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Three essays on agricultural insurance and farm real estate investment

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Three essays on agricultural insurance and farm real estate investment

by

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The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after a degree is conferred.

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DEDICATION

I would like to dedicate this dissertation to my wife Wei Zhou, to my parents Yunhua Zhao and Duanxi Feng, without whose support I would not have been able to complete this work.

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ABSTRACT

The subject of this dissertation includes studies on agricultural insurance and farm real estate investment. Portfolio risk in crop insurance due to the systemic nature of crop yield losses has inhibited the development of private crop insurance markets. Government subsidy or reinsurance has therefore been used to support crop insurance programs. The first essay investigates the possibility of converting systemic crop yield risk into "poolable" risk. The results indicate that the systemic risk in crop insurance can be eliminated by combining crop insurance policies across crops and countries. The second essay investigates farmland portfolio returns from a forward-looking perspective taking into account time-varying return and serial correlation. The results indicate that it takes a number of years for the expected return to reach the long-term equilibrium. From a forward-looking perspective, the attractive average return level observed historically can only be attained over a long investment period. The risk involved in the long investment period, however, is also considerably higher than the historical sample volatility. The third essay examines the predictive power of macroeconomic risk factors for farmland asset returns. Farmland value slightly increased in 2017 even though farm income was lower. This development suggests the rate of return required by investors for farmland asset has been reduced. One possible explanation for the decreasing required rate of return is an increased money supply. Previous research suggests that the money supply affects several macroeconomic risk factors through different transmission channels, which in turn influence investor behaviors and asset returns. Both linear and neural network models are used in this study to predict farmland returns. The forecast accuracy is compared across different models. The results indicate that farmland return prediction

is significantly improved by adding capital market excess return as an explanatory variable. Adding additional risk factors, however, does not improve the prediction with the sample used in this study.

CHAPTER 1. OVERVIEW

This dissertation consists of three essays discussing topics on agricultural insurance and farm real estate investment. The first essay focuses on diversifying systemic risk in crop insurance portfolios. Portfolio risk in crop insurance due to the systemic nature of crop yield losses has inhibited the development of private crop insurance markets. Government subsidy or reinsurance has therefore been used to support crop insurance programs. We investigate the possibility of converting systemic crop yield risk into poolable risk. Specifically, we examine whether it is possible to remove the co-movement as well as tail dependence of crop yield variables by enlarging the risk pool across different crops and countries. Hierarchical Kendall copula models are used to allow for potential non-linear correlations of the high-dimensional risk factors. A Bayesian estimation approach is applied to account for estimation risk in the copula parameters. The results indicate that the systemic risk in crop insurance can be eliminated by combining crop insurance policies across crops and countries.

The second essay attempts to provide an explanation for the high return-low risk paradox in farmland investment. We investigate both the nominal and real returns of a farmland portfolio from a forward-looking perspective. Land values and cash rents are slow to adjust and therefore the return from owning land is likely to be time-varying and serially correlated. Time-series and copula modeling techniques are used to construct the optimal portfolio and to evaluate the risk-return profile. The results indicate that it takes a number of years for the expected return to reach the long-term equilibrium. From a forward-looking perspective, the attractive average return level observed historically can only be attained over a long investment period. The risk involved in the long investment period, however, is also considerably higher than the historical sample volatility. This is due to autocorrelation in the return series. These findings help explain the high return and low risk puzzle observed in historical farmland returns.

The third essay examines the predictive power of capital market risk factors for farmland returns. Farmland value slightly increased in 2017 even though farm income was lower. This development suggests the rate of return required by investors for farmland asset has been reduced. A similar phenomenon has been observed in the equity market which also suggests reduced equity risk premium. One possible explanation for the decreasing required rate of return is an increased money supply. Previous research suggests that the money supply affects several macroeconomic risk factors through different transmission channels, which in turn influence investor behaviors and asset returns. This article examines the predictive power of these risk factors for farmland asset returns. Both linear and neural network models are used and the forecast accuracy is compared across different models. The results indicate that farmland return prediction is significantly improved by adding capital market excess return as an explanatory variable. Adding additional risk factors, however, does not improve the prediction with the sample used in this study.

CHAPTER 2. DIVERSIFYING SYSTEMIC RISK IN AGRICULTURE

2.1 Introduction

There is substantial systemic risk in crop yield losses due to natural disasters such as area-wide droughts. This systemic risk has been a potential obstacle for the development of private crop insurance markets. Miranda and Glauber (1997) find that the portfolio risk faced by US crop insurers is about ten times larger than that of conventional insurance lines. The US government provides reinsurance via the USDA Risk Management Agency and this helps explain the wide variety of crop insurance products available to US crop producers. In countries where there is no government reinsurance, private crop insurers would need to charge premiums well in excess of fair value so as to accumulate large enough reserves, or purchase expensive international reinsurance. To date, this has impeded the introduction of multi-peril crop insurance outside of the United States. If systemic risk in agriculture could be managed with more efficient techniques and tools, crop insurance could potentially be made available on a worldwide basis.

A central question addressed here is whether the systemic risk is inherently non-diversifiable as its name suggests, or it is because the risk pool is too small to be well diversified. The systemic risk comes from spatial positive correlations among crop yields caused by widespread weather events. In addition, the correlations are characterized by "state-dependent" structure. Empirical evidence has shown that the correlations tend to be stronger during large crop loss periods when diversification opportunities are most needed (Goodwin, 2001). This suggests that there is potentially non-zero dependence existing in the lower tail of the distributions of crop yield variables. Systemic risk may be underestimated without taking this lower-tail dependence into account[1].

Previous studies have explored diversification opportunities for systemic risk in crop insurance. Wang and Zhang (2003) use a spatial statistics approach to investigate the extent of correlation among county-level yields for corn, soybeans, and wheat in the United States. The results indicate that the correlation of yield losses decreases with distance, and the authors conclude systemic risk can be diversified if the risk pool is large enough. However, the authors do not consider lower-tail dependence and implicitly assume constant correlation levels. This may understate the magnitude of the risk as Goodwin (2001) indicates that the spatial correlation decays with increasing distance with different patterns for different weather events. In normal years, a faster spatial decay is observed while in extreme weather years, the rate of decay is much reduced, suggesting an increased correlation of crop yields in extreme years. Zhu *et al.* (2008) and Goodwin and Hungerford (2015), also indicate that "state-dependent" correlation structure exists using model selection criteria when fitting crop yield variables. These studies suggest that alternative correlation structures can have significant implications for quantifying the magnitude of systemic risk and that a linear correlation structure may seriously underrate the risk.

Recent studies have started to account for lower-tail dependence with the applications of copula models for the evaluation of diversification effect. Xu *et al.* (2010) use three copulas from the Archimedean copula family to determine the correlation of weather indices across different regions in Germany. Okhrin *et al.* (2012) employ the hierarchical Archimedean copula (HAC) models to explore the possibility of spatial diversification of systemic weather risk in China. These two studies conclude that the possibility of reducing systemic risk by increasing the size of risk pool is limited for index-based crop insurance within Germany and China, respectively.

[1] For a formal definition of tail dependence, see McNeil *et al.* (2015).

This article enlarges risk pool across different crop types and countries, and investigate the diversification effect within the pool[2]. Two large agricultural producing countries — the United States and China — and five major crops in each country are considered. A synthetic area-yield insurance portfolio across the countries and crop types at the state/province-level is used to evaluate the diversification effect[3][4]. It has been shown that the pooling effect is greatest if area-yield insurance is provided at such a level (Miranda and Glauber, 1997). Our empirical analysis demonstrates that the systemic nature — both positive correlation and lower-tail dependence — of crop yield risks has been removed by pooling the spatially diversified crops across the two countries.

Another contribution of this article is the application of the hierarchical Kendall copula (HKC) in the context of agricultural risks. A variety of advanced copulas have been used in agriculture due to their superiority over basic copulas in multivariate modeling. Examples are vine copulas and the hierarchical Archimedean copula (HAC) (Goodwin and Hungerford, 2015; Okhrin *et al.*, 2012). Given the high dimensionality of the problem at hand, we use the hierarchical Kendall copula (HKC), a recent innovation in copula modeling. Compared to other advanced copulas, the HKC achieves both flexibility and parsimony in modeling the joint distribution of high dimensional variables. The hierarchical structure of the HKC ensures that it is parsimonious in terms of the number of copula parameters. In addition, the basic copula at each hierarchical level of the HKC is not limited to any copula class.

The HKC parameters are estimated using a sequential procedure. Parameters of the basic copulas at the lowest hierarchical level are estimated first. Parameters of the copu-

[2] Risk pooling across different countries may need an intermediary agency working on regulatory conditions. An example of such an agency is the World Bank, which has initiated the African Risk Capacity (ARC) project which pools systemic extreme climate risks across African countries.

[3] An area-yield insurance contract indemnifies the owner based on the shortfall of average crop yields in a specific area indicated in the contract.

[4] Area-yield insurance at the state/province-level would result in yield basis risk for farmers. If the insurer were to offer individual coverage this yield basis risk would be eliminated. The additional risk taken on by offering this additional coverage would, by definition be poolable and would not impact the systemic risk that is the focus of this paper.

las at higher levels are estimated consecutively by plugging-in the estimates of the copula parameters from lower levels. In cases where the choice of the appropriate copula might influence the results, we provide a sensitivity analysis using alternative models. A Bayesian approach is applied to take into account estimation risk. This approach avoids accumulating estimation errors at each sequential estimating step. With the estimated HKC model, the risk associated with the insurance portfolio can be assessed from the predictive distribution of joint insurance loss.

2.2 Basic Copulas

The copula was first introduced by Sklar (1959). Sklar's theorem states that if F is an arbitrary k-dimensional joint continuous distribution function, then the associated copula is unique and defined as a continuous function $C : [0,1]^k \to [0,1]$ that satisfies the equation

$$F(x_1, \ldots, x_k) = C[F_1(x_1), \ldots, F_k(x_k)], \qquad x_1, \ldots, x_k \in \mathbb{R}, \tag{2.1}$$

where $F_1(x_1), \ldots, F_k(x_k)$ are the respective marginal distributions.

Basic copula families are generally composed of parametric and nonparametric copulas. Empirical studies typically use parametric copulas because of their superiority in simulation. The most frequently used parametric copulas are elliptical copulas and Archimedean copulas, each of which implies different correlation structures (Power *et al.*, 2009; Cooper *et al.*, 2012; Larsen *et al.*, 2013). These will serve as the building blocks for more complicated correlation structures used later in this study.

2.3 Hierarchical Copulas

Basic copulas are effective for problems with low dimensionality, such as with pair-wise correlation. For higher dimensions, the correlation structure implied by basic copulas is inflexible and restrictive. For example, some Archimedean copulas, while allowing for asymmetric tail dependence, imply symmetry of the permutation of variables within the copula function. That is, the order of the marginal distributions is exchangeable. This restriction is strict and implausible. In addition, Archimedean copulas represent the multivariate correlation structure with only one single parameter, this may lead to large estimation errors (Okhrin *et al.*, 2012).

Recent research has developed advanced copula models for situations with high dimensionality. These include vine copulas and hierarchical Archimedean copulas. A vine copula is built by decomposing the joint multivariate density into a product of pairwise-copulas, thereby allowing for considerable flexibility in higher dimensions. However, the extremely large number of copula parameters (at least $n(n-1)/2$ parameters for n dimensions) may severely affect the usefulness of vine copulas from a computational viewpoint. Hierarchical Archimedean copulas are much more parsimonious in terms of the number of copula parameters ($n-1$ parameters for n dimensions). However, the building blocks are restricted to the class of Archimedean copulas, which, as mentioned earlier, are not appropriate in some applications.

Hierarchical Kendall copulas have both flexibility and parsimony when modeling the correlation structure in high dimensional situations. A hierarchical Kendall copula is built by a hierarchy of basic copulas that limits the number of copula parameters. In contrast to hierarchical Archimedean copulas, the choice of the basic copula at each hierarchical level is not limited to any copula class. In other words, the building blocks of a hierarchical Kendall copula can be basic copulas of any arbitrary type.

2.3.1 Kendall Distribution Functions

An important component of hierarchical Kendall copulas is the Kendall distribution function. Following Genest and Rivest (1993) and Brechmann (2014), for $U := (U_1, \ldots, U_d)' \sim C$, where C is a d-dimensional copula, the Kendall distribution function $K^{(d)}$ is defined as

$$K^{(d)}(t) := P(C(U) \leq t), \qquad t \in [0,1]. \tag{2.2}$$

For $t \in [0,1]$, it holds that $t \leq K^{(d)}(t) \leq 1$ and $K^{(d)}(0-) = 0$. By definition, the Kendall distribution function is the univariate distribution function of the random variable $Z := C(U)$. Thus, it holds that $K^{(d)}(Z) \sim U(0,1)$.

It is in general complicated to derive the Kendall distribution function in explicit form for a given copula. A recursive formula is given by Imlahi *et al.* (1999):

$$K^{(d)}(t) = K^{(d-1)}(t) + \int_t^1 \int_{C_{u_1}^{-1}(t)}^1 \cdots \int_{C_{u_1,\ldots,u_{d-2}}^{-1}(t)}^1 \int_0^{C_{u_1,\ldots,u_{d-1}}^{-1}(t)} c(u_1, \ldots, u_d) du_d \ldots du_1, \tag{2.3}$$

where $K^{(d)}$ denotes the Kendall distribution function of the d-dimensional copula C with the density c, and $K^{(d-1)}$ denotes the Kendall distribution function of the $(d-1)$-dimensional margin of the first $d-1$ variables. The formula is also involved with the inverse of the copula quantile function $C_{u_1,\ldots,u_r}^{-1}$ which is defined as

$$C(u_1, \ldots, u_r, C_{u_1,\ldots,u_r}^{-1}(z), 1, \ldots, 1) = z, \tag{2.4}$$

for $r = 1, \ldots, d-1$, and $C_{\emptyset}^{-1}(z) := z$ for $z \in (0,1)$.

The usually high-dimensional integration and lack of a copula quantile function in explicit form make it impossible to determine the Kendall distribution function explicitly. One exception is with Archimedean copulas. Barbe *et al.* (1996) derive the Kendall distribution function for a d-dimensional Archimedean copula with generator φ as

$$K^{(d)}(t) = t + \sum_{i=1}^{d-1} \frac{(-1)^i}{i!} \varphi(t)^i (\varphi^{-1})^{(i)}(\varphi(t)), \tag{2.5}$$

for $t \in (0, 1]$, where $(\varphi^{-1})^{(i)}(\cdot)$ denotes the i-th derivative of φ^{-1}.

Brechmann (2014) defines the hierarchical Kendall copula from the Kendall distribution function as follows:

"Let $u_1, \ldots, u_n \sim U(0,1)$ and let $C_0, C_1, \ldots, C_d$ be copulas of dimensions $d, n_1, \ldots, n_d$, respectively, where $n_i \geq 1$, $i = 1, \ldots, d$, and $n = \sum_{i=1}^{d} n_i$. Let $K_1, \ldots, K_d$ denote the Kendall distribution functions corresponding to $C_1, \ldots, C_d$, respectively. Define $m_i = \sum_{j=1}^{i} n_j$ for $i = 1, \ldots, d$, and $m_0 = 0$ as well as $U_i := (u_{m_{i-1}+1}, \ldots, u_{m_i})'$ and $V_i := K_i(C_i(U_i))$ for $i = 1, \ldots, d$. Under the assumptions that

$A_1 : U_1, \ldots, U_d$ are mutually independent conditionally on $(V_1, \ldots, V_d)'$, and

A_2 : the conditional distribution of $U_i \mid (V_1, \ldots, V_d)'$ is the same as the conditional distribution of $U_i \mid V_i$ for all $i = 1, \ldots, d$, that is, $F_{U_i|V_1,\ldots,V_d} = F_{U_i|V_i}$ $\forall i \in \{1, \ldots, d\}$,

the random vector $(u_1, \ldots, u_n)'$ is said to be distributed according to the hierarchical Kendall copula C_K with nesting copula C_0 and cluster copulas $C_1, \ldots, C_d$ if

(i) $U_i \sim C_i$ $\forall i \in \{1, \ldots, d\}$,

(ii) $(V_1, \ldots, V_d)' \sim C_0$."

As Brechmann (2014) notes, the two assumptions can be interpreted as: conditional on the information of the nesting variables $V_1, \ldots, V_d$, the clusters $U_1, \ldots, U_d$ are independent of each other and also independent of other nesting variables. That is, while the correlations within each cluster U_i are explained by the joint behavior of the random variables $u_{m_{i-1}+1}, \ldots, u_{m_i}$ for $i = 1, \ldots, d$, the correlations among the clusters $U_1, \ldots, U_d$ are explained through the joint behavior of the unobserved factors $V_1, \ldots, V_d$, each of which has a uniform distribution because $C_i(U_i) \sim K_i$ for all $i = 1, \ldots, d$.

The above definition from Brechmann (2014) shows how a two-level hierarchical Kendall copula is built. The construction can be extended to an arbitrary number of levels. Figure 2.1 is an illustration of a simple three-level hierarchical Kendall copula. For more detailed descriptions of the general k-level hierarchical Kendall copulas, reference can be found in Brechmann (2014).

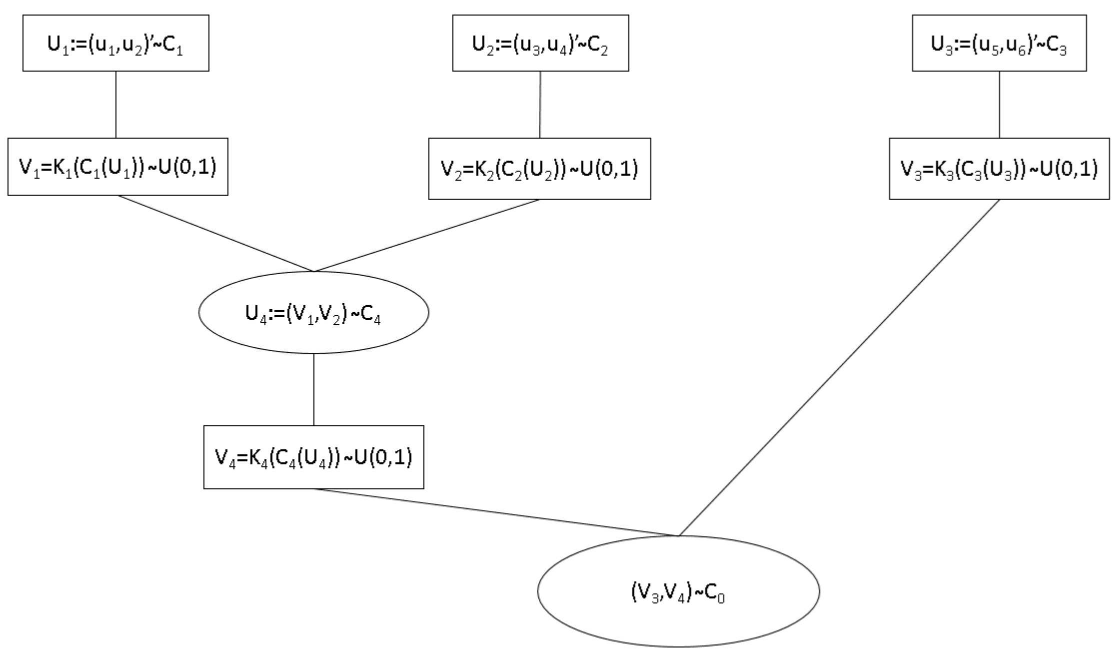

Figure 2.1 Illustration of A Three-level Hierarchical Kendall Copula

2.4 Empirical Application

Here we apply hierarchical Kendall copulas to examine the effectiveness of diversifying systemic risk across five crops—corn, cotton, rice, soybeans, and wheat—in the United States and China. The study area includes ten major producing states in the United States and ten major producing provinces in China for each crop except rice. Only six US states have sufficient data for rice. The US crop yield data include 46-dimensional state-level historical yields, and the Chinese crop yield data include 50-dimensional province-level historical yields. The US crop yield data, covering a 44-year period from 1970 to 2013, are taken from the USDA's National Agricultural Statistics Service (NASS) databases. The Chinese crop yield data, covering a 31-year period spanning from 1979 to 2009, are taken from China Statistical Yearbook.

