# The Bridge Between Worlds: Relating Position and Disposition in the Mathematical Field

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University of Edinburgh

2016

# Declaration

I, Lorenzo Lane, declare that this thesis has been composed solely by myself and that it has not been submitted, in whole or in part, in any previous application for a degree. Except where states otherwise by reference or acknowledgment, the work presented is entirely my own.

(Lorenzo Lane)

## Abstract

Using ethnographic observations and interview based research I document the production of research mathematics in four European research institutes, interviewing 45 mathematicians from three areas of pure mathematics: topology, algebraic geometry and differential geometry. I use Bourdieu's notions of habitus, field and practice to explore how mathematicians come to perceive and interact with abstract mathematical spaces and constructions. Perception of mathematical reality, I explain, depends upon enculturation within a mathematical discipline. This process of socialisation involves positioning an individual within a field of production. Within a field mathematicians acquire certain structured sets of dispositions which constitute habitus, and these habitus then provide both perspectives and perceptual lenses through which to construe mathematical objects and spaces.

I describe how mathematical perception is built up through interactions within three domains of experience: physical spaces, conceptual spaces and discourse spaces. These domains share analogous structuring schemas[1], which are related through Lakoff and Johnson's notions of metaphorical mappings and image schemas. Such schemas are mobilised during problem solving and proof construction, in order to guide mathematicians' intuitions; and are utilised during communicative acts, in order to create common ground and common reference

[1] A schema is a term taken from the cognitive sciences. It describes a mental structure or framework for organising, relating and categorising phenomena. Schemata shape the ways in which we perceive reality, influencing what we attend to as well as how we attend to certain things (thus shaping our dispositions towards phenomena).

frames. However, different structuring principles are utilised according to the contexts in which the act of knowledge production or communication take place. The degree of formality, privacy or competitiveness of environments affects the presentation of mathematicians' selves and ideas. Goffman's concept of interaction frame, front-stage and backstage are therefore used to explain how certain positions in the field shape dispositions, and lead to the realisation of different structuring schemas or scripts.

I use Sewell's qualifications of Bourdieu's theories to explore the multiplicity of schemas present within mathematicians' habitus, and detail how they are given expression through craftwork and bricolage. I argue that mathematicians' perception of mathematical phenomena are dependent upon their positions and relations. I develop the notion of social space, providing definitions of such spaces and how they are generated, how positions are determined, and how individuals reposition within space through acquisition of capital.

# Acknowledgements

I would like to thank my supervisors Professor Donald MacKenzie and Dr Jane Calvert for their help and support during the writing of this thesis. Without their guidance and advice the following thesis would not have been possible. I would like to thank Professor Ursula Martin for her help and encouragement, and her role in establishing my social network in mathematics. Special thanks must be given to Professor Andrew Ranicki, Dr John Toland, and especially to Professor Catherine Goldstein, all of whom have helped and supported me a great deal during my studies. Finally I would like to thank the many mathematicians interviewed during the course of my fieldwork. Thank you for your friendship and for sharing your experiences in mathematics with me.

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# Chapter 1: Introductions

## 1.0 Overview

In this introductory chapter I provide an overview of the thesis, and justify my choice of field sites, areas of study, and theorists used. In section 1.2 I provide the reader with a literature review, which covers a discussion of key works associated with some of the main themes addressed within the thesis' body. In section 1.3 I review the main methods used during the course of my fieldwork. In section 1.4 I detail my use of Bourdieu, Goffman and Lakoff and Johnson, providing definitions of the terms and theories I will use later on in the thesis. Finally, in 1.5 I provide chapter summaries to better guide the reader through the text.

## Section 1.1: Motivations

### 1.11 Introduction: Bridging Worlds

This thesis concerns the production of mathematical knowledge by research mathematicians, within the contexts of mathematics institutes. This study does not focus on specific local engagements with mathematical problems or controversies, but rather its aim is to document the broader, everyday practices undertaken by a range of different individuals in the course of producing mathematics. I am not specifically concerned with the production of proof, but rather explore a range of activities involved in research mathematicians' work lives, activities such as collaborating, teaching, supervising, sketching, reflecting, reading, socialising, even escaping from their work. My study does not narrow down to one specific mathematical sub-discipline, but rather I study individuals within algebraic geometry, topology and differential geometry. The reason for this generality was not only a result of access and availability of researchers, but because, within research institutes, a variety of mathematicians, from a range of disciplines, share spaces, resources and ideas. Many researchers collaborate across sub-disciplines, and adopt tools and techniques from other fields. The boundaries between disciplines is often porous, and thus delineating a group who are pure algebraic geometers or pure topologists can be difficult.

Maintaining this level of generality therefore is a way of not reifying the divisions between mathematical disciplines, and reflects better the fuzzy boundaries between social groupings in mathematics. Using a multi-sited ethnographic approach provides a breadth, rather than a depth, to this study.

Focusing on the everyday work-practices, and on the social lives of mathematicians, likewise provides a feel for what work and life is like in these disciplines. I argue that this approach can complement the in-depth work conducted in the sociology of mathematics by, for example, MacKenzie (1999), Livingston (1998) and Bloor (1978). Such breadth-work provides a glimpse of the wider contexts for mathematical knowledge production, providing an overview of the working environments and practices of knowledge workers. Such mundane worlds can be taken for granted, but this thesis argues that it is from the mundane that the complex, abstract mathematical worlds are built. I will use the background habits, the mundane daily practices, as the starting point for discussing the processes by which knowledge is experienced, validated and constructed.

### 1.12 What areas of mathematics am I studying?

The mathematical field is divided into a number of sub-fields, which correspond to populations of individuals working on certain problems, objects or concepts of a mathematical nature. The sub-fields of mathematics are often grouped together into two broad areas of "applied" and "pure" mathematics. Applied mathematicians apply their research to practical problems in other disciplines, such as engineering, natural sciences, computer science, etc. Pure mathematicians on the other hand deal with abstract objects and concepts, which may or may not be related to real world problems. The areas this thesis deals with: algebraic geometry, topology and differential geometry, fall into this category of "pure" mathematics, although many ideas from these fields have been applied to problems outside of the field. Let me now give some brief overviews of the three disciplines I am engaging with in this thesis.

The first discipline, algebraic geometry, uses techniques from abstract algebra to solve geometric problems. Algebraic geometers study solutions to systems of polynomial equations, through use of geometric representations of such equations, in objects called "algebraic varieties". These varieties are divided into classes of objects which share certain features, for example they may take the form of plane curves, lines, parabolas, ellipses, etc. The work of the algebraic geometer is not only to classify such objects, but also to explore them, documenting their features, how they are generated, and what relationships exist between objects of different classes.

Our second community, of topologists, study the changing properties of objects as they are continuously distorted or deformed. Topologists convert objects under investigation into collections of sets, which allow such objects to be treated as "topological spaces". Through translating objects into topological spaces, the properties, relationships, and generating principles of the objects can be explored. Different fields of topology explore how topological spaces are grouped together or connected (algebraic topology), how spaces are embedded in other spaces (geometric topology), or how spaces are affected by differentiable functions (differential topology).

Finally, the third discipline of differential geometry, deals with the application of calculus and algebra to studying geometric problems. Differential geometers analyse curves and surfaces, exploring their properties and relationships. Through such analysis they classify such objects, and provide explanations for their properties, and the ways in which they are constructed.

### 1.13 Why study mathematical communities?

Mathematicians will often borrow tools and techniques from other mathematical disciplines, as well as collaborate across disciplinary boundaries, making such boundaries somewhat porous. However, as Restivo (1994) explains, the boundaries between disciplines have emerged over time, through processes of professionalization and specialisation. Such processes serve to differentiate aspects of mathematics. Through such differentiation different communities are bounded and constituted as separate disciplines:

> Specialization, professionalization, and bureaucratization are aspects of the organizational and institutional history of modern mathematics. These processes occurred in earlier mathematics traditions but their scope, scale, and continuity in modern times are unparalleled. Their effect is to generate closure in mathematics worlds. As closure increases, the boundaries separating mathematics worlds from each other and from other social words thicken and become increasingly impenetrable. Specialized languages, symbols, and notations are some of the things that thicken the boundaries around mathematics worlds (Restivo, 1994:214)

The mathematical worlds that Restivo presents are inherently social worlds, which are the product of histories, politics and social change. Schoenfeld (1992) also acknowledges mathematics as a social practice, which is learned through becoming socialised within the mathematical field:

> Mathematics is an inherently social activity in which a community of trained practitioners (mathematical scientists) engages in the science of patterns - systematic attempts based on observation, study, and experimentation to determine the nature or principles of regularities in systems defined axiomatically or theoretically or models of systems abstracted from real world objects. The tools of mathematics are abstraction, symbolic

> representation, and symbolic manipulation. However, being trained in the use of these tools no more means that one thinks mathematically than knowing how to use shop tools makes one a craftsman. Learning to think mathematically means (a) developing a mathematical point of view - valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure - mathematical sense-making (Schoenfeld, 1992:335)

This process of socialisation, according to Schoenfeld, concerns assimilating the individual within the field of knowledge production, through providing them with a viewpoint on the field, as well as the tools necessary to contribute to the field's reproduction. Restivo argues that these "viewpoints" and "tools of the trade" have become ever more specialised within mathematics and its sub-disciplines, as well as ever more abstracted from the day-to-day realities we experience. Restivo argues that the disciplines of mathematics form their own separate worlds, whose reference objects are not "picturable" in terms of objects from the phenomenal world. Rather, referents in mathematics are internal to the mathematical language within which they are constituted. Restivo (1994) writes:

> As generational continuity is extended and closure proceeds in a mathematical community, mathematicians work more and more in and less and less out of their mathematics worlds. As a result, their experiences become progressively more difficult to ground and discuss in terms of generally familiar everyday world experiences. The worlds they leave behind are pictured worlds, landscapes of identifiable things. Mathematics worlds are worlds of specialized symbols and notations. This is the social and material foundation of so-called `pictureless' mathematics. But mathematical experiences in highly specialized mathematics worlds are not literally picture-less. The resources being manipulated and imagined in mathematics worlds are so highly refined that they are not picturable in terms of everyday reality; the referents for mathematical objects are increasingly mathematical objects and not objects from the everyday world. Since closure is never perfect, some degree of everyday picturing does occur even in the most abstract mathematical work; and, in any case, everyday pictures are almost inevitably produced as mathematicians move back and forth between mathematics and other social worlds. At the same time, new

> picturing experiences and processes lead to the development of new pictures, *mathematics world pictures*. (Restivo, 1994:216)

Mathematics becomes abstracted from the "picturable" world, meaning that linguistic systems are generated, whose referents lie in abstract conceptual spaces, rather than in physical space. To perceive and "picture" mathematical objects, an individual must learn mathematical languages, and acquire a vocabulary of mathematical objects by which to classify the mathematical world. The perception of mathematical reality is then mediated by the categories that are constructed within a mathematical language, and this language is, in turn, shaped through consensus between members of the community of practice. The abstract pictures which mathematicians use to visualise and perceive mathematical reality are the products of the social group and, therefore, in order to understand mathematical perception, we must explore the social mechanisms by which categories in language are produced. Restivo writes that we must understand mathematics as a form of social reproduction. This involves documenting the production processes involved in mathematical knowledge work, and the mechanisms involved in the production of the mathematical field itself. For such a reason we must ask:

> [W]hat do scientists produce, and how do they produce it; what re-sources do they use and use up; what material by-products and wastes do they produce; what good is what they produce, in what social contexts is it valued, and who values it; what costs, risks, and benefits does scientific work lead to for individuals, communities, classes, societies, and the ecological foundations of social life ... What is the relationship be-tween scientists and various publics, clients, audiences, patrons; how do scientists relate to each other, their families and friends, their colleagues in other walks of life; what is their relationship as workers to the owners of the means of scientific production; what are their self-images, and how do they fit into the communities they live in; what are their goals, visions, and motives? (Restivo, 1988:218)

From the questions above, we see that there are many opportunities to study mathematics from a sociological perspective. Knowledge production in mathematics is a social process, and through exploring not only how and where knowledge is produced, but also why knowledge is produced, we can begin to understand the extent to which knowledge is socially shaped. Restivo's questions provide much of the motivation for the following thesis, and in part explain the more general approach I have taken during the course of my questioning. One of my aims, in this thesis, is to orient the reader within the field of mathematics, through situating mathematical production within its wider social frameworks.

### 1.14 Why were these field sites selected?

In order to frame mathematical production I must ground production practices within specific contexts. Mathematics, after all, is produced by individuals within certain physical and social environments, at certain times, according to certain rules, using certain tools and techniques. In this thesis I chose to study mathematicians working at mathematics research institutes. Such institutes were designed specifically to facilitate the production of mathematics, and recruit populations of mathematicians for this purpose. These mathematicians, recruited according to the areas of mathematics they worked in, and from a variety of different institutions and backgrounds, provided representative samples of the wider population of elite research mathematicians. This sample of mathematicians however were already pre-selected, in the sense that the institutes themselves recruited individuals according to their own, institution-specific, selection criteria.

Studying mathematical production in institutes allowed me to explore spaces which have been specifically designed to facilitate mathematical knowledge production. Such spaces were thus "mathematical spaces" in the sense that they were pre-tooled for use by mathematicians, and afforded certain resources for effective production of mathematics. Within the confines of the institute, mathematicians were more likely to communicate with one another, form collaborations, and thus be open to talking about their research. Since mathematicians were already expected to speak about their ideas and collaborate, I reasoned that the institute would be a good place to observe how knowledge is performed and made public[2] and visible.

The institutes themselves were chosen on the basis of a number of criteria: access to the institutes and the mathematicians working there, existing contacts within the institutes, availability of funding for travel and accommodation, reputation of the institutes, as well as the areas of mathematics such institutes specialised in. Before choosing my field sites I built up a network of mathematicians through personal correspondences, attending conferences and talks in the relevant fields, and through collaborating with individuals working within mathematics and computer science. I then obtained a number of scholarships from the institutions

[2] The question of what constitutes "public" must be addressed at this point. Public is a relational concept and depends on the perceived position an individual occupies with respect to other individuals within a given environment in which an individual presents themselves. The public realm is a space in which individuals present their "social" face to the world. Public spaces are places where socially meaningful symbols and values are communicated between socialised individuals. In mathematics these spaces may comprise of physical spaces of institutes as well as virtual spaces on forums, personal websites or publications. The public is any space where the individual becomes visible to the evaluations of others, and where they must present a "face" or position by which they wish their social selves to be viewed.

themselves, along with permission from the institutes' directors, which allowed me to undertake 1-2 month long fieldwork at each of the sites.

My aim was to narrow down my investigations to studying mathematics within elite, well-known institutions. The reason for this was to study knowledge production in its most "visible" and "public" form. Such institutions select mathematicians who are often well-published, well-cited, well-known, well-educated and highly competitive. Such a population, I believed, would likely be leaders within their respective fields, and thus would have the ability to influence the directions in which their fields developed. Studying how these change-agents produced knowledge, how they performed and presented their knowledge, and how they assimilated such knowledge into the mathematical field, would therefore allow me to better understand the production processes within the mathematical field itself. However, in the future, to extend this study I believe it is necessary to explore knowledge production within "middle and lower tier" institutions, and get a sense of the role that individuals with lower capital accumulations play in shaping the field of production.

**1.15 Why use Bourdieu?**

The next set of decisions I need to justify concern the use of Bourdieu's ideas on practice, habitus and field, which act as the core theoretical grounding for this thesis. Within the sociology of scientific knowledge and sociology of science and technology community Bourdieu's ideas have had little influence, indeed they are somewhat unpopular, and therefore neglected[3]. In this thesis, however, Bourdieu's theories take centre stage. There are several reasons for this.

My first reason for using Bourdieu is because his theories provide a coherent framework by which to structure my fieldwork observations and interview materials. Explaining the complexity involved in research mathematics involves studying phenomena on a range of spatial and temporal scales, which takes place between different groups of researchers, within different social contexts; these researchers use complex languages and complex representations to communicate and produce knowledge. Much of their work takes place in inaccessible spaces of the mind, and are therefore opaque to observations. Confronted with such complexity within multiple dimensions of reality, within physical space, conceptual space, linguistic space and discourse space, requires theories which take such complexity into account. Bourdieu's notion of field, habitus and practice do just this, encompassing these different realms of experience, and demonstrating the interconnectivity between the structuring principles in each. Using Bourdieu thus allows me to relate phenomena within these different dimensions.

Bourdieu provides a wide body of work from which to draw insight and inspiration, covering ethnographic work conducted among the Berber of Algeria, textual studies of 19th century French literature, studies of academic life, studies in linguistics, as well as work on the production of knowledge in science and art. This wide range of interests is united by a common framework for analysis, which

[3] Bourdieu’s unpopularity within STS stems from a series of harsh critiques levelled at leading figures within the field, including Harry Collins, David Bloor, Karen Knorr-Cetina and Bruno Latour (see Sismondo, 2011:86). Bourdieu unfairly describes the work of these authors as “post-modern rantings” which undermine public confidence in science.

demonstrates the similarities between the organisation and functioning of social systems, at different levels of experience. Bourdieu looks not only at local contexts, at the structuring principles of the Berber house, for example, but also explores wider social systems - in the generation of academic fields, for instance. At different scales of social organisation there are common structuring principles, and the role of the sociologist becomes that of relating the local fields to the global fields which encompass them, and of understanding how meaning, power, and resources may be transformed or reproduced at different scales, locations, or periods of time. Within mathematics I was confronted with the problem of relating the local work of solving problems, to the wider field in which those problems are constituted. Through Bourdieu's ideas I was able to understand how perception of mathematical problems is shaped by the wider field of discourse, by structuring individual's dispositions towards mathematical phenomena.

As a result of the interpretive flexibility of Bourdieu's definitions (especially of habitus), and of the analogical nature of Bourdieu's concept of structure, I was able to weave in other theorists such as Goffman and Lakoff and Johnson, who I believe complement Bourdieu's thought. Goffman's work on theatres of performance fits in well with Bourdieu's notion of practice, and Lakoff and Johnson's work on metaphor works well with Bourdieu's analogical, multi-dimensional conception of habitus. Although such modularity may make this thesis appear eclectic and amorphous in its structure, I believe that such additions are necessary to fully understand how mathematical perception is shaped, and how knowledge is produced in mathematics.

My final justification for using Bourdieu concerns the notion of social topology. Bourdieu refers to this notion variously as a "social physics", or a "social calculus", but social topology works rhetorically within this thesis, as I work with topologists themselves. The simplicity of Bourdieu's ideas around social topologies is what attracted me to this mode of analysis, which simply involves describing properties of position, relation, differentiation, integration and orientation within social systems. I found that this simple method could be developed into a tool for describing the mathematical fields I studied, and a way to conceptualise how different dimensions of experience could be integrated within a common interpretive frame. I will however leave further explanation of my interpretation of Bourdieu's idea of topology until the end of the thesis, during the discussion, but I ask that the reader keep such ideas in mind when progressing through the main body of the thesis.

### 1.16 What aspects of Bourdieu's analytical framework will be adopted?

Standard uses of Bourdieu's analytical framework will focus on the role that social position (class conditions), education and early socialisation play in shaping an individual's position and disposition. The purpose of this is to situate or "objectivate" the individual within certain classes within a field, as well as determine their social position and capital accumulations. To follow the Bourdieusian framework precisely a researcher must therefore study in great detail the socio-economic backgrounds of the participants, exploring their parent's occupations, their nationality, ethnicity, religious affiliation, up-bringing and early childhood education. After this has been determined so the participants' research history, education and career trajectory, and history of thought can be explored.

In this thesis the socio-economic and educational backgrounds of participants were explored, however my thesis diverges from tradition by not privileging this data in the later presentation. Although it can be said that socio-economic background does play a role in shaping later career trajectories, through influencing the schools attended, the membership to elite social networks, etc., it appeared that subsequent socialisation within mathematical and academic fields had more of an influence on the subsequent development of mathematicians' ideas.

In this thesis I use the method of objectivation as a means of understanding mathematicians' positionings within institutional spaces. I use ethnographic observations to study work-habits and social interaction, studying how they change as a function of the formality and visibility of space. I then use objectivation during interviews, asking interviewees to locate themselves and their ideas within the field, through prompting them to outline their educational histories, the major influences on their thought, and the relationships they may have with collaborators, peers and supervisors. Understanding how the individual is positioned and how they position themselves within the field, through exploring goals, motivations and strategies, then allows me to study the "field effects" experienced by mathematicians.

As a result of the complexities involved in implementing Bourdieu's ideas on social matrices, I have chosen to neglect some of the finer details of "objectivation".

To construct a social matrix would have required exploring a number of parameters (for example family life, leisure pursuits, media consumed etc.), listing each parameter within the rows and columns of a matrix, and then measuring the effects of each parameter on every other parameter (taking partial derivatives). Understanding how each dimension of the social matrix functions in relation to every other dimension is possible, but is difficult to realise in the confines of a thesis. Instead I have chosen to study the multi-dimensional properties of habitus, through objectivating the individual and their ideas within physical, conceptual and discourse spaces, and showing the similar structuring principles present within each space.

Finally I must state that my thesis diverges significantly from the Bourdieusian framework by its lack of focus on inequality. Social stratification within the mathematical field is significant, especially when it comes to interactions between PhD's and tenured professors, the allocation of prizes and positions, the presentation of knowledge during chalk-talks, and acceptance, validation and citation of published works. I briefly describe the tensions between friendship and competition, the pressures of productivity and the influence of certain individuals in the field, however limitations of space prevent inequality being discussed in any great detail.

**1.17 What are some of the criticisms levelled at Bourdieu?**

Bourdieu's analytical framework should not be employed naively to studying the production of the mathematical field. Indeed, as Sismondo (2011) and Camic (2011) argue, there are some underlying problems within Bourdieu's thought, which present challenges to developing Bourdieu's sociology of science. According to Sismondo, Bourdieu constructs an overly rationalised view of science, which neglects the messiness of knowledge production and decision-making in practice (Sismondo, 2011:89). Bourdieu is seen to neglect the materiality of scientific knowledge production, focusing on the more theoretical, "thinking" sciences, such as theoretical physics and mathematics, rather than the practical, "wet" biological sciences (Sismondo, 2011:92). This bias privileges the explicit, formalisable aspects of knowledge over those of the tacit and sensual. However, although it can be said that Bourdieu viewed Actor Network Theory in a negative light, it cannot be said that Bourdieu neglects the role that the material world, and the role that the tacit dimensions of experience, play in shaping the production and perception of knowledge. In this thesis I shall demonstrate these material aspects of knowledge-production, and show how they are incorporated within Bourdieu's notion of habitus. Materiality, I explain, far from being neglected by Bourdieu, was in fact a central part of his notion of habitus and field.

The next apparent problem associated with Bourdieu's work lies in the varying notions of fields and their boundaries. According to Camic (2011), Bourdieu presents two antagonistic notions of field: on the one hand as discrete entities with their own internal logic, and on the other as interconnected, intersecting spaces, whose boundaries are permeable (Camic, 2011:281-3). Such varying definitions for the field, according to Camic, are symptoms of Bourdieu's "cleft" sociology, which emerges from the antagonisms between his theoretical and empirical works. This duality in interpretation leads some researchers, such as Sismondo (2011), to conclude that Bourdieu reifies the boundaries around fields, and abstracts such fields from their wider socio-cultural contexts and histories of formation (Sismondo, 2011:92). However, such an interpretation, as Camic argues, neglects the multi-scale nature of Bourdieu's analytical framework, which seeks to understand how fields embed and interact within systems of fields (Camic, 2011:290).

These interactions and intersections between fields, can be difficult to articulate and define explicitly. This difficulty arises from the complexities and interdependencies involved in networked fields' interactions. The field effects of one field upon another therefore are hard to measure and characterise in a single sociological study. For such a reason a Bourdieu-inspired analysis requires multiple investigations, at different scales of magnification, beginning at the level of local field, or local collection of fields, and building out to encompass a global field of relations. This initial focus on the local field can result in the reification of the boundaries of a field, making fields appear as independent entities. But this reification of a field is a necessary evil, imposed by the limits of sociological analysis. Ideally all relevant forces would be studied simultaneously, in order to characterise a social system. However the complexities involved in such analysis would create a single body of work too vast to be practical.

For this reason it is better to adopt a modular approach to sociological analysis, which takes the local field, or collection of local fields, as independent units of analysis, and then builds together different units, in separate sociological studies, into more complex systems of relations. Within this thesis I take a collection of sub-fields[4] in mathematics as my unit of analysis, and focus on defining how individuals position themselves at different spatial levels within these fields. The end result is not to fully characterise individuals' positioning within the mathematical field, but rather to begin a process of locating the individual within a social system. From this starting point it will then be possible to change my objective frames of reference, zooming in or out to interactions on different scales of magnification. These different framings, I explain in the conclusion, can then be added together to present a more coherent understanding of an individual's position within a social system, and the effect of such position on their dispositions and knowledge-production practices.

### 1.18 How has the Bourdieusian framework been used in the history of mathematics?

A Bourdieusian style framework of analysis has been adopted by a number of researchers within the history of mathematics, notably Goldstein et al. (2007a, 2007b) on the intellectual histories of Gauss and Einstein, Brian (1994) on the socio-cultural histories of statistics in 18th-19th century France, Ehrhardt (2010, 2012, 2016) on adaptation of Galois' work in 19th century France, and Gingras (2001) on the mathematization of physics during the 18th century.

[4] A sub-field I take to be a sub-set of a field. The field of Algebraic geometry, for example, is a sub-field of the Mathematical field, it is therefore contained within the wider field of mathematics, which, in turn, is contained within the wider field of "Academia". Fields therefore can be nested within other fields, and thus can be defined as "sub-fields" according to their relative positions with respect to other fields or sets of fields. Being a sub-field of a field, so the sub-field bears certain qualities possessed by the field, but also has unique qualities with respect to other areas of the field, for example possessing specialised capital in the form of languages, tools, techniques, histories, etc. not shared with other parts of the wider field. These sub-fields are separate spaces for competition, in which the quality of capital can only be perceived through socialisation within such specific contexts.

Goldstein and Schappacher's (2007a) articles on the publication and dissemination of Gauss' *Disquisitiones Arithmeticae* (1801) utilise an implicit Bourdieusian style of analysis, which positions Gauss' work within its historical, intellectual and social contexts. Goldstein et al. study how the work was related to existing bodies of knowledge, how it was accepted and assimilated within the intellectual community, as well as how the work generated a new field of enquiry within mathematics. According to Goldstein and Schappacher, to objectivate the position of Gauss' ideas requires:

> …[M]ore than just a "thicker description" of such milestones; it requires that light be shed on other patterns of development, other readers, other mathematical uses of the book – it requires a change in our questionnaire. We need to answer specific questions, such as: What happened to the book outside Germany? What were the particularities, if any, of its reception in Germany? Which parts of it were read and reworked? And when? Which developments, in which domains, did it stimulate – or hamper? What role did it play in later attempts to found mathematics on arithmetic? (Goldstein and Schappacher 2007: 4-5)

Goldstein and Schappacher trace out the connections between *Disquisitiones Arithmeticae* (DA) and other works of the time, through following citations referenced in Gauss' book. Through building up a network of citations so they were able to trace the social relations between ideas incorporated into DA, as well as the social relations between individuals who were working on similar problems at the time. As they write below, this map of objective relations (in the form of citations), was mirrored by the personal correspondences taking place between authors, and reflected the boundaries which formed around the field of arithmetic algebra at that time:

> More specifically, we have called arithmetic algebraic analysis the domain of research directly connected with the D.A. that knit together reciprocity laws, series with arithmetical interpretations, elliptic functions and algebraic equations. We argue that it constituted a (research) field, in the sense that "all the people who are engaged in [this] field have in common a certain number of fundamental interests, viz., in everything that is linked to the very existence of the field," and that one can uncover "the presence in the work of traces of objective relations … to other works, past or present, [of the field]."As we have noticed, its main actors were indeed linked by a dense communication network, both personal and mathematical. Their published papers would meet with prompt reactions; quite a number of these papers were in fact excerpts of letters addressed to another mathematician working in the domain

This process of tracing out objective relations between works was precisely what Bourdieu intended to accomplish through his objectivation process. Such a process of objectivation requires analysis at different levels of magnification, from the local level of individual interpretations and negotiations, to the global level of the field of discourse. Ehrhardt (2016) in her discussion of the posthumous reimagining of Évariste Galois' work balances this study of local representations with an analysis of the socio-intellectual contexts within which Galois' ideas were popularised. Below, for example, Ehrhardt objectivates the mathematician Arthur Cayley within the wider historical context of 19th century Cambridge, in order to understand his interpretation of Galois' theories:

> Therefore, Cayley's interpretation implied a complete conceptual change, as well as new representations associated with Galois's memoir. If we take a closer look at Cayley's intellectual and social environment, Cambridge University in the Victorian age, we can gain a better understanding of this phenomenon. In the nineteenth century, Cambridge was much more than place of education: it was home to a specific mathematical tradition that structured a whole scientific community, both socially and culturally (Warwick 2003, 49–285). In the 1850s it was certainly the place in Europe where algebra was the least synonymous with the theory of equations. From the 1810s onward, mathematics in Cambridge was profoundly renewed by a group of mathematicians often called the English Algebraic

> School… In other words, this choice of notation was linked to a specific mathematical practice that was typical of the scholarly culture Cayley belonged to. Moreover, the mathematicians of the English Algebraic School had an epistemological way of thinking about the "essence" of algebra that they put into practice in mathematical works that developed new objects and new methods. Cayley's generation may have been less concerned with seeking the foundations of algebra, but the fact that Cayley saw the concept of the group as a kind of generic concept that would enable mathematicians to unify many particular situations reveals that he also tried to uncover the "true reasons" of these situations that were concealed by their specificity. (Ehrhardt, 2016:332).

Ehrhardt (2010) builds up an historical frame in which to position the individual, and through which we can understand how Galois' ideas were being construed. Through studying mathematicians' syllabuses and text-books from the period she also develops a sense of the conceptual habitus that Galois' work would have been related to, and through which it was interpreted by readers of the period:

> Analyzing the standard mathematical textbooks helps us to paint this picture with a finer brush by showing the practical skills attached to the theory of equations. These books not only inform us about the kind of knowledge that was to be passed on, but also elucidate the contours of each sub-discipline, the way knowledge was introduced and used in examples and applications. They embody cognitive and methodological preferences, which will be adopted by the later generations in the course of their training, and which will be considered as the right way to ask questions and to solve problems (Schubring 1987). Thus opening Lacroix's textbooks allows us to see Galois's paper through the eyes of its readers. (Ehrhardt: 2010:95)

Ehrhardt explicitly makes use of Bourdieu's notions of habitus and attempts to locate Galois within the mathematical field at the time, through comparing his position to other authors of similar position, she writes:

> To understand how algebraic research relied, in a practical way, on the nineteenth century French mathematical habitus, and then what Galois's readers expected to find in his paper, we will compare this paper to another. But as every mathematical research is unique by definition, we will first explain what this means. First, we need a paper that could have been

> seen by the academicians as one that resembles Galois's with respect to the author's position in the community and to its subject. (Ehrhardt, 2010:102)

Through comparisons with other authors in similar socio-economic positions she locates Galois within a dominated strata of the field. Below she discusses how this position influenced how his ideas were received by the establishment:

> Thus, the scientific field offered Galois a “space of possibilities” much larger than what was labeled by the Academy of Sciences. Nevertheless, Galois's student position was at the bottom of the scale of positions within this field – in Bourdieu's words, Galois was in a “dominated” position. Still, Galois thought that he deserved more than a teacher's job in province and for this, the highest scientific rewards, he needed the Academy of Sciences whose members were in a “dominating” position within the scientific field through an institution whose role in evaluating research meant that it still embodied the scientific temperament (Ehrhardt, 2010:107)

Erhard uses this Bourdieusian style of analysis to better understand how disposition, and validation of knowledge vary as a function of social position. Like Goldstein (2007) and Brian (1994) Erhard demonstrates the power of objectivating positions within a field, and of situating the individual within specific socio-historical frameworks. In the following thesis I will carry on this process of objectivation, however, rather than focusing on an historical context, I will attempt to objectivate mathematicians within the present-day field of mathematical production.

**1.19 What does this study contribute to sociology?**

My hope is that this study can add to the growing body of ethnographic work undertaken with mathematicians by Heintz (1999), Greiffenhagen (2011) and Barany (2012). My thesis attempts to answer some of the questions posed by Restivo (1982), through documenting the processes of production in mathematics, and exploring mathematics as a form of craftwork. Like Livingston (2007) I attempt to explore the local contexts of production, but I attempt to situate such production within the wider mathematical field. I use Bourdieu (1978, 2004) to understand the dialectics between local and global, agency and structure, the individual and society, and the body of this thesis follows Bourdieu's process of analysing social systems. Through use of Bourdieu I demonstrate how mathematical perception is constructed, and how it functions on three spatial levels, encompassing physical, conceptual and discourse spaces. In developing Bourdieu's notion of a social topology I present the potential to construct mathematical models for representing social systems, which can be developed as a tool for comparing and understanding the production of different social fields.

### 1.19.1 What is my main thesis?

The mathematical field is a social field which consists of socialised individuals bearing properties of position, relation, differentiation, integration, and orientation. These spatial relations are structured according to the possession of social capital, which is unequally distributed through the field. An individual's position within the field generates habitus - systems of durable, transposable dispositions - and such habitus structure individual's perception of mathematical phenomena and of the mathematical field itself. I demonstrate how mathematical perception is built through orienting the individual within the mathematical field. I describe how, by understanding how individuals position themselves within the field of production, through practice and performance, we can understand how they become oriented or disposed towards mathematical phenomena. I argue that these operations of position, orientation, differentiation, integration and relation are present within three different spatial dimensions: conceptual, physical and discourse spaces. I demonstrate that, because these spaces bear similar properties, they are analogically related through metaphor[5]. Finally I propose that these analogical relationships are themselves the result of shared generative schemas, which allow us to constitute the field of production in terms of Bourdieu's "social topology".

[5] Although analogy implies approximation, there is also another sense in which metaphor can be interpreted, which is as homology or isomorphy (equality). In mathematics there are direct homologues to certain concepts within different disciplines. One object can thus have many manifestations depending on the language used to describe it, the frames of reference used to view it, or the objects or examples used to relate it to.

## Section 1.2: Literature Review

### 1.21 Introduction

In this literature review I shall contextualise this thesis within the existing literature on the sociology of mathematics. I will divide this review into four main sections, each discussing a major theme present within the body of my thesis. The first theme concerns the nature of objectivity and the social shaping of knowledge in mathematics. The second theme explores the presentation and performance of knowledge in mathematics. The third theme concerns the mutability of categories through dialogue and debate, as well as the stabilisation and socialisation of meaning. The fourth theme involves understanding mathematical practices themselves, exploring work involving the local contexts of knowledge production.

### Theme 1: Nature of Objectivity

#### 1.22: How is objectivity in mathematics socially shaped?

Bloor (1984) argues that what is considered objective and factual in any scientific discipline, mathematics included, is a product of its social context. Objectivity is produced through "institutionalisation" of concepts. According to Bloor:

> [O]bjectivity is social. What I mean by saying that objectivity is social is that the impersonal and stable character that attaches to some of our beliefs, and the sense of reality that attaches to their reference, derives from these beliefs being social institutions (Bloor, 1984:229)

The requirements of being objective are that beliefs are non-subjective, stable, and shared, as Bloor states:

> [A] belief that is objective is one that does not belong to any individual. It does not fluctuate like a subjective state or a personal preference. It is not mine or yours, but can be shared. It has an external, thing-like aspect to it (Bloor, 1984:229)

Objectivity gives object-like stability to the things we believe in, makes rules and standards compelling, and gives the appearance of being external to the individual. These criteria for objectivity, according to Bloor, are met by social institutions. He writes:

> My claim is that these specifications are met by social institutions. The taken-for-granted practices sanctioned by a group have just this quality of being external to the individual. They have a stability far greater than the individual's changing desires. They are the common ground where individuals meet. They are shared. So institutions satisfy the general conditions for objectivity. The second step is to seize the opportunity presented by this interesting fact, and to identify the objective with the social. The second step does not, of course, follow from the first step. It is an act of theory formation: a conjecture, not a deduction (Bloor, 1984:229-30)

Bloor argues that because social institutions satisfy the criteria of creating objectivity, that objectivity itself is the product of "social institutions". Such social institutions can take many forms: religious organisations, nation-states, languages, or academic disciplines. In relation to this thesis "social institution" encompasses the community of mathematical practitioners within mathematical institutes and mathematical disciplines. Within these institutions mathematicians are constituted as mathematicians, through sharing common languages, common beliefs, conceptual tools, rules and standards, and methods for constructing and evaluating knowledge. As a result of objectivity being located within such social institutions, the members of institutions are also able to define what constitutes as objective and valid. The consequence of the social construction of objectivity is that objectivity is then influenced by the social contexts within which it is generated.

The objectivity of certain beliefs, and the constitution of beliefs as "factual", therefore depends upon the systems of beliefs within which such knowledge is constituted, and the community of believers who share these beliefs. Restivo (1994) uses Durkheim's concept of "thought collective" to characterise these systems of beliefs and believers. An individual's thoughts are seen as being shaped by their social contexts, by Bloor's social institutions. The institution therefore defines the entire field of possibilities for action and thought, which makes an individual's thoughts a product of the collective consciousness. This is explained by Restivo (1994) in the following:

> Durkheim (1961:485) argues that individualized thoughts can only be understood and explained by attaching them to the social conditions they depend on. Thus, ideas become communicable concepts only when, and to the extent that, they can be and are shared... The apparently purest concepts, logical concepts, take on the appearance of objective and impersonal concepts only to the extent that, and by virtue of, the fact that they are communicable and communicated, that is, only insofar as they are collective representations. All concepts, then, are collective representations and collective elaborations because they are conceived, developed, sustained, and changed through social work in social contexts. In fact, all contexts of human thought and action are social. The next intellectual step is to recognize that 'work', 'context', 'thought', and 'action' are inseparable; concepts, then, are not merely social products, they are constitutively social. This line of thinking leads to the radical conclusion that it is social worlds or communities that think, not individuals. Communities as such do not literally think in some superorganic sense. Rather, individuals are vehicles for expressing the thoughts of communities or 'thought collectives'. Or, to put it another way, minds are social structures. (Restivo, 1994: 209)

Objectivity then is external to the individual but internal to the system of representations and beliefs within which it is defined. The community therefore determines what constitutes knowledge through processes of evaluation and negotiation, and such knowledge is then incorporated within the community's store of beliefs about the world. The validity of beliefs is dependent upon the social frames of reference from which the belief is interpreted. Different social contexts will produce different reference frames and different sets of beliefs. Restivo characterises these different social frames as "worlds", which he describes below:

> This first step awakens us to 'mathematics worlds', networks of human beings communicating in arenas of conflict and cooperation, domination and subordination. Here we begin to experience mathematics as social practice, and to identify its connections to, and interdependence with, other social practices. (Restivo, 1994: 211)

Within these "worlds" a "world-view" is generated. Such worldviews produce perspectives from which to perceive certain phenomena and constitute them as objects. Such objects, because they are constructed by a world view, are subject to changes within that world-view. Restivo explains below that mathematical objects also change as a product of their social histories:

> Mathematical forms or objects increasingly come to be seen as sensibilities, collective formations, and worldviews. The foundations of mathematics are not located in logic or systems of axioms but rather in social life. Mathematical forms or objects embody mathematics worlds. They contain the social history of their construction. They are produced in and by mathematics worlds. It is, in the end, mathematics worlds, not individual mathematicians, that manufacture mathematics (Restivo, 1994:212)

Restivo goes on to argue that objects within mathematics, because they are socially constructed, can be studied in a similar way to material artefacts. Social histories of their use can be "unpacked" from objects and their biographies charted out. Through mapping out a mathematical concept’s genealogy it is then possible to understand its relationship to the social world, as well as to the wider system of representations within which it is entangled:

> Explaining the 'content' of mathematics is not a matter of constructing a simple causal link between a mathematical object such as a theorem and a social structure. It is rather a matter of unpacking the social histories and social worlds embodied in objects such as theorems. Mathematical objects are and must be treated literally as objects, things that are produced by, manufactured by social beings. There is no reason that an object such as a theorem should be treated any differently than a sculpture, a teapot, or a skyscraper. Only alienated and alienating social worlds could give rise to the idea that mathematical objects are independent, free-standing creations, and that the essence of mathematics is realized in technical talk. Notations and symbols are tools, materials, and in general resources that are socially constructed around social interests and oriented to social goals. They take their meaning from the history of their construction and usage, the ways they are used in the present, the consequences of their usage inside and outside of mathematics, and the network of ideas that they are part of. The sociological imperative, especially when informed by the sociological imagination, is a tool for de-alienation and for uncovering the images and values of workers and social worlds in mathematics. (Restivo, 1994: 219)

Restivo grounds mathematical objectivity within the social contexts of "mathematical worlds". Such worlds are similar to Bloor's idea of social institution in so far as both concepts blur the boundaries between subjectivity and objectivity, and relativize the concept of validity and factuality. Both authors argue that objectivity is a product of perception, with this perception being shaped through adopting and being enculturated within certain socio-cultural reference frames. Such frames of reference, called variously "worlds" or "institutions", involve understanding the social milieus of individuals within these communities, as well as understanding the biographies of these communities.

Restivo's concept of mathematical worlds as thought-collectives does, however, present some problems. A certain degree of agency is taken away from individuals, who are considered as avatars for the wider collective consciousness. Individuals are described as vehicles for expressing the thoughts of communities, who become akin to Garfinkel's (1967) cultural dopes: simply replicating cultural scripts given by their world-views. The question must also be asked as to what the boundaries are between mathematical worlds, or between such worlds and other social worlds - the worlds of language, art, religion, family, state, politics and the everyday. Drawing a boundary around such mathematical worlds thus seems problematic, and artificial, which means that the "collective" nature of such collective representations is difficult to establish.

Nevertheless Bloor's notion of "social institution" and Restivo's "mathematical worlds" do provide the starting point from which to question what effect social life has on the production of knowledge. Such concepts allow us to understand that objectivity is to some degree a product of perception, that perception itself is socially shaped, and that the nature of perception, and therefore of objectivity, can evolve over time. In both Bloor and Restivo's work, knowledge is considered as being valid relative to the systems within which it is produced. Yet, despite this relativism, neither are denying the existence of a reality outside of human artifice. Rather the argument is one of *perception* of reality, and of phenomena, and the resultant classifications which are considered as being socially shaped. In my own thesis I do not concern myself with the nature of "truth" or existence, but rather I am only dealing with perception. I deal with the practices and performances of validation and classification, rather than attempting to show that x or y concept is true, or an arbitrary social construction. Within this thesis I do not make use of Bloor's social institution or Restivo's mathematical worlds. Rather I choose to adopt Bourdieu's (1978) notions of field, habitus and practice, and use Sewell's (1992) refinements of social structure and agency.

## Theme 2: Visibility of ideas

### 1.23 How are ideas presented within mathematics?

From the social shaping of objectivity let us turn towards how such ideas are made visible and presented to mathematical audiences. When I speak of the visibility of ideas I am referring to the ways in which ideas are transformed in the process of being communicated and made public. Often-times there is a difference between what we hold as private belief and what we communicate and make public. Through the process of making belief public we must transform such belief in such a way as to be accepted and understood by another individual. This process involves reformulating concepts, making tacit understandings explicit, using certain prescriptive modes of presentation, or even using rhetorical techniques to make arguments more persuasive.

Making an idea visible is thus a process of making the idea socially acceptable and surveyable to the community of practitioners. We witness this process of rendering mathematics visible during seminar presentations, during collaborations between mathematicians, during lectures, or through publication of mathematical proofs. Hersh (1991) argues that, during these processes of presenting and performing mathematics, the mathematical constructions or arguments may be transformed. Hersh differentiates the mathematics that is constructed in private from that which is performed in public, using Goffman's (1956) dramaturgical metaphors of the frontstage and backstage to understand how ideas change as a function of their social contexts. Ideas in the backstage are informal, unfinished, fuzzy and multi-threaded, whilst ideas presented in the public frontstage are formalised, bounded and coherent.

Greiffenhagen (2011) and Barany (2012) provide some insight into the differences between backstage and frontstage presentations of mathematics, and make use of blackboard inscriptions to trace out the "materialisation" of mathematical concepts. Merz and Knorr-Cetina (1997) similarly, in the context of studying theoretical physicists at work at CERN in Geneva, study the process of communicating and constructing mathematical objects. Much like mathematicians, theoretical physicists conduct their research around boards, through pen and paper, orally, or via email. As a result, Merz and Knorr-Cetina adapt their methodology to studying communicative practices, and interactions with representations and texts. As they write below, the traditional ethnographic methods used for studying laboratory life need to be adapted to study "thinking sciences":

> [T]he laboratory approach had to be adapted to the obdurateness of the field: the study is based rather less on the observation of physicists' activities than on one analyst's capability to exploit her physics training and interact with participants as a member of their culture. It is also anchored in a close 'reading' of physicists' personal-professional communications (their e-mail correspondence;), their calculation protocols, and their explanations to us which invariably involved paper and pencil. The close 'reading' was adopted to gain access to the ethnomethods implicated in doing theoretical physics work. Our approach yielded layers of methodical policies, "*ansatze*", tricks and other devices, which are piled into doing a theoretical computation. The policies, *ansatze*, tricks and devices were mutually embedded in one another within a sequential interactional system involving disembodied objects, several physicists and competing teams (Merz & Knorr-Cetina, 1997:74)

In both theoretical physics and mathematics researchers are involved in manipulating and reconfiguring what Merz and Knorr-Cetina call "disembodied objects", which may comprise of equations, models, constructions or diagrams (Merz & Knorr-Cetina, 1997:75). Such manipulations are part of the informal crafting processes which take place behind the scenes, in the backstage of knowledge-work. Within this backstage researchers are often dealing with quasi-objects, which lack clear definitions, or which possess multiple interpretations. Such ill-formed concepts are concealed from public display, remaining on scratch paper, or written and erased from white-boards, rather than published in journals.

The objectivity of such hypothetical-objects is, therefore, not completely determined, as they have not yet been presented for evaluation by the wider community of practitioners. Rather such objects are transacted within smaller private networks of friends, close collaborators or co-workers, where they are experimented and tinkered with. A concept within mathematics or theoretical physics will therefore undergo certain transformations as it moves from circulation within informal, private social networks to formal, public displays, during presentations and publications. When it enters the public sphere it is still subject to change, to reformulation and re-presentation, as it moves to different stages of performance. Such stages may vary in their degrees of formality, audiences, openness, visibility and prestige, and so the concept itself may change as a function of such spaces.

In the context of the sociology of logic, Rosental (2003, 2008) weaves together these themes of display, performance, demonstration and debate in his work on the presentation of a theorem in the field of fuzzy-logic. Rosental describes his work as analysing:

> [T]he concrete modalities of the production and certification of a particular logical theorem in the field of artificial intelligence in the 1990s. I investigate, in particular, how this theorem was collectively accredited in practice-what "recognition" it and its author received…I successively consider the different stages in the emergence of the theorem, from its earliest drafts, to its first publication, and then to the author's writing of several new versions in response to the critiques it triggered.(Rosental, 2003:623-4)

Using ethnographic, textual, and interview-based research Rosental follows the presentations, debates and re-presentations of a proof by Charles Elkan, which explored the paradoxes of fuzzy logic (Rosental, 2003:626). Rosental presents the controversies which emerged around Elkan's proof, documenting the different types of performances taking place on online forums, within journals, at conferences, and through personal exchanges with Elkan. Over the course of the controversy a few dominant viewpoints emerge, and it is around such views that the debate eventually stabilises. Through the course of the study Rosental asks the following questions:

> What does the expression of a point of view in the public and private spheres mean and require for the actors? Who has the resources to express him or herself, and in which cases? What individual and collective representations emerge from such interventions in the debate and how? Who reads what, how, and why? How are symbolic languages appropriated, and do they generate univocal readings of proofs? As a result, do the actors necessarily reach a consensus at a certain point through a simple victory of one side over the other? Or are misunderstandings possible, and can these misunderstandings contribute to the building of specific forms of agreement? (Rosental, 2003:626)

In this thesis I will be asking similar questions concerning the distinction between public and private performances, the production of shared resources and representations, as well as the process of translating and reformulating ideas. For such a reason I will discuss in greater detail the relevant findings from Rosental's work, and demonstrate the relationship such work has to my own study of mathematicians.

The first relevant aspect of Rosental's work to my own concerns positioning of debates and individuals within certain theatres of performance. Different theatres come with different rules and expectations, different levels of formality and visibility. Private email exchanges for example were often informal, friendly, and not subject to outside scrutiny or pressure to respond or perform. The forum upon which the controversy had been debated also remained informal, open, with low barriers to entry. However, as the debate moved to more formal settings in journals and at conferences, so the presentation of knowledge, the character of communication and the composition of the interlocutors changed, as Rosental writes:

> Thus, several months after the conference, the center of debate shifted from the electronic forum to journals that specialized in artificial intelligence. This shift of exhibited interaction to other arenas was accompanied by a radical transformation in the time-scale of debates and a substantial rise in the barriers to be surmounted to "stay in the game." Making a point of view public now required authors to produce polished texts and to subject them to editorial constraints. It also required full investment in a milieu in which interindividual relations and reputations were essential in the processes of selecting (and often commissioning) articles (Rosental, 2003:625)

As performances entered the frontstage, and as visibility of ideas increased, so the presentation of the arguments were adapted to suit these formalised domains. Rosental states that presentations became more "polished" and increased effort was required in presentation. Within these formalised domains personal and institutional reputations and statuses were on display, as much as the ideas themselves. This passage returns us again to the concept of objectivity, in the sense that such ideas are not neutral, or self-evident, but rather they are positioned within social systems. Ideas are associated with the individuals who created them, as well as the institutions such individuals themselves are affiliated with. Ideas, thus, rather than being objective, are subject to the judgements of the individuals present within the systems in which it is produced. Ideas therefore change as a function of their social contexts.

As ideas move between different social contexts so different resources and schemas are utilised. In this process of context switching so certain individuals are excluded from participating, as a result of their positioning within the field. Rosental (2008) discusses this idea of shared schemas and resources in his discussion of modes of reading:

> In fact, the gaps between the different modes of reading a given demonstration can be apprehended as a problem of replication, the difficulty of duplicating the specific mode of reading that the author anticipates. The difficulty stems in particular from the fact that both writer and reader need to have acquired in advance the same large set of specific tacit skills…Similarly the replication of a given type of reading of a demonstration by a reader necessitates that the reader and writer share numerous and sometimes very unusual skills. In each case, obtaining a replication is more the exception than the rule, given the improbability that such conditions will be met. (Rosental, 2008:105-6).

Rosental explains that to successfully read a demonstration or proof an individual must share certain sets of tacit skills (what I classify as schemas). Such tacit skills are built up through individuals sharing similar knowledge bases, similar educational backgrounds, and similar experiences in presenting and demonstrating their knowledge. These tacit skills generate a perspective and a perceptual frame through which an individual can construe and interpret phenomena. Through possessing similar reference frames, reference objects and resources, so individuals are able to co-ordinate their perspectives; such coordination, Rosental writes, takes place around shared texts:

> [This] approach allows us to grasp the way de-monstrative action, which involves the collective of readers and authors participating in the news-group, is coordinated by the presence of textual devices; this action is distributed among the inscriptions, the tacit practices of causing to appear, and the acts of visual verification of symbolic manipulations. (Rosental, 2008:100)

Within these texts, arguments and ideas are both shown (monstrated) and explained (demonstrated). These different modes of presenting are different strategies by which points of view are communicated, and through which individuals are persuaded to adopt a given perspective. As Rosental states below, these different modes of presenting are used variously by different actors who possess a multiplicity of different resources and schemas for interpreting a given phenomenon:

> Demonstrative practices will be interrogated in relation to the potentially multiple exercises in which the actors are involved, but also in relation to the varied resources and registers used by researchers to support their own points of view - whether these elements are presented in the texts of proofs, brought into various types of oral and written interventions, or shaped during interactions. On this occasion, it will be crucial to ask to what

> extent a dichotomy generally operated between showing (*montrer*) and demonstrating (*demontrer*) is pertinent, and to what extent the formalist activity studied can be identified with an activity of putting into form, or formalisation. (Rosental, 2008: 54)

The multiplicity of perspectives from which to construe phenomena leads to the problem of how to create consensus and produce certified knowledge. Through the online forum the number of different interpretations of Elkan's proof multiplied, as individuals began producing new demonstrations:

> In the messages addressed to comp.ai.fuzzy, we note that the participants make abundant efforts to produce new demonstrations - or counterdemonstrations - of the theorem in order to convince their interlocutors of the correctness - or the absence of correctness - of Elkan's proof. Indeed, we observe a veritable inflation in the texts through whose mediations the writers seek to make themselves spokespersons for Elkan's article. The writers present their own messages as reformulations of the original proof, a proof to which they attribute recourse to specific resources (assorted logical principles and notions, implicit axiomatics, demonstrative mechanisms); they offer the forum's readers substitutes for the articles Elkan published in the proceedings of the 1993 conference on artificial intelligence. (Rosental, 2008:96)

Individuals, in demonstrating their reformulations of Elkan's proof, were drawing on different schemas and resources through which to en-frame Elkan's ideas. Such reformulations created parallel arguments which could act as substitutes for Elkan's original formulations. Such substitutions similarly took place with arguments running counter to Elkan's proof. What such parallel arguments created was a system of associations which linked multiple viewpoints together, and therefore provided a more holistic frame through which to view the original arguments. Objectivity was thus being constructed through making multiple frames of reference agree on common reference points. It is through creating such common reference points that classification and judgements are stabilised.

### 1.24 How does the presentation of ideas affect their perceived objectivity?

Individuals create associations between ideas through a process of staging their presentations so as to appear self-evident. Previously accepted beliefs are linked together with novel propositions so as to legitimise the novel propositions through association:

> The staging of this "appearance" is constituted by the linking of inscriptions, and it is guided by the search for agreement among the forum's readers as to the legitimacy of the equivalences advanced. In other words, the writer is seeking to win acceptance for the step he is taking in associating one inscription with the other, in order ultimately to emphasise the original author's recourse to the logical principle identified. (Rosental, 2008:101).

