

2012

Essays on Investments in New Technologies --- Policy, Information, and Learning

Ruiqing Miao
Iowa State University

Follow this and additional works at: <https://lib.dr.iastate.edu/etd>

 Part of the [Economics Commons](#)

Recommended Citation

Miao, Ruiqing, "Essays on Investments in New Technologies --- Policy, Information, and Learning" (2012). *Graduate Theses and Dissertations*. 12407.
<https://lib.dr.iastate.edu/etd/12407>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Graduate Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

Essays on Investments in New Technologies
— Policy, Information, and Learning

by

Ruiqing Miao

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:

David A. Hennessy, Major Professor

Bruce A. Babcock

Dermot J. Hayes

Hongli F. Hennessy

Sergio H. Lence

Iowa State University

Ames, Iowa

2012

Copyright © Ruiqing Miao, 2012. All rights reserved.

DEDICATION

To Jian, Anna and John

TABLE OF CONTENTS

LIST OF TABLES	iv
LIST OF FIGURES	v
ACKNOWLEDGEMENTS	vii
GENERAL ABSTRACT	viii
CHAPTER 1. OVERVIEW	1
CHAPTER 2. INVESTMENT IN CELLULOSIC BIOFUEL REFINERIES: DO WAIV- ABLE BIOFUEL MANDATES MATTER?	3
CHAPTER 3. ECONOMIC VALUE OF INFORMATION: SEGREGATING WHEAT BY PROTEIN CONCENTRATION	48
CHAPTER 4. TO LEARN OR TO CHANGE: OPTIMAL R&D INVESTMENTS UNDER UNCERTAINTIES IN THE CASE OF CLIMATE CHANGE	122

LIST OF TABLES

Table 3.1	Summary Statistics for HRW and HRS Wheat Prices ,Protein Premia, and Stocks	77
Table 3.2	Estimates of the Transformed LA/IAIDS Model	78
Table 3.3	Estimated Quantity Elasticities (or Flexibilities) at the Sample Mean	79
Table 3.4	Summary Statistics of HRW Wheat Testing Results (Observations: 538)	80
Table 3.5	Equilibrium Prices, Quantities and Operating Profit of MS Technology under Various Marginal Cost and Market Structure Combinations	81
Table 3.6	Adopters' Average WTP and the HRW and HRS Wheat Stock Value under Various Marginal Cost and Market	82
Table 3.7	Protein Premia under Various Marginal Cost and Market Structure Combina- tions of M&S Technology (cents/bu.)	83
Table 3.A.1	Trial Locations and Names of Top Varieties of HRW Wheat in the State of Washington (1997-2009)	108
Table 3.A.2	A Sample of HRW Variety Performance Observations .	109
Table 3.A.3	Estimates of the Coefficients of Years, Varieties, and Locations for	110
Table 3.A.4	Estimates of the Coefficients of Years, Varieties, and Locations for HRS Wheat Protein	111
Table 4.1	Results of Comparative Static Analysis for Case 1 and Case 3	147
Table 4.2	Results of Comparative Static Analysis for Case 2 and Case 4 in Scenario 1 .	148
Table 4.3	Results of Comparative Static Analysis for Case 2 and Case 4 in Scenario 2 .	149
Table 4.4	Results of Comparative Static Analysis for Case 2 and Case 4 in Scenario 3 .	150

LIST OF FIGURES

Figure 2.1	The value of z as a function of v_2	27
Figure 2.2	RIN price as a function of X under NWM Policy and WM Policy when $p_2 = 0$	28
Figure 2.3	Cumulative distribution function of v_2 under <i>laissez-faire</i> , NWM Policy, and WM Policy	29
Figure 3.1	A Three-Step Price Schedule	84
Figure 3.2	A Protein Distribution such that $t_2 \leq l_1$	85
Figure 3.3	A Protein Distribution such that $t_2 > l_1$ and $\hat{l}_1 > 0$	86
Figure 3.4	A Protein Distribution such that $t_2 > l_1$ and $\hat{l}_1 = 0$	87
Figure 3.5	Annual Average Protein Concentration of HRW and HRS Wheat Produced between 1979 and 2009 in the United States	88
Figure 3.6	Protein Premia of HRW Wheat	89
Figure 3.7	Protein Premia of HRS Wheat	90
Figure 3.8	HRW Wheat Stocks by Protein Concentration	91
Figure 3.9	HRW Wheat Stocks by Protein Concentration	92
Figure 3.10	The Histogram of Protein Concentration Residuals	93
Figure 3.11	Demand Curve for the M&S Technology	94
Figure 3.12	Price Schedule which Turns from being Concave to Convex	112
Figure 3.13	Price Schedule which Turns from being Convex to Concave	113
Figure 3.14	Points I , A , and J Form a Concave Price Schedule	114
Figure 3.15	Points E , B , and F Form a Convex Price Schedule	115
Figure 3.16	Points E , Z , and D Form a Convex Price Schedule	116
Figure 3.17	Points E , B , and $[l_j, p(l_j)]$ Form a Convex Price Schedule	117

Figure 3.18	Points Z , C , and $[l_j, p(l_j)]$ Form a Concave Price Schedule	118
Figure 3.19	Points C , J , and D Form a Convex Price Schedule	119
Figure 4.1	The Social Planner's Decision Tree	151
Figure 4.2	Possible Cases when $q_{lc} < q_c$ (Panel A) and when $q_{lc} < q_c$ (Panel B)	152
Figure 4.3	Values of c^* and l^* when $\eta = 0$ and $q_{lc} > q_c$	153
Figure 4.4	Values of c^* and l^* when $\eta = 0$ and $q_{lc} \leq q_c$	154
Figure 4.5	Possible Cases in Scenario 1 ($\alpha \in [0, \alpha_1]$)	155
Figure 4.6	Values of c^* and l^* when $\eta > 0$ under Scenario 1	156
Figure 4.7	Values of c^* and l^* when $\eta > 0$ under Scenario 2	157
Figure 4.8	Possible Cases in Scenario 3 ($\alpha \in [\alpha_2, \alpha_3]$) and in Scenario 4 ($\alpha > \alpha_3$)	158
Figure 4.9	Values of c^* and l^* when $\eta > 0$ under Scenario 3	159
Figure 4.10	Values of c^* and l^* when $\eta > 0$ under Scenario 4	160

ACKNOWLEDGEMENTS

I am grateful to my advisor, Dr. David Hennessy, for his guidance, encouragement, and support throughout the entire research work of this thesis. I am very appreciative of the time and effort Dr. Hennessy invested in developing my research and communication skills. The training he provided to me will benefit all of my future work. Meanwhile, Dr. Hennessy's enthusiasm, availability, and genuine care of students' professional growth have set a great example of advisor to follow.

I would like to express my sincere gratitude to Dr. Bruce Babcock, Dr. Dermot Hayes, Dr. Hongli Hennessy, and Dr. Sergio Lence for serving on my committee. I thank Dr. Babcock for his great input to the first essay in this thesis, and for his generous support in my third year of study at ISU. I thank Dr. Hayes for being so generous with his time when he was my mentor for the Preparing Future Faculty Program. I thank Dr. Hongli Hennessy for her constant help in my research and teaching. I also thank Dr. Lence for his constructive comments on the second essay of my thesis.

I acknowledge Dr. Cheng Wang for encouraging and helping me to come to the U.S. Here I would like to quote a sentence from my advisor's dissertation acknowledgement that was written nineteen years ago because it is true to me as well, "I am appreciative of the opportunities that the U.S., still the land of hope, has made available to me."

I also would like to thank Dr. Peter Orazem for his advising during my first two years at ISU and thank Dr. John Schroeter for his generous help. I thank my past office mates Ranojoy Basu, Xiaodong Du, and Na Jin for their help and company on the journey of pursuing a PhD.

I would like to express my deep appreciation to my parents and parents-in-law for their love, sacrifice, and support. In the past three years they flew thousands of miles several times to help take care of my children, experiencing language barrier, loneliness, and many other inconveniences. I thank my wife Jian for her love and support as well as for putting up with me. I would also like to thank my son John and my daughter Anna who make the home full of laughters.

GENERAL ABSTRACT

The general theme of this dissertation is investment in new technologies, with a focus on the impact of policy, information, and learning. The first essay investigates the impacts of the waivable biofuel mandates on investment in cellulosic biofuel refineries. This essay contributes to the understanding of waivable mandates by specifying conditions under which even a waivable mandate can stimulate investment in cellulosic biofuel refineries. Focusing on wheat markets, the second essay estimates (1) growers' willingness to pay for a new technology that can segregate wheat grain after measuring grain protein concentration, (2) the new technology's impacts on U.S. wheat markets, and (3) the technology's market prospect. The third essay studies optimal investments in two lines of research activities under uncertainties of climate change and research outcomes. These two lines of research activities are (1) research to mitigate the possible negative impact of climate change; and (2) research to investigate the true impact of climate change.

CHAPTER 1. OVERVIEW

The essays in this dissertation, though different in specific issues addressed, are linked through the common theme of investing in new technologies. The new technologies discussed in this dissertation are of interest because they are policy and market related. As of February 2012, climate change was still a concern to the world and much resource had been directed to (1) research activities to reduce the greenhouse gas (GHG) emission or to better adapt to climate change, and (2) research activities to investigate the true state of climate change (i.e., its causes and impacts). Partly because of their benefit of GHG emission reduction, cellulosic biofuels were mandated to be consumed in the United State starting from 2010. However, due to the uncompetitive production cost, there was not enough production to meet the mandate. Therefore, the U.S. Environmental Protection Agency had waived the mandate at the third time in 2012. While corn cobs and stover were attracting increasing attention because they can be feedstock for cellulosic biofuel, wheat growers were facing an opportunity to better capture wheat protein premia by adopting a new technology that can segregate wheat grain according to protein concentration. This dissertation consists of three essays that focus on cellulosic biofuels, wheat grain segregation technology, and investment in R&D to mitigate climate change's impact.

In the first essay, "Investment in Cellulosic Biofuel Refineries: Do Waivable Biofuel Mandates Matter," we develop a conceptual model to study the impact of mandate policies on stimulating investment in cellulosic biofuel refineries. In a two-period framework, we compare the first-period investment level (FIL) under three scenarios: *laissez-faire*, non-waivable mandate (NWM) policy, and waivable mandate (WM) policy. Results show that when plant-level marginal costs are increasing then both NWM policy and WM policy may stimulate FIL. The WM policy has a smaller impact than does the NWM policy. When the plant-level marginal costs are constants, however, WM policy does not increase FIL but does increase the expected profit of more efficient investors.

The second essay, "Economic Value of Information: Segregating Wheat by Protein Concentration,"

is focused on the wheat grain segregation technology. This new technology provides growers with opportunities to identify grain that can be directed to premium markets. We study wheat growers' willingness to pay (WTP) for the technology and the technology's impact on wheat market values. The market prospect of the new technology is analyzed as well. Depending on the technology's market structure and marginal costs, (1) the average WTP of adopters for the technology ranges between 14 and 22 cents per bushel, and (2) upon the adoption of this new technology, market value of Hard Red Winter wheat will decrease by 0.2% to 2.3%, but market value of Hard Red Spring wheat will increase by 0.3% to 3.3%. The aggregate wheat market value decreases by a slight amount (0.0002% to 0.004%). If the technology is supplied by a monopoly firm, then demand from Hard Red Winter and Hard Red Spring wheat growers in the United States for the technology will provide the firm with an annual operating profit of between \$5.9 million and \$7.7 million.

In the first essay, "To Learn or To Change: Optimal R&D Investments under Uncertainties in the Case of Climate Change," we investigate the optimal investment in R&D to increase the society's ability to face challenges from climate change (termed as "research to change" or RTC). When studying RTC, current literature overlooks the existence of purchased learning (termed as "research to learn" or RTL) in which new information on climate change is acquired by investment in research activities. Since RTL absorbs substantial research resources in climate change research, it cannot be ignored when seeking to optimize resource allocation for addressing climate change problems. In this article we explore the interactions between investments in RTC and RTL under uncertainties of climate change. Here uncertainties include uncertainty about how serious climate change's damage is, and uncertainty about when the research activities succeed. We find that 1) if the success of RTL and RTC are statistically independent, then it is almost never optimal to invest in RTL and RTC simultaneously; 2) if the success of RTL accelerates the success of RTC, then RTC and RTL are substitutes; 3) if the success of RTL accelerates the success of RTC and if the cost of RTL is small enough, then it is never optimal to invest in RTC only. Factors that affect the optimal investment levels in RTC and RTL are studied as well.

CHAPTER 2. INVESTMENT IN CELLULOSIC BIOFUEL REFINERIES: DO WAIVABLE BIOFUEL MANDATES MATTER?

Abstract

We develop a conceptual model to study the impact of mandate policies on stimulating investment in cellulosic biofuel refineries. In a two-period framework, we compare the first-period investment level (FIL) under three scenarios: *laissez-faire*, non-waivable mandate (NWM) policy, and waivable mandate (WM) policy. Results show that when plant-level marginal costs are increasing then both NWM policy and WM policy may stimulate FIL. The WM policy has a smaller impact than does the NWM policy. When the plant-level marginal costs are constants, however, WM policy does not increase FIL but does increase the expected profit of more efficient investors.

Key words: cellulosic biofuels, investment, Renewable Identification Numbers, waivable mandate

JEL classification: D24, L52, Q48

Introduction

The U.S. Energy Independence and Security Act of 2007 (EISA) that was passed into law in December 2007 mandates U.S. consumption of 21 billion gallons of advanced biofuels by 2022. Of this, 16 billion gallons are to come from cellulosic feedstocks. Mandates for cellulosic biofuels begin at 0.1 billion gallons in 2010, increasing to 16 billion gallons in 2022. However, it is not yet clear as of 2011 which technology platform will prove to be the most efficient at producing cellulosic biofuels. It is also unclear when, if ever, the market value of cellulosic biofuels will cover production costs. As of June 2011, cellulosic biofuel costs are not competitive with corn ethanol costs (D’Amico, 2011). As a result of technology uncertainty and poor financial competitiveness, no commercial-scale cellulosic biofuel refinery has been built as of June 2011. EISA’s Renewable Fuel Standard (RFS), along with biofuel tax credits, aims to support investment in biofuel refineries.

The RFS mandates a floor on the amount of biofuels being consumed in every calendar year. Trade in Renewable Identification Numbers (RINs) is the market mechanism by which the mandates are to be met. Each batch, or gallon, of biofuel is assigned a RIN after it is produced or imported. As long as biofuels are blended with gasoline and made ready for consumption, the RIN attached to the biofuels can be separated and can then be traded on RIN markets. Obligated parties (i.e., producers or importers of motor fuel) must give the Environmental Protection Agency (EPA) enough RINs to meet their RFS mandate every year. They can obtain RINs either through the purchase of biofuels or by entering the RIN market and buying RINs. Since the price of RINs will be reflected in the price of biofuels, the RFS would seem to lower the risk of investing in cellulosic biofuel refineries. This is because when cellulosic biofuel production is lower than the mandate, the RIN price will rise to reflect the scarcity of biofuels.

However, EISA allows for waivers of mandates, as specified in Section 202 of EISA:

“(D) Cellulosic Biofuel. – (i) For any calendar year for which the projected volume of cellulosic biofuel production is less than the minimum applicable volume established under paragraph (2)(B), . . . , the Administrator shall reduce the applicable volume of cellulosic biofuel required under paragraph (2)(B) to the projected volume available during that calendar year.”

Waivers have occurred for cellulosic biofuels. In March 2010 the EPA waived the 2010 cellulosic biofuel mandate from 100 million gallons, as listed in EISA, to 6.5 million ethanol equivalent gallons (EPA 2010a). Again, in November 2010 the EPA waived the 2011 cellulosic biofuel mandate from 250 million gallons to 6 million ethanol equivalent gallons (EPA 2010b). In June 2011, EPA proposed the 2012 cellulosic biofuel standard as 3.55-15.7 million ethanol equivalent gallons, which indicated that a waiver for cellulosic biofuel will occur for the third time in 2012 (EPA 2011). The reason for these waivers was that there was not enough production capacity to meet the mandates (EPA 2010a,b; EPA 2011; Yacobucci 2010).

The purpose of this study is to determine the impact of biofuel mandates established by RFS on the incentive to invest in a cellulosic biofuel refinery when the mandates are waivable. The literature on the effects of biofuel mandates has not yet addressed this question. Some studies (Gallagher *et al.* 2003; McPhail and Babcock 2008a,b; and Lapan and Moschini 2009) modeled the mandate as a floor on the ethanol consumption. However, Althoff, Ehmke, and Gray (2003) and de Gorter and Just (2009) analyzed the mandate as an upward shift of the fuel supply curve because, they argued, the price per gallon of fuel would be increased by mandating that biofuel be blended with gasoline. Roberts and Schlenker (2010) studied the effects of U.S. biofuel mandates on world food prices by assuming that the mandates would require 5 percent of world calories to be devoted to biofuel production. Gardner (2007) modeled the mandate by adding the mandate quantity directly to the corn demand. Taheripour and Tyner (2007) studied the impacts of the mandate on the distribution of ethanol subsidies by assuming that the mandate, in addition to limited ethanol production capacity, made the short run supply curve for ethanol vertical. All of these studies implicitly assumed that the mandate will be met and did not consider the possibility that a mandate could be waived.

The contribution of this article is threefold. First, it emphasizes the waivability aspect of the mandates and studies this aspect's investment effects. The results show that under certain conditions even the waivable mandate can stimulate investments. Second, it discovers that mandate policies have the effect of rewarding more efficient investors or refineries. Third, policy implications are derived from the results of this article.

We first construct a two-period real option model in which an investor can either invest in the current period or wait and decide whether to invest in the future. We then compare first-period investment levels

in three scenarios: (1) *laissez-faire*, (2) non-waivable mandate (NWM) policy, and (3) waivable mandate (WM) policy.

We find that the investment impact of biofuel mandates depends on investors' marginal costs, and also on the distribution of the cellulosic biofuel's price in the second period. When the price distribution is such that almost surely every realization is sufficiently high, then neither NWM policy nor WM policy affect the first-period investment level. This is because under this price distribution condition the expected net profit of investors who are break-even in the *laissez-faire* scenario is not affected by mandates. If this condition on the price distribution does not hold and if marginal costs are increasing, then both NWM policy and WM policy can stimulate the investment level in the first period. This is because they increase the expected profit of investors who are break-even in the *laissez-faire* scenario. Moreover, the WM policy has a smaller impact than the NWM policy does. However, if marginal costs are constants then WM policy has no effect on the investment level no matter what the price distribution is. But WM policy still can increase, at least weakly, the expected profit of more efficient investors.

In what follows, we first develop a conceptual model of a potential investor's decision problem. Then we discuss RIN prices under both NWM policy and WM policy. Based on RIN prices, we compare period-one investment levels under scenarios of *laissez-faire*, NWM policy, and WM policy. The last section provides concluding remarks.

Model

In this section we develop the conceptual model of investors' investment decisions. In a two-period world, there is a unit mass continuum of potential investors in the cellulosic biofuel industry. The investors are risk-neutral. Each investor chooses whether to invest in period one, or in period two, or not invest at all. The option to invest expires at the end of period two. We denote the action set for each investor in period one as $\{I_1, NI_1\}$. Here I_1 and NI_1 mean investing and not investing in period one, respectively. To invest is to build a biofuel refinery. Once the refinery is built, the cost of doing so is sunk. Refineries that are built in period one can produce in both period one and period two. We normalize each refinery's capacity to one unit.

For simplicity, we assume that an investor's cost function has a quadratic form. Specifically, if an

investor invests and produces in period $i = 1, 2$, then the cost function is

$$C_i(q_i) = \begin{cases} a_i q_i^2 + s q_i + f_i & \text{if } q_i \in [0, 1] \\ \infty & \text{if } q_i > 1, \end{cases} \quad (2.1)$$

where q_i is the quantity of output in period i ; a_i and s are non-negative parameters; and $f_i > 0$ is the one-time sunk setup cost of a refinery. As the slope of marginal cost is likely to be small within the range of capacity while the fixed costs of a cellulosic biofuel refinery are likely to be large, we assume that $a_i \leq f_i$. Assuming that $a_i \leq f_i$ also admits constant marginal costs as one can set $a_i = 0$. Allowing $a_i > f_i$ extracts no extra insight from the article but requires a somewhat distended analysis that arrives at essentially the same conclusions. To capture the fact that technology advances and learning-by-doing may decrease production costs of cellulosic biofuels over time, we have $x_2 < x_1$, where $x \in \{a, f\}$. For simplicity we assume that a_i and f_i are common across investors in period i . Parameter s , however, is allowed to vary over investors to capture investor heterogeneities. Therefore, s also serves as an index of investor efficiency. The higher the s is, the lower the investor's efficiency. Let $G(s)$ denote the distribution function of s with support $[0, \infty)$. We assume that $G(s)$ is continuous and $dG(s)/ds > 0$. We could allow s to vary over periods. But since i) the decreasing costs over time have been captured by decreasing a and f , and ii) the focus of this article regards the effects of mandates, instead of technology advances, on investment in cellulosic biofuel, allowing s to vary will not add much insight. Moreover, since s acts as an index of investors as well, allowing it to vary will require two distributions of investors in our model, one for each period. And we need to make further assumptions about the relationship between these two distributions so that the aggregate capacity built up in period one can be consistently measured. To avoid such complications we let s be constant over time.

If a refinery that is set up in period one continues producing in period two, then in period two its cost function has parameter a_2 instead of a_1 , which means that in period two refineries that are built in period one can benefit from technology advances and learning-by-doing to reduce their variable production costs. This fits the cellulosic industry well because some major inputs into producing cellulosic biofuel, such as enzymes, are expected to become less expensive in light of technology advances (Geddes, Nieves, and Ingram 2011). Based on the cost function, an investor's marginal cost in period $i = 1, 2$ can

be written as

$$C'_i(q_i) = \begin{cases} 2a_i q_i + s & \text{if } q_i \in [0, 1] \\ \infty & \text{if } q_i > 1. \end{cases} \quad (2.2)$$

Hereafter, whenever we mention “an investor with s ” or simply “investor s ” without further explanation, then we mean an investor with marginal cost as in equation (2.2). Naturally, “a refinery with s ” means the refinery built by investor s . We assume that each investor’s cost function is common knowledge and all investors have rational expectations.

Since there is a continuum of investors, each investor’s production has no effect on total production. So an investor will be a price taker after she enters the cellulosic biofuel industry. One may argue that, since there is a biofuel blending mandate and a new refinery will take about two years to build, the available refineries can charge an arbitrarily high price for their products. But if producers charge a very high price, then the obligated party can petition the EPA to grant a waiver according to EISA. Therefore an arbitrarily high price is unlikely. Therefore, if an investor with s is in the cellulosic biofuel industry in period i , then her optimal production level will follow the schedule

$$q_i^* = \min \left[1, \max \left[0, \frac{v_i - s}{2a_i} \right] \right], \quad (2.3)$$

where v_i is the value of one unit of cellulosic biofuel in period $i = 1, 2$. Specifically, $v_1 = p_1$ and $v_2 = p_2 + p^{RIN}$ where p_1 and p_2 are the prices of cellulosic biofuel in periods one and two, respectively; and p^{RIN} is the price of RINs in period two. To save on notation, here we let p_1 include the RIN value in the first period. Since renewable biofuel is only a small part of the fuel market, it is reasonable to assume that p_1 and p_2 are exogenously determined by the price of gasoline (Feng and Babcock, 2010). One can view p_2 as the value of energy in one unit of cellulosic biofuel that is exogenously determined by gasoline price; and view p^{RIN} as a price reflecting scarcity of cellulosic biofuel under mandate policies in period two. The value of one unit of cellulosic biofuel in period two, v_2 , should include both p_2 and p^{RIN} .

If investor s has invested in period $i = 1, 2$, then her maximum operating profit in period i is

$$\begin{aligned}\pi_i(v_i; s) &= v_i q_i^* - a_i (q_i^*)^2 - s q_i^* \\ &= \begin{cases} 0 & \text{whenever } v_i \leq s \\ \frac{(v_i - s)^2}{4a_i} & \text{whenever } s \leq v_i \leq s + 2a_i \\ v_i - s - a_i & \text{whenever } v_i \geq s + 2a_i. \end{cases}\end{aligned}\quad (2.4)$$

For a refinery with s , by equation (2.4) we know that when $v_i \geq s + a_i + f_i$ then the fixed cost, f_i , can be covered by the operating profit in period i , which means that under this situation investment is profitable. By equation (2.3) we can readily check that the same refinery will run at full capacity whenever $v_i \geq s + 2a_i$ and will shut down whenever $v_i \leq s$. Therefore, in period i for investor s , we define investment price, full-capacity-running price, and shut down price as $s + a_i + f_i$, $s + 2a_i$, and s , respectively.

Let $\beta \in [0, 1]$ denote the discount factor. For investor s , the expected present value of profit from investing in period one (i.e., from taking action I_1) is

$$B(I_1; s) = \pi_1(p_1; s) - f_1 + \beta \int_0^\infty \pi_2(p_2 + p^{RIN}; s) dJ(p_2), \quad (2.5)$$

where $J(p_2)$ is the distribution of stochastic period two price, p_2 , with support $[0, \infty)$. We assume that $J(p_2)$ is continuous and $dJ(p_2)/dp_2 > 0$.

If an investor does not invest in period one then she receives nothing in period one but still has the option to invest in period two. In period two, after observing p_2 and p^{RIN} , the investor makes a decision that maximizes her profit. She will invest whenever $\pi_2 - f_2 \geq 0$. Therefore, for investor s the expected present value of profit from not investing in period one (i.e., from taking action NI_1) is

$$B(NI_1; s) = \beta \int_0^\infty \max[\pi_2(p_2 + p^{RIN}; s) - f_2, 0] dJ(p_2). \quad (2.6)$$

In period one, investors choose I_1 or NI_1 to maximize their expected total present value of profit from both periods. We define the profit difference between choosing I_1 and NI_1 for a potential investor with s as

$$\begin{aligned}\Delta(s) &\equiv B(I_1; s) - B(NI_1; s) \\ &= \pi_1(p_1; s) - f_1 + \beta \int_0^\infty \min[\pi_2(p_2 + p^{RIN}; s), f_2] dJ(p_2).\end{aligned}\quad (2.7)$$

Here $B(NI_1; s)$ can be seen as the option value that is forgone by choosing action I_1 , so it should be part of the opportunity cost of choosing I_1 (p. 6, Dixit and Pindyck, 1994). An investor will invest in the first period if $\Delta(s) \geq 0$. Investors with s such that $\Delta(s) = 0$ are indifferent between I_1 and NI_1 . We refer to such investors as period-one break-even investors hereafter and assume that they invest in period one.

Let $z(v_2) \equiv \min[\pi_2(p_2 + p^{RIN}; s), f_2]$. Since $v_2 \equiv p_2 + p^{RIN} \in [0, \infty)$ and consequently $\pi_2 \in [0, \infty)$, we have $z \in [0, f_2]$. The dark four-part curve in Figure 2.1 shows the value of z as a function of v_2 . So for a fixed s , the upper bound of $\Delta(s)$ is $\pi_1(p_1; s) - f_1 + \beta f_2$. Formally, we define the upper bound of $\Delta(s)$ as

$$\bar{\Delta}(s) \equiv \pi_1(p_1; s) - f_1 + \beta f_2. \quad (2.8)$$

We further define \bar{s} such that $\bar{\Delta}(\bar{s}) = 0$. It is clear that when $\pi_i > 0$ then π_i is strictly decreasing in s , which tells us that investors with $s > \bar{s}$ will never invest in period one. Therefore, $G(\bar{s})$ can be seen as the upper bound of the capacity built in period one. In addition, $\bar{\Delta}(\bar{s}) = \pi_1(p_1; \bar{s}) - f_1 + \beta f_2 = 0$ together with $f_2 < f_1$ imply $\pi_1(p_1; \bar{s}) > 0$. Therefore, we know that $\pi_1(p_1; s) > 0$ for all $s \leq \bar{s}$. Together with $\partial \pi_2 / \partial s \geq 0$, we can conclude that for all $s \in [0, \bar{s}]$, $\Delta(s)$ is strictly decreasing in s . This says that for investors who may invest in period one (i.e., investors with $s \leq \bar{s}$), low efficiency investors have less incentive to invest in period one. This is because the profit difference between choosing I_1 and choosing NI_1 in both periods decreases as investors' efficiency decreases.

If $\bar{\Delta}(0) < 0$, which means that even the most efficient investor will never invest in period one, then no potential investor will invest in period one. This could be an appropriate approximation to the cellulosic biofuel industry in 2010 and 2011 when waivers were granted: low prices and high investment costs make commercial-scale cellulosic biofuel refineries unviable. Were $\bar{s} = 0$, which means that no investor will invest in period one even if the expected market situation in period two is very good, then mandate policies will have no effect on period one investment levels. Therefore, in the rest of this article we assume

Assumption 2.1. *The most efficient investor will invest in period one, i.e., $\Delta(0) > 0$.*

Assumption 1 implies $\bar{s} > 0$. This is because for any $p_1 \in [0, \infty)$ there exist an $s > 0$ such that $\pi_1(p_1; s) = 0$ and $\bar{\Delta}(s) = -f_1 + \beta f_2 < 0$. Since $\bar{\Delta}(0) \geq \Delta(0) > 0$, by applying the intermediate value

theorem we know there must be a $\bar{s} > 0$ such that $\bar{\Delta}(\bar{s}) = 0$. We summarize the conclusions so far as Remark 1.

Remark 2.1. *i) Investors with s greater than \bar{s} will never invest in period one. Therefore, ii) the upper bound of period one investment level is $G(\bar{s})$. iii) For all $s \in [0, \bar{s}]$, $\Delta(s)$ is strictly decreasing in s . That is, the less efficient an investor is, the less likely she will invest in period one.*

Suppose that $J(\bar{s} + a_2 + f_2) = 0$, i.e., almost surely the realization of p_2 is higher than $\bar{s} + a_2 + f_2$. Then by equation (2.4) and the definition of \bar{s} we know that $\Delta(\bar{s}) = \bar{\Delta}(\bar{s}) = 0$. From Remark 2.1 we can conclude that if $J(\bar{s} + a_2 + f_2) = 0$ then investors with $s \leq \bar{s}$ will invest in period one. The intuition is as follows. If $J(\bar{s} + a_2 + f_2) = 0$, then investors with $s \leq \bar{s}$ that had not already invested in period one will almost surely invest in period two anyway because p_2 will be higher than the investment price of investor s . Therefore, the benefit of deferring investment from period one to period two is just to save one period of interest on the fixed cost, which is $(1 - \beta)f_1$; and the decrease in the present value of fixed cost as time moves into period two, which is $\beta(f_1 - f_2)$. The opportunity cost of this benefit is π_1 . For investors with $s < \bar{s}$ the cost is greater than the benefit, so they will not delay investment into period two. We then have Remark 2.2.

Remark 2.2. *If $J(\bar{s} + a_2 + f_2) = 0$ then the period one investment level will reach its upper bound, $G(\bar{s})$.*

Since trade in RINs is the market mechanism by which the mandates are to be met, the price of RINs plays a key role in both NWM policy and WM policy. Therefore, before we study the effects of mandate policies, we need to first discuss price of RINs. We assume that under NWM policy the government's commitment is credible. If the commitment is not credible, investors would expect that waivers will occur whenever production capacity is less than the mandate level. Then a NWM policy effectively becomes a WM policy.

RIN Price

The analysis in this section focuses on RIN price in period two because we assume that p_1 , which includes period-one RIN price, is given at the very beginning of period one. Under NWM policy (WM policy), the RIN price will start rising when p_2 is not high enough to ensure that the mandate (waived

mandate) is met. Therefore, p^{RIN} in period two is determined by three factors. These are mandate level, M ; capacity built up in period one, X ; and the production level under p_2 . The mandate policies essentially provide investors with a put option whose strike price ensures that the mandate (or waived mandate under the WM policy) is met. Therefore, the mandate policies are valuable to investors only if p_2 is lower than the strike price. RIN prices perform differently under NWM policy and WM policy because of the waivability feature that distinguishes these two policies. In this section we first study RIN price under NWM policy and then RIN price under WM policy.

RIN price under NWM policy

Under NWM policy, the production level of cellulosic biofuel in period two must be no less than the mandate level, M . Therefore, when $X < M$ then $v_2 = p_2 + p^{RIN}$ will be high enough to induce new investment in period two. When $X \geq M$, however, then new investment is not needed in order to meet the mandate level. Therefore, the RIN price when $X < M$ may differ from the RIN price when $X \geq M$. So we discuss RIN price in two cases, based on whether or not $X < M$.

Case 1. RIN price under NWM policy when $X < M$

In this case, in order to meet the mandate investors with $s \in (G^{-1}(X), G^{-1}(M)]$ will be driven to invest in period two by the NWM policy. From equation (2.4) we know that to induce investment from investors with $G^{-1}(M)$ requires that the cellulosic biofuel in period two, v_2 , should be such that $v_2 \geq G^{-1}(M) + a_2 + f_2$. If $p_2 < G^{-1}(M) + a_2 + f_2$, then the RIN price will rise to a level such that $p_2 + p^{RIN} = G^{-1}(M) + a_2 + f_2$. If $p_2 \geq G^{-1}(M) + a_2 + f_2$, then the mandate will be met and hence $p^{RIN} = 0$. Therefore, when $X < M$ then the RIN price under NWM policy is $p^{RIN} = \max[G^{-1}(M) + a_2 + f_2 - p_2, 0]$.

Case 2. RIN price under NWM policy when $X \geq M$

In this case, to meet the mandate no new investment in period two is necessary. Based on the magnitude of X , there are two subcases to consider. To find the critical value of X that differentiates the two subcases, we need one more piece of notation. Let function $A(X)$ denote the production level in period two when period one investment level is X and when $v_2 = G^{-1}(X)$. Clearly when $v_2 = G^{-1}(X)$ then refineries with $s \geq G^{-1}(X)$ will shut down; If $v_2 = G^{-1}(X) \geq 2a_2$ then refineries with $s \in [0, G^{-1}(X) - 2a_2]$ will run at full capacity and refineries with $s \in (G^{-1}(X) - 2a_2, G^{-1}(X))$ will

run at partial capacity. If $v_2 = G^{-1}(X) < 2a_2$ then refineries with $s \in [0, G^{-1}(X))$ will run at partial capacity. Specifically, we have

$$A(X) = G\left(\max[G^{-1}(X) - 2a_2, 0]\right) + \int_{\max[G^{-1}(X) - 2a_2, 0]}^{G^{-1}(X)} \frac{G^{-1}(X) - s}{2a_2} dG(s), \quad (2.9)$$

where the first (second) term on the right side of the equation stands for production from investors who run their refineries at full (partial) capacity. Item A of the Appendix shows that $A(0) = 0$, $A(1) = 1$, and $A(X)$ is strictly increasing in X .

The two subcases are differentiated based on whether or not $A(X) > M$ (i.e., $X > A^{-1}(M)$). The reason is as follows. Condition $A(X) > M$ means that the period two production level is higher than the mandate level when period one investment level is X and when the cellulosic biofuel value in period two is $v_2 = G^{-1}(X)$. This tells us that when period one investment level is X then to ensure the mandate is just met requires a value of v_2 lower than $G^{-1}(X)$, which implies that some refineries built in period one will shut down in period two if the mandate is just met. However, when $A(X) \leq M$ then to ensure the mandate is just met requires a value of v_2 higher than $G^{-1}(X)$. This means that all refineries built in period one will run (at full or partial capacity) in period two if the mandate is just met. Since the purpose of RIN price is to drive v_2 to a level that the mandate can just be met whenever p_2 is not high enough, RIN prices differ based on whether or not $A(X) > M$ (i.e., $X > A^{-1}(M)$).

Sub-case 1. $X \in (A^{-1}(M), 1]$

To ease the exposition, hereafter we define v_2^M such that when $v_2 = v_2^M$ and when the period one investment level is $X \geq M$ then the mandate level, M , is just met. As we have discussed above, in this sub-case we must have $v_2^M < G^{-1}(X)$ and some refineries from period one will shut down in period two when $v_2 = v_2^M$. So v_2^M is determined by

$$G\left(\max[v_2^M - 2a_2, 0]\right) + \int_{\max[v_2^M - 2a_2, 0]}^{v_2^M} \frac{v_2^M - s}{2a_2} dG(s) = M, \quad (2.10)$$

where the first (second) term on the left side of the equation stands for the production from investors who run their refineries at full (partial) capacity. We define $\widehat{v_2^M}$ as the solution to equation (2.10). Therefore, in this sub-case $p^{RIN} = \max[\widehat{v_2^M} - p_2, 0]$. Item B of the Appendix shows that $\widehat{v_2^M} = G^{-1}(A^{-1}(M))$. We can see that $\widehat{v_2^M}$ does not change with X . This is because when $A^{-1}(M) \leq X \leq 1$ then to just meet the mandate level refineries with s higher than a certain value will be shut down. For any X such that

$A^{-1}(M) \leq X \leq 1$, an increase of X just means that more refineries with s higher than this certain value are built in period one. These added refineries will be shut down anyway in this sub-case and will not affect the value of v_2 that is required to make the mandate level just be met.

Sub-case 2. $X \in [M, A^{-1}(M)]$

This sub-case is relevant only if $a_2 > 0$, i.e., investors' marginal costs are strictly increasing. This is because when $a_2 = 0$ then, by the definition of $A(X)$, we have $A(X) = X$ and hence $A^{-1}(M) = M$, which means that the range of $[M, A^{-1}(M)]$ will shrink to a point, M . As we have discussed above, in this sub-case we have $v_2^M \geq G^{-1}(X)$. This means that if $A(X) < M$ and if $v_2 = v_2^M$, then no refinery is shut down but some refineries do not run at full capacity. Hence v_2^M , the period two cellulosic biofuel market value under which the mandate is just met, is determined by equation

$$G\left(\max[v_2^M - 2a_2, 0]\right) + \int_{\max[v_2^M - 2a_2, 0]}^{G^{-1}(X)} \frac{v_2^M - s}{2a_2} dG(s) = M, \quad (2.11)$$

where the first (second) term on the left side of the equation stands for the production from investors who run their refineries at full (partial) capacity. Let \widetilde{v}_2^M denote the solution of equation (2.11). Remark 2.3 summarizes the properties of \widetilde{v}_2^M .

Remark 2.3. When $X \in [M, A^{-1}(M)]$ then \widetilde{v}_2^M exists and is unique. Moreover, \widetilde{v}_2^M is decreasing in the period one investment level, X . When $X = M$ then \widetilde{v}_2^M reaches its maximum value, $G^{-1}(X) + 2a_2$. When $X = A^{-1}(M)$ then \widetilde{v}_2^M reaches its minimum value, $G^{-1}(A^{-1}(M))$.

The derivation of Remark 2.3 can be found in Item C of the Appendix. The reason that \widetilde{v}_2^M is decreasing in X is because when $M \leq X < A^{-1}(M)$ then to meet the mandate all refineries built in period one will run. Therefore, if X is increasing then it means that more refineries are running and hence a lower value of v_2 is required to make the production level reach the mandate. We summarize the analysis of RIN price under NWM policy as

Remark 2.4. Given mandate level, M , then the RIN price under NWM policy is

$$p^{RIN} = \begin{cases} \max[G^{-1}(M) + a_2 + f_2 - p_2, 0] & \text{whenever } 0 \leq X < M \\ \max[\widetilde{v}_2^M - p_2, 0] & \text{whenever } M \leq X \leq A^{-1}(M) \\ \max[G^{-1}(A^{-1}(M)) - p_2, 0] & \text{whenever } A^{-1}(M) < X \leq 1. \end{cases} \quad (2.12)$$

A visual presentation of these three branches of RIN price in equation (2.12) can be found in Figure 2.2. In Figure 2.2 we set $p_2 = 0$ so that the patterns of the RIN prices can be better presented. Overall, under the NWM policy, the RIN price schedule is decreasing, at least weakly, in the period one investment level, X . From Figure 2.2 we can see that a fall in the RIN price schedule under NWM policy occurs at $X = M$. This fall results from the existence of fixed costs and the characteristics of NWM. Under NWM policy, if $X < M$ and if p_2 is not high enough to ensure that the mandate is met, then in order to meet the mandate the RIN price will rise to a level such that $p_2 + p^{RIN} = G^{-1}(M) + a_2 + f_2$, the investment price of an investor with $s = G^{-1}(M)$. When $X = M$ then there are enough refineries available to ensure that mandate is met. Therefore, when $X = M$ then the fixed costs have nothing to do with the RIN price and the RIN price only needs to rise to a level such that $p_2 + p^{RIN} = G^{-1}(M) + 2a_2$. The magnitude of this fall is $f_2 - a_2$. Clearly, when $a_2 = f_2$ there is no discontinuity at $X = M$. This is because when $a_2 = f_2$ then an investor's investment price is equal to the full-capacity-running price so the fixed cost can be covered under the full-capacity-running price. The discontinuity caused by the RIN price fall when $a_2 < f_2$ means that equilibrium X under the NWM policy may not exist. This matter will be discussed in detail when we study the effects of mandate policies in the next section.

RIN price under WM policy

Under WM policy, we assume that when the second period production induced by p_2 is lower than the mandate level, then the mandate will be waived to the available *production capacity* under p_2 . This assumption fits waivers well in reality as the EPA waived (or proposed to waive in the case of 2012) the mandates of 2010, 2011, and 2012 to the projected production capacity in these years (EPA 2010a,b; EPA 2011; Yacobucci 2010). If the mandate level is waived to the *production* level under p_2 , then p^{RIN} will always be zero since the waived mandate is met by the available production under p_2 . Therefore, the WM policy has no effect on any investors. The basic conclusions in this article still hold when the mandate is assumed to be waived to a level between available production under p_2 and the original mandate. We also assume that the waived mandate level must be met by the cellulosic biofuel production in period two. EISA allows the obligated parties to use waiver credits as a substitute for RINs whenever a waiver happens. For simplicity of exposition and to focus on the effects of mandates, we assume this away in our model. If $X \geq M$ then the mandate level will not be waived under WM policy.

So there is no difference between RIN prices under NWM policy and WM policy whenever $X \geq M$. Therefore, in this subsection we only need to discuss the RIN price under WM policy when $X < M$. We have Remark 2.5 as follows.

Remark 2.5. *Under the WM policy, when the period one investment level is less than the mandate level (i.e., $X \in [0, M)$), then the RIN price will place a floor on v_2 at $G^{-1}(X) + 2a_2$. That is, under the WM policy when $X \in [0, M)$ then $p^{RIN} = \max[G^{-1}(X) + 2a_2 - p_2, 0]$.*

The proof of Remark 2.5 is in Item D of the Appendix. By assumption $a_2 \leq f_2$ we know that for an investor with s the investment price (i.e., $s + a_2 + f_2$) is no less than the full-capacity-running price (i.e., $s + 2a_2$). This means that whenever an investor is induced to invest in period two, then her refinery will run at full capacity after she invests. In other words, whenever there is some new investment in period two, the production level in period two will be equal to the capacity. Therefore, under the WM policy, when $X < M$ and when there is some new investment under p_2 (i.e., when $p_2 > G^{-1}(X) + a_2 + f_2$), then the RIN price will be zero because either the mandate or the waived mandate (if a waiver occurs) can be met by p_2 . When there is no new investment induced by p_2 (i.e., when $p_2 \leq G^{-1}(X) + a_2 + f_2$), the mandate will be waived to X , the period one investment level. Under this situation the purpose of the RIN price is to place a floor on v_2 at $G^{-1}(X) + 2a_2$ so that the production level in period two can reach X to meet the waived mandate, which means that $p^{RIN} = \max[G^{-1}(X) + 2a_2 - p_2, 0]$. Then the whole schedule of RIN price under the WM policy can be summarized as

Remark 2.6. *Given mandate level, M , then the RIN price under WM policy is*

$$p^{RIN} = \begin{cases} \max[G^{-1}(X) + 2a_2 - p_2, 0] & \text{whenever } 0 \leq X < M \\ \max[\widetilde{v}_2^M - p_2, 0] & \text{whenever } M \leq X \leq A^{-1}(M) \\ \max[G^{-1}(A^{-1}(M)) - p_2, 0] & \text{whenever } A^{-1}(M) < X \leq 1. \end{cases} \quad (2.13)$$

The three branches of the RIN prices under the WM policy are depicted in Figure 2.2 as well. When $X < M$ then p^{RIN} is increasing in X under the WM policy. This is because the larger the value of X , the higher the value of v_2 that is required to meet the waived mandate level, X . When $X \geq M$ then the RIN price schedule under the WM policy coincides with that under the NWM policy. From Figure 2.2 we also can see that the RIN price schedule under the WM policy is continuous and the maximum value of p^{RIN} is obtained at $X = M$. Moreover, when $X < M$ then the RIN price under the WM policy is

lower than the RIN price under the NWM policy. This is intuitive because the non-waivability under the NWM policy will drive RIN price to rise high enough to ensure the mandate level is met.

Effects of NWM and WM Policies

In this section we discuss mandate policies' profit effects and period one investment effects on potential investors. To do so we compare the expected profits and period one investment levels under three scenarios: *laissez-faire*, NWM policy, and WM policy. In the *laissez-faire* scenario the government does not impose a mandate. Hence a RIN market does not exist and p^{RIN} can be set as 0. Intuition would tell us that the NWM policy will increase expected profits and stimulate period one investment level while the WM policy does not have such effects. This is because the WM policy basically is a non-credible commitment by government. However, our analysis in this section shows that this intuition is only partially true.

Several pieces of notation are needed for further analysis. Hereafter we let superscript l , n , and w denote variables or functions under scenarios of *laissez-faire*, NWM policy, and WM policy, respectively. For example, we denote the equilibrium period one investment levels in the *laissez-faire* scenario, NWM policy scenario, and WM policy scenario as X^l , X^n , and X^w , respectively. We then define $s^u \equiv G^{-1}(X^u)$ where $u \in \{l, n, w\}$. That is, s^u indexes period one break even investors under the corresponding scenario. We further define $\Delta^u(\cdot)$, $u \in \{l, n, w\}$, as the $\Delta(\cdot)$ function described in equation (2.7) under the corresponding scenario. Then by definition we have $\Delta^u(s^u) = 0$ for all $u \in \{l, n, w\}$. Since mandate policies affect investors' period one investment decisions by changing the expected present value of profit, we first study the mandate policies' profit effects before we study their period one investment effects.

Profit Effects

RIN prices under NWM and WM policies effectively change the distribution of cellulosic biofuel's value in period two because RIN prices place a floor on the value of cellulosic biofuel in that period. We define the distribution of v_2 under *laissez-faire*, NWM policy, and WM policy as $J^l(v_2)$, $J^n(v_2)$, and $J^w(v_2)$, respectively, which are depicted in Figure 3. Distribution function $J^l(v_2)$ is the same as

$J(p_2)$ because under the *laissez-faire* scenario $v_2 = p_2$. Essentially, $J^n(v_2)$, and $J^w(v_2)$ are censored distributions of $J^l(v_2)$ with censoring points at \underline{v}_2^n and \underline{v}_2^w , respectively. Here \underline{v}_2^w and \underline{v}_2^n denote the value floors of v_2 under the WM policy and the NWM policy, respectively. By equations (2.12) and (2.13) we have $\underline{v}_2^w \leq \underline{v}_2^n$. In Figure 3, when $v_2 < \underline{v}_2^w$ then $J^u(v_2) = 0$; and when $v_2 \geq \underline{v}_2^w$ then the curves of $J^l(v_2)$ and $J^u(v_2)$ coincide. Here the superscript $u \in \{n, w\}$.

From the RIN prices in equations (2.12) and (2.13) we know that $J^l(v_2) \geq J^w(v_2) \geq J^n(v_2)$ for all $v_2 \in [0, \infty)$. That is, $J^n(v_2)$ first order stochastically dominates $J^w(v_2)$; and $J^w(v_2)$ first order stochastically dominates $J^l(v_2)$. We are indebted to an anonymous referee for comments that led to this approach. Since π_2 is a non-decreasing function of v_2 , we have $B^n(I_1) \geq B^w(I_1) \geq B^l(I_1)$ and $B^n(NI_1) \geq B^w(NI_1) \geq B^l(NI_1)$, where $B^u(\cdot)$ is function $B(\cdot)$ as described in equations (2.5) and (2.6) under scenarios $u \in \{l, n, w\}$. This means that the mandate policies can increase the expected present value of profit from period two. Moreover, the increase is larger under the NWM policy than under the WM policy.

The profit increase caused by mandate policies is not evenly distributed among investors in that only efficient investors can benefit from the mandate policies. As we have defined above, s^u , $u \in \{l, n, w\}$, indexes period one break even investors under the corresponding scenario. Under the WM policy, investors that are less efficient than investor s^w will not benefit from the policy. This is because the value floor fixed by the RIN price under the WM policy is always no higher than $s^w + 2a_2$, which is lower than the investment prices (i.e., $s + a_2 + f_2$) of investors with $s > s^w$. In other words, whenever the RIN market under the WM policy is working then investors with $s > s^w$ are not in the cellulosic biofuel market. Therefore, they do not benefit from the WM policy. For the same reason we know that under the NWM policy, when $X^n < M$ then investors with $s > G^{-1}(M)$ will not benefit from the NWM policy; however, when $X^n \geq M$ then investors with $s > s^n$ will not benefit from the NWM policy. We summarize the mandate policies' profit effects as follows.

Proposition 2.1. *The NWM and WM policies can increase efficient investors' expected present value of profit from period two. The increase is larger under the NWM policy. Under the WM policy, investors who are less efficient than period one break even investors will not benefit from the policy. Under the NWM policy, when $X^n \geq M$ then investors who are less efficient than period one break even investors will not benefit from the policy; and when $X^n < M$ then investors with $s > G^{-1}(M)$ will not benefit from*

the policy.

Proposition 1 identifies a transfer implication of mandate policies. While a mandate is “revenue neutral” as shown in Lapan and Moschini (2009), it is not “transfer neutral.” The more efficient an investor is, the more likely it will benefit from mandate policies. This means that mandate policies could encourage investors to adopt more cost-efficient production technologies, a matter that is beyond the scope of this article and may require future research. Moreover, the profit increase under the mandate policies may provide a “cash cushion” for these efficient investors and prevent them from shutting down when the price of cellulosic ethanol is low. In 2009 we did observe that some grain-based ethanol plants shut down because they hit cash flow problems (Wisner, 2009).

