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Three essays on modeling biofuel feedstock supply

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Three essays on modeling biofuel feedstock supply

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DEDICATION

I would like to dedicate this thesis to my mother Zhengqiao Wang, my father Jianyuan Zhou and to my boyfriend Xiaoguang Feng without whose support I would not have been able to complete this work.

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CHAPTER 1. INTRODUCTION

The general theme of this dissertation is modeling biofuel feedstock supply. All three essays focus on different topics relating to the theme. The first essay considers supply of corn stover for commercial production of a cellulosic ethanol plant. My second essay analyzes the impact of ethanol mandates on corn prices. The third essay presents a new model of agricultural supply which combines Positive Mathematical Programming (PMP) with the rational expectations storage model.

The first essay "Optimal Cellulosic Ethanol Plant Size under Uncertainty" determines the optimal size of a cellulosic biorefinery under uncertain future corn stover yields and cellulosic ethanol prices. Using a two period model, the capacity is determined in the first period when taking into account all the production decisions under various corn yields and gasoline prices. We also analyze the impacts on optimal plant sizes under two payment schemes for corn stover procurement. Payment scheme one considers the plant owner as a monopsony who pays for all transportation costs and all farmers receive the same price per ton of feedstock. Payment scheme two assumes that farmers pay for transportation costs and the plant owner pays a uniform per ton price for all delivered stover at the plant. Farmers close to the plant are able to capture transportation-related rents in the second scheme while they can't in the first scheme. Our results show that the optimal capacity under both certainty and uncertainty increases with a rise in land fraction distributed to biomass or a fall in feedstock transportation cost, plant scale factor. The sensitivity analysis shows that optimal capacities are almost the same under certainty and uncertainty when changing parameter in a possible range. Comparing the plant size under two corn stover payment schemes, the optimized plant size under certainty or uncertainty decreases when shifting from plant pays the transportation cost scheme to farmers pay the transportation cost.

The second essay “Estimating the Impact of Ethanol on Corn Prices Using the Competitive Storage Model” examines the impact of biofuel mandates on the prices of commodities used to produce food. We built a model that can capture important attributes of commodity markets and the mechanism by which mandates are enforced. US biofuel mandates are enforced using a system of tradable permits, called RINs. RIN market dynamics are important because they can be banked for future use. Rational expectations competitive storage models are best suited to capture the dynamic behavior of commodity markets. Such a model is developed for corn and RIN markets to estimate the impacts of alternative future ethanol mandate levels. The model considers corn use for ethanol, storage and all other uses in each period, accounting for two random variables: oil prices and corn yields. Borrowing and banking provisions of the Renewable Fuels Standard (RFS) mandate are also integrated into the model. The impact of mandates are estimated by comparing model solutions under two scenarios. The first scenario assumes that EPA allows mandates to increase to 15 billion gallons, which is the cap on RFS mandates that can be met with corn ethanol. The second scenario assumes that EPA keeps mandates at approximately 10 percent of US gasoline consumption. At this level practically all fuel in the United States will contain 10% ethanol, which is called E10. The impacts are estimated with a new competitive storage model of RINs and corn. The sources of uncertainty in the model are variable corn growing conditions which leads to ethanol supply uncertainty and uncertain gasoline prices which cause ethanol demand uncertainty. The resulting stochastic dynamic programming model is solved through the 2019 crop year using USDA projections of corn demand along with trend yield adjustments. The solved model is used to simulate future corn and RIN price distributions that show the impact of increasing mandates. Results indicate that increasing mandates have rather modest impacts on future corn prices but large impacts on RIN price distributions.

The third essay “Endogenous Price in a Dynamic Model for Agricultural Supply Analysis” presents a new model of agricultural supply which combines Positive Mathematical Programming (PMP) with the rational expectations storage model and compares methods to solve the combined model. We first follow Mrel and Bucaram (2010) to calibrate a leontief-quadratic model of agricultural supply to a certain allocation level and a set of supply elasticities. Then we combined the supply model with competitive storage model to endogenize the crop prices. PMP is widely used in policy analysis but the model does not endogenize expected output prices. On the other hand, competitive storage model endogenizes expected prices by assuming futures prices formed by agents are realized expected prices given all optimal decisions about storage, acreage and consumption. The combination of these two models will create a more powerful tool for policy analysis. However, the multi-crop competitive storage model becomes more difficult to solve using collocation method when the number of crop increases. In solving the model, we compare generalized stochastic simulation algorithm (GSSA) and the Smolyak collocation method. We also consider approximations of different solution functions such as storage rule approximation and expected price approximation. We use (1) GSSA with storage rule approximation, (2) Smolyak collocation with storage rule approximation and (3) Smolyak collocation with parameterized expected price. The results show that Smolyak collocation method performs better than GSSA considering computational time and accuracy in solving the multi-crop storage model.

CHAPTER 2. OPTIMAL CELLULOSIC ETHANOL PLANT SIZE UNDER UNCERTAINTY

2.1 Introduction

Extensive research has been conducted on the optimal size of biofuel plants using deterministic models. The basic tradeoff is between economies of scale in per-unit processing cost and diseconomies in feedstock costs due to transportation costs (Aden et al. (2002); Huang et al. (2008); Rosburg (2012)). Most studies have found the optimal plant size by maximizing the net present value or profits or by minimizing total costs. The optimal plant capacity is determined by equating marginal benefit of increased capacity with the marginal cost of increasing feedstock costs (Nguyen et al. (1995); Kaylen et al. (1999); Leboreiro et al. (2010); Cameron et al. (2006), Gan et al. (2010); Rosburg (2012)). Sensitivity analysis of how key factors affect the choice of plant size is usually included. Factors that decrease biomass transportation costs (or the distance that biomass must travel) or that decrease per-unit production costs increase the optimal plant size (Nguyen et al. (1995); Leboreiro and Hilaly (2010); Huang et al. (2008), Gan et al. (2010)). One factor that has not been extensively studied is the impact of uncertainty on optimal plant size. For example, some of the first cellulosic biofuel plants will use corn stover as a feedstock. But it is well documented that corn yields vary from year to year because of variations in weather. When stover yields are low, feedstock will have to be brought from a longer distance hence the cost of feedstock will increase. Similarly when stover yields are high, feedstock costs may be low.

Stochastic models have been used to compare the impact of uncertainty on profits for different plant sizes. For example, Dal-Mas et al. (2011) compares the expected net present value for four different plant sizes under several scenarios of feedstock purchase costs and ethanol market prices. But the impact of uncertainty on optimal plant size has not been studied. It is well known that an increase in uncertainty will impact optimal decisions if the uncertainty impacts costs or profits nonlinearly (Rothschild and Stiglitz). Nonlinearities can be caused by either nonlinear objective functions in static models or by nonlinearities due to the dynamic nature of the problem. Two stage stochastic models are widely used in system or capacity planning studies because investment decisions must be made before the level of the uncertain variable that affects production levels is resolved (Bok et al. (1998); Santoso et al. (2005); Yang (2010)). Hence, uncertainty in feedstock costs may have a significant impact on the optimal plant size because of the dynamic nature of the problem.

This paper explores the impact of uncertainty on optimal plant size using a two period model. The problem considered is that of an investor who wants to build an ethanol plant using corn stover as the only feedstock. The first stage of the problem is to choose a plant size. Assuming that the plant has the flexibility to purchase corn stover as needed after yield is known, the second stage decision is to choose a production level conditional on the plant size being chosen and on what corn stover yield turns out to be. When yield is abundant, the production level is high. When biomass is short, the production level is low. The other way to think about the story is that the plant decides the plant size in the first period and contracts with farmers for the corresponding acreage of biomass without any contract fee in the first period and any costs for not using all contracted acreages in the second period. The plant will contract enough acreage to make the contracted acreages not a constraint in the second period. Thus the plant also has the flexibility to get any amount of stover in the second period. If the contract fee is positive, the story is completely different and beyond the scope of this paper.

The optimization problem is to maximize expected profits in the first stage taking into account the second stage decision rule. The approach follows Turnovsky (1973) who used general cost functions to model firms' production plans under price uncertainty. In this paper we use specific cost functions to investigate the firm's optimal plant size under feed stock yield uncertainty.

An additional contribution of this paper is that we consider two payment schemes for corn stover procurement. Corn stover procurement cost includes farm gate cost and transportation cost. Payment scheme one (P1) considers the plant owner as a monopsony and who pays for all transportation costs and a uniform price at the farm gate to all suppliers. Payment scheme two (P2) assumes that the plant owner pays a uniform per ton price for all delivered stover at the plant so that farmers pay for transportation costs. To our knowledge the impacts of these two payment alternatives on optimal plant size have not been studied.

2.2 The Model

Suppose there is only one ethanol plant in a geographic region where corn stover is evenly distributed. The ethanol plant with capacity Q that limits the gallons of ethanol can be produced in a year and a production level V which is the actual amount of ethanol produced to maximize profit conditional on a realized per acre yield. Assume that the plant can get the amount of stover needed after observing stover yield and ethanol price. The market price for ethanol is exogenously given and denoted by p_e. Therefore p_eV is revenue generated per gallon of ethanol. Assume the conversion rate of corn stover is fixed at Y_e, so V gallons of ethanol is produced by $\frac{V}{Y_e}$ tons of corn stover. Cost for the plant has two components: feedstock procurement cost (FPC) and production cost (PC).

Considering the plant's problem under uncertainty, the risk neutral ethanol plant investor wishes to maximize expected profits from building a biorefinery with corn stover as the only biomass feedstock. Plant capacity cannot be changed once the plant is built and the ethanol refinery wishes to maximize profits from producing ethanol given plant capacity. The timeline of the decision making is illustrated in figure 1.

In period 1, plant capacity is chosen. Per acre yield of corn stover and output price are revealed at the beginning of period 2, and then the optimal production level and the capture radius is chosen under fixed capacity. Assume that the fraction of land devoted to stover that will be supplied to the plant is constant. The plant collects all the available biomass in the circular region of the optimal capture radius. The two period optimization problems can be solved using backward induction. The second period problem is first solved by choosing the production level given plant capacity and corn stover yield, then the investor makes the optimizing first period capacity decision by taking into account all ex post possibilities. The ex post model follows Huang et. al(2009), Leboreiro(2010, 2013), Gan(2010), Rosberg(2012) with modifications for stochastic considerations.

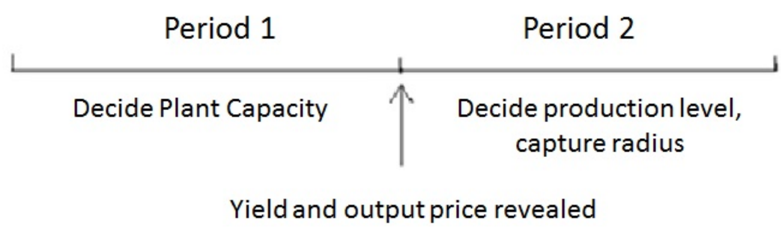

Figure 2.1: 2 periods

2.2.1 Feedstock Procurement Cost

Feedstock procurement cost (FPC) includes all the costs that occur before stover arrives at the plant gate. Biomass needs to be bought from farmers in the collection region, stored in storage depots and transported to the plant. Thus we assume the total feedstock procurement cost includes farm gate cost (FGC), storage cost (S) and transportation cost (TC) (Leboreiro and Hilaly (2010), Rosburg (2012)). Transportation costs will vary across farmers according to how far they are away from the plant. The storage cost is constant per ton of stover collected.

The farm gate cost is defined as the minimum selling price of stover at the farm gate. It will be positive because leaving corn stover in a field reduces soil erosion, maintains soil carbon, and restores nutrients. Thus its removal incurs costs. In addition, stover has to be harvested, baled and hauled to the edge of the field. Assume that the total farm gate cost includes nutrient replacement cost and corn stover collection cost. Denote the per ton farm gate cost as c_f.

Consider there are two payment schemes for FPC. Under the first payment scheme, the plant is assumed to be a price-discriminating monopsony. The plant owner pays the transportation cost and storage cost while all farmers receive the same price per ton of feedstock. The net cost of feedstock to the plant owner varies by farmers according to the distance between each farm and the plant. Thus the farmers close to the plant are not able to capture transportation-related rents.

Under the other FPC payment scheme, the plant owner lists a total feedstock procurement price excluding storage cost to all stover suppliers who pay transportation costs themselves.

Because all farmers would like to supply corn stover if the listed price can at least cover the total transportation cost plus farm gate cost, the listed price equals to the total feedstock procurement cost for the stover supplier farthest from the plant. The total price paid by the plant is constant across all farmers so that the nearby farmers receive a higher net price (net of transportation cost) than farmers farther from the plant. Thus farmers capture location rents in this case. Because the price paid by the plant under the second scheme must equal farm gate cost plus transportation cost of the farthest away farmer, average feedstock cost to the plant from the second payment scheme will be higher than under the first payment scheme.

2.2.1.1 Plant Owner Pays transportation Cost

Total transportation cost paid by the plant is determined by the amount of corn stover delivered to the plant, the per unit transportation cost, and the distance the corn stover must travel. Following Overend (1982), Nguyen and Prince (1995)'s approach, suppose the plant is located at the center of a circular collection region, which has a size that adjusts to meet the annual corn stover demand with no adjustment cost. Annual demand equals total yearly production. Consider a production of V gallons a year. Let r be the radius of the circular region, f is the fraction of land around the plant devoted to feedstock production multiplied by the fraction of stover supplied by farmers. Assume the willingness to supply is constant for any price level. Define y as per acre yield of corn stover that is measured in tons per acre. Then the collection radius can be calculated from $\frac{V}{Y_e} = \pi r^2 y f$, so that

$$r = \sqrt{\frac{V}{Y_e \pi y f}} \tag{2.1}$$

Assuming that all transportation activities are carried out by semi-truck with flat bed. The total amount of stover at distance between x and $(x - dx)$ from the plant is $2\pi x y f dx$. The number of the trips for hauling the product from distance x to plant is the total amount of corn stover divided by the load per truck, which is $2 \cdot (2\pi y f dx/N)$ while N is the load per truck. We consider round-trip transportation cost (Leboreiro and Hilaly (2010), Morey et. al. (2010)), thus there is a 2 in front of the formula. Per mile shipping cost is c_{tN}. The per trip cost is $x c_{tN}$. Considering the actual distance is not a straight-line, then the per trip cost is

$\tau c_{tN} x$. The total variable transportation cost from shipping stover at distance between x and $(x - dx)$ from the plant is given by the following integral:

$$TC = \int_0^r 2(\frac{2\pi xyf}{N})\tau c_{tN} x dx = \frac{4\pi\tau c_{tN} dyr^3}{3N} \quad (2.2)$$

Substituting $\frac{V}{Y_e} = \pi r^2 dy$, the total variable transportation cost of gathering corn stover in the circular region can be rewritten as:

$$TC = \frac{4\tau c_{tN} r}{3N} \frac{V}{Y_e} \quad (2.3)$$

c_{tN} can also be seen as the per mile per load transportation cost. $\frac{c_{tN}}{N}$ is the transportation cost per ton per mile. We use c_t to denote the per ton per mile transportation cost in the rest of the paper. Comparing with the variable transportation cost calculated from one-way trip by Nguyen and Prince (1995), our transportation cost is doubled. Substituting (1) into (3) yields a form without the distance variable r:

$$TC = \frac{4}{3}\tau c_t \frac{1}{\sqrt{\pi yd}}(\frac{V}{Y_e})^{\frac{3}{2}} \quad (2.4)$$

This is the variable transportation cost required to deliver tons of corn stover to the plant which under the payment scheme is paid for by the plant owner. As the amount of corn stover processed increases, per ton DVC increases at a decreasing rate. The DVC increases because an additional unit of corn stover needed by the plant has to be hauled from far away. Also, DVC decreases when corn stover is abundant because the plant can find any given amount of stover closer to the plant than when yields are low. Consider per ton fixed distance cost (loading and unloading cost) is h. Also we assume all stover are stored on farm, stacked as a pyramid shape with tarp on the top (Wright and Brown (2011)). Storage cost is assumed to be constant per ton corn stover with representing per ton storage cost. Total feedstock procurement cost is

$$FPC = TC + S + FGC = (\frac{4\tau c_t r}{3} + h + s + c_f)\frac{V}{Y_e} = \frac{4\tau c_t}{3}\frac{1}{\sqrt{\pi yf}}(\frac{V}{Y_e})^{\frac{3}{2}} + (h + s + c_f)\frac{V}{Y_e} \quad (2.5)$$

2.2.1.2 Farmers Pay Transportation Cost

Now assume that the plant lists a corn stover procurement price at the plant that is paid to all suppliers who deliver corn stover to the plant. In this scheme, farmers are responsible for paying transportation costs. The price that is paid for stover by the plant will equal the farm gate price plus the cost of transporting corn stover from the farthest point of the draw area. Per ton shipping cost for corn stover per ton to the biorefinery at the edge of the circular region with radius r is $2rc_t$ and the per ton farm gate cost is same as P1, so total procurement cost for all corn stover ($V = \pi r^2 yd$) in the region is:

$$FPC = TC + S + FGC = (2rc_t + s + c_f)\frac{V}{Y_e} = 2rc_t\frac{1}{\sqrt{\pi yd}}(\frac{V}{Y_e})^{\frac{3}{2}} + (h + s + c_f)\frac{V}{Y_e} \tag{2.6}$$

Comparing the two payment schemes, procurement cost is higher under the second scheme than under the first scheme. Notice that the variable portion of transportation feedstock costs when the plant owner pays transportation costs is one-third lower than when the plant posts a price at the plant for all delivered stover. Because of this constant relationship we first solve for the optimal plant size when the plant owner pays transportation cost and then show how a 50 percent increase in c_t affects plant size. The resulting change in plant size from this 50 percent increase in c_t measures the impact from moving from a plant owner paying transportation costs to farmers paying transportation costs[1].

2.2.2 Production Cost

Cellulosic biofuel production cost (PC) includes capital cost (PC_c) and operating cost (PC_o). Capital cost depends on biofuel plant capacity and exhibits economies of scale. Total capacity cost for equipment in process industry has the following characteristics (Aden et. al. (2002), Huang et. al. (2008), Leboreiro et. al. (2010), Gan et. al.(2010), Rosburg(2012)):

$$PC_c = c_c(\frac{Q}{Q_0})^{\alpha},\ 0 < \alpha < 1 \tag{2.7}$$

[1] The transportation cost could be overestimated for the second payment scheme because the supplied stover per square mile is assumed to be constant in the analysis. In reality, the farms which located nearer to the plant keep more location rents and are willing to supply more stover. In that case, increases with smaller distance between farms and the plant. Thus the capture region is smaller and transportation cost decreases.

c_c is the total capital cost for plant size Q_0 and the new capital cost for plant with capacity Q is determined by the exponential scaling expression above. α is the scaling exponent and is strictly less than 1, implying decreasing marginal capital cost. The marginal capital cost function is

$$\frac{dPC_c}{dQ} = \frac{c_c}{Q_0^\alpha}\alpha Q^{\alpha-1} \tag{2.8}$$

Assume that operating cost doesn't exhibit economies of scale, goes to infinity at capacity level and the per gallon operating cost is approximately constant below capacity level. A possible variable production cost expression could be:

$$PC_o = c_o\frac{V}{(Q-V)^\beta},\ \beta > 0 \tag{2.9}$$

c_o and β are the variable production cost factors. Strictly positive β implies that the denominator goes to zero at capacity level and total operating cost goes to infinity. Thus production could approach but never reach capacity. We assume β is small enough to make $(Q-V)^\beta \approx 1$ when production is below capacity, then $PC_o \approx c_oV$ where c_o represents a constant per gallon operating cost. Therefore, the total production cost is:

$$PC = c_c(\frac{Q}{Q_0})^\alpha + c_o\frac{V}{(Q-V)^\beta} \tag{2.10}$$

2.2.3 Optimal Plant Size When Plant Owner Pays Transportation

In the following model sections, we'll only consider the optimal plant size decision when plant owner pays transportation cost (payment scheme 1). The difference in optimal plant sizes for the two payment schemes will be shown in the simulation results. A two stage decisions model is built in the section and optimal decision making under yield certainty is analyzed in section 2.3.1. The optimal capacity is found to be determined by economies of scale in fixed production cost and diseconomies of scale in transportation cost (equation (15)). In section 2.4.1, the model is extended for stochastic yield and the proposition for decisions under increasing risk developed by Roschild and Stigliz is used for comparing optimal capacity under certainty and uncertainty.

In the second period, the plant has the flexibility to purchase corn stover under certain capacity when per acre yield is known and makes a decision on the optimal production level. The profit maximization problem is:

$$\underset{V}{Max}\Pi = P_e V - FPC - PC \tag{2.11}$$

$$\underset{V}{Max}\Pi = p_e V - (\frac{4\tau c_t}{3}\frac{1}{\sqrt{\pi y d}}(\frac{V}{Y_e})^{\frac{3}{2}} + (h+s+c_f)\frac{V}{Y_e}) - (c_c(\frac{Q}{Q_0})^{\alpha} + c_o\frac{V}{(Q-V)^{\beta}}) \tag{2.12}$$

p_e is per gallon price of cellulosic ethanol. Suppose p_e is at least higher than marginal total cost at an infinitesimal quantity, then the second period first-order condition for optimal production given a plant capacity ($V^*(Q)$) is

$$p_e - (\frac{2\tau c_t V^*(Q)^{\frac{1}{2}}}{\sqrt{Y_e \pi y d}} + h + s + c_f)\frac{1}{Y_e} - (\frac{c_o}{(Q - V^*(Q))^{\beta}} + \frac{c_p \beta V^*(Q)}{(Q - V^*(Q))^{\beta+1}}) = 0 \tag{2.13}$$

Marginal cost is monotonically increasing in V, hence (13) determines a unique level of production. Because $V^*(Q)$ is chosen after y and p_e are observed, variability in y implies variability in $V^*(Q)$.

2.2.3.1 Optimal Capacity Under Yield Certainty

In the first period, the plant makes its capacity decision taking into account the second period decision problem. Under yield certainty, optimal capacity and the optimal production in second period have the following relationship according to envelope theorem:

$$\Pi_Q = -\frac{c_c \alpha}{Q_0^{\alpha}} Q^{*\alpha-1} + \frac{c_o V^*(Q^*)\beta}{(Q^* - V^*(Q^*))^{\beta+1}} = 0 \tag{2.14}$$

Combining (3.14) and (2.14), we can write total marginal cost in terms of only optimal capacity Q^*. Optimal capacity is determined by:

$$p_e - c_o - \frac{c_c \alpha}{Q_0^\alpha} Q^{*\alpha-1} = c_f + s + (\frac{2\pi c_t}{\sqrt{Y_e \pi y d}} \sqrt{(\frac{c_c \alpha Q^{*\alpha-1}}{c_c \alpha Q^{*\alpha-1} + c_o \beta Q_0^\alpha}) Q^* + h}) \frac{1}{Y_e} \tag{2.15}$$

The LHS of the first order condition (3.16) is the net marginal revenue of producing ethanol from corn stover accounting for the cost of converting corn stover to ethanol. The RHS of (3.16) is the marginal cost of acquiring corn stover, including per ton farm gate cost, per ton storage cost and the marginal cost of transporting corn stover. The expression under the square root sign is the optimal production for the given capacity $((\frac{c_c \alpha Q^{*\alpha-1}}{c_c \alpha Q^{*\alpha-1} + c_o \beta Q_0^\alpha}) Q^* = V^*(Q))$. The optimal plant size is found by equating the net marginal returns to production to the marginal cost of corn stover. With economies of scale, a larger plant size increases net marginal returns to production. But increased plant size also increases the marginal cost of stover because the marginal cost of transportation increases. Thus the solution to (3.16) defines the optimal capacity.