As Rosental argues, the staging of appearance does not seek to justify (demonstrate) claims but simply shows relationships, making it appear self-evident that relationships exist between propositions. Showing or "monstration" is thereby made to appear as explanation or demonstration:

> The de-monstration is thus presented as a linkage presupposed by the writer to be self-sufficient, one that does not require complementary developments. Each stage seems to be perceived as transparent: all its elements are purportedly there to be seen, and thus it would convey its own self-evidence. Since at each stage the writer is content precisely to present a display, we can say that this de-monstration is deployed in a series of showings, or "monstrations". This term offers an evocative characterisation of a quite specific practice that can be related to a formalist project of making everything visible, a practice that consists in linking up textual fragments that the writer believes he can simply exhibit without further staging. (Rosental, 2008:103)

Self-evidence and objectivity of arguments thus can change as a function of their mode of presentation. Exhibiting without explaining creates the appearance of self-evidence, however the self-evidential nature of such statements are challenged through demands for justification:

> He exhibits an equivalence without explaining why he attributes this status to the pair of statements in question. The structure of the passage illustrates quite well the extent to which the de-monstration is composed of a series of monstrations, that is, moments in which, implicitly, the author considers it sufficient to show, without providing justifications for what he shows as such. Challenging a step in a de-monstration, as we have seen on comp.ai.fuzzy, consists precisely in demanding justifications, or in producing a textual fragment intended to refute what is presented as "self evident". (Rosental, 2008:213)

Modes of presenting ideas, as well as the visibility of ideas, therefore effects the degree to which they are considered as self-evident or objective. The stabilisation of interpretations around a given proof is dependent upon managing the proof's presentation and interpretation by different groups of actors. As the quote below states, stabilisation of interpretation was achieved through coordinating different viewpoints, and reformulating the proof to counter criticisms levelled against it:

> Moreover, the formation of representations of Elkan's theorem was not a sum of strictly individual approaches. It was very much a question of collective actions involving a substantial amount of coordination and was set in the struggles between coalitions of actors, the configuration of which evolved partly in line with viewpoints expressed during the debate. This coordination of view- points and the management of their visibility by means of various mechanisms was one of the elements clearly showing that the practice of logic did not simply amount to the production of reasoning, whether oral or written (Rosental, 2008:640)

The end result of such negotiations was for Elkan to adapt his proof to better suit its social climate. The proof thus was tinkered with and reassembled so as to present more nuanced arguments, as Rosental writes:

> Elkan thus had the opportunity to elaborate and test several different reformulations on diverse interlocutors. He adapted his talks to suit his interlocutors and the forums at which he presented, adjusting his presentations in a differentiated, evolving, and sometimes personalized way. (Rosental, 2003:637)

Through presenting, debating and reformulating within different presentational contexts so different versions of the proof are generated:

> The different versions of Elkan's text form a record of negotiations. As objections arose, the proof problematized more and more the de-monstrative steps and logical notions. It became more resistant because less direct, more nuanced and better accompanied. The theorem, the proof, and the logical objects incorporated were transformed simultaneously. From one version to the next, certain elements of the proof were abandoned, others were brought to light and exploited in new de-monstrative sequences. By following the steps in the de-monstrative production, we observe that the work of proving put into play skilful bricolages.
>
> (Rosental, 2008:236-7)

Even after such skilful bricolages, which transform the presentation of the proof, the negotiations and controversies surrounding the proof persisted. However, through re-presenting the proof through secondary presentations and articles, the primary text of the proof could be re-construed and re-evaluated, as Rosental writes:

> Because his article, a singular material device launched in the world, eventually proved somewhat ineffective in countering criticism, Elkan added to it by producing new texts and new speeches. He thus provided new instructions for his theorem's interpretation and general comprehension, thereby forging new tools for changing readers' relationship to his original text. Such adjustments also helped to stabilize debate because they limited

> disagreements by making them appear, retrospectively, and at least partly, as misunderstandings (which differed, of course, depending on the interlocutors and the publics).(Rosental, 2003:637)

What Rosental's work demonstrates is that even formal presentations within logic are subject to social processes of evaluation. The presentations of ideas change as a function of the social contexts within which such presentations take place. Objectivity is thus never a given, but rather it is constructed through processes of formulation, debate and reformulation. Knowledge, as DeMillo, Lipton and Perlis (1972) argue below, must be subjected to the judgement of the community before it can be certified and perceived as valid:

> The proof by itself is nothing; only when it has been subjected to the social processes of the mathematical community does it become believable…Mathematical proofs increase our confidence in the truth of mathematical statements only after they have been subjected to the social mechanisms of the mathematical community (DeMillo, Lipton and Perlis, 1972:275).

Such knowledge, even in mathematics, is therefore only certified or validated through becoming assimilated within existing systems of knowledge. Ideas must be socialised and related to what a community already understands and accepts as being valid. As DeMillo, Lipton and Perlis state below, the presentation of formal, deductive arguments is not enough to qualify its objectivity, rather objectivity is bestowed upon the proof through its subjectivity, that is to say through it becoming a subject of the existing field of discourse:

> After enough internalization, enough transformation, enough generalization, enough use, and enough connection, the mathematical community eventually decides that the central concepts in the original theorem, now perhaps greatly changed, have an ultimate stability. If the various proofs feel right and the results are examined from enough angles,

> then the truth of the theorem is eventually considered to be established. The theorem is thought to be true in the classical sense--that is, in the sense that it could be demonstrated by formal, deductive logic, although for almost all theorems no such deduction ever took place or ever will (DeMillo, Lipton and Perlis, 1972:274)

## Theme 3: Mutability of Mathematics

### 1.25 How is knowledge stabilised through debate?

So far we have seen that objectivity and self-evidence are products of social institutions. We have observed that the presentations and perceived validity of beliefs change as a function of their position within social fields. Different social contexts thus transform the relationships that individuals have with a given belief, affecting the perspectives from which they may view it, and their perception of it as being objective or not. The ways in which knowledge is performed also shapes our perceptions of it, as presentations must conform to certain expectations, and serve to make visible certain aspects of a belief rather than others. There is therefore a certain degree of context-dependency, indeterminacy, multiplicity and flexibility in the beliefs that we constitute as knowledge, despite our experience of such knowledge as being stable and objective.

In mathematics the process of validating and, therefore, of stabilising knowledge, is achieved mainly through proof. Such proof techniques and their reasons for use can vary between different mathematical cultures. MacKenzie (1999) for example compares different cultures of proving within the formal verification community to those of pure mathematics. Within formal verification proof can be comprised of mechanised proof carried out by a computer, or it can involve formalised proof carried out by hand. Proofs may also need to fulfil certain criteria, for example providing "insight", being "rigorous", "beautiful", or "surveyable" (See Heintz 2003:930). MacKenzie (1999, 2001) explores how different definitions of proof came into conflict, as a result of Kenneth Appel and Wolfgang Haken's 1976 proof of the 4-Colour Theorem. This theory dealt with the minimum number of colours required to fully colour contiguous regions of a map, so that no two adjacent regions would share the same colour. The controversy associated with the proof concerned its use of a computer program to check that a given map possessed certain properties.

For Tymoczko (1979), traditional proofs in mathematics must fulfil certain criteria of being convincing, surveyable and formalizable (Tymoczko, 1979: 59), he characterises these attributes as follows:

> (a) Proofs are convincing. This fact is key to understanding mathematics as a human activity. It is because proofs are convincing to an arbitrary mathematician that they can play their role as arbiter of judgment in the mathematical community.
>
> (b) Proofs are surveyable. Proofs are the guarantees of mathematical knowledge and so they must be comprehended by mathematicians. A proof is a construction that can be looked over, reviewed, verified by a rational agent. We often say that a proof must be perspicuous, or capable of being checked by hand. It is an exhibition, a derivation of the conclusion, and it needs nothing outside of itself to be convincing. The mathematician surveys the proof in its entirety and thereby comes to know the conclusion.
>
> (c) Proofs are formalizable. A proof, as defined in logic, is a finite sequence of formulas of a formal theory satisfying certain conditions. It is a deduction of the conclusion from the axioms of the theory by means of the axioms and rules of logic. Most mathematicians and philosophers believe that any acceptable proof can be formalized. We can always find an appropriate formal language and theory in which the informal proof can be embedded and "filled out" into a rigorous formal proof. Formal proofs carry with them a certain objectivity. That a proof is formalizable, that the formal proofs have the structural properties that they do, explains in part why proofs are convincing to mathematicians. (Tymoczko, 1979: 59-60).

According to Tymoczko, it is the property of surveyability that is of utmost importance to a proof in mathematics, with mathematicians organising their proofs to facilitate it being surveyed by others:

> Surveyability is an important subjective feature of mathematical proofs which relates the proofs to the mathematicians, the subjects of mathematical investigations. It is in the context of surveyability that the idea of 'lemma' fits. Mathematicians organize a proof into lemmas to make it more perspicuous. The proof relates the mathematical known to the mathematical knower, and the surveyability of the proof enables it to be comprehended by the pure power of the intellect-surveyed by the mind's eye, as it were. (Tymoczko, 1979:60)

Through organising the proof clearly into lemmas the mathematician structures the proof so that another reader can follow what is being presented. Formal proofs which may be difficult to survey can be mediated by related surveyable proofs which have already been verified:

> Hence it begins to appear that, in practice, at least, mathematicians come to know formal proofs only through the mediation of surveyable proofs. Either the formal proofs are simple enough to be surveyed themselves and verified to be proofs, or their existence is established by means of informal surveyable arguments. (Tymoczko, 1979:62)

However in the case of the 4-Colour Theorem, there was no surveyable proof to act as a mediator between the formalisations and the steps conducted by the computer program. For such a reason, Tymoczko argues that the 4 Colour Theorem is a departure from the traditional proving process in mathematics:

> In summary, the proof of the 4CT, although much like a traditional proof, differs in certain key respects. It is convincing, and there is a formal proof. But no known proof of the 4CT is surveyable, and there is no known proof that a formal proof exists. The crucial difference between the 4-Color proof and traditional proofs is that the 4-color proof requires the appeal to

> computers to fill the gap in an otherwise traditional proof. The work of the computer is itself not surveyable (Tymoczko, 1979: 73).

As Tymoczko argues below, if the Appel and Haken proof of the 4-Colour Theorem were accepted (which it was), then the community's notion of what a proof was would have to change to accommodate the use of computers:

> Has the 4CT a surveyable proof? Here the answer is no. No mathematician has surveyed the proof in its entirety; no mathematician has surveyed the proof of the critical reducibility lemma. It has not been checked by mathematicians, step by step, as all other proofs have been checked. Indeed, it cannot be checked that way. Now Appel, Haken, and Koch did produce something that was surveyable in the sense that it could be looked over. Their work, as we have said, is very much like a surveyable proof with a lacuna where a key lemma is justified by nontraditional means-by computer. Incidentally, we must be wary of verbal entanglements here. Of course, if we call the appeal to computers a "new method of proof" in the strictest sense, then, trivially, the 4CT will have a surveyable proof. But the notion of proof itself will have shifted to accommodate the new method. (Tymoczko, 1979:70)

Surveyability of proof still remains core to many mathematicians' notion of proof as " [A] construction that can be looked over, reviewed, verified, by a rational agent.." (Tymoczko, 1979: 54). Such a definition returns us to DeMillo, Litpon and Perillis (1977) and their argument that the social processes of reviewing and debating proofs makes them valid knowledge. These processes aim to convince audiences and generate a consensus, as Kleiner (1991) argues:

> The truth of a theorem, then, has a certain probability, usually < 1, attached to it. The probability increases as more mathematicians read, discuss, and use the theorem. In the final analysis, the acceptance of a theorem (i.e., the acceptance of the validity of its proof) is a social process and is based on the confidence of the mathematical community in the social systems that it has established for purposes of validation. (Kleiner, 1991: 310)

This process of socialising and stabilising mathematical knowledge does not only take place at the point of certification of proofs by the community. Rather these processes of negotiation and debate take place during the construction of the proofs themselves. As Lakatos (1976) explains, mathematical knowledge is not immutable and infallible, but rather theorems are subject to a constant examination and reformulation, through a process of conjecture, proof, counter-example and refinement. The boundaries around mathematical categories are therefore subject to change, as the proof is used as a tool for testing the precision of definitions, as well as constructing those definitions.

The proof thus makes mathematical objects visible and surveyable, in the sense that it provides a framework by which objects can be clearly defined and related to other objects. As Bloor explains, mathematicians do not start off with definitions and then derive theorems and proofs, rather the definitions are constructed within the framework of a given proof:

> Everyone remembers the mathematical textbook which begins with long and complicated definitions, announces a surprising theorem, and then develops an austerely compelling proof. Definition; theorem; proof; QED. No, says Lakatos; this is all upside down. What really come at the beginning are not definitions, but problems and conjectured solutions to them. Theorems are conjectures. Like all conjectures they need testing, and proofs, odd though this may sound, are attempts to test them. (Bloor, 1978:246)

Proofs are built up through testing conjectures through presenting theorems, providing examples and counter-examples, and then reformulating the theorem so as to encompass or exclude certain elements which contradict it:

> Proofs start with a 'thought-experiment', or exploit some quasiempirical procedure to break down the problematic conjecture, embedding it into what may be a quite distinct body of knowledge. Each step in this decomposition of the theorem becomes a possible source of error. It will fail if exceptions are found to it. Exceptions to the steps of the proof Lakatos calls 'local' counterexamples; exceptions to the original conjecture are 'global' counterexamples. (Bloor, 1978:246)

Through negotiating with these local and global counter examples to the proof, through barring certain "monsters" from it, so clearer boundaries are created between the categories of objects that the proof is seeking to relate to. When certain phenomena fall outside of these boundaries, then either the boundaries themselves are redrawn or a new set of classifications and boundaries are constructed. But the important point to take away from Lakatos, according to Bloor, is that these boundaries can only be tested through encountering certain test objects, and through individuals negotiating the bounds of definitions through use of such test objects:

> We have seen that a proof begins with the invention of a technique or procedure, like stretching or triangulating. This can be carried out on a limited number of familiar figures, but everything surrounding this narrow area of accomplishment is, at first, simply darkness. The accomplishment is mute about its own scope and about the broader range of contingencies to which it may come to be related. It says nothing about whether such things as nested cubes or twin tetrahedra do, or even can, exist; or whether they have any relevance to the study of polyhedra.
>
> This approach to proofs may be called 'finitist'. The point is that a proof procedure does not have a set of preordained implications outside the immediate context of use. How it comes to be accorded these implications as that context of use is extended is precisely what Lakatos is investigating. He is not saying that the implications pre-exist but we do not know what they are: the implications await our creation. In particular, the question of whether there are counterexamples to a proof procedure is not settled in advance. (Bloor, 1978:247-8)

The boundaries around categories are thus only determined through their encounter with specific examples. The boundaries around categories are then redefined in relation to such examples, and redrawn accordingly:

> We draw the boundary lines. Classification is our achievement and our problem. Nothing is to be gained by seeing different boundary lines as more or less corresponding to the 'real' ones. But this is not all. For Lakatos the world is so densely populated by objects of all shapes and sizes, and there are so many imaginable procedures that can be based on them, that there is an indefinitely large number of different boundaries that we might reasonably draw. (Bloor, 1978:248)

Our experience of categories as objective, therefore, are products of our socialisation within systems of classifications, which construct conceptual boundaries around phenomena. As Bloor explains, the objective nature of our categories are products of our habituation to certain classifications, which construct our objects of perception:

> The belief in a fixed basic vocabulary of perfectly understood terms is an illusion created by our verbal habits. We become habituated to a certain usage in a particular context; it becomes 'obvious', transparent, and direct. We think that we will know exactly how to use the word in all future cases, as if there were a unique and natural way of extending it outside its old range. This is wrong, because new proof procedures can decompose any idea, however simple. They bring to it a new context, suggest new connotations and hence endow it with a new, inner complexity. What the Cauchy proof procedure did for our idea of polyhedra could be done for any concept including point and line. Our concepts can always be 'stretched'. (Bloor, 178:249)

As Bloor explains, our categories become naturalised and gain a self-evident quality. However the future use of our categories are somewhat underdetermined in practice. As a result, the boundaries around categories are constantly being redrawn. New objects assimilated into a category subtly distort the original framework by which the class was originally constructed, and thus all objects become transformed by the redefinition. Bloor argues that this process of redrawing and reformulating is integral to the production of knowledge in mathematics:

> This fails to meet Lakatos' point. He is saying that concept-stretching and the redrawing of classificatory boundaries is an integral part of mathematical reasoning. Trying out wider and different applications of concepts, and making the consequent adjustments to theorems and definitions is something that is going on all the time. Changing the meaning of concepts in this way is not a subterfuge to be shrugged off, as if the counterexamples it created were unimportant.' This is because our intellectual judgements are guided by the properties of our overall system of thought, not by its isolated elements. In the interests of overall coherence any particular achievement may be subverted and any theorem may have to be modified: You cannot separate refutations and proofs on the one hand and changes in the conceptual, taxonomical, linguistic framework on the other. (Bloor, 1978:250)

There is therefore an inherent instability within all classification, whether they be in the context of natural language or mathematics. Rather, categories are constantly in the process of redefinition, as classifications come into contact with a reality of almost infinite variation. Such instability in definitions, however, is nullified through consensus between the members of a community of practice. Through agreeing on definitions, and certifying proofs, so the community creates shared frames of reference, which are internally coherent, through which phenomena can be construed. As Bloor concludes, there is no final truth regarding proofs, rather there exists only the system of interlocking claims and counter claims in which knowledge is connected and stabilised:

> Lakatos is saying that the stability and scope of every theorem is precarious: critical argument and adjustment is in principle endless; there is no final truth to reveal, only a ramified and interlocking network of claims and counterclaims to be balanced and stabilized....If the stream of potential counterexamples is endless, then the processes whereby we accord, or fail to accord, recognition of them must also be endlessly at work. Without their remorseless operation and that of the forces which govern them, there would be neither order nor coherence in mathematical knowledge. Its classifications, its counterexamples and its theorems would have no agreed relations to one another. The great significance of Lakatos' work is that it makes the forces which govern the response to anomaly constitutive of mathematical knowledge: they are a necessary part of that knowledge. (Bloor, 1978:251)

## Theme 4: Mathematics in practice

**1.26 How is mathematical reality constructed through local negotiations?**

The process of proving, refuting and reformulating concepts is an ongoing process through which knowledge in mathematics is constructed and validated. Such processes take place through an individual's situated engagements with problems within a given field. The concept-user encounters problems in the course of their daily practices, and must define the boundaries around the concept for themselves, according to their understanding of the concept and the wider system of relations within which that concept is defined:

> Each and every instance of concept application takes place under the impact of local contingencies, and among these local contingencies will be sociological variables. I have already listed some of them: traditions, precedents, authorities, goals and interests. The fit of our concept to reality is, therefore, not just a matter for reality; it is always a matter (collectively) for the concept users as well. (Bloor, 1996:853)

Each local engagement constitutes an "instance" within which the concept is given a specific set of referents. Such referents form reference points around which a reference frame for the concept is generated. Through this reference frame the wider category, of which the concept is a member, is generated. The instance thus serves as an anchor around which the boundaries of a category are related, and from which later instances are related to.

Such local negotiations of categorical boundaries are theorised within Barnes, Bloor and Henry's category of "finitism", the core principle Barnes defines as follows:

> [Finitism's] core assertion is that proper usage is developed step by step, in processes involving successions of on the spot judgements. Every instance of use, or of proper use, of a concept must in the last analysis be accounted for separately, by reference to specific, local, contingent determinants. Finitism denies that inherent properties or meanings attach to concepts and determine their future correct applications. (Barnes, 1982:30)

Finitism thus argues that meaning is developed through use within certain situations. There is no absolute rule which individuals follow, rather future uses of a term are underdetermined, and will be subject to change, being re-created on a case-by-case basis; as Barnes, Bloor and Henry (1996) write: '…*there is nothing in the meaning of a term, or its previous use, or the way it has previously been defined, which will serve to fix its future proper use"* (1996:78). Indeed Barnes, Bloor and Henry (1996) claim that finitism has 5 main implications:

> 1. The future applications of terms are open-ended;
>
> 2. No act of classification is ever indefeasibly correct;
>
> 3. All acts of classification are revisable;
>
> 4. Successive applications of a term are not independent; and
>
> 5. The applications of different kinds of terms are not independent of each other (Barnes, Bloor & Henry, 1996:55)

The boundaries of categories are thus somewhat fluid, as the prototypical reference points for each individual actor can vary. Each individual therefore will possess a specific point of view, according to the collection of referents which constitute their reference frame for any given concept. Such reference points, however, can be calibrated through dialogue and debate, through which common referents are determined. For each individual the categories are not totally underdetermined. Rather, past experience, use, and co-ordination with other socialised individuals, gives a certain degree of structure to their perception, as Bloor writes:

> When an individual confronts a putative new instance of a term, he confronts an array of similarities and differences, between the new and the past instances, and among the past instances. Formally, his assertion that an instance falls under a term is only his contingent judgement to the effect that similarity outweighs difference. Past usage offers precedents for this usage, but is not sufficient to fix it because there is no natural or universal scale for the weighing of similarity against difference (Bloor, 1982: 28–9)

Barnes, Bloor and Henry (1996: 103) explain that training provides resources for orienting one's perspective, in the form of exemplars and solved problems. Such past experiences are used as heuristic models through which analogies can be generated between past precedent and novel phenomena. They argue that education and experience:

> [R]enders the unknown in terms of the known, and hence allows calculations about the unknown to be made by analogy with calculations about the known. Once the connection between the known and the unknown is made, inductive inference can flow from the former to the latter directly, without passing through any 'general theory', and expectations can be developed about the unknown. Thus, knowledge can develop from case to case piecemeal … (Barnes, Bloor & Henry, 1996: 103).

Such heuristic frameworks are built up through experience with mathematical objects and constructions, developed through encountering them within their day to day work lives. Such experience can take the form of explicit knowledge formulated in text books, or in lecture or seminar presentations, as well as implicit or tacit knowledge, which may comprise of motor skills and schemas built up through solving problems, crafting proofs, and through personal engagements with other researchers and their ideas (Collins, 1974: 1981). Such tacit knowledge, MacKenzie (1995) writes, is situated knowledge, developed within local contexts:

> Because tacit knowledge is transmitted person to person, there are greater barriers to the spread of competence than the traditional view might lead us to expect. If science rests upon specific, hard-to-acquire, tacit skills, then there is a sense in which scientific knowledge is always local knowledge. It is, for example, often small "core sets," rather than wider scientific communities, that resolve scientific controversies (MacKenzie, 1995:46)

Within such local contexts individuals develop rhythms and routines by which they constitute the day-to-day worlds of work, and through which they develop intimate understandings of the problems and the proofs they work with. Merz and Knorr-Cetina (1997) explore the situated-ness of knowledge production in theoretical physics, through the local communities of practice that develop between friends, colleagues and supervisor and student:

> Suffice it to say that these relationships can be analyzed along the lines of their rhythm, their sequencing, their logic and dynamics. Theoretical physicists seemingly learn how to collaborate in early contacts with their thesis supervisor and with fellow students, as they learn how to handle objects. Like these teacher-student relationships, the 'thought alliances' physicists form later also contain an element of consultation. However, thought alliances are also sustained by and embedded in 'friend-ships' which develop from physicists spending time together in one place. (Merz and Knorr-Cetina, 1997:84)

Through situated engagements with individual problems so individuals develop an intimacy with the objects they study. By getting "stuck" on a problem so physicists and mathematicians are forced to confront definitions and conceptual boundaries around categories and knowledge. They manipulate objects in order to gain new perspectives from which to view them, deconstructing and expanding classifications in the process. Merz and Knorr-Cetina describe this process:

> Working toward a solution does not mean simply doing a calculation, but finding ways out of being "stuck". Physicists "get stuck" many times while doing a computation. But not only do the physicists get stuck: so does the computation. Physicists and computations are stuck together against the resistance of an equation. In the beginning of a computation, the object asserts itself forcefully, and ways must be found for physicists to "gain control". Gaining control rather literally means reconfiguring the object, through the process of deconstruction. The equation is changed by becoming "reduced", "converted", "divided up", "cut", "split" or "decomposed". "Reducing" or "converting" is done by "cutting" or "splitting" the computation into smaller components, which have to be "combined" or "recombined" later to arrive at final results. The concrete form of such deconstructions will become visible in subsequent sections. While physicists try to "reduce" an overbearing equation to manageable size in order to prevent it from getting "totally out of hand", they also expand the equation into new elements which can be rearranged, substitute other equations for it and supplement it by studying exemplary cases in other contexts. Physicists' "cutting" and "dividing" vocabulary conceals the expansionary character of the actual deconstruction (Merz and Knorr-Cetina, 1997:82-3)

The boundaries around categories are deliberately dissolved in order to discover new view-points from which to perceive a given problem. Researchers attempt to relate a given concept to other objects within their vocabulary, they use models to structure objects, and use conceptual schemas to translate and transmute categories. Below Merz and Knorr-Cetina document this process:

> Model objects, on the other hand, are related to the main object by analogy. As a consequence, they can be exploited in more flexible ways along the lines of their analogous relationship - as models for the object, for the

> problem specification, for difficult technical steps, or for possible results. One can take advantage of a suitable analogy by adapting known results to the new context. In addition, one can use the model as a playground for new attempts to deduce old results - for trying out new calculations in a safe and controlled environment. For example, one might try to find a new proof for a solved problem to test a proposal for how to solve the main problem. This work with the model as a 'laboratory' for trying out new calculations is very important and distinguishes physicists' use of models from pure reasoning by analogy or from doing 'exemplars' (Merz and Knorr-Cetina, 1997:102)

We see again the mutability of objects in theoretical physics and mathematics. At such local scales the boundaries around concepts remain flexible, as multiple heuristic frameworks are applied, in order to construe objects in certain ways, so as to fit certain expected patterns. Here metaphor and polysemy are exploited as tools for generating new perspectives on problems. In order to solve a problem individuals must be able to "see", or perceive, its solutions through creating interpretive lenses, or schemas, by which to view the problem. Individuals must interpret the phenomena as some category, or through use of some category, in order for that phenomena to be cognitively grasped. Such a construal of phenomena are not however inherent to the phenomena itself, rather they are induced within phenomena through the situated engagement of the individual and the object at hand. Livingston (1999) explains that the process of reasoning often involves use of mathematical intuition, which become written out of the arguments detailed in the published proof:

> Mathematical reasoning, both in its concrete detail and in its transcendence of that concreteness as an organization of those details, is analogous to these features of gestalt perception. The details of a written mathematical argument are seen in terms of the reasoning that argument describes, yet that reasoning goes beyond the literal details of the written argument. (Livingston, 1999:868)

The process of understanding a proof is thus concerned with reconstructing this intuition and reconstituting the frame of mind of the author. Livingston argues that understanding is a process of rediscovering or perceiving the "perceptual gestalts" which give the elements of the proof structure and coherence:

> In analogy with perceptual gestalts, mathematical proofs are discovered, and rediscovered on subsequent occasions, as organised totalities of reasoning and practice: the material detail of an argument articulates a coherent "whole" of reasoning that is not present in any of the argument's individual details. (Livingston, 1999:869)

In the process of constructing the proof, the prover must construct the proof in such a way as to allow other members of the community to follow their lines of reasoning. The prover must provide sufficient detail to allow the reader to re-construct the perceptual gestalt utilised by the author. The author must build up material detail for the reader, enough so that it evokes an authentic context similar to that inhabited by the author (see Becker 2000). Livingston explains that such context building takes the form of a craft process:

> As such partial arguments are produced, they are subject to any number of operations: they are inspected, integrated, discarded, revised, compared, combined and reworked, thereby embedding within the material detail of the written argument a dense texture of reasoning. In such layered arguments, provers look for the coherence of reasoning - the gestalt - of which the projected proof consists, at the same time that they are stabilizing within their work the communally recognised practices of proving. When provers arrange, rearrange and rework the material details of a prospective and developing proof, they are, in fact, orienting to and composing the cultural substance of their work. (Livingston, 1999:880)

In the process of crafting, however, the author artificially reconstructs the history of the proof to make its conclusions appear self-evident. Through following the proof, the reader is confronted with a linearized narrative, purged of its dead-ends and multiple plot-lines:

> A proof is used to justify claims about how that proof was discovered. Second, when mathematicians engage in discovery work, they continually look forward, prospectively, to the proof that lies on the horizon of their efforts. They are trying to prove something, and what they are doing now is viewed in terms of what that work will come to later. In this way, as provers work on a proof, they continually 'rewrite' what they are doing and what they have done. Retrospectively, the false paths, mistakes and failures to see what now seems 'obvious' all appear to be idiosyncratic failings of an individual prover. Looking back, such circumstances are viewed as unaccountable errors of judgment and ability; they appear to be 'without reason' and, therein, despite their prevalence wherever and whenever mathematicians are at work, are seen as impediments to, and obfuscations of, actual mathematical practice (Livingston, 2006:60-1)

The re-writing of proof, according to Livingston (2006), distorts what the lived-work of proving actually looks like in practice. According to Livingston, the prover is confronted by a field of possibilities which constantly shifts as an individual engages with the specificities of the problem at hand:

> Rather than being a static background of familiar techniques (for example, the introduction of an auxiliary line) and known facts (such as the Side-Angle- Side Congruence Theorem), that context is a dynamically changing horizon of a prover's current work. It is also a changing texture of detail and of the perceived relevance of that detail as part of the prospective horizon of that activity (Livingston, 2006:46)

According to Livingston, each activity is constituted by a horizon of possibilities, which is altered prospectively by the search for solutions, and retrospectively by the previous lines of enquiry already conducted. Within this field there are multiple potential directions that the proof-work can take. The prover must adopt and adapt strategies through which to best reach their goal. They must also present their path through the field of possibilities in such a way as to allow readers to follow, as well as for such a path to appear as a natural course:

> This other, thematically developing approach to proving the theorem was interwoven at times with the first, the prospective argument being reengaged and cultivated as different 'pieces of a puzzle' came more clearly into view, and then being cast aside when they did not seem to offer a proof that they had again seemed to promise. The idea that I was continually working on one approach and that, even though the direction of my work changed, that work was a straightforward progression to an eventual proof, is incorrect. More generally, provers are located in a field of possible directions, in a field of possibly relevant proof-specific, proof-relevant details, in a field of potential relevancies of that detail and of possible organizations of those details as possible courses of proving. Clarifying and developing one of those possible organizations of details and, therein, the field of conditional, situated relevancies of detail, provide the immediate, conditional horizon of a prospective course of proving. (Livingston, 2006:59)

The lived work of proving and problem solving in mathematics is thus a process of negotiating with the possibilities of the field. It is a form of craft work which is situated within local contexts and local engagements with problems. These contexts are constantly changing, as certain approaches fail, or as new relationships are brought to awareness. The path that the proof eventually ends up taking, and the end structure of a proof, is thus the product of negotiation and selection from a field of possibilities. However, through crafting the proof and reassembling it with the form of a narrative, so the multiple threads of arguments and multiple meanings of concepts are filtered out. What is left is an argument presented as natural and self-evident, and it is through such modes of presentation that we lose sight of this lived, embodied work.

### 1.28 Conclusion

This literature review has presented a small fraction of the work that has taken place within the sociology of mathematics and logic. The works presented were intended to highlight some of themes that will appear within the main body of this thesis. Such themes included objectivity, presentation of knowledge, mutability and stability of knowledge, as well as the performances and practices involved in constructing knowledge. I have shown that the social world[6] does have an influence on the shaping of knowledge in mathematics; I demonstrated that the boundaries around knowledge can change as a function of the contexts of their use; I also illustrated the different craft processes which are involved in producing mathematics.

In the theory section that is to follow (section 1.4) I want to bring these themes together into a more coherent theory of production of mathematical knowledge. I will use Bourdieu's theories on structure, field, habitus and practice and demonstrate how they can be used effectively to understand mathematical practice.

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[6] Here, by "social world", I mean the wider, day to day world of person to person interactions and social relations. "Social" is here meant to mean that which is held in common, whose value is bestowed through negotiation and communication between individuals in groups or communities. The "social world" is a subjective world, comprised of individual subjects, who are imbued with agency to effect change, as well as interpretive potential by which to associate meaning to events or phenomena. The "social world" is thus defined in opposition to the "natural world", which stands prior to and outside of human sociality. However as Berger (1967) and Searle (1996) would argue, the natural world is always already a social world in the sense that it is already interpreted, penetrated and structured by language.

## Section 1.3: Methodology

### 1.31 Overview

The following thesis is based upon 6 months of ethnographic fieldwork at four research institutes across Europe. I have anonymised the institutes in order to conceal the identities of the individuals involved within the study, within this thesis I shall refer to the institutes by pseudonyms. These institutes are referred to as: UK-A – a well-respected institute associated with a prestigious university in the UK; UK-B – comprised of a group of mathematicians located within the mathematics department of a well-respected UK university; Germany-A – a prestigious institution located within a German city; Paris-A – an institute in Paris associated with a prestigious French university.

### 1.32 Research methods

This project is based upon both observations and interviews conducted with mathematicians at the institutes outlined above. 45 interviews were undertaken during the course of fieldwork, each lasting between one to two hours in length. Interviewees were mathematicians from algebraic geometry, topology, and differential geometry. Interviews took place with researchers at different stages of their career, and included PhD students, Post-doctoral researchers, lecturers, research fellows, associate and assistant professors, full professors, directors of institutes, and retired professors. This wide range of interview subjects spanned the age range of 22-72, but the sample of female mathematicians was small, comprising 25% (11 individuals) of the sample (although female mathematicians are generally under-represented within many mathematics departments globally).

The selection of interviewees depended upon the willingness and availability of researchers to participate in the study. Directors, senior professors, and individuals already taking part in the study helped recruit participants for the project. Professor C in Paris-A was integral in recruiting individuals in Paris, offering her contacts at the mathematics institute. At the UK-A institute, Director J introduced the project to the mathematicians present, and helped recruit participants. Likewise, at the UK-B Institute, a senior professor made introductions, and encouraged others to participate in this study. At the Germany-A institute no formal introductions were given, and a network was instead built up independently. In all field sites a network of participants was developed through myself becoming assimilated into the mathematician's professional social networks.

Ethnographic observations were also conducted at all institutes. This process involved occupying common spaces, or private office spaces, and recording social interactions and work routines. Certain collaborative groups and individuals were shadowed during their daily routines, and the observations of such shadowing were recorded through a photo journal and through extensive fieldwork notes. Such shadowing involves moving with the working groups through the different spaces of the institute, from common areas to seminar rooms, from libraries to private offices, and observing how the activities and communicative practices are transformed through movement between different spaces. I also observed intra and inter-group interactions in order to understand how different communicative modalities are implemented across diverse social situations. Throughout this process of shadowing I collected notes, diagrams, sketches and examples, in order to understand the evolution of mathematical concepts, and the means by which they are constructed.

### 1.33 Interview Protocol

Interviews lasted between one to two hours in length and spanned a range of themes including: personal biographies, education within mathematics, daily work practices, life-histories of ideas, representation and perception of mathematics, as well as individuals' motivations and career goals. Interviews began with situating the individual within a wider social frame, through understanding their socio-economic background. Introductory questions explored their parents' backgrounds and occupations, their early experiences in mathematics, and their education within mathematics. Questioning then aimed to situate them within the mathematical field, exploring their career trajectories, and the influences on their research. Interviews moved on to discussing the particularity of an individual's research, exploring PhD problems, current problems, and the evolution of their thought. Such questions aimed to understand how individuals position their knowledge within the mathematical field, and how concepts change as a function of such positioning.

Institutional contexts were then explored, to discover what effect space has on day to day routines. Questions regarding work routines, habits, and production practices were explored, with interviewees being asked to walk the interviewer through work-routines, narrate notebooks or black-board inscriptions, and to describe in detail any current research. During this stage of the interview the researcher was questioned about how research was presented and communicated to others, as well as the process of solving problems as part of collaborations. The intention of such questioning was to understand how collaborative work practices differed from solo-work, as well as how knowledge changed in the process of presentation and communication. Towards the end of the interview I wished to explore mathematicians' perceptions of the wider field. As a result questions regarding aesthetics, validation of proofs, competition and metaphor were discussed.

### 1.34 Research tools

Altogether 45 interviews were conducted, producing 65 hours of audio material. These were recorded using a Sony ICD-PX240 Voice Recorder and a back-up voice recorder mobile app. These recordings were then transcribed by myself, with sections being classified in Nvivo according to relevant themes.

During the course of fieldwork I kept a regular account of my experiences in 4 pocket-sized fieldwork notebooks. I also kept a separate, digital fieldwork reflection journal, which is recorded in Microsoft OneNote. Throughout the process I used a small, pocket 16 mega-pixel Nikon COOLPIX S9900 camera, and my Lumina 535's 3.5 mega-pixel camera, to record a regular photo-journal of my fieldwork experiences. Using these cameras I recorded scratch paper notes, blackboard writing and other ephemera of daily life. Using the note-taking app Evernote I kept a backup of my interviews, photos and notebooks, which were automatically synced to a secure cloud server.

### 1.35 Training and background

Over the course of the PhD I have increased my understanding of mathematics, mainly through private study of mathematical text books, and through participating in online mathematics and physics courses. Online material has mainly been provided through MOOCs on Coursera, MIT open courses, and EdX. During the first year of my PhD I introduced myself to mathematics using textbooks produced by Dover publications. During my second and third years of PhD I then moved to reading the Springer Undergraduate Mathematics series of textbooks. Through such texts I gained a grounding in the following topics in mathematics:

> Linear Algebra, Set theory, Calculus, Multi-variable calculus, Analysis of complex numbers, Functional analysis, Advanced algebra, Advanced calculus, Topology, Algebraic geometry, Number theory, Group theory, and Galois theory.

This grounding in mathematics, however, does not furnish me with sufficient knowledge to participate in the creation of research mathematics. Nor is this background sufficient to understand a modern proof within my target fields. This background only provides me with a basic understanding of certain constructions, processes and objects used within the disciplines I work with.

### 1.36 Research sites

The four sites were chosen mainly because of their excellent reputation within the mathematical community, as well as the ease by which they could be accessed. The target community of differential geometers, algebraic geometers and topologists were also present within these institutes at the time of the studies, making them ideal locations in which to undertake fieldwork.

A total of 6 months was spent at all 4 research institutes. Time was divided as follows: UK-B Institute: 8 weeks; UK-A Institute: 5 weeks; Germany-A Institute: 4 weeks; Paris-A institute: 8 weeks.

Research took place between March-May 2014 at the UK-A institute, October 2014 at Germany-A Institute, September-November 2015 at UK-B Institute and January-March 2016 at the Paris-A Institute.

**1.37 Existing ethnographic work in mathematics**

For the purpose of this particular study I have selected the Mathematical research institute as my research site because of the relative ease of access to mathematicians, and because of the greater likelihood of witnessing communication of concepts between mathematicians. Such research institutes attract mathematicians for the purpose of collaborating and this by definition requires communication and the communal production and sharing of knowledge. Two key researchers: Bettina Heintz and Christian Grieffenhagen have already begun undertaking work within research institutes. My thesis supports and builds upon their findings, demonstrating the importance Heintz places on trust and friendship in validating proof. I go on to explain the importance Grieffenhagen attaches to inscription practices on black-boards, as well as the use of the body in communicating and materialising mathematics.

The stimulus for this kind of ethnographic investigation into scientific knowledge-production stems in part from Latour and Woolgar's 1979 *Laboratory life*. The aim of their project was to understand the internal workings of scientific knowledge construction, as they explain:

> Although our knowledge of the external effects and reception of science has increased, our understanding of the complex activities which constitute the internal workings of scientific activity remains undeveloped (Latour and Woolgar 1979:281)

Their study aimed to understand the day-to-day realities of life within a lab at the Salk Institute in California. By utilising ethnographic fieldwork techniques, they sought to make the familiar strange and study the scientific community as if they were a "tribe", as they explain:

> Partly as a result of our dissatisfaction, and in an effort both to penetrate the mystique of science and to provide a reflexive understanding of the detailed activities of working scientists, we decided to construct an account based on the experiences of close daily contact with laboratory scientists over a period of 2 years (Latour and Woolgar, 1979:312)

Their employment of ethnographic techniques is now common practice within sociology of scientific knowledge, but such widespread adoption is perhaps a testament to the power of the technique in gaining insight into the situated practices by which "scientific facts" are generated. Merz and Knorr Cetina (1997) are an example of how "Laboratory Life" has been extended out into other fields of scientific enquiry. In Merz and Knorr-Cetina's case they explore the production of cutting-edge research in theoretical physics at CERN in Geneva. The application of Latour and Woolgar's techniques to the "thinking" science (as opposed to "wet" or hands-on science of biology) is perhaps closer to home with regards to my own study, as the problems of how you observe knowledge-production become more obvious when most of the production process is located within the minds of individual physicists, indeed Merz and Knorr-Cetina found themselves asking:

> When theoretical physicists do a calculation, when they compute the BRST cohomology of the W-algebra, when they grade by the ghost number or face the Tb γγ term or when they discuss the advantage of doing $H(M\otimes M^*)$ over $H(F\otimes F)$, students of science usually look the other way. Are these sorts of operations still within the scope of our interest? More important perhaps, are they even within our reach? Can we study them ethnographically or observationally? What, if one could study them, might one find out?" (1997:73)

As with mathematics, they found that most of the work was not subject to observation, consisting of desk-work involving just a pen and paper. For this reason Merz and Knorr-Cetina adapted their technique:

> It also has to be admitted that the laboratory approach had to be adapted to the obdurateness of the field: the study is based rather less on the observation of physicists' activities than on one analyst's capability to exploit her physics training and interact with participants as a member of their culture. It is also anchored in a close 'reading' of physicists' personal-professional communications (their e-mail correspondence […]), their calculation protocols, and their explanations to us, which invariably involved paper and pencil. The close 'reading' was adopted to gain access to the ethno-methods implicated in doing theoretical physics work. (Ibid 1997:74)

Such ethnographic fieldwork involved analysing email communication, collecting artefacts in the form of notes and examples of diagrams. They studied small work-groups, attended group meetings and shadow teams throughout their work-day. Greiffenhagen (2008) followed a similar practice, as he explored the contexts within which mathematics is communicated. He observed mathematicians at lectures, at seminars, and during graduate students' interactions with their advisors. In his work on student-teacher communication, Grieffenhagen explains his research as follows:

> [L]ooking at mathematical lectures only gives us access to existing mathematical knowledge (and only the demonstration of this knowledge for a particular set of recipients, namely graduate students). The weekly supervision meetings between a supervisor and his doctoral students were therefore chosen as a second site. In these supervision meetings doctoral students present the current state of their research to the supervisor and they collaboratively try to find solutions for problems for which there are as yet no solutions. In these discussions, mathematicians explain to each other their reasons for why a particular way of proceeding might be successful (or not) and thereby have to explicitly formulate some of the strategies, tricks, and competences that they employ when working out novel mathematics (Grieffenhagen 2008:12)

During Grieffenhagen's (2008) fieldwork at a UK research institute, and from seminar lectures and professor-student dialogues within office spaces, he highlighted the importance that written communication on blackboards played in the construction and presentation of proof. Indeed Grieffenhagen's analysis of video-footage from mathematicians interacting at blackboards showed the ways in which mathematical concepts were constructed and unfolded through time. In my own study, communication around, and presentation upon, blackboards was a primary focus as a site for mathematical knowledge-production.

Such blackboards, and indeed research institutes in general, are sites where mathematical knowledge-production is made visible, as they are spaces where mathematical communication is necessary. Sites such as the UK-A Institute and Germany-A institute were thus selected for this study because the intentions of social actors within these institutional contexts are to communicate concepts, to explain and justify, and thus make mathematical concepts visible and subject to scrutiny. It is such "visibility" and "survey-ability" of mathematical practice that makes the research institute and the study of mathematical collaborations subject to sociological investigation. Such investigations have been conducted successfully in the past, by Heintz (2003, 1998).

Heintz conducted her sociological investigations at a research Institute in Germany. Her (1998) work consisted of a 4 month ethnographic study in 1996, during which time she conducted 20 ethnographic interviews and observed the mathematicians interacting during seminars, coffee breaks and work in offices and library spaces. The core findings from her research, relevant to this thesis concerned the social aspects involved in validating proof. Heintz described how new technologies and conceptual tools were often adopted by other mathematicians on the basis of trust.

Heintz observed that complete understanding of the conceptual machinery of a proof was not necessary for a tool to be used or accepted. Rather the character, status and standing of a mathematician within the community of practice was enough for a proof or technology to be trusted and validated. Heintz also outlined the role that aesthetics played in determining the truth of an argument, arguing that problem solving was a product of *Erleuchtung, or* enlightenment, which took the form of aesthetic feelings, intuition, and insight (Heintz, 2003:149-150).

Heintz' work highlights the role that ethnographic observations and interview based research can play in understanding the social nature of mathematical production. Such methods allow us to contextualise the production of mathematical knowledge within the day to day worlds of mathematicians. In the following thesis I shall expand on Heintz's and Greiffenhagen's work and provide a holistic account of the knowledge production process.

## Section 1.4: Theoretical Background

### 1.41 Introduction

Bourdieu's theories on the field, habitus and practice are critical components of this thesis. I use these ideas to construct a theoretical lens through which to view the mathematical communities I am studying. Through such a theoretical framework I define certain components to the mathematical production system: group, field, capital, habitus, and practice. I use Bourdieu's ideas to show how these components are formed, how they interact, and how they change over time. Although I use terms such as system, production, components, my intention is not to construct an overly mechanised vision of production in mathematics, or to reify the boundaries around groups, categories or social structures. I simply use the terms as a means for conceptualising and describing the social phenomena encountered.

Bourdieu's ideas will not be used without question, rather I will later qualify Bourdieu's definitions of habitus and field, through use of Sewell's (1992) revisions. Bourdieu's ideas, although they have stood the test of time, have not been without their critics. Bourdieu's earlier work, for example was criticised as being overly structuralist in his analysis. His work on the Berber house Bourdieu himself describes as "perhaps the last work I wrote as a blissful structuralist" (Bourdieu, 1990:9). Such a structuralist approach neglected the ambiguities and contradictions involved in interpreting symbols in concrete social situations (McAllister, 2004: 118). As Davison (1988) argues, it is the situated actions between individuals and material culture which produce symbolic meaning. The symbolic oppositions built into the organisation of space in the Berber house therefore may be manifested in different ways, according to the contextualised performances of different social actors within such spaces (Davison, 1988:100). A more nuanced understanding of Bourdieu would therefore consider how agency is exerted through performances in space, through use of tactics and strategies for manipulating symbols in order to influence relationships, structure and perceptions, as McAllister (2004) writes:

> [T]hese principles can be manipulated, adapted, selected from, ignored, reversed, etc., by the actors, to recreate and reinforce relationships or to signal changes in them, in terms of the interests and strategies of those involved. (McAllister, 2004:119)

The later works of Bourdieu take into account the mutability of structure, and study strategies and tactics for manipulating them. Bourdieu uses the term "Practice" or the "logic of practice" to describe the mechanisms by which different scripts or schemes are adapted to the practical needs of individuals. Structures are seen not only to constrain, but also to enable, and equally to be manipulated according to needs of the individual. This modification to his theories invests more agency and awareness in the individual, and presents a more nuanced vision of structure, as a set of organising principles which are reproduced by social actors through social interactions, but also which are altered by the individual through such interactions.

Another criticism of Bourdieu's theories, especially of the habitus, concerned their lack of definition. Multiple interpretations can be drawn from the definitions provided by Bourdieu and, as a result, his work is liable to be misinterpreted or misrepresented. In the following review of Bourdieu's theories I will present as clear definitions as I can, using excepts from Bourdieu's writings to substantiate the interpretations that I am presenting. With such preliminaries dealt with, I will now proceed to provide definitions of the key ideas I will be adapting from Bourdieu's theoretical corpus.

### 1.42. Defining objectivity

Let us begin by discussing Bourdieu's notion of objectivity. Much like Bloor (1984), Bourdieu locates objectivity within the social field. Objectivity within the sciences, according to Bourdieu (2004), is produced within the specific contexts of a discipline, through agreements between members of that discipline:

> Objectivity is an intersubjective product of the scientific field: grounded in the presuppositions shared within this field, it is the result of the intersubjective agreement within the field. Each field (discipline) is the site of a specific legality (a *nomos*), a product of history, which is embodied in the objective regularities of the functioning of the field and, more precisely, in the mechanisms governing the circulation of information, in the logic of the allocation of rewards etc. and in the scientific habitus produced by the field, which are the condition of the functioning of the field. (Bourdieu, 2004:83)

For Bourdieu there is no distinction between objectivity and subjectivity, rather objectivity is subject to intersubjective negotiations within the social field. The field constitutes certain phenomena as objective, legitimising certain phenomena as "facts". Facts are therefore constructed entities produced through consensus between individuals socialised within a given field. The fact is not a given, rather Bourdieu explains:

> The fact is won, constructed, observed, in and through the dialectical communication among subjects, that is to say through the process of verification, collective production of truth, in and through negotiation, transaction, and also homologation, ratification by the explicitly expressed consensus - homologein - (and not only in the dialectic between hypothesis and experiment). A fact truly becomes a scientific fact only if it is recognised.

> The construction is socially determined in a twofold way: on the one hand, by the position of the laboratory or scientist within the field; on the other hand, by the categories of perception associated with the position of the receiver (with the effect of imposition, authority, being that much greater the lower this position is in relative terms). (Bourdieu, 2004:73)

A fact can only be classified as such if it is recognised by members of a field. The fact, Bourdieu writes is created through recognition, through the act of discerning and discriminating. The means by which such facts are agreed upon, by communities of practice, are through creating rules for verifying knowledge. For an individual to participate in the creation of knowledge they must be cognisant of the rules and have skill at manipulating them. Bourdieu characterises this mastery of rules as possessing a "feel for the game", as he describes below:

> Epistemological rules are the conventions established for settling controversies: they govern the confrontation of the scientist with the external world, that is, between theory and experiment, but also with other scientists, enabling him to anticipate criticism and refute it. A good scientist is someone who has a sense of the scientific game, who can anticipate criticism and adapt in advance to the criteria defining acceptable arguments, thus advancing the process of recognition and legitimation; who stops experimenting when he thinks that the experimentation conforms to the socially defined norms of his science and when he feels sufficiently assured to confront his peers. Scientific knowledge is the set of propositions that have survived objections. (Bourdieu, 2004:83)

### 1.43 Defining social position

Epistemological rules thus define the forms that acceptable knowledge in the field can take. Such rules also shape individual's perception of knowledge, by creating positions or stances from which individuals can relate to knowledge and relate knowledge together:

> Epistemological rules are nothing other than the social rules and regularities inscribed in structures and/ or in habitus, particularly as regards the way of conducting a discussion (the rules of argumentation) and settling a conflict. Researchers put an end to their experimentation when they think that their experiment is consistent with the norms of their science, and can confront the expected criticism. (Bourdieu, 2004:71)

These standpoints are defined by an individual's own position within the field, which is determined relative to other members of the field. The sum of possible positions in a field defines the social space of the field, with different stand-points, or positions, linked through operations of relation and differentiation, as Bourdieu explains:

> But a point of view is also a point in a space (*Standpunkt*), a point of space where one stands in order to take a view, a point of view in the first sense, on that space: to conceive the point of view in this way is to conceive it differentially, relationally, in terms of the possible alternative positions to which it is opposed in different respects (income, qualifications, etc.). (Bourdieu, 2004: 31)

Positions, or standpoints, within social space, generate different viewpoints from which to view the space of social relations, as well as the objective facts of the field. Each view creates different perspectives of the field, and thus affects their perception of the field. The result of this position-dependency is that the epistemological rules, as well as the schemas of perception inherited from habitus, will have different expression as a function of the individual’s position in the field. As Bourdieu explains, there is no absolute point of view of the field, rather all views are anchored, and their summation produces the field of view, through dialogue and debate:

> A point of view is first of all a view taken from a particular point (*Gesichtspunkt*), a particular position in space and, in the sense in which I shall mean it here, in the social space: to objectivate the subject of objectivation, the (objectivating) point of view, is to break with the illusion of the absolute point of view, which is characteristic of every point of view; it is therefore also a perspective view (*Schau*): all perceptions, visions, beliefs, expectations, hopes, etc., are socially structured and socially conditioned and they object a law which defines the principle of their variation, the law of the correspondence between positions and position-takings. Individual A's perception is to individual B's perception as A's position is to B's position, with the habitus making the connection between the space of positions and the space of points of view (Bourdieu, 2004: 95)

**1.44 Defining social spaces**

Each scientific discipline generates its own social spaces, which are composed of local, hierarchically ordered spaces of institutes, departments, collaborative groups, etc. The spaces generated by disciplines are themselves ordered hierarchically in relation to the total set of spaces which constitute "Science" in its generality. Mathematical disciplines such as algebraic geometry are thus related to other disciplines, such as topology, within the set of disciplines which constitutes the field of "mathematics". As Bourdieu explains below, individual work-sites, such as laboratories, form local spaces or microcosms, which are related to other laboratories to constitute a discipline. Within each individual laboratory there will be a given positioning of individual scientists, these local positions are further structured according to the laboratory's position amidst the wider collection of laboratories:

> But it is immediately clear that a laboratory is a social microcosm, itself situated in a space containing other laboratories, these together constituting a discipline (itself situated in a hierarchized space, that of the disciplines), and that is derives a major part of its properties from the position it occupies within that space. If one ignores this series of structural

> interlockings, this (relational) position and the associated effects of position, one is likely, as in the case of the village monograph, to look in the laboratory for explanatory principles which in fact lie outside it, in the structure of the space within which it is located. Only through an overall theory of the scientific space, which understands it as a space structured according to both generic and specific logics, is it possible truly to understand a given point in this space, where a particular laboratory or an individual researcher. (Bourdieu, 2004, 32-3)

The position of the individual within social space is thus determined simultaneously at the local level of the laboratory or department, and at the global level of the discipline. The relationships between the local and the global spaces constitutes the field of objective relationships. An individual's place within this field then shapes their dispositions towards phenomena, as well as their performances of their selves and their ideas, as Bourdieu explains:

> The notion of the field marks a first break with the interactionist approach, inasmuch as it takes note of the existence of this structure of objective relationships among laboratories and among researchers which governs or orients practices. It makes a second break, inasmuch as the relational or structural approach that it introduces is associated with a dispositionalist philosophy, which breaks with the finalism, allied to a naïve intentionalism, which sets agents - in this case researchers - as rational calculators seeking not so much the truth as the social profits accruing to those who appear to have discovered it. (Bourdieu, 2004:33)

**1.45 Defining scientific capital**

Position and therefore disposition (habitus) within the field is part determined by the possession of capital. Capital is always already social capital, because it has been constituted and consecrated within social systems. Capital can take the form of economic capital (money, possessions, labour power, earning power), moral capital (religious standing), symbolic capital (status, position, recognition), knowledge capital, political capital (ability to influence decision making), etc. In the case of the sciences and mathematics Bourdieu creates the notion of "scientific capital". According to Bourdieu scientific capital is derived from the category of symbolic capital, and relates to the recognition individuals receive as a result of producing accepted knowledge within a scientific discipline:

> Symbolic capital is a set of distinctive properties which exist in and through the perception of agents endowed with the adequate categories of perception, categories which are acquired in particular through experience of the structure of the distribution of this capital within the social space or a particular social microcosm such as the scientific field. Scientific capital is a set of properties which are the product of acts of knowledge and recognition performed by agents engaged in the scientific field and therefore endowed with the specific categories of perception that enable them to make the pertinent distinctions, in accordance with the principle of pertinence that is constitutive of the nomos of the field. This diacritical perception is only accessible to those who possess sufficient incorporated cultural capital. (Bourdieu, 2004:55)

To produce accepted knowledge an individual has to have incorporated the schemas of perception necessary to discern and discriminate novel phenomena within the field. Through making a recognised contribution to the stock of knowledge, so a scientist adds to their stock of scientific capital and distinguishes themselves relative to their peers:

> To exist scientifically is to have a "plus" in terms of the categories of perception prevailing within the field, that is to say, for one's peers (to have contributed something), to have distinguished oneself (positively) by a distinctive contribution. In scientific exchange, the scientist makes a "contribution" for which he is recognised by acts of public recognition such as citation among the sources of the knowledge used. Thus scientific capital is the product of recognition by competitors (and an act of recognition brings capital to the extent that the person who makes it is himself autonomous and rich in specific capital). (Bourdieu, 2004:55)

Scientific capital, like any form of capital, is only legitimised through recognition. Only individuals socialised within the field possess sufficient understanding of the domain of knowledge to be able to recognise contributions to knowledge. For such a reason recognition of scientific capital is valid only within the limits of the field in which that capital is constituted:

> Scientific capital functions as a symbolic capital of recognition that is primarily, sometimes exclusively, valid within the limits of the field (although it can be converted into other kinds of capital, economic capital in particular). (Bourdieu, 2004:55)

Only within the bounds of the field can the value of a contribution be judged and given weight. The more distinctive a contribution is the more recognition a concept gains, and therefore a contribution and its producer gain in visibility as a result:

> A scientist's symbolic weight tends to vary with the distinctive value of his contributions and the originality that his competitor-peers recognise in his distinctive contribution. The notion of visibility, used in the American university tradition, accurately evokes the differential value of this capital which, concentrated in a known and recognised name, distinguishes its bearer from the undifferentiated background into which the mass of anonymous researchers merges and blurs. (Bourdieu, 2004:56)

To increase one's scientific capital an individual must therefore increase the value of their contributions and their visibility within the field. Individuals thus act in such a way as to increase their standing in the social field, through maximising the acquisition and effectiveness of their capital. To accomplish this task individuals adopt social strategies, or positions, which maximise the profit they receive from their capital investments:

> These position-takings are the product of the relationship between a position in the field and the dispositions (the habitus) of its occupant. Every scientific choice - the area of research, the methods used, the place of publication, the choice, well described by Hagstrom (1965:100), of rapid publication of partially verified findings or later publication of fully checked findings - is also a social strategy of investment oriented towards maximisation of the specific, inseparably social and scientific profit offered by the field and determined by the relationship between position and dispositions that I have just set out. (Bourdieu, 2004:59).

