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Essays on infinite-variance stable errors and robust estimation procedures

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Essays on infinite-variance stable errors and robust estimation procedures

by

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DEDICATION

I would like to dedicate this dissertation to my parents Öznur and Nihat Serttaş.

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ABSTRACT

Gaussian normal error assumption and OLS-based regression estimations form the basis of co-integration testing techniques. However, many studies have found evidence that financial and some important economic time series data such as exchange rate returns and inflation rates are subject to high variability. In particular, their innovations exhibit the features of skewness, excessive peakness around the mean and heavier tails than those of the Gaussian normal distribution. Stable distributions (which are used to model high variability in data and infinite-variance processes) provide more realistic distributional assumptions than the Gaussian distribution for heavy-tailed financial and economic time series. Least Absolute Deviation (LAD) based estimators often yield robust results for heavy-tailed data compared to least squares based estimators. In this dissertation, as a natural extension of the extant studies, we consider a new robust residual-based co-integration test under the assumption of infinite-variance errors which are in the domain of attraction of a stable law. We implement the least absolute deviation (LAD) procedure in our regression estimations.

In part I, the new co-integration tests are proposed. The test is parametric: the critical values of the test statistic depend on the stability index of the stable distribution from which the errors are driven. The unit root test statistic we consider under the null of no co-integration is taken from Samarakoon and Knight (2009, *Econometric Reviews*, 28, 314–334). We find the critical values of these new co-integration tests through Monte Carlo simulations and observe that the null convergence of the test statistic is faster for lighter tails. Size and power comparisons are included to evaluate the performance of the new residual-based tests relative to conventional OLS-based ones which are due to Caner (1998, *J. of Econometrics*, 86, 155–175). We observe that the LAD-based tests have power advantages over the OLS-based tests

as the sample size gets larger and the tails get heavier with infinite-variance error assumption, yet there are more size distortions associated with LAD-based tests especially for small sample sizes compared to OLS-based ones.

The new tests are employed to test for forward rate unbiasedness hypothesis (FRUH) with daily frequency data for a sample of eight currencies (Australian dollar, Canadian dollar, French franc, German mark, Italian lira, Japanese yen, Swiss franc and U.K. pound) against the U.S. dollar for 1-month, 3-month, 6-month and 1-year forward contracts.[1] We also run fully-modified ordinary least squares (FM-OLS) and fully-modified least absolute deviation (FM-LAD) estimators on the co-integrating regressions to test the coefficient restrictions which are implied by the FRUH. We observe that tests involving longer maturity forward contracts (6-month and 1-year) and LAD-based co-integration tests mostly provide the evidence that is inconsistent with FRUH.

In part II, weak and strong-form purchasing power parity (PPP) relations are re-examined by using LAD-based procedures under infinite-variance error assumption. LAD-based co-integration tests that are proposed in part I are used to test for weak-form PPP. FM-OLS and FM-LAD procedures are used to test for the strong-form PPP hypothesis. The results from LAD-based estimations are compared to their OLS-based counterparts. Monthly exchange rate (per U.S. dollar) and PPI data for a sample of eight countries (Austria, Canada, Denmark, Germany, Japan, Netherlands, Sweden and the U.K.) from 1973:1 to 2009:12 are used for estimation purposes. Neither weak-form nor strong-form PPP relations can be justified empirically regardless of the estimation procedure. Results from the new residual-based co-integration tests give slightly more support of the weak-form PPP.

[1]We thank Professor Dermot Hayes for providing the data to us and helping us with Datastream.

PART I

RESIDUAL-BASED TESTS OF CO-INTEGRATION WITH INFINITE-VARIANCE ERRORS: SIMULATION ANALYSIS AND AN EMPIRICAL APPLICATION

CHAPTER 1. INTRODUCTION

1.1 Introduction and background

The behaviors of financial and some other important economic time series data such as exchange rate returns and inflation rates are more extremal than what the Gaussian normal distribution can predict. Those kinds of data variables are more frequently exposed to large fluctuations, crashes and booms that their distributions can be better approximated by heavier-tailed distributions. Errors describing such time series are also often asymmetric and have high peaknesses around their means. Stable family of distributions provide a better and more flexible alternative than the normal distribution for heavy-tailed financial and economic time series. Several parameters characterize the shape of a stable distribution. The parameter that determines the tail heaviness (stability index or characteristic exponent) provides a benchmark to distinguish between finite-variance and infinite-variance stable distributions. Normal distribution is a special case in the stable distribution family with finite-variance and a stability index that equals to 2. A stability index less than 2 corresponds to infinite-variance case.

Stable distributions are appealing to researchers not only in the applications of finance and economics fields but also in other application areas including network traffic modeling, signal processing and oceanography (Xiaohu et al. (2003); Nikias and Shao (1995); Pierce (1997)). There are two unique properties of stable distributions that make them desirable for researchers in many diverse fields: stability property and generalized central limit theorem. "Stability property" makes stably distributed variables closed under linear combinations. Generalized central limit theorem (GCLT) states that stable distributions are the only possible limit law for sums of independent and identically distributed random variables when they are properly normalized and centered. This result is similar to the result of ordinary central limit theorem (CLT) involving the normal distribution but CLT requires a finite second moment whereas GCLT can be generalized to the infinite-variance case.[1]

The error terms in regression and time series models can be hypothesized as the sum of a large number of factors. If those factors are independent with finite-variance and none of them is "too large" compared to others, then each error term is normally distributed by CLT (Griffiths et al., 1993, ch. 3). However, in practice, we observe that the behaviors of many random processes do not conform well with the characteristics of the normal distribution. Hence emphasis has shifted towards the stable distributions as they explain such processes in a much better way and are justified by the GCLT.

The "stable" name is given due to Lévy (1924) who has introduced the class of stable or α-stable distributions. The early recognition of stable distributions in the field of economics appears in Mandelbrot (1963). Mandelbrot studies the cotton prices in the United States and shows that logarithmic price changes behave more like an infinite-variance stable distribution than a normal distribution. Fama (1965) studies the stock prices and arrives at a similar conclusion strengthening Mandelbrot' s claims. Mandelbrot (1967) provides additional empirical evidence for wheat and railroad stock price changes, as well as for interest and exchange rate returns. He concludes that they possess a large number of extreme values and approximating them with a normal distribution is not appropriate.

Many empirical papers propose several other distributions to explain the observed leptokurtosis of financial returns. Those distributions include Student' s t-distribution, mixture of normals and Weibull distribution (Boothe and Glassman (1987); Hall et al. (1989); Mittnik and Rachev (1993)). The main problem with such alternative distributions is that they do not satisfy the "stability property" and the GCLT does not apply to them. Financial returns are believed to be the aggregation or summation of vast number of independent small shocks which are influenced by the arriving of new information and the decisions of market participants (McCulloch, 1996). GCLT says that the only possible limiting distributions of financial

[1] See Gnedenko and Kolmogorov (1968) and Uchaikin and Zolotarev (1999) on the central limit theorems.

returns (which includes stock price, exchange rate and interest rate changes) have to be the stable ones (Rachev et al. (2005), ch. 7; Rachev et al. (2007), ch. 14).

Infinite-variance stable distributions have been used in modeling foreign exchange rates in numerous papers including Westerfield (1977), Bagshaw and Humpage (1986), So (1987), Fofack and Nolan (2001) and Falk and Wang (2003). Interest rate returns and inflation rates with stable distributions are considered in McCulloch (1985), Charemza et al. (2005) and Bidarkota and McCulloch (1998). Falk and Wang (2003) calculate the stability indices of inflation rates for 12 industrialized countries and find infinite-variance behavior.

There is an active research area on the testing of unit-roots and co-integration under the assumption of infinite-variance errors. Unit root tests with OLS estimator are studied in Chan and Tran (1989), Phillips (1990), Rachev et al. (1998), Horváth and Kokoszka (2003), Patterson and Heravi (2003) and Martins (2009). Extensions to M-estimators are made in Knight (1989, 1991), Shin and So (1999) and Samarakoon and Knight (2009). OLS-based co-integration tests for infinite-variance case are considered in Caner (1998), Paulauskas and Rachev (1998), Mittnik et al. (2001), Chen and Hsiao (2010) and Fasen (2010).[2]

In literature, although there are many studies that consider unit root and co-integration tests while assuming infinite-variance errors, the emphasis is mostly on least-squares (OLS) based methods. OLS estimator is widely used in applications because it has a closed form solution and it is easy to implement. However, when the error structure is heavy-tailed, OLS estimator will only focus on few large errors and tend to ignore the rest of the data, resulting in inferior estimates (Wilson (1978); Calder and Davis (1998)). For that purpose, LAD method is used in practice as a better alternative when the errors have heavy tails or large outliers. The outliers do not dominate the minimization procedure as they do for the OLS estimator.

The asymptotic theory of LAD estimator in non-stationary co-integrated regressions is studied in Phillips (1995). He shows that with finite-variance errors, similar to OLS, LAD is super-consistent—i.e. converging to the true parameter at the rate $O_p(T^{-1})$ rather than the

[2] There are two most popular approaches for co-integration tests: residual-based type (Engle and Granger (1987); Phillips and Ouliaris (1990)) and Johansen (1988, 1991) type. Caner (1998) also examines the Johansen approach. In this dissertation, we only consider the residual-based approach.

usual rate $O_p(T^{-1/2})$.[3] Phillips (1995) also considers the consistency of OLS and LAD with infinite-variance errors and shows that LAD estimator has a faster convergence rate ($O_p(T^{-a})$) than OLS ($O_p(T^{-1})$), where $a > 1$ for infinite-variance stable noise. However, because of endogeneity of the regressors, LAD and OLS estimators both suffer from a second order bias. They are both first order or mean unbiased but not second order or median unbiased.[4] Phillips (1995) corrects that bias and develops fully modified versions of LAD and M-estimators. Bias corrected fully modified OLS estimator is first studied in Phillips and Hansen (1990).

Xiao (2009) considers quantile regression (which includes LAD estimator as a special case) in the co-integration context. He develops fully-modified quantile co-integrating regression estimators with endogenous regressors to remove the second order bias and nuisance parameters that occur in the asymptotic theory. In a Monte Carlo experiment, he shows the efficiency gain of the LAD estimator over the OLS estimator with errors generated from the finite-variance t-distribution (see Tables 1a and 1b in the same study). As an extension, he suggests considering quantile regression in a co-integration setting with infinite-variance errors. Knight and Samarakoon (2009) derive the limit distributions of M-estimators in co-integrated models with infinite-variance errors for Johansen approach and show that the convergence of M-estimators they consider are faster than the OLS.

Samarakoon and Knight (2009) propose new unit root tests under stably distributed infinite-variance noise structure. They develop the asymptotic theory of M-estimators for Dickey-Fuller type unit root tests. M-estimators are a broad class of estimators which include least-squares (OLS) and least absolute deviation (LAD) methods as special cases. In particular, for a simple regression as given below,

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t \tag{1.1}$$

The parameters β_0 and β_1 are estimated by minimizing,

[3] On the super-consistency of OLS in a finite-variance error structure see Stock (1987) and Phillips and Durlauf (1986).

[4] On the property of median unbisedness see Birnbaum (1964).

$$\sum_{t=1}^{n} \rho(Y_t - \beta_0 - \beta_1 X_t) \tag{1.2}$$

where ρ is a suitably chosen loss function. The LAD estimator is found by setting $\rho(x) = |x|$ and the OLS estimator is found by setting $\rho(x) = x^2$.

In part I, we study new types of residual-based co-integration tests based on the LAD estimator and based on the unit root test statistics considered in Samarakoon and Knight (2009). We tabulate the critical values of our co-integration tests through Monte Carlo simulations. We assume that the error terms in the regression equations come from the α-stable distribution family. In section 1.2 there is a brief summary of the properties of α-stable distributions. Chapter 2 contains the details of the LAD-based co-integration tests that we propose and gives an overview of the relevant literature of unit root tests and residual-based co-integration tests. Monte Carlo simulations that we run in order to derive the critical values of our tests are also described in section 2.3 of chapter 2.

We apply the new LAD-based tests together with OLS-based ones of Caner (1998) to forward exchange rate market and test for the forward rate unbiasedness hypothesis (FRUH). The dataset we consider includes 1-month, 3-month, 6-month and 1-year maturities for the forward contracts. We observe that FRUH does not get support from the data mostly with LAD-based co-integration tests and the tests involving 6-month and 1-year forward contracts. The details of the empirical analysis are given in chapter 3. Chapter 4 concludes.

1.2 Stable distributions

An α-stable distribution is described by four parameters: $\alpha, \beta, \sigma, \mu$. If a random variable X has an α-stable distribution then it is denoted as $X \sim S_\alpha(\beta, \sigma, \mu)$. $\alpha \in (0, 2]$ is called the stability index and it determines the tail shape. The case of $\alpha = 2$ corresponds to the Gaussian normal distribution and when $\alpha < 2$, the variance of X does not exist. As α gets smaller tails get heavier, if $\alpha \leq 1$ tails are so heavy that even the mean does not exist. $\beta \in [-1, 1]$ is the skewness parameter and it determines the degree and sign of asymmetry, $\sigma > 0$ is the scale parameter and $\mu \in \mathbf{R}$ is the location parameter.

Infinite-variance means that the variance of the distribution is not defined. It is somehow counter-intuitive to argue for an infinite-variance as in practice we deal with finite sample size and for any finite set of data if we apply the formula of variance, we will get a finite value. But the financial return data are defined over an infinite range (from negative infinity to positive infinity). Over that range, finite-variance distributions assign so little probability to the tails that they have finite variances. Some distributions on the other hand assign much higher probabilities to the tails and have infinite variances (Kaplan, 2009). For an α-stable distribution with $0 < \alpha < 2$ we can write,[5]

$$E|X|^p < \infty \text{ for } 0 < p < \alpha$$

$$E|X|^p = \infty \text{ for } p \geq \alpha$$

where $E|X|^p = \int_{-\infty}^{\infty} |x|^p f(x)dx$, p is any real number and $f(.)$ is the probability density function of X. Figure 1.1 compares simulated stable variables ($\alpha = 1.7$) with standard normal variables ($\alpha = 2$). From the figure we can see that stable distribution allows for more volatility than the normal distribution. Other features of stable distribution include excessive peakness around the mean and heavy-tails.

Probability density functions of α-stable distributions exist and are continous, but their closed form solutions are not known except for three special cases; Cauchy distribution: ($\alpha = 1, \beta = 0$), Gaussian distribution: ($\alpha = 2, \beta = 0$), Lévy distribution: ($\alpha = 0.5, \beta = \pm 1$). Stable random variables are uniquely characterized through their characteristic function,

[5] The reader is referred to Samorodnitsky and Taqqu (1994) and Nolan (2010) for a detailed analysis of stable distributions.

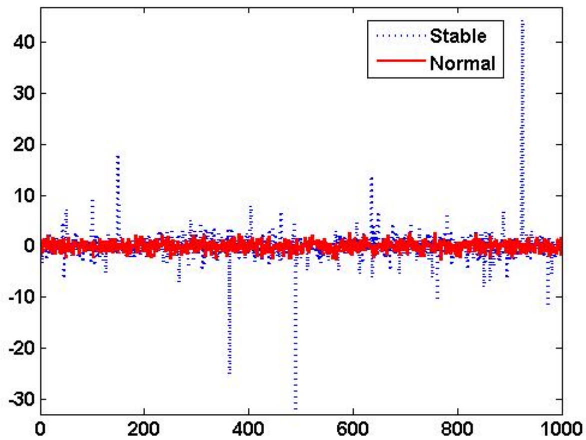


Figure 1.1: Simulated $S_{1.7}(0,1,0)$ variables with $N(0,1)$ variables

$$E\exp\{i\tau X\} = \left\{ \begin{array}{ll} \exp\left\{ -\sigma^{\alpha}|\tau|^{\alpha}[1+i\beta(sign(\tau)\tan(\frac{\pi\alpha}{2}))] + i\mu\tau\right\} & \alpha \neq 1 \\ \exp\left\{ -\sigma|\tau|[1+i\beta(\frac{2}{\pi})(sign(\tau))\ln|\tau|] + i\mu\tau\right\} & \alpha = 1 \end{array} \right\} \tag{1.3}$$

In practice the values of stable parameters are unknown and have to be estimated. Maximum likelihood estimation (MLE) method gives accurate and reliable estimates of stable parameters. The method is briefly explained next.

1.2.1 Estimating the stable parameters

Maximum likelihood procedure is one way of estimating the stable parameters. It has been advocated by McCulloch (1997) and Fofack and Nolan (1999) primarily against the Hill (1975) estimator which is a widely used stability index estimator in literature. Recently, Borak et al. (2005) compares several different estimation techniques and mentions MLE as being "almost always the most accurate, in particular, with respect to the skewness parameter". The procedure involves deriving a numerical approximation of the density function of the stable distribution and estimating $(\alpha, \beta, \sigma, \mu)$ by maximizing the likelihood function (Nolan, 2001). The following equation describes a relation between the probability density function

$f(x)$ and the characteristic function $\varphi(t)$,

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\{-itx\}\varphi(t)dt \tag{1.4}$$

Given this relation one can calculate the density function $f(x)$ for all x. A highly accurate method for that is discussed in Nolan (1997). The likelihood function can then be found given a sample $X = (X_1, X_2, ..., X_T)$. Maximum likelihood estimation applies a quasi-Newton method to numerically maximize the likelihood function with respect to the unknown parameters $(\alpha, \beta, \sigma, \mu)$ by using the quantile estimator of McCulloch (1986) as an initial approximation. In this study, we implement Nolan' s approach as mentioned in estimating $(\alpha, \beta, \sigma, \mu)$.[6]

Let the parameter vector be denoted by $\vec{\theta} = (\alpha, \beta, \sigma, \mu)$. The parameter space is $\Theta = (0, 2] \times [-1, 1] \times (0, \infty) \times (-\infty, \infty)$. The likelihood function for an i.i.d. stable sample $X = (X_1, X_2, ..., X_T)$ is given by,

$$\ell(\vec{\theta}) = \sum_{i=1}^{T} \log f(X_i \backslash \vec{\theta}) \tag{1.5}$$

Consistency and asymptotic normality of the ML estimator are established in DuMouchel (1973). Covariance matrix of ML estimator is denoted by $T^{-1}B$, where T is the sample size, $B = (b_{ij})$ is the inverse of 4×4 Fisher information matrix I. The components of I are given by,

$$I_{ij} = \int_{-\infty}^{\infty} \frac{\partial f}{\partial \theta_i} \frac{\partial f}{\partial \theta_j} \frac{1}{f} dx \tag{1.6}$$

Large sample confidence intervals for $\theta_i \in \Theta$ are given by,

[6] MLE parameters, density functions and distribution functions of stable distributions as well as stable random variables can be derived by using the STABLE program for MATLAB which was written and provided by J. P. Nolan. For more information see http://academic2.american.edu/~jpnolan/stable/stable.html.

$$\theta_i \pm z_{\phi/2}\frac{\sigma_{\hat{\theta}_i}}{\sqrt{T}} \tag{1.7}$$

where $\sigma_{\hat{\theta}_1}, ..., \sigma_{\hat{\theta}_4}$ are the square roots of the diagonal entries of B, $(1-\phi)$ is the percentage of the confidence interval, and $z_{\phi/2}$ corresponds to the tabular value of standard normal distribution that comes closest to the specified percentile $\phi/2$. The standard theory applies when the parameters are in the interior of the parameter space $\vec{\theta}$.

CHAPTER 2. TESTING FOR CO-INTEGRATION

2.1 Unit root tests

Many of the economic and financial time series are non-stationary and they possess a single unit root—i.e. they are I(1) or difference stationary processes. There are many forms of stationarity, here stationarity denotes weak stationarity. A unit root process violates the conditions of being a weak stationary process (Shumway and Stoffer, 2006, p. 22). If a time series sequence $\{x_t\}$ is non-stationary but becomes stationary after differencing d times then it is integrated of order d or is an I(d) process. One important feature of a unit root variable is that, the persistence of shocks to that variable will be infinite. This in turn will have important implications while testing for co-integration so unit root variables are discussed briefly. If we consider the simple AR(1) process,

$$x_t = \delta x_{t-1} + \varepsilon_t,\ \varepsilon_t \sim \text{i.i.d.} \tag{2.1}$$

After successive substitutions with an initial condition x_0, one can write equation (2.1) as,

$$x_t = \delta^t x_0 + \delta^{t-1}\varepsilon_1 + \delta^{t-2}\varepsilon_2 + \ldots + \delta^2\varepsilon_{t-2} + \delta\varepsilon_{t-1} + \varepsilon_t \tag{2.2}$$

The series x_t is stationary when $|\delta| < 1$ and the effects of past shocks die out as $t \to \infty$. Conversely, the series is non-stationary if $|\delta| = 1$ and the effects of past shocks have a non-decaying effect on x_t.[1] Stationary variables tend to revert back to a long-run mean over time,

[1] The non-stationary case when $|\delta| > 1$, is an unappealing and not so common case where the effects of past shocks propagate over time. It is usually not considered in practice.

whereas non-stationary unit root variables have no tendency to be mean reverting and wander randomly no matter how much we go further along the time index.

Testing for unit roots enables us to distinguish between stationary and non-stationary cases. In this paper, we apply Phillips and Perron (1988) and Samarakoon and Knight (2009) type unit root tests to test for a unit root in our data. Phillips-Perron tests are designed for normally distributed errors and involve running OLS estimations. Since we have infinite-variance error assumption and consider LAD estimations, we also implement Samarakoon and Knight (2009) type unit root tests, which require infinite-variance stable errors and use M-estimators with LAD being a special case.

Earlier versions of unit root tests for normal errors and OLS-based regressions are studied in Dickey and Fuller (1979). They consider three different versions of equations to test for a unit root,

$$\Delta x_t = \gamma x_{t-1} + \varepsilon_t, \tag{2.3}$$

$$\Delta x_t = \mu + \gamma x_{t-1} + \varepsilon_t, \tag{2.4}$$

$$\Delta x_t = \mu + \gamma x_{t-1} + \beta t + \varepsilon_t \tag{2.5}$$

The null hypothesis $H_0 : \gamma = 0$, tests for a unit root in x_t sequence against the alternative $H_1 : \gamma < 0$, where the sequence is stationary.[2] The resulting t-statistic, $t_{\hat{\gamma}} = \hat{\gamma}/\text{s.e.}(\hat{\gamma})$, is compared to the appropriate Dickey-Fuller critical values.[3] The tests require the error structure to be normal and independent and identically distributed (i.i.d.). Otherwise an augmented version of Dickey-Fuller tests can be used by including the p lag values of x_t say in equation (2.3),

[2]Note from the hypothesis that we are testing for a positive unit root. Assuming a positive unit root is typical in practice when dealing with economic data because economic data are often positively correlated.

[3]The values can be found in Enders (2004), p. 439.

$$\Delta x_t = \phi x_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta x_{t-i+1} + \varepsilon_t \tag{2.6}$$

Phillips-Perron type unit root tests allow for serial correlation and heteroskedasticity in the innovation structure and involve running one of the three OLS regressions in the form,

$$x_t = \alpha x_{t-1} + u_t, \tag{2.7}$$

$$x_t = \mu + \alpha x_{t-1} + u_t, \tag{2.8}$$

$$x_t = \mu + \beta(t - T/2) + \alpha x_{t-1} + u_t \tag{2.9}$$

where T denotes the sample size. The innovation series u_t do not have to be i.i.d.. In models (2.7), (2.8) and (2.9), the null hypothesis of a unit root is $H_0 : \alpha = 1$, and is tested against the alternative $H_1 : \alpha < 1$. Phillips-Perron adjusted t-statistics for the above three models are given by,

$$Z_t = (S_0/S_{Tl})t_{\hat{\alpha}} - 1/2(S_{Tl}^2 - S_0^2)\left[T^{-1}S_{Tl}\left(\sum_{t=2}^{T} x_{t-1}^2\right)^{1/2}\right]^{-1} \tag{2.10}$$

$$Z_{t_\mu} = (S_0/S_{Tl})t_{\hat{\alpha}} - (1/2S_{Tl})(S_{Tl}^2 - S_0^2)\left[T^{-2}\sum_{t=2}^{T}\left(x_{t-1} - \bar{X}_{-1}\right)^2\right]^{-1/2} \tag{2.11}$$

$$Z_{t_\tau} = (S_0/S_{Tl})t_{\hat{\alpha}} - \left(T^3/(4\sqrt{3}D_x^{1/2}S_{Tl})\right)(S_{Tl}^2 - S_0^2) \tag{2.12}$$

where Z_t, Z_{t_μ} and Z_{t_τ} are the test statistics for models (2.7), (2.8) and (2.9) respectively. S_0 is the regression standard error, $t_{\hat{\alpha}} = (\hat{\alpha} - 1)/\text{s.e.}(\hat{\alpha})$, $\bar{X}_{-1}$ is the mean of x_{t-1}, D_x is the determinant of $(X'X)$ with X denoting the matrix of explanatory variables in model (2.9). S_{Tl}^2 is given by,

Table 2.1: 5% critical values of Z_t, Z_{t_μ}, Z_{t_τ} statistics for Caner unit root test

α	standard[a]	constant[b]	constant and trend[c]
1	-1.68	-3.22	-3.56
1.5	-1.83	-2.98	-3.45
1.6	-1.87	-2.96	-3.44
1.7	-1.88	-2.91	-3.44
1.8	-1.88	-2.90	-3.42
1.9	-1.90	-2.89	-3.42

[a] Phillips-Perron critical value ($\alpha = 2$) at 5% significance is -1.95.
[b] Phillips-Perron critical value ($\alpha = 2$) at 5% significance is -2.86.
[c] Phillips-Perron critical value ($\alpha = 2$) at 5% significance is -3.41.

$$S_{Tl}^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_t^2 + 2T^{-1}\sum_{s=1}^{l}\omega_{sl}\sum_{t=s+1}^{T}\hat{u}_t\hat{u}_{t-s} \tag{2.13}$$

where $\omega_{sl} = l - s/(l+1)$, l is the number of autocorrelations of $\hat{u}_t$. Five percent critical values of Phillips and Perron (1988) tests can be found in Hamilton (1994), p. 763.

Caner (1998) considers Phillips-Perron type unit root test with OLS regressions but assumes errors that are in the domain of attraction of a stable law. He derives the critical values of Phillips-Perron unit root tests with infinite-variance errors. His study includes the critical values for $\alpha = 0.5$, $\alpha = 1$ and $\alpha = 1.5$ only. We calculated the 5% critical values for other α levels ($\alpha = 1.6, 1.7, 1.8, 1.9$), through Monte Carlo simulations. The results of those simulations are presented in Table 2.1 along with the 5% critical values of Phillips-Perron test. Similar to Caner (1998), the number of iterations is set to 20,000 and the sample size is set to 1,000. Standard, constant, constant and trend cases correspond to models (2.7), (2.8) and (2.9) respectively.

Samarakoon and Knight (2009) unit root tests extend Dickey-Fuller type unit root tests to infinite-variance errors and M-estimators. Tests involve running three regressions,

$$\Delta x_t = \phi x_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta x_{t-i+1} + v_t, \tag{2.14}$$

$$\Delta x_t = \mu + \phi x_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta x_{t-i+1} + v_t, \tag{2.15}$$

$$\Delta x_t = \mu + \beta(t-1) + \phi x_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta x_{t-i+1} + v_t \tag{2.16}$$

where v_t is i.i.d. stable with $\alpha < 2$. Unit root hypotheses $H_0 : \phi = 0$ are tested against the alternative $H_1 : \phi < 0$.

Equations (2.14), (2.15) and (2.16) can be estimated via LAD estimator. Then the corresponding test statistics for each equation is given by,

$$\pi_\phi = \left(\sum_{t=p+1}^{T} x_{t-1}^2 \right)^{1/2} \hat{\phi}_{\mathrm{LAD}} / \left(2f(\theta)\right)^{-1}, \tag{2.17}$$

$$\pi_\mu = \left(\sum_{t=p+1}^{T} \left(x_{t-1} - \bar{x}\right)^2 \right)^{1/2} \hat{\phi}_{\mathrm{LAD}} / \left(2f(\theta)\right)^{-1}, \tag{2.18}$$

$$\pi_\tau = C_T^2 \, \hat{\phi}_{\mathrm{LAD}} / \left(2f(\theta)\right)^{-1} \tag{2.19}$$

where $f(.)$ is the probability density function of v_t, θ is the median of v_t and $\bar{x}$ is the mean of $x_{p+1}, \ldots, x_T$. C_T^2 is defined as,

$$\begin{aligned} C_T^2 \;\; = \;\; & \sum_{t=p+1}^{T} \Bigg[x_{t-1} + \frac{6t}{T} \sum_{s=p+1}^{T} \left(1 - \frac{2s}{T}\right) x_{s-1} \\ & + \sum_{s=p+1}^{T} \left(2 - \frac{3s}{T}\right) x_{s-1} \Bigg]^2 \end{aligned} \tag{2.20}$$

Samarakoon and Knight (2009) have shown that the asymptotic distributions of π_ϕ, π_μ and π_τ statistics are standard normal.

2.2 Residual-based co-integration tests

A vector of time series z_t are co-integrated of order d, b, denoted $z_t \sim CI(d, b)$, if all the components of z_t are $I(d)$ and there exists a linear combination of them such that $\beta' z_t = u_t$; u_t is $I(d-b)$, $b > 0$, $\beta \neq 0$ is the co-integrating vector (Engle and Granger, 1987).

In economic applications, it is typical to consider the case: $d = 1, b = 1$, because the co-integrating combination is treated as an "equilibrium" relationship (Banerjee et al., 1993, ch. 5). The existence of a long-run equilibrium among I(1) economic variables requires those variables to be co-integrated. When such an equilibrium exists, the deviations from the equilibrium are only temporary. The components of z_t vector can be interpreted as economic time series variables with an equilibrium condition: $\beta' z_t = u_t$ and u_t as the deviation from that equilibrium. If there is co-integration among the components of z_t, the economy reverts back towards equilibrium whenever it moves away and u_t is I(0). If on the other hand, u_t series is I(1), then the effects of past shocks to u_t will never die out and the long-run equilibrium can not be maintained (Enders, 2004, ch. 6).

One popular way of testing for co-integration is through residual-based methods. Residual-based tests of co-integration appear in an early study of Engle and Granger (1987). The method is implemented by choosing one of the variables of z_t as the dependent variable; say y_t, and the rest as explanatory variables, say vector x_t. Then it involves two steps; deriving the OLS residuals from the regression below,

$$y_t = \beta' x_t + u_t \tag{2.21}$$

As a next step, applying a unit root test on the residuals through running OLS to the regression,

$$\Delta \hat{u}_t = \phi \hat{u}_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta \hat{u}_{t-i+1} + v_t \tag{2.22}$$

Based on their power comparisons Engle and Granger (1987) suggest performing an augmented Dickey-Fuller test to test for a unit root in the residuals. The null hypothesis is $H_0 : \phi = 0$ and the alternative hypothesis is $H_1 : \phi < 0$. The null hypothesis corresponds to the hypothesis of no co-integration among the components of z_t series and the alternative corresponds to co-integration. Test statistic used to test for this hypothesis is in the form of the usual t-statistic, $t = \hat{\phi}_{\text{OLS}}/\text{s.e.}(\hat{\phi}_{\text{OLS}})$.

Engle and Granger (1987) assume that the error processes driving x_t and y_t series in equation (2.21) are i.i.d. normal. Phillips and Ouliaris (1990) assume the error terms to be finite-variance and strict stationary processes. They consider running OLS regressions in the form of equation (2.21) and OLS regressions on the residuals,

$$\hat{u}_t = \delta \hat{u}_{t-1} + v_t \tag{2.23}$$

The null hypothesis to test for no co-integration is then $H_0 : \delta = 1$ against the alternative of co-integration $H_1 : \delta < 1$. Phillips and Ouliaris (1990) derive the asymptotic theory for Phillips and Perron (1988) Z_t statistic among other statistics. They also tabulate the critical values of those test statistics by assuming an i.i.d. normal error structure for x_t and y_t series in equation (2.21). The critical values change whether a constant or a trend is included in the equation.

Here we mention Z_t statistic because it is the test statistic that we will use for our comparisons later. The asymptotics of Z_t statistic are the same as the asymptotics of augmented Dickey-Fuller statistic (Phillips and Ouliaris (1990); Caner (1998)). Z_t statistic is already studied in section 2.1. Here it is re-written in another form to be consistent with the formulation in Phillips and Ouliaris (1990). Equation (2.10) and below formulation are equivalent.

Z_t can also be written as,

$$Z_t = \left(\sum_{t=2}^{T} \hat{u}_{t-1}^2 \right)^{1/2} (\hat{\delta}_{\text{OLS}} - 1)/s_{tl} - (1/2)(s_{tl}^2 - s_k^2) \left[s_{tl} \left(T^{-2} \sum_{t=2}^{T} \hat{u}_{t-1}^2 \right)^{1/2} \right]^{-1} \tag{2.24}$$

where,

$$s_{tl}^2 = T^{-1} \sum_{l}^{T} \hat{v}_t^2 + 2T^{-1} \sum_{s=1}^{l} \omega_{sl} \sum_{t=s+1}^{T} \hat{v}_t \hat{v}_{t-s} \tag{2.25}$$

$$s_k^2 = T^{-1} \sum_{l}^{T} \hat{v}_t^2 \tag{2.26}$$

and $\omega_{sl} = l - s/(l+1)$, l is the number of autocorrelations of $\hat{v}_t$.

Caner (1998) extends the asymptotic theory studied in Phillips and Ouliaris (1990) to infinite-variance case and assumes that errors that derive x_t and y_t in equation (2.21) come from α-stable distribution with $\alpha < 2$. Similar to the studies of Engle and Granger (1987) and Phillips and Ouliaris (1990), he runs OLS-based regressions. He also simulates the asymptotic critical values of some of the test statistics that are considered in Phillips and Ouliaris (1990).

