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# Three essays in economics of the environment

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**Three essays in Economics of the Environment**

by

Subhra Bhattacharjee

A dissertation submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of  
DOCTOR OF PHILOSOPHY

Major: Economics

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# CHAPTER 1. A KUHN-TUCKER MODEL OF MICRO-LEVEL HOUSEHOLD DEMAND: TEMPORAL STABILITY OF RECREATION PREFERENCES USING PANEL DATA

## 1.1 Introduction

Recreation demand models are one of the primary tools of trade for environmental economists seeking to estimate welfare changes from improved environmental quality. These estimates are used in benefit-cost analysis and evaluation of policy that affects environmental amenities. Recreational usage provides one way to measure the use value of environmental amenities and thus allows us to capture welfare changes through changes in usage when the quality of an environmental amenity changes. The data collected for analysis in these models is among the most detailed microeconomic data available for demand estimation. Thus these data make an ideal laboratory for testing and evaluating state of the art economic demand systems. Particularly challenging econometric aspects of these micro-level household data include the fact that there are many recreation sites to choose from, a large percentage of households take no trips at all, and a small proportion of households make multiple trips, generally to a small subset of sites available. The choice behavior underlying these data consists of both discrete and continuous components - whether to make a trip or not and if taking a trip, which sites to visit and how many trips to take to the chosen sites. The techniques developed to estimate recreation demand

models can be used to estimate demand for any micro-level household data.

Common recreation demand models are random utility maximization (RUM) models (Yen *et al.*, 1994; Herriges *et al.*, 1997), count data models (Egan *et al.*, 2006; Herriges *et al.*, 2008) and Kuhn-Tucker models (Phaneuf *et al.*, 2000; Von Haefen *et al.*, 2004). Each of these addresses a specific subset of characteristics of the data. This paper focuses on Kuhn-Tucker models.

The Kuhn-Tucker models are utility-theoretic and the estimating equations for them are derived directly from the underlying optimization problem. Further, these models can address all but one of the typical characteristics of recreation demand data, *viz.*, their count nature. However, the Kuhn-Tucker models are highly non-linear and the presence of corners, the situation where a consumer does not consume one or more goods available, makes them analytically challenging. Phaneuf *et al.* (2000) presented one of the first estimates of the Kuhn-Tucker model in conjunction with welfare estimates for hypothetical changes in site quality and prices. von Haefen *et al.* (2004) extended that work by developing an algorithm for estimating a Kuhn-Tucker model when the choice set is large, *i.e.* when there are a very large number of possible combinations of recreation sites to visit and, therefore, “corners” that each individual can choose among.

While significant strides have been made in accurate econometric estimation of Kuhn-Tucker models of recreation demand, the application of these models is still rare. For instance, very little is known about the temporal stability of parameter estimates and associated welfare estimates of environmental quality improvements. Typical studies collect a year of data, fit demand equations and use them to estimate welfare measures for policy relevant changes in environment quality. But what if parameter estimates are not stable over time? Is it possible to use welfare estimates from a single year of data to characterize welfare changes for all future years? Or do parameters vary largely from year to year,

necessitating the use of multiple years of data to accurately pin down preferences for environmental quality changes? These are questions that have not been asked in the literature before. This paper addresses these questions using a four-year panel dataset with information on trips taken by households in Iowa to 127 major lakes. The availability of such a dataset enables us to ask and answer these questions since we can observe how demand for recreational trips have changed with changes in site quality attributes over the four years.

This paper estimates a Kuhn-Tucker model with a large choice set using a four-year panel dataset for close to 1300 households. This is the first such estimation in the literature, and only the second estimation of corner solutions with panel data. Chakir *et al.*(2004) estimated a model with corner solutions for four possible choices and two years of data. However, that model is not utility theoretic as the Kuhn-Tucker model since it arbitrarily posits a demand function and adds an error term to it. We present estimates with four years of data and a choice set consisting of 127 sites.

There have been very few panel data studies in the recreation demand literature. Those that use panel methods (Englin and Cameron, 1996; Egan and Herriges, 2006) combine revealed preference and stated preference data from individuals at the same point of time, but do not have multi-year data. In addition there are no prior studies that combine the triad of a Kuhn Tucker model, a large choice set and a panel dataset. This research fills that gap. Specifically, the contribution of this paper lies in presenting a method to incorporate correlations in preferences over time within the Kuhn-Tucker model for a large choice set and using that method to examine whether preferences of households for outdoor recreation are stable over time.

The paper is organized as follows. Section 2 presents a brief overview of the precursors of the Kuhn-Tucker models and discusses the place and the importance of this approach

in the context of recreation demand models; sets up the model; describes the likelihood functions for different assumptions about the distributions of the regression error term; and presents the method for estimating the model with correlation across time. Section 3 describes the data and examines the stability of reported trips across different years. Section 4 presents the estimation results and the final section concludes.

## 1.2 The Kuhn-Tucker Approach and Model Specification

The Kuhn-Tucker model is the latest in a line of approaches that model the downward censored nature of recreation demand data, i.e the fact that trip demand cannot be negative. It is a development from the Amemiya-Tobin approach (Amemiya, 1974; Tobin, 1958). The Kuhn-Tucker formulation improves on the previous approaches in that it is utility consistent while the earlier approaches are not. Utility consistency implies that the decision to undertake a trip and the site choice decision are both derived from the same underlying utility function. However, until recently these models were not extensively used because of their analytical intractability. Recent advances in computational resources have made it much easier to estimate these models.

Phaneuf *et al.* (2000) used the Kuhn Tucker model to estimate demand for fishing in the Wisconsin Great Lakes region and offered a method for estimating the expected welfare effects proxied by compensating variation associated with hypothetical policy changes in the Great Lakes region. The welfare estimation process involved computing the demands at every possible corner of the budget-constrained choice space and choosing the one that maximized utility. With the choice set consisting of four different sites, for each individual there were 16 ( $2^4$ ) different corners at which demands had to be estimated. This was computationally tractable. However, this method becomes unwieldy for large choice sets which are typical for many kinds of recreational choices. Von Haefen *et al.* (2004) offered

a method for estimating changes in welfare systems for large demand systems. All of these studies worked with data for a single time period.

We extend the Phaneuf *et al.* (2000) and the von Haefen *et al.* (2004) work by accounting for the correlation in household preferences over time and implement it using a panel dataset for trips to 127 lakes in Iowa. This also enables us to look at the stability of the parameter estimates from the Kuhn-Tucker model over time.

For the model we assume that there are T periods and a total of M sites each having K characteristics. The utility function for a household  $i$  in period  $t$  is given by

$$U_{it} = \sum_{j=1}^M \exp(\delta' S_{it} + \eta_{ijt}) \ln[\exp(\gamma' q_{jt}) x_{ijt} + \theta] + \ln(z_{it}), \quad (1.1)$$

where

$x_{ijt}$  = Number of trips taken by the  $i^{th}$  individual to site  $j$  in period  $t$ ,

$Q_t = [q_{1t}, q_{2t} \dots q_{Mt}]$ , where  $q_{jt}$  is a K by 1 vector of quality variables associated with site  $j$ ,

$S_{it}$  = the set of demographic characteristics for the  $i^{th}$  household in period  $t$

$\theta$  = parameter allowing for corner solutions,

$\eta_i$  = an MT by 1 matrix of error terms for the  $i^{th}$  individual,

$P_{ijt}$  = the  $i^{th}$  individual's cost of visiting site  $j$  in period  $t$ ,

$z_{it}$  = a composite of all other goods (the numeraire and a necessary good),

$\gamma$  and  $\delta$  = parameters of the model.

$\eta_{ijt}$  represents heterogeneity in preferences consisting of factors that are assumed known to the individual but unobserved by the researcher. Associating the error term with preferences rather than with the demand makes this model consistent with McFadden's random utility maximization framework. This specification of the utility function is additively separable. Further, it assumes that every good (site in this case) is a normal good and all goods are Hicksian substitutes. The utility function also assumes weak complementarity,

meaning that quality attributes of a site do not affect the total utility of the individual if the site is not visited (Maler, 1974). In other words, the individual cares about the quality attributes of only those sites that s/he visits. The budget constraint for the individual in period  $t$  is

$$Y_{it} = \sum_{j=1}^M P_{ijt}x_{ijt} + z_{it}; x_{ijt} \geq 0; z_{it} > 0 \forall i, j, t. \quad (1.2)$$

The decision variables for individual  $i$  are  $x_{ijt}$  and  $z_{it}$ . The number of trips to any lake must be non-negative while the expenditure on the numeraire must be strictly positive. The Kuhn-Tucker first order condition for the  $i^{th}$  individual, in period  $t$  with respect to the  $j^{th}$  site is given by

$$\frac{\partial U_{it}}{\partial x_{ijt}} = \exp(\delta' S_{it} + \eta_{ijt}) \frac{\gamma' q_{it}}{[\exp(\gamma' q_{it})x_{ijt} + \theta]} \leq \frac{P_{ijt}}{z_{it}}, \quad (1.3)$$

with a complementarity slackness condition which requires that

$$x_{ijt}(\exp(\delta' S_{it} + \eta_{ijt}) \frac{\gamma' q_{it}}{[\exp(\gamma' q_{it})x_{ijt} + \theta]} - \frac{P_{ijt}}{z_{it}}) = 0 \quad (1.4)$$

This in turn implies that

$$\eta_{ijt} \leq \ln\left(\frac{P_{ijt}}{z_{it}}\right) + \ln\left[x_{ijt} + \frac{\theta}{\exp(\gamma' q_{it})}\right] - \delta' S_{it} = g(x_{ijt}, P_t, z_{it}, S_{it}, q_{jt}; \beta), \quad (1.5)$$

where  $\beta$  is the vector of parameters to be estimated. For each individual  $i$  in each period  $t$ , there are  $M$  such equations - one for each site. These equations together with the assumed distribution of the error term define a likelihood function which can then be used to estimate the parameters and welfare changes from hypothetical changes in site quality attributes.

The error term for each individual and for each site are allowed to be correlated across time. The error term for the  $i^{th}$  individual and  $j^{th}$  site in period  $t$  consists of two components - one that is unique to the individual, the site and the time period - a purely

stochastic term - and another which is unique to the individual but remains constant over time. The second component is the one that induces correlation in preferences across time. This component could be further broken down into two components - one that is constant across time and sites and another that is constant across time but different for each site. All three components of the error term are random. Errors are uncorrelated across individuals. Formally, this can be written as:

$$\eta_{ijt} = u_i + \tau_{ij} + \varepsilon_{ijt}$$

$u_i$ ,  $\tau_{ij}$  and  $\varepsilon_{ijt}$  are all drawn from different distributions with zero location parameters and scale parameters given by  $\sigma_u^2$ ,  $\sigma_\tau^2$ , and  $\sigma_\varepsilon^2$ , respectively. They are all uncorrelated with each other and across individuals. This specification of the error term nests two sub-specifications given by

$$\eta_{ijt} = u_i + \varepsilon_{ijt}$$

$$\eta_{ijt} = \tau_{ij} + \varepsilon_{ijt}$$

$u_i$  indicates correlation across time and sites. When the correlation across time is constant across sites as well, it indicates that individuals who are more likely to take trips to one site are also more likely to take trips to other sites. It would capture the fact that some people are avid trip takers, boat owners or fishers and others are not into outdoor recreation - factors that are invariant across sites for a given individual.  $\tau_{ij}$  indicates correlation across time but not across sites. A correlation term like this captures the fact that people like some sites more than others and visit the same sites over and over again.

For the present paper, we assume that  $\varepsilon_{ijt}$ ,  $u_i$  and  $\tau_{ij}$  are all drawn from different normal distributions.

The first order conditions of the Kuhn-Tucker model together with the complementarity



slackness conditions require that

$$\begin{aligned}\eta_{ijt} &= g(x_{it}, P_t, z_{it}, S_{it}, q_{jt}; \beta) = g_{ijt} \text{ if } x_{ijt} > 0 \text{ and} \\ \eta_{ijt} &\leq g(x_{it}, P_t, z_{it}, S_{it}, q_{jt}; \beta) = g_{ijt} \text{ if } x_{ijt} = 0.\end{aligned}$$

Let us assume that the  $i^{th}$  consumer makes no trips to the first  $n$  sites and a positive number of trips to the rest. Further let  $[P_t, z_{it}, S_{it}, q_{jt}] = R_{it}$ . Then the likelihood function for individual  $i$  in period  $t$  is given by

$$L_{it} | \beta = \int_{-\infty}^{g(x_{i1t}; R_{it}; \beta)} \dots \int_{-\infty}^{g(x_{int}; R_{it}; \beta)} f(\eta_{i1t}, \dots, \eta_{int}, g_{i(n+1)t}, \dots, g_{iMt}) | J | d\eta_{i1t} \dots d\eta_{int} \quad (1.6)$$

where  $| J |$  is the Jacobian determinant for the transformation of the error term,  $f(\eta_{it})$  is the assumed probability density function of the error and  $F(\eta_{it})$  is the corresponding distribution function. If preferences are not correlated over time so that the error term has only one component i.e  $\eta_{ijt} = \varepsilon_{ijt}$  then we can use this likelihood function to compute the likelihood function for the entire sample.

However, we have data for more than one period, and we need to take into account the term that for each individual induces correlation across time periods. There are two ways that we can incorporate the correlation in our estimation process. If the correlation term is given by  $u_i$ , the first order condition can be written as

$$\begin{aligned}u_i + \varepsilon_{ijt} &= g(x_{it}, R_{it}; \beta) \text{ if } x_{ijt} > 0, \text{ and} \\ u_i + \varepsilon_{ijt} &\leq g(x_{it}, R_{it}; \beta) \text{ if } x_{ijt} = 0.\end{aligned}$$

These, in turn imply

$$\begin{aligned}\varepsilon_{ijt} &= g(x_{it}, R_{it}; \beta) - u_i \text{ if } x_{ijt} > 0 \\ \varepsilon_{ijt} &\leq g(x_{it}, R_{it}; \beta) - u_i \text{ if } x_{ijt} = 0\end{aligned}$$

Taking into account this correlation term, the likelihood function in equation (5) is conditional on a given draw of  $u_i$ . Further, an individual's likelihood function must include the contributions for each year of the panel. Conditional on a given draw of the correlation term  $u_i$ , the likelihood functions for an individual in different years are independent. Hence, the conditional likelihood function for individual  $i$  is given by the product of his/her conditional likelihood functions in each year.

$$L_i | \beta; u_i = \prod_{t=1}^T (L_{it} | \beta; u_i). \quad (1.7)$$

The expected value of the likelihood for individual  $i$  is given by

$$E(L_i | \beta) = \int_{-\infty}^{\infty} \prod_{t=1}^T (L_{it} | \beta, u_i) f_u(u_i) du_i. \quad (1.8)$$

The estimation process will involve the following steps. First we define the likelihood function for each individual in each period, conditional on a draw of the correlation term  $u_i$ . The conditional likelihood functions for that individual for each year are multiplied to get the conditional likelihood function for that individual for a given draw of  $u_i$ . Integrating this over the range of  $u_i$  gives us the expected likelihood function for that individual. This integration will have to be done numerically since the functional form of this likelihood function is not analytically tractable. If the correlation term is given by  $\tau_{ij}$ , individual  $i$ 's conditional likelihood function is given by

$$L_i | \beta; \tau_{ij} = \prod_{t=1}^T (L_{it} | \beta; \tau_{i,j}). \quad (1.9)$$

so that her likelihood function is given by

$$E(L_i | \beta) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \prod_{t=1}^T (L_{it} | \beta, \tau_i) f_{\tau}(\tau_i) d\tau_{i1} \dots d\tau_{iM}. \quad (1.10)$$

Using the maximum likelihood approach we can estimate the distribution parameters of the term that induces correlations in preferences and test if these parameters are significantly different from zero. An alternate approach is to model the correlation in preferences through a method of bootstrapping. To implement the bootstrapping method, we take a set of random draws with replacement from the sample of households and estimate the model for for each of the four years separately for the same set of individuals. Each time we take 1280 draws with replacement from our sample of 1280 and repeat the process 1000 times. Since the single year estimates use data for the exact same individuals for each of the four years, the correlation in preferences is absorbed into the other parameters of the model and we do not use a separate parameter for correlation. Using a large number of repeated draws (1000) we can establish the 95% confidence intervals for the parameter and welfare estimates and also test the differences in these estimates across the years.