### 1.46 Defining the field

The field I have discussed so far is a field of objective relations between individuals, determined by the distribution of social capital. This field defines an individual's position relative to others, and their position shapes their dispositions and stances towards structures determined by the field. As Bourdieu writes, an individual's position and consequent stance will therefore constitute the field of possibilities for potential action within the social world:

> In reality, the dispositions durably inculcated by the possibilities and impossibilities, freedoms and necessities, opportunities and prohibitions inscribed in the objective conditions (which science apprehends through statistical regularities such as the probabilities objectively attached to a group or class) generate dispositions objectively compatible with these conditions and in a sense pre-adapted to their demands. (Bourdieu, 2004:54)

The total space of positions is a field which generates the field of possibilities for social action. Such possibilities, however, can only be perceived through situated stances (dispositions) which are structured by the habitus. Through use of a habitus (a system of schemas structuring dispositions) the individual generates a vision of the field of possible actions:

> The space of positions, when perceived by a habitus adapted to it, functions as a space of possibles, the range of possible ways of doing science, among which one has to choose; each of the agents engaged in the field has a practical perception of the various realizations of science, which functions as a problematic. This perception, this vision, varies according to the agent's dispositions, and is more or less complete, more or less extensive; it may rule out some sectors disdaining them as uninteresting or unimportant. The relationship between the space of possibles and dispositions can function as a system of censorship, excluding some directions and means of research de facto without even stating any restrictions. This narrowing effect is strongest for those agents who have least symbolic capital and specific cultural capital. (Bourdieu, 2004:59-60)

The possibilities for action in the field are limited by individuals' position in the field, which is determined by their possession of capital. The possibility of an individual acting freely and exerting agency are limited by their possession of capital. Through unequal distribution of capital within fields, so inequality is generated, which creates positions of relative domination and subordination within any given social field:

> Even in the absence of direct interaction, intervention or manipulation, the structure of the field, defined by the unequal distribution of capital, that is of the specific weapons or assets, bears on all the agents within it, restructuring more or less the space of possibles that is open to them, depending on how well placed they are within the field, that is within this distribution. A dominant agent is one who occupies a place within the structure such that the structure works in his favour. (Bourdieu, 2004:34)

Individuals will therefore possess varying degrees of agency by which to change their position within the field, or produce desired actions within the field. This ability to effect change is characterised as the "force" exerted by the individual within the field. Such force is proportional to the volume or mass of accumulated capital they possess:

> The force attached to an agent depends on his various "assets", differential factors of success which may give him an advantage in the competition, that is to say, more precisely, the volume and structure of the capital in its various forms that he possesses…The structure of the distribution of capital determines the structure of the field, in other words the relations of force among the scientific agents: possession of a large quantity of capital gives a power over the field, and therefore over agents (relatively) less endowed with capital (and over the price of entry to the field) and governs the distribution of the chances of profit. (Bourdieu, 2004:33-4)

Possessing capital affects an individual's power over the field, with large accumulations of capital possessing more force by which to influence decision making, tastes, and allocation of resources. An individual is thus motivated to maximise their acquisition of capital, in order to exert more force and hence more influence in the field of production. The field thus is not only a field of objective relations between individuals, but is also a field of struggles within which individuals compete for the resources necessary to exert their agency:

> [T]he field as a field of struggles, a socially constructed field of action in which agents endowed with different resources confront one another to conserve or transform the existing power relations. Agents undertake actions there, the ends, means and efficacy of which depend on their position within the field of forces, their position within the structure of the distribution of capital. (Bourdieu, 2004:34-5)

Researchers adopt strategies by which to increase their volume of capital, but they are at the same time constrained in their strategic intentions by their positions within the field itself, as Bourdieu explains:

> Rather than being deployed in the context of a universe without gravity or inertia, where they would be able to develop without restriction, researchers' strategies are oriented by the objective constraints and possibilities implied in their respective position and by the representation (itself linked to their position) they are able to form of their position and those of their rivals, on the basis of their information and their cognitive structures. (Bourdieu, 2004:34-5)

Such positionings are themselves defined by the structure of the field and the distribution of capital within a given field. Fields, as Bourdieu describes, are characterised by structures of domination and sub-ordination, which vary according to the concentration of capital:

> The room for manoeuvre available to their strategies will depend on the structure of the field, characterised for example by a more or less high degree of concentration of capital (ranging from near monopoly…to a virtually equal distribution among all competitors); but it will always be organised around the principle opposition between the dominant and the dominated, the challengers. The former are able, often effortlessly, to impose the representation of science most favourable to their interests, that is to say, the "correct" legitimate way to play and the rules of the game and therefore the participation in the game. Their interests are bound up with the established state of the field and they are the natural defenders of the "normal science" of the day. They enjoy decisive advantages in the competition, one reason being that they constitute an obligatory reference point for their competitors, who, whatever they do, are…required actively or passively to take up a position in relation to them. (Bourdieu, 2004:35)

**1.46 Defining the boundaries of the field**

To be part of a field is to be part of a game in which individuals compete against one another for scarce resources of capital. To be part of the game an individual must accept certain rules, as well as the limits within which the game can be played. The limits of the game however are not clear cut, as the boundaries around the field merge with other social fields. The field of mathematics for example may be contained within wider fields of sociality: within universities, nation states, the scientific community, etc. Or they may exist parallel to other social fields: politics, family life, religion, etc. The boundaries around the field are thus ill defined, as the structure of the field itself is subject to change as a result of external pressures and internal tensions. Fields will have different degrees of autonomy according to how the pressures and tensions are balanced, as well as how the boundaries around the fields are maintained:

> In fact, the field is subject to (external) pressures and contain tensions, in the sense of forces that act so as to drive apart, separate, the constituent parts of a body. To say that the field is relatively autonomous with respect to the encompassing social universe is to say that the system of forces that are constitutive of the structure of the field (tension) is relatively independent of the forces exerted on the field (pressure). It has, as it were the "freedom" it needs to develop its own necessity, its own logic, its own *nomos*. (Bourdieu, 2004:47)

The varying degrees of autonomy of the field, that is to say the boundaries of the field, are thus constructed through controlling the production and possession of capital, through creating entry conditions to the field itself:

> The process of autonomization is linked to the rise in the implicit or explicit price of entry. This price of entry is competence, scientific capital, incorporated and turned into a "sense of the game", but it is also the propensity to take part in the game, *the libido scientifica*, the *illusio,* the belief not only in the stakes but also in the game itself, the idea that the game is…worth playing. (Bourdieu, 2004:50)

In the case of fields within mathematics such entry conditions take the form of restricting access and production of knowledge through professionalization of the discipline:

> Mathematization first produces an effect of exclusion from the field of discussion. With Newton (I would add Leibniz), the mathematization of physics tended increasingly, from the mid-eighteenth century, to set up a very strong social separation between professionals and amateurs, insiders and outsiders. Mastery of mathematics became the price of entry and reduced not only the number of potential readers but also of potential producers. The Boundaries of the space were slowly redefined in such a way that the potential readers were more and more limited to the potential contributors, having the appropriate training. In other words,

> mathematization contributed to the formation of an autonomous scientific field" (Gingras 2001:24). (Bourdieu, 2004:48)

### 1.47 Defining habitus[7]

Any given field is thus relatively autonomous to other fields, depending on the field's boundary conditions and how they are maintained. However, even within fields, there is an internal order generated by the distribution of capital. Individuals

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[7] Bourdieu's notion of habitus is taken primarily from two sources: the work of the art historian Erwin Panofsky (1976 [1951]) and the anthropologist Marcel Mauss (1973 [1934]). Mauss himself adapts the concept from Aristotle's "hexis" (acquired state of being). Mauss, however, uses the term habitus to describe the "techniques of the body", which are shaped through socialisation (through education and apprenticeship), he traces out the concept of habitus in the following :

> I have had this notion of the social nature of the 'habitus' for many years. Please note that I use the Latin word-it should be understood in France-habitus. The word translates infinitely better than 'habitude' (habit or custom), the 'hexis', the 'acquired ability' and 'faculty' of Aristotle (who was a psychologist). It does not designate those metaphysical habitudes, that mysterious 'memory', the subjects of volumes or short and famous theses. These 'habits' do not just vary with individuals and their imitations, they vary especially between societies, educations, proprieties and fashions, prestiges. In them we should see the techniques and work of collective and individual practical reason rather than, in the ordinary way, merely the soul and its repetitive faculties. (1973:73).

Mauss' concept of habitus combines notions of body schemas: the techniques necessary for shaping bodily action – with evaluative schemas: the means by which skills are perceived, valued and judged by other socialised individuals. The body thus, in Mauss' thought, is a socialised body, which is both structured on a physiological level, as well as at a moral or aesthetic level.

For Panofsky the notion of habitus concerned the "habits of the mind", and how mental structures could become translated into physical architecture through adopting a certain *modus operandi* (way of operating). Panofsky studied the emergence of Gothic architecture in and around Paris during the 12th and 13th centuries, exploring architecture of this period as a cultural product inspired by the philosophies of the medieval scholastics (Panofsky, 1976:4-5). Masons of the period developed their craft through adopting the "mental habits" of the scholastics of the period (1976:20-21). By adopting certain motifs and principles of clarity, symmetry, totality, and hierarchy so a unified visual logic was generated which reflected the logic of the scholastics (p.68). These generative principles for structuring cultural productions were thus part of a wider habitus shared by educated elites of the time, and reflected, as Hanks (2005) writes, the "spirit of the age" (Hanks, 2005:72).

Through translating Panofsky's work in 1976 Bourdieu adopted and adapted Panofsky's notions of habitus as a means of explaining how certain socio-cultural-material systems were generated, and how physical, mental, and discursive structures could emerge which shared certain ordering principles (practical logic) (See Hanks, 2005: 71-2).

with similar capital accumulations will be located in similar objective positions within the field, and hence neighbourhoods are created, which Bourdieu constitutes as classes. Members of such classes, because they share similar objective positions within the field, will tend to share similar norms, beliefs and dispositions. Such class dispositions Bourdieu defines as "habitus", which are characterised within the following two quotes, taken from two separate, but similar characterisations by Bourdieu:

> The structures constitutive of a particular type of environment (e.g. the material conditions of existence characteristic of a class condition) produce habitus, systems of durable, transposable dispositions, structured structures predisposed to function as structuring structures, that is, as principles for the generation and structuring of practices and representations which can be objectively “regulated” and “regular” without in any way being the product of obedience to rules, objectively adapted to their goals without presupposing a conscious aiming at ends or an express mastery of the operations necessary to attain them and, being all this, collectively orchestrated without being the product of the orchestrating action of a conductor (Bourdieu, 1977: 72)

> The conditionings associated with a particular class of conditions of existence produce habitus, systems of durable, transposable dispositions, structured structures predisposed to function as structuring structures, that is, as principles which generate and organise practice and representations that can be objectively adapted to their outcomes without presupposing a conscious aiming at ends or an express mastery of the operations necessary in order to attain them. Objectively "regulated" and "regular" without being in any way the product of obedience to rules, they can be collectively orchestrated without being the product of the organizing action of a conductor. (Bourdieu, 1990:53)

Habitus, we find, are "durable" in the sense that they persist within social groups over time, they are thus resistant to change. Habitus are transposable in the sense that certain schemes are flexible and can be used within multiple contexts. They are also structured structures - this reflects the schematic or program-like nature of habitus, which act to give order to experience, much like the notion of a gestalt. Such schemes work at multiple levels of experience, from shaping the body's movements and practical actions, to shaping representations in linguistic and conceptual spaces, to shaping social interactions between individuals. This is made more explicit by Bourdieu in the following quotes:

> [A] subjective but not individual system of internalised structures, schemes of perception, conception, and action common to all members of the same group or class. (Bourdieu, 1977:86).
>
> Habitus is both a system of schemes of production of practices and a system of perception and appreciation of practices… (Bourdieu, 1989: 19- 20).

Within these quotes we see the perceptual, conceptual and behavioural aspects of habitus. Bourdieu thus does not want us to limit habitus' operations to the social world, but he argues that such schemes organise every facet of our existence. We think, and act, and socialise through internalising certain social programs, which then run in the background of our consciousness. The habitus, as Bourdieu explains, constitutes the practical world of action, through creating "motivating structures" which present possibilities for action in the world:

> The practical world that is constituted in the relationship with the habitus, acting as a system of cognitive and motivating structures, is a world of already realised ends - procedures to follow, paths to take - and of objects endowed with a "permanent teleological character", in Husserl's phrase, tools or institutions. This is because the regularities inherent in an arbitrary condition tend to appear as necessary, even natural since they are the basis of the schemes of perception and appreciation through which they are apprehended. (Bourdieu, 1990:53-4)

The schemes of a habitus thus creates perspectival frames through which phenomena are construed, given order and categorised. The individual is presented with possibilities for action, which are organised as strategies, not through the strategic intentions of the individual, but rather through path-dependent operations programmed by the habitus:

> The habitus contains the solution to the paradoxes of objective meaning without subjective intention. It is the source of these strings of "moves" which are objectively organised as strategies without being the product of a genuine strategic intention - which would presuppose at least that they be apprehended as one among other possible strategies. If each stage in the sequence of ordered and oriented actions that constitute objective strategies can appear to be determined by anticipation of the future, and in particular, of its own consequences (which is what justifies the use of the concept of strategy), it is because the practices that are generated by the habitus and are governed by the past conditions of production of their generative principle are adapted in advance to the objective conditions whenever the conditions in which the habitus functions have remained identical, or similar, to the conditions in which it was constituted. (Bourdieu, 1990:54)

Certain actions thus become sedimented within the individual's habits, they become naturalised as "intuition" (what Bourdieu calls the "feel for the game"). The possibilities for action are experienced as a practical sense, which is manifested as choices for directing the individual:

> Practical sense is a quasi-bodily involvement in the world which presupposes no representation either of the body or of the world, still less of their relationship. It is an immanence in the world through which the world imposes its imminence, things to be done or said, which directly govern speech and action. It orients "choices" which, though not deliberate, are no less systematic, and which, without being ordered and organised in relation to an end, are none the less charged with a kind of retrospective finality. A particularly clear example of practical sense as a proleptic adjustment to the demands of a field is what is called, in the language of sport, a "feel for the game". This phrase gives a fairly accurate idea of the almost miraculous encounter between the habitus and a field, between incorporated history and an objectified history, which makes possible the near-perfect anticipation of the future inscribed in all the concrete configurations on the pitch or board. (Bourdieu, 1990:66)

### 1.48 Defining body hexis

This "feel for the game", or intuition, is a product of being socialised within the field and specifically with certain sub-classes of the field. Position within classes determines the sets of schemas, or habitus, an individual is exposed to, and these schemas, in turn, shape an individual's practices, strategies and perception of the field. Habitus themselves are manifested in the dispositions of individuals. Such dispositions can take the form of physical dispositions: ways of speaking, using the body, using space, interacting with others; conceptual dispositions: the theories, concepts, classifications one adopts; aesthetic dispositions: tastes, personal preferences, aesthetic sensibilities etc. Disposition affects how an individual interacts with others, who they interact with, what they discuss, what they consume, what they believe and aspire to be; in short such dispositions involve every facet of an individual's life-world. The schemes of the habitus relate phenomena together, and may serve to construe one set of categories in terms of another, thereby creating analogues between domains of experience:

> The habitus, as "a matrix[8] of perceptions, appreciations and actions," allows for "the achievement of infinitely diversified tasks, thanks to analogical transfers of schemes permitting the solution of similarly shaped problems" (Bourdieu 1977:83).

Schemas thus are transferred between domains of experience through analogical transfers and metaphorical mappings, through use of schemas or heuristics. The

[8] This concept of Matrix is another way of representing the multi-dimensional nature of habitus, instead of the more geometrical representation depicted within this thesis. The habitus is seen to exist as an “n row by n column” matrix (in which n is an arbitrarily large number), in which different dimensions of the field are represented and related together within the rows and columns of the matrix.

origins of these analogues lies in the phenomenal world, in the physical structures of inhabited space, which serve to ground all experience. Bourdieu, in his discussion of the Berber house, identifies inhabited space, the house in particular, as the "privileged site of the objectification of the generative schemes":

> Inhabited space - starting with the house - is the privileged site of the objectification of the generative schemes, and, through the divisions and hierarchies it establishes between things, between people and between practices, this materialized system of classification inculcates and constantly reinforces the principles of the classification which constitutes the arbitrariness of a culture. Thus, the opposition between the sacred of the right hand and the sacred of the left hand, between *nif* and *h'aram*, between man, invested with protective and fertilizing powers, and woman, who is both sacred and invested with maleficent powers, is materialised in the division between masculine space, with the assembly place, the market or the fields, and female space, the house and the garden, the sanctuaries of *h'aram*; and, secondarily, in the opposition which, within the house itself, assigns regions of space, objects and activities either to the male universe of the dry, fire, the high, the cooked, the day, or the female universe of the moist, water, the low, the raw, the night. (Bourdieu, 1990:76)

Inhabited space comes to mirror the inner worlds of thought, and acts as a tool for externalising this internal world, in the form of physical structures. Through dialogue between the resources of the material world and the schemes of structure, so the habitus comes to be objectified, and social structure externalised, within the physical surroundings. The habitus then is mirrored or repeated within the physical, conceptual and social spaces, which it constitutes as analogically connected. Bourdieu describes the metaphorical nature of the habitus in the following:

> The world of objects,  is a kind of object, a kind of book in which each thing speaks metaphorically of all others and from which children learn to read the world, is read with the whole body, in and through the movements and displacements which define the space of objects as much as they are defined by it. The structures that help to construct the world of objects are

> constructed in the practice of a world of objects constructed in accordance with the same structures. The "subject" born of the world of objects does not arise as a subjectivity facing an objectivity: the objective universe is made up of objects which are the product of objectifying operations structured according to the same structures that the habitus applies to them. The habitus is a metaphor of the world of objects, which is itself an endless circle of metaphors that mirror each other ad infinitum. (Bourdieu, 1990:76-7)

It is in dialogue with this externalised structure, the objective, physical manifestations of the habitus, that the habitus itself is replicated by individual inhabitants of the field. Through inhabiting socialised spaces so individuals develop the habits and dispositions which constitute a viewpoint on the world. Bourdieu writes that, through structuring positions, relations and displacements within space, so structure is read by the body and enacted within day to day practice:

> The house, an *opus operatum*, lends itself as such to a deciphering, but only to a deciphering which does not forget that the book from which the children learn their vision of the world is read with the body, in and through the movements and displacements which make the space within which they are enacted as much as they are made by it. (Bourdieu, 1977:90)

Actions performed within structured spaces thus are invested with social meaning and serve to educate the body's dispositions to phenomena, and construct the categories by which and individual perceives reality:

> But in fact all the actions performed in a structured space and time are immediately qualified symbolically and function as structural exercises through which practical mastery of the fundamental schemes is constituted. Social disciplines take the form of temporal disciplines and the whole social order imposes itself at the deepest level of the bodily dispositions through a particular way of regulating the use of time, the temporal distribution of collective and individual activities and the appropriate rhythm with which to perform them. (Bourdieu, 1990:75)

This art of educating the body and gaining practical mastery of the schemes of perception Bourdieu refers to as *Body hexis*:

> *Body hexis* speaks directly to the motor function, in the form of a pattern of postures that is both individual and systematic, because linked to a whole system of techniques involving the body and tools, and charged with a host of social meanings and values: in all societies, children are particularly attentive to the gestures and postures which, in their eyes, express everything that goes to make an accomplished adult. (Bourdieu, 1977: 87)

**1.49 Defining practice**

The body acts as a bridge between worlds: between physical, conceptual and social spaces. Through the body's actions, so common structuring frameworks are created between these multiple dimensions. The body thus grounds the individual within a context and, through being conditioned within social spaces, so it is oriented to the world in a certain way. Bodies which are domesticated within similar structured spaces will have their habits shaped in similar ways, sharing similar references and thus viewpoints on the world. Individuals within similar contexts thus come to co-ordinate their reference frames, sharing common ground and common habitus:

> The habitus is precisely this immanent law, *lex insita*, inscribed in bodies by identical histories, which is the precondition not only for the co-ordination of practices but also for practices of co-ordination. The corrections and adjustments the agents themselves consciously carry out presuppose mastery of a common code; the undertakings of collective mobilisation cannot succeed without a minimum of concordance between the habitus of the mobilising agents and the dispositions of those who recognise themselves in their practice or words, and above all, without the inclination towards grouping that springs from the spontaneous orchestration of dispositions. (Bourdieu, 1990:59)

As a result of sharing common ground, and hence common reference frames, so individuals will also share common practices, that is to say common dispositions towards phenomena: they may, for example, come to adopt similar habits, forms of speech, or dress, or deportment. However, such practices are not merely replicated whole-sale by each individual, rather they are enacted and negotiated during situated engagements with the world. Practical action thus is context-dependent, and under-determined by the schemes which generate it. Within the immanence of the act there is therefore a degree of uncertainty in the outcome, as Bourdieu explains:

> The generative formula which enables one to reproduce the essential features of the practices treated as an *opus operatum* is not the generative principle of the practices, the *modus operandi*. If the opposite were the case, and if practices had as their principle the generative principle which has to be constructed in order to account for them, that is, a set of independent and coherent axioms, then the practices produced according to perfectly conscious generative rules would be stripped of everything that defines them distinctively as practice, that is, the uncertainty and "fuzziness" resulting from the fact that they have as their principle not a set of conscious, constant rules, but practical schemes, opaque to their possessors, varying according to the logic of the situation, the almost invariably partial viewpoint which it imposes, etc. Thus, the procedures of practical logic are rarely entirely coherent and rarely entirely incoherent. (Bourdieu, 1990:12)

The individual replicates the structure of habitus in practical ways, according to their own judgement and the specificities of the context in which the action takes place. They apply the schemas of the habitus in practical ways, according to their needs and the resources they have at hand. As a result of this pragmatism, the logic by which a system functions will always have elements which are inconsistent and incoherent, and thus which are difficult to model explicitly:

> In other words, symbolic systems owe their practical coherence - that is, on the one hand, their unity and their regularities, and on the other, their "fuzziness" and their irregularities and even incoherences, which are both equally necessary, being inscribed in the logic of their genesis and

> functioning - to the fact that they are the product of practices that can fulfil their practical functions only in so far as they implement, in the practical state, principles that are not only coherent - that is, capable of generating practices that are both intrinsically coherent and compatible with the objective conditions - but also practical, in the sense of convenient, that is, easy to master and use, because they obey a "poor" and economical logic. (Bourdieu, 1990:86)

This "practical logic" is economic in the sense that a few generative principles are used across multiple different domains of experience:

> This practical logic - practical in both senses - is able to organize all thoughts, perceptions and actions by means of a few generative principles, which are closely interrelated and constitute a practically integrated whole, only because its whole economy, based on the principle of the economy of logic, presupposes a sacrifice of rigour for the sake of simplicity and generality and because it finds in "polythesis" the conditions required for successful use of polysemy. (Bourdieu, 1990:86)

The use of schemas across multiple domains of experience as heuristic devices mean that, in practice, individuals seek out equivalences between categories, and hence analogies between categories, in order to match novel phenomena to those categories they already understand. Practical equivalences are hence drawn between phenomena, in order to construct practical models by which to organise the world. As Bourdieu explains, the body serves as the original reference from which all practical equivalences are constructed. Bodily understanding of position, orientation, distinction, relation are the basis from which all other structuring and classifying operations are drawn:

> When the properties and movements of the body are socially qualified, the most fundamental social choices are naturalised and the body, with its properties and its movements, is constituted as an analogical operator establishing all kinds of practical equivalences among the different divisions of the social world - divisions between the sexes, between the age groups and between the social classes - or, more precisely, among the meanings

> and values associated with the individuals occupying practically equivalent positions in the spaces defined by these divisions. (Bourdieu, 1990:44)

### 1.410 Defining craft

The context-dependency of habitus mean that the practical application of rules and schemas are localised and underdetermined in practice. As with our previous discussion on finitism, the local reference frames for action are specific to the individual and the context within which the action takes place. Knowledge production or action in the social world thus are not the product of conscious, explicit, applications of rules and schemas, but rather are tacit, crafting processes by which knowledge is organically built up through local engagements with concrete problems:

> To reintroduce the idea of the habitus is to set up as the principle of scientific practices, not a knowing consciousness acting in accordance with the explicit norms of logic and experimental method, but a "craft", a practical sense of the problems to be dealt with, the appropriate ways of dealing with them, etc. (Bourdieu, 2004:38)

This notion of craftwork extends not only to the production of knowledge, but also to the process of domesticating the individual within the field of knowledge production. The individual is crafted by the habitus as much as they themselves craft out their own viewpoint from within the habitus. Through such craftwork the individual gains a practical mastery of the schemes and resources of the field. Such schemes are incorporated and embodied within the individual, through being educated within the field. This education then manifests itself through gaining an "educated eye", which allows the individual to discern and recognise certain distinctions and relationships between categories of experience. Within scientific fields, Bourdieu explains, the educated eye is crafted through situated engagements with examples, through problem solving and experimenting, not simply through learning theory. The practical mastery of the field thus is more akin to hands on craft work, that is to say, science is more akin to art:

> There is always an implicit, tacit dimension, a conventional wisdom engaged in evaluating scientific works. This practical mastery is a kind of "connoisseurship" which can only be communicated through example, and not through precepts; it is not so different from the art of recognising a good picture, or identifying its period and author, without necessarily being able to articulate the criteria that one is applying. "Scientific research - in short - is an art" (Polanyi, 1957:57). (Bourdieu, 2004:38)

**1.411 Incorporating Sewell's reformulations of Bourdieu**

In Bourdieu's use of practice and craft there is an attempt to counter some of the critiques levelled against him, which argued that he reified social structure and neglected the agency exerted by individuals and material artefacts. In the above definitions of Bourdieu's theories I have tried to present a clear picture of Bourdieu's ideas, using, where possible, Bourdieu's own words. However, for the purpose of this thesis, I will modify Bourdieu's ideas slightly, incorporating Sewell's critiques and reformulations on Bourdieu, Lakoff and Johnson's work on metaphor, and Goffman's work on performance. These additions will provide a stronger theoretical basis for explaining the mechanisms by which individuals' positions in the mathematical field shape their dispositions within, and perceptions of, mathematical spaces. Such additions will also allow me to explain Bourdieu's notion of social topology at the end of this thesis.

I will first turn towards Sewell, and his interpretation of Bourdieu. Sewell's reformulations of structure and habitus will allow me to discuss the multiplicity of structures, which better accounts for the role of polysemy and metaphor. Through discussing Sewell I can introduce Lakoff and Johnson's work on metaphor. Metaphor, we shall see, explains better the role that body hexis and physical space play in structuring conceptual and social spaces. Metaphor links into Bourdieu's notions of analogy and the multi-dimensional properties of the habitus. Finally Goffman grounds this study in practices and performances, and demonstrates the context-dependency of actions, as well as the multiplicity of structures.

As Sewell explains, there is a problem with our notion of structure, in the fact that it is overly rigid and reified. Such use of structure privileges the structures themselves, and neglects the role that the individual plays in constituting structure:

> What tends to get lost in the language of structure is the efficacy of human action-or "agency," to use the currently favoured term. Structures tend to appear in social scientific discourse as impervious to human agency, to exist apart from, but nevertheless to determine the essential shape of, the strivings and motivated transactions that constitute the experienced surface of social life. A social science trapped in an unexamined metaphor of structure tends to reduce actors to cleverly programmed automatons. (Sewell, 1992:2)

Bourdieu's notion of structure strongly privileges stability and reproduction of social structures, and does not provide strong arguments by which to account for social change, as Sewell explains:

> A second and closely related problem with the notion of structure is that it makes dealing with change awkward. The metaphor of structure implies stability. For this reason, structural language lends itself readily to explanations of how social life is shaped into consistent patterns, but not to explanations of how these patterns change over time. In structural

> discourse, change is commonly located outside of structures, either in a telos of history, in notions of breakdown, or in influences exogenous to the system in question. Consequently, moving from questions of stability to questions of change tends to involve awkward epistemological shifts. (Sewell, 1992:2-3)

To address some of these issues Sewell begins with using Giddens' notion of the duality of structure. Structures are thus "*both the medium and the outcome of the practice which constitute social systems*" (Giddens, 1981:27). Structures constitute social life and practice, but they are also reproduced through social action by knowledgeable agents. These structures comprise of rules and resources which are only realised or instantiated through human actions (Giddens, 1984:377). Social systems therefore are brought into existence through the patterned social practices of structure. Both the social system and the structure thus only have potential or "virtual" existence prior to their moment of instantiation (Giddens, 1984:17).

Sewell, however, clarifies the term "virtual" to apply only to the "rules" (what Sewell refers to as schemas), and not to resources. Resources, rather, are "actual" but equal parts of structure; both virtual and actual parts are in constant dialogue with each other, and both are necessary to generate social systems (Sewell, 1992:11). Sewell characterises the dual nature of structure in the following:

> If structures are dual in this sense, then it must be true that schemas are the effects of resources, just as resources are the effects of schemas… A factory is not an inert pile of bricks, wood, and metal. It incorporates or actualizes schemas, and this means that the schemas can be inferred from the material form of the factory. The factory gate, the punching-in station, the design of the assembly line: all of these features of the factory teach and validate the rules of the capitalist labour contract…In short, if resources are instantiations or embodiments of schemas, they therefore inculcate and justify the schemas as well. Resources, we might say, are read like texts, to recover the cultural schemas they instantiate. (Sewell, 1992:13)

The durability of structure depends on the persistence of the relationship between resource and schemas sets. Schemes require resources in order to be reproduced over time; without such a resource base such schemes are forgotten:

> If resources are effects of schemas, it is also true that schemas are effects of resources. If schemas are to be sustained or reproduced over time - and without sustained reproduction they could hardly be counted as structural - they must be validated by the accumulation of resources that their enactment engenders. Schemas not empowered or regenerated by resources would eventually be abandoned and forgotten, just as resources without cultural schemas to direct their use would eventually dissipate and decay. Sets of schemas and resources may properly be said to constitute structures only when they mutually imply and sustain each other over time. (Sewell, 1992:13)

Sewell relates the schema-resource sets to Bourdieu's notion of habitus:

> Bourdieu recognizes the mutual reproduction of schemas and resources that constitutes temporally durable structures - which he calls "habitus." His discussion of habitus powerfully elaborates the means by which mutually reinforcing rule-resource sets constitute human subjects with particular sorts of knowledge and dispositions. (Sewell, 1992:15)

This notion of habitus, Sewell argues does not account well for social change. The habitus, Sewell argues, privileges the replication of structure, and thus of the reproduction of existing social institutions. Sewell, for such a reason, modifies Bourdieu's terminology, introducing a number of modifications:

> It is my conviction that a theory of change cannot be built into a theory of structure unless we adopt a far more multiple, contingent, and fractured conception of society - and of structure. What is needed is a conceptual vocabulary that makes it possible to show how the ordinary operations of structures can generate transformations. To this end, I propose five key axioms: the multiplicity of structures, the transposability of schemas, the unpredictability of resource accumulation, the polysemy of resources, and the intersection of structures. (Sewell, 1992:16)

The first of Sewell's axioms is the "multiplicity of structures". This term takes into account the variety of contexts within which social life unfolds, such contexts generate different assemblages of schemes and resources (habitus). Structure in practice therefore is manifested in multiple ways, depending on the local contexts of actions, as Sewell explains:

> Societies are based on practices that derive from many distinct structures, which exist at different levels, operate in different modalities, and are themselves based on widely varying types and quantities of resources. While it is common for a certain range of these structures to be homologous… it is never true that all of them are homologous. Structures tend to vary significantly between different institutional spheres, so that kinship structures will have different logics and dynamics than those possessed by religious structures, productive structures, aesthetic structures, educational structures, and so on…The multiplicity of structures means that the knowledgeable social actors whose practices constitute a society are far more versatile than Bourdieu's account of a universally homologous habitus would imply: social actors are capable of applying a wide range of different and even incompatible schemas and have access to heterogeneous arrays of resources. (Sewell, 1992:16-17)

The next axiom to discuss focuses on "transposability". This term, adapted from Bourdieu (1977), refers to the generalisability of schemes and their extension to different contexts and different sets of resources. Such schemes are transposed through analogically relating different categories together. Sewell argues that the recognition of analogies cannot be determined in advance, rather they are determined through situated engagements with problems:

> Whether a given problem is similarly shaped enough to be solved by analogical transfers of schemes cannot be decided in advance by social scientific analysts, but must be determined case by case by the actors, which means that there is no fixed limit to the possible transpositions… To say that schemas are transposable, in other words, is to say that they can be applied to a wide and not fully predictable range of cases outside the context in which they are initially learned. (Sewell, 1992:17)

As Sewell argues competence within a domain comes through correct application of rules to unfamiliar cases. Agency thus is the act of applying schemas creatively and effectively within novel contexts:

> Whether we are speaking of rules of grammar, mathematics, law, etiquette, or carpentry, the real test of knowing a rule is to be able to apply it successfully in unfamiliar cases. Knowledge of a rule or a schema by definition means the ability to transpose or extend it - that is, to apply it creatively. If this is so, then agency, which I would define as entailing the capacity to transpose and extend schemas to new contexts, is inherent in the knowledge of cultural schemas that characterizes all minimally competent members of society. (Sewell, 1992:18)

A consequence of the underdetermined nature of schema application is that the consequences of structure are never fully predictable. This brings Sewell's next axiom of unpredictability to bear:

> But the very fact that schemas are by definition capable of being transposed or extended means that the resource consequences of the enactment of cultural schemas is never entirely predictable…Moreover, if the enactment of schemas creates unpredictable quantities and qualities of resources, and if the reproduction of schemas depends on their continuing validation by resources, this implies that schemas will in fact be differentially validated when they are put into action and therefore will potentially be subject to modification. (Sewell, 1992:18)

As a result of the context-dependency, unpredictability, and differential validity of schema application, there is a degree of interpretation involved in construing certain schema-resource sets within certain categories. Multiple interpretations can thus be derived from any schema-resource set, depending on the dispositions of the interpreting agent:

> The term polysemy (or multiplicity of meaning) is normally applied to symbols, language, or texts. Its application to resources sounds like a contradiction in terms. But, given the concept of resources I am advocating

> here, it is not. Resources, I have insisted, embody cultural schemas. Like texts or ritual performances, however, their meaning is never entirely unambiguous…Any array of resources is capable of being interpreted in varying ways and, therefore, of empowering different actors and teaching different schemas. Again, this seems to me inherent in a definition of agency as the capacity to transpose and extend schemas to new contexts. Agency, to put it differently, is the actor's capacity to reinterpret and mobilize an array of resources in terms of cultural schemas other than those that initially constituted the array. (Sewell, 1992:19)

Finally the axiom of intersection of structures refers to the ways in which resources can act as boundary objects and have different, intersecting schemas applied to them by different actors from different viewpoints. Schemas may also be transferred between different contexts, or be incorporated from the schemas of other social systems (for example in the appropriation of linguistic terms from another language, or in the adoption and adaptation of new technologies):

> The intersection of structures, in fact, takes place in both the schema and the resource dimensions. Not only can a given array of resources be claimed by different actors embedded in different structural complexes (or differentially claimed by the same actor embedded in different structural complexes), but schemas can be borrowed or appropriated from one structural complex and applied to another. (Sewell, 1992:19)

Through use of Sewell's five axioms, Bourdieu's notions of structure can be adapted so as to better account for change and agency. Using Sewell's axioms we can explain how change is built into structure, and enacted by individuals in their negotiations between schemes and resources. Reproduction of social systems is never automatic, but rather there is a degree of indeterminacy, through the interpretive and dialectical nature of schema-resource sets (habitus), as Sewell explains:

> Structures, then, are sets of mutually sustaining schemas and resources that empower and constrain social action and that tend to be reproduced by that social action. But their reproduction is never automatic. Structures are at risk, at least to some extent, in all of the social encounters they shape-because structures are multiple and intersecting, because schemas are transposable, and because resources are polysemic and accumulate unpredictably. Placing the relationship between resources and cultural schemas at the center of a concept of structure makes it possible to show how social change, no less than social stasis, can be generated by the enactment of structures in social life. (Sewell, 1992:19)

### 1.412 Relating Bourdieu to metaphor

Through Sewell's reformulations we notice the polysemic and indeterminate nature of structure. Structures are multiple and applied across multiple domains of experience, and it is the shared structuring principles used to order these diverse realms of experience, which generates analogies and resemblances between them. In this thesis I shall discuss the ordering of conceptual spaces in mathematics. In order to perceive mathematical objects within such conceptual spaces an individual must internalise certain schemas through which to structure their perception. These schemas are part of a habitus, which shape individuals' dispositions, or orientations, towards phenomena. I explore how these schemes of perceiving mathematical spaces are adapted from schemes organising physical spaces. I show that our body's understanding of position, relation, orientation, are adapted so as to structure abstract conceptual spaces and constitute such spaces as "landscapes". In order to understand this process I therefore insert Lakoff and Johnson's ideas on Metaphor into Bourdieu's concept of schema and habitus. I use ideas on metaphor to demonstrate how certain ways of organising perception become analogically transferred, so as to organise abstract mathematical spaces.

Metaphor, as Lakoff and Johnson describe it, is a linguistic device for structuring one domain of experience in terms of another. In mathematics, metaphor can be seen to play a role in relating one's bodily experiences and intuitions within physical spaces to analogous processes taking place in mathematical spaces. In this mapping process the intuitions that one has of the mechanics of the physical world are used as schemas for interpreting operations taking place within mathematical spaces. For example, movement, orientation, speed, acceleration and distance are all schemas for organising and measuring physical spaces, which can be applied to the organisation of abstract mathematical spaces. In utilising such schemas, and applying them metaphorically to mathematical spaces, so the semantic and somatic associations linked to the operations are also transferred. Abstract conceptual space is then understood as if it were an analogue of physical space, and the body relates to it and explores it as if it were physical space.

Metaphor serves as a scaffolding device for making systematic comparisons between domains of knowledge. The ultimate point of reference for all such comparisons lies in the body's sensory-perceptual experience of the physical world. Lakoff and Johnson (1980) argue that such bodily experiences are structured into perceptual, or "embodied" schemas, which form orderings of recurrent patterns of behaviours, stimuli or dispositions. They write: *"[Embodied schemas] are a primary means by which we construct or constitute order and are not mere passive receptacles into which experience is poured*." (Lakoff & Johnson, 1980:29-30). Such schemas are built up from our experience of inhabiting and interacting with our environments, and go on to actively shape our perception.

The schema comprises cognitive frameworks for organising experience and filtering perception, for ease of processing information. As Lakoff and Nunez (1997) write:

> Johnson (1987) considers image schemata to be the fundamental means by which we build or constitute order in our experiences: … in addition to propositional comprehension, understanding is an evolving process or activity in which image schemata (as organizing structures) partially order and form our experience and are modified by their embodiment in concrete experiences. (Lakoff and Nunez, 1997: 30)

Such Image schemata are seen to link different domains of knowledge together, allowing for mapping between different concepts and generating novel ideas as a result.

> These image schemata can facilitate mathematical reasoning because their internal structure can be extended figuratively to develop understanding of formal relations among concepts and propositions. This is achieved through metaphorical projections, that is, metaphors (cf. analogies) serve to map image schemata (which structure space) into abstract models (which structure concepts). Because image schemata are directly understood in terms of physical experience, the metaphors used in the mapping process are motivated by the structures of these experiences. (Lakoff and Nunez, 1997:263)

These schemata gain their power through linking physical experiences to abstract concepts - the metaphor thus relates the world of ideas to that of physical experience. It grounds knowledge within the body and the experience of the world. In using metaphor individuals thus are using their existing experiences and knowledge of physical reality to scaffold their knowledge of more abstract structures such as mathematics:

> Our experience of physical objects and substances provides a further basis for understanding—one that goes beyond mere orientation. Understanding our experiences in terms of objects and substances allows us to pick out parts of our experience and treat them as discrete entities or substances of a uniform kind. Once we can identify our experiences as entities or substances, we can refer to them, categorize them, group them, and quantify them—and, by this means, reason about them. (Lakoff and Johnson, 2003: 508)

Organising reality through such experiential gestalts synthesises abstract concepts into larger bodies of knowledge, providing them with a coherent structure, through creating relational correspondences between different knowledge-systems (Lakoff and Johnson, 2003:152). Such mappings, Lakoff and Johnson argue, are not arbitrary, but are motivated by previous experiences. In the case of metaphor in mathematics Lakoff and Nunez (1997) present the following examples:

> The notion of an algorithm is, of course, based on the machine metaphor. An algorithm, as a metaphorical machine performs operations sequentially on input objects to yield output objects. Since the machine is metaphorical and not real, the operations and objects are conceptual in nature, and they always apply perfectly in exactly the same way, since imperfections of physical objects are not mapped onto conceptual objects by the metaphor (Lakoff and Nunez, 1997:1046)
>
> A number line is a conceptual blend formed from the superimposition of the source domain (Geometry) of the Arithmetic as Geometry metaphor onto the target domain (Arithmetic). The entities in this blend are number-points—numbers that are metaphorically points. The blend combines truths of geometry with truths of arithmetic, to yield new inferences about the target domain, arithmetic. Using the metaphorical concept of the number line, we can construct a much more complex metaphorical concept, the Cartesian Plane. (Lakoff and Nunez, 1997:1093)

Mapping across domains is commonplace within mathematics, but the question is: does the creation of analogies or metaphors between different domains of knowledge in mathematics actually enhance understanding or allow for developing new knowledge? English (1997) argues that it does, giving the example of the application of blocks in teaching mathematics in primary school:

> Both analogy and metaphor are important not only in the initial construction of these mental models but also in their subsequent development and refinement. For example, children's explorations with tens- and ones- blocks assist them in constructing a basic model of the relationships within two-digit numbers. This mental model then becomes the source for children's exploration of the relations in multi-digit whole numbers (via manipulation of the concrete representation), and subsequently, for their introduction to decimal fraction ideas. Of course, a complete understanding of numeration entails many other relations, effectively developed through analogical reasoning. (English, 1997:181)

Metaphors are hence useful in structuring knowledge and shaping the discovery process. Once the ability to manipulate, communicate and reason using analogies is grasped, the individual has a powerful tool through which to build new frameworks through which to structure novel experience. In mathematics, the use of discourses on beauty, and the enculturation of aesthetic appreciation of mathematical concepts and proofs are other means by which these schemas are built up. As Pimm (2007) argues:

> Aesthetic considerations concern what to attend to (the problems, elements, objects), how to attend to them (the means, principles, techniques, methods) and why they are worth attending to (in pursuit of the beautiful, the good, the right, the useful, the ideal, the perfect or, simply, the true). (Pimm, 2007:160)

The process of socialising mathematicians to appreciate certain phenomena as being "truthful" or "beautiful" is the means through which social values and evaluation principles are communicated within the mathematical community. Aesthetic discourse determines what should be attended to and how, and thus provides the prototypical examples against which the validity of discoveries is measured. Aesthetic objects thus provide the measure for evaluating proofs and also provide the markers for orienting future action and application of certain rules or definitions. In establishing a discourse on aesthetic appreciation certain standards are generated and hence classifications are more likely to be oriented towards such pre-existing structures, as momentum has already been generated through their wide-spread adoption and use.

### 1.413 Defining performance and frames of reference

Goffman is not someone we usually associate with Bourdieu's ideas, however, there are some elements of Goffman's work which complement Bourdieu's own. I have already provided several additions to Bourdieu's notion of structure and habitus, demonstrating their multiplicity, ambiguity and transposability, through use of Sewell (1992). Introducing metaphor also explained the mechanisms by which transposition takes place. Through Goffman's work I wish to add another clarification of Bourdieu's concepts, by exploring how schemas are applied in practice, how such practices are context dependent, and how reference frames are generated within situated encounters between individuals.

The first thing to note is that Goffman's socialised individual is much like Bloor's, in that they apply rules according to the specific, local contexts of interactions (which Goffman calls "encounters"). These rules are moral principles impressed on the individual from an externalised "social" field. The individual is constructed through such rules in relation to other social actors, who bear an understanding of the rules at play. Rules provide a means of evaluating and recognising one's own and others' actions, and such rules allow them to determine the character (what Goffman calls "face") of themselves and others' "selves". Encounters between individuals are a means of mobilising these rules, thus establishing social frameworks by which social selves are manifested and made visible, as Goffman explains:

> Universal human nature is not a very human thing. By acquiring it, the person becomes a kind of construct, built up not from inner psychic propensities but from moral rules that are impressed upon him from without. These rules, when followed, determine the evaluation he will make of himself and of his fellow-participants in the encounter, the distribution of his feelings, and the kinds of practices he will employ to maintain a specified and obligatory kind of ritual equilibrium. The general capacity to be bound by moral rules may well belong to the individual, but the particular set of rules which transform him into a human being derives from requirements established in the ritual organisation of social encounters. (Goffman, 1955:246)

Goffman's "rules" bear certain similarities with the schemes of Bourdieu's habitus: rules are objective; they are shared tools by which to perceive social categories; they generate social selves; and they are performed and manifested through practice. The resemblances between these terms however are limited, but they provide the starting point from which to assimilate certain elements from Goffman into Bourdieu's work. I will therefore highlight other similarities between terms, in order to create a series of equivalences and begin the process of assimilation.

The second equivalence relevant to this thesis is that between "status" and "social capital". Status, for Goffman, is determined by a person's acquisition of rights and privileges; the possession of certain statuses defines an individual's position and role within the social field. Such status is defined through performances of status, and having such status recognised by other socialised individuals, as Goffman explains:

> The terms status, position, and role have been used interchangeably to refer to the set of rights and obligations which governs the behaviour of persons acting in a given social capacity. In general, the rights and obligations of a status are fixed through time by means of external sanctions enforced by law, public opinion, and threat of socio-economic loss, and by internalized sanctions of the kind that are built into a conception of self and give rise to guilt, remorse, and shame. A status may be ranked on a scale of prestige, according to the amount of social value that is placed upon it relative to other statuses in the same sector of social life. An individual may be rated on a scale of esteem, depending on how closely his performance approaches the ideal established for that particular status. (Goffman, 1951:294)

Individuals possessing similar status are located within similar social positions, and such positions generate dispositions from which to experience the world. Individuals present their social positions through use of status symbols which serve to "place", or classify, actors within the social field:

> Persons in the same social position tend to possess a similar pattern of behaviour. Any item of a person's behaviour is, therefore, a sign of his social position. A sign of position can be a status symbol only if it is used with some regularity as a means of " placing" socially the person who makes it. (Goffman, 1951:295)

The status symbol thus acts as a "sign vehicle" for conveying socially relevant information between individuals:

> By definition, then, a status symbol carries categorical significance, that is, it serves to identify the social status of the person who makes it. But it may also carry expressive significance, that is, it may express the point of view, the style of life, and the cultural values of the person who makes it, or may satisfy needs created by the imbalance of activity in his particular social position. (Goffman, 1951:295)

Using such status symbols to place other individuals sets up a frame for the interactions that are to follow. An individual forms a stance, or frame, through which to construe the interaction, which sets up certain expectations and dispositions to adopt, according to the relative statuses of the actors involved:

> Information about the individual helps to define the situation, enabling others to know in advance what he will expect of them and what they may expect of him. Informed in these ways, the others will know how best to act in order to call forth a desired response from him. (Goffman, 1951:135-6)

Individuals use a number of social cues, or signs, to classify other persons. Ways of speaking, ways of dressing, their deportment, the social situation are all taken into consideration when "placing" the individual. Using such information the behaviour of the individual is inferred and thus an appropriate interactional frame is generated, to guide the resulting interaction:

> For those present, many sources of information become accessible and many carriers (or "sign vehicles") become available for conveying this information. If unacquainted with the individual, observers can glean clues from his conduct and appearance which allow them to apply their previous experience with individuals roughly similar to the one before them or, more importantly, to apply untested stereotypes to him. They can also assume from past experience that only individuals of a particular kind are likely to be found in a given social setting. They can rely on what the individual says about himself or on documentary evidence he provides as to who and what he is. If they know, or know of, the individual by virtue of experience prior to the interaction, they can rely on assumptions as to the persistence and generality of psychological traits as a means of predicting his present and future behaviour. (Goffman, 1951:136)

The performance of status symbols, and their role in framing interactions, demonstrates another equivalence between Bourdieu's ideas and Goffman's, this relationship is based upon the idea of distinction. In Bourdieu's thought, distinction is integral to the concept of the field. Distinction is a product of the distribution of capital, or statuses; through recognising distinctions between social categories, so groups are either differentiated or integrated within classes. Through this process of integration and differentiation so the social field is given structure. Goffman describes how this process of distinguishing and classifying others is carried out through symbolic interactions between individuals:

> Co-operative activity based on a differentiation and integration of statuses is a universal characteristic of social life. This kind of harmony requires that the occupant of each status act toward others in a manner which conveys the impression that his conception of himself and of them is the same as their conception of themselves and him. A working consensus of this sort therefore requires adequate communication about conceptions of status. The rights and obligations of a status are frequently ill-adapted to the requirements of ordinary communication. Specialized means of displaying ones position frequently develop. Such sign-vehicles have been called status symbols. They are the cues which select for a person the status that is to be imputed to him and the way in which others are to treat him. Status symbols visibly divide the social world into categories of persons, thereby helping to maintain solidarity within a category and hostility between different categories. Status symbols must be distinguished from collective symbols which serve to deny the difference between categories in order that members of all categories may be drawn together in affirmation of a single moral community. (Goffman, 1951:294-5)

Such distinction is enforced through use of schemas or rules which order interactions. Such rules form, as Goffman describes, systems of practices and conventions which structure an individual's dispositions and perceptions - these systems are thus equivalent to Bourdieu's habitus. Such schemes shape the flows of conversations, providing frames of reference by which individuals can orient their behaviours and thus successfully participate in symbolic exchanges, as Goffman explains:

> In any society, whenever the physical possibility of spoken interaction arises, it seems that a system of practices, conventions, and procedural rules come into play which functions as a means of guiding and organising the flow of

> messages. An understanding will prevail as to when and where it will be permissible to initiate talk, among whom, and by means of what topics of conversation. A set of significant gestures is employed to initiate a state of communication and as a means for the persons concerned to accredit each other as legitimate participants. When this process of reciprocal ratification occurs, the persons so ratified are in what might be called a state of talk - that is, they have declared themselves officially open to one another for purposes of spoken communication and guarantee together to maintain a flow of words. A set of significant gestures is also employed by which one or more new participants can officially join the talk, by which one or more accredited participants can officially withdraw, and by which the state of talk can be terminated. (Goffman, 1955:239)

These interactions between individuals are divided into local frames or "episodes" of interaction, which form the situated instances from which social systems are constructed:

> These rules of talk pertain not to spoken interaction considered as an ongoing process, but to an occasion of talk or episode of interaction as a naturally bounded unit. This unit consists of the total activity that occurs during the time that a given set of participants have accredited one another for talk and maintain a single moving focus of attention. (Goffman, 1955:240)

Within these episodes the individual actively negotiates with the schemes of the habitus, practicing their "mastery" of such schemes. Individuals position themselves in relation to the interlocutors through having evaluated the social roles at play within the practical context. Within these contexts of practice, so the individual, according to Goffman, adopts a stance from which to relate to their interlocutor, such a stance Goffman characterises as a "line", he explains:

> Every person lives in a world of social encounters, involving him either in face-to-face or mediated contact with other participants. In each of these contacts, he tends to act out what is sometimes called a line - that is, a pattern of verbal and nonverbal acts by which he expresses his view of the situation and through this his evaluation of the participants, especially himself. Regardless of whether a person intends to take a line, he will find

> that he has done so in effect. The other participants will assume that he has more or less wilfully taken a stand, so that if he is to deal with their response to him he must take into consideration the impression they have possibly formed of him. (Goffman, 1955:222)

Along with the line the individual also adopts a "face", or persona, which is the image of themselves they want their interlocutor to have of them:

> The term face may be defined as the positive social value a person effectively claims for himself by the line others assume he has taken during a particular contact. Face is an image of self, delineated in terms of approved social attributes - albeit an image that others may share. (Goffman, 1955:222)

A person performs using such a face, as an actor performs when playing the role of a character within a play. The "face" presents a coherent image of the individual to others, and emerges as a set of dispositions during the course of an encounter. The habitus of internalised schemas therefore manifests itself during the course of encounters in the form of socially meaningful dispositions, whose intent is to communicate certain traits relevant to the individual's communicative intent:

> A person may be said to have, or be in, or maintain face when the line he effectively takes presents an image of him that is internally consistent, that is supported by judgements and evidence conveyed by other participants, and that is confirmed by evidence conveyed through impersonal agencies in the situation. At such times the person's face clearly is something that is not lodged in or on his body, but rather something that is diffusely located in the flow of events in the encounter and becomes manifest only when these events are read and interpreted for the appraisals expressed in them. (Goffman, 1955:223)

To maintain this face, and keep it consistent with the persona an individual is trying to convey, so an individual must monitor the performances of themselves, and maintain what Goffman refers to as an "expressive order":

> As an aspect of the social code of any social circle, one may expect to find an understanding as to how far a person should go to save his face. Once he takes on a self-image expressed through face he will be expected to live up to it. In different ways in different societies he will be required to show self-respect, abjuring certain actions because they are above or beneath him, while forcing himself to perform other even though they cost him dearly. By entering a situation in which he is given a face to maintain, a person takes on the responsibility of standing guard over the flow of events as they pass before him. He must ensure that a particular expressive order is sustained - an order that regulates the flow of events, large or small, so that anything that appears to be expressed by them will be consistent with his face. (Goffman, 1955:224)

These face-saving operations for maintaining the expressive order are constructed within frames (Goffman, 1974:7). These frames create boundaries around encounters, and help define for the individual the face or dispositions to adopt, as well as the expectations and rules that parties involved should have. Such frames however are not always clearly defined. Applying the wrong frame, or changing ("re-keying") frame, can lead to embarrassment and loss of face. In such framing the social action is thus perceived within a certain light, as Sewell would argue, the schema-resource sets can shift as a function of the reference frame.

