 + \Phi_{11})\Gamma_1 + c_1\Phi_{12}\Gamma_2 \end{pmatrix}
\end{aligned} \tag{3.8}$$

The transfer function for the augmented model can be computed as follows:

$$\begin{aligned}
TF &= \bar{C} (zI - \bar{\Phi}_d)^{-1} \Gamma_d = (c_1 \quad 0 \quad 0) \begin{pmatrix} z - \Phi_{11} & -\Phi_{12} & 0 \\ -\Phi_{21} & z - \Phi_{22} & 0 \\ -c_1 & 0 & z - 1 \end{pmatrix}^{-1} \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ 0 \end{pmatrix} \\
&= \frac{\bar{c}_2 z^2 + \bar{c}_3 z + \bar{c}_1}{z^3 + \bar{\Phi}_1 z^2 + \bar{\Phi}_2 z + \bar{\Phi}_3}
\end{aligned} \tag{3.9}$$

Where

$$\begin{aligned}
\bar{\Phi}_1 &= -(\Phi_{11} + \Phi_{22} + 1) = -(2e^{-\lambda T} \cosh(\psi T) + 1) \\
\bar{\Phi}_2 &= \Phi_{11}\Phi_{22} + \Phi_{11} + \Phi_{22} - \Phi_{12}\Phi_{21} = 2e^{-\lambda T} \cosh(\psi T) + e^{-2\lambda T} \\
\bar{\Phi}_3 &= -\Phi_{11}\Phi_{22} + \Phi_{12}\Phi_{21} = -e^{-2\lambda T} \\
\bar{c}_1 &= c_1\Gamma_1 \\
\bar{c}_2 &= \left(\frac{c_1 b_2}{a_2} \right) (a_1\Phi_{12} - \bar{\Phi}_3 - 1) \\
\bar{c}_3 &= \left(\frac{c_1 b_2}{a_2} \right) (\bar{\Phi}_3 + \Phi_{22})
\end{aligned} \tag{3.30}$$

The system (3.27) can be rearranged in CCF as:

$$\begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \\ \bar{x}_a(k+1) \end{pmatrix} = \begin{pmatrix} -\bar{\Phi}_1 & -\bar{\Phi}_2 & -\bar{\Phi}_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_a(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u(k) \quad (3.10)$$

$$y = (\bar{c}_1 \quad \bar{c}_2 \quad \bar{c}_3) \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_a(k) \end{pmatrix}$$

The controllability matrix for the augmented model that found in CCF is :

$$C_{mx} = (\bar{\Gamma} \quad \bar{\Phi}\bar{\Gamma} \quad \bar{\Phi}^2\bar{\Gamma}) = \begin{pmatrix} 1 & -\bar{\Phi}_1 & \bar{\Phi}_1^2 - \bar{\Phi}_2 \\ 0 & 1 & -\bar{\Phi}_1 \\ 0 & 0 & 1 \end{pmatrix} \quad (3.11)$$

The inverse of the matrix in the last equation can be derived as:

$$C_{mx}^{-1} = \begin{pmatrix} 1 & \Phi_1 & \Phi_2 \\ 0 & 1 & \Phi_1 \\ 0 & 0 & 1 \end{pmatrix} \quad (3.12)$$

Then, the transformation matrix between two forms can be calculated as follows:

$$T_c = \begin{pmatrix} \Gamma_1 & \bar{c}_2 & \bar{c}_3 \\ \Gamma_2 & c_1 & c_1 \\ 0 & \bar{c}_1 & -\bar{c}_3 \end{pmatrix} \quad (3.13)$$

The inverse of the transformation matrix is obtained by:

$$T_c^{-1} = \frac{1}{\Gamma_2(3\bar{c}_1\bar{c}_3 + \bar{c}_2\bar{c}_3 - \bar{c}_1^2)} \begin{pmatrix} c_1\Gamma_2(2\bar{c}_3 - \bar{c}_1) & \bar{c}_3(\bar{c}_1 + \bar{c}_2) & \Gamma_2(\bar{c}_3 - \bar{c}_1) \\ c_1\Gamma_2\bar{c}_3 & -c_1\Gamma_1\bar{c}_3 & \Gamma_2(\bar{c}_3 - \bar{c}_1) \\ c_1\Gamma_2\bar{c}_2 & -c_1\Gamma_1\bar{c}_1 & \Gamma_2(\bar{c}_3 - \bar{c}_1) \end{pmatrix} \quad (3.35)$$

Where:

$$|T_c| = \frac{\Gamma_2}{c_1} (3\bar{c}_1\bar{c}_3 + \bar{c}_2\bar{c}_3 - \bar{c}_1^2)$$

The corresponding state feedback controller for the augmented model defined by eq(3.31) takes the form:

$$u = -L_{xd}\bar{x}_d = -L_x\bar{x} - L_a\bar{x}_a = -L_{x1}\bar{x}_1 - L_{x2}\bar{x}_2 - L_a\bar{x}_a \quad (3.36)$$

Substitute for u from the above equation into eq(3.31), the closed loop system for the augmented model which is found in CCF becomes:

$$\begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \\ \bar{x}_a(k+1) \end{pmatrix} = \begin{pmatrix} -(\bar{\Phi}_1 + L_{x1}) & -(\bar{\Phi}_2 + L_{x2}) & -(\bar{\Phi}_3 + L_{xa}) \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_a(k) \end{pmatrix} \quad (3.37)$$

The characteristic equation of the above model is:

$$z^3 + (\bar{\Phi}_1 + L_{x1})z^2 + (\bar{\Phi}_2 + L_{x2})z + (\bar{\Phi}_3 + L_{xa}) \quad (3.38)$$

By comparing eq (3.12) with the last equation, we obtain:

$$\begin{aligned} L_{x1} &= d_1 - \bar{\Phi}_1 \\ L_{x2} &= d_2 - \bar{\Phi}_2 \\ L_{x3} &= d_3 - \bar{\Phi}_3 \end{aligned}$$

The state feedback gains vector for the transformed model (3.31) can be expressed as:

$$L_{xd} = (L_x \quad L_{xa}) = (d_1 - \bar{\Phi}_1 \quad d_2 - \bar{\Phi}_2 \quad d_3 - \bar{\Phi}_3) \quad (3.39)$$

The gains vector for the original system described in eq(3.27) can be found as follows:

$$\begin{aligned} L_{zd} &= L_{xd}T_c^{-1} = (L_x \quad L_{xa})T_c^{-1} = \\ & \frac{(d_1 - \bar{\Phi}_1 \quad d_2 - \bar{\Phi}_2 \quad d_3 - \bar{\Phi}_3)}{\Gamma_2(3\bar{c}_1\bar{c}_3 + \bar{c}_2\bar{c}_3 - \bar{c}_1^2)} \begin{pmatrix} c_1\Gamma_2(2\bar{c}_3 - \bar{c}_1) & \bar{c}_3(\bar{c}_1 + \bar{c}_2) & \Gamma_2(\bar{c}_3 - \bar{c}_1) \\ c_1\Gamma_2\bar{c}_3 & -c_1\Gamma_1\bar{c}_3 & \Gamma_2(\bar{c}_3 - \bar{c}_1) \\ c_1\Gamma_2\bar{c}_2 & -c_1\Gamma_1\bar{c}_1 & \Gamma_2(\bar{c}_3 - \bar{c}_1) \end{pmatrix} \end{aligned} \quad (3.40)$$

Where

$$L_{zd} = (L_{z1} \quad L_{z2} \quad L_{za})$$

3.5 Digital Output Feedback Controller

In section (3.4) we discussed the design state feedback method that utilizes the feedback of all state variables to form the required control input to achieve the desired performance specifications. As described in chapter 2, in practice, not all state variables are available for direct measurements, only a few states are measurable. The lack of measurements of some states can be compensated by including an observer or an estimator subsystem in the feedback controller.

3.5.1 Digital Observer

The equation of the closed-loop observer for discrete time system is:

$$\hat{x}(k+1) = \Phi \hat{x}(k) + \Gamma u(k) + H(y(k) - \hat{y}(k)) \quad (3.41)$$

The error vector can be defined as:

$$e(k) = x(k) - \hat{x}(k) \quad (3.42)$$

Then, the difference equation of $e(k)$ can be evaluated as:

$$\begin{aligned} e(k+1) &= x(k+1) - \hat{x}(k+1) \\ &= (\Phi - HC)e(k) \end{aligned} \quad (3.43)$$

If the matrix H is designed such that the matrix $(\Phi - HC)$ has eigenvalues inside the unit circle, then the estimated states will converge to the actual states of the plant and the error dynamics will be reduced to zero with sufficient speed.

3.5.2 Designing Digital Observer For A Special Case Of 2nd Order System

The observability matrix of the system described in (3.7)

$$O_{mz} = \begin{pmatrix} C \\ C\Phi \end{pmatrix} = \begin{pmatrix} c_1 & 0 \\ c_1\Phi_{11} & c_1\Phi_{12} \end{pmatrix} \quad (3.44)$$

The system can be transformed into observable canonical form as:

$$\begin{aligned} \bar{x}(k+1) &= \begin{pmatrix} -\bar{\Phi}_1 & 1 \\ -\bar{\Phi}_2 & 0 \end{pmatrix} \bar{x}(k) + \begin{pmatrix} \bar{\Gamma}_1 \\ \bar{\Gamma}_2 \end{pmatrix} u(k) \\ y(k) &= (1 \ 0) \bar{x}(k) \end{aligned} \quad (3.14)$$

Where

$$\begin{aligned} \bar{\Phi}_1 &= -2e^{-\lambda T} \cosh(\psi T) \\ \bar{\Phi}_2 &= e^{-2\lambda T} \\ \bar{\Gamma}_1 &= \left(\frac{c_1 b_2}{a_2} \right) (1 - \cosh(\psi) - (\lambda/\psi) \sinh(\psi)) \\ \bar{\Gamma}_2 &= \left(\frac{c_1 b_2}{a_2} \right) (e^{-2\lambda T} - \cosh(\psi) + (\lambda/\psi) \sinh(\psi)) \end{aligned} \quad (3.15)$$

The observability matrix of the transformed model (3.45) can be calculated as:

$$O_{mx} = C_{mx}^T = \begin{pmatrix} 1 & 0 \\ -\bar{\Phi}_1 & 1 \end{pmatrix} \quad (3.47)$$

The transformation matrix between the two forms is obtained by:

$$T_o = O_{mz}^{-1} O_{mx} = \begin{pmatrix} c_1 & 0 \\ c_1 \bar{\Phi}_{11} & c_1 \bar{\Phi}_{12} \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ -\bar{\Phi}_1 & 1 \end{pmatrix} = \frac{1}{c_1 \bar{\Phi}_{12}} \begin{pmatrix} \bar{\Phi}_{12} & 0 \\ \bar{\Phi}_{22} & 1 \end{pmatrix} \quad (3.48)$$

Define $H_x = (h_{x1} \quad h_{x2})^T$

The closed loop observer $(\bar{\Phi} - H_x \bar{C})$ for system (3.45) can be found as:

$$(\bar{\Phi} - H_x \bar{C}) = \begin{pmatrix} -\bar{\Phi}_1 & 1 \\ -\bar{\Phi}_2 & 0 \end{pmatrix} - (1 \quad 0) \begin{pmatrix} h_{x1} \\ h_{x2} \end{pmatrix} = \begin{pmatrix} -(\bar{\Phi}_1 + h_{x1}) & 1 \\ -(\bar{\Phi}_2 + h_{x2}) & 0 \end{pmatrix} \quad (3.49)$$

Then, the characteristic equation of the closed loop observer (3.49) is:

$$|sI - (\bar{\Phi} - H_x \bar{C})| = \begin{vmatrix} s + (\bar{\Phi}_1 + h_{x1}) & -1 \\ (\bar{\Phi}_2 + h_{x2}) & s \end{vmatrix} = s^2 + (\bar{\Phi}_1 + h_{x1})s + \bar{\Phi}_2 + h_{x2} \quad (3.50)$$

Comparing the coefficients of the last characteristic equation to the coefficients of equal power of z in eq(3.14), we require that

$$\begin{aligned} P_1 &= \bar{\Phi}_1 + h_{x1} \\ P_2 &= \bar{\Phi}_2 + h_{x2} \end{aligned} \quad (3.51)$$

From equation (3.51) we get:

$$\begin{aligned} h_{x1} &= P_1 - \bar{\Phi}_1 \\ h_{x2} &= P_2 - \bar{\Phi}_2 \end{aligned} \quad (3.52)$$

Substituting from eq(3.15) for p_1 and p_2 , and from eq(3.46) for $\bar{\Phi}_1$ and $\bar{\Phi}_2$ into eq(3.52), we get:

$$\begin{pmatrix} h_{x1} \\ h_{x2} \end{pmatrix} = \begin{pmatrix} 2(e^{-\lambda T} \cos h(\psi T) - e^{-\beta_0 \zeta \omega_n T} \cos(\beta_0 \omega_d T)) \\ e^{-2\beta_0 \zeta \omega_n T} - e^{-2\lambda T} \end{pmatrix} \quad (3.16)$$

The observer gain matrix for the original system is obtained as follows:

$$\begin{aligned}
H_z &= T_o H_x \\
&= O_{mz}^{-1} C_{mx}^T H_x = \frac{1}{c_1} \begin{pmatrix} H_{x_1} \\ H_{x_2} + \Phi_{22} H_{x_1} \\ \Phi_{12} \end{pmatrix}
\end{aligned} \tag{3.17}$$

3.5.3 Effects of The Addition of The Observer In A Closed-Loop System

As described in chapter 2, for the output feedback control, the estimated states were used rather than the actual states for the purpose of feedback.

Consider the completely states controllable and completely observable system defined by the equation:

$$x(k+1) = \Phi x(k) + \Gamma u(k) \tag{3.18}$$

The state feedback control based on the observed states can be written as:

$$u(k) = -L\hat{x}(k) \tag{3.19}$$

By substituting of the control input u that found in eq(3.56) into eq(3.55), the closed loop system can be written as:

$$x(k+1) = \Phi x(k) - \Gamma L\hat{x}(k) \tag{3.20}$$

From equation(3.42) ,we have:

$$\hat{x}(k) = x(k) - e(k) \tag{3.21}$$

Substituting for $\hat{x}(k)$ in equation (3.57) , the state equation becomes:

$$x(k+1) = (\Phi - \Gamma L)x(k) + \Gamma L e_x(k) \tag{3.22}$$

The augmented state vector for the interconnected system is:

$$z(k) = \begin{pmatrix} x(k) \\ e_x(k) \end{pmatrix}$$

Note that the observer error difference equation was given by eq(3.43), repeated here:

$$e(k+1) = (\Phi - HC)e(k) \tag{3.60}$$

Combining eq(3.59) and eq(3.60), the augmented state model due to introducing the observer in the feedback controller can be written as:

$$\begin{pmatrix} x(k+1) \\ e_x(k+1) \end{pmatrix} = \begin{pmatrix} \Phi - \Gamma L & \Gamma L \\ 0 & \Phi - HC \end{pmatrix} \begin{pmatrix} x(k) \\ e_x(k) \end{pmatrix} \quad (3.23)$$

Because the matrix is an upper-triangular, the closed loop poles of the observed-state feedback control consist of the poles $(\Phi - \Gamma L)$ plus the poles $(\Phi - HC)$. This means that the state feedback control design and the observer design are independent of each other. They can be designed separately and combined together to form an output feedback control.

3.5.4 Designing An Observer Based Integral Controller

The block diagram of an observer-based integral controller in the discrete control system can be described as shown in fig.(3.9) below:

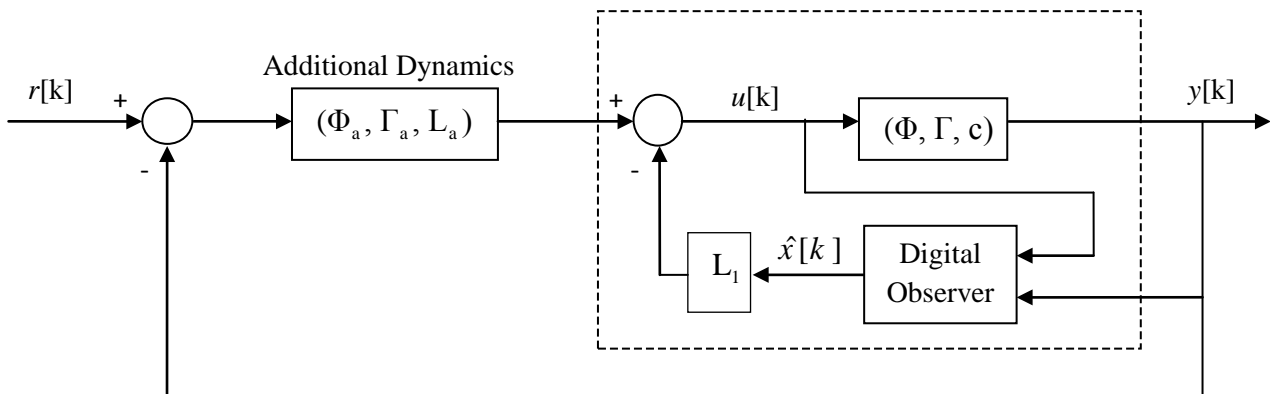


Figure 3.9: An observer based integral controller.

As described previously, The configuration of such controller shown in fig.(3.9) has the advantage that the design of the state feedback regulation and the observer are separated.

For the state feedback design, all the state variables are assumed to be measured and the same result of designing the tracking controller which is derived in eq(3.40) can be used.

For the observer design, only the output is assumed to be measured and also the same result of designing the observer gains vector which is derived in eq(3.54) can be used.

3.6 Digital Gain Scheduled Controller For A Special Case System

Consider the special case 2nd order nonlinear system described by:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x_1, x_2, u) \\
y &= h(x_1)
\end{aligned} \tag{3.24}$$

From chapter 2, The linearization of the nonlinear system (3.62) about a family of parameterized operating points $(x_{ss}(\alpha), u_{ss}(\alpha))$ can be written as:

$$\begin{aligned}
\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -a_2(\alpha) & -a_1(\alpha) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2(\alpha) \end{pmatrix} u \\
y &= (c_1(\alpha) \quad 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}
\end{aligned} \tag{3.63}$$

Using eq(3.7), the parameterized discrete time state space equation is:

$$\begin{aligned}
\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} \Gamma_1(\alpha) \\ \Gamma_2(\alpha) \end{pmatrix} u(k) \\
y(k) &= (c_1(\alpha) \quad 0) \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix}
\end{aligned} \tag{3.25}$$

For designing a digital gain scheduled observer-based integral controller, we can use the same results of gains that derived in eq(3.40) and eq(3.54) by setting their parameters as a function of the scheduling variables α .

Then, the state feedback scheduled gains vector can be written as:

$$L_{zd}(\alpha) = (d_1 - \bar{\Phi}_1(\alpha) \quad d_2 - \bar{\Phi}_2(\alpha) \quad d_3 - \bar{\Phi}_3(\alpha)) T_c^{-1}(\alpha) \tag{3.26}$$

And the scheduled gains vector for an observer can be expressed as:

$$H_z(\alpha) = \frac{1}{c_1(\alpha)} \begin{pmatrix} H_{x_1}(\alpha) \\ \frac{H_{x_2}(\alpha) + \Phi_{22}(\alpha) H_{x_1}(\alpha)}{\Phi_{12}(\alpha)} \end{pmatrix} \tag{3.27}$$

CHAPTER 4

4 TRACKING A NONLINEAR SYSTEM VIA NETWORK

4.1 Introduction

In modern manufacturing control systems, the common bus network architecture is becoming more dominant than point-to-point architecture due to developments in communication capabilities, improvements in network infrastructures, small reconfiguration cost, better resource utilization, and reduced maintenance cost[3]. The utilization of a wired or wireless communication network in control applications provides several benefits, but in the same time, faces many difficulties. The main difficulty with the design of such control loop is the presence of sensing and actuation delays introduced by the communication networks. Unlike conventional time delay systems, the type of delays introduced by the network are time-varying, since they depend on the traffic currently on the network. The main objective of this chapter is to present a methodology in order to design a dynamic output feedback gain scheduled controller to control a nonlinear system via NCS that is characterized by time-varying control and constant delays in the utilized of two transmission channels.

