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Essays on energy economics: Microeconomic and macroeconomic dimensions

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Essays on energy economics: Microeconomic and macroeconomic dimensions

by

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ABSTRACT

The 2000's rise in oil prices has reignited the interest of economists about the influence of these swings on both specific good markets and macroeconomic fluctuations. In the macroeconomic arena, the attention of policymakers and economists has been posed the effects of oil price swings on the economic domestic cycle. In this work, we explore a question under scrutiny among economists and policymakers: the use of fiscal policy as an instrument to accommodate the domestic fluctuations caused by oil price shocks. In particular, we study a floating oil-tax rate used as an instrument to reduce the pass-through of the international price of oil into the domestic economy. We develop two works in this topic. The first paper study a small open economy calibrated to Chile with perfect foresight and finite horizon. This model is applied to two types of policy: a Ramsey problem and an ad-hoc stabilization rule. For both cases, we find that the social planner finds optimal to reduce the pass-through of oil price shocks into the domestic economy. Also, we compute the welfare gains and losses that arise from both types of policy compared to a competitive equilibrium economy where the government plays a passive role. The second paper recasts the model developed in the first study. The salient features of the new model are: uncertainty; infinite horizon; the model is calibrated to two representative economies: developed and developing countries; and the optimal taxation policy is analyzed considering both complete and incomplete markets for the bonds issued by the government. We find significative differences in how the optimal oil-tax rate responds whether the economy is calibrated to a developed country or a developing economy, or if public debt is traded in complete or incomplete markets.

In the microeconomic field, we study the ethanol market. Aukayanagul and Miranowski (2009) show that the price of ethanol closely follows the price of oil. Given the sharp rise of oil

and ethanol prices in the last years a question has been under scrutiny in the ethanol market: When is profitable to invest in an ethanol plant? In the third paper, we develop a model using the real options approach to answer this question. We incorporate the market structure of the ethanol industry to our model, in which entry and exit of new firms affect the ethanol gross margin (price of ethanol plus co-product value minus purchases of corn). The solution of the model gives threshold values of the ethanol gross margin that indicate when is optimal to both invest in an ethanol plant and to exit the market for an active firm.

WHAT CAN FISCAL POLICY DO TO CURB
THE EFFECT OF OIL SHOCKS?

Leandro G. Andrián and P. Marcelo Oviedo

1   Introduction

The recent sharp rise in oil prices has reignited the interest of policymakers around the world to find policies capable of alleviating the financial burden that skyrocketing gasoline prices have imposed on business and households. The strength of oil shocks to cause macroeconomic fluctuations in U.S. was forcefully documented econometrically by Hamilton (1983) in an influential paper that showed that all but one U.S. recessions after the World War II were preceded by oil price hikes. Hamilton's paper initiated an insightful debate that among other things showed that oil shocks were commonly followed by monetary tightenings intended to curb the inflationary pressures induced by the shocks. The debate about the destabilization consequences of oil shocks and the optimal policy responses has rarely extended outside the *monetary* policy arena and has almost exclusively referred to the U.S. economy. On the contrary, this paper focuses on the optimal *fiscal* response to oil shocks in the context of an oil-dependent, small open economy, although some of the paper's conclusions can be spanned to other economies.

Figure 1 documents that not only in the U.S. oil shocks are a source of macroeconomic destabilization by showing the responses to an oil price hike of the GDP growth rates in eight countries. These responses arise after estimating country-specific, annual versions of the VAR model that Hamilton (2008) estimated on quarterly basis to restate the relationship between oil shocks and macroeconomic growth in U.S.. The figure illustrates that oil shocks cause

highly similar output responses around the globe without distinguishing between industrial and emerging market countries.[1]

We focus on a particular fiscal stabilization policy that has been under scrutiny among policymakers, namely, the management of an oil floating tax by which the fiscal authority administers the fuel tax rates so as to prevent the full passing through of world oil-price swings into the domestic economy. The policy has been debated in major industrial countries and largely applied in developing countries recently; Mati (2008), for instance, reports that less than half of a sample of 42 developing and emerging market countries fully passed through the 2007 high oil prices.[2] Our main policy-evaluation question asks about the optimal degree of pass-through of shocks to the world price of oil into an oil importing small open economy. We find that a floating tax that prevents the full passing through of oils shocks is optimal from the viewpoint of a "restricted" Ramsey planner. The restrictions we consider for the Ramsey planner problem are motivated by actual fiscal policies and consist in the impossibility of changing frequently all tax rates. Restricting the planner's problem allows us to focus on the optimal management of oil tax rates, which could be easier to adjust at high frequency than income or consumption taxes.

Oil tax revenues account for a non-trivial share of total tax revenues around the world. The average share of oil tax revenues in total fiscal revenues was equal to 5.5 percent for the period 1988-2007 among OECD countries and as high as 11, 12, and 14 percent in Portugal, Greece, and Korea in 1998.[3] Outside the OECD, in the Chilean tax system for instance, energy taxes accounted for 7.7 percent of fiscal revenues during the 1998-2007 period, on average, and energy taxes represents 33 and 76 percent of the final domestic prices of diesel and gasoline.

We study the optimality of an oil-floating tax by resorting to the neoclassical growth theory to model a small open economy where the government collects fiscal revenues by taxing income,

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[1]Blanchard and Gali (2007) conduct a similar exercise for G7 countries and find results like ours.

[2]The debate in industrial countries includes the view of France president, Mr. Sarkozy, who was quoted by the Herald Tribune (May 27, 2008) when asking: "If the barrel continues to rise, must we maintain a VAT rate that is proportional to the price in the same conditions?" The same question was debated in the primaries and presidential campaigns in the US during the same time and was sometimes informally referred as oil-tax holidays.

[3]See Joumard (2002) who refers to environmentally-related taxes although she states that "motor fuel and vehicle taxes account for the bulk of these [tax] revenues".

consumption of non-energy goods, and purchases of oil. Our departing point is a taxation system in which constant ad-valorem oil taxes provide the government a nontrivial source of fiscal revenues although the consumption and income taxes remain as the main sources. As in the neoclassical theory, we assume the existence of an aggregate production function where non-energy goods are produced from inputs of capital and labor, but departing from the standard, we add energy as an additional input of the production process. Input and output markets are perfectly competitive and energy goods are imported from the rest of the world. Except for our inclusion of oil and our dealing with a nonmonetary economy, our modeling strategy is similar to Cooley and Hansen (1992) at focusing on a finite number of periods of a perfect-foresight economy.

Economic theory was puzzled for some time to account for the aggregate fluctuations caused by oil shocks in a competitive model because the typical share of energy in output is about 5 percent. Kim and Loungani (1992), for instance, showed that in a standard perfect competitive, business cycle model where oil cooperates with labor and capital through a CES production function, oil-price shocks could explain no more than a modest fraction of the observed U.S. business cycles. Later, Finn (2000) demonstrated that oil shocks are a meaningful source of business cycles under perfect competition when energy is modeled as an essential input for the utilization of capital, which in turns affects the depreciation rate.[4]

Because of our perfect competition assumption, we follow Finn (2000) to model the way oil enters into the production function. One advantage of her formulation of the perfect competitive model is to make the theory consistent with the idea that an increase in the price of energy works just like a negative productivity shock, as suggested by Hall (1988, 1990), and more recently by Kehoe and Ruhl (2008) who take a broader view and refer to the terms of trade of an open economy.

On the household side of the economy, as in the standard theory, we assume that people are willing to substitute between a basket of consumption of goods and leisure, both intertem-

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[4]Before Finn's paper, the first passage to success at formulating a theory capable of explaining the effect of oil price shocks was advanced by Rotemberg and Woodford (1996), although they had to depart from the assumption of perfect competition.

porally and intratemporally. We consider that the household's consumption basket includes both energy and non-energy goods.

We calibrate our model to the 1996-2007 period of the Chilean economy, which is a typical oil-importing small open economy. We set our model to adopt the observed cyclical components of the Chilean government expenditures and we fix the model tax rates on consumption and income equal to the average tax rates in Chile; in the competitive equilibrium, the model also mimics the Chilean oil tax rate. The model also incorporates the dynamics of external assets in Chile and the productivity shocks are backed out to mimic the observed dynamics of output.

Once we obtain the competitive equilibrium consistent with the evolution of the Chilean output in the calibrated period, we investigate the pass through of changing world oil prices into the domestic economy from two different perspectives. We first conduct a *normative* analysis that appeals to the standard optimal taxation criterion in the public finance literature to find welfare-maximizing taxes. Namely, we find the optimal oil tax rates by solving a "restricted" Ramsey optimal taxation problem in which the planner is free to choose time and state contingents tax rates on the household's and firm's purchases of oil while taking the consumption and income tax rates as given. The advantage of this approach is that absent any energy shock, oil tax rates would remain constant at their steady-state values. So, it is the presence of shocks what makes the Ramsey planner to choose whether or not allow for a full pass-through of international oil prices and to issue public debt to accommodate the changes in fiscal revenues induced by both, the oil shocks and the oil tax policy.

For the 16-year period to which we calibrate the model, we find that the household would be willing to give up 0.017 percent of its consumption if it were allowed to switch from a constant oil-tax regime to a regime where the Ramsey planner sets the oil taxes. Beyond this modest welfare gain, we find that oil taxes should be reduced by almost 0.70 percentage point for every percentage point increase in the price of oil. The Ramsey policy delivers domestic oil prices that are 78 percent as volatile as world oil prices and produce GDP and consumption cycles that are less volatile than the competitive equilibrium's cycles obtained with fixed oil taxes.

De-Miguel and Manzano (2006) make an ample discussion of Ramsey-optimal oil taxes under several oil and tax settings in an infinite-horizon model in which oil enters into the production function as suggested by Kim and Loungani (1992). They first prove and then illustrate by using impulse-response functions that when oil is used for consumption and production purposes and there is a sole tax on oil, oil-tax rates are different from zero. Our model shows their results also hold when the incompleteness of the tax system arises by the impossibility of choosing non-oil taxes. Furthermore, we quantify the optimal responses of oil tax rates, their effects on fiscal balances, and their macroeconomic stabilization consequences in a model where oil enters into the production function as suggested by Finn (2000).

Our second perspective to discuss the management of oil taxes in an oil importing small open economy belongs to the terrain of the *positive* analysis and is motivated by the typical stabilization role of macroeconomic policy, or more specifically, of monetary policy. In monetary policy models, the role of the central bank is summarized by a loss function that penalizes deviations of output from its trend and of inflation from a target rate.[5] As inflation plays no role in our nonmonetary model, the role of the stabilization policy should be minimizing the output gap. But, if we consider aggregate fluctuations driven by oil shocks, minimizing the output gap in our model is tantamount to fixing the domestic price of oil since doing so fully smoothes out the oil-driven output swings. In other words, the policymaker can fully stabilize output by fixing the domestic gross price of oil.

This zero pass-through policy of oil shocks reduces the household utility although the welfare cost is minimal: the household would demand a 0.03 percent increase in the level of consumption to voluntarily accept the allocations of the full stabilization policy. The result leads us to think that if a stable macroeconomic environment generates gains that are not completely captured by a representative agent model, the inefficiency caused by stable domestic oil prices could be nil.

Oil-floating taxes have been criticized on the grounds that they promote economic inefficiency and generate excessive fiscal costs. The results of the Ramsey and zero pass-through

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[5]The tension between these policy goals when the economy is subject to oil shocks is analyzed by Bernanke et al. (1997).

regimes cast doubts on the efficiency criticism. The dynamics of the model's fiscal revenues and stock of public debt also challenge the fiscal-cost criticism. To consider the fiscal effect of an oil tax policy, it should be taken into consideration that the demand for oil is highly price inelastic —around -0.05 according to cross country estimates by Cooper (2003), so oil-tax revenues tend to increase with oil-price hikes. Therefore, it is not necessarily true that a reduction in the tax rate in response to an oil shock leads to tumbling fiscal revenues.

The rest of the paper is organized as follows. In the next section we present our small open economy model extended to include the utilization of oil in consumption and production and three tax rates. We calibrate the model to the Chilean economy in Section 3 and solve for its competitive equilibrium in Section 4. We discuss the restricted Ramsey problem in Section 5 and the positive analysis of oil taxes in Section 6. Section 7 contains some concluding remarks.

2   The Economy

We model a perfect competitive economy without uncertainty, i.e., agents have perfect foresight. Our modeling strategy is similar to Cooley and Hansen's (1992) although we depart from them by modeling a nonmonetary open economy that imports oil from the rest of the world. Furthermore, instead of specifying a stationary economy that stays in place after a tax reform as in Cooley and Hansen, we focus on a finite horizon economy. This forces us to specify the initial and terminal conditions of the model's state variables but releases us from working the stationary part that follows to the period of interest.

The economy is inhabited by infinitely many identical households that rank consumption and work hours, h_t, according to

$$\sum_{t=-1}^{T+1} \beta^t \left[\log(\hat{c}_t) + \omega \log(1 - h_t) \right], \tag{1}$$

where there is an endowment of time normalized to 1 to be divided between work and leisure. Here $\beta \in (0, 1)$ is the discount factor and $\hat{c}_t$ a consumption basket defined as a CES aggregator of non-energy and energy goods, denoted as c_t and e_t^h, that takes the following form:

$$\hat{c}_t = \left[(c_t)^{-\eta} + \phi(e_t^h)^{-\eta} \right]^{-1/\eta}$$

where $(1 + \eta)^{-1}$ is the elasticity of substitution between energy and non-energy goods and $\phi > 0$ is a share parameter. Hereafter, U_t denotes the time t value of the utility function between brackets in (2) and its partial derivatives are denoted by adding the corresponding variable to the subscript. The same notation is used below to refer to the production function and its derivatives.

Without loss generality, there exists a single firm owned by the household that produces non-energy goods by operating the following constant returns to scale technology:

$$y_t = F(u_t k_{t-1}, h_t, z_t) = A \exp(z_t) h_t^{1-\alpha} (u_t k_{t-1})^\alpha \tag{2}$$

where u_t is the capital utilization rate; k_{t-1} the aggregate stock of capital carried over from period $t-1$; z_t an exogenous productivity shock equal to zero in the steady state; A a productivity scale parameter and α the share of effective capital in output. As in Greenwood et al. (1988), the depreciation rate is an increasing, convex function of u. That is:

$$\delta(u_t) = \frac{\delta_0 u_t^{\delta_1}}{\delta_1}; \quad \delta(\cdot) \in (0,1), \ \delta_0 > 0, \delta_1 > 1 \tag{3}$$

Next, we follow Finn (2000) to link the capital utilization rate and the use of energy in production, e_t^y, as follows:

$$e_t^y = k_{t-1} a(u_t), \ a(u_t) = \frac{\nu_0 u_t^{\nu_1}}{\nu_1}; \quad \nu_0 > 0, \nu_1 > 0 \tag{4}$$

Equation (5) states that the energy requirement in production increases with the capital utilization rate. We can think how energy affects u_t, by reexpressing (5) as:

$$u_t = \left(\frac{e_t^y}{k_{t-1}}\right)^{\frac{1}{\nu_1}} \left(\frac{\nu_1}{\nu_0}\right)^{\frac{1}{\nu_1}} \tag{5}$$

which permits to see that given the capital stock inherited from the previous period, there is a one-to-one relationship between the choices of energy and capital utilization rate. If we plug the precedent expression into the production function (3) we obtain:

$$\tilde{A} \exp(z_t) h_t^{1-\alpha} \left[(k_{t-1})^{1-\frac{1}{\nu_1}} (e_t^y)^{\frac{1}{\nu_1}} \right]^\alpha \tag{6}$$

where $\tilde{A} \equiv A \left(\nu_1/\nu_0\right)^{\frac{\alpha}{\nu_1}}$; this reexpression of the production function shows that the capital and energy shares in output are $\alpha \left(1 - 1/\nu_1\right)$ and $\alpha \left(1/\nu_1\right)$, respectively. Finn (2000) uses (6) to

show the *direct* effect of energy in output. She also includes and *indirect* effect, which operates through capital utilization rate; given that rate, a hike in p_t^e raises the marginal cost of using capital; that is $p_t^e(1 + \tau_t^e)a(u_t)$, prompting further reductions in output.

Without loss of generality, we assume that it is the household and not the firm who chooses the optimal capital utilization rate (and consequently the utilization of energy e_t^y). The competitive firm, in turns, faces a market rental price of effective capital r_t and optimally decides the quantity of $u_t k_{t-1}$ to rent in every period.

The third agent in our economy is a government that provides public goods g_{1t} and extend transfers g_2 to the household, although none of g_{1t} and g_2 is a choice variable. Non-energy goods can be freely transformed into public goods, and the amount of public and private non-energy goods consumed domestically come from operating the production technology and from net purchases from the rest of world. Energy goods, however, have to be imported at the exogenous price p_t^e. The government collects fiscal revenues from taxing income and the consumption of energy and non-energy goods.

2.1   Asset Markets and Resource Constraints

We assume the existence of two financial assets in our economy, (a) an international bond, b_t^i, traded by the household and whose price q_t^i is exogenous to the domestic economy, and (b) a domestic bond, b_t^g, issued by the government, held exclusively by domestic households, and priced at q_t^g. The following ad-hoc limits are imposed on the asset positions[6]:

$$\underline{b}^h \leq (b_t^i + b_t^g) \leq \bar{b}^h, \qquad \text{and} \qquad \underline{b}^g \leq b_t^g \leq \bar{b}^g \tag{7}$$

but we are going to design our computable analysis so that these asset limits are never binding. Consequently, the returns on public and household bonds are arbitraged and $q_t^g = q_t^i$, $\forall t$[7]. The assumption that only the household borrows and lends abroad is made without loss of generality. The optimal allocations of the present arrangement would also arise optimally in

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[6]See Aiyagari et al. (2002) and Ljunqvist and Sargent (2004) for a discussion of natural and ad-hoc debt limits.

[7]If the bounds were binding there would be necessary to consider the cases in which $q_t^i \neq q_t^g$ as well as cases where b_t^i has a shadow price different from q_t^i.

another arrangement where both the government and the household issue bonds in international financial markets. To see this, define $b^h \equiv b^i + b^g$ as the household net asset position. If the government wants to issue more bonds but the household is not willing to modify its net asset position, b^i should fall by the same amount b^g increases, which is tantamount to saying that the government is directly borrowing abroad.

In every period $t \in [-1, T+1]$, the household and the government must observe their budget constraints, respectively:

$$I_t(1 - \tau^I) + (b^i_{t-1} + b^g_{t-1}) + g_2 \geq (q^i_t b^i_t + q^g_t b^g_t) + c_t(1 + \tau^c) + (e^h_t + e^y_t)p^e_t(1 + \tau^e_t) + i_t, \quad (8)$$

and

$$I_t\tau^I + c_t\tau^c + (e^h_t + e^y_t)p^e_t\tau^e_t + q^g_t b^g_t \geq b^g_{t-1} + g_{1t} + g_2 \quad (9)$$

Here, $i_t = (k_t - k_{t-1})p^k_t$ denotes the household's gross investment and $I_t \equiv w_t h_t + [r_t u_t - \delta(u_t)p^k_t]k_{t-1}$ its taxable income. The wage rate and the rate of return of an effective unit of capital are w_t and $r_t u_t$. The household takes as given the price of new capital, p^k_t, and this price depends on the speed of adjustment of the aggregate capital stock K_t as follows:[8]

$$p^k_t = 1 + \frac{\mu}{2}(K_t - K_{t-1})^2 + \mu(K_t - K_{t-1}), \quad \mu \geq 0 \quad (10)$$

We follow the standard, neoclassical small-open-economy literature at including adjustment costs of capital to distinguish between financial and physical capital and to avoid a variability of the investment rate that largely exceeds what is observed in practice. Notice that by setting the adjustment cost parameter μ to zero, consumption and capital goods become homogeneous, as in the standard closed economy version of the neoclassical model.

As for the taxation policy, τ^I is the income tax rate, and τ^c and τ^e_t are the tax rates on the consumption of non-energy and energy goods; the latter two encompass both sale and value added taxes in actual economies. Observe that the definitions of I_t and i_t imply that the depreciation is tax deductible. As in De-Miguel and Manzano (2006), we assume that the government cannot secern whether oil is purchased for consumption or production purposes and therefore has to tax all purchases at the same rate τ^e_t. Notice that our strategy to study

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[8]See Mendoza (2006) for the reasons to adopt this functional form.

the taxation policies aimed at curbing the domestic impact of oil shocks fixes the income and consumption tax rates and will consider alternative settings of the tax rate on energy goods.

Finally, the household and government budget constraints (6) and (7) imply the following aggregate resource constraint:

$$F(u_t k_{t-1}, h_t, z_t) + b^i_{t-1} \geq q^i_t b^i_t + c_t + g_{1t} + p^k_t i_t + \delta p^k_t k_{t-1} + p^e_t(e^h_t + e^y_t), \qquad t \in [-1, T+1]. \quad (11)$$

2.2 The Competitive Equilibrium

This subsection contains definitions and a characterization of the competitive equilibrium.

DEFINITION 1. A *government fiscal policy* comprises "exogenous components" on one side, and a debt and an oil taxation policy on the other. The *exogenous components of the fiscal policy* are the fixed tax rates on income and consumption τ^I and τ^c, the fixed transfer g_2, and the sequence of government expenditures $\{g_{1t}\}^{T+1}_{t=-1}$. The sequence of oil tax rates $\{\tau^e_t\}^{T+1}_{t=-1}$ and the debt positions $\{b^g_t\}^{T+1}_{t=-1}$ are the *endogenous components of the fiscal policy* and our object of study.

DEFINITION 2. A *price system* is a an exogenous sequence of oil and international bond prices $\{p^e_t, q^i_t\}^{T+1}_{t=-1}$, a sequence of wage rates, rates of returns on capital and public bond prices $\{w_t, r_t, q^g_t\}^{T+1}_{t=-1}$, and a sequence of the capital price $\{p^k_t\}^{T+1}_{t=-1}$.

DEFINITION 3. Given the initial and terminal asset positions b^i_{-2} and b^i_{T+1}, the initial and terminal stocks of capital $\{k_{-2}, k_{T+1}, K_{-2}, K_{T+1}\}$, and the exogenous variables $\{p^e_t\}^{T+1}_{t=-1}$ and $\{z_t, q^i_t, g_{1t}\}^{T+1}_{t=-1}$, a *feasible allocation* is a sequence $\{c_t, k_t, K_t, h_t, e^h_t, e^y_t, u_t, b^i_t\}^{T+1}_{t=-1}$ that satisfies the aggregate resource constraint (12).

Before defining the competitive equilibrium it is convenient to characterize the household's and firm's optimality conditions. The household chooses processes $\{c_t, h_t, e^h_t, u_t\}^{T+1}_{t=-1}$ and $\{k_t, b^i_t, b^g_t\}^T_{t=-1}$ to maximize (2) subject to the sequence of budget constraints (6), taken as given the price system, the fiscal policy, and the initial and terminal conditions of the capital stock and the two bonds in the economy, i.e., $\{k_{-2}, K_{-2}, b^i_{-2}, b^g_{-2}, k_{T+1}, K_{T+1}, b^i_{T+1}, b^g_{T+1}\}$. The necessary and sufficient conditions for optimality are the budget constraint (6) holding

with equality at $t \in [-1, T+1]$ and the following:

$$-\frac{U_{c,t}}{U_{h,t}} = \frac{(1+\tau^c)}{w_t(1-\tau^I)}, \quad t \in [-1, T+1] \tag{12}$$

$$\frac{U_{c,t}}{U_{e,t}} = \frac{(1+\tau^c)}{p_t^e(1+\tau_t^e)}, \quad t \in [-1, T+1] \tag{13}$$

$$p_t^e(1+\tau_t^e)a'(u_t) = (1-\tau^I)[r_t - \delta'(u_t)p_t^k], \quad t \in [-1, T+1] \tag{14}$$

$$U_{c,t}q_t^i = \beta U_{c,t+1}, \quad t \in [-1, T] \tag{15}$$

$$U_{c,t}q_t^g = \beta U_{c,t+1}, \quad t \in [-1, T] \tag{16}$$

$$U_{c,t}p_t^k = \beta \left\{ U_{c,t+1} \left[(1-\tau^I)(r_{t+1}u_{t+1} - \delta(u_t)p_{t+1}^k) + \right. \right.$$
$$\left. \left. p_{t+1}^k - p_{t+1}^e(1+\tau_{t+1}^e)a(u_{t+1}) \right] \right\}, \quad t \in [-1, T] \tag{17}$$

According to (13) the marginal rate of substitution of consumption for leisure equates the relative price of consumption in terms of labor. In (14) the marginal rate of substitution of non-energy for energy goods equates the relative price of these goods. The marginal benefit and marginal cost of increasing the utilization of capital are equated in equation (15); the cost comes from the extra oil necessary to raise utilization when oil is priced at $p_t^e(1+\tau_t^e)$; the benefit is the income (net of tax and depreciation) from renting an additional unit of effective capital. Equations (16) and (17) are the standard Euler conditions for international and government bonds, and equation (18) is the Euler equation for capital; the latter equates the loss of utility at buying a unit of capital on the left-hand side, to the discounted utility gain at $t+1$ of owning and renting an extra unit of capital given the utilization decisions at times t and $t+1$, where the capital prices are implied by the adjustment costs.

In a finite horizon problem like ours, care should be exercised at specifying the terminal conditions. Equations (16) to (18) and the discussion that precedes them reveal that the household does not choose any of k_{T+1}, b_{T+1}^i, and b_{T+1}^g. If open to choice, the household would choose minus infinity values for the three of them. To make our problem meaningful we force the household to start and finish with specific values of these three assets as well as to accept specific terminal asset prices. For the stock of public debt, we force the household to hold the terminal stock of debt the government must issue to balance its intertemporal budget

constraint. For the other two assets, we request the household to start and finish with the steady values of k and b^i. In short, we impose the following terminal conditions:

$$q^i_{T+1} b^i_{T+1} = q^i_{ss} b^i_{ss}, \quad k_{T+1} = k_{ss}, \quad \text{and} \quad q^g_{T+1} = q^i_{ss}; \tag{18}$$

in addition to the following initial conditions:

$$b^i_{-2} = b^i_{ss}, \quad k_{-2} = k_{ss}, \quad \text{and} \quad b^g_{-2} = b^i_{ss}. \tag{19}$$

Switching to the production side of our model, at time $t \in [-1, T+1]$, the firm equates the marginal productivity of the factors of production to their respective market prices:

$$F_{h,t} = w_t, \qquad \text{and} \qquad F_{uk,t} = r_t, \tag{20}$$

and the linear homogeneity of the production function makes profits equal to zero.

Notice that under the current setting of our model, while the firm only cares about the effective level of capital $u_t k_{t-1}$, the household optimally chooses k_t and u_t separately. Beyond this way of organizing production, we can still think about the firm's optimal use of energy. Denoting by $F_{e,t}$ to the marginal product of oil, optimality requires setting $F_{e,t} = p^e_t(1 + \tau^e_t)$. Combining this with the household optimality condition (15), we obtain the following equilibrium condition

$$F_{e,t} = p^e_t(1 + \tau^e_t) = (1 - \tau^I)[r_t - \delta'(u_t)p^k_t]/a'(u_t), \tag{21}$$

which is used in Section 5 to characterize the restricted Ramsey problem.

The competitive equilibrium of our model is defined as follows:

DEFINITION 4. Given (a) the initial positions of the capital stock and the two bonds, (b) the terminal positions of the stock of capital and the international bond; (c) the sequence of oil prices, productivity shocks and price of international bonds $\{z_t, p^e_t, q^i_t\}^{T+1}_{t=-1}$, and (d) the restriction on q^g_{T+1}, a *competitive equilibrium* is a feasible allocation, a government fiscal policy, and a price system such that:

i. The household maximizes (2) subject to (6) holding with equality and the initial and terminal conditions (18) and (19); in the solution, equations (13) to (18) hold.

ii. The firm employs the profit-maximizing allocations of efective capital ($u_t k_{t-1}$) and labor, that is, equations (19) hold.

iii. The representative household's stock of capital coincides with the aggregate stock of capital in every period, that is, $k_t = K_t$, $\forall t \in [-1, T+1]$.

iv. The government's budget constraint (7) holds with equality in every period. This condition, along with the equality of the household budget constraint (6), implies that the aggregate resource constraint (12) also holds with equality in every period.

v. The assets' limits on (7) are never binding.

3   Model Calibration

We assign values to the parameters of preferences, technology, and fiscal policy so that the model's stationary competitive equilibrium matches key averages of the Chilean economy at annual frequency for the period 1992-2007, when all model shocks are set to their mean values. Except for some statistics for which we report our data sources, all other Chilean data are annual figures from the Central Bank of Chile (CBC). The results of our calibration are summarized in Table 1.

Our calibration strategy has five steps. The starting point is a few normalizations. Second, we take some parameter values from the related literature. Third, we impose a set of key macroeconomic ratios and the value of some model variables so as to mimic key Chilean macroeconomic and fiscal data. Fourth, we use the model equations to derive the values of some parameters. And fifth, Chilean data are used to jointly specify the model's dynamics and two model parameters.

The normalizations in the first step include setting the value of output (or GDP) $y = 1$; given the unitary endowment of time, we set $h = 0.3$ to match the well known fact that households spend around 1/3 of their time working. We also set $e^y = 1$, leaving the price of energy goods p^e to adjust to match the Chilean use of energy-to-GDP ratio. Finally, it can be seen from equation (10) that $p^k = 1$ in the steady state.

Next, we take parameter values from the related literature. Following Duval and Vogel (2008), de Fiore et al. (2006), and Backus and Crucini (2000), the elasticity of substitution in consumption between non-energy and energy goods is $(1+\eta)^{-1} = 0.09$. As we do not have an estimation of the Chilean e^h/e^y ratio, we take the U.S. ratio reported by Dhawan and Jeske (2008) which is equal to 0.88.

The remaining calibration steps are the most cumbersome. The third starts with the aggregate macroeconomic ratios. Our model mimics the Chilean annual average ratios to GDP of consumption, investment, and government expenditures, which are equal to 0.63, 0.21, 0.12, respectively, as well as the average 1992-2005 ratio $k/y = 2.13$. In the model, the ratio to GDP of the expenditures on energy goods is equal to $p^e(e^h + e^y)/y$; in the data, we obtain this ratio as the product $(M_e/M) \times (M/GDP)$, where M_e and M in the first term are the U.S. dollar values of imports of oil and total imports, and M and GDP in the second term correspond to the value in constant, domestic currency of imports and GDP. This procedure gives a Chilean ratio equivalent to the model's $p^e(e^h + e^y)/y$ ratio equal to 0.052. The average effective capital share in output, α, was equal to 0.36 during the period 1996-2007. Capacity utilization, u, is at 0.74 according to monthly data available for the period 2003-2005.

As we do not have information on the tax rates, we estimate them from data of the CBC and the Chilean Budgeting Office (Dipres). We calculate the consumption-tax rate following Mendoza et al. (1994) and obtain $\tau^c = 12\%$. Given the homogeneity of the production function, the steady-state value of the model's income tax revenue-to-GDP ratio is $TR^I/y = \tau^I(1 - \frac{\delta(u)k}{y})$; the data value of this ratio at 4.6% permits recovering $\tau^I = 5.9\%$; we compute the Chilean TR^I/y ratio using data on income-tax revenues (TR^I) and total tax revenues (TR); $TR^I/y = TR^I/TR \times TR/y$; then, the TR^I/y ratio, along with the model investment $i = \delta(u)k$, obtains $\tau^I = (TR^I/y)/(1 - i/y) = 0.06$. And the oil tax rate is estimated as follows. The Chilean Energy Agency repots tax burdens on the final domestic price of several fuels for the period 2000-2007; these tax burdens, along with the final fuel prices, permit recovering annual averages of implicit tax rates for several fuels. Then, using Chilean energy balance sheets, we calculate the weights of these fuels in total consumption of energy goods; $\tau^e = 49\%$ is obtained as the

weighted average of the implicit tax rates. We also use data from Dipres to compute the ratio of government debt-to-GDP ratio, (b^g/y), which is equal 0.15 on average for the period 1992-2007.

