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Essays on food assistance program participation and demand for food

Ariun Ishdorj
Iowa State University

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Ariun Ishdorj

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Program of Study Committee:
Helen H. Jensen, Co-major Professor
Justin L. Tobias, Co-major Professor
Alicia L. Carriquiry
Joseph A. Herriges
Peter F. Orazem

Iowa State University

Ames, Iowa

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Household food demand and choices over food products are constantly evolving. Therefore better understanding of the relationship among household socioeconomic characteristics, expenditures, foods and nutrient choices of consumers and food prices is important to food producers, health professionals, policymakers and educators. This dissertation is a collection of three papers, each analyzing a particular issue related to consumer behavior. The first two papers explore two important issues related to the Special Supplemental Nutrition Program for Women, Infants, and Children (WIC) program that have not been extensively addressed in the past. First, although the WIC program is primarily devised with the intent of improving the nutrition of "targeted" children and mothers, it is possible that WIC may also change the consumption of foods by non-targeted individuals within the household. Second, although WIC eligibility status is predetermined, participation in the program is voluntary and therefore potentially endogenous. Although the two papers address similar topics, they differ in empirical approach. The first paper uses a two-stage instrumental variables approach and the second paper uses a Bayesian approach in order to handle the endogeneity of WIC program participation. Findings from these two papers indicate that based on the specification of the empirical model the choice of the estimation method can play an important role on the final outcome of the research. The third paper of this dissertation examines consumer demand for grain products. Given the public health interest in increased consumption of whole grains, demand for different types of cereals, both refined and whole grain is estimated. Bayesian methods are employed in the estimation accounting for the censoring of the dependent variables. Results show that demand for all types of cereals is inelastic to changes in prices. The expenditure elasticities do not vary widely in the magnitude. The expenditure elasticity is slightly above unity for the whole grain ready-to-eat cereals suggesting that as the expenditure on cereals increases households will allocate proportionally more on whole-grain ready-to-eat cereals and less on other cereals.

1. GENERAL INTRODUCTION

Introduction

Household food demand and choices over food products are constantly evolving. In the last several decades, the U.S. food sector has gone through a rapid transformation in response to rising incomes, changing demographics, increased labor force participation by women, new product introduction and innovation, scientific advances and understanding of the role of diet in achieving good health. Today there is increased domestic production, greater availability and diversity of products through trade and improvements in technology that enhance the nutritional quality of products and provide other desirable product attributes.

The relationship between diet and health continues to be a question of interest in many studies. Today, many consumers are aware of the link between nutrition and good health. In addition, public programs and nutrition education are being designed to promote healthy food choices. Knowledge about food components or introduction of new products, for example, food products with more variety, improved taste, flavor and health attributes, can lead to changes in consumers' diets. Also, scientific advances in nutrition underpin revisions of dietary guidelines. Such guidance and new information may result in changes in consumer dietary behavior and lead to increased demand for products that combine healthy attributes with convenience and attractive taste profiles.

Knowledge of demand structure and consumer behavior is important for a wide range of questions that arise in public health, marketing and behavioral contexts. Better understanding of the relationship between household socioeconomic characteristics, expenditures, foods and nutrient choices of consumers and food prices is important to food producers, health professionals, policymakers and educators. For example, basic demand parameters give information needed for effective design and targeting of food assistance programs, as well as for evaluating the impact of economic changes on households and the general well-being of the population.

This dissertation consists of three essays that address standard but important issues related to consumer behavior: (1) food choice and the evaluation of the effectiveness of a targeted food assistance program; and (2) analysis of the demand for food. The three essays are the explorations of the two phenomena that are important to better understanding of consumer and household behavior as well as for public policy. Both of these topics have a long history of empirical analysis, yet still hold considerable research interest.

The first and second papers consider the design and effectiveness of food assistance programs. Each year the federal government spends a large amount of money on major food assistance programs. These programs help to provide food and meet nutritional requirements for individuals and households that are vulnerable due to low income or other circumstances. As a result, there is a great deal of interest in evaluating the effectiveness of these programs in helping to improve health and nutritional status of this population.

The Special Supplemental Nutrition Program for Women, Infants, and Children (WIC) is one of the government's targeted food assistance programs, designed to improve the nutrition and health of qualifying women, infants and young children in the household through the use of vouchers for selected foods. The papers analyse the effectiveness of the WIC program by addressing two important issues related to the program that have not been extensively addressed in past. First, an important problem faced in evaluating the effect of the WIC program on dietary intakes is that the receipt of WIC foods by the

WIC-recipient may change consumption of foods by non-targeted individuals within the household. As little is known about the allocation of food once it reaches the household, there is also little information available on how the provision of WIC-approved foods might affect intra-household allocation decisions. Reallocation of program benefits in response to a program targeted towards individuals would lead to smaller than expected gains to the recipient of the transfer and larger than expected intake by non-targeted individuals in the program household. Hence, spillover of program benefits may affect the measurement of program effectiveness and impact. We formally address this issue by comparing the impact of WIC participation on both targeted household members and non-targeted household member.

A second problem addressed in this paper is that there is potential endogeneity associated with the WIC program. Households that choose to participate in WIC may also have strong preference for health promoting foods. As a result, they choose to consume more of the WIC-approved foods. This problem is addressed through the econometric specification and estimation. To handle the potential endogeneity of WIC participation we employ a treatment-response model and estimate it using Bayesian methods.

Although the two papers address similar topics, they differ in empirical approach. The first paper uses a two-stage instrumental variables approach in order to handle the endogeneity of WIC program participation. The two-stage method is a commonly used single equation estimation method in the empirical literature that uses instrumental variables (IV) that are uncorrelated with the disturbances to obtain predicted values for the endogenous variables. Readily available software packages and ease of computation made this method very attractive to researchers. WIC-approved foods are very important in the diets of not only children but also adults. After controlling for the participation decision, results of the first paper show that participation in WIC is associated with increased intake of calcium through milk for WIC targeted individuals. There was no evidence of spillover of program provided dairy benefits. The second paper uses a Bayesian approach to address the endogeneity of program participation. Overall, we find little direct evidence that speaks to the efficacy of WIC. Instead, most of the benefits that might potentially be attributed to the program seem to arise from differences in unobservables across WIC and non-WIC families. Furthermore, we find little evidence associated with possible “spillover” or “leakage” benefits that have been suggested in the literature, as non-targeted members of WIC households have consumption patterns that are quite consistent with non-targeted members of non-WIC households. Findings from these two papers indicate that after the specification of the empirical model, the choice of the estimation method can play an important role on the final outcome of the research. The Bayesian approach supports careful evaluation of the underlying structural relationships and allows us to separate the effects of unobservables from direct effects. By providing a careful analysis of intra-household allocation decision processes and related evidence on beneficiary outcomes, the findings of the research can improve program evaluation, design and policy analysis.

New methods and data allow further extensions in studies of demand. Innovations in data collection also make available extensive and detailed information on product expenditures. The wider availability of scanner-based consumer data, for example, has allowed the collection of detailed purchase information with relatively little respondent burden. Household level panel scanner data contain detailed demographic information which allows handling of heterogeneous preferences. Also the large sample size provides sufficient information to estimate a large demand system. Scanner data enable researchers to examine consumer purchase behavior with extensive product detail along with expenditure and price data. The detailed data do hold some new challenges. The household level data require the use of techniques for handling the observations with “zero” values in the dependent variable. Although the estimation of a censored demand for a single good can be done easily by using a Tobit type model, estimation of the censored demand for multiple products is not straight forward.

The third paper of this dissertation estimates a demand system for a selected product group, cereal products, using the Almost Ideal Demand System (AIDS) model. The 2005 Dietary Guidelines emphasize the importance of consumption of whole grain products. Given the public health interest in increased consumption of whole grains, we consider demand for different types of cereals, both refined and whole grain. Cereal products are a major source of whole grains in the diet. This paper accounts for the censoring of the dependent variables and estimates the demand system using Bayesian methods. Results show that demand for all types of cereals is inelastic to changes in prices. The expenditure elasticities do not vary widely in the magnitude. The expenditure elasticity is slightly above unity for the whole grain ready-to-eat cereals suggesting that as the expenditure on cereals increases households will allocate proportionally more on whole-grain ready-to-eat cereals and less on other cereals. By providing estimates of how food consumption is likely to change with changes in prices and income, the research provides important inputs into food and related nutritional policy initiatives.

Dissertation Organization

This dissertation is organized as follows. Following this introduction, there are the three chapters. Each chapter analyzes a particular issue related to consumer behavior. Although each chapter in this dissertation is meant to stand alone, there are some common factors that connect them. All three essays address questions related to the consumer food and nutrient choices. Second, all three essays extend the current research in the area by using methods to improve estimation.

2. INTRA-HOUSEHOLD ALLOCATION AND CONSUMPTION OF WIC-APPROVED FOODS: A TWO-STAGE APPROACH

Abstract

One of the primary objectives of most food assistance and nutrition programs is to improve health and nutritional status of vulnerable sections of population, particularly women and children. Better understanding of the effectiveness of these targeted programs is crucial for policy analysis. The Special Supplemental Nutrition Program for Women, Infants, and Children (WIC) provides vouchers for food items to qualifying women, infants and young children in a household. The amount of foods provided is designed to enhance the intake of key nutrients needed by the targeted individuals. Although the vouchers are issued to an individual, once acquired, the food items are available to share in the household. Little is known about intra-household reallocation of WIC-provided food benefits. The overall goal of the research is to develop information on targeting of food benefits and the "spillover" of food program effects within the household, information that is needed for policy analysis and evaluation of the effectiveness of WIC. Empirical analysis is performed using data from the USDA Continuing Survey of Food Intake by Individuals (CSFII) 1994-96, 1998. Individuals are classified by WIC income-eligibility, program targeted group, and participation status. A Tobit model with endogenous program participation is estimated using two-stage method in order to compare WIC-approved food intakes of targeted individuals in WIC households with non-targeted individuals in the same households, targeted individuals in non-WIC households and non-targeted individuals in non-WIC households. The findings imply that targeted individuals in WIC households consume more of the WIC-approved foods than individuals in other groups.

Keywords: nutrition, WIC, two-stage method, intrumental variables.

JEL Classification: C31; C34; I38.

Introduction

The U.S. Department of Agriculture spent nearly \$41.6 billion in FY2003 on 15 food assistance programs. The Special Supplemental Nutrition Program for Women, Infants, and Children (WIC) was the third-largest food assistance program in terms of total expenditures in that year. The WIC program serves participants by providing supplemental foods, nutrition education, breastfeeding support and referrals to health and social services in order to of improving birth outcomes, support the growth and development of infants and children, and to promote long-term health of all WIC participants. In FY2003, WIC provided \$3.2 billion in supplemental food to participants with estimated average monthly food package cost per person of \$35.28 and today serves over half of all infants born in the United States, 25 percent of all U.S. children ages 1 through 4 years, along with many of their mothers (USDA/FNS 2005; USDA/ERS 2005; IOM 2005). The WIC program provides benefits as in-kind transfers, nutritional education and social support to a vulnerable population.

The WIC program provides nutritious foods to supplement the diets of infants, children up to age five, and pregnant, breastfeeding and postpartum women. The program benefits, usually in the form of checks or vouchers, allow participants to obtain specific "packages" of foods, foods that currently include infant formula, milk, cheese, eggs, juice, cereals, and peanut butter/dried beans, and, for fully breast-feeding mothers, carrots and tuna. Table 2.1 contains the list of foods and the maximum monthly allowances for young children as an example. In April 2005, an Institute of Medicine (IOM) committee recommended substantial changes to the foods offered in the packages to be consistent with new scientific knowledge about nutrition and to make it easier for participants to improve their diets and health.

There is a well established literature on the effect of WIC participation on health and nutrient intake of infants, toddlers and children (Burstein, et al. 2000; Oliveira and Gundersen, 2000; Ponza, et al. 2004; Rose, et al. 1998; Siega-Riz, et al. 2004; Oliveira and Chandran, 2005). Considerable work has also been done to evaluate the effect of the WIC participation on pregnant women (Kowaleski-Jones and Duncan, 2002, Bitler and Currie, 2005). Oliveira and Chandran (2005) find that participation in the WIC program increases the consumption for at least some types of WIC-approved foods for children participating in the WIC program compared to eligible nonparticipating children living in non-WIC households, eligible nonparticipating children living in WIC households, and children living in households with income too high to be eligible for WIC (income greater than 185% of the poverty threshold).

An important problem faced in evaluating the effect of the WIC program on dietary intakes is that the receipt of program foods by the WIC-recipient may change the consumption of foods by non-targeted individuals within the household. This has been referred to as “spillover” (Oliveira and Chandran 2005) or “leakage” (Barrett 2002) of benefits. As Oliveira and Chandran note, this might be done if 1) receipt of WIC benefits frees up food dollars for use to benefit other, nonparticipating children; 2) nutrition education changes food selection for all members; and 3) WIC foods are shared with non-WIC household members. Little is known about the degree to which this occurs, and in general, about food allocation processes within households. However, for the purposes of policy analysis and evaluation of the effectiveness of WIC, better understanding of intra-household allocation is critical.

The overall goal of the research reported here is to better understand targeting of food benefits and spillover of food program effects within the household. The specific application is to the WIC program and household allocation of WIC approved foods. The empirical analysis uses data from the USDA Continuing Survey of Food Intake by Individuals (CSFII) 1994-96, 1998. Although there are more recent dietary intake data available, the CSFII is the most current, publicly available national data that allows tracking food consumption of targeted WIC recipients and other members of the same household.

Hypotheses

We hypothesize that participation in the WIC program will have a positive effect on the quality of diets of not only WIC participating individuals in the household, but also those of other members of the household. More specifically,

a. qualifying/targeted individuals in households that participate in nutrition programs such as WIC are (in dietary intake) better off than those who are in non-WIC households, holding all else equal

b. The non-targeted individuals in WIC households are better off (in dietary intake) compared to similar individuals in non-WIC households

c. The dietary intake of non-targeted individuals in WIC households is less improved compared to that of the targeted individuals in WIC households.

We consider milk and cheese intake, measured in calcium equivalence.

The paper is proceeds as follows. The theoretical model of household resource allocation and the empirical specification are outlined, followed by the description of the data used in the analysis. Results are presented and discussed. The paper concludes with the summary of the findings and suggestions for future research.

Theoretical Model

Traditional literature of consumer theory like Deaton and Muellbauer (1980) treat household members all alike. The same applies to most of the empirical applications (Blundell and Walker, 1986; Browning and Meghir, 1991). This method is referred to in the literature as the unitary model. Although household characteristics may be accounted for through scaling and translating parameters, these parameters adjust household preferences and do not enter the utility directly. The unitary model does not allow analysis of intra-household allocation of consumption and consequently welfare.

Early attempts that account for the fact that households may consist of different individuals with their own preferences are Samuelson (1956) and Becker (1974). Samuelson modeled the household decision making problem by letting the individuals' utility functions enter as subutility functions in the model. Becker on the other hand models the household decision making problem by assuming that the head of the household takes into account the preferences of all household members.

Other two approaches that explicitly take into account several decision-makers in a household use a game theoretic approach. The first of these approaches models household behavior in a non-cooperative framework (Browning, 2000; Chen and Wooley, 2001). In these models household members are assumed to maximize their utility, taking the other individuals' behavior as given. One potential drawback of non-cooperative models is that they do not result in Pareto efficient intra-household allocation. That is, it is possible to make one person better off without making other household members worse off. The second approach is the one developed by Manser and Brown (1980) and McElroy and Horney (1981). They incorporated bargaining theory into a household decision making model in a cooperative framework. An important criticism of the approach of choosing a particular bargaining approach to model household behavior is that when empirical tests are rejected it is hard to determine whether the particular choice is rejected or the bargaining setting in general rejected.

A valuable improvement to the above mentioned model is made by Chiappori (1988, 1992) and Apps and Rees (1988). The only assumption they make is that intra-household decisions are Pareto efficient. This model referred in the literature as the collective model to household behavior. The collective approach takes account of the fact that multi-person households consists of several members which may have different preferences.

In the collective model, individual preferences lead to collective choices, for example, how resources are accumulated and how resources are spent by individual household members. This model treats individuals as the decision-making agents and takes an axiomatic approach for determining intra-household allocation. The model assumes that allocations satisfy the following conditions: (i) efficiency - the outcome of the household decision process is Pareto efficient, and (ii) uniqueness of the solution. In the collective model case, changes in others' non-labor resources (i.e., program cash and non-cash benefits) may affect household allocation decisions.

We will consider the preferences of members of the household in two different states: 1) the household participates in the WIC program; and 2) the household does not participate in the WIC program. We assume that households will allocate food differently depending on whether they participate in the WIC program. Households participating in WIC receive an in-kind transfer in addition to other income, and this term will enter their budget constraint.

Consider a household with two individuals $i = NT, T$ where each individual consumes two private goods. Also assume that each individual consumes one good exclusively, with x_{NT} for individual NT and for individual T. Both individuals consume the third private good f, where $f = f_{NT} + f_T$. For simplicity assume goods prices are normalized to one. Total household income y is exogenous and a fixed supply of labor. The bundle $(x_{NT}, x_T, f_{NT}, f_T)$ is a Pareto optimal allocation of consumption within the household if it is a solution to the following maximization problem:

$$max_{x_{NT},x_T,f_{NT},f_T} U^{NT}(x_{NT}, f_{NT})$$

$$\text{s.t.} \quad y + sP_w = x_{NT} + x_T + f_{NT} + f_T$$

$$U^T(x_T, f_T) \geq \bar{U}^T,$$

where s stands for gains from social (in-kind) transfer to program participants. This can be represented as a difference between the benefits of participating in the program and the costs associated with program participation. P_w is an indicator equal to one if household participates in the program and 0 otherwise, and $\bar{U}_T$ is some required utility level for individual T. Then the solution to individual NT's maximization problem, yields the optimal level of y_i's and f_i's , $i = NT, T$. The optimal choices are:

$$x_i = g_i(y, P_w, Z),$$

$$f_i = w_i(y, P_w, Z),$$

where Z is the vector of other explanatory variables.

Empirical Analysis

Model and Estimation

The 2,421 individual from 1,018 households were assigned into one of the four mutually exclusive groups (Table 2.2). For the analysis we chose WIC targeted individuals as children of age one through four and pregnant, lactating or breastfeeding women and non-targeted individuals as other adults and children of age five and older. Although all households in the sample are WIC eligible by income and have at least one targeted individual living with them 558 households with 1,386 individuals are not participating in WIC and 460 households with 1035 individuals are participating in WIC. WIC participating households have WIC targeted individuals and non-targeted individuals of different ages. Among targeted individuals in these households 423 are WIC recipients and 95 non-recipients. For our initial analysis we will combine non-recipients in WIC households with non-targeted individuals in the same households and form a group of non-targeted individuals in WIC households.

Two dummy variables:

$WIC_h = 1$ if households is participating in WIC, 0 otherwise,

$T_{ih} = 1$ if WIC targeted individual, 0 otherwise, will help us to create each group of individuals. Let D_1, D_2, D_3 and D_4 be the dummies representing each of the groups as specified in Table 2.2. Then

$D_1 = WIC_h * T_{ih}$,

$D_2 = WIC_h * T_{ih} = (1 - WIC_h)T_{ih}$,

$D_3 = (1 - WIC_h) * T_{ih}$, and

$D_4 == (1 - WIC_h)(1 - T_{ih}$.

The econometric model is a single equation model of demand for a specific food:

$$c_{ih} = X_{ih}\beta + u_{ih} \tag{2.1}$$

where the dependent variable is the amount (milligrams calcium) consumed by individual i in household h from WIC-approved foods (milk, cheese, 'milk and cheese'), X_{ih} contains group identifyers D_2 through D_4 along with individual and household specific characteristics.

The dependent variable, c_{ih}, in equation (2.1) is zero if an individual does not consume the food, and positive if it does. Zero consumption is censored by an unobservable latent variable. We cannot use OLS regression since it is known that estimated coefficients are inconsistent when only observed positive consumption data are used in the estimation. A Tobit model will be used to correct for zero consumption, defined as follows:

$$c_{ih} = c^*_{ih} = X_{ih}\beta + u_{ih} \quad \text{if} \quad c^*_{ih} > 0 \tag{2.2}$$

$$c_{ih} = 0 \quad \text{if} \quad c^*_{ih} \leq 0 \tag{2.3}$$

where u_{ih} are residuals that are independently and normally distributed, with mean zero and a common variance σ^2; and c^*_{ih} is an unobservable latent variable.