2.3 Residual-based co-integration tests via LAD estimation

2.3.1 Critical values through Monte Carlo simulations

Unlike the previous studies that employ OLS based estimations, here we apply residual-based tests of co-integration through LAD estimation procedures. We consider the data generating processes of x_t and y_t in the form,

$$y_t = y_{t-1} + \varepsilon_{1t}, \;\; \varepsilon_{1t} \sim \text{i.i.d. } S_\alpha(0,1,0), \; \alpha < 2 \tag{2.27a}$$

$$x_t = x_{t-1} + \varepsilon_{2t}, \;\; \varepsilon_{2t} \sim \text{i.i.d. } S_\alpha(0,1,0), \; \alpha < 2 \tag{2.27b}$$

Following regressions are run through LAD,

$$y_t = \beta_0 + \beta_1 x_t + u_t \tag{2.28}$$

$$y_t = \beta_0 + \beta_1 t + \beta_2 x_t + u_t \tag{2.29}$$

Another LAD regression is run on the residuals from equations (2.28) and (2.29),

$$\Delta\hat{u}_t = \phi\hat{u}_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta\hat{u}_{t-i+1} + v_t,\ v_t \sim \text{i.i.d.} \tag{2.30}$$

Testing the null hypothesis of no co-integration corresponds to testing the null hypothesis of a unit root in the residual $\hat{u}_t$ series; $H_0 : \phi = 0$ against the alternative of co-integration $H_1 : \phi < 0$.

As mentioned in section 2.1, Samarakoon and Knight (2009) derive the asymptotics of LAD-based test statistics for Dickey-Fuller type unit root tests under the existence of infinite-variance stably distributed errors. They show that under the null of unit root of a *raw* time series say u_t from a regression as given below,

$$\Delta u_t = \phi u_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta u_{t-i+1} + \xi_t,\ \xi_t \sim \text{i.i.d. } S_\alpha(\beta, \sigma, \mu),\ \alpha < 2 \tag{2.31}$$

The statistic π_ϕ has an asymptotic standard normal distribution,

$$\pi_\phi = \frac{\left\{ \sum_{t=p+1}^{T} u_{t-1}^2 \right\}^{1/2} \hat{\phi}_{\text{LAD}}}{(2f(\theta))^{-1}} \tag{2.32}$$

Since u_t is a raw time series, the asymptotic critical values for π_ϕ can not be used in testing the unit root hypothesis of residual series $\hat{u}_t$ in equation (2.30) (see for example Hamilton

(1994), p. 592). The critical values of π_ϕ statistic for testing $H_0 : \phi = 0$ in equation (2.30), are calculated through our Monte Carlo simulations. We consider two co-integrating regressions, one with constant and one with constant and trend.

Tables 2.2 through 2.6 present the 5% critical values for different stability indices (α = 1.5, 1.6, 1.7, 1.8, 1.9) and for different sample sizes. The number of iterations is 50,000. Five percent critical values correspond to the 5-th percentile of the simulated distribution of π_ϕ under the null hypothesis of no co-integration. Our simulation results for the critical values depend only on α but not on other stable parameters. Although the data generating processes assume that x_t and y_t are AR(1), the critical values are robust to higher autoregressive orders.

Table 2.2: 5% critical values for α=1.5

T	constant	constant and trend
50	-3.19	-3.84
100	-2.90	-3.37
500	-2.62	-3.01
1,000	-2.52	-2.92
2,000	-2.45	-2.84
3,000	-2.40	-2.81
5,000	-2.34	-2.78
7,000	-2.30	-2.75
15,000	-2.24	-2.73
∞	-2.15	-2.77

Table 2.3: 5% critical values for α=1.6

T	constant	constant and trend
50	-3.08	-3.63
100	-2.92	-3.34
500	-2.71	-3.06
1,000	-2.64	-2.97
2,000	-2.59	-2.90
3,000	-2.53	-2.87
5,000	-2.50	-2.83
7,000	-2.47	-2.78
15,000	-2.38	-2.74
∞	-2.30	-2.67

We mainly consider cases with $\alpha \geq 1.5$.[4] Convergence of the critical values to their asymptotic values tends to be faster for lighter tails. As α approaches 2—i.e. the error distribution gets similar to the normal distribution, the critical values approach to their large sample values faster. An equation to find the asymptotic critical values for the constant without trend case can be estimated by OLS as below,

[4]We do not consider smaller stability indices because many empirical economic data possess α levels that are greater or equal to 1.5 (see Kurz-Kim and Loretan (2007), p. 17).

Table 2.4: 5% critical values for α=1.7

T	constant	constant and trend
50	-3.05	-3.53
100	-2.96	-3.32
500	-2.80	-3.12
1,000	-2.75	-3.07
2,000	-2.70	-3.02
3,000	-2.67	-2.99
5,000	-2.65	-2.95
7,000	-2.63	-2.91
15,000	-2.56	-2.85
∞	-2.48	-2.76

Table 2.5: 5% critical values for α=1.8

T	constant	constant and trend
50	-3.02	-3.44
100	-2.99	-3.33
500	-2.89	-3.22
1,000	-2.86	-3.18
2,000	-2.81	-3.14
3,000	-2.80	-3.12
5,000	-2.77	-3.07
7,000	-2.76	-3.05
15,000	-2.69	-3.00
∞	-2.65	-2.93

Table 2.6: 5% critical values for α=1.9

T	constant	constant and trend
50	-3.01	-3.41
100	-3.01	-3.36
500	-3.00	-3.34
1,000	-2.99	-3.32
2,000	-2.98	-3.31
3,000	-2.97	-3.29
5,000	-2.95	-3.29
7,000	-2.95	-3.27
15,000	-2.93	-3.25
∞	-2.89	-3.21

$$\text{c.v.} = \underset{(0.15)}{0.62} - \underset{(0.09)}{1.83}\alpha \qquad R^2 = 0.99 \tag{2.33}$$

Similarly, asymptotic critical values for the constant and trend case can be estimated by OLS as below,

$$\text{c.v.} = -18.15 + 19.26\alpha - 6.00\alpha^2 \qquad R^2 = 0.99 \tag{2.34}$$
$$(1.65) \quad (1.92) \quad (0.57)$$

where c.v. is the abbreviation for critical value and the numbers in parantheses are the standard errors of the coefficients.

Caner (1998) is an influential study that extends the widely used OLS-based co-integration tests to infinite-variance errors. We will make size and power comparisons between the residual-based co-integration tests that are studied in Caner (1998) and residual-based co-integration tests through LAD estimation procedures that we propose. We consider Phillips and Perron (1988)' s Z_t statistic. The asymptotics of Z_t statistic are derived in Phillips and Ouliaris (1990) for residual-based co-integration tests and extended to the infinite-variance case in Caner (1998).

2.3.2 Size comparisons

Tables 2.7 and 2.8 show the size comparisons between Caner test and our test for no trend and trend cases (equations (2.28) and (2.29) respectively). While making the size comparisons, asymptotic critical values are used. Comparisons are made for α=1.5, 1.65, 1.9 with 50, 100, 500, 1,000 and 5,000 sample sizes with 20,000 iterations. The framework used for size calculations are as follows. First we consider a VAR(1) system of y_t and x_t where both series are simulated as unit root I(1) processes as in the system (2.27) with equal stability indices:

$$y_t = a_{10} + a_{11}y_{t-1} + a_{12}x_{t-1} + \varepsilon_{1t} \tag{2.35}$$

$$x_t = a_{20} + a_{21}y_{t-1} + a_{22}x_{t-1} + \varepsilon_{2t} \tag{2.36}$$

In each iteration, coefficients in equations (2.35) and (2.36) are estimated via OLS (LAD) method when we are calculating the size for Caner (our) test. Next, the residuals $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$

are taken and the stability indices (α) are estimated from each residual series. We find that especially for small sample size, the stability indices from the two residual series are not always found to be equal to each other.[5] In that case we follow a conservative approach: for Caner test we pick the smallest α level to determine the critical value and for our test we pick the largest α level to determine the critical value. The reason for that is because Caner test' s critical values decrease as α decreases for any size and our test' s critical values increase as α decreases. Last, we calculate the rejection rate with the asymptotic critical value corresponding to the estimated α to calculate the size of the test for each sample size.[6]

As the size comparison results indicate, both tests have size distortions for small sample sizes ($T = 50, 100$), but our test has more compared to Caner test. For larger samples, Caner test' s empirical sizes equal to the nominal size (5%), whereas our test has higher empirical size than 5%, which means the probability of making a type I error is greater than the predetermined 5% level.[7] Our test' s size distortions are not so severe as α approaches 2. Size evaluation results of no trend and trend cases are similar. It is also worthwhile to mention that, for Caner test, the convergence of the test statistic (Z_t) is faster: approximately 500 sample size is enough for Z_t to attain its asymptotic values. Our test, on the other hand, requires much larger sample sizes. Even though the convergence is faster for $\alpha > 1.7$, still a sample size of approximately 20,000 is needed for π_ϕ to attain its asymptotic values. For $1.5 \leq \alpha \leq 1.7$, the sample size required for convergence is around 30,000. For that purpose, the usage of size-corrected critical values is recommended for our test especially for small sample sizes and small α levels around 1.5.

2.3.3 Power comparisons

Decisions based on statistical tests can be wrong due to two kinds of errors: type I and type II errors. Type I error is the probability of rejecting the null when the null hypothesis is true. It denotes the significance level of a test and as a rule of thumb it is set equal to 1%, 5% or 10%.

[5] We make use of MLE procedure to estimate the stability indices.

[6] Asymptotic critical values for intermediate α levels can be found from equations (2.33) and (2.34).

[7] See section 2.3.3 on type I error.

Table 2.7: Empirical sizes for 5% nominal size (constant)

	α=1.5		α=1.65		α=1.9	
T	Caner test	Our test	Caner test	Our test	Caner test	Our test
50	0.07	0.18	0.07	0.12	0.07	0.07
100	0.06	0.15	0.06	0.11	0.06	0.07
500	0.05	0.12	0.05	0.10	0.05	0.07
1,000	0.05	0.11	0.05	0.09	0.05	0.06
5,000	0.05	0.08	0.05	0.07	0.05	0.07

Table 2.8: Empirical sizes for 5% nominal size (constant and trend)

	α=1.5		α=1.65		α=1.9	
T	Caner test	Our test	Caner test	Our test	Caner test	Our test
50	0.07	0.16	0.08	0.13	0.08	0.07
100	0.06	0.12	0.06	0.11	0.06	0.07
500	0.05	0.07	0.06	0.10	0.05	0.06
1,000	0.05	0.05	0.05	0.08	0.05	0.06
5,000	0.05	0.05	0.05	0.07	0.05	0.06

Here we will fix the size to 5%. Type II error is the probability of not rejecting the null when the null hypothesis is false. Power of a statistical test is equal to the probability of rejecting the null when it is false.[8] Power comparison results we present are based on size-adjusted power.

For power comparisons the data generating processes (DGPs) are chosen to be,

$$u_t = \rho u_{t-1} + v_t,\ v_t \sim \text{i.i.d. } S_\alpha(0,1,0), \tag{2.37a}$$

$$x_t = x_{t-1} + \varepsilon_t,\ \varepsilon_t \sim \text{i.i.d. } S_\alpha(0,1,0), \tag{2.37b}$$

$$y_t = a_1 + a_2 x_t + u_t \tag{2.37c}$$

where $a_1 = 0.9$ and $a_2 = 0.5$. After generating x_t and y_t series as above, the following regressions are run,

[8] Power = 1 - type II error.

$$y_t = \beta_0 + \beta_1 x_t + e_t \tag{2.38}$$

$$y_t = \beta_0 + \beta_1 t + \beta_2 x_t + e_t \tag{2.39}$$

Note that x_t and y_t are co-integrated for $\rho < 1$ and not co-integrated for $\rho = 1$ in the system (2.37). This DGP is similar to the ones in Blangiewicz and Charemza (1990) (DGP 2) and Engle and Yoo (1987). Graphs of co-integrated and non-co-integrated series that are derived from the system (2.37) with $a_1 = 0$ are illustrated in Figure 2.1. Panels (a), (c) and (e) are produced by simulating stable random variables with $\alpha = 1.7$, $\beta = 0$, $\sigma = 1$ and $\mu = 0$ for the errors in the DGP. Panels (b), (d) and (f) are produced by simulating standard normal random errors. For three-variable case, two explanatory variables x_{1t} and x_{2t} are generated as I(1). The DGPs in that case are as follows,

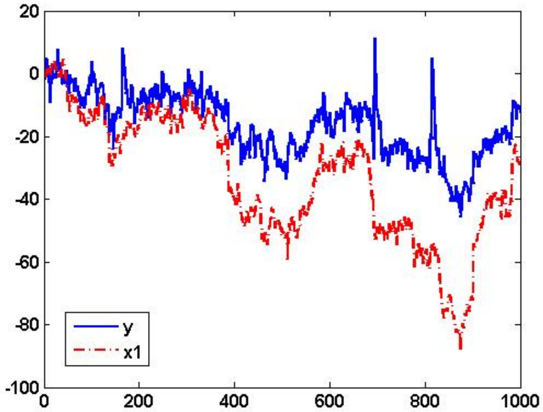
$$u_t = \rho u_{t-1} + v_t,\ \ v_t \sim \text{i.i.d. } S_\alpha(0,1,0), \tag{2.40a}$$

$$x_{i,t} = x_{i,t-1} + \varepsilon_{i,t},\ \ \varepsilon_{i,t} \sim \text{i.i.d. } S_\alpha(0,1,0),\ \ i = 1,2 \tag{2.40b}$$

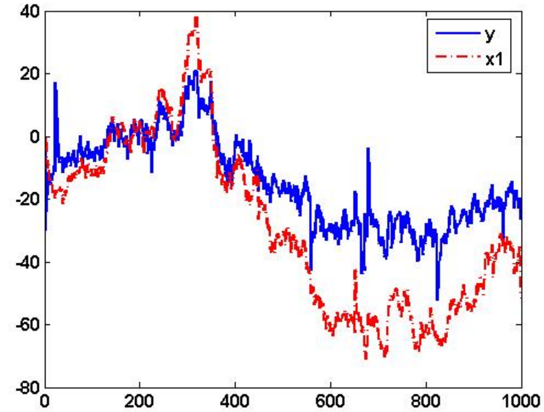
$$y_t = 0.5x_{1t} + 0.5x_{2t} + u_t \tag{2.40c}$$

For power comparisons, a unit root test is applied on the residuals ($\hat{e}_t$) from equations (2.38) and (2.39): no trend and trend included in co-integrating equation. There are two test statistics considered: Z_t and π_ϕ. Z_t is calculated for OLS estimations and π_ϕ is calculated for LAD estimations. The rejection rate for the unit root tests are calculated for different values of $\rho = 1, 0.9, 0.8$,...etc.. The number of iterations is 20,000. Some results from power comparisons are given in Figures 2.2 (constant) and 2.3 (constant and trend). In the figures, T denotes the sample size and α is the stability index.

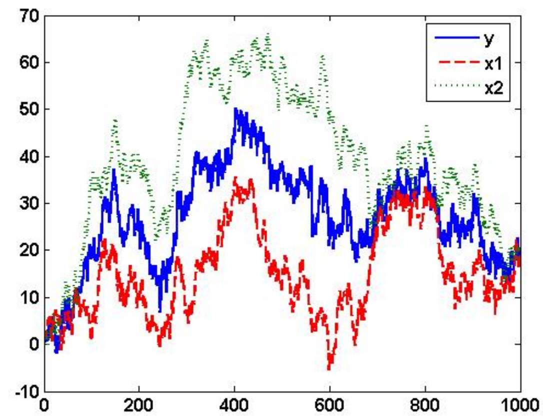
Results of power comparisons show that as the sample size gets larger and tails of the distribution gets heavier, LAD-based co-integration tests that we consider have superior power over the conventional OLS-based ones with infinite-variance errors. If the sample size gets



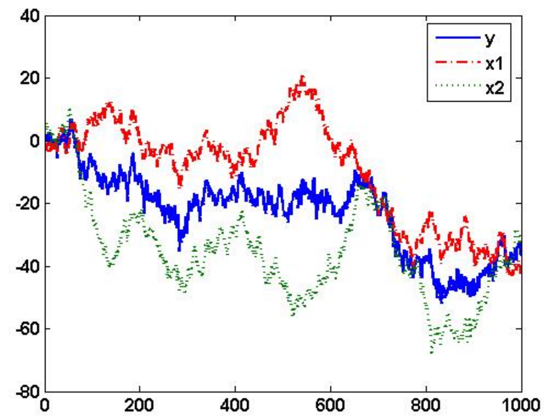
(a) Two co-integrated series with $S_{1.7}(0,1,0)$ errors



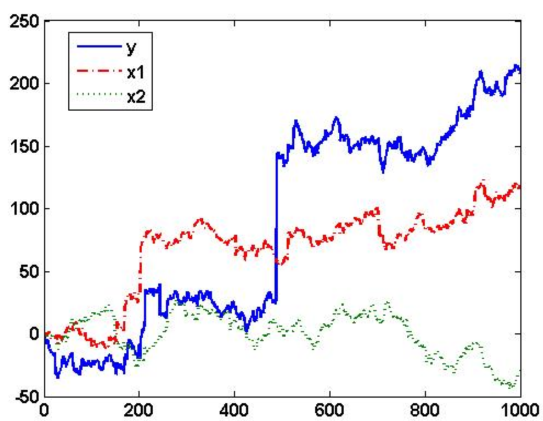
(b) Two co-integrated series with $N(0,1)$ errors



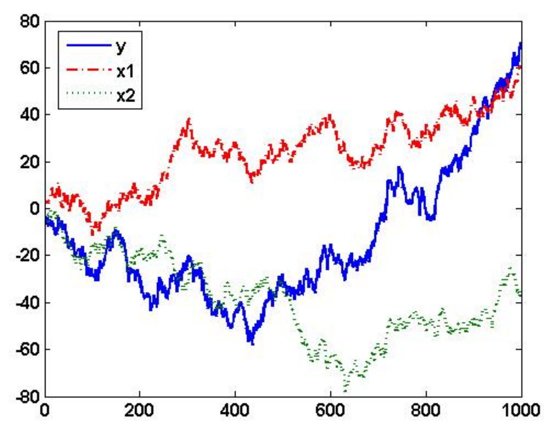
(c) Three co-integrated series with $S_{1.7}(0,1,0)$ errors



(d) Three co-integrated series with $N(0,1)$ errors

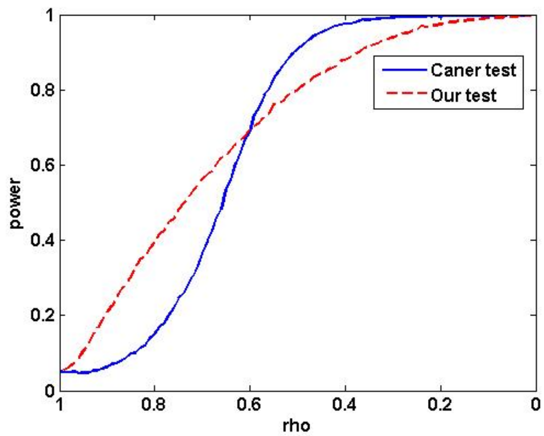


(e) Three non-co-integrated series with $S_{1.7}(0,1,0)$ errors

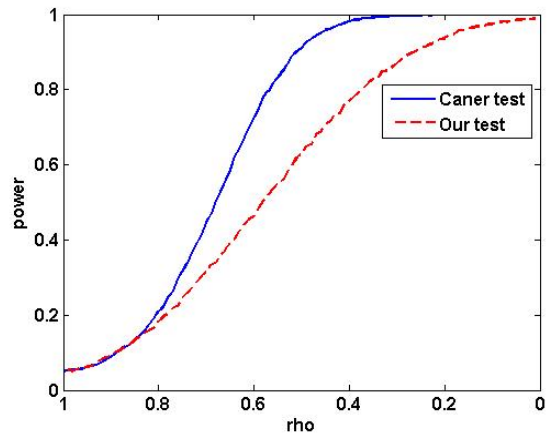


(f) Three non-co-integrated series with $N(0,1)$ errors

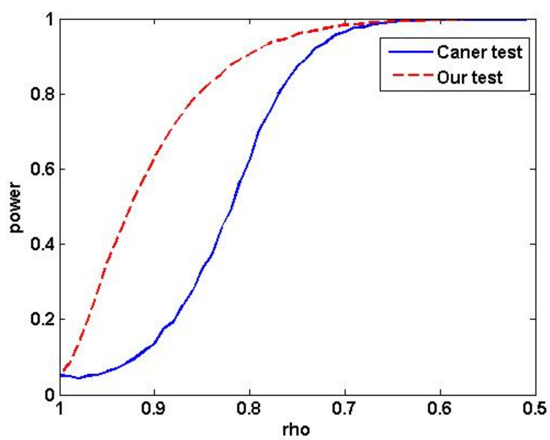
Figure 2.1: Graphs of co-integrated and non-co-integrated series with stable and normal errors derived from systems (2.37) (with $a_1 = 0$) and (2.40)



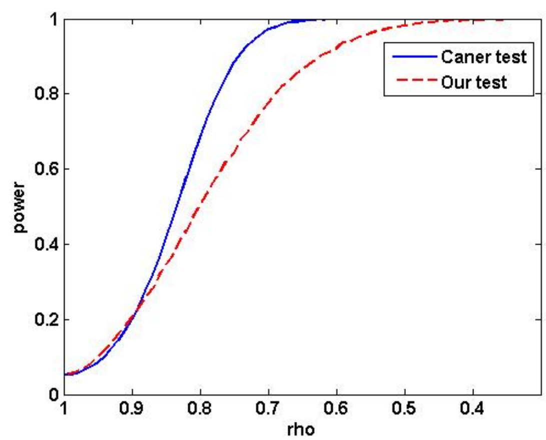
(a) $\alpha = 1.5$, $T = 50$



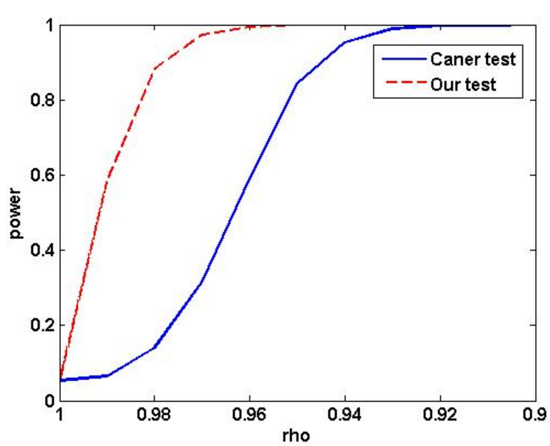
(b) $\alpha = 1.9$, $T = 50$



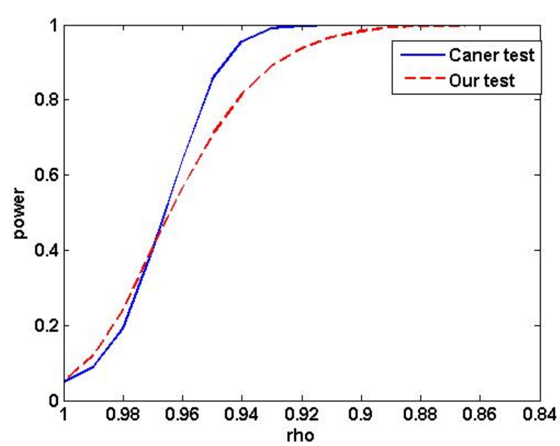
(c) $\alpha = 1.5$, $T = 100$



(d) $\alpha = 1.9$, $T = 100$

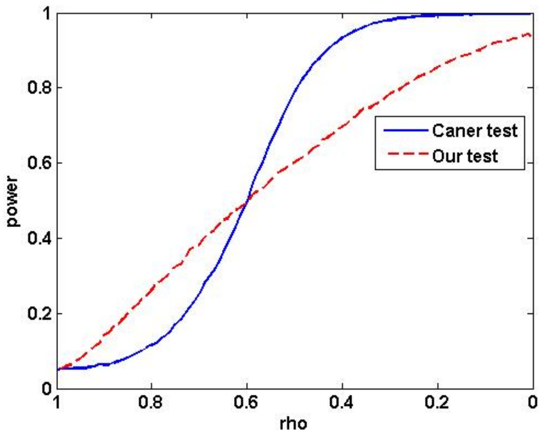


(e) $\alpha = 1.5$, $T = 500$

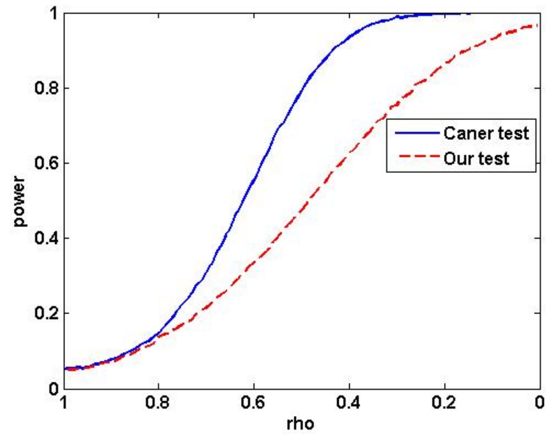


(f) $\alpha = 1.9$, $T = 500$

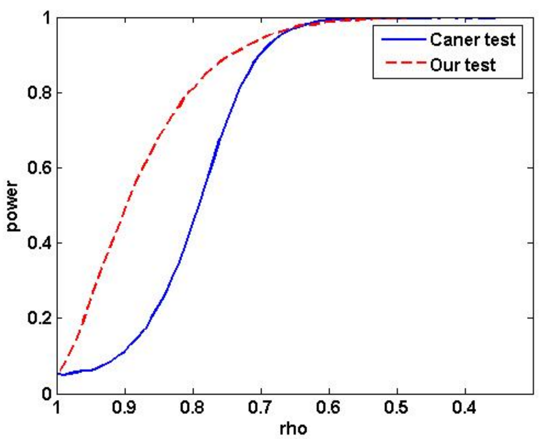
Figure 2.2: Size-adjusted power comparisons (constant)



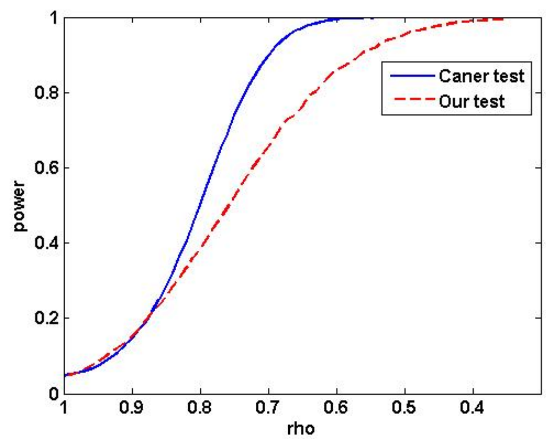
(a) $\alpha = 1.5$, $T = 50$



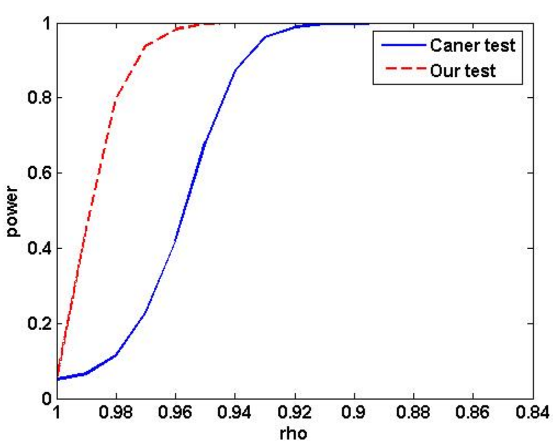
(b) $\alpha = 1.9$, $T = 50$



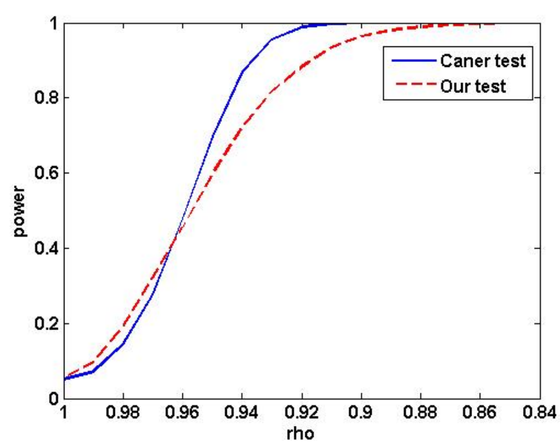
(c) $\alpha = 1.5$, $T = 100$



(d) $\alpha = 1.9$, $T = 100$



(e) $\alpha = 1.5$, $T = 500$



(f) $\alpha = 1.9$, $T = 500$

Figure 2.3: Size-adjusted power comparisons (constant and trend)

large, empirical power of both tests converge to 100% fast for all α levels (see panels (e) and (f) in Figures 2.2 and 2.3). For infinite-variance case, our test has superior power as the sample size exceeds 50. Furthermore, as α approaches 2 and sample size gets large, our test still has some power advantages if the co-integrating regression' s residuals are near unit root. The effects of past errors do not die out fast for near unit root processes. If the errors are stably distributed with $\alpha < 2$, their distributions are subject to more volatility compared to normal distribution. In that case, LAD-based tests have a slightly better performance in distinguishing between co-integration and no co-integration (see panels (f) in Figures 2.2 and 2.3).

CHAPTER 3. EMPIRICAL APPLICATION

In this chapter, we test the forward rate unbiasedness hypothesis (FRUH) for our data through residual-based co-integration tests and apply the new LAD-based test from chapter 2. The dataset contains Australian dollar, Canadian dollar, French franc, German mark, Italian lira, Japanese yen, Swiss franc and U.K. pound spot exchange rates as well as 1-month, 3-month, 6-month, 1-year forward exchange rates of daily frequency. Data span runs from 01/01/1985 to 11/30/2007 for Australia and Canada ($T = 5,979$); from 01/02/1984 to 11/30/2007 for Japan, Switzerland and U.K. ($T = 6,240$) and from 01/01/1997 to 11/30/2007 for Germany, France and Italy ($T = 2,848$). Data is taken from Datastream. Australia, Canada, Japan, Switzerland and U.K. data source is Barclay Bank PLC. The data source of Germany, France and Italy is WM/Reuters. Following Caner (1998), if we let Y_t be a k-vector integrated process:

$$Y_t = Y_{t-1} + \varepsilon_t, \;\; \varepsilon_t \sim \text{i.i.d. } S_\alpha(0,1,0), \; \alpha < 2 \tag{3.1}$$

Then partitioning Y_t vector into scalar Y_{1t} and m-vector Y_{2t},[1] the co-integrating regressions can be written as,

$$Y_{1t} = \hat{\beta}' Y_{2t} + u_t, \tag{3.2}$$

$$\hat{u}_t = \hat{\rho}\hat{u}_{t-1} + v_t \tag{3.3}$$

[1] $Y_t = (Y_{1t}, Y_{2t}')'$.

Equations (3.2) and (3.3) are utilized with OLS estimations to apply Caner (1998) test. Following equations are utilized with LAD estimations to apply our test,

$$Y_{1t} = \hat{\beta}' Y_{2t} + u_t, \tag{3.4}$$

$$\Delta \hat{u}_t = \phi \hat{u}_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta \hat{u}_{t-i+1} + v_t,\ v_t \sim \text{i.i.d.} \tag{3.5}$$

3.1 Diagnostic checks

As a first step in empirical analysis we investigate whether or not an infinite-variance stable assumption is reasonable to describe the errors. We start by estimating the stable parameters of the residuals. By Caner (1998)' s assumptions, we assume that logged values of spot and forward exchange rates are I(1) processes and can be represented by a finite-order bivariate VAR process. The estimated residuals from those VARs can be replaced by the unobserved error terms if the correct model is selected and the VAR coefficients are consistent. Calder and Davis (1998) has shown that OLS and LAD estimators are consistent for ARMA(p, q) models under the infinite-variance assumption. Since we are assuming finite-order VARs, the consistency results of Calder and Davis (1998)' s study can be extended to our case for each of the VAR equations (see Falk and Wang (2003)).

The procedure we choose for estimating stable parameters is the maximum likelihood estimation (MLE) procedure. The procedure requires data to be i.i.d.. Thus before implementing the MLE method, appropriate VAR models should be selected. Standard Box and Jenkins (1976) approach of model selection assumes Gaussian errors but the model selection tools that are used in Gaussian case can also be generalized to infinite-variance stable case though one needs to be more cautious (Adler et al., 1998).

3.1.1 Model selection

Box-Jenkins model selection involves fitting a linear ARMA(p, q) model to a time series x_t,

$$x_t - \phi_1 x_{t-1} - \ldots - \phi_p x_{t-p} = \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t,\ \varepsilon_t \sim \text{i.i.d.} \tag{3.6}$$

where ε_t is either normally or stably distributed. After fitting a model, i.i.d. tests are applied on the residuals. Depending on which distribution the errors come from, the limit theorems of test statistics of i.i.d. tests change (Runde (1997); Adler et al. (1998); Lin and McLeod (2008)).

One of the stylized empirical facts of exchange rate series of high frequency data is the fact that the returns exhibit very little serial correlation and the series itself behave as a random walk (see Cont (2001, 2007)). Following this stylized fact in literature, we start with a VAR(1) model for the spot and forward exchange rates. Since we have 4 different forward rates (1-month, 3-month, 6-month and 1-year), we consider 8 residual series for each country, in total yielding 64 residual series. VARs considered are,

$$s_t = a_{10} + a_{11} s_{t-1} + a_{12} f^i_{t-1} + \varepsilon_{1t} \tag{3.7a}$$

$$f^i_t = a_{20} + a_{21} s_{t-1} + a_{22} f^i_{t-1} + \varepsilon_{2t} \tag{3.7b}$$

where s_t and f^i_t are log spot rate and log forward rate respectively with i being the index that determines the maturity of the forward rate. In particular, $i = 1, 2, 3, 4$ correspond to 1-month, 3-month, 6-month and 1-year maturities respectively. Each residual series is checked for serial independence.