2.4.1 Detrending the Yield Data

The observed yield data are detrended by fitting a simple linear trend model[5]:

$$y_t = \beta_0 + \beta_1 t + \epsilon_t, \tag{2.6}$$

where $t = 1970, \ldots, 2013$ for the US data, and $t = 1979, \ldots, 2009$ for the Chinese data. The corresponding yield trends are calculated as the predicted yields from the above regression:

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 t, \tag{2.7}$$

Detrended yields to 2009-equivalents are generated as:

$$y_t^{det} = y_t \frac{\hat{y}_{2009}}{\hat{y}_t}. \tag{2.8}$$

where $t = 1970, \ldots, 2013$ for the US, and $t = 1979, \ldots, 2009$ for China. All the yield data used in the remainder of this article are composed of the detrended 2009-equivalent yields.

[5]More complex detrending procedures include log-linear (Woodard and Garcia, 2008) and linear spline models (Harri *et al.*, 2011).

2.4.2 Estimation

A commonly used two-step procedure is adopted to estimate the parameters of the hierarchical Kendall copula model. This method is called inference for margins (IFM) (Joe and Xu, 1996). In the IFM method, the parameters of the marginal distributions are estimated first. Next, the copula parameters are determined given the estimated margins $\hat{F}_{X_i}$, $i = 1, \ldots, n$. That is, the parameters of the hierarchical Kendall copula are estimated based on the uniform variables derived as $\hat{u}_{ji} = \hat{F}_{X_i}(x_{ji})$, $j = 1, \ldots, t$, $i = 1, \ldots, n$. As demonstrated by Joe (2005), the IFM method provides consistent estimators of copula parameters, with very little loss in accuracy when compared to joint estimation of the parameters of marginal and copula functions.

The marginal distribution for each yield variable is estimated independently. As Goodwin (2015) notes, there is little guidance for selecting an optimal distribution for crop yield density among various parametric alternatives. The beta distribution has found favor in many studies to capture potential skewness of crop yields (Babcock and Hennessy, 1996; Nelson and Preckel, 1989). The beta density function of a variable y can be written as:

$$Beta(y \mid \alpha, \beta, y_{min}, y_{max}) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(y - y_{min})^{\alpha-1}(y_{max} - y)^{\beta-1}}{y_{max}^{\alpha+\beta-1}}, \; y_{min} \leq y \leq y_{max}, \quad (2.9)$$

where $\Gamma(\cdot)$ denotes the gamma function, y_{min} and y_{max} are parameters for the lower and upper limits of the feasible range for y respectively, and α and β are shape parameters.

A separate beta distribution is fitted to the historical data for each yield variable y_i. For each y_i, the lower and upper limits are set as $y_{min_i} = 0$ and $y_{max_i} = y_i^U + 1.5\sigma_i$, where y_i^U denotes the maximum observation of y_i, and σ_i is the sample standard deviation of y_i. The shape parameters α_i and β_i are estimated by the maximum likelihood method.

The second step of the IFM method involves the estimation of copula functions. This is based on the uniform variables $u_i = \hat{F}_i(y_i)$, where $\hat{F}_i$ is the cumulative distribution function

(CDF) of the estimated beta distribution for yield variable y_i from the first step. The visual patterns of the correlations between the variables are shown in Figure 2.2. Each of the scatterplots displays a sample of the uniform variables from randomly selected pairs of US states or Chinese provinces. From the plots, it is observed that while risk-pooling across different crop types within the United States mitigates the magnitude of lower-tail dependence (bottom-left mass), combining the two countries appears to eliminate the systemic nature of the risks altogether.

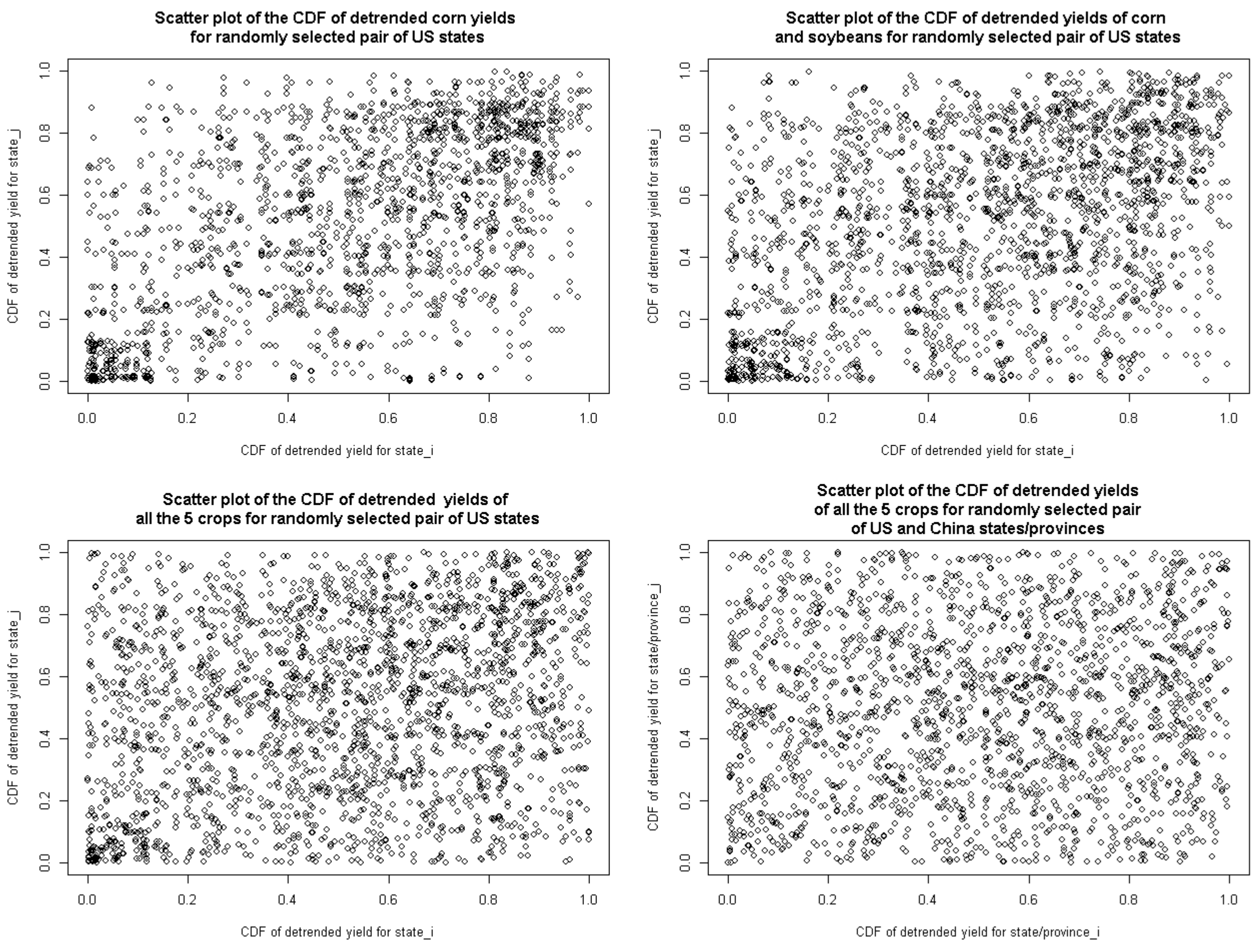

Figure 2.2 Scatter Plots of the CDF of Randomly Paired Detrended Yields in the Same Year

The hierarchical copula representation of the uniform variables depends on the specific hierarchical structure. A natural clustering takes place where each of the two countries forms a geographical cluster. Within each country/cluster, the structure is built by aggregating a group of variables using one cluster copula at each hierarchical level. Archimedean copulas are used as cluster copulas at all hierarchical levels[6]. This is done because an explicit form of the Kendall distribution function is available only for this case (Brechmann, 2014). The cluster copulas are all bivariate so that all the substructure of the correlations can be revealed. All cluster copulas are selected to be of the same type so as to limit model complexity. The commonly used Gaussian copula is used as a benchmark model to evaluate the fit of the cluster copulas within each country/cluster.

To specify the hierarchical structure within each cluster, a metric is needed to measure the distance between (groups of) variables. Variables with shorter distance are aggregated at a lower hierarchical level. The distance between two variables is determined by the level of correlation between them. The higher the correlation, the shorter the distance. As the copula parameter is increasing with the associated (rank) correlation level for both survival Gumbel and Clayton copulas, the distance can be appropriately measured by the value of the copula parameters.

Following Okhrin and Ristig (2012), the hierarchical structure within each cluster is constructed using the following recursive approach. At the lowest level, a bivariate cluster copula is fitted to every possible couple of the variables u_i using the maximum likelihood method. We select the pair of variables (denoted as $\hat{u}_1$ and $\hat{u}_2$) with the highest level of correlation and denote the estimated bivariate copula and its parameter as C_1 and $\hat{\theta}_1$,

[6]For high dimensions, there is a trade-off between revealing the substructure of correlations and fully exploiting the flexibility of the HKC within a cluster. While bivariate cluster copulas display the complete substructure of correlations, the large number of possible combinations of bivariate Archimedean copula types requires simplification at the cost of underusing the flexibility of the HKC. While exchangeable multivariate Archimedean copulas reduce the number of the possibilities, the substructure is hidden and the implied permutation symmetry is not plausible (Okhrin *et al.*, 2012).

respectively. Then we introduce a pseudo-variable $Z_1 = C_1\left(\hat{u}_1, \hat{u}_2; \hat{\theta}_1\right)$. At the next step, the remaining variables and the pseudo-variable compose a new set. We proceed in the same way considering this new set of variables. The most highly correlated couple of variables are selected from this new set and another pseudo-variable is obtained from the corresponding estimated bivariate copula. This procedure is repeated until the whole hierarchical structure is determined.

No obvious tail dependence is observed in the scatterplots of the correlations between the two countries/clusters. Therefore, the Gaussian copula and the Frank copula, both of which imply no tail dependence, are chosen as our alternatives for the nesting copula. The absence of tail dependence is confirmed by a sensitivity analysis provided later.

In summary, four different hierarchical Kendall copulas are used to model the entire correlation structure. These are: (a) survival Gumbel clustering with Gaussian nesting; (b) Clayton clustering with Gaussian nesting; (c) survival Gumbel clustering with Frank nesting; and (d) Clayton clustering with Frank nesting (see Figure 2.3).

2.4.3 Estimation of the Copula Parameters in the Presence of Estimation Risk

The parameters of a hierarchical Kendall copula with specified structure can be estimated with the sequential maximum likelihood procedure provided by Brechmann (2014). To improve the quality of the estimation of the copula parameters, our estimation strategy takes estimation risk into account. The approach, based on Bayes' criterion, allows us to incorporate parameter uncertainty (model uncertainty) and avoid accumulating estimation errors when sequentially estimating the hierarchical Kendall copula parameters[7]. In addition, credible intervals of the correlation level, and by extension the insurance loss can be constructed from the Bayesian framework.

[7] We focus on the copula parameters which determine the correlation level and the diversification effect. Estimation risk for the marginal distributions is not considered here because it would not influence the level of systemic risk.

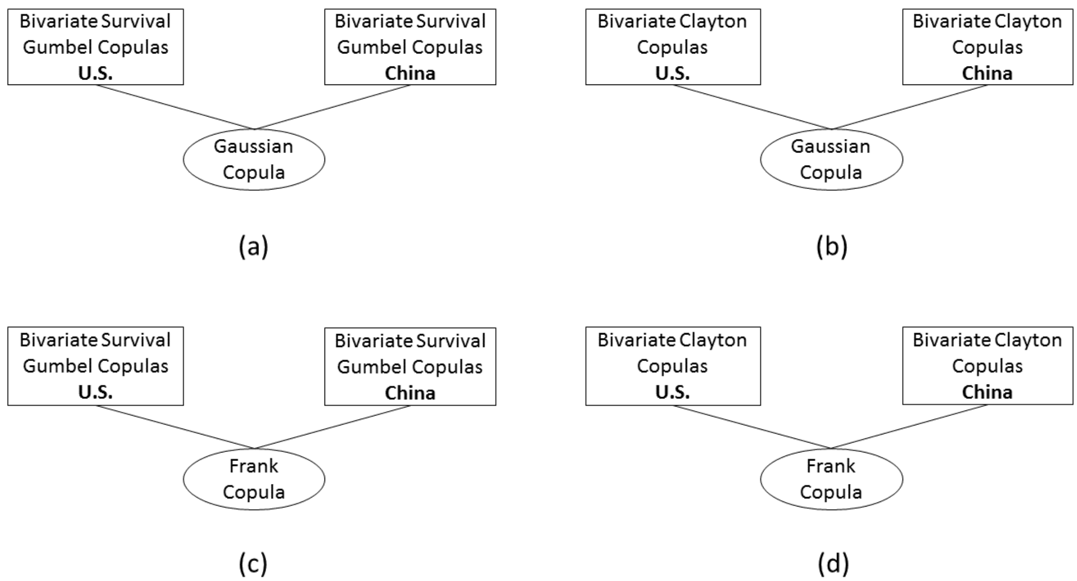

Figure 2.3 Alternative Hierarchical Kendall Copula Models for the Entire Correlation Structure

For a two-level hierarchical Kendall copula C_K with nesting copula C_0 and cluster copulas $C_1, \ldots, C_d$, let $(u_{j,1}, \ldots, u_{j,n})'_{j=1,\ldots,t}$ be a sample of the hierarchical Kendall copula C_K, where $C_0, C_1, \ldots, C_d$ are copulas of dimensions $d, n_1, \ldots, n_d$, respectively. Define $m_i = \sum_{l=1}^{i} n_l$ for $i = 1, \ldots, d$, and $m_0 = 0$. The estimates of the parameters $\theta_0, \theta_1, \ldots, \theta_d$ of the copulas $C_0, C_1, \ldots, C_d$, respectively, are obtained by

(i) estimating θ_i based on $(u_{j,m_{i-1}+1}, \ldots, u_{j,m_i})'_{j=1,\ldots,t}$ using the Bayesian method and obtaining the posterior density $p(\theta_i \mid u_{j,m_{i-1}+1}, \ldots, u_{j,m_i})$ for θ_i, for $i = 1, \ldots, d$, and

(ii) estimating θ_0 based on the pseudo observations

$$\hat{v}_{j,i} := \int_{\Theta} K_i(C_i(u_{j,m_{i-1}+1}, \ldots, u_{jm_i}; \theta_i); \theta_i) p(\theta_i \mid u_{j,m_{i-1}+1}, \ldots, u_{j,m_i}) d\theta_i,$$

$$i = 1, \ldots, d, \; j = 1, \ldots, t, \quad (2.10)$$

using the Bayesian method. This procedure can be generalized for estimating k-level hierarchical Kendall copulas by proceeding with more iterations.

Our hierarchical Kendall copula models are estimated using the above procedure. The cluster and nesting copulas have all been specified as bivariate copulas. The posterior density for the bivariate copula parameter θ conditional on a sample of two variables (u_1, u_2), is the product of the likelihood function $f(u_1, u_2 \mid \theta)$ and the prior distribution $\pi(\theta)$ normalized by an appropriate constant:

$$p(\theta \mid u_1, u_2) = \frac{f(u_1, u_2 \mid \theta)\, \pi(\theta)}{\int f(u_1, u_2 \mid \theta)\, \pi(\theta)\, d\theta}. \tag{2.11}$$

Given a sample $(u_{j,1}, u_{j,2})'_{j=1,\ldots,t}$ of the variables (u_1, u_2), the likelihood function is given by

$$f(u_1, u_2 \mid \theta) = \prod_{t=1}^{T} f(u_{1t}, u_{2t} \mid \theta) = \prod_{t=1}^{T} c(u_{1t}, u_{2t} \mid \theta), \tag{2.12}$$

where $c(\cdot,\cdot)$ is the copula density function derived by taking derivatives of the copula function C:

$$c(u_1, u_2 \mid \theta) = \frac{\partial^2 C(u_1, u_2 \mid \theta)}{\partial u_1 \partial u_2}. \tag{2.13}$$

For the bivariate survival Gumbel copula, the density function is derived as

$$\begin{aligned} \bar{c}^G(\bar{u}_1, \bar{u}_2 \mid \theta) = {} & \frac{\left[(-\ln \bar{u}_1)^\theta + (-\ln \bar{u}_2)^\theta\right]^{\frac{1}{\theta}} + \theta - 1}{\bar{u}_1 \bar{u}_2} \\ & \cdot \exp\left\{-\left[(-\ln \bar{u}_1)^\theta + (-\ln \bar{u}_2)^\theta\right]^{1/\theta}\right\} \left[(-\ln \bar{u}_1)^\theta + (-\ln \bar{u}_2)^\theta\right] (-\ln \bar{u}_1)^{\theta-1} (-\ln \bar{u}_2)^{\theta-1}, \end{aligned} \tag{2.14}$$

where $\bar{u}_i = 1 - u_i$ for $i = 1, 2$ are "survival variables."

For the bivariate Clayton copula, the density function is derived as

$$c^{C}\left(u_1,u_2\right)=\left(\theta+1\right)\left(u_1^{-\theta}+u_2^{-\theta}-1\right)^{-\frac{1}{\theta}-2}u_1^{-\theta-1}u_2^{-\theta-1}. \tag{2.15}$$

For the bivariate Frank copula, the density function is derived as

$$c^{F}\left(u_1,u_2\right)=-\frac{\theta exp(-\theta u_1)exp(-\theta u_2)[exp(-\theta)-1]}{\{[exp(-\theta u_1)-1][exp(-\theta u_2)-1]+exp(-\theta)-1\}^2}. \tag{2.16}$$

For the bivariate Gaussian copula, the density function is derived as

$$c^{N}\left(v_1,v_2\mid\Sigma\right)=|\Sigma|^{-\frac{1}{2}}\,exp\{-\frac{\left[\Phi^{-1}(v_1),\Phi^{-1}(v_2)\right]^{'}\left(\Sigma^{-1}-I\right)\left[\Phi^{-1}(v_1),\Phi^{-1}(v_2)\right]}{2}\}, \tag{2.17}$$

where Φ_Σ is a two-dimensional normal distribution with zero mean and correlation matrix Σ, Φ^{-1} is the inverse distribution function of the standard normal distribution, and I is the identity matrix.