4.2 Delays Analysis In The Control Loop

Since the NCS operates over a network, data that transfers between the controller and the remote system will induce network delays in addition to the controller processing delay. Network delays in a NCS can be categorized from the direction of data transfers as the sensor-to-controller (forward channel) delay and the controller-to-actuator (backward or feedback channel) delay. These network delays can be longer or shorter than the sampling time T . There are several sources of delays in NCS. Not only the network dynamics affect the total delay, but also the signal processing and computational delays that depend on the scheduling policies should be taken into account. Network delays are modeled and analyzed in various ways depending on the network type and protocols. They can be modeled as a constant delay (timed buffer), independent random delay and delay with known probability distribution governed by Markov chain model. Different types of time delay compensation schemes are used

to compensate the time delays caused by the network such as: PID controller, smith predictor, optimal controller, fuzzy controller, robust control, sliding mode controller, and adaptive controller. The typical construction of networked control systems (NCSs) in the discrete time model is described in fig(4.1), and the timing diagram of network delay propagation is also described in fig(4.2).

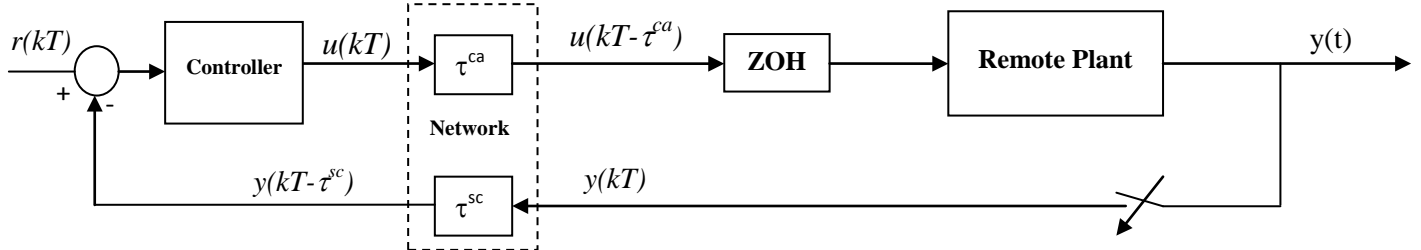


Figure 4.1: General NCS configuration and network delays in discrete-time formulation

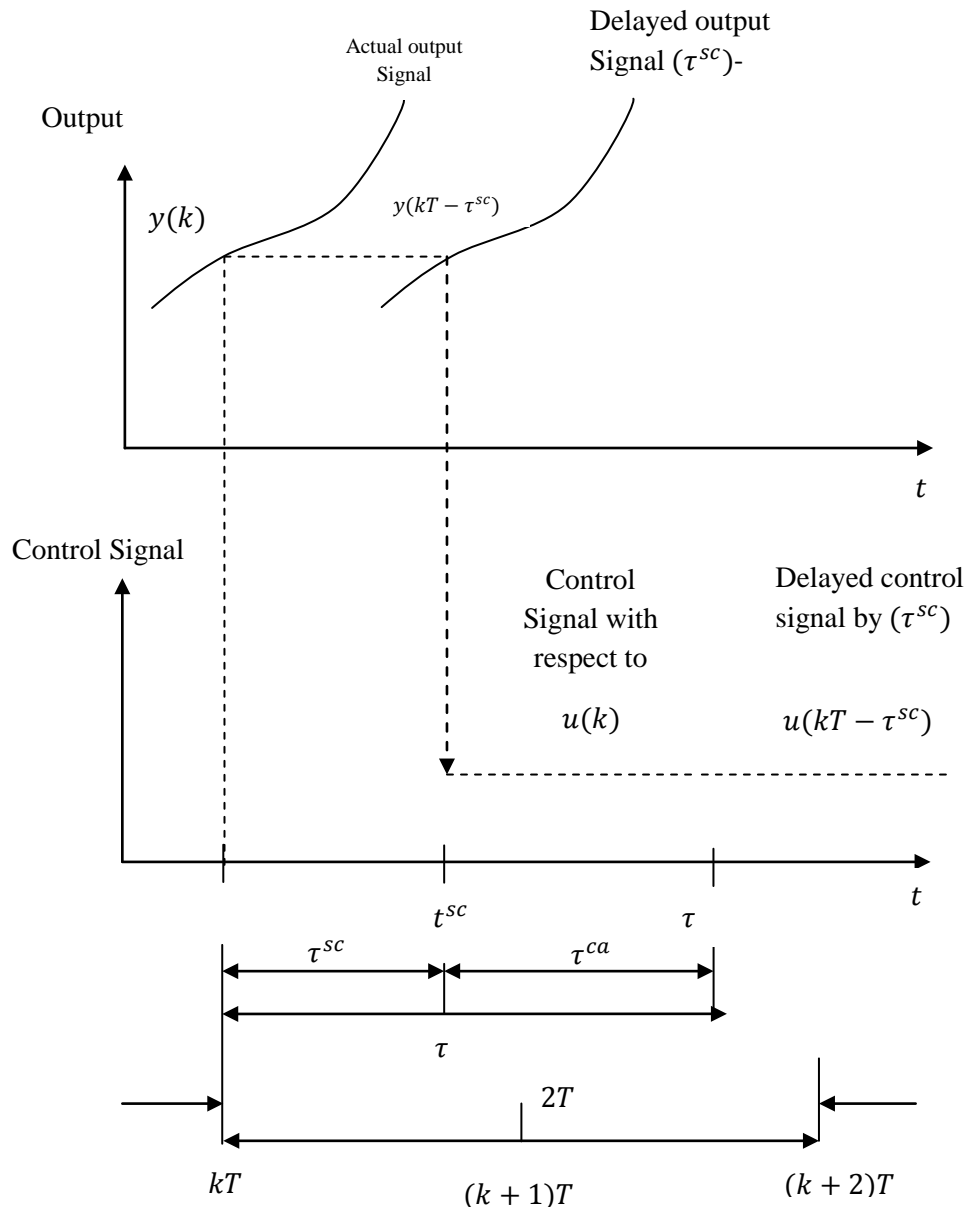


Figure 4.2: Timing diagram of network delay propagation

4.3 Effects Of Delays In The Control Loop

As described previously, the network induced delays are brought into the loop of the control system along with the inserted communication network, which not only degrade the control system performances, but may be bring the whole control system unstable. Thus, any control system constructed without compensation for these delays have a low performance and reliability.

4.4 Network Time Delay Compensation

Networked Control System (NCS) is a type of closed loop control system with real time communication networks imported into the forward control channel and the feedback channel. Then, the network-induced delays have two parts: forward channel delay and feedback channel delay.

For the sake of simplicity, the scheme of compensation for network-induced time delay in the NCS under randomly varying delay in the forward channel and also deterministically delay in the feedback channel can be decomposed into two steps of delay compensation. These step are: Compensation for forward time delay τ^{ca} and compensation for feedback time delay τ^{sc} .

In the case of forward delay compensation, we assume that the network is applied only between the controller output and the plant input as shown in fig.(4.3), and the main proposed idea to handle the network delays here is to augment the system model by including delayed variables as additional states and designing an observer to estimate the state variables of the plant and delay, so a closed-loop output feedback tracking controller for the augmented model can be built.

After the forward delay is compensated, the network will be considered into the feedback channel as shown in fig(4.4), and the designed observer for the NCS must be modified to compensate for the feedback channel time delay τ^{sc} by rebuilding the non-delayed plant estimates, which is more accurate for the controller to generate a control signal. This type of observers is also called *Predictor Observer* or *States Predictor*.

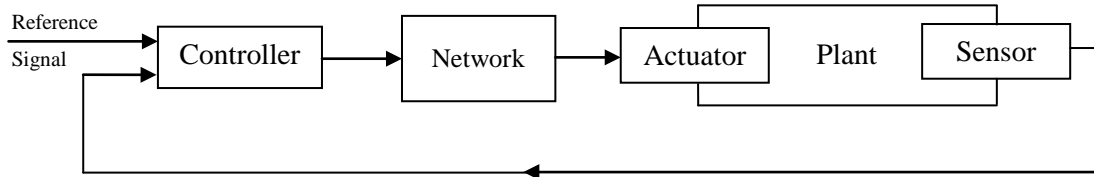


Figure 4.3: Network applied in the forward channel only

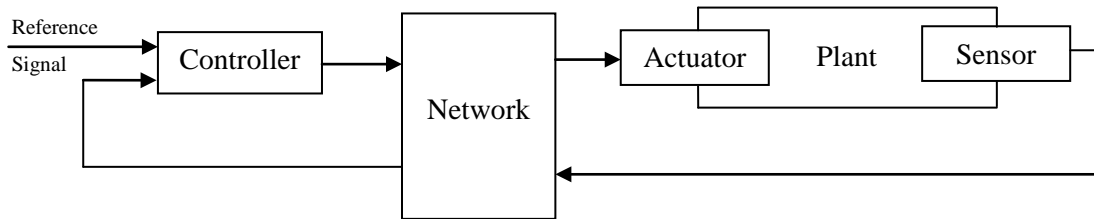


Figure 4.4: Network with feedback channel is considered

4.5 Forward Channel Time Delay Compensation

The continuous state space model of the system with time delay induced by the forward network channel can be written as:

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t - \tau^{ca}) \\ y &= Cx(t) \end{aligned} \quad (4.1)$$

Where τ^{ca} is the forward channel time delay or controller-to-actuator delay. Assume that the value of the state vector is known at time t_1 , then the control input $u(t)$ has to be known over the interval $t_1 \leq t \leq t_1 + \tau^{ca}$.

Since the control input contains a continuum of values over this interval, the state space representation of delayed system will need an infinite number of the state variables to summarize the information about the system at time t_1 . Thus, a continuous time systems with time delays are infinite dimensional.

A finite dimensional description of a system can be obtained by sampling the continuous time system, such that, the infinite dimensional continuous-time system has been reformulated to the finite dimensional discrete-time system.

4.5.1 ZOH Equivalent Model Of A System With Time Delay

It is however easy to handle systems with time delays with ZOH reconstruction, because the control signal is constant between the sampling instants, which makes the sampled-data system finite dimensional.

In general, the time delay will equal some integer number (possibly zero) of sampling intervals plus a fractional part of a sampling interval. Thus, any time delay can be represented by the following form:

$$t_d = mT + \gamma \quad (4.2)$$

Where $0 < \gamma \leq T$ and $m \geq 0$

The updated formula for the state vector at sampling instants will be obtained by integrating the state equation over one sampling period as described in the following equation:

$$x(kT + T) = e^{AT} x(kT) + \int_{kT}^{kT+T} e^{A(kT+T-t)} B u(t - \tau^{ca}) dt \quad (4.3)$$

In order to perform the integration in eq(4.3), the value of $u(t - \tau^{ca})$ is needed to be known over the interval kT to $kT + T$.

From the delay form described in eq(4.2), the delayed input signal will take two different values during the interval (kT) to $(kT + T)$ as shown in fig(4.5). Thus, to perform the integration in eq(4.3), the integral must be split in two parts, where the delayed input is constant over each part.

$$u(t - \tau^{ca}) = \begin{cases} u(kT - (m+1)T), & kT \leq t < kT + \tau^{ca} \\ u(kT - mT), & kT + \gamma \leq t < kT + T \end{cases} \quad (4.4)$$

Substituting the above expression for the delayed input, and splitting the integral in eq(4.3) into two parts, the integration of the state equation becomes:

$$x(kT + T) = e^{AT} x(kT) + \int_{kT}^{kT+\tau^{ca}} e^{A(kT+T-t)} B u(kT - mT - T) dt + \int_{kT+\tau^{ca}}^{kT+T} e^{A(kT+T-t)} B u(kT - mT) dt \quad (4.5)$$

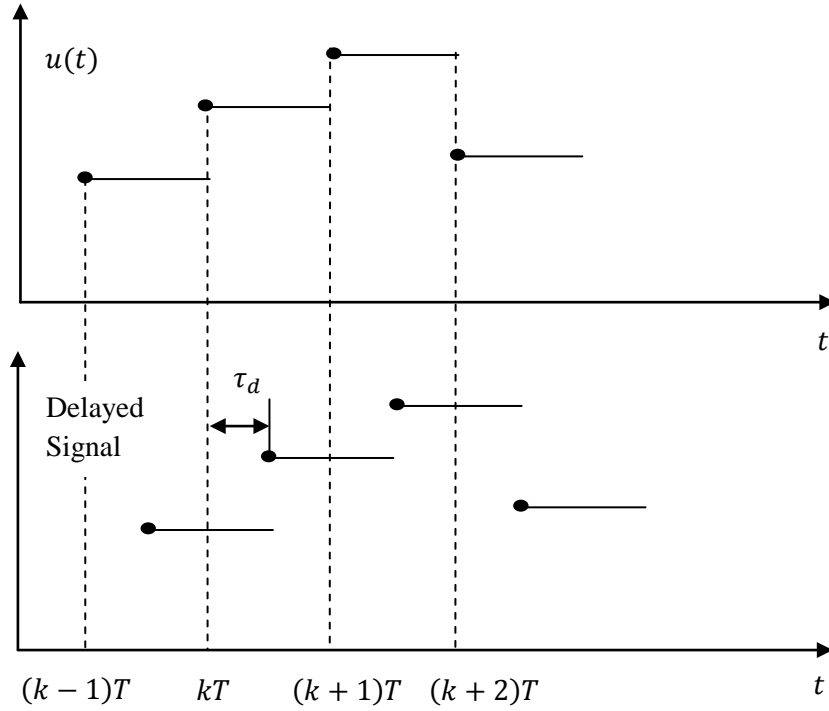


Figure 4.5: The relationship among $u(t)$, the delayed signal $u(t - \tau^{ca})$, and the sampling instants.

Let $s = kT + T - t$

Substitution for s in eq(5.5) , yield

$$\begin{aligned}
 x(kT + T) = e^{AT} x(kT) + \int_{T - \tau^{ca}}^T e^{As} B u(kT - mT - T) ds \\
 + \int_0^{T - \tau^{ca}} e^{As} B u(kT - mT) ds
 \end{aligned} \tag{4.6}$$

Eq(4.5) can be written in discrete notation as:

$$x(kT + T) = \Phi x(kT) + \Gamma_{d0}(\tau^{ca}) u(kT - mT) + \Gamma_{d1}(\tau^{ca}) u(kT - mT - T) \tag{4.7}$$

Where:

$$\begin{aligned}
 \Phi &= e^{AT} \\
 \Gamma_{d0}(\tau^{ca}) &= \int_0^{T - \tau^{ca}} e^{As} B ds \\
 \Gamma_{d1}(\tau^{ca}) &= \int_{T - \tau^{ca}}^T e^{As} B ds = e^{A(T - \tau^{ca})} \int_0^{\tau^{ca}} e^{As} B ds
 \end{aligned}$$

From thesis assumptions, It is assumed that the time delay that induced by the forward channel is less than one sampling period. When the time delay is less than or equal one sampling period ,we have:

$$\begin{aligned}
 m &= 0 \\
 \tau^{ca} &= \gamma, \quad \gamma \leq T
 \end{aligned}$$

Substituting for the value of m in eq(4.7), then the state space equation of the delayed system can be rewritten as:

$$x(k+1) = \Phi x(k) + \Gamma_{d0}(\tau^{ca}) u(k) + \Gamma_{d1}(\tau^{ca}) u(k-1) \quad (4.8)$$

From the above equation, a new state variable $u(k-1)$ is appeared, which represent the past value of the control signal. Thus, the complete state updated equation for a system with forward channel time delay can be obtained by augmenting the extra state variable $u(k-1)$ into the full system state model and is given by:

$$\begin{pmatrix} x(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} \Phi & \Gamma_{d1}(\tau^{ca}) \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ u(k-1) \end{pmatrix} + \begin{pmatrix} \Gamma_{d0}(\tau^{ca}) \\ 1 \end{pmatrix} u(k) \quad (4.9)$$

4.5.2 ZOH Equivalent Model For Nonlinear System With Time Delay:

From chapter 3, the ZOH equivalent model for the linearized system with no time delay and which is parameterized by the scheduling variable (α) is given by:

$$\begin{pmatrix} x_1(k+1) \\ x_1(k+1) \end{pmatrix} = \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_1(k) \end{pmatrix} + \begin{pmatrix} \Gamma_1(\alpha) \\ \Gamma_2(\alpha) \end{pmatrix} u(k) \quad (4.10)$$

$$y(k) = \begin{pmatrix} c_1(\alpha) & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_1(k) \end{pmatrix}$$

Where:

$$\Phi_{11}(\alpha) = e^{-\lambda(\alpha)T} \left(\cosh(\psi(\alpha)T) + (\lambda(\alpha)/\psi(\alpha)) \sinh(\psi(\alpha)T) \right)$$

$$\Phi_{12}(\alpha) = \frac{e^{-\lambda(\alpha)T} \sinh(\psi(\alpha)T)}{\psi(\alpha)}$$

$$\Phi_{21}(\alpha) = \frac{-a_2(\alpha) e^{-\lambda(\alpha)T} \sinh(\psi(\alpha)T)}{\psi(\alpha)}$$

$$\Phi_{22}(\alpha) = e^{-\lambda(\alpha)T} \left(\cosh(\psi(\alpha)T) - (\lambda(\alpha)/\psi(\alpha)) \sinh(\psi(\alpha)T) \right)$$

And

$$\Gamma(\alpha) = \begin{pmatrix} \Gamma_1(\alpha) \\ \Gamma_2(\alpha) \end{pmatrix} = \begin{pmatrix} \frac{b_2(\alpha)}{a_2(\alpha)} [1 - \Phi_{11}(\alpha)] \\ b_2(\alpha) \Phi_{12}(\alpha) \end{pmatrix}$$

If the time delay in the forward channel is taken into account, the values of $\Gamma_{d0}(\alpha, \tau_k^{ca})$ and $\Gamma_{d1}(\alpha, \tau_k^{ca})$ for a special case of 2nd order system should be computed, then:

$$\begin{aligned}
\Gamma_{d0}(\alpha, \tau_k^{ca}) &= \int_0^{T-\tau_k^{ca}} e^{A(\alpha)s} ds B(\alpha) = \int_0^{T-\tau_k^{ca}} \begin{pmatrix} \Phi_{11}(\alpha, s) & \Phi_{12}(\alpha, s) \\ \Phi_{21}(\alpha, s) & \Phi_{22}(\alpha, s) \end{pmatrix} \begin{pmatrix} 0 \\ b_2(\alpha) \end{pmatrix} ds \\
&= b_2(\alpha) \int_0^{T-\tau_k^{ca}} \begin{pmatrix} \Phi_{12}(\alpha, s) \\ \Phi_{22}(\alpha, s) \end{pmatrix} ds \\
&= b_2(\alpha) \begin{pmatrix} \frac{1-\Phi_{11}(\alpha, T-\tau_k^{ca})}{a_2} \\ \Phi_{12}(\alpha, T-\tau_k^{ca}) \end{pmatrix} = \begin{pmatrix} \Gamma_{d01}(\alpha, \tau_k^{ca}) \\ \Gamma_{d02}(\alpha, \tau_k^{ca}) \end{pmatrix}
\end{aligned} \tag{4.11}$$

$$\begin{aligned}
\Gamma_{d1}(\alpha, \tau_k^{ca}) &= \int_{T-\tau_k^{ca}}^T e^{A(\alpha)s} B(\alpha) ds = e^{A(\alpha, T-\tau_k^{ca})} \int_0^{\tau_k^{ca}} e^{A(\alpha)s} B(\alpha) ds \\
&= \begin{pmatrix} \Phi_{11}(\alpha, T-\tau_k^{ca}) & \Phi_{12}(\alpha, T-\tau_k^{ca}) \\ \Phi_{21}(\alpha, T-\tau_k^{ca}) & \Phi_{22}(\alpha, T-\tau_k^{ca}) \end{pmatrix} \int_0^{\tau_k^{ca}} \begin{pmatrix} \Phi_{12}(\alpha, s) \\ \Phi_{22}(\alpha, s) \end{pmatrix} ds \\
&= b_2(\alpha) \begin{pmatrix} \frac{(\Phi_{11}(\alpha, T-\tau_k^{ca}) - \Phi_{11}(\alpha, T))}{a_2} \\ \Phi_{12}(\alpha, T) + \Phi_{12}(\alpha, T-\tau_k^{ca}) \end{pmatrix} = \begin{pmatrix} \Gamma_{d11}(\alpha, \tau_k^{ca}) \\ \Gamma_{d12}(\alpha, \tau_k^{ca}) \end{pmatrix}
\end{aligned} \tag{4.12}$$

Let

$$\begin{pmatrix} \Phi_{13}(\alpha, \tau_k^{ca}) \\ \Phi_{23}(\alpha, \tau_k^{ca}) \end{pmatrix} = \begin{pmatrix} \Gamma_{d11}(\alpha, \tau_k^{ca}) \\ \Gamma_{d12}(\alpha, \tau_k^{ca}) \end{pmatrix} \tag{4.13}$$

Then, the complete ZOH model for a special case of 2nd order system including the forward channel time delay at any k th sampling period can be written as:

$$\begin{aligned}
\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ u(k) \end{pmatrix} &= \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) & \Phi_{13}(\alpha, \tau_k^{ca}) \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) & \Phi_{23}(\alpha, \tau_k^{ca}) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \end{pmatrix} + \begin{pmatrix} \Gamma_{d01}(\alpha, \tau_k^{ca}) \\ \Gamma_{d02}(\alpha, \tau_k^{ca}) \\ 1 \end{pmatrix} u(k) \\
y &= (c_1(\alpha) \ 0 \ 0) \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \end{pmatrix}
\end{aligned} \tag{4.14}$$

Based on the last equation, for every k th sampling period, the discrete state space equation of the augmented model is parameterized as a function of two variables: α and τ_k^{ca} . Since the controller to actuator τ_k^{ca} is random, the system in this case is time varying.

