

2012

# Essays on the environmental effects of agricultural production

Juan Francisco Rosas  
*Iowa State University*

Follow this and additional works at: <https://lib.dr.iastate.edu/etd>



Part of the [Economics Commons](#)

---

## Recommended Citation

Rosas, Juan Francisco, "Essays on the environmental effects of agricultural production" (2012). *Graduate Theses and Dissertations*. 12737.  
<https://lib.dr.iastate.edu/etd/12737>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Graduate Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact [digirep@iastate.edu](mailto:digirep@iastate.edu).

**Essays on the environmental effects of agricultural production**

by

**Juan Francisco Rosas Pérez**

A dissertation submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of

**DOCTOR OF PHILOSOPHY**

Major: Economics

Program of Study Committee:  
Dermot J. Hayes, Major Professor  
Sergio H. Lence  
Bruce A. Babcock  
GianCarlo Moschini  
Keri L. Jacobs

Iowa State University

Ames, Iowa

2012

Copyright © Juan Francisco Rosas Pérez, 2012. All rights reserved.

**DEDICATION**

I would like to dedicate this thesis to my wife Karina Crosignani. Your endless sacrifice and tolerance made possible that I reach this objective. No words or actions are sufficient to thank you for all that you have done for me.

## TABLE OF CONTENTS

DEDICATION	ii
LIST OF FIGURES	v
LIST OF TABLES	vi
GENERAL DISSERTATION ABSTRACT	vii
CHAPTER 1. GENERAL DISSERTATION INTRODUCTION	1
1. References	8
CHAPTER 2. DUALITY THEORY ECONOMETRICS: HOW RELIABLE IS IT WITH REAL-WORLD DATA?	9
1. Introduction	11
2. Lau's Hessian identities	15
3. Model of a single firm	17
4. Simulation of panel data	21
4.1 Random generation of true production parameters: $\mathbf{a}_f^*$	23
4.2 Random generation of quasi-fixed netput quantities: $\mathbf{K}_{ft}^*$	26
4.3 Random generation of expected variable netput prices: $\mathbf{p}_{ft}^{**}$ and $\mathbf{p}_{ft}^*$	27
4.4 Random generation of initial wealth: $\mathbf{W}_{0,ft}$	32
4.5 Simulation of noiseless dataset	33
4.6 Simulation of noisy dataset	34
4.6.1 Maximization of expected utility	34
4.6.2 Realized shocks of production and prices	35
4.6.3 Omitted variable netputs	37
4.6.4 Aggregation across netputs	37
4.6.5 Measurement error in prices and quantities	38
4.7 Unobserved Firm Heterogeneity.	38
5. Data for estimation	39
6. Estimation	41
7. Results	43
7.1 Noiseless data estimation	43
7.2 Noisy data estimation	46
8. Conclusions	50
9. Appendix	65
9.1 Appendix I. Random generation of the firm-specific sets $\mathbf{a}_f^*$	65
9.2 Appendix II. Random generation of market shocks $\Phi_t$	68
9.3 Appendix III: Random generation of initial wealth $\mathbf{W}_{0,ft}$	69
10. References	71
CHAPTER 3. RESPONSE OF CROP YIELDS TO OUTPUT PRICES: A BAYESIAN APPROACH TO COMPLEMENT THE DUALITY THEORY ECONOMETRICS	74
1. Introduction	76

2. Literature review on yield elasticities	80
3. Model	82
4. Data	86
5. Empirical application	91
5.1 The dual demand-supply system	91
5.2 The production function model	96
5.3 Elasticities of interest	98
5.4 The Bayesian estimation approach	99
5.4.1 The dual system	100
5.4.2 Direct production function estimation	105
5.4.3 Estimation of corn yield response to input use	107
5.4.4 Metropolis algorithm steps	108
6. Results	110
7. Conclusions	115
8. Appendix	125
8.1 Appendix I: Additive general error model for profit function	125
8.2 Appendix II: Cross-equation parameter restrictions	126
8.3 Appendix III: Integration properties of supply-demand system	131
9. References	132
 CHAPTER 4. NITROUS OXIDE EMISSION REDUCTIONS FROM CUTTING EXCESSIVE NITROGEN FERTILIZER APPLICATIONS	 136
1. Introduction	137
2. Farmer's Optimization Problem	139
3. Outline of the Model	143
4. The Simulation Exercise	146
4.1 N <sub>2</sub> O Emissions and the N Application Rate	146
4.2 Estimation of a Conditional Yield Distribution	148
4.3 Simulation of Correlated Yields and Price Draws	149
4.4 Maximization of Expected Utility of Profits	150
5. Simulation Results for Nitrogen Application Rate	151
6. Estimation of a Distribution of Emission Reductions	153
6.1 Distribution of Rainfall and Temperature	153
6.2 Weather Effects on N <sub>2</sub> O Emissions	153
7. Simulation Results for the Expected Reduction in Emissions	154
8. Conclusions	156
9. References	170
 CHAPTER 5. GENERAL DISSERTATION CONCLUSIONS	 174
 ACKNOWLEDGEMENTS	 178

## LIST OF FIGURES

### CHAPTER 2:

Figure 1. Production shock as a function of firm's average variable netput quantity ( $y_{ft_0}$ ) on time $t = t_0$ , for selected netputs.	53
Figure 2. DGP of noiseless and noisy datasets used in estimation.	54
Figure 3. Comparison between true and estimated elasticities for noiseless and noisy datasets.	54
Figure 4. Elasticities of variable netput quantities with respect to prices. True versus estimated values with noiseless data.	55
Figure 5. Elasticities of variable netput quantities with respect to quasi-fixed netputs. True versus estimated values with noiseless data.	56
Figure 6. Own- and cross-price elasticities of variable netput quantities. True versus estimated values with noisy data.	57
Figure 7. Elasticity of variable netput quantities with respect to quasi-fixed netputs. True versus estimated values with noisy data.	58

### CHAPTER 3:

Figure 8. Corn yield elasticities with respect to selected prices.	117
Figure 9. Corn yield elasticities with respect to quantity of intermediate inputs and quantity of fertilizers.	118
Figure 10. Corn yield elasticities with respect to selected prices. Comparison between proposed approach (light blue) and dual approach (blue).	119
Figure 11. Soybean yield elasticities with respect to selected prices. Comparison between proposed approach (light blue) and dual approach (blue).	120

### CHAPTER 4:

Figure 12. Average N <sub>2</sub> O emissions as a function of N rates	158
Figure 13. Optimal N application and N <sub>2</sub> O emissions reductions.	159
Figure 14. Offset payment structure as a function of the optimal nitrogen application rate $N^*$	160
Figure 15. Parametric estimation of a conditional beta probability density function of Iowa corn yields for different N rates	161
Figure 16. Expected marginal value product (EMVP) curve and marginal cost curves.	162
Figure 17. Random weather and response of N <sub>2</sub> O emissions	163
Figure 18. Histograms of N <sub>2</sub> O emission reductions (kg of carbon equivalent per hectare) for random weather.	164

## LIST OF TABLES

### CHAPTER 2:

Table 1. Parameter estimates of fixed effects model, equation (11), to calibrate production function parameter variation, and realized weather shocks on netput quantities.	59
Table 2. Estimation results of the OLS regression model used to generate random exogenous “national” prices from equation (13).	60
Table 3. Parameter estimates of initial wealth and the form of its heteroskedsticity, equation (19).	61
Table 4. Comparison of estimated elasticities versus moments of the distribution of true elasticities (noiseless data).	62
Table 5. Comparison of estimated elasticities ( $\hat{E}_{ij}$ ) versus median of true elasticities distribution ( $\bar{\bar{E}}_{ij}$ ) in the case of noisy data.	63
Table 6. Sensitivity analysis. Comparison of estimated elasticities ( $\hat{E}_{ij}$ ) versus median of true elasticities distribution ( $\bar{\bar{E}}_{ij}$ ) in the case of noisy data, and different sources of noise.	64
Table 7. Calibrated parameter values of market shocks ( $\phi_{nt}$ ) in equation (18)	68

### CHAPTER 3:

Table 8. Literature review of estimated elasticities of yield with respect to corn price	121
Table 9. Corn yield elasticities with respect to selected prices and quantities.	122
Table 10. Corn and soybean yield elasticities with respect to selected prices and quantities.	123
Table 11. Sensitivity analysis: the case of corn seed hybrids. Corn yield elasticities with respect to selected prices and quantities.	124

### CHAPTER 4:

Table 12. Estimation Results of Emissions Curve: $e(N)$	165
Table 13. Yields (tons per hectare) from Continuous Corn Field Experiments in Iowa	166
Table 14. Maximum Likelihood Estimation of Beta Parameters	167
Table 15. Results of the N <sub>2</sub> O Emissions Reductions Incentive Program (per hectare)	168
Table 16. Sensitivity of N <sub>2</sub> O Emissions to Changes in Precipitation and Temperature	169

## **GENERAL DISSERTATION ABSTRACT**

This dissertation is devoted to the study of environmental effects of agricultural production. Recent periods of high demand for agricultural products and the increase of world commodity prices result, in part, from the implementation of biofuel policies and the growth of per-capita income in developing countries. The extent to which food, feed, and fuel demands are satisfied depends on the ability of agricultural supply to react to these events. In economics, supply response models are used as the framework to analyze these types of problems in providing estimated magnitudes of the mentioned effects. The accuracy with which these magnitudes are calculated impacts the measurement of environmental effects of agricultural production, such as green-house gas emissions and land use change at a global scale, having important consequences on country-level accountings. Chapter 2 analyzes the econometric applications of the Neoclassical duality theory of the firm intended to measure the response of production quantities to price changes. We find that the use of real-world market-based data, which is typically available to practitioners but includes features that contradict some hypothesis of the theory, induces bias in the estimated supply response values. In light of these results, Chapter 3 proposes an alternative approach that overcomes the problems encountered when duality theory is applied to real-world data. This novel approach combines market-based data with information about production functions, which are simultaneously used in the econometric estimation of the



supply response parameters. The methodology employs Bayesian econometric methods and bases the complementarity among the various datasets on underlined theoretical relationships. An application of this approach to U.S. agriculture provides updated measures of crop yield elasticities with respect to prices. Chapter 4 takes on the issue of direct environmental effects from agricultural production. In particular, it documents and quantifies the effects on nitrous oxide emissions from cutting nitrogen fertilizer applications when farmers face a market instrument intended to discourage the excessive use of nitrogen in soils. An expected utility maximization problem is specified where the farmer chooses the optimal nitrogen application facing a nonlinear market instrument. The nonlinearity captures the nonlinear relationship between nitrogen applications and nitrous oxide emissions and is arguably more efficient than linear schemes. Simulation results for U.S. corn show that farmers are induced to significantly reduce their fertilization (and consequently emissions) with only minor effects on expected crop yields.

## **CHAPTER 1. GENERAL DISSERTATION INTRODUCTION**

Recent developments in the world economy and described by the economics literature have emphasized the environmental effects of agricultural production. Issues that have emerged include; land use change at a global scale as induced by biofuels policies, the additional greenhouse gas (GHG) emissions generated by such policies, and the consequent increase in food prices due to the requirement of a higher production to satisfy the demand for biofuel feedstocks, as well as food and feed (Searchinger et al., 2008; Dumortier et al., 2011).

Searchinger et al. (2008) showed that current mandates on the utilization of biofuels are capable of inducing land use changes in the U.S. and foreign agriculture that result in a longer payback period of GHG emissions than previous estimates. Fargione et al. (2008) showed that the effects of biofuel policies on carbon savings are very sensitive to the type of land and feedstock used to produce renewable fuels. Righelato and Spracklen (2007) concluded that if the objective of biofuels policies is to mitigate global warming induced by carbon-dioxide, policy makers should concentrate first on improving energy use efficiency. This is true because a small substitution of fossil fuels by renewable sources requires the conversion of large areas of pasture and forests.

It has been argued that increased demand for feedstocks by the biofuels industry and food from developing countries has driven higher food prices. The extent to which this extra demand will be satisfied in the near future with more or less land conversion depends on how yields react to these price changes (Keeney and Hertel, 2009).

In this context, the following elements are important: first, small changes in crop yields have a great impact on the payback period of GHG emissions induced by agriculture, and also on the quantity of new land that is brought into agriculture to satisfy an increasing demand of

agricultural products (Dumortier et al., 2011). Second, the allocation of land to competing enterprises (cash crops, pasture, forestry and other land uses) is very sensitive to price shocks.

Therefore, the environmental effects of biofuel and climate policies, as well as the successive years of sustained growth in the demand for food and feed from developing countries, are inherently related to both the change in commodity prices and the supply of agricultural products. This focuses the attention on supply response, especially on price-induced yield response.

Supply response models in agriculture evaluate both the intensive and extensive margins. The intensive margin, or “intensification,” accounts for the increase in production due to reallocation of inputs without changing the area dedicated to each crop; i.e., an increase of agricultural yields. The extensive margin, or “extensification,” measures the change in production derived from the reallocation of land among different crops; this is known as land-use change.

This work presents three essays devoted to understanding important environmental implications of agricultural production. The first and second essays address this topic from the point of view of agricultural crop yields and crop yields measurements considering that they not only directly impact the supply of feedstocks for the food, feed, and fuel industry, but also have direct and indirect influences on land use change and GHG emissions. The third essay deals with direct GHG fluxes coming from the application of chemical fertilizers in agricultural soils.

The first essay (Chapter 2), “Duality theory econometrics: How reliable is it with real-world data?”, looks at the dual theorem of the Neoclassical theory of the firm, in particular, its use in empirical studies seeking to calculate supply elasticities with respect to prices. Practical applications of this approach usually consist of econometric estimations of a system of input

demands and output supplies using available market data on input and output prices and quantities. We argue that relevant assumptions required by the dual theorem do not hold in real-world datasets available to practitioners, and as a consequence, the methodology provides biased estimations. In light of Dumortier et al. (2011) results, the use of these figures for policy evaluation and for projection of GHG inventories and other economic aggregates may have important consequences on the accuracy of such calculations (Keeney and Hertel, 2008).

In this Chapter 2, by means of Monte Carlo simulations, we generate a panel dataset of input and output prices and quantities over successive periods of time. The data are from a population of technologically heterogeneous firms that belong to different regions. The calibration is pursued so as to replicate the main data properties encountered in datasets that have been widely employed in estimations relying in duality theory. These properties involve price and quantity variability, length of time series, sample size, aggregation across heterogeneous firms, unexpected shocks in production and prices, decisions under uncertainty, omitted variables, individual netputs aggregation into single categories, and measurement error.

Parameters determining each feature are calibrated within the model in order to provide a sufficiently close representation of production and market data. By estimating elasticities applying econometric methods on the simulated data and by knowing the true elasticity values that generated such data we can analyze the degree of accuracy with which duality theory can recover these underlying parameters. We find that the approach at hand provides estimates of output and input price elasticities that are, on average, 71% different from their true elasticity values.

The second essay (Chapter 3) is titled “Response of Crop Yields to Output Prices: A Bayesian Approach to Complement the Duality Theory Econometrics.” Its contribution to the

literature is twofold. First, given results from the simulation exercise on duality theory in Chapter 2, we propose an alternative methodology for calculating elasticities that seeks to complement the information provided by market datasets with additional information coming from other sources. Second, we aim to calculate new and updated estimates of crop yield elasticities with respect to netput prices. The proposed approach considers that, on the one hand, market data on input and output prices and quantities has embedded information about the elasticities we are trying to estimate, out of which, duality theory is able to retrieve only a portion. On the other hand, considering that price elasticities are dictated by certain features of the underlying production technology, we can use data on known physical relationships to estimate relevant parameters of the production function.

The data on physical relationships comes purely from experimental data measuring output response to the use of relevant inputs. A key point of the approach is the existence of certain technology parameters that can be recovered with the use of both data sources, that is, market-based and experimental. Bayesian econometric methods allow us to jointly estimate all model parameters by simultaneously using all datasets available for the study. Then we come up with the final estimated values employing a weighting structure in which the weights are based on how likely it is that these parameters are generated from each dataset, i.e. by the likelihood functions. We apply this methodology to the case of Iowa corn yields and find an own-price elasticity estimate with mean of 0.29, implying that farmers react to expected higher output prices by improving their management practices. The elasticity of corn yields with respect to input prices has the expected negative sign. A sensitivity analysis shows that results are robust to the incorporation of other sources of information; for example, about the yields response to the use of other inputs.

The third essay (Chapter 4), “Nitrous Oxide Emissions Reductions from Cutting Excessive Nitrogen Fertilizer Applications,” considers the environmental effects of agricultural production from a different perspective. It seeks to measure the effects of reducing nitrogen (N) fertilizers on expected nitrous oxide emissions ( $\text{N}_2\text{O}$ ) as well as on expected crop yields when an observed nonlinear relationship between  $\text{N}_2\text{O}$  emissions and N applications is taken into account. Nitrous oxide is a GHG with a global warming potential 310 times higher than that of carbon dioxide ( $\text{CO}_2$ ). According to EPA GHG inventory report, the agricultural sector accounts for 6.3% of total U.S. GHG emissions or 428.4 million metric tons of  $\text{CO}_2$  equivalent, out of which, 52% correspond to  $\text{N}_2\text{O}$  and 48% to methane (U.S. EPA 2011). Their importance on the GHG national accountings makes these emissions a relevant issue.

The nonlinearity between N applications and  $\text{N}_2\text{O}$  fluxes from agricultural soils has its origin on the N cycle (both in soils and in the atmosphere) and its connection with the uptake from crops or grasses growing on them. Plants compete with  $\text{N}_2\text{O}$ -producing microbes for the use of N in soils in such a way that  $\text{N}_2\text{O}$  production is limited until crop N uptake has been completely satisfied. This implies that  $\text{N}_2\text{O}$  emissions will be low as the crop prevails in the use of N, but emissions will increase more rapidly once the crop’s N demand is satisfied. Our model explicitly treats this nonlinearity by the use of a nonlinear market instrument aimed at incentivizing farmers to reduce their nitrogen fertilizer applications. It is generally accepted that, ex-post, U.S farmers apply more fertilizers than required by the agronomically optimum. The reason behind this behavior is that the response of crop yields to N fertilizer and weather conditions is such that when one acts as a limiting nutrient, yields rapidly decrease their growth. Because weather is uncertain and produces nutrient losses during the growing season, and because nitrogen is not excessively expensive as compared to their effect on expected revenues

(through increased yields) it is not costly for farmers to behave this way. However, accounting for this uncertainty, evidence in the literature suggests that these decisions are ex-ante optimum (Babcock, 1992; Sheriff, 2005).

We calibrate a farmer's expected utility model to the case of Iowa corn, a crop characterized by its intensive use of N, and use Monte Carlo simulations to show that the nonlinear market instrument prompts farmers to cut N fertilizer applications with a significant impact on both expected and actual N<sub>2</sub>O emissions but without significantly harming expected or actual yields. Results are conditional on society's valuation of air pollution that we operationalize by an exogenous carbon market that accepts carbon credits from agriculture. We conduct a sensitivity analysis with different carbon prices because they are influential of final results. Finally, results are robust to several risk aversion levels.

The present dissertation encompasses topics in both theoretical and applied microeconomics. In particular, our applications deal with problems of agent's decisions under uncertainty set up as expected utility and expected profit maximization programs. As becomes apparent from previous paragraphs, these topics belong to the fields of agricultural economics, environmental and resource economics, and applied microeconometrics. Econometric models set up in this work include simultaneous systems of equations (seemingly unrelated regressions), fixed-effects regressions, and single-equation multivariate regressions. Parameters from these models are estimated using various estimation methods such as iterated three-stage least squares, iterated feasible generalized least squares, and ordinary least squares minimization, as well as maximum likelihood estimation and Monte Carlo Markov Chains in Bayesian estimation methods. Datasets used in estimation of these models consists of cross-sectional, time-series, and panel data, which in some cases are real-world data and in others are originated from simulations

calibrated to represent real-world features present in the data. Numerical methods used include numerical optimization, numerical integration (by Monte Carlo methods and by Gaussian quadratures), pseudo-random number generation from various (independent and correlated) probability distributions, parametric and non-parametric fitting of probability distributions.



## 1. References

- Babcock, B. A. 1992. "The Effects of Uncertainty on Optimal Nitrogen Applications." *Review of Agricultural Economics* 14(2): 271-280
- Dumortier, J., D. J. Hayes, M. Carriquiry, F. Dong, X. Du, A. Elobeid, J. Fabiosa, and S. Tokgoz. 2011. "Sensitivity of Carbon Emission Estimates from Indirect Land-Use Change." *Applied Economic Perspectives and Policy* 33(3): 428-448.
- Fargione, J., J. Hill, D. Tillman, S. Polasky, and P. Hawthorne. 2008. "Land Clearing and the Biofuel Carbon Debt." *Science* 319.
- Keeney, R. and T. W. Hertel. 2008. "Yield Response to Prices: Implications for Policy Modeling." *GTAP Working Paper* 08 - 13.
- Keeney, R. and T. W. Hertel. 2009. "The Indirect Land Use Impacts of United States Biofuel Policies: The Importance of Acreage, Yield, and Bilateral Trade Responses." *American Journal of Agricultural Economics* 91: 895-909.
- Righelato, R., and D.V. Spracklen. 2007. "Carbon Mitigation by Biofuels or by Saving and Restoring Forests?" *Science* 317: 902.
- Searchinger, T. 2008. "Use of U.S. Croplands for Biofuels Increases Greenhouse Gases Through Emissions from Land-Use Change." *Science* 319: 1157-1268.
- Sheriff, G. 2005. "Efficient Waste? Why Farmers Over-Apply Nutrients and the Implications for Policy Design." *Review of Agricultural Economics* 27(4): 542-557.
- U.S. Environmental Protection Agency (U.S. EPA). 2011. Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990 – 2009. Washington, DC.

## **CHAPTER 2. DUALITY THEORY ECONOMETRICS: HOW RELIABLE IS IT WITH REAL-WORLD DATA?**

### **Abstract**

The Neoclassical theory of production establishes a dual relationship between the profit value function of a competitive firm and its underlying production technology. This relationship, commonly referred to as duality theory, has been widely used in empirical work to estimate production parameters without the requirement of explicitly specifying the technology. We analyze the ability of this approach to recover the underlying production parameters and its effects on estimated elasticities when the data available for estimation features typical characteristics found in real-world data: unobserved firm heterogeneity, decisions under uncertainty, unexpected production and price shocks, endogenous prices, output and input aggregation, measurement error in variables, and omitted variables. We compute the data generating process by Monte Carlo simulations such that the true technology parameters are known. A careful calibration of the data generating process yields a dataset that features the main characteristics of U.S. agriculture and levels of noise typically found in the data. The use in calibration of widely employed datasets guarantees that the levels of noise introduced are realistic. By construction, this noise prevents duality theory from holding exactly. Our findings show that the true production parameters are not precisely recovered and therefore the elasticities are inaccurately estimated. We compare the estimated production parameters with the true (and known)

parameters by means of the identities between the Hessians of the production and profit functions. The deviation of the own- and cross-price elasticities from their true values, given our parameter calibration, ranges between 6% and 247%, with an average of 71%. Also, own-price elasticities are as imprecisely recovered as cross-price elasticities. Sensitivity analysis shows that results still hold for different sources and levels of noise, as well as sample size used in estimation.

Keywords: duality theory, firm's heterogeneity, measurement error, data aggregation, omitted variables, endogeneity, uncertainty, Monte Carlo simulations.

*JEL Codes: Q12, D22, D81, C18*

## 1. Introduction

The Neoclassical theory of production establishes that a competitive firm's optimization problem is characterized by a dual relationship between the value function (profit, cost, or revenue function) and the underlying production function (e.g., Mas-Colell, Winston, and Green, Ch. 5, 1995). This implies that a given functional form of the production function determines a specific form of the profit, cost, or revenue function. Alternatively, for a given functional form used to approximate the firm's value function, there exists an underlying production function wherein the value function parameters appear in a specific way.

This dual relationship has been widely used in empirical work as a tool to estimate production parameters without explicitly specifying the technology. Shumway (1995) and Fox and Kivanda (1994) list more than one hundred applications of duality theory in nine agricultural economics journals. Typically, empirical studies consist of

- i. Approximating the value function (profit, cost, or revenue function) by a parametric functional form.
- ii. Deriving a set of input demand and output supply equations by applying Shephard's lemma or Hotelling's lemma.
- iii. Using econometric methods to jointly estimate the parameters of the system described in (ii). In some instances, value function parameters are estimated together with those of the input and output supply system.
- iv. Using estimated parameters from (iii) to draw conclusions about substitution elasticities, price elasticities, and/or returns to scale.

Conclusions from duality applications may be influenced by the choice of specific functional forms. As a result, a large number of studies intend to test the validity of duality

theory and focus on investigating the most preferable (flexible) functional forms (FFF) for empirical purposes (Guilkey, Lovell and Sickles, 1983; Dixon, Garcia and Anderson, 1987; Thompson and Langworthy, 1989). Analyses of this type usually consist of the following steps. First, a parametric functional form is selected to approximate the production technology. Several parameter scenarios are chosen, and simulated observations corresponding to the “true” production data generating process (DGP) are obtained for each scenario. Second, a set of input and output prices is computed under the assumption of profit maximization. Third, depending on the objective, the profit or cost function is approximated by a FFF and the resulting system of input demands and output supplies is calculated. Fourth, econometric methods are applied to estimate the set of parameters of the approximated system, which are finally compared with the true, known production parameters.

The aforementioned studies focusing on FFFs assume the basic tenets underlying duality theory, including perfect competition, profit maximizing behavior, and certainty. Therefore, these studies only consider empirical deviations from duality theory stemming from the functional form choice. However, the DGP used to recover the production parameters in this type of analysis is free from problems commonly encountered in data available to practitioners. As a result, these studies provide little guidance regarding how well duality theory applies to empirical analysis of real world data.

In this paper, we propose to analyze the ability of the duality theory approach to recover underlying production parameters from data with commonly observed problems. Among other realistic properties, the simulated data includes (i) optimization under uncertainty; (ii) prediction errors in prices and quantities of variable netputs; (iii) omitted variable netputs; (iv) output and input data aggregation; (v) measurement errors in the observed variables; (vi) unobserved

heterogeneity across firms; and (vii) endogenous output and input prices. For meaningful analysis, we calibrate the simulated data to capture realistic magnitudes of the noise arising from each source. Knowing the true technology parameters, Monte Carlo simulations are used to compute the necessary price and quantity variables. While calibrated to represent typical datasets encountered in practice, the levels of noise embedded in these variables affect the data used in estimation, preventing duality theory from holding exactly. Hence, the true production parameters may not be recovered with enough precision, and the estimated elasticities measurements may be more inaccurate than expected.

Early efforts to test the validity of duality theory in practice (Burgess, 1975; Appelbaum, 1978) failed to identify the source of the discrepancy between conclusions from the primal and dual approaches. The authors used real-world data which are expected to suffer from the aforementioned problems, and therefore they did not know the true DGP. As a result, when the primal and dual approaches led to conflicting results, the authors could not establish which approach was preferable. In addition, the authors did not use a self-dual functional form to approximate both the production and the cost function (translog). This prevented them from attributing the whole divergence in the estimated parameters to a failure of duality, because there is at least some difference attributable to functional specification. An exception is the study by Lusk et al. (2002) who analyzed the empirical properties of duality theory by simulating various datasets representing scenarios of price variability, length of time series, and measurement error. They found that small sources of measurement error translate in to large errors in estimated parameters, emphasizing the necessity of high-quality data to estimate empirical models.

Since we are not interested in testing different functional forms, we use a quadratic production function for convenience to generate a “true” production dataset, or input and output

quantities, using Monte Carlo simulations. Key advantages of the quadratic production function for present purposes include (i) being a self-dual FFF and (ii) having second derivatives dependent only on parameters and not variables, which greatly facilitates the analysis. We obtain the set of input and output quantities by assuming profit maximization, conditional on randomly generated prices.

We analyze two cases. First, we use a generated panel of price and quantity variables over time based on firms with heterogeneous technology, free from the problems described above, that we aggregate over firms to construct a time-series to be used in estimation. The majority of studies applying duality theory use country-, state- or county- level data as if it belonged to a single firm; however, such a firm does not exist. We are interested in answering the following question: Whose production parameters are we recovering when we pool together production data from several heterogeneous firms? We aim to identify the consequences of such estimation that assumes a “representative” firm deciding for several heterogeneous firms.

Second, we add noise to the generated panel of price and quantity variables to replicate the aforementioned real-world problems found in data used by practitioners. We aim at generating noise comparable to that encountered in widely used datasets, such as the one constructed and maintained by Eldon Ball for US input/output price and quantities (USDA-ERS), the USDA-ARMS database, the U.S. Agricultural Census database, and the Chicago Mercantile Exchange (CME) future prices database. We chose the first dataset because it is publicly available and it has been used for applications of duality theory in several widely cited papers (Ball, 1985; Ball, 1988; Baffes and Vasavada, 1989; Shumway and Lim, 1993; Chambers and Pope, 1994). The remaining two datasets are data sources which provide useful information for calibrating cross-sectional parameters. We seek to calibrate parameters and noise levels directly observed (e.g.,

price variability and length of time series) and also unobserved (e.g., measurement error, endogeneity of output prices, production and price shocks). These three datasets provide useful information to calibrate model parameters. We adopt the criteria of calibrating parameter values to favor recovery of true production parameters, especially for those unobservable.<sup>1</sup>

We set up the expected profit function and derive the system of input demands and output supplies, to then econometrically estimate its parameters for comparison with the true (and known) production parameters. Comparisons are performed using Lau's (1976) Hessian identities between production and restricted profit functions, which are straightforward under the advocated quadratic specification.

## 2. Lau's Hessian identities

Consider a producer who chooses the level of netputs<sup>2</sup> to maximize profits. The producer's problem can be described as follows:

$$\max_{[y, y_0]} \{\mathbf{p}'\mathbf{y} + y_0\} \quad (1)$$

where  $\mathbf{y}$  is a choice vector of  $n$  variable netput quantities,  $\mathbf{p}$  is a vector of  $n$  variable netput prices normalized by  $p_0$  or the price of the numeraire commodity  $y_0$ . The augmented vector  $[y_0, \mathbf{y}', \mathbf{K}']$  is referred to as the production plan of the production possibilities set  $S$  which is a subset of  $R^{1+n+m}$ , with  $m$  equal to the number of quasi-fixed netputs (denoted as the vector  $\mathbf{K}$ ) that constrain the production possibilities set.<sup>3</sup>

---

<sup>1</sup> In this study, we generate a panel data of observations across firms and over time. We focus here on the properties of duality theory applications using time series data. The analysis of applications with cross-sectional data is as relevant as the one pursued here. We leave it for future research. The properties of duality theory using panel data can be studied with the data generated, but they are less frequent in the literature because these datasets are not as readily available.

<sup>2</sup> We use the standard definition of netput, where a positive value represents a net output and a negative value represents a net input.

<sup>3</sup> The properties of the set  $S$  include: i) the origin belongs to  $S$ ; ii)  $S$  is closed; iii)  $S$  is convex; iv)  $S$  is monotonic with respect to  $y_0$ ; and v) non-producibility with respect to at least one variable input, which implies at least one



Jorgenson and Lau (1974) showed existence of a one-to-one correspondence between the set  $S$  (with properties described in footnote 2) and a production function  $G$  defined as:

$$G(\mathbf{y}, \mathbf{K}) = -\max \{y_0 / [y_0, \mathbf{y}', \mathbf{K}'] \in S\} \quad (2)$$

We follow the convention that  $\max\{\emptyset\} = -\infty$ , where  $\{\emptyset\}$  is defined as the empty set, such that the value of the production function is positive infinity if a production plan is not feasible.<sup>4</sup>

The set of quasi-fixed netputs that constrains the set  $S$  also constrains the production function  $G$ .

The maximization problem can be rewritten as:

$$\max_{[\mathbf{y}]} \{\mathbf{p}'\mathbf{y} - G(\mathbf{y}, \mathbf{K})\} \quad (3)$$

The solution to problem (3) is a set of netput demand equations  $\mathbf{y}^*(\mathbf{p}, \mathbf{K})$  and a restricted profit function  $\pi_R(\mathbf{p}, \mathbf{K})$  which are dependent on the vector of normalized netput prices and the vector of quasi-fixed netputs.

Lau (1976) derived the relationships between the Hessian of the production function  $G(\mathbf{y}, \mathbf{K})$  and the Hessian of the restricted profit function  $\pi_R(\mathbf{p}, \mathbf{K})$  under the assumption of convexity and twice continuously differentiability of both functions. Omitting the arguments of each function to simplify notation, the identities are as follows:

$$\begin{bmatrix} \frac{\partial^2 \pi_R}{\partial \mathbf{p}^2} & \frac{\partial^2 \pi_R}{\partial \mathbf{p} \partial \mathbf{K}} \\ \frac{\partial^2 \pi_R}{\partial \mathbf{K} \partial \mathbf{p}} & \frac{\partial^2 \pi_R}{\partial \mathbf{K}^2} \end{bmatrix} \equiv \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad (4)$$

$$B_{11} = \left[ \frac{\partial^2 G}{\partial \mathbf{y}^2} \right]^{-1}$$

---

commodity is freely disposable and can only be a net input in the production process (a primary factor of production).

<sup>4</sup> The properties of the production function  $G$  are: i) the domain is a convex set of  $R^{n+m}$  that contains the origin; ii) the value of  $G$  at the origin, say  $G(0)$ , is non-positive; iii)  $G$  is bounded; iv)  $G$  is closed; and v)  $G$  is convex in  $\{\mathbf{y}, \mathbf{K}\}$ . Convexity is required because of the convention used in Lau (1976) that  $y_0 = -G(\mathbf{y}, \mathbf{K})$ .

$$B_{12} = (B_{21})' = - \left[ \frac{\partial^2 G}{\partial \mathbf{y}^2} \right]^{-1} \left[ \frac{\partial^2 G}{\partial \mathbf{y} \partial \mathbf{K}} \right]$$

$$B_{22} = - \left[ \frac{\partial^2 G}{\partial \mathbf{K}^2} \right] - \left[ \frac{\partial^2 G}{\partial \mathbf{K} \partial \mathbf{y}} \right] B_{11} \left[ \frac{\partial^2 G}{\partial \mathbf{y} \partial \mathbf{K}} \right]$$

By defining, in a similar fashion, the production function Hessian sub-matrices as  $A_{ij}$ , the identities can be rewritten in the following more compact form:

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} [A_{11}]^{-1} & -[A_{11}]^{-1}[A_{12}] \\ -[A_{21}][A_{11}]^{-1} & -[A_{22}] - [A_{21}][A_{11}]^{-1}[A_{12}] \end{bmatrix} \quad (5)$$

The Hessian relationships allow us to “transform” the estimated parameters of the restricted profit function into parameters of the underlying production function, and then compare these transformed parameters with the true parameters of the production function. The Hessian of the restricted profit function contains the information necessary to calculate the matrix of input demand and output supply elasticities with respect to own and cross prices, and with respect to quantities of quasi-fixed netputs. Ultimately, the Hessian identities allow us to conclude how precisely we estimate demand and supply elasticities.

### 3. Model of a single firm

The model setup and assumed throughout the analysis is problem (3), which has been reformulated to include uncertainty faced by the firm in the decision process. Dropping the firm and time subscripts, the firm’s problem is:

$$\begin{aligned} \max_{[\mathbf{y}]} \{EU(\tilde{W}_1)\} &= \max_{[\mathbf{y}]} \{EU(W_0 + \tilde{\pi})\} \\ &= \max_{[\mathbf{y}]} \{EU(W_0 + \tilde{\mathbf{p}}'\tilde{\mathbf{y}} - G(\tilde{\mathbf{y}}, \mathbf{K}; \boldsymbol{\alpha}))\} \end{aligned} \quad (6)$$

where  $U$  is a strictly increasing and twice-continuously differentiable concave utility function of terminal wealth ( $\tilde{W}_1$ ), defined as initial wealth ( $W_0$ ) plus uncertain end-of-period profits ( $\tilde{\pi}$ ). The tilde ( $\sim$ ) indicates a random variable and  $E$  is the expectation operator that integrates over the

uncertainty of random variables ( $\tilde{\mathbf{p}}$  and  $\tilde{\mathbf{y}}$ ). The degree of concavity of the utility function determines the degree of risk aversion. The solution to problem (6) is a vector of expected variable netput quantities ( $\mathbf{y}^*$ ) conditional on the set of expected prices ( $\mathbf{p}$ ) and model parameters.

The treatment of risk and uncertainty in the duality theory framework with profit functions includes the work by Pope (1982), Coyle (1992), Coyle (1999), Pope and Just (2002). In the case of cost functions, developments are due to Pope and Chavas (1994), Pope and Just (1996), Pope and Just (1998), Chambers and Quiggin (1998), Moschini (2001), and Chavas (2008), among others.

To make this problem operational, we assume a constant absolute risk aversion (CARA) utility function of the form  $U(\tilde{\pi}) = -e^{-\lambda \tilde{W}_1}$  with  $\lambda$  representing the coefficient of absolute risk aversion, defined as  $\lambda = \frac{U''}{U'}$  where  $U'$  and  $U''$  are the first and second derivatives, respectively, of the utility function with respect to the random terminal wealth. We assume a quadratic FFF for the production function  $G(\mathbf{y}_{ft}, \mathbf{K}_{ft}; \boldsymbol{\alpha}_f)$ :

$$G(.) = \mathbf{y}'_{ft} A_{1f} + \mathbf{K}'_{ft} A_{2f} + \frac{1}{2} \mathbf{y}'_{ft} A_{11f} \mathbf{y}_{ft} + \mathbf{y}'_{ft} A_{12f} \mathbf{K}_{ft} + \mathbf{K}'_{ft} A_{22f} \mathbf{K}_{ft} - \psi_{ft} \quad (7)$$

where  $A_{1f}$  and  $A_{2f}$  are  $(n \times 1)$  and  $(m \times 1)$  vectors of  $\alpha_{i,f}$  coefficients,  $A_{11f}$  is a symmetric and nonsingular  $(n \times n)$  matrix, and  $A_{12f}$  and  $A_{22f}$  are  $(n \times m)$  and  $(m \times m)$  matrices of firm  $f$ .

Submatrices  $A_{11f}$ ,  $A_{12f}$  and  $A_{22f}$  form a symmetric and positive semi-definite  $((n + m) \times (n + m))$  matrix  $A_f$  of  $\alpha_{ij,f}$  coefficients.<sup>5</sup> We collectively denote all  $\alpha_{i,f}$  and  $\alpha_{ij,f}$  coefficients as  $\boldsymbol{\alpha}_f$ .

---

<sup>5</sup> Positive semi-definiteness is required because of the convention used in Lau (1976) that  $y_0 = -G(\mathbf{y}, \mathbf{K})$ .

The quadratic functional form is selected for three reasons. First, it is self-dual—the functional form of the constrained or unconstrained profit function that is consistent with this production function is also quadratic. This favors recovery of the true production parameters because the estimation is free from errors arising from functional form specification. Second, the Hessian matrices of both the production and profit functions are only functions of parameters; this proves to be useful because the comparison of the profit and production function Hessians does not depend on the set of model variables at which Hessians are evaluated. Third, the normalized quadratic profit function is widely used in empirical analysis (Schuring, Huffman and Fan, 2011; Arnade and Kelch, 2007; Lusk et al., 2002; Lim and Shumway, 1993; Huffman and Evenson, 1989; Thompson and Langworthy, 1989).

Uncertainty in a farmer's decision process comes from events such as random weather, pests, and selling prices not known with certainty at the time of making allocation decisions, among others. In particular, the farmer optimizes by choosing the quantity of expected output at the end of the growing season. We model production uncertainty by introducing a mean zero, heteroskedastic production shock denoted by  $\psi_{ft}$  for each firm  $f$  and time  $t$ . The functional form is as follows:

$$\begin{aligned}\psi_{ft} &= g(\mathbf{y}_{ft}; \mathbf{D})\mathbf{v}_{ft} \\ &= \frac{2}{8} \left[ \mathbf{D} \cdot (\mathbf{y}'_{ft})^{\frac{3}{2}} \right] \mathbf{v}_{ft}\end{aligned}\tag{8}$$

where  $\mathbf{D}$  is a  $(1 \times n)$  row vector of constants, “ $\cdot$ ” is the dot product, and  $\mathbf{v}$  is an  $(n \times 1)$  random vector. The entries of  $\mathbf{v}$  corresponding to variable outputs are distributed as:  $\mathbf{v} \sim -1 + 2 \times \text{Beta}(2,2)$ , or an independent and identically distributed (*iid*) symmetric shock with mean zero in the interval  $[-1,1]$ . Elements corresponding to variable inputs are zero. While this is consistent

with firms facing output quantity uncertainty, the jointly specified technology induces uncertainty on all choice variable netputs. There are at least two reasons for choosing functional form in (8). First, it guarantees a heteroskedastic production error with a standard deviation increasing at a decreasing rate, consistent with the assumption that bigger firms are less exposed to uncertain events (weather) because a bad draw is more likely to be offset by a good draw within the same firm. Second, the multiplicative constants, the beta distribution parameters, and the random error jointly induce a production shock ranging from plus or minus 10% to 60% of the average quantity produced.<sup>6</sup> The shock enters the solution of variable netput quantities in its first derivative and premultiplied by  $(A_{11})^{-1}$ . To achieve the desired level of variability in each netput quantity, and to reduce variability induced by other netputs (especially in the case of inputs), entries of  $\mathbf{D}$  are set equal to the inverse of the main diagonal of  $(A_{11})^{-1}$ .

Figure 1 shows selected production shocks computed for netput 1 (output) and netput 8 (input) for all firms  $f$  and a given time  $t_0 = 1$ . In the top panels, the distribution of the netput quantity faced by each firm ( $\tilde{y}_{ft_0}$ ) is plotted against the firm's average netput quantity  $\bar{y}_{ft_0}$ . The middle panels show the minimum (green), mean (blue), and maximum (red) of the production shock ( $\tilde{\psi}$ ) as a percentage of the firm's average netput quantity. For firms producing or using more, the minimum and maximum shock represents a lower percentage of the average quantity, ranging between 10% and 50% depending on the netput. As a consequence, the coefficient of variation is decreasing in netput quantity which is consistent with the desired production shock heteroskedasticity (bottom panels).

---

<sup>6</sup> For comparison, a pooled panel of farm-specific corn yield over a period of five years shows that the 2.5<sup>th</sup> and 97.5<sup>th</sup> percentile are respectively 60% lower and 40% higher than the average yields in the Corn Belt region, 60% lower and 42% higher in the Lake States region, and 80% lower and 70% higher in the Northern Plains region.

Firms also face end-of-period output price uncertainty, modeled as a log-normal deviation from the firm-specific price  $\mathbf{p}_{ft}^*$ , or:

$$\log(\tilde{\mathbf{p}}_{ft}) = \log(\mathbf{p}_{ft}^*) + \mathbf{e} \quad (9)$$

where  $\mathbf{e}$  is an  $(n \times 1)$  random vector. Entries associated with outputs are *iid* normally distributed shocks with mean zero and standard deviation of 0.2 (Lence 2009). Entries corresponding to inputs are zero assuming input prices are known at the decision moment.

We induce correlation between the levels of output prices and quantities by the Iman and Conover (1982) method. We assume production shocks have an impact on prices of the opposite sign and set the correlation coefficient to -0.30 based on observed correlations of these variables for the U.S. Further, because commodity prices tend to move together, we impose a strong positive correlation of 0.90 among commodity prices. Similarly, we assume output quantity shocks are positively correlated among them because weather is likely to affect all crops; therefore we set the correlation coefficient to 0.90.

#### 4. Simulation of panel data

The data generation process (DGP) considers variability of prices and quantities over time within three regions composed of heterogeneous firms. Heterogeneity across regions is assumed to be higher than heterogeneity of firms within each region. The DGP consists of generating a panel of  $F = 10,000$  farms, in  $R = 3$  regions,  $T = 50$  years ( $R \times F \times T = 1.5$  million) for each variable of the vector  $[\mathbf{y}_{ft}, \mathbf{p}_{ft}, \mathbf{K}_{ft}; \mathbf{a}_f^*]$ , where  $f$  and  $t$  index firms and time periods (years) respectively,<sup>7</sup> conditional on the true (\*) value of the production parameters set  $\mathbf{a}_f^*$ .

---

<sup>7</sup> Corresponds to roughly about one-fifth of the quantity of farms in a given state of the Corn Belt, Lake States and Northern Plains regions in the U.S. (Corn Belt states: IA, IL, IN, MO, OH; Lake States: MI, MN, WI; and Northern Plains states: KS, ND, NE, SD). State-level time-series datasets with information on prices and quantities of agricultural outputs and inputs are available for no more than 50 years in the U.S.

