A MATHEMATICAL SIMULATION OF
ASIAN OPTIONS ON THE TOKYO GRAIN EXCHANGE

A Thesis presented to the Faculty of the Graduate School
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In Partial Fulfillment
Of the Requirements for the Degree

Masters of Science

by
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The undersigned, appointed by the dean of the Graduate School,

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A Mathematical Simulation of Asian Option on the Tokyo Grain Exchange

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A candidate for the degree of

Master of Science

And hereby certify that, in their opinion, it is worthy of acceptance.

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Dedicated to T.C.

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CHAPTER 1: MOTIVATION

The volatile nature of commodities has become a serious problem for commodity-dependent producers and countries. An estimated two billion people receive the income to maintain livelihoods from primary commodities (UNCTAD, 2005). On a macro level, 95 of 141 (~67%) developing countries receive at least half of foreign exchange profits from commodities (South Center, 2005). Commodity prices can suffer from extreme volatility in the short term changing as much as 50% in one year (Cashin and McDemermott, 2002). A prime example of this volatility is evident in the United States soybean market. In January of 2010, soybean prices were 913 dollars per ton. Slightly one year later, in February of 2011, the price of soybeans increased 159% as the price spiked up to 1455 dollars per ton (Tradingcharts.com).

The volatility of commodities has become a more pressing matter in the last four decades. There have been as many price shocks since 1970 than all the price shocks combined in the previous 75 years (Cashin and McDermott, 2002)

Due to the high prices received by some commodities, the high volatility and risk of other commodities has been disguised and forgotten. This is a problematic approach to dealing with volatility. In 2006 Stephen Roach warned of a "commodity bubble" he said, "It's not a matter of if the bubble bursts – but when (Spence)."

Brown, Crawford, and Gibson's 2008 paper for the International Institute for Sustainable Development (IISD) lists the following factors which affect commodity income volatility: business cycles in key markets, changing weather patterns, conflicts in producing countries, exchange rate fluctuations, price speculations, export dumping, and food aid. The IISD paper then cited five ways to help ease commodity price volatility: supply management, national revenue management, market-based price risk management, compensatory finance, and alternative trade initiatives.

This paper will address the Brown, Crawford, and Gibson (2008) volatility affecting factor, price speculation and the economic tool, market-based price risk management.

The true price changes that occur because of shifts in supply and demand can be amplified by the price speculation of investors and brokerage house funds that use commodity futures and derivatives in investments (Brown, Crawford, and Gibson, 2008). The agricultural market is moving into an era where cash transactions are being replaced with contracts. In 2007 it was reported that 20-50% of the agricultural commodities (i.e. wheat, live hogs, cattle, and corn) were used in “Wall Street” investment funds. These companies' investment portfolios rarely deal in actual deliveries. Therefore at contract maturation, the portfolio must rollover into a new contract and re-hedge. This action can lead to changes in demand for commodities in the financial sector that do not exist in the production world (Barrionuevo and Anderson, 2007). The ability of investment firms to deal with a larger number of commodity contracts than the individual producer can lead to price manipulation. Therefore one needs a financial tool to deal with both high volatility and price manipulation.

Market-based price risk management is a financial tool that helps producers reduce the risk in the price received for their product. This financial tool helps producers transfer some of the price risk over to investors in the commodity market. The idea is that the strategies provided by this tool will give producers better income predictability. Having better income predictability allows individuals to make wiser decisions, and procure better credit terms (Rutten and Youssef, 2007). The most common market-based risk management strategy is a futures market with possible financial derivatives. Here buyers and sellers can exchange contracts, with predetermined quantity and quality measures, to lock into an agreed upon price for future sale. One drawback to a futures market is individual producers may be unable to afford the financial obligations to interact, i.e margins and margin calls. Possible solutions to this dilemma are an over-the-counter contract (which can be defaulted on) or a less costly option, such as an Asian or averaging option (Kemna & Vorst, 1990).

In 2005 the United Nations Conference on Trade and Development (UNCTAD) stated, “As a result of recent and expected developments in demand for commodities, now is the best opportunity in many decades for improving the economies of commodity-depend developing countries. This requires action by developing-county governments and the international community (UNCTAD, 2005).” UNCTAD (2005) also notes that market-based risk management decreases volatility and that Asian options are cheaper than plain vanilla options(Kemna &Vorst, 1990). Therefore the purpose of this paper is to look at an exotic option that would accommodate individual's ability to purchase, decrease income volatility, and deal with price manipulation.

“In the past three decades, we have witnessed the phenomenal growth in the trading of financial derivatives and structured products in the financial market around the globe and the surge in research on derivative pricing theory (Kwok, 2008, vii).” The TGE offers no option market, yet there is a need. In 2008 GM Ready Soybeans covered 92% of US soybean acreage (Non-GMO Report). However for the 2009 planting season Greg Lickteig, senior group manager, The Scoular Company reported, "we are seeing a record number of non-GMO soybean production contracts being written this spring." Many farmers are switching over to non-GMO soybeans because of the premiums over the Chicago Board of Exchange. With increased excitement over non-GMO soybeans and the lack of a futures market in the US, the TGE offers an interesting market to study non-GMO soybean trading. Also with the premiums ranging from one dollar to two dollars and seventy-five cents for the non-GMO beans Asian options are an interesting tool for hedging against volatility (The Organic and Non-GMO Report).

CHAPTER 2: INTRODUCTION

2.1 Contracts

The Tokyo Grain Exchange's (TGE's) non-genetically modified organism (non-GMO) soybean futures is a mature niche market. This contract has lower trading volume than its counter soybean contract and, it has large swings in volatility at contract maturation. These three factors make the non-GMO soybean futures contract a prime subject for this research.

Sibler (1981) suggests that 66-75% of futures contracts fail due to inadequate volume. Kolb (1991) finds that only 30% of new futures contracts will be profitable. In 1954 Irwin stated that a stable futures contract will develop slowly. However the rapid, successful growth of the non-GMO soybean futures contract refutes Irwin's (1954) findings. Moreover the TGE non-GMO soybean futures contract has enough volume to engage hedgers, ample volatility to captivate speculators and the arbitrage is comparable with a developed futures contract (Parcell, 2004).

Lence and Hayes (2001) found that if a large enough price differentiation occurred between GMO and non-GMO crops then production would move to the crop with higher prices. The difference in price between non-GMO and GMO soybeans can be thought of as the marginal cost of producing non-GMO soybeans (Parcell, 2004). Figure 2.1.1 illustrates the price difference (in Yen per 1000 kg) in non-GMO soybeans and GMO soybeans in January through April 15, 2011.

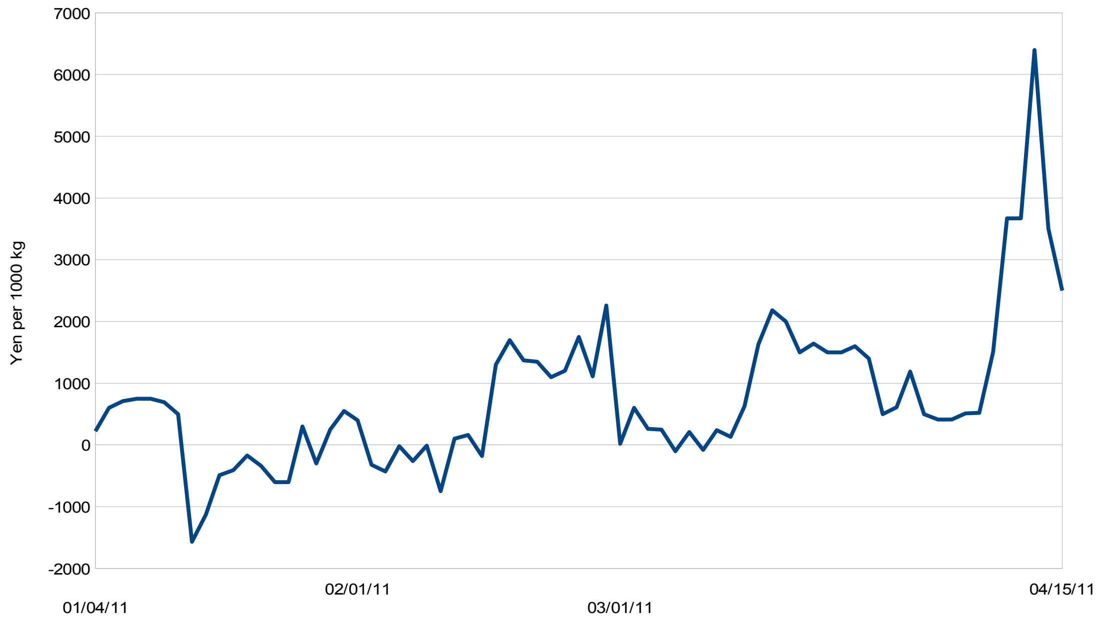

Figure 2.1.1: non-GMO Soybean Premium 01/04-4/15

Source: Tokyo Grain of Exchange

There is a large price difference in the GMO and non-GMO soybean contracts. The existence of this premium for non-GMO soybeans shows the need for a non-GMO soybean contract (Parcell, 2004). Moreover there is a need and demand for non-GMO soybean futures.

A well written contract is essential of the success of a futures contract (Powers, 1967). Sykuta and Parcell (2003) found the following three criteria define a well written contract:

1) Allocation of value

2) Allocation of risk

3) Allocation of decision rights.

Allocation of value is the profits from trade. Allocation of risk is the uncertainty in profit that is taken on by the buyer and seller. Allocation of decision rights, is the proprietorship of terms of trade by the buyer and the seller (Sykuta and Parcell, 2003). One can think of value as arbitrage in the futures market, and think of risk as meeting the contract deliver grade specifications.

There were concerns over the quality of non-GMO soybeans in the early days of the futures market (Nill, 2000). Thompson, Garcia, and Wildman (1996) note that for delivery to occur the delivered product must be close in quality to the underlying monetary value. Before April 2001 mislabeling of non-GMO soybean occurred and delivered lots with a 5% tolerance level of "GMO Free" could be labeled as non-GMO. However in April of 2001 under mandatory labeling laws and The Law Concerning Standardization and Proper Labeling of Agricultural and Forestry Products (Law No. 175 of 1950) food quality soybeans began being delivered. After these rulings the market regained power. Under Sykuta and Parcell's (2003) definition of a well written contract, the risk of meeting contract delivery specifications was decreased. Through the TGE's constant efforts to better the contract specifications there has been the successful emergence of a new viable non-GMO soybeans contract.

Some may argue that there is no need for the non-GMO soybean market and that this market can be replaced with cross-hedging of traditional soybeans. However, Parcell (2004) found that traditional soybeans and non-GMO soybeans are only weak substitutes with a cross-hedge coefficient of .84. While information sharing may occur there is statistically a large enough difference that cross hedging will not be effective, so there is a need for this thinly traded niche market.

The non-GMO-soybean contract is more thinly traded than the conventional soybean contract. For example on January 31, 2011 the open interest[1] for the conventional contract was 55,705 (20,466,017 bu.) contracts, while the non-GMO-soybean open interest was 795 (292,083 bu.) contracts (TGE). Moreover the volume[2] for the conventional soybeans on January 31, 2011 was 5,725 (2,103,365 bu.) contracts, while the non-GMO soybean volume was only 220 (80,828 bu.) contracts (TGE). Roughly, the non-GMO soybean futures market has 3.8% of the volume of the traditional soybean contract. Volume of contracts is not the only difference in the non-GMO soybean and conventional soybean contracts. Table 2.1.1 and Table 2.1.2 show the key differences (yellow) and similarities (white) in the two different contracts. Notice the contract unit in the both the non-GMO soybean futures contract and conventional soybean futures contract is 10,000 kilograms. The TGE board of directors decreased the conventional soybean contract from 50,000 kilograms to 10,000 kilograms from October 2009 onward (TGE).

Table 2.1.1: non-GMO Soybean Contract Specifications

Date Trading Began	May 18, 2000
Contract Unit	10,000 kilograms (10 metric tons)
Trading Hours	(Continuous Trading)

1 Open interest: “The total number of options and/or futures contracts that are not closed or delivered on a particular day (Investopedia).”
2 Volume: “The number of shares or contracts traded in a security or an entire market during a given period of time (Investopedia).”

<table>
<tr><td></td><td>Day Session (8:30 ~) 9:00 ~ 15:30
Night Session (16:45 ~) 17:00 ~ 23:00
The time in the parenthesis is when orders are accepted.
* ~ 15:30 on the last trading day of the spot month.</td></tr>
<tr><td>Contract Months</td><td>February, April, June, August, October and December within a twelve-month period</td></tr>
<tr><td>Price Quotation</td><td>Yen per 1,000 kilograms</td></tr>
<tr><td>Minimum Fluctuation</td><td>10 yen per 1,000 kilograms (100 yen per contract)</td></tr>
<tr><td>Position Limits</td><td>Maximum long or short positions for each contract month:
<table>
<tr><th></th><th>Members</th><th>Non-Members</th><th>Managed Funds</th><th>Omnibus Accounts[3]</th></tr>
<tr><td>Spot Month</td><td>300</td><td>300</td><td>300</td><td>300</td></tr>
<tr><td>1 month prior to nearby month</td><td>600</td><td>600</td><td>600</td><td>600</td></tr>
<tr><td>2nd month</td><td>1500</td><td>1500</td><td>1500</td><td>1500</td></tr>
<tr><td>3rd month</td><td>3000</td><td>3000</td><td>3000</td><td>6000</td></tr>
<tr><td>4th and onward months</td><td>3000</td><td>3000</td><td>6000</td><td>9000</td></tr>
</table></td></tr>
<tr><td>Last Trading Day</td><td>Two business days prior to the delivery day.</td></tr>
<tr><td>Delivery Day</td><td>One business day prior to the last business day of the delivery month. December 24th for December contract; if not a business day, the delivery day is moved up to the nearest business day.</td></tr>
<tr><td>Delivery System</td><td>Physical delivery against designated warehouse receipt</td></tr>
<tr><td>Standard Grade</td><td><Effective from April 2009 and onward months>
Identity preserved non-genetically modified organism (non-GMO) No. 2 yellow soybeans of the growths of Indiana, Ohio, Michigan, Iowa, Illinois, Wisconsin, Minnesota, North Dakota, South Dakota, Nebraska, Kansas, Missouri and Arkansas in the U.S.A. (Stored in silo, without screening and sorting processing).</td></tr>
<tr><td>Deliverable Grades</td><td>Identity preserved yellow soybeans produced in Canada and the People's Republic of China that satisfy the terms and conditions stipulated in the Exchange rules (Stored in silo, without screening and sorting processing).</td></tr>
<tr><td>Delivery Points</td><td>Designated warehouses in the Tokyo metropolitan area and the prefectures of Kanagawa, Chiba and Saitama.</td></tr>
</table>

Source: Tokyo Grain of Exchange

Table 2.1.2: Conventional Soybean Contract Specifications

Date Trading Began	March 1, 1984
Contract Unit	10,000 kilograms (10 metric tons)
Trading Hours	(Continuous Trading) Day Session (8:30 ~) 9:00 ~ 15:30 Night Session (16:45 ~) 17:00 ~ 23:00 The time in the parenthesis is when orders are accepted.

[3] Omnibus Accounts: "An account between two futures merchants (brokers). It involves the transaction of individual accounts which are combined in this type of account, allowing for easier management by the futures merchant (Investopedia)."

<table>
<tr><td></td><td>* ~ 15:30 on the last trading day of the spot month.</td></tr>
<tr><td>Contract Months</td><td>February, April, June, August, October and December within a twelve-month period</td></tr>
<tr><td>Price Quotation</td><td>Yen per 1,000 kilograms</td></tr>
<tr><td>Minimum Fluctuation</td><td>10 yen per 1,000 kilograms (100 yen per contract)</td></tr>
<tr><td>Position Limits</td><td>Maximum long or short positions for each contract month:
<table>
<tr><th></th><th>Members</th><th>Non-Members</th><th>Managed Funds</th><th>Omnibus Accounts</th></tr>
<tr><td>Spot Month</td><td>500</td><td>500</td><td>500</td><td>500</td></tr>
<tr><td>1 month prior to nearby month</td><td>2000</td><td>2000</td><td>2000</td><td>2000</td></tr>
<tr><td>2nd month</td><td>5000</td><td>5000</td><td>5000</td><td>5000</td></tr>
<tr><td>3rd month</td><td>10000</td><td>10000</td><td>10000</td><td>20000</td></tr>
<tr><td>4th and onward months</td><td>10000</td><td>10000</td><td>20000</td><td>30000</td></tr>
</table></td></tr>
<tr><td>Last Trading Day</td><td>Fifteenth calendar day of the delivery month; if that day is not a business day, then the last trading day is moved up to the nearest business day.</td></tr>
<tr><td>First Delivery Day</td><td>Three business days following the last trading day.</td></tr>
<tr><td>Last Delivery Day</td><td>The last business day of the delivery month. For December contract, the last delivery day is three business days prior to the last business day of December.</td></tr>
<tr><td>Delivery System</td><td>Physical delivery against designated warehouse receipt</td></tr>
<tr><td>Standard Grade</td><td>GMO, GMO mixed and GMO non-segregated No. 2 or better yellow soybeans produced in the U.S.A. (Stored in silo, without screening and sorting processing).</td></tr>
<tr><td>Deliverable Grades</td><td>GMO, GMO mixed and GMO non-segregated No. 2 yellow soybeans produced in the U.S.A. and yellow soybeans produced in the Federative Republic of Brazil and the Republic of Paraguay that satisfy the terms and conditions stipulated in the Exchange Rules (Stored in silo, without screening and sorting processing).</td></tr>
<tr><td>Delivery Points</td><td>Designated warehouses in the Tokyo metropolitan area and the prefectures of Kanagawa, Chiba Saitama and Ibaraki.</td></tr>
</table>

Source: Tokyo Grain of Exchange

Tables 2.1.1 and Table 2.1.2 confirm that the non-GMO soybean contract has much smaller position limits than the TGE's regular soybean contract. Table 2.1.3 below shows how many more bushels of soybeans can be traded on the conventional soybean contract over the non-GMO soybean contract.

Table 2.1.3: Additional # of Bushels From Trade on the Conventional Soybean Contract

	Members	Non-Members	Managed Funds	Omnibus Accounts
Spot Month	73,480	73,480	73,480	73,480
1 month prior to nearby month	514,360	514,360	514,360	514,360
2nd month	1,285,900	1,285,900	1,285,900	1,285,900
3rd month	2,571,800	2,571,800	2,571,800	5,143,600
4th and onward months	2,571,800	2,571,800	5,143,600	7,715,400

Source: Tokyo Grain of Exchange

The difference is quite large, Table 2.1.3 above shows just how thinly traded the non-GMO-Soybean contract is. With fewer bushels being traded, traders may be weary of price manipulation at contract maturation. When markets are highly volatile, the Tokyo Grain Exchange attempts to deal with price manipulation by keeping the contract limits lower and by imposing higher margins.

2.2 Inefficient Markets

Shao and Roe (2002) look at Asian options in the hog finishing sector. They note that thin markets may lead to market power, which in turn can create an inefficient market. Shao & Roe (2002) , Kemna & Vorst (1990), as well as Turnbull & Wakeman (1991) note that Asian options can protect against market power and price manipulation. Bergman made the same findings in 1985 when he found Asian options attractive due to the fact that they specifically deal with this near maturity manipulation. Turnbull & Wakeman (1991) state the reason Asian options can deal with price manipulation is that by design there is more than one date that matters for the final payout.