Although mandate policies can increase investors’ expected present value of profit from period two, one cannot conclude that mandate policies will then stimulate investment levels in period one. This is because the investment decision in period one is determined by the difference between $B(I_1; s)$ and $B(NI_1; s)$ rather than their absolute values. This matter will be discussed next.

Period One Investment Effects

Since the aggregate potential capacity is normalized to 1, whenever the mandate level, M , is greater than 1 then it will never be met. Moreover, if the mandate level is less than or equal to the period one investment level under *laissez-faire*, X^l , then it will never be waived. Therefore, we focus on $M \in (X^l, 1]$. By Remark 1 we know that the upper bound of period one investment level is $G(\bar{s})$. Therefore, when $M \in (G(\bar{s}), 1]$ then we always have $X < M$. However, when $M \in (X^l, G(\bar{s})]$ then we may have either $X \geq M$ or $X < M$. So we divide our discussion into two cases. In the first case $M \in (G(\bar{s}), 1]$ and in the second case $M \in (X^l, G(\bar{s})]$. In each case we compare X^l , X^n , and X^w , the equilibrium period one investment levels under corresponding scenarios. However, the comparisons presuppose the existence of X^l , X^n , and X^w , which is addressed in Proposition 2.2.

Proposition 2.2. *Regarding the existence of X^l , X^n , and X^w , we have i) X^l and X^w exist and are unique; ii) when $M \in (G(\bar{s}), 1]$ then X^n exists and is unique; and iii) when $M \in (X^l, G(\bar{s})]$ then X^n exists if and only if $\Delta^n(G^{-1}(X)) \geq 0$ at $X = M$; and whenever X^n exists it is unique.*

Please see Item E of the Appendix for the proof. The results in i) and ii) of Proposition 2.2 are

not surprising because the RIN price schedules in the two items are continuous. Under the *laissez-faire* scenario p^{RIN} is set as zero so the RIN price schedule is continuous. In item *iii*) when $M \in (X^l, G(\bar{s})]$ then the RIN price schedule under the NWM policy is not continuous. This discontinuity of the RIN price with respect to the period one investment level causes the non-existence of X^n in some cases. If $\Delta^n(G^{-1}(X)) \geq 0$ whenever $X = M$, which means that investors with $s = G^{-1}(M)$ invest in period one when period one investment level equals the mandate level, then this discontinuity will not cause non-existence of X^n . The reason is that when investors with $s = G^{-1}(M)$ invest in period one then by Remark 2.1 we know that $X^n \geq M$. From Figure 2.2 we know that RIN price is continuous whenever $X \geq M$. That is, if $\Delta^n(G^{-1}(M)) \geq 0$ then the part of RIN price schedule that is discontinuous does not determine the equilibrium period one investment level. Therefore, under this condition the discontinuity does not cause nonexistence of X^n . If $\Delta^n(G^{-1}(M)) < 0$, however, the part of the RIN price schedule that is discontinuous becomes relevant and X^n does not exist. Now let us start comparing X^l , X^n , and X^w .

Case 1. $M \in (G(\bar{s}), 1]$

In this case we only need to focus on the RIN price schedules when $X < M$ because here we have $X \leq G(\bar{s}) < M$. By Proposition 2.2 we know that in this case X^l , X^n , and X^w exist and are unique. The comparisons between X^l , X^n , and X^w are provided in Proposition 2.3.

Proposition 2.3. *Suppose the mandate level is higher than the upper bound of period one investment but not higher than the aggregate potential capacity, i.e., $M \in (G(\bar{s}), 1]$. Then, i) both NWM and WM policies have positive investment effects but the effect of the WM policy is not higher than that of the NWM policy (i.e., $X^n \geq X^w \geq X^l$); ii) the equilibrium period one investment level under the NWM policy reaches the upper bound of period one investment level (i.e., $X^n = G(\bar{s})$); iii) $X^w > X^l$ if and only if a) $a_2 \in (0, f_2]$ and b) $J(s^w + 2a_2) > 0$; and iv) $X^w < X^n$ if and only if a) $a_2 \in [0, f_2)$ and b) $J(s^w + a_2 + f_2) > 0$.*

The proof is in Item F of the Appendix. The result in item *i*) of Proposition 2.3 is intuitive because the RIN price under the WM policy places a floor on the market value of cellulosic biofuel in period two, but such a floor is lower than the one under the NWM policy. One reason to delay investment from

period one to period two is to mitigate price uncertainties by learning p_2 at the beginning of period two. The value floors under the mandate policies reduce uncertainties about cellulosic biofuel values in period two. Therefore, *ceteris paribus*, they reduce the incentive to delay investment from period one to period two and increase, at least weakly, the period one investment levels under mandate policies.

Item *ii*) of Proposition 2.3 indicates that under the NWM policy, investors with $s \leq \bar{s}$ will invest in period one whenever $M \in (G(\bar{s}), 1]$. Since the NWM policy fixes period two cellulosic biofuel value at $p_2 + p^{RIN} = \max\{s^M + a_2 + f_2, p_2\}$, by Remark 2.2 this result immediately follows.

Item *iii*) of Proposition 2.3 specifies the necessary and sufficient conditions under which the WM policy can strictly stimulate period one investment level when compared with the *laissez-faire* scenario. The conditions are a) the marginal cost of refineries are strictly increasing, and b) the distribution of p_2 is such that there is a positive probability that the realization of p_2 is small enough (i.e., $J(s^w + 2a_2) > 0$). The intuition here is as follows. In the *laissez-faire* scenario, the refineries of investors with $s = s^w$ will be shut down whenever $p_2 < s^w$ because the operating profit is zero. However, in the WM policy scenario, because the RIN price places a floor at $s^w + 2a_2$, the investor with $s = s^w$ can obtain an operating profit, a_2 , even when $p_2 < s^w$ in period two. However, if the marginal cost is constant (i.e., $a_2 = 0$) then this operating profit is 0, so that the investor with $s = s^w$ is indifferent between continuing to run her refinery and shutting it down. That is, when $a_2 = 0$ then the WM policy has no effect on an investor with $s = s^w$. This is why a WM policy may stimulate more investment in period one when the marginal cost is increasing but fails to achieve this when the marginal cost is constant.

One can also interpret this difference from the perspective of intensive and extensive margins. When investors' marginal costs are increasing, then the RIN price can improve the intensive margin and consequently increase the profit of a running plant. That is, the incentive to invest is enhanced. Therefore, more investors invest in period one and the extensive margin is enlarged as well. However, when investors' marginal costs are constant, the RIN price cannot improve the intensive margin of the break-even investor. Therefore, it has no effect on the extensive margin either.

Under the WM policy if $J(s^w + 2a_2) = 0$ then the distribution of v_2 is exactly the same as the distribution of v_2 under the *laissez-faire* scenario. This is because $s^w + 2a_2$ is no less than the value floor of v_2 placed by the RIN price under the WM policy. Therefore, the RIN price has no effect on v_2 , or on the profit and period one investment level.

Item *iv*) of Proposition 2.3 identifies necessary and sufficient conditions under which the WM policy has a smaller period one investment effect than does the NWM policy. Were $a_2 = f_2$ or were $J(s^w + a_2 + f_2) = 0$, then under WM policy the floor of v_2 in period two will be so high that the fixed cost of an investor with s^w can be covered by the operating profit. Therefore, an investor with s^w will certainly invest in period two if she does not invest in period one. This means that under the WM policy we have $\Delta^w(s^w) = \pi_1 - f_1 + \beta f_2 = 0$, which implies that $s^w = \bar{s}$ and hence $s^w = s^n$.

In this case X^n exists because $M \in (G(\bar{s}), 1]$. But X^n may not always exist in the second case where the mandate level is lower than or equal to the upper bound of period one investment level, or more precisely, $M \in (X^l, G(\bar{s})]$. Our comparison between investment levels will be conditional on the existence of X^n .

Case 2. $M \in (X^l, G(\bar{s})]$

Clearly this case requires that the equilibrium period one investment level under the *laissez-faire* scenario is less than the upper bound of period one investment level (i.e., $X^l < G(\bar{s})$). From Remark 2.2 we know that $X^l < G(\bar{s})$ implies $J(\bar{s} + a_2 + f_2) > 0$. So, in this case we assume that $J(\bar{s} + a_2 + f_2) > 0$. When $M \in (X^l, G(\bar{s})]$ then either $X < M$ or $X \geq M$ is possible. Therefore, in this case the RIN price schedule under the NWM policy is not continuous in X (Figure 2.2). Proposition 2.4 presents the results of this case.

Proposition 2.4. *Suppose the mandate level is higher than the equilibrium period one investment level under the laissez-faire scenario but not higher than the upper bound of period one investment level, i.e., $M \in (X^l, G(\bar{s})]$. Then we have i) when the equilibrium period one investment level under the NWM policy, X^n , exists then $X^n \in [M, A^{-1}(M))$ and $X^n = X^w > X^l$; and ii) when X^n does not exist then $X^w \geq X^l$; specifically, $X^w > X^l$ if and only if a) $a_2 \in (0, f_2]$ and b) $J(s^w + 2a_2) > 0$.*

The proof is in Item G of the Appendix. Intuitively, in item *i*) of Proposition 2.4 the reason that the period one investment level under the NWM policy is no less than the mandate level (i.e., $X^n \geq M$) is as follows. Suppose $X^n < M$. Then the non-waivability of the NWM policy will place a floor for period two cellulosic biofuel value at a level which is so high that investors with $s = G^{-1}(M)$ will invest in period one. This means that the realized period one investment level will be no less than the mandate

level, M , which contradicts the premise that $X^n < M$. The reason for $X^n < A^{-1}(M)$ in item i) is that were $X^n \geq A^{-1}(M)$ then the RIN price is so low (see Figure 2.2) that investors with $s = G^{-1}(X^n)$ will not have incentive to invest in period one. Since the WM policy is exactly the same as the NWM policy whenever the period one investment level is no less than the mandate level, the conclusion $X^n = X^w > X^l$ follows naturally from the results that $X^n \geq M > X^l$ in this item. Item ii) of Proposition 2.4 compares X^w and X^l when X^n does not exist in this case. The intuition that we have discussed for item iii) of Proposition 2.3 applies here too.

Concluding Remarks

In this article we study the impact of mandates on stimulating investment in cellulosic biofuel refineries. In a two-period model, the first-period investment levels in three scenarios are compared. These scenarios are (1) *laissez-faire*, (2) NWM policy, and (3) WM policy. We find that the investment impact of mandates depends on investors' marginal costs and the distribution of the price of cellulosic biofuels in the second period. When the price distribution is such that almost surely every realization is sufficiently high, then neither the NWM policy nor the WM policy affect the investment level in the first period. If this condition on the price distribution does not hold and if marginal costs are increasing, then both NWM policy and WM policy can stimulate the investment level in the first period. But the WM policy has a smaller impact than does the NWM policy. If marginal costs are constant, then the WM policy has no effect on the investment level. However, the policy still can increase, at least weakly, the expected profit of more efficient investors.

We emphasize the waivability aspect of the mandates and study the effects of the waivable mandate. There may be a political economy issue concerning what the waiver is and who determines it. Many studies about the effects of U.S. biofuel mandates, such as the ones we reviewed in this paper, implicitly assumed that a mandate is non-waivable. However, if a mandate can be waived (as did occur for cellulosic biofuels in 2010 and 2011 and will occur again in 2012), then policymakers should re-evaluate the conclusions of these studies when making further biofuel policies. Moreover, we show that WM policy has the effect of rewarding more efficient investors or refineries, which will encourage the adoption of cost-reducing technologies in the cellulosic biofuel industry. However, a tax credit policy may

not have such an effect because it subsidizes refineries based on quantity of output (gallons of biofuels produced). That is, two refineries producing equal quantities of biofuels will get the same amount of tax credits, no matter how their production efficiencies differ. From this perspective, a mandate may be preferable to a tax credit as an instrument to promote long-run growth in the biofuel industry.

Moreover, that a waivable mandate may or may not induce investment in biofuels refineries raises the question of how the EISA objective of 36 billion gallons of biofuels by 2022 is going to be met. At least some backers of EISA have likely believed that even a waivable mandate would induce investment because if a plant comes on line, then the RIN price will increase enough to keep it running. But this is true only if plant-level marginal costs are increasing. This article demonstrates that if the refineries' marginal costs are constants, then a waivable mandate does not impact the marginal profit of break-even investors. Thus, aggregate investment may not increase. Therefore, in order to quantify the magnitude of the mandate policies' impacts on investment, studies about the refinery-level marginal costs may be in order.

This article contributes to the understanding of waivable mandates by specifying a condition (i.e., the increasing marginal cost) under which even the waivable mandate can stimulate the period one investment level. However, the exploration in this article does not exhaust all possible conditions under which a waivable mandate will matter. Such conditions could include, but may not be limited to, risk aversion, learning-by-doing, uncertainties in production cost, and asymmetric information. Research on the effects of waivable mandate under these conditions may further expand our understanding of mandate policies.

References

- Althoff, K., C. Ehmke, and A.W. Gray. 2003. "Economic Analysis of Alternative Indiana State Legislation on Biodiesel." West Lafayette, Indiana: Center for Food and Agricultural Business, Department of Agricultural Economics, Purdue University.
- D'Amico, E. 2011. "Next Generation Biofuels still Faces Challenges Heading towards Launch." *ICIS Chemical Business*, June 23. Available at <http://www.icis.com/Articles/2011/06/23/9472252/Next-generation-biofuels-still-faces-challenges-heading->

- towards.html (accessed on 07/06/2011).
- de Gorter, H., and D.R. Just. 2009. "The Economics of a Blend Mandate for Biofuels." *American Journal of Agricultural Economics* 91:738-750.
- Dixit, A.K., and R.S. Pindyck. 1994. *Investment under Uncertainty*. Princeton NJ: Princeton University Press.
- Environmental Protection Agency. 2010a. "Regulation of Fuels and Fuel Additives: Changes to Renewable Fuel Standard Program; Final Rule." *Federal Register* Vol. 75, No. 58, March 26, 2010.
- Environmental Protection Agency. 2010b. "Regulation of Fuels and Fuel Additives: 2011 Renewable Fuel Standards." *Federal Register* Vol. 75, No. 236, December 9, 2010.
- Environmental Protection Agency. 2011. "Regulation of Fuels and Fuel Additives: 2012 Renewable Fuel Standards." *Federal Register* Vol. 76, No. 127, July 1, 2011.
- Feng, H., and B.A. Babcock. 2010. "Impacts of Ethanol on Planted Acreage in Market Equilibrium." *American Journal of Agricultural Economics* 92(3):789-802.
- Gallagher, P.W., H. Shapouri, J. Price, G. Schamel, and H. Brubaker. 2003. "Some Long-run Effects of Growing Markets and Renewable Fuel Standards on Additives Markets and the US Ethanol Industry." *Journal of Policy Modeling* 25:585-608.
- Gardner, B.L. 2007. "Fuel Ethanol Subsidies and Farm Price Support" *Journal of Agricultural & Food Industrial Organization* Vol. 5: Issue 2, Article 4.
- Geddes, C.C., I.U. Nieves, and L.O. Ingram. 2011. "Advances in Ethanol Production." *Current Opinion in Biotechnology* 22(3):312-319.
- Lapan, H., and G. Moschini. 2009. "Biofuels Policies and Welfare: Is the Stick of Mandates Better than the Carrot of Subsidies?" Working Paper No. 09010, Department of Economics, Iowa State University.
- McPhail, L.L., and B.A. Babcock. 2008a. "Ethanol, Mandates, and Drought: Insights from a Stochastic Equilibrium Model of the U.S. Corn Market." Working Paper 08-WP 464, Center for Agricultural and Rural Development, Iowa State University.
- . 2008b. "Short-Run Price and Welfare Impacts of Federal Ethanol Policies." Working Paper 08-WP 468, Center for Agricultural and Rural Development, Iowa State University.
- Roberts, M., and W. Schlenker. 2010. "Identifying Supply and Demand Elasticities of Agricultural

Commodities: Implications for the US Ethanol Mandate” Working Paper 15921, National Bureau of Economic Research, Cambridge, MA.

Taheripour, F., and W.E. Tyner. 2007. “Ethanol Subsidies, Who Gets the Benefits?” Paper presented at Biofuels, Food, & Feed Tradeoffs Conference Organized by the Farm Foundation and USDA, St. Louis, MO, 12-13 April.

Wisner, R. 2009. “Ethanol Economic Crisis: Potential Impact on Corn Use & Ethanol Production vs. Mandated Ethanol Use.” *AgMRC Renewable Energy Newsletter*, Agricultural Marketing Resource Center, Iowa State University.

Yacobucci, B.D. 2010. “Waiver Authority under the Renewable Fuel Standard (RFS).” CRS Report for Congress. Congressional Research Service. August 20, 2010.

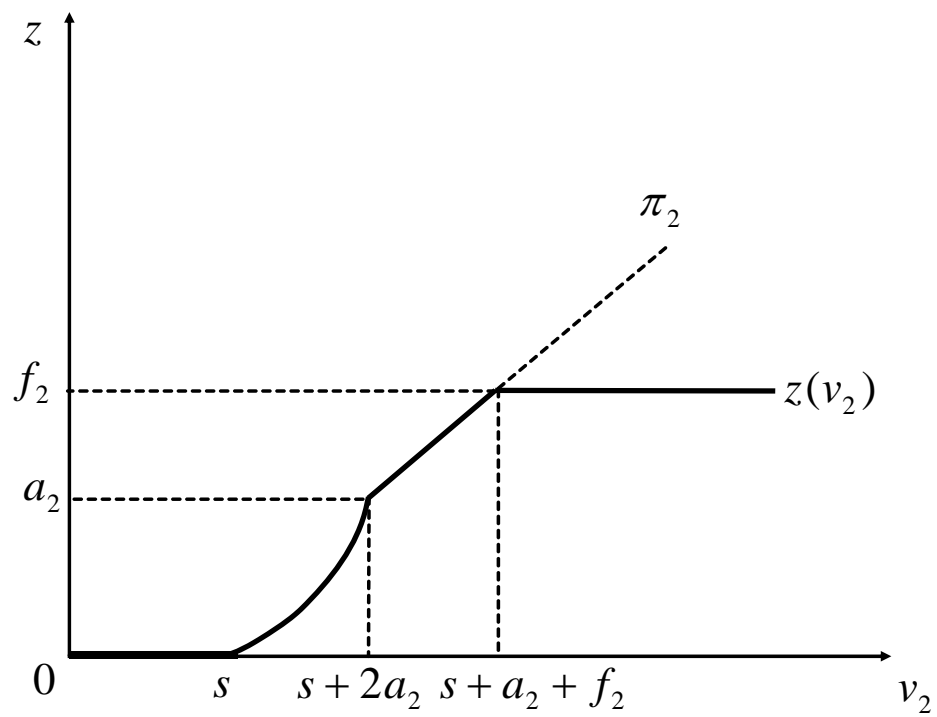


Figure 2.1: The value of z as a function of v_2

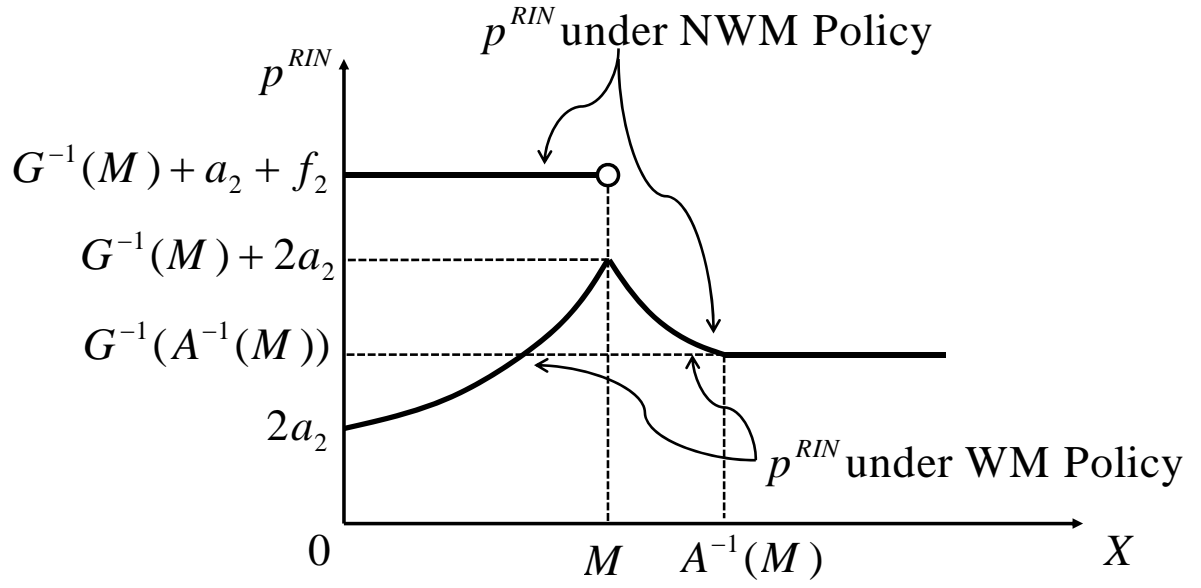


Figure 2.2: RIN price as a function of X under NWM Policy and WM Policy when $p_2 = 0$

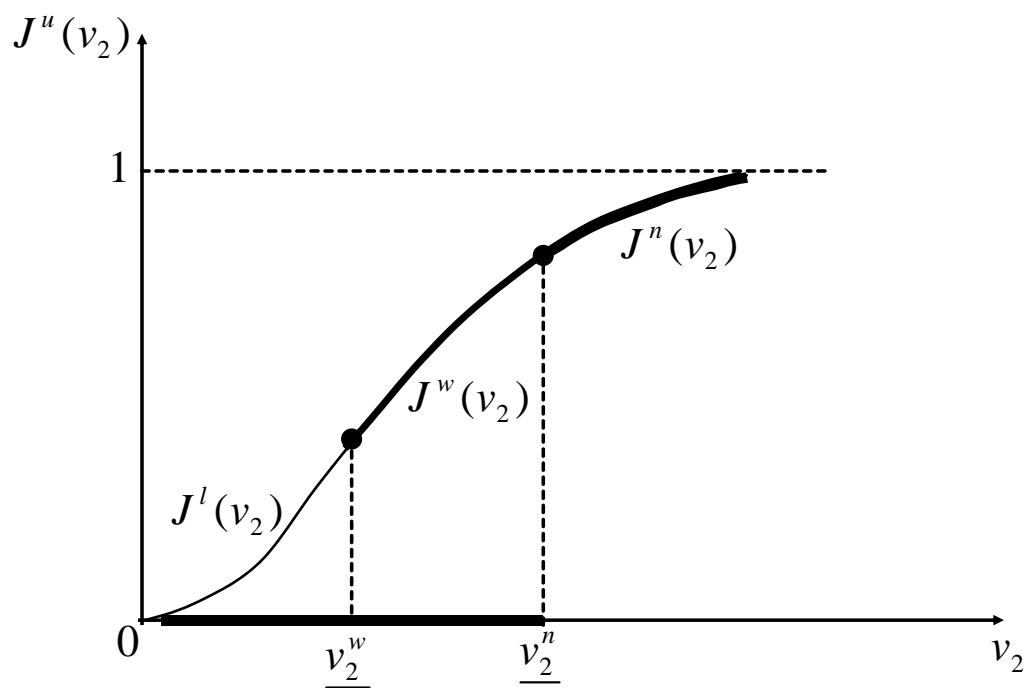


Figure 2.3: Cumulative distribution function of v_2 under *laissez-faire*, NWM Policy, and WM Policy

Appendix

Item A

In this item we show that the function $A(X)$ defined in equation (2.9) has properties as follows: (i) $A(0) = 0$ and $A(1) = 1$; and (ii) $A(X)$ is strictly increasing in X . It is trivial to show that when $a_2 = 0$ then $A(X) = X$ and consequently (i) and (ii) are true. In what follows of this item we focus on $a_2 > 0$.

Proof. Step 1. We prove (i). We first show that

$$A(X) = \int_{\max[G^{-1}(X)-2a_2, 0]}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds. \quad (2.14)$$

When $G^{-1}(X) - 2a_2 \geq 0$, then by equation (2.9) and by integration by parts we have

$$\begin{aligned} A(X) &= G(G^{-1}(X) - 2a_2) + \int_{G^{-1}(X)-2a_2}^{G^{-1}(X)} \frac{G^{-1}(X) - s}{2a_2} dG(s) \\ &= G(G^{-1}(X) - 2a_2) + \frac{G^{-1}(X) - s}{2a_2} G(s) \Big|_{G^{-1}(X)-2a_2}^{G^{-1}(X)} + \\ &\quad \int_{G^{-1}(X)-2a_2}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds \\ &= G(G^{-1}(X) - 2a_2) + \frac{G^{-1}(X) - G^{-1}(X)}{2a_2} X - \\ &\quad G(G^{-1}(X) - 2a_2) + \int_{G^{-1}(X)-2a_2}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds \\ &= \int_{G^{-1}(X)-2a_2}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds. \end{aligned} \quad (2.15)$$

When $G^{-1}(X) - 2a_2 < 0$, then by equation (2.9) and by integration by parts we have

$$\begin{aligned} A(X) &= \int_0^{G^{-1}(X)} \frac{G^{-1}(X) - s}{2a_2} dG(s) \\ &= \frac{G^{-1}(X) - s}{2a_2} G(s) \Big|_0^{G^{-1}(X)} + \int_0^{G^{-1}(X)} \frac{G(s)}{2a_2} ds \\ &= \frac{G^{-1}(X) - G^{-1}(X)}{2a_2} X - \frac{G^{-1}(X) - 0}{2a_2} G(0) + \int_0^{G^{-1}(X)} \frac{G(s)}{2a_2} ds \\ &= \int_0^{G^{-1}(X)} \frac{G(s)}{2a_2} ds. \end{aligned} \quad (2.16)$$

Therefore, by equations (2.15) and (2.16) we have equation (2.14). From equation (2.14) we know that $A(X)$ is continuous.

When $X = 0$ then clearly $G^{-1}(0) = 0$ and $\max[G^{-1}(0) - 2a_2, 0] = 0$. So by equation (2.14) we have $A(0) = 0$.

When $X = 1$ then $G^{-1}(X) = \infty$ and $\max[G^{-1}(X) - 2a_2, 0] = G^{-1}(X) - 2a_2$. Define $x = G^{-1}(X)$, here $X \in [0, 1]$. Therefore, by equation (2.14) we have

$$\begin{aligned} A(1) &= \int_{G^{-1}(1)-2a_2}^{G^{-1}(1)} \frac{G(s)}{2a_2} ds \\ &\leq \int_{G^{-1}(1)-2a_2}^{G^{-1}(1)} \frac{1}{2a_2} ds \\ &= 1. \end{aligned} \tag{2.17}$$

Since $G(s)$ is increasing with s and $G(\infty) = 1$, we have

$$\begin{aligned} A(1) &= \int_{G^{-1}(1)-2a_2}^{G^{-1}(1)} \frac{G(s)}{2a_2} ds \\ &\geq \lim_{x \rightarrow \infty} \int_{x-2a_2}^x \frac{G(x-2a_2)}{2a_2} ds \\ &= \lim_{x \rightarrow \infty} G(x-2a_2) \\ &= 1. \end{aligned} \tag{2.18}$$

Therefore, we have $A(1) = 1$. This finishes the proof of item (i).

Step 2. We prove (ii). Define $g(X)$ as the density function of distribution $G(X)$. When $G^{-1}(X) - 2a_2 \geq 0$ then by equation (2.14) and by Leibniz's formula we have

$$\begin{aligned} \frac{dA(X)}{dX} &= \frac{X}{2a_2 g(X)} - \frac{G(G^{-1}(X) - 2a_2)}{2a_2 g(X)} \\ &= \frac{X - G(G^{-1}(X) - 2a_2)}{2a_2 g(X)} \\ &> 0, \end{aligned} \tag{2.19}$$

where the inequality holds because $X = G(G^{-1}(X)) > G(G^{-1}(X) - 2a_2)$. Similarly, when $G^{-1}(X) - 2a_2 < 0$ then we have

$$\frac{dA(X)}{dX} = \frac{X}{2a_2 g(X)} > 0. \tag{2.20}$$

Therefore, item (ii) is proved. □

Item B

In this item we show that when $X \in (A^{-1}(M), 1]$ then the solution of equation (2.10) is $\widehat{v_2^M} = G^{-1}(A^{-1}(M))$.

Proof. By the similar procedure of showing equation (2.14) we can simplify equation (2.10) as

$$\int_{\max[v_2^M - 2a_2, 0]}^{v_2^M} \frac{G(s)}{2a_2} ds = M. \quad (2.21)$$

We define $V \equiv G(v_2^M)$. Then clearly, from equation (2.14) we know that the left side of equation (2.21) is function $A(V)$. By items (i) and (ii) in Item A and by applying intermediate value theorem we know that for any $M \in [0, 1]$ there is a unique V such that $A(V) = M$. Therefore, we have $V = A^{-1}(M)$. By the definition of V we have $\widehat{v_2^M} = G^{-1}(A^{-1}(M))$. \square

Item C

In this item we derive the results in Remark 2.3. That is, when $X \in [M, A^{-1}(M)]$ then (i) $\widetilde{v_2^M}$, the solution of equation (2.11), exists and is unique; (ii) $\widetilde{v_2^M}$ is decreasing with X ; (iii) when $X = M$ then $\widetilde{v_2^M} = G^{-1}(X) + 2a_2$; and (iv) when $X = A^{-1}(M)$ then $\widetilde{v_2^M} = G^{-1}(A^{-1}(M))$.

Proof. Step 1. We show that equation (2.11) can be equivalently written as

$$\frac{v_2^M - G^{-1}(X)}{2a_2} X + \int_{\max[v_2^M - 2a_2, 0]}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds = M. \quad (2.22)$$

When $v_2^M - 2a_2 \geq 0$ then equation (2.11) becomes

$$\begin{aligned} & G(v_2^M - 2a_2) + \int_{v_2^M - 2a_2}^{G^{-1}(X)} \frac{v_2^M - s}{2a_2} dG(s) = M \\ \Leftrightarrow & G(v_2^M - 2a_2) + \frac{v_2^M - s}{2a_2} G(s) \Big|_{v_2^M - 2a_2}^{G^{-1}(X)} + \int_{v_2^M - 2a_2}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds = M \\ \Leftrightarrow & G(v_2^M - 2a_2) + \frac{v_2^M - G^{-1}(X)}{2a_2} X - G(v_2^M - 2a_2) + \int_{v_2^M - 2a_2}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds = M \\ \Leftrightarrow & \frac{v_2^M - G^{-1}(X)}{2a_2} X + \int_{v_2^M - 2a_2}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds = M. \end{aligned} \quad (2.23)$$

When $v_2^M - 2a_2 < 0$ then equation (2.11) becomes

$$\begin{aligned}
 & G(0) + \int_0^{G^{-1}(X)} \frac{v_2^M - s}{2a_2} dG(s) = M \\
 \Leftrightarrow & \frac{v_2^M - s}{2a_2} G(s) \Big|_0^{G^{-1}(X)} + \int_0^{G^{-1}(X)} \frac{G(s)}{2a_2} ds = M \\
 \Leftrightarrow & \frac{v_2^M - G^{-1}(X)}{2a_2} X + \int_0^{G^{-1}(X)} \frac{G(s)}{2a_2} ds = M.
 \end{aligned} \tag{2.24}$$

By equations (2.23) and (2.24) we can obtain equation (2.22).

Step 2. We show that $G^{-1}(X) \leq v_2^M \leq G^{-1}(X) + 2a_2$. If $v_2^M < G^{-1}(X)$ then production level under v_2^M will be less than M . This is because in this item we have assumed that $A(X) \leq M$ (see Remark 2.3). If $v_2^M > G^{-1}(X) + 2a_2$ then at least X amount of refineries will be running at full capacity. Therefore the production level will be higher than M because $X \geq M$. This violates the definition of v_2^M that under v_2^M the production level is exactly M . In sum, we have $G^{-1}(X) \leq v_2^M \leq G^{-1}(X) + 2a_2$.

Step 3. We show that $\widetilde{v_2^M}$ exists and is unique. We let function $H(v_2^M)$ denote the left side of equation (2.22). That is

$$H(v_2^M) = \frac{v_2^M - G^{-1}(X)}{2a_2} X + \int_{\max[v_2^M - 2a_2, 0]}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds. \tag{2.25}$$

Clearly $H(v_2^M)$ is a continuous function. So if we show that $H(G^{-1}(X)) \leq M$, $H(G^{-1}(X) + 2a_2) \geq M$, and $H(v_2^M)$ is strictly increasing with $v_2^M \in (G^{-1}(X), G^{-1}(X) + 2a_2)$, then by applying the intermediate value theorem we can conclude that there is a unique solution to equation (2.22) on $[G^{-1}(X), G^{-1}(X) + 2a_2]$.

When $v_2^M = G^{-1}(X)$ then

$$\begin{aligned}
 H(v_2^M) &= H(G^{-1}(X)) \\
 &= \int_{\max[G^{-1}(X) - 2a_2, 0]}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds \\
 &= A(X).
 \end{aligned} \tag{2.26}$$

Since in this item we have $X \leq A^{-1}(M)$ (i.e., $A(X) \leq M$), we have $H(G^{-1}(X)) \leq M$.

When $v_2^M = G^{-1}(X) + 2a_2$ then

$$\begin{aligned}
 H(v_2^M) &= H(G^{-1}(X) + 2a_2) \\
 &= X + \int_{G^{-1}(X)}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds \\
 &= X.
 \end{aligned} \tag{2.27}$$

Since in this item $X \geq M$, we have $H(G^{-1}(X) + 2a_2) \geq M$.

Now let us show that $H(v_2^M)$ is strictly increasing with $v_2^M \in (G^{-1}(X), G^{-1}(X) + 2a_2)$. When $v_2^M - 2a_2 \geq 0$ then by equation (2.25) and by Leibniz's formula we have

$$\frac{dH(v_2^M)}{dv_2^M} = \frac{X - G(v_2^M - 2a_2)}{2a_2} > 0, \quad (2.28)$$

where the inequality holds because $v_2^M < G^{-1}(X) + 2a_2$.

Similarly, when $v_2^M - 2a_2 < 0$ then we have

$$\frac{dH(v_2^M)}{dv_2^M} = \frac{X}{2a_2}, \quad (2.29)$$

which implies that $\frac{dH(v_2^M)}{dv_2^M} > 0$ as long as $X > 0$. Since we have assumed $X \geq M$ in this item, when $X = 0$ then we must have $M = 0$ and hence $\widetilde{v_2^M} = 0$. Here we assume this trivial case away.

Therefore, we can conclude that $H(v_2^M)$ is strictly increasing with $v_2^M \in (G^{-1}(X), G^{-1}(X) + 2a_2)$. This finishes the proof that $\widetilde{v_2^M}$ exists and is unique.

Step 4. We show that $\widetilde{v_2^M}$ is decreasing in X . From Step 3 we know that $\widetilde{v_2^M}$ is the unique solution of equation $H(v_2^M) = M$. The value of $\widetilde{v_2^M}$ can either be greater than or not greater than $2a_2$. We define implicit function $Y(v_2^M, X)$ as

$$Y(v_2^M, X) = \frac{v_2^M - G^{-1}(X)}{2a_2}X + \int_{\max[v_2^M - 2a_2, 0]}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds - M = 0. \quad (2.30)$$

If we can show that for all $X \in (M, A^{-1}(M))$, a) if $v_2^M - 2a_2 \geq 0$ then we have $\frac{\partial v_2^M}{\partial X} < 0$ and b) if $v_2^M - 2a_2 < 0$ then we have $\frac{\partial v_2^M}{\partial X} < 0$ as well, then we can prove that $\widetilde{v_2^M}$ is decreasing in $X \in [M, A^{-1}(M)]$. This is because by showing a) and b) we show that $\widetilde{v_2^M}$ is decreasing in $X \in [M, A^{-1}(M)]$ whether or not $\widetilde{v_2^M} \geq 2a_2$.

When $v_2^M - 2a_2 \geq 0$ then

$$Y(v_2^M, X) = \frac{v_2^M - G^{-1}(X)}{2a_2}X + \int_{v_2^M - 2a_2}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds - M = 0. \quad (2.31)$$

By the implicity function theorem we have

$$\begin{aligned}
\frac{\partial v_2^M}{\partial X} &= -\frac{\partial Y / \partial X}{\partial Y / \partial v_2^M} \\
&= -\frac{\frac{v_2^M - G^{-1}(X)}{2a_2} + \frac{X}{2a_2} \frac{dG^{-1}(X)}{dX} - \frac{X}{2a_2} \frac{dG^{-1}(X)}{dX}}{\frac{X - G(v_2^M - 2a_2)}{2a_2}} \\
&= -\frac{\frac{v_2^M - G^{-1}(X)}{2a_2}}{\frac{X - G(v_2^M - 2a_2)}{2a_2}} < 0,
\end{aligned} \tag{2.32}$$

where the inequality holds because $G^{-1}(X) < v_2^M < G^{-1}(X) + 2a_2$ is implied by $X \in (M, A^{-1}(M))$.

When $v_2^M - 2a_2 < 0$ then

$$Y(v_2^M, X) = \frac{v_2^M - G^{-1}(X)}{2a_2} X + \int_0^{G^{-1}(X)} \frac{G(s)}{2a_2} ds - M = 0. \tag{2.33}$$

By the implicit function theorem we have

$$\begin{aligned}
\frac{\partial v_2^M}{\partial X} &= -\frac{\partial Y / \partial X}{\partial Y / \partial v_2^M} \\
&= -\frac{\frac{v_2^M - G^{-1}(X)}{2a_2} + \frac{X}{2a_2} \frac{dG^{-1}(X)}{dX} - \frac{X}{2a_2} \frac{dG^{-1}(X)}{dX}}{\frac{X}{2a_2}} \\
&= -\frac{\frac{v_2^M - G^{-1}(X)}{2a_2}}{\frac{X}{2a_2}} < 0,
\end{aligned} \tag{2.34}$$

where the inequality holds because $G^{-1}(X) < v_2^M < G^{-1}(X) + 2a_2$ is implied by $X \in (M, A^{-1}(M))$.

This finishes the proof that $\widetilde{v_2^M}$ is decreasing with X .

Step 5. We show that when $X = M$ then $\widetilde{v_2^M} = G^{-1}(X) + 2a_2$. Plugging $X = M$ and $v_2^M = G^{-1}(X) + 2a_2$ into equation (2.22) we easily check that the equation holds. That is,

$$\begin{aligned}
&\frac{v_2^M - G^{-1}(X)}{2a_2} X + \int_{\max[v_2^M - 2a_2, 0]}^{G^{-1}(X)} \frac{G(s)}{2a_2} ds \\
&= \frac{G^{-1}(M) + 2a_2 - G^{-1}(M)}{2a_2} M + \int_{G^{-1}(M)}^{G^{-1}(M)} \frac{G(s)}{2a_2} ds \\
&= M.
\end{aligned} \tag{2.35}$$

Since we have shown that equation (2.22) has only one solution, we can conclude that $X = M$ then $\widetilde{v_2^M} = G^{-1}(X) + 2a_2$.

Step 6. We show that when $X = A^{-1}(M)$ then $\widetilde{v_2^M} = G^{-1}(A^{-1}(M))$. In Item B of the supplemental material we have shown that $\widetilde{v_2^M} = G^{-1}(A^{-1}(M))$. Therefore, if $X = A^{-1}(M)$, then $G^{-1}(X) =$

$G^{-1}(A^{-1}(M)) = \widehat{v}_2^M$. By plugging $v_2^M = \widehat{v}_2^M$ and $G^{-1}(X) = \widehat{v}_2^M$ into equation (2.22) we have

$$\int_{\max[\widehat{v}_2^M - 2a_2, 0]}^{\widehat{v}_2^M} \frac{G(s)}{2a_2} ds = M, \quad (2.36)$$

which has exactly the same form as equation (2.21). We know that \widehat{v}_2^M solves equation (2.21). Therefore \widehat{v}_2^M solves equation (2.36). Because \widehat{v}_2^M is unique, then we conclude that when $X = A^{-1}(M)$ then $\widehat{v}_2^M = \widehat{v}_2^M = G^{-1}(A^{-1}(M))$. \square

Item D

In this item we prove the conclusions in Remark 2.5. That is, under the WM policy when $X \in [0, M)$ then $p^{RIN} = \max[G^{-1}(X) + 2a_2 - p_2, 0]$.

Proof. Under the WM policy, the mandate level, M , will be waived to the available production capacity under p_2 in period two whenever this available production capacity is lower than M . According to the magnitude of p_2 , we have three cases to consider.

Case 1. $p_2 \geq G^{-1}(M) + a_2 + f_2$. In this case p_2 is greater than the investment price of investor with $s = G^{-1}(M)$. That is, p_2 is high enough to ensure that the mandate is met. Therefore, in this case no waiver happens. In addition, refineries with $s = G^{-1}(M)$ in period two will run at full capacity. This is because p_2 is higher than the price needed for running at full capacity for such refineries, i.e., $p_2 \geq G^{-1}(M) + a_2 + f_2 \geq G^{-1}(M) + 2a_2$. Therefore, the production level in period two is higher than the mandate level as well. So in this case $p^{RIN} = 0$.

Case 2. $p_2 \in [G^{-1}(X) + a_2 + f_2, G^{-1}(M) + a_2 + f_2)$. In this case some new investment will be induced by p_2 . This is because $p_2 \geq G^{-1}(X) + a_2 + f_2$. Let \tilde{s} be the value of s for the break-even investor under p_2 . Then $G(\tilde{s}) = G(p_2 - a_2 - f_2) \in [X, M)$. This means that the new investment together with the investment in period one cannot meet the mandate level. Therefore, the mandate will be waived to the available production capacity, which is $G(\tilde{s})$. Here we have $p_2 = \tilde{s} + a_2 + f_2 > \tilde{s} + 2a_2$, which implies that p_2 can keep all available refineries running at full capacity. Therefore, in this case the RIN price is zero as well.

Case 3. $p_2 \in [0, G^{-1}(X) + a_2 + f_2)$. In this case no new investment is induced under p_2 in period two. Therefore, the mandate will be waived to X , the available capacity in period two. To meet the

waived mandate (i.e., X), RIN price will put a floor of v_2 at $G^{-1}(X) + 2a_2$, under which the production level is exactly the waived mandate level, X . That is, when $p_2 \geq G^{-1}(X) + 2a_2$ then the floor of v_2 is surpassed and then the RIN market is dormant so $p^{RIN} = 0$. When $p_2 < G^{-1}(X) + 2a_2$ then the RIN price needs to rise so that v_2 can reach the floor. Therefore, in this case we have $p^{RIN} = \max[G^{-1}(X) + 2a_2 - p_2, 0]$.

By summarizing the RIN price in these three cases we conclude that when $X < M$ then RIN price under WM policy is $p^{RIN} = \max[G^{-1}(X) + 2a_2 - p_2, 0]$. \square

Item E

In this item we prove Proposition 2.

Proof. Step 1. We show item i), X^l and X^w exist and are unique. Here we only demonstrate the existence and uniqueness of X^w . Proof of the existence and uniqueness of X^l follows the same way but is much simpler.

From equation (2.13) we know that under the WM policy for each period one investment level $X \in [0, 1]$ there is a corresponding distribution of v_2 in period two. The equilibrium period one investment level, X^w , should be such that when every investor expects X^w as the period one investment level during their decision making process then the finally realized period one investment level is X^w as well. That is, given the distribution of v_2 determined by X^w , we have $\Delta^w(s^w) = 0$. By Remark 2.1 we know that under the distribution of v_2 determined by X^w , for any $s < s^w$ we have $\Delta^w(s^w) > 0$, which means that investors who are more efficient than investor s^w do invest in period one. Similarly, for any $s \geq s^w$ we have $\Delta^w(s^w) < 0$, which means that investors who are less efficient than investor s^w do not invest.

Given a period one investment level $X \in [0, 1]$, there are infinite combinations of investors who form this investment level. For example, this investment level may be built up from investors with $s \in [0, G^{-1}(X)]$; or from investors with $s \in [G^{-1}(1 - X), \infty)$. From Remark 2.1 we know that $\Delta(s)$ is strictly decreasing in $s \in [0, \bar{s}]$. That is, investors who have smaller s have higher incentive to invest in period one. Therefore, we only need to consider one combination of investors who build up X , i.e., investors with $s \in [0, G^{-1}(X)]$.

Let $\Delta^w(G^{-1}(X))$ denote the value of the $\Delta(\cdot)$ function in equation (2.7) under the WM policy where

the p^{RIN} is determined by X . By Assumption 1 we know that $\Delta^w(0) > 0$. If we can show that a) $\Delta^w(G^{-1}(X))$ is decreasing in X over the range of $[0, 1]$; and b) $\Delta^w(G^{-1}(X)) < 0$ whenever $X \geq A^{-1}(M)$, then we can conclude that X^w exists and is unique.

Sub-step 1-1. We first prove that $\Delta^w(G^{-1}(X))$ is decreasing in X over the range of $[0, 1]$. We know that $\Delta^w(G^{-1}(X))$ is continuous on $[0, 1]$ because the RIN price schedule is continuous under the WM policy. Since there are two kinks on the RIN price schedule (Figure 2.2), the function $\Delta^w(G^{-1}(X))$ may not be differentiable on $[0, 1]$. However, if we can show that $\frac{d\Delta^w(G^{-1}(X))}{dX} \leq 0$ on $(0, M)$, $(M, A^{-1}(M))$, and $(A^{-1}(M), 1]$, then we still can conclude that $\Delta^w(G^{-1}(X))$ is decreasing with X on $[0, 1]$. We define $j(p_2)$ as the probability density function of p_2 .

When $X \in (0, M)$, then $v_2 = p_2 + p^{RIN} = \max[G^{-1}(X) + 2a_2, p_2]$. Therefore, we have

$$\begin{aligned}
 & \Delta^w(G^{-1}(X)) \\
 = & \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[\int_0^{G^{-1}(X)+2a_2} \pi_2(G^{-1}(X) + 2a_2; G^{-1}(X)) dJ(p_2) + \right. \\
 & \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + \int_{G^{-1}(X)+a_2+f_2}^{\infty} f_2 dJ(p_2) \right] \\
 = & \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[\int_0^{G^{-1}(X)+2a_2} a_2 dJ(p_2) + \right. \\
 & \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + f_2 (1 - J(G^{-1}(X) + a_2 + f_2)) \right] \\
 = & \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[a_2 J(G^{-1}(X) + 2a_2) + \right. \\
 & \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + f_2 (1 - J(G^{-1}(X) + a_2 + f_2)) \right].
 \end{aligned} \tag{2.37}$$

For equation (2.37), by differentiating with respect to X we obtain

$$\begin{aligned}
 & \frac{d\Delta^w(G^{-1}(X))}{dX} \\
 = & \frac{\partial \pi_1(p_1; G^{-1}(X))}{\partial X} + \beta \frac{dG^{-1}(X)}{dX} \left[a_2 j(G^{-1}(X) + 2a_2) + \right. \\
 & f_2 j(G^{-1}(X) + a_2 + f_2) - a_2 j(G^{-1}(X) + 2a_2) - \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} 1 dJ(p_2) - \\
 & \left. f_2 j(G^{-1}(X) + a_2 + f_2) \right] \\
 = & \frac{\partial \pi_1(p_1; G^{-1}(X))}{\partial X} - \beta \frac{dG^{-1}(X)}{dX} \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} dJ(p_2) \leq 0,
 \end{aligned} \tag{2.38}$$

where the inequality holds because $\partial \pi_1(p_1; G^{-1}(X))/\partial X \leq 0$, $dG^{-1}(X)/dX \geq 0$, and $\int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} dJ(p_2) \geq 0$.

When $X \in (M, A^{-1}(M))$, then $v_2 = p_2 + p^{RIN} = \max[\widetilde{V}_2^M, p_2]$. Since $G^{-1}(X) \leq \widetilde{V}_2^M \leq G^{-1}(X) + 2a_2$, we have

$$\begin{aligned}
& \Delta^w(G^{-1}(X)) \tag{2.39} \\
= & \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[\int_0^{\widetilde{V}_2^M} \frac{(\widetilde{V}_2^M - G^{-1}(X))^2}{4a_2} dJ(p_2) + \right. \\
& \int_{\widetilde{V}_2^M}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))^2}{4a_2} dJ(p_2) + \\
& \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + \int_{G^{-1}(X)+a_2+f_2}^{\infty} f_2 dJ(p_2) \right] \\
= & \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[J(\widetilde{V}_2^M) \frac{(\widetilde{V}_2^M - G^{-1}(X))^2}{4a_2} + \right. \\
& \int_{\widetilde{V}_2^M}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))^2}{4a_2} dJ(p_2) + \\
& \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + f_2 (1 - J(G^{-1}(X) + a_2 + f_2)) \right].
\end{aligned}$$

For equation (2.39), by differentiating with respect to X we obtain

$$\begin{aligned}
& \frac{d\Delta^w(G^{-1}(X))}{dX} \\
&= \frac{\partial \pi_1(p_1|G^{-1}(X))}{\partial X} + \beta \left[j(\widetilde{V}_2^M) \frac{\partial \widetilde{V}_2^M}{\partial X} \frac{(\widetilde{V}_2^M - G^{-1}(X))^2}{4a_2} + \right. \\
& \quad J(\widetilde{V}_2^M) \frac{\widetilde{V}_2^M - G^{-1}(X)}{2a_2} \left(\frac{\partial \widetilde{V}_2^M}{\partial X} - \frac{dG^{-1}(X)}{dX} \right) + \\
& \quad \frac{dG^{-1}(X)}{dX} a_2 j(G^{-1}(X) + 2a_2) - j(\widetilde{V}_2^M) \frac{\partial \widetilde{V}_2^M}{\partial X} \frac{(\widetilde{V}_2^M - G^{-1}(X))^2}{4a_2} - \\
& \quad \frac{dG^{-1}(X)}{dX} \int_{\widetilde{V}_2^M}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))}{2a_2} dJ(p_2) + \\
& \quad \frac{dG^{-1}(X)}{dX} f_2 j(G^{-1}(X) + a_2 + f_2) - \\
& \quad \left. \frac{dG^{-1}(X)}{dX} a_2 j(G^{-1}(X) + 2a_2) - \frac{dG^{-1}(X)}{dX} \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} 1 dJ(p_2) - \right. \\
& \quad \left. f_2 j(G^{-1}(X) + a_2 + f_2) \frac{dG^{-1}(X)}{dX} \right] \\
&= \frac{\partial \pi_1(p_1|G^{-1}(X))}{\partial X} + \beta \left[J(\widetilde{V}_2^M) \frac{\widetilde{V}_2^M - G^{-1}(X)}{2a_2} \left(\frac{\partial \widetilde{V}_2^M}{\partial X} - \frac{dG^{-1}(X)}{dX} \right) - \right. \\
& \quad \frac{dG^{-1}(X)}{dX} \int_{\widetilde{V}_2^M}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))}{2a_2} dJ(p_2) - \\
& \quad \left. \frac{dG^{-1}(X)}{dX} \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} dJ(p_2) \right] \leq 0,
\end{aligned} \tag{2.40}$$

where the inequality holds because $\partial \pi_1(p_1; G^{-1}(X))/\partial X \leq 0$, $\partial \widetilde{V}_2^M/\partial X \leq 0$, $dG^{-1}(X)/dX \geq 0$, and $G^{-1}(X) \leq \widetilde{V}_2^M \leq G^{-1}(X) + 2a_2$.