In our fourth calibration step we use the model equations in steady state and the parameter values found in the previous three steps. Once we have the values of τ^I, τ^e, α, and the values of the ratios i/y, k/y, $p^e(e^h + e^y)/y$, and e^h/e^y, we can find the values of A, β, and the depreciation rate $\delta(u)$. From the household's optimality condition for k in (18), along with the firm's optimality condition for effective capital (19), the homogeneity of the production function, and the optimal use of energy in equation (5), we can solve for β:

$$\beta = \left\{\left(1 - \tau^I\right)\left(\alpha - i/y\right) + k - \left(1 + \tau^e\right)\left[p^e(e^h + e^y)/y\right]\left(1 + e^h/e^y\right)^{-1}\right\}/k$$

Then, equation (16) implies that $q^i = \beta$. The model's steady-state investment gives $\delta(u) = (i/y)k$; the productivity scaling parameter is $A = y/h^{1-\alpha}\left(u_t k\right)^\alpha$. The normalizations of y and e^y, and the data ratios $p^e(e^h + e^y)/y$ and e^h/e^y determine the values of p^e and e^h. The aggregate resource constraint (12) implies that:

$$b^i = \left\{1 - [c + g_1 + \delta k + p^e(e^h + e^y)]\right\}/\left(q^i - 1\right)$$

and obtains $b^i = 0.4444$, while the government budget constraint (7) solves for $g_2 = 0.0175$ as follows:

$$g_2 = TR - g_1 + \left(q^g - 1\right)b^g$$

where TR is the tax revenue. The adopted value of g_2 is approximately equal to the difference in the data between fiscal outlays different from the expenditures included in g_1 (mainly subsidies and transfers) and non-traditional fiscal revenues (including those coming from cooper and other taxes). Equations (13) and (14) pin down the preference parameters ω and ϕ.

It remains to find the values of δ_0, δ_1, ν_0, v_1 and the capital adjustment cost parameter μ. The values of the latter two depend on the model's shocks and will be discussed in subsection 3.1 below. For a given value of ν_1, the values of the first three are found by solving a system of three nonlinear equations, namely equations (4) and (5) and the following:

$$\left(1 - \tau^I\right)\left(\alpha - \delta_1 i/y\right) = \left(1 + \tau^e\right)\left[p^e(e^h + e^y)/y\right]\left(1 + e^h/e^y\right)^{-1}v_1$$

where the latter is a reexpression of (15) that uses the properties of the production function and functional forms of $\delta(u)$ and $a(u)$.

The ad-hoc asset limits in (7) are $\underline{b}^h = -\bar{b}^h = b_{ss}^h - 0.2$ and $\underline{b}^g = -\bar{b}^g = b_{ss}^g - 0.1$, which are arbitrarily chosen to never bind in our numerical results.

3.1 Sources of Dynamics and the Values of ν_1 and μ

Our model's sources of dynamics are the cyclical government expenditures and shocks to the interest rate, oil-price, and productivity. The series of government expenditures, as well as the oil and interest-rate shocks are taken from the data, while the productivity shocks are backed out from the model to replicate the Chilean cyclical GDP.

Let x_{ss} and x_t denote the steady-state and time t values of x in the model, where x represents government expenditures, oil prices and the interest rate. From the values of the x_{ss}'s discussed above and summarized in Table 1 we obtain $x_t = x_{ss}(1 + x_t^c)$, where x_t^c is the cyclical component of x in the data, defined as the deviations in logs of the corresponding series from their corresponding HP trends.

The data sources used to derive x_t^c are the following. Government expenditures are from the CBC. Oil nominal prices are from the U.S. Energy Information Administration and we deflate them with the U.S. Producer Price Index reported by the U.S. Labor Bureau of Statistics. The implicit gross, world interest rate is $(1+r_{ss}^*) = 1/q_{ss}^i$. To get the shocks to this rate we first use annual data for the period 1999-2007 to compute $(1 + r^*) = (1 + r^{US})(1 + \rho)$, where r^{US} denotes the real three-month U.S. treasury bill rates (from the IMF International Financial Statistics) deflated with the U.S. expected inflation, and ρ is the country risk as measured by the Chilean EMBI spread. Expected inflation in period t is approximated as the simple average of the inflation rates in period t to $t-1$, as suggested by Uribe and Yue (2006) for quarterly data. For the period 1992-1998, the Chilean EMBI spread is not available and we approximate the Chilean interest rate with the Chilean lending rate in US dollars (from the IMF IFS).

For a specific value of ν_1; and given oil, interest and government expenditures shocks,

we first estimate $(\mu; \{A \exp(z_t)\}_{t=1}^{16})$ jointly so that we match the dynamics of the cyclical component of the Chilean GDP and the volatility of the Chilean investment. Then, using the μ estimated in the first step, we solve our competitive equilibrium model with just oil shocks and adjust the value of ν_1 so that oil shocks explain around 35 percent of the variability of the cyclical component of GDP estimated by Blanchard and Gali (2007) for G7 countries. With the new value of ν_1, we repeat the procedure referred above until the values of μ and ν_1 jointly satisfy the criteria explained above, we obtain $\nu_1 = 0.26$[9] and $\mu = 0.36$. This strategy makes that the model and Chilean cyclical GDP become identical.

These sources of dynamics of our model are shown in Figure 2. Figure 3, in turns, shows the model's GDP in levels, and the cyclical component of GDP, which by construction, matches the Chilean. Combining the information in the two figures, it can be seen that during the 1992-2007 period, the Chilean government expenditures displayed low volatility and procyclicality, increasing during the 1995-1997 boom and falling during the 2002-2003 slowdown. On the other hand, the standard deviation of the cyclical component of oil prices is equal to 0.17, more than 10 times higher than the standard deviation of the Chilean cyclical GDP, something that reflects the high volatility of the oil price in recent years. Constructing annual oil prices and shocks from average data, however, prevents seing the absolute peak prices that arise in high-frequency data; for this reason, the 2007 value of the series does not fully capture the recently observed hike in oil prices.

4   Oil Shocks in The Competitive Equilibrium

The government plays no active role in the competitive equilibrium of our model; it just collects fiscal revenues from the consumption, oil, and income taxes and spends on delivering the public goods g_{1t} and extending the transfers g_2. Whereas in the calibrated steady state, the primary fiscal surplus is exactly equal to the interest services the government pays for

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[9]Our calibrated value of ν_1 is much lower than the 1.66 used by Finn (2000). The deterministic nature of our model makes the indirect effect of the oil price much more powerful than in Finn's stochastic model. While in her model, the indirect effect depends on the persistence of the oil shocks, in our model (see equation (15)) households know exactly the price of oil in every period and are therefore able to adjust the utilization of capital with full response to the shocks. In our setting the elasticity of output with respect to an oil shock is approximately equal -0.04; if we use Finn's $\nu_1 = 1.66$ the elasticity rises to -0.18; in Kim and Loungani (1992), $\nu_1 \to 0$ and the elasticity falls to -0.01.

its stock of public debt (equal to 15% of GDP), activating the shocks and the expenditures gives the public debt a dynamics as the debt buffers the ensuing overall deficits and surpluses. Consequently, the final stock of public debt depends on the sequences of shocks and government expenditures.

As we are interested in the effects of oil shocks, we shut down all but the oil-price shocks and solve the competitive equilibrium to study their fiscal and macroeconomic consequences. In this equilibrium, an oil-price hike is fully passed through to the domestic economy and induces the household to increase its ratio (c_t/e_t^h) while lowering its consumption of both goods. The reason why (c_t/e_t^h) increases can be seen by reexpressing equation (14) using the CES form of the utility function:

$$\frac{c_t}{e_t^h} = \omega_t^{\frac{1}{1+\eta}}, \qquad \text{where } \omega_t \equiv \left[\frac{(1+\tau_t^e)p_t^e}{(1+\tau^c)\phi}\right]. \tag{22}$$

An increase in p_t^e raises ω_t and therefore the ratio on the left-hand side. On the other hand, recalling that $q_t^i = \beta$ in the steady state, the Euler equation (16) indicates that the marginal utility of consumption is constant when there are no interest-rate shocks; using (22), it can be shown that $U_{c,t} = c_t[1 + \omega_t(c_t/e_t^h)]$. Therefore, the increases in ω_t and (c_t/e_t^h) following an oil shock are consistent with a constant $U_{c,t}$ only if c_t falls. In sum, an oil shock pulls down the consumption of all goods and affects proportionally more to the consumption of energy goods.

As for the use of oil in production, two things happen. On one side, firms seek to economize on the use of oil by substituting it with capital and labor (the direct effect discussed earlier); on the other hand, it becomes convenient to reduce the utilization of capital to lower its associated requirement of oil.

The responses to an oil-price change of both, the utilization of oil and the oil-tax revenue, defined as $e_t \equiv e_t^h + e_t^f$ and $TR_t^e \equiv \tau_t^e p_t^e e_t$ respectively, depend on the price elasticity of the demand for oil, $\eta^e = (de_t/dp_t^e)(p_t^e/e_t)$. When τ_t^e is constant, for η^{TR^e} denoting the elasticity of TR_t^e with respect to the oil price, it is easy to see that $\eta^{TR^e} = (1 + \eta^e)$.[10] Thus, the value of η^e becomes crucial for determining the fiscal consequences of oil price shocks.

Using a partial-adjustment econometric model, Cooper (2003) estimates the short-run price

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[10]If the demand for oil is infinitely inelastic, that is $\eta^e = 0$, then a 1% change in p_t^e makes the TR^e to increase by 1%.

elasticity of the demand for oil in 23 countries.[11]   His average estimate is equal to -0.0486.

Solving our competitive model shutting all but the oil shocks and using the solution to estimate

Cooper's econometric model obtains an elasticity equal to -0.0792. These estimations imply

that in the model, a ten-percent hike in p_t^e leads to an approximate 9.2 percent increase in the

oil-tax revenue.[12]

To investigate further the effects of an oil shock in our artificial economy and to provide a

useful benchmark to understand the results of the next two sections, we solve the competitive

equilibrium with 18 periods and under two particular sequences of p_t^e. In both sequences p_t^e

remains at its steady-state value during the first 8 years, and then jumps up one standard

deviation in the 9th year. The sequences differ in years 10 to 18. In the first one, the price of

oil returns to its steady-state value, so that $p_t^e = 0.0276$ in every year but the 9th when it is

equal to 0.0323. In the second sequence, p_9^e is also equal to 0.0323, but then the deviations of

p_t^e from its steady-state value fade down according to a first-order autoregressive process with

persistence parameter equal to 0.5.

Figure 4 summarizes the responses of the competitive equilibrium of the artificial economy

to the one-time oil shock. The dynamics of the household's variables are the simplest ones.

When oil is more expensive in the 9th year, according to (22), the household consumes less

of all goods, including energy goods. In all other periods, before and after the oil shock, the

consumption of both goods remains constant. The access to perfect capital markets allows the

household to fully stabilize $U_{c,t}$ over the whole period while adjusting c_t and e_t in year 9 as

discussed before. Finally, work hours respond in a similar way than the use of oil in production,

as discussed next.

The dynamics on the production and fiscal side of the economy are more interesting and

encompasses two types of responses. One happens in period 9 when oil is more expensive and

the direct and indirect effect of the oil price in output kick in. In that period, the use of oil e_t^y

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[11]All countries in Figure 1 but Chile are included in Cooper's sample of countries; his point estimates of the elasticities are: U.S., -0.068; France, -0.069; Italy, -0.071; Spain, -0.087; Portugal, -0.023; New Zealand, -0.054; and Korea, -0.094.

[12]The correlation between TR_t^e and p_t^e equal to 0.99 and the value of this correlation in Chile is equal to 0.42; here, it must be noticed that while the quantity demanded of oil in actual countries is affected by several factors, here we are considering a version of our model where only the oil price is changing.

is reduced at its minimum level of the 18 years. That also requires bringing the utilization of capital to its own global minimum. And because the demand for capital is at its lowest level in year 9, the capital stock is at its minimum as well. The existence of capital adjustment costs and boundary constraints, however, prevents sudden adjustments in the capital stock and explains the "v" shape of the dynamics of k_{t-1}. The increment of an input price (oil) reduces the optimal level of output (i.e., GDP falls) and also induces the firm to substitute capital and labor for oil. Furthermore as, ceteris paribus, using capital becomes more expensive, firms reduce the utilization of capital, which is the indirect effect that helps to explain the fall in e_t^y and GDP in year 9.

The second type of responses of the economy to the oil shock is due to our perfect foresight assumption and the existence of capital adjustment costs. Agents anticipate the 9th year's shock and prepare in advance to weather its effect. Capital is adjusted gradually starting from period 2 and, as capital is falling, its utilization is increasing so as to avoid a large output fall when the oil price has not changed yet. The rise in utilization is, however, less important than the fall in the capital stock when it comes to explain the use of oil, and therefore e_t^y falls before the shock. Once the oil price returns to its steady-state value in period 10, the capital stock begins to recover slowly towards its steady-state value and so do e_t^y and GDP. The recovering capital stock again becomes more important for the demand for oil than the declining capital utilization.

On the fiscal side, as output and the use of energy in production are declining before the shock, fiscal revenues are shrinking as well. Although the consumption tax revenue is constant, the government collects less revenue from the income and oil taxes, specially from the latter. Declining revenues and constant expenditures give a positive slope to the stock of public debt during the first 8 years. At the shock year, however, public debt jumps down. The low price elasticity of the demand for oil makes p_t^e and TR_t^e to comove closely so that the increase in TR_9^e that follows that of p_t^e allows the government to reduce its debt.

Finally, international assets first rise because the current account surplus that follows the fall in the use of energy and investment (notice the fall in k_{t-1}) overcompensates the GDP

decline. The fall in period 10 is due to a household's portfolio re-adjustment aimed at undoing the increment in b_9^g. From period 9 on, there is a reversal in the current account that (qualitatively) mirrors the rise in periods 1 to 8.

Figure 5 shows the dynamics of the same variables included in Figure 4 when oil shocks display persistence. The qualitative responses are very similar to those arising in the one-time shock discussed in the precedent paragraphs. When the economy is subject to the oil shocks observed in the data, the responses are more difficult to track although the intuition behind the variables' dynamics remains unchanged. We will come back to this case in the next section when we discuss the Ramsey planner's management of the oil tax.

5 A Ramsey Approach to the Optimal Taxation of Oil

Here we take the viewpoint of the standard theory of public finance and formulate a restricted Ramsey planner problem where the planner takes the consumption and income tax rates as given and chooses over competitive equilibria. We assume that the government can commit itself to follow the optimal endogenous component of its fiscal policy. We appeal to the primal approach to optimal taxation and find the sequence of optimal oil tax rates and government bonds that attain the maximum household welfare given the government's expenditures and transfers to be financed with tax revenues.

Operatively, the problem is to choose a feasible allocation that maximizes:

$$\sum_{t=0}^{T} \beta^t \left[\log(\hat{c}_t) + \omega \log(1 - h_t) \right], \tag{23}$$

subject to (a) an implementability constraint built from the household's and firm's optimality conditions of the competitive equilibrium, (b) the initial and terminal conditions on oil taxes, and (c) four additional constraints arising because we are dealing with an "incomplete" tax system.

The implementability constraint is standard (see Chari and Kehoe, 1998) and gives:

$$
\begin{aligned}
- & \sum_{t=0}^{T} \beta^t \left(c_t U_{c,t} + h_t U_{h,t} + e_t^h U_{e,t} \right) + \sum_{t=0}^{T} \beta^t \frac{U_{c,t}}{1+\tau^c} g_2 \\
+ & \frac{U_{c,0}}{1+\tau^c} \left\{ k_{-1}[(1-\tau^I)\left(F_{k,0}u_0 - \delta(u_0)p_0^k + p_0^k - F_{e,0}a(u_0)\right) + b_{-1}^i + b_{-1}^g] \right\} \\
- & \frac{U_{c,T}}{1+\tau^c}[q_T^i(b_T^i + b_T^g) + p_T^k k_T] = 0
\end{aligned}
\tag{24}
$$

where we use the equilibrium condition (20) at $t = 0$ when we write $F_{e,0}$.

If we do not restrict the choices of τ_t^e in $t = 0$ and $t = T$, the government would choose τ_0^e and τ_T^e to indirectly tax heavily the initial and final capital stock. The following constraints prevent that behavior:

$$
\begin{aligned}
U_{c,-1}p_{-1}^k = \beta \Big\{ U_{c,0} \Big[(1-\tau^I)(F_{uk,0}u_0 - \delta(u_0)p_0^k) + \\
p_0^k - F_{e,0}a(u_0) \Big] \Big\}]
\end{aligned}
\tag{25a}
$$

$$
\begin{aligned}
U_{c,T}p_T^k = \beta \Big\{ U_{c,T+1} \Big[(1-\tau^I)(F_{uk,T+1}u_{T+1} - \delta(u_{T+1})p_{T+1}^k) + \\
p_{T+1}^k - F_{e,T+1}a(u_{T+1}) \Big] \Big\}]
\end{aligned}
\tag{25b}
$$

where the allocations dated at $t = -1$ and $t = T + 1$ are the ones belonging to the competitive equilibrium. Constraints (25) prevent that in periods 0 and T, when the shocks are at their steady-state values for $t \in [-1, T+1]$, the oil tax rate must also take its steady-state value at $t = 0$ and $t = T$. Furthermore, to make the results of the Ramsey planner comparable with those of the competitive equilibrium, we force the planner to have the same initial and terminal values of k, b^g, and b^i as in that equilibrium, that is the boundary conditions $\{k_{-1}, k_T, b_{-1}^g, b_T^g, b_{-1}^i, b_T^i\}$ that faces the Ramsey planner are the optimal allocations from the competitive equilibrium at periods $t = -1$ and $t = T + 1$.[13] This will allow us to compare two endogenous fiscal policies whose starting and finishing conditions are the same across regimes so that when one delivers a higher welfare than the other, all welfare gains can be attributed to the endogenous components of the fiscal policy. It must be notice that the Ramsey problem will be solved for $t \in [0, T]$, while the competitive equilibrium will be obtained for $t \in [-1, T + 1]$, the reason is

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[13]Operatively, we substitute these restrictions for the variables dated at 0 and T in (24).

the inclusion of equations (25) as additional constraints in the Ramsey problem that are not imposed to the competitive equilibrium.

As we are dealing with an incomplete tax system, we still need to impose four additional constraints to the Ramsey problem. An incomplete tax system arises whenever for at least one pair of goods in the economy, the government has no policy instrument to drive a wedge between their marginal rate of substitution and the corresponding marginal rate of transformation (see Chari and Kehoe, 1998). The incompleteness of our tax system is due to the following: a) we have two uses of oil but a sole oil tax; b) the same fixed tax rate is applied to capital and labor in every period; c) there is a fixed tax rate on consumption. Only in presence of a complete tax system it is possible to back out the proper tax rates so that satisfying the implementability constraint implies satisfying the optimality conditions of the competitive equilibrium. In presence of an incomplete tax system other constraints are required to decentralize the allocations of the Ramsey problem as a competitive equilibrium.

First, we need an additional constraint because there is one oil tax and two uses of oil. To derive this constraint we focus on the the marginal rates of substitution (technical and in consumption) between energy and labor and their corresponding relative prices. On the production side, it can be seen from equation (19) that firms set $F_{e,t}/F_{h,t} = (1 + \tau_t^e)p_t^e/w_t$, where we use (15) to obtain $F_{e,t}$. Due to the income tax distortion, the household and the firm face different relative prices of energy to labor; in the household's case, it can be seen from (13) and (14) that optimality requires setting $-U_{e,t}/U_{h,t} = (1 + \tau_t^e)p_t^e/[w_t(1 - \tau^I)]$. Combining the household's and firm's equalizations of the marginal rates of substitutions to their respective relative prices permits obtaining the following additional constraint to the Ramsey problem:

$$-\frac{U_{h,t}}{U_{e,t}} = \frac{F_{h,t}}{F_{e,t}}(1 - \tau^I) \tag{26}$$

If the government were allowed to tax the two uses of oil at different rates the right-hand side of (26) would include the ratio of the two oil tax rates, say $(1+\tau_t^{e_y})/(1+\tau_t^{e_h})$. In that case, (26) would be a redundant constraint because any feasible allocation satisfying the implementability constraint (24) would be consistent with (26) for a proper choice of the $\tau_t^{e_y}$ and $\tau_t^{e_h}$.[14]

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[14]De-Miguel and Manzano (2006) also impose a constraint like (26) in one version of their model and they

Second, the absence of taxes on bonds and the constancy of the income tax rate requires imposing (16) (repeated here for convenience) and a version of (18) in every $t \in [0, T-1]$:[15]

$$U_{c,t}q_t^i = \beta U_{c,t+1}, \tag{27a}$$

$$\left[\frac{U_{c,t}p_t^k}{\beta U_{c,t+1}} - p_{t+1}^k + a(u_{t+1})F_{et+1} \right] \frac{1}{F_{uk,t+1}u_{t+1} - \delta(u_t)p_{t+1}^k} = \left(1 - \tau^I\right), \tag{27b}$$

where in the second equation we use $F_{uk,t} = r_t$ and $F_{e,t} = p_t^e(1 + \tau_t^e)$. Finally, because our consumption tax rate is constant we have to impose the following:

$$\frac{U_{c,t}}{U_{h,t}}F_{h,t} = \frac{1 + \tau^c}{1 - \tau^I} \tag{28}$$

where $F_{h,t} = w_t$.

We are now in conditions of defining the Ramsey problem.

DEFINITION 5. A *Ramsey problem with uniform oil-tax rates and fixed income- and consumption-tax rates*, shortly referred as the *restricted Ramsey problem*, chooses the competitive equilibrium that attains the maximum household lifetime discounted utility, i.e., it maximizes (23) subject to (12), (24) to (28), and the initial and terminal positions on bonds and the stock of capital of the competitive equilibrium.

5.1 The Ramsey Planner Stabilizes the Domestic Price of Oil

As common with most Ramsey-planner problems, it is difficult to track the intuition behind the planner's decisions from his optimality conditions. The difficulty arises because of our dealing with distorted economies; it is known from Lancaster and Lipsey's (1956) theory of the second best that: "in general nothing can be said about the direction or the magnitude of the secondary departures from optimum conditions made necessary by the original non-fulfillment of one condition". Dealing with a restricted planner problem makes the intuition deduction even more difficult due to the four additional constraints of our problem. Despite

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show that the optimal Ramsey allocation would lead to avoiding taxing the firm's utilization of oil, i.e., $\tau_t^{ey} = 0$, $\forall t \in [0, T]$.

[15]Correia (1996) considers a case where a constraint like (27a) has to be imposed because the government cannot tax the returns on international bonds; and Chari and Kehoe (1998) consider a case where a constraint like (27b) has to be imposed because the Ramsey planner has to choose a single τ^I for all periods.

these difficulties, we are able to compare the pass throughs of oil shocks of the competitive equilibrium with those of the Ramsey problem.

This is done in Figure 6, which shows the dynamics of the model under three oil-tax regimes when only the oil shocks are active; the first regime corresponds to the fixed tax rate or full pass-through, as in the competitive equilibrium; the second corresponds to the tax rates administered by the Ramsey planner; and the third one is the full stabilization regime or zero pass-through to be discussed in the next section.[16] The top-left pane of the figure shows that the gross price of oil, that is $p_t^e(1 + \tau_t^e)$, fluctuates more in the competitive equilibrium than in the Ramsey problem. During the 16-year period where oil-shocks are allowed to deviate from its steady-state value, the coefficient of variation of $p_t^e(1 + \tau_t^e)$ in the Ramsey problem is equal 0.78 percent of the value of that coefficient in the competitive equilibrium. In other words, the planner finds it optimal to avoid a full pass-through to the domestic economy of the world oil shocks and the domestic economy should face smoother prices of oil than what is implied by world prices.

The top-right pane of Figure 6 shows that the planner reduces the volatility of the domestic price of oil by setting an oil tax rate that is negatively correlated with oil shocks. To measure the reaction of the tax to the these shocks we regress the logarithm of the tax rate on a constant and the logarithm of the world oil price. The coefficient on the latter reveals that the optimal elasticity of the tax to the oil shocks is equal to -0.70; that is, every percentage point increase in the oil price in world markets should be approximately followed by a 0.70 percentage points reduction in the domestic tax rate on oil.

The panes in the second row of Figure 6 show that the optimal management of the oil tax stabilizes output and consumption fluctuations, a direct implication of the smoother gross price of oil that affects household's and firms' decisions (see, for instance, the dynamics of capital utilization and of oil used in production). What is more surprising is that public debt fluctuations are smaller under the planner's policy than under the full pass-through approach

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[16] Operatively we solve first the competitive equilibrium for $t \in [-1, T+1]$. Where $T = 18$, actual shocks on oil prices are present in $t \in [1, T-1]$ and remain in steady state for the rest of periods. Then, the Ramsey problem and the full stabilization regimes are solved for $t \in [0, T]$ using as initial and terminal conditions the optimal allocations found in the competitive equilibrium for $t = -1$ and $t = T+1$.

of the competitive equilibrium. Thus, if a restricted Ramsey planner would have managed the oil tax rate during the considered period, the economy would have started and finished with the same stock of public debt as the observed ones and would have smoothed out domestic fluctuations and the stock of public debt. The debt smoothing effect arises from the constancy of government expenditures and the enhanced stability of the three tax bases.

The results of this section challenge the view that either implementing oil-tax holidays in the aftermath of runaway world oil prices, or smoothing oil price fluctuations, damages the sustainability of the fiscal policy or generates macroeconomic instability. Furthermore, since we have solved a Ramsey problem, we also know that the household's welfare in the floating tax regime is not lower than what it is attainable in the competitive equilibrium. We quantify these welfare differences in the next section.

6   A Positive View of Oil Taxation as a Stabilization Policy

Implicit in our normative analysis of oil taxation of the precedent section and justifying its focus on a representative agent was the assumption of either the strict homogeneity of the households or the existence of complete financial markets that makes any household heterogeneity irrelevant. Lucas (1987) has shown that in this context, other things equal, stabilizing business cycles and therefore consumption does not raise significantly the welfare of the household. The Ramsey planner is therefore more concerned with the tax distortions than with the stability of business cycles.

To a large extent, the planner's concern contrasts with the goals of actual policymakers who are at least as much, if not more, interested in achieving macroeconomic stability as in minimizing tax distortions. Indeed, macroeconomic stabilization and price stability are the two most common goals pursued by actual central banks and this viewpoint extends beyond policymakers. Woodford (2003), for instance, observes the existence of "a fair amount of consensus in the academic literature that a desirable monetary policy is one that achieves a low expected value of a discounted-loss function, where the losses each period are a weighted average

of terms quadratic in the deviation of inflation from a target rate and in some measure of output relative to potential". This prominent role assigned to the stabilization policy motivates the study of optimal management of oil taxes from a positive perspective. In our nonmonetary model the focus should be on stabilizing output fluctuations and the question is what can fiscal policy do to stabilize an economy buffeted by oil-shocks.

Recalling that τ_{ss}^e and p_{ss}^e are the steady-state values of the oil price and tax rate, the government can fully neutralize the effect of oil shocks by designing tax policy such that $\tau_t^e = (1 + \tau_{ss}^e)(p_{ss}^e/p_t^e) - 1$. When oil shocks are the only driving force of business cycles, this policy of zero pass-through of oil-price shocks guarantees a full stabilization of aggregate fluctuations, as it is shown in Figure 6. The top-right pane of the figure shows that the tax rate should display the same qualitative responses as in the Ramsey problem but with a much wider amplitude. Now the elasticity of τ_t^e with respect to p_t^e is equal to -2.60, much higher in absolute value than the Ramsey elasticity at -0.70.

While the Ramsey oil taxes were designed to match the stock of public debt of the competitive equilibrium at periods 0 and T, the same matching is not possible here. The stock of public debt will increase or decrease over time depending on the oil shocks buffeting the economy. In the particular case of the 1992-2007 oil shocks that lead to the results in Figure 6, the stock of public debt falls over time. It is possible that under a different sequence of shocks, and/or under a longer time horizon than ours, the stock of public debt hits any of its bounds in (7). Although the study of this case is beyond the scope of this paper, it is possible to think that by changing the mean tax rates we could do this case away.

If fiscal sustainability is a major concern for the oil floating tax policy, it would be interesting to know the value of the elasticity of the tax rate with respect to the price of oil that less harms fiscal accounts. Extending our discussion on the elasticity of tax revenues held on page 18 to the case where τ_t^e is variable, it is easy to see that $\eta^{TR} = 1 + \eta^e + \eta^{\tau^e}$, where η^{τ^e} is the policy-determined elasticity of the oil-tax rate to the price of oil. If we consider the -0.05 average price-elasticity of demand estimated by Cooper (2003), we can see by setting $\eta^{TR} = 0$ that an oil taxation policy with $\eta^{\tau^e} = -0.95$ would make fiscal revenues approximately constant.

What is the efficiency cost of the zero pass-through policy? And, how worse than the Ramsey policy it can be? To answer these questions, we resort to a finite-horizon version of Lucas' (1987) welfare computations and ask how the three pass-through policies of the oil price (full pass-through, Ramsey pass-through, and zero pass-through) affect the welfare of the household in the considered period. We define the utility function W as:

$$W\left(\lambda, \{\hat{c}_t, h_t\}_{t=0}^T\right) \equiv \sum_{t=0}^T \beta^t \left\{\log[(1+\lambda)\,\hat{c}_t] + \omega \log(1-h_t)\right\},$$

and use $\lambda \in (-1,1)$ to compute the compensations in consumptions needed to equate the welfare of the household across the three policies. Next, we use an asterisk, a bullet, and a circle to refer to the variables arising from the Ramsey, the full, and the zero pass-through policies, respectively. We normalize the W of the full pass-through of the competitive equilibrium as $W^\bullet = W\left(0, \{\hat{c}_t^\bullet, h_t^\bullet\}_{t=0}^T\right)$, and find that the equalities

$$W\left(\lambda^*, \{\hat{c}_t^*, h_t^*\}_{t=0}^T\right) = W^\bullet = W\left(\lambda^\circ, \{\hat{c}_t^\circ, h_t^\circ\}_{t=0}^T\right)$$

are satisfied for $\lambda^* = -0.000008$ and $\lambda^\circ = 0.0004$. In words, the household would demand a 0.04 percent increase in the level of consumption to voluntarily accept the allocations of the zero pass-through policy instead of the allocations of the competitive equilibrium. On the other hand, the household would be willing to give up 0.0008 percent of its consumption if it is able to access to the allocations of the Ramsey policy..[17] From these results we can conclude that if there are benefits derived from macroeconomic stability that are not captured in full by a representative-agent economy, the efficiency costs that should be paid to reach those benefits are trivial. Certainly, the optimal policy in a tax-distorted economy requires avoiding a full pass-through of oil shocks into the domestic economy.

7 Concluding Remarks

Since the early 1970's, the volatility of oil prices has increased considerably and each significant positive shock has been followed by a renewed interest in designing macroeconomic stabilization policies capable of mitigating the effects of oil shocks. The discussions invariably

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[17]If we consider the welfare for the 16 periods where the oil price shocks are active we have $\lambda^* = -0.00017$ and $\lambda^\circ = 0.00027$

pore over what monetary policy can and cannot do. The reactions of monetary policy are, however, subject to both internal and external lags and can lead to undesired recessive or inflationary outcomes. On the contrary, we have shown in this paper that fiscal policy could be better endowed to fight against destabilizing oil shocks: oil floating taxes could do the job.