Following Bokusheva (2011), a non-informative uniform distribution is selected as the prior for the parameters of all the bivariate Archimedean copulas[8]. The bounds are selected so that a wide range of the support for the parameter is covered: $\theta\sim U\left(1,100\right)$ for the survival Gumbel copula, $\theta\sim U\left(0,100\right)$ for the Clayton copula, and $\theta\sim U\left(-1000,1000\right)$ for the Frank copula.

Following Smith (2011), the prior for the correlation matrix of the bivariate Gaussian copula is selected based on a Cholesky decomposition. The correlation matrix Σ can be decomposed as

$$\Sigma=diag(\Omega)^{-\frac{1}{2}}\Omega diag(\Omega)^{-\frac{1}{2}}, \tag{2.18}$$

[8]Informative priors can be used to incorporate expert knowledge into this type of model, as demonstrated in Shen *et al.* (2016). The use of such priors would tighten the distribution and reduce systemic risk. In order to be conservative, we assume non-informative priors.

where Ω is a positive definite matrix. And the matrix Ω can be decomposed as

$$\Omega = RR', \tag{2.19}$$

where $R = \{r_{i,j}\}$ is a lower triangular Cholesky factor. To ensure the decompositions are unique, we set $r_{i,i} = 1$, for $i = 1, 2$. Then the correlation matrix Σ can be written as

$$\Sigma = \begin{bmatrix} 1 & \frac{r_{2,1}}{\sqrt{1+r_{2,1}^2}} \\ \frac{r_{2,1}}{\sqrt{1+r_{2,1}^2}} & 1 \end{bmatrix}. \tag{2.20}$$

A non-informative prior distribution of $r_{2,1}$ is assigned as $r_{2,1} \sim U(-100, 100)$ to cover a wide range of the support for the correlation matrix.

The posterior distribution of the copula parameters is simulated using Markov Chain Monte Carlo (MCMC) methods. For each parameter, three chains of 600,000 iterations are run for different sets of initial values. The first 300,000 iterations are then discarded. In order to reduce autocorrelation, every 30th iteration of each chain is saved. Adequate convergence is confirmed by both the Monte Carlo error and the Gelman-Rubin (1992) test[9].

Note that within each of the two clusters, Bayesian inference is based on all the available data. Within the US, it is based on the historical yield data covering 44 years (1970—2013), and results in the pseudo observations $\hat{v}_{1970,us}, \ldots, \hat{v}_{2013,us}$. Within China, it is based on the historical yield data covering 31 years (1979—2009), and results in the pseudo observations $\hat{v}_{1979,ch}, \ldots, \hat{v}_{2009,ch}$. As Chinese historical yield data is not available for the time before year 1979 or after year 2009, Bayesian inference for the nesting copula parameter is based on part of the US pseudo observations and all the Chinese pseudo observations: $\hat{v}_{1979,us}, \ldots, \hat{v}_{2009,us}$ and $\hat{v}_{1979,ch}, \ldots, \hat{v}_{2009,ch}$. This again demonstrates the flexibility of the hierarchical Kendall copula model. Although the information contained in the US data before 1979 or after

[9]The Monte Carlo error is less than 1/1000 and the potential scale reduction factor is less than 1.1 for all parameters.

2009 cannot be used for estimating the nesting copula because of lack of the corresponding Chinese data, it can still be used for estimating the cluster copulas within the US. With such a copula model, all the relevant information in the available data can be exploited.

The Kendall's correlation coefficient is selected as a measure of association between the correlation level and the copula parameter because it is indifferent to nonlinear monotonic transformations[10]. Following Nelsen (2006), the Kendall's τ associated with Archimedean copulas can be expressed as $\tau = 1 - \frac{1}{\theta}$ for the survival Gumbel copula, $\tau = \frac{\theta}{\theta+2}$ for the Clayton copula, and $\tau = 1 - \frac{4}{\theta} + \frac{4}{\theta^2 \int_0^\theta \frac{t}{\exp(t)-1} dt}$ for the Frank copula. The Kendall's τ associated with the Gaussian copula can be written as $\tau(v_i, v_j) = \frac{2}{\pi} \arcsin(\epsilon_{ij})$, where ϵ_{ij} is the $(i, j)\,th$ element of the correlation matrix Σ of the Gaussian copula.

Within the US, the posterior mean of the Kendall's τ associated with the cluster copulas ranges from 0.111 to 0.698 as implied by the survival Gumbel copula, and from 0.091 to 0.638 as implied by the Clayton copula. Within China, the posterior mean of the Kendall's τ associated with the cluster copulas ranges from 0.072 to 0.613 as implied by the survival Gumbel copula, and from 0.046 to 0.575 as implied by the Clayton copula. This indicates that there is positive correlation existing among the yield variables within each country.

The fit of the cluster copulas within each country is compared to that of a multivariate Gaussian copula. The Gaussian copula models are estimated using maximum likelihood. While the formal Vuong test is mainly conducted for pairwise comparison, the AIC and BIC criteria have been commonly used in the literature to compare the goodness-of-fit of multiple models (Hungerford and Goowin, 2014; Larsen *et al.*, 2015). Table 2.1 presents the log-likelihood and AIC/BIC values of different copula fits for each country. It is to be expected that the Gaussian copula model will have a larger log-likelihood value due to the large number of parameters of the model. However, according to the AIC and BIC values,

[10] A reviewer pointed out that crop yield distributions are nonstationary with respect to mean and variance. It is therefore possible that our assumption of a constant structure of dependencies across time is incorrect. Investigating whether there is structural change in this structure is beyond the scope of this paper.

the hierarchical Kendall copula model, with either survival Gumbel clustering or Clayton clustering, provides superior fit for within-country correlations for both the US and China. The survival Gumbel clustering leads to a better fit than the Clayton clustering. These results indicate that taking into account lower-tail dependence provides a better fit of the correlations of crop yields within each country. This is consistent with the visual patterns of the correlations observed from the scatterplots.

Table 2.1 Goodness-of-fit Statistics of Different Copula Models for Within-country Correlations of Crop Yields in the US and China

	Log-lik.	# of Par.	AIC	BIC
U.S.				
Hier. Kendall (Survival Gumbel)	504.94	45	-919.88	-839.60
Hier. Kendall (Clayton)	445.80	45	-801.60	-721.31
Gaussian	1308.27	1035	-546.53	1300.11
China				
Hier. Kendall (Survival Gumbel)	331.51	49	-565.01	-471.32
Hier. Kendall (Clayton)	267.29	49	-436.59	-342.90
Gaussian	1160.96	1225	128.07	1884.71

The between-country correlation is calculated from the nesting copula. The posterior distribution of the Kendall's τ associated with the nesting copula is plotted in Figure 2.4. From the graph, the posterior mean and median are both very close to zero under all of the four models. In addition, with a posterior probability of 95%, the correlation is less than 0.151 (0.067) for Gaussian nesting with survival Gumbel (Clayton) clustering, and less than 0.152 (0.003) for Frank nesting with survival Gumbel (Clayton) clustering, all indicating that there is little positive correlation in crop yields between the two countries.

2.5 Simulation Study

Next, we use a simulation study to investigate the degree of diversification that can be expected. Using simulated copula parameters from the full posterior distribution and the

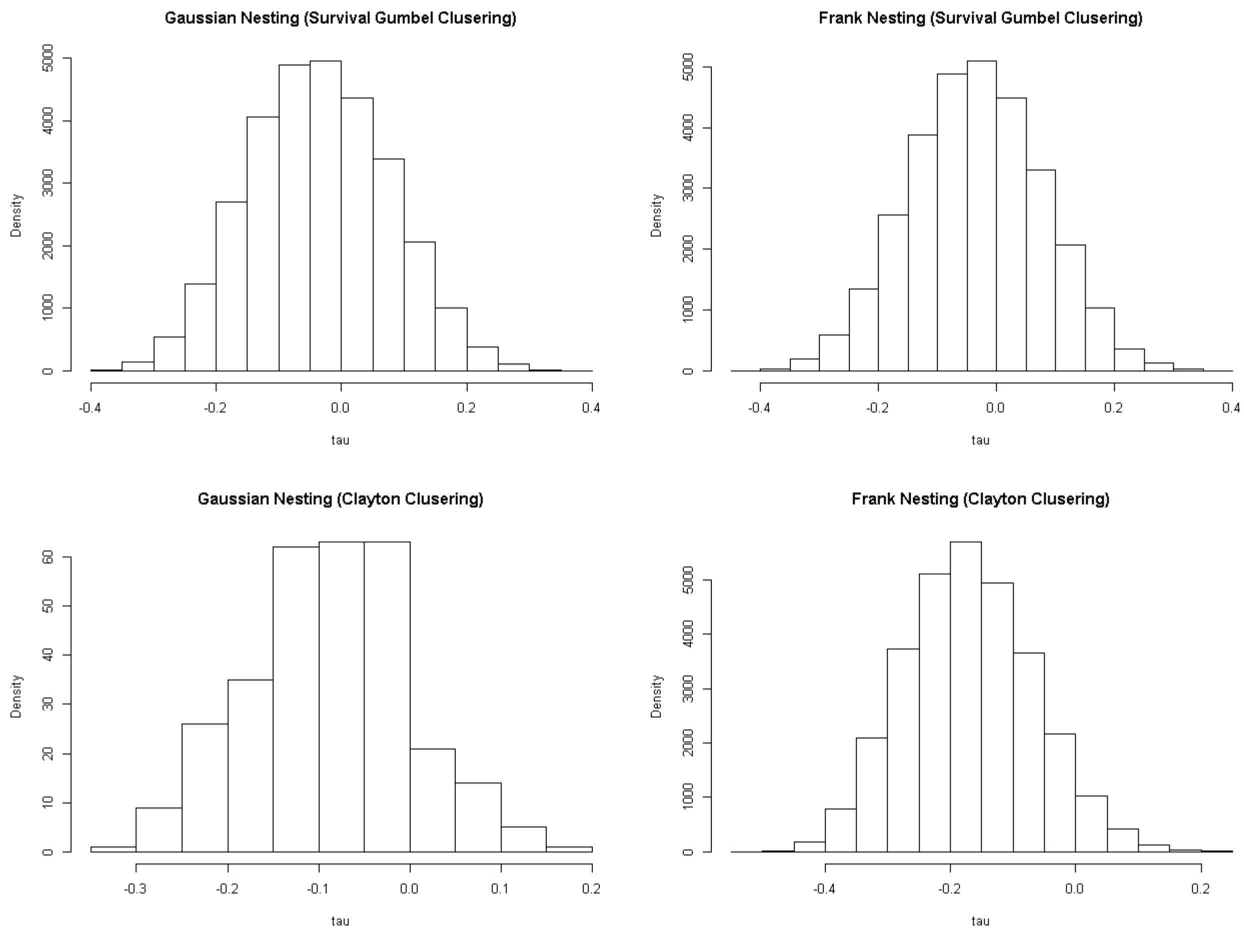

Figure 2.4 Posterior Distribution of Kendall's Correlation Coefficient Associated with the Nesting Copulas

estimated Beta margins, a sample of predicted crop yields can be generated. The predictive distribution of aggregated net insurance income at different coverage levels can then be obtained based on the predicted crop yields. The systemic risk associated with an insurance portfolio is assessed by the statistics of the predictive distribution of net insurance income.

According to Bayes' theorem, the posterior predictive distribution of a new unknown observable $\tilde{u}$ conditional on the observed data u is

$$p(\tilde{u} \mid u) = \int p(\tilde{u}, \theta \mid u) d\theta = \int p(\tilde{u} \mid \theta, u) p(\theta \mid u) d\theta = \int p(\tilde{u} \mid \theta) p(\theta \mid u) d\theta. \tag{2.21}$$

Recall that we have independently simulated a sample of $30,000$ values from the posterior distribution for each copula parameter. So a sample of the uniform variables u_i can be generated by simulating the sth observation of the uniforms from the corresponding hierarchical Kendall copula with the sth simulated copula parameters, for $s = 1, \ldots, 30000$. Sampling from a given hierarchical Kendall copula is conducted with the algorithm provided by Brechmann (2014). Given all the simulated uniform variates from the copula model, predicted crop yields are obtained based on previously estimated Beta marginal distributions.

We consider insurance portfolios composed of state (province)-level area yield insurance contracts. For each state (province)-level insurance contract i, the indemnity paid for one unit area takes the form of

$$I_i = p_i * max[\lambda y_i^e - y_i, 0], \tag{2.22}$$

where y_i denotes the realized crop yield, $y_i^e = E(y_i)$ denotes the expected crop yield, λ denotes the coverage level, and p_i denotes the base price of the crop associated with insurance contract i, which represents the expected harvest time price at planting time. We consider two coverage levels (CL) 70% and 90%. The base price is specified as the average US futures price during the planting month for futures contract expiring at harvest time[11]. The actuarially fair premium π_i is equal to the expected indemnity payment:

$$\pi_i = E(I_i) = p_i * Emax[\lambda y_i^e - y_i, 0] = p_i * E[(\lambda y_i^e - y_i)I(y_i \leq \lambda y_i^e)], \tag{2.23}$$

where I is the indicator function. The net insurance loss of contract i can then be written as

$$L_i = I_i - \pi_i. \tag{2.24}$$

The aggregated net insurance loss L associated with an insurance portfolio is therefore the weighted average of L_i:

$$L = \sum w_i L_i, \tag{2.25}$$

where w_i denotes the weight of insurance contract i in the portfolio, which is assumed to be proportional to the planting area that insurance contract i covers.

2.5.1 Diversification Effect across the Crops within Each Country

We first investigate the diversification effect across the five crops (corn, cotton, rice, soybeans, and wheat) in the United States. The simulated net insurance losses under three different scenarios are compared: (a) insuring the five crops separately ignoring any diversification effect across the five crops (separately); (b) insuring them jointly assuming the estimated correlation structure of crop yields (jointly); and (c) insuring them jointly assuming independence of crop yields across the five crops (benchmark). Table 2.2 presents the

[11] Ideally, Chinese futures price should be used as the base price for Chinese crops. However, reliable records of Chinese futures prices do not exist for the five crops for the entire time period. Therefore, US futures price is used as an approximate base value for Chinese crops. This, will impose downward bias on the estimated diversification effect.

statistics of the distribution of net insurance income under the three scenarios[12]. Recall that two kinds of cluster copulas (survival Gumbel and Clayton) are evaluated to measure the within-country correlations. Here we report simulation results from the survival Gumbel copula. Results from the Clayton copula are similar.

Table 2.2 Simulated Distribution of Net Insurance Income ($/acre) over Multiple Crops in the US Using Survival Gumbel Clustering

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	2.45	-59.12	-10.40	-2.05
Jointly	1.77	-52.66	-7.01	-1.68
Benchmark	1.54	-38.59	-6.09	-1.68
90% Coverage Level				
Separately	12.47	-134.48	-52.72	-24.67
Jointly	9.64	-122.73	-41.15	-18.05
Benchmark	8.01	-77.40	-32.54	-15.53

Since we are assuming that companies charge a fair premium, and because we ignore other transaction charges, the mean of net insurance income is zero for all scenarios. At the 70% coverage level, the standard deviation decreases from $2.45/acre to $1.77/acre when moving from "separately" scenario to "jointly" scenario, and falls to $1.54/acre under "benchmark" scenario assuming independence. With the survival Gumbel clustering correlation structure, the minimum observed net income, and value at risk at 1% and 5% levels (VaR(1%) and VaR(5%), respectively) are all improved by combining the insurance policies across the five crops, but not as much when compared to "benchmark" scenario. At the 90% coverage level, the results are consistent to those at the 70% coverage level. These simulation results taken together show that although there is some diversification when combining multiple crops, it is not as significant as if risks were independent among the crops. This also demonstrates that positive correlation exists among the yields of these five US crops.

[12] The statistics under "separately" scenario are simply the weighted average of the statistics for the five crops since any diversification effect is ignored.

The diversification effect (DE), which is defined as the percentage of the risk diversified away in "jointly" or "benchmark" scenario relative to "separately" scenario, is presented in Figure 2.5:

$$DE = \frac{S_s - S_j}{S_s}, \tag{2.26}$$

where S_s is a statistic (standard deviation, minimum value, or VaR's) in "separately" scenario and S_j is the statistic in "jointly" or "benchmark" scenario. For relatively small risks, such as losses around \$2/acre, the difference between the DE under "jointly" scenario and "benchmark" scenario is insignificant. As the risks become larger, the difference is more obvious. For example, for losses around \$10/acre, the difference is about 10%. For losses of \$50/acre, the difference is about 20%. When losses are more than \$100/acre, the DE difference reaches almost 50%. These results indicate that diversifying risks across multiple crops becomes more difficult as the size of the loss increases. This is due to lower-tail dependence among the crop yields.

Similar results are found for the diversification effect across the five crops in China. Risks diminish by insuring the crops jointly. However, just as in the United States, the diversification effect is not as significant as if risks were independent among crops. This again demonstrates the existence of positive correlation and systemic risk of crop yields within a country. The same pattern of diversification effect is also found in China; that is, losses are more difficult to diversify when they are larger.

2.5.2 Diversification Effect across Countries

We now compare the net insurance loss under three scenarios: (a) insuring the crops in the United States and China separately (separately); (b) insuring the crops in these two countries jointly assuming the estimated correlation structure (jointly); and (c) insuring

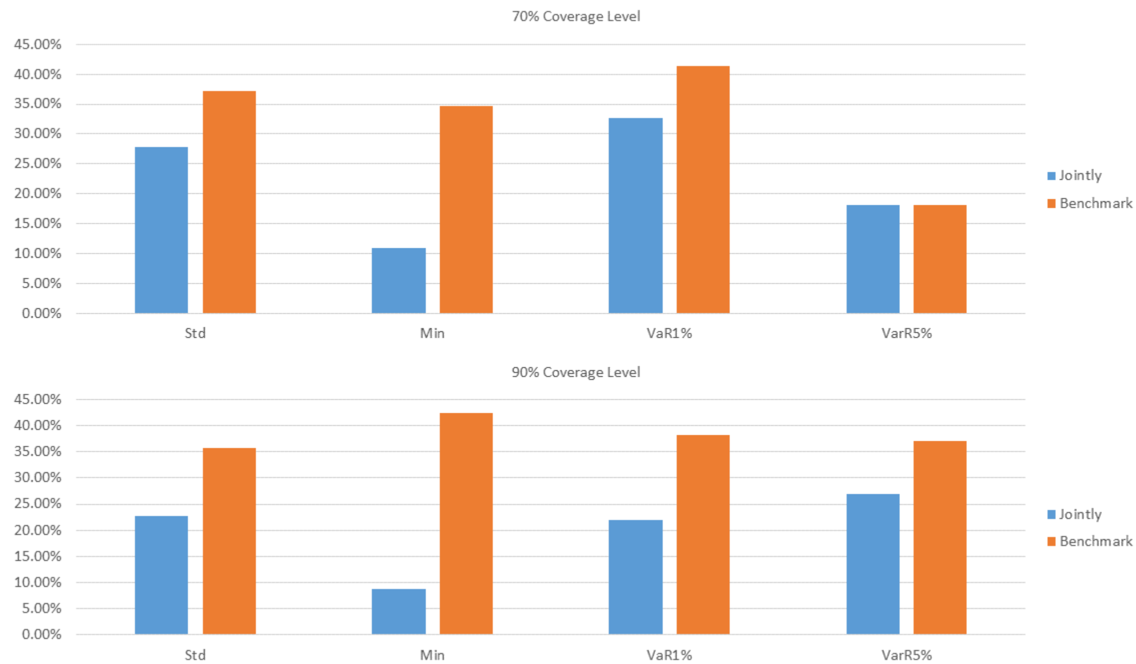

Figure 2.5 Diversification Effect (DE) Achieved by Combining Insurance Contracts across Multiple Crops in the US Using Survival Gumbel Clustering

them jointly assuming independence of crop yields between the two countries (benchmark). Recall that two kinds of the nesting copulas (Gaussian and Frank) are used to model the between-country correlation. Along with the two kinds of cluster copulas, there are 4 (2 * 2) different copula models for the whole correlation structure of all the crop yield variables in the two countries. We report the results observed from the survival Gumbel clustering and Gaussian nesting model. Table 2.3 and Figure 2.6 show the statistics of the distribution of net insurance income and the diversification effect, respectively. Results from the other models are consistent.