Many institutions have started offering Asian style options. The Brazilian coffee market uses Asian options to help farmers hedge against low prices. The farmers needed an Asian option because by its nature the Asian Option gave farmers protection over the steady reduction in coffee prices (Nelken, 1996, 179). The CME Group had begun working on over-the-counter Asian options for corn, wheat, soybeans, soybean meal, and soybean oil. However the project is currently postponed until future notice, as the CME continues clarify regulations with their regulator. Because freight options depend on the average price over a set time, Imarex has begun offering forward freight agreements that are Asian. Specifically Imarex is using Turnbull and Wakemans 1991 formulas (Attikouris, 2005).

Asian options are also offered over-the-counter as a means to hedge foreign currencies. Asian options have become quite popular in this setting because 1) Asian options are cheaper than European options and 2) Asian options better suit a treasurer's trading wants. The alternative to an Asian option is entering into a strip of individual options; however, with multiple options this trading strategy can become rather expensive (Levy, 1992). Also expensive, as noted by Bollman, Garcia, and Thompson (2003), is large transaction costs in a thinly traded market. Kemna & Vorst (1990) note Asian options are also usually less costly that European options, due to the lower volatility on the price of an average.

The Figure 2.2.1 shows the percentage change in daily values for the 2005 non-GMO TGE soybean December contract. As can be seen in the last days of the month, there is a large 8.29 % daily change. The yearly volatility from the first 30 days of the contract is 17.75% however the yearly volatility from the last 30 days is 33.97%. As can be seen at certain times the market is prone to extreme volatility. Levy (1992) notes Asian options are especially useful in this type of market because the averaging acts as a

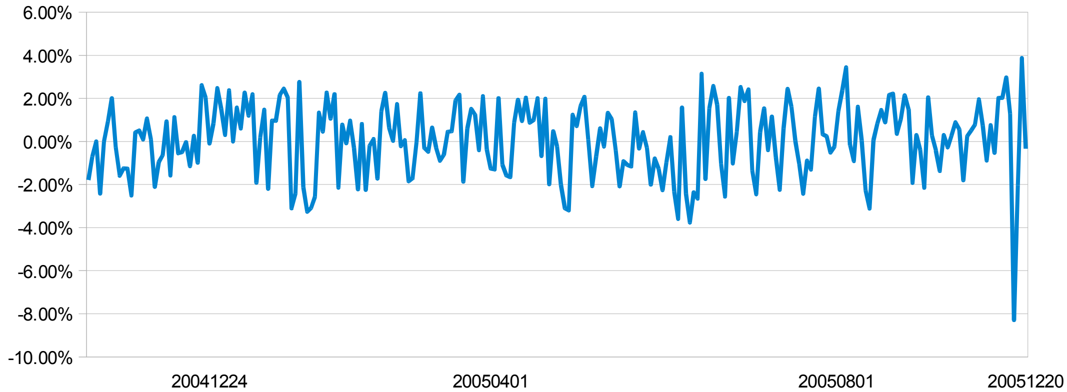

Figure 2.2.1: Percentage Daily change in the 2005 December Contract

Source: Tokyo Grain of Exchange

smoothing operation.

One reason the TGE non-GMO soybean futures contract was chosen for analysis was because of year-to-year volatility level changes. Table 2.2.1 provides a summary of the annual volatility, and the last column represents the percentage change from the previous year. Figure 2.2.2 represents the visual representation of the annual volatility. For the years 2001, 2004, 2008, 2009, 2010 the percentage change in year-to-year volatility was relatively large. The extreme changes in volatility level motivate the need for further assessment of the potential for options trading.

Table 2.2.1: TGE Yearly non-GMO Soybean Volatility

Year	Volatility	% Change
2000	23.69%	~
2001	16.79%	29.12%
2002	15.78%	6.05%
2003	17.53%	11.12%
2004	29.02%	65.56%
2005	31.20%	7.51%
2006	29.43%	5.68%
2007	25.38%	13.76%
2008	60.77%	139.46%
2009	35.74%	41.19%
2010	16.98%	52.50%

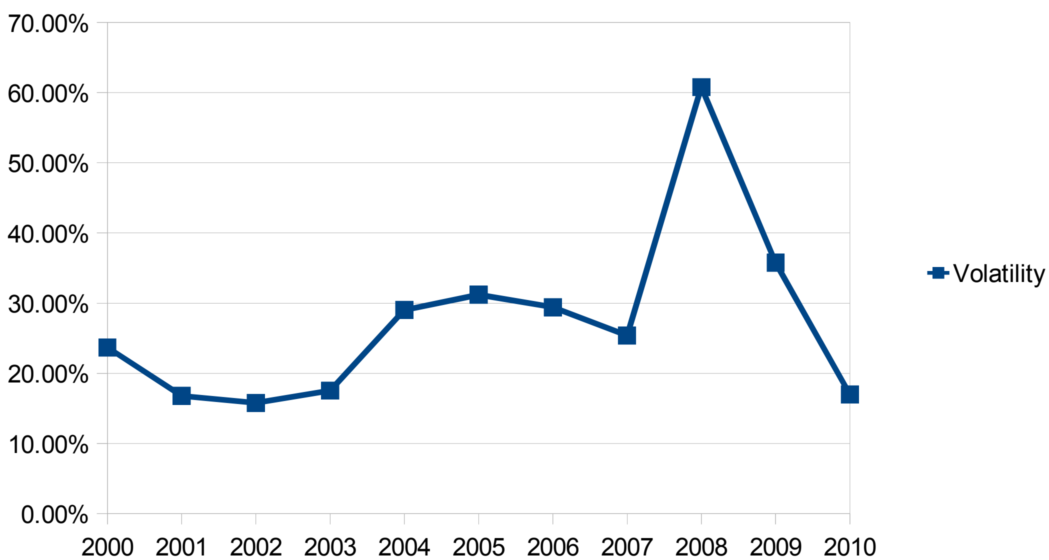

Figure 2.2.2: TGE Yearly non-GMO Soybean Volatility

Since there is no thick underlying cash market for the non-GMO soybean futures, the market behaves somewhat like that of the stock market. When deciding how a stock will perform investors read the news, look into the stock prices of the company's complementary and supplementary companies, or look at the overall performance of the stock market. Unlike many commodities, the underlying product of a stock does not have a well defined cash market. For example, one cannot pick up the paper and find the prices of widgets in major cities around the country. With soybeans, one can look at prices in different cities or countries and make a speculation of how one thinks the prices will move. However, this cannot be done with the non-GMO soybean contract, which makes the investment slightly unsettling. With such a lack of available pricing information on non-GMO soybeans, another buying option for this market is needed--one that is cheaper with less volatility. The research involves pricing a fixed strike call to acquire the price of floating strike put. The motivation of the research is to provide producers with additional tools to help smooth their incomes; hence, the choice of the put option.

CHAPTER 3: LITERATURE AND METHODOLOGY

3.1 Asian/Averaging Options

Asian options were first used by Banker's Trust Tokyo in 1987 for pricing average crude oil contracts, thus the name “Asian option” (Global-Derivatives). Such options allow investors to hedge against the average price of the asset rather than the end-of-period price (Kwok, 2008, 212). That is, the terminal payoff of an Asian option depends on the average price of the underlying asset. Thus Asian options are also referred to as averaging options. The averaging period can be for the entirety of the option, a specified partial time period of the option, or in rare instances even longer than the life of the option. Asian options are typically less expensive than plain vanilla options (Hull, 2009, 564). This observation is particularly apparent in comparisons against European options as the volatility of the underlying asset rises.

Asian options come in two forms; fixed strike (average rate) whose ending payout is $\max(A_T-X,0)$ and floating strike (average strike) whose terminal payout is $\max(S_T-A_T,0)$, where A_T is the average asset price, S_T is the asset expiry value, and X is the strike price. Fixed strike options are the most common (Kowk, 2008, 226), and much research had been conducted on fixed strike Asian options. Considerably less attention has been paid to floating strike Asian options, notably due to difficulty in pricing (Henderson, Hobson, Shaw, and Wojakowski, 2004). However Henderson and Wokjakowski (2002) made the important discovery that a fixed strike Asian option can be used to price a floating strike Asian option. Henderson, Hobson, Shaw, and Wojakowski (2004, 4) say it eloquently, "Given a favorite method of pricing a fixed-strike call, the problem of pricing a floating strike option is essentially solved, or to put it another way, the problems of pricing fixed and floating options are equally difficult."

3.1.1 Average the options

The fixings of an Asian option can either be arithmetically or geometrically averaged. Also the fixing can be of either a discrete or continuous nature. The data collected from the TGE is of a discrete nature, so this paper will focus on discrete averaging procedures. The formulas for discrete averaging are as follows (Kwok, 2008, 212):

Discrete arithmetic:

Equation 3.1.1 $A_t = \frac{1}{n} \sum_{i=1}^{n} S_{t_i}$

Discrete geometric:

Equation 3.1.2 $A_t = \left[\prod_{i=1}^{n} S_{t_i} \right]^{\frac{1}{n}}$,where S_{t_i} is the discrete asset price at time $t_i = 1,2,3 \ldots . n$ .When *n* approaches infinity, the average becomes continuous.

Academic research solving for explicit Asian option formulas like those examined here is important for averaging options because while other methods may be slightly more accurate, those methods are also very time intensive and expensive (Levy, 1992). Researchers (e.g. Levy [1992], Turnbull and Wakeman [1991], and Kemna and Vorst [1990]) often focus on call options since it is obvious that if a call can be priced, so too can a put. However, rational in this research is different. Since the research is from the view of the producer, in order to price a floating put one must price a fixed call. This is very convenient, as fixed strike Asian call options have received much academic research over the last 10 years (Henderson, Hobson, Shaw, and Wojakowski, 2004). The starting price for a fixed strike call is:

Equation 3.1.3 $C_{fix} = \exp(-rT) E\left[\left(\left(A_T - X\right), 0\right)\right]$.

The initial price for a floating strike put is:

Equation 3.1.4 $P_{float} = \exp(-rT) E\left[max\left(\left(A_T - S_T\right), 0\right)\right]$.

where r is the risk free rate X is the strike and T is the end time (Kwok, 2008, 220).

3.1.2 Arithmetic Averaging Options

Turnbull and Wakeman

Turnbull and Wakeman (1991) propose "A Quick Algorithm for Pricing European Average Options" which notes that average options "provide a way to ameliorate any possible price distortions that might arise because of a lack of depth in the market of the underlying asset (336])." Turnbull and Wakeman (1991) find that pricing this sort of option creates certain difficulties. The value of the option is dependent on the history of the asset, if the asset in hand has a lognormal distribution, then the arithmetic average will not be of a lognormal nature and will not have an explicit representation (Levy,1992). Turnbull and Wakeman (1991) use a Edgeworth Series expansion in the Asian option pricing model.

The Assumptions made by Turnbull and Wakeman (1991) and are as follow:

1. “No transactions costs, no differential taxes, no borrowing or lending restrictions, and trading takes place continuously.”

2. “The term structure of the interest rate is flat and non-stochastic.”

3. “It is assumed that the stock price S is described by a log-normal distribution: $ds = S\alpha dt + S\theta dz$ “

where *dz* is a generalized Weiner process; α is the “constant instantaneous mean” or as stated by Hull (2009, 226) the expected rate of return, and θ being the volatility.

While this paper is not using Turnbull and Wakeman's (1991) Asian option formulas, review of their work is still needed. Levy's (1992) arithmetic averaging options use the first term of the Edgeworth expansion, and therefore, these principles are detailed below.

Edgeworth Series Expansion

Kwok (2008, 226-228) in “Mathematical Models of Financial Derivatives” reviews an Edgeworth Series expansion (Jarrow and Rudd, 1982 and Turnbull and Wakeman, 1991). This is one of the methods that will be used in the research. If the true probability distribution is $F_t(s)$, one needs to find an approximation of distribution; call it $F_a(s)$. Both equations are in a class which is differentiable with respect to *s*, so both have continuous density functions. Frst define the following:

1. the j^{th} moment of *F*:

Equation 3.1.2.1 $a_j(f)=\int_{(-\infty)}^{(\infty)} s^j f(s)ds$.

2. the j[th] central moment of distribution *F*:

Equation 3.1.2.2 $\mu_j(F)=\int_{(-\infty)}^{(\infty)} [s-\alpha_1(F)]^j f(s)ds$.

3. characteristic function of *F*:

Equation 3.1.2.3 $\phi(F,t)=\int_{(-\infty)}^{(\infty)} e^{its} f(s)ds \quad i=\sqrt{-1}$.

Assume the moments for $\alpha_j(F)$ exist for $j \leqslant n$. The log of a moment generating function is known as a cumulant generating function. The cumulants k_i are given by:

Equation 3.1.2.3 $\ln \phi(F,t)=\sum_{(j=1)}^{(n-1)} k_j(F)\frac{(it)^j}{j!}+o(t^{n-1})$.

Therefore the first four cumlulants (Nielsen, 2001) are:

Equation 3.1.2.4 $k_1(F)=\alpha_1(F)=E(X)$,

Equation 3.1.2.5 $k_2(F)=\mu_2(F)=E[(X-E[X])^2]$,

Equation 3.1.2.6 $k_3(F)=\mu_3(F)=E[(X-E[X])^3]$, and

Equation 3.1.2.7 $k_4(F)=\mu_4(F)-3(\mu_2(F))^2=E[(X-E[X])^4]-3(E[(X-E[X])^2])^2$.

Now one needs the assumption that the derivatives $\frac{d^j F_a(s)}{ds^j}$ for $j \leqslant m$ exists. If N=min(*n,m*) then the difference in the true distribution and approximated distribution,

$\ln \phi(F_t, t)$ and $\ln \phi(F_a, t)$ is:

Equation 3.1.2.8 $\ln \phi(F_t, t) = \sum_{(j=1)}^{(N-1)} \left[k_j(F_t) - k_j(F_a) \right] \frac{(it)^j}{j!} + \ln \phi(F_a, t) + o(t^{N-1})$.

Next one must take the exponential of the above equation (noting that $e^{o(t^{N-1})} = 1 + o(t^{N-1})$):

Equation 3.1.2.9 $\phi(F_t, t) = \exp\left(\sum_{(j=1)}^{(N-1)} \left[k_j(F_t) - k_j(F_a) \right] \frac{(it)^j}{j!} \right) \phi(F_a, t) + o(t^{n-1})$.

Now, if one expands the exponential term into a power series then the following equation is:

Equation 3.1.2.10 $\exp\left(\sum_{(j=1)}^{(N-1)} \left[k_j(F_t) - k_j(F_a) \right] \frac{(it)^j}{j!} \right) = \sum_{(j=0)}^{(N-1)} \left(\frac{(it)^j}{j!} \right) + o(t^{N-1})$.

This form of the equations looks like an Asian option. The zero through third cumulants are:

Equation 3.1.2.11 $E_0 = 1$

Equation 3.1.2.12 $E_1 = k_1(F_t) - k_1(F_a)$

Equation 3.1.2.13 $E_2 = \left[k_2(F_t) - k_2(F_a) \right] + E_1^2$

Equation 3.1.2.14 $E_3 = \left[k_3(F_t) - k_3(F_a) \right] + 3 E_1 \left[k_2(F_t) - k_2(F_a) \right] + E_1^3$ etc.

Now equation Equation 3.1.2.9 can be written as:

Equation 3.1.2.15 $\phi(F_t, t) = \left[\sum_{(j=1)}^{(N-1)} E_j \frac{(it)^j}{j!} \right] \phi(F_a, t) + o(t^{N-1})$.

When one takes the inverse Fourier transformation of the above and using the following relations one will finally be able to show the Edgeworth series expansion as:

Equation 3.1.2.16 $f_t(s)=\frac{1}{2\Pi}\int_{(-\infty)}^{(\infty)} e^{-its}\Phi\left(F_t, t\right)dt$.

Equation 3.1.2.17 $\left(-1\right)^j\frac{d^j f_a(s)}{ds^j}=\frac{1}{2\Pi}\int_{(-\infty)}^{(\infty)} e^{-its}(it)^j\Phi\left(F_a, t\right)dt$ where j=0,1,...,N-1.

Finally, the Edgeworth series expansion is:

Equation 3.1.2.18 $f_t(s)=f_a(s)+\sum_{j=1}^{N-1} E_j\frac{(-1)^j}{j!}\frac{d^j f_a(s)}{ds^j}+\in(s, N)$,

such that

$\in(s, N)=\frac{1}{2\Pi}\int_{(-\infty)}^{(\infty)} e^{-its}o\left(t^{N-1}\right)dt$ where ∈(s,n) exists for all s one finds

$\lim_{(N\to\infty)}\left|\in(s, N)\right|=0$ for all s.

The Edgeworth series expansion is important to price an averaging option because it finds the approximate price formula similar to the Black-Scholes (1973) type formula. Once one finds variables for the first and second moments that have a lognormal approximate distribution equal to the true distribution ($\alpha_1\left(F_t\right)=\alpha_1\left(F_a\right)$ and $\mu_2\left(F_t\right)=\mu_2\left(F_a\right)$) then the two term Edgeworth series expansion is $f_t(s)=f_a(s)+\in(s, 3)$.

Through some algebraic manipulation and with Equation 3.1.2.4 and Equation 3.1.2.5 one finds the cumulants E_1 and E_2 are zero which yields a simplified version.

While Turnbull and Wakeman's (1991) research is interesting, it is included in this paper because it is the base of Levy's (1992) arithmetic averaging option model, which is used in the current study.

Levy's Discrete Average Rate Options

The Edgeworth Series Expansion is very important in average options. In 1992 Emond Levy made a notable contribution to the field of pricing Asian options. Levy notes Turnball and Wakeman "overlook the fact that when only the first two moments are taken into account in the approximation, the accuracy of the log normal assumption is acceptable making redundant the need to include additional terms in the expansion involving higher moment." Both Levy and Turnbull and Wakemans methods are important.

Through risk neutral evaluation, the price of a fixed strike Asian call is $c(S,A,t)=e^{-r\tau}E_Q\left[max\left(A\left(t_n\right)-X,0\right)\right]$ where $\tau=t_n-t$.

First, define $\tilde{A}\left(t_n;t\right)$ and X^* to average the asset over a time period $\left[t_0,t_n\right]$ at discrete points $t_i=t_0+i\Delta t$ where $t=1,2,\ldots.n$ then

Equation 3.1.2.18 $\Delta t=\frac{\left(t_n-t_0\right)}{n}$,

and at the current time *t* the rolling average denoted by A_t is:

$A(t)=\frac{1}{(m+1)}\sum_{i=0}^{m}S_{t_i}\;\; 0\leqslant m\leqslant n\quad where\;\; t_m\leqslant t<t_{m+1}$ (Kwok, 2008, 225-226).

So that $A(t)=0$ when $t\leq t_0$. Also let t_n be the time of expiry so that the payoff function is $max\left(A_{t_n} - X, 0\right)$. If $t = t_0$ then time zero and the moment the option begins averaging are the same if $t < t_0$ the current time is before the averaging period.