In the bootstrap method we do not estimate a parameter relating to the correlation term. Neither do we need to make any assumptions about the nature of the correlation, unlike the error components approach, which induces a specific form of correlation. If the model with the error components is correctly specified, then the bootstrap method will be less efficient in terms of the parameter estimates. However, it will be much more flexible both in terms of the number of parameters and in terms of the form of the correlation.

### 1.3 Data

For four consecutive years from 2002 to 2005, a survey questionnaire about trips to lakes in Iowa was sent out to a random sample of households within the state. Each

year about 4000 completed surveys were returned. Of the returned surveys, information on some individuals could not be used because the surveys were incomplete. Excluding those individuals for whom complete information on trips taken was not available, we have about 3500 completed surveys in each year except in 2003 for which we have about 4500 surveys. The sample was expanded in 2003 with surveys being sent out to an additional sample of households who did not receive the survey in 2002. There was an attrition from the balanced panel as households dropped out over the years. Some returned surveys did not have complete trip information and were dropped from the dataset. Further, some households claimed to have made over hundred trips. They were typically households who lived on or close to a lake and drove by the lake everyday as they went by their daily activities. Our purpose is to model recreational visit to lakes and hence these households were dropped from the dataset. A balanced four-year panel with complete trip information is available for 1621 observations.

One of the first questions in the survey asked respondents if they had visited any lakes in Iowa in the previous year. The purpose of this paper is to model single-day trips taken to lakes for recreational purposes. To that end, the data for visits include only day trips and not overnight trips. Table 1 presents a year-wise summary of the returns received, the number and percentage of respondents indicating that they did not visit any lake in Iowa during that year (columns 3 and 4 titled “Novisits”) and the average number of lakes visited by all individuals (column 5). The fourth column in table 1 indicates that 33-35% of respondents in 2003, 2004 and 2005 did not visit any lake in Iowa. 41% of respondents in 2002 did not visit any lakes. The average number of lakes visited by all respondents ranged between 1.8 and 2.5. The table also presents this information for a two-year panel, a three-year panel and the full four-year panel. All three panels are for consecutive years. Comparing the single year statistics with the panels we can see that the sample size

decreases as the years in the panel increase. This reflects the fact that people tend to drop out of the sample over the years.

Comparing across the different panels in Table 1 we find that the percentage of respondents who did not visit any lakes in any year decreases in 2002 and 2003 but not for 2004 and 2005, compared with the single year data as the number of years in the panel increases. This indicates that people who make trips to lakes i.e. lake users are not any more likely than non-users to continue responding to the survey over the years. The average number of lakes visited by each individual is also similar for the four-year panel as compared with the single year averages. This indicates that the four year balanced panel is fairly representative of the larger dataset. An anomalous finding is that the maximum number of lakes visited is lower for the four-year panel as compared with the single years or the two year-panel.

Table 2 presents the summary of the trips taken by the respondents to lakes in Iowa. The table presents two sets of statistics - one for the entire sample in each year or panel and the second for only those respondents who took one or more trips. For the second set of statistics we exclude individuals who did not take any trips in that year. The average number of trips for all individuals ranged between 6.4 and 7.5 (column 3). Between 64% and 66.5% of all respondents in 2003, 2004 and 2005 took one or more trips during the year. The proportion of positive trip takers in 2002 was lower at around 59%. The average trips of those who took positive trips moved around 11 in each of the four years. The proportion of positive trip takers was higher in the four-year panel as compared with the single year data for all years. However, the average trips for all respondents was not systematically different for the four-year panel as compared with the single year data. This was because the average trips of positive trip takers was lower, albeit marginally, in the four-year panel as compared with the single year data. The maximum trips of all trip takers is curtailed

at 52 because we exclude individuals who report taking more than 52 trips in a year. While this is an arbitrary threshold, it helps to exclude those that live close to a lake and happen to pass the lake in course of other daily activities. The purpose of the paper is to model recreational day trips to lakes and this arbitrary cutoff provides us with a set of observations whose trips closely resemble what we are trying to model.

Table 3 presents pair-wise comparison across years of mean trips taken by all individuals and lake users and the percentage of trip takers in the sample. The differences that show up in tables 2 and 3 are statistically tested in tables 4 and 5. Table 4 presents the t-statistic for testing the differences across years in (i) trips of all respondents; (ii) trips of lake users; (iii) the number of lakes visited and (iv) the percentage of non-users in the sample. For each pair of years, the statistic is calculated using a sample of all individuals who responded in both of those years but not necessarily in any of the other years. Table 5 presents the t-statistics calculated using information for individuals who responded to the survey in all four years. Table 4 indicates that the mean trips of all respondents were significantly different between 2003 and 2004 but not for the other years. The mean trips of trip takers were not significantly different for any pair of years.

Table 1.1 Summary of returns Respondents who do not visit any lake & average number of lakes visited

Year	Returns <sup>1</sup>	Novisits <sup>2</sup>	% Novisits	Lakes visited <sup>34</sup>		
				Mean	STDev	Max
2002	3897	1607	41.2	1.8	2.5	31
2003	4579	1548	33.8	2.4	2.9	22
2004	3826	1356	35.4	2.4	3.1	30
2005	3516	1177	33.4	2.5	3.2	32
Individuals who responded in both 2002 and 2003						
2002	2644	1014	38.4	1.9	2.6	31
2003		918	34.7	2.3	2.9	22
Individuals who responded in 2002, 2003 and 2004						
2002	2033	758	37.3	2.0	2.5	19
2003		673	33.1	2.4	2.9	22
2004		740	36.4	2.3	3.0	30
Individuals who responded every year of the survey 2002-05						
2002	1621	584	36.0	2.0	2.5	19
2003		524	32.3	2.4	2.9	20
2004		575	35.5	2.4	3.1	30
2005		542	33.4	2.5	2.9	24

1. Number of respondents in each year
2. Number of individuals who stated that they did not visit any lake in that year
3. Number of lakes that individuals report visiting in that year
4. Those who reported more than 52 trips in any year excluded from the sample

Table 1.2 Number of Trips to All Lakes<sup>1,2</sup>

Year	Returns	Trips		Trips of positive trip-takers			Max
		Mean	Stdev	% Taking trips	Mean	Stdev	
2002	3897	6.4	10.1	58.8	10.8	11.2	52
2003	4579	7.3	10.2	66.2	11.1	10.7	52
2004	3826	6.9	9.9	64.6	10.8	10.6	52
2005	3516	7.5	10.4	66.5	11.2	11.0	52
Individuals who responded in both 2002 and 2003							
2002	2644	6.5	9.8	61.6	10.5	10.7	51
2003		7.0	9.9	65.3	10.7	10.5	52
Individuals who responded in 2002, 2003 and 2004							
2002	2033	6.5	9.6	62.7	10.4	10.4	51
2003		7.2	9.9	66.9	10.8	10.4	52
2004		6.3	9.0	63.6	9.9	9.5	51
Individuals who responded every year of the survey 2002-05							
2002	1621	6.6	9.4	64.0	10.3	10.0	51
2003		7.3	9.8	67.7	10.8	10.3	52
2004		6.5	9.0	64.5	10.1	9.5	51
2005		6.9	9.4	66.6	10.5	9.9	52

1. Excluding individuals who took trips but did not disclose the number of trips taken

2. Excluding individuals who took more than 52 trips in any year



Table 1.3 Pair-wise Comparison across Years of Trips to All Lakes<sup>1</sup>

Pair	Returns	Year	Trips		Trips of positive trip takers		
			Mean	Stdev	Mean	Stdev	% pos <sup>2</sup>
2002/2003	2644	2002	6.5	9.8	10.5	10.7	61.6
		2003	7.0	9.9	10.7	10.5	65.3
2002/2004	2278	2002	6.4	9.8	10.5	10.7	60.8
		2004	6.3	9.1	10.0	9.7	63.2
2002/2005	2097	2002	6.6	9.9	10.6	10.7	61.6
		2005	7.0	10.0	10.8	10.6	65.4
2003/2004	3382	2003	7.4	10.1	10.9	10.5	67.8
		2004	6.8	9.6	10.5	10.2	64.6
2003/2005	3142	2003	7.5	10.2	11.1	10.7	67.6
		2005	7.2	9.9	10.8	10.4	66.5
2004/2005	2960	2004	7.1	9.9	10.8	10.4	65.8
		2005	7.4	10.0	10.8	10.5	67.9

1. Excluding individuals who took trips but did not disclose the number of trips taken
2. Percentage of respondents who took positive trips

Table 1.4 T-Statistic to Test for Pairwise Differences using Two-Year Panels<sup>1</sup>

Years	Sample Size	T-Statistic			
		Trips	Positive Trips <sup>2</sup>	Lakes visited	Novisits
2002-03	2644	-1.76	-0.51	-4.73	2.74
2002-04	2278	0.29	1.73	-4.44	1.65
2002-05	2097	-1.57	-0.36	-5.11	2.53
2003-04	3382	2.52	1.59	0.52	-2.73
2003-05	3142	1.27	1.11	-0.43	-0.97
2004-05	2960	-0.92	-0.04	-0.52	1.74

1. Sample size is different for each pair of years and consists of the same individuals for each pair but not necessarily across pairs.
2. Excluding individuals who took trips but did not disclose the number of trips taken

Table 1.5 T-Statistic to Test for Pairwise Differences for Full Four-year Panel <sup>1</sup>

	Trips	Positive Trips <sup>2</sup>	Lakes visited	No visits
2002-03	-1.99	-1.21	-4.16	2.22
2002-04	0.25	0.62	-3.73	0.33
2002-05	-1.06	-0.36	-4.34	1.55
2003-04	2.27	1.85	0.25	-1.89
2003-05	0.95	0.86	-1.15	-0.67
2004-05	-1.32	-1.00	0.40	1.22

1. The sample consists of the same 1621 individuals for every year

2. Excluding individuals who took trips but did not disclose the number of trips taken

Table 1.6 Summary Statistics for Independent Variables

Years	Income(\$1000s)			Age			Travel Cost			Secchi Depth		
	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max
2002	57.9	5	170	52.2	17	80	136.0	0.5	750.2	1.2	0.1	5.7
2003	58.8	5	170	53.1	21.5	80	140.1	0.5	934.8	1.5	0.2	8.1
2004	61.8	5	170	54.0	17	80	144.5	0.5	937.2	1.1	0.2	5.1
2005	61.8	5	170	54.8	17	80	149.2	0.5	944.5	1.2	0.1	5.7

Column 4 of table 4 indicates that the number of lakes visited by each individual in 2002 differed significantly from the number of lakes visited by all individuals in 2003, 2004, 2005. The number of lakes visited by each individual were not significantly different in 2003, 2004 and 2005 (column 5). The proportion of non-users in the sample was significantly higher in 2002 as compared with 2003 and 2005. The proportion of non-users in 2004 was significantly higher in 2004 as compared with 2003.

Table 5 confirms the significant differences in number of lakes visited in 2002 and the three following years. It also confirms in the significant differences in the proportion of

non-users in 2002 as compared with 2003.

Table 6 presents the summary statistics for the other regressors of the model. For the purposes of this paper, the only demographic variable used was age and the only water quality measure used was secchi transparency, also known as secchi depth. Secchi depth is a measure of clarity of the water in a lake - it indicates how deep into the water one can see. It is one of the most easily observed and understood water quality measures. The other variables of interest are income and travel cost. Travel distance and time for each household to each lake was estimated using the PC Miler software. Travel cost was computed using a fixed amount per mile and the imputed cost of time. The per mile cost was the same for each household and was fixed at \$0.25, \$0.26, \$0.265 and \$0.28 for 2002, 2003, 2004 and 2005, respectively. The increasing mileage cost is meant to reflect rising gasoline prices and general inflation. However, the increases are arbitrary and not derived from a consumer price index or oil price index.

The opportunity cost of time for each household depends on the average wage rate for that household. It is derived by dividing the yearly income by 2000, which is average number of hours worked in a year after accounting for two weeks of leave (50 weeks at 40 hours a week). This is an average measure that allows us to translate the income in the household to the cost of its time. It is not meant to substitute for exact information on wage rates. For the purposes of this paper, the opportunity cost of time is given by one-third of the wage rate computed this way. This is a fairly standard method of calculating the time cost in recreation demand. However, there have been many studies arguing against the use of income or wage rate as a reliable measure of the opportunity cost of time (Shaw, 1992; Feather and Shaw, 1999; McKean *et al.*, 2003). An obvious problem is that it does not reflect the opportunity cost of time for students, the unemployed, the retired, or homemakers. Further, salaried individuals usually do not have the option to

work greater hours for more pay or fewer hours for less pay. Without the choice to work more or fewer hours, the wage rates for salaried individuals do not reflect the opportunity cost of their time either. Working time forgone is not the only opportunity cost of travel time to recreational sites. Other alternative uses for travel time include voluntary work and other recreational activities (Beal, 1995). Shaw (1992) suggested that the value of time need not equal the cost of time. For instance, a person with a low wage rate may place very high value on her time. Further, it is usually assumed that the travel to a recreational site does not provide any utility in itself. That is a questionable assumption because individuals may enjoy driving to a lake some distance away. In this case, the opportunity cost of time will be lower than what is measured by the wage rate.

In practice, however, a fraction of the wage rate has often been used as a proxy for the opportunity cost of time (Phaneuf *et al.*, 2000; von Haefen *et al.*, 2004; Egan *et al.*, 2009). There has been some discussion on what the appropriate fraction of wage rate should be. There appears to be a consensus in the literature on 25% being the lower bound and 100% being the upper bound (Parsons *et al.*, 2003). In this range, 33% is the most commonly used fraction (Hellerstein, 1993; Moeltner, 2005). Differences in travel cost are not the focus of this paper. Hence, we choose to approximate opportunity cost of time by one-third of the wage rate, even as we recognize the limitations of this approach.

We have complete trip information for a full four-year panel of 1621 observations. Complete information on income and other demographic variables is not available for all of these individuals. We have complete information required for our model for all four years for 1280 individuals. The information in table 6 is for these 1280 observations.

As Table 6 indicates, secchi depth of all lakes ranged between less than 20 centimeters to more than 5.5 meters in three of the four years. In 2003 there was an increase in the maximum secchi depth of Lake Okoboji, the clearest lake in the state, to over eight meters.

The average secchi depth was between one and 1.5 meters. Travel cost ranged from 45 cents to over \$940 for a two-way trip with the mean around \$140 in all four years.

Survey respondents were asked to indicate which of 14 income and seven age categories they belonged to. Everyone belonging to an age category was assigned the median age for that category. The same was done for income. The mean income in all four years was close to \$60,000 per annum and increased every year. The mean age in all four years was around 53 years.

## 1.4 Results

As a first step, the Kuhn-Tucker model was estimated separately for each year. Table 7 presents the parameter estimates from this estimation process. All of the parameters of this simplified model are highly significant for each of the four years. Further, the parameters have similar signs and are of the same order of magnitude over the years suggesting that preferences were consistent over the time considered. The coefficient for secchi depth is positive as expected. The coefficient of age is positive too. This may be counterintuitive and conceal two different effects. Initially, when young adults grow older, they get jobs, have higher incomes, acquire families and tend to take more trips. After reaching middle age, trips taken tend to plateau and then fall as they grow older. The number of trips taken may increase after retirement and then fall off as the household grows older. The number of recreational trips, therefore, are expected to be positively correlated with age up to a point and then negatively correlated with age.

The single year estimates do not account for correlation in preferences across time. To model correlation in preferences we adopt two approaches - (i) bootstrapping from the sample and estimating the model for each of four years for each draw. Thus the model is estimated for each year for exactly the same individuals, which incorporates the correlation

across time implicitly into our estimates. The second approach is to model the correlation explicitly using a parameter of correlation.

#### 1.4.1 Bootstrap Exercise

Bootstrapping offers a way to model the correlation in preferences across time in our model without using a parameter of correlation. To implement this method we draw with replacement, a random sample of 1280 households from our dataset. We estimate a single year model without correlation for each of the four years for this sample. Since the data for each of the four years consists of the same set of households, the correlation in preferences is subsumed in the other parameters of the model. We do this 1000 times and compute the mean willingness to pay for an improvement in Secchi depth of all lakes at least to the level of lake Okoboji (5.66 meters) in each year for each of the 1000 draws. We then compute the differences in parameters and welfare estimates across the years and check to see if the 95% confidence intervals for each of these differences contain zero.