Under certainty, optimal capacity increases with feedstock yields and output price ($\frac{\partial Q^*}{\partial y} > 0$, $\frac{\partial Q^*}{\partial p} > 0$). An increase in per acre yield causes a decline in marginal transportation cost because more feedstock is available closer to the plant. Higher output price leads to higher per ton revenue generated and results in a larger optimal capacity.

Increases in other parameters such as economies of scale ($\alpha - \beta$), per ton per mile transportation cost (c_t), production cost factor (c_o) all lead to higher total marginal cost and therefore lead to a lower optimal capacity. This means that the optimal plant size is smaller when the farmer pays transportation costs than when the plant pays transportations costs.

2.2.3.2 Optimal Capacity Under Uncertainty

We now consider the impact of uncertain corn stover yield, specifically, how a mean preserving spread in yield affects optimal capacity. Let per-acre yield y have a density function $f(y)$ and cumulative distribution function $F(y)$ with lower bound y_L and upper bound y_H. The second period decision follows equation (3.14) when yield is known. Taking the ex-post adjustment V^* into the first period, the expected profit for the biorefinery is given by

$$\underset{Q}{Max}\, E\Pi = \int\limits_{y} \Pi(Q,y)dF(y) = \int\limits_{y} (p_e V^*(Q,y) - (\frac{4\pi c_t}{3}\frac{1}{\sqrt{\pi y d}}(\frac{V^*(Q,y)}{Y_e})^{\frac{3}{2}} + (h+s+c_f)\frac{V^*(Q,y)}{Y_e}) \tag{2.16}$$

$$-(c_c(\frac{Q}{Q_0})^{\alpha} + c_o\frac{V^*(Q,y)}{(Q-V^*(Q,y))^{\beta}}))dF(y)$$

The optimal capacity under uncertainty $\widetilde{Q}^*$must satisfy

$$\int\limits_{y} (-\frac{c_c\alpha}{Q_0^{\alpha}}Q^{*\alpha-1} + \frac{c_o V^*(Q^*)\beta}{(Q^*-V^*(Q^*))^{\beta+1}})dF(y) = E\Pi_Q(\widetilde{Q}^*, y) = 0 \tag{2.17}$$

When comparing the optimal decision under certainty and uncertainty, it's impossible to get an answer from the first order conditions given by equation (3.16) and equation (2.17). The proposition given by Rothchild and Stiglitz for analyzing the economic consequences under increasing risk is used to see the impacts of yield to the optimal plant capacity.

Where Q^* is a unique solution to (4.19) in the neighborhood of Q^*, $\Pi_Q(Q,y)$ is monotone decreasing in Q. If $\Pi_Q(Q,y)$ is concave in y, a mean preserving spread in (an increase in riskiness) will decrease Q^*. When $\Pi_Q(Q,y)$ is convex y, a mean preserving spread in increases Q^*. The effect of increaing in risk is ambiguous when $\Pi_Q(Q,y)$ is neither convex nor concave (Rothschild and Stiglitz).

To see why convex marginal profit leads to a larger plant under yield uncertainty than under certainty, consider an ethanol producer who faces yield uncertainty and is considering building a plant size when yield is nonstochastic at its mean level $E(y)$. The optimal capacity under certainty corresponds to the mean yield is Q^*. Building a unit greater than Q^* leads to higher profits when yield is higher than average but lower profits when yield is less than average. Convex marginal profit means that the expected marginal profit of capacity (Π_Q) is greater than zero at the capacity level Q^*, which implies that an additional increase in capacity would increase expected profits. The capacity increases until marginal expected profits falls to zero. Thus, the optimal capacity under uncertainty is greater than that under certainty. Similarly, if marginal profit is concave in yield, then expected marginal profit due to a change in capacity is negative when evaluated at Q^*, hence yield uncertainty leads to a smaller plant size.

The Rothschild-Stiglitz result indicates that a sufficient condition for signing the effects of yield uncertainty on plant size is to check if the $\Pi_Q(Q, y)$ is concave or convex in y. The second derivative of $\Pi_Q(Q, y)$ with respect to y is:

$$\underbrace{\frac{\partial^2 \Pi_Q}{\partial y^2}}_{?} = \underbrace{(\frac{\partial V}{\partial y})^2[\frac{(1+\beta)(Q-V(Q))^\beta}{(Q-V(Q))^{2(\beta+1)}} + \frac{1+\beta+(1+\beta)(2+\beta)(Q-V(Q))^{\beta+1}V(Q)}{(Q-V(Q))^{2(\beta+2)}}]}_{+} \quad (2.18)$$
$$+\underbrace{\frac{\partial^2 V}{\partial y^2}}_{?}\underbrace{[\frac{1}{(Q-V(Q))^{\beta+1}} + \frac{(1+\beta)V(Q)}{(Q-V(Q))^{\beta+2}}]}_{+}$$

Because $\frac{\partial V^*}{\partial y}$ is positive $\frac{\partial^2 V^*}{\partial y^2}$ is indeterminate. The sign of $\frac{\partial^2 \Pi_Q}{\partial y^2}$ is indeterminate. The mathematical proofs for the indeterminate sign of $\frac{\partial^2 \Pi_Q}{\partial y^2}$ is provided in the appendix. Simulations are used to examine the conditions under which plant size increases or decreases with yield uncertainty.

2.3 Simulation

In this section, we first illustrate the computational strategies of deriving the optimal plant size under uncertainty and certainty in the baseline case and then we show the parameter values we are using in the simulation. The estimates for the parameters are all found in other research papers or estimated using USDA data.

In the baseline case under yield and ethanol price uncertainty, we discretize the possible continuous optimal capacity space from 20 million gallon to 200million gallon into 300 points. For each capacity value belonging to the 300 points, we find expected profits given the capacity. The capacity that gives the highest expected profits is the optimal capacity that we find. To obtain the expected profits for a certain capacity, we first generate the combinations of yield and price values together with the corresponding probability using Gaussian quadrature method. For each combination of yield and price revealed in the second period, the first order condition in equation (3.14) is used to get the optimal production. Then we take each optimal production back into the first period and calculate the profit according to equation (2.12). The expected profit for the certain capacity is calculated by multiplying each profit by its corresponding probability.

Assume that the per acre yield of corn stover follows a beta distribution to capture the uncertainty in yields. Because one dry ton of grain equals one dry ton of stover, the corn stover yield can be captured as the beta distribution using Iowa corn yields data from 1970 to 2012 from the website of USDA NASS. Using a linear trend, the corn yield time series data are detrended to 2007 technology because the biomass conversion technology is in 2007. The maximum yield is set to be 1.2 of the maximum yield in the detrended data. y_{max} is the maximum yield and $y_{max} = 244$ bushels per acre. Minimum yield is assumed to be zero. Fit the data to a beta distribution yields: $y \sim beta(\alpha_y, \beta_y)\, \alpha_y = 21.9206$, $\beta_y = 10.6065$. Under certainty, the expected yield value is used. The expected yield equals the mean of the above beta distribution which is $E(y) = \frac{244\alpha_y}{\alpha_y+\beta_y} = 164$ bushels/acre = 4.6 tons/acre.

Function integration in two dimensional spaces is approximated by using Gaussian quadrature method. Matlab code by Miranda and Fackler is used for generating quadrature nodes and weights to approxiate a joint distribution while yield follows beta distribution and gasoline prices follow a lognormal distribution. Since corn stover yield and gasoline prices are uncorrelated, there is no correlation between the yield and gasoline distributions. We select 3 quadrature nodes from each distribution.

Assume that the harvested corn stover moisture content is 15% (Morey (2010)). The total fraction of land devoted to stover production near the cellulosic ethanol plant is assumed to be 0.6 (Wright and Brown (2007)). The percentage of corn stover that farmers are willing to supply varies from 0.1 to 0.37 in literature (Gan and Smith (2011), Leboreiro and Hilaly (2013), Huang et. al. (2009)). In this paper, we assume 30% of corn stover are supplied to the biorefinery.

The willingness to sell corn stover at the farm gate is determined by nutrient replacement cost and stover collection cost. The nutrient removal per ton of feedstock has been estimated to be about 20 lb of N, 5.9 lb P_2O_5 and 25.0 lb of K_2O[2]. 2011 year fertilizer cprices are used and

[2]Nutrient removal per ton of corn stover data is from Iowa state university extension

indexed to 2007 year price[3]. With a price of \$0.377 per lb N, \$0.45 per lb P_2O_5 and \$0.33 per lb K_2O, per ton nutrient removal price would be \$7.50 for N, \$2.65 for P_2O_5, \$8.25 for K_2O. The total per ton corn stover nutrient replacement cost is \$18.40. Beside nutrition replacement cost, baling and staging cost per ton of corn stover are assumed to be \$24.33 (Huang et al. (2009)). Then the total farm gate cost is \$42.73 per ton.

Different ways for storage result in different cost. The assumption for storage follows Thompson and Tyner (2011). It is assumed that the stover is stored on farm for up to 12 months and the bales are stacked in a pyramid formation with a tarp on the top. The per ton storage cost at 15% moisture is \$14.7[4] according to the estimate by Thompson and Tyner (2011). The loading and unloading cost is \$6.9 per dry ton (Huang (2009)). The variable cost of transporting corn stover depends on the price of diesel fuel. Transportation cost per dry ton per mile in the baseline case is assumed to be 0.71 as in Brown and Wright (2007) and Rosburg (2012). Tortuosity factor is 1.5 (Brown and Wright (2007)).

Production cost parameters are estimated using NREL Aspen Model which is described by Humbird et al. (2011). Dilute Acid Pretreatment with Enzymatic Hydrolysis and Co-Fermentation is used in the ethanol production process in this report. The expression for variable capital cost is $c_o \frac{V}{(Q-V)^\beta}$ where $(Q-V)^\beta \approx 1$ when $V \neq Q$. Thus it's reasonable to set $\beta = 0.0001$ because $(Q-V)^\beta = 1.002$ for $Q = 700$ million and $V = 0$. c_o can be approximated as per gallon constant operating cost including the enzymes cost and non-enzyme conversion cost. Enzymes cost is \$0.34/gal and non-enzyme conversion cost is \$1.08/gal, thus c_o is \$1.42/gal. For the fixed production cost part, the total capital cost from the Aspen model is \$422.5 million for an ethanol plant with a production level of 61 million gallon per year. Amortized the total capital cost over 20 years with 8% interest rate (Wright and Brown (2007)), per year payment c_c is \$43 million. The scale factor of fixed cost α ranges from 0.6 to 0.9 and is assumed to be constant for all capacity levels in literatures (Aden et al (2002); Huang et al. (2008); Gan et al. (2010); Leboreiro (2011)). Here we assume the scale factor is 0.75 in the baseline model.

http://www.extension.org/pages/26618/corn-stover-for-biofuel-production

[3]By using fertilizer price index from USDA. http://www.ers.usda.gov/data-products/fertilizer-use-and-price.aspx

[4]The per ton storage cost is indexed to 2007 value by using mean hourly wage of agricultural equipment operators from Bureau of Labor Statistics.

Optimal plant size under certainty	Optimal production	Radius	Per gallon feedstock cost	Per gallon production cost
48.7 mgy	48.69 mgy	21 miles	1.1893 (2007$)	2.1646 (2007$)

Table 2.1: Optimal plant size under certainty and related statistics

Ethanol yield is 79.0 gallons per year in the Aspen model.

Cellulosic ethanol price has a relationship with gasoline price according to the following formula:

$$p_e = 0.667p_g + \text{tax credit} \tag{2.19}$$

While the conventional ethanol tax credit has already expired, the cellulosic ethanol tax is extended and set to be $1.01 per gallon. Using the present RBOB gasoline price value, cellulosic ethanol mean price cannot support a positive operating margin of a cellulosic ethanol plant for the baseline data parameters for all plant capacity levels. In order to make the baseline case in our simulation for a breakeven plant at maximized capacity, a mean value of gasoline price equal to $3.515/gallon (in 2007 dollar) is used. The variance of lognormal distribution is assumed to be the same as the standard deviation for monthly New York harbor RBOB gasoline price data from 2011 to 2013. The standard deviation is 0.22. Under certainty, the gasoline price is $3.515/gallon.

2.4 Results

2.4.1 Baseline Case Optimal Plant Sizes for Plant Pay Transportation Cost

Optimal plant size is 48.7 million gallons per year for baseline case under certainty. The optimal production is almost the same as the optimal capacity. Corn stover collection radius for such a plant is 21 miles. Simulated per gallon feedstock procurement cost including transportation cost, storage cost and payment for stover is approximately $1.1893. Per gallon production cost including both amortized payment for capital investment cost and variable production cost is $2.1646.

Plant capacity is optimized at 47.5 million gallons per year under both yield and price uncertainties. An increase in corn stover yield increases optimal production towards capacity (47.5 million gallons per year) and decreases optimal biomass collection radius. Figure 2 shows the second period decision making under different yields and gasoline prices for the 47.5 million gallons per year plant. For all gasoline prices ranging from 2.2 $/gallon to 4 $/gallon, optimal productions increase with higher yield values. If the gasoline price is at the mean value (3.515 $/gallon), optimal productions increase to almost capacity level when yield is less than 2 tons per acre. For all yield values, optimal productions increase with higher gasoline price values. When yield is at its mean value (4.6 tons per acre), optimal productions reach almost capacity level when gasoline price is less than 3 $/gallon.

Figure 3 shows the capture radiuses with different yields and gasoline prices in the ex post decision making. When yield goes high, capture region shrinks for any gasoline price. At mean gasoline price, the capture radius shifts from 30 miles to 10 miles when the yield increases from 1ton per acre to 6 tons per acre. The capture radius goes up when gasoline price increases for any yield value. At mean yield, capture radius rises from 0 to 21 miles when gasoline price increases from 2.2 $/gallon to 4 $/gallon.

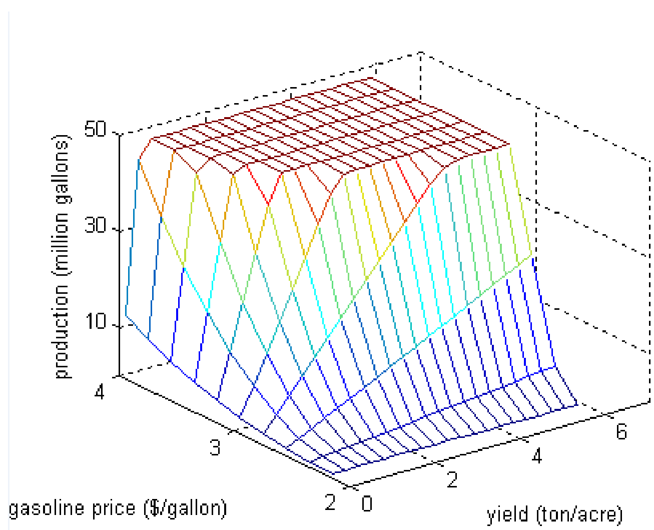

Figure 2.2: optimal production with different yields and gasoline prices for 47.5 mgy plant

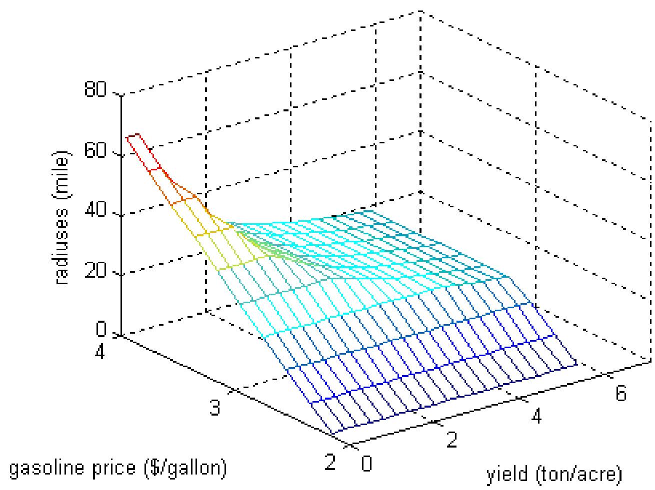

Figure 2.3: optimal radiuses with uncertain factors for 47.5 mgy plant

2.4.2 Sensitivity Analysis

The sensitivity of optimal plant capacity under certainty and uncertainty to a change in scale factor, variable transportation cost and available land is shown in Figure 4. All black lines and red lines shown in figure 4 show the relationship between optimal plant capacities and different factors under certainty and uncertainty respectively.

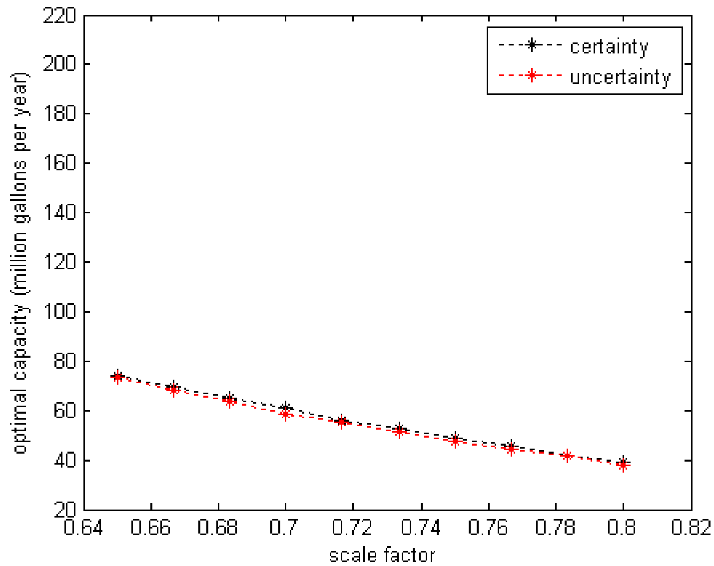

Figure 2.4: Sensitivity to scale factor of optimal plant size (mgy) under certainty and uncertainty

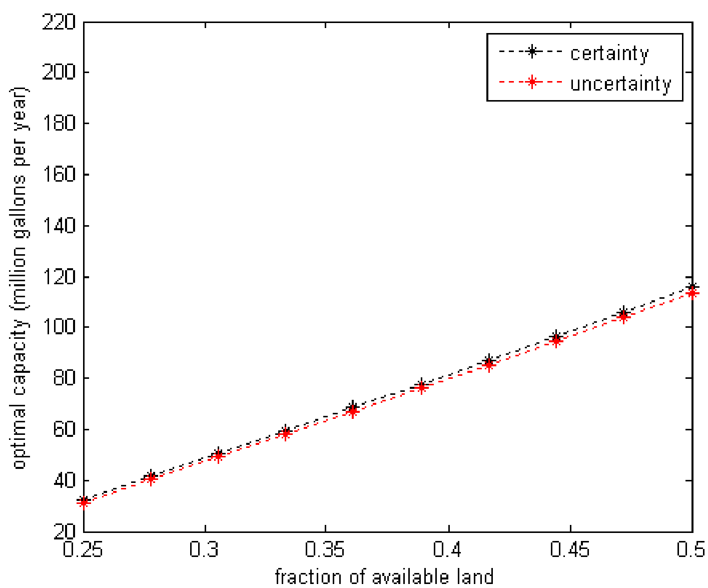

Figure 2.5: Sensitivity to available land of optimal plant size (mgy) under certainty and uncertainty

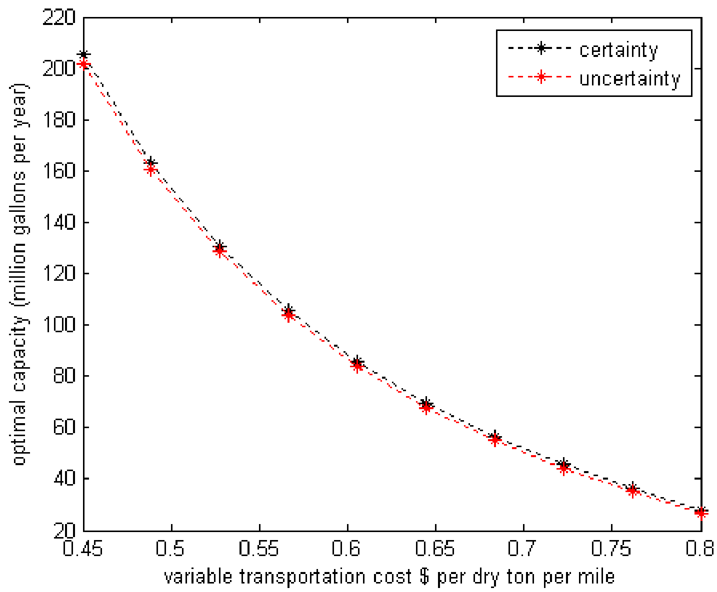

Figure 2.6: Sensitivity to transportation cost of optimal plant size (mgy) under certainty and uncertainty

Economies of scale

The optimal plant size decreases with scale factor (α) as shown in figure 4(a). Higher α increases total capital cost for all capacity levels and the optimal plant size decreases. $\alpha = 0.75$ is assumed in the baseline and we get the optimal plant capacity values for α varies from 0.65 to 0.8. Optimal plant capacity decreases from 75 million gallons per year to 40 million gallons per year when scale factor rises from 0.65 to 0.8. Optimal capacity under uncertainty also decreases with higher scale factor value and the difference between optimal capacity under certainty and uncertainty is only 1 to 2 million gallons per year (figure 4(1)).

Land fraction devoted to stover

The fraction of land devoted to corn stover around a plant has an impact on the variable transportation cost. The fraction of land for biomass differs in different regions. It is assumed that 30% of the land is available for stover production in the baseline case. More land devoted to corn stover has the same effect as a higher corn stover yield. Thus higher available land leads to a bigger plant. Optimal plant capacity under certainty and uncertainty increases almost linearly as available land rises as shown by figure 4(2).

Transportation cost

We test the variable transportation cost from 0.45 dt^{-1}mile^{-1} to 0.8 dt^{-1}mile^{-1} when our baseline value is 0.71 dt^{-1}mile^{-1}. The optimal plant capacity goes down considerably with variable transportation cost as shown in figure 4(c) especially when cost is small. Increase in variable transportation cost from 0.45 dt^{-1}mile^{-1} to 0.8 dt^{-1}mile^{-1} decreases the optimal capacity size by more than 170 million gallons per year. The plant capacity under uncertainty is still only 1 or 2 million gallons per year smaller than that under certainty for all transportation cost values. In all cases examined above, all cost factors changes the optimal plant capacity both under certainty and uncertainty significantly. However, the optimal capacities under certainty are almost the same as that under uncertainty in all sensitivity analysis.

2.4.3 Comparing Plant Size by Different Transportation Payment Schemes

The sensitivity of optimal plant size to transportation costs implies that the optimal plant size will also be greatly impacted by whether the plant owner pays transportation costs or whether the farmer pays. Recall that the variable portion of transportation cost when the farmer pays transportation costs is 50 percent higher than when the plant owner pays transportation costs. This means that the impact on the optimal plant size from moving from a system whereby the plant owner pays transportation cost to a system whereby the farmer pays can be measured by comparing the optimal plant size when $c_t = \$0.50dt^{-1}mile^{-1}$ to the optimal plant size when $c_t = \$0.75$ dt^{-1}mile^{-1}in Figure 4(3). When $c_t = \$0.50$ dt^{-1}mil$e^{-1}$and the transportation payment plan changing from plant pays to a system whereby the farmer pays, the optimal plant size decreases dramatically. Figure 4(3) shows that P2's plant size shrinks to less than one third of P1's size, implying that plant size is much smaller when farmers pay for the transportation cost. When plant manages the feedstock transportation itself, the transportation cost is cheaper, resulting in a bigger optimal capacity. As explained in footnote 1, the plant size under P2 is overestimated if changes in farmers' willingness to supply are taken into account.