Presentations of self therefore can change depending on the reference frame one adopts, that is to say, depending on how a given encounter is interpreted. The performances of certain faces is a product of an individual's judgement within the situated contexts of the encounter itself. During encounters individuals choose to adopt certain stances, make visible certain information rather than others, and dramatically highlight certain aspects of their persona or knowledge (Goffman, 1957:40). Such performances, however, are not completely underdetermined, rather a role is practiced beforehand. Through the process of rehearsing, routinizing, and “playing about” with the possibilities of the role, an individual becomes comfortable with a way of acting, and so they adopt patterns of behaviour which constitute an "idiom", rather than prescribed sets of behaviours which follow rules (Goffman, 1957:80-82). Through this process of adopting the role, and acting out the role, Goffman argues that we "dramatically realise" the role in practice. Such realisations take place within different "theatres" of performance. Such theatres vary in their degree of visibility, from public spaces to private spaces. Goffman refers to public spaces as being "front of stage"; within these spaces the individual performs the roles that they have practiced, presenting their social face to the world. The private, "backstage", spaces are where the individual rehearses their role in preparation for presenting it in a public theatre.

**1.414 Conclusion**

Notions of frontstage and backstage, the encounter, and interactional frame, are used in this thesis as a way of positioning individuals within social spaces, and of “framing” the encounters between individuals within the context of the field. In this thesis I use three different frames of reference: conceptual, physical and discourse frames. Each of these frames present different theatrical stages for mathematical encounters, and for the production of mathematical selves and ideas. I use these spatial frames as tools for situating the individual within social space, and for explaining how dispositions and perceptions are generated as a function of position within social space.

In what follows I shall ground the reader within these different reference frames, guiding the reader through the situated practices undertaken by mathematicians, across these different spatial dimensions. Through exploring these different frames I hope to show how they are related, analogically, through Bourdieu’s notion of habitus. I then wish to demonstrate how these spatial dimensions are assembled, as a tools for perceiving mathematical phenomena, as well as for producing the mathematical field.

## Section 1.5: Overview of Thesis

### 1.51 Chapter Summaries

Chapter 2 discusses the physical contexts within which mathematical research is produced. Section 2.1 compares the physical architectures of two institutes, and explores how such architectures shape the activities which take place within them. The next section discusses how individuals domesticate space and develop routines within space. In this section we will come to understand how work habits structure thinking in mathematics. In the final section of this chapter the thesis explores how physical spaces provide different stages on which to perform and produce mathematics. The thesis focuses on comparing performances in public and private settings, as well as formal and informal settings, and demonstrates how ideas change as they are presented within different contexts.

Chapter 3 discusses how conceptual spaces are crafted by mathematicians. The first section explores the process of developing intuitions about mathematical concepts, and the role that supervisors play in guiding a student's vision of the field. Through use of exemplar objects and constructions a student develops perceptual schemas, which allow them to perceive mathematical structures. In section 3.2 we explore how mathematicians learn to manipulate these schemas through a process of experimentation. Such experiments take the form of physical engagements with mathematical concepts through sketches on blackboards and paper. In the final section (3.3) of chapter 3 the thesis explores how such visualisations and representations on blackboards and paper act as framing devices for manipulating and materialising mathematical constructions. Through such perceptual frames, I argue that abstract spaces become structured in terms of our experience of physical space.

The penultimate chapter, chapter 4, concerns discourse space in mathematics. It explores the role that the social world plays in shaping taste and judgement in mathematics. The first section (4.1) explores how individuals coordinate their reference frames, and come to share a common vocabulary of mathematical terms. The second section (4.2) explores the biographies of mathematical concepts, and demonstrates how such concepts change as they move within social networks. The final section (4.3) explores how authority figures influence the adoption and development of concepts in mathematics, returning to ideas concerning habitus developed in chapter 2.

In the final, concluding chapter (Chapter 5), I provide a summary of the main arguments articulated within the body of the thesis. I also return to Bourdieu's ideas of field and habitus, and integrate these concepts within his notion of social topology. I will explain how such topologies are created, through positioning individuals within social systems, and explore how these positions within topological spaces generate dispositions and perspectives by which to view the world. I argue that social topologies allow us to study not only field effects (social forces and influences on behaviour), but also effects of position, differentiation, integration and orientation. Finally I provide an outline of how social topologies may be of use to sociological analysis of complex social systems, such as mathematical disciplines.

# Chapter 2: Physical Spaces

## 2.0: Overview

In this chapter I will establish the physical frames within which the mathematical field is produced. I show how mathematicians position themselves within physical space, and how such positionings shape their dispositions. I document how spaces are socialised and come to possess certain properties of privacy and formality. I then show how the presentations of self and ideas change as a function of these properties.

The spatial resources of institutes are transformed through mathematicians inhabiting them. Each mathematicians' encounter with a spatial resource, I argue, is uniquely shaped by their positions within the mathematical field. The ways in which resources are utilised, and the self is performed, will therefore depend upon the unique engagements between an individual's dispositions, and the social structures by which a space is organised prior to the encounter. In the following chapter I document how such engagements unfold in practice, and provide a sense of the lived realities of research mathematics.

## Section 2.1: Spaces for Thought

### 2.11 Overview of Section

In this section I will explore how physical space is used as a starting point for grounding and materialising abstract thinking in mathematics. This initial section (2.1) will serve to frame some of the later discussions, within the local contexts of mathematical production. I will explore how physical spaces in the institute are inhabited, and how physical spaces are used as machines for thinking and assembling mathematical concepts. I will begin the chapter by surveying two of my research sites, at UK-A and Germany-A. I will present descriptive accounts of the institutional spaces, and then build into these accounts the working practices and routines of the mathematicians who inhabit these spaces. Through such accounts I will position mathematical production within the mathematical field, and demonstrate how certain dispositions and orientations emerge through situated engagements between the schemas and resources of habitus.

### 2.12 Research Site 1: The UK-A Institute

Let us begin then by building up a picture of the institutional habitats within which my ethnographic observations took place. My first research site was located in a small city in England, for the purposes of this thesis I refer to it as UK-A. This institute is part of a network of 27 European research centres on mathematics distributed throughout Europe. The UK-A itself was founded in 1992 with a mandate to promote the development of interdisciplinary collaborations between mathematicians and researchers from other sciences. Such collaborations take place during organised research programmes, which range in time scale from 4 week short programmes to longer 6 month projects.

My research focused on a short, 4 week-long programme in Metagenomics. The aim of this research programme was to bring together biologists, computer scientists and mathematicians to develop statistical and mathematical tools for analysing and classifying large, complex data sets of sequenced genetic data. This type of inter-disciplinary programme is common at the UK-A Institute, but other more intra-mathematical programmes also took place at the UK-A, such as the 6 month long "Free Boundary problems" programme which also took place in the institute during my study (this programme mainly involved mathematicians in differential geometry and analysis). Exchanges between programmes, which run in parallel, are encouraged, but the degree of communication between programmes is dependent on there being shared interests between participants.

The institute is isolated from the centre of the city, located approximately one mile north-west of the city centre. The institute is part of a larger building complex associated with the mathematics department of the nearby university which, as can be seen from the map below (labelled A-H), takes up a sizable portion of the science park's endowment lands. The university buildings house the offices of the mathematics faculty of the university, the main library for mathematics, as well as lecturing and leisure facilities for graduate and undergraduate students. Although my observations do extend out into this wider environment, I will, for the purposes of this thesis, limit my analysis to activities taking place within the UK-A

Institute proper, and the adjoining gatehouse building, where the bulk of my study took place.

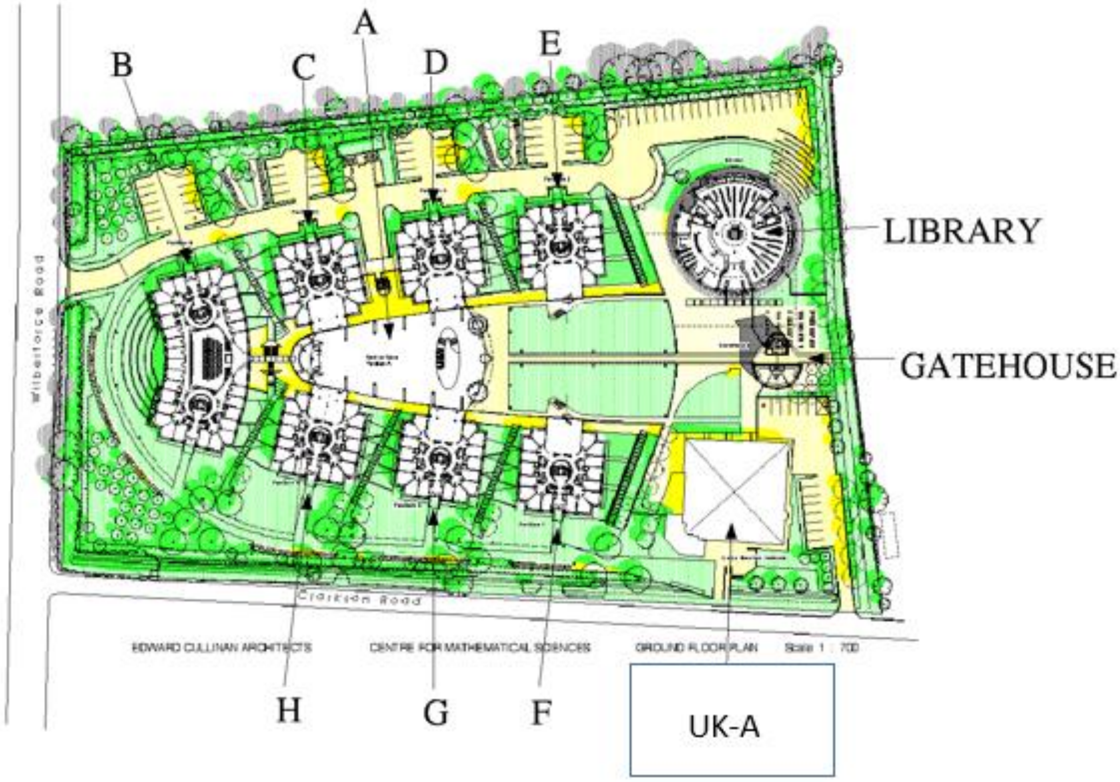


*Figure 1 Map of the UK-A's endowment lands. Buildings A-H indicate the Mathematics department's facilities*

Let me now give a sense of the physical environment of UK-A by providing some detailed descriptions from my fieldwork journal:

> The orange-brown bricked building is typical of the town, without a sign the building would go on, unnoticed, but the sign transforms it somehow. Within, the dental-whites, light, piny-browns and coarse-reds speak to the eye. Glass latticed with wood and concrete and steel. Looking up one glimpses the pine roof on high, looking straight ahead one sees into the lecture hall and to the right, one glimpses the library, through glass. It is a building made to be open, visible and on display; a building almost invisible on the outside, but of complete visibility and exposure within.

Just to the left of the entrance lies the reception staffed by two receptionists, one eying the stairway, with a mirror on the top of her desk to spy behind, and the other outward-facing to the entryway. A touch screen occupies space at one end of the reception's counter, on it a list of participant's faces flash one moment, and an itinerary the next. Behind reception glares a room, glass-fronted, manned and busy with administrative staff going about their chores; behind this stands a closed door to the private office of the director. Running through the centre of the building is a large, angled staircase leading up to the office space above. Behind the stair, to the left of reception, is a more private, but still commonly held area, lined with a glass windows and doors which open out onto a disused patio. A movable partition doubles as a notice board and blocks off some of the space from the door to the Director's room: it is a cosy, quiet corner populated by little more than chairs and coffee tables and at one end a large, green chalkboard. (Day1, UK-A)

*Figure 2 Images of the UK-A institute, clockwise, from top-left we see the entrance, view from the main stairway, common area, library, view overlooking common space, view to ceiling*

There are two seminar rooms in all, which can be booked through reception as required. The larger of the two is designed for lectures and presentations, and boasts four large, two-tiered blackboards, and an impressive three pull-down projector screens. To the right, there is an array of windows with glass doors accessing the courtyard to the mathematics department beyond.

The room is lit by nature, brightened by the sky. Six tiers of desks occupy the theatre-space, flanked by aisles either side - each tier offering berth for ten or so chairs. Overhead are suspended three large projectors and a number of microphones threaded from cables, dangling. Cameras peak out between the tiles. Circular loud-speakers spot the ceiling tiles, while a pair of oblong speakers stand like sentinels above each aisle. At the back of the theatre hides a small room containing monitors, recording equipment, and the unobtrusive, bespectacled Alonzo who works soundless, behind the scenes. To find the second seminar room we cross the courtyard and enter a building called the Gatehouse. Flighting the steps one comes across a small, semi-circular room without windows, with three half-moons of ten chairs congregating around a large two-tier blackboard. A small "On Air" sign hangs to the right of the board, lighting up only for the publically broadcast lectures, consumed online.

Back to the main building, on the ground floor, through a glass-paned wall the discussion room is made visible. One wall is filled with three large, two tiered blackboards, a projector screen is angled in one corner, and large cupboards, locked and contents unknown, occupy the opposite end of the room. At the room's centre eight desks make a square with an empty centre. (Day1, UK-A)

*Figure 3 Discussion room in use for large presentation and pair blackboard work afterwards, UK-A*

The library neighbours the discussion room, mirroring it with its glass-frontage and foyer-facing gaze. At the entrance the opening hours read "9.00 - 17:00 Mondays-Fridays". With a card you can swipe through and enter to find the large windows opposite, letting in the light. The librarian watches the screen of her computer, and screens the entrance with a welcome smile. Across from her are four low-to-the-ground arm-chairs circling neatly arranged newspapers on a coffee table. Hugging the pine-framed-glass-wall one confronts a display of UK-A-branded merchandise for sale. A small stand beside this bears the latest scientific journals. A small seating area complete with flip-chart occupies the far left of the room, as a flight of stairs leads its way to the bookshelves of the upper gallery. In the centre of the room is a long desk, partitioned into four sections on either side, with a computer terminal snuggly fitted into each niche. To the far right, in one corner is a small, private space with a lonely chair, opposite this is a larger space with two desks, which six chairs congregate around.

Exiting the library, back to the foyer is the main staircase. Ascending one encounters a mezzanine floor, directly ahead are pigeon-holed shelves which act as post-boxes. Names of participants bottom each slot. On top of the post-boxes rests a small plastic board containing pictures and names of the members of staff. Turning around, beside the IT office, is an LCD screen presenting faces and names of participants from the programmes most current. Four offices occupy this middle ground, directly above this level another floor contains a further eight offices, which look down on to a large expanse of common space. This common ground is populated by five tables, three low to the ground and two of larger size. Four chairs cluster around each. Tables range in size with two smaller coffee tables, and one larger, all topped with glass and paired with low-lying, lounging chairs. Two large, green chalk boards take pride of place beside pillars supporting the roof. At one end of the common space opposite and to the left of the main stair-way is the "kitchen" area, comprised of two arrow-shaped counters separated by a walking space. A microwave, dishes, cutlery, fridges, snack-selection and coffee machine are provided for, but snacks come with a price. The hulk of the coffee machine surveys its kingdom from the far side. (Day1, UK-A)

*Figure 4 Main common space of UK-A*

Eleven office doors confront the common space. Offices come shared two to three to a room, except for programme organisers who retain their privacy. Each possess a large window and comes equipped with blackboard, personal computer terminal, two low-lying lounge chairs, a notice board, and a filing cabinet and an anonymised uniformity. (Day1, UK-A)

*Figure 5 Pair work in offices, UK-A*

**2.13 Research Site 2: The Germany-A Institute**

My second research site is the Germany-A Institute, located within a mid-sized German city. Like the UK-A, Germany-A is part of the group of European research centres on mathematics. Unlike the UK-A Institute, the Germany-A caters mainly to pure mathematicians, and is not particularly concerned with interdisciplinary collaborations. Originally the institute was founded by a famous German mathematician in 1980, and emerged in its current manifestation in 1992. The institute is run by a board of directors who approve the applications of individual researchers who may undertake a sabbatical lasting a few weeks to several months. Both graduate students as well as professors may make applications to the Institute; full-time, salaried, post-doctoral positions are also available. The institute specialises in a range of mathematical specialisms including Algebraic Groups, Arithmetic Geometry, Number theory, Representation Theory, Algebraic and Complex Geometry, Differential Geometry and Topology, Algebraic Topology, Dynamical systems, Non-commutative geometry, Analysis and Mathematical Physics. Most researchers work independently on personal projects whilst at the Institute, however many take the opportunity to network and find collaborators.

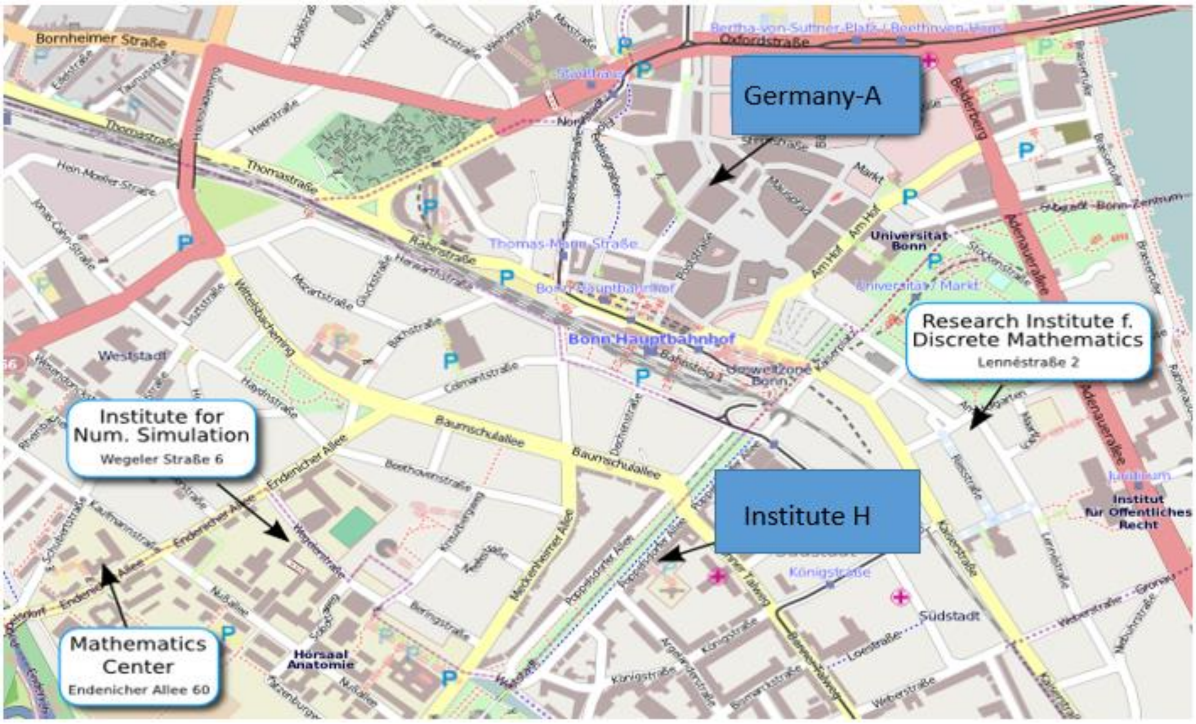


*Figure 6 Locations of Germany-A and affiliated institutions,*

Unlike the UK-A institute, the Germany-A is situated in the heart of the city, located close to the cathedral, central shopping district and train and bus stations. The Institute is also well connected with the mathematics department of the nearby university, with many of the scientific staff and researchers supervising graduate students at the university. The Institute also has a close relationship with the nearby Institute H for Mathematics, the Mathematics Centre, the Research Institute for Discrete Mathematics, and the Institute for Numerical Simulation. But let me now describe some of the physical spaces of the institute by using passages from my fieldwork journal and post-fieldwork reflections journal:

> The entrance to the institute is unassuming, I had walked right by it a couple of minutes before. A sign besides a glass-panelled, wooden door bears the words: "Germany-A Institut Für Mathematik". Pressing the button on the intercom I am ushered up to the 3rd floor. (Day 1, Germany-A)

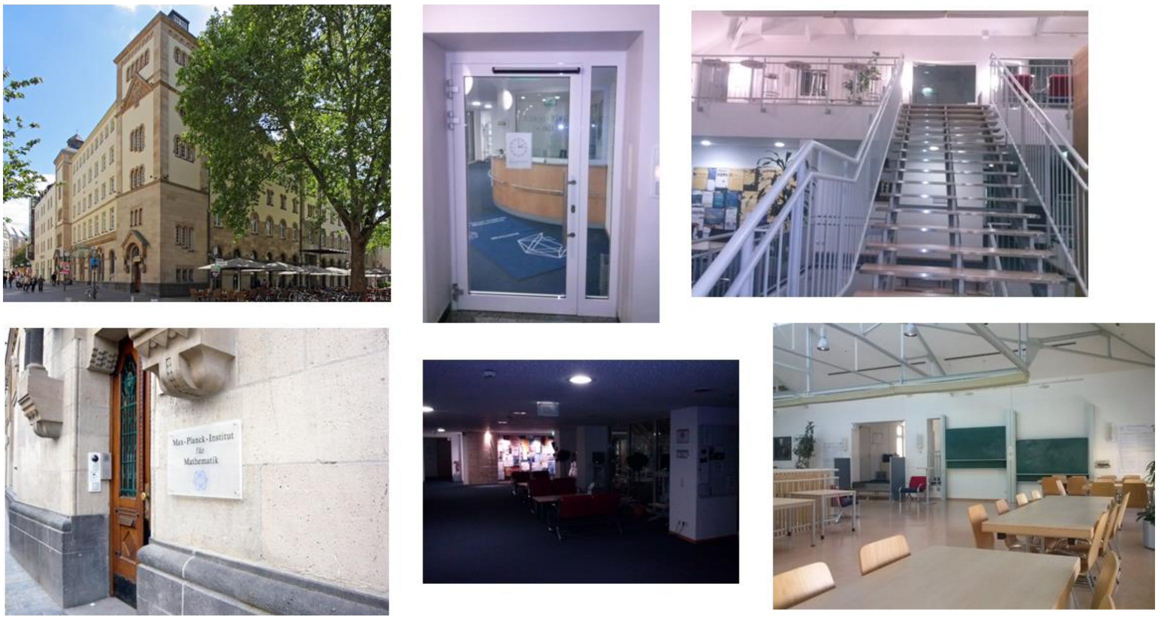


*Figure 7 Images of Germany-A, featuring entrances, main stair, reception and main dining hall*

> Another unassuming door, of glass and steel construction opens of its own accord at my approach. A receptionist grins and says hello. "Ah - first time?" I nod "I have presents!" she says, handing over a stack of forms "I also need one from you - a little photograph perhaps?" She takes it and shows me; it looks awful, but I smile and say "I'm not getting any prettier". The reception counter is shielded by glass on the side facing the main door to cut out the draught. It forms a semi-circular shape by which to better sight its surroundings: the corridor to the right, the stairs and the entryway to the left. Opposite is a lounge area - two coffee tables and two sets of low-to-the-ground chairs surrounding each. Leaflets, brochures, magazines sprawl across the tables and populate the stands to the right. A couple of computer terminals press up against a glass wall looking on to the main stair. Two estranged orange plants pot about to the sides, separated by the computer screens. At the far end of the space glows a notice board, heaving with posters, pine-fronted lockers close in beside. A dark space looms to the left of this. Hidden from view, are a couple of offices. The left hand wall

bears four doors, all closed, a map of City, and more posters about events coming soon, or having passed. (Day 1, Germany-A)


*Figure 8 Reception and noticeboard at Germany-A*

Following the glass wall to the right, one comes across a grey cupboard filled with white coffee cups, a coffee machine beside, and a small kitchen with a lonely kettle. Behind this are some bar stools and a small, high, circular metal table, upon which a group of three now comfortably perch their cups. A frosted glass board nearby still holds onto its ink. To the right of the kitchen stretches another corridor, lined with artwork, office doors, whiteboards and notices. Opposite is the main stair - steel painted white and grey and pine planks for the steps. Pigeon holes populate the space beside and another computer screen sits at the foot of the stair. Directly ahead, beside the reception counter I spy a notice board containing the lecture schedules for the weeks ahead, as well as a picture board with the faces of all the staff and participants currently at the institute.

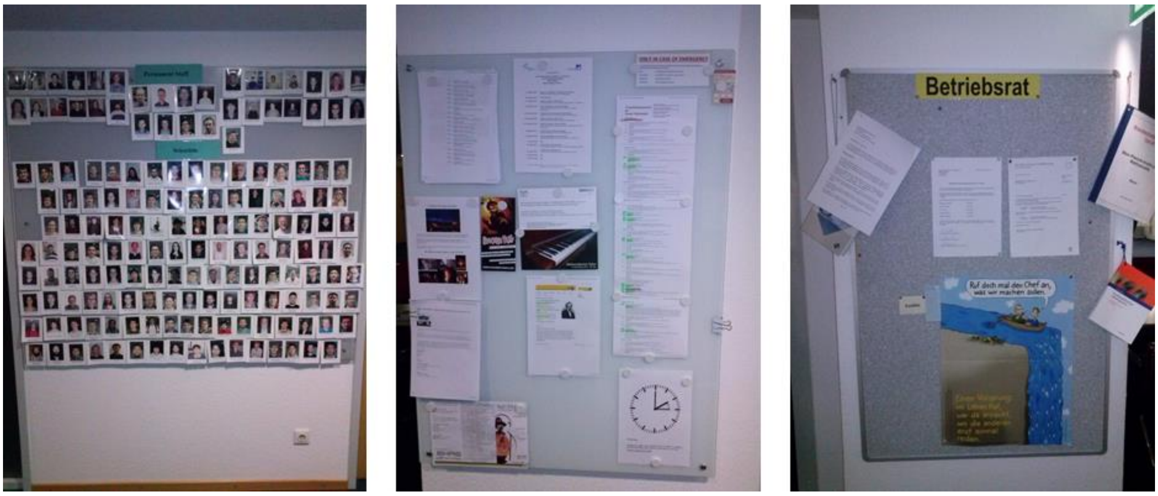


*Figure 9 Notice boards and wall of photographs of institute members*

Looking up the stairs, upon ascent, an oil painting of the founder of the institute stares down. More paintings, notice boards, photographs adorn the walls. Whiteboards, frosted-glass boards punctuate the space. Offices, doors ajar or closed, face out onto the landing. A couple of desks, eight chairs press up against the railings of the stair. Another flight leads up, another corridor opens out to the library in one direction, another leads to the Horsalle lecture hall in the other. (Day1, Germany-A)

*Figure 10 Main spaces at Germany-A, clockwise from top left we see the main seminar room, discussion room, conference room and dining hall.*

The main lecture theatre is a cavernous room, made airy by the tall arched windows flanking either side of the space. At the far end of the hall is a two-paned, three-tiered, green chalk-board. The central space bears ten rows of desks, eight fold-down red chairs to a desk, a spiral staircase wends its way to a balcony with a further set of seats - all in all some 120 people could sit comfortably within.

Exploring along the third floor one comes across numerous side spaces, inhabited by a few chairs, whiteboards, windows or computer terminals. They make quiet spaces to escape to, to ignore and to be ignored. Along a corridor lined with photographs of the who's-who of the mathematical world: Gauss, Lovelace, Poincaré, broken up by the open doors of offices. On the opposite wall are windows with plaster sculptures of manifolds on their sills which look out onto a small courtyard. Beyond a set of doors, to the left is the elevator and a small 10ft x 10ft room. It boasts a small metal table, two metal chairs, a view of the courtyard and the kitchen opposite, and two large whiteboards. People often use this space for skype conversations. I go here to talk quietly to people every now and again. Escaping the space, an oak staircase leads down to the conference room. But continuing further left one reaches the library (Day 1, Germany-A).

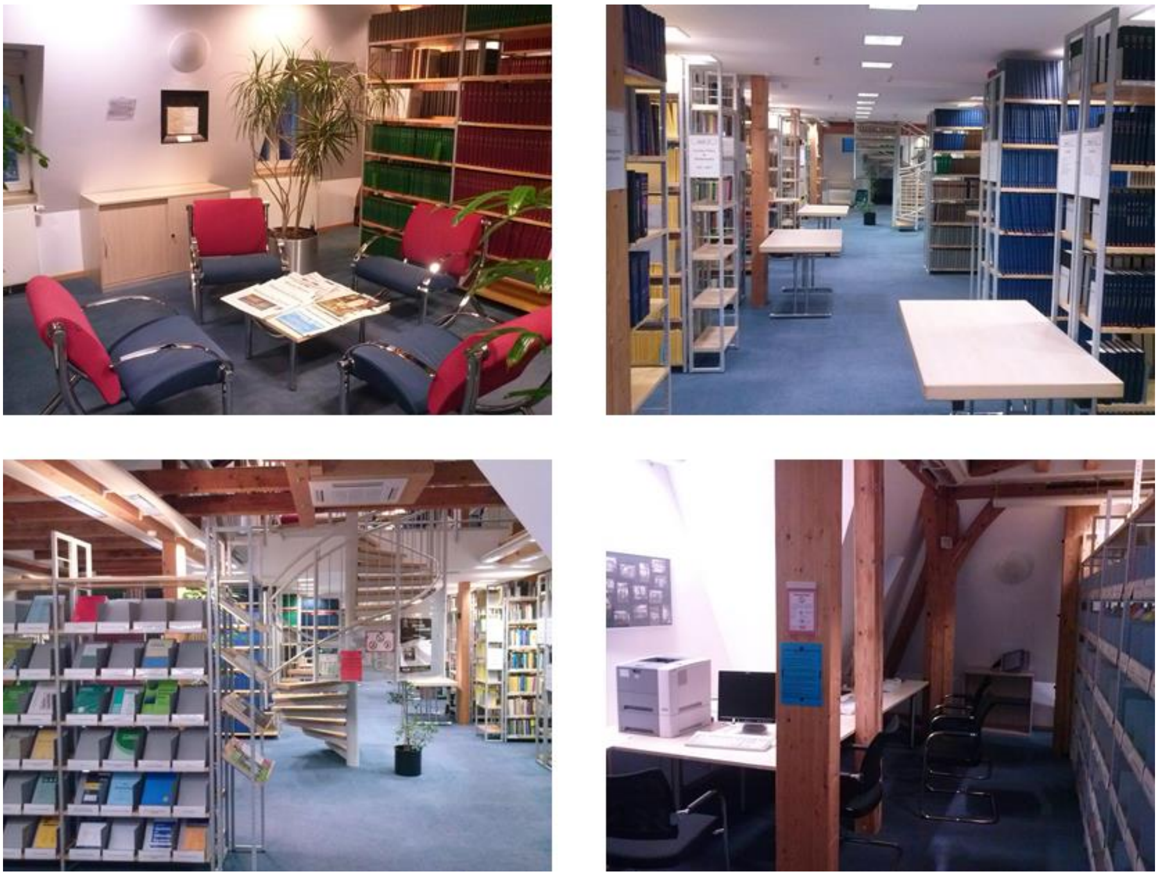

*Figure 11 Library spaces at the Germany-A Institute*

From a small glass-fronted room the librarian looks out. One cannot enter without being checked in. It is spacious, a spiral staircase winds up to an upper floor with lounge-chairs and coffee-tables. The lower floor is lined with rank after rank of book shelves. In between a seat emerges, a potted plant, a table, a window complete with window seat. Mid way through a cluster of four chairs surround a newspaper covered coffee table, palm-plants potter around.

*Figure 12 Corridors and escape spaces of Germany-A*

Exiting, heading down the oak-stair one finds corridor after corridor filled with offices, a warren broken up by nodes of private spaces with chairs and boards, or kitchen areas. One room is open to the air, intended for smokers, complete with whiteboard, another is tucked away accessible via a separate walkway, again it is not complete without a whiteboard.

Through the warren, back to the main stair, and up to the fourth floor one finds the main dining area. Two small, two tier boards stand off to the right, a passage-way heads off dead-centre, leading to more offices. The left wall is cut by six frosted-glass doors of offices. A table with coffee paraphernalia stands opposite the stair, with bar-tables opposite this, against the banisters of the staircase. Posters of graduate students' PhD research plaster the walls, broken up only by the complexities of abstract art, classical sculptures, and photographs. Three long tables hug the area off to the right, beside windows looking out onto roof-tops. It is to the far table that I make my nest - I watch the day unfold, jot down anecdotes, record. (Day1, Germany-A)

## 2.14 Inhabiting and assembling space: How physical spaces are used

I have discussed the spaces of the institutes abstracted from the people who inhabit them, but now let me add another layer of description to these studies, by exploring how spaces are used and interacted with by mathematicians. Here my intentions are to give ethnographic snap shots of different activities being undertaken within the various spaces of the institutes. Through exploring these varying activities in spaces I wish to demonstrate how different spaces are used in different ways throughout the institutes, and how these spaces are assembled together into a machine for thinking mathematically.

There are a number of different classifications of space I have identified within the institutes. These are as follows:

1. Common and private space
2. Formal and informal spaces
3. Productive and relaxation space

Each of these different spaces are involved in shaping mathematician's dispositions towards conceptual spaces in mathematics, and production of mathematical concepts proceeds through interactions (between people and mathematical phenomena) within the different physical spaces of the institute. The institutes each have different expectations of the mathematicians who enter their spaces, with the UK-A Institute emphasising the production of collaborative mathematics, and the Germany-A Institute having a more laissez-faire attitude, leaving the decisions up to the individual researcher. These differing expectations of work create different scripts, or schemes, for organising and inhabiting spaces, as well as for organising the production of knowledge within these spaces. These schemes affect the performances which take place with such spaces and the dispositions of the mathematicians who inhabit them.

Let me begin with discussing the differences between common and private spaces at the institutes. At the UK-A there is a greater volume of space dedicated to common areas compared to private or personal space. Such common spaces are visually accessible through the open and connected architecture of the institute. All three floors of offices are connected to the central staircase of the institute, which gives access to the common space at the centre of the main building. Offices are thereby visually accessible from this common area, and one can quickly survey the space to determine who is present. Access to participants is facilitated by the researchers themselves, who, as a norm, have decided to keep the doors to shared offices open (This convention is not so prevalent at Germany-A, only occurring in shared office spaces of three or more individuals). The degree of openness of doors is influenced by a number of factors, such as whether an office mate is present or absent, the desire to locate or interact with another participant, the individual's workload, their personal predilections towards collaboration, as well as their availability to be contacted. Individual's rarely close doors, except for when holding private meetings, personal skype conversations, or when undertaking tasks which require focus and concentrated effort.

The accessibility to the common space allows for individuals to feel connected to the activities of the wider group. Through the ringing of a bell during the start of lectures and meetings, individuals are assembled. Participants also make regular checks of the common space through "listening in", or "peeking out" into common spaces from their offices; participants will also make regular transitions to the coffee/ tea machine to scout out the common areas and improve the chances of encounters. The following excerpt from my fieldwork report highlights some of the main activities which take place between transitions between offices and the common area:

> The common area is a highly changeable space, as people move through it throughout the day. People transition through the space to move between offices, get refreshments from the kitchen area, rest in the lounging chairs, undertake desk-work on the high desks with pen and paper or laptop, or else casually read printed out articles on the lounging chairs. The common space is also a conversational space where people stage chance encounters with other researchers around the coffee machine, or else around blackboards. However, because office doors are often left open, people will talk quietly so as not to disturb the other participants - the common space is thus better described as a "muted conversational space" during working hours. The blackboards are used on a regular basis by participants, they are sites around which people gather, chat socially, or else explain their research to one another. Often gatherings around blackboards attract other participants and the material presented will be described and narrated to newcomers so that they can join in on the conversation (Day 20, UK-A)

The excerpt highlights some of the qualities of open common spaces. We see that the common space affords interactions and chance meetings between participants, the space assembles and connects individuals and serves as the main hub around which social life at the institute unfolds. The chalkboards and coffee machine become central sites around which individuals interact and share ideas, as well as social niceties. The common area is not however a site devoted purely to social interaction, as people bring their laptops and notebooks into this area to work.

The multiple purposes to which the common space can be employed, from meeting place, discussion space, networking place, thinking space, social space, work-space, leisure space and dining area mean that there are multiple competing demands for the space, which can create friction between different individuals. These activities generate different levels of noise and thus have effects on the activities of surrounding individuals or groups. Discussions, debates, presentations, networking and social events all naturally generate noise. Work which requires intense concentration, however, demands silence. Individuals engaged in solo work will either complain about the noise, relocate, or close office doors. The net result of this activity is to scale down the size of social groups and to drive larger groups to discussion or seminar rooms, or else to locations outside the institute. Such activities also reduces groups' sizes in general and lead to more private conversations in office spaces. Over time this led to a breakdown in group cohesion and a trend towards smaller social groups of 3-4 individuals.

At the Germany-A institute there is less of a problem with these competing demands for common space, because the common spaces themselves are less centralised compared with the UK-A. The main common space is located on the 4th floor, in the dining area at the top of the main stair case. This area on occasion hosts receptions and after lecture drinks during notable events, but, on a day to day basis, the dining area is used by a few individuals who regularly work on the desks, or by individuals collaborating using the main boards.

4pm heralds tea time and, at this point, individuals throughout the institute gather for refreshments and biscuits, staying for up to one hour to converse and collaborate around the main chalkboards on the 4th floor. Although there are offices on this floor, they tend to have their doors firmly closed throughout the course of the day, and are not overly disturbed by events unfolding within the dining hall. Because the coffee machine lies on the 1st floor there is less traffic moving through the large common space, and so fewer encounters take place on the 4th floor, except for at tea time. Whereas the UK-A Institute privileged visibility (and the staging of encounters), the Germany-A relies on movement and chance encounters to locate individuals. One must move about Germany-A, bumping into people along corridors, or chancing them around the coffee machine. The many different, smaller, distributed common areas throughout the building, and throughout the 4 floors of the institute, mean that groups tend to remain in pairs or 3 at most. Only during social events, or during tea time, do group sizes exceed 4 people.

There is less of a problem with noise at Germany-A because of the rhizomic structure to common spaces. There is less need to seek out quiet spaces, as corridors tend to be quiet and office doors are more easily closed (except for PhD offices which have a tendency to hold up to 8 individuals). At the UK-A institute, however, the centralised, dendritic structure of space leads to the accumulation of noise, and the intensification of social life around the main common area. Periodically individuals will escape this intensity and enter escape spaces such as the library or lower common area, or else move to the nearby university mathematics buildings - here individuals seek out peace and quiet and a place where they can concentrate on solo work, as the following extract from my fieldwork diary indicates:

> **11.01am:** Marina and Carlos are in the library, hiding away in a small corner, tucked away from the hustle and bustle of the institute beyond. Marina has her head-phones in, she's working on paper. **11.04am:** Carlos has his laptop open, a folder in front of him, looking intensely at his screen. The library is perfectly quiet, the only sound being my pen scribbling away and Marina clicking through papers on her screen, Carlos turns from the screen, studying a paper. In another corner is a new mathematician (tall guy) with head phones in, concentrating hard. All is perfectly quiet, still, peaceful. It is a perfect refuge from the noise and exposure of the rooms outside. The quiet mathematicians, the outsiders, come here to work and think, and escape. This is a place of calm in the rush and movement, a place to hide. It is by definition an anti-social space. But nevertheless it has a purpose: Escape. More such spaces like this are needed. I feel very calm here, safe, relaxed, I can feel myself a little less observed (Day 14, UK-A).

Individuals at the UK-A use the library as a private space to escape to. Head phones are equipped and Facebook is often opened up, as individuals take the time to escape their daily routine and indulge in non-work related activities, away from the gaze of fellow mathematicians. As the quote from my fieldwork report below indicates, the library space is also colonised by the staff who, lacking a staff room, take their breaks in the quiet of the library, socialising away from the main common space which is reserved for participants:

> The library, when it is empty of participants, also acts as a meeting point for the staff (IT staff, caterer and the librarian). The librarian and IT staff will often meet for informal chats and drink coffee or tea. It is a space where they unwind, laugh and converse fairly freely. For the most part though the library is a quiet space. It’s quiet, calm atmosphere can be an attraction for many of those who say they want to escape the constant flux of people through the common areas. There are a number of little niches in the library where one can be unobserved and can work in peace if required. Marina and Carlos come to the library to work, placing head-phones in and working at their computers. When asked why they work there, their answer is that they don't have a private office, although they agree that they prefer the quiet of the library to working in an office. (Day 24, UK-A)

As the above quote highlights, there is sometimes a need to become invisible within institutes, in order to escape and refresh oneself. When at the institute many individuals described to me how they feel that they must demonstrate themselves as being productive. As a result they will carry lap-tops or notepads whenever possible, and keep doors to offices open, in an effort to show that they are working, as well as open to collaborating. By making themselves invisible in escape spaces, such as the library, they can freely dip into the news, blogs, or Facebook without feeling guilty that they are not 'working'.

At Germany-A, perhaps because the duration of their stay is longer, and therefore the pressure to be productive is less intense, individuals are more relaxed about checking Facebook, and social media platforms whilst in their offices. This degree of informality at Germany-A does not carry into certain spaces, such as the seminar and discussion rooms, or for that matter around certain shared chalk-boards. Seminars are naturally formalised events, where presentations have been practiced and honed over time. The formal presentations of research are important for recruiting collaborators. Individuals must make good impressions and try to communicate their research as simply as possible, in order for them to make contacts and build their professional network. Formal presentations present ideas in finished forms, as they would appear in publications, and thus they are not opportunities to experiment or conjecture.

Within formal settings, such as the seminar room or the discussion room, participants sit within ordered spaces, around tables or in rows, facing a chalk-board or projector screen. This ordered space, according to some participants, leads them to act in certain ways, and prevents a full and frank discussion of the research being presented. Below one participant, Luke, explains why one formal presentation in the discussion room wasn't successful:

> LU: Yes, so for instance, like the…the presentation that Fred and I did just briefly – that was even too formal, because you know how, at the end is Daniel is really asking everybody: what's wrong with this, you know? Identify these kinds of things! We both really want people to identify the holes, and noticed that it was only the really senior people who even said something; it would have been nice to actually really get… You know, because everybody has a critique… so it would have been nice to actually get some free flowing critique, but… So I think the discussion room is good for at least three quarters of the way finished ideas, kind of thing, you know? You're all sitting round in a circle with a board, there's one person talking at a time, you know, that kind of thing? It's less of a place to just bat around ideas. That's more the… you know, around the coffee machine, around the beer, dinner, whatever, late at night.

The social norms which certain formal spaces demand inhibit the free-flow and free-critique that Luke was demanding. Formal spaces create expectations for how to interact within them; they are imbued with social expectations and these social scripts (schemes) interfere with the discussion process.

I finally turn towards the last categorisation of space, which is productive and relaxation space. This classification of space is more difficult to pin down, as these are not well defined physical spaces as such, but rather they are spaces which individuals themselves create and assemble during the course of inhabiting institutes. These spaces are adapted to suit a given need, and they thereby have their frames (schemes) "re-keyed" (Goffman, 1957) according to the situated encounter between the individual and the space at hand.

There are of course certain physical affordances (resources), which makes space subject (predisposed) to transformation into one of these spaces, for example because of the privacy, formality, acoustics of a space; but such spaces are produced relative to individual's needs. The common space of UK-A, for instance, can be transformed into a relaxation space during quiet periods, such as the lunch-time lull, or after the hours of 5pm. At Germany-A, the main dining floor, or many of the other common areas, become spaces where mathematicians pace and quietly reflect to themselves. Reflective or relaxation spaces are spaces within which people can step back and think deeply on a given issue or else take the time to escape from their work and play games or socialise.

The quiet, productive space of offices, or quiet corners of Germany-A, are transformed into leisure spaces usually after 7pm. Mathematicians who do not have internet access in their apartments will escape to social media platforms, or watch a movie or TV show in the comfort of their offices, or in a secluded nook or cranny of the institute.

At the UK-A Institute the usual productive spaces of the common areas are transformed into leisure spaces after hours, with a Pizza night every Wednesday seeing the mathematicians unwind and relax. They play board games, drink beer, and use the boards to present puzzles and quizzes. The architecture of the institute is thus repurposed (re-keyed) as a relaxation space, as the previous regime of productivity no longer needs to be enforced beyond the hours of the working day. Within working hours common spaces are similarly repurposed by both staff and participants, during lunch breaks. Here the regimes of visibility created through the open architecture are undermined through creating auditory private spaces, by plugging in ear phones and listening to music, or watching programmes on their mobile phones or tablets. The quote from my fieldwork notebook highlights one example of people escaping into their own reflection spaces:

> Its lunch time and I'm in the open area again just looking at interactions. It's empty though, many of the free-boundary people are still away, and the other metagenomics people have gone to Wolfson for lunch. The secretary is still in her corner eating secretly, away from the camera, she's checking her phone…I think - I can't really see her…Lunch time at the institute is generally characterised by silence – it's kind of interesting, you would think that people could bring their own lunches, but people seem to want to escape their place of work more than anything else (Day 4, UK-A).

Here individuals create their own escape spaces through hiding away, relocating, listening to music, or becoming oblivious to the world through focusing their attentions on their phones, or computer screens. Individuals screen out the world through this process of gazing at the screen, and in this way they become invisible to others who may be surveying the space. The regime of productivity and visibility is thus undermined through this mode of escape. Through this method of screening the individual thus transforms public spaces into private spaces, using technology to reassemble the spaces around them to better suit their needs.

Such re-purposing of space demonstrates the multiple schemes which can be applied to the spatial resources of the institute. Such schemes of informality, productivity and privacy are able to be transposed to different spatial contexts, depending on the affordance of the spatial resource and the individual's practical necessities. Within situated practice, we therefore see the mutability of the structural schemes of habitus. We see that individuals produce multiple realisations of habitus, by adapting schemes according to the specificities of the encounter at hand. In the next chapter I shall explore these situated encounters in greater detail, and outline how engagements with spatial resources produce certain dispositions and habits.

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## Section 2.2: Production of Space

### 2.20 Overview

The following section will discuss how individuals inhabit space. I will demonstrate how different habits and routines shape and are shaped by institutional spaces, and how these practices structure not only one's daily routines, but also one's way of thinking in mathematics. I will show how physical order and personal habits give structure to a mathematician’s work and ways of thinking, through generating schemes for shaping practice.

### 2.21 Introduction: How are social spaces produced?

As we saw in the previous section (2.1), space is not simply the passive backdrop for social life, but rather it shapes and is shaped by the individuals who live within it. The rooms of the institutes provide spatial resources for organising work and thought, and are selectively manipulated in order to support an individual's plans and activities. In the previous section I introduced the idea of assembling space and adapting spatial schemas, but now let me explore how space is "produced" by mathematicians at the institutes.

The idea of producing space seeks to emphasise the role of the individual inhabitant of space as the agent involved in generating and scripting action within space. Physical architecture is inscribed with "scripts" (schemes), which influence how space should be used. These scripts, however, are adapted by the inhabitants of a given architecture, and transformed so as to support their needs. Different individuals may inhabit space in a variety of ways, and thereby develop their own personal scripts for acting within given spaces. Such scripting unfolds through the process of becoming habituated to a space. At my field sites I observed how, with growing familiarity, mathematicians developed patterns and routines of work, and learned to domesticate the spaces they inhabited. Through the act of working, mathematicians at the institute produce not only ideas, but architectures for thinking and producing these ideas.

At any one time there are a multiplicity of schemes being executed within the architecture of the institute, as each inhabitant goes about their daily routine. Individuals move between offices, transition across common spaces, escape, return, relax, reflect, congregate around boards, coffee machines, computer screens. The following quote from my fieldwork journal captures a snap-shot of this complex unfolding:

> **5.37**: I can see Constantine in his office - hand on head, pen in hand, paper in front of him. He gets up, grabs a book from the book shelf. Meanwhile three mathematicians, along with Tall, climb the stairs with their cups of 50 cent coffee cupped in hands. They gather around the blackboard in their room and chat, each stepping back and letting one speak, then write, taking turns to present and question. The Russian guy in the room opposite has headphones in, I think he's skyping again[9]. The Russian lady is speaking on the phone, loudly and in Russian. The Red-shirt man beside me has left his desk for the past 30 minutes with his laptop plugged in and notebook open, a LaTex scripting interface opened up before him. (Day 13, Germany-A).

[9] The composition of Germany-A was an even mix of young researchers (under 35) to older researchers (35 – 70s). Researchers at many different levels of their career were present, from PhD students to tenured professors. Social media and skype use spans all age ranges and degree of use depends on the individual predilections, however use more prevalent and intensive amongst the younger generation (PhD-Post-doctoral researchers especially), older professionals however tend to use phone-calls rather than skype to communicate with individuals not in the immediate vicinity. Younger members of the community tend to occupy public spaces to a greater degree and are more anxious and on the look-out for potential collaborators. Older professors adopt a more relaxed attitude to forming collaborations and are not overly concerned with their productivity to the same extent as those researchers under 35.

The ethnographer's eye catches only a glimpse of the happenings within the institute. From the extract above we see the difference between people who I know and are named and those who are quickly jotted down and reduced to adjectives or nationalities: "Russian", "Tall", "red-shirt". Space for me at this point is not yet familiar, and neither are the people who inhabit it. This is much the same for those mathematicians first entering the institute and still getting used to it. They must learn how to use the space, to position themselves within it, and they also must learn how to perceive each space's affordances. They must know where to look to find a book, a collaborator, or an unclaimed board they can write on. In the above extract Constantine puzzles over a problem in private, while four other mathematicians take their turns at solving a problem collaboratively on a white board. Other individuals keep in touch with friends, family, or colleagues, via skype or phone. Some are scribbling away some half-formed idea on scratch paper, while others read papers on already solved proofs, or else perfect their own ideas in a word processor. Ideas lie about the institute in various levels of construction: some are completed, consumed as Pdf's on a screen, or else in physical print; other ideas are only just forming, sketched out on boards or on paper; still others are being made ready for publication, crafted in LaTex (a mathematical scripting program) or debated amongst colleagues. But the process of assembling these thoughts are distributed throughout the institute itself, in a complex assembly line of thought.

**2.22 Rhythm and routine: Understanding how spaces shape habits**

At the UK-A there are similar processes of assembling people and ideas in space. The work-day is more formally structured by the programme organisers, which leads to individuals in the programme becoming synchronised in their work activities. Their daily routines are concentrated in the common area on the second floor, unlike in Germany-A, where activity is distributed about four floors. Below I provide some transcripts of activity recorded in my fieldwork notebooks, as well as passages from my fieldwork report which illustrate a typical work day at the UK-A Institute:

## Morning Start

**7am - 9.15am:** From 7am a few participants begin arriving at the Institute, with early morning people grabbing a quick coffee before heading to their offices. Typically the majority of participants will arrive between 8.45-9.15am. Participants usually head straight up to their respective offices and at the beginning of the day they will close their doors if their room-mate has not yet arrived. During this arrival period they will check emails before exiting their office to grab coffee or tea. After the initial period around 9.30 doors are generally left open.

> **9.10:** Rodney is in his room, the door is firmly closed. **9.26:** William, Stan, Alfred and Andrew keep their doors open, I spy inside and see them all tapping away on keyboards, or scrolling down a PDF. **10.14:** Mathew arrives, gives a nod in my direction, but, without nonsense, heads straight to his office, the door closes. Nigel meanwhile is working away on his laptop on a high table, close to the stair. Beside the coffee machine Alfred has popped out for a chat with Vas. He says he's sceptical of finding anything worthwhile on the 4th domain research. **10.46**: Donald chats with Luke, asking what time the meeting will be. Donald is unsure and says he'll ask Grant. **10.48:** Sam is in William's room, puzzling over a blackboard. **11.34:** The common space is filled with Free boundary people. Three tables are occupied with

> some eight individuals from the other research programme. Felix comes up the staircase looking around, shocked. He comes past and says with a smile: "They've taken over" - he enters his room and closes the door (Day 17, UK-A).

## Late Morning

**9.15-11.00am:** Morning sees movement through common spaces based around the search for coffee. Participants will sometimes get coffee and then return immediately to their rooms, or else linger in the common area, pacing or sitting in the lounge chairs. It is during this break that chance coffee-time encounters occur. Occasionally a morning meeting will be scheduled, and participants will be attracted by the sound of a bell, rung from the foyer below. Participants quickly assemble, bring laptops or note-pads from their offices. The morning is dedicated to "normal", non-UK-A institute work, received during the early morning email-check. Such work can comprise checking over data sent from a lab, looking over PhD theses and making corrections to papers participants have written.

> **10.56**: I'm in the open area scouting out. Luke has arrived. Han and Vas are still talking about their data-set, off to the side of the coffee machine. Felix has his head-phones in, listening to music, ignoring the world. William is sending an email to the group. **10.59**: Annie talks to Roger and an unknown man I haven't seen before - a guest perhaps. **11.00**: Felix walks down with a notepad and knocks on M2 - Zen is inside. They close the door and come out a moment later in search of coffee. Two minutes later and they return. Penelope arrives a little later - car trouble on the way up from London. **11.08**: Vas leads the unknown man to his room for a chat, and Roger heads up to his office on the mezzanine floor (Day 11, UK-A).

> **12.20**: I sat in on Luke and Felix, in their office. I found a perch on the window sill and merged into the background. Felix jumped after a while as he turned - he had forgotten about me - he says I'm like the Cheshire cat. Luke had prepared some graphs to show Felix. He had worked on them overnight after both had chatted back at their apartment block.

Felix is keen to interrogate the data and asks specific questions about the results. Luke clarifies. "What does this one mean?" say Felix scratching his head, peering down at the plot. Luke describes what he's seeing, giving a break-down of his approach. He turns to the computer and begins playing about with the heat-map plot, changing the python code he's working on to generate a new visualisation. He brings up another plot and says to Felix to look at the comparison with another program he's used called metaphlan. Felix is concerned at the comparison. He begins to compare metaphlan to different levels of K-mer size[10] - 30, 50 etc. Felix looks disheartened, it's not what he expected. Luke is more optimistic and tells him that in quality, speed and range their method is better at distinguishing between the different species in the data set. Felix asks whether they're ready to present (Day 17, UK-A).

## Lunchtime

**12.00-2.00pm:** Lunch is usually eaten away from the institute, usually at a nearby college, or else in the main building of the university mathematics department. Usually only staff will eat their lunch at the institute in the common spaces upstairs, which have been vacated by the participants. Participants will either leave in pairs, knocking on each other's doors to talk on a one-to-one basis, or they will leave as part of a larger group, recruited on an ad-hoc basis.

**13.41**: Back in open area after I took Alfred, Luke and Carl out to lunch. It was a good opportunity to learn about their research. Sam, William, Grant, and Cat ate lunch together - they sat on the table beside us. Returning to the common room William asks Cat if he could explain his problem as simply as possible. Cat suggests that Sam and Sarah should be in on the talk as well. **1.49:** Alfred heads to Rogers room to talk about their project. Meanwhile Donald visits Grant in his office (Day 17, UK-A).

[10] A Unit of measurement for DNA fragments from a sequenced genome.

## Afternoon

**2.00-5.00pm:** On returning to the institute, participants will often break down into smaller groups and continue conversations over coffee, with others simply returning to their rooms to check emails or return to writing papers. Much of the work conducted at the boards takes place during this period of the day, as they are more willing to focus on collaborative work. After lunch we see movement between rooms as participants ask questions, and exchange information. Often participants will present data to each other by moving between rooms with a printout, or their own laptops, or else take other participants to their own rooms and show them their data there.