Table 8 presents the 95% confidence intervals for the differences in the parameter estimates. All of the differences are obtained by subtracting the estimate for the later year from the estimate for the earlier year. If the 95% confidence interval for the difference in the parameter estimate for a pair of years does not contain zero within it, it indicates that the difference is significantly different from zero. In other words, it indicates that the estimates for that parameter for the two years are significantly different from each other. Table 9 presents the percentage of the differences in parameter estimates that were positive. Percentages above 90% and below 10% indicate significant differences in the parameter estimates for that pair of years.

As tables 8 and 9 indicate, there are significant differences in the parameter estimates across years. In other words, parameter estimates do not appear to be stable over time.

To check whether these estimates are substantively different, we look at welfare estimates for each year for each of these 1000 draws. Welfare estimates are computed for a policy that increases the secchi depth of all 127 lakes to at least 5.6 meters which is the secchi depth of lake Okoboji, the clearest lake in Iowa. Table 10 presents the differences in welfare estimates across years in terms of 95% confidence intervals and the percentages of differences that are positive. Table 10 indicates that welfare estimates are significantly different between all pairs of years except 2002 and 2005, and between 2003 and 2005.

The bootstrap approach is more flexible compared to the error components approach presented in the following subsection. This is because the bootstrap approach does not impose a structure on the correlation and does not require any assumptions about the distribution of the correlation term. Further, the bootstrap approach can accommodate a greater number of variables without getting computationally intractable. In the error components approach, adding one variable will require four parameters to be estimated for the four years in the panel, which will greatly increase the computing time required for the estimation to converge. However, if the error components model is correctly specified, its parameter estimates will be more efficient as compared with estimates from the bootstrap approach.

#### **1.4.2 Correlation Term Constant Across Sites and Distributed Normally with Mean Zero**

The next step was to estimate the model over a panel dataset taking into account correlations across time and sites for each individual. The unobservable individual-specific effect is given by  $u_i$  which is drawn from a normal distribution with mean zero and a standard deviation given by  $\sigma_u$ . Two models were estimated. The first model restricts the parameters to be the same for each year in the panel. The second model removes that

restriction and allows different parameters for each year in the panel. Table 11 presents the results of the restricted model for a two-year panel (2002-03) and a four-year panel (2002-05). The parameter estimates from the two-year panel are of similar sign and order of magnitude as the single year estimates. In the two-year panel, the coefficient of age is not significant. All other parameters are highly significant for both panels. One parameter of interest here is the scale parameter associated with the correlation term. It is highly significant indicating the presence of correlations across time and site for each individual. Table 11 presents the results of the unrestricted model for a two-year panel.

A likelihood ratio test (Table 12) rejects the restricted model in favor of the unrestricted model. This indicates that even though parameters are mostly consistent in sign across the years, there are changes in their magnitude that are statistically significant. We have estimated a simple specification of the Kuhn-Tucker model. However, if our specification of the model is correct, then the above result indicates that preferences are not stable over time. This is something that policy-makers need to take into account while computing welfare changes from possible policy measures. Alternatively, it is possible that the differences in the parameter estimates reflect the effect of omitted variables that affect preferences and changed across the four years in question.

#### **1.4.3 Correlation Term Not Constant Across Sites and Normally Distributed**

Next we consider the case when the error component that induces correlation across time is not constant across sites. The correlation term is given by  $\tau_{ij}$  which is drawn from a normal distribution with mean zero and a variance given by  $\sigma_{\tau}^2$ . The restricted model is estimated for a two-year and a four-year panel. The unrestricted model is estimated for a two year and a four year panel. Table 14 presents the parameter estimates for



the restricted model with a two-year and a four-year panel dataset. As before, all the parameters of the model are highly significant. In particular the variance of the correlation term is highly significant indicating that preferences are correlated across time. Table 15 presents the parameter estimates for the two-year unrestricted model. The parameter estimates, which are all highly significant, are also of the same sign and roughly the same magnitude for the two years.

A likelihood ratio test (Table 16) rejects the restricted model in favor of the unrestricted model for the two-year panel dataset. This again confirms the fact that parameters of the model vary across years. The likelihood ratio test establishes that parameter estimates for the Kuhn-Tucker model are not stable over time for our dataset. To check if the variation in parameter estimates translates into substantive changes in welfare estimates, we compute welfare estimates for four years using the estimates from the restricted and unrestricted models when the correlation terms is given by  $\tau_{ij}$ . Mean willingness to pay is computed for an improvement in the secchi depth of all 127 lakes to the level of the clearest lake, *i.e.* to 5.6 meters. Table 19 presents the welfare estimates for this change for both the restricted and unrestricted models. Table 19 indicates that there is a substantive difference in the welfare estimates derived from the unconstrained and the constrained model. This indicates that instability of parameter estimates over time translate into substantive differences in the respective welfare estimates. However, the welfare estimates are not systematically biased in one way or another between the two models.

The recent interest in Kuhn-Tucker models has been enabled by the availability of greater computing power. There have been large improvements in the computing ability and speed in the ten years since Phaneuf et al. 2000 used numerical integration to implement the Kuhn-Tucker models. Allowing for correlation across time required a second degree of integration which pushed the limits of our computing capacity. The estimations

were done using the Maxlik package on GAUSS. The four year unrestricted models for the two versions of the error term converged in 102 and 117 iterations respectively. These iterations took, respectively, 178.1 hours and 213.6 hours to run. This is one of the reasons for choosing a very parsimonious model for this paper. Adding one extra variable to the model would require the program to optimize over four additional parameters when estimating the four year unconstrained model. This would increase the computing time by close to 20% over the present setup where the four year unconstrained model requires optimizing over 21 parameters.

## 1.5 Conclusion

This paper provided a method for incorporating correlation in preferences across time in a Kuhn-Tucker model of recreation demand with a large choice set. The method was implemented using a panel dataset of trips made by a random sample of Iowa households to 127 lakes in Iowa and was used to examine if preferences are stable across time. This is a logical next step from Phaneuf *et al.* (2000) and von Haefen *et al.* (2004) in extending the application of the Kuhn-Tucker approach for modeling recreation demand data. The Kuhn-Tucker model deals with a large number of corner solutions typical of recreation demand data, in an internally consistent utility theoretic framework. The decision to take a trip or not, the selection of the site and number of trips, if trips are taken are derived from the same model. The availability of a panel dataset with information on trips taken in four consecutive years allowed us to examine if parameter estimates in the Kuhn-Tucker framework are stable over time. Most policy relating to provision of outdoor recreational and other environmental amenities involve costs spread over an extended time period and expect to create welfare gains over more than just a single period. If parameter estimates are not stable over time, projecting welfare estimates computed using a single year of data

may be misleading for cost benefit analysis.

We find that household preferences are significantly correlated across time. In addition, for our specification of the model and dataset, parameter estimates are not stable across time. The instability of parameter estimates across the years translates into substantive differences in the corresponding welfare estimates. This implies that using a single year of data to evaluate medium or long-term welfare changes from a policy measure may be misleading. The instability of parameter estimates over time could be driven by changes in other demographic variables not included in the model or even macroeconomic variables. Consequently, models that are parsimonious, such as the one used in this paper, may be more likely to yield parameter estimates that are not stable over time. Even models that include a large number of variables may yield parameter estimates that vary over time, if there are substantial differences in the macroeconomic environment from year to year. Therefore, greater deliberation needs to go into projecting welfare estimates from one year into other years. In particular, adjustments need to be made to account for changes in macroeconomic environment. Further, if it is known beforehand that one year's welfare estimates are going to be projected into future years, it might be useful to choose a more detailed model over a parsimonious one.

While the model in the current paper includes only one demographic variable and one water quality variable, the framework can accommodate any number of demographic and site attribute variables. All the models in this paper were estimated using a balanced panel of 1280 individuals. Using a balanced panel requires dropping a number of observations from the dataset. Estimating the restricted and unrestricted models with an unbalanced panel will allow us to use the information in the complete dataset. These constitute the agenda for future work.

Table 1.7 Single Year Estimates of Simple Model

	2002	2003	2004	2005
Constant	-8.24** (0.06)	-8.86** (0.07)	-8.47** (0.06)	-8.47** (0.06)
Age	4.98** (0.84)	11.22** (0.85)	5.44** (0.82)	7.94** (0.80)
Secchi Depth	9.37** (1.05)	9.76** (0.86)	13.30** (1.18)	8.26** (1.00)
Theta <sup>1</sup>	1.36** (0.03)	1.26** (0.03)	1.24** (0.03)	1.23** (0.03)
Sigma <sup>2</sup>	0.24** (0.01)	0.36** (0.01)	0.30** (0.01)	0.27** (0.01)
No. of Obs	1280	1280	1280	1280
Mean log lik	-12.84	-15.41	-14.94	-14.95

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal distribution

\*\* : Significant at 1% level

Table 1.8 Differences in Parameter Estimates across Years: 95% Confidence Intervals

Years	Constant	Age	Secchi Depth	Theta <sup>1</sup>	Sigma <sup>2</sup>
2002-03	0.39 – 0.82	-9.13 – -3.23	-2.70 – 1.66	0.03 – 0.16	-0.16 – -0.07
2002-04	-0.03 – 0.47	-3.92 – 3.05	-6.60 – 1.16	0.06 – 0.20	-0.10 – -0.01
2002-05	-0.02 – 0.45	-6.68 – -0.62	-1.34 – 3.46	0.06 – 0.20	-0.08 – 0.02
2003-04	-0.62 – -0.16	2.80 – 8.75	-5.75 – -1.19	-0.04 – 0.09	0.01 – 0.10
2003-05	-0.60 – -0.19	0.25 – 6.19	-0.84 – 3.80	-0.03 – 0.10	0.04 – 0.12
2004-05	-0.21 – 0.20	-5.59 – 0.71	2.28 – 7.49	-0.05 – 0.06	-0.01 – 0.07

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal distribution for the error

Table 1.9 Differences in Parameter Estimates across Years: Percentage Positive

Years	Constant	Age	Secchi Depth	Theta <sup>1</sup>	Sigma <sup>2</sup>
2002-03	100	0.0	36.0	99.8	0.0
2002-04	96.4	40.9	0.1	99.9	0.4
2002-05	95.7	5.9	80.1	100.0	7.8
2003-04	0.0	100.0	0.1	79.1	99.0
2003-05	0.0	98.4	89.6	84.2	100.0
2004-05	49.2	6.3	100.0	58.7	92.2

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal distribution for the error

Table 1.10 Differences in Welfare Estimates across Years: 95% Confidence Intervals and Percentage Positive

Years	95% Confidence Interval	Percentage positive
2002-03	-55.61 – 9.35	9.5
2002-04	-156.22 – -53.14	0.0
2002-05	-51.33 – 26.66	25.8
2003-04	-127.48 – -34.66	0.1
2003-05	-28.96 – 50.09	66.2
2004-05	37.80 – 143.63	99.9

Table 1.11 Restricted Model Estimated for Two-Year and Four-year Panels

	Two-year Panel (2002-03)	Four-year Panel (2002-04)
Constant	-7.9** (0.08)	-7.80** (0.06)
Age	5.28** (1.38)	3.05* (1.04)
Secchi Depth	9.42** (0.57)	9.09** (0.43)
Theta <sup>1</sup>	1.44** (0.02)	1.38** (0.01)
Sigma <sup>2</sup>	0.03** (0.01)	-0.03** (0.01)
Sigmatcorr <sup>3</sup>	-0.27** (0.03)	-0.24** (0.02)
No. of Obs	1280	1280
Mean log lik	-26.02	-52.85

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal error
3. Log of the standard deviation for the correlation term

\* and \*\* indicate significance of levels 5% and 1%, respectively

Table 1.12 Unrestricted Model Estimated for a Two-Year Panel

	Parameter Estimates for	
	2002	2003
Constant	-7.60** (0.09)	-8.14** (0.09)
Age	1.84 (1.48)	7.72** (1.49)
Secchi Depth	8.74** (0.91)	9.03** (0.75)
Theta <sup>1</sup>	1.48** (0.03)	1.39** (0.03)
Sigma <sup>2</sup>	-0.05** (0.01)	0.09** (0.01)
Sigmatcorr <sup>3</sup>	-0.25** (0.02)	
No. of Obs	1280	1280
Mean log lik	-25.95	

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal error
3. Log of the standard deviation for the correlation term

\* and \*\* indicate significance of levels 5% and 1%, respectively

Table 1.13 Unrestricted Model Estimated with Full Four-Year  
Panel  $u_i \sim N(0, \sigma_\tau^2)$

	2002	2003	2004	2005
Constant	-7.50** (0.07)	-7.98** (0.07)	-7.63** (0.05)	-7.72** (0.07)
Age	-1.40 (0.94)	4.34** (0.95)	0.05 (0.35)	2.32** (0.91)
Secchi Depth	8.82** (0.92)	9.00** (0.75)	12.00** (1.01)	7.62** (0.88)
Theta	1.46** (0.03)	1.38** (0.03)	1.36** (0.02)	1.34** (0.03)
Sigma	-0.17** (0.01)	0.10** (0.01)	-0.00** (0.01)	0.01** (0.01)
Sigmatcorr	-0.23** (0.02)			
No. of Obs	1280			
Mean log lik	-52.76			

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal distribution

\*: Significant at 1% level

Table 1.14 Likelihood Ratio Test between Restricted and Unrestricted  
Models

Year	Mean log-likelihood		LR Stat	Crit. Value— at 1%
	Unrestr. Model	Restr. Model		
2002-03	-25.95	-26.01	161.02	15.09
2002-05	-52.76	-52.85	238.85	30.58



Table 1.15 Restricted Model Estimated for Two-Year and Four-year Panels:  $\tau_{ij} \sim N(0, \sigma_\tau^2)$

	Two-year Panel (2002-03)	Four-year Panel (2002-04)
Constant	-8.57** (0.05)	-8.54** (0.03)
Age	8.42** (0.59)	7.72** (0.41)
Secchi Depth	10.07** (0.66)	9.66** (0.49)
Theta <sup>1</sup>	1.32** (0.02)	1.26** (0.01)
Sigma <sup>2</sup>	0.30** (0.01)	0.27** (0.01)
Sigmacorr <sup>3</sup>	-1.71** (0.08)	1.26** (0.03)
No. of Obs	1280	1280
Mean log lik	-28.28	-57.93

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal error
3. Log of the standard deviation for the correlation term

\* and \*\* indicate significance of levels 5% and 1%, respectively

Table 1.16 Unrestricted Model Estimated for a Two-Year  
 Panel:  $\tau_{ij} \sim N(0, \sigma_\tau^2)$

	Parameter Estimates for	
	2002	2003
Constant	-8.24** (0.06)	-8.85** (0.07)
Age	4.96** (0.83)	11.20** (0.84)
Secchi Depth	9.18** (1.05)	9.73** (0.86)
Theta <sup>1</sup>	1.36** (0.03)	1.26** (0.03)
Sigma <sup>2</sup>	0.23** (0.01)	0.35** (0.01)
Sigmacorr <sup>3</sup>	-1.70** (0.08)	
No. of Obs	1280	1280
Mean log lik	-28.24	

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal error
3. Log of the standard deviation for the correlation term

\* and \*\* indicate significance of levels 5% and 1%, respectively

Table 1.17 Unrestricted Model Estimated with Full Four-Year  
Panel:  $\tau_{ij} \sim N(0, \sigma_\tau^2)$

	2002	2003	2004	2005
Constant	-8.31** (0.06)	-8.89** (0.07)	-8.50** (0.06)	-8.52** (0.06)
Age	-5.13** (0.83)	11.43** (0.84)	5.52** (0.82)	8.03** (0.80)
Secchi Depth	8.75** (1.06)	9.52** (0.86)	13.39** (1.18)	7.83** (1.01)
Theta	1.33** (0.03)	1.24** (0.03)	1.22** (0.03)	1.20** (0.03)
Sigma	0.20** (0.01)	0.31** (0.01)	-0.26** (0.01)	0.23** (0.01)
Sigmatcorr	-0.89** (0.02)			
No. of Obs	1280			
Mean log lik	-57.53			

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
2. Log of the standard deviation of the normal distribution

\*: Significant at 1% level

Table 1.18 Likelihood Ratio Test between Restricted and Unrestricted  
Models when  $\tau_{ij} \sim N(0, \sigma_\tau^2)$

Year	Mean log-likelihood		LR Stat	Crit. Value— at 1%
	Unrestr. Model	Restr. Model		
2002-03	-28.24	-28.28	122.62	15.09
2002-05	-57.53	-57.94	1036.03	30.58

Table 1.19 Year-wise Welfare Estimates in Dollars for an Improvement in Secchi Depth of all Lakes to at least 5.6m:  $\tau_{ij} \sim N(0, \sigma_\tau^2)$

	Constrained Model	Unconstrained Model
2002	268.26	201.24
2003	261.07	285.99
2004	304.05	441.22
2005	294.45	239.27
No. of Obs	1280	

## CHAPTER 2. THE EFFECT OF THE SOURCE OF ERROR ON WELFARE MEASURES IN A KUHN-TUCKER MODEL

### 2.1 Introduction

As discussed in the previous chapter, one of the main goals of recreation demand models is to estimate changes in welfare from changes in the quality and availability of environmental goods. Benefits estimates computed from these studies inform decisions of policymakers on the optimal allocation of resources for the provision and maintenance of public goods. The typical way these models are used is to examine whether a proposed change in the availability, quality or price of a public good will result in a net increase in welfare over all individuals affected.