When $X \in (A^{-1}(M), 1)$, then $v_2 = p_2 + p^{RIN} = \max[\widehat{V}_2^M, p_2]$. Item B has shown that $\widehat{V}_2^M = G^{-1}(A^{-1}(M))$. When $X \in (A^{-1}(M), 1)$ then we have $G^{-1}(A^{-1}(M)) < G^{-1}(X)$. Therefore, at the value floor of \widehat{V}_2^M , investors with $s = G^{-1}(X)$ will be shut down, which indicates that this value floor has no effect on such

investors. So we have

$$\begin{aligned}
& \Delta^w(G^{-1}(X)) \\
= & \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[\int_0^{G^{-1}(X)} 0 dJ(p_2) + \right. \\
& \int_{G^{-1}(X)}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))^2}{4a_2} dJ(p_2) + \\
& \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + \int_{G^{-1}(X)+a_2+f_2}^{\infty} f_2 dJ(p_2) \right] \\
= & \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[\int_{G^{-1}(X)}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))^2}{4a_2} dJ(p_2) + \right. \\
& \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + f_2(1 - J(G^{-1}(X) + a_2 + f_2)) \right].
\end{aligned} \tag{2.41}$$

For equation (2.41), by taking derivative with respect to X we obtain

$$\begin{aligned}
& \frac{d\Delta^w(G^{-1}(X))}{dX} \\
= & \frac{\partial \pi_1(p_1; G^{-1}(X))}{\partial X} + \beta \frac{dG^{-1}(X)}{dX} \left[a_2 j(G^{-1}(X) + 2a_2) - \right. \\
& \int_{G^{-1}(X)}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))}{2a_2} dJ(p_2) + f_2 j(G^{-1}(X) + a_2 + f_2) - \\
& \left. a_2 j(G^{-1}(X) + 2a_2) - f_2 j(G^{-1}(X) + a_2 + f_2) \right] \\
= & \frac{\partial \pi_1(p_1; G^{-1}(X))}{\partial X} - \beta \frac{dG^{-1}(X)}{dX} \int_{G^{-1}(X)}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))}{2a_2} dJ(p_2) \\
\leq & 0,
\end{aligned} \tag{2.42}$$

where the inequality holds because $\partial \pi_1(p_1; G^{-1}(X))/\partial X \leq 0$, $dG^{-1}(X)/dX \geq 0$, and

$$\int_{G^{-1}(X)}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))}{2a_2} dJ(p_2) \geq 0.$$

Sub-step 1-2. Now let us prove that $\Delta^w(G^{-1}(X)) < 0$ whenever $X \in [A^{-1}(M), 1]$. That is, if the period one investment level is higher than $A^{-1}(M)$, then investors with $G^{-1}(X)$ will not invest in period one. Under the *Laissez-faire* scenario, if $X \in [A^{-1}(M), 1]$ then investors with $s = G^{-1}(X)$ will not invest

in period one because $G^{-1}(X) \geq A^{-1}(M) \geq M > b^l$. That is,

$$\begin{aligned}
& \Delta^l(G^{-1}(X)) \\
&= \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[\int_0^{G^{-1}(X)} 0 dJ(p_2) + \right. \\
&\quad \left. \int_{G^{-1}(X)}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))^2}{4a_2} dJ(p_2) + \right. \\
&\quad \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + \int_{G^{-1}(X)+a_2+f_2}^{\infty} f_2 dJ(p_2) \right] \\
&= \pi_1(p_1; G^{-1}(X)) - f_1 + \beta \left[\int_{G^{-1}(X)}^{G^{-1}(X)+2a_2} \frac{(p_2 - G^{-1}(X))^2}{4a_2} dJ(p_2) + \right. \\
&\quad \left. \int_{G^{-1}(X)+2a_2}^{G^{-1}(X)+a_2+f_2} (p_2 - a_2 - G^{-1}(X)) dJ(p_2) + f_2 (1 - J(G^{-1}(X) + a_2 + f_2)) \right] \\
&< 0.
\end{aligned} \tag{2.43}$$

By comparing equations (2.41) and (2.43) we find that they are the same, which implies that $\Delta^w(G^{-1}(X)) < 0$ whenever $X \in [A^{-1}(M), 1]$. The reason they are the same is because when $X \in [A^{-1}(M), 1]$ then the value floor placed by RIN price is so low that investors with $s = G^{-1}(X)$ will shut down whether or not a floor is in place. Therefore, the value floor under the WM policy does not have any effect on the decision of investors with $s = G^{-1}(X)$. Since when $X \geq M$ then RIN prices under the WM policy and the NWM policy are the same, we also can conclude that $\Delta^n(G^{-1}(X)) < 0$ whenever $X \in [A^{-1}(M), 1]$.

Step 2. We show item ii), i.e., when $M \in (G(\bar{s}), 1]$ then X^n exists and is unique. Under NWM policy the RIN price in period two guarantees the lowest market value of cellulosic biofuel at $s^M + a_2 + f_2$. Therefore, investors with $s < s^M$ will run their refineries at full capacity in period two, because $s + 2a_2 < s^M + a_2 + f_2$. Therefore, for investors with $s < s^M$ we have

$$\begin{aligned}
\Delta^n(s) &= \pi_1(p_1; s) - f_1 + \beta \int_0^{\infty} \min[\pi_2(v_2; s), f_2] dJ(p_2) \\
&= \pi_1(p_1; s) - f_1 + \\
&\quad \beta \left[\int_0^{s^M+a_2+f_2} \min[s^M + a_2 + f_2 - a_2 - s, f_2] dJ(p_2) + \right. \\
&\quad \left. \int_{s^M+a_2+f_2}^{\infty} \min[p_2 - a_2 - s, f_2] dJ(p_2) \right] \\
&= \pi_1(p_1; s) - f_1 + \beta f_2.
\end{aligned} \tag{2.44}$$

By definition we have $\Delta^n(s^n) = 0$. Since function $\Delta(s)$ is strictly decreasing in s (Remark 2.1) and we have defined that $\pi_1(p_1; \bar{s}) - f_1 + \beta f_2 = 0$, we have $s^n = \bar{s}$. This means that when $M \in (G(\bar{s}), 1]$ then

X^n exists and is unique. In addition, $X^n = G(\bar{s})$.

Step 3. We show item iii). That is, when $M \in (X^l, G(\bar{s})]$ then X^n exists if and only if $\Delta^n(G^{-1}(X)) \geq 0$ at $X = M$; and whenever X^n exists it is unique.

We first show that when $M \in (X^l, G(\bar{s})]$ then X^n cannot be less than M . Suppose that $X^n < M$. Then by equation (2.12) we have $p^{RIN} = \max[G^{-1}(M) + a_2 + f_2 - p_2, 0]$. Plugging this RIN price into equation (2.7) we have

$$\begin{aligned} \Delta^n(s) &= \pi_1(p_1; s) - f_1 + \beta \left[\int_0^\infty \min[\pi_2(v_2; s), f_2] dJ(p_2) \right] \\ &= \pi_1(p_1; s) - f_1 + \beta \left[\int_0^{G^{-1}(M) + a_2 + f_2} f_2 dJ(p_2) + \right. \\ &\quad \left. \int_{G^{-1}(M) + a_2 + f_2}^\infty f_2 dJ(p_2) \right], \\ &= \pi_1(p_1; s) - f_1 + \beta f_2. \end{aligned} \tag{2.45}$$

Because in this case we have $M \leq G(\bar{s})$, by Remark 2.1 we have $\Delta^n(G^{-1}(M)) \geq 0$. This means that when $M \in (X^l, G(\bar{s})]$ and when $X^n < M$ then the realized period one investment level will be no less than M , which contradicts the premise that $X^n < M$. So we can conclude that when $M \in (X^l, G(\bar{s})]$ and when X^n exists, then we must have $X^n \geq M$.

In Sub-step 1-2 we have shown that $\Delta^n(G^{-1}(X)) < 0$ whenever $X \in [A^{-1}(M), 1]$. Since $\Delta^n(G^{-1}(X))$ is continuous in X , if we can show that $\Delta^n(G^{-1}(X))$ is decreasing in $X \in (M, 1)$ then item iii) follows by applying the intermediate value theorem. The reason is as follows. For necessity, we want to show that when $M \in (X^l, G(\bar{s})]$ and when X^n exists then we must have $\Delta^n(G^{-1}(M)) \geq 0$. We have shown that when $M \in (X^l, G(\bar{s})]$ and when X^n exists then $X^n \geq M$. That is, by the definition of X^n , equilibrium period one investment level under the NWM policy, there is an $X^n \geq M$ such that $\Delta^n(G^{-1}(X^n)) = 0$. If $\Delta^n(G^{-1}(X))$ is decreasing in $X \in (M, 1)$, then we can conclude that $\Delta^n(G^{-1}(M)) \geq 0$. Sufficiency follows immediately by applying the intermediate value theorem and does not require that $\Delta^n(G^{-1}(X))$ be decreasing in $X \in (M, 1)$.

Now let us show that $\Delta^n(G^{-1}(X))$ is decreasing in $X \in (M, 1)$. In Sub-step 1-1 we have shown that $\Delta^w(G^{-1}(X))$ is decreasing with $X \in [M, 1]$. Since when $X \in [M, 1]$ then the RIN prices under the WM policy and the NWM policy are the same, we have $\Delta^n(G^{-1}(X)) = \Delta^w(G^{-1}(X))$ whenever $X \in [M, 1]$. Therefore, $\Delta^n(G^{-1}(X))$ is decreasing with $X \in [M, 1]$. This finishes the proof of item iii).

□

Item F

In this item we prove Proposition 3.

Proof. Step 1. We prove item i) of Proposition 3, i.e., $X^n \geq X^w \geq X^l$.

If we can show that $\Delta^l(s^w) \leq \Delta^w(s^w) \leq \Delta^n(s^w)$ then we know that $\Delta^l(s^w) \leq 0$ and $\Delta^n(s^w) \geq 0$. This is because by definition we have $\Delta^l(s^l) = \Delta^w(s^w) = \Delta^n(s^n) = 0$. From Remark 2.1 we know that for all $s \in [0, \bar{s}]$, $\Delta(s)$ is strictly decreasing with s . Therefore, $\Delta^l(s^w) \leq 0$ and $\Delta^n(s^w) \geq 0$ imply $s^l \leq s^w \leq s^n$. Now let us show that $\Delta^l(s^w) \leq \Delta^w(s^w) \leq \Delta^n(s^w)$.

When $M \in (G(\bar{s}), 1]$ then we have $X < M$ because $X \leq G(\bar{s})$. By equations (2.12) and (2.13) we can obtain the market value of cellulosic biofuel in period two, v_2 , as

$$\begin{aligned} v_2 &= p_2 + p^{RIN} \\ &= \begin{cases} p_2 & \text{laissez-faire scenario} \\ \max[G^{-1}(M) + a_2 + f_2, p_2] & \text{NWM policy scenario} \\ \max[G^{-1}(X) + 2a_2, p_2] & \text{WM policy scenario.} \end{cases} \end{aligned} \quad (2.46)$$

Then plugging v_2 in equation (2.46) into equation (2.7) we can obtain $\Delta(s^w)$ under the three scenarios.

They are

$$\begin{aligned} \Delta^l(s^w) &= \pi_1(p_1; s^w) - f_1 + \beta \left[\int_0^\infty \min[\pi_2(p_2; s^w), f_2] dJ(p_2) \right] \\ &= \pi_1(p_1; s^w) - f_1 + \beta \left[\int_0^{s^w+2a_2} \pi_2(p_2, s^w) dJ(p_2) + \right. \\ &\quad \left. \int_{s^w+2a_2}^{s^w+a_2+f_2} (p_2 - a_2 - s^w) dJ(p_2) + \int_{s^w+a_2+f_2}^\infty f_2 dJ(p_2) \right], \end{aligned} \quad (2.47)$$

$$\begin{aligned} \Delta^w(s^w) &= \pi_1(p_1; s^w) - f_1 + \beta \left[\int_0^\infty \min[\pi_2(v_2; s^w), f_2] dJ(p_2) \right] \\ &= \pi_1(p_1; s^w) - f_1 + \beta \left[\int_0^{s^w+2a_2} \pi_2(s^w + 2a_2; s^w) dJ(p_2) + \right. \\ &\quad \left. \int_{s^w+2a_2}^{s^w+a_2+f_2} (p_2 - a_2 - s^w) dJ(p_2) + \int_{s^w+a_2+f_2}^\infty f_2 dJ(p_2) \right], \end{aligned} \quad (2.48)$$

and

$$\begin{aligned} \Delta^n(s^w) &= \pi_1(p_1; s^w) - f_1 + \beta \left[\int_0^\infty \min[\pi_2(v_2; s^w), f_2] dJ(p_2) \right] \\ &= \pi_1(p_1; s^w) - f_1 + \beta \left[\int_0^{s^w+2a_2} f_2 dJ(p_2) + \right. \\ &\quad \left. \int_{s^w+2a_2}^{s^w+a_2+f_2} f_2 dJ(p_2) + \int_{s^w+a_2+f_2}^\infty f_2 dJ(p_2) \right], \end{aligned} \quad (2.49)$$

where equation (2.49) holds because when $v_2 = G^{-1}(M) + a_2 + f_2$ and when $G^{-1}(M) > s^w$ then $\pi_2(v_2; s^w) = v_2 - a_2 - s^w > f_2$.

By comparing equations (2.47) and (2.48) we can see that the only difference between these two equations is the first term in the squared brackets. This is because when $p_2 \geq s^w + 2a_2$ then under the WM policy $p^{RIN} = 0$ and hence the profit of investor with s^w under the WM policy is the same as that under the *Laissez-faire* scenario. When $p_2 < s^w + 2a_2$, however, under WM policy $p^{RIN} > 0$ and v_2 is fixed at $s^w + 2a_2$. So π_2 under the WM policy scenario is higher than that under *Laissez-faire* scenario. Therefore, we have

$$\int_0^{s^w+2a_2} \pi_2(p_2, s^w) dJ(p_2) \leq \int_0^{s^w+2a_2} \pi_2(s^w + 2a_2; s^w) dJ(p_2), \quad (2.50)$$

which implies that $\Delta^l(s^w) \leq \Delta^s(s^w)$.

Since $\min[\pi_2, f_2] \leq f_2$, we can readily obtain that $\Delta^w(s^w) \leq \Delta^n(s^w)$. This finishes the proof of item i).

Step 2. We show item ii), i.e., $X^n = G(\bar{s})$.

The proof here is the same as the proof in Step 2 of Item E.

Step 3. We show item iii), i.e., $X^w > X^l$ if and only if a) $a_2 \in (0, f_2]$ and b) $J(s^w + 2a_2) > 0$.

First, we prove sufficiency. If $a_2 \in (0, f_2]$ and if $p_2 < s^w + 2a_2$, then we must have $\pi_2(p_2; s^w) < \pi_2(s^w + 2a_2; s^w)$. Then $J(s^w + 2a_2) > 0$ implies that

$$\int_0^{s^w+2a_2} \pi_2(p_2; s^w) dJ(p_2) < \int_0^{s^w+2a_2} \pi_2(s^w + 2a_2; s^w) dJ(p_2), \quad (2.51)$$

where the left (right) side of the inequality is the first term in the squared brackets of equation (2.47) (equation (2.48)). Therefore, inequality (2.51) implies that $\Delta^l(s^w) < \Delta^w(s^w)$ and hence $X^l < X^w$.

Second, we prove necessity. Here we employ the method of contrapositive proof.

If $a_2 = 0$ then the left side of inequality (2.51) becomes $\int_0^{s^w} \pi_2(p_2; s^w) dJ(p_2) = 0$. The right side becomes $\int_0^{s^w} \pi_2(s^w + 2a_2; s^w) dJ(p_2) = 0$. Clearly, when $a_2 = 0$ then equality holds in inequality (2.51). Therefore, we have $\Delta^l(s^w) = \Delta^w(s^w)$ and hence $X^l = X^w$.

If $J(s^w + 2a_2) = 0$, then p_2 has no mass in the range of $[0, s^w + 2a_2]$. So we will have $\int_0^{s^w+2a_2} \pi_2|_{v_2=p_2} dJ(p_2) = \int_0^{s^w+2a_2} \pi_2|_{v_2=s^w+2a_2} dJ(p_2) = 0$. Therefore, we have $\Delta^l(s^w) = \Delta^w(s^w)$ and hence $X^w = X^l$ as well.

Step 4. We show item iv), i.e., $X^w < X^n$ if and only if a) $a_2 \in [0, f_2)$ and b) $J(s^w + a_2 + f_2) > 0$.

First, we prove sufficiency. If $a_2 \in [0, f_2)$ then we have $s^w + 2a_2 < s^w + a_2 + f_2$. In addition, by equation (2.4), we know that $\pi_2(s^w + 2a_2; s^w) = a_2$. Together with $J(s^w + a_2 + f_2) > 0$ we have either

$$\begin{aligned} & \int_0^{s^w+2a_2} \pi_2(s^w + 2a_2; s^w) dJ(p_2) \\ &= \int_0^{s^w+2a_2} a_2 dJ(p_2) \\ &< \int_0^{s^w+2a_2} f_2 dJ(p_2), \end{aligned} \tag{2.52}$$

or

$$\int_{s^w+2a_2}^{s^w+a_2+f_2} (p_2 - a_2 - s^w) dJ(p_2) < \int_{s^w+2a_2}^{s^w+a_2+f_2} f_2 dJ(p_2), \tag{2.53}$$

or both. The situation under which both inequalities (2.52) and (2.53) hold is when $J(s^w + a_2 + f_2) > J(s^w + 2a_2) > 0$. By equations (2.48) and (2.49) we know that inequalities (2.52) and (2.53) imply $\Delta^w(s^w) < \Delta^n(s^w)$. Therefore, we can conclude that if $a_2 \in [0, f_2)$ and if $J(s^w + a_2 + f_2) > 0$ then $X^w < X^n$.

Second, we prove necessity. Here we employ the method of contrapositive proof. If $a_2 = f_2$, then the first term in the squared brackets of equation (2.48) becomes $\int_0^{s^w+2a_2} f_2 dJ(p_2)$ and the second term becomes 0. Therefore, we have $\Delta^w(s^w) = \Delta^n(s^w)$ and hence $X^w = X^n$. If $J(s^w + a_2 + f_2) = 0$ then the first two terms in the squared brackets of both equations (2.48) and (2.49) become 0, which implies $\Delta^w(s^w) = \Delta^n(s^w)$ and hence $X^w = X^n$. \square

Item G

In this item we prove Proposition 4, which provides a comparison of period one investment levels of the three scenarios in the case under which the mandate level is higher than the equilibrium period one investment level under the *laissez-faire* scenario but not higher than the upper bound of period one investment level, i.e., $M \in (G(s^l), G(\bar{s})]$.

Proof. Step 1. We show item i). That is, when $M \in (G(s^l), G(\bar{s})]$ and when X^n exists then $X^n \in [M, A^{-1}(M))$ and $X^n = X^w > X^l$. In Step 1 of Item E we have shown that under the WM policy, investors with $s = G^{-1}(X)$ will not invest in period one whenever $X \in [A^{-1}(M), 1]$. The same conclusion holds under the NWM policy because the RIN prices under the NWM policy and the WM policy are the same whenever $X \in [A^{-1}(M), 1]$. In Step 3 of Item E we have shown that when $M \in (X^l, G(\bar{s})]$ then $X^n \geq M$. Therefore, we can conclude that $X^n \in [M, A^{-1}(M))$ whenever $M \in (X^l, G(\bar{s})]$ and X^n exists.

Now let us prove that $X^n = X^w$ when $M \in (G(s^l), G(\bar{s})]$ and when X^n exists. This is equal to prove that when $M \in (G(s^l), G(\bar{s})]$ and when X^n exists then the equilibrium period one investment under the WM policy, X^w , is such that $X^w \in [M, A^{-1}(M))$. This is because a) the RIN prices under the WM policy and the NWM policy are the same whenever $X \leq M$; b) X^w is unique (item *i*) of Proposition 2.2); and c) X^n is unique whenever it exists (item *iii*) of Proposition 2.2). By item *iii*) of Proposition 2.2) we know that when $M \in (G(s^l), G(\bar{s})]$ and when X^n exists then $\Delta^n(G^{-1}(M)) \geq 0$. Since $\Delta^w(G^{-1}(X)) = \Delta^n(G^{-1}(X))$ whenever $X \in [M, 1]$, we have $\Delta^w(G^{-1}(M)) \geq 0$. In Step 1 of Item E we have shown that $\Delta^w(G^{-1}(X)) < 0$ whenever $X \in [A^{-1}(M), 1]$. Therefore, by applying the intermediate value theorem we have $X^w \in [M, A^{-1}(M))$. Therefore, we have $X^w = X^n$ and then $X^w > X^l$ follows immediately because $X^w = X^n \geq M > X^l$.

Step 2. We show item *ii*), i.e., when X^n does not exist then $X^w \geq X^l$; specifically, the strict inequality (i.e., $X^w > X^l$) holds if and only if a) $a_2 \in (0, f_2]$ and b) $J(s^w + 2a_2) > 0$. The proof in this step is the same as the proof of items *i*) and *iii*) of Proposition 3 demonstrated in Item F. \square

CHAPTER 3. ECONOMIC VALUE OF INFORMATION: SEGREGATING WHEAT BY PROTEIN CONCENTRATION

Abstract

A technology that can measure grain protein concentration and then segregate grain according to the measurement is on the horizon. This new technology provides growers with opportunities to identify grain that can be directed to premium markets. We study wheat growers' willingness to pay (WTP) for the technology and the technology's impact on wheat market values. The market prospect of the new technology is analyzed as well. Depending on the technology's market structure and marginal costs, (1) the average WTP of adopters for the technology ranges between 14 and 22 cents per bushel, and (2) upon the adoption of this new technology, market value of Hard Red Winter wheat will decrease by 0.2% to 2.3%, but market value of Hard Red Spring wheat will increase by 0.3% to 3.3%. The aggregate wheat market value decreases by a slight amount (0.0002% to 0.004%). If the technology is supplied by a monopoly firm, then demand from Hard Red Winter and Hard Red Spring wheat growers in the United States for the technology will provide the firm with an annual operating profit of between \$5.9 million and \$7.7 million.

Key words: economic value of information, market structure, protein, wheat

JEL classification: Q13, Q16, L63.

Introduction

Product heterogeneity prevails among agricultural commodities. Wheat, the world's second largest crop by annual production between 1999 and 2009, is a typical example (Goodwin and Smith 2009).¹ In the United States, wheat is divided into six classes based on genetic characteristics and in each class there are multiple grades (U.S. Wheat Associates). Besides factors such as dockage and moisture, protein concentration is one of the major factors that affect prices of wheat. Wheat with higher protein concentration often receives protein premiums due to the favorable end-use properties added by the higher protein level. A detailed description of how protein concentration affects wheat grading and prices in major wheat producing countries can be found in Popper, Schäfer, and Freund (2007).

Wheat that is harvested from the same field can have different protein concentrations due to various reasons, such as local soil quality and growing condition (Long, Engel, and Siemens 2008). Technological advances in near infrared (NIR) sensors make measuring grain protein, and hence segregating grains according to the protein concentration, possible (Long, Engel, and Siemens 2008; Taylor *et al.* 2005). According to Long, Engel, and Siemens (2008), an on-combine NIR sensor can measure the protein concentration at 0.5Hz measurement rate (i.e., once every two seconds) with standard error of prediction lower than 5.0g/kg.² Not only does this technology provide wheat growers with accurate information about protein concentration distribution, it also provides them with an opportunity to segregate their harvest into loads with different protein concentrations to better capture protein premium.³ For example, suppose wheat with 13% or higher protein receives a protein premium in the market and other wheat receives the base price. Also suppose a wheat grower harvests 2,000 bushels of wheat from his field in which 1,000 bushels are with protein concentration at 13% and the other 1,000 bushels are with protein concentration at 11%. Without measuring and segregating, the grower can only receive the base price because the average protein concentration level of his wheat is below 13%. However, if

¹Data source: Food and Agricultural Organizations of the United Nations (<http://faostat.fao.org/site/567/default.aspx#ancor>), accessed on October 19th, 2010.

²According to protein data from *Crop Quality Report* (U.S. Wheat Associates 1980-2010), the average protein concentration levels for Hard Red Winter wheat and Hard Red Spring wheat are 12.2% and 14.3% over 1979-2009, respectively. Therefore, a standard error lower than 5.0g/kg means that the magnitude of measurement error is smaller than 4% of the real protein concentration.

³The segregation can be achieved by letting a device control the direction of grain stream according to the protein information provided by a NIR sensor (Long, Engel, and Siemens 2008).

the grower measures and segregates his wheat, then half of his wheat can receive the protein premium and the rest can still receive the base price. Therefore, if the protein premium obtained can justify the measurement and segregation costs, then an individual wheat grower has the incentive to adopt the measuring and segregating technology (M&S technology hereafter).

Were many wheat growers to adopt the technology, then the price of high protein wheat would be expected to be affected. This is because the wide spread adoption of the new technology will boost the supply of wheat with high protein concentration. Moreover, the prices of ordinary wheat may be affected as well because when wheat growers segregate high quality wheat out from ordinary wheat then the supply of ordinary wheat decreases. Therefore, the adoption of this new technology may generate far-reaching impacts on wheat markets. Also, the impact will not be evenly distributed among wheat growers. Those who typically produce high quality wheat do not have the incentive to adopt the technology and hence their profit may decline due to the technology as the protein premium falls. For wheat growers whose wheat quality is so poor that even the new technology does not help very much (i.e., no high quality wheat can be segregated out), other growers' adoption of the technology may benefit them because the supply of ordinary wheat goes down due to wheat segregation. For growers who adopt the new technology, their profit may or may not be higher than under the original pre-technology situation. Therefore, the welfare impact of the new technology on wheat growers is ambiguous without further information.

Since structure in the M&S technology market determines the technology's price, it partly determines the extent of the technology's adoption, and hence the technology's impacts. For instance, if growers' welfare is decreasing (increasing) in the adoption rate, then an increase in the number of technology's suppliers (assuming Cournot competition) may increase (decrease) wheat growers' welfare. Therefore, when studying the value and impacts of the M&S technology, the effect of the market structure should come under scrutiny.

The purpose of this article is threefold. First, we quantify wheat growers' willingness to pay (WTP) for the M&S technology, i.e., the value of the technology to an individual wheat grower. Second, we study the welfare impact of the technology for the wheat industry. Third, we analyze the impact of the M&S technology's market structure on wheat growers' welfare and the technology's market prospects under various market structures. In order to fulfill these three goals, we first develop a microeconomic

optimization model of an individual wheat grower's segregating and blending decisions. Then we analyze the changes in wheat market equilibrium due to the adoption of the M&S technology. In order to do so, we utilize U.S. wheat prices and stocks to estimate a wheat demand system. This allows us to establish the effects of changes in the protein profile of wheat stocks on protein premiums. Then the simulation section combines the results from the microeconomic optimization model and from the econometric estimations to simulate wheat growers' WTP for the technology. For the M&S technology market, a standard n -player Cournot competition model is applied to determine the equilibrium price and quantity of the technology. The welfare impacts on the wheat industry are analyzed under various technology market structures.

In this article we find that when the new technology's market structure is more competitive (i.e., more producers in the market) then the adopters' average WTP for the technology becomes lower. For example, when the M&S technology's market structure is monopoly (perfect competition) and when the cost of measuring and segregating is 0.4 cents/bushel, then adopters' average WTP for the technology is about 22.2 (14.4) cents per bushel. Aggregate wheat market value is slightly decreased by the adoption of this new technology, no matter what the market structure is. This decrease is larger when the technology market is more competitive. However, the technology's impact on wheat market value varies over different wheat markets. Upon the adoption of this new technology, market value of Hard Red Winter wheat will decrease by 0.2% to 2.3%, but market value of Hard Red Spring wheat will increase by 0.3% to 3.3%. For the technology's market prospect, when the market structure is monopoly, then the aggregate operating profit related with the U.S. HRW and HRS wheat is about \$5.87 to \$7.67 million per year, depending on how high the production cost is. When there are more producers in the technology market, the aggregate operating profit becomes lower.

The contributions of this article lie in three aspects. First, to the authors' best knowledge, this article is the first to attempt to quantify wheat growers' WTP for M&S technology and to analyze the technology's welfare impact on wheat growers. Second, it shows that the advent of an information technology may not improve the welfare of adopters. This finding is consistent with that in Lave (1963) and Babcock (1990), who showed that farmers' welfare would be decreased by better weather information. Third, we develop a general microeconomic optimization models of an individual wheat grower's segregating and blending decisions under various price schedules and under any protein distributions.

Previous studies about optimal segregating and blending decisions either utilized linear programming to approximate (e.g., Vercammen 2011; Giannakas, Gray, and Lavoie 1999) or misspecified the optimal decision model (e.g., Sivaraman, Lyford, and Brorsen 2002). The method developed in this article can be readily transferred to other commodities such as barley, soybeans, or rice.

This article enriches three strands of literature in agricultural economics. First, this article complements the literature about the economic value of information in the line of Lave (1963) and Babcock (1990) arguing that more information may hurt farmers' welfare. In this article we find that the adoption of the M&S technology will decrease the market values of wheat at the industry-level even though some growers may benefit from the adoption. Intuitively, this is because non-cooperative activities among farmers generate a price externality that lowers the value of wheat at the market-level.

The second strand lies in quality control (through blending or segregation) in grain markets. Quality control in grain market has been a focus of economic research for a long period. Vercammen (2011) provides an overview of the economics of blending. Pirrong (1995) asserts that gains to elevators from blending were so large as to impede a private market system of grain grading in 1860s. Hennessy (1996) and Hennessy and Wahl (1997) discuss the optimal decisions on blending and cleaning commodities that can be separated completely into high quality and low quality constituents. Giannakas, Gray, and Lavoie (1999) study the impact of increasing the number of protein increments in a wheat grading system on blending revenue in the Canadian wheat industry. Adam, Kenkel, and Anderson (1994) analyze the benefits and costs of cleaning wheat utilizing an engineering/programming model. Yoon, Brorsen, and Lyford (2002) compare the cost and benefit of increasing kernel uniformity for millers and find that the benefit cannot justify the cost. Johnson and Wilson (1993) develop a non-linear programming model on an elevator's optimal cleaning and blending decisions. However, all of these articles focus on elevator-level or industry-level grain processing decisions and do not consider the market equilibrium effect of such decisions. Our article extends the quality control literature by focusing on the farm-level grain processing decision and considering the market equilibrium effect of this decision. Moreover, our exercise contributes to the technology adoption literature by providing an example of quantifying the WTP for a new technology and of analyzing the impact of a technology's market structure on the adoption of the technology.

The rest of this article is structured as follows. In Section 2 we develop a conceptual model of a

typical wheat grower's optimal segregating and commingling decisions facing various protein premium schedules. Section 3 contains a conceptual market equilibrium model of technology adoption. While Section 4 estimates a wheat protein demand system, Section 5 studies field-level protein variations. Section 6 combines the wheat demand system and the conceptual models to simulate the WTP for the sorting technology and then discusses the results. Section 7 concludes.

The Value of the M&S Technology to a Single Wheat Grower

In this section we develop a microeconomic optimization model of a wheat grower's segregating and blending decisions according to wheat protein concentrations. The wheat grower's goal is to maximize profit from selling her wheat by optimally segregating and blending given a protein premium schedule and a distribution of wheat grain's protein concentrations. We assume that the wheat grower produces one unit of wheat per year. Her decision problem can be divided into two stages: (1) deciding whether or not to adopt the M&S technology; and (2) if she adopts the technology, then determining what the profit maximizing segregation strategy is. By backward induction, we solve the stage two decision problem first. For simplicity, we assume that once the technology is adopted, the marginal cost of measuring and segregating is zero.⁴ Therefore, the profit maximizing goal is equal to maximizing the revenue from selling wheat.

Processing wheat with various protein levels is different from processing wheat with various dockage rates. Since it is part of a wheat kernel, protein within one load of wheat does not have the perfect separability that dockage has. For example, 1,000 bushels of grain with 10% average protein level cannot be segregated into 900 bushels of zero percent protein wheat and 100 bushels of 100% protein wheat. This means that the segregating results will be constrained by the distribution of protein concentration.

Terms

Before the M&S technology is adopted, the wheat grower sells her wheat as one load at a single price. After adopting the technology, then she has the freedom to segregate her wheat according to protein concentration to optimize her revenue. To facilitate exposition, a series of definitions about

⁴Assuming marginal processing cost to be a constant is common in the grain quality control literature (e.g., Hennessy and Wahl 1997; and Giannakas, Gray, and Lavoie 1999).

the wheat grower's activities (i.e., measuring, blending, segregating, and processing) are necessary. *Measuring* refers to obtaining the protein concentration information by applying the M&S technology. *Blending* is the act of mixing loads so that any sample of the mix has the same protein concentration. *Segregating* one load is the act of separating the load into two or more sub-loads with different protein concentrations. *Completely segregating* one load refers to separating the load into as many as possible sub-loads such that each sub-load only contains wheat with the same protein concentration where the protein concentration of each sub-load differs. To *Process* one load of wheat is to measure the load of wheat and then either blend or segregate.

Model Analysis

Suppose a wheat grower has one load of wheat with mean protein level μ . The mass of this load is normalized to one. The protein concentration distribution in this load is $F(l)$ with density function $f(l)$ and support $[0, \bar{L}]$. Here $\bar{L} \leq 1$ is the upper bound of grain protein concentration in the load. We assume there are no atoms on the protein concentration distribution (i.e., no points of discontinuity on $F(l)$). For simplicity we assume the grower knows the protein distribution before she adopts the M&S technology. If she does not know the protein distribution until she utilizes the technology, then the estimation of willingness to pay for the technology would require assumptions on the farmer's belief about protein distributions of her harvest. In the situation that farmers only have a belief about the protein concentration distribution, our analysis in optimal processing decisions is still necessary. This is because for any given protein concentration distribution under a belief our analysis can be used to obtain the optimal processing decisions.

In light of their large number, in our model the grower is assumed to be a price taker. Let the non-decreasing wheat price function facing the farmer be $p(l)$, where l is the level of protein concentration of one unit of wheat. The protein premium is imbedded in the price schedule because high protein wheat receives high prices. In wheat markets the price schedules are often of the step function form. Therefore, we study a grower's optimal processing decisions when price schedules are in three-step form. For optimal processing decisions under uniformly curved schedules and nonuniformly curved schedules, please see Items A and B in Appendix 1, respectively. When the price schedule is an N -step schedule ($N > 3$), unfortunately, we cannot obtain an elegant uniformly concave or convex effective

price schedule by eliminating dominated discontinuous points on the step price schedule like Hennessy and Wahl (1997) did. This is because wheat does not have perfect separability in the protein dimension.⁵

Suppose the three-step price schedule is

$$p(l) = \begin{cases} p_1 & \text{if } \underline{L} \leq l < l_1 \\ p_2 & \text{if } l_1 \leq l < l_2 \\ p_3 & \text{if } l_2 \leq l \leq \bar{L}, \end{cases} \quad (3.1)$$

where $l \in [\underline{L}, \bar{L}]$ is wheat protein concentration; $p_3 > p_2 > p_1 > 0$ are prices; and l_1 and l_2 are constants such that $\underline{L} \leq l_1 < l_2 \leq \bar{L}$. Figure 3.1 depicts this three-step price schedule. In this subsection we further assume that the mean protein concentration of one load, μ , is such that $\mu \in (0, l_2)$. If $\mu \geq l_2$, then it will receive the highest price and hence its owner has no incentive to process further.

The grower's problem is to maximize her revenue by optimally processing her wheat through segregating and blending activities based on knowledge of the protein concentration distribution. Because the price schedule has a three-step function form, the farmer's problem is to optimally segregate her wheat into three sub-loads, namely S_1 , S_2 , and S_3 , to maximize revenue. Let μ_i , p_i , and q_i be the mean protein concentration, the price received, and the quantity of sub-loads S_i , $i \in \{1, 2, 3\}$, respectively. By construction we have $\mu_1 \in [0, l_1)$, $\mu_2 \in [l_1, l_2)$, $\mu_3 \in [l_2, \bar{L}]$, and $\sum_{i=1}^3 q_i \mu_i = \mu$ (please recall that the total quantity is normalized to 1). Then the farmer's problem is to maximize $\sum_{i=1}^3 p_i q_i$ subject to certain constraints. These constraints need to reflect the fact that wheat protein is imperfectly separable, as discussed above. To find these constraints is the major task of specifying an appropriate form for the grower's programming problem. Many other studies (e.g., Adam, Kenkel and Anderson 1994; Johnson and Wilson 1993; and Vercammen 2011) use linear programming to characterize the optimization problem when assuming a discrete protein distribution or to approximate the optimal solution when assuming a continuous protein distribution. However, the linear programming approach often involves solving many unknowns, which provides little insight on how the optimization mechanism works. Below we provide a simple non-linear programming framework which directly generates the optimal allocation under three-step price schedules for any continuous protein distribution without atoms. For the next result, several definitions are necessary.

⁵However, we do solve out the optimal processing strategies under a four-step price schedule. The results are available upon request from the authors.

Definition 3.1. We define t_1 and t_2 as the least non-negative constants such that $\int_{t_1}^{\bar{L}} f(l)dl/(1 - F(t_1)) \geq l_1$ and $\int_{t_2}^{\bar{L}} f(l)dl/(1 - F(t_2)) = l_2$, respectively.

Definition 3.1 states that the average protein level of the mix of all wheat with protein level higher than t_1 is greater than or equal to l_1 ; and the average protein level of the mix of all wheat with protein level higher than t_2 is equal to l_2 . Clearly we have $t_2 < l_2$ and $t_1 < l_1$. It is also readily checked that the maximum amount of wheat with average protein level at l_2 (or no lower than l_1) that can be segregated from the original load of wheat is $1 - F(t_2)$ (or $1 - F(t_1)$). That is, the maximum values of q_2 and q_3 are $1 - F(t_1)$ and $1 - F(t_2)$, respectively. Furthermore, when $t_2 > l_1$, we have a definition as follows.

Definition 3.2. When $t_2 > l_1$, then \hat{l}_1 is defined as the minimum non-negative constant that satisfies $\int_{\hat{l}_1}^{t_2} f(l)dl/(F(t_2) - F(\hat{l}_1)) \geq l_1$.

Definition 3.2 states that \hat{l}_1 is the minimum non-negative constant such that the average protein level of wheat distributed on $[\hat{l}_1, t_2]$ is no less than l_1 . Here \hat{l}_1 identifies the maximum value of q_2 given that q_3 is maximized.

Let $M \equiv \{q_i: q_i \geq 0, \text{ and } \sum_{i=1}^3 q_i = 1\}$, where $i \in \{1, 2, 3\}$. We present the following two non-linear programming problems as follows.

$$\begin{aligned} & \max_{q_i \in M} \sum_{i=1}^3 p_i q_i \\ \text{s.t.} \quad & \int_0^{F^{-1}(q_1)} f(l)dl + q_2 l_1 + q_3 l_2 = \mu, \quad F(t_1) \leq q_1 \leq F(t_2). \end{aligned} \quad (3.2)$$

$$\begin{aligned} & \max_{q_i \in M} \sum_{i=1}^3 p_i q_i \\ \text{s.t.} \quad & \int_0^{F^{-1}(q_1)} f(l)dl + q_2 l_1 + q_3 l_2 = \mu, \quad F(t_1) \leq q_1 \leq F(\hat{l}_1). \end{aligned} \quad (3.3)$$

For the optimal processing outcomes under three-step price schedules, we have the following proposition.

Proposition 3.1. Suppose one wheat load's mean protein concentration, μ , is such that $\mu \in (0, l_2)$. Then for this load, (i) when $t_2 \leq l_1$ then the optimal processing outcomes are the solutions of problem (3.2); (ii) when both $t_2 > l_1$ and $\hat{l}_1 > 0$ then the optimal processing outcomes are the solutions of problem (3.3); and (iii) when both $t_2 > l_1$ and $\hat{l}_1 = 0$ then $q_1^* = 0$, $q_2^* = F(t_2)$, and $q_3^* = 1 - F(t_2)$.

Proof. Please see Item C in Appendix 1. □

Visual presentations of the three items in Proposition 3.1 are depicted in Figures 3.2 to 3.4, respectively. The first constraints in problems (3.2) and (3.3) are the same. This constraint states that the segregation should not change the load's total protein quantity. It also states that sub-load S_1 consists of wheat distributed on a continuous section of the left tail of protein distribution. Without this constraint the programming becomes a linear programming and the mean protein level in sub-load S_1 will always be driven down to zero. This is because if the protein concentration of sub-load S_1 is zero then all the protein will be directed to sub-loads S_2 and S_3 so that the quantities of S_2 and S_3 , and hence the revenue, can increase.

The second constraint in each of problems (3.2) and (3.3) concerns the range of q_1 . It is intuitive that $q_1 \geq F(t_1)$ because if not then the average protein level of the blend from sub-loads S_2 and S_3 will be less than l_1 , which cannot be true. When $t_2 \leq l_1$, then it is not optimal to put wheat with protein level higher than t_2 into sub-load S_1 because they can always be part of sub-loads S_2 or S_3 to receive a higher price (Figure 3.2). When $t_2 > l_1$ and when $\hat{l}_1 > 0$, then it is not optimal to put wheat with protein level higher than \hat{l}_1 into sub-load S_1 (Figure 3.3). The reason is that wheat with protein level higher than \hat{l}_1 can be put into sub-load S_2 to receive a protein premium. Moreover, the constraints of problems (3.2) and (3.3) guarantee that the optimal solutions are achievable under wheat's non-perfect separability in the protein dimension. To better illustrate how the constraints in problems (3.2) and (3.3) work let us see an example.

Example 1. Suppose the grain protein concentrations of one HRW wheat load are uniformly distributed on $[0.06, 0.15]$, i.e., $l \sim U(0.06, 0.15)$. Then the average protein level of this wheat load is 0.105. The price schedule for HRW wheat market is assumed to be

$$p(l) = \begin{cases} 3 & \text{if } 0.06 \leq l < 0.12 \\ 4 & \text{if } 0.12 \leq l < 0.13 \\ 7 & \text{if } 0.13 \leq l \leq 0.15, \end{cases} \quad (3.4)$$

where the price unit is \$/bushel. Then in this example we have $l_1 = 0.12$ and $l_2 = 0.13$. By Definition 3.1 we can calculate that $t_1 = 0.09$ and $t_2 = 0.11$. This means that the average protein level of wheat grain with protein level on interval $[0.09, 0.15]$ (or $[0.11, 0.15]$) is 0.12 (or 0.13). Since $t_2 < l_1$,

there is no value of \hat{l}_1 . Therefore, according to Proposition 3.1 we know that problem (3.2) should be applied to this example. Let $F(l)$ denote the distribution function of the protein concentrations. Based on the assumption of protein concentration distribution, we have $F(t_1) = 1/3$, $F(t_2) = 5/9$, and $\int_{0.06}^{F^{-1}(q_1)} f(l)dl = 0.06q_1 + 0.045q_1^2$. Then the optimization problem can be written as

$$\begin{aligned} & \max_{q_i \in M} \sum_{i=1}^3 p_i q_i \\ \text{s.t.} \quad & 0.06q_1 + 0.045q_1^2 + q_2l_1 + q_3l_2 = 0.105, \quad 1/3 \leq q_1 \leq 5/9. \end{aligned} \quad (3.5)$$

Solving problem (3.5) gives us the optimal processing results $(q_1^*, q_2^*, q_3^*) = (5/9, 0, 4/9)$ and maximized revenue at $R^* = 3q_1^* + 4q_2^* + 7q_3^* = 43/9$.

Now let us study the implications of deleting the two constraints in problem (3.5). Since the mean protein level in this load is 0.105, which is lower than $l_1 = 0.12$, the threshold protein value of receiving protein premium, we must have $q_1 > 0$ in the optimal processing results. If not, i.e., if $q_1 = 0$, then the average of protein level of wheat in sub-loads S_2 and S_3 will be equal to 0.105, which contradicts the requirement that mean protein level of wheat in sub-load S_2 (or S_3) should be no lower than $l_1 = 0.12$ (or $l_2 = 0.13$). If $q_1 > 0$, then in the optimal processing results we must have the average protein levels of sub-loads S_2 and S_3 , i.e., μ_1 and μ_2 , such that $\mu_2 = 0.12$ and $\mu_3 = 0.13$. Otherwise the revenue can be increased by moving some wheat from sub-load S_1 to sub-loads S_2 or S_3 . Therefore, if we delete the two constraints in problem (3.5), then the wheat grower's optimization problem can be written as

$$\begin{aligned} & \max_{q_i \in M} \sum_{i=1}^3 p_i q_i \\ \text{s.t.} \quad & q_1\mu_1 + q_2l_1 + q_3l_2 = 0.105, \quad \mu_1 \geq 0.06, \end{aligned} \quad (3.6)$$

where μ_1 is the average protein level of sub-load S_1 . Solving problem (3.6) we have $(\hat{q}_1, \hat{q}_2, \hat{q}_3) = (5/14, 0, 9/14)$ and $\mu_1 = 0.06$. The corresponding revenue is $\hat{R} = 3\hat{q}_1 + 4\hat{q}_2 + 7\hat{q}_3 = 78/14$. It is clear that $\hat{R} > R^*$. But given the protein distribution in this example, the solution $(\hat{q}_1, \hat{q}_2, \hat{q}_3) = (5/14, 0, 9/14)$ cannot be achieved. This is because from this wheat load one cannot obtain a sub-load of wheat with quantity at 5/14 and average protein level at 0.06. Therefore, we can see that the first constraint in problem (3.5) is necessary to find out the real optimal solution for the grower's segregation problem. \square

After we obtain the optimal processing results, (q_1^*, q_2^*, q_3^*) , then a grower's WTP for the M&S technology can be computed as $\text{WTP} = \sum_{i=1}^3 p_i q_i^* - p(\mu)$, where $p(\mu)$ is the price received without

segregation. The grower will adopt the technology if and only if her WTP is greater than the cost of the M&S technology. Now let us move on to study the value of the M&S technology to the wheat industry.

Value of M&S Technology to Wheat Industry

In the previous section we assumed that an individual grower is a price taker and her adoption decision does not affect wheat prices. However, when all growers face the same decision problem then the aggregate effect of these decisions may change prices. In this section we first define an adoption equilibrium under which prices and quantities of wheat with different protein levels and growers' adoption decisions are determined. Then we provide a measure of the M&S technology's value to wheat industry.

Assume there are N wheat growers. Each grower produces one unit of wheat with protein distribution $d \in D$, where d is density function and D is the set of density functions of all wheat growers. Let $Q(l)$, $l \in [\underline{L}, \bar{L}]$, denote the wheat quantity distribution in the market before the M&S technology is available. Let $\mathbf{P}(\cdot)$ be a function system describing the inverse demand for wheat with different protein levels. Then the wheat price schedule before the technology is available is $p(l) = \mathbf{P}[Q(l)]$. We assume the technology is adopted gradually, starting with the grower who has the highest WTP for the technology under prices $p(l)$. Wheat prices are affected when growers adopt the technology, in a way to be discussed in detail in Section 6.2. Let $p^n(l)$ denote wheat price schedule after the n^{th} grower adopts the technology. The adoption process stops whenever the WTP of growers who have not adopted the technology under $p^n(l)$ is less than the price of the technology, w . Hence, for a given technology price, w , we can identify the quantity of wheat segregated and a wheat price schedule at which the adoption process stops. We say that the adoption process reaches the equilibrium when it stops. We then define wheat price and quantity schedule in the adoption equilibrium as $p^*(l)$ and $Q^*(l)$, respectively.

Once we have price and quantity schedules in the adoption equilibrium, the value of the M&S technology to the wheat industry can be readily written as $\int_{\underline{L}}^{\bar{L}} [p^*(l)Q^*(l) - p(l)Q(l)]dl$. Please note that $p^*(l)$, and hence the number of growers who adopt the technology, are determined by the price of the technology. Therefore, a demand curve for the sorting technology can be obtained. We let $D(\cdot)$ denote the inverse demand function for the M&S technology. Because an individual wheat grower's

adoption decision is also determined by the price of the new technology, w , we should expect that the price of the new technology would affect the value of the M&S technology to the wheat market. The M&S technology price is determined by the technology's market structure and production cost. We assume that there are I ($I \geq 1$) identical firms producing the technology with constant marginal cost c .⁶ Denote z_i as firm i 's production level and $Z \equiv \sum_{i=1}^I z_i$ as aggregate production. Then standard analysis in the Cournot competition model with identical firms will generate the optimal aggregate production level, Z^* , that is implicitly determined by

$$D(Z^*) + \frac{Z^*}{I} D'(Z^*) = c. \quad (3.7)$$

The effect of the M&S technology's market structure can be analyzed by varying the number of firms, I .