4.5.3 Additional Dynamics For Tracking

As described in the previous chapter, to track a unit step or ramp input with zero steady state error, an additional dynamics must be included as a part of the controller. The new updating of the augmented model that generated from the introduced additional dynamics becomes:

$$\begin{aligned} \begin{pmatrix} x(k+1) \\ u(k) \\ x_a(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi(\alpha) & \Gamma_{d1}(\alpha, \tau_k^{ca}) & 0 \\ 0 & 0 & 0 \\ C(\alpha) & 0 & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} + \begin{pmatrix} \Gamma_{d0}(\alpha, \tau_k^{ca}) \\ 1 \\ 0 \end{pmatrix} u(k) \\ y &= (C(\alpha) \ 0 \ 0) \begin{pmatrix} x(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} \end{aligned} \quad (4.15)$$

Then, the new augmented model for a special case of 2nd order system can be derived as:

$$\begin{aligned} \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ u(k) \\ x_a(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) & \Phi_{13}(\alpha, \tau_k^{ca}) & 0 \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) & \Phi_{23}(\alpha, \tau_k^{ca}) & 0 \\ 0 & 0 & 0 & 0 \\ c_1(\alpha) & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} + \begin{pmatrix} \Gamma_{d01}(\alpha, \tau_k^{ca}) \\ \Gamma_{d02}(\alpha, \tau_k^{ca}) \\ 1 \\ 0 \end{pmatrix} u(k) \\ y &= (c_1(\alpha) \ 0 \ 0 \ 0) \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} \end{aligned} \quad (4.16)$$

4.5.4 Making of NCS to be Time-Invariant:

When the induced delay is varying in a random fashion, the system becomes time varying, and the theoretical results for analysis and design for time-invariant systems cannot be used directly.

One way to reshape the induced random delay by the forward channel to a constant delay is done by introducing the network delay compensator (NDC) at the input of actuator node as shown in fig(4.6), such that the augmented model becomes time-invariant. The NDC acts as a clocked buffer by holding the arrived control signal within $\Delta_{\tau,k}$ time-varying interval during the k th sampling period, where:

$$\Delta_{\tau,k} = t_d - \tau_k^{ca}$$

And t_d is a specified constant time delay in which it is less than or equal one sampling period.

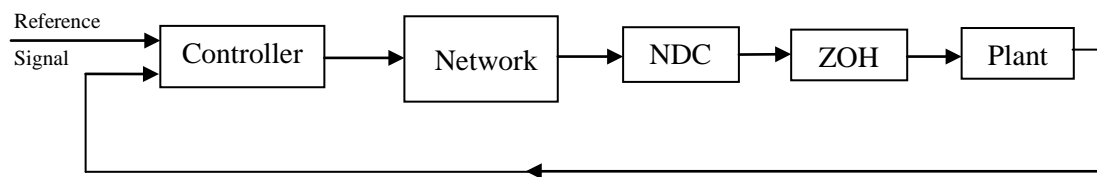


Figure 4.6: Making of NCS to be time invariant by introducing NDC at the actuator node input

Since τ_k^{ca} is assumed to be less than one sampling period, and for ease of analysis, the best scenario for the deterministic time delay occurs when the time delay is equal to one sampling period.

In this case, the proposed structure of the NDC in a standard sampled data control system is shown in fig(4.7) below.

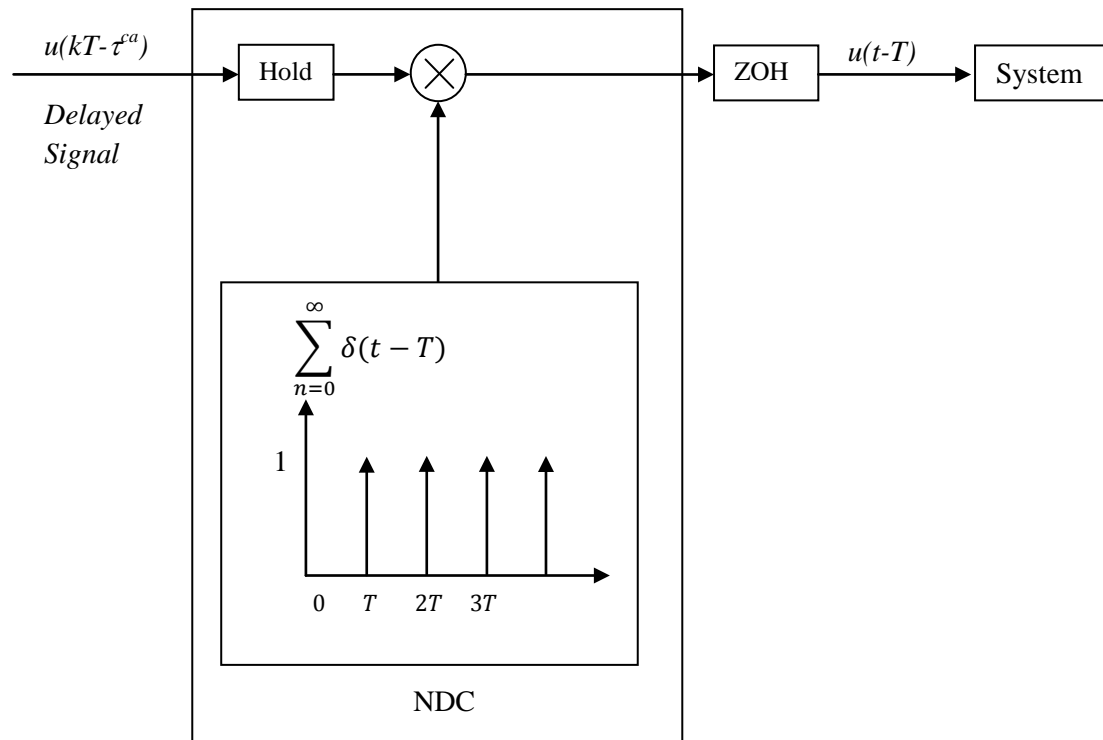


Figure 4.7: Structure of Network Delay Compensator in discrete time model

Fig(4.7) shows that due to including the NDC, the control signal is applied to the plant after a time delay equal to one sampling period is occurred.

4.5.5 Augmented Model With Time Delay Equal To One Sampling Period

As mentioned above, after introducing the NDC into the control system, the induced delay by the forward channel becomes constant and equal to one sampling period.

When the time delay induced by the network equal to one sample time in such a case, the independent random delay model is justified. For this case the best scenario for the deterministic time delay occurs because it will be simplified the derivation of the ZOH equivalent model for the system with time delay.

After substituting for $\tau_k^{ca} = T$ into eq(4.11), and (4.12) the results are :

$$\Gamma_{d0}(\tau_k^{ca} = T) = 0, \quad \Gamma_{d1}(\tau_k^{ca} = T) = \Gamma \quad (4.17)$$

where Γ is the input vector for the ZOH model without time delay.

Substituting For the above equation into eq(4.8), we get,

$$x(kT + T) = \Phi x(kT) + \Gamma u(k-1) \quad (4.18)$$

Substituting for eq(4.17) into the augmented model(4.9), the result is:

$$\begin{pmatrix} x(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} \Phi(\alpha) & \Gamma(\alpha) \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ u(k-1) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k) \quad (4.19)$$

After introducing an additional dynamic as a part of controller, the complete time invariant augmented model becomes:

$$\begin{pmatrix} x(k+1) \\ u(k) \\ x_a(k+1) \end{pmatrix} = \begin{pmatrix} \Phi(\alpha) & \Gamma(\alpha) & 0 \\ 0 & 0 & 0 \\ C(\alpha) & 0 & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} u(k) \quad (4.20)$$

$$y = (C(\alpha) \quad 0 \quad 0) \begin{pmatrix} x(k) \\ u(k-1) \\ x_a(k) \end{pmatrix}$$

In the case of one sampling period delay, we obtained the following results for a special case of 2nd order nonlinear system:

$$\Gamma_{d0}(\alpha, \tau_k^{ca} = T) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \Gamma_{d1}(\alpha, \tau_k^{ca} = T) = \begin{pmatrix} \Gamma_1(\alpha) \\ \Gamma_2(\alpha) \end{pmatrix} = \begin{pmatrix} \Phi_{13}(\alpha) \\ \Phi_{13}(\alpha) \end{pmatrix} \quad (4.21)$$

Substitute from eq(4.21) into eq(4.14), the time invariant discrete state space model of the augmented system can be derived as:

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) & \Gamma_1(\alpha) \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) & \Gamma_2(\alpha) \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(k) \quad (4.22)$$

$$y = \begin{pmatrix} c_1(\alpha) & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \end{pmatrix}$$

After introducing an additional dynamics as a part of controller, the complete time invariant discrete state space of the augmented model becomes:

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ u(k) \\ x_a(k+1) \end{pmatrix} = \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) & \Gamma_1(\alpha) & 0 \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) & \Gamma_2(\alpha) & 0 \\ 0 & 0 & 0 & 0 \\ c_1(\alpha) & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} u(k) \quad (4.23)$$

$$y = \begin{pmatrix} c_1(\alpha) & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \\ x_a(k) \end{pmatrix}$$

4.5.6 Design Of An Observer For The Forward Channel Time Delay

Compensation

The scheme of compensation for network induced delay by the forward channel is based on designing an observer to estimate the state variables of the augmented model that includes the state variables of the delayed control input. Since the observer is a part of the output feedback controller, the design process of the observer will be illustrated in the design of the dynamic output feedback controller lately.

Two types of such controllers will be mentioned in the next section which are: Time invariant gain scheduled controller and time varying gain scheduled controller.

4.6 Output Feedback Gain Scheduled Controller

From a control perspective, the control system with varying delays will no longer be time-invariant. As described in previous section, the theoretical results for analysis and design for time-invariant systems cannot be used directly. It can be used after introduction of NDC at the input of the actuator node.

Since the gain scheduling technique is widely used for controlling certain classes of nonlinear or linear time varying systems, this technique can be extended to be applicable for time varying and/or nonlinear system whose linearized dynamics are approximated by an affine parameter dependent model (e.g) as described in eq(4.20).

Based on the above, two strategies for designing a gain scheduled controller can be used to track a nonlinear system via NCS, these strategies are:

- 1- The first strategy is to make the system time invariant by converting the random fashion of the induced delay into deterministic fashion and designing time invariant gain scheduled controller, in which the scheduling parameters of such controller are the tracking trajectory parameters.
- 2- The second strategy is to incorporate the current measurement of time delay into the scheduling parameters and designing a time varying gain scheduled controller in which the scheduling parameters of such controller became the current measurement of time delay and the tracking trajectory parameters. In this strategy, the controller compares the time-stamp value with its local clock, and then it can compute the total time delay value easily.

4.6.1 Time Invariant Output Feedback Gain Scheduled Controller

As described in the previous chapters, for this type of controller, the separation principle can be applied, such that the designs of state feedback gains and observer gains can be performed separately.

Before designing the controller, its assumed that the deterministic time delay which makes the system time invariant is equal to one sampling period. The augmented model that includes the state variables of the additional dynamics and delayed signal into the full system state model and described in eq(4.20), can be rewritten as:

$$\begin{aligned} x_{td}(k+1) &= \Phi_{td}(\alpha)x_{td}(k) + \Gamma_{td}(\alpha)u(k) \\ y(k) &= C_{td}(\alpha)x_{td}(k) \end{aligned} \quad (4.24)$$

Where:

$$\begin{aligned} \Phi_{td}(\alpha) &= \begin{pmatrix} \Phi(\alpha) & \Gamma(\alpha) & 0 \\ 0 & 0 & 0 \\ C(\alpha) & 0 & 0 \end{pmatrix}, \Gamma_{td}(\alpha) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\ C_{td}(\alpha) &= (C(\alpha) \ 0 \ 0), \quad x_{td} = \begin{pmatrix} x(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} \end{aligned} \quad (4.25)$$

The time-invariant output feedback gain scheduled controller as a combination of an observer and a state feedback controller is described in fig(4.8) below:

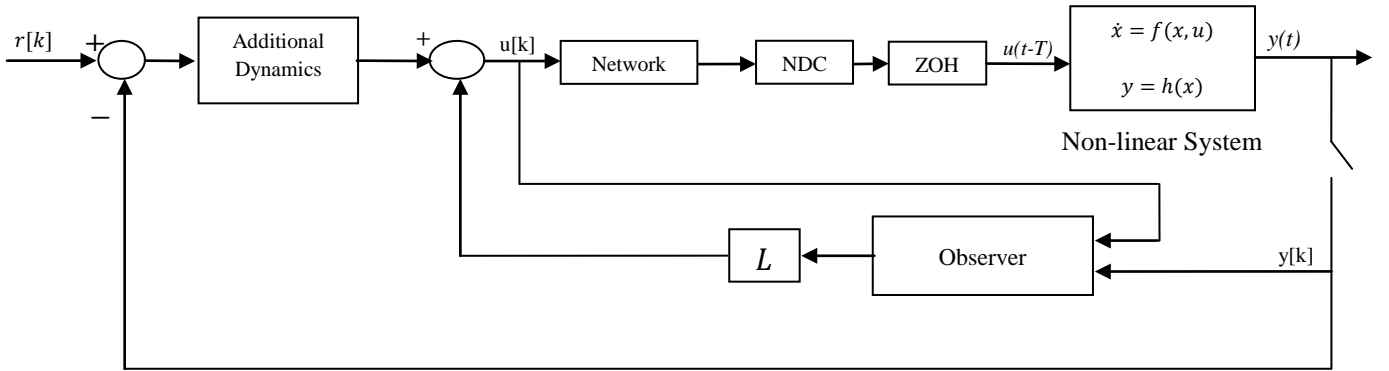


Figure 4.8: Output feedback gain scheduled tracking controller via network

4.6.1.1 Time Invariant State Feedback Gain Scheduled Controller

The tracking task here for a nonlinear system via a network will be satisfied by designing a time invariant state feedback gain scheduled regulator to the time invariant augmented model (4.24). The time invariant vector of designed gains $L_{ztd}(\alpha)$ that assigns the poles of the closed loop system $(\Phi_{td}(\alpha) - \Gamma_{td}(\alpha)L_{ztd}(\alpha))$ at the desired location inside the unit circle has a dimension of $(n+2)$ and it can be partitioned into three parts as follows:

$$L_{ztd}(\alpha) = [L_z(\alpha) \quad L_d(\alpha) \quad L_a(\alpha)]$$

Where $L_z(\alpha)$ is the time invariant feedback gains vector for the designed model of the nonlinear system which has a dimension of n , L_d is the time invariant feedback gain for the delayed state variable, and L_a is the time invariant feedback gain for the state of an additional dynamic system.

4.6.1.2 Time Invariant Gain Scheduled Observer

The equation of the time invariant observer for the discrete time system described in eq(4.19) can be written as

$$\hat{x}_t(k+1) = \Phi_t(\alpha)\hat{x}_t(k) + \Gamma_t(\alpha)u(k) + H_{zt}(\alpha)(y_t(k) - \hat{y}_t(k)) \quad (4.26)$$

Where

$$\Phi_t(\alpha) = \begin{pmatrix} \Phi(\alpha) & \Gamma(\alpha) \\ 0 & 0 \end{pmatrix}, \Gamma_t(\alpha) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \hat{x}_t(k) = \begin{pmatrix} \hat{x}(k) \\ \hat{u}(k-1) \end{pmatrix}$$

The error vector can be defined as:

$$e_t(k) = x_t(k) - \hat{x}_t(k) \quad (4.27)$$

Then, the difference equation of the $e_t(k)$ can be evaluated as:

$$\begin{aligned} e_t(k+1) &= x_t(k+1) - \hat{x}_t(k+1) \\ &= (\Phi_t(\alpha) - H_{zt}(\alpha)C_t(\alpha))e_t(k) \end{aligned} \quad (4.28)$$

If the scheduled matrix $H_{zt}(\alpha)$ designed such that the matrix $(\Phi_t(\alpha) - H_{zt}(\alpha)C_t(\alpha))$ has eigenvalues inside the unit circle, then the estimated states will converge to the actual states of the plant and the error dynamics will reduce to zero with sufficient speed.

The time invariant observer gains $H_{zt}(\alpha)$ that assigns the poles of the closed loop observer $(\Phi_t(\alpha) - H_{zt}(\alpha)C_t(\alpha))$ at the desired location inside the unit circle has a dimension of $(n+1)$ and it can be partitioned into two parts as follows:

$$H_{zt}(\alpha) = [H_z(\alpha) \quad H_d(\alpha)]$$

Where $H_z(\alpha)$ is the observer gains vector for the design model and $H_d(\alpha)$ is the observer gain for the delayed signal.