In general, if a finite-variance Gaussian series x_t is i.i.d. noise then,

$$\gamma_x(h) = \begin{cases} \sigma_x^2 & \text{if } h = 0 \\ 0 & \text{if } h \neq 0 \end{cases}$$

where $\gamma_x(h) = \text{cov}(x_{t+h}, x_t)$. Autocorrelation function at lag h is defined as,

$$r_x(h) = \frac{\gamma_x(h)}{\gamma_x(0)} \tag{3.8}$$

Sample autocorrelation function (SACF) is calculated as,

$$\hat{r}_x(h) = \frac{\sum_{t=1}^{T-h} x_t x_{t+h}}{\sum_{t=1}^{T} x_t^2} \tag{3.9}$$

Mean-corrected version of sample autocorrelations is,

$$\tilde{r}_x(h) = \frac{\sum_{t=1}^{T-h}(x_t - \bar{x})(x_{t+h} - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} \tag{3.10}$$

Under the null hypothesis of i.i.d. noise,

$$H_0 : r(1) = r(2) = \ldots = r(m) = 0$$

against

$$H_1 : \text{At least one of the autocorrelations is not zero}$$

all autocorrelations must be zero for $|h| > 0$ and,

$$(\sqrt{T}\widetilde{r}(1), \sqrt{T}\widetilde{r}(2), \ldots, \sqrt{T}\widetilde{r}(m)) \xrightarrow{d} N(0_m, I_m) \text{ as } T \to \infty$$

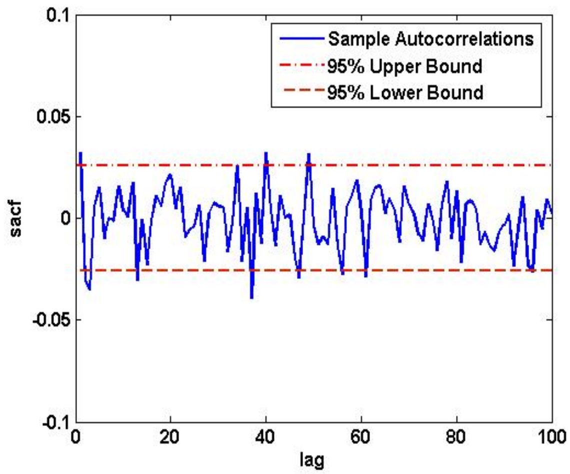
where $\xrightarrow{d}$ denotes convergence in distribution, 0_m is m-dimensional zero vector and I_m is $m \times m$ identity matrix. Thus in practice with a sample size T, if one plots the sample autocorrelations $\tilde{r}(h)$ against h, 95% of them should lie between the 95% Gaussian confidence bounds: $\pm 1.96\, T^{-1/2}$.

In addition to SACFs, sample partial autocorrelations are also checked to determine independence. Autocorrelations calculate the correlations between two values of the time series which are separated by h time periods. The partial autocorrelation function (SPACF) on the other hand, calculates the correlation between the values of the time series separated h time periods after removing the effects of the intervening time periods. In other words, the indirect effects of correlations are not present in the SPACF structure.

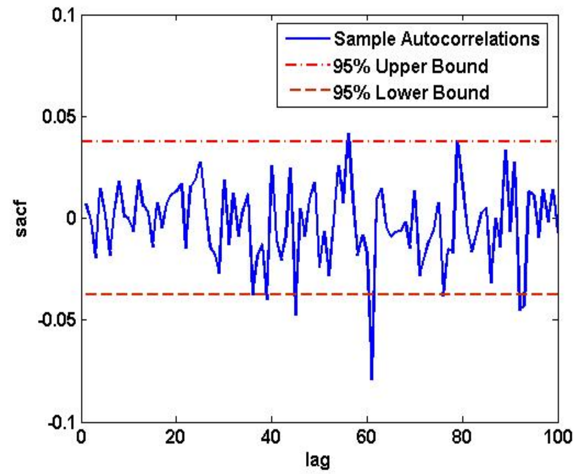
When there is heavy-tailed structure in the data and variances and even means may be infinite, the mathematical correlations do not exist. However, SACFs and SPACFs still can be used as useful tools in diagnostic checks, because they both have limiting distributions. Quantiles of those limiting distributions have been obtained in Adler et al. (1998), section 3. Adler et al. (1998) run Monte Carlo simulations to construct confidence intervals for model identification to test for three different distributions' (a stable distribution with correct α, Cauchy distribution and normal distribution) performances in selecting the correct model. They suggest using Cauchy bounds for "small" sample sizes near 1,000, as they perform better than the other two distributions' bounds in selecting the correct model. However, as the sample size increases and as $\alpha \geq 1.7$, Gaussian bounds also have a good performance in model identification. The conventional Gaussian bounds can still be employed to check for independence of the residuals.

Mikosch (1998) also suggests the usage of classical confidence bounds with Box-Jenkins way of model selection under stably distributed error assumption but in a more conservative sense. Following Adler et al. (1998) and Mikosch (1998) studies, Tokat et al. (2003) use more conservative confidence bounds of $\pm 2.57\,T^{-1/2}$ in model selection for variables with stable indices less than 1.7 and conventional Gaussian bounds for variables with stable indices greater than 1.7. Based on Tokat et al. (2003), the SACF and SPACF structure of bivariate first-order VAR residuals are also demonstrated with more conservative bounds of $\pm 2.57\,T^{-1/2}$ in addition to the classical Gaussian bounds.

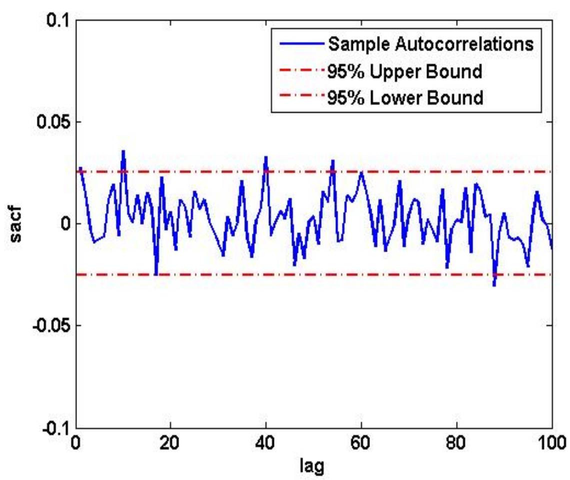
SACFs and SPACFs of the LAD-based VAR(1) residuals with $i = 1$ in equation (3.7a) for Australia, France, Japan and U.K. are given in Figures 3.1, 3.2, 3.3 and 3.4. Similar results



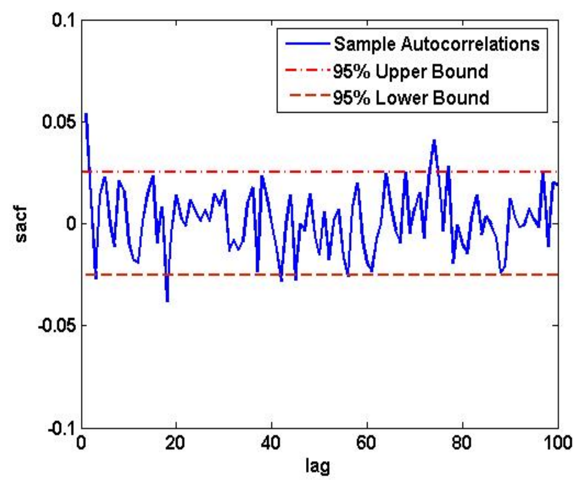
(a) Australian residuals



(b) French residuals

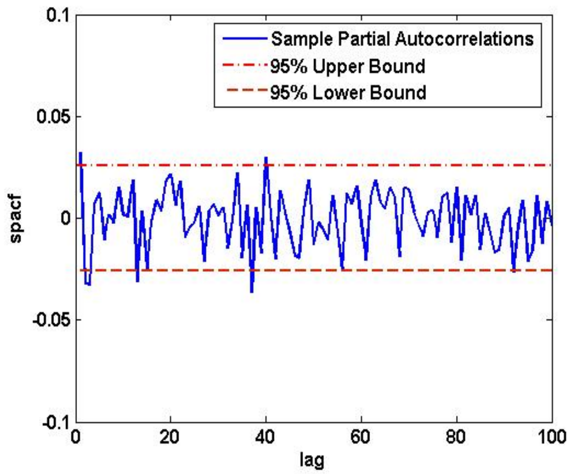


(c) Japanese residuals

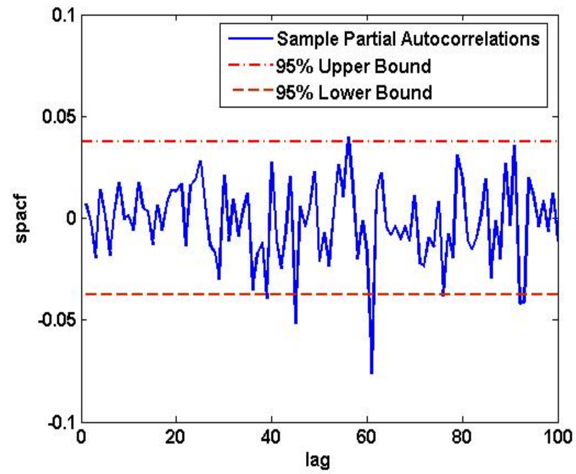


(d) U.K. residuals

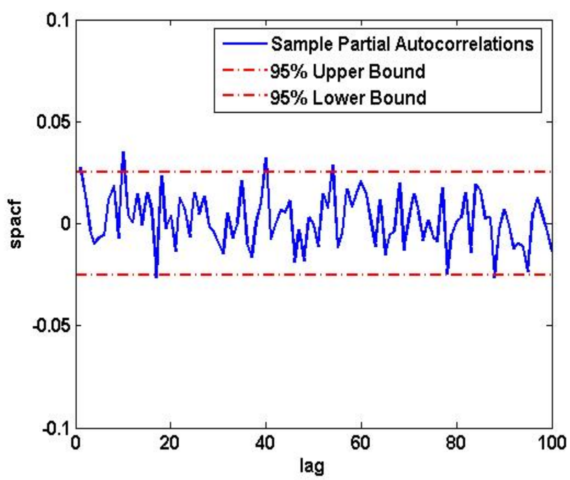
Figure 3.1: SACFs with Gaussian confidence bounds: $\pm 1.96\frac{1}{\sqrt{T}}$



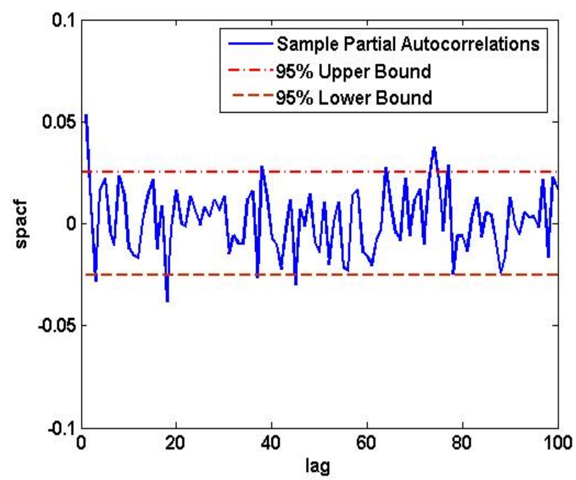
(a) Australian residuals



(b) French residuals

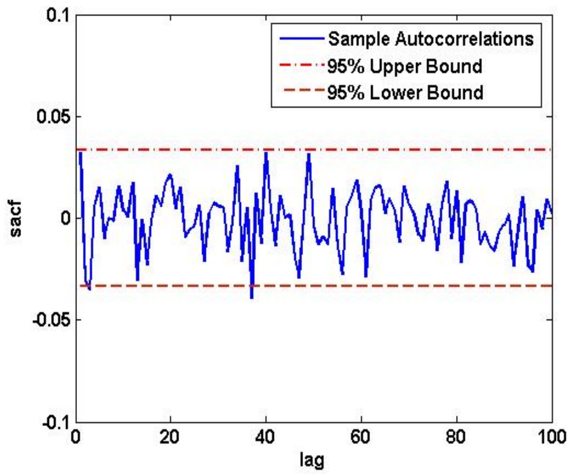


(c) Japanese residuals

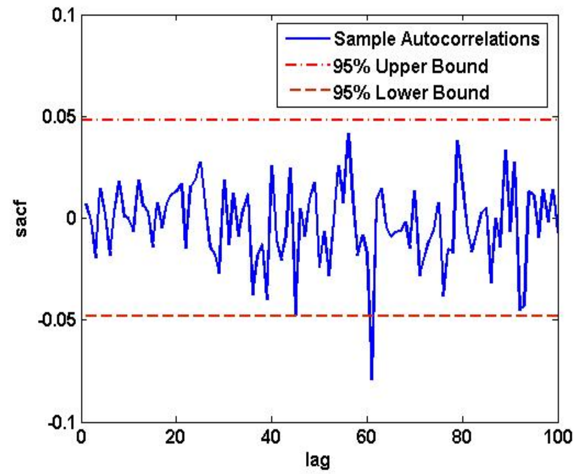


(d) U.K. residuals

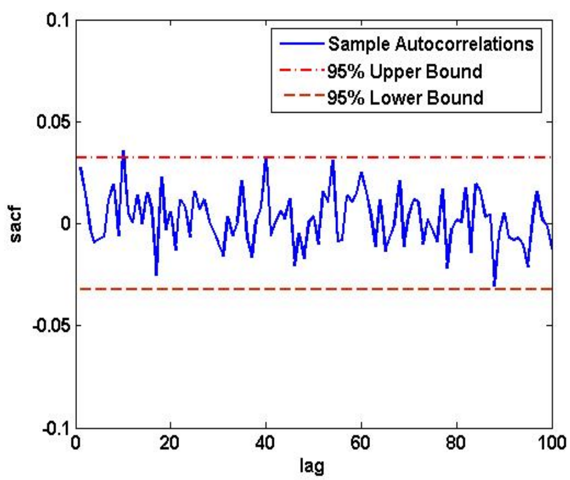
Figure 3.2: SPACFs with Gaussian confidence bounds: $\pm 1.96\frac{1}{\sqrt{T}}$



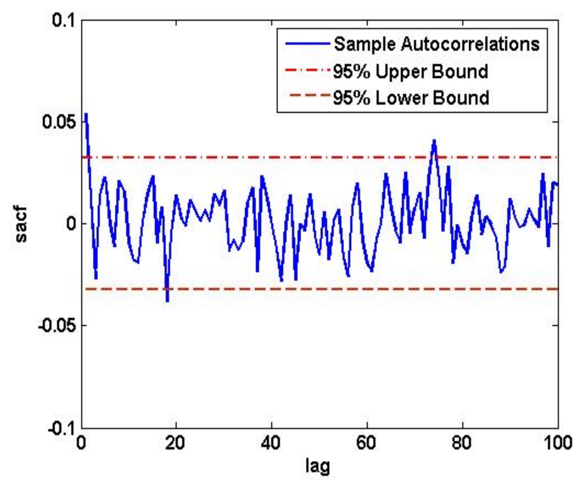
(a) Australian residuals



(b) French residuals

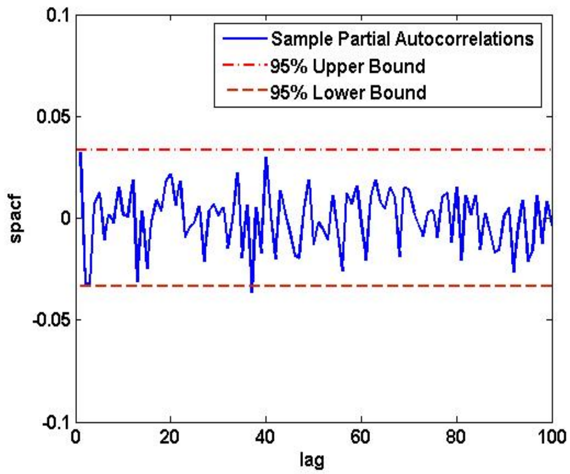


(c) Japanese residuals

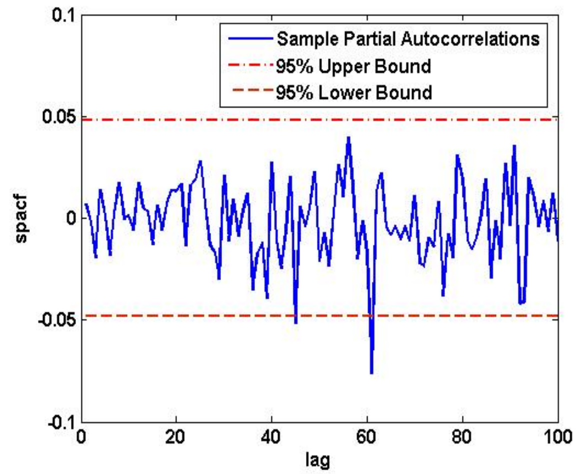


(d) U.K. residuals

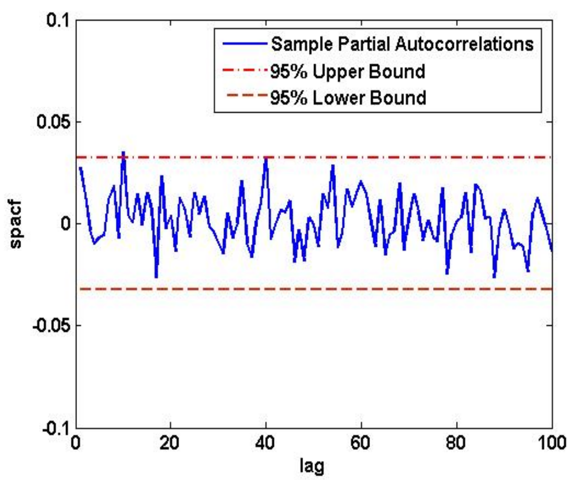
Figure 3.3: SACFs with conservative confidence bounds: $\pm 2.57\frac{1}{\sqrt{T}}$



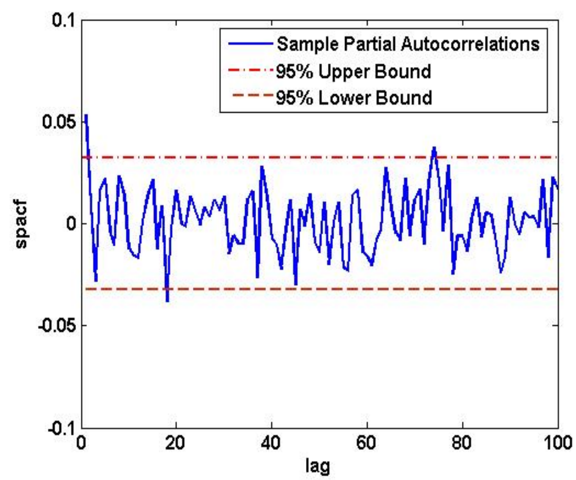
(a) Australian residuals



(b) French residuals



(c) Japanese residuals



(d) U.K. residuals

Figure 3.4: SPACFs with conservative confidence bounds: $\pm 2.57\frac{1}{\sqrt{T}}$

are derived for OLS-based VARs' residual series and other equations. Based on the SACF and SPACF structures, accepting the i.i.d. hypothesis seems like a reasonable conclusion to make as all the SACFs and SPACFs are close to zero and lie between the confidence bounds approximately 95% of the time. Especially with the more conservative confidence bounds, the autocorrelations and partial autocorrelations indicate a random structure in the residuals. However, in order to make a certain decision on independence, residual series are tested for randomness via Portmanteau type tests. Gallagher (2001) has shown that, as α approaches 2, the large sample distribution of SACF provides a poor approximation and the accuracy of the tests depending on the limit theorems of SACFs is low under the stable distribution assumption. Thus we proceed with other i.i.d. tests to reach a more definite conclusion.

3.1.2 Portmanteau type tests

We further investigate the independence of our residuals by using Portmanteau type tests for infinite-variance stable variables which are proposed in a recent study of Lin and McLeod (2008).[2] They consider tests that are stable analogues of Box and Pierce (1970) and Peña and Rodriguez (2002) tests.

In Gaussian case, with i.i.d. assumption, Box-Pierce statistic,

$$\tilde{Q}_{T,m} = T \sum_{h=1}^{m} \tilde{r}^2(h), \;\; h = 1, \ldots, m \tag{3.11}$$

will have an asymptotic $\chi^2_{(m)}$ distribution, with m being the number of lags.

Peña-Rodriguez test statistic,

$$\tilde{D}_m = T(1 - |\tilde{R}_m|^{1/m}) \tag{3.12}$$

where $|\tilde{R}_m|$ is the determinant of the residual correlation matrix of order m,

[2] See http://www.stats.uwo.ca/faculty/aim/2007/LinMcLeod/.

$$\tilde{R}_m = \begin{bmatrix} 1 & \tilde{r}(1) & \dots & \tilde{r}(m) \\ \tilde{r}(1) & 1 & \dots & \tilde{r}(m-1) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{r}(m) & \tilde{r}(m-1) & \dots & 1 \end{bmatrix}$$

will have an asymptotic $Gamma(a, b)$ distribution with parameters defined as,

$$a = \frac{3m[(m+1) - 2(p+q)]^2}{2[2(m+1)(2m+1) - 12m(p+q)]} \tag{3.13}$$

and

$$b = \frac{3m[(m+1) - 2(p+q)]}{2(m+1)(2m+1) - 12m(p+q)} \tag{3.14}$$

For stable case, the limit distributions of $\tilde{Q}_{T,m}$ and $\tilde{D}_m$ test statistics are studied in Runde (1997) and Lin and McLeod (2008).[3] The convergence of both test statistics to their limiting distributions are very slow and require very large sample sizes in practice for those tests to be reliable. Therefore Lin and McLeod (2008) develop new Monte Carlo tests (or parametric bootstraps) which work as stable analogues of Box and Pierce (1970) and Peňa and Rodriguez (2002) tests for small sample size. Their study also includes new Monte Carlo tests that are developed for the same test statistics with normally distributed errors.

Lin and McLeod (2008) investigate the robustness of Box and Pierce (1970) and Peňa and Rodriguez (2002) tests with normally distributed errors to infinite-variance errors and show that the tests are not robust to infinite-variance assumption. As an empirical illustration, they test the daily S&P 500 stock returns and monthly CRSP value-weighted index returns for independence. They show that Portmanteau type tests studied in Box and Pierce (1970) and Peňa and Rodriguez (2002) more often leads to a rejection of i.i.d. hypothesis if one mistakenly

[3] Peňa-Rodriguez test has better power than Box-Pierce test both for normal and stable errors (Peňa and Rodriguez (2002); Lin and McLeod (2008)).

assumes that the returns are normally distributed instead of being stably distributed. Also for a large enough lag length, the randomness assumption of daily stock returns is strongly rejected under the normality assumption but not rejected under the stable distribution assumption.

We use Lin-McLeod Monte Carlo tests (both for normal and stable cases), to test for independence. P-values of the Lin-McLeod i.i.d. tests are given in Tables 3.1 and 3.2 for Australia and France respectively. The conclusions drawn from the tests are similar for the exchange rate data of France, Germany, Italy, Japan and Switzerland: the i.i.d. hypothesis is not rejected no matter which distributional assumption is made since all p-values are unambigously greater than 0.05. On the other hand, for Australia, Canada and U.K. data, i.i.d. assumption can not be rejected only for the stable case. The hypothesis is generally rejected if we make the normality assumption and use "normal" versions of Peňa-Rodriguez and Box-Pierce tests. Test results are considerably different for Australia and U.K., at lags 50 and 100. P-values under normality assumption tend to be very low, rejecting the i.i.d. error hypothesis, whereas the opposite occurs under stable distribution assumption. In Tables 3.1 and 3.2, it is worthwhile noting that the new Monte Carlo tests of Lin-McLeod using normal random variables yield very similar results as the Box-Pierce tests with chi-square asymptotic distribution. Tables 3.1 and 3.2 present the results of LAD residuals. We have a similar structure for OLS residuals.

3.1.3 Descriptive statistics and normality tests

In this section we explore data characteristics of exchange rate returns. Some descriptive statistics are presented in Tables 3.3 through 3.10. The return series seem to be asymmetric (mostly negatively skewed except for Australian, Canadian and U.K. log returns) and have means zero. Excess kurtosis or kurtosis greater than 3 (which is the kurtosis level associated with normal distribution) is observed for all data series. Jarque and Bera (1980) test, which tests the null hypothesis of normality based on the sample skewness and kurtosis, rejects the null hypothesis since all p-values are less than 0.05. Jarque-Bera test statistic is,

Table 3.1: P-values of randomness tests for Australian exchange rates from VAR(1) model

Spot Rate & 1-month Forward Rate					
lag	P-R Stable[a]	P-R Normal[b]	B-P Stable[c]	B-P Normal[d]	B-P Chi-square[e]
25	0.039	0.002	0.056	0.022	0.021
50	0.049	0.003	0.061	0.005	0.005
100	0.069	0.005	0.113	0.049	0.052
1-month Forward Rate & Spot Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.039	0.002	0.056	0.022	0.021
50	0.049	0.003	0.061	0.005	0.005
100	0.069	0.005	0.113	0.049	0.052
Spot Rate & 3-month Forward Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.040	0.002	0.058	0.022	0.020
50	0.048	0.003	0.061	0.005	0.005
100	0.070	0.005	0.114	0.049	0.052
3-month Forward Rate & Spot Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.043	0.003	0.050	0.009	0.007
50	0.046	0.001	0.056	0.001	0.002
100	0.063	0.002	0.106	0.030	0.029
Spot Rate & 6-month Forward Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.038	0.002	0.058	0.022	0.019
50	0.047	0.003	0.060	0.004	0.005
100	0.070	0.005	0.113	0.049	0.050
6-month Forward Rate & Spot Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.019	0.000	0.030	0.000	0.000
50	0.032	0.000	0.054	0.001	0.001
100	0.055	0.000	0.102	0.023	0.022

Table 3.1: (Continued)

Spot Rate & 1-year Forward Rate					
lag	P-R Stable[a]	P-R Normal[b]	B-P Stable[c]	B-P Normal[d]	B-P Chi-square[e]
25	0.039	0.001	0.058	0.021	0.019
50	0.046	0.003	0.060	0.004	0.005
100	0.069	0.005	0.114	0.048	0.048
1-year Forward Rate & Spot Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.075	0.049	0.088	0.077	0.068
50	0.098	0.094	0.120	0.143	0.145
100	0.122	0.148	0.172	0.270	0.278

[a] Peňa-Rodriguez test statistic with stably distributed variables.
[b] Peňa-Rodriguez test statistic with normally distributed variables.
[c] Box-Pierce test statistic with stably distributed variables.
[d] Box-Pierce test statistic with normally distributed variables.
[e] Box-Pierce chi-square test statistic with normally distributed variables.

$$JB = \frac{T}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \tag{3.15}$$

where S is sample skewness and K is sample kurtosis defined as,

$$S = \frac{\frac{1}{T}\sum_{i=1}^{T}(x_i - \bar{x})^3}{\left(\frac{1}{T}\sum_{i=1}^{T}(x_i - \bar{x})^2\right)^{3/2}} \tag{3.16}$$

$$K = \frac{\frac{1}{T}\sum_{i=1}^{T}(x_i - \bar{x})^4}{\left(\frac{1}{T}\sum_{i=1}^{T}(x_i - \bar{x})^2\right)^{2}} \tag{3.17}$$

where $\bar{x}$ is the sample mean of the series which is being tested for normality. JB statistic is asymptotically chi-squared with 2 degrees of freedom.

Table 3.2: P-values of randomness tests for French exchange rates from VAR(1) model

Spot Rate & 1-month Forward Rate					
lag	P-R Stable[a]	P-R Normal[b]	B-P Stable[c]	B-P Normal[d]	B-P Chi-square[e]
25	0.831	0.995	0.743	0.983	0.985
50	0.685	0.972	0.434	0.758	0.760
100	0.319	0.564	0.131	0.196	0.237
1-month Forward Rate & Spot Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.782	0.987	0.716	0.977	0.978
50	0.665	0.962	0.393	0.709	0.715
100	0.276	0.491	0.109	0.136	0.162
Spot Rate & 3-month Forward Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.823	0.995	0.739	0.983	0.985
50	0.682	0.970	0.427	0.752	0.754
100	0.308	0.548	0.129	0.181	0.221
3-month Forward Rate & Spot Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.790	0.987	0.717	0.977	0.978
50	0.667	0.960	0.385	0.688	0.693
100	0.268	0.478	0.097	0.113	0.141
Spot Rate & 6-month Forward Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.823	0.995	0.743	0.986	0.986
50	0.683	0.970	0.421	0.739	0.748
100	0.305	0.538	0.127	0.173	0.211
6-month Forward Rate & Spot Rate					
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.802	0.990	0.732	0.980	0.981
50	0.676	0.962	0.392	0.683	0.689
100	0.267	0.474	0.097	0.109	0.137

Table 3.2: (Continued)

	Spot Rate & 1-year Forward Rate				
lag	P-R Stable[a]	P-R Normal[b]	B-P Stable[c]	B-P Normal[d]	B-P Chi-square[e]
25	0.823	0.995	0.755	0.987	0.988
50	0.682	0.970	0.414	0.723	0.735
100	0.291	0.519	0.120	0.155	0.192
	1-year Forward Rate & Spot Rate				
lag	P-R Stable	P-R Normal	B-P Stable	B-P Normal	B-P Chi-square
25	0.825	0.995	0.772	0.989	0.991
50	0.697	0.976	0.401	0.708	0.714
100	0.281	0.507	0.101	0.120	0.153

[a] Peña-Rodriguez test statistic with stably distributed variables.
[b] Peña-Rodriguez test statistic with normally distributed variables.
[c] Box-Pierce test statistic with stably distributed variables.
[d] Box-Pierce test statistic with normally distributed variables.
[e] Box-Pierce chi-square test statistic with normally distributed variables.

Table 3.3: Descriptive statistics and normality tests of Australian log returns

	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.005	0.005	0.007	0.007	0.007
Skewness	0.722	0.719	0.611	0.716	0.600
Kurtosis	11.059	9.163	11.361	9.189	8.599
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

Table 3.4: Descriptive statistics and normality tests of Canadian log returns

	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.004	0.004	0.004	0.004	0.004
Skewness	0.102	0.116	0.111	0.126	0.158
Kurtosis	6.185	6.186	6.144	6.092	7.098
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

Table 3.5: Descriptive statistics and normality tests of French log returns

	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.006	0.006	0.006	0.006	0.006
Skewness	-0.214	-0.205	-0.202	-0.198	-0.195
Kurtosis	4.086	4.172	4.166	4.184	4.182
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

Table 3.6: Descriptive statistics and normality tests of German log returns

	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.006	0.006	0.006	0.006	0.006
Skewness	-0.209	-0.195	-0.192	-0.188	-0.182
Kurtosis	4.066	4.159	4.153	4.170	4.163
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

Table 3.7: Descriptive statistics and normality tests of Italian log returns

	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.006	0.006	0.006	0.006	0.006
Skewness	-0.206	-0.208	-0.204	-0.200	-0.198
Kurtosis	4.084	4.099	4.088	4.102	4.092
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

Table 3.8: Descriptive statistics and normality tests of Japanese log returns

	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.007	0.007	0.007	0.007	0.007
Skewness	-0.543	-0.515	-0.546	-0.557	-0.589
Kurtosis	7.718	7.957	7.877	7.804	8.435
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

Table 3.9: Descriptive statistics and normality tests of Swiss log returns

	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.007	0.007	0.007	0.007	0.007
Skewness	-0.199	-0.172	-0.175	-0.176	-0.144
Kurtosis	5.269	5.332	5.379	5.325	5.478
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

Table 3.10: Descriptive statistics and normality tests of U.K. log returns

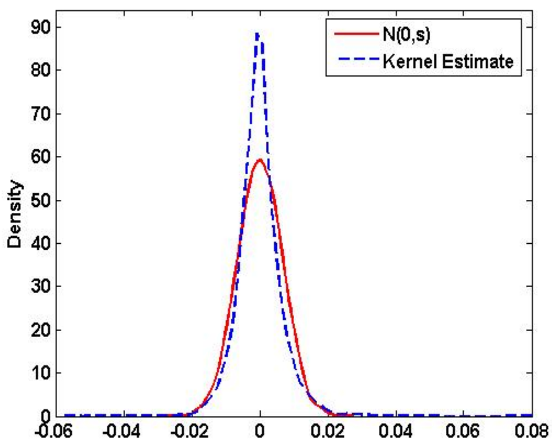
	Spot rate	1-month fwd.	3-month fwd.	6-month fwd.	1-year fwd.
Mean	-0.000	-0.000	-0.000	-0.000	-0.000
Median	0.000	0.000	0.000	0.000	0.000
St. Deviation	0.006	0.006	0.006	0.006	0.006
Skewness	-0.095	-0.110	-0.121	-0.165	-0.088
Kurtosis	6.952	7.015	6.939	7.426	7.086
JB Test[a]	0.000	0.000	0.000	0.000	0.000

[a] P-values of Jarque-Bera normality test.

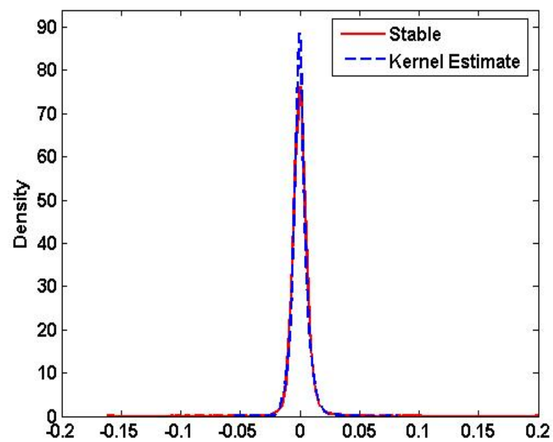
3.1.4 Graphical investigation

Kernel density estimates of LAD-based VAR(1) residuals with $i = 1$ from equation (3.7a) are compared to normal densities and stable densities. Figure 3.5 gives the densities of Australian and French residuals. Stable and normal ($N(0, s)$ with s denoting the sample standard deviation) densities are estimated by simulating stable and normal random variables with parameters imposed from the actual data series. We observe that stable density provides a better fit than the normal density for the high peaked series. In order to better visualize the heavy-tailed structure, quantile quantile (QQ) plots are also provided.