We have shown that in a restricted Ramsey problem with fixed income and tax rates, it is not optimal to fully passing through oil price shocks into an oil importing, small open economy. Oil tax rates should be reduced when oil is more expensive in international markets. This floating tax policy is perfectly consistent with fiscal sustainability. Our positive viewpoint of oil taxes, on the other hand, indicates that by a proper adjustment of the oil tax rates, the government could fully stabilize domestic prices and oil driven business cycles without rising serious concerns over the fiscal consequences of the policy.

We have obtained our results with a finite horizon, non-stochastic model where the demand for oil quickly adjusts to oil-price shocks. It is known that the short- and long-run elasticities of oil differ significantly due to the difficulties to substitute for oil in the short run. Bodenstein et al. (2007) incorporate these difficulties in a model where consumption, including oil, is subject to external habits and where it is also costly to adjust the oil intensity in the production of non-energy goods. Future research could investigate how the optimal oil taxes respond when the model takes another dose of realism at incorporating these substitution difficulties. And future research could also investigate how the results in this paper change in a stochastic setting when the government has only access to non-state contingent debt as in Aiyagari et al. (2002).

OPTIMAL OIL-TAX RATE UNDER INCOMPLETE MARKETS: A DIGRESSION FOR DEVELOPED AND DEVELOPING COUNTRIES

Leandro G. Andrián and P. Marcelo Oviedo

1   Introduction

In our previous essay, from now on Andrian and Oviedo (2009), we developed a model for an oil importing small open economy (SOE) where a Ramsey planner accommodates the oil-tax rate in order to reduce the pass through of the oil price swings. In that model, the Ramsey planner finds optimal to significantly reduce the oil-tax rate when an oil price hike has occurred. Among other results, the Ramsey planner changes in average 7 percentage points for each 10 percentage points that the oil price varies. Also, Andrian and Oviedo (2009) found that the welfare gains from switching from a competitive equilibrium where the government plays a passive role (i.e. the oil-tax rate is fixed) to the Ramsey's competitive equilibrium are minimal, which is a typical outcome obtained in real business cycle models (RBC) (see Lucas (1987)).

The paper of Andrian and Oviedo (2009) focused in a finite horizon problem with perfect foresight, so the oil price shocks where known in advance for the agents of the economy and the Ramsey planner chooses the whole sequence of tax rates from the beginning of the horizon period. Also, the Chilean case was chosen as a typical oil importing SOE. In this paper we introduce uncertainty and we expand the analysis in several dimensions. First, the oil price shocks are stochastic and they are the only source of uncertainty in this economy. Second, we study the optimal fiscal policy for two representative economies with significant differences:

developed and developing countries[1]. Finally, we explore the variations in the optimal tax rate and the dynamics of the public debt that arise when we consider a Ramsey problem under incomplete versus complete markets for public bonds.

The size of the fiscal sector and the tax structure between developed and developing economies differ significantly. For example, the implicit income tax-rates that we get in our calibration section shows that capital and labor are more heavily taxed in industrial countries than in the emerging ones. There is also a significant difference between advanced and emerging economies in terms of oil taxation; in average, the implicit tax-rates applied to gasoline and diesel in developed countries practically doubles the observed ones in developing countries. Thus, according to the International Fuel Prices report of 2007 and using own calculations, we find that developed economies apply an implicit tax-rate on fuels equal to 141%, while for developing economies this tax-rate is 77%. Our main numerical results show that the Ramsey planner will choose to decrease the oil-tax rate around 8.4 and 7.4 percentage points (industrial and emerging economies respectively) for each 10 percentage points that the price of oil increases in the first period.

We also solve our restricted Ramsey problem with complete markets for public debt. Our results show that this time, the social planner chooses an oil tax rate that varies in 9 and 7.7 percentage points (industrial and emerging countries respectively) for each 10 percentage points that the oil price changes in the first period. Despite of the additional complexities that arise in a Ramsey problem with an incomplete tax system, we are able to track some of the economic intuition that is behind the differences between the two models (i.e. complete and incomplete markets). Although the differences in the optimal tax rate are no as large as they seem, we find substantial differences in the optimal debt management. The exercise of comparing complete and incomplete markets is per se interesting; it explores, in this relative new topic of the literature of optimal taxation, how optimal taxes respond for a particular good when the Ramsey planner is constrained to maintain fixed the tax-rate of the other goods.

The rest of the paper is organized as follows. In the next section we present our oil importing

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[1]The terms rich, industrial and advanced will be used as synonymous of developed, while for developing the synonymous will be emerging and poor.

SOE model. We characterize its competitive equilibrium in Section 3 and we calibrate the model for both developed and developing countries in Section 3. Section 5 states the restricted Ramsey problem for incomplete markets and present our first results; and for the complete markets case the problem and the results are shown in Section 6. The sensitivity analysis is presented in Section 7. Section 8 contains some concluding remarks.

2 The Economy

We recast the model from Andrian and Oviedo (2009) for a small open economy with perfect competitive markets where there is an energy good (i.e. oil) that has to be imported at an exogenous price $p^e(s^t)$. Denote $s^t = (s_0, ..., s_t)$ the history of events up to including period t, and s_t is a particular state of the world at time t. The probability as of time 0 of any particular history s^t is denoted by $\pi(s^t)$. The only source of uncertainty in our model is the price of oil, p^e, that follows an AR(1) process:

$$\log(p^e(s^{t+1})) = (1 - \rho)log(p^e_{ss}) + \log(p^e(s^t)) + \varepsilon(s^{t+1}), \tag{1}$$

where p^e_{ss} is the steady-state value of p^e, $\rho \in [0,1]$ is the autocorrelation coefficient, and $\varepsilon(s^{t+1})$ is an error term with a normal distribution with mean zero and constant variance σ. The economy is inhabited by infinitely many identical households that rank consumption and work hours, $h(s^t)$, according to

$$\sum_{t=0,s^t} \beta^t \pi(s^t) [\log(\hat{c}(s^t)) + \omega \log(1 - h(s^t))], \tag{2}$$

where there is an endowment of time normalized to 1 to be divided between work and leisure. Here $\beta \in (0,1)$ is the discount factor and $\hat{c}(s^t)$ a consumption basket defined as a CES aggregator of non-energy and energy goods, denoted as $c(s^t)$ and $e^h(s^t)$ respectively, that takes the following form:

$$\hat{c}(s^t) = \left[(c(s^t))^{-\eta} + \phi(e^h(s^t))^{-\eta} \right]^{-1/\eta}$$

where $(1 + \eta)^{-1}$ is the elasticity of substitution between energy and non-energy goods and $\phi > 0$ is a share parameter. Hereafter, $U(s^t)$ denotes the time t and history (s^t) value of

the utility function between brackets in (2) and its partial derivatives are denoted by adding the corresponding variable to the subscript. The same notation is used below to refer to the production function and its derivatives.

Also, there exists a single firm owned by the household that produces non-energy goods by operating the following constant returns to scale technology:

$$y(s^t) = F(u(s^t)k(s^{t-1}), h(s^t)) = Ah(s^t)^{1-\alpha} \left(u(s^t)k(s^{t-1})\right)^{\alpha} \tag{3}$$

where $u(s^t)$ is the capital utilization rate; $k(s^{t-1})$ the aggregate stock of capital carried over from period $t-1$; A a productivity scale parameter and α the share of effective capital in output. As in Greenwood et al. (1988), the depreciation rate is an increasing, convex function of u. That is:

$$\delta(u(s^t)) = \frac{\delta_0 u(s^t)^{\delta_1}}{\delta_1}; \quad \delta(\cdot) \in (0,1), \ \delta_0 > 0, \delta_1 > 1 \tag{4}$$

Next, we follow Finn (2000) to link the capital utilization rate and the use of energy in production, $e(s^t)^y$, as follows:

$$e^y(s^t) = k(s^{t-1})a(u(s^t)), \ a(u(s^t)) = \frac{\nu_0 u(s^t)^{\nu_1}}{\nu_1}; \quad \nu_0 > 0, \nu_1 > 0 \tag{5}$$

Equation (5) states that using capital and its rate of utilization in production requires certain amount of energy, and this use of oil in production is increasing in both factors. Without loss of generality, we assume that it is the household and not the firm who chooses the optimal level of investment and capital utilization rate (and consequently the utilization of energy $e^y(s^t)$). The competitive firm, in turns, faces a market rental price of effective capital $r^k(s^t)$ and optimally decides the quantity of $u(s^t)k(s^{t-1})$ to rent in every period.

The third agent in our economy is a government that provides non-energy public goods g_1 and extend transfers g_2 to the household. The government collects fiscal revenues from taxing income and the consumption of energy and non-energy goods.

As in De-Miguel and Manzano (2006) and Andrian and Oviedo (2009) we assume that is the household who has access to an international bond credit market, $b^i(s^t)$, at an exogenous interest rate r. While the government trades a domestic bond, $b^g(s^t)$, with the household, the return that will pay a public bond issued at history s^t in period $t+1$ is a non state-contingent

interest rate $r^g(s^t)$. Hence, the government commits itself at history s^t to pay a pre-announced interest at period $t+1$ regardless the state of the world s_{t+1}.

In every period t and history (s^t), the household and the government must observe their budget constraints, respectively:

$$
\begin{aligned}
& I(s^t)(1-\tau^I) + b^i(s^{t-1})(1+r) + b^g(s^{t-1})(1+r^g(s^{t-1})) + g_2 \geq \\
& b^i(s^t) + b^g(s^t) + c(s^t)(1+\tau^c) + (e^h(s^t) + e^y(s^t))p^e(s^t)(1+\tau^e(s^t)) \\
& + i(s^t) + AC_k(s^t) + AC_{bh}(s^t),
\end{aligned}
\tag{6}
$$

and

$$
\begin{aligned}
& I(s^t)\tau^I + c(s^t)\tau^c + (e^h(s^t) + e^y(s^t))p^e(s^t)\tau^e(s^t) + b^g(s^t) \geq \\
& b^g(s^{t-1})(1+r^g(s^{t-1})) + g_1 + g_2 + AC_{bg}(s^t),
\end{aligned}
\tag{7}
$$

where $i(s^t) = (k(s^t) - k(s^{t-1}))$ denotes the household's gross investment and $I(s^t) \equiv w(s^t)h(s^t) + [r^k(s^t)u(s^t) - \delta(u(s^t))]k(s^{t-1})$ its taxable income. The wage rate and the rate of return of an effective unit of capital are $w(s^t)$ and $r^k(s^t)u(s^t)$. The terms $AC_k(s^t)$, $AC_{bh}(s^t)$ and $AC_{bg}(s^t)$ refer to adjustment costs in capital, private bonds and public bonds respectively, their functional forms are given by:

$$
AC_k(s^t) = \kappa_1/2(k(s^t) - k(s^{t-1}))^2, \kappa_1 > 0
\tag{8}
$$

$$
AC_{bh}(s^t) = \kappa_2/2(b^i(s^t) - \bar{b}^i)^2, \kappa_2 > 0
\tag{9}
$$

$$
AC_{bg}(s^t) = \kappa_2/2(b^g(s^t) - \bar{b}^g)^2
\tag{10}
$$

where $\bar{b}^i$ and $\bar{b}^g$ are the steady state levels of private and public bonds, respectively. As explained by Andrian and Oviedo (2009), the inclusion of capital adjustment costs responds to the necessity of distinguishing between financial and physical capital and to avoid a variability of the investment rate that largely exceeds what is observed in practice. Following Riascos and Vegh (2003) and Espada (2006) among others, the presence of adjustment costs in international and public bonds guarantees that the model has a stationary steady-state (see Schmitt-Grohé and Uribe (2003) for a detailed treatment of solving the problem of stationarity in small open economies models).

In addition to the their budget constraints, household and government must observe the following no-Ponzi-game conditions:

$$\lim_{t\to\infty}\prod_{j=0}^{t}(\frac{1}{1+r})b^i(s^{t+1}) \geq 0 \ \forall t, s^t, \qquad \text{and} \qquad \lim_{t\to\infty}\prod_{j=0}^{t}(\frac{1}{1+r^g(s^j)})b^g(s^{t+1}) \leq 0 \ \forall t, s^t, \quad (11)$$

Finally de economy as a whole is subject to the following aggregate resource constraint:

$$y(s^t) + b^i(s^{t-1})(1+r) + k(s^{t-1})(1-\delta(u(s^t))) \geq$$
$$b^i(s^t) + c(s^t) + (e^h(s^t) + e^y(s^t))p^e(s^t) + k(s^t)+ \qquad (12)$$
$$AC_k(s^t) + AC_{bh}(s^t) + AC_{bg}(s^t),$$

3 The Competitive Equilibrium

In this section we characterize the competitive equilibrium. In order to do so, we start with the following definitions:

DEFINITION 1. A *government fiscal policy* comprises "exogenous components" on one side, and a debt and an oil taxation policy on the other. The *exogenous components of the fiscal policy* are the fixed tax rates on income τ^I and consumption τ^c, the fixed transfer g_2, the government expenditures g_1 and the sequence of pre-announced interest rate $\{r^g(s^{t-1})\}_{t=0}^{\infty}$. The state-contingent sequence of oil tax rates $\{\tau^e(s^t)\}_{t=0}^{\infty}$ and the debt positions $\{b^g(s^t)\}_{t=0}^{\infty}$ are the *endogenous components of the fiscal policy*.

DEFINITION 2. A *price system* is a an exogenous stochastic process of the price of oil $\{p^e(s^t)\}_{t=0}^{\infty}$, an exogenous international interest rate r, and a state-contingent sequence of wage rates, rates of returns on capital and interest rates on $b^g(s^t)$ $\{w(s^t), r^k(s^t), r^g(s^{t-1})\}_{t=0}^{\infty}$.

DEFINITION 3. Given the initial financial asset positions $b^i(s^{-1})$ and $b^g(s^{-1})$, the initial stock of capital $k(s^{-1})$, and the exogenous stochastic process for the price of oil $\{p^e(s^t)\}_{t=0}^{\infty}$, a *state-contingent feasible allocation* is a sequence $\{c(s^t), k(s^t), h(s^t), e^h(s^t), e^y(s^t), u(s^t), b^i(s^t),$ $b^g(s^t)\}_{t=0}^{\infty}$ that satisfies the aggregate resource constraint (12) in every period t and history s^t.

Before defining the competitive equilibrium we characterize the household's and firm's optimality conditions. The household chooses processes $\{c(s^t), h(s^t), e^h(s^t), u(s^t)\}_{t=0}^{\infty}$ and $\{k(s^t), b^i(s^t), b^g(s^t)\}_{t=0}^{\infty}$ to maximize (2) subject to the sequence of budget constraints (6),

taken as given the price system, the fiscal policy, the initial conditions of the capital stock and the two bonds in the economy, and the no-Ponzi-game conditions eqs (11). The necessary and sufficient conditions for optimality are the budget constraint (6) holding with equality and the following:

$$-\frac{U_c(s^t)}{U_h(s^t)} = \frac{(1 + \tau^c)}{w(s^t)(1 - \tau^I)} \tag{13}$$

$$\frac{U_c(s^t)}{U_e(s^t)} = \frac{(1 + \tau^c)}{p^e(s^t)(1 + \tau^e(s^t))}, \tag{14}$$

$$p^e(s^t)(1 + \tau^e(s^t))a'(u_t) = \left(1 - \tau^I\right)\left[r^k(s^t) - \delta'(u(s^t))\right], \tag{15}$$

$$U_c(s^t)(1 + \kappa_2(b^i(s^t) - \bar{b}^i)) = \beta E U_c(s^{t+1})(1 + r), \tag{16}$$

$$U_c(s^t) = (1 + r^g(s^{t-1}))\beta E U_c(s^{t+1}), \tag{17}$$

$$U_c(s^t)(1 + \kappa_1(k(s^t) - k(s^{t-1}))) = \beta E \left\{ U_c(s^{t+1}) \left[\left(1 - \tau^I\right) \left(r^k(s^{t+1})u(s^{t+1}) - \delta(u(s^{t+1}))\right) + \right. \right.$$

$$\left. \left. 1 + \kappa_1(k(s^{t+1}) - k(s^t)) - p^e(s^{t+1})(1 + \tau^e(s^{t+1}))a(u(s^{t+1}))\right]\right\}, \tag{18}$$

According to (13), the marginal rate of substitution of consumption for leisure equates the relative price of consumption in terms of labor. In (14) the marginal rate of substitution of non-energy for energy goods equates the relative price of these goods. The marginal benefit and marginal cost of increasing the utilization of capital are equated in equation (15); the cost comes from the extra oil necessary to raise utilization when oil is priced at $p^e(s^t)(1 + \tau^e(s^t))$, and the benefit is the income (net of tax and depreciation) from renting an additional unit of effective capital. Equations (16) and (17) are the standard Euler conditions for international and domestic bonds respectively. It is worth to noting that arbitrage implies that $1 + r^g(s^{t-1}) = (1 + r)/(1 + \kappa_2(b^i(s^t) - \bar{b}^i))$, i.e. the gross return of a public bond is equal to the gross return of an international bond adjusted by the effect of household's adjustment costs on b^i. Finally, equation (18), the Euler equation for capital, equates the loss of utility at buying a unit of capital (after the effect of capital adjustment costs) on the left-hand side to the expected discounted utility gain at $t+1$ of owning and renting an extra unit of capital given the utilization decisions at times t and $t + 1$.

Switching to the production side of our model, at time t history s^t, the firm equates the marginal productivity of the factors of production to their respective market prices:

$$F_h(s^t) = w(s^t), \qquad \text{and} \qquad F_{uk}(s^t) = r^k(s^t), \tag{19}$$

and the linear homogeneity of the production function makes profits equal to zero.

Notice that denoting the marginal product of oil by $F_e(s^t)$, optimality requires setting $F_e(s^t) = p^e(s^t)(1 + \tau^e(s^t))$. Combining this with the household optimality condition (15), we obtain the following equilibrium condition:

$$F_e(s^t) = p^e(s^t)(1 + \tau^e(s^t)) = (1 - \tau^I)[r^k(s^t) - \delta'(u(s^t))]/a'(u(s^t)), \tag{20}$$

which is used in Sections 5 and 6 to characterize the restricted Ramsey problem.

The competitive equilibrium of our model is defined as follows:

DEFINITION 4. Given (a) the initial conditions of the capital stock and the two bonds, and (b) the exogenous stochastic process of oil prices $\{p^e(s^t)\}_{t=0}^{\infty}$, a *competitive equilibrium* is a state-contingent feasible allocation, a state-contingent government fiscal policy, and a price system such that:

i. The household maximizes (2) subject to (6) holding with equality, the initial conditions on the stock of capital and bonds, and equation (11); in the solution, equations (13) to (18) hold.

ii. The firm employs the profit-maximizing allocations of efective capital $(u(s^t)k(s^{t-1}))$ and labor, that is, equations (19) hold.

iii. The government's budget constraint (7) holds with equality in every period t and state of the world s_t. This condition, along with the equality of the household budget constraint (6), implies that the aggregate resource constraint (12) also holds with equality in every period and state of the world.

iv. The assets' limits on (11) are never binding.

4 Model Calibration and Numerical Method

We calibrate the model for both developing and developed countries. The economies belonging to each category are classified according to the income classification from the World Development Indicators (WDI). From the energy balances of 2006 reported by the International Energy Agency (IEA), we classified as importing oil economies those countries with more than 90 percent of the Total Primary Energy Supply[2] from crude oil and petroleum products were imported[3].

The calibration strategy is similar to the one used in Andrian and Oviedo (2009). In fact, our criterium is to mimic annual average values observed in developed and developing countries during the period 1990-2008 so that the model's stationary competitive equilibrium matches key averages of both types of economies. Except for some statistics for which we report our data sources, all other data are annual figures from the WDI. The results of our calibration are summarized in Tables 2 and 3.

Our calibration strategy has four steps. The starting point is a few normalizations. Second, we take some parameter values from the related literature. Third, we impose a set of key macroeconomic ratios and the value of some model variables so as to mimic key macroeconomic and fiscal data from both developing and developed countries. And fourth, we use the model equations to derive the values of some parameters.

For both economies, the normalizations in the first step include setting the value of output (or GDP) $y = 1$; given the unitary endowment of time, we set $h = 0.3$ to match the well known fact that households spend around 1/3 of their time working. We also set $p^e = 0.025$. The parameter α is equal to 0.35. In steady state we assume that the depreciation rate, δ_{ss}, is

---

[2]Total Primary Energy Supply is defined as: domestic production + imports - exports - international marine bunkers +/- stock changes.

[3]The selected developed countries are: Austria, Belgium, Cyprus, Finland, France, Germany, Greece, Hong Kong, Israel, Iceland, Ireland, Italy, Japan, Korea Republic of, Luxembourg, Malta, Netherlands, New Zealand, Portugal, Singapore, Slovenia, Spain, Sweden and Switzerland. While the selected developing countries are: Armenia, Bangladesh, Belarus, Benin, Bosnia and Herzegovina, Botswana, Bulgaria, Cambodia, Chile, Costa Rica, Croatia, Czech Republic, Dominican Republic, El Salvador, Eritrea, Estonia, Ethiopia, Georgia, Ghana, Guatemala, Haiti,Honduras, Hungary, India, Jamaica, Jordan, Kenya, Korea Democratic People's Republic of, Latvia, Lebanon, Lithuania, Macedonia, Moldova, Mongolia, Morocco, Mozambique, Namibia, Nepal, Nicaragua, Panama, Paraguay, Philippines, Poland, Senegal, Serbia, Slovak Republic, South Africa, Sri Lanka, Tanzania, Togo Turkey, Uruguay, Zambia and Zimbabwe

equal to 0.1.

Next, we take parameter values from the related literature. Following Backus and Crucini (2000), Duval and Vogel (2008), and de Fiore et al. (2006), the elasticity of substitution in consumption between non-energy and energy goods is $(1 + \eta)^{-1} = 0.09$. As we do not have an estimation for both types of countries of the e^h/e^y ratio, we take the U.S. ratio reported by Dhawan and Jeske (2008) which is equal to 0.88.

The third step starts with the aggregate macroeconomic ratios. Our model mimics, in average and for the selected countries, annual average ratios to GDP of consumption, investment, government expenditures, and public debt, which are equal to 0.25, 0.56, 0.19 and 0.61 for industrial countries, and for developing countries these average ratios are 0.22, 0.72, 0.15 and 0.53. In the model, the ratio to GDP of the expenditures on energy goods is equal to $p^e(e^h + e^y)/y$; in the data and according to U.S. Energy Information Administration, the annual oil consumption to GDP at 2000 U.S. dollars is 0.024 and 0.045 for developed and developing countries respectively.

As in Andrian and Oviedo (2009), we calculate the consumption-tax rate following Mendoza et al. (1994) and obtain τ^c equal to 13% and 11% for developing and developed economies, respectively. Given the homogeneity of the production function, the steady-state value of the model's income tax revenue-to-GDP ratio is $TR^I/y = \tau^I(1 - \frac{\delta(u)k}{y})$; the data value of this ratio for industrial countries is 8.2% while for developing countries is 4.4%. The values of this ratio permit recovering $\tau^I = 11.0\%$ and $\tau^I = 5.6\%$ for industrial and emerging economies. The oil tax rate is estimated as follows. First, using the International Fuel Prices 2007 Report we obtain for the selected countries the after-tax prices for diesel and gasoline at 2006 U.S. dollars, we define these prices as the domestic price and equal to $p_d^e = p^e(1 + \tau^e)$. Second in order to obtain the prices of both fuels net of taxes (p^e), we take the U.S. prices of gasoline and diesel and we subtract the federal and state taxes (available in the Tax Foundation). And third, we obtain the oil-tax rate for each country as $\tau^e = p_d^e/p^e - 1$. Averaging across countries and fuels we have that τ^e is equal to 141% and 77% for developed and developing countries, respectively. The parameters that govern the rate of growth of p^e (equation (1)) were obtained

as the autocorrelation ($\rho = 0.73$) and standard deviation ($\sigma = 0.013$) of the HP filter cycle for the annual period 1990-2008 of the price of oil deflated by the U.S. CPI.

In our fourth calibration step we use the model equations in steady state and the parameter values found in the previous three steps. In steady-state we have that $i = \delta(u)k$, using the ratio i/y, and $\delta_{ss} = 0.1$ we get that k is equal to 2.53 and 2.22 for industrial and emerging countries respectively. Once we have the values of τ^I, τ^e, α, and the values of the ratios i/y, k/y, $p^e(e^h + e^y)/y$, and e^h/e^y, we can find the values of A, and β. From the household's optimality condition for k in (18), along with the firm's optimality condition for effective capital (19), the homogeneity of the production function, and the optimal use of energy in equation (5), we can solve for β:

$$\beta = \left\{ \left(1 - \tau^I\right)(\alpha - i/y) + k - (1 + \tau^e)\left[p^e(e^h + e^y)/y\right](1 + e^h/e^y)^{-1} \right\}/k$$

The value of β is 0.98 for developed and 0.97 for developing economies[4]. Then, equations (16) and (17) imply that $r = 1/\beta - 1 = r^g$. The productivity scaling parameter is $A = y/h^{1-\alpha}(u_t k)^\alpha$. The normalizations of y and p^e, and the data ratios $p^e(e^h + e^y)/y$ and e^h/e^y determine the values of e^y and e^h. The aggregate resource constraint (12) implies that:

$$b^i = \left\{1 - [c + g_1 + \delta k + p^e(e^h + e^y)]\right\}/(r)$$

and obtains $b^i = 1.20$ and $b^i = 3.99$ for developed and developing economies respectively, while the government budget constraint (7) solves for $g_2 = -0.011$ and $g_2 = -0.019$ as follows:

$$g_2 = TR - g_1 - rb^g$$

where TR is the tax revenue from consumption, income and oil purchases. Equations (13) and (14) pin down the preference parameters ω and ϕ.

In order to find the values of δ_0, δ_1, ν_0 and ν_1 we follow Finn (2000). There are two main steps to the solution here. First we solve for δ_1 and ν_1. Noting the properties of the production, δ and a functions, equation (15) in steady state can be written as:

$$p^e(1 + \tau^e)\nu_1 ak = (1 - \tau^I)(\alpha - \delta_1\delta_{ss}k) \tag{21}$$

---

[4]The values of the discount factor is consistent with the common view that emerging economies care less the future (see Mendoza (2006) for a detailed explanation).

Let us rewrite the functions δ and a as:

$$\delta = \hat{u}^{\delta_1}/\delta_1 \tag{22}$$

$$a = \hat{u}^{\nu_1}/\nu_1 \tag{23}$$

where $\hat{u}$ is a scalar (denoted by m) of u. Since we know the values of ak and $\delta_{ss}k$, equations (21) - (23) form a system of three nonlinear equations in the three unknowns: δ_1, ν_1 and $\hat{u}$. The value for δ_1 is equal to 1.25 and 1.42 for industrial and emerging markets. While the value of ν_1 is 0.98 and 0.78. In the second step we obtain δ_0, ν_0, noting that

$$\delta_0 u^{\delta_1}/\delta_1 = \hat{u}^{\delta_1}/\delta_1 \tag{24}$$

$$\nu_0 u^{\nu_1}/\nu_1 = \hat{u}^{\nu_1}/\nu_1 \tag{25}$$

$$m = \hat{u}/u \tag{26}$$

These last three equations may be solved to find m (not of importance), δ_0 and ν_0. The values of δ_0 are 0.17 and 0.20 for developed and developing countries; and the values of ν_0 are 0.24 and 0.40. It remains to determine the values of the capital and bonds adjustment cost parameters, κ_1 and κ_2. These two parameters are set according to the literature. Thus, we set $\kappa_1 = 0.028$ as in Mendoza (1991) and Schmitt-Grohé and Uribe (2003); and $\kappa_2 = 0.1$ (close to the value of Riascos and Vegh (2003)). Regarding the value of the last parameter a sensitivity analysis will be conducted.

Finally, as in Riascos and Vegh (2003) and Espada (2006), once we have a recursive representation of the models (one for industrial countries and one for emerging countries), we are going to linearize them around their non-stochastic stationary steady states. Then, we will analyze the dynamic properties of such models using some statistics and the impulse response functions of the endogenous variables (measured as percent deviations from their steady state values) when a shock has occurred in period $t = 0$.

5   Optimal Taxation with Incomplete Markets

In this optimal taxation problem, the government chooses state-contingent feasible allocations that can be implementable as a competitive equilibrium. Formally, this Ramsey problem consists in maximizing (2) subject to (6), (7), (13) to (19), taking as given the initial conditions and the no-Ponzi-game conditions (11).

This restricted Ramsey problem has a non-recursive structure since decision rules will depend not only on the current state but also on future information based on agents' decision rules. This is because the imposition of equations (16) and (18) makes that the current allocations depend on plans for future variables[5]. Following Riascos and Vegh (2003) and Espada (2006) for the case of a SOE, we write the government's problem in a recursive framework by using the recursive contracts approach of Marcet and Marimon (1994), also used in Aiyagari et al. (2002) for optimal taxation with incomplete markets for a closed economy. In a non-recursive notation we have that the objective function of the Ramsey planner can be written as:

$$
\begin{aligned}
\sum_{t=0,s^t} \beta^t \pi(s^t) \{ & U(c(s^t), e^h(s^t), h(s^t)) \\
& + \mu_1(s^t)[U_c(s^t) - \beta E_t U_c(s^{t+1}) \tilde{R}(s^{t+1})] \\
& + \mu_2(s^t)[U_c(s^t) - \beta E_t U_c(s^{t+1}) R_k(s^{t+1})] \}
\end{aligned}
\tag{27}
$$

subject to (6), (7), (13) to (15), the initial conditions and (11). Where $\tilde{R}_{t+1} = (1+r)/(1+\kappa_2(b^i(s^t)-\bar{b}^i))$ and $R_k(s^t) = [(1-\tau^I)(r^k(s^t)u(s^t)-\delta(u(s^t)))+1+\kappa_1(k(s^t)-k(s^{t-1}))-p^e(s^t)(1+\tau^e(s^t))a(u(s^t))]/(1 + \kappa_1(k(s^t) - k(s^{t-1})))$. Using the law of iterated expectations, we can transform the government's objective function (27) to the following equivalent representation:

$$
\sum_{t=0,s^t} \beta^t \pi(s^t) \{ U(c(s^t), e^h(s^t), h(s^t)) + U_c(s^t)/(1+\tau^c)(\gamma_1(s^t) + \gamma_2(s^t)) \}
\tag{28}
$$

$$
\mu_1(s^t) = \tilde{R}_t(s^t)\mu_1(s^{t-1}) + \gamma_1(s^t), \mu_1(s^{-1}) = 0
\tag{29}
$$

---

[5]Equation (16) is the essence of the non-recursive nature of the Ramsey problem (see Aiyagari et al. (2002) for more detail). However, in our particular case, a fixed income-tax rate on capital imposes equation (18) as an additional constraint that generates an identical problem of non-recursion as the Euler for bonds.

$$\mu_2(s^t) = R_k(s^t)\mu_2(s^{t-1}) + \gamma_2(s^t), \mu_2(s^{-1}) = 0 \tag{30}$$

Notice that this problem does not have any future variables in the constraints and that all the functions in the constraints are known. Moreover, the solution to the original Ramsey problem must also be a solution to the problem of maximizing (28) subject to (6), (7), (13) - (15), (29) - (30), and taking as given the initial conditions and (11).