An important component of all regression analyses is the regression error. In the context of recreation demand models, the source of the regression error can be of prime importance. This is because, the way welfare estimates are computed for these models may differ depending on what the regression error represents. Most recreation demand models assume the error term to represent heterogeneity in preferences - arising out of some factor that is known to the consumer but not to the researcher. Hence, the expected consumer surplus from hypothetical changes in price and quality attributes of a site is estimated by drawing randomly from the assumed distribution of the errors and averaging the estimates over all draws. However, other sources of error have been discussed in the literature.

The error term has variously been traced to socioeconomic factors that is known to the individual but not to the researcher (Gum and Martin, 1975), randomness in human behavior (Hanemann,1983), measurement error in the dependent variable (Hiett and Worral, 1977) and measurement errors in the explanatory variables (Brown et. al, 1983). Bockstael and Strand (1987), discuss four possible sources for the error in regression: (i) omitted variables; (ii) randomness in preferences; (iii)error in the measurement of the dependent variable and (iv) error in the measurement of one or more independent variables. The heterogeneity in preferences as modeled in standard recreation demand models, derives from omitted variables as discussed by Bockstael and Strand (1987). It is important to note that regression error arising out of the first three sources can still satisfy the Gauss-Markov conditions, while error on account of wrongly measured independent variables will not be orthogonal to the independent variables. Hence, estimating a model where the regression error refers to measurement errors in the independent variable will require corrections that make the estimation process different from the process for models where the error derives from the first three sources of error.

Bockstael and Strand (1987) consider only the first three sources of the regression error in the context of linear or log linear demand functions and show that different sources of error imply different welfare measures - higher welfare estimates associated are associated with the regression error arising from heterogeneity in preferences as compared to randomness in preferences. Their findings cannot be extended automatically to non-linear models. For a range of recreation demand models, including the Kuhn-Tucker models, the error term enters the demand function in a more complex manner than the linear and log-linear functions and it is not possible to isolate and sign the effect of the error term on the welfare measure analytically. The purpose of this paper to explore the issue empirically.

There have been studies that assumed the regression error arising from sources other

than heterogeneity in preferences in the context of non-linear models (Herriges *et al.*, 1999; Whitehead *et al.* 2000; Von Haefen *et al.*, 2004), but none that conduct a systematic empirical study of the welfare estimates under different sources of the error, especially in the context of the highly non-linear Kuhn-Tucker framework. This paper fills that gap by computing the welfare measures when the regression error arises from two different sources in a Kuhn-Tucker model with non-linear demand functions. In case of linear or semi-log demand functions Bockstael and Strand (1987) found that welfare estimates derived under the assumption that the errors represent heterogeneity in preferences are likely to be larger as compared with the estimates that are derived under the assumption that errors represent randomness in preferences. If this result extends to the non-linear functions too, it would suggest that careful consideration needs to go into the assumptions about the source of the regression error to ensure that the welfare measure computed is truly indicative of the benefits from a policy measure.

Kuhn-Tucker models are particularly suitable for studying site choice behavior when the consumer chooses among several options. This is because the Kuhn-Tucker framework is utility theoretic and can model the downward censored nature of trip data. In addition, the high degree of non-linearity of the Kuhn-Tucker framework makes it a good case study for undertaking the Bockstael and Strand (1987) kind of comparison in a non-linear framework.

In this paper, we estimate welfare measures using the Kuhn-Tucker model applied to a large and unique data set: four years of recreation trips from a random sample survey of Iowa residents regarding their use of 127 lakes. This is the same dataset that is used in chapter 1. Both compensating and equivalent variation is estimated for the sample separately for each of the four years - 2002-2005. In constructing the welfare measures, we consider two interpretations of the error term - first, that it represents heterogeneity in preferences on account of factors that are known to the individual but not to the researcher

and second, that it arises from an inherent randomness in human behavior or circumstances that cannot be predicted even by the individual in advance. Note, in both cases the error term is associated with the demographic variable and enters the utility function in an identical manner. The process of estimating the parameters are identical in the two cases, but the process of computing welfare estimates are not. In both cases, the model is correctly specified. Further, in both cases the regression error can safely be assumed to satisfy identical regularity conditions. This is what makes them especially suitable for comparing the effect of the source of error on welfare estimates.

Unlike Bockstael and Strand (1987), we do not focus on the case where the regression error represents measurement errors in the dependent variable - the number of trips taken in this case. In the linear demand functions studied by Bockstael and Strand (1987), the error term was associated linearly with the demand function and was identical for the three sources of regression error that they considered. In the Kuhn-Tucker model, the error enters the utility function in a non-linear fashion in association with the demographic terms for the first two cases, *viz.*, heterogeneity in preferences and randomness in preferences. As a result the regression error enters the demand function non-linearly too. But when the regression error originates from incorrectly measured trips taken, it can no longer be associated with the demographic variables and must enter the demand function linearly. This will make the equations estimated different from the equations estimated in the first two cases. Hence, we do not consider the case where error originates from measurement errors in trip data.

The rest of the paper is organized as follows. Section 2 describes the Kuhn-Tucker model for estimating the parameters of the utility and demand functions. Section 3 discusses the way the different welfare measures are estimated under the different interpretations of the error term. Section 4 provides a brief overview of the data, section 5 discusses the results



and section 6 concludes.

## 2.2 The Model

Assume that there are  $T$  periods and a total of  $M$  sites each having  $K$  characteristics. The utility function for a household  $i$  in period  $t$  is given by

$$U_{it} = \sum_{j=1}^M \exp(\delta' S_{it} + \eta_{ijt}) \ln[\exp(\gamma' q_{jt}) x_{ijt} + \theta] + \ln(z_{it}), \quad (2.1)$$

where

$x_{ijt}$  = Number of trips taken by the  $i^{th}$  individual to site  $j$  in period  $t$ ,

$Q_t = [q_{1t}, q_{2t} \dots q_{Mt}]$ , where  $q_{jt}$  is a  $K$  by 1 vector of quality variables associated with site  $j$ ,

$S_{it}$  = the set of demographic characteristics for the  $i^{th}$  household in period  $t$

$\theta$  = parameter allowing for corner solutions,

$\eta_i$  = an  $MT$  by 1 matrix of error terms for the  $i^{th}$  individual,

$P_{ijt}$  = the  $i^{th}$  individual's cost of visiting site  $j$  in period  $t$ ,

$z_{it}$  = a composite of all other goods (the numeraire and a necessary good),

$\gamma$  and  $\delta$  = parameters of the model.

In the standard Kuhn-Tucker model as estimated in chapter 1,  $\eta_{ijt}$  would represent heterogeneity in preferences - demographic or other systematic factors that are known to the individual but unobserved by the researcher. In this case it could also stand for a random error that is unknown to both the individual and the researcher. This specification of the utility function is additively separable. Further, it assumes that every good (site in this case) is a normal good and all goods are Hicksian substitutes. The utility function also assumes weak complementarity meaning that quality attributes of a site do not affect the total utility of the individual if the site is not visited (Maler, 1974). In other words,

the individual cares about the quality attributes of only those sites that s/he visits. The budget constraint for the individual in period t is

$$Y_{it} = \sum_{j=1}^M P_{ijt}x_{ijt} + z_{it}; x_{ijt} \geq 0; z_{it} > 0 \forall i, j, t. \quad (2.2)$$

The decision variables for individual i are  $x_{ijt}$  and  $z_{it}$ . The number of trips to any lake must be non-negative while the expenditure on the numeraire must be strictly positive. The Kuhn-Tucker first order condition for the  $i_{th}$  individual, in period t with respect to the  $j_{th}$  site implies

$$\begin{aligned} \exp(\delta' S_{it} + \eta_{ijt}) \frac{\gamma' q_{it}}{[\exp(\gamma' q_{it})x_{ijt} + \theta]} - \frac{P_{ijt}}{z_{it}} &\leq 0, \text{ and} \\ (\exp(\delta^T S_{it} + \eta_{ijt}) \frac{\exp(\gamma^T q_{jt})}{[\exp(\gamma^T q_{jt})x_{ijt} + \theta]} - \frac{P_{ijt}}{z_{it}})x_{ijt} &= 0 \end{aligned}$$

This in turn implies that

$$\eta_{ijt} \leq \ln\left(\frac{P_{ijt}}{z_{it}}\right) + \ln\left[x_{ijt} + \frac{\theta}{\exp(\gamma' q_{it})}\right] - \delta' S_{it} = g(x_{it}, P_t, z_{it}, S_{it}, q_{jt}; \beta),$$

where  $\beta$  is the vector of parameters to be estimated. This also implies a demand function of the form

$$x_{ijt} = \text{Max} \left( 0, \frac{\exp[\delta^T S_{it} + \eta_{ijt}] \frac{z_{it}}{P_{jt}} \exp(\gamma^T q_{jt}) - \theta}{\exp(\gamma^T q_{jt})} \right) \quad (3)$$

For each individual i in each period t, there are M such equations - one for each site. These equations together with the assumed distribution of the error term define a likelihood function which can then be used to estimate the parameters of the model.

### 2.3 Welfare Measures

We consider a policy which improves one or more site quality attributes in one or more sites and may involve corresponding changes in the access prices of those sites. Compensating variation is given by the change in income required to ensure that the individual has the same utility after the change as she did before. Thus, for a policy that improves one or more site quality attributes, the compensating variation would be a measure of the individual's willingness to pay to obtain that change. Equivalent variation is given by the change in income required to ensure that the individual does at least as well under the current regime as she would under the new regime. For a policy change that worsens one or more site quality attributes, the equivalent variation would indicate the willingness to accept payment to allow that change.

Denoting the original price-quality configuration by  $(p^0, q^0)$  and the new price-quality configuration by  $(p^1, q^1)$  compensating variation for the  $i$ th individual for this change is defined as

$$CV_i = y_i - e(p^1, q^1; u_i^0)$$

where

$y_i$  : income of the  $i$ th individual

$u_i^0$  : maximum total utility of the  $i$ th individual at the  
original price-quality configuration

$e(p^1, q^1; u_i^0)$  : minimum expenditure required for the individual to attain  $u_i^0$   
at the new price-quality configuration

Similarly equivalent variation for the individual is defined as

$$EV_i = e(p^0, q^0; u_i^1) - y_i$$

where

$u_i^1$  : maximum total utility that the individual can attain with the current income at the new price-quality configuration

The compensating and equivalent variations can then be averaged over all individuals in the sample to get an average measure of the welfare change. When the error term arises from heterogeneity in preferences that is known to the individual, then the error term is like an explanatory variable or a composite effect of explanatory factors. This is because the error term stands in for some variable that affects utility and demand functions in a systematic way. It should, therefore, be included in the calculation of demand and total utility because excluding it will result in an omitted variable bias. On the other hand, when the error term reflects a randomness in tastes and circumstances which affect trip demand but cannot be predicted by the individual, the researcher is better off focusing on the systematic portion of the trip demand. In this case, therefore, only the mean of the error term should be incorporated into either trip demand or total utility. For the two sources of error that we consider, the process for estimating the parameters of the model are identical. The only difference is in the computation of welfare estimates. If we were to consider, a model where the error term reflects inaccurate reporting of trips taken or one of the explanatory variables, the process of estimating the parameters of the model would be different since the corresponding regression equations would be different. For this reason, it is useful in our context, to focus only on the first two sources of error which provides us with a way to compare two cases that are nearly identical except in the last

step, i.e. computation of welfare estimates. This is different from Bockstael and Strand (1987) where the error term was associated with the demand function.

In the Kuhn-Tucker model with our specification of the utility function, there is no analytical solution for either the compensating variation or the equivalent variation. Both must be computed numerically. When the error term is to be included in the welfare computation, for each individual we take 500 draws from the distribution of the error and calculate demand at the initial and new regimes for each draw of the error. Thus for each draw we have an estimate of the compensating and equivalent variations. The compensating variation and equivalent variation for each individual are the averages of each measure over all 500 error draws.

## 2.4 Data

The dataset for this analysis is the same as used in chapter 1. For four consecutive years from 2002 to 2005, a survey questionnaire about trips to lakes in Iowa was sent out to a random sample of households within the state. Each year about 4000 completed surveys were returned. Of the returned surveys, information on some individuals could not be used because the surveys were incomplete. Excluding those individuals for whom complete information on trips taken was not available, we have about 3500 completed surveys in each year except in 2003 for which we have about 4500 surveys. A complete four-year panel is available for 1621 observations.

As discussed in chapter 1, 33-35% of respondents in 2003, 2004 and 2005 did not visit any lake in Iowa. 41% of respondents in 2002 did not visit any lakes. The average number of lakes visited by all respondents ranged between 1.8 and 2.5.

The average number of trips for all individuals ranged between 6.4 and 7.5. Between 64% and 66.5% of all respondents in 2003, 2004 and 2005 took one or more trips during

the year. The proportion of positive trip takers in 2002 was lower at around 59%. The average trips of those who took positive trips moved around 11 in each of the four years. The proportion of positive trip takers was higher in the four-year panel as compared with the single year data for all years. However, the average trips for all respondents was not systematically different for the four-year panel as compared with the single year data. This was because the average trips of positive trip takers was lower, albeit marginally, in the four-year panel as compared with the single year data. As in Chapter 1, we are modeling single-day trips to lakes for recreational purposes and do not consider overnight trips taken. The maximum trips of all trip takers is curtailed at 52 because we exclude individuals who report taking more than 52 trips in a year. While this is an arbitrary threshold, it helps to exclude those that live close to a lake and happen to pass the lake in course of other daily activities. Table 1 reproduces the information in Table 6 of chapter 1 to provide summary statistics for the independent variables.

Table 2.1 Summary Statistics for Independent Variables

Years	Income(\$1000s)			Age			Travel Cost			Secchi Depth		
	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max	Mean	Min	Max
2002	57.9	5	170	52.2	17	80	136.0	0.5	750.2	1.2	0.1	5.7
2003	58.8	5	170	53.1	21.5	80	140.1	0.5	934.8	1.5	0.2	8.1
2004	61.8	5	170	54.0	17	80	144.5	0.5	937.2	1.1	0.2	5.1
2005	61.8	5	170	54.8	17	80	149.2	0.5	944.5	1.2	0.1	5.7

Table 1 presents the summary statistics for the regressors of the model. For the purposes of this paper, the only demographic variable used was age and the only water quality measure used was secchi depth. Secchi depth is one of the most easily observed and understood water quality measures. The other variables of interest are income and travel cost. Travel distance and time for each household to each lake was estimated using the PC

Miler software. Travel cost was computed using a fixed amount per mile and the imputed cost of time. The per mile cost was the same for each household and was fixed at \$0.25, \$0.26, \$0.265 and \$0.28 for 2002, 2003, 2004 and 2005, respectively. The imputed cost of time for each household depends on the average wage rate for that household calculated by the yearly income divided by 2000, which is average number of hours worked in a year after accounting for two weeks of leave. This is an average measure that allows us to translate the income in the household to the cost of its time. It is not meant to substitute for exact information on wage rates. We have complete trip information for a full four-year panel of 1621 observations. Complete information on income and other demographic variables is not available for all of these individuals. We have complete information required for our model for all four years for 1280 individuals. The information in Table 1 is for these 1280 observations.