> **2.01**: Luke and Felix head off to Luke's room. Alfred speaking with Indiana and Anna, giving a summary of his research interests. William tries to reinterpret what Alfred is telling them by saying "let me try to fully understand what you're saying", before rephrasing what Alfred has told him. **2.21**: Eventually Anna leaves and Carl comes to spectate, sitting back in his chair, listening in.
>
> **2.26:** I'm in Luke's room now, observing. Felix and Luke look intensely at the screen. Luke is trying to figure out what visualisations to include in the PowerPoint presentation they're giving. Felix asks to flick back to one of the plots. "Look there - what's this structure? Funny right?" He points to the screen. Luke looks puzzled and tries to make sense of it. **2.40:** William sits down to talk to Felix in Alan's room. Felix is at the board drawing something out, William meanwhile draws out a grid with green chalk and starts filling out a table of amino acid sequences (Day 14, UK-A).

## After Work

**5.00-5.30pm**: The institute quickly clears around this time. Some participants will come by each other's doors, give a knock and chat before departing. On occasion participants will leave together, other times they will form larger groups to travel into town, whether it be to a pub or a restaurant.

**5.30-8pm**: For those who remain, the late hours are spent either playing games like "Go", or chess, or else just casually checking emails or watching lectures. Skype calls can be heard from closed doors. Every Wednesday around 5.30 the free boundary people will have their pizza and beer party at which point the institute is transformed to a buzz of activity, laughing, spontaneous conversation and merriment.

> **5.07:** Anna knocks on William's door. I call over to Nigel. He's going to read up on some papers on the 4th domain work, he heads up to his office. **5.13**: The free boundary mathematician in office M14 leaves. Grant goes over to William's room. **5.24:** Henry stops by Penelope's room: she's having trouble with a piece of software Henry's written. **5.26:** William visits Anna's office. Blue shirt and Yellow shirt come up from the downstairs common room, they've been collaborating around the main board there, leaving the "please leave sign" in place. **5.48:** Felix, William, Alfred, Henry and Simon congregate outside Felix's office, they tell me they're going to the theatre to see a play. Felix says "It's Shakespeare…I think". Luke leaves separately, saying he'll meet them there just after he returns to his apartment. **5.50:** Vas and Charlotte leave; Vas doesn't look too happy. Maybe see them in the pub later**. 6.12:** Anna Skyping her student in Germany; he's been waiting for her to send back a report he's written. **6.26:** All quiet in the Institute. Penelope returns to her office to use the WiFi. She says she's not sure where the theatre is and needs to google map it.
>
> **6.28:** Anna leaves, we have a quick chat. **6.34:** The free-boundary mathematician in M11 leaves. **6.35**: Simon returns to drop off his laptop in his room. **6.46:** Italian Free-boundary mathematician from M11 begins taking pictures of the common space. Takes 3 pictures in all: one of his office, one of the coffee machine and then one of the LCD screen showing the faces of mathematicians present in the institute. He locks his office door then leaves. I scuttle off to the balcony and watch as he takes pictures of the

> library, reception, the main stair and then the lecture theatre. He leaves then. **6.46:** Penelope leaves in a hurry. The office mate of M11 returns. **6.58**: Blue shirt comes down to use the photocopier. **7.04**: Blue shirt leaves. **7.24:** M11's office mate leaves. **7.41:** Yellow shirt leaves from M12. **7.54:** IT comes past me to get a cup of coffee, blanks me (Day 10, UK-A).
>
> **5.50 - 6.27**: I'm in the common area. Felix and I are chatting about his new dance class called 5 Rhythms. I tease him for a while, saying I hadn't realised he was into the New Age kind of stuff. Felix laughs and says that he finds it therapeutic, but quickly changes the subject when Luke arrives. Luke says "Let's talk normalisation" and Felix jumps up and both head to the large chalkboard beside us. Felix writes down the problem in a spidery hand, starting at the top left of the board. Luke looks on, making comments, finger on lips as he listens to Felix's explanation (Day 12, UK-A).

From these observations of day-to-day routines we can begin to see the complexity of interactions taking place within the institute, as well as the different assembly points for people and ideas. The office space features prominently in individuals' work-routines, with individuals holding private meetings, skype conversations, or black-board presentations within the confines of their offices. The common area is also a place for work, a place to meet, present ideas, chat and reflect, and is adapted for these different purposes throughout the work day.

Different sites outside of the institute are also important: such as the college dining halls, where the mathematicians eat lunch, socialise as well as share ideas. Note-pads or lap-tops are brought to lunch and are kept close at hand, as any activity is an opportunity to present one's ideas or to forge a new collaboration. The fact that individuals carry their notebooks and laptops or tablets around means that their workplace is necessarily mobile, and can travel to different spaces in the institute.

Because of the coming and going of individuals at the institute it is necessary to stay visible in order to contact others, and open oneself up for collaborations. Staking out common spaces and remaining conspicuous is therefore important to some early career researchers, who will make "nests" in the common areas, choosing a favourite chair or table to set up as their work-space. High visibility sites are sometimes chosen, which include positions facing main entrances and staircases, heavily trafficked sites near to the coffee machine, or sites near to main chalkboards. This siting is sometimes due to individuals not possessing a private office, but in other cases the siting is more deliberate (they are positions from which to stage encounters with others): in order to catch someone's eye, or ambush a would-be collaborator with questions. The open doors of offices are also a sign that one is open to collaborating. Sometimes doors are left completely open to allow individuals to monitor the common spaces, as well as to show that they are welcome and open to collaborating or chatting.

The workday is broken up by the ringing of a bell which summons individuals to a meeting or a seminar, or else by gatherings in the common spaces. Such gatherings will either bring individuals out of their offices, or see doors firmly closed to screen out the noise. Coffee breaks are a regular habit and take place every couple of hours throughout the day: they are an opportunity to look at who is "in", or "around", or just to sit in the common space on the lounge chairs and relax for a while. Usually during these periods individuals come out and engage in an informal "chit-chat" with me, or with any other common-space inhabitant.

On a day to day basis individuals will engage in home-institution and administrative activities, such as checking emails, reviewing PhD theses, writing and editing existing publications, skyping with existing collaborators, and organising other events outside of UK-A. Time devoted to UK-A work can often become radically reduced, as pre-existing deadlines loom and normal work takes precedence. Oftentimes, the mathematicians inform me, this prioritisation of their normal work leads to them feeling guilty that they are not collaborating enough. This is another factor leading to the open office doors and conspicuous work activities. Individuals tell me that it is a privilege to be at the institute, and so say that they feel that they must be seen to be working and being productive to justify being there.

The conspicuous nature of work, and the need to appear productive during working hours, influences how individuals use spaces and present themselves. However these scripts for how to use space are no longer in effect after 5pm. As mentioned in the previous sub-section, individuals escape work by going to the pub, or dancing, or going back to their residences, or else they transform the institute into a more social space by playing games, streaming television programmes on their laptops, or just chatting socially. Time of day thus serves to demarcate the temporal boundaries of work, and provides another structure by which work and thought are ordered within the institute.

At Germany-A mathematicians provide their own structure to the workday. Only presentations by members of the institute or lectures provide formal structures to their work routine. 4pm tea time on the 4th floor provides an opportunity for individuals to gather and chat socially, or else explain ideas using the two large two-tier boards in the main hall. The following quote from my fieldwork journal documents the scene:

> A man and a woman took their mugs and perched on some stools around a high, round bar-table, next to the stair railings; they chatted casually. 4pm hits and there is a mad rush for coffee - people seem to be programmed to come out at that time because the place quickly fills up and there is a buzz - something I had not encountered in the mornings.
>
> The groups slowly moved out from the centre around the coffee pots, forming bands of concentric circles, all oriented inwards towards the coffee and the conversation around it. They spontaneously formed pairs and groups of 3-4; many others were itinerant, just listening in, floating between groups and interesting conversations (Day 7, Germany-A).

Life at Germany-A follows a different rhythm to that of UK-A. Tea time creates large gatherings of up to 20 people for an hour, but for the better part of the day interactions are mainly in pairs or 3-4 people at most. These interactions are dispersed across all 4 floors of the institute. The institute is quiet throughout the day, and so interruptions in the silence bring people out of their offices to investigate, as the following extract from my fieldwork journal describes:

> **1.06**: A Russian woman is working away beside me, tapping on the keys of the keyboard one at the time using only one finger, looking intensely at the keyboard at the different keys as she logs away….
>
> Another Russian man across the landing, in a room opposite stepped out of his room to have a Skype conversation; he left the room he was sharing with another guy in order not to disturb him, but sitting in the common space, and concealing himself in a corner he was able to have a conversation (in Russian), despite the other people within this common space. The idea of privacy through both physical concealment and isolation via language thus allows people to create a privacy bubble around themselves, without actually being in private. In this case being in an open, common area with people present.
>
> **2.43:** It’s perfectly quiet in the institute: people are in their rooms, mainly at their laptops or desktops, reading articles. Interestingly most of

the work is done at desk with pen and paper. Almost no work is done using the blackboard: the blackboard just appears to be a social thing (Day 8, Germany-A).

Grey T-shirt is back and is just surfing the internet. Indeed the internet does appear to keep a lot of people at the institute. Outside, in their apartments for example, internet is not provided (If you're smart you can pick it up outside in a quiet café; it’s patchy though). They are thus almost forced into spending time at the institute. But the work of a modern research mathematician, I should say a young research mathematician, does depend on constant internet access, just in terms of accessing email and research papers. But also there is a social aspect to this: connection to Facebook, distractions from work via blogs and news reports. These small things are built in to the day to day work practices and thinking practices of research mathematicians. Indeed periods of escape and socialising seem essential in this process of thinking. People need to step outside of their thought spaces, beyond their computer screens, or papers and think about something else. Sometimes they literally step out: they pace outside their offices, they grab coffee, or else they go for a walk around The city. On returning they are able to re-engage, and get back into work mode. The constant process of escape and return is thus important in building coherent concepts, as it means that the information originally written down needs to be refreshed, rethought out, and reconstructed.

Another person steps out of the institute for lunch. **1.30:** Grey shirt steps out. A tall German guy steps out of his office and begins pacing the dining area, back and forth, arms crossed. Looking at the different posters on the wall, pausing, returning to his room, and then finally settling back at his desk: a short escape, but also breathing room to think. The Russian woman is on the phone again; it’s loud and echoes, so the professor from Taiwan in the far office closes her door. Constantine comes out of his office and looks up the stairs to see who is speaking, a look of annoyance on his face. My office mate has a similar aversion to noises (Day 10, Germany-A).

From the above passages we get the sense that Germany-A is a place of retreat from one's day to day work at home institutions. Mathematicians come and go, moving between other mathematical venues in the city, taking breaks in coffee shops, restaurants, libraries. They may stay in the city or else use their travel stipends to meet collaborators in other cities, or even institutions in other countries. The institute also encourages members to invite guests, providing funding for researchers to visit in order to collaborate. The institute thus serves as a tool for individual members to build their own collaborative networks and their own personalised thinking spaces. Since there is no compulsion to attend seminars or events, one's work day is structured according to one's own personal choices. Such choices give members of the institute more flexibility in how they allocate time to work and social activities. In the above passages we see that skype and telephone conversations with collaborators, friends and family play a major role in one's work routine. There is a constant movement into and out of the institute, there is also movement within the institute, which takes the form of pacing, coffee-breaks, or Skype-breaks in secluded spaces.

Freedom, however, is not an absolute, as can be seen by the fact that individuals retreat to the institute to use the internet, because many apartments do not come with a broadband connection. Individuals thus spend leisure time at the institute as well, watching movies, TV shows, or using social media platforms such as Facebook. The private, focused work also necessitates silence, which again restricts many people's behaviour in private spaces or shared offices. To have more animated conversations one moves to unoccupied offices, or else discussion rooms, or the open-air lounge room on the second floor.

### 2.23 Creating order, structuring thought: Balancing structure and agency

Structuring one's work life thus is a product of both personal choice and institutional influence, we see that it is a balance between freedoms and constraints. However, what I now want to explore is how individuals’ rhythms and routines structure their work and thinking. Through such explorations we will get a sense of the flexibility of mathematicians’ work routines at the institute:

> Spoke with Morry - he's a German mathematician working in topology. We began by speaking about my research on collaboration among mathematicians, which really opened the door to speaking. He stressed that mathematicians are not necessarily geniuses; in fact they are people who have understood the fundamentals of the language and have been able to use it effectively to think with. Morry says that most mathematicians are lazy and they like the freedom of having no rules: they get up when they want, work when they want, and produce what they want. He says the only thing they need to do is to write applications for their research. He says he doesn't like rules, but, he says, some rules are necessary to continue living the life he leads. But he says that applicability of the research shouldn't be what motivates mathematics. People should write papers and then see what happens, the applicability is not something which should be a concern for them.
>
> People seem to have different rhythms. Saul said he prefers to take the morning off and read some articles and "books" which he's "acquired" online (he was a little nervous of saying where he acquired them, but finally relented by saying a Russian website). Saul said he doesn't usually come in until 2.30pm; mainly because there's no advantage to coming in earlier, as it appears that the mathematicians he speaks to start work late and finish sometimes around 6pm. Tonight we're staying until 7.30 because we're heading to a pub called James Joyce with the rest of the post-docs. (Day 16, Germany-A)

We see above how certain mathematicians enjoy the freedom and flexibility that life at the institute provides. The temporal boundaries of the day are relaxed at the Germany-A, with some individuals coming in after lunch or mid-afternoon, in time for tea-time. Other mathematicians will work 9am-5pm, as at the UK-A; still others work late into the night, with individuals even known to sleep overnight at the institute.

One's work, thought and habits are thus shaped through a negotiated practice between the formal structures of institutional contexts and the agency of the individual. Individuals shape situations in order to better suit their agendas. Some mathematicians use the institute as a chance to escape from their day to day routines, and undertake focused work on a given problem, as Simon comments below:

> Si: I'm able to be more focused, so I can work longer hours. The fact that the place where I'm staying is very close to where I work means that I'm not commuting for a half an hour or an hour each way. Yes, it's really the focus and the fact that I want to get something before I go that really gives me the drive to finish things. And the fact that because I'm somewhere else, I can consciously focus on something else rather than have the, you know, the various duties of [my home] institution to distract me.

As researchers at the institute informed me, the geographical location of the UK-A, away from the city centre, makes it a perfect retreat from normal city living. The proximity to their accommodation also reduces their daily commute, and means that more time can be allocated to their work. Being "somewhere else" also allows individuals to escape their normal frames of reference and look at problems with fresh eyes, as well as escape their day to day administrative routines. They are able to craft new routines and tailor their work-life to a single problem. The short duration of this escape period also means that individuals are motivated to try and get the task completed within their allotted time, as they feel they will not get another opportunity for prolonged and concentrated work. William says the following:

> Wm: So over the years, probably seven years at least, I've been playing around with these other ideas and they're gradually coming together. It's taken a long time, but you need a lot of pure, concentrated thought. You can't do that in fragments of time, so what I did, I… beginning last year, I resigned half my job and I'm on half pay, and the idea was I would have more time to do research on the things I thought were important because otherwise, I couldn't find such time. You know, you really need months on end to think about these things uninterrupted, and you simply don't get that in a university setting.

So I had to basically go part time in order to get time to work on these things. That's what I've done, although it hasn't quite happened yet because I had so many… you know, as an academic, when you decide to stop something, you can't just stop it. It's like a juggernaut. It's all these things that you're kind of committed to, this programme being one of them, that you can't just say, that's it, chap, I'm off, you know, and the whole thing goes down the tubes. You can't do that. So it's taken me 18 months to actually… working more than full time, to actually deal with all the things that I needed to deal with, but when I get back to X the hope is that I actually will then have some real quality time to really concentrate on a few things that I really want to concentrate on without any distractions. That's what I'm planning to do when I get back.

Mathematicians require time to be creative, to play around with ideas and assemble the fragments of thoughts into one coherent concept. To nurture such creativity requires that day to day pressures and stresses be suspended, and that the burden of productivity be lifted. The institute, as a liminal space, outside of their normal work life, gives individuals the opportunity to step back, to think deeply, reflect, and collect their thoughts[11]. This process of crafting ideas is integrated into the practices associated with developing routines and becoming socialised within institutional spaces.

From the excerpts above we see how these routines vary as a function of the individual, as well as the institutional contexts they occupy. The architectures of the institutes, as well as the expectations of the institution's directors, funders and administrators, shape the dispositions of the individuals present within institutional spaces. The individual inhabitants themselves bear certain pre-dispositions,

[11] This phrase means that the institute provides a liminal space, outside of the contexts of one's home department, within which the individual is given the opportunity to work freely on research. Such freedom however is not absolute, as their departmental responsibilities are replaced by the specific institutional responsibilities, requiring an individual to "be productive", by writing papers or forming collaborations.

expectations and motivations before they even enter these institutional contexts, and these also serve to shape their interactions, habits and dispositions. The patterns of behaviour, thought, and production which emerge within the institutes are thus a complex product of these different sets of structuring principles.

In the next sub-section I shall discuss how these patterns of behaviour and dispositions are manifested through individual's performances of their mathematical selves and ideas. Such performances are functions of the situated engagements between individuals' habitus and the affordances of the spatial resources encountered in practice.

## Section 2.3: Performance Spaces

**2.30 Overview**

The following section will explore how individuals present themselves and their ideas within institutional contexts. I will explore the importance of such performances of the self in constructing collaborative networks and successfully disseminating ideas. Such performances take place within both the public, visible theatres of Goffman's frontstage, and also within the more private, informal backstage areas. The movement between front and backstage is important in successful performances of individuals' mathematical selves and ideas. Such transitions between frontstage and backstage, we shall see, are important in creating "common ground" between researchers, which serves to build bonds of friendship, trust and free intellectual exchange.

**2.31 Introduction: Performance spaces**

Individuals escape the normal, departmental contexts of mathematical production at their home department, only to escape into other regimes of work and productivity at the mathematics institutes. Individuals are freed from the obligation to teach, but not from the need to present their research and to produce papers. This pressure to be productive comes from a number of sources: from research councils, institutes, university administrators, heads of department, and from competition with colleagues and fellow researchers. Such social pressures form fields of force, which generate habitus that influence the predilections, dispositions and actions of individuals who inhabit these mathematical fields.

In the following section we shall see how these pressures to perform and to be productive are manifested in the behaviours and interactions between researchers at the mathematical institutes under study. We have already seen how the environments of mathematical institutes are socialised and assembled into productive spaces, and we have seen how these spaces position individuals within the mathematical field, and how such positions generate habits and dispositions. What I now want to turn towards is how different spaces are utilised in the presentation of ideas and the self within mathematics.

I shall use Goffman's idea of *frontstage* and *backstage* in order to explore how different areas of the institute become adapted as "performance spaces" for "fronting", or foregrounding, mathematical identities. I shall show how the pressure to be productive manifests itself in an increasing need to make one's mathematics visible, measurable and presentable. Such need to perform foregrounds the more formal, visible, frontstage at the expense of the backstage of production. But I shall demonstrate that the more informal, private, backstage world is equally as important in shaping the mathematical self and the mathematical habitus.

### 2.32 Life on stage: How the self is performed in mathematics

The *frontstage* is the place where mathematics is presented and made visible. It includes virtual spaces, such as online, on personal web-pages, blogs or forums; it can include literary spaces through published proofs or pre-prints; but, for our purposes, I focus on the physical sites for presentation at institutes: the lecture, seminar, discussion rooms and public spaces. It is within these spaces that proofs and ideas are presented formally, in more refined, coherent states. These formal, public spaces are where an individual's main purpose is to present information, to debate ideas, and to keep people up to date on research in a given area. Within spaces such as lecture halls, seminar rooms, and discussion rooms, individuals are placed within a hierarchically ordered space, where one or more individuals are constructed as performers, who face another group of individuals who take their positions as an audience.

The audience, however, is not in a passive or submissive role; instead the interaction between audience and presenter is much more dynamic, with questions often being asked by audience members throughout the course of a presentation. Questions can have the aim of clarifying an issue, asking for more description, examples, or counter-examples; or questions can be aimed to probe the veracity of a statement, or the strength of an argument. Statements which provide details, examples, counter-examples, counter-arguments, or opinions are also offered to the presenter, as well as errors being pointed out, or "corrections" provided. *PowerPoint* presentations are rarely given at Germany-A, but are encouraged by UK-A, in order to facilitate knowledge transfer between participants, as well as facilitate dissemination of knowledge online (via their website). At Germany-A, presentations take the form of "chalk-talks", where presenters write out their research on chalk-boards, narrating an argument or proof as they go along. Below is an example of such a chalk talk, recorded in my fieldwork journal:

> I sat in on one of Constantine's seminars on "translating theoretical physics concepts to mathematicians". Constantine's presentation was mainly focused around developing examples which he thought the audience would understand. He set out at the beginning with some definitions from theoretical physics, which he then translated into mathematical terms. The examples were incredibly detailed and involved a lot of differentiation of

> terms, which at times could be difficult to follow, as the board quickly filled with a zoo of terms, which stood for different physical phenomena.
>
> During his differentiating he mistakenly placed some terms on the boards, which the audience picked up on and periodically commented on, saying: "That term chi, shouldn't be there", "I think you are mistaken", "Remove the double dot", etc. They then went through and tried to find more holes: one person thought he had seen something, but Constantine quickly countered, leading to the person backing down quite quickly, apologising.
>
> One professor at the front continued to ask questions about what some of the things presented actually meant. Constantine would go through and explain: "yes it’s a tensor bundle", or "No it’s a flabby space". At the end Owen asked a "technical" question, on whether the manifold was "fine and soft and not flabby". Constantine was stumped, he didn't know how to interpret the terms "fine" or "flabby", and asked why it was important. Another professor however stepped in and said that the fine-ness was important in this case. Constantine called a break in order to talk to the student and the professor in private. Others gathered around to listen in; the rest broke into their respective cliques and began talking amongst themselves (Day 11, Germany-A).

In the extract above we get a sense of the difficulties in presenting material to a mixed audience of experts and novices. The intention of the seminar had been to introduce concepts in theoretical physics to novices, however experts in the area had turned up in order to "audit" the course. The result was that Constantine was set upon by a host of technical questions and error corrections. To give such a presentation, in which one is constantly interrupted, does sound somewhat daunting, however such presentations are common ways of testing one's knowledge, as well as one's rigor. To survive such onslaughts allows one to grow in esteem. The trial by seminar, which younger mathematicians go through, thus acts as a proving ground for mathematicians, giving them a chance to hone their skills at arguing and presenting their ideas.

On a separate occasion I had talked to Constantine about his Tuesday lectures. My fieldwork journal records the event:

> We talked about his Tuesday lectures. He said that he was often worried about them because directors and senior mathematicians - as he put it "actual" differential geometers - would turn up to “observe”. Sometimes, although the lectures were tailored to graduates with a low level of knowledge, when a senior person arrived, Constantine explained to me that he and his fellow mathematicians felt that they had to "bump up" the rigor of the talk and include more detailed, complex information. As a result of this, the graduate audience could easily become lost and not follow what was going on in the talk.
>
> Constantine spoke of how mathematicians who came to the institute, although they were masters of their given field, also wanted to spend the time learning something new. Often though they would be too embarrassed to admit they didn't know a given concept in a different field, and so would not ask questions. They did however want to understand a little about that field, say physics for example. The seminars - on mathematical physics - were thus aimed to communicate certain ideas in physics to mathematicians, in ways they could understand. Deep knowledge of the physics was not required, according to Constantine. Rather what was sought was a motivation for the mathematicians to understand why certain questions could be interesting for them to study (Day 12, Germany-A).

The extract gives us a glimpse behind the scenes, and highlights some of the anxieties that mathematicians have about performing their mathematics. Short-cuts, short-hands, or "loose" language is left by the wayside, and instead precise definitions are provided, which often requires that the audience possesses a sophisticated vocabulary, in order to be able to fully grasp what is happening.

The chalk-board is the preferred medium of presentation because it allows individuals to adapt their exposition depending on the audience's questions, or level of understanding. The mutability of chalk allows for individuals to rub out incorrect statements, to elaborate on concepts, and to show their working clearly. The rate at which a presenter writes on the board also sets a pace which allows for the audience to follow along and make their own notes in parallel. The act of writing out and unfolding a presentation through chalk-work thus keeps the audience attentive and engaged, allowing them to return to definitions on a previous board, and re-trace their steps if they become lost. By rubbing out an entire board the presenter provides a much needed pause, which allow for both presenter and audience to take a break and collect their thoughts together.

Both at Germany-A and UK-A the act of presenting research is important in the social processes of institutional life. The initial presentation is a rite of passage which introduces researchers' thought and themselves to the group. Without such presentations researchers find it difficult to find collaborators, as the presentation provides "excuses" for individuals to approach presenters, to ask questions, and introduce themselves and their own research, as one researcher, Sarah, explains:

> SA: I'm very shy. I don't... even if I read a paper, I find a paper very enlightening, I don't usually email the person. I have a lot of colleagues who do, but I am much too shy. It's usually if I meet them. I have to meet somebody and say, oh, actually I have this question, it's a silly question, but... and that's how it starts.

Individuals make first contact through approaching presenters and asking "silly" questions, to break the ice. They then begin informal investigations into an individual's research, and attempt to establish common ground. Sarah goes on to describe the importance of giving talks, for the purpose of attracting potential collaborators:

> SA: At the beginning it was a little bit funny because when they invited me to come, they didn't invite me to give a talk, and I had to beg them to let me give a talk, and it was very strange. It was a very strange interaction, that is, they had invited me quite a long time ago - more than a year ago - to come,

> and I said yes, and then sent out a programme with the talks, and I wasn't on the programme and they hadn't asked me to give a talk, and I... and I found the interaction weird, and then I said, well, I'm willing to give a talk. If you have time, I'd really like to give a talk, because I know that giving a talk, people always come and talk to you after they've heard your talk. And until you've given a talk, they won't talk to you, and so it was very important to me the first week to give a talk... but I, sort of, had to struggle with that. They didn't... they hadn't programmed that... so that was a little strange. But the Institute, you know, the facilities and the way we have coffee and things like that, that was good. And the way of doing a week of talks at the beginning, as I say, that's the way to get to know people. They have to give a talk and then you know, oh, that person's doing this, that person's doing that.

The talk thus is a way of setting up shop, and displaying intellectual wares. In giving a talk presenters become open to talk afterwards, and to show that they are accessible and open to collaborating. The talk also allows for individuals to classify one another according to what they do, and what problems they are interested in. The talk thus is the starting point from which individuals can order ideas, by putting names and ideas to faces. Through the presentation, or "talk", presenters *gives face* to concepts, and begins the process of socialising themselves and their ideas.

**2.33 Behind the scenes: How performances are practiced in privacy**

By "socialising ideas" I simply mean that concepts become part of assemblages of people and things. Ideas become grounded in day to day life through their attachment to people. At the institute people speak of Sarah's technique, Penelope's data, Luke's theorem, and use these names to act as handles for searching MathSciNet, Arxiv.org, or google scholar, for related material by which to trace the origins of the ideas.

Knowledge in mathematics is embodied. Individuals act as repositories of knowledge, and many researchers prefer to ask a question directly to an expert, in order to find out more about a problem or idea, rather than consulting written material. The oral transmission of knowledge in mathematics is a means through which information contained in publications can be expanded and summarised, so as to provide the key points, motivations and overall structures of proofs. Interviewees highlight the importance of asking experts about a problem, paper or approach, which can serve a number of purposes:

1. As a time-saving device.
2. As a means of acquiring interesting techniques and problems.
3. As a way of getting up to speed in an area.
4. As a means to socialise and share interests in mathematics.

The following gives an example of why oral communication is important to one researcher, Nemo:

> N: So I'm not fast in oral communication. So I like to discuss maths with others, but at the level that is of motivation and general interests, questions. But when I want to think really deep, I need a pen, and I need some loneliness at some point to think through a question. When someone is watching me I am not so good at thinking in front of somebody. But I need communication and exchange about questions, in fact to be interested about a problem I need to understand why we should all care, as a community, about this question, not just "Oh it's a hot problem - you should try this".
>
> IN: So you use people to get a problem and interrogate them to find out why the problem is interesting?
>
> N: Yes I use people in that way, and also I see it as a cultural thing also. So I like to exchange, and see what the vision of mathematics we have…Doing maths with others is all about what questions we can work on between ourselves. It's not just "Oh lets go to the blackboard and solve a

> problem", it’s rather: "Oh have you heard of that, and look I can do this, describe some simple examples and so on". At some point exchanging around this, and maybe having a small idea or small example, and then afterwards have more quiet and just think about the natural places where these ideas should sit.

The informal, back-channels of communicating are important in shaping researchers’ interests. Informal chats in private offices, over coffee, or in common rooms, help researchers to disseminate their own research interests, as well as gain new insight or perspectives from their interlocutors. Many interviewees spoke of needing to "pick the brains" of experts, of "gaining insight" on a problem, trying different approaches to solving a problem, or else moving in different research directions as a result of informal chats with fellow mathematicians.

Often times the process involves an exchange of research interests, at a general, "motivational" level, before moving deeper and asking more specific questions, through probing each other's knowledge of a proof. Once a researcher understands how and why a given proof or construction is useful, they will go about the process of "reading around" a subject. Such "reading around" involves obtaining key words, names and references from their interlocutors, which act as handles for searching archives, personal webpages and research databases.

As many mathematicians explained to me, it's important to "know who is who and who knows what". They must know where to access certain information, who to speak to about a problem or proof, and who to follow to keep up to date on developments in a field. People thus are "repositories" of knowledge; they are living archives or databases, whose knowledge can be accessed through simply asking questions.  Below, Adrian describes how an individual researcher must learn their own limitations, and must extend their social networks so as to compensate for their lack of knowledge or expertise in a given area:

> AD: Of course it’s like pond life - I mean if you are in a pond you know everybody. But on the other hand the really good people tend not to be so narrow. I mean number theorists have had to learn algebraic geometry, algebraic geometers have had to learn number theory and topologists have to know everything. Not that I know everything being a topologist. But nevertheless I have to have some feeling, if I can’t do something I should know it or I have to be friends with someone who knows these things. I tend to use my collaborators who I talk to as repositories of knowledge. It’s very important to know who knows what…. You have to talk to people, you have to know what they can do, what they can't do. You have to have a feeling for the knowledge, for where the knowledge repositories are - there's a lot in published papers but that's not enough, you really have a little bit more of a fluid approach.

Adrian notes that reading the formalised, published papers is sometimes not enough. Rather a "fluid" approach is necessary, by which an individual can question a person who is a “knowledge-repository” on specific questions related to a concept or proof, in order to "get the bare-bones" or the "gist" of an argument. This is an important task to undertake before a researcher decides to fully commit to investing time and mental energy in reading a given paper. The informal back-channels of questioning knowledge-repositories serves as a means of scoping out potential problems or useful concepts, narrowing down the search parameters needed for later literature reviews and google-scholar/ arxiv.org searches.

*Figure 13 Over-the-shoulder work at Germany-A*

*Figure 14 Developing personal spaces at UK-A*

Such back-channels of communication are where the problem-finding/ problem-solving, experimentation and proof-construction take place. Behind closed doors, or in quiet corners, a great deal of one-to-one work takes place, at a blackboard or shoulder to shoulder around a sheet of paper. Prior to such engagements, individuals will spend time reading up on a mathematician's work, accessing their personal web-pages, or skimming through their papers on the

Arxiv.org website. Before any presentation, such preparatory work takes place. Individuals will ask what they call their "stupid" or "silly" questions before face-to-face encounters, utilising close friends or colleagues, or else will use the question and answer site MathOverflow.com. Then when it comes to asking the expert, or giving an exposition, one does so effortlessly.

Such displays of effortlessness are important in showing that a researcher possesses "natural" ability within their field, and thus they are important in developing their reputation as a competent, rigorous, trustworthy researcher. In public, much of the hard work of learning is made invisible. Through downplaying the effort involved in understanding concepts, so individuals create distance between themselves and their audience. Some individuals, I am told, are perceived as "naturals", "geniuses", "gods" because of their effortless ability to perform and demonstrate their understanding of mathematics. When many such "naturals" talk in private between themselves, or when being interviewed, the struggles involved in grasping concepts are revealed.

Individuals may talk of being "demoralised" by not understanding a presentation, or struggling to understand a concept after repeated attempts, or even of feelings of inadequacy of their cognitive faculties, in comparison to that of their colleagues. One example comes from a night when four post-docs and myself decide to tour the pubs of the city. It was fairly early in the evening when we settled in to one venue, and began chatting casually:

Andre snatched away a little booklet Paolo was holding.

"Hey give that back!" Paolo says, but Andre is already flicking through. "Derived categories…" Andre mutters "I never understood this".

"Now is your chance" Paolo smirks, as Andre flicks through.

"I was wondering what this was" Andre continues, "you've been cradling it all day and this was my chance to nab it".

Paolo says that he needed to learn it. Already he had gone through it in detail, reading and re-reading the text. Indeed the printed booklet had obviously been flicked through multiple times, and it was covered in notes, under-linings and coffee stains.

"Tell me what you know about it" Paolo asks Andre. Andre is a little flustered and gives a really high level overview, Paolo laughs in response.

"You shouldn't bother going to lectures! What good will it do you when that's all you know? You need to read this. From the first chapter" Paolo says, as Andre is on Chapter two.

"Hey, I know that stuff" Andre retorts, looking a little hurt. Paolo is insistent. Paolo continues to mock him and asks if Simon can help explain to Andre what the topic is about. Simon backs away and says that he's only used it once to prove something.

"I could, but I wouldn't be able to do it well" Simon says shyly. Paolo persists on the topic, poking fun at Andre's lack of knowledge for the next ten minutes, making most of us at the table uncomfortable. Andre is downcast as he sips on his beer, flicking through the battered little book. Eventually Paolo seems to become aware of the change of mood, and he moves the conversation towards Perelman's proof of the Poincaré conjecture, telling us why it should have been more widely publicised in the popular press.

"It should have been written out in full on the New York times!" Paolo cries, we all giggle, leaving Paolo to continue the rant (Day 26, Germany-A).

The above anecdote reveals the tension between the formal and the informal. In this instance, what Andre had considered an informal moment of play or light-heartedness, was transformed by Paolo into an opportunity to interrogate Andre on his knowledge, and make visible Andre's lack of knowledge. The mood around the table changed, from one of intimacy to that of distance, as Paolo began to perform his knowledgeability, and challenge others on what they knew.

Such "outings" of one's lack of knowledge are not uncommon at tea-times at Germany-A, when in groups of four or five, or during chalk-talks. They are less common during informal beer-nights, or when senior mathematicians are present, but, none-the-less, the public and the private do sometimes intersect, leading to the backstage becoming the frontstage, and individuals finding themselves having to perform their knowledge in front of others. Such pressures to perform contribute to many mathematicians preferring to avoid tea times at Germany-A altogether, as well as avoid discussing mathematics with colleagues in common rooms, for fear of being "outed" as being ignorant of certain papers, problems or proofs.

**2.34 Setting the stage: Constructing public representations**

It is because of this pressure to perform and the fears around visibility, that it becomes important in investing in backstage and backchannel communication networks. Keeping the backstage private and invisible is achieved by filtering out those individuals who are anti-social and overly competitive. To facilitate such filtering, a researcher needs to invest in socialising with other researchers. Through socialising, individuals get a sense of other people's research interests, their degree of shared interests, their personality, and whether they are "compatible" for collaborating with. Socialising in common spaces, around coffee machines, or outside of the institute, thus are ways of building rapport and trust between researchers. Such rapport is important in reducing the formality of interactions, and trust is important in being able to more freely exchange ideas during informal presentations.

When inhabiting foreign spaces, such as institutes, individuals must begin the process of domesticating these new spaces, making them their own through personalising their surroundings. By decorating space, furnishing walls, desks, office doors or notice-boards with their own things, according to their own tastes, so individuals appropriate their office spaces, making them more "at home", more "domestic" and thus more informal.  The pictures below contrast an office space within an institute, which has relatively low levels of personal investment, compared to a private office at a mathematician's home department, which is fully personalised according to their own tastes.

*Figure 15 Shared office, Germany-A, demonstrating lack of personalisation*

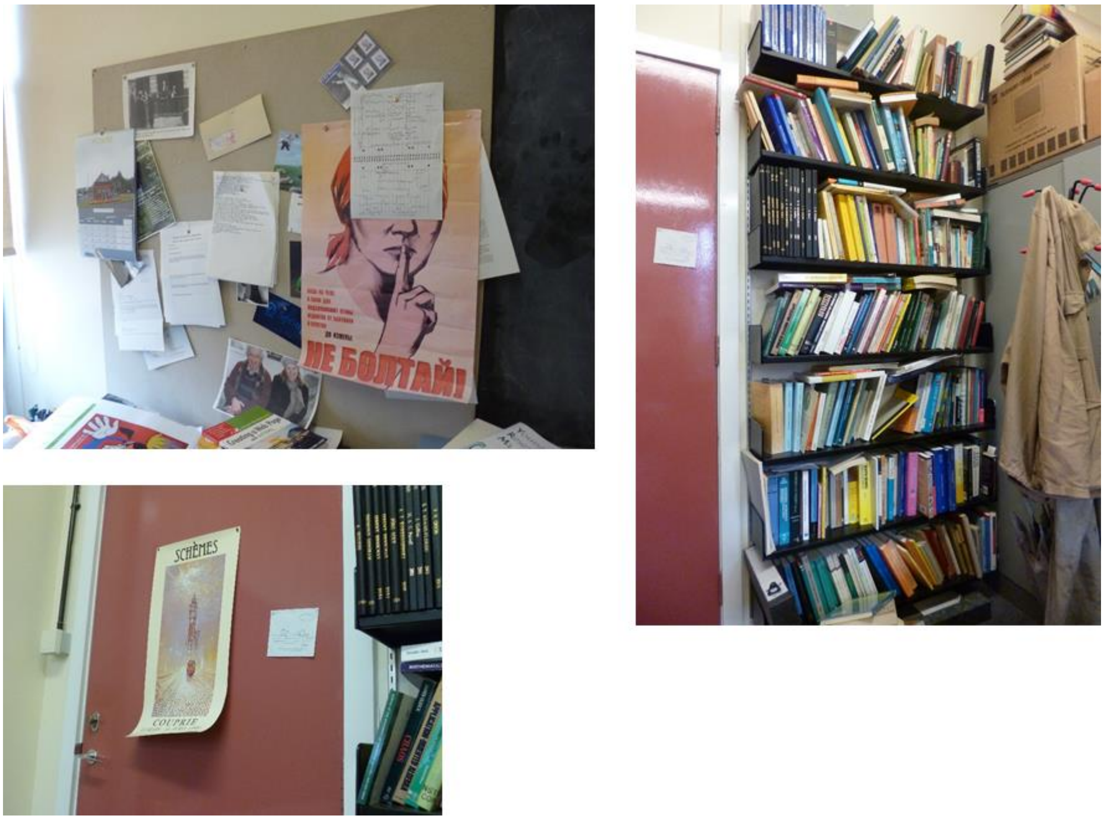



*Figure 16 Personal investment in departmental office space, UK-B*

Individuals entering as visitors to institutes, once they have acquired their office, will only hesitantly begin the process of personalisation. The reasons for lack of investment in space, at both UK-A and Germany-A, are that individuals will often share office spaces, and so their spaces are not completely private. The short duration of visits, from one month to one year, also means that there is less likelihood that they invests in modifying their working environments. Institutional administrations themselves can also discourage personal investment in space, stressing cleanliness of workspaces, and a desire to minimize clutter and untidiness. This is especially important within institutes which are accessible to the public, where untidiness of spaces is viewed as a sign of disorganisation among the administrators.


*Figure 17 Biographical information outside of personal offices, Germany-A*


*Figure 18  Personalising space and displaying the self at Germany-A*

The socialisation of space, however, does occur to varying degrees, through the habitual occupation of certain common spaces; through the personal ownership of chalk/ white boards throughout the institute; and through the furnishing of office doors and notice boards. At Germany-A the notice-boards outside of offices have short research biographies posted outside them. Such biographies contain information about individual's research interests and the areas they're looking to collaborate in. They may also contain a small picture of the researcher and an example paper, result, or favourite visualisation that the researcher wants to show-case. These notices can be as detailed as the researcher likes, and in some cases may sprawl into several pages of text. Such personalisation can thus quickly become another way of presenting one's ideas, and performing one's abilities and achievements in mathematics. Many PhD researchers may thus become intimidated by such displays, with their biographies reading simply: "Come inside and find out more!"

The degree to which an individual feels at home in a space, that is to say the degree to which they domesticate a given space, is therefore dependent on how comfortable an individual is in performing their mathematics, and engaging in such displays of knowledge. For many researchers, displaying their ideas is a way of being "outed" as ignorant, and so many are naturally reluctant to reveal too much about themselves in public. Other researchers are more adept at communicating their research, and more confident in displaying their intellectual wares. For the less confident researchers, their strategy of sharing information relies on developing informal relationships through socialising, and then sharing information, one on one, during private exchanges.

*Figure 19 Small group exchanges, UK-A, built up by the end of the 3rd week.*

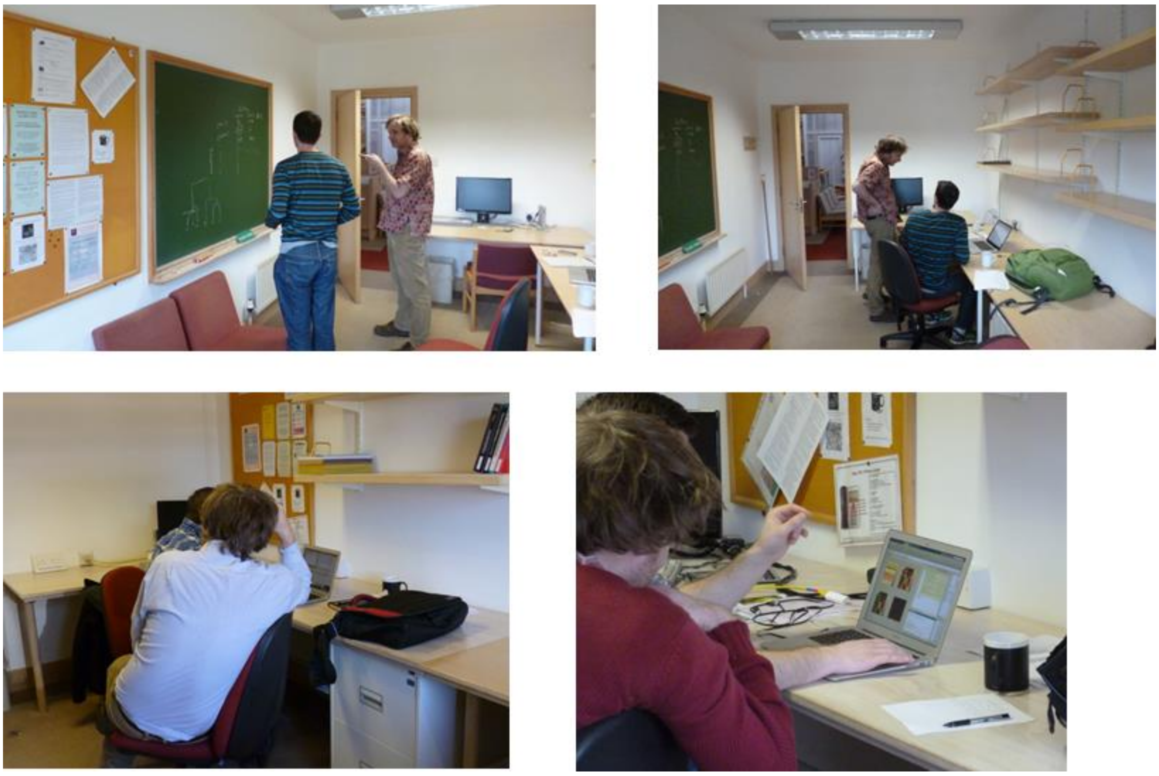


*Figure 20 Close work, in private spaces, UK-A*

Most individuals interviewed stressed the importance of trust, friendship and rapport in creating informal spaces within which they can test ideas and communicate their thoughts. Many commented that collaborators were also friends, highlighting the importance of friendship in making mathematics a pleasurable activity. Oftentimes research can be a lonely, isolating experience, but the ability to communicate, socialise and work at the same time makes collaboration an attractive alternative to solo-work. As geometer Jason explained to me, the social aspects are his main motivations for creating collaborations:

> J: Yes So I really enjoy collaborating, and it is absolutely fundamental for me. So maybe the only time that I did not collaborate was during my PhD. And as soon as I discovered this interaction I completely decided that it was my way of doing mathematics, and it's definitely changed the type of questions that I'm interested in, so when I really need ideas and motivation I really need to find someone to work with. I had a lot of such collaborations, maybe, well I never counted, but maybe 10 people. And obviously they provide very different motivations and change the kind of questions that I'm interested in. And sometimes I even choose to work with them on their problems, this can happen. And usually my pleasure is, that when I've understood something, I will go to the office of my collaborator and explain it to him. It is the way I like to do mathematics. For instance it's difficult for me to have long distance collaborations, I really need people to be here. I did it, but only after working with the person. Afterwards we need to fill in the details, finalise the project, but the active part of the collaboration was really together.
>
> And another part you may be interested in is the fact that all my collaborators are my friends… I believe that doing maths should be a pleasure. In some sense this is broken if you don't have a good relationship with someone. I don't know how it works…My motivation when we start collaborating, we spend a lot of time at the blackboard saying: "I don't know how to do that, I don't understand that" - and usually when I come back to my office if I have the motivation to spend a lot of time trying to solve this specific problem because I want to help, to give this pleasure to this collaborator. It's really a kind of service. For that I want is to give something back, you understand that?

Above we see how collaboration becomes more than just seeking new ideas or problems, or increasing productivity through co-authored articles. The collaboration sometimes is not simply a means to an end - the production of papers or grant proposals - but rather it becomes an end in itself, through the creation of friendships. Jason speaks of the pleasure of collaborating, and of how producing papers serves to strengthen the relationships with his collaborators. Jason describes how his motivations to solve a collaborative problem are not necessarily because of a need to produce papers, but rather to strengthen the bonds of friendship between his collaborators. Such collaborations, he argues, are based on reciprocity, and the exchange of gifts between himself and his collaborators.

The case above stresses the importance of the inter-personal, informal, aspects of knowledge production in mathematics. The emphasis is on developing friendships and furnishing the private, back-stage world. Here researchers are motivated by working with others and building friendships, rather than just publishing to survive within the competitive world of mathematics. Of course the interviewee recognises that the pressures to be productive do exist, and that such pressures do force an individual to collaborate with people they would not consider to be friends. However sometimes such collaborations are unsuccessful, as Jason tells me in the following:

> J: I cannot imagine having collaboration without this very close relationship. I had once a collaborator who didn't work out because of an incompatibility of character, or something - it just didn't go well - it was a very bad moment for me and I really saw that it was impossible for me to do maths with somebody that I wasn't really friends with. But I have a lot of other friends who are mathematicians
>
> I just had this bad experience and I had to just stop completely. And I had to say “no”; and it was hard for me to continue, and I had to say to this person "I'm sorry but I just can't go further". Well we had this small problem, but maybe for him it was something he could put to one side and go on working, because we had a lot of things to do, but for me it was too hard. I really noticed that I couldn't collaborate anymore, and I had to say to him directly, because he didn't understand that. It was very hard but I decided to go and say "I'm sorry but…" maybe I didn't really say the exact reason, but I had to say to him directly that I had to stop the collaboration. It was really, completely emotional in some sense, nothing to do with mathematics. And at the same time collaboration, scientific collaboration is much to do with relationships, maybe.

Here the reason for collaborating was oriented towards publication, rather than the fostering of social relations. Ultimately the market nature of the exchange conflicted with Jason's social need to create friendships, leading to the termination of the collaboration. The pressure to be productive influences many individuals in their decisions to collaborate. Often the case is that collaboration is undertaken not out of pleasure, but out of necessity. Collaborating becomes a strategy for producing more articles, and fulfilling certain funding criteria, which often succeed through using key words such as "interdisciplinary" or "collaborative" in their proposals.

Indeed the rhetoric around collaboration, and the focus on measuring one's productivity, through successful grant applications or published articles, leads to "collaboration" being valued not as an end in itself, but rather as a means to produce "collaborative" publications. The field thus transforms this informal, private, gift exchange into a much more visible, public, market exchange of ideas. What had once happened spontaneously, in a researcher’s home department, in the home, or around a coffee, beer, or lunch, now has become operationalised within institutes as a means of making the process of collaborative knowledge-production more predictable, efficient and measurable. In this movement from the backstage to the frontstage the field of production and competition becomes visible. The seeming disinterestedness of knowledge-for-knowledge’s-sake gives way to the self-interestedness of the game of capital pursuits. The visibility of ideas thus makes knowledge public and thus transforms it into a form of social capital, a status symbol, whose possession leads to a repositioning of the individual within the field of production.

**2.35 Creating common ground: Negotiating performances of the self.**

Ideas are becoming ever more global, visible and accessible since the advent of the internet. Researcher productivity is likewise being subject to measurement, with research output itself being more easily measured. With increasing information on researcher outputs: from lectures given, collaborations formed, awards obtained, conferences attended, or papers produced, so individuals are more easily compared against one another, and ranked as a result. The by-products of this visibility, accessibility and measurability are that individuals are better able to compete against one another for scarce resources, such as grants and research positions. A researcher's intellectual and professional life is increasingly available for public consumption, and thus individuals feel a need, or pressure, to perform their mathematics. But this increased visibility and competition is of course not completely negative, as Benjamin comments below:

> BN: You can now, for example, 20 years ago, when you heard a name, unless they were very famous, like super famous people like Atiyah, Grothendiek, Fields medal winners, of course you would know who they were, but of course there were people who were lower down the foodchain, you really had no idea what they did. Now you can go to mathematics reviews and you see everything that they did, or use the Arxiv. So its enormously competitive, so that when you hear a name you don't know, and you want to know what they did, you can find out. And it's very good. It's more competitive, but I mean in a good way. There's much much more information. So if I want to talk to somebody who knows x, y or z then I can. In the old days this was a big problem, you had a graduate student and you gave him or her a problem and they worked hard at it, and they did it or didn't do it or whatever, and then you find out that someone somewhere else has done this - this cannot happen now. I mean it really is the supervisor's fault. In the old days you could be ignorant - there was no way of finding out who wrote PhD theses - now you can of course hide from the internet, if you want to. But by and large, as I say to my students, the first thing you do when you write your thesis is that you post it on the internet, the second is that you publish it in a journal.

Benjamin highlights that this increased visibility does make the field more competitive, but such competition has positive aspects. Visibility, through the internet, makes the once invisible work of some mathematicians more accessible and visible through search engines. Individuals can now better understand where in the field they can make a contribution, and can measure their impact on their field and on other researchers, through measuring citation counts, or other bibliometric data. Individuals can access potential collaborators through googling their names, or else obtaining information on personal web-pages, or on reference databases. The field of competition and display thus is moved from local contexts, to become global in character. Such a move towards globalised knowledge-production and competition presents opportunities for some individuals to perform their knowledge, but disadvantages others who may be less confident, or less willing, to present their research, or those who are more modest, or introverted in disposition.

Many see the mathematical field as a competitive field, a battlefield, where they must compete against other researchers for scarce resources. Researchers are taught earlier on in their careers, during their PhDs, to have papers in mind for publication, to be on the look-out for potential collaborators, as well as to hone their performances during chalk-talks. Increasingly, as information flows freely through blogs, forums, social media, friendship networks, through supervisors, or through institutions, so individuals become habituated to becoming expert performers of their knowledge, as well as expert strategists in planning their professional development.

Such an awareness of the field appears to be the result of the visibility and survey-ability of knowledge-production within the community. Among the researchers interviewed, there is an awareness of their place within the community, of who their competitors are, of who knows what, and what actions they need to take to develop their careers, connections and knowledge. The very fact that the individuals were present at the institutes marked them already as having found success along their mathematical career paths. Such success, however, was achieved through individual's own personal negotiations with the field of production, through their own personal strategies for managing the ways in which they present and position themselves and their knowledge.

The presentation of self within mathematics institutes provides us, as social scientists, with some insight into the day-to-day processes by which knowledge is communicated and constructed. We see that not all performances are designed for public display. Rather, successful performances are based upon balancing action within the public, frontstage, with the private, backstage. Both these theatres for action are important stages upon which social actors can perform their identities. It is through the assembling various spaces of institutes into stages for performing mathematics that individuals carve out a niche for themselves and their ideas within the wider mathematical field. Within institutes we have seen how individuals adopt and adapt their surrounding habitus to their own personal needs.

At the institutes we are provided with microcosms for studying the different realisations of mathematical habitus. We come to see that institutional spaces do not simply imprint their socio-cultural scripts upon the individual, but rather the process of domestication of space is dialectical in nature, and negotiated through practice.

Individuals assemble spaces for their own needs, personalising them, investing them with meaning, constructing the backstage and frontstage, producing formal and informal spaces according to how they wish to present themselves and their ideas. The processes of assembling space and performing one's self and one's knowledge within such spaces are selective, experimental, and evolve over time. Certain stages are important earlier on in the process of concept development, such as the informality of the backstage, whereas, as concepts gain their strength, so they are tested in more public arenas, such as during chalk-talks, which then can provide opportunities for growing a researcher's social network and increasing their reputation and esteem (social capital) amongst the community.

The path from problem to proof thus is a process of negotiating the formal and informal, the visible and the invisible, the public and the private. The assembly of physical spaces is an important part of this negotiation process, as is the assembly of a researcher's social network. The institute provides a common space within which the mechanisms for knowledge production can be assembled together; the institute enframes the theatres of performance, the actors and their activities. We see that the sites, the physical spaces and architectures of the institute, shape mathematical production.

We have also seen how dispositions are generated by inhabiting and domesticating space, and how movement between public and private spaces alters the presentation and performances of a researcher's ideas. But the questions that I want to address, in the following chapter, will be around how these ideas of the frontstage and backstage, the private and the public, the visible and the invisible, shape the conceptual spaces which mathematicians inhabit. I want now to move on to a discussion of how the processes of crafting and domesticating conceptual spaces bears certain family resemblances to the ways in which physical space is given structure and order. In the next chapter we move from the performance of mathematics, to the ways in which it is perceived and represented.

# Chapter 3: Conceptual Spaces

## 3.0 Overview

Here I establish how mathematicians position themselves within conceptual spaces. I chart out how perspectival frames are generated through becoming socialised within the mathematical field. The perspective a mathematician adopts depends upon their position within the field of production, which are products of education, social relations, and possession of social capital. Such positioning shapes individuals' orientations towards phenomena, and thus shapes the reference frames by which mathematical phenomena are construed.

I argue that perceptual frames are crafted through a mathematician's situated engagements with mathematical problems. Through such engagements mathematicians construct frameworks of reference objects, by which to orient their perspective of the field. These reference frames serve as bases for differentiating, relating and categorising mathematical phenomena. Operations of position, differentiation and integration are used in perceiving and classifying mathematical spaces. I shall argue that the schemas used in organising perception of conceptual spaces are analogous to those used to structure physical space.

## Section 3.1: Shaping the Conceptual Habitus

### 3.10 Overview of Section

The following section explores the ways in which mathematicians become habituated to thinking about mathematical objects and spaces. Using interviews taken from across my field sites I will analyse how mathematicians gain intuition about mathematical phenomenon. I will study how this intuition is built up through becoming familiarised with certain viewpoints, techniques and canonical examples. We shall see how PhD problems serve as starting points for exploration within mathematical spaces, how such PhD problems are inherited from supervisors, and how these initial problems go on to shape a mathematician's later research interests.