2.5 Conclusions

By maximizing the profit and expected profit of cellulosic ethanol production, we develop a framework for determining optimal biorefinery plant capacity under both certainty and uncertainty, which in turn leads to the derivation of the maximum profit possible plant. Data from the published literature and USDA data are used to determine the optimal biorefinery capacity under both certainty and uncertainty.

Optimized biorefinery plant capacity under certainty is found to be 48.7 million gallons per year in the baseline case. The optimal plant size is 47.5 million gallons per year in baseline case when considering uncertain stover yields and gasoline prices. For a plant under uncertainty, its optimal production goes up with a rise in per gallon gasoline price or per acre stover yield. The optimal stover supply radius decreases with an increase in stover yield and increases with a rise in per gallon gasoline price. The optimal capacity under both certainty and uncertainty increases with a rise in land fraction distributed to biomass or a fall in feedstock transportation cost, plant scale factor. The sensitivity analysis shows that optimal capacities are almost the same under certainty and uncertainty for all cases. We also evaluate the impact of two corn stover payment schemes on the optimal biorefinery plant capacity. The optimized plant size under certainty or uncertainty decreases when shifting from plant pays the transportation cost scheme to farmers pay the transportation cost.

The results have several important implications for bioenergy development. The results show that only mean values for stover yield and gasoline price need to be considered when planning the plant size. Other than the findings by previous research that optimal plant size can be affected by different factors such as plant scale factor and transportation cost parameters, optimal plant size is also impacted by transportation payment plans. Hence, decisions on plant capacity and the corresponding corn stover supply radius should be made carefully with consideration of corn stover procurement, biomass conversion, mean values for stover yield and gasoline price, per gallon revenue and different stover payment schemes.

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2.6 Appendix

1. Prove that second order sufficient condition for optimal capacity under certainty is satisfied.

$$\frac{\partial^2\Pi}{\partial Q^2}(Q^*) = -\frac{c_c\alpha}{Q_0^\alpha}(\alpha-1)Q^{*\alpha-2} - \frac{\tau c_t}{Y_e\sqrt{Y_e\pi yd}}\left(\frac{c_c\alpha Q^{*\alpha}}{c_c\alpha Q^{*\alpha}+c_o\beta Q_0^\alpha}\right)^{-1/2}\frac{c_c^2\alpha^2Q^{*2\alpha-2}+c_cc_o\beta\alpha^2Q_0^\alpha Q^{*\alpha-1}}{(c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha)^2} < 0 \tag{2.20}$$

Thus the capacity is optimized at Q^*.

2. Prove that the signs of $\frac{\partial^2\Pi_Q}{\partial y^2}$ and $\frac{\partial^2\Pi_Q}{\partial p_e^2}$ are undetermined.

$$\underbrace{\frac{\partial^2\Pi_Q}{\partial p_e^2}}_{?} = \underbrace{\left(\frac{\partial V}{\partial p_e}\right)^2\left[\frac{(1+\beta)(Q^*-V^*(Q^*))^\beta}{(Q^*-V^*(Q^*))^{2(\beta+1)}} + \frac{1+\beta+(1+\beta)(2+\beta)(Q^*-V^*(Q^*))^{\beta+1}V^*(Q^*)}{(Q^*-V^*(Q^*))^{2(\beta+2)}}\right]}_{+}$$

$$+\underbrace{\frac{\partial^2V^*}{\partial p_e^2}}_{?}\underbrace{\left[\frac{1}{(Q^*-V^*(Q^*))^{\beta+1}} + \frac{(1+\beta)V^*(Q^*)}{(Q^*-V^*(Q^*))^{\beta+2}}\right]}_{+} \tag{2.21}$$

$$\frac{\partial^2V^*}{\partial p_e^2} = \left(\frac{\partial V}{\partial p_e}\right)^2\underbrace{\left[\frac{\tau c_t}{2Y_e^2}\frac{1}{\sqrt{Y_e\pi yd}}V^{-3/2} - \frac{\beta c_o\left(\frac{1}{(Q^*-V^*(Q^*))^{2(\beta+1)}} + \frac{1+\beta}{(Q^*-V^*(Q^*))^{\beta+2}} + \frac{V^*(1+\beta)(2+\beta)}{(Q^*-V^*(Q^*))^{\beta+3}}\right)}{\frac{\tau c_t}{Y_e}\frac{1}{\sqrt{Y_e\pi yd}}V^{-1/2} + c_o\beta\frac{1}{(Q^*-V^*(Q^*))^{\beta+1}} + \frac{V(1+\beta)}{(Q^*-V^*(Q^*))^{\beta+2}}}\right]}_{\text{?but negative for all possible parameter values}} \tag{2.22}$$

$$\underbrace{\frac{\partial^2\Pi_Q}{\partial y^2}}_{?} = \underbrace{\left(\frac{\partial V}{\partial y}\right)^2\left[\frac{(1+\beta)(Q-V(Q))^\beta}{(Q-V(Q))^{2(\beta+1)}} + \frac{1+\beta+(1+\beta)(2+\beta)(Q-V(Q))^{\beta+1}V(Q)}{(Q-V(Q))^{2(\beta+2)}}\right]}_{+} \tag{2.23}$$

$$+\underbrace{\frac{\partial^2V}{\partial y^2}}_{?}\underbrace{\left[\frac{1}{(Q-V(Q))^{\beta+1}} + \frac{(1+\beta)V(Q)}{(Q-V(Q))^{\beta+2}}\right]}_{+} \tag{2.24}$$

$$\frac{\partial^2V^*}{\partial y^2} =$$

$$\underbrace{\left(\frac{\partial V^*}{\partial y}\right)^2\left[\frac{\beta c_o\left(\frac{2(1+\beta)}{(Q^*-V^*(Q^*))^{\beta+2}} + \frac{V^*(1+\beta)(2+\beta)}{(Q^*-V^*(Q^*))^{\beta+3}} - \frac{\tau c_t}{2Y_e}\frac{1}{\sqrt{Y_e\pi yd}}y^{-1/2}V^{*-3/2}\right)}{\frac{\tau c_t}{Y_e}\frac{1}{\sqrt{Y_e\pi yd}}y^{-1/2}V^{-1/2} + c_o\beta\frac{1}{(Q^*-V^*(Q^*))^{\beta+1}} + \frac{V^*(1+\beta)}{(Q^*-V^*(Q^*))^{\beta+2}}}\right] - V^*(Q)^{-1}\left(\frac{\partial V^*}{\partial y}\right)^2 + \frac{3}{2}y^{-1}\frac{\partial V^*}{\partial y}}_{?} \tag{2.25}$$

All of the signs are indeterminate.

4. Prove that optimal capacity is increasing in per acre yield and output price.

To get $\frac{\partial Q}{\partial y}$ from the implicit function

$$p_e - c_o - \frac{c_c\alpha}{Q_0^\alpha}Q^{*\alpha-1} = c_f + s + \left(\frac{2\tau c_t}{\sqrt{Y_e\pi yd}}\sqrt{\left(\frac{c_c\alpha Q^{*\alpha-1}}{c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha}\right)Q^*} + h\right)\frac{1}{Y_e} \tag{2.26}$$

Take derivative w.r.t on both sides yields:

$$\frac{2\pi c_t}{Y_e\sqrt{Y_e\pi dy}}\sqrt{\frac{c_c\alpha^2 Q^{*\alpha-1}(c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha) - c_c^2\alpha^2 Q^{*(2\alpha-2)}(\alpha-1)}{(c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha)^2}}\left(\frac{\partial Q^*}{\partial y}\right) + \frac{c_c\alpha}{Q_0^\alpha}(\alpha-1)Q^{*(\alpha-2)}\frac{\partial Q^*}{\partial y}$$
$$= \frac{\tau c_t y^{-3/2}}{Y_e\sqrt{Y_e\pi dy}}\sqrt{\frac{c_c\alpha Q^{*\alpha}}{c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha}} \tag{2.27}$$

Under square root, $\frac{c_c\alpha^2 Q^{*\alpha-1}(c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha) - c_c^2\alpha^2 Q^{*(2\alpha-2)}(\alpha-1)}{(c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha)^2} > 0$ because the scale factor $\alpha < 1$, $\frac{\partial Q^*}{\partial y}$ has to be greater than 0 to make the whole thing under square root positive. Take derivative w.r.t on both sides yields:

$$\frac{2\pi c_t}{Y_e\sqrt{Y_e\pi dy}}\sqrt{\frac{c_c\alpha^2 Q^{*\alpha-1}(c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha) - c_c^2\alpha^2 Q^{*(2\alpha-2)}(\alpha-1)}{(c_c\alpha Q^{*\alpha-1}+c_o\beta Q_0^\alpha)^2}}\left(\frac{\partial Q^*}{\partial p_e}\right) + \frac{c_c\alpha}{Q_0^\alpha}(\alpha-1)Q^{*(\alpha-2)}\frac{\partial Q^*}{\partial p_e} = 1 \tag{2.28}$$

For the same reason, $\frac{\partial Q^*}{\partial p_e} > 0$.

CHAPTER 3. ESTIMATING THE IMPACT OF ETHANOL MANDATE ON CORN PRICES USING THE COMPETITIVE STORAGE MODEL

3.1 Introduction

The impact of increasing biofuel consumption has been the subject of intense study over the last few years because of the concern that policies that support biofuels may have unintended impacts on impact food prices (World Bank), greenhouse gas emissions (Rajagopal and Plevin (2013)), or fuel prices (CBO (2014)). This paper presents a new way to model biofuel and feedstock markets that explicitly connects the US policy mechanism used to support biofuels with the market for corn–the world's most commonly used biofuel feedstock, and the commodity that is usually associated with excess greenhouse gas emission and higher food prices. Biofuel mandates in the Renewable Fuels Standard (RFS) is the primary policy tool that supports biofuel consumption in the United States so attention has recently focused on the impact of these mandates on biofuel production. The compliance mechanism for the RFS is that fuel producers and importers must obtain sufficient Renewable Identification Numbers (RINs) to show that they have met their biofuel obligations. RINs are produced when biofuels are produced. Their 38-digits facilitate traceability of each biofuel batch. Obligated parties obtain RINs either by buying biofuel from producers or by buying RINs in the market. Optimizing firms will choose to buy RINs if the price of RINs is less than their net cost of buying and blending biofuels that have the RINs attached. The net cost of buying and blending biofuels is the difference between the market price of the biofuel and its value in the fuel market. In theory, the price of RINs in the market will reflect this net cost. When RFS mandates push biofuel beyond the level that market forces alone would support, then the price of RINs will increase. RIN prices will approach zero when mandates are lower than the quantity that market forces support. Thus RIN prices provide a market measure of the impact of the RFS on biofuel consumption and production as well as providing a measure of marginal compliance costs.

To lower the cost of complying with biofuel mandates the Environmental Protection Agency (EPA) allows RINs to be banked or borrowed. Thus the price of RINs will not only reflect current net costs of buying and blending biofuels but also anticipated future net costs. Rubin (1996) developed temporal arbitrage conditions for tradable environmental permits that can be banked or borrowed. McPhail (2010) applied these conditions to the RIN market and solved the optimal conditions for how many RINs to store based on no-arbitrage profit conditions. However, she ignores the 20

The contribution of this paper is estimation of the future impact of RFS ethanol mandates using a rational expectations competitive storage model. The impact of mandates are estimated by comparing model solutions under two scenarios. The first scenario assumes that EPA allows mandates to increase to 15 billion gallons, which is the cap on RFS mandates that can be met with corn ethanol. The second scenario assumes that EPA keeps mandates at approximately 10 percent of US gasoline consumption. At this level practically all fuel in the United States will contain 10

3.2 Model

Our model consists of perfectly competitive markets, an obligated party who produces gasoline under constant cost conditions and who blends ethanol and gasoline to make fuel, a representative profit-maximizing corn farmer with an upward sloping supply curve that depends on the expected future price of corn, and a representative biofuel producer who combines corn and other inputs to produce ethanol. Each year the farmer chooses acreage based on next year's expected corn price. Harvested corn is consumed as ethanol or feed or put into storage. Storage levels are such that no arbitrage profits are possible. The obligated gasoline producer and blender decides how much ethanol to buy each year and how many RINs to borrow or bank to minimize discounted lifetime costs of compliance with mandated ethanol volumes.[1]

[1]As defined by RFS rule, obligated parties are any refineries producing gasoline or diesel fuel within the 48 contiguous states or Hawaii, or any importer that imports gasoline or diesel fuel into the 48 contiguous states or Hawaii. Blenders, who simply buy ethanol and gasoline and blend ethanol into gasoline, are not obligated

The corn farmer bases current year's planting decisions on next year's post-harvest expected price as shown in equation (3.1). A_{t+1} is acreage harvested in $t+1$. $A(\cdot)$ is a concave function of $E_t(p_{ct+1})$, i.e., $A'(\cdot) > 0$ and $A''(\cdot) < 0$.

$$A_{t+1} = A(E_t(p_{ct+1})) \tag{3.1}$$

Equilibrium storage satisfies the following no-arbitrage condition

$$\beta E_t(p_{ct+1}) = p_{ct} + SC_t(x_{t+1}), \tag{3.2}$$

where x_{t+1} is beginning corn stock in time $t+1$ and per-bushel storage costs is denoted by SC_t. Storage cost includes convenience yield which goes to negative infinity when stock level approaches zero. The discount factor β equals $\frac{1}{1+r}$ where r is the interest rate. Equilibrium corn storage is where the expected gains from holding corn to the next period equals storage costs. The expected revenue from storing a bushel of corn is the discounted expected price of corn. The cost of storing one bushel of corn is the per bushel corn price at time t plus the storage cost. If there is positive economic profit from holding corn, firms and individuals will store it. With more corn stored for $t+1$, total corn consumption decreases in t and total corn supply increases in $t+1$. The current corn price goes up and the expected corn price goes down. When expected gains from storage equal storage costs the incentive to store corn disappears and a no-arbitrage condition is reached. When stock level approaches zero, marginal storage cost goes to negative infinity. Thus, the expected gain can never be less than the cost of holding one unit of stock. This specification of marginal storage cost eliminates stock-out conditions.

The non-ethanol demand for corn is referred to as feed demand which is denoted by $D_c(p_{ct})$, where p_{ct} is the corn price at time t. Corn demand by ethanol producers is a derived demand which depends on the US ethanol demand for ethanol. Ethanol demand includes both the demand for ethanol in E10 and consumer's demand for E85 is taken from Pouliot and Babcock (2014) . Total U.S. ethanol disappears according to an ethanol usage curve $D_e(p_{et}^d, p_{gt})$, where

parties under the RFS. However, some blenders also own oil refineries such as BP and ConocoPhilips and so are considered obligated parties. The mandate for each obligated party is determined as a percentage of the total gasoline they sell in the United States. We simplify by combining blenders and obligated parties into one entity.

p_{et}^d and p_{gt} are the ethanol price and exogenous gasoline price at which an amount $D_e(p_{et}^d, p_{gt})$ of ethanol is used. We assume that gasoline supply is perfectly elastic and that changes in the gasoline price shift the demand curve for ethanol.

Our model focuses on corn and ethanol so we simplify the gasoline sector by assuming a representative gasoline producer with costs that are proportionate to random oil prices. The supply of gasoline is therefore perfectly elastic and random. The gasoline producer is an obligated party who minimizes the lifetime cost of meeting ethanol mandates. The cost of meeting the mandate in each period is $c_t(e_t - \varepsilon_t)$, where e_t is total ethanol consumed in time t and ε_t is the amount of ethanol that would be consumed without a mandate. Thus $c_t(\cdot)$ measures compliance cost. The blender can borrow or bank RINs up to 20% of the next period's total mandate.[2] Let B_t be the beginning stock of RINs and M_t be the mandate in each period t. The obligated party minimizes life time compliance cost:

$$\min_{e_t, \forall t} \sum_{t=0}^{\infty} \beta^t E_0(c_t(e_t - \varepsilon_t)) \tag{3.3}$$

$$s.t. \qquad B_{t+1} = min(B_t + e_t - M_t, 0.2M_{t+1}) \tag{3.4}$$

$$B_t + e_t - M_t \geqslant -0.2M_t \tag{3.5}$$

where $0.2M_{t+1}$ is the upper bound of banked RINs and $-0.2M_{t+1}$ is the lower bound of borrowed RINs. Equation (3.4) says that the carryover B_{t+1} cannot exceed the maximum carryover allowed at time t. Available RINs stock is the sum of RIN stock and RINs generated less the mandate, $B_t + e_t - M_t$. If available RIN stock is not greater than the banking limit, $B_t + e_t - M_t$ will be banked. If available RIN stock exceeds the banking limit, only $0.2M_{t+1}$ can be used in the next period and the rest of the available RINs will expire. Borrowing constraint (3.5) says that carryover RIN stock should be greater than -0.2 of next period's mandate. RIN stock is a state variable in ethanol producer's problem.

[2] RFS rules specify that RINs are valid for only two years and forbids RIN borrowing in consecutive two years. To simplify model solutions we allow borrowing in consecutive years when economic incentivce to do so exists.

Let

$$V_t(B_t, T) = \underset{e_t, \forall t}{min} \sum_{t=T}^{\infty} \beta^t E_0(c_t(e_t - \varepsilon_t)). \tag{3.6}$$

The Bellman equation for solving the problem with borrowing and banking constraints is

$$V_t(B_t,\, t) = \underset{e_t}{min} c_t(e_t - \varepsilon_t) + \beta E_t V_{t+1}(min(B_t + e_t - M_t, 0.2M_{t+1}),\, t+1) + \lambda_t(-0.2M_{t+1} - (B_t + e_t - M_t)), \tag{3.7}$$

where λ_t is Karush-Kuhn-Tucker (K.K.T) multiplier. The K.K.T. conditions are:

$$e_t:\ c_t^{'}(e_t - \varepsilon_t) + \beta \frac{\partial E_t V_{t+1}(min(B_t + e_t - M_t,\, 0.2M_{t+1}),\, t+1)}{\partial e_t} - \lambda_t \geqslant 0,\ \frac{\partial V_t(B_t,\, t)}{\partial e_t} \cdot e_t = 0, \tag{3.8}$$

$$\lambda_t:\ -0.2M_{t+1} - (B_t + e_t - M_t) \leqslant 0,\ (-0.2M_{t+1} - (B_t + e_t - M_t)) \cdot \lambda_t = 0,\ \lambda_t \geqslant 0. \tag{3.9}$$

The envelope condition is

$$\frac{\partial V_t(B_t,\, t)}{\partial B_t} = \beta \frac{\partial E_t V_{t+1}(min(B_t + e_t - M_t, 0.2M_{t+1}),\, t+1)}{\partial B_t} - \lambda_t. \tag{3.10}$$

Without RINs borrowing and banking, the ethanol producer has to produce at least the mandated level in each period. After introducing borrowing and banking provisions, the ethanol producer can produce less than or more than the mandated level to minimize the total cost of meeting the obligated volume for all time periods. (3.8) means that if total ethanol production is not zero then $\frac{\partial V_t(B_t,t)}{\partial e_t} = 0$. Ethanol produced, e_t, cannot be zero because at least 80% of the mandate has to be met by RINs generated in this period. Thus the K.K.T condition with respect to e_t (3.8) becomes

$$e_t:\ c_t^{'}(e_t - \varepsilon_t) + \beta \frac{\partial E_t V_{t+1}(min(B_t + e_t - M_t, 0.2M_{t+1}),\, t+1)}{\partial e_t} - \lambda_t = 0. \tag{3.11}$$

If the borrowing and banking constraints are not binding, then marginal cost of the borrowing constraint equals zero ($\lambda_t = 0$) and all available RINs will be banked $B_{t+1} = B_t + e_t - m_t$. Equation (3.11) becomes

$$c_t^{'}(e_t - \varepsilon_t) = -\beta \frac{\partial E_t V_{t+1}(min(B_t + e_t - M_t, 0.2M_{t+1}), t+1)}{\partial e_t}, \tag{3.12}$$

and together with equation (3.10), we have the marginal compliance cost in time t equals the discounted expected marginal compliance cost in the next period:

$$c_t^{'}(e_t - \varepsilon_t) = \beta E_t(c_{t+1}^{'}(e_{t+1} - \varepsilon_{t+1})). \tag{3.13}$$

The relationship between marginal compliance cost in t and expected marginal compliance cost in $t+n$ can be derived from equation (3.13),

$$c_t^{'}(e_t - \varepsilon_t) = \beta^n E_t(c_{t+n}^{'}(e_{t+n} - \varepsilon_{t+n})). \tag{3.14}$$

Equation (3.14) says that the expected marginal compliance cost increases at the interest rate. If the expected marginal compliance cost increases at a rate greater than interest rate, the gasoline producer would have an incentive to bank RINs. With more ethanol produced and more RINs banked, current marginal compliance cost increases and expected marginal cost decreases. No more RINs will be banked until current marginal compliance cost equals the discounted expected marginal compliance cost. If the expected marginal compliance cost grows less than interest rate, ethanol producer will borrow RINs from future, resulting in a decrease in current marginal compliance cost and a rise in expected marginal compliance cost. An equilibrium is achieved when current marginal compliance cost is equal to the discounted expected marginal compliance cost.

If the maximum banking limit is reached, λ_t is zero, $min(B_t + e_t - M_t, 0.2M_{t+1}) = 0.2M_{t+1}$, and (3.11) becomes

$$c_t^{'}(e_t - \varepsilon_t) = -\beta \frac{\partial E_t V_{t+1}(min(B_t + e_t - M_t, 0.2M_{t+1}), t+1)}{\partial e_t} = \beta \frac{\partial E_t V_{t+1}(0.2M_{t+1})}{\partial e_t} = 0. \tag{3.15}$$

Marginal cost is zero only if no extra ethanol is consumed and

$$e_t = \varepsilon_t. \tag{3.16}$$

In this case, the available RIN stock could be greater than the number of RINs that can be banked so that additional RINs have no value. The optimal condition for ethanol consumption requires that the marginal compliance cost of meeting the mandate is zero which implies by (3.16) that consumption produced is the same as if there is no mandate and no banking and borrowing of RINs.

When the borrowing constraint is binding at time t , $e_t - (M_t - B_t) = -0.2M_{t+1}$, $\lambda_t > 0$. Then we have

$$c_t^{'}(e_t - \varepsilon_t) = \beta E_t(c_{t+1}^{'}(e_{t+1} - \varepsilon_{t+1})) + \lambda_t. \tag{3.17}$$

Equation (3.17) says that the expected marginal compliance cost grows less than the rate of interest, the ethanol producer would have an incentive to borrow additional RINs from the future. However there is a physical constraint restricting the maximum borrowing, resulting in a binding borrowing constraint.

From the discussions of the constraints, we know that the expected marginal cost compliance costs increases at the rate of interest whenever the constraints on borrowing and banking are not binding. Marginal compliance cost grows by less than the interest rate when the borrowing constraint binds and marginal compliance cost is zero when the banking constraint binds.