So far we have discussed the value of the M&S technology to an individual grower and to the wheat industry. In order to quantify this value, we need to estimate a wheat demand system, which is the content of the next section.

Wheat Demand System

In this article we focus on HRW wheat and HRS wheat where each class of wheat is broken down to three types based on protein concentrations. Let vectors

$$\mathbf{p}^0 = (p_1^0, p_2^0, p_3^0, p_4^0, p_5^0, p_6^0) \text{ and } \mathbf{Q}^0 = (Q_1^0, Q_2^0, Q_3^0, Q_4^0, Q_5^0, Q_6^0)$$

contain prices and quantities of wheat of the six protein levels in HRW and HRS wheat market, respectively. Here superscript 0 means prices and quantities before the M&S technology is available. Subscripts 1, 2, and 3 mean HRW wheat with protein level lower than 12%, no lower than 12% but lower than 13%, and no lower than 13%, respectively. We name these types of wheat as HRW low, middle, and high protein wheat, respectively. Subscripts 4, 5, and 6 mean HRS wheat with protein level lower than 13%, no lower than 13% but lower than 14%, and no lower than 15%, respectively. We name these wheat as HRS low, middle, and high protein wheat, respectively.

⁶For simplicity and given that only a few major firms produce agricultural machinery in the U.S., here we assume that there is no free entry in the M&S technology industry.

We apply the Inverse Almost Ideal Demand System (IAIDS) model to estimate the wheat demand system. Compared with the Almost Ideal Demand System (AIDS) model under which prices are assumed to be predetermined, the IAIDS model is more appropriate when analyzing demand for commodities whose prices adjust to clear markets (Eales and Unnevehr 1994; Eales, Durham, and Wessells 1997; Grant, Dayton, and Foster 2010). Wheat falls into this category because wheat protein concentration is mainly determined by varieties and growing conditions (irrigation and precipitation). Wilson (1983) shows that annual average protein concentration has not displayed any long term trend between 1961 and 1980. Data from the U.S. Wheat Associates between 1979 and 2009 also support this conclusion (Figure 3.5). Moreover, Goodwin and Smith (2009) showed that shocks in protein availability significantly affect wheat prices.

A system of inverse demand functions of m types of wheat ($m = 6$ in this article) with different protein levels can be specified as

$$r_{i,t} = \alpha_i + \sum_{j=1}^m \gamma_{ij} \ln(Q_{j,t}) + \beta_i \ln(I_t) + e_{i,t}, \quad (3.8)$$

where $r_{i,t}$ is type i wheat's revenue share at time t ; $Q_{j,t}$ is type j wheat's quantity at time t , ($i, j \in \{1, \dots, m\}$ and $t \in \{1, \dots, T\}$, where T is the length of the time series in the data). Here α_i , β_i , and γ_{ij} are parameters; $e_{i,t}$ is an error term; and $\ln(I_t)$ is a quantity index defined as

$$\ln(I_t) = \alpha_0 + \sum_{j=1}^m \alpha_j \ln(Q_{j,t}) + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \gamma_{ij} \ln(Q_{i,t}) \ln(Q_{j,t}). \quad (3.9)$$

The homogeneity constraints, symmetry constraints, and adding-up across the share equations require $\sum_{i=1}^m \alpha_i = 1$, $\sum_{i=1}^m \gamma_{ij} = 0$, $\sum_{i=1}^m \beta_i = 0$, and $\gamma_{ij} = \gamma_{ji}$.

Let ε_{ij} denote the percentage change of good i 's price caused by one percent change in the quantity of good j , i.e., the cross-quantity elasticities. Once the IAIDS model is estimated, then according to Eales and Unnevehr (1994) ε_{ij} can be calculated by

$$\varepsilon_{ij} = -\delta_{ij} + \frac{\gamma_{ij} + \beta_i(r_j - \beta_j \ln I_t)}{r_i}, \quad (3.10)$$

where δ_{ij} is the Kronecker delta that equals one whenever $i = j$. When applying the IAIDS model, it is common to replace the quantity index $\ln(I_t)$ with Stone's quantity index defined as $\ln(I_t^*) = \sum_{j=1}^m r_{j,t} \ln(Q_{j,t})$ (Moschini and Vissa 1994; Grant, Dayton, and Foster 2010), which gives us the linear-

approximate IAIDS model (i.e., LA/IAIDS model)

$$r_{i,t} = \alpha_i + \sum_{j=1}^m \gamma_j \ln(Q_{j,t}) + \beta_i \ln(I_t^*) + e_{i,t}, \quad (3.11)$$

For the LA/IAIDS model, according to Moschini and Vissa (1994) the quantity elasticity is

$$\varepsilon_{ij} = -\delta_{ij} + \frac{\gamma_j + \beta_i r_j}{r_i}. \quad (3.12)$$

Since the data utilized in our study are time-series data, ignoring auto-correlation in the error term may render the standard errors and test statistics of estimates invalid. We allow the error term to be first-order auto-correlated:

$$e_{i,t} = \rho e_{i,t-1} + \xi_t, \quad (3.13)$$

where ξ_t is assumed to be independent across time t . With the auto-correlation structure in equation (3.13), the variables in model (3.11) can be transformed into:

$$\tilde{x}_{i,t} = x_{i,t} - \rho x_{i,t-1}, \quad (3.14)$$

where $x_{i,t}$ denotes variables in model (3.11), including the dependent variable, independent variables, and constants. For example, $\widetilde{\ln(Q_{j,t})} = \ln(Q_{j,t}) - \rho \ln(Q_{j,t-1})$. Therefore, the transformed LA/IAIDS model can be written as

$$\tilde{r}_{i,t} = (1 - \rho)\alpha_i + \sum_{j=1}^m \gamma_j \widetilde{\ln(Q_{j,t})} + \beta_i \widetilde{\ln(I_t^*)} + \xi_t, \quad (3.15)$$

Instead of estimating model (3.11), we estimate model (3.15) and then apply equation (3.12) to obtain the flexibilities.

Data

We focus on Hard Red Winter (HRW) wheat and Hard Red Spring (HRS) wheat because they account for more than 60% of wheat production in the United States and their prices based on protein concentrations are well documented (U.S. Wheat Associates). The daily cash price data based on protein concentration of HRW wheat and HRS wheat in the Pacific Northwest (PNW) region are obtained from Montana Wheat and Barley Committee (<http://wbc.agr.mt.gov/>). These prices are averages of daily cash prices reported from elevators in the PNW region between 1980 and 2010. According to the

Law of One Price, we believe that these cash prices are good approximations for transportation cost adjusted wheat cash prices in the U.S. The prices in the data set follow a step-function schedule. To facilitate exposition, we define HRW wheat with protein levels in ranges [11%,12%), [12%,13%), and [13%,100%) as HRW low, middle, and high protein wheat, respectively. For HRS wheat, we define HRS low, middle, and high protein wheat as HRS wheat with protein levels in ranges [13%,14%), [14%,15%), and [15%,100%), respectively. We then define p_1 , p_2 , and p_3 as prices of HRW low, middle, and high protein wheat, respectively; and define p_4 , p_5 , and p_6 as prices of HRS low, middle, and high protein wheat, respectively.

Our analysis utilizes monthly averages of these daily price data. Since the time range is from 1980 to 2010, we expect to have 1,116 (i.e., 31 years times 12 months times 3 prices per month) price observations for HRW wheat and another 1,116 price observations for HRS wheat. For HRW wheat, there are 16 missing values out of these 1,116 price observations. The missing value ratio is about 0.01. For HRS wheat prices, there are 24 missing values and the missing value ratio is about 0.02. Since the missing value ratios are very low, for simplicity we apply cubic spline interpolation to fill the missing values (Baltazar and Claridge, 2006). Table 3.1 shows summary statistics for the six price variables.

Protein premia are of interest because they are the driving force of segregating wheat according to protein concentrations. We define $p_{21} \equiv p_2 - p_1$, which stands for protein premium of HRW middle protein wheat over HRW low protein wheat. Similarly, we define $p_{31} \equiv p_3 - p_1$, $p_{54} \equiv p_5 - p_4$, and $p_{64} \equiv p_6 - p_4$. Summary statistics for these protein premia are presented in Table 3.1 as well. The means of p_{21} and p_{31} are 12.9 and 30.6 cents/bushel, respectively. This means that on average HRW middle protein wheat (or HRW high protein wheat) receives 12.9 (or 30.6) cents more per bushel than HRW low protein wheat. For HRS wheat, protein premia are even higher. The means of p_{54} and p_{64} are 30.6 and 51.1 cents per bushel, respectively. The variances of protein premia are large as well, which means that protein premia are very volatile. From Figures 3.6 and 3.7 we can see this.⁷ Since prices of HRW high protein wheat are always higher than prices of HRW middle protein wheat, in Figure 3.6 the line for p_{31} is always above that for p_{21} . The distance between the two lines measures the premium for HRW high protein wheat over HRW middle protein wheat. The same interpretation follows for Figure

⁷In the figures monthly wheat prices start in August 1980 because the monthly wheat stock data starts in August 1980.

3.7. Figures 3.6 and 3.7 have similar patterns in terms of the changes of protein premia over time.

Quarterly stocks of all wheat between 1980 and 2010 are obtained from National Agricultural Statistics Service (NASS) of the U.S. Department of Agriculture (USDA). Here “all wheat” means wheat in all six wheat classes, not just HRW and HRS wheat.⁸ Monthly stocks of HRW wheat and HRS wheat with different protein levels are calculated by the authors using a procedure that is similar to the one in Goodwin and Smith (2009). The procedure is presented in Appendix 1. Let s_1 , s_2 , and s_3 denote the monthly stocks of HRW low, middle, and high protein wheat, respectively. Let s_4 , s_5 , and s_6 denote the monthly stocks of HRS low, middle, and high protein wheat, respectively. Once we obtain the monthly cash prices and stocks of the six types of HRW and HRS wheat, then the revenue share of each type of wheat is readily to calculate by $r_i = p_i s_i / \sum_{j=1}^6 p_j s_j$, $i \in \{1, \dots, 6\}$. Figures 3.8 and 3.9 depict the monthly HRW wheat and HRS wheat stocks at different protein levels. Summary statistics for the monthly stocks are presented in Table 3.1 as well.

By studying Figures 3.6 and 3.8 we can find that protein premia decrease when stocks of high protein wheat increase. For example, from August 1989 to August 1991, the protein premia were close to zero (Figure 3.6). From Figure 3.8 we can see that in this period s_3 was either higher than s_2 and s_1 or close to them. However, during the period of August 2007 to August 2010 when s_3 is significantly lower than s_1 and s_2 , we observe premium spikes. For HRS wheat, the same pattern follows.

Estimation Results and Elasticities

The demand system in equation (3.15) is estimated by using seemingly unrelated regressions. Table 3.2 reports the parameter estimates for the transformed LA/IAIDS model presented in equation (3.15). The coefficients of transformed logarithmic quantities are all statistically significant at 1% level. However, the coefficients of transformed Stone’s quantity index are not significant in the equations of HRW low and middle protein wheat. Table 3.2 also contains the measure of fit for the equations and Durbin-Watson (DW) statistics. Each equation in all the three models explains more than 80% of the variation in revenue shares. The DW statistics are close to 2, which indicate the autocorrelation problem is largely eliminated.

⁸According to the U.S. Wheat Associates, the six classes of wheat are: Hard Red Winter, Hard Red Spring, Hard White, Soft White, Soft Red Winter, and Durum.

Table 3.3 presents the quantity elasticities (or flexibilities) calculated from the estimates by using equation (3.12). For example, the number -0.1748 (row “HRW Low” and column “HRW Middle” in Table 3.3) means that when the stocks of HRW middle protein wheat increases by 1% then the price of HRW low protein wheat will decrease by 0.1748%. From Table 3.3 we can see that all the own-quantity and cross-quantity elasticities are negative. This is intuitive because different types of wheat are likely to substitute for each other. We also find that all the own-quantity elasticities are much lower than one in absolute value, which means that the demand for wheat is very inflexible (or elastic). Moreover, almost all the cross-quantity elasticities are smaller than own-quantity elasticities, and in-class cross-quantity elasticities are much higher than between-class cross-quantity elasticities. This is reasonable because the quantity change of one type of wheat will mainly affect its own price. Since consumers (i.e., bakers or millers) are more likely to look for substitutes within the same wheat class rather than in other wheat classes, we expect that the quantity change of one type of HRW wheat has higher impacts on HRW wheat prices than on HRS wheat prices.

We apply the bootstrap technique to approximate the standard errors of the flexibilities. We first transform the variables by using equation (3.14) to obtain a transformed data set. Then we draw from the transformed data set with replacement to form 500 samples of size that is equal to the size of the transformed data during bootstrapping. Standard errors of the flexibilities, which are reported in Table 3.3 as well, are calculated from estimates based on these 500 samples.

Field-Level Protein Concentration Distribution

In order to estimate how much wheat with different protein levels can be sorted out from a given load of wheat, we need to know the protein concentration distribution of this load of wheat. In this section we estimate field-level protein concentration distributions. The Washington State University Extension Cereal Variety Testing Program (<http://variety.wsu.edu/>) provides wheat variety test data that can be traced back to 1997. The data report different varieties’ yield, test weight and protein concentrations. Here we discuss the data and method to identify field-level protein concentration by using HRW wheat as an example. For HRS wheat the sampling method and data structure are similar.

From 1997 to 2009, the Cereal Variety Testing Program tested 194 HRW varieties in 15 locations

across the State of Washington. Even though many varieties were tested, only a few were widely planted by wheat growers in the state. According to data from USDA NASS, in each crop year the top ten varieties usually accounted for more than 90 percent of planted area for the same class of wheat in the State of Washington.⁹ Therefore, in our analysis we only focused on the top 10 varieties in a crop year. According to this standard, 16 HRW wheat varieties were chosen from 1997 to 2009. The names of varieties and locations are listed in Table 3.A.1 in Appendix 1

Top varieties varied from year to year. One variety may be popular in some years but disappeared from the list of top ten varieties in another year. In addition, not every variety was tested in every location every year. We admit a variety's performance observations into our data set whenever: a) it is in the list of top varieties in some years between 1997 and 2009; and b) for at least one location it was tested in some years between 1997 and 2009. Using this screen we collected 538 observations. These 538 observations include 16 varieties planted at 15 locations over the 13 years. For each observation, we know the variety's name, yield, test weight, protein level, trial location where it was tested, and year when it was tested. Table 3.A.2 presents a sample of variety performance observations.

Table 3.4 presents summary statistics for the 538 observations on HRW wheat. For protein concentration, the sample mean is 11.87%. Its maximum and minimum values are 16.6% and 7.4%, respectively. The sample standard deviation is 0.017. However, this sample standard deviation is not a satisfactory estimator of the variability of HRW wheat protein concentration. This is because it includes between-location variation, between-year variation and between-variety variation. But for wheat segregation, what the grower encounters is field-level variation for a given variety in a given year. If we accept the sample standard deviation as the estimator of protein variability, then we will over-estimate protein variability. To obtain a better estimate, we need to control for the effect of varieties, locations and years.

We applied regression analysis to estimate the protein variability given a location, variety, and year. The idea is that once we control for location, variety, and year, then the variation that cannot be explained by the control variables is an approximation of protein variation conditional on location,

⁹Data source: http://www.nass.usda.gov/Statistics_by_State/Washington/Historic_Data/smallgrains/whtvar.pdf (accessed on September 9, 2011).

variety, and year.¹⁰ The regression can be written as

$$y = c + \sum_{i=2}^{13} \alpha_i yr_i + \sum_{j=2}^{16} \beta_j var_j + \sum_{k=2}^{15} \gamma_k loc_k + u, \quad (3.16)$$

where y denotes protein concentration; yr_i , var_j , and loc_k stand for year i , variety j , and location k ; α_i , β_j , and γ_k are parameters to be estimated; and u is the error term. An unbiased estimator for the variance of u is $s^2 = e'e/(n - K - 1)$ where e is the least square residuals when estimating equation (3.16), n is the number of observations, and K is the number of independent variables.

The results of regression (3.16) are listed in Table 3.A.3 of Appendix 1. The estimated standard deviation (i.e., s) is 0.01. Figure 3.10 shows a histogram of the residuals, from which we can see that the residuals have a normal-formed distribution. Therefore, in our simulation we assume that the field-level protein concentration follows a normal distribution. Regarding HRS wheat, the data set includes variety testing results of 13 years, 15 varieties, and 28 locations in the State of Washington. The estimates are presented in Table 3.A.4 in Appendix 1. The estimated standard deviation for HRS wheat is 0.012.

Value of the M&S Technology: Simulation

So far we have had all necessary elements for simulating wheat growers' WTP and a demand curve for the M&S technology. These elements are 1) the optimal segregating algorithm for the individual grower; 2) an equilibrium technology adoption at market-level; 3) a wheat demand system; and 4) field-level protein distributions. In this section we first summarize our simulation procedures and then present the simulation results, including the value of the M&S technology to wheat growers, wheat industry, and potential producers of the M&S technology.

Optimal Segregating Algorithm for an Individual Wheat Grower

Since we focus on three-step price schedules, the optimal segregating strategies for an individual wheat grower are presented in Proposition 3.1. In our simulations Proposition 3.1 is implemented as follows. First, given a protein distribution, we calculate protein concentration thresholds t_1 , t_2 , and \hat{l}_1 that are defined in Definition 3.1 and Definition 3.2. Second, whenever $t_2 > l_1$ and $\hat{l}_1 = 0$, then we have

¹⁰Since trials in one location are very close to each other, we can see these trails as a reasonable sample from an individual field. For example, in Connell Washington from year 2005 to 2009, the shortest distance between two trials is only 200 feet; and the longest distance is 1.4 miles.

$q_1^* = 0$, $q_2^* = F(t_2)$, and $q_3^* = 1 - F(t_2)$. Third, whenever $t_2 \leq l_1$, then we numerically solve problem (3.2). Fourth, whenever $t_2 > l_1$ and $\hat{l}_1 > 0$, then we numerically solve problem (3.3).

The way to numerically solve problems (3.2) and (3.3) is as follows. Here we use problem (3.2) as an example to illustrate the procedure but the same approach applies when solving problem (3.3). The idea is a straightforward application of grid search. Step 1). Since the second constraint in problem (3.2) states that $q_1 \in [F(t_1), F(t_2)]$, we choose 1,000 values of q_1 that are evenly distributed on interval $[F(t_1), F(t_2)]$. Step 2). Given each q_1 , solve out q_2 and q_3 such that $q_1 + q_2 + q_3 = 1$ and $\int_0^{F^{-1}(q_1)} f(l)dl + q_2 l_1 + q_3 l_2 = \mu$. Step 3). For each (q_1, q_2, q_3) from Step 2) we calculate $\sum_{i=1}^3 p_i q_i$ and then choose (q_1, q_2, q_3) that maximizes $\sum_{i=1}^3 p_i q_i$.

Simulate the Demand Curve for the M&S Technology

In this sub-section we summarize the simulation procedure to obtain a demand curve for the M&S technology. That is, for a given price of the M&S technology, we simulate the equilibrium quantity demanded for the technology. Since we do not know the processing capacity per device of the M&S technology, we utilize the quantity of wheat that is produced by wheat growers who adopt the technology as the measure for the technology's quantity demanded.

Based on HRW (HRS) protein data obtained from Washington State University Extension Cereal Variety Testing Program that have been described in Section 5, we estimate an empirical distribution of mean protein levels of HRW (HRS) wheat by using kernel density estimation performed by Matlab function “ksdensity” setting bandwidth at 0.5. Then we draw M ($M = 2,000$ in our simulation) protein means from the estimated HRW protein mean distribution. For HRS wheat growers, we draw δM protein means from the estimated HRS protein mean distribution, where $\delta = 0.62$ is the ratio of HRS stock over HRW stock evaluated at the sample mean. The reason that we multiply M by δ for HRS wheat is to reflect the fact that the stock quantity of HRS wheat is smaller than that of HRW wheat. We let each draw stand for the average protein level of the wheat produced by a wheat grower. Therefore, in our simulation we have 2,000 HRW wheat growers and 1,240 HRS wheat growers. We further assume each of them produce the same amount of wheat. For HRW (HRS) wheat the standard deviation of protein concentration is 0.01 (0.012) according to the results we present in Section 5.

Since growers who produce HRW (HRS) high protein wheat do not have any incentive to segregate

their wheat, we only need to focus on wheat growers who produce middle or low protein wheat. The simulation procedure is as follows.

Step 1): we calculate each wheat grower's WTP under the initial wheat prices and sort all WTP values in descending order. Let W^{max} denote the maximum among these WTP values. Then the effective range of the technology's price is $[0, W^{max}]$. This is because whenever the technology's price is higher than W^{max} then no wheat grower will adopt the technology.

Step 2): for a technology price, $w \in [0, W^{max}]$, we let the grower with the highest WTP in Step 1) adopt and segregate. A quantity shock, ΔQ^1 , is generated by her adoption and segregation. Given ΔQ^1 , a new vector of wheat prices can be calculated by using $\mathbf{p}^1 = E \times \Delta Q^1 \times \mathbf{p}^0 + \mathbf{p}^0$, where E is the flexibility matrix estimated in Section 4 and \mathbf{p}^0 is the initial wheat prices. A new vector of wheat quantities can be calculated by using $\mathbf{Q}^1 = \mathbf{Q}^0 + \Delta Q^1$. We then update the flexibility matrix to E^1 under the new prices and quantities by using equation (3.12).

Step 3): under prices \mathbf{p}^1 , calculate the WTPs of growers who have not adopt the M&S technology. If none of these WTPs is no less than the technology price, w , then the technology adoption process stops and an adoption equilibrium is reached. If the maximum of these WTPs is greater than w , then we let the grower who has the maximum WTP adopt the technology. Therefore, as in Step 2), this adoption generate a quantity shock, ΔQ^2 , a new vector of prices, \mathbf{p}^2 , a new vector of quantities, \mathbf{Q}^2 , and a new flexibility matrix, E^2 .

Step 4): repeat Step 3) (replace \mathbf{p}^1 with the newest prices) until the technology adoption process stops under technology price, w .

In the simulation we use mean stocks and mean prices over the period 1980-2010 as the initial wheat stocks and prices (Table 3.1). Under the initial prices, the maximum WTP by the wheat growers is \$321 for segregating one thousand bushels of wheat. To simulate the demand curve for the M&S technology, we select 100 prices that are evenly distributed on the range of $[0, 321]$. Then for each of these technology prices we simulate equilibrium adoption rate based on the procedure we just have discussed in this sub-section. Therefore, the simulation procedure generates 100 simulated price-quantity data points of the technology demand curve. In order to utilize equation (3.7) to identify the optimal production level for each firm, we first utilize the locally weighted regression approach to obtain smoothed data points for the demand curve. Then we apply the shape-preserving piecewise cubic Hermite interpolating

method onto the smoothed data points to fit the demand curve. This interpolating method is performed by Matlab command “fit.” We refer readers to Moler (2008) for a detailed discussion on this interpolating method. Figure 3.11 depicts the simulated data points and the fitted demand curve. Based on the fitted demand curve, we analyze the value of the M&S technology to wheat growers, wheat industry, and M&S technology producers in the next section.

Value of the M&S Technology: Simulation Results

Since the technology is at the beginning of commercialization, there are no production cost data available yet. Therefore, we can only assume a range for the marginal cost of a M&S technology device. In our calculation we assume that the marginal cost for an additional M&S technology device, c , takes values between \$2,000 and \$20,000. We further assume that each device can process a maximum amount of 500,000 bushels of wheat.¹¹ Therefore, if we define one unit of the M&S technology as the amount of the technology that can process 1,000 bushels of wheat, then the marginal cost for one unit of M&S technology is between \$4 and \$40. Given the marginal cost per unit, we numerically solve the first order condition of a firm’s production problem presented in equation (3.7).

Table 3.5 presents the optimal quantity supplied of the M&S technology under various marginal cost and market structure combinations. From this table we can see that, intuitively, the equilibrium quantity (price) of the M&S technology decreases (increases) when the marginal cost becomes higher or when the number of producers becomes smaller. Specifically, when the technology market is monopoly and when the marginal cost of the technology is \$4 per unit, then the equilibrium technology price is \$149.3 for segregating 1,000 bushels of wheat and there are 52.8 million bushels of wheat to be segregated. When the marginal cost increases to \$40 per unit then the monopolistic technology price increases to \$164.1 and the quantity demanded for the technology decreases to segregating 47.3 million bushels of wheat. For the monopolistic producer, the annual operating profit is \$7.7 (\$ 5.9) million when marginal cost is \$4 (\$40) per unit. Suppose the interest rate is 5% and assume that the equilibrium prices and quantities are constants over time. When marginal cost is \$4 and when market structure is monopoly, the net present value of the aggregate operating profit over time is \$153.5 million. However, if the

¹¹Suppose an on-combine M&S device’s lifespan is 5 years. Further assume a combine can harvest 2,500 acres per year and the yield is 40 bushels per acre. Then we can obtain the M&S device’s processing capacity by $5 \times 2,500 \times 40 = 500,000$ bushels.

market structure is duopoly while the marginal cost is \$4 per unit, then the net present value of an individual producer's aggregate operating profit over time becomes \$60 million. If the fixed cost can be justified by the net present value of the aggregate operating profit, then the M&S technology is a viable investment choice for potential producers.

Table 3.6 shows adopters' average WTP for the M&S technology and wheat market values upon the technology adoption. Both the average WTP and the wheat market values are evaluated under various marginal cost and market structure combinations. The upper-left panel in Table 3.6 shows that adopters' average WTP ranges from \$144 to \$222, where the lower bound (i.e., \$144) is obtained when the technology market is perfectly competitive and when the marginal cost for the M&S technology is \$4 per unit; and the upper bound (i.e., \$222) is obtained when the technology market is monopolistic and when the technology's marginal cost is \$40 per unit. The upper-right panel in Table 3.6 shows the aggregate U.S. HRW and HRS wheat stock values under various marginal cost and market structure combinations. The technology adoption slightly decreases the wheat stock values by 0.0002% to 0.004%. This is because the technology adoption has opposite impacts on HRW and HRS wheat stock values. Specifically, upon the adoption of this new technology, market value of Hard Red Winter wheat will decrease by 0.2% to 2.3%, but market value of Hard Red Spring wheat will increase by 0.3% to 3.3%.

The reason for why the impacts of technology adoption on HRW wheat value and HRS wheat value are opposite should be traced back to the elasticity matrixes, the quantity shock when a grower adopts the technology, and the initial wheat stocks. However, since we have a six-dimension demand system and the elasticity matrix changes whenever one more grower adopts the technology, it is difficult to identify the complete reason. Here, we provide an intuitive explanation focusing on the initial wheat stocks and the starting elasticity matrix in Table 3.3. The initial low, middle, and high protein HRW wheat stocks are 207, 193, and 179 million bushels, respectively. The initial low, middle, and high protein HRS wheat stocks are 100, 126, 133 million bushels, respectively. Please notice that for HRW wheat, the higher the protein level, the smaller the wheat stock. The opposite is true for HRS wheat. Therefore, for instance, when 1.93 million bushels of wheat is directed from HRW middle protein wheat stock to high protein wheat stock by segregation, then the decrease of HRW middle protein wheat stock is 1% but the increase of high protein wheat stock is higher than 1%. For HRS wheat,

when 1.26 million bushels of wheat is directed from middle protein wheat stock to high protein wheat stock, then the decrease of middle protein wheat stock is 1% but the increase of high protein wheat stock is lower than 1%. Everything else equal, this difference between HRW and HRS wheat makes that when moving 1% middle protein wheat stock to high protein wheat stock, then the percentage increased of HRW middle protein wheat price is less than that of HRS middle protein wheat price but the percentage decreased of HRW high protein wheat price is larger than that of HRS high protein wheat price. Moreover, by comparing the own-flexibilities of HRW and HRS high protein wheat in Table 3.3 we find that the absolute value of HRS high protein wheat's own-flexibility is much smaller than that of HRW high protein wheat, which means that price of HRW high protein wheat is more sensitive to quantity changes than the price of HRS high protein wheat is. That is, when stocks of high protein wheat increases as segregation continues, then price of HRW high protein wheat decreases faster than the price of HRS high protein wheat does. Therefore, the quantity relationship between wheat stocks with different protein levels as well as the difference between flexibility values can partly explain why the technology adoption's impacts on HRW and HRS wheat market values are opposite.

Table 3.7 presents equilibrium protein premia under various technology market structure and marginal cost combinations. An overall pattern is that protein premia are increasing in the marginal cost of the M&S technology but is decreasing in the number of producers. This is intuitive because higher marginal cost of the technology or less producers in the technology market imply the technology price will be high and the quantity of wheat segregated will be low. Therefore, the wheat stock shocks caused by the M&S technology will be smaller. This means that the quantity of high protein wheat segregated out from low or middle protein wheat will be lower and hence the decrease in protein premium caused by the M&S technology will be smaller. From Table 3.7 we also observe some violations for this overall protein premium pattern. For example, when technology market is monopolistic, then the protein premium of high protein HRW wheat over middle protein HRW wheat and the protein premium of middle protein HRS wheat over low protein HRS wheat are decreasing in the marginal cost of the M&S technology. The reason for these violations is that protein premia keep changing as segregation continues and the changing is determined by the types of wheat that are segregated, the flexibility matrix, and wheat stocks. For example, when the technology price is very high, then only some growers of HRS low protein wheat can afford the M&S technology. Therefore, we should expect that the protein pre-

mium of HRS middle protein wheat over low protein wheat decreases as some HRS middle or high protein wheat is segregated from low protein wheat. When the technology price becomes lower, however, then some growers of HRS middle protein wheat can afford the technology, which will increase the protein premium of HRS middle protein wheat over low protein wheat because now the quantity of HRS middle protein wheat stock is decreasing and the quantity of HRS low protein wheat stock is increasing or unchanged.

Concluding Remarks

Two important and related trends in food markets are a) growth in demand for differentiated products, and b) capacity to distinguish between quality attributes at the commercial level. U.S. planted wheat acres are declining in the face of stiff international competition in premium product markets and demand for crop acres from biofuels (NASS 2011, p. 139). A segregating technology could allow wheat growers to better identify grains that can be directed to premium markets while also increasing consumer surplus. Our work provides a coherent methodology for evaluating the benefits of a farm-level information technology. A microeconomic optimization model of wheat growers' segregating and blending decisions is developed. Then wheat growers' WTP for the sorting technology is simulated using U.S. HRW and HRS wheat prices and stocks based on an estimation of a wheat demand system. The impact of the M&S technology's market structure on wheat growers' WTP for the technology and on the value of the technology to the wheat industry is studied as well. We conclude that the adoption of the M&S technology will decrease the market values of wheat at industry-level even though some growers may benefit from the adoption. The profitability of the M&S technology to potential producers is studied under various market structure and marginal cost combinations.

This article can be extended in several aspects. First, if wheat growers and elevators face the same protein premium schedule, then once farmers adopt the technology to exploit arbitrage opportunities there will be very little benefit for elevators to process wheat according to protein concentration levels. Therefore, the technology will re-allocate the segregating and blending benefit from elevators to wheat growers. The magnitude of this shift in rents may be of interest for future research. As discussed in Pirrong (1995), in the 1860s the benefit of blending for elevators were so important as to impede a

private market system of grain grading. Even nowadays protein concentration still has not been one of the criteria that determine wheat grades in the United States. If the adoption of the M&S technology transfers the blending benefit from elevators to farmers, then grain merchandisers may be motivated to implement a more comprehensive grading system that includes protein concentration. Second, in this article we confine our analysis to assessing wheat growers' welfare, wheat market value, and the M&S technology's market prospects. What we omit is wheat consumers' welfare change caused by the technology. Therefore, analyzing the technology's impacts on wheat consumers' surplus could be another direction of extension.

References

- Adam, B.D., P. Kenkel, and K. Anderson. 1994. "The Economics of Cleaning Winter Wheat for Export: An Evaluation of Proposed Federal 'Clean Grain' Standards." *Journal of Agricultural and Resource Economics*. 19(2):280-298.
- Babcock, B.A. 1990. "The Value of Weather Information in Market Equilibrium." *American Journal of Agricultural Economics*. 72:63-72.
- Baltazar, J. and D.E. Claridge. 2006. "Study of Cubic Splines and Fourier Series as Interpolation Techniques for Filling in Short Periods of Missing Building Energy Use and Weather Data." *Journal of Solar Energy Engineering*. 128(2):226-230.
- Eales, J.S. and L.J. Unnevehr. 1994. "The Inverse Almost Ideal Demand System." *European Economic Review*. 38:101-115.
- Eales, J., C. Durham, and C.R. Wessells. 1997. "Generalized Models of Japanese Demand for Fish." *American Journal of Agricultural Economics*. 79: 1153-1163.
- Giannakas, K., R. Gray, and N. Lavoie. 1999. "The Impact of Protein Increments on Blending Revenues in the Canadian Wheat Industry." *International Advance in Economic Research*. 5(1):121-136.
- Goodwin, B.K. and V.H. Smith. 2009. "Harvest-Time Protein Shocks and Price Adjustment in U.S. Wheat Market." *Journal of Agricultural and Resource Economics*. 34(2):237-255.
- Grant, J.H., L.M. Dayton, and K.A. Foster. 2010. "A Seasonal Inverse Almost Ideal Demand System

- for North American Fresh Tomatoes.” *Canadian Journal of Agricultural Economics*. 58:215-234.
- Hennessy, D.A. 1996. “The Economics of Purifying and Blending,” *Southern Journal of Economics*. 63:223-32.
- Hennessy, D.A. and Wahl, T.I. 1997. “Discount Schedules and Grower Incentives in Grain Marketing.” *American Journal of Agricultural Economics*. 79: 888-901.
- Johnson, D.D. and W.W. Wilson. 1993. “Wheat Cleaning Decisions at Country Elevators.” *Journal of Agricultural and Resource Economics*. 18:198-210.
- Lave, L.B. 1964. “The Value of Better Weather Information to the Raisin Industry.” *Econometrica*. 31:151-164.
- Long, D.S., Engel, R.E., and Siemens, M.C. 2008. “Measuring Grain Protein Concentration with In-line Near Infrared Reflectance Spectroscopy.” *Agronomy Journal*. 100(2):247-252.
- Moler, C.B. 2008. *Numerical Computing with MATLAB, Revised Reprint*. Society for Industrial and Applied Mathematics, Rev Rep edition (July 25, 2008).
- Moschini, G. and A. Vissa. 1992. “A Linear Inverse Demand System.” *Journal of Agricultural and Resource Economics*. 17(2):294-302.
- National Agricultural Statistics Service (NASS). 2011. *Crop Production — Historical Track Records*. Washington DC: U.S. Department of Agriculture, April.
- Pirrong, S.C. 1995. “The Efficient Scope of Private Transactions-Cost-Reducing Institutions: The Successes and Failures of Commodity Exchanges.” *The Journal of Legal Studies*, 24(1):229-255.
- Popper, L., Schäfer, W., and Freund, W. (eds.). *Future of Flour - A Compendium of Flour Improvement*. Erschienen: Jan 2007. (available online at: <http://www.muehlenchemie.de/english/know-how/future-of-flour.html>) (Accessed on Oct. 9th, 2010).
- Sivaraman, E., Lyford, G.P., and Brorsen, W.B. 2002. “A General Framework for Grain Blending and Segregation.” *Journal of Agribusiness*, 20(2), 155-61.
- Taylor, J., Whelan, B., Thylén, L., Gilbertsson, M., and Hassall, J. “Monitoring Wheat Protein Content On-Harvester: Australian Experiences.” In J.V. Stafford, J.V. ed. *Precision Agriculture '05, Proceedings of the 5th European Conference on Precision Agriculture*. Wageningen, The Netherlands: Wageningen Academic Publishers, pp.369-375.
- U.S. Wheat Associates. *Crop Quality Report*. Issues from 1980 to 2010. (available online at:

<http://www.uswheat.org/reports/cropQuality>) (Accessed on Jan. 22th, 2012).

Vercammen, J. 2011. *Agricultural Marketing: Structural Models for Price Analysis*. London: Routledge.

Wilson, W.W. 1983. "Price Relationships between Hard Red Spring and Hard Red Winter Wheat." *North Central Journal of Agricultural Economics*. 5(2):19-26.

Yoon, B., B.W. Brorsen, and C.P. Lyford. 2002. "Value of Increasing Kernel Uniformity." *Journal of Agricultural and Resource Economics*. 27(2):481-494.

Table 3.1: Summary Statistics for HRW and HRS Wheat Prices ,Protein Premia, and Stocks

	Variables	Mean	Std. Dev.	Min	Max
prices (cents/bu.)	HRW low protein wheat (p_1)	452	135	279	1226
	HRW middle protein wheat (p_2)	465	141	283	1290
	HRW high protein wheat (p_3)	482	148	289	1373
	HRS low protein wheat (p_4)	489	188	278	2057
	HRS middle protein wheat (p_5)	520	191	331	2062
	HRS high protein wheat (p_6)	541	197	345	2062
protein premia (cents/bu.)	HRW middle over low protein wheat (p_{21})	12.9	14.3	0	89.7
	HRW high over low protein wheat (p_{31})	30.6	30.3	0	199.7
	HRS middle over low protein wheat (p_{54})	30.6	31.0	0.34	160.4
	HRS high over low protein wheat (p_{64})	51.1	54.6	1.73	266.7
monthly stocks (million metric tons)	HRW low protein wheat (s_1)	5.63	3.88	0.35	16.10
	HRW middle protein wheat (s_2)	5.25	2.95	0.73	17.90
	HRW high protein wheat (s_3)	4.88	3.42	0.28	18.80
	HRS low protein wheat (s_4)	2.73	1.58	0.14	7.38
	HRS middle protein wheat (s_5)	3.43	1.78	0.38	8.29
	HRS high protein wheat (s_6)	3.61	2.48	0.20	17.20

Table 3.2: Estimates of the Transformed L.A/AIDS Model

Equations	Constants	transformed logarithmic quantities						Stone	R^2	DW
		HRW Low	HRW Middle	HRW High	HRS Low	HRS Middle	HRS High	Quantity Index		
HRW Low	0.027 (0.007)	0.114 (0.003)	-0.035 (0.002)	-0.046 (0.002)	-0.005 (0.002)	-0.016 (0.002)	-0.013 (0.002)	0.00068 (0.00244)	0.85	1.75
HRW Middle	0.018 (0.003)	-0.035 (0.002)	0.154 (0.004)	-0.041 (0.002)	-0.018 (0.002)	-0.022 (0.002)	-0.038 (0.002)	0.00018 (0.00143)	0.83	1.95
HRW High	0.007 (0.003)	-0.046 (0.002)	-0.041 (0.002)	0.127 (0.002)	-0.005 (0.001)	-0.019 (0.001)	-0.016 (0.002)	0.00593 (0.00178)	0.95	1.70
HRS Low	0.029 (0.004)	-0.005 (0.002)	-0.018 (0.002)	-0.005 (0.001)	0.076 (0.002)	-0.028 (0.002)	-0.019 (0.001)	-0.00336 (0.00139)	0.82	1.64
HRS Middle	0.019 (0.002)	-0.016 (0.002)	-0.022 (0.002)	-0.019 (0.001)	-0.028 (0.002)	0.115 (0.002)	-0.029 (0.001)	-0.00197 (0.00133)	0.83	1.36

Note: 1. Numbers in parentheses are standard errors. 2. Low, Middle, and High mean low, middle, and high protein wheat. 3. All the coefficients of transformed logarithmic quantities are significant at the 1 percent level.

Table 3.3: Estimated Quantity Elasticities (or Flexibilities) at the Sample Mean

Equations	Constants	transformed logarithmic quantities						Stone	R^2	DW
		HRW Low	HRW Middle	HRW High	HRS Low	HRS Middle	HRS High	Quantity Index		
HRW Low	0.027 (0.007)	0.114 (0.003)	-0.035 (0.002)	-0.046 (0.002)	-0.005 (0.002)	-0.016 (0.002)	-0.013 (0.002)	0.00068 (0.00244)	0.85	1.75
HRW Middle	0.018 (0.003)	-0.035 (0.002)	0.154 (0.004)	-0.041 (0.002)	-0.018 (0.002)	-0.022 (0.002)	-0.038 (0.002)	0.00018 (0.00143)	0.83	1.95
HRW High	0.007 (0.003)	-0.046 (0.002)	-0.041 (0.002)	0.127 (0.002)	-0.005 (0.001)	-0.019 (0.001)	-0.016 (0.002)	0.00593 (0.00178)	0.95	1.70
HRS Low	0.029 (0.004)	-0.005 (0.002)	-0.018 (0.002)	-0.005 (0.001)	0.076 (0.002)	-0.028 (0.002)	-0.019 (0.001)	-0.00336 (0.00139)	0.82	1.64
HRS Middle	0.019 (0.002)	-0.016 (0.002)	-0.022 (0.002)	-0.019 (0.001)	-0.028 (0.002)	0.115 (0.002)	-0.029 (0.001)	-0.00197 (0.00133)	0.83	1.36

Note: 1. Numbers in parentheses are standard errors. 2. Low, Middle, and High mean low, middle, and high protein wheat. 3. All the coefficients of transformed logarithmic quantities are significant at the 1 percent level.

Table 3.4: Summary Statistics of HRW Wheat Testing Results (Observations: 538)

	Mean	standard deviation	Maximum	Minimum
Protein	11.87%	0.017	16.6%	7.40%
Yield (bu./acre)	63.25	32.56	165.90	9.90
Test Weight (lb/bu.)	60.48	2.48	64.50	47.10

Table 3.5: Equilibrium Prices, Quantities and Operating Profit of MS Technology under Various Marginal Cost and Market Structure Combinations

equilibrium technology prices ^b					equilibrium technology quantities ^c				
		technology market structure			technology market structure				competition
		monopoly	duopoly	triopoly	perfect	monopoly	duopoly	triopoly	
marginal	4	149.27	65.00	49.48	4	52.81	98.37	110.92	146.91
cost^a	16	154.35	72.64	58.74	16	50.96	92.19	103.42	138.79
(dollar)	28	159.28	79.87	67.69	28	49.13	86.26	96.20	128.65
	40	164.09	121.46	76.16	40	47.31	62.95	89.31	118.67
annual operating profit of an individual technology firm (million dollars)					NPV of operating profit over time for a single technology producer when interest rate is 5% (million dollars)				
		technology market structure			competition	technology market structure			
		monopoly	duopoly	triopoly		monopoly	duopoly	triopoly	perfect
marginal	4	7.67	3.00	1.68	0	153.45	60.01	33.63	0
cost^a	16	7.05	2.61	1.47	0	141.00	52.22	29.47	0
(dollar)	28	6.45	2.24	1.27	0	128.99	44.74	25.46	0
	40	5.87	2.56	1.08	0	117.41	51.28	21.53	0

Note: ^a marginal cost for producing the quantity of M&S technology that can process 1,000 bushels of wheat.

^b dollars per unit of M&S technology that can process 1,000 bushels of wheat. ^c measured by equilibrium quantity of wheat to be segregated (unit: million bushels).

Table 3.6: Adopters' Average WTP and the HRW and HRS Wheat Stock Value under Various Marginal Cost and Market Structure Combinations of M&S Technology

		adopters' average WTP ^b (unit: \$/sorting 1,000 bushels of wheat)				HRW and HRS wheat stock value ^b (unit: million dollars)			
		technology market structure			perfect competition	technology market structure			perfect competition
		monopoly	duopoly	triopoly		monopoly	duopoly	triopoly	
marginal cost^a	4	218.97	178.72	167.93	144.14	4,560.425	4,560.339	4,560.314	4,560.267
	16	219.91	183.26	175.62	145.06	4,560.425	4,560.349	4,560.331	4,560.272
(dollar)	28	220.81	188.20	180.81	146.19	4,560.426	4,560.362	4,560.343	4,560.278
	40	222.26	211.07	185.72	146.71	4,560.432	4,560.411	4,560.356	4,560.282
		HRW wheat stock value ^b (unit: million dollars)				HRS wheat stock value ^b (unit: million dollars)			
		technology market structure			perfect competition	technology market structure			perfect competition
		monopoly	duopoly	triopoly		monopoly	duopoly	triopoly	
marginal cost^a	4	2,687.912	2,673.297	2,663.996	2,634.896	1,872.512	1,887.042	1,896.318	1,925.371
	16	2,687.329	2,677.226	2,670.831	2,634.748	1,873.096	1,883.123	1,889.500	1,925.524
(dollar)	28	2,686.734	2,681.249	2,675.221	2,634.602	1,873.692	1,879.113	1,885.122	1,925.676
	40	2,686.599	2,690.160	2,679.240	2,634.553	1,873.833	1,870.251	1,881.116	1,925.729

Note: ^a marginal cost for producing the quantity of M&S technology that can process 1,000 bushels of wheat. ^b The initial HRW wheat stock value, HRS wheat stock value, and total wheat stock value before M&S technology adoption are \$2,695.726 million, \$1864.717 million, and \$4,560.443 million, respectively.

Table 3.7: Protein Premia under Various Marginal Cost and Market Structure Combinations of M&S Technology (cents/bu.)

	HRW			HRS			HRW			HRS		
	Middle over Low ^b	High over Middle ^c	Middle over Low ^b	Middle over Low ^b	High over Middle ^c	Middle over Low ^b	Middle over Low ^b	High over Middle ^c	Middle over Low ^b	Middle over Low ^b	High over Middle ^c	Middle over Low ^b
initial protein premia	13.0	17.7	17.7	30.9	20.7	20.7	13.0	17.7	30.9	20.7	20.7	20.7
	monopoly			duopoly			duopoly			duopoly		
marginal cost^a	4	15.2	9.9	42.7	15.6	15.6	10.0	7.3	38.8	7.8	7.8	7.8
(dollar)	16	15.6	9.8	42.5	15.7	15.7	10.2	8.1	40.1	8.1	8.1	8.1
	28	16.1	9.7	42.4	15.9	15.9	10.4	8.9	41.4	8.9	8.9	8.9
	40	16.8	9.3	41.9	17.1	17.1	12.3	11.1	44.0	12.6	12.6	12.6
	triopoly			perfect competition			perfect competition			perfect competition		
marginal cost^a	4	8.5	6.3	35.7	6.0	6.0	0	8.1	26.4	0.6	0.6	0.6
(dollar)	16	9.6	7.0	38.0	7.0	7.0	0	7.8	26.6	1.9	1.9	1.9
	28	10.1	7.8	39.5	7.7	7.7	0	7.3	26.9	3.7	3.7	3.7
	40	10.3	8.5	40.7	8.5	8.5	0	7.1	27.0	4.6	4.6	4.6

Note: ^a marginal cost for producing the quantity of M&S technology that can process 1,000 bushels of wheat. ^b “Middle over Low” means the protein premium of middle protein wheat over low protein wheat. ^c “High over Middle” means the protein premium of high protein wheat over middle protein wheat.