4.6.2 Time Varying Output Feedback Gain Scheduled Controller

The time varying augmented model that includes the state variables of the additional dynamics and delayed signal into the full system state model and described in eq(4.15), can be rewritten as:

$$\begin{aligned} x_{td}(k+1) &= \Phi_{td}(\alpha, \tau_k^{ca})x_{td}(k) + \Gamma_{td}(\alpha, \tau_k^{ca})u(k) \\ y(k) &= C_{td}(\alpha)x_{td}(k) \end{aligned} \quad (4.29)$$

Where:

$$\begin{aligned} \Phi_{td}(\alpha, \tau_k^{ca}) &= \begin{pmatrix} \Phi(\alpha) & \Gamma_{d1}(\alpha, \tau_k^{ca}) & 0 \\ 0 & 0 & 0 \\ C(\alpha) & 0 & 0 \end{pmatrix}, \quad \Gamma_{td}(\alpha, \tau_k^{ca}) = \begin{pmatrix} \Gamma_{d0}(\alpha, \tau_k^{ca}) \\ 1 \\ 0 \end{pmatrix} \\ C_{td}(\alpha) &= (C(\alpha) \ 0 \ 0), \quad x_{td} = \begin{pmatrix} x(k) \\ u(k-1) \\ x_a(k) \end{pmatrix} \end{aligned} \quad (4.30)$$

The tracking task for a nonlinear system can be achieved by designing a time varying state feedback gain scheduled regulator to the time varying augmented model (4.29). The time varying vector of designed gains $L_{ztd}(\alpha, \tau_k^{ca})$ that assigns the poles of the closed loop system $(\Phi_{td}(\alpha, \tau_k^{ca}) - \Gamma_{td}(\alpha, \tau_k^{ca})L_{ztd}(\alpha, \tau_k^{ca}))$ at the desired location inside the unit circle has two scheduling variables α , and τ_k^{ca} , and also can be partitioned into three parts as follows:

$$L_{ztd}(\alpha, \tau_k^{ca}) = [L_z(\alpha, \tau_k^{ca}) \quad L_d(\alpha, \tau_k^{ca}) \quad L_a(\alpha, \tau_k^{ca})]$$

To build a time varying observer, the time varying scheduled matrix $H_{zt}(\alpha, \tau_k^{ca})$ should be designed such that, the poles of the closed loop observer $(\Phi_t(\alpha, \tau_k^{ca}) - H_{tz}(\alpha, \tau_k^{ca})C_t(\alpha))$ are placed to desired locations inside the unit circle.

4.7 Designing A Time-Invariant Controller With Forward time Delay

The time invariant augmented model for a special case of 2nd order system which described in eq(4.23), can be expressed as:

$$\begin{aligned}x_{id}(k+1) &= \Phi_{id}(\alpha)x_{id}(k) + \Gamma_{id}(\alpha)u(k) \\y(k) &= C_{id}(\alpha)x_{id}(k)\end{aligned}\quad (4.31)$$

Where:

$$\begin{aligned}\Phi_{id} &= \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) & \Gamma_1(\alpha) & 0 \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) & \Gamma_2(\alpha) & 0 \\ 0 & 0 & 0 & 0 \\ c_1(\alpha) & 0 & 0 & 0 \end{pmatrix}, \Gamma_{id} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \\ C_d &= (c_1(\alpha) \ 0 \ 0 \ 0), \quad x_{id} = \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \\ x_a(k) \end{pmatrix}\end{aligned}\quad (4.32)$$

For the sake of brevity, the subscript α from the following matrices in this section will be omitted.

4.7.1 Desired Closed-Loop Pole Locations

The dominant poles for the prototype 2nd order linear system are given by:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_d \quad (4.33)$$

Where:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

When the new two state variables generated from inserting a network in the forward loop of the control system, and introducing an additional dynamic system, two extra desired poles should be included in the desired specifications which are:

$$\begin{aligned}s_3 &= -\beta_1\zeta\omega_n \\s_4 &= -\beta_2\zeta\omega_n\end{aligned}\quad (4.34)$$

Where:

$$\beta_1 \ \& \ \beta_2 \geq 4$$

By using the ZOH POLE-mapping formula, the desired discrete characteristic equation can be derived as :

$$d(z) = z^4 + d_1z^3 + d_2z^2 + d_3z + d_4 \quad (4.35)$$

Where:

$$\begin{aligned}
d_1 &= -\left(2e^{-\zeta\omega_n T_s} \cos(\omega_d T_s) + e^{-\beta_1 \zeta \omega_n T_s} + e^{-\beta_2 \zeta \omega_n T_s}\right) \\
d_2 &= \left(e^{-(\beta_1 + \beta_2) \zeta \omega_n T_s} + e^{-2\zeta \omega_n T_s} + 2\left(e^{-(\beta_1 + 1) \zeta \omega_n T_s} + e^{-(\beta_2 + 1) \zeta \omega_n T_s}\right) \cos(\omega_d T_s)\right) \\
d_3 &= -\left(e^{-(\beta_1 + 2) \zeta \omega_n T_s} + e^{-(\beta_2 + 2) \zeta \omega_n T_s}\right) - 2e^{-(\beta_1 + \beta_2 + 1) \zeta \omega_n T_s} \cos(\omega_d T_s) \\
d_4 &= e^{-(\beta_1 + \beta_2 + 2) \zeta \omega_n T_s}
\end{aligned}$$

The desired poles of the observer must be chosen so that the transient response of the observer is much quicker than the response of the 2nd order controlled system.

The observer desired poles for the 2nd order linear system in s domain are:

$$s_{1,2} = -\beta_3 \zeta \omega_n \pm j \beta_3 j \omega_d \quad (4.36)$$

The extra desired pole which needs to be included in the desired specifications due to augmenting the delayed input into the full state space is:

$$s_3 = -\beta_4 \zeta \omega_n \quad (4.37)$$

Where:

$$\beta_4 \geq 4\beta_3$$

The desired polynomial in a discrete time model for the observer, can be derived as:

$$P_{obs}(z) = z^3 + P_1 z^2 + P_2 z + P_3 \quad (4.38)$$

Where:

$$\begin{aligned}
P_1 &= -\left(2e^{-\beta_3 \zeta \omega_n T} \cos(\beta_3 \omega_d T) + e^{-\beta_4 \zeta \omega_n T}\right) \\
P_2 &= 2e^{-(\beta_3 + \beta_4) \zeta \omega_n T} \cos(\beta_3 \omega_d T) + e^{-2\beta_3 \zeta \omega_n T} \\
P_3 &= e^{-(2\beta_3 + \beta_4) \zeta \omega_n T}
\end{aligned}$$

4.7.2 Designing A Time-Invariant State Feedback Gain Scheduled Controller

The controllability matrix for the original augmented model (4.31) can be derived as:

$$C_{mz} = \begin{pmatrix} 0 & \Gamma_1 & \Phi_{11}\Gamma_1 + \Phi_{12}\Gamma_2 & (\Phi_{11}^2 + \Phi_{12}\Phi_{21})\Gamma_1 + (\Phi_{11}\Phi_{12} + \Phi_{12}\Phi_{22})\Gamma_2 \\ 0 & \Gamma_2 & \Phi_{21}\Gamma_1 + \Phi_{22}\Gamma_2 & (\Phi_{11}\Phi_{21} + \Phi_{21}\Phi_{22})\Gamma_1 + (\Phi_{22}^2 + \Phi_{12}\Phi_{21})\Gamma_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & c_1\Gamma_1 & c_1(\Phi_{11}\Gamma_1 + \Phi_{21}\Gamma_2 + \Gamma_1) \end{pmatrix} \quad (4.39)$$

The transfer function of that augmented model can be evaluated as:

$$\begin{aligned}
TF &= C_d (zI - \Phi_d)^{-1} \Gamma_d \\
&= \frac{\left[c_1 \Gamma_1 z^2 - \left(\frac{c_1 b_2}{a_2} \right) (a_1 \Phi_{12} - \bar{\Phi}_3 - 1) z + \left(\frac{c_1 b_2}{a_2} \right) (\bar{\Phi}_3 + \Phi_{22}) \right]}{z^4 - (2e^{-\lambda T} \cosh(\psi T) + 1) z^3 + (2e^{-\lambda T} \cosh(\psi T) + e^{-2\lambda T}) z^2 - e^{-2\lambda T} z} \quad (4.40)
\end{aligned}$$

Let

$$\begin{aligned}
\bar{\Phi}_1 &= -(2e^{-\lambda T} \cosh(\psi T) + 1) \\
\bar{\Phi}_2 &= 2e^{-\lambda T} \cosh(\psi T) + e^{-2\lambda T} \\
\bar{\Phi}_3 &= -e^{-2\lambda T}
\end{aligned}$$

And

$$\begin{aligned}
\bar{c}_1 &= c_1 \Gamma_1 \\
\bar{c}_2 &= -\left(\frac{c_1 b_2}{a_2} \right) (a_1 \Phi_{12} - \bar{\Phi}_3 - 1) \\
\bar{c}_3 &= \left(\frac{c_1 b_2}{a_2} \right) (\bar{\Phi}_3 + \Phi_{22})
\end{aligned}$$

Then, the transfer function (4.40) can be rewritten as:

$$TF = \frac{\bar{c}_1 z^2 + \bar{c}_2 z + \bar{c}_3}{z^4 + \bar{\Phi}_1 z^3 + \bar{\Phi}_2 z^2 + \bar{\Phi}_3 z} \quad (4.41)$$

From this transfer function, the state space representation of the augmented model in the CCF can be obtained as:

$$\begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \\ \bar{x}_3(k+1) \\ \bar{x}_4(k+1) \end{pmatrix} = \begin{pmatrix} -\bar{\Phi}_1 & -\bar{\Phi}_2 & -\bar{\Phi}_3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \\ \bar{x}_4(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u(k) \quad (4.42)$$

The controllability matrix of the transformed model (4.42) can be derived as:

$$C_{mx} = \begin{pmatrix} 1 & -\bar{\Phi}_1 & \bar{\Phi}_1^2 - \bar{\Phi}_2 & \bar{\Phi}_1 \bar{\Phi}_2 - \bar{\Phi}_1 (\bar{\Phi}_1^2 - \bar{\Phi}_2) - \bar{\Phi}_3 \\ 0 & 1 & -\bar{\Phi}_1 & \bar{\Phi}_1^2 - \bar{\Phi}_2 \\ 0 & 0 & 1 & -\bar{\Phi}_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4.43)$$

Where:

$$C_{mx}^{-1} = \begin{pmatrix} 1 & \bar{\Phi}_1 & \bar{\Phi}_2 & \bar{\Phi}_3 \\ 0 & 1 & \bar{\Phi}_1 & \bar{\Phi}_2 \\ 0 & 0 & 1 & \bar{\Phi}_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4.44)$$

Then, the transformation matrix between the two forms can be derived as:

$$T_c(\alpha) = C_{mz} C_{mx}^{-1} = \begin{pmatrix} 0 & \Gamma_1 & \frac{\bar{c}_2}{c_1} & -\frac{\bar{c}_3}{c_1} \\ 0 & \Gamma_2 & -2\Gamma_2 & \Gamma_2 \\ 1 & \bar{\Phi}_1 & \bar{\Phi}_2 & \bar{\Phi}_3 \\ 0 & 0 & \bar{c}_1 & \bar{c}_3 \end{pmatrix} \quad (4.45)$$

Also, the inverse of the transformation matrix can be calculated as:

$$T_c^{-1}(\alpha) = \begin{pmatrix} \frac{a_2((\Phi_3 - \Phi_1)\bar{c}_1 + (2\Phi_1 + \Phi_2)\bar{c}_3)}{c_1 b_2^2 (2 + \Phi_1 - \Phi_3)^2} & \frac{-a_2((\bar{\Phi}_2 \bar{c}_3 + \bar{\Phi}_3 \bar{c}_1)\bar{c}_1 + \Phi_1 \bar{c}_3^2)}{c_1^2 b_2^2 \Gamma_2 (2 + \Phi_1 - \Phi_3)^2} & 1 & \frac{a_2}{c_1 b_2 (2 + \Phi_1 - \Phi_3)} \\ \frac{-a_2^2 (2\bar{c}_3 - \bar{c}_1)}{c_1 b_2^2 (2 + \Phi_1 - \Phi_3)^2} & \frac{a_2^2 \bar{c}_3^2}{c_1^2 b_2^2 \Gamma_2 (2 + \Phi_1 - \Phi_3)^2} & 0 & \frac{a_2}{c_1 b_2 (2 + \Phi_1 - \Phi_3)} \\ \frac{-a_2^2 \bar{c}_3}{c_1 b_2^2 (2 + \Phi_1 - \Phi_3)^2} & \frac{a_2^2 \Gamma_1 \bar{c}_3}{c_1 b_2^2 \Gamma_2 (2 + \Phi_1 - \Phi_3)^2} & 0 & \frac{a_2}{c_1 b_2 (2 + \Phi_1 - \Phi_3)} \\ \frac{-a_2^2 \bar{c}_1}{c_1 b_2^2 (2 + \Phi_1 - \Phi_3)^2} & \frac{a_2^2 \Gamma_1^2}{b_2^2 \Gamma_2 (2 + \Phi_1 - \Phi_3)^2} & 0 & \frac{a_2}{c_1 b_2 (2 + \Phi_1 - \Phi_3)} \end{pmatrix} \quad (4.46)$$

The corresponding state feedback gain scheduled controller for the augmented model that defined in CCF takes the form:

$$u = -L_{xtd} \bar{x}_{td} = -L_{x1} \bar{x}_1 - L_{x2} \bar{x}_2 - L_{x3} \bar{x}_3 - L_{x4} \bar{x}_4 \quad (4.47)$$

Where:

$$L_{xtd} = (L_{x1} \quad L_{x2} \quad L_{x3} \quad L_{x4})$$

Substituting for u from above equation into eq(4.42), the closed loop system for the augmented model which is found in CCF becomes:

$$\begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \\ \bar{x}_3(k+1) \\ \bar{x}_4(k+1) \end{pmatrix} = \begin{pmatrix} -(\bar{\Phi}_1 + L_{x1}) & -(\bar{\Phi}_2 + L_{x2}) & -(\bar{\Phi}_3 + L_{x3}) & -L_{x4} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \\ \bar{x}_4(k) \end{pmatrix} \quad (4.48)$$

The characteristic equation of closed loop system (4.48) can be computed as:

$$z^4 + (\bar{\Phi}_1 + L_{x1})z^3 + (\bar{\Phi}_2 + L_{x2})z^2 + (\bar{\Phi}_3 + L_{x3})z + L_{x4} \quad (4.49)$$

By comparing the eq (4.35) with the last equation, we obtain

$$\begin{aligned} L_{x1} &= d_1 - \bar{\Phi}_1 \\ L_{x2} &= d_2 - \bar{\Phi}_2 \\ L_{x3} &= d_3 - \bar{\Phi}_3 \\ L_{x4} &= d_4 \end{aligned}$$

The state feedback gains vector for the transformed model (4.42) can be expressed as:

$$L_{xtd}(\alpha) = (d_1 - \bar{\Phi}_1 \quad d_2 - \bar{\Phi}_2 \quad d_3 - \bar{\Phi}_3 \quad d_4) \quad (4.50)$$

Finally, the scheduled gains vector for the original system described in eq(4.31) can be evaluated as follows:

$$L_{ztd}(\alpha) = L_{xtd}(\alpha)T_c^{-1}(\alpha) \quad (4.51)$$

4.7.3 Designing A Time Invariant Gain Scheduled Observer

The augmented model which needs to design an observer for compensating the forward time delay is :

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \Gamma_1 \\ \Phi_{21} & \Phi_{22} & \Gamma_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \end{pmatrix} + b_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(k) \quad (4.52)$$

The observability matrix of the model described above, can be found as:

$$O_{mz} = \begin{pmatrix} C \\ C\Phi \\ C\Phi^2 \end{pmatrix} = \begin{pmatrix} c_1 & 0 & 0 \\ c_1\Phi_{11} & c_1\Phi_{12} & c_1\Gamma_1 \\ c_1(\Phi_{11}^2 + \Phi_{12}\Phi_{21}) & c_1(\Phi_{11}\Phi_{12} + \Phi_{12}\Phi_{22}) & c_1(\Phi_{11}\Gamma_1 + \Phi_{12}\Gamma_2) \end{pmatrix} \quad (4.53)$$

System (4.52) can be transformed into observable canonical form as:

$$\begin{aligned} \begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \\ \bar{x}_3(k+1) \end{pmatrix} &= \begin{pmatrix} -\bar{\Phi}_1 & 1 & 0 \\ -\bar{\Phi}_2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \end{pmatrix} + \begin{pmatrix} \bar{\Gamma}_1 \\ \bar{\Gamma}_2 \\ \bar{\Gamma}_3 \end{pmatrix} u(k) \\ y(k) &= (1 \ 0 \ 0) \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \end{pmatrix} \end{aligned} \quad (4.54)$$

The observability matrix of the transformed system (4.54) can be found as:

$$O_{mx} = C_{mx}^T = \begin{pmatrix} \bar{C} \\ \bar{C}\bar{\Phi} \\ \bar{C}\bar{\Phi}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\bar{\Phi}_1 & 1 & 0 \\ \bar{\Phi}_1^2 - \bar{\Phi}_2 & -\bar{\Phi}_1 & 1 \end{pmatrix} \quad (4.55)$$

Then, the transformation matrix T_o between the two forms is obtained by:

$$T_o(\alpha) = O_{mx}^{-1} O_{mz} = \begin{pmatrix} c_1^2 \Phi_{12} (\Phi_{12} \Gamma_2 - \Phi_{22} \Gamma_1) & 0 & 0 \\ c_1^2 \left(-b_2 \Phi_{12}^2 - \frac{b_2}{a_2} (\bar{\Phi}_2 - \Phi_{22}) \bar{\Phi}_1 + \Gamma_1 \bar{\Phi}_2 \right) & \frac{c_1^2 b_2}{a_2} (\bar{\Phi}_2 - \Phi_{22}) & -c_1^2 \Gamma_1 \\ 0 & 0 & c_1^2 \Phi_{12} \end{pmatrix} \quad (4.56)$$

$$\frac{c_1^3 \Phi_{12} (\Phi_{12} \Gamma_2 - \Phi_{22} \Gamma_1)}{c_1^2 \Phi_{12} (\Phi_{12} \Gamma_2 - \Phi_{22} \Gamma_1)}$$

Define $H_{xt}(\alpha) = (h_{x1} \ h_{x2} \ h_{x3})^T$

The closed loop observer $(\bar{\Phi}_t - H_{xt} \bar{C})$ for the system (4.54) can be found as:

$$(\bar{\Phi}_t - H_{xt} \bar{C}) = \begin{pmatrix} -\bar{\Phi}_1 & 1 & 0 \\ -\bar{\Phi}_2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} - (1 \ 0 \ 0) \begin{pmatrix} h_{x1} \\ h_{x2} \\ h_{x3} \end{pmatrix} = \begin{pmatrix} -(\bar{\Phi}_1 + h_{x1}) & 1 & 0 \\ -(\bar{\Phi}_2 + h_{x2}) & 0 & 1 \\ -h_{x3} & 0 & 0 \end{pmatrix} \quad (4.57)$$

Then, the characteristic equation of the closed loop observer (4.57) is:

$$|sI - (\bar{\Phi}_t - H_{xt} \bar{C})| = z^3 + (\bar{\Phi}_1 + h_{x1})z^2 + (\bar{\Phi}_2 + h_{x2})z + h_{x3} \quad (4.58)$$

Comparing the coefficients of the last characteristic equation with the coefficients of equal power of z in eq(4.38), we require:

$$\begin{aligned}
P_1 &= \bar{\Phi}_1 + h_{x_1} \\
P_2 &= \bar{\Phi}_2 + h_{x_2} \\
P_3 &= h_{x_3}
\end{aligned} \tag{4.59}$$