Marginal expected compliance cost equals the expected RIN price which is the difference between the marginal cost of producing ethanol and marginal benefit of using ethanol (McPhail (2010)). The marginal cost of producing an extra gallon of ethanol equals feedstock cost plus a constant conversion cost. Corn is assumed to be the feedstock. Let Y_e be the net corn use for producing a gallon of ethanol accounting for byproducts produced per bushel of corn processed in ethanol production[3]. Per gallon conversion cost is assumed to be a constant c_e. Assume that the technology for producing ethanol from corn is constant for all time periods so that Y_e and c_e do not change with t. Thus $\frac{p_{ct}}{Y_e} + c_e$ is the marginal cost of ethanol production. With more corn devoted to produce ethanol, corn price rises and the marginal production cost increases. This specification connects the ethanol market with the corn market. The marginal benefit of

[3]The details of modeling the net corn production for ethanol can be found in Lapan and Moschini (2012) page 227.

blending ethanol equals the ethanol price p_{et}^d along the ethanol demand curve. Thus the RIN price in any period equals $\frac{p_{ct}}{Y_e} + c_e - p_{et}^d$.

Our model treats RIN as a storable commodity. Just as storing stabilizes corn prices under yield uncertainty, RIN storage serves the same function, helping to stabilize ethanol prices and corn prices under an ethanol mandate in a stochastic world. The carryover of corn is determined by the no-arbitrage equation that compares the value of storing one unit in this period and the value of consuming one unit in this period. Without borrowing and banking constraints, RIN storage is determined by the no-arbitrage condition that equates the marginal cost of meeting the mandate in this period to the discounted value of the expected cost of meeting the mandate in the next period. The difference between the use of corn storage and RIN storage is that corn carry-over can be consumed but RIN storage cannot be consumed as ethanol in the next period. Because stored RINs can be used to meet the next period's mandate, they affect equilibrium ethanol production. If ethanol production is greater than the mandate, total ethanol production will be consumed and the extra generated RINs will be stored for use in the next period up to the banking constraint.

Markets are in equilibrium when corn supply equals corn demand, when ethanol production equals ethanol consumption and when there are no arbitrage profit possibilities from storing RINs or corn. The total supply of corn at time t is denotes as $TS_t = A_t y_t + x_t$, where A_t is the acreage harvested at time t which is decided by farmers in time $t-1$: $A_t = A(E_{t-1}(p_t))$ and where y_t is the corn yield at time t. Total corn demand is the sum of feed demand $D_c(p_{ct})$, corn for ethanol $\frac{e_t}{Y_e}$ and storage x_{t+1}. Equilibrium conditions are sequences of quantities $\{e_t,\, D_{ct},\, x_{t+1},\, A_{t+1}\}_{t=0}^{\infty}$ and prices $\{p_{ct},\, p_{et}\}_{t=0}^{\infty}$ such that (i) the quantities solve the arbitrage conditions for different agents given the sequence of prices, (ii) the corn market clearing condition is met as shown by equation (3.18), (iii) and the ethanol market clears through equation (3.19).

$$\forall t,\ A(E_{t-1}(p_{ct}))y_t + x_t = D_c(p_{ct}) + \frac{e_t}{Y_e} + x_{t+1} \tag{3.18}$$

$$e_t = D_e(p_{et}^d,\, p_{gt}) \tag{3.19}$$

3.3 Calibration

Wholesale Gasoline Price

The ethanol demand curve is taken from Pouliot and Babcock (2014) which includes both ethanol demanded in E10 by blenders and ethanol demanded in E85 by consumers. There is little data for U.S. ethanol demand beyond the blend wall. Pouliot and Babcock (2014) use Brazilian data to estimate the consumers willingness to pay for E85, making the assumption that US consumer preferences for E85 are identical to Brazilian preferences.

Both the ethanol demand in E10 and E85 depend on the gasoline price. Ethanol in E10 complements gasoline, whereas ethanol in E85 substitutes for gasoline. Denote the ethanol demand functions for E10 and E85 as $Q_e^{10} = D_e^{10}(p_{et}, p_{gt})$, $Q_e^{85} = D_e^{85}(p_{85t}^r, p_{gt}^r)$, where p_{gt}, p_{et}, p_{85t}^r and p_{gt}^r are the wholesale gasoline price, wholesale ethanol price, retail E85 price and retail gasoline price in time t. Horizontal summation of the two inverse demand curves gives rise to the total inverse demand of ethanol. Details of the functional forms is provided as follows.

To allow the model to be solved at different gasoline prices, both the ethanol demand in E10 and E85 are approximated using piecewise linear functions:

$$Q_e^{10} = \begin{cases} 13 & 0 < \frac{p_{et}}{p_{gt}} < 0.686777 \\ 14.2178 - 1.7731\frac{p_{et}}{p_{gt}} & 0.686777 < \frac{p_{et}}{p_{gt}} \leqslant 1.074941 \\ 18.5193 - 5.7748\frac{p_{et}}{p_{gt}} & 1.074941 \leqslant \frac{p_{et}}{p_{gt}} \leqslant 1.145516 \\ 11.9042 & \frac{p_{et}}{p_{gt}} = 1.145516 \end{cases} \tag{3.20}$$

The maximum ethanol demanded in E10 is 13 billion gallons to reflect the blend wall.

Let x be the ratio of retail E85 price to retail gasoline price, $x = \frac{p_{85t}^r}{p_{gt}^r}$. With the current fleet of flex vehicle and new E85 stations, ethanol demand in E85 as a function of price ratio with no new E85 stations is as follows.

$$Q_e^{85} = \begin{cases} 1.6979 - 0.7773x & 0.409 \leqslant x < 0.62 \\ 3.4383 - 3.5843x & 0.62 \leqslant x < 0.92 \\ 0.7814 - 0.6964x & 0.92 \leqslant x < 1.1 \end{cases} \tag{3.21}$$

With 2500 new E85 stations, ethanol demand in E85 is:

$$Q_e^{85} = \begin{cases} 3.22 - 1.5283x & 0.409 \leqslant x < 0.688 \\ 7.308 - 7.47x & 0.688 \leqslant x < 0.9188 \\ 2.4482 - 2.1808x & 0.9188 \leqslant x < 1.0864 \end{cases} \tag{3.22}$$

Ethanol demand in E85 with 5000 new E85 stations is:

$$Q_e^{85} = \begin{cases} 4.3108 - 1.249x & 0.409 \leqslant x < 0.67 \\ 11.2651 - 11.6285x & 0.67 \leqslant x < 0.93 \\ 2.3139 - 2.0035x & 0.93 \leqslant x < 1.1 \end{cases} \tag{3.23}$$

The demand for E85 is small when the price of E85, adjusted on a cost per mile basis, is higher than E10. But demand becomes elastic when E85 prices become competitive with E10. Eventually demand becomes quite inelastic due to limits on the number of stations that sell E85. The range of E85 quantities where demand is elastic increases if additional E85 fueling stations become available because the bottleneck limiting demand for E85 is access to fueling stations not the number of flex vehicles. In this analysis, it is assumed that E85 contains 75 percent ethanol, the wholesale E85 price is the weighted average price of ethanol and gasoline, retail E85 ethanol price is \$0.75 per gallon higher than wholesale price and retail gasoline price is \$0.75 per gallon higher than the wholesale gasoline price (Pouliot and Babcock (2014)).

Thus the price ratio x can also be stated as $\frac{0.75p_{gt}+0.25p_{et}+0.75}{0.75+p_{gt}}$. This value can be substituted into (3.21), (3.22), (3.23). Adding the quantity of ethanol in E10 and E85 using (3.20), (3.21), (3.22), (3.23). Then we get the inverse ethanol usage function $e_t = D_e(p_{et}^d, p_{gt})$ for each investment with each wholesale gasoline price. The inverse ethanol usage function is $p_{et}^d = D_e^{-1}(e_t, p_{gt})$.

3.3.1 Wholesale gasoline price

Wholesale gasoline prices are assumed to be log-normally distributed. Mean wholesale gasoline price in 2014/15 is set to be $2.68/gallon which is the average of RBOB gasoline futures prices from September 2014 to August 2015 in August 13th 2014. The values of RBOB gasoline futures prices are taken from CME Group. The RBOB future gasoline price is falling in the following years and we assume the mean value for 2015/16 is $2.60/gallon and is $2.50/gallon for all years in the future. The standard deviation is assumed to be 20% of the mean price.

3.3.2 Mandate

The RFS operates on a calendar-year basis while our model operates on a marketing year basis. So we assume that the marketing year mandate is one-third of one year and two-thirds of the next year. Thus for the scenario that allows the mandate to increase to 15 billion gallons, we set the 2014/15 marketing year mandate at 14.8 billion gallons. The mandate is set at 15 billion gallons for all subsequent years.

3.3.3 Beginning RIN stock

The potential carry in of RINs in 2014 is estimated to be 0.997 billion gallons when 2013 yearly mandate is set to be 13.8 billion gallons (Paulson (2014)). Because the marketing year mandate for 2012/13 is 13.6 billion gallons, the carry-in RIN in 2013/14 is assumed to be 0.2 more than Paulson's estimate. Thus the beginning RIN stock is set to be 1.2 billion gallons in 2013/14. According to WASDE report in August 2014, 5,075 million bushels of corn are devoted to ethanol production and a bushel of corn yields 2.8 gallons of ethanol. Ethanol yield from corn is taken from monthly profitability of ethanol by Iowa State University[4]. Thus the generated ethanol production is 5,075*2.8=14.21 billion gallons. The 2013/14 mandate is 14.2 billion gallons. The carryover stock is then calculated as the potential carry-in in 2013/14 plus the generation less the 2013/14 mandate. Carryover stock is 1.2+14.21-14.2=1.408 billion gallons.

[4] http://www.extension.iastate.edu/agdm/refirst.html

3.3.4 Corn Demand

In this study, we use a constant elasticity non-ethanol, non-storage demand function.

$$p_c = a_1 D_c^{a_2}$$

When producing ethanol from corn, a valuable by-product called DDGS (dry distillers grains plus solubles) is also produced. DDGS is a close substitute for corn in livestock feed. If the by-product's price is proportional to corn, then it is legitimate to assume that less corn can be used for producing the same amount of ethanol (Lapan and Moschini (2012)). Here, we assume that the price of DDGS is 91% of the price of corn (Anderson, Anderson and Sawyer (2010)). 56 lbs of corn (1 bushel) that is processed into ethanol production will produce 17 lbs of DDGS. The net corn used for producing 2.8 gallons of ethanol is calculated as $1 - 0.91 * 17/56$. That is, the yield of one bushel of corn is $\frac{2.8}{1-0.91*17/56} = 3.87$ gallons of ethanol. It is assumed that the corn feed demand elasticity is fixed at -0.44 (Adjemian and Smith (2012)). D_c includes all non-ethanol use except corn storage. The value of a_1 in 2014/15 is calibrated using the average received corn price by farmers and the non-ethanol, non-storage quantity demanded in August 2014 WASDE report. The non-ethanol use for corn is calculated by subtracting the net corn use for ethanol from the total use of corn (Total corn use(without storage) - corn use for ethanol *(1-0.91*17/56) = non-ethanol use). The positions of demand curves from 2015/16 to 2019/20 are based on USDA's long term agriculture projections in February 2014.[5] From 2012/13 to 2015/16, the values of a_1 are different. a_1 is assumed to be constant after 2015/16.

3.3.5 Corn Yield

Corn yield is assumed to be beta-distributed with a linear trend. US corn yields from 1970 to 2013 reported by National Agricultural Statistics Service (NASS) of the U.S. Department of Agriculture are used to estimate the trend. Then we scaled up all the yield realizations from 1970-2013 to 2013 trend yield levels. Detrended yield data is used for estimating the parameters of a beta distribution that represents corn yield distribution during 2013/14 marketing year.

[5] USDA long term projection 2014 can be found here
http://usda.mannlib.cornell.edu/MannUsda/viewStaticPage.do?url=http://usda.mannlib.cornell.edu/usda/ers/94005/./2014/i

The estimated corn yield is assumed to have an upper bound $y_u = 200$ bushels per acre, lower bound $y_l = 90$ bushels per acre. The beta distribution in 2013/14 is $beta(7.3766, 4.7497)$. According to the corn yield trend line, the per acre mean yield value increases by approximately two bushels per year. Thus we assume that the whole corn yield distribution shifts out two bushels per acre until 2019/20. After 2019/20, assume that corn yield distribution remains the same as in 2019/20. The calibrated mean yield values are documented in table 6. Corn yield in 2014 is set to be 167.4 bushels per acre according to August WASDE 2014. Corn yields are impacted mainly by weather, so we assume that corn yields are independent of gasoline prices.

3.3.6 Other Variable Cost

Per gallon ethanol conversion cost (c_e) is assumed to be constant. This cost includes the cost of natural gas used in the production process and variable costs. We fix the non-corn ethanol production cost 50 cents per gallon (Hofstrand (2014).

3.3.7 Harvested Acres

Farmers make planting decisions according to the expected corn price. We assume that harvested acres has a constant elasticity functional form as

$$A = \delta_1 E(p_c)^{\delta_2}. \tag{3.24}$$

The elasticity of harvested acres is assumed to be 0.2. This supply elasticity is roughly consistent with that of Roberts and Schlenker (2010) which is 0.14. δ_1 is determined by the expected price for 2013/14 and the harvested acres in 2013/14. The per bushel expected price is represented by the average marketing year's December futures price in 2013 from September 1st 2012 to August 31st 2013. The expected price in 2013/14 is \$5.68 per bushel. The harvested acres is 0.877 100 million acres. We have $\delta_1 = 0.62$. We use harvested acres in August WASDE 2014 to be the real harvested acres in 2014. After 2014, the acreage decision follows (3.24).

3.3.8 Storage Cost and Convenience Yield

The storage cost per unit includes per unit observed cost and per unit unobserved cost. The observed part of the storage cost (OSC) is a constant physical per unit storage cost paid by the storer. We assume that the observed per bushel storage cost is 3 cents per bushel per month (Peterson and Tomek (2010)). The yearly observed storage cost is thus \$0.36 per bushel, $OSC_t = 0.36$. One component of unobserved storage cost is the opportunity cost that increases with stock level when stock levels are large. This is because holding more stock of one crop decreases the opportunity of holding other more profitable crops (Paul (1970)). The other unobserved storage cost is the marginal convenience yield. The unobserved storage cost should be increasing with stock level and it is negative when stock level s_t is small and positive when stock level is high. Rui and Miranda (1995) uses a logarithmic function to achieve it. $USC_t = \eta_1 + \eta_2 log(s_{it+1})$ where η_1 and η_2 are two parameters needed to be calibrated. We calibrate this USC_t to two points. One point has low ending stock and the other has abundant ending stock in recent years. The chosen low ending stock point is 2012/13. Both 2008/09 and 2009/10 have high ending stocks. Thus we use the average of the stock level and the average of unobserved storage cost in those two years as the other point. We can get unobserved storage cost from the storage no-arbitrage condition: $USC_t = \beta E_t(p_{t+1}) - p_t - OSC_t$. We collect our data including current price p_t, expected price $E_t(p_{t+1})$ in 2008/09, 2009/10 and 2012/13. Use average price received by the farmer in each marketing year from USDA NASS for the current year price. The average of December corn futures price from September 1st to August 31st is used as the yearly expected price. The discount factor, β, is defined as $\frac{1}{1+r}$ where r is the interest rate. We use the return for 1-year treasury constant maturities as risk free interest rate. for all interest rates from 2014, we set the value to 0.13%, which is the same as the annual 1-year treasury constant maturities in 2013. We have $\eta_1 = -1.65$, $\eta_2 = -2.8926$. It is also assumed that the per bushel storage cost goes to infinity when approaching the storage capacity 4.0 billion bushels. Thus we have $USC_t = \begin{cases} -1.65 - 2.8926 log(s_{t+1}) & 0 < s_{t+1} < 0.4 \\ \infty & s_{t+1} \geqslant 0.4 \end{cases}$.

3.4 Solution Methods

Corn stock and RIN stock are the two state variables in our model. The model is solved by finding the expected corn price and expected ethanol price functions which depend on these two state variables. These unknown functions need to meet three conditions: acreage must maximize farmer expected profit; no-arbitrage profits for corn storage; and no-arbitrage profits for RIN storage. The collocation method is used to solve the problem. This method has been applied to solve other commodity storage models including Miranda (1997), Peterson and Tomek (2005) and Gouel (2013). In our model, we assume a stationary world after 2019/2020 in that we assume that non-ethanol demand and the distributions of the stochastic variables remain constant over time. Before 2019/20 mean yield increases by two bushels per acre per year and non-ethanol demand changes are calibrated to USDA projections.

The collocation method approximates an unknown function P using a linear combination of functions ϕ_1, ϕ_2,... , ϕ_n, called the basis functions (Judd 1998). If there is only one dimension

$$P(x) \approx \sum_{j=1}^{n} c_j \phi_j(x) \tag{3.25}$$

The unknown coefficients c_1, c_2,... , c_n are determined when the approximated function satisfies the model's equilibrium conditions at n points x_1, x_2,... , x_n chosen in the space of x, $[\underline{x}, \bar{x}]$. All possible values of x should be in $[\underline{x}, \bar{x}]$. The n points are called the collocation nodes.

The expected prices of corn and ethanol (EP_c, EP_e) can be represented as two dimensional polynomials of given degrees of approximation. The approximation of expected corn prices and ethanol prices functions at each collocation node i are given by

$$EP_k(x_i, B_i) = \sum_{j_1=1}^{n_1} \sum_{j_2=1}^{n_2} c_{kj_1j_2} \phi_{j_1}(x_i) \phi_{j_2}(B_i) \tag{3.26}$$

$$k = c, e,\ j_1 = 1, 2, ...n_1,\ j_2 = 1, 2, ..., n_2$$

The steps used to solve for these functions are as follows:

(0) Initial Step:

Select the degree of approximation in each dimension n_i, for $i = 1, 2$. Select the cubic spline basis functions ϕ_1,..., $\phi_{n_1 n_2}$. Select the collocation nodes x_i, B_i, for $i = 1, 2, ...n_1 n_2$. Guess initial values of $c_{cj_1j_2}$,$c_{ej_1j_2}$, $j_1 = 1, ..., n_1$, $j_2 = 1, ..., n_2$, where n_1, n_2 are the selected degrees of approximation in each dimension. We use $n_1 = 30$, $n_2 = 10$ to show the results. Then we determine the state spaces for each state variable. Let x lie in the interval [0, 4] in units of billion bushels and B is chosen in [−3, 3] in units of billion gallons. $n_1 n_2$ spline collocation points are chosen to be evenly distributed in each dimension.

Gaussian quadrature is used to replace the continuous yield and gasoline price distributions by m_1-point and m_2-point discrete distributions. The discrete yield values are y_1, y_2, ... , y_{m_1} with the associated probabilities w_{k_1} for $k_1 = 1, 2, ..., m_1$. The values p_{g1}, p_{g2}, ... , p_{gm_2} are assumed to be the discrete demand shocks and w_{k_2} with $k_2 = 1, 2, ..., m_2$ are the corresponding probabilities. We pick 8 quadrature nodes for both the beta distribution and log-normal distribution. The Matlab codes given in Miranda and Fackler's book are used to generate quadrature nodes and the corresponding probabilities.

(1) Solution Step:

Get the total supply ($A_{ik_1k_2}$) for each collocation node (i) and Gaussian quadrature node (k_1, k_2). The acreage harvested in t can be written as $A_{ik_1k_2} = A(\sum_{j_1=1}^{n_1} \sum_{j_2=1}^{n_2} c_{cj_1j_2} \phi_{j_1}(x_i)\phi_{j_2}(B_i))$, so the total supply at each collocation node (x_i, B_i) given y_{k_1} and p_{gk_2} is

$$TS_{ik_1k_2} = x_i + A_{ik_1k_2} y_{k_1}. \tag{3.27}$$

Given the total supply, solve the corn storage arbitrage condition and RIN storage arbitrage conditions described in (3.28), (3.29) and (3.30) to get corn storage ($x_{ik_1k_2}$) and quantity of ethanol ($e_{ik_1k_2}$) at each collocation node i for $i = 1, 2, ..., n_1 n_2$ and each Gaussian quadrature node $k_1 = 1, 2, ..., m_1$, $k_2 = 1, 2, ..., m_2$.

$$\beta EP_c(x_{ik_1k_2}, B_{ik_1k_2}) - p_c(TS_{ik_1k_2} - x_{ik_1k_2} - \frac{e_{ik_1k_2}}{Y_e}) - SC(x_{ik_1k_2}) = 0. \tag{3.28}$$

Let

$$F = \frac{1}{Y_e} p_c(TS_{ik_1k_2} - x_{ik_1k_2} - \frac{e_{ik_1k_2}}{Y_e}) + ce - p_e^d(e_{ik_1k_2}, p_{gk_2})$$
$$-\beta[\frac{1}{Y_e} E(P_c(x_{ik_1k_2}, B_{ik_1k_2})) + ce - E(P_e(x_{ik_1k_2}, B_{ik_1k_2})))$$

with $B_{ik_1k_2} = min(B_i + e_{ik_1k_2} - M, 0.2M)$,

$$e_{ik_1k_2} = -0.2M + M - B_i,\ F > 0$$

$$-0.2M + M - B_i \leqslant e_{ik_1k_2} \leqslant 0.2M + M - B_i,\ F = 0 \tag{3.29}$$

$$e_{ik_1k_2} = 0.2M + M - B_i,\ F < 0.$$

The nonlinear equation system (3.28) to (3.29) can be solved using $PATH\ Solver$[6]. Whenever the ethanol consumption binds by $e_{ik_1k_2} = 0.2M + M - B_i$, we need to solve equations (3.28) together with equation (3.30) for the unknowns $x_{ik_1k_2}$, $e_{ik_1k_2}$,

$$\frac{1}{Y_e} p_c(TS_{ik_1k_2} - x_{ik_1k_2} - \frac{e_{ik_1k_2}}{Y_e}) + ce - p_e^d(e_{ik_1k_2}, p_{gk_2}) = 0. \tag{3.30}$$

Equation (3.30) says that if the banking constraint binds, we assume that ethanol production is the same as it would be with no borrowing and banking limit and only the maximum level of banked RIN stock will be carried to the next period.

(2) Update Step:

update the coefficients $\hat{c}_{cj_1j_2}$,$\hat{c}_{ej_1j_2}$ that solve the equation system (3.31), (3.32):

$$\sum_{j_1=0}^{n_1}\sum_{j_2=0}^{n_2} \hat{c}_{cj_1j_2}\phi_{j_1}(x_i)\phi_{j_2}(B_i) = \sum_{k1=1}^{m_1}\sum_{k2=1}^{m_2} w_{k_1}w_{k_2}p_c(TS_{ik_1k_2} - x_{ik_1k_2} - \frac{e_{ik_1k_2}}{Y_e}),\ i = 1, ..., n_1n_2 \tag{3.31}$$

$$\sum_{j_1=0}^{n_1}\sum_{j_2=0}^{n_2} \hat{c}_{ej_1j_2}\phi_{j_1}(x_i)\phi_{j_2}(B_i) = \sum_{k1=1}^{m_1}\sum_{k2=1}^{m_2} w_{k_1}w_{k_2}p_e^d(e_{ik_1k_2}, p_{gk_2}),\ i = 1, ..., n_1n_2 \tag{3.32}$$

[6]Path solver for Matlab can be downloaded from http://pages.cs.wisc.edu/˜ferris/path.html.