The vector  $\mathbf{a}_f^*$  does not depend on time, which implies the assumption that technology remains unchanged from period one through  $T$ . This assumption favors the recovery of true production parameters because the estimation is free from misspecification that may arise from the evolution of technology over time. This is equivalent to postulate a specific form of netput technological change and proceed to estimation by exactly specifying its form as if the econometrician knew it with certainty. A different model specification of the mentioned technical change would only add a higher level of noise in the estimation process. The study of productivity changes over time, their measurement, and their effects on the recovery of true production parameters is a relevant research topic which is beyond the scope of this paper and is left for future research.

To analyze the empirical properties of duality theory, we generate a noiseless and a noisy dataset. The noiseless dataset is not only used to illustrate the ability of duality to recover true production parameters when data is free from common problems, but also to show the implications on parameters recovery when data is aggregated across firms with heterogeneous technology. The noisy dataset allows us to document the effects on production parameter estimation from using duality theory when the dataset features realistic problems.

Figure 2 shows the data simulation process. For both datasets, we start by creating the variables conditioning the firm's decisions problem in (6). First, we generate the set of true production parameters  $\mathbf{a}_f^*$  and the quasi-fixed netputs  $\mathbf{K}_{ft}^*$ . Second, conditioning on these values, we draw expected variable netput prices that are exogenous in the case of the noiseless dataset  $\mathbf{p}_{ft}^{**}$ , and endogenous in the noisy data  $\mathbf{p}_{ft}^*$ . For the latter, calibrated values of initial wealth  $W_{0,ft}$  and the coefficient of absolute risk aversion  $\lambda_f$  are provided for the maximization problem's objective function. Data generation of  $\mathbf{a}_f^*$ ,  $\mathbf{K}_{ft}^*$ ,  $\mathbf{p}_{ft}^{**}$ ,  $\mathbf{p}_{ft}^*$ ,  $W_{0,ft}$ , and  $\lambda_f$  are explained

in sections 4.1 through 4.4. Third, for the noiseless dataset, we solve an expected profit maximization problem to obtain the expected variable netput quantities  $\mathbf{y}_{ft}^{**}$  (section 4.5). This study focuses on time-series estimation and therefore we aggregate variables across heterogeneous firms before proceeding to estimation (section 4.7). The result is a set of estimated production parameters denoted as  $\check{\mathbf{a}}_f$ .

In the case of the noisy data (section 4.6), we assume a risk-averse individual who maximizes utility of end-of-period terminal wealth to obtain optimal expected netput quantities  $\mathbf{y}_{ft}^*$  (subsection 4.6.1). Before proceeding to estimation, we disturb the data with the following sources of noise: shocks in production and expected output and price (section 4.6.2), omitted variables (section 4.6.3), aggregation across netputs (section 4.6.4), and measurement error in price and quantity variables (section 4.6.5). Finally, the variables are aggregated over unobserved heterogeneous firms to conduct time-series estimation (section 4.7). The expected netput quantity and prices are denoted as  $\mathbf{y}_{ft}$  and  $\mathbf{p}_{ft}$ , respectively. Estimation results include the set of production parameters denoted as  $\hat{\mathbf{a}}_f$ .

#### 4.1 Random generation of true production parameters: $\mathbf{a}_f^*$

The value of  $\mathbf{a}_f^*$  characterizes the firm's technology and is unobserved, making its simulation more challenging. From (7),  $\mathbf{a}_f^*$  consists of the submatrices  $A_{1f}$ ,  $A_{2f}$ , and  $A_f$  (formed in turn by  $A_{11f}$ ,  $A_{12f}$  and  $A_{22f}$ ). As we mentioned above, firm heterogeneity exists both within and across regions, such that technology is more similar between firms in the same region than across regions. Hence, we select values of the elements of  $\mathbf{a}$  for a “generic” firm such that the symmetric  $((n + m) \times (n + m))$  matrix  $A$  is positive-semidefinite. To induce variation across regions we obtain “regional”  $\mathbf{a}_r$  sets as deviations from  $\mathbf{a}$ . Then, firm heterogeneity within a



region comes from generating parameters in the firm-specific set  $\alpha_f$  as deviations from their corresponding regional  $\alpha_r$ . To assure the matrix  $A_f$  and its inverse are positive-semidefinite we draw the entries of the upper triangular matrix  $C_f$ , the Cholesky decomposition of matrix  $(A_f)^{-1}$ , such that the latter is formed as the matrix product  $C_f' C_f$ .

The size, dispersion, and skewness of the elements in  $\alpha_f$  determine the size, dispersion, and skewness of the netput quantity variables,  $y_{ft}^*$ , according to the first-order conditions (FOCs) of the firm's optimization problem. Therefore, these elements must be calibrated so as to yield a realistic distribution of quantities produced and used. We rely on the 2002 U.S. Agricultural Census, the USDA-ARMS databases, and weather data from PRISM to accomplish this objective. See Appendix I for further details.

We calibrate the skewness of the firm-specific deviations from “regional”  $\alpha_r$  by fitting a standard beta distribution to the county-level data of the Census variable “Total sales, Value of sales, number of farms” which serves as a proxy for firm size. The shape parameters are estimated by maximum likelihood, yielding a positive skewed distribution. This is consistent with the higher proportion of small firms observed in each region.

The size of the elements in  $\alpha_f$  is tackled by inducing positive rank correlation among the beta random shocks, such that a firm producing high levels of output, is likely to use more inputs.

Finally, to calibrate the unobserved dispersion of  $\alpha_f$  from  $\alpha_r$ , we assume that observed yield dispersion in a region is a function of unobserved technology heterogeneity and observed random weather shocks. If all firms used the same technology, the observed yield variability would come only from weather shocks. At the other extreme where firms all differ but no

weather shocks occur, all yield dispersion comes from heterogeneity across firms. Most likely the reality is somewhere in the middle. We intend to calculate the portion of yield variation attributable to heterogeneity across firms. To this end we use a panel of firm-specific crop yields from USDA-ARMS database and county-specific weather data (growing season precipitation and temperature) from PRISM over five years, and estimate a fixed-effects model.

Yields are specified as a function of a county-specific constant (the fixed effect) representing the average county's technology and cumulative precipitation and average temperature over the growing season, assuming the constant is correlated with the weather variables. The objective is to isolate the between effects, or the variation in yields across counties not attributable to weather, from the within effects or the variation in yields within a county over time. Firm-level yields are specified as follows:

$$y_{ft} = b_{0c} + b_1 W_{1ct} + b_2 W_{2ct} + b_3 D_{1t} + \dots + b_6 D_{4t} + \epsilon_{1ft} \quad (10)$$

where  $c$ ,  $f$  and  $t$  index counties, firms, and time respectively. Variables  $W_1$  and  $W_2$  are precipitation and temperature, respectively, for the county, and  $D_1$  through  $D_4$  are year dummy variables (2001 through 2004 respectively, with year 2000 as the base). The parameter  $b_{0c}$  represents county-level technology and is the focus of our interest. Because we presume it to be correlated with weather variables, we estimate a fixed-effects model where parameters  $b_1$  through  $b_6$  are estimated by demeaning the data (means taken for each county and over time), resulting in the following model (Greene 2003):

$$\ddot{y}_f = b_1 \ddot{W}_{1c} + b_2 \ddot{W}_{2c} + b_3 \ddot{D}_1 + \dots + b_6 \ddot{D}_4 + \epsilon_{2f} \quad (11)$$

with “ $\ddot{\phantom{x}}$ ” indicating demeaned variables, estimated by OLS. The county-specific parameter  $b_{0c}$  is then recovered by calculating the following equation:

$$\check{b}_{0c} = \bar{y}_c - \hat{b}_1 \bar{W}_{1c} - \hat{b}_2 \bar{W}_{2c} - \hat{b}_3 \bar{D}_1 - \dots - \hat{b}_6 \bar{D}_4 \quad (12)$$

where the “ $-$ ” indicates means over time (used in demeaning the model) and the “ $\hat{\phantom{x}}$ ” indicates the point estimate of the parameters. Table 1 provides estimation results.

Finally, the coefficient of variation of  $\check{b}_{0c}$ , representing variation across counties, serves to calibrate the unobserved dispersion of the production parameters  $\mathbf{a}_f$  around the regional mean  $\mathbf{a}_r$  that are not attributable to weather changes.<sup>8</sup> Note that this coefficient of variation does not represent the estimation standard error of the parameter but the production coefficients variation across counties.

#### 4.2 Random generation of quasi-fixed netput quantities: $\mathbf{K}_{ft}^*$

We obtain the vector  $\mathbf{K}_f^*$  of quasi-fixed netputs by drawing  $R \times F$  beta distributed random deviates. The beta distribution is chosen because it can mimic the different levels of skewness observed in the distribution of these variables at the firm level. Because we choose to represent farm size as the quasi-fixed netput, we use the 2002 U.S. Agricultural Census variable “Farms & land in farms, approximate land area” to calibrate the parameters of the beta distribution for each region.<sup>9</sup> This shows a relative abundance of small-sized farms, implying a positively skewed standard beta distribution. Region-specific distributions include:  $\mathbf{K}_{f,r=1}^* \sim \text{Beta}(0.5679, 6.9707)$ ;  $\mathbf{K}_{f,r=2}^* \sim \text{Beta}(0.6026, 9.0446)$ ; and  $\mathbf{K}_{f,r=3}^* \sim \text{Beta}(0.4929, 2.9624)$ .

Because both  $\mathbf{K}_f^*$  and  $\mathbf{A}_f$  determine size of netput quantities, we generate the vector of quasi-fixed netputs imposing positive correlation with the production function parameters. We use the Iman and Conover (1982) method to impose rank correlation.

---

<sup>8</sup> We calibrate the production parameter variation equal to variation between counties as opposed to between firms. Firstly, we do not have firm-specific weather data to calculate the between firms effects. Secondly, the county (and more aggregated) data is likely to have a smaller variation than at the firm level in a given region, favoring parameter recovery.

<sup>9</sup> It is common practice to include land as a quasi-fixed output.

Next, we generate time variation in each firm's quasi-fixed netput quantity by means of a multiplicative and independent shock centered at one and uniformly distributed. That is,

$K_{ft}^* = K_f^* \epsilon_{ft}$ , where  $\epsilon_{ft} \sim \text{Uniform}[0.90, 1.10]$ . The narrow interval implies low variation in firm size over time, which is meant to represent the observed low dispersion over time of aggregate agricultural area in a region.<sup>10</sup>

### 4.3 Random generation of expected variable netput prices: $p_{ft}^{**}$ and $p_{ft}^*$

We generate two sets of firm-specific expected prices for each region. Prices are exogenous in the first case ( $p_{ft}^{**}$ ) and endogenous with respect to the aggregated netput quantity produced in the second one ( $p_{ft}^*$ ). The former is used to test duality theory with noiseless data, and the latter to evaluate its properties in empirical work when using more realistic data.

In the exogenous case, we begin by simulating “national” netput prices to match the properties (mean, standard deviation, and serial autocorrelation) of those found in a time series of future crop prices from the CME and of input prices from Eldon Ball's dataset. We assume firms base their production decisions on future output prices and current input prices.

We model netput prices as lognormally distributed and behaving according to an AR(1) processes:

$$\log(p_{nt}) = \theta_{n0} + \theta_{n1} \log(p_{n,t-1}) + \zeta_n \quad (13)$$

where “ $n$ ” indexes netputs and  $\zeta_n$  is an error term distributed  $N(0, \sigma_{\zeta_n}^2)$ . Parameters  $\theta_n$  are estimated by OLS regressions. Table 2 shows results for each of the  $n$  regressions. Dropping the “ $n$ ” subscript to ease notation, in the long run, the logarithm  $p_t$  and  $p_{t-1}$  converge to  $\bar{p}$  and therefore we can calculate long run expected prices as  $\log(\bar{p}) = \theta_0/(1 - \theta_1)$ . The variance of

---

<sup>10</sup> This creates, for each time period, a distribution of quasi-fixed netput quantities for each firm that is not necessarily the regional Beta (it is Beta with other parameters), but still maintains the required skewed shape due to the lower dispersion of firm size over time.

the error term in (13) can be calibrated from observed price variation of Eldon Ball's datasets:

$\sigma_{\log(\mathbf{p})}^2 = \theta_1^2 \sigma_{\log(\mathbf{p})}^2 + \sigma_\zeta^2$  which implies that  $\sigma_\zeta^2 = (1 - \theta_1^2) \sigma_{\log(\mathbf{p})}^2$ . In this case, we calibrate price variation from a combination of data observed variance and regression results, and not exclusively from the latter.

To draw exogenous log-normal netput prices, we fit (13) with the estimated parameters, set  $\log(p_{s=0}) = \theta_0 / (1 - \theta_1)$ , and take a draw from a  $N(0, (1 - \theta_1^2) \sigma_{\log(\mathbf{p})}^2)$  random variable, yielding a netput price for each  $n$  in the first iteration, i.e.  $\log(p_{s=1})$ . We repeat this procedure  $S=10,000$  times; we keep the last 50 iterations for the set of exogenous “national” netput prices and burn the remaining iterations.

The case of endogenous “national” prices is more involved. Price endogeneity arises as prices respond to changes in aggregated quantities produced or demanded. In a competitive market it is realistic to assume each firm is a price taker, because the netput quantity decisions of any single firm do not affect price levels. This is usually modeled as the firm facing exogenous and fixed netput prices (i.e., a perfectly horizontal demand for outputs and supply for inputs). For an aggregation of firms, this is not necessarily the case. On aggregate, firms face downward sloping demand curves for their outputs and upward sloping supplies for inputs. In these cases, changes in netput quantities at the aggregate level result in market-level price changes.

We introduce a system of isoelastic market demands and supplies faced by firms in period  $t$ , described by  $\mathbf{Q}_t = \mathbf{\Phi}_t \mathbf{p}_t^\eta$ . The  $n$ -dimensional vector  $\mathbf{Q}_t$  is the aggregate market demand of output or the aggregate market supply of input  $n$  faced by firms;  $\mathbf{p}_t^\eta$  denotes a  $n$ -dimensional vector of  $p_{nt}$  netput market prices, each raised to the power of  $\eta_n$  (the calibrated netput-specific demand or supply own price elasticity); and  $\mathbf{\Phi}_t$  is an  $(n \times n)$  diagonal matrix of

supply and demand netput-specific shocks  $\phi_{nt}$  coming from the market. All Greek letters represent calibrated parameters.

The objective is to find a vector of netput prices  $\mathbf{p}_t^*$  where the optimal vector of netput quantities aggregated across firms ( $\mathbf{y}_t^* = \sum_f \mathbf{y}_{ft}^*$ ) equals the vector of market quantities ( $\mathbf{Q}_t$ ). As will become apparent when we set up the firm's maximization problem, we can write the optimal quantity of variable netputs as follows:

$$\mathbf{y}_t^* = \sum_f ((\mathbf{X}_f)(\mathbf{p}_{ft}) + \boldsymbol{\varphi}_{ft}) \quad (14)$$

where  $\mathbf{X}_f$  is a time-invariant matrix of production coefficients summarizing the elements of  $\mathbf{a}_f$ ,  $\mathbf{p}_{ft}$  is the vector of firm-specific prices received (defined as  $\mathbf{p}_{ft} = \mathbf{p}_t \varepsilon_{ft}$  and explained below), and  $\boldsymbol{\varphi}_{ft}$  is a vector of production errors, such as optimization mistakes, weather shocks, deviation of prices from expected values, etc. These errors depend on the firm's production parameters due to the claimed heteroskedasticity given by function  $g(\cdot)$  in (8). By substituting for the firm-specific production coefficients and prices we have:

$$\mathbf{y}_t^* = \sum_f (\mathbf{X}\mu_f) (\mathbf{p}_t \varepsilon_{ft}) + g_0(\mathbf{X}\mu_f \mathbf{p}_t \varepsilon_{ft}) v_{ft} \quad (15)$$

where  $\mathbf{X}$  is the analog of the set of production coefficients  $\mathbf{a}$ ,  $\mathbf{p}_t$  is the vector of “national” prices, and  $g_0(\cdot)$  is the analog of function  $g(\cdot)$ . With a sufficiently high number of farms ( $F$ ) and by independency of the random variables  $\mu_f$ ,  $\varepsilon_{ft}$  and  $v_{ft}$  (the *iid* shocks in  $\psi_{ft}$  and  $\boldsymbol{\varphi}_{ft}$ ),  $\mathbf{y}_t^*$  converges in distribution by the law of large numbers to a normal random variable whose mean is:

$$\bar{\mathbf{y}}_t = F\mathbf{X}\mathbf{p}_t + F\bar{\boldsymbol{\varphi}}_t \quad (16)$$

The expression in (16) depends only on the known “average” production parameters and “national” time- $t$  prices  $\mathbf{p}_t$ , which in fact are the same as those on the isoelastic demand or supply function faced by firms.

Therefore, the vector of time- $t$  netput prices is the  $\mathbf{p}_t^*$  which clears the market ( $\mathbf{Q}_t = \bar{\mathbf{y}}_t$ ), or in other words, the one which implicitly solves the following system for each  $t$ :

$$\mathbf{\Phi}_t \mathbf{p}_t^\eta = F\mathbf{X}\mathbf{p}_t + F\bar{\mathbf{\phi}}_t. \quad (17)$$

The system in (17) is nonlinear in  $\mathbf{p}_t$  and is conditional on known values—the set of known production parameters  $\mathbf{X}$  and time-specific systematic shocks  $\mathbf{\Phi}_t$ . We obtain the desired vector of “national” netput prices  $\mathbf{p}_t^*$  by numerically solving this system for each time  $t$ , given a random market shock  $\mathbf{\Phi}_t$ .

This requires generating values of  $\mathbf{\Phi}_t$ . We model the systematic shocks coming from the market  $\mathbf{\Phi}_t$  as auto-correlated and behaving according to a log-normal distribution:

$$\log(\phi_{nt}) = \rho_{n0} + \rho_{n1} \log(\phi_{n,t-1}) + \xi_n \quad (18)$$

where  $\xi_n \sim \text{Normal}(0, \sigma_{\xi_n}^2)$ . For each netput  $n$ , the three parameters  $\rho_{n0}$ ,  $\rho_{n1}$ , and  $\sigma_{\xi_n}$  are calibrated such that the resulting vector of average prices  $\mathbf{p}_t^*$  have volatility values comparable to those of Eldon Ball’s dataset. Appendix II provides calibration details.

The systematic shocks coming from the market,  $\phi_{nt}$ , are generated for each  $t$  and  $n$  as follows. Dropping the “ $n$ ” subscript again to ease notation, in the long run, the logarithms of  $\phi_t$  and  $\phi_{t-1}$  converge to  $\bar{\phi}$  and therefore we can calculate the long-run expected shocks as  $\log(\bar{\phi}) = \rho_0 / (1 - \rho_1)$ . To generate the desired systematic log-normal shocks, we set  $\log(\phi_{s=0}) = \rho_0 / (1 - \rho_1)$ , take a draw from a  $N(0, \sigma_{\xi_n}^2)$  random variable, and use (18) to

calculate the systematic shock for the first iteration, i.e.  $\log(\phi_{s=1})$ . This procedure is repeated  $S=10,000$  times.

Next, we plug the vector  $\phi_s$  for each iteration into (17) and use the MATLAB function *fsolve* to solve this system 10,000 times for the vector of “national” prices that simultaneously clears the  $n$  markets in each  $t$ .<sup>11</sup> We keep the last 50 solutions which constitute the vector of endogenous “national” netput prices  $\mathbf{p}_t^*$ .

Finally, for both the exogenous and endogenous case, we generate firm-specific netput prices  $\mathbf{p}_{ft}^*$  (and  $\mathbf{p}_{ft}^{**}$ ) as deviations from the “national” market price, deviations that are small relative to  $\mathbf{p}_t^*$  (and  $\mathbf{p}_t^{**}$ ) to acknowledge for the contemporaneous low variability of prices firms receive and pay. A regional average is first calculated as  $\mathbf{p}_{rt}^* = \mathbf{p}_t^* d_r \varepsilon_{rt}$ ,<sup>12</sup> where  $d_r$  is a regional indicator with mean one across regions<sup>13</sup> and  $\varepsilon_{rt}$  is a mean one symmetric shock distributed as  $\varepsilon_{rt} \sim [0.95 + 0.1\text{Beta}(2,2)]$ . Random variables  $d_r$  and  $\varepsilon_{rt}$  are symmetric and independently distributed. The indicator implies prices of region  $r$  are on average  $(d_r - 1)\%$  away from the national average, and the  $\varepsilon_{rt}$  allows for non-constant deviations over time.

From the regional prices, we generate  $F$  firm-specific random prices per region as deviations from the regional average:  $\mathbf{p}_{ft}^* = \mathbf{p}_{rt}^* \varepsilon_{ft}$ , where  $\varepsilon_{ft}$  is a symmetric mean one shock distributed as follows:  $\varepsilon_{ft} \sim [0.80 + 0.40\text{Beta}(2,2)]$ . Shocks  $\varepsilon_{rt}$ ,  $\varepsilon_{ft}$ , and  $d_r$  are independent. Values for  $\varepsilon_{ft}$  are calibrated using prices from the USDA-ARMS dataset, such that they yield a coefficient of variation of 0.08, which is twice as much as the one observed in the USDA-ARMS dataset.

---

<sup>11</sup> Price variability is a key element for recovering production parameters, because a high dispersion contributes to the identification of a bigger portion of the production function. Random draws from  $N(0, \sigma_{\xi_n})$  are independent from each other, and therefore systematic shock are as well; however, when plugged into system (17) correlation between national prices is induced through matrix  $\mathbf{X}$ . This DGP ultimately generates national netput prices with higher temporal variance and with lower correlation between netput prices than Eldon Ball’s dataset. These two aspects favor identification in estimation when prices are explanatory variables, as it is our case.

<sup>12</sup> The same procedure and shocks are used for  $\mathbf{p}_{ft}^{**}$ .

<sup>13</sup> The values of  $d_r$  are 0.90, 1.00, and 1.10 for regions 1 through 3 respectively.



For simulation, netput prices are correlated with quantities at the aggregate level, but independent at the firm level. While it is possible prices received and paid could be correlated with firm size, we assume independence. This assumption favors parameter identification. Also, observed prices in USDA-ARMS show the majority of firm-level prices are concentrated at four or fewer different prices in each region; however, we generate a “continuum” of firm-specific prices to favor identification.

#### 4.4 Random generation of initial wealth: $W_{0,ft}$

In the noisy dataset, each firm  $f$  at time  $t$  is assumed to be an expected utility maximizer. The argument of the utility function is end-of-period terminal wealth calculated as initial wealth ( $W_{0,ft}$ ) plus random profits. Based on the strong correlation observed between total net assets ( $TNA_{ft}$ ) and value of production ( $VP_{ft}$ ) in the USDA-ARMS database, we model initial wealth as a function of each firm’s value of production.<sup>14</sup> The following model is used for each firm  $f$  at time  $t$ :

$$TNA = \gamma_0 + \gamma_1 VP + \gamma_2 VP^2 + \tau \quad (19)$$

where  $\tau$  is an heteroskedastic error term distributed  $N(0, \sigma_\tau^2)$  which accounts for the non-constant variation observed in total net assets as a function of value of production. We seek to estimate the  $\gamma$  parameters as well as the form of the heteroskedasticity. Following Wooldridge (2003), we model heteroskedasticity as follows:  $\hat{u}^2 = \exp(\delta_0 + \delta_1 VP + \delta_2 VP^2)\kappa$ , where  $\hat{u}^2$  is the estimated variance of  $\tau$  and  $\kappa$  is a mean one multiplicative error term. This implies  $\log(\hat{u}^2) = \delta_0 + \delta_1 VP + \delta_2 VP^2 + e$  or a linear regression for which  $e \sim N(0, \sigma_e^2)$ .

---

<sup>14</sup> Total net assets are calculated as “value of total farm financial assets” minus “total farm financial debt.” Value of production is calculated as “all crops – value of production” plus “all livestock – value of production.” These two variables from USDA-ARMS database constitute the dataset used to estimate the model in equation (19).

We estimate model parameters with USDA-ARMS data. Table 3 shows results. Using these parameter estimates and the value of production coming from our model, we generate the initial wealth for each firm  $f$  in time  $t$  (details in Appendix III).

#### 4.5 Simulation of noiseless dataset

The noiseless dataset is formed by variable netput quantities and prices, and quasi-fixed netputs:  $[\mathbf{y}_{ft}^{**}, \mathbf{p}_{ft}^{**}, \mathbf{K}_{ft}^*]$ . We first solve the problem in (6) assuming all farmers are risk neutral (or expected profit maximizers) and prices received or paid are exogenous ( $\mathbf{p}_{ft}^{**}$ ). These results are used to test the accuracy of duality theory in recovering production technology using time-series data whose only source of noise is aggregation across heterogeneous firms. This constitutes the minimum possible noise when interested in applying duality theory with time series. Under the normalized quadratic production function  $G(\mathbf{y}_{ft}^{**}, \mathbf{K}_{ft}^*; \mathbf{a}_f)$  in (20), the FOCs are:

$$\mathbf{p}_{ft}^{**} - A_{1f} - A_{11f}\mathbf{y}_{ft}^{**} - A_{12f}\mathbf{K}_{ft}^* = 0 \quad (20)$$

This system is jointly solved for the vector of optimal variable netput quantities  $\mathbf{y}_{ft}^{**}$  as a function of the vector of variable netput prices  $\mathbf{p}_{ft}^{**}$ , the vector of quasi-fixed netput quantities  $\mathbf{K}_{ft}^*$ , and the production parameters  $\mathbf{a}_f^*$ . The solution is:

$$\mathbf{y}_{ft}^{**}(\mathbf{p}_{ft}^{**}, \mathbf{K}_{ft}^*; \mathbf{a}_f^*) = A_{11f}^{-1}(\mathbf{p}_{ft}^{**} - A_{1f} - A_{12f}\mathbf{K}_{ft}^*) \quad (21)$$

This produces a panel dataset of  $(R \times F)$  firms over  $T$  time periods that can be used to recover production parameters using time-series or cross-section with noiseless data. We denote this dataset as follows:

$$[\mathbf{y}_{ft}^{**}, \mathbf{p}_{ft}^{**}, \mathbf{K}_{ft}^*] \quad (22)$$

## 4.6 Simulation of noisy dataset

In this section, we explain how we generate data to mimic the features faced by practitioners when working with real-world data. It contains variable netput quantities and prices, and quasi-fixed netputs:  $[y_{ft}, p_{ft}, K_{ft}^*]$ . We assume risk-averse firms that maximize expected utility, and the data is subject to omitted variables, aggregation across netputs, measurement error, and aggregation across heterogeneous firms.

### 4.6.1 Maximization of expected utility

We solve the problem in (6) for the vector of expected variable netput quantities

$y_{ft}^*(p_{ft}^*, K_{ft}^*, \lambda_f, W_{0,ft}; \mathbf{a}_f^*)$  conditional on expected netput prices, quasi-fixed netput quantities, the level of absolute risk aversion ( $\lambda_f$ ), initial wealth, and the true production parameters. Values of  $\lambda_f$  are consistent with a relative risk aversion coefficient uniformly distributed in the interval  $[2.0, 4.0]$  (Pennacchi, 2008 pp. 16). This constitutes a source of noise because duality theorem assumes a deterministic problem whose solution is generally different from the expected utility case.

We solve the problem using numerical methods and employing Gaussian quadratures. Using four nodes for each output price and quantity random variable we are guaranteed to exactly approximate the problem's objective function up to the seventh moment. For this application, the numerical integration of the objective function has to take into account its multi-dimensions, that nodes behave according to nonstandard distributions, and that they are correlated with each other. Given these problem requirements, we create a routine to calculate nodes and weights used in the objective function approximation. First, based on the MATLAB functions *qnwnorm* (Miranda and Fackler, 2011), that calculates standard normal nodes and weights, we generate four independent log-normally distributed nodes and weights for each of

the three output price random variables. Similarly, based on the function *qnbeta*, that calculates standard beta nodes and weights, we generate four independent nodes and weights distributed beta in the interval of interest for the three output quantity random variables. Second, using the Iman and Conover (1982) method, we impose directly to the nodes, negative correlation between output prices and quantities (correlation coefficient equal to -0.30) and positive correlation within them (coefficient of 0.90). These transformations do not affect the weights. Third, we use the MATLAB function *fmincon* to optimize the approximated objective function. We pass the FOCs and SOC to the optimization routine, respectively, as equality and inequality constraints. Based on the normalized quadratic production function in (6), the FOCs are:

$$E[U'(\tilde{W}_1)(\tilde{\mathbf{p}}^* - A_1 - A_{11}\tilde{\mathbf{y}} - A_{12}\mathbf{K} + \mathbf{g}'(\mathbf{y}) \cdot \mathbf{v})] = 0 \quad (23)$$

with  $U'(\tilde{W}_1) = \lambda e^{-\lambda \tilde{W}_1}$ . The SOC is:

$$\begin{aligned} E[U''(\tilde{W}_1)(\tilde{\mathbf{p}}^* - A_1 - A_{11}\tilde{\mathbf{y}} - A_{12}\mathbf{K} + \mathbf{g}'(\mathbf{y}) \cdot \mathbf{v})^2 \\ + U'(\tilde{W}_1)(-A_{11} + \mathbf{g}''(\mathbf{y}) \cdot \mathbf{v})] \leq 0 \end{aligned} \quad (24)$$

where  $U''(\tilde{W}_1) = -\lambda^2 e^{-\lambda \tilde{W}_1}$ .

The optimal solution is the vector of expected netput quantities for each farm and time that we denote as  $\mathbf{y}_{ft}^*$ .

#### 4.6.2 Realized shocks of production and prices

Farmers solve the maximization problem given a set of output prices, which reflect their expectations of harvest prices. It is commonly accepted that prediction errors make this difference relevant. Even in the case of locking in the production with instruments such as forward contracts, it might be the case that not all of the production is sold under this type of arrangement. In the case of input prices, some prices might not be known at the beginning of the production period, especially for inputs purchased during the season. In either case, deviations

from the true expected price induce bias and inconsistency in the parameter estimates of a model when prices are regressors, and consequently in the elasticities of interest.

We obtain the observed price  $\mathbf{p}_{ft}$  by drawing from the distribution in equation (9) and adding that draw to the price used in optimization:

$$\log(\mathbf{p}_{ft}) = \log(\mathbf{p}_{ft}^*) + \mathbf{e}_t \quad (25)$$

where  $\mathbf{e}_t$  is a realization of a  $N(0, \sigma_e = 0.2)$  random variable for the case of outputs (Lence, 2009) and a  $N(0, \sigma_e = 0.1)$  for the inputs, implying price deviation from decision values is lower for inputs. These shocks are systematic and affect all firms by the same proportion in a given time.

We also claim that data on netput quantities are different from planned quantities or the optimal solution of (6) due to uncertain events in agricultural production, such as weather. We model the observed quantities as follows:

$$\mathbf{y}_{ft} = \mathbf{y}_{ft}^* + g(\mathbf{y}_{ft}^*)\mathbf{v}_{ft} \quad (26)$$

where the shock  $\mathbf{v}_{ft}$  is a realization of the random variable controlling production errors ( $\psi_{ft}$ ) given by (8). We assume this shock has two components; one is systematic given by  $\mathbf{v}_t \sim -1 + 2Beta(2,2)$ , and the other is idiosyncratic modeled as:  $\mathbf{v}_{ft} = \mathbf{v}_t * U[0.87, 1.13]$ . This allows weather variables to not only affect production quantities over time but also have different local effects in a given year. To calibrate the width of the interval, we rely on the fixed-effects model estimates of (11). We measure the contribution of weather variables to yield variation by fitting a “restricted” model with only the weather and time-dummy variables using the estimated parameters. The coefficient of variation of the fitted yields provides the dispersion of weather shocks  $\mathbf{v}_{ft}$ .

Finally, we introduce contemporaneous negative correlation between quantity and price shocks with a coefficient equal to -0.3 (Rosas, Babcock, Hayes, 2012), and positive correlation with a coefficient of 0.9 within quantities and within prices.

#### 4.6.3 Omitted variable netputs

Production takes place with several netputs, but the econometrician rarely observes them all. This situation can arise due to a misreporting of data from a surveyed producer in which one or more than one netputs are omitted, or when some inputs are not part of the surveyed set. In either case, while the producer optimally chooses a set of  $n$  variable netputs to maximize profits, the econometrician only observes a subset of them.

#### 4.6.4 Aggregation across netputs

Technology processes employ a variety of inputs to produce several outputs; however, data available to practitioners is usually not as disaggregated. In some cases, even if data is available for several inputs and outputs, they are aggregated because they are not the objective of the study and/or to not excessively penalize the degrees of freedom during estimation. We aggregate netput quantities and prices at the firm and for each time. Dropping the  $f$  and  $t$  subscripts, this is done as follows:

$$\begin{aligned} y_{n^i} &= \sum_{n^{ij} \in \Omega_i} w_{n^{ij}} y_{n^{ij}} \\ p_{n^i} &= \sum_{n^{ij} \in \Omega_i} w_{n^{ij}} p_{n^{ij}} \end{aligned} \tag{27}$$

where  $\Omega_i$  is a subset  $i$  of netputs,  $n^{ij}$  is the  $j^{th}$  netput in subset  $\Omega_i$ , and  $n^i$  denotes a new netput formed by the aggregation of those in subset  $\Omega_i$ . The pooling of variable netput quantities ( $y_{n^i}$ ) in set  $\Omega_i$  is performed by a weighted average of the quantities in the set, with weights given by each netput value ( $p_{n^{ij}} y_{n^{ij}}$ ) share on the total value of the set  $\Omega_i$ ; that is

$w_{n^{ij}} = (p_{n^{ij}} y_{n^{ij}}) (\sum_{n^{ij} \in \Omega_i} p_{n^{ij}} y_{n^{ij}})^{-1}$ . The procedure to aggregate netput prices is analogous.

#### 4.6.5 Measurement error in prices and quantities

Measurement error is a common problem in datasets available to researchers and induces bias and inconsistency in parameter estimation. Efforts to quantify the level of errors in the data include Morgenstern (1963), who identifies a 10% standard error in the national income data, and reports that the U.S. Department of Commerce in the state-level Food and Kindred Products data has an 8% measurement error in input and output figures. Lusk et al. (2002) study the consequences of applying duality theory using variables measured with error, and Lim and Shumway (1992a, 1992b) analyze violations of maintained hypotheses such as profit maximization, convex technology, and regressive technical change. Based on the mentioned literature, we calibrate an error with a standard deviation of 0.05 around the “true” value for netput prices, 0.08 for variable netput quantities, and 0.10 for quasi-fixed netputs. Calibration values are less than or equal to those reported in the literature, especially in the case of prices. The error is distributed as standard  $Beta(2,2)$  and we modify its interval to yield the desired standard deviation.

Added noise described in previous subsections implies the following panel for firm  $f$  and time  $t$  which we can use to recover production parameters applying either time-series or cross-section analysis:

$$[\mathbf{y}_{ft}, \mathbf{p}_{ft}, \mathbf{K}_{ft}; \boldsymbol{\alpha}_f^*] \quad (28)$$

#### 4.7 Unobserved Firm Heterogeneity.

Finally, in agreement with this study’s objective of testing duality theory using time-series data, before estimation we proceed to aggregate across the  $F=10,000$  heterogeneous firms as if data came from a single firm. This aggregation is performed on both the noiseless data described in (22) and the noisy data in (28). If the objective were to study empirical properties of duality

under a cross-sectional dataset, we would have taken one year of the panel and conducted the analysis without aggregating across firms. This is left for future research.

For each period  $t$ , we aggregate the subvector  $[\mathbf{y}_{ft}, \mathbf{p}_{ft}, \mathbf{K}_{ft}]$  across firms to obtain observations over  $T=50$  time periods (years) of a “single firm”  $[\mathbf{y}_t, \mathbf{p}_t, \mathbf{K}_t]$ . For netput quantities, we aggregate by adding across firms since they are homogeneous commodities. The  $n^{\text{th}}$  netput price at period  $t$  ( $p_{nt}$ ) is a quantity-weighted average of the firm-specific netput prices.

$$\begin{aligned}\mathbf{y}_t &= \sum_f \mathbf{y}_{ft} \\ \mathbf{K}_t &= \sum_f \mathbf{K}_{ft} \\ p_{nt} &= (y_{nt})^{-1} \sum_f p_{nft} y_{nft}.\end{aligned}\tag{29}$$

The time-series noiseless dataset used in estimation is denoted as follows:

$$[\mathbf{y}_t^{**}, \mathbf{p}_t^{**}, \mathbf{K}_t^{*}]\tag{30}$$

and the noisy dataset used in estimation is the following:

$$[\mathbf{y}_t, \mathbf{p}_t, \mathbf{K}_t].\tag{31}$$

## 5. Data for estimation

Noiseless data in (30) includes all  $n = 8$  netput quantities and prices, and  $m = 1$  quasi-fixed netput. Variable netput prices are exogenous from quantities but have serial autocorrelation. The DGP yields 500 thousand observations for each of the three regions ( $F=10,000$  firms in the region over  $T=50$  years). We aggregate the 10,000 heterogeneous firms at each time  $t$ , resulting in a dataset of 50 observations for each variable per region that we use to estimate a system of netput demands and supplies in (33). To avoid the addition of another source of noise coming from heterogeneous technology across regions, we select region 1 to conduct the estimation, and compare results with the true parameters of that same region. The consequences of pursuing estimation incorporating data from other more heterogeneous regions to capture a broader area



and increase the sample size, which is common in these applications, is shown as a sensitivity analysis.

In the case of noisy data in (31), the dataset includes  $n' = 4$  netputs due to the omission of one input and one output, the pooling of two variable outputs into one, and pooling of two variable inputs into one. There is also one quasi-fixed netput because we did not consider the case of omitting the quasi-fixed netput. The figure below represents the structure of the noisy data.

Variable netput	1	2		4	5	6	7	
--------------------	---	---	---	---	---	---	---	---

Noisy data is also subject to endogeneity between netput prices and quantities, maximization under expected utility of risk-averse farmers, production and price unexpected shocks, and measurement error. We take region's population of 10,000 heterogeneous firms and draw 100 samples of 6,000 observations. We aggregate over the heterogeneous firms resulting in a time-series dataset of 50 observations for each variable and, for each sample, conduct econometric estimation of the system in (33). We sample from the population to avoid final results to be dependent on a single sample<sup>15 16</sup>. For the same reasons stated above, we select region 1 to conduct the estimation. The case of pooling observations from heterogeneous regions as a way of increasing the number of observations in estimation, and its effects on parameters recovery is presented as sensitivity analysis.

<sup>15</sup> Given that the population size in each region is relatively large, we do not require too many samples to achieve robust results. Also, the sample size within a region (6,000) is sufficiently high if compared to real-world datasets used to construct state-level aggregates. For example, the 2004 ARMS dataset consists of samples that range between 48 and 1600 firms depending on the state, with an average of 428 firms.

<sup>16</sup> For comparison, estimation was also conducted using the entire population in the region and aggregating across all the heterogeneous firms, which implies only one time-series dataset to be estimated. Results were very similar to the case of the 100 samples from the population.

## 6. Estimation

We approximate the restricted profit function  $\pi_R(\mathbf{p}, \mathbf{K})$ , solution to the problem in (3), by the following normalized quadratic flexible functional form:

$$\pi_R(\mathbf{p}, \mathbf{K}; \boldsymbol{\beta}) = \mathbf{p}' B_1 + \mathbf{K}' B_2 + \frac{1}{2} \mathbf{p}' B_{11} \mathbf{p} + \mathbf{p}' B_{12} \mathbf{K} + \mathbf{K}' B_{22} \mathbf{K} + \mathbf{p}' \boldsymbol{\kappa} \quad (32)$$

where  $B_1$  and  $B_2$  are  $(n \times 1)$  and  $(m \times 1)$  vectors of  $\beta_i$  coefficients,  $B_{11}$  is a symmetric  $(n \times n)$  matrix, and  $B_{12}$  and  $B_{22}$  are  $n \times m$  and  $m \times m$  matrices. Submatrices  $B_{11}$ ,  $B_{12}$ , and  $B_{22}$  form a symmetric  $((n + m) \times (n + m))$  matrix  $B$  of  $\beta_{ij}$  coefficients, which in the case of the NQ profit function, is exactly the Hessian matrix with respect to  $(\mathbf{p}, \mathbf{K})$ . All  $\beta_i$  and  $\beta_{ij}$  coefficients collectively form the set  $\boldsymbol{\beta}$ . The error structure  $\mathbf{p}' \boldsymbol{\kappa}$  is consistent with McElroy's (1987) additive general error model (AGEM) applied to the case of profit functions. The  $(n \times 1)$  vector of random variables  $\boldsymbol{\kappa}$  is jointly normally distributed with mean equal to a  $(n \times 1)$  vector of zeros and an  $(n \times n)$  covariance matrix  $\boldsymbol{\Sigma}_\kappa$ . This covariance matrix induces contemporaneous correlation between the equations. Also, the DGP of netput prices—both exogenous and endogenous—was constructed as an AR(1) process, implying serial autocorrelation in the independent variables that needs to be accounted for in the estimation.

We derive the set of input demands and output supplies by Hotelling's lemma, yielding the system to be estimated:

$$\mathbf{y}(\mathbf{p}, \mathbf{K}; \boldsymbol{\beta}) = B_1 + B_{11} \mathbf{p} + B_{12} \mathbf{K} + \boldsymbol{\kappa}. \quad (33)$$

We conduct estimation by iterated SUR, which converges to maximum likelihood, and is the most common method employed in empirical works based on duality theory. We impose symmetry cross-equation restrictions ( $\beta_{ij} = \beta_{ji}, i \neq j$ ) in matrix  $B_{11}$ . We do not estimate the parameters of the profit function because the parameters needed to evaluate the production parameters of interest are present in the demands and supplies.

*Treating Mean-Independence Violations in Estimation.* First, an inspection of the autocorrelation and partial autocorrelation functions of the noiseless and noisy time series suggests first differentiation of the data for estimation. This is a consequence of the DGP of price data as AR(1) processes.

Second, for the case of the noisy dataset only, we employ an instrumental variables approach to treat the omitted price variables. We use the same omitted variables as instruments which are regarded as the best instrument possible.

Third, we also use instruments to account for the endogeneity of the explanatory variables in the case of the noisy dataset. Endogeneity is present because the independent variables (prices) in the output supplies and input demands system are correlated with the error term  $\kappa$  as a consequence of the systematic shocks  $\phi_{nt}$  in the market. Instruments have to be correlated with prices but uncorrelated with the error term. Given that we know the source of the endogeneity (i.e., shocks  $\phi_{nt}$ ), we construct instruments by regressing each price on its systematic shock:  $p_{nt} = \lambda_0 + \lambda_1 \phi_{nt} + iv_{nt}$ . The error term ( $iv_{nt}$ ) is, by construction, orthogonal to  $\phi_{nt}$  but correlated with  $p_{nt}$  accounting for the variation of prices not explained by the systematic shocks, constituting the ideal instrument. There is one instrument for each netput price, as well as one instrument for each omitted variable.

The estimated values of matrix  $B_{11}$  and vector  $B_{12}$  are the focus of our attention; they are, respectively, the marginal effects of prices and quasi-fixed netputs on netput quantities, and therefore they are the base to construct the estimated profit function Hessian matrix  $[\hat{B}]$  and the elasticities matrix of netput quantities with respect to own price, cross prices, and quasi-fixed

netputs  $[\hat{E}]$ .<sup>17</sup> As described in Figure 3 we obtain matrices  $[\check{B}]$  and  $[\hat{B}]$  from estimation using noiseless and noisy datasets respectively, that are then transformed into elasticity matrices in a straightforward way. In order to compare estimated elasticities with true values, we proceed as follows. We begin from the true and known firm-specific production function Hessian matrix  $[A]_f$  and convert it into the corresponding profit function Hessian  $[B]_f$  using Lau's Hessian identities. We further transform the true profit function Hessian into the true matrix of own- and cross- price elasticities and quasi-fixed elasticities of netput quantities  $[E]_f$ . Finally, as indicated in Figure 3, we compare the true  $[E]_f$  versus the estimated values ( $[\hat{E}]$  and  $[\check{E}]$ ) to evaluate how precisely we recover the true price and quasi-fixed netput elasticities under duality theory, both in the case of noiseless and noisy data. Note that this comparison implies that the true values are represented by a distribution of each firm's true parameters, while the estimated values consist of a point estimate and its confidence interval.

## 7. Results

Estimation results for the noiseless and noisy data are presented separately.

### 7.1 Noiseless data estimation

Figure 4, Figure 5, and Table 4 summarize the results from estimating output supplies and input demands parameters in (33).