The most common type of Asian option is a fixed strike with arithmetic averaging (Kwok, 2008, 225), which is handy since it is equal to a floating strike put (Henderson and Wokjakowski, 2002). However, the pricing of these options becomes difficult due to the assumption that the underlying asset follows a geometric Brownian process. With a geometric Brownian process, one must deal with the problem that the sum of log normal components have no explicit formulation (Kwok, 2008, 226). Therefore $A\left(t_n\right)$ and its approximation $\tilde{A}\left(t_n ;t\right)$ have no explicit representation. The best approximation for these analytical prices is found by using the Edgeworth series expansion.

Now one sees a fixed Asian strike can also be written as

Equation 3.1.2.19 $c\left(S,A,t\right) = e^{-r\tau} E_Q\left[max\left(\tilde{A}\left(t_n\right) - X^*, 0\right)\right]$.

The expectation is subject to the risk neutral *Q* assumption, where *Q* is conditional on $S_t = S$, $A\left(t\right) = A$, and $\tau = t_n - t$ (Kwok, 2008, 228). Now to find an approximation for $\tilde{A}\left(t_n ;t\right)$. As previously mentioned, the assumption must be found with a log normal distribution $\mu\left(t\right)$ as the mean and $\sigma\left(t\right)^2$ as the variance.

With the approximating log normal distribution the first two moments are:

Equation 3.1.2.20 $\alpha_1\left(F_a\right) = \mu\left(t\right) + \frac{\sigma\left(t\right)^2}{2}$, and

Equation 3.1.2.21 $\alpha_2(F_a)=2\mu(t)+2\sigma(t)^2$.

This current research uses the two term Edgeworth Series approximation, so as stated previously, one must solve for variables that make $\alpha_1(F_T)=\alpha_1(F_A)$ and $\mu_2(F_t)=\mu_2(F_a)$.

By substitution one finds:

Equation 3.1.2.22 $\mu(t)+\frac{\sigma(t)^2}{2}=lnE_Q[\tilde{A}(t_n;t)]$, and

Equation 3.1.2.23 $2\mu(t)+2\sigma(t)^2=\ln E_Q\left[\tilde{A}(t_n;t)^2\right]$.

Solving for $\mu(t)$ and $\sigma(t)^2$ one finds:

Equation 3.1.2.24 $\mu(t)=2\ln E_Q\left[\tilde{A}(t_n;t)\right]-\frac{1}{2}lnE_Q\left[\tilde{A}(t_n;t)^2\right]$, and

Equation 3.1.2.25 $\sigma(t)^2=lnE_Q\left[\tilde{A}(t_n;t)^2\right]-2\,lnE_Q\left[\tilde{A}(t_n;t)^2\right]$.

If one assumes $\ln A(t_n;t)$ to be a log normal distribution with mean $\mu(t)$ and variance $\sigma(t)^2$, then the formula for the fixed strike Asian call option is:

Equation 3.1.2.26 $c(S,A,t)=e^{-rt}\{E_Q[\tilde{A}(t_n;t)]N(d_1)-X^* N(d_2)\}$. Note that if $A(t)=0,\ then\ X^*\ is\ just\ X$.

Levy's ARO d_1 and d_2 are unlike the d_1 and d_2 in Black-Scholes (1973) Model

Equation 3.1.2.27 $d_1=\frac{\mu(t)+\sigma(t)^2-lnX^*}{\sigma(t)}$ and

Equation 3.1.2.28 $d_2=d_1-\sigma(t)$.

Now the task of finding $\tilde{A}(t_n;t)$ and $\tilde{A}(t_n;t)^2$. If one assumes S_t is the asset price at time *t* then when $t \leqslant t_0$ the following formula is used

Equation 3.1.2.29 $E_Q\left[\tilde{A}(t_n;t)\right]=\frac{S_t}{(n+1)}e^{r(t_0-t)}\left[\frac{1-e^{r(n+1)\Delta t}}{1-e^{r\Delta t}}\right]$, $t \leq t_0$.

Similarly one finds the equations for $\tilde{A}(t_n;t)^2$ when $t \leq t_0$.

Now after what Levy (1992, 489) calls "some tedious algebra" one can evaluate $\tilde{A}(t_n;t)^2$ with the following:

Equation 3.1.2.30 $$E_Q\left[\tilde{A}(t_n;t)^2\right]=\frac{S_t^2}{(n+1)^2}e^{(-2r+\sigma^2)(t_0-t)}(B_1-B_2+B_3-B_4)$$,

where

Equation 3.1.2.31 $B_1=\frac{1-e^{(2r+\sigma^2)(n+1)\Delta t}}{1-e^{r\Delta t}\left[1-e^{(2r+\sigma^2)\Delta t}\right]}$,

Equation 3.1.2.32 $B_2=\frac{e^{r(n+1)\Delta t}-e^{(2r+\sigma^2)(n+1)\Delta t}}{\left(1-e^{r\Delta t}\right)\left[1-e^{(r+\sigma^2)\Delta t}\right]}$,

Equation 3.1.2.33 $B_3=\frac{e^{r\Delta t}-e^{r(n+1)\Delta t}}{\left(1-e^{r\Delta t}\right)\left[1-e^{(r+\sigma^2)\Delta t}\right]}$, and

Equation 3.1.2.34 $B_4=\frac{e^{(2r+\sigma^2)\Delta t}-e^{(2r+\sigma^2)(n+1)\Delta t}}{\left[1-e^{(r+\sigma^2)\Delta t}\right]\left[1-e^{(2r+\sigma^2)\Delta t}\right]}$.

3.1.3 Geometric Averaging Options

Discrete Geometric Averaging Fixed Strike Options

Beckenbach and Bellman (1971) prove that a geometric average is always lower than an arithmetic average. Therefore, similar to Kemna and Vorst (1990), the preset research will also have a geometric average in the calculations. The current study will be using the geometric average (GA) that follows Boyle (1993).

Begin by defining a running geometric average and the time units *t*. Say the running geometric average is:

Equation 3.1.3.1 $G_k = \left[\prod_{i=1}^{k} S_{t_i} \right]^{\frac{1}{k}}$ where $k = 1, 2, \ldots, n$.

The time units *t* need to be at equally distributed times such that $t_i = i\Delta t$ where $i = i, 2, \ldots, n$ and Δt is the equal change in time. The terminal payoff of a geometric average option is $max(G_n - X, 0)$. Assume the asset is a geometric Wiener process, so

that if one takes the ratio $R_i = \frac{S_{t_i}}{S_{t_{i-1}}}$, $i = 1, 2, \ldots, n$ is log-normally distributed. Applying the risk neutral system *Q:*

$\ln R_i \sim N((r - \frac{\sigma^2}{2}) \Delta t, \sigma^2 \Delta t)$ $i = 1, 2, \ldots, n$ where *r*=risk free interest rate and *N* is a normal distribution $N(\mu, \sigma^2)$. Like all the other options the price of GA option depends on whether *t* is before or during the averaging period. Since this thesis is only dealing with $t \leqslant t_0$ the following formula applies.

Equation 3.1.3.2 $\frac{G_n}{S_t}=\frac{S_{t_0}}{S_t}\left\{\frac{S_{t_n}}{S_{t_{n-1}}}\left[\frac{S_{t_{n-1}}}{S_{t_{n-2}}}\right]^2\ldots\left[\frac{S_{t_1}}{S_{t_0}}\right]^n\right\}^{\frac{1}{n}}$

such that

Equation 3.1.3.3 $\ln\frac{G_n}{S_t}=\ln\frac{S_{t_0}}{S_t}+\frac{1}{n}\left[\ln R_n+2\ln R_{n-1}+\ldots+n\ln R_1\right]\quad t\leqslant t_0$

Since $\ln R_i$ where $i=1,2,\ldots,n$ and $\ln\frac{S_{t_0}}{S_t}$ are independent Wiener increments, that are not in the same time intervals, they are each normally distributed and independent, which

implies $\ln\frac{G_n}{S_t}$ is normally distributed with mean:

Equation 3.1.3.5 $\left(r-\frac{\sigma^2}{2}\right)\left(t_0-t\right)+\frac{1}{n}\left(r-\frac{\sigma^2}{2}\right)\Delta t\sum_{i=1}^{n}i \;=\; \left(r-\frac{\sigma^2}{2}\right)\left[\left(t_0-t\right)+\frac{n+1}{2\mathrm{n}}\left(T-t_0\right)\right]$

and variance

Equation 3.1.3.6 $\sigma^2\left(t_0-t\right)+\frac{1}{n^2}\sigma^2\Delta t\sum_{i=1}^{n}i^2 \;=\; \sigma^2\left[\left(t_0-t\right)+\frac{(n+1)(2\mathrm{n}+1)}{6n^2}\left(T-t_0\right)\right].$

Again let $\tau=T-t$ through

Equation 3.1.3.7 $\sigma_G^2\,\tau=\sigma^2\left\{\tau-\left[1-\frac{(n+1)(2\mathrm{n}+1)}{6\mathrm{n}^2}\right]\left(T-t_0\right)\right\}$ and

Equation 3.1.3.8 $\left(\mu_G-\frac{\sigma_G^2}{2}\right)\tau=\left(r-\frac{\sigma^2}{2}\right)\left[\tau-\frac{n+1}{2\mathrm{n}}\left(T-t_0\right)\right]$

with the transition density function of G_n, with price S_t, at time *T*, and applying the risk neutral approach one finds a GA European fixed strike call option price is:

Equation 3.1.3.9 $c_G(S_t, t) = e^{-r\tau}\left[S_t e^{\mu_G \tau} N(d_1) - XN(d_2)\right]$ *where* $t \leqslant t_0$,

Equation 3.1.3.10 $d_1 = \dfrac{\ln \dfrac{S_t}{X} + \left(\mu_G + \dfrac{\sigma_G^2}{2}\right)\tau}{\sigma_G \sqrt{\tau}}$

and Equation 3.1.3.11 $d_2 = d_1 - \sigma_G \sqrt{\tau}$.

3.1.4 Options Equivalence of Fixed Strike and Floating Asian options

Henderson and Wojakowski (2002) found that, via symmetry, a fixed strike Asian option could be used to price a floating strike Asian option. Henderson and Wojakowski (2002) found that a floating strike call can be priced using a fixed strike put, and similarly, a floating strike put can be priced using a fixed strike call. Henderson, Hobson, Wojakowski and Shaw (2004) give a very good explanation of how the symmetry works. If one prices a floating strike put $p_{float}(S,K)$ then when the the fixed strike call formula is used, reverse *S* and *K* in the pricing so $c_{fix}(K,S)$.

3.2 European Options

Black-Scholes Model

It is questionable whether or not Asian options are applicable in the non-GMO-Soybean futures market. To test the theory that Asian options are appropriate, one needs to compare these options to a different type of option. This study will compare the averaging options to a European put. Black-Scholes 1973 model for will be employed for the European pricing formula as follows:

Equation 3.2.1 $P_{BS} = Xe^{(-r\tau)} N(-d_2) - S_t N(-d_1)$

where

Equation 3.2.2 $d_1 = \frac{(\ln F_0 / K) + (r + \sigma^2/2)T}{\sigma\sqrt{(T)}}$ and Equation 3.2.3 $d_2 = d_1 - \sigma\sqrt{T}$.

All variables are as defined previously.

3.3 Volatility from Historical Data

Figlewski (1997) finds implied volatility to be superior because implied volatility uses the actual market's forecast of volatility. However, since there currently are no options trading on the TGE non-GMO soybean contract one must use a different method to obtain volatility. The historical volatility will be calculated as explained by Hull (2009). To find the historical volatility the futures price is observed at equal discrete units in time.

From Hull's (2008, 282-284) explanation of historical volatility, define:

$n+1$: as the number of observations

$S_{(i)}$: as the futures price at the end of the *i*th interval, where $i = 1, 2, \ldots, n$

τ: as the length of time in years,

then:

Equation 3.3.1 $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ for $i = 1, 2, \ldots, n$.

Now the standard deviation of u_i is calculated as:

Equation 3.3.2 $s=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i-\bar{u})^2}$.

The standard deviation of u_i is $\sigma\sqrt{\tau}$ such that s is an estimate of $\sigma\sqrt{\tau}$. Therefore, one can say that $\hat{\sigma}$ is an estimate of σ and $s=\hat{\sigma}\sqrt{(\tau)}$. The rational in using historical volatility is the lognormal properties and the simplistic nature of its calculation.

Rolling Over Contracts

Due to the limited life span of a contract, academic hypotheses and trading rules can become complicated. To deal with this complication, a linking of individual contracts commonly known as rolling over contracts occurs to create a longer artificial time line.

There are two major issues to deal with when rolling over contracts 1) the rollover date and 2) price adjustment (Ma, Mercer, & Walker, 1992). If liquidity is of concern, then contracts are rolled over at some arbitrary time before the delivery date (Ma, Mercer, & Walker, 1992). Junkus (1986), when dealing with stock index futures, excluded prices from the delivery month to evade non-stationary returns created by the increased variance from decreasing time to maturity. Moreover it is a widely accepted concept that due to the erratic volume and volatile prices, data from the last month of a futures contract can be worthless (Ma, Mercer, & Walker, 1992). This research will be following Junkus'(1986) method and rolling over at the end of the month.

The next issue is price adjustment. When rolling over from one contract to the next there can be faux price jumps created by differences in the mismatched contracts. These faux jumps manufacture large price jumps which can create unnaturally high volatility (Ma, Mercer, & Walker, 1992).

As Ma, Mercer, and Walker identify in their 1992 article, one common way to deal with price adjustment at the rollover date is to take the price from the previous contract and subtract it from the new contract. Then continue subtracting that result from all new prices in the series (or add the differences to the previous prices). This is the rollover method that is used in the research.

CHAPTER 4: PROCEDURES

To examine the reduction in income volatility, a mathematical trading simulation was enacted for the years 2001 through 2010. First data was collected from the Tokyo Grain of Exchange. End of day data is available for download since the launch of the non-GMO soybean futures. For volatility end of day data was used. To calculate the volatility for a simulation option contract, the historical volatility from the previous year was found using Hull's (2009) historical volatility method and the rolling over of contract methods previously explained (Junkus, 1986) . Starting in January on the first day of trading until the last day of trading in December end of day data was used from the nearest expiring contract nearby, excluding prices from the delivery month. Each contract was rolled over at the end of the month prior to the expiring contract's expiration month. The data was then price adjusted, as described in the roll-over-contracts section (Ma, Mercer, and Walker, 1992).

After the volatility was computed the values were entered into Levy's (1992) arithmetic averaging options, Black-Scholes (1973) option, and the geometric averaging option formulas (Boyle, 1993). Two simulation option contracts were conducted A) a six month contract option bought six months in advance ($t<t_0$) and B) a six month contract option bought at the contract's beginning ($t=t_0$). The first assumption was the option holder always executed the option, then the research examined an income maximizing option holder.

Next one needed to find S_{t_i}, the asset price at a discrete time for pricing and purchasing an option. For the year long contract this research used the first trading day of the year, and simulated purchasing an option or a futures contract for December that would mature on the last trading day of the year. For the six month long contract, this study used the first trading day in July and again, simulated purchasing an option or a futures contract for December that would mature on the last trading day of the year.

On November 14, 2008 the Board of Directors of the TGE decided to move the non-GMO soybean futures from a call session trading platform (Itayose[4]) to a continuous trading platform[5]. However on June 29, 2009 the TGE Board of Directors decided to move the non-GMO soybeans back to a call trading system (Itayose[1]). The non-GMO soybean futures stopped being traded continuously at the end of September 2009. For ease and continuity in this analysis end of day data was downloaded from the TGE and continuous trading was not considered. For the futures purchasing price the opening price or first reported price on the first day of the contract was used; and for selling the settlement price on the last day the contract was traded was used.

A TGE non-GMO soybean futures contract is 13.6 times smaller than a Chicago Mercantile Exchange (CME) soybeans futures contract. Based on the 2008-2011 Japanese Yen US Dollar exchange rates, adding 3,000 Yen to one TGE non-GMO futures contract would be approximately the same as adding 10 cents to a CME soybean futures

4 An explanation of the Itayose method can be found at the Tokyo Stock Exchange website (http://www.tse.or.jp/about/books/trading_methodology.pdf)

5 Continuous trading involves the immediate execution of orders upon their reception by market makers and specialists (Investopedia).

contract. Since this research is from the standpoint of a soybean producer the research assumed the producer owned the soybeans. Therefore the hedging strategy is comparable to a protective put (Hull, 2008, 220). The producer owns the soybeans so if prices do decrease it would be attractive to sell the soybeans for 3,000 Yen more than the day the option was written. So for a strike price, *K*, 300 Yen were added to the fixed asset price.

Next one would need an interest rate for the option formulas. For simulation *A* the twelve month Japanese Yen LIBOR was employed and for simulation *B* the the six month Japanese Yen LIBOR was used. This data was collected from global-rates.com.

Four different contracts were simulated as a means for hedging risk; Levy's arithmetic averaging options(1992), Black-Scholes option(1973), geometric averaging options(Boyle,1993), and a futures contract. This research also looked at the income that would have been received if a hypothetical no hedge strategy was used. In the two different types of Asian options, three different settlement averaging[6] price periods were constructed, n=4 and n=12 as well as n=daily. This follows previous works like Turnbull and Wakeman (1991), Kemna and Vorst (1990), Levy (1992), and Curran (1994). After the contract prices were determined, the payoff was calculated that would have been received had an individual invested and delivered one of the simulated options.

Profits for the averaging options were derived from the average prices selected by n. The value n tells how many times to divide the averaging period by. So for example if there were 120 observed prices in the averaging period for n=4 one would average the

[6] The different averaging periods and fixings are explained in Section 3.1.1 Average the Options

30th, 60th, 90th, 120th prices. After the average income earnings were determined, the cost of the option for the final transaction free income was subtracted. The profit from using Black-Scholes option was calculated by subtracting the option's purchase price from the strike price. For the income from using a futures contract, the price at which the futures contract was entered into was used. For the hypothetical no-hedge plan, the income was determined by assuming the producer sold at the contract maturation price. Transaction costs were ignored in all simulations. First the research examined if the producer always executed the option. Then following the same procedures described above (if the producer had always executed the option) the research assessed the income from a money maximizing producer. A money maximizing producer would not execute the option if more money could be made from selling at the ending day price.

In all simulations the standard deviations, variance, profits and costs of the contracts and options were compared.

CHAPTER 5: RESULTS

5.1 Prologue and No Hedge

This section summarizes the findings associated with simulation of options trading in the TGE non-GMO soybean futures contract. The information presented in this section represents the income a producer would have received from each investment plan, in Yen per 1000 kg. In the first section the producer was assumed to always execute the option. A comparison was conducted of the different option strategies to a futures contract as well as a no hedge scenario, i.e., selling at the market price without ever purchasing a futures contract or an option. Also considered was the income stream a producer would have received if he made the money maximizing decision. The futures contract and options scenarios discussed in this section were bought six months in advance of the averaging period, and in entirety lasted 12 months until contract expiration. Also a simulation of a six month futures contract or option bought at the start of the averaging period was conducted (see Appendix A for later results).