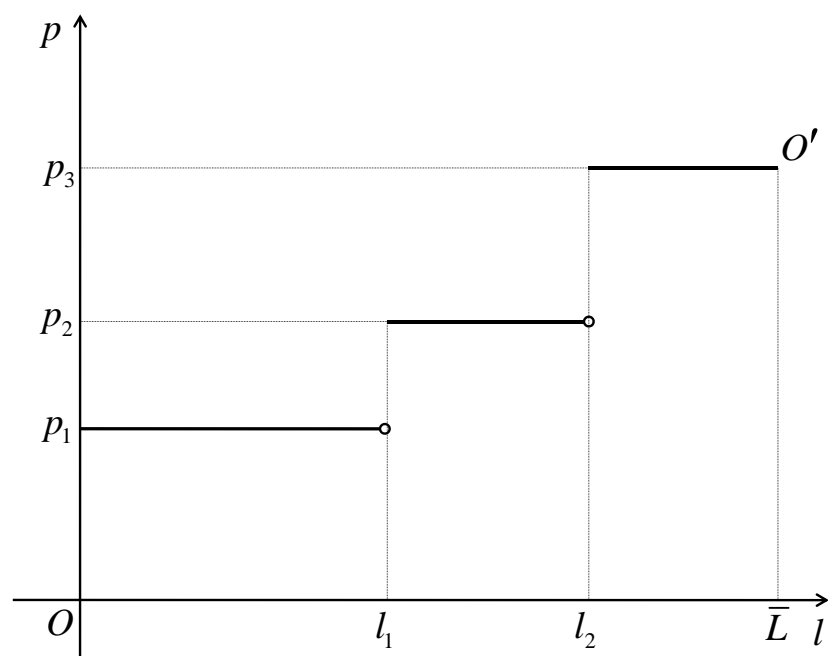


Figure 3.1: A Three-Step Price Schedule

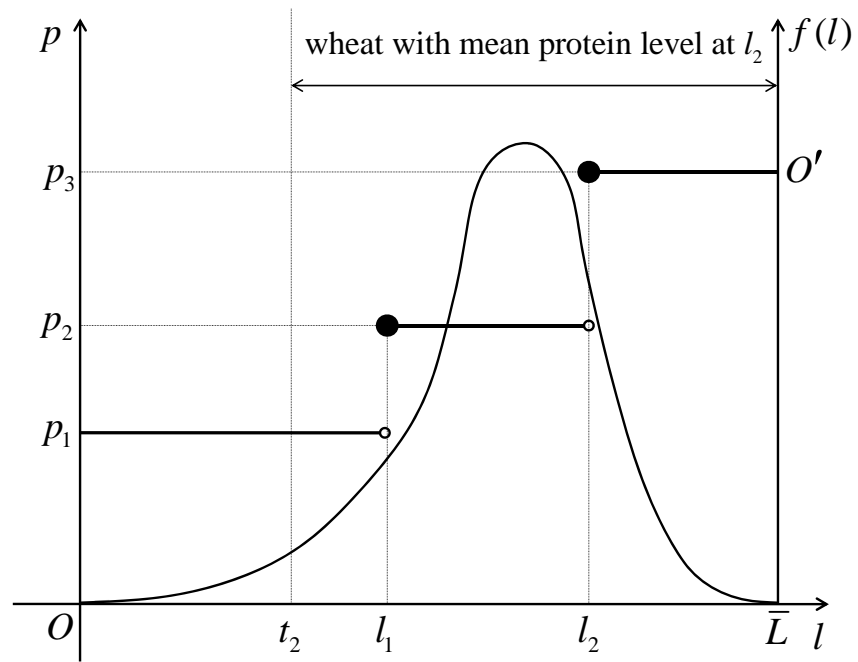


Figure 3.2: A Protein Distribution such that $t_2 \leq l_1$

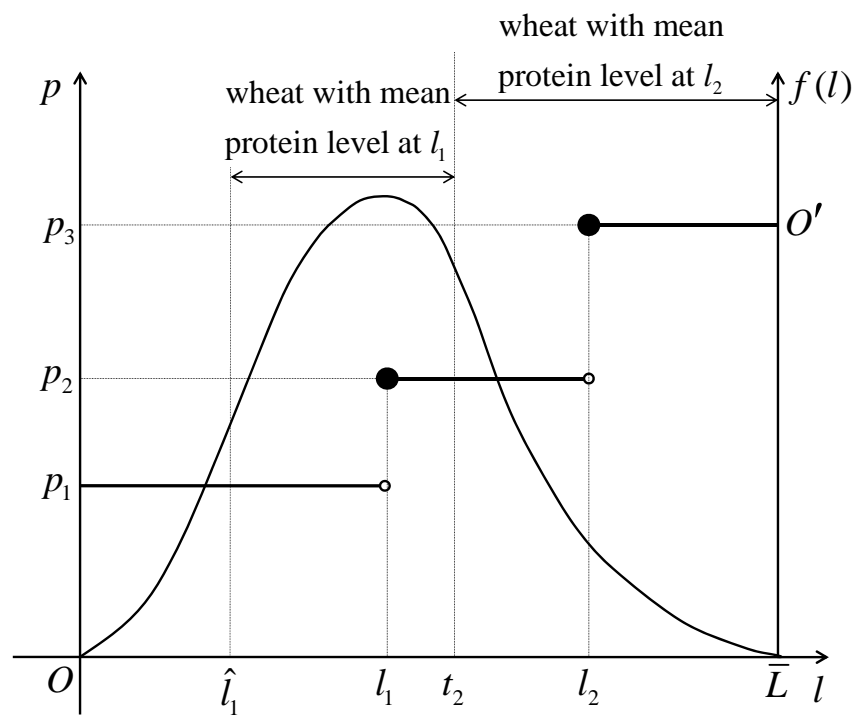


Figure 3.3: A Protein Distribution such that $t_2 > l_1$ and $\hat{l}_1 > 0$

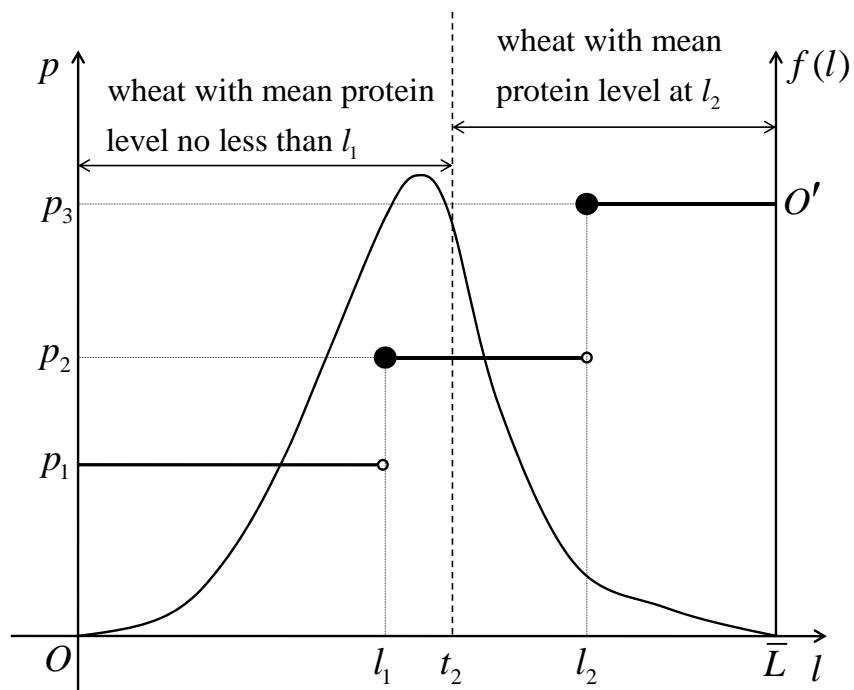
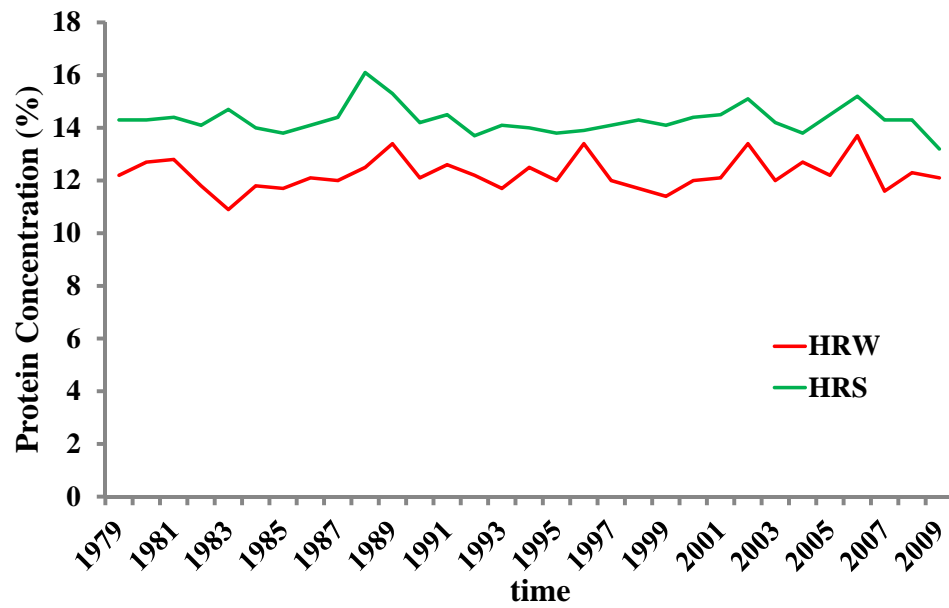
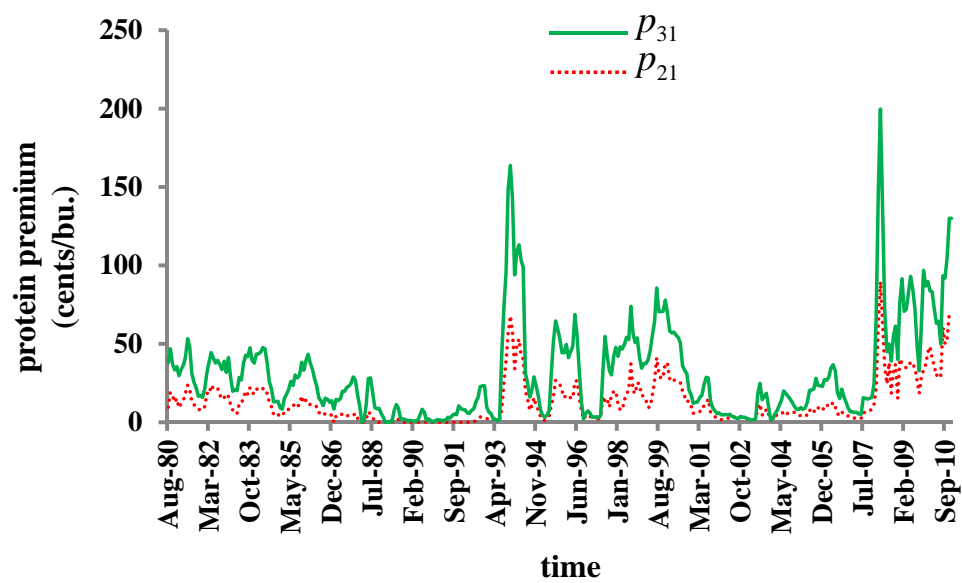


Figure 3.4: A Protein Distribution such that $t_2 > l_1$ and $\hat{l}_1 = 0$



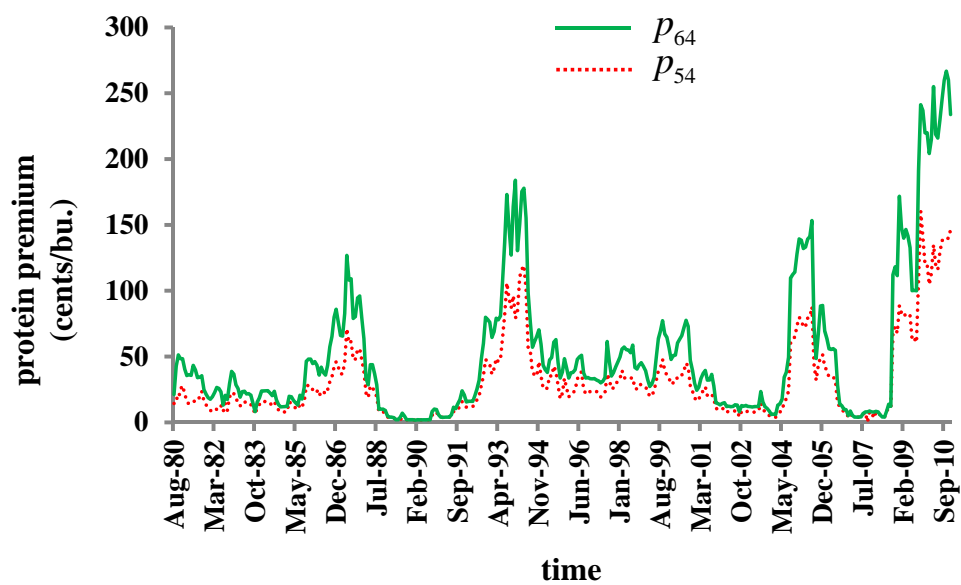
Source: *Crop Quality Report*, U.S. Wheat Associates, issues from 1980 to 2010.

Figure 3.5: Annual Average Protein Concentration of HRW and HRS Wheat Produced between 1979 and 2009 in the United States



Note: p_{31} is protein premium for HRW high protein wheat over HRW low protein wheat, and p_{21} is protein premium for HRW middle protein wheat over HRW low protein wheat.

Figure 3.6: Protein Premia of HRW Wheat



Note: p_{64} is protein premium for HRS high protein wheat over HRS low protein wheat, and p_{54} is protein premium for HRS middle protein wheat over HRS low protein wheat.

Figure 3.7: Protein Premia of HRS Wheat

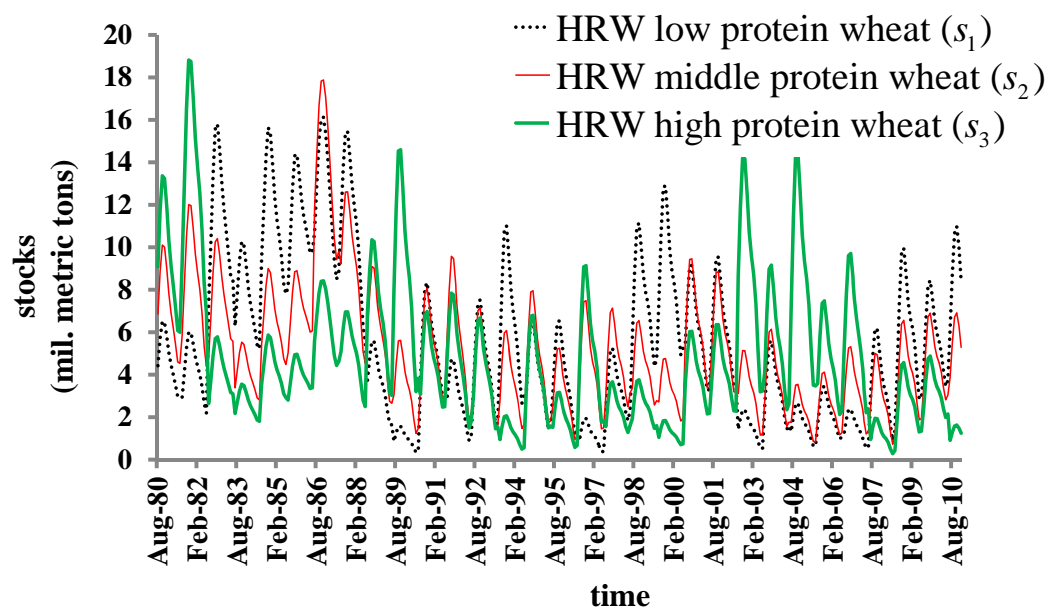


Figure 3.8: HRW Wheat Stocks by Protein Concentration

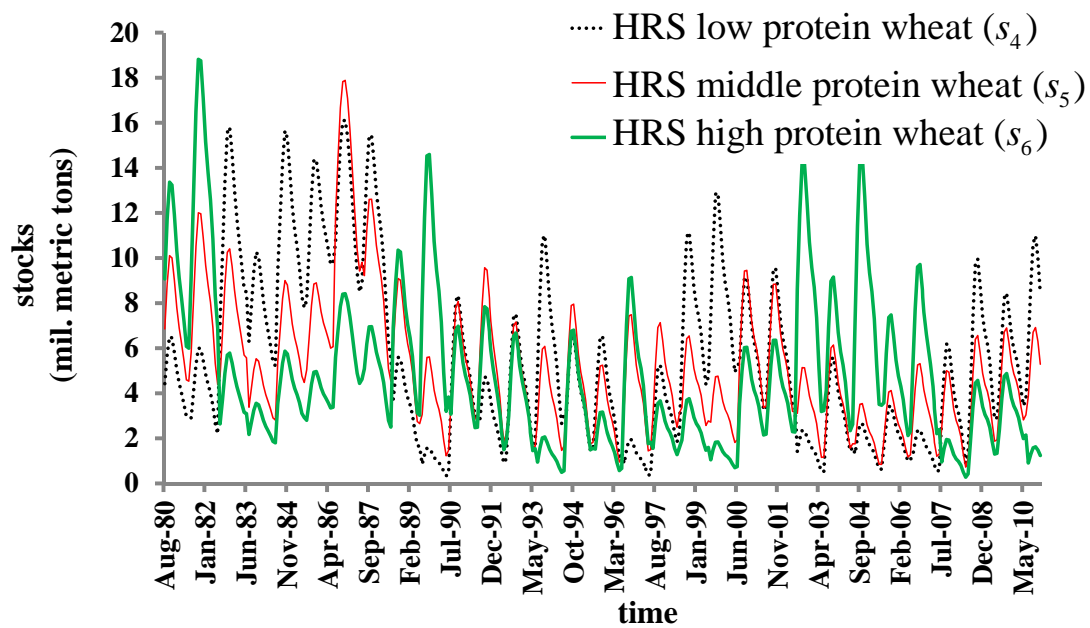


Figure 3.9: HRW Wheat Stocks by Protein Concentration

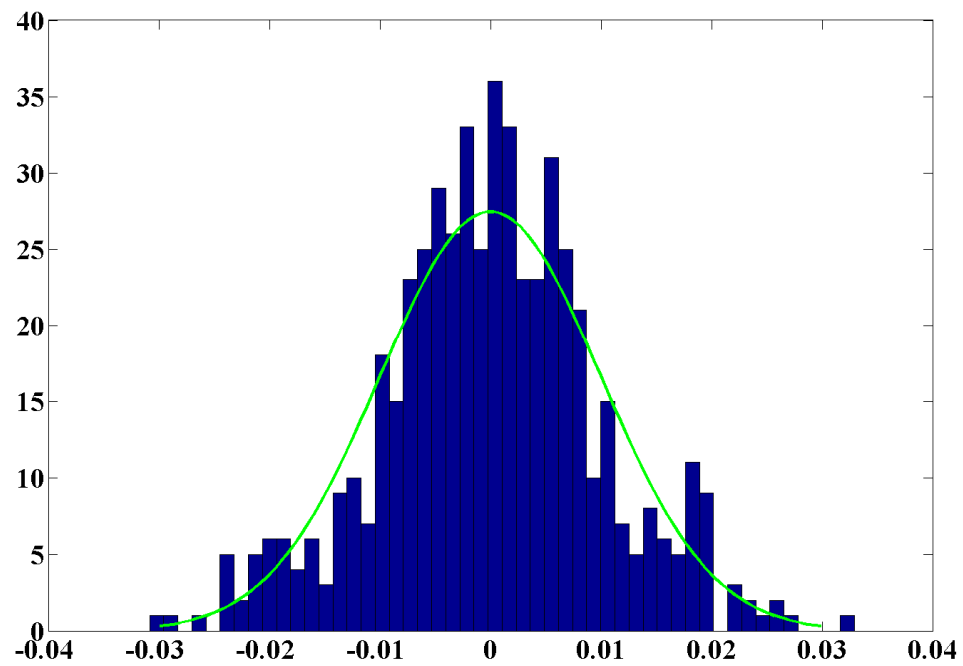


Figure 3.10: The Histogram of Protein Concentration Residuals

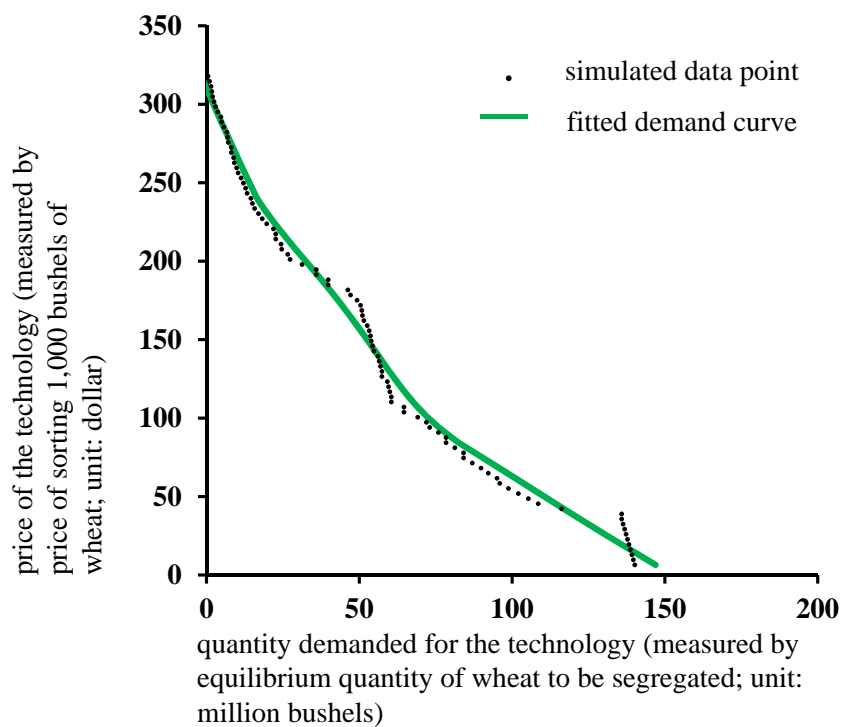


Figure 3.11: Demand Curve for the M&S Technology

Appendices

Appendix 1

Item A

In this item we discuss optimal processing decisions under uniformly curved price schedules. Incentives to segregate and blend grain with different dockage when the price schedule is uniformly curved (i.e., concave or convex) have been studied in Hennessy and Wahl (1997). Regarding wheat with different protein concentrations under uniformly curved price schedules, incentives to segregate and blend is similar as what is in Hennessy and Wahl (1997). Therefore, we just demonstrate the results here and refer readers to Hennessy and Wahl (1997) for the proof.

Proposition 3.2. *If the price schedule is concave, then no segregating is needed to the load. That is, this load of wheat will be sold as it is. If the price schedule is convex, then the load should be completely segregated.*

From Proposition 3.2 we have the following corollary.

Corollary 3.1. *Blending any two loads increases (decreases) a wheat grower's revenue when the price schedule is concave (convex). Segregating one load into two or more sub-loads increases (decreases) a wheat grower's revenue when the price schedule is convex (concave).*

Item B

In this item we discuss optimal processing decisions under non-uniformly curved price schedules. Instead of being uniform curves, price schedules may have nonuniform curves (Hennessy and Wahl 1997). Figures 3.12 and 3.13 present two possibilities of these schedules. Figure 3.12 shows a price schedule that is concave at low protein concentration levels and convex at high protein concentration levels. Figure 3.13 shows a price schedule that is convex at low protein concentration levels and concave at high protein concentration levels. Following Hennessy and Wahl (1997) we call schedules with the curvature of Figure 3.12 as shape type I and schedules with the curvature of Figure 3.13 as shape type II. Without loss of generality it is assumed that $p(0) = 0$, so the schedules pass through the origin. In Figure 3.12 and Figure 3.13, points O and O' are two ends of the respective price schedules. Point B is

the inflexion point where the schedule changes from being concave (convex) to being convex (concave) in Figure 3.12 (Figure 3.13).

In Figure 3.12, there are two tangent lines of price curve OO' that are of critical interest. Line $O'A$ is the tangent of the price curve with tangency point at A . If there is no tangency point, then set point A as origin O . Line CD is defined as in Definition 3.3. Let the coordinates of points A , B , C , and D be $[l_A, p(l_A)]$, $[l_B, p(l_B)]$, $[l_C, p(l_C)]$, and $[l_D, p(l_D)]$, respectively.

Definition 3.3. Line CD in Figure 3.12 is defined as: (1) Line CD is a tangent of curve OB with tangency point at C . (2) Line CD intersects curve $O'B$ at point D . (3) The l -coordinates of points C and D is such that $l_A < l_C < l_B$ and $\int_0^{l_D} f(l)dl / \int_0^{l_D} f(l)dl = l_C$.

Definition 3.3 indicates that the commingle of wheat with protein concentration no higher than l_D has mean protein concentration l_C . Recall that μ is the mean protein concentration of the initial load of wheat. Therefore, if $\mu \leq l_A$, then line CD does not exist.

Similarly, in Figure 3.13, there are two critical tangent lines as well. Line OA is the tangent of the price curve with tangency point at A . If there is no tangency point, then set point A as O' . Line CD in Figure 3.13 is defined as in Definition 3.4.

Definition 3.4. Line CD in Figure 3.13 is defined as: (1) Line CD is a tangent of curve $O'B$ with tangency point at C . (2) Line CD intersects curve OB at point D . (3) The l -coordinates of points C and D are such that $l_B < l_C < l_A$ and $\int_{l_D}^{\bar{L}} f(l)dl / \int_{l_D}^{\bar{L}} f(l)dl = l_C$.

Definition 3.4 indicates that the blending of wheat with protein concentration no less than l_D has mean protein concentration l_C . Therefore, if $\mu \geq l_A$, then line CD does not exist.

For schedules with shape type I and schedules with shape type II, the optimal processing arrangements are presented in next the proposition.

Proposition 3.3. For shape type I schedules, (i) when $\mu \leq l_A$, then no processing is needed in the optimal arrangements; and (ii) when $\mu > l_A$, then in the optimal arrangements wheat in this load with protein concentration higher than l_D should be completely segregated and the remaining wheat should be completely blended. Here l_D is the l -coordinate of point D defined in Definition 3.3.

For shape type II schedules, (i) when $\mu \geq l_A$, then no processing is needed in the optimal arrangements; and (ii) when $\mu < l_A$, then in the optimal arrangements wheat in this load with protein level

higher than l_D should be completely blended and the remaining wheat should be completely segregated. Here l_D is the l -coordinate of point D defined in Definition 3.4.

Before we formally prove the Proposition 3.3, let us first discuss the intuition underlying it. Here we focus on shape type *I* schedules. For type *II* schedules, similar intuition applies. Under type *I* schedule, when the mean protein concentration of the load of wheat is no higher than l_A (i.e., $\mu \leq l_A$), then the effective price schedule is concave. If no wheat in this load has protein level higher than l_A , then it is trivial to show that the effective price schedule for this load is concave. Now suppose there is one sub-load with protein concentration level higher than l_A . More specifically, let us denote this sub-load's protein level as l_x and $l_x > l_A$. Then from Figure 3.12 we can see that for point A , and an arbitrary point on the left of point A , and another arbitrary point on the right of point A , these three points form a concave schedule. Since $\mu \leq l_A$, for wheat with protein level at l_x , one can always find some wheat with protein level lower than l_A to blend with; otherwise the premise $\mu \leq l_A$ will be violated. Therefore, according to Proposition 3.2 when $\mu \leq l_A$ this load of wheat does not need any segregation. When $\mu > l_A$, however, we can see that the effective price schedule for wheat with protein concentration higher than l_D is convex, which indicates segregating; for the remaining wheat the effective price schedule is concave, which indicates blending. Now let us to prove Proposition 3.3.

We first introduce a lemma that will be used repeatedly in the proof. Suppose one unit of wheat with protein concentration α is a commingle of wheat with α_1 protein concentration and wheat with α_2 protein concentration. And suppose this unit of wheat is segregated into two sub-loads, namely A and B , with mean protein concentration l_A and l_B , respectively. Without loss of generality, we assume that $\alpha_1 < \alpha_2$ and $l_A < l_B$. Then the following two items are true:

Lemma 3.1. (i) $\alpha_1 \leq l_A < l_B \leq \alpha_2$. (ii) l_A (or l_B) can be any value on the interval of $[\alpha_1, \alpha)$ (or $(\alpha, \alpha_2]$).

The proof of Lemma 3.1 is trivial. Based on Lemma 3.1 we can prove Proposition 3.3. Here we only prove the results for shape type *I* schedules. The same procedure applies when proving results related with shape type *II* schedules.

Proof. Part A. In this part we prove that under shape type *I* schedules item (i) is true. Suppose $\mu \leq l_A$. And suppose in the optimal arrangement the load is segregated into $n \geq 2$ sub-loads with different protein concentrations. Then there must be at least one sub-load, say sub-load i , with protein concentration

less than l_A . If not, then μ would be greater than l_A , which contradicts $\mu \leq l_A$ in item (i). Next we are going to show that the wheat grower can increase her revenue by commingling sub-load i with any other sub-load $j \neq i$.

Let l_i and l_j be the protein concentration of sub-loads i and j , respectively. If $l_j \leq l_A$, then sub-load i and sub-load j are under the segment of the price schedule that is uniformly concave, therefore, according to Corollary 3.1 the grower can always increase her revenue by commingling sub-load i and sub-load j . If $l_j > l_A$, then from Figure 3.14 we see that points $[l_i, p(l_i)]$ (i.e., point I), A , and $[l_j, p(l_j)]$ (i.e., point J) form a concave price schedule. It is easy to show that for any two points E and F on the price curve, if point E (F) is on the left (right) of point A , then points E , A , and F form a concave price schedule. Similarly, by Corollary 3.1 the grower can always increase her revenue by commingling sub-load i and sub-load j . In sum, for shape type I schedules, when $\mu \leq l_A$, then no sorting is needed in the optimal arrangements.

Part B. Now let us show that for shape type I schedules item (ii) is true. In Step 1 we show that when $\mu > l_A$, then the tangent line CD defined in Definition 3.3 exists and is unique. Step 2 shows that in the optimal arrangement there is one and only one sub-load that has protein concentration lower than l_B . Let Z be the name of this sub-load. Step 3 shows that sub-load Z has protein concentration l_C . Step 4 shows that in sub-load Z there is no wheat with protein concentration higher than l_D . Step 5 concludes the proof.

Step 1. In this step we show that when $\mu > l_A$, then line CD defined in Definition 3.3 exists and is unique. Let us start from the tangent line AO' . Imagine that line AO' is rotated in a clockwise direction while the tangency point between the line and the curve OB moves rightward from point A . Let $[l_j, p(l_j)]$ denote the coordinates of the tangency point, J . And let $[l_k, p(l_k)]$ denote the intersection point, K . During the rotation the value of $\int_0^{l_k} f(l)dl / \int_0^{l_k} f(l)dl$ (i.e., the mean protein concentration of the commingle of wheat with protein concentration no higher than l_k) is decreasing and the value of l_j is increasing. At point B we have $l_j = l_k$ and $\int_0^{l_k} f(l)dl / \int_0^{l_k} f(l)dl < l_j$.

When the coordinate of the tangency point is $[l_j, p(l_j)]$, then the slope of the tangent is $p'(l_j)$. Hence the equation of the tangent is $p = p'(l_j)l + [p(l_j) - p'(l_j)l_j]$. Then given l_j , the l -coordinate of

the intersection point, $l_k > l_j$, can be determined by an equation system as follows

$$\begin{cases} p = p(l_k), \\ p = p'(l_j)l_k + [p(l_j) - p'(l_j)l_j], \end{cases} \quad (3.17)$$

where $l_k > l_j$. From equation system (3.17) we can obtain that the relationship between l_j and l_k is determined implicitly by

$$H(l_k; l_j) \equiv p(l_k) - p'(l_j)l_k - [p(l_j) - p'(l_j)l_j] = 0. \quad (3.18)$$

By the implicit function theorem we have

$$\begin{aligned} \frac{dl_k}{dl_j} &= -\frac{\partial H / \partial l_j}{\partial H / \partial l_k} \\ &= -\frac{-p''(l_j)l_k - p'(l_j) + p'(l_j) + p''(l_j)l_j}{p'(l_k) - p'(l_j)} \\ &= -\frac{-p''(l_j)l_k + p''(l_j)l_j}{p'(l_k) - p'(l_j)} \\ &= \frac{p''(l_j)(l_k - l_j)}{p'(l_k) - p'(l_j)} \end{aligned} \quad (3.19)$$

Since curve OB is concave, we have $p''(l_j) < 0$. Together with $l_k > l_j$ we have $p''(l_j)(l_k - l_j) < 0$ in equation (3.19). Because at point $[l_k, p(l_k)]$ the slope of curve $O'B$ is greater than the slope of the line JK , it is true that $p'(l_k) - p'(l_j) > 0$. Therefore, we show $dl_k/dl_j < 0$.

When $l_j = l_B$, which means the tangency point is at point B , then we have $l_k = l_B$ as well because point B is the inflection point. This implies that when $l_j = l_B$ then the tangency point and the interception point coincide with point B .

Let us construct a function

$$M(l_j) = \int_0^{l_k(l_j)} f(l)dl / \int_0^{l_k(l_j)} f(l)dl - l_j, \quad (3.20)$$

where $l_k(\cdot)$ is a function of l_j implicitly determined in equation (3.18). When $l_j = l_A$, then $M(l_j) > 0$, which is because $\mu = \int_0^{O'} f(l)dl / \int_0^{O'} f(l)dl > l_A$. When $l_j = l_B$, then $M(l_j) < 0$, which is because $\int_0^{l_B} f(l)dl / \int_0^{l_B} f(l)dl < l_B$. Therefore, according to the intermediate value theorem, there must be an $l_C \in (l_A, l_B)$ such that $M(l_C) = 0$. That is $\int_0^{l_k(l_C)} f(l)dl / \int_0^{l_k(l_C)} f(l)dl = l_C$. This shows that when $\mu > l_A$, then line CD defined in Definition 3.3 exists.

Now we show that the line defined in Definition 3.3 is unique. The uniqueness will be proved if we show $dM(l_j)/dl_j < 0$.

$$\begin{aligned}
\frac{dM(l_j)}{dl_j} &= \frac{f(l_k)l_k l'_k(l_j) \int_0^{l_k(l_j)} f(l)dl - f(l_k)l'_k(l_j) \int_0^{l_k(l_j)} f(l)ldl}{\left(\int_0^{l_k(l_j)} f(l)dl\right)^2} - 1 \\
&= \frac{f(l_k)l_k l'_k(l_j) \left(\int_0^{l_k(l_j)} f(l)dl\right) \left(1 - \frac{\int_0^{l_k(l_j)} f(l)ldl}{l_k \int_0^{l_k(l_j)} f(l)dl}\right)}{\left(\int_0^{l_k(l_j)} f(l)dl\right)^2} - 1 \\
&< 0.
\end{aligned} \tag{3.21}$$

The inequality in expression (3.21) holds because $f(l_k)l_k l'_k(l_j) \left(\int_0^{l_k(l_j)} f(l)dl\right) < 0$ and $\int_0^{l_k(l_j)} f(l)ldl / l_k \int_0^{l_k(l_j)} f(l)dl < 1$.

Step 2. In this step we show that in the optimal arrangement there is one and only one sub-load that has mean protein concentration less than l_B . We denote this unique sub-load as Z . Suppose there are two or more sub-loads that have protein concentration less than l_B . Since curve OB is concave, according to Corollary 3.1 the grower can increase her revenue by commingling these sub-loads. Therefore, having more than one sub-loads that are with protein concentration less than l_B is not optimal.

If there is not any sub-load that has protein concentration lower than l_B , then there must be one sub-load, namely sub-load J , with protein concentration $l_j \geq l_B$ that is a commingle of wheat with protein concentration $l_i < l_B$ and wheat with protein concentration $l_k > l_B$. If $l_j > l_B$, then since curve $O'B$ is convex, by Proposition 3.2 sub-load J should be completely segregated. If $l_j = l_B$, then for any point, say point E , with l -coordinate l_E such that $l_B < l_E < l_k$, we can always find a point, say point F with l -coordinate l_F such that $l_i < l_F < l_B$, so that points E , B , and F form a convex price schedule (Figure 3.15). By Lemma 3.1, the sub-load with protein concentration l_j can be segregated into two smaller sub-loads. One is with protein concentration at l_E and the other one with protein concentration l_F . Again, by Proposition 3.2 sub-load J should be segregated. Therefore, having no sub-load whose protein concentration is less than l_B is not optimal either. In sum we know that in the optimal processing result there is one and only one sub-load whose protein concentration is less than l_B .

Step 3. In this step we show that the protein concentration of sub-load Z , l_Z , is equal to l_C . Here l_C is the l -coordinate of point C defined in Definition 3.3. Suppose in the optimal arrangement we have $l_Z > l_C$. Then there are two types of configuration of this unique sub-load Z . The first one is that there is some wheat with protein concentration $l_j > l_D$ in sub-load Z ; the second one is that some wheat with

protein concentration lower than l_Z is not included in sub-load Z . These two types of configuration exist because $\int_0^{l_D} f(l)dl / \int_0^{l_D} f(l)dl = l_C$. Intuitively, since the mean protein concentration of wheat with protein concentration lower than l_D is l_C , then to form a sub-load with protein concentration higher than l_C one needs either to include some wheat with protein concentration higher than l_D in the sub-load or to exclude some wheat with protein concentration lower than l_Z , or both. The two types of configuration are not mutual exclusive.

Now we show the first configuration is not optimal. Suppose sub-load Z has some wheat with protein concentration $l_j > l_D$. Since the mean protein concentration of this sub-load is equal to l_Z , this sub-load must have some wheat with protein concentration lower than l_Z . Draw a line that connects points Z and D (see Figure 3.16). Then we can always find a point, say point E , that is very close to point Z from the left side so that $l_E > l_C$, here l_E is the l -coordinate of point E . Points E , Z , and D form a convex price schedule. By Lemma 3.1, sub-load Z can be segregated into two smaller loads, one is with protein concentration at l_D ; and the other one is with protein concentration l_E . By Corollary 3.1, segregating sub-load Z increases the grower's revenue. Therefore, the first type of configuration is not optimal.

Now we show the second one is not optimal either. If some wheat with protein concentration lower than l_Z is not included in sub-load Z , then this wheat must be blended with some wheat with protein concentration higher than l_B to form a sub-load with mean protein concentration no less than l_B . If not, then there are at least two sub-loads that have protein concentration lower than l_B , which has been shown not to be optimal in Step 2. Suppose that wheat with protein concentration equal to $l_i < l_Z$ is blended with wheat with protein concentration equal to $l_j > l_B$ to form a sub-load K with protein concentration equal to $l_k \geq l_B$. If $l_k > l_B$, then it is always beneficial to segregate sub-load K because curve $O'B$ is convex. If $l_k = l_B$, then (with the same argument we made in Step 2) we can always find a point, say E , which is very close to point B from the left side, so that points E , B , and $[l_j, p(l_j)]$ form a convex price schedule (See Figure 3.17). According to Corollary 3.1, however, segregating this sub-load is beneficial. Therefore, the second type is not optimal either.

Now we show the unique sub-load Z cannot have $l_Z < l_C$. If $l_Z < l_C$, then there must be some wheat with protein concentration at l_j such that $l_Z \leq l_j \leq l_D$ that is not in sub-load Z . Otherwise the mean protein concentration of sub-load Z will be l_C or higher. However, the three points, Z , C , and $[l_j, p(l_j)]$,

form a concave price schedule (See Figure 3.18). According to Corollary 3.1 the grower can increase her revenue by commingling wheat in sub-load Z with wheat that has protein concentration l_j .

Step 4. This step shows that in sub-load Z there is no wheat with protein concentration higher than l_D . Here l_D is the l -coordinate of point D defined in Definition 3.3. Suppose this is not true, then sub-load Z contains some wheat with protein concentration $l_k > l_D$. Therefore, there must be some wheat with protein concentration l_j such that $l_C \leq l_j \leq l_D$ that is not in sub-load Z . This is because if all wheat with protein concentration between l_C and l_D is in sub-load l_Z , then together with some wheat with protein concentration higher than l_D being in sub-load l_Z as well, the mean protein concentration of sub-load Z must be higher than l_C . Please recall that the mean protein concentration of wheat with protein concentration less than l_D is l_C . We name the sub-load that contains wheat with protein concentration l_j as sub-load J . By the result in Step 2 we know the mean protein concentration of sub-load J is no less than l_B . We claim that sub-load J only contains wheat with protein concentration at l_j . If sub-load J is a commingle of wheat with different protein concentrations and if its mean protein concentration is higher than l_B , then according to Corollary 3.1 it is profitable to segregate sub-load J . If sub-load J is a commingle of wheat with different protein concentrations and if its mean protein concentration is equal to l_B , then on the price curve we can always find two points, say E and F , such that (1) E is on the left of point B and F is on the right of point B ; and (2) points E , B , and F form a convex price shape. According to Corollary 3.1, under this situation segregating sub-load J is profitable.

From sub-load Z we can separate out one unit of wheat with mean protein level l_j that is a mix of wheat with protein concentration l_k and some wheat with mean protein concentration l_C . Exchanging this unit of mix separated from sub-load Z with one unit wheat from sub-load J does not affect the mean protein concentrations of both sub-load Z and sub-load J . Therefore, the total revenue is not affected by this exchange. However, the grower can increase her revenue by segregating the unit of mix originally from sub-load Z but now in sub-load J . One way to do so is to segregate the mix into two groups, where one group has mean protein concentration at l_C and the other group has mean protein concentration at l_D . The three points, C , J , and D , form a convex price schedule (See Figure 3.19). Therefore, according to Corollary 3.1 the grower can increase her revenue by segregating the unit of mix.

Step 5. We have shown in Step 1 that line CD defined in Definition 3.3 is unique and exists when $\mu > l_A$. We also have shown that there is one and only one sub-load, namely sub-load Z , that has mean

protein concentration less than l_B but equal to l_C in Step 2 and Step 3. In Step 4 we showed that there is no wheat with protein concentration higher than l_D in sub-load Z , which implied that sub-load Z is a commingle of wheat with protein concentration no higher than l_D . Because wheat with protein concentration higher than l_D is under a convex price schedule and there is no commingling opportunity for such wheat, these wheat will be completely segregated according to Proposition 3.2. In sum, for the type I price schedules, when $\mu > l_A$ then in the optimal arrangements the grower should have wheat with protein concentration higher than l_D completely segregated and the remaining wheat completely blended. This finishes the proof. \square

Item C

In this item we prove Proposition 3.1 where the optimization problem under three-step price schedule is discussed. To prove the proposition, several lemmas are necessary.

Lemma 3.2. *For a load of wheat with protein distribution $F(l)$ and mean $\mu \in (0, l_2)$, the maximized amount of wheat with mean protein concentration no lower than l_2 (or l_1) that can be segregated out from the initial load is $1 - F(t_2)$ (or $1 - F(t_1)$).*

Proof. The proof is quite straightforward according to Definition 3.1. Suppose now all wheat with protein concentration that is no less than t_2 is segregated into sub-load S_3 . By the definition of t_2 we know that the mean protein concentration of sub-load S_3 is l_2 . In order to increase the weight of sub-load S_3 , one must add some of the remaining wheat into sub-load S_3 . However, the remaining wheat now has protein concentration lower than t_2 , which is lower than l_2 . Adding such wheat into sub-load S_3 will make the mean protein concentration in the sub-load lower than l_2 . Therefore, $1 - F(t_2)$ is the largest amount of wheat with protein concentration at l_2 . The same argument applies when proving the other part of this lemma. \square

Lemma 3.3. *In the optimal arrangements, (i) if the quantity of sub-load S_1 is strictly positive (i.e., $q_1^* > 0$), then the mean protein levels of sub-loads S_2 and S_3 are equal to l_1 and l_2 , respectively; (ii) if the quantity of sub-load S_2 or S_3 is strictly positive (i.e., if $q_2^* > 0$ or if $q_3^* > 0$), then the mean protein level of sub-load S_3 is l_2 .*

Proof. The proof is completed by simple arbitrage arguments. For item (i), if $q_1^* > 0$ but $\mu_2 > l_1$, then the grower can always increase her revenue by blending some wheat from sub-load S_1 to sub-load S_2 as long as $\mu_2 \geq l_1$. This is because the wheat that is moved from sub-load S_1 to sub-load S_2 now is sold at price p_2 instead of price p_1 and the price of wheat initially in sub-load S_2 is not affected. The same argument applies for $\mu_3 = l_2$ of item (i) and for the first part of item (ii). If $q_3^* > 0$, then $q_2^* > 0$ or $q_1^* > 0$, or both. This is because by assumption we have $\mu < l_2$. By item (i) and the first part of item (ii) we know that $\mu_3 = l_2$. \square

Let $l_{S_1}^{max}$ denote the protein concentration of wheat that has the highest protein in S_1 . Let $l_{S_2}^{min}$ (or $l_{S_3}^{min}$) denote the protein concentration of wheat that has the lowest protein in sub-load S_2 (or S_3). The next lemma can be stated as

Lemma 3.4. *In the optimal arrangements, we have $l_{S_1}^{max} \leq \min[l_{S_2}^{min}, l_{S_3}^{min}]$. That is, any wheat with protein concentration no higher than $l_{S_1}^{max}$ is in sub-load S_1 and hence $q_1^* = F(l_{S_1}^{max})$.*

Proof. Suppose in the optimal arrangement we have $l_{S_1}^{max} > \min[l_{S_2}^{min}, l_{S_3}^{min}]$. That is, protein concentration of some wheat in sub-load S_1 is higher than protein concentration of some wheat in sub-load S_2 or in sub-load S_3 . We first study the case in which $\min[l_{S_2}^{min}, l_{S_3}^{min}] = l_{S_2}^{min}$. In this case the grower can increase her revenue by doing: step (1), exchanging 1 unit of $l_{S_1}^{max}$ wheat from sub-load S_1 with 1 unit wheat with protein concentration lower than $l_{S_1}^{max}$ from sub-load S_2 ; and step (2) moving δ amount of wheat with protein concentration lower than l_1 from sub-load S_1 to sub-load S_2 as long as μ_2 is no less than l_1 . By doing step (1), μ_2 is increased and but the revenue is not affected; by doing step (2), q_2 is increased by δ and q_1 is decreased by δ . So is the revenue is increased by $\delta(p_2 - p_1)$.

When $\min[l_{S_2}^{min}, l_{S_3}^{min}] = l_{S_3}^{min}$, then the grower can increase her revenue by doing: step (1), exchanging 1 unit of $l_{S_1}^{max}$ wheat from sub-load S_1 with 1 unit wheat with protein concentration lower than $l_{S_1}^{max}$ from sub-load S_3 ; and step (2), moving δ amount of wheat with protein concentration lower than l_1 from sub-load S_1 to sub-load S_3 as long as μ_3 is no less than l_2 . By doing step (1), μ_3 is increased and but the revenue is not affected; by doing step (2), q_3 is increased by δ and q_1 is decreased by δ . So is the revenue is increased by $\delta(p_3 - p_1)$.

Therefore, we have shown that if $l_{S_1}^{max} > \min[l_{S_2}^{min}, l_{S_3}^{min}]$ then the grower can increase her revenue by rearranging wheat between the three sub-loads. So in the optimal arrangement we must have $l_{S_1} \leq$

$\min[l_{S_2}^{min}, l_{S_3}^{min}]$. This finishes the proof. \square

Lemma 3.5. *In the optimal arrangements, (i) if $t_2 \leq l_1$, then the highest protein concentration of wheat grain in sub-load S_1 is in the interval $[t_1, t_2]$ (i.e., $l_{S_1}^{max} \in [t_1, t_2]$); (ii) if $t_2 > l_1$, then the highest protein concentration of wheat grain in sub-load S_1 is in the interval $[t_1, \hat{l}_1]$ (i.e., $l_{S_1}^{max} \in [t_1, \hat{l}_1]$).*

Proof. First, we show that $l_{S_1}^{max} \geq t_1$. If $t_1 = 0$, then $l_{S_1}^{max} \geq t_1$ is trivial. Now we consider the situation where $t_1 > 0$. If $l_{S_1}^{max} < t_1$, then the mean protein concentration of the commingle of wheat in sub-load S_2 and wheat in S_3 will be lower than l_1 , which is inconsistent with $\mu_2 \in [l_1, l_2)$ and $\mu_3 \geq l_2$.

Second, we show that if $t_2 \leq l_1$ then $l_{S_1}^{max} \leq t_2$. Suppose we have $l_{S_1}^{max} > t_2$. Then according to Lemma 3.4 the mean protein concentration of the commingle of wheat in sub-load S_2 and wheat in S_3 will be higher than l_2 , which contradicts that in the optimal arrangements $\mu_2 \in [l_1, l_2)$ and $\mu_3 = l_2$ (Lemma 3.3).

Third, we show that if $t_2 > l_1$ then $l_{S_1}^{max} \leq \hat{l}_1$. Please note that \hat{l}_1 is defined when $t_2 > l_1$. Suppose $l_{S_1}^{max} > \hat{l}_1$ when $t_2 > l_1$. Therefore we have $q_1^* = F(l_{S_1}^{max}) > 0$. Taking q_1^* as fixed, to maximize the revenue is equal to maximize q_3 under the constraint of $\mu_2 \geq l_1$. The maximized q_3 is $1 - F(t_2)$. Since $t_2 > l_1$ and $l_{S_1}^{max} > \hat{l}_1$, we must have $\mu_2 > l_1$, which is not optimal (Lemma 3.3). \square

Lemma 3.6. *In the optimal arrangements, if any one of the following two conditions holds, then we have $\mu_2 = l_1$. These two conditions are (i) $t_2 \leq l_1$; and (ii) both $t_2 > l_1$ and $\hat{l}_1 > 0$.*

Proof. If $q_1^* > 0$, then according to Lemma 3.3 we have $\mu_2 = l_1$. Now we prove that the claim is true when $q_1^* = 0$.

If $q_1^* = 0$, then we must have $q_2^* > 0$ and $q_3^* \geq 0$ because we assume that the mean protein is such that $\mu < l_2$. This implies that $\mu \geq l_1$. If $t_2 \leq l_1$, then the initial load of wheat can be seen as a commingle of wheat with l_2 protein concentration and wheat with l_0 protein concentration, where $l_0 \equiv \int_0^{t_2} f(l)l dl / \int_0^{t_2} f(l) dl < l_1$. According to Lemma 3.1, the initial load of wheat can be segregated into two sub-loads with one sub-load having protein concentration at l_1 and the other sub-load having protein concentration at l_2 . Given $q_1^* = 0$, this segregation is optimal. It is because that if $q_1^* = 0$, then the optimal segregation should be to maximize q_3 while keeping $\mu_2 \geq l_1$. Some algebra can show that q_3 is not maximized when $\mu_2 > l_1$.

If $q_1^* = 0$, then q_2^* and q_3^* should be such that

$$\begin{cases} q_2^* + q_3^* = 1 \\ q_2^* \mu_2 + q_3^* l_2 = \mu. \end{cases} \quad (3.22)$$

Solving (3.22) we obtain $q_3^* = \frac{\mu - \mu_2}{l_2 - \mu_2}$. Then we have

$$\frac{dq_3^*}{d\mu_2} = \frac{\mu - l_2}{(l_2 - \mu_2)^2} < 0. \quad (3.23)$$

Therefore, in the optimal arrangement μ_2 must be equal to l_1 if $t_2 \leq l_1$. The same procedure follows when proving condition (ii) implies $\mu_2 = l_1$. \square

Now let us prove Proposition 3.1.

Proof. To optimally process one load of wheat is to explore the benefit of segregation based on the information from measuring protein concentration. Since the grower's goal is to find out the optimal q_1 , q_2 , and q_3 to maximize her revenue, the objective function is $\max_{q_i} \sum_{i=1}^3 p_i q_i$. The major work of specifying an appropriate form of programming problem for the grower is to correctly deal with the non-linear segregating property imposed by protein concentration distribution. In this proof we show that the constraints specified in problem (3.2) and problem (3.3) achieve this goal. We only need to focus on q_1 because q_1 and q_2 are determined by the quantity and protein sum-up constraints whenever q_1 is fixed.

Constraint $\int_0^{F^{-1}(q_1)} f(l) dl + q_2 l_1 + q_3 l_2 = \mu$ says two things. First, it says that the total protein quantity is not affected by segregation. Second, it says that sub-load S_1 consists of wheat distributed on a continuous section of protein distribution (Lemma 3.4). Without this constraint the program becomes a linear program and the mean protein level in sub-load S_1 will always be driven down to zero. Constraint $F(t_1) \leq q_1 \leq F(t_2)$ is determined by Lemma 3.5. At last, constraints $q_i \geq 0$ and $\sum_i q_i = 1$ are needed for clear reasons.