(3) Convergence Check:

If $\mid \hat{c}_{cjj_2} - c_{cj_1j_2} \mid < \epsilon$ and $\mid \hat{c}_{ej_1j_2} - c_{ej_1j_2} \mid < \epsilon$ for all j_1, j_2 and some convergence tolerance ϵ, set $\hat{c}_{cj_1j_2} = c_{cj_1j_2}$ and $\hat{c}_{ej_1j_2} = c_{ej_1j_2}$; otherwise set $c_{cj_1j_2} = \hat{c}_{cj_1j_2}$ and $c_{ej_1j_2} = \hat{c}_{ej_1j_2}$ for all j_1, j_2 and return to step (1).

Let T=2019/20. In the non-stationary world before 2019/20, we solve for approximated expected price functions in $T-1$ taking $T's$ expected price functions as given. After solving approximated expected price functions in $T-1$, the coefficients for approximated expected prices can be solved backwards in $T-2$, $T-3$, ... in the same way.

3.5 Results

If we are in the nth state of world in 2014/15, we can solve for storage decision in 2014/15 ($x_{15/16}$), harvest decision in next year ($A_{15/16}$), beginning RIN stocks in the next period ($B_{15/16}$), quantity of ethanol ($e_{14/15}$) and corn feed consumption ($TS_{14/15} - \frac{e_{14/15}}{Y_e} - x_{14/15}$) given the beginning corn stock ($x_{14/15}$), beginning RIN stock ($B_{14/15}$), acreage harvested ($A_{14/15}$) in 2014/15. Thus all prices are obtained in 2014/15. Starting from all known values of 2014/15 and if we are in the nth state of the world in 2015/16, first we need to know the values of all state variables. The beginning corn stock in 2015/16 is $x_{15/16}$ and the beginning RIN stock is $B_{15/16}$. Then we solve for all variables of interest in 2015/16 when the nth state happens and all prices are known for 2015/16. The same method is used for solving the prices in 2016/17. After solving for the unknown variables for each of the 5,000 sequences of states we obtain distributions of prices in each period.

Table 1 shows the average of the simulations for the scenario in which EPA lets the ethanol mandate increase to 14.4 billion gallons in 2014 and 15 billion gallons in 2015. Because this model is solved on a marketing year basis, the mandates that are imposed on the model solutions are 14.8 billion gallons for the 2014/15 marketing year, and 15 billion gallons thereafter. To allow these mandates to be met, 2,500 additional stations that sell E85 are installed in the 2014/15 marketing year and another 2,500 additional stations are built in the following marketing year.

The results show that the increased ethanol mandates can be met with the 2,500 additional

Table 3.1: Average Model Solutions with Increased Mandates*

	14/15	15/16	16/17	17/18	18/19	19/20
Ethanol Mandate	14.8	15	15	15	15	15
New E85 Stations	2,500	2,500	0	0	0	0
Harvested Acreage	83.80	82.14	82.30	82.47	82.70	82.91
Corn Production	1.40	1.32	1.34	1.36	1.38	1.40
Corn Price	3.86	4.05	4.12	4.18	4.22	4.28
Ending Corn Stocks	1.58	1.53	1.52	1.52	1.53	1.49
Ethanol Demand Price	0.96	1.00	1.02	1.03	1.05	1.06
Ethanol Production	14.36	15.08	15.00	14.97	14.96	14.95
RIN Price	0.5412	0.5436	0.5449	0.5452	0.5452	0.5437
Beginning RIN Stock	1.408	0.969	1.051	1.018	1.003	0.968

*Units are billion gallons for ethanol mandate, ethanol production, and beginning RIN stock; million acres for harvested acreage $ per bushel for corn prices, $ per gallon for ethanol price and RIN price; and billion bushels for ending corn stocks and corn production.

stations in the first two marketing years through a combination of expanded ethanol consumption and production and a drawdown in the number of banked RINs. The first-year drawdown of banked RINs is about 0.44 billion RINs to meet the 14.8 billion gallon mandate. Thus about 14.36 billion gallons of ethanol are actually consumed. Thereafter, ethanol production and consumption are much more closely aligned, with the average size of the RIN bank staying around zero in the following periods. Average corn prices rise modestly through the projection period. This modest rise hides the actual volatility in the model solutions caused by yield variability. Average harvested corn acreage falls from its high mark of 83.8 million acres in 2014, stabilizing at an average level of 82.5 million acres. Average RIN prices are slightly below 55 cents per gallon which implies that ethanol mandates push average ethanol consumption higher than what market demand would dictate in the absence of mandates. This level of RIN prices would likely incentivize additional investment in stations that sell E85 (or E15) which would then results in lower RIN prices.

Average model solutions with reduced mandates and no investment in E85 stations are shown in Table 2. Corn prices and production are modestly lower due to decreased demand for ethanol. Average RIN prices are close to zero which implies that the 13 billion gallon ethanol mandate is largely irrelevant to ethanol production and consumption levels. Because

Table 3.2: Average Model Solutions with Reduced Mandates*

	14/15	15/16	16/17	17/18	18/19	19/20
Ethanol Mandate	13.00	13.00	13.00	13.00	13.00	13.00
New E85 Stations	0.00	0.00	0.00	0.00	0.00	0.00
Harvested Acreage	83.80	81.06	81.25	81.43	81.68	81.91
Corn Production	1.40	1.31	1.32	1.34	1.36	1.38
Corn Price	3.61	3.80	3.87	3.92	3.97	4.02
Ending Corn Stocks	1.58	1.52	1.51	1.52	1.52	1.48
Ethanol Demand Price	1.42	1.48	1.49	1.51	1.52	1.53
Ethanol Production	13.20	13.37	13.27	13.25	13.24	13.23
RIN Price	0.0113	0.0063	0.0064	0.0071	0.0069	0.0083
Beginning RIN Stock	1.408	1.603	1.927	2.026	2.061	2.066

*Units are billion gallons for ethanol mandate, ethanol production, and beginning RIN stock; million acres for harvested acreage $ per bushel for corn prices, $ per gallon for ethanol price and RIN price; and billion bushels for ending corn stocks and corn production.

the average price of RINs is so low, the average bank of RINs grows and is used to buffer the effects of short corn crops. At the end of the projection period the bank of RINs grows to about 2 billion on average.

The impact of reduced mandates can be measured by comparing the Table 2 results with the Table 1 results. Both the absolute difference in average results and the percent difference are shown in Table 3. Corn prices drop about 6 percent from reduced mandates or about 25 cents per bushel. Corn production drops by about 17 million bushels which is between 1.2 and 1.3 percent. Ethanol production drops by about 11.5 percent from reduced mandates. Corn prices would decrease even more from this drop in demand except that the decrease in corn supply from lower planted acreage boosts average prices a bit.

There are two ways of viewing these results. The rather modest decrease in corn prices from relaxing the mandates could be viewed as evidence that the agricultural crop sector will not suffer too much from a reduction in ethanol mandates. An alternative view is that a reduction in mandates would not be a panacea for livestock organizations or anti-hunger groups who want to see corn prices decrease by even more than they have dropped since September, 2013. The low RIN prices in Table 2 also suggest that corn prices would not move any lower even if mandates were eliminated, because the mandate is not increasing the production of ethanol. This result

Table 3.3: Table 3. Impact of Reduced Ethanol Mandates*

	14/15	15/16	16/17	17/18	18/19	19/20
Corn Production	0	-0.017415	-0.01711815	-0.01713816	-0.01702584	-0.01689
	0.00%	-1.31%	-1.28%	-1.26%	-1.23%	-1.21%
Corn Price	-0.2557	-0.2545	-0.2573	-0.257	-0.2536	-0.2521
	-6.62%	-6.28%	-6.24%	-6.15%	-6.01%	-5.90%
Ending Corn Stocks	-0.008	-0.009	-0.008	-0.008	-0.008	-0.009
	-0.51%	-0.59%	-0.53%	-0.53%	-0.52%	-0.60%
Ethanol Demand Price	0.4624	0.4712	0.4717	0.4715	0.4726	0.4701
	48.17%	46.93%	46.20%	45.59%	45.22%	44.30%
Ethanol Production	-1.166	-1.714	-1.731	-1.72	-1.72	-1.719
	-8.12%	-11.36%	-11.54%	-11.49%	-11.50%	-11.50%
RIN Price	-0.5299	-0.5373	-0.5385	-0.5381	-0.5383	-0.5354
	-97.91%	-98.84%	-98.83%	-98.70%	-98.73%	-98.47%
Beginning RIN Stock	0	0.634	0.876	1.008	1.058	1.098

*Units are billion gallons for ethanol production and beginning RIN stock; million acres for harvested acreage $ per bushel for corn prices, $ per gallon for ethanol price and RIN price; and billion bushels for ending corn stocks and corn production.

hinges on the assumption that oil companies would continue to find it profitable to blend inexpensive ethanol with low-octane gasoline blendstock to create 87 regular gasoline. In either case, it is difficult to argue that a change in corn prices provides an over-riding justification for either reducing mandates or letting them grow because the impacts of a reduction are modest.

Before concluding it is useful to consider how a reduction in mandates would affect the distribution of corn and RIN prices. One justification for lower mandates is that mandates can exacerbate corn price spikes caused by short crops. Figure 1 shows the distribution of corn prices for the 2017/18 marketing year for the two scenarios considered. The distribution with the increased mandate is shifted to the right, which represents a higher average corn price, and it is slightly flatter, which seems to indicate a bit more price variability. But the coefficient of variation of price in the two distrbutions are approximately equal. The increase in price variability is not greater because of the role that RIN and corn stocks play in buffering the effects of low corn yields. Corn stocks are drawn down in low yield years as are RIN buffer stocks. Due to the ability to borrow RINs from future years, the RIN stock can actually turn negative, further buffering the effects of low corn yields.

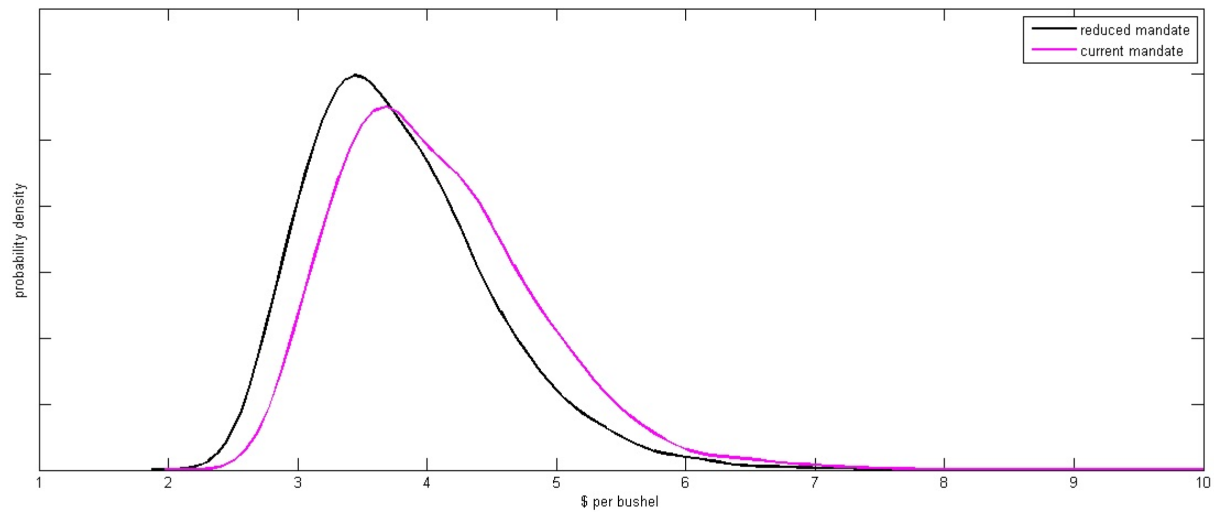

Figure 3.1: Distributions of Corn Prices in 2015/16

Figure 2 shows the two distributions of RIN price solutions in 2017/18. The distribution with reduced mandates shows that 90 percent of the RIN price solutions are less than one cent. This represents a return to the situation that mostly prevailed between 2008 and 2011 when RIN prices were quite low. If mandates are increased and if 5,000 new E85 stations are built, then about 95 percent of RIN price solutions are between 50 and 80 cents.

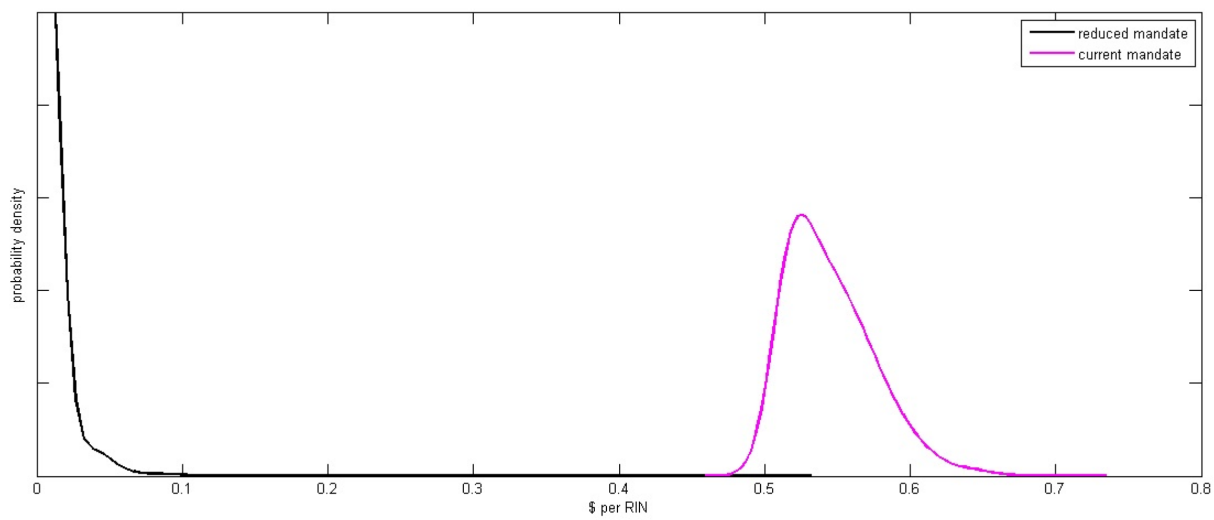

Figure 3.2: Distributions of RIN Prices in 2015/16

3.6 Policy Implications

A substantial part of the opposition to biofuel subsidies and mandates is the impact they have on the price of food. This opposition was fueled by the coincident increases in corn prices and biofuel production since 2006. Corn prices reached record high levels early in 2013 due in part to strong demand for ethanol as well as a record drought that affected the primary US corn growing regions. For example, the average price received by corn farmers in March of 2013 was $7.13 per bushel. Since that peak the price of corn has dropped dramatically. In July 2014 the average price received by farmers for corn was $3.80 per bushel (NASS 2014). In September 2014, cash prices for corn in Central Iowa had dropped to $2.80 per bushel, a level not seen since 2006. In contrast to the large year-to-year swings we have seen in corn prices, the results presented here indicate that EPA's mandate decisions going forward will impact corn prices by an average of about 22 cents per bushel or by between 5 and 6 percent. Furthermore, the results indicate that the volatility of corn prices measured by the coefficient of variation of price within a year is unaffected by mandate levels. Volatility increases are limited because corn and RIN stocks buffer the impact of yield shocks on corn prices. These relatively modest impact suggests that whether ethanol mandates should be reduced to levels that can be easily met with 10 percent blends or increased above those levels should be determined by factors other than the impact on corn prices and subsequent food prices.

Two stated objectives of the RFS are to reduce greenhouse gas emissions and to reduce petroleum imports. Economists are nearly unanimous that the best way to cut emissions is with a carbon tax because the cost of reducing emissions is minimized when a tax is applied equally to all major emission sources. Similarly, the most efficient way of reducing oil imports is to tax imports. But politicians rarely agree with economists' prescriptions so second-best policy instruments such as the RFS that only apply to liquid transportation fuels to meet policy objectives are utilized.

Increasing RFS mandates above levels that can be met with E10 does increase US biofuel consumption. This increase likely results in decreased petroleum imports. And unless expanded mandates result in large unintended increases in greenhouse gas emission (Rajagopal and Plevin (2013)), substitution of gasoline with ethanol will reduce emission levels, particularly if future ethanol mandates exceed the 15 billion gallon level that can be met with corn ethanol. Thus the RFS, however inefficiently, will likely meet its stated objectives.

The question facing EPA and Congress is whether the costs of maintaining support for biofuels through the RFS are too high for the benefits that are obtained. If the costs are too great or if a more efficient policy is available, then policy should be changed as quickly as possible because a quicker decision to withdraw support for biofuels will allow investment dollars to be redirected to other enterprises. However, if a withdrawal of support for biofuels is not forthcoming, then an EPA decision to set mandates at levels that lead to low RIN prices sends exactly the wrong signal to investors because without investment, increased consumption of biofuels will never occur. The results presented here demonstrate that corn and food price considerations should not be important factors in the debate.

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Table 3.4: Parameter values in 2014/15

parameter	value	source or explanation
ethanol yield	2.8	Monthly Profitability of Ethanol Production by Iowa State University
DDGS yield	17/56	
DDGS price	91% of corn price	Anderson, Anderson and Saweyer(2008)
other ethanol production cost c_e	50 cents	Monthly Profitability of Ethanol Production by Iowa State University
constant storage cost per bushel per year within capacity	36 cents	Peterson and Tomek(2005)
beginning corn stock	0.1181 10 million bushels	May 2014 WASDE Report
beginning RIN stock	0.1408 10 billion gallon	Calculated
non-ethanol demand elasticity	-0.44	Adjemian and Smith (2012)
supply elasticity	0.2	Roberts and Schlenker (2012) Increased a little bit
supply factor	0.62	Chicago board of trade
gasoline price distribution	log normal(0.9185,0.2722)	fit yield data from USDA from 1970 to 2012

Table 3.5: Parameters in other years

parameters	14/15	15/16	16/17	17/18	18/19	19/20	Source
demand factor(α_1)	3.95	3.45	3.62	3.81	4.07	4.36	USDA
mean yield	165.3	160.94	162.94	164.94	166.94	168.94	trend line value
							except 14/15
mean gasoline price	2.68	2.6	2.5	2.5	2.5	2.5	RBOB gasoline

CHAPTER 4. ENDOGENOUS PRICES IN A DYNAMIC MODEL FOR AGRICULTURAL SUPPLY ANALYSIS

4.1 Introduction

The purpose of this paper is to present a new model of agricultural supply which combines Positive Mathematical Programming (PMP) with the rational expectations storage model. PMP is an approach widely used for calibrating mathematical models with multiple agricultural outputs. The PMP approach is able to generate optimal production plans that replicate a reference allocation. Popularized by Howitt (1995), PMP has been developed by researchers both in calibration using exogenous supply elasticities (Hechkelei and Britz (2005), Mrel and Bucaram (2010), Mrel et al. (2011)) or estimation using multiple data points (Britz and Heckelei (2000), Jansson and Heckelei (2011)). Most of these models assume exogenous output prices. Arfini et al. (2008) first incorporate endogenous prices in PMP by modeling 'farm level' demand functions and cost functions while taking the demand functions into profit maximization problem. Later, Arfini and Donati point out that their old approach is inappropriate because their model assumes that individual farms are not price takers so it does not fit the competitive market. Their new approach introduces endogenous prices by maximizing the difference between the total value of the output and the total cost of variable inputs subjected to the aggregating constraints and individual farm's constraints. However, existing PMP models do not consider that the acreage decisions made by forward-looking farmers are determined by expected revenues. One way to include this behavior is to incorporate with a rational expectations storage model

The rational expectations storage model has emerged as a powerful tool in crop price analysis and policy analysis (Williams and Wright (1991), Miranda and Glauber (1993), Gouel (2013a)). It endogenizes expected price by assuming futures prices formed by agents are realized expected prices given all optimal decisions about storage, acreage and consumption. Cafiero et al. (2011) validate the model empirically and raise an important issue of the quality of solutions properties of generated prices. Miranda (1997) compares different numerical methods for solving the storage model, including collocation methods, least squares, space discretization and linearization when approximating current price functional form. He finds that the collocation method with Chebyshev or spline polynomials outperforms the other methods. Gouel (2013b) compares methods for approximating various functions including the value function, the expected crop price function and the storage rule. He claims that expected price function approximation leads to the most accurate result because the expected price function is smooth and close to linear. At the same time, expected price approximation is also the most time consuming method. However, considering the desired accuracy to achieve, the author recommends parameterize expected price algorithm especially for several state variables. Comparing the results given by the storage model with and without convenience yield, he finds that convenience yield smooths the approximated functions and generally gives higher accuracy for all methods.

However, computational cost of the collocation method increases exponentially with the number of state variables. To extend the storage model to multiple crops, we need to employ new numerical methods.

The perturbation method is widely used in solving dynamic stochastic general equilibrium (DSGE) models in macroeconomics. The perturbation method linearizes the solution at the steady state and uses the solution to infer results away from steady states. Because it only requires solving a system of linear equations, it can be easily applied to models with multiple state variables. However, the perturbation approximation performs poorly away from the steady state. Because the economy is usually not around the steady state, this method cannot be applied to storage model (Gouel (2013b), Miranda (1997)).

Kollman et.al (2006) compares several methods to solve stochastic neoclassical growth models with multiple countries. They suggest using Smolyak collocation method to interpolate solution functions. Invented by a Russian Mathematician Sergey Smolyak (1963), Smolyak grid is used instead of tensor gird to interpolate and represent multivariate functions. First adopted by Krueger and Kubler (2004) to solve a dynamic over-lapping generation model in economics, the Smolyak collocation method is also used in many other applications. Malin et. al (2011) apply it to solve a multiple country international real business cycle model. Judd et. al (2013) extended the method by using grid construction and a non-derivative fixed point algorithm.

Besides the Smolyak method, generalized stochastic simulation algorithm (GSSA) (Judd et al. (2011)) improves stochastic simulation algorithm (Haan and Marcet (1990)) by replacing inaccurate Monte-carlo and unstable standard least square methods to solve high dimensional dynamic models. GSSA solves the model using a relatively small number of points that are visited in equilibrium rather than by using the collocation method which requires larger domains (Judd (1992)). GSSA has been shown to be numerically stable even with a large number of state variables in multi-country neoclassical growth models (Judd et al. (2011)), an 80-period overlapping generation model (Hasanhodzic and Kotlikoff (2013)) and a search and hiring model with heterogeneous workers and hiring selectivities (Villena-Roldn (2013)).

In this paper, we present a new way of formulating a three crop competitive storage model. In each period, crops can be consumed or stored for future use. A representative farmer maximizes expected profit using the expected crop prices and a cost function. It is assumed that the only production input is land. The cost function is calibrated to conditions in a base year and the implied expected land elasticities coincide with exogenous supply elasticities (Mrel and Bucaram (2010)). The model is calibrated to three crops: corn, soybean and all other crops.

We incorporate convenience yield in our model. The idea of convenience yield is first introduced by Kaldor (1939) to explain backwardation. Backwardation is a phenomenon that positive stocks exist when spot price is less than the next to expire futures price. Convenience yield is often motivated by the option value of stoage. For example, the producing firms need to meet a sudden increase in demand to keep consumer's satisfied. Thus they need to keep a certain amount of storage even in backwardation. The convenience yield is greater when stock on hand is smaller. A recent work by Joseph et al. (2011) validate the existence of convenience yield for CBOT corn, soybean and wheat markets using 1990 to 2010 data.