### 3.11 Introduction

Much of a mathematician's day to day work is invisible, not subject to measure, not counted as productive, nor articulated as important. This invisible work takes the form of emails sent and received; conversations with colleagues or students; references written; applications sent, received, reviewed; meetings attended; lectures given. Such work falls to the background of the day-to-day, and yet even more work remains hidden from view: in the hours spent in silence, thinking; in the sketching of a figure on a scratch-paper note; in the endless figuring and erasing at the blackboard. Within departments, within institutes, there is much work that is not measured, but still is essential to producing and disseminating knowledge within mathematics. The contours of the working day thus change as the mathematician balances competing demands: a meeting, a lecture, an email. As differential geometer Ivan says below, there are no average days, but within such busy working days there are moments of escape, where he can turn towards mathematics, not only as work, but also as leisure:

> I: So my favourite periods are the periods when I can just think, sit down and think peacefully. Well it happens, but sometimes I need to write something. In particular it can happen that I need to write down some parts of an article, in this case I just try to write, but there are also other things that one has to write from time to time, there are also some recommendation letters, other letters and other administration duties, so it really depends on what I need to do at any particular time. From time to time I need to, after exams, I need to grade. And this also takes some time. It's probably, for me, it's an important advantage of this profession in that, in a sense all the days are different. I don't really know what my average day is.
>
> There are many various things that should be done and I try to mix them somehow, I try not to spend too much time on my emails, but on the other hand if I don't look at my email box for the whole day, then the next day, at least at the beginning of the next day is a nightmare as I have 50 messages to treat. So I don't know how to answer the question of an average day, and I think it's a good thing. I don't really like the idea of some typical day. And well from time to time I need to work more intensively because when I have some idea I prefer to think and not to be disturbed by other things. In this case I can think more or less in any place, it's not important for me to be in my office, I can think at home, it's completely OK. From time to time, when I'm tired and something doesn't work, [phone rings] I'm sorry.

Above we see the burdens of invisible work, as well as the ways in which Ivan balances the competing demands on his time. Only a small fraction of the narrative relates to the writing up of a paper, the rest is devoted to administrative work, and most importantly to the work of "thinking". Such knowledge work is again mostly invisible, and so it is difficult to measure, but it forms the basis of work as a researcher.

Research consists of reading and writing papers, books, proofs; attending and giving lectures; creating and giving presentations; discussing and collaborating with other researchers; exploring and experimenting with ideas, as well as generating problems and solutions. To seasoned mathematicians and researchers, these qualifications of what research entails may appear obvious. These activities have become naturalised into one's work routines and they have ceased to be pondered, or spoken about. But for a graduate student the meaning of "research" is not yet fully understood. What it means to experiment, find a problem, produce a paper, construct a proof - all these constituents to mathematical activity are known in principle, but are alien in practice. So how then does a mathematician learn to produce mathematics, and to think "mathematically"? We shall explore this process of learning to become a mathematician by attempting to understand the means by which graduates learn the tools of the trade, and become familiarised (socialised) within mathematical spaces. We start at the beginning, in the act of finding a good supervisor and research problem.

### 3.12 Learning to see: How are research questions found in mathematics?

The first stage in a graduate mathematician's career begins with selecting a supervisor. Oftentimes such supervisors are chosen on the basis of recommendations from other students or professors; through google searches on departmental websites; through familiarity with published works or previous teaching experience; or through simply random choices within a general area of interest. Another guiding factor in choosing one's supervisor is the degree of familiarity an individual has with a given area of mathematics. They may have enjoyed classes or readings in algebraic topology, analysis, or arithmetic geometry etc. Such preferences help define their choice of research area to work in. But, as is often the case, graduates do not go into their graduate degree having concrete questions or problems in mind. Rather their level of understanding of a given field is rather limited, and thus a concrete and interesting problem is not yet within their grasp.

Because of the graduate's limited vision of the field of enquiry, the supervisor must help guide the student's research and their interests in certain directions, in order to narrow down their research questions. According to the mathematicians interviewed, narrowing the field of possibilities can take the form of providing "interesting" papers, or, as is mostly the case, of directly gifting a problem to a PhD student. Such a gift helps orient the researcher within the field, and puts some limits on the boundaries of their explorations. The limited vision of the young researcher means, as Martin says below, that the PhD must “advance in the dark”, trusting that their supervisor has furnished them with a concrete, precise and interesting enough question:

> M: When you're going to start a PhD in mathematics it has to be very specific. If you are going to start a PhD in pure mathematics someone starts by saying that this is an interesting question, and you have no way of knowing and understanding "what does that mean, what is that question?" You have to advance in the dark.
>
> In order to do your own research you have to understand what the state of the art is. And this can be Impossible. So that’s the kind of contradiction to start with, you don’t even know what people are doing, so how can you contribute?

Due to the rapid pace of developments within mathematics, and the abundance of publications, a young researcher can quickly become overwhelmed by the vastness of the intellectual landscape. For Martin this vast landscape is conceived of as a desert, which lacks familiar landmarks, is hostile and dangerous:

> M: Because, of course, after a while, it may be typical of the PhD, but you have to cross the desert, and once you've done that, you have some experience, and you have your own questions. And in the end the rich thing that you've developed are that you have your own questions - the questions that came from you.

Crossing the intellectual "desert" is an act of taming or domesticating the foreign problem that the researcher first confronted, making a once strange and alien landscape habitable and familiar. Through this intellectual struggle, Martin describes how he gained the skills and experience to find his own problems, and survive within the wider mathematical world. Martin below describes the experience of choosing his thesis advisor and receiving a PhD question:

> M: Knot theory is kind of beautiful, but also a specific field of geometry and topology, and so in the lab where I did my first year before I had to choose a PhD advisor, there was a Romanian guy who was extremely vague all the time. Kind of vague about many things, and in many aspects had ways of thinking about mathematics which are opposite mine. As, in order to understand something, I need some precision of mind. But he had obviously a very broad knowledge of what was going on, so I went to him and said "Please give me a PhD question", and he said "OK, so here is your question!"
>
> And I had no idea of what that meant, what the picture was behind everything, but I had the trust that he knew what kind of things I liked, and so there was the confidence that he could give me a question that would fit somehow. There is a lot of danger in this procedure of choosing the right PhD advisor. There is no solution to this problem, you just have to try and see what comes afterwards I think. And when I started to understand the question years later, then it turned out to be a question that I liked very much. And it was successful enough to get me a position.

The quote above highlights the degree of trust invested with the supervisor, as the student does not have sufficient knowledge to evaluate the quality of the questions gifted to them. It is only years later, towards the end of his PhD, that Martin recognises the problem as being a good one to have tackled. But for other researchers there are many dangers associated with being gifted a problem, as Martin recounts:

> M: Sometimes I still meet regularly my PhD advisor and we were talking about someone of my age, comparing careers, and everything, and he told me: "Oh this guy never got out of his PhD question", and I told him: "Come on, you know I never did either!", and he said: "No, no you did things completely different" -that's very subjective.

Some students in mathematics are unable to find a solution to their PhD problems, and thus are unable to complete their studies in mathematics. Martin too recalls how he could not directly tackle the question he had been given, after he discovered the history and difficulties in developing a proof for it:

> M: I did not answer this question [laughs]. Because I couldn't do that. First of all I started to try and understand what was already known about this guy, and this goes back to Poincaré. And there's a lot of literature to read about this stuff, and there's an enormous amount to read. And this guy has a finite number of connected components and one of these connected components is called a Teichmüller space, and it's very classical. And actually the question of my thesis, which I didn't know at first, was to try and understand to what extent a Teichmüller space could be transposed to the other connected components… So I started out - I don't even know why - but to understand if for that very specific thing I could extend something to the other connected components, and actually at the end of my PhD it was bad, because I had no result and my PhD advisor was not happy any more.
>
> M: In the end I tried to prove that something you could say about Teichmüller spaces could be said about this space. And I used something relatively simple, that would be simple to prove. And I could get my PhD and have it over with. So I took this property and I tried to prove that it could be extended. So I took a plan, propositions, etc. to make a proof. And then I had something, and I was requested to do talks at conferences. This was

really the end of my PhD, so I had to hurry and write something. And then I started to sit down and prove the lemmas, and realised that there was a major gap in the lemma that I thought was true. And this gap became bigger and bigger and gave me counter examples to the whole thing. And so this very reasonable thing that you thought "OK, this is going to extend", seeing that this does not is so surprising that, in the end, it made a very interesting PhD; and now look, I'm here! [At the institute]

M: This happens a lot in mathematics: that you think that something is true and you start to write the proof and, at some point, you see that there is a gap, and a counter example to the whole thing might very well come from this gap. But this was a big piece of luck… This very first question is still open, and I had no idea before starting my PhD, but the very same question has recurrently been the question of many PhD students, starting from the 80s. All these people who tried to solve the same problem, they all did something else, or maybe they didn't do anything. But yeah, that's a good problem, in the sense that it's asking about something that we don't really understand. But we understand it enough to be able to ask a lot of questions, and some of then we have no idea, and that's a good source of research because that means that's a good place to dig.

For Martin the initial problem he started with was not the final problem he ended up proving. This is indeed common among many research mathematicians. Through the process of engaging with a question they discover different problems, new approaches, dead-ends or counter-examples. Their problem evolves as their knowledge of mathematics matures. As Martin experienced, starting off with a large, open question, he eventually developed a simpler, more specific question, which related to the original problem. When first approaching the problem, the researcher is somewhat naïve, and does not have a sophisticated enough understanding of the mathematical landscape to see the full complexity of their problem. But, over time, a researcher's vision matures, and they are able to see the relatedness of their problem to other problems, and to other objects within the mathematical landscape. This "double vision" is highlighted in the following quote from Martin, when asked to explain his PhD research problem in detail:

> M: So that's math. Can I use the board? [Erases board]. So I can try to tell you, from the perspective of the student, what the question was; or I can try to explain from the viewpoint, now, what the question is. They are different - completely. It's because when you ask a question, if it's not too naive a question, then there are many connections to many problems. If you have no idea, then it's just a bare question. Once you understand the relations, which requires you have read the literature, you see this from above. Then one question is linked to other similar questions which, to the student, are very different questions, but which are essentially the same question.

As the quote above suggests, there are multiple perspectives from which to view research problems. A mathematician's vision of the problem changes as they learn a given mathematical language, and learn the necessary vocabulary by which certain phenomena can be discriminated and discerned. Through developing their mathematical vocabulary, they are able to "see" certain objects as belonging to certain conceptual categories. By being able to identify such objects, the researcher is then able to relate them to similar objects, which are already present in their classificatory system. The researcher is then able to "see", or perceive, objects or problems as belonging to certain categories, because of their shared sets of features, or family resemblances.

### 3.13 The guiding hand: What influences research directions?

The ways in which mathematical objects are perceived are shaped by the mathematical tools, objects and literature that a mathematician is exposed to during their research life. This corpus of ideas forms the backdrop against which their research questions are given meaning and related to. The ways in which a researcher confronts a problem is thus mediated by the conceptual tools and vocabulary that they have acquired throughout the course of their education.

This mathematical corpus forms part of the habitus: the system of scheme-resource sets which produce orientations and dispositions towards phenomena. The conceptual machinery that a mathematician acquires thus serves to produce a certain way of perceiving mathematical objects and spaces, and also generates dispositions by which they are able to confront problems within certain mathematical domains. Through becoming familiarised with certain techniques, objects, languages or papers, mathematicians furnish their habitus, and this, in turn, re-shapes the perceptual lenses by which they are able to perceive a problem.

For a mathematician to be able to discern features in the mathematical landscape, they must first be able to give structure to their perceptual stimuli. These structures are provided through the mathematical languages they learn, through the rule systems they adopt, and through the exemplar objects they use to ground their understandings. Mathematicians spoke to me about the importance of having examples, which ground their understanding of a theory or proof. Such examples provide concrete demonstrations of how a proof or technique works, or what a given object "looks like" or "relates to". For a PhD student these exemplar objects form part of the working vocabulary they will need to solve their problem. The first examples they grasp form fixed points, or landmarks, within the landscape, which anchor their vision of the field and allow them to have some bearings within their surroundings.

For many PhD researchers the supervisor provides much of the motivation and conceptual machinery for tackling a problem. Some advisors have a more laissez-faire approach to supervision, encouraging their students to explore and think independently. Other, less experienced researchers, however, require more guidance, as oftentimes they may come from outside of a specific research domain, and have not yet developed a working vocabulary adapted to tackle their PhD question. The advisor, in this case, takes a more active role in the student's research, providing papers, problems, and explanations to guide the researcher's enquiries. As one PhD student, Jenny, tells me:

> J: So I didn't really have this knowledge before. My advisor - I think you met my advisor - so he knows a lot of things, so he explained to me a lot of things in topology and dynamical systems. So in these kind of things there are good books to read. So actually it's really hard for me to read math, and so it's good to have somebody who can explain stuff. That's why my advisor is good. But there are a lot of things that I don't know. Sometimes he's telling me that I should read that and I print it and I think - "uh-huh" - and then you just look at this thing for 2 hours and it turns out you've just read one page.
>
> J: So yes he told me to read this one article and said I would understand things. But it was not well written, it was something like 30 pages. He said "OK, so if you read it in the first year it will be good for you". I said "30 pages in one year?…yes I can do that [laughs]". But this was really hard. For every word you have to understand a lot of things. You can read that, that, that, that and you just get used to the different notions.
>
> He had a lot of time for me, so I could just discuss about things, and he explained a lot of things to me. It's easier to read when someone explains the story before. I mean you don't have the details but you know where you're going, and you just fill in the details when you read.

Here Jenny explains how her advisor influences her research directions, by providing exemplar proofs and papers containing certain tools and or objects related to her research interests. The advisor, in providing such material, creates anchor points in the mathematical landscape around which her future explorations will be oriented. The 30 page paper she speaks of provides a concrete starting point, from which she can begin building up her mathematical vocabulary. Every word needs to be unpacked and explored, in order to understand the bigger picture that is being discussed. Through such an unpicking and unpacking process, she can start to familiarise herself with the mathematical landscape she finds herself in. Through the advisor providing the "story", or the motivation, behind the paper, giving summaries and explanations, Jenny is able to orient herself and knows the research direction she's going in.

This form of guided discovery in mathematics, is mirrored in another interview with a PhD student called Marie. Below we see how Marie's supervisor (Estevan) introduces her to a new domain in mathematics called "Contactology", and how, eventually, she is able to build up her own vision of the mathematical landscape, developing her own questions and interests in the process:

> M: I think, in principle, he wants me to be free and encourages me to do what I like. It's not over-protective, but he holds my hand all the time. I can see him every day in a week for discussion - he's really present, you know. I don’t know if he's really orienting me, but he's there to speak about anything I want to talk about. It was a little risky, Estevan was not in the same domain that I was working in when I started my PhD - I was doing algebraic topology and knot theory, so I didn’t know about contact topology, or differential topology. So for the first 6 months of my PhD he taught me everything about that. So I didn’t know any of the questions in that domain. So he helped form that domain and presented to me all my objects and questions, and now they're natural for me. He presented the questions by himself and others were coming from me. Maybe I'm able to formulate an interesting question just now, over this one year maybe, before it was always suggested by Estevan.
>
> M: I wasn’t saying "Why is that interesting?" I was careful, I didn't know him - the student must be interested all the time. Now I'm asking questions all the time, saying "why am I doing that?" Before I was not asking questions, because I did not want him to think that I am so stupid. So there

> were many complexities and I didn’t want him to think that I was so stupid. Now that I know him, I don’t mind asking stupid questions. Even if I try to hide it, he knows that I'm stupid, so it's OK. The manner to catch me was through the blackboard, and through the discussion. I need to have a person motivated to present to me the ideas and the drawings. Really he was able to draw contact topology, and so I came into the contact world through the drawings, because the formulas are absolutely not able to speak to me.

The above interview with Marie also shows the evolution of the relationship between student and advisor over the course of the PhD. Marie above starts off as a passive receiver of information, simply learning the language and absorbing the tools that she has been given. However, as the PhD progresses, and she gains familiarity over the tools and the domain, she is better able to interrogate the information that her advisor is providing her with. As she becomes more confident so she becomes less fearful of appearing ignorant and, instead, more actively wants to interrogate the objects and the supervisor himself. Through both the student and advisor inhabiting shared conceptual spaces they are, over time, able to establish some common ground by which to discuss shared objects. Marie, below, discusses this process of building up her "tool-box" of concepts, and expanding her tool-box according to the needs of the problem at hand:

> M: I can learn a language if I need it. I learn the first toolbox to survive, and it’s the same with mathematics. I learn a small tool box to survive and then I play with it. But to really master the discipline or master English, that is another step for which I need another strong motivation to go there. I don’t need this motivation in English because we speak a lot of French here. And I don’t have a strong motivation in symplectic geometry either, because I'm still satisfied with my little tools. I'm happy like that right now.

Here we see how Marie's vision of the mathematical landscape is expanded incrementally, through "playing around" with a small set of tools and objects. Her motivations are not to develop a big picture of the field of symplectic geometry and how it relates to the wider field of differential geometry, but rather it is to equip herself with tools by which she is better able to explore her given objects, with finer and finer detail. In this case the wider field falls out of focus, and it is the specific features within the landscape attract her attention.

**3.14 Shaping perception: How are habitus constructed?**

This question of what to attend to in the mathematical landscape is a question again of perception and discernment. In the case of the relationship between advisor and student, the advisor is teaching his students how to see features in the mathematical landscape. For Marie and Jenny, their objectives were to develop a working vocabulary by which to discern such features, and to speak about them with their supervisors. In the day to day life of research mathematicians, this need to establish a working vocabulary is equally important. As Martin tells me, researchers need constantly to communicate with other mathematicians and learn new things, in order to participate in the research community:

> M: You have the danger of dying. You have to keep alive. You have to keep doing things that you want so much to know the answers to. You must be able to hurt yourself to know the answer. That's being alive. If you have too much teaching, that's the danger in research. You need to keep alive, but also you want to try to talk with people around you. And you want to get out with something. I think it's difficult, during the first years. Of course PhD is difficult because you have to cross the desert, but even afterwards you have to survive. The fact is that you change completely the environment [when entering a new institute], and the mathematical objects that other people around you are familiar with change too. And you have to work with these new things. And I'm sure that many people don't even see these difficulties because they are easy. I think it's easy for some people.

In order to “keep alive” within the mathematical landscape, a researcher has to adapt to their changing environments, and learn new languages by which to communicate with their colleagues. Over time, they need to learn new vocabularies and gain access to new ways of inhabiting and exploring the mathematical landscape. This process of developing a shared working vocabulary is not confined to the office, the institute, or the seminar room, but as the following conversation with Han and Bernie indicates, the process of building a shared knowledge base is continuous, happening throughout the working day:

> H: This is very important: the word vocabulary. Because this vocabulary has built up not only through working one day a week, but this vocabulary has been created when going to take a drink, or when walking, or when talking about another subject. This vocabulary pops up in math, so it's difficult to say. Every day there is new input into this vocabulary. It took us really one month to have a common vocabulary, but this vocabulary has become richer and richer with time.
>
> B: But it was about one month, and we could speak to one another, and we knew what the other guy was talking about. But we did other things. During this month we were explaining and asking and saying "I do not understand could you explain in more detail?"
>
> H: This is like what we said at lunch, he was speaking German and I was speaking in French...

The above dialogue reveals that one’s dispositions and orientations towards phenomena are continuously being shaped throughout one's working life. Habitus thus are not bounded and discretised within the confines of a mathematical institute, but rather expands out, in a continuous fashion, into one's day-to-day world. The conceptual space thus intersects with one's social spaces. The same is true for the PhD student who is likewise affected by their environment. As topologist Nigel says below, the influences on his research, and on the shaping of his perception, came not only from his supervisor, but also from other PhD students, as well as other members of the department:

> N: In my case I'm not sure so much, because it turns out that I didn't really work on his [his advisor's] subject. So he worked on the topology of spaces with 3 dimensional coordinates. But the conjecture he mentioned to me was for any dimension. And it turned out that I spent much of my time working on higher dimensional space, related to arithmetic constructions, so coming from number theory. So I got quite early, quite far from his own interests. So of course he shaped some part of my work. But [it was] the direction of the entire department - where other members played a role, not only my official advisor. I discussed quite a lot with other PhD students and other researchers, and so the whole was important.

Although the advisor does have a central role in shaping the development of a researcher's perception of the field, they are not solely responsible for the intellectual maturation of a researcher. Often, when researchers are questioned, there are rarely defining, eureka moments, where they can pin-point the origins of a concept. Rather the process of proving is articulated as existing along a spectrum of events, from which ideas evolve. The student must assemble knowledge together from multiple domains, and select certain elements from the continuous stream of stimuli into something coherent. The wider environment within which the researcher is placed thus creates a field of force, which shapes the researcher's thoughts in a given direction. But, ultimately, it is as a consequence of the researcher's choices that certain elements are selected and assembled together into a system of thought, which goes on to produce a proof or publication.

The systems of dispositions which constitute habitus are thus constructed through both passive and active processes of adopting and assimilating new tools and vocabularies. The process of learning to see the mathematical landscape is, at first, guided by the student either finding their own research problem or being gifted one by their advisor. Such initial problems form the starting point from which they can begin their mathematical explorations. At first these explorations prove difficult, as researchers advance in the dark, unable to fully grasp the relevance of their investigations. The mathematical landscape appears foreign and hostile to the researcher, as they begin to cross the mathematical desert. But, over time, their vision of the mathematical landscape changes as their knowledge grows, as landmarks appear, and as they become more familiar with the spaces they are inhabiting.

We shall see in the next section how, through the mastery of technique, the tools and language of mathematics are moved to the background of consciousness. The perception of the mathematical landscape becomes second nature, and the researcher learns to use their intuition. We shall explore the processes by which the language of mathematics becomes naturalised and backgrounded, so that working memory is freed up, in order to focus on the process of discovery, creativity, and assembly. Mathematics, we shall see, becomes a craft, and the mathematician is transformed into the bricoleur.

## Section 3.2: The Mathematician's Craft

### 3.20 Overview

The following section will explore the practices involved in assembling and experimenting with mathematical constructions. Mathematics we shall see can be thought of as a form of craftwork, involving very physical engagements with material artefacts. Mathematics thus is not a purely abstract system of symbolic manipulation, but rather it is also a subjective process rooted in the body and extended out into the physical world. This role of the body is played out through the practices involved in constructing mathematical proofs. It is through manipulating material representations by assembling, selecting, sorting, ordering, tinkering and relating that the mathematician builds up the machinery of proof.

### 3.21 Introduction

Once mathematicians have gained proficiency in a language, once they have built up a certain set of conceptual tools and working habits, so they are able to start perceiving features of the mathematical landscape. With perception comes the ability to explore the landscape, and to identify new features within it. But discovery is not a passive process, as we have seen in the previous chapter, it is an active process of interrogation of a problem.

Through such guided questioning the map of the landscape becomes more detailed, and the mathematician becomes more attuned to the possible paths they can take to navigate around problems. This process of navigating through the mathematical landscape involves first rooting this abstract space within the physical world. Through inscribing mathematical concepts on blackboards and paper the mathematician objectifies mathematical reality, and makes such reality visible and accessible to manipulation. In the following section we shall see how this process of assembling and manipulating mathematical reality takes place. Through studying such construction processes in terms of craft-work, we shall see how the mathematician comes to resemble a bricoleur.

### 3.22 Practicing mathematics: Documenting the mathematician's craft

We have seen already how different work-spaces are assembled throughout the working day, as well as the different routines which take place within such spaces. Now we turn towards exploring the work processes themselves, and the life-histories of problems and proofs. The creative work of the mathematician extends out of the office spaces, following the mathematician wherever they go. My interviews suggest that the problem is worked away in the backstage of the unconscious mind, as well as on the frontstage of consciousness. As one mathematician, Gordon, explains to me, thinking is process of rumination, of chewing over and digesting ideas; and such ideas follow the mathematician around, both in their minds, as well as in their notebooks:

> G: In the last 40 years I simply never spent a week without thinking about mathematics, a day yes, but not a week. It's like ruminating you know, we're like cows but what we eat is mathematics. It's a long rumination. Some people work late at night, and usually the best are like that and I'm not one of the best. For me it's like a long rumination, OK, mathematics is with you whatever you do. You can go and something else, of course, but it is always escorting you around. I was mentioning these notebooks, they are with me essentially always, and sometimes I don't touch them for a week because nothing happens but they are with me anyhow, where ever I am.

The thought process extends out into the world, and the origins of ideas often cannot be neatly packaged into a single event or eureka moment. Rather thinking is a messy process of assembling knowledge together, of digesting it, chewing it over, processing and reprocessing it, searching for patterns, or else experimenting with different possibilities, until finally thoughts are distilled and ordered into some argument or construction. The dislocations and disordering of the thinking process gains visibility in office spaces, with the various scratch paper notes, articles, chalked-out blackboards, inked whiteboards, and window-laden computer screens giving testimony to the eclectic and multi-faceted nature of research.


*Figure 21 Desk spaces at Germany-A, UK-B and Paris-A*

The mathematician's notebook also bears testimony to the eclectic nature of thinking. As Gordon tells me, his notebook is always present, in order to jot down a thought or to sketch something out. Notebooks collect such fleeting thoughts, and form an assemblage of heterogeneous entities, bringing together elements such as calculations, sketches of geometric objects, idle scribblings, to-do lists, lecture notes, theorems, definitions, or references. The notebooks can contain fully developed proofs, or they may just house experiments or vague ideas. There is thus no "typical" notebook, only individual examples within the genre. Such notebooks are very personal to the mathematician and they reflect idiosyncratic ways of "figuring out" problems. Below, Nemo guides me through one of his own notebooks, walking me through certain representations:

> N: I have mathematical notes but it's still vague, when I write down, it's still at the early stage and it's really badly handwritten. It's more like a collection of "Oh this can work like that" or else just sentences to myself. But I can show you this. So currently [flicks through notebooks] it's a mixture of papers that I have printed, and notes like this where I'm working on some random walks in some spaces, and these are the random walks. These are the kinds of stuff that I write. This is really not that precise. And when it's crystallised into being something special like this, you have a statement labelled lemma and you can see that there is no proof behind, because when I'm at the level of proof then I will start writing things down the paper. And you can see stuff like this, where we can relate these two theories and these are notes from a discussion with a colleague. Not very precise, but quite helpful. At some stage it's written in the computer and at some stage I will just chuck these. Maybe I will take this same paper and write down something else on it. At some point when I'm thinking I just need to write something down to help me think. I don't keep these notes. The notes I keep are already type-set.
>
> In fact seeing it like this - only a very few people will it be meaningful for. I think what is interesting is to see that even if the paper has no picture, but I've wrote a picture for myself here… So one important thing in writing is to help the mind to concentrate. Sometimes it's not important if it's right. I explained this to a collaborator once: that I just need to write. It's not strictly useful to write down, but it helps to think. And also the

mathematician does not want to be naked in public. You don't want to show that you were really dumb at some point.

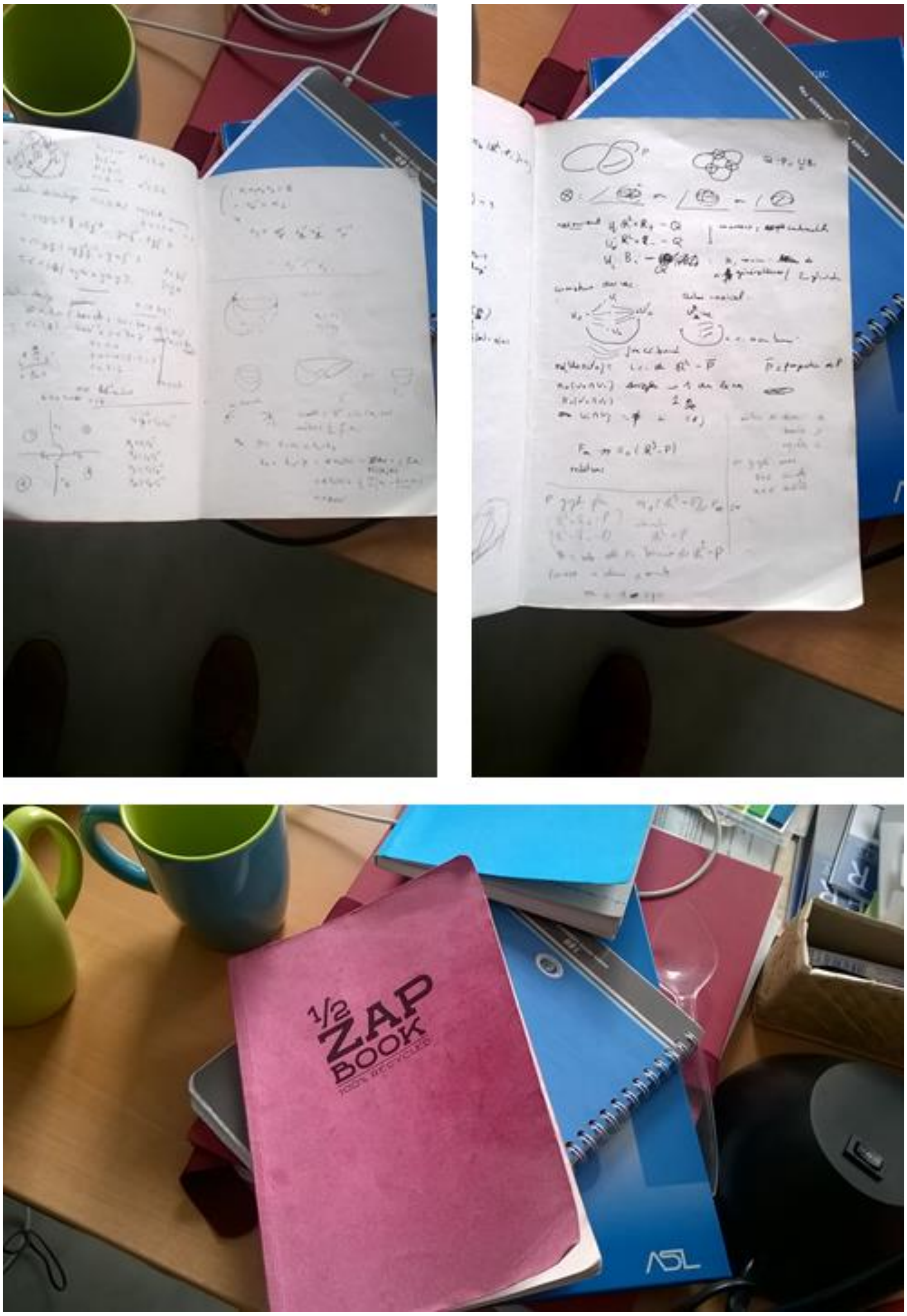



*Figure 22 Examples of notebooks from researchers at Paris-A*

Writing, as Nemo tells us, helps him concentrate, to reformulate and to re-figure concepts. Such thoughts start off as vague, but over time they are shaped into something much more concrete, "crystallising" into definitions or lemmas. Mathematical writing thus provides a physical scaffold for crafting ideas. For many researchers this scaffolding is a way of offloading certain processes from working memory, distributing memory onto an external storage device. In the following interview with geometer Aaron, we see this use of the notebook as a memory aid:

> A: And so sometimes after some scribble I understand something and then I can write down the proof. Another one [flicks through] maybe it's not here, and sometimes, here it's mainly questions. But sometimes I'm reading a paper and I take notes on the paper. That happens, it's easy to forget otherwise. Because when you have a long paper it's hard to remember all the definitions, you can turn all the pages, or you can go back in the pdf, but sometimes you need to write it down. So I take notes if I needed to teach something, or I take notes in the subway of what I will do. And then when I take it in the other way round [refers to back of book] it means that I'm doing something else. So here I was just thinking about music, this was Indian music, this was some rock and roll. This was a meeting.
>
> IN: There's little sketches, crossing out,
>
> A: So you have an example, stuff that does not work…And sometimes I repeat myself, maybe it's not visible here, sometimes there's the same thing on three pages, and again on the next three pages, it's because sometimes I need to start again and I start the same thing.
>
> IN: Is that because you need to go back to ideas?
>
> A: So for the last three or four of them I know that I need to go back. I have another blue book like that one where I have some information that I haven't used yet. So I don't want to throw it out. And the other I don't throw it away because I don't throw away anything. I don't know precisely why. I have many notebooks which I haven't reopened.
>
> IN: Is there examples where you go from these scattered thoughts to a paper?
>
> A: The problem is that I don't write papers by hand. So I go straight from that to the computer. Sometimes I keep intermediate versions, but usually not. But then on the typesetting programme there is a lot of reworking. When you write a sentence or you write a paragraph you look at it and you write it again because it's not a good way to write the sentence or

> the proof, so you change everything. You write a mistake and you remove the mistake, so you take your notebook again just to remember what you should have done.

The passage highlights the role of notebooks as memory aids which help a mathematician to return to ideas, clarify issues, and rectify mistakes. They return to the notebook in order to reconstruct a thought process, rediscovering the threads of thoughts which lay at the root of an argument. They return to the notebook to re-fresh their working memory and re-collect their thoughts. The process of recollecting the past is thus sometimes literally a process of re-collecting ideas, which are present in written form within the notebook. The process of proof construction thus becomes as much a physical task as a mental one, involving collecting papers, proofs and notes together into an assemblage of thoughts. From this mass of information the mathematician then begins the process of finding, ordering, experimenting, sorting, selecting and storing information. These processes are what I refer to as the craft of mathematics.

### 3.23 Crafting mathematics: Documenting the assembly process

To start the crafting process one needs raw materials. These are mined from books, articles, lectures or conversations with colleagues, and they collect as notes on scratch paper, in notebooks, in LaTex documents, or on blackboards. Such material accumulates upon the desk, on a hard-drive, or on cloud-storage, and it can take on the appearance of a mass, as can be seen below:

*Figure 23 Collections of notes and papers, UK-B*

Such accumulations are often scanned into pdf files and stored on the computer. By scanning documents, one researcher, Alfred, says such documents can more easily be searched for and sorted:

> A: No I do have to scan things, I use the computer as a kind of Index file - an exo-memory - actually that is the word - have you come across the word? It's an outside memory. So people use this all the time , you forget words or you forget something or you forget what you did - it's all in the computer. Unfortunately it's not linear but nevertheless…my correspondent - over the years we have had many email exchanges. My emails are always much longer than his answers …But I used the computer a lot as a storage device. I tend not to compute. I tend just to store information - books, papers, ideas - I mean whatever scans. It has changed my life for the better - enormously.

The archive indexes ideas and makes them accessible through searches by date or name-tag. Alfred refers to this archive as his "exo-memory" - his external store of ideas, which provides the source of inspiration for his proofs and publications. Other researchers prefer to keep physical notes, collating related material in one place. The material is sorted manually, reading through each note and either keeping or discarding what is considered relevant for a paper. Geometer Mark describes this process below:

> M: Very often I keep my drafts. Just pictures, rough pictures and everything, related to a project I put it all in the same folder. I put them together, and I do this absolutely all the time... So I keep my drafts until I solve the problem. So after that, everything that is not relevant to the solution, I'm going to drop it. But not until the project is finished.

This process of sweeping through, sorting and selecting, is a repeated action in the process of constructing proofs. Like the sculptor, the act of shaping the proof is a reductive process of paring the argument down to its simplest, most elegant form. With this process of selection and sorting comes the process of ordering ideas, giving them some structure within the framework of a proof. Alfred describes how this process of constructing proofs resembles that of building a house:

> A: I have the intuition and then I have to make sure that the examples fit into the theoretical model. It's a lot like building a house - you have all kinds of jumbled things and you build a house and you want to make sure that all these jumbled things fit into it - that there's enough room in the house. Suppose I had a brand new office and suppose I wanted to reorder this office - if I just had a replica of this office, that's just not good enough. I have to have a better ordering to the one I've got now. But I don't have a great theoretical foundation to what I do, I just do things. If I just wanted to do theory - I perhaps would not have become a pure mathematician

The ways in which the muddle of ideas is given order is through developing an awareness of, or an aesthetic appreciation of, the things that fit together. Alfred describes how the process of sorting occurs through the development of an intuition of mathematical spaces:

> A: Instinct. Instinct. The things I've been doing for the last day, week, month, year, decade work. At the moment with these kinds of things which attract recognition - they are not so frequent. It has happened. And you see very

> clearly either where you have gone wrong, or not deep enough, and suddenly there is a shift, and you kind of suddenly things fit into place
>
> A: When everything fits together, you just have an instinct. That this is the right way of doing things. And of course you can test it against other examples. In fact it's one of the things I dislike when reading other people's papers is when they have a theory and no examples.

Like the craftsperson, the mathematician must decide whether certain ideas fit, or "work", together. The proof must be well proportioned, aesthetically pleasing, and must accomplish the tasks that it was built for. As a way of proving that a given proof performs its desired tasks, the mathematician then tests the proof against known examples. The examples provide a training data-set for testing the strength of a given mathematical construction, through comparing the obtained results from those which are expected. Such experiments take place throughout the process of producing proofs, as one researcher Martin explains:

> M: …You experiment all the time, you experiment with points on the board. Once you've figured out what you want to do, you experiment with the way you write it. You keep experimenting on the formalisms. Once you're done with the formalism it takes a week to write out the full article. The writing of the proof is itself the result of an experiment.

The act of tinkering never ceases, taking place on scratch paper notes, blackboards, on paper drafts or in LaTex files. The results of such tinkering can be seen in the sketches below:

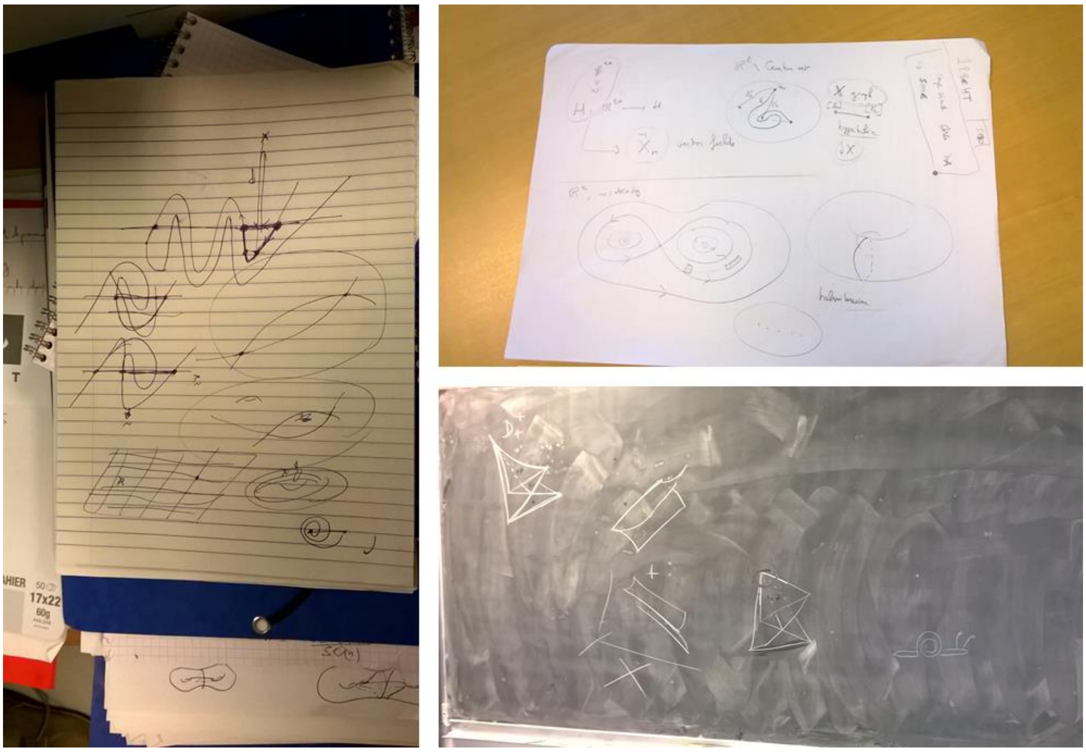


*Figure 24 Sketches on scratch paper and blackboard, Paris-A*

The craftwork of mathematics sees mathematicians moving between different representational mediums: from paper, to word-processor, to black/white board. Looking at one white board below we see the zoo of terms and diagrams that emerge through the process of "figuring out" a mathematical construction. The board shows an assemblage of different geometrical shapes, operations and equations/inequalities. The same object is broken up into separate "views" or representations, which give access to varying perspectives from which to visualise the operations being performed on the object. Narrating the board, in this form, is difficult, because of the many erasures and over-writing that has occurred over the course of the thinking process. To the outside observer, even a mathematician, the meaning of such a board is difficult to reconstruct. What we observe is a mess of different shapes and symbols; but to the craftsperson themselves, they see a pattern or a structure taking shape, refined through the act of figuring out on the board.

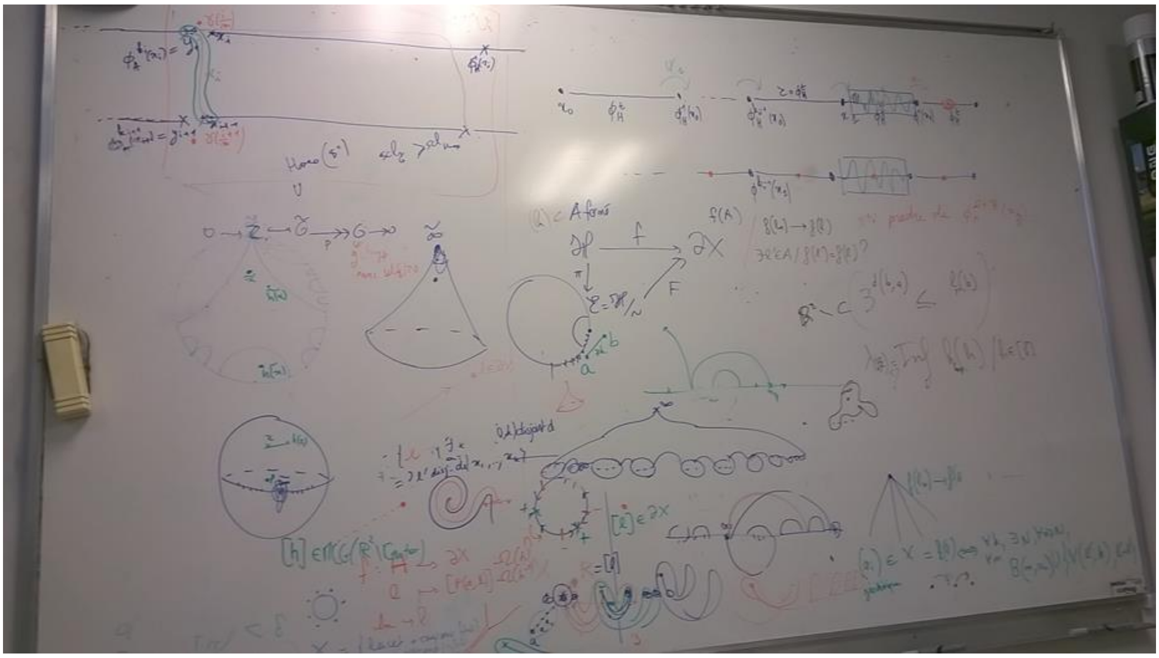


*Figure 25 Preparatory sketches and concept formulation on white board, at Germany-A*

From the board the mathematician returns to scratch-paper and will often "re-work", or "chew over", the information on the board: condensing, clarifying, and re-ordering the material so that it follows a certain logic. Through removing certain elements, and reducing the "bushiness" of the concept tree, the mathematician narrows the scope of the proof to following a single line of enquiry. The construction thus is transformed from something that was unstructured and rhizomic in nature, to something that is structured and dendritic, with definite premises and conclusions. The process of crafting the proof is thus to reduce the number of interconnected concepts, to limit the possible story-lines and create one single, coherent, dominant narrative. The process of crafting proof is thus designed to make it more palatable for an audience to digest. Below we see the processes by which the messy thoughts begins to be classified and structured into boxes:

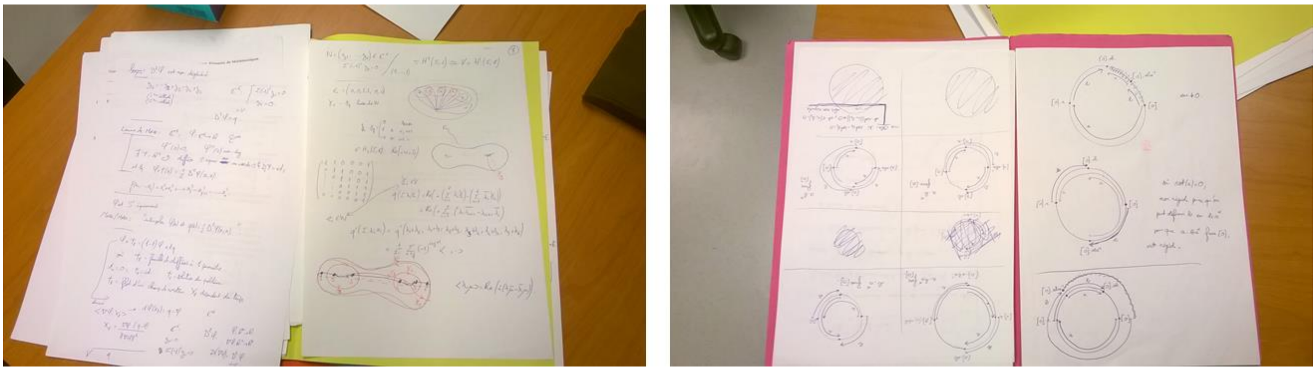


*Figure 26 Classification and structuring process in paper notes, at Paris-A*

At the point at which the paper is written it still undergoes a long process of re-ordering, selection and re-formulation. As can be seen in the re-working of a paper below:

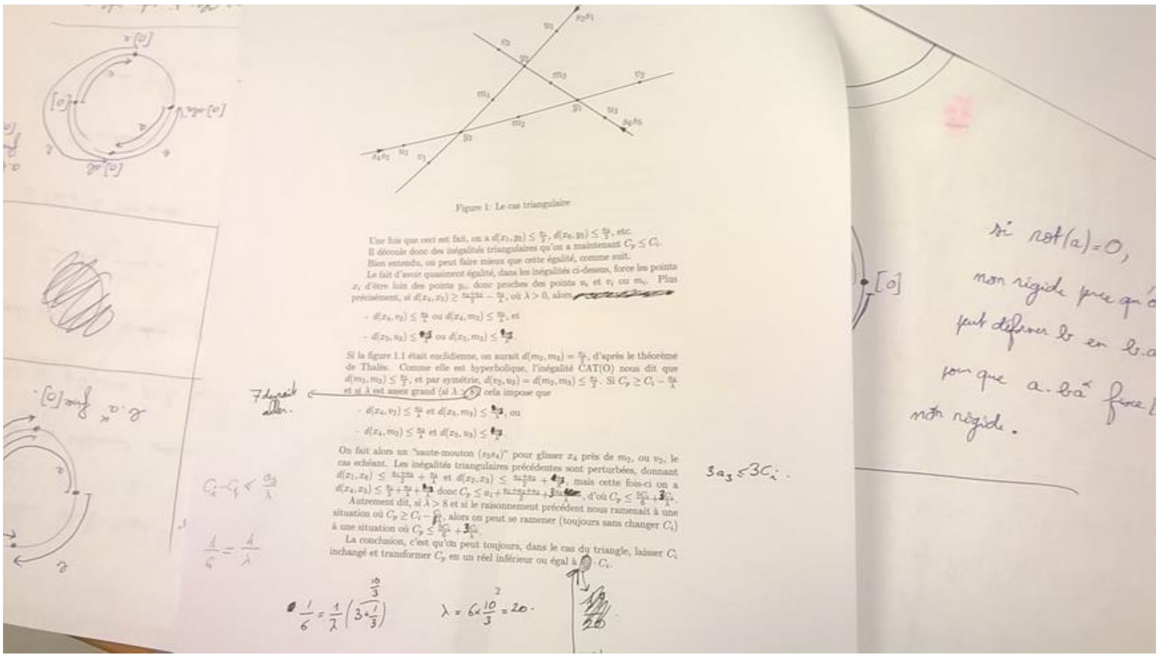


*Figure 27 Reworking printed articles in preparation for publication, at Paris-A*

In crafting a proof, the mathematician remains for a long time in the backstage, informally manipulating information in note-books, scratch paper and on the blackboard. The various elements are configured in a variety of ways before their structure is formalised and given coherence. This vagueness and informality at the start of the thinking process allows for a wide field of possibilities to be explored and manipulated on the blackboard.

The blackboard or whiteboard are ideal mediums for mediating this discovery process, for a number of reasons: they allow for elements to be presented at once, on one viewing surface; elements can be quickly linked by arrows and related to one another; or elements can be isolated through drawing boxes around them; finally elements can be re-ordered through changing their positions, which can then be quickly re-configured through erasure. The adaptability of this medium thus allows for different ways of seeing mathematical spaces to be simulated, as well as for objects within such spaces to be easily manipulated. Topologist Norman explains to me how the board is a "synthetic" medium for assembling and relating ideas, and crafting one's perception of mathematical spaces:

> N: I prefer general feeling, and discussion when you know "this should be like that because" and I like to have an explanation, at least at a certain level why things should behave like that. And so at the blackboard you can be very synthetic. And when it comes the time where you need a really rigorous statement, that's the time when I already trying to write down the proof. There is already this level, it can stay for longer, the level where you are very vague, during the discussion, where we are pretty sure that it should work and when it is something that comes to a certain maturation where we say "OK, it should work for a certain amount of times now we should try to write it down" and so I go and try to write down a proof. That's the way I remember things in my mind. This is where I sketch out an idea. There are many things that I don't know how to prove, or maybe I just haven't paid attention to, and it can turn out to be a real problem, but that's not the way - I need to have a big light which gives a reason to believe that. And that's only at the stage of writing that I see that "Oh this thing that I always considered completely obvious, or it didn't matter, and then you have to confront it - that usually comes later. But I first like to have a big picture and then go to that stage.

The sketching out that takes place on boards and scratch paper is thus similar to the processes involved in artists sketching out and experimenting with forms in their sketchbooks (See Gombrich 1977: 147; Verstijnen et al. 1998: 521; Goldschmidt 1991). The process involves testing out different ways of representing and perceiving spaces and objects. Through such informal sketching the mathematician gets a "feel" for what a space should look like, or what a tool should do. They become familiar with the space or object, and thus they can begin to probe how it will react when they tinker with it some way. Through manipulating equations and shapes on blackboards mathematicians begin to build up an intuition about how their mathematical "machines" function. Through successive manipulations on the blackboard the kernels of proofs start to emerge. Below we see the start of the process of formalising a natural language statement into a mathematical statement. We see the process of linking through arrows, boxing, and relating geometric (coloured red), natural language (in yellow) and algebraic objects (in green) together.

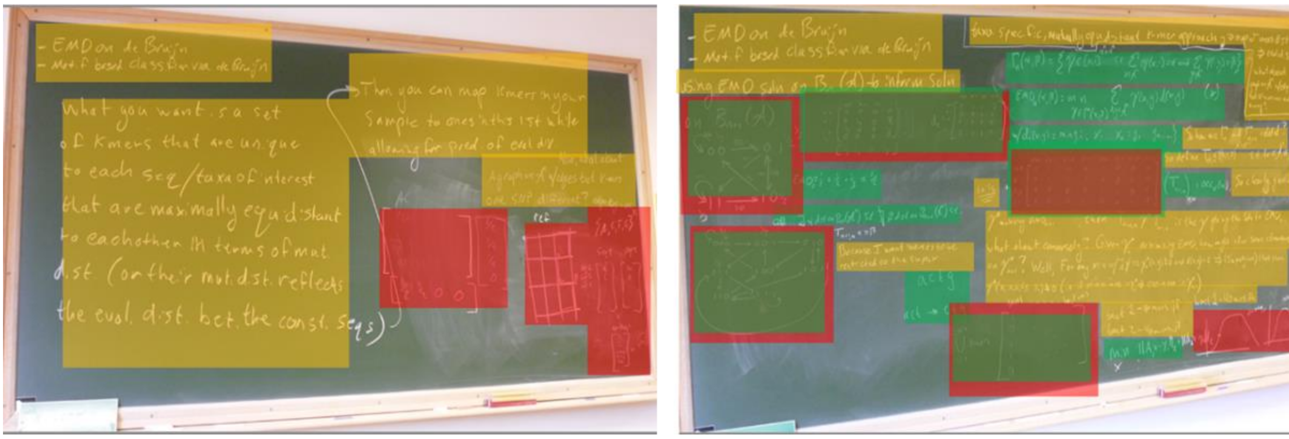


*Figure 28 Highlighting the different perspectives involved in producing mathematics, taken at UK-A*

### 3.24 Mathematician as Bricoleur: Documenting how perception is crafted

Constructing proofs involves a continuous process of formulation and refinement. Ideas are sketched out on paper or on blackboards to test out ways of representing or envisioning them, and these sketches serve as scaffolds for building up clearer definitions, orderings and arguments. The act of writing out mathematics serves to objectify concepts, offloading mental processing costs from working memory onto a physical, "exo-memory", which takes the form of notebooks, paper and digital archives; as well as computer screens and blackboards. The inscription process thus serves to extend working memory and frees up the brain's processing power, allowing for higher-order thought processing such as pattern detection, comparison operations and argument construction[12]. Such higher order thought processes take place through visual inspections and physical manipulations of mathematical representations on blackboards, paper or computer screens.

Like Levi-Strauss' bricoleur, the way of constructing mathematics is a matter of assembling, selecting, sorting and structuring isolated elements of language into systems of thought, which serve to articulate arguments. Through experimenting with the order and composition of mathematical assemblages, the mathematician as bricoleur is able to tailor their own mathematical habitus through which they are able to perceive the mathematical landscape. Sketching out objects and spaces allows mathematicians to craft ways of seeing such objects, making abstract concepts visually accessible and open to manipulation. "Picturing" a concept on paper or on the blackboard thus serves to objectify abstract mathematics, allowing concepts to be apprehended by the body's senses. Below, Gordon explains to me this process of picturing mathematics and the role such picturing has in crafting ideas:

> G: Not a real picture, sometimes again you work with actual pictures for one problem or another, but the object is like a concept, somehow it is a concept actually. Let us think about, let me take the concept of freedom for example, because I mentioned it before, you have some feeling of what freedom means. Sometimes you have pictures or sentences that go with it, it can be a painting, it can be liberte, egalite, fraternite, I don’t know, but

[12] This process relates to work in cognitive science on distributed cognition in which the cognitive processes of individuals can be offloaded and extended out into the environment and to members of a social group (Hutchings 1995). In this thesis I explore processes of memory management and extension of working memory using blackboards, archives and scratch paper notes as a form of “exo-memory”. I also explore how cognition is distributed within mathematicians’ social networks, through individuals acting as knowledge-repositories.

> those pictures do not reduce or explain fully the concept of freedom. So mathematical objects are really concepts in a sense. And the way you see them, or the way you remember them it's a little hard to tell. At least, since I used the word to see, our actual perceptions, that is vision and hearing play a role in mathematics, the other senses, to the best of my knowledge, play no role. But vision and the ability to hear play a role in the way we perceive and work in mathematics. So it's really I would say, seeing objects, rather than hearing them, or any other perception. It goes, and this is at a different level, with the way you remember things. For instance I remember better what I have seen written on a blackboard, or on paper. And sometimes I remember if it was written on the left or on the right of the blackboard, even very long after, OK. SO at least for me, the relationship between mathematics and vision is very strong.