Once the problem is linearized around its steady state, we get the impulse response functions (IRF) and some variables' statistics. This framework allows us to describe the dynamics of the economy when the social planner adjusts period by period the oil-tax rate and the stock of public debt after an oil price hike has taken place. Starting with the first numerical exercise, figure 7 shows the dynamics of the optimal solution to this Ramsey problem using the IRF to a 1 percent increment in p^e[6] for both industrial and emerging countries; and tables 4 and 5 exhibit statistical moments for selected variables. These results are in line with ones of Andrian and Oviedo (2009). Comparing the response of the oil tax rate, we observe that if the price of oil increases 10%, the Ramsey planner finds optimal to decrease τ^e, at $t = 0$[7]. in 8.41% and 7.44% for developed and developing countries, respectively; that is, τ^e falls 12 and 5.5 percentage points for advanced and emerging economies, respectively. Given the large degree of pass through of oil price shocks that it is allowed in emerging economies, the allocations of such countries exhibit larger variability than the ones observed in industrial countries (see tables 4 and 5). Thus, at 1% increment in p^e, GDP decreases a 0.05% for developing countries, while in developed countries output falls 0.04%. Consumption of non-energy goods, c, displays a variation of -0.02% and -0.03% for developed and developing economies, respectively. Productive variables -such as rate of utilization of capital, stock of capital, and use of oil in production (e^y)- also fall with a higher drop in developing countries than in developed countries. Thus, u falls 0.075% and 0.088%, $k(s^t)$ decreases 0.014% and 0.03%, and e^y display percentage variations of -0.073% and -0.069% for developing and industrial economies respectively.

The behavior of the financial assets displays the other side of the coin regarding real allocations. Thus, the drop of tax revenues from income and consumption are overcompensated

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[6]Unless is strictly necessary we drop the time history (s^t) from the variables

[7]All comparisons will be made at the moment that the shock in p^e occurs, i.e. in $t = 0$.

by the climb of oil tax revenue, which it turns out in a rise of total tax revenues and a fall of the stock of public debt. However, the ratio public debt to GDP, $b^g(s^t)/GDP(s^t)$, increases 0.044% and 0.047% for rich and developing economies; that is because the fall in GDP is higher than the improvement registered in the stock of debt. Also, the household's international net asset position as percentage of GDP, $b^i(s^t)/GDP(s^t)$, displays an increment due to a current account surplus. This results from an improvement in the trade balance explained by a fall in both the domestic absorption $(c + g + i + AC_k + AC_{bh} + AC_{bg})$ and oil imports $(p^e(e^h + e^y))$ that overcompensate the drop in the GDP. The percentage increment for $b^i(s^t)/GDP(s^t)$ is 0.05% and 0.06% for developed and developing countries respectively.

Also, from tables 4 and 5 we observe several interesting results. First, as explained before, the optimal oil-tax rate for rich countries displays a higher volatility than for emerging countries. As a result of a higher pass trough that the Ramsey planner allows in developing economies, the optimal allocations $(GDP, c, e^y, u, k, b^i, b^g)$ exhibit a higher degree of variability compared to developed economies. Second, it is notable the large degree of correlation between real allocations and price of oil. Again, the higher response of τ^e in rich countries implies that the correlations between the allocations are lower than in poor countries.

It remains to answer what elements explain the difference in the optimal tax policies between both type of countries. We are going to try to develop an answer in sections 6 and 7, when we introduce an additional analysis in debt management and we conduct sensitivity analysis of some parameters and variables.

6   Optimal Taxation with Complete Markets and Debt Managment

There are several ways in which we can introduce complete markets for public bonds in our model. Complete markets can be modeled with a state-contingent interest rate (say $R_b(s^t)$) and a single bond (Chari et al. (1994) for a closed economy and De-Miguel and Manzano (2006) for SOE). Another alternative is by using state-contingent bonds $(b_g(s_{t+1}|s^t))$ that have to be bought at prices $q(s_{t+1}|s^t)$ (Aiyagari et al. (2002)). Besides, either the whole set of financial assets (private and public bonds) or just the bonds issued by the public sector can

be traded in complete markets[8]. Schmitt-Grohé and Uribe (2003), in a model without public sector, "close" a SOE with complete markets using state-contingent bonds because the shock affecting the economy is domestic (i.e. a productivity shock). Therefore, under the assumption of small economy, the household can completely hedge the domestic fluctuations by issuing the right amount of state-contingent bonds for each state of the world. This fact implies that the marginal utility of consumption is constant across states and time, "closing" the small open economy. Using this approach Riascos and Vegh (2003) arrive at the same outcome of a constant marginal utility of consumption as Schmitt-Grohé and Uribe (2003) for an optimal taxation problem with complete markets for a small open endowment economy. In Riascos and Vegh (2003), the shocks are of a domestic nature and there is only one consumption-tax rate. In our case, the oil price shock is a foreign type. The empirical evidence shows that oil price hikes affect negatively the whole world (or at least a non trivial number of countries) as recently documented by Blanchard and Gali (2007) and Hamilton (2008), among others[9]. Therefore, domestic economies cannot completely hedge from the oil price swings. In consequence, we follow De-Miguel and Manzano (2006) and we assume that only the government can issue bonds with a state-contingent return.

We introduce a complete markets environment following Chari et al. (1994) for a closed economy and De-Miguel and Manzano (2006) for a SOE. In particular, the government is allowed to pay a state-contingent return of bonds, $r^g(s^t)$, over the debt issued in the precedent period, $b^g(s^{t-1})$. Hence, the household's and government's budget constraints are modified as follows[10]:

$$
\begin{aligned}
& I(s^t)(1 - \tau^I) + b^i(s^{t-1})(1 + r) + b^g(s^{t-1})(1 + r^g(s^t)) + g_2 \geq \\
& b^i(s^t) + b^g(s^t) + c(s^t)(1 + \tau^c) + (e^h(s^t) + e^y(s^t))p^e(s^t)(1 + \tau^e(s^t)) \\
& + i(s^t) + AC_k(s^t) + AC_{bh}(s^t),
\end{aligned}
\tag{31}
$$

---

[8]An interesting work is the one of Angyridis (2007), where he model a SOE with complete markets in international bonds and a public domestic bond that is issued in incomplete markets

[9]see Andrian and Oviedo (2009) for a detailed review of this literature

[10]Although convergence is ensured in public bonds due to the complete markets environment, we kept positive adjustment costs, $AC_{bg}(s^t)$, so the results of this model can be fully compared with the ones of incomplete markets.

and

$$I(s^t)\tau^I + c(s^t)\tau^c + (e^h(s^t) + e^y(s^t))p^e(s^t)\tau^e(s^t)$$
$$+ b^g(s^t) \geq b^g(s^{t-1})(1 + r^g(s^t)) + g_1 + g_2 + AC_{bg}(s^t) \tag{32}$$

The first order condition for public bonds becomes[11]:

$$U_c(s^t) = \beta E U_c(s^{t+1})(1 + r^g(s^{t+1})), \tag{33}$$

In this complete markets environment, the Ramsey planner maximizes equation (2) subject to the households's optimality conditions (equations (13) to (16)), equation (18) and equation (33), the firm's optimality conditions (equation (19)), the agents' budget constraints (equations (31) and (32)), the initial conditions in the stock of the two bonds and capital, and equation (11).

Figures 8 (industrial countries) and 9 (emerging countries) compare the IRFs of selected variables for incomplete versus complete markets. For both types of economies, there are remarkable differences between the adjustment path that displays both types of models. In fact, the initial percent deviation of τ^e respect its steady state is, in absolute values, higher in the complete markets case than the incomplete markets case (0.04 and 0.03 percentage points for developed and developing countries, respectively). That is, for each 10 percent increment in the price of oil, oil-tax rate falls 8.9 and 7.7 percent for developed and developing countries, respectively; This higher response of the oil-tax rate to a hike in the oil price for complete markets shows that real allocations, such as GDP and the ratio c/e^h, depict a smoother path trough their steady state values. In this way, we observe that the volatility of GDP, consumption on non-energy goods and the ratio c/e^h are lower in complete markets compared to incomplete markets (see tables 6 and 7).

Another main difference between complete markets (CM) and incomplete markets (IM) relies in the public sector, in particular the public debt management. Using figures 8 and 9 and tables 6 and 7, we observe that while the primary surplus shows a deeper fall in CM -due to a higher drop in oil tax revenues compared to IM- the percentage variation in the total

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[11]note the slightly difference between complete and incomplete markets, in the case under analysis the government can choose at period t and history s^t the interest rate $r^g(s^t)$

surplus in the first period is positive. In fact, under CM the government "partially defaults" the interest payments in $t = 0$ (r^g decreases 0.0125 and 0.006 percentage points for developed and developing countries, respectively). The later allows the government not only to reduce more the pass through of oil price shocks (first pane in pictures 8 and 9) into the domestic economy in CM compared to IM (i.e. the response of τ^e in CM is bigger than IM), but also to generate an overall public surplus. After the first period, the dynamics of the public sector show that public debt increases again, but at a lower rate than in incomplete markets. The paths of b^g/GDP displayed in figures 8 and 9 may result inconsistent to the reader after the last explanations, but are in line with the aforementioned in the IM section. Thus, the percentage increment in this ratio obeys to the fact that GDP is falling at a higher rate than b^g in the first period. Then, when public debt increases, it does it at a higher rate than the recovery of output. Finally, both the variability and the autocorrelation displayed by the primary surplus and government debt are lower in CM.

7 Sensitivity Analysis

In this Section we conduct sensitivity analysis to evaluate how public debt and the optimal oil-tax rate respond when we vary some key steady-state variables and parameters. Primarily, the objective of this Section is to evaluate what parameters explain the wedge in the optimal fiscal policy between developed and developing countries. Therefore, only the parameters that have shown significant changes between the two country's models are shown. In particular we analyze some of the factors that affect the government's budget constraint, the adjustment cost on debt, and the tax structure. In order to simplify we use the developed economy with incomplete markets as the benchmark model. As it is common with most restricted Ramsey-planner problems, it is difficult to track the intuition behind the planner's decisions from his optimality conditions. Despite this fact, we are able to compare the pass through of oil shocks into the domestic economy of the different sensitivity analysis exercises with the one of the baseline case.

7.1 Sensitivity Analysis: Financial Adjustment Costs (κ_2)

What is the effect of the presence of adjustment costs in the optimal fiscal policy? How does debt management change if adjustment costs is much lower than the baseline case? Thus, our first experiment consists on modifying the financial adjustment cost parameter. We compare the benchmark case, where $\kappa_2 = 0.1$, with a smaller value of this parameter, $\kappa_2 = 0.0008$, which is closer to the values used by Schmitt-Grohé and Uribe (2003) and Espada (2006). Figure 10 displays the IRFs to 1 percentage increase in the price of oil. There is a monotonic relationship between oil-tax rate and κ_2; as the value of this parameter decreases, the optimal response of τ^e increases with the value of κ_2 (at $t = 0$ we have that τ^e falls 0.88% and 0.84% when κ_2 is equal to 0.0008 and 0.1 respectively).

Since the government smoothes distortions over time, a lower bond's adjustment cost allows the government to divert resources that instead of being used in adjustment costs are used in supporting a higher loss in oil-tax revenue (i.e. via reduction of τ^e). This additional drop of tax revenues is financed with a greater level of public debt compared to the benchmark case. Looking at tables 4 and 8 we note that real allocations display less variability when financial adjustment costs are almost nil. Moreover, given the functional form of $AC_{bg}(s^t)$, with a small value of κ_2 (i.e. small adjustment costs), the Ramsey planner is capable of rollover large levels of debt in every period.

There is a particular feature across the adjustment path of real allocations: even though the IRFs with $\kappa_2 = 0.0008$ display a smoother pattern than the IRFs with $\kappa_2 = 0.1$, they tend to cross each other. This phenomenon is explained by two effects that low adjustment costs has on the optimal fiscal policy. First, as we state before, time series tend to be smoother for small κ_2 explained by low degree of pass through of oil price shocks and less variability of marginal utility of consumption across time. Second, a low value of κ_2 enables the government to rollover an amount of debt different than the steady state value at a lower cost. Therefore, marginal utility of consumption tends to be different from its steady state value for a longer period of time compared to the benchmark case. Thus, we have lower variability but higher persistence in the adjustment paths.

7.2 Sensitivity Analysis: The Tax Structure

In this section we modify one by one the tax rates on income and oil[12] (τ^I and τ^e), so we can explore the effects of each tax-rate in the optimal response of the Ramsey planner to changes in the price of oil. It is worth to noting that by modifying some of these parameters will vary the steady state value of other parameters and endogenous variables.

7.2.1 Income-Tax rate (τ^I)

We reduce τ^I from its baseline value to the one reported for developing economies. Figure 11 shows the IRFs for selected variables. A decrement in τ^I does not produce significant changes of the optimal response of τ^e to shocks in the oil price. In fact, compared to the benchmark case, the percentage deviation of τ^e is 0.01 percentage points smaller (in absolute values) in the first period. Neither GDP nor consumption of non-energy goods evidence significant responses to change made in τ^{I}[13].

The production factors display almost imperceptible variations compared to the benchmark case. However, these changes follow the expected direction, ceteris paribus a small income tax-rate increases the rate of wage and the rate of return of effective capital. Therefore, GDP and productive factors decrease less to an oil price shock in this exercise than in the benchmark case. Figure 11 shows that the paths of these variables for a small τ^I are always above than the ones in the baseline case.

7.2.2 Sensitivity Analysis: Oil-Tax Rate (τ^e)

In this exercise, we reduce the steady state value of τ^e from 1.41 to 0.77. Figure 12 displays the IRFs for selected variables to 1 percent increment in p^e. We have that changing the steady state value of the oil-tax rate generate some differences between the benchmark case and the modified one. In fact, the IRF for τ^e displays a lower response compared to the baseline case (0.08 percentage points). This change in the optimal response of the Ramsey planner to hikes in the price of oil is remarkable similar to the results of Section 5 when the cases for industrial

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[12]The consumption-tax rate, τ^c, is not analyzed because it is very similar in both type of countries

[13]given the similarity of both type of models, we do not report the table of statistics.

versus emerging economies were studied. However, the adjustment paths of this economy to its steady state are different from the ones for emerging economies. In fact, the variables in this exercise trend to closely follow the behavior of the benchmark case. Also, tables 5 and 10 show that the variability of the real allocations is lower in this case than the benchmark case, given the higher response of τ^e at period 0.

7.3   Sensitivity Analysis: the oil requirement parameter ν_1

The last exercise consists on modifying the value of the parameter that affects the requirements of oil in production, e^y. Thus, we reduce the value of ν_1 from 0.98 (baseline case) to 0.78 (developing countries case). Here, the numerical solution shows that the optimal response of τ^e decreases in absolute value from -0.84% to -0.81%. As explained before, the higher pass through allowed in the modified case is reflected in higher variations in real allocations, such as consumption, GDP, capital, and rate of utilization of capital (see figure 13). However, the variability displayed in this case is smaller compared to the benchmark case (see table 11).

Looking at the results of Section 7.2.2, we can intuitively note that the effects of ν_1 and the steady state value of τ^e complement each other in reproducing the dynamics of the benchmark case. On one hand, while reducing the value of τ^e (keeping ν_1 at the baseline value) changes the optimal response of itself to the levels observed in developing economies, does not significantly modify the adjustment paths of the rest of the variables in the the model. On the other hand, a change in ν_1 (keeping τ^e at the baseline value) produces a lower response of the oil-tax rate, but the trajectories of the variables are markedly affected. Thus, we observe that the differences between developed and developing economies can be explained in some degree by differences observed in ν_1 and τ^e.

8   Conclusion

In this paper we extend the analysis made in Andrian and Oviedo (2009). In particular, we introduce uncertainty in the oil price shocks and we study the optimal fiscal policies under complete and incomplete markets for public debt. We have motivated our analysis wondering

how the Ramsey planner responds to an oil price hike when the economy was either a developed or a developing country. We have centered this study on three facts. First, how the optimal policy rules vary from industrial to emerging economies; second, what model's parameter may help to explain such differences; and third, how the solutions for a Ramsey problem differ for complete and incomplete markets.

We found that the social planner in a developed country decrease the oil-tax rate by 8.5% for every 10% oil price hike, while for a developing country the optimal response is 7.4%. Another finding shows that the differences between incomplete and complete markets were not as significant as we expected. However, the exercise was per se interesting. By taking advantage of the possibility of "partially" default its debt and lowering in a higher degree the oil tax-rate, the Ramsey planner in a complete markets environment decreases more the pass through of oil price shocks than the planner in incomplete markets.

Future research could investigate how oil should be taxed in a less restrictive Ramsey problem. That is, the model should be expanded so the social planner have the possibility of modifying more than one tax rate levied on other goods (De-Miguel and Manzano (2006)) with an incomplete markets environment for public debt.

ETHANOL PLANT INVESTMENT UNDER UNCERTAINTY

Leandro G. Andrián and John Miranowski

1 Introduction

Before 2005, most dry-mill ethanol plant investments were made to support up to 50 million gallons per year (MGY) capacity operations. Many of these plants were financed by local investors, who presumably were interested in capturing higher returns on their corn, land, businesses, and adding value and jobs to the local community. After 2005 and 2006, an economic evolution occurred in the ethanol industry, with characteristics including: increasing plant size, increasing output per worker, and ultimately, decreasing employment per gallon of ethanol output. Larger capacity ethanol plants have smaller local job creation impacts per gallon of production than smaller sized plants (see Miranowski et al. (2008) and Rajagopal and Zilberman (2007) for a detailed literature review on these topics). We anticipate that most future ethanol plant investments, in contrast to pre-2005 investments, will be larger scale plants, primarily in the 100-120 MGY plant size.

We motivate this work by asking the economic factors driving the decision to invest in an ethanol plant. In this analysis we evaluate the decision to invest in a 100MGY plant. In order to accomplish this task, we will start with the rationale behind the investment in an ethanol plant, where ethanol is traded in a competitive market. We model the investment decision for an ethanol plant under risk using a real options framework. The advantage of this approach over the traditional net present value (NPV) approach is that it provides a systematic way of capturing price fluctuations and incorporating this information into the analysis of the optimal timing of investment under risk. Alternatively, it compares whether to invest now or to wait

for more information, given the current evolution of market prices and quantities. Therefore, this method is superior to NPV because it not only captures the costs and benefits of waiting to invest, but it also reflects the uncertainty associated with the investment project.

An issue that arises when we construct our model is whether we need to evaluate the optimal decision to invest in different production processes. There are two kinds of production processes in the corn-based ethanol industry, dry and wet mills. Wet mill plants require higher investment costs and tend to have very large capacity. Yet, in the last twenty years, there has not been a new wet mill plant constructed. Since the objective of this paper is to model the investment decision for current ethanol plant projects, we exclude the wet mill alternative.

An important contribution of this paper is the use of actual data to estimate capital costs and the cost function while accounting for the effects of economies of scale in production and diseconomies of transportation costs. Given the number of ethanol plant projects in construction or to be constructed in the 2006-2007 period, capital costs have increased dramatically. The increase in installation and construction costs changes capital costs in a significant way, and this fact has been underestimated in previous works. We use a standard revenue-cost spreadsheet (Swenson (2006), Tiffany and Eidman (2003)) and related literature (Miranowski et al. (2007) and Miranowski and Rosburg (2010)) to avoid underestimating costs and include the vast majority factors that influence the revenue and cost functions in the ethanol industry.

Related to the existing literature of ethanol plant investment (Schmit et al. (2008)), we incorporate in our model the effects of market structure, including the endogenous effect of firm entry and exit on ethanol gross margin (ethanol revenue minus purchases of corn). Among other things, we find that the threshold gross margin (2007 prices) that triggers entry of new firms in the market ranges between \$1.22 and \$1.67 and between \$0.30 and \$0.46 for exit of existing firms. The differences in the threshold gross margins is due to, among other things, the assumed functional forms, the assumed own elasticity demand for corn, variability of costs across firms, and how the stochastic shock affects the gross margin.

For robustness, we tested the results using different functional forms for the components of the gross margin as the gross margin may be affected by the stochastic shock (linear versus

non-linear effects). We also incorporate the possibility that the production of ethanol may affect the price of corn [1] (Gustafson (2002), Ferris and Joshi (2004), McNew and Griffith (2005) and Fortenbery and Park (2008)).

We conduct sensitivity analysis to understand the relevance of several of the assumptions made in the baseline case. From this analysis, we found that modeling the gross margin shocks as geometric Brownian motion was robust to all the specifications made in the baseline case and in the sensitivity analysis exercises. Finally, our numerical simulations using the existing capital and production subsidy schemes help explain why smaller sized ethanol plants were competitive with larger plants in the development of the current ethanol industry, and why the industry has evolved in a particular way.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 shows the empirical estimations used to calibrate the model. Section 4 shows the main results of the benchmark case calibrated in section 3. Section 5 contains sensitivity analysis. Section 6 contains a simple digression on the effects of capital and production subsidies in the ethanol industry. Finally, section 7 draws conclusions and discusses possible extensions.

2 The Model

2.1 Characterization of the Problem and Assumptions

The model is recast from the model of Dixit (1989b). In his model, Dixit analyzes the optimal number of foreign firms in a competitive domestic market for a particular good. Foreign firms face a price in foreign currency P equal to the exchange rate e times the residual demand function that they face $(p(q))$ denominated in domestic currency, where q is the residual supply that foreign firms face. Thus, the price of the good in foreign currency is $P = ep(q)$. The source of uncertainty is the exchange rate. Foreign firms face irreversible costs for entry and exit. In order to explain the optimal number of existing foreign firms in the market, Dixit solves the model looking at the threshold values of P that trigger entry or exit.

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[1]According to Fortenbery and Park (2008) the ethanol's domestic industry used a record 13 percent of domestic corn production in 2005 and a 20 percent in the 2006/2007 marketing year. See also, these authors for a detailed literature review of the effects of ethanol production in corn's prices.

The model of Dixit (1989b) is the particular interest because the mathematical and economic structure is identical to the model that we are trying to solve. In our model, instead of working with the price of a particular good and foreign firms, we use the per unit (gallon) gross margin of an ethanol plant.[2] The gross margin is defined as the price of ethanol per gallon plus co-product value (DDG[3]) less corn purchases. This variable is composed of a stochastic component and an endogenous element that is a function of the aggregate ethanol supply. As in previous literature we introduce uncertainty by a multiplicative shock on the gross margin (Dixit and Pindyck (1994) pp. 261-265, Leahy (1993) and Dixit (1989a)).

Because of the high correlation between the price of DDG[4] and the price of corn, we can express the price of DDG as:

$$P_{DDG,t} = \gamma_0 + \gamma_1 P_{c,t} \text{ with } \gamma_1 > 0, \tag{1}$$

where $P_{DDG,t}$ and $P_{c,t}$ are the observed prices of DDG and corn, respectively. Let x_{DDG} be the co-product per gallon of ethanol for DDG, and x_c be the input requirement per gallon of ethanol for corn (i.e. the quantities of DDG obtained and corn used per gallon of ethanol produced). Note that the contribution to the gross margin of DDG and corn is equal to $x_{DDG}P_{DDG,t} - x_c P_{c,t}$. Then, using equation (1), we can define two constants $X_c = x_c - \gamma_1 x_{DDG}$ and $\gamma_0 x_{DDG}$, such that $x_{DDG}P_{DDG,t} - x_c P_{c,t} = X_c P_{c,t} + \gamma_0 x_{DDG}$. Now, we define R_t as the gross margin in moment t and equal to:

$$R_t = Y_t \hat{R}(q_t) = Y_t(D(q_t) - X_c S_c(q_t) + \gamma_0 x_{DDG}) \text{ with } D'(q_t) < 0, S_c'(q_t) > 0 \tag{2}$$

where Y_t is the stochastic component that affects the gross margin (its characterization will be defined later), $\hat{R}(q_t)$ is the part of R_t that depends on the aggregate supply of ethanol (q_t), and $Y_t D(q_t)$ and $Y_t S_c(q_t)$ are the market demand for ethanol and the residual supply of corn,[5] respectively. Note that, at any instant t, the area under the demand function measures the social surplus in the ethanol market, and the area under the supply's function of corn measures

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[2]Schmit et al. (2008) also use the gross margin as the relevant variable in their analysis.

[3]DDG stands for distiller's dry grain and it is a substitute for corn in livestock feed.

[4]The sample correlation for period 1995-2008 is around 0.81.

[5]I.e. $Y_t S_c(q_t)$ is the residual supply of corn that face the ethanol's industry

the producer's revenue of corn that is obtained from the sales to the ethanol industry. Then, there are functions $U(q)$ and $\Pi(q)$ such that $U'(q) = Y_t D(q_t)$ and $\Pi'(q) = Y_t S_c(q_t)$. Following Dixit (1989b), the functional forms for $D(q_t)$ and $S_c(q_t)$ are:

$$D(q) = \psi_{d1} - q/\psi_{d2} \text{ and } S_c = D(q) = \psi_{s1} + q\psi_{s2}, \text{ with } \psi_{ij} > 0 \ i = d, s, \ j = 1, 2 \quad (3)$$

Before stating the problem and developing the model, we begin with a brief characterization of the ethanol industry and the stochastic process that affects R_t. The ethanol industry displays increasing returns to scale in the use of labor and capital. However, economies of scale are limited and eventually offset by the diseconomies of scale in transportation costs (especially in corn) at some scale or plant size. The optimal size of an ethanol plant is outside the scope of this paper and given the diversity of factors that may affect the optimal plant size not consider in this paper, we assume that a 100MGY[6] is the efficient scale of a plant (i.e. where total average costs are minimized). We assume that the rest of the inputs that compose the operating costs display a fixed coefficient technology that is independent of the scale of the plant. We assume that the plant lives forever. This assumption is not completely unrealistic given that an ethanol plant has a productive life of at least 20 to 30 years.

Operating costs (C) are assumed to be constant over time. Assuming that active firms produce at their annual capacity, the profit per gallon in a given year (t) is:

$$\pi_t = R_t - C, \quad (4)$$

Given the characteristics of market supply and demand for a homogeneous good with an increasing returns technology that is offset by increasing transportation costs, we treat the ethanol industry as competitive.[7]

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[6]From 2005 vast majority of ethanol plants that had been constructed or under construction are 100-120 MGY plants (see Renewable Fuels Association).

[7]There are other factors that lead us to model this market as a competitive industry. First, currently there are many plants installed in US and owned for several companies (according to the Renewable Fuels Association there are more than 170 operational plants in 26 states). Second, given the higher degree of substitution of ethanol with gasoline monopolistic and/or oligopolistic strategies are unlikely to happen in this market beyond government mandated use of ethanol (Aukayanagul and Miranowski (2009) show that the dynamics of the price of ethanol closely follows the price of gasoline or oil).

It is worth noting that the characterization and assumptions of the model imply that changes in the market supply of ethanol will only occur by entry of new firms or exit of active ones. However, since there is an optimal plant size, increments in the total supply of ethanol can only be characterized with discrete changes in q. We normalize 100MGY as one unit of output; thus, the number of firms, n, is equal to the total supply of ethanol, q.

In order to solve the model, we need to know how the stochastic component of the gross margin evolves over time, or equivalently, how the stochastic process is characterized. This type of process is modeled either as a geometric Brownian motion (GBM) or as a geometric mean-reverting process (GMR).[8] In order to obtain the process that best fits equation (2), we check for the presence of a unit root in an AR(1) model for Y. The presence of a unit root rules out the possibility that can be modeled as a GMR. We use the augmented Dickey-Fuller and the Phillips-Perron for unit root tests. In both cases, we cannot reject the null hypothesis that Y follows a random walk representation at a 90% percent confidence level (see Appendix 1). Therefore, we do not reject the presence of a unit root, and assume that the process that governs the stochastic shock follows the following GBM[9] representation:

$$dY = \alpha Y dt + \sigma Y dz, \tag{5}$$

Firms are assumed to have rational expectations about the underlying process of R. Also, we assume that exist a maximum number of (N) potential firms exist that can be operating in the industry. Finally, firms must incur a sunk cost I in order to produce one unit of output and must pay a cost E if they want to exit the industry. In principle $E < 0$, the exit cost can be positive if some part of the investment is not irreversible. We assume that $I + E > 0$, so we rule out, in terms of Dixit and Pindyck (1994), a "money machine" of rapid cycles of investment and abandonment. The assumption of irreversibility relies on the fact that most of the plant and other equipment is specific to the ethanol plant.[10] Also, instead of assuming risk neutral investors, we modify the model of Dixit (1989b) by incorporating a risk-adjusted

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[8]See Dixit and Pindyck (1994) for more details.

[9]Schmit et al. (2008) also found that the gross margin follows a GBM.

[10]Note that in this model, firms do not have the option to shut down and a restart and idle firm (see Schmit et al. (2008) for a model that considers this issue for the ethanol industry).

expected rate as in the model of Dixit and Pindyck (1994) pags. 215-219. Thus we impose:

$$\mu = r + \phi\rho\sigma_x, \tag{6}$$

where $\phi = (r_m - r)/\sigma_m$ is the market price of risk, with r_m and σ_m are the expected return and standard deviation of the market portfolio, the parameter ρ is the correlation between the asset under valuation and the market portfolio, and σ_x is the standard deviation of the asset. Following Dixit and Pindyck (1994) (pag. 149), let us define the convenience yield δ as:

$$\delta = \mu - \alpha, \tag{7}$$

So far we have described some aspects of the ethanol industry but have not stated the problem under question. That is, we want to establish the threshold values of R that triggers the decision to enter the market by investing in a new ethanol plant and the decision for an active firm to exit the market. Therefore, we find the entry gross margin that maximize the value of a new plant entering in a competitive market or the exit gross margin that maximize the value of an active plant leaving that market.

2.2 Solving the model

Except for the unique characteristics of the model such as the composition of the gross margin, the residual supply of corn and the inclusion of a risk-adjusted return as discount factor; the solution closely follows the explanation of Dixit (1989b). The nth firm's contribution to utility less corn revenues is defined as:

$$R_n = [U(n) - U(n-1)] - [\Pi_c(n) - \Pi_c(n-1)] + Y[n - (n-1)]\gamma_0 x_{DDG}, \tag{8}$$

Imposing $U(0) = 0$ and $\Pi_c(0) = 0$, define SS_n as the area under the demand curve for ethanol minus the area under the residual supply of corn up to the nth firm:

$$SS_n = U(n) - \Pi_c(n) = \sum_{j=1}^{n} R_j, \tag{9}$$

In a similar way, define W_n as the sum of the operating costs up to the nth firm:

$$W_n = \sum_{j=1}^{n} C_j, \tag{10}$$

With a constant C across firms equation (10) simplifies to $W_n = nC$. Since firms are competitive and have rational expectations, we can find the industry equilibrium by maximizing the expected discounted present value of the overall surplus of this industry:

$$E\left\{ \int_o^\infty \{SS_n - W_n\} e^{-\mu t} dt - \sum_i \left(I[\Delta n_i]_+ + E[\Delta n_i]_-\right) e^{-\mu i} \right\}, \tag{11}$$

where at instants $t = i$ the number of active firms change, and $[\Delta n_i]_+$ and $[\Delta n_i]_-$ denote an increase and a decrease of active firms, respectively. Following Dixit (1989b), the solution to this dynamic programming problem (equation (11)) can be seen as a sequence of option princing problems. We denote as $V_n(R)$ the value to the social planner when n firms are active.[11]

Thus, for each n we can regard the collection of the first n firms as an asset. The asset produces dividends in the form of the current flow of net surplus as well as capital gains since the value changes with the stochastic movements of R. Also, this asset is an option to buy other assets, namely collections of $(n + 1)$ and $(n - 1)$ firms or new entry or exit, respectively.[12],[13] The exercise prices of the respective options are I and E. At the values of R where it becomes optimal to exercise these options, $V_n(R)$ is linked to $V_{n+1}(R)$ and $V_{n-1}(R)$ by the standard option pricing. Thus, we need to determine $V_n(R)$ for all n simultaneously in order to solve the problem.