In Figure 4 we show how the estimated own- and cross- price elasticity of netput quantities ( $\check{E}_{ij}$ ) compare with the distribution of true firm-specific elasticities, the mean of the distribution ( $\bar{E}_{ij}$ ) and its median ( $\bar{\bar{E}}_{ij}$ ), for the 64 entries of the  $8 \times 8$  elasticity matrix. The vertical axis represent the mean of the distribution of true elasticities and the horizontal axis show descriptive statistics of the distribution of true elasticities (represented by the horizontal

---

<sup>17</sup> In the case of noiseless data they are denoted as  $[\check{B}]$  and  $[\check{E}]$  respectively.

line<sup>18</sup>), the mean of the distribution (diamond), median of the distribution (filled square), and the SUR estimated elasticity (circle). Therefore all the means (diamonds) are along the 45° line. The median (filled square) is to the left or to the right of the mean depending on the skewness of the distribution. The elasticity point estimates (circle) and the 95% confidence intervals (vertical lines) are in all cases within the support of the true distribution. This implies that estimation with a noiseless dataset, constructed as the aggregation across heterogeneous firms (as if it belonged to a representative firm), is able to recover elasticities that are not only within the relevant range of the distribution but also fairly close to the median and the mean.

A second conclusion arises by noting that the point estimates are closer to the median of the distribution than to the mean. In the case of noiseless data, the representative firm is better described by the median of the distribution than the mean. The root mean squared error (RMSE) helps illustrate this conclusion. The RMSE is the average difference between each entry of the estimated elasticity matrix versus its corresponding true elasticity, expressed in elasticity units. We show two alternative values to describe the true elasticity: the median of the true firm-specific elasticity distribution and its mean. When compared to the median of the distribution, the RMSE is:

$$RMSE = \left[ \frac{1}{64 \times S} \sum_i \sum_j \sum_s (\bar{\bar{E}}_{ij,s} - \check{E}_{ij,s})^2 \right]^{1/2} \quad (34)$$

where  $S = 10,000$  is the number of draws from the limiting distribution of the SUR parameter estimates and the subscript  $s$  indicates the  $s^{\text{th}}$  draw of the  $ij^{\text{th}}$  parameter. For comparison with the mean we substitute  $\bar{\bar{E}}_{ij}$  by  $\bar{E}_{ij}$ . The RMSE averages over all the  $64 \times S$  squared differences. We also provide a measure of its dispersion by calculating the standard deviation of these  $64 \times S$

---

<sup>18</sup> The horizontal line represents the 90% highest probability density interval of the true distribution.

values before averaging over them. The RMSE standard deviation contains two sources of variation or error. One is due to the SUR estimation error within each of the 64 parameters and the other is associated with the variation of the difference between the estimated and the true value of the elasticity across the 64 parameters.

As shown in Table 4, RMSE is 0.048 in the case of the median and more than double (0.111) for the mean. To put these values into perspective, we calculate the percentage deviation of the RMSE with respect to the descriptive statistics of the true distribution of elasticities. Relative to the median it yields a difference of 12.4% and, as expected, it is higher relative to the mean, 26.3%.

The RMSE standard deviation is 0.078 for the median and a higher value (0.196) for the mean. Given the SUR estimation provides only a minor source of error because the point estimates are all highly significant due to the use of noiseless data,<sup>19</sup> the majority of the RMSE standard deviation is attributed to the deviations between the estimated and the true value across elasticities.

Figure 5 illustrates the estimated results of the eight netput quantity elasticities with respect to the quasi-fixed input. The SUR estimated elasticities (circles) are within the interval of true elasticity distribution for all cases, and similar to the variable netputs case, closer to the median of the distribution than to its mean. As Table 4 indicates, the RMSE is 0.035 in the case of the media, and 0.071 for the mean. The size of the RMSE standard deviation also suggests high variation (of their dispersion relative to the true value) across the 8 elasticities. Our estimated elasticities are 7.5% apart from the median absolute value of the true elasticity and 14.7% from the mean absolute value.

---

<sup>19</sup> Results are available upon request.

## 7.2 Noisy data estimation

Estimation with noisy data consists of 16 own- and cross-price elasticity values of variable netput quantities and 4 elasticities with respect to quasi-fixed netputs. Figure 6 shows the distribution of the true firm-specific price elasticities and its corresponding SUR point estimates indicated with a red circle (and its 95% confidence interval with a red “+” sign). After estimation we take 10,000 draws from the parameters asymptotic distribution of each of the 100 samples, transform them into elasticities, and calculate their mean, standard deviation, and confidence interval over the 1,000,000 values. Except for the case of entries (2 2), (2 3), (3 2), and (3 3), all other distributions involve more than one true elasticities due to the aggregation of netputs, as described in section 4.6.4. In these cases, and in order to compare with the SUR estimated elasticities, we construct a “new true” elasticity distribution as a revenue weighted average of the original true elasticities.

In light of previous section conclusions, we measure the accuracy in recovering the true elasticity by comparing the estimated value to the median of the true distribution.<sup>20</sup> Figure 6 shows that duality theory provides an apparent good approximation to the true distribution in some cases, and a bad one in others, when comparing where the estimated value falls relative to where the true distribution accumulates more mass.

However, as Table 5 shows, the percentage difference between the median of the true distribution ( $\bar{\bar{E}}_{ij}$ ) and the estimated value ( $\hat{E}_{ij}$ ) is high for the majority of the entries in the elasticity matrix. The difference ranges between 6% and 247%, and is less than 12% in only one entry. The estimation of the own price elasticities (main diagonal) are not recovered with sufficient precision given that the differences range between 15% and 44%. Moreover, entries (2

---

<sup>20</sup> A comparison using the mean of the true distribution was conducted and provided less accurate results.

2) and (3 3) which correspond to netputs 4 and 5 (which are not aggregated with other netputs) are more imprecisely estimated than the other main diagonal elements which do arise as aggregated netputs. In the case of the off-diagonal elements (or the cross-price elasticities) as their recovery requires more information, they are, as expected, more inaccurately estimated than the main diagonal entries.

As a summary measure of the dispersion in recovering the true elasticities, we calculate the RMSE of the difference between the median of the true distribution and the SUR estimated values for all 16 estimated elasticities. As shown in Table 6 under case 1, it yields a value of 0.22 in elasticity units. The average value of all true elasticities (calculated as the mean absolute value of all the medians of the true distributions) is 0.31. Therefore, by comparing both values we conclude that duality theory recovers elasticities which are, on average, 71% deviated from the true elasticity. These results provide evidence that the econometric approach of duality is unable to deliver precise estimates of underlying production parameters when employed with data featuring real-world characteristics.

The estimation of variable netput elasticities with respect to quasi-fixed netputs is even more inaccurate. Results are shown in Figure 7. Each panel titled as  $E_{ik}$  is the elasticity of netput  $i$  with respect to the quasi-fixed netput. The SUR point estimates of the elasticities are within the support of the true distribution except for  $E_{4k}$  in which case, the estimated elasticity has the reversed sign. As a similar summary measure, the RMSE relative to the median of the true distribution is 0.67 expressed in elasticity units, and the average value of the elasticities is calculated at 0.54. These results, shown in Table 6 under case 1, imply that there is an inaccuracy in recovering the true elasticities, averaging over the 4 netputs, reaching 123%.



We explore the robustness of noisy data estimation results to changes in the sources and levels of noise. Estimations that consider different sets of omitted netputs and different sets of aggregated netputs provide similar results. For example, two estimations with the noisy data structure shown below yields that econometric applications of duality theory yields price elasticity estimates with respect to variable netputs that are 57% and 69% deviated from the true price elasticities. Elasticities with respect to quasi-fixed netputs differ 76% and 120% relative to their true values.

Variable netput	<del>1</del>	2	3	<del>4</del>	5	6	7	8
Variable netput	1	2	<del>3</del>	4	5	6	<del>7</del>	8

These values, shown in Table 6 under case 2 and case 3 respectively, are quantitatively similar to our previous conclusions, suggesting that other combinations of omitted netputs and aggregated netputs would provide results that are at least qualitatively similar.<sup>21</sup>

We regularly encounter empirical applications of duality theory with time-series data where observations from different regions or states are pooled together for estimation; examples are Schuring, Huffman and Fan (2011) and O'Donnell, Shumway and Ball (1999). While on the one hand it increases the sample size favoring the degrees of freedom, which is especially advantageous in the presence of several explanatory variables, on the other, it implies considering observations from states that are likely to have different technology. We conduct a sensitivity analysis to study the consequences of such practice. This practice implies seeking to recover production parameters from firms that are more heterogeneous than in the case of a single state usually by adding regional- (or state-) level dummy variables. For this sensitivity analysis we employ our noisy simulated data from regions 1, 2, and 3 in (28). From each region

---

<sup>21</sup> We present only a few cases due to the computational burden of such analysis

and in each of the 50 time-periods, we take five samples of 2,000 observations representing samples of firms from five states within the region. We aggregate each sample across its heterogeneous firms to obtain the time-series data for each state. We stack the observations by state first, and then by region, resulting in a dataset composed by 750 observations. Note that results above are derived with 50 observations. We estimate model in (33), and as it is done in the mentioned studies, we add dummy variables for observations in region 1 and 2, leaving region 3 as the base. We transform the estimated parameters into netput elasticities with respect to variable netput prices and with respect to quasi-fixed netputs, and compare them with the true elasticities. These are represented by the distribution of true firm-specific elasticities, as it was in the previous analysis, but in this case, is the distribution over the firms in the three regions.

We find that the RMSE relative to the median of such distribution of true elasticities, and averaged over the 16 elasticities calculated is 53%, which is very similar to the findings above. This is formed as the ratio between the RMSE relative to the median of the true distribution (0.18) and the median of the true distribution of elasticities (0.35). Divergence from true elasticities ranges between 11% and 209% depending on which of the 16 entries of the elasticities matrix we consider. Standard errors of the estimated elasticities are lower than in the previous analysis, which in part, is a consequence of an increased number of observations. However that does not contribute to reduce the bias in the estimated parameters and elasticities relative to the true ones. Results when RMSE is calculated with respect to the mean of the distribution and results of netput quantity elasticities with respect to quasi-fixed netputs also indicate inaccuracy in recovering production parameters. Therefore, the practice of incorporating data from other regions, characterized by a more heterogeneous technology than within the

region, only contributes to reducing the standard error of the point estimates but does not help in reducing the bias relative to the true elasticity values.

## **8. Conclusions**

The dual relationship between the production function and the profit or cost function established by the Neoclassical theory of the firm has been widely applied in empirical work with the objective of obtaining price elasticities, substitution elasticities, and return to scale estimates. This empirical method, usually referred to as “duality theory approach” has the advantage of providing the mentioned features of the production function using market data on input and output prices and quantities, without the requirement of explicitly specifying the technology relationships. However, the duality theorem requires a set of assumptions, which we claim they fail to hold in practice; or in other words, market data typically employed in this type of studies bears levels of noise that prevent the theorem from holding exactly. If this is the case, the elasticity estimates will be biased with respect to their true values.

In this paper we analyze the ability of the approach to recover the technology features when the dataset taken to estimation reflects real-world characteristics comparable to those found by practitioners in empirical applications. We start by selecting a parametric form of the production technology and choosing its set of parameter values. Using Monte Carlo simulations, we generate observations of netput prices and quantities such that they are comparable to those found in data on U.S. agriculture. More precisely, we generate a panel of production and price data for successive periods of time, coming from a population of technologically heterogeneous firms that belong to different regions. In this regard, the data generation process incorporates optimization under uncertainty, prediction errors in prices and quantities of variable netputs, endogenous prices, omitted variable netputs, output and input data aggregation, measurement

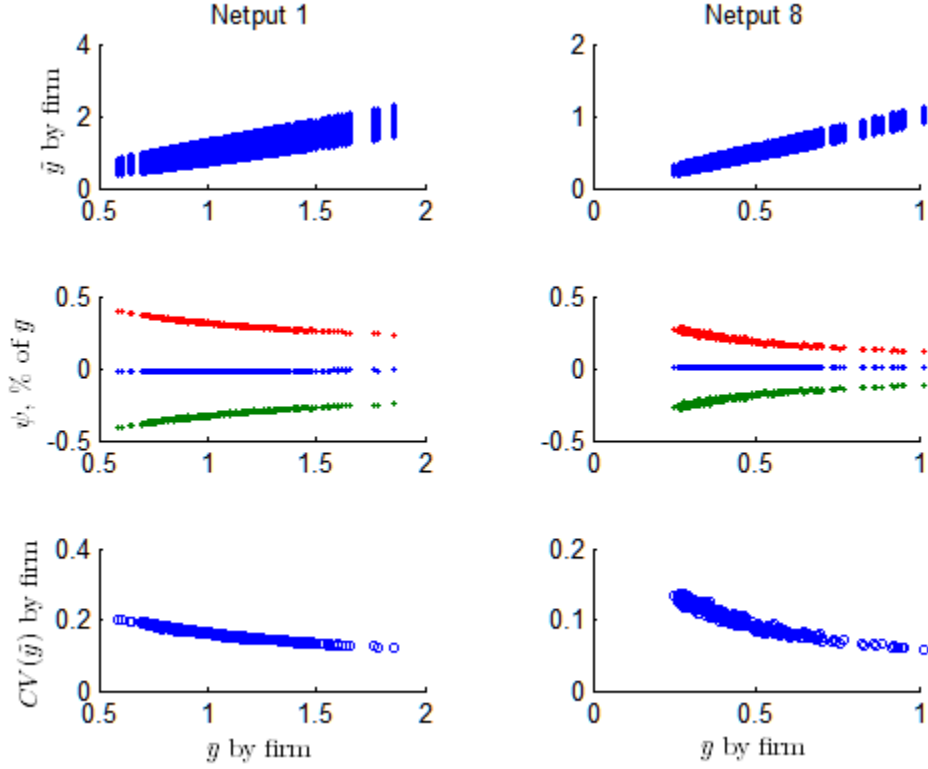
errors in the observed variables, and unobserved heterogeneity across firms. We calibrate model parameters using datasets (both time-series and cross-sectional) widely employed in empirical applications.

Estimated parameters (and resulting elasticities) come from applying econometric methods to a system of input demands and output supplies with the simulated data. Because the true parameters are known from the outset, we can judge the degree with which the dual approach is able to recover these parameters. Also, because we know the existing sources of noise in the data, we explicitly treat them in estimation. We deal with serial autocorrelation by estimating the model with data in its first differences. We tackle omitted variables employing an instrumental variables approach, in which our instruments are precisely the variables we omit in the first place. Similarly, instruments are used to consider the presence of endogeneity in aggregate prices. In this case, we also know the source of endogeneity and therefore we can construct the best set of instruments possible. Comparison between true and recovered parameters relies on the use of Lau's (1976) Hessian identities.

Results show that the dual approach applied on a time-series dataset bearing the minimum noise possible, i.e., only the aggregation across technologically heterogeneous firms is able to recover elasticities that not only are within the support of the distribution of true elasticities, but also considerably close to the mean and median of such distribution. We conclude that this approach applied to aggregated (county-, state-, or country-level) data as if it belonged to a representative firm optimizing for the entire region, will recover technology features that are close to the firm in the median of the distribution, provided that the data is not mined with other sources of noise.

Second, the use of noisy data prevents the dual approach from providing parameter estimates that are sufficiently close to their true values. Both own- and cross-price elasticities are inaccurately recovered even when it is the case that the former require less information from the data than the latter to be estimated with the same level of precision. The root mean squared error, measuring the average deviation of the estimated elasticities from their true values, is calculated at 71%, implying that the dual approach estimates elasticities are, on average, 71% away from the true values. The case of netput elasticities with respect to quasi-fixed netputs is even more inaccurate. Results are robust to different calibrations of the data structure, specifically, the omission and aggregation of different sets of netputs, as well as the sample of firms used in estimation. Also, sensitivity analysis shows that a common practice of pooling data from different states and/or regions in order to increase the degrees of freedom in estimation produces a similar bias in the estimated elasticities as in the case of considering a single and more technologically homogeneous state.

## Figures



**Figure 1. Production shock as a function of firm's average variable netput quantity ( $\bar{y}_{ft_0}$ ) on time  $t = t_0$ , for selected netputs.**

Note: Top panels: distribution of netput quantities ( $\bar{y}_{ft_0}$ ) faced by each firm. Middle panels: minimum, mean, and maximum shock as percentage of firm's average quantity  $\bar{y}_{ft_0}$ . Bottom panels: coefficient of variation of the distribution of quantities  $CV(\bar{y}_{ft_0})$  by firm.

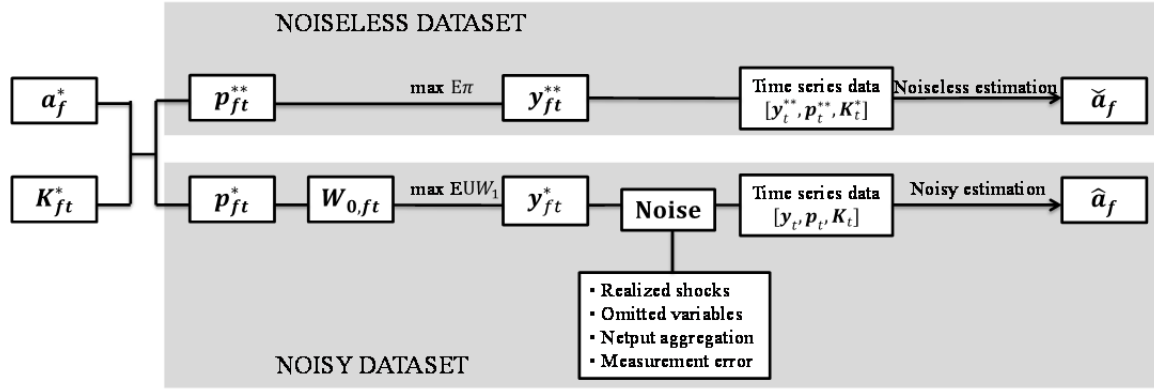


Figure 2. DGP of noiseless and noisy datasets used in estimation.

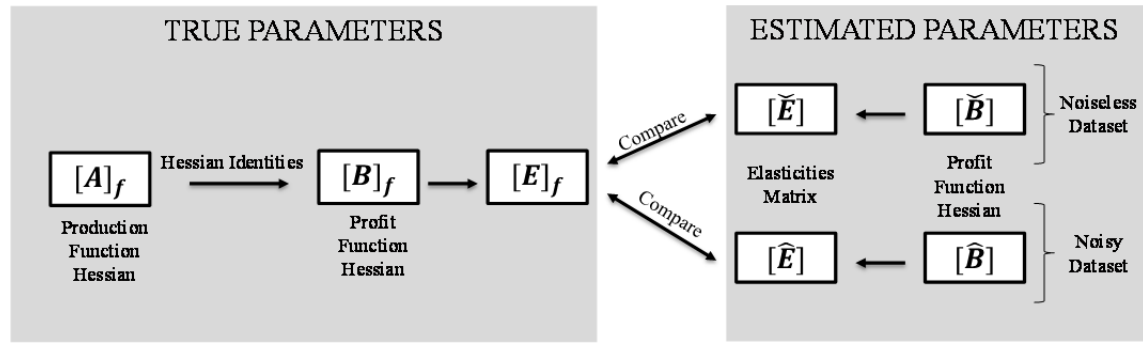
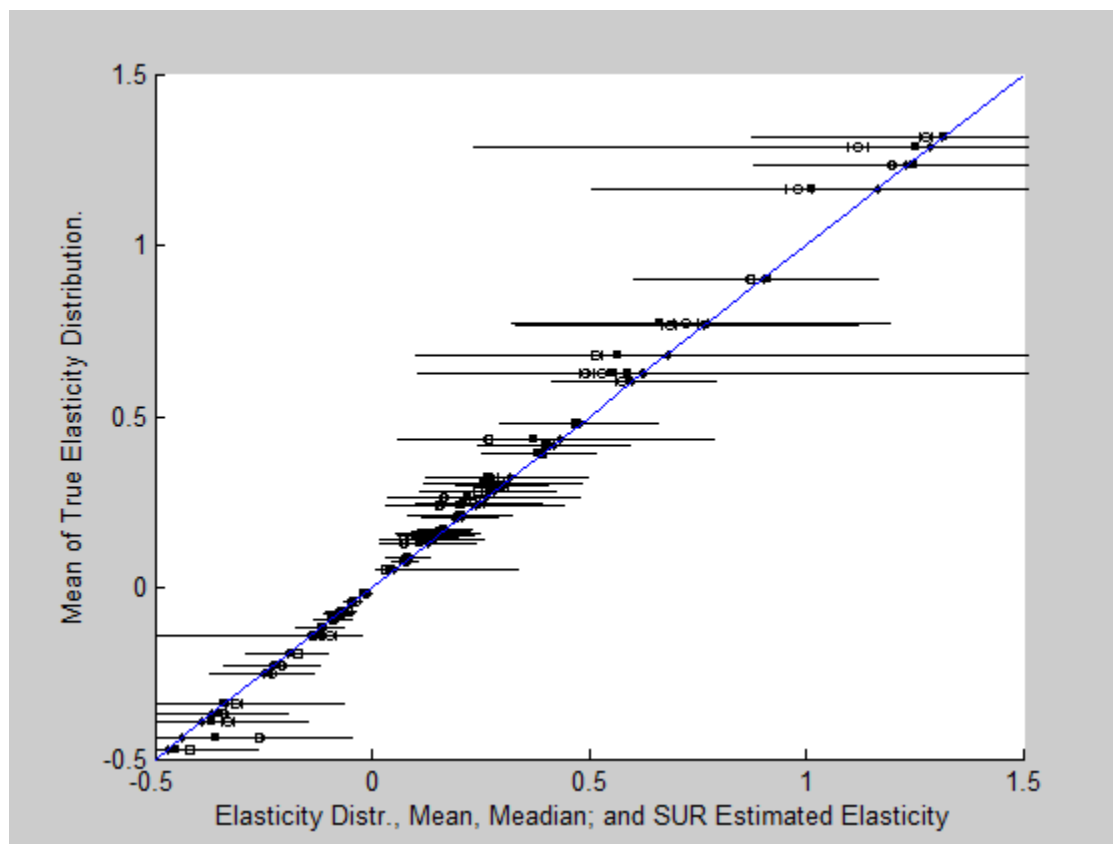
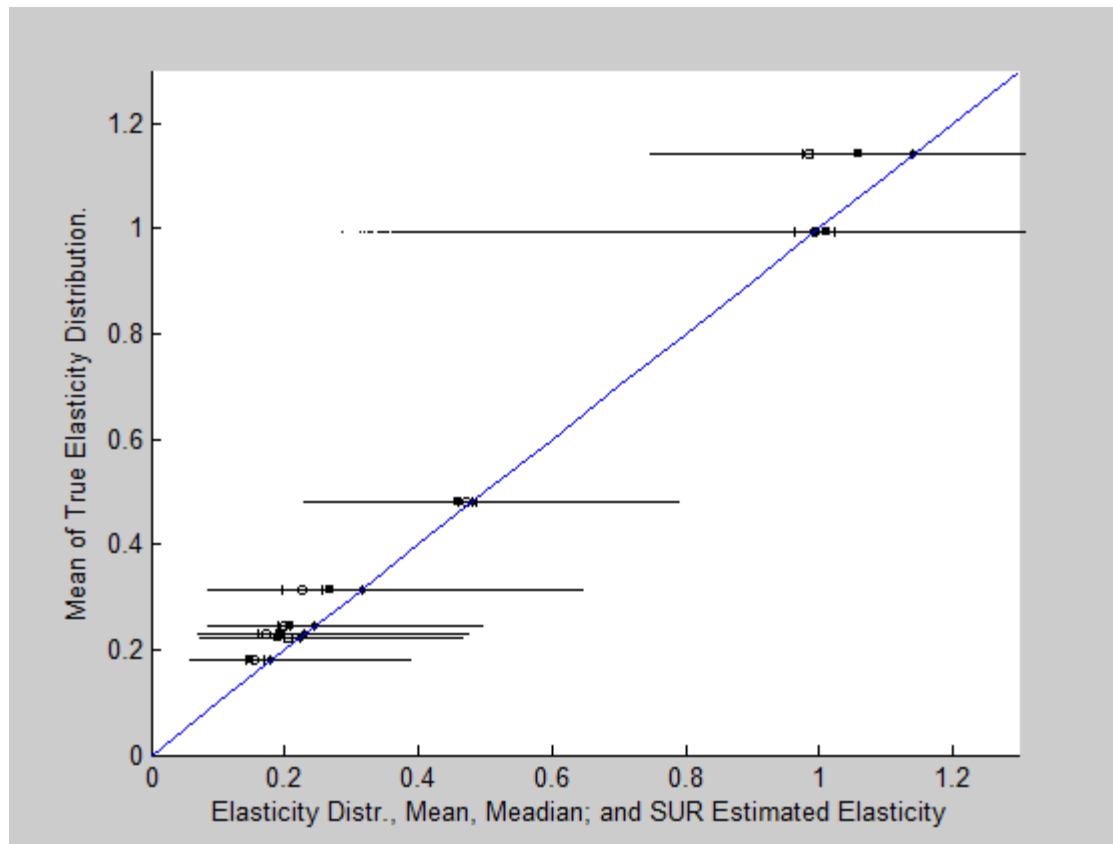


Figure 3. Comparison between true and estimated elasticities for noiseless and noisy datasets.

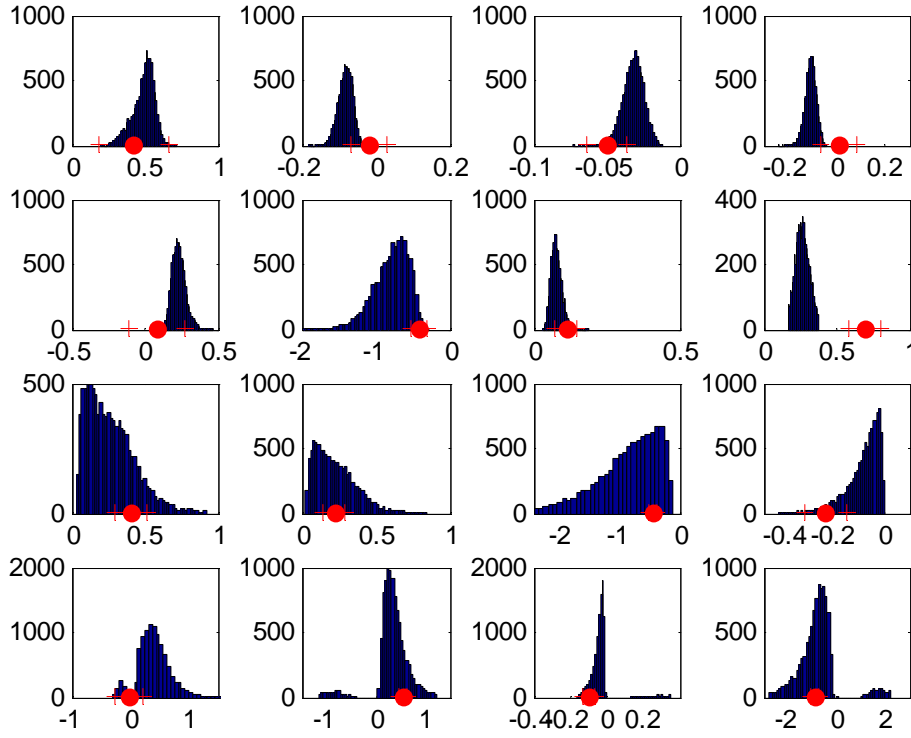


**Figure 4. Elasticities of variable netput quantities with respect to prices. True versus estimated values with noiseless data.**



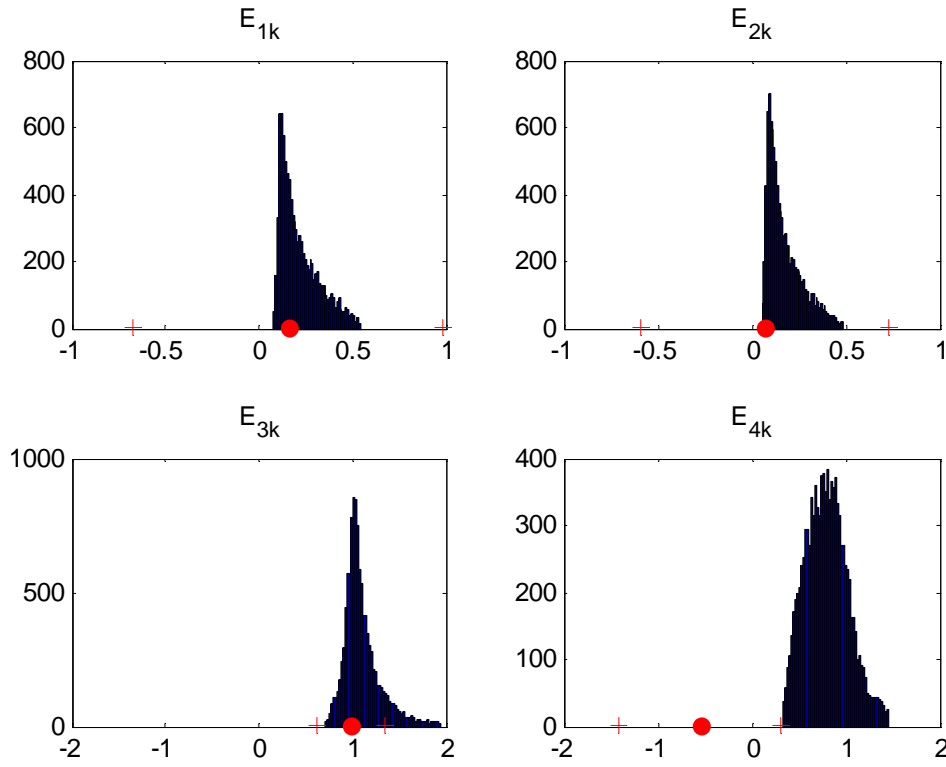


**Figure 5. Elasticities of variable netput quantities with respect to quasi-fixed netputs. True versus estimated values with noiseless data.**



**Figure 6. Own- and cross-price elasticities of variable netput quantities. True versus estimated values with noisy data.**

Note. Each  $ij$  panel is the  $ij$  entry of the 4x4 own- and cross-price elasticity matrix  $E$  in the case of noisy data. The elasticity value is in the horizontal axis and histogram frequency in the vertical. The histograms are the distribution across firms of the true elasticity ( $E_{ij}$ ). Red dot is the SUR estimated elasticity ( $\hat{E}_{ij}$ ) and red “+” sign is the 95% confidence interval.



**Figure 7. Elasticity of variable netput quantities with respect to quasi-fixed netputs. True versus estimated values with noisy data.**

Note. Each  $E_{ik}$  panel is the elasticity of netput  $i$  with respect to the quasi-fixed input in the case of noisy data. The elasticity value is in the horizontal axis and histogram frequency in the vertical. The histograms are the distribution across firms of the true elasticity ( $E_{ik}$ ). Red dot is the SUR estimated elasticity ( $\hat{E}_{ik}$ ) and red “+” sign is the 95% confidence interval.

**Table 1. Parameter estimates of fixed effects model, equation (11), to calibrate production function parameter variation, and realized weather shocks on netput quantities.**

Dependent variable: $\ddot{y}_f$	Region 1	Region 2	Region 3
Explanatory variables	Parameter estimates: $b_i, i = 1, \dots, 6$		
Precipitation: $\ddot{W}_{1c}$	-0.0002 (0.0008)	0.0040 (0.0007)	0.0019 (0.0014)
Temperature: $\ddot{W}_{2c}$	-0.3202 (0.0322)	-0.0408 (0.0229)	0.0028 (0.0359)
Year 2001: $\ddot{D}_1$	5.7192 (7.2434)	-12.5194 (2.0513)	4.7602 (3.0744)
Year 2002: $\ddot{D}_2$	-17.5728 (5.1586)	0.2315 (1.6542)	-7.8958 (3.2074)
Year 2003: $\ddot{D}_3$	-17.5792 (4.1648)	2.7325 (1.8828)	-2.5457 (2.7899)
Year 2004: $\ddot{D}_4$	8.7423 (4.1253)	1.6293 (1.8068)	27.9887 (4.1110)
Firm heterogeneity contribution to yield variation: $CV(\ddot{b}_{0c})$	0.0578	0.1702	0.4276
Weather variables contribution to yield variation (CV)	0.0726	0.1263	0.4040

Note:  $\ddot{y}_f$  denotes demeaned farm-specific crop yields. Accent character “ .. ” represent a demeaned variable. Standard errors in parenthesis.

**Table 2. Estimation results of the OLS regression model used to generate random exogenous “national” prices from equation (13).**

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
$\theta_{n0}$	-0.0306 (0.0376)	-0.0645 (0.0324)	-0.0124 (0.0304)	-0.0308 (0.0638)	-0.0012 (0.035)	-0.0566 (0.0347)	0.0413 (0.024)	0.0008 (0.0366)
$\theta_{n1}$	0.6802 (0.0942)	0.3443 (0.1437)	0.6711 (0.1128)	0.9015 (0.0785)	0.8613 (0.0799)	0.6029 (0.1166)	0.8428 (0.0804)	0.9225 (0.0536)
$\bar{p}$	0.9087	0.9063	0.9629	0.7317	0.9913	0.8672	1.3007	1.01
$\log(\bar{p})$	-0.0958	-0.0984	-0.0378	-0.3123	-0.0088	-0.1425	0.2629	0.0099
$\sigma_{\zeta}^2$	0.068	0.0342	0.0372	0.034	0.0439	0.0392	0.0207	0.0237

Note: Standard errors in parenthesis.

**Table 3. Parameter estimates of initial wealth and the form of its heteroskedsticity, equation (19).**

<b>Dependent variable: <math>TNA</math></b>	<b>Region 1</b>	<b>Region 2</b>	<b>Region 3</b>
<b>Explanatory variables</b>	<b>Parameter estimates: <math>\gamma_i, i = 0, 1, 2</math></b>		
Constant	0.724 (0.040)	0.8429 (0.058)	0.8917 (0.058)
$VP$	1.279 (0.064)	1.1375 (0.062)	0.5743 (0.042)
$VP^2$	-0.066 (0.008)	-0.019 (0.002)	-0.0096 (0.001)
$R^2$	0.16	0.1947	0.0922
<b>Dependent variable: <math>\log(\hat{u}^2)</math></b>	<b>Region 1</b>	<b>Region 2</b>	<b>Region 3</b>
<b>Explanatory variables</b>	<b>Parameter estimates: <math>\delta_i, i = 0, 1, 2</math></b>		
Constant	-2.062 (0.045)	-1.925 (0.058)	-1.5069 (0.048)
$VP$	1.544 (0.071)	0.9639 (0.063)	0.416 (0.035)
$VP^2$	-0.105 (0.008)	-0.0264 (0.002)	-0.0061 (0.001)
$R^2$	0.16	0.1131	0.0805
$\hat{\sigma}_e$	2.084	2.170	2.0104

Note:  $TNA$ : Total Net Assets.  $VP$ : Value of Production. Standard errors in parenthesis.

**Table 4. Comparison of estimated elasticities versus moments of the distribution of true elasticities (noiseless data).**

<b>Elasticities with respect to</b>		<b>Moment of True Distribution</b>	
		Median	Mean
<b>Netputs Prices</b>	Variable RMSE	0.048	0.111
	Std. error	0.078	0.196
	% deviation	12.4	26.3
<b>Quasi-fixed</b>	RMSE	0.035	0.071
<b>Netputs</b>	Std. error	0.044	0.089
<b>Quantities</b>	% deviation	7.5	14.7

**Table 5. Comparison of estimated elasticities ( $\hat{E}_{ij}$ ) versus median of true elasticities distribution ( $\bar{\bar{E}}_{ij}$ ) in the case of noisy data.**

		<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>1</b>	$\bar{\bar{E}}_{ij}$	0.505	-0.081	-0.031	-0.106
	$\hat{E}_{ij}$	0.424	-0.019	-0.050	0.009
	Std. Dev.	(0.125)	(0.024)	(0.007)	(0.038)
	Interval	0.178 0.670	-0.066 0.027	-0.064 -0.036	-0.066 0.084
	% diff.	16%	76%	60%	108%
<b>2</b>	$\bar{\bar{E}}_{ij}$	0.230	-0.757	-0.076	0.263
	$\hat{E}_{ij}$	0.078	-0.423	0.112	0.688
	Std. Dev.	(0.097)	(0.056)	(0.019)	(0.055)
	Interval	-0.111 0.268	-0.532 -0.312	0.074 0.150	0.580 0.796
	% diff.	66%	44%	247%	162%
<b>3</b>	$\bar{\bar{E}}_{ij}$	0.244	0.206	-0.810	-0.075
	$\hat{E}_{ij}$	0.399	0.217	-0.457	-0.246
	Std. Dev.	(0.057)	(0.038)	(0.035)	(0.045)
	Interval	0.288 0.510	0.144 0.291	-0.525 -0.389	-0.334 -0.158
	% diff.	64%	6%	44%	229%
<b>4</b>	$\bar{\bar{E}}_{ij}$	0.397	0.332	-0.035	-0.782
	$\hat{E}_{ij}$	-0.028	0.547	-0.101	-0.895
	Std. Dev.	(0.124)	(0.044)	(0.018)	(0.069)
	Interval	-0.271 0.215	0.461 0.633	-0.136 -0.065	-1.031 -0.759
	% diff.	107%	65%	190%	15%

Note: Interval is the 95% confidence interval of the point estimate  $\hat{E}_{ij}$



**Table 6. Sensitivity analysis. Comparison of estimated elasticities ( $\hat{E}_{ij}$ ) versus median of true elasticities distribution ( $\bar{\bar{E}}_{ij}$ ) in the case of noisy data, and different sources of noise.**

Elasticities with respect to		RMSE relative to Median		
		case 1	case 2	case 3
<b>Variable Netput Prices</b>	RMSE	0.22	0.26	0.20
	Median	0.31	0.46	0.29
	% deviation	71%	57%	69%
<b>Quasi-fixed Netput Quantity</b>	RMSE	0.67	0.44	0.42
	Median	0.54	0.57	0.35
	% deviation	123%	76%	120%

Note: Each case consists of a different set of omitted netputs and a different set of netputs aggregated together. In case 1, netputs 3 and 8 are omitted, and netputs 1 and 2, and 6 and 7 are aggregated. In case 2, netputs 1 and 4 are omitted, and netputs 2 and 3, and 7 and 8 are aggregated. In case 3, netputs 3 and 7 are omitted, and netputs 1 and 2, and 5 and 6 are aggregated.

## 9. Appendix

### 9.1 Appendix I. Random generation of the firm-specific sets $\mathbf{a}_f^*$

We start from the “generic” set  $\mathbf{a}$  which is composed by the vectors  $A_1, A_2, A_{11}, A_{12}, A_{21}, A_{22}$ , and matrix  $A_{11}$ , where

$$A_1 = [0.007 \quad 0.007 \quad 0.004 \quad -0.059 \quad -0.314 \quad -0.028 \quad -0.017 \quad -0.081]'$$

$$A_2 = -70.620$$

$$A_{11} = \begin{bmatrix} 1.569 & 1.062 & 0.027 & 2.245 & 1.986 & 1.378 & 0.742 & 3.120 \\ 1.062 & 1.899 & 0.051 & 3.025 & 2.840 & 1.985 & 1.143 & 4.435 \\ 0.027 & 0.051 & 3.154 & 4.059 & 1.030 & 1.530 & 3.356 & 1.646 \\ 2.245 & 3.025 & 4.059 & 16.789 & 7.045 & 5.359 & 7.293 & 10.778 \\ 1.986 & 2.840 & 1.030 & 7.045 & 17.592 & 4.210 & 2.169 & 9.662 \\ 1.378 & 1.985 & 1.530 & 5.359 & 4.210 & 5.624 & 3.481 & 6.311 \\ 0.742 & 1.143 & 3.356 & 7.293 & 2.169 & 3.481 & 7.016 & 3.539 \\ 3.120 & 4.435 & 1.646 & 10.778 & 9.662 & 6.311 & 3.539 & 19.629 \end{bmatrix}$$

$$A_{12} = (A_{21})' = [5.493 \quad 8.967 \quad 8.221 \quad 35.195 \quad 36.758 \quad 19.172 \quad 20.705 \quad 38.203]$$

$$A_{22} = 150.640$$

These values are based on profit function estimated parameters  $\mathbf{B}_{ij}$  found in literature using Eldon Ball’s dataset (Schuring, Huffman, and Fan 2011). We transform the original estimates to meet the desired convexity properties and convert them to production function parameters using Hessian identities. This provides us with a first approximation of the parameters size.

*Vectors  $A_{1,r}$ ,  $A_{2,r}$ ,  $A_{1,f}$  and  $A_{2,f}$ :* For  $r = \{1,2,3\}$ , we obtain the regional vectors  $A_{1,r}$  and  $A_{2,r}$ , by respectively affecting each entry of  $A_1$  and  $A_2$  by independent multiplicative shocks  $v_a \sim \text{Uniform}[0.60, 1.40]$ . Then the farm-specific vectors within each region  $A_{1,f}$  and  $A_{2,f}$ , are obtained from each regional value also by inducing variation with correlated and multiplicative shocks  $\mu_a$  distributed beta. These shocks determine the size, dispersion and skewness of the netput quantities produced, so they need to be calibrated accordingly. To control for the

skewness, we use the county-level variable “Total sales, Value of sales, number of farms” of the 2002 U.S. Agricultural Census as a proxy of firm’s size, to fit a standard beta distribution for each region; the results are: a Beta(0.3062, 2.5654) for region 1; a Beta(0.2810, 2.4012) for region 2; and a Beta (0.3315, 2.1364) for region 3. To obtain the desired variability of the firm-specific parameters we modify the beta distributions interval widths to [0.90, 1.40] [0.90, 2.00], and [0.90, 4.20] for regions 1 through 3 respectively, so they match the coefficient of variation of the technology parameters estimated by the fixed-effects regression using USDA-ARMS and PRISM data and described in the text. Because parameters determine firm size, we impose a positive correlation of 0.9 between the shocks, so that firms producing high output quantities also use more inputs. In all cases, correlation is imposed by the Iman and Conover (1982) method.

*Matrices  $A_{11,r}$  and  $A_{11,f}$ :* We generate the inverse of the regional and firm-specific matrices  $A_{11,r}$  and  $A_{11,f}$ , because the latter is the one entering the FOCs of the firm’s problem. First, we perturb each entry of an upper triangular matrix  $C$  representing the Cholesky factorization of the “generic” positive-semidefinite matrix  $(A_{11})^{-1}$ , such that  $(A_{11,r})^{-1} = C_r' C_r$ . This guarantees the matrices of interest are positive-semidefinite in each iteration. The regional deviations come from using an independent and multiplicative shock denoted as  $v_b$  and distributed Uniform [0.70, 1.30]. Then, to obtain the firm-specific submatrices  $(A_{11,f})^{-1}$  in each region we induce variation on the Cholesky factors of  $(A_{11,r})^{-1}$  with correlated and multiplicative beta shocks  $\mu_b$  with shape parameters mentioned in the previous paragraph, but over the intervals [0.90, 1.20], [0.90, 1.60] and [0.80, 2.60] for regions 1, 2 and 3 respectively. Again, we set the interval width so that the coefficient of variation of the parameters matches

that from the fixed effects regression for each region. Also, we impose positive correlation among the parameters of the matrix to control for firm size.

*Vectors  $A_{12,f}$  and  $A_{22,f}$ :* Similar to the case of matrices  $A_{11,r}$  and  $A_{11,f}$ , we construct these vectors, as well as the “generic” vectors  $A_{12}$  and  $A_{22}$ , starting from the “generic” profit function parameters  $B_{12}$  and  $B_{22}$ , and using Hessian identities in (4). This is done not only to guarantee theoretically consistent values of the vectors of interest, but also because profit function parameters are readily available in the literature. We respectively shock each entry of  $B_{ij}$  by independent multiplicative deviations  $v_c \sim \text{Uniform}[0.95, 1.05]$ , obtaining regional  $B_{ij,r}$ . The corresponding firm-specific values ( $B_{ij,f}$ ) within a region come from deviations of the regional  $B_{11,r}$ ,  $B_{12,r}$  and  $B_{22,r}$  by means of multiplicative and correlated shocks beta  $\mu_c$ , and then transformed into  $A_{12,f}$  and  $A_{22,f}$  using the Hessian identities in (4). Note that in this process we do not directly generate regional vectors  $A_{12,r}$  and  $A_{22,r}$ . The Beta distribution shape parameters are the ones stated above and the intervals for  $B_{12,f}$  are set at  $[0.90, 2.00]$ ,  $[0.90, 2.20]$ , and  $[0.80, 3.60]$  for each region, and at  $[0.90, 1.10]$  for all regions in the case of  $B_{22,f}$ . The narrow interval in the latter case is due to the fact that enough variation is already induced on  $A_{22,f}$  by  $B_{11,f}$ ,  $B_{12,f}$  and  $B_{22,f}$  through the Hessian relationship. Finally, we impose positive correlation between the entries of  $A_{12,f}$  and  $A_{22,f}$  to take care of firm size.