Gordon explains that the picture sketched out on the blackboard is more than just a depiction of an object, like in a photograph; rather it is an interpretation of an abstract concept which is given expression within a painting. The painting, depending on the skill of the artist, can encode within it a number of concepts (Gombrich 1977: 44). Understanding what information is encoded within an artwork depends on one's understanding of the style and language of the artist. The act of sketching out objects thus is a means of condensing mathematical concepts into a visual short-hand, which provides a further scaffold for constructing complex arguments. Through condensing information into a visual format, arguments or concepts become subject to visual inspection and interrogation, which facilitates the process of pattern discovery and the synthesis of information, as Marie explains below:

> M: Yes. For example I've found out in my research a kind of singularity that I was able to draw and I was sure that it was the same singularity listed by Arnold in one of his books. And there is this List, and my object should be part of this list, or it's a deformation, it's a neighbour of one of these things in this list. First I had to see if it's one of the listed objects. It appears that it's not on the list. But just by looking at it, no formulas, just looking at it. It was not the same. And then, so if it's not the same it's a deformation - but which one. And all this with my PhD advisor - he was able to convince me that it

> was not the same on the list and we spent a lot of time just trying to figure out what was the difference and all without making any calculus. Just looking at it, looking at it [laughs]. It's funny. That's a big part of my work, just looking at objects.

Visual perception thus plays a role in apprehending the mathematical landscape. The picturing process allows for mathematical concepts to be projected into physical space, and perceived using one's sensory apparatus. Through this process of objectification and the analogy made with physical space, the mathematician is able to experience abstract mathematical objects as if they were real, physical objects. Reification then allows for one's in-born intuitions of the physical world to be repurposed to intuiting over abstract mathematical spaces. It is for such reasons that the first attempts to understand mathematical objects or spaces involve first drawing them out, and manipulating them by hand, as Jessica explains below:

> J: I first draw pictures and then go and try to explain it in a mathematical way and I ask "what is this picture really trying to say". That's the really hard problem. It's like: "OK I have to write every details…ahaha" How do I do this? SO it has to be a real proof so everybody says "yes this is true"...For me I like speaking with my hands. But then we have to write rigorously... But if you look at my drafts it's like picture, picture, picture.

Oftentimes mathematicians begin by working with pictures, building up intuitions about objects through manipulating such pictures. It is only later on in the proof construction process that these physical intuitions are erased, as the pictures are abstracted away, in the formal process of "writing up". In writing up much of the informal processes of experimentation, as well as the physical intuitions on which arguments are based, get written out of the end publication. However, in erasing pictures and examples from the published proofs, it can often become difficult for other mathematicians to follow a proof. Much of the working processes, the many choices, and other potential narratives, are made invisible in the act of formalisation. This serves to distance the author from the finished product, and to make the end result appear less of a product of human artifice.

When we show mathematicians' working practices we begin to see the complexities involved in constructing mathematical habitus. We see how the mathematician tinkers with objects and representations, and how they work and rework their proofs, formalising and abstracting out superfluous elements, in an effort to attain an aesthetic ideal of elegance and simplicity. In the pursuit of such simplicity, the choices, intuitions and experiments are written out or erased. The embodied processes of crafting mathematics on paper or on a blackboard are forgotten. However, these embodied processes, as we shall see in the next section, are integral in constructing mathematical habitus, and in the experience and perception of the mathematical landscape.

## Section 3.3: Frames of Reference

### 3.30 Overview

In the following section we will explore the ways in which the mathematical landscape is visualised and perceived. We shall see how paper and blackboards act as tools for enframing mathematical objects and spaces. Such frames are synthetic tools for generating relationships between different mathematical representations, and they serve to give the mathematical landscape structure and coherence. I argue that constructing boards and paper as reference frames allows mathematicians to perceive, explore and manipulate mathematical constructions.

### 3.31 Introduction

Pictorial representations play a major role in constructing the mathematical landscape. Without pictures, abstract spaces are difficult to visualise and interpret. Drawing, as we have seen, provides means of grounding understanding in the material world, and results in malleable, manipulable objects being created, which extend one's working memory. By writing and drawing on blackboards and paper mathematicians create perspectival frames for viewing and structuring the mathematical landscape. The board and paper, for the mathematician, act in much the same way as the canvas does for the landscape artist: they are representational mediums for presenting a perspective or viewpoint on reality. For the mathematician this "reality" takes the form of an abstract, mathematical landscape. In the following section we shall explore the processes by which the mathematical landscape is framed and perceived.

### 3.32 Landscapes of thought: Understanding mathematical perception

We can imagine mathematical reality as composed of spaces which contain certain structures, objects and processes which can be described and related using mathematical languages. These mathematical spaces are what I will refer to as mathematical "landscapes", and they form the structural frameworks within which thinking in mathematics takes place. Landscapes, at their simplest, are composed of landmarks or features, which are related to one another by measures of distance. In our mathematical landscape we can imagine the landmarks to be numbers such as pi or e, or perhaps shapes or surfaces, or equations; they are, in short, objects or concepts which have some mathematical identity and description. The relationships between these conceptual landmarks manifests itself in our notion of distance. Similar concepts are closer together in the mathematical landscape; more challenging concepts may be represented by height, with tall peaks representing concepts which require a great deal of effort to understand.

To begin to understand the landscape, the mathematician must first learn to identify and perceive the features present within the landscape. This first requirement involves learning definitions, and building up a core vocabulary of mathematical objects, tools and concepts. This core vocabulary forms a key landmark within one's mathematical landscape, and is the local fixed point from which one relates to one's surroundings. From this origin point the mathematician can gauge distance to other landmarks, creating a point of reference within the landscape. It is against this reference point that artefacts are compared, based upon the features shared in common. In this way the mathematician’s perception of the mathematical landscape is shaped by the landmarks that they are already familiar with.

This set of local landmarks is a product of one's education in mathematics (as discussed in section 3.1), and is often based around the examples one becomes familiar with during graduate studies. These landmarks provide a perceptual frame for viewing the rest of the landscape, and shape one's future orientations and interactions with the wider mathematical landscape. For many mathematicians, their positioning within the landscape is shaped by their ability to construe mathematical phenomena in terms of pictures. They then base subsequent interpretations of the landscape upon this pictorial frame of reference. One mathematician, Alex, below describes how she bases her understanding of mathematics upon a "geometrical" frame of reference:

> A: When I chose mathematics it was because of geometry. You can reason on figures, for example, if I cannot draw a picture I cannot even understand what I'm doing. It's clear for me that without a picture I cannot prove anything. I'm sure not all mathematicians are like that. I know many who are interested in very abstract things, it's a conflict all the time. They can be very abstract and I cannot follow them. If I do not have a visual support I cannot understand their language.

Construing mathematical concepts as geometrical objects allows Alex to "reason on figures". Mathematical concepts are visualised as shapes which undergo certain changes, as a result of functions being applied to them. Because of the abstract nature of mathematics, mathematicians require visual supports, as Alex claims: "*Everybody is visualising something, they have their own way of approaching the abstract thing*". This sentiment is echoed by other mathematicians, but the basis of this "visualisation" is not always geometric in origin, as one topologist, Max informs me:

> M: Everyone must have an intuition, or manner to visualise things in their mind. I think some people are seeing some formula, and the numbers speak to them. For me, absolutely not. I don't know if it's because of me, or my past, because I've done algebraic topology. I think it's because of me, because I was oriented towards algebraic topology, I was motivated to draw things. But even for people who prefer to work with algebra and number etc. they have their own kind of visualisation.

These modes of visualising mathematical reality are referred to as gaining "intuition". This "intuition" is what we refer to here as a perceptual frame. This perceptual frame is built up over the course of one's career as a mathematician, and comprises of the vocabulary of concepts that an individual has internalised within long-term memory. To perceive such objects first requires an active phase of "picturing" the object, bringing it to the foreground of one's attention. Through repeatedly reconstructing such pictures they eventually can become naturalised within their mind, and so fall to the background of attention. One topologist, Jim, describes this process of becoming habituated to certain representations:

> J: Yes…hmmm well I'm a topologist, in knot theory and it's very visual in some sense. But maybe I'm trying to get into more algebraic constructions. Maybe at the beginning I was very visual, I had a lot of pictures…it also depends on the kinds of questions you ask. And I feel that the older I get the less I need these pictures. I have made the same pictures 1000 times, and after a while you don't really need it, but it's clear that I have it in mind, in some sense. I definitely have pictures in mind, but I don't write them out so much.

### 3.33 Exploring structure: Relating perception to habitus

Mathematicians thus use pictures as visual supports for building up their intuition of the mathematical landscape. Such pictures can often be recalled without writing them out, as they become naturalised within one's habitus. However, in the early stages of understanding concepts, the mathematician must figure them out at a blackboard or on scratch paper. The blackboard or paper act as tools for visualising and exploring the architecture of concepts, allowing elements to be related to one another and assembled together. One mathematician, David, describes the importance of visually exploring mathematical structures and gluing elements together on blackboards:

> D: I am a visual thinker but the math I do - I need some structure. In some sense I am almost afraid because sometimes if I have to write down the proof and if there is not enough structure I'm afraid I won't be able to…in a sense if the mathematics is too soft it's a bit frightening, because I feel I will have a hard time in writing down, completely, the details. So I like really the mathematics where there is a lot of structure, but where you can visualise the structure and know that you will need it and you will be able to write down the proof at a table.
>
> So you have a lot of things that can be done, but that doesn't mean that I know how to do it, if you ask me just right now. But I know that I will be able to do it if there is the kind of structure which makes things glue together. And because I know these things are here I can have the visual thinking of the theorem I want to prove; and when I have the theorem I come back to the more structured thing and painfully write the proof. And so, to summarise, the mathematics that I am doing is very structured, so there is a lot of heavy things in there. But I like to think that the object that I want to understand is very geometric, and I use this structure to say something geometric, and that it can be described on this object.

Understanding a mathematical construction becomes a process of visualising its component parts, and seeing how they function within the larger mathematical mechanism. The drawing provides a visual scaffold for extending working memory and for exploring abstract constructions; also, concepts in picture form become observable, referenceable objects, which can be reasoned about. But perhaps the most important aspect of visual representations is the fact that they simplify complex constructions into visual short-hands. They are able to condense complex information into symbols, signs and graphics, which represent complex processes and phenomena. Through simplification and condensation into two and three dimensional drawings or algebraic symbols, so the background structure of the construction becomes more apparent. A topologist, Peter, highlights the role of "picturing" concepts, in order to reveal their underlying structuring principles:

> P: Yes, yes, completely yes. I do have a picture. Of course I can only draw 2-D and 3-D pictures, and it's very helpful for me to have them. I think I understand really an argument when I can have a picture of it. In fact there are several aspects in my work. The most geometrical aspect, the one I would say that I most understand is when I can draw a picture. And then there are more algebraic aspects, where it's more about where I understand how the structure is built together. But to make a proof I need to be able to summarise it, in my head, into a few steps, because if that's not the case I feel that I don't really understand what's going on.

### 3.34 Framing the landscape: How metaphor functions as a reference frame

Creating visual analogues to abstract mathematical constructions transforms concepts into perceptible objects. The visual representation approximates the structure of the mathematical construct, sharing with it certain characteristics. Visual representations in mathematics are used in similar ways to how metaphors are used in natural language. The visual representation bears certain resemblance to, or shares certain features with, the mathematical construct that it refers to. The visual representation thus acts as a schema for organising the abstract mathematical construct. Modifying the visual representation becomes analogous to transforming the mathematical construct in some way.

Visual representations serve as frames for viewing and organising the abstract, mathematical landscape. The visualisations are not the mathematical objects themselves, but merely approximations of these objects, projected into our physical reality. They serve as a means for us to relate to abstract objects and spaces through our understanding of the physical world we inhabit. Often a mathematician must become habituated to using such representations, learning how to translate the picture in terms of the mathematical construct it stands for. Such ways of thinking about or translating mathematical concepts are called "tools" by mathematicians, and they provide a way of educating their perceptions into being able to perceive the mathematical landscape. One Algebraic geometer, Gillian, tells me how learning to see an area of mathematics through such a visual tool transforms the way in which she experiences the mathematical landscape:

> G: For instance I was teaching group theory in the first semester. Sometimes it's not always possible to draw pictures for groups. But one of my colleagues told me that one way of understanding groups via the subgroups, and you draw the sub groups and when I saw that I said: "Yeah, that's what I was looking for". When I don't have the right tool, I'm looking for it. Maybe it's also a handicap. Whenever I cannot draw pictures anymore I'm a little handicapped. Now mathematics has become very abstract, with category theory, functors, etc. Maybe these people have a secret drawing behind the way they think.

Over time a mathematician will learn more visual schemas for organising their experiences within the mathematical landscape. They will learn the function of these tools, and how to use them to manipulate abstract concepts. Through acquiring these representational tools they gain an intuition of the mathematical landscape, and understand the possibilities for acting within it. As a topologist, Margret, explains, one's intuition of the mathematical landscape grows as one becomes adept at using representational tools:

> M: For this it’s a problem of experience. He [her supervisor] has more experience than me. It’s more clear for him the notion of co-dimension of generic objects. It’s more clear for him what can happen and what cannot happen in some way. [Goes to board] For example, if I pass from this drawing to this one you can imagine that you can take the tangent lines and force the tangents - you take this put this here. So when you do this you have a film of singularities from this one to this one. So in some way this one is an exceptional situation of this one. As soon as the tangency is not there you have this. If you push this more you will not have the tangency again. So what can generically happen… it's one of experience. Actually the genericity I'm speaking about is about moving from this to this, but staying within an object like this. If I just push those objects without asking anything it will not stay like this, this singularity will spin off. So what am I doing? I'm trying to go from this thing to this thing, in this class of objects, and asking is that a big constraint or not? You understand? And for Erik those things are more clear. For me it’s slowly, slowly more clear.

### 3.35 The Horizon of vision: Exploring the limits of pictorial representations

Using visual representations as perceptual lenses for framing and viewing the mathematical landscape does have its limits. The limitations stem from the fact that the visual representations are only approximations to the abstract mathematical concepts themselves. Pictures distort certain features of the mathematical landscape, emphasising one aspect with respect to others. In some sense, as Gillian explains below, visual representations are "bad pictures", in so far as they are not completely accurate depictions of spaces or objects:

> G: The problems that I work on are geometrical. It's about varieties, draw-able objects. But it's because of my taste for drawings. I'm also interested in what other people have to offer. To see their way of thinking, I will adapt. But my definite taste is for something I can express with an image. For instance in the library here we have an exhibition of surfaces. I wasn’t even

aware that surfaces could be presented like that. When we're studying surfaces we don’t draw the right drawing. For example with a singularity we have a local drawing and you don’t really see what's happening around. We do projections or sections. But when the computer is showing you exactly what it looks like, you are quite amazed. But they say that geometry is the art of reasoning well on bad pictures, so I quite agree with it. Because when the picture is too good you get confused, you don’t know what to prove any more. It's so obvious from the picture that you don’t realise that you have to prove it.

When an object or space is computed, and a computer visualisation generated, the mathematician comes to understand how their own visualisations deviate from the "truth". But, as Gillian explains, the bad pictures mathematicians draw are representations which emphasise certain features, and motivate certain actions or interpretations. For Gillian, her visualisations serve to emphasise what aspects of the mathematical landscape need to be explored and explained through proof. However, sometimes a picture can not only distort the underlying mathematical reality, it can also mislead the mathematician, as Nemo explains:

N: You mean can a picture be misleading? Yes, I'm not so sure I have a good example in my own work, but certainly. It happens sometimes that I see a picture, when I'm reading somebody's work, and I make a picture which is not the right picture and it can take me several times before I realise that I have been completely misled by this picture which was not what he was thinking. There are also famous examples in my field of theorems that were not considered possible, because the way we thought of things, in terms of pictures, which were naturally occurring, and were showing the opposite of the theorem. In general these are the most interesting theorems, because you really understand why your picture was not the right one.

The pictures which mathematicians use to frame the mathematical landscape structure perception so as to construe reality in a certain way. Such interpretations of mathematical reality can often turn out to be false, but, as Nemo argues, the very fact that they are false provides insight. In other instances, visual representations of complex or higher dimensional spaces are not always possible. Sometimes such objects can be represented through projections into 2 or 3 dimensional space, or else through slicing the objects and studying their cross sections. But as Algebraic geometer Eleanor explains below, such projections or slices are not always able to render the complexity of certain objects or spaces:

> E: Yes so I have my own ideas. Buts it's very difficult to visualise. So what I'm working on is in complex dimension 2. Which means that it is real dimension 4, so it's already a mess to visualise. So you develop an intuition of your own which has not much to do with objective slices, 3 dimensional slices, etc. So I developed an intuition of this space, which is one more dimension from the space we are living in. And there, more or less, I know what's going in my area. So its intuition. It’s a way of seeing things, but I don't see it in the way I see you for example. And if I want to combine it with real vision which means slicing and getting 2 dimensional slices, or 3 dimensional slices, the truth is that it rarely helps me. It doesn't help me you see, you need other avenues.
>
> There is a very famous conjecture in my field which is called the exception of minimal set for formations of $\mathbb{C}^2$ on the complex projective space. So you have a complex space which is of dimension 2, which is compact and which is the projective space, which is the simplest case which one can imagine, and you study global foliations with singular points on this object. And the conjecture is that there is no invariant minimal set. So people started using geometric ideas, it didn't work. And then they had this idea of using pictures, so using computers…and it doesn't help at all [laughs]. So doing pictures, trying to see if there is a minimal point "how could it be? where could it be?" so you get a huge amount of pictures, which of course are not 4 dimensional, which are just projections, and then you don’t know how to deal with them. It doesn't help much in my field. Touching the reality through computations etc. doesn't help much, its more just abstract geometrical arguments.

### 3.36 Translating thought: How metaphor structures knowledge domains

Some spaces resist interpretation and representation within 2 and 3 dimensional space. As a result some mathematical spaces and objects cannot easily be presented in the form of visual representations. Rather, mathematicians construe the mathematical landscape within another kind of representational medium, within an algebraic language for example, reducing its dimensions to another form of symbolic short hand. One mathematician, Alfred, explains that spaces or objects can be interpreted not as shapes, but as maps or graphs:

> A: Not really. This space for example, is a space of maps. So I visualise moving into the space by having a map, say a translation and you deform it. So the question was how to deform this space.

Transforming objects thus becomes encoded as movements between different nodes of a graph, or as functional mappings between different algebraic symbols. Indeed the process of representing objects can take the form of natural language statements, algebraic equations or inequalities, graphs, or maps. The mathematical landscape is thus open to multiple interpretations (framing from multiple schemas), many of which are layered on top of one another. As mathematicians learn to construe objects in different ways, through different frames of reference, so they learn to move more fluidly between different perspectival frames, or "intuitions". One algebraic topologist, Jessica, explains to me how mathematical objects are transformed by the mathematician, and how the mathematician learns to move fluidly between different frames of reference:

J: I think there's lots of different layers to this. In my head I think of a knot as a three dimensional thing that's sitting inside space. Obviously when I'm working I will draw a 2 dimensional picture of it and work with that particular diagram. And if I think about 4 dimensional geometry then suddenly it becomes very algebraic and I don’t really look at the pictures any more. A 4 dimensional space is just 4 coordinates, it doesn’t matter whether you work in n dimensions; it doesn’t make any difference to have n coordinates. So the more abstract it is, you move to different techniques to study it. Depending on the context you will think of one object in many different ways.

I guess even when I'm thinking of something really abstract. Like a 4 dimensional knot I just have to relate it back to something that I already know. So if it’s a sphere I can't possibly think of how a sphere can be knotted in 4 dimensions. But I sort of know how a circle can be knotted in 3 dimensions. So you sort of have this picture in your mind of a sphere that when you take a cross section you get some sort of knot. Although you can't quite picture it, it gives you enough of a concept that you can start to work with it.

Or an example that I’ve used more concretely, for kids to think in 4 dimensions. If I were to draw a picture of a cube for you on the paper. So If I draw something like this, most people would tell you that you're looking at a cube, or you're looking into a box. So this is the back of the box, this is the front of the box and these are the 4 sides. Most people have no problem with the idea that that's a cube. So this in real life, this strange trapezoid is actually a square. And in real life the small square at the back of the box is the same size as the big square at the front. So people have no difficulty in seeing how a 3 dimensional shape becomes distorted when we put it on 2 dimensional paper. So then you can say, "So what does a 4 dimensional cube look like? How can we generalise this? So instead of having a square in a square, you have a cube inside a cube, and now I'm drawing a 4 dimensional object. You have a big cube on the outside and a small cube on the inside and the corners of the cubes are joined together. Something like this. And in 4 dimensions the cube on the inside is the same size as the one on the outside, but one is further away in the 4th dimension, so it looks smaller. And all these shapes on the corners they're all cubes, but they've been distorted by the way we're observing it from the outside.

So even though your brain cannot possibly picture what a 4 dimensional cube looked like, you can use the analogy of something that you already understand to try to at least have an idea of the structure of how it would work. And then from there you can't really talk about the 5th dimension but you can start saying things like. Well we know the coordinates of all these points are, so in 4 dimensions it will have 4 coordinates, in 3 dimensions there are 8 vertices of the cube and the coordinates are: they all have zeros and ones. And all the combinations of zeros and ones gives you all the vertices of the cube. And in 4 dimensions you have the same thing. So there are 4 coordinates so you have 16

> dimensions and so on. So it's very easy to generalise to any dimension you want. So you're sort of moving between the idea of the abstract coordinates, the algebra, to the geometrical picture, to the projections of things and it's all happening at the same time in your head.

The mathematical landscape thus is not something that is fixed and unchanging. Rather the terrain is contested, multi-layered and multiply interpreted. The landscape can be viewed from many different perspectives, and mathematicians perceive it according to their existing knowledge and experience (that is to say according to their positioning within the field). Such multidimensionality of the landscape indicates its metaphorical nature. Like metaphor in natural language, the mathematical landscape can be interpreted in multiple ways, and mathematicians are able to fluidly move between these different points of view. It is towards the metaphorical nature of mathematical concepts that we will next turn, as we explore how mathematicians become habituated to certain visions of the mathematical landscape. We shall see how those visions are shaped, how they evolve, and how they are influenced by the community of mathematical practitioners.

# Chapter 4: Discourse Spaces

## 4.0 Overview

In this final chapter I position mathematical perception within the wider social field of mathematical production. This social field is shaped through the distribution of social capital, which generates fields of force which influence individual's decisions and orientations within the field. Mathematicians' dispositions towards phenomena are products of their positions within this social field, and their socialisation within certain habitus.

I explore the field as a competitive field, in which individuals are motivated by gaining social capital, through making their knowledge visible and recognisable. Ideas, in order to be assimilated within the wider field, and constituted as capital, must be structured so as to fit certain socially valued characteristics. Knowledge therefore must be coordinated and related to existing bodies of knowledge, in order to be accepted and assimilated within the wider field. I explore this process of coordinating local frames of reference to global reference frames, and show how individuals shape their ideas so as to conform to certain standards and exemplars.

## Section 4.1: Sharing Visions

### 4.10 Overview of Section

In the following section I explore how an individual's personal vision of the mathematical landscape is coordinated with that of the community of practice. Private thought is transformed into public knowledge through conforming to certain structuring principles. Such structures order knowledge into categories which the wider community utilise and, in this way, new ideas are assimilated into the existing lexicon.

### 4.11 Introduction

To communicate their private vision of the mathematical landscape, a mathematician must refer to objects and concepts that other mathematicians already recognise. They must co-ordinate their personal understanding with that of the community's, and generate a shared frame of reference. Using this shared reference frame, individuals can refer to objects, and have them recognised by others as bearing certain qualities and meanings.

The following section will explore the processes by which individuals come to share the same reference frames, and the means by which private thoughts are translated into common knowledge. I will argue that such reference frames are built up around a core of fundamental concepts and objects, which form key reference points within the mathematical landscape. Such reference points act as conceptual bridges between different knowledge domains, and serve to coordinate individual perspectives. I will demonstrate that all reference points ultimately converge to a common origin, not at infinity, but rather within our own physical world. Such a convergence, I argue, is manifested in mathematicians' ideas around mathematical intuition.

**4.12 Between worlds: How are domains of knowledge are connected?**

The first stage of understanding a novel concept or object is to relate that unknown entity to things one is already familiar with. Mathematicians thus explore unknown objects and spaces, in relation to the features or qualities which objects within their existing lexicon already possess. Once familiar features are identified, so a systematic process of comparison operations can proceed, between unknown concept and familiar concept. The mathematician is then able to notice similarities between different entities. The process of assimilating new information into one's existing concept categories can be summarised in the steps below:

1. Feature detection and recognition
2. Feature mapping and systematic comparison
3. Classification and ordering

By this simple process, a mathematician relates the unknown features of the mathematical landscape to the features they are already familiar with. New elements thus accrete around existing conceptual structures within the lexicon, becoming dependent upon the originating reference objects for interpretation. Through this mapping procedure the mathematician creates analogies between categories of objects or concepts. These analogies then influence how the mathematician interacts with the foreign concept, by creating certain expectations about its characteristics and functions. Mathematicians thus use their knowledge of a familiar domain to give order to the foreign domain. We see this phenomena when mathematicians undertake interdisciplinary collaborations, or when a mathematician branches out into another field, as Amos describes below:

> IN: So you are very much crossing different fields.
>
> A: Yes I like that. For example, a big project that we are doing with a colleague...we asked for a number-theoretical reason. It's important to study some variants of analytic spaces that look like usual spheres and so on, and to make differential calculus on them. But those spaces come from a strange part of number theory and strictly speaking they are totally discontinuous, so it's more like scattered sand, or something. So it's not a usual vision of what a differential form is. There are some analogies that are possible, we developed a full kind of calculus that works and allows you to speak of objects the same way that you speak of them in the classical world, so I wanted to push forward the analogy and create objects with this.
>
> IN: But in order to have the analogy you have to have…
>
> A: A basic knowledge of the other field. Yes, yes. Also a basic understanding of why the technology that is used in the other field is important in that field. And how it is supposed to interact with mine. But for example, with previous work in number theory I had already observed that some machinery that works in analysis was useful for me, but it was useful for me that I knew that some analogue was working or should be working in the other so the idea was to try to cross the border.
>
> IN: When you've determined that something is an analogous object in one field, do you use the techniques from one field to explore the object in the other field?
>
> A: Yes and no, for example, concerning the subject I was just mentioning. For 10 years I really just used the tools and the theorems from the other field, and then for the last project, we tried to develop or create objects which satisfies the same rules as the other field. But the proof of the rules, that the object satisfies these rules, is not really the proof that comes from the other field. So the statements of the theorems were inspired by the other field, but sometimes the proof, we need to have new ones.
>
> IN: Can you go backwards to exporting the object from one field and reimport it back into the other?
>
> A: In principle it could happen. But it has not happened yet. Probably because our analogy is not yet deep enough, and has not yet produced enough results, so there's nothing to pull back to the other field yet. Except that, for example, we were mixing between two fields called Tropical

geometry and differential-form-occurrence-distribution theory and so what we did, so I was motivated by distribution theory and at some point we observed that the tools from tropical geometry were useful, but I had no interest at all in tropical geometry, except that I used it. And it happened that our work has implications for tropical geometry. But it was not planned it was really a surprise. We proved a very general formula, and we understood that it generalised some formula in tropical geometry, but we were not so much interested in that, but people in tropical geometry were interested to know that we had another proof.

Identifying analogues allows for tools and techniques, used to describe one domain of knowledge, to be applied systematically to studying the analogical domain. The degree to which the conceptual "machinery" can be adapted to studying the analogue is, however, limited by the degree of similarities between the knowledge domains. Amos speaks about "deepening" the analogy between the concepts, in order to extend the techniques from one area into the other. The reference domain thus serves as a guide to structuring the analogical domain, influencing how the mathematician experiments with and interrogates the foreign entity.

### 4.12 Anchoring thought: How shared reference frames are constructed

The reference domain is often built around a small set of familiar objects, concepts or techniques. Such reference points furnish the mathematician with an intuition about comparable spaces or objects, motivating them to ask certain questions, use certain tools or techniques, or interpret the object in a certain way. Below, Eleanor describes how her mathematical intuition is shaped by her "favourite objects", which serve as guide-posts for navigating around the mathematical landscape:

IN: How do you develop this intuition? Do you have specific tools and objects in mind?

E: Yes, that's right, I have some favourite objects [laughs]. I'm very often testing my ideas on very simple cases, which doesn't mean much

> because it may work in this case, but not in generality. But that's true, and I like to make small computations so to grab the object more, rather than just speculate. I cannot just be satisfied with a purely computational argument. The starting point, in many cases, is an example. Yes, it's an example. It's an example I'm working on or somehow things that I've heard in a seminar that I relate to what I'm doing. So from a particular case I start to investigate and from that it grows into a theory.
>
> IN: You then test the theory on more and more examples and try to make it more and more general in some way?
>
> E: Yes, so if my idea doesn't work, in a basic example, then it means that there's something that I've missed obviously. So I'm trying not to resolve the problem for that specific example, but to think of what could be going on in that particular question that I didn't see. So that's where more abstract arguments become involved. And ultimately I can make a statement that yes it works like that in that example - because people like to have examples, they're not satisfied with just pure theory. It comes and goes. I start from the example, then add new ideas, and then I ultimately go back to the examples. Not to be sure, but just for fun, I check on the examples just for fun.

The mathematician tests novel phenomena against their exemplar objects, in order to determine the features held in common. The exemplar thus serves to anchor the mathematician's investigations around a specific set of questions. These questions narrow down the possible categories to which the object can belong to, and thus they serve to organise the novel phenomena within a given class of object. Through incorporating an object within a given category, the mathematician is then able to subject the object to the same interpretive framework that other objects in the class are subject to. In this way, the novel object becomes semantically anchored to the contents of the class.

As a mathematician builds up their library of exemplar objects, furnishing and diversifying their classificatory systems, so they are able to detect a greater range of mathematical phenomena, and categorise them accordingly. Greater experience leads to a more developed mathematical intuition, which results in experienced mathematicians being able to more easily identify possible solutions to problems. As one researcher, Gordon, explains, the experienced mathematician is able to identify the relationships between objects in a similar way to the chess player is able to identify possible moves on a chess board:

> G: It’s a little hard to describe actually. Giving a sense of what intuition is, is getting a little involved. What I can say is that mathematical objects live their own lives. And for a mathematician they are as real as a rock can be for a geologist or a star can be for an astronomer, they are actual objects and you get to know how they behave. It’s a vision of objects and the relationships between these objects, OK. On the one hand you have the mathematical objects themselves, on the other hand you have the techniques, proofs, this theorem, that theorem, which gives you direction, *ligne de fuite* we say in French, in which that particular technique can be efficient. I would compare it with chess, ok. When you see one bishop for instance on a chess board. Inexperienced people see the bishop itself on a given square, experienced players see the lines on which the bishop is actually efficient. And for experienced mathematicians it is exactly the same. You see where and how the technique can be applied and then usually colleagues see it as well as you do, but sometimes you can say did you use these types of things and sometimes the idea did not even occur to the person you were talking with.

Mathematical intuition manifests itself as an ability to "see" certain connections, or possible solutions to problems (that is to say that mathematicians gain a practical mastery of the field and thus are able to construe the field as a field of possibilities for action). Solutions appear as visual representations of the mathematical referents, and come to mind in the form of mental "pictures", which can often be sketched on paper or on the blackboard. With increasing experience, so the mathematician is able to bring to mind a greater number and variety of mental pictures, which serve to structure how the novel concept is being perceived and interpreted. Topologist, Marie, explains how her supervisor is able to discern a greater range of mathematical phenomena, and thus is able to guide her own perception of the mathematical landscape:

> M: For this Estevan has more intuition than me because he knows all the different guys on the list and he had many more rules from which to choose the deformation that I do not have. I know the list but I have ideas of how to use the different pieces of the theory and the different images, but I'm not sure if they're true. But I don't have as much knowledge as Estevan, so I have to use him a lot.
>
> M: Maybe there are some singularities that I wasn't seeing, because I didn't have them in mind. Maybe there was a bifurcation I was not able to imagine. But I'm still not sure how to go from this to this. For Estevan it's not a problem.

**4.13 The Bridge between worlds: How is meaning translated between domains?**

The quote from Marie demonstrates how, once a suitable reference object is obtained, the solution to a problem can be more easily perceived. The referent provides the necessary characteristics to motivate certain comparison operations, which more easily classify the novel object within a more well-defined semantic space. These reference objects thus can be shared between mathematicians, providing common reference points by which to orient individual's perspectives. The reference domain acts as a bridge between different knowledge domains. Once this bridge linking different domains is recognised, so it can be used as an interface for transferring meaning between them. A geometer, Brian, explains to me the process by which semantic bridges are generated:

> B: I had found some inequalities on multiplicities in commutative algebra, that's one thing and at some point I realised that they bore a very close formal resemblance with inequalities from the theory of convex sets, convex bodies in $\mathbb{R}^n$. So formally the two things are quite distinct, but as it turned out - also this was a stroke of luck because the theory of toric varieties was being born, more or less at that time. And I found that through the theory of toric varieties you could make a direct bridge. So the question then arose

that, if these inequalities are true for toric varieties, are they true in general for any algebraic variety? So this kind of problem you see comes up naturally once you have some attitude. Like when you want to understand specifically why these inequalities are so similar and intuitively you realise that there has to be some connection, and then you try to make it work. And that sometimes gives you some very nice connections. Of course you can't say that this works always. But that is one way in which you can have problems, which at least I find interesting.

IN: So when you see the analogy between two domains can you use the tools you use to describe one object to describe the object in the other domain?

B: No in fact one of the interesting things is that sometimes the tools of proof are completely different but you must build a bridge. As I say, one of the things I was interested in was to prove isoperimetric inequalities from algebraic geometry using this bridge. Of course the way in which one proves isomperimetric inequalities is rather different, within the theory of convex bodies let us say.

But in fact what you use in theory - so you prove inequalities in algebraic geometry which imply those isoperimetric inequalities - but the two methods, in fact, deep down, they have something in common which is something about the theory of elliptic operators. So deep down there is something in common, but in fact that came after, once you realise that the proofs in that field can be connected by a link, then you start saying "well there has to be something common to these two proofs" and then if you look for it, then in fact you see that its more or less apparent [laughs] in the theory of convex bodies.

The bridge acts as a decryption key for translating meaning between different domains. When corresponding structures in one domain are discovered or created within the receiver domain, so objects from one can be transformed into objects within the other. Once a mathematician understands how the decryption key functions, then the process of translating between domains becomes second nature. Indeed the process of translation becomes naturalised, and falls into the realm of the unconscious, intuitive aspects to mathematical work. As the concepts evolve, so the conceptual bridges become part of the background infrastructure to the field, and they are used without comment. The analogical nature of such semantic bridges are thus obscured, as constructions become formalised; thus novel features to the landscape are domesticated, familiarised and forgotten.

**4.14 Affective frames: How is mathematics grounded in the phenomenal world?**

This process of formalisation, familiarisation, and forgetting is common practice in natural language. We use metaphors, foreign words, place names, slang, idioms, without even being aware of it; in mathematical languages the same is true. Novelty becomes banality over time, as concepts are assimilated into a language, and as they are used more frequently. The key reference points within the landscape become shared references by the community of practice. Thus the reference frames through which the mathematical landscape is viewed become common to the community.

This common, invisible, conceptual infrastructure is built upon over time, becoming increasingly formalised and abstracted away from its original referents. But for some mathematicians, in order to gain an intuition of a concept, they must seek reference points outside of conceptual spaces, and ground their understanding within their experiences of the physical world. Indeed the physical world becomes the source of inspiration for many mathematicians, who use physical processes as analogies for operations within abstract spaces. As a topologist, Estevan tells me:

> E: Sometimes you just need to put your hands in the mud and make computations or drawings and you prove something without such a high level of abstraction. Sometimes you even need some intuition from Physics.

One differential geometer, Vincent, goes a step further, and argues that all mathematical intuition has its grounding in an understanding of the physical world:

> V: I like to use this image which is perhaps a bit shocking to you, but I say that you understand a field when you have explained it to your primate. That means that your basic intuition of the world is what the primate has intuition of space, intuition of progression, of size and things like that. Ultimately, when you try to learn a subject you slowly, slowly teach your intuition to work in that subject. But to think of some kind of movement in some abstract space, but which is similar to something that you understand. For example if you learn mechanics, ultimately you understand that in a configuration space a movement is like throwing a stone. So any mechanical object, however complicated, you could spin tops, you could make a big top and put little tops on top of it, and you can make the whole thing turn, and so you have a very complicated mechanical system, but in the end in the right configuration space the trajectory is just a line, it's just a curve, OK. And then that speaks to your intuition of the world because you know that when you throw a stone it describes a curve.
>
> So when you manage to reduce, when you have the mathematical language to reduce complicated objects like that to some relatively simple intuition, then you can make progress. You can imagine problems, and imagine connections and then you become fluent in the field. Even though you may not master all the techniques, you become somehow fluent in the field, you become able to understand proofs.
>
> V: Understanding proof is never something you do by formal reduction, it's always something you do by explaining to your primate how it works. Of course your primate has to be educated. That's what it means to become a mathematician. You have painfully, through the years, taught your primate to understand some things which are actually written in scientific papers. But basically you must understand that when you read the word "hyperplane sections" your primate says "Oh yes, I know that, it's like that", and when you read the world, I don't know - "raising to the square, or raising to some power" your primate says "Oh yes I know that too because you have taught me what It means". Or even "approximation", because your primate has an idea of the real line and he knows what it means to be near or nearer, or not so near and so on.

Vincent argues that mathematicians' intuitions of mathematical spaces are derived from an understanding of processes in the physical world. Vincent's "primate" refers to the embodied, primitive mind of our early ancestors. Abstract thought, he argues, emerges through adapting cognitive structures specialised for visual perception, spatial thinking and planning. For him, constructions, objects and spaces in mathematics bear certain resemblances to physical constructions, objects and spaces. The act of intuiting becomes a way of relating, analogically, the processes occurring in the abstract space to processes which occur in physical contexts. The ultimate reference frame for mathematical reality thus becomes the body's experience of the physical world.

This view does not reduce mathematical reality to physical reality, but rather it allows us to think about the common organising principles which structure both domains of experience. We have seen that analogies lie at the root of human experience, serving to link together elements within the mathematical landscape, as well as structures in physical and conceptual realities. Analogies serve to structure objects by mapping the features of one source, or reference, domain onto a receiver domain. The semantic bridges created between reference and receiver domains then facilitate further transfers of structure and content between them. Through individuals sharing reference frames with a community of language users, so a shared conceptual landscape is created and reified.

The mathematical landscape is perceived as "natural" and objective, outside of the realms of human artifice, yet, as we have seen, these conceptual landscapes are subject to change. We have witnessed how certain objects, concepts and techniques come to play dominant roles as points of reference in the landscape, and how certain proofs and semantic bridges have become formal, invisible parts of the conceptual infrastructure. Shared reference frames, we saw, serve to orient individual's visions of the mathematical landscape. Such frames organise experience, giving structure to the field of view, but, at the same time, they filter out certain elements, and distort one's vision of the field.

The very language that a mathematician uses to describe mathematical reality is thus not neutral, but rather it is the product of history, politics and culture (Restivo, 1994:212). I argue that, like natural languages, mathematical languages possess discourse structures which influence individuals' orientations and dispositions to reality. Mathematical languages form fields of force which help organise experience, and shape the direction of thought. In the final two sections I shall explore this role that language plays in shaping perception. I shall chart the social lives that concepts in mathematics experience, following the biographies of concepts. Finally I shall demonstrate how political and social forces play a role in shaping taste, beauty and truth, through the use of "aesthetic discourse".

## Section 4.2: Socialising the Landscape

### 4.20 Overview

In the following section I shall explore the life histories of mathematical concepts. I shall provide examples of how concepts emerge, how they develop, and how they are shaped by the social world (through "field effects"). The mathematical landscape, we will see, is part of the socio-cultural[13] landscape, and subject to the structuring principles of habitus. In what follows we will come to understand how individual mathematicians negotiate social structures, and exert their own agency.

### 4.21 Introduction

I argue that the mathematical landscape is interwoven with the socio-cultural landscape within which it is created. I will use mathematicians' narratives about their research histories to chart out the biographies of ideas. We shall see that ideas are products of the social contexts within which a mathematician lives. Mathematical concepts are given meaning through their assimilation into pre-existing bodies of knowledge. Ideas become subject to evaluation and classification in their quest to be incorporated within a mathematical language.

For a mathematical proof to be accepted as valid it must, therefore, conform to the conventions of a given mathematical genre, and must ultimately be accepted or reshaped by a community of practitioners. The perceived validity of a

[13] By cultural contexts I am referring to the specific mathematical cultures and traditions of practice which emerge within national, regional, or specific institutional contexts. Such traditions are transmitted through processes of socialisation and lead to specific styles or schools of thought, as well as certain styles for presenting and representing knowledge in mathematics.

proof therefore depends upon the contexts in which it is presented. In what follows I shall demonstrate how ideas are transformed as they move between contexts. Ideas, we shall see, have social lives, forming social relationships with people and other ideas, as they become assimilated and accepted by communities of practice.

### 4.22 Socialising thought: Understanding the social lives of ideas

Mathematicians' social networks[14] are integral to the development, dissemination, and acceptance of their ideas. Supervisors provide guidance, direction and initial problems, which can influence the trajectory of a student's career. Collaborators provide a means of extending a researcher's interests, tools and techniques, and provide opportunities to proof-read papers and test ideas. Colleagues, likewise, provide a source of new ideas, and the latest news from the wider community. Mathematicians' social spheres coordinate their actions, influencing the ideas they consume, as well as the knowledge they produce.

During a mathematician's career certain individuals will have a large impact on the subsequent questions that they ask, as well as the research tools they may choose to adopt. One topologist, Lawrence, describes how chance encounters with two professors shaped his entire research career in mathematics:

> L: I was extremely lucky. I had amazing luck, I had two supervisors from Cambridge. One was a big shot called FA... So FA was a rather forbidding character, and very, very capable, but rather frightening. And on my first day as a graduate student I went to see him saying I want to be your student, but also possibly involving AC. He listened politely and he gave me three possible topics, and I kind of didn't like any of them. I didn't say it quite so

[14] Here a mathematical "social network" is taken to mean the system of relationships an individual forms with other individuals or organisations within the mathematical field. The social network of a mathematician comprises of relationships with colleagues, collaborators, students, supervisors, directors of institutes, members of funding bodies, as well as affiliations with institutions, such as universities, awards bodies, publishers, etc.

brutally because he was the big professor and I was the first day graduate student, let alone first year. Anyway he sensed that I was not enthusiastic. And then a miracle happened. He always came to tea, he was very rigorous about that, and I sat with him at tea and he said he had just been to a big international conference for mathematicians in Nice, in France, and the great Russian mathematician Novikov had been forbidden by the Russians to come and collect his field's medal and Michenko - a young Russian colleague went to give Novikov's lecture for him. And he took notes - AC - and maybe I would be interested in reading them and working in that field. And then a miracle happened, and I've been working on these problems ever since.

If I had said yes to one of these three problems in the morning who knows what would have happened. But I held out till the afternoon. [The problem I chose] was something AC could help me with - it's called Surgery theory. So I was very, very fortunate that AC didn't work in surgery theory himself - he worked in Homotopy theory - a neighbouring subject. But he kind of sensed that I would never be a Homotopy theorist. He used to say that I couldn't even pronounce the word correctly - I put the accent on the wrong syllable. But he did point me in the right direction, he was very kind to me. Maybe because I wasn't really working within his own field. I mean his own students didn't manage to get PhDs by and large. At least I managed to get my PhD. He was a great man and, much later, I dedicated one of my books to him. But sadly 25 years ago, he died in a car accident, but I've cherished his memory.

The narrative reveals a number of important sociological processes which are involved in the replication of mathematical habitus. The first is the apprentice-master relationship between student and supervisor. Oftentimes a mathematics PhD will have much more close supervision and mentoring in comparison to students from other disciplines. The internationalisation of research networks reveals itself in AC travelling to Nice for a lecture by Novikov. The role of informal conversations and social exchanges at Tea-time and coffee breaks becomes apparent, as well as the continued friendships which persist between Lawrence and AC, even after he graduates. The fact that AC is not a competitor to Lawrence, because he does not work in Surgery Theory, also plays a role in the Lawrence's success within the field, as well as his continued friendship with AC. Finally the gift of a dedication to AC in Lawrence's book is a way of reciprocating AC's initial gift of the Novikov lecture notes early on in his career.

The process of becoming socialised within the mathematical community is similar in other cultural contexts. Below for example Bertrand gives his own personal experiences of becoming enculturated within the French education system:

> B: So while I was in the Class Preparatoire I became interested in number theory, and then I was a student at the Ecole Polytechnique. And at the Ecole Polytechnique I was still interested in number theory. But there it was very difficult to study anything. Everything went very fast and you didn't have much time to concentrate. Anyway, when I came out I was still interested in number theory, especially additive number theory, at that time... But I had the good fortune at some point to have lectures by Zeriski. And then I became really enthusiastic about what he was doing, and what he was talking about. I did not understand anything about what he was talking about, but it just looked great. From his personality, from the way he spoke about it, as I say I couldn't really understand anything that was going on.
>
> Especially at the Ecole Polytechnique we learned analysis and practically no algebra to speak of, only Linear algebra, and that was terrible. So I didn't know what a ring was, I didn't know what an ideal was, none of that stuff. And so, to me, what Zeriski was talking about was a kind of beautiful poetry. But, as I say, I couldn't understand anything. But then I started working on this with the help of Pierre Samuel, who was really encouraging. And then I had a second stroke of luck, which was meeting Hironaka - and he then became my teacher.
>
> And so then I became interested in singularities, first listening to Zeriski, then meeting Hironaka. So it was a great thing at that time because you could get a position without, practically, having done anything. [Laughs] I mean I entered the CNRS practically straight after the Ecole Polytechnique. I didn't have a thesis, I didn’t have anything. I just had some recommendations from my teachers at the Ecole Polytechnique, and that was that.
>
> So at the beginning, as I said, I didn't know what a ring was. I remember that the first sentence of Zariski's paper was "Let O be a one dimensional local ring…" and so every word was new to me [laughs]. So, after several months, we worked at it, and finally understood something, and gave some talks finally at the seminar of Samuel. So that was the

> influence of Samuel: to always be very encouraging and open, and so we all have a debt to him. I mean we were all nothing to him. He was not at all obliged to help us...Yes I have a very fond memory of him.

Mathematics in France is a highly competitive field to enter into. Individuals who are academically gifted find that the natural course for them is to continue through the top schools, through preparatory schools, and onwards to university, eventually becoming researchers or professors. The school a mathematician attends has a large bearing on subsequent career moves they can make. However, association with well-known figures in the field can serve as a way to step around some obstacles. In the above narrative Bertrand tells us how his advisors' recommendations allowed him to acquire a job at the CNRS, despite having no publications, and little experience in research.

It is perhaps because of such kindness from members of the community, or through the results of fortuitous encounters or events, that many mathematicians attribute their success not necessarily to their own skill, but rather to their own good luck. Success, they argue, is never an inevitability; it is never just the product of genius, rather it is a result of navigating successfully through the social field. In order to navigate across the social landscape an individual requires guides. In Bertrand's narrative such guides take the form of influential lecturers, such as Zariski, advisors and mentors, such as Hironaka and Pierre Samuel, as well as fellow students such as Monet. These guides provide access to new ideas, to social networks, to employment, as well as to social capital. These influences from the social world therefore shape the directions that a mathematician takes during their career, and this has an effect on the concepts they consume and produce as a result.

**4.23 Social histories of thought: How the mathematical field is reproduced**

If we turn now to exploring the life-histories of ideas, we see that they are inextricably intertwined with the lives of their creators. The origins of ideas are often hard to pin down, rather they emerge unexpectedly during the course of one's life. Lawrence, recalls his first unexpected encounter with the Geometric Hopf invariant, a concept which was to influence his subsequent research in topology:

> L: So that was all very important, but not mathematical. The next day I went to a book shop, right there on Mallet Street, which I used to go to as a student. And this was a bookshop - I've forgotten the name - but it's the - it was in those days. In the 60s when I was young, the most important bookshop for mathematics books in London.
>
> Now - and there was a book - just came out by - not just by MC - but by MC and his supervisor called EJ, who was at that time one of the main professors at Oxford. And most of the book - I looked at it and I spent about 5 minutes looking at it, and most of it, to be honest was slightly dull. But there was one page when I realised: this is very interesting. And I've been - so this one page - contained something called the Hopf invariant - the geometric Hopf Invariant - which I realised was going to be very important for me. And so ever since, I came back here, and I asked MC whether we could work on this together. And so we did - and so I have this, unfortunately, rather long manuscript. Which is now, however many pages it is [flicking through].

On first encountering the geometric Hopf invariant Lawrence contacts the author and begins a conversation with MC, which eventually leads to a long-term collaboration. Lawrence later explains how this concept of the geometric Hopf invariant had itself been passed to MC from MC's supervisor J:

> L: I was trying to understand…well to use technical words - in my early work I showed how to go from something called the stable map of spaces to

> quadratic functions. It had been known for a long time that stable maps give quadratic functions in algebra, but what he did - in the geometric Hopf invariant - was to associate to a stable map a quadratic function in geometry. So he had a very concrete construction which goes back to something his supervisor had done. His supervisor, professor J, had done very good work in the 1950s, and some of it is forgotten. But there was one idea there that was crucial for MC and for me. It's a very simple idea - it's called the difference construction. If you take - I mean I can even say it in one sentence - we have two maps of spaces we cannot add or subtract them. [But] you can with stable maps.
>
> I mean that sounds rather stupid, but it has been long known that stable maps have better addition and subtraction properties…it's a simple idea, but I didn't know about it to be honest, and I wish I had had the gumption to ask myself - but I didn't. But that's alright, MC did have the gumption. But what he didn't know is how to apply it to my - what I call "Surgery theory" - that's where I come in. He was doing it for his own purposes. And I could see that there were other purposes. So that is very good, you always want to do mathematics that grabs somebody else. Of course you can try to hammer somebody and say: "I have solved this great problem". But he wasn't solving a great problem, he was just developing a nice technique which was quite useful for him, and I suspect that I'm one of the few people that have read his books. I may be wrong. But it's really an acquired taste.

Through collaborating with MC, Lawrence is introduced to this new set of concepts which MC had inherited from his supervisor J. Lawrence's contribution to the collaboration was to make MC's technique more accessible, and useful, within the context of Surgery theory. As Lawrence indicates, few people read MC's books because they are perhaps too specialised. But, in order for such ideas to become more well-known, they must be written in an accessible format. Below, Lawrence continues his narrative, explaining that MC's writing style can be inaccessible because of its clipped, condensed, "telegraphic" style:

> L: In fact he (MC) wrote much shorter papers, in telegraphic style, 40 years ago, 30 years ago, and the reviewer wrote: "This is a master piece of telegraphic mathematics" [Laughs]. So his comments to me, on our project, are a bit similar. Actually to be fair to him, earlier this year he has written all of 15 pages of text, which I'm struggling at the moment to work out what to

> do with, because you can't just slot it in. The way he writes you can't just take it and put it as a chapter of the book, and off you go…So I'm struggling with it a lot, and I may have to invite him to come here, or I'll go there. But I mean he's obviously a very good collaborator in the sense that he's right. A very bad collaborator in that he doesn't…he's repressed [laughs]. And a repressed mathematician is a very tough customer - but they're always right… You would like a little bit of explanation, a little bit of the human touch - a little bit of almost encouragement. But I'm slightly being unkind.

The style of one's presentation of a proof in mathematics can thus be just as important as the content of the proof. For an idea to have some success, to influence other mathematician's work, it must be presented in a conventional, accessible way. Ideas in mathematics can thus bear similar characteristics to their creators - being social or antisocial, introverted or extroverted. In the above example, MC's proofs are an acquired taste, requiring specialised knowledge in order to be fully understood.

Other researchers, however, are more easily able to move between different social and conceptual contexts: crossing fields and adapting their conceptual apparatus to translate meaning between knowledge worlds. Andre, a geometer, tells me how he prefers to assimilate tools from many different contexts:

> A: So my PhD was already the kind of stuff like that. I had a very strange question, from my advisor, which said that "someone has proved this for elliptic curves and can you prove this for Abelian varieties which are generalisations of elliptic curves?" And the tools employed by the other people were clearly not applicable in the general case...Elliptic curves is a very nice subject because it is linked with very abstract theory, but also its very concrete with many equations which you can write down explicitly. So you can mix them. So I just tried to work on that. Incorporating what I knew from other fields, and other points of view. I was beginning in what is called Arithmetic geometry, which is already a geometrical way to look at number theory, that mixes algebra, geometry, number theory and analysis. Everything is there already, so that was my kind of basic training field, for

my Master's thesis. And then the question that was asked to me might have had some relation with that point of view or not. But I thought of it in this point of view, and gradually I understood what happened, and how one should prove the results I was interested in. And, ultimately, it appeared that what I did was not so unrelated to the case of the elliptic curves.

A: In fact, after my PhD, I tried to solve another problem in which I learned a new set of techniques. Not really learned, but used. Back during my PhD I was close to another colleague and we talked about math and he used these kinds of techniques of distributions that I referred to. And so we discussed this together, just because it was interesting. So I followed very closely what he did in his PhD thesis, and it was clear that those tools were relevant for what I did at some point, so I absorbed them, and used them. And those tools - it is precisely motivated by those tools - that I tried recently to develop the geometric theory. And so, in some sense, what I have inherited from my PhD times is some kind of point of view, which I try to impose on problems that I want to solve.

By moving between fields, and assimilating knowledge and styles from a number of contexts Andre has been able to make his ideas much more accessible. As a result his papers are more likely to be read, and therefore passed onto a wider community of practitioners.

### 4.24 From Private thought to public practice: How do ideas become part of habitus?

These issues of accessibility confront the mathematician upon publication, and during presentation. When ideas move from the private to the public sphere, so their structure needs to change in order to conform to the conventions of a community. An algebraic topologist, Jessica, describes her experiences of presenting and translating her ideas to a non-specialist audience:

J: So the problem is, if you want to describe curve here, I needed to describe what it means to "Begin like alpha 2". When you draw a picture you say: "Oh look, it begins like alpha 2!", but then, when you think about that, it's up to isotopy. And if you just say: "Begin like", your alpha 2 can go anywhere in the surface, because you can move it. So I can make a code, just like this:

> you choose an equator and you put a name on every segment between the points of the Cantor set. Imagine it is like that, so you have a name for every segment, 1234, then you say: "This is like beta". And so this is a picture of beta. But then you say beta is like 3, because you go to 3, 1,4 and three; and there is only one way to do that. And when you write it this way its rigorous, because there's only one way to do that. And then I went to Mexico and people read my paper and they said: "We didn't understand what you said about the coding". They were like: "What are you talking about?" [Laughs]. Yeah because when you write just the results, I try to write it pedagogically, but I think I failed. When you write it at the board, taking the time and everything, people are like: "OK I understand that". But when you just write it with the code they're like: "Wow. What just happened?" We have to do it this way because picture is not a proof.

When ideas move from one's private thoughts to a public setting so they must be formalised, conventionalised and stylised to suit the audience one is presenting to. Ideas thus are transformed to fit the classifications of the genre, and are formalised so as to create a familiar context for others to understand them. A common conceptual ground is thus created, through ideas being related to a certain history and tradition of thought.