From equation above, we get,

$$\begin{aligned}
h_{x_1} &= P_1 - \bar{\Phi}_1 \\
h_{x_2} &= P_2 - \bar{\Phi}_2 \\
h_{x_3} &= P_3
\end{aligned} \tag{4.60}$$

Finally, the observer gain matrix for the original system (4.51) is obtained as follows:

$$\begin{aligned}
H_z(\alpha) &= T_o(\alpha) H_x(\alpha) \\
&= \frac{\begin{pmatrix} c_1^2 \Phi_{12} (\Phi_{12} \Gamma_2 - \Phi_{22} \Gamma_1) & 0 & 0 \\ c_1^2 \left(-b_2 \Phi_{12}^2 - \frac{b_2}{a_2} (\bar{\Phi}_2 - \Phi_{22}) \bar{\Phi}_1 + \Gamma_1 \bar{\Phi}_2 \right) & \frac{c_1^2 b_2}{a_2} (\bar{\Phi}_2 - \Phi_{22}) & -c_1^2 \Gamma_1 \\ 0 & 0 & c_1^2 \Phi_{12} \end{pmatrix} \begin{pmatrix} P_1 - \bar{\Phi}_1 \\ P_2 - \bar{\Phi}_2 \\ P_3 \end{pmatrix}}{c_1^3 \Phi_{12} (\Phi_{12} \Gamma_2 - \Phi_{22} \Gamma_1)} \tag{4.61}
\end{aligned}$$

4.8 Designing A Time-Varying Output Feedback Gain Scheduled Tracking Controller

4.8.1 Time Varying State Feedback Gain Scheduled Controller

The time varying augmented model for a special case of 2nd order system described in eq(4.16), can be repeated here as:

$$\begin{aligned}
x_{td}(k+1) &= \Phi_{td}(\alpha, \tau_k^{ca}) x_{td}(k) + \Gamma_{td}(\alpha, \tau_k^{ca}) u(k) \\
y(k) &= C_{td}(\alpha) x_{td}(k)
\end{aligned} \tag{4.62}$$

Where

$$\Phi_{td}(\alpha, \tau_k^{ca}) = \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) & \Phi_{13}(\alpha, \tau_k^{ca}) & 0 \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) & \Phi_{23}(\alpha, \tau_k^{ca}) & 0 \\ 0 & 0 & 0 & 0 \\ c_1(\alpha) & 0 & 0 & 0 \end{pmatrix}, \Gamma_{td}(\alpha, \tau_k^{ca}) = \begin{pmatrix} \Gamma_{d01}(\alpha, \tau_k^{ca}) \\ \Gamma_{d02}(\alpha, \tau_k^{ca}) \\ 1 \\ 0 \end{pmatrix} \quad (4.63)$$

$$, C_d(\alpha) = (c_1(\alpha) \ 0 \ 0 \ 0) \ , \ x_{td} = \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \\ x_a(k) \end{pmatrix}$$

For the sake of brevity, the subscripts α and τ_k^{ca} from the following matrices will be omitted.

The characteristic equation of the augmented model (4.62) can be evaluated as:

$$|zI - \Phi_{td}| = \begin{vmatrix} z - \Phi_{11} & -\Phi_{12} & -\Phi_{13} & 0 \\ -\Phi_{21} & z - \Phi_{22} & -\Phi_{23} & 0 \\ 0 & 0 & z & 0 \\ -c_1 & 0 & 0 & z - 1 \end{vmatrix} \quad (4.64)$$

$$z^4 - (\Phi_{11} + \Phi_{22} + 1)z^3 + (\Phi_{11} + \Phi_{22} + \Phi_{11}\Phi_{22} - \Phi_{12}\Phi_{21})z^2 - (\Phi_{11} + \Phi_{22} - \Phi_{12}\Phi_{21})z$$

After performing some simplifications for the above characteristic equation, the result becomes:

$$|zI - \Phi_{td}| = z^4 + \bar{\Phi}_1 z^3 + \bar{\Phi}_2 z^2 + \bar{\Phi}_3 z \quad (4.65)$$

In which:

$$\begin{aligned} \Phi_1 &= -(\Phi_{11} + \Phi_{22} + 1) \\ \Phi_2 &= \Phi_{11}\Phi_{22} + \Phi_{11} + \Phi_{22} - \Phi_{12}\Phi_{21} \\ \Phi_3 &= -(\Phi_{11} + \Phi_{22} - \Phi_{12}\Phi_{21}) \end{aligned}$$

From the resulted characteristic equation (4.65), it can be seen that the characteristic equations for both time varying and time invariant augmented models are similar. So, when we want to transform the time varying model into a controllable conical form, we arrive to the same result that derived for the time invariant model described in(4.50). Thus, we can use the designed state feedback scheduled gains that derived in eq(4.50) to get the state feedback scheduled gains for the original time varying model described in eq(4.62).

Therefore, the design of a time varying state feedback gain scheduled controller for the time varying model needs only an evaluation of the time varying transformation matrix $T_c(\alpha, \tau_k^{ca})$ that transforms the gains of the augmented model that found in CCF to a time varying scheduled gains of the original augmented model.

The time varying transformation matrix can be derived as:

$$T_c(\alpha, \tau_k^{ca}) = C_{mz}(\alpha, \tau_k^{ca})C_{mx}^{-1}(\alpha) = \begin{pmatrix} \Gamma_{d01} & \Phi_{12}\Gamma_{d02} + \Phi_{13} - \Gamma_{d01}(\Phi_{22} + 1) & \Phi_{22}\Gamma_{d01} - \Phi_{12}\Gamma_{d02} - (1 + \Phi_{22})\Phi_{13} + \Phi_{12}\Phi_{23} & \Phi_{22}\Phi_{13} - \Phi_{12}\Phi_{23} \\ \Gamma_{d02} & \Phi_{21}\Gamma_{d01} + \Phi_{23} - \Gamma_{d02}(\Phi_{11} + 1) & -\Phi_{21}\Gamma_{d01} + \Phi_{11}\Gamma_{d02} - (1 + \Phi_{11})\Phi_{23} + \Phi_{21}\Phi_{13} & \Phi_{11}\Phi_{23} - \Phi_{21}\Phi_{13} \\ 1 & \bar{\Phi}_1 & \bar{\Phi}_2 & \bar{\Phi}_3 \\ 0 & c_1\Gamma_{d01} & c_1(-\Phi_{22}\Gamma_{d01} + \Phi_{12}\Gamma_{d02} + \Phi_{13}) & c_1(\Phi_{12}\Phi_{23} - \Phi_{22}\Phi_{13}) \end{pmatrix}$$

Then, the time varying scheduled gains can be evaluated as:

$$L_{ztd}(\alpha, \tau_k^{ca}) = L_{xtd}(\alpha)T_c^{-1}(\alpha, \tau_k^{ca}) \quad (4.66)$$

Where

$L_{xtd}(\alpha)$ is the time invariant scheduled gains vector for the model that found in CCF.

The goal of ensuring that the system (4.62) is stable can be achieved by using this type of compensation scheme, because at every sampling instant, the closed loop system has the same transfer function, which means that the poles of the closed loop system will be placed at every sampling instant to the same desired locations inside the unit circle.

4.8.2 Designing A Time Varying Gain Scheduled Observer

The time varying augmented model which needs to design time varying observer for compensating the forward time delay is :

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ u(k-1) \end{pmatrix} + b_2 \begin{pmatrix} \Gamma_{d1} \\ \Gamma_{d2} \\ 1 \end{pmatrix} u(k) \quad (4.67)$$

The observability matrix of the original model can be derived as:

$$O_{mz}(\alpha, \tau_k^{ca}) = \begin{pmatrix} c_1 & 0 & 0 \\ c_1 \Phi_{11} & c_1 \Phi_{12} & c_1 \Phi_{13} \\ c_1 (\Phi_{11}^2 + \Phi_{12} \Phi_{21}) & c_1 (\Phi_{11} \Phi_{12} + \Phi_{12} \Phi_{22}) & c_1 (\Phi_{11} \Phi_{13} + \Phi_{12} \Phi_{23}) \end{pmatrix} \quad (4.68)$$

The augmented model (4.67) has the same representation in OCF to that of time invariant model described in eq(4.52), so there is only a need to evaluate the time varying transformation matrix $T_o(\alpha, \tau_k^{ca})$ that transforms the designed observer gains for the time invariant augmented model that found in OCF to the time varying observer scheduled gains for the original model(4.67).

The transformation matrix between two forms can be derived as:

$$T_o = O_{mz}^{-1} O_{mx} = \begin{pmatrix} c_1 & 0 & 0 \\ c_1 \Phi_{11} & c_1 \Phi_{12} & c_1 \Phi_{13} \\ c_1 (\Phi_{11}^2 + \Phi_{12} \Phi_{21}) & c_1 (\Phi_{11} \Phi_{12} + \Phi_{12} \Phi_{22}) & c_1 (\Phi_{11} \Phi_{13} + \Phi_{12} \Phi_{23}) \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 & 0 \\ -\bar{\Phi}_1 & 1 & 0 \\ \bar{\Phi}_1^2 - \Phi_2 & -\bar{\Phi}_1 & 1 \end{pmatrix} \quad (4.69)$$

$$\begin{pmatrix} \begin{pmatrix} 1 \\ c_1 \end{pmatrix} & 0 & 0 \\ \frac{a_2 \Phi_{12} \Gamma_2 + b_2 (\Phi_2 - \Phi_{22}) \Phi_1 + \Gamma_1 \Phi_2}{c_1 \Phi_{21} (\Phi_{12} \Gamma_2 - \Phi_{22} \Gamma_1)} & \frac{\Phi_{22} - \bar{\Phi}_2}{c_1 \Phi_{21} (\Phi_{12} \Gamma_2 - \Phi_{22} \Gamma_1)} & \frac{-\Gamma_1}{c_1 \Phi_{12} (\Phi_{12} \Phi_{23} - \Phi_{22} \Phi_{13})} \\ 0 & 0 & \frac{1}{c_1 (\Phi_{12} \Phi_{23} - \Phi_{22} \Phi_{13})} \end{pmatrix}$$

CHAPTER 5

5 MAGNETIC BALL LEVITATION CE152

5.1 Introduction To Magnetic Levitation System

The Magnetic levitation system is an example of nonlinear, open loop unstable system with fast dynamics. For these properties of the Magnetic levitation system, modeling and mainly control design is very difficult. However, Magnetic levitation system has wide application in various industries than high-speed trains, frictionless bearing, etc and therefore this field of research is devoted significant effort in recent years[21]. Magnetic levitation system has the advantages of being contact-free, can eliminate the mechanical components, reduce the mechanical alignment and satisfies the environmental demands[22]. Magnetic levitation system has been also used for educational purpose in teaching students on the concept of feedback control. Thus a lot of studies have been conducted for the control of magnetic levitation systems[23].

5.2 Magnetic Levitation Model CE 152

The CE 152 Magnetic Levitation Model is one of the range of educational scale models offered by Humusoft for teaching system dynamics and control engineering principles. The Magnetic Levitation Model and the associated manual are teaching aid for control engineering students at all levels and the experiments cover wide range of problems which appear in the industry. The CE 152 Magnetic Levitation Model is one of a unique range of products designed for the theoretical study and practical investigation of basic and advanced control engineering principles. This includes system dynamics modeling, identification, analysis and various controllers design by classical and modern methods[24], the CE 152 Magnetic Levitation apparatus is shown in fig(5.1).



Figure 5.1: CE152 magnetic ball levitation.

The objective of this chapter is to derive the nonlinear model of the Magnetic Ball Levitation CE 152 and using the linearization technique to analyze the nonlinear model of the device, fig(5.2) shows the principal scheme of the magnetic levitation model[24].

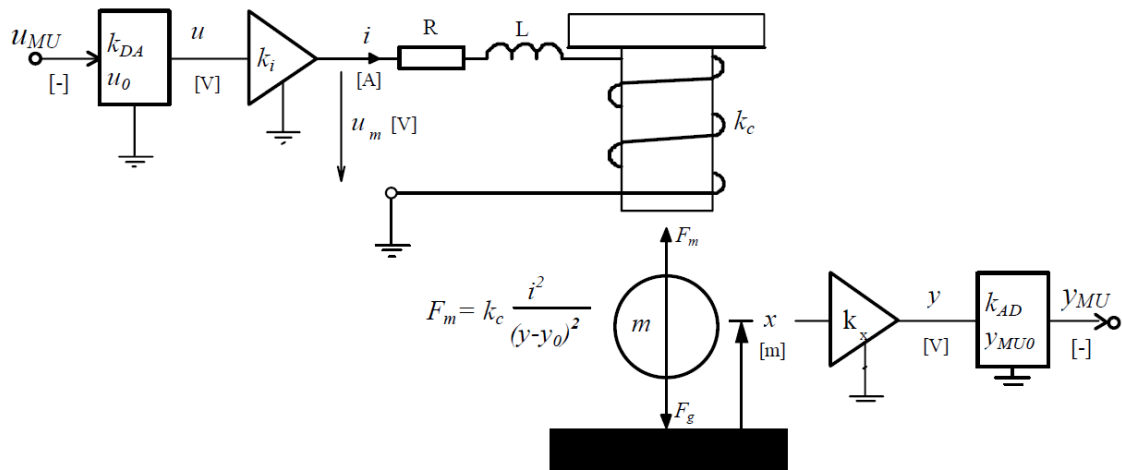


Figure 5.2: Principal scheme of the magnetic levitation model.

The CE152 model, shown in Fig. (5.2) consists of the following blocks[24]:

- D/A converter.
- Power amplifier.
- Ball & coil subsystem.
- Position sensor.
- A/D converter.

5.2.1 Power Amplifier Equation

As described above, the power amplifier is one of the five parts of the magnetic ball levitation model which can be considered as a source of constant current. The simplified equation of this part that needed for modeling the Maglev apparatus is[24]:

$$i = k_i * u \quad (5.1)$$

5.3 State Space Representation

There is no general analysis techniques are applicable to all nonlinear systems, so, the linearization tool is used to derive the linear state space model for the nonlinear differential equation of the magnetic levitation CE152 to simplify the design process of the controller.

From[4], the nonlinear motion equation is given by,

$$m\ddot{x} + k_{fv}\dot{x} = \frac{i^2 k_c}{(x - x_0)^2} - mg \quad (5.2)$$

Assume the state variables x_1 and x_2 are defined by the equations

$$x_1(t) = x \quad (5.3)$$

$$x_2(t) = \dot{x} \quad (5.4)$$

Substituting from eq(5.3) and (5.4) into eq(5.2), then the nonlinear state equations of the system are expressed as,

$$\frac{dx_1(t)}{dt} = x_2(t) = f_1(x_1, x_2, u) \quad (5.5)$$

$$\frac{dx_2(t)}{dt} = \frac{i^2 k_c}{m(x_1 - x_0)^2} - \frac{k_{fv}}{m} x_2 - g = f_2(x_1, x_2, u) \quad (5.6)$$

Where the functions f_1 and f_2 are defined by these equations. The general vector form of these equations can be expressed as:

$$\dot{x} = f(x, u) \quad (5.7)$$

Where

$$f = (f_1 \quad f_2)^T$$

Let $p=(x_{ss},u_{ss})$ is an operating point of the nonlinear system (5.5)-(5.6) where $x_{ss}=(x_{1ss},x_{2ss})$. If the Taylor series is valid about the operating point (x_{ss},u_{ss}) , the equations (5.5) and (5.6) can be linearized and the linearized equations can be written in the standard vector matrix format,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (5.8)$$

Where

$$\mathbf{A} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{x_{ss}, u_{ss}} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix}_{x_{1ss}, x_{2ss}, u_{ss}} \quad (5.9)$$

$$\mathbf{B} = \left. \frac{\partial \mathbf{f}}{\partial u} \right|_{x_{ss}, u_{ss}} = \begin{pmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{pmatrix}_{x_{1ss}, x_{2ss}, u_{ss}} \quad (5.10)$$

Where the matrices (5.2) and (5.10) are called the *Jacobian matrices*.

By setting the equations (5.5) and (5.6) to be zero , we get

$$x_{2ss} = 0 \quad (5.11)$$

$$u_{ss} = \sqrt{\frac{mg}{k_f k_{DA}^2}} (x_{1ss} - x_0)^2 = \frac{mg}{k_f k_{DA}^2} (x_0 - x_{1ss}) \quad (5.12)$$

After applying the linearization about the equilibrium point that found in eq(5.11) and (5.12), we have:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ \frac{2g}{(x_{L0} - x_{1ss})} & -\frac{k_{fv}}{m_k} \end{pmatrix} \quad (5.13)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ \frac{2k_{DA} \sqrt{k_F g}}{\sqrt{m_k} (x_{L0} - x_{1ss})} \end{pmatrix} \quad (5.14)$$

From [4], the system output can be written as:

$$y = k_x k_{AD} x_1 \quad (5.15)$$

Then, the linear state space model for the magnetic ball levitation can be written as :

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{2g}{(x_{L0} - x_{1ss})} & -\frac{k_{fv}}{m_k} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{2k_{DA}\sqrt{k_F g}}{\sqrt{m_k}(x_{L0} - x_{1ss})} \end{pmatrix} U_{MU} \quad (5.16)$$

$$Y_{MU} = \begin{pmatrix} k_x k_{AD} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Using the values of parameters in table (5.1) of the magnetic levitation CE152, the linearized model becomes:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{19.62}{.00826 - x_{1ss}} & -2.381 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{.53206}{(.00826 - x_{1ss})} \end{pmatrix} U_{MU} \quad (5.17)$$

$$Y_{MU} = \begin{pmatrix} 159.49206 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The linear state space model for the magnetic ball levitation described in (5.17) is similar to a special case of the second order system that considered in the thesis assumption as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2 \end{pmatrix} u \quad (5.18)$$

$$y = \begin{pmatrix} c_1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Where $a_2 = \frac{-19.62}{.00826 - x_{1ss}}$, $a_1 = 2.381$, $b_2 = \frac{.53206}{(.00826 - x_{1ss})}$, $c_1 = 159.49206$

From system model (5.18), we conclude that all approaches introduced in this thesis for designing controllers to several configurations of the control system such as : continuous, discrete and networked control system models is valid to be applied for the magnetic ball levitation CE152.