As Table 1 indicates, secchi depth of all lakes ranged between less than 20 centimeters to more than 5.5 meters in three of the four years. In 2003 there was an increase in the maximum secchi depth of Lake Okoboji, the clearest lake in the state, to over eight meters. The average secchi depth of all lakes ranged between one and 1.5 meters in the four years under consideration. Travel cost ranged from 45 cents to over \$940 for a two-way trip with the mean around \$140 in all four years.

Survey respondents were asked to indicate which of 14 income and seven age categories they belonged to. Everyone belonging to an age category was assigned the median age for that category. The same was done for income. The mean income in all four years was close to \$60,000 per annum and increased every year. The mean age in all four years was around 53 years.

## 2.5 Results

In estimating the model we assume that the error term follows a normal distribution with mean zero and a variance that is estimated from the data. The errors are independently and identically distributed for each individual and each site within a year. Table 2 presents the parameter estimates for each of the four years. All of the parameters of this simplified model are highly significant for each of the four years. Further, the parameters have similar signs and are of the same order of magnitude over the years suggesting that preferences were consistent over the time considered. The coefficient for secchi depth is positive as expected. The coefficient of age is positive too.

For the purposes of computing welfare estimates, two policy measures were considered. In the first case, the secchi depth of all lakes are increased to at least 5.67 meters which is the secchi depth of the clearest lake (Lake Okoboji) in three of the four years. Note that this is an exceptionally large improvement meant to drive a large willingness to pay. For many of the lakes in the dataset, increasing secchi depth to 5.67 meters would amount to an improvement of more than 500%. This is obviously not observed in reality. The access price for each of the lakes remains unchanged. In this case, the willingness to pay should be positive and the compensating variation should exceed the equivalent variation measure. In the second case, 10 of the 127 sites were removed from the choice set. This was done by raising the access price of each of these 10 lakes above the choke price, i.e. the price at which all demand for trips to that lake falls to zero. The access prices for the other lakes were left unchanged. In this case, the willingness to pay should be negative and the equivalent variation should exceed the compensating variation. The welfare measures for the policies were computed at the point estimates presented in Table 2 for two interpretations of the error term. Tables 3 and 4 present the welfare estimates for the two policy measures.



As expected, the equivalent variation in each case is of similar magnitude to the compensating variation. If the error term is interpreted as heterogeneity in preferences, the magnitude of the willingness to pay is substantially higher than if the error term is interpreted as randomness in preferences. This is true for both policies and for both compensating and equivalent variation measures of willingness to pay. For instance, the willingness to pay for an improvement in secchi depth of all lakes to 5.67 meters ranged between \$221 and \$460 when the error was assumed to come from heterogeneity in preferences. When the error was assumed to arise from randomness in preferences, the corresponding willingness to pay ranged between \$0.40 and \$1.30. Thus the benefits from policy 1 are substantially lower if we assume the error term arising from randomness in preferences. This implies that depending on the cost of undertaking such an improvement, assumptions about the source of the error term can make a critical difference to whether a project is considered worth undertaking or not.

Similarly for policy 2, the willingness to accept compensation to allow the removal of 10 lakes from the choice set ranged between \$27 to \$51, when the error was assumed to represent heterogeneity in preferences. When the error was assumed to represent randomness in behavior, the corresponding willingness to accept compensation ranged between \$0.01 and \$0.04. This indicates that the assumption about the source of the error can make a crucial difference to the total benefit estimate and hence benefits net of costs of a project that improves one or more sites.

The stark differences in the welfare measures associated with the two sources of regression errors observed particularly in Table 3, can be traced to the way that the error enters the utility function. From section 2, the error term enters the demand function exponentially. Thus  $x_{ijt}$  is increasing and convex in  $\eta_{ijt}$ . Consider a matrix  $\eta_{it}$  of  $N$  draws of 127 error terms drawn from a mean zero distribution, for individual  $i$  in period  $t$ . Let

$\hat{x}_{it}$  be the corresponding Nx127 matrix of trips for this individual in the same period given by

$$\hat{x}_{ijt} = \frac{\exp[\delta^T S_{it} + \eta_{ijt}] \frac{z_{it}}{P_{jt}} \exp(\gamma^T q_{jt}) - \theta}{\exp(\gamma^T q_{jt})} \quad (2.3)$$

For the purposes of this thought experiment, we are not requiring the  $\hat{x}_{ijt}$  to be non-negative. Thus the matrix  $\hat{x}_{it}$  consists of both negative and positive numbers. Let  $\bar{\hat{x}}_{it}$  be the mean of the matrix  $\hat{x}_{it}$  over the N draws. Thus  $\bar{\hat{x}}_{it}$  is a 1x127 vector. The mean of  $\eta_{it}$ ,  $\text{bar}\eta_{it}$  is a 127x1 vector of numbers that are very close to zero because the errors are drawn from a mean zero distribution. The untruncated trip vector corresponding to  $\text{bar}\eta_{it}$  is a 1x127 vector, which we will call  $\tilde{\hat{x}}_{it}$ . Let  $x_{it}$  be the truncated trip vector corresponding to  $\eta_{it}$  and given by

$$x_{ijt} = \text{Max} \left( 0, \frac{\exp[\delta^T S_{it} + \eta_{ijt}] \frac{z_{it}}{P_{jt}} \exp(\gamma^T q_{jt}) - \theta}{\exp(\gamma^T q_{jt})} \right) \quad (3)$$

Let  $\bar{x}_{it}$  be the mean of  $x_{it}$  over the error draws.  $x_{it}$  is of dimension Nx127, while  $\bar{x}_{it}$  is of dimension 1x127. Since  $x_{ijt}$  is increasing and convex in  $\eta_{ijt}$ , by Jensen's inequality  $\bar{x}_{it} > \tilde{\hat{x}}_{it}$ .  $\hat{x}_{it}$  contains both positive and negative terms while  $x_{it}$  is left-truncated. Hence  $\bar{x}_{it} \geq \tilde{\hat{x}}_{it}$ . Together, these two inequalities imply that  $\bar{x}_{it} \geq \tilde{\hat{x}}_{it} > \hat{x}_{it}$ . This explains why mean trips in our model are not just higher but are substantially higher when we average over the error draws than when we compute them at the mean of the error draws which is zero. The predicted trips under the two conditions before and after the improvement on account of policy 1 are presented in Table 5. As before, when the error reflects heterogeneity in preferences, the predicted trips should be averaged over all error draws. When the error reflects randomness in preferences, the predicted trips should include only the mean of the error draws, which, in this case is zero. As Table 5 shows, error reflecting heterogeneity in preferences is associated with much higher predicted trips than error reflecting randomness in preferences. This is true both before and after the improvement. The large differences

in welfare estimates associated with the two error interpretations is driven largely by this difference in predicted trips. But the effect of the error term on welfare estimates is further exaggerated by the fact that the error term enters the indirect utility function in two ways - directly with the demographic variables and indirectly through the trip demand. The indirect utility function in our model is given by

$$U_{it} = \sum_{j=1}^M \exp(\delta' S_{it} + \eta_{ijt}) \ln[\exp(\gamma' q_{it}) \text{Max} \left( 0, \frac{\exp[\delta^T S_{it} + \eta_{ijt}] \frac{z_{it}}{P_{jt}} \exp(\gamma^T q_{jt}) - \theta}{\exp(\gamma^T q_{jt})} \right) + \theta] + \ln(z_{it}) \quad (2.4)$$

This further amplifies the impact of large positive error draws on the indirect utility function. The effect of large negative draws are muted by the fact that for large negative draws, trip demand is zero - the same as for smaller negative draws. Thus non-linearity of the indirect utility function in the error term, together with the left-ward truncated nature of trip demand amplifies the effect of large positive error draws and mutes the effect of large negative error draws, thus resulting in welfare estimates which are substantially higher than the estimates computed with the error draw set at zero.

## 2.6 Conclusion

This chapter sought to examine if the source of the error term in a Kuhn-Tucker model of recreation demand makes a material difference to welfare estimates from hypothetical changes to site quality attributes. Earlier studies have examined this question in the context of linear and semi-log demand functions where the error term is associated additively with trip demand. There have been no systematic empirical studies to answer this question in the context of non-linear demand functions. The Kuhn-Tucker model is highly non-linear and so it is not possible to analytically map the source of the error to the size of the welfare estimate in such a model. Using four years of data on a sample of Iowa house-

holds' demand for trips to lakes in Iowa, I examine the welfare estimates associated with two interpretations of the error term, viz., heterogeneity in preferences and randomness in behavior. I find that the source of the error does make a difference to the magnitudes of both compensating variation and equivalent variation from a change in some site quality attributes in one or more lakes. Specifically, benefits estimates associated with error arising out of heterogeneity in preferences are systematically larger than benefits estimates associated with the error arising out of randomness in behavior. This is consistent with the findings of Bockstael and Strand (1987) with linear and semi-log demand functions. It is, therefore, important to carefully consider the source of the error when using these measures to determine if a project is worth undertaking or not.

Table 2.2 Single Year Estimates of Simple Model

	2002	2003	2004	2005
Constant	-8.24** (0.06)	-8.86** (0.07)	-8.47** (0.06)	-8.47** (0.06)
Age	4.98** (0.84)	11.22** (0.85)	5.44** (0.82)	7.94** (0.80)
Secchi Depth	9.37** (1.05)	9.76** (0.86)	13.30** (1.18)	8.26** (1.00)
Theta <sup>1</sup>	1.36** (0.03)	1.26** (0.03)	1.24** (0.03)	1.23** (0.03)
Sigma <sup>2</sup>	0.24** (0.01)	0.36** (0.01)	0.30** (0.01)	0.27** (0.01)
No. of Obs	1280	1280	1280	1280
Mean log lik	-12.84	-15.41	-14.94	-14.95

Figures in brackets indicate standard errors

1. Log of the parameter that allows corner solutions
  2. Log of the standard deviation for the normal distribution
- \*\* : Significant at 1% level

Table 2.3 Mean CV and EV in dollars under Policy 1: Secchi Depth of all lakes improved to at least 5.67 m

	Error reflects	2002	2003	2004	2005
CV	Heterogeneity	221.64	302.60	447.50	250.70
	Randomness	0.70	0.42	1.20	0.72
EV	Heterogeneity	225.26	308.02	457.90	254.18
	Randomness	0.79	0.44	1.29	0.76

Table 2.4 Mean CV and EV in dollars under Policy 2: Ten lakes are removed from the choice set

	Error reflects	2002	2003	2004	2005
CV	Heterogeneity	-30.48	-50.64	-39.89	-41.72
	Randomness	-0.04	-0.01	-0.02	-0.03
EV	Heterogeneity	-26.55	-44.88	-36.47	-38.39
	Randomness	-0.01	-0.01	-0.01	-0.03

Table 2.5 Projected Mean Trips before and after Improvement: Policy 1

		2002	2003	2004	2005
Before Improvement	Heterogeneity	9.35	11.04	10.09	10.36
	Randomness	0.45	0.29	0.36	0.44
After Improvement	Heterogeneity	12.44	14.04	14.69	13.15
	Randomness	0.73	0.46	0.74	0.68

## CHAPTER 3. GREEN CONSUMERISM AND THE POLLUTION-GROWTH NEXUS

### 3.1 Introduction

In recent decades, there has been a worldwide surge in awareness of the enormity of global environmental problems and the role of human activity in creating these problems. That awareness has produced a kind of environmental consciousness – living in a way so as to lower one’s negative impact on the environment. This heightened consciousness has been documented in numerous surveys. For example, the 2009 Cone Consumer Environmental Survey finds that 35% of Americans report greater interest in the environment compared to the previous year.<sup>1</sup> 70% of the Americans surveyed indicate that they are paying attention to the current activities of companies with regard to the environment. A very similar picture emerges from the 2008 NHK “Survey of Attitudes Toward the Environment” in Japan.

The heightened environmental consciousness has translated into concrete action at the level of individuals. The NHK survey found that more than 50% of Japanese have adopted relatively easy energy-saving efforts, such as setting air conditioners and heating units at moderate temperatures, turning off lights when not needed, and bringing their own bags

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<sup>1</sup>The 2009 Cone Consumer Environmental Survey presents the findings of an online survey conducted January 29-30, 2007 by Opinion Research Corporation among a sample of 1,087 adults comprising 518 men and 569 women 18 years of age and older. The margin of error associated with a sample of this size is  $\pm 3\%$ .

with them when shopping (*eko baggu*); 83% took out old newspapers and empty bottles and cans for goods-recycling or reusable waste collection. When asked what is necessary to solve environmental problems, 26% answered “effort of individuals”.<sup>2</sup> A large number of Americans too are now environmental “doers” – in 2007, almost half have purchased environmentally-friendly products, of whom 62% bought products with recycled content, 56% made energy-efficient home improvements, 24% bought organic or other third-party certified foods/beverages, and 13% bought energy-efficient cars.<sup>3</sup> Despite the economic downturn, in 2009, 34% of Americans were more likely to buy environmentally responsible products than they were a year back. Only 14% did not shop with the environment in mind. I label all such private action aimed at reducing one’s negative impact on the environment as voluntary environmentalism or green consumerism.<sup>4</sup>

Green behavior consists of a set of lifestyle choices based on a concern for the environment. It manifests itself in activities such as, more energy conservation than is warranted by simple cost savings, broad-based recycling, purchase and use of biodegradable products, greater use of public transportation and bicycles (in spite of the associated inconveniences), purchasing fuel-efficient cars with dubious pecuniary benefits, and so on. Green consumerism also manifests in patronizing businesses and purchasing products of companies that are known to be environmentally responsible. Undertaken by enough indi-

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<sup>2</sup>As for the question “How willing would you be to accept cuts in your standard of living in order to protect the environment?” as few as 3 percent said “very willing,” while 37 percent said “fairly willing,” 25 percent “fairly unwilling,” and 12 percent “very unwilling.” When asked “How willing would you be to pay higher taxes in order to protect the environment?” only 1 percent said “very willing,” while 30 percent said “fairly willing,” 27 percent “fairly unwilling,” and 18 percent “very unwilling.” [<http://www.nhk.or.jp/bunken/english/pdf/090228-07.pdf>]

<sup>3</sup>Back in 2003, *Business Week* reported on the greening of China: “But, as in much of the rest of the world, the rise in living standards is also leading to calls among members of the new middle class for greater attention to the environment. Newspapers are writing about the problem more often, and independent environmental groups are springing up. As Beijing’s leaders try to balance the needs of development with the imperative to clean up, it may well be the citizens who lead the way.”

<sup>4</sup>I use the phrase voluntary environmentalism or mainstream environmentalism to distinguish it from the more-militant environmental movement. Such environmentalism consists of private lifestyle and consumption choices as distinct from political activism and lobbying that marks the environmental movement.



viduals, these activities collectively will likely result in observable differences in the quality of the environment, but the activities of any one individual are too miniscule to make any difference to the environment as a whole.

In this paper, I introduce green consumerism into an otherwise-standard, neoclassical growth model, and use it to study the well-known pollution-growth nexus. Green behavior is modeled by assuming that all agents derive a warm glow (in the sense of Andreoni, 1990) from their green activities. That is, the fact that people voluntarily incur private costs intended to increase the supply of a public good such as environmental quality, is motivated by the private benefit of a warm glow and is unrelated to the actual supply of the public good. More specifically, I study a two-period overlapping-generations model (a “Diamond model”) with production where agents receive direct utility from consumption in each period of life and from their pollution-abatement expenditures made when young. Agents save for standard life-cycle reasons, and the collective savings of a generation become the start-of-period capital stock in the following period. The current pollution level is assumed to depend on its immediate-past level, capital use in the current period, and the combined pollution-mitigation expenditures made by these green agents. Pollution affects the lives of agents in that it reduces a scale factor attached to second-period felicity; *ceteris paribus*, more pollution reduces the attractiveness of old-age consumption. The assumption that pollution affects the marginal utility from old-age consumption, i.e., pollution is not additively-separable from consumption in its impact on utility is of crucial importance in mapping the impact of pollution on human choices. What is also crucial is that private agents do not internalize the effect of their green activities on pollution, and in turn, its effect on their lives.