The information presented below represents the different options and futures contract scenarios for the years 2001 through 2010. The price reported is in Yen per 1000 kilograms, so to find the payout per contract one would multiply by 10 for the 10,000 kilogram TGE non-GMO futures contract. Also reported was the variance and standard deviation of the options and futures contract scenarios. Currently, the TGE offers a futures contract and delivery points. Since there is no underlying cash market, only speculation can be used as for a non-hedging producer. To create this scenario, the closing price at maturity was used as a proxy for a cash sale. This hypothetical scenario is referred to as the no-investment plan or no-hedge plan throughout this thesis. Illustrated in Figure 5.1.1 is the hypothetical no-investment plan. The income is more volatile than for the futures scenario; and this constant changing in income can lead hedgers to make unwise decisions in certain years. Hence, these decisions may have harsh repercussions.

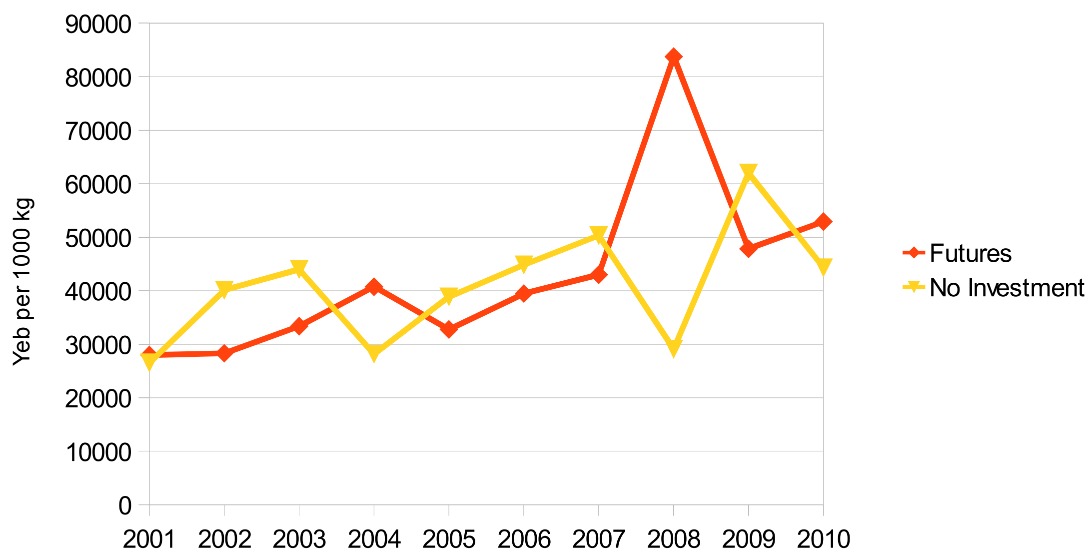

Figure 5.1.1: Yearly Income for Futures Contract and the Hypothetical No Hedge

As detailed in Table 5.1.1 the percentage change from yearly income is much larger in the hypothetical no-hedge plan. Also the no-investment plan has a higher standard deviation of percentage change in income from year to year. One of the purposes of this research is to find investment plans that help stabilize investors' income. As can be seen, a no hedge postion cannot accomplish this goal.

Table 5.1.1 Percentage Change in Yearly Income for Futures and Hypothetical No Hedge Scenarios

	Futures	% Change	No Investment	% Change
2001	26850		26520	
2002	30140	12.25%	40120	51.28%
2003	32780	8.76%	43990	9.65%
2004	38470	17.36%	28150	-36.01%
2005	36530	-5.04%	38870	38.08%
2006	35660	-2.38%	44850	15.38%
2007	53780	50.81%	50320	12.20%
2008	84530	57.18%	29060	-42.25%
2009	52000	-38.48%	61980	113.28%
2010	45820	-11.88%	44300	-28.53%
Standard Deviation of % Change		0.3		0.49

5.2 Futures Contract vs. Black-Scholes Options

The table below represents the payout a hedger or speculator would have received by either investing in the Black-Scholes option premium or assuming a futures contract position.

Table 5.2.1: Black-Scholes vs. Futures Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Black-Scholes	25575	26559	31408	38080	29119	34733	38284	75899	36559	45752
Futures	28000	28300	33350	40770	32750	39470	43000	83740	47850	52920

The standard deviation shows how dispersed the data is from the sample mean, while the variance shows how dispersed the data is from the mean squared.

Table 5.2.2: Black Scholes vs. Futures Standard Deviation and Variance of Yearly Incomes

	Standard Deviation	Variance
Black-Scholes	14592	212942997
Futures	16460	270931450

The Black-Scholes options pricing model derived premium decreases variance in income. The payout variance from executing the Black-Scholes option was 79% of the variance of the futures contract. This is a noticeable decrease in income variance.

Figure 5.2.1 represents the income a producer would have received each year for the simulation. If an individual expects to make *X* amount of income each year, then the line graph shows how much the individual's income varies by choosing between the investment plans or having no investment plan.

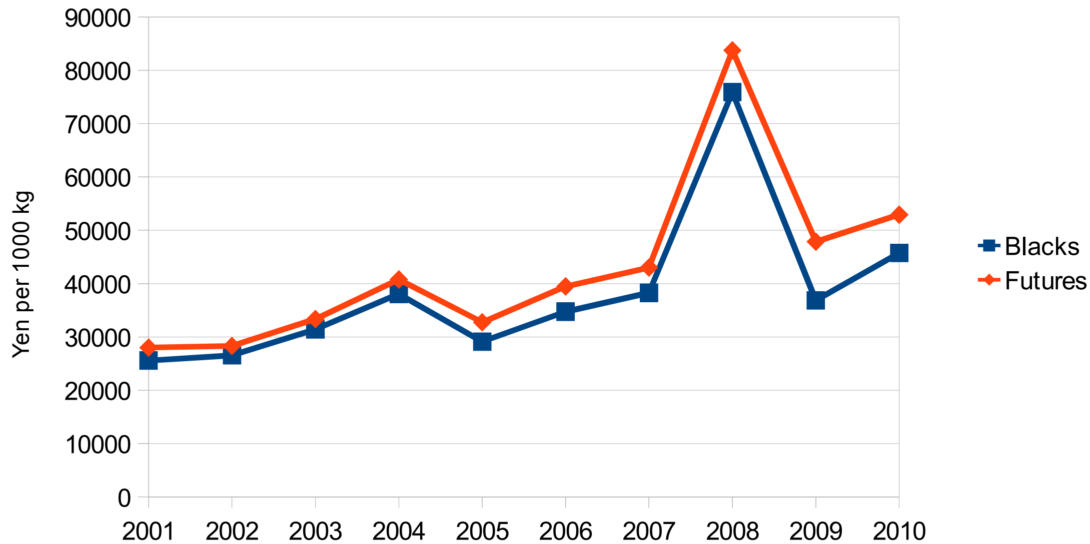

Figure 5.2.1: Yearly Incomes for Black-Sholes Option and a Futures Contract

The visual representation of changes in income each year yields interesting results. The Black-Scholes option pricing model income and the futures contract income were similar. In both investment strategies the income appears to be decently stable from year to year except in 2008.

5.3 Futures Contract vs. Levy's Arithmetic Asian options

Table 5.3.1 displays the different payouts an individual would have received by investing in strictly a futures contract compared to the three different versions of Levy's arithmetic Asian options premium. The difference in the three different options is the averaging increments. One option is averaged daily, the second option is averaged 12 times in the six-month period, and the last contract is averaged four times in the six month period.

Table 5.3.1: Three Alternatives for Levy's ARO vs. Futures Contract Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Levy n=daily	25596	30076	35005	32650	32162	31240	53959	53205	40828	42506
Levy n=12	25582	30521	35178	32033	32323	31808	53814	50139	41355	42497
Levy n=4	25363	31361	36160	29855	32159	32634	52846	43765	43029	42768
Futures	28000	28300	33350	40770	32750	39470	43000	83740	47850	52920

Figure 5.3.1 represents the different income streams that would have been received by each investment plan. As can be seen, the futures contract scenario has the most year to year change. For Levy's options scenarios, n=daily has the least year-to-year income change, and as n increases, the income stream increases in variation. However, the variation in the different values of n is relatively small compared to the futures contract. The futures contract has much greater variability than any of the three Levy option pricing alternatives.

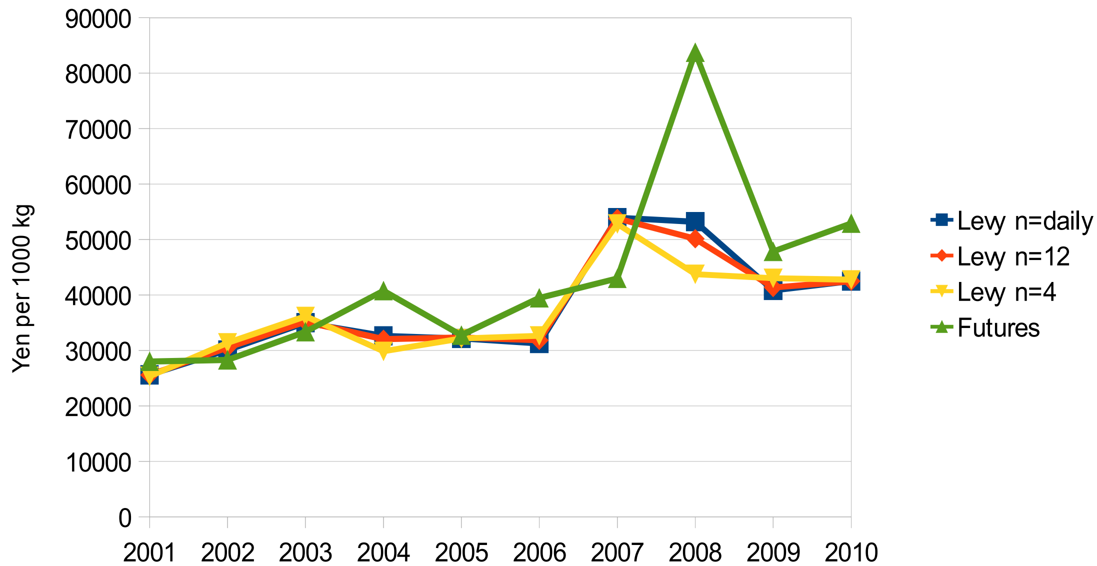

Figure 5.3.1: Yearly Incomes for Levy's ARO and a Futures Contract

Table 5.3.2 represents a comparison of the standard deviation and variance of the four different contracts. The first column shows the contract type *n*=daily is Levy's arithmetic averaging option with daily averaging of the non-GMO soybean prices; *n*=12 is twelve equally spaced end-of-day prices averaged, and *n*=4 is four equally spaced end-of day-prices averaged. As can be seen, in the Futures Variance/Levy's Variance column, the futures contract has greater variance than any of the three alternative arithmetic averaging options.

Table 5.3.2: Levy's ARO vs. Futures Standard Deviation and Variance

	Standard Deviation	Variance	Futures Variance/Levy's Variance
Levy n=daily	9703	94161230	2.88
Levy n=12	9134	83439141	3.25
Levy n=4	8353	69773184	3.88
Futures	16460	270931450	

5.4 Futures Contract vs. Geometric Averaging Options

Figure 5.4.1 represents the different revenues that would be made from purchasing a geometric averaging option or a holding futures contract position. Consistent with the arithmetic averaging findings above, the option alternatives provide for a more stable income flow than does the futures contract position.

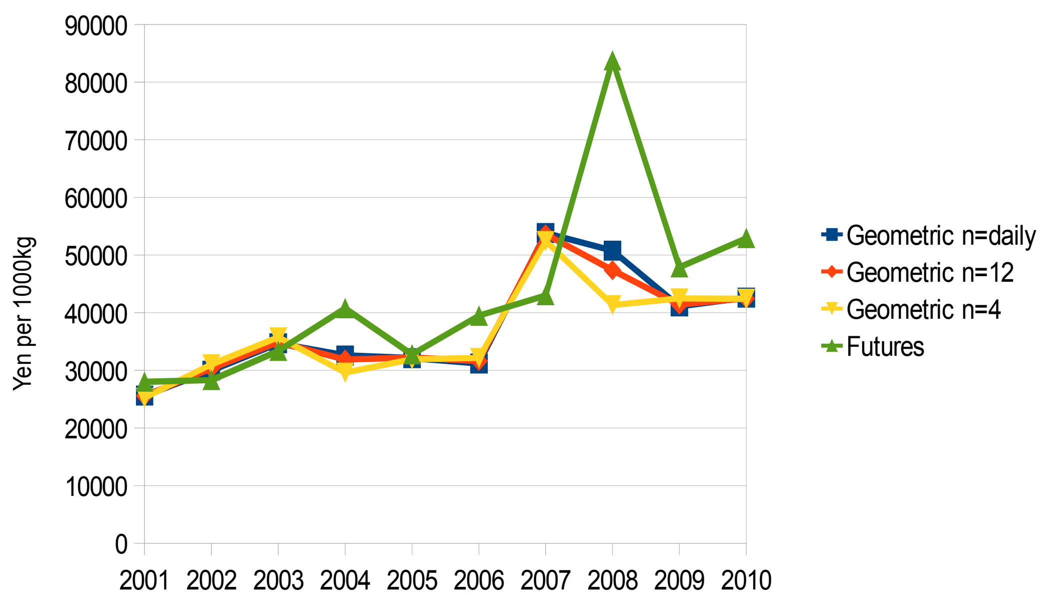

Figure 5.4.1: Yearly Incomes for Geometric ARO and a Futures Contract

Table 5.4.1 reports the numerical results from Figure 5.4.2

Table 5.4.1: Geometric ARO vs. Futures Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Geometric n=daily	25624	29955	34681	32576	32115	31182	53818	50745	41090	42620
Geometric n=12	25562	30329	34851	31873	32213	31606	53599	47364	41391	42472
Geometric n=4	25251	31040	35726	29597	31913	32163	52415	41324	42488	42413
Futures	28000	28300	33350	40770	32750	39470	43000	83740	47850	52920

Compare now the relative standard deviation and variance of a futures contract and geometric averaging options (Table 5.4.2). In the most extreme case, a futures contract has 4.12 times the variance of the geometric ARO n=4. Also, with the geometric ARO, like Levy's ARO, the variance decreases as n decreases.

Table 5.4.2: Geometric ARO vs. Futures Standard Deviation and Variance

	Standard Deviation	Variance	Futures Variance/Geometric Variance
Geometric n=daily	9309	86652921	3.13
Geometric n=12	8758	76696237	3.53
Geometric n=4	8109	65753417	4.12
Futures	16460	270931450	

5.5 Black-Scholes options vs. Levy's Arithmetic Averaging Options

Table 5.5.1 represents the yearly income a hedger or speculator would have received from executing either Levy's ARO or Black-Scholes option position. Levy's ARO option position had a steadier income stream than the Black-Scholes option position (see also Figure 5.5.1).

Table 5.5.1: Levy's ARO vs. Black-Scholes Income stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Levy n=daily	25596	30076	35005	32650	32162	31240	53959	53205	40828	42506
Levy n=12	25582	30521	35178	32033	32323	31808	53814	50139	41355	42497
Levy n=4	25363	31361	36160	29855	32159	32634	52846	43765	43029	42768
Black Scholes	25576	26559	31408	38081	29120	34734	38284	75900	36560	45752

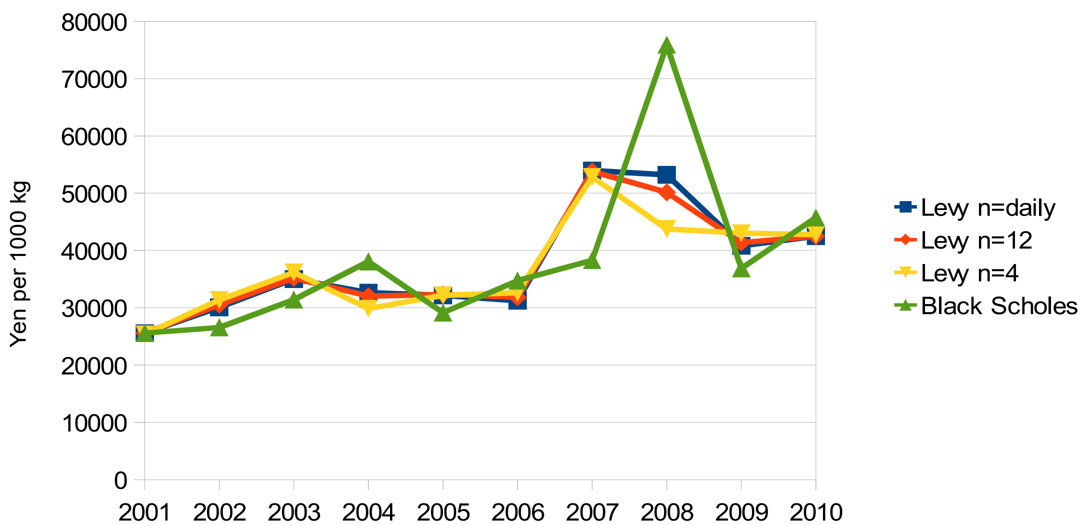

Figure 5.5.1: Yearly Incomes for Levy's ARO and Black-Scholes Option

The results reported in Table 5.5.1 have subtracted the cost of the initial option premium from the income stream. Levy (1992) noted an Asian option will always be cheaper than a plain vanilla option; therefore, it is important to compare the different costs of the option pricing strategies. Table 5.5.2 and Figure 5.5.2 represents each of

Levy's ARO alternatives subtracted from the Black-Sholes option scenario. The relative difference varies from 321 Yen per 1000 kg more expensive, over Levy's daily ARO, in 2002 to 1687 Yen per 1000 kg more expensive, over Levy's *n*=4 ARO, in 2009. In terms of a per contract difference, the 2009 Black-Scholes options pricing scenario would have cost the producer an additional 16,870 Yen per contract, or an additional 45 Yen per bushel. Figure 5.5.2 is an illustration of the data reported in table 5.5.2.

Table 5.5.2: The Additional Cost of Black-Scholes Option Over Each of Levy's AROs

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Black-Sholes – Levy n=daily	352	321	357	489	665	854	632	1162	1587	1044
Black-Sholes – Levy n=12	361	328	364	499	678	870	649	1191	1623	1070
Black-Scholes – Levy n=4	377	340	377	517	702	901	680	856	1687	1115

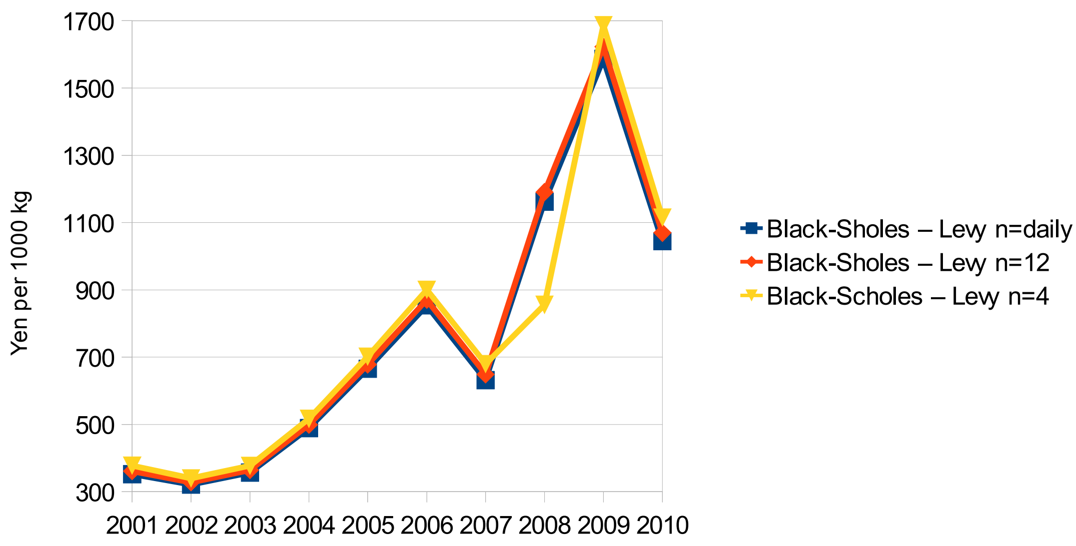

Figure 5.5.2: A Graphical Representation of the Additional Cost of Black-Scholes Option Over Each of Levy's AROs in Yen per 1000 kg

Table 5.5.3 expresses the actual standard deviation and variation of the different options.