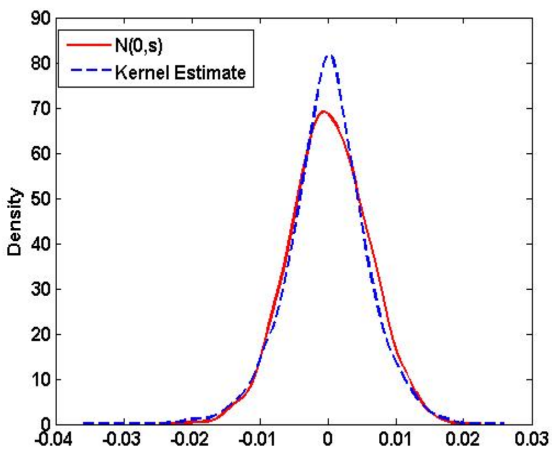
QQ plots indicate a heavy-tailed structure for the residuals. Each residual series is standardized: the mean is subtracted and the result is divided by the standard error of the series. Figure 3.6 provides the plots of the VAR(1) residuals of Australia, France, Japan and U.K. from equation (3.7a) with $i = 1$. QQ plot displays a plot of the sample quantiles of the residuals versus theoretical quantiles from a standard normal distribution. If the residuals come



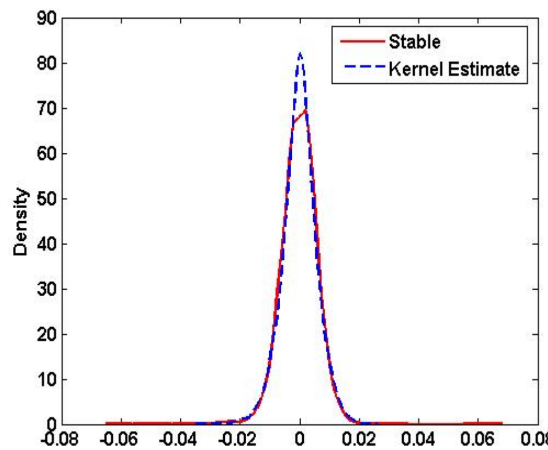
(a) Australian residuals



(b) Australian residuals



(c) French residuals



(d) French residuals

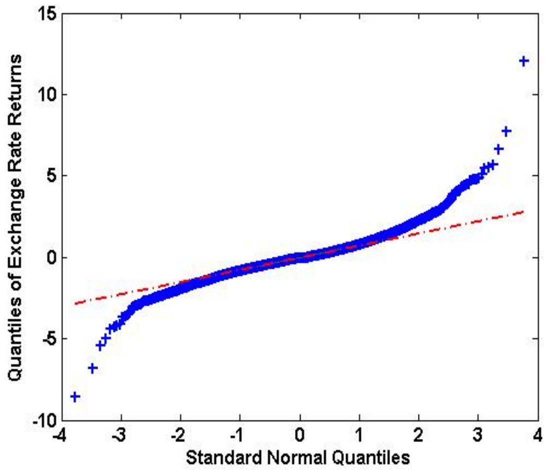
Figure 3.5: Data based kernel densities versus stable and normal densities

from a normal distribution then the plot should be approximately linear. The S-shape from the figures gives evidence that the tails of the distribution that residuals are drawn from are heavier than the tails of the normal distribution. French residuals have the least deviation from the 45-degree line among the series.

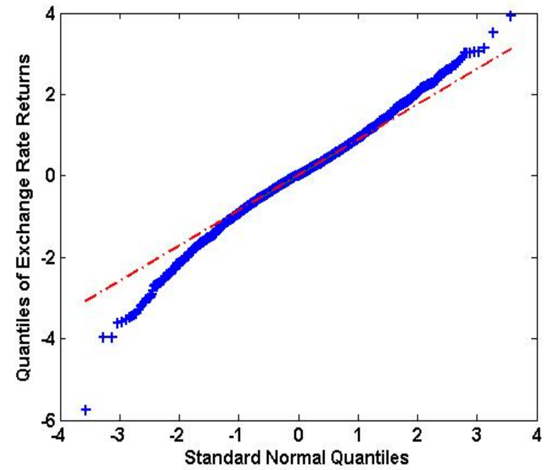
3.2 Estimation of stable parameters

In this section we estimate the stable parameters as we are convinced (by the results in section 3.1) that the empirical evidence provide enough to conclude that the residuals come from an i.i.d. process. Table 3.11 gives the estimation results for stable parameters (α, β) for each country along with the 95% confidence intervals. We can see from Table 3.11 that all α estimates are less than 2 and all confidence intervals exclude 2, which conforms well with the infinite-variance assumption made for error terms. Furthermore, skewness parameter estimates show that there is asymmetry in the errors which again does not conform with the symmetric normal distribution assumption. Although not presented here, negative skewness has been found for most of the exchange rate returns (especially for those of Canada, France, Germany, Italy, Japan and Switzerland).[4] Negative skewness means that extreme negative returns are more frequent than extreme positive returns and there is a long-run tendency towards appreciation of these currencies against the U.S. dollar. Fofack and Nolan (2001) apply the maximum likelihood procedure and examine the behavior of exchange rate returns in African countries against the U.S. dollar. They find that those exchange rate returns are positively skewed, indicating a long-run tendency towards depreciation. In our case, we are considering the exchange rate changes in industrialized countries and negative skewness provide more realistic distributional assumptions.

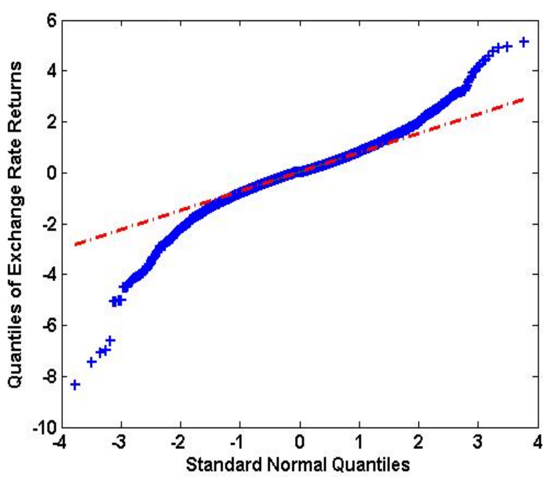
[4] Exchange rate returns are calculated as the first-order log differences of exchange rates.



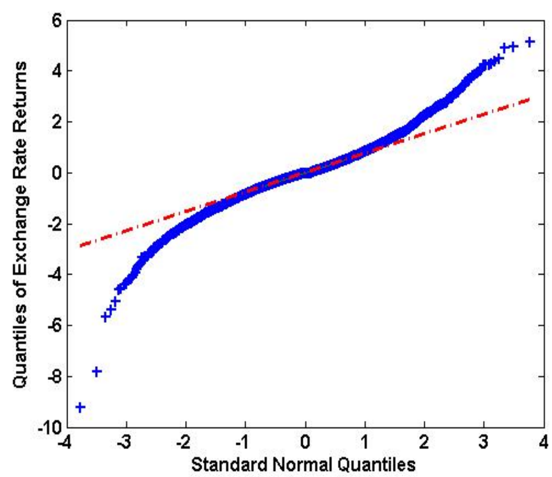
(a) Australian residuals



(b) French residuals



(c) Japanese residuals



(d) U.K. residuals

Figure 3.6: Quantiles of (standardized) residuals versus the quantiles of $N(0, 1)$ distribution

Table 3.11: Maximum Likelihood Estimation of stable parameters

Country	Equation of Residuals	Parameter	Forward Rate			
			1-month	3-month	6-month	1-year
Australia	Equation (3.7a)	$\hat{\alpha}$	1.68±0.04	1.67±0.04	1.67±0.04	1.67±0.04
		$\hat{\beta}$	0.26±0.10	0.26±0.10	0.26±0.10	0.26±0.10
	Equation (3.7b)	$\hat{\alpha}$	1.66±0.04	1.64±0.04	1.66±0.04	1.63±0.04
		$\hat{\beta}$	0.25±0.10	0.23±0.09	0.24±0.10	0.22±0.09
Canada	Equation (3.7a)	$\hat{\alpha}$	1.61±0.04	1.61±0.04	1.61±0.04	1.61±0.04
		$\hat{\beta}$	-0.03±0.09	-0.03±0.09	-0.03±0.09	-0.03±0.09
	Equation (3.7b)	$\hat{\alpha}$	1.61±0.04	1.62±0.04	1.64±0.04	1.64±0.04
		$\hat{\beta}$	-0.03±0.09	-0.03±0.09	-0.02±0.10	-0.02±0.10
France	Equation (3.7a)	$\hat{\alpha}$	1.87±0.04	1.87±0.04	1.87±0.04	1.87±0.04
		$\hat{\beta}$	-0.36±0.29	-0.36±0.29	-0.35±0.28	-0.35±0.28
	Equation (3.7b)	$\hat{\alpha}$	1.87±0.04	1.87±0.04	1.87±0.04	1.87±0.04
		$\hat{\beta}$	-0.32±0.28	-0.32±0.28	-0.31±0.28	-0.31±0.28
Germany	Equation (3.7a)	$\hat{\alpha}$	1.88±0.04	1.87±0.04	1.87±0.04	1.87±0.04
		$\hat{\beta}$	-0.36±0.29	-0.36±0.29	-0.36±0.29	-0.35±0.29
	Equation (3.7b)	$\hat{\alpha}$	1.87±0.04	1.87±0.04	1.87±0.04	1.87±0.04
		$\hat{\beta}$	-0.32±0.28	-0.32±0.28	-0.31±0.28	-0.31±0.28
Italy	Equation (3.7a)	$\hat{\alpha}$	1.88±0.04	1.88±0.04	1.88±0.04	1.88±0.04
		$\hat{\beta}$	-0.36±0.29	-0.35±0.29	-0.34±0.29	-0.34±0.29
	Equation (3.7b)	$\hat{\alpha}$	1.88±0.04	1.88±0.04	1.88±0.04	1.88±0.04
		$\hat{\beta}$	-0.34±0.29	-0.34±0.29	-0.33±0.29	-0.33±0.29

Table 3.11: (Continued)

Country	Equation of Residuals	Parameter	Forward Rate			
			1-month	3-month	6-month	1-year
Japan	Equation (3.7a)	$\hat{\alpha}$	1.67±0.04	1.67±0.04	1.67±0.04	1.67±0.04
		$\hat{\beta}$	-0.14±0.10	-0.14±0.10	-0.14±0.10	-0.14±0.10
	Equation (3.7b)	$\hat{\alpha}$	1.64±0.04	1.66±0.04	1.66±0.04	1.66±0.04
		$\hat{\beta}$	-0.13±0.10	-0.14±0.10	-0.15±0.10	-0.15±0.10
Switzerland	Equation (3.7a)	$\hat{\alpha}$	1.81±0.03	1.82±0.03	1.82±0.03	1.82±0.03
		$\hat{\beta}$	-0.15±0.15	-0.15±0.15	-0.15±0.15	-0.15±0.15
	Equation (3.7b)	$\hat{\alpha}$	1.78±0.04	1.78±0.04	1.78±0.04	1.77±0.04
		$\hat{\beta}$	-0.05±0.13	-0.08±0.13	-0.10±0.13	-0.08±0.13
U.K.	Equation (3.7a)	$\hat{\alpha}$	1.69±0.04	1.69±0.04	1.68±0.04	1.68±0.04
		$\hat{\beta}$	0.09±0.10	0.09±0.10	0.09±0.10	0.09±0.10
	Equation (3.7b)	$\hat{\alpha}$	1.67±0.04	1.67±0.04	1.67±0.04	1.66±0.04
		$\hat{\beta}$	0.10±0.10	0.09±0.10	0.09±0.10	0.09±0.10

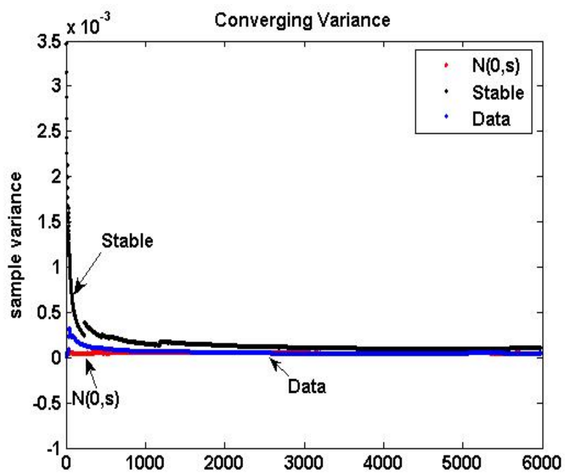
Note: 95% confidence intervals are given along with parameter estimates of α and β.

3.3 Converging variance test

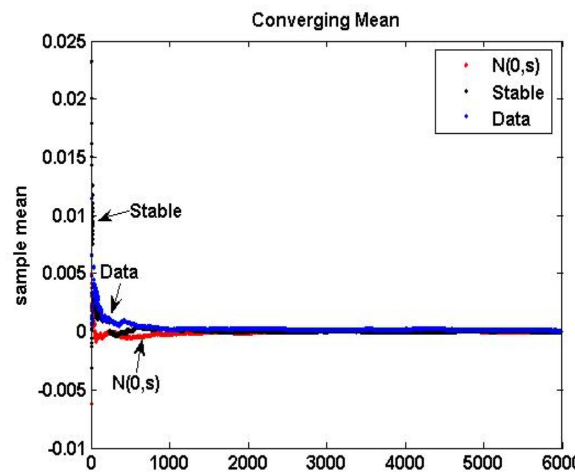
Granger and Orr (1972) have proposed the "converging variance test" for detecting infinite variance in a series. The conclusions can only be based on visual inspections. The test involves plotting the first n realizations of the sample variance for $t \leq n \leq T$,

$$s_n^2 = \frac{1}{n} \sum_{t=1}^{n} (x_t - \bar{x}_n)^2 \tag{3.18}$$

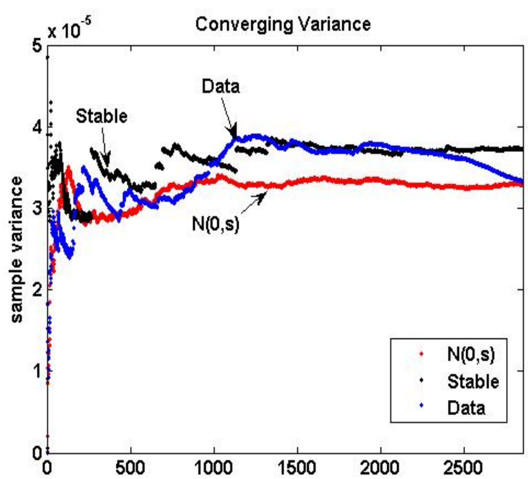
against n. If x_t comes from a finite-variance population, s_n^2 sequence converges as n increases, otherwise it diverges. The same type of test can be applied to sample mean ($\bar{x}_T$) as well. If the population mean of x_t is finite then the sample mean sequence should converge as we plot the first n realizations of it against n. Figure 3.7 presents the converging variance test results for sample variances as well as means for Australian residuals from equation (3.7a), $i = 1$ (panels (a) and (b)) and French residuals from equation (3.7a), $i = 1$ (panels (c) and (d)). Panels (a) and (b) plot the first n sample variances and means of Australian residuals respectively against n. Panels (c) and (d) plot the first n sample variances and means of French residuals respectively against n. Sample means converge fast for both countries. Thus finite-mean structure can be unambigously determined. Sample variances calculated from the data, on the other hand, do not show an unambigous finite-variance structure. It is known that sample variances of stable distributions are prone to several jumps. Sample variances of normal variables converge very fast without exhibiting any jumps. From Figure 3.7 (panels (a) and (c)), if we observe the s_n^2 sequence for Australian and French series over n, we can see that our data can be better described by a stable distribution than a normal distribution. The behavior of the sample variances from the data mimics the behavior of the sample variances from the stable distribution more than the normal distribution.



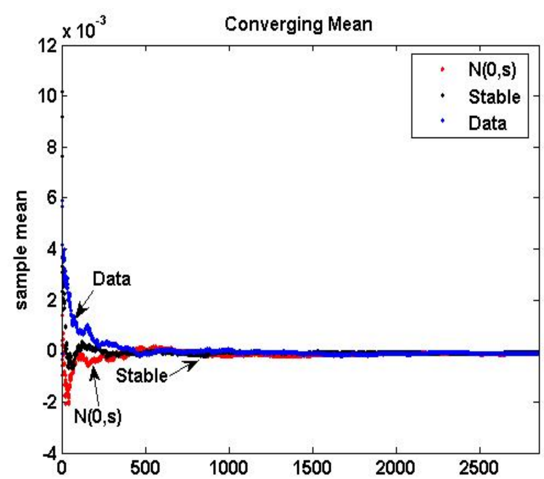
(a) Australian residuals



(b) Australian residuals



(c) French residuals



(d) French residuals

Figure 3.7: Converging variance and mean tests

3.4 Testing for forward rate unbiasedness hypothesis

3.4.1 Economic background of the hypothesis

One form of market efficiency asserts that the current forward rate is an unbiased predictor of the future spot rate, which is also known as the forward rate unbiasedness hypothesis. If FRUH holds one can write,

$$E_t(s_{t+k}) = f_{t,k} \tag{3.19}$$

where s_{t+k} and $f_{t,k}$ are logarithms of future spot rate and current forward rate respectively and $E_t(.)$ is the rational expectations formed at time t.[5] Spot rate, which is also interpreted as the current exchange rate, is the rate of a foreign exchange contract for immediate delivery.[6] Forward rate is the exchange rate that is quoted and traded today but for delivery and payment on a specific future date.[7]

Equality in equation (3.19) ensures that there are no arbitrage oppurtunities in the market. Otherwise there would be room for the investors to make arbitrarily large profits through speculating in forward foreign exchange markets (Razzak, 1999). To be more specific, consider a speculator who bought foreign currency at the price $f_{t,k}$ to be delivered at period $t+k$. Since the foreign currency can be sold at s_{t+k} in the spot market, the speculator can make a profit (loss) if $s_{t+k} > f_{t,k}$ ($s_{t+k} < f_{t,k}$).

Equation (3.19) makes two assumptions about the behaviors of agents in the market. First, it assumes that the agents are risk neutral (unconcerned about risk) so that the risk premium is zero and second, it assumes that all agents behave according to what the rational expectations hypothesis states. Under the assumption of rational expectations one can write,

[5] The reason why logs are considered instead of levels is because of Siegel (1972)' s paradox. For details on Siegel' s paradox, see Obstfeld and Rogoff (1999), p. 586.
[6] Investopedia, A Forbes Digital Company.
[7] Wikipedia, The Free Encyclopedia.

$$s_{t+k} - E_t(s_{t+k}) = \varepsilon_{t+k} \tag{3.20}$$

where ε_{t+k} is a serially independent forecast error with mean zero. Rational expectations assume that agents in the market use all available information at time t to make decisions about the future. Although future is not predictable, their expectations are not systematically biased so that "on average" their decisions are correct.

Combining equations (3.19) and (3.20), one can get,

$$s_{t+k} = f_{t,k} + \varepsilon_{t+k} \tag{3.21}$$

Fama (1984) specifies the forward exchange rate as the sum of expected future spot rate and a risk premium,

$$E_t(s_{t+k}) + rp_t = f_{t,k} \tag{3.22}$$

where the risk premium rp_t is the compensation for perceived risk that might arise from holding different currencies (Razzak, 2002). If the agents are risk-neutral then $rp_t = 0$. It is conventional to assume for the exchange rate market that the agents are risk-neutral and the risk premium is zero. The following equation has been widely used in literature to test for FRUH,

$$s_{t+k} = a + bf_{t,k} + \varepsilon_{t+k} \tag{3.23}$$

where the parameter a is a constant risk premium (Razzak, 1999). Provided that s_t and f_t are both I(1) variables, FRUH requires that there be a linear combination of them which is stationary.[8] Thus by running the "co-integrating regression" of equation (3.23), and estimating $\hat{a}$ and $\hat{b}$, one can test for residuals for a unit root and determine whether co-integration exists between forward and spot rates.

It should, however, be noted that with the assumptions of rational expectations and risk-neutrality, in order for the FRUH to hold, s_{t+k} and $f_{t,k}$ have to be co-integrated with the co-integrating vector $(1, -1)'$—i.e. $a = 0$ and $b = 1$ (see for example Delcoure et al. (2003)). Thus formal tests should be applied to jointly test for $a = 0$ and $b = 1$. Also given the fact that the exchange rate series are heavy-tailed, one should use estimation methods that are robust to heavy-tails. Empirical studies that follow similar methodologies to test for FRUH include Phillips and McFarland (1997) and Phillips et al. (1996).

When testing for $a = 0$ and $b = 1$, conventional tests are not aymptotically valid for the coefficients estimated via LAD and OLS methods because of non-stationarity of the data. Tests that involve fully modified OLS (FM-OLS) and fully modified LAD (FM-LAD) estimators should be applied to test for the joint hypothesis $a = 0$ and $b = 1$ as well as the individual hypotheses of $H_0 : a = 0$ and $H_0 : b = 1$. FM-OLS and FM-LAD estimators are explained in section 3.4.4 in detail.

3.4.2 Unit root tests

One needs to test for co-integration between s_{t+k} and $f_{t,k}$ because a lack of co-integration would be inconsistent with the FRUH. In order to test for co-integration, we need to show that forward and spot rates are both I(1). Unit root tests that are due to Caner (1998) and Samarakoon and Knight (2009) are applied for heavy-tailed data to determine the degree of integratedness.

Table 3.12 shows the unit root test results of Caner test. Table 3.13 shows the unit root test results of Samarakoon-Knight test. Graphing the natural logarithm of exchange rates against time shows that a constant term or a trend term is not necessary to be included in the equations while testing for a unit root (see panels (a) and (b) in Figure 3.8 for Australian and

[8] See Hakkio and Rush (1989).

Table 3.12: Z_t statistic for a unit root test in the logarithms of spot and forward rates

Country	Spot rate		1-month fwd. rate		3-month fwd. rate		6-month fwd. rate		1-year fwd. rate	
	x_t	Δx_t	x_t	Δx_t	x_t	Δx_t	x_t	Δx_t	x_t	Δx_t
Australia	-0.73	-74.8*	-0.71	-74.3*	-0.72	-76.2*	-0.71	-74.3*	-0.69	-77.2*
Canada	-0.94	-77.8*	-0.94	-77.8*	-0.94	-77.3*	-0.94	-76.9*	-0.94	-78.5*
France	-0.51	-52.9*	-0.51	-53.0*	-0.50	-53.0*	-0.48	-53.1*	-0.44	-53.1*
Germany	-0.55	-52.9*	-0.55	-53.0*	-0.54	-53.0*	-0.53	-53.1*	-0.50	-53.1*
Italy	-0.47	-52.6*	-0.47	-53.6*	-0.48	-52.6*	-0.47	-52.6*	-0.49	-52.6*
Japan	-1.49	-76.8*	-1.52	-77.8*	-1.54	-77.4*	-1.53	-77.1*	-1.52	-78.6*
Switzerland	-1.91*	-78.3*	-1.91*	-79.0*	-1.89*	-78.9*	-1.85	-79.0*	-1.79	-79.6*
U.K.	0.46	-74.9*	0.47	-75.0*	0.48	-75.1*	0.41	-76.0*	0.43	-76.8*

* Rejects the null hypothesis of a unit root at 5% significance level. Critical values are taken from Table 2.1 (standard).

Note: x_t denotes the log exchange rate series in levels and Δx_t denotes the first difference of the series.

Table 3.13: π_ϕ statistic for a unit root test in the logarithms of spot and forward rates

Country	Spot rate		1-month fwd. rate		3-month fwd. rate		6-month fwd. rate		1-year fwd. rate	
	x_t	Δx_t	x_t	Δx_t	x_t	Δx_t	x_t	Δx_t	x_t	Δx_t
Australia	-0.08	-78.9*	-0.06	-79.8*	-0.09	-82.7*	-0.09	-80.2*	-0.11	-82.6*
Canada	-0.04	-80.7*	-0.07	-80.4*	-0.05	-79.8*	-0.06	-78.9*	-0.05	-79.9*
France	0.08	-45.5*	0.22	-45.8*	0.15	-45.8*	0.11	-45.8*	0.14	-45.8*
Germany	0.52	-45.4*	0.43	-45.8*	0.46	-45.7*	0.28	-45.8*	0.33	-46.0*
Italy	0.06	-45.4*	0.08	-45.5*	0.08	-45.4*	0.07	-45.4*	0.08	-45.4*
Japan	0.05	-80.6*	0.04	-83.1*	0.06	-82.5*	0.06	-82.0*	0.05	-83.3*
Switzerland	0.02	-71.3*	0.01	-72.3*	0.02	-72.5*	0.01	-72.6*	0.02	-73.2*
U.K.	0.08	-77.9*	0.08	-78.7*	0.09	-78.6*	0.10	-79.5*	0.09	-79.8*

* Rejects the null hypothesis of a unit root at 5% significance level. Critical value at 5% significance is -1.64.

Note: x_t denotes the log exchange rate series in levels and Δx_t denotes the first difference of the series.

French spot exchange rates). Similar graphs can be produced for all countries and exchange rates.

Based on Caner unit root test, all exchange rate series appear to be I(1): we can not reject the null hypothesis of a unit root in levels at 5% significance, but we can reject the unit root hypothesis in differences.[9] A similar conclusion is reached for Samarakoon-Knight unit root test. All exchange rate series appear to be I(1).

3.4.3 Residual-based co-integration tests

In order to test for co-integration, the residuals from equation (3.23) should be checked whether they are I(0) or not. If the residuals are I(0) then we can reject the null hypothesis that spot and forward rates are not co-integrated. We run four different "co-integrating regressions" for spot rates against 1-month, 3-month, 6-month and 1-year forward rates for each country. The value of k is taken to be 22, 66, 132 and 264 for 1-month, 3-month, 6-month and 1-year forward rates respectively.[10] The procedure for determining Z_t and π_ϕ statistics are as follows; estimate equation (3.23) and get the residuals. For Z_t, estimate the equation via OLS and run OLS on the residuals,

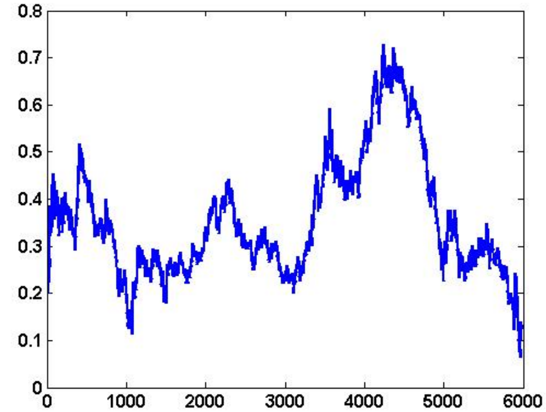
$$\hat{\varepsilon}_t = \delta \hat{\varepsilon}_{t-1} + v_t \tag{3.24}$$

then test for the null hypothesis $H_0 : \delta = 1$ against $H_1 : \delta < 1$.

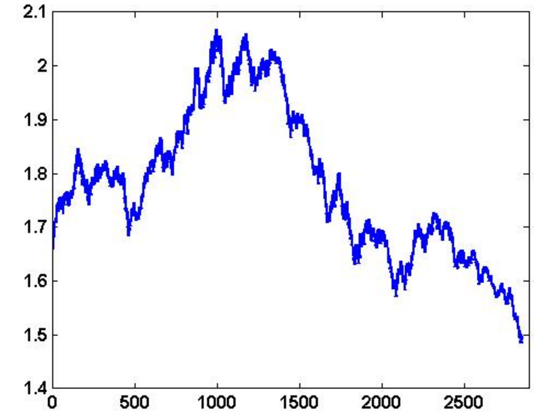
For π_ϕ, estimate the equation by LAD and run LAD on the residuals,

[9] Caner test results of Swiss spot rate, 1-month forward rate and 3-month forward rate reject the null of a unit root in levels as well (critical value is -1.88 for α=1.8) but the difference between the computed test statistics and the critical value are slight. If we assumed normally distributed returns for Swiss data and applied Phillips-Perron test, the critical value for rejecting the null hypothesis of a unit root would be -1.95 and with that value we would not reject the unit root hypothesis in Swiss spot rate, 1-month and 3-month forward rates in levels. For Z_t statistic, order of serial correlation (l) is selected according to $l = \text{int}(4(T/100)^{1/4})$.

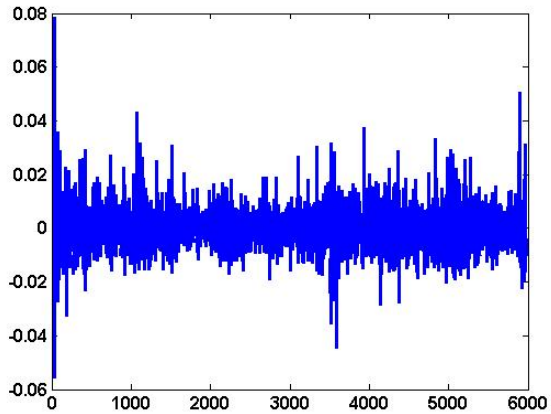
[10] Here k values are chosen according to the procedure mentioned in Phillips et al. (1996). Our data do not include saturdays and sundays.



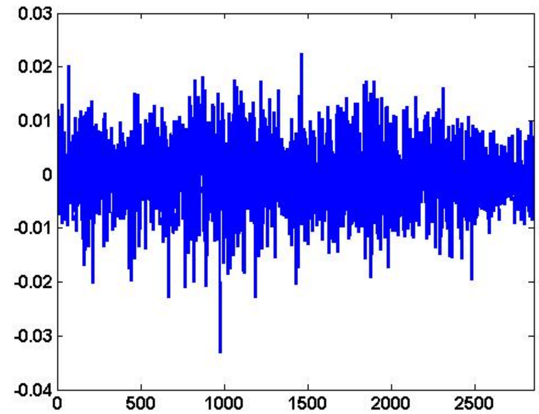
(a) Australian dollar log spot rate



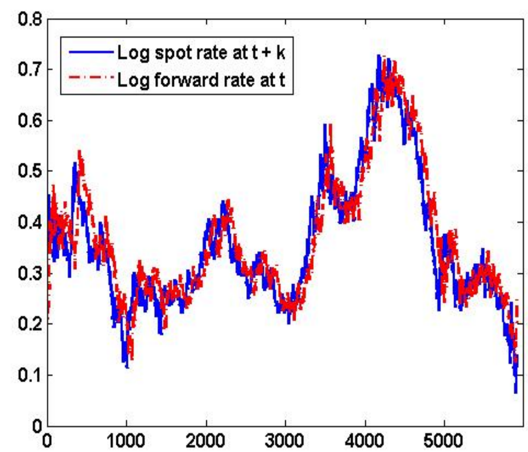
(b) French franc log spot rate



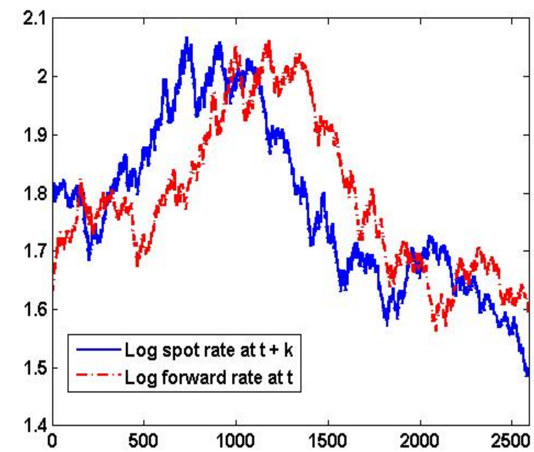
(c) Australian dollar spot returns



(d) French franc spot returns



(e) Australian dollar. Log spot and 3-month fwd. rate



(f) French franc. Log spot and 1-year fwd. rate

Figure 3.8: Australian dollar: 1985-2007 and French franc: 1997-2007

Table 3.14: 5% critical values of Z_t statistic for Caner co-integration test ($n = 1$)

α	constant[a]	constant and trend[b]
1	-3.76	-4.10
1.5	-3.45	-3.88
1.6	-3.43	-3.86
1.7	-3.40	-3.84
1.8	-3.38	-3.83
1.9	-3.37	-3.80

[a] Phillips-Ouliaris critical value ($\alpha = 2$) at 5% significance is -3.37.
[b] Phillips-Ouliaris critical value ($\alpha = 2$) at 5% significance is -3.80.
Note: n is the number of regressors in co-integrating regression.

$$\Delta\hat{\varepsilon}_t = \phi\hat{\varepsilon}_{t-1} + \sum_{i=2}^{p} \lambda_i \Delta\hat{\varepsilon}_{t-i+1} + v_t \tag{3.25}$$

then test for the null hypothesis $H_0 : \phi = 0$ against $H_1 : \phi < 0$.

Results of the co-integration tests are presented in Tables 3.15 through 3.18. Five percent critical values for Z_t are -3.43 for Canada (α=1.60); -3.40 for Australia, Japan (α=1.65) and U.K. (α=1.7); -3.38 for Switzerland (α=1.8); -3.37 for France, Germany and Italy (α=1.9). Z_t critical values are calculated through simulations based on 1,000 sample size with 20,000 iterations following Caner (1998). Results of those simulations are in Table 3.14. Five percent critical values of π_ϕ are -2.30 for Canada (α=1.6); -2.48 for Australia, Japan and U.K. (α=1.7); -2.65 for Switzerland (α=1.8); -2.89 for France, Germany and Italy (α=1.9). π_ϕ critical values are the asymptotic critical values and are present in Tables 2.2 through 2.6.

Tables 3.15 and 3.16 indicate that there is evidence for co-integration for all markets for shorter maturities (1-month and 3-month forward contracts) no matter which test is used. This result is consistent with FRUH. Results of 6-month and 1-year contracts however, depend on which type of co-integration test is applied. OLS-based co-integration tests find co-integration between spot and 6-month forward rates for Australia, Canada, Japan, Switzerland and U.K.. Only Japan, Switzerland and U.K. data show evidence of co-integration between spot and 1-year forward exchange rates via OLS-based tests. If we assumed normal errors and used

Table 3.15: Co-integration tests between logarithms of spot and 1-month forward rates

	OLS coefficients		LAD coefficients			
Country	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$	Z_t	π_ϕ
Australia	0.005	0.979	0.005	0.972	-12.133*	-9.658*
Canada	-0.004	1.008	-0.004	1.009	-12.236*	-9.419*
France	0.014	0.992	0.006	0.997	-8.622*	-5.592*
Germany	0.043	0.991	0.002	0.997	-8.635*	-5.578*
Italy	0.066	0.991	0.019	0.997	-8.638*	-5.632*
Japan	0.114	0.977	0.079	0.984	-11.866*	-8.107*
Switzerland	0.007	0.981	0.007	0.985	-12.475*	-9.358*
U.K.	-0.017	0.970	-0.008	0.989	-11.794*	-8.426*

* Rejects the null hypothesis of no co-integration at 5% significance level.
Z_t critical values are taken from Table 3.14 (constant).
π_ϕ critical values are taken from Tables 2.2 through 2.6 (constant).

Table 3.16: Co-integration tests between logarithms of spot and 3-month forward rates

	OLS coefficients		LAD coefficients			
Country	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$	Z_t	π_ϕ
Australia	0.013	0.939	0.010	0.934	-7.301*	-4.026*
Canada	-0.010	1.017	-0.013	1.029	-6.856*	-4.709*
France	0.066	0.962	-0.003	1.001	-4.117*	-3.034*
Germany	0.019	0.961	-0.002	1.002	-4.097*	-3.074*
Italy	0.270	0.963	0.041	0.994	-4.172*	-3.197*
Japan	0.361	0.926	0.401	0.918	-5.937*	-3.907*
Switzerland	0.023	0.936	0.033	0.905	-5.624*	-4.984*
U.K.	-0.055	0.904	-0.033	0.949	-5.595*	-4.211*

* Rejects the null hypothesis of no co-integration at 5% significance level.
Z_t critical values are taken from Table 3.14 (constant).
π_ϕ critical values are taken from Tables 2.2 through 2.6 (constant).