I go on to characterize optimal green expenditures by agents. I show that in a sufficiently capital-poor economy, agents do not indulge in green consumerism, that is unless

an economy becomes rich enough, agents do not invest in pollution-abatement expenditures. This result conforms well to popular notions that green lifestyles among people are more commonly seen in developed countries, and not so much in countries such as India and China, which nevertheless face dire environmental problems. I also show that capital rich economies which engage in green consumerism do not necessarily enjoy lower long run levels of pollution than economies which do not engage in green consumerism.

Next, I consider dynamic, competitive equilibria both in settings where private agents do and do not make pollution-abatement expenditures. In the former case, the evolution equation for pollution reduces to a one-dimensional dynamical system, making the analysis analytically tractable. I can show that, depending on parameter restrictions, the economy admits either a unique steady state or two steady states. The unique steady state may be attracting but the path to it involves oscillatory pollution dynamics – periods of high pollution are followed by those of low pollution. If there are two steady states, the low-pollution steady state is locally stable, indicating that, once agents in an economy start to make green expenses, the long-run level of pollution is low. In the case where private agents do not make pollution-abatement expenditures, the evolution equation for pollution and capital forms a two-dimensional dynamical system. In this case, I can prove there is a unique non-trivial steady state. I can show that economies that do not engage in green consumerism, the dirtier technology as reflected in higher emissions per unit capital, is associated with higher pollution and lower capital. Further, higher tolerance for pollution is associated with higher pollution and higher capital. For future use, note a certain self-destructing nature to pollution: as pollution increases, old-age consumption becomes less attractive, and this causes agents to smoothen lifecycle consumption by raising young-age consumption via reduction in saving, which, in turn, helps to reduce future pollution.

I go on to characterize the set of allocations that would be chosen by a benevolent social

planner. Such a planner would internalize the effect of pollution-mitigation expenses on pollution, something that the market economy (described above) would not. For standard reasons, the market economy may or may not accumulate more capital than the planner would. Here, in addition to that route for inefficiency, it is possible that the market economy may allocate more or less resources to pollution-abatement. The young consumption associated with green consumerism in a market economy is higher than the young consumption associated with green consumerism that emerges from the planning solution. I can show that, given the level of development of the economy, the planner may choose not to allocate any resources to pollution-abatement. I can further show that in the planning solution young and old consumption as well as capital are higher when mitigation is positive as compared with the situation where it is zero. However, pollution will not necessarily be lower in presence of mitigation than in absence of it.

Using numerical methods, I proceed to compare long-run levels of various variables of interest across the market and planned economies. The upshot of this comparison is this. Relative to the competitive outcome, the planner allocates less to both young and old-age consumption, but uses these freed-up resources to finance more pollution-mitigation expenses. The result is lower pollution and lower capital than what the market would have chosen.

My analysis departs from previous work in this area in several ways. First, many models (such as, in the seminal piece by John and Pecchenino, 1994, and many others in that line), studying the pollution-growth nexus incorporate environmental quality as an argument of utility, and allow private agents to influence it either directly via their own pollution-mitigation investments or indirectly by a government via tax collections. In contrast, I explicitly model the idea that actions taken by an atomistic private agent could not possibly influence the overall quality of the environment. These studies implicitly

assume that the quality of the environment does not impact the marginal utility from consumption of non-environmental goods. This additive-separability of the environment and consumption buys them tractability but is restrictive in that the impact of human choices on the environment is studied but the reverse, the impact of the environmental on human behavior, is ignored. My paper is part of a short, recent line of work that incorporate the bi-directional link between individual choices and pollution. In Jouvét et al. (2007), Varvarigos (2008), and Mariani et al. (2010), pollution reduces longevity, and hence saving (which, in turn, affects pollution via production); however, the strength of the empirical link – see Bloom et al. (2003), for example – between longevity and saving is not very strong. My formulation avoids this potential pitfall. Finally, to the best of my knowledge, the current paper is the first to study the new and evolving phenomenon of green consumerism and its connection to the pollution-growth nexus.

Rest of the paper is organized as follows. Section 2 describes the model and examines the competitive equilibrium, section 3 details the benchmark planning problem, and Section 4 discusses some comparative static results comparing the competitive equilibrium with the planning solutions in the long run. Section 5 concludes.

### 3.2 The model

Consider an economy populated by an infinite sequence of two-period lived overlapping generations and an initial old generation. At each date,  $t = 1, 2, 3, \dots$ , a new generation is born; each consists of a continuum of agents with mass 1. Each agent, starting with Generation 1, has one unit of time when young and retires when old.

The sole final good of the economy is produced using a production function  $F(K_t, L_t)$ , where  $K_t$  denotes the capital input and  $L_t$  denotes the labor input at  $t$ . Let  $k_t \equiv K_t/L_t$  denote the capital-labor ratio (capital per young agent). Output per young agent at time

$t$  may be expressed as  $y_t = f(k_t)$ ;  $f(k_t) \equiv F(K_t/L_t, 1)$  is the intensive form. For most of what I do below, I assume  $f(k) = Ak^\theta$ ,  $\theta \in (0, 1)$ . The final good is either consumed in the period it is produced or it can be stored to yield capital the following period. For analytical tractability, capital is assumed to depreciate entirely between periods.

Let  $c_{1t}$  ( $c_{2t+1}$ ) denote the consumption of the final good at date  $t$  (date  $t + 1$ ) by a representative young (old) agent born at  $t$ . Let  $m_t$  denote the pollution-mitigating expenditures at date  $t$  by a young agent. All such agents have preferences representable by the time-separable utility function

$$U(c_{1t}, c_{2t+1}, m_t) \equiv \ln(c_{1t}) + \varphi \ln(c_{2t+1}) + \gamma m_t; \quad \gamma > 0$$

where  $\varphi$  is a factor that scales old-age felicity. Agents take  $\varphi$  as given. In actuality,  $\varphi$  is assumed to depend on the level of pollution in the economy; specifically,

$$\varphi \equiv \varphi(P_t) = \beta \left( 1 - \frac{P_t}{\hat{P}} \right)$$

where  $P_t$  is pollution at date  $t$ . The parameter  $\hat{P}$  provides an upper bound on the pollution tolerable. Intuitively, it stands for that level of pollution at which agents attach zero weight on second period consumption and do not save for the future. In that sense it is the maximum tolerable pollution. The parameter  $\beta$  is positive. For future reference note, in my overlapping-generations setup, it is strictly speaking, not necessary to have  $\varphi \leq 1$ .

Intuitively,  $\varphi$  can be thought of as a scaling factor that weighs utility from second period consumption relative to that from first period consumption. In recent studies that examine the impact of pollution on longevity,  $\varphi$  is the probability of the agent surviving to old age (Jouvet et al., 2007; Varvarigos, 2008; and Mariani et al. 2010).<sup>5</sup> In other studies examining health expenditure in the context of growth models this has been interpreted

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<sup>5</sup>Exposure to fine particulate matter in air has long been associated with increased morbidity and mortality. For instance, particulate matter (PM) air pollution has been known to increase risks of cardiovascular incidents. Even short-term exposure to PM2.5 over a few hours can trigger myocardial infarctions, cardiac

as the fraction of the second period that the agent is alive (Bhattacharya and Xiao, 2005). In its present formulation,  $\varphi$  could be analogous to quality-adjusted life years, indicating that young-age exposure to pollution reduces the ability to enjoy consumption in old age. Alternatively, absence of pollution or cleanliness of the environment may be thought of as complementary to old age consumption.

It deserves mention that utility is assumed to be linear in mitigation. This is a deliberate choice and serves two purposes. First, it permits corner solutions in mitigation expenses. This is important in the present context because green consumerism, as yet, is observed primarily in the developed world. Second, linearity of utility in mitigation greatly enhances the analytical tractability of the model. The thrust of our results will not change if this linearity is abandoned.

Since young agents do not value leisure, they supply their unit labor endowments inelastically in competitive labor markets, earning a wage of  $w_t$  at time  $t$ , where

$$w_t \equiv w(k_t) = f(k_t) - k_t f'(k_t) = (1 - \theta) A k_t^\theta$$

and  $w'(k_t) > 0$ . In addition, capital is traded in competitive capital markets, and earns a gross real return of  $R_{t+1}$  between  $t$  and  $t + 1$ , where

$$R_{t+1} \equiv R(k_{t+1}) = f'(k_{t+1}) = \theta A k_{t+1}^{\theta-1}$$

with  $R'(k_{t+1}) < 0$ .

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ischemia, arrhythmias, heart failure, stroke, exacerbation of peripheral arterial disease, and sudden death. Chronic exposure to moderately elevated levels also enhances the risk for developing a variety of cardiovascular diseases, possibly including hypertension and systemic atherosclerosis (Brook, 2007) On the other side of the coin is the documented improvement in longevity on account on improvements in air quality. In a study of 211 county units in the 51 U.S. metropolitan areas for two periods between late 1970s and early 1980s and the late 1990s and early 2000s, Pope et al. (2009) found that a decrease of 10  $\mu\text{g}$  per cubic meter in the concentration of fine particulate matter was associated with an estimated increase in mean life expectancy of 0.61 years after adjusting for changes in socioeconomic and demographic variables and in proxy indicators for the prevalence of cigarette smoking. The study concluded that reductions in air pollution accounted for 15% of overall increases in life expectancy in the study areas. It is important to note that the impact of air and water pollution goes beyond mortality rates.

Finally, it remains to describe the process of evolution for pollution. Pollution in period  $t$  is given by

$$P_t = (1 - \alpha) P_{t-1} + bk_t - \nu m_t \quad (1)$$

where  $\alpha < 1$  is the fraction of pollutants that is naturally absorbed by the biosphere in one period. Thus  $(1 - \alpha) P_{t-1}$  is the pollution from period  $t - 1$  that persists into period  $t$ . New emissions from production (capital use  $k$ ) add to the pollution and mitigation ( $m$ ) activities neutralize some of the emissions and persisting pollution.

The representative agent's budget constraints are given by

$$\begin{aligned} w_t &= c_{1t} + s_t + m_t \\ c_{2t+1} &= R_{t+1} s_t \end{aligned}$$

where  $s$  denotes saving. Wage income is allocated to consumption, savings and potentially to mitigation investment. Savings are loaned out to firms who use them as capital for production. Wages are paid at the beginning of each period. Interest on capital is paid at the end of each period. At the end of each period, young agents retire and receive their interest earnings which they consume fully as old agents in the next period.

### 3.2.1 Competitive Equilibrium

Atomistic agents take the environment as given and do not internalize the impact of their consumption or mitigation decisions on the environment. As a result they take the discount factor  $\varphi$  as given. Similar to the standard Diamond model, agents also take the market interest rate as given. The first-order conditions of the representative agent's problem are given by

$$\frac{1}{c_{1t}} = \varphi \frac{1}{s_t} \geq \gamma.$$

where  $\gamma$  is the constant marginal utility of mitigation expenditure. The standard Kuhn-Tucker necessary conditions require that  $m_t = 0$  as long as the marginal utility of young (and old) consumption exceeds  $\gamma$ , the constant marginal utility of mitigation and  $m_t > 0$  when the marginal utility of young consumption equals  $\gamma$ .

A competitive equilibrium is a sequence  $\{c_{1t}, c_{2t+1}, m_t, s_t, k_{t+1}, P_t\}_{t=0}^{\infty}$  such that the agents' first order conditions are satisfied, the goods market and the capital market both clear, and pollution evolves according to (1). In equilibrium  $s_t = k_{t+1}$  so that the agent's first order conditions imply

$$k_{t+1} \frac{1}{\varphi(P_t)} = c_{1t} \leq \frac{1}{\gamma}$$

This condition will hold with equality when mitigation is positive. This tells us that an economy will engage in mitigation expenditure only when it is rich enough in the sense that its savings and young age consumption are high enough. The economy will not mitigate till its consumption equals  $\frac{1}{\gamma}$ . Since  $\varphi'(P_t) < 0$ , it may be tempting to suggest that an economy will engage in mitigation expenditure only when its pollution level is high enough. But this line of argument may be inaccurate since current mitigation also affects current pollution as given by (1) and the two variables are simultaneously determined. It is useful to look at the competitive equilibrium in terms of two regimes. viz. when mitigation is positive and when it is at zero. As can be anticipated from here, the path of the economy where mitigation expenditure is zero will be determined by a different set of conditions from the path of an economy where mitigation expenditure is positive. In other words, a poor economy's dynamics will be different from those of a rich economy. This means that in terms of the dynamics of an economy the transition from poverty to prosperity will not be smooth.



### 3.2.1.1 Mitigation Expenditure is zero

In an economy that is not rich enough for green consumerism to have started, the competitive equilibrium is determined by

$$k_{t+1} \frac{1}{\varphi(P_t)} = c_{1t}$$

Market clearing requires that

$$w(k_t) = c_{1t} + k_{t+1}$$

which translates into

$$A(1 - \theta) k_t^\theta = \left(1 + \frac{1}{\varphi(P_t)}\right) k_{t+1} \quad (2)$$

The law of motion for pollution is given by

$$P_t = (1 - \alpha) P_{t-1} + b k_t \quad (3)$$

Equations (2) and (3) together define the dynamics of the economy over time. This is a two dimensional system so that the standard analysis of the time paths will not be possible. Steady states for the economy are given by

$$\begin{aligned} A(1 - \theta) k^\theta &= \left(1 + \frac{1}{\varphi(P)}\right) k \\ \alpha P &= b k \end{aligned}$$

**Proposition 1** *The economy admits two steady states, a trivial one where  $k$  and  $P$  both equal zero, and another positive steady state.*

**Proof.** The two equations above can be collapsed into one to give

$$A(1 - \theta) k^\theta = \left(1 + \frac{1}{\varphi\left(\frac{b}{\alpha} k\right)}\right) k$$

This obviously has a solution at  $k = 0$  since  $\varphi\left(\frac{b}{\alpha}0\right) = \beta$ . Consider the case where  $k \neq 0$ . Then we have

$$A(1 - \theta)k^{\theta-1} = \left(1 + \frac{1}{\varphi\left(\frac{b}{\alpha}k\right)}\right) \quad (4)$$

$LHS = A(1 - \theta)k^{\theta-1}$  is decreasing and concave in  $k$ .  $RHS = 1 + \frac{1}{\beta\left(1 - \frac{b}{hP}k\right)}$  is increasing and convex in  $k$ . Therefore, there can be at most one intersection between the two, implying that there can be only one steady state solution at which both  $k$  and  $P$  are positive. Such an intersection will exist only if  $\lim_{k \rightarrow 0} LHS < \lim_{k \rightarrow 0} RHS$ .

$$\begin{aligned} \lim_{k \rightarrow 0} LHS &= \lim_{k \rightarrow 0} A(1 - \theta)k^{\theta-1} = \infty \\ \lim_{k \rightarrow 0} RHS &= 1 + \lim_{k \rightarrow 0} \frac{1}{\beta\left(1 - \frac{b}{\alpha P}k\right)} = 1 + \frac{1}{\beta} < \infty \end{aligned}$$

This implies that there does exist an intersection between the LHS and the RHS. ■ Let the positive steady state equilibrium defined by this intersection be given by  $k^0$ , and let the corresponding level of pollution be given by  $P^0$ .

**Lemma 1** *There is an upper bound on the unique positive steady state level of pollution of an economy which does not engage in green consumerism. It is given by  $P^0 \leq \frac{\beta}{\frac{\alpha\gamma}{b} + \frac{\beta}{P}}$ . The corresponding upper bound on capital is given by  $k^0 \leq \frac{\alpha}{b} \frac{\beta}{\frac{\alpha\gamma}{b} + \frac{\beta}{P}}$ .*

**Proof.** See Appendix 1 ■

At this positive steady state we can examine the comparative static results with respect to some of the parameters of the model.