Table 5.5.3: Levy's ARO vs. Black-Scholes Option Standard Deviation and Variance

	Standard Deviation	Variance	Black's Variance/Levy's Variance
Levy n=daily	9704	94161230	2.26
Levy n=12	9135	83439141	2.55
Levy n=4	8353	69773185	3.05
Blacks	14593	212942998	

Table 5.5.3 shows that Black-Scholes Option creates more variance in income streams. However, after seeing the variance multiplier from Levy's ARO compared to a futures contract, the Black-Sholes variance multiplier is less in all cases. Therefore, options do decrease variance. Since it is already known that Levy's ARO n=daily has the most variation of Levy's AROs it is of no surprise that it has the smallest multiplier to create the Black-Scholes' income variance.

5.6 Black-Scholes Options vs. Geometric Averaging Options

From the similarities in Levy's ARO and geometric ARO's, and the similarities in Black-Scholes options and a futures contract, one can speculate how the Black-Scholes options perform in comparison to a geometric ARO. Table 5.6.1 outlines the income a producer would have received from using either the Black-Scholes option or a geometric ARO. In Table 5.6.1, one can see that the geometric AROs give a more uniform level of income. While the geometric ARO's may fluctuate from year to year, there are no drastic jumps like that from 2008 to 2009 observed for the Black-Scholes option.

Table 5.6.1: Black-Scholes Option vs. Geometric ARO Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Geometric n=daily	25624	29955	34681	32576	32115	31182	53818	50745	41090	42620
Geometric n=12	25562	30329	34851	31873	32213	31606	53599	47364	41391	42472
Geometric n=4	25251	31040	35726	29597	31913	32163	52415	41324	42488	42413
Blacks	25576	26559	31408	38081	29120	34734	38284	75900	36560	45752

Figure 5.6.1 is a visual representation of the numerical income results in Table 5.6.2. Now one can see the stability in a geometric ARO compared to Black-Scholes option. For the Black-Scholes option there is a huge income spike in 2008, yet the years prior and after have practically the same lower value. The income from executing the Black-Scholes option in 2008 is 198% of 2007 income. The income change for the geometric ARO from 2007 to 2008 is much less drastic. The 2008 income as a percentage of 2007 income for using the different options is 79% for n=4, 88% for n=12, and 94% for n=daily. Income like that received in 2008 can entice farmers or producers to overspend and possibly make careless decisions. These illogical choices, followed by a year of lower income may be very difficult for the farmer or producer.

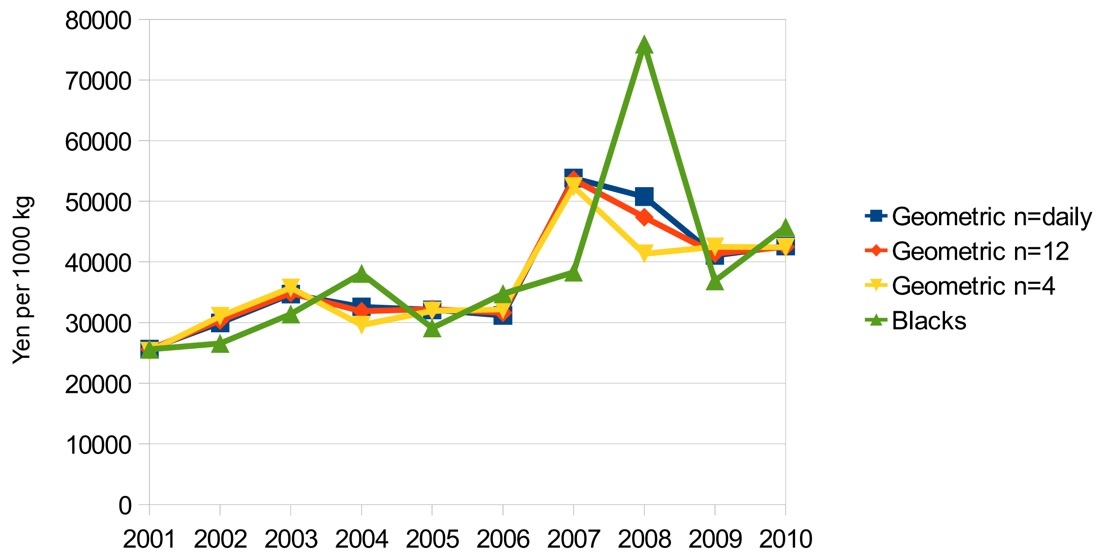

Figure 5.6.1: Yearly incomes for Black-Scholes Option and Geometric AROs

Table 5.6.2 gives the standard deviation, variance, and Black-Scholes variance divided by each the different geometric ARO's variance. The Black-Scholes variance is always two to three times larger than the geometric variance. This illustrates that the income received from executing the Black-Scholes option will create larger swings in yearly income than if a producer had used a geometric ARO. However, Black-Scholes variance compared to a geometric ARO, in relation to a futures contract compared to a geometric ARO, is much lower.

Table 5.6.2: Black-Scholes Option vs. Geometric ARO Standard Deviation and Variance

	Standard Deviation	Variance	Black-Scholes Variance/Geometric ARO Variance
Geometric n=daily	9308	86652921	2.46
Geometric n=12	8757	76696236	2.78
Geometric n=4	8108	65753417	3.24
Black-Scholes	14592	212942997	

As stated before, the earning for each year already account for the cost of the option. A Black-Sholes option is more expensive than a geometric ARO. It is important to realize how much more expensive Black-Sholes option is. The additional cost and variance of Black-Scholes option makes an ARO much more attractive. Table 5.6.3 shows the added cost of Black-Scholes option over each of the different geometric AROs depending on *n* (in Yen per 1000 kg). Black-Scholes option ranges from being 261 Yen per 1000 kg of soybeans more expensive for the geometric ARO *n*=4 in 2002 to 2,029 Yen per 1000 kg more expensive for the geometric ARO *n*=daily in 2009. For the extreme case in 2009 the Black-Scholes option is 20,290 Yen more expensive per contract or an additional 55 yen per bushel. Figure 5.6.2 is the graphical interpretation of the data from Table 5.6.3.

Table 5.6.3: The Additional Cost of Black-Scholes option Over Each Geometric ARO (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Black-Sholes – Geometric n=daily	385	337	373	513	725	937	713	1276	2029	1196
Black-Sholes – Geometric n=12	353	314	348	479	680	879	651	1168	1890	1104
Black-Scholes – Geometric n=4	275	261	288	399	572	739	501	524	1548	883

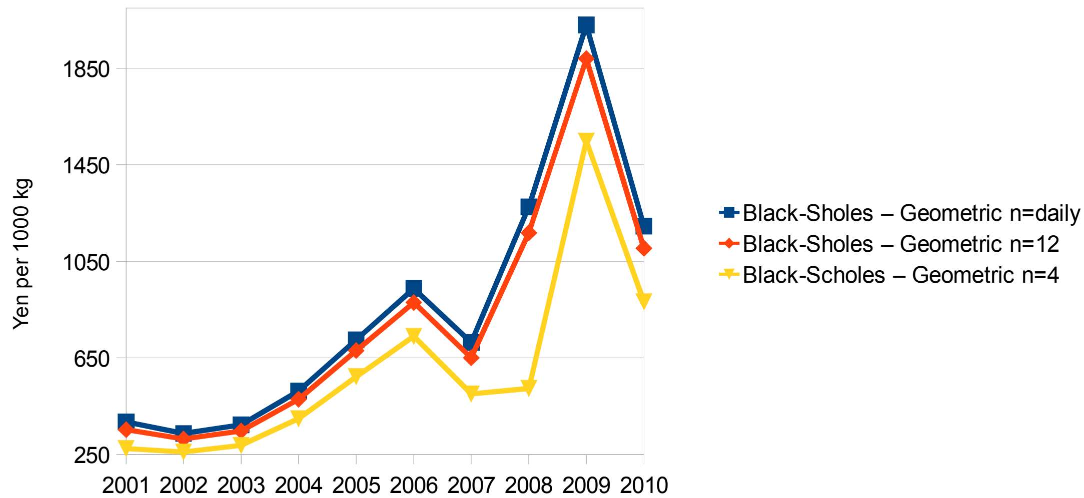

Figure 5.6.2: A Graphical Representation of the Additional Cost of Black-Scholes Option over each Geometric ARO (in Yen per 1000 kg)

5.7 Impacts of Changing n in Averaging Rate options

Before one compares Levy's ARO and the geometric ARO the difference the level of *n* has in these two AROs is be examined. First a portrayal of the differences in Levy's AROs will be shown and then the differences in the geometric AROs.

An interesting observation is that the magnitude of the variances and price of option increases as *n* increases. Levy found in his 1992 paper that as *n* increases, the prices of his arithmetic option also increase. This is because a larger amount of price risk exposure is removed in the first fixing in averaging options with smaller *n*. Since *n*=4 is the cheapest of Levy's ARO, one must subtract it from *n*=12 and *n*=daily. Figure 5.7.1 displays in Yen per 1000 kilograms how the various values of *n* increase the cost of

Levy's ARO.

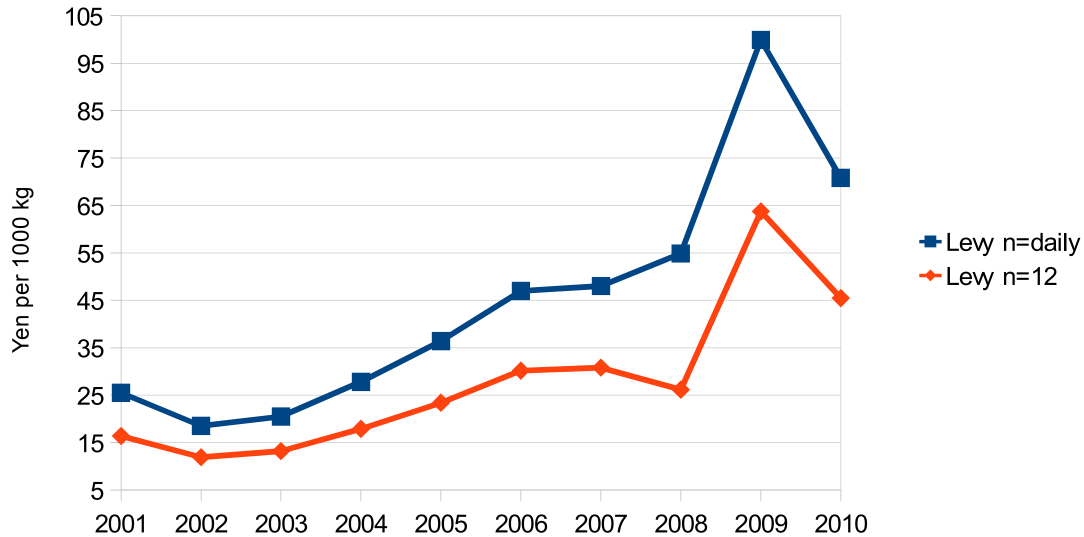

Figure 5.7.1: A Comparison of Increasing Option Price as the Level n Increases

Table 5.7.1 reports the numerical representation of Figure 5.3.1.

Table 5.7.1: Numerical Results of Increasing Option Price as the Level n Increases

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Levy n=daily	25.49	18.52	20.51	27.82	36.42	46.97	48.02	54.88	99.93	70.82
Levy n=12	16.38	11.9	13.19	17.89	23.41	30.17	30.81	26.15	63.75	45.45

The option price and variance increase as *n* increases. However, income streams from increasing *n* should also be examined. From Table 5.3.1, the simulation *n*=daily has the highest income in years 2001, 2004, 2007, 2008, and 2010. The simulation *n*=12 has the highest income stream in 2005. The simulation *n*=4 has the highest income stream in 2002, 2003, 2006, and 2009. Table 5.7.2 reveals the income a producer would have received summing up all 10 years.

Table 5.7.2: Ten Year Summed Income in Levy's ARO

	Sum
Levy n=daily	377228
Levy n=12	375250
Levy n=4	369941

Table 5.7.3: The Simulation n=daily Variance Divided by the Respective Level of n

Levy n=12	1.13
Levy n=4	1.35

Table 5.7.3 displays daily variance divided by the simulation *n*=12 variance in the first row and the simulation *n*=4 variance in the second row. The simulation *n*=daily variance has 35% more variance than *n*=4 variance. However interestingly Table 5.7.2 illuminates that the simulation *n*=daily ARO creates the largest income stream. This initiates a dilemma in that while the simulation *n*=daily has the highest 10 year income stream, it also has the most expensive options and has the most variance. This leads to more research which needs to be done on how the different values of *n* affect variance and income.

Just like Levy's options the variance in a geometric ARO decreases as *n* decreases. However unlike Levy's ARO the option price decreases as *n* increases. This is because as *n* increases μ_G decreases, and when taking the exponential function to a smaller number the result decreases, which gives a smaller option price. Also as *n* increases the negative d2 (see equation 3.1.4.11) increases and the positive d1 (see equation 3.1.4.10) decreases. Recall the normal distribution, when entering decreasing values of a positive nature, or increasing value of a negative nature, the result increases. Since the simulation *n*=daily is the cheapest of the geometric AROs, this value has been subtracted from the simulation *n*=4 option price and the simulation *n*=12 option price. Figure 5.7.2 represents these differences in prices in Yen per 1000 kg.

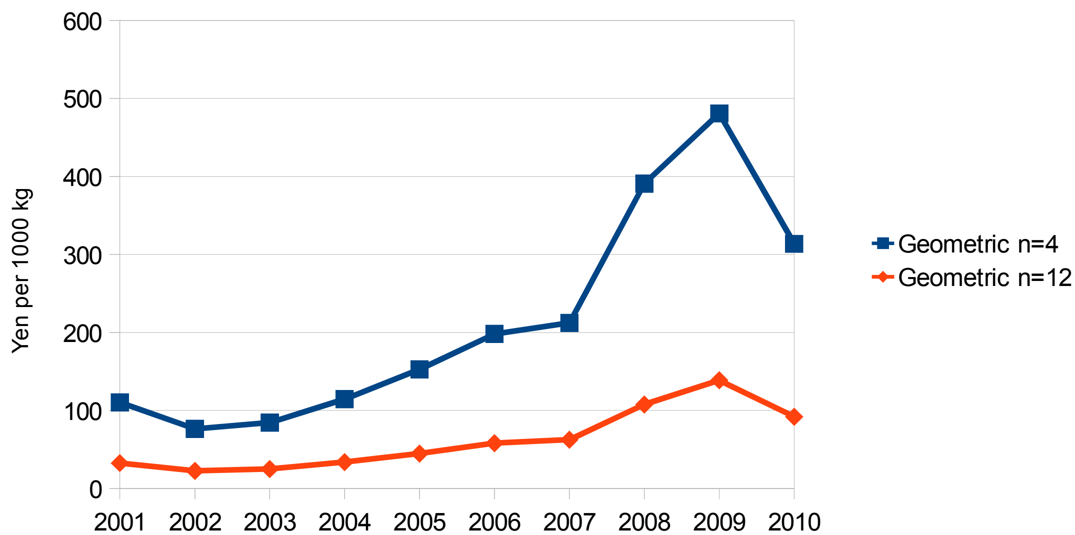

Figure 5.7.2: A Comparison of Increasing Option Price as the Level of n Decreases

Below is the numerical representation of the Figure 5.4.1.

Table 5.7.4: Numerical Results of Increasing Option Price as the Level of n Decreases

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Geometric n=4	110.42	76.37	84.39	114.58	152.71	198.14	212.23	390.81	480.79	313.54
Geometric n=12	32.65	22.65	25.01	33.93	44.89	58.21	62.66	107.7	138.76	92.01

Table 5.7.5 The Simulation n=daily Variance Divided by the Respective Level of n

Geometric n=4	1.15
Geometric n=12	1.06

The Table 5.7.5 illustrates the increased variance a daily geometric ARO has over the simulation *n*=12 and the simulation *n*=4 geometric ARO. Just like Levy's ARO the simulation of *n*=4 option has the least variance. However an interesting difference is that the daily geometric ARO has only 15% more variance than the simulation *n*=4 geometric ARO option; and this is petite in comparison to the different *n* simulations for Levy's ARO.

Table 5.7.6: Ten Year Geometric ARO Summed Income

	Sum
Geometric n=daily	374405
Geometric n=12	371258
Geometric n=4	364330

From Table 5.4.1 the simulation *n*=daily has the highest income streams in years 2001, 2004, 2007, 2008, and 2010. Also the simulation *n*=12 has the highest income in 2005 and the simulation *n*=4 has the highest income in 2002, 2003, 2006, 2009. The geometric ARO creates a different dilemma from Levy's ARO in Table 5.7.6 as the simultation *n*=daily has the highest ten year income stream. Also, recall Table 5.7.4 where the simulation *n*=daily has the lowest option cost. However, from Table 5.4.2 the simulation *n*=daily has the highest variance. Table 5.7.5 displays the increasing variance of the simulation *n*=daily over the simulations *n*=4 and *n*=12. The geometric ARO also leads to more research needing to be done on *n* in relation to Asian options, and in deciding how to deal with income, option price, and income variance.

5.8 Levy's Arithmetic Averaging Options vs. Geometric Averaging Options

This research has already proven that an ARO yields the least variance, and most constant income stream. Now the research turns to investigating how two different AROs compare.

Table 5.8.1 conveys the different earnings depending on *n* for Levy's ARO and a geometric ARO for the years 2001-2010. The last column gives the 10-year summation of income. All results are in Yen per 1000 kg.

Table 5.8.1: Geometric ARO vs. Levy's ARO Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Sum
Geometric n=daily	25624	29955	34681	32576	32115	31182	53818	50745	41090	42620	374405
Geometric n=12	25562	30329	34851	31873	32213	31606	53599	47364	41391	42472	371259
Geometric n=4	25251	31040	35726	29597	31913	32163	52415	41324	42488	42413	364330
Levy n=daily	25596	30076	35005	32650	32162	31240	53959	53205	40828	42506	377228
Levy n=12	25582	30521	35178	32033	32323	31808	53814	50139	41355	42497	375250
Levy n=4	25363	31361	36160	29855	32159	32634	52846	43765	43029	42768	369941

There is little difference in the payouts. In all cases Levy's ARO gives higher 10-year income earnings than the geometric ARO. For the simulation *n*=daily, the difference is 2,823 yen or .75%; and for the simulation *n*=12, the difference is 3,991 yen or 1.06%. For the simulation *n*=4, the difference is 5,610 yen or 1.52%. Figure 5.8.1 is the graphical representation of the data in Table 5.8.1

Figure 5.8.1: A Comparison Levy's ARO and Geometric ARO Income Streams

While Levy's ARO always has a higher cumulative payout the geometric ARO will always have lower variance. Table 5.8.2 denotes the differences in variance in the contracts. In all cases Levy's ARO has higher variance than the geometric ARO. For both the simulations *n*=daily and *n*=12, the variance in income from executing Levy's ARO is 9% larger, and for the simulation *n*=4, it drops down to being 6% larger.