Table 3.17: Co-integration tests between logarithms of spot and 6-month forward rates

	OLS coefficients		LAD coefficients			
Country	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$	Z_t	π_ϕ
Australia	0.028	0.874	0.025	0.868	-4.548*	-2.718*
Canada	-0.013	1.008	-0.027	1.064	-5.109*	-3.706*
France	0.148	0.915	0.049	0.967	-3.260	-1.269
Germany	0.046	0.913	0.009	0.967	-3.238	-1.141
Italy	0.598	0.919	0.215	0.969	-3.186	-1.178
Japan	0.758	0.844	1.092	0.774	-4.162*	-1.882
Switzerland	0.053	0.857	0.067	0.783	-4.326*	-2.545
U.K.	-0.118	0.786	-0.104	0.819	-4.131*	-2.415

* Rejects the null hypothesis of no co-integration at 5% significance level.
Z_t critical values are taken from Table 3.14 (constant).
π_ϕ critical values are taken from Tables 2.2 through 2.6 (constant).

Table 3.18: Co-integration tests between logarithms of spot and 1-year forward rates

	OLS coefficients		LAD coefficients			
Country	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$	Z_t	π_ϕ
Australia	0.100	0.653	0.093	0.635	-2.969	-0.695
Canada	-0.002	0.943	-0.040	1.069	-3.156	-1.922
France	0.457	0.739	0.311	0.809	-1.656	0.022
Germany	0.144	0.736	0.084	0.806	-1.674	0.194
Italy	1.869	0.748	1.277	0.825	-1.588	-0.216
Japan	1.640	0.660	2.228	0.536	-3.545*	-1.310
Switzerland	0.132	0.635	0.163	0.512	-3.415*	-1.655
U.K.	-0.239	0.555	-0.269	0.481	-3.593*	-0.389

* Rejects the null hypothesis of no co-integration at 5% significance level.
Z_t critical values are taken from Table 3.14 (constant).
π_ϕ critical values are taken from Tables 2.2 through 2.6 (constant).

Phillips-Ouliaris tests (critical value=-3.37), our conclusions would not change. LAD-based co-integration tests, on the other hand, find no evidence of co-integration for 1-year maturity forward and spot rates. Except for the cases of Australia and Canada, 6-month forward and spot rates do not seem to be co-integrated with LAD-based tests. No co-integration result is not consistent with FRUH, because we fail to find a long-run relationship between forward and corresponding future spot rates.

The results of OLS and LAD-based co-integration tests are similar except for Japan, Switzerland and U.K. for 6-month and 1-year contracts and for them the results are in favor of FRUH for OLS-based tests and not in favor of FRUH for LAD-based ones. Also, regardless of the estimation technique, FRUH is rejected more for longer maturity forward contracts (6-month and 1-year). This is an indication that the decisions of the agents in predicting the future spot rate from the current forward rate tend to be biased as time to maturity of the forward contract increases. This bias is more observed in the results of the LAD-based tests.

3.4.4 Fully-modified estimations

Fully-modified OLS and LAD estimators are developed in Phillips and Hansen (1990) and Phillips (1995) respectively. The estimators correct the OLS and LAD estimators in a co-integrating regression such as equation (3.23) for possible autocorrelation and heteroskedasticity in the error terms and for endogeneity bias that occurs because the regressors and the regression error are correlated. In econometrics, the term "endogeneity" is used in a broad sense to describe any situation where the explanatory variable is correlated with the error term. If the explanatory variable is not correlated with the equation error then it is said to be exogenous (Wooldridge, 2002, p. 50).

FM-OLS and FM-LAD estimators are developed for estimating co-integrating equations with non-stationary data and making inference. Specifically, in a co-integration system given by,

$$y_t = x_t'\beta + u_{0t} \tag{3.26a}$$

$$\Delta x_t = u_{xt} \tag{3.26b}$$

where $\Delta x_t = x_t - x_{t-1}$ and $u_t = (u_{0t}, u_{xt}')'$, $t = 2, \ldots, T$, is a stationary m-vector time series ($m = 1 + m_x$) with long-run covariance matrix,[11]

$$\Omega_{uu} = \sum_{j=-\infty}^{\infty} E(u_0 u_j') = \begin{bmatrix} \Omega_{00} & \Omega_{0x} \\ \Omega_{x0} & \Omega_{xx} \end{bmatrix} \tag{3.27}$$

where Ω_{xx} is assumed to be a positive definite matrix and the partition of Ω_{uu} is conformable with vector u_t. The one-sided long-run covariance matrix of u_t is,

$$\Delta_{uu} = \sum_{j=0}^{\infty} E(u_0 u_j') = \begin{bmatrix} \Delta_{00} & \Delta_{0x} \\ \Delta_{x0} & \Delta_{xx} \end{bmatrix} \tag{3.28}$$

In equation (3.26a), endogeneity bias occurs due to $cov(x_t, u_{0t}) \neq 0$. Although the OLS and LAD coefficients estimated from a co-integrating regression are both super-consistent, they suffer from a second order bias or are not median unbiased because of the existence of endogeneous regressors.[12] Also the standard testing procedures such as t-tests can not be used in making inference because either the existence of endogeneity bias or autocorrelation can lead to standard errors that are incorrect (Lim and Martin, 1995, p. 7). Fully modified estimation methods get rid of the second order bias and come up with an asymptotic theory that allows one to make use of standard testing procedures such as t-tests and Wald tests.

OLS estimator is written in the form,

[11]See Appendix on how to consistently estimate the long-run covariance matrices.

[12]Median unbiasedness is another term used for second order unbiasedness (see for example Taniguchi (1983)). If the median of the limiting distribution is equal to the true parameter, the estimator is median unbiased. Similarly if the mean of the limiting distribution is equal to the true parameter, the estimator is mean unbiased or unbiased in the usual sense.

$$\beta_{OLS} = (X'X)^{-1}(X'y) \tag{3.29}$$

The corrected FM-OLS estimator can be written as,

$$\beta^{+}_{OLS} = (X'X)^{-1}(X'y^{+} - T\hat{\Delta}^{+}_{x0}) \tag{3.30}$$

with

$$y^{+} = y - \Delta X'\hat{\Omega}^{-1}_{xx}\hat{\Omega}_{x0} \tag{3.31}$$

where $\hat{\Omega}^{-1}_{xx}$, $\hat{\Omega}_{x0}$ and $\hat{\Delta}^{+}_{x0}$ are consistent estimates of Ω^{-1}_{xx}, Ω_{x0} and $\Delta^{+}_{x0} = \Delta_{x0} - \Delta_{xx}\Omega^{-1}_{xx}\Omega_{x0}$ respectively.[13]

The asymptotic distribution of FM-OLS can be approximated with a normal distribution,

$$\beta^{+}_{OLS} \overset{a}{\sim} N\left(\beta, \omega_{00.x}(X'X)^{-1}\right) \tag{3.32}$$

where $\omega_{00.x} = \Omega_{00} - \Omega_{0x}\Omega^{-1}_{xx}\Omega_{x0}$. If one can consistently estimate $\omega_{00.x}$ then standard t-tests and Wald tests can be applied for testing linear restrictions on the FM-OLS coefficients. The fully modified t-ratios for tests about individual coefficients are given by,

$$t_i = \frac{(\beta^{+}_{OLS} - \beta_i)}{s_i} \tag{3.33}$$

and

[13] $\hat{u}_{0t}$ is the estimate of u_{0t}. It is the residual series from the equation (3.26a) via OLS (LAD) when estimating FM-OLS (FM-LAD).

$$s_i = \sqrt{[\hat{\omega}_{00.x}(X'X)^{-1}]_{ii}} \qquad i = 1, ..., k \tag{3.34}$$

where $\hat{\omega}_{00.x} = \hat{\Omega}_{00} - \hat{\Omega}_{0x}\hat{\Omega}_{xx}^{-1}\hat{\Omega}_{x0}$ is a consistent estimate of $\omega_{00.x}$ and k is the number of coefficients estimated in equation (3.26a). The t-ratios are asymptotically $N(0,1)$ so that inference can be made as in stationary linear regression.

When testing hypothesis about two or more linear restrictions ($H_0 : R\beta - r = 0$ against $H_0 : R\beta - r \neq 0$) on FM-OLS coefficients, Wald statistic can be used,

$$W^+ = \left(R\beta^+_{OLS} - r\right)' \left\{R\hat{\omega}_{00.x}(X'X)^{-1}R'\right\}^{-1} \left(R\beta^+_{OLS} - r\right) \tag{3.35}$$

The limiting distribution of the statistic W^+ under the null is χ^2_q with q number of restrictions.

In order to estimate the FM-LAD estimator, we transform the error $v_t = sign(u_{0t})$ ($v_t = 1$ for $u_{0t} \geq 0$ and $v_t = -1$ for $u_{0t} < 0$). The long-run covariance matrix of $w_t = (v_t, u'_{xt})'$, $t = 2, \ldots, T$, is given as,

$$\Omega_{ww} = \sum_{j=-\infty}^{\infty} E(w_0 w'_j) = \begin{bmatrix} \Omega_{vv} & \Omega_{vx} \\ \Omega_{xv} & \Omega_{xx} \end{bmatrix} \tag{3.36}$$

The one-sided long-run covariance matrix is given as,

$$\Delta_{ww} = \sum_{j=0}^{\infty} E(w_0 w'_j) = \begin{bmatrix} \Delta_{vv} & \Delta_{vx} \\ \Delta_{xv} & \Delta_{xx} \end{bmatrix} \tag{3.37}$$

FM-LAD estimator is from the same family as FM-OLS and has the same features in a regression with non-stationary regressors. In addition to having the same features, FM-LAD is robust and outlier resistant when applied to heavy-tailed data. FM-LAD estimator can be derived as,

$$\beta^+_{LAD} = \beta_{LAD} - \left[2\hat{f}(0)(X'X)\right]^{-1} \left[X'\Delta X\hat{\Omega}_{xx}^{-1}\hat{\Omega}_{xv} + T\hat{\Delta}^+_{xv}\right] \tag{3.38}$$

In equation (3.38), $\hat{f}(0)$ is a consistent estimator of $f(0)$: the probability density of u_{0t} at the origin. $\hat{\Omega}_{xx}$, $\hat{\Omega}_{xv}$ are consistent estimates of Ω_{xx} and Ω_{xv}. $\hat{\Delta}^{+}_{xv}$ is a consistent estimate of one-sided long-run covariance matrix,

$$\Delta^{+}_{xv} = \sum_{k=0}^{\infty} E(u_{x0}v^{+}_{k}) = \Delta_{xv} - \Delta_{xx}\Omega^{-1}_{xx}\Omega_{xv} \tag{3.39}$$

To estimate Δ^{+}_{xv}, we first need to estimate v^{+}_{t},

$$v^{+}_{t} = v_t - \Omega_{vx}\Omega^{-1}_{xx}\Delta x_t \tag{3.40}$$

Since $\hat{v}_t = sign(\hat{u}_{0t})$, by using consistent estimates $\hat{\Omega}_{vx}$ and $\hat{\Omega}_{xx}$, we can construct $\hat{v}^{+}_{t}$. Then Δ^{+}_{xv} can be estimated by the one sided long-run covariance matrix of v^{+}_{t} and u_{xt}.

The limiting distribution of FM-LAD estimator can be approximated as normal,

$$\beta^{+}_{LAD} \overset{a}{\sim} N\left(\beta, [1/(2f(0))]^2\omega_{vv.x}(X'X)^{-1}\right) \tag{3.41}$$

where $\omega_{vv.x} = \Omega_{vv} - \Omega_{vx}\Omega^{-1}_{xx}\Omega_{xv}$. The t-statistics and Wald statistics for testing linear restrictions about FM-LAD coefficients can be constructed as in the FM-OLS case,

$$t_i = \frac{\left(\beta^{+}_{LAD} - \beta_i\right)}{s_i} \tag{3.42}$$

$$s_i = \sqrt{[\hat{\omega}_{vv.x}(X'X)^{-1}]_{ii}} \qquad i = 1, ..., k \tag{3.43}$$

$$W^{+} = \left(R\beta^{+}_{LAD} - r\right)' \left\{R\hat{\omega}_{vv.x}(X'X)^{-1}R'\right\}^{-1} \left(R\beta^{+}_{LAD} - r\right) \tag{3.44}$$

where $\hat{\omega}_{vv.x} = \hat{\Omega}_{vv} - \hat{\Omega}_{vx}\hat{\Omega}^{-1}_{xx}\hat{\Omega}_{xv}$ is a consistent estimate of $\omega_{vv.x}$.

Phillips (1995) considers the limit distribution of FM-LAD with infinite-variance stably distributed error terms in the system (3.26). For infinite-variance case, equations (3.41), (3.42), (3.43) and (3.44) still hold but Ω_{vv} replaces $\omega_{vv.x}$.[14] Also with infinite-variance errors in u_t, the long-run covariance matrices Ω and Δ are not well-defined. However, it is still possible to proceed in the same way as in the finite-variance case and calculate the covariances in the usual way with a finite sample of data (see Phillips et al. (1996) and the references therein).

Tables 3.19 through 3.22 present FM-OLS and FM-LAD coefficients. FM-LAD‡ row denotes the FM-LAD estimator with infinite-variance errors. The t-statistics for testing the hypotheses, $H_0 : a = 0$ and $H_0 : b = 1$ and Wald statistics for testing the joint hypothesis, $H_0 : a = 0, b = 1$ of market efficiency are also presented. Wald statistic has an asymptotic χ^2_2 distribution.

From Tables 3.19 through 3.22, it is observed that there are differences between FM-OLS and FM-LAD regression coefficients. In general, FM-OLS estimates of a and b are closer to 0 and 1 respectively when compared to FM-LAD ones. The differences are small for regressions involving shorter maturity (1-month and 3-month) forward contracts, but are more obvious for regressions involving longer maturity (6-month and 1-year) contracts except for French, German and Italian data, for which cases, FM-LAD coefficients of a and b are closer to 0 and 1. It is also observed that a and b estimates of FM-OLS and FM-LAD both tend to get farther away from 0 and 1 as the contract maturity increases.

If the current forward rate is an unbiased predictor of the future spot rate then there should exist co-integration between future spot and current forward rates. In addition to that, we test whether the co-integrating vector is $(1, -1)'$—i.e. whether the null hypothesis $H_0 : a = 0, b = 1$ holds or not. For 1-month and 3-month contracts, co-integration exists for OLS and LAD-based regressions and the co-integrating vector is generally found to be $(1, -1)'$ with both FM-OLS and FM-LAD coefficients (exceptions occur with Australia, Canada and U.K.). Regarding the estimations involving longer maturity contracts: joint chi-square and individual t-test results are much less in favor of FRUH for FM-OLS, FM-LAD and FM-LAD‡.

[14] See Phillips and McFarland (1997), p. 889.

Table 3.19: Empirical estimates of equation (3.23) for spot and 1-month forward rates

		Parameters, t-ratios and $\chi^2_{(2)}$ statistic				
Country	Method	$\hat{a}$	$t_a = \hat{a}/s_a$	$\hat{b}$	$t_b = (\hat{b}-1)/s_b$	Joint test
Australia	FM-OLS	-0.003	-1.327	0.999	-0.046	16.790[c]
	FM-LAD	-0.002	-0.410	0.990	-1.001	17.365[c]
	FM-LAD‡	-0.002	-0.274	0.990	-0.670	7.772[c]
Canada	FM-OLS	-0.003	-3.123[a]	1.004	1.132	36.458[c]
	FM-LAD	-0.004	-1.775	1.005	0.829	8.923[c]
	FM-LAD‡	-0.004	-1.194	1.005	0.558	4.039
France	FM-OLS	0.001	0.123	0.999	-0.224	1.773
	FM-LAD	-0.009	-0.408	1.005	0.421	0.202
	FM-LAD‡	-0.009	-0.252	1.005	0.260	0.077
Germany	FM-OLS	0.000	-0.069	0.999	-0.221	1.535
	FM-LAD	-0.003	-0.385	1.005	0.433	0.213
	FM-LAD‡	-0.003	-0.238	1.005	0.268	0.082
Italy	FM-OLS	0.004	0.115	0.999	-0.146	2.885
	FM-LAD	-0.043	-0.455	1.006	0.456	0.209
	FM-LAD‡	-0.043	-0.288	1.006	0.288	0.084
Japan	FM-OLS	0.017	0.889	0.996	-0.894	0.805
	FM-LAD	0.002	0.073	1.000	0.038	5.484
	FM-LAD‡	0.002	0.046	1.000	0.024	2.181
Switzerland	FM-OLS	-0.000	-0.460	1.000	0.085	0.809
	FM-LAD	-0.000	-0.067	1.002	0.283	0.274
	FM-LAD‡	-0.000	-0.038	1.002	0.161	0.088
U.K.	FM-OLS	-0.006	-2.088[a]	0.995	-0.940	27.778[c]
	FM-LAD	0.002	0.483	1.011	1.089	7.706[c]
	FM-LAD‡	0.002	0.319	1.011	0.718	3.350

[a] Rejects the null: $H_0 : a = 0$ against $H_1 : a \neq 0$ at 5% significance.
[b] Rejects the null: $H_0 : b = 1$ against $H_1 : b \neq 1$ at 5% significance.
[c] Rejects the null: $H_0 : a = 0, b = 1$ against H_1 : Otherwise at 5% significance.
Five percent critical values for t-statistics (two-sided) are $\pm$ 1.96.
Five percent critical value for $\chi^2_{(2)}$ statistic is 5.99.
FM-LAD‡ denotes the FM-LAD estimator with infinite-variance errors.

Table 3.20: Empirical estimates of equation (3.23) for spot and 3-month forward rates

		Parameters, t-ratios and $\chi^2_{(2)}$ statistic				
Country	Method	$\hat{a}$	$t_a = \hat{a}/s_a$	$\hat{b}$	$t_b = (\hat{b}-1)/s_b$	Joint test
Australia	FM-OLS	-0.010	-1.032	1.002	0.090	8.498[c]
	FM-LAD	-0.005	-0.442	0.978	-0.680	11.615[c]
	FM-LAD‡	-0.005	-0.360	0.978	-0.554	7.713[c]
Canada	FM-OLS	-0.012	-3.235[a]	1.023	1.818	24.048[c]
	FM-LAD	-0.016	-2.248[a]	1.037	1.610	7.376[c]
	FM-LAD‡	-0.016	-1.878	1.037	1.345	5.149
France	FM-OLS	0.012	0.371	0.992	-0.445	1.139
	FM-LAD	-0.061	-0.736	1.033	0.712	0.620
	FM-LAD‡	-0.061	-0.595	1.033	0.576	0.405
Germany	FM-OLS	0.002	0.218	0.992	-0.436	1.006
	FM-LAD	-0.021	-0.764	1.033	0.706	0.611
	FM-LAD‡	-0.021	-0.612	1.033	0.574	0.403
Italy	FM-OLS	0.034	0.211	0.995	-0.232	1.426
	FM-LAD	-0.204	-0.560	1.027	0.588	0.719
	FM-LAD‡	-0.204	-0.498	1.027	0.489	0.497
Japan	FM-OLS	0.116	1.412	0.976	-1.413	1.998
	FM-LAD	0.196	1.941	0.960	-1.901	4.418
	FM-LAD‡	0.196	1.405	0.960	-1.376	2.313
Switzerland	FM-OLS	0.003	0.333	0.989	-0.612	0.641
	FM-LAD	0.016	1.389	0.948	-1.990	4.886
	FM-LAD‡	0.016	1.041	0.948	-1.491	2.744
U.K.	FM-OLS	-0.025	-2.096[a]	0.968	-1.349	13.901[c]
	FM-LAD	-0.011	-0.641	0.995	-0.128	5.305
	FM-LAD‡	-0.011	-0.530	0.995	-0.105	3.616

[a] Rejects the null: $H_0 : a = 0$ against $H_1 : a \neq 0$ at 5% significance.
[b] Rejects the null: $H_0 : b = 1$ against $H_1 : b \neq 1$ at 5% significance.
[c] Rejects the null: $H_0 : a = 0, b = 1$ against H_1 : Otherwise at 5% significance.
Five percent critical values for t-statistics (two-sided) are ± 1.96.
Five percent critical value for $\chi^2_{(2)}$ statistic is 5.99.
FM-LAD‡ denotes the FM-LAD estimator with infinite-variance errors.

Table 3.21: Empirical estimates of equation (3.23) for spot and 6-month forward rates

		Parameters, t-ratios and $\chi^2_{(2)}$ statistic				
Country	Method	$\hat{a}$	$t_a = \hat{a}/s_a$	$\hat{b}$	$t_b = (\hat{b}-1)/s_b$	Joint test
Australia	FM-OLS	-0.009	-0.368	0.974	-0.399	5.840
	FM-LAD	0.000	0.005	0.936	-0.895	8.077[c]
	FM-LAD[‡]	0.000	0.004	0.936	-0.782	6.165[c]
Canada	FM-OLS	-0.019	-2.103[a]	1.028	0.958	14.920[c]
	FM-LAD	-0.031	-2.258[a]	1.078	1.701	6.993[c]
	FM-LAD[‡]	-0.031	-2.037[a]	1.078	1.534	5.687
France	FM-OLS	0.059	0.766	0.964	-0.828	1.344
	FM-LAD	-0.009	-0.056	0.999	-0.008	0.731
	FM-LAD[‡]	-0.009	-0.045	0.999	-0.006	0.477
Germany	FM-OLS	0.017	0.644	0.964	-0.835	1.264
	FM-LAD	-0.009	-0.172	0.999	-0.016	0.696
	FM-LAD[‡]	-0.009	-0.140	0.999	-0.013	0.458
Italy	FM-OLS	0.224	0.638	0.969	-0.656	1.423
	FM-LAD	-0.001	-0.002	0.999	-0.015	0.950
	FM-LAD[‡]	-0.001	-0.002	0.999	-0.013	0.694
Japan	FM-OLS	0.393	2.196[a]	0.919	-2.194[b]	4.820
	FM-LAD	0.766	3.562[a]	0.842	-3.570[b]	12.755[c]
	FM-LAD[‡]	0.766	3.031[a]	0.842	-3.039[b]	9.239[c]
Switzerland	FM-OLS	0.018	1.007	0.945	-1.312	1.899
	FM-LAD	0.045	2.264[a]	0.839	-3.486[b]	16.506[c]
	FM-LAD[‡]	0.045	1.685	0.839	-2.595[b]	9.143[c]
U.K.	FM-OLS	-0.076	-2.840[a]	0.875	-2.230[b]	13.915[c]
	FM-LAD	-0.068	-1.891	0.896	-1.388	7.759[c]
	FM-LAD[‡]	-0.068	-1.627	0.896	-1.195	5.758

[a] Rejects the null: $H_0 : a = 0$ against $H_1 : a \neq 0$ at 5% significance.
[b] Rejects the null: $H_0 : b = 1$ against $H_1 : b \neq 1$ at 5% significance.
[c] Rejects the null: $H_0 : a = 0, b = 1$ against H_1 : Otherwise at 5% significance.
Five percent critical values for t-statistics (two-sided) are ± 1.96.
Five percent critical value for $\chi^2_{(2)}$ statistic is 5.99.
FM-LAD[‡] denotes the FM-LAD estimator with infinite-variance errors.

Table 3.22: Empirical estimates of equation (3.23) for spot and 1-year forward rates

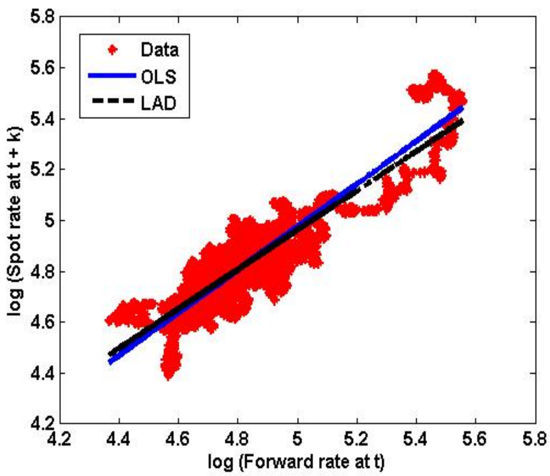
		Parameters, t-ratios and $\chi^2_{(2)}$ statistic				
Country	Method	$\hat{a}$	$t_a = \hat{a}/s_a$	$\hat{b}$	$t_b = (\hat{b}-1)/s_b$	Joint test
Australia	FM-OLS	0.044	0.651	0.797	-1.228	4.571
	FM-LAD	0.058	1.072	0.723	-2.081[b]	13.775[c]
	FM-LAD‡	0.058	0.973	0.723	-1.890	11.363[c]
Canada	FM-OLS	-0.016	-0.714	0.988	-0.164	8.305[c]
	FM-LAD	-0.042	-1.307	1.075	0.723	4.692
	FM-LAD‡	-0.042	-1.116	1.075	0.617	3.421
France	FM-OLS	0.379	3.227[a]	0.783	-3.322[b]	12.441[c]
	FM-LAD	0.273	0.793	0.831	-0.881	2.170
	FM-LAD‡	0.273	0.607	0.831	-0.675	1.272
Germany	FM-OLS	0.117	2.941[a]	0.781	-3.262[b]	11.847[c]
	FM-LAD	0.070	0.615	0.829	-0.889	2.115
	FM-LAD‡	0.070	0.478	0.829	-0.691	1.276
Italy	FM-OLS	1.565	3.339[a]	0.789	-3.365[b]	13.565[c]
	FM-LAD	1.156	0.803	0.841	-0.825	2.348
	FM-LAD‡	1.156	0.579	0.841	-0.596	1.223
Japan	FM-OLS	1.196	3.916[a]	0.752	-3.918[b]	15.350[c]
	FM-LAD	1.945	6.655[a]	0.595	-6.695[b]	45.242[c]
	FM-LAD‡	1.945	5.837[a]	0.595	-5.872[b]	34.812[c]
Switzerland	FM-OLS	0.080	2.163[a]	0.770	-2.624[b]	7.135[c]
	FM-LAD	0.136	3.682[a]	0.583	-4.774[b]	25.025[c]
	FM-LAD‡	0.136	2.895[a]	0.583	-3.754[b]	15.475[c]
U.K.	FM-OLS	-0.202	-4.778[a]	0.636	-4.021[b]	31.504[c]
	FM-LAD	-0.235	-4.060[a]	0.556	-3.572[b]	19.678[c]
	FM-LAD‡	-0.235	-3.816[a]	0.556	-3.358[b]	17.383[c]

[a] Rejects the null: $H_0 : a = 0$ against $H_1 : a \neq 0$ at 5% significance.
[b] Rejects the null: $H_0 : b = 1$ against $H_1 : b \neq 1$ at 5% significance.
[c] Rejects the null: $H_0 : a = 0, b = 1$ against H_1 : Otherwise at 5% significance.
Five percent critical values for t-statistics (two-sided) are $\pm$ 1.96.
Five percent critical value for $\chi^2_{(2)}$ statistic is 5.99.
FM-LAD‡ denotes the FM-LAD estimator with infinite-variance errors.

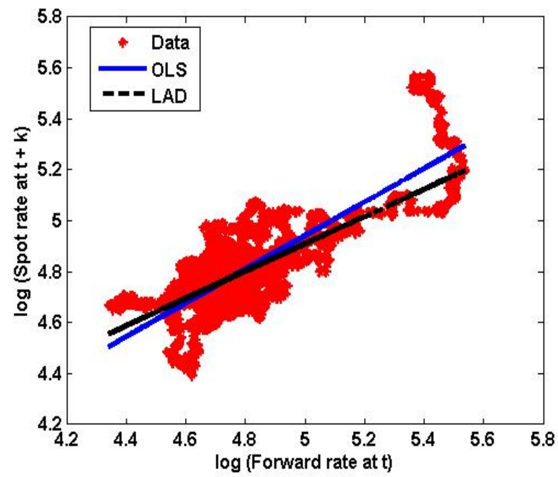
The conclusions drawn from the test results involving FM-LAD and FM-LAD‡ are similar except for Canada (1-month, 3-month and 6-month contracts) and U.K. (1-month and 6-month contracts). For those cases, FM-LAD‡ chi-square tests do not reject the null hypothesis that $a = 0$ and $b = 1$ whereas FM-LAD ones reject it. Thus with infinite-variance assumption, FRUH restrictions are less rejected for the FM-LAD estimator.

FM-LAD coefficient estimates in general tend to give the results that are less in favor of the FRUH as the forward contract maturity increases, especially with the data from Japan, Switzerland and U.K.. Figure 3.9 provides the graphs of OLS, LAD, FM-OLS and FM-LAD regressions for Japanese exchange rate data, where outliers are more pronounced for regressions with 6-month and 1-year forward contracts. LAD and FM-LAD based regression coefficients are not as influenced by the outliers as OLS and FM-OLS coefficients.

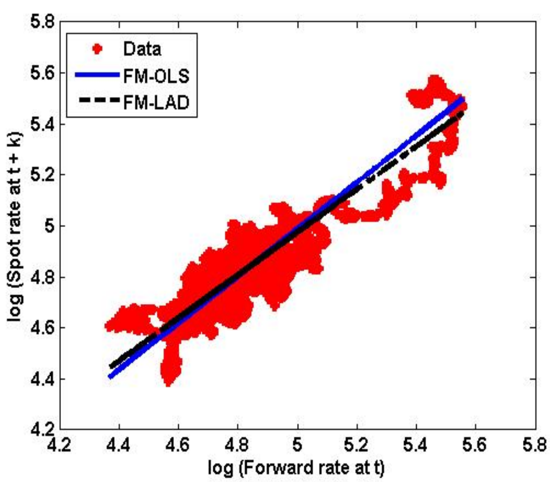
Phillips and McFarland (1997) apply tests of FRUH with daily frequency data of spot, 1-month and 3-month forward Australian exchange rates over the period 1984-1991. They observe that tests based on FM-OLS accept the joint hypothesis of market efficiency whereas tests based on FM-LAD tend to reject it and the chi-square test statistics get larger in the direction of rejecting the hypothesis more for 3-month case than the 1-month case. Regarding the individual coefficients, FM-OLS coefficients of a and b are closer to 0 and 1 than the FM-LAD ones and the magnitude of the coefficients get farther away from 0 and 1 as the forward contract expiration is longer. Chi-square test statistic strongly rejects the joint hypothesis with FM-LAD estimations and accepts it with FM-OLS estimations. Phillips et al. (1996) test for FRUH with daily data involving 1-month forward contracts of Belgian and French franc, Italian lira and U.S. dollar measured against British pound during 1920s. Their market efficiency (chi-square) test results yield the same conclusions for FM-OLS and FM-LAD based tests but the FM-LAD coefficient estimates of a and b are closer to 0 and 1.



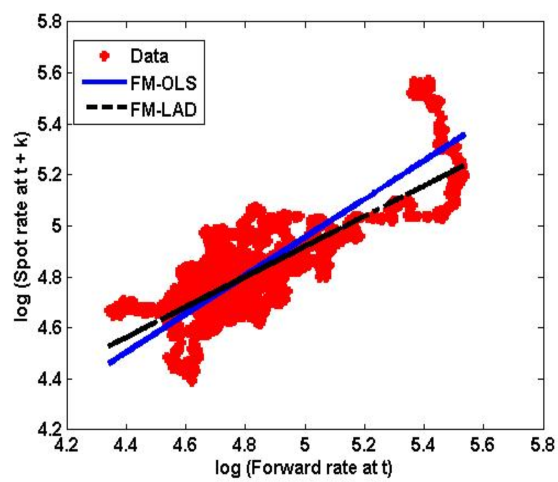
(a) OLS and LAD regressions: 6-month contracts



(b) OLS and LAD regressions: 1-year contracts



(c) Fully-modified regressions: 6-month contracts



(d) Fully-modified regressions: 1-year contracts

Figure 3.9: Estimations of co-integrating regressions for Japanese exchange rates

CHAPTER 4. CONCLUDING REMARKS

In many empirical papers, error terms of some important economic and financial data are shown to have heavier-tails than the tails of Gaussian normal distribution. Stable distribution family is a better fit for such data than Gaussian distribution. In part I, we consider residual-based tests of co-integration with infinite-variance errors that come from the stable distribution family. As an alternative to conventional OLS-based tests that is widely studied, we propose LAD-based co-integration tests for infinite-variance heavy-tailed data. We evaluate the size and power of our new tests by comparing them with Caner (1998) test. Caner (1998) extends the residual-based tests of Phillips and Ouliaris (1990) to infinite-variance stable errors.

Size comparisons indicate significantly more size distortions for the new LAD-based tests for small sample sizes especially around 50 and 100, compared to OLS-based tests. Those distortions decrease for our test as the tail structure gets closer to the normal distribution assumptions—i.e. as α approaches 2.

Power comparisons between the two types of tests show that the new residual-based co-integration tests are superior to the conventional OLS-based ones when the tails of the error distribution get heavier and the sample size gets larger. For stability indices (α) less than 1.7 and for sample sizes larger than 50, LAD-based tests perform better than the conventional OLS-based ones. As the stability index gets closer to 2, conventional OLS-based tests perform better especially with small sample size. For large sample size, which test is superior over the other depends on how far away the structure is from the null of no co-integration. Our test still has some power advantages in distinguishing the null from the alternative when the residual series are "close" to being non-stationary or the near unit root case. For the near unit root case, the effects of past shocks tend to be high and if those shocks come from a heavy-tailed stable distribution, the new LAD-based co-integration tests have slightly higher empirical power.

As an empirical illustration, we apply OLS and LAD-based co-integration tests and associated statistical tests to test for the forward rate unbiasedness hypothesis (FRUH) with daily data for a sample of eight countries: Australia, Canada, France, Germany, Italy, Japan, Switzerland and U.K.. Their currencies are measured relative to the U.S. dollar for spot rate and 1-month, 3-month, 6-month, 1-year forward exchange rates. We find that the infinite-variance stable error distribution assumption is a reasonable assumption to make for all cases we consider. There is also evidence of negative skewness in the exchange rate returns indicating a long-run tendency of those currencies towards appreciation against the U.S. dollar.