The results show that systemic risk is significantly reduced by diversifying across the two countries. At the 70% coverage level, the standard deviation falls from \$1.46/acre to \$1.09/acre. The minimum observed net income is increased from a loss of \$37.40/acre

Table 2.3 Simulated Distribution of Net Insurance Income ($/acre) over the US and China Using Survival Gumbel Clustering and Gaussian Nesting

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	1.46	-37.40	-5.93	-1.70
Jointly	1.09	-23.50	-4.61	-1.47
Benchmark	1.04	-24.47	-4.22	-1.43
90% Coverage Level				
Separately	8.16	-104.13	-34.93	-15.03
Jointly	6.02	-64.30	-23.39	-11.89
Benchmark	5.82	-61.12	-22.38	-11.28

to a loss of $23.50/acre. The value at risk at 1% (5%) of net income is improved from a loss of $5.93/acre ($1.70/acre) to a loss of $4.61/acre ($1.47/acre). Similar results are obtained at the 90% coverage level. Most striking is that the diversification effect across the countries is comparable to that in the benchmark case of independence. In addition, the diversification effect remains significant no matter how large the yield losses are. Unlike the results from diversifying across crops within a country, diversifying across countries has removed the systemic nature — both positive correlation and lower-tail dependence — of the risks associated with crop yields.

Another thing to note is that the results do not vary significantly across the different copula models. The only difference is that models with the Clayton clustering imply a larger maximum net loss than models with the survival Gumbel clustering. However, the difference tends to disappear when looking at the diversification effect. This robustness suggests that the results are quite stable across the different copula modeling methods. This confirms the reliability of the estimated correlation structure as well as the diversification effect.

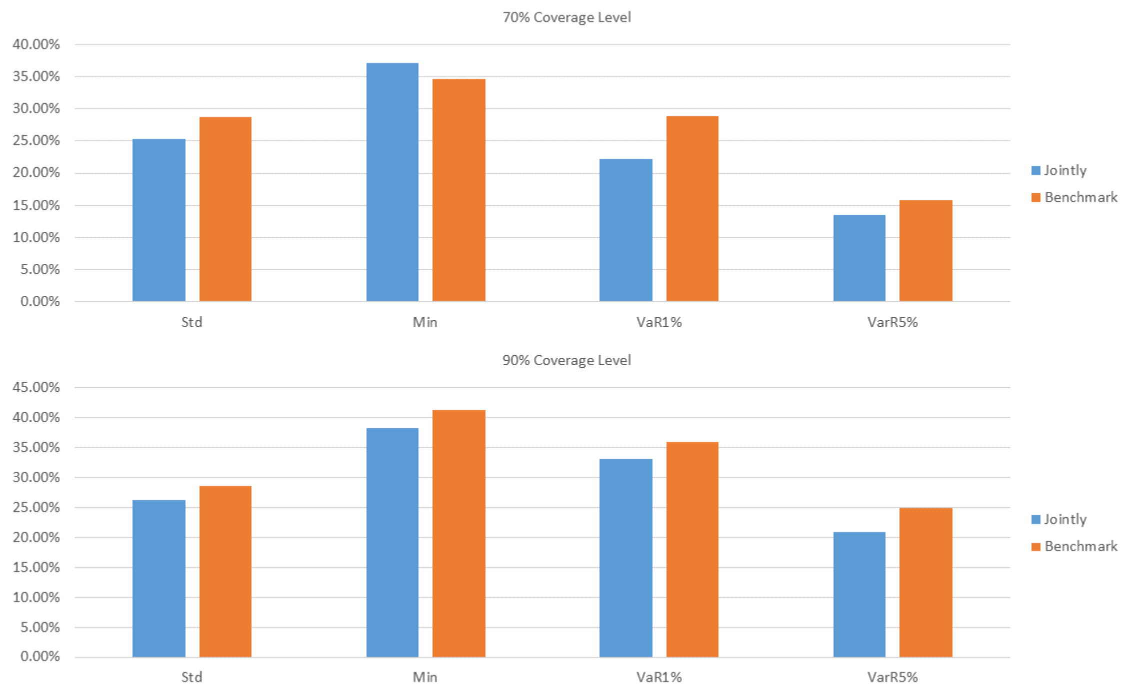

Figure 2.6 Diversification Effect (DE) Achieved by Combining Insurance Contracts across the US and China Using Survival Gumbel Clustering and Gaussian Nesting

2.6 Comparison among Models

Although the results of the diversification effect show robustness across the different copula models, it is worthwhile to compare the models in terms of how they fit the historical data. Table 2.4 summarizes the log-likelihood and AIC/BIC values of the different models. It shows that when survival Gumbel clustering is applied to model the within-country correlation, Gaussian nesting is slightly superior to Frank nesting in modeling the between-country correlation. However, if Clayton clustering is used to model the within-country correlation, Frank nesting is a better choice for the between-country correlation. Overall, the survival Gumbel clustering and Gaussian nesting model provides the best fit.

Table 2.4 Goodness-of-fit Statistics of Alternative Hierarchical Kendall Copula Models

	Log-lik.	# of Par.	AIC	BIC
(Survival Gumbel, Gaussian)	836.50	95	-1482.99	-1346.77
(Survival Gumbel, Frank)	836.49	95	-1482.99	-1346.76
(Clayton, Gaussian)	713.53	95	-1237.05	-1100.83
(Clayton, Frank)	714.34	95	-1238.68	-1102.46

2.7 Sensitivity Analysis for the Selection of the Nesting Copula

To rule out the possibility that the selection of the nesting copula has a significant effect on the estimated diversification effect, we model the between-country correlation with nesting copulas that allow for lower-tail dependence. The empirical results are then compared to the results obtained before using nesting copulas with no tail dependence.

Results from the survival Gumbel clustering and survival Gumbel nesting model are reported in Table 2.5. Even when using the survival Gumbel nesting copula that allows for lower-tail dependence, the simulated statistics and diversification effect in "jointly" scenario are comparable to "benchmark" scenario. Consistent results are found for the Clayton clustering and Clayton nesting model.

Table 2.5 Simulated Distribution of Net Insurance Income ($/acre) over the US and China Using Survival Gumbel Clustering and Survival Gumbel Nesting

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	1.46	-37.40	-5.93	-1.70
Jointly	1.13	-22.47	-4.77	-1.47
Benchmark	1.04	-24.47	-4.22	-1.43
90% Coverage Level				
Separately	8.16	-104.13	-34.93	-15.03
Jointly	6.01	-60.49	-23.67	-11.79
Benchmark	5.82	-61.12	-22.38	-11.28

The sensitivity analysis suggests that results obtained before are quite resilient to alternative nesting copulas. When using a nesting copula that allows for lower-tail dependence, a similar diversification effect is found. This confirms the assumption that no tail dependence exists for the between-country correlation.

2.8 Policy Implications

To the best of our knowledge there are no crop insurance companies operating in both China and the US. However, reinsurance companies operate on a worldwide basis and could be used to eliminate residual systemic risk. In order to evaluate the practical implications of our results, we ran a simulation where the reinsurance company paid all losses in excess of five times the premium on the 180 million crop acres in the ten US states and 200 million crop acres in the ten Chinese provinces. The fair value of this reinsurance is \$0.39 per acre for the US and \$0.16 per acre for China. The fair reinsurance premium for both countries is \$102.2 million. The average gross written premium for the top ten reinsurance companies in 2012 was \$15.5 billion. The addition of a new \$102.2 million premium to the average of these reinsurers would result in only a 0.659% increase in the total book value. This would not represent a significant increase in total exposure.

2.9 Conclusions

Systemic risk in crop yields has been a long-standing problem in agriculture. The systemic risk has been hard to diversify due to the "state-dependent" correlations among crop yields. The correlations are stronger when crop losses are greatest which is when diversification opportunities are most needed. Consequently, private insurers have to rely on Government reinsurance. This study provides a potential solution for this problem. We look at the effectiveness of diversifying systemic risk across multiple crops and countries. A hierarchical Kendall copula (HKC) model is applied to estimate the correlation structure of the yield variables. The HKC is superior in that it achieves both flexibility and parsimony in modeling correlations. The flexibility makes the HKC model appropriate to represent various correlation structures precisely. This can be useful in modeling the complicated correlation structure among crop yields. The parsimony of the HKC makes it computationally efficient in modeling high-dimensional correlation structure. However, the flexibility of the HKC may raise questions regarding about the choice of the best model. Therefore, several alternative HKC models with different building blocks are evaluated. There is no significant difference across the models in terms of the key diversification results. The results show that systemic crop yield risks can been removed and that complete diversification can be achieved.

The copula-based correlation modeling proposed in this study can also be applied in other financial areas to develop risk management tools and insurance products for both agricultural and non-agricultural applications. The copula model separates the marginal distributions and the correlation of joint variables. Thus, the model can be applied to variables with arbitrary marginal distributions. The flexibility and parsimony of the copula model also ensure that it can represent different kinds of correlation structures among potentially high dimensional variables.

2.10 Appendix

Table 2.6 Simulated Distribution of Net Insurance Income ($/acre) over Multiple Crops in the US Using Clayton Clustering

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	2.95	-103.64	-10.74	-1.87
Jointly	2.41	-98.46	-7.42	-1.71
Benchmark	1.76	-50.75	-6.52	-1.72
90% Coverage Level				
Separately	12.97	-180.92	-53.48	-25.32
Jointly	10.44	-175.69	-42.40	-19.38
Benchmark	8.02	-90.28	-32.08	-15.45

Table 2.7 Simulated Distribution of Net Insurance Income ($/acre) over Multiple Crops in China Using Survival Gumbel Clustering

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	1.75	-35.89	-8.07	-1.92
Jointly	1.17	-31.06	-4.94	-1.72
Benchmark	0.89	-13.49	-3.78	-1.70
90% Coverage Level				
Separately	8.89	-94.90	-36.25	-17.80
Jointly	6.79	-86.90	-29.17	-12.24
Benchmark	4.21	-34.73	-14.43	-8.14

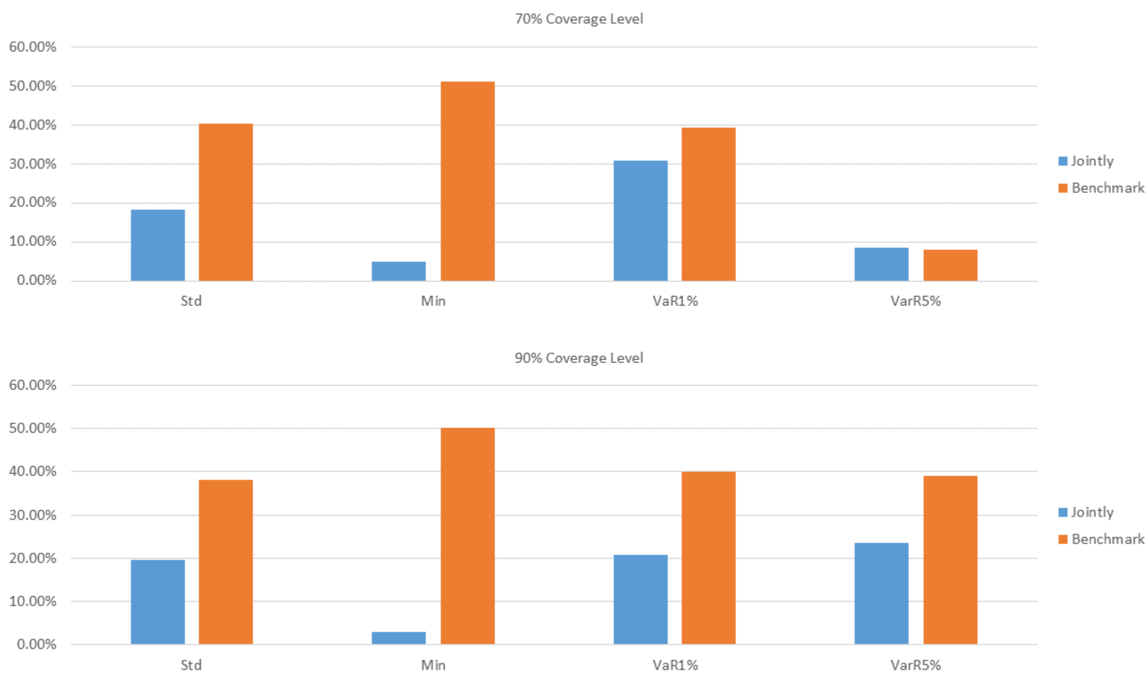

Figure 2.7 Diversification Effect (DE) Achieved by Combining Insurance Contracts across Multiple Crops in the US Using Clayton Clustering

Table 2.8 Simulated Distribution of Net Insurance Income ($/acre) over Multiple Crops in China Using Clayton Clustering

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	2.54	-96.40	-8.83	-1.85
Jointly	1.98	-80.73	-5.32	-1.75
Benchmark	1.24	-30.36	-4.66	-1.81
90% Coverage Level				
Separately	9.87	-180.13	-38.66	-19.02
Jointly	7.66	-163.15	-29.10	-12.82
Benchmark	4.68	-52.27	-15.41	-8.72

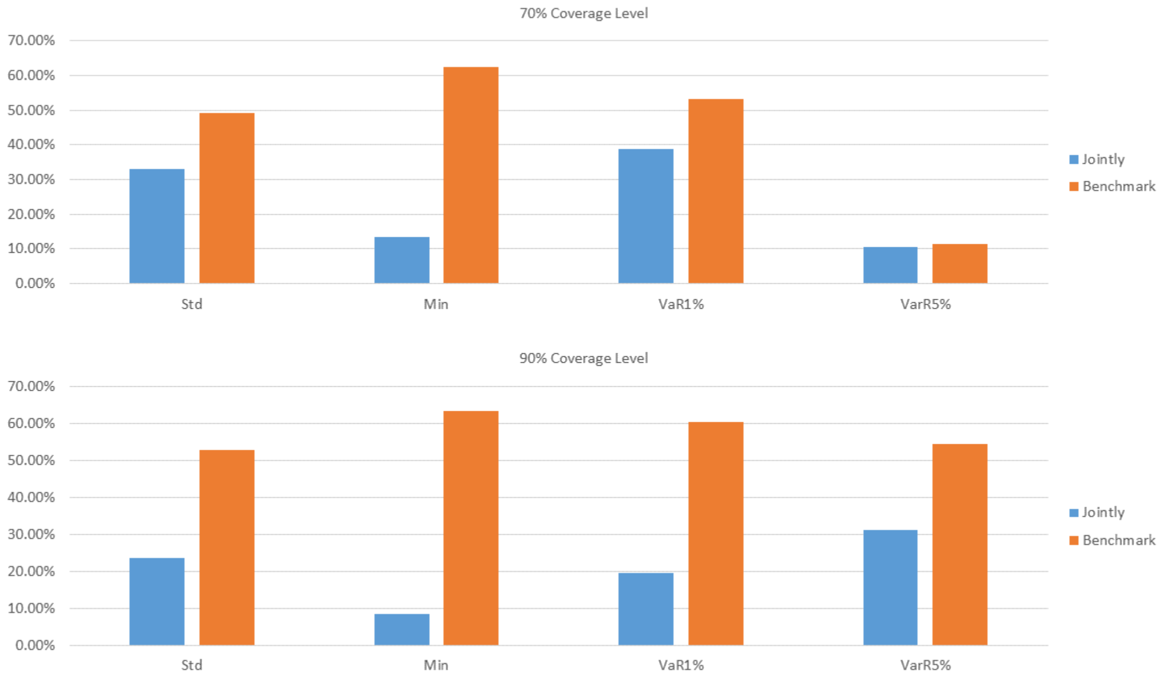

Figure 2.8 Diversification Effect (DE) Achieved by Combining Insurance Contracts across Multiple Crops in China Using Survival Gumbel Clustering

Table 2.9 Appendix D1. Simulated Distribution of Net Insurance Income ($/acre) over the US and China Using Clayton Clustering and Gaussian Nesting

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	2.19	-89.25	-6.33	-1.73
Jointly	1.44	-51.44	-4.89	-1.53
Benchmark	1.55	-47.09	-4.80	-1.51
90% Coverage Level				
Separately	9.00	-169.18	-35.50	-15.97
Jointly	6.20	-100.01	-23.69	-11.66
Benchmark	6.42	-110.10	-24.08	-11.81