These four constraints define a space that the optimal solutions must be in. Next we show that any point of (q_1, q_2, q_3) satisfying these constraints is achievable given the wheat distribution $F(l)$. If $F(t_1) \leq q_1 \leq F(t_2)$ and if $t_2 \leq l_1$ then the blend of wheat in sub-loads S_2 and S_3 is no less than l_1 and this blend can be viewed as a mix of wheat with mean protein at l_2 and wheat with mean protein at level that is lower than l_1 . Then by Lemma 3.1 we know that this mix can always be segregated into wheat

with protein l_2 and wheat with protein l_1 . That is, given any $q_1 \in [F(t_1), F(t_2)]$, the quantities q_2 and q_3 of sub-loads S_2 and S_3 solved out from $\Sigma_i q_i = 1$ and $\int_0^{F^{-1}(q_1)} f(l)ldl + q_2l_1 + q_3l_2 = \mu$ are achievable.

A similar procedure is followed when proving item (ii). For item (iii), when $t_2 > l_1$ and $\hat{l}_1 = 0$, then according to Lemma 3.5 we have $q_1^* = 0$. Then in the optimal arrangements q_3 must be maximized under the constraint $\mu_2 \geq l_1$. By the definition of \hat{l}_1 we have $\int_{\hat{l}_1}^{t_2} f(l)ldl / \int_{\hat{l}_1}^{t_2} f(l)dl \geq l_1$. By Lemma 3.2 we know that $q_3^* = 1 - F(t_2)$. Therefore, $q_2^* = F(t_2)$. This concludes the proof. \square

Sivaraman *et al.* (2002) claim that their method applies to step premium schedules (page 157, Case 4). However, their claim is not correct. They assume that in the optimal outcomes the protein levels in one bin are continuous, (i.e., $D_i = [d_{i-1}, d_i]$ in the last paragraph on page 156). But this may not be true. Here is an example. Suppose the protein concentrations of one load of wheat is uniformly distributed on $[11.4\%, 13.6\%]$. Then the average protein level of this load is 12.5%. The price schedule has wheat with protein level higher than or equal to 13% receiving a high price; wheat with protein level lower than 12% receiving a low price; and the remaining wheat receiving a middle price. Suppose wheat prices encourage blending and the optimal solution is that $q_1^* = 0$, $q_2^* > 0$, and $q_3^* > 0$, here q_1^* , q_2^* , and q_3^* are quantities of wheat that receive low, middle, and high price, respectively. Then $q_2^* = 0.5$ and $q_3^* = 0.5$ can be solved by

$$\begin{cases} q_2^* + q_3^* = 1 \\ 0.12q_2^* + 0.13q_3^* = 0.125. \end{cases} \quad (3.24)$$

Based on the uniform distribution, how should one achieve $q_2^* = 1/2$ and $q_3^* = 1/2$? Is it possible to find $d \in [11.4\%, 13.6\%]$ such that $(d - 11.4\%)/(13.6\% - 11.4\%) = 1/2$ and $(d + 11.4\%)/2 = 12\%$? The answer is no. One procedure that can make $q_2^* = 1/2$ and $q_3^* = 1/2$ is as follows: Step 1. Put wheat with protein levels between 12.4% and 13.6% into one bin, say bin A, and mix them completely; so the average protein level in bin A is 13%; Step 2. Put wheat with protein level between 11.4% and 12.4% into another bin, say bin B; so the average protein level in bin B is 11.9%. Step 3. Move some wheat (with average protein level 13%) from bin A to bin B until the average protein level in bin B reaches 12%. Clearly protein levels of wheat in bin B is not continuous. For example, Bin B could includes wheat with protein levels between 11.4% and 12.4% and wheat with protein levels at 13%.

Table 3.A.1: Trial Locations and Names of Top Varieties of HRW Wheat in the State of Washington (1997-2009)

Locations	(1) Almira (2) Bickleton (3) Connell (4) Coulee city (5) Dayton	(6) Dusty (7) Finley (8) Horse heaven (9) Lamont (10) Lind	(11) Pullman (12) Reardan (13) Ritzville (14) St andrews (15) Walla walla
Varieties	(1) Agripro paladin (2) Bauermeister (1) (3) Boundary (4) Buchanan (5) Columbia – 1 (6) Declo	(7) Eddy (8) Estica (9) Finley (10) Hatton (11) Quantum hybrid 542	(12) Residence (13) Semper (14) Symphony (15) Wanser (16) Weston

Note: The numbers in the parentheses are IDs, which are used in Table 14 to Table 16.

Table 3.A.2: A Sample of HRW Variety Performance Observations

Variety	Year	Location	Protein (%)	Yield (bu./acre)	Test Weight (lbs/bu.)
Buchanan	2006	Almira	9.2	112.4	60.1
Buchanan	2005	Connell	13.2	41.9	59.3
Eddy	2009	Ritzville	12	50	62.3
Finley	2009	Ritzville	11.2	58.6	63.3

Table 3.A.3: Estimates of the Coefficients of Years, Varieties, and Locations for HRW Wheat Protein

variable	coefficient	t value	variable	coefficient	t value
constant	0.1153	22.6973	variety10	-0.0084	-1.8672
year2	-0.0165	-6.5310	variety11	-0.0033	-0.7456
year3	0.0038	1.3690	variety12	-0.0102	-1.9260
year4	-0.0042	-1.3678	variety13	-0.0082	-1.5482
year5	0.0137	5.4812	variety14	-0.0011	-0.2564
year6	0.0028	1.0606	variety15	-0.0058	-1.1062
year7	-0.0034	-1.2555	variety16	0.0016	0.3564
year8	0.0080	2.9421	location2	-0.0184	-6.3658
year9	-0.0007	-0.2265	location3	0.0088	3.5184
year10	0.0052	1.7054	location4	-0.0006	-0.1847
year11	0.0127	3.9615	location5	0.0004	0.1355
year12	0.0145	4.6577	location6	0.0067	1.2968
year13	0.0099	3.1529	location7	0.0119	3.0240
variety2	-0.0095	-2.2997	location8	0.0209	8.8342
variety3	-0.0079	-1.9563	location9	-0.0080	-2.4570
variety4	-0.0141	-3.2322	location10	0.0150	6.3996
variety5	0.0002	0.0295	location11	0.0015	0.6535
variety6	0.0007	0.1719	location12	0.0059	1.8062
variety7	-0.0033	-0.8035	location13	0.0035	1.3373
variety8	-0.0089	-1.9462	location14	-0.0105	-4.4670
variety9	-0.0067	-1.6678	location15	0.0056	2.0292

Table 3.A.4: Estimates of the Coefficients of Years, Varieties, and Locations for HRS Wheat Protein

variable	coefficient	t value	variable	coefficient	t value
constant	0.1185	43.57	location2	0.0067	1.82
year2	0.0051	2.77	location3	0.0055	1.11
year3	0.0148	7.72	location4	0.0039	1.79
year4	0.0233	11.44	location5	0.0202	8.54
year5	0.0318	15.41	location6	0.0031	1.48
year6	0.0357	17.19	location7	0.0073	3.13
year7	0.0321	15.43	location8	0.0084	2.67
year8	0.0350	17.72	location9	-0.0135	-5.83
year9	0.0400	19.79	location10	0.0006	0.27
year10	0.0332	16.53	location11	0.0130	6.27
year11	0.0344	16.34	location12	0.0033	1.59
year12	0.0356	17.07	location13	0.0165	5.27
year13	0.0381	18.37	location14	0.0234	7.25
variety2	-0.0060	-2.72	location15	0.0090	3.34
variety3	-0.0100	-2.93	location16	0.0259	9.24
variety4	-0.0105	-3.86	location17	-0.0007	-0.32
variety5	-0.0089	-5.28	location18	0.0083	3.19
variety6	-0.0035	-1.97	location19	0.0107	3.56
variety7	-0.0062	-2.39	location20	-0.0008	-0.37
variety8	-0.0096	-5.79	location21	0.0058	1.21
variety9	0.0040	1.54	location22	0.0061	2.93
variety10	-0.0005	-0.13	location23	-0.0029	-1.31
variety11	-0.0114	-6.85	location24	0.0158	6.13
variety12	-0.0097	-4.09	location25	0.0144	5.04
variety13	-0.0080	-4.44	location26	0.0010	0.50
variety14	-0.0043	-2.48	location27	0.0059	1.54
variety15	-0.0106	-2.77	location28	0.0030	1.36

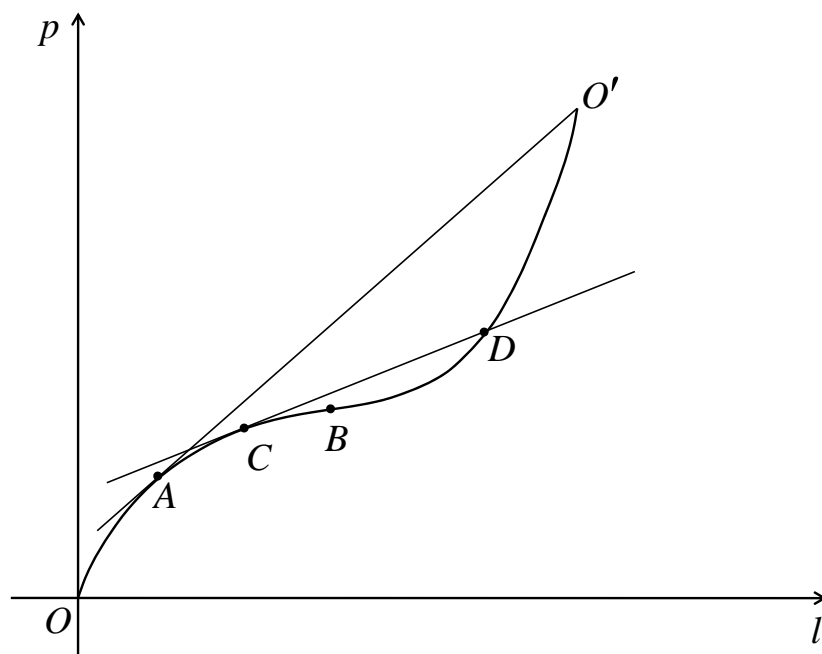


Figure 3.12: Price Schedule which Turns from being Concave to Convex

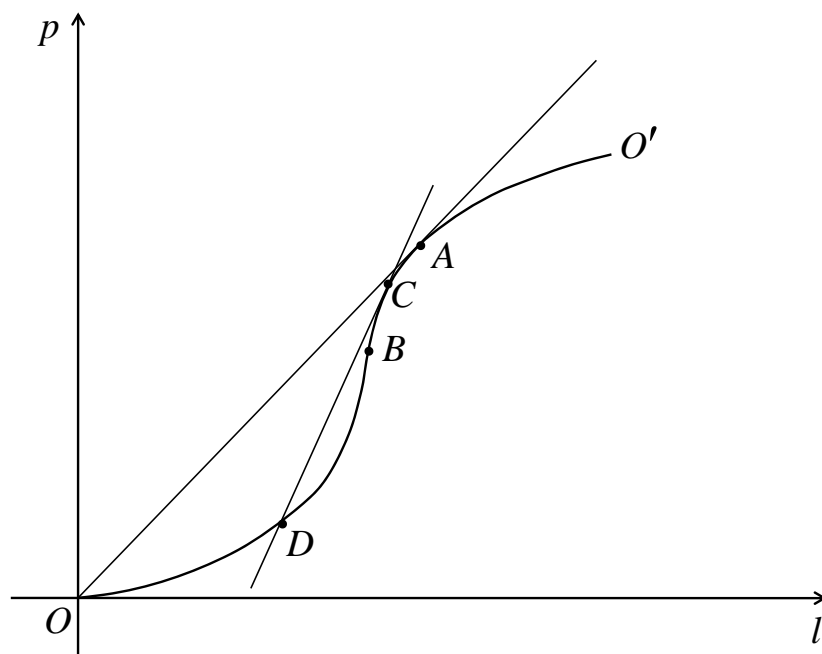


Figure 3.13: Price Schedule which Turns from being Convex to Concave

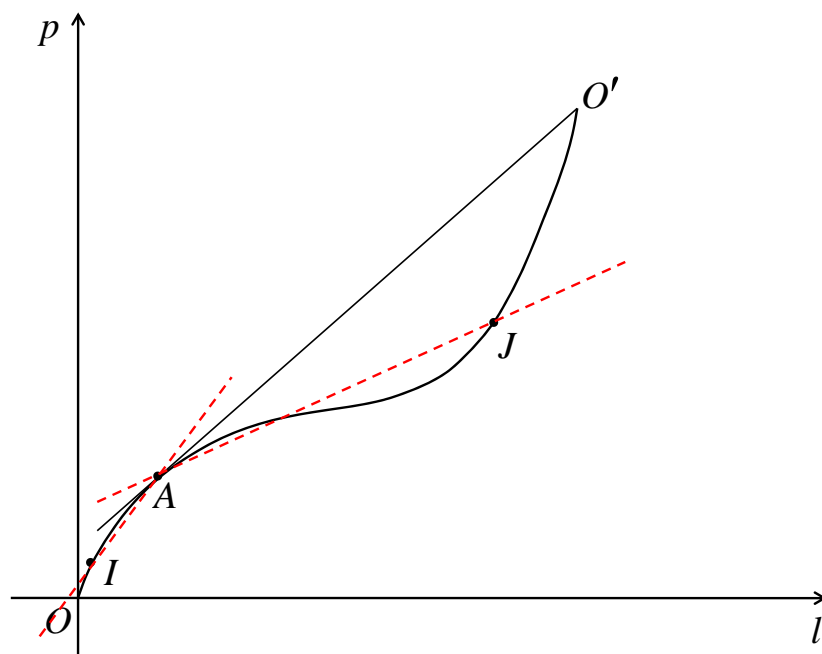
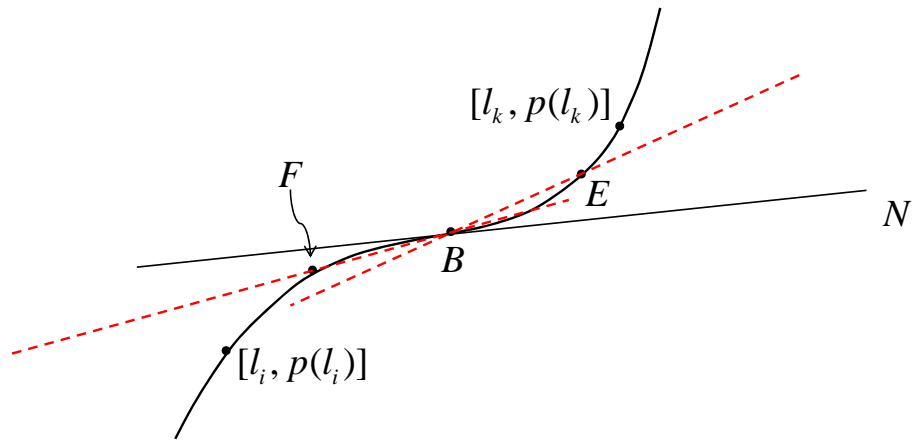


Figure 3.14: Points I , A , and J Form a Concave Price Schedule



Note: The slope of line BN is the same as the slope of price schedule curve at point B .

Figure 3.15: Points E , B , and F Form a Convex Price Schedule

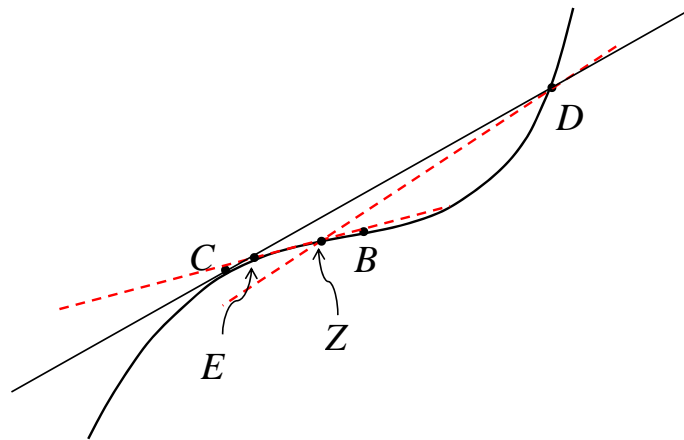
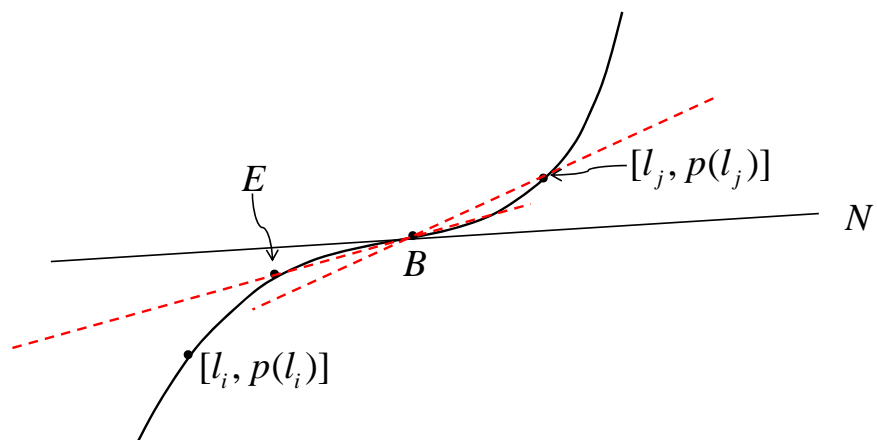


Figure 3.16: Points E , Z , and D Form a Convex Price Schedule



Note: The slope of line BN is the same as the slope of price schedule curve at point B .

Figure 3.17: Points E , B , and $[l_j, p(l_j)]$ Form a Convex Price Schedule

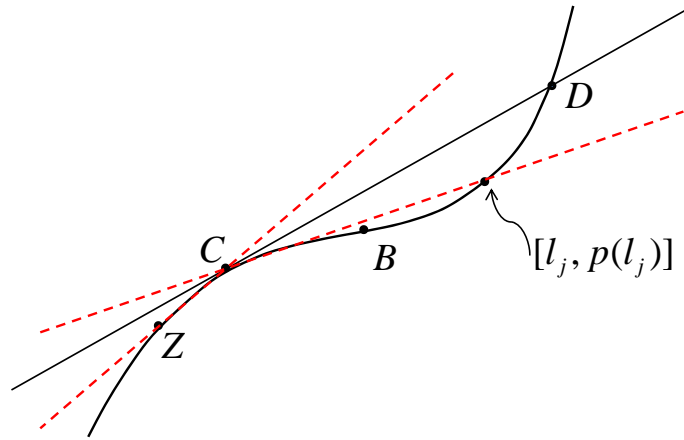


Figure 3.18: Points Z, C, and $[l_j, p(l_j)]$ Form a Concave Price Schedule

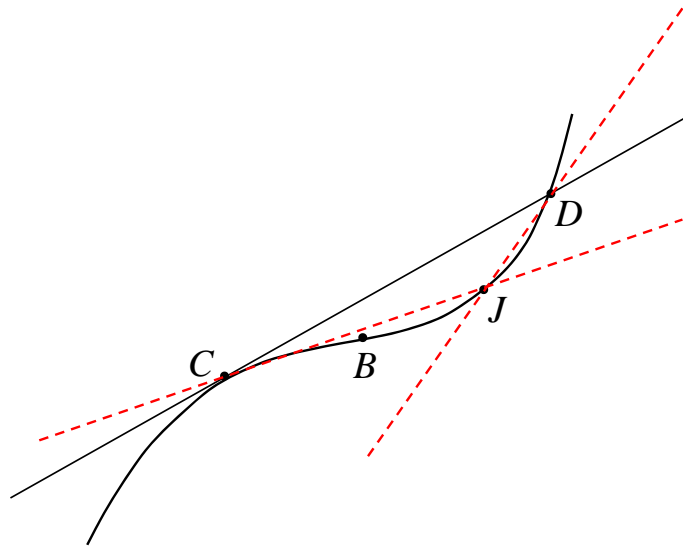


Figure 3.19: Points C , J , and D Form a Convex Price Schedule

Appendix 2

This appendix demonstrates the procedure to obtain monthly HRW and HRS wheat stocks at different protein concentration levels. Here we use HRW wheat as an example. The same procedure applies for HRS wheat. The procedure here is similar to the one on pages 239-240 of Goodwin and Smith (2009), in which the authors constructed monthly protein availability by utilizing quarterly wheat stocks data obtained from USDA NASS and protein content data obtained from the U.S. Wheat Associates.

Step 1): aggregate quarterly stocks for all wheat (1980-2010) are obtained from USDA NASS; here “all wheat” means wheat in all six wheat classes, not just HRW and HRS wheat.

Step 2): calculate the percentage of HRW wheat stock among all wheat stock using data of “ending stocks by class” from the annual *Crop Quality Report* published by U.S. Wheat Associates during 1980-2010.

Step 3): all wheat quarterly stocks in step 1) are multiplied by the percentage in step 2) to get the quarterly HRW wheat stock.

Step 4): use cubic spline interpolation to quarterly stocks obtained in step 3) to get monthly HRW wheat stocks.

Step 5): the percentages of HRW wheat with different protein levels in every year from 1980 to 2010 are obtained from *Crop Quality Report* published by U.S. Wheat Associates during 1980-2010. For HRW wheat the crop year is assumed to start on July 1st since most HRW wheat growing regions finish harvest in July. We also assume that the percentages of HRW wheat with different protein levels within one crop year do not change. For HRS wheat the crop year is assumed to start on August 1st and the percentages of HRS wheat with different protein levels within one crop year do not change either. Therefore, in our data set the time range from monthly wheat stocks is between August 1980 and December 2010.

Step 6): use data from Step 5) to calculate the percentages of the following three categories of HRW wheat in total HRW wheat stock: HRW wheat with protein level less than 12%, HRW wheat with protein level higher than or equal to 12% but less than 13%, HRW wheat with protein level higher than or equal to 13%.

Step 7): monthly HRW wheat stocks from step 4) were multiplied by percentages in step 6) to get

the monthly stocks of the three types of HRW described in step 6).

CHAPTER 4. TO LEARN OR TO CHANGE: OPTIMAL R&D INVESTMENTS UNDER UNCERTAINTIES IN THE CASE OF CLIMATE CHANGE

Abstract

When studying investment in R&D to increase the society's ability to face challenges from climate change (termed as "research to change" or RTC), current literature overlooks the existence of purchased learning (termed as "research to learn" or RTL) in which new information on climate change is acquired by investment in research activities. Since RTL absorbs substantial research resources in climate change research, it cannot be ignored when seeking to optimize resource allocation for addressing climate change problems. In this article we explore the interactions between investments in RTC and RTL under uncertainties of climate change. Here uncertainties include uncertainty about how serious climate change's damage is, and uncertainty about when the research activities succeed. We find that 1) if the success of RTL and RTC are statistically independent, then it is almost never optimal to invest in RTL and RTC simultaneously; 2) if the success of RTL accelerates the success of RTC, then RTC and RTL are substitutes; 3) if the success of RTL accelerates the success of RTC and if the cost of RTL is small enough, then it is never optimal to invest in RTC only. Factors that affect the optimal investment levels in RTC and RTL are studied as well.

Key words: climate change; R&D investments; uncertainties

JEL classification: Q28, D83.

Introduction

Uncertainties often arise when faced with emerging problems whose possible impacts on human welfare are not conclusively understood, such as climate change or an outbreak of certain animal diseases. These uncertainties are on: (1) the magnitude of the problems' human welfare impact, and (2) the future date when this unknown impact becomes clear. Responding to these two dimensions of the uncertainty surrounding a possible hazard is likely to require two distinct lines of research. Take climate change as an example. To mitigate possible negative impact on humans, much research has been devoted to greenhouse gas (GHG) emission abatement technologies, energy efficient technologies, renewables, and adaptation technologies. We term this research category as "research to change" (RTC). On the other hand, research is also devoted to studying the uncertain impact of climate change itself. Will it be a manageable 2°C or a 4°C change? Or is climate change caused by greenhouse gas accumulation or something else, such as solar activity (Svensmark and Calder, 2008)? We term this category as "research to learn" (RTL). RTL can accelerate the resolution time of the uncertain impact so as to improve decisions on extent of resources to be put into RTC. If in the future climate change is proved to have only a mild effect, some of the RTC investment will have turned out to have been wasted. However, if climate change proves disastrous, we would have wanted more RTC. Or if it turns out that the real reason for climate change is something else other than GHG accumulation, then the tremendous effort to reduce GHG emission would be mis-targeted. RTL decreases the probability of making these mistakes. Since RTL is costly as well, optimal decisions on RTC investment should take into account the interaction between RTL and RTC.

Even though our analysis will focus on climate change, the message in this study can be applied to many other cases. For instance, while lacking firm evidence, some scientists believe that Crohn's disease in humans can be caused by Johne's disease in cattle (Uzoigwe *et al.*, 2007). In this case, RTC includes research into preventing or treating Johne's disease or into technologies that can cut off channels by which Johne's disease affects humans. RTL may include research to find out the true relationship between these two diseases. The interaction between RTC and RTL should be considered when allocating resources to research regarding Johne's disease. If Johne's disease does cause Crohn's disease, then more RTC will be justified. But if it does not, then these research investments will turn

out to be a waste. Therefore, RTL is favored in the sense that it can prevent this kind of waste.

Our model can also be applied in the decision process at the company level. When the concept of a new product that has a potential to be profitable becomes available, a company in a related industry can either invest into market analysis of this new product to find the true state of its profitability (i.e., RTL) or invest into activities to study how to accommodate this new product into their existing production lines or even invest into building a new production line (i.e., RTC). In this scenario the study of the interaction between RTL and RTC is of especial interest since the study can help firms make the right decision.

Research outcomes are also uncertain. For RTC, it could either end up with a failure or a breakthrough. By breakthrough we mean hereafter that the RTC reaches its goals and potential problems are solved. For instance, if we had a breakthrough in greenhouse emission abatement or alternative energy technologies like biofuels, wind or solar, the possible welfare effect of climate change will be largely eliminated. The outcome of RTL is also uncertain. If it is successful, it can accelerate the resolution date of the welfare impact uncertainty of climate change. Under these uncertainties, what is the optimal allocation of scarce research resources between RTL and RTC?

Moreover, RTL and RTC are not necessarily independent of each other. The reason is that research is a such complex activity that the output of one research may have positive externalities for the success of other research. Therefore it is reasonable to assume that the success of RTL can contribute to breakthrough in RTC. For example, in the case of Johne's disease and Crohn's disease, during the process of RTL (i.e., research to find out the true relationship between these two diseases), some knowledge may be generated that can help the RTC (i.e., research to prevent or treat Johne's disease). A breakthrough of RTC may also accelerate the success in RTL. Again in the example of Johne's disease and Crohn's disease, the success of research into treating Johne's disease may help scientists better understand the true relationship between these two diseases.

This article explores the interaction between RTL and RTC as well as the optimal allocation of research resources in the face of uncertainties. The model we develop in this paper can be summarized as follows. At time $t = 0$ a social planner has to make choices to respond to an emerging problem whose effects are uncertain. She can either invest in RTL or RTC or both. But when these research will be successful is uncertain. Let τ_1 denote the time when the true impact of climate change is identified

(either by RTL or only by the passage of time (i.e., autonomous learning)). Here τ_1 is a random variable. If at τ_1 the RTC has not succeeded yet, the social planner will make the investment decision on RTC again based on new information from RTL at that time. Moreover, in this case further RTC may benefit from the success of RTL. If the success of RTC happens before τ_1 , the social planner will apply the outcome of this research immediately after the success so that the possible negative welfare impact is eliminated. In our model we assume there is no switch cost for applying the outcome of RTC. For the discussion of the effects of switch cost on optimal decisions we refer readers to Hennessy and Moschini (2006). In this setting we study the interactions between these two kinds of research and the factors that affect the optimal allocation of resources devoted to them.

This article lies in the strand of literature that studies the effect of learning on an irreversible decision. Research investment decisions fit into the theoretical framework developed by this literature because the research cost cannot be redeemed later. Classic examples of early work in this literature includes Weisbrod (1964), Arrow and Fisher (1974), Henry (1974) and Hanemann (1989). Their studies show that the possibility of learning in the future that can mitigate the uncertainties will encourage more preservation in the present decision. Later work in this literature argued that if there are two opposing irreversibilities associated with the decision, the effect of uncertainty and learning on the irreversible decision is ambiguous. Examples are Olson (1990), Kolstad (1996), and Marwah and Zhao (2007).

By applying the model in Hanemann (1989), Schimmelpfennig (1995) argues that in order to keep the option of using energy efficient technologies open, learning in the future will encourage decision makers to invest more on those technologies. Baker *et al.* (2006) study optimal decisions on investment in R&D programs under climate uncertainty. But in that study Baker *et al.* assume the outcome of R&D programs is certain. Baker and Adu-Bonnah (2008) study investment in risky R&D programs under climate uncertainty. Baker and Shittu (2008) provide a comprehensive review of investment into R&D in climate models under uncertainties.

The above literature implicitly assumes that uncertainties are only resolved by autonomous learning (i.e., the passage of time) instead of by purchased learning (i.e., RTL).¹ In reality, however, purchased learning does consume a significant part of research resource.² Hennessy and Moschini (2006) study the

¹Kolstad (1996) discussed three types of learning related with climate change. (1) Active learning, (2) Purchased learning (R&D), and (3) Autonomous learning.

²For example, the federal obligations for research in environmental sciences in FY 2006 was

optimal scientific research (purchased learning) on the damage that a certain practice could cause when a social planner is considering whether to ban this practice. We expand their model into a continuous choice model that takes uncertainty about research outcomes into account. Utilizing this model we explore interactions between RTC and RTL. Factors that affect the optimal allocation of resources to RTC and RTL are studied as well.

The rest of this article is structured as follows. Section 2 outlines the basic optimization model of a social planner. Section 3 analyzes the interaction between RTC and RTL when these two lines of research are independent (i.e., the success of one line of research does not affect the success of the other line of research). Section 4 studies the interaction between RTC and RTL when these two lines of research are not independent. Section 5 conducts a comparative statics analysis of the optimal RTC and RTL decisions. Section 6 concludes with a discussion of possible extensions of this research.

Model

A social planner seeks to minimize the expected negative welfare impact of climate change by investing in RTC and RTL. At time $t = 0$, the climate change's impact is uncertain to her. For simplicity we assume there are only two future states of nature: a true state T with probability $q \in [0, 1]$ in which climate change imposes an constant instantaneous damage D and a false state F with probability $1 - q$ in which it does not impose any damage. Uncertainty about this welfare impact will be resolved at time $t = \tau_1$, which has an exponential distribution with density function $f(\tau_1) = (l_0 + l)e^{-(l_0 + l)\tau_1}$, where $l_0 > 0$ and $l \geq 0$ are distribution parameters. Here l_0 is fixed but l is determined by the social planner's investment in RTL. The higher the investment is, the larger the value of l . If there is no investment in RTL, then $l = 0$. Suppose that α amount of investment in RTL will increase l by one unit. At time $t = 0$ the social planner can also conduct RTC to respond to the potential damage of climate change. That is, the social planner can take a precautionary action to respond to the possible damage from climate change. We assume that the success of RTC will happen at time $t = \tau_2$, which is also exponentially

\$3.4 billion (Data source: National Science Foundation (NSF)). Environmental sciences are defined by NSF as, "Environmental sciences (terrestrial and extraterrestrial) are, with the exception of oceanography, concerned with the gross nonbiological properties of the areas of the solar system that directly or indirectly affect human survival and welfare. " Available at: <http://www.nsf.gov/statistics/nsf10303/tables/tab23.xls> (accessed on 1/21/2012).

distributed with density function $g(\tau_2) = ce^{-c\tau_2}$. By choosing c , the social planner can govern the expectation of the success time $1/c$. In order to increase c by one, β units of investment in RTC is needed. We also assume τ_1 and τ_2 are independently distributed. Figure 4.1 shows a visual presentation of the social planner's decision problem. From Figure 4.1 we can see the social planner's decision problem consists of four sub-problems, A) to D), which are studied next.

Sub-problem A) happens when a RTL breakthrough occurs before success in RTC and the true state of the world is T . In this sub-problem, since the RTC has not been successful yet when the true state is revealed, an acceleration of the success time of RTC may be desirable. Therefore at time τ_1 the social planner will choose $c' \geq 0$ to minimize the total costs (i.e., the damage of climate change plus research costs). Here c' is the added investment into RTC at time τ_1 . Since success in RTL may contribute to a breakthrough in RTC, then at time τ_1 the breakthrough time for RTC has a new probability density function. For notational clarity we utilize τ_3 to denote the breakthrough time of RTC when it happens after τ_1 . Then the probability density function of τ_3 is

$$g'(\tau_3) = (c + c' + \eta)e^{-(c+c'+\eta)(\tau_3-\tau_1)}, \quad (4.1)$$

where $\tau_3 > \tau_1$ and $\eta \geq 0$.³ Here η measures the magnitude of the contribution of the success of RTL to RTC. The smaller the η is, the less the contribution. If $\eta = 0$, then there is no such contribution. We assume that $\eta \geq 0$ and η is only defined when $\tau_2 > \tau_1$. This means that only the outcome (the success of RTL), not the input, of RTL can affect the breakthrough time of RTC.

Mathematically, Sub-problem A) can be written as

$$V^A(c, l) = \beta c + \alpha l + \min_{c' \geq 0} \left\{ \beta c' e^{-r\tau_1} + E_{\tau_3} \left[\int_0^{\tau_3} D e^{-rt} dt \right] \right\}, \quad (4.2)$$

where $V^A(c, l)$ denotes the minimized total cost in sub-problem A) given the value of c and l . Here r is the continuous time discount rate. We assume that $\eta < \sqrt{D/\beta} - r$. This assumption eliminates the possibility that the contribution of RTL to RTC (i.e., η) is so high that the social planner will find

³This density function can be motivated in the following way. We let u_1 denote the breakthrough time for RTC governed by the newly added investment into RTC, c' ; and let u_2 denote the breakthrough time for RTC governed by the outcome of RTL. Their density functions are $h_1(u_1) = c'e^{-c'(u_1-\tau_1)}$ and $h_2(u_2) = \eta e^{-\eta(u_2-\tau_1)}$, respectively. Here we have $u_1 > \tau_1$ and $u_2 > \tau_1$. The density of τ_2 conditional on $\tau_2 > \tau_1$ is $f(\tau_2 | \tau_2 > \tau_1) = ce^{-c(\tau_2-\tau_1)}$. We assume u_1 , u_2 , and τ_2 are independent. Then we define $\tau_3 \equiv \min\{(\tau_2 | \tau_2 > \tau_1), u_1, u_2\}$. It is easy to check that τ_3 has density function $(c + c' + \eta)e^{-(c+c'+\eta)(\tau_3-\tau_1)}$, where $\tau_3 > \tau_1$.

it is optimal to not invest into RTC at time τ_1 even if the true state is proved as T . This assumption implies that $\sqrt{D/\beta} - r > 0$, which means that were climate change harmful for sure (i.e., $q = 1$) then the optimal c will be greater than 0. Justification for this assumption is provided in Item A of Appendix. We formally state this assumption as

Assumption 4.1. *The contribution of RTL's success to RTC's success will never be so high that the social planner will find it optimal to not invest into RTC at time τ_1 even if the true state is proved as T . That is $\eta < \sqrt{D/\beta} - r$.*

Assuming an interior solution for c' , we show that

$$V^A(c, l) = \beta c + \alpha l + \frac{D}{r} + (2\sqrt{\beta D} - \beta(c + r + \eta) - \frac{D}{r})e^{-r\tau_1}. \quad (4.3)$$

The algebra to obtain equation (4.3) is presented in Item B of Appendix.

Sub-problems B) and D) are straightforward. If at time τ_1 it is proved that climate change is not harmful (i.e., State F), then the social planner will not put more investment into RTC. Therefore the total cost for the social planner in sub-problem B) or D) is only

$$V^B(c, l) = V^D(c, l) = \beta c + \alpha l. \quad (4.4)$$

When the RTC breakthrough happens not later than the RTL success (i.e., $\tau_2 \leq \tau_1$), then the social planner will adopt the outcome of RTC immediately even though at time τ_2 the true state of the world has not been realized yet.⁴ If the true state of the world is T , then the social planner faces sub-problem C). In this case the total cost is

$$V^C(c, l) = \beta c + \alpha l + \int_0^{\tau_2} D e^{-rt} dt. \quad (4.5)$$

Therefore, at time 0 the social planner's problem is to choose $c \geq 0$ and $l \geq 0$ to minimize the total cost $V(c, l)$, which is

$$\begin{aligned} V(c, l) = & \int_0^\infty \int_{\tau_1}^\infty [qV^A(c, l) + (1 - q)V^B(c, l)]g(\tau_2)f(\tau_1)d\tau_2d\tau_1 \\ & + \int_0^\infty \int_0^{\tau_1} [qV^C(c, l) + (1 - q)V^D(c, l)]g(\tau_2)f(\tau_1)d\tau_2d\tau_1, \end{aligned} \quad (4.6)$$

⁴Please notice that we assume there is no switch cost. For a study of the role of switch cost in the optimal decisions when facing uncertainty, we refer readers to Hennessy and Moschini (2006).

where the first (second) term on the right-hand side of equation (4.6) is the expected cost when $\tau_2 > \tau_1$ ($\tau_2 \leq \tau_1$). After some algebra, which is shown in Item C of Appendix, we can simplify problem (4.6) as

$$V(c, l) = \alpha l + \beta c + \frac{q}{r + l_0 + l + c} \{D + [2\sqrt{D\beta} - \beta(c + r + \eta)](l_0 + l)\}. \quad (4.7)$$

The first two terms on the right-hand side of equation (4.7) (i.e., αl and βc) are the research investments into RTL and RTC, respectively. The third term is the expected damage due to climate change when investment into RTC and RTL are c and l , respectively. It is easy to check that if $q = 0$ then the damage would be 0. Suppose $c = l = l_0 = 0$, then the third term becomes qD/r , which is the expected negative welfare impact of the climate change when the social planner does nothing and when autonomous learning will take an infinite amount of time to reveal the true state of climate change's impact. An observation is that the cost of RTL, α , has no effect on the third term given (c, l) . This is because the decision on RTL only happens at time 0. Unlike the decision on RTC, once the decision on RTL is made then the social planner will no longer need to make further decisions on RTL. Therefore, given (c, l) , changing α does not affect the expected damage due to climate change. The social planner's problem can be written as

$$\min_{c, l \geq 0} V(c, l), \quad (4.8)$$

where $V(c, l)$ is described in equation (4.7). The first order conditions (FOCs) of problem (4.8) are:

$$\frac{\partial V}{\partial c} = \beta - q \frac{(\sqrt{D} + \sqrt{\beta}(l_0 + l))^2 - \beta\eta(l_0 + l)}{(r + l_0 + l + c)^2} \geq 0 \quad (4.9)$$

$$\frac{\partial V}{\partial l} = \alpha - q \frac{(\sqrt{D} - \sqrt{\beta}(c + r))^2 + \beta\eta(r + c)}{(r + l_0 + l + c)^2} \geq 0. \quad (4.10)$$

Algebra to obtain the FOCs are shown in Item D of Appendix, in which we also show that under Assumption 1 $V(c, l)$ is convex. Therefore, the values (c^*, l^*) that satisfies FOCs (4.9) and (4.10) are the optimal solutions to problem (4.6). In the following sections we first analyze the model by setting $\eta = 0$ (i.e., RTC and RTL are independent). This baseline model has closed-form solutions and provides a benchmark for later analysis. Then we analyze the model with $\eta > 0$, i.e., the success of RTL will positively contribute to the breakthrough in RTC.

Benchmark: RTC and RTL are independent

When RTC and RTL are independent (i.e., $\eta = 0$) then the FOCs from equations (4.9) and (4.10) become

$$\frac{\partial V}{\partial c} = \beta - q \left\{ \frac{\sqrt{D} + \sqrt{\beta}(l_0 + l)}{r + l_0 + l + c} \right\}^2 \geq 0 \quad (4.11)$$

$$\frac{\partial V}{\partial l} = \alpha - q \left\{ \frac{\sqrt{D} - \sqrt{\beta}(c + r)}{r + l_0 + l + c} \right\}^2 \geq 0. \quad (4.12)$$

According to whether or not c^* and l^* are strictly positive, there are four possible cases. They are Case 1, $c^* = 0$ and $l^* = 0$; Case 2, $c^* = 0$ and $l^* > 0$; Case 3, $c^* > 0$ and $l^* = 0$; and Case 4, $c^* > 0$ and $l^* > 0$. We discuss these four cases immediately.

Case 1. $c^* = 0$ and $l^* = 0$.

If $c^* = 0$ and $l^* = 0$, then from the FOCs (4.11) and (4.12) we have

$$q \leq \beta \left\{ \frac{r + l_0}{\sqrt{D} + \sqrt{\beta}l_0} \right\}^2 \equiv q_c \quad (4.13)$$

$$q \leq \alpha \left\{ \frac{r + l_0}{\sqrt{D} - \sqrt{\beta}r} \right\}^2 \equiv q_l. \quad (4.14)$$

Equations (4.13) and (4.14) tell us if $q \leq \min\{q_c, q_l\}$, then optimal c and l will be 0. That is, when climate change is very unlikely to have a negative welfare impact, then there is no incentive to do either RTL or RTC. Here $\min\{q_c, q_l\}$ can be viewed as a probability threshold to determine whether or not a positive amount of investment in RTC or RTL is optimal. By Assumption 1, we have $q_c = (r + l_0)^2 / (\sqrt{D/\beta} + l_0)^2 < 1$. Intuitively, q_c (or q_l) is the probability at which the marginal cost of increasing one unit of investment into RTC (or RTL) equals the marginal benefit (i.e., the deduction of expected negative impact of climate change) of doing so when evaluated at $(c, l) = (0, 0)$.

From equations (4.13) and (4.14) we can see that increasing the cost of RTC, β , will increase both q_c and q_l . However, increasing the cost of RTL, α , increases q_l but will not affect q_c . Here, that q_c (or q_l) is increasing with β (or α) is quite intuitive because the higher the cost of RTC (or RTL), the higher will be the probability thresholds for making an investment in RTC or RTL. But why q_l is increasing with β and why q_c is not affected by α need some explanation. We know that one benefit of investing into RTL is to accelerate the realization of the true state of the world, so that an accurate decision on RTC can be made sooner to reduce the expected negative impact of climate change. But an increase in β will

decrease the incentive to invest into RTC, and hence decrease the incentive to invest into RTL. In an extreme case, if investment into RTC is impossible (say, β is extremely high), then there is no point to invest into RTL at all to get more information about the state of the world. This is why q_l is increasing with β . The cost of RTL, α , does not affect q_c in this case because the decision on RTL only happens at time 0. Unlike the decision on RTC, once the decision on RTL is made, then the social planner will no longer need to make further decisions on RTL. Therefore, given (c, l) , changing α does not affect the marginal benefit of increasing RTC. This is why q_c is not affected by α .

Case 2. $c^* = 0$ and $l^* > 0$.

If $c^* = 0$ and $l^* > 0$, then the FOCs of equations (4.11) and (4.12) are:

$$\frac{\beta}{q} \geq \left\{ \frac{\sqrt{D} + \sqrt{\beta}(l_0 + l^*)}{r + l_0 + l^*} \right\}^2; \quad (4.15)$$

$$\frac{\alpha}{q} = \left\{ \frac{\sqrt{D} - \sqrt{\beta}r}{r + l_0 + l^*} \right\}^2. \quad (4.16)$$

From equation (4.16) we have $l^* = \sqrt{q/\alpha}(\sqrt{D} - \sqrt{\beta}r) - l_0 - r = (\sqrt{q/q_l} - 1)(r + l_0)$. Since $l^* > 0$, then we must have $q > q_l$. Plugging l^* into inequality (4.15) brings us $q \leq (1 - \sqrt{\alpha/\beta})^2 \equiv q_{lc}$. Here q_{lc} is the probability at which given the investment pair $(0, l^*)$ on RTC and RTL, the marginal cost of adding one more unit investment in RTC is equal to the marginal expected benefit of doing so. Therefore, equations (4.15) and (4.16) tell us that if $q \in (q_l, q_{lc}]$, then it is optimal to only invest in RTL. The intuition here is that if the q is neither very high nor very low, i.e., the belief about the welfare impact is “ambiguous,” then investing in RTL only is more favorable. This explains why a much-debated issue usually attracts more research resources, but for establishing its significance and not for solving it. We also find that an increase in the cost of RTL will shrink the range of q supporting this case. This is because that when RTL becomes more expensive, the social planner may either switch research resources from RTL to RTC or just invest nothing in these two activities.

Since the quantitative relationships between q_c , q_l , and q_{lc} are important during the analysis of this paper, here we provide a remark that shows these relationships.

Remark 4.1. *If $q_{lc} > q_c$, then $q_c > q_l$ and hence $q_{lc} > q_c > q_l$; if $q_{lc} \leq q_c$, then $q_c \leq q_l$ and hence $q_{lc} \leq q_c \leq q_l$.*

The proof of Remark 4.1 is shown in Item E of Appendix. The intuition here is that if the cost of

RTL is low enough when compared with the cost of RTC (i.e., $\alpha < \beta \left\{ \frac{\sqrt{D}-\sqrt{\beta}r}{\sqrt{D}+\beta l_0} \right\}^2$), then the probability required for investing into RTL will be lower than the probability required for investing into RTC. If the cost of RTL is high enough, however, then the opposite holds.

Case 3. $c^* > 0$ and $l^* = 0$.

If $c^* > 0$ and $l^* = 0$, then the FOCs from equations (4.11) and (4.12) are

$$\frac{\beta}{q} = \left\{ \frac{\sqrt{D} + \sqrt{\beta}l_0}{r + l_0 + c^*} \right\}^2; \quad (4.17)$$

$$\frac{\alpha}{q} \geq \left\{ \frac{\sqrt{D} - \sqrt{\beta}(r + c^*)}{r + l_0 + c^*} \right\}^2. \quad (4.18)$$

From equation (4.17) we have $c^* = (\sqrt{q/q_c} - 1)(r + l_0)$. Since $c^* > 0$, then $q > q_c$. Plugging c^* into inequality (4.18) we get

$$q \geq \left(1 - \sqrt{\frac{\alpha}{\beta}} \right)^2 \equiv q_{cl}. \quad (4.19)$$

Here $q_{cl} = (1 - \sqrt{\alpha/\beta})^2$ is the probability at which given the investment pair $(c^*, 0)$ on RTC and RTL, the marginal cost of adding one more unit of investment in RTL is equal to the marginal benefit of doing so. Comparing with q_{lc} in Case 2 we find that $q_{cl} = q_{lc}$. This means that at probability $q_{lc} = q_{lc} = (1 - \sqrt{\alpha/\beta})^2$ the benefit of investing one dollar into RTC starting at $(0, l^*)$ is equal to the benefit of investing one dollar into RTL starting at $(c^*, 0)$.

We can see that when $q > \max\{q_c, q_{cl}\}$ then $c^* > 0$ and $l^* = 0$. This tells us that when q is big enough, then there is no need to do RTL but only invest in RTC. In this case $c^* = \sqrt{q}(\sqrt{D/\beta} + l_0) - l_0 - r$ is not affected by the cost of RTL, α . Here we have $\partial c^*/\partial l_0 < 0$, which means that if the resolution date is coming sooner, the investment on RTC will be less and hence “wait and see” will be more preferable. This is consist with Result 1 in Hennessy and Moschini (2006).

Case 4. $c^* > 0$ and $l^* > 0$.

If $c^* > 0$ and $l^* > 0$, then the FOCs from equations (4.11) and (4.12) are:

$$\frac{\beta}{q} = \left\{ \frac{\sqrt{D} + \sqrt{\beta}(l_0 + l^*)}{r + l_0 + l^* + c^*} \right\}^2; \quad (4.20)$$

$$\frac{\alpha}{q} = \left\{ \frac{\sqrt{D} - \sqrt{\beta}(r + c^*)}{r + l_0 + l^* + c^*} \right\}^2. \quad (4.21)$$

In this case equations (4.20) and (4.21) are identical if $q = q_{lc}$. They are

$$c^* = \sqrt{\frac{qD}{\beta}} - (1 - \sqrt{q})(l_0 + l^*) - r, \quad (4.22)$$

where $q = q_{lc}$. From equation (4.22) we can see that RTL and RTC are perfect substitutes for each other. Moreover, if $l^* = 0$ then $c^* = (\sqrt{q/q_c} - 1)(r + l_0)$, which is the optimal RTC investment in Case 3. If $c^* = 0$ then $l^* = (\sqrt{q/q_l} - 1)(r + l_0)$, which is the optimal RTL investment in Case 2.

By analyzing equations (4.20) and (4.21) we can obtain Remark 4.2 as follows:

Remark 4.2. *The FOCs in equations (4.20) and (4.21) imply that $q > \max\{q_c, q_l\}$. Moreover, the existence of a solution (c^*, l^*) such that equations (4.20) and (4.21) requires $q = q_{lc}$.*

The proof of Remark 4.2 is shown in Item F of Appendix. By Remark 4.1, if both $q > \max\{q_c, q_l\}$ and $q = q_{lc}$ hold, then we must have $q_{lc} \geq q_c$. Remark 4.2 shows that Case 3 is a “knife-edge” situation which happens only when the cost of RTL is sufficient low and the probability of the state T is equal to q_{lc} . This case tells us that when the success of RTL does not contribute to a RTC breakthrough (i.e., $\eta = 0$), it is almost always not optimal to carry out RTC and RTL simultaneously. This can explain why in the business world we rarely observe a company conducting market analysis (RTL) and investment into production lines (RTC) for a potential product at the same time. Since the only benefit from investing into RTL is to make better decisions on RTC so that statistical type I and II errors can be prevented, the expected negative impact of climate change will not decrease just because of the success in RTL. In the next section we will see that when the success of RTL can contribute to a breakthrough in RTC (i.e., $\eta > 0$), then there will be a range of q in which the social planner carries out RTC and RTL simultaneously.

We summarize the above analysis as Result 4.1.