From the co-integration tests, we find that co-integration exists between the current forward ($f_{t,k}$) and the future spot rate (s_{t+k}) for shorter maturity (1-month and 3-month) contracts with both OLS and LAD-based co-integration tests. Tests involving LAD estimations reject the hypothesis that there is co-integration between forward and spot rates more for longer maturity (6-month and 1-year) contracts. Furthermore, joint hypotheses tests for testing a co-integrating vector of $(1, -1)'$ between s_{t+k} and $f_{t,k}$ are pursued with FM-OLS and FM-LAD based methods. In general, regardless of the estimation method, the restrictions on the co-integrating vectors hold when shorter maturity (1-month and 3-month) forward contracts are considered. This reconciles with FRUH. For longer maturity (6-month and 1-year) contracts, however, the restrictions are generally rejected with both methods. This finding contradicts with FRUH and suggests that the decisions of the agents in predicting the future spot rate from the current forward rate tend to be biased as time to maturity of the forward contract increases.

PART II

TESTING WEAK-FORM AND STRONG-FORM PPP VIA ROBUST ESTIMATION PROCEDURES

CHAPTER 5. INTRODUCTION

The law of one price (LOP) states that the same good should cost the same price everywhere if all prices are measured in terms of one common currency. The theory of purchasing power parity (PPP), which is a logical extension the law of one price, states that apart from transportation costs and trade barriers, similar goods should sell for similar prices, if prices are measured in terms of one common currency. On a broader sense, PPP states that once converted to a common currency, national price levels should be equal (Rogoff, 1996).

PPP theory has important implications in international finance and is an important relation in exchange rate determination. Thus testing the validity of the PPP through different econometric methods has been an important issue for econometricians for many years. LOP is the basic building block of PPP. One can put LOP into equations by the following equational form for any good i,

$$\frac{E.P_i^*}{P_i} = 1 \tag{5.1}$$

where E is the nominal exchange rate defined as the amount of domestic currency needed to purchase one unit of foreign currency, P_i^* is the foreign price of good i and P_i is the domestic price of good i.

LOP is the simplest form of PPP. Absolute PPP theory provides a broader measure for the international price differentials by taking the aggregate price levels into account and considers a "basket" of goods instead of one single good. The following equation form can be used in formulating the PPP theory,[1]

[1]See Rogoff (1996).

$$\frac{E.P^*}{P} = 1 \tag{5.2}$$

where E is the nominal exchange rate defined as the amount of domestic currency needed to purchase one unit of foreign currency, P^* is the foreign aggregate price level and P is the domestic aggregate price level. Equation (5.2) can be summarized as follows: a basket of goods in the domestic country should be of the same price as the same basket of goods in the foreign country once the foreign prices are expressed in terms of the domestic currency. Another implication of equation (5.2) is that the acceptance of PPP requires the exchange rate between two currencies to be equal to the relative price of these two countries. In a time series context the relationship of PPP can be written as,

$$s_t = p_t - p_t^* + \varepsilon_t \tag{5.3}$$

where s_t, p_t, p_t^* are the log of nominal exchange rate, domestic price level and foreign price level respectively. An error term is added to the equation to generalize the approach and capture the shocks that affect the system. Note that in the equation we are assuming a linear form.

Equation (5.3) is often known as the strong-form PPP. It has been one of the common equations in testing PPP. However, strong-form PPP theory suffers from some serious problems. Therefore, another form of PPP, which is called weak-form PPP, has captured the attention of economists as a more realistic form.[2] In this study, we test whether weak-form and strong-form PPP holds for a sample of eight countries (Austria, Canada, Denmark, Germany, Japan, Netherlands, Norway and the United Kingdom) with monthly bilateral exchange rates against U.S. dollar between January 1973 and October 2009. The innovations describing exchange rate returns and inflation rates are heavy-tailed in nature and their densities can be better approximated by stable distributions other than the Gaussian normal distribution (Falk and Wang, 2003). Based on this fact, this study has two main contributions to the existing PPP testing literature. First, we test weak-form PPP through LAD-based co-integration tests

[2]Strong-form and weak-form PPP are further explored in chapter 6.

that are introduced in part I of this dissertation. LAD estimator has superior properties over OLS estimator with heavy-tailed data structure. Secondly, strong-form PPP is tested through fully-modified least absolute deviation (FM-LAD) procedure by considering the heavy-tailed structure of exchange rate returns and inflation rates.

There exists a contrasting empirical evidence on determining whether PPP theory holds or not. The evidence from different empirical studies will be shown in chapter 6, which accounts for a literature review on the subject. In chapter 7, empirical analysis is performed: data variables are explored and the estimation results are presented. Chapter 8 concludes.

CHAPTER 6. LITERATURE REVIEW

Many macroeconomic time series are non-stationary processes which have trend and cyclical components. Early works of time series econometrics have devoted significant attention to the problems that occur when dealing with regressions that include non-stationary variables.[1] A necessary condition for a set of non-stationary variables to have a long-run equilibrium relationship is that they are co-integrated. A vector of time series v_t, which is formed of n individual time series, is co-integrated if those individual series are difference stationary and their linear combination $\delta' v_t$ is stationary.[2] In other words, v_t is co-integrated with the co-integrating vector δ if a linear combination of the series forming v_t (which is $\delta' v_t$) is integrated with a lower order than the individual series. It is possible to have more than one co-integrating vectors for $n > 2$. In that case, it can be assumed that there are $r \leq n-1$ linearly independent co-integrating vectors and the number of co-integrating vectors is the co-integrating rank of v_t. The reason why we can use co-integration methods to test for PPP is because PPP can be considered as a long-run equilibrium concept. Note that in the case of PPP, the time series components that form the vector v_t are s_t, p_t, p_t^* and $n = 3$.

As mentioned in section 1.1, it is possible to categorize the most popular approaches in testing co-integration into two: residual-based co-integration tests (Engle and Granger, 1987; Phillips and Ouliaris, 1990) and likelihood ratio tests (Johansen, 1988, 1991). Both type of tests can be used in testing the weak-form PPP relation. The proposal of the residual-based approach is to estimate the co-integrating relationship and use one of the unit root tests to check for the stationarity of the residuals. We use the residual-based approach to test the weak-form PPP relation.

[1] See for example Granger and Newbold (1974) and Phillips (1986).

[2] A formal definition of co-integration due to Engle and Granger (1987) was also given in section 2.2.

In testing long-run PPP through co-integration methods, it is important to mention different estimation procedures together with different forms of PPP. The first form of PPP that will be considered is the strong-form PPP. If the co-integrating vector δ is known a priori through economic theory, one can decide whether co-integration holds by testing if the linear combination $\delta' v_t$ is stationary or not. To understand the strong-form PPP it helps to re-write the relationship of equation (5.3) in the following form,

$$s_t - p_t + p_t^* = v_t \tag{6.1}$$

If strong-form PPP holds then one would expect the components of the v_t series represented in equation (6.1) to be tied together in the long-run although many developments could cause permanent changes in individual s_t, p_t and p_t^* series. If v_t is a stationary process in equation (6.1) then strong-form PPP is said to hold. In particular, if v_t series is stationary (I(0)) even though s_t, p_t and p_t^* are all I(1) then the null hypothesis of no co-integration can be rejected. Testing the strong-form PPP through equation (6.1) implies the restriction of the co-integrating vector δ among s_t, p_t and p_t^* series to $[1 - 11]'$. Strong-form PPP has homogeneity as the underlying assumption. Homogeneity assumption says that if the prices are multiplied by the same constant, PPP remains unchanged.

Corbae and Ouliaris (1988) is one of the early studies that test for the validity of strong-form PPP for U.S., West Germany, U.K., Japan, Italy, Canada and France. They apply the unit root tests: augmented Dickey-Fuller and Phillips-Perron, for monthly averages of daily data of U.S. dollar, Deutsche mark, Japanese yen, British pound, Italian lira, Canadian dollar and French franc to test whether real exchange rates are stationary. Their test results led to the rejection of long-run PPP for the period between July 1973 and September 1986. Lothian and Taylor (1996) analyze the long-run mean reversion properties of real exchange rates of France and U. S. against the sterling beginning in 1791 and ending in 1990. Their results are mostly in favor of the strong-form PPP hypothesis. By applying Dickey-Fuller and Phillips-Perron unit root tests, they can reject the unit root hypothesis of their real exchange rates for the full sample period. When they consider smaller subperiods, however, they can not reject the existence of a unit root in the real exchange rates. They also estimate AR(1) equations for dollar-sterling and franc-sterling exchange rates both over the full sample period and the period excluding the years after 1973. The estimated coefficients are found to be close but less than unity.

Studies by Taylor (2002) and Lopez et al. (2005) use more recent unit root tests to test whether the real exchange rates are stationary or not. Taylor (2002) investigates the PPP theory for 20 countries (16 developed and 4 developing) over 100 years by performing augmented Dickey-Fuller (ADF) and generalized least squares versions of Dickey-Fuller (DF-GLS) tests of Elliot et al. (1996), which are more powerful than the ADF tests. He finds that ADF tests do not reject the null hypothesis of a unit root for most cases but there is strong support for PPP when DF-GLS tests are applied. Thus he concludes that long-run PPP holds in the twentieth century.[3] Lopez et al. (2005) challenges Taylor' s results and lag selection methods for ADF and DF-GLS tests.[4] They re-test the stationarity of real exchange rates for the same sample that Taylor (2002) used excluding the developing countries and focusing on developed ones by again performing ADF and DF-GLS tests. But they employ proper lag selection procedures: general to specific procedure of Hall (1994) for ADF tests and modified Akaike information criterion (MAIC) of Ng and Perron (2001) for DF-GLS tests. They find that the unit root hypothesis is rejected only for 9 out of 16 real exchange rates and the differences in the ADF and DF-GLS test results are almost eliminated, suggesting that there is no sufficient evidence to conclude that long-run PPP held in the twentieth century.

Many researchers prefer to test a less restrictive version of PPP than the strong-form PPP. In reality, price series have substantial measurement error and contain important non-tradeable elements constructed differently from country to country which makes it a relatively difficult and restrictive theory to justify. Especially with time series data, other problems arise as new goods emerge over time. Also the tastes of consumers and the quality of goods are different for different countries. Thus it is difficult to identify a relevant consumption basket and to

[3]Taylor (2002) also tests PPP through Johansen-type co-integration tests but can not find a strong supportive evidence and mentions that it might be due to the low power problem of those type of tests.

[4]Taylor (2002) chooses the lag lengths by a Lagrange multiplier criterion.

assign the right weights to the goods in the basket (Rogoff (1996); Levanoni and Darnell (1999)). Weak-form PPP is a less restrictive version of strong-form PPP and it has started getting attention among economists due to its more realistic nature (Patel (1990); MacDonald (1993)). Strong-form PPP restricts the co-integrating vector among economic series s_t, p_t and p_t^* to $[1 - 11]'$. Weak-form PPP allows for *any* kind of co-integration and tests whether there is an arbitrary relation among the three series. In a time series context, weak-form PPP can be tested in a general trivariate form by the following equation,

$$s_t = \beta_0 + \beta_1 p_t + \beta_2 p_t^* + \varepsilon_t \tag{6.2}$$

Patel (1990) argues that restricting β_1 and β_2 to 1 and - 1 is inappropriate because of measurement errors and differing weights for different countries in constructing price indices. Restricting $\beta_1 = 1$ and $\beta_2 = -1$ is related to the differences in price index weights and also to the presence of non-tradeable goods. If this restriction holds then the homogeneity assumption is said to hold (Hallwood and MacDonald (2000), p. 144).[5] In practice, the aggregate price indices such as the CPI or PPI contain many non-tradeable goods prices, which are less likely to be equalized by international trade. Dutton and Strauss (1997) find that non-traded goods relative prices are in fact an important determinant of real exchange rate behavior and are an important source of persistent deviations of the real exchange rate away from its PPP value.

Patel (1990) makes use of the testing methods of Engle and Granger (1987) and Stock and Watson (1988) to test for weak-form PPP for 6 developed and relatively free-market economies (U.S., U.K., Canada, West Germany, Netherlands and Japan) between 1974 and 1986 with quartely data. His results do not present enough evidence in accepting the PPP theory in the long-run, as he finds support for PPP only in 5 out of 15 country pairs. Enders (1988) tests for PPP through error correction models and residual-based tests as proposed in Engel and Granger (1987). His monthly data is formed of wholesale price index based real exchange rates between U.S.A. and her trading partners (Canada, Japan and Germany) for two different periods: January 1960-April 1971 (period of fixed exchange rates) and January 1973-November

[5] It is also known as the proportionality condition (Sosvilla-Rivero and Garcia, 2006).

1986 (period of flexible exchange rates). He tests for PPP by making the symmetry assumption ($\beta_1 = -\beta_2$). Although he finds strong support for PPP between U.S. and Japan during the fixed exchange rates period and weak support for PPP between U.S. and Canada during the flexible exchange rates period, the data overall does not indicate strong evidence to accept the PPP theory.

Residual-based tests can test for the existence of co-integration, likelihood-ratio-based tests allow us to test for the number of co-integrating vectors. MacDonald (1993) tests weak-form PPP with Johansen' s method for Canada, France, U.K., Germany and Japan through January 1974 and June 1990. His data gives supportive evidence to a long-run PPP relationship between the sample countries' bilateral exchange rates with U.S. dollar and their corresponding relative prices. He also tests and rejects the existence of strong-form PPP for his data. Salehizadeh and Taylor (1999) apply the Johansen co-integration procedure for 27 emerging/developing economy countries against the U.S. dollar with monthly data covering the period 1975:01-1997:09. They find out that only 14 out of 27 countries show significant evidence in support of the weak-form PPP. When they test for strong-form PPP with ADF unit root tests, their results are not in favor of real exchange rate stationarity except for Mexico.

Based on the study of Phillips and Hansen (1990), some researchers test the strong-form PPP through fully-modified OLS (FM-OLS) procedure (Dutt and Ghosh (1995); Braha and Anoruo (2002); Crownover et al. (1996)). Dutt and Ghosh (1995) and Crownover et al. (1996) make the symmetry assumption. Dutt and Ghosh (1995) test for both weak-form and strong-form PPP. They use Phillips and Ouliaris (1990) residual-based co-integration tests to test for weak-form PPP (necessary condition for PPP) and FM-OLS Wald tests to test for strong-form PPP (necessary and sufficient condition for PPP). Their monthly data span the period from 1973:1 to 1992:12 and are formed of the countries of the members of EMS (European Monetary System) and participants to the European Exchange Rate Mechanism (ERM): Belgium, Denmark, France, Italy, Netherlands. Bilateral exchange rates are studied *vis-á-vis* the anchor currency Deutschemark. Their results support the weak-form PPP and show evidence that there is co-integration between the exchange rate and price ratio series (only exception is with the Belgian franc/German mark exchange rate series). However, the strong-form PPP is not supported by the data and strongly rejects the null of joint restrictions for the EMS. Crownover et al. (1996) also test for both weak-form and strong-form PPP and make use of FM-OLS method for annual data running from 1927 to 1992. They consider six countries: Canada, France, Germany, Italy, U.K. and U.S.A. with bilateral exchange rates. While testing for the weak-form PPP, they investigate whether there is a unit root in FM-OLS residuals or not. They find that weak-form PPP holds for 10 out of 15 country pairs. When it comes to strong-form PPP tests by FM-OLS Wald tests, for only 5 country pairs they can not reject the null of joint restrictions. A more recent study by Braha and Anoruo (2002) tests weak-form PPP and strong-form PPP through Harris-Inder co-integration tests and FM-OLS method respectively for Asian countries (Philippines, Singapore, Thailand and Malaysia) with quarterly data from 1973:1 to 1999:2. While the results are mostly in favor of the weak-form PPP, strong-form PPP does not get much support from their data.

Residual-based and Johansen-type likelihood ratio tests assume that exchange rates and inflation rates follow normal or at least near normal behavior in the tails. But when the distributional aspects of the exchange rates and inflation rates are taken into account, many studies indicate that exchange rate returns and inflation rates are subject to high volatility and heavy-tails. Furthermore, their innovations exhibit Pareto-like infinite-variance behavior in the tails. In particular, the studies that find evidence of infinite-variance in exchange rate returns are Koedijk et al. (1990), Koedijk and Kool (1992), Akgiray et al. (1988), Fofack and Nolan (2001) and Basterfield et al. (2003). Bidarkota and McCulloch (1998) and Charemza et al. (2005) find evidence of infinite-variance behavior in inflation rates.

A recent study by Falk and Wang (2003), tests PPP with the infinite-variance error assumption. While applying the theoretical findings of Caner (1998) to test for weak-form PPP through residual-based tests of Phillips and Ouliaris (1990) and (Johansen, 1988, 1991) type tests, Falk and Wang (2003) assume that the log levels of exchange rate, domestic price level, foreign price level are finite-order, trivariate VARs with i.i.d. disturbances that are α-stable processes sharing a common stability index α. They consider the bilateral exchange rates and price levels (CPI) of 12 industrialized countries: Belgium, Canada, Denmark, France, Germany, Italy, Japan, the Netherlands, Norway, Spain, Sweden and U.K.. All exchange rates are measured relative to the U.S. dollar. They estimate the stability indices of the innovation series by Nolan (1997)' s maximum likelihood procedure for each country with monthly data (1973:1-1999:12). Their results indicate that all α levels are less than 2 (except for the Canadian exchange rate), supporting the infinite-variance assumption. Their results are slightly less supportive of the PPP theory under the infinite-variance error assumption.

Overall evidence from the previous empirical studies that investigate the heavy-tailed structure of exchange rate returns and inflation rates suggests that it is not appropriate to assign a normal distributional structure to the exchange rate returns and inflation rates. Based on this evidence, in our empirical application to test for weak-form and strong-form PPP, we assume that the error distributions of our exchange rate and price series come from infinite-variance stable distributions. We utilize a general trivariate form as in equation (6.2) for our estimation purposes. Our contributions to the literature are twofold: first to test for weak-form PPP, we make use of the new robust residual-based tests that are proposed in part I, and secondly, to test for strong-form PPP, we make use of FM-LAD estimator, which is more robust to heavy-tails than the FM-OLS estimator.[6]

[6] As mentioned above, one study that uses the FM-OLS estimator to test for strong-form PPP is Crownover et al. (1996). They also investigate the possibility of thick tails for exchange rate data and find little evidence of thick tails with annual frequency. Therefore, they proceed with FM-OLS calculations. However, they apply the Hill (1975) estimator to estimate the tail indices. Hill (1975)' s method has been criticized by many studies for not giving accurate results (see for example McCulloch (1997) and Fofack and Nolan (1999) and the references therein.). Moreover, they apply the Hill (1975) estimator to the logarithms of exchange rate data directly. It is known that Hill (1975) estimator is only optimal when applied to i.i.d. data (Resnick and Stărică (1997); Ling and Peng (2004)).

CHAPTER 7. EMPIRICAL ANALYSIS

7.1 Data

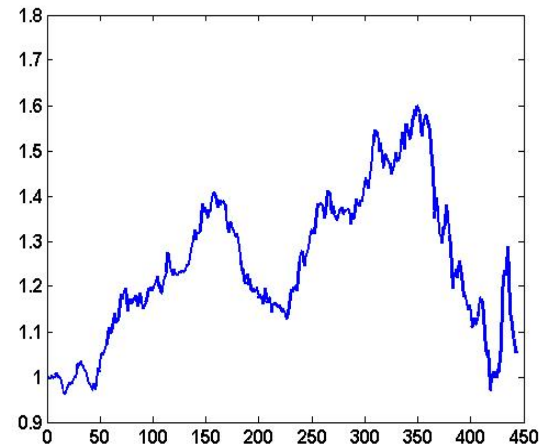
Data are taken from the International Monetary Fund' s International Financial Statistics (IFS). Data span the period of flexible exchange rate regimes. A sample of eight countries is chosen including Austria, Canada, Denmark, Germany, Japan, Netherlands, Sweden and the United Kingdom. Frequency of the data is monthly between January 1973 and December 2009. Exchange rate data are domestic currencies per unit of the United States Dollar: Austria (Schilling), Canada (Canadian Dollar), Denmark (Krone), Germany (Deutsche Mark), Japan (Yen), Netherlands (Guilder), Sweden (Krona) and U.K. (Pound). Exchange rate data are from the line RF (period average) in IFS tapes.[1] Producer price indices (PPIs) have been collected to be the price levels. All PPI series have been indexed to year 2005=100.

Graphs of Canadian exchange rate and PPI data together with the exchange rate returns and inflation rates (log difference of PPIs) against time are presented in Figure 7.1. Similar graphs are given for U.K. in Figure 7.2.

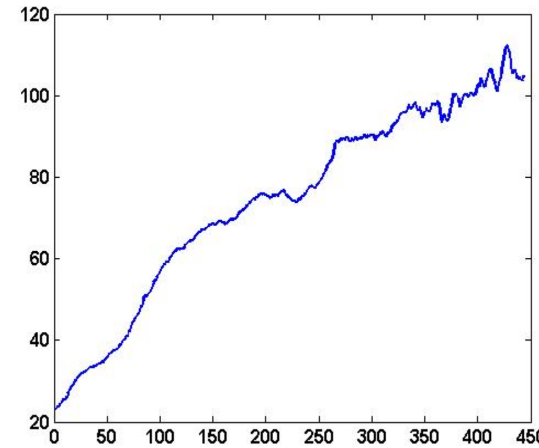
Similar to Caner (1998) and Falk and Wang (2003), s_t, p_t and p_t^* series are assumed to follow a finite-order trivariate VAR system with infinite-variance stable innovations that share a common stability index α.[2] We check whether weak-form PPP holds or not through residual-based co-integration tests. Both OLS-based and LAD-based co-integration tests will be used. Next, fully-modified coefficient estimates from equation (6.2) will be used to test whether strong-form PPP holds or not through Wald tests.

[1] The exchange rate data for U.K. are from the line RH. The reciprocal of the published data is used.

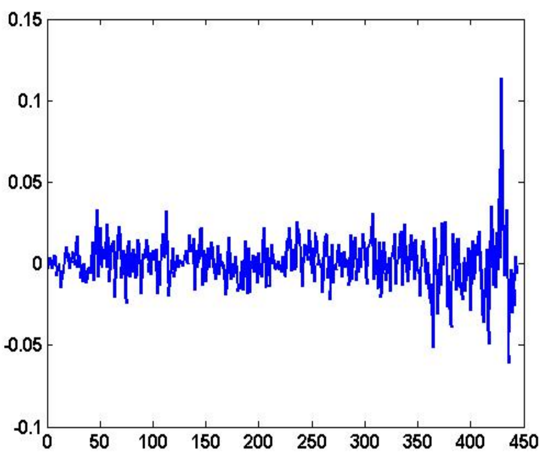
[2] s_t, p_t and p_t^* series in our case are log nominal exchange rate, log PPI and log US PPI respectively for each country.



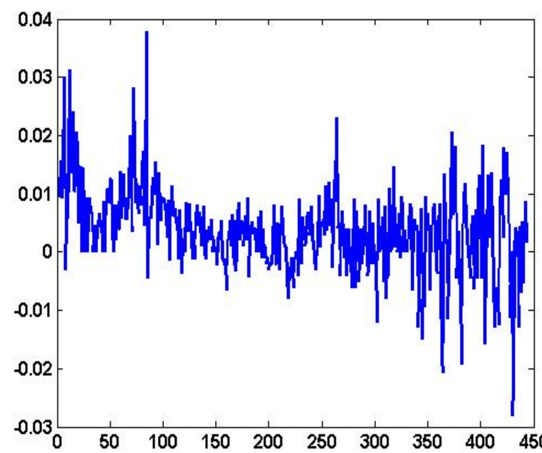
(a) Canadian dollar exchange rate



(b) Canadian Producer Price Index

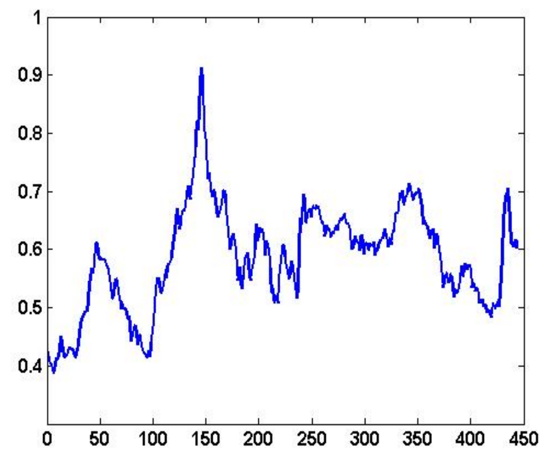


(c) Canadian dollar exchange rate returns

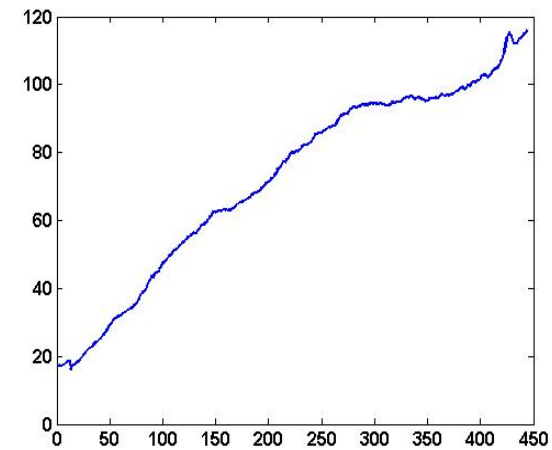


(d) Canadian inflation rates

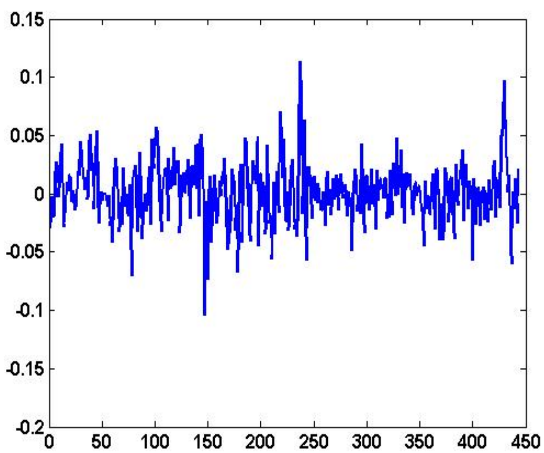
Figure 7.1: Canadian dollar exchange rate and PPI: 1973:1-2009:12



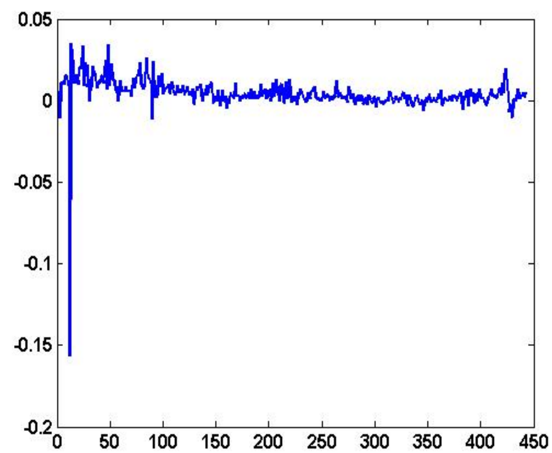
(a) U.K. pound exchange rate



(b) U.K. Producer Price Index



(c) U.K. pound exchange rate returns



(d) U.K. inflation rates

Figure 7.2: U.K. pound exchange rate and PPI: 1973:1-2009:12

7.1.1 Estimation of stable parameters

In this section, we use Box-Jenkins methods of model selection to fit VAR(q) models to s_t, p_t and p_t^*. We run a system of regressions that are in the form:

$$s_t = \mu_1 + a_{11}s_{t-1} + b_{11}p_{t-1} + c_{11}p_{t-1}^* + \ldots + a_{1q}s_{t-q} + b_{1q}p_{t-q} + c_{1q}p_{t-q}^* + \varepsilon_{1t} \qquad (7.1a)$$

$$p_t = \mu_2 + a_{21}s_{t-1} + b_{21}p_{t-1} + c_{21}p_{t-1}^* + \ldots + a_{2q}s_{t-q} + b_{2q}p_{t-q} + c_{2q}p_{t-q}^* + \varepsilon_{2t} \qquad (7.1b)$$

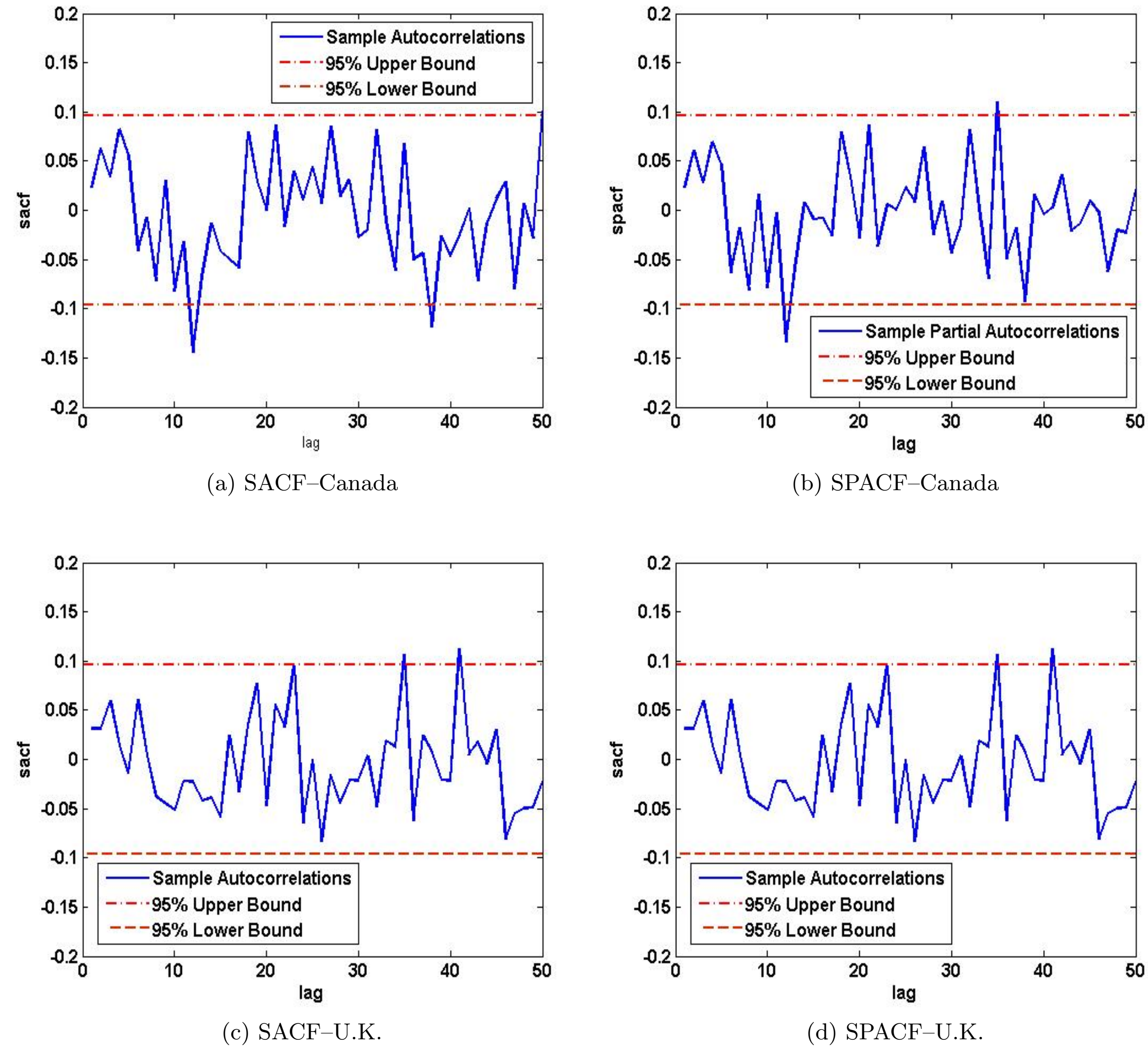
$$p_t^* = \mu_3 + a_{31}s_{t-1} + b_{31}p_{t-1} + c_{31}p_{t-1}^* + \ldots + a_{3q}s_{t-q} + b_{3q}p_{t-q} + c_{3q}p_{t-q}^* + \varepsilon_{3t} \qquad (7.1c)$$

OLS and LAD procedures are used for estimation purposes for Caner test and our test respectively. We also include 11 seasonal dummies to VAR equations.[3] Then we test the residual series from those VAR(q) models for independence to determine whether they come from an i.i.d. distribution or not. In total there are 24 residual series from 8 countries.

Lag selection of the VARs are utilized by checking the SACF & SPACF structures of the residuals. SACFs and SPACFs for some of the Canadian and U.K. residuals from the fitted VARs are presented in Figures 7.3 through 7.8. VAR(12) models are fitted for Canada and U.K.. Lin-McLeod i.i.d. tests are applied on the residuals after we identified a model. Table 7.1 shows all the lag orders chosen for the VARs together with the Lin-McLeod i.i.d. test results. In order to consume less space, only the results from LAD-based VARs are presented for i.i.d. tests and estimates of stable parameters. Lag length selection and stability index estimation are important to decide about the critical values. Table 7.9 shows the critical values chosen for OLS-based Caner test in addition to our test which is LAD-based.

In Table 7.1, p-values of Lin-McLeod tests need to be greater than 0.05 for us not to reject the i.i.d. hypothesis at 5% significance level. From Table 7.1, we can see that the selected VAR models perform well enough to decide that the innovations are in fact i.i.d. processes. Next, we estimate the stable parameters (α, β) for each residual series and present the 95% confidence intervals. The results are shown in Table 7.2.

[3]See for example MacDonald (1993) and Falk and Wang (2003).