5.4 Simulation

The final interconnected blocks of the magnetic levitation model CE 152 are built on MATLAB SIMULINK as shown in fig.(5.3),

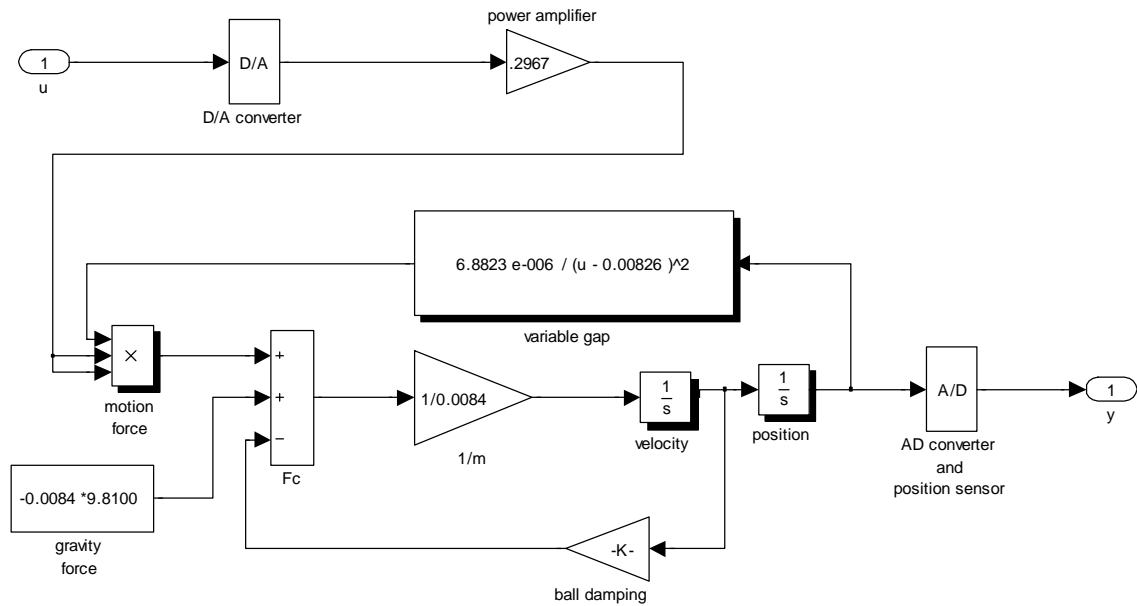


Figure 5.3: The interconnected blocks of magnetic levitation CE152

The simplify the use of magnetic ball levitation plant in the simulation process, we built a simulation model to the called maglev CE152 that includes all interconnected blocks as shown in fig(5.4)

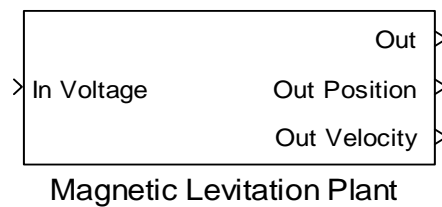


Figure 5.4: The block diagram of magnetic levitation CE152

The closed loop system without controller is built on MATLAB SIMULINK tool as shown in fig(5.5). it's step response is also shown in fig(5.6).

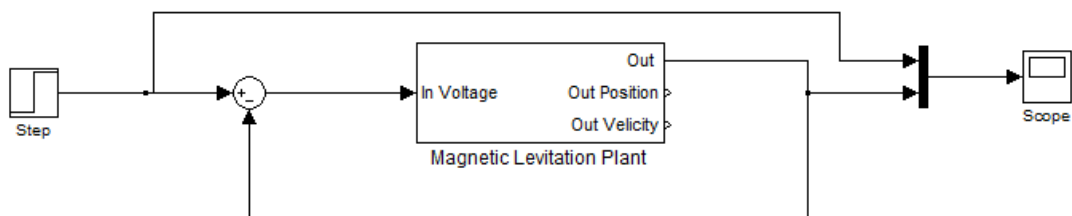


Figure 5.5: The closed loop of the magnetic ball levitation.

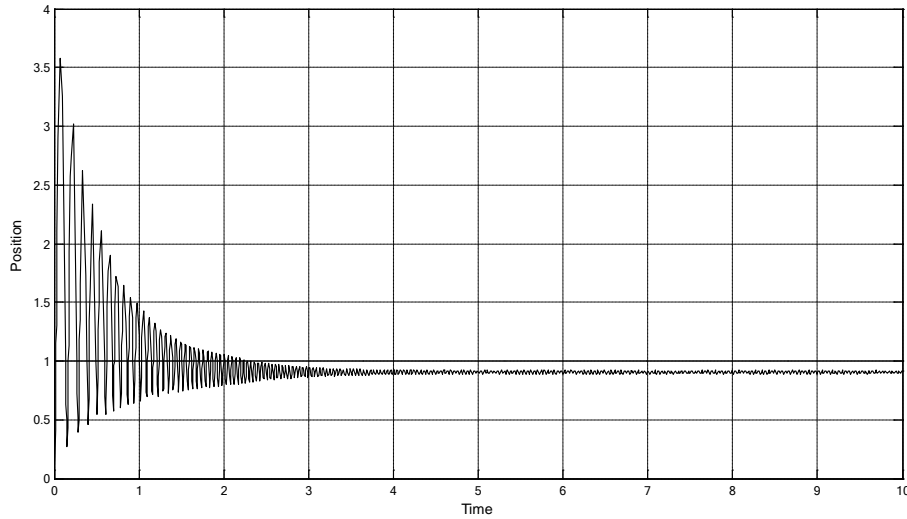


Figure 5.6: Step response of the closed loop transfer function of the magnetic ball levitation.

From the step response in the figure above, we note that it is necessary to design a controller to obtain the desired characteristic that a closed loop control system should have.

Table 5.1: Parameters of magnetic ball levitation CE 152

Parameter	Symbol	Value
ball diameter	Dk	12.7x10 ⁻³ m
ball mass	mk	0.0084 kg
distance from the ground and the edge of the magnetic coil	Td	0.019 m
distance of limits= 0.019 - Dk	L	0.0063 m
gravity acceleration constant	g	9.81 m.s ⁻²
maximum DA converter output voltage	U_DAm	5 V
coil resistance	Rc	3.5 Ω
coil inductance	Lc	30 x10 ⁻³ H
current sensor resistance	Rs	0.25 Ω
current sensor gain	Ks	13.33
power amplifier gain	K_am	100
maximum power amplifier output current	I_am	1.2 A
amplifier time constant= $Lc/((Rc+Rs)+Rs*Ks*K_am)$	Ta	1.8694 x10 ⁻⁵ s

amplifier gain= $K_{am} / ((R_c+R_s)+R_s*K_s*K_{am})$	k_i	0.2967
viscose friction	KFv	0.02 N.s/m
converter gain	k_{DA}	10
Digital to Analog converter offset	u_0	0 V
Analog to Digital converter gain	k_{AD}	0.2
Analog to Digital converter offset	y_{MU0}	0 V
position sensor constant	k_x	797.4603
coil bias	x_0	8.26×10^{-3} m
Aggregated coil constant	k_f	0.606×10^{-6} N/V
coil constant = $k_f/(k_i)^2$	k_c	6.8823×10^{-6} N/V

6 SIMULATION AND RESULTS

6.1 Introduction

In this chapter, the proposed methodology for tracking a nonlinear system via a network has been tested on a simulation model for the magnetic ball levitation CE152. The simulation work has been performed on four cases of the control system using MATLAB/SIMULINK tool. These cases are:

- 1- Continuous output feedback control system.
- 2- Digital output feedback control system.
- 3- Digital output feedback control system without time delay compensation.
- 4- Digital output feedback control system with time delay compensation.

6.2 Continuous Output Feedback Gain Scheduled Control System

Now we want to apply the procedures for designing a continuous output feedback gain scheduled tracking controller that derived in chapter 2 for a special case of 2nd order nonlinear system to the magnetic ball levitation CE152 apparatus.

From the previous chapter, the linearized state space model that derived for the magnetic ball levitation CE152 is described by:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{19.62}{(0.00826 - x_{1ss}(\alpha))} & -2.381 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{0.53206}{(0.00826 - x_{1ss}(\alpha))} \end{pmatrix} u \quad (6.1)$$

$$y = (159.49206 \quad 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

When $r = \alpha = \text{constant value}$, the controlled output y follows the reference input and in this case, we have

$$x_{1ss}(\alpha) = \frac{y}{c_1} = \frac{\alpha}{c_1} = \frac{\alpha}{159.49206} \quad (6.2)$$

Substituting for $x_{1ss}(\alpha)$ from (6.2) into eq(6.1), the parameterized state space model becomes,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{3129.234217}{(1.3174-\alpha)} & -2.381 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{84.86}{(1.3174-\alpha)} \end{pmatrix} u \quad (6.3)$$

$$y = (159.49206 \quad 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

And the scheduled steady state input $u_{ss}(\alpha)$ is expressed by :

$$u_{ss}(\alpha) = \sqrt{\frac{mg}{k_F}} (x_{1ss}(\alpha) - x_{L0}) = 368.755 (x_{1ss}(\alpha) - 0.00826) \quad (6.4)$$

The linearized state space model for a special case of 2nd order nonlinear system is described by eq(2.35) in chapter 2, repeated here:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_2(\alpha) & -a_1(\alpha) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_2(\alpha) \end{pmatrix} u \quad (6.5)$$

$$y = (c_1(\alpha) \quad 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

When equating system (6.3) with (6.5), we get

$$\begin{aligned} a_1(\alpha) &= 2.381 \\ a_2(\alpha) &= \frac{-3129.234217}{(1.3174-\alpha)} \\ b_2(\alpha) &= \frac{84.86}{(1.3174-\alpha)} \\ c_1(\alpha) &= 159.49206 \end{aligned} \quad (6.6)$$

The augmented state space model of the magnetic ball levitation CE152 due to introducing an integral control to the forward path of the control system, becomes:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\sigma} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ \frac{3129.234217}{(1.3174-\alpha)} & -2.381 & 0 \\ 159.49206 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \sigma \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{84.86}{(1.3174-\alpha)} \\ 0 \end{pmatrix} u \quad (6.7)$$

$$y = (159.49206 \quad 0 \quad 0) \begin{pmatrix} x_1 \\ x_2 \\ \sigma \end{pmatrix}$$

6.2.1 Desired Performance Specifications

The performance specifications for the magnetic ball levitation CE 152 are assumed to be:

- 1- Maximum overshoot $OS \leq 5$ percent.
- 2- Settling time $t_s \leq 0.1$ sec.

From these selected specifications, we can evaluate the values of the natural frequency ω_n and the damping ratio ζ that are needed in the calculations of gains vectors for both state feedback controller and observer.

$$\zeta = \frac{-\ln(OS \%)}{\sqrt{\pi^2 + \ln(OS \%)^2}} = 0.6901 \quad (6.8)$$

$$\omega_n = \frac{4}{\zeta t_s} = 57.862$$

From chapter 2, the desired characteristic equations for the closed loop system and the observer are given by:

$$P(s) = s^3 + (\beta_1 + 2)\zeta\omega_n s^2 + (1 + 2\beta_1\zeta^2)\omega_n^2 s + \beta_1\zeta\omega_n^3 \quad (6.9)$$

$$P_o(s) = s^2 + 2\beta_0\zeta\omega_n s + \beta_0^2\omega_n^2 \quad (6.10)$$

We have selected the factor $\beta_1 = 10$, which means that the third pole is located at ten times where the real part of the roots of a prototype 2nd order polynomial are located, and to ensure that the desired transient response of the observer is much faster than that of the plant, we selected the factor $\beta_0 = 10$.

After substituting for the values β_0, β_1, ζ , and ω_n into eq(6.9) and eq(6.10), then the desired characteristic equation for the closed loop system and observer become,

$$P(s) = s^3 + 480s^2 + 35360s + 1343865.854 \quad (6.11)$$

$$P_o(s) = s^2 + 800s + 335970 \quad (6.12)$$

6.2.2 Continuous State Feedback Gain Scheduled Controller

The designed state feedback scheduled gains for a special case of 2nd order system that derived in chapter 2 are repeated here:

$$k_{z1}(\alpha) = \frac{(1 + 2\beta_1\zeta^2)\omega_n^2 - a_2(\alpha)}{b_2(\alpha)} \quad (6.13)$$

$$k_{z2} = \frac{(\beta_1 + 2)\zeta\omega_n - a_1(\alpha)}{b_2(\alpha)} \quad (6.14)$$

$$k_{\sigma} = \frac{\beta_1\zeta\omega_n^3}{c_1b_2(\alpha)} \quad (6.15)$$

After substituting the value of β_1 and the values from (6.6), and (6.8) into (6.13), (6.14), and (6.15), the scheduled gains for the state feedback controller are evaluated as:

$$\begin{aligned} k_{z1}(\alpha) &= 416.6856(1.4059 - \alpha) \\ k_{z2}(\alpha) &= 5.62836(1.3174 - \alpha) \\ k_{\sigma}(\alpha) &= 99.2927(1.3174 - \alpha) \end{aligned} \quad (6.16)$$

6.2.3 Gain Scheduled Observer

The designed scheduled gains of the observer that derived in chapter 2 are repeated here:

$$h_{z1}(\alpha) = \frac{2\beta_0\zeta\omega_n - a_1}{c_1} \quad (6.17)$$

$$h_{z2}(\alpha) = \frac{\beta_0^2\omega_n^2 - a_2(\alpha) - 2a_1\beta_0\zeta\omega_n + a_1^2}{c_1} \quad (6.18)$$

After substituting the value of β_0 and the values from (6.6), and (6.8) into (6.17), and (6.18), the observer scheduled gains are computed as:

$$h_{z1}(\alpha) = 5.0010 \quad (6.19)$$

$$h_{z2}(\alpha) = 2094.6 + \frac{19.62}{(1.3174 - \alpha)} \quad (6.20)$$

6.2.4 MATLAB Simulation For The Continuous Output Feedback Control

System

The gain scheduled output feedback control has been implemented on MATLAB SIMULINK Program for the magnetic ball levitation CE152 simulation model as shown in fig.(6.1). Different types of reference signals can be applied to the implemented closed loop system, the responses of some signals are described below :

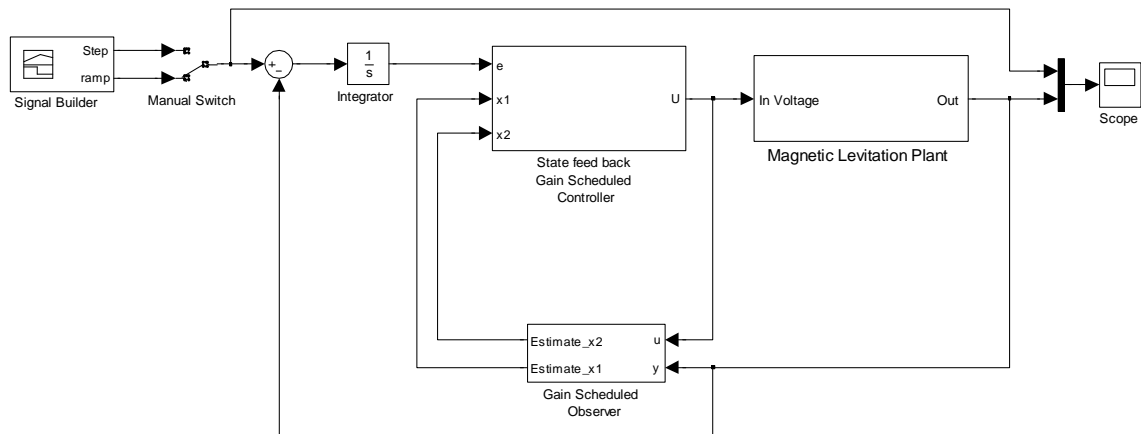


Figure 6.1: Block diagram of output feedback gain scheduled control for maglev CE152

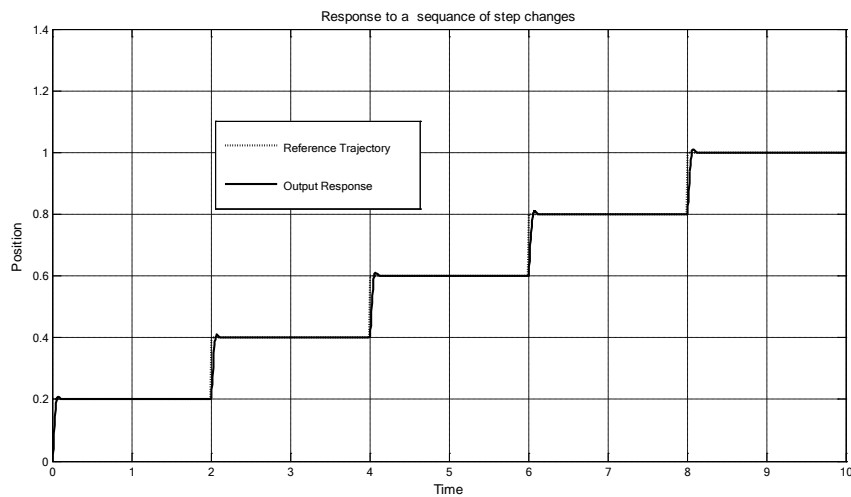


Figure 6.2: Output response to a sequence of step changes in the reference signal for Maglev CE 152

Fig.(6.2) shows how the controlled output tracks the sequence of step changes in the reference signal to reach the maximum value of position.

An alternative method to change the reference set point to move slowly from one point to another is that using a slow ramp signal as an input, Fig.(6.3) shows the

response of the closed loop system under slowly varying scheduling variable with slope =0.1.

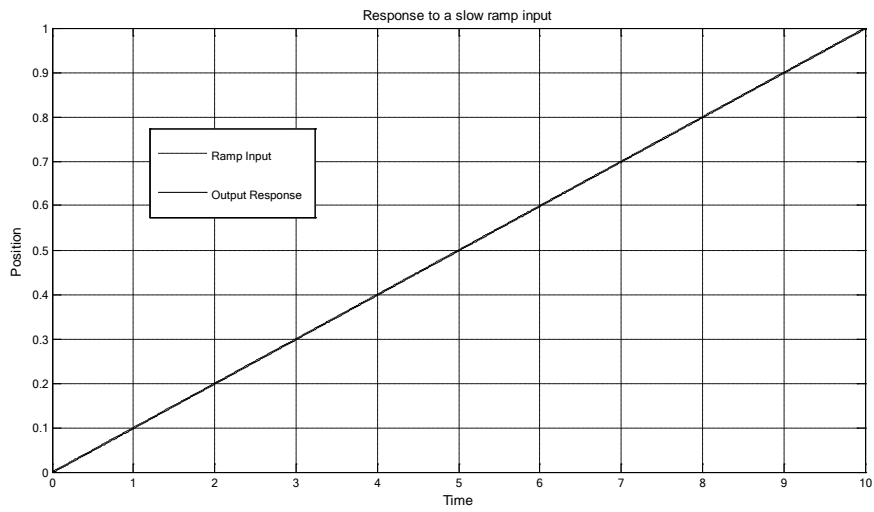


Figure 6.3: The slow ramp input and the output response of output feedback gain scheduled control for Maglev CE 152

Fig.(6.4), shows that how the output response signal can follow the square wave reference input.

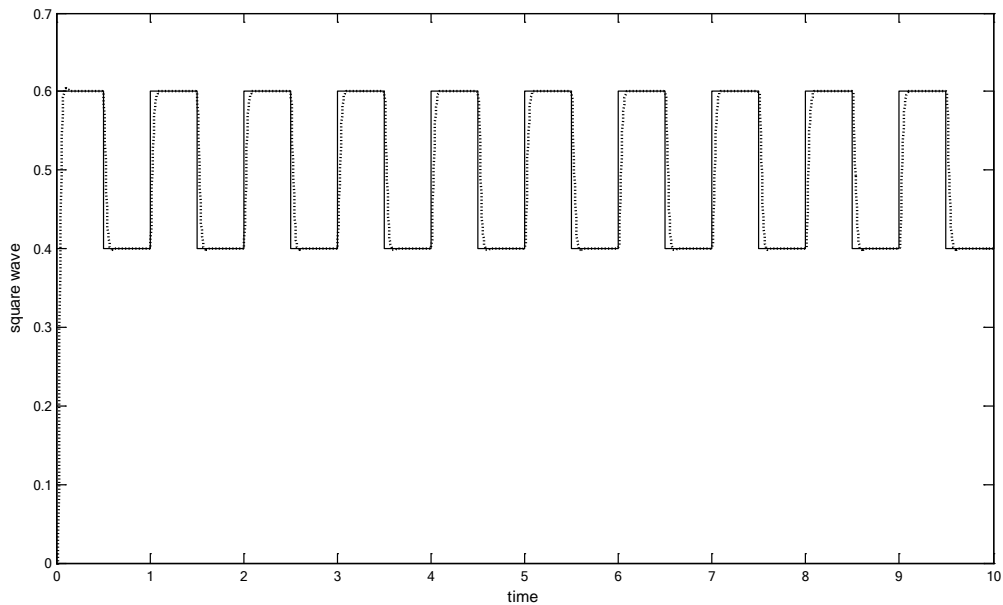


Figure 6.4: The square wave input (solid) and output (dashed) of gain scheduled output feedback control for Maglev CE 152

6.3 Digital Output Feedback Gain Scheduled Control System

To illustrate the effectiveness and usefulness of the work presented for designing a digital output feedback controller for a special case of 2nd order system. The procedure of designing such controller is applied on a simulation model for the magnetic ball levitation CE152 apparatus, and the results are shown subsequently.