We use standard techniques for option pricing, constructing a portfolio with suitable combinations of the asset to be valued (in our case V_n) and the underlying asset that spans the

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[11]Dixit (1989b) solves the problem in terms of Y rather than R. However given the functional forms of R and the GBM follows by Y, the results are identical.

[12]Given the concavity of SS_n (equation (9)), the convexity of W_n (equation (10)) and the linearity of I and E, there is no need to consider transitions from n to $(n + i)$ or $(n - i)$ with $i > 1$ (see Dixit (1989b) for more detail).

[13]In this model firms do not have the option to mothball a plant and restart operations again like in Schmit et al. (2008). If this were the case, we will need a social planner that takes account of these options also. That is, the planner needs to consider opening a new plant versus restarting a mothballed one.

valued asset (R_n).[14],[15] Over an interval of time or values of R where n remains fixed, the evolution of R and V_n are given by Ito's Lemma:

$$dR = R(n)(\alpha dt + \sigma dz), \tag{12}$$

$$dV_n = \frac{V_n'(R)dR + 0.5V_n''(R)\sigma^2 R^2 dt}{\left[\alpha R V_n'(R) + 0.5V_n''(R)\sigma^2 R^2\right] dt + \sigma R V_n'(R)dz}, \tag{13}$$

Now, consider a portfolio consisting of a long position for one unit of V_n and a short position for V_n' units of R_n. Then, the return of this portfolio in an interval of time dt is capital gains and dividends:

$$[(r - \delta)RV_n'(R) + 0.5V_n''(R)\sigma^2 R^2 + SS_n - W_n]dt, \tag{14}$$

Since this return is riskless, equation (14) must equal to the risk free return in the interval dt or:

$$[(r - \delta)RV_n'(R) + 0.5V_n''(R)\sigma^2 R^2 + SS_n - W_n] = r(V_n(R) - V_n'(R)), \tag{15}$$

where we cancel dt on both sides of the equation. Following Dixit (1989b) and Dixit and Pindyck (1994), equation (15) has a known general solution given by:

$$V_n(R) = A_n R^{\beta_1} + B_n R^{\beta_2} + SS_n(R)/\delta - W_n/r \ , \beta_1 < 0 \text{ and } \beta_2 > 0, \tag{16}$$

where A_n and B_n are constants to be determined. The coefficients β_1 and β_2 are the two solutions for the following quadratic expression:

$$0.5\beta\sigma^2(\beta - 1) + \beta(r - \delta) - r = 0 \tag{17}$$

The economic interpretation of equation (16) is the following. The last two terms of equation (16) give the expected present discounted value of maintaining exactly n firms forever,

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[14]See Dixit and Pindyck (1994) chapters 4 to 6 for a detailed treatment of these types of problems with GBM, and chapters 7 and 8 for a solution to a problem very similar to the one at hand.

[15]Four purposes of simplification we will denote $R(n)$ as R_n.

starting with a gross margin R. The first term of equation (16), $A_n R^{\beta_1}$, is the value of the option to shut down some existing firms and the second term, $B_n R^{\beta_1}$, is the option of adding some new firms. Equation (16) has the following boundary conditions for endpoints $n = 0$ and $n = N$. If $n = 0$, then $A_0 = 0$. In words, when there are no firms established, the shutdown option has zero value. Similarly, if $n = N$ there are no firms available to enter the market, hence $B_N = 0$(i.e. the option value of adding an additional firm has zero value).

The next step is to determine the value-matching and smooth-pasting conditions for those values of R where V_n, V_{n+1} and V_{n-1} are linked. First, let R_n^E denote the value of R where the social planner finds it optimal to introduce the nth firm when $(n-1)$ firms previously existed. Then, it must be true that:

$$V_{n-1}(R_n^E) = V_n(R_n^E) - I, \tag{18a}$$

$$V'_{n-1}(R_n^E) = V'_n(R_n^E), \tag{18b}$$

where equations (18a) and (18b) are the value-matching and smooth-pasting conditions, respectively. Similarly, let R_n^e the value of R when it is optimal to shut-down a firm starting from n. Thus we have:

$$V_n(R_n^e) = V_{n-1}(R_n^e) - E, \tag{19a}$$

$$V'_n(R_n^e) = V'_{n-1}(R_n^e), \tag{19b}$$

If we substitute equations (16) into equations (18) (or equations (19)), we get the following relationships:

$$a_n = A_n - A_{n-1}, \ b_n = B_{n-1} - B_n, \tag{20a}$$

$$A_n = \sum_{j=1}^{n} a_j, \ B_n = \sum_{j=n+1}^{N} b_j \tag{20b}$$

Using equations (2), (8) - (10) and (20), equations (18) and (19) become:

$$a_n R_n^{E\beta_1} + b_n R_n^{E\beta_2} + R_n^E/\delta - C_n/r - I = 0, \tag{21a}$$

$$\beta_1 a_n R_n^{E\beta_1 - 1} + \beta_2 b_n R_n^{E\beta_2 - 1} + 1/\delta = 0, \tag{21b}$$

$$a_n R_n^{e\beta_1} + b_n R_n^{e\beta_2} + R_n^e/\delta - C_n/r - E = 0, \tag{21c}$$

$$\beta_1 a_n R_n^{e\beta_1 - 1} + \beta_2 b_n R_n^{e\beta_2 - 1} + 1/\delta = 0, \tag{21d}$$

That is, for each $n \in [0, N]$, we have a system of four equations (21) with four unknowns, R_n^E, R_n^e, a_n and b_n. Solving for these unknowns we obtain the solution for the variables of interest, that is the thresholds R_n^E and R_n^e. Note that the imposition of an arbitrary maximum number of firms,[16] N; does not affect the solution to equations (21) (i.e. equations (21) are only functions of n). The arbitrary maximum number of firms only affects the number of independent systems (21) to be solved, and the value of the social planner function (equation (16)), but there is no effect on the threshold values of the gross margin, which are the variables of interest.

3 Calibration

We start by describing the parameters that are taken from historic data of prices and quantities and the literature. All data were seasonal adjusted and deflated at 2007 prices. Technical values for R and costs are taken from Swenson (2006), Tiffany and Eidman (2003), Roe et al. (2006), Miranowski et al. (2007) and the Securities and Exchange Commission (SEC) database.[17]

We have available data from the SEC database for four firms with 100MGY capacity that started production in 2007 or later. The average start-up capital cost for these plants was $171.9 million at 2007 prices or $I = 1.72$. We assume as in Schmit et al. (2008) that the exit cost E is equal to $0.2I$, or $E = 0.344$.

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[16] In the calibration Section, the value of N will respond to an economic intuition. Basically, note that for a fixed value of the shock Y, say Y_{ss}, the gross margin $R(n, Y_{ss})$ is a decreasing function of n. Then, N will be chosen as the maximum number of firms that support positive gross margins, i.e. $R(N, Y_{ss}) = \min R(n, Y_{ss})_{n=0}^{\infty} > 0$, where Y_{ss} is the calibration value of the shock.

[17] All publicly listed ethanol firms are required by the Securities and Exchange Commission to provide initial and periodic financial data on their operations. Typically, for larger, multi-plant firms provide less detailed information, but newer operations provide more detailed financial data each ethanol plant.

We divide operating costs, C, in four components: fixed transportation costs (FTC), variable transportation costs (VTC), tax credit per Gallon (TC), and remaining operating costs (C_0). Therefore, total operating cost is equal to:

$$C = C_0 + FTC + VTC - TC, \qquad (22)$$

The cost C_0 is composed as follows. We assume a total labor force of 45 employees for a 100MGY plant. We assume, as in Swenson (2006), that the management costs are 10% of the total labor force cost, while Tiffany and Eidman (2003) estimate management costs of 33%. Adding other costs, we find that operating costs net of transportation and tax credits costs are equal to $C_0 = 0.91$ (See Table 12).

Tax credit per gallon, TC, is equal to \$0.45 beginning January 2009 based on the Food and Energy Security Act of 2007, and referred to as the Volumetric Ethanol Excise Tax Credit (VEETC). Variable transportation costs are equal to:

$$VTC = C_m D, \qquad (23)$$

where C_m is the cost per mile of transporting corn and D is the average hauling distance to the plant. Following Miranowski and Rosburg (2010), this distance is calculated for a circular supply area with a square road grid based on the formulation by French (1960) and is equal to:

$$D = 0.4789\sqrt{Sx_c/(640Y_B B)}, \qquad (24)$$

where S is the size of the plant (Sx_c is the total annual requirement of bushels of corn), Y_B is the yield per acre of corn equal to 160 bushels according to USDA estimates, and B is the biomass (bushels of corn) density available for supply to the ethanol plant, equal to 15% according to estimates of Perlack and Turhollow (2002) and own calculations.[18] Hence, the average distance (D) is equal to 23.3 miles.

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[18]I am grateful to Alicia Rosburg for helping me in the calculations of the variable transportation costs.

The cost per mile, C_m, is calibrated such that the scale of plant has the lowest total average cost (without FTC), $C = C_0 + VTC - TC + rI$. The cost per mile is calculated as follows. We calculate the investment cost using the power formula (see Kumarappan (2003)) for sizes equal to $(100 + \epsilon)$ and $(100 - \epsilon)$ and $\epsilon = 0.1$. Thus, we have $K_S = (S/100)^\theta K_{100}$ for $\theta \in [0.6, 0.7]$, where S stands for the plant size $(100 + \epsilon)$ and $(100 - \epsilon)$, K_S is the capital cost for a plant of size S and K_{100} is the cost of the 100MGY plant. The resulting value of C_m is 0.22. This value is lower than the one estimated by Miranowski and Rosburg (2010) which is 0.35, but in the range of the estimates of Kumar et al. (2005). Fixed transportation costs (FTC) are obtained from Kumar et al. (2005) and it is equal 0.092.

$$R_t = Y_t(P_{e,t} + 0.0033P_{DDG,t} - 0.3374P_{c,t}), \tag{25}$$

where $P_{e,t}$ is the price of ethanol. Using monthly data for the period 1995-2008,[19] we estimate the assumed linear relationship between the price of corn and DDG for the period 1995-2008 (equation (1)) with coefficients $\gamma_0 = 9.38$ and $\gamma_1 = 38.11$; and we find $X_c = 0.263$ and $\gamma_0 x_{DDG} = 0.096$. From Luchansky and Monks (2009), we obtain the own-price elasticity demand of ethanol, ϵ_E, equal to 1.65 in absolute value. Also from Fortenbery and Park (2008) we use an elasticity of the price of corn with respect to ethanol, ϵ_c, equal to 0.16. Based to the ethanol plant information from the Renewable Fuels Association we calculate an initial number of plants $n = 85$ (total ethanol capacity of production in MGY at 2008 divided 100MGY). For the 1995-2008 period at prices of 2007, we calculate $P_{ess} = 2.04$ and $P_{css} = 3.38$ for average data of the prices ethanol and corn, respectively. Then, from equation (3) we calculate:

$\psi_{d2} = n/P_{ess}\lvert\varepsilon_E\rvert$	56.84
$\psi_{d1} = n/\psi_{d2} + P_{ess}$	2.98
$\psi_{s2} = P_{css}/n\varepsilon_c$	0.0076
$\psi_{s1} = P_{css} - n\psi_{s2}$	2.59

The number of potential firms, N, is calibrated such that $\hat{R}(N) = \min \hat{R}(n)_{n=0}^{\infty} > 0$, and equal to 118. The risk free interest rate, r, is equal to 0.032 annual according to the 10-year

[19] All prices were deflated using the producer price index (PPI) with base 2007. Also, data on prices and production of ethanol were seasonally adjusted.

T-Bill real interest rate (source: Federal Reserve). After obtaining the historical series of shocks (Y) as explained in Appendix A, we calibrate the parameters of equation (5) to equal $\alpha = 0.0044$ and $\sigma = 0.11$,[20],[21] where we use the fact that σ is equal to the sample standard deviation of $\ln(Y_t/Y_{t-1})$ and $\alpha - 0.5\sigma^2$ is equal to sample mean of $\ln(Y_t/Y_{t-1})$.[22] Finally, the risk-adjusted return (μ) for the ethanol industry was calculated using historical data of the S&P 500 index (as the portfolio market) and Archer-Daniels-Midland Co. (as the proxy for the ethanol industry), using monthly data for the 1995-2008 period we have that μ equals 0.0063.[23]

4 Numerical Results

In this section we analyze the numerical results of the baseline model developed in Sections 2 and 3. The results presented here will be used as a point of comparison with several exercises in the next two sections. There are two significant results shown in Table 13 and Figure 14, where we illustrate the threshold values of R for different numbers of plants. First, the threshold values of the gross margin are independent of the number of plants in the market. The constancy of the threshold values of R is due to several factors: Y follows a GBM (Lipsey and Lancaster (1956) and Dixit (1989b)), the gross margin is linear in the shocks, and operating and investment costs are independent of the number of firms. Second, as the optimal number of plants increases, the threshold values of the shocks (Y) for entering and exiting go up. Let Y_n^E and Y_n^e be the values of the shock Y when entry and exit of the nth firm occurs, respectively. Therefore, the changes in Y_n^E and Y_n^e exactly compensate the changes in $\hat{R}(n)$,[24] such that

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[20]Following Odening et al. (2007) for their simulated exercise, the parameters of equation (5) were calculated taking the values of the gross margin inside the interval $[\overline{R}^o \pm \sigma^o]$, where $\overline{R}^o$ is the sample mean of the observed gross margin and σ^o its standard deviation.

[21]It is worth to make a digression about the parameters of equation (5); the parameter *alpha* is highly sensitive to the standard deviation σ. In fact the ratio of the σ to the estimated mean of $\ln(Y_t/Y_{t-1})$, depending of the data transformation, is in a range of 12 to 35 times. Also, the standard deviation of $\ln(Y_t/Y_{t-1})$ is relatively stable (in a range of 0.13 to 0.11) to different data transformation (seasonal adjusted method and deflator used); and also, the mean of $\ln(Y_t/Y_{t-1})$ changes in small proportions (in a range of -0.0022 to 0.007). Thus, the estimated drift parameter α varies considerably regarding the transformation data, while the standard deviation σ remains relatively stable.

[22]The parameters estimated are close to the ones found by Schmit et al. (2008).

[23]The value of the discount rate is similar to the one found by Kumarappan (2003) and the one used by Schmit et al. (2008).

[24]Remember from equation (2) that $R(n) = Y\hat{R}(n)$.

$R^E(n)$ and $R^e(n)$ remain constant across n.

Note that the NPVs for entry and exit are $C + rI$ and $C - rE$, respectively. Our optimal rule solutions, $R^E(n)$ and $R^e(n)$, exceed the NPV rules. We find that $R^E(n)$ is 50% higher than $C + rI$ (for all n), while and $R^e(n)$ is 54% lower than $C - rE$. This fact is explained by Dixit (1989b) as an economic hysteresis,[25] since the actual number of firms does not change if the market's gross margin, R, hits the NPV gross margins either for entry or exit (i.e. $C + rI$ and $C - rE$, respectively). Thus, n only varies when R reaches the thresholds values $R^E(n)$ and $R^e(n)$.

In our model, the gross margin that induces one or more firms to enter, $R^E(n)$, is four times higher than the gross margin that induces exit, $R^e(n)$, while this gap for the NPV approach is just 10% ($[C + rI/(C - rE) - 1] \times 100$). Note that the values of $R^E(n)$ and $R^e(n)$ are compatible with any number of firms. What it is important to understand in this model is that entry or exit only occurs if R hits the corresponding bounds.

Our values are for $R^E(n)$ and $R^e(n)$ are 1.44 and 0.35, respectively. Schmit et al. (2008) found that $R^E(n) = 1.33$, while $R^e(n) = 0.38$. The differences, are due to several reasons. First, Schmit et al. (2008) uses a model where temporary suspension is allowed resulting in a significantly lower value. Second, they calibrate their model using a shorter time series (from 1998 to June of 2008). Finally, and most importantly, the authors model a totally myopic firm. In this regard, the firm does not take into account the effect of its entry/exit into the market and the decisions of possible competitors (myopia in the sense of Leahy (1993)). Also, the firm is myopic by assuming that changes in gross margin are purely stochastic, reducing equation (2) to and affecting the estimation of the parameters of equation (5).

We compare the model's results with the observed data. In Figure 15, for the period 1995-2008, we compare average annual data for R, say R_t^o, with the results of the model. In general, the observed data remains under the threshold values of the model. Major differences are found when we compare R_n^E and R_t^o. For the year 2006, we find that R_t^o is 50% higher than R_n^E.

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[25]Hysteresis is defined by Dixit (1989b) as an effect that persists after the cause that brought it about has been removed.

The constancy of the threshold gross margins across n in our model does not allow us to determine the optimal number of firms. However, it is worth noticing two key elements of this model. First, as mentioned above, the objective of this model is to determine the threshold gross margins that trigger changes in the number of active firms, which were normalized to be equal to the ethanol supply. Second, but related to the first point, is that the entrance of new firms depends heavily on the size of the shock Y. It is easy to think that any time that R hits its bounds; an unlimited number of firms may enter or exit from the market. This conclusion is wrong. In order to fully understand the dynamics of the model, let us consider an example with a sequential entry of firms. Assume that the actual number of firms is n_1. Let Y_{n+1} be the shock such that $R(Y_{n+1}, n_1 + 1) = R^E$. If more than one firm $(n + i, i \geq 2)$ enters into the market, then $R(Y_{n+1}, n_1 + i) < R^E$, because the increased supply of ethanol will lower the market equilibrium price of ethanol, and the price of corn will increase via the increased demand for corn. Thus, the result of unlimited entry is inconsistent with the rule of the optimal gross margin threshold for entry for a number of firms larger than $n + 1$. In this way, the magnitude of the shock to Y is critical to the number of new firms entering the industry or active firms exiting the market.

To clarify the ideas, consider an example provided in Table 13. Assume that the initial number of firms is $n = 70$, the actual shock is equal to 1 and the gross margin is $R(1, 70) = 1.02$. If in the immediate interval of time dt, the shock Y goes to 1.21, then exactly ten new firms will enter the market such that the gross margin is equal to $R^E(80)$(new size of the market equals 80). With a shock equal to 1.38, the size of the market will increase to 90 firms. On the other hand, if Y falls to 0.24, ten firms will exit the market until the gross margin is equal to $R^e(60)$.

A salient feature of our model is the extra rents, as defined by Leahy (1993), that arise by the discrete changes in the supply of ethanol. Let ϵ a small positive constant (i.e. $\epsilon \to 0$) and define $\widetilde{Y}_n^E = Y_n^E - \epsilon$ and $\widetilde{Y}_n^e = Y_n^e + \epsilon$. Also, define the gross margins $R_{n-1}^{max} = \widetilde{Y}_n^E \hat{R}(n-1)$ and $R_{n+1}^{min} = \widetilde{Y}_n^e \hat{R}(n+1)$. Note that by definition of gross margin (equation (2)) and the optimal threshold rule for entry and exit, we have that $R_{n-1}^{max} > R_n^E$ and $R_{n+1}^{min} > R_n^e$. That is, the n

active firms in the market are able to capture extra rents beyond (below) $R_n^E(R_n^e)$ until the point that the shock Y reaches the level of $Y_{n+1}^E(Y_{n+1}^e)$. Figure 16 depicts the extra rents; the first pane depicts the maximum gross margin attainable for the n active firms. These gross margins increase with the number of active firms because the value of the shocks Y that triggers entry increase with n. Finally, if the change in the supply of ethanol tends to be continuous, the extra rents tend to disappear; in the limit as dq (change in supply) is continuous our model will be identical to the models of Leahy (1993) and Dixit and Pindyck (1994) pags. 261-264.

5 Sensitivity Analysis

5.1 Variable costs

In this section we run two exercises: we consider a change in operating costs and investment costs. In the first exercise, operating costs vary by 10 percent around the mean value across firms, resulting in:

$$C_n = C - 0.1 + n0.1/nss, \tag{26}$$

where nss is equal to the number of firms used in the calibration section. One reason for an increase in operating costs is if firms have to settle in marginal areas where the yield per acre of corn is lower[26] increasing transportation costs, or firms pay more for natural gas, or there could be a local saturation of ethanol plants. Table 14 and Figure 17 show the numerical results for the increase in operating costs. With an increase in operating costs, the thresholds $R^E(n)$ and $R^e(n)$ grow with n, and the increments are linear. Also, when C_n is given by equation (26) and $n = 1$ we have that $R^E(1)$ and $R^e(1)$ are lower compared with the baseline case (10% and 17%, respectively); and at $n = N$, $R^E(N)$ and $R^e(N)$ are higher relative to the benchmark case (8% and 14%, respectively).

The second sensitivity exercise allows entry (I) and exit (E) costs to vary in a similar fashion to equation (26), except that I and E vary 15 percent around the mean value. Thus, we modify irreversible investment and exit costs in the following way:

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[26]Note that we still assume that a 100MGY plant is the most cost efficient one.

$$I_n = I - 0.15 + n0.15/nss \text{ and } E_n = E - 0.15 + n0.15/nss \tag{27}$$

This exercise allows us to capture, in a nave[27] (or primitive) way, characteristics of the 2005-07 period when several firms wanted to enter in the market. The demand for plant construction and installation substantially increased capital cost during this period. We assume that C_n is given by equation (26) and that the industry initially has nss firms. Table 15 and Figure 18 illustrate the results of this exercise. If R was initially equal to 1.44, and Y increases by 1.5%, only one firm will find it profitable to enter the industry. If Y increases by 4.7%, three firms will enter the ethanol market. Finally, a 17% positive shock to Y will result in ten more firms entering the market.

5.2 Changing functional forms of ethanol demand and corn supply

How sensitive are the results to different functional forms for $D(q)$ and $S_c(q)$? To test the sensitivity to functional forms, we change the linear functional forms from equation (3) to constant elasticity functions:

$$D(q) = \psi_d 1/q^{\psi_{d2}} \text{ and } S_c(q) = \psi_s 1q^{\psi_{s2}} \tag{28}$$

The values of ψ_{d2} and ψ_{s2} are equal to the values reported in the calibration section for the elasticities of the own demand for ethanol and the price of corn with respect to the supply of ethanol. The parameters ψ_{d1} and ψ_{s1} match the value of R from the calibration section. In this example, the model uses the functional forms in equation (28) in place of equation (3) and the observed stochastic shock (Y) changes. These factors lead to a modification in the parameters estimated in equation (5) and a possible modification of the GBM assumption. However, if the stochastic process remains GBM and all costs are constant across n, the threshold values of R will be constant and independent from the number of firms.

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[27]We call nave this exercise, because we are not assuming that investment (and exit) costs only increase for new plants. If this was the case, the problem will be more complex because the social planner would have to simultaneously evaluate option values for n, $(n-1)$, $(n-2)$, and $(n+1)$, $(n+2)$, rather than options values only for n, $(n-1)$ and $(n+1)$.

With the functional forms in equation (28), the stochastic process that governs Y is still GBM as in equation (5), except $\alpha = 0.0044$ and $\sigma = 0.12$.[28] Table 16 and Figure 19 display the outcomes for this exercise. With the constant elasticity demand for ethanol and residual supply of corn, $R^E(n)$ is a 15.8% higher compared with the benchmark case, while $R^e(n)$ is 6% higher. Thus, for the entry gross margin, we find, in some degree, that there is a significant difference in both types of functional forms.

5.3 Changing initial state of n

So far the model has been calibrated for the average values for the 1995-2008 period, with the initial n equal to nss in the calibration section. How will results change if we calibrate the model to the 2008 situation? That is, how the results change if we set nss to the total capacity of the industry in 2008. Recall that any modification in the calibration that affects $R_t = Y_t \hat{R}(n_t)$, will change the parameters' value in equation (5) and henceforth the solutions to the model. Therefore, we can expect that modifying the evolution of Y_t may lead to significant changes in the values of the drift and variance of equation (5) and changes in the numerical solutions.

The exercise can be viewed from the standpoint of an investor at the beginning of 2009 who is trying to decide when to invest in the ethanol industry if he bases his actions with the information available at 2009. The parameters of equation (5) are now $\alpha = 0.01$ and $\sigma = 0.12$. The threshold values for $R^E(n)$ and $R^e(n)$ are slightly higher than in the benchmark case (see Table 17 and Figure 20). The percentage changes are 5.6% and 4% for the upper and lower bounds, respectively. Therefore, the results are not significantly different when the model is calibrated using average values from 1995-2008 compared to using 2008 values only.

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[28]We use the same procedure as in the calibration section. However, because of the exponential increment in the ethanol production, which gives unrealistic values for the shock Y in the early years; we calculate the parameters for the 2000-08 period. Otherwise, we get a drift parameter significantly large and equal to a 70% annual increment.

5.4 The elasticity of demand for ethanol

In this section we analyze the effect of the own price elasticity of ethanol. We conduct two different exercises. In the first exercise, we cut the value of elasticity reported by Luchansky and Monks (2009) in half from -1.60 to -0.8.[29] This numerical example allow us to analyze the effect of the elasticity of demand for ethanol on the size of the shocks (Y) needed to cause entry of new firms in the industry.

We compare two types of functional forms: the baseline case (linear functions) and the one constant elasticity function (5.2, equation (28)). In order to track the relevance of the elasticity of demand, we keep the same calibrated values for dY from Section 3 for the linear case and from Section 5.2 for the constant elasticity case. Therefore, only the sizes of the shocks $Y^E(n)$ and $Y^e(n)$ change, while the gross margin thresholds remain the same. The calibrated parameters are $\psi_{d1} = 4.1$ and $\psi_{d2} = 28.4$ for the linear demand and $\psi_{d1} = 333.3$ for the constant elasticity case. Figure 21 displays the main result. With a lower elasticity, the growth rate of the shocks is higher with respect to the baseline cases. For example, using tables 13, 16 and 18, and taking $n = 80$, now $Y^E(n)$ is 56% higher in the linear case and a 20% larger for the constant elasticity case with a lower elasticity demand.

Besides the effects of the residual supply of corn, the economic intuition behind the results above is as follows. Shocks on Y that cause displacements in an elastic demand are translated into quantities rather than prices. Therefore, only small perturbations on Y are needed to cause movements in the equilibrium number of firms. To clarify concepts, we develop a simple example for the linear case. For simplicity, we only consider the upper barrier case $R^E(n)$. Let $e1$ and $e2$ the subscripts representing the elastic and inelastic gross margins[30] functions, respectively. Assume that the initial market equilibrium has 65 firms ($n_1 = 65$) and a gross margin equal to R^E. This equilibrium is displayed in Figure 22, where the gross margins functions $D1_{e1}$ (elastic case) and $D1_{e2}$ (inelastic case) intersect the threshold R^E at $n = n_1$. Now we ask: how large of shocks are needed to bring the equilibrium of firms to 70 ($n_2 = 70$)?

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[29]In the early stages of the ethanol industry the ethanol's demand was inelastic rather than elastic, Rask (1998) for the period 1984-1993 reported an own demand elasticity equal to -0.37.

[30]The only difference between the two is the elasticity of the demand for ethanol.

Let Y_{e1}^E and Y_{e2}^E denote the shocks such that the gross margin functions induce displacement respectively from $D1_{e1}$ and $D1_{e2}$ and from $D2_{e1}$ and $D2_{e2}$, respectively. The new market equilibriums are reached at the value R^E and n_2 number of firms. From Figure 22 we can see that the elastic gross margin function (subscript $e1$) requires a smaller shock in order to induce 10 new firms to enter the industry compared to the inelastic gross function (subscript $e1$); therefore, $Y_{e1}^E > Y_{e2}^E$.

The second exercise explores the effects of introducing the price elasticity into the stochastic process of R. So far we have assumed that the shock Y affects the gross margin linearly (i.e. $R_t = Y_t \hat{R}(q_t)$). Now, we assume that equation (2) is changed such that:

$$R_t = \hat{R}(q_t, Y_t) = (D(q_t, Y_t) - X_c S_c(q_t, Y_t) + Y_t \gamma_0 x_{DDG}), \tag{29}$$

where the demand for ethanol and the residual supply for corn take the following functional forms:

$$D(q, Y) = \psi_{d1}(Y/q)^{\psi_{d2}} \text{ and } S_c(q, Y) = \psi_{s1}(Yq)^{\psi_{s2}} \tag{30}$$

By Ito's lemma dR is equal to

$$dR = R(n)(\hat{\alpha}dt + \hat{\sigma}dz), \tag{31}$$

where the drift and the variance of equation (31) are giving by:

$$\hat{\alpha} = (\psi_{d2} - X_c\psi_{s2})\alpha + 0.5\sigma^2[(\psi_{d2} - 1)\psi_{d2} - X_c(\psi_{s2} - 1)\psi_{s2}], \tag{32}$$

$$\hat{\sigma} = (\psi_{d2} - X_c\psi_{s2} + \gamma_0 x_{DDG})\sigma, \tag{33}$$

The theoretical solution of the model does not change because R still displays a GBM process. Parameters calibration remains unchanged from section 3. However, we need to recalibrate the parameters of equation (5) using the observed data and equations (29) to (33). It is noted that the observed value of Y cannot be obtained in the same straightforward method as in sections 3 and 5.2 since Y does not have a closed form solution (see Appendix A).

We will refer the model from Section 5.2 as case 1 and the model described by equations (29) to (33) as case 2. In Table 19 we report the thresholds values for the gross margin and the shock for case 2. The upper threshold gross margin is 16% lower in case 2 compared to case 1, while the lower threshold gross margin is 32% higher for case 2 relative to case 1. In fact, the gap between $R^E(n)$ and $R^e(n)$ is narrower in case 2 than case 1 (see second pane Figure 23). What is interesting from this last example is that, the stochastic shock that affects the gross margin (via demand and supply elasticities) narrows the significantly the gap between the two threshold values. Also, we still observe that Y is driven by a GBM and that there are no significant numerical differences between model specifications.

6   Investing in small plants: the effects of subsidies

The numerical simulations in the previous sections were calculated using the VEETC (see section 3). This production subsidy applies for any plant size and was created to promote the ethanol industry as a whole. In this section, we consider the capital costs and production subsidies[31] that would be needed to induce an investor to construct a small scale plant, say 50MGY. In particular, we are interested in the level of capital subsidies under two types of schemes. In the first case we consider no additional subsidies beyond the VEETC, while in the second case, we consider an additional production subsidy for small capacity plants.

We start with a subsidy on capital costs and ask what capital subsidy is needed for an investor to be indifferent between installing a 50MGY plant or a 100MGY plant? When attempting to answer this question, a model limitation arises: the increasing returns to scale in the ethanol industry. This characteristic allows only discrete changes in the ethanol supply. Thus, we cannot address different size plants in our model that restricts discrete changes in supply equal to one unit of output.[32]

In fact, if we model a Ramsey problem where a social planner tries to find the subsidy that maximizes the expected social surplus, we should find that the optimal subsidy is zero.

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[31]See Miranowski (2007) and Schumacher (2006) for a detailed explanation of different ethanol subsidy schemes for US for the ethanol subsidy.

[32]Remember that 1 unit of output is equal to a 100MGY plant.