Peterson and Tomek (2005) calibrate convenience yield and embeded it in a rational expectations storage model for U.S. corn market and find that the model generate similar price pattern of actual commodity prices. In another recent research of storage model, Roberts and Tran (2012) generate too little storage comparing to the real world storage levels. One explaination could be that they do not consider the convenience yield. Later, Roberts and Tran (2013) use calibrated negative constant storage cost to represent the existence of convenience yield. In our model we will calibrate a convenience yield similar to Peterson and Tomek's approach.

To solve the model, the expected revenue functions are approximated in order to get the acreage decisions in farmer's problem. Besides that, either storage, expected price or current price needs to be approximated as functions of all state variables. If storage function is approximated, the expected price can be calculated using the known storage rule. If the expected price function is approximated as storage levels of all crops, then the storage levels can be solved using Euler equations.

In solving the model, we use both GSSA and the Smolyak collocation method. Then we need to choose which function to approximate. We are considering storage rule or expected price function approximations. Storage rule approximation takes less time than the expected price function approximation, however, the latter way gives more accurate results. Now we have four candidate approaches: (1) GSSA with storage rule approximation, (2) GSSA with expected price approximation, (3) Smolyak collocation with storage rule approximation and (4) Smolyak collocation with parameterized expected price. GSSA with expected price function is too time-consuming, thus we try the rest three approaches (1), (3) and (4).

Storage levels for all three crops are state variables in GSSA. Expected crop price are approximated as functions of storage levels for all crops. We simulate the model with draws generated from correlated crop yields to get the state space that is visited in equilibrium. The solutions are computed using the simulated points. Monomial integration is used instead of quadrature or Monte Carlo. Monomial rules make the integration possible for many random variables and it is applied for all three methods.

The other two methods use Smolyak collocation methods. One approximates the storage rule while the other approximates expected crop prices. The steps for Smolyak collocation method are: (1) Descretize the continuous state space. Finite points are used to approximate a continuous function. Those chosen finite points are called grid points. (2) Find basis functions and the collocation matrix. The approximated function is constructed by unknown coefficients and basis functions. (3) At each grid point, solve the true values of the approximated functions. (4) Solve for the unknown coefficients. Compared to a tensor grid, use of a Smolyak grid requires fewer supporting points to approximate a multivariate function, thus making it feasible to represent higher dimensional functions.

For Smolyak collocation with storage rule approximation, total supply for each crop is a state variable. Storage rule is approximated as a function of total supply. This is a fixed point approach associated with a sparse grid. The fixed point approach requires only direct calculations and it should require less computational time (Judd et. al (2013)).

For Smolyak collocation with parameterized expected price, all storage levels are state variables. Expected prices are represented as functions of storage. This method requires time iteration which means it approximates future prices and expected revenues and solve the current storage decisions using a numerical solver at each grid point. Thus it takes more computational time compared to a fixed point approach, but expected price approximation is shown to be the most accurate algorithm by Gouel (2013).

To date, rational expectations storage models are usually used to solve with one crop and no resource constraint. Our model with multiple crops and land constraint is more suitable for policy analysis. Because of the curse of dimensionality, PMP supply models have never been combined with rational expectations storage model. With the new methods, we show that it is developed to analyze agricultural supply model in a realistic way that incorporates forward-looking rational agents[1].

In the rest part of the paper, we first introduce the basic model with three crops. The computational approaches are presented and the candidate solution qualities are tested next. At the end, we show some simulation results from the model.

4.2 The Model

The model is a three-crop, rational expectations competitive storage model. Each agents optimization problem is described below:

4.2.1 Farmer

A representative farmer maximizes time t expected profits from planting three crops given a land constraint. Crop yields are realized in t+1. Land is the only input and production technology for each activity is Leontief . The farmer's maximization problem follows Mrel and Bucaram (2010) with some modifications:

[1] The supply side of the model will be further developed to a regional supply model with ten crops in US. The whole US is divided into 10 regions while each region has its own supply elasticities. So the model is calibrated into 10 PMPs. There is a total demand for each crop in the model. Besides the supply side, demand and storage remains the same as the simple model illustrated here. We will simulate the model 5000 times with 5000 sequences of 10 years' crop yields, then we can get crop price distributions for 10 consecutive years. This model will be used for policy analysis similar to the FAPRI model is used to do.

$$\underset{x_{it+1}}{Max}\sum_{i=1}^{3}P_{it}x_{it+1}-(\lambda_{2i}-\gamma_i\overline{x}_i)x_{it+1}-\frac{1}{2}\gamma_i x_{it+1}^2 \tag{4.1}$$

$$s.t.\quad \sum_{i=1}^{3}x_{it+1}\leq A$$

For crop i, x_{it+1} is the planted acreage at time t, $\bar{x}_{i1}$ is the observed acreage level in the base year calibration ($t=0$). Let $y_{it+1}$and p_{it+1} be time $t+1$ yield and price for crop i. The discounted expected revenue at the time of planting for each crop i is $\delta E_t(p_{it+1}y_{it+1})$. Assume that $\delta=\frac{1}{1+r}$ where r is the interest rate. C_{it} is the per acre observed cost. $P_{it}=\delta E_t(p_{it+1}y_{it+1})-C_{it}$ is the gross margin. $\gamma=[\gamma_1,\ \gamma_2,\ \gamma_3]$ is the coefficient vector needed to be calibrated so that the model's elasticities are equal to the exogenously determined elasticities. λ_{2i} is used for exact calibration purposes with $\lambda_{2i}=P_{i0}-\bar{\lambda}$ where $\bar{\lambda}$ is the shadow price for the binding land constraint. The value of land rent is suggested to be used for $\bar{\lambda}$ (Gohin and Chantreuil (1999)). The constraint says that the total acreage for three crops is not greater than A where $A=\sum_{i=1}^{3}\overline{x}_{i1}$. In each period, it is assumed that the representative farmer faces the same land constraint.

Instead of using the land supply elasticity with respect to price (Mrel and Bucaram (2010)), we calibrate the model to land supply elasticity with respect to per-acre expected revenue. Let $\bar{\eta}_i$ for $i=1,2,3$ be the expected revenue elasticities in the base year $t=0$, we have

$$\bar{\eta}_i=\frac{dx_{i1}}{dE_0(p_{i1}y_{i1})}\frac{E_0(p_{i1}y_{i1})}{\overline{x}_{i1}} \tag{4.2}$$

Using (4.2), the acreage response to per acre gross margin is

$$\frac{dx_{i1}}{dP_{i0}}=\frac{dx_{i1}}{d(\delta E_0(p_{i1}y_{i1})-C_{it})}=\frac{1}{\delta}\frac{\bar{x}_{i1}\bar{\eta}_i}{E_0(p_{i1}y_{i1})} \tag{4.3}$$

$\bar{\eta}_i$ and $\bar{x}_{i1}$ are known. If base year revenue for each crop $E_0(p_{i1}y_{i1})$ is known, we can get the acreage response and calibrate the unknown parameters γ in the cost function.

Solving the farmer's constrained optimization problem[2],

[2]The detailed procedure can be found in Mrel and Bucaram (2010) page 399-402.

$$\frac{dx_{i1}}{dP_{i0}} = \frac{1}{\gamma_i}(1 - \frac{\partial \lambda_1}{\partial P_i}) = \frac{1}{\gamma_i}(1 - (\sum_{i=1}^{3}\frac{1}{\gamma_i})^{-1}\frac{1}{\gamma_i}),\ i = 1, 2, 3 \tag{4.4}$$

Where λ_1 is the Lagrange multiplier associated with the land constraint. Let $w_i = \frac{dx_{i1}}{dP_{i0}}$, if $w_i < \sum_{j \neq i} w_j$, then we can get positive values of γ_i, $i = 1, 2, 3$ by solving three unknowns from three equations (4.4). This condition requires at least three crops in the calibration system and the response of one crop should not be greater than the sum of responses of the other crops.

Note that another assumption here for getting the γ is that the base year expected revenues are known. In the later algorithm section, we will approximate the expected revenue for each crop as function of state variables in order to solve farmer's problem in each period. If the base year $E_0(p_{i1}y_{i1})$ is pre-determined, it may not be the same as what is implied by the model. Thus we treat it as endogenous and approximate it in each iteration using the approximated value. More details will be provided in the algorithm section.

4.2.2 Storer

A representative storer maximizes his/her profit from storing crops. At time t, the revenue from storing is the expected crop price in time $t+1$. The cost of storing is storage cost plus the opportunity cost from not selling the crop in time t. Equilibrium storage satisfies the following no-arbitrage condition for crop i

$$\delta E_t(p_{it+1}) - p_{it} - SC_{it} = 0\ i = 1, 2, 3 \tag{4.5}$$

where $E_t(p_{it+1})$ is the crop i's expected price, per bushel storage cost of crop i is denoted by SC_{it}. The marginal storage cost includes marginal convenience yield which goes to negative infinity when stock level approaches zero. This specification of marginal storage cost will eliminate stock-out conditions.

4.2.3 Consumer

A represented consumer maximizes his/her utility by consuming three crops and a numeraire good m_t given a budget constraint. Assume the utility function is separable. The price for crop i is denoted by p_{it} and total income is denoted by I.

$$\underset{\{c_{it}\}_{i=1,2,3}}{Max} \sum_{t=1}^{\infty} (\sum_{i=1}^{3} U_i(c_{it}) + m_t) \tag{4.6}$$

$$\sum_{i=1}^{3} p_{it} c_{it} + m_t = I$$

The optimization condition gives us the inverse demand function for each crop i,

$$p_{it} = U_i^{'}(c_{it}) = D_i^{-1}(c_{it}) \tag{4.7}$$

4.2.4 Equilibrium Condition

In each period, total supply is the sum of total production in time t and carryover stocks. $TS_{it} = h_i x_{it} y_{it} + s_{it}$, where h_i is the harvest rate for crop i. Total supply is then consumed in time t or stored for future use,

$$TS_{it} = c_{it} + s_{it+1},\ i = 1, 2, 3 \tag{4.8}$$

4.3 The Algorithms

Monomial integration is used to descretize multi-normal yield distribution in all algorithms. Formulas of monomial rule used in the paper is described as the second formula in supplementary material to Judd et. al (2011). We used code provided by Judd (2011) to generate monomial nodes and weights. If there are three crops, the total number of nodes There are $3^3 + 1 = 19$. Monomial nodes are denoted by a $N \times 3$ matrix $[y_1, y_2, y_3]$, where y_i is an $N \times 1$ vector representing the all monomial nodes for crop i. w is the weight vector where jth element is the probability for $[y_1(j), y_2(j), y_3(j)]$.

4.3.1 Method 1. GSSA With Storage Rule Approximation

GSSA is an algorithm developed by Judd et al. (2011) the matlab code can be found online. The model requires solving (4.1), (4.5), (4.7), (4.8). We approximate the storage rules and expected revenues for crop i as functions of all state variables $s_{it} = f_i(TS_{1t}, TS_{2t}, TS_{3t})$ and $E_t(p_{it+1}y_{it+1}) = g_i(TS_{1t}, TS_{2t}, TS_{3t})$, respectively. Flexible functional forms $\psi_i(TS_{1t}, TS_{2t}, TS_{3t}, a_i)$, $\phi_i(TS_{1t}, TS_{2t}, TS_{3t}, b_i)$ and vectors of coefficients a_i and b_i for $i = 1, 2, 3$ are chosen such that $f_i(TS_{1t}, TS_{2t}, TS_{3t}) \approx \psi_i(TS_{1t}, TS_{2t}, TS_{3t}, a_i)$ and $g_i(TS_{1t}, TS_{2t}, TS_{3t}) \approx \phi_i(TS_{1t}, TS_{2t}, TS_{3t}, b_i)$. The detailed steps are as follows:

Initialization: Choose initial guesses $a_i^{(1)}$, $b_i^{(1)}$, $i = 1, 2, 3$. Choose the initial state $(TS_{10}, TS_{20}, TS_{30})$ for simulations. Choose a simulation length T, draw a sequence of crops yields $\{y_{it}\}_{t=1,\ldots,T}, i = 1, 2, 3$. The steps for generating correlated crop yields are from (i) to (iv).

(i) Get the variance and covariance matrix for 3 crops and denote it as M. Let L be the Cholesky decomposition of M.

(ii) Generate a $T \times 1$ vector of random normal deviates for three yields independently. Each vector is denoted by z_1, z_2 and z_3.

(iii) Impose the correlation by Cholesky decomposition matrix. So $[z_1, z_2, z_3] \times L$.

(iv) Impose the real mean yields. y_i is the ith column of $[z_1, z_2, z_3] \times L$ plus mean yield of crop i.

Step 1. At iteration p, use $\left\{a_i^{(p)}\right\}_{i=1,2,3}$, $\left\{b_i^{(p)}\right\}_{i=1,2,3}$, calibrate for γ and simulate the the model T periods forward.

(1i) Calibration for γ.

The base year total supplies for corn, soybean and all the others are TS_{10}, TS_{20}, TS_{30}, respectively.

Expected revenue at base year is $\phi_i(TS_{10}, TS_{20}, TS_{30}; b_i^{(p)})$, $i = 1, 2, 3$.

Acreage response for crop i at base year is: $\frac{dx_{i1}}{dP_{i0}} = \frac{1}{\delta}\frac{\bar{x}_{i1}\bar{\eta}_i}{E_0(p_{i1}y_{i1})} = \frac{1}{\delta}\frac{\bar{x}_{i1}\bar{\eta}_i}{\phi_i(TS_0^1, TS_0^2, TS_0^3; b_i^{(p)})}$. Then solve for γ using systems of equations (4.4). As $b_i^{(p)}$ converges, the expected revenue at the point of base year total supplies converges and so does γ. In this way γ is then calibrated in the whole algorithm. If expected revenues in the base year are fixed and γ is fixed at the beginning of all iterations, then expected revenue is not the same as one implied by the model.

(1ii) When expected revenues are known, solve the farmer's constrained optimization problem (4.1) to get x_{it+1}, $i = 1, 2, 3$.

(1iii) Total supplies, stock levels and expected revenues in $t + 1$ are:

$$TS_{it+1} = \psi_i(TS_{1t}, TS_{2t}, TS_{3t}; a_i^{(p)}) + h_i x_{it+1} y_{it+1} \tag{4.9}$$

$$s_{it+1} = \psi_i(TS_{1t+1}, TS_{2t+1}, TS_{3t+1}; a_i^{(p)}) \tag{4.10}$$

$$E_{t+1}^{(p)}(p_{it+2} y_{it+2}) = \phi_i(TS_{1t+1}, TS_{2t+1}, TS_{3t+1}, b_i^{(p)}),\ i = 1, 2, 3 \tag{4.11}$$

The model then can be simulate T periods forward using (4.9), (4.10), (4.11).

Step 2. The storage can be approximated as:

$$z_{it}^s = TS_{it} - D_i(\delta E_t(P_{it+1}) - SC_{it}) \tag{4.12}$$

Expected revenue for crop i in time t can be approximated as (4.13) using monomial nodes and weights:

$$z_{it}^{epy} = \sum_{j=1}^{N} w(j) D_i^{-1}(TS_{it+1,j} - s_{it+1,j}) y_i(j) \tag{4.13}$$

Where the next period total supply vector for all yield nodes for crop i , the next period storage rule for each crop i at each monomial node j and Expected price for crop i in time t are defined by

$TS_{it+1,j} = s_{it} + h_{it} x_{it+1} y_i(j)$

$s_{it+1,j} = \psi_i(TS_{1t+1,j}, TS_{2t+1,j}, TS_{3t+1,j}, a_i^{(p)})$

$E_t(p_{it+1}) = \sum_{j=1}^{N} w(j) D_i^{-1}(TS_{it+1,j} - s_{it+1,j})$

Step 3. Find $\{\hat{a}_i\}_{i=1,2,3}$ and $\{\hat{b}_i\}_{i=1,2,3}$ that minimize the errors ϵ_{it}, ζ_{it} in the regression equation using LAD method as described in Judd et. al (2011).

$$z_{it}^{s} = \psi_i(TS_{1t}, TS_{2t}, TS_{3t}, a_i^{(p)}) + \epsilon_{it},\ i = 1, 2, 3 \tag{4.14}$$

$$z_{it}^{epy} = \phi_i(TS_{1t}, TS_{2t}, TS_{3t}, b_i^{(p)}) + \zeta_{it},\ i = 1, 2, 3 \tag{4.15}$$

Step 4. Check the convergence and end (2) if

$$\frac{1}{T}\sum_{1}^{T}\sum_{i=1}^{3}\left(\left|\ \frac{s_{it}^{(p)} - s_{it}^{(p-1)}}{s_{it}^{(p-1)}}\ \right| + \left|\ \frac{E_t^{(p)}(p_{it}y_{it}) - E_t^{(p-1)}(p_{it}y_{it})}{E_t^{(p-1)}(p_{it}y_{it})}\ \right|\right) < \varepsilon \tag{4.16}$$

where $s_{it}^{(p)}$,$s_{it}^{(p-1)}$, $E_t^{(p)}(p_{it}y_{it})$, $E_t^{(p-1)}(p_{it}y_{it})$ are the storage and expected revenue series obtained on iteration p and $p-1$.

Step 5. Compute $a_i^{(p+1)}$ and $b_i^{(p+1)}$ for iteration $(p+1)$ for $i = 1, 2, 3$.

$$a_i^{(p+1)} = (1 - \xi)a_i^{(p)} + \xi\widehat{a}_i \tag{4.17}$$

$$b_i^{(p+1)} = (1 - \xi)b_i^{(p)} + \xi\widehat{b}_i \tag{4.18}$$

where $\xi \in (0, 1]$ is a damping parameter. Go to (2) with new coefficients $a_i^{(p+1)}$, $b_i^{(p+1)}$ and stop the iteration until convergence criterion is reached.

4.3.2 Smolyak Collocation Method

The Smolyak method was found by Smolyak (1963) to approximate multivariate functions. Compared to the use of standard tensor grids, Smolyak grids require fewer support nodes. For example, if we want to approximate an N dimensional function. The number of tensor nodes is 5^N if using 5 points for each dimension. When $N = 10$, the total number of collocation nodes equals 9,765,625, in which case we will face the curse of dimensionality. Smolyak nodes are constructed by the levels of approximation. Higher approximation level leads to higher accuracy. For 10 dimension with 2nd, 3rd or 4th level approximation, the Smolyak grid requires 221 points, 1581 points and 8801 points respectively. Thus, the Smolyak method makes it feasible to solve high dimensional models.

4.3.3 Method 2 Smolyak Collocation with Storage Rule Approximation

Step 1: Define the interval for state variables. Let the total supply of crop i lies in $[TS^i_{min}, TS^i_{max}]$, $i = 1, 2, 3$. The minimum and maximum total supply values should not be violated in iterations.

At each Smolyak grid point $(TS_{1k}, TS_{2k}, TS_{3k})$, $k = 1, ...K$, guess $K \times 1$ coefficient vectors $a_i = [a_{i1}, ..., a_{iK}]$ and $b_i = [b_{i1}, ..., b_{iK}]$, $i = 1, 2, 3$. Storage levels are approximated as $s_{ik} = f^{d,\mu}(TS_{1k}, TS_{2k}, TS_{3k}; a_i)$, $i = 1, 2, 3$, expected revenues are approximated as $epy_{ik} = f^{d,\mu}(TS_{1k}, TS_{2k}, TS_{3k}; b_i)$, $i = 1, 2, 3$. $f^{d,\mu}$ will be defined later by (4.23). d is the number of dimensions and μ is the approximation level.

Equidistant grid points performs worse than Chebyshev-based nodes for interpolation, therefore Chebyshev-Gauss-Lobatto grid is used as suggested in Klimke (2006). Chebyshev-Gauss-Lobatto grid is a kind of sparse grid using extrema of Chebyshev polynomials. The details of forming uni-dimension Chebyshev-Gauss-Lobatto grid can be found in Judd et al. (2013) appendix A. We use the sparse grid interpolation toolbox developed by Andreas Klimke (2007) to obtain the Chebyshev-Gauss-Lobatto points.

To see how the Smolyak grids are constructed. We first show the nodes constructed in unidimentional. Then we show a special case used in our algorithm, three dimensional with three approximation level.

The part of forming grid points and basis functions follows Malin et. al (2007) and Judd et al. (2013), the set of grid point X_i is defined as the set of the extrema of the Chebyshev polynomials with a number of $m(i)$ points in each set. $m(i) = 2^{i-1} + 1$ when $i \geqslant 2$ and $m(1) = 1$.

The formula for extrema of the Chebychev polynomials is $X_i = -cos(\frac{\pi(i-1)}{m(i)-1})$ $i = 1, 2, ..., m(i)$.

The set for extrema of the Chebyshev polynomials are:

$i = 1,\ X_1 = \{0\}$;

$i = 2,\ X_2 = \{-1, 0, 1\}$;

$i = 3,\ X_3 = \{-1, -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 1\}$;

when $i = 5$, there are 17 points in the set, the set of grid points $X_5 = -cos(\frac{\pi(i-1)}{17-1}),\ i = 1, 2, ..., 17$.

From the construction, we can see that X_i is a subset of X_j when $j > i$.

For higher dimensions, the Chebyshev-Gauss-Lobatto grid is formed as follows:

In the three dimensional case, we must select tensor products of points selected from unidimension according to

$d \leqslant i_1 + i_2 + i_3 \leqslant d + \mu$.

For example, in the three dimensional case, $d = 3$.

If $\mu = 1$, $3 \leqslant i_1+i_2+i_3 \leqslant 4$. Thus the sets for $\{(i_1, i_2, i_3)\} = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1)\}$.

Given a value of i_1, i_2, i_3, find the set of extrema of the Chebychev polynomials (X_i) in each dimension with $i = i_1$, $i = i_2$, $i = i_3$.

When $i_1 = 1$, $i_2 = 1$, $i_3 = 1$, then the tensor product of the unidimentional nodes of each dimension is $\{(0, 0, 0)\}$.

When $i_1 = 1, i_2 = 1, i_3 = 2$, we have X_2 for the third dimension and X_1 for the other dimensions. The tensor product of the points are $\{(0, 0, 0), (0, 0, 1), (0, 0, -1)\}$.

Doing this for all $\{i_1, i_2, i_3\}$, we have seven points in the first level approximation are

$\{(0, 0, 0), (0, 0, 1), (0, 0, -1), (0, 1, 0), (0, -1, 0), (1, 0, 0), (-1, 0, 0)\}$.

If $\mu = 2$, $3 \leqslant i_1+i_2+i_3 \leqslant 5$, There are several combinations of i_1, i_2, i_3 that satisfy this restriction: $\{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)\}$.