Table: 5.8.2 Geometric ARO vs. Levy's ARO Standard Deviation and Variance

	Standard Deviation	Variance	Levy's Variance / Geometric Variance when n=n	Geometric Variance / Levy's Variance when n=n
Geometric n=daily	9309	86652921		0.92
Geometric n=12	8758	76696237		0.92
Geometric n=4	8109	65753417		0.94
Levy n=daily	9704	94161230	1.09	
Levy n=12	9135	83439141	1.09	
Levy n=4	8353	69773185	1.06	

In Levy's ARO, as *n* increased the price of the option increased; remember however, in the geometric ARO as *n* increased, the option price decreased. Table 5.8.3 shows the geometric ARO subtracted from Levy's ARO for each corresponding *n.* Figure 5.8.2 is the graph of the data from Table 5.8.3.

Table 5.8.3: The Difference in Price Between Levy's and a Geometric ARO (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
n=daily	-33.5	-15.75	-15.9	-24.29	-59.7	-83.88	-81.63	-113.54	-441.88	-152.03
n=12	8.26	13.52	16.43	19.57	-1.81	-8.87	-1.75	22.89	-266.94	-34.65
n=4	102.4	79.14	88.99	118.11	129.43	161.23	178.63	332.15	138.83	232.33

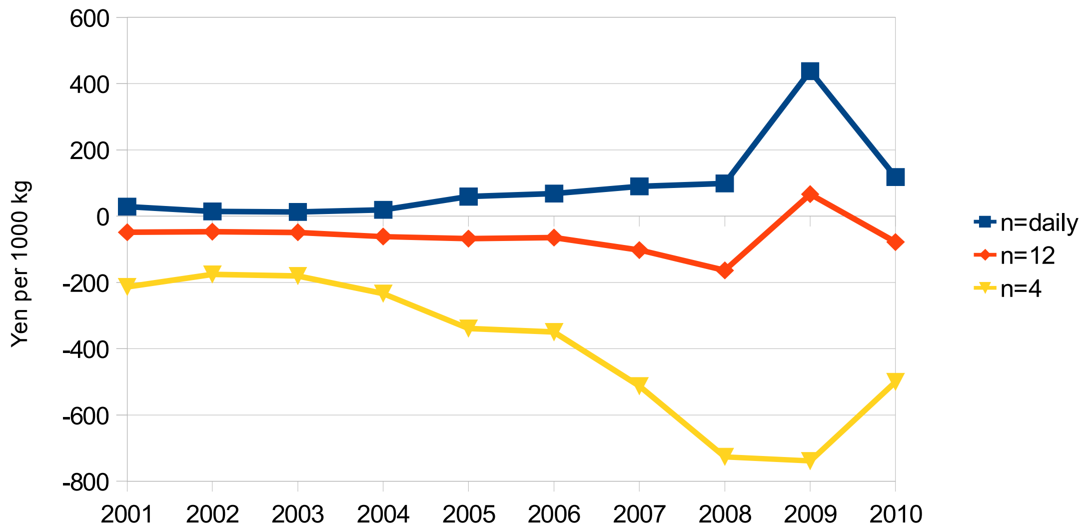

Figure 5.8.2: The Difference in Price Between Levy's and a Geometric ARO (in Yen per 1000 kg)

In all cases (futures contracts, Black-Scholes options, and the hypothetical no-hedge) the AROs had less variance and a steadier income stream. However, the comparison of Levy's ARO and a geometric ARO leads to many interesting questions. Most institutions use arithmetic AROs, and as Kowk (2008) stated, arithmetic AROs are the most popular. The income variance in using Levy's ARO (9% and 6%) is barely above the geometric ARO; however, the geometric ARO does have less variance. From the comparison above this research leads to many more questions in this field of study. Some issues of interest could be the importance of: averaging days verses lowest variance verses option price vs highest income.

Clearly the AROs are the best choice for lower variance in this futures market but because the two AROs are so similar, more research needs to be done on which ARO is the "best."

5.9 A Comparison of all Investments

This paper has outlined the results and comparison of the futures contracts, Black-Scholes option and the ARO options. However an interesting illustration is Figure 5.9.1, which shows all the different income streams, as well as the hypothetical no-hedge income stream, together. As illustrated in the figure the income streams with the least variability are those that involve an ARO.

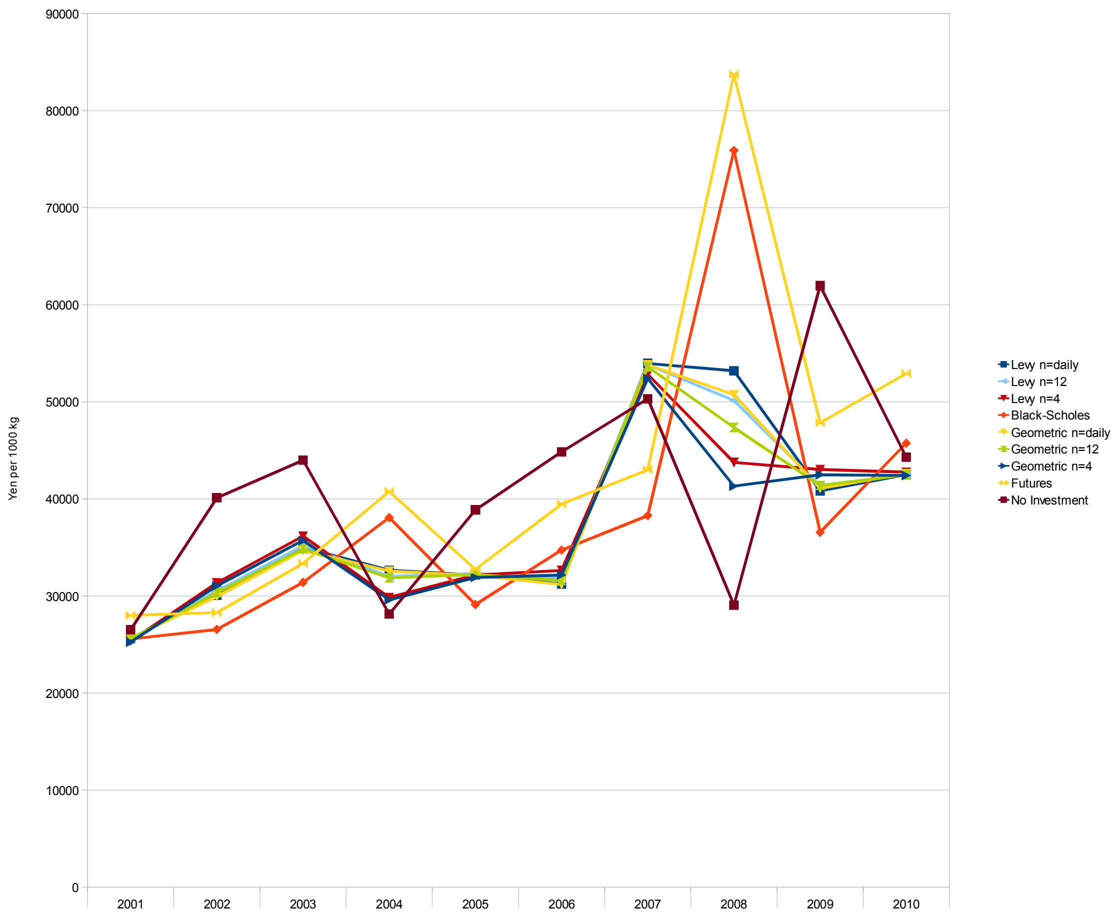

Figure 5.9.1: A Comparison of All Income Streams

	Levy n=daily	Levy n=12	Levy n=4	Black-Scholes	Geometric n=daily	Geometric n=12	Geometric n=4	Futures	No Investment
Levy n=daily	1	1.13	1.35	0.44	1.09	1.23	1.43	0.35	0.78
Levy n=12	0.89	1	1.2	0.39	0.96	1.09	1.27	0.31	0.69
Levy n=4	0.74	0.84	1	0.33	0.81	1.1	1.06	0.26	0.58
Black	2.26	2.55	3.05	1	2.46	2.78	3.24	0.79	1.77
Geometric n=daily	0.92	1.04	1.24	0.41	1	1.13	1.32	0.32	0.72
Geometric n=12	0.81	0.92	1.1	0.36	0.89	1	1.17	0.28	0.64
Geometric n=4	0.7	0.79	0.94	0.31	0.76	0.86	1	0.24	0.55
Futures	2.88	3.25	3.88	1.27	3.13	3.53	4.12	1	2.25
No Investment	1.28	1.44	1.73	0.57	1.39	1.57	1.83	0.44	1

Table 5.9.1 A Comparison of Variance in All Income Streams

Table 5.9.1 displays the magnitude of the variation in each different income stream in all cases. In Table 5.9.1, each cell shows the row heading divided by the column heading. It is interesting to note that in this mathematical simulation, entering into a year-long futures contract has the most variance, even more than no investment plan. However, in Figure 5.9.1 this does not seem to look visually correct. Moreover in Figure 5.9.1 the year 2008 seems to be an outlier. Had it not been for the erratic prices in 2008, the hypothetical no-investment income variance would be 1.56 times more than the futures contract income variance.

5.10 The Money Maximizing Investor Income Stream

An income maximizing producer or investor would not have exercised the option premium if at contract maturation the closing price was higher. However, since the option premium has already been purchased, the cost of the option premium must be subtracted from the ending price. The year 2001 had very interesting results.

The closing price for the December 2001 contract was 26,520 yen per 1,000 kg, which was higher than the payout from any of the options. However, once the option cost was subtracted from the maturity closing price more income was made by exercising the option. Table 5.10.1 represents the income that would have been received in for each scenario. Column one shows the different options, column two is the option price subtracted from the closing price (S_t), and column three shows the income from exercising the option.

Table 5.10.1: Price Received from Option Operations (in Yen per 1000 kg)

	St-Option price	Exercising the option
Levy n=daily	24147.6	25596.25
Levy n=12	24156.71	25581.71
Levy n=4	24173.08	25363.08
Black-Scholes	23795.84	25575.84
Geometric n=daily	24181.1	25623.61
Geometric n=12	24148.45	25561.96
Geometric n=4	24070.68	25251.28

Figure 5.10.1 represents the income stream a producer would have received if money maximizing strategies were used. The ARO scenarios have the least variance.

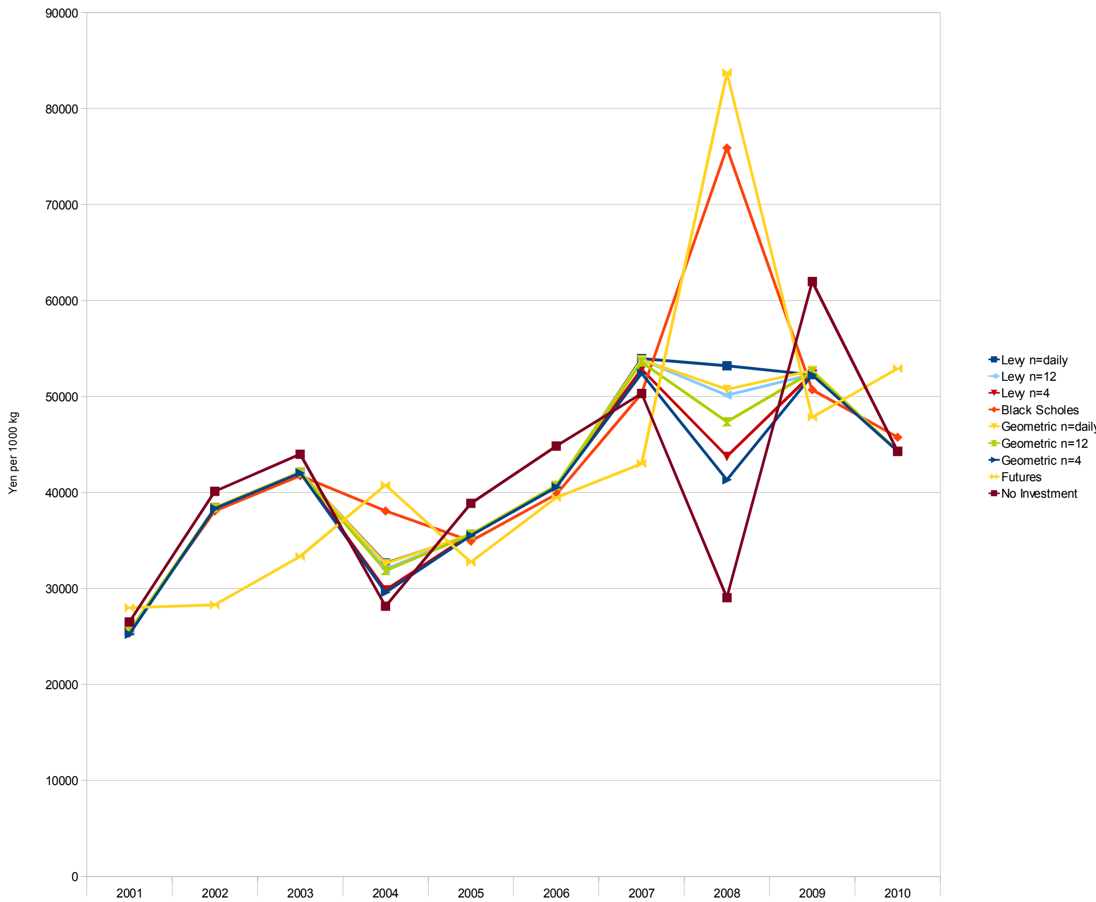

Figure 5.10.1: A Comparison of All Income Streams when the Producer is Money Maximizing

Table 5.10.1 is of the same nature as Table 5.9.1 in that it conveys the variance value represented by the row heading divided by the variance value represented by the column heading. It is of no surprise that the variability of all the different investment scenarios is closer related for the money maximizing alternative. In the years 2002, 2003, 2005, 2006, and 2009 the option would have expired worthless. These five instances make all of the options and the no-investment plan closer in relation.

	Levy n=daily	Levy n=12	Levy n=4	Black-Scholes	Geometric n=daily	Geometric n=12	Geometric n=4	Futures	No Investment
Levy n=daily	1	1.07	1.14	0.49	1.06	1.13	1.17	0.32	0.73
Levy n=12	0.93	1	1.07	0.46	0.99	1.05	1.09	0.3	0.68
Levy n=4	0.87	0.94	1	0.43	0.93	0.98	1.02	0.28	0.64
Black	2.05	2.19	2.34	1	2.17	2.31	2.39	0.66	1.49
n=dailyGeometric	0.94	1.01	1.08	0.46	1	1.06	1.1	0.31	0.69
Geometric n=12	0.89	0.95	1.02	0.43	0.94	1	1.04	0.29	0.65
Geometric n=4	0.86	0.92	0.98	0.42	0.91	0.96	1	0.28	0.62
Futures	3.08	3.3	3.53	1.51	3.27	3.47	3.6	1	2.25
No Investment	1.37	1.47	1.57	0.67	1.45	1.54	1.6	0.44	1

Table 5.10.2: A Comparison of Variance in All Income Streams When the Producer is Money Maximizing

CHAPTER 6: CONCLUSIONS

This research began due to concern from the volatile nature of commodities markets in the last four decades. Volatile commodity markets can lead to unpredictable income revenue and possible unwise decisions by investors and producers. It was decided to tackle this problem with market based price risk management, in the form of options. The hypothesis was that AROs would lead to less income volatility over the 10-year data span of the research.

The TGE non-GMO soybean futures market was used for price data. First the previous year's volatility was found for the option pricing. The next step was to plug this volatility into Black-Scholes formula, Levy's discrete arithmetic option and a geometric averaging option. For the AROs, fixings of four, twelve and daily were used. The research calculated the different incomes an individual would have received from the hypothetical no-hedge plan (i.e.., selling at market price without a futures contract or option), entering into a futures contract, buying Black-Sholes option, or purchasing either an arithmetic or geometric ARO. Two mathematical simulations were made one using a six-month averaging option bought at averaging activation the other a 12-month investment bought six months prior to the averaging period. In both cases the profits were calculated based on whether the option was always exercised or if the investor was money maximizing.

It is important to note this is a mathematical simulation, and some hypothetical examples have been procured from the data. However, the estimations in this study support the idea that Asian options on the TGE would decrease income volatility.

The AROs consistently had lower income volatility as well as increased stability in yearly fluctuations over the actual futures contract. This study has also displayed that thinly traded markets can have large variances in the futures price near-contract maturation. Recall from Figure 2.2.1, the yearly volatility for the last 30 days of the December contract was 33.97%. This study has shown with the high fluctuation in soybean prices near maturation, an ARO can help protect against possible market manipulation.

As previously noted, in 2006, Stephen Roach warned of a “commodity bubble.” He said, “It's not a matter of if the bubble bursts – but when (Spence).” Asian options are one way to tackle the current commodity dilemma. This research has unquestionably shown that Asian options would decrease income volatility. However, this study leads to many more interesting questions in the field of options. One possible deviation from this research would be to determine if different markets perform better with particular exotic options over plain vanilla options.

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APPENDIX A: RESULTS FOR SIX MONTH INVESTMENTS

Futures Contract vs. Black-Scholes Options

Table A-1: Black-Scholes vs. Futures Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Black-Scholes	25234	28859	31467	36715	33687	32670	49553	78846	43422	41407
Futures	26850	30140	32780	38470	36530	35660	53780	84530	52000	45820

Table A-2: Black Scholes vs. Futures Standard Deviation and Variance of Yearly Incomes

	Standard Deviation	Variance
Black-Scholes	15395	236991581
Futures	16907	285834027

Futures variance divided by Black-Scholes Variance = 1.21.