To see this point, imagine that there is no exit costs[33] and that a Ramsey planner can choose between a 50MGY and a 100MGY plant and a level of subsidy such that the investor is indifferent between both plant sizes[34] (or strictly prefers the small plant). Note two things: first, any investment subsidy decreases the total social surplus as long as consumers finance it; and second, a Ramsey planner will try to choose a firm size such that economies of scale are exhausted (i.e. a 100MGY plant) so the social surplus is maximized. Since both effects go in the same direction, there is no incentive for a social planner to optimally choose a positive subsidy in a competitive market.

In order to get a positive subsidy, we need a model where such a subsidy is imposed ad-hoc or where other elements affect the social surplus (such as an ethanol plant that serves a community and hires local labor). However, such models create complications because the social planner needs to compare several option values. For example, an option for a 100MGY plant against a 50MGY plant or two 50MGY plants.

Instead of pursuing such a complicated model, which is beyond the scope of this paper, we ask what capital subsidy is needed to induce the installation of two 50MGY[35] plants (i.e. equal to one unit of output in our model) such that equations (21) hold at the threshold values found for the benchmark case. Let θ_n^E and θ_n^e be the subsidies (in percentage) such that:

$$a_n R_n^{E\beta_1} - b_n R_n^{E\beta_2} + R_n^E/\delta - C_{50,n}/r - I_{50}(1 - \theta_n^E) = 0, \tag{34a}$$

$$a_n R_n^{e\beta_1} - b_n R_n^{e\beta_2} + R_n^e/\delta - C_{50,n}/r + E_{50}(1 - \theta_n^e) = 0, \tag{34b}$$

where $C_{50,n}$, I_{50} and E_{50} are the operating, entry and exit costs of two 50MGY plants when the total supply of the industry is n. From Swenson (2006) and calculations of transportation costs using equation (24), we obtain $C_{50,n} = 1.06$. This value is 2.5 cents higher compared to the 100MGY case ($C_{100,n} = 1.01$) due to the net effect of the diseconomies of scale in labor and the diseconomies of scale in transportation costs. Following Kumarappan (2003)

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[33]This assumption allow us to simplify our explanation.

[34]Otherwise, if the indifference condition does not hold no rational firm will chose freely to invest in a small plant.

[35]Note that the model allows only discrete changes in the ethanol supply and equal to one unit of output. Thus we "approximate" the solution for the subsidies treating two 50MGY plants as one unit of output.

and using the power formula as explained in the calibration section, we find $I_{50} = 2.19$ and we set $E_{50} = 0.2I_{50}$ as in the benchmark case.

Because we know that optimal thresholds are constant across n (and therefore a_n and b_n), we only need to calculate θ_n^E and θ_n^e once. The values that satisfy equations (34) are $\theta_n^E = 0.50$ and $\theta_n^e = -1.17$. Thus, while plant investment cost (per gallon) is 27% higher than in a 100MGY, the incentive required to induce investment in small size plant is 51%.[36]

Finally, we consider the addition of a production subsidy to small plants. In particular, we incorporate a federal subsidy known as the Small Producer Tax Credit (SPTC), equal to \$0.10 for the first 15 million gallons of production per year for 60MGY plants or smaller. We again ask what capital subsidy is needed for an investor to be indifferent between installing a 50MGY plant or a 100MGY plant? When the addition of the SPTC, the operating costs for a 50MGY plant decreases to a value equal to 1.03. We repeat the exercise of finding the rates of capital subsidy that satisfy equations (34) at the thresholds gross margins of the baseline case. With this operating cost, we find that subsidy-rates are $\theta_n^E = 0.07$ and $\theta_n^e = 0.91$. Thus, a capital subsidy equal to 7% is necessary for an investor to be indifferent between a 100MGY plant and a 50MGY. That is, the effect of the subsidy on production for small plants is so powerful, that only a small capital subsidy is needed to promote investment in 50MGY plants. In addition, if we also consider subsidy programs that are no longer in effect, such as the USDA Commodity Credit Corporation, or local and state subsidy programs for small plants, then we will find evidence in support of 50MGY and smaller plants being preferable to 100MGY plants by investors in the past.

7   conclusions

We modeled the decision for a firm operating in competitive a market to enter or exit the ethanol industry. We find that the thresholds that trigger changes in the ethanol supply for the industry largely exceed the traditional NPV boundaries. Also, the model is relatively stable to changes in assumptions and functional forms. Changing the functional forms of the demand

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[36]The linearity of investment and operating costs imposed by the model implies that the subsidy is the same for one or two plants.

for ethanol or the residual supply of corn may affect in some degree the results.

Thus, our results are robust among several specifications of the model. One of the main findings is that we never reject the null hypothesis of unit roots for the shocks that hit the gross margin; the shocks were modeled as a GBM. Also, we find that there is a stable relationship for the lower threshold gross margins. For the upper bound, R fluctuates between \$1.22 and \$1.67, while the lower bound of R fluctuates around \$0.30 and \$0.35. We conduct several sensitivity tests in support of these results.

Economies of scale in ethanol plants make it optimal to invest in large scale plants until transportation diseconomies offset scale economies. All other things equal, expected returns to large sized plants exceed returns to small size plants and thus, large sized plants will prevail in long run equilibrium. However, prior to 2005, while oil price remain low, small plants were the norm in the ethanol industry due to government subsidies to small scale plants, such as the SPTC. The capital and production subsidy schemes "leveled the playing field" for small and presumably local investors. Coupled with the fact that the local investors were able to capture an extra local rent (e.g., higher cash rents, increased local jobs, increased local incomes, improved personal consumption) from investment in a small size ethanol plant, provided the impetus for the initial expansion in the industry.

Several extensions can be made to this work. First, this paper was initiated when oil and gasoline prices were high, and a large number of new ethanol plants were under consideration. But under current oil and gasoline prices, what are the implications for the ethanol industry as a whole? An interesting extension to this model would be to consider firms that temporarily mothball production operations. In this case, the model would be modified to consider a social planner who faces several options, such as entry of new plants versus activating idle plants, and mothballing existing plants versus exit of existing plants.

Second, what is the future of new technology that uses biomass or cellulose as feedstock for ethanol? The ethanol industry is moving in the direction of using new technology to produce cellulosic ethanol, especially under the revised Renewable Fuels Standard (RFS.2) contained in the Food and Energy Security Act of 2007. How will the cellulosic ethanol industry evolve?

What are the economic costs to society of this policy prescription? It is possible to answer these questions using the methodological framework of this paper?

Finally, how will the new fuel composition mandate affect the profitability of both the corn and cellulosic ethanol industry? The US mandate on fuel composition requires an increasing proportion of ethanol in LTF fuels until 2022.[37] According to the Renewable Fuel Association, the new Renewable Fuel Standard (RFS) mandate, passed in 2007, will require 36 billion gallons of renewable and alternative fuels in 2022 with a target of 20 billion gallons for 2015. How will these incremental changes in the demand for biofuels affect the optimal decision to invest in the ethanol industry? Even more, how will the RFS mandate affect the optimal investment decisions for alternative technologies taking into account that a considerable proportion of the future ethanol production will have to come from cellulose feedstock.[38]

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[37]From both types of technologies, ethanol derived from corn and ethanol derived from biomass or cellulose.
[38]Of the total 36 billion gallons, 16 billion gallons will have to come from cellulose feedstock.

APPENDIX A.   Tests for Unit roots

Tests for Unit roots of Y for the baseline case

In order test for the presence of unit root in $\ln(Y)$, we construct the series $\ln(Y)$ of this variable following Odening et al. (2007). Given the functional form of R equation (2) and the specific demand for ethanol and supply equation (3), we obtain Y_t as:

$$Y_t = R_t^o/\hat{R}_t(q_t) \tag{A.1}$$

where the superscript o refers to the observed values in monthly data for the period 1995 - 2008. All prices were deflated using PPI with base 2007. Also, data on prices and production of ethanol were seasonally adjusted. The parameters values for the demand and supply function as well as the ones of equation (1) can be seen in the calibration section.

Augmented Dickey-Fuller test for unit root:

Regression	Test Statistic	Critical values		
		1%	5%	10%
$\ln Y_t - \ln Y_{t-1} = \rho \ln Y_{t-1}$	-2.181	-3.490	-2.886	-2.576

Phillips - Perron test for unit root:

Regression	Test Statistic	Critical values		
		1%	5%	10%
$\ln Y_t - \ln Y_{t-1} = \rho \ln Y_{t-1}$	-1.840	-3.490	-2.886	-2.576

Data source of prices:

- Ethanol price: Renewable Fuel News (weekly news): average of US ethanol price.

- DDG price: USDA.

- Corn price: USDA.

- Ethanol production: USDA.

Tests for Unit roots of non-linear shocks

Let R_t^o and n_t^o the observed values at period t, then the observed value of the shock, Y_t^o, can be recovered as a non linear function of equation (29):

$$R_t^o = \hat{R}(n_t^o, Y_t^o) = D(n_t^o, Y_t^o) - X_c S_c(n_t^o, Y_t^o) + Y_t^o \gamma_0 x_{DDG}, \tag{A.2}$$

where $D(n_t^o, Y_t^o)$ and $S_c(n_t^o, Y_t^o)$ are given by equation (30). The test for unit roots does not reject the null hypothesis.

Augmented Dickey-Fuller test for unit root:

Regression	Test Statistic	Critical values		
		1%	5%	10%
$\ln Y_t - \ln Y_{t-1} = \rho \ln Y_{t-1}$	-1.039	-3.490	-2.886	-2.576

Phillips - Perron test for unit root:

Regression	Test Statistic	Critical values		
		1%	5%	10%
$\ln Y_t - \ln Y_{t-1} = \rho \ln Y_{t-1}$	-0.992	-3.490	-2.886	-2.576

Table 1   Model Calibration: Parameters and Steady-State Value of Key Variables (Annual Frequency)

Preferences		Technology		Fiscal Policy		Key Variables	
β	0.96	δ_{ss}	0.10	τ^c	0.12	k	2.13
η	10.11	A	1.85	τ^I	0.06	e^y	1
ϕ	1.58	δ_0	0.27	τ^e	0.49	e^h	0.88
ω	1.91	δ_1	1.62	g_1	0.12	q^h	0.6
		ν_0	0.14	g_2	0.02	h	0.3
		ν_1	0.26	b^g	0.15	p^e	0.0276
		μ	0.36	q^h	0.96	p^k	1
						c	0.63
						b^h	0.41
						y	1

Table 2 Developed Countries - Model Calibration: Parameters and Steady-State Value of Key Variables (Annual Frequency)

Preferences		Technology		Fiscal Policy		Key Variables	
β	0.98	δ_{ss}	0.10	τ^c	0.13	k	2.53
η	10.11	A	1.71	τ^I	0.11	e^y	0.5
ϕ	0.0036	δ_0	0.17	τ^e	1.41	e^h	0.44
ω	2.03	δ_1	1.25	g_1	0.19	r^i	0.98
		ν_0	0.24	g_2	-0.01	h	0.3
		ν_1	0.98	b^g	0.61	p^e	0.025
		μ	0.36	r^i	0.022	c	0.56
		α	0.35			b^i	1.20
						y	1

Table 3 Developing Countries - Model Calibration: Parameters and Steady-State Value of Key Variables (Annual Frequency)

Preferences		Technology		Fiscal Policy		Key Variables	
β	0.97	δ_{ss}	0.10	τ^c	0.10	k	2.22
η	10.11	A	1.79	τ^I	0.06	e^y	0.95
ϕ	0.22	δ_0	0.20	τ^e	0.77	e^h	0.84
ω	1.71	δ_1	1.42	g_1	0.15	q^i	0.97
		ν_0	0.40	g_2	-0.02	h	0.3
		ν_1	0.78	b^g	0.53	p^e	0.025
		μ	0.36	r^i	0.035	c	0.72
		α	0.35			b^i	3.99
						y	1

Table 4   Incomplete Markets and Developed Countries - statistical Moments

Variable (x)	Std dev %	Corr(p^e, x)	Autocorr
Oil-tax rate (τ^e)	1.214	-0.997	0.721
GDP (y)	0.061	-0.993	0.709
Consumption non-energy goods (c)	0.049	-0.909	0.865
Use of Oil in production (e^y)	0.116	-0.993	0.771
Utilization of capital (u)	0.103	-0.758	0.655
Capital (k)	0.089	-0.586	0.983
Public Debt / GDP (b^g/GDP)	0.068	0.998	0.748
International bonds / GDP (b^i/GDP)	0.068	0.958	0.691

Table 5   Incomplete Markets and Developing Countries - statistical Moments

Variable (x)	Std dev %	Corr(p^e, x)	Autocorr
Oil-tax rate (τ^e)	1.142	-0.995	0.757
GDP (y)	0.074	-0.994	0.716
Consumption non-energy goods (c)	0.070	-0.898	0.877
Use of Oil in production (e^y)	0.136	-0.942	0.860
Utilization of capital (u)	0.116	-0.655	0.597
Capital (k)	0.134	-0.697	0.970
Public Debt / GDP (b^g/GDP)	0.064	0.982	0.674
International bonds / GDP (b^i/GDP)	0.082	0.982	0.701

Table 6 Complete vs Incomplete Markets and Developed Countries - statistical Moments

	Complete markets			Incomplete markets		
Variable (x)	Std dev %	Corr(p^e, x)	Autocorr	Std dev %	Corr(p^e, x)	Autocorr
τ^e	1.299	-0.999	0.730	1.214	-0.997	0.721
$(1+\tau^e)p^e$	0.713	0.999	0.739	0.764	0.998	0.747
GDP	0.056	-0.988	0.697	0.061	-0.993	0.709
c/e^h	0.064	0.999	0.739	0.069	0.998	0.747
Primary surp	0.179	-0.961	0.804	0.109	-0.301	0.929
b^g/GDP	0.043	0.992	0.688	0.068	0.998	0.748
b^i/GDP	0.063	0.944	0.681	0.068	0.958	0.691

Table 7 Complete vs Incomplete Markets and Developing Countries - statistical Moments

	Complete markets			Incomplete markets		
Variable (x)	Std dev %	Corr(p^e, x)	Autocorr	Std dev %	Corr(p^e, x)	Autocorr
τ^e	1.180	-0.995	0.757	1.142	-0.995	0.757
$(1+\tau^e)p^e$	0.961	0.999	0.722	0.978	0.999	0.722
GDP	0.072	-0.994	0.715	0.074	-0.994	0.716
c/e^h	0.086	0.999	0.722	0.088	0.999	0.722
Primary surp	0.167	0.283	0.217	0.192	0.982	0.213
b^g/GDP	0.055	0.973	0.655	0.064	0.982	0.674
b^i/GDP	0.080	0.980	0.700	0.082	0.982	0.701

Table 8　Sensitivity analysis - $\kappa_2 = 0.0008$ - statistical Moments

Variable (x)	Std dev %	Corr(p^e, x)	Autocorr
Oil-tax rate (τ^e)	1.331	-0.996	0.746
GDP (y)	0.070	-0.967	0.769
Consumption non-energy goods (c)	0.035	-0.884	0.836
Use of Oil in production (e^y)	0.120	-0.994	0.782
Utilization of capital (u)	0.088	-0.867	0.509
Capital (k)	0.063	-0.880	0.929
Public Debt / GDP (b^g/GDP)	0.181	0.568	0.961
International bonds / GDP (b^i/GDP)	0.432	0.101	0.982

Table 9　Sensitivity analysis - $\tau^I = 0.056$ - statistical Moments

Variable (x)	Std dev %	Corr(p^e, x)	Autocorr
Oil-tax rate (τ^e)	1.226	-0.999	0.731
GDP (y)	0.056	-0.987	0.695
Consumption non-energy goods (c)	0.047	-0.922	0.856
Use of Oil in production (e^y)	0.106	-0.996	0.761
Utilization of capital (u)	0.095	-0.750	0.642
Capital (k)	0.080	-0.607	0.981
Public Debt / GDP (b^g/GDP)	0.054	0.987	0.675
International bonds / GDP (b^i/GDP)	0.064	0.937	0.678

Table 10   Sensitivity analysis - $\tau^e = 0.77$ - statistical Moments

Variable (x)	Std dev %	Corr(p^e, x)	Autocorr
Oil-tax rate (τ^e)	1.096	-0.994	0.721
GDP (y)	0.055	-0.992	0.708
Consumption non-energy goods (c)	0.047	-0.914	0.863
Use of Oil in production (e^y)	0.106	-0.992	0.774
Utilization of capital (u)	0.092	-0.770	0.643
Capital (k)	0.079	-0.605	0.981
Public Debt / GDP (b^g/GDP)	0.059	0.998	0.731
International bonds / GDP (b^i/GDP)	0.065	0.944	0.687

Table 11   Sensitivity analysis - $\nu_1 = 0.78$ - statistical Moments

Variable (x)	Std dev %	Corr(p^e, x)	Autocorr
Oil-tax rate (τ^e)	1.216	-0.999	0.747
GDP (y)	0.051	-0.984	0.687
Consumption non-energy goods (c)	0.044	-0.913	0.857
Use of Oil in production (e^y)	0.086	-0.964	0.820
Utilization of capital (u)	0.088	-0.645	0.653
Capital (k)	0.085	-0.615	0.980
Public Debt / GDP (b^g/GDP)	0.047	0.983	0.658
International bonds / GDP (b^i/GDP)	0.064	0.945	0.681

Table 12   Operating Costs - C_0

Concept(1)	x*P(2)
Water (gal) (plus treatment)	0.0344
KwH Electricity	0.0477
Natural Gas (btu* bushel*alcohol yield)	0.3017
Enzymes	0.0593
Yeasts	0.0272
Chemicals: processing and antibiotics	0.0247
Chemicals: boiling and cooling	0.0062
Denaturants	0.0433
Waste management	0.0206
Maintenance	0.0143
Transportation (rail only - ethanol)	0.1834
All other unspecified	0.0870
Labor	0.0319
Management	0.0064
Indirect taxes	0.0234
Total operative costs	0.9116

Notes: (1) not included corn and return to investors
(2) input requirement per ethanol gallon times input price

Table 13   Baseline Case: threshold values of Y_n and $R(n)$

N	Y_n^E	Y_n^e	$R^E(n)$	$R^e(n)$
10	0.65	0.16	1.44	0.35
20	0.69	0.17	1.44	0.35
30	0.75	0.18	1.44	0.35
40	0.81	0.20	1.44	0.35
50	0.88	0.22	1.44	0.35
60	0.97	0.24	1.44	0.35
70	1.08	0.26	1.44	0.35
80	1.21	0.30	1.44	0.35
90	1.38	0.34	1.44	0.35
100	1.61	0.39	1.44	0.35
110	1.93	0.47	1.44	0.35

Table 14   Sensitivity Analysis: Variable operating costs: Y_n and $R(n)$

N	Y_n^E	Y_n^e	$R^E(n)$	$R^e(n)$
10	0.58	0.13	1.31	0.30
20	0.63	0.15	1.32	0.31
30	0.69	0.16	1.34	0.31
40	0.76	0.18	1.36	0.32
50	0.84	0.20	1.38	0.33
60	0.94	0.22	1.40	0.34
70	1.06	0.26	1.42	0.34
80	1.20	0.29	1.44	0.35
90	1.39	0.34	1.45	0.36
100	1.64	0.41	1.47	0.36
110	1.99	0.50	1.49	0.37

Table 15   Sensitivity Analysis: Variable entry / exit costs: Y_n and $R(n)$

N	Y_n^E	Y_n^e	$R^E(n)$	$R^e(n)$
10	0.56	0.14	1.26	0.31
20	0.61	0.15	1.28	0.31
30	0.68	0.17	1.31	0.32
40	0.74	0.18	1.33	0.33
50	0.83	0.20	1.36	0.33
60	0.93	0.23	1.38	0.34
70	1.05	0.26	1.41	0.34
80	1.20	0.29	1.43	0.35
90	1.39	0.34	1.46	0.36
100	1.65	0.40	1.48	0.36
110	2.01	0.49	1.51	0.37

Table 16   Sensitivity Analysis: Constant elasticity functions: Y_n and $R(n)$

N	Y_n^E	Y_n^e	$R^E(n)$	$R^e(n)$
10	0.03	0.01	1.67	0.37
20	0.09	0.02	1.67	0.37
30	0.18	0.04	1.67	0.37
40	0.30	0.07	1.67	0.37
50	0.46	0.10	1.67	0.37
60	0.66	0.15	1.67	0.37
70	0.92	0.21	1.67	0.37
80	1.27	0.29	1.67	0.37
90	1.76	0.39	1.67	0.37
100	2.45	0.55	1.67	0.37
110	3.58	0.80	1.67	0.37

Table 17   Sensitivity Analysis: changing nss: Y_n and $R(n)$

N	Y_n^E	Y_n^e	$R^E(n)$	$R^e(n)$
10	0.67	0.16	1.52	0.37
20	0.71	0.17	1.52	0.37
30	0.75	0.18	1.52	0.37
40	0.80	0.19	1.52	0.37
50	0.86	0.21	1.52	0.37
60	0.92	0.22	1.52	0.37
70	0.99	0.24	1.52	0.37
80	1.07	0.26	1.52	0.37
90	1.17	0.28	1.52	0.37
100	1.29	0.31	1.52	0.37
110	1.44	0.35	1.52	0.37

Table 18   Sensitivity Analysis: Elasticity demand for ethanol - 1st exercise

Function	Linear		Constant elasticity	
N	Y_n^E	Y_n^e	Y_n^E	Y_n^e
10	0.16	0.09	0.44	0.26
20	0.29	0.17	0.48	0.29
30	0.43	0.25	0.53	0.32
40	0.57	0.33	0.59	0.35
50	0.72	0.41	0.67	0.40
60	0.89	0.51	0.78	0.46
70	1.07	0.61	0.92	0.55
80	1.28	0.73	1.13	0.67
90	1.51	0.86	1.45	0.86

Table 19   Sensitivity Analysis: Non-linear shocks:

N	Y_n^E	Y_n^e	$R^E(n)$	$R^e(n)$
10	15.28	6.22	1.22	0.46
20	24.53	11.27	1.22	0.46
30	30.38	15.11	1.22	0.46
40	35.47	18.15	1.22	0.46
50	39.14	20.65	1.22	0.46
60	42.14	22.76	1.22	0.46
70	44.66	24.58	1.22	0.46
80	46.84	26.17	1.22	0.46
90	48.74	27.59	1.22	0.46
100	50.42	28.86	1.22	0.46
110	51.93	30.02	1.22	0.46

Figure 1   Cross Country Responses of Growth Rates to Oil Shocks; Annual data for the period 1960-2007
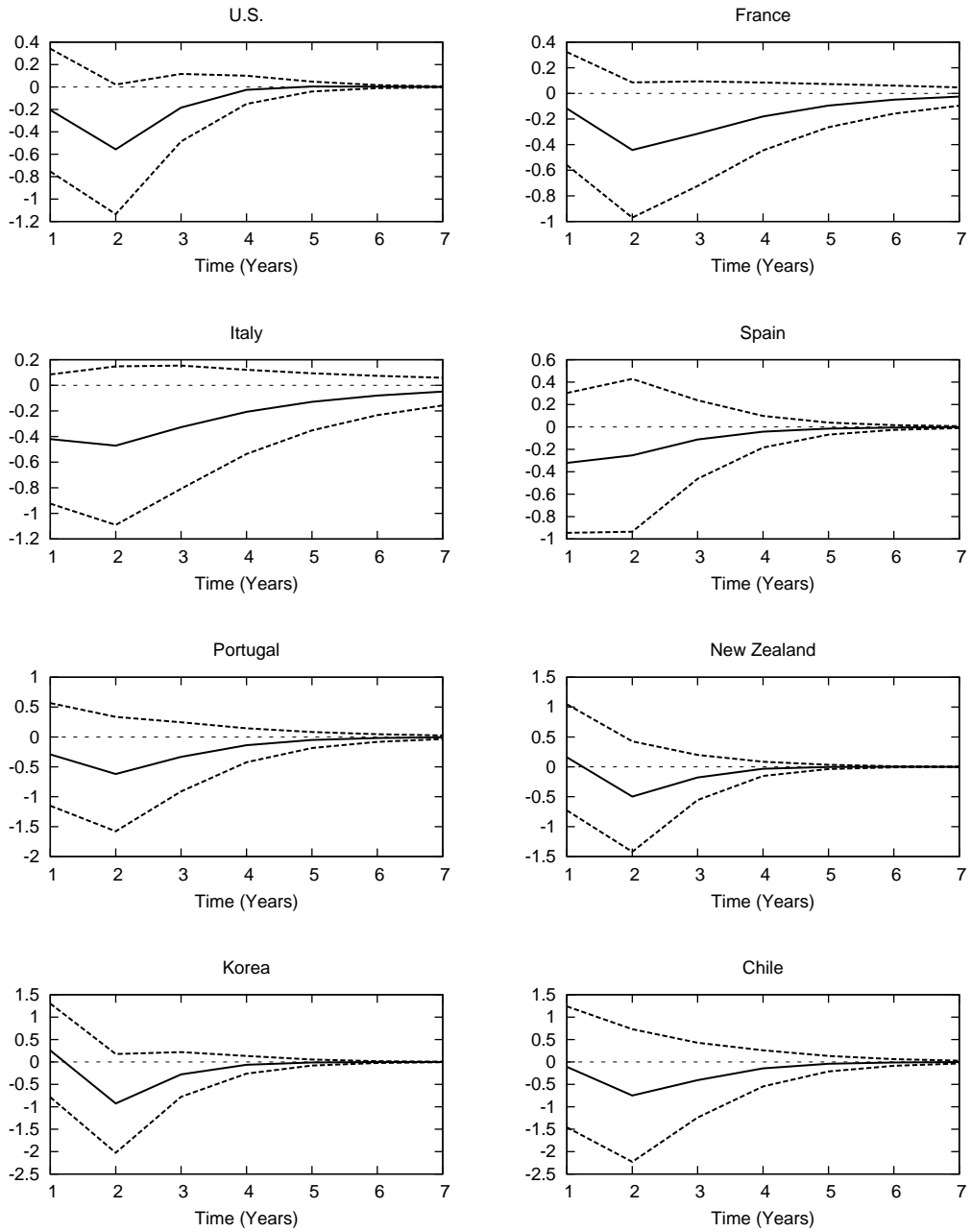
Notes: The plots show the responses of GDP growth rates, measured in percentages, to the Choleski, one-standard deviation innovation in α, where $\alpha \equiv \log(\text{Oil Price}_t) - \log(\text{Oil Price}_{t-1})$. Oil prices are measured in real terms and the source of data for the growth rates is the World Bank.

Figure 2   Sources of Fluctuations in the Model
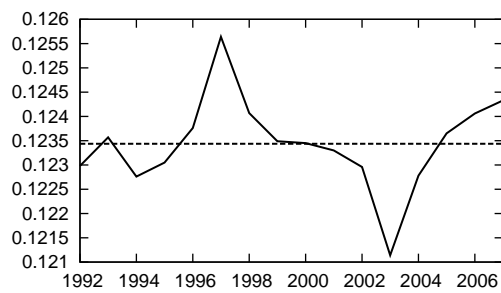
(a) Government Expenditures (g_{1t})
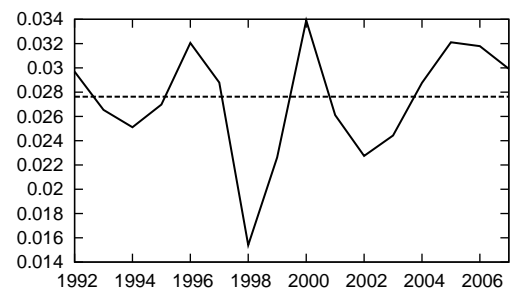
(b) Oil Prices (p_t^e)
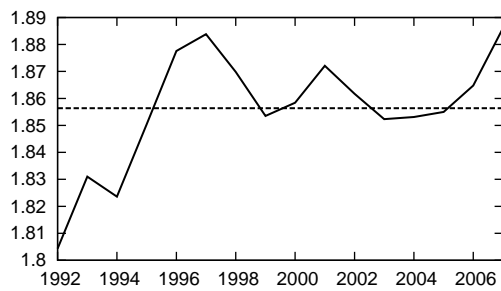
(c) Productivity ($A\exp(z_t)$)
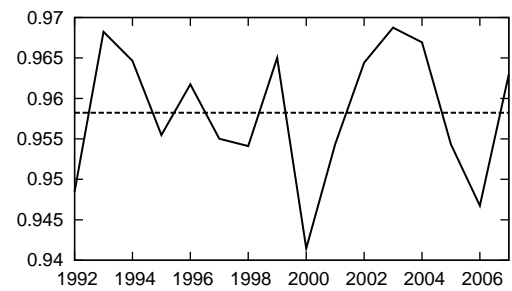
(d) Bond Prices (q_t^i)

Notes: Cycles computed with HP filter and smoothing parameter $\lambda = 6.25$. For each source of dynamics, the dashed lines indicate the steady-state level of the corresponding variables.

Figure 3 Data and Model GDP's
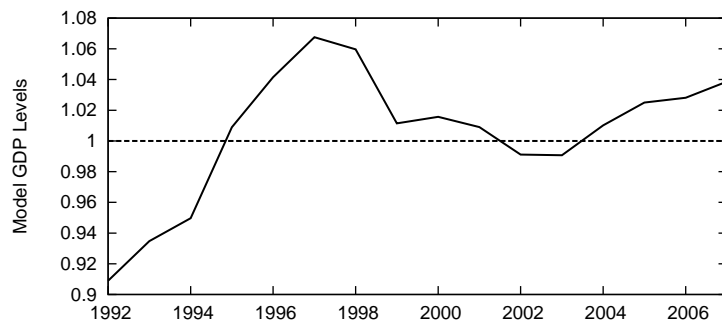
(a) Level of GDP in the Model
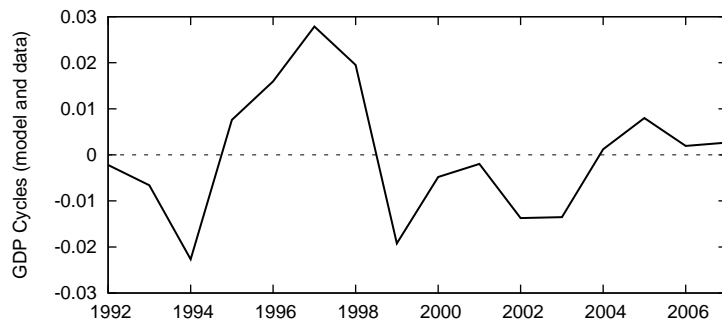
(b) Cycles of GDP, Data and Model

Notes: Cycles computed with HP filter and smoothing parameter $\lambda = 6.25$. By design, the data and model GDP cycles coincide. Dashed lines indicate the steady-state level of the corresponding variables.