WIC is the Special Supplemental Nutrition Program that targets only women, infants and children eligible for the program. Therefore, although the WIC eligibility status can be predetermined, since WIC is not a mandatory program, participation in the program is endogenous. Ignoring the endogeneity will lead to a biased estimate.

We adopt Nelson and Olson's (1978) two-stage instrumental variable estimation method in our analysis. In the first stage probit choice model of WIC participation will be estimated:

$$WIC_h^* = z_h\gamma + \epsilon_h, \quad \epsilon_h \sim N(0,1) \tag{2.4}$$

$$WIC_h = 1 \quad \text{if} \quad WIC_h^* > 0 \tag{2.5}$$

$$WIC_h = 0 \quad \text{if} \quad WIC_h^* \leq 0 \tag{2.6}$$

where z_h contains intruments which are correlated with WIC participation decision but uncorrelated with u_{ih}.

Parents choose to enroll or not to enroll eligible individuals of the households in WIC. There could be several reasons why they choose not to participate even though they are eligible. Participation in WIC can carry some costs associated with applying for WIC, visiting WIC office regularly for re-qualification and check-up, picking up vouchers monthly, and taking nutrition education classes. Stigma associated with participation in food assistance program or transportation issues can also affect households' participation decision. Also, it is possible that parents are not motivated enough to improve targeted individuals' nutrition or these targeted individuals are not at low nutritional risk. While all of the above mentioned reasons are important, we do not observe most of them. So for the empirical analysis, these factors are included the error term. The standard errors are complicated by the first-stage imputation, so we will bootstrap the standard errors.

The specification discussed above will allow us to make several comparisons among the four groups for estimating the effects of WIC on consumption of WIC-approved foods. Conditional on having a positive intake, we can obtain the predicted values of individual i's food intake conditional on being in one of the four group individual.

The Data

The ideal data set would include information on program eligibility, multiple program participation, and dietary intake on all members of the household. The best available data, and the data planned for use in this research, are from the 1994-96, 1998 Continuing Survey of Food Intake by Individuals (CSFII). The 1998 survey is a supplemental children's survey and only contains data from children age 10 and under. It will be used only when children are compared in tabular analysis. Although data from the NHANES 1999-2002 are more recent, the survey does not collect information on intakes of other household members and for this reason, is not sufficient for our analysis.

The CSFII data contain basic demographic information for each household and household member, and the survey uses a randomization strategy to select certain members to participate in a complete food intake survey. That is, even though there is not food intake data available for all members, the full composition of the household is known. For each of these sample persons questions on twenty four hour recall of food intake were conducted on two nonconsecutive days. The respondents report both the types and amounts of food consumed during this period.

Some foods are eaten as a single food and some as an ingredient in a meal. For our variable of interest, amount of milk or cheese consumed, CSFII 94-96, 98 will give us the amount in grams of milk/cheese if the food is classified as a dairy product. There are other foods that use milk and/or cheese as an ingredient. To identify the amount of milk or cheese used as an ingredient in non-dairy foods we use the Pyramid Servings Database for USDA survey food codes used in processing national surveys between 1994 and 2002. This dataset includes both Pyramid servings food and intake data files by 30 Pyramid food groups for food codes used to process intakes from CSFII 94-96, 1998. It also includes the information on serving of milk or cheese per 100 gram of food. We use this information to obtain the total amount of milk and cheese consumed as a single food or as an ingredient in meal by individual per day.

Since we compare food intakes of four different groups, each group may have both adults and children. Adults and children have different dietary requirements (Dietary Reference Intakes). In order to make the individuals comparable we convert the grams of food of interest into a calcium equivalent measure.

To be eligible for WIC, individuals must be in a WIC-qualifying population group (pregnant, up to 6 months post-partum; non-breastfeeding woman up to 6 months post-partum; a breastfeeding woman up to 1 year postpartum; an infant under 1 year of age; or a child up to his/her 5th birthday). The household's income must be at or below 185 percent of the Federal poverty guidelines or participate in qualifying programs (Medicaid, Food Stamps, Temporary Assistance for Needy Families (TANF)). And, applicants must be at nutritional risk, as determined by a health professional. Although it is not possible to determine individuals that are at nutritional risk from the CSFII data, nearly all U.S. women and children meet the criteria by failing to meet the Dietary Guidelines (IOM 2002). Thus, we define WIC eligible individuals as those living in households with income of 200 percent of the poverty guidelines (the same population considered in Oliveira and Chandran). Individuals will be identified as: WIC income-eligible or not; being a WIC-recipient or not/ and being a WIC targeted individual.

Derivation of the Dependent Variables

For the initial analysis we consider three foods: milk, cheese, and 'milk and cheese'. In order to be able to compare the food intake of individuals of different age and gender we convert the dependent variable 'amount of food' consumed (grams) to a calcium equivalent measure. CSFII 94-96, 98 contains information on grams of milk and/or cheese consumed as a single food or an ingredient in the dairy food, but this is not enough for our analysis. In addition to consumption of the dairy foods, we would like to know the amount of milk and/or cheese consumed as an ingredient in the preparation of the multi-ingredient dishes. For this reason we use the Pyramid Servings Database that provides the servings of milk or cheese from different foods consumed. The Pyramid Foods database applies to individuals 2 years of age and older, but for our analysis we are also interested in milk and cheese consumption of 1 year children. Since from the dataset we can find the number of servings of milk per 100 gram of different foods consumed we use this information to find the number of servings consumed by one year old children. A serving of liquid milk is defined as 1 cup which is 244gr and a serving of cheese is measured in ounces, which is 28.35 grams. By using the calcium conversion factor we can convert the grams of milk and/or cheese consumed into milligrams of calcium received from the food consumed. When the reported foods containing milk and/or cheese as an ingredient did not report specifically what kind of milk or cheese was used, a calcium conversion factor based on the most frequently consumed milk (whole milk) and cheese (American cheddar type cheese) was used.

The Dietary Reference Intake for calcium children of ages one through three (500mg of calcium/day) was used as the base or reference amount to convert all other individuals' calcium intake into an age and gender equivalent measure. The dependent variable is measured in milligrams (mg) of calcium received from the food consumed.

Table 2.3 contains the description of WIC-approved milk and cheese selected for the analysis. We followed closely the selection of WIC-approved foods done by Oliveira and Chandran (2005). Consumption of milk and cheese is analyzed regardless of whether the food was actually purchased through WIC.

Independent Variables

Table 2.4 contains definitions of independent variables used in the analysis. The demographic variables of individuals include age, gender, race, and education level. The education variable reports the household's main meal preparer's education. Other variables that show household characteristics and are useful in determining the WIC participation decision are household size, income, region, rural/urban, an indicator of having cash assets of less than \$5,000, participation in food stamp, number of young age children, presence of pregnant, breastfeeding, or lactating women.

Households that do not satisfy WIC eligibility requirements are excluded from the analysis. These are the households that have income of more that the 200 percent of the poverty guidelines, and do not have children under the age of 5 or pregnant, lactating and breastfeeding women. The resulting sample consists of 2421 individuals.

Table 2.5 reports the means of dependent and independent variables for the whole sample and each of the group of interest. Households that participate in WIC have lower income and less likely to have cash assets of \$5,000 compared to non-participating households. WIC households are more likely to receive food stamp, live in central city or rural area and be Black or Hispanic.

Empirical Results

Table 2.6 reports the results of first-stage probit estimation. Valid instruments in this setting must be correlated with the WIC participation variables but uncorrelated with the error term in the consumption equation. An instrument we chose is 'having cash of less than \$5,000'. Having cash assets of more than \$5,000 decreases the probability of participation in WIC. Household size, presence of infants, participation in food stamp program, being Hispanic all have positive significant effect on the probability of being in WIC. While presence of children under age of 5, having high school education, being Black, living in Northwest, Midwest and West, and residing in suburbs decrease the probability of being in WIC.

A total of three Tobit consumption models were estimated, one for each of the three types of foods: milk, cheese, milk and cheese and the results are presented in Table 2.7. The parameters of primary interest are the coefficients on different groups of individuals.

Compared with non-targeted individuals in WIC households and targeted and non-targeted individuals in non-WIC households, targeted individuals in WIC households consume significantly more milk, holding other factors constant. The same result holds for the consumption of 'milk and cheese'.

There is no statistically significant difference in the consumption of cheese between these groups. Most of the variables in the estimation of the milk and the 'milk and cheese' equation have the same sign and significance level, except for household size and some college education. Household size had a significant negative effect on the consumption of both cheese and 'milk and cheese' and some college education has a positive and significant effect on the consumption of milk and has no statistically significant effect on the consumption of 'milk and cheese'.

Breastfeeding, pregnant and lactating women consumed significantly less milk and 'milk and cheese' than others. Children of age five through eight consumed significantly more milk, cheese, and 'milk and cheese' than children in the age group 1-4. Individuals of age 9 through 18 and 19-50 consumed significantly more milk, cheese and 'milk and cheese' compared to very young children of age under five. Compared to Whites, Blacks consumed significantly less of WIC-approved milk and 'milk and cheese' and Hispanics and other race/ethnicity consumed significantly less of cheese. Individuals with some college education consumed significantly more WIC-approved milk and cheese than individual with some high school education. Compared to South, living in Midwest was associated with the increased consume of milk, cheese and 'milk and cheese' and living in the West with increased consumption of milk and 'milk and cheese'. Living in the rural has negative significant effect on consumption of milk and 'milk and cheese' compared to living in the central city.

Table 2.8 contains the values of the predicted conditional means. Targeted individuals in WIC households consume 65 percent more milligrams of calcium from milk than non-targeted individuals in the same households compared to 60 percent of difference in consumption between same individuals in non-WIC households. Also targeted individuals in WIC households consume 11 and 65 percent more milligrams of calcium from milk than targeted and non-targeted in non-WIC households, respectively. Non-targeted individuals in WIC and non-WIC household consumed similar amount calcium from milk. A similar pattern is observed for the consumption of 'milk and cheese' except non-targeted individuals in WIC households consumed less 'milk and cheese' than similar individuals in non-WIC households. Since parameter estimates for cheese were not significant, we did not calculate the predicted conditional means for cheese.

Conclusions and Directions for Future Research

WIC-approved foods are very important in the diets of not only children but also adults. After controlling for the participation decision we found that participation in WIC is associated with increased consumption of milk and 'milk and cheese' for the targeted individuals. In summary, we find:

a. targeted individuals in WIC households consume more milk and 'milk and cheese' than similar individuals in non-WIC households. More specifically, WIC was associated with an increase of 11% in the amount of calcium individuals get from milk compared to targeted individuals in non-WIC households.

b Consumption of milk for non-targeted individuals in WIC households was similar to those individuals in non-WIC households. We could not say the same for the consumption of 'milk and cheese'. Non-targeted individuals in WIC households consumed about 2% less calcium from 'milk and cheese' than similar individuals in non-WIC households.

c Non-targeted individuals in WIC households consumed less milk and 'milk and cheese' than targeted individuals in the same households. More specifically, targeted individuals in WIC households consumed about 65 % and 60 % more milk and 'milk and cheese' respectively, than non-target individuals in the same households.

This research provided some insight for understanding the intra-household allocation of WIC-approved foods. We did not detect any spillover effect within households. There is relatively little differentiation in the types of milk and cheese available through WIC in terms of brand and types of WIC-approved products; WIC also puts relatively few restrictions on the milk and cheese products. The earlier study by Oliveira and Chandran (2005) found that the effect of WIC food package on participants' food consumption differed by type of food. They found that WIC participation had a little effect on the consumption of foods, such as milk, cheese, eggs, dry beans/peas and peanut butter, where WIC participants' food choices were less constrained. They also found that WIC has a large impact on the consumption of some foods such as WIC-approved cereal and juices. It would be of interest to consider intakes of WIC-approved foods such as juices and cereal, where there are more restrictions on the types and brand of foods that consumers can purchase using their vouchers. WIC-approved foods include iron-fortified and low-sugar cereal and 100 percent juice. There are several limitations to the approach used in the analysis and its findings. First of all the dataset we use for the analysis is not recent. Advances in technology, new health related research findings, new dietary guidelines and invention of new foods affect consumers' preferences toward food. Although CSFII 94-96, 98 is the most currently available national data that allows tracking food consumption of WIC and WIC-eligible individuals and other members of the same household, the use of more current data would be preferred.

When doing the two-stage estimation we used predicted values from the first stage estimation and imputed in the second stage. Since we are using dummy variables to classify individuals into one of the groups it would be of interest to estimate two equations simultaneously. Also we ignore the fact that the unit of observation in the first equation is individual and in the second equation is household. For estimation simplicity we replicate the household observations to obtain the same number of households as individuals.

Although, the estimation approach used here provides some insight about intra-household allocation of WIC approved foods, additional work is required for the development of the estimation technique that will account for the difference in the units of observations in two estimated equations and for the correlation of individuals within the household.

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Table 2.1 Maximum Monthly Allowances of WIC Food Packages for Children

Fruit Juices	288 fl oz of vitamin C-rich juice (about 9 fl oz per day)
Milk and Alternatives	24 quarts of milk (about 3 cups per day)
	with some allowed substitutions
Grains	36 ounces of iron-fortified cereal
Meat and Alternatives	2-2.5 dozen eggs
	1 pound of dried beans or peas
	or
	18 ounces of peanut butter

Source: Brief Report, April 2005, Institute of Medicine of the National Academies

Table 2.2 Number of Individuals in Each Group by WIC Status

No. of Individuals	Group	WIC Status
423	D_1	Targeted individuals in WIC household
612	D_2	Non-targeted individuals in WIC household
706	D_3	Targeted individuals in non-WIC household
680	D_4	Non-targeted individuals in non-WIC household
2421		Total individuals

Table 2.3 WIC-approved Milk and Cheese Used in the Analysis

WIC-approved Milk	Includes: fluid cows milk (whole, 2%, 1%, skim), low-lactose milk, buttermilk, dry (powdered) milk, evaporated milk, acidophilus milk, milk flavored after purchase, milk added to cocoa mix,the milk added in preparing the food at home. Excludes: human milk, calcium-fortified milk, condensed milk, soy or rice milk, imitation milk, milk beverages, milk drinks such asYoo-Hoo, milk shakes, milk purchased already flavored.

Table 2.4 Variables and Definitions

Variables	Definitions
D_1	Targeted recipients in WIC household (omitted)
D_2	Non-targeted & non-recipient individuals in WIC household
D_3	Targeted individuals in non-WIC household
D_4	Non-targeted individuals in non-WIC household
Income	Household income
Household size	Household size
Children ages 1-5	1 if children of age 1-5 are present, 0 otherwise
Infant	1 if infant present, 0 otherwise
Food Stamp	1 if household receives food stamp, 0 otherwise
PregLactPost	1 if female is breastfeeding, pregnant or lactating, 0 otherwise
Age 1-4	1 if age is between 1&4, and 0 otherwise (omitted)
Age 5-8	1 if age is between 5&8, and 0 otherwise
Age 9-18	1 if age is between 9&18, and 0 otherwise
Age 19-50	1 if age is between 19&50, and 0 otherwise
Age 51-up	1 if age is between 51& older, and 0 otherwise
White	1 if race is white, 0 otherwise (omitted)
Black	1 if race is black, 0 otherwise
Hispanic	1 if race is Hispanic, 0 otherwise
Other	1 if race is black, 0 otherwise
Male	1 if male, 0 otherwise
Female	1 if female, 0 otherwise (omitted)
Edushs	1 if main meal preparer's education is less than high school
Eduhs	1 if main meal preparer's education is high school, 0 othewise
Eduscol	1 if main meal preparer's education is some college, 0 otherwise
No edu	1 if main meal preparer's education is not reported, 0 otherwise
Cash less 5000	1 if household has savings of cash assets of more than 5000
Northwest	1 if household is located in the Northwest, 0 otherwise
Midwest	1 if household is located in the Midwest, 0 otherwise
South	1 if household is located in the South, 0 otherwise (omitted)
West	1 if household is located in the West, 0 otherwise
Central City	1 if household is located in Central City, 0 otherwise (omitted)
Suburbs	1 if household is located in suburbs, 0 otherwise
Rural	1 if household is located in rural area, 0 otherwise

Table 2.5 Variables and Mean Values (weighted)

Variable	Sample	WIC Targeted	WIC Non-Targeted Non-Targeted	Non-WIC Targeted	Non-WIC Non-Targeted
N	2421	423	612	706	680
mg milk	233.70	454.32	147.90	402.30	162.37
mg cheese	68.10	66.41	60.41	76.71	70.31
mg milk cheese	292.65	509.92	200.43	466.35	224.64
Income	18600	14210	17381	19522	20222
Household size	4.99	4.48	5.33	4.71	5.00
Children ages 1-5	0.91	0.95	0.77	0.98	0.98
Food Stamp	0.37	0.57	0.53	0.26	0.24
PregLactPost	0.04	0.15	0.01	0.14	0.00
Infant	0.01	0.01	0.02	0.00	0.00
Age 1-4	0.28	0.87	0.07	0.87	0.00
Age 5-8	0.13	0.00	0.14	0.00	0.21
Age 9-18	0.13	0.01	0.20	0.01	0.18
Age 19-50	0.44	0.12	0.57	0.12	0.58
Age 51-up	0.03	0.00	0.03	0.00	0.04
White	0.46	0.40	0.39	0.52	0.50
Black	0.22	0.26	0.22	0.20	0.21
Hispanic	0.26	0.27	0.34	0.22	0.22
Other	0.04	0.05	0.03	0.05	0.05
Male	0.46	0.46	0.45	0.46	0.47
Female	0.54	0.54	0.55	0.54	0.53
Edushs	0.12	0.15	0.15	0.09	0.11
Eduhs	0.56	0.56	0.56	0.58	0.55
Eduscol	0.29	0.26	0.26	0.31	0.32
No edu	0.02	0.03	0.03	0.01	0.02
Northwest	0.18	0.16	0.19	0.16	0.18
Midwest	0.21	0.20	0.18	0.23	0.22
South	0.35	0.38	0.36	0.32	0.34
West	0.27	0.26	0.27	0.29	0.27
Central City	0.39	0.45	0.43	0.38	0.35
Suburbs	0.39	0.29	0.34	0.41	0.44
Rural	0.22	0.26	0.23	0.21	0.21
Instrument					
Cash less 5000	0.92	0.97	0.94	0.91	0.90

Means provided for individuals characteristics are weighted by the CSFII sampling weights

Table 2.6 First-Stage Probit Estimates

Parameter	Estimate	Std Error
Intercept	-0.48^{***}	0.18
Household size	0.09^{***}	0.02
Children ages 1-5	-0.26^{***}	0.04
Infant	0.76^{***}	0.27
Food Stamp	0.90^{***}	0.06
PregLactPost	0.01	0.16
Eduhs	-0.18^{**}	0.09
Eduscol	-0.14	0.10
No edu	0.06	0.19
Black	-0.16^{**}	0.08
Hispanic	0.38^{***}	0.08
Other	-0.03	0.14
Northwest	-0.18^{**}	0.08
Midwest	-0.25^{***}	0.08
West	-0.36^{***}	0.08
Suburbs	-0.21^{***}	0.07
Rural	0.12	0.08
Instrument		
Cash less 5000	0.22^{**}	0.12
Ln L	-1460	
N	2421	

Notes: *** =significance at 1% level;** =significance at 5% level

Table 2.7 Second-Stage Tobit Estimates for Consumption of Milk and Cheese

	Milk		Cheese		Milk&Cheese	
Variable	Estimate	Std Err	Estimate	Std Err	Estimate	Std Err
Intercept	472.82***	43.97	−36.15	28.84	554.22***	42.43
D_2	−318.92***	36.30	−27.10	24.33	−302.31***	35.36
D_3	−87.51***	22.97	19.32	15.22	−67.50***	22.47
D_4	−325.12***	41.11	−28.35	27.10	−314.37***	39.85
Income	0.001	0.00	0.001**	0.00	0.001	0.001
Household size	−4.72	5.24	−10.57***	3.48	−10.69**	5.05
Food Stamp	−13.19	19.18	13.18	12.44	−0.147	18.52
PregLactPost	−140.03***	49.54	−23.88	32.47	−175.05***	48.16
Age 5-8	99.59***	39.32	47.12*	26.08	93.44***	38.27
Age 9-18	−80.02**	39.15	51.93**	25.53	−66.79*	37.82
Age 19-50	−193.31***	34.75	60.88***	22.83	−118.09***	33.52
Age 51-up	−220.15***	56.35	−30.56	37.31	−201.49***	53.68
Black	−92.04***	22.49	−20.70	14.32	−96.64***	21.60
Hispanic	7.90	21.39	−43.01***	13.77	−16.52	20.62
Other	−1.34	40.17	−129.89***	29.16	−46.73	38.98
Male	8.51	15.34	8.13	9.83	18.61	14.79
Eduhs	34.02	25.72	8.96	16.74	12.80	24.57
Eduscol	54.49**	28.67	34.15*	18.44	41.87	27.39
No edu	46.89	54.78	−21.47	37.05	25.34	52.89
Northwest	27.88	23.54	−14.20	15.28	19.42	22.73
Midwest	34.48*	21.44	26.92**	13.40	46.45**	20.61
West	83.94***	21.47	18.43	13.78	86.79***	20.76
Suburbs	15.60	18.50	15.21	11.99	21.53	17.90
Rural	−56.02***	21.39	20.15	13.62	−37.18*	20.57
Ln L		-14345		-8389		-15801
N		2421		2421		2421

Table 2.8 Predicted Conditional Means of Milk, Milk & Cheese Intake (base 500mg)

Group	Milk (mg calcium)	Milk and Cheese (mg calcium)
D_1=Targeted recipient WIC household	503 (1.06)	555 (1.11)
D_2=Non-targeted in WIC household	180 (0.36)	238 (0.48)
D_3=Targeted in non-WIC household	447 (0.89)	514 (1.03)
D_4=Non-targeted in non-WIC household	179 (0.36)	242 (0.48)

3. INTRA-HOUSEHOLD ALLOCATION AND CONSUMPTION OF WIC-APPROVED FOODS: A BAYESIAN APPROACH

Abstract

WIC, the Special Supplemental Nutrition Program for Women, Infants, and Children, is a widely studied public food assistance program that aims to provide foods, nutrition education and other services to at-risk, low-income children and pregnant, breastfeeding and postpartum women. From a policy perspective, it is of interest to assess the efficacy of the WIC program - how much, if at all, does the program improve the nutritional outcomes of WIC families? In this paper we address two important issues related to the WIC program that have not been extensively addressed in the past. First, although the WIC program is primarily devised with the intent of improving the nutrition of "targeted" children and mothers, it is possible that WIC may also change the consumption of foods by non-targeted individuals within the household. Second, although WIC eligibility status is predetermined, participation in the program is voluntary and therefore potentially endogenous. We make use of a treatment-response model in which the dependent variable is the requirement-adjusted calcium intake from milk consumption and the endogenous variable is WIC participation, and estimate it using Bayesian methods. Using data from the CSFII 1994-1996, we find that the correlation between the errors of our two equations is strong and positive, suggesting that families participating in WIC have an unobserved propensity for high calcium intake. The direct "structural" WIC parameters, however, do not support the idea that WIC participation leads to increased levels of calcium intake from milk.