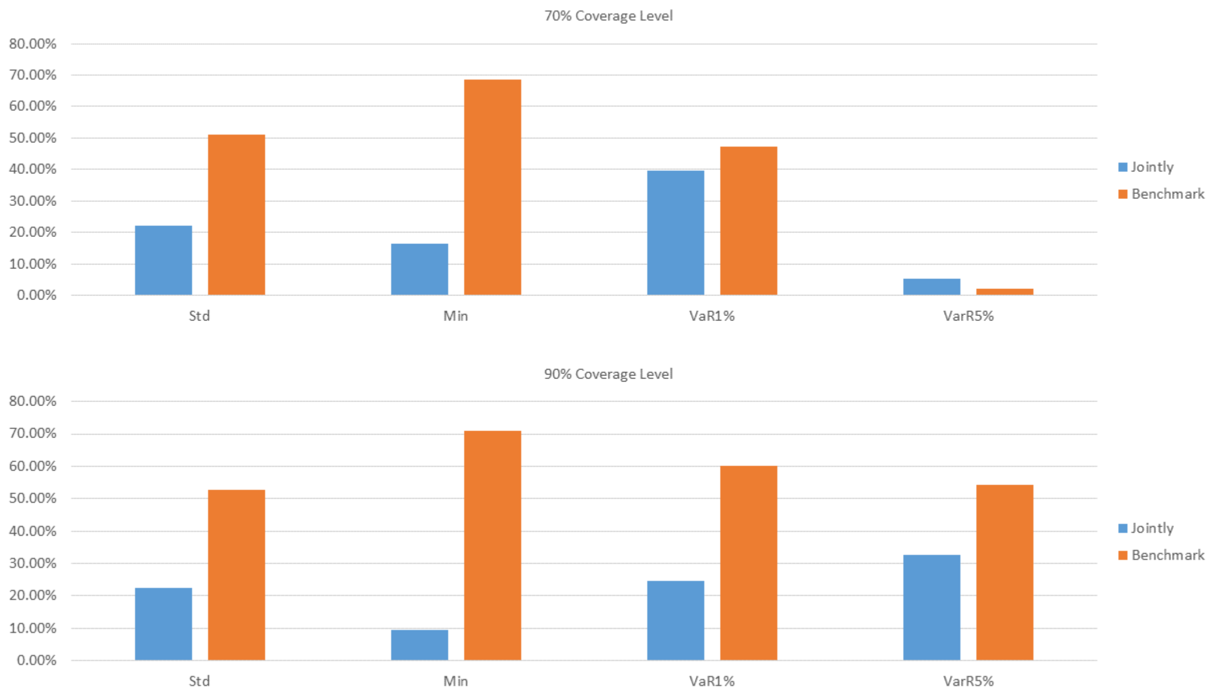

Figure 2.9 Diversification Effect (DE) Achieved by Combining Insurance Contracts across Multiple Crops in China Using Clayton Clustering

Table 2.10 Simulated Distribution of Net Insurance Income ($/acre) over the US and China Using Survival Gumbel Clustering and Frank Nesting

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	1.46	-37.40	-5.93	-1.70
Jointly	1.13	-25.32	-4.63	-1.44
Benchmark	1.04	-24.47	-4.22	-1.43
90% Coverage Level				
Separately	8.16	-104.13	-34.93	-15.03
Jointly	6.05	-60.83	-24.01	-11.68
Benchmark	5.82	-61.12	-22.38	-11.28

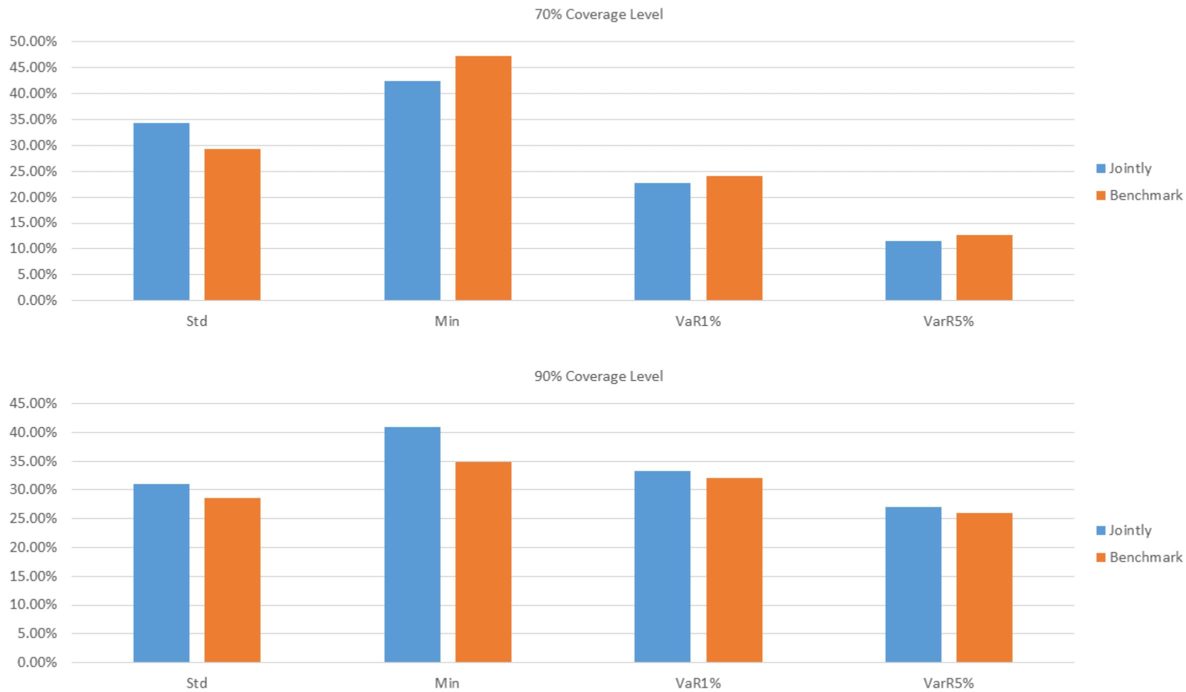

Figure 2.10 Diversification Effect (DE) Achieved by Combining Insurance Contracts across the US and China Using Clayton Clustering and Gaussian Nesting

Table 2.11 Simulated Distribution of Net Insurance Income ($/acre) over the US and China Using Clayton Clustering and Frank Nesting

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	2.19	-89.25	-6.33	-1.73
Jointly	1.35	-47.25	-4.83	-1.48
Benchmark	1.55	-47.09	-4.80	-1.51
90% Coverage Level				
Separately	9.00	-169.18	-35.50	-15.97
Jointly	6.10	-81.86	-23.43	-11.65
Benchmark	6.42	-110.10	-24.08	-11.81

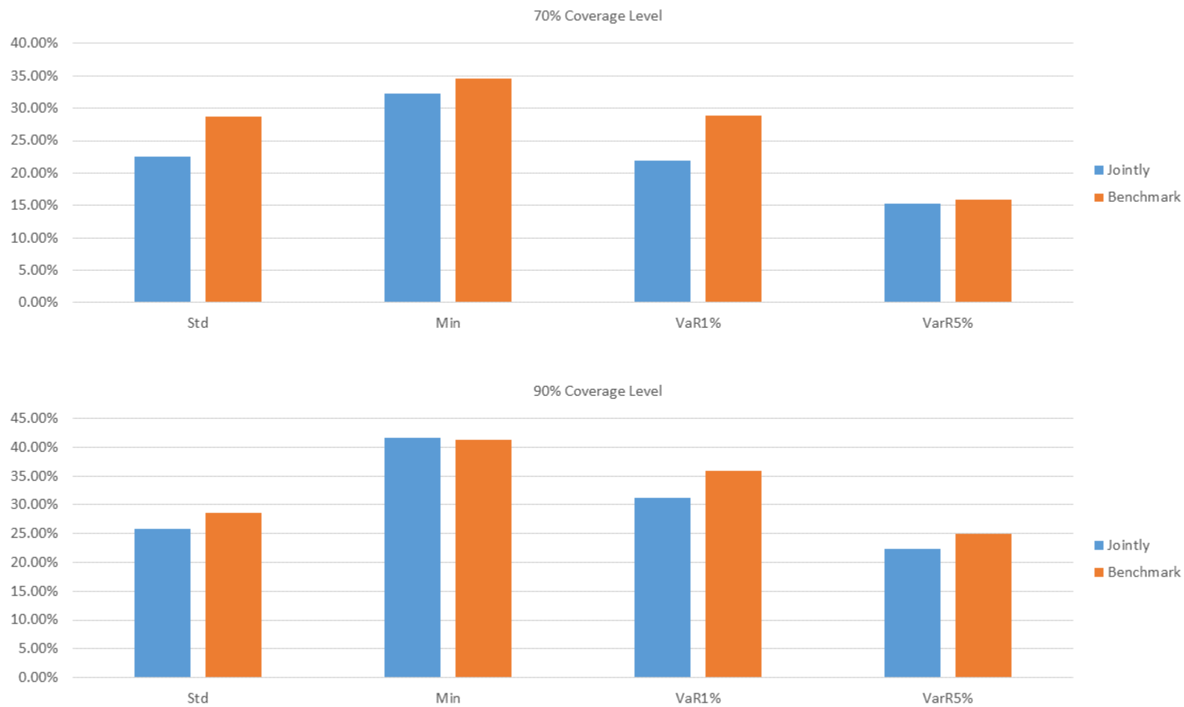

Figure 2.11 Diversification Effect (DE) Achieved by Combining Insurance Contracts across the US and China Using Survival Gumbel Clustering and Frank Nesting

Table 2.12 Simulated Distribution of Net Insurance Income ($/acre) over the US and China Using Clayton Clustering and Clayton Nesting

	Std	Min	VaR(1%)	VaR(5%)
70% Coverage Level				
Separately	2.19	-89.25	-6.33	-1.73
Jointly	1.37	-46.23	-4.68	-1.48
Benchmark	1.55	-47.09	-4.80	-1.51
90% Coverage Level				
Separately	9.00	-169.18	-35.50	-15.97
Jointly	6.22	-91.29	-23.55	-11.71
Benchmark	6.42	-110.10	-24.08	-11.81

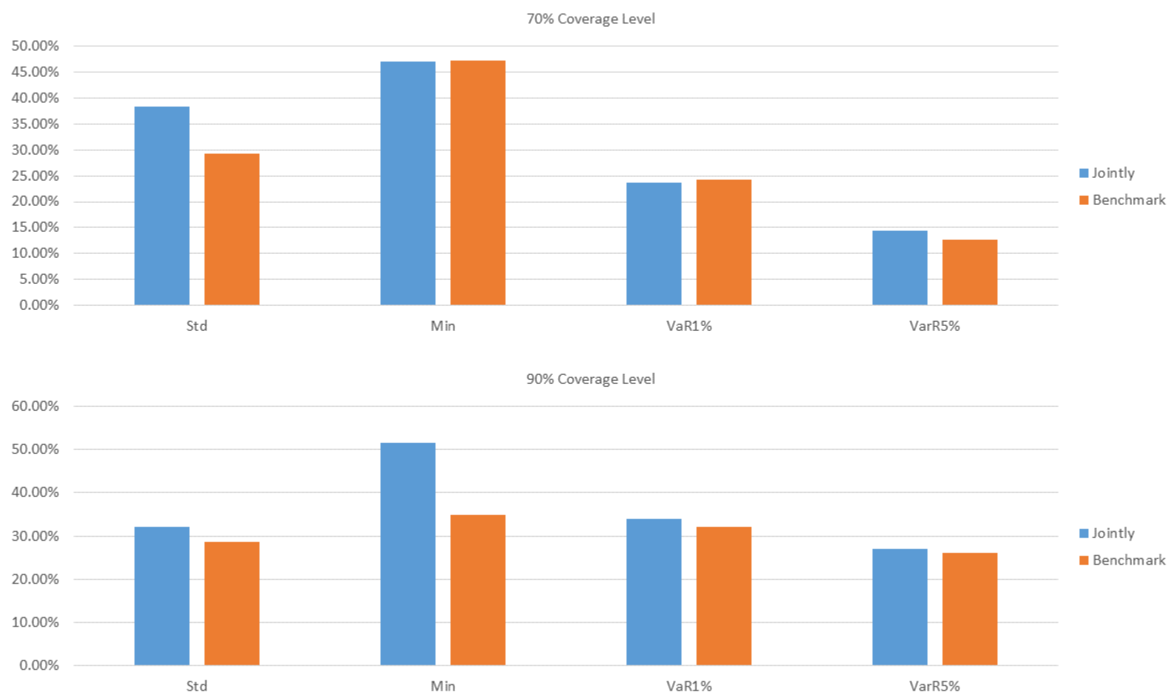

Figure 2.12 Diversification Effect (DE) Achieved by Combining Insurance Contracts across the US and China Using Clayton Clustering and Frank Nesting

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CHAPTER 3. FARMLAND INVESTMENT CHARACTERISTIC FROM A FORWARD-LOOKING PERSPECTIVE: AN EXPLANATION FOR THE HIGH RETURN-LOW RISK PARADOX

3.1 Introduction

Farmland portfolios have been shown to exhibit high expected returns, low risk, diversification benefits, and an inflation-hedge ability. Kaplan (1985) showed that farmland asset class had equity-like return, bond-like volatility, and a low return correlation with traditional asset classes. The author also indicated that farmland returns were positively correlated with the CPI. Barry (1980) found that farmland returns added mostly nonsystemic risk to a well-diversified portfolio of stocks and bonds. More recent studies, including Irwin, Forster, and Sherrick (1988), Hennings, Sherrick, and Barry (2005), Noland *et al.* (2011), Sherrick, Mallory, and Hopper (2013), and Baker, Boehlje, and Langemeier (2014) among others, found similar results in terms of the superior characteristics of farmland returns.

With changing economic conditions, the expected farmland return will vary over time (Bjornson, 1995). As a result, past performance will not be an indicator of future performance. To the best of our knowledge, this feature of farmland returns has been ignored by the literature. One reason that previous studies on farmland portfolio performance did not account for time-varying expected returns is that they were performed under the Markowitz's mean-variance (M-V) framework (Markowitz, 1968). The M-V model requires the mean and variance-covariance of asset returns as model inputs and does not allow the expected return to vary over time. These inputs are usually obtained from historical sample statistics.

Moss, Featherstone, and Baker (1987) provide evidence showing that, unlike stock asset returns, farmland asset returns are correlated over time. The authors also indicate that failing to account for this autocorrelation may result in inappropriate estimates of the variance-covariance of asset returns. The Moss *et al.* research accounts for autocorrelation and shows the implication of this autocorrelation on multi-period investments. However, they assume constant expected return by imposing steady state initial conditions on the asset return variables.

The objective of this article is to account for both autocorrelation and time-varying expected return for the modeling of multivariate farmland returns and to use the model to make forward-looking estimations and predictions of farmland portfolio investment risk and return. Both nominal and real returns (inflation excluded) are considered. Time-series techniques are used to model individual land return series. Correlations among multiple return series are estimated by copulas (Sklar, 1959). An advantage of using copulas is that marginal distributions are not limited to normality and can thus be extended to allow for potential fat tails existing in farmland returns.

The results suggest that the forward-looking farmland investment risk-return profile is significantly different from the risk and return observed historically. As of July 2017, the expected return is low in the short term, and it then recovers over a longer period. It takes multiple years for the expected return to reach the long-term equilibrium. The forward-looking expected return varies across different holding periods and the large average return level observed historically can only be attained through long-term investments. Also, because of autocorrelation in the return series, the diversification effect across time is much smaller than if the annual returns were independent. As a result, the amount of risk involved in long-term investment is considerably larger than the historical sample volatility. This shows that while superior return can be expected through long-term investments, the risk involved in the long holding period is also substantial. These findings may help explain the "high return and low risk" puzzle in farmland investment and provide potential investors and policymakers with a better understanding of the characteristics of this non-traditional asset class.

3.2 Empirical Framework

3.2.1 Capitalization Theory

Assuming a constant discount rate and expected growth rate, the relationship between cash rents and land values can be represented by the capitalization model (Featherstone, Taylor, and Gibson, 2017).

$$L_t = C_t/(r - g), \tag{3.1}$$

where L_t is the land value at time t, C_t the cash rent to land at time t, r the discount rate, and g the income growth rate. The holding period return from owning land can be calculated as

$$R_t = \frac{L_t - L_{t-1} + C_{t-1}}{L_{t-1}} = \frac{L_t}{L_{t-1}} - 1 + r - g \tag{3.2}$$

The annual growth rate of farmland prices ($\frac{L_t}{L_{t-1}} - 1$) is significantly autocorrelated as shown by Lence (2014). Therefore, it is reasonable to assume that farmland returns, according to equation (3.2), are also correlated over time. This assumption is consistent with the empirical observations provided by Moss, Featherstone, and Baker (1987). In this article, the autocorrelation in farmland return series is accounted for using time-series techniques.

3.2.2 Time-Series Models

Time-series models have long been used in describing economic and financial data (Cochrane, 2005; Tsay, 2005; Adhikari, Ratnadip, and Agrawal, 2013). The autoregressive-moving-average (ARMA) process (Box *et al.*, 2015) is particularly useful for modeling time-series data and for predicting future values based on past observations. The ARMA model accounts for potential autocorrelation in the series and allows for time-varying expected values.

Defining the lag operator L as $Lx_t = x_{t-1}$, an ARMA(p,q) process can be written as

$$(1 - \sum_{i=1}^{p} \phi_i L^i) y_t = (1 + \sum_{j=1}^{q} \theta_j L^j) \varepsilon_t, \tag{3.3}$$

or $\phi(L)y_t = \theta(L)\varepsilon_t$, where $\phi(\cdot)$ and $\theta(\cdot)$ are p^{th} and q^{th} lag polynomials respectively.

An appropriate ARMA model for a specific time-series dataset can be selected and estimated by Box-Jenkins methodology (Box *et al.*, 2015). The methodology follows a three-step procedure: model identification, parameter estimation, and diagnostic checking. Specifically, the orders p and q of an ARMA (p, q) model are first tentatively selected. The parameters of the model are then estimated from the data. Finally, diagnostic tests are performed to check the adequacy of the estimated model in describing the data. This three-step procedure is iterated until the satisfactory model is identified.

3.2.3 Copulas

Correlations among multiple time series can be modeled by copulas. Copulas were first introduced by Sklar (1959). Sklar's theorem states that if F is an arbitrary k-dimensional joint continuous distribution function, then the associated copula is unique and defined as a continuous function $C : [0,1]^k \to [0,1]$ that satisfies the equation

$$F(x_1, \ldots, x_k) = C\left[F_1(x_1), \ldots, F_k(x_k)\right], \qquad x_1, \ldots, x_k \in \mathbb{R}, \tag{3.4}$$

where $F_1(x_1), \ldots, F_k(x_k)$ are the respective marginal distributions. Let c denote the density function of the copula C, which can be described as

$$c(u_1, \ldots, u_k) = \frac{\partial^k C(u_1, \ldots, u_k)}{\partial u_1 \cdots \partial u_k}, \tag{3.5}$$

There are different basic parametric copula families. The most frequently used are elliptical copulas and Archimedean copulas (Power, Vedenov, and Hong, 2009; Cooper *et al.*, 2012). The standard Gaussian copula from the elliptical family is used in this article.

The Gaussian copula takes the form of

$$C^N(u_1, \ldots, u_k \mid \Sigma) = \Phi_\Sigma\left[\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_k)\right], \tag{3.6}$$

where Φ_Σ is a k-dimensional normal distribution with zero mean and correlation matrix Σ, and Φ^{-1} is the inverse distribution function of the standard normal distribution.

3.3 Empirical Application

Our dataset consists of annual state-level nominal cash rents and land values for cropland in 15 major agricultural producing states in the United States. The dataset spans from 1967 to 2015. All the data are taken from the USDA National Agricultural Statistics Service (NASS) database. The data were collected by the NASS in annual surveys on cash rents and cropland values. Real cash rents and land values are calculated by dividing the nominal values by the corresponding consumer price index. The annual rate of return is calculated as the sum of cash rents and capital gain divided by land value[1].