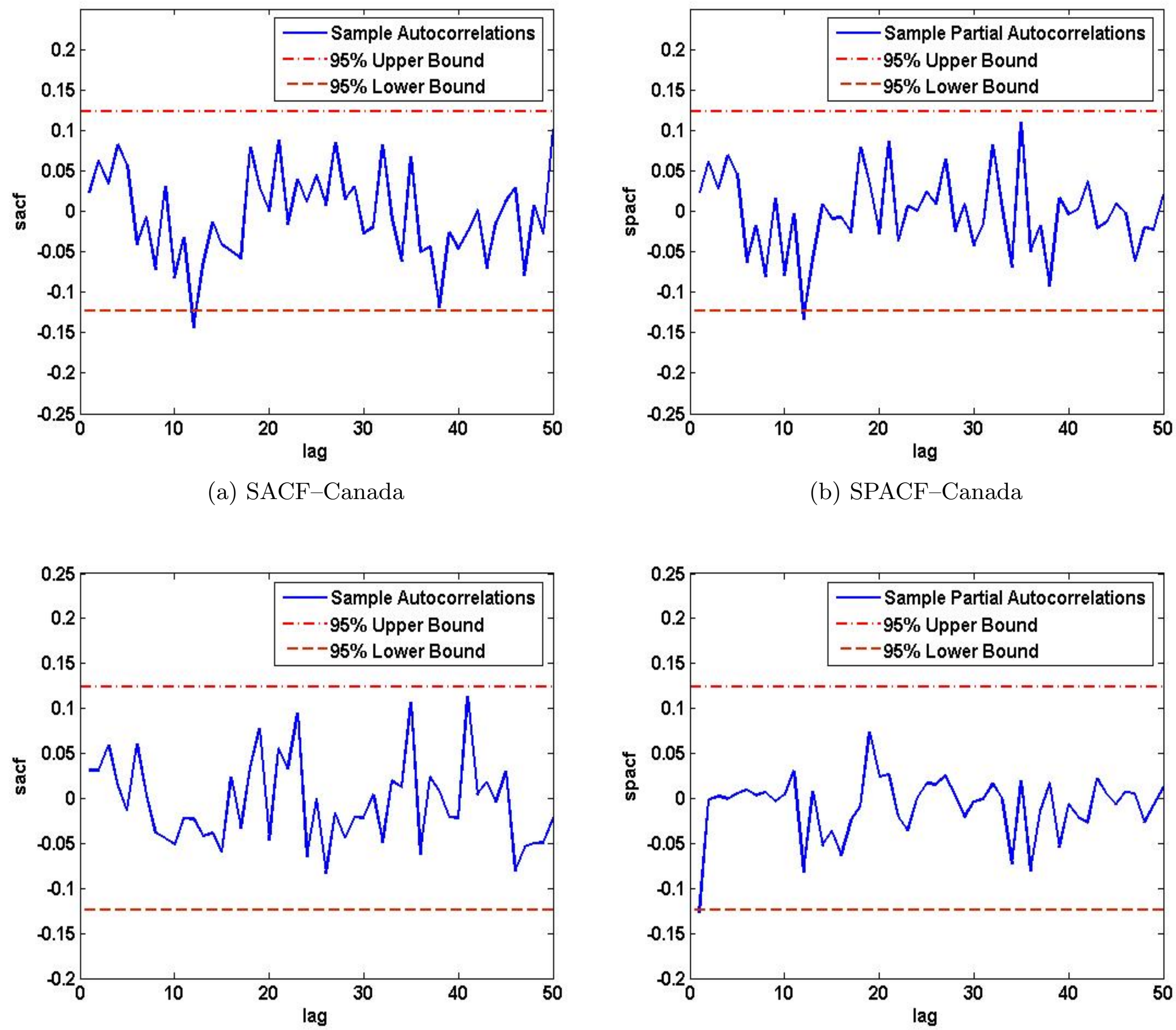
(a) SACF–Canada

(b) SPACF–Canada

(c) SACF–U.K.

(d) SPACF–U.K.

Figure 7.3: SACFs and SPACFs of the residuals from equation (7.1a) with Gaussian confidence bounds: $\pm 1.96\frac{1}{\sqrt{T}}$



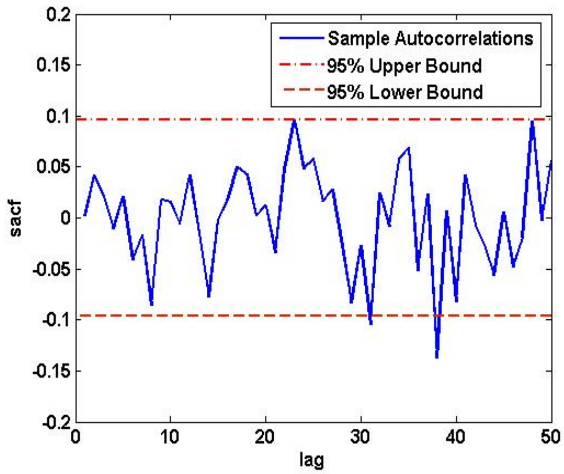
(a) SACF–Canada

(b) SPACF–Canada

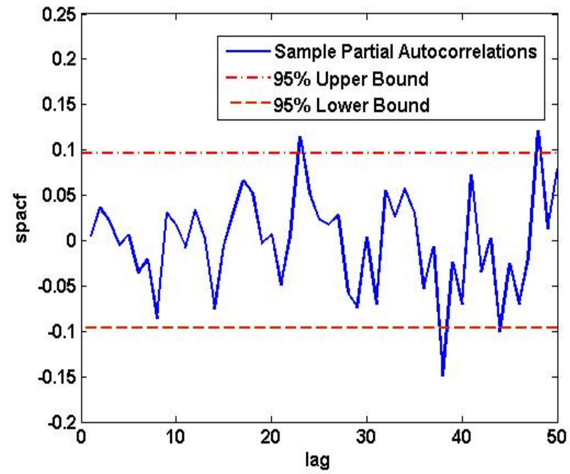
(c) SACF–U.K.

(d) SPACF–U.K.

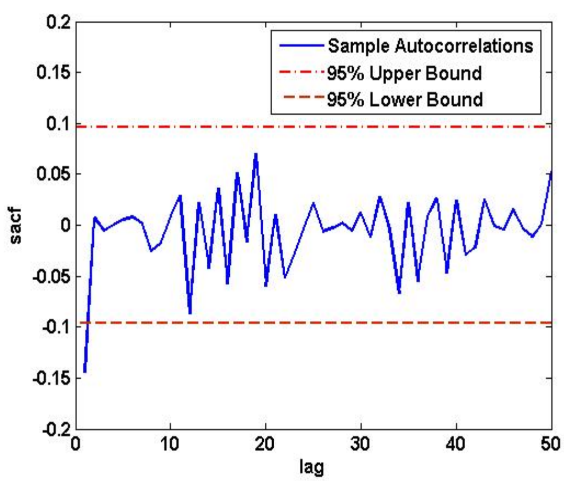
Figure 7.4: SACFs and SPACFs of the residuals from equation (7.1a) with conservative confidence bounds: $\pm 2.57 \frac{1}{\sqrt{T}}$



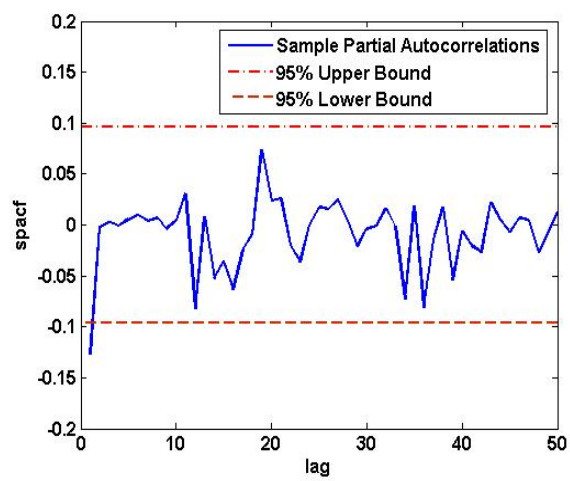
(a) SACF–Canada



(b) SPACF–Canada

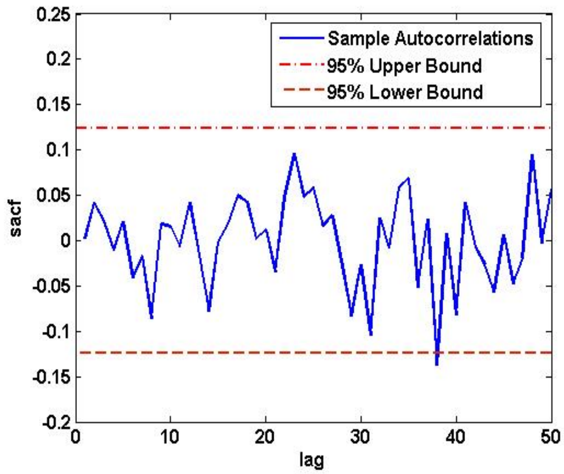


(c) SACF–U.K.

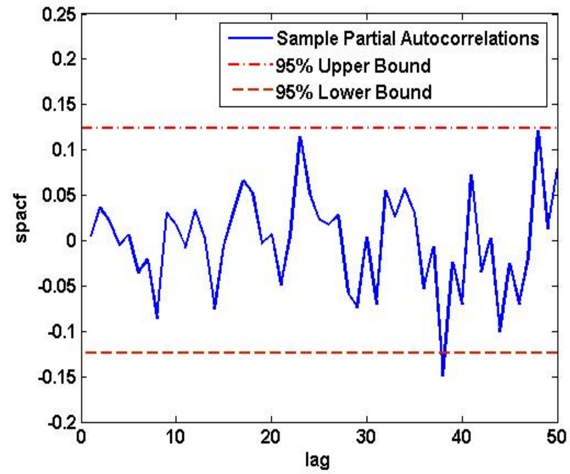


(d) SPACF–U.K.

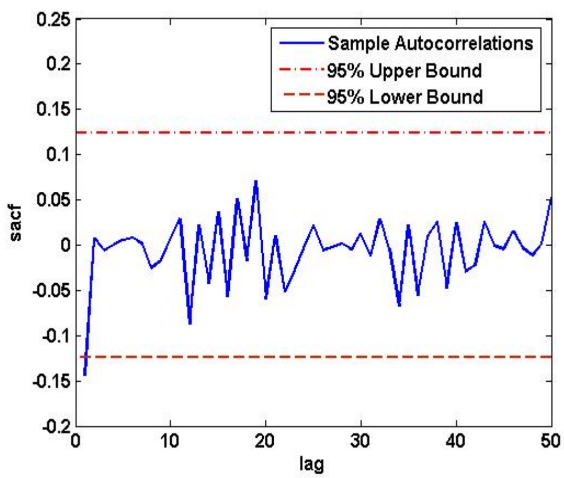
Figure 7.5: SACFs and SPACFs of the residuals from equation (7.1b) with Gaussian confidence bounds: $\pm 1.96\frac{1}{\sqrt{T}}$



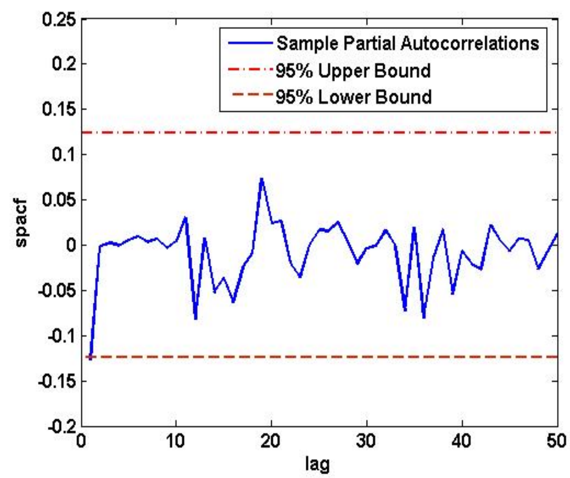
(a) SACF–Canada



(b) SPACF–Canada

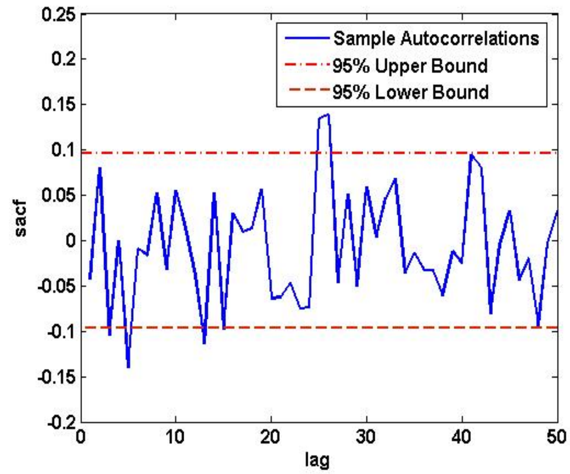


(c) SACF–U.K.

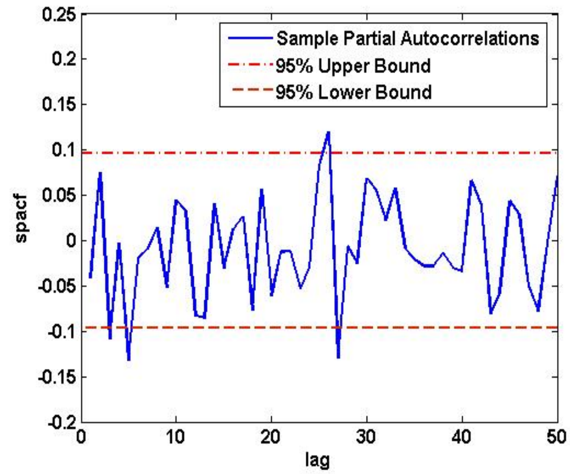


(d) SPACF–U.K.

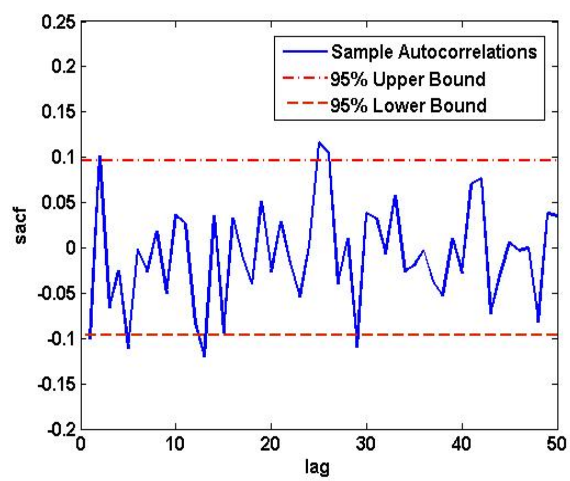
Figure 7.6: SACFs and SPACFs of the residuals from equation (7.1b) with conservative confidence bounds: $\pm 2.57\frac{1}{\sqrt{T}}$



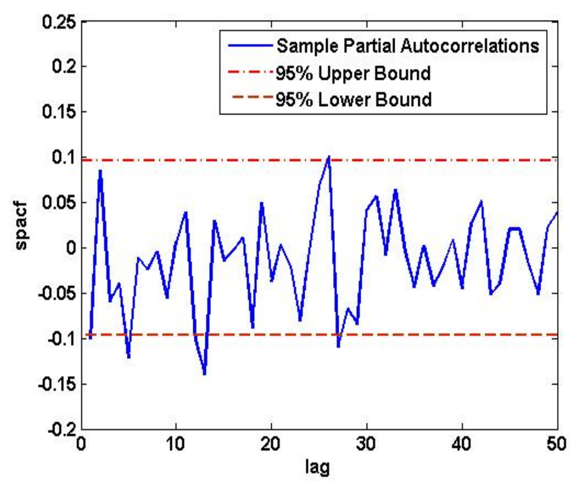
(a) SACF–Canada



(b) SPACF–Canada

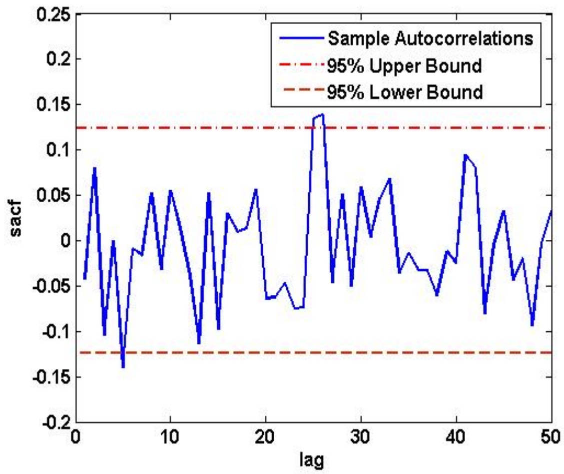


(c) SACF–U.K.

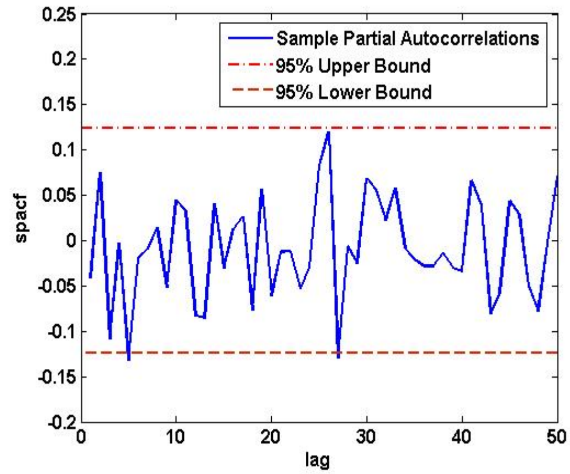


(d) SPACF–U.K.

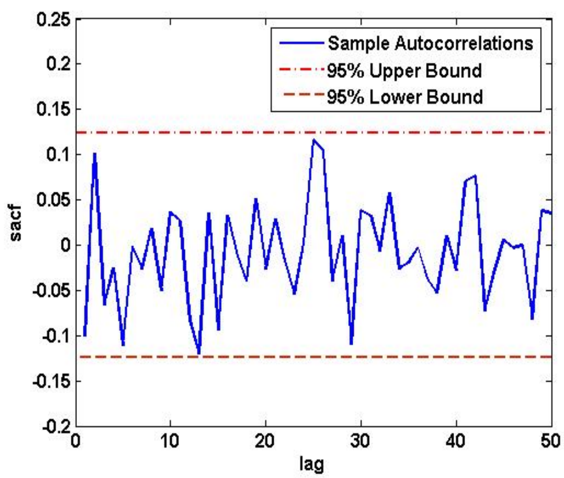
Figure 7.7: SACFs and SPACFs of the residuals from equation (7.1c) with Gaussian confidence bounds: $\pm 1.96 \frac{1}{\sqrt{T}}$



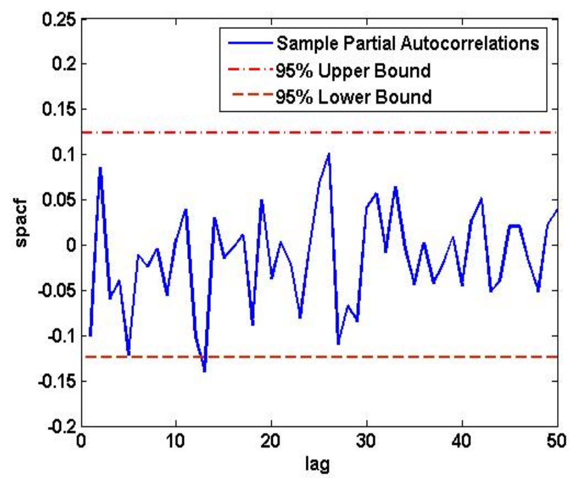
(a) SACF–Canada



(b) SPACF–Canada



(c) SACF–U.K.



(d) SPACF–U.K.

Figure 7.8: SACFs and SPACFs of the residuals from equation (7.1c) with conservative confidence bounds: $\pm 2.57\frac{1}{\sqrt{T}}$

Table 7.1: P-values for Lin-McLeod tests of randomness

Country	Fitted Model	Variable	lag	P-R Stable[a]	P-R Normal[b]	B-P Stable[c]	B-P Normal[d]	B-P Chi-square[e]
Austria	VAR(13)	Exchange Rate	25	0.517	0.751	0.553	0.799	0.837
			50	0.596	0.835	0.311	0.468	0.617
			65	0.551	0.804	0.364	0.548	0.747
		PPI	25	0.223	0.492	0.065	0.069	0.072
			50	0.145	0.257	0.061	0.063	0.083
			65	0.159	0.286	0.035	0.033	0.045
		US PPI	25	0.079	0.077	0.100	0.123	0.157
			50	0.064	0.037	0.061	0.047	0.061
			65	0.058	0.033	0.039	0.027	0.039
Canada	VAR(12)	Exchange Rate	25	0.116	0.155	0.052	0.063	0.058
			50	0.127	0.165	0.086	0.099	0.142
			65	0.159	0.225	0.074	0.091	0.158
		PPI	25	0.520	0.901	0.486	0.862	0.896
			50	0.284	0.571	0.142	0.230	0.342
			65	0.168	0.271	0.084	0.124	0.222
		US PPI	25	0.062	0.035	0.061	0.036	0.032
			50	0.047	0.006	0.043	0.027	0.038
			65	0.044	0.005	0.031	0.028	0.039
Denmark	VAR(12)	Exchange Rate	25	0.528	0.782	0.696	0.929	0.952
			50	666	0.914	0.590	0.886	0.943
			65	0.665	0.917	0.610	0.894	0.966
		PPI	25	0.109	0.142	0.200	0.324	0.376
			50	0.260	0.481	0.492	0.884	0.941
			65	0.324	0.633	0.484	0.887	0.962
		US PPI	25	0.094	0.120	0.095	0.104	0.110
			50	0.070	0.032	0.054	0.041	0.053
			65	0.060	0.028	0.051	0.035	0.055

Table 7.1: (Continued)

Country	Fitted Model	Variable	lag	P-R Stable[a]	P-R Normal[b]	B-P Stable[c]	B-P Normal[d]	B-P Chi-square[e]
Germany	VAR(12)	Exchange Rate	25	0.575	0.778	0.646	0.826	0.868
			50	0.590	0.793	0.349	0.510	0.659
			65	0.547	0.756	0.364	0.547	0.742
		PPI	25	0.472	0.911	0.424	0.841	0.883
			50	0.458	0.900	0.465	0.922	0.969
			65	0.418	0.864	0.375	0.817	0.927
		US PPI	25	0.068	0.038	0.070	0.036	0.032
			50	0.054	0.007	0.045	0.025	0.033
			65	0.049	0.008	0.034	0.025	0.035
Japan	VAR(12)	Exchange Rate	25	0.494	0.740	0.475	0.732	0.768
			50	0.338	0.553	0.224	0.334	0.473
			65	0.305	0.509	0.289	0.473	0.677
		PPI	25	0.167	0.335	0.272	0.646	0.681
			50	0.321	0.747	0.516	0.974	0.993
			65	0.391	0.870	0.551	0.989	0.999
		US PPI	25	0.069	0.039	0.065	0.040	0.035
			50	0.047	0.005	0.047	0.030	0.040
			65	0.044	0.005	0.034	0.025	0.035
Netherlands	VAR(12)	Exchange Rate	25	0.170	0.274	0.218	0.293	0.336
			50	0.317	0.456	0.092	0.104	0.148
			65	0.344	0.493	0.121	0.141	0.257
		PPI	25	0.043	0.010	0.089	0.097	0.098
			50	0.083	0.099	0.177	0.296	0.432
			65	0.106	0.127	0.312	0.624	0.810
		US PPI	25	0.047	0.007	0.067	0.028	0.027
			50	0.047	0.001	0.055	0.027	0.038
			65	0.042	0.001	0.037	0.025	0.035

Table 7.1: (Continued)

Country	Fitted Model	Variable	lag	P-R Stable[a]	P-R Normal[b]	B-P Stable[c]	B-P Normal[d]	B-P Chi-square[e]
Sweden	VAR(12)	Exchange Rate	25	0.387	0.643	0.310	0.506	0.552
			50	0.309	0.535	0.476	0.797	0.888
			65	0.284	0.477	0.506	0.824	0.932
		PPI	25	0.519	0.791	0.556	0.811	0.853
			50	0.389	0.671	0.327	0.558	0.700
			65	0.385	0.663	0.231	0.338	0.543
		US PPI	25	0.084	0.076	0.091	0.089	0.090
			50	0.063	0.027	0.075	0.088	0.122
			65	0.060	0.029	0.066	0.085	0.149
U.K.	VAR(12)	Exchange Rate	25	0.620	0.852	0.387	0.600	0.623
			50	0.491	0.716	0.233	0.349	0.492
			65	0.409	0.625	0.200	0.286	0.477
		PPI	25	0.281	0.602	0.398	0.816	0.860
			50	0.461	0.904	0.567	0.981	0.996
			65	0.552	0.973	0.675	0.999	0.999
		US PPI	25	0.047	0.004	0.063	0.022	0.022
			50	0.045	0.000	0.049	0.025	0.035
			65	0.045	0.001	0.031	0.013	0.018

[a] Peña-Rodriguez test statistic with stably distributed variables.
[b] Peña-Rodriguez test statistic with normally distributed variables.
[c] Box-Pierce test statistic with stably distributed variables.
[d] Box-Pierce test statistic with normally distributed variables.
[e] Box-Pierce chi-square test statistic with normally distributed variables.

7.1.2 Diagnostic checks for normality

Residuals from the LAD-based VAR models are visually checked to determine how much they deviate from the normality assumption.[4] The empirical densities of some of the residuals can be found in Figures 7.9 and 7.10. Stable and normal ($N(0, s)$ with s denoting the sample standard deviation) densities are drawn by simulating random variables from stable and normal distributions with parameters estimated from actual data. From the figures it can be inferred that the stable distribution assumption on the errors is more suitable than the normality assumption.

Figure 7.11 contains the QQ plots for the VAR residuals of exchange rates for Austria, Canada, Germany and U.K.. Figure 7.12 contains the QQ plots for the same countries for the VAR residuals of log price levels. QQ plots show that heavy-tailedness is an important issue for our data especially for the inflation rates.

Converging mean and variance tests are also applied to the residuals. Figure 7.13 and 7.14 present the results for Canadian and Swedish data. Panels (a) and (b) plot the first n sample variances and means of Canadian residuals respectively against n. Panels (c) and (d) plot the first n sample variances and means of Swedish residuals respectively against n. In the figures, "$N(0, s)$" (s: sample standard deviation) and "Stable" denote the simulated normal and stable random variables respectively with parameters imposed from actual data. From what we observe in Figures 7.13 and 7.14, stable distribution with infinite-variance and finite-mean seems to be an appropriate assumption on the errors rather than the finite-variance and finite-mean Gaussian normal assumption.

7.2 Unit root tests

First, the order of integration of all variables are checked since the necessary condition for PPP is that all variables are I(1). Tables 7.3 and 7.4 report the results of the OLS-based Caner and LAD-based Samarakoon-Knight unit root tests for the nominal exchange rate series. The results of PPI series are shown in Tables 7.5 and 7.6. Order of serial correlations for Caner

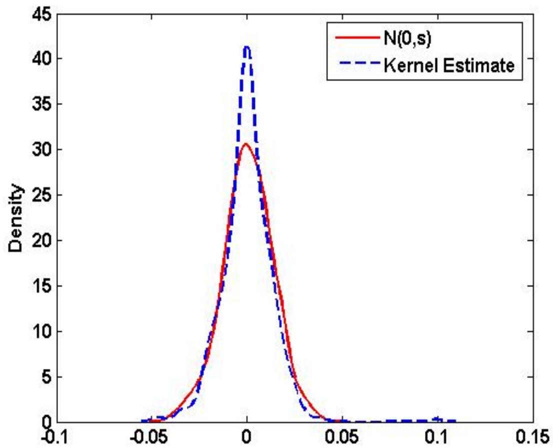
[4] We have a similar structure for OLS-based VAR residuals.

Table 7.2: Maximum Likelihood Estimation of stable parameters (PPP)

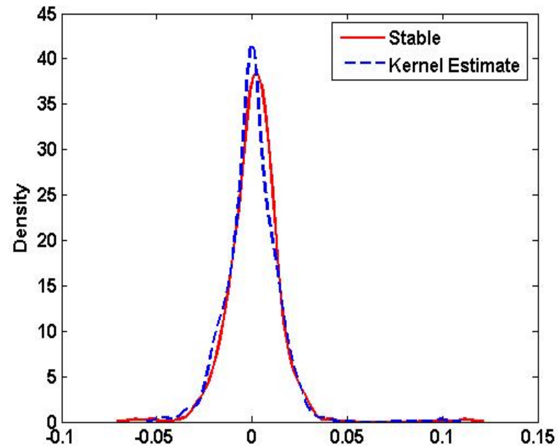
Country	Parameter	Variable		
		Exchange Rate	PPI	US PPI
Austria	$\hat{\alpha}$	1.70±0.14	1.53±0.15	1.25±0.14
	$\hat{\beta}$	-0.08±0.40	0.07±0.30	0.07±0.21
Canada	$\hat{\alpha}$	1.86±0.12	1.59±0.15	1.33±0.14
	$\hat{\beta}$	-0.32±0.69	0.09±0.32	0.12±0.22
Denmark	$\hat{\alpha}$	1.86±0.12	1.59±0.15	1.33±0.14
	$\hat{\beta}$	-0.32±0.69	-0.27±0.32	0.06±0.20
Germany	$\hat{\alpha}$	1.83±0.12	1.36±0.15	1.24±0.14
	$\hat{\beta}$	-0.17±0.63	0.02±0.24	0.11±0.21
Japan	$\hat{\alpha}$	1.86±0.10	1.34±0.15	1.30±0.14
	$\hat{\beta}$	-0.99±0.43*	0.03±0.23	0.03±0.22
Netherlands	$\hat{\alpha}$	1.80±0.13	1.47±0.15	1.18±0.14
	$\hat{\beta}$	-0.18±0.52	0.16±0.27	0.15±0.19
Sweden	$\hat{\alpha}$	1.86±0.11	1.80±0.13	1.26±0.14
	$\hat{\beta}$	0.31±0.70	0.18±0.52	-0.01±0.22
U.K.	$\hat{\alpha}$	1.85±0.12	1.31±0.14	1.11±0.13
	$\hat{\beta}$	0.27±0.68	0.17±0.22	-0.01±0.18

* 95% confidence interval is given for $\beta = -0.9$, since $\beta = -0.99$ is very close to the boundary.

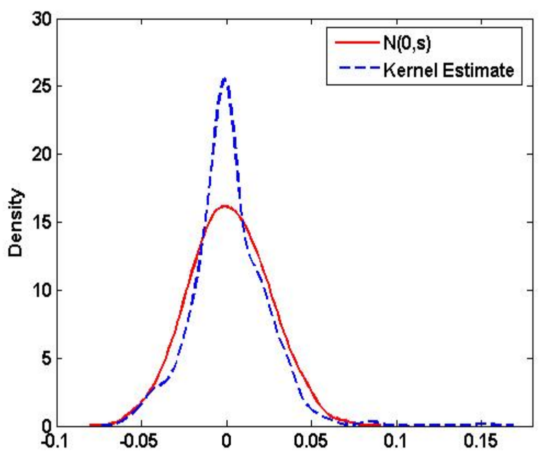
Note: The results are based on LAD estimations.