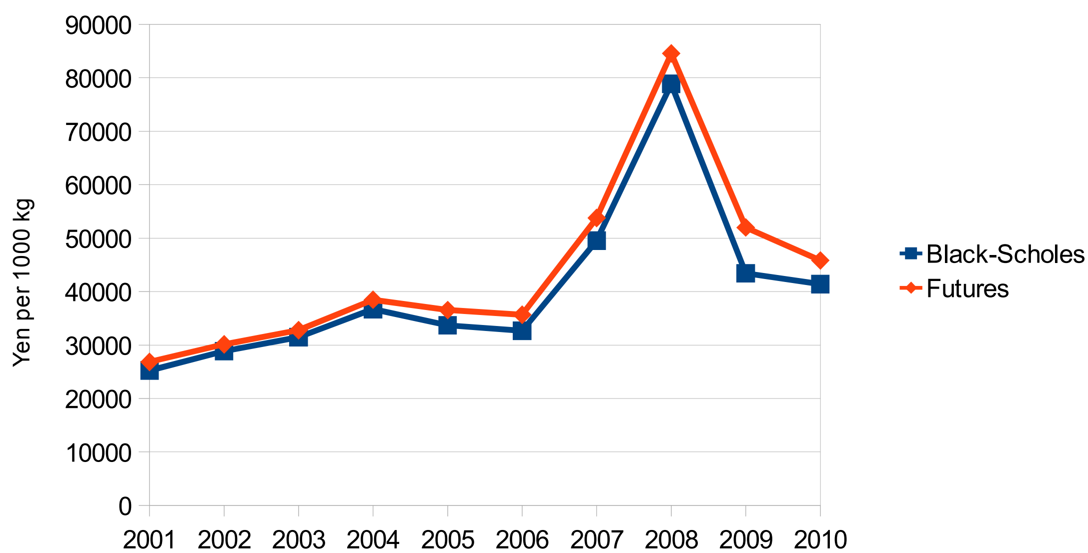

Figure A-1: Yearly Incomes for Black-Sholes Option and a Futures Contract

Futures Contract vs. Levy's Arithmetic Asian options

Table A-3: Three Alternatives for Levy's ARO vs. Futures Contract Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Levy n=daily	26754	30806	35884	33890	33538	33447	55569	56802	45155	46077
Levy n=12	26748	31259	36064	33281	33715	34029	55450	53766	45727	46087
Levy n=4	26545	32113	37060	31120	33581	34881	54531	47474	47482	46394
Futures	26850	30140	32780	38470	36530	35660	53780	84530	52000	45820

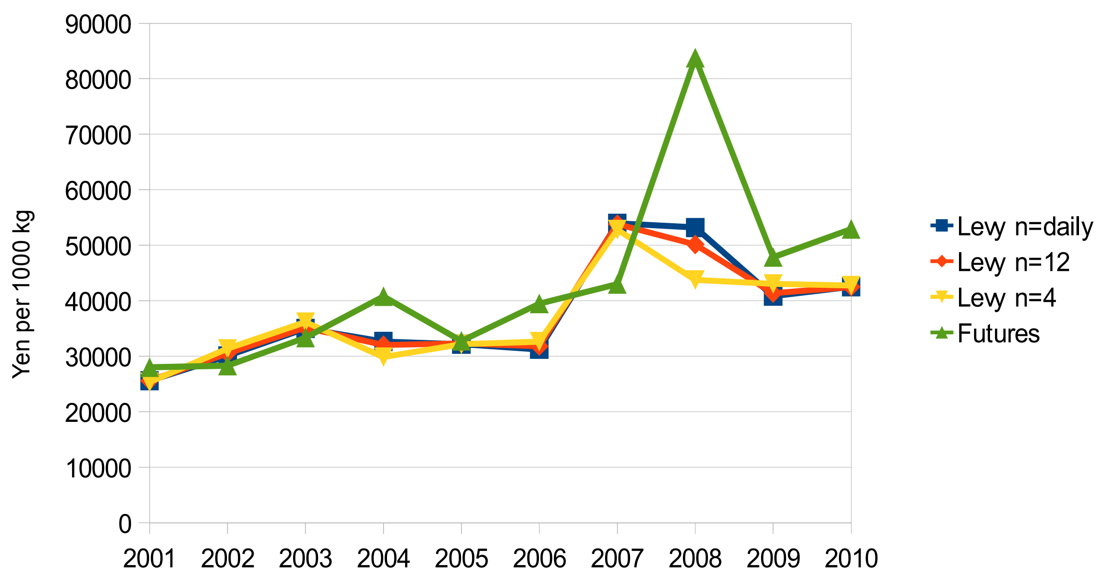

Figure A-2: Yearly incomes for Levy's ARO and a Futures Contract

Table A-4: Levy's ARO vs. Futures Standard Deviation and Variance

	Standard Deviation	Variance	Futures Variance/Levy's Variance
Levy n=daily	10471	109636567	2.61
Levy n=12	9913	98262997	2.91
Levy n=4	9154	83804804	3.41
Futures	16907	285834027	

Futures Contract vs. Geometric Averaging Options

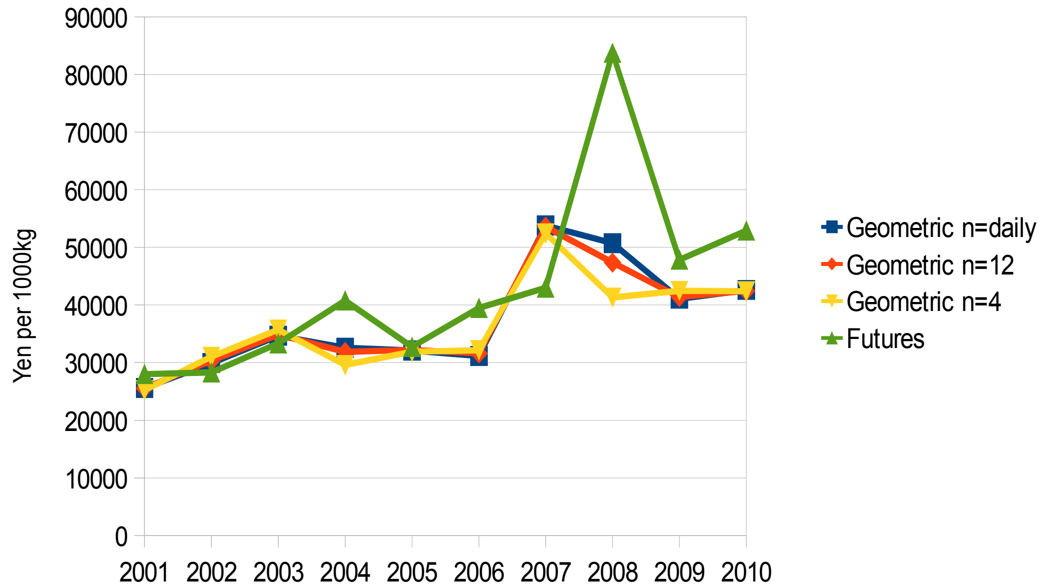

Figure A-3: Yearly incomes for Geometric ARO and a Futures Contract

Table A-5: Geometric ARO vs. Futures Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Geometric n=daily	26776	30683	35557	33810	33489	33373	55436	54326	45413	46157
Geometric n=12	26688	31033	35704	33079	33536	33753	55131	50850	45562	45949
Geometric n=4	26322	31695	36533	30747	33126	34222	53766	44638	46341	45772
Futures	26850	30140	32780	38470	36530	35660	53780	84530	52000	45820

Table A-6: Geometric ARO vs. Futures Standard Deviation and Variance

	Standard Deviation	Variance	Futures Variance/Geometric Variance
Geometric n=daily	10079	101585732	2.81
Geometric n=12	9488	90020345	3.18
Geometric n=4	8759	76725509	3.73
Futures	16907	285834027	

Black-Scholes options vs. Levy's Arithmetic Averaging Options

Table A-7: Levy's ARO vs. Black-Scholes Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Levy n=daily	26754	30806	35884	33890	33538	33447	55569	56802	45155	46077
Levy n=12	26748	31259	36064	33281	33715	34029	55450	53766	45727	46087
Levy n=4	26545	32113	37060	31120	33581	34881	54531	47474	47482	46394
Black-Scholes	25234	28859	31467	36715	33687	32670	49553	78846	43422	41407

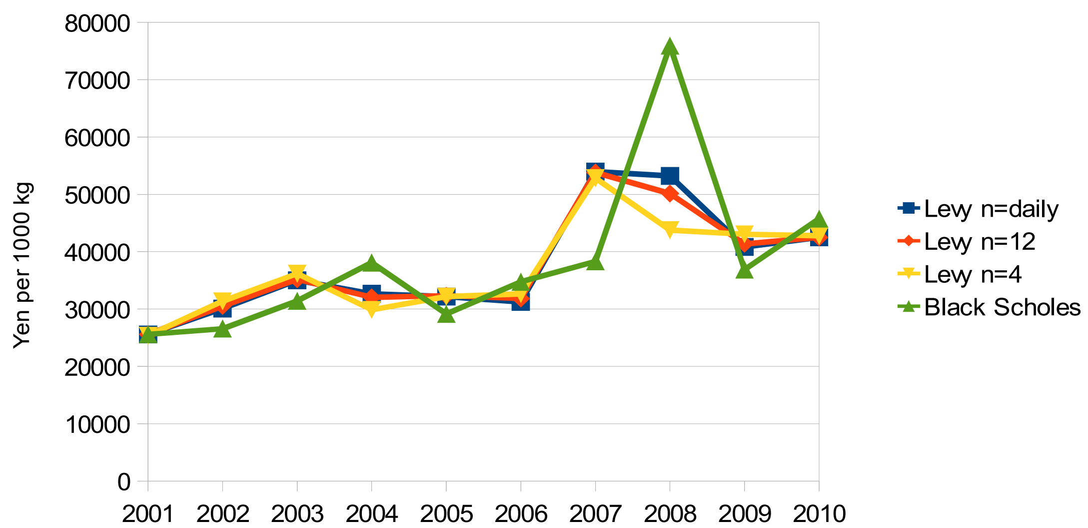

Figure A-4: Yearly Incomes for Levy's ARO and Black-Scholes Option

Table A-8: The Additional Cost of Black-Scholes Option Over Each of Levy's AROs

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Black-Sholes – Levy n=daily	702	591	606	794	1254	1314	1753	2241	3501	1861
Black-Sholes – Levy n=12	719	605	621	813	1284	1345	1796	2300	3582	1906
Black-Scholes – Levy n=4	751	631	647	847	1337	1401	1876	2408	3727	1987

Minimum=591 Maximum=3727

For the most extreme case Levy's ARO with 4 fixings in 2009 Black-Scholes option would cost an additional 37270 Yen per contract or an additional 101 Yen per bushel.

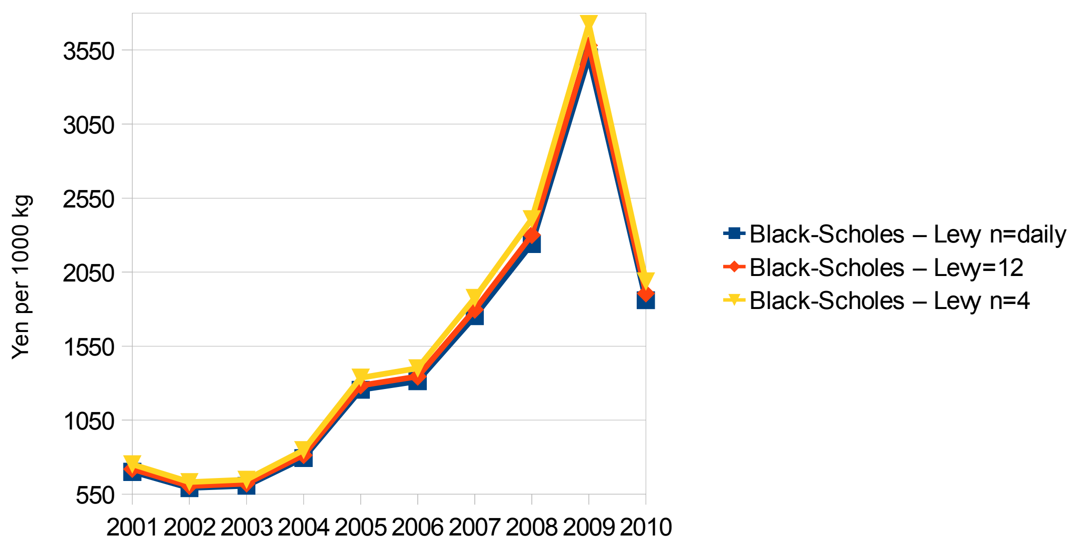

Figure A-5: A Graphical Representation of the Additional Cost of Black-Scholes Option Over Each of Levy's AROs (in Yen per 1000 kg)

Table A-9: Levy's ARO vs. Black-Scholes Option Standard Deviation and Variance

	Standard Deviation	Variance	Black's Variance/Levy's Variance
Levy n=daily	10471	109636567	2.16
Levy n=12	9913	98262997	2.41
Levy n=4	9154	83804804	2.83
Blacks	15395	236991581	

Black-Scholes options vs. Geometric Averaging Options

Table A-10: Black-Scholes Option vs. Geometric ARO Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Geometric n=daily	26776	30683	35557	33810	33489	33373	55436	54326	45413	46157
Geometric n=12	26688	31033	35704	33079	33536	33753	55131	50850	45562	45949
Geometric n=4	26322	31695	36533	30747	33126	34222	53766	44638	46341	45772
Blacks	25234	28859	31467	36715	33687	32670	49553	78846	43422	41407

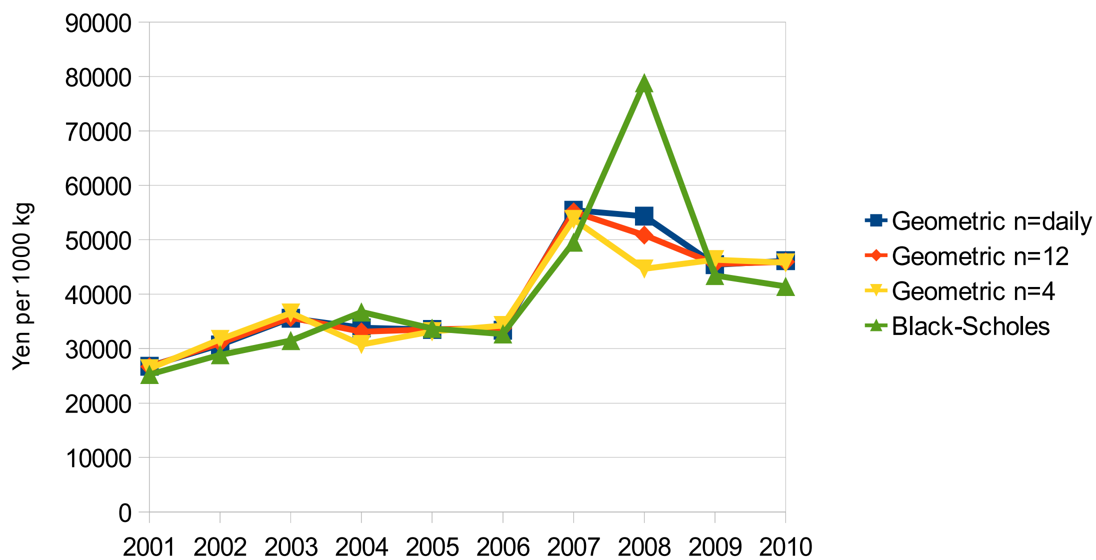

Figure A-6: Yearly Incomes for Black-Scholes Option and Geometric AROs

Table A-10: Black-Scholes Option vs. Geometric ARO Standard Deviation and Variance

	Standard Deviation	Variance	Black-Scholes Variance/Geometric ARO Variance
Geometric n=daily	10079	101585732	2.33
Geometric n=12	9488	90020345	2.63
Geometric n=4	8759	76725509	3.09
Black-Scholes	15395	236991581	

Table A-11: The Additional Cost of Black-Scholes Option Over Each Geometric ARO (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Black-Sholes – Geometric n=daily	730	605	619	813	1313	1382	1842	2340	3939	1979
Black-Sholes – Geometric n=12	671	559	572	751	1216	1280	1694	2136	3648	1827
Black-Scholes – Geometric n=4	538	456	466	614	998	1051	1363	1682	2989	1487

Minimum=455 Maximum=3939

For the most extreme case a geometric ARO with daily fixings in 2009 Black-Scholes option would cost an additional 39,390 Yen per contract or an additional 107 Yen per bushel.

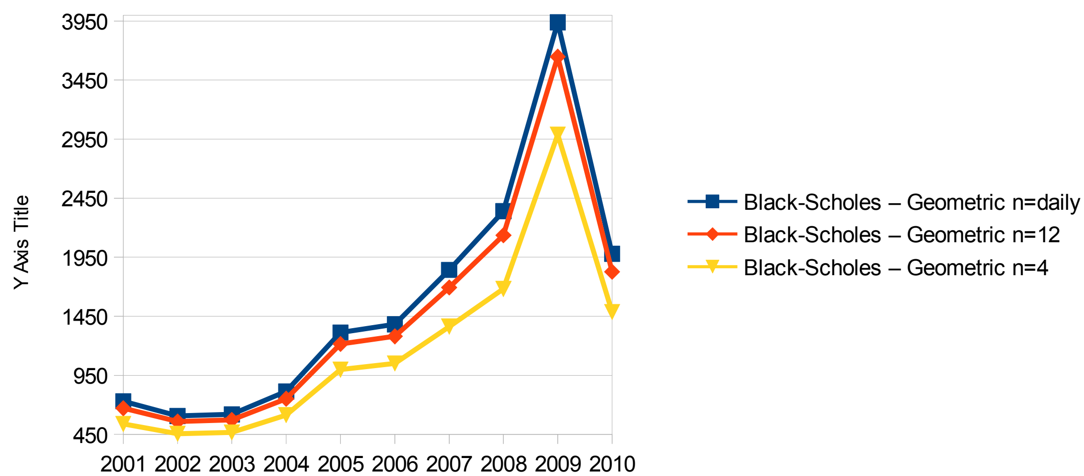

Figure A-7: A Graphical Representation of the Additional cost of Blakc-Scboles Option Over Each Geometric ARO (in Yen per 1000 kg)

Impacts of Changing n in Averaging Rate options

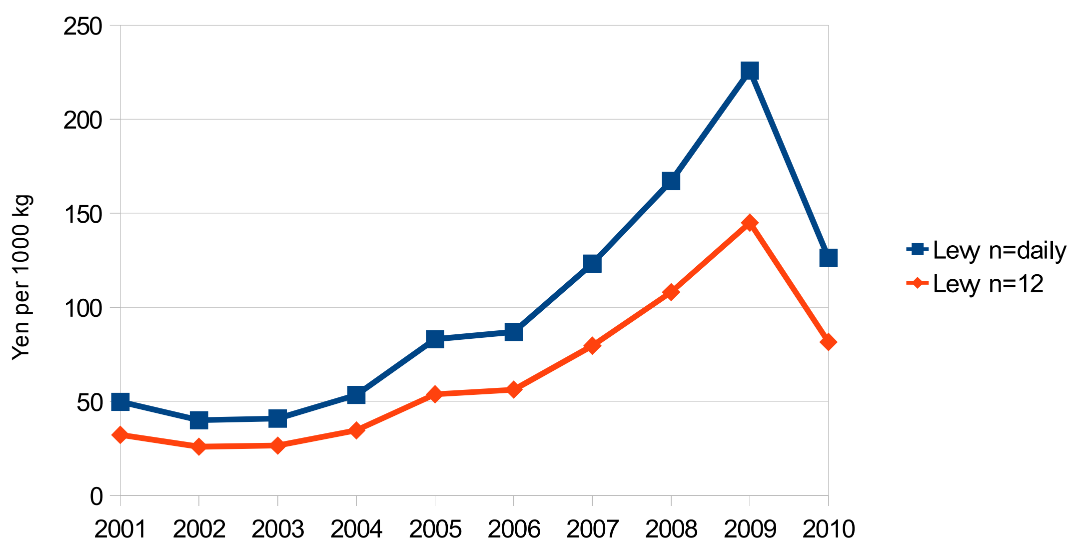

Figure A-8: A Comparison of Increasing Option Price as the Level of n Increases

Table A-12: Numerical Results of Increasing Option Price as the Level of n Increases