Each of the marginal return series is first modeled using time-series techniques. The correlation structure of the residuals from the marginal time series is then constructed by

[1]Changes in the portion of rented farmland may impose bias on the cash rent data. However, as the landowners who rent out farmland account for the majority of all landowners in the US (Zhang, 2015), it seems unlikely that the potential bias can render the analysis in this article invalid.

copulas. A Ljung-Box test (Ljung and Box, 1978) is used to check if autocorrelation exists in the individual return series. The test results indicate that the null hypothesis is rejected for both the nominal and real return series in all the 15 states. This indicates that there are significant autocorrelations for the return series. The individual return series are fitted using four candidate models, namely, ARMA(1,1), AR(1), MA(1), and white noise models [2]. The Student's t distribution is used to account for potentially heavy tails. If the estimated degree of freedom of the Student's t error distribution is greater than 10, a normal distribution is used. The Bayesian Information Criterion (BIC) is used to select the best model. Model sufficiency is tested by performing one-sample Kolmogorov-Smirnov (K-S) goodness-of-fit test on the standardized residuals. The p-value from the K-S test is greater than 5% for all the return series; this confirms the fitness of the residual distributions. A Ljung-Box test is performed again on the residuals; this shows that there is no autocorrelation in the residual series. Table 3.1 presents fitted time-series model for each of the return series. The correlation structure of the residuals is estimated by the Gaussian copula using maximum likelihood [3].

With estimated copula models and marginal time series, an optimal investment portfolio can be constructed by the following procedure:

(1) A sample of the standardized residuals is simulated from the copula model.

(2) Individual returns for the next period are projected using the simulated residuals and the respective marginal time series.

(3) The portfolio return is calculated as the weighted average of individual returns.

(4) Portfolio weights are optimized by maximizing the portfolio return with a given risk level.

[2] Unit root is checked for all the return series using Augmented Dickey Fuller (ADF) test. The null hypothesis is rejected at the 5% significance level for 13 of the 15 nominal return series, and at the 10% significance level for all the 15 nominal return series. The null hypothesis is rejected at the 5% significance level for all the 15 real return series. Therefore, we assume no unit root for the return series.

[3] To account for any potential tail dependence, Student's t copula is also used as an alternative model to estimate the correlations. There is no significant impact on investment portfolio's risk-return profile.

Table 3.1 Fitted Time-Series Models for Farmland Returns

	Nominal Return			Real Return		
	Model	Constant	Parameter	Model	Constant	Parameter
Arkansas	AR(1)	0.047	0.610	AR(1)	0.038	0.551
Illinois	AR(1)	0.059	0.567	AR(1)	0.043	0.549
Indiana	AR(1)	0.036	0.744	AR(1)	0.028	0.718
Iowa	AR(1)	0.048	0.678	AR(1)	0.035	0.687
Kansas	AR(1)	0.037	0.676	AR(1)	0.025	0.684
Louisiana	AR(1)	0.041	0.671	AR(1)	0.042	0.500
Michigan	AR(1)	0.037	0.684	AR(1)	0.028	0.623
Minnesota	AR(1)	0.034	0.761	AR(1)	0.024	0.780
Mississippi	AR(1)	0.029	0.772	AR(1)	0.031	0.663
Missouri	AR(1)	0.063	0.586	AR(1)	0.052	0.535
North Dakota	AR(1)	0.055	0.666	AR(1)	0.041	0.659
Ohio	AR(1)	0.049	0.607	AR(1)	0.037	0.597
South Dakota	AR(1)	0.082	0.561	AR(1)	0.065	0.539
Texas	AR(1)	0.043	0.640	AR(1)	0.027	0.602
Wisconsin	AR(1)	0.033	0.764	AR(1)	0.027	0.719

3.3.1 Risk-return Profile of Nominal Farmland Returns

Future farmland asset returns are simulated in order to construct the forward-looking optimal portfolios. Table 3.2 shows the projected mean and standard deviation of state-level nominal returns with different holding periods, as well as the historical sample mean and standard deviation. Due to the recent poor performance, the one-year-out expected return is much lower than the historical average. When we extend the holding period, the expected return recovers. This recovery speed varies across different states. The expected return exceeds the historical level in a ten-year holding period for Louisiana. For other states, the expected return within ten years is lower than the historical average. The standard deviation of the returns is either higher or lower with a longer holding period. This is because a longer period entails larger uncertainty with the existence of autocorrelation; however, the diversification effect over time tends to reduce the total risk within the entire holding period.

These two offsetting effects cause an ambiguous relationship between volatility and the length of holding period.

Table 3.2 Statistics of Farmland Nominal Returns in Individual States

	Projected - 1Yr		Projected - 5Yr		Projected - 10Yr		Historical	
	E(r)	σ(r)	E(r)	σ(r)	E(r)	σ(r)	E(r)	σ(r)
Arkansas	8.92%	7.30%	10.75%	6.17%	11.33%	5.09%	13.22%	8.94%
Illinois	5.79%	10.96%	10.34%	9.75%	11.83%	7.76%	13.58%	11.26%
Indiana	5.54%	10.49%	9.19%	11.10%	11.11%	10.22%	14.49%	11.71%
Iowa	5.00%	11.95%	9.89%	11.00%	12.04%	9.58%	16.01%	13.57%
Kansas	0.70%	8.86%	5.71%	8.56%	8.08%	7.31%	13.36%	10.48%
Louisiana	9.74%	7.85%	10.97%	7.09%	11.53%	6.08%	10.92%	9.23%
Michigan	4.64%	7.68%	8.14%	7.00%	9.70%	6.11%	12.56%	9.55%
Minnesota	5.35%	11.75%	9.04%	12.45%	11.06%	12.27%	16.66%	13.36%
Mississippi	8.08%	8.20%	9.96%	8.57%	11.03%	8.01%	13.88%	10.51%
Missouri	7.45%	10.08%	11.91%	8.36%	13.39%	6.76%	15.57%	10.68%
North Dakota	3.29%	9.17%	9.71%	8.36%	12.56%	7.18%	17.31%	12.36%
Ohio	5.82%	11.75%	9.66%	11.34%	10.99%	9.72%	13.19%	10.90%
South Dakota	6.95%	14.46%	13.71%	14.38%	15.95%	11.01%	19.34%	11.78%
Texas	7.40%	9.82%	9.75%	8.78%	10.64%	7.45%	10.88%	8.43%
Wisconsin	8.58%	8.07%	10.68%	8.39%	11.81%	7.83%	16.22%	11.85%

Figure 3.1 shows the efficient frontiers of forward-looking farmland portfolio nominal returns with different holding periods derived from the ARMA-copula model. For a given risk level, the expected return increases with the holding period. For the minimum-risk portfolios, the expected returns are 7.34%, 10.05%, and 11.27% for the one-year, five-year, and ten-year holding periods, respectively[4]. The minimum risk decreases with a longer holding period. The minimum volatilities are 5.99%, 5.50%, and 4.68% for the one-year, five-year, and ten-year holding periods, respectively.

Figure 3.2 compares the efficient frontiers of nominal returns derived from the ARMA-copula model and from the M-V approach for one-year, five-year, and ten-year holding periods. The inputs for the M-V approach are a historical sample mean and variance-

[4]Note we do not take property tax or transaction costs into account.

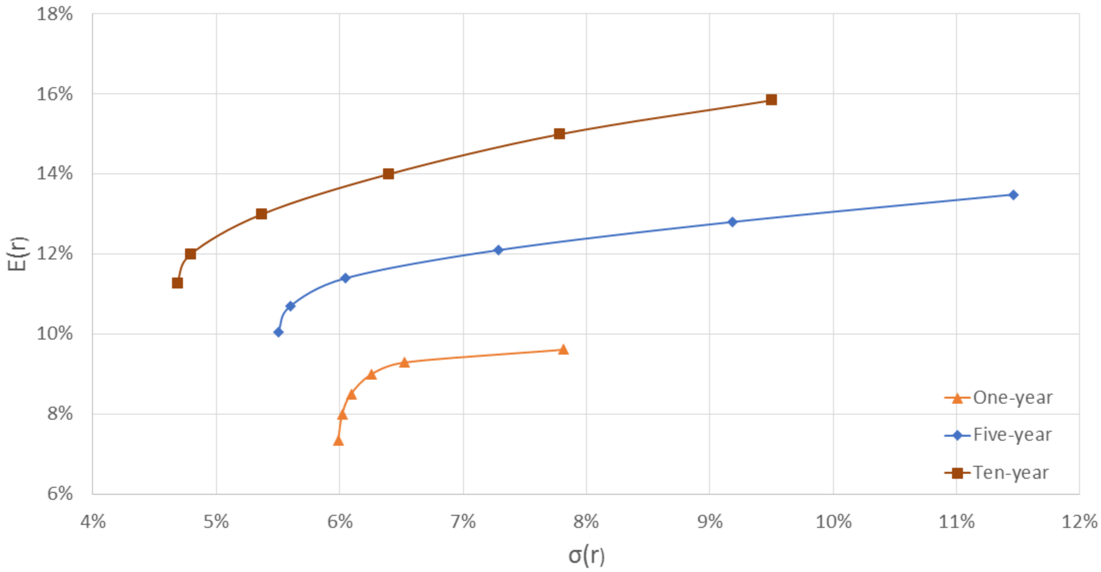

Figure 3.1 Efficient frontiers of nominal farmland return with different holding periods

covariance with the implicit assumption that returns are independent across time. The expected return based on the M-V approach, is identical across different holding periods. The standard deviation decreases with a longer holding period, due to diversification over time, as measured by the square root of the length of the holding period. By contrast, the expected return, implied by the ARMA-copula model, varies across different holding periods due to the time-varying expected return. The standard deviation for a longer holding period does not decrease as much as in the M-V approach because of the autocorrelation in the return series. These results indicate that the time-varying expected return and autocorrelation in the return series have important implications on the risk-return profile of farmland investment.

The ARMA-copula results show that the superior performance predicted by the M-V approach should be treated with caution. First, the high expected return can only be

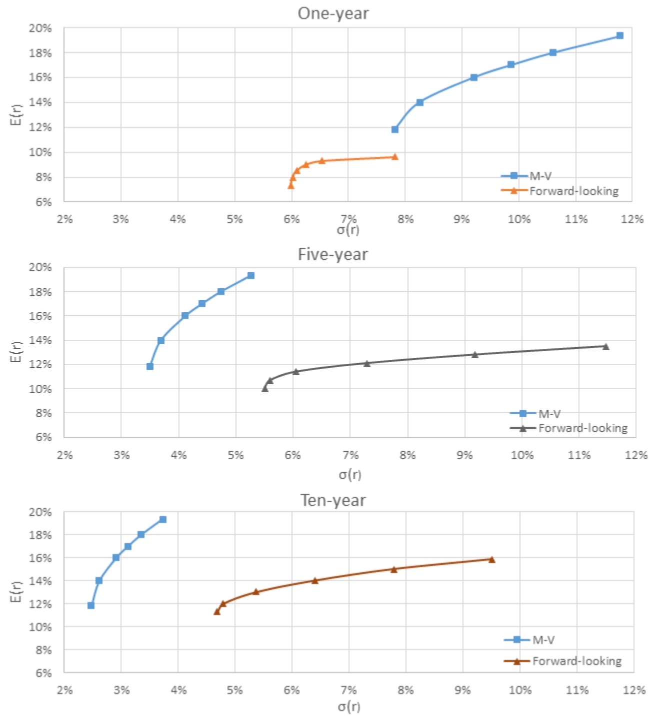

Figure 3.2 Efficient frontiers of nominal farmland return as implied by the ARMA–copula model and the M-V method

achieved with a long holding period. The graphs in Figure 3.2 show that for a one-year holding period, the forward-looking expected return is much lower than the M-V approach implies. The forward-looking expected return is closer to the historical average for a five-year holding period and becomes comparable for a ten-year holding period. Second, the forward-looking risk involved in farmland investment is not as low as implied by the M-V approach for long holding periods. This is because the diversification effect over time is offset by the autocorrelation in the return series. Figure 3.2 shows that the forward-looking standard deviation is lower than the M-V value for a one-year holding period; however, it is higher than the M-V value for the five-year and ten-year holding periods. In summary, from a forward-looking perspective, while the high expected return can only be achieved with long-term investment, the risk involved with the long holding period is also relatively high.

This may explain why farmland assets comprise a small portion of investors' portfolios, in spite of the superior historical performance.

3.3.2 Risk-return Profile of Real Farmland Returns

Similar results are observed for real farmland returns. Table 3.3 shows the real-return statistics for individual states. The one-year-out expected real return is lower than the historical average for all the states except Louisiana. When the holding period is longer, the expected real return improves, as was the case for the nominal return. For all states except Louisiana, the expected real return with a ten-year holding period is below the historical average. The relationship between volatility and the length of holding period is also ambiguous for real returns. While a longer period entails larger uncertainty due to autocorrelation, the diversification effect over time reduces the overall risk within the entire holding period.

Table 3.3 Statistics of Farmland Real Returns in Individual States

	Projected - 1Yr		Projected - 5Yr		Projected - 10Yr		Historical	
	E(r)	σ(r)	E(r)	σ(r)	E(r)	σ(r)	E(r)	σ(r)
Arkansas	7.04%	6.06%	7.87%	4.75%	8.16%	3.83%	8.95%	7.28%
Illinois	3.80%	8.84%	7.05%	7.21%	8.18%	5.73%	9.28%	9.82%
Indiana	3.85%	10.51%	6.28%	12.90%	7.67%	10.37%	10.16%	10.12%
Iowa	3.02%	11.50%	6.74%	11.66%	8.51%	10.10%	11.66%	12.09%
Kansas	-1.29%	7.60%	3.00%	7.13%	5.05%	6.24%	9.10%	9.17%
Louisiana	7.69%	7.42%	8.31%	8.99%	8.38%	5.70%	6.66%	7.53%
Michigan	2.93%	6.51%	5.34%	5.70%	6.29%	4.75%	8.27%	8.23%
Minnesota	3.36%	13.00%	6.17%	11.27%	7.75%	11.24%	12.26%	11.57%
Mississippi	6.61%	7.45%	7.95%	6.61%	8.53%	5.71%	9.57%	8.58%
Missouri	5.81%	8.25%	9.00%	6.69%	10.09%	5.14%	11.26%	9.00%
North Dakota	1.25%	8.00%	6.55%	7.21%	9.02%	6.22%	12.96%	10.78%
Ohio	3.95%	10.23%	6.70%	9.08%	7.79%	7.35%	8.87%	9.28%
South Dakota	4.46%	13.32%	10.11%	9.82%	11.97%	8.10%	15.00%	10.58%
Texas	4.82%	7.25%	5.90%	6.40%	6.30%	5.51%	6.65%	7.27%
Wisconsin	6.81%	7.38%	8.07%	7.01%	8.76%	6.45%	11.86%	10.35%

Figure 3.3 shows the efficient frontiers of forward-looking real returns with different holding periods. For a given risk level, the expected real return increases with the holding period. For the minimum-risk portfolios, the expected returns are 5.08%, 7.00%, and 8.48% for the one-year, five-year, and ten-year holding periods, respectively. The minimum risk decreases with a longer holding period. The minimum volatilities are 4.87%, 4.18%, and 3.58% for the one-year, five-year, and ten-year holding periods, respectively. These observations are consistent with what has been observed for nominal returns.

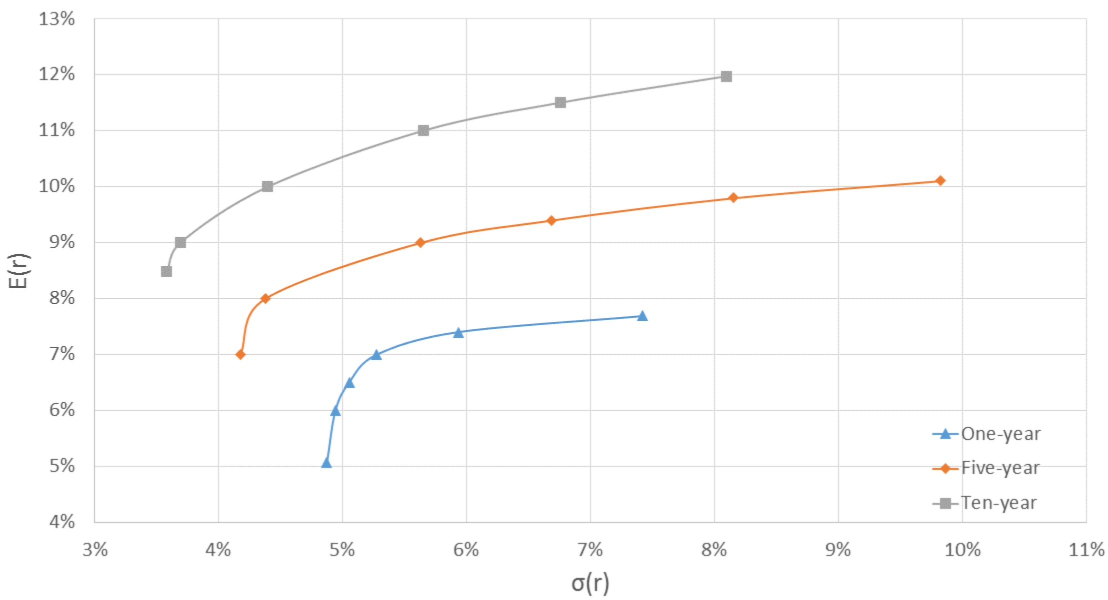

Figure 3.3 Efficient frontiers of real farmland return with different holding periods

Figure 3.4 compares the forward-looking efficient frontiers of real farmland return and the efficient frontiers as implied by the M-V approach. As was true with nominal returns, the forward-looking expected real return varies across different holding periods. For a given expected return level, volatility decreases for longer holding periods, but to a lesser degree as in the M-V approach. This reduced time diversification effect is due to the autocorrelation in the return series. These results indicate that the time-varying expected return and autocorrelation in the return series have similar implications on the risk-return profile for real returns as for nominal returns. That is, while the superior expected return predicted by the M-V approach can only be attained through a long holding period from a forward-looking perspective, the risk involved in the long holding period is much higher than implied by the M-V approach.

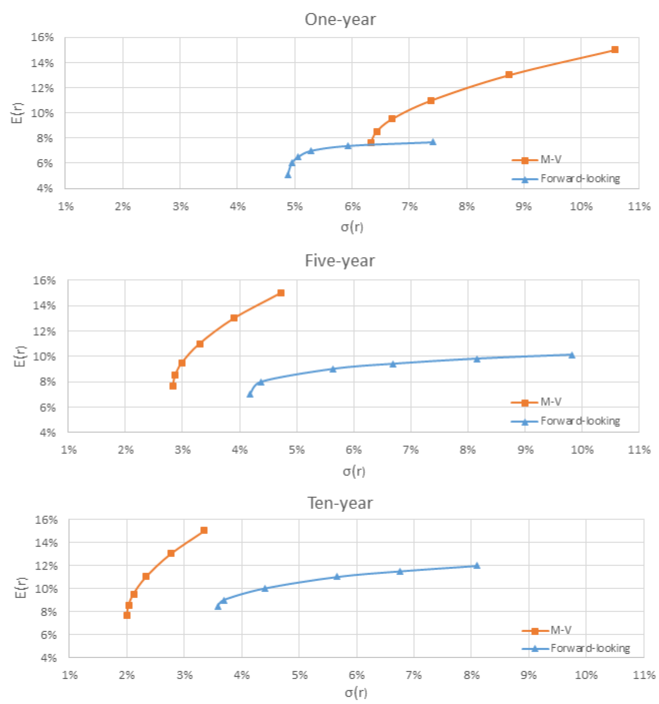

Figure 3.4 Efficient frontiers of real farmland return as implied by the ARMA-copula model and the M-V method

3.4 Conclusions

The ARMA-copula model proposed in this article can serve as a tool for forward-looking farmland portfolio management by taking into account autocorrelation and time-varying patterns in farmland returns. The optimal portfolio is constructed based on projected future returns instead of historical values. We show that the forward-looking risk-return profile is significantly different than the historical profile for both nominal and real returns. The results shed light on the high return-low risk paradox in the existing land value literature. The results may help farmland investors gain a better understanding of expected farmland returns and to optimize their portfolios for better future performance.