Result 4.1. *Suppose RTC and RTL are independent (i.e., $\eta = 0$). i) When climate change is very unlikely to be harmful (i.e., $q < \min\{q_c, q_l\}$), then the social planner invests in neither RTC nor RTL (Case 1). ii) When the probability of a harmful climate change is moderate (i.e., $q \in (q_l, q_{lc})$), then the social planner invests in RTL only (Case 2). iii) When the probability of a harmful climate change is big enough (i.e., $q > \max\{q_c, q_{lc}\}$), then the social planner invests in RTC only (Case 3); and iv) the social planner will almost never invest in both RTC and RTL.*

The four cases in the benchmark setting can be summarized in Figure 4.2. Panel A in Figure 4.2 shows the possible cases when $q_{lc} > q_c$. We can see that when $q_{lc} > q_c$ then every case is possible. Resorting to simulation, Figure 4.3 shows an example of the four cases when $q_{lc} > q_c$. When $\alpha <$

$\beta((\sqrt{D} - \sqrt{\beta}r)/(\sqrt{D} + \sqrt{\beta}l_0))^2$, then every case is possible. Increasing α will expand interval $[0, q_l]$ and interval $[q_{lc}, 1]$. So an increase in the cost of RTL will make the social planner more likely do nothing (Case 1) or only invest in RTC (Case 3). Consequently the interval (q_l, q_{lc}) which supports Case 2 will shrink due to an increase in α . That is, the social planner will be less likely to conduct RTL only when the cost of RTL increases. When the cost of RTC, β , increases, then interval $[0, q_l]$ will expand and interval $[q_{lc}, 1]$ will shrink. That means the social planner will be more likely to do nothing (Case 1) and less likely to invest into RTC only (Case 3). The effect on the interval that supports Case 2 is ambiguous. Surprisingly, in this scenario an increase in D does not affect interval $[q_{lc}, 1]$ but shrinks interval $[0, q_l]$. This means that an increased damage rate will make the social planner less likely to do nothing and more likely to invest in RTL only. The likelihood of conducting RTC only is not affected. This is because given $(c^*, 0)$, the marginal benefit of adding one more unit of RTL is not affected by D (D canceled out when calculating the marginal benefit of one more unit of RTL at time 0). But the optimal RTC, c^* , increases with D .

Interestingly, when $q_{lc} < q_c$ only Case 1 and Case 3 are possible. This means that when the cost of RTL is high enough, i.e., $\alpha > \beta((\sqrt{D} - \sqrt{\beta}r)/(\sqrt{D} + \sqrt{\beta}l_0))^2$, then $l^* > 0$ will never be the case. This implies that the decision problem with only autonomous learning could be a special case of our model when the cost of RTL is high enough. Panel B in Figure 4.2 provides a visual presentation of this scenario. An numerical example can be found in Figure 4.4.

Extension: The Success of RTL Accelerates the Breakthrough of RTC

In this section we extend the benchmark model to allow success in RTL to positively contribute to a RTC breakthrough (i.e., $\eta > 0$). This extension admits the situation in which RTL and RTC could happen simultaneously on a range of probabilities of the true state (i.e., values of q). Following Section 4 we first discuss the four cases according to the values of c^* and l^* . Then we discuss these four cases in different scenarios. For simplicity, but without losing generality, in this section we assume $l_0 = 0$. This implies that the true state will never be known if the social planner does not invest into RTL. Even though we let $l_0 = 0$, autonomous learning still can be viewed as a specific instance of our model when we fix the investment level into RTL.

Case 1. $c^* = 0$ and $l^* = 0$.

If $c^* = 0$ and $l^* = 0$, then from the FOCs in (4.9) and (4.10) we can obtain

$$q \leq \frac{\beta r^2}{D} \equiv \hat{q}_c \quad (4.23)$$

$$q \leq \frac{\alpha r^2}{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta \eta r} \equiv \hat{q}_l \quad (4.24)$$

The above two conditions tell us that whenever $q \leq \min\{\hat{q}_c, \hat{q}_l\}$ then $(c^*, l^*) = (0, 0)$. By Assumption 1 we can check that $\hat{q}_c < 1$. Here \hat{q}_c (or \hat{q}_l) is the counterpart of q_c (or q_l) in Section 4. We find that \hat{q}_l is decreasing in η . This is because a larger η will increase the marginal benefit of RTL given the level of q . Therefore, if $\hat{q}_c > \hat{q}_l$ then an increase in η will shrink the range of q that supports this case. Therefore, if the success of RTL could contribute to RTC at a larger magnitude, then it is less likely for the social planner to do nothing. However, if $\hat{q}_c < \hat{q}_l$, then an increase in η will not affect the range of q that supports Case 1. The reason is as follows. When $l^* = 0$ and $l_0 = 0$, from the expression of \hat{q}_c we can see that η does not affect \hat{q}_c , which implies that η does not affect the marginal benefit of RTC. We know that η can affect the breakthrough of RTC only if success in RTL happens before RTC. If $l^* = 0$ and $l_0 = 0$, then success in RTL will never happen. Therefore, increasing η will not affect a breakthrough in RTC and hence the marginal benefit of RTC. If $l_0 > 0$, then an increase in η will increase the range of q that supports Case 1. The reason is as follows. When $\hat{q}_c < \hat{q}_l$, then the cost of RTL must be relatively high. Since we have $l_0 > 0$, which means the success of RTL will be expected to happen at time $1/l_0$ even when there is no investment into RTL, then the increased impact of RTL on RTC will decrease the incentive to invest into RTC. Therefore the range of q in which the social planner does nothing is enlarged.

Case 2. $c^* = 0$ and $l^* > 0$.

When $c^* = 0$ and $l^* > 0$ then we must have

$$\beta - q \frac{(\sqrt{D} + \sqrt{\beta}l^*)^2 - \beta \eta l^*}{(r + l^*)^2} \geq 0, \quad (4.25)$$

$$\alpha - q \frac{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta \eta r}{(r + l^*)^2} = 0. \quad (4.26)$$

From equation (4.26) we can obtain

$$l^* = \left(\sqrt{\frac{q}{\hat{q}_l}} - 1 \right) r. \quad (4.27)$$

Since in this case $l^* > 0$ then we must have $q > \hat{q}_l$. Plugging $l^* = (\sqrt{q/\hat{q}_l} - 1)r$ into inequality (4.25) we get $q \leq \hat{q}_{lc}$, where

$$\hat{q}_{lc} = \hat{q}_l \frac{\left\{ r + \frac{1}{2\beta} [(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/\hat{q}_l)}] \right\}^2}{r^2}. \quad (4.28)$$

The algebra to show equation (4.28) is presented in Item G of Appendix, in which we also show that the existence of \hat{q}_{lc} requires that $\alpha \leq \frac{\beta}{D}((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r) \equiv \alpha_3$. It is readily checked that $\hat{q}_{lc} = (1 - \sqrt{\alpha/\beta})^2$ when $\eta = 0$.

Case 3. $c^* > 0$ and $l^* = 0$.

If $c^* > 0$ and $l^* = 0$, then the FOCs in (4.9) and (4.10) are:

$$\beta - q \frac{D}{(r + c^*)^2} = 0 \quad (4.29)$$

$$\alpha - q \frac{(\sqrt{D} - \sqrt{\beta}(r + c^*))^2 + \beta\eta(r + c^*)}{(r + c^*)^2} \geq 0. \quad (4.30)$$

From equation (4.29) we can obtain

$$c^* = \left(\sqrt{\frac{q}{\hat{q}_c}} - 1 \right) r. \quad (4.31)$$

Since in this case $c^* > 0$, then we must have $q > \hat{q}_c$. Plugging $c^* = (\sqrt{\frac{q}{\hat{q}_c}} - 1)r$ into inequality (4.30), we obtain

$$q + \left(\sqrt{\frac{\beta}{D}}\eta - 2 \right) \sqrt{q} + 1 - \frac{\alpha}{\beta} \leq 0. \quad (4.32)$$

In order to make Case 3 occur, inequality (4.32) must be satisfied by some q . This requires that $(\sqrt{\frac{\beta}{D}}\eta - 2)^2 - 4(1 - \frac{\alpha}{\beta}) \geq 0$, i.e., $\alpha \geq \beta(\eta\sqrt{\beta/D} - \beta\eta^2/4D) \equiv \alpha_1$, from which we can see that Case 3 happens only when α (i.e., the cost of RTL) is big enough.

If we put

$$q + \left(\sqrt{\frac{\beta}{D}}\eta - 2 \right) \sqrt{q} + 1 - \frac{\alpha}{\beta} = 0, \quad (4.33)$$

and let q_1 and q_2 denote the two solutions of equation (4.33), where $q_1 \leq q_2$, then it is easy to check that $q_1 = (1 - \sqrt{\alpha/\beta})^2$ and $q_2 = (1 + \sqrt{\alpha/\beta})^2 > 1$ when $\eta = 0$. The range of q that supports Case 3 is $[\max\{\hat{q}_c, q_1\}, \min\{1, q_2\}]$. We can show that when q_1 and q_2 exist, we always have $q_2 > \hat{q}_c$. The proof is in Item H of Appendix.

According to the values of q_1 and q_2 , the interval $[\max\{\hat{q}_c, q_1\}, \min\{1, q_2\}]$ can take one of the following three possibilities.

Possibility 1: $[q_1, q_2]$. This possibility requires $q_1 \geq \hat{q}_c$ and $q_2 \leq 1$. We can check that if $q_2 \leq 1$ then $q_1 \geq \hat{q}_c$. Furthermore, $q_1 \geq \hat{q}_c$ and $q_2 \leq 1$ imply that $\alpha_1 \leq \alpha \leq \beta\eta\sqrt{\beta/D} \equiv \alpha_2$. The algebra to obtain this is shown in Item I, which also includes the algebra to get the ranges of α in Possibilities 2 and 3 below.

Possibility 2: $[q_1, 1]$. This possibility requires $q_1 \geq \hat{q}_c$ and $q_2 > 1$. Therefore α must be such that $\alpha_2 \leq \alpha \leq \frac{\beta}{D}[(\sqrt{D} - \sqrt{\beta}r)^2 + r\eta\beta] \equiv \alpha_3$.

Possibility 3: $[q_c, 1]$. This possibility requires $q_1 < \hat{q}_c$ and $q_2 > 1$. Therefore α must be such that $\alpha > \alpha_3$.

Case 4. $c^* > 0$ and $l^* > 0$.

If $c^* > 0$ and $l^* > 0$, the FOCs in (4.9) and (4.10) are

$$\beta - q \frac{(\sqrt{D} + \sqrt{\beta}l^*)^2 - \beta\eta l^*}{(r + l^* + c^*)^2} = 0 \quad (4.34)$$

$$\alpha - q \frac{(\sqrt{D} - \sqrt{\beta}(r + c^*))^2 + \beta\eta(r + c^*)}{(r + l^* + c^*)^2} = 0 \quad (4.35)$$

By the same procedure in Item F of Appendix we can show that Case 4 requires $q > \max\{\hat{q}_c, \hat{q}_l\}$. Due to the complexity of equations (4.34) and (4.35) we cannot explicitly solve (c^*, l^*) out. But since Cases 1 to 4 are mutually exclusive and form a partition of all possible outcomes for (c^*, l^*) , then the range of q that supports Case 4 contains any q that does not support Case 1 to 3. Therefore, after we figure out the intervals of q that supports Case 1 to 3, we can find out the range of q that supports Case 4 naturally by subtracting the intervals of q supporting Case 1 to 3 from interval $(0, 1)$.

Scenario Analysis

In this sub-section we analyze the possibilities for Case 2 and Case 3 according to the values of RTL cost (i.e., α). Since the quantitative relationships between \hat{q}_c , \hat{q}_l , \hat{q}_{lc} , and q_1 are important to the analysis that follows, here we summarize the relationships in a remark. The proof this remark is shown in Item J of Appendix.

Remark 4.3. Whenever i) $\alpha \leq \alpha_3$, then $\hat{q}_l \leq \hat{q}_c \leq \hat{q}_{lc} \leq q_1$; 2) $\alpha \geq \alpha_3$, then $\hat{q}_l \geq \hat{q}_c \geq q_1$, and the equalities hold when $\alpha = \alpha_3$.

According to the values of α , we have four scenarios to analyze.

Scenario 1. $\alpha \in [0, \alpha_1)$. In this scenario Case 3 does not happen because there is no q satisfying inequality (4.30). By Remark 4.3 we know that $\hat{q}_l < \hat{q}_c < \hat{q}_{lc}$. Then Case 1 will happen if $q \in [0, \hat{q}_l]$; Case 2 will happen if $q \in (\hat{q}_l, \hat{q}_{lc}]$; Case 4 will happen if $q \in [\hat{q}_{lc}, 1]$. Figure 4.5 includes a visual presentation of this scenario. Therefore, we can see that if 1) the cost of RTL is low enough, and 2) the success of RTL can contribute to a breakthrough in RTC, then it is never optimal to only invest into RTC. Figure 4.6 provides a numerical example of this scenario. One interesting observation is that in Case 4 ($c^* > 0$ and $l^* > 0$) the optimal level of RTL is first decreasing in q and then increasing in q as q increases from \hat{q}_{lc} to 1. One explanation is that when climate change is very likely harmful (i.e., q is big) and when the cost of RTL is low enough, then the social planner may want to invest more in RTL expecting that success in RTL could accelerate RTC's success. The optimal level of RTC in Case 4, however, is always increasing in q .

Scenario 2. $\alpha \in [\alpha_1, \alpha_2]$. In this scenario every case is possible. A visual presentation is shown in Figure 4.5. We can see that Case 1 happens if $q \in [0, \hat{q}_l]$; Case 2 happens if $q \in (\hat{q}_l, \hat{q}_{lc}]$; Case 3 happens if $q \in [q_1, q_2]$. One interesting thing is that Case 4 happens if $q \in [\hat{q}_{lc}, q_1]$ or $q \in [q_2, 1]$. In range $[\hat{q}_{lc}, q_1]$, c^* is increasing with q but l^* is decreasing with q . In the range of $q \in [q_2, 1]$, however, both c^* and l^* are increasing with q . Figure 4.7 provides a numerical example of this scenario.

Scenario 3. $\alpha \in [\alpha_2, \alpha_3]$. In this scenario every case is possible as well. A visual presentation of this scenario is shown in Panel A of Figure 4.8. We can see that Case 1 happens if $q \in [0, \hat{q}_l]$; Case 2 happens if $q \in (\hat{q}_l, \hat{q}_{lc}]$; Case 3 happens if $q \in [q_1, 1]$; and Case 4 happens if $q \in [\hat{q}_{lc}, q_1]$. Figure 4.8 includes a visual presentation of this scenario. Figure 4.9 provides a numerical example.

Scenario 4. $\alpha > \alpha_3$. In this scenario only Cases 1 and 3 are possible. Case 1 happens if $q \in [0, \hat{q}_c]$ and Case 3 happens if $q \in (\hat{q}_c, 1)$. Therefore, if $\alpha > \alpha_3$ then $l^* > 0$ will never be optimal. This because if the cost of RTL is too high, then there is no point in conducting RTL. When the probability of state T is lower than \hat{q}_c , the social planner needs to do nothing. When the probability of state T is higher than \hat{q}_c , the social planner will only conduct RTC. Figure 4.8 includes a visual presentation of this scenario. Figure 4.10 provides a numerical example.

We summarize the analysis in this section as Result 4.2.

Result 4.2. *Suppose success in RTL can accelerate success in RTC. i) When the cost of RTL is low*

enough (i.e., $\alpha < \alpha_1$), then it is never optimal to only invest in RTC. ii) When the cost of RTL is moderate (i.e., $\alpha \in [\alpha_1, \alpha_2]$) and when the probability of a harmful climate change is either very high or moderate (i.e., $q \in [q_2, 1]$ or $q \in [\hat{q}_{lc}, q_1]$), then the social planner invests in both RTC and RTL. iii) In Case 4 under both Scenarios 1 and 2, the optimal RTL investment level decreases and then increases in q .

From Result 4.2 we can see that whether to follow the “precautionary rule” (favors investing in RTC as early and as much as possible) or the “learn-then-act rule” (favors delaying the investment in RTC) regarding investment into R&D of new technologies depends on the costs of research activities and the probability distribution of climate change’s damage. Therefore, the model provides an explicit resolution to the debate between the advocates for these two rules. We are also interested in how the change in an exogenous parameter affects optimal RTC and RTL, which is studied in the next section by utilizing numerical methods.

Comparative Statics of the Extension Model

The effect of the exogenous parameters (i.e., α , β , η , q , r , and D) on \hat{q}_c , \hat{q}_l , and c^* in Case 1 and Case 3 can be easily determined after some algebra. The results are shown in Table 4.1. However, the parameters’ effects on \hat{q}_{lc} , q_1 , q_2 , c^* , and l^* of Cases 2 and 4 of scenarios 1 to 3 are not so straightforward. For these effects we resort to numerical method to conduct the comparative statics. During the numerical analysis, the initial values of these exogenous parameters are set as: $\beta = 15,000$, $\eta = 0.08$, $q = 0.23$, $r = 0.05$, and $D = 300$. By these parameters we calculate the critical values of α : α_1 , α_2 , and α_3 . We then specify the four scenarios according to the values of α_1 , α_2 , and α_3 . Since Cases 2 and 4 do not happen in Scenario 4, we conduct the numerical analysis only for Scenarios 1 to 3. The corresponding results are shown in Tables 4.2 to 4.4, respectively. We discuss these effects in detail next.

Effects on c^* and l^*

From Tables 4.1 and 4.2 we can see that a marginal change on any parameters will not affect c^* in Cases 1 and 2 (or l^* in Cases 1 and 3) because c^* (or l^*) is zero in Cases 1 and 2 (or Cases 1 and 3). In Case 3, an increase in α has no effect on c^* . This is because when $c^* > 0$ and $l^* = 0$, then RTL will

never succeed and hence RTL has no effect on RTC. Therefore, changing the cost of RTL will not affect the optimal level of RTC. For the same reason, increasing η , the contribution of the success of RTL to a RTC breakthrough, has no effect on c^* either. The policy implication here is that when Case 3 happens then a program that subsidizes RTL will have no effect on either RTL or RTC. It is quite intuitive that increasing the cost of RTC will decrease c^* but increasing damage rate D or the probability that this damage occur, q , will increase c^* .

From Tables 4.2 to 4.4 we see that an increase in α (or β) decreases the optimal level of investment in RTL (or RTC). We also find that in Case 4 of Scenarios 1 throughout 3, an increase in α (or β) will always increase the optimal investment in RTC (or RTL), which means that in Case 4 the RTL and RTC are substitutes. However, in Case 2 of Scenarios 1 to 3, the effect of β on l^* is ambiguous. The intuition is as follows. Suppose a success of RTL contributes to a breakthrough in RTC. Then on the one hand, RTL complements RTC; on the other hand, RTL substitutes for RTC. The effect of β on l^* will depend on whether the substitute effect is bigger than the complement effect or not. In the benchmark model, since the success of RTL does not contribute to a breakthrough of RTC, to minimize the expected negative effects of climate change the social planner cannot resort to RTL only. To this extent in the benchmark model RTL and RTC are complements. But in the extension model, the success of RTL can contribute to a breakthrough of RTC. This means that to some extent RTL substitutes for RTC. Therefore, in Case 2 of Scenarios 1 to 3 we observe the ambiguous effect of β on l^* .

Now let us study the effect of η , the magnitude of the contributions of the success of RTL to a RTC breakthrough, on c^* and l^* . In Case 1 and Case 3 through all four scenarios, η has no effect on l^* and c^* . This is because l^* is zero in Cases 1 and 3. Since $l^* = 0$, the success of RTL will almost surely never happen. Therefore, an increase in η will not affect c^* . In Case 2 and Case 4 throughout Scenarios 1 to 3, η has a positive effect on l^* and a negative effect on c^* . This is quite intuitive because an increased η will make RTL more “valuable” hence the social planner is going to “purchase” more RTL and less RTC.

The effect of q can be clearly seen in the numerical examples provided in Figures 4.3, 4.4, 4.6, 4.7, 4.9, and 4.10. From these figures and Tables 4.1 to 4.4 we can see an increase in q will always increase (even though not necessarily strictly) the optimal investment level into RTC. This is intuitive because when the damage is more likely to be true, the social planner wants to put more resources into RTC to

respond to this possible damage. However, the effect of q on l^* is more complicated. In Case 2 of all Scenarios 1 to 3, l^* is increasing with q . This is because when $c^* = 0$, then an increase in the possibility of the damage being true leads the social planner to put more resources into RTL in order to accelerate the success of RTC at the second stage in case the true state of climate change is harmful. In Case 4 the effect of q on l^* is ambiguous. Therefore, 1) when q is relatively small then increasing q will decrease l^* ; 2) when q is large enough then increasing q will increase l^* . A tentative explanation is that when the likelihood of the damage is small and when the cost of RTL is low enough ($\alpha \in [\alpha_1, \alpha_2]$), then the increasing c^* substitutes l^* out as q is increasing. However, if the likelihood of the damage is large and the cost of RTL is still low enough ($\alpha \in [\alpha_1, \alpha_2]$), then an increasing q will cause the social planner to put more resources into both RTC and RTL. Under this situation the cost of RTL is reasonable and the damage is so likely that the social planner can benefit from increasing both RTL and RTC.

The continuous time discount rate, r , always has a negative effect on c^* and l^* . This is because when the future welfare is less important, it is always better for the social planner to invest less into RTL and RTC, whose benefit is only reaped in the future.

An increase in the damage rate, D , always increase c^* . This is easy to understand because when the damage is bigger the social planner has more incentive to investigate the method to respond to this damage. However, D has different effects on l^* in different cases. In Case 2, an increase in D will bring a higher l^* . This is because a bigger D means a higher marginal benefit from one more unit of RTL in Case 2. In Case 4, however, an increase in D will always decrease l^* . The intuition here is that when both RTL and RTC are conducted at the same time, an increase in D will put more “weight” on RTC because RTC is a direct response to the negative welfare impact while RTL is not.

We summarize the above analysis as Result 4.3

Result 4.3. *Regarding the exogenous parameters' effects on the optimal RTC and RTL investment (i.e., c^* and l^* , respectively), we have conclusions as follows. i). c^* is increasing in RTL cost, α , the probability of having a harmful climate change, q , and the damage rate, D , but is decreasing in RTC cost, β , the impact magnitude of RTL's success on RTC's success, η , and the discount rate, r . ii). When $c^* = 0$ and $l^* > 0$ (i.e., Case 2) then l^* is decreasing in α (i.e., RTL cost) and r , but is increasing in η , q , and D . The impact of RTC cost (i.e., β) on l^* is ambiguous. iii). When $c^* > 0$ and $l^* > 0$ (i.e., Case 4) then l^* is decreasing in α , r , and D , but is increasing in β and η . The impact of q on l^* is ambiguous.*

Effects on q -Intervals Supporting Different Cases

In this subsection we study the exogenous parameters' effects on \hat{q}_c , \hat{q}_l , \hat{q}_{lc} , q_1 , and q_2 , the critical values of q that support different cases. This allows us to see how the ranges of q that supports different cases change.

Case 1. $c^* = 0$ and $l^* = 0$.

Case 1 occurs when $q \leq \min\{\hat{q}_l, \hat{q}_c\}$. From Remark 4.3 we know that if $\alpha < \alpha_3$ then $\hat{q}_l < \hat{q}_c$. From Table 4.1 we see that a higher cost of RTL can increase \hat{q}_l but has no effect on \hat{q}_c . This means that if $\alpha < \alpha_3$ then a higher RTL cost will enlarge the range of q supporting Case 1; if $\alpha > \alpha_3$, then a higher RTL cost will not affect this range of q supporting Case 1. It is intuitive because in Case 1 when α is small enough then an increase in α will expand the range of q in which the social planner conducts neither RTL nor RTC. If α is high enough (i.e., $\alpha > \alpha_3$), then only Case 1 and Case 3 are possible. Since in Case 1 and Case 3 the optimal RTL is 0 and an increase in α does not affect \hat{q}_c , then an increase in α will not affect the range of q that supports Case 1.

From Table 4.1 we can see that \hat{q}_l and \hat{q}_c is increasing with β , which means that an increase in β will enlarge the interval of q that supports Case 1. The discount rate r has the same effect as β does. But D has the opposite effect. All of these effects can be intuitively explained as follows. When the damage is smaller (or the future is less important, or the cost of doing RTC is higher), the social planner will have a higher incentive to do nothing. Therefore the range of q that supports Case 1 is enlarged.

By equation (4.24) we know that \hat{q}_l is decreasing in η . Therefore, if $\alpha < \alpha_3$ then an increase in η will shrink the range of q that supports Case 1. This is because when RTL is more valuable in the sense that its success can contribute to a RTC breakthrough at a larger magnitude and when RTL is relatively inexpensive, then the social planner will be more likely to conduct some RTL to respond to the possible hazard. Consequently the range of Case 1 shrinks. However, if $\alpha \geq \alpha_3$ then an increase in η will not affect the range of Case 1. Intuitively, this is because a small increase in η will not trigger the social planner's investment in RTL when the RTL cost is very high.

Case 2. $c^* = 0$ and $l^* > 0$.

Case 2 happens if $q \in [\hat{q}_l, \hat{q}_{lc}]$. From Tables 4.1 to 4.4 we see that an increase in α (or a decrease in η) will shrink this range. This is because when RTL is more costly (i.e., α increases) or RTL is less helpful to RTC's success then the social planner is less likely to conduct RTL only. Other parameters' effects on this range are ambiguous.

Case 3. $c^* > 0$ and $l^* = 0$.

Case 3 only occurs under Scenarios 2 to 4. In Scenarios 2 and 3, Case 3 occurs when $q \in [q_1, q_2]$ and when $q \in [q_1, 1]$, respectively. From Table 4.3 and Table 4.4 we can see that 1) an increase in α or D can expand this range; 2) an increase in either β or η can shrink this range; and 3) a change in continuous time discount rate, r , has no effect on this range. Here item 1) is true because an increase in the cost of RTL investment (i.e., α) or the damage from climate change (i.e., D) will lower the social planner's probability threshold for q over which the RTC investment occurs (i.e., q_1). Item 2) is true because when the cost of RTC becomes high and the positive impact of RTL's success on RTC's success becomes larger, then the social planner wants to increase the threshold of q that triggers investment in RTC. The reason for item 3) is as follows. Given the optimal RTC investment level (i.e., c^*), the benefit of conducting RTL is not affected by the discount rate. This is because the optimal RTC has fully responded to the discount rate.

In Scenario 4, Case 3 occurs when $q \in [\hat{q}_c, 1]$. From equation (4.23) we can conclude that 1) an increase in β or r can shrink this range; 2) an increase in D will expand this range; and 3) a change in α or η will not affect this range. Here items 1) and 2) are intuitive but item 3) needs some explanation. In Scenario 4, l^* is always equal to 0. Moreover, the effect of α or η on c^* works through RTL by changing RTC's marginal benefit. Therefore, if there is no RTL investment, then a change in α or η will not affect the marginal benefit of RTC (see Table 4.1), and hence will not affect the range of q that supports Case 3.

Case 4. $c^* > 0$ and $l^* > 0$.

Case 4 only occurs under Scenarios 1 to 3. In Scenario 1, the range of q that supports Case 4 is $q \in [\hat{q}_{lc}, 1]$. From Table 4.2 we can see that \hat{q}_{lc} is decreasing in α and D but increasing in β , η , and r . This means that under Scenario 1 an increase in α or D will expand the interval of q under which Case 4 occurs. However, an increase in β , η , or r will shrink this range. In Scenarios 2 and 3, Case 4 occurs when $q \in [\hat{q}_{lc}, q_1] \cup [q_2, 1]$ and when $q \in [\hat{q}_{lc}, q_1]$, respectively. From Table 4.3 and Table 4.4 we can see that only r has an unambiguous effect on these ranges. When r increases, then these ranges will shrink. Other parameters' effects are ambiguous.

We summarize the analysis in this sub-section as Result 4.4:

Result 4.4. *Regarding the exogenous parameters' effects on ranges of q that support Cases 1 to 4, we conclude as follows. i). The range of q supporting Case 1 is enlarged (at least weakly) by an increase in RTL cost, α , RTC cost, β , and discount rate, r , or by a decrease in the impact magnitude of RTL's success on RTC's success, η , and the damage rate, D . ii). The range of q supporting Case 2 is shrunk by an increase in RTL cost, α , but is enlarged by an increase in η . However, other parameters' effects on this range are ambiguous. iii). The range of q supporting Case 3 is enlarged (at least weakly) by an increase in α and D , or by a decrease in β , η , and r . iv). Under Scenario 1, the range of q supporting Case 4 is enlarged (at least weakly) by an increase in α and D , or by a decrease in β , η , and r . Under Scenarios 2 and 3, however, the range of q supporting Case 4 is enlarged by a decrease in r but other parameters' effects are ambiguous.*

Conclusions and Future Research

How to face the challenge of climate change will be the focus of international policies before the world clearly understands the magnitude of the welfare impact of climate change or before the world is confident that the technologies available could handle any possible effects of climate change. In other words, the world will be concerned about climate change until one or two of the uncertainties discussed in this article disappear. The two uncertainties are on the magnitude of the problems' human welfare impact and on the future date when this unknown impact becomes clear. This article investigate the optimal investment decisions on mitigating these two uncertainties. The results show that RTC

can complement with or substitute for RTL depending on the probability distribution of the damage of climate change. If the outcomes of RTL and RTC are statistically independent, then it is almost never optimal to conduct both RTL and RTC simultaneously. If the success of RTL increases the probability of a RTC breakthrough and the cost of RTL is small enough, then it is never optimal to conduct RTC only. We also show that whether to follow the “precautionary rule” or the “learn-then-act rule” regarding investment into R&D about new technologies depends on the costs of research activities and the probability distribution of damage due to climate change. Therefore, the article provides an explicit resolution to the debate between the advocates for these two rules. It also provides a framework about optimal R&D decisions facing the uncertainties of climate change.

There are at least two possibilities to extend this research. The first one is to generalize the analysis into a formal Bayesian decision framework. The current analysis is an special case of a general Bayesian decision framework. But we expect that the generalization would cause challenging technical problems. The second possibility for extension, also a more interesting one, is to calibrate the current model and simulate what the optimal RTL and RTC should be, which could provide policy implications on optimally allocating the scarce research resources.

References

- Arrow, Kenneth J. and Anthony C. Fisher. (1974). “Environmental Preservation, Uncertainty, and Irreversibility,” *The Quarterly Journal of Economics*, 88, 312-319.
- Baker, Erin, Leon Clarke and John Weyant. (2006). ”Optimal Technology R&D in the Face of Climate Uncertainty,” *Climate Change*, 78: 157-179.
- Baker, Erin and Kwame Adu-Bonnah. (2008). ”Investment in Risky R&D programs in the Face of Climate Uncertainty,” *Energy Economics*, 30: 465-486.
- Baker, Erin and Ekundayo Shittu. (2008). ”Uncertainty and Endogenous Technical Change in Climate Policy Models,” *Energy Economics*, 30: 2817-2828.
- Hanemann, W. Michael. (1989). “Information and the Concept of Option Value,” *Journal of Environmental Economics and Management*, 16, 23-37.
- Hennessy, David A. and GianCarlo Moschini. (2006). ”Regulatory Actions under Adjustment Costs

- and the Resolution of Scientific Uncertainty,” *American Journal of Agricultural Economics* 88(2): 308-323.
- Henry, Claude. (1974). “Option Values in the Economics of Irreplaceable Assets,” *The Review of Economics Studies*, 41: 89-104.
- Kolstad, Charles D. (1996). “Fundamental Irreversibilities in Stock Externalities,” *Journal of Public Economics*, 60, 221-233.
- Marwah, Shikha and Jinhua Zhao. (2007). “Double Irreversibilities and Endogenous Learning in Land Conversion Decisions,” unpublished manuscript, Department of Economics, Iowa State University.
- Olson, Lars J. (1990). “Environmental Preservation with Production,” *Journal of Environmental Economics and Management*, 18: 88-96.
- Schimmelpfennig, David. (1995). “The Option Value of Renewable Energy,” *Energy Economics*, 17: 311-317.
- Svensmark, H. and N. Calder. 2008. *The Chilling Stars: A Cosmic View of Climate Change*. Blue Ridge Summit, PA: Totem Books.
- Uzoigwe, Jacinta C., Margaret L. Khaita, and Penelope S. Gibbs. (2007). “Epidemiological Evidence for Mycobacterium Avium Subspecies Paratuberculosis as a Cause of Crohn’s disease,” *Epidemiology and Infection*, 135: 1057-1068.
- Weisbrod, Burton A. (1964). “Collective Consumption Services of Individual Consumption Goods,” *Quarterly Journal of Economics*, 77: 471-477.

Table 4.1: Results of Comparative Static Analysis for Case 1 and Case 3

Exogenous Variable (j)	Case 1		Case 3	
	$\frac{\partial c^*}{\partial j}$	$\frac{\partial l^*}{\partial j}$	$\frac{\partial c^*}{\partial j}$	$\frac{\partial l^*}{\partial j}$
α	0	0	0	0
β	0	0	-	0
η	0	0	0	0
q	0	0	+	0
r	0	0	-	0
D	0	0	+	0

Table 4.2: Results of Comparative Static Analysis for Case 2 and Case 4 in Scenario 1

Exogenous Variable (j)	Case 2		Case 4			
	$\partial c^*/\partial j$	$\partial l^*/\partial j$	$\partial c^*/\partial j$	$\partial l^*/\partial j$	$\partial \hat{q}_{lc}/\partial j$	$\partial q_1/\partial j$ $\partial q_2/\partial j$
α	0	-	+	-	-	/
β	0	-/+	-	+	+	/
η	0	+	-	+	+	/
q	0	+	+	-/+	0	/
r	0	-	-	-	+	/
D	0	+	+	-	-	/

Note: Since q_1 and q_2 do not exist in this scenario, the comparative statics is not applicable to them, which is denoted by a “\”.

Table 4.3: Results of Comparative Static Analysis for Case 2 and Case 4 in Scenario 2

Exogenous Variable (j)	Case 2		Case 4		Case 4			
	$\frac{\partial c^*}{\partial j}$	$\frac{\partial l^*}{\partial j}$	$\frac{\partial c^*}{\partial j}$	$\frac{\partial l^*}{\partial j}$	on the left of Case 3	$\frac{\partial c^*}{\partial j}$	on the right of Case 3	$\frac{\partial l^*}{\partial j}$
α	0	-	+	-	+	-	-	+
β	0	-/+	-	+	-	+	+	-
η	0	+	-	+	-	+	+	-
q	0	+	+	-	+	+	0	0
r	0	-	-	-	-	-	+	0
D	0	+	+	-	+	-	-	+

Table 4.4: Results of Comparative Static Analysis for Case 2 and Case 4 in Scenario 3

Exogenous Variable (j)	Case 2		Case 4			
	$\frac{\partial c^*}{\partial j}$	$\frac{\partial l^*}{\partial j}$	$\frac{\partial c^*}{\partial j}$	$\frac{\partial l^*}{\partial j}$	$\frac{\partial \hat{q}_{lc}}{\partial j}$	$\frac{\partial q_1}{\partial j} \quad \frac{\partial q_2}{\partial j}$
α	0	-	+	-	-	- +
β	0	-/+	-	+	+	+ -
η	0	+	-	+	+	+ -
q	0	+	+	-/+	0	0 0
r	0	-	-	-	+	0 0
D	0	+	+	-	-	- +

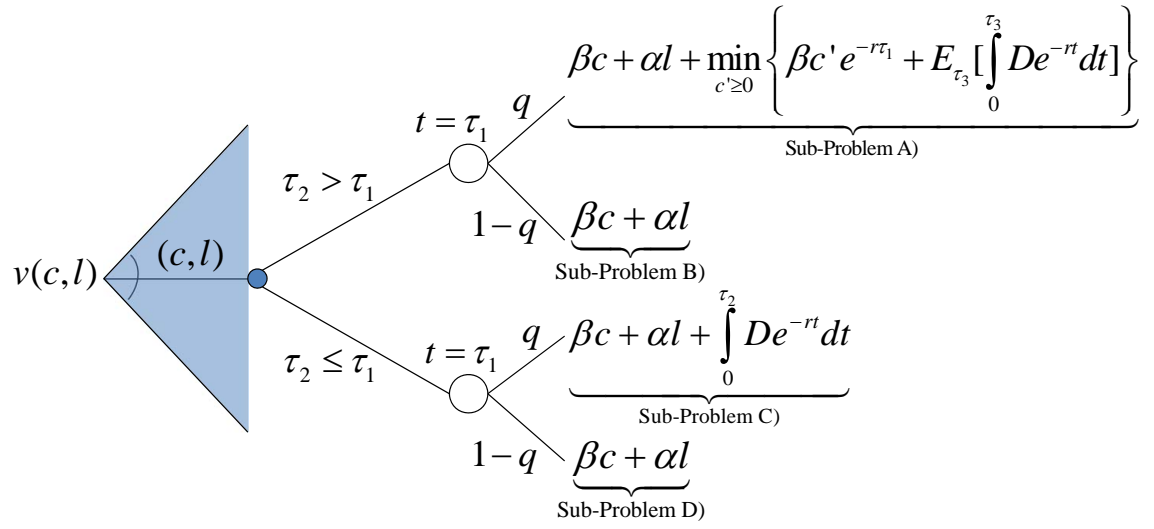


Figure 4.1: The Social Planner's Decision Tree

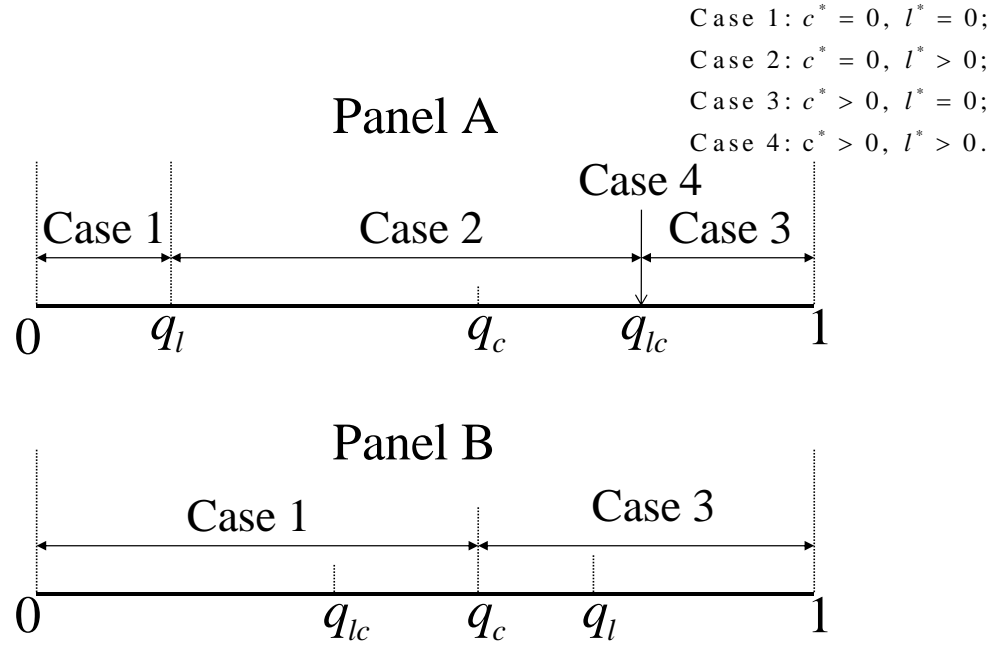


Figure 4.2: Possible Cases when $q_{lc} < q_c$ (Panel A) and when $q_{lc} < q_c$ (Panel B)

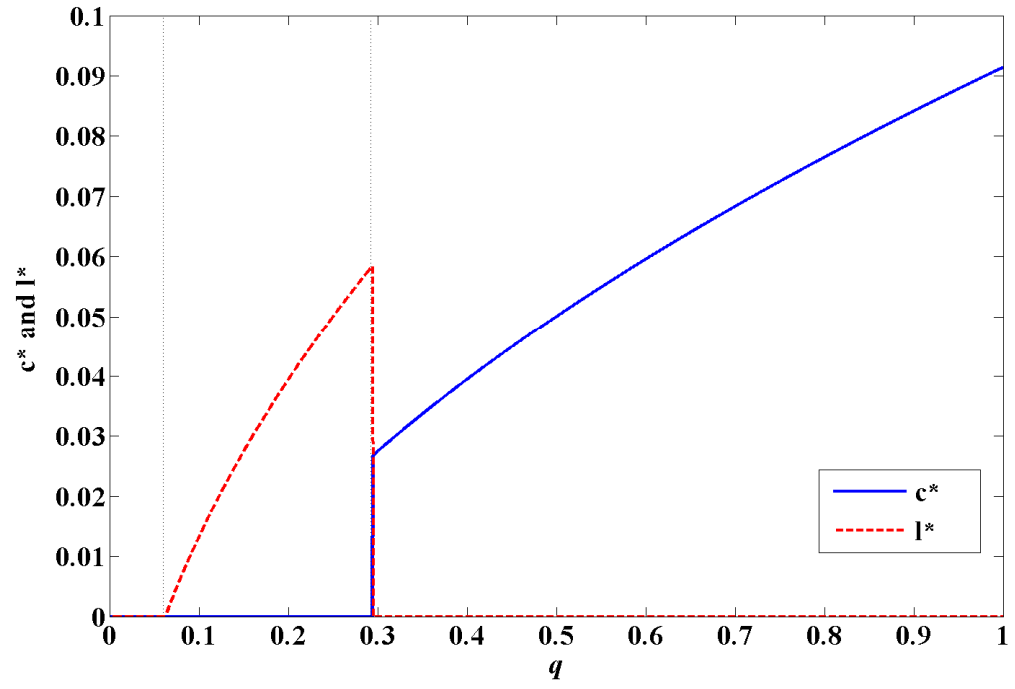


Figure 4.3: Values of c^* and l^* when $\eta = 0$ and $q_{lc} > q_c$

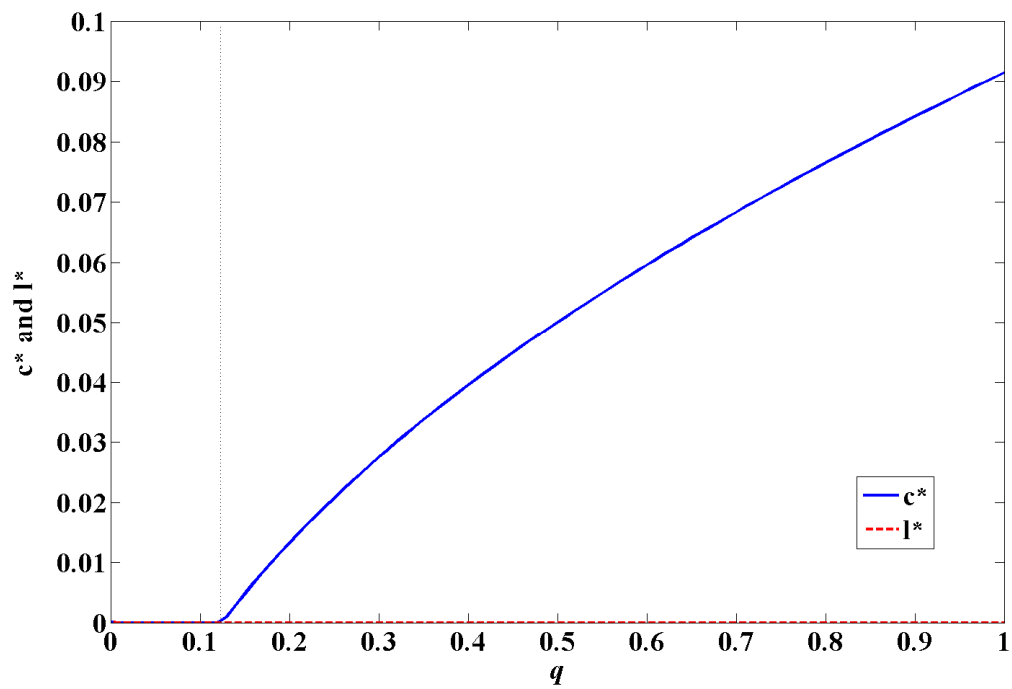


Figure 4.4: Values of c^* and l^* when $\eta = 0$ and $q_{lc} \leq q_c$

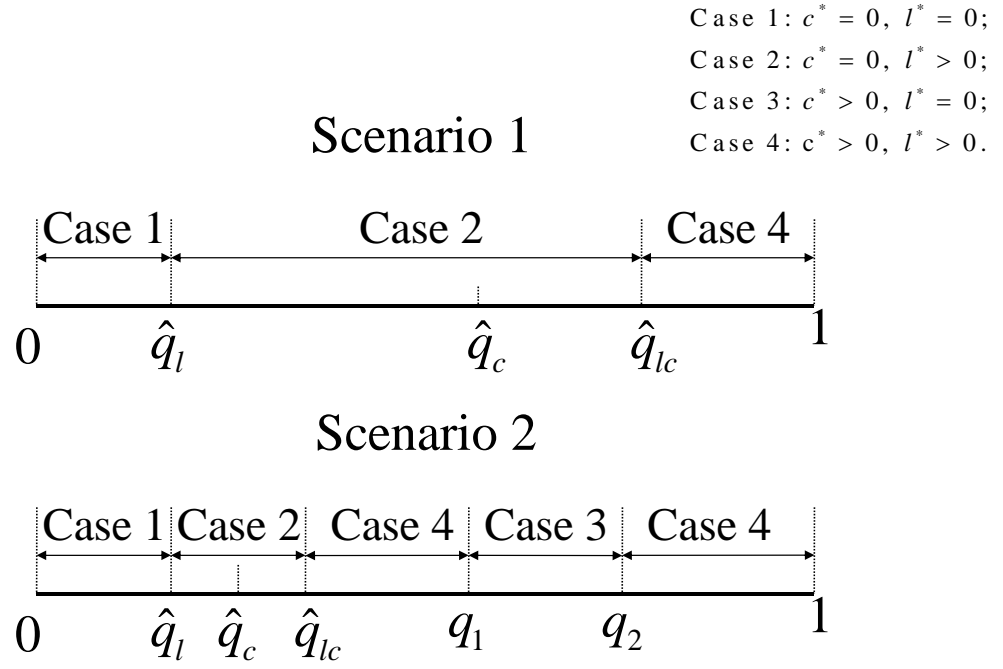


Figure 4.5: Possible Cases in Scenario 1 ($\alpha \in [0, \alpha_1]$)

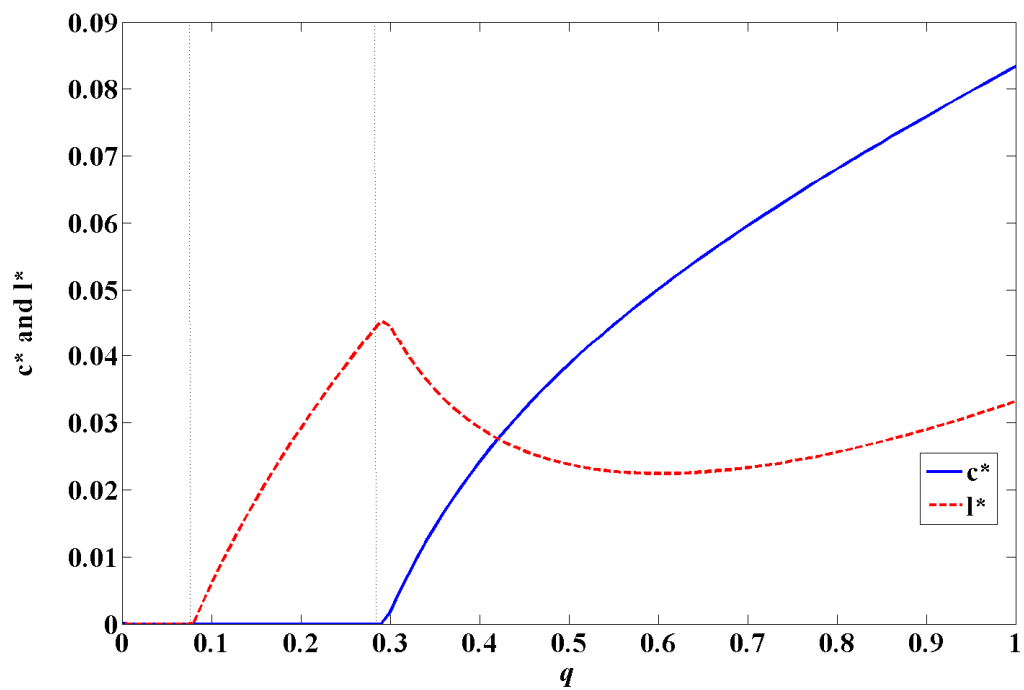


Figure 4.6: Values of c^* and l^* when $\eta > 0$ under Scenario 1

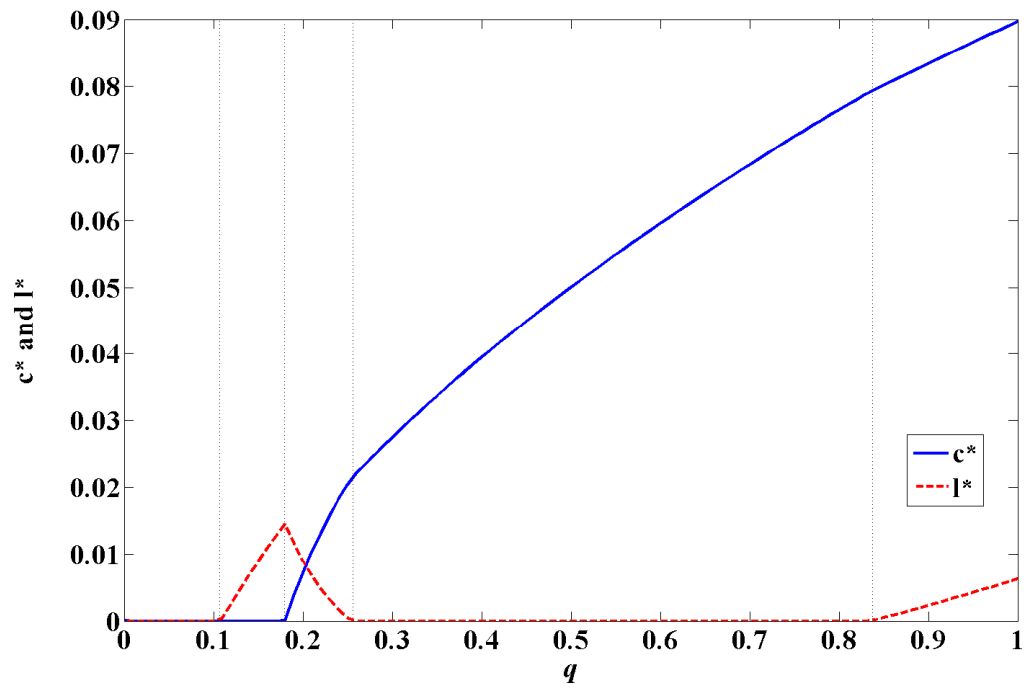


Figure 4.7: Values of c^* and l^* when $\eta > 0$ under Scenario 2

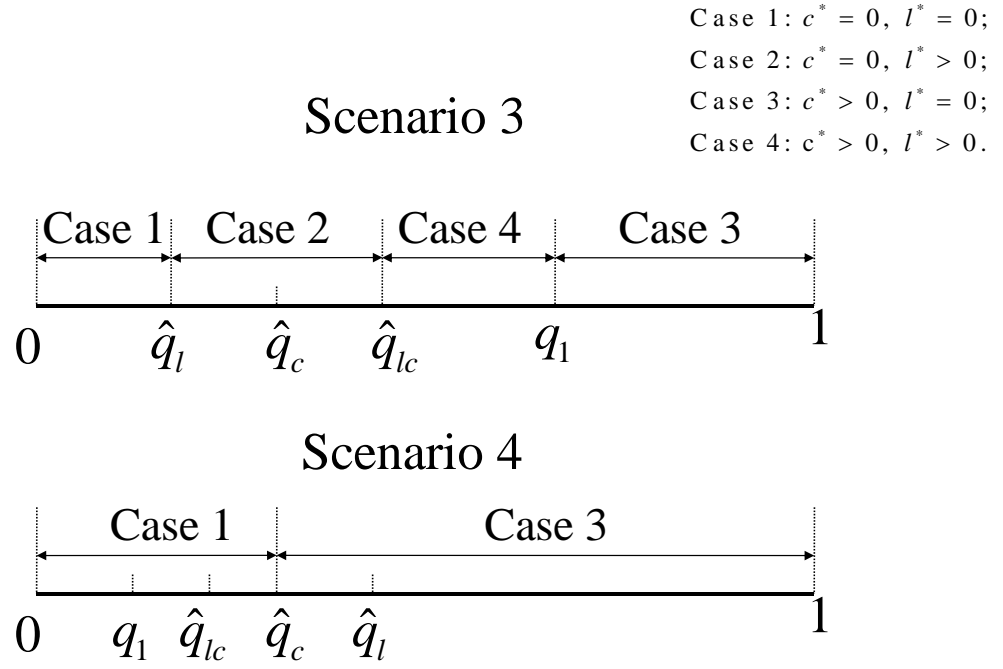


Figure 4.8: Possible Cases in Scenario 3 ($\alpha \in [\alpha_2, \alpha_3]$) and in Scenario 4 ($\alpha > \alpha_3$)