We calibrate the width of the beta intervals enumerated above by trial and error such that they yield a set of firm-specific production parameters  $\alpha_f^*$  in each region whose coefficient of variation is consistent with  $\check{b}_{0c}$  estimated with the fixed-effects model. These are 0.06, 0.17, and 0.43 for regions 1 through 3 respectively.

## 9.2 Appendix II. Random generation of market shocks $\Phi_t$

The vector of market elasticities of the isoelastic demand or supply ( $Q_t = \Phi_t p_t^\eta$ ) faced by firms is the following (FAPRI Elasticities Database and other sources):

$$\eta = [-0.25 \quad -0.21 \quad -0.75 \quad 0.90 \quad 0.87 \quad 0.85 \quad 0.83 \quad 0.80]$$

The calibrated parameter values of market shocks ( $\phi_{nt}$ ) in equation (18) used to generate random endogenous “national” prices are in the table below. We calculate these values as follows.

**Table 7. Calibrated parameter values of market shocks ( $\phi_{nt}$ ) in equation (18)**

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
$\rho_{n1}$	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
$\log(\bar{\phi})$	10.2193	10.5644	10.3588	9.528	8.5391	8.9875	8.9013	9.2518
$\sigma_{\log(\phi)}^2$	0.2372	0.104	0.1874	0.4071	0.4807	0.6218	0.9402	0.3587
$\rho_{n0}$	5.1096	5.2822	5.1794	4.764	4.2696	4.4937	4.4506	4.6259
$\sigma_{\xi_n}^2$	0.1779	0.078	0.1406	0.3053	0.3605	0.4664	0.7051	0.269

We generate data for variables  $\phi_{n,t}$  in equation (17). These shocks are not observed directly so we approach the problem as follows. What we do observe are time series of netput prices (as in Eldon Ball’s dataset), which are related to the market shock by equation (17). So we first, plug values of randomly generated netput prices (10,000 time periods described in Appendix II) into the system, and solve for starting values of  $\phi_{n,t}$  with MATLAB function *fsolve*. This yields a time series of  $\phi_{n,t}$  that allows us to “learn” about its moments conditional on the AR(1) log-normally distributed prices, “average” production parameters, and elasticities.

From (18), the long run mean of market shocks is  $\log(\bar{\phi}) = \rho_0/(1 - \rho_1)$ , and the long run variance is  $\sigma_{\log(\phi)}^2 = \rho_1^2 \sigma_{\log(\phi)}^2 + \sigma_{\xi}^2$ , or  $\sigma_{\xi}^2 = (1 - \rho_1^2) \sigma_{\log(\phi)}^2$  if we solve for the variance

of the error term. Both the mean and the variance are directly obtained from the time series of starting values of  $\phi_{n,t}$ . Therefore we have two equations and three parameters ( $\rho_0, \rho_1$ , and  $\sigma_\xi^2$ ) implying that we need to fix one. We fix  $\rho_1 = 0.5$ , and from the mean and variance of the mentioned time series ( $\log(\bar{\phi})$  and  $\sigma_{\log(\phi)}^2$  respectively) we calculate  $\rho_0$  and  $\sigma_\xi^2$  for each netput  $n$  (as reported in **Table 7**).

Then we set  $\log(\phi_{s=0}) = \rho_0 / (1 - \rho_1)$ , draw a random deviate from a  $N(0, \sigma_\xi^2)$ , and using (18) and  $\rho_0$  and  $\rho_1$  we calculate market shock for the initial period,  $\log(\phi_{s=1})$ . We iterate over this procedure  $S=10,000$  times and for each netput  $n$ , obtaining a time series of market shocks. These are finally plugged in system (17) to solve for the vector of “national” netput prices using MATLAB function *fsolve*, as explained in the text.

### 9.3 Appendix III: Random generation of initial wealth $W_{0,ft}$

Based on parameter estimates in **Table 3**, the firm- and time-specific initial wealth  $W_{0,ft}$  is generated as follows:

STEP 1: Obtain the value of production of firm  $f$  in time  $t$ , calculated as:  $VP_{ft} = \mathbf{y}_{ft}^{***'} \mathbf{p}_{ft}^*$ .

Endogenous prices  $\mathbf{p}_{ft}^*$  are those from section 4.3. We approximate firm's netput quantities  $\mathbf{y}_{ft}^{***}$  as the solution of problem (6) under risk-neutrality. We do not assume risk-aversion at this stage because solving the expected utility problem requires conditioning on initial wealth which is what we are trying to calculate. The value of  $\mathbf{y}_{ft}^{***}$  allows us to calculate a proxy for firm's value of production and initial wealth and is used exclusively in this step and nowhere else in the analysis.

STEP 2: Take a draw from  $\kappa \sim \text{logn}(-\frac{\hat{\sigma}_e^2}{2}, \hat{\sigma}_e^2)$ . Together with  $\hat{\delta}$  and  $VP_{ft}$  find the estimated variance of  $\tau$  using the equation for  $\hat{u}^2$ . Because  $e$  is normal then  $\kappa$  is log-normal; since the mean of  $\kappa$  is one, the mean of  $e$  is  $-\frac{\hat{\sigma}_e^2}{2}$ . The standard notation is that parameters in the log-normal are mean and variance of the underlying normal distribution.

STEP 3: Take a draw from a  $N(0, \hat{u}^2)$  for the error term  $\hat{\tau}$ .

STEP 4: Calculate the initial wealth as:  $W_{0,ft} = \hat{\gamma}_0 + \hat{\gamma}_1 VP_{ft} + \hat{\gamma}_2 VP_{ft}^2 + \hat{\tau}$ .

## 10. References

- Appelbaum, E. 1978. "Testing Neoclassical Production Theory" *Journal of Econometrics* 7(1):87-102
- Arnade, C. and D. Kelch. 2007. "Estimation of Area Elasticities from Standard Profit Function". *American Journal of Agricultural Economics*. 89 (3): 727-737.
- Baffes, J. and U. Vasavada. 1989. "On the Choice of Functional Forms in Agricultural Production Analysis" *Applied Economics* 21:1051-1061
- Ball, V.E. 1985. "Output, Input, and Productivity Measurement in U.S. Agriculture, 1948-79." *American Journal of Agricultural Economics* 67(3):475-486
- Ball, V.E. 1988. "Modeling Supply Response in a Multiproduct Framework." *American Journal of Agricultural Economics* 70:813-25
- Burgess, D. F. 1975. "Duality Theory and Pitfalls in the Specification of Technologies" *Journal of Econometrics* 3(2):105-121
- Chambers R. G. and R. D. Pope. 1994. "A Virtually Ideal Production System: Specifying and Estimating the VIPS Model" *American Journal of Agricultural Economics* 76(1):105-113
- Chambers R. G. and J. Quiggin. 1998. "Cost Functions and Duality for Stochastic Technologies" *American Journal of Agricultural Economics* 80(2):288-295
- Chavas J. -P. 2008. "A Cost Approach to Economic Analysis Under State-Contingent Production Uncertainty" *American Journal of Agricultural Economics* 90(2):435-446
- Coyle, B. T. 1992. "Risk Aversion and Price Risk in Duality Models of Production: A Linear Mean-Variance Approach" *American Journal of Agricultural Economics* 74(4):849-859
- Coyle, B. T. 1999. "Risk Aversion and Yield Uncertainty in Duality Models of Production: A Mean-Variance Approach" *American Journal of Agricultural Economics* 81(3):553-567
- Dixon, B. L., P. Garcia and M. Anderson. 1987. "Usefulness of Pretests for Estimating Underlying Technologies Using Dual Profit Functions" *International Economic Review* 28(3):623-633
- Fox, G. and L. Kivanda. 1994. "Popper or Production?" *Canadian Journal of Agricultural Economics* 42(1):1-13
- Greene, W. H. 2003. *Econometric Analysis*. 5<sup>th</sup> Edition. Prentice Hall. Upper Saddle River, N.J.
- Guilkey, D. K., C. A. K. Lovell and R. C. Sickles. 1983. "A Comparison of the Performance of Three Flexible Functional Forms" *International Economic Review* 24(3):591-616



- Huffman, W. E. and R. E. Evenson. 1989. "Supply and Demand Functions for Multiproduct U.S. Cash Grain Farms: Biases Caused by Research and Other Policies" *American Journal of Agricultural Economics* 71(3):761-773
- Iman, R. L. and W. J. Conover. 1982. "A Distribution-free Approach to Inducing Rank Correlation among Input Variables" *Communications in Statistics - Simulation and Computation* 11(3): 311-334
- Jorgenson, D. W. and L. J. Lau. 1974. "The Duality of Technology and Economic Behaviour" *The Review of Economic Studies* 41(2):181-200
- Lau, L. J. 1976. "A Characterization of the Normalized Restricted Profit Function" *Journal of Economic Theory* 12(1):131-163
- Lence, S. H. 2009. "Joint Estimation of Risk Preferences and Technology: Flexible Utility or Frutility?" *American Journal of Agricultural Economics* 91(3):581-598
- Lim, H. and C. R. Shumway. 1992a. "Separability in State-Level Agricultural Technology" *American Journal of Agricultural Economics* 74(1):120-131
- Lim, H. and C. R. Shumway. 1992b. "Profit Maximization, Returns to Scale, and Measurement Error" *The Review of Economics and Statistics* 74(3):430-438
- Lim, H. and C. R. Shumway. 1993. "Functional Form and U.S. Agricultural Production Elasticities" *Journal of Agricultural and Resource Economics* 18(2):266-276
- Lusk, J. L., A. M. Featherstone, T. L. Marsh and A. O. Abdulkadri. 2002. "Empirical Properties of Duality Theory" *The Australian Journal of Agricultural Economics* 46(1):45-68
- Mas-Colell, A., M. D. Winston, and J. R. Green. "Microeconomic Theory" Oxford University Press. 1995
- McElroy, M. B. 1987. "General Error Models for Production, Cost, and Derived Demand or Share Systems" *The Journal of Political Economy* 95(4):737-757
- Miranda M. J. and P. L. Fackler. 2002. "Applied Computational Economics and Finance." The MIT Press. Cambridge, Massachusetts.
- Morgenstern, O. 1963. "On the Accuracy of Economic Observations", Second Edition Princeton University Press
- Moschini GC. 2001. "Production Risk and the Estimation of Ex-ante Cost Functions." *Journal of Econometrics* 100:357-380
- O'Donnell, C. J., C. R. Shumway, and V. E. Ball. 1999. "Input Demands and Inefficiency in U.S. Agriculture" *American Journal of Agricultural Economics* 81(4): 865-880

- Pennacchi, G. 2008. "Theory of Asset Pricing." Pearson Addison Wesley. Boston, Massachusetts.
- Pope, R. D. 1982. "Expected Profits, Price Change, and Risk Aversion" *American Journal of Agricultural Economics* 64(3):581-584
- Pope, R. D. and J. -P. Chavas. 1994. "Cost Functions Under Production Uncertainty" *American Journal of Agricultural Economics* 76(2):196-204
- Pope, R. D. and R. E. Just. 1996. "Empirical Implementation of ex ante Cost Functions" *Journal of Econometrics* 72:231-249
- Pope, R. D. and R. E. Just. 1998. "Cost Function Estimation Under Risk Aversion" *American Journal of Agricultural Economics* 80(2):296-302
- Pope, R. D. and R. E. Just. 2002. "Random Profits and Duality" *American Journal of Agricultural Economics* 84(1):1-7
- PRISM Climate Group. 2011. Oregon State University. Available at: <http://www.prism.oregonstate.edu/>
- Rosas, F., D. J. Hayes, and B. A. Babcock. 2011. "A Nonlinear Offset Program to Reduce Nitrous Oxide Emissions Induced by Excessive Nitrogen Application." CARD Working Paper Series: 11-WP 521. Iowa State University, Ames IA, U.S.
- Schuring, J., W. E. Huffman, and X. Fan. 2011. "The Impact of Public and Private R&D on Farmers' Production Decisions: Econometric Evidence for Midwestern States, 1960-2004." Department of Economics Working Paper Series. WP #10021. Iowa State University, Ames IA, U.S.
- Shumway, R. C. and H. Lim. 1993. "Functional Form and U.S. Agricultural Production Elasticities" *Journal of Agricultural Resource Economics* 18(2) 266-276
- Shumway, R. C. 1995. "Recent Duality Contributions in Production Economics" *Journal of Agricultural and Resource Economics* 20(1):178-194
- Thompson, G. D. and M. Langworthy. 1989. "Profit Function Approximations and Duality Applications to Agriculture" *American Journal of Agricultural Economics* 71(3):791-798
- U.S. Department of Agriculture, Economic Research Service (USDA-ERS). "Agricultural Productivity in the U.S." Available at: <http://www.ers.usda.gov/Data/AgProductivity/>
- U.S. Department of Agriculture, National Agricultural Statistics Service (USDA-NASS). "2002 U.S. Agricultural Census." Available at: <http://www.agcensus.usda.gov/Publications/2002/index.asp> (accessed April 2011).
- Wooldridge, J. M. "Introductory Econometrics. A Modern Approach." Second Edition. Thomson South-Western. 2003

### **CHAPTER 3. RESPONSE OF CROP YIELDS TO OUTPUT PRICES: A BAYESIAN APPROACH TO COMPLEMENT THE DUALITY THEORY ECONOMETRICS**

#### **Abstract**

Sustained high demand for agricultural products and the increase in world commodity prices, result in part, from the growth of per-capita income in developing countries and from the application of biofuel and climate change policies around the globe. The ability of agricultural production to satisfy this increasing demand is inherently related to the response of crop yields to the mentioned events, in particular to price changes. Accurately measuring these effects is crucial for policy evaluation and for projecting economic aggregates. Two are the contributions of this paper to the literature. Firstly, given results in Chapter 2 about the empirical properties of econometric applications based on duality theory, we propose an alternative methodology for calculating elasticities that seeks to complement the information provided by market-based datasets with additional information coming from other sources. Secondly, we aim to calculate new and updated estimates of crop yield elasticities with respect to netput prices. The dual approach is able to retrieve only a portion of the information embedded in market-based datasets about the production technology. The proposed approach considers that price elasticities are dictated by certain features of the underlying production technology, and therefore additional data on crop response to the use of relevant inputs can provide extra information about the parameters of interest. Bayesian econometric methods naturally fit our requirement of jointly estimating

all model parameters in such a way that we simultaneously use all datasets available for the study. Then likelihood functions give the weights with which we measure each data source contribution to the estimated value of the parameters. Our application shows that, in the case of Iowa corn yields, the own-price elasticity estimate has a mean of 0.29 and ranges only on positive values. One possible explanation of this result is that farmers react to expected higher output prices by applying better management practices so as to improve yields and possibly profit from the expected price scenario. We also find elasticities of corn yields with respect to input prices with the expected negative sign. According to our sensitivity analysis the incorporation of other sources of information, such as yields response to the use of other inputs, has minor impacts on final results.

Keywords: yield elasticities, duality theory, Bayesian econometrics

*JEL Codes: Q12, D22, D81, C11*

## 1. Introduction

Agricultural production plays a relevant role in the set of human practices that impact the environment and the use of natural resources. Evidence in the literature has shown that these effects have recently increased. For example, increased demand for agricultural products originating from biofuels policies and higher per-capita income in developing countries is said to have induced land-use changes at a global scale. These land-use changes have in turn generated additional direct and indirect greenhouse gas (GHG) emissions (Righelato and Spracklen, 2007; Fargione et al., 2008; Searchinger et al., 2008; Dumortier et al., 2011).

A direct consequence of the higher pressure on agricultural supply is an increase in commodity prices in international markets. The quantity of new land that is required to satisfy this extra demand depends critically on the ability of crop yields to react to these higher prices (Keeney and Hertel, 2009). Furthermore, small changes in crop yields have a large impact on the payback period of GHG emissions induced by agriculture, and on the quantity of new land that is brought into agriculture to satisfy the increasing demand (Dumortier et al., 2011). Finally, the allocation of land to competing enterprises (cash crops, pasture, forestry and others land uses) is very sensitive to price shocks. What are needed, therefore, are accurate and updated estimations of how crop yields respond to prices.

Supply response models, which in agriculture must consider both the intensive and extensive margins, provide a framework to analyze this effect. The intensive margin, or “intensification,” is commonly associated with changes in agricultural yields, and the extensive margin, or “extensification,” to the direct and indirect land-use change.

This paper is centered on the study of the intensive margin, a topic that has been virtually ignored in the last 30 years. In what follows, our focus on intensive margin is pursued through the yield elasticities with respect to crop and input prices.

The two main approaches in the literature to estimate yield elasticities are based on the Neoclassical theory of the firm; they are the primal approach in which elasticities are calculated after a direct estimation of the production technology, and the dual approach in which elasticities are obtained from the estimation of output supply function equations. The latter is consistent with an indirect recovery of the underlying production parameters.

The primal approach typically starts by specifying a parametric form of the production technology, and estimating its parameters using econometric methods with data on output and input quantities. It ultimately identifies the engineering relationships that underline the production process without any reliance on their prices. Conditional on this estimation, price data is used to calculate yield elasticities from (usually nonlinear) supply equations that come from the first-order conditions (FOC) of the firm's (expected) profit maximization problem. Attempts in the literature to identify this relationship involve studies using aggregate, as well as micro-level market-based data on input and output quantities. Examples include Houck and Gallagher (1976), Menz and Pardey (1983), Love and Foster (1990), Choi and Helmberger (1993), Kaufmann and Snell (1997), and Huang and Khanna (2010) which used aggregate country-, state-, or county-level corn yield data to econometrically estimate a response curve to inputs use. In certain cases (Houck and Gallagher, 1976; Menz and Pardey, 1983; Huang and Khanna, 2010) supply functions are specified by means of reduced form equations.

The drawbacks of this approach, as enumerated by Colman (1983) and Just (1993), include the econometric identification and efficiency lost by ignoring information that prices can

provide about the production function, as well as problems of multicollinearity and simultaneity, due to the employment of market-generated data. Closely related to this type of analysis, given that they also involve a direct estimation of technological relationships, are the biochemical computer simulation models calibrated to represent the daily crop growth process as a function of the agro-ecosystem. The agro-ecosystem environment is described by model input variables such as soil type, climatic variables, crop rotations, and management practices. Model parameters are calibrated using field-level data, usually coming from tailored field experiments or from exhaustive literature reviews. Examples of such simulation models are the Environmental Policy Integrated Climate (EPIC 2012), and the Denitrification Decomposition (DNDC 2012) models.

The dual approach seeks to recover these production technology relationships through the use of market data on both quantities and prices. Duality theory was introduced in the mid-1950s through the seminal work of Ronald Shephard (Shephard, 1953), and since then its use in the economics science has been extensive. The contribution of Christensen, Jorgenson and Lau (1971), Diewert (1971), Diewert and Wales (1987), Lau (1974), and McFadden (1978), allowed for several types of flexible functional forms to be used in describing the data.

Dual approaches in production theory, using profit or cost functions, are general enough to treat different economic environments: perfect competition, monopoly, and non-profit objectives (Pope and Chavas, 1994); and by using input prices instead of input quantities as explanatory variables in estimation, a potential source of simultaneity is removed (Moschini, 2001). Dual approaches allow researchers to analyze multi-output technologies in a straightforward fashion, and the flexible approximation of the problem's value function yields a more tractable system than primal approaches. In the latter, the profit maximizing output

supplies and input demands are derived from a set of usually highly nonlinear FOC associated with a production function previously specified (Just, 1993).

Based on this discussion, dual approaches are preferable over primal estimations, which is consistent with the pattern observed in the more recent literature. However, the nature of agriculture is characterized by a heavy influence of uncertain events during the production process (weather and other unexpected factors such as pests). These are translated into high levels of spatial and temporal production variability commonly observed in real-world data. When the focus is the estimation of crop yield responses to price changes, the dual approach may not be able to recover all features of the technology because identification is based on price variation, which may not be sufficient to identify changes in yields caused by weather shocks. Moreover, in light of the results from Chapter 2, the level of noise embedded in available datasets induces bias in the estimated technology parameters of interest.

In order to overcome these challenges, we propose an approach in which information about production parameters provided by the dual approach is complemented with production function data. Two types of datasets are readily available for the present study. First, a time-series dataset on input and output quantities and prices allows us to econometrically estimate the production parameters employing the dual approach. Second, this is complemented with experimental datasets on input and output quantities that can be used to directly estimate production functions parameters. Though independent from each other, both datasets are capable of providing information about the same feature of the production technology. In this paper we propose an approach that can simultaneously make use of both to recover the parameters of interest.



The direct estimation of the production technology is possible due to available experimental data on quantities of output produced and inputs used. Experiments can be designed to describe specific features of the production technology which in our particular case consists of the response of crop yields to the application of fertilizers and use of seeds.

This paper is organized as follows. In the next section we review the literature available on estimation of yield elasticities with respect to prices. The theoretical model employed in this analysis is in section 3. Section 4 describes the data used for estimation. We present the econometric approach to conduct the estimation of the desired elasticities in section 5. Results are discussed in section 6, and finally, section 7 has the concluding remarks.

## **2. Literature review on yield elasticities**

While yield responses to output prices was a widely analyzed topic in the 70s and 80s, providing several empirical estimations, this issue has been essentially ignored in recent decades. Table 8 summarizes the studies mentioned below. Pioneering work by Houck and Gallagher (1976) found clear evidence of positive own-price corn yield elasticity in the period 1951–1971 in the United States. Choi and Helmberger (1993) also found a positive relationship for U.S. corn, soybean and wheat in the period 1964–1988. However, Menz and Pardey (1983) pursued an analysis similar to that of Choi and Helmberger but for a longer period, and found that the yield-price elasticity was not significant for the following 10-year period. Lyons and Thompson (1981) used cross-country data and found a positive and significant response of yields to corn price. Arnade and Kelch (2007) estimated yield elasticities for corn, soybeans, and other grains in Iowa using a dual approach that includes shadow land price equations, obtaining a positive response of yields to price changes. Huang and Khanna (2010) used U.S. county-level data on yields,

acreage, and weather variables, and also found a positive response to price increases employing a reduced form model of crop yields.

Some studies have also been pursued in other countries. Pomareda and Samayoa (1979) studied corn yield and area response to prices in Guatemala, and find positive responses of both variables to changes in corn prices. The yield price elasticity is in the range proposed by Houck and Gallagher (1976). Guyomard, Baudry, and Carpenter (1996) jointly estimated supply elasticities for several crops to study the effects of the CAP (Common Agricultural Policy) in Europe, and found supply to be responsive to crop prices when land allocation is held fixed. Stout and Alber (2004) used a price elasticity of coarse grain of 0.15 for Canada and 0.18 for Mexico in the ERS/PSU trade model, a partial equilibrium, multiple-commodity, multiple-region model of agricultural policy and international agricultural trade.

However, some studies have encountered minimal or negative response of yields to corn prices in the U.S., including Reed and Riggins (1982) and Ash and Lin (1987). Keeney and Hertel (2008) suggest that a possible explanation for the lack of response found in these studies could be the plateau-like relationship between yields and fertilizer, especially for those studies which rely heavily on a primal specification of the technology, such as Ash and Lin (1987). Also, the estimation of supply response in single equation models, as in Reed and Riggins (1982), fails to acknowledge the effect of land substitution by other crops. Kaufmann and Snell (1997) modeled U.S. corn yields as a function of a large group of climatic and economic variables, finding results consistent with those of Houck and Gallagher, but with a value close to zero. Arnade, Kelch and Leetmaa (2002) estimated yield elasticities for a variety of crops in France, Germany, and the U.K. They found a negative response of corn yield to price changes.

Stout and Alber (2004) used a low yield price elasticity of 0.02 in the case of U.S. coarse grains in the ERS/PSU trade model.

Keeney and Hertel (2008) reviewed the literature on yield-price elasticities of several crops, and highlighted the lack of recent estimations. For example Keeney and Hertel (2009) approximated the long run corn yield-price elasticity by the average of a series of studies from the 70s and 90s, and showed how land use changes are highly sensitive to the yield elasticity assumption. Estimations for other crops, such as soybeans and wheat, are even more difficult to find.

### 3. Model

Assume there exists a representative farmer whose problem is to maximize the expected value of uncertain profits at planting ( $\tilde{\pi}$ ) from producing  $k$  outputs using  $n$  inputs, with uncertainty arising from the stochastic nature of agricultural production and unobserved output prices. The farmer's problem is as follows:

$$\max_{[x]} E(\tilde{\pi}) = \max_{[x]} \{E[\tilde{\mathbf{p}}'\tilde{\mathbf{y}} - \mathbf{w}'\mathbf{x}]\} \quad (35)$$

where  $\tilde{\mathbf{p}} = \{\tilde{p}_1, \dots, \tilde{p}_J\}$  is a  $(J \times 1)$  vector denoting stochastic output prices,  $\tilde{\mathbf{y}} = \{\tilde{y}^1, \dots, \tilde{y}^J\}$  is a  $(J \times 1)$  vector of stochastic output quantities,  $\mathbf{x} = \{x_1, \dots, x_I\}$  is an  $(I \times 1)$  choice vector of inputs used in production, and  $\mathbf{w} = \{w_1, \dots, w_I\}$  is an  $(I \times 1)$  vector comprising their observed prices.<sup>22</sup> Expectations  $E[.]$  are taken over the randomness of the production technology and output prices, which for exposition, we assume to be independent from each other.<sup>23</sup>

<sup>22</sup> Throughout the analysis, we adopt the notation that a superscript in a quantity variable or in a parameter indexes the output itself, or the output in which the input is used.

<sup>23</sup> This independence assumption is consistent with considering input and output prices as independent (right-hand-side) variables during estimation. Alternatively, if independence is not assumed, endogeneity between prices and quantities has to be explicitly treated in estimation.

Assume that the production function  $h_j(\cdot)$  determines the technology for each output  $j$ , where  $\tilde{y}^j = h_j(\mathbf{x}^j, \mathbf{K}; \boldsymbol{\alpha}_0^j, \tilde{\eta}^j)$ . The vector  $\mathbf{x}^j = \{x_1^j, \dots, x_I^j\}$  represents all inputs used in the production of output  $j$ ,  $\mathbf{K}$  is a vector of non-allocatable quasi-fixed netputs that constrains the production technology,  $\boldsymbol{\alpha}_0^j$  is a set of production function parameters, and  $\tilde{\eta}^j$  is the random error driving the stochastic technology. We assume separable technologies, which require that input allocations to one output do not affect the technology of the other outputs. One reason for this assumption is to overcome data restrictions. In particular, the yield effects on one crop from inputs used in other crops is unknown and little information exists on this. However, the analysis proposed here is also valid in case a joint technology is assumed; but the data requirements will be larger.

The profit of producing output  $j$  is given by:  $\tilde{\pi}_j = \tilde{p}_j h_j(\mathbf{x}^j, \mathbf{K}; \boldsymbol{\alpha}_0^j, \tilde{\eta}^j) - \mathbf{w}' \mathbf{x}^j$ , and problem (35) can be rewritten as:

$$\begin{aligned} \max_{[\mathbf{x}^j]} E(\tilde{\pi}) &= \max_{[\mathbf{x}^j]} E \left[ \sum_j \tilde{\pi}_j \right] \\ &= \max_{[\mathbf{x}^j]} E \left[ \sum_j (\tilde{p}_j h_j(\mathbf{x}^j, \mathbf{K}; \boldsymbol{\alpha}_0, \tilde{\eta}) - \mathbf{w}' \mathbf{x}^j) \right] \end{aligned} \quad (36)$$

The solution to the problem in (36) is a set of output-specific Marshallian input demands

$\mathbf{x}^{j*} = \{x_1^{j*}, \dots, x_I^{j*}\}$ . The following system constitutes the solution of the multi-output firm:

$$\begin{aligned} x_1^{j*} &= x_1^j(\bar{p}_j, w_1, \dots, w_I, \mathbf{K}; \boldsymbol{\beta}_0) & \text{for all } j = 1, \dots, J \\ &\vdots \\ x_I^{j*} &= x_I^j(\bar{p}_j, w_1, \dots, w_I, \mathbf{K}; \boldsymbol{\beta}_0) & \text{for all } j = 1, \dots, J \end{aligned} \quad (37)$$

The  $j^{th}$  expected output supply  $\bar{y}^{j*} = E[\tilde{y}^j] = y^j(\bar{p}_j, w_1, \dots, w_I, \mathbf{K}; \boldsymbol{\beta}_0)$ , and the expected profit value function  $\pi_j^* = \pi(\bar{p}_j, w_1, \dots, w_I, \mathbf{K}; \boldsymbol{\beta}_0)$ , are dependent on all variable input

prices, quasi-fixed netputs, a set  $\beta_0$  of parameters, and due to our separability assumption, they are functions of only the own  $j^{th}$  expected output price ( $\bar{p}_j = E[\tilde{p}_j]$ ).

Finally, the aggregate expected profit value function is the sum over the crop-specific profits  $\pi^* = \sum_j \pi_j^* = \pi(\bar{p}_1, \dots, \bar{p}_J, w_1, \dots, w_I, K; \beta_0)$ . Note that while the crop-specific value function depends only on the own-price, the aggregate profit function depends on all output prices. The parameter  $\beta_0$  conditioning the solution system is related to the parameters  $\alpha_0$  of the underlying production function. We make explicit use of this theoretical relationship throughout the analysis, which proves to be useful in the estimation process.

Note that for estimating (37), we require data on crop-specific allocations for all inputs. This data is not generally available for all inputs. We can overcome this issue in a straightforward fashion by rewriting the system so as to aggregate all the inputs for which allocations are not available, in which case the demand depends on all input and output prices. That is,  $x_i^* = x_i^{1*} + \dots + x_i^{J*} = x_i(\bar{p}_1, \dots, \bar{p}_J, w_1, \dots, w_I, K; \beta_0)$  for all  $i$  for which no allocation data available.

The focus of our attention is the calculation of output price elasticities. The  $j^{th}$  output equation can be re-written as follows:

$$\begin{aligned} \bar{y}^{j*} &= E \left[ h_j[x^{j*}, K; \alpha_0^j, \tilde{\eta}^j] \right] \\ &= E \left[ h_j[x_1^{j*}, \dots, x_I^{j*}, K; \alpha_0^j, \tilde{\eta}^j] \right] \\ &= E \left[ h_j[x_1^{j*}(\bar{p}_j, w_1, \dots, w_I, K; \beta_0), \dots, x_I^{j*}(\bar{p}_j, w_1, \dots, w_I, K; \beta_0), K; \alpha_0^j, \tilde{\eta}^j] \right] \end{aligned} \tag{38}$$

Then, applying the chain-rule, the  $j^{th}$  output marginal effects with respect to own-price and with respect to the  $i^{th}$  input price are:

$$\begin{aligned}
\frac{\partial \bar{y}^{j*}}{\partial \bar{p}_j} &= E \left[ \frac{\partial h_j(x^{j*}, K; \alpha_0^j, \tilde{\eta}^j)}{\partial x_1^{j*}} \cdot \frac{\partial x_1^{j*}}{\partial \bar{p}_j} + \dots + \frac{\partial h_j(x^{j*}, K; \alpha_0^j, \tilde{\eta}^j)}{\partial x_l^{j*}} \cdot \frac{\partial x_l^{j*}}{\partial \bar{p}_j} \right] \\
&= E \left[ \sum_i \frac{\partial h_j(x^{j*}, K; \alpha_0^j, \tilde{\eta}^j)}{\partial x_i^{j*}} \cdot \frac{\partial x_i^{j*}}{\partial \bar{p}_j} \right] \\
\frac{\partial \bar{y}^{j*}}{\partial w_{i'}} &= E \left[ \frac{\partial h_j(x^{j*}, K; \alpha_0^j, \tilde{\eta}^j)}{\partial x_1^{j*}} \cdot \frac{\partial x_1^{j*}}{\partial w_{i'}} + \dots + \frac{\partial h_j(x^{j*}, K; \alpha_0^j, \tilde{\eta}^j)}{\partial x_l^{j*}} \cdot \frac{\partial x_l^{j*}}{\partial w_{i'}} \right] \\
&= E \left[ \sum_i \frac{\partial h_j(x^{j*}, K; \alpha_0^j, \tilde{\eta}^j)}{\partial x_i^{j*}} \cdot \frac{\partial x_i^{j*}}{\partial w_{i'}} \right]
\end{aligned} \tag{39}$$

Equation (39) illustrates how we calculate our output and input price elasticities. An output's marginal effect is the sum of the production function's partial derivatives with respect to each input (evaluated at the optimal solution) times the partial derivative of the optimal input choice with respect to the output price (or the input price). In this regard, this methodology can consider both the direct estimation of production technologies and the dual approach of production theory. The former is present in the production function's marginal effect with respect to the input quantity, and the latter in the optimal input demand's marginal effect with respect to the output (or input) price. This can also be seen from the fact that our marginal effects of interest depend on both sets of parameters,  $\alpha_0$  and  $\beta_0$ .

In order for this procedure to be theoretically consistent, we require i) that the functional form of the system of input demands and output supplies (and the profit function) be the dual of the production function functional form; and ii) that the relationship between the parameters of the production function and its dual profit function be explicitly established. For the first, we use quadratic approximations for the profit and production functions, and for the second we rely on Lau's (1976) "Hessian identities."

Defining  $y_0 \equiv H(\mathbf{y}, \mathbf{x}, \mathbf{K}; \boldsymbol{\alpha}_0)$  as the farmer's constrained multi-output technology,  $\{\mathbf{y}, -\mathbf{x}\}$  the vector of variable netput quantities,  $\{\mathbf{p}, \mathbf{w}\}$  the vector of variable netput prices normalized by the price of netput  $y_0$ , and  $\pi^*(\mathbf{p}, \mathbf{w}, \mathbf{K})$  the normalized restricted profit function, Lau's Hessian relationships are:

$$\begin{bmatrix} \frac{\partial^2 \pi^*}{\partial \{\mathbf{p}, \mathbf{w}\}^2} & \frac{\partial^2 \pi^*}{\partial \{\mathbf{p}, \mathbf{w}\} \partial \mathbf{K}} \\ \frac{\partial^2 \pi^*}{\partial \mathbf{K} \partial \{\mathbf{p}, \mathbf{w}\}} & \frac{\partial^2 \pi^*}{\partial \mathbf{K}^2} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$B_{11} = \left[ \frac{\partial^2 H}{\partial \{\mathbf{y}, -\mathbf{x}\}^2} \right]^{-1} \quad (40)$$

$$B_{12} = (B_{21})' = -B_{11} \left[ \frac{\partial^2 H}{\partial \{\mathbf{y}, -\mathbf{x}\} \partial \mathbf{K}} \right]$$

$$B_{22} = - \left[ \frac{\partial^2 H}{\partial \mathbf{K}^2} \right] - \left[ \frac{\partial^2 H}{\partial \mathbf{K} \partial \{\mathbf{y}, -\mathbf{x}\}} \right] B_{11} \left[ \frac{\partial^2 H}{\partial \{\mathbf{y}, -\mathbf{x}\} \partial \mathbf{K}} \right]$$

In the estimation process we make use of these identities to find explicit equations in which parameters of the production function are written as a function of profit function parameters.

#### 4. Data

The empirical approach makes use of various independent datasets for Iowa agriculture. The dataset we use in the dual approach application consists of time-series from 1960 to 2004 of input and output quantities and prices in Iowa. The data we use in the direct estimation of the production technology contains panel observations of per-hectare Iowa corn yields response to nitrogen and phosphate fertilizer application rates, as well as response to seed population per hectare and seed hybrids use.

The first dataset is composed by four variable outputs (corn, soybeans, other crops, and livestock products), three variable inputs (hired labor, intermediate inputs, and fertilizer), five quasi-fixed netputs (farm land, agricultural capital, family labor, CRP land, and farm related output), and a time trend. We define livestock products as the numeraire good. Data on quantities and prices of inputs and quantities of quasi-fixed netputs (except for CRP land) were provided by Eldon Ball at USDA-ERS. Complete information about the methods used to construct this dataset is available on USDA-ERS (2012a).

In the Ball dataset the quantity of hired labor is a weighted index of hours worked and hourly compensation, such that the more productive hours (wages) are given a higher weight than those with lower marginal productivity. The weighting structure is possible because these hours are classified for each state by sex, age, education, and employment class. Intermediate input is an aggregate variable including seeds, pesticides, energy (petroleum fuels, natural gas, and electricity), and other purchased intermediate inputs (contract labor services, custom machine services, machine and building maintenance and repairs, and irrigation). Pesticide prices come from hedonic price functions incorporating the contribution to productivity of the different types and qualities of pesticides. Hedonic price functions allow the construction of a pesticide price index, and the corresponding quantity index is calculated as the ratio between total pesticide expenditures and the price index. Energy quantities are the ratio between total expenditures and the price index of the individual fuels. Other purchased inputs are calculated in a similar fashion.

Implicit quantity index and price index of fertilizer products come from hedonic price functions similar to pesticides. We take Ball's fertilizer quantities and further divide them into



their allocation to different crops based on data available on fertilizer (by nutrient) use by crops by state (USDA-ERS 2012b).

Farmland quantity consists of an index expressed in constant-quality units. It is calculated as the county total value of land divided by an intertemporal price index of land. Land quality heterogeneity is considered by calculating relative price of land from hedonic functions.

Agricultural capital input quantity is a weighted sum of the different assets, with weights given by their own rental rates, adjusted for changes in input quality. The capital input implicit price index is the ratio between the total dollar value of capital flows and the quantity index. Self-employed and family labor compensations are not observed, so their opportunity cost is calculated by applying the mean wage earned by hired workers of similar demographic characteristics to the reported worked hours.

Output quantity data on corn, soybean and other crops (wheat, oat, hay, silage corn, rye, and barley) come from USDA-NASS (USDA-NASS 2012). We calculate quantities of other crops as the weighted average of each production quantity, with weights given by the revenue generated by each crop (Arnade and Kelch, 2007). Prices for corn and soybeans are from the Chicago Mercantile Exchange (CME) futures markets reports. The price of corn equals the average of the March 15<sup>th</sup> and March 30<sup>th</sup> of the December delivery price of each year. The price of soybeans equals the average of the March 15<sup>th</sup> and March 30<sup>th</sup> of the November delivery price (Choi and Helmberger, 1993). Livestock prices are from Ball's livestock products price index. While livestock futures prices exist, they do not exist for all categories, implying that an index would include a mixture of future and current prices, still providing measurement error with respect to the expected price. Future prices for the outputs included in other crops are not

available and therefore we use their current-year price; then a price index for other crops is the ratio between the total revenue generated by these crops to the weighted average of production.

We include two sources of information to control for farm programs in supply response. First, we consider conservation reserve programs (CRP) land as a quasi-fixed input because farmers may be prompted to change their decision about their land enrollment as they observe changes in expected output prices. The dataset consists of CRP acres enrolled in Iowa for the period of the analysis. Second, we take into account price floors imposed by federal farm programs under the successive Farm Bills. In particular, we use the maximum between the loan rate and the CME future prices as the expected output price that farmers take into account in their optimization problem.<sup>24</sup>

The second dataset consists of per-hectare yield responses to nitrogen and phosphate fertilizer applications, to seed density, and to seed hybrids. These are regarded as the inputs farmers can most easily change in response to commodity price changes. Data comes from two different sources. Yield response to fertilizer and seed density come from experimental data using the EPIC model, a biochemical simulation model of agro-ecological systems, capable of describing crop growth over time given a set of input variables (weather, field management practices, and soil characteristics) and a set of model parameters (EPIC 2012). Crop growth is simulated taking into account leaf interception of solar radiation, conversion to biomass, division of biomass into roots, above ground mass, and economic yield, root growth, water use, and nutrient uptake. Regarding the yield response to fertilizer applications on the one hand, and to plant population on the other, the model embeds crop-specific yield equations as a function of

---

<sup>24</sup> During the late 1980s loan rates acted as floor prices. As an alternative way to consider the effects of farm programs in crop prices, we could include the Direct Payments and Target Price and add them to the loan rates to calculate the floor price corresponding to the base acres. Because farmers do not have the same base acres, prices would be farm-specific. Including loan rates only allows us to consider the minimum price that all farmers observe.

the input variables (weather, management practices, and soil) and parameters. Model parameters are calibrated using actual agronomic field experiments over a long period of time. The dataset used in this analysis consists of corn yield response curves to nitrogen fertilizer, phosphate fertilizer, and plant population, for 30 years, and for the 22 most representative soil types of Iowa. For example, in the case of nitrogen, each run of the EPIC model consists of the following. Given a soil type and a nitrogen application rate from a grid ranging from 0 to 300 kilograms per hectare (kg/ha), the model runs and calculates, among other variables, corn yields for each of the 30 years. In each year, crops receive successive and typical management practices as well as climatic shocks given by weather variables from an Iowa weather station located in an area characterized by the soil type in the run. Looked by the nitrogen application rate, the data describes one-dimensional corn production functions composed by 200 pairs of yield and input quantities (ranging from 0 to 300 kilograms kg/ha), for each soil type (22) and year (30), leaving all other inputs constant.<sup>25</sup> Therefore, the dataset consists of  $S = 132,000$  observations of yields-nitrogen pairs. This dataset is also accompanied by the area of each soil type in Iowa which allows us to weight a type by how representative it is in the state. The datasets on yield response to phosphate fertilizer and seed density are generated in a similar fashion.

The other source of yield response data is with respect to seed hybrids. This dataset comes from the Iowa Crop Improvement Association at Iowa State University. It consists of corn yields for three Iowa districts, five years, six trait segments, and several brands (ISU-ICIA 2012). Seed brands are then matched with their market price. We argue that when farmers expect higher

---

<sup>25</sup> The EPIC model parameters are calibrated for a continuous corn rotation with mulch tillage. Fertilizer applications are on April 18<sup>th</sup>; mulch tillage on May 2<sup>nd</sup>; planting on May 9<sup>th</sup>; and harvest on October 19<sup>th</sup>. The response curve for an input assumes the other inputs to be at their optimum; these are: nitrogen applications at 148 kg/ha, phosphate at 75 (kg/ha), and seed population at 85,000 plants per hectare. In all cases potash fertilizer applications are fixed at 88 kg/ha. These values are the 2001 to 2005 average of Iowa's nutrient application rate (USDA-ERS 2012b) and Iowa State University extension recommendations (Duffy 2012).

crop prices, they react not only by adding more seed and fertilizer per hectare but also by buying better and more expensive seed genetics. If this is the case, a yield response curve to hybrids price can be included as complementary information for the production function direct estimation.

## **5. Empirical application**

Independently generated datasets are available for this study which can provide information about the same feature of agricultural production technology. The objective is to aid the dual approach (which uses market-based data) with other sources of data that help identify specific features of the underlying production technology. Using each dataset, we set up distinct models whose parameters are to be econometrically estimated, such that all sources of information (datasets) simultaneously contribute to the estimation of these parameters.

### **5.1 The dual demand-supply system**

Empirical applications of duality theory approximate the multi-output profit (value) function by a flexible functional form.<sup>26</sup> Hotelling's lemma is used to obtain a system of input demand and output supply equations which is then jointly econometrically estimated. However, this procedure does not allow us to make use of available data on allocations of inputs among the different crops, because when differentiating the value function with respect to an input price, it gives us the aggregate demand for that input and not the crop-specific input demand.

However, in our estimation and in order to make use of input allocation data, we take a slightly different approach, and instead we directly approximate the input demands and output

---

<sup>26</sup> Our objective is to analyze the price response of output supplies. Therefore, by approximating the profit function instead of the cost function, equations conveniently have prices as arguments facilitating the calculation of elasticities. In the case of cost functions, we would require the use the profit maximizing condition to make the output supplies function of output prices (Moschini, 2001), which for some widely used functional forms (such as Normalized Quadratic, Translog, Generalized Leontief, among others) induces high nonlinearities in the system to be estimated.

supplies arising from a standard expected profit maximization problem. Antecedents of this approach in the agricultural production literature include O'Donnell, Shumway, and Ball (1999), and Chambers and Pope (1994), and is referred to as the virtually ideal production system (VIPS). In the consumer demand theory Deaton and Muellbauer (1980) termed it as the almost ideal demand system (AIDS); some applications are in Vartia (1983), LaFrance and Hanemann (1989), and von Haefen (2007), among others.

Therefore, instead of approximating the profit function, we approximate each of the demand and supply equations in (37) by a functional form with expected output prices, input prices, and quasi-fixed netput quantities as arguments. For those inputs where no allocation data is available, we approximate the aggregate input use as a function of the mentioned arguments. Below we show that this is consistent with approximating the profit function by a flexible functional form with certain characteristics, depending on the approximation used and for which crop-specific inputs data is available.