3.5 Appendix

Table 3.4 Optimal Forward-looking Farmland Portfolio Weights for One-year Holding Period with Nominal Returns

$E(r)$	7.34%	8.00%	8.50%	9.00%	9.30%	9.62%
$\sigma(r)$	5.99%	6.02%	6.09%	6.26%	6.53%	7.81%
Arkansas	0.21	0.27	0.32	0.40	0.43	0.00
Illinois	0.00	0.00	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.00	0.00	0.00	0.00	0.00	0.00
Louisiana	0.14	0.16	0.19	0.31	0.47	1.00
Michigan	0.14	0.05	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.10	0.12	0.13	0.05	0.00	0.00
Missouri	0.00	0.00	0.00	0.00	0.00	0.00
North Dakota	0.12	0.08	0.04	0.00	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.00	0.00	0.00	0.00	0.00	0.00
Texas	0.20	0.18	0.16	0.07	0.00	0.00
Wisconsin	0.09	0.14	0.16	0.17	0.09	0.00

Table 3.5 Optimal Forward-looking Farmland Portfolio Weights for Five-year Holding Period with Nominal Returns

$E(r)$	10.05%	10.70%	11.40%	12.10%	12.80%	13.48%
$\sigma(r)$	5.50%	5.60%	6.05%	7.29%	9.18%	11.46%
Arkansas	0.44	0.56	0.62	0.46	0.17	0.00
Illinois	0.00	0.00	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.00	0.00	0.00	0.00	0.00	0.00
Louisiana	0.10	0.12	0.10	0.00	0.00	0.00
Michigan	0.14	0.00	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.00	0.00	0.00	0.00	0.00	0.00
Missouri	0.00	0.00	0.00	0.07	0.13	0.00
North Dakota	0.14	0.08	0.00	0.00	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.00	0.06	0.24	0.47	0.70	1.00
Texas	0.16	0.11	0.00	0.00	0.00	0.00
Wisconsin	0.02	0.07	0.04	0.00	0.00	0.00

Table 3.6 Optimal Forward-looking Farmland Portfolio Weights for Ten-year Holding Period with Nominal Returns

$E(r)$	11.27%	12.00%	13.00%	14.00%	15.00%	15.85%
$\sigma(r)$	4.68%	4.79%	5.37%	6.39%	7.78%	9.50%
Arkansas	0.51	0.58	0.53	0.27	0.00	0.00
Illinois	0.02	0.00	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.00	0.00	0.00	0.00	0.00	0.00
Louisiana	0.13	0.11	0.00	0.00	0.00	0.00
Michigan	0.11	0.00	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.00	0.00	0.00	0.00	0.00	0.00
Missouri	0.00	0.00	0.14	0.26	0.34	0.00
North Dakota	0.12	0.10	0.01	0.00	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.00	0.13	0.30	0.47	0.66	1.00
Texas	0.12	0.08	0.01	0.00	0.00	0.00
Wisconsin	0.00	0.00	0.00	0.00	0.00	0.00

Table 3.7 Optimal Farmland Portfolio Weights as Implied by the M-V Approach with Nominal Returns

$E(r)$	11.82%	14.00%	16.00%	17.00%	18.00%	19.34%
$\sigma(r)$	7.82%	8.26%	9.21%	9.86%	10.60%	11.78%
Arkansas	0.22	0.35	0.30	0.27	0.20	0.00
Illinois	0.11	0.00	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.05	0.00	0.00	0.00	0.00	0.00
Louisiana	0.04	0.00	0.00	0.00	0.00	0.00
Michigan	0.00	0.00	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.00	0.00	0.00	0.00	0.00	0.00
Missouri	0.00	0.00	0.00	0.00	0.00	0.00
North Dakota	0.00	0.00	0.00	0.00	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.00	0.27	0.52	0.65	0.76	1.00
Texas	0.58	0.38	0.18	0.08	0.00	0.00
Wisconsin	0.00	0.00	0.00	0.00	0.04	0.00

Table 3.8 Optimal Forward-looking Farmland Portfolio Weights for One-year Holding Period with Real Returns

$E(r)$	5.08%	6.00%	6.50%	7.00%	7.40%	7.69%
$\sigma(r)$	4.87%	4.94%	5.05%	5.28%	5.93%	7.42%
Arkansas	0.23	0.32	0.37	0.44	0.39	0.00
Illinois	0.00	0.00	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.00	0.00	0.00	0.00	0.00	0.00
Louisiana	0.07	0.11	0.14	0.24	0.57	1.00
Michigan	0.10	0.00	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.10	0.13	0.14	0.10	0.00	0.00
Missouri	0.00	0.00	0.00	0.00	0.00	0.00
North Dakota	0.16	0.10	0.03	0.00	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.00	0.00	0.00	0.00	0.00	0.00
Texas	0.26	0.21	0.18	0.05	0.00	0.00
Wisconsin	0.08	0.14	0.15	0.17	0.04	0.00

Table 3.9 Optimal Forward-looking Farmland Portfolio Weights for Five-year Holding Period with Real Returns

$E(r)$	7.00%	8.00%	9.00%	9.40%	9.80%	10.11%
$\sigma(r)$	4.18%	4.38%	5.63%	6.68%	8.16%	9.82%
Arkansas	0.42	0.62	0.32	0.07	0.00	0.00
Illinois	0.02	0.00	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.00	0.00	0.00	0.00	0.00	0.00
Louisiana	0.03	0.05	0.00	0.00	0.00	0.00
Michigan	0.11	0.00	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.06	0.05	0.00	0.00	0.00	0.00
Missouri	0.00	0.00	0.35	0.49	0.28	0.00
North Dakota	0.12	0.00	0.00	0.00	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.00	0.11	0.33	0.44	0.72	1.00
Texas	0.22	0.09	0.00	0.00	0.00	0.00
Wisconsin	0.02	0.08	0.00	0.00	0.00	0.00

Table 3.10 Optimal Forward-looking Farmland Portfolio Weights for Ten-year Holding Period with Real Returns

$E(r)$	8.48%	9.00%	10.00%	11.00%	11.50%	11.97%
$\sigma(r)$	3.58%	3.69%	4.40%	5.65%	6.76%	8.10%
Arkansas	0.71	0.60	0.31	0.00	0.00	0.00
Illinois	0.00	0.00	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.00	0.00	0.00	0.00	0.00	0.00
Louisiana	0.17	0.10	0.00	0.00	0.00	0.00
Michigan	0.00	0.00	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.02	0.00	0.00	0.00	0.00	0.00
Missouri	0.00	0.14	0.42	0.52	0.25	0.00
North Dakota	0.00	0.00	0.00	0.00	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.07	0.14	0.27	0.48	0.75	1.00
Texas	0.00	0.00	0.00	0.00	0.00	0.00
Wisconsin	0.03	0.02	0.00	0.00	0.00	0.00

Table 3.11 Optimal Farmland Portfolio Weights as Implied by the M-V Approach with Real Returns

$E(r)$	7.62%	8.50%	9.50%	11.00%	13.00%	15.00%
$\sigma(r)$	6.33%	6.43%	6.69%	7.38%	8.73%	10.58%
Arkansas	0.29	0.38	0.41	0.40	0.30	0.00
Illinois	0.10	0.04	0.00	0.00	0.00	0.00
Indiana	0.00	0.00	0.00	0.00	0.00	0.00
Iowa	0.00	0.00	0.00	0.00	0.00	0.00
Kansas	0.01	0.00	0.00	0.00	0.00	0.00
Louisiana	0.12	0.01	0.00	0.00	0.00	0.00
Michigan	0.00	0.00	0.00	0.00	0.00	0.00
Minnesota	0.00	0.00	0.00	0.00	0.00	0.00
Mississippi	0.00	0.00	0.00	0.00	0.00	0.00
Missouri	0.00	0.00	0.00	0.00	0.00	0.00
North Dakota	0.00	0.10	0.08	0.02	0.00	0.00
Ohio	0.00	0.00	0.00	0.00	0.00	0.00
South Dakota	0.00	0.02	0.17	0.40	0.64	1.00
Texas	0.47	0.44	0.34	0.18	0.00	0.00
Wisconsin	0.00	0.00	0.00	0.00	0.06	0.00

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CHAPTER 4. PREDICTING FARMLAND ASSET RETURNS USING CAPITAL MARKET RISK FACTORS

4.1 Introduction

Farmland accounts for over 80 percent of the total value of a typical farmer's portfolio (Economic Research Service, 2016). Farmland returns therefore greatly impact the financial well-being of the nation's agricultural sector. This important role of farmland assets has generated a large literature on farmland returns. Barry (1980) first analyzed farmland returns using market portfolio return as the only risk factor. The author found that farmland assets added insignificant systematic risk to a well-diversified investment portfolio. Several studies extended Barry's work by applying multifactor asset pricing models that included a set of risk factors beyond market portfolio return (see, among others, Arthur, Carter, and Abizadeh, 1988; Irwin *et al.*, 1988; Bjornson, 1995; Kuethe, Hubbs, Morehart, 2014). The risk factors are either explicit macroeconomic variables or implicit principle components extracted from observable variables. Moss and Katchova (2005) summarized the findings on the relationship between farmland return and market factors, concluding that while farmland assets exhibit low systematic risk, farmland return is sensitive to several financial market risk factors.

After a decline in 2016, average US farmland value increased slightly in 2017. Farm income, however, has declined and is still under stress following a record high in 2013. The relationship between farmland price and the income produced by the farmland can be expressed by the capitalization model $P = I/(r - g)$, where P represents the farmland price, I the farmland income, r the discount rate, and g the income growth rate. When the farmland income is decreasing, an increase in farmland price, according to the capitalization model, can potentially be caused by a lowered discount rate. The discount rate represents the opportunity cost or required rate of return for investors to invest in farmland. This rate can be thought of as the risk-free rate plus a risk premium associated with farmland investment. Given that the risk-free rate has been low after the 2007-09 financial crisis, the decreased discount rate perhaps indicates that investors now require lower rate of return or risk premium on farmland investment. The same "low income high asset price" phenomenon has also been observed in the equity market. The S&P 500 index level has increased significantly in 2017 as has the price to earnings ratio[1]. This shows that investors are accepting of a lower risk premium in equity investments.

At the end of year 2008 when federal funds rate as the traditional monetary policy target reached its lower bound of zero, the Federal Reserve started to implement a large-scale asset purchase program by purchasing substantial quantities of medium- and long-maturity assets. Holdings of the assets by private investors were replaced by short-term, risk-free bank reserves which increased the total money supply in the economy. Figure 4.1 and 4.2 show the broad money supply (M2) and the money to GDP ratio in the US starting from 2007 respectively. The increased money supply potentially impacted several macroeconomic factors and potentially changed investor behavior. According to Gagnon *et al.* (2010), the first macroeconomic factor being impacted by increased money supply is the risk premium on the assets that the Fed purchased as well as other related assets via the portfolio balance effect (Tobin, 1958). By purchasing a particular asset, the Federal Reserve reduces the aggregate supply for particular assets. In market equilibrium, the risk premium then must adjust to match aggregate demand with the decreased aggregate supply. In order for investors to adjust their required rate of return, the expected return on the purchased and other assets has to fall. As Emmerling *et al.* (2015) indicate, the purchases by the Federal Reserve bid up the price and the realized return of the purchased assets, which in turn lowered the expected

[1] Historical data of the S&P 500 index level, dividend yield, and P/E ratio are available at www.multpl.com

return of the asset. Investors' required rate of return therefore has to decline. These effects are not only imposed on the assets being purchased but also spill over onto other assets through portfolio rebalancing by investors. Besides the risk premium, the increased money supply also has impacts on other macroeconomic risk factors such as the interest rate term spread and corporate credit spread (Gagnon *et al.*, 2010; Gilchrist and Zakrajsek, 2013).

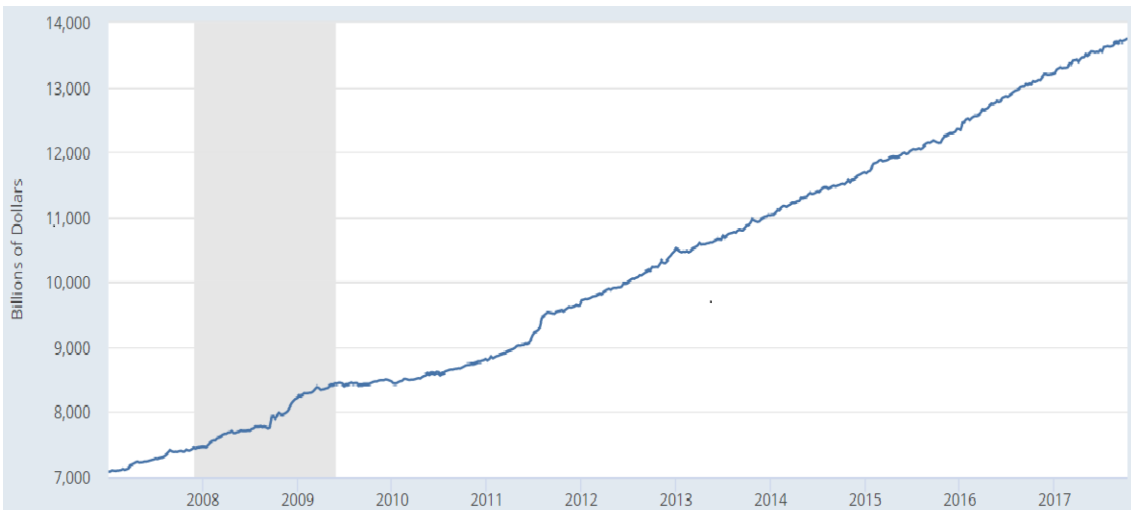

Figure 4.1 Broad money supply (M2) in the US. Source: Board of Governors of the Federal Reserve System (US)

In this article, we investigate the predictive power of these risk factors for farmland asset returns. We assume farmland returns follow an autoregressive process with expected farmland return as the mean value. Investors' demand drives the expected return towards required rate of return which is affected by the macroeconomic risk factors. We first examine three linear models: a univariate time-series model, a one-factor model using capital market risk premium as the single risk factor[2], and a three-factor model taking on additional risk factors that are also affected by increased money supply. In addition, the forecast accuracy of the three linear models is compared with that of their artificial neural network counterparts that use the same predictors. The neural network model imitates the nonlinear, parallel

[2]This factor is referred to as "excess market return" for the rest of the article.

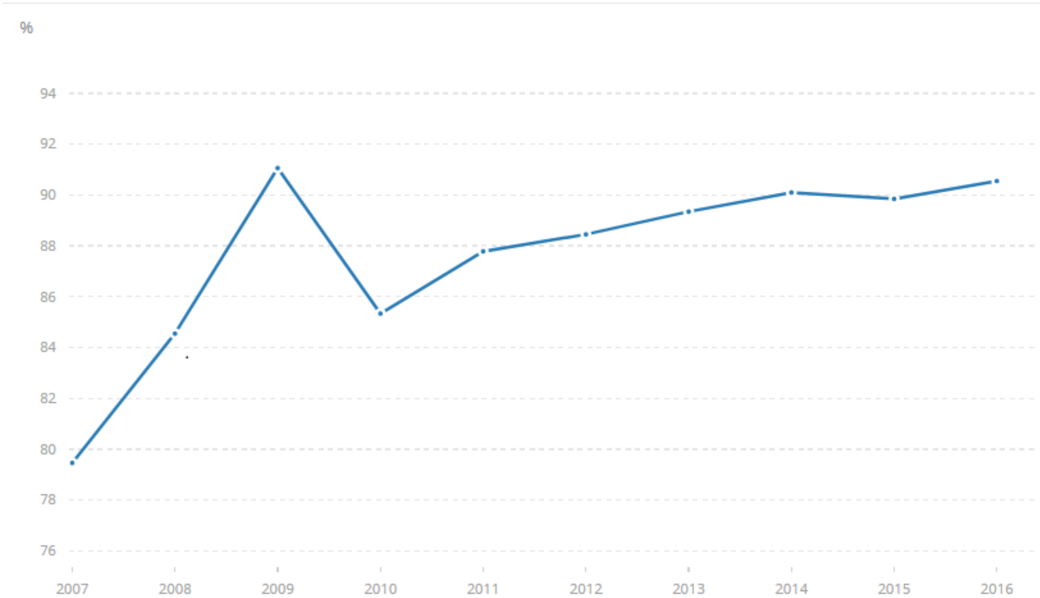

Figure 4.2 Broad money supply (M2) as a percentage of total GDP in the US. Source: World Bank Open Data

information processing structure of human brain network (McNelis, 2005). By allowing for a more flexible functional relationship between predicted variables and predictors, the neural network model relaxes the linearity assumption and captures potentially undetected regularities in asset return movements (White, 1989).

Our study focuses on state-level cropland returns across 15 major agricultural producing states in the US. The annual land return, spanning the period 1968-2016, is calculated as the sum of cash rental income and capital appreciation divided by land value. A rolling estimation forecasting procedure is used to generate forecasts of annual land returns for each of the 15 states. Each forecast is based on models estimated using the data from previous 41 years. Forecasts of land returns from 2008 to 2016 are generated and compared to actual returns. The mean squared error (MSE) is calculated to assess forecast accuracy for each state. Results indicate that each neural network model provides lower MSE than the corresponding linear model for all the states. More importantly, while adding market risk factors seems not to help for some states using linear models, it becomes helpful and improves the prediction under the neural network framework.

A formal comparison of the forecast accuracy between different models is conducted using a paired sample t-test. It turns out that the one-factor model with the excess market return factor included is the best performing model under both linear and neural network frameworks. In addition, the one-factor neural network model significantly outperforms its linear counterpart. These results indicate that the excess market return factor could provide information that significantly improves farmland return prediction. This may offer help for farmers to plan future agricultural production and decision makers to resolve agricultural policy issues given the significant role of farmland assets in the agricultural sector. There is no empirical evidence, however, that the other two risk factors add useful information to predict farmland returns.