Thus we have 25 Smolyak grid points,

$$\{(0, 0, 0), (0, 0, 1), (0, 0, -1), (0, 1, 0), (0, -1, 0), (1, 0, 0), (-1, 0, 0), (0, 0, -\frac{1}{\sqrt{2}}), (0, 0, \frac{1}{\sqrt{2}}),$$
$$(0, -\frac{1}{\sqrt{2}}, 0), (0, \frac{1}{\sqrt{2}}, 0), (-\frac{1}{\sqrt{2}}, 0, 0), (\frac{1}{\sqrt{2}}, 0, 0), (0, -1, -1), (0, -1, 1), (0, 1, -1), (0, 1, 1),$$
$$(-1, 0, -1), (-1, 0, 0), (1, 0, -1), (1, 0, 1), (-1, -1, 0), (-1, 1, 0), (1, -1, 0), (1, 1, 0)\}$$

If $\mu = 3$, $3 \leqslant i_1 + i_2 + i_3 \leqslant 6$. The i_1, i_2, i_3 satisfy for the restriction are

$$\{(i_1, i_2, i_3)\} = \{(1,1,1),(1,1,2),(1,2,1),(2,1,1),(1,1,3),(1,3,1),(3,1,1),(1,2,2),$$
$$(2,1,2),(2,2,1),(1,1,4),(1,4,1),(4,1,1),(1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,2,1),(3,1,2)\}.$$

and there are 69 Smolyak grid points.

Step 2: Construct Smolyak Chebyshev basis functions. Here we only provide the details for 3 dimensions (d) with 3rd approximation level (μ) . The Smolyak polynomial function is given by a general form:

$$f^{d,\mu}(TS_{1k},...,TS_{nk};a) = \sum_{max(d,\mu+1)\leq|i|\leq d+\mu} (-1)^{d+\mu-|i|}\begin{pmatrix} d-1 \\ d+\mu-|i| \end{pmatrix} p^{|i|}(TS_{1k},...,TS_{nk}) \tag{4.19}$$

$(-1)^{d+\mu-|i|}\begin{pmatrix} d-1 \\ d+\mu-|i| \end{pmatrix} = (-1)^{d+\mu-|i|}\frac{(d+\mu-|i|)!}{(d-1)!(\mu-|i|+1)!}$ is a counting coefficient to insure that there are no repeated basis functions. n is the number of state variables. A tensor product operator $p^{|i|}(TS_{1k},...,TS_{nk})$ is defined as

$$p^{|i|}(TS_{1k},...,TS_{nk}) = \sum_{i_1+...+i_d=|i|} p^{i_1,...,i_d}(TS_{1k},...,TS_{nk}) \ ,$$

where $p^{i_1,...,i_d}$ is defined as

$$p^{i_1,...,i_d}(TS_{1k},...,TS_{nk}) = \sum_{l_1=1}^{m(i_1)} ... \sum_{l_d=1}^{m(i_d)} a_{l_1...l_d}\psi_{l_1}(TS_{1k})...\psi_{l_d}(TS_{nk}) \ ,$$

where $m(i_j) = 2^{i_j-1}+1$, $i_j \geq 2$, $m(1) = 1$. In our example we have $d = 3$, $\mu = 3$, $n = 3$. Counting factor is $(-1)^{3+3-|i|}\begin{pmatrix} 3-1 \\ 3+3-|i| \end{pmatrix}$, $|i| = 4,5,6$, $a_{l_1...l_d}$ are the coefficients. We have

$$p^4 = p^{1,1,2} + p^{1,2,1} + p^{2,1,1} \tag{4.20}$$

$$p^5 = p^{1,1,3} + p^{1,3,1} + p^{3,1,1} + p^{1,2,2} + p^{2,2,1} + p^{2,1,2} \tag{4.21}$$

$$p^6 = p^{1,1,4} + p^{1,4,1} + p^{4,1,1} + p^{2,1,3} + p^{2,2,2} + p^{3,1,2} \tag{4.22}$$

let $c_{j_1j_2j_3}$ represents $a_{l_1l_2l_3}\psi_{l_1}(TS_{1k})\psi_{l_2}(TS_{2k})\psi_{l_3}(TS_{3k})$. $\psi_{l_1}(TS_{1k})\psi_{l_2}(TS_{2k})\psi_{l_3}(TS_{3k})$ is a basis function where $\psi(\cdot)$ is Chebyshev polynomial basis function.

The Smolyak polynomial function is:

$$\begin{aligned} f^{3,3}(TS_{1k}, TS_{2k}, TS_{3k}; a) = {} & c_{111} + c_{112} + c_{113} + c_{114} + c_{115} + c_{116} + c_{117} + b_{118} + c_{119} + c_{121} \\ & + c_{131} + c_{141} + c_{151} + c_{161} + c_{171} + c_{181} + c_{191} + c_{211} + c_{311} + b_{411} \\ & + c_{511} + c_{611} + c_{711} + c_{811} + c_{911} + c_{122} + c_{123} + c_{124} + c_{125} + b_{132} \\ & + c_{133} + c_{134} + c_{135} + c_{142} + c_{143} + c_{152} + c_{153} + c_{212} + c_{213} + b_{214} \qquad (4.23) \\ & + c_{215} + c_{312} + c_{313} + c_{314} + c_{315} + c_{412} + c_{413} + c_{521} + c_{531} + b_{221} \\ & + c_{231} + c_{241} + c_{251} + c_{321} + c_{331} + c_{341} + c_{351} + b_{421} + c_{431} + b_{521} \\ & + c_{531} + c_{222} + c_{223} + c_{232} + c_{322} + c_{333} + c_{332} + c_{333} + c_{323} + c_{323} + c_{233} \qquad (4.24) \end{aligned}$$

The set of Chebyshev polynomial basis functions are defined recursively as follows: $\psi_1(x) = 1$, $\psi_2(x) = x$, $\psi_n(x) = 2x\psi_{n-1}(x) - \psi_{n-2}(x)$. The Chebyshev matrix is denoted by Φ. In this case Φ is a 69×69 matrix.

Approximate storage and expected revenues for each crop i as follows:

$s_{ik} = f^{d,\mu}(TS_{1k}, ..., TS_{nk}; a_i)$.

$epy_{ik} = f^{d,\mu}(TS_{1k}, ..., TS_{nk}; b_i)$, where $f^{d,\mu}$ is defined as equation (4.19).

Step 3. At iteration p, use $\left\{a_i^{(p)}\right\}_{i=1,2,3}$, $\left\{b_i^{(p)}\right\}_{i=1,2,3}$. Calibrate for γ.

(3i) The base year total supplies for corn, soybean and all the others are TS_{10}, TS_{20}, TS_{30}. All Chebyshev-Gauss-Lobatto grid points are normalized to $[-1, 1]$, so we need to normalize total supplies in the base year before constructing the base year expected revenues.

$TS_0^1 = \frac{2(TS_{10} - TS_{min}^1)}{(TS_{max}^1 - TS_{min}^1)} - 1$, $TS_0^2 = \frac{2(TS_{20} - TS_{min}^2)}{(TS_{max}^2 - TS_{min}^2)} - 1$, $TS_0^3 = \frac{2(TS_{30} - TS_{min}^3)}{(TS_{max}^3 - TS_{min}^3)} - 1$.

(3ii) Expected revenues at base year are $f^{3,3}(TS_0^1, TS_0^2, TS_0^3; b_i^{(0)})$, $i = 1, 2, 3$.

Acreage responses at base year are: $\frac{dx_{i1}}{dP_{i0}} = \frac{1}{\delta}\frac{\bar{x}_{i1}\overline{\eta}_i}{E_0(p_{i1}y_{i1})} = \frac{1}{\delta}\frac{\bar{x}_{i1}\overline{\eta}_i}{f^{3,3}(TS_0^1, TS_0^2, TS_0^3; b_i^{(0)})}$, $i = 1, 2, 3$. Then solve γ using (4.4).

Step 4. At each grid point k, solve the optimal acreage decisions x_{ik}, $i = 1, 2, 3$, solve storage and expected revenue at each grid point using monomial integration of expectations. The expected revenues are

$$z_{ik}^{epy} = \sum_{j=1}^{N} w(j) D_i^{-1}(TS_{ikj} - f^{3,\mu}(TS_{1kj}, TS_{2kj}, TS_{3kj}; a_i^{(p)})) y_i(j), \; i = 1, 2, 3, \; k = 1, 2, ...K. \tag{4.25}$$

The storage levels are

$$z_{ik}^{s} = TS_{ik} - D_i(\delta ep_{ik} - SC_{ik}), \; i = 1, 2, 3, \; k = 1, 2, ..., K. \tag{4.26}$$

where the expected prices are

$$ep_{ik} = \sum_{j=1}^{N} w(j) D_i^{-1}(TS_{ikj} - f^{3,\mu}(TS_{1kj}, TS_{2kj}, TS_{3kj}; a_i^{(p)})), \; l = 1, 2, 3, \; i = 1, 2, 3. \tag{4.27}$$

The next period total supply for crop i with yield $y_i(j)$ is $TS_{ikj}^{n} = f^{3,\mu}(TS_{1k}, TS_{2k}, TS_{3k}; a_i^{(p)}) + h_i x_{ik} y_i(j)$. Because we need to transform our nodes which normalized in the interval $[-1, 1]$ to the interval $[TS_{min}^{i}, TS_{max}^{i}]$, each crop i will be $TS_{ikj} = \frac{(TS_{ikj}^{n}+1)}{2}(TS_{max}^{i} - TS_{min}^{i}) + TS_{min}^{i}$

Step 5. See if the approximated storage and expected revenue have converged

$$\frac{1}{K}\sum_{1}^{K}\sum_{i=1}^{3}(\mid \frac{s_{ik}^{(p)} - s_{ik}^{(p-1)}}{s_{ik}^{(p)}} \mid + \mid \frac{epy_{ik}^{(p)} - epy_{ik}^{(p-1)}}{epy_{ik}^{(p-1)}} \mid) < \varepsilon \tag{4.28}$$

Step 6. Update the coefficients if the convergence criterion is not satisfied.

z_i^s is a $K \times 1$ vector with kth element equals to z_{ik}^s. z_i^{epy} is a $K \times 1$ vector with kth element equals to z_{ik}^{epy}.

$$a_i^{(p+1)} = (1 - \xi) a_i^{(p)} + \xi \Phi^{-1} z_i^s, \; i = 1, 2, 3. \tag{4.29}$$

$$b_i^{(p+1)} = (1 - \xi) b_i^{(p)} + \xi \Phi^{-1} z_i^{epy}, \; i = 1, 2, 3. \tag{4.30}$$

Step 7. go to step 3 until the condition in step 5 is satisfied.

4.3.4 Method 3. Smolyak Collocation with Expected Price Approximated

Initialization. storage levels of three crops are state variables. Pick grid points and construct basis functions as described in method 2. The Chebyshev matrix is denoted by Φ.

Step 1. Define the intervals for state variables. The storage levels of three crops is contained in $[0, s^i_{max}]$ for $i = 1, 2, 3$.

At each collocation node (s_{1k}, s_{2k}, s_{3k}), expected prices are approximated as $ep_{ik} = f^{3,\mu}(s_{1k}, s_{2k}, s_{3k}; a_i^{(0)})$, expected revenues are approximated as $epy_{ik} = f^{3,\mu}(s_{1k}, s_{2k}, s_{3k}; b_i^{(0)})$ where $\left\{a_i^{(0)}\right\}_{i=1,2,3}$, $\left\{b_i^{(0)}\right\}_{i=1,2,3}$ are coefficient vectors with initial guesses.

Step 2. At iteration p, use $a_i^{(p)}$, $b_i^{(p)}$. Calibrate γ. The base year observed storage levels for corn, soybean and all the others are s_{10}, s_{20}, s_{30}. Normalize storage levels to $[-1, 1]$.

$s^1_0 = \frac{2s_{10}}{s^1_{max}} - 1$, $s^2_0 = \frac{2s_{20}}{s^2_{max}} - 1$, $s^3_0 = \frac{2s_{30}}{s^3_{max}} - 1$.

Expected revenues at base year are $f^{3,3}(s^1_0, s^2_0, s^3_0; b_i^{(0)})$, $i = 1, 2, 3$.

Acreage responses at base year are: $\frac{dx_{i1}}{dP_{i0}} = \frac{1}{\delta}\frac{\bar{x}_{i1}\overline{\eta}_i}{E_0(p_{i1}y_{i1})} = \frac{1}{\delta}\frac{\bar{x}_{i1}\overline{\eta}_i}{f^{3,3}(s^1_0, s^2_0, s^3_0; b_i^{(0)})}$, $i = 1, 2, 3$. Then solve γ using systems of equations (4.4).

Step 3. For each collocation node, solve the optimal acreage decision x_{ik}, $i = 1, 2, 3$.

Solve the storage decision at each grid point and each monomial node, where s_{ikj} is the storage decision solved from the non-arbitrage condition:

$$\delta f^{3,\mu}(s_{1kj}, s_{2kj}, s_{3kj}; a_i) - D_i^{-1}(TS_{ikj} - s_{ikj}) - SC_{ikj} = 0,\ i = 1, 2, 3 \tag{4.31}$$

where total supply for crop i is $TS_{ikj} = s_{ik} + h_i x_{ik} y_i(j)$.

Thus the expected revenues are

$$z^{epy}_{ik} = \sum_{j=1}^{N} w(j) D_i^{-1}(TS_{ikj} - s_{ikj}) y_i(j),\ i = 1, 2, 3, k = 1, 2, ..., K \tag{4.32}$$

The expected prices are

$$z^{ep}_{ik} = \sum_{j=1}^{N} w(j) D_i^{-1}(TS_{ikj} - s_{ikj}),\ i = 1, 2, 3,\ k = 1, 2, ..., K \tag{4.33}$$

Step 5. See if the approximated expected prices and expected revenues converges

$$\frac{1}{K}\sum_{1}^{K}\sum_{i=1}^{3}(\mid \frac{ep_{ik}^{(p)} - ep_{ik}^{(p-1)}}{ep_{ik}^{(p)}} \mid + \mid \frac{epy_{ik}^{(p)} - epy_{ik}^{(p-1)}}{epy_{ik}^{(p-1)}} \mid) < \varepsilon \tag{4.34}$$

Step 6. If not, update the coefficients:

$$a_i^{(p+1)} = (1-\xi)a_i^{(p)} + \xi\Phi^{-1}z_i^{epy},\ i = 1,2,3. \tag{4.35}$$

$$b_i^{(p+1)} = (1-\xi)b_i^{(p)} + \xi\Phi^{-1}z_{ik}^{ep},\ i = 1,2,3. \tag{4.36}$$

z_i^{epy} is a $K \times 1$ vector with $z_i^{epy}(k) = z_{ik}^{epy}$, z_i^{ep} is a $K \times 1$ vector with $z_i^{ep}(k) = z_{ik}^{ep}$.

step 6. go to step 2 until the condition in step 5 is satisfied.

4.4 Calibration for the Model

The model is calibrated to corn, soybeans and "others" which include wheat and cotton for simplicity in this example.

4.4.1 Land Allocation

We calibrate the land allocation problem to exogenous elasticities and endogenous expected revenue so that the optimal allocation is the same as what was projected to happen in 2013/14 according to WASDE 2013/14 January report. The acreages allocated to corn, soybean and other in the base year are 95.4 million acres, 74.5 million acres and 66.61 million acres, respectively. Thus, total acreage is 238.51 million acres for all time periods. The exogenous supply elasticities for corn, soybean and all others are assumed to be 0.25, 0.2, 0.2 as illustration purposes. Land rent is assumed to be \$200 per acre, we'll have $\bar{\lambda} = 200$.

4.4.2 Yield Distributions

National crop yields from 1970 to 2013 given by USDA NASS are employed to get the yield distributions. All crop yield distributions are assumed to be normal for illustration of the approach. Detrended data for each crop is used separately to fit a normal distribution with mean μ and standard deviation σ, $N(\mu, \sigma^2)$, we have corn yield in bushel per acre, $y_1 \sim N(157, 15.4^2)$, soybean yield in bushel per acre, $y_2 \sim N(43.6, 3.18^2)$. The yield data for other in each year is the average yield of wheat and cotton weighted by output. Thus yield distribution for others is $y_3 \sim N(1.1801, 0.0818^2)$ with tons per acre as the unit. The correlation between crop yield variables are also calculated using the same detrended crop yield data. The covariance between crop i and crop j is estimated as

$$COV_{ij} = \frac{1}{N}\sum_{t=1}^{N}(y_{it} - \overline{y}_i)(y_{jt} - \overline{y}_j) \tag{4.37}$$

where N is the total number of observations, y_{it} is the detrended yield for crop i at year t and $\bar{y}_i$ is the average yield for crop i for all observations. The covariance between crops are $COV_{12} = 21.34$, $COV_{13} = 0.20704$, $COV_{23} = -0.02897$. With the above information, we can construct the variance covariance matrix for the multivariate normal distribution for all three crops. Using Cholesky decomposition, the variance covariance structure is imposed on the simulated yields.

4.4.3 Demand Functions

Demand functions are assumed to be constant elasticity:

$$D^{-1}(\cdot) = \alpha_{1i} c_i^{-\alpha_{2i}}, \; i = 1, 2, 3 \tag{4.38}$$

The demand parameters are calibrated to the total use and average prices received by farmers for 2013/14. Total use of corn including food, feed, export and ethanol is 13.150 billion bushels. Total use of soybean is 3.3040 billion bushels. Average prices received by farmers for corn and soybean are \$4.4 per bushel and \$12.5 per bushel. Total use and price of wheat and cotton, defined as average of wheat and cotton weighted by output, are 68.36 million tonnes and \$312.4 /ton. In our program, we set the units for quantities of corn and soybean to be 10 billion bushels. The units for all the other to be 100 million tonnes. The price units are dollar per bushel for corn and soybean and dollar per tonne for the other crops. Demand elasticities for corn, soybean -0.44 (Adjemian and Smith. (2012)) and -0.236 (Roberts and Schlenker (2013)). The other crop demand elasticity is assumed to be -0.1 as an illustration purpose. Thus we have $\alpha_{11} = 8.192456$, $\alpha_{21} = 2.27$, $\alpha_{12} = 0.114537$, $\alpha_{22} = 4.237288$, $\alpha_{13} = 0.069614$, $\alpha_{23} = 10$.

4.4.4 Storage Cost and Marginal Convenience Yield

The storage cost per unit includes per unit observed cost and per unit unobserved cost. The observed part of the storage cost (OSC) is a constant physical storage cost paid by the storer. We assume the observed per bushel storage cost is 3 cents per bushel per month (Peterson and Tomek (2010)). The yearly observed storage cost is thus \$0.36 per bushel, $OSC = 0.36$. One component of unobserved storage cost is the opportunity cost that increases with stock level when stock levels are large. This is because holding more stock of one crop decreases the opportunity of holding other more profitable crops (Paul (1970)). The other unobserved storage cost is the marginal convenience yield. We can get unobserved storage cost from the storage non-arbitrage condition: $USC_{it} = \delta E_t(p_{it+1}) - p_{it} - OSC_{it}$. We collect our data including current price p_t, expected price p_{t+1} from 2001/2002 to 2011/2012. Use average price received by the farmer in each marketing year from USDA NASS for the current year price. For corn, the December corn futures price from September 1st to August 31st is used as the average of yearly expected price. The discount factor, δ, is defined as $\frac{1}{1+r}$ where r is the interest rate. We use the return for 1-year treasury constant maturities as risk free interest rate. The unobserved storage cost should be increasing with stock level and it is negative when stock level s_t is small and positive when stock level is high. Rui and Miranda (1995) uses a logarithmic function to achieve it. In Tomek and Peterson (2005), they incorporate expected total supply in marginal convenience yield function and successfully avoid stockout condition using that specification. Because current price indicates relative shortage of the crop (Na jin(2013)), we assume the unobserved cost is also increasing in price when it is positive, $UOC_{it} = p_{it}log(a_i + b_i s_{it+1})$ where a_i and b_i are parameters needed to be calibrated. For each crop i, when stock level is low, $log(a_i + b_i s_{it+1})$ is negative, current price is positive, unobserved storage cost is negative. When stock level is large, the unobserved storage cost is positive.

For soybean and the other crops, the soybean expected price is the average of November Soybean futures from September 1st to August 31st. We use wheat unobserved cost to represent the expected price of other crops. Wheat expected price is the average of July wheat futures from June 1st to May 31st.

We use two points to calibrate the parameters a_i and b_i in the unobserved marginal cost function. One point is the unobserved cost/current price and end year stock in 2012/13. The other point is the (average unobserved cost)/price and average ending stock from 2001/02 to 2012/13. Thus we find that $a_i = 0.5229,\ b_i = 0.2772.$ for soybean $a_i = 0.5488,\ b_i = 0.1593$, for all others $a_i = 0.1799,\ b_i = 0.1286$.

4.4.5 Beginning Stock

The base year is set to be 2013/14, beginning stocks for corn and soybean are 0.0821 10 billion bushels and 0.0141 10 billion bushels. The other stock is the sum of wheat stock and cotton stock. Wheat stock is 718 million bushels and cotton stock is 3.9×480 pounds. Then the total stock is 0.203897 100 million tons. The total supply for corn, soybean and all the others are 1.4781 10 billion bushels, 0.3454 10 billion bushels and 0.8550 100 million ton.

4.4.6 Harvest Rate

The harvested acres is less than the acres planted depending on weather conditions. The harvest rate is defined as the ratio of harvested acreage to the planted acreage. The harvest rates for corn, soybean and all other crops are assumed to be 92%, 99% and 85% respectively.

4.5 Accuracy Test for Algorithms 1, 2 &3

The purpose of this part is to subject the candidate solutions to an independent and stringent test and compare the the quality of the solutions among the three algorithms. GSSA and Smolyak collocation method both solving the approximated solutions function forms using a finite set of points. In the accuracy check, we want to see how the candidate solutions perform for other points in the state space.

Euler equation (EE) error developed in Judd (1992) is used to evaluate how accurate the solutions are. The accuracy tests require to check how far the Euler equations for both storage and acreage decisions holds from zero when using the approximated solution functions. This bounded rationality measure allows us to evaluate the one period optimization error or how much resource is wasted using the approximated solution in a unit free form.

Below shows how to conduct an accuracy test for GSSA. Using simulation to generate points in the state space for the test. Total supplies for three crops in 2013/14 are used as the starting point. Construct another set of crop yields $\{y_{i\tau}\}_{\tau=1,\ldots,T^{test}}, i = 1, 2, 3$ from the joint distribution of three crop yields, with the length of period $T^{test} = 10,000$. Using the solved rules for storage and expected prices to simulate a time series of total supply of all crops for 10,000 periods.

EE error is developed from Euler equation for storage

$$TS_{i\tau} - s_{i\tau}(TS_{1\tau}, TS_{2\tau}, TS_{3\tau}) = D_i(\delta E_t(p_{i\tau+1}) - SC_{i\tau}) \tag{4.39}$$

The left hand side of (4.39) is today's consumption given today's storage decision. The right hand side is what today's consumption would be if the represented storer using storage rule in the next period which determines $E_t(p_{i\tau+1})$. The EE error shows how much the storer's deviation from the optimization rule. The Euler Equation error is then defined in a unit free way as shown in (4.40).

$$EE_{si\tau} = 1 - \frac{D_i(\delta E_t(p_{i\tau+1}) - SC_{i\tau})}{TS_{i\tau} - s_{i\tau}(TS_{1\tau}, TS_{2\tau}, TS_{3\tau})} \tag{4.40}$$

The subscript s means EE for storage Euler equation, i denotes a specific crop. log10 of EE is used to show the error. To interpret the error, $EE = -1$ means the consumer makes a 1 dollar mistake in consumption when spending 10 dollars. $EE = -4$ means the consumer makes a 1 dollar mistake when spending 10000 dollars.