The model is empirically specified with four variable outputs ( $\mathbf{y} = \{y^1, \dots, y^4\}$ : corn, soybeans, other crops, and livestock products); three variables inputs ( $\mathbf{x} = \{x_1, x_2, x_3\}$ : hired labor, intermediate inputs, and fertilizer); four quasi-fixed inputs and one quasi-fixed output ( $\mathbf{K} = \{K_1, \dots, K_5\}$ : agricultural capital, family labor, farm related output, CRP land, and a time trend). For fertilizers, allocation data is available for each of the three crops, and we denote the vector as  $\mathbf{x}_3 = \{x_3^1, x_3^2, x_3^3\}$ , with the superscript indicating the output where it is used; corn, soybeans and other crops, respectively. Observed variable input prices are  $\mathbf{w} = \{w_1, w_2, w_3\}$ , and expected output prices are  $\bar{\mathbf{p}} = \{\bar{p}_1, \dots, \bar{p}_4\}$ .

We make a first-order (linear) approximation of the system of input demands and output supplies in (37). We impose homogeneity of degree zero of the input demands and output

supplies by normalizing input and output prices by the price of livestock products, which is the numeraire good. Producer theory also imposes homogeneity of degree one of the profit function and symmetry. We drop the livestock output equation in estimation to avoid singularity of the estimated error variance-covariance matrix; however, its parameters can be recovered by means of the parameter restrictions and the maximization problem objective's function. The system of  $N=8$  equations that we obtain is the following:

$$\begin{aligned}
 -x_1 &= \mathbf{a}_1 \mathbf{w} + \mathbf{b}_1 \bar{\mathbf{p}} + \mathbf{c}_1 \mathbf{K} + \varepsilon_1 \\
 -x_2 &= \mathbf{a}_2 \mathbf{w} + \mathbf{b}_2 \bar{\mathbf{p}} + \mathbf{c}_2 \mathbf{K} + \varepsilon_2 \\
 -\mathbf{x}_3 &= \mathbf{A}_3 \mathbf{w} + \mathbf{B}_3 \bar{\mathbf{p}} + \mathbf{C}_3 \mathbf{K} + \boldsymbol{\varepsilon}_3 \\
 \mathbf{y} &= \mathbf{B}_Y \mathbf{w} + \mathbf{D}_Y \bar{\mathbf{p}} + \mathbf{F} \mathbf{K} + \boldsymbol{\varepsilon}_y
 \end{aligned} \tag{41}$$

which follows the standard netput notation (inputs represented as negative quantities). Each equation has  $T$  observations indexing time. The first two equations represent hired labor and intermediate inputs aggregated over the use in all outputs, and therefore are dependent on all output prices. Parameter vectors  $\mathbf{a}_1$ ,  $\mathbf{b}_1$ ,  $\mathbf{a}_2$ , and  $\mathbf{b}_2$ , are  $(1 \times 3)$ , and  $\mathbf{c}_1$  and  $\mathbf{c}_2$  are  $(1 \times 5)$ , because we include a time trend to control for technology changes over time. The third equality ( $\mathbf{x}_3$ ) is fertilizer use, and is, in turn, composed of three equations, one for the fertilizer used in each of the three crops. Similarly, the last one corresponds to the three crop supplies. This implies that  $\mathbf{A}_3$ ,  $\mathbf{B}_3$ ,  $\mathbf{B}_Y$ , and  $\mathbf{D}_Y$  are of  $(3 \times 3)$  dimension. Matrices  $\mathbf{C}_3$  and  $\mathbf{F}$  are  $(3 \times 5)$ . Matrices  $\mathbf{B}_3$  and  $\mathbf{D}_Y$  are diagonal due to the nonjointness assumption. Parameters  $\mathbf{a}_1$ ,  $\mathbf{b}_1$ ,  $\mathbf{a}_2$ ,  $\mathbf{b}_2$ ,  $\mathbf{c}_1$ ,  $\mathbf{c}_2$ ,  $\mathbf{A}_3$ ,  $\mathbf{B}_3$ ,  $\mathbf{B}_Y$ ,  $\mathbf{C}_3$ ,  $\mathbf{D}_Y$ , and  $\mathbf{F}$  all belong to the set  $\boldsymbol{\beta}_0$ .

Variables  $x_1$ ,  $x_2$ ,  $\mathbf{x}_3$ ,  $\mathbf{y}$ , and  $\mathbf{K}$  are all expressed in per acre terms. This is a consequence of the constant returns to scale assumption embedded in the construction of Ball's dataset.

Consistent with the focus of estimating yield elasticities, this assumption proves to be useful

because we can directly associate the estimated crop supply response to a yield response and plug it in equation (39).

Each equation in (41) has a disturbance term reflecting unknown factors to the econometrician, but not necessarily unobserved by the firm. The implied error structure is consistent with the McElroy (1987) additive general error model (AGEM) applied to the case of profit function (see appendix I for a sketch of the proof).

A set of parameter restrictions are implied by the properties of aggregate profit function, input demands, and expected output supplies, and by the availability of input data allocation (see appendix II for a list of the restrictions). We incorporate these restrictions in estimation, as explained in the next section.

Production theory indicates that the system in (41) implies a certain functional form for the aggregate profit function and certain parameter restrictions. First, if we integrate back input demands with respect to input prices, and output supplies with respect to output prices (which is the opposite to applying Hotelling's lemma to an aggregate profit function), and then sum over the results, we derive the form of the underlying profit. In our case, the result is a normalized quadratic profit function with some specific parameter restrictions (see appendix III for details). If one chooses to approximate the aggregate profit function by a flexible functional form, the parameters corresponding to the crop-specific input use equations ( $x_3^1, x_3^2, x_3^3$  in our case) cannot be recovered by applying Hotelling's lemma. This is true because they enter only as a summation and not individually, and it is the reason why we proceed to directly approximate the demand and supply system. Also note that this procedure is equivalent to taking each crop-specific profit value function of problem (36), approximating them by a normalized quadratic profit function, and then adding them up.

As we mentioned above, the proposed approach intends to aid the dual estimation with (independent) information about the production technology obtained from other non-market sources. In order to make this approach theoretically consistent, we need to explicitly describe the underlying production function implied by the functional form approximation in the dual model.

We note that the functional form dual to the normalized quadratic profit function specified above is also quadratic. The quadratic dual production function  $y_0 = H(\mathbf{y}, \mathbf{x}, \mathbf{K}; \boldsymbol{\alpha}_0)$  describing farmer's multi-output separable technology is:

$$\begin{aligned}
 y_0 &= H(\mathbf{y}, \mathbf{x}, \mathbf{K}; \boldsymbol{\alpha}_0) \\
 &= \frac{1}{2} \sum_{j=1}^{j=3} \sum_{i=1}^{i=3} \sum_{i'=1}^{i'=3} \gamma_{ii'}^j x_i^j x_{i'}^j + \frac{1}{2} \sum_{j=1}^{j=3} \delta_{jj} (y^j)^2 \\
 &\quad - \frac{1}{2} \sum_{j=1}^{j=3} \sum_{i=1}^{i=3} \lambda_{ij}^j x_i^j y^j - \boldsymbol{\varrho}_x \mathbf{x}' \mathbf{K} + \boldsymbol{\varrho}_y \mathbf{y}' \mathbf{K}
 \end{aligned} \tag{42}$$

where  $i$  and  $j$  index inputs and outputs respectively;  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $\mathbf{K}$  as defined above; and parameters  $\gamma$ ,  $\lambda$ ,  $\boldsymbol{\varrho}_x$ , and  $\boldsymbol{\varrho}_y$  all belong to the set of production function parameters  $\boldsymbol{\alpha}_0$ . Netput  $y_0$  is the numeraire. The separable technology assumption implies that cross-effects between outputs and between inputs used for producing different outputs are zero.

Everything else the same, the marginal effects of inputs  $i$  on (corn) output  $y^1$ , denoted by  $\phi_{i1}$ , are:



$$\begin{aligned}
\phi_{11} &\equiv \frac{dy^1}{dx_1^1} = -\frac{\frac{\partial H}{\partial x_1^1}}{\frac{\partial H}{\partial y^1}} = -\frac{\sum_i \gamma_{1i}^1 x_i^1 - \lambda_{11}^1 y^1 - \mathbf{q}_{x_1^1} \mathbf{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11} y^1 + \mathbf{q}_{y^1} \mathbf{K}} \\
&= -\frac{\gamma_{11} x_1 + \gamma_{12} x_2 + \gamma_{13}^1 x_3^1 - \lambda_{11}^1 y^1 - \mathbf{q}_{x_1} \mathbf{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11} y^1 + \mathbf{q}_{y^1} \mathbf{K}} \\
\phi_{21} &\equiv \frac{dy^1}{dx_2^1} = -\frac{\frac{\partial H}{\partial x_2^1}}{\frac{\partial H}{\partial y^1}} = -\frac{\sum_i \gamma_{2i}^1 x_i^1 - \lambda_{21}^1 y^1 - \mathbf{q}_{x_2^1} \mathbf{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11} y^1 + \mathbf{q}_{y^1} \mathbf{K}} \\
&= -\frac{\gamma_{12} x_1 + \gamma_{22} x_2 + \gamma_{23}^1 x_3^1 + \lambda_{21}^1 y^1 - \mathbf{q}_{x_2} \mathbf{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11} y^1 + \mathbf{q}_{y^1} \mathbf{K}} \\
\phi_{31} &\equiv \frac{dy^1}{dx_3^1} = -\frac{\frac{\partial H}{\partial x_3^1}}{\frac{\partial H}{\partial y^1}} = -\frac{\sum_i \gamma_{3i}^1 x_i^1 - \lambda_{31}^1 y^1 - \mathbf{q}_{x_3^1} \mathbf{K}}{-\sum_i \lambda_{i1}^1 x_i^1 + \delta_{11} y^1 + \mathbf{q}_{y^1} \mathbf{K}}
\end{aligned} \tag{43}$$

Which are technological relationships representing, respectively, the effect of hired labor, intermediate inputs, and fertilizers on corn yields. Note that these physical production function relationships are recovered from the demand-supply system by means of information on prices and quantities.

These marginal effects are the center of our attention because, as we describe next, they can also be recovered from direct estimation of the production function; therefore, they are the “link” through which we incorporate new and independent information to the dual estimation.

## 5.2 The production function model

We set up a production response relationship for corn yields as a function of fertilizer applications. The availability of datasets on yield response to nitrogen on the one hand, to phosphate, seed density, and seed hybrids on the other, which in turn are independent from each other, implies that we can set up a model for each dataset. Given our discussion about functional

forms, we require a quadratic specification of the technology in which per-hectare corn yields ( $Q$ ) are a function of per-hectare quantity of fertilizer used ( $F$ )<sup>27</sup> and a set of dummy variables:

$$\begin{aligned}
 Q &= g(F_i, \mathbf{D}; \boldsymbol{\eta}) \\
 &= \mathbf{F}_i \boldsymbol{\eta}' + \zeta_i \\
 &= \eta_0 + \eta_1 F_i + \eta_2 (F_i)^2 + \eta_3 F_i \mathbf{D}_1 + \eta_4 F_i \mathbf{D}_2 + \eta_{D_1} \mathbf{D}_1 + \eta_{D_2} \mathbf{D}_2 + \zeta_i
 \end{aligned} \tag{44}$$

where  $i = \{N, P\}$  index the fertilizer product (nitrogen or phosphate),  $\mathbf{D}_1$  is a set of 21 site-specific dummy variables given by the soil type of where the crop is grown, and  $\mathbf{D}_2$  is a set of 29 time-dummy variables corresponding to the crop year. In each set of dummies we drop one to avoid perfect collinearity. Interaction terms in the quadratic specification capture the observed different yield response curvatures as we change soil types or time periods.

We assume variables  $F$ ,  $\mathbf{D}_1$  and  $\mathbf{D}_2$  are independent due to the experimental design of the data. The data generating process, for each soil type and time period consists of marginally increasing the application rate from 0 to 300 kg/ha for nitrogen and from 0 to 200 kg/ha in the case of phosphate fertilizer. As a consequence of the independence of the explanatory variables, we estimate the model as a pooled regression (Greene 2003, p. 285).

The error term  $\zeta_i$  is assumed to be normally distributed with mean zero and variance  $\sigma_{\zeta_i}^2$ . Importantly, error terms  $\zeta_N$  and  $\zeta_P$  are independent from each other and independent from  $\boldsymbol{\varepsilon}$  in the dual system, which as will become clear in the next sections, facilitates the estimation of parameters  $\phi_{ij}$  which are common to both models.

From this model, we can obtain the parameter that is common to the dual specification, i.e. the marginal effect of corn yields with respect to fertilizer applications, that is:

---

<sup>27</sup> Below we explain the case of yield response to seed density and seed hybrids.

$$\phi_{31,i} \equiv \frac{\partial Q}{\partial F_i} = \eta_1 + 2\eta_2 F_i + \eta_3 \mathbf{D}_{1i} + \eta_4 \mathbf{D}_{2i} \quad (45)$$

We use a similar approach for the case of  $\phi_{21}$  representing the response of corn yields to intermediate inputs. Given our data on corn yield response to seed density (SD) and to seed hybrids price (SH), that are independent from each other and independent from the market-based data, we set up a model equivalent to (44) and derive the marginal effect as in (45).

### 5.3 Elasticities of interest

Yield elasticities with respect to output and input prices are the focus of this analysis. For example, the own price marginal effect of output one (corn), from (39) and (41) is:

$$\frac{\partial \bar{y}^{1*}}{\partial \bar{p}_1} = \frac{\partial h_1}{\partial x_1^*} \frac{\partial x_1^*(\mathbf{w}, \bar{\mathbf{p}}, \mathbf{K})}{\partial \bar{p}_1} + \frac{\partial h_1}{\partial x_2^*} \frac{\partial x_2^*(\mathbf{w}, \bar{\mathbf{p}}, \mathbf{K})}{\partial \bar{p}_1} + \frac{\partial h_1}{\partial x_3^{1*}} \frac{\partial x_3^{1*}(\mathbf{w}, \bar{\mathbf{p}}_1, \mathbf{K})}{\partial \bar{p}_1} \quad (46)$$

where  $h_1$  is evaluated at  $\bar{\eta}$ . Note that we use  $\partial h_1 / \partial x_i^*$  instead of  $\partial h_1 / \partial x_i^{1*}$  for  $i = \{1, 2\}$  due to the lack of allocation data for inputs 1 and 2, implying that their effect on corn yields is that of the aggregated input use and not that of the portion used exclusively in corn. Using (41), (43), and (45) we can rewrite equations in (46) in terms of parameters and marginal effects; that is:

$$d_{11} = \phi_{11} b_{11} + \phi_{21} b_{21} + \phi_{31} b_{31} \quad (47)$$

with  $\phi_{ij}$  is the marginal effect of input  $i$  on output  $j$ . This equation shows how we can write our elasticities of interest incorporating information from the production function parameters, and from the parameters of the (dual) demand and supply system. Recall that we can express  $\phi_{ij}$  in terms of the dual parameters by using Hessian identities. Corn output marginal effects with respect to the  $n = \{1, 2, 3\}$  input prices are:

$$\begin{aligned}\frac{\partial \bar{y}^{1*}}{\partial w_n} &= \frac{\partial h_1}{\partial x_1^*} \frac{\partial x_1^*(\mathbf{w}, \bar{\mathbf{p}}, \mathbf{K})}{\partial w_n} + \frac{\partial h_1}{\partial x_2^*} \frac{\partial x_2^*(\mathbf{w}, \bar{\mathbf{p}}, \mathbf{K})}{\partial w_n} + \frac{\partial h_1}{\partial x_3^{1*}} \frac{\partial x_3^{1*}(\mathbf{w}, \bar{\mathbf{p}}_1, \mathbf{K})}{\partial w_n} \\ b_{11} &= \phi_{11}a_{11} + \phi_{21}a_{12} + \phi_{31}a_{13} \\ b_{21} &= \phi_{11}a_{12} + \phi_{21}a_{22} + \phi_{31}a_{23} \\ b_{31} &= \phi_{11}a_{13} + \phi_{21}a_{23} + \phi_{31}a_{33}\end{aligned}\tag{48}$$

In light of the discussion above about the availability of datasets on netput prices and quantities on the one hand, and experimental data on crop yields response, on the other, both data sources can be used to estimate values of  $\phi_{i1}$ . With the experimental data, the marginal effect of crop yields with respect to input quantities can be calculated by estimating the parameters of a production function. With market-based data on netput prices and quantities, we can also recover these marginal effects by estimating the parameters of the dual problem and using Hessian identities and equations in (43). Consistency of this approach, as discussed above, requires that the profit and the production function functional forms are dual to each other, and that the relationships between each other's parameters be explicitly established.

#### 5.4 The Bayesian estimation approach

A Bayesian approach for estimation is a convenient choice for this particular application. It allows that the estimation of all model parameters be influenced by the information from the different datasets available for the study. While this is particularly important for those parameters that are common to both problems, we note that those not common to both problems are also affected by the introduction of independent information. This information enters as restrictions in model parameters, and the Bayesian approach is especially suited for imposing these constraints not deterministically, but in such a way that takes into account the degree of information that each dataset provides to the recovery of common parameters.

Specifically, the common parameters are given by  $\phi_{i1}$ ,  $i = \{1,2,3\}$  because, on the one hand, they can be estimated from (43) using the market data on netput prices and quantities and applying the Hessian identities, and on the other, from (45) by means of data on yield response to inputs. In this application, given the data available on corn yield response to intermediate inputs and fertilizers, the values of  $\phi_{21}$  and  $\phi_{31}$  can be calculated from both sources. The lack of data on yield response to hired labor prevents us from estimating  $\phi_{11}$  from sources other than the dual approach, and therefore it is recovered by means of the first equation in (43).

#### 5.4.1 The dual system

This estimation treats the  $N = 8$  input demand and output supply equations in system (41) as a seemingly unrelated regression (SUR) model that can be estimated using the time-series dataset of output and input prices and quantities ( $T = 45$ ). We re-write the system with stacked variables as follows:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (49)$$

where  $\mathbf{Y}$  is an  $((NT) \times 1)$  vector of stacked dependent variables, including both input and output quantities;  $\mathbf{X}$  is an  $((NT) \times k)$  block-diagonal matrix of explanatory variables with  $N$  diagonal blocks composed by the matrix  $\mathbf{X}_n$  of explanatory variables of the  $n^{\text{th}}$  equation;  $\boldsymbol{\beta}$  is a  $(k \times 1)$  vector of unknown parameters belonging to the set  $\boldsymbol{\beta}_0$ ; and  $\boldsymbol{\varepsilon}$  is an  $((NT) \times 1)$  stacked vector of random disturbances. We assume there is no autocorrelation within equations, but that there is contemporary correlation among the equation errors, that is:  $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \boldsymbol{\Omega} = \boldsymbol{\Sigma} \otimes \mathbf{I}_T$ , where  $\boldsymbol{\Sigma}$  is a  $(T \times T)$  matrix and  $\otimes$  is the Kronecker delta. The assumption of autocorrelation absence arises from the fact that, prior to the estimation, we take pseudo second-differences of the time-series to remove serial autocorrelation found in the time-series (Greene 2003, p. 272). We further assume that  $\boldsymbol{\varepsilon} \sim MVN(\mathbf{0}, \boldsymbol{\Omega})$ . Therefore, (49) can be regarded as an SUR model.

The system of equations is constrained by a set of equality constraints that we represent as  $\mathbf{R}\boldsymbol{\beta} = \mathbf{r}$ , where  $\mathbf{R}$  is a  $(q \times k)$  matrix, and  $\mathbf{r}$  is a  $(q \times 1)$  vector,  $q$  being the number of constraints, and  $k$  the total number of parameters.

In this application, the dual system has two sources of restrictions. First, the cross-equation restrictions given by symmetry conditions, and second the restrictions imposed by the knowledge of  $\phi_{21}$  and  $\phi_{31}$  from other independent sources of information, i.e., the dataset on crop yield response to input quantities. The form of these restrictions is given by the second and third equations in (43). As a consequence, we can classify the set of dual parameters to be estimated in the following groups:

1. free, denoted as  $\boldsymbol{\beta}^*$
2. constrained by symmetry, denoted as  $\boldsymbol{\gamma}_2$
3. “impacted” by the knowledge of  $\phi_{21}$  and  $\phi_{31}$ , denoted as  $\boldsymbol{\gamma}_3$
4. constrained by the knowledge of  $\phi_{21}$  and  $\phi_{31}$ , denoted as  $\boldsymbol{\gamma}_4$

We explain how we estimate each subset in the rest of this section.

The first step is to estimate the free parameters in group 1, conditional on the symmetry restrictions and parameters in group 3 and 4. In terms of model set up, we can view these conditioning parameters as we view those constrained by symmetry; therefore we can use the usual SUR tools to impose constraints. Following Giles (2003) and Amemiya (1985 p.22), the SUR model constraints can be equivalently re-written as follows:

$$\begin{aligned} \mathbf{r} &= \mathbf{R}\boldsymbol{\beta} \\ &= [\mathbf{R}_1 \quad \mathbf{R}_2] \begin{bmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta}^* \end{bmatrix} \end{aligned} \tag{50}$$

where  $\mathbf{R}_1$  and  $\mathbf{R}_2$  are  $(q \times q)$  and  $(q \times (k - q))$  submatrices, and  $\boldsymbol{\gamma}$  and  $\boldsymbol{\beta}^*$  are  $(q \times 1)$  and  $((k - q) \times 1)$  vectors of parameters. The new vector of parameters has the same dimension as  $\boldsymbol{\beta}$ , but with its entries reordered in such a way that vector  $\boldsymbol{\gamma}$  contains the constrained parameters (subsets 2, 3 and 4), while  $\boldsymbol{\beta}^*$  contains the unconstrained or free parameters (subset 1). Consistent with this specification, we also rewrite model (49) in the following partitioned form:

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \\ &= [\mathbf{X}^1 \quad \mathbf{X}^2] \begin{bmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\beta}^* \end{bmatrix} + \boldsymbol{\varepsilon} \end{aligned} \quad (51)$$

where,  $\mathbf{X}^1$  and  $\mathbf{X}^2$  are  $(NT \times q)$  and  $(NT \times (k - q))$  submatrices. This step also requires reordering the columns of matrix  $\mathbf{X}$  of explanatory variables in such a way that is consistent with the reordering of the rows of  $\boldsymbol{\beta}$ . The new matrix of explanatory variables loses its block diagonal structure. Solving (50) for  $\boldsymbol{\gamma}$  we have:

$$\boldsymbol{\gamma} = \mathbf{R}_1^{-1}(\mathbf{r} - \mathbf{R}_2\boldsymbol{\beta}^*) \quad (52)$$

and plugging  $\boldsymbol{\gamma}$  into (51) we have:  $\mathbf{y} - \mathbf{X}^1\mathbf{R}_1^{-1}\mathbf{r} = (\mathbf{X}^2 - \mathbf{X}^1\mathbf{R}_1^{-1}\mathbf{R}_2)\boldsymbol{\beta}^* + \boldsymbol{\varepsilon}$ , which written in compact form becomes:

$$\mathbf{y}^* = \mathbf{X}^*\boldsymbol{\beta}^* + \boldsymbol{\varepsilon} \quad (53)$$

where  $\mathbf{y}^* = \mathbf{y} - \mathbf{X}^1\mathbf{R}_1^{-1}\mathbf{r}$  is the new  $(NT \times 1)$  vector of dependent variables,  $\mathbf{X}^* = \mathbf{X}^2 - \mathbf{X}^1\mathbf{R}_1^{-1}\mathbf{R}_2$  is the new  $(NT \times (k - q))$  matrix of explanatory variables, and  $\boldsymbol{\beta}^*$  and  $\boldsymbol{\varepsilon}$  as defined above. Properties of  $\boldsymbol{\varepsilon}$  remain unchanged as compared to those in equation (49) (Giles, 2003), so this system of equations constitutes the new unconstrained SUR model whose  $(k - q)$  parameters we seek to estimate.

The Bayesian estimation starts by setting the likelihood function that resumes the information given by the data, conditional on the parameters. Given our assumption about the error term, the likelihood is as follows:

$$\begin{aligned} f(\mathbf{y}^*|\boldsymbol{\beta}^*, \boldsymbol{\Sigma}) &\propto |\boldsymbol{\Sigma}|^{-\frac{NT}{2}} \exp\{-\mathbf{0.5}(\mathbf{y}^* - \mathbf{X}^*\boldsymbol{\beta}^*)'(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I}_T)(\mathbf{y}^* - \mathbf{X}^*\boldsymbol{\beta}^*)\} \\ &\propto |\boldsymbol{\Sigma}|^{-\frac{NT}{2}} \exp\{-\mathbf{0.5}\text{tr}(\mathbf{A}\boldsymbol{\Sigma}^{-1})\} \end{aligned} \quad (54)$$

where  $\propto$  means “proportional to,”  $\text{tr}$  is the trace operator, and  $\mathbf{A}$  is an  $(N \times N)$  symmetric matrix formed by elements  $a_{nn'} = (\mathbf{y}_n^* - \mathbf{X}_n^*\boldsymbol{\beta}^*)'(\mathbf{y}_{n'}^* - \mathbf{X}_{n'}^*\boldsymbol{\beta}^*)$ . Then we define the priors’ joint probability density function that collects our beliefs about the unknown parameters  $\boldsymbol{\beta}^*$  and  $\boldsymbol{\Sigma}$ . We choose the following non-informative prior:

$$\begin{aligned} f(\boldsymbol{\beta}^*, \boldsymbol{\Sigma}) &\propto f(\boldsymbol{\beta}^*)f(\boldsymbol{\Sigma})\mathbf{I}(\boldsymbol{\Theta}) \\ &\propto |\boldsymbol{\Sigma}|^{-\frac{N+1}{2}}\mathbf{I}(\boldsymbol{\Theta}) \end{aligned} \quad (55)$$

where  $f(\boldsymbol{\beta}^*)$  is proportional to a real-valued constant in  $\mathcal{R}^1$ ,  $f(\boldsymbol{\Sigma}) \propto |\boldsymbol{\Sigma}|^{-(N+1)/2}$  is the limit of an inverted Wishart density defined over the support of positive-definite matrices, and  $\mathbf{I}(\boldsymbol{\Theta})$  is an indicator function taking the value one if the set of parameters falls into the set  $\boldsymbol{\Theta}$ , and zero otherwise. The set  $\boldsymbol{\Theta}$  allows us to impose further restrictions, such as monotonicity, on the estimated parameters (Giles, 2003). By Bayes theorem, the joint posterior density function is then:

$$\begin{aligned} f(\boldsymbol{\beta}^*, \boldsymbol{\Sigma}|\mathbf{y}^*) &\propto f(\mathbf{y}^*|\boldsymbol{\beta}^*, \boldsymbol{\Sigma})f(\boldsymbol{\beta}^*, \boldsymbol{\Sigma}) \\ &\propto |\boldsymbol{\Sigma}|^{-\frac{NT+N+1}{2}} \exp\{-\mathbf{0.5}(\mathbf{y}^* - \mathbf{X}^*\boldsymbol{\beta}^*)'(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I}_T)(\mathbf{y}^* - \mathbf{X}^*\boldsymbol{\beta}^*)\}\mathbf{I}(\boldsymbol{\Theta}) \\ &\propto |\boldsymbol{\Sigma}|^{-\frac{NT+N+1}{2}} \exp\{-\mathbf{0.5}\text{tr}(\mathbf{A}\boldsymbol{\Sigma}^{-1})\}\mathbf{I}(\boldsymbol{\Theta}) \end{aligned} \quad (56)$$

The Bayesian approach seeks to estimate the marginal posterior density functions of the parameters  $\boldsymbol{\beta}^*$ . To this end, we use a Gibbs sampler to generate random draws from these



marginal posteriors (Casella and George, 1992). Implementation of the Gibbs sampler requires knowing the form of the conditional posteriors of the parameters. For the parameter  $\beta^*$ , viewing  $\Sigma$  as a constant, and using (56), we have:

$$f(\beta^*|\Sigma, y^*) \propto \exp\{-0.5(y^* - X^*\beta^*)'(\Sigma^{-1} \otimes I_T)(y^* - X^*\beta^*)\}I(\Theta) \quad (57)$$

which is proportional to a multivariate normal with mean  $(X^{*'}(\Sigma^{-1} \otimes I_T)X^*)^{-1}X^{*'}(\Sigma^{-1} \otimes I_T)y^*$  and covariance matrix  $[X^{*'}(\Sigma^{-1} \otimes I_T)X^*]^{-1}$ . In the case of  $\Sigma^{-1}$ , viewing  $\beta^*$  as a constant and using (56), the marginal posterior is:

$$f(\Sigma|\beta^*, y^*) \propto |\Sigma|^{-\frac{NT+N+1}{2}} \exp\{-0.5\text{tr}(A\Sigma^{-1})\}I(\Theta) \quad (58)$$

Note that estimation of  $\beta^*$  and  $\Sigma$  is conditioned on the parameters vector  $\gamma$ , out of which  $\gamma_3$  and  $\gamma_4$  implicitly bring information about the underlying production function through  $\phi_{21}$  and  $\phi_{31}$  so as to complement the information that duality theory can recover from the technology.

Upon estimation of  $\beta^*$  parameters, elements of subsets 2, 3, and 4 can be recovered from equation (52). Out of these, only those constrained by symmetry are regarded as draws from their marginal posterior, because they maintain their marginal distributions given by (57).

However, those in subsets 3 and 4 cannot be regarded as such because their conditional posteriors are no longer given by (57), due to the (highly nonlinear) expression in (43). As a consequence, we require an alternative method to draw from their “unknown” conditional posterior distribution. We explain estimation of group 3 parameters in the next paragraph, and group 4 in the next section.

We use a Metropolis-Hastings algorithm to take random draws from the unknown distributions of group 3 parameters (Chib and Greenberg, 1996). In particular, we employ the so-called t-walk algorithm, which is a general purpose sampling algorithm (Christen and Fox, 2010; Lieberman, 2012). This algorithm requires an objective function to evaluate whether to accept or

reject proposed values of the parameter; we use the log-likelihood function of the dual problem evaluated at the proposed parameters and conditional on the last draw of all remaining parameters. Note that equalities (47) and (48) provide equations to calculate four parameters without the requirement of drawing from their marginal posterior distributions. These, together with the ones drawn using t-walk, constitute the set of impacted parameters ( $\boldsymbol{\gamma}_3$ ).

#### 5.4.2 Direct production function estimation

We estimate the model in (44) with the output and input quantities datasets both for nitrogen and phosphate yield response.<sup>28</sup> The ability to obtain information of the corn yield response to fertilizer applications  $\phi_{3i}$ , both from this model as well as from other sources of information, implies constraints on the parameters of model (44). In this particular case we classify the parameters to be estimated in two groups: 1) free parameters, and 2) constrained parameters. In fact, according to equation (45) only one parameter is constrained; we arbitrarily select  $\eta_1$  as the constrained parameter (group 2), and the remaining are all free parameters.

Solving for  $\eta_1$  in (45), plugging in (44), and rearranging terms, we can rewrite the model as follows:

$$\begin{aligned} Q_i &= \eta_0 - \eta_2(F_i)^2 + \eta_{D_1}\mathbf{D}_1 + \eta_{D_2}\mathbf{D}_2 + \phi_{31}F_i + \zeta_i \\ Q_i - \phi_{31}F_i &= \eta_0 - \eta_2(F_i)^2 + \eta_{D_1}\mathbf{D}_1 + \eta_{D_2}\mathbf{D}_2 + \zeta_i \\ Q_i^* &= \eta_0 - \eta_2(F_i)^2 + \eta_{D_1}\mathbf{D}_1 + \eta_{D_2}\mathbf{D}_2 + \zeta_i \end{aligned} \tag{59}$$

which constitutes the new model to be estimated. These substitutions do not affect the distribution of the error term  $\zeta_i$ . Let  $\mathbf{F}$  denote the set of  $k_F$  explanatory variables for each observation, and similarly,  $\boldsymbol{\eta}$  the set of model parameters.

---

<sup>28</sup> We estimate the model of yield response to seed density and seed hybrids in a similar fashion.

The Bayesian approach starts by noting that the error term is normally distributed:

$\zeta_i \sim N(0, \sigma_{\zeta_i}^2)$ . Dropping the  $i$  subscript denoting the fertilizer product, the likelihood function can be written as:

$$l(Q^*|\boldsymbol{\eta}, \sigma_{\zeta}^2) \propto \sigma^{-S} \exp\{-1/2\sigma^2(Q^* - \mathbf{F}\boldsymbol{\eta}')'(Q^* - \mathbf{F}\boldsymbol{\eta}')\} \quad (60)$$

with  $S$  being the number of observations as defined above. Assuming a joint non-informative prior distribution for  $\boldsymbol{\eta}$  and  $\sigma_{\zeta}^2$  of the form  $l(\boldsymbol{\eta}, \sigma_{\zeta}^2) \propto \sigma^{-1}$ , the joint posterior density function for the parameters of interest conditional on the sampling data is the following:

$$l(\boldsymbol{\eta}, \sigma_{\zeta}^2 | \phi_{31}, Q^*) \propto \sigma^{-(S+1)} \exp\{-1/2\sigma^2(Q^* - \mathbf{F}\boldsymbol{\eta}')'(Q^* - \mathbf{F}\boldsymbol{\eta}')\} \quad (61)$$

Then, conditional on the value of  $\sigma_{\zeta}^2$  and  $\phi_{31}$ , the marginal conditional posterior distribution of the  $\boldsymbol{\eta}$  parameters  $l(\boldsymbol{\eta} | \sigma_{\zeta}^2, \phi_{31}, Q^*)$  is distributed multivariate normal:

$$l(\boldsymbol{\eta} | \sigma_{\zeta}^2, \phi_{31}, Q^*) \sim MVN\{[(\mathbf{F}'\mathbf{F})^{-1}(\mathbf{F}'Q^*)], \sigma_{\zeta}^2(\mathbf{F}'\mathbf{F})^{-1}\} \quad (62)$$

Conditional on the value of  $\boldsymbol{\eta}$  and  $\phi_{31}$ , the marginal conditional posterior distribution of  $\sigma_{\zeta}^2$ ,  $l(\sigma_{\zeta}^2 | \boldsymbol{\eta}, \phi_{31}, Q^*)$ , can be obtained from the posterior of the precision  $\tau = \sigma_{\zeta}^{-2}$ , that is:

$$l(\tau | \boldsymbol{\eta}, \phi_{31}, Q^*) \sim \chi_{(S-k_F)}^2 / (S - k_F) v^2 \quad (63)$$

where  $v^2 = (S - k_F)^{-1}(Q^* - \mathbf{F}\boldsymbol{\eta}')'(Q^* - \mathbf{F}\boldsymbol{\eta}')$  is a consistent estimator of  $\sigma_{\zeta}^2$ .

With the objective of estimating the marginal probability functions of the parameters  $\boldsymbol{\eta}$  and  $\sigma_{\zeta}^2$ , we use a Gibbs sampler to draw random numbers from the mentioned conditional posteriors.

Note that the above production function model setup and estimation procedure, explained for the cases of nitrogen and phosphate fertilizers, is also valid for the other production function models i.e., the corn yield response to seed density and to seed hybrid expenditure. For these cases, we change  $\phi_{31}$  for  $\phi_{21}$ , and  $\mathbf{F}_i$  for  $\mathbf{SD}$  (seed density) and  $\mathbf{SH}$  (seed hybrids).

### 5.4.3 Estimation of corn yield response to input use

Focusing first on yield response to fertilizer applications, the value of the marginal effect of fertilizer on corn yields ( $\phi_{31}$ ) that conditions models in sections 5.4.1 and 5.4.2, can be estimated by means of Monte Carlo Markov Chain methods. In particular, we use the Metropolis-Hastings algorithm in which knowledge of the conditional posterior distribution functional form is not required. This procedure also allows model estimation combining more than one dataset.

We use the t-walk algorithm to draw random deviates from the unknown posterior density of this parameter. Because information about this parameter can be provided by each of the available datasets, which in turn are independent from each other, we use the sum of the log-likelihoods from each model, conditional on the value of all their remaining parameters, as the t-walk objective function. This serves as an objective function not only because it gives the joint probability that the proposed value of  $\phi_{31}$  be generated from these datasets, but also it will more often accept candidates that come from the dataset that provides higher likelihood.<sup>29</sup>

The iterations of  $\phi_{31}$  within the Metropolis algorithm, and the weighting structure given by this objective function, imply that acceptance of candidates is dictated by how likely each dataset is generated from this parameter candidate. It also implies that constraints given by the knowledge of  $\phi_{31}$  are not deterministically imposed; they are imposed with uncertainty to the econometrician, with weights given by how likely the data is given the parameter.

Once a candidate of  $\phi_{31}$  is accepted, constrained parameters in the dual model (group 4) and constrained parameters in the production function model (group 2) can be calculated using, respectively, equations (43) and (45).

---

<sup>29</sup> The independence of these datasets, given by the independence between the errors  $\epsilon$ ,  $\zeta_N$ , and  $\zeta_P$  allows us to evaluate the candidate using only the sum of the log-likelihood, and not requiring their cross-products.

It is important to note that parameter  $\phi_{31}$ , recovered from the dual model, represents the yield response to all fertilizer products (nitrogen, phosphate, and potash), while from the direct estimation of production function we can estimate the yield response to the individual nutrients, represented as  $\phi_{31,N}$  and  $\phi_{31,P}$ .<sup>30</sup> This is taken into account in estimation by considering  $(\phi_{31,N} + \phi_{31,P})$  as the yield response to fertilizer use from the direct estimation of production function.

The procedure employed to estimate the yield response to intermediate inputs  $\phi_{21}$  closely follows the one for fertilizers. In this case, we use yield response to seed purchases (seed quantity per hectare and seed hybrids) as a proxy for the yield response to intermediate inputs. We conduct sensitivity analysis to evaluate the consequences of omitting other intermediate inputs when estimating  $\phi_{21}$ . These are shown in the results section.

#### 5.4.4 Metropolis algorithm steps

Using a Bayesian approach with the following steps, we conduct estimation of the model in (53) and models in (59) (for each nitrogen, phosphate, seed density, and seed hybrids inputs).

STEP 1: Select starting values for the parameters:  $\sigma_{\zeta_N}^{2(0)}$ ,  $\sigma_{\zeta_P}^{2(0)}$ ,  $\sigma_{\zeta_{SD}}^{2(0)}$ ,  $\sigma_{\zeta_{SH}}^{2(0)}$ ,  $\Sigma^{(0)}$ ,  $\phi_{21}^{(0)}$ ,  $\phi_{31}^{(0)}$ ,  $\gamma_3^{(0)}$ , and  $\gamma_4^{(0)}$ . Set  $r = 1$ .

STEP 2: Conditional on  $\sigma_{\zeta_N}^{2(r-1)}$  and  $\phi_{31}^{(r-1)}$ , generate a draw of  $\eta_N^{(r)}$  from  $l(\eta_N | \sigma_{\zeta_N}^2, \phi_{31}, Q^*)$  by Gibbs sampling using equation (62) with  $i = N$ .

STEP 3: Conditional on  $\eta_N^{(r)}$  and  $\phi_{31}^{(r-1)}$ , generate a draw of  $\sigma_{\zeta_N}^{2(r)}$  from  $l(\sigma_{\zeta_N}^2 | \eta_N, \phi_{31}, Q^*)$  by Gibbs sampling using equation (63) with  $i = N$ .

---

<sup>30</sup> The yield response to potash is assumed to be zero. For example, the EPIC model shows no response to potash fertilizer applications.

STEP 4: Conditional on  $\sigma_{\zeta_P}^{2(r-1)}$  and  $\phi_{31}^{(r-1)}$ , generate a draw of  $\boldsymbol{\eta}_P^{(r)}$  from  $l(\boldsymbol{\eta}_P | \sigma_{\zeta_P}^2, \phi_{31}, Q^*)$  by Gibbs sampling using equation (62) with  $i = P$ .

STEP 5: Conditional on  $\boldsymbol{\eta}_P^{(r)}$  and  $\phi_{31}^{(r-1)}$ , generate a draw of  $\sigma_{\zeta_P}^{2(r)}$  from  $l(\sigma_{\zeta_P}^2 | \boldsymbol{\eta}_P, \phi_{31}, Q^*)$  by Gibbs sampling using equation (63) with  $i = P$ .

STEP 6: Conditional on  $\sigma_{\zeta_{SD}}^{2(r-1)}$  and  $\phi_{21}^{(r-1)}$ , generate a draw of  $\boldsymbol{\eta}_{SD}^{(r)}$  from  $l(\boldsymbol{\eta}_{SD} | \sigma_{\zeta_{SD}}^2, \phi_{21}, Q^*)$  by Gibbs sampling using equation (62) with  $i = SD$ .

STEP 7: Conditional on  $\boldsymbol{\eta}_{SD}^{(r)}$  and  $\phi_{21}^{(r-1)}$ , generate a draw of  $\sigma_{\zeta_{SD}}^{2(r)}$  from  $l(\sigma_{\zeta_{SD}}^2 | \boldsymbol{\eta}_{SD}, \phi_{21}, Q^*)$  by Gibbs sampling using equation (63) with  $i = SD$ .

STEP 8: Conditional on  $\sigma_{\zeta_{SH}}^{2(r-1)}$  and  $\phi_{21}^{(r-1)}$ , generate a draw of  $\boldsymbol{\eta}_{SH}^{(r)}$  from  $l(\boldsymbol{\eta}_{SH} | \sigma_{\zeta_{SH}}^2, \phi_{21}, Q^*)$  by Gibbs sampling using equation (62) with  $i = SH$ .

STEP 9: Conditional on  $\boldsymbol{\eta}_{SH}^{(r)}$  and  $\phi_{21}^{(r-1)}$ , generate a draw of  $\sigma_{\zeta_{SH}}^{2(r)}$  from  $l(\sigma_{\zeta_{SH}}^2 | \boldsymbol{\eta}_{SH}, \phi_{21}, Q^*)$  by Gibbs sampling using equation (63) with  $i = SH$ .

STEP 10: Conditional on  $\boldsymbol{\Sigma}^{(r-1)}$ ,  $\phi_{21}^{(r-1)}$ ,  $\phi_{31}^{(r-1)}$ ,  $\boldsymbol{\gamma}_3^{(r-1)}$ , and  $\boldsymbol{\gamma}_4^{(r-1)}$  generate a draw of  $\boldsymbol{\beta}^{*(r)}$  from  $\boldsymbol{f}(\boldsymbol{\beta}^* | \boldsymbol{\Sigma}, \boldsymbol{y}^*)$  in equation (57) by Gibbs sampling. Recover parameters constrained by symmetry restrictions  $\boldsymbol{\gamma}_2^{(r)}$  using (51). Form vector  $\boldsymbol{\beta}^{(r)} = [\boldsymbol{\beta}^{*(r)}, \boldsymbol{\gamma}_2^{(r)}, \boldsymbol{\gamma}_3^{(r-1)}, \boldsymbol{\gamma}_4^{(r-1)}]$

STEP 11: Conditional on  $\boldsymbol{\beta}^{*(r)}$ ,  $\phi_{21}^{(r-1)}$ ,  $\phi_{31}^{(r-1)}$ ,  $\boldsymbol{\gamma}_3^{(r-1)}$ , and  $\boldsymbol{\gamma}_4^{(r-1)}$  generate a draw of  $\boldsymbol{\Sigma}^{(r)}$  from  $\boldsymbol{f}(\boldsymbol{\Sigma} | \boldsymbol{\beta}^*, \boldsymbol{y}^*)$  in equation (58) by Gibbs sampling.

STEP 12: Conditional on  $\boldsymbol{\beta}^{*(r)}$ ,  $\boldsymbol{\Sigma}^{(r)}$ ,  $\phi_{21}^{(r-1)}$ ,  $\phi_{31}^{(r-1)}$ , and  $\boldsymbol{\gamma}_4^{(r-1)}$ , generate a draw of  $\boldsymbol{\gamma}_3^{(r)}$  using the t-walk algorithm and (54) as t-walk objective function.

STEP 13: Conditional on  $\boldsymbol{\beta}^{*(r)}$ ,  $\boldsymbol{\Sigma}^{(r)}$ ,  $\boldsymbol{\gamma}_3^{(r)}$ ,  $\boldsymbol{\eta}_{SD}^{(r)}$ ,  $\sigma_{\zeta_{SD}}^{2(r)}$ ,  $\boldsymbol{\eta}_{SH}^{(r)}$ , and  $\sigma_{\zeta_{SH}}^{2(r)}$  generate a draw of  $\phi_{21}^{(r)}$ , using the t-walk algorithm with objective function given by the log-likelihood function  $\log L = \log f(.) + \log l_{SD}(.) + \log l_{SH}(.)$  in (54) and (60). Recover corresponding parameters in vector  $\boldsymbol{\gamma}_4^{(r)}$ ; as well as  $\eta_{1,SD}$  and  $\eta_{1,SH}$  using (45).