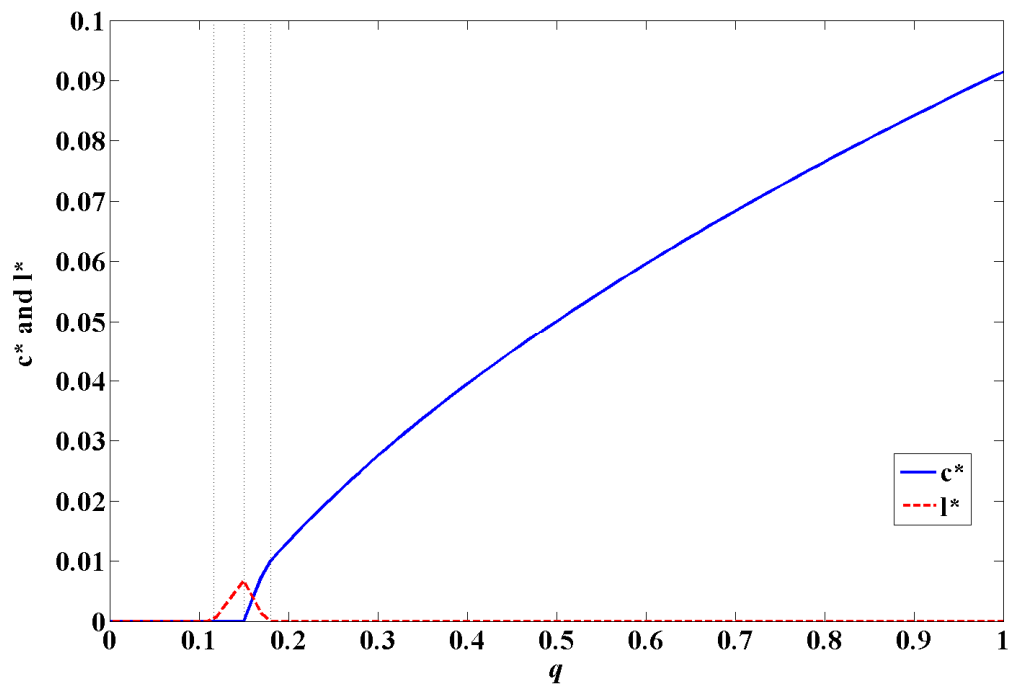


Figure 4.9: Values of c^* and l^* when $\eta > 0$ under Scenario 3

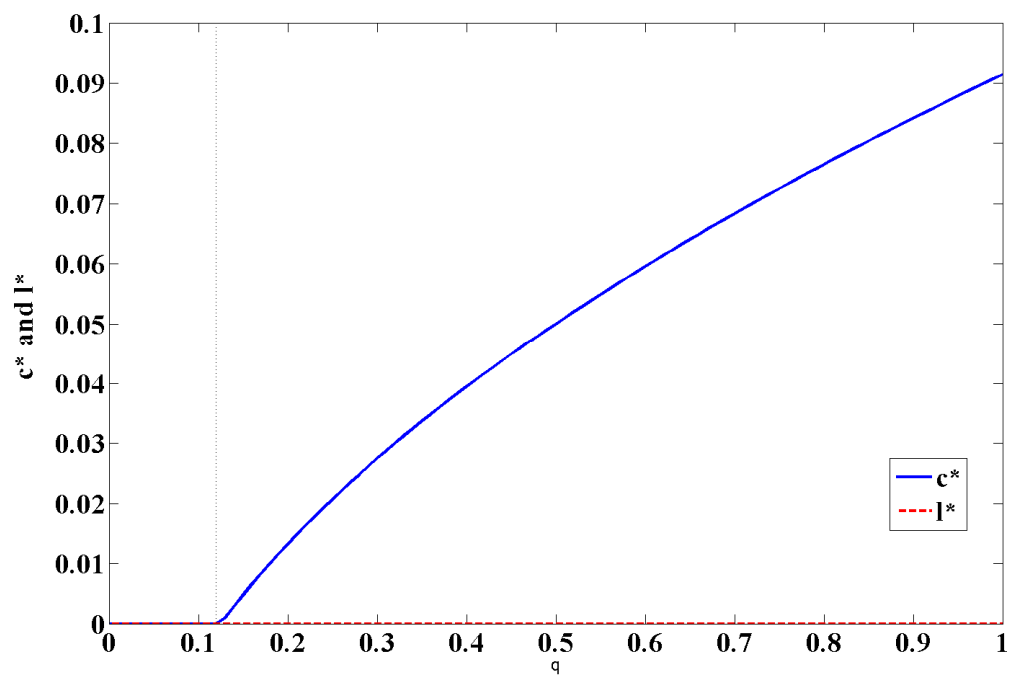


Figure 4.10: Values of c^* and l^* when $\eta > 0$ under Scenario 4

Appendix

Item A

In this Item we are going to show the justification of $\sqrt{D/\beta} - r - \eta > 0$ and $\sqrt{D/\beta} - r > 0$ in Assumption 1. Here we only need to justify $\sqrt{D/\beta} - r - \eta > 0$ because $\sqrt{D/\beta} - r > 0$ will naturally follow when $\sqrt{D/\beta} - r - \eta > 0$ holds.

If at time 0 there is no investment into RTC and at time τ_1 the true state is proved to be T , then at time τ_1 the social planner's problem is to choose an investment level, $c' \geq 0$, on RTC so that the damage from climate change plus research costs are minimized.

$$\min_{c' \geq 0} \beta c' e^{-r\tau_1} + \int_0^{\tau_1} D e^{-rt} dt + E_{\tau_3} \left[\int_{\tau_1}^{\tau_3} D e^{-rt} dt \right] \quad (4.36)$$

Since the density function of τ_3 is $g'(\tau_3) = (c' + \eta) e^{-(c' + \eta)(\tau_3 - \tau_1)}$, where $\tau_3 > \tau_1$, we have

$$\begin{aligned} E_{\tau_3} \left[\int_{\tau_1}^{\tau_3} D e^{-rt} dt \right] &= \int_{\tau_1}^{\infty} \frac{D}{r} (e^{-r\tau_1} - e^{-r\tau_3}) (c' + \eta) e^{-(c' + \eta)(\tau_3 - \tau_1)} d\tau_3 \\ &= \frac{D}{r} \left[e^{-r\tau_1} - \int_{\tau_1}^{\infty} e^{-r\tau_3} (c' + \eta) e^{-(c' + \eta)(\tau_3 - \tau_1)} d\tau_3 \right] \\ &= \frac{D}{r} \left(1 - \frac{c' + \eta}{r + c' + \eta} \right) e^{-r\tau_1} \end{aligned}$$

Then the social planner's problem becomes

$$\min_{c' \geq 0} \beta c' e^{-r\tau_1} + \int_0^{\tau_1} D e^{-rt} dt + \frac{D}{r} \left(1 - \frac{c' + \eta}{r + c' + \eta} \right) e^{-r\tau_1}, \quad (4.37)$$

which is equivalent to

$$\min_{c' \geq 0} \beta c' e^{-r\tau_1} + \frac{D}{r} \left(1 - e^{-r\tau_1} \frac{c' + \eta}{r + c' + \eta} \right). \quad (4.38)$$

The first order condition for an interior solution is

$$\beta - \frac{D}{(c'^* + r + \eta)^2} = 0 \quad (4.39)$$

So $c'^* = \sqrt{D/\beta} - r - \eta$. The interior solution requires $\sqrt{D/\beta} - r - \eta > 0$. Naturally $\sqrt{D/\beta} - r > 0$ follows.

Item B

If at time $t = \tau_1$ it is proved that climate change is harmful and the RTC has not been successful yet, then the social planner will add more investment, $c' \geq 0$, in this research to accelerate the coming of the breakthrough time. Therefore, at time $t = \tau_1$ the social planner's problem is:

$$V^A(c, l) = \min_{c' \geq 0} \beta c' e^{-r\tau_1} + E_{\tau_3} \left[\int_0^{\tau_3} D e^{-rt} dt \right]. \quad (4.40)$$

We know that at time $t = \tau_1$ the random variable τ_3 becomes possible. Its probability density function is $g'(\tau_3) = (c + c' + \eta) e^{-(c+c'+\eta)(\tau_3-\tau_1)}$, where $\tau_3 \geq \tau_1$. Therefore

$$\begin{aligned} E_{\tau_3} \left[\int_0^{\tau_3} D e^{-rt} dt \right] &= E_{\tau_3} \left[\frac{D}{r} (1 - e^{-r\tau_3}) \right] \\ &= \int_{\tau_1}^{\infty} \frac{D}{r} (1 - e^{-r\tau_3}) (c + c' + \eta) e^{-(c+c'+\eta)(\tau_3-\tau_1)} d\tau_3 \\ &= \frac{D}{r} [1 - e^{-r\tau_1} \int_{\tau_1}^{\infty} (c + c' + \eta) e^{-(r+c+c'+\eta)(\tau_3-\tau_1)} d\tau_3] \\ &= \frac{D}{r} \left(1 - \frac{c + c' + \eta}{r + c + c' + \eta} e^{-r\tau_1} \right) \end{aligned}$$

Then at time τ_1 the social planner's problem becomes

$$\min_{c' \geq 0} \beta c' e^{-r\tau_1} + \frac{D}{r} \left(1 - \frac{c + c' + \eta}{r + c + c' + \eta} e^{-r\tau_1} \right) \quad (4.41)$$

The first order condition is

$$\beta - \frac{D}{(r + c + c'^* + \eta)^2} \geq 0 \quad (4.42)$$

So the interior solution is

$$c'^* = \sqrt{D/\beta} - c - r - \eta.$$

The interior solution requires $\sqrt{D/\beta} - c - r - \eta > 0$ and hence $\eta < \sqrt{D/\beta} - r - c \leq \sqrt{D/\beta} - r$. Plug the interior solution into the objective function (4.41), we get

$$\begin{aligned} V^A(c, l) &= \beta (\sqrt{D/\beta} - c - r - \eta) e^{-r\tau_1} + \frac{D}{r} \left(1 - \frac{\sqrt{D/\beta} - r}{\sqrt{D/\beta}} e^{-r\tau_1} \right) \\ &= \frac{D}{r} + [\sqrt{\beta D} - \beta(c + r + \eta) - \frac{D}{r} \frac{\sqrt{D/\beta} - r}{\sqrt{D/\beta}}] e^{-r\tau_1} \\ &= \frac{D}{r} + (2\sqrt{\beta D} - \beta(c + r + \eta) - \frac{D}{r}) e^{-r\tau_1} \end{aligned}$$

Item C

In this Item we are going to show how to obtain equation (4.7).

Plugging equations (4.3), (4.4), and (4.5) into equation (4.6), we get

$$\begin{aligned}
 V(c, l) &= \int_0^\infty \int_{\tau_1}^\infty \left\{ q \left[\frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1} \right] + (1-q)(\beta c + \alpha l) \right\} g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
 &\quad + \int_0^\infty \int_0^{\tau_1} \left\{ q \left[\frac{D}{r} (1 - e^{-r\tau_2}) \right] + (1-q)(\beta c + \alpha l) \right\} g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
 &= \beta c + \alpha l + q \int_0^\infty \int_{\tau_1}^\infty \left[\frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1} \right] g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
 &\quad + q \int_0^\infty \int_0^{\tau_1} \left[\frac{D}{r} (1 - e^{-r\tau_2}) \right] g(\tau_2) f(\tau_1) d\tau_2 d\tau_1,
 \end{aligned}$$

where $f(\tau_1) = (l_0 + l)e^{-(l_0+l)\tau_1}$, $g(\tau_2) = ce^{-c\tau_2}$.

Since

$$\begin{aligned}
 &\int_0^\infty \int_{\tau_1}^\infty \left[\frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1} \right] g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
 &= \int_0^\infty e^{-c\tau_1} \left[\frac{D}{r} + (2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r})e^{-r\tau_1} \right] (l_0 + l)e^{-(l_0+l)\tau_1} d\tau_1 \\
 &= \frac{D}{r} \int_0^\infty e^{-c\tau_1} (l_0 + l)e^{-(l_0+l)\tau_1} d\tau_1 + \int_0^\infty [2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r}] e^{-r\tau_1} (l_0 + l)e^{-(l_0+l)\tau_1} d\tau_1 \\
 &= \frac{D(l_0 + l)}{r(l_0 + l + c)} + [2\sqrt{\beta D} - \beta(c+r+\eta) - \frac{D}{r}] \frac{l_0 + l}{r + l_0 + l + c},
 \end{aligned}$$

and

$$\begin{aligned}
& \int_0^\infty \int_0^{\tau_1} \left[\frac{D}{r} (1 - e^{-r\tau_2}) \right] g(\tau_2) f(\tau_1) d\tau_2 d\tau_1 \\
&= \int_0^\infty \int_0^{\tau_1} \left[\frac{D}{r} (1 - e^{-r\tau_2}) \right] c e^{-c\tau_2} (l_0 + l) e^{-(l_0+l)\tau_1} d\tau_2 d\tau_1 \\
&= \frac{D}{r} \left[\int_0^\infty \int_0^{\tau_1} c e^{-c\tau_2} (l_0 + l) e^{-(l_0+l)\tau_1} d\tau_2 d\tau_1 - \int_0^\infty \int_0^{\tau_1} c (l_0 + l) e^{-(l_0+l)\tau_1} e^{-(r+c)\tau_2} d\tau_2 d\tau_1 \right] \\
&= \frac{D}{r} \left[\frac{c}{l_0 + l + c} - \frac{c(l_0 + l)}{r + c} \int_0^\infty e^{-(l_0+l)\tau_1} (1 - e^{-(r+c)\tau_1}) d\tau_1 \right] \\
&= \frac{D}{r} \left[\frac{c}{l_0 + l + c} - \frac{c(l_0 + l)}{r + c} \left[\int_0^\infty e^{-(l_0+l)\tau_1} d\tau_1 - \int_0^\infty e^{-(r+l_0+l+c)\tau_1} d\tau_1 \right] \right] \\
&= \frac{D}{r} \left[\frac{c}{l_0 + l + c} - \frac{c(l_0 + l)}{r + c} \left[\frac{1}{l_0 + l} - \frac{1}{r + l_0 + l + c} \right] \right] \\
&= \frac{D}{r} \left[\frac{c}{l_0 + l + c} - \frac{c}{r + c} + \frac{c(l_0 + l)}{(r + c)(r + l_0 + l + c)} \right] \\
&= \frac{D}{r} \left[\frac{c}{l_0 + l + c} + \frac{c(l_0 + l) - c(r + l_0 + l + c)}{(r + c)(r + l_0 + l + c)} \right] \\
&= \frac{D}{r} \left[\frac{c}{l_0 + l + c} - \frac{c}{r + l_0 + l + c} \right] \\
&= \frac{Dc}{(l_0 + l + c)(r + l_0 + l + c)},
\end{aligned}$$

then

$$\begin{aligned}
V(c, l) &= \beta c + \alpha l + q \left\{ \frac{D(l_0 + l)}{r(l_0 + l + c)} + [2\sqrt{\beta D} - \beta(c + r + \eta) - \frac{D}{r}] \frac{l_0 + l}{r + l_0 + l + c} \right. \\
&\quad \left. + \frac{Dc}{(l_0 + l + c)(r + l_0 + l + c)} \right\} \\
&= \beta c + \alpha l + q \left\{ \frac{D(l_0 + l)}{r} \left(\frac{1}{l_0 + l + c} - \frac{1}{r + l_0 + l + c} \right) \right. \\
&\quad \left. + [2\sqrt{\beta D} - \beta(c + r + \eta)] \frac{l_0 + l}{r + l_0 + l + c} + \frac{Dc}{(l_0 + l + c)(r + l_0 + l + c)} \right\} \\
&= \beta c + \alpha l + q \left\{ \frac{D(l_0 + l) + Dc}{(l_0 + l + c)(r + l_0 + l + c)} + [2\sqrt{\beta D} - \beta(c + r + \eta)] \frac{l_0 + l}{r + l_0 + l + c} \right\} \\
&= \beta c + \alpha l + q \left\{ \frac{D}{r + l_0 + l + c} + [2\sqrt{\beta D} - \beta(c + r + \eta)] \frac{l_0 + l}{r + l_0 + l + c} \right\} \\
&= \beta c + \alpha l + \frac{q}{r + l_0 + l + c} \left\{ D + [2\sqrt{\beta D} - \beta(c + r + \eta)](l_0 + l) \right\}
\end{aligned}$$

This is equation (4.7).

Item D

In this Item we derive the FOCs of problem (4.7) and show the function $v(c, l)$ is convex if $0 \leq \eta \leq \sqrt{D/\beta} - r$. FOCs are:

$$\begin{aligned}
 \frac{\partial v(c, l)}{\partial c} &= \beta - \frac{q\beta(l_0 + l)(r + l_0 + l + c) + q\{D + [2\sqrt{D\beta} - \beta(c + r + \eta)](l_0 + l)\}}{(r + l_0 + l + c)^2} \\
 &= \beta - q \frac{\beta(l_0 + l)(r + l_0 + l + c) + D + 2\sqrt{D\beta}(l_0 + l) - \beta(c + r + \eta)(l_0 + l)}{(r + l_0 + l + c)^2} \\
 &= \beta - q \frac{\beta(l_0 + l)^2 + 2\sqrt{D\beta}(l_0 + l) + D - \beta\eta(l_0 + l)}{(r + l_0 + l + c)^2} \\
 &= \beta - q \frac{(\sqrt{D} + \sqrt{\beta}(l_0 + l))^2 - \beta\eta(l_0 + l)}{(r + l_0 + l + c)^2} \geq 0
 \end{aligned}$$

and

$$\begin{aligned}
 \frac{\partial v(c, l)}{\partial l} &= \alpha + q \frac{[2\sqrt{D\beta} - \beta(c + r)](r + l_0 + l + c) - \{D + [2\sqrt{D\beta} - \beta(c + r + \eta)](l_0 + l)\}}{(r + l_0 + l + c)^2} \\
 &= \alpha + q \frac{[2\sqrt{D\beta} - \beta(c + r + \eta)](r + c) - D}{(r + l_0 + l + c)^2} \\
 &= \alpha - q \frac{\beta(r + c)^2 - 2\sqrt{D\beta}(r + c) + D + \beta\eta(c + r)}{(r + l_0 + l + c)^2} \\
 &= \alpha - q \frac{(\sqrt{D} - \sqrt{\beta}(c + r))^2 + \beta\eta(r + c)}{(r + l_0 + l + c)^2} \geq 0.
 \end{aligned}$$

Next we are going to show $v(c, l)$ is convex when $0 \leq \eta \leq \sqrt{D/\beta} - r$. To do this we need to show that if $0 \leq \eta \leq \sqrt{D/\beta} - r$, then $V''_{cc} \geq 0$, $V''_{ll} \geq 0$, and $V''_{cc}V''_{ll} - (V''_{lc})^2 \geq 0$. We have

$$\begin{aligned}
 V''_{cc} &= 2q \frac{(r + l_0 + l + c)[(\sqrt{D} + \sqrt{\beta}(l_0 + l))^2 - \beta\eta(l_0 + l)]}{(r + l_0 + l + c)^4} \\
 &= 2q \frac{(\sqrt{D} + \sqrt{\beta}(l_0 + l))^2 - \beta\eta(l_0 + l)}{(r + l_0 + l + c)^3} \\
 V''_{ll} &= 2q \frac{(\sqrt{D} - \sqrt{\beta}(r + c))^2 + \beta\eta(r + c)}{(r + l_0 + l + c)^3} \\
 V''_{lc} &= -q \frac{(r + l_0 + l + c)[-2\sqrt{\beta}(\sqrt{D} - \sqrt{\beta}(r + c)) + \beta\eta] - 2[(\sqrt{D} - \sqrt{\beta}(r + c))^2 + \beta\eta(r + c)]}{(r + l_0 + l + c)^3} \\
 &= -q \frac{-2[\sqrt{D} - \sqrt{\beta}(r + c)][\sqrt{D} + \sqrt{\beta}(l_0 + l)] + \beta\eta[(l_0 + l) - (r + c)]}{(r + l_0 + l + c)^3} \\
 &= q \frac{2[\sqrt{D} - \sqrt{\beta}(r + c)][\sqrt{D} + \sqrt{\beta}(l_0 + l)] + \beta\eta[(r + c) - (l_0 + l)]}{(r + l_0 + l + c)^3}.
 \end{aligned}$$

Define $\phi \equiv q^2/(r+l_0+l+c)^6$. Then we have

$$\begin{aligned}
& V''_{cc}V''_{ll} - (V''_{lc})^2 \\
&= \phi \left\{ 4[(\sqrt{D} + \sqrt{\beta}(l_0+l))^2 - \beta\eta(l_0+l)][(\sqrt{D} - \sqrt{\beta}(r+c))^2 + \beta\eta(r+c)] \right. \\
&\quad \left. - \{2[\sqrt{D} - \sqrt{\beta}(r+c)][\sqrt{D} + \sqrt{\beta}(l_0+l)] + \beta\eta[(r+c) - (l_0+l)]\}^2 \right\} \\
&= \phi\beta\eta \left\{ 4(r+c)(\sqrt{D} + \sqrt{\beta}(l_0+l))^2 - 4(l_0+l)(\sqrt{D} - \sqrt{\beta}(r+c))^2 \right. \\
&\quad - 4[(r+c) - (l_0+l)][\sqrt{D} + \sqrt{\beta}(l_0+l)][\sqrt{D} - \sqrt{\beta}(r+c)] \\
&\quad \left. - \beta\eta[4(l_0+l)(r+c) + ((r+c) - (l_0+l))^2] \right\} \\
&= \phi\beta\eta \left\{ 4(r+c)(\sqrt{D} + \sqrt{\beta}(l_0+l))[(\sqrt{D} + \sqrt{\beta}(l_0+l)) - (\sqrt{D} - \sqrt{\beta}(r+c))] \right. \\
&\quad + 4(l_0+l)(\sqrt{D} - \sqrt{\beta}(r+c))[(\sqrt{D} + \sqrt{\beta}(l_0+l)) - (\sqrt{D} - \sqrt{\beta}(r+c))] \\
&\quad \left. - \beta\eta(r+l_0+l+c)^2 \right\} \\
&= \phi\beta\eta \left\{ 4(r+c)(\sqrt{D} + \sqrt{\beta}(l_0+l))[\sqrt{\beta}(r+l_0+l+c)] \right. \\
&\quad + 4(l_0+l)(\sqrt{D} - \sqrt{\beta}(r+c))[\sqrt{\beta}(r+l_0+l+c)] \\
&\quad \left. - \beta\eta(r+l_0+l+c)^2 \right\} \\
&= \phi\beta\eta \left\{ 4[\sqrt{\beta}(r+l_0+l+c)] \left[(r+c)(\sqrt{D} + \sqrt{\beta}(l_0+l)) + (l_0+l)(\sqrt{D} - \sqrt{\beta}(r+c)) \right] \right. \\
&\quad \left. - \beta\eta(r+l_0+l+c)^2 \right\} \\
&= \phi\beta\eta \left\{ 4\sqrt{D\beta}(r+l_0+l+c)^2 - \beta\eta(r+l_0+l+c)^2 \right\} \\
&= \phi\beta\eta \left\{ 4\sqrt{D\beta} - \beta\eta \right\} (r+l_0+l+c)^2 \\
&= \phi\beta^2\eta \left\{ 4\sqrt{\frac{D}{\beta}} - \eta \right\} (r+l_0+l+c)^2 \\
&\geq 0 \quad \text{if } 0 \leq \eta \leq \sqrt{D/\beta} - r.
\end{aligned}$$

It is obvious that $V''_{ll} \geq 0$. Now we only need to check $V''_{cc} \geq 0$ when $0 \leq \eta \leq \sqrt{D/\beta} - r$. We have

$$\begin{aligned}
 V''_{cc} &= 2q \frac{(\sqrt{D} + \sqrt{\beta}(l_0 + l))^2 - \beta\eta(l_0 + l)}{(r + l_0 + l + c)^3} \\
 &\geq 2q \frac{(\sqrt{D} + \sqrt{\beta}(l_0 + l))^2 - \beta(\sqrt{D/\beta} - r)(l_0 + l)}{(r + l_0 + l + c)^3} \\
 &= 2q \frac{D + 2\sqrt{D\beta}(l_0 + l) + \beta(l_0 + l)^2 - \sqrt{D\beta}(l_0 + l) + r\beta(l_0 + l)}{(r + l_0 + l + c)^3} \\
 &= 2q \frac{D + (\sqrt{D\beta} + r\beta)(l_0 + l) + \beta(l_0 + l)^2}{(r + l_0 + l + c)^3} \\
 &\geq 0.
 \end{aligned}$$

Hence we have shown that $V(c, l)$ is convex when $0 \leq \eta \leq \sqrt{D/\beta} - r$.

Item E

This Item proves Remark 4.1. Here we just prove that if $q_{lc} > q_c$, then $q_c > q_l$. The other part of the remark can be proved by the same procedure.

Proof.

$$\begin{aligned}
 q_{lc} > q_c &\Rightarrow 1 - \sqrt{\alpha/\beta} > \sqrt{\beta} \frac{r + l_0}{\sqrt{D} + \sqrt{\beta}l_0} \\
 &\Rightarrow 1 - \sqrt{\alpha/\beta} > 1 - \frac{\sqrt{D} - \sqrt{\beta}r}{\sqrt{D} + \sqrt{\beta}l_0} \\
 &\Rightarrow \sqrt{\alpha/\beta} < \frac{\sqrt{D} - \sqrt{\beta}r}{\sqrt{D} + \sqrt{\beta}l_0} \\
 &\Rightarrow \frac{\sqrt{\alpha}}{\sqrt{D} - \sqrt{\beta}r} < \frac{\sqrt{\beta}}{\sqrt{D} + \sqrt{\beta}l_0} \\
 &\Rightarrow \frac{\sqrt{\alpha}(r + l_0)}{\sqrt{D} - \sqrt{\beta}r} < \frac{\sqrt{\beta}(r + l_0)}{\sqrt{D} + \sqrt{\beta}l_0} \\
 &\Rightarrow \sqrt{q_l} < \sqrt{q_c} \\
 &\Rightarrow q_l < q_c
 \end{aligned}$$

□

Item F

In this Item we prove Remark 4.2.

Proof. From FOC (4.20) we have

$$\begin{aligned}
 q &= \beta \left\{ \frac{r + l_0 + c^* + l^*}{\sqrt{D} + \sqrt{\beta}(l_0 + l^*)} \right\}^2 \\
 &> \beta \left\{ \frac{r + l_0 + l^*}{\sqrt{D} + \sqrt{\beta}(l_0 + l^*)} \right\}^2 \quad \text{by } c^* > 0 \\
 &= \left\{ \frac{\sqrt{D} + \sqrt{\beta}(l_0 + l^*) - (\sqrt{D} - \sqrt{\beta}r)}{\sqrt{D} + \sqrt{\beta}(l_0 + l^*)} \right\}^2 \\
 &= \left\{ 1 - \frac{\sqrt{D} - \sqrt{\beta}r}{\sqrt{D} + \sqrt{\beta}(l_0 + l^*)} \right\}^2 \\
 &> \left\{ 1 - \frac{\sqrt{D} - \sqrt{\beta}r}{\sqrt{D} + \sqrt{\beta}l_0} \right\}^2 \quad \text{by } l^* > 0 \\
 &= q_c.
 \end{aligned}$$

From FOC (4.21) we have

$$\begin{aligned}
 q &= \alpha \left\{ \frac{r + l_0 + c^* + l^*}{\sqrt{D} - \sqrt{\beta}(r + c^*)} \right\}^2 \\
 &> \alpha \left\{ \frac{r + l_0 + c^*}{\sqrt{D} - \sqrt{\beta}(r + c^*)} \right\}^2 \quad \text{by } l^* > 0 \\
 &> \alpha \left\{ \frac{r + l_0}{\sqrt{D} - \sqrt{\beta}r} \right\}^2 \quad \text{by } c^* > 0 \text{ and } \sqrt{D} - \sqrt{\beta}(r + c^*) > 0 \\
 &= q_l.
 \end{aligned}$$

Therefore we have $q > \max\{q_c, q_l\}$. Next we show that the existence of a solution (c^*, l^*) such that FOCs (4.20) and (4.21) in Case 3 requires $q = q_{lc}$.

Define $X \equiv l_0 + l^*$ and $Y \equiv r + c^*$. The the FOCs (4.20) and (4.21) in Case 3 become

$$\sqrt{\frac{\beta}{q}} = \frac{\sqrt{D} + \sqrt{\beta}X}{X + Y} \quad (4.43)$$

$$\sqrt{\frac{\alpha}{q}} = \frac{\sqrt{D} - \sqrt{\beta}Y}{X + Y}, \quad (4.44)$$

where $X > l_0$ and $Y > r$. Form equation (4.43) and (4.44) we have

$$Y = \sqrt{\frac{qD}{\beta}} - (1 - \sqrt{q})X \quad (4.45)$$

$$Y = \frac{\sqrt{D}}{\sqrt{\frac{\alpha}{q}} + \sqrt{\beta}} - \frac{\sqrt{\alpha/q}}{\sqrt{\frac{\alpha}{q}} + \sqrt{\beta}}X, \quad (4.46)$$

where $X > l_0$ and $Y > r$. When $q = q_{lc}$, then

$$\begin{aligned}
 \sqrt{q} &= 1 - \sqrt{\alpha/\beta} \\
 \Rightarrow \sqrt{q} + \sqrt{\alpha/\beta} &= 1 \\
 \Rightarrow \sqrt{q\beta} + \sqrt{\alpha} &= \sqrt{\beta} \\
 \Rightarrow \sqrt{\beta} + \sqrt{\alpha/q} &= \sqrt{\beta/q}.
 \end{aligned}$$

Therefore, equation (4.46) becomes

$$\begin{aligned}
 Y &= \frac{\sqrt{D}}{\sqrt{\beta/q}} - \frac{\alpha/q}{\sqrt{\beta/q}} X \\
 &= \sqrt{\frac{qD}{\beta}} - \sqrt{\frac{\alpha}{\beta}} X \\
 &= \sqrt{\frac{qD}{\beta}} - (1 - \sqrt{q})X \quad \text{by } \sqrt{q} = 1 - \sqrt{\alpha/\beta}.
 \end{aligned}$$

We can see that if $q = q_{lc}$ then equations (4.43) and (4.44) are identical. So any solution (c^*, l^*) such that equation (4.43) must be such that equation (4.44). Next we are going to show if $q \neq q_{lc}$ there is no solution (c^*, l^*) such that the equation system consisting of equations (4.43) and (4.44).

First, we consider the scenario in which $q > q_{lc}$. If $q > q_{lc}$, then $\sqrt{\beta} + \sqrt{\alpha/q} > \sqrt{\beta/q}$. Therefore $\frac{\sqrt{D}}{\sqrt{\frac{\alpha}{q}} + \sqrt{\beta}} < \sqrt{\frac{qD}{\beta}}$. This means the Y-intercept of function (4.45) is smaller than the Y-intercept of function (4.46). If we can show the absolute value of the slope of the curve of function (4.45) is greater than the slope of the curve of function (4.46) then we can conclude that the two curves have no intersection point in the first quadrant. The following algebra shows this is true.

$$\begin{aligned}
 \frac{\sqrt{\alpha/q}}{\sqrt{\frac{\alpha}{q}} + \sqrt{\beta}} &> 1 - \sqrt{q} \\
 \Leftrightarrow \sqrt{\alpha/q} &> (1 - \sqrt{q})(\sqrt{\frac{\alpha}{q}} + \sqrt{\beta}) \\
 \Leftrightarrow \sqrt{\alpha/q} &> \sqrt{\frac{\alpha}{q}} + \sqrt{\beta} - \sqrt{\alpha} - \sqrt{q\beta} \\
 \Leftrightarrow 0 &> \sqrt{\beta} - \sqrt{\alpha} - \sqrt{q\beta} \\
 \Leftrightarrow 0 &> 1 - \sqrt{\alpha/\beta} - \sqrt{q} \\
 \Leftrightarrow \sqrt{q} &> 1 - \sqrt{\alpha/\beta} \\
 \Leftrightarrow q &> q_{lc}.
 \end{aligned}$$

By the same procedure we can show that if $q > q_{lc}$ then there is no solution (c^*, l^*) such that the equation system consisting of equations (4.43) and (4.44) either. The following figure shows the relationship between these two curves when $q \neq q_{lc}$. In this figure $a \equiv \frac{\sqrt{D}}{\sqrt{\frac{q}{\alpha}} + \sqrt{\beta}}$ and $b \equiv \frac{\sqrt{qD}}{\sqrt{\beta}}$. □

Item G

In this Item we show how to obtain \hat{q}_{lc} . Plugging $l^* = (\sqrt{\frac{q}{\hat{q}_l}} - 1)r$ into inequality (4.25) we get

$$\begin{aligned} \beta r^2 &\geq \hat{q}_l [D + (2\sqrt{D\beta} - \beta\eta)l^* + \beta l^{*2}] \\ \Rightarrow \beta \left(\sqrt{\frac{q}{\hat{q}_l}} r - r \right)^2 + (2\sqrt{D\beta} - \beta\eta) \left(\sqrt{\frac{q}{\hat{q}_l}} r - r \right) + D - \frac{\beta r^2}{\hat{q}_l} &\leq 0 \\ \Rightarrow \beta y^2 + (2\sqrt{D\beta} - \beta\eta)y + D - \frac{\beta r^2}{\hat{q}_l} &\leq 0, \end{aligned} \quad (4.47)$$

where $y \equiv \sqrt{\frac{q}{\hat{q}_l}} r - r$. Then the solutions of equation

$$\beta y^2 + (2\sqrt{D\beta} - \beta\eta)y + D - \frac{\beta r^2}{\hat{q}_l} = 0$$

are

$$y_{1,2} = \frac{-(2\sqrt{D\beta} - \beta\eta) \pm \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/\hat{q}_l)}}{2\beta}.$$

The existence of the solutions requires that

$$(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/\hat{q}_l) \geq 0,$$

from which we can get

$$\begin{aligned} (2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/\hat{q}_l) &\geq 0 \\ \Rightarrow (2\sqrt{D\beta} - \beta\eta)^2 - 4\beta D + 4\frac{\beta^2}{\alpha}((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r) &\geq 0 \\ \Rightarrow 4\frac{\beta^2}{\alpha}[(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r] &\geq 4\beta\eta\sqrt{D\beta} - \beta^2\eta^2 \\ \Rightarrow \alpha &\leq \frac{4\beta^2[(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r]}{4\beta\eta\sqrt{D\beta} - \beta^2\eta^2} \\ \Rightarrow \alpha &\leq \frac{\beta[(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r]}{D(\eta\sqrt{\beta/D} - \frac{1}{4}\eta^2\beta/D)} \\ \Rightarrow \alpha &\leq \frac{\beta[(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r]}{D(\frac{\eta}{\sqrt{D/\beta}})(1 - \frac{1}{4}\frac{\eta}{\sqrt{D/\beta}})} \equiv \alpha_l. \end{aligned}$$

Therefore the inequality (4.47) requires $y \in [y_2, y_1]$. By Assumption 1 we have $-(2\sqrt{D\beta} - \beta\eta)/2\beta < 0$. Due to $q > \hat{q}_l$, we must have $y_{1,2} > 0$. It is easy to check $y_2 < 0$. Therefore y_2 is not the solution we want. To guarantee $y_1 > 0$, we must have $4\beta(D - \beta r^2/\hat{q}_l) < 0$, that is $\alpha < \frac{\beta}{D}((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r) \equiv \alpha_3$. By Assumption 1 we have $\alpha_3 < \alpha_l$. Hence the existence of a positive y_1 requires $\alpha \leq \alpha_3$.

$$\begin{aligned}
y &\leq y_1 \\
\Rightarrow \sqrt{\frac{q}{\hat{q}_l}}r - r &\leq y_1 \\
\Rightarrow \sqrt{q} &\leq \frac{\sqrt{\hat{q}_l}(r + y_1)}{r} \\
\Rightarrow q &\leq \hat{q}_l \left(\frac{r + y_1}{r}\right)^2 \\
\Rightarrow q &\leq \hat{q}_l \frac{\left\{ r + \frac{1}{2\beta} [(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/\hat{q}_l)}] \right\}^2}{r^2} \equiv \hat{q}_{lc}.
\end{aligned}$$

Item H

In this Item we show that when q_1 and q_2 exist and $q_2 \geq q_1$, we always have $q_2 > \hat{q}_c$. By equation (4.33) we have

$$q_2 = \left\{ \frac{(2 - \eta\sqrt{\beta/D}) + \sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}}{2} \right\}^2.$$

Then

$$\begin{aligned}
\sqrt{q_2} &\geq \frac{2 - \eta\sqrt{\beta/D}}{2} \\
&= 1 - \frac{1}{2}\eta\sqrt{\frac{\beta}{D}} \\
&\geq 1 - \frac{1}{2}\sqrt{\frac{\beta}{D}}(\sqrt{\frac{D}{\beta}} - r) \quad \text{by Assumption 1} \\
&= 1 - \frac{1}{2}(1 - r\sqrt{\frac{\beta}{D}}) \\
&= \frac{1}{2} + \frac{1}{2}r\sqrt{\frac{\beta}{D}} \\
&\geq r\sqrt{\frac{\beta}{D}} \quad \text{by } r/\sqrt{D/\beta} < 1 \text{ from Assumption 1} \\
&= \sqrt{\hat{q}_c}.
\end{aligned}$$

Therefore $q_2 > \hat{q}_c$.

Item I**Possibility 1**

First we show that $q_1 \geq \hat{q}_c$ as long as $q_2 \leq 1$. Since

$$q_{1,2} = \left\{ \frac{(2 - \eta\sqrt{\beta/D}) \mp \sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}}{2} \right\}^2,$$

then $q_2 \leq 1$ implies that

$$\begin{aligned} & \frac{(2 - \eta\sqrt{\beta/D}) + \sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}}{2} \leq 1 \\ \Rightarrow & -\eta\sqrt{\beta/D} + \sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)} \leq 0 \\ \Rightarrow & \sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)} \leq \eta\sqrt{\beta/D}. \end{aligned}$$

Then

$$\begin{aligned} \sqrt{q_1} &= \frac{(2 - \eta\sqrt{\beta/D}) - \sqrt{(2 - \eta\sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}}{2} \\ &\geq \frac{(2 - \eta\sqrt{\beta/D}) - \eta\sqrt{\beta/D}}{2} \\ &= 1 - \eta\sqrt{\frac{\beta}{D}} \\ &= \sqrt{\frac{\beta}{D}}(\sqrt{\frac{\beta}{D}} - \eta) \\ &\geq \sqrt{\frac{\beta}{D}}r \quad \text{by Assumption 1} \\ &= \sqrt{\hat{q}_c}. \end{aligned}$$

Second we show that $q_2 \leq 1$ requires $\beta(\eta\sqrt{\beta/D} - \beta\eta^2/4D) \leq \alpha \leq \beta\eta\sqrt{\beta/D}$. Since the existence of q_2 requires $\alpha \geq \beta(\eta\sqrt{\beta/D} - \beta\eta^2/4D)$, then we only need to show $q_2 \leq 1$ requires that

$$\alpha \leq \beta \eta \sqrt{\beta/D}.$$

$$\begin{aligned}
& q_2 \leq 1 \\
\Rightarrow & \frac{(2 - \eta \sqrt{\beta/D}) + \sqrt{(2 - \eta \sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}}{2} \leq 1 \\
\Rightarrow & -\eta \sqrt{\beta/D} + \sqrt{(2 - \eta \sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)} \leq 0 \\
\Rightarrow & \sqrt{(2 - \eta \sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)} \leq \eta \sqrt{\beta/D} \\
\Rightarrow & (2 - \eta \sqrt{\beta/D})^2 - 4(1 - \alpha/\beta) \leq \frac{\beta}{D} \eta^2 \\
\Rightarrow & 4 - 4\eta \sqrt{\beta/D} + \frac{\beta}{D} \eta^2 - 4 + 4\frac{\alpha}{\beta} \leq \frac{\beta}{D} \eta^2 \\
\Rightarrow & \frac{\alpha}{\beta} \leq \eta \sqrt{\beta/D} \\
\Rightarrow & \alpha \leq \beta \eta \sqrt{\beta/D}.
\end{aligned}$$

Possibility 2

Now we are going to show that $q_1 \geq \hat{q}_c$ and $q_2 > 1$ requires that $\beta \eta \sqrt{\beta/D} \leq \alpha \leq \frac{\beta}{D}[(\sqrt{D} - \sqrt{\beta}r)^2 + r\eta\beta]$. By the algebra in Possibility 1 we know that $q_2 > 1$ implies $\alpha \geq \beta \eta \sqrt{\beta/D}$. Now we only need to show $q_1 \geq \hat{q}_c$ implies $\alpha \leq \frac{\beta}{D}[(\sqrt{D} - \sqrt{\beta}r)^2 + r\eta\beta]$.

$$\begin{aligned}
& q_1 \geq \hat{q}_c \\
\Rightarrow & \frac{(2 - \eta \sqrt{\beta/D}) - \sqrt{(2 - \eta \sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)}}{2} \geq r \sqrt{\beta/D} \\
\Rightarrow & \sqrt{(2 - \eta \sqrt{\beta/D})^2 - 4(1 - \alpha/\beta)} \leq (2 - \eta \sqrt{\beta/D}) - 2r \sqrt{\beta/D} \\
\Rightarrow & (2 - \eta \sqrt{\beta/D})^2 - 4(1 - \alpha/\beta) \leq (2 - \eta \sqrt{\beta/D})^2 - 4r \sqrt{\beta/D}(2 - \eta \sqrt{\beta/D}) + 4r^2 \frac{\beta}{D} \\
\Rightarrow & -4 + 4\alpha/\beta \leq -4r \sqrt{\beta/D}(2 - \eta \sqrt{\beta/D}) + 4r^2 \frac{\beta}{D} \\
\Rightarrow & \frac{\alpha}{\beta} \leq 1 - 2r \sqrt{\beta/D} + r^2 \beta/D + \beta r \eta/D \\
\Rightarrow & \alpha \leq \frac{\beta}{D}[(\sqrt{D} - \sqrt{\beta}r)^2 + r\eta\beta].
\end{aligned}$$

Item J

In this Item we prove Remark 4.3.

First, we are going to show that if $\alpha \leq \alpha_3$ then $\hat{q}_l \leq \hat{q}_c$.

$$\begin{aligned} \hat{q}_l &\leq \hat{q}_c \\ \Leftrightarrow \frac{\alpha r^2}{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r} &\leq \frac{\beta r^2}{D} \\ \Leftrightarrow \alpha &\leq \frac{\beta}{D}[(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r] = \alpha_3 \end{aligned}$$

It is easy to check that the equality holds when $\alpha = \alpha_3$.

Then we are going to show that if $\alpha < \alpha_3$ then $\hat{q}_c \leq \hat{q}_{lc}$. To show this we first show when $\alpha = \alpha_3$, we have $\hat{q}_c = \hat{q}_{lc}$. Then we are going to show that if $\alpha < \alpha_3$ then $\frac{\partial \hat{q}_{lc}}{\partial \alpha} < 0$. Since $\frac{\partial \hat{q}_c}{\partial \alpha} = 0$ then the result follows.

We have shown that if $\alpha = \alpha_3$, then $\hat{q}_l = \hat{q}_c$. Therefore, when $\alpha = \alpha_3$ we will have

$$\begin{aligned} &\hat{q}_{lc} \\ = &\hat{q}_c \frac{\left\{ r + \frac{1}{2\beta} [(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \beta r^2/\hat{q}_c)}] \right\}^2}{r^2} \\ = &\hat{q}_c \frac{\left\{ r + \frac{1}{2\beta} [(\beta\eta - 2\sqrt{D\beta}) + \sqrt{(2\sqrt{D\beta} - \beta\eta)^2 - 4\beta(D - \frac{\beta r^2}{\beta r^2/D})}] \right\}^2}{r^2} \\ = &\hat{q}_c. \end{aligned}$$

Plugging $\hat{q}_l \frac{\alpha r^2}{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r}$ into equation (4.28) we have

$$\begin{aligned} &\hat{q}_{lc} \\ = &\frac{\alpha}{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r} \left\{ r + \frac{(\beta\eta - 2\sqrt{D\beta}) + \sqrt{\beta^2\eta^2 - 4\beta\eta\sqrt{D\beta} + \frac{4\beta^2}{\alpha}((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r)}}{2\beta} \right\}^2 \\ = &\frac{1}{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r} \left\{ \left(r + \frac{\eta}{2} - \sqrt{\frac{D}{\beta}} \right) \sqrt{\alpha} + \sqrt{\alpha} \sqrt{\frac{\eta^2}{4} - \eta\sqrt{\frac{D}{\beta}} + \frac{4}{\alpha}((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r)} \right\}^2 \\ = &\frac{1}{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r} \left\{ \left(r + \frac{\eta}{2} - \sqrt{\frac{D}{\beta}} \right) \sqrt{\alpha} + \sqrt{\left(\frac{\eta^2}{4} - \eta\sqrt{\frac{D}{\beta}} \right) \alpha + 4((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r)} \right\}^2 \end{aligned}$$

Define $\Phi \equiv (r + \frac{\eta}{2} - \sqrt{\frac{D}{\beta}})\sqrt{\alpha} + \sqrt{(\frac{\eta^2}{4} - \eta\sqrt{\frac{D}{\beta}})\alpha + 4((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r)}$. Therefore we have

$$\frac{\partial \hat{q}_{lc}}{\partial \alpha} = \frac{2\Phi \frac{\partial \Phi}{\partial \alpha}}{(\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r}.$$

We can check that $\Phi \geq 0$ when $\alpha \leq \alpha_3$. So $\text{sign}(\frac{\partial \hat{q}_{lc}}{\partial \alpha}) = \text{sign}(\frac{\partial \Phi}{\partial \alpha})$. Next we need to show $\partial \Phi / \partial \alpha < 0$.

$$\frac{\partial \Phi}{\partial \alpha} = (r + \frac{\eta}{2} - \sqrt{\frac{D}{\beta}}) \frac{1}{2\sqrt{\alpha}} + \frac{\frac{\eta^2}{4} - \eta\sqrt{\frac{D}{\beta}}}{2\sqrt{(\frac{\eta^2}{4} - \eta\sqrt{\frac{D}{\beta}})\alpha + 4((\sqrt{D} - \sqrt{\beta}r)^2 + \beta\eta r)}}$$

By Assumption 1 we have $r + \eta/2 - \sqrt{D/\beta} < 0$ and $\eta^2/4 - \eta\sqrt{D/\beta} < 0$. Therefore $\partial \Phi / \partial \alpha < 0$ when $\alpha < \alpha_3$.

In Item I we show that $q_1 > \hat{q}_c$ is equivalent to $\alpha < \alpha_3$. The part that “if $\alpha \leq \alpha_3$ then $\hat{q}_{lc} \leq q_1$ ” has not been analytically proved yet. Numerical method was employed and no violation was found.