We can get EE errors for acreage decision Euler equations in the same way. From the F.O.C. of the farmer's maximization problem, we have

$$x_{i\tau} = \frac{1}{\gamma_i}(\delta E_t(p_{i\tau+1}y_{i\tau+1}) - E_0(p_{i1}y_{i1}) + \overline{\lambda}) + \bar{x}_i - \frac{1}{\gamma_i}\lambda_\tau \tag{4.41}$$

Where λ_τ is the Lagrangian multiplier with land constraint in time τ. As the expected revenues are approximated, for any given total supply, the Euler equation holds. Define the unit free Euler equation error for acreage planted as:

$$EE_{xi\tau} = 1 - \frac{1}{x_{i\tau}}(\frac{1}{\gamma_1}(\delta E_t(p_{i\tau+1}y_{i\tau+1}) - E_0(p_{i1}y_{i1}) + \overline{\lambda}) + \bar{x}_i - \frac{1}{\gamma_i}\lambda_\tau) \tag{4.42}$$

The subscript x means EE for acreage Euler equation. The $log_{10}|EE_H|$ shows the mistake made by making the acreage decision. -1 mean the farmer make 1 acre mistake by planting 10 acres. -4 means the representative profit optimizing farmer make 1 acre mistake when planting 10,000 acres.

For each period τ, compute $EE_{si\tau}$, $EE_{xi\tau}$, $i = 1, 2, 3$. We evaluate the quality of a candidate solution by computing the maximum and mean of the $EE_{si\tau}$ and $EE_{xi\tau}$ for $\tau \in [1,\ T^{test}]$.

GSSA with 700 time series and 3rd order polynomial basis functions takes 944 seconds, however the accuracy is the worst among the three for all EE errors. The maximum EE errors for for corn and soybean are above -2. Because of the low approximation quality, we don't show the EE errors for each for other Euler equation in details. When changing the basis functions to Chebyshev polynomials or increase the basis functions to 4th order polynomials, the maximum Euler equation error across all equations doesn't decrease below -2.

Table 1-2 below show both the maximum errors and mean errors across 10,000 periods for sparse grid with storage approximation and sparse grid with expected price approximation. To read the data from the tables below, -2 means the maximum or mean error is 10^{-2}.

Sparse grid with storage rule approximation and Smolyak grid to 3rd order. Computational time is 236 seconds. Except for soybean storage, all maximum Euler equation error lies below -3 which mean a maximum 1 dollar error for a 1000 dollars consumption. The maximum error in soybean is $10^{-2.95} \approx 0.0011$. Maximum Euler equation error is largest in soybean Euler equations across 10,000 periods indicating that soybean storage rule is more nonlinear comparing to corn and other crop sotrage rules. This can be seen from Figure 1.1, Figure 2.1 and Figure 2.2.

The same approach with 4th level approximation takes 912 seconds and the absolute maximum EE across drops by 0.5 from the 3rd level aproximation. The maximum mean value of EE errors across all Euler euqations is -4.57.

Table 4.1: Euler equation errors for sparse grid with storage rule approximation

			Storage	Euler	Equations	Acreage	Euler	Equations
		EE	EE_c	EE_s	EE_o	EE_{xc}	EE_{xs}	EE_{xo}
μ	3rd level	max	-2.95	-2.88	-3.21	-3.77	-3.70	-3.01
		mean	-4.26	-3.82	-4.20	-4.99	-4.72	-4.57
	4th level	max	-3.70	-3.46	-3.64	-4.66	-4.54	-4.07
		mean	-4.99	-4.85	-4.95	-5.60	-5.69	-5.01

Computational time for sparse grid with expected price approximation is 3747 seconds. The EE errors for storage Euler equation are smaller than those using method 2. However, the EE errors for accreage decisions are greater than that using method 2. Maximum absolute EE error across all Euler equations is -3.28. Parameterized expected price takes more time but performs better than storage rule approximation with the same number of grid points. This findings are the same as Gouel (2013).

Table 4.2: Euler equation errors for sparse grid with expected price approximation

EE	EE_c	EE_s	EE_o	EE_{xc}	EE_{xs}	EE_{xo}
max	-4.02	-4.04	-3.93	-3.84	-3.82	-3.28
mean	-5.21	-5.22	-4.81	-5.21	-5.20	-4.43

4.6 Simulation Results

We use the solution functions got from algorithm 2 with 4th level approximation to simulate the model. Various functions of total supply of corn are shown by Figure 1.1-1.4. In each graph, the dotted lines, solid lines and dashed lines represent functions given low supplies, medium supplies and high supplies of soybean and other crops respectively. Low supplies are defined as $TS_s = 2.7$ billion bushels, $TS_o = 70$ million tons. Medium supplies are defined as $TS_s = 3.454$ billion bushels, $TS_o = 85.59$ million tons. High supplies are defined as $TS_s = 4$ billion bushels, $TS_o = 110$ million tons. The medium supplies for all three crops are set to be the real total supplies in 2013/14 marketing year.

Figure 1.1 shows that stock level rises with higher supply level. For higher supplies of both soybean and other crops, the whole storage curve shifts to the right. To see why, when total supplies for soybean and other crops rise in this period, the acreage decisions for other two crops except corn decrease. As land constraint binds, corn acreage must increase and the corn storage decreases at each total supply level because there is not need to store as much when expected supply increases.

Both the expected price and the acreage level of corn fall with higher corn supply given fixed soybean and other crops levels as described by Figure 1.2 and 1.3 respectively. The reason is that for a certain total corn supply, higher supplies of soybean and other crops result in lower acreage level for these two. In corresponse, corn acreage rises in the binding constraint in Figure 1.3 and thus expected corn price decreases as shown by figure 1.2.

The current corn price decreases with an increasing rate as corn supply rises for fixed values of soybean and other crop total supplies as shown in Figure 1.4. Different total supplies of the other two crops shift little of the current prices.

Figure 1.1 The Relationship Between Corn Stock Level and Corn Total Supply

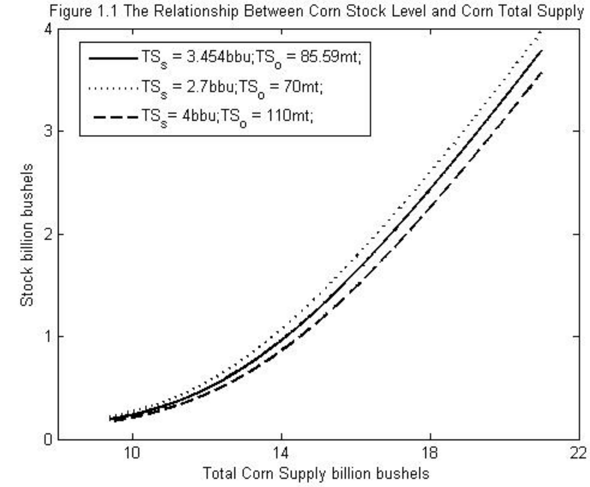


Figure 1.2 The Relationship Between Expected Corn Price and Corn Total Supply

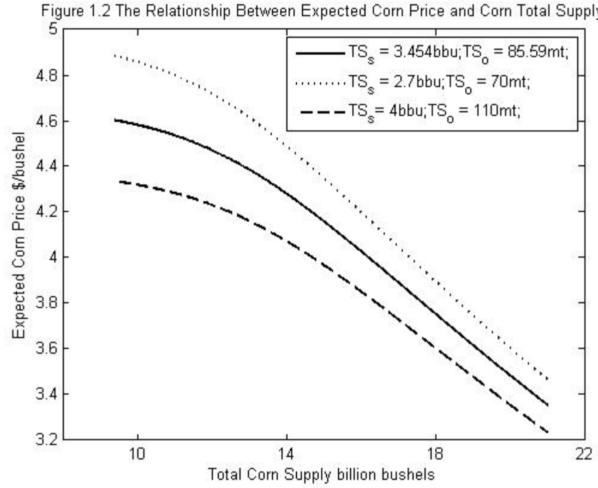


Figure 1.3 The Relationship Between Corn Acreage and Corn Total Supply

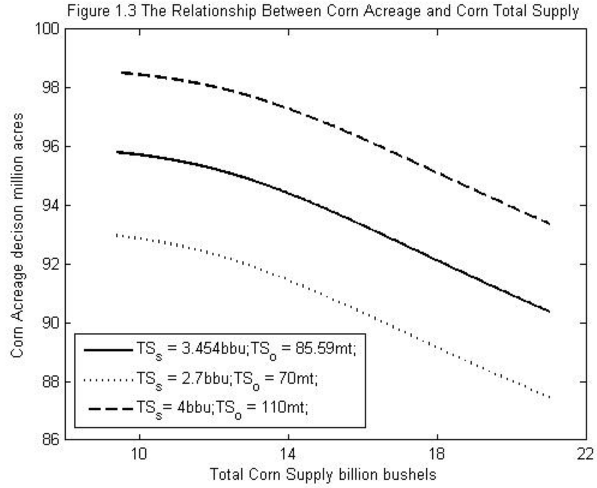


Figure 1.4 The Relationship Between Other Current Corn Price and Corn Total Supply

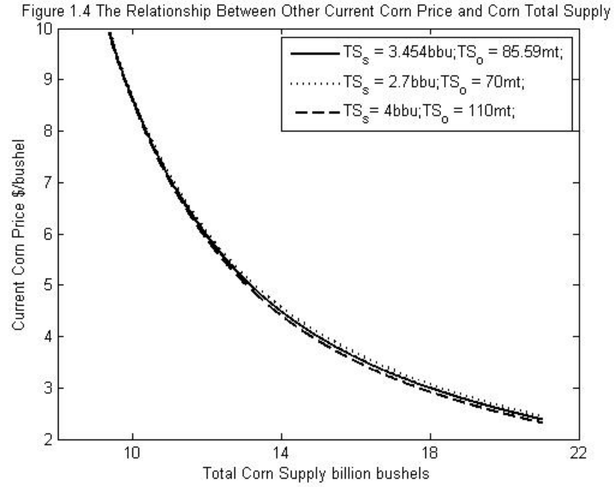


Figure 2.1 The Relationship Between Soybean Stock Level and Soybean Total Supply

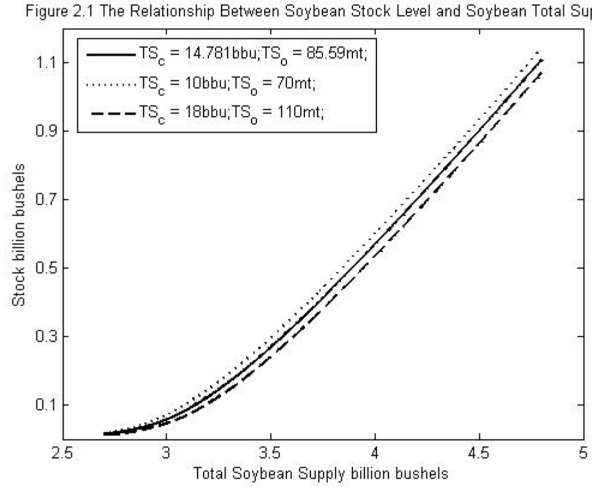
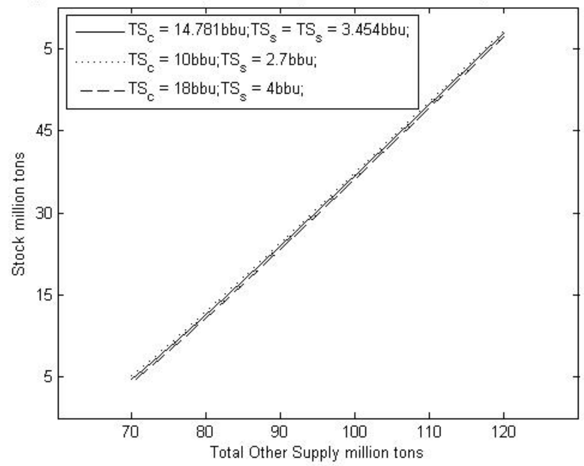


Figure 2.2 The Relationship Between Soybean Stock Level and Soybean Total Supply



In order not to bore the readers, we will only show the storage curve for soybean and all the others as their own total supply rises given low medium and high corn and other supplies in Figures 2.1 and Figure 2.2 using dotted lines, solid lines and dashed lines. Low corn supply is defined as $TS_c = 10$ billion bushels, medium corn supply is defined as $TS_c = 14.781$ billion bushels, high corn supply is defined as $TS_o = 18$ billion bushels. Soybean storage curve looks similar to corn's. Considering the scales of stock levels for soybean and corn in Figure 2.1 and 1.1, the soybean storage function is more nonlinear. The storage curve for all other crops is almost linear in the graph, and the curve does not respond much to different supply levels of the other two crops. The more linearity in the other stock curve leads to higher accuracy in storage rule approximation.

4.6.1 Shocks of Corn Yield

In this part, we show how decisions and prices respond to a high yield, mean yield and low yield in the second period. High corn yield, medium corn yield and low corn yield are defined as 170 bu/acre, 157 bu/acre and 120 bu/acre. As shown in Table 3, the yield shock only happens in the second period in all three periods. Corn yield stays at 157 bu/acre for the rest two periods. Yields of soybean and all others are at the mean levels for all periods. Mean yield values of soybean all other crops are 43.8 bushels per acre and 1.18 tons per acre.

Table 4.3: Corn Yield Shock in 2nd Period

Period	1	2	3
mean	157	157	157
low	157	120	157
high	157	170	157

Figure 3.1-3.4 describe how storage decisions, expected prices, harvested acres and current prices of corn change with yield shock respectively. Total supply in 2nd period rises with an increase in crop yield. Thus the stock level is highest among the three cases as shown in Figure 3.1. Figure 3.2 says that the expected prices rise and fall in the opposite direction with total supply. With bumper crop, expected corn prices fall. As shown by Figure 3.3 and 3.4, the acreage decisions and current prices perform in the same pattern as expected prices. Changes of corn acreage is associated with expected revenue. Corn yield is low, the expected corn price will be higher, then the acreage harvested will be higher.

To see the impacts of yield shock for decisions and prices of crops other than corn, we use soybean as an example and the results are shown by Figure 4.1-4.4. Figure 4.1 depicts that all else being equal, if the corn supply is relatively higher, soybean stock levels will be lower. To explain it, considering an increase in corn supply due to the yield shock, the expected revenue decreases and hence the corn acreage for next year decreases. The decrease in the corn acreage will lead to an increase in soybean acreage (Figure 4.3), implying a fall in expected prices of soybean (Figure 4.2) and the level of stock levels of the two stocks (Figure 4.1). The total supplies of soybean are the same in 2nd period with the same soybean yields and acreage levels. Thus current prices of soybean are only determined by the storage decisions. As we can see from Figure 4.4, higher carryover stock leads to higher current soybean prices while lower carryover stock result in lower current soybean price.

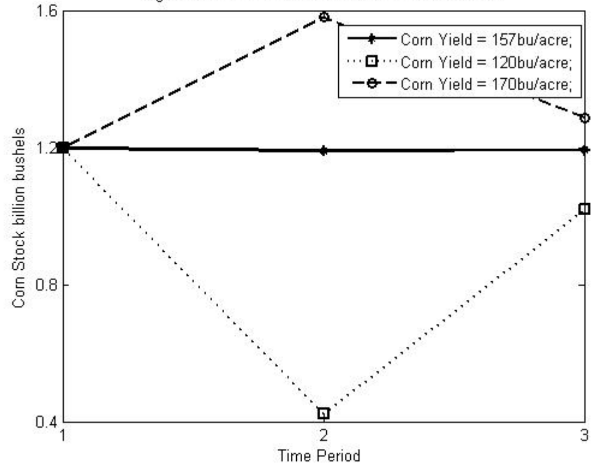


Figure 3.1 Corn Stock Levels with Corn Yield Shock

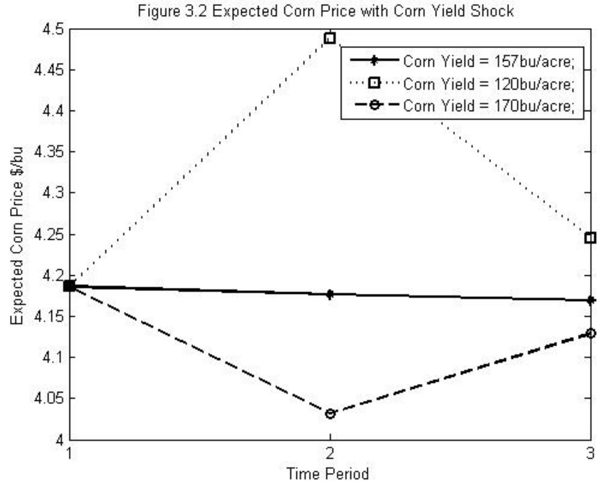


Figure 3.2 Expected Corn Price with Corn Yield Shock

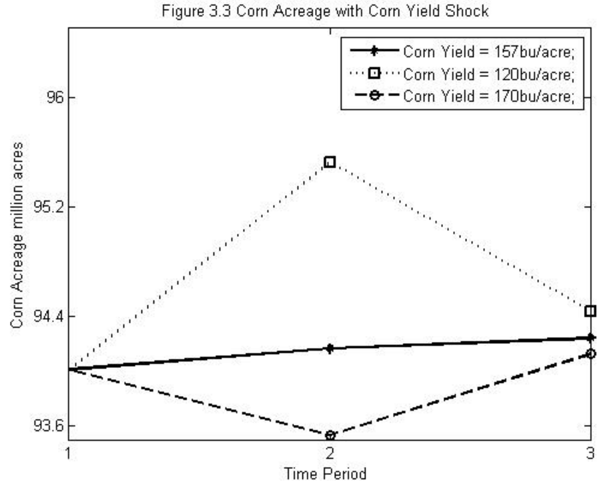


Figure 3.3 Corn Acreage with Corn Yield Shock

Figure 3.4 Current Corn Price with Corn Yield Shock

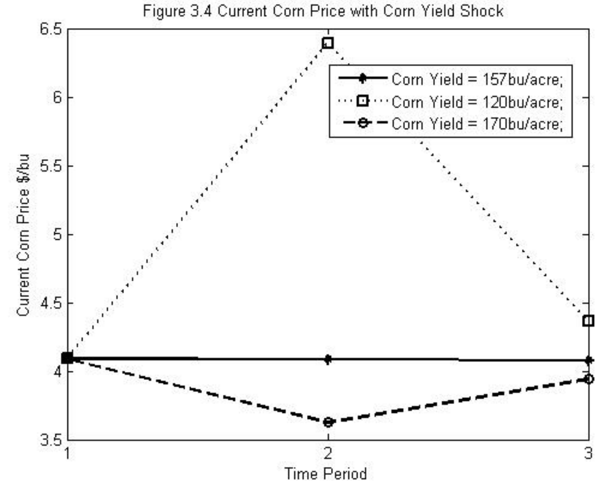


Figure 4.1 Soybean Stock Levels with Corn Yield Shock

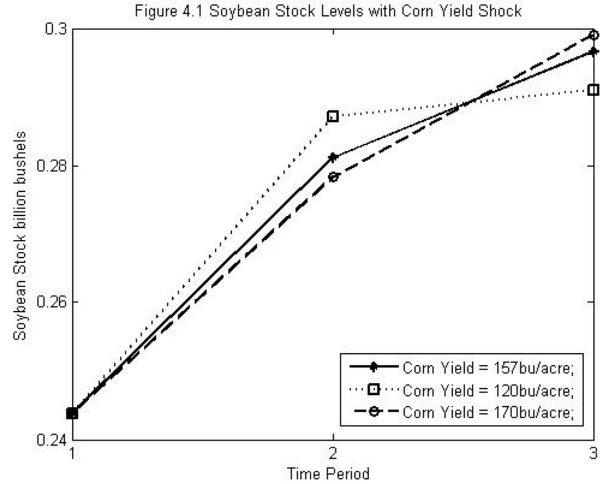
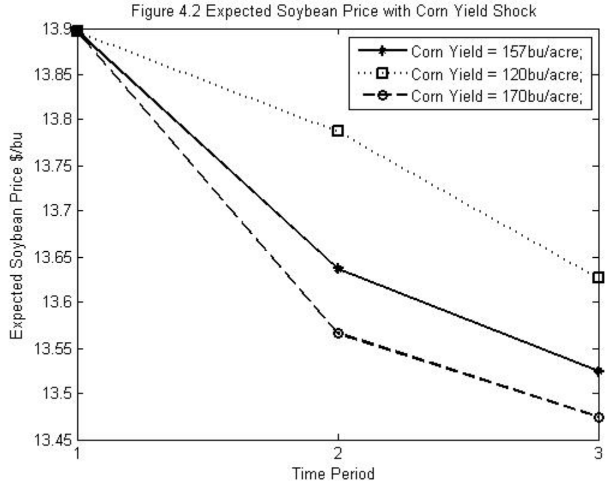
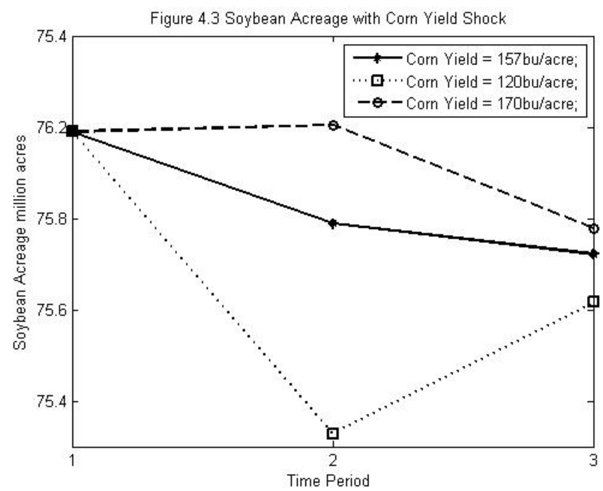
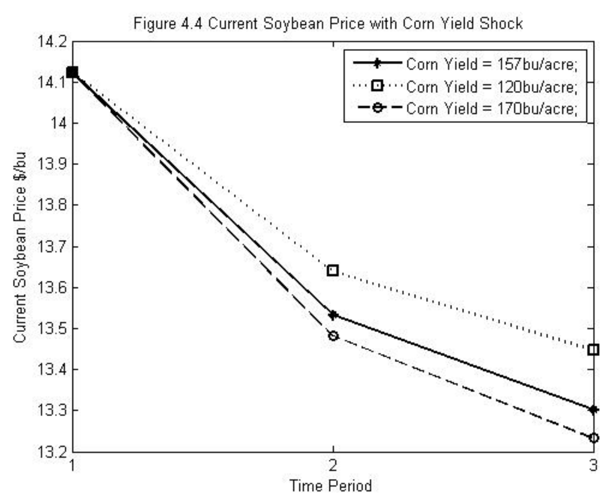


Figure 4.2 Expected Soybean Price with Corn Yield Shock







4.7 Conclusions

Smolyak collocation methods perform better than GSSA considering computational time and accuracy in solving multi-crop storage model.

The most promising approach for solving a extended model with crops more than three crops is Smolyak collocation method with storage rule approximation. At the same time, there are also several approaches to improve the current method as described by Judd et. al (2013), e.g. put more grid points to those dimensions that are most important for overall quality of approximation. In our case, this means we may put more grid points to descretize soybean total supply space in method 2.

Another condition for making the second method favorable is the assumption of convenience yield without which there will be stock out conditions and a kink in the storage function. The difficulties of approximating a function with kink will bring the accuracy for all the second algorithm down to the unacceptable level. In this case, the 3rd algorithm that approximate more smooth expected price functions could be used instead.

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