STEP 14: Conditional on  $\boldsymbol{\beta}^{*(r)}$ ,  $\boldsymbol{\Sigma}^{(r)}$ ,  $\boldsymbol{\gamma}_3^{(r)}$ ,  $\boldsymbol{\eta}_N^{(r)}$ ,  $\sigma_{\zeta_N}^{2(r)}$ ,  $\boldsymbol{\eta}_P^{(r)}$ , and  $\sigma_{\zeta_P}^{2(r)}$  generate a draw of  $\phi_{31}^{(r)}$ , using the t-walk algorithm with objective function given by the log-likelihood function  $\log L = \log f(.) + \log l_N(.) + \log l_P(.)$  in (54) and (60). Recover corresponding parameters in vector  $\boldsymbol{\gamma}_4^{(r)}$ ; as well as  $\eta_{1,N}$  and  $\eta_{1,P}$  using (45).

STEP 15: If  $r$  equals maximum number of iterations, STOP, otherwise set  $r = r + 1$ , and return to STEP 2.

## 6. Results

The results section is organized as follows. First, we present estimated values of corn yield elasticities with respect to prices. Second, we compare results from both estimation methods, i.e., our mixed approach and the traditional dual approach. Third, we present sensitivity analysis on some features of the model that we believe may drive final results of the parameters estimation.

Figure 8 shows the corn yield elasticity estimates with respect to selected prices. These are the parameters that are influenced the most by the data included to aid the duality approach estimation. They are based on equations (47) and (48), and histograms represent the 95% highest probability density interval (HPDI), also known as most credible interval, of the marginal posterior density function of each elasticity. In particular, based on the median<sup>31</sup> elasticity calculated at 0.29, the own-price elasticity is not only positive, but also the interval does not

---

<sup>31</sup> We choose to report the posterior medians due to the observed skewness of most posterior distributions.

include zero or negative values. A positive own-price elasticity can be caused by farmers expecting higher prices for their corn and, as a consequence, reacting by improving farm management practices, such as applying more fertilizer, planting more and better seeds per hectare, and hiring more labor, among others. According to Table 8 this value also falls in the range of previous estimates from the literature, but it is largest among the most recent figures.

The corn yield elasticities with respect to input prices are also of the expected negative sign, consistent with observing farmers cutting their input use as their prices increase. According to the posterior distribution medians, shown in Figure 8 and in Table 9, corn yields are more responsive to changes in fertilizer and intermediate inputs (with median elasticities of -0.15 and -0.17 respectively) than to changes in wages (-0.12). The fact that the elasticity with respect to hired labor is lower may be caused by the fact that on-farm labor demand is more difficult to cut (inelastic) than reducing other inputs such as fertilizer or seeds. In the cases of hired labor and fertilizer we cannot reject the hypothesis that yields are non-responsive to input use given that zero is within the elasticity interval.

Another set of estimated parameters that are heavily influenced by the outside sources of data are the corn yield response to the quantity used of intermediate inputs and fertilizers. These are based, respectively on parameters  $\phi_{21}$  and  $\phi_{31}$ . Recall that these parameters, which can be recovered from the dual approach, and also by directly estimating the underlying production function, serve as the “bridge” through which both approaches complement each other. Figure 9 shows histograms of the marginal posterior density of both elasticities, and the median and 95% HPDI are at the bottom of Table 9. As expected they are positive, implying that at the optimum, the higher their use the higher corn yields are. The response of corn yield with respect to intermediate inputs and with respect to fertilizer use is in both cases 0.42. The narrow interval in



both cases is a consequence of the fact that the information about the same parameters provided by the direct estimation of production function using experimental data is not only highly significant, but also “dominates” the one provided by the dual approach based on market data. This is a consequence of the relative sizes of the log-likelihood function of each model that are used to “weight” the information from each source.

The elasticity of corn yields with respect to hired labor quantity is estimated with a median of 0.19 as shown Table 9. A relatively lower value of this yield elasticity indicates that the technology reaches to a point of low responsiveness at the optimal value of labor use, and as a result, might be an explanation for labor substitution by other inputs. This elasticity is based on parameter  $\phi_{11}$ . Given that no data on corn yield response to labor is available for this study, which would allow us to employ a similar procedure as with intermediate inputs and fertilizer, we recover the parameter from the underlying production technology according to the dual theorem and using Hessian identities. This explains its wider HPDI, because recovering this parameter involves equation (43) which is a highly nonlinear function of several estimated parameters.

We compare results of using the proposed approach in which information from different sources complement each other to estimate model parameters, versus the estimation with the standard dual approach. In the latter case, we conduct a Bayesian estimation of the SUR model represented by the system of input demands and output supplies in (49) but with a matrix  $\mathbf{R}$  of constraints that impose only symmetry restrictions. Therefore we have only two groups of parameters: free parameters and those constrained by symmetry. We modify equations (50)

through (58) to accommodate the new set of restrictions and estimate the model parameters by a Gibbs sampler on the marginal posterior distributions given by equations (57) and (58).<sup>32</sup>

Results show that the two approaches provide different results. We present two cases: first, the corn yield elasticities, which are the ones most impacted by the independent sources of information, and secondly present soybean yields elasticities which are less impacted by new information.

Figure 10 shows the case of corn yields in which the dual approach provides, on average, lower estimated values. This is consistent with the conclusions of Chapter 2, in which we found that parameter estimates using this approach can suffer from attenuation bias (Greene, 2003 p.85) when the data used is subject to sources of noise that prevent the dual theorem from holding exactly. Table 10 compares the dual and the proposed approach showing the median of the posterior distribution of the yield elasticities with respect to the prices of interest. It also shows the lower and upper bounds of the HPDI of such posterior distributions. Not only the dual approach elasticity estimates are lower, but also the hypothesis of non-responsiveness cannot be rejected in all cases. Regarding soybean yields (Figure 11 and Table 10), in which no new information about soybean production technology is used, both methods also provide different results and with the same pattern as in corn, i.e., lower values of the parameter estimates in the case of the dual approach. This shows how the introduction of new information has effects on all parameters of the dual model.

Finally we present sensitivity analysis to evaluate the consequences of omitting other intermediate inputs when estimating  $\phi_{21}$ , i.e., the model parameter that measures the yield

---

<sup>32</sup> A classical econometric method could also be used to estimate the dual model, but we argue that Bayesian methods are more appropriate for this comparison because they were also used as the estimation method in the proposed approach.

response to them. The omission of an input implies assuming that yields are not responsive to its use. In particular, we compare results when only seed density is included versus when both seed density and seed hybrids are considered.

Two caveats. First, the lack of data on yield response to other intermediate inputs prevents us from conducting other type of sensitivity analysis, such as the consequences of omitting a third or more intermediate inputs. However, the one proposed here is informative because it can tell where in the estimation such omission impacts the most and by how much. Second, the inputs considered in this analysis (hired labor, seed density, seed hybrids, nitrogen, and phosphate) account for 62% of all pre-harvest input costs. The remaining costs include pesticides, herbicides, potash, lime, machinery, insurance, interests, and miscellaneous (Duffy, 2012). We argue that omitted inputs do not represent a high portion of input costs and also that some of them clearly do not affect crop yields directly.

Table 11 shows the results when we consider only seed density in  $\phi_{21}$  estimation; that is, we removed from our model seed hybrids as an intermediate input to study the consequences of such omission. It can be seen that the estimated corn yield elasticity with respect to intermediate inputs quantities is 0.36, when it was 0.42 in the previous case. As expected, this elasticity is lower because before it was calculated as  $\phi_{21} = (\phi_{21,SD} + \phi_{21,SH})$  and now it is only  $\phi_{21} = \phi_{21,SD}$ . However, when we observe the consequences on the yield response to prices the changes are very minor (corn yield own-price elasticity, given by equation (47), decreases from 0.29 to 0.28). The case of yield response to input prices is also marginally affected. As a consequence, this provides support to our claim that the intermediate inputs that might have been omitted have small impacts on final results; the reason being both their relatively small share of the total input costs, and/or that we are already considering all inputs with the highest impacts on yields.

## 7. Conclusions

In this paper we study the yield elasticities (or intensive margin) with respect to prices. Yield elasticities have become a center of discussion based on observed periods of sustained high commodity prices caused by a combination of factors including biofuel policies, and increased demand from developing countries, among others. Precise measures of production responses are important in all circumstances; but the case of crop yields is of high relevance, given that evidence in the literature (Dumortier et al., 2011; Keeney and Hertel, 2009) shows that small deviations in the values assumed for these elasticities have great impacts on a country's GHG emissions accounting and land-use change evaluations.

We propose an estimation approach for calculating crop yields elasticities with respect to output and input prices. We start by noting that the two preferred methods used to calculate elasticities (the primal and dual) both have their drawbacks (Colman, 1983; Just, 1993) and can provide biased results (Chapter 2) when employed with market-based datasets. In particular, the dual approach that is usually preferred over the primal still has its problems, as described in Chapter 2. The proposed approach consists of incorporating data on production response in order to complement the estimation using duality theory on market data. The different datasets coming from various sources of information are independent from each other.

We set up an expected profits maximization problem, noting that some of its parameters can be simultaneously recovered by means of other model setups that require information that is available for this study. We then specify the conditions required for using more than one source of information to estimate parameters that are common to more than one model. Specifically, the ability to estimate the same parameters from different models implies certain parameter

constraints. The inclusion of outside information to aid the parameter recovery seeks to overcome the identification problems found in empirical applications of duality theory.

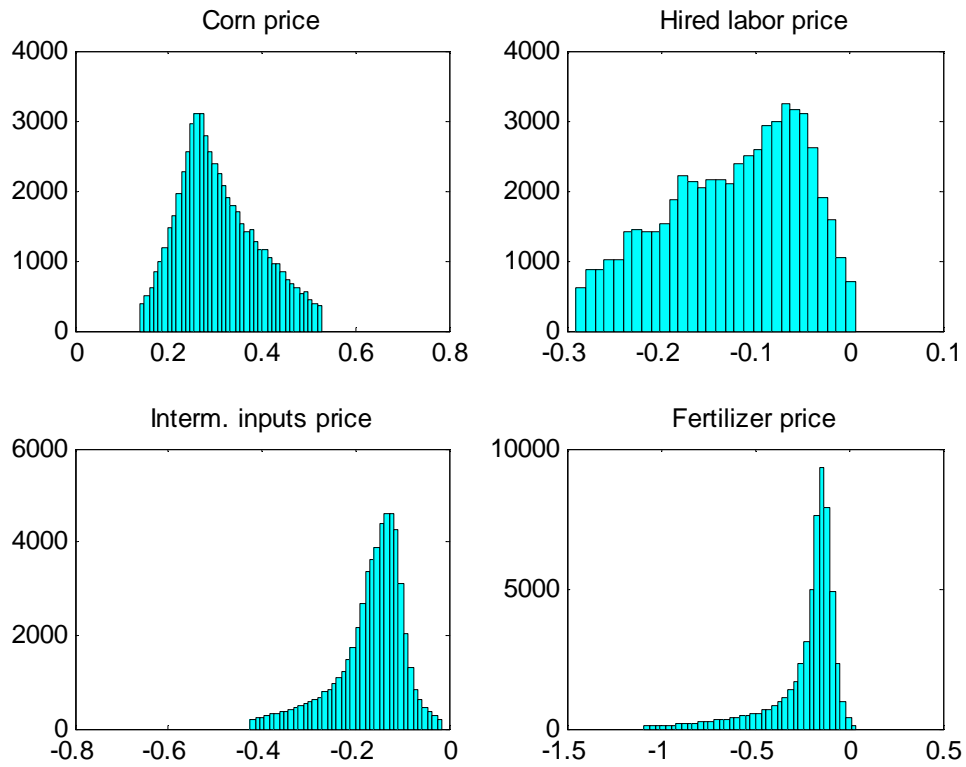
Bayesian estimation methods are a natural choice for model estimation because they allow incorporating the mentioned information in a straightforward way, without the requirement of imposing these constraints deterministically. In fact, the constraints are imposed with uncertainty, and we use the log-likelihood function of each model to indicate the relative weight assigned to each source of information. Bayesian estimation is performed by means of a Metropolis-in-Gibbs algorithm; in particular, we use a routine called t-walk that allows drawing samples from the marginal posteriors, especially when their functional form is unknown.

Results show that own-price corn yield elasticity falls within the interval of empirical studies found in the literature. This elasticity, as well as the elasticity with respect to input prices, are all of the expected sign (yields increasing with own-price and decreasing with input prices).

Final results are driven by the inclusion of other independent data. This is true because an estimation of the model without any further information, other than the market-based data, yields very different results.

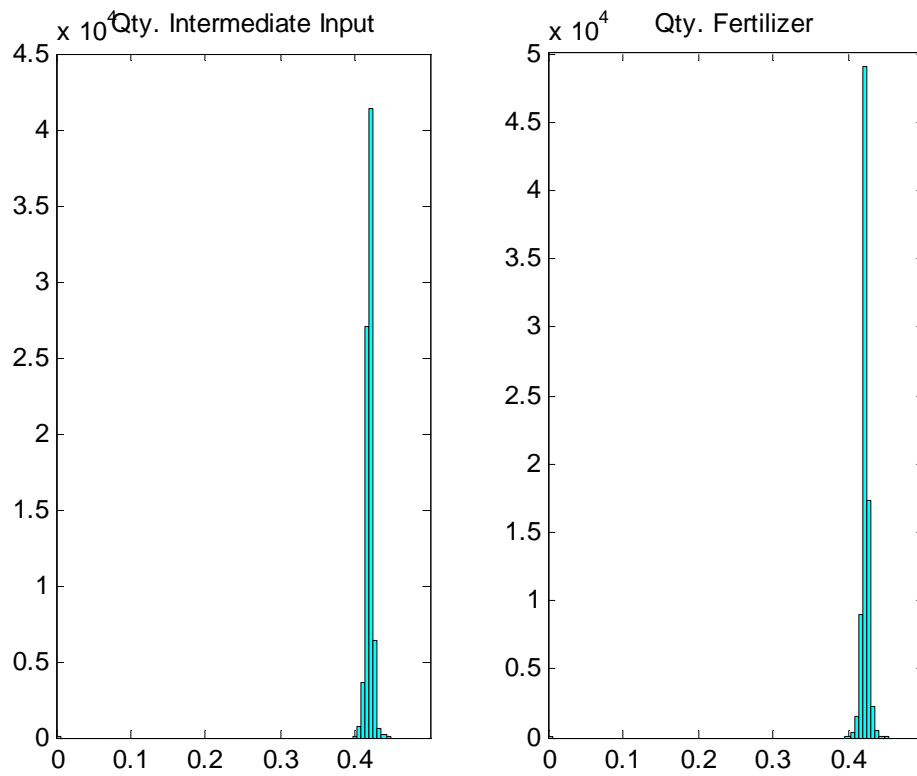
Finally we study the sensitivity of the approach to the inputs not considered in the analysis, in particular the case of intermediate inputs. Our results suggest that this omission has very little impact on yield elasticities. This supports the argument that the inputs already considered represent a large fraction of total inputs and that the remaining inputs do not appear to induce much yield response.

## Figures



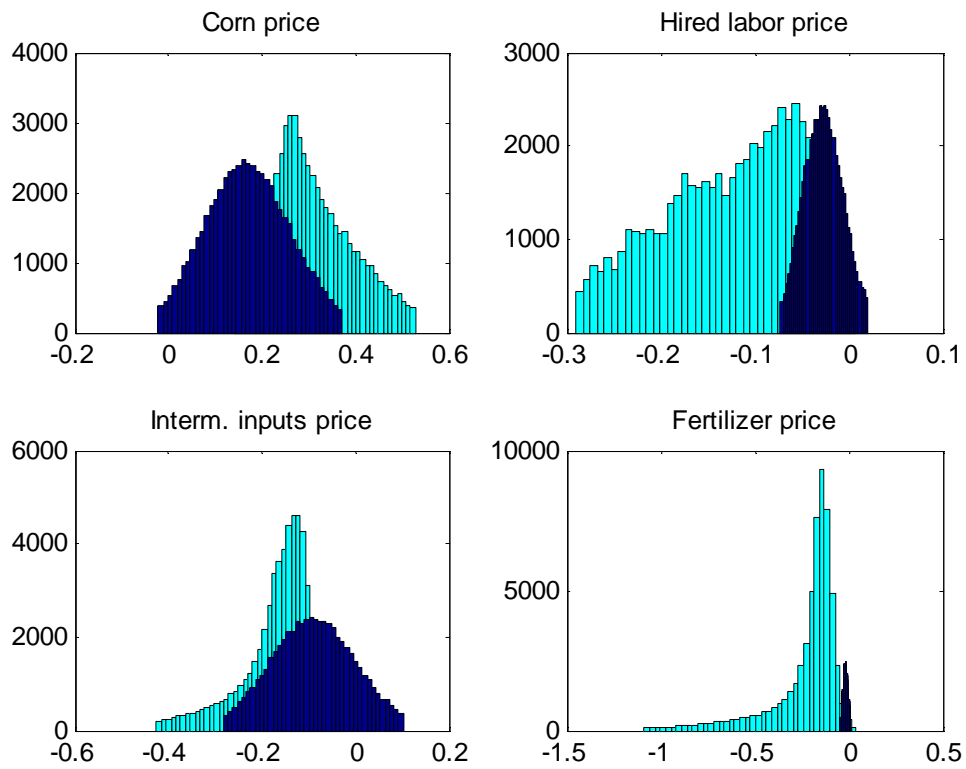
**Figure 8. Corn yield elasticities with respect to selected prices.**

Note: Histograms show highest probability density intervals at the 95%.



**Figure 9. Corn yield elasticities with respect to quantity of intermediate inputs and quantity of fertilizers.**

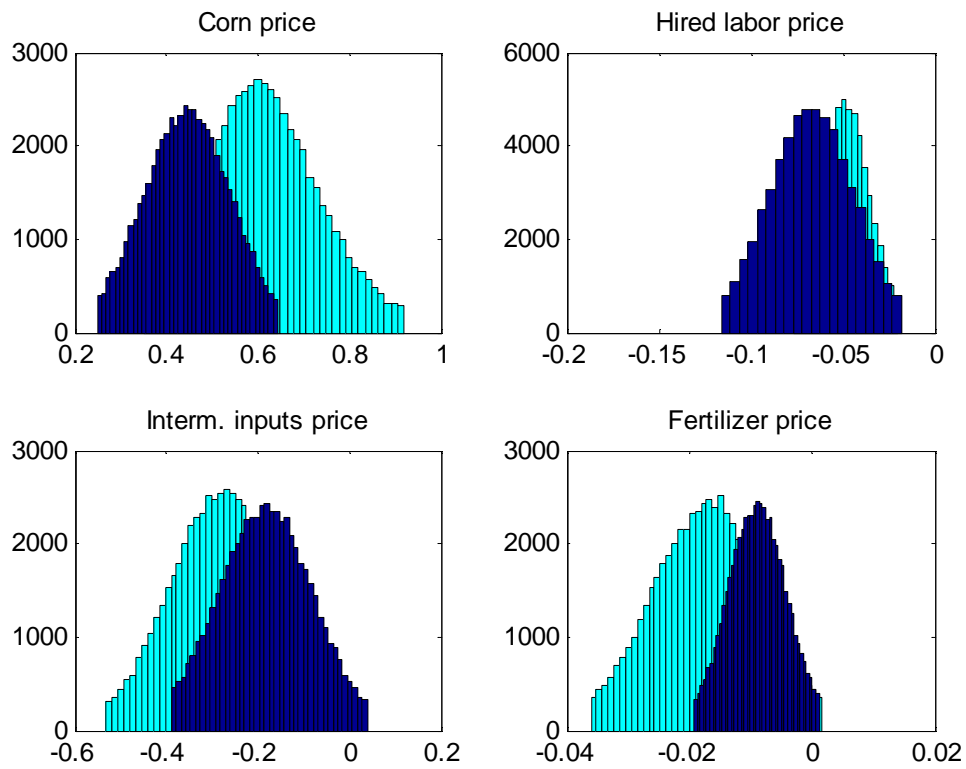
Note: Histogram of marginal posterior density function.



**Figure 10. Corn yield elasticities with respect to selected prices. Comparison between proposed approach (light blue) and dual approach (blue).**

Note: Histograms show highest probability density intervals at the 95%.





**Figure 11. Soybean yield elasticities with respect to selected prices. Comparison between proposed approach (light blue) and dual approach (blue).**

Note: Histograms show highest probability density intervals at the 95%.

**Tables****Table 8. Literature review of estimated elasticities of yield with respect to corn price**

Authors	Time Period	Data	Elasticity	t-statistic
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.76	6.33
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.69	6.32
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.28	3.59
Houck and Gallagher	1951-1971	Time series of U.S. yields	0.24	3.11
Menz and Pardey	1951-1971	Time series of U.S. yields	0.61	5.17
Menz and Pardey	1972-1980	Time series of U.S. yields	0.44	(*)
Choi and Helmberger	1964-1988	Time series of U.S. yields	0.27	2.80
Lyons and Thompson	1961-1973	Time series (14 countries)	0.22	3.13
Pomareda & Samayoa		Time Series Guatemala	0.50	
Kaufmann and Snell	1969-1987	Time series of U.S. yields	~ 0	
Stout and Alber		U.S.	0.02	
Stout and Alber		Canada	0.15	
Stout and Alber		Mexico	0.18	
Reed and Riggins	1960-1979	Time series Kentucky	Neg.	(*)
Arnade and Kelch	1960-1999	Time series Iowa	0.19	1.63
Madhu and Kahna	1994-2007	Panel data of U.S. yields	0.15	

Note: The t-values (excepting Lyons and Thompson) are the reported t-values for the price coefficient from the estimated model. In Lyons and Thompson is the elasticity with respect to the relative price of corn to nitrogen. (\*): Based on a parameter estimate that is not statistically different from zero.

**Table 9. Corn yield elasticities with respect to selected prices and quantities.**

	<b>Lower bound</b>	<b>Median</b>	<b>Upper bound</b>
<b>Elasticity of corn yields with respect to:</b>			
Corn price	0.14	0.29	0.53
Hired Labor price	-0.29	-0.12	0.01
Intermediate Inputs price	-0.43	-0.15	-0.01
Fertilizer price	-1.09	-0.17	0.04
Hired Labor quantity	0.000	0.190	0.461
Intermediate Inputs quantity	0.412	0.420	0.429
Fertilizer quantity	0.413	0.422	0.431

Note: Lower and upper bounds represent extremes of the 95% highest probability interval of the marginal posterior density function of each elasticity.

**Table 10. Corn and soybean yield elasticities with respect to selected prices and quantities.**  
**Corn yield elasticity with respect to:**

	Lower bound	Median	Upper bound
	<b>Proposed Approach</b>		
Corn price	0.14	0.29	0.53
Hired Labor price	-0.29	-0.12	0.01
Intermediate Inputs price	-0.43	-0.15	-0.01
Fertilizer price	-1.09	-0.17	0.04
	<b>Dual Approach</b>		
Corn price	-0.02	0.17	0.37
Hired Labor price	-0.07	-0.03	0.02
Intermediate Inputs price	-0.28	-0.09	0.10
Fertilizer price	-0.05	-0.02	0.01

	Lower bound	Median	Upper bound
	<b>Proposed Approach</b>		
Soybean price	0.35	0.61	0.92
Hired Labor price	-0.08	-0.05	-0.02
Intermediate Inputs price	-0.53	-0.28	-0.02
Fertilizer price	-0.04	-0.02	0.00
	<b>Dual Approach</b>		
Soybean price	0.25	0.45	0.64
Hired Labor price	-0.12	-0.07	-0.02
Intermediate Inputs price	-0.39	-0.18	0.04
Fertilizer price	-0.02	-0.01	0.00

Note: Lower and upper bounds represent extremes of the 95% highest probability interval of the marginal posterior density function of each elasticity.

**Table 11. Sensitivity analysis: the case of corn seed hybrids. Corn yield elasticities with respect to selected prices and quantities.**

	Lower bound	Median	Upper bound
<b>Elasticity of corn yields with respect to:</b>			
Corn price	0.13	0.28	0.53
Hired Labor price	-0.28	-0.10	0.05
Intermediate Inputs price	-0.45	-0.16	0.02
Fertilizer price	-1.09	-0.16	0.03
Hired Labor quantity	0.00	0.20	0.46
Intermediate Inputs quantity	0.36	0.36	0.37
Fertilizer quantity	0.41	0.42	0.43

Note: Lower and upper bounds represent extremes of the 95% highest probability interval of the marginal posterior density function of each elasticity.

## 8. Appendix

### 8.1 Appendix I: Additive general error model for profit function

McElroy (1987) derived the additive general error model (AGEM) for the case of cost functions. Below we show that a similar error structure follows for profit functions. Consider the following expected profit maximization problem:  $\max_{[\mathbf{x}]} E(\tilde{\pi}) = \max_{[\mathbf{x}]} \{E[\tilde{\mathbf{p}}'\tilde{\mathbf{y}} - \mathbf{w}'\mathbf{x}]\}$  where  $\tilde{\mathbf{p}}$  is the  $(J \times 1)$  vector denoting unobserved output prices,  $\tilde{\mathbf{y}}$  is the  $(J \times 1)$  vector of stochastic output quantities,  $\mathbf{x}$  is a  $(I \times 1)$  choice vector of inputs used in production of all outputs, and  $\mathbf{w}$  are their observed prices. Expectation  $E[.]$  are taken over the randomness of production technology and of unobserved prices, which for simplicity we assume to be independent from each other. The solution is a set of input demands  $x_i^* = x_i(w, \bar{p}, \mathbf{K})$  for  $i = \{1, \dots, I\}$  representing the use in all outputs, a set of expected output supplies  $\bar{y}_j^* = \bar{y}_j(w, \bar{p}, \mathbf{K})$  for  $j = 1, \dots, J$ , and a value function  $\pi^* = \pi(w, \bar{p}, \mathbf{K}) = \sum_j \bar{p}_j \bar{y}_j^*(w, \bar{p}, \mathbf{K}) - \sum_i w_i x_i^*(w, \bar{p}, \mathbf{K})$ . However in reality the true input demands and output supplies, while are observed with certainty by the producer, are observed with an error by the econometrician. We claim that we observe the following:

$$\begin{aligned} x_i &= x_i^*(w, \bar{p}, \mathbf{K}) + \varepsilon_i \text{ for } i = 1, \dots, I \\ \bar{y}_j &= \bar{y}_j^*(w, \bar{p}, \mathbf{K}) + \mu_j \text{ for } j = 1, \dots, J \end{aligned} \tag{64}$$

where the left hand side variables are observed values. Therefore the profit value function consistent with these observed input demands and output supplies is obtained by substituting the observed, instead of the true values, in the objective function; that is:

$$\begin{aligned}
\pi^* &= \pi(w, \bar{p}, K) \\
&= \sum_j \bar{p}_j \bar{y}_j(w, \bar{p}, K) - \sum_i w_i x_i(w, \bar{p}, Z) \\
&= \sum_j \bar{p}_j [\bar{y}_j^*(w, \bar{p}, Z) + \mu_j] - \sum_i w_i [x_i^*(w, \bar{p}, K) + \varepsilon_i] \\
&= \sum_j \bar{p}_j \bar{y}_j^*(w, \bar{p}, K) - \sum_i w_i x_i^*(w, \bar{p}, K) + \sum_j \bar{p}_j \mu_j - \sum_i w_i \varepsilon_i
\end{aligned} \tag{65}$$

from which it can be clearly seen that the error structure arising from using the observed input demands and output supplies is the same as the error structure in the aggregate profit value function in (67).

## 8.2 Appendix II: Cross-equation parameter restrictions

The system to be estimated, written with symmetry restrictions given by Young's theorem, is the following:

$$\begin{bmatrix} -x_1 \\ -x_2 \\ -x_3^1 \\ -x_3^2 \\ -x_3^3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3^1 \\ a_3^2 \\ a_3^3 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13}^1 & a_{13}^2 & a_{13}^3 & b_{11} & b_{12} & b_{13} \\ a_{12} & a_{22} & a_{23}^1 & a_{23}^2 & a_{23}^3 & b_{21} & b_{22} & b_{23} \\ a_{13}^1 & a_{23}^1 & a_{33}^1 & 0 & 0 & b_{31}^1 & b_{32}^1 & b_{33}^1 \\ a_{13}^2 & a_{23}^2 & 0 & a_{33}^2 & 0 & b_{31}^2 & b_{32}^2 & b_{33}^2 \\ a_{13}^3 & a_{23}^3 & 0 & 0 & a_{33}^3 & b_{31}^3 & b_{32}^3 & b_{33}^3 \\ b_{11} & b_{21} & b_{31}^1 & b_{31}^2 & b_{31}^3 & d_{11} & d_{12} & d_{13} \\ b_{12} & b_{22} & b_{32}^1 & b_{32}^2 & b_{32}^3 & d_{12} & d_{22} & d_{23} \\ b_{13} & b_{23} & b_{33}^1 & b_{33}^2 & b_{33}^3 & d_{13} & d_{23} & d_{33} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_3 \\ w_3 \\ \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \end{bmatrix} + CK + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3^1 \\ \varepsilon_3^2 \\ \varepsilon_3^3 \\ \varepsilon_{y_1} \\ \varepsilon_{y_2} \\ \varepsilon_{y_3} \end{bmatrix} \tag{66}$$

where  $w_3$  repeated three times reflects the fact that is the same input allocated among the three outputs. We do this to clarify the explanation of the system setup and parameter restrictions. The negative sign in front of the variables  $X$ 's follows the standard netput notation (negative in the case of inputs). Integration of the system of input demands and output supplies yields the following underlying profit function:

$$\begin{aligned}
\pi(\mathbf{w}, \bar{\mathbf{p}}, \mathbf{K}; \boldsymbol{\beta}_0) &= \sum_{i=1}^3 a_i w_i + \sum_{i=1}^3 d_i \bar{p}_i \\
&+ \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 a_{ij} w_i w_j + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 d_{ij} \bar{p}_i \bar{p}_j + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} w_i \bar{p}_j \\
&+ \sum_{i=1}^3 \sum_{j=1}^5 c_{ij} w_i K_j + \sum_{i=1}^3 \sum_{j=1}^5 f_{ij} \bar{p}_i K_j \\
&+ \sum_{i=1}^3 w_i \varepsilon_i + \sum_{i=1}^3 \bar{p}_i \mu_i
\end{aligned} \tag{67}$$

where, due to the existence of allocation data on input 3, we have that:

$$\begin{aligned}
a_3 &= \sum_{k=1}^3 a_3^k \\
a_{3j} &= \sum_{k=1}^3 a_{3j}^k \text{ for } j = \{1, 2, 3\} \\
b_{3j} &= \sum_{k=1}^3 b_{3j}^k \text{ for } j = \{1, 2, 3\} \\
c_{3j} &= \sum_{k=1}^3 c_{3j}^k \text{ for } j = \{1, 2, 3, 4, 5\}
\end{aligned} \tag{68}$$

Equalities in (68) arising from the existence of input allocation data and symmetry restrictions given by Young's theorem, imply that the following constraints can be imposed in estimation:

$$\begin{aligned}
a_{13} &= a_{13}^1 + a_{13}^2 + a_{13}^3 \\
a_{23} &= a_{23}^1 + a_{23}^2 + a_{23}^3 \\
b_{31} &= b_{31}^1 + b_{31}^2 + b_{31}^3 \\
b_{32} &= b_{32}^1 + b_{32}^2 + b_{32}^3 \\
b_{33} &= b_{33}^1 + b_{33}^2 + b_{33}^3
\end{aligned} \tag{69}$$

where  $a_{13}$ ,  $a_{23}$ ,  $b_{31}$ ,  $b_{32}$ , and  $b_{33}$  denote, respectively, the derivatives of  $-x_1$ ,  $-x_2$ ,  $y_1$ ,  $y_2$ , and  $y_3$  with respect to  $w_3$ .



Separable technology implies that there are no cross-price effects in the case of outputs, and that output prices affect only their own use of inputs; therefore matrices  $\mathbf{D}_Y$  and  $\mathbf{B}_3$  become:

$$\mathbf{D}_Y = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \quad (70)$$

$$\mathbf{B}_3 = \begin{bmatrix} b_{31}^1 & b_{32}^1 & b_{33}^1 \\ b_{31}^2 & b_{32}^2 & b_{33}^2 \\ b_{31}^3 & b_{32}^3 & b_{33}^3 \end{bmatrix} = \begin{bmatrix} b_{31}^1 & 0 & 0 \\ 0 & b_{32}^2 & 0 \\ 0 & 0 & b_{33}^3 \end{bmatrix} \quad (71)$$

Together with symmetry we further have that:

$$\begin{aligned} b_{31} &= b_{31}^1 \\ b_{32} &= b_{32}^2 \\ b_{33} &= b_{33}^3 \end{aligned} \quad (72)$$

which means that changes in fertilizer prices affect crop supply only through its own use, and not through fertilizer used in other crops. In terms of the underlying profit function, they imply that the following terms can be rewritten as:

$$\begin{aligned} \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 d_{ij} \bar{p}_i \bar{p}_j &= \frac{1}{2} \sum_{i=1}^3 d_{ii} \bar{p}_i \bar{p}_i \\ \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} w_i \bar{p}_j &= \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^3 b_{ij} w_i \bar{p}_j + \sum_{k=1}^3 b_{3k}^k w_3 \bar{p}_k \end{aligned} \quad (73)$$

In summary, plugging (69) through (72) in system (66), the matrix of coefficients that we take to estimation is:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & b_{11} & b_{12} & b_{13} \\ a_{12} & a_{22} & a_{23} & 0 & 0 & b_{21} & b_{22} & b_{23} \\ a_{13}^1 & a_{23}^1 & a_{33}^1 & 0 & 0 & b_{31} & 0 & 0 \\ a_{13}^2 & a_{23}^2 & 0 & a_{33}^2 & 0 & 0 & b_{32} & 0 \\ a_{13} - a_{13}^1 - a_{13}^2 & a_{23} - a_{23}^1 - a_{23}^2 & 0 & 0 & a_{33}^3 & 0 & 0 & b_{33} \\ b_{11} & b_{21} & b_{31} & 0 & 0 & d_{11} & 0 & 0 \\ b_{12} & b_{22} & 0 & b_{32} & 0 & 0 & d_{22} & 0 \\ b_{13} & b_{23} & 0 & 0 & b_{33} & 0 & 0 & d_{33} \end{bmatrix} \quad (74)$$

In fact, (74) is exactly the profit function Hessian sub-matrix that produces system (66) with respect to variable input and output prices. By Lau's Hessian relationships and considering the sub-matrix with respect to quasi-fixed inputs, the production function Hessian with respect to variable input and output quantities is calculated as:

$$\left[ \frac{\partial^2 h}{\partial \{-\mathbf{x}, \mathbf{y}\}^2} \right] = \left[ \left( \frac{\partial^2 \pi}{\partial \{\mathbf{w}, \bar{\mathbf{p}}\}^2} \right)^{-1} \left( - \left( \frac{\partial^2 \pi}{\partial \{\mathbf{w}, \bar{\mathbf{p}}\}^2} \right)^{-1} \frac{\partial^2 \pi}{\partial \{\mathbf{w}, \bar{\mathbf{p}}\} \partial \mathbf{K}} \right) \right] \quad (75)$$

Given the specified normalized quadratic profit function, the dual production function is also quadratic; that is:

$$\begin{aligned} y_0 = H(\mathbf{x}, \mathbf{y}, \mathbf{K}; \boldsymbol{\alpha}_0) = & - \sum_{i=1}^9 \gamma_i x_i + \sum_{i=1}^3 \delta_i y_i \\ & + \frac{1}{2} \sum_{i=1}^9 \sum_{j=1}^9 \gamma_{ij} x_i x_j + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \delta_{ij} y_i y_j - \frac{1}{2} \sum_{i=1}^9 \sum_{j=1}^3 \lambda_{ij} x_i y_j - \boldsymbol{\varrho}_x \mathbf{x}' \mathbf{K} \\ & + \boldsymbol{\varrho}_y \mathbf{y}' \mathbf{K} \end{aligned} \quad (76)$$

where  $\mathbf{x} = \{x_1, \dots, x_9\}$ . The first three are the uses in  $y_1$ , the second three the uses in  $y_2$ , and the remaining the uses in  $y_3$ . Netput  $y_0$  is the numeraire. Considering that only input 3 allocation data is available and the assumed separable technology, the production function Hessian can be written as:

$$\left[ \frac{\partial^2 h}{\partial \{-\mathbf{x}, \mathbf{y}\}^2} \right] = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13}^1 & \gamma_{13}^2 & \gamma_{13}^3 & \lambda_{11}^1 & \lambda_{12}^2 & \lambda_{13}^3 \\ \gamma_{12} & \gamma_{22} & \gamma_{23}^1 & \gamma_{23}^2 & \gamma_{23}^3 & \lambda_{21}^1 & \lambda_{22}^2 & \lambda_{23}^3 \\ \gamma_{13}^1 & \gamma_{23}^1 & \gamma_{33}^1 & 0 & 0 & \lambda_{31}^1 & 0 & 0 \\ \gamma_{13}^2 & \gamma_{23}^2 & 0 & \gamma_{33}^2 & 0 & 0 & \lambda_{32}^2 & 0 \\ \gamma_{13}^3 & \gamma_{23}^3 & 0 & 0 & \gamma_{33}^3 & 0 & 0 & \lambda_{33}^3 \\ \lambda_{11}^1 & \lambda_{21}^1 & \lambda_{31}^1 & 0 & 0 & \delta_{11} & 0 & 0 \\ \lambda_{12}^2 & \lambda_{22}^2 & 0 & \lambda_{32}^2 & 0 & 0 & \delta_{22} & 0 \\ \lambda_{13}^3 & \lambda_{23}^3 & 0 & 0 & \lambda_{33}^3 & 0 & 0 & \delta_{33} \end{bmatrix} \quad (77)$$

The superscript  $j$  in  $\gamma_{ii'}$  indicates the mutual effects of inputs  $i$  and  $i'$  when used in output  $j$ , and the superscript  $j$  in  $\lambda_{ij}$  represents the effect of inputs  $i$  on output  $j$  when input is used in producing output  $j$ . By separability we only have  $\lambda_{ij}^j$ . The zeros represent the separability or lack of effects between netputs involved in producing different outputs.

We use the Hessian relationships in (75) and the profit function Hessian matrix in (74) to back out the values of parameters in (77) in order to explicitly write the form of the constraints arising from the use of datasets describing features of the production function. From these datasets we can calculate the marginal effect on output  $Y_1$  (corn) of the input uses,  $\phi_{i1}$ , which everything else equal, can be calculated as follows:

$$\begin{aligned} \phi_{11} &\equiv \frac{dy^1}{dx_1^1} = -\frac{\frac{\partial H}{\partial x_1^1}}{\frac{\partial H}{\partial y^1}} \\ \phi_{21} &\equiv \frac{dy^1}{dx_2^1} = -\frac{\frac{\partial H}{\partial x_2^1}}{\frac{\partial H}{\partial y^1}} \\ \phi_{31} &\equiv \frac{dy^1}{dx_3^1} = -\frac{\frac{\partial H}{\partial x_3^1}}{\frac{\partial H}{\partial y^1}} \end{aligned} \quad (78)$$

### 8.3 Appendix III: Integration properties of supply-demand system

We show the form of the profit function that is consistent with first-order linear approximation of a system of demands and supplies in which some inputs are specified by their allocation to the outputs of the technology. Without loss of generality, we assume that one input (input  $N$ ) is specified in its allocation. By integrating each input demand as well as each output supply with respect to their own prices (which is the opposite to applying Hostelling's lemma to a profit function) we can derive the profit function implied by the system :

$$\begin{aligned}
 \pi(\mathbf{w}, \bar{\mathbf{p}}, \mathbf{Z}; \boldsymbol{\theta}) = & + \sum_{i=1}^{N-1} a_i w_i + (a_N^1 + \dots + a_N^K) w_N + \sum_k d_k \bar{p}_k + \sum_{i=1}^{N-1} \sum_{j=1}^N a_{ij} w_i w_j + \\
 & \sum_{j=1}^N (a_{Nj}^1 + \dots + a_{Nj}^K) w_N w_j + \sum_{i=1}^{N-1} \sum_k b_{ik} w_i p_k + \sum_k (b_{Nk}^1 + \dots + b_{Nk}^K) w_N \bar{p}_k + \\
 & \sum_k \sum_m d_{km} \bar{p}_k \bar{p}_m + \sum_{i=1}^{N-1} \sum_{r=1}^R c_{ir} w_i Z_r + \sum_{r=1}^R (c_{Nr}^1 + \dots + c_{Nr}^K) w_N Z_r + \\
 & \sum_k \sum_r f_{kr} \bar{p}_k Z_r + \sum_{i=1}^{N-1} w_i \varepsilon_i + w_N (\varepsilon_N^1 + \dots + \varepsilon_N^K) + \sum_k \bar{p}_k \varepsilon_{y_k}
 \end{aligned} \tag{79}$$

The profit function is a normalized quadratic with the special features that some coefficients are the summation of the parameters across the  $K$  crops.

From this expression is clearly seen that if one chooses to second-order approximate a profit function by a functional form, the parameters corresponding to crop-specific input uses ( $N$  in this example), cannot be recovered because they enter only as a summation and not individually. This is the reason we proceeded to directly approximate the input demands.

## 9. References

- Amemiya, T. 1985. *Advanced Econometrics*. Harvard University Press. Cambridge, MA
- Arnade, C. and D. Kelch. 2007. "Estimation of Area Elasticities from Standard Profit Function". *American Journal of Agricultural Economics*. 89 (3): 727-737.
- Arnade, C., D. Kelch, and S. Leetmaa. 2002. "Supply Response in France, Germany, and the U.K.: Technology and Price." *Agricultural and Applied Economics Association Annual Meetings, Long Beach, California*.
- Ash, M.S., and W. Lin. 1987. *Regional Crop Yield Response for U.S. Grains*. Agricultural Economic Report No. 577, Economic Research Service, USDA, Washington, DC.
- Casella, G. and E. I. George. 1992. "Explaining the Gibbs Sampler" *The American Statistician* 46(3): 167-173
- Chambers R. G. and R. D. Pope. 1994. "A Virtually Ideal Production System: Specifying and Estimating the VIPS Model" *American Journal of Agricultural Economics* 76(1):105-113
- Chib, S. and E. Greenberg. 1996. "Markov Chain Monte Carlo Methods in Econometrics" *Econometric Theory*. (12): 409-431
- Choi J. S. and P. Helmberger. 1993. "How Sensitive are Crop Yields to Price Changes and Farm Programs?" *Journal of Agriculture and Applied Economics* 25:237-244.
- Christen, J.A. and Fox, C. (2010), "A General Purpose Sampling Algorithm for Continuous Distributions (the t-walk)", *Bayesian Analysis* 5(2): 263-282.
- Christensen, L. R., D. W. Jorgenson and L. J. Lau. 1971. "Transcendental Logarithmic Production Frontiers". *The Review of Economics and Statistics*. 55 (1): 28-45
- Colman, D. 1983. "A Review of the Arts in Supply Response Analysis." *Review of Marketing and Agricultural Economics*. 51(3):201-230
- Deaton A. S., and J. Muellbauer. 1980. "An Almost Ideal Demand System". *The American Economic Review* 70:312-326
- Diewert, W.E. 1971. "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function." *Journal of Political Economy* 79:481-507.
- Diewert, W.E. and T.J. Wales. 1987. "Flexible Functional Forms and Global Curvature Conditions." *Econometrica* 55: 43-68.
- Duffy, M. 2012. Estimated Costs of Crop Production in Iowa—2012. *Ag Decision Maker*, Iowa State University Extension. January.