4.2 Theoretical Framework

Due to the potential autocorrelation in farmland return series (Moss, Featherstone, and Baker, 1987), we start by assuming farmland return follows an autoregressive process with lag order 1 (AR(1))[3].

$$R_t - E(R) = \beta_0 + \varphi * [R_{t-1} - E(R)] + \varepsilon_t, \tag{4.1}$$

[3]ARMA(1,1), AR(1), MA(1), and white noise time-series models were all examined for fitness for the sample of farmland return series in each of the 15 states selected in this study. The AR(1) model was the best model for all the 15 states in terms of the Bayesian information criterion (BIC). Model sufficiency was also confirmed using the Kolmogorov-Smirnov (K-S) test and the Ljung-Box test.

where R_t represents the farmland return at time t and $E(R)$ represents expected farmland return. By rearranging equation (4.1) we obtain

$$R_t = \beta_0 + (1 - \varphi) * E(R) + \varphi * R_{t-1} + \varepsilon_t, \tag{4.2}$$

We then assume that farmland asset supply is fixed at Q_s. Farmland asset is demanded by farmers to produce crop products and by investors to gain investment returns. Farmers derive their demand function from expected farmland return, denoted by $Q_{d,f}(E(R))$[4]. As indicated by Frankel and Dickens (1983), investors' demand for an asset is related to the expected return of the asset and their risk premium for risky asset investment. Following this approach, we assume an investor's demand function for farmland takes the form of $Q_{d,i}(E(R), RP)$, where RP is the risk premium associated with farmland investment. Under equilibrium where total demand equals supply, the following equation holds,

$$Q_s = Q_{d,f}(E(R)) + Q_{d,i}(E(R), RP). \tag{4.3}$$

Supposing $E(R)$ can be solved out from equation (4.3), the following equation can be obtained,

$$E(R) = f(Q_s, RP). \tag{4.4}$$

For any risky asset investment, a risk premium would be needed to compensate an investor for holding the risky assets. Excess money supply could change an investor's risk premium through the portfolio balance effect. In the absence of data on investors' risk premium level, we use realized capital market excess return in the previous period as an indicator for current risk premium level required by investors. By bidding up asset price, the excess money supply

[4]Farmland return to farmers who derive income from farming might be different from the return to investors who collect cash rents. However, empirical evidence (Du, Hennessy, and William, 2007) shows that the difference is rather small in the long-run.

increases realized return and reduces investors' risk premium over future periods (Emmerling *et al.*, 2015). We then rewrite equation (4.4) as

$$E(R) = f(Q_s, R_m - R_f), \tag{4.5}$$

where $R_m - R_f$ is realized excess market return in the previous period.

Substituting equation (4.5) into equation (4.2) we have

$$R_t = \beta_0 + (1 - \varphi) * f(Q_s, (R_m - R_f)_{t-1}) + \varphi * R_{t-1} + \varepsilon_t. \tag{4.6}$$

Besides the risk premium for risky asset investments, increased money supply has been shown to have impacts on other macroeconomic risk factors such as the interest rate term spread and corporate credit spread (Gagnon *et al.*, 2010; Gilchrist and Zakrajsek, 2013). Assuming an investor's demand for farmland is also driven by these risk factors, we can rewrite an investor's demand function as $Q_{d,i}(E(R), RP, \gamma_1, \ldots, \gamma_n)$, where $\gamma_1, \ldots, \gamma_n$ represent the additional n macroeconomic risk factors. Following this new demand function, equation (4.5) and (4.6) can then be rewritten as

$$E(R) = f(Q_s, R_m - R_f, \gamma_1, \ldots, \gamma_n), \tag{4.7}$$

$$R_t = \beta_0 + (1 - \varphi) * f(Q_s, (R_m - R_f)_{t-1}, \gamma_{1,t}, \ldots, \gamma_{n,t}) + \varphi * R_{t-1} + \varepsilon_t. \tag{4.8}$$

4.2.1 Models

Three linear models and three artificial neural network models are considered and compared for their predictive ability. The three linear models consist of a univariate time-series model, a one-factor model, and a three-factor model. The three neural network models use exactly the same predictors as the linear models but have a different non-linear functional form.

The first linear model is built from equation (4.2) by assuming farmland return simply follows an AR(1) process with a constant expected value,

$$R_t = \beta_0 + \varphi R_{t-1} + \varepsilon_t, \tag{4.9}$$

where R_t denotes the farmland return at time t and R_{t-1} denotes the farmland return at time $t-1$. For convenience, this model is labeled as Model 1.

The second linear model is derived from equation (4.6) by assuming the farmer's demand function $Q_{d,f}(\cdot)$ and the investor's demand function $Q_{d,i}(\cdot)$ are both linear. As a result, the function $f(\cdot)$ in equation (4.6) is in a linear form. Denoting the excess market return at time t as $\gamma_{m,t}$, the model can be described as

$$R_t = \beta_0 + \varphi R_{t-1} + \beta_m \gamma_{m,t-1} + \varepsilon_t. \tag{4.10}$$

This one-factor linear model is referred to as Model 2.

The third linear model is derived from equation (4.8) by assuming again the farmer's demand function and the investor's demand function are both linear. Function $f(\cdot)$ is therefore also linear and two additional risk factors beyond excess market return are included. The two additional risk factors are the bond term spread (TERM) and the economy-wide default risk (DEFAULT). The TERM factor is calculated as the difference between ten-year and one-year Treasury bond yields. The DEFAULT factor is calculated as the difference between long-term BAA- and AAA-rated corporate bond yields. The model can be written as

$$R_t = \beta_0 + \varphi R_{t-1} + \beta_m \gamma_{m,t-1} + \beta_1 \gamma_{1,t} + \beta_2 \gamma_{2,t} + \varepsilon_t, \tag{4.11}$$

where $\gamma_{1,t}$ denotes the TERM factor and $\gamma_{2,t}$ the DEFAULT factor at time t. We refer this three-factor linear model as Model 3.

Neural network models relax the linearity assumption and potentially increase the flexibility of the demand functions. Similar to linear models, a neural network builds up functional relationships between a set of input variables and one or more output variables. The difference between linear models and neural network models is that the neural network model uses hidden layers to link input and output variables. Input variables are transformed by an activation function in the hidden layer and then connected to the output variables, in a similar way that the human brain processes information. This hidden layer processing method represents an efficient way to model nonlinear relationships (McNelis, 2005). In addition, it has been shown that given sufficiently many hidden-layer units, a three-layer (input, output, and hidden layers) neural network can approximate any continuous functions arbitrarily well (Bengio, Goodfellow, and Courville, 2015). For a three-layer neural network, each input-layer variable has a weighted connection to each hidden-layer unit, and each hidden-layer unit has a weighted connection to the output-layer variable. Using the logistic activation function to transform the input variables, the generic three-layer neural network model can be formally written as

$$n_{j,t} = \beta_{j,0} + \sum_{i=1}^{i^*} \beta_{j,i} x_{i,t}, \tag{4.12}$$

$$N_{j,t} = logsig(n_{j,t}) = \frac{1}{1 + exp(-n_{j,t})}, \tag{4.13}$$

$$y_t = \alpha_0 + \sum_{j=1}^{j^*} \alpha_j N_{j,t}, \tag{4.14}$$

where y is the output variable, $[x_1, \ldots, x_{i^*}]$ is the vector of input variables, $[N_1, \ldots, N_{j^*}]$ is the vector of hidden-layer units, $[\alpha_0, \alpha_1, \ldots, \alpha_{j^*}]$ is the vector of weights connecting the hidden-layer units to the output variable, and $[\beta_{j,0}, \beta_{j,1}, \ldots, \beta_{j,i^*}]$ is the vector of weights connecting input variables to the jth hidden-layer unit.

The neural network models that correspond to the three linear models defined previously can thus be described as

$$R_t = \alpha_0 + \sum_{j=1}^{j^*} \alpha_j logsig(\beta_{j,0} + \varphi_j R_{t-1}) + \varepsilon_t, \tag{4.15}$$

$$R_t = \alpha_0 + \sum_{j=1}^{j^*} \alpha_j logsig(\beta_{j,0} + \varphi_j R_{t-1} + \beta_{j,m}\gamma_{m,t-1}) + \varepsilon_t, \tag{4.16}$$

$$R_t = \alpha_0 + \sum_{j=1}^{j^*} \alpha_j logsig(\beta_{j,0} + \varphi_j R_{t-1} + \beta_{j,m}\gamma_{m,t-1} + \beta_{j,1}\gamma_{1,t} + \beta_{j,2}\gamma_{2,t}) + \varepsilon_t, \tag{4.17}$$

These three neural network models are labeled as Model 4, Model 5, and Model 6 respectively.

4.3 Empirical Analysis

We focus on annual state-level cropland returns in 15 major agricultural producing states in the US. The land return is calculated as the sum of cash rental income and capital appreciation divided by land value. Historical data of the cash rental income and land value were collected from the USDA National Agricultural Statistics Service (NASS) databases, spanning from 1968 to 2016. The excess market return data were gathered from the Center for Research in Security Prices on Kenneth French's website[5]. The maturity risk (TERM) and default risk (DEFAULT) data were obtained from the Federal Reserve Bank of St Louis database.

[5]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

4.3.1 Mean Squared Error

A rolling estimation forecasting procedure is used to generate forecasts of land returns in 2009-2016 for each state. We generate each forecast based on models estimated using the data from previous 41 years. That is, we use the data from 1968 to 2008 to generate forecast of land return in 2009, the data from 1969 to 2009 to generate forecast for 2010, and so on. The Mean Squared Error (MSE) is used as a measure to assess forecast accuracy of different models:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{R}_i - R_i)^2, \tag{4.18}$$

where n is the total number of forecasts generated, $\hat{R}_i$ is the ith predicted return, and R_i is the ith realized return.

Table 4.1 reports the MSE of predicted farmland returns for each of the 15 states using different models. Among the linear models (Model 1 through Model 3), adding market factors improves forecast accuracy for eight out of the 15 states. Specifically, the one-factor model provides the most accurate prediction for seven states and the three-factor model outperforms the one-factor model for only one state among the eight states for which market factors improve the prediction. For the remaining seven states, the best linear model is the univariate time-series model.

All the neural network models provide lower MSE than their linear counterparts for all the 15 states. More importantly, the market factors turn out to play more significant roles within the neural network models. Forecast accuracy is improved for 12 out of the 15 states after market factors being included. In addition, the three-factor neural network model outperforms the one-factor neural network model for eight states among the 12 states for which market factors improve the prediction. These results show that the more flexible neural network models seem to capture market information more efficiently than linear models.

While market factors seem not useful for predicting farmland return in some states using traditional linear models, market factors become helpful under the neural network framework.

Table 4.1 The mean squared error of predicted farmland returns for individual states

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Arkansas	0.0046	0.0059	0.0092	0.0039	0.0051	0.0048
Illinois	0.0672	0.0675	0.0683	0.0541	0.0591	0.0498
Indiana	0.0690	0.0632	0.0758	0.0671	0.0575	0.0587
Iowa	0.1216	0.1153	0.1336	0.1160	0.1012	0.1087
Kansas	0.0678	0.0680	0.0785	0.0645	0.0622	0.0601
Louisiana	0.0223	0.0230	0.0301	0.0207	0.0199	0.0198
Michigan	0.0336	0.0337	0.0339	0.0315	0.0296	0.0295
Minnesota	0.0638	0.0616	0.0915	0.0625	0.0544	0.0594
Mississippi	0.0191	0.0248	0.0499	0.0180	0.0210	0.0195
Missouri	0.0246	0.0235	0.0288	0.0237	0.0198	0.0187
North Dakota	0.1561	0.1576	0.1939	0.1310	0.1317	0.1352
Ohio	0.0386	0.0371	0.0390	0.0378	0.0358	0.0315
South Dakota	0.0934	0.0926	0.1207	0.0904	0.0844	0.0855
Texas	0.0393	0.0393	0.0355	0.0352	0.0304	0.0299
Wisconsin	0.0234	0.0209	0.0235	0.0216	0.0178	0.0173

4.3.2 Paired Sample t-test

A paired sample t-test (Ramsey and Schafer, 2012) is conducted to formally compare the forecast accuracy of different models using samples from all the 15 states as a whole. The paired sample t-test is a statistical procedure used to determine whether the mean difference between two sets of observations is zero. The null hypothesis assumes the true mean difference is equal to zero.

The first sample group we use for the t-test is the squared deviations of predicted returns from the corresponding realized returns, normalized by the square of realized returns to account for varying return levels across states. Specifically, for the ith forecast of farmland return in the jth state, the squared error is calculated as $[(\hat{R}_{ij} - R_{ij})/R_{ij}]^2$, where $\hat{R}_{ij}$ is the predicted farmland return and R_{ij} is the realized return. To evaluate if there is any incremental value by adding market risk factors, the following null hypotheses are tested.

H1: The univariate time-series model and the one-factor model provide no forecasting accuracy difference under the linear framework.

H2: The one-factor model and the three-factor model provide no forecasting accuracy difference under the linear framework.

H3: The univariate time-series model and the one-factor model provide no forecasting accuracy difference under the neural network framework.

H4: The one-factor model and the three-factor model provide no forecasting accuracy difference under the neural network framework.

Table 4.2 presents the t-test results. Among the linear models, the one-factor model outperforms the univariate time-series model significantly. However, the difference between the one-factor model and the three-factor model is not significant. The t-score shows that the three-factor linear model provides even lower forecast accuracy than the one-factor linear model, although the difference is not statistically significant. This indicates that the additional risk factors beyond excess market return may not add more useful information for predicting future farmland returns. Based on these results, hypothesis H1 can be rejected but there is not enough support to reject hypothesis H2.

For the neural network models, similar results are observed. Adding a single excess market return factor lowers forecast error and improves the forecast accuracy significantly. While adding the two additional market risk factors also leads to lower forecast error, the difference is not statistically significant. Again, we reject hypothesis H3 but fail to reject hypothesis H4. All these results indicate that while the excess market return factor adds useful information for farmland return prediction, the other two market risk factors seem not helpful under either linear or neural network framework. Therefore, in terms of which market factors provide the most accurate forecast, the best model turns out to be the one-factor model with the excess market return factor included.

Table 4.2 Paired t-test results on forecast accuracy between different models with the first sample set

	t-score	p-value
Linear Model Comparisons		
Model 1 vs. Model 2	2.5148	0.0132*
Model 2 vs. Model 3	-1.3866	0.1682
Neural Network Model Comparisons		
Model 4 vs. Model 5	2.2279	0.0278*
Model 5 vs. Model 6	0.7947	0.4284

To evaluate the effectiveness of the neural network models relative to their linear counterparts, the following hypotheses are also tested.

H5: The linear univariate time-series model and the corresponding neural network model provide no forecasting accuracy difference.

H6: The linear one-factor model and the corresponding neural network model provide no forecasting accuracy difference.

H7: The linear three-factor model and the corresponding neural network model provide no forecasting accuracy difference.

Table 4.3 presents the t-test results for the above hypotheses. A comparison between the linear and the neural network models shows that all the neural network models provide higher forecast accuracy than the corresponding linear models. However, only for the one-factor model does the neural network improve the forecast accuracy significantly. Thus only hypothesis H6 is rejected while hypotheses H5 and H7 cannot be rejected. As the one-factor model has shown to be the best model in terms of which risk factors to be included, the implication is that the use of neural networks improves farmland return forecast accuracy compared to linear models. This demonstrates the superiority of the neural network approach relative to linear models in predicting farmland asset returns.

Table 4.3 Paired t-test results on forecast accuracy between linear and neural network models with the first sample set

	t-score	p-value
Linear versus Neural Network Model Comparisons		
Model 1 vs. Model 4	0.6791	0.4984
Model 2 vs. Model 5	2.0750	0.0401*
Model 3 vs. Model 6	1.3055	0.1942

The second sample set used for the t-test is the absolute value of normalized deviations of predicted returns from the corresponding realized returns. For the ith forecast of farmland return in the jth state, the absolute error is calculated as $\left|(\hat{R}_{ij} - R_{ij})/R_{ij}\right|$, where $\hat{R_{ij}}$ is the predicted farmland return and R_{ij} is the realized return. The same null hypotheses are tested with this new sample set.

Table 4.4 presents the t-test results with the new sample set. Again, the linear one-factor model outperforms the linear univariate time-series model significantly. In addition, the linear one-factor model also outperforms the linear three-factor model significantly. This indicates that while adding the excess market return factor improves the forecast accuracy, adding the additional risk factors actually hurts the forecast accuracy. Based on these results, hypothesis H1 and H2 are rejected; the best linear model is the one-factor model with the excess market return factor included.

In terms of the neural network models, adding a single excess market return factor lowers forecast error and improves the forecast accuracy significantly. Adding the two additional market risk factors seems to lower the forecast error, but the difference is not statistically significant. Thus, we reject hypothesis H3 but fail to reject hypothesis H4. The testing conclusion with the second sample set is again that while the excess market return factor improves farmland return prediction, the other two risk factors do not help. Under both the linear and the neural network framework, the best model is the one-factor model with the excess market return factor included.

Table 4.4 Paired t-test results on forecast accuracy between different models with the second sample set

	t-score	p-value
Linear Model Comparisons		
Model 1 vs. Model 2	2.7199	0.0075*
Model 2 vs. Model 3	-3.3811	0.0010*
Neural Network Model Comparisons		
Model 4 vs. Model 5	3.1570	0.0020*
Model 5 vs. Model 6	0.1830	0.8551

To evaluate the effectiveness of the neural network models relative to their linear counterparts, the following hypotheses are also tested.

H5: The linear univariate time-series model and the corresponding neural network model provide no forecasting accuracy difference.

H6: The linear one-factor model and the corresponding neural network model provide no forecasting accuracy difference.

H7: The linear three-factor model and the corresponding neural network model provide no forecasting accuracy difference.

Table 4.5 presents the t-test results for the above hypotheses with the second sample set. The comparison between the linear and the neural network models again shows that all the neural network models provide higher forecast accuracy than the corresponding linear models. For both the one-factor model and the three-factor model, the neural network approach improves the forecast accuracy significantly. That is, hypotheses H6 and H7 are rejected in support of the neural network models. There is, however, not sufficient evidence to reject hypotheses H5. The testing results using the two sample sets consistently show the superiority of the neural network approach in farmland return predictions.

Table 4.5 Paired t-test results on forecast accuracy between linear and neural network models with the second sample set

	t-score	p-value
Linear versus Neural Network Model Comparisons		
Model 1 vs. Model 4	1.2884	0.2001
Model 2 vs. Model 5	3.5720	0.0005*
Model 3 vs. Model 6	2.9368	0.0040*

4.4 Conclusions

This article examines whether capital market risk factors are useful in farmland return prediction. We evaluate the predictive ability of the market risk factors using both traditional linear models and advanced neural network approach. Our results indicate that the one-factor model including excess market return significantly improves the forecast accuracy compared to the univariate time-series model that uses lagged farmland return as the only predictor. However, there is no significant difference in forecast accuracy between the one-factor model and the three-factor model with additional risk factors included. These observations apply consistently to both linear and neural network framework. In addition, we find that neural network models provide higher forecast accuracy than the corresponding linear models with the same explanatory variables. For the one-factor model, the improvement in forecast accuracy offered by neural network is statistically significant. This could be because the non-linear neural network structure captures market information in a more flexible and potentially more efficient way. For practical implications, the findings in this article suggest that farmers as well as policy makers can take advantage of market risk factors to make better future farmland return predictions.

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