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Levy n=daily	49.78	39.98	40.87	53.42	83.07	86.91	123.25	167.13	225.85	126.28
Levy n=12	32.2	25.87	26.47	34.6	53.75	56.2	79.61	108.1	144.98	81.58

Table A-13: Ten Year Summed Income in Levy's ARO

	Sum
Levy n=daily	397921
Levy n=12	396126
Levy n=4	391181

Table A-14: The Simulation n=daily Variance Divided by the Respective Level of n

Levy n=12	1.12
Levy n=4	1.31

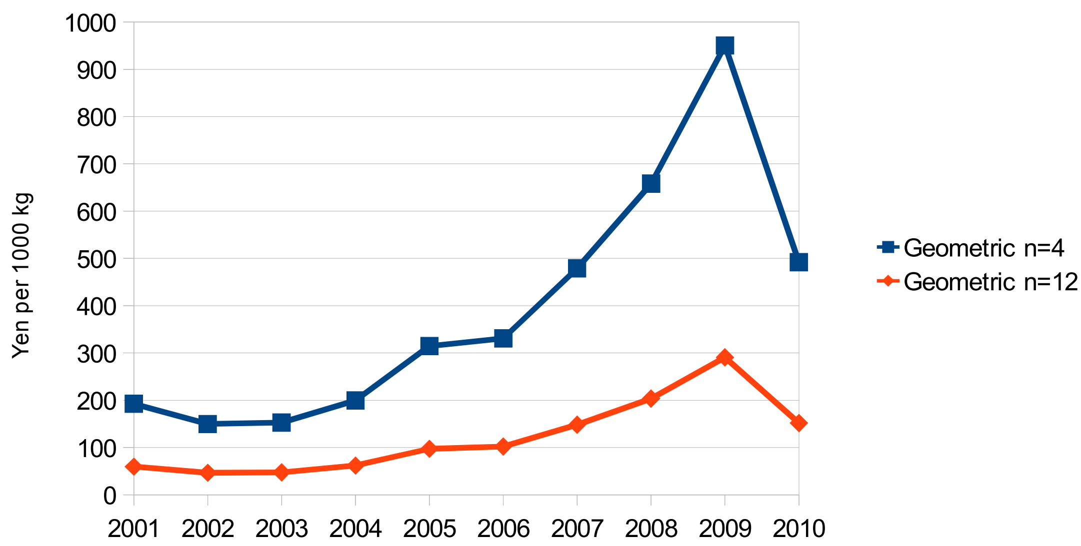

Figure A-9: A Comparison of Increasing Option Price as the Level of n Decreases

Table A-15: Numerical Results of Increasing Option Price as the Level of n Decreases

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Geometric n=4	59.53	46.44	47.3	61.81	97.17	102.12	148.21	203.49	290.68	151.7
Geometric n=12	192.39	149.73	152.66	199.48	314.78	330.8	478.91	658.04	950.46	492.03

Table A-16: The Simulation n=daily Variance Divided by the Respective Level of n

Geometric n=4	1.13
Geometric n=12	1.32

Table A-17: Ten Year Geometric ARO Summed Income

	Sum
Geometric n=daily	395022
Geometric n=12	391285
Geometric n=4	383161

Levy's Arithmetic Averaging Options vs. Geometric Averaging options

Table A-18: Geometric ARO vs. Levy's ARO Income Stream (in Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Sum
Geometric n=daily	26776	30683	35557	33810	33489	33373	55436	54326	45413	46157	374405
Geometric n=12	26688	31033	35704	33079	33536	33753	55131	50850	45562	45949	371259
Geometric n=4	26322	31695	36533	30747	33126	34222	53766	44638	46341	45772	364330
Levy n=daily	26754	30806	35884	33890	33538	33447	55569	56802	45155	46077	377228
Levy n=12	26748	31259	36064	33281	33715	34029	55450	53766	45727	46087	375250
Levy n=4	26545	32113	37060	31120	33581	34881	54531	47474	47482	46394	369941

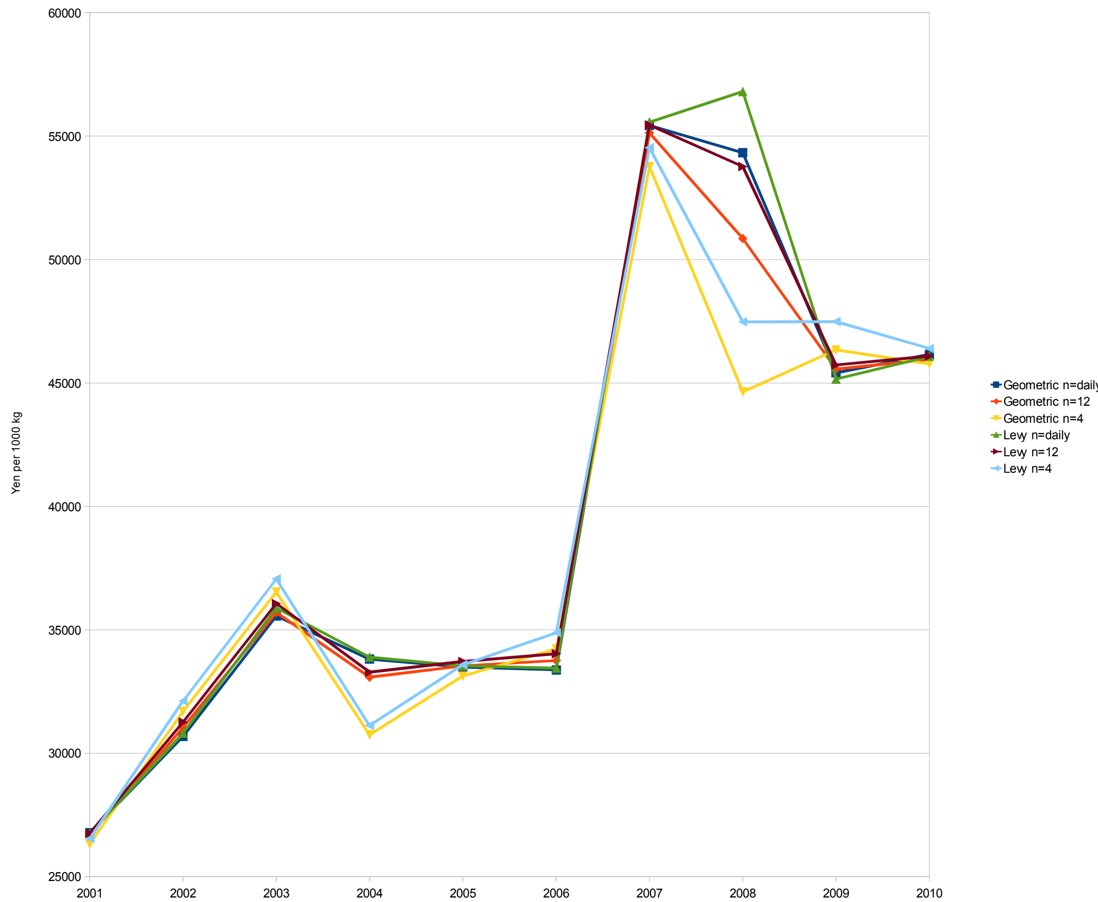

Figure A-10: A Comparison Levy's ARO and Geometric ARO Income Streams

Table A-19: Geometric ARO vs. Levy's ARO Standard Deviation and Variance

	Standard Deviation	Variance	Levy's Variance / Geometric Variance when n=n	Geometric Variance / Levy's Variance when n=n
Geometric n=daily	10079	101585732		0.93
Geometric n=12	9488	90020345		0.92
Geometric n=4	8759	76725509		0.92
Levy n=daily	10471	109636567	1.08	
Levy n=12	9913	98262997	1.09	
Levy n=4	9154	83804804	1.09	

Table A-20: The Difference between Levy's and a Geometric ARO in (Yen per 1000 kg)

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
n=daily	28.57	14.02	12.8	18.98	58.81	67.93	89.55	98.74	437.74	118.08
n=12	-48.53	-46.53	-48.9	-61.65	-67.67	-64.9	-102.3	-163.78	66.2	-78.32
n=4	-213.59	-175.69	-180.73	-233.92	-339.03	-349.77	-512.62	-726.43	-738.56	-500.23

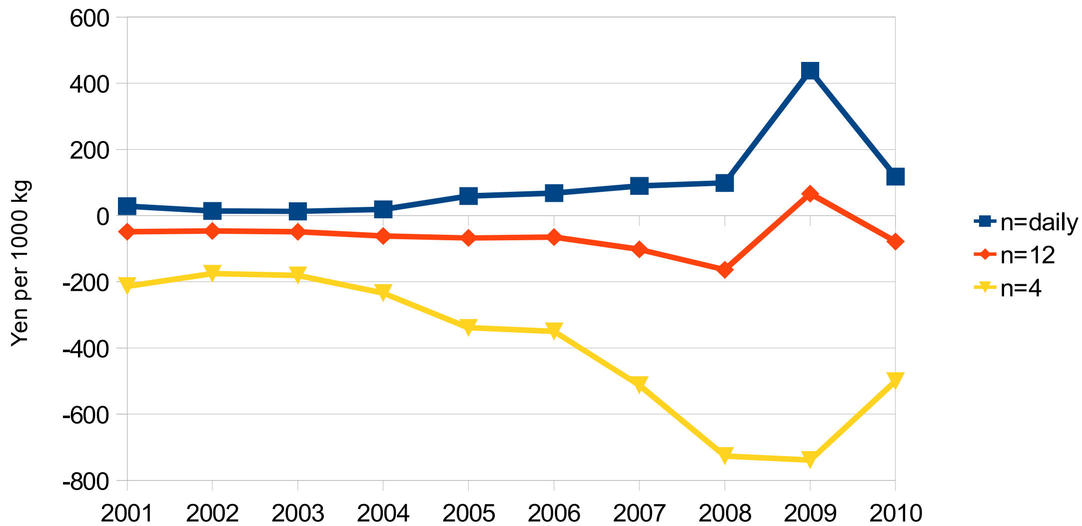

Figure A-11: The Difference in Price Between Levy's and a Geometric ARO in (Yen per 1000 kg)

A Comparison of all Investments

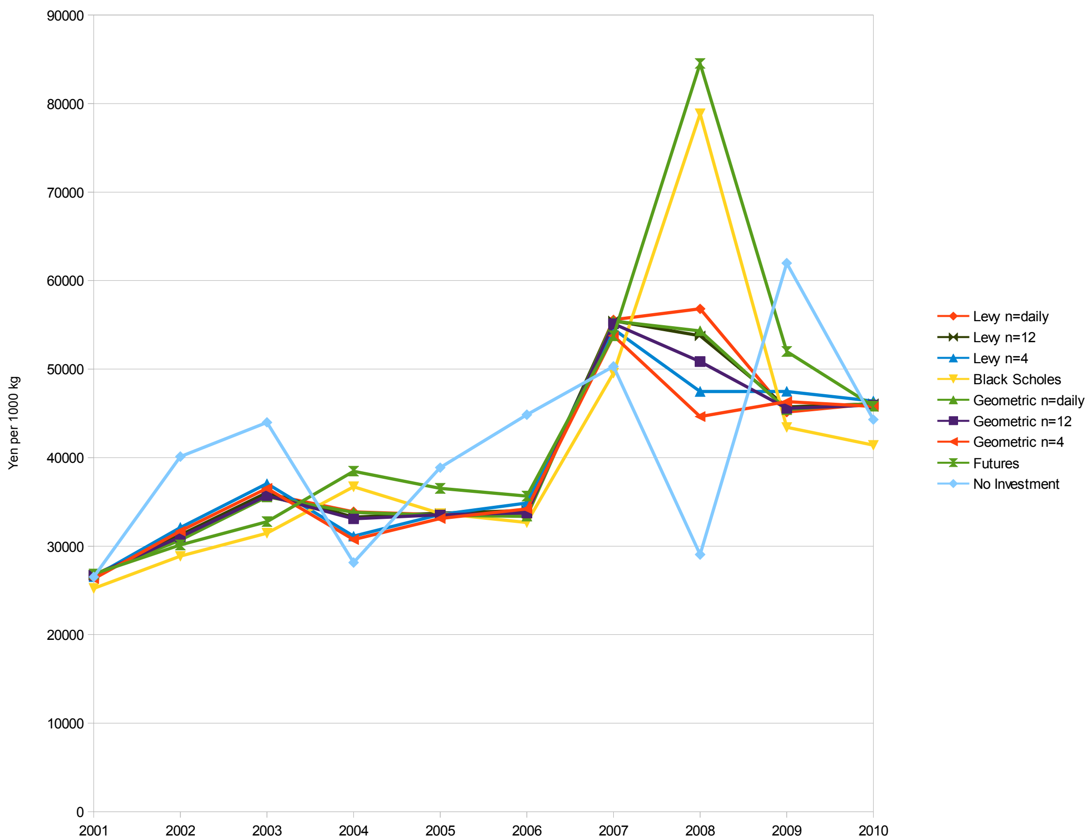

Figure A-12: A Comparison of All Income Streams

	Levy n=daily	Levy n=12	Levy n=4	Black-Scholes	Geometric n=daily	Geometric n=12	Geometric n=4	Futures	No Investment
Levy n=daily	1	0.9	0.76	2.16	0.93	0.82	0.7	2.61	1.1
Levy n=12	1.12	1	0.85	2.41	1.03	0.92	0.78	2.91	1.23
Levy n=4	1.31	1.17	1	2.83	1.21	1.07	0.92	3.41	1.44
Black	0.46	0.41	0.35	1	0.43	0.38	0.32	1.21	0.51
Geometric n=daily	1.08	0.97	0.82	2.33	1	0.89	0.76	2.81	1.19
Geometric n=12	1.22	1.09	0.93	2.63	1.13	1	0.85	3.18	1.34
Geometric n=4	1.43	1.28	1.09	3.09	1.32	1.17	1	3.73	1.57
Futures	0.38	0.34	0.29	0.83	0.36	0.31	0.27	1	0.42
No Investment	0.91	0.82	0.7	1.97	0.84	0.75	0.64	2.37	1

A-21: A Comparison of Variance in All Income Streams

Table A-22: Price Received from Option Operations(in Yen per 1000 Kg)

	St-Option price	Exercising the option
Levy n=daily	25305.16	26753.82
Levy n=12	25322.74	26747.74
Levy n=4	25354.94	26544.94
Black-Scholes	24603.52	25233.52
Geometric n=daily	25333.74	26776.25
Geometric n=12	25274.21	26687.73
Geometric n=4	25141.35	26321.95

The producer or investor would not use Black-Scholes option or either of the AROs in 2002, 2003, 2005, and 2006.

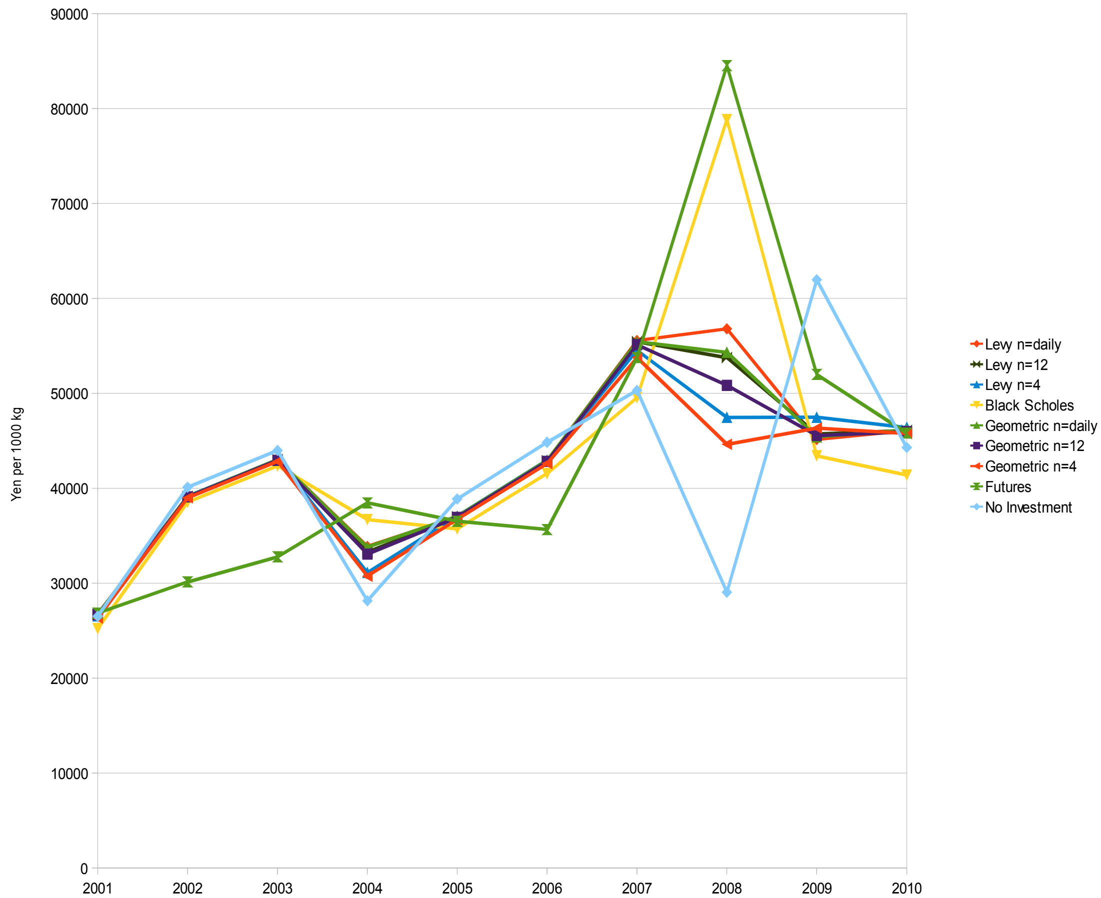

Figure A-13: A Comparison of All Income Streams when the Producer is Money Maximizing

	Levy n=daily	Levy n=12	Levy n=4	Black-Scholes	Geometric n=daily	Geometric n=12	Geometric n=4	Futures	No Investment
Levy n=daily	1	0.91	0.82	2.32	0.91	0.83	0.76	3.39	1.43
Levy n=12	1.1	1	0.9	2.54	1	0.91	0.83	3.71	1.57
Levy n=4	1.21	1.11	1	2.81	1.11	1	0.92	4.11	1.73
Black	0.43	0.39	0.36	1	0.39	0.36	0.33	1.46	0.62
Geometric n=daily	1.1	1	0.9	2.54	1	0.91	0.83	3.71	1.56
Geometric n=12	1.21	1.1	1	2.8	1.1	1	0.92	4.09	1.72
Geometric n=4	1.32	1.2	1.09	3.05	1.2	1.09	1	4.46	1.88
Futures	0.3	0.27	0.24	0.68	0.27	0.24	0.22	1	0.42
No Investment	0.7	0.64	0.58	1.62	0.64	0.58	0.53	2.37	1

Table A-23: A Comparison of Variance in All Income Streams When the Producer is Money Maximizing