- Dumortier, J., D. J. Hayes, M. Carriquiry, F. Dong, X. Du, A. Elobeid, J. Fabiosa, and S. Tokgoz. 2011. "Sensitivity of Carbon Emission Estimates from Indirect Land-Use Change." *Applied Economic Perspectives and Policy* 33(3): 428-448.
- The Denitrification-Decomposition Model (DNDC). Institute for the Study of Earth, Oceans, and Space. Complex Systems Research Center. University of New Hampshire. Available at <http://www.dndc.sr.unh.edu/> (accessed on March 2012).
- The Environmental Policy Integrated Climate (EPIC). 2012. Center for Agricultural and Rural Development. Resource and Environmental Policy. Available at [http://www.card.iastate.edu/environment/interactive\\_programs.aspx](http://www.card.iastate.edu/environment/interactive_programs.aspx) (accessed April 2012)
- Fargione, J., J. Hill, D. Tillman, S. Polasky, and P. Hawthorne. 2008. "Land Clearing and the Biofuel Carbon Debt." *Science* 319.
- Giles, D. E. A. 2003. Computer-Aided Econometrics. Marcel Dekker Inc., New York, NY
- Greene, W. H. 2003. Econometric Analysis. 5<sup>th</sup> Edition. Prentice Hall. Upper Saddle River, NJ
- Guyomard, H., M. Baudry and A. Carpenter. 1996. "Estimating Crop Supply Response in the Presence of Farm Programmes: Application to the CAP". *European Review of Agricultural Economics*. 23: 401-420.
- Houck, J.P., and P.W. Gallagher. 1976. "The Price Responsiveness of U.S. Corn Yields," *American Journal of Agricultural Economics* 58: 731-734.
- Huang, H. and M. Khanna. 2010. "An Econometric Analysis of U.S. Crop Yield and Cropland Acreage: Implications for the Impact of Climate Change." *Agricultural and Applied Economics Association Annual Meetings*, Denver, Colorado
- Iowa State University, Iowa Crop Improvement Association (ISU-ICIA). 2012. Crop Testing. Ames, IA. Available at <http://www.croptesting.iastate.edu/> (accessed on June 2012).
- Just, R. E. 1993. "Discovering Production and Supply Relationships: Present Status and Future Opportunities". *Review of Marketing and Agricultural Economics*. 61:11-40.
- Kaufman, R.K., and S.E. Snell. 1997. "A Biophysical Model of Corn Yield: Integrating Climatic and Social Determinants," *American Journal of Agricultural Economics*, 79: 178-190.
- Keeney, R. and T. W. Hertel. 2008. "Yield Response to Prices: Implications for Policy Modeling." *GTAP Working Paper* 08 - 13.
- Keeney, R. and T. W. Hertel. 2009. "The Indirect Land Use Impacts of United States Biofuel Policies: The Importance of Acreage, Yield, and Bilateral Trade Responses." *American Journal of Agricultural Economics* 91:895-909.
- LaFrance, J. T. and W. M. Hanemann. 1989. "The Dual Structure of Incomplete Demand Systems" *American Journal of Agricultural Economics* 71(2):262-274

- Lau, L. J. 1974. "Comments on Applications of Duality Theory", in: M. D. Intriligator and D. A. Kendrick, eds., *Frontiers of Quantitative Economics*, Vol. II, North-Holland, Amsterdam, 176-199.
- Lau, L. J. 1976. "A Characterization of the Normalized Restricted Profit Function" *Journal of Economic Theory* 12(1):131-163
- Lieberman, C. 2012. "t-walk MATLAB code". Available at: <http://web.mit.edu/celieber/www/software/twalk.m> (accessed April 2012)
- Love, H. A. and W. E. Foster. 1990. "Commodity Program Slippage Rates for Corn and Wheat." *Western Journal of Agricultural Economics* 15(2):272-281
- Lyons, D.C., and R.L. Thompson. 1981. "The Effect of Distortions in Relative Prices on Corn Productivity and Exports: A Cross-Country Study," *Journal of Rural Development* 4: 83-102.
- McElroy, M. 1987. "Additive General Error Models for Production, Cost, and Derived Demand or Share Systems". *Journal of Political Economy*. 95:737-757.
- McFadden, D. 1978. "The General Linear Profit Function" in *Production Economics: A Dual Approach to Theory and Applications*, Vol. 1, ed. By M. Fuss and D. McFadden. Amsterdam: North-Holland. 269-286.
- Menz, K.M., and P. Pardey. 1983. "Technology and U.S. Corn Yields: Plateaus and Price Responsiveness". *American Journal of Agricultural Economics* 65: 558-562.
- Moschini G. 2001. "Production Risk and the Estimation of Ex-ante Cost Functions." *Journal of Econometrics* 100: 357-380.
- O'Donnell, C. J., C. R. Shumway, and V. E. Ball. 1999. "Input Demands and Inefficiency in U.S. Agriculture" *American Journal of Agricultural Economics* 81(4): 865-880
- Pomareda, C., and O. Samayoa. 1979. "Area and Yield Response to Price Policy: A Case Study in Guatemala, C. A." *American Journal of Agricultural Economics* 61: 683-686.
- Pope R.D. and J.-P. Chavas. 1994. "Cost Functions Under Production Uncertainty". *American Journal of Agricultural Economics* 76 (2): 196-204.
- Reed, M. R., and S. K. Riggins. 1982. "Corn Yield Response: A Micro-Analysis," *North Central Journal of Agricultural Economics* 4: 91-94.
- Righelato, R., and D.V. Spracklen. 2007. "Carbon Mitigation by Biofuels or by Saving and Restoring Forests?" *Science* 317: 902.
- Searchinger, T., R. Heimlich, R.A. Houghton, F. Dong, A. Elobeid, J. Fabiosa, S. Tokgoz, D. Hayes, T-H. Yu. 2008. "Use of U.S. Croplands for Biofuels Increases Greenhouse Gases Through Emissions from Land-Use Change." *Science* 319: 1157-1268.

- Shephard, R. W. 1953. *Cost and Production Functions*. Princeton University Press, Princeton, NJ.
- Stout, J. and D. Alber. 2004. ERS/Penn State Trade Model Documentation.  
[http://trade.aers.psu.edu/pdf/ERS\\_Penn\\_State\\_Trade\\_Model\\_Documentation.pdf](http://trade.aers.psu.edu/pdf/ERS_Penn_State_Trade_Model_Documentation.pdf)
- U.S. Department of Agriculture, Economic Research Service (USDA-ERS). 2012a. Agricultural Productivity in the United States. Washington, DC. Available at <http://www.ers.usda.gov/Data/AgProductivity/> (accessed on April 2012).
- U.S. Department of Agriculture, Economic Research Service (USDA-ERS). 2012b. Fertilizer Use and Price. Washington, DC. Available at <http://www.ers.usda.gov/Data/FertilizerUse/> (accessed on April 2012).
- U.S. Department of Agriculture, National Agricultural Statistics Service (USDA-NASS). 2012. Quick Stats. Washington, DC. Available at <http://quickstats.nass.usda.gov/> (accessed on April 2012).
- Vartia, Y. O. 1983. "Efficient Methods of Measuring Welfare Change and Compensated Income in Terms of Ordinary Demand Functions" *Econometrica* 51(1): 79-98
- von Haefen, R. H. 2007. "Empirical strategies for incorporating weak complementarity into consumer demand models" *Journal of Environmental Economics and Management* 54(1): 15-31



## CHAPTER 4. NITROUS OXIDE EMISSION REDUCTIONS FROM CUTTING EXCESSIVE NITROGEN FERTILIZER APPLICATIONS

### Abstract

On average, U.S. farmers choose to apply nitrogen fertilizer at a rate that exceeds the ex post agronomically optimal rate because the technology underlying the yield response to nitrogen rewards producers who over apply in years when rainfall is excessive. The overapplication of nutrients can lead to negative environmental consequences such as nitrous oxide ( $\text{N}_2\text{O}$ ) emissions and/or water pollution. Here we consider a nonlinear market instrument targeting farmer's nitrogen use, and by solving for the optimal expected utility nitrogen reduction, we evaluate the induced  $\text{N}_2\text{O}$  emission reductions that are consistent with the instrument introduced. The market instrument is nonlinear because of the nonlinear relationship between  $\text{N}_2\text{O}$  and nitrogen application rates. Our simulations show that, by taking into account nonlinearity, payments will induce participation in the program and will have a significant impact on both expected and actual  $\text{N}_2\text{O}$  emissions without significantly harming expected or actual yields. We also show the beginning-of-season probability distribution of emission reductions induced by this incentive scheme.

*Key words:* carbon offsets, nitrogen fertilizer, nitrous oxide, pollution, uncertainty.

*JEL Codes:* Q12, Q18, Q51, Q53, Q54, D8

## 1. Introduction

The nitrogen (N) fertilizer application decision is made under uncertainty because the N available to the crop during the growing season is affected by weather conditions (especially rainfall and temperature). Also, there is evidence in the literature that, ex post and on average, U.S. farmers apply more N fertilizer than the agronomic optimum. The reason behind this behavior is that the technology underlying the response of yields to both N fertilizer and weather conditions is such that the nutrient provided in the smallest amount becomes the limiting nutrient. This gives farmers the incentive to apply more N fertilizer, expecting the growing season to be either wet or warm. However, evidence in the literature suggests that this overapplication of nutrients is ex ante optimal (Babcock 1992; Sheriff 2005).

The overapplication of N has environmental effects, such as volatilization of nitrous oxide ( $\text{N}_2\text{O}$ ), water pollution, and other indirect effects on human health (Townsend et al. 2003; Galloway et al. 2008). We focus here on  $\text{N}_2\text{O}$ , a greenhouse gas (GHG) with global warming potential (GWP) 310 times higher than that of carbon dioxide ( $\text{CO}_2$ ) over a 100-year time period.

Several studies using calibrated  $\text{N}_2\text{O}$  emissions models (Maggi et al. 2008; Del Grosso et al. 2006; Grant et al. 2006; and Li, Narayanan, and Harriss 1996), field experiment data (Hoben et al. 2011; Chen, Huang, and Zou 2008; McSwiney and Robertson 2005; Chantigny et al. 1998; Izaurralde et al. 2004; and Yamulki et al. 1995), thorough literature reviews of peer-reviewed studies (Snyder et al. 2009; and Bouwman, Boumans, and Batjes 2002), or conceptual models of N input saturation on ecosystems (Townsend et al. 2003) have documented that low  $\text{N}_2\text{O}$  emissions occur when N is applied at or below the optimal crop requirement, but that higher emissions are consistent with N rates greater than that threshold. This literature suggests that crops compete with  $\text{N}_2\text{O}$ -producing microbes for the use of N in soil, limiting  $\text{N}_2\text{O}$  production

until crop N uptake has been satisfied. If the crop uses all available N in the soil, N<sub>2</sub>O emissions will be low. Emissions will increase rapidly once the crop's N demand is satisfied.

Consequently, a nonlinear relationship exists between N<sub>2</sub>O emissions and the N application rate. For illustration, Figure 12 is an expression of N<sub>2</sub>O emissions as a function of N rate based on more than 900 measures reviewed by Bouwman, Boumans, and Batjes (2002). They come from peer-reviewed studies comprising different types and rates of fertilizer, crops, soil types/qualities, and lengths of experiments. The literature is not conclusive as to whether emissions increase at an increasing or at a decreasing rate once the crop N requirement has been passed.

The response of yields to increasing nitrogen application has also been widely documented in the literature. Different functional forms have been employed to describe this relationship (quadratic, linear response and plateau, quadratic response and plateau, and Mitscherlich, among others). Berck and Helfand (1990) and Tembo et al. (2008) provide thorough overviews and discussions about the different production functions estimated in the literature. Most of these studies have also considered the stochastic nature of agricultural production due to uncertain weather, pests, and soil qualities.

The nonpoint source (NPS) nature of N<sub>2</sub>O emissions implies that the most effective way to address the environmental consequences is by altering the use of the input that ultimately causes the pollution (Hansen 1998; Shortle and Abler 1997; Xepapadeas 1997; and Segerson 1988).

The main contribution of this article is the establishment of a connection between two aspects of the literature that have been, thus far independently studied. The first one refers to the optimal fertilization decision when farmers face a market instrument targeting their N applications. The second is the quantification of N<sub>2</sub>O emission reductions coming from cutting N

fertilizer applications. In this article we calculate, for the first time, the magnitudes of N<sub>2</sub>O emission reductions that are induced by a market instrument imposed on N fertilizer applications, for different CO<sub>2</sub> prices. These prices reflect the social value assigned to air quality. The yield and nonlinear N<sub>2</sub>O response curves to N applications are estimated using field-level data. Considering the mentioned nonlinearity is crucial in at least two aspects throughout the paper: first, in designing a market instrument that transmits price signals to farmers that are consistent with the damage they cause to air quality and that prompt them to reduce N applications; and second, in driving the important result that modest fertilization reductions have big impacts in actual and expected N<sub>2</sub>O emission reductions with minor crop yield penalties. Failure to consider this nonlinearity induces an underestimation of true emission reductions and discourages the application of N<sub>2</sub>O reducing policies. The response curves are also used to calculate the distribution of emission reductions at the beginning of the planting season produced by this market instrument.

## 2. Farmer's Optimization Problem

Consider a farmer who maximizes expected utility of per hectare profits by choosing the optimal level of N fertilizer application rate (in kilograms per hectare). The farmer's problem is:

$$\max_{[N]} EU(\tilde{\pi}) = \max_{[N]} EU(\tilde{P}\tilde{y}(N; \theta) - P_N N) \quad (80)$$

where  $U(\cdot)$  is a strictly increasing and concave utility function, and  $\tilde{y}(N; \theta)$  is the yield response to N, a concave function of random weather during the growing season and of a set of farm-specific characteristics  $\theta$  (soil type, fertilizer type, tillage, and irrigation practices). Unknown output price at harvest time is denoted by  $\tilde{P}$  and the observed price of the fertilizer input is  $P_N$ . Expectations ( $E$ ) are taken over the two random variables, that is output prices and quantities.

Assuming, for now, that  $U(\cdot)$  is linear (i.e., risk neutrality) and that yield and output prices

are independent random variables,<sup>33</sup> and denoting expected values with a bar, the first-order condition (FOC) that solves the optimization problem is  $\bar{P} \frac{\partial \bar{y}(N; \theta)}{\partial N} = P_N$ . We denote its optimal solution as  $N_0$ . Panel (a) of Figure 13 shows the expected  $N_2O$  emissions associated with the optimal fertilizer application, and panel (b) shows the optimal  $N_0$  at the intersection between the decreasing expected marginal value product curve and the constant observed marginal cost  $P_N$  (point A). The expected marginal value product curve is consistent with the shape of the production function and we plot only its relevant portion, i.e. in the neighborhood of the intersection point.

Suppose that society assigns a value to the environmental damage caused by farmer's  $N_2O$  emissions. The damage value is a function of the N rate and is calculated as  $\phi(N; \theta) = 0.310 P_c e(N; \theta)$ , where  $e(N; \theta)$  is the expected quantity of  $N_2O$  emitted as a function of N and the set of farm-specific characteristics  $\theta$ ,  $P_c$  is the exogenous market price of  $CO_2$ , and 0.310 is the GWP equivalence between tons of  $CO_2$  and kilograms of  $N_2O$ . In this regard, suppose there exists a regulatory agency that aims to incentivize farmers to reduce N fertilizer applications by, for example, distributing offsets (credits) for the carbon equivalent value of his direct  $N_2O$  emission reductions.<sup>34</sup> Below we show that the exact same conclusions can be obtained with a tax.

The incentive payment structure accounts for the increasing and nonlinear relationship between  $N_2O$  emissions and N applications, for a given value of  $\theta$ . In panel (a) of Figure 13,

<sup>33</sup> In the next section we remove the linearity and independence assumptions and solve the expected utility problem under risk aversion and correlated random deviates of yield and prices.

<sup>34</sup> We restrict the social damage to the direct effect on air quality; however considering water pollution and other indirect human health effects caused by N applications in soils would increase the social price and imply higher payments per unit of N reduced. Further extensions of this work could include these aspects. For example, emerging water markets can be a reference to price the effects on water quality such as reduction in fish kill and the Gulf of Mexico's "dead zone", or other forms of eutrophication.

expected emissions (in kg N<sub>2</sub>O/ha/year) is plotted against the fertilizer rate (in kg N/ha). Based on this curve, we calculate the curve representing the market value of total damage, denoted by  $\phi(N; \theta)$ , as explained above.<sup>35</sup>

When farmers are paid for their emissions (or application) reductions with offsets, the optimization problem becomes:

$$\max_{[N]} EU(\tilde{\pi}) = \max_{[N]} EU(\tilde{P}\tilde{y}(N; \theta) - P_N N + \phi(N_0; \theta) - \phi(N; \theta)) \quad (81)$$

where  $[\phi(N_0; \theta) - \phi(N; \theta)]$  is the dollar payment received by the farmer for reducing nitrogen applications from  $N_0$  to  $N$ . Note that reductions are measured relative to  $N_0$ , usually called the business-as-usual (BAU) or baseline rate, which is what this farmer would have applied in the absence of the incentive payment. With the mentioned per hectare payoff structure, the participating farmer receives a payment equal to zero when application equals  $N_0$  (because these applications imply zero N<sub>2</sub>O emission reductions), and the payment increases nonlinearly as the farmer reduces the applications. With our assumptions of linear utility and uncorrelated yield and output prices, the FOC is  $\bar{P} \frac{\partial \tilde{y}(N; \theta)}{\partial N} = P_N + \phi'(N; \theta)$ . The farmer's maximization is achieved when expected marginal value product equals marginal cost plus the value of emissions from a marginal unit of fertilizer applied,  $\phi'(N; \theta)$ . The term  $\phi'(\cdot)$  increases the marginal cost of applying nitrogen (i.e., shifts the marginal damage curve up as shown in panel (b) of Figure 13) because it represents the marginal dollar amount that the farmer forgoes for each kilogram of N that is applied. The solution, denoted as  $N^*$ , is shown as point B, and the associated quantity of

---

<sup>35</sup> Alternatively, the problem can be set up with a risk-averse social planner which values (positively) farmers' utility and (negatively) the uncertain emissions curve, and that maximizes its utility by choosing the socially optimal parameters of the payoff structure. Then this optimal payment structure is faced by farmers in their expected utility maximization problem. The outcome of such set up would imply higher payments per unit of N reduced than in the risk-neutral case, because the regulator, due to its risk-aversion, assigns more weight to (uncertain) emissions that are higher than the average. This is consistent with a shift up of the expected damage curve  $e(N; \theta)$  relative to the risk-neutral case. The sensitivity analysis on the value of  $e(N; \theta)$  serves as illustration.

N<sub>2</sub>O emissions is shown in panel (a). Therefore, given the decreasing marginal value product, the new optimality implies a reduction in N rates and N<sub>2</sub>O emissions.

The marginal damage curve  $\phi'(N; \theta)$  faced by a farmer under the program can shift for two reasons: (i) a change in the price of carbon, and (ii) a change in the emission function  $e(N; \theta)$  due to changes in  $\theta$  for a given N application. In the first case, a higher price of carbon implies a higher opportunity cost of applying fertilizer, because the fertilizer application reductions are more valuable, so we should expect greater reductions in fertilizer applications (and emissions). In the second case, different farm management practices (tillage; fertilizer type, depth, and timing; manure application) induce different emissions for the same quantity of N fertilizer applied. So if the farmer changes to a management practice that emits more, the value of  $\phi'(N; \theta)$  will be higher, the opportunity cost of fertilizer applications will increase, and we should expect the farmer to apply less fertilizer. However, in this case, the expected marginal product might also change because the production function  $y(N; \theta)$  is shifted by the use of a different management practice, and therefore we cannot unambiguously say the direction of the fertilizer applications change. As a result we condition on the value of  $\theta$  throughout the rest of the analysis.

This offset payment will induce the same N application reductions as a tax imposed on the purchases of the N input, provided the following conditions are met. (i) The tax structure, which according to panel (a) of Figure 13 implies an increasing (progressive) tax rate, has a revenue curve (as a function of the N rate) that is equal to the total value damage curve  $\phi(N; \theta)$ . (ii) The tax rate has to adjust to the annual changes in the market price of carbon. In this case, the farmer's problem is  $\max_{[N]} EU(\tilde{\pi}) = \max_{[N]} EU(\tilde{P}\tilde{y}(N; \theta) - P_N N - \phi(N; \theta))$ . With the linear utility and uncorrelated yield and output prices assumptions, the FOCs are  $\bar{P} \frac{\partial \tilde{y}(N; \theta)}{\partial N} = P_N +$

$\phi'(N; \theta)$ , which are the same as in the offset program. Therefore, the solution (the optimal  $N^*$ ) and the nitrogen application reduction are the same, so the results are not driven by the mechanism chosen to induce the N fertilizer reductions. Clearly, however, the distributive or welfare effects of each policy are different.

### 3. Outline of the Model

The offset structure takes into account two important factors. First, the input decision is made under uncertainty coming from both the stochastic production function and output prices. Second, the market value of N<sub>2</sub>O emissions as a function of N fertilizer application rates  $\phi(N)$  and its first derivative  $\phi'(N)$  are nondecreasing and nonlinear.

Emission reductions are measured relative to a BAU rate that we calculate as follows. At the beginning of the planting season a farmer maximizes expected utility of per hectare profits<sup>36</sup> by choosing the optimal nitrogen application rate,  $N$ . He solves the following problem:<sup>37</sup>

$$\max EU(\tilde{\pi}_0) = \max_{[N]} \int_0^\infty \int_a^b U(\tilde{\pi}_0) h(P) f(y|N) dy dP \quad (82)$$

where  $\tilde{\pi}_0 = \tilde{P}\tilde{y} - P_N N$  is the farmer's random profit, randomness coming from uncertain output prices  $\tilde{P}$  and uncertain yields  $\tilde{y}$ . Yields behave according to a conditional density function  $f(y|N)$  whose support is the non-negative closed interval  $[a, b]$ ,  $a$  and  $b$  representing the minimum and maximum yield possible, respectively. Output prices are governed by a probability density function  $h(P)$  where  $P \in [0, \infty]$ . Expectations ( $E$ ) are taken with respect to both random variables, and  $U(\cdot)$  is a concave twice continuously differentiable utility function. The FOC are

$$\int_0^\infty \int_a^b \left[ U'(\pi_0)(-P_N) h(P) f(y|N) + U(\pi_0) h(P) \frac{\partial f(y|N)}{\partial N} \right] dy dP = 0. \text{ The solution is the farm-}$$

<sup>36</sup> In the previous section, for exposition, we assumed a farmer who maximizes under a linear utility. In what follows, we assume a utility that can accommodate different degrees risk aversion. However results are very similar for different risk aversion levels.

<sup>37</sup> To save notation we omit the conditioning parameter  $\theta$  in  $f(y|N; \theta)$ .



specific BAU rate which we denote by  $N_0$  and is a function of  $P_N$  and the set of parameters of the distributions  $f(y|N; \theta)$  and  $h(P)$ . We assume that the second derivative evaluated at  $N_0$  is negative.

When the farmer faces the offset payment, the expected utility problem becomes:

$$\max EU(\tilde{\pi}) = \max_{[N]} \int_0^\infty \int_a^b U(\tilde{\pi}_1) h(P) f(y|N) dy dP \quad (83)$$

where  $\tilde{\pi}_1 = \tilde{P}\tilde{y} - P_N N + [\phi(N_0) - \phi(N)]$ .<sup>38</sup> Therefore, he maximizes a standard expected utility problem but incorporating the mentioned payoff structure. The FOC are

$$\int_0^\infty \int_a^b \left[ U'(\pi_1)(-P_N - \phi'(N)) h(P) f(y|N) + U(\pi_1) h(P) \frac{\partial f(y|N)}{\partial N} \right] dy dP = 0$$

whose solution, denoted by  $N^*$ , is a function of  $N_0$ ,  $P_N$ , and the set of parameters of the function  $\phi(N; \theta)$  and the distributions  $f(y|N; \theta)$  and  $h(P)$ . We assume that the second derivative evaluated at  $N^*$  is negative. With  $N^*$ , we are able to analyze the consequences of introducing this nonlinear offset payment on the tradeoff between N rates and yields, and on the farmer's profitability.

The implementation of this program requires knowledge of certain field characteristics (such as soil type slope) and management practices (crop rotation, tillage, quantity and type of fertilizer) to determine the payment function, baseline, and optimal N application rates. Some can be observed by visual inspection but others, such as type and quantity of fertilizer applied, cannot. Moreover, farmers may have the incentive to misreport these values in order to claim more offsets than those consistent with the true N reductions. This poses an implementation challenge not new to NPS pollution studies in agriculture. For example, the literature on water quality as affected by nitrogen, phosphorous, and pesticide pollution has acknowledged the issue of finding a cost-effective mechanism to monitor, verify, and enforce programs aimed to reduce

---

<sup>38</sup> Similarly, to save notation we omit the conditioning parameter  $\theta$  in  $\phi(N; \theta)$ .

on-farm nutrient use (Metcalf et al. 2007; Huang et al. 2001; Chowdhury and Lacewell 1996; Huang and LeBlanc 1994; Thomas and Boisvert 1994; and Huang and Lantin 1993). While implementation of the program we describe is beyond the scope of this study, recent initiatives (Millar et al. 2010; MSU-EPRI 2010; Government of Alberta 2010; DEFRA 2008) propose and use available state-of-the-arts technology to monitor and verify N application rates and compliance to the program. They include a combination of (preseason and late season) soil nitrate tests, late-season stalk nitrate tests, chlorophyll meter readings, remote sensing of soil and crop canopy properties, soil electrical conductivity maps, and on-site crop test strips with different fertilizer rates. Complementary to these are the so-called Best Management Practices (BMP) in the use of fertilizers, such as the “Right Source-Rate-Time-Place (4R) Nutrient Stewardship” proposed by the International Plant Nutrition Institute (IPNI 2011) and the Nutrient BMP Endorsement for Crop Revenue Coverage Insurance (USDA-RMA 2003), which would reduce uncertainty about on-farm practices. In particular, a program that allows farmers to earn carbon credits for their quantifiable and verifiable N application reductions, has been introduced in Alberta, Canada. It uses, depending on the target reductions, ammonium-based fertilizers, slow/controlled release fertilizers, or inhibitors (right source); injected or band applications (right place); split applications in spring, and fall applications only if slow/controlled released fertilizers or inhibitors are used (right time); and applications based on field variability requirements and nitrogen balance (right rate) quantified by digitalized soil maps, landscape position, grid soil sampling, satellite imagery, in-season stalk nitrate tests, and overviewed by a program accredited professional advisor (Government of Alberta 2010, CFI 2011).

An alternative that may simplify the implementation of the nonlinear scheme would imply finding the farmers-specific socially optimal N application rate under the proper nonlinear

scheme and then impose a linear payoff that would induce the farmer to apply the socially optimal rate. The system set up with the linear payoff would be less complicated to implement and the final results would be the same.

#### 4. The Simulation Exercise

We assume that a farmer owns one hectare of land, plants it on a continuous corn rotation, and evaluates the decision to participate in the offset program to reduce N<sub>2</sub>O emissions. The farmer solves the expected utility model described above. There exists an environmental regulatory agency that oversees the offset program and distributes carbon credits for N<sub>2</sub>O emission reductions, reductions measured relative to the farm-specific BAU nitrogen rate  $N_0$ .

##### 4.1 N<sub>2</sub>O Emissions and the N Application Rate

Measures of N<sub>2</sub>O emissions as a function of N application rates were collected from corn field experiments conducted in the northern U.S. and Canada. They consist of more than 20 studies summarized by Rochette et al. (2008); Grant et al. (2006); Li, Narayanan, and Harriss (1996); Bouwman (1996); and Thornton and Valente (1996). A list is available upon request. We fit the following emissions curve to the data:

$$e(N) = \begin{cases} \sum_{i=0}^{i=3} a_i (N_m)^i & N \leq N_m \\ \sum_{i=0}^{i=3} a_i N^i & N > N_m \end{cases} \quad (84)$$

where  $a_i$  are estimated parameters (shown in Table 12) from a regression model applied to the mentioned data, and  $N_m$  is the nitrogen rate at which the estimated curve has slope equal to zero.<sup>39</sup> This is consistent with the nonlinear relationship between emissions and the N rate shown in Figure 12.

---

<sup>39</sup> For the estimated curve  $N_m$  equals 141 kg/ha.

Assuming there exists a market price for CO<sub>2</sub> ( $P_c$ ), that is, GHG emissions are negatively valued by society, this emissions curve is used by the regulatory agency to construct the offset payment structure that rewards N reductions by the market value of their environmental damage. This value is  $\phi(N) = 0.310 P_c e(N)$ , expressed in dollars per hectare. Therefore, the payment structure as a function of the optimal N (shown in Figure 14), which pays reductions relative to the BAU rate, is  $[\phi(N_0) - \phi(N_m)]$  for  $N \leq N_m$  and  $[\phi(N_0) - \phi(N)]$  for  $N > N_m$ . It implies that given  $N_0$ , per hectare payments increase from zero up to their maximum  $[\phi(N_0) - \phi(N_m)]$  as the farmer reduces the optimal N rate.<sup>40</sup> This nonlinear payment structure should give more efficient results because if the objective is to reward emissions reductions, a “flat” payoff to all application rates as suggested by IPCC-Tier 1<sup>41</sup> or a per unit nitrogen tax will not capture the implicit emissions behavior and thus will not provide correct signals to farmers. We compare both schemes in the results section.

It has to be noted that for a given N rate, different weather conditions will generate different levels of emissions. However, when determining the marginal payment structure, the regulator uses emissions at average weather conditions allowing the farmer to optimize under a known payment structure.<sup>42</sup> The optimization under an uncertain incentive scheme is treated by Segerson (1988). We present a sensitivity analysis of how results are driven by the estimation of this emissions curve. Also, we revise this assumption in the last section.

---

<sup>40</sup> Payment no longer increases for reductions beyond  $N_m$  because the incentive program has a superior objective of not harming crop yields excessively. If carbon prices turn high enough, it could be optimal for the farmer to apply an extremely low N fertilizer rate (high N reduction), affecting crop yields and possibly food and feed supply.

<sup>41</sup> The Intergovernmental Panel on Climate Change (IPCC) assumes that N<sub>2</sub>O emissions are a constant proportion of 1.25 +/- 1% of N applications (Bouwman 1996).

<sup>42</sup> The rationale of this assumption is that if we average a farmer's emission reductions over several years, they will be consistent with the incentive payment received in each year.

## 4.2 Estimation of a Conditional Yield Distribution

The yield response to nitrogen was estimated using 1987 to 1991 data from field-plot experiments on continuous corn conducted on four different farms spread throughout Iowa. Yields were updated to 2010 levels using a proportional yield adjustment based on Iowa corn yield growth.

One of the objectives of the experiment was to isolate the effect on yields of increasing nitrogen application rates, leaving everything else constant. This dataset was also used in past studies by Babcock and Hennessy (1996) and Roosen and Hennessy (2003). The experiment consisted of 10 nitrogen application rates (0, 25.06, 56.10, 84.14, 112.18, 140.23, 168.28, 224.37, 280.46, 336.55 kg N/ha)<sup>43</sup> with three replications on each of the four farms (sites) and in each of the five years. So there are 600 observations or 60 observations for each N application rate. Table 13 shows mean and standard deviation of corn yields by site and by year.

Following Babcock and Hennessy (1996), we assume that, conditional on a given N application rate, yields behave according to a beta distribution with shape parameters  $p$  and  $q$ . We further assume that yield randomness comes from the interaction of factors that are unobserved by the researchers (such as weather or pests). The beta distribution is specified because it describes the nonsymmetric historical behavior of yields with respect to these unobservables.

The moments of the yield distribution depend on the N application rate, and given that moments of the beta distribution are completely defined by the shape parameters, we specify them as a function of N rate, that is,  $p(N)$  and  $q(N)$ . Then the conditional beta distribution can be written as follows:

---

<sup>43</sup> These are, respectively, 0, 25, 50, 75, 100, 125, 150, 200, 250, and 300 pounds per acre.

$$f(y|N) = \frac{\Gamma[p(N) + q(N)]}{\Gamma[p(N)]\Gamma[q(N)]} \frac{(y - y_{min})^{p(N)-1} (y_{max} - y)^{q(N)-1}}{(y_{max} - y_{min})^{p(N)+q(N)-1}} \quad (85)$$

where  $\Gamma$  is the Gamma function,  $p(N) = p_0 + p_1 N^{0.5} + p_2 N$  and  $q(N) = q_0 + q_1 N^{0.5} + q_2 N$ .

Parameters  $p_0, p_1, p_2, q_0, q_1, q_2$  are estimated by maximum likelihood (shown in Table 14). By feeding (85) with one value of N rate, we obtain the distribution of yields conditional on that N.

Figure 15 shows how the first two moments of the estimated yield density change with the N rate. From equation (85) we draw beta deviates for any given nitrogen application rate using the inversion method.

### 4.3 Simulation of Correlated Yields and Price Draws

The optimization problem is to maximize expected utility of profits, where uncertainty comes from both random yields and random output prices. Random corn prices were generated assuming a lognormal distribution. This is a standard assumption given that the percentage change of commodity prices can be approximated by a normal distribution with certain mean and variance, and therefore the variable in levels (the commodity price) is lognormally distributed (Hull 2009, p. 271). So the price vector  $P \sim \log N(\mu, \sigma^2)$  is generated from the equation  $P_r = e^{\mu + \sqrt{\sigma^2} Z_{1r}}$ , where  $\mu = \log(E(P)) - \frac{\sigma^2}{2}$ , and  $\sigma^2 = \log(volat^2 + 1)$ .<sup>44</sup>  $P_r$  is the  $r^{\text{th}}$  commodity price deviate generated;  $Z_{1r}$  indicates the  $r^{\text{th}}$  deviate from the random variables  $Z_1$  distributed standard normal;  $E(P)$  is the mean of corn prices; and  $volat = \frac{\sqrt{V(P)}}{E(P)}$  is the volatility of corn prices interpreted as the percentage change of prices with respect to their mean. The mean of corn prices  $E(P)$  was set equal to \$151.69 per ton, which is the average of the Chicago Mercantile Exchange (CME) quotation on April 1 and April 15 of the December futures price for 2010. Price volatility was calibrated at 0.29 and calculated using the implied volatility from

---

<sup>44</sup> Proof is available upon request.

Blacks with an “at the money” call option on corn futures on the same days.

We remove the independence assumption between corn prices and yields of the previous section by following Johnson and Tenenbein (1981) to generate correlated draws from these two distributions. Given a target level of correlation  $\rho$ , the method consists of generating draws from two standard normal random variables  $Z_2$  and  $Z_3$  and creating another random variable  $Z_1$  as a linear combination of the previous two. The linear combination is what creates correlation between  $Z_1$  and the other variables. The linear combination weight is optimally selected so that the target correlation is achieved. By plugging  $Z_1$  into the random price generator formula and by substituting  $Z_2$  by a vector of randomly generated corn yields, we obtain correlated corn and yield draws.<sup>45</sup>

#### 4.4 Maximization of Expected Utility of Profits

First, we solve the case where emission are not valued by society, i.e. the BAU case with solution denoted as  $N_0$ ). To this end, we generate  $R=1000$  random draws of correlated yields and corn prices and use a line-search algorithm to find a value of  $N$  that maximizes the expression:

$$EU(\tilde{\pi}) = \frac{1}{R} \sum_{r=1}^R U(P_r y_r - P_N N) \quad (86)$$

where  $y_r$  and  $P_r$  are the  $r^{\text{th}}$  draw of yield and corn prices, respectively;  $P_N$  is a known price of nitrogen; and  $U(\cdot)$  is assumed to be a constant absolute risk aversion (CARA) utility function of the form  $U(\tilde{\pi}) = -e^{-ra\tilde{\pi}}$ , where  $ra = -\frac{U''}{U'}$  is the coefficient of absolute risk aversion. The risk aversion coefficient was set as a value consistent with a risk premium equal to 0%, 25%, and

---

<sup>45</sup> We selected two levels of correlation, one negative and one positive. Negative correlation would exist because when corn prices increase, farmers have the incentive to plant more corn, substituting land away from other uses. If that new corn land is of lower productivity, we can expect a yield decrease. Positive correlation might occur if higher prices induce changes in management practices with the objective of obtaining higher yields (using high-yielding seeds or different types of fertilizers or herbicides).

50% of the standard deviation of profits.<sup>46</sup> Other studies (Babcock, Choi, and Feinerman 1993; Babcock and Hennessy 1996; Hennessy, Babcock, and Hayes 1997) have set the risk aversion coefficient such that the risk premium is equal to a certain percentage of revenue. As this percentage increases, the individual is willing to pay more money to avoid the risk, implying a more risk-averse agent.

We then solve the problem when emissions are negatively valued. The farmer takes as given the level of  $N_0$ ,  $P_N$ , and the payoff structure  $\phi(\cdot)$ , and maximizes the expected utility of profits conditional on  $R$  correlated draws of yields and corn prices. Then, the expression to be maximized by the farmer is:

$$EU(\tilde{\pi}) = \begin{cases} \frac{1}{R} \sum_{r=1}^R U[P_r y_r - P_N N + \phi(N_0) - \phi(N_m)] & \text{if } N \leq N_m \\ \frac{1}{R} \sum_{r=1}^R U[P_r y_r - P_N N + \phi(N_0) - \phi(N)] & \text{if } N > N_m \end{cases} \quad (87)$$

We again use a line-search algorithm to find the maximum and denote the solution as  $N^*$ . With  $N_0$  and  $N^*$ , we can find the nitrogen application reduction, and also the payment the farmer receives from the program.

## 5. Simulation Results for Nitrogen Application Rate

We present in Table 15 results of the expected utility optimal application rate induced by participating in the offset program ( $N^*$ ), the BAU nitrogen application rate ( $N_0$ ), the reduction of N applied, the yield loss for applying less N, the incentive payment received by the farmer, and the change in the farmer's profits due to participation. We use carbon prices of \$15, \$30, and \$45 per ton, and various risk-aversion coefficients and price-yield correlations.<sup>47</sup>

<sup>46</sup> The risk premium (RP) is the dollar amount an individual is willing to pay to avoid a risky bet and receive a certain profit. For our utility function, the risk premium is found to be  $RP = E(\tilde{\pi}) + \frac{1}{ra} \log[E(e^{-ra\tilde{\pi}})]$ .

<sup>47</sup> Throughout the estimation we assumed a nitrogen price of \$726/ton, equivalent to \$0.33/lb suggested by Iowa



With a carbon price of \$30/ton, a participating farmer whose absolute risk-aversion coefficient is consistent with a risk premium equal to 25% of the standard deviation of profits optimally reduces his nitrogen applications by 16.00 kg/ha for participating in the program and obtains an incentive payment of \$7.91 per hectare. The increase in profits is \$4.54 per hectare because lower variable costs are offset by a yield penalty.<sup>48</sup> Therefore the offset program induces N rate reductions of 7% but a yield penalty of less than 1%.

Results are driven by the estimated parameters of the emissions curve  $e(N)$ . A 90% confidence interval for this curve illustrates how results would change. With  $P_c$  at \$30, the risk premium at 25%, and using the emissions curve at the lower extreme of the interval, a farmer reduces optimal N applications by 11 kg/ha and receives a payment of \$3.75; while at the upper extreme, the N reduction is 19 kg/ha and the offset payment is \$12.18.

Farmer's risk preferences have little influence on final results. For levels ranging from risk neutrality to high levels of risk aversion, farmer's optimal N application reductions are very similar, as shown in Table 15. In comparative statics results, that for reasons of space are not presented here, we show that, first, the more risk-averse farmer optimally applies a lower nitrogen rate than the less risk-averse farmer. It can be shown that at this level, N is a risk-increasing input. Second, the more risk-averse farmer optimally makes a higher reduction in applications when participating in the offset program. This comes from the fact that, abstracting from the incentive payment, profits of the more risk-averse farmer are lower because of the yield penalty; therefore, when faced with a certain offset payment, he will reduce the N applications

---

State University Extension for continuous corn (Duffy 2009).

<sup>48</sup> We also solved the model with linear utility which is equivalent to a risk premium equal to zero. Results, shown in Table 15, are very similar: for carbon prices of \$30, optimal N application reductions are 15.19 kg N/ha (237-222) with a payment of \$7.83, so the degree of risk aversion does not affect the main conclusions. Figure 16 shows the estimation of what is discussed in Figure 13: N application reductions of 15.19 kg N/ha (237-222) at carbon prices of \$30, and of 20.4 kg N/ha (237-217) at \$45.

by a greater amount because this certain payment represents a higher proportion of the uncertain profits.<sup>49</sup>

## 6. Estimation of a Distribution of Emission Reductions

We need to know, before the planting season starts, the distribution of emission reductions that will be induced by the program. Actual end-of-season N<sub>2</sub>O emission reductions depend on random weather (rainfall and temperature in our particular case). To this end, we simulate the weather effects on the N<sub>2</sub>O emissions induced by the optimal N application reduction ( $N_0 - N^*$ ) of the participant farmer.<sup>50</sup>

### 6.1 Distribution of Rainfall and Temperature

To simulate random weather, we fit nonparametric density functions to Iowa rainfall and temperature time series (1895-2008) from the National Climate Center at the National Oceanic and Atmospheric Administration using an Epanechnikov kernel (DiNardo and Tobias 2001).<sup>51</sup> Rainfall is the total annual precipitation for the state and is measured in centimeters per year (cm/yr).<sup>52</sup> Temperature is the annual average temperature for the state measured in degrees Celsius. Both rainfall and temperature densities are bell-shaped. They respectively average 89 cm/yr (range 56 to 122) and 8.8 degrees Celsius (7 to 11). Random draws were generated from both densities. Because the correlation between the two series is virtually zero, we draw independently from both distributions.

### 6.2 Weather Effects on N<sub>2</sub>O Emissions

Based on our data collection on applications of N fertilizer and N<sub>2</sub>O emissions, we estimate a

---

<sup>49</sup> We solved the model with a positive correlation ( $\rho=30$ ), and results were very similar.

<sup>50</sup> For this simulation we select the scenario of  $P_c = 30$ ,  $RP = 0.25$  and  $\rho = -0.30$ .

<sup>51</sup> Bandwidth =  $h = 0.9 \left( \min \left\{ \hat{\sigma}, \frac{IQR}{1.34} \right\} \right) n^{-\frac{1}{5}}$ ;  $\hat{\sigma}$  is the sample standard deviation; IQR is the interquartile range (difference between the 75<sup>th</sup> and 25<sup>th</sup> percentile), and  $n$  is the number of observations.

<sup>52</sup> One inch of rain equals 2.5 cm.

response curve using the following regression model:  $\epsilon(N) = \sum_{i=0}^3 a_i N^i$ .<sup>53</sup> Given that data from these studies covered different years, we assume that the fitted curve represents the behavior of emissions for average weather conditions. From this curve we calculate the levels of emissions at  $N^*$  and  $N_0$ . The effects of precipitation and temperature on  $N_2O$  emissions are obtained by running the Denitrification-Decomposition (DNDC) model calibrated for a continuous corn rotation in Iowa (Li, Narayanan, and Harriss 1996) with different N application rates<sup>54</sup>. Table 16 and the upper panels of Figure 17 (blue circles) show the response of  $N_2O$  emissions to changes in precipitation (temperature), holding temperature (precipitation) fixed at its average,  $N^* = 220$  kg N/ha, and holding all other variables at baseline levels.<sup>55</sup>

The levels of emissions for each draw of the weather variables are obtained by a cubic spline interpolation<sup>56</sup> of the results of the DNDC runs. The continuous lines in the upper panels of Figure 17 show both interpolated curves at  $N^*$  and  $N_0$ . With these functions we obtain the level of emission reduction induced by the optimal N application reduction ( $N_0 - N^*$ ) for each draw of the weather variables.

## 7. Simulation Results for the Expected Reduction in Emissions

The results are presented as histograms in Figure 18, for both precipitation and rainfall, in kilograms of carbon dioxide equivalent (kg CO<sub>2</sub>e). Panel (a) shows the distribution of the per

---

<sup>53</sup> Not restricting the response curve for values of N less than  $N_m$  does not affect the results because the portion of interest of the curve is to the right of  $N_m$ .

<sup>54</sup> DNDC is a computer simulation model of carbon and nitrogen biochemistry in agro-ecosystems. It takes soil, climate, crop, and management practices (including N application rates) as inputs, and return crop production, nutrient losses as leaching and runoff, and gas fluxes (including  $N_2O$ ) as outputs for a given parcel and year.

<sup>55</sup> Baseline scenario values are N concentration in rainfall, 1.6 mg N/liter; soil texture loam, clay 19%; pH, 6.0; bulk density, 1.4 g/c.c.; soil organic carbon, 0.025 kg C/kg; fertilizers, 37.5 kg nitrate-N, 37.5 kg ammonium-N, 75.0 kg anhydrous ammonia-N/ha applied on April 25 at surface; soil tilled with disks on April 15 and with moldboard on October 15; neither manure nor irrigation applied.

<sup>56</sup> This method fits a piecewise cubic function between each of the given points (knots).

hectare emission reductions for random precipitation holding temperature at the average.

Average emission reductions are 323 kg of CO<sub>2</sub>e, ranging between 227 and 490. The particular shape of this distribution is associated with the behavior of emissions as precipitation changes, as is shown in the upper panel of Figure 17. The dollar value of the average emission reduction is \$9.70 per hectare, which is comparable to the \$7.91 received by the farmer. This suggests that the nonlinear offset program provides the correct price signals to farmers.<sup>57</sup>

The distribution of N<sub>2</sub>O emission reductions as affected by random temperature shown in panel (b) of Figure 18 also has a shape determined by how emissions are affected by temperature. It has an average value of 257 kg of CO<sub>2</sub>e per hectare per year, ranging between 232 and 322. Its dollar value is \$7.70.