Figure 4   One-Time Shock to Oil in the Competitive Equilibrium without
Persistence

(a) Oil Price (p_t^e)

(b) Consumption (c_t)

(c) Oil in Production (e_t^y)

(d) Capital Utilization (u_t)

(e) Capital Stock (k_{t-1})

(f) GDP

(g) Public Debt (b_t^g)

(h) International Assets (b_t^i)
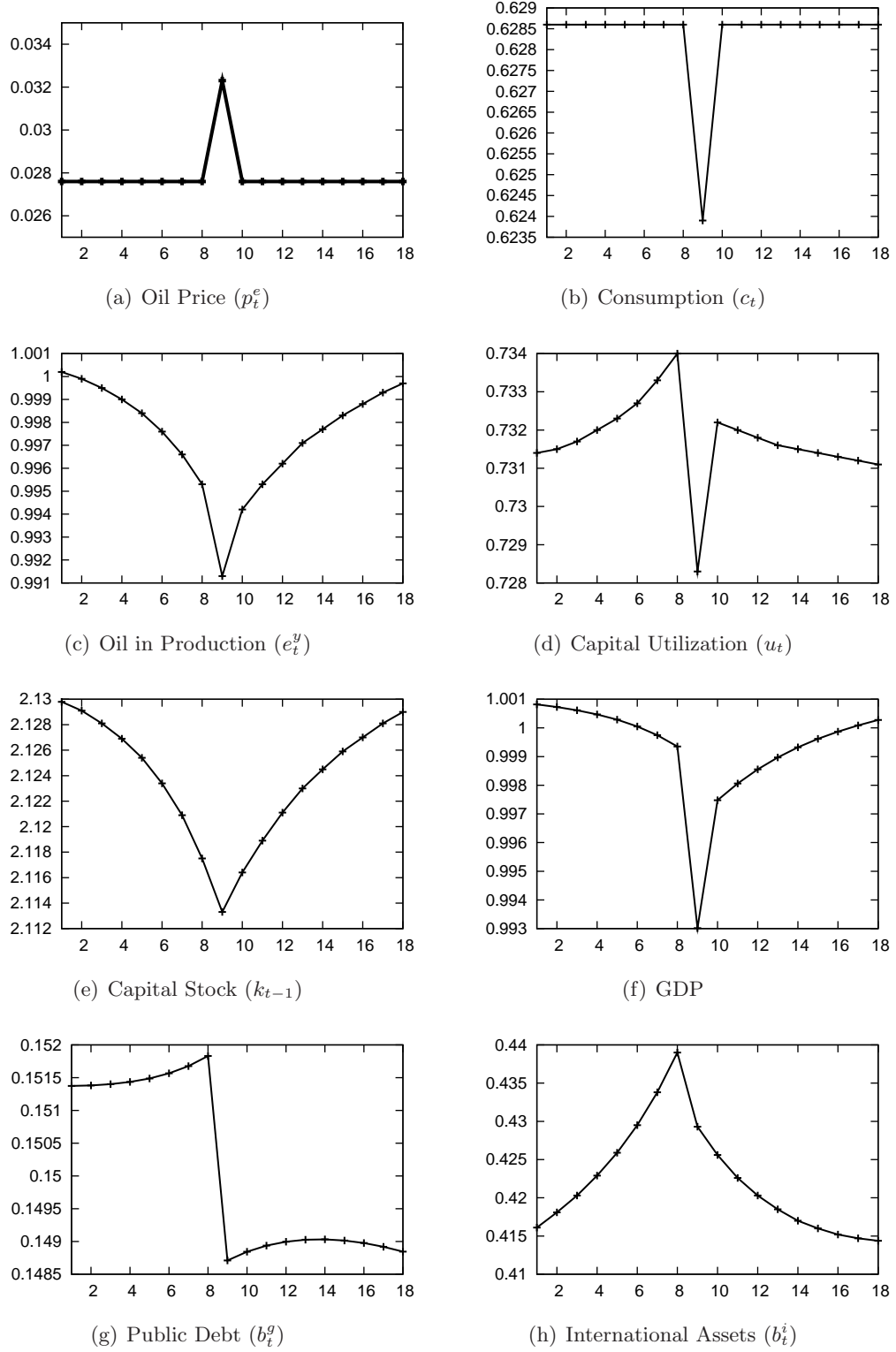
Figure 5   One-Time Shock to Oil in the Competitive Equilibrium with Persistence
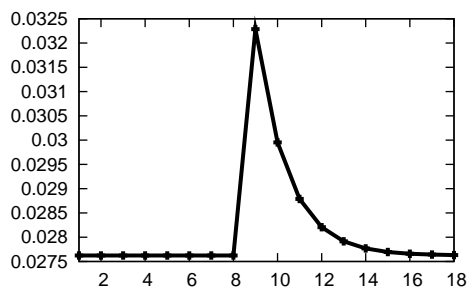
(a) Oil Price (p_t^e)
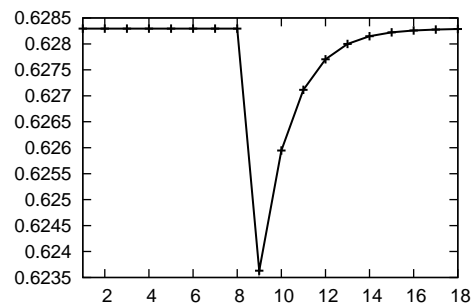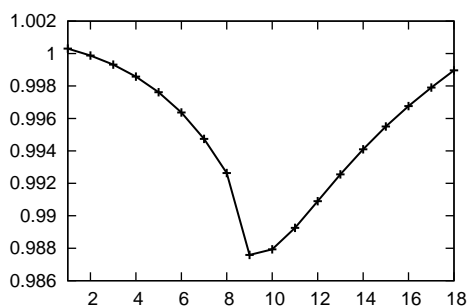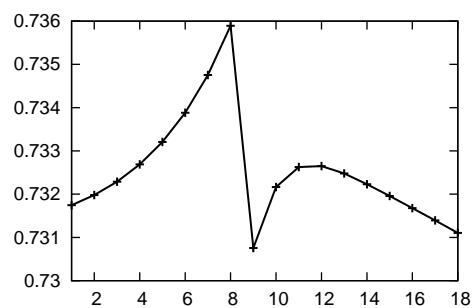
(b) Consumption (c_t)

(c) Oil in Production (e_t^y)

(d) Capital Utilization (u_t)
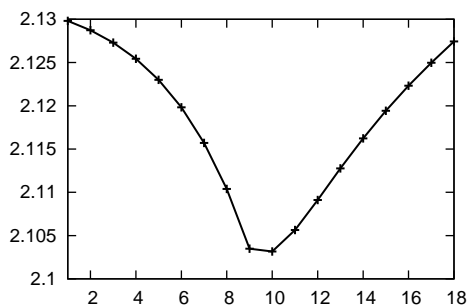
(e) Capital Stock (k_{t-1})
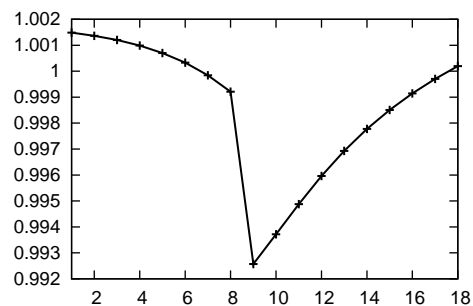
(f) GDP
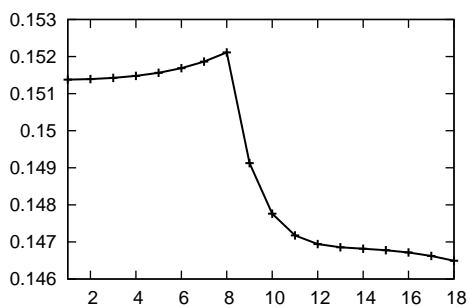
(g) Public Debt (b_t^g)
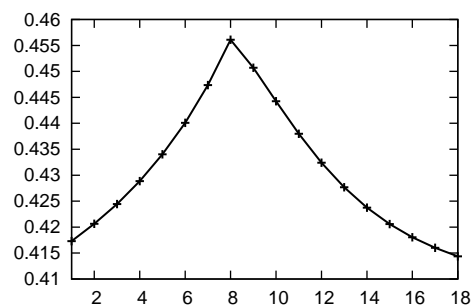
(h) International Assets (b_t^i)

Figure 6   Oil Tax Policies and Model Dynamics
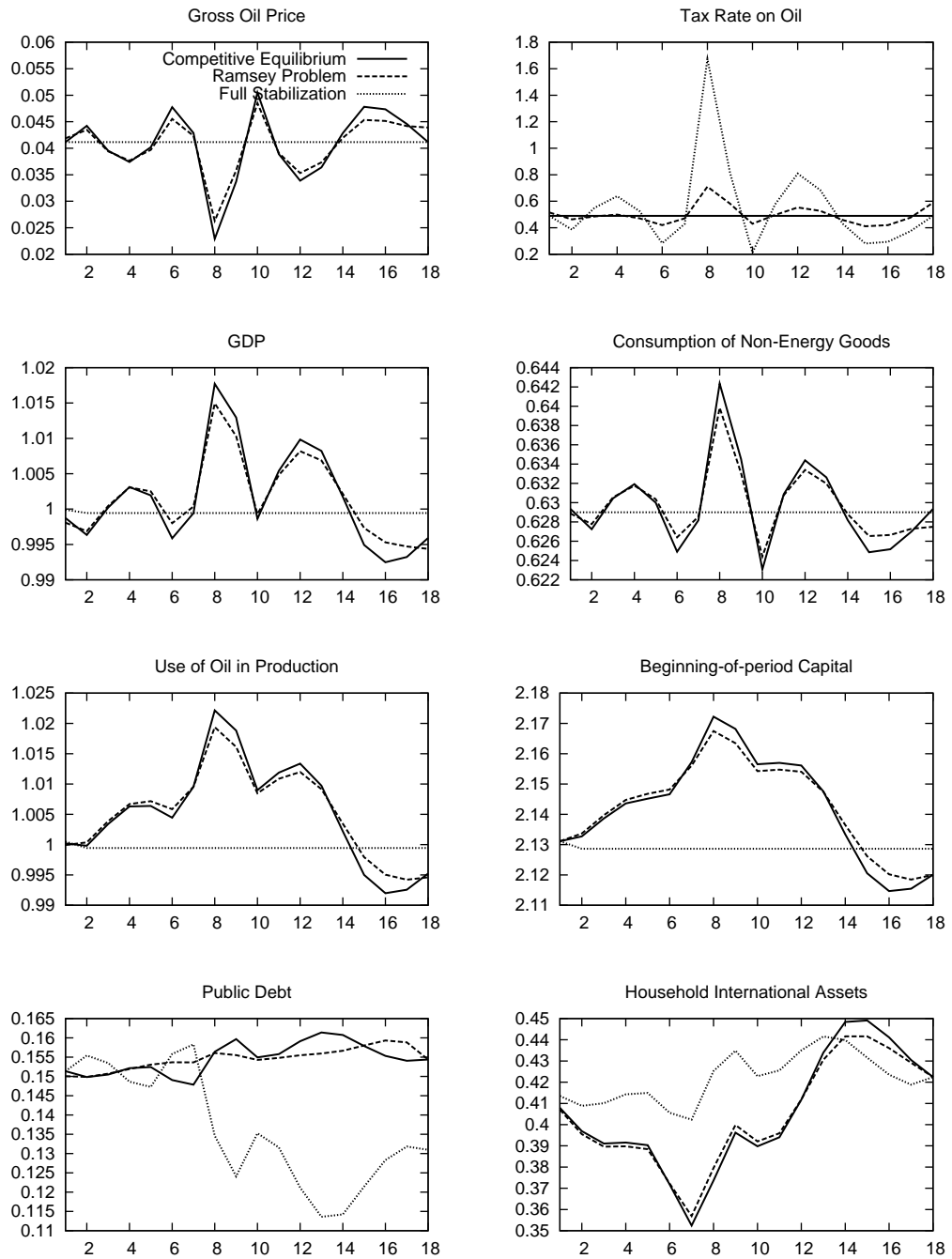
Notes: The top-left pane shows the gross price of oil $p_t^e(1 + \tau_t^e)$ in the competitive equilibrium, the Ramsey problem, and under full-stabilization. The other panes show how some key model variables behave in each case.

Figure 7   IRFs Ramsey Problem under Incomplete Markets - Developed
and Developing Countries
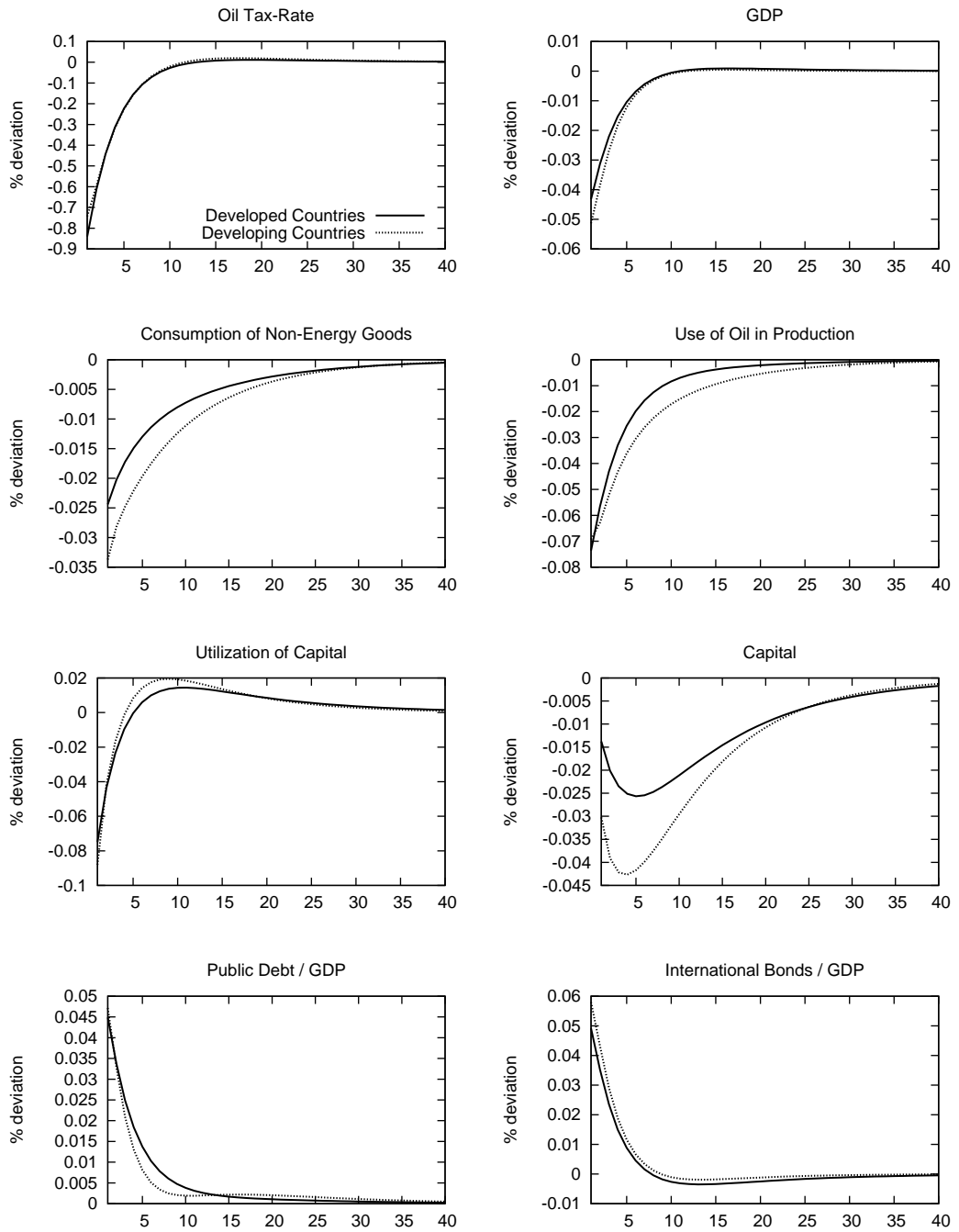
Figure 8 IRFs Ramsey Problem for Incomplete vs Complete Markets -
Developed Countries
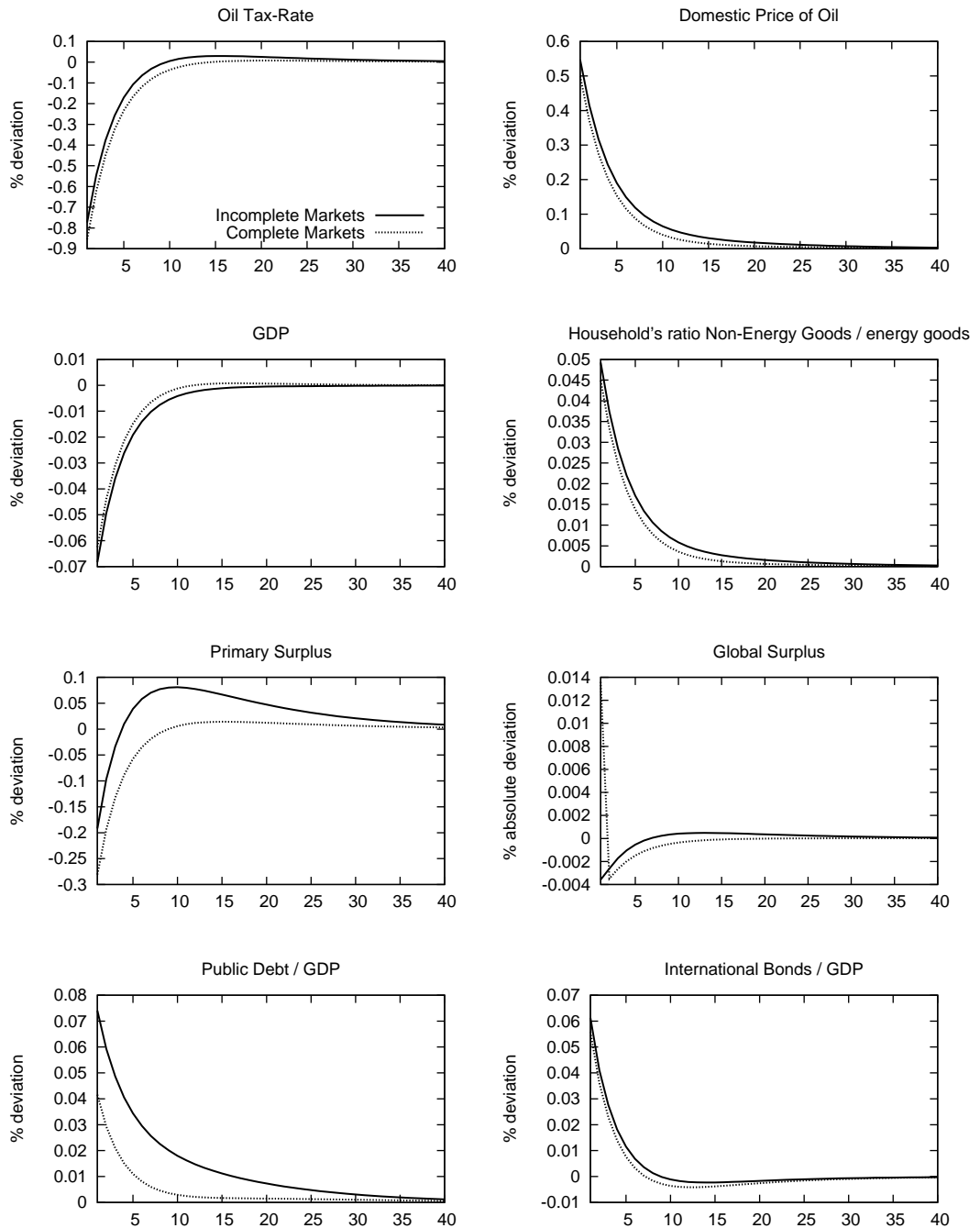
Figure 9 IRFs Ramsey Problem for Incomplete vs Complete Markets - Developing Countries
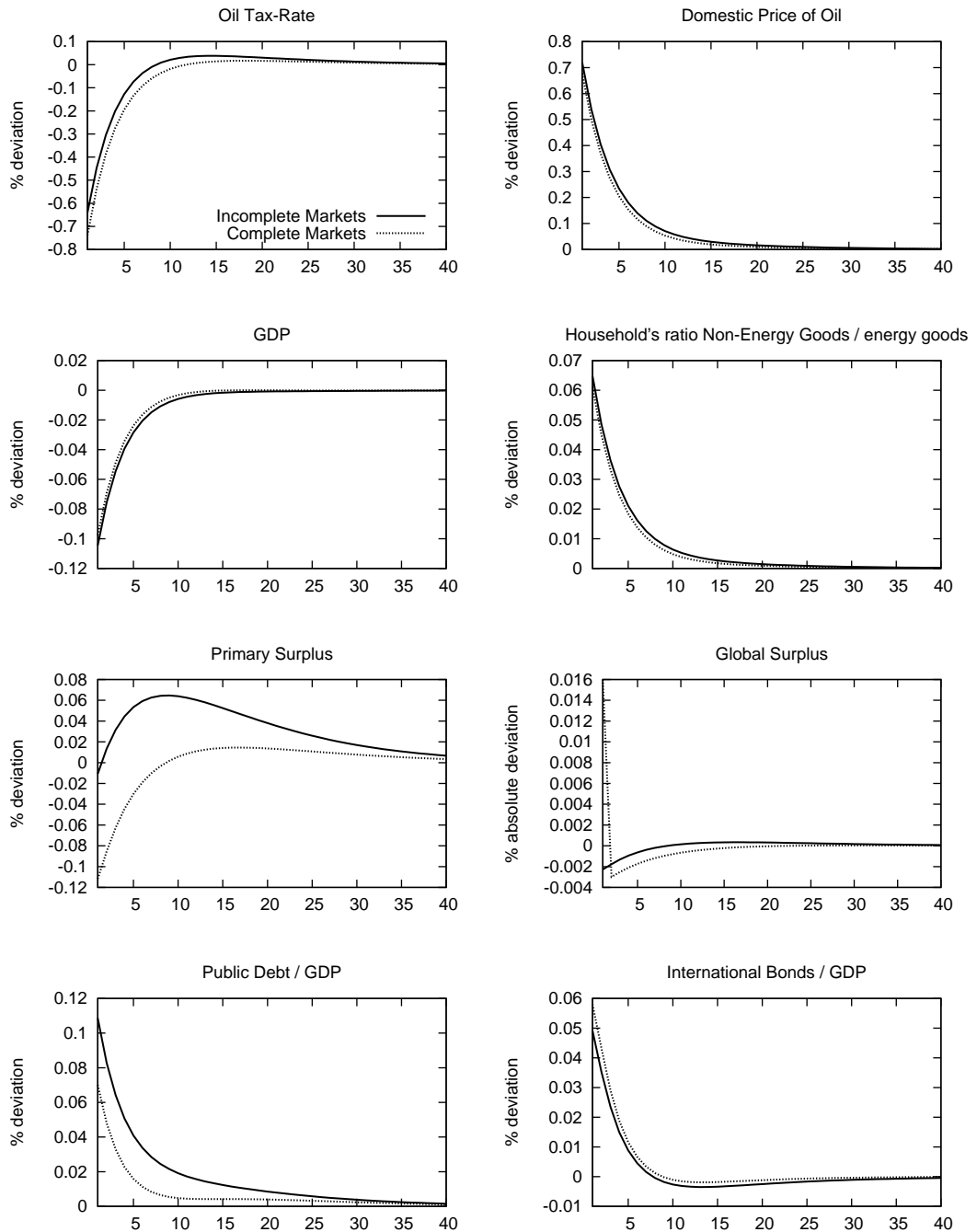
Figure 10   IRFs Sensitivity Analysis - κ_2
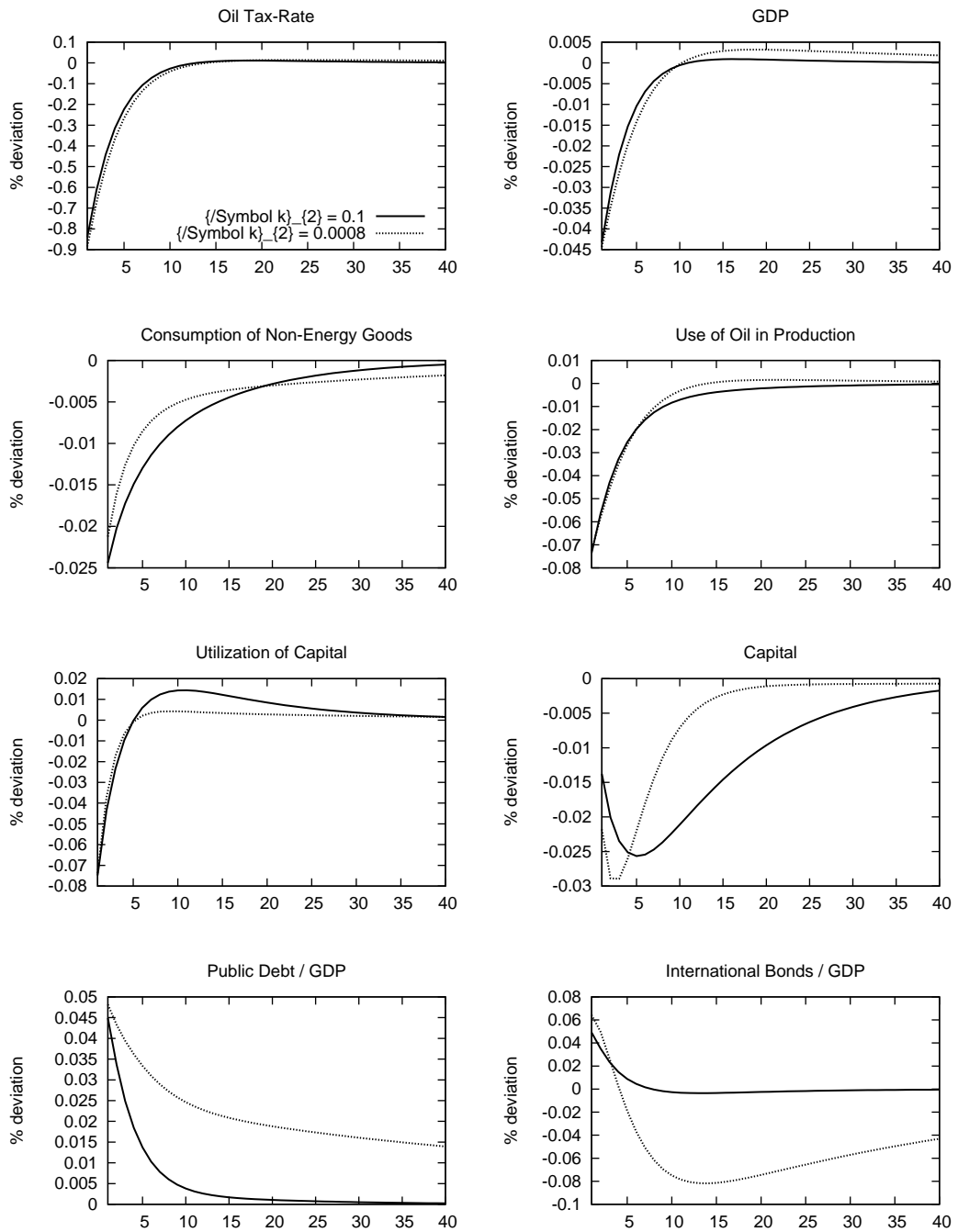
Figure 11   IRFs Sensitivity Analysis - τ^I
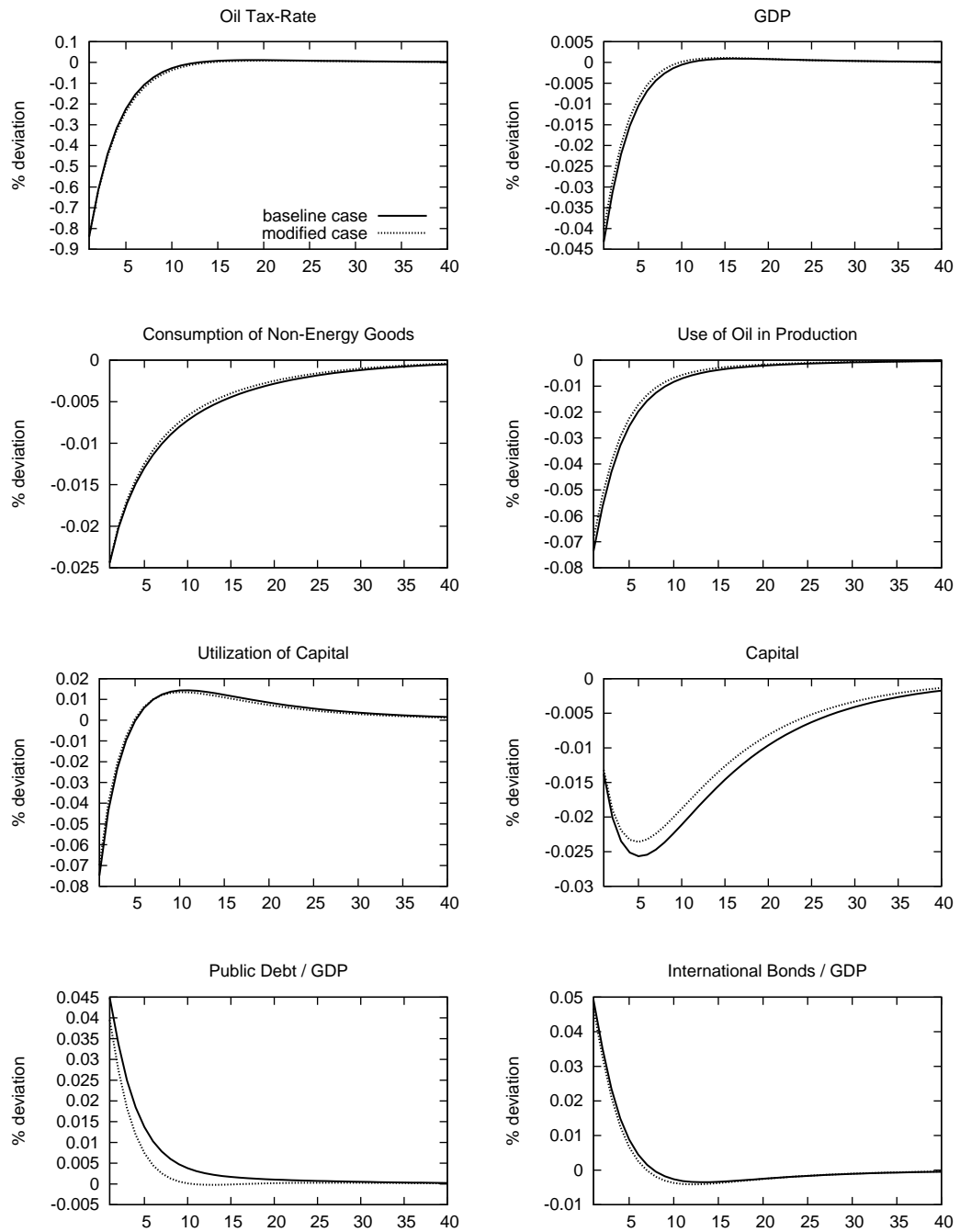
Figure 12   IRFs Sensitivity Analysis - τ^e
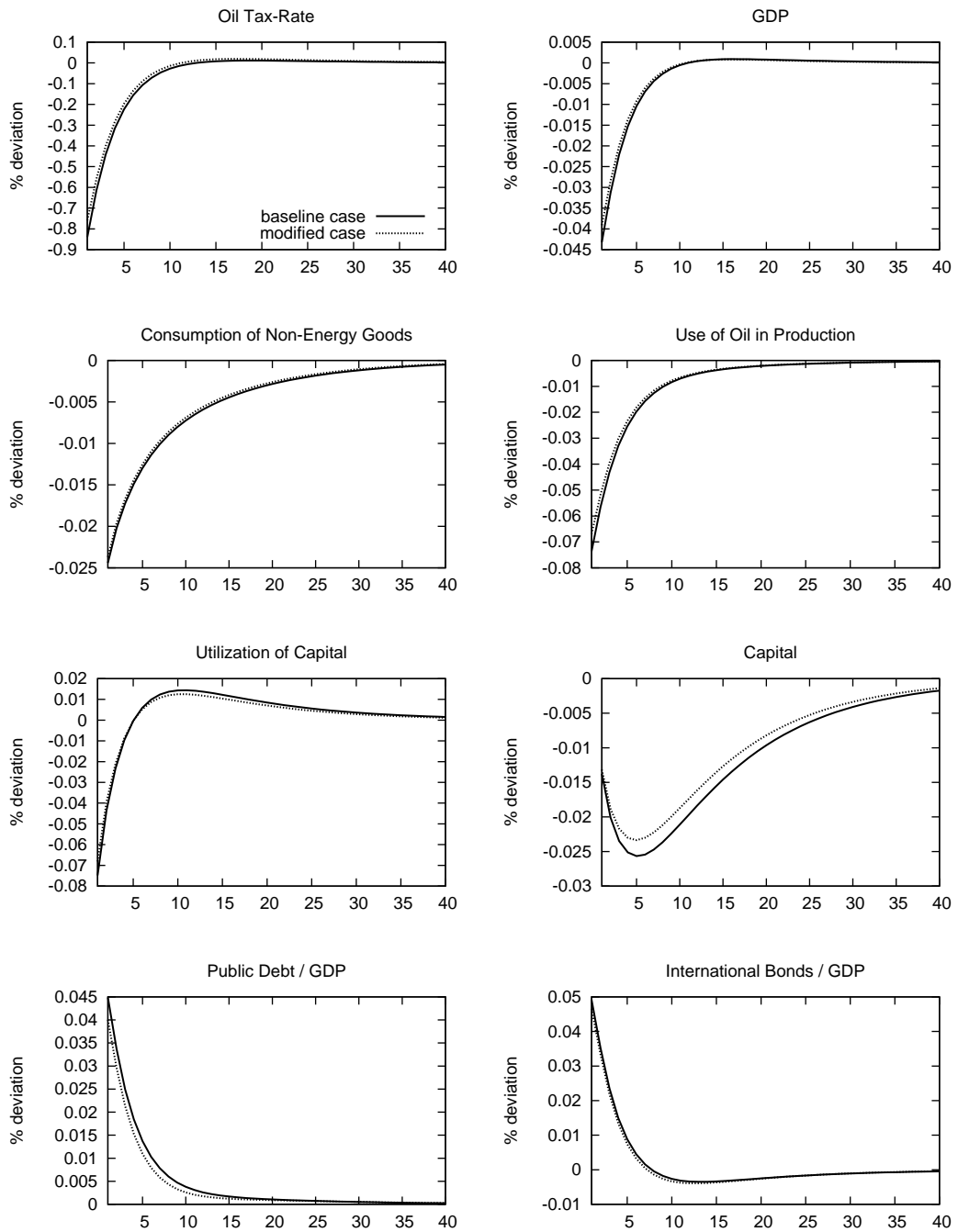
Figure 13   IRFs Sensitivity Analysis - ν_1
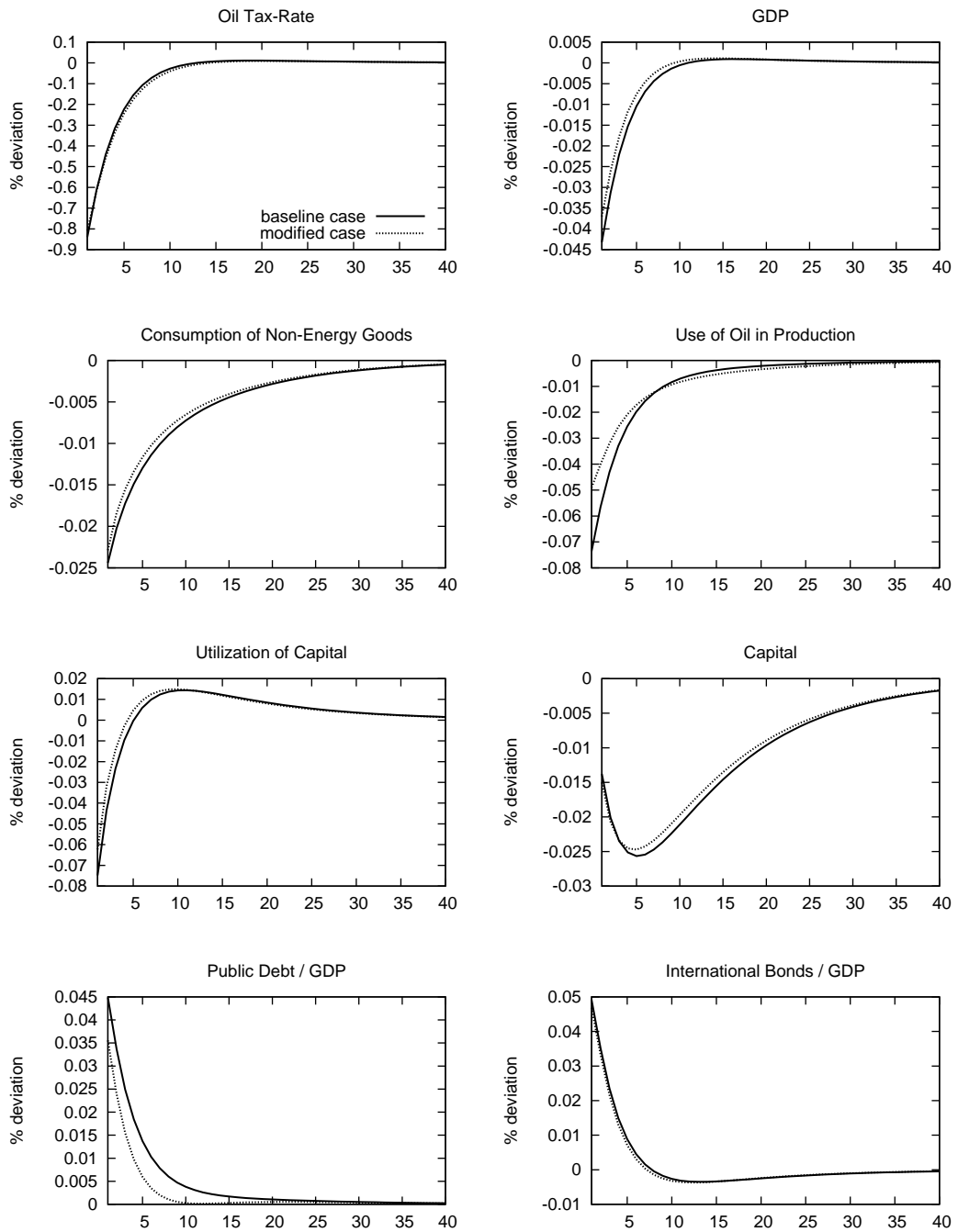
Figure 14   Baseline case: threshold values of Y_n and $R(n)$
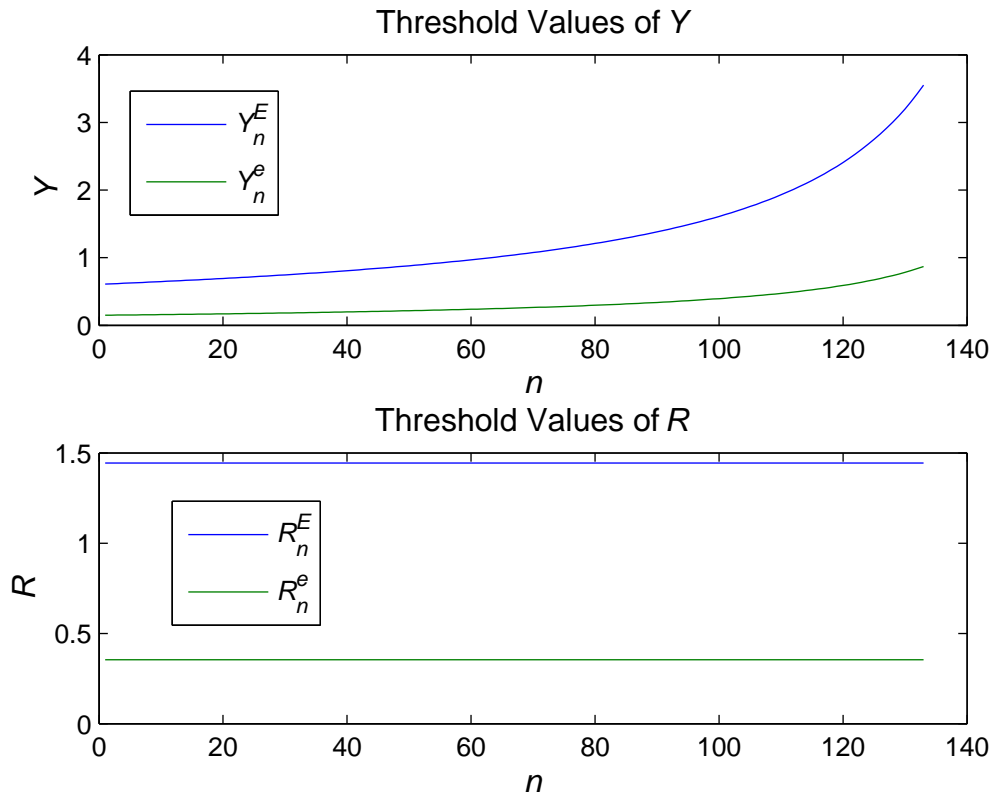
Figure 15 R_t annual average vs $R^E(n)$ and $R^e(n)$
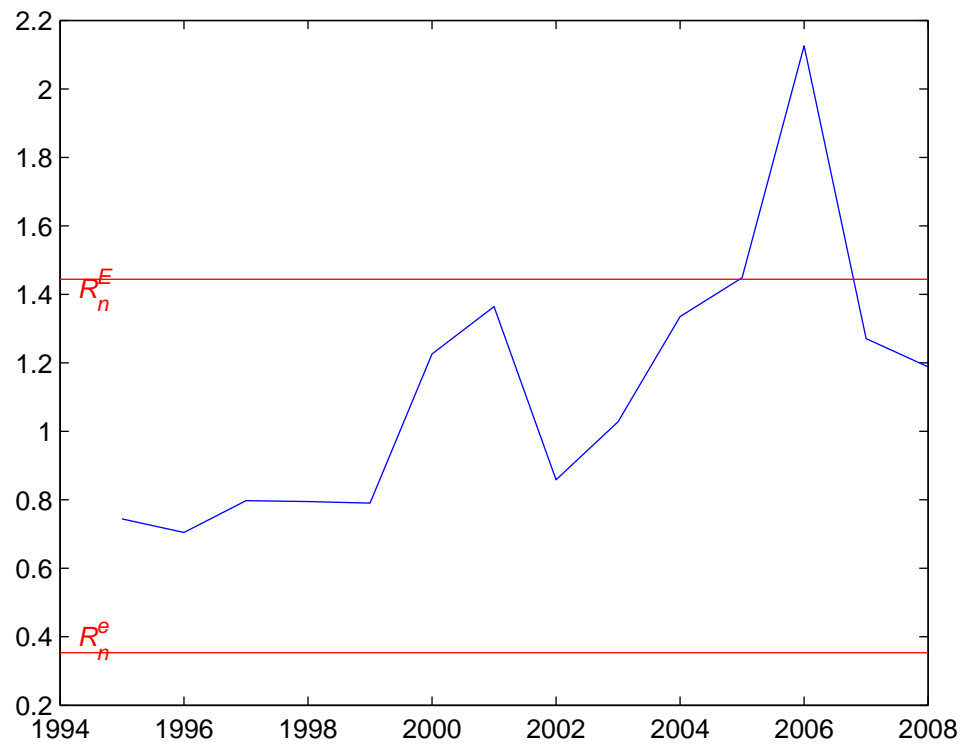
Figure 16   Extra Rents
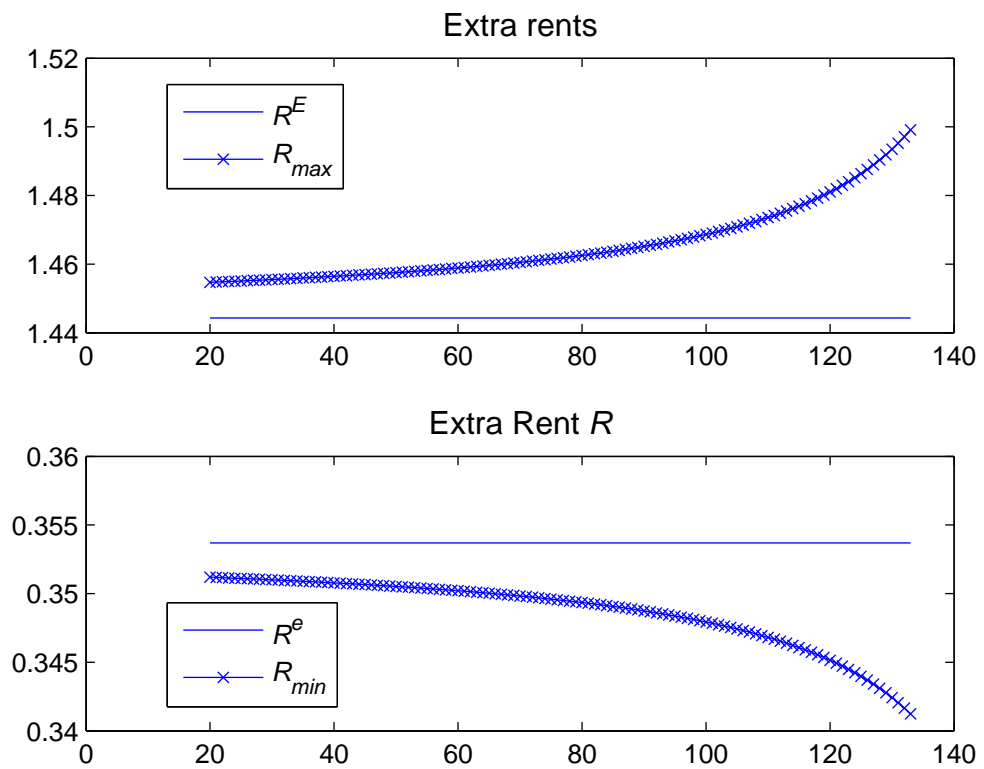
Figure 17   Sensitivity Analysis: Variable operative costs
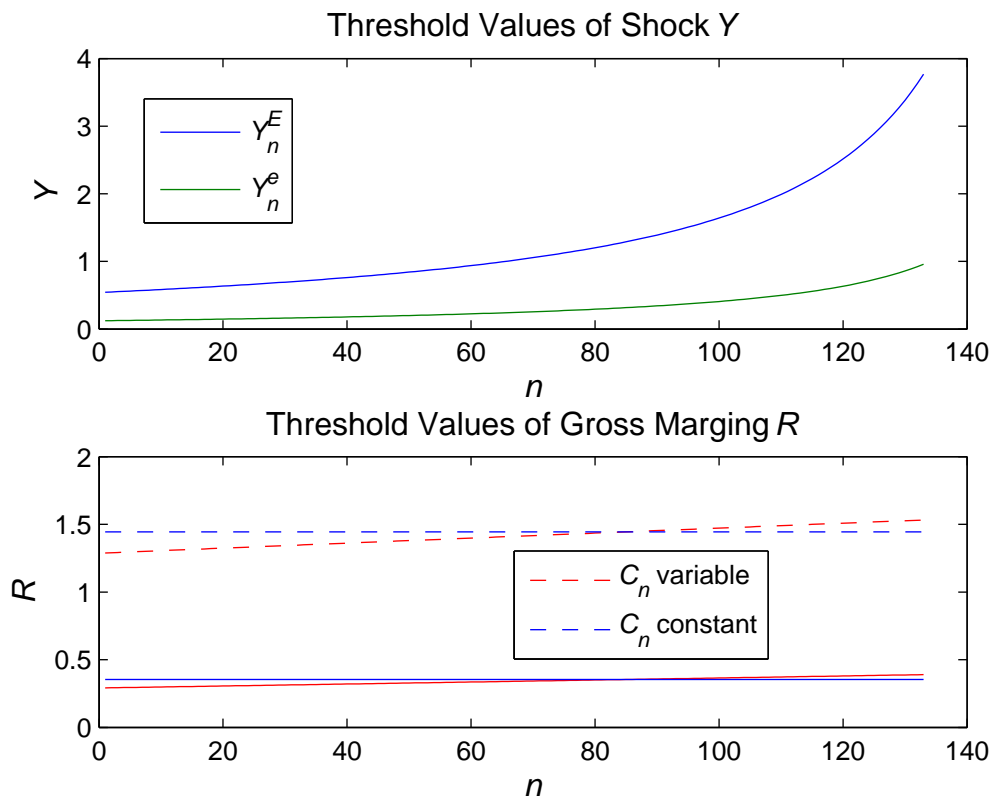
Notes: Dashed lines are for R_n^E and continuous lines are for R_n^e.