6.3.1 ZOH Equivalent Model For The Magnetic Ball Levitation CE152

From chapter 3, the parameterized discrete state space model for a special case of 2nd order system is given by:

$$\begin{aligned} \begin{pmatrix} x_1(k+1) \\ x_1(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_1(k) \end{pmatrix} + \begin{pmatrix} \Gamma_1(\alpha) \\ \Gamma_2(\alpha) \end{pmatrix} u(k) \\ y(k) &= (c_1(\alpha) \ 0) \begin{pmatrix} x_1(k) \\ x_1(k) \end{pmatrix} \end{aligned} \quad (6.21)$$

Where:

$$\begin{aligned} \Phi_{11}(\alpha) &= e^{-\lambda(\alpha)T} \left(\cosh(\psi(\alpha)T) + (\lambda(\alpha)/\psi(\alpha)) \sinh(\psi(\alpha)T) \right) \\ \Phi_{12}(\alpha) &= \frac{e^{-\lambda(\alpha)T} \sinh(\psi(\alpha)T)}{\psi(\alpha)} \\ \Phi_{21}(\alpha) &= \frac{-a_2(\alpha) e^{-\lambda(\alpha)T} \sinh(\psi(\alpha)T)}{\psi(\alpha)} \\ \Phi_{22}(\alpha) &= e^{-\lambda(\alpha)T} \left(\cosh(\psi(\alpha)T) - (\lambda(\alpha)/\psi(\alpha)) \sinh(\psi(\alpha)T) \right) \\ \Gamma(\alpha) &= \begin{pmatrix} \Gamma_1(\alpha) \\ \Gamma_2(\alpha) \end{pmatrix} = \begin{pmatrix} \frac{b_2(\alpha)}{a_2(\alpha)} [1 - \Phi_{11}(\alpha)] \\ b_2(\alpha) \Phi_{12}(\alpha) \end{pmatrix} \end{aligned} \quad (6.22)$$

And

$$\lambda = \frac{a_1}{2}, \quad \psi = \frac{1}{2} \sqrt{a_1^2 - 4a_2} \quad (6.23)$$

Substituting from eq(6.6) for $a_1(\alpha)$ and $a_2(\alpha)$ into eq(6.23), the result is:

$$\lambda = 1.1905 \quad \text{and} \quad \psi(\alpha) = \sqrt{\frac{3131.10 - 1.41729\alpha}{(1.3174 - \alpha)}} \quad (6.24)$$

After substituting for the above result into eq(6.22), we have:

$$\begin{aligned}
\Phi_{11}(\alpha) &= 0.98816558 \left(\cosh \left(\frac{\psi(\alpha)}{100} \right) + (1.1905/\psi(\alpha)) \sinh \left(\frac{\psi(\alpha)}{100} \right) \right) \\
\Phi_{12}(\alpha) &= 0.98816558 \left(\frac{\sinh(\psi(\alpha)/100)}{\psi(\alpha)} \right) \\
\Phi_{21}(\alpha) &= \frac{3092.201558}{(1.3174-\alpha)} \left(\frac{\sinh(\psi(\alpha)/100)}{\psi(\alpha)} \right) \\
\Phi_{22}(\alpha) &= 0.98816558 \left(\cosh \left(\frac{\psi(\alpha)}{100} \right) - (1.1905/\psi(\alpha)) \sinh \left(\frac{\psi(\alpha)}{100} \right) \right) \\
\Gamma_1(\alpha) &= 0.027 \left[1 - 0.988166 \left(\cosh \left(\frac{\psi(\alpha)}{100} \right) + (1.1905/\psi(\alpha)) \sinh \left(\frac{\psi(\alpha)}{100} \right) \right) \right] \\
\Gamma_2(\alpha) &= \frac{83.85573147}{(1.3174-\alpha)} \left(\frac{\sinh(\psi(\alpha)/100)}{\psi(\alpha)} \right)
\end{aligned} \tag{6.25}$$

From [25], the value of α is varied within the interval $[0,1]$. The transfer function of this model at any value of α can be computed by using the following matlab code:

```

a=.2;% scheduling parameter
Ts=.01; % sampling time
A=[0 1;3129.2342172/(1.3174044156-a) -2.381];
B=[0;84.86/(1.3174044156-a)];
C=[159.49206 0]
a2=-A(2,1);
a1=-A(2,2);
c1=C(1);
b2=B(2);
r=a1/2;
u=(sqrt((a1^2)-4*a2))/2;
PH11=exp(-r*Ts)*(cosh(u*Ts)+((r/u)*sinh(u*Ts)));
PH12=exp(-r*Ts)*sinh(u*Ts)/u;
PH21=-a2*exp(-r*Ts)*sinh(u*Ts)/u;
PH22=exp(-r*Ts)*(cosh(u*Ts)-((r/u)*sinh(u*Ts)));
PH=[PH11 PH12;PH21 PH22];
TH=[(b2/a2)*(1-PH11);B(2)*PH12];
[NdDd]=SS2TF(PH,TH,C,0);
Sys_discrete =Tf(Nd,Dd,Ts)

```

The result from matlab when $\alpha = 0.2$ is:

$$\frac{0.615 z + 0.6102}{z^2 - 2.26 z + 0.9765} \quad (6.26)$$

For model validating, we find the discrete transfer function form the continuous version at $\alpha = 0.2$ by using the following matlab workspace commands:

```
a=0.2;
Ts=.01;
A=[0 1;3129.2342172/(1.3174044156-a) -2.381];
B=[0;84.86/(1.3174044156-a)];
C=[159.49206 0]
[num den]=SS2TF(A,B,C,0);
Sc=tf(num,den);
Sd=C2D(Sc,Ts,'zoh')
```

The result from matlab is the transfer function:

$$\frac{0.615 z + 0.6102}{z^2 - 2.26 z + 0.9765} \quad (6.27)$$

It can be seen easily that the two resulted transfer functions (6.26), and (6.27) are the same, which means that the derived ZOH model for a special case of 2nd order system that derived in this thesis is an accurate model.

When introducing the additional dynamics system that described in eq(3.27) as a part of the compensator to achieve zero steady state error, the augmented discrete state space model for the magnetic ball levitation can be written as:

$$\begin{aligned} \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_a(k+1) \end{pmatrix} &= \begin{pmatrix} \Phi_{11}(\alpha) & \Phi_{12}(\alpha) & 0 \\ \Phi_{21}(\alpha) & \Phi_{22}(\alpha) & 0 \\ 159.49206 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_a(k) \end{pmatrix} + \begin{pmatrix} \Gamma_1(\alpha) \\ \Gamma_2(\alpha) \\ 0 \end{pmatrix} u(k) \\ y &= (159.49206 \ 0 \ 0) \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_a(k) \end{pmatrix} \end{aligned} \quad (6.28)$$

Where the values $\Phi_{11}(\alpha)$, $\Phi_{12}(\alpha)$, $\Phi_{21}(\alpha)$ and $\Phi_{22}(\alpha)$ are found in eq(6.25)

From eq(3.31) in chapter 3, the CCF representation for the augmented model is derived as:

$$\begin{pmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \\ \bar{x}_a(k+1) \end{pmatrix} = \begin{pmatrix} -\bar{\Phi}_1 & -\bar{\Phi}_2 & -\bar{\Phi}_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_a(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u(k) \quad (6.29)$$

$$y = \begin{pmatrix} \bar{c}_1 & \bar{c}_2 & \bar{c}_3 \end{pmatrix} \begin{pmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_a(k) \end{pmatrix}$$

Where:

$$\begin{aligned} \bar{\Phi}_1 &= -(\Phi_{11} + \Phi_{22} + 1) = -(2e^{-\lambda T} \cosh(\psi T) + 1) \\ \bar{\Phi}_2 &= \Phi_{11}\Phi_{22} + \Phi_{11} + \Phi_{22} - \Phi_{12}\Phi_{21} = 2e^{-\lambda T} \cosh(\psi T) + e^{-2\lambda T} \\ \bar{\Phi}_3 &= -\Phi_{11}\Phi_{22} + \Phi_{12}\Phi_{21} = -e^{-2\lambda T} \\ \bar{c}_1 &= c_1\Gamma_1 \\ \bar{c}_2 &= \left(\frac{c_1 b_2}{a_2}\right)(a_1\Phi_{12} - \bar{\Phi}_3 - 1) \\ \bar{c}_3 &= \left(\frac{c_1 b_2}{a_2}\right)(\bar{\Phi}_3 + \Phi_{22}) \end{aligned} \quad (6.30)$$

After substitution, the CCF parameters for the magnetic ball levitation can be derived as:

$$\begin{aligned} \bar{\Phi}_1 &= \left(1.97633 \cosh\left(\frac{\psi(\alpha)}{100}\right) - 1\right) \\ \bar{\Phi}_2 &= 1.97633 \cosh\left(\frac{\psi(\alpha)}{100}\right) + 0.9764712 \\ \bar{\Phi}_3 &= -0.9764712 \end{aligned} \quad (6.31)$$

And

$$\begin{aligned} \bar{c}_1 &= 4.3063 \left[1 - 0.988166 \left(\cosh\left(\frac{\psi(\alpha)}{100}\right) + (1.1905/\psi(\alpha)) \sinh\left(\frac{\psi(\alpha)}{100}\right) \right) \right] \\ \bar{c}_2 &= 10.1781 \left(\frac{\sinh(\psi(\alpha)/100)}{\psi(\alpha)} \right) - 0.10176538 \\ \bar{c}_3 &= 4.274 \left(\cosh\left(\frac{\psi(\alpha)}{100}\right) - (1.1905/\psi(\alpha)) \sinh\left(\frac{\psi(\alpha)}{100}\right) \right) - 4.2234 \end{aligned} \quad (6.32)$$

6.3.2 Digital State Feedback Gain Scheduled Controller Design

The designed state feedback gains vector for the transformed model (6.29) is given by:

$$L_{xd} = (L_x \quad L_{xa}) = (d_1 - \bar{\Phi}_1 \quad d_2 - \bar{\Phi}_2 \quad d_3 - \bar{\Phi}_3) \quad (6.33)$$

By using the ZOH-mapping formula and selecting a sampling period of $T = 0.01$, the desired characteristic equation (6.11) becomes,

$$P(z) = z^3 - 1.2427z^2 + 0.4718z - 0.0082 \quad (6.34)$$

Then, we have:

$$\begin{aligned} d_1 &= -1.2427 \\ d_2 &= 0.4718 \\ d_3 &= -0.0082 \end{aligned} \quad (6.35)$$

After substituting for $d_1, d_2, d_3, \bar{\Phi}_1, \bar{\Phi}_2$ and $\bar{\Phi}_3$ from eq(6.35) and eq(6.31) into eq (6.33), the state feedback scheduled gains vector for the original model for the magnetic ball levitation CE152 can be derived as:

$$L_{zd}(\alpha) = \begin{pmatrix} -1.2427 + \left(1.97633 \cos h \left(\frac{\psi(\alpha)}{100} \right) + 1 \right) \\ -1.97633 \cos h \left(\frac{\psi(\alpha)}{100} \right) - 0.5047 \\ 0.9682 \end{pmatrix}^T T_c^{-1}(\alpha) \quad (6.36)$$

Where $T_c^{-1}(\alpha)$ is the transformation matrix between the two forms.

6.3.3 Digital Gain Scheduled Observer Design

The desired characteristic equation of the closed loop observer that found in eq(6.12) maps into the z-domain as:

$$P(z) = z^2 + 0.0181z + 0.0003 \quad (6.37)$$

Then, we get:

$$\begin{aligned} P_1 &= 0.0181 \\ P_2 &= 0.0003 \end{aligned} \quad (6.38)$$

From eq(3.52) in chapter 3, the scheduled observer gains vector for the system in OCF is given by:

$$\begin{aligned} h_{x_1} &= P_1 - \bar{\Phi}_1 \\ h_{x_2} &= P_2 - \bar{\Phi}_2 \end{aligned} \quad (6.39)$$

After substituting for P_1 , P_2 , $\bar{\Phi}_1$, $\bar{\Phi}_2$ from eq(6.31) and eq(6.35) into eq (6.39), the observer scheduled gains vector for the system that represented in OCF can be derived as:

$$\begin{pmatrix} h_{x_1}(\alpha) \\ h_{x_2}(\alpha) \end{pmatrix} = \begin{pmatrix} \left(1.9763 * \cos h \left(\frac{\psi(\alpha)}{100} \right) + 0.0181 \right) \\ -0.9761 \end{pmatrix} \quad (6.40)$$

The transformation matrix between the two forms can be calculated as:

$$T_o(\alpha) = 0.0063 * \begin{pmatrix} 1 & 0 \\ \Phi_{22}(\alpha)/\Phi_{12}(\alpha) & 1/\Phi_{12}(\alpha) \end{pmatrix} \quad (6.41)$$

The scheduled observer gains vector for the original model can be derived as:

$$H_z(\alpha) = \begin{pmatrix} 1 & 0 \\ \Phi_{22}(\alpha) & 1 \\ \Phi_{12}(\alpha) & \Phi_{12}(\alpha) \end{pmatrix} \begin{pmatrix} \left(0.0124 * \cos h \left(\frac{\psi(\alpha)}{100} \right) + 1.1363 \times 10^{-4} \right) \\ -0.9761 \end{pmatrix} \quad (6.42)$$

Where :

$\Phi_{22}(\alpha)$ and $\Phi_{12}(\alpha)$ are defined in eq(6.25).

6.3.4 Matlab Simulation Of The Digital Output Feedback Control

The digital gain scheduled output feedback control has been built on Matlab/Simulink tool as shown in fig.(6.5) below.

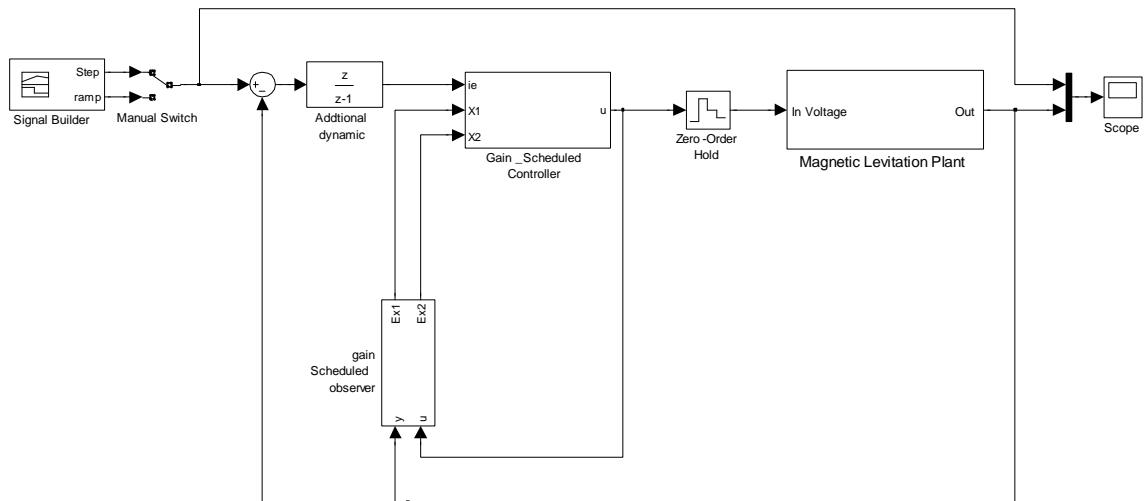


Figure 6.5: Digital gain scheduled output feedback control built on Matlab/Simulink

The response of the closed loop system to a sequence of step changes in a reference signal is shown in fig.(6.6).

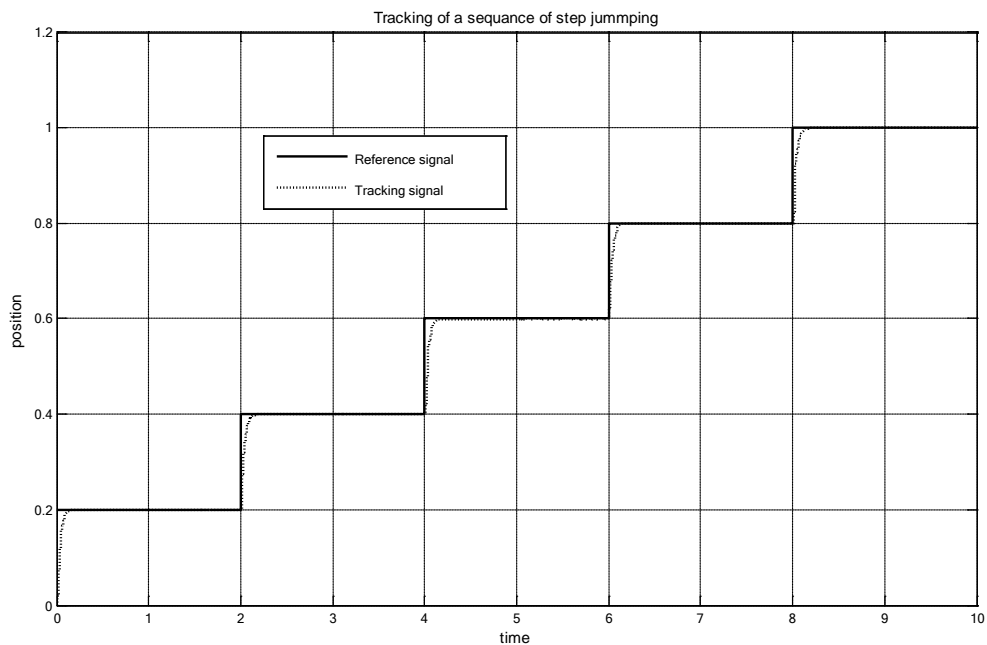


Figure 6.6: Response to a sequence of step changes in the digital output feedback control

Figure (6.6) shows the output performance with comparison to the desired signal, which converges to the desired signal quickly and meets the performance specifications.

The response of the closed loop system to a slow ramp input that takes the set point from zero to one over a period of 10 seconds is shown in fig.(6.7).

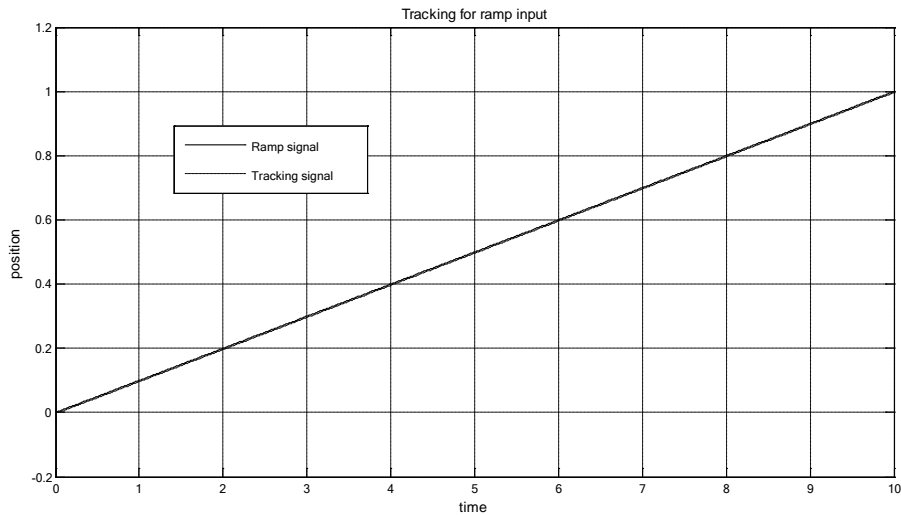


Figure 6.7: Response to a slow ramp with slope 0.1 in the digital output feedback control

6.4 Digital Output Feedback Control System Without Time Delay Compensation.

The digital gain scheduled output feedback control system without time delay compensation has been built on Matlab/Simulink tool as shown in fig.(6.8) below.