**4.25 Contextualising thought: Understanding the domestication of ideas**

For ideas to be adopted and accepted by a community they must be presented in an acceptable manner. Ideas must be fitted into the rest of the field, through creating relationships with other concepts and problems in the field. To give order to new ideas one must fit them into existing categories, and construct historical narratives around them, which legitimise their presence in such categories. Vincent, an algebraic topologist, exemplifies this process, when he traces out the lineage of his idea:

> V: Alright - so my thesis was all to do with chain complexes. So these are purely algebraic objects, and they have quadratic structure. So one of the big subjects in mathematics is quadratic forms…And originally the modern version of quadratic forms works with rings and modules and that's fine, that's kind of abstract algebra. It has a history going back to Lagrange, in the 18th century, Silvest in the 19th. Between Gauss in the early 19th - many, many famous mathematicians have worked on quadratic forms. That's a standard topic of number theory, and also has very important application in topology.

Other researchers build upon the ideas of contemporary influential individuals. Bertrand, below, describes this process of simplifying, generalising, clarifying and formalising the ideas of Hironaka, in order to make them more accessible to other members of the field:

> B: You know I think that, as far as I'm concerned, you learn things, you try to understand them, and problems sort of jump at you. You suddenly see that if things are this way they would be simpler, and it would clarify things, and then problems suddenly jump at you. Of course you can choose one of the well-known problems, but I don't think that's ever the way it works. In fact Clochet and I went to Harvard to work with Hironaka and, at that time, we were helping in the reduction of his work on singularities. But that was Hironaka's problem, Hironaka's ideas.
>
> But as we worked we tried to simplify, and, in trying to simplify, we raised questions and problems. In the end it was very fruitful. We found several things which were motivated by the fact that we had to try and understand some of the techniques that Hironaka used. We wanted to perfect them, to make them more intuitive, and that's how problems arose. And then later, well, there is another way, of course, to find problems, which is to make analogies, or to understand some operation. For example, a little later, I tried to understand what the effect of hyperplane sections had on singularities in a precise way. And this raised a whole series of questions.

This process of socialising ideas within the field, of creating relationships to other, existing, accepted concepts, is the means by which novel concepts become domesticated within the existing field. Novel ideas become adapted and formalised so as to fit the contexts in which they are being presented. Through this process of contextualisation, they are incorporated and related to the pre-existing habitus. Novel ideas are sewn into an historical narrative, related to other authority figures within the field, and thus given a place within the existing social and conceptual landscape. In the next section we shall explore how a common conception of fields within mathematics are created through this process of socialising ideas and individuals. We shall see, through creating a system of rules, standards and conventions, so systems of evaluating merit and ascribing status emerge.

## Section 4.3: The Field of Mathematical Production

### 4.30 Overview

In this final section I shall discuss the role that authority figures play in shaping mathematician's visions of the mathematical landscape. I will explore how authority figures help shape discourse and influence the directions of thought in mathematics, as well as the means by which points of view and dispositions (habitus) are reproduced and inherited by subsequent generations of researchers. Through habitus, we shall see, mathematicians come to experience a field of force which shapes their tastes, their values and judgements, as well as their visions of mathematical reality.

### 4.31 Introduction

I argue that mathematical habitus reproduce the social order, through replicating certain values, habits and dispositions within communities of practice. New members of the community are assimilated within it through adopting and adapting the linguistic, conceptual and social machinery needed to produce the mathematical field. Through becoming socialised within mathematical cultures, so individuals adopt the discourse structures which give order to their worlds.

Discourse in mathematics takes the form of talk about truth, beauty, value and meaning. Such discourse serves to transmit the rules and conventions of the group, shaping how phenomena are evaluated and interpreted by members of the community. Such discourse thus serves to structure how individuals perceive and relate to the mathematical landscape they encounter, and shapes the community's interests and orientations within the field. In the following we shall see how influential individuals shape the field of discourse, and influence the values, interests and tastes of other members of the mathematical community.

**4.32 Footprints of the field: Understanding how authority figures shape discourse**

In mathematics, the impact of individual agency on the structure of the language becomes evident in the naming conventions for concepts, techniques and objects. There are Euler numbers, Lagrangian operators, Wiles' Proof, Conway's Thrackle conjecture, etc. The naming conventions provide some indication of the origins of certain ideas, and of the individuals involved in generating those ideas. These individuals are immortalised in the named features of the mathematical landscape, and they serve not only as guide-posts for navigating the field, but also as goal-posts for the achievements mathematicians should aspire to in their careers. Such influence is highlighted in the quote from an algebraic geometer, Claude, below, which discusses the influences on his research in Homological algebra:

> IN: How did you become interested in Homological algebra?
>
> C: It was in the air. Maurice Alessandra came for one year in Paris and gave a series of seminars and he was a tremendous person. He was a very creative and elegant mind. And a friend of mine, he's in New York now, we got very interested in what he was saying and explaining. And that's how I began with that. But he was invited by our advisor Pierre Samuel - so there is some kind of continuity. But at that time you chose your subject, you chose your PhD subject. I mean there's nothing original in that, everybody was doing that, with or without great successes.
>
> The trouble is that I changed my research subject. I moved to, not completely, but I moved to classical geometry, compact geometry. And also that was taught to you - if you wanted to progress you had to extend your results, if you've found a reasonable group of results that has been recognised by publication in journals, although you didn't solve, in my case, the main conjectures, still you made a way through a group of conjectures

> which have been accepted as important conjectures. You have published in decent journals and at some point, in my case, it was 10 years in the same subject and it was good enough.
>
> And I slowly moved, changed collaborators, and moved to classical geometry, which at that time was a very active part of mathematics, classical projective geometry, which is of course not the case now. Under the influence of Grothendiek of course as was my first subject, which was clearly under the influence of Grothendiek. So I moved to that, published some good papers, one which had, historically, a very impressive success, numerous citations, I don't know how many. Which doesn't impress me that much by the way because I was lucky at that.

Above Claude explains the roles played by a number of influential individuals during his career. For many mathematicians in algebraic geometry Alexander Grothendiek plays a leading role in their narratives, with their intellectual lineages being traced back to his foundational works on schemes, etale cohomology, and k-theory. Many mathematicians confess to having an intellectual genitor or "hero", who inspires their work and career directions, as Nico explains below:

> N: That's true. So I agree with you, I have some mathematical heroes and I want to see their notes. So my main mathematical hero is Poincaré and what is unusual about him is that even his writing style, in his papers, he never returns to what he said, so there are a lot of mistakes, a lot of dead ends, even in his published papers. But there are not so many Poincarés. I'm sure there are people like Gromov today, I'm sure there are mathematicians who have kept their notes and I'm sure there is enough for an army of historians and sociologists. I'm 40 now, I'm no Poincaré. I think I'm a decent mathematician, but there are plenty of mathematicians like me. It's not like I'm going to change the face of mathematics.

Nico uses Poincaré as a point of reference for his own work. Despite his own considerable achievements, Nico believes he pales in comparison to Poincaré. Living in the shadow of giants thus leads to many mathematicians adopting a discourse of modesty when talking about their work and achievements. Their heroes, on the other hand, have their reputations and histories embellished and mythologised. Narratives are constructed about famous people, which add to their sense of mystique and genius. As algebraic topologist, Evan, explains, famous individuals accrue status and fame, as ideas seek to become popularised through association:

> E: Most ideas are attributed to people because they are well known as they had made the ideas transmissible. But they were not the original authors of the ideas. When an idea appears in general it is not understandable. And it appears in that context and you have to find it. The use of it comes after. The guy who comes up with the idea should be credited with it, but that's not always the case.

By associating certain ideas with certain names and persons of influence, ideas can thus themselves become more visible within the field (they can accumulate more social capital). Status can be transferred from people to ideas, and thus ideas themselves can become influential and widely recognisable. It is perhaps for that reason that many mathematicians prefer to work in the shadows of giants, and to construct their intellectual lineages in descent of great men and women. Such histories of ideas assimilate individuals into wider narratives, and serve to legitimise their own ideas, through deferring questions of authority and legitimacy to their intellectual forebears. However, such deference to authority figures does lead to a certain degree of distortion of the past, and the figures of the past, as differential geometer Michael explains:

> M: There is no contest between people. People like Thom are original, his ideas are his ideas. Some people say he had not proved things, which is quite often true, but quite often he gave the main ideas for the proof. That's what happened in singularity theory.
>
> Usually the process is that you have a really good mathematician exploring an unknown area and they do almost everything that is interesting

> and what is left is the bones, so to speak. And people call that someone's conjecture and it becomes a prize.  But it's maybe not as interesting as the interesting things that have been done before. But it’s a matter of taste. There are many conjectures like that, which are still unsolved and may be considered interesting, but it's not obvious that all of them are.

As Michael points out, great individuals are not infallible. Their work may later be scrutinised and found to be flawed, untrue, or incomplete. The contribution of the great individual however is not only the result of creating accurate proofs, but rather it is through their originality of vision or approach, or through their leadership, which distinguishes them. However, when we start to dissociate authority and legitimacy from truth, then we begin to see, as Michael points out, how there is a certain degree of arbitrariness in significations. Prizes, status and position have a lot to do with taste, as much as they have to do with correctness.

**4.33 Visions of truth and beauty: Understanding how discourse shapes perception**

Despite the arbitrary nature of taste, in mathematical discourse there are perceived to be certain absolutes about mathematical proofs and structures. Beauty is one of these absolutes, and manifests itself in aesthetic discourse. Such discourse naturalises certain values and dispositions towards features of the mathematical landscape, shaping how we come to perceive and relate to them. Discourse around "truth", similarly, is used as a means of legitimising certain ways of viewing the mathematical field. Below geometer Carl gives a description of what an ideal or "true" mathematician should be:

> C: A true mathematician?  A true mathematician is somebody who pursues truth with passion, with no compromise. He is obsessed by truth - what is truth? And you understand by how I live that I have no problem with compromise, because in life things are based on compromising. But that is a

very non-mathematical approach to life. Many true mathematicians - they don't even understand that question. They pursue truth with elegance and passion and they don't believe that the rest has so much value. So this obsession with finding the truth takes all their strength. I think that's a decent definition of a true mathematician. It's a test for truth and truth does not exist outside of mathematics. Even in physics it is unclear, but in mathematics it is a unique community in the world where the definition of quality is essentially accepted and shared by all the community. Go visit a physicist they are ready for war every day. Mathematicians more or less agree - it's a very special group.

This idealisation of the mathematician is no doubt based upon certain cultural tropes, stereotypes and prototypes from the popular mathematical imagination. Certain famous individuals such as Leonard Euler, Henri Poincaré, or Isaac Newton, become the reference points by which to guide mathematician's perception of success within the field. These utopian visions of the "True" mathematician are part of the schemas which comprise mathematical habitus. The true mathematician becomes part of the imagined field of mathematics, along with the ideals of truth and beauty, and a belief in consensus and harmony across the field. Such images of the ideal mathematician structure individuals' conceptions of the field of mathematics by cultivating certain sets of values and judgements. Cultivating a certain style of thought, or habitus, influences how an individual relates to mathematical reality, but, as Topologist Estevan, below, explains, entertaining certain viewpoints or ideologies can narrow one's vision of the field as much as it can expand it:

E: The mainstream is still very much in the Bourbaki style[15], which is very abstract, not drawing too much. It's a French tradition and very strong here. Most people in this corridor they think like me. We work a lot with

[15] Nicolas Bourbaki was a pseudonym adopted by a group of primarily French mathematicians operating out of Paris during the 1930's. Their preoccupation was on creating a set-theoretical foundation for all of mathematics.

> theoretical physicists. People in quantum gravity for example need our work. There's been a change in the world, now the concrete people are mathematicians and the abstract people are the physicists. It's the physicists who are inventing very complex objects to describe relativity and quantum mechanics. Mathematicians we don't pretend to understand this. We just walk along a very small part of this field. It's a very interesting observation that in the middle of the 20th century the Bourbaki viewpoint has led to a very important set of results.
>
> The Bourbakist idea works like this, if you think of some problem you can make it very general, very abstract and then it becomes clear and simple. It's a fact that sometimes this approach works. If you find the right structure and ignore everything else, if you keep the important structure then everything becomes clear and understandable. It's wonderful to see this. To transform a problem that was very complicated, once you put in some complicated language then it becomes simple. So there was a kind of dream that everything could be handled this way. This was the Bourbakist's dream. But this is only partially true. There are always people like Thom or isolated French mathematicians, Russians, Americans, who were very suspicious about that. This approach leads nowhere - just to pure abstraction, with no deep result.

One's vision of the mathematical landscape, we see, is never pristine. Rather it is influenced by habitus, by the values and dispositions generated by certain ideologies or "styles" of thought. The ideals of truth and beauty are likewise the product of ideology, taste and fashion. Such value-categories therefore are not fixed, and absolute, but rather they are fluid, and change with time. The importance of certain proofs and constructions is thus subject to the judgements of the community of practitioners, and is therefore influenced by the dominant tastes and aesthetic sensibilities of the social group. Through discourse on truth and beauty the individual mathematician is influenced in the directions their research takes, the questions they ask, as well as ways in which their ideas are presented. As topologist Lawrence explains, the directions of the field are constantly changing, as tastes and interests evolve over time:

> L: As I said he was a great man. But that was my, as I said, rather unexceptional history. But I was fortunate in that the field has developed relatively differently. It used to be very popular in the late 60s, and then in the 1970s, just as I got into it, it stopped dead, and so I've kind of been a bit like the last of the Mohicans - everybody has moved on to something else and I'm still working on surgery theory. But it's alright I can live with the shame. In fact when people in more fashionable fields need to know something about this theory they come to me. At any rate I have my speciality - it's not so narrow. It is somewhat narrow. It's not as fashionable as it used to be, but it hasn't completely gone out of fashion either. It's a kind of interesting field and it's quite difficult for outsiders to get into as well. That's a problem. So I have to spend a lot of time explaining to my students what it is and so on, there's quite a lot of background.

**4.34 The Value of visionaries: How agency affects the structure of the field**

If we accept that tastes change over time, then we come to understand that the ideal of an absolute sense of truth and beauty in mathematics does not exist, but rather is a product of the beliefs of the present community. These beliefs, in turn, are shaped by influential individuals and groups within the field. Such social entities exert their influence through their control over both the means of knowledge production, and the distribution of social capital. Through editors controlling what is published in journals, or committees allocating prizes; from directors deciding where to allocate funds, to the heads of departments choosing who to hire, so there are a number of decision-making entities, at local levels, who sum to affect the directions of the global field.

Decision-makers help to shape the tastes of others, by assigning status (implicitly or explicitly) to certain problems, questions, or ideas. Such assignments focus attention on one part of the field and therefore not on others, and, as a result, resources flow to some areas and not to others. By assigning values and statuses to problems and ideas, so the field is stratified into different value categories. Through such stratification, a structure is generated by which individuals working on certain problems or proofs accrue capital. The importance of problems are often justified through defining them as "difficult", but, as Bertrand explains, the "difficulty" of problems is itself relative to one's viewpoint:

> B: So there is not much philosophy in that, but still it's the source of problems. You have to choose the right "a implies b" sometimes the question is not so interesting and sometimes it becomes like the Poincare conjecture - it becomes one of the major questions. If you look at how the problem was born it's just "I know that a implies b, but does b imply a".
>
> So I don't think that there should be a hierarchy of problems, I think that the sources of problems are very diverse. But I think for example that it's totally wrong to say that some problems are worth a million dollars[16] and other problems are not worth anything, or maybe 10,000 dollars [laughs]. I think it's totally absurd because to me it's a sign of arrogance. Because many problems are solved because someone makes a small shift in viewpoint and suddenly a solution appears.
>
> It's very arrogant to say "this is a very difficult problem". It just means that we don't have the right viewpoint. And if we had not had the work of the 19th century mathematicians on elliptic curves, which was really I think governed by an aesthetic need to understand those complicated elliptic functions from many diverse viewpoints. This occupied many of the best mathematicians in the late 19th and early 20th century. If we didn't have that, then Fermat's theorem would not have been proved because, basically, all the theory that went into it took its roots, after the discovery of Gauch, it took its theory from the classical theory of elliptic curves. Of course it was made arithmetical and made closer to Galois Theory in many ways. But, nevertheless, without this sort of background in elliptic curves maybe Fermat's theorem would not have been proved.
>
> But who can say which theory is going to be useful in solving which problem? In 1950 nobody would have, before Gauch, have thought that behind Fermat's problem was a difficult problem on elliptic curves so as I say I think that we should be more modest, that is what history teaches us. It also teaches us that a problem like that which does not look so interesting in the end has something fairly deep behind it perhaps, but we don't know because Fermat's conjecture has been solved, but the ABC conjecture is still not solved. It may be solved by an elegant change in viewpoint. But who

[16] Bertrand refers here to the Clay Mathematics Institute's 7 Millennium Prize problems. These problems include the Riemann hypothesis and the Poincare conjecture among others. Successful proof of these problems comes with a $1 million prize.

> knows - someone has claimed to have solved it. Mochizucki has - but apparently no one understands.

Bertrand argues that, by creating hierarchies of problems, mathematicians narrow down their fields of view, and neglect certain viewpoints. The effect of popularising certain ideas over others can also influence the career trajectories of researchers, as Alicia explains:

> A: I was a little lucky when it comes to my degree. When I was doing my masters there was the proof of Fermat's conjecture. In [19]92. It changed everything in this field. Before the proof of Wiles everyone said that it was a very bad subject and we cannot prove anything in it - it's too difficult. After Wiles there were a lot of new questions, and a lot of money for us to go to conferences. Everybody was excited. It was a great time to begin in this field. So I'm lucky in that respect. At the time I only tried to learn a little bit about Fermat's theorem. So I tried to understand a little part of the theorem and to use the theorem to understand some other parts. It was a very nice time.
>
> I was also lucky because since there is this proof of Fermat, the community received a lot of money. So in Algebraic geometry we arranged a lot of conferences. Beside these we had a semester at the Institut Henri Poincaré. So it was a great chance for me because every mathematician in the field was there. So I met everyone at this conference. I began to discuss with CB. And it was very nice because he is a wonderful person. You know in mathematics there are people who create new mathematics, and he is one of these people.
>
> At the beginning he was quite sad. CB is a student of Fontane's - he was not "CB" at this time, he hadn't yet got his personality. His title was "the student of Fontane". He didn't think at the time that people believed his theory.
>
> At the time I shared an office with him, but I believed his theory. I'm not a mathematician like him, I don’t have so many good ideas, but I like to make computations. I enjoy computing. I did a lot of computation for CB and it was a wonderful collaboration, because he had so many good ideas. But he was quite sad because he thought that nobody quite believed his ideas. But I believed, and did a lot of computation for him.
>
> He had a big influence on many people. He created an entire field of research. I knew how to compute a mathematical object which is called a deformation ring and he knows this theory of Fontane, because he was a

> student of Fontane. And if you put everything together you obtain our conjecture. This conjecture, now called the CB-AM conjecture, it's in fact the first symptom of the P-adic Langland's theory. It’s the first time we see the two different worlds are related. It was a numerical aspect of the P-adic Langlands programme. It was the first computation which shows that the two worlds should be related. So a lot of people are working in this field, and he created this…CB is very original. He was one of the first people to have the idea for finding a path.

Being part of a dominant discipline gives a researcher access to resources: to funding, research positions, statuses and prizes. Being part of a dominant area of the field is empowering, through providing individuals with opportunities to travel to conferences, to present their work, to build their networks, and to enhance their profile and reputation. A famous problem in a popular area can therefore create famous problem solvers. The same is true for more visible parts of the field, which make individuals who work in such areas more visible. To become visible within mathematics thus requires not only that an individual presents "important" ideas, but also that they present such ideas within visible fields, in manners fitting to the field, and have their work recognised and valued by others within such a field. To be valued as a "visionary" is thus dependent on there being a suitable audience to recognise one as such.

**4.35 The fields of force: How social status shapes the perceptual field**

Authorities invested with status, influence and social capital shape the mathematical field; through the forces generated by capital accumulations, so individuals gain the ability to shape perceptions within the field; but such forces exerted by capital-rich individuals also have an effect on other individuals who comprise the field. Such forces, however, are often invisible, hidden in discourse, and made to appear natural. These forces, as we have seen, can make certain viewpoints more visible, they can make ideas more recognisable, or individuals more reputable. But equally the forces generated by capital-accumulations can also destroy visions, careers, and ideas. Individuals who wield these forces can have a lasting influence on the field and the lives of ideas and individuals, as Martin explains below:

> M: You must realise that as a student of Thom, I was much frustrated by the end of the 70s because you know that in the 70s you had Thom's books and you had Zeeman's papers. And so it was very fashionable to do catastrophe theory and lots of people were saying absolutely stupid things about the subject. And at the end of the 70s Smale had decided to write a paper in the Bulletin of the American Mathematical Society, which was strongly against catastrophe theory. For me it was a catastrophe, so to speak, because that's what I wanted to do. And in fact, as I had to earn a living for my family, I was a coward and stopped.
>
> I started doing something like catastrophe theory and I only started doing catastrophe theory in 2005. I have always taught bifurcations and so on and it's closely related to invariant manifolds so I cannot say I didn't do it at all, but nevertheless I started doing things in that domain in 2005, and then what I tried to do was something considered impossible. Because one of the reasons why catastrophe theory was considered a failure was that the catastrophe theory for dynamical systems is not something as clear as for differentiable maps.
>
> In fact what I started doing was what Thom calls the natural stratification of a space of functions. I started doing a stratification for the space of dynamical systems. So it's not exactly the space of dynamical systems, it's the product of the space of dynamical systems with the manifold on which they are acting. And so in fact it works. So I started to carry the things a little further than what was known, in particular I proved an extension of the Hopf bifurcation - this is what happens when an equilibrium becomes a periodic orbit, and this is rather a brutal process, it happens all at once. And you have the same phenomenon, for maps, if you have a fixed point, it becomes an invariant circle all at once and it has a translation for vector fields. You have a periodic solution that becomes quasi periodic, that is a torus. But what I proved was that if you allow more than one parameter then you have lots of invariant compact manifolds arising. You have spheres, you have all kinds of intersections of quadrics, you have tori which have larger dimensions than what you imagine and so on. And all this takes some room, meaning that it happens in open regions. It's a strange thing.

IN: Did Smale's article make catastrophe theory more unpopular amongst the mathematical community?

M: Yes, of course. It was strikingly effective. The problem is that at the time, in Europe, even in the United Kingdom, research was not paid for by public funds, based on projects. You did not have to propose a project to get money. This concept in mathematics did not exist at all. So if someone in Europe wanted to say something absolutely stupid in catastrophe theory it didn't cost one cent to the tax payer. But unfortunately in the United States it was already like it is now everywhere, that is it costs a lot to the tax payer. That is the reason that Smale felt compelled to say that lots of the funding allotted to catastrophe theory was not well placed. He was right, but on the other hand, I still remember it because that was when I became a mathematician. And it's quite remarkable how it made people from different parts of mathematics and even different parts of science communicate. For example, people doing meteorology learnt about the Hopf bifurcation at the time, because they discussed it with mathematicians. And it has become impossible again after Smale's paper, and of course the funding for the research in Catastrophe theory went down drastically. And in fact I still believed it was a scientific error. Of course there were absolutely silly things done under the name of catastrophe theory. But there were very deep ideas that needed to be developed and extended. And this didn't completely stop at the time, but only people like Forest Takens in the Netherlands or Vladimir Arnold in the Soviet Union were strong, or isolated enough, to go on with that. Because in other parts of the world it became not only unfashionable but it almost a dirty word to say "catastrophe theory". It was really hard for people like me, because I was too young to have gone on with it and convince people. But now, I have no career and it doesn't matter.

We see that Smale's article acted as a catalyst which hindered the development of an entire field of enquiry within mathematics. Martin's account exemplifies the role that the social world plays in influencing knowledge production in mathematics, and particularly the agency that famous individuals can exert on the structure of the field.

The social world, we see, penetrates the mathematical landscape, shaping its contours, its features and its boundaries. The social world, we see, is a field of force which produces and reproduces the field of production, through the generative schemas of habitus.

In ending we then see that thinking of the notions of truth and beauty as absolutes, can no longer be entertained. Rather, such ideals are part of value categories constructed within the realm of discourse. Such values are generated within a social field, by social entities, for the purpose of reproducing certain visions of the world. In the above I have attempted to give a sense of how such visions and narratives come to dominate the field of discourse in mathematics. I have attempted to show that tastes and ideas evolve as a function of one's social context. And finally, through the example of catastrophe theory, I have tried to show how individual agency can have a lasting effect on social structure.

# Chapter 5: Discussion and Conclusions

## 5.0 Overview

Within this thesis I have introduced three different "frames", which serve to shape mathematicians' perceptions of the mathematical field. In the first frame, the physical frame (chapter 2), I demonstrated how space is utilised to structure routines, and performances of self and ideas within mathematical institutes. In the second, conceptual frame (chapter 3), I demonstrated how reference frames are created within physical space which serve, analogically, to model abstract conceptual spaces. In the third, discourse frame (chapter 4), I demonstrated the role that social relations play in shaping perceptions of taste, aesthetics, and importance in the field.

Throughout I have focused on positioning the mathematician within the mathematical field. This positioning, I explained, takes place within multiple dimensions, as a result of analogous structuring principles operating simultaneously across these dimensions. A mathematician's dispositions and perceptions of the field, I argued, are products of positioning, and such positioning, in turn, shape how mathematics is produced and performed in practice. The mathematical field is thus a field of positionings, and a field of relations between positions. This field is structured according to the distribution of socially valued capital, and this capital serves as a means of differentiating and integrating certain members of the field, that is to say, possession of shared capital resources creates objective relationships between individuals, through structuring them within the same neighbourhoods within the field.

Within these neighbourhoods individuals are subjected to similar local forces from the field, which shape their dispositions and orientations to mathematical phenomena in similar ways. The structured sets of dispositions which result are called habitus and, in mathematics, these may consist of shared examples, concepts, techniques, histories, and notions of taste specific to a given discipline. In practice, individuals each bear their own unique instantiations of habitus, which have emerged through their own situated engagements with mathematical problems, through experiments and craftwork (Section 3.1). These habitus are also subject to change, as a function of the social and physical contexts in which a mathematician performs their mathematics. As we saw in Section 2.3, instantiations of habitus varied depending on the formalities and visibility of the stages of performance.

### 5.1 Position, perspective, perception: How does position shape disposition?

The structure of habitus, because of their variability, multiplicity, mutability, and metaphorical nature, make them somewhat under-determined in practice. However, it is precisely because of these properties that I utilise the notion of habitus. The dispositions which constitute habitus do not solely refer to bodily dispositions, manifested in ways of gesturing, speaking, etc. They also refer to perceptual dispositions, aesthetic dispositions, mental dispositions, etc. The habitus thus is a way of relating different realms of experience to processes of socialisation and domestication.

Habitus embeds all human experience, all knowledge-production, all objectivity, within the realm of the social, and thus mathematical disciplines can be analysed as any other system of social relations. Such systems of social relations, within the theoretical framework I have used, are referred to as fields. Such fields, as Bourdieu characterises them, are contested terrains in which individuals struggle for the possession of capital. Capital distorts the field and creates fields of force, which influence the decision making powers of other actors in the field. Such forces shape habitus, and thus shape the perception of phenomena, as Bourdieu explains below:

> [T]he scientific field, like other fields, is a structured field of forces, and also a field of struggles to conserve or transform this field of forces. The first part of the definition (a field of forces) corresponds to the physicalist stage of sociology conceived as a social physics. The agents, isolated scientists, teams or laboratories, create, through their relationships, the very space that determines them, although it only exists through the agents placed in it, who, to use the language of physics, "distort the space in their neighbourhood", conferring a certain structure upon it. It is in the relationship between the various agents (conceived as "field sources") that the field and the relations of force that characterize it are generated (a specific symbolic relation of force, given the "nature" of the force capable of being exerted in this field - scientific capital, a form of symbolic capital which acts in and through communication). More precisely, it is the agents, that is to say the isolated scientists, teams or laboratories, defined by the volume and structure of the specific capital they possess, that determine the structure of the field that determines them, in other words the state of the forces that are exerted on scientific production, on the scientist's practice. The weight associated with an agent, who undergoes the forces of the field at the same time as he helps to structure it, depends on all the other agents, all the other points in the space, and the relations among those points, that is to say, the whole space. (Bourdieu, 2004:33)

In Bourdieu's notion of field, individuals are weighted according to their relative capital accumulations, and such weight determines the extent to which they distort the field, and exert a force on other social actors. But such forces can only be felt if all the actors involved share certain principles and values, that is to say, that they share certain definitions of what constitutes social or symbolic capital. Such definitions are generated by habitus, which provide the perceptual lenses through which individuals can discern, discriminate, recognise and evaluate socially meaningful categories and values. The individual's ability to recognise and discriminate, to have a discerning eye, is specific to their given position within the field. There are thus many habitus present within the field, which are adopted and adapted by the individual according to their place in the field. As Bourdieu explains, the ability to recognise, to discriminate certain phenomena, bestows upon the individual their sense of place, relative to others within the field:

> Thus the representations of agents vary with their position (and with the interest associated with it) and with their habitus, as a system of schemes of perception and appreciation of practices, cognitive and evaluative structures which are acquired through the lasting experience of a social position. Habitus is both a system of schemes of production of practices and a system of perception and appreciation of practices. And, in both of these dimensions, its operation expresses the social position in which it was elaborated. Consequently, habitus produces practices and representations which are available for classification, which are objectively differentiated; however, they are immediately perceived as such only by those agents who possess the code, the classificatory schemes necessary to understand their social meaning. Habitus thus implies a "sense of one's place" but also a "sense of the place of others." (Bourdieu, 1989:19)

Positions shape dispositions (perceptions) and also the strategies individuals adopt in their practice of day to day life. An individual's place within the field thus becomes manifested in the food they eat, the sports they enjoy, the places they visit, etc. Social position is symbolised in ways of speaking, acting, consuming products; and these symbols of status are made visible and displayed through performing them in public places. Through such presentations of self, the individual creates the distinctions which symbolise social position, as Bourdieu explains:

> This formula, which might seem abstract and obscure, states the first condition for an adequate reading of the analysis of the relation between social positions (a relational concept), dispositions (or habitus), and stances ("position taking"), that is, the "choices" made by the social agents in the most diverse domains of practice, food, or sport, music or politics, and so on. It is a reminder that comparison is possible only from system to system, and that the search for direct equivalence between features seized in isolation, whether, appearing at first sight different, they prove to be "functionally" or technically equivalent or nominally identical, risks unduly identifying structurally different properties or wrongly distinguishing structurally identical properties. The very title Distinction serves as a reminder that what is commonly called distinction, that is, a certain quality of bearing and manners, mostly considered innate is nothing in fact but difference, a gap, a distinctive feature, in short, a relational property existing only in and through its relation with other properties. (Bourdieu, 1996:10-11)

In the mathematics institute I demonstrated how individuals position themselves within spaces: occupying common spaces, staging encounters, making themselves visible or invisible. I observed different presentations of self and ideas in the frontstage and backstage, and documented the transformations ideas undergo as they move from private to public realms (see section 2.3). As ideas enter public stages, so they must conform to certain standards and values; and, through others recognising such properties, so ideas may be positioned in relation to existing knowledge, and assimilated into the field. Within mathematics, however, this process of accepting or validating knowledge is also dependent upon the social position of the individual knowledge-producer within the field of production. An individual's reputation and social standing in the community facilitates an idea's chances of being accepted, trusted, and made visible. Recognition of ideas therefore is as much a product of the idea's merit, as it is the force exerted by the social position of its author (See section 4.3).

Position within social space therefore generates certain perceptions or preconceptions about ideas, which invests them with aesthetic qualities, such as being "truthful" or "beautiful". The ability to recognise or discern such qualities defines one's sense of place within the field, and such place is experienced as being "natural", as Bourdieu explains:

> [T]he sense of the position occupied in social space (what Erving Goffman calls the "sense of one's place") is the practical mastery of the social structure as a whole that reveals itself through the sense of the position occupied within that structure. The categories of perception of the social world are, as regards their most essential features, the product of the internalization, the incorporation, of the objective structures of social space. Consequently, they incline agents to accept the social world as it is, to take it for granted, rather than to rebel against it, to counterpose to it different, even antagonistic, possibles (Bourdieu, 1985:728)

In this thesis I demonstrated how this sense of place manifests itself in the ways in which mathematicians talk about their own place within, and contributions to, the mathematical field. Many position themselves in relation to the "great mathematicians": Euler, Poincaré, Grothendiek, etc. Others characterise themselves as "lucky" at having reached their current position, and do not attribute their status to their own abilities. Some position themselves in relation to their supervisors, and demonstrate the role played by others in shaping their careers and ideas. However some individuals position themselves outside of the mainstream, preferring to remain invisible, or silent, or powerless, and limit their presentations of self during seminars, or conferences. For such individuals the field can appear alienating, disempowering and frustrating.

**5.2 Distinguishing oneself: How is the social field reproduced?**

The senses of self and place within the field therefore affect the presentations of self and ideas. Position affects how visible ideas become, how widely they are accepted or adopted, or how much recognition they receive. The sense of position also shapes the social networks and collaborations an individual establishes, the institutions or conferences they attend, the journals they publish in. An individual in a certain position is likely to associate with other individuals of similar status, and thus publish in similar journals, and receive similar levels of recognition for their work (in terms of citation counts for example). As a result of these similarities in social position and knowledge-production practices, we can develop a means of measuring the degrees of relatedness between individuals in social space:

> The idea of difference is at the basis of the very notion of space, that is, a set of distinct and coexisting positions which are exterior to one another and which are defined in relation to one another through relations of proximity, vicinity, or distance, as well as through order relations, such as above, below, and between; certain properties of members of the bourgeoisie or petit-bourgeoisie can, for example, be deduced from the fact that they occupy an intermediate position between two extreme positions, without it being possible objectively to identify them and without their subjectively identifying themselves, either with one or the other position. (Bourdieu, 1996:11)

As Bourdieu writes above, the positions of individuals allows us to deduce the behaviours and characteristics they may manifest. Such positions generate patterns of behaviour, consumption, and dispositions, which manifest themselves as "styles" of thinking and acting, which are shared by members of a given category (class) of individuals, as Bourdieu explains:

> The second opposition, like the first is the source of differences in dispositions and, therefore, in "positions", which can differ in their contents according to period and society or can appear under an identical form, such as the opposition between intellectuals and proprietors which, in post-war France and Japan alike, is translated, in politics, into an opposition between left and right, and so on. More broadly, the space of social positions is retranslated into a space of "position takings" by the mediation of the space of dispositions (or habitus); or, in other words, the system of differential deviations in agents' properties (or in the properties of constructed classes of agents), that is, in their practices and in the goods they possess, corresponds to the system of differential deviations which defines the different positions in the two major dimensions of social space. Habitus, which are the products of the social conditioning associated with the corresponding condition, make a systematic set of goods and properties, united by an affinity of style, correspond to each class of positions. (Bourdieu, 1996:14-15)

These styles of thinking or acting, or "lifestyles", are distinguishing markers for members of a given social grouping. Such markers of distinction become part of the symbolic machinery used to convey information about position and possession of capital, and such signs, when they are performed effectively, establish the interactional frames within which knowledge is produced, and ideas presented. Perception of distinctions between individuals and phenomena are thus mechanisms for dividing up the social field. The perception of distinctions serves both to differentiate and integrate phenomena within certain classes, creating differences or relationships between aspects of the field, as Bourdieu explains:

> Distinction -in the ordinary sense of the word -is the difference inscribed in the very structure of the social space when perceived through categories adapted to that structure; and the Weberian *Stand*, which is often contrasted with the Marxist class, is the class constructed by an adequate division of social space, when perceived through categories derived from the structure of that space. Symbolic capital -another name for distinction -

> is nothing other than capital, in whatever form, when perceived by an agent endowed with categories of perception arising from the internalization (embodiment) of the structure of its distribution, i.e., when it is known and recognized as self-evident. Distinctions, as symbolic transfigurations of de facto differences, and, more generally, ranks, orders, grades, and all other symbolic hierarchies, are the product of the application of schemes of construction that, like (for example) the pairs of adjectives used to utter most social judgements, are the product of the internalization of the structures to which they are applied; and the most absolute recognition of legitimacy is nothing other than the apprehension of the everyday world as self-evident that results from the quasi-perfect coincidence of objective structures and embodied structures. (Bourdieu, 1985:731)

In relation to mathematics, we explored how mathematicians learn to distinguish mathematical phenomena, through mathematical craftwork and problem solving (section 3.2); we also explored how mathematicians adopt certain perceptual frameworks from supervisors, and through being socialised within the field (section 4.1); and finally we explored how aesthetic discourse within mathematics orients the individual to certain key texts, exemplars, and role models within the field (section 4.3). Through using these frameworks, exemplars, and role-models as reference frames, mathematicians are then able to orient themselves within the field, and establish stances, or points of view, on the mathematical field itself. Such points of view produce sets of positions and possibilities individuals can take within the field, and these shape the choices available to individuals in the future. Knowing one's place in the field thus gives the individual a knowledge of the field's static elements (its present condition), as well as its dynamics (its future directions):

> To have knowledge of the structure is to acquire the means of understanding the state of the positions and position-takings, but also the probable evolution, the future, of those positions and position-takings. In short…analysis of the structure, the statics, and analysis of change, the dynamics, are indissociable. (Bourdieu, 2004:61)

By understanding one's position relative to others, and understanding the possibilities for position-taking, so an individual comes to understand the structure of the field, as well as their own abilities to effect change within that structure. Built into this notion of point of view, or disposition, is therefore a notion of agency, or the ability to alter one's position within social space. In mathematics this agency manifests itself in the problems mathematicians choose to address, the positions they apply for, or the collaborations they form. These choices emerged through an understanding of their position within the field, and a perception of the possibilities for action based upon their positionings.

### 5.3 Constructing a social topology: How can we represent social fields?

One's position is determined through reference to other positions held by other individuals, and these collections of positions constitute a space of relations between positions. This space of relative positions is what Bourdieu refers to as a topological space. The role of the sociologist is that of constructing such social topologies, through defining the relationships between individuals' positions within social space. It is for this reason that Bourdieu refers to sociology as "social topology" - that is to say, the study of social topologies:

> Sociology, in its objectivist moment, is a social topology, an analysis situs as they called this new branch of mathematics in Leibniz's time, an analysis of relative positions and of the objective relations between these positions. (Bourdieu, 1989:16)

Bourdieu asks us to think of these topologies, or social spaces, as geographic spaces (topographical spaces), which bear properties of position, distance and orientation. Individuals closer together in this social space will, therefore, share similar features and be integrated within the same classes:

> At this point of the discussion, we can compare social space to a geographic space within which regions are divided up. But this space is constructed in such a way that the closer the agents, groups or institutions which are situated within this space, the more common properties they have; and the more distant, the fewer. (Bourdieu, 1989:16)

Within these classes individuals share similar dispositions and viewpoints from which to view the field:

> On the basis of knowledge of the space of positions, one can separate out classes, in the logical sense of the word, i.e., sets of agents who occupy similar positions and who, being placed in similar conditions and subjected to similar conditionings, have every likelihood of having similar dispositions and interests and therefore of producing similar practices and adopting similar stances. (Bourdieu, 1985:725)

These dispositions serve to constitute the group within the class, and act as distinction operators, by which other individuals in the field identify themselves and others as belonging to certain positions in social space:

> One of the functions of the notion of habitus is to account for style unity, which unites both the practice and goods of a singular agent or a class of agents. Habitus are these generative and unifying principles which retranslate the intrinsic and relational characteristics of a position into a unitary life-style, that is, a unitary set of persons, goods, practices. Like the positions of which they are the product, habitus are differentiated, but they are also differentiating. Being distinct and distinguished, they are also distinction operators, implementing different principles of differentiation or using differently, the common principles of differentiation. (Bourdieu, 1996:15)

Through situating individuals within these topological spaces Bourdieu argues that we can understand how individuals are shaped by, and how they shape, the social fields within which they are situated. Representing social space as a topological space, which bears certain properties of distance and differentiability, allows us to better situate individuals within the field, and make predictions about the choices they will make, and the beliefs they will hold, as a function of their positions within such spaces.

One of the roles of the sociologist is thus to develop social topologies for the social systems we encounter. We do this through understanding the positionings of individuals within social spaces, and then objectifying these positions within topological representations of social space. To undertake such a process of positioning individuals within the field, we must understand the space of positions which are specific to a given field. Such a process of positioning the individual involves contextualising them within different frames of experience, from the local level of an individual's department, to the wider discipline within which they are entangled:

> This work of objectification of the subject of objectivation must be carried out at three levels: one first has to objectify the position of the subject of objectivation in the overall social space, his or her original position and trajectory, his or her membership of and commitment to social and religious groups ; then one has to objectivate the position he or she occupies within the field of specialists, each discipline having its own traditions and national particularities, its obligatory problematics, its habits of thought, its shared beliefs and self-evidences, its rituals and consecrations, its constraints as regards publication of findings, its specific forms of censorship, not to mention the whole set of presuppositions inscribed in the collective history of the speciality (the academic unconscious); thirdly, one has to objectivate everything that is linked to membership of the scholastic universe, paying particular attention to the illusion of the absence, of the pure, absolute, "disinterested" point of view. The sociology of intellectuals brings to light the particular form of interest which is the interest in disinterestedness. (Bourdieu, 2004:94)

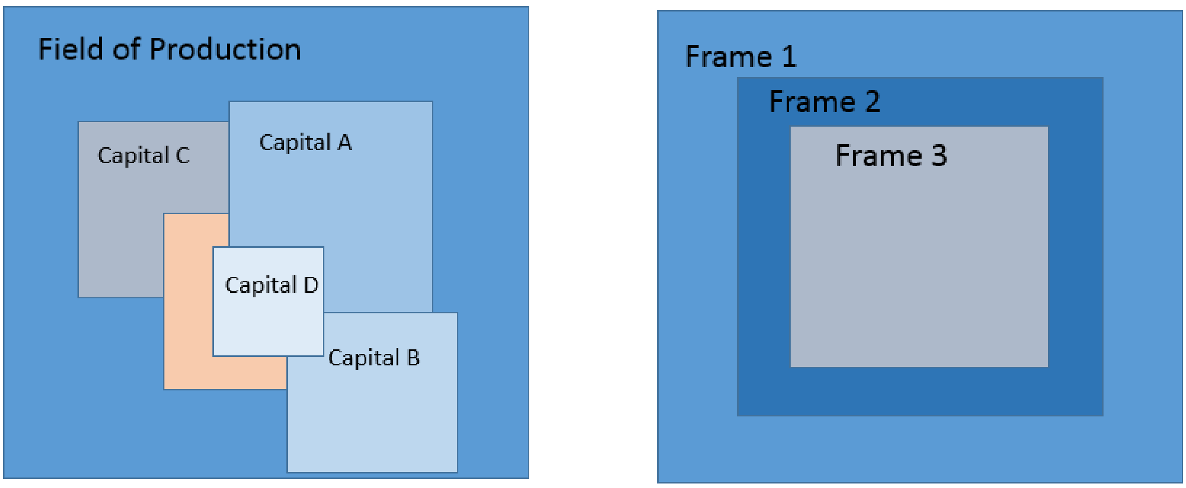
Through this process of positioning, or "objectivating", the individual we are better able to understand how a particular point of view, or set of dispositions, is generated. We can understand how such perspectives are produced as products of socialisation and conditioning within certain habitus, and also determine the extent to which these perspectives are shared between members of a class. As Bourdieu explains below, when the sociologist attempts to position the individual, they are attempting to reconstruct the socialisation process by which the individual was themselves constituted within a social space. The sociologist thus aims to reconstruct the perspectival frame, by which the individual construes the world, by attempting to locate the enframing mechanisms within certain positions within the social topology:

> A point of view is first of all a view taken from a particular point (*Gesichtspunkt),* a particular position in space and, in the sense in which I shall mean it here, in the social space: to objectivate the subject of objectivation, the (objectivating) point of view, is to break with the illusion of the absolute point of view, which is characteristic of every point of view; it is therefore also a perspective view (*Schau*): all perceptions, visions, beliefs, expectations, hopes, etc., are socially structured and socially conditioned and they object a law which defines the principle of their variation, the law of the correspondence between positions and position-takings. Individual A's perception is to individual B's perception as A's position is to B's position, with the habitus making the connection between the space of positions and the space of points of view. (Bourdieu, 2004: 95)

This role of positioning the individual and creating a space of relationships between individuals, in the form of a social topology, thus provides the sociologist with a means of understanding how position within social space shapes dispositions and perceptions. Understanding positionings within the field, likewise, allow us to see how individuals within the same mathematical discipline come to share similar reference frames and objects, similar tools and languages, as well as similar abilities to perceive the mathematical landscape. Such similarities are the products of socialisation within the specific local contexts of a social space, which generate similar habitus, and styles for thinking and acting.

**5.4 Representing social topologies: How useful is the concept of social topology?**

Within this thesis I have merely provided the background necessary to understand Bourdieu's notion of social topology, and have not fully constructed a topological representation of the mathematical disciplines I have studied. I have begun the process of situating mathematical production and perception within three different realms of experience: physical, conceptual and discourse spaces. I use these spaces as frameworks by which to position the reader within the mathematical field, and to give the reader a sense of the complexities involved in producing and perceiving mathematics. What I have not done is to demonstrate how the sociologist goes about the process of positioning individuals in the field, and building topological representations of mathematical fields. In this final section I will describe how this process of objectivation (Goffman's "framing") can proceed, as well as the possible representations which may result:




1. Dividing the field according to types of capital

2. Constructing different frames by which to position the individual within the field

*Figure 29 Dividing the field according to varieties of capital and creating frames for positioning individuals within the field.*

- To begin to transform a field of production into a social topology we first need to divide the field into different social frames, which are relevant to determining the position of the individual.
- These frames correspond to the different varieties of capital that are relevant to the positioning of the individual in the field. The distribution of capital, after all, is what constitutes the field of production in the first place.
- Once we have determined the relevant grades of capital, be they economic, symbolic, political, etc., we can begin to view the field as a whole, and locate different groups or classes co-existing within the field.
- We now need to understand the general contours of the social space. To do this we must understand how different forms of capital are distributed within the field. In relation to mathematics this could be through studying bibliometric data, studying impact, or through looking at funding levels according to discipline.
- Our next task is to understand the sites of capital accumulation and knowledge-production. To do this we must locate the various departments or institutes within which mathematics is produced. We must ask how these

institutions are connected and perhaps measure productivity and reputation as a function of output (of publications, PhD students, prizes won, etc.).

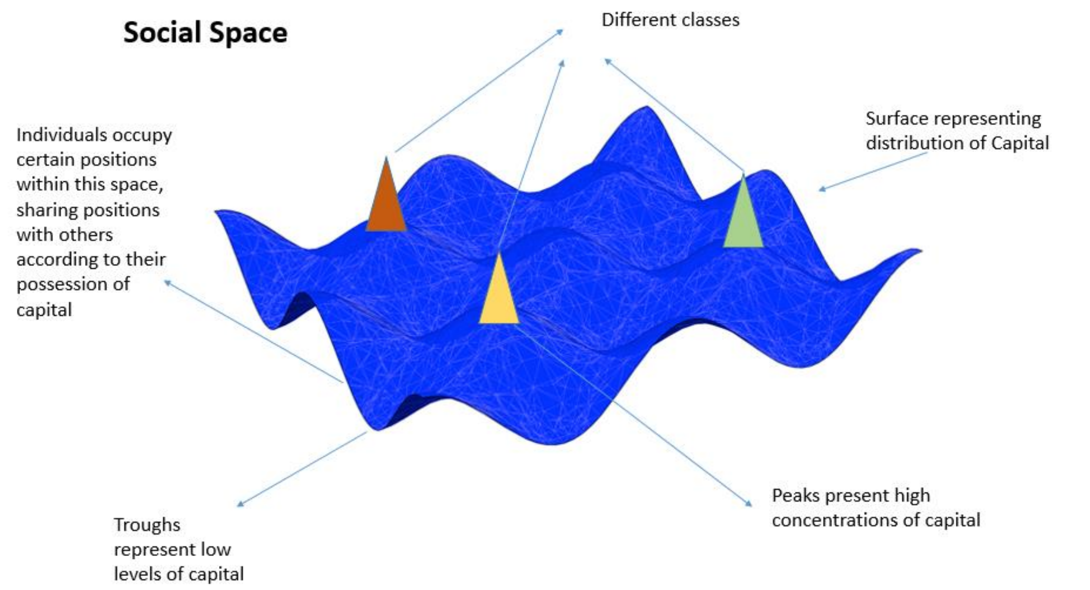



*Figure 30: Visualising different classes which emerge as a result of shared capital accumulations, as represented on a topological space*

- Once these macro level interactions and distributions of capital have been defined it is possible to explore the finer scale interactions, at the level of institutes, or individuals' social network.
- Using such maps it is then possible to identify key individuals within a social network. These individuals can act as nodes for transferring information within or between networks, or can be influential individuals who shape the

opinions or ideas of others. Such individuals are given certain weights which change the topology or shape of the network itself.

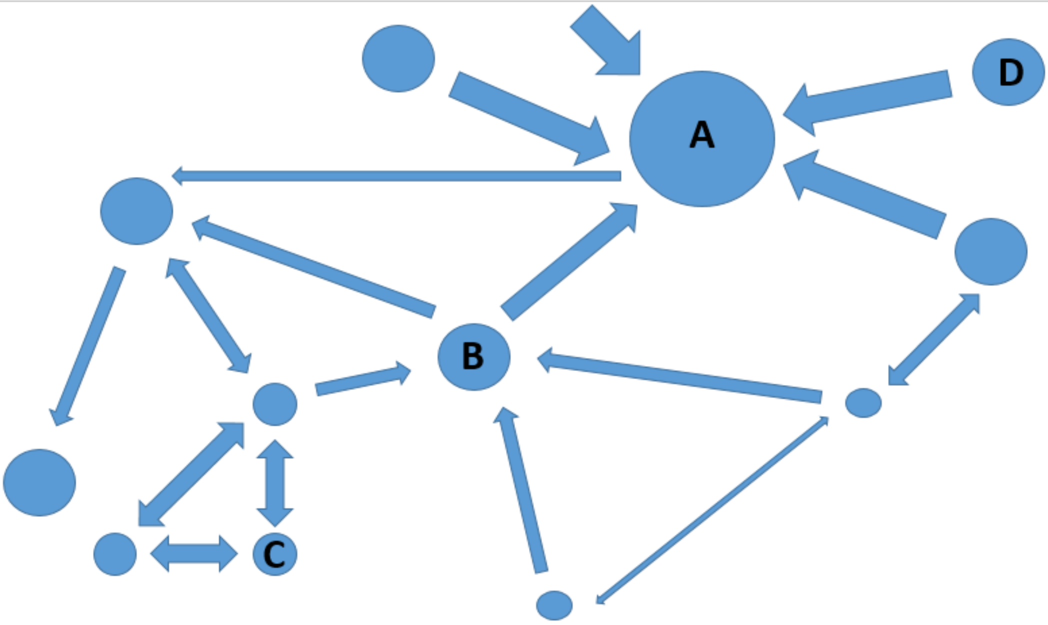



*Figure 31 Graph representation of an individual's social network. Large nodes represent influential individuals with large capital accumulations, arrows determine flow of capital or direction of influence.*

- After building these social network topologies it is possible to position individuals within specific reference frames: within nations, within institutions, within local communities, etc. Micro scale, social spaces of individuals can then be constructed, which chart out the individual's biography - for example education or career trajectories.
- An individual's history of ideas and the influences on such ideas can be charted, through asking the individual to describe their research histories, and the origins of their ideas.
- The individual’s performances within different institutional frameworks can be documented through shadowing individuals over extended periods, and understanding how they present themselves and ideas within different spatial contexts. The motivations, expectations, perceptions of individuals

can be studied, in order to understand how an individual positions themselves within the field, and how such a field is perceived by them.

- Finally the life-cycles of individual ideas can be explored, from conception to publication, through following the material remains of concepts in notebooks, and at blackboards. At this level we will explore decision making processes - of how to present an argument, where to publish the paper, etc.; as well as the changes ideas undergo, as they are communicated within different contexts and to different individuals.
- Through a process of framing, at different scales of magnification, we can thus build up a clear sense of an individual's position, orientations and dispositions within a field. We can then explore how such orientations affect the knowledge produced, and how and where such knowledge is presented. By means of this system of framing, or "objectivating", the individual and their ideas, we can build up a social topology, and a sense of how the individual's ideas change as a function of their position and movement within social space.

This task of constructing social topologies is beyond the scope of this thesis. However, my future work will involve developing these concepts further and building tools by which social spaces and habitus can be represented mathematically. However, I hope that the above outline gives a sense of how such social topologies may be built, through following Bourdieu’s procedures on framing and positioning. The justifications for creating such social topologies may not seem entirely clear from the above descriptions, however I believe that being able to situate individuals within social space can provide analytical tools by which to:

1. Study the effects of unequal distributions of capital on social groups.
2. Understand social mobility as a function of social topology
3. Model different effects of capital accumulations.
4. Compare the topologies of different social systems.
5. Understand how positioning effects dispositions and world view.
6. Predict behaviour of individuals within certain framings.

This process of defining social spaces in terms of topological spaces, however, lies in my future work. What I have presented in this thesis is a foundation for understanding how such an analysis can proceed. This foundation is based primarily on Bourdieu's theoretical framework of capital, field, habitus and practice. I have used such terminology to define how individuals position themselves, and how they are positioned within, mathematical fields. Such positionings, I have explained, shape individuals' dispositions towards mathematical phenomena, which in turn affect how they perceive mathematical spaces and produce knowledge in the mathematical field.

The forces of fields and habitus, I have explained, affect dispositions and behaviour on multiple dimensions of experience. I have demonstrated the effects at the levels of physical, conceptual and discourse spaces, and have showed how similar ordering principles exist within each of these different spatial dimensions. My argument was that habitus act as bridges between worlds, and that the schemes of habitus structure different dimensions of experience in similar ways, through the processes of analogical relation and metaphorical projection. I explained that individuals are socialised within habitus according to their positions within social fields, and that these positions then serve to shape dispositions towards the world.

**5.5 Contribution to Knowledge**

Although such a Bourdieu-inspired analysis has been undertaken within the history of mathematics (by Goldstein 2007, Ehrhardt 2010, 2016 and Brian 1994), within the sociology of mathematics such an analysis still retains some novelty. In my discussion of the tensions between competition and friendship I do present nuance to the traditional view of the field as a purely competitive field, or battlefield. Rather than the field being conflictual I suggest there are other strategies that can be adopted, based upon reciprocity or altruism. Within these modalities the notion of distinction and differentiation becomes less important, and the ideal of integration comes more to the fore.

My observations on craftwork extend existing work conducted by Barany (2011) and Greifenhagen (2012). I provide greater detail on the physical processes involved in organising and producing mathematics, and demonstrate the selecting, sorting, relating and differentiating processes that ideas undergo as they move from informal to formal stages of presentation. Relating knowledge-production practices to physical processes I believe presents the embodied, affective aspects to knowledge work in mathematics.

### 5.6 Future Directions

One aspect of my future work will involve studying mathematical genealogies, and exploring the mechanisms by which knowledge is transmitted and transformed as it is inherited. To understand inheritance and kinship networks I intend on extending my studies outside of institutional contexts, to explore mathematics education within university mathematics' departments. My aim will be to document the socialisation process by following mathematicians' careers from undergraduate to post-graduate levels. To examine the effects of different cultural traditions on mathematical practice and knowledge production I will attempt to gain access to departments in China, Japan or Russia and compare them with education at European or North American institutions.

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