**Proposition 2** *In the positive steady state of an economy that does not does not mitigate  $\frac{dk}{dP} > 0$ ,  $\frac{dk}{db} < 0$ ,  $\frac{dP}{dP} > 0$ , and  $\frac{dP}{db} > 0$*

**Proof.** See Appendix 2 ■

A higher tolerance for pollution as manifested in a high value of  $\hat{P}$  translates into a higher value of the discount factor for second period consumption. This makes old age consumption more valuable and hence encourages savings which in turn implies higher levels of capital so that  $\frac{dk}{d\hat{P}} > 0$ . Higher levels of savings (capital) correspond with higher levels of pollution in absence of mitigation implying  $\frac{dP}{d\hat{P}} > 0$ . The parameter  $b$  is the amount of emissions per unit capital used for production. A dirtier technology manifesting in a high value of  $b$  implies higher levels of pollution, which through the discount factor lowers attractiveness of old age consumption and hence savings (capital). This indicates that technology transfer to less developed economies for lowering emissions per unit capital use are useful for lowering their long run pollution levels.

### 3.2.1.2 Mitigation Expenditure is Positive

When the economy is engaging in green consumerism, meaning  $m_t > 0$  holds, its competitive equilibrium is defined by  $k_{t+1} = \frac{\varphi(P_t)}{\gamma}$  or  $k_t = \frac{\varphi(P_{t-1})}{\gamma}$  and  $c_{1t} = \frac{1}{\gamma}$ . This is the primary benefit of assuming quasi-linearity; the marginal utility of young-age consumption is a constant, and hence, young-age consumption is fixed. Using the agent's budget constraint, it follows that

$$w(k_t) = \frac{1}{\gamma} + \frac{\varphi(P_t)}{\gamma} + m_t \text{ such that}$$

$$m_t = w\left(\frac{\varphi(P_{t-1})}{\gamma}\right) - \frac{1}{\gamma} - \frac{\varphi(P_t)}{\gamma}$$

Substituting this into the law of motion for pollution, I get

$$P_t = (1 - \alpha) P_{t-1} + b \frac{\varphi(P_{t-1})}{\gamma} - \nu \left[ w\left(\frac{\varphi(P_{t-1})}{\gamma}\right) - \frac{1}{\gamma} - \frac{\varphi(P_t)}{\gamma} \right]$$

The constant marginal utility of mitigation allows us to capture the dynamics of the economy in a single equation in lagged values of  $P_t$ . Expanding  $\varphi(P_t)$  into its specified func-

tional form we can express  $P_t$  as a function of  $P_{t-1}$  and the parameters of the model as

$$P_t = \frac{(1 - \alpha) P_{t-1} + b \frac{\varphi(P_{t-1})}{\gamma} - \nu w \left( \frac{\varphi(P_{t-1})}{\gamma} \right) + \frac{\nu(1+\beta)}{\gamma}}{\left( 1 + \frac{\nu\beta}{\hat{P}\gamma} \right)} \quad (5)$$

As discussed earlier,  $\hat{P}$  is the maximum tolerable pollution, beyond which the atomistic agent has no incentive to save for old age. This is an extreme situation and it is useful to focus on the conditions under which agents do save for old age. So far, there is nothing that prevents pollution in period  $t$  from exceeding  $\hat{P}$ . As it happens, we need to impose some restrictions on parameters of the model to ensure that pollution in any period does not exceed  $\hat{P}$ .

**Lemma 2**  $\frac{\nu}{\gamma} \leq \alpha \hat{P}$  is necessary and sufficient condition to ensure that  $P_t$  never exceeds  $\hat{P}$

**Proof.** See Appendix 3 ■

Call this condition C1.

$$\frac{\nu}{\gamma} \leq \alpha \hat{P} \quad (C1)$$

Differentiating with respect to  $P_{t-1}$  we get

$$\begin{aligned} \frac{dP_t}{dP_{t-1}} &= \frac{(1 - \alpha) - \frac{\beta}{\hat{P}\gamma} \left[ b - \nu A \theta (1 - \theta) \left( \left( 1 - \frac{P_{t-1}}{\hat{P}} \right) \frac{\beta}{\gamma} \right)^{\theta-1} \right]}{\left( 1 + \frac{\nu\beta}{\hat{P}\gamma} \right)} \text{ and} \\ \frac{d^2 P_t}{dP_{t-1}^2} &= \frac{\nu A \theta (1 - \theta)^2 \left( \left( \hat{P} - P_{t-1} \right) \right)^{\theta-2} \left( \frac{1}{\hat{P}} \frac{\beta}{\gamma} \right)^\theta}{\left( 1 + \frac{1}{\hat{P}} \frac{\beta g}{\gamma} \right)} > 0 \end{aligned}$$

The slope  $\frac{dP_t}{dP_{t-1}}$  is increasing in  $P_{t-1}$ . This slope could be positive throughout or negative initially and then bottom out before becoming positive. *A priori*, we cannot say which of these would materialize for an economy but we can impose certain necessary and/or conditions on the parameters associated with each scenario.

**Lemma 3** (i)  $1 - \alpha - \frac{b}{\hat{P}} \frac{\beta}{\gamma} + \nu A \theta (1 - \theta) \left( \frac{\beta}{\gamma} \right)^{\theta-1} \frac{1}{\hat{P}} > 0$  is sufficient to ensure that  $\frac{dP_t}{dP_{t-1}} > 0$  always. (ii)  $1 - \alpha - \frac{b}{\hat{P}} \frac{\beta}{\gamma} + \nu A \theta (1 - \theta) \left( \frac{\beta}{\gamma} \right)^{\theta-1} \frac{1}{\hat{P}} < 0$  is a necessary condition for  $\frac{dP_t}{dP_{t-1}} < 0$  at least for a stretch.

**Proof.** (i) Since  $\frac{dP_t}{dP_{t-1}}$  is increasing in  $P_{t-1}$ ,  $\frac{dP_t}{dP_{t-1}}$  is at its lowest when  $P_{t-1} = 0$ . Hence, if  $\frac{dP_t}{dP_{t-1}} \geq 0$  when  $P_{t-1} = 0$ , it must be positive for all  $P_{t-1} > 0$ . When  $P_{t-1} = 0$ ,

$$\frac{dP_t}{dP_{t-1}} = \frac{1 - \alpha - \frac{\beta}{\hat{P}\gamma} \left[ b - \nu A \theta (1 - \theta) \left( \frac{\beta}{\gamma} \right)^{\theta-1} \right]}{\left( 1 + \frac{\nu\beta}{\hat{P}\gamma} \right)}$$

Hence  $1 - \alpha - \frac{\beta}{\hat{P}\gamma} \left[ b - \nu A \theta (1 - \theta) \left( \frac{\beta}{\gamma} \right)^{\theta-1} \right] \geq 0$  implies and is implied by  $\frac{dP_t}{dP_{t-1}} \geq 0$  at  $P_{t-1} = 0$ . (ii) Since  $1 - \alpha - \frac{\beta}{\hat{P}\gamma} \left[ b - \nu A \theta (1 - \theta) \left( \frac{\beta}{\gamma} \right)^{\theta-1} \right] \geq 0$  is a sufficient condition for  $\frac{dP_t}{dP_{t-1}}$  to be positive always, its violation is a necessary condition to ensure that  $\frac{dP_t}{dP_{t-1}} < 0$  at least for a stretch. ■

Note, if  $1 - \alpha - \frac{b}{\hat{P}} \frac{\beta}{\gamma} + \nu A \theta (1 - \theta) \left( \frac{\beta}{\gamma} \right)^{\theta-1} \frac{1}{\hat{P}} < 0$  then  $\frac{dP_t}{dP_{t-1}} < 0$  for very small values of  $P_{t-1}$  and  $\frac{dP_t}{dP_{t-1}} > 0$  for larger values of  $P_{t-1}$ .

$\frac{d^2 P_t}{dP_{t-1}^2} > 0$  implies that the slope of the path of pollution is continually rising and if  $P_t$  is initially falling in  $P_{t-1}$ , it will eventually bottom out and then rise with  $P_{t-1}$ . In other words, if there is a stationary point on this path, it is a minimum. We call this necessary condition C2.

$$1 - \alpha - \frac{b}{\hat{P}} \frac{\beta}{\gamma} + \nu A \theta (1 - \theta) \left( \frac{\beta}{\gamma} \right)^{\theta-1} \frac{1}{\hat{P}} < 0 \quad (\text{C2})$$

The steady states for this economy are given by

$$\alpha P + \frac{b + \nu}{\gamma} \beta \frac{P}{\hat{P}} = -\nu w \left( \frac{\beta}{\gamma} \left( 1 - \frac{P}{\hat{P}} \right) \right) + \frac{\nu}{\gamma} + \frac{b + \nu}{\gamma} \beta$$

**Proposition 3** *If condition C1 holds, then  $\frac{\nu}{\gamma} + \frac{b+\nu}{\gamma}\beta - \nu A(1-\theta)\left(\frac{\beta}{\gamma}\right)^\theta < 0$  is necessary and sufficient to ensure that there exists a unique steady state for this economy when mitigation is positive.*

**Proof.** See Appendix 4 ■

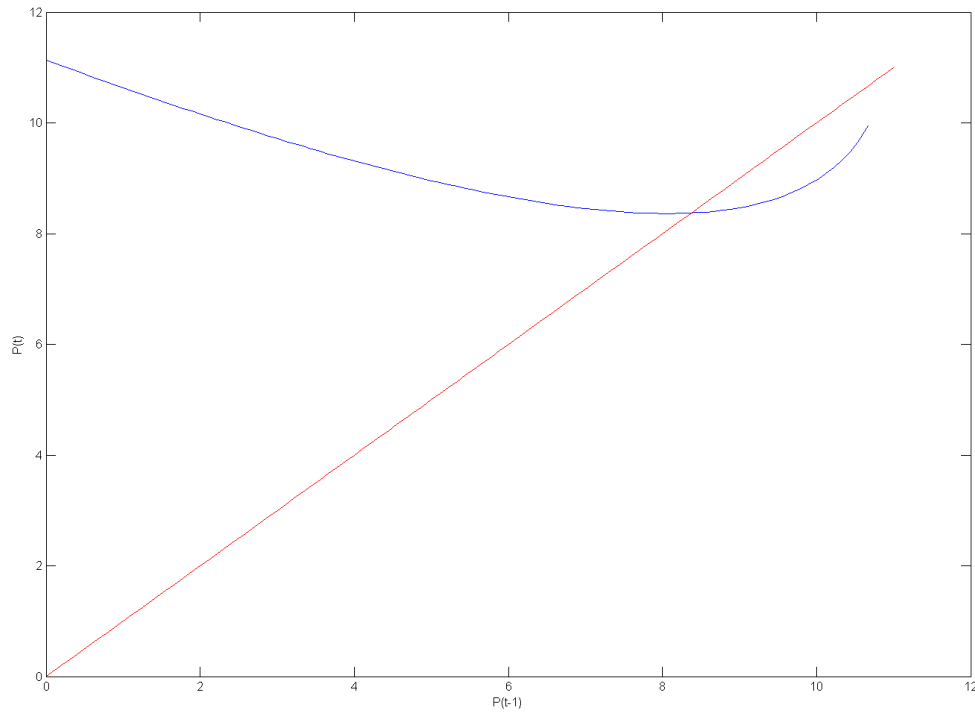


Figure 3.1 Law of Motion with a unique positive steady state

Let  $A = 12$ ,  $\theta = 0.39$ ,  $\gamma = 0.2$ ,  $\beta = 1$ ,  $\alpha = 0.1$ ,  $b = 2.9$ ,  $\nu = 2$ , and  $\hat{P} = 10$ . Figure 1 illustrates the relation between  $P_t$  and  $P_{t-1}$  under these parameter restrictions so that condition C2 is satisfied. As can be seen from the figure,  $\frac{dP_t}{dP_{t-1}}$  is negative initially and then becomes positive. This is an instance when the path of pollution cuts the 45 degree line

from above. An economy approaching this steady state will display cycles with periods of low pollution alternating with periods of high pollution. The steady state shown in figure 1 is stable if the slope of the path of pollution is less than one in absolute value at the steady state. Slope of the path of pollution at a steady state is given by

$$\left| \frac{dP_t}{dP_{t-1}} \right| = \frac{\alpha + \frac{b}{\hat{P}} \frac{\beta}{\gamma} - 1 - \nu A \theta (1 - \theta) \left( \left( 1 - \frac{P}{\hat{P}} \right) \frac{\beta}{\gamma} \right)^{\theta-1} \frac{1}{\hat{P}} \frac{\beta}{\gamma}}{\left( 1 + \frac{\beta \nu}{\hat{P} \gamma} \right)}$$

The denominator is greater than one and so a necessary condition for the absolute value of this derivative to be positive is that  $\alpha + \frac{b}{\hat{P}} \frac{\beta}{\gamma} > 1 + \frac{\beta \nu}{\hat{P} \gamma}$  or  $b > \nu + (1 - \alpha) \frac{\hat{P} \gamma}{\beta}$ . If this condition does not hold, it implies that  $\left| \frac{dP_t}{dP_{t-1}} \right| < 1$  and the steady state is stable. Thus a sufficient condition for the steady state to be stable is

$$b > \nu + (1 - \alpha) \frac{\hat{P} \gamma}{\beta} \tag{C3}$$

$b$  is the emission per unit capital usage in production.  $\nu$  is the pollution abated per unit expenditure on mitigation. Hence, for the steady state to be unstable, the abatement potential of mitigation investment has to be less than the emission potential for new production. In other words, mitigation has to be inefficient. For an economy on this path, the cycles will eventually die down and converge to the steady state. However, in the transition period it will periodically seem to regress on its path to lower pollution. The possibility of a steady state like this indicates that just because an economy is mitigating does not mean it will be on a path of monotonically declining pollution.

Another possible time path of pollution could arise when condition C2 does not hold. Let  $A = 15$ ,  $\theta = 0.4$ ,  $\gamma = 0.3$ ,  $\beta = 1.5$ ,  $\alpha = 0.1$ ,  $b = 5$ ,  $\nu = 1$ , and  $\hat{P} = 11$ . Figure 2 illustrates the relation between  $P_t$  and  $P_{t-1}$  under these parameter restrictions. In this instance, the slope of the curve is positive throughout. There are two steady states

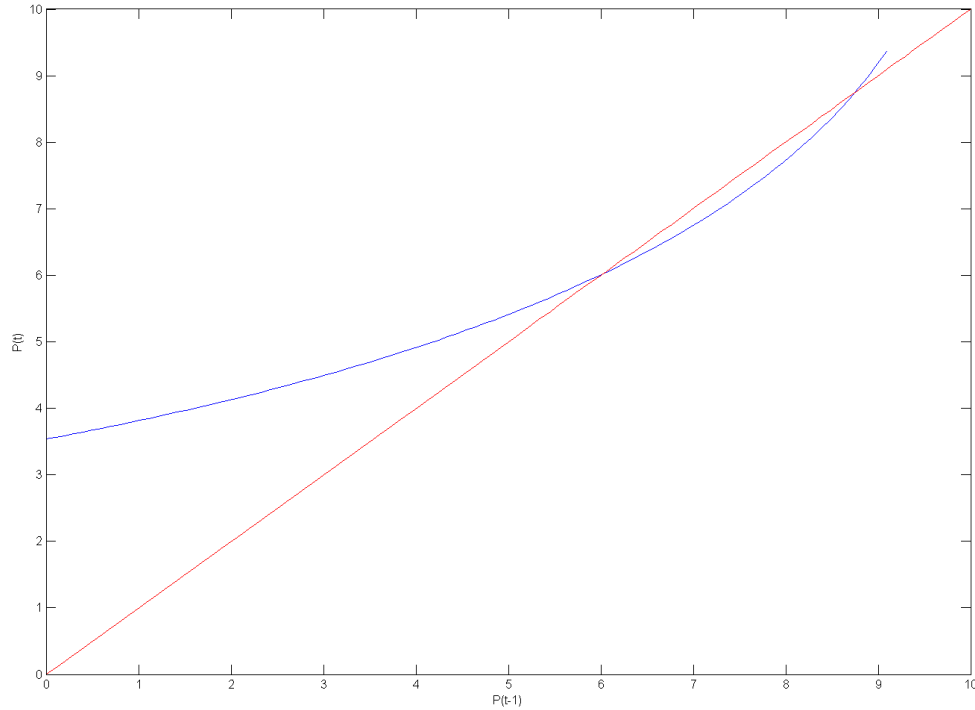


Figure 3.2 Law of Motion with two positive steady states

In this case the lower steady state is stable while the higher one is not. It indicates that when an economy is engaging in green consumerism, it will not converge to a high pollution steady state in the long run. Convergence to the low pollution steady state will be monotonic.