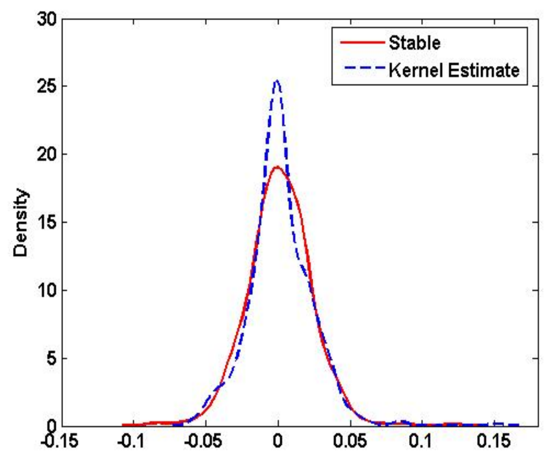
(a) Canadian residuals from VAR(12) ($\alpha = 1.86$)



(b) Canadian residuals from VAR(12)

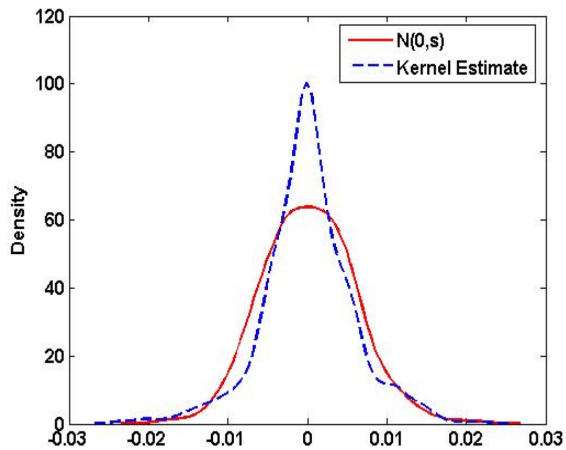


(c) Swedish residuals from VAR(12) ($\alpha = 1.86$)

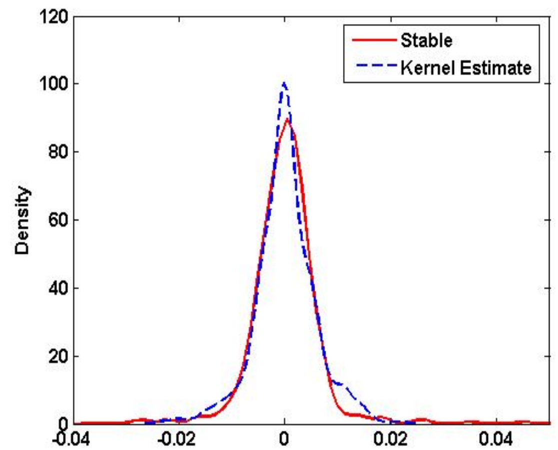


(d) Swedish residuals from VAR(12)

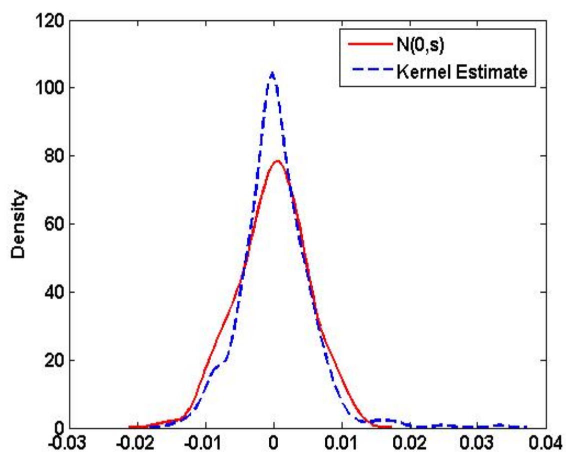
Figure 7.9: Data based kernel densities of the residuals from the fitted VARs (log exchange rates) versus stable and normal densities



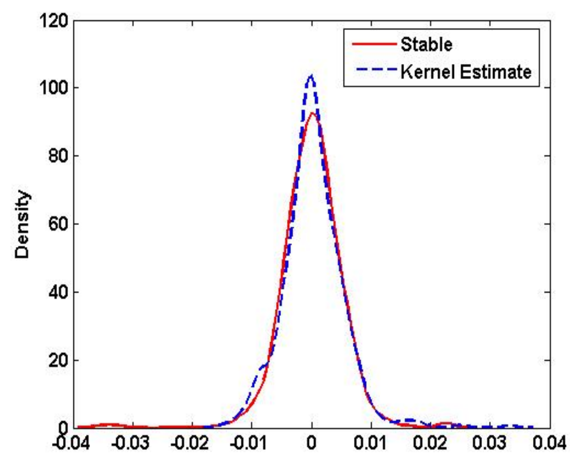
(a) Canadian residuals from VAR(12) ($\alpha = 1.59$)



(b) Canadian residuals from VAR(12)

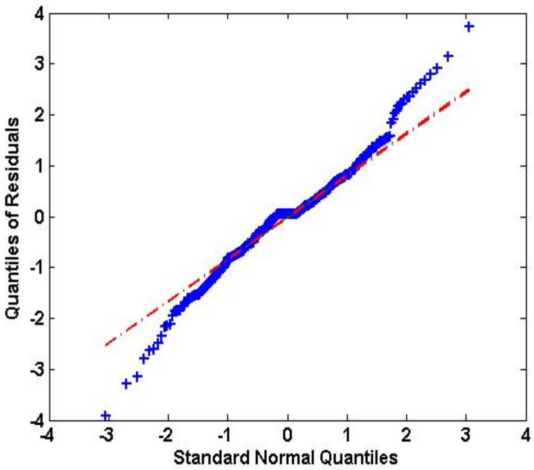


(c) Swedish residuals from VAR(12) ($\alpha = 1.80$)

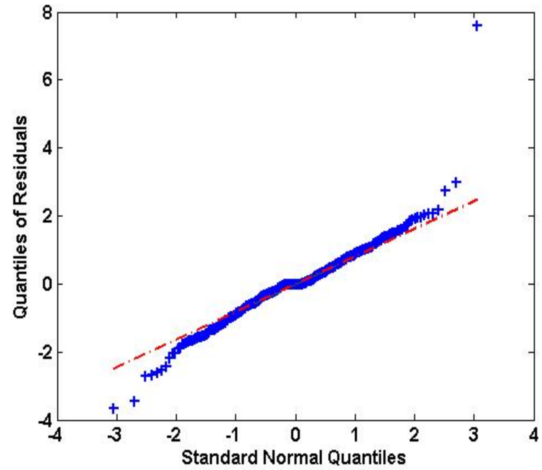


(d) Swedish residuals from VAR(12)

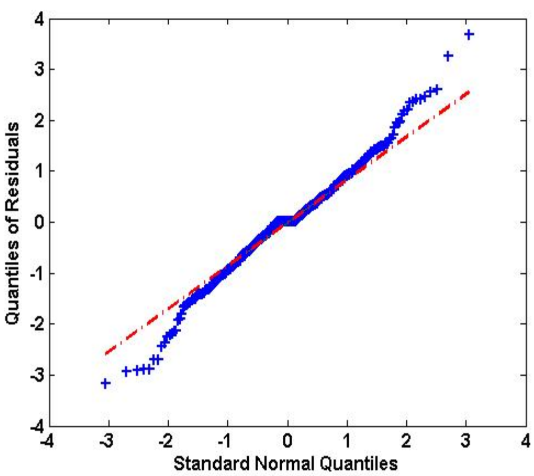
Figure 7.10: Data based kernel densities of the residuals from the fitted VARs (log PPIs) versus stable and normal densities



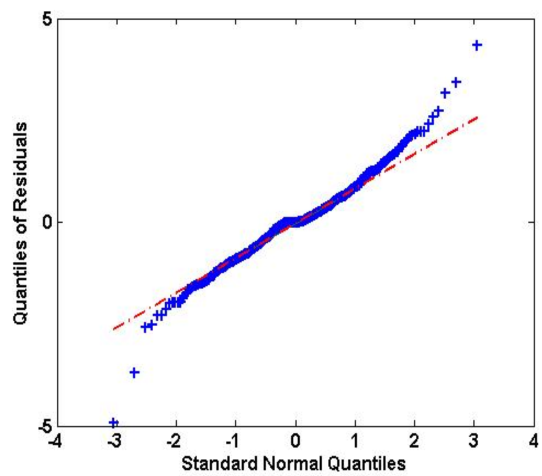
(a) Austrian residuals from VAR(13) ($\alpha = 1.70$)



(b) Canadian residuals from VAR(12) ($\alpha = 1.86$)

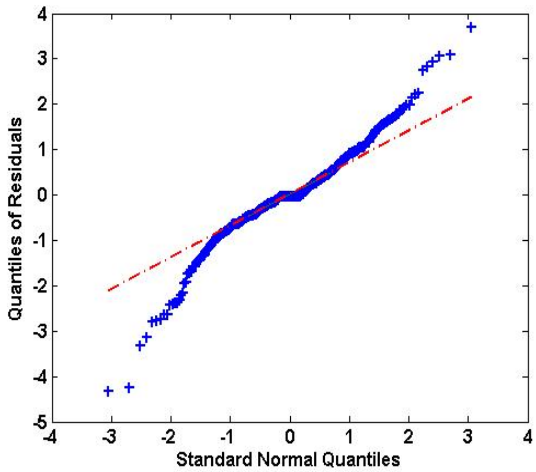


(c) German residuals from VAR(12) ($\alpha = 1.83$)

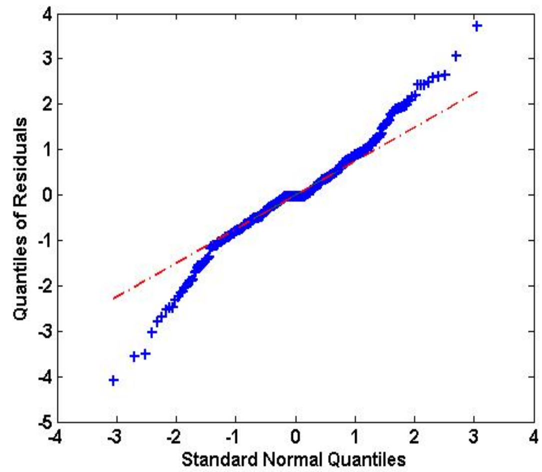


(d) U.K. residuals from VAR(12) ($\alpha = 1.85$)

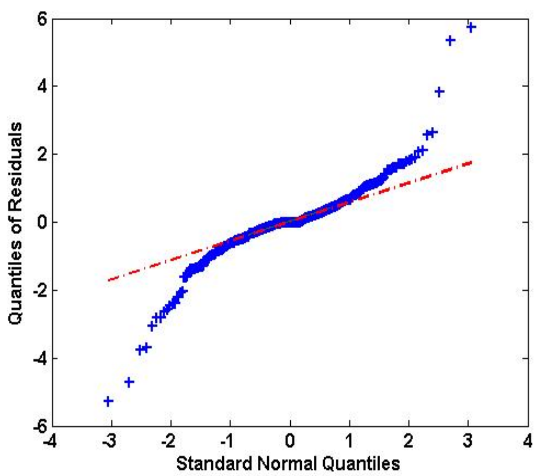
Figure 7.11: Quantiles of (standardized) residuals from the fitted VARs (log exchange rates) versus the quantiles of $N(0,1)$ distribution



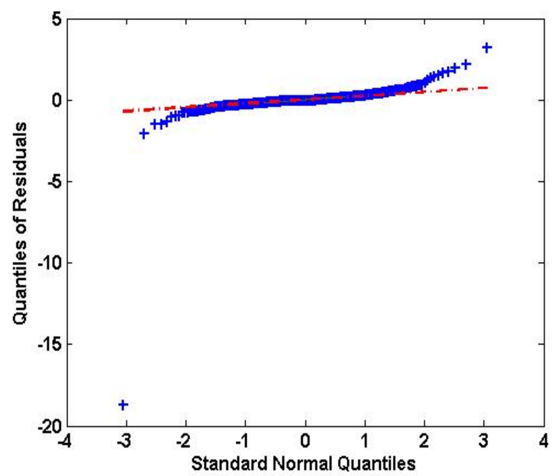
(a) Austrian residuals from VAR(13) ($\alpha = 1.53$)



(b) Canadian residuals from VAR(12) ($\alpha = 1.59$)

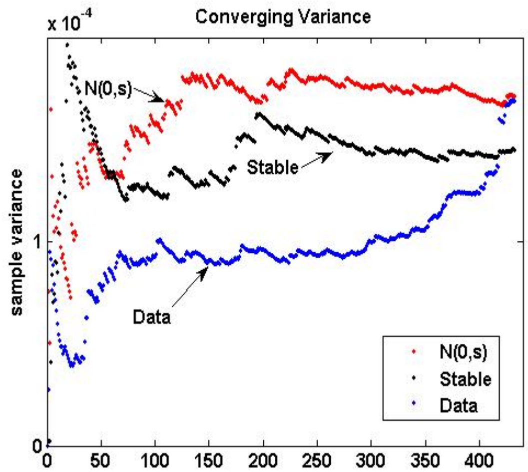


(c) German residuals from VAR(12) ($\alpha = 1.36$)

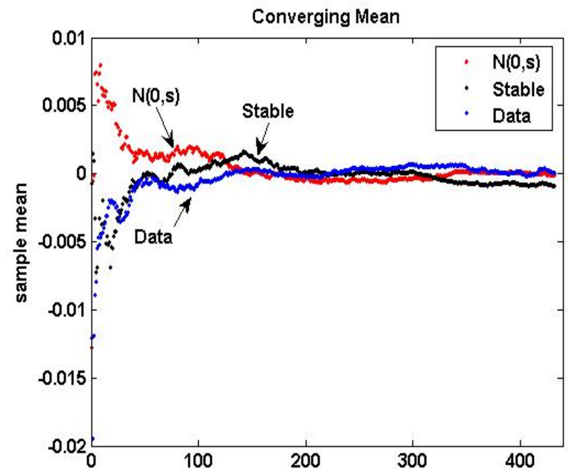


(d) U.K. residuals from VAR(12) ($\alpha = 1.31$)

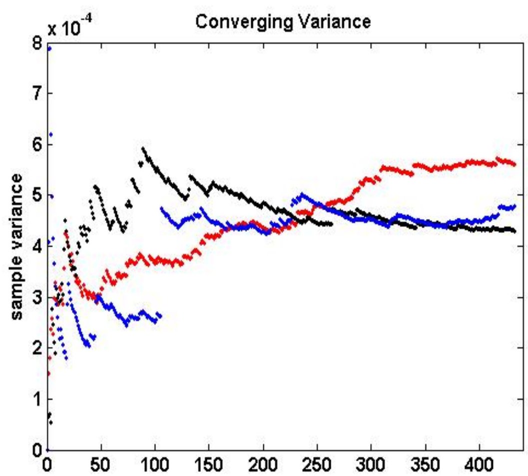
Figure 7.12: Quantiles of (standardized) residuals from the fitted VARs (log PPIs) versus the quantiles of $N(0, 1)$ distribution



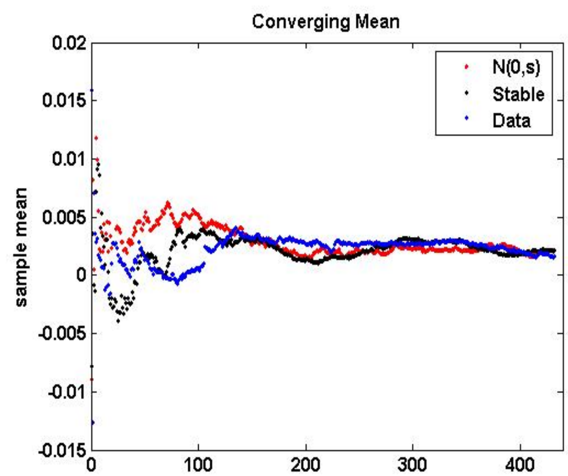
(a) Canadian residuals from VAR(12) (α=1.86)



(b) Canadian residuals from VAR(12)

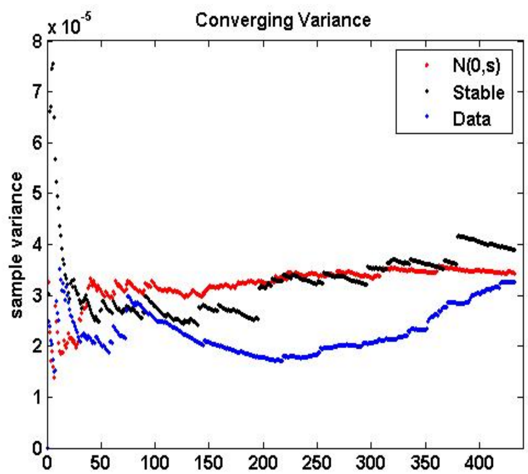


(c) Swedish residuals from VAR(12) (α=1.86)

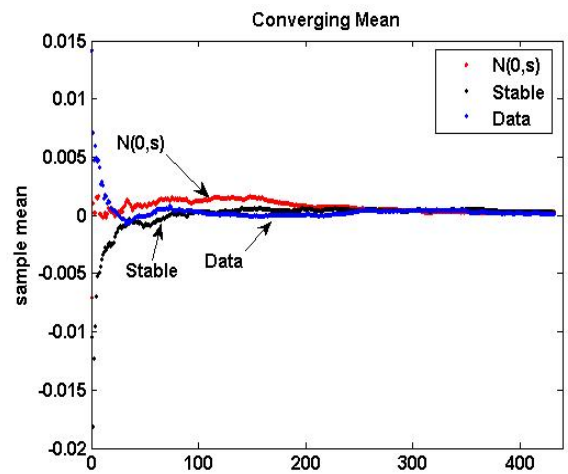


(d) Swedish residuals from VAR(12)

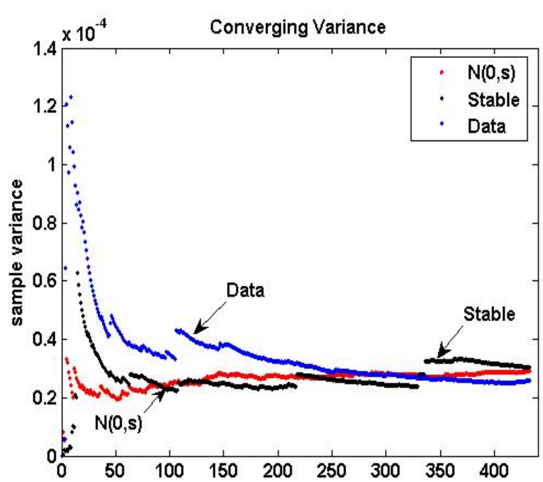
Figure 7.13: Converging variance and mean tests of residuals from the fitted VARs (log exchange rates)



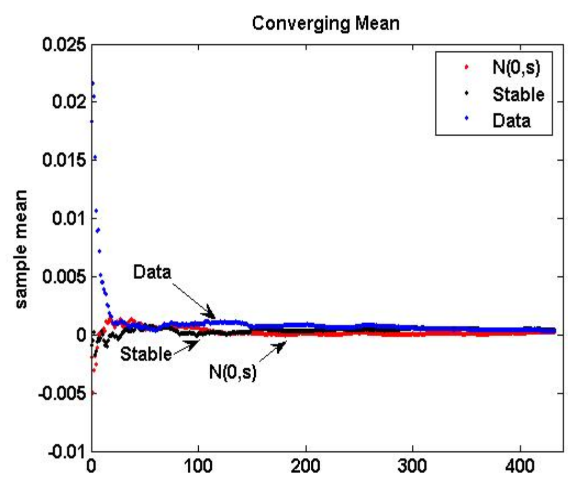
(a) Canadian residuals from VAR(12) (α=1.59)



(b) Canadian residuals from VAR(12)



(c) Swedish residuals from VAR(12) (α=1.80)



(d) Swedish residuals from VAR(12)

Figure 7.14: Converging variance and mean tests of residuals from the fitted VARs (log PPIs)

test is chosen by the following criteria,

$$l = \text{int}(4(T/100)^{1/4}) \tag{7.2}$$

where T is the sample size and "int" function returns the nearest integer. For Samarakoon-Knight tests, lag lengths have been chosen by the AIC and diagnostic checks with SPACF and SACF structure.

When the results of OLS-based unit roots are considered: all nominal exchange rate series (except for Austrian and Japanese exchange rates) appear to have a unit root in levels and have no unit root in first differences. For Austria and Japan, the nominal exchange rates are found to be I(0) both in levels and first differences. The conclusions hold regardless of whether we make infinite-variance or finite-variance assumption. The results from LAD-based tests show that all exchange rate series appear to be I(1).

When OLS-based unit root tests are applied on the price indices, nearly half of the price series seem to be I(0) both in levels and in first differences; exceptions occur with Germany, Netherlands, Sweden and U.K., price series of which appear to be I(1) (see Table 7.5). If we make finite-variance assumption, U.K. PPI seems to be I(0). The results of the LAD-based Samarakoon-Knight unit root tests with PPI series are shown in Table 7.6. The price indices are I(1) except for those of Canada, Japan and U.K.. PPIs of Canada, Japan and U.K. are again found to be I(0) both in levels and first differences.

We need all exchange rate and price series to be I(1). There seem to be some deviations from that assumption especially when we consider OLS-based unit root tests. The number of series that appear to be I(1) is greater when we consider LAD-based unit root tests. Although we have mixed results from the two types of unit root tests, we proceed to the next estimations by assuming that all series that we have are I(1).

7.3 Residual-based co-integration tests

Weak and strong-form PPP can be tested through a general equation as in equation (6.2). Here we consider testing the weak-form PPP through residual-based co-integration tests. We

Table 7.3: Caner unit root test in exchange rates

	Level	First Difference
Country	Z_t †	Z_t
Austria	-1.97*	-15.93*
Canada	-0.03	-16.12*
Denmark	-0.68	-15.90*
Germany	-1.88	-15.78*
Japan	-2.41*	-15.70*
Netherlands	-1.58	-15.79*
Sweden	-1.31	-14.67*
U.K.	0.32	-14.28*

* Rejects the null hypothesis of a unit root at 5% significance level.

† For $\alpha = 2$, critical value is -1.95 (standard). Critical values are taken from Table 2.1.

Table 7.4: Samarakoon-Knight unit root test in exchange rates

	Level		First Difference	
Country	Lag†	π_ϕ	Lag	π_ϕ
Austria	2	-0.38	1	-14.09*
Canada	5	0.35	4	-12.60*
Denmark	2	0.55	2	-14.22*
Germany	2	-0.16	1	-14.23*
Japan	4	-0.53	3	-17.07*
Netherlands	2	-0.87	1	-13.32*
Sweden	4	0.70	3	-14.50*
U.K.	4	1.27	3	-16.03*

† Selected by AIC and SACF & SPACF structure.

* Rejects the null hypothesis of a unit root at 5% significance level. 5% critical value is -1.64.

Table 7.5: Caner unit root test in price indices

	Level	First Difference
Country	Z_{t_μ} †	Z_t ‡
Austria	-3.68*	-15.26*
Canada	-3.60*	-14.26*
Denmark	-3.54*	-10.92*
Germany	-2.69	-14.21*
Japan	-5.87*	-10.26*
Netherlands	-1.22	-19.23*
Sweden	-2.10	-14.51*
U.K.	-2.91	-13.90*
U.S.	-3.27*	-22.45*

* Rejects the null hypothesis of a unit root at 5% significance level.

† For $\alpha = 2$, critical value is -2.86 (constant).

‡ For $\alpha = 2$, critical value is -1.95 (standard). Critical values are taken from Table 2.1.

Table 7.6: Samarakoon-Knight unit root test in price indices

	Level		First Difference	
Country	Lag†	π_μ	Lag	π_ϕ
Austria	15	-1.11	14	-14.02*
Canada	2	-2.85*	1	-9.68*
Denmark	3	-2.50	2	-8.54*
Germany	6	-0.65	5	-12.26*
Japan	3	-4.56*	2	-18.12*
Netherlands	4	-0.34	3	-19.03*
Sweden	14	-0.43	13	-6.84*
U.K.	4	-3.99*	3	-12.94*
U.S.	7	0.52	6	-11.94*

† Selected by AIC and SACF & SPACF structure.

* Rejects the null hypothesis of a unit root at 5% significance level. 5% critical value is -1.64.

implement both OLS and LAD-based tests. Our LAD-based tests' critical values are presented in section 2.3.1 of part I with one regressor in the co-integrating regression. Table 7.8 presents the critical values of our test with two regressors in the co-integrating regression (as needed in testing PPP) and Table 7.7 shows the critical values for Caner test with two regressors. Z_t critical values are calculated through simulations based on 1,000 sample size with 20,000 iterations following Caner (1998). π_ϕ critical values are the large sample critical values that are found with 50,000 iterations.

Table 7.7: 5% critical values of Z_t statistic for Caner co-integration test (n=2)

α	constant[a]	constant and trend[b]
1	-4.22	-4.54
1.5	-3.85	-4.21
1.6	-3.81	-4.19
1.7	-3.81	-4.17
1.8	-3.79	-4.17
1.9	-3.78	-4.16

[a] Phillips-Ouliaris critical value ($\alpha = 2$) at 5% significance is -3.77.
[b] Phillips-Ouliaris critical value ($\alpha = 2$) at 5% significance is -4.16.
Note: n is the number of regressors in the co-integrating regression.

Table 7.8: 5% critical values of LAD-based co-integration tests ($n = 2$)

Stability index	constant	constant and trend
1.5	-2.25	-2.72
1.6	-2.44	-2.74
1.7	-2.65	-2.90
1.8	-2.90	-3.14
1.9	-3.13	-3.41

Note: n is the number of regressors in the co-integrating regression.

Note that the stability indices of the errors that drive the s_t, p_t and p_t^* series are not the same for each country. However, both Caner test and our test require homogeneous stability indices. For Caner test, the critical values decrease as α decreases for any given size. Therefore, following a conservative approach, we choose the critical values of Caner test according to the smallest α of the three innovation series. For our test, the critical values increase as α decreases for any given size. Thus if we want to follow a conservative approach, we should select the critical values according to the largest α of the three innovation series. By doing this our aim is to make the actual size of the tests less than the nominal size.[5] Five percent critical values chosen for Z_t and π_ϕ are given in Table 7.9. In the Table, minimum α levels are found by MLE procedure from OLS-based estimations and maximum α levels are found by the same procedure from LAD-based estimations.

Table 7.9: 5% critical values of Z_t and π_ϕ

			Z_t		π_ϕ	
Country	Minimum α	Maximum α	No Trend	Trend	No Trend	Trend
Austria	1.58	1.70	-3.81	-4.19	-2.65	-2.90
Canada	1.60	1.86	-3.81	-4.19	-3.13	-3.41
Denmark	1.61	1.86	-3.81	-4.19	-3.13	-3.41
Germany	1.52	1.83	-3.85	-4.21	-2.90	-3.14
Japan	1.55	1.86	-3.81	-4.19	-3.13	-3.41
Netherlands	1.80	1.87	-3.79	-4.17	-2.90	-3.14
Sweden	1.50	1.86	-3.85	-4.21	-3.13	-3.41
U.K.	1.45	1.85	-3.85	-4.21	-3.13	-3.41

Note: Z_t critical values are taken from Table 7.7 and π_ϕ critical values are taken from Table 7.8.

Residual-based co-integration test results are shown in Tables 7.10 (OLS-based) and 7.11 (LAD-based). Regardless of making the finite-variance or infinite-variance assumption, OLS-based co-integration tests fail to provide evidence of the weak-form PPP hypothesis. When our LAD-based test is applied, we see that the hypothesis is again rejected for all countries except for Denmark and U.K. for no trend case.

7.4 Fully-modified estimations

Considering equation (6.2), if the strict interpretation of long-run PPP theory (or strong-form PPP) applies, joint parameter restrictions of the null hypothesis $H_0 : \beta_0 = 0, \beta_1 = 1, \beta_2 = -1$ should hold. In addition to the joint restrictions, here individual coefficient restrictions are also tested. In summary, we test the following joint and individual hypotheses:

[5]Our conjecture that the actual test size will be less than the nominal size can be proven by Monte Carlo simulations.

Table 7.10: OLS-based co-integration tests in PPP

Country	No Trend	Trend
Austria	-3.07	-3.19
Canada	-2.79	-2.76
Denmark	-2.87	-2.98
Germany	-2.36	-2.55
Japan	-1.56	-1.42
Netherlands	-1.91	-3.36
Sweden	-2.17	-2.12
U.K.	-2.71	-2.52

5% critical values are from Table 7.9.

Table 7.11: LAD-based co-integration tests in PPP

Country	Fitted Model†	No Trend	Trend
Austria	AR(13)	-2.20	-2.34
Canada	AR(12)	-2.14	-2.10
Denmark	AR(12)	-3.22*	-3.40
Germany	AR(12)	-2.84	-2.65
Japan	AR(12)	-2.45	-2.66
Netherlands	AR(12)	-1.80	-2.80
Sweden	AR(12)	-2.83	-2.93
U.K.	AR(12)	-3.27*	-3.09

† Model fitted for residuals from Equation 6.2. Selected according to VAR order of section 7.1.1.

* Rejects the null hypothesis of no co-integration at 5% significance level.

5% critical values are taken from Table 7.9.

$$
\begin{aligned}
H_0 &: \beta_0 = 0, \beta_1 = 1, \beta_2 = -1 \\
H_1 &: \text{Otherwise}
\end{aligned}
$$

$$H_0 : \beta_0 = 0$$

$$H_1 : \beta_0 \neq 0$$

$$H_0 : \beta_1 = 1$$
$$H_1 : \beta_1 \neq 1$$

$$H_0 : \beta_2 = -1$$
$$H_1 : \beta_2 \neq -1$$

We can see from Table 7.12 that using fully-modified Wald tests to justify the strong-form PPP empirically turns out to be a disappointment. All the p-values are very small strongly rejecting the joint hypothesis: $H_0 : \beta_0 = 0, \beta_1 = 1, \beta_2 = -1$. We also explore the results from the individual coefficient tests: $H_0 : \beta_0 = 0$, $H_0 : \beta_1 = 1$ and $H_0 : \beta_2 = -1$. The individual restrictions are mostly rejected with FM-OLS regressions. But even through FM-LAD regressions, the coefficient estimates are not close to the predictions of the strong-form PPP hypothesis. The results associated with FM-LAD and FM-LAD‡ based tests are similar.

Table 7.12: Empirical estimates of equation (6.2)

		Parameters, t-ratios and P-values for joint test						
Country	Method	$\hat{\beta}_0$	t_{β_0}	$\hat{\beta}_1$	t_{β_1}	$\hat{\beta}_2$	t_{β_2}	P-value
Austria	OLS	-1.16		2.62		-1.85		
	FM-OLS	-1.48	-1.11	2.67	2.80[b]	-1.83	-2.60[c]	0.00[d]
	LAD	-0.41		2.27		-1.67		
	FM-LAD	-0.79	-0.48	2.44	1.95	-1.76	-1.93	0.00[d]
	FM-LAD‡	-0.79	-0.37	2.44	1.52	-1.76	-1.50	0.00[d]
Canada	OLS	0.28		1.35		-1.37		
	FM-OLS	0.18	1.21	1.30	2.32[b]	-1.29	-1.94	0.00[d]
	LAD	0.30		1.40		-1.43		
	FM-LAD	0.15	0.84	1.37	2.37	-1.35	-1.95	0.00[d]
	FM-LAD‡	0.15	0.76	1.37	2.17	-1.35	-1.78	0.00[d]
Denmark	OLS	2.55		2.04		-2.21		
	FM-OLS	2.11	6.20[a]	1.73	2.09[b]	-1.80	-2.07[c]	0.00[d]
	LAD	2.39		1.77		-1.91		
	FM-LAD	2.22	8.41[a]	1.73	2.68[b]	-1.83	-2.75[c]	0.00[d]
	FM-LAD‡	2.22	7.37[a]	1.73	2.35[b]	-1.83	-2.41[c]	0.00[d]
Germany	OLS	-3.11		3.08		-2.31		
	FM-OLS	-2.58	-1.84	2.60	2.13[b]	-1.94	-2.05[c]	0.00[d]
	LAD	-2.40		2.67		-2.07		
	FM-LAD	-2.38	-1.18	2.66	1.54	-2.05	-1.61	0.00[d]
	FM-LAD‡	-2.38	-1.01	2.66	1.32	-2.05[c]	-1.37	0.00[d]
Japan	OLS	4.77		1.10		-1.14		
	FM-OLS	4.57	4.98[a]	1.09	0.41	-1.09	-1.28	0.00[d]
	LAD	5.22		0.97		-1.10		
	FM-LAD	5.03	2.32[a]	0.94	-0.11	-1.03	-0.20	0.00[d]
	FM-LAD‡	5.03	2.23[a]	0.94	-0.10	-1.03	-0.20	0.00[d]

Table 7.12: (Continued)

Country	Method	Parameters, t-ratios and P-values for joint test						
		$\hat{\beta}_0$	t_{β_0}	$\hat{\beta}_1$	t_{β_1}	$\hat{\beta}_2$	t_{β_2}	P-value
Netherlands	OLS	0.22		1.68		-1.60		
	FM-OLS	0.75	0.69	1.47	0.57	-1.50	-0.84	0.00[d]
	LAD	0.62		1.30		-1.31		
	FM-LAD	0.52	0.54	1.57	0.81	-1.56	-1.09	0.00[d]
	FM-LAD‡	0.52	0.47	1.57	0.69	-1.56	-0.94	0.00[d]
Sweden	OLS	1.50		1.06		-0.96		
	FM-OLS	1.35	4.62[a]	1.05	0.37	-0.91	0.51	0.00[d]
	LAD	1.57		1.09		-1.01		
	FM-LAD	1.35	1.91	0.98	-0.08	-0.84	0.36	0.00[d]
	FM-LAD‡	1.35	1.89	0.98	-0.08	-0.84	0.35	0.00[d]
U.K.	OLS	0.03		0.66		-0.79		
	FM-OLS	0.14	-0.45	0.60	-3.49[b]	-0.69	1.74	0.00[d]
	LAD	0.16		0.67		-0.83		
	FM-LAD	0.07	0.18	0.59	-2.92[a]	-0.73	1.23	0.00[d]
	FM-LAD‡	0.07	0.15	0.59	-2.40[b]	-0.73	1.01	0.00[d]

[a] Rejects the null: $H_0 : \beta_0 = 0$ against $H_1 : \beta_0 \neq 0$ at 5% significance.
[b] Rejects the null: $H_0 : \beta_1 = 1$ against $H_1 : \beta_1 \neq 1$ at 5% significance.
[c] Rejects the null: $H_0 : \beta_2 = -1$ against $H_1 : \beta_2 \neq -1$ at 5% significance.
[d] Rejects the null: $H_0 : \beta_0 = 0, \beta_1 = 1, \beta_2 = -1$ against H_1 : Otherwise at 5% significance.
Five percent critical values for t-statistics (two-sided) are ± 1.96.
Five percent critical value for $\chi^2_{(3)}$-statistic is 7.82.
FM-LAD‡ denotes the FM-LAD estimator with infinite-variance errors.

CHAPTER 8. CONCLUDING REMARKS

Purchasing Power Parity is an important condition in international finance as exchange rate determination is of crucial importance to understand the links between domestic and foreign economies. In part II, weak-form and strong-form PPP relationships are re-examined for a sample of eight countries (Austria, Canada, Denmark, Germany, Japan, Netherlands, Sweden and the United Kingdom) with monthly data on exchange rate (per U.S. dollar) and PPI series from January 1973 to December 2009.

As a contribution to the extant literature, weak-form PPP is tested through residual-based co-integration tests by utilizing the least absolute deviation (LAD) estimator and strong-form PPP through fully-modified least absolute deviation (FM-LAD) method while considering the heavy-tailed error structure of exchange rate returns and inflation rates. LAD method is known to be more robust to heavy-tailed data than the OLS estimator. The errors driving our exchange rates and inflation rates show characteristics that are consistent with those of the α-stable distributions with infinite-variance. LAD-based weak-form PPP test results are compared to those of conventional OLS-based co-integration tests and strong-form PPP test results of FM-LAD regressions are compared to those of fully-modified ordinary least squares (FM-OLS) regressions.

Weak-form PPP does not get support from the data with both OLS-based co-integration tests. On the other hand, only for 2 cases out of 16, we accept weak-form PPP with LAD-based tests. Strong-form PPP is tested through both FM-OLS and FM-LAD based chi-square tests. Strong-form PPP is strongly rejected for all countries with both methods regardless of the finite-variance or infinite-variance assumption. When the individual coefficients are considered, it is observed that the parameters are in general not close to the predictions of the strict PPP theory neither with FM-OLS nor with FM-LAD estimations.

The results show that weak-form and strong-form PPP can not be justified empirically. Even the weak-form PPP does not hold (except for 2 cases) although we consider the heavy-tailed structure of errors and apply LAD-based co-integration tests. However, there are limitations to our study. Our residual-based tests assume that all the variables that are tested for co-integration are driven by errors with the same stability index. There is no test for testing the equality of stability indices. In part I, we tested for forward rate unbiasedness hypothesis. Stability indices of the errors from the spot and forward exchange rates for the same country turned out to be the same with only very slight differences. In part II, with PPP data, stability indices of the errors of exchange rate, PPI and US PPI tend not to be that similar even though the differences are not too big. Nevertheless, we can still follow a conservative approach both for Caner (1998) test and our test to reach a conclusion.

APPENDIX
LONG-RUN COVARIANCE MATRIX ESTIMATION

Long-run covariance matrices Ω and Δ can be consistently estimated from the residuals $\hat{u}_t = (\hat{u}_{0t}, u'_{xt})'$, $t = 2, \ldots, T$, via a kernel. Kernel estimates have the general form,[1]

$$\hat{\Omega} = \sum_{j=-T+1}^{T-1} W(j/M)\Gamma(j) \tag{A.1}$$

and

$$\hat{\Delta} = \sum_{j=0}^{T-1} W(j/M)\Gamma(j) \tag{A.2}$$

where $\Gamma(j) = (1/T)\sum_{t=j+1}^{T} \hat{u}_{t-j}\hat{u}'_t$ with $\Gamma(-j) = \Gamma(j)'$, $W(.)$ is a weight function (kernel) and M is a lag truncation or bandwidth parameter. In equations (A.1) and (A.2), truncation occurs when $W(j/M) = 0$ for $|j| \geq M$. A bandwidth parameter is chosen according to the data-based automatic bandwidth selection method of Andrews (1991). A Parzen kernel is chosen,

$$W(x) = \begin{cases} 1 - 6x^2 + 6|x|^3 & \text{for } 0 \leq |x| \leq 1/2 \\ 2(1-|x|)^3 & \text{for } 1/2 \leq |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $W(0) = 1$.

[1] See for example Hansen (1992).

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