Keywords: nutrition, WIC, Bayesian econometrics, treatment-response.

JEL Classification: C11; C31; C34; I38.

Introduction

In fiscal year 2006, the United States Department of Agriculture (USDA) spent nearly \$53 billion on food assistance programs (Oliveira 2007). The third largest of these programs, the Special Supplemental Nutrition Program for Women, Infants, and Children (commonly and henceforth denoted as WIC), has been widely studied in the health and nutrition literatures and aims to serve the public by providing supplemental foods, nutrition education and other services to foster the growth, development and long-term health of participating individuals.

For families that qualify for WIC participation, the program provides access to nutritious foods to supplement the diets of infants, children up to age five, and pregnant, breastfeeding and postpartum women. The program benefits, usually in the form of checks or vouchers, allow participants to obtain specific "packages" of foods. These foods include infant formula, milk, cheese, eggs, juice, cereals, peanut butter/dried beans, and, for fully breast-feeding mothers, these also include carrots and tuna.

From a policy perspective, it is of primary interest to assess the efficacy of the WIC program - how much, if at all, does the program improve the nutritional outcomes of WIC families? In this paper we employ a Bayesian methodology to address this question and estimate the impact of WIC participation on a specific nutritional outcome - calcium intake via milk consumption. Our study is certainly not the first in this regard, as other efforts using different models and maintained assumptions have been conducted in the past. For example, Oliveira and Chandran (2005) find that participation in the WIC program increases consumption for some types of WIC-approved foods for WIC children compared to eligible nonparticipating children and children living in households with income too high to be eligible for WIC (income greater than 185% of the poverty threshold). Other efforts in this regard include the studies of Rose et al. (1998), Burstein, et al. (2000), Oliveira and Gundersen, (2000) Ponza, et al. (2004) and Siega-Riz, et al.

(2004), who generally find positive impacts associated with the WIC program.

There are, however, two important issues related to the WIC program that have not been extensively addressed in past work, and we seek to address these issues in the current paper. First, although the WIC program is primarily devised with the intent of improving the nutrition of "targeted" children and mothers, it is possible that WIC may also change the consumption of foods by non-targeted individuals within the household. This has been referred to as "spillover" (Oliveira and Chandran 2005) or "leakage" (Barrett 2002) of WIC benefits. As Oliveira and Chandran note, this might occur if 1) receipt of WIC benefits frees up food dollars for use to benefit other, nonparticipating children; 2) nutrition education changes food selection for all members; or 3) WIC foods are shared with non-WIC household members. Little is known about the degree to which this occurs. In the current paper, we formally address this issue by comparing the impact of WIC participation on both targeted household members as well as non-targeted members of WIC families.

Second, the previous literature on this topic has certainly been aware of the potential endogeneity of WIC participation and, in some cases, has interpreted the obtained results with caution in light of this concern. To our knowledge, however, no study in the literature has dealt with this problem extensively. To address this endogeneity issue, we make use of a treatment-response model in which the dependent variables are the requirement-adjusted calcium intake from milk consumption and the decision to participate in WIC. We estimate this two equation system jointly and handle the endogeneity issue by introducing covariates that affect WIC participation directly but (presumably) are conditionally uncorrelated with levels of calcium intake. These instruments include indicators of household assets as well as variables exploiting regional variation in requirements for WIC participation. Ostensibly, WIC participation will lead to increased calcium intake from milk, though in the presence of endogenous participation, this need not be the case. For example, families who choose to participate in WIC may simultaneously (and unobservably) be quite concerned regarding the nutritional intake of each family member, and thus members of households participating in WIC may have high calcium intake even in the absence of WIC. Moreover, freed resources enable families to consume calcium through other sources, so that WIC could actually lead to a reduction in calcium intake through milk.

In terms of our posterior predictive distributions of calcium intake from milk, we find results consistent with our prior expectations and the majority of past work on this topic. That is, WIC targeted individuals have higher levels of calcium intake than their non-WIC counterparts. However, the posterior predictives combine two sources of information: what we might term the “structural” effect of WIC participation as well as an unobserved correlation between the errors of the participation and outcome equations. As one might suspect, we find that the correlation between errors in the WIC participation and calcium consumption equations is strong and positive, suggesting that families participating in WIC have an unobserved propensity for high calcium intake. What drives the intuitive ordering among the posterior predictives is primarily the selection effect - those families in WIC would have had large levels of calcium intake in the absence of the program. The direct “structural” WIC parameters do not directly support the idea that WIC participation leads to increased levels of calcium intake, a finding that is, to our knowledge, new to this literature. Indeed, these families may be substituting away from milk and toward other preferred alternatives, a finding that has significant implications for the selection of foods within the WIC program.

The paper proceeds as follows. The next section describes the model specification and the associated Bayesian posterior simulator. The data used in the analysis are described followed by a description of empirical results. The paper concludes with a summary of the findings.

The Model, Posterior Simulator and Posterior Predictives

We first let w_h be a binary variable equal to one if household h participates in WIC and equal to zero otherwise. Within a given household, some members, including children under five and pregnant/breastfeeding mothers, will be *targeted* individuals, i.e., those family members the WIC program is primarily designed to serve. To this end, we will let T_{ih} be an exogenous binary variable denoting if individual i in household h is a WIC targeted individual. The construction of these two variables leads to the categorization of all individuals in our sample into four mutually exclusive groups:

$G_{1,ih} = w_h * T_{ih}$ (targeted individual in a WIC participating household),

$G_{2,ih} = w_h * (1 - T_{ih})$ (non-targeted individual in a WIC participating household),

$G_{3,ih} = (1 - w_h) * T_{ih}$ (targeted individual in a WIC eligible but non-participating household),

$G_{4,ih} = (1 - w_h) * (1 - T_{ih})$ (non-targeted individual in a WIC eligible but non-participating household).

Our outcome variable of interest is requirement-adjusted calcium intake through milk consumption. We represent this variable as c_{ih}. Importantly, there is a censoring problem associated with calcium intake in our data, since approximately 16% of our sample has identically zero consumption values. To this end, we follow Chib (1992) and Albert and Chib (1993) and work with latent consumption c^*_{ih}, which is assumed to be generated by:[1]

$$c^*_{ih} \quad = \quad \boldsymbol{x_{ih}\alpha} + \epsilon_{ih}, \tag{3.1}$$

and

$$c_{ih} \quad = \quad max\{0, c^*_{ih}\}. \tag{3.2}$$

[1]We follow standard conventions of using capital letters to denote matrix quantities and bold script to denote vectors or matricies.

The group identifiers $G_2 \rightarrow G_4$ [2] above together with other relevant demographic variables such as age, income, gender indicators, etc., are included in the vector $\boldsymbol{x_{ih}}$. By comparing the α coefficients across these groups, we can determine if WIC participation has an important effect on calcium intake, and, moreover, we can test for the presence of the hypothesized "spillover" effects within a WIC household. That is, we can determine whether or not non-targeted members in WIC households have higher levels of calcium intake through milk consumption than non-targeted members of non-WIC households.

As stressed in the introduction of this paper, WIC participation is voluntary, and thus the binary indicator w_h (and associated group identifiers $G_2 \rightarrow G_4$) is not necessarily exogenous in (3.1). That is, household heads choosing to participate in WIC could, for example, be very concerned about the nutritional intakes of its constituents, thus leading to higher levels of calcium intake for these families on average. To this end, we first consider the household-level decision to participate in WIC:

$$w_h^* = \boldsymbol{z_h}\boldsymbol{\beta} + \nu_h, \quad \nu_h \overset{iid}{\sim} N(0,1) \tag{3.3}$$

where

$$w_h = \begin{cases} 1 & if \quad w_h^* > 0 \\ 0 & if \quad w_h^* \leq 0 \end{cases} \tag{3.4}$$

and $\boldsymbol{z_h}$ is a vector of instruments and household specific characteristics.

To account for the potential endogeneity of WIC participation, we allow the errors of (3.1) and (3.3) to be correlated. That is, household-level unobservables that make a family more likely to participate in WIC may also make that family more likely to have high levels of calcium intake. We choose to accommodate this type of correlation by including a household-specific error term in (3.1) and allowing this error to be correlated with ν_h in (3.3). The intuition behind this modeling assumption is that a household head who chooses to participate in WIC will also tend to guide meal preparation in

[2] Here G_1 (targeted individuals participating in WIC) is the excluded category.

the household and monitor the nutritional habits of the household members. Thus, unobservable factors affecting WIC participation will likely spill over and correlate with the nutritional intakes of *all* the family members and should probably correlate in a similar way across each member. To this end, we consider the following model:

$$c^*_{ih} = \boldsymbol{x_{ih}\alpha} + \psi s^*_{ih} + u_h + \epsilon_{ih}, \tag{3.5}$$

$$w^*_h = \mathbf{z}_h\boldsymbol{\beta} + \nu_h, \tag{3.6}$$

where

$$\left[\begin{array}{c} u_h \\ \nu_h \end{array}\right] \Bigg| \, \mathbf{x}, \mathbf{z}, \mathbf{s}^* \overset{iid}{\sim} N\left[\left(\begin{array}{c} 0 \\ 0 \end{array}\right), \left(\begin{array}{cc} \sigma^2_u & \sigma_{uv} \\ \sigma_{uv} & 1 \end{array}\right)\right], \tag{3.7}$$

and

$$\epsilon_{ih} | \, \mathbf{x}, \mathbf{z}, \mathbf{s}^* \overset{iid}{\sim} N(0, \sigma^2_\epsilon). \tag{3.8}$$

Equations (3.5) and (3.6) now represent a standard two-equation treatment-response model using only observed rather than potential outcomes.[3] However, we note that equation (3.5), unlike its counterpart in (1), has included a latent variable s^*_{ih}. This latent variable is included, like the model of Chen, Dey and Shao (1999), to capture possible skew in the distribution of calcium intake.[4] These latent variables are specified to be generated from a known distribution with one-sided support, thereby introducing the possibility of accommodating skew in the outcome distribution beyond what is implied by normality (given that $c^*_{ih} > 0$). A rather standard choice in this regard, as employed in Chen, Dey and Shao (1999), is to assume that s^*_{ih} is generated from a *half-normal*

[3] For more on related posterior simulators for such models, see Koop and Poirier (1997), Chib and Hamilton (2000, 2002), Poirier and Tobias (2003) and Chib (2007).

[4] Note that, unlike adopting the log specification, the model in (3.5) introduces skew without having to address potential issues such as taking the log of negative values (and simultaneously considering the mass point at zero consumption), or introducing an additional "hurdle" or "threshold" to the analysis. This representation is, of course, not as flexible as other alternatives such as Gaussian mixtures or Dirichlet processes but is a simpler alternative that may be adequately flexible to capture the salient features of a given problem.

distribution,

$$s_{ih}^{*}|\,\mathbf{x},\mathbf{z} \overset{iid}{\sim} TN_{(0,\infty)}(0,1),$$

with $TN_{(a,b)}(\mu,\sigma^2)$ denoting a normal distribution with mean μ and variance σ^2 truncated to the interval (a,b). When integrating the conditional density for c_{ih}^{*} (given s_{ih}^{*}) over this half-normal for s_{ih}^{*}, it can be shown that, marginally, c_{ih}^{*} will have a *skew-normal* distribution (e.g., Azzalini and Dalla Valle [1996], Chen, Dey and Shao [1999] and Branco and Dey [2002]). The sign of the parameter ψ governs the direction of the skew (i.e., positive values produce a distribution with a right-skew, conversely for negative values of ψ, and $\psi = 0$ reduces to joint normality). Since the potential for such skew exists in both the conditional and unconditional distributions of calcium intake (Figure 3.1), we adopt the above procedure for handling this issue. As shown in our empirical results section, the data strongly support the hypothesis of $\psi \neq 0$ so that the default assumption of joint normality is not appropriate for this data. This is suggested in the following graph of the raw calcium intake data:

With the formulation in (3.5), the composite error term $\psi s_{ih}^{*} + u_h + \epsilon_{ih}$ is not mean zero since s_{ih}^{*} is not mean zero. Though this shift will be "absorbed" by the intercept parameter, this creates a muddled interpretation of the parameter ψ and may lead to slower mixing of the posterior simulations.[5] We handle this issue by simply shifting the distribution of s_{ih}^{*} back by its mean, $\sqrt{(2/\pi)}$. Thus, in our analysis, we specify[6]

$$s_{ih}^{*}|\,\mathbf{x},\mathbf{z} \overset{iid}{\sim} TN_{(-\sqrt{2/\pi},\infty)}(-\sqrt{2/\pi},1) \tag{3.9}$$

and the model is given by (3.5)-(3.8) together with the revised distributional assumption on s_{ih}^{*} given in (3.9).

[5]This issue has been pointed out by Pewsey (2000) and others.
[6]In generated data experiments, this simple transformation seemed to improve the mixing of the posterior simulations.

The Joint Posterior

For the implementation of the posterior simulator, it will be instructive to work with the population expectation of u_h given ν_h. Given the joint normality assumption above, we can write

$$u_h = \sigma_{uv}\nu_h + \eta_h,$$

where

$$\eta_h \stackrel{iid}{\sim} N(0, \sigma_\eta^2), \quad \text{and} \quad \sigma_\eta^2 \equiv \sigma_u^2 - \sigma_{uv}^2.$$

Thus, we can re-write our initial equation system in the following way:

$$\begin{aligned} c_{ih}^* &= \boldsymbol{x_{ih}}\boldsymbol{\alpha} + \psi s_{ih}^* + \sigma_{uv}\nu_h + \eta_h + \epsilon_{ih} \\ w_h^* &= \boldsymbol{z_h}\boldsymbol{\beta} + \nu_h \end{aligned}$$

where

$$\begin{aligned} \epsilon_{ih} &\stackrel{iid}{\sim} N(0, \sigma_\epsilon^2) \\ \nu_h &\stackrel{iid}{\sim} N(0, 1) \\ \eta_h &\stackrel{iid}{\sim} N(0, \sigma_\eta^2). \end{aligned}$$

Thus, conditioned on the common ν_h, the consumption and WIC participation equations are independent.

Let

$$\boldsymbol{\delta} = [\boldsymbol{\alpha}' \ \boldsymbol{\beta}' \ \psi \ \sigma_{uv} \ \sigma_\epsilon^2 \ \sigma_\eta^2]'$$

denote the parameters of our model other than the random effects $\boldsymbol{\eta}$. In addition, let n_h denote the number of individuals in household h, H denote the total number of households in the sample, $NH \equiv \sum_{h=1}^{H} n_h$, k denote the number of explanatory variables and, finally, define

$$\mathbf{c}_h^* = \begin{bmatrix} c_{1h}^* \\ c_{2h}^* \\ \vdots \\ c_{n_h h}^* \end{bmatrix}, \quad \mathbf{X}_h = \begin{bmatrix} \boldsymbol{x_{1h}} \\ \boldsymbol{x_{2h}} \\ \vdots \\ \boldsymbol{x_{n_h h}} \end{bmatrix}, \quad \mathbf{s}_h^* = \begin{bmatrix} s_{1h} \\ s_{2h} \\ \vdots \\ s_{n_h h} \end{bmatrix},$$

$$\mathbf{c}^* = \begin{bmatrix} \mathbf{c}_1^* \\ \mathbf{c}_2^* \\ \vdots \\ \mathbf{c}_H^* \end{bmatrix}, \quad \mathbf{s}^* = \begin{bmatrix} \mathbf{s}_1^* \\ \mathbf{s}_2^* \\ \vdots \\ \mathbf{s}_H^* \end{bmatrix}, \quad \mathbf{w}^* = \begin{bmatrix} \mathbf{w}_1^* \\ \mathbf{w}_2^* \\ \vdots \\ \mathbf{w}_H^* \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\eta} = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_H \end{bmatrix},$$

where $\boldsymbol{x_{ih}}$ is a $1 \times k$ covariate vector for agent i, $\boldsymbol{X_h}$ is the $NH \times k$ matrix of stacked covariate data and $\boldsymbol{c^*}$, $\boldsymbol{s^*}$, $\boldsymbol{w^*}$ and $\boldsymbol{\eta}$ are $NH \times 1$ vectors. As in Albert and Chib (1993), we will include the latent $\boldsymbol{c}^*$, $\mathbf{w}^*$, $\boldsymbol{s}^*$ and vector of random effects $\boldsymbol{\eta}$ into our posterior and thus will work with an augmented posterior of the form

$$\begin{aligned} p(\mathbf{c}^*, \mathbf{w}^*, \mathbf{s}^*, \boldsymbol{\delta}, \boldsymbol{\eta} | \mathbf{c}, \mathbf{w}) &\propto p(\mathbf{c}, \mathbf{w} | \mathbf{c}^*, \mathbf{w}^*, \mathbf{s}^*, \boldsymbol{\delta}, \boldsymbol{\eta}) p(\mathbf{c}^*, \mathbf{w}^*, \mathbf{s}^* | \boldsymbol{\delta}, \boldsymbol{\eta}) p(\boldsymbol{\eta} | \boldsymbol{\delta}) p(\boldsymbol{\delta}) \\ &= p(\boldsymbol{\delta}) \Big[\prod_{i=1}^{H} p(w_h | w_h^*) p(\mathbf{c}_h^*, w_h^* | \mathbf{s}_h^*, \boldsymbol{\delta}, \eta_h) p(\eta_h | \boldsymbol{\delta}) \\ &\quad \times \left(\prod_{i \in h} p(c_{ih} | c_{ih}^*) p(s_{ih}^*) \right) \Big]. \end{aligned}$$

In the first line above, we write the posterior as proportional to the full joint distribution (of parameters, latent and observed data), and decompose this joint distribution into a sequence of conditionals times marginals. The densities $p(\boldsymbol{\eta}|\boldsymbol{\delta})$ and $p(\boldsymbol{\delta})$ denote prior distributions for these parameters, and, in the second line of the above, we incorporate the assumed (conditional) independence across households. Finally, in regard to the density $p(\mathbf{c}, \mathbf{w} | \mathbf{c}^*, \mathbf{w}^*, \mathbf{s}^*, \boldsymbol{\delta}, \boldsymbol{\eta})$, we note that the distribution of w_h depends only on w_h^* (and is degenerate given its value), and, likewise, the distribution of c_{ih} depends only on c_{ih}^* (and is degenerate given its value). That is,

$$p(w_h | w_h^*) = I(w_h = 1) I(w_h^* > 0) + I(w_h = 0) I(w_h^* \leq 0)$$

and

$$p(c_{ih}|c_{ih}^*) = I(c_{ih} = c_{ih}^*)I(c_{ih}^* > 0) + I(c_{ih} = 0)I(c_{ih}^* \leq 0).$$

As for the joint distribution of household h's calcium intake, $\boldsymbol{c_h^*}$, and WIC participation,w_h^*, note that

$$\begin{bmatrix} \mathbf{c}_h^* \\ w_h^* \end{bmatrix} \Big| \mathbf{s}_h^*, \boldsymbol{\delta}, \eta_h \overset{ind}{\sim} N\left[\begin{pmatrix} \mathbf{x}_h\boldsymbol{\alpha} + \psi\mathbf{s}_h^* + \eta_h\boldsymbol{\iota}_{n_h} \\ \mathbf{z}_h\boldsymbol{\beta} \end{pmatrix}, \begin{pmatrix} \sigma_\epsilon^2\mathbf{I}_{n_h} + \sigma_{uv}^2\boldsymbol{\iota}_{n_h}\boldsymbol{\iota}_{n_h}' & \sigma_{uv}\boldsymbol{\iota}_{n_h} \\ \sigma_{uv}\boldsymbol{\iota}_{n_h}' & 1 \end{pmatrix}\right],$$

where $\boldsymbol{\iota}_{n_h}$ is an $n_h \times 1$ vector of ones, and, likewise, $\mathbf{I}_{n_h}$ is the identity matrix of dimension n_h.