From the first order conditions of the market economy we surmised that the economy will have a higher level of young consumption when mitigation expenditure is positive than when it is not mitigating. We can take the comparison between the two regimes in a market economy further. Let us denote the unique positive steady state when mitigation



expenditure is positive by  $(k^*, P^*)$ . As before, the unique positive steady state when the mitigation expenditure is zero is given by  $(k^0, P^0)$ .

**Proposition 4**  $k^* \geq k^0$ ;  $P^* \begin{matrix} \geq \\ \leq \end{matrix} P^0$

**Proof.** See Appendix 5 ■ This tells us that an economy will start engaging in green consumerism only when it is sufficiently capital rich. However, just because it is engaging in green consumerism does not mean that an economy will have lower levels of pollution than the time when it was capital poor but was not engaging in green consumerism. This is because the higher levels capital result in higher emissions which may or may not be entirely neutralized by the green consumerism. Total pollution in the  $m_t > 0$  is lower as compared to the  $m_t = 0$  regime if the pollution abatement on account of green consumerism more than neutralizes the entire additional emissions on account of the higher capital.

It is useful to study the steady states of this economy in comparison with the first best. The planner's problem provides the first best steady states. As stated earlier, atomistic agents do not internalize the impact of their savings decisions either on the interest rate or on the level of pollution. Similarly, atomistic agents do not internalize the impact of their green consumerism on the level of pollution and they take the scale factor  $\phi$  as given. It is interesting to see what happens in the planner's where these three factors are internalized in making the consumption, savings and mitigation decisions.

### 3.3 Planner's Problem

The planner maximizes the discounted sum of the utilities of all current and future generations subject to a resource constraint and the law of motion for pollution. The Lagrangian function for the planner is given by

$$L = u_0 + \sum_{t=0}^{\infty} \delta^t \left[ \ln(c_{1t}) + \beta \left(1 - \frac{P_t}{\hat{P}}\right) \ln(c_{2t+1}) + \gamma m_t + \lambda_t \left[ Ak_t^\theta - c_{1t} - c_{2t} - k_{t+1} - m_t \right] + \mu_t [P_t - (1 - \alpha) P_{t-1}] \right]$$

where  $\delta$  is the intergenerational discount rate.  $\lambda_t$  and  $\mu_t$  are Lagrange multipliers. Note,  $-\mu_t$  is the marginal disutility of emissions so that  $\mu_t$  is positive. The first order conditions for the planner are given by

$$\begin{aligned} \frac{1}{c_{1t}} - \lambda_t &= 0 \\ \beta \left(1 - \frac{P_{t-1}}{\hat{P}}\right) \frac{1}{c_{2t}} - \delta \lambda_t &= 0 \\ \gamma - \lambda_t + \mu_t \nu &\leq 0 \\ \delta \lambda_{t+1} A \theta k_{t+1}^{\theta-1} - \lambda_t - \delta \mu_{t+1} b &= 0 \\ -\frac{\beta}{\hat{P}} \ln(c_{2t+1}) + \mu_t - (1 - \alpha) \delta \mu_{t+1} &= 0 \\ Ak_t^\theta - c_{1t} - c_{2t} - k_{t+1} - m_t &= 0 \\ P_t - (1 - \alpha) P_{t-1} - bk_t + \nu m_t &= 0 \end{aligned}$$

The planner internalizes four different factors which the atomistic agent does not. First, the planner recognizes impact of pollution on the discount factor for old age consumption. Second, the planner recognizes that pollution is persistent and higher pollution in any one period translates into higher residual pollution in the next period. Third, she recognizes the impact of capital accumulation on pollution. For this reason, the planner perceives a lower marginal utility of savings than the market does. Fourth, the planner internalizes the impact of mitigation on the level of pollution. This is reflected in the first order condition with respect to mitigation. For the market, the marginal utility of mitigation is given by  $\gamma$ , reflecting only the warm-glow on account of green consumerism. For the planner, the marginal utility of mitigation expenditure equals  $\gamma + \mu_t \nu$ .

In the steady state the planner's first order conditions are given by

$$\begin{aligned} \frac{1}{c_1} - \lambda &= 0 \\ \gamma - \lambda + \mu\nu &\leq 0 \\ \delta\lambda A\theta k^{\theta-1} - \lambda - \delta\mu b &= 0 \\ -\beta \left( \frac{1}{\hat{P}} \right) \ln(c_2) + \mu - (1 - \alpha)\delta\mu &= 0 \\ Ak^\theta - c_1 - c_2 - k - m &= 0 \\ \alpha P - bk + \nu m &= 0 \end{aligned}$$

Several things stand out. First, that we can put a lower bound on old age consumption and an upper bound on capital in the first best solution.

The first order condition with respect to pollution in the planner's problem implies

$$[1 - (1 - \alpha)\delta]\mu = \frac{\beta}{\hat{P}} \ln(c_2)$$

Note, negative  $\mu$  is the shadow price of net emissions or the marginal utility of net emissions. Typically we would expect  $\mu$  to be positive. If  $\mu$  is always positive, it implies that  $\ln(c_2)$  is positive or  $c_2 > 1$ .

From the first order condition with respect to capital we have

$$\left( \delta A\theta k^{\theta-1} - 1 \right) \lambda = \delta b\mu$$

where  $\lambda$ , the shadow price of capital, is positive. This implies that  $\delta A\theta k^{\theta-1} - 1$  is positive meaning  $k < (\delta A\theta)^{\frac{1}{1-\theta}}$

Like the competitive equilibrium, the planner's problem too needs to be studied under two regimes, one where there is no mitigation going on and another where the mitigation expenditure is positive.

### 3.3.1 When mitigation is zero

As long as  $c_{1t} \leq \frac{1}{\gamma + \mu_t \nu}$ , the planner will not allocate resources to mitigation investment. The steady state for the planner when mitigation expenditure is zero is described by the following equations.

$$\begin{aligned} c_2 &= \frac{\beta}{\delta} \left(1 - \frac{P}{\hat{P}}\right) c_1 \\ Ak_t^\theta - k &= \left(1 + \frac{\beta}{\delta} \left(1 - \frac{P}{\hat{P}}\right)\right) c_1 \\ \alpha P &= bk \\ \mu &= \left(\delta A \theta k^{\theta-1} - 1\right) \frac{1 + \frac{\beta}{\delta} \left(1 - \frac{bk}{\alpha \hat{P}}\right)}{(Ak_t^\theta - k) \delta b} \\ [1 - (1 - \alpha) \delta] \mu &= \frac{\beta}{\hat{P}} \ln(c_2) \end{aligned}$$

Together they can be written as a single equation in  $k$  as

$$[1 - (1 - \alpha) \delta] \left(\delta A \theta k^{\theta-1} - 1\right) \frac{1 + \frac{\beta}{\delta} \left(1 - \frac{bk}{\alpha \hat{P}}\right)}{(Ak_t^\theta - k) \delta b} = \frac{\beta}{\hat{P}} \ln \left[ \frac{\beta}{\delta} \left(1 - \frac{bk}{\alpha \hat{P}}\right) \frac{Ak_t^\theta - k}{\left(1 + \frac{\beta}{\delta} \left(1 - \frac{bk}{\alpha \hat{P}}\right)\right)} \right]$$

### 3.3.2 Mitigation is positive

The planner allocates resources to mitigation investment as soon as young consumption equals  $\frac{1}{\gamma + \mu_t \nu}$ . A market economy, will not engage in green consumerism unless young age consumption equals  $\frac{1}{\gamma}$ . In other words, the first best solution would require mitigation expenditure to start at lower levels of young age consumption than the levels at which market economy starts mitigating. In this regime, the planner's steady state equilibrium is given by

$$c_2 = \frac{\beta}{\delta} \left(1 - \frac{P}{\hat{P}}\right) c_1$$

$$\begin{aligned}
\frac{1}{c_1} - \lambda &= 0 \\
\gamma - \lambda + \mu\nu &= 0 \\
\delta\lambda A\theta k^{\theta-1} - \lambda - \delta\mu b &= 0 \\
-\beta \left( \frac{1}{\hat{P}} \right) \ln(c_2) + \mu - (1 - \alpha)\delta\mu &= 0 \\
Ak^\theta - c_1 - c_2 - k - m &= 0 \\
\alpha P - bk + \nu m &= 0
\end{aligned}$$

Rearranging these conditions, we can define the steady state in terms of three equations in the variables  $c_1$ ,  $k$  and  $P$ .

$$\begin{aligned}
c_1 &= \frac{1}{\gamma} - \left[ \delta A\theta k^{\theta-1} - 1 \right] \frac{\nu}{\delta b \gamma} \\
Ak^\theta - k - \left( 1 + \frac{\beta}{\delta} \left( 1 - \frac{P}{\hat{P}} \right) \right) c_1 &= \frac{b}{\nu} k - \frac{\alpha}{\nu} P \\
\frac{\beta}{\hat{P}} \ln \left( \frac{\beta}{\delta} \left( 1 - \frac{P}{\hat{P}} \right) c_1 \right) c_1 &= \frac{(\delta A\theta k^{\theta-1} - 1)}{\delta b} [1 - (1 - \alpha)\delta]
\end{aligned}$$

This can be further reduced to two equations in  $k$  and  $P$  after substituting out  $c_1$ .

As in the case for the market economy, the planning solution too admits a higher young consumption under the positive mitigation regime than under the zero mitigation regime. This derives directly from the first order conditions with respect to young consumption and mitigation which imply that  $c_1 \leq \frac{1}{\gamma + \mu\nu}$  when resources are not allocated to mitigation and  $c_1 = \frac{1}{\gamma + \mu\nu}$  when resources are allocated to mitigation expenditure. Further, the steady state capital in the presence of mitigation expenditure exceeds steady state capital in the no-mitigation regime. Consider a case where there is a unique, non-trivial steady state in

each of the two regimes. Let  $\{c_1^0, c_2^0, k^0, P^0\}$  indicate the steady state levels of young-age consumption, old-age consumption, capital and pollution in the no-mitigation regime and let  $\{c_1^*, c_2^*, k^*, P^*\}$  indicate the corresponding steady state levels in the positive mitigation regime. As discussed before  $c_1^* \geq c_1^0$ . Further, let  $\mu^0$  and  $\mu^*$  be the steady state levels of  $\mu$  in the two regimes.

**Proposition 5**  $c_2^* \geq c_2^0, k^* > k^0, P^* \begin{matrix} \geq \\ \leq \end{matrix} P^0$

**Proof.** From the first order conditions for young consumption and mitigation we have

$$\begin{aligned}\mu^0 &\leq \gamma - \frac{1}{c_1^0} \\ \mu^* &= \gamma - \frac{1}{c_1^*}\end{aligned}$$

$c_1^* \geq c_1^0$  implies that  $\mu^* \geq \mu^0$ . From the first order conditions for pollution we have

$$[1 - (1 - \alpha)\delta]\mu = \frac{\beta}{\bar{P}} \ln(c_2)$$

so that

$$\frac{\beta}{\bar{P}} \ln(c_2^*) \geq \frac{\beta}{\bar{P}} \ln(c_2^0)$$

which in turn implies that  $c_2^* \geq c_2^0$ . The resource constraints in the two regimes can be written as

$$\begin{aligned}f(k^0) - k^0 &= c_1^0 + c_2^0 \\ f(k^*) - k^* &= c_1^* + c_2^* + m^*\end{aligned}$$

This, together with  $c_1^* \geq c_1^0$  and  $c_2^* \geq c_2^0$  implies that  $f(k^*) - k^* > f(k^0) - k^0$ . By Lemma 3,  $f'(k) > 1$ . This is true for capital in both regimes. Hence  $f(k^*) - k^* > f(k^0) - k^0$  implies that  $k^* > k^0$ .  $k^* > k^0$  means that emissions in under positive mitigation are higher.

But the higher emissions will be neutralized to some extent by positive mitigation. In the end

$$P^* = \frac{b}{\alpha}k^* - \frac{\nu}{\alpha}m^* \begin{matrix} \geq \\ \equiv \\ < \end{matrix} \frac{b}{\alpha}k^0 = P^0$$

■

Comparing the planner's solution with that of the market we find that the planner starts allocating resources to mitigation at a lower level of young age consumption than the market. From the first order conditions with respect to young consumption and mitigation of the we have,  $c_{1p} \leq \frac{1}{\gamma + \mu_p \nu} < \frac{1}{\gamma} = c_{1mk}$ , where the subscript  $p$  stands for the planner and the subscript  $mk$  stands for the market economy. This implies that the planner's level of young consumption is associated with positive mitigation is lower than the young consumption in a market economy at which mitigation begins.

### 3.4 Comparative Statics

The steady states of the planner and the market cannot be compared analytically but we can examine the comparative statics of the two numerically. The purpose of this is to examine the responses of the market economy relative to the first best responses, when a parameter of the models is changing. In other words, it allows us to examine the long run equilibria of the market economy relative to the first best for a continuum of changes in the values of one parameter, holding other parameters unchanged. As a first instance, it might be useful to look at the comparative statics with respect to the maximum tolerable pollution or  $\hat{P}$ . To examine the comparative statics for the competitive equilibrium and the planning solution with respect to  $\hat{P}$  To to this, we set the parameters of the model as follows  $A = 8.5$ ,  $\theta = 0.3$ ,  $\gamma = 0.2$ ,  $\beta = 1$ ,  $\alpha = 0.5$ ,  $b = 3.5$ ,  $\nu = 2$ , and  $\delta = 0.9999$ . The parameter of interest,  $\hat{P}$  ranged between 21 and 30. Figure 3 presents the comparative

static results for the planner and the market economy with respect to  $\hat{P}$ .

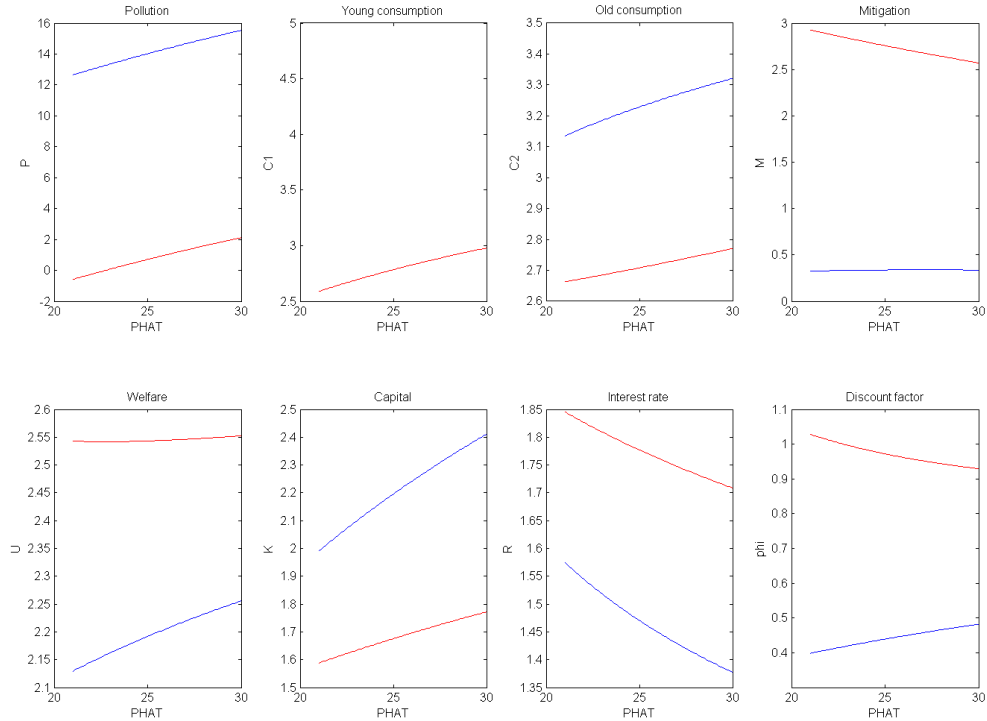


Figure 3.3 Comparative Statics of the Planner and Market with respect to  $\hat{P}^*$

\*The planning solutions are indicated in red and the competitive equilibria in blue

The red lines