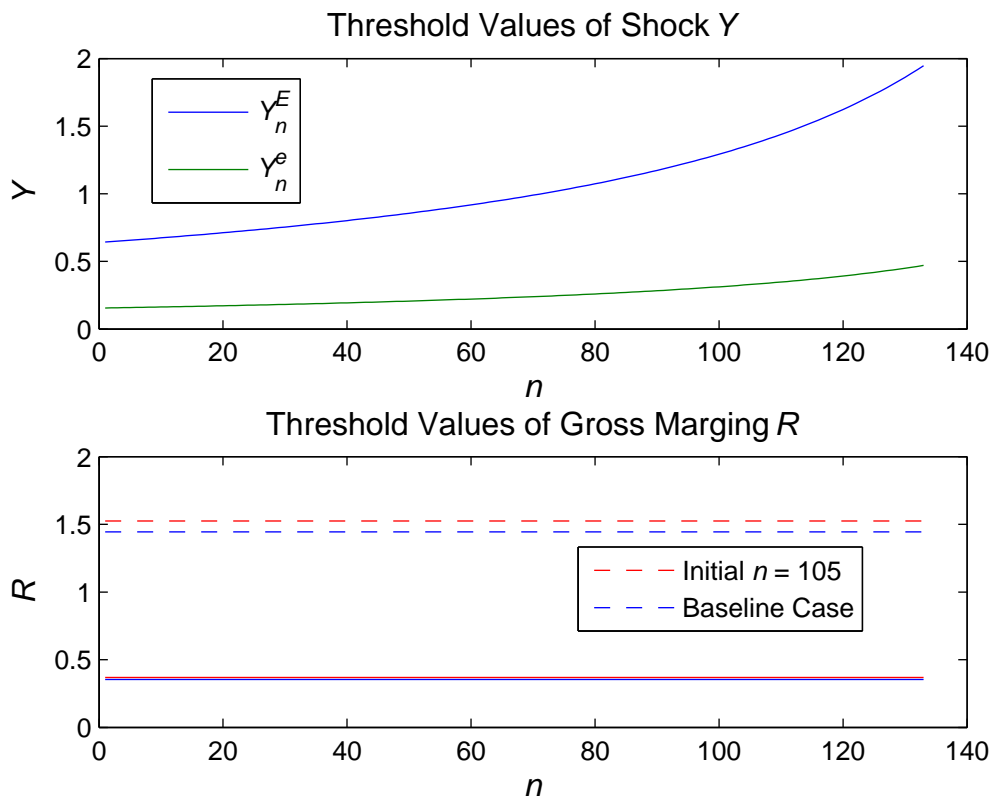Figure 18   Sensitivity Analysis: Variable entry / exit costs
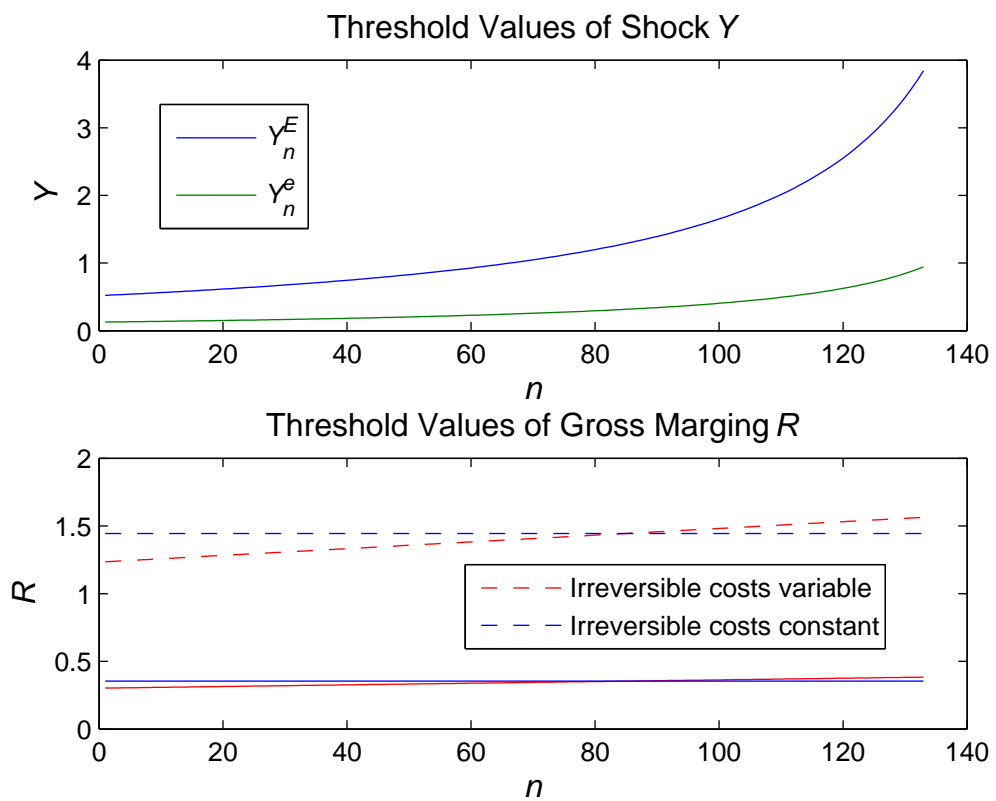
Notes: Dashed lines are for R_n^E and continuous lines are for R_n^e.

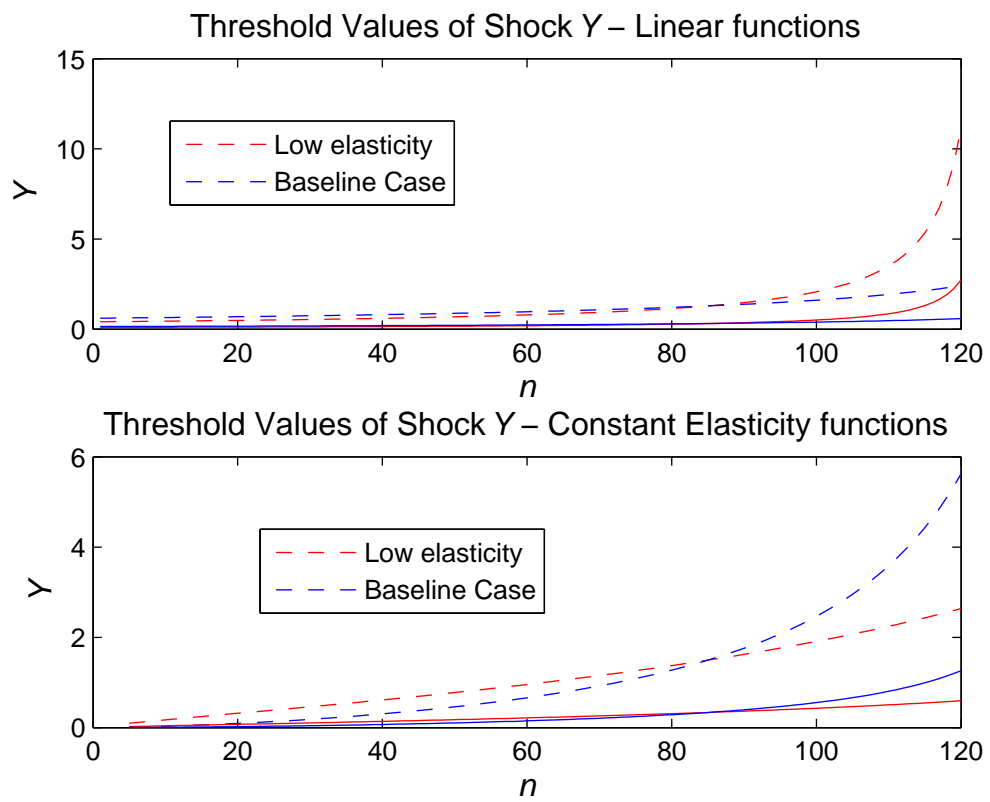Figure 19   Sensitivity Analysis: Constant elasticity functions
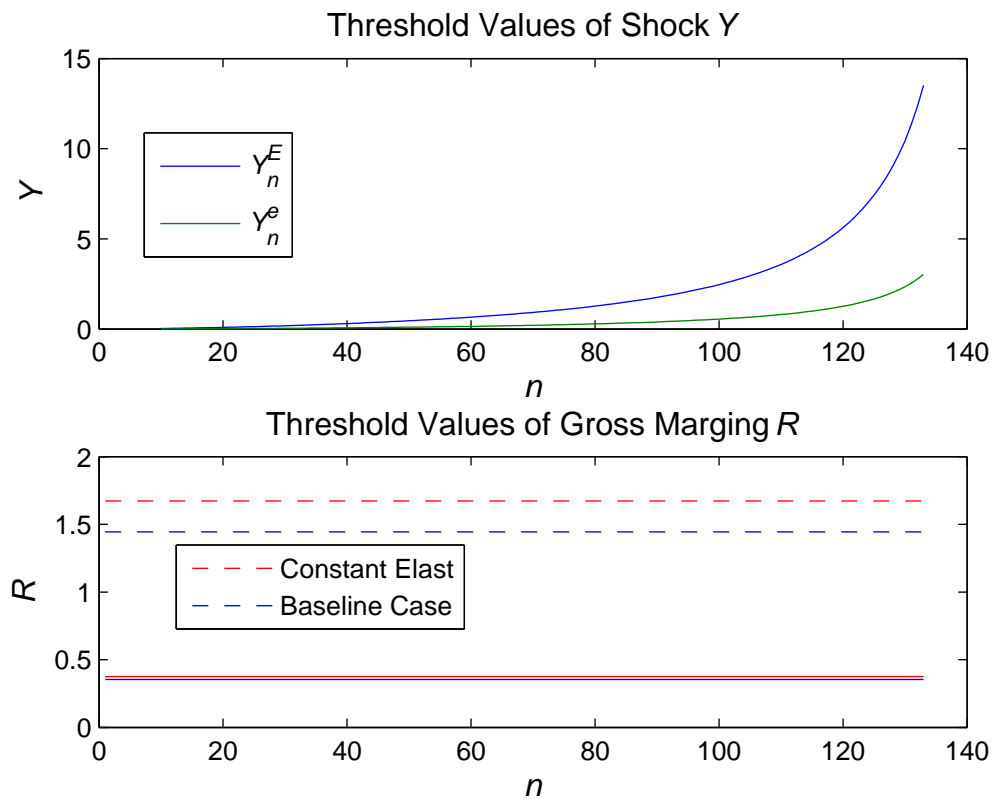
Notes: Dashed lines are for R_n^E and continuous lines are for R_n^e.
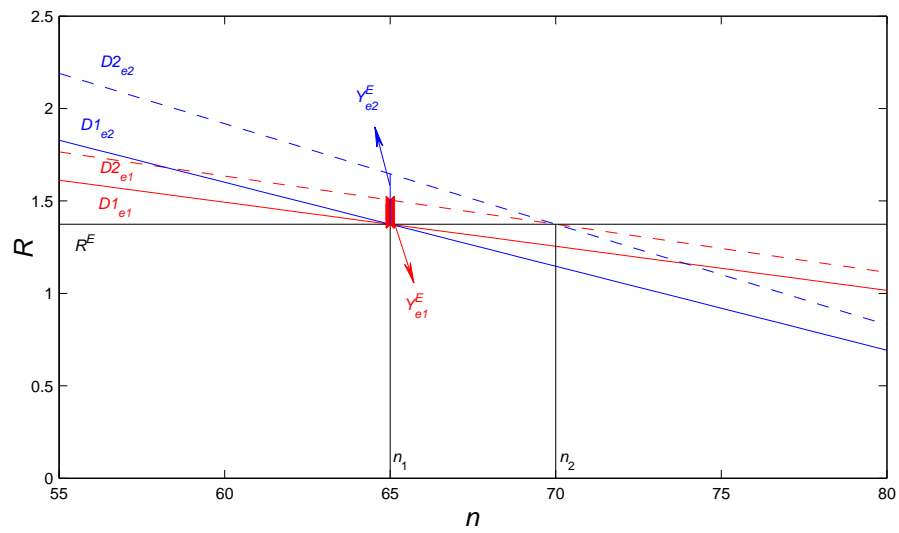
Figure 20   Sensitivity Analysis: Changing nss

Notes: Dashed lines are for R_n^E and continuous lines are for R_n^e.

Figure 21   Sensitivity Analysis: elasticity of demand for ethanol - 1st exercise
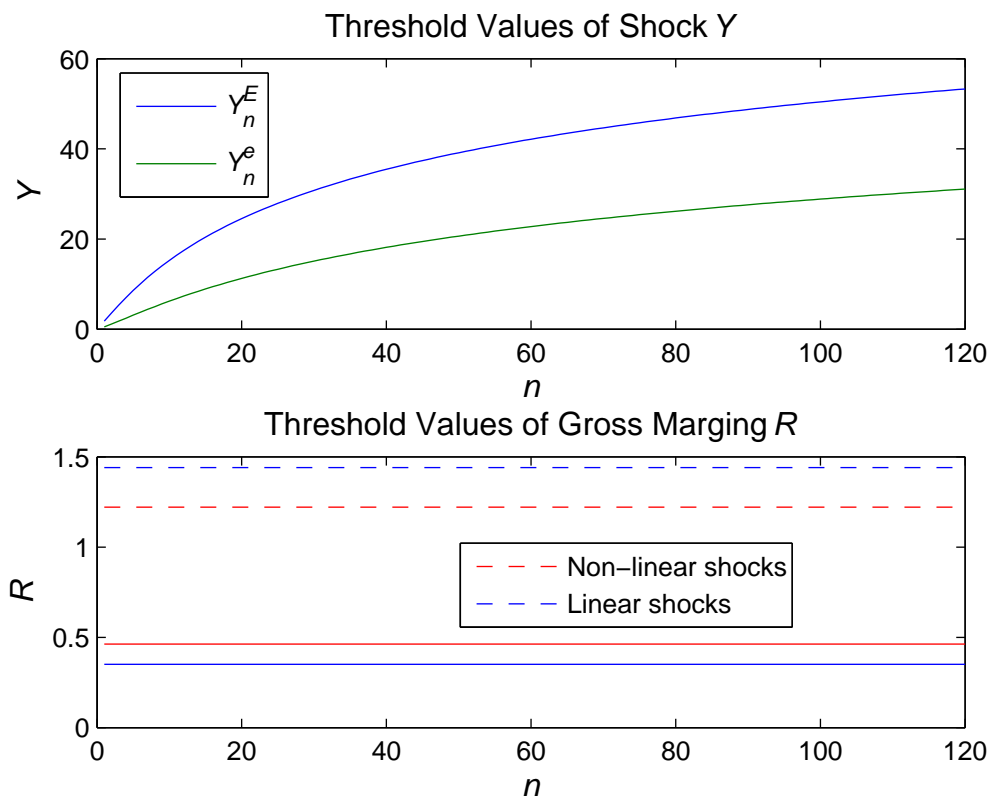

Notes: Dashed lines are for R_n^E and continuous lines are for R_n^e.

Figure 22   Static comparative - elasticity of demand for ethanol

Notes: 1) blue lines represent the inelastic gross margins and red lines represent the elastic gross margins.
2) continue lines represent equilibrium at n_1 and dashed lines at n_2.

Figure 23   Sensitivity Analysis: elasticity of demand for ethanol - 2st exercise

Notes: Dashed lines are for R_n^E and continuous lines are for R_n^e.

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