To complete our Bayesian analysis we must also introduce our priors. To this end, we let

$$\boldsymbol{\gamma} \equiv \begin{bmatrix} \boldsymbol{\alpha} \\ \psi \\ \boldsymbol{\beta} \end{bmatrix}$$

and specify priors of the form

$$\begin{aligned} \boldsymbol{\gamma} &\sim N(\boldsymbol{\mu}_\gamma, \mathbf{V}_\gamma) && (3.10) \\ \sigma_{uv} &\sim N(\mu_{uv}, V_{uv}) && (3.11) \\ \sigma_\epsilon^2 &\sim IG(a_\epsilon, b_\epsilon) && (3.12) \\ \sigma_\eta^2 &\sim IG(a_\eta, b_\eta). && (3.13) \end{aligned}$$

The Posterior Simulator

We fit this model using recent advances in Markov Chain Monte Carlo (MCMC) techniques, namely, the Gibbs sampler (e.g., Gelfand et al [1990], Casella and George [1992], Albert and Chib [1993]). Implementation of the Gibbs sampler involves deriving and then iteratively simulating from the conditional posterior distributions of the model's parameters. The sequence of simulations produced from this sampling procedure forms a Markov chain that, under certain regularity conditions, converges to the targeted distribution (i.e., the joint posterior). To mitigate the effect of initial conditions on this chain, an initial set of pre-convergence or "burn-in" simulations is discarded, and the remaining set of simulations is then used to calculate posterior features of interest.

Our complete Gibbs algorithm consists of 8 steps, and the first two of these form a *blocking step* (e.g., Chib and Carlin [1999]), where the parameters $\boldsymbol{\gamma} = [\boldsymbol{\alpha}' \ \psi \ \boldsymbol{\beta}']'$ and random effects $\boldsymbol{\eta}$ are sampled in a single block. We do this via the *method of composition.* That is, we first sample $\boldsymbol{\gamma}$ from its conditional posterior, where the random effects $\boldsymbol{\eta}$ have been integrated out. We then sample $\boldsymbol{\eta}$ by drawing each η_h independently from its complete conditional posterior. For simplicity in notation below, we let $\boldsymbol{\Gamma} = [\boldsymbol{\delta}' \ \mathbf{c}^{*\prime} \ \mathbf{w}^{*\prime} \ \mathbf{s}^{*\prime}]'$ and let $\boldsymbol{\Gamma}_{-x}$ denote all parameters other than x.

Step 1: $\boldsymbol{\gamma}|\boldsymbol{\Gamma}_{-\boldsymbol{\gamma}}, \mathbf{c}, \mathbf{w}$.

First, define

$$\overline{\mathbf{X}}_h \equiv \begin{bmatrix} \mathbf{X}_h & \mathbf{s}_h^* & 0 \\ 0 & 0 & z_h \end{bmatrix}, \quad \overline{\mathbf{c}}_h^* \equiv \begin{bmatrix} \mathbf{c}_h^* \\ w_h^* \end{bmatrix},$$

and

$$\boldsymbol{\Sigma}_h \equiv \begin{bmatrix} [\sigma_\epsilon^2 \mathbf{I}_{n_h} + (\sigma_\eta^2 + \sigma_{uv}^2)\boldsymbol{\iota}_{n_h}\boldsymbol{\iota}_{n_h}'] & \sigma_{uv}\boldsymbol{\iota}_{n_h} \\ \sigma_{uv}\boldsymbol{\iota}_{n_h}' & 1 \end{bmatrix}.$$

It follows that

$$\boldsymbol{\gamma}|\boldsymbol{\Gamma}_{-\boldsymbol{\gamma}}, \mathbf{c}, \mathbf{w} \sim N(\mathbf{D}_\gamma \mathbf{d}_\gamma, \mathbf{D}_\gamma), \tag{3.14}$$

where

$$\mathbf{D}_\gamma = \left[\sum_h (\overline{\mathbf{X}}_h' \boldsymbol{\Sigma}_h^{-1} \overline{\mathbf{X}}_h) + \mathbf{V}_\gamma^{-1}\right]^{-1} \quad \text{and} \quad \mathbf{d}_\gamma = \sum_h (\overline{\mathbf{X}}_h' \boldsymbol{\Sigma}_h^{-1} \overline{\mathbf{c}}_h^*) + \mathbf{V}_\gamma^{-1}\boldsymbol{\mu}_\gamma.$$

Step 2: $\eta_h|\boldsymbol{\Gamma}_{-\eta_h}, \mathbf{c}, \mathbf{w}$

$$\eta_h|\Gamma_{-\eta_h}, \mathbf{c}, \mathbf{w} \overset{ind}{\sim} N(D_{\eta_h} d_{\eta_h}, D_{\eta_h}), \quad h = 1, 2, \cdots, H, \tag{3.15}$$

where

$$D_{\eta_h} = \frac{\sigma_\eta^2 \sigma_\epsilon^2}{n_h \sigma_\eta^2 + \sigma_\epsilon^2}$$

$$d_{\eta_h} = \frac{1}{\sigma_\epsilon^2} \sum_{i \in h} \left(c_{ih}^* - \mathbf{x}_{ih}\boldsymbol{\alpha} - s_{ih}^*\psi - \sigma_{uv}[w_h^* - \mathbf{z}_h\boldsymbol{\beta}] \right).$$

Step 3: $\sigma_{uv}|\Gamma_{-\sigma_{uv}}, \mathbf{c}, \mathbf{w}$

First, define the $NH \times 1$ vectors V and η as follows:

$$\mathbf{V} \equiv \begin{bmatrix} \boldsymbol{\iota}_{n_1}[w_1^* - \mathbf{z}_1\boldsymbol{\beta}] \\ \boldsymbol{\iota}_{n_2}[w_2^* - \mathbf{z}_2\boldsymbol{\beta}] \\ \vdots \\ \boldsymbol{\iota}_{n_H}[w_H^* - \mathbf{z}_H\boldsymbol{\beta}] \end{bmatrix}, \quad \overline{\boldsymbol{\eta}} \equiv \begin{bmatrix} \boldsymbol{\iota}_{n_1}[\eta_1] \\ \boldsymbol{\iota}_{n_2}[\eta_2] \\ \vdots \\ \boldsymbol{\iota}_{n_H}[\eta_H] \end{bmatrix}.$$

It follows that

$$\sigma_{uv}|\Gamma_{-\sigma_{uv}}, \mathbf{c}, \mathbf{w} \sim N\left(D_{\sigma_{uv}} d_{\sigma_{uv}}, D_{\sigma_{uv}}\right), \tag{3.16}$$

where

$$D_{\sigma_{uv}} = \left(\mathbf{V}'\mathbf{V}/\sigma_\epsilon^2 + V_{uv}^{-1}\right)^{-1}, \quad d_{\sigma_{uv}} = \mathbf{V}'(\mathbf{c}^* - \mathbf{X}\boldsymbol{\alpha} - \mathbf{s}^*\psi - \overline{\boldsymbol{\eta}})/\sigma_\epsilon^2 + V_{uv}^{-1}\mu_{uv}.$$

Step 4: $\sigma_\epsilon^2|\Gamma_{-\sigma_\epsilon^2}, \mathbf{c}, \mathbf{w}$

$$\sigma_\epsilon^2|\Gamma_{-\sigma_\epsilon^2}, \mathbf{c}, \mathbf{w} \sim IG\left(\tfrac{NH}{2} + a_\epsilon, \left[b_\epsilon^{-1} + \tfrac{1}{2}\textstyle\sum_{i=1}^{NH}(c_{ih}^* - \mathbf{x}_{ih}\boldsymbol{\alpha} - s_{ih}^*\psi - \eta_h - \sigma_{uv}[w_h^* - \mathbf{z}_h\boldsymbol{\beta}])^2\right]^{-1}\right). \tag{3.17}$$

Step 5: $\sigma_\eta^2|\Gamma_{-\sigma_\eta^2}, \mathbf{c}, \mathbf{w}$

$$\sigma_\eta^2|\Gamma_{-\sigma_\eta^2}, \mathbf{c}, \mathbf{w} \sim IG\left(\frac{H}{2} + a_\eta, \left[b_\eta^{-1} + \frac{1}{2}\sum_{h=1}^{H}(\eta_h^2)\right]^{-1}\right). \tag{3.18}$$

Step 6: $\mathbf{w}^*|\Gamma_{-\mathbf{w}^*}, \mathbf{c}, \mathbf{w}$

Each of the latent variables in the WIC participation equation are sampled independently as follows:

$$w_h^*|\Gamma_{-\mathbf{w}^*}, \mathbf{c}, \mathbf{w} \sim \begin{cases} TN_{(0,\infty)}(\mu_{w_h^*}, \sigma_{w_h^*}^2) & \text{if } w_h = 1 \\ TN_{(-\infty,0]}(\mu_{w_h^*}, \sigma_{w_h^*}^2) & \text{if } w_h = 0 \end{cases}, \tag{3.19}$$

where

$$\mu_{w_h^*} = \mathbf{z}_h\boldsymbol{\beta} + \sigma_{uv}\boldsymbol{\iota}_{n_h}' \left[\sigma_\epsilon^2 \mathbf{I}_{n_h} + \sigma_{uv}^2 \boldsymbol{\iota}_{n_h}\boldsymbol{\iota}_{n_h}'\right]^{-1} (\mathbf{c}_h^* - \mathbf{X}_h\boldsymbol{\alpha} - \mathbf{s}_h^*\psi - \eta_h\boldsymbol{\iota}_{n_h}),$$

and

$$\sigma_{w_h^*}^2 = 1 - \sigma_{uv}^2\boldsymbol{\iota}_{n_h}' \left[\sigma_\epsilon^2 \mathbf{I}_{n_h} + \sigma_{uv}^2 \boldsymbol{\iota}_{n_h}\boldsymbol{\iota}_{n_h}'\right]^{-1} \boldsymbol{\iota}_{n_h}.$$

Step 7: $\mathbf{c}^*|\Gamma_{-\mathbf{c}^*}, \mathbf{c}, \mathbf{w}$

Note that, conditioned on η_h and the remaining parameters of the model, each latent c_{ih}^* can be sampled independently from its conditional posterior:

$$c_{ih}^*|\Gamma_{-c_{ih}^*}, c, w \sim TN_{(-\infty,0)}(\mu_{c_{ih}^*}, \sigma_\epsilon^2) \quad \text{if } c_{ih} = 0, \tag{3.20}$$

where

$$\mu_{c_{ih}^*} = \mathbf{x}_{ih}\boldsymbol{\alpha} + s_{ih}^*\psi + \eta_h + \sigma_{uv}(w_h^* - \mathbf{z}_h\boldsymbol{\beta}).$$

When $c_{ih} > 0$, the conditional posterior for c_{ih}^* is degenerate around the observed c_{ih} and does not need to be simulated.

Step 8: $s_{ih}^*|\Gamma_{-s_{ih}^*}, \mathbf{c}, \mathbf{w}$

The assumptions of our model imply that each s_{ih}^* can be sampled independently from its complete conditional posterior. Completing the square in s_{ih}^* yields a posterior conditional of the form:

$$s_{ih}^*|\Gamma_{-s_{ih}^*}, \mathbf{c}, \mathbf{w} \overset{ind}{\sim} TN_{(-\sqrt{2/\pi},\infty)}(\mu_{s_{ih}^*}, \sigma_{s^*}^2), \quad i = 1, 2, \cdots, NH, \tag{3.21}$$

where

$$\mu_{s_{ih}^*} = \frac{\psi\big(c_{ih}^* - \mathbf{x}_{ih}\boldsymbol{\alpha} - \eta_h - \sigma_{uv}[w_h^* - \mathbf{z}_h\boldsymbol{\beta}]\big) - \sqrt{2/\pi}\sigma_\epsilon^2}{\sigma_\epsilon^2 + \psi^2}$$

and

$$\sigma_{s^*}^2 = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \psi^2}.$$

A Gibbs sampler proceeds by iteratively sampling from (3.14)-(3.21).

Posterior Predictive Intake Distribution

In our empirical application we are primarily concerned with the calculation and comparison of intake distributions for individuals in each of the four groups of interest. To this end, we focus on posterior prediction and fix the exogenous covariates' values for simplicity. Given our model, the posterior predictive intake distribution for such a representative agent with fixed covariates and $w_h = 1$, conditioned on the model parameters Γ, is given as

$$\begin{aligned} p\left(c^*_{n+1,h}|w_h = 1, \mathbf{\Gamma}\right) &= p\left(c^*_{n+1,h}|w^*_h > 0, \mathbf{\Gamma}\right) \\ &= \left[\Pr\left(w^*_h > 0|\mathbf{\Gamma}\right)\right]^{-1} \int_0^\infty p\left(c^*_{n+1,h}, w^*_h|\mathbf{\Gamma}\right) dw^*_h, \end{aligned}$$

where the $n+1$ subscript is used to denote an out-of-sample, "representative" agent. After some manageable algebra, we perform the required integration and obtain:

$$\begin{aligned} &p\left(c^*_{n+1,h}|w_h = 1, \mathbf{\Gamma}\right) \\ &\qquad = \Phi\left[\frac{\boldsymbol{z_h\beta} + [\sigma_{uv}/(\sigma^2_{uv} + \sigma^2_\epsilon)]\left(c^*_{n+1,h} - \boldsymbol{x_{n+1,h}\alpha} - \psi s^*_{n+1,h} - \eta_h\right)}{\sqrt{\sigma^2_\epsilon/[\sigma^2_\epsilon + \sigma^2_{uv}]}}\right] \\ &\qquad \times \frac{\phi\left(c^*_{n+1,h}; \boldsymbol{x_{n+1,h}\alpha} + \psi s^*_{n+1,h} + \eta_h, \sigma^2_\epsilon + \sigma^2_{uv}\right)}{\Phi\left(\boldsymbol{z_h\beta}\right)}. \end{aligned} \tag{3.22}$$

The density in (3.22) is not of an immediately recognizable form, though the steps leading to its derivation suggest a method of obtaining draws directly from this density. Specifically, draws from (3.22) can be obtained from the following procedure:

First, sample

$$w^*_{n+1,h} \sim TN_{(-\boldsymbol{z_h\beta},\infty)}\left(0,1\right).$$

Then, set

$$c^*_{n+1,h} = \pi_{0,n+1} + \pi_1 w^*_{n+1,h} + \pi_2\epsilon \tag{3.23}$$

where

$$\begin{aligned}\epsilon &\sim N(0,1)\\ \pi_{0,n+1} &= \boldsymbol{x_{n+1,h}\alpha} + \psi s^*_{n+1,h} + \eta_h,\\ \pi_1 &= \sigma_{uv}\\ \pi_2 &= \sigma_\epsilon.\end{aligned}$$

It can be shown that $c^*_{n+1,h}$ has the density given in (3.22). The proof of this fact is reasonably straightforward, noting that $p(c^*_{n+1,h}) = \int p(c^*_{n+1,h}|w^*_{n+1,h})p(w^*_{n+1,h})dw^*_{n+1,h}$ and substituting in the formulas above to perform the necessary integration.

Since this procedure obtains a draw from the posterior predictive for a given vector of parameters $\mathbf{\Gamma}$, the influence of these parameters can be marginalized out of the predictive by noting:

$$p(c^*_{n+1,h}|w_h = 1, \boldsymbol{c}, \boldsymbol{w}) = \int p(c^*_{n+1,h}|w_h = 1, \mathbf{\Gamma})p(\mathbf{\Gamma}|\boldsymbol{c}, \boldsymbol{w})d\mathbf{\Gamma}. \tag{3.24}$$

Thus, for every post-convergence $\mathbf{\Gamma}$ draw produced from the simulator, we apply the above procedure to obtain a draw from the posterior predictive. Though the details are suppressed here, similar steps can be used to obtain the posterior predictive associated with the event that $w_h = 0$. Finally, calcium intake is linked to the latent $c^*_{n+1,h}$ by noting: $c_{n+1,h} = \max\{0, c^*_{n+1,h}\}$, which is calculated for each iteration of the sampler.

The Data

Our empirical analysis makes use of data from the USDA 1994-96 Continuing Survey of Food Intake by Individuals (CSFII). The CSFII is a nationally representative, cross-sectional survey of individuals in households in the United States. The survey uses a randomization strategy to select certain members of the household to participate in a complete food intake survey; thus, not all members of a WIC household are present in our sample. For each of the sampled individuals, questions involving a 24-hour recall of food intake were conducted on two nonconsecutive days. Importantly for our purposes, respondents report milk consumption and the consumption of milk-containing foods during this period.

Household and individual characteristics can be used to identify WIC eligible households, and we focus only on those individuals and households that are WIC eligible in our analysis. To be eligible for WIC, at least one individual in the household must be in a WIC-qualifying population group (women who are pregnant; non-breastfeeding women up to six months postpartum; breastfeeding women up to one year postpartum; infants under one year of age; or children from one year old up to the child's fifth birthday). The household's income must also be at or below 185% of the federal poverty guidelines, or the household must participate in other qualifying programs (Medicaid, Food Stamps, Temporary Assistance for Needy Families [TANF]). Finally, individual applicants must be at nutritional risk, as determined by a health professional. Although it is not possible to determine individuals that are at nutritional risk from the CSFII data, nearly all U.S. women and children meet this criterion (IOM 2002) so that, in practice, this additional constraint can be assumed to apply to all eligible individuals. Finally, we follow Oliveira and Chandran (2005) and define eligible households as those with incomes within 200% of the federal poverty guidelines.

Our final sample consists of 2,372 individuals from 1,036 households. As discussed in the previous section, these individuals were assigned into one of the four mutually exclusive groups (Table 3.1). For our analysis we define WIC targeted individuals as children of ages one through four and pregnant, breastfeeding and postpartum women, and non-targeted individuals as children or adults in the household age five and older. [7] All households in our final sample are identified as WIC eligible by meeting the income

[7] Infants of age less than one year old are not included in the analysis because of their unique dietary requirements and intakes.

criterion and having at least one targeted individual living in the household.