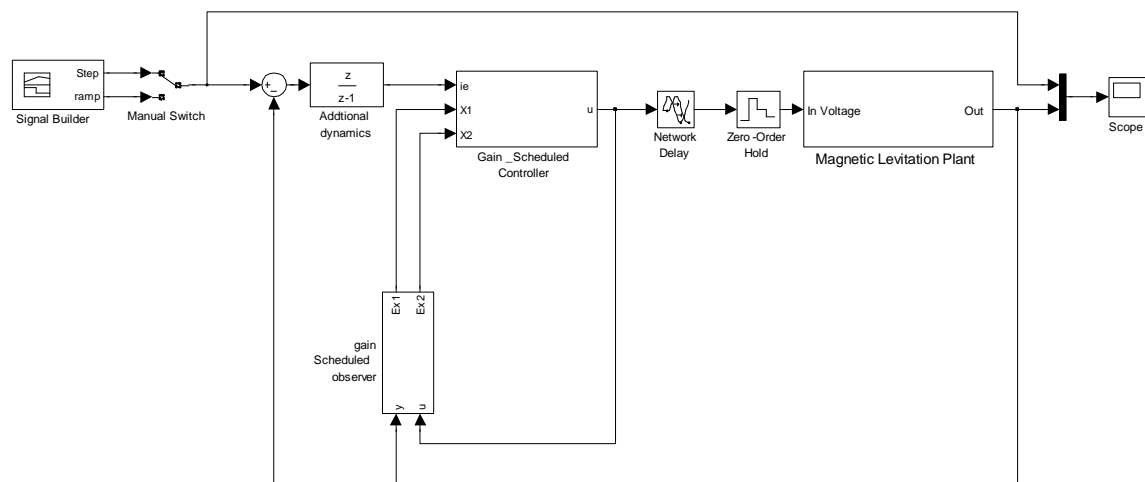


Figure 6.8: Digital output feedback control system with no time delay compensation built on Matlab/Simulink

The step response of the closed loop system without taking the time delay into consideration in the design of an output feedback controller is shown in fig.(6.9).

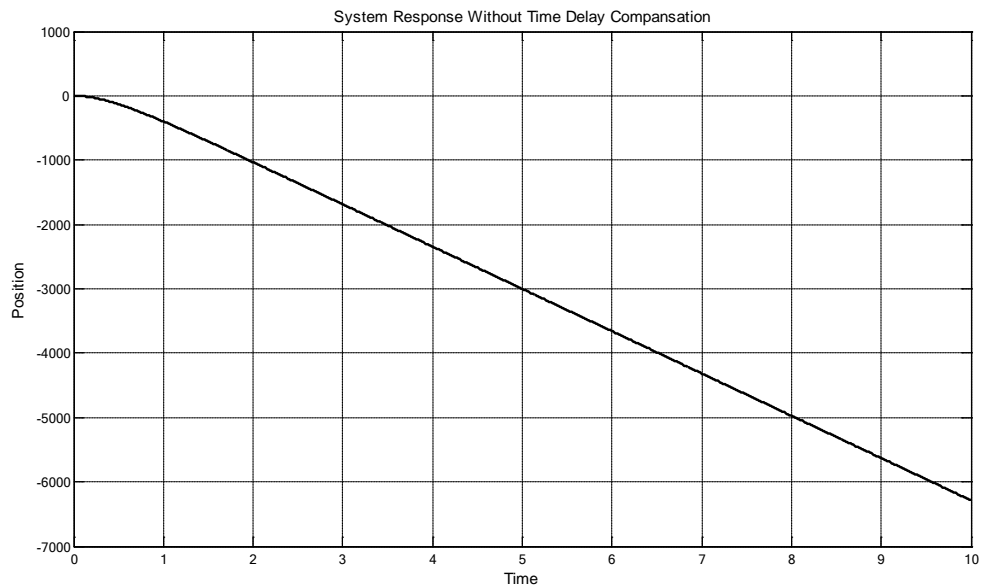


Figure 6.9: Output response of the uncompensated control system for the magnetic ball levitation CE152

From fig(6.9) , we can clearly note that the magnetic ball levitation CE152 system becomes unstable due to the effect of network delay.

6.5 Digital Output Feedback Control System With Time Delay

Compensation

The time-invariant and time varying gain scheduled output feedback control system with time delay compensation have been built on Matlab/Simulink tool as shown in the following figure:

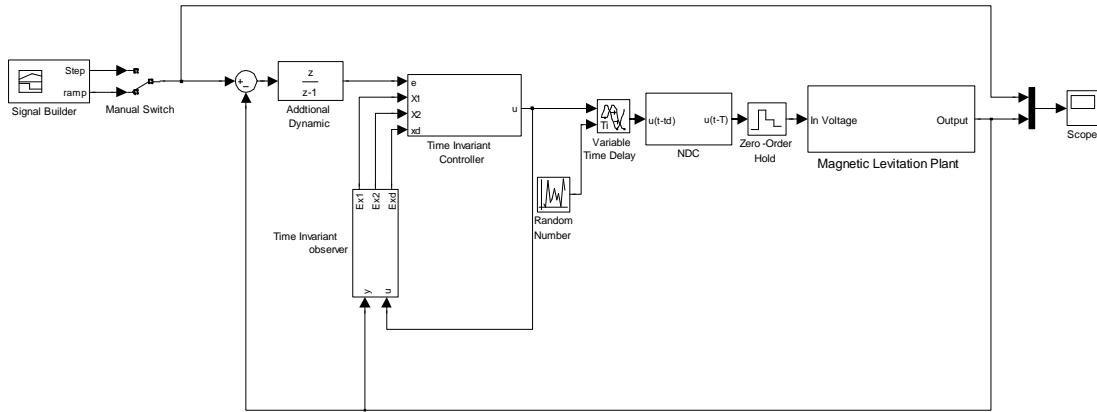


Figure 6.10: Time invariant digital output feedback control system with time delay compensation built on Matlab/Simulink

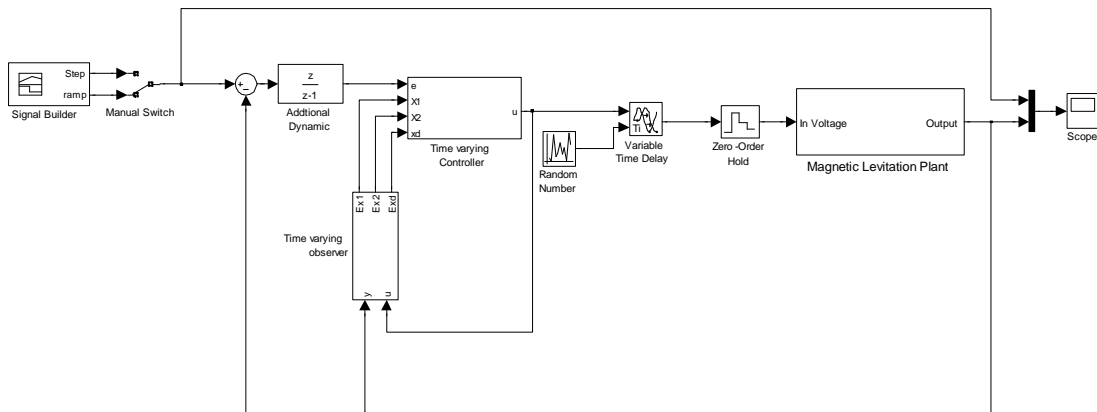


Figure 6.11: Time varying digital output feedback control system with time delay compensation built on Matlab/Simulink

The two types of gain scheduled controllers are achieved similar levels of performance for the magnetic ball levitation CE152.

The response of the closed loop system for both types of controllers for a sequence of step changes in a reference signal is shown in figure(6.12).

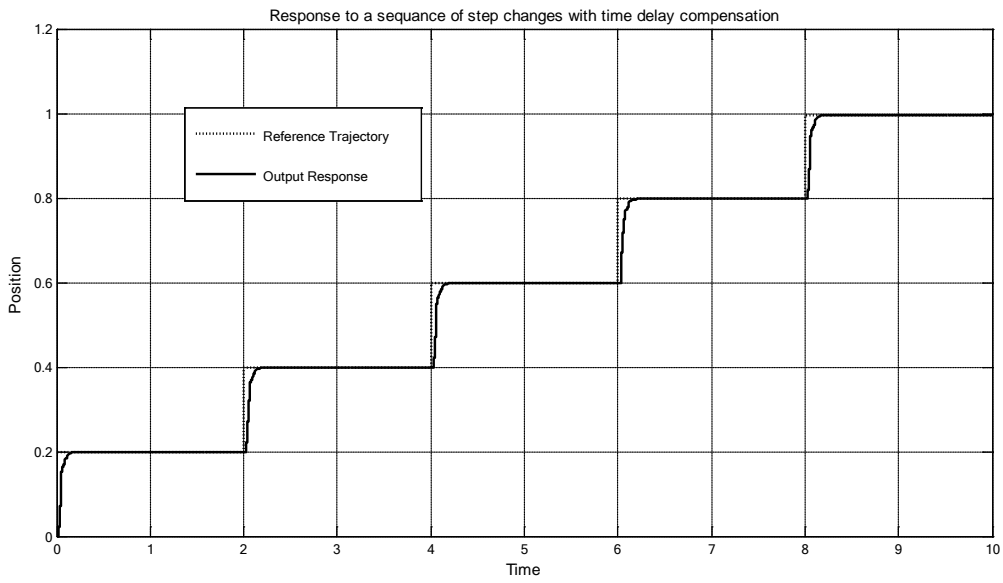


Figure 6.12: Response to a sequence of step changes in the digital output feedback control with time delay compensation

Also, the response of the closed loop system to a slow ramp with slope 0.1 is shown in fig.(6.13).

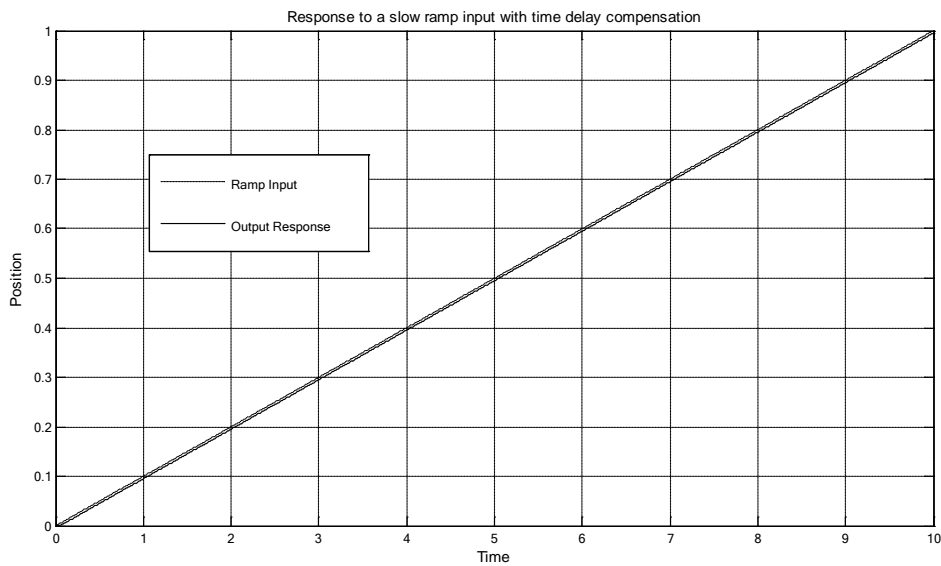


Figure 6.13: Response to a slow ramp with slope 0.1 in the digital output feedback control with time delay compensation

Comparisons between the responses of the compensated system signal with the system with no time delay signal are shown in figures (6.14) and (6.15) below:

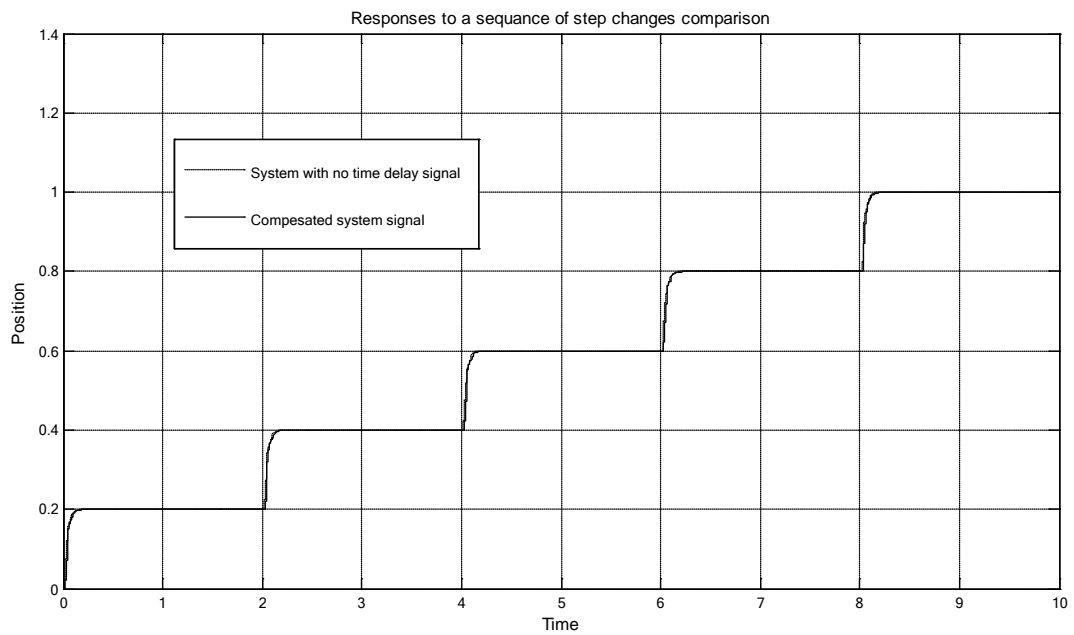


Figure 6.14: Comparison of the responses to a sequence of step changes for the system with no time delay with the compensated time delay system

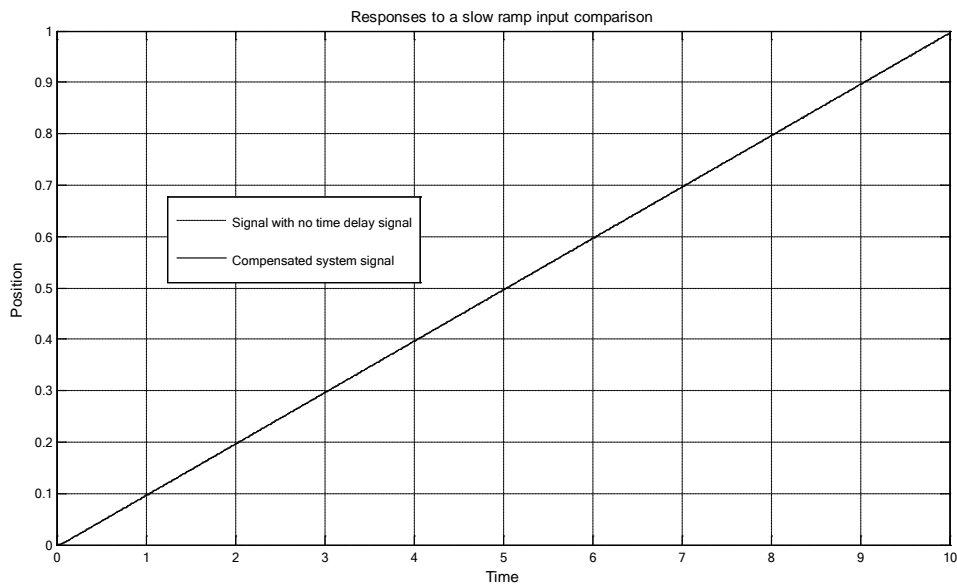


Figure 6.15: Comparison of the responses to a slow ramp input for the system with no time delay with the compensated time delay system.

6.6 Discussion Of The Results

- 1- From figures (6.14) and (6.15) it can be seen that the extension of the control system to compensate the network induced delay lead to system responses close to the responses of the digital control system with no time delay. The improved responses occurred due to the controller structure capability in compensating the induced time delay .
- 2- The two types of gain scheduled control system achieved similar levels of performance due to the existence of equivalent transfer functions ate every kth sampling period. Then, the time varying gain scheduled controller is more efficient than the time invariant gain scheduled controller since there is no need to introduce any hardware components at the actuator node input.
- 3- Fig.(6.9) showed that the system became unstable when the forward channel induced delay is not taken into account during the design of such controller.
- 4- To reach the maximum position of the magnetic ball levitation CE152, we selected the reference signal to be ramp signal or a sequence of step changes in the reference signal, because the stability reasons of gain scheduled technique that require slowly varying reference trajectory with respect to the system dynamics[26].
- 5- In [25], an advanced fuzzy gain scheduling methodology was developed for the same special case of 2nd order nonlinear systems that assumed in this thesis with no time delay. Table 6.1 shows a comparison between the results of this thesis with the work proposed in [25].

Table 6.1: Parameters of magnetic ball levitation CE 152

Controller	Presence of time delay	Overshoot	Settling Time
Fuzzy Gain Scheduling Controller[25]	No	0%	0.09
Output Feedback Gain Scheduling Controller	Yes	0%	0.11

Although in this thesis the time delay was taken into account, table 6.1 shows that our results are close to the results of the advanced methodology proposed in [25], this indicates that the developed Output Feedback Gain Scheduling Controller has achieved a good performance with compensating the effect of time delay.

CHAPTER 7

7 CONCLUSION AND FOR FUTURE WORK

A networked control architecture has many advantages compared to the traditional point-to-point design, but there are some problems due to the presence of a communication network. The communication network introduces time delays in the control loop and these delays have effects on system stability and performance.

In continuous-time systems, the delay is expressed as infinite dimension term which prevents applying different traditional design methods directly. In discrete system, though the delay can be expressed as a finite-dimension term, it increases the dimension and hence the complexity of the system significantly.

The insertion of the communication network into the forward channel of a control system increases the complexity of the analysis and design for NCS. For the sake of simplicity, three steps for designing a tracking controller via a communication network was considered. In the first step, a continuous output feedback controller for the system with no time delay was designed using a gain scheduling technique. In the second step, the continuous system was sampled, and a discrete output feedback gain scheduled controller for the system with no time delay was also designed. In the last step, the communication network in the forward channel was taken into account, and two types of extended gain scheduled output feedback controllers were designed for the discrete model of NCS: Time invariant and Time varying controllers.

All the designed controllers in this thesis were implemented on MATLAB SIMULINK and were applied to the magnetic ball levitation CE152 simulation model. From the simulation results, it has demonstrated that the proposed methodology has achieved the desired performance as well as treating the effect of time delays induced by the network.

A general recommendation for future work is to extend the proposed methodology to compensate for the feedback channel induced delays, also we can handle the effect of data packet losses during communication in NCSs. Cyclic service network type was assumed in the thesis, other types of networks such as random access network can be considered for future work.

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