These high levels of N<sub>2</sub>O emission reductions are driven by the nonlinear payoff scheme. If we were to consider a linear payoff structure such as that proposed by the IPCC Tier-1 (where the N<sub>2</sub>O response to N is approximated by a linear curve with a slope of 0.0125), a participating farmer would reduce N applications by 5 kg N/ha (from 236 to 231), receiving a payment of less than \$1. Or in order to make a comparable emission reduction of 323 kg of CO<sub>2</sub>e (that in the nonlinear scheme is achieved by an N application reduction of 16 kg/ha, and a yield penalty of 0.77%), under this linear scheme, farmers would have to inadvertently reduce applications by 84 kg/ha, inducing a yield penalty of 888 kg/ha (or 7%).

To show how much these emission reductions represent, consider first that an approximation of the continuous corn area in Iowa in 2010 was about 1.6 million hectares. Then, using the range obtained for random rainfall and assuming that all Iowa continuous corn farmers participate in the offset program, we get a reduction between 349,000 and 754,000 tons of CO<sub>2</sub>e,

---

<sup>57</sup> The difference arises because, at the optimum, the slope of the emissions curve used by the regulatory agency, 0.046, is slightly lower than that obtained with the interpolation, 0.065.

with an average of 497,000. The EPA Inventory of GHG for 2009 (U.S. EPA 2011) calculates  $\text{N}_2\text{O}$  emissions from the application of synthetic fertilizers on U.S. cropland and grassland at 40.8 million tons of  $\text{CO}_2\text{e}$ . Therefore, Iowa reductions based only on 2010 continuous corn would have been 1.2% of the total emissions from the application of synthetic fertilizer on U.S. cropland and grassland. Given that continuous corn rotation represents about 30% of total Iowa corn in 2010,  $\text{N}_2\text{O}$  reductions from other rotations such as corn-soybeans might also be significant. Also, we restrict the analysis to direct  $\text{N}_2\text{O}$  emissions from N applications but the damage value from water pollution and other indirect health effect could also be substantial.

## 8. Conclusions

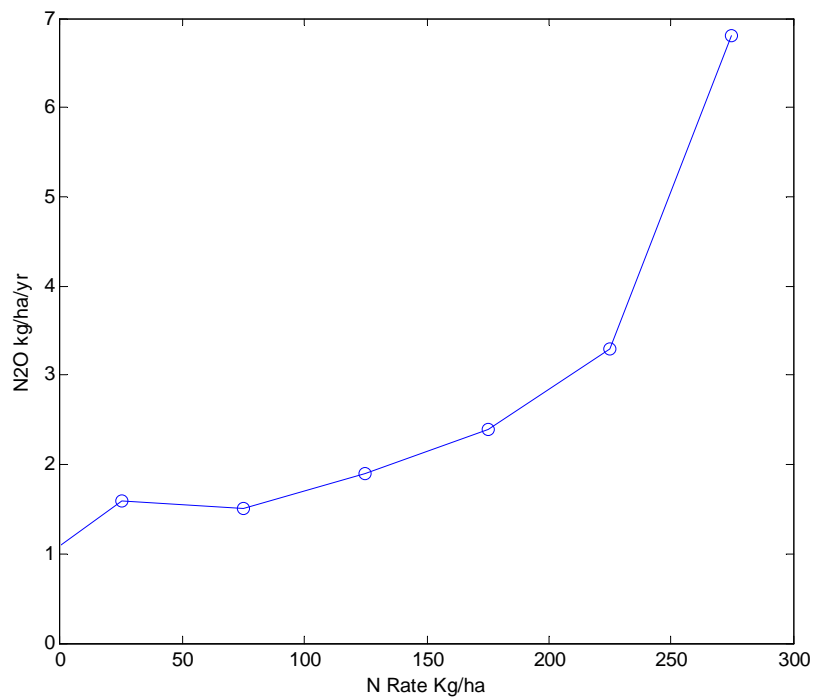
The overapplication of nitrogen by corn growers, while optimal from an ex ante perspective, has negative environmental consequences. In this article, we document the  $\text{N}_2\text{O}$  emission reductions that are consistent with a market instrument imposed on nitrogen fertilizer applications in order to induce a lower use of nutrients. The instrument targets the nitrogen applications because of the NPS nature of the emissions. We consider a farmer maximizing expected utility of per hectare profits and choosing the optimal nitrogen application rate. Reductions are measured relative to the farm-specific BAU nitrogen rate. We use a nonlinear payoff structure that is consistent with the nonlinear relationship between  $\text{N}_2\text{O}$  emissions and nitrogen application rates. This instrument is far more efficient than traditional linear schemes because it transmits price signals that are aligned with the true  $\text{N}_2\text{O}$  behavior and the ultimate objective of the program.

The key insight in the article is driven by the simulation results. These show that with a very modest carbon price of \$30, a farmer reduces his nitrogen applications by about 7% as a result of an offset payment of \$7.91 per hectare. The lower nitrogen induces only a minimal expected yield penalty (about 1%) because the program targets nitrogen applications that in most years are

surplus relative to crop needs. Therefore, taking into account the mentioned nonlinearity, we find that the true impact on N<sub>2</sub>O emission reductions is significant but yields are only slightly harmed. A linear scheme aiming to achieve the same N<sub>2</sub>O emission reductions would inadvertently require an N application reduction of 35% with an associated yield penalty of 7%. Therefore, failure to consider this nonlinearity may render an N<sub>2</sub>O emission reduction policy unattractive.

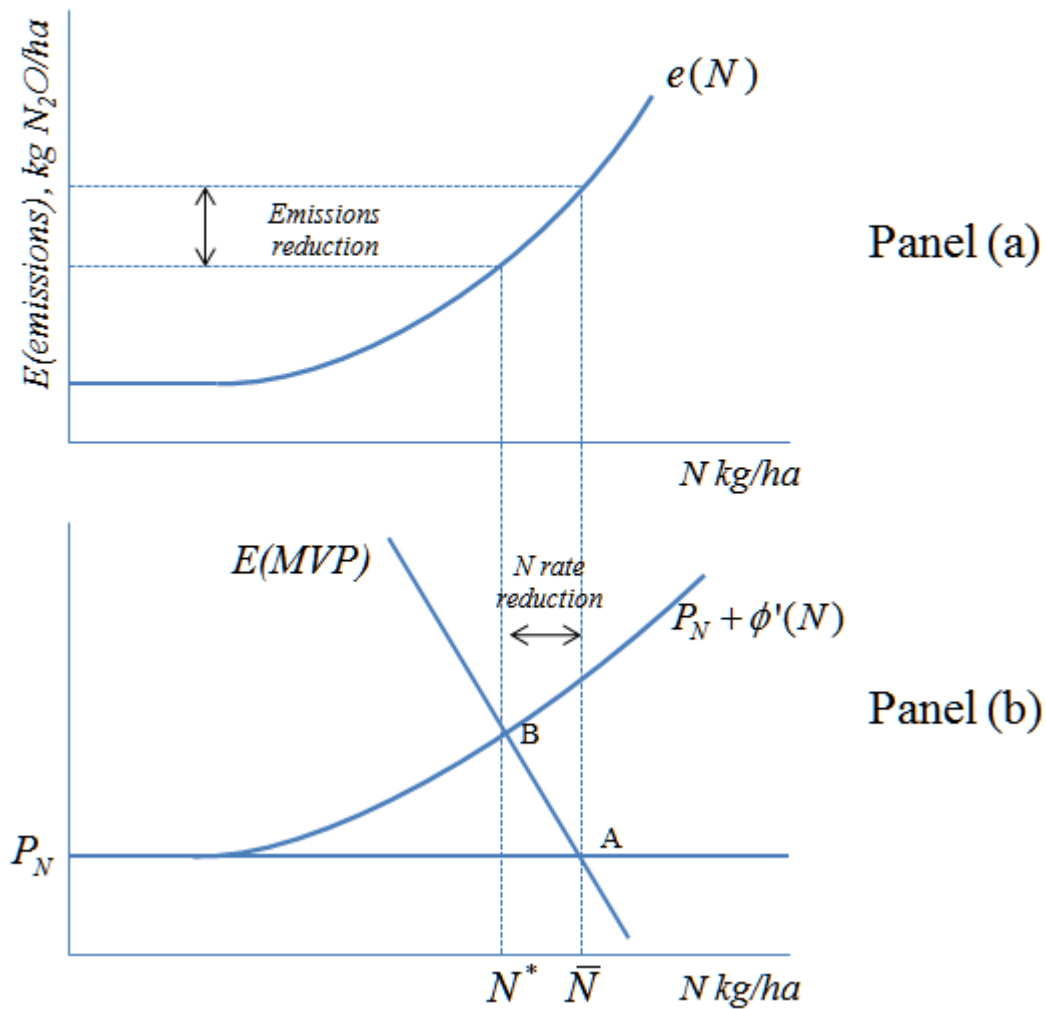
Results are robust to different levels of risk aversion. We find that a more risk-averse farmer applies less nitrogen because, for these particular application rates, nitrogen fertilizer is a risk-increasing input. We also find that the more risk-averse farmer makes a higher application reduction because the certain offset payment represents a higher proportion on the uncertain profits that are in turn decreased by the relatively lower use of nitrogen.

We also present the distribution of emission reductions induced by this market instrument that takes into account a priori unknown weather variables. We find that, for random rainfall and fixed temperature, the distribution of emission reductions averages 323 kg CO<sub>2</sub>e per hectare, with a shape depending on how emissions respond to rainfall. For random temperature and fixed rainfall, the average reduction is 257 kg CO<sub>2</sub>e per hectare.

**Figures**

**Figure 12. Average N<sub>2</sub>O emissions as a function of N rates**

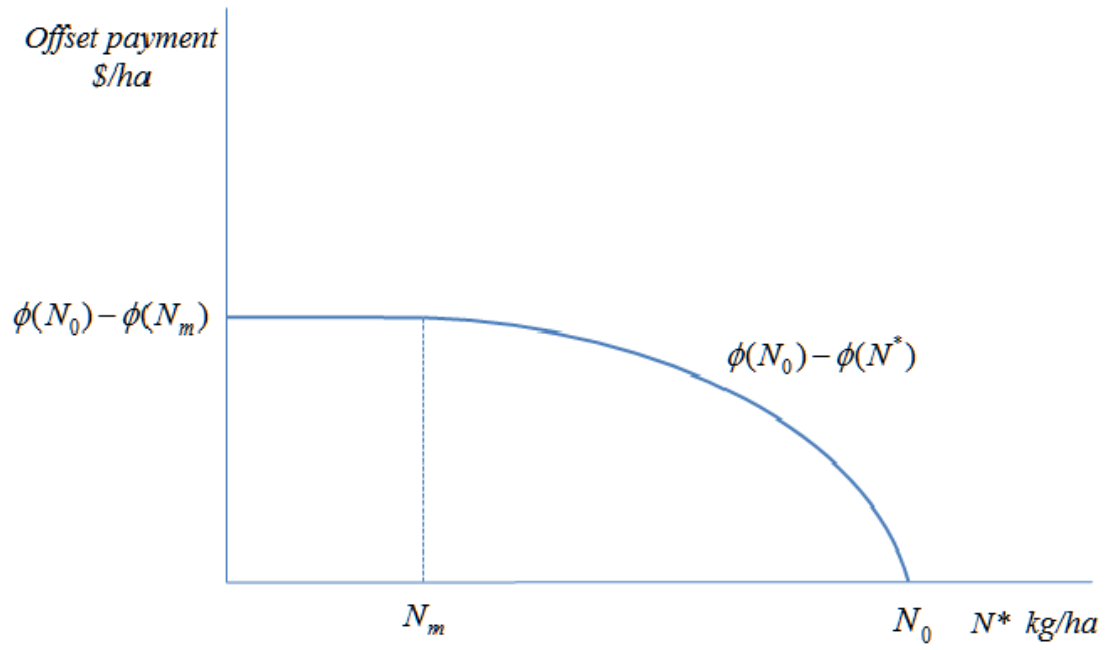
Source: Based on table 5, Bouwman, Boumans, and Batjes (2002)



**Figure 13. Optimal N application and N<sub>2</sub>O emissions reductions.**

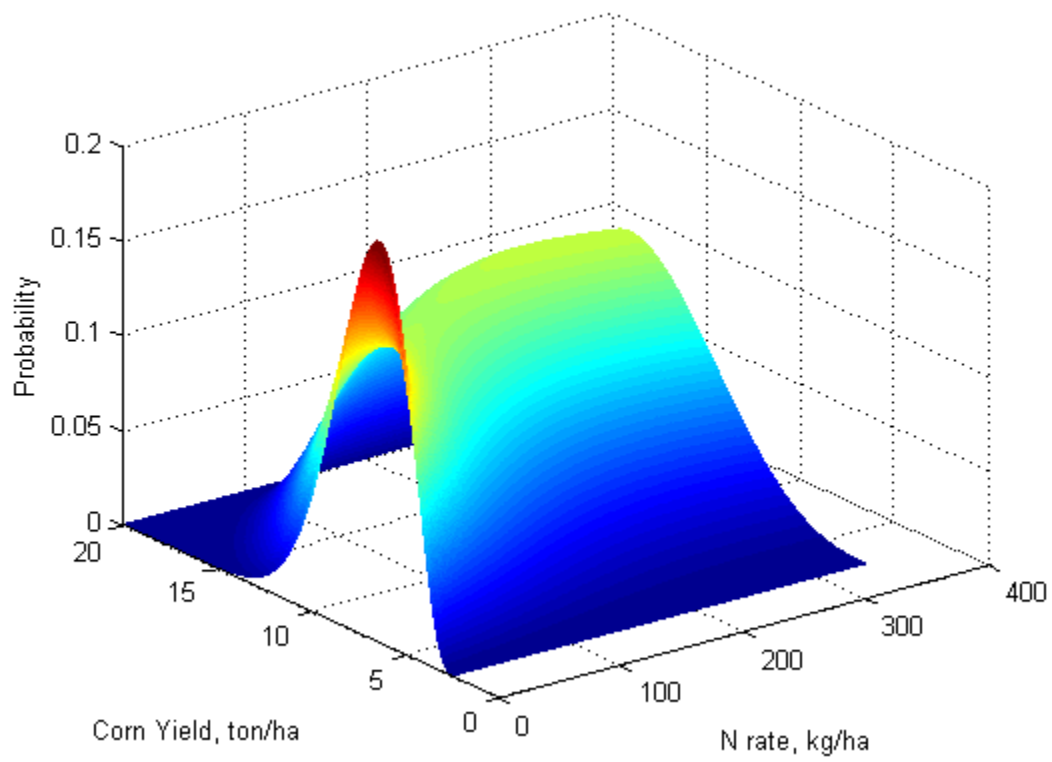
Note: Panel (a): N<sub>2</sub>O emissions as a nonlinear function of nitrogen rates, and N<sub>2</sub>O emission reductions from the incentive program. Panel (b): Profit maximizing nitrogen application rates in the business-as-usual case (point A) and the case of N<sub>2</sub>O emissions negatively valued by society (point B), and optimal nitrogen application reductions



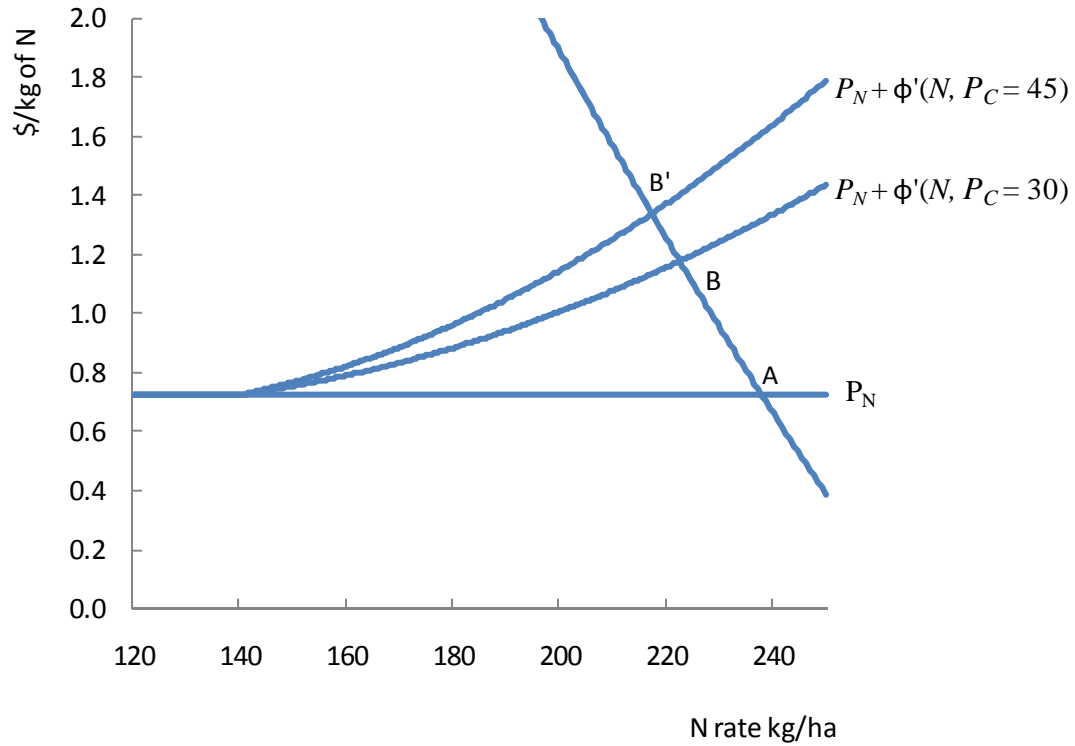


**Figure 14. Offset payment structure as a function of the optimal nitrogen application rate  $N^*$**

Note: The set of farm-specific characteristics  $\theta$  in the function  $\phi(N; \theta)$  is omitted to save notation.

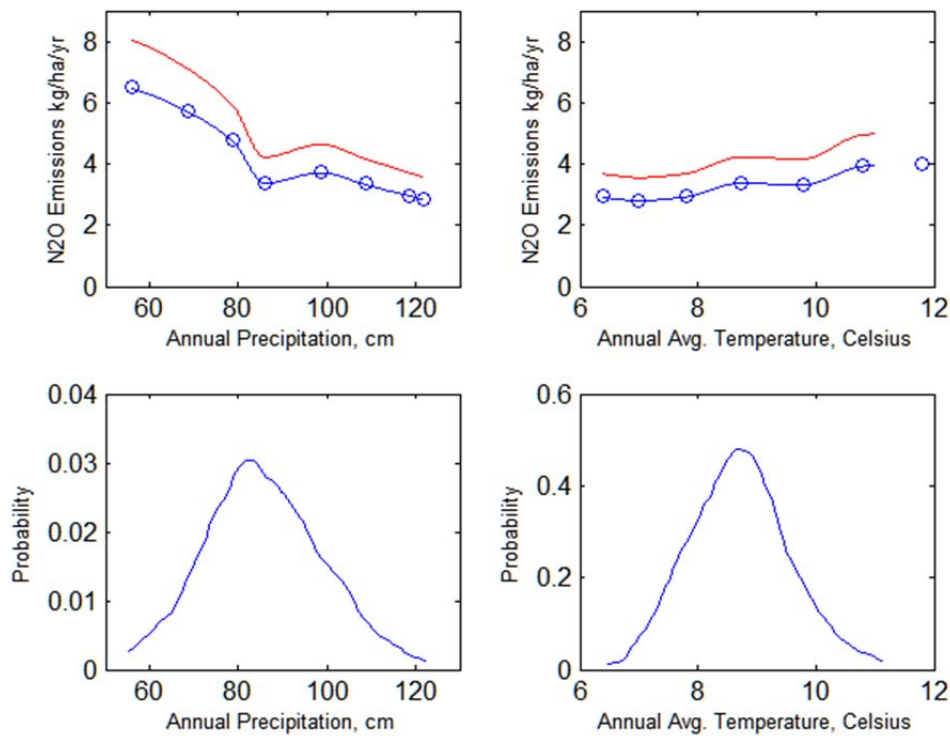


**Figure 15. Parametric estimation of a conditional beta probability density function of Iowa corn yields for different N rates**



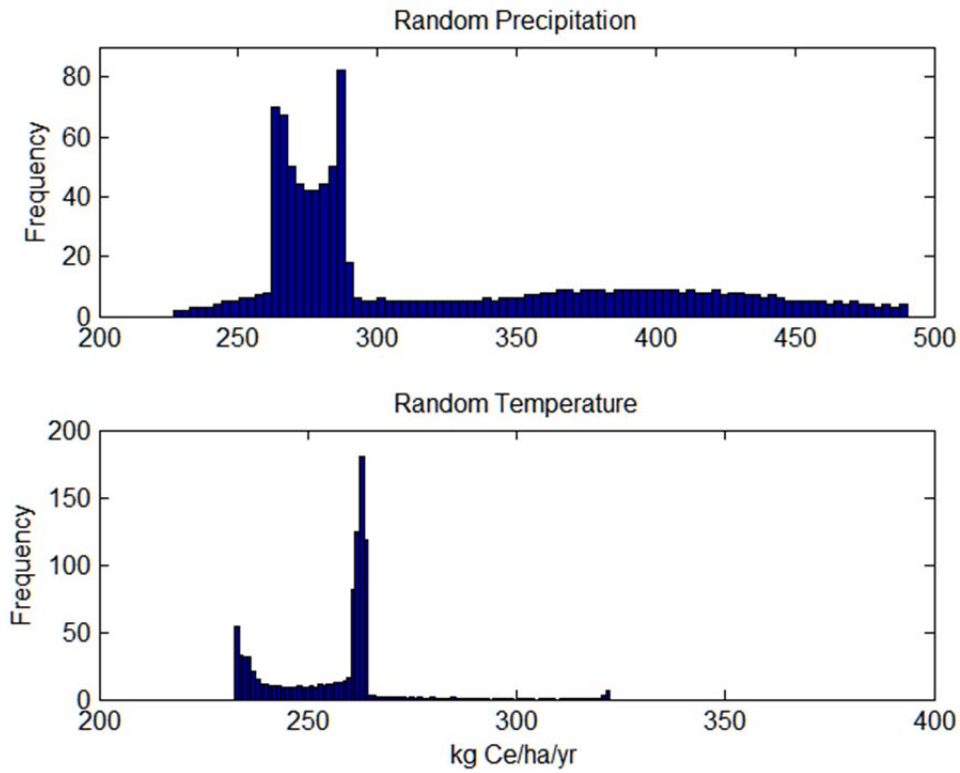
**Figure 16. Expected marginal value product (EMVP) curve and marginal cost curves.**

Note: Scenarios: carbon price  $P_C$  is zero, \$30, and \$45. The intersections show the optimal solution of the linear utility maximization problem for the different  $P_C$  scenarios: A = (237, 0.727), B = (222, 1.051) and B' = (217, 1.159).



**Figure 17. Random weather and response of N<sub>2</sub>O emissions**

Note: Upper panels show the variation of emissions as a function of average precipitation and average temperature at  $N^* = 220$  kg N/ha (blue) and  $N_0 = 236$  kg N/ha (red). Circles show available data and the blue line the interpolated values. Lower panels represent nonparametric probability density function of precipitation and temperature using an Epanechnikov kernel.



**Figure 18. Histograms of N<sub>2</sub>O emission reductions (kg of carbon equivalent per hectare) for random weather.**

Note: Precipitation (panel a) and random temperature (panel b) induced by an optimal N application reduction of  $(N_0 - N^*) = 16.00$  kg N/ha, when  $P_c = \$30/\text{ton CO}_2$

## Tables

**Table 12. Estimation Results of Emissions Curve:  $e(N)$**

	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$
Coefficient	1.09801	0.03640	-3.9874E-04	1.2758E-06
Standard error	0.33864	0.01713	1.8446E-04	5.1955E-07
t-stat	3.24243	2.12474	-2.16164	2.45554

**Table 13. Yields (tons per hectare) from Continuous Corn Field Experiments in Iowa**

Yields by site					Yields by year				
	1	2	3	4	1987	1988	1989	1990	1991
Mean	11.57	11.71	12.46	11.11	13.28	7.41	12.69	12.89	12.30
	(3.28)	(4.21)	(4.28)	(3.12)	(3.00)	(2.60)	(3.66)	(3.31)	(2.84)

Note: Standard deviation in parentheses.

**Table 14. Maximum Likelihood Estimation of Beta Parameters**


---

Functional forms:  $p(N) = p_0 + p_1N^{0.5} + p_2N$ ;  $q(N) = q_0 + q_1N^{0.5} + q_2N$

---

$\hat{p}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{q}_0$	$\hat{q}_1$	$\hat{q}_2$
4.160	-0.114	0.005	12.832	-1.377	0.043
(0.515)	(0.094)	(0.005)	(1.416)	(0.205)	(0.008)

---



**Table 15. Results of the N<sub>2</sub>O Emissions Reductions Incentive Program (per hectare)**

<b>Carbon Price, <math>P_c = \\$15/\text{ton CO}_2</math></b>						
<b>RP (%)</b>	<b><math>N^*</math> (kg)</b>	<b><math>N_0</math>(kg)</b>	<b>N reduct.</b>	<b>Yield loss(%)</b>	<b>\$ payoff</b>	<b><math>\pi</math> increase</b>
~0	229	237	8.67	0.35	2.36	1.27
25	226	236	9.20	0.40	2.41	1.31
50	223	233	9.80	0.46	2.45	1.34
<b>Carbon Price, <math>P_c = \\$30/\text{ton CO}_2</math></b>						
<b>RP (%)</b>	<b><math>N^*</math> (kg)</b>	<b><math>N_0</math> (kg)</b>	<b>N reduct.</b>	<b>Yield loss(%)</b>	<b>\$ payoff</b>	<b><math>\pi</math> increase</b>
~0	222	237	15.19	0.69	7.83	4.46
25	220	236	16.00	0.77	7.91	4.54
50	216	233	16.89	0.88	7.95	4.60
<b>Carbon Price, <math>P_c = \\$45/\text{ton CO}_2</math></b>						
<b>RP (%)</b>	<b><math>N^*</math> (kg)</b>	<b><math>N_0</math> (kg)</b>	<b>N reduct.</b>	<b>Yield loss(%)</b>	<b>\$ payoff</b>	<b><math>\pi</math> increase</b>
~0	217	237	20.40	1.00	15.08	8.95
25	214	236	21.37	1.11	15.12	9.07
50	211	233	22.42	1.25	15.05	9.13

Note: Risk premium (RP) is the % of the standard deviation of profits. The corn price is \$151.70/ton, and the N price is \$726.87/ton. Yield and corn price correlation  $\rho = -0.30$ .

**Table 16. Sensitivity of N<sub>2</sub>O Emissions to Changes in Precipitation and Temperature**

<b>Annual Precipitation</b>		<b>Annual Average Temperature</b>	
<b>Precipitation</b>	<b>N<sub>2</sub>O-N</b>	<b>Temperature</b>	<b>N<sub>2</sub>O-N</b>
<b>cm/year</b>	<b>kg/ha/yr</b>	<b>°C/year</b>	<b>kg/ha/yr</b>
56.0	7.62	-	-
68.7	6.73	7.0	2.81
78.7	5.61	7.8	2.93
86.0	3.98	8.7	3.38
98.7	4.38	9.8	3.33
108.7	3.93	10.8	3.94
118.7	3.48	11.8	3.99

## 9. References

- Alberta, Canada, Government of. 2010. Quantification Protocol for Agricultural Nitrous Oxide Emissions Reductions. Version 1.0.
- Babcock, B. A. 1992. The Effects of Uncertainty on Optimal Nitrogen Applications. *Review of Agricultural Economics* 14(2): 271-280.
- Babcock, B. A.; E. K. Choi, and E. Feinerman. 1993. Risk and Probability Premiums for CARA Utility Function. *Journal of Agricultural and Resource Economics* 18(1): 17-24.
- Babcock, B. A., and D. A. Hennessy. 1996. Input Demand Under Yield and Revenue Insurance. *American Journal of Agricultural Economics* 78(2): 416-427.
- Berck, P., and G. Helfand. 1990. Reconciling the von Liebig and Differentiable Crop Production Functions. *American Journal of Agricultural Economics* 72(4): 985-996.
- Bouwman, A. F. 1996. Direct Emissions of Nitrous Oxide from Agricultural Soils. *Nutrient Cycling in Agroecosystems* 46: 53-70.
- Bouwman, A. F., L. J. M. Boumans, and N. H. Batjes. 2002. Emissions of N<sub>2</sub>O and NO from Fertilized Fields: Summary of Available Measurement Data. *Global Biogeochemical Cycles* 16(4): 1058.
- Canadian Fertilizer Institute (CFI). 2011. "The Nitrous Oxide Emission Reduction Protocol (NERP)" Available at: <http://www.cfi.ca/nitrousoxideemissionreductionprotocol/> (Accessed on March 2012)
- Chantigny, M. H., D. Prevost, D. A. Angers, R. R. Simard, and F.-P. Chalifour. 1998. Nitrous Oxide Production in Soils Cropped to Corn with Varying N Fertilization. *Canadian Journal of Soil Science* 78: 589-596.
- Chen, S., Y. Huang, and J. Zou. 2008. Relationship between Nitrous Oxide Emissions and Winter Wheat Production. *Biology and Fertility of Soils* 44: 985-989.
- Chowdhury, M. E. and R. D. Lacewell. 1996. Implications of Alternative Policies on Nitrate Contamination of Groundwater. *Journal of Agricultural and Resource Economics* 21(1): 82-95
- Department for Environment, Food and Rural Affairs, UK (DEFRA). 2008. The Nitrate Pollution Prevention Regulations. Available at: <http://www.legislation.gov.uk/ukxi/2008/2349/contents/made> (accessed on March 2012)
- Del Grosso, S. J. Parton, A. R. Mosier, M. K. Walsh, D. S. Ojima, and P. E. Thornton. 2006. DAYCENT National-Scale Simulations of Nitrous Oxide Emissions from Cropped Soils in the United States. *Journal of Environmental Quality* 35: 1451-1460.
- DiNardo, J. and J. Tobias. 2001. Nonparametric Density and Regression Estimation. *Journal of*

*Economic Perspectives* 15(4): 11-28.

The Denitrification-Decomposition Model (DNDC). Institute for the Study of Earth, Oceans, and Space. Complex Systems Research Center. University of New Hampshire. Available at: <http://www.dndc.sr.unh.edu/> (accessed on March 2012).

Duffy, M. 2009. Estimated Costs of Crop Production in Iowa—2010. *Ag Decision Maker*, Iowa State University Extension. December.

Galloway, J. N., A. R. Townsend, A. W. Erismann, M. Bekunda, Z. Cai, J. R. Freney, L. A. Martinelli, S. P. Seitzinger, and M. A. Sutton. 2008. Transformation of the Nitrogen Cycle: Recent Trends, Questions, and Potential Solutions. *Science* 320(5878): 889-892.

Grant, R. F., E. Pattey, T. W. Goddard, L. M. Kryzanowski, and H. Puurveen. 2006. Modeling the Effect of Fertilizer Application Rate on Nitrous Oxide Emissions. *Soil Science Society of America Journal* 70: 235-248.

Hoben, J. P., R. J. Gehl, N. Millar, P. R. Grace, and G. P. Robertson. 2011. Nonlinear Nitrous Oxide (N<sub>2</sub>O) Response to Nitrogen Fertilizer in On-Farm Corn Crops of the US Midwest. *Global Change Biology* 17(2): 1140-1152.

Huang, W.-J., R. G. Heifner, H. Taylor, and N. D. Uri. 2001. Using Insurance to Enhance Nitrogen Fertilizer Application to Reduce Nitrogen Losses to the Environment. *Environmental Monitoring and Assessment* 68: 209-233.

Huang, W.-J., and R. M. Lantin. 1993. A Comparison of Farmers' Compliance Costs to Reduce Excess Nitrogen Fertilizer Use under Alternative Policy Options. *Review of Agricultural Economics* 15(1): 51-62.

Huang, W.-J., and M. LeBlanc. 1994. Market-Based Incentives for Addressing Non-Point Water Quality Problems: A Residual Nitrogen Tax Approach. *Applied Economics Perspectives and Policy* 16(3): 427-440.

Hull, J. C. 2009. *Options, Futures, and Other Derivatives*. Upper Saddle River, NJ: Prentice-Hall.

Hennessy, D. A., B. A. Babcock and D. J. Hayes. 1997. Budgetary and Producer Welfare Effect of Revenue Insurance. *American Journal of Agricultural Economics* 79(3): 1024-1034.

IPNI – International Plant Nutrition Institute. 2011. 4R Nutrient Stewardship. Available at <http://www.ipni.net/4r> (accessed on March 2012).

Izaurrealde, R. C., R. L. Lemke, T. W. Goddard, B. McConkey, and Z. Zhang. 2004. Nitrous Oxide Emissions from Agricultural Toposequences in Alberta and Saskatchewan. *Soil Science Society of America Journal* 68: 1285-1294

Johnson, M. E., and A. Tenenbein. 1981. A Bivariate Distribution Family with Specified

- Marginals. *Journal of the American Statistical Association* 73(373): 198-201.
- Li, C., V. Narayanan, and R. Harriss. 1996. Model Estimates of Nitrous Oxide Emissions from Agricultural Lands in the United States. *Global Biogeochemical Cycles* 10(2): 297-306.
- Maggi, F., C. Gu, W. J. Riley, G. M. Hornberger, R. T. Ventura, T. Xu, N. Spycher, T. Steefel, N. L. Miller, and C. M. Oldenburg. 2008. A Mechanistic Treatment of the Dominant Soil Nitrogen Cycle Processes: Model Development, Testing and Application. LBNL Paper LBNL-486E, Lawrence Berkeley National Laboratory: Lawrence Berkeley National Laboratory, Berkeley, CA.
- McSwiney, C. P., and G. P. Robertson. 2005. Nonlinear Response of N<sub>2</sub>O Flux to Incremental Fertilizer Addition in a Continuous Maize (*Zea mays* L.) Cropping System. *Global Change Biology* 11(10): 1712-1719.
- Metcalf, T., D. J. Bosch, J. W. Pease, M. M. Alley, S. B. Phillips. 2007. Yield Reserve Program Costs in the Virginia Coastal Plain. *Agricultural and Resource Economics Review* 36(2): 197-212.
- Millar, N., G. P. Robertson, P. R. Grace, R. J. Gehl, and J. P. Hoben. 2010. Nitrogen Fertilizer Management for Nitrous Oxide (N<sub>2</sub>O) Mitigation in Intensive Corn (Maize) Production: An Emissions Reduction Protocol for US Midwest Agriculture. *Mitigation and Adaptation Strategies for Global Change* 15(2): 185-204.
- MSU-EPRI – Michigan State University – Electric Power Research Institute. 2010. Quantifying N<sub>2</sub>O Emissions Reductions in US Agricultural Crops through N Fertilizer Rate Reduction. Available at <http://www.v-c-s.org/methodologies/quantifying-n2o-emissions-reductions-us-agricultural-crops-through-n-fertilizer-rate-0> (accessed on March 2012)
- National Oceanic and Atmospheric Administration, National Climate Center. Database. Available at <http://www.ncdc.noaa.gov/oa/climate/climatedata.html> (accessed on August 2009).
- Rochette, P., D.E. Worth, R.L. Lemke, B.G. McConkey, D.J. Pennock, C. Wagner-Riddle, and R. L. Desjardins. 2008. Estimation of N<sub>2</sub>O Emissions from Agricultural Soils in Canada. I: Development of a Country-Specific Methodology. *Canadian Journal of Soil Science* 88: 641-654.
- Roosen, J., and D. A. Hennessy. 2003. Tests for the Role of Risk Aversion on Input Use. *American Journal of Agricultural Economics* 85(1): 30-43.
- Segerson, K. 1988. Uncertainty and Incentives for Non-point Pollution Control. *Journal of Environmental Economics and Management* 15: 87-98.
- Sheriff, G. 2005. Efficient Waste? Why Farmers Over-Apply Nutrients and the Implications for Policy Design. *Review of Agricultural Economics* 27(4): 542-557.

- Shortle, J., and D. Abler. 1997. *The International Yearbook of Environmental and Resource Economics 1997/1998*. H. Folmer and T. Tietenberg, eds. Aldershot, UK: Edward Elgar Publishing.
- Snyder, C. S., T. W. Bruulsema, T. L. Jensen, and P. E. Fixen. 2009. Review of Greenhouse Gas Emissions from Crop Production Systems and Fertilizer Management Effect. *Agriculture, Ecosystems and Environment* 133: 247-266.
- Tembo, G., B. W. Brorsen, F. M. Epplin, and E. Tostao. 2008. Crop Input Response Functions with Stochastic Plateaus. *American Journal of Agricultural Economics* 90(2): 424-434.
- Thomas, A., and R. N. Boisvert. 1994. Bioeconomics of Regulating Nitrates in Groundwater: Taxes, Quantity Restrictions, and Pollution Permits. Working Paper 94-07, Department of Agricultural, Resource, and Managerial Economics. Cornell University.
- Townsend, A.R., R. W. Howarth, F. A. Bazzaz, M. S. Booth, C. C. Cleveland, S. K. Collinge, A. P. Dobson, P. R. Epstein, E. A. Holland, D. R. Keeney, M. A. Mallin, C. A. Rogers, P. Wayne, and A. H. Wolfe. 2003. Human Health Effects of a Changing Global Nitrogen Cycle. *Frontiers in Ecology and the Environment* 1: 240-246.
- U.S. Department of Agriculture, Risk Management Agency (USDA-RMA). 2003. Nutrient BMP Endorsement for CRC Policy. Washington, DC. Available at [http://www.rma.usda.gov/ftp/Policies/2003/n-bmp/pdf/N-BMP\\_ur.pdf](http://www.rma.usda.gov/ftp/Policies/2003/n-bmp/pdf/N-BMP_ur.pdf) (accessed on March 2012).
- U.S. Environmental Protection Agency (U.S. EPA). 2011. Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990 – 2009. Washington, DC.
- Thornton, F.C., and R. J. Valente. 1996. Soil Emissions of Nitric Oxide and Nitrous Oxide from No-Till Corn. *Soil Science Society of America Journal* 60(4): 1127-1133.
- Xepapadeas, A. 1997. *Advanced Principles in Environmental Policy*. Aldershot, UK: Edward Elgar.
- Yamulki, S., R. M. Harrison, K. W. T. Goulding, and C. P. Webster. 1995. Effect of Fertilizer Application on NO and N<sub>2</sub>O Fluxes from Agricultural Fields. *Journal of Geophysical Research* 100(12): 25923-25931.

## CHAPTER 5. GENERAL DISSERTATION CONCLUSIONS

The environmental effects of agricultural production have been highlighted in the economics literature due to recent events in the world economy. They include the land-use change and greenhouse gas emissions induced by biofuel policies, as well as the increase in commodity prices resulting from such policies and from the higher global demand of food and feed.

Satisfaction of the increased demand for agricultural products becomes a relevant issue, and therefore, the study of how agricultural supply responds to these events is inherently related to this matter. Accurately measuring these effects is important to precisely quantify the environmental effects of management practices employed in agricultural production.

The first two essays of this dissertation (Chapters 2 and 3) are devoted to the analysis and measurement of the supply response to price changes. The third essay provides a quantification of the direct effect of agricultural management practices (the application of nitrogen fertilizers in soils) on greenhouse gas emissions.

In Chapter 2 we analyze the empirical applications of duality theory, which is the preferred empirical method to estimate price elasticities, substitution elasticities, and other supply response measurements. We argue that this approach provides biased estimated results as a consequence of using real-world data that is usually available to researchers. The reason is that these datasets contain certain features that are not consistent with the assumptions of the theory. We use Monte Carlo simulations to generate a dataset of netput prices and netput quantities that are comparable to those encountered in databases typically used by practitioners. More precisely, we generate a panel of observations for successive periods of time and coming from a population of technologically heterogeneous firms that belong to different regions. The data incorporates realistic features like optimization under uncertainty, prediction errors in prices and quantities,

endogenous prices, omitted variable netputs, output and input data aggregation, measurement errors in the observed variables, and unobserved heterogeneity across firms. These features prevent the theory from holding exactly. To make sure that we assign magnitudes that are realistic and also comparable to those in U.S. agricultural data, we make use of datasets widely employed in empirical applications.

We find that the dual approach fails to provide parameter estimates that are sufficiently close to their true values. Both own- and cross-price elasticities are inaccurately recovered. Sensitivity analysis shows that results are robust to different sources and levels of noise, and also robust to the sample of firms used in estimation. Furthermore, we find that the usual practice of pooling observations from different states and/or regions in order to increase the degrees of freedom in estimation produces similarly biased results.

Based on both the inability of the dual approach to precisely calculate supply response values, and on the relevance of this issue with regard to the measurement of the environmental effects of agricultural production, we propose in Chapter 3 an alternative estimation approach. This essay is focused on crop yield response to price changes due to the importance of crop yields as a component of the supply response, and the impact that they have in the accountings of GHG emissions and land use change. The proposed approach consists of simultaneously incorporating the estimation procedure datasets from different sources. Given that elasticities formulae are a function of underlying production function parameters, and that these parameters can be equivalently recovered from the theory in at least two ways, we make use of these theoretical relationships to feed the estimation procedure with the various sources of information about the same production function parameter. In this application, information comes from U.S.



agriculture market-based datasets on the one hand, and on the other, production functions relationships datasets.

Bayesian estimation methods allows us to incorporate different sources of information, such that the weight with which each source contributes to the parameter-estimated value is given by how much information that source provides. More precisely, the weighting structure is based on log-likelihood functions.

Besides presenting a new approach to calculate supply response measures, we also provide updated values of crop yield elasticities with respect to input and output prices in an application to Iowa agriculture. Results show that own-price corn yield elasticity has the expected positive sign and the yield elasticity with respect to input prices has the expected negative sign; that is, yields increasing with own-price and decreasing with input prices.

Chapter 4 looks at the direct environmental effects of certain agricultural management practices, in particular, the application of nitrogen fertilizers in soils. The excessive use of nitrogen in soils has negative environmental consequences, such as the emission of nitrous oxide, the runoff of nutrients to the waterways, as well as indirect health related effects. In this essay we document the nitrous oxide emission reductions that are consistent with a market instrument imposed on nitrogen fertilizer applications in order to induce a lower use of nutrients. We assume an expected utility maximizer farmer that chooses the optimal rate of nitrogen application, conditional on the uncertainty given by production and price shocks, and a market instrument that targets the applications of nitrogen. The market instrument is nonlinear due to the nonlinearity that exists between nitrous oxide fluxes and the use of nitrogen fertilizer. This is more efficient than a linear incentive scheme because it provides the farmers the correct price

signals; that is, a higher incentive to reduce applications in the region where these applications induce higher emissions per unit.

Results are obtained by solving the model using simulated data that is calibrated for the case of corn in Iowa. We find that for modest carbon prices, farmers are induced to make significant reductions of nitrogen applications while only marginally penalizing crop yields. We show that a linear scheme is likely to cause the incentive policy to be unattractive because it would require much larger fertilization reductions (and yield penalties) to achieve a comparable reduction of emissions. Sensitivity analysis shows that different levels of risk aversion have almost no impact on final results; however, optimal nitrogen reductions are highly dependent on the prevailing market price of carbon, i.e., the value at which nitrous oxide emissions are valued by society.

## ACKNOWLEDGEMENTS

First, I am indebted to my major professor Dr. Dermot J. Hayes. Without your economic intuition, encouragement, guidance, support, and advice, I would not have been able to achieve this objective.

Second, I want to thank my committee members Dr. Sergio H. Lence, Dr. Bruce A. Babcock, Dr. GianCarlo Moschini, and Dr. Keri L. Jacobs, for continuously pushing my work towards excellence.

Then, the following individuals contributed directly or indirectly with the work I present here and/or with the work I did during the PhD program. Their contribution includes, but it is not limited to, providing data bases, economic advice, editorial work, and helpful comments and suggestions. Dr. Sergio H. Lence, Dr. Alicia S. Rosburg, Dr. Eldon Ball, Dr. Keri L. Jacobs, Dr. Jim R. Rouse, Dr. Changsheng Li, Dr. Bruce A. Babcock, Dr. Chad E. Hart, Dr. Katherine L. Kling, Dr. Phil W. Gassman, Todd Campbell, PhD(c) Adriana Valcu, Dr. Juan Dubra, Dr. Marcelo Caffera, Sandra Clarke, Kristin L. Senty-Brown, Nathan M. Cook, and PhD(c) Nadia Crosignani.

Finally, I want to thank my wife and kids Karina, Manuela, and Mateo; my parents Héctor and Charo; my brothers and sisters Nicolás, Carolina, and Gastón for unconditionally standing by me even in the most difficult moments. Also, Maro Ibarburu and Andrea Sibils, and Miguel Carriquiry and Alejandra Pedraja for always having their doors open for me and my family.