Each of the four population groups described in Table 3.1 may have both adults and children. In order to compare the food intakes of individuals of varying age and gender, we convert the dependent variable, amount of milk consumed (grams), to a calcium-equivalent measure and then normalize the consumption in terms of the individuals' dietary requirement for calcium. This is accomplished in several steps. First, the CSFII 94-96 data set contains information on grams of milk consumed as a *single* food or an ingredient in a food containing *dairy* products. However, milk is commonly included as an ingredient in other non-dairy foods, and it is important to capture this aspect of calcium intake in the construction of our dependent variable. To this end, we consult the Pyramid Servings Database for USDA Survey Food Codes, Version 2.0, which provides information on the amount of milk per 100 grams contained within a variety of different foods.[8] The amounts of milk from all foods consumed by an individual are then added together to produce the total amount of calcium intake from milk and milk product consumption by the individual.

The Dietary Reference Intake (DRI) value expresses the average intake of calcium required by a given population subgroup (i.e., children age one to three years old) (IOM 1997). The calcium requirement for children of ages one through three (500 mg of calcium/day) was used as the base or reference amount to normalize consumption by other population groups. That is, the calcium intake of the surveyed individuals was converted into an age and gender equivalent measure. Thus, the dependent variable is measured as a requirement-adjusted amount of calcium (mg) received from the foods consumed. For example, if a young child reported an intake of 600 mg per day of calcium, their reported intake of 600 mg would be measured relative to their DRI (500 mg) and converted to a 500mg reference value 600 mg: (=[600 mg / 500 mg] * 500 mg). For an adult with a DRI of 1000 mg, an actual intake of 600 mg is converted to a

[8]For reference, one cup of liquid milk is set equal to 244 grams.

requirement-adjusted intake of 300 mg (=[600 mg / 1000 mg] * 500 mg).

Table 3.2 lists a summary of the data for the total sample and for each of the four groups of interest observed at the individual and at the household levels. The individual-level controls that are used in the analysis include household income, household size, an indicator if an individual is currently receiving food stamps, an indicator if an individual is currently lactating or postpartum, and a set of dummies for age, main food preparer's education level, urban residence, gender and race. The household-level controls include household income, household size, an indicator for the presence of lactating or postpartum women in the household, an indicator for the presence of an infant, an indicator denoting if the household receives food stamps, and a set of dummies for the main food preparer's education and race.

In order to deal with the potential endogeneity of WIC program participation in our model, it is useful to have an instrument. This instrument must affect the household's WIC participation decision but not be correlated with unobservables in the consumption equation. Our choice of instrument in this regard is to exploit state-level institutional characteristics of the WIC program in which the individuals reside. Specifically, we make use of information regarding whether or not the state WIC program allows participants to self-declare their income in order to prove eligibility. Less strict states (i.e., those that allow individuals to self-declare) should generally be associated with higher participation rates in WIC. However, allowing households to self-declare income in order to establish WIC eligibility should play no structural role in calcium intake, conditioned on WIC participation.[9] We also make use of a second instrument, which is an indicator denoting if household savings are less than $5000. Our argument in this regard is that families with little savings may be more likely to participate in WIC, while levels of

[9]Owing to confidentiality concerns, our data do not provide state identifiers but do provide region identifiers. To this end, we obtain an average of state policies within each region, using the fraction of WIC participants in state s within region r to weight the policy associated with state s. This instrument is not ideal but should still provide some overall degree of conditional correlation with WIC participation to aid identification. Empirically, we find that this is the case.

asset accumulation should have little to do with calcium intake, conditioned on current income, WIC participation, education and other demographic controls.

Empirical Results

Using the algorithm of section 3.2, we fit our model, running the Gibbs sampler for 100,000 iterations and discarding the first 10,000 as the burn-in period. The prior hyperparameters used in the calculations are $\mu_\gamma = \mathbf{0}_{k_\gamma}$, $\mathbf{V}_\gamma = 100\mathbf{I}_{k_\gamma}$, $\mu_{uv} = 0$, $V_{uv} = 100$, $a_\epsilon = 3$, $b_\epsilon = 1/(2 * .3)$, $a_\eta = 3$ and $b_\eta = 1/(2 * .3)$. Generated data experiments were also performed with large sample sizes to suggest that our code performs well and that our algorithm can adequately recover parameters of the data generating process in these cases. Parameter posterior means, standard deviations and probabilities of being positive associated with the model in (3.5) - (3.9) are reported in Table 3.3.

With respect to WIC participation, the results shown in Table 3.3 are generally consistent with our prior expectations. Larger households with smaller incomes and infants present in the house are clearly more likely to participate in WIC. Similarly, our instruments appear to play strong roles in the WIC participation decision and operate in the direction that we expect *a priori*. That is, families living in regions where self-reports of income are more likely to provide sufficient proof of WIC eligibility are associated with higher probabilities of WIC participation. Similarly, families with relatively small amounts of savings are also associated with higher probabilties of WIC participation.

We also conduct a variant of the standard "overidentification" test to investigate an aspect of the instrument's validity. That is, conditioned on the assumption that self-reports of income is a valid instrument, our savings indicator is superfluous in the sense that it is not needed for identification. To this end, we re-estimate the model and include this variable in the latent calcium consumption equation. Doing this, we find a posterior mean (and posterior standard deviation) associated with the Savings $<$ 5000 coefficient equal to -.15 (.32), and an associated posterior probability of being positive equal to .32. Thus, we do not see a strong role for our asset accumulation variable in the calcium consumption equation. Moreover, we calculate the relevant Bayes factor (via the Savage-Dickey density ratio) which, under equal prior odds and under the employed priors, gives the posterior odds in favor of the model imposing that ($\beta_{asset} = 0$). The Bayes factor in this case turns out to be (approximately) 22.7, again providing evidence that the asset accumulation variable can be omitted from the calcium consumption equation.

With respect to calcium intake, few variables emerge as strong predictors. Larger households tend to consume more calcium through milk while households with higher incomes tend to consume less calcium through milk. Of course, the most important of the coefficients in Table 3.3 are the coefficients associated with the group identifiers $G_2 \rightarrow G_4$.[10] These findings first suggest, quite sensibly, that non-targeted members living in WIC households (G_2) have a lower (adjusted) calcium intake through milk than targeted members of WIC households (G_1). Surprisingly, however, the results also suggest that non-WIC members, both targeted and non-targeted, receive more calcium intake through milk than their WIC counterparts.

Although these results might seem startling, and potentially suggest that the WIC program is ineffective, this is not necessarily the correct interpretation. Individuals participating in WIC may, in fact, use the benefits provided by the WIC program to purchase other products and receive an adequate level of calcium intake through the consumption of these alternative products. What the results do tell us, however, is that the WIC program does not appear to be effective at increasing calcium intake through milk. In short, the coefficients associated with the group identifiers do not necessarily call into question the effectiveness of the WIC program, but at the same time, and unlike past studies in the literature, they cannot be used to speak to its virtues. At a minimum, we

[10]Given that G_1 (targeted individuals participating in WIC) represents the excluded category, the coefficients on $G_2 \rightarrow G_4$ should be interpreted relative to this base group.

find that the presence of the WIC program leads to repackaging of consumption bundles and a substitution away from milk consumption toward other possible foods providing calcium. To our knowledge, these results represent a new contribution to the existing literature on this topic.

Table 3.3 also shows significant evidence of skew through positive values associated with the skewness parameter ψ. The table also shows, quite interestingly, a large, positive value associated with the correlation parameter ρ. This suggests, consistent with our prior views, that unobservable factors making a family more likely to participate in WIC also lead that family to consume higher levels of calcium through milk.

Table 3.4 presents posterior predictive means and standard deviations associated with calcium intake levels, as described in section 2.3, while Figure 3.2 plots the entire posterior predictive calcium distributions for each of the four groups. When performing these calculations, we set the continuous covariates at sample mean values specific to the "targeted" or "non-targeted" populations. (Setting age, for example, to the overall mean of 12.6 would seem inconsistent with both the targeted and non-targeted populations, leading us to set the covariates to group-specific means for this exercise). For the binary indicators, we round the targeted-/non-targeted-specific sample means to either zero or one.

Since these posterior predictive densities account for both the "structural" impacts of WIC participation as well as the role of unobserved confounding, we would expect these predictives to match, to a reasonable degree, the means found in the raw data. A comparison of the entries of Tables 3.2 and 3.4 shows that this is (approximately) the case - targeted members of WIC households and targeted members of non-WIC households have the highest levels of calcium intake with posterior means equal to 470 and 387 milligrams, respectively. Similarly, non-targeted WIC and non-WIC members have lower levels of adjusted calcium intake with posterior means equal to 192 and 183, respectively, which is also broadly consistent with the mean intake levels found in the raw data. Figure 3.2 and Table 3.4 also offer little evidence in favor of the potential "leakage" or "spillover" benefits associated with the WIC program; the posterior predictives for the non-targeted WIC (G_2) and non-targeted non-WIC (G_4) individuals are very similar and nearly indistinguishable in Figure 3.2. Finally, the posterior standard deviations of Table 3.4 and plots in Figure 3.2 also reveal considerable heterogeneity associated with the calcium intakes for each of these four groups, with targeted individuals associated with the highest levels of uncertainty.

While inspection of just the "structural" WIC coefficients in Table 3.3 would appear to suggest that targeted non-WIC individuals will have more calcium intake through milk than targeted WIC individuals, the posterior predictives tell a different story. Like the raw data, these posterior predictives reveal that targeted WIC individuals will have the highest levels of calcium intake through milk. What is responsible for this finding is the role of unobserved correlation - those families that select into WIC possess unobserved factors that also strongly correlate with calcium intake. This finding is broadly consistent with the idea that the families participating in WIC, holding all else constant, also take great care in the nutritional intakes of their children and thus would likely consume relatively high levels of calcium even in the absence of WIC. What we have offered in this paper, which to our knowledge is new to this literature, is a model that seeks to separate the influences of unobservables and direct "structural" impacts. When combining these influences, we generate predictions that are consistent with the raw data and the findings of past work on this topic. When separating them, we produce no direct evidence that WIC itself is responsible for increases in calcium intake and improved overall nutrition. Again, we must interpret this finding with care, as it is certainly possible that the WIC program leads individuals to substitute away from traditional consumption bundles and meet necessary nutritional requirements through other foods. If true, this result does not seem to have been documented in the literature and has important implications for designing efficient mechanisms for achieving desired nutrient intake levels.

Conclusion

In this paper we have described a Bayesian posterior simulator for fitting a two-equation treatment-response model and employed this method to investigate the effectiveness of a widely used food assistance program. This program, commonly denoted as WIC, seeks to improve the nutrition of at-risk low-income children and pregnant/breastfeeding mothers. We evaluate this program by focusing on calcium intake through milk consumption and comparing such intake levels across WIC and non-WIC households and individuals. Though this metric is, admittedly, rather narrow, we also recognize that adequate calcium intake is one of the primary focuses of the WIC program, and milk is a primary vehicle through which calcium is consumed.

Overall, we find little direct evidence that speaks to the efficacy of WIC. Instead, most of the benefits that might potentially be attributed to the program seem to arise from differences in unobservables across WIC and non-WIC families. Furthermore, we find little evidence associated with possible "spillover" or "leakage" benefits that have been suggested in the literature, as non-targeted members of WIC households have consumption patterns that are quite consistent with non-targeted members of non-WIC households. We must interpret our results with caution, however, as it remains possible that WIC benefits lead individuals to substitute away from milk and toward other goods that also provide adequate nutrition. To our knowledge, no studies in the area have attempted to separate the effects of unobservables and direct impacts, yet doing so has clearly been quite important in the context of our application.

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Figure 3.1 Distribution of Positive Calcium Intake

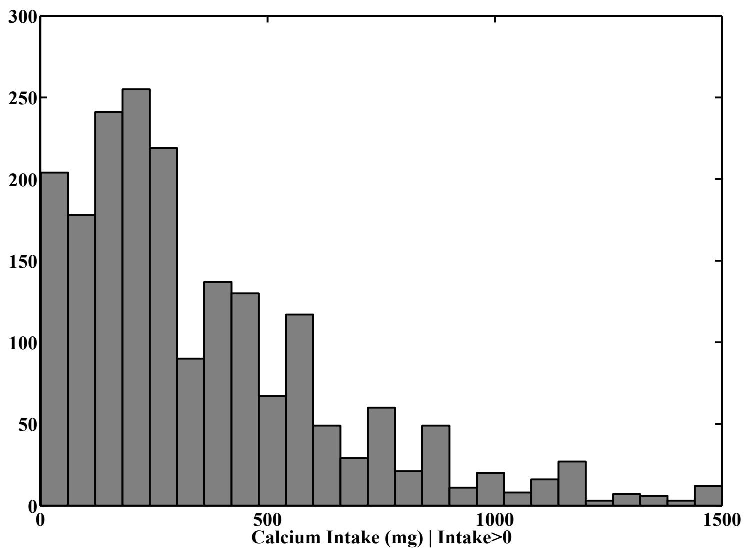

Figure 3.2 Predictive Posterior Intake Distributions for Four Groups

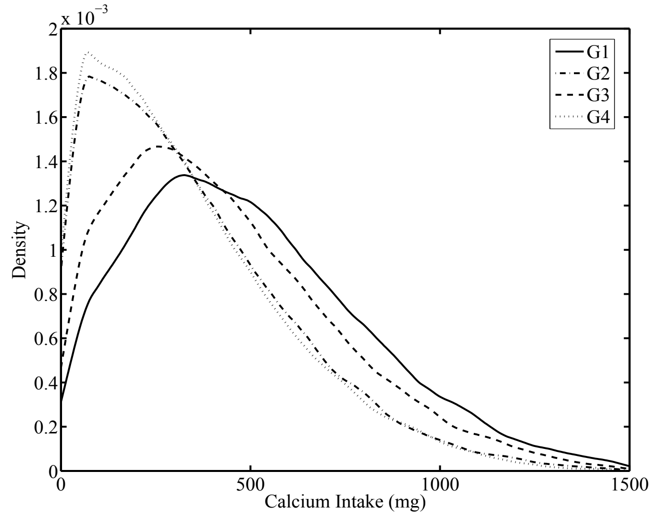

Table 3.1 Number of Individuals in Each Group by WIC Status

No. of Individuals	Group	WIC Status
526	G_1	Targeted individuals in WIC household
488	G_2	Non-targeted individuals in WIC household
712	G_3	Targeted individuals in non-WIC household
646	G_4	Non-targeted individuals in non-WIC household
2372		Total individuals

Table 3.2 Variables and Sample Mean Values

Variable	Sample	WIC Targeted	WIC Non-Targeted	Non-WIC Targeted	Non-WIC Non-Targeted
Individual					
Number of indiv.	2372	526	488	712	646
Milk/100g	3.17	4.68	1.49	4.42	1.82
Income/$1000	17.83	15.37	16.63	18.68	19.81
Household size	4.96	4.84	5.45	4.62	5.07
Food stamp indiv.	0.38	0.56	0.52	0.28	0.24
PregLactPost indiv.	0.03	0.07	0.00	0.05	0.00
Age	12.61	3.31	23.63	3.42	22.01
College	0.30	0.27	0.26	0.32	0.32
Urban	0.76	0.76	0.75	0.77	0.76
White	0.47	0.40	0.35	0.56	0.49
Male	0.49	0.48	0.53	0.48	0.49
Household					
Number of hhlds.	1036				
Income/$1000	17.19	15.07	15.21	18.69	18.72
Household size	4.56	4.64	4.85	4.48	4.71
Food stamp present	0.44	0.60	0.54	0.33	0.34
College	0.30	0.27	0.29	0.33	0.34
Children ages 1-5	0.94	0.98	0.76	0.99	0.98
Urban	0.76	0.75	0.73	0.78	0.78
White	0.60	0.54	0.51	0.64	0.61
Infant present	0.20	0.25	0.42	0.08	0.08
PregLactPost present	0.15	0.20	0.16	0.12	0.11
Self-report income	0.16	0.16	0.17	0.16	0.15
Savings less $5,000	0.94	0.98	0.95	0.91	0.91

Table 3.3 Posterior Means, Standard Deviations and Probabilities of Being Positive

Variable	E(·|y)	Std(·|y)	Pr(· > 0|y)
Consumption Equation			
Intercept	3.40	0.42	1.00
G_2	-1.37	0.24	0.00
G_3	1.21	0.37	1.00
G_4	0.50	0.39	0.90
Household size	0.07	0.06	0.94
Income/$1000	-0.02	0.01	0.06
Food stamp indiv.	-0.09	0.20	0.31
PregLactPost indiv.	-0.51	0.34	0.07
Age	-0.07	0.01	0.00
White	0.21	0.18	0.88
Male	-0.03	0.11	0.61
College	-0.10	0.20	0.29
Urban	0.50	0.22	0.99
Participation Equation			
Intercept	0.31	0.35	0.81
Household size	0.06	0.03	0.99
Income/$1000	-0.02	0.01	0.00
Infant present	0.89	0.11	1.00
Food stamp present	0.45	0.10	1.00
PregLactPost present	0.12	0.12	0.84
College	-0.03	0.09	0.37
Children ages 1-5	-0.95	0.22	0.00
White	-0.13	0.09	0.06
Urban	-0.15	0.10	0.07
Savings less $5,000	0.35	0.18	0.97
Self-report income	0.60	0.31	0.98
Covariance Matrix and Skew Parameters			
ρ	0.53	0.10	1.00
σ_ϵ^2	0.15	0.07	1.00
σ_u^2	3.45	0.43	1.00
ψ	4.55	0.10	1.00

Table 3.4 Posterior Predictive Statistics Associated with Calcium Intake for Four Groups

Group	$E(\cdot\|y)$	Std$(\cdot\|y)$
G_1	470	337.1
G_2	192	337.6
G_3	387	324.0
G_4	183	324.2

4. BAYESIAN ESTIMATION OF A CENSORED AIDS MODEL FOR WHOLE GRAIN PRODUCTS

Abstract

When using household-level data to examine consumer demand it is common to find that consumers purchase only a subset of the available goods, setting the demand for the remaining goods to zero. Ignoring such censoring of the dependent variables can lead to estimators with poor statistical properties and estimates that lead to poor policy decisions. In this paper we investigate household demand for four types of grain products using a censored Almost Ideal Demand System (AIDS) and estimate the parameters of the model via Bayesian methods. Using 2006 ACNielsen Homescan data we find that demand for all types of cereals is inelastic to changes in prices. The expenditure elasticity is slightly above unity for the whole grain ready-to-eat cereals suggesting that as the expenditure on cereals increases households will allocate proportionally more on whole-grain ready-to-eat cereals and less on other cereals.

Keywords: AIDS model, Bayesian econometrics, censored, cereals, whole grains

JEL Classification: C11; C34; D12

Introduction

The U.S. Department of Health and Human Services (USDHHS) and the U.S. Department of Agriculture (USDA) have published the Dietary Guidelines for Americans since 1980. The Guidelines provide dietary recommendations to aid the development of nutrition programs and to help and encourage consumers to choose diets that meet their nutritional needs and improve their health. The Guidelines are revised every 5 years based on findings from available research. The 2005 Dietary Guidelines for Americans put new emphasis on whole grain consumption by recommending consumption of at least three 1-ounce-equivalent [1] servings of whole grains [2] per day. In the Guidelines, whole grains are described as follows: "Whole grains, as well as foods made from them, consist of the entire grain seed, usually called the kernel. The kernel is made of three components - the bran, the germ and the endosperm. If the kernel has been cracked, crushed, or flaked, then it must retain nearly the same relative proportion of bran, germ, and endosperm as the original grain in order to be called whole grain" (US DHHS and USDA 2005). Consumption of diets high in whole grains have been reported to have a number of beneficial health effects including reduced risk of cancer (Jacobs, et al. 1998), cardiovascular disease (Truswell, 2002; Liu et al. 1999), diabetes (Fung et al. 2002; Liu et al. 2000), blood pressure (Hallfrisch et al. 2003) and cholesterol (Lumpton, et al. 1994).

The U.S. Food and Drug Administration (FDA), which regulates U.S. nutrition labeling of most foods and authorizes the use of nutrient and health claims, has allowed three health claims related to grain intakes (FDA, 2008). A specific claim for whole grain foods allows the statement that diets rich in whole grain foods and other plant foods and low in total fat, saturated fat, and cholesterol may reduce the risk of heart

[1] In general, 1-ounce slice of bread; one cup of ready-to-eat cereal, or $\frac{1}{2}$ cup of cooked rice, cooked pasta, or cooked cereal can be considered as one-ounce-equivalent from the grains group (http://www.mypyramid.gov).

[2] see Table 4.1 for the list of whole grains

disease and some cancer. The release of the 2005 Dietary Guidelines and FDA's consideration of health-related claims gave whole grain product manufacturers the opportunity to differentiate their products from refined grain products and the incentive to produce more whole grain products or reformulate the existing products to meet the whole grain requirements. While FDA has no mandatory labeling requirements regarding whole grains, manufacturers can use nutrient labels such as "100 percent whole grain" or "10 grams of whole grain" on the label of their products as long as the statements are not false or misleading (FDA, 2008).

Mandatory labeling provides greater information and therefore more informed consumer choices. However, in the absence of mandatory labeling it is common for third-party labeling service to emerge. In the case of grain products, the Whole Grain Council (WGC), a nonprofit organization, promotes consumption of whole grains through a packaging symbol, a Whole Grain Stamp [3], indicating whole grain content. The Stamp serves as a tool to help consumers easily indentify whole grain products.

Although the lack of clear labeling makes it more difficult for consumers to identify whole grain food products, the availability and consumption of whole grain products are likely to increase (Buzby, Farah and Volke 2005). Policymakers use recommendations from the 2005 Dietary Guidelines in the development of food program guidance. One example is the recently revised food packages for the Supplemental Nutrition Program for Women, Infants and Children (WIC), which include provisions to allow participants to obtain whole grain products effective in 2009.

There are relatively few recent studies of grain consumption. Evidence from food intake surveys indicates that Americans consume less whole grain than recommended. On average, individuals were eating 10 servings of grains a day in 2003, more that

[3]Two types of stamps can be awarded, based on the product ingredients and amount of whole grains in the food. Products must contain at least 8 grams of whole grain per labeled serving in order to use the basic Stamp and at least 16 grams of whole grain and where all grains are whole grain to the 100 percent Whole Grain Stamp

recommended daily allowance, of which whole grain accounted for just over 1 serving (Mancino and Buzby 2005). Similar results were found by Lin and Yen (2007). Using data from 1994-96 and 1998 Lin and Yen compared grain consumption of individuals by economic and demographic characteristics and found that individuals consumed more than the recommended daily amount of all grain, while consuming only 34 percent of the amount of whole grain recommended by the 2005 Dietary Guidelines. Analysis of 1999-2000 National Health and Nutrition Examination survey (NHANES) data shows only 15 % of all grains consumed by individuals are whole grain, and most whole grains come from crackers and snacks and from cereals. More specifically, whole grain crackers and snacks account for 5 % of the total grains consumed by individuals, where as ready-to-eat cereals account for 3 % (Mancino and Buzby 2005).

Given the public health interest in increased consumption of whole grains, it is important to have a good understanding of basic demand parameters for grain and cereal products. We consider demand for cereals, one of the major sources of whole grains in the diet, and estimation based on household level data.

When using household-level data to examine consumer food demand, it is common to find that consumers choose only a subset of the available goods, leaving observed demand for some of the goods to be zero. Ignoring such censoring of the dependent variables can lead to estimators with poor statistical properties and estimates that lead to poor policy decisions. Hence, we carefully address the issue of censoring in a demand system framework. There exist a number of estimation procedures that handle this censoring problem (Wales and Woodland 1983; Lee and Pitt 1986). Although theoretically consistent, these approaches suffer from the drawback that in the case of many non-consumed goods for some households, evaluation of multiple integrals is necessary. An alternative approach is an Amemiya-Tobin approach, which is the generalization of Tobin's (1958) limited dependent variable model proposed by Amemiya (1974) and implemented by Wales and Woodland (1983). However, the use of Amemiya-Tobin type estimators is also complicated by the need for evaluating multiple integrals in cases where censoring is severe. Due to the complexity of estimating the models above, a two-step procedure based on the Amemiya-Tobin approach has sometimes been used to estimate censored demand systems (Shonkwiler and Yen (1999)). This method has been widely used in the applied literature. Although the two-step procedure holds an advantage in its ability to estimate large systems, the two-step procedures are known to be inefficient and overlook the adding-up condition of the observed shares.

A number of papers have used variations of the Amemiya-Tobin approach to deal with the issues of censoring in food demand (e.g. Yen and Roe (1989), Perali and Chavas (2000), Golan, Perloff and Shen (2001), Yen, Kan and Su (2002) and Yen (2005)). Advances in simulation methods that allow approximations of high-dimensional integrals have been used in the estimation of the censored demand system (Yen, Lin and Smallwood (2003), Dong, Gould and Kaiser (2004)).

We propose a Bayesian procedure for estimating the censored demand system using the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980). Estimating an AIDS model with a Bayesian approach avoids the need to evaluate the multiple probability integrals. The marginal distribution of model parameters and latent shares are simulated by numerical methods. Specifically, we fit the model using the Gibbs sampler. The method developed is used to examine the demand for different types of breakfast cereals. We use data from 2006 ACNielsen Homescan household level scanner data files.

The estimation focuses on cereal (whole grain and other, ready-to-eat and hot) products which form a product group widely consumed in the United States. Lin and Yen (2007) found that breakfast was a good source of whole grain. Individuals consumed 40 percent of whole grain at breakfast, compared with 23 percent at lunch and 17 percent at dinner and the rest provided by snack foods. Although scanner data provide information on foods purchased only for at home consumption, cereals are generally purchased in retail food stores, and in case of the breakfast cereals, generally consumed at home. Hence, scanner data are well suited for estimating demand relationships for this product group.

In estimating the demand system for cereal products we assume that demand for cereal is separable from the demand for other goods in the consumer budget. In a multistage budgeting framework, it is usually assumed that consumers first allocate their expenditures to broad aggregate commodity groups. Subsequently, consumer's decisions are based on group expenditures and commodity prices within each group. Hence, by weak separability we focus on a demand structure in which cereal expenditures are allocated to various types of cereals.

The paper is organized as follows. The next section describes the AIDS model and the associated Bayesian posterior simulator. Then data used in the analysis are described, followed by a description of empirical results. The paper concludes with a summary of the findings and the directions for the future research.

AIDS Model and Posterior Simulator

The Model

The AIDS model of Deaton and Muellbauer (1980) can be expressed in the latent expenditure share form as: [4]

$$s^*_{ih} = \alpha_i + \boldsymbol{z_{ih}\delta_i} + \sum_{j=1}^{n} \gamma_{ij} ln\,(p_{jh}) + \beta_i ln\,(y_h/P_h) + \epsilon_{ih}, \quad i,j = 1,...,n, \quad h = 1,..,H \quad (4.1)$$

and

$$s_{ih} = \begin{cases} s^*_{ih} & if \quad s^*_{ih} > 0 \\ 0 & if \quad s^*_{ih} \leq 0 \end{cases} \quad (4.2)$$

[4]Matrices and vectors are denoted by bold letters.

where, s^*_{ih} and s_{ih} are the latent and observed expenditure shares, respectively, for good i of household h, p_{jh} is the price of the jth good, $\mathbf{z_{ih}}$ is a set of household specific characteristics, y_h represents total expenditure of household h on all n goods and P_h is a price index defined as:

$$lnP_h = \alpha_0 + \sum_{i=1}^{n} \alpha_i ln(p_{ih}) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} ln(p_{ih}) ln(p_{jh}). \tag{4.3}$$

In this paper we use the Stone price index $lnP_h = \sum_i s_{ih} ln(p_{ih})$ instead of nonlinear price index given above and estimate the so-called LA/AIDS (Linear Approximate AIDS) model.

The theoretical properties of the demand function given by equation (4.1) can be imposed by the following equality restrictions on the parameters [5]:

adding-up: $\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = \sum_i \beta_i = \sum_i \boldsymbol{\delta_i} = 0;$

homogeneity: $\sum_j \gamma_{ij} = 0$ and

symmetry: $\gamma_{ij} = \gamma_{ji}, \quad i \neq j, \quad i, j = 1, ..., n.$

For each household h stacking (4.1) over $i = 1, ...n$ we obtain:

$$\mathbf{s_h^*} = \boldsymbol{\alpha} + \mathbf{Z_h}\boldsymbol{\delta} + \mathbf{ln(p_h)}\boldsymbol{\gamma} + \boldsymbol{\beta}\mathbf{ln(y_h/P_h)} + \boldsymbol{\epsilon_h}, \tag{4.4}$$

where

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}, \quad \boldsymbol{\delta} = \begin{bmatrix} \boldsymbol{\delta_1} \\ \boldsymbol{\delta_2} \\ \vdots \\ \boldsymbol{\delta_n} \end{bmatrix}, \quad \boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{\gamma_1} \\ \boldsymbol{\gamma_2} \\ \vdots \\ \boldsymbol{\gamma_n} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix},$$

$$\mathbf{s_h^*} = \begin{bmatrix} s^*_{1h} \\ s^*_{2h} \\ \vdots \\ s^*_{nh} \end{bmatrix}, \quad \mathbf{Z_h} = \begin{bmatrix} \boldsymbol{z_{1h}} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{z_{2h}} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{z_{nh}} \end{bmatrix},$$

[5] Here, we are not imposing the adding up to unity restriction, $\sum_i s^*_{ih} = 1$, on the latent shares.

$$\mathbf{ln(p_h)} = \begin{bmatrix} \boldsymbol{ln(p_{1h})} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{ln(p_{2h})} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \boldsymbol{ln(p_{nh})} \end{bmatrix},$$

and

$$\mathbf{ln(y_h/P_h)} = ln(y_n/P_h)\mathbf{i_n},$$

We can rewrite (4.4) as:

$$\mathbf{s_h^*} = \mathbf{X_h}\boldsymbol{\theta} + \boldsymbol{\epsilon_h}, \tag{4.5}$$

where $\mathbf{X_h} = [\mathbf{I}\ \mathbf{Z_h}\ \mathbf{ln(p_h)}\ \mathbf{ln(y_h/P_h)}]$ is the $n \times k$ matrix of stacked covariate data, $k = \sum_{i=1}^{n} k_i$, k_i denotes the number of explanatory variables, $\boldsymbol{\theta} = [\boldsymbol{\alpha'}\ \boldsymbol{\delta'}\ \boldsymbol{\gamma'}\ \boldsymbol{\beta'}]'$ is $k \times 1$ vector and $\boldsymbol{\epsilon_h} \overset{iid}{\sim} N(\mathbf{0}, \boldsymbol{\Sigma})$ where Σ is $n \times n$.

Stacking (4.5) over $h = 1, .., H$ we obtain:

$$\mathbf{s}^* = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\epsilon} \tag{4.6}$$

where $\mathbf{X}$ is $nH \times k$, $\boldsymbol{\epsilon} \overset{iid}{\sim} N(\mathbf{0}, \boldsymbol{\Omega})$ and $\boldsymbol{\Omega}$ is $\mathbf{I_H} \otimes \boldsymbol{\Sigma}$ matrix.

The AIDS model specified above is a Seemingly Unrelated Regression (SUR) model proposed by Zellner (1962) on the latent data $\mathbf{s}^*$, with the same regressors in each equation. Since the expenditure shares are censored we follow Huang et al.(1987) and estimate a SUR Tobit model.

To impose the parameter restrictions in the estimation of (4.6) we follow the method specified in Griffiths, O'Donnell and Tan Cruz (2000). Let J, where $J < k$, be the number of equality restrictions imposed on the parameters of the model, then

$$\mathbf{R}\boldsymbol{\theta} = \mathbf{r}, \tag{4.7}$$

where $\mathbf{R}$ is $J \times k$ and $\mathbf{r}$ is $J \times 1$.

As an example, suppose we want to estimate the following two equation system:

$$s_1^* = \alpha_1 + \gamma_{11}x_{11} + \gamma_{12}x_{12} + \epsilon_1$$

$$s_2^* = \alpha_2 + \gamma_{21}x_{21} + \gamma_{22}x_{22} + \epsilon_2$$

and the linear restrictions that we want to impose are

$$\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = 0 \quad \text{and} \quad \gamma_{12} = \gamma_{21}.$$

Then $\mathbf{R}\boldsymbol{\theta} = \mathbf{r}$ in this case will be:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \gamma_{11} \\ \gamma_{12} \\ \alpha_2 \\ \gamma_{21} \\ \gamma_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

These restrictions imply that some of the parameters of the model are redundant and can be recovered from the estimated parameters and imposed parameter restrictions. We will rearrange the elements of $\boldsymbol{\theta}$ and partition it into vectors of redundant and free parameters, denoted $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$, respectively, where $\boldsymbol{\theta_1}$ is $J \times 1$ and $\boldsymbol{\theta_2}$ is $(k - J) \times 1$. Accordingly, we partition $\mathbf{X}$ by reordering its columns so that equations (4.6) and (4.7) can be written as:

$$\mathbf{s}^* = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\epsilon} = \begin{bmatrix} \mathbf{X_1} & \mathbf{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta_1} \\ \boldsymbol{\theta_2} \end{bmatrix} + \boldsymbol{\epsilon}, \tag{4.8}$$

and

$$\mathbf{R}\boldsymbol{\theta} = \begin{bmatrix} \mathbf{R_1} & \mathbf{R_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta_1} \\ \boldsymbol{\theta_2} \end{bmatrix} = \mathbf{r}, \tag{4.9}$$

where $\mathbf{X_1}$ and $\mathbf{X_2}$ are $nH \times J$ and $nH \times (k-J)$ submatrices of $\mathbf{X}$, respectively, $\mathbf{R_1}$ is $J \times J$, $\mathbf{R_2}$ is $J \times (k-J)$ and $\mathrm{rank}(\mathbf{R_1}) = J$. In this notation the covariate matrix is no longer block-diagonal. For the example mentioned above

$$\boldsymbol{\theta_1} = \begin{bmatrix} \gamma_{12} \\ \alpha_2 \\ \gamma_{21} \\ \gamma_{22} \end{bmatrix}, \quad \boldsymbol{\theta_2} = \begin{bmatrix} \alpha_1 \\ \gamma_{11} \end{bmatrix}, \quad \mathbf{R_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}, \quad \mathbf{R_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{X_1} = \begin{bmatrix} x_{12} & 0 & 0 & 0 \\ 0 & 1 & x_{21} & x_{22} \end{bmatrix} \quad \text{and} \quad \mathbf{X_2} = \begin{bmatrix} 1 & x_{11} \\ 0 & 0 \end{bmatrix}.$$

As mentioned earlier, we only need to estimate $\boldsymbol{\theta_2}$, since $\boldsymbol{\theta_1}$ is redundant and can be recovered from $\boldsymbol{\theta_2}$ and imposed restrictions. Solving for $\boldsymbol{\theta_1}$ from (4.9) we get:

$$\boldsymbol{\theta_1} = \mathbf{R_1^{-1}}(\mathbf{r} - \mathbf{R_2}\boldsymbol{\theta_2}). \tag{4.10}$$

By substituting $\boldsymbol{\theta_1}$ into (4.8) and rearranging terms we get

$$\tilde{\mathbf{s}}^* = \tilde{\mathbf{X}}\boldsymbol{\theta_2} + \boldsymbol{\epsilon} \tag{4.11}$$

where $\tilde{\mathbf{s}}^* = \mathbf{s}^* - \mathbf{X_1}\mathbf{R_1^{-1}}\mathbf{r}$ and $\tilde{\mathbf{X}} = \mathbf{X_2} - \mathbf{X_1}\mathbf{R_1^{-1}}\mathbf{R_2}$. Thus, (4.11) is a latent variable SUR model with no restrictions on $\boldsymbol{\theta_2}$.

The Augmented Posterior

For computational simplicity, we follow Albert and Chib (1993) and treat the latent data $\tilde{\mathbf{s}}^*$ as additional parameters of the model. The augmented posterior $p(\tilde{\mathbf{s}}^*, \boldsymbol{\theta_2}, \boldsymbol{\Sigma}|\mathbf{s})$ is then proportional to

$$
\begin{aligned}
p(\tilde{\mathbf{s}}^*, \boldsymbol{\theta_2}, \boldsymbol{\Sigma}|\mathbf{s}) &\propto p(\mathbf{s}|\tilde{\mathbf{s}}^*, \boldsymbol{\theta_2}, \boldsymbol{\Sigma})p(\tilde{\mathbf{s}}^*|\boldsymbol{\theta_2}, \boldsymbol{\Sigma})p(\boldsymbol{\theta_2}, \boldsymbol{\Sigma}) && (4.12)\\
&\propto p(\boldsymbol{\theta_2}, \boldsymbol{\Sigma})\left(\prod_{h=1}^{H} p(\mathbf{s_h}|\tilde{\mathbf{s}}^*_{\mathbf{h}})p(\tilde{\mathbf{s}}^*_{\mathbf{h}}|\boldsymbol{\theta_2}, \boldsymbol{\Sigma})\right) && (4.13)\\
&\propto p(\boldsymbol{\theta_2}, \boldsymbol{\Sigma})\left[\prod_{h=1}^{H} p(\tilde{\mathbf{s}}^*_{\mathbf{h}}|\boldsymbol{\theta_2}, \boldsymbol{\Sigma})\left(\prod_{i=1}^{n} p(s_{ih}|\tilde{s}^*_{ih})\right)\right], && (4.14)
\end{aligned}
$$

where

$$p(s_{ih}|\tilde{s}^*_{ih}) = I(s_{ih} = \tilde{s}^*_{ih})I(\tilde{s}^*_{ih} > c_h) + I(s_{ih} = 0)I(\tilde{s}^*_{ih} \leq c_h),$$

and c_h is the h^{th} element of $-\mathbf{X_1R_1^{-1}r}$.

From (4.4), the sampling density of the latent data, $\tilde{\mathbf{s}}^*$, is given as:

$$p(\tilde{\mathbf{s}}^*_{\mathbf{h}}|\boldsymbol{\theta_2}, \boldsymbol{\Sigma}) \propto |\boldsymbol{\Sigma}|^{-\frac{H}{2}} \exp\left(-\frac{1}{2}\sum_{h=1}^{H}(\tilde{\mathbf{s}}^*_{\mathbf{h}} - \tilde{\mathbf{X}}_{\mathbf{h}}\boldsymbol{\theta_2})'\boldsymbol{\Sigma^{-1}}\sum_{h=1}^{H}(\tilde{\mathbf{s}}^*_{\mathbf{h}} - \tilde{\mathbf{X}}_{\mathbf{h}}\boldsymbol{\theta_2})\right) \tag{4.15}$$

To implement a Bayesian analysis, we must introduce the priors. We assume that the priors are independent and of the conditionally conjugate forms:

$$
\begin{aligned}
\boldsymbol{\theta_2} &\sim N(\mu_{\boldsymbol{\theta_2}}, \boldsymbol{V_{\theta_2}}) && (4.16)\\
\boldsymbol{\Sigma}^{-1} &\sim W(\underline{\mathbf{A}}, \underline{\nu}), && (4.17)
\end{aligned}
$$

where W denotes a Wishart distribution (Koop, Poirier and Tobias, 2007, pg. 339).

The Posterior Simulator

In this section we introduce our posterior simulator for fitting the demand model given by (4.11) together with the priors in (4.16)-(4.17). We use the Gibbs sampling algorithm to iteratively draw values from the posterior distribution of each parameter conditional on other parameters of the model. Those posterior conditionals are enumerated below.

Step 1: $\boldsymbol{\theta_2}|\mathbf{s}, \boldsymbol{\Sigma}$

$$\boldsymbol{\theta_2}|\mathbf{s}, \boldsymbol{\Sigma} \sim N(\mathbf{D_{\theta_2}d_{\theta_2}}, \mathbf{D_{\theta_2}}), \tag{4.18}$$

where

$$\mathbf{D}_{\boldsymbol{\theta_2}} = \left(\tilde{\mathbf{X}}'(\boldsymbol{\Sigma^{-1}} \otimes \mathbf{I_H})\tilde{\mathbf{X}} + V_{\boldsymbol{\theta_2}}^{-1}\right)^{-1}$$

$$\mathbf{d}_{\boldsymbol{\theta_2}} = \left(\tilde{\mathbf{X}}'(\boldsymbol{\Sigma^{-1}} \otimes \mathbf{I_H})\tilde{\mathbf{s}^*} + \mathbf{V}_{\boldsymbol{\theta_2}}^{-1}\boldsymbol{\mu}_{\boldsymbol{\theta_2}}\right)$$

<u>**Step 2:**</u> $\boldsymbol{\Sigma^{-1}}|\boldsymbol{\theta_2}, \mathbf{s}$

$$\boldsymbol{\Sigma^{-1}}|\boldsymbol{\theta_2}, \mathbf{s} \sim W\left(\overline{\mathbf{A}}, \overline{\nu}\right) \tag{4.19}$$

where

$$\overline{\nu} = H + \underline{\nu}$$

and

$$\overline{\mathbf{A}} = \left[\underline{\mathbf{A}}^{-1} + \sum_{h=1}^{H}\left(\tilde{\mathbf{s}}_{\mathbf{h}}^{*} - \tilde{\mathbf{X}}_{\mathbf{h}}\boldsymbol{\theta_2}\right)\left(\tilde{\mathbf{s}}_{\mathbf{h}}^{*} - \tilde{\mathbf{X}}_{\mathbf{h}}\boldsymbol{\theta_2}\right)'\right]^{-1}$$

<u>**Step 3:**</u> $\tilde{s}_{ih}^{*}|\mathbf{s}, \boldsymbol{\theta_2}, \boldsymbol{\Sigma}$

From (4.14) the posterior conditional of $\tilde{\mathbf{s}}_{\mathbf{h}}$ is multivariate truncated normal. We therefore follow Geweke (1991) and draw each latent, $\tilde{s}_{ih}^{*}$ from a univariate truncated normal density.

Let ω_{ij} denote the (i, j) element of Σ^{-1} and c_h be the h^{th} element of $-\mathbf{X_1}\mathbf{R_1^{-1}}\mathbf{r}$ as defined before. For each household h we can idependently sample each of the n goods, $i = 1, ..., n$ as follows [6]:

$$\tilde{s}_{ih}^{*}|\mathbf{s}, \boldsymbol{\theta_2}, \boldsymbol{\Sigma} \sim TN_{(-\infty, c_h)}\left(\mu_{i|-i}, \omega_{ii}^{-1}\right), \quad \text{if } \tilde{s}_{ih} = 0, \tag{4.20}$$

where

$$\mu_{i|-i} = \mu_i - \omega_{ii}^{-1}\sum_{i \neq j}\omega_{ji}(\tilde{\mathbf{s}}_{-i}^{*} - \boldsymbol{\mu_{-i}})$$

then repeat for $h = 1, 2, \ldots, H$.

[6] The way the dependent variables are specified in our model it is possible that the observed shares are clustered both at zero and at one. Accounting for the two-sided censoring in the specification of the model is appropriate. However, only 5%, 4%, 3% and 2% of observed shares in our data are clustered at one. Hence, in this analysis we consider only the case when the observed shares are clusterd at zero.

In the above, $TN_{(a,b)}(\mu, \sigma^2)$ denotes a normal density with mean μ and variance σ^2 truncated to the interval (a, b), μ_i is the i^{th} row element of μ, μ_{-i} denotes all the elements of μ other than μ_i.

The posterior simulator involves iteratively drawing from (4.18)-(4.20).

A Generated Data Experiment

In this section we conduct a generated data experiment to demonstrate the performance of our posterior simulator. A sample of 10,000 households is generated from the following demand model:

$$\begin{aligned}
s^*_{1h} &= \alpha_1 + \gamma_{11} ln(p_{1h}) + \gamma_{12} ln(p_{2h}) + \gamma_{13} ln(p_{3h}) + \gamma_{14} ln(p_{4h}) + \beta_1 ln(y_h/P_h) + \epsilon_{1h} \\
s^*_{2h} &= \alpha_2 + \gamma_{21} ln(p_{1h}) + \gamma_{22} ln(p_{2h}) + \gamma_{13} ln(p_{3h}) + \gamma_{24} ln(p_{4h}) + \beta_2 ln(y_h/P_h) + \epsilon_{2h} \\
s^*_{3h} &= \alpha_3 + \gamma_{31} ln(p_{1h}) + \gamma_{32} ln(p_{2h}) + \gamma_{33} ln(p_{3h}) + \gamma_{34} ln(p_{4h}) + \beta_3 ln(y_h/P_h) + \epsilon_{3h} \\
s^*_{4h} &= \alpha_4 + \gamma_{41} ln(p_{1h}) + \gamma_{42} ln(p_{2h}) + \gamma_{43} ln(p_{3h}) + \gamma_{44} ln(p_{4h}) + \beta_4 ln(y_h/P_h) + \epsilon_{4h}
\end{aligned}$$

where $ln(p_{ih})$ and $ln(y_h/P_h)$ are drawn independently from a $N(0, 1)$ and the error terms $[\epsilon_{1h} \quad \epsilon_{2h} \quad \epsilon_{3h} \quad \epsilon_{4h}]'$ are drawn jointly from the multivariate Normal distribution:

$$\begin{bmatrix} \epsilon_{1h} \\ \epsilon_{2h} \\ \epsilon_{3h} \\ \epsilon_{4h} \end{bmatrix} \overset{iid}{\sim} N \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} .5 & -.45\sqrt{.5}\sqrt{.3} & .5\sqrt{.5}\sqrt{.1} & -.35\sqrt{.5}\sqrt{.6} \\ -.45\sqrt{.5}\sqrt{.3} & .3 & -.2\sqrt{.3}\sqrt{.1} & .4\sqrt{.3}\sqrt{.6} \\ .5\sqrt{.5}\sqrt{.3} & -.2\sqrt{.3}\sqrt{.1} & .1 & -.5\sqrt{.1}\sqrt{.6} \\ -.35\sqrt{.5}\sqrt{.6} & .4\sqrt{.3}\sqrt{.6} & -.5\sqrt{.1}\sqrt{.6} & .6 \end{pmatrix} \right]$$

Some of the variables in our actual data have high degree of censoring. To imitate the actual data as close as possible we generate the data with 30 %, 21 %, 56 % and 70 % of censoring.

We fit our model using the posterior simulator described in previous section, ran the algorithm 100,000 iterations, and discarded the first 30,000 draws as the burn-in period.

Table 4.2 and Figures 4.1 and 4.2 summarize the results of the generated data experiment. We plot the lagged autocorrelations up to order 8 for several selected parameters: γ_{14}, α_2, γ_{33}, γ_{41}, ρ_{12}, ρ_{24}, σ_1^2 σ_4^2, and ρ_{13}. From the plots we can see that the Gibbs sampler displays good mixing of the parameters.

In Table 4.2 we report the estimates of the posterior means, standard deviations and probabilities of being positive from the generated data along with their true values. As we can see from the table, all the parameters have been estimated with reasonable accuracy and the estimated results are quite close to their true values.

The Data

Household Data

We use data from the ACNielsen 2006 Homescan survey of households. The data come from a nationally representative sample of U.S. households that scan their purchased foods at home after each shopping occasion using a scanning device and report the results to the collection firm once a week. The dataset includes product modules of dairy department purchase data, dry grocery department purchase data, produce, meat and frozen departments purchase data and a module for random-weight purchase data for the year of 2006. Each product module and the random-weight data includes product codes that identify brand, size, flavor, form, formula, container, style, type and variety. Each food item was represented by a unique UPC or product number. The data also contain information on purchase date, quantity purchased, total expenditures on the item, whether the price was paid with a deal or not and the coupon value used if any.

The 2006 Homescan data include information from over 37,000 households, although only 7,534 households reported purchases of both random-weight and UPC coded food items. Of these, 7,415 households reported purchases for at least 10 months in 2006.

Our final sample comes from the household panel and consists of 7,081 households that had expenditures on ready-to-eat and hot cereals at some time during the year.

We matched the household purchases with the household demographic data. The household characteristics include household size, income, age of household head, education and employment of female and male heads, marital status, race, presence of children and region of residence.

Whole Grains Identification

We constructed a dataset for purchases of four cereal types: whole grain ready-to-eat, non-whole grain ready-to-eat, whole grain hot, and non-whole grain hot cereals. Although the 2005 Dietary Guidelines recommend that Americans eat three or more one-ounce-equivalent servings of whole grains per day, the government offers no straightforward way for consumers to identify whole grain products, and guidance to the industry on labeling is still not mandated by the Food and Drug Administration. Manufacturers have begun to label their products on whole grain content and the Whole Grains Council provides an approved stamp to indicate products that are good sources of whole grain. ACNielsen provided information on the grain type of some products reported in the HomeScan files. We used these three sources to identify cereals as whole grain and non-whole grain: the Whole Grains Council listing; manufacturers' sites; and the ACNielsen indicator of grain content.

Where information on whole grain content was lacking from the Whole Grain Council, we verified manufacturers' websites and specifically checked if the product was claimed as a whole grain or contained whole grain as a first ingredient. In most cases we were able to identify whole grain products. For example, all General Mills ready-to-eat cereals carry a whole grain claim and listed whole grain as a first ingredient. Many websites had information on ingredients. In some cases, when we were not able to find a manufacturer's whole grain claim, we identified cereals as whole grain if the first ingredient listed was whole grain. Again we found some discrepancies in whole grain coding, but resolved them based on evidence from similar products.

Table 4.3 shows the total number of UPC's by cereal type in our data set and number and percent of cereals identified as whole grain from the three sources: scanner data "grain type" variable, Whole Grain Council and manufacturer's claim. As indicated in the table, we considered 3810 unique UPC product types; most were in the ready-to-eat cereal category. Included in the data were UPC codes for a large number of private label cereals. Private labels represent 61%, and 68% of total UPC's of ready-to-eat cereals and hot cereals, respectively. Without a manufacturer site, we needed to assign these products to whole grain and non-whole grain product groups. We developed two approaches to classification. In the first, we coded cereals as whole grain if they (a) carried the Whole Grain Council stamp or (b) were identified as a whole grain product by the manufacturer. The remaining products were coded as non-whole grain. In the second approach, we coded products as whole grain if they (a) carried a Whole Grain Stamp, or (b) were identified as a whole grain product by the manufacturer, and the remaining products, including the private labels, were assigned to whole grain if the majority of the observations in the grain type variable were identified as whole grain. That is, if the private label hot cereal indicated the grain type was "rolled oats", then the private label hot cereal was classified as "whole grain".

The two resulting classifications are shown in Table 4.4. As we can see, there are substantial differences in the number of whole grain UPC's identified by the two classifications. From the total of 2,850 different UPCs available for ready-to-eat cereals, only 18 % is identified as whole grain by classification 1 and almost double of this amount is identified as whole grain by classification 2. With respect to hot cereals, 91% of all UPCs available are identified as whole grain by classification 2, compared to only 22% by classification 1. Compared to classification 1, which assignes all private labels to non-whole grain group, classsification 2 seems more reasonable. Although some concerns may be raised regarding the sensitivity of the analysis to the classifications used, it is clear that estimating a demand system using classification 1 can could lead to unreliable results.

Variables and Descriptive Statistics

The data include repeated expenditures and quantities for each purchased item. The price of each commodity was calculated as the unit value, defined as the aggregated household expenditure for the product divided by quantity purchased in ounces (reported for the year). The household's expenditure was calculated by subtracting the value of any coupons used during the purchase from the amount paid. We also calculated average regional prices. The dataset provides information on 52 Scantrack markets and rural areas. We derived average prices for all four commodities by 52 Scantrack markets and rural areas. For households not purchasing a particular product, we replaced missing prices with the average prices (unit values) based on prices paid by the purchasing households for the household's corresponding market area.

Table 4.5 presents purchase frequencies, mean expenditure shares, mean expenditures, quantities purchased and unit values for the purchasing households for the commodities used in the analysis. Whole grain ready-to-eat cereal was consumed by the majority of the households and also had the highest mean expenditure and expenditure share among different types of cereals. Ready-to-eat non-whole grain cereal was next most frequently purchased by the households in our sample.

Table 4.6 presents the definitions of the variables used in the analysis along with the means and standard deviations of the variables for the whole sample. The average household income was \$59,270. The average household size was 2.34, 23 percent of the sample were households with children, and 59 percent were married couple households. For the analysis reported in this paper, the estimates were unweighted.

Table 4.7 presents the means and standard deviations for the variables used in the model for the four commodities. As indicated in Table 4.7, not much difference exists among the mean values of the variables across product categories, except for some variables of households purchasing non-whole grain hot cereals. These households were more likely to have lower income, be over the age of 65 and be married compared to the other three groups.

Empirical Results

A system of four equations was estimated using data based on classification 2 in Table 4.4 (the classification that assigns whole grain values to private label items). We fit our model using the algorithm specified in previous section. We ran our posterior simulator for 100,000 iterations and discarded the first 30,000 as the burn-in. For our prior hyperparameters, we set $\mu_{\boldsymbol{\theta_2}}$ equal to a zero vector of the dimension $(k - J) \times 1$, $\boldsymbol{V}_{\boldsymbol{\theta_2}}$ and $\underline{\mathbf{A}}$ to identity matrices of the appropriate dimensions and $\underline{\nu} = 5$.

Tables 4.8 and 4.9 present the posterior means, posterior standard deviations and probabilities of being positive for the demographic, price and expenditure related parameters for whole grain and non-whole grain ready-to-eat and hot cereals, respectively. We find that larger households are less likely to consume either type of whole-grain cereals and more likely to consume non-whole grain cereals, both ready-to-eat and hot. Households with higher income tend to consume more whole grain ready-to-eat and non-whole grain hot cereals and less non-whole grain ready-to-eat and whole grain hot cereals. Households with children present tend to consume both types of ready-to-eat cereals and are less likely to consume both types of hot cereals. There are some race/ethnic differences. Ready-to-eat cereal is a prevalent food in the diets of Americans, especially children (Song and et al. 2006).

Estimated parameters were used to calculate price and cereal expenditure elasticities in order to examine the responsiveness of the consumers to economic incentives (Table 4.10). The uncompensated and compensated own-price elastiticities are all negative, as expected for normal goods for which demand responds negatively to increases in prices. The uncompensated (Marshallian) price elasticities include an income effect as well as price effect.

The values of uncompensated own-price elasticities range from -0.89 for non-whole grain hot cereals to -0.44 for non-whole grain ready-to-eat. All are price inelastic, with largest (absolute) values being for whole grain cereals.

The mean unit prices for hot cereals reported in Table 4.5, especially for non-whole grain, were relatively smaller compared to mean unit prices for both ready-to-eat cereals and whole-grain hot cereals. Most of the (Hicksian) cross-price elasticities are positive indicating substitutability among the cereal types. Results indicate that most of the cross-price elasticities are small; the largest one is between the ready-to-eat and hot whole grain cereals. Relatively lower values (in absolute terms) for the cross-price effects indicate that consumers are more responsive to own-price rather than prices of the other goods.

The total expenditure elasticities do not vary widely in the magnitude. The total expenditure elasticity is slightly above unity for the whole grain ready-to-eat cereals suggesting that as the expenditure on cereals increases households will allocate proportionally more on whole-grain ready-to-eat cereals and less on other cereals.

Discussion and Conclusion

This paper describes a procedure for estimating a censored AIDS model using Bayesian methods. ACNielsen 2006 scanner data are used in estimating the demand for breakfast cereals. We disaggregate the cereals by grain type and by type of cereal and estimate the system of four equations. Within the cereal groups demand we find demand for all four cereals to be price inelastic. Demand for whole grain hot cereals (which includes rolled oats) is the most sensitive to price changes. Cross price elasticities indicate consumers substitute among the four types, although the cross-price subsitution effects (elasticities) are small.

Using information from several different sources we were able to classify ready-to-eat and hot cereals into whole grain and non-whole grain product groups. However, results of this research can be sensitive to this classification, since more that 50 percent of cereals in our data carry private labels. In additional extensions to our work, we plan to improve the classfications proposed in this paper an examine how sensitive the results are to the classifications.

Although the observed shares for the four products we analyzed do add-up to unity, by construction, the estimation method we used does not account for the adding-up to unity of the latent expenditure shares. Further work is needed to specify a model that imposes an adding-up to unity restriction on both the latent and observed expenditure shares. Nonetheless, this is the first attempt in estimating the censored AIDS model using Bayesian methods and addresses an important issue in empirical demand estimation.

Appendix

Let y be the total expenditure on some or all of n goods. These goods can be bought in non-negative quantities $\mathbf{q_i} = (q_1, q_2, ..., q_n)$, $i = 1, \ldots, n$, at given prices $\mathbf{p_i} = (p_1, p_2, ..., p_n)$. The budget contraint of the household is $\sum_{i=1}^{n} p_i q_i = y$. Defining the utility function as $u(\mathbf{q})$, the households aim is to maximize the utility subject to the budget constraint:

$$max \quad u(\mathbf{q}) \quad \text{subject to} \quad \sum_{i=1}^{n} p_i q_i = y$$

The solution of this maximisation problem leads to the Marshallian (uncompensated) demand functions $q_i = g_i(p, x)$.

Alternatively, the consumers problem can be defined as the minimization of the total expenditure necessary to attain a specified level of utility u^*, at given prices:

$$min \quad \sum_{i=1}^{n} p_i q_i = y \quad \text{subject to} \quad u(\mathbf{q}) = u^*$$

The solution to this minimization problem leads to the Hicksian (compensated) demand function $q_i = f_i(\mathbf{p}, u)$. Therefore, a cost function can be defined as

$$C(\mathbf{p}, u) = \sum_{i=1}^{n} p_i f_i(\mathbf{p}, u) = y$$

The AIDS model specify the following cost function:

$$lnC(\mathbf{p}, u) = a(\mathbf{p}) + ub(\mathbf{p})$$

where $a(p) = \alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2} \sum_i \sum_j \gamma_{ij} ln(p_i) ln(p_j)$ and $b(\mathbf{p}) = \beta_0 \prod_i p_i^{\beta_i}$

The derivative of $lnC(\mathbf{p}, u)$ with respect to lnp_i is:

$$\frac{\partial lnC(\mathbf{p}, u)}{\partial lnp_i} = \alpha_i + \sum_j \gamma_{ij} ln(p_j) + u\beta_i\beta_0 \prod_i p_i^{\beta_i}$$

Since $C(p,u) = y \Leftrightarrow lnC(p,u) = lny \Rightarrow lny = a(p) + ub(p)$. From here solving for u we get

$$u = \frac{lny - a(\mathbf{p})}{b(\mathbf{p})}.$$

Substituting u in $\frac{\partial lnC(\mathbf{p},u)}{\partial lnp_i}$ we get

$$\frac{\partial lnC(\cdot)}{\partial ln(p_i)} = \frac{\partial C(\cdot)}{\partial p_i}\frac{p_i}{C(\cdot)} = f_i(\cdot)\frac{p_i}{C(\cdot)} = \frac{p_i q_i}{y} = \alpha_i + \sum_j \gamma_{ij} ln(p_j) + \beta_i(lny - a(\mathbf{p}))$$

If we set a price index P such that $lnP = a(\mathbf{p})$, then

$$\frac{\partial C(\mathbf{p},u)}{\partial ln(\mathbf{p_i})} = \alpha_i + \sum_j \gamma_{ij} ln(p_j) + \beta_i(lny - lnP)$$

or

$$s_i = \alpha_i + \sum_j \gamma_{ij} ln(p_j) + \beta_i ln\left(\frac{y}{P}\right),$$

where $lnP = a(p) = \alpha_0 + \sum_i \alpha_i ln(p_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij}]n(p_i)ln(p_j)$.

Elasticities

The following formulae are used in the calculation of the elasticities: Expenditure elasticity: $e_i = \frac{\beta_i}{s_i} + 1$;

Uncompensated own-price elasticities: $\eta_{ii} = \frac{\gamma_{ii}}{s_i} - \beta_i - 1$;

Uncompensated cross-price elasticities: $\eta_{ij} = \frac{\gamma_{ii}}{s_i} - \beta_i \frac{s_j}{s_i}$;

Compensated own-price elasticities: $\eta^*_{ii} = \eta_{ii} + e_i s_i$;

Compensated cross-price elasticities: $\eta^*_{ij} = \eta_{ij} + e_i s_j$.

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Figure 4.1 Lagged Autocorrelations for γ_{14}, α_2, γ_{33} and γ_{41}

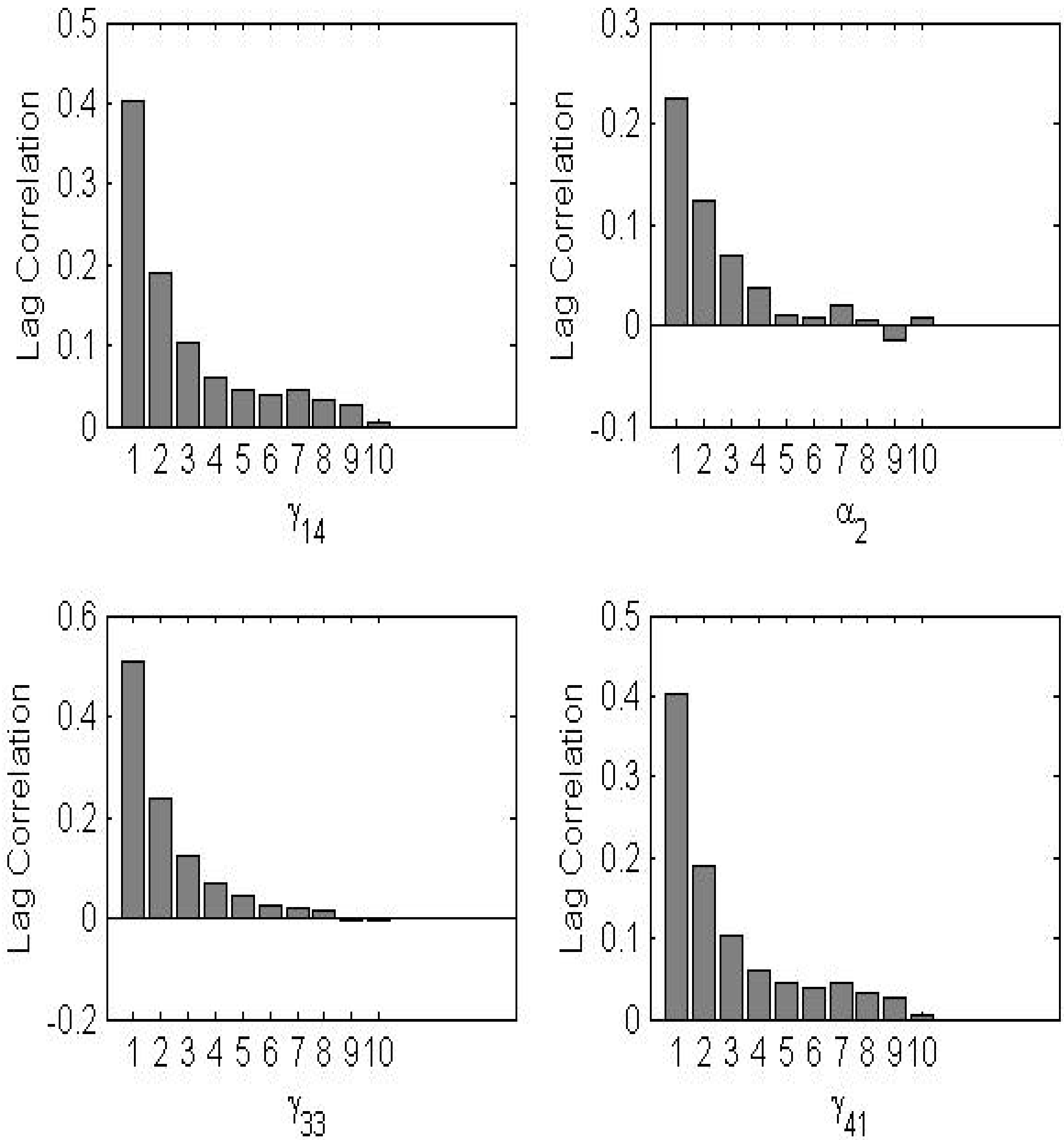

Figure 4.2 Lagged Autocorrelations for ρ_{12}, ρ_{24}, σ_1^2 and σ_4^2

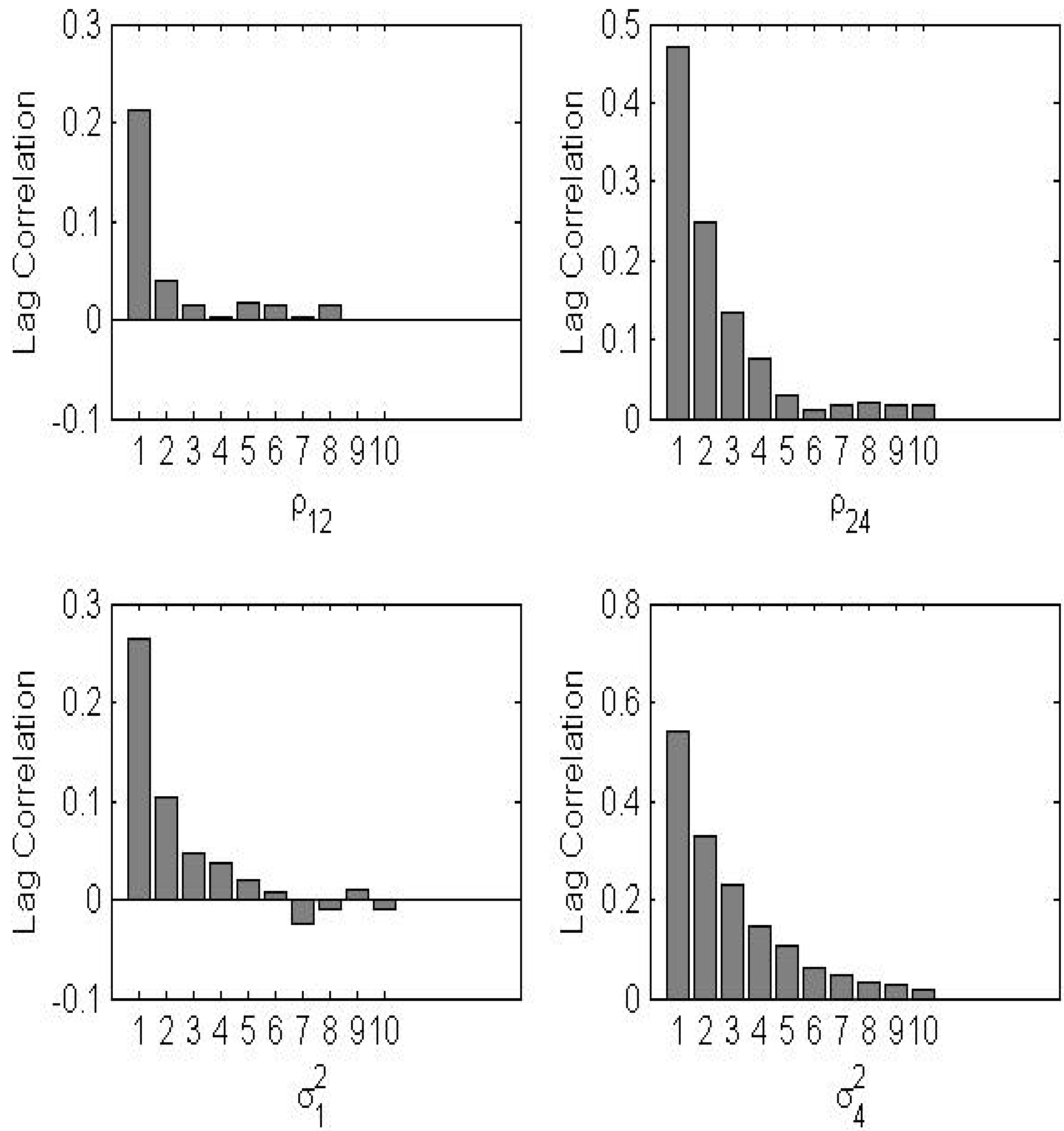

Table 4.1 Examples of Whole Grains

Brown rice
Buckwheat
Bulgur (cracked wheat)
Millet
Popcorn
Quinoa
Sorghum
Triticale
Whole-grain barley
Whole-grain corn
Whole-oats/oatmeal
Whole rye
Whole wheat
Wild rice

Source: Dietary Guidelines for Americans.

Table 4.2 True Values and Posterior Estimates of the Parameters

		Posterior Estimates	
Variable	True Value	$E(\cdot\|y)$	$Std(\cdot\|y)$
Regression Parameters			
α_1	0.64	0.6323	0.0066
γ_{11}	0.35	0.3547	0.0039
γ_{12}	0.39	0.3852	0.0028
γ_{13}	-0.53	-0.5297	0.0019
γ_{14}	-0.21	-0.2103	0.0031
β_1	-0.49	-0.494	0.0046
α_2	0.93	0.9311	0.0049
γ_{21}	0.39	0.3852	0.0028
γ_{22}	0.3	0.3023	0.0035
γ_{23}	0.2	0.1997	0.0018
γ_{24}	-0.89	-0.8872	0.0029
β_2	0.25	0.2515	0.0041
α_3	-0.12	-0.1213	0.0031
γ_{31}	-0.53	-0.5297	0.0019
γ_{32}	0.2	0.1997	0.0018
γ_{33}	0.1	0.0992	0.0021
γ_{34}	0.23	0.2308	0.0019
β_3	0.34	0.3404	0.0024
α_4	-0.45	-0.4421	0.0063
γ_{41}	-0.21	-0.2103	0.0031
γ_{42}	-0.89	-0.8872	0.0029
γ_{43}	0.23	0.2308	0.0019
γ_{44}	0.87	0.8668	0.0041
β_4	-0.1	-0.098	0.0048
Covariance Matrix Parameters			
ρ_{12}	-0.45	-0.4543	0.0079
ρ_{23}	-0.2	-0.1997	0.0095
ρ_{13}	0.5	0.5034	0.0075
ρ_{14}	-0.35	-0.3405	0.0088
ρ_{24}	0.4	0.3967	0.0085
ρ_{34}	-0.5	-0.4899	0.0076
σ_1^2	0.5	0.505	0.0071
σ_2^2	0.3	0.3038	0.0043
σ_3^2	0.1	0.1008	0.0014
σ_4^2	0.6	0.5982	0.0085

Table 4.3 Cereals Identified as Whole Grain from Different Sources

	Total UPC's	Manufacturer		WG Council		By Grain Type	
	N	N	%	N	%	N	%
Ready-to-eat	2850	514	18.0	198	6.9	603	21.2
Hot Cereal	960	212	22.1	60	6.3	633	65.9
All	3810						

Table 4.4 Classification of Cereals into Whole Grain

	Total UPC's	Classification 1		Classification 2	
	N	N	%	N	%
Ready-to-eat	2850	519	18.2	938	32.9
Hot Cereal	960	212	22.1	877	91.4
All	3810				

Table 4.5 Distribution of Purchasing Households and Sample Mean Values of Selected Variables

Product Category	No. of Hhlds	% of Hhlds	Mean Expenditure Share	Mean Quantity (ounces)	Mean Expenditure ($)	Mean Unit Value ($/ounce)
Sample	7081	100.0				
Ready-to-eat WG	6382	90.1	0.48	255.80	39.43	0.16
Ready-to-eat Non-WG	5960	84.2	0.34	183.08	27.60	0.16
Hot Cereal WG	4414	62.3	0.14	99.00	12.40	0.14
Hot Cereal Non-WG	1922	27.1	0.04	84.42	6.47	0.10

Table 4.6 Definition of Variables, Sample Mean Values and Standard Deviations

Variable	Definition	Mean	Std.
N	Number of households	7081.00	
Income/$1000	Household income/$1000	59.27	39.02
Household size	Household size	2.34	1.29
Age of Head<30	1 if household heads age is under 30	0.01	0.09
30≤Age of Head ≤49	1 if household heads age is between 30&49	0.31	0.46
50≤Age of Head≤64	1 if female heads age is between 50&64	0.40	0.49
65≤Age of Head	1 if female heads age is 65 and older	0.28	0.45
Presence of children	1 if household has children	0.23	0.42
Male head employed	1 if the male head is employed	0.66	0.47
Female head employed	1 if the female head is employed	0.59	0.49
≤ High school (male)	1 if the male heads education is high school	0.27	0.44
Some college (male)	1 if the male heads education is some college	0.31	0.46
College + (male)	1 if the male heads education is college	0.43	0.49
≤ High school (female)	1 if female heads education is high school	0.27	0.44
Some college (female)	1 if the female heads education is some college	0.31	0.46
College (female)	1 if female heads education is college	0.41	0.49
Married	1 if married	0.59	0.49
White	1 if race is white	0.77	0.42
Black	1 if the race is black	0.13	0.34
Other	1 if race is other	0.10	0.30
Hispanic	1 if Hispanic	0.07	0.26
East	1 if the household lives in the East region	0.22	0.42
Central	1 if the household lives in the Central region	0.17	0.37
South	1 if the household lives in the South region	0.38	0.49
West	1 if the household lives in the West region	0.23	0.42
Urban	1 if the household lives in urban area	0.87	0.34

Table 4.7 Variables and Sample Mean Values (N=7081)

Variable	Ready-to-Eat (n=6875) WG	Ready-to-Eat (n=6875) Non-WG	Hot Cereal (N=5031) WG	Hot Cereal (N=5031) Non-WG
N	6382	5960	4414	1922
Income/$1000	60.19	59.43	60.82	55.76
Household size	2.40	2.45	2.43	2.47
Age of Head$<$30	0.01	0.01	0.01	0.01
30$\leq$Age of Head $\leq$49	0.32	0.33	0.30	0.30
50$\leq$Age of Head$\leq$64	0.40	0.39	0.39	0.36
65$\leq$Age of Head	0.28	0.27	0.30	0.33
Presence of Children	0.24	0.25	0.24	0.25
Male Head Employed	0.67	0.67	0.65	0.62
Female Head Employed	0.59	0.59	0.58	0.54
$\leq$ High School (male)	0.27	0.28	0.26	0.29
Some College (male)	0.31	0.31	0.31	0.30
College + (male)	0.43	0.41	0.42	0.41
$\leq$ High School (female)	0.27	0.28	0.27	0.29
Some College (female)	0.32	0.32	0.33	0.32
College + (female)	0.41	0.40	0.40	0.39
Married	0.61	0.62	0.63	0.65
White	0.78	0.77	0.78	0.79
Black	0.13	0.13	0.13	0.13
Other	0.10	0.10	0.10	0.08
Hispanic	0.08	0.08	0.08	0.08
East	0.23	0.23	0.23	0.21
Central	0.17	0.17	0.16	0.20
South	0.38	0.38	0.38	0.38
West	0.23	0.22	0.23	0.21
Urban	0.87	0.87	0.87	0.83

Table 4.8 Ready-to-Eat Cereals: Posterior Means and Probabilities of Being Positive

	Ready-to-Eat					
	WG			Non-WG		
Variable	$E(\cdot\|y)$	$Std(\cdot\|y)$	$Pr(\cdot > 0\|y)$	$E(\cdot\|y)$	$Std(\cdot\|y)$	$Pr(\cdot > 0\|y)$
Demographic Characteristics						
Intercept	0.2333	0.0041	1	0.3559	0.0038	1
Income/ $1000	0.0006	0	1	-0.0005	0	0
Household size	-0.0201	0	0	0.0266	0	1
Age of Head<30	0.0506	0.0003	1	0.0641	0.0003	1
$30 \leq$ Age of Head ≤ 49	-0.0244	0.0001	0	0.0629	0.0001	1
$50 \leq$ Age of Head ≤ 64	-0.0005	0.0001	0	0.0126	0.0001	1
Presence of Children	0.0076	0.0001	1	0.0051	0.0001	1
Male Head Employed	0.0126	0.0001	1	0.001	0.0001	0.8
Female Head Employed	-0.0084	0.0001	0	0.0087	0.0001	1
$\leq$ High School (male)	-0.0245	0.0001	0	0.0354	0.0001	1
Some College (male)	-0.0137	0.0001	0	0.0176	0.0001	1
$\leq$ High School(female)	-0.0037	0.0001	0	0.0171	0.0001	1
Some College (female)	-0.0063	0.0001	0	0.011	0.0001	1
Married	0.0105	0.0001	1	-0.0093	0.0001	0
White	0.0547	0.0002	1	-0.0389	0.0002	0
Black	-0.0262	0.0003	0	0.0188	0.0003	1
Hispanic	-0.0197	0.0003	0	0.0171	0.0003	1
East	0.0229	0.0002	1	-0.0022	0.0001	0
Central	-0.0015	0.0003	0	0.0092	0.0002	1
South	-0.0074	0.0003	0	0.0145	0.0002	1
Urban	0.0164	0.0002	1	-0.0142	0.0002	0
Price Coefficients						
RTE WG	0.0909	0.0008	1	-0.0536	0.0006	0
RTE NWG	-0.0536	0.0006	0	0.0909	0.0004	1
Hot WG	0.0044	0.0004	1	-0.0066	0.0003	0
Hot Non-WG	-0.0417	0.0017	0	-0.0307	0.0013	0
Total Expenditure						
Expenditure	0.0388	0.0001	1	-0.0107	0.0001	0

Table 4.9 Hot Cereals: Posterior Means and Probabilities of Being Positive

	Hot Cereal					
	WG			Non-WG		
Variable	$E(\cdot\|y)$	$Std(\cdot\|y)$	$Pr(\cdot > 0\|y)$	$E(\cdot\|y)$	$Std(\cdot\|y)$	$Pr(\cdot > 0\|y)$
			Demographic Characteristics			
Intercept	0.4335	0.0035	1	-0.0227	0.0114	0
Income/ $1000	-0.0001	0	0	0.001	0	1
Household size	-0.0079	0	0	0.0014	0	1
Age of Head<30	-0.1059	0.0003	0	-0.0088	0.0005	0
$30 \leq$ Age of Head ≤ 49	-0.0346	0.0001	0	-0.0039	0.0001	0
$50 \leq$ Age of Head ≤ 64	-0.0107	0.0001	0	-0.0014	0.0001	0
Presence of Children	-0.0102	0.0001	0	-0.0025	0	0
Male Head Employed	-0.0136	0.0001	0	0.0009	0.0002	1
Female Head Employed	0.0027	0.0001	1	-0.003	0.0003	0
$\leq$ High School (male)	-0.0124	0.0001	0	0.0015	0.0001	1
Some College (male)	-0.0075	0.0001	0	0.0037	0.0002	1
$\leq$ High School(female)	-0.0102	0.0001	0	-0.0032	0	0
Some College (female)	-0.005	0.0001	0	0.0002	0.0001	1
Married	-0.0017	0	0	0.0006	0	1
White	-0.0291	0.0002	0	0.0133	0.0005	1
Black	-0.0152	0.0003	0	0.0226	0.0009	1
Hispanic	-0.0137	0.0003	0	0.0163	0.0008	1
East	-0.0262	0.0001	0	0.0056	0.0004	1
Central	-0.0218	0.0002	0	0.0141	0.0006	1
South	-0.0208	0.0002	0	0.0137	0.0007	1
Urban	0.0072	0.0001	1	-0.0095	0.0005	0
			Price Coefficients			
Hot WG	0.0235	0.0002	1	-0.0213	0.001	0
Hot Non-WG	-0.0213	0.001	0	0.0937	0.004	1
			Total Expenditure			
Expenditure	-0.024	0.0001	0	-0.0042	0.0003	0

Table 4.10 Estimated Demand Elasticities

	Ready-to-Eat				Hot Cereals			
	WG		Non-WG		WG		Non-WG	
Variable	$E(\cdot\|y)$	$Pr(\cdot > 0\|y)$	$E(\cdot\|y)$	$Pr(\cdot > 0\|y)$	$E(\cdot\|y)$	$Pr(\cdot > 0\|y)$	$E(\cdot\|y)$	$Pr(\cdot > 0\|y)$
Marshallian Elasticity								
RTE WG	-0.7764	0	-0.166	0	-0.0161	0	-0.1505	0
RTE Non-WG	-0.2879	0	-0.4415	0	-0.016	0	-0.1894	0
Hot WG	0.0485	1	-0.0062	0	-0.8926	0	-0.066	0
Hot Non-WG	-0.2363	0	-0.1761	0	-0.1082	0	-0.4425	0
Hicksian Elasticity								
RTE WG	-0.3842	0	0.2262	1	0.3762	1	0.2418	1
RTE Non-WG	-0.1309	0	-0.2844	0	0.1411	1	-0.0324	0
Hot WG	0.3133	1	0.2586	1	-0.6278	0	0.1988	1
Hot Non-WG	-0.0504	0	0.0099	1	0.0778	1	-0.2566	0
Expenditure Elasticity								
	1.109	1.000	0.9348	1.000	0.9163	1.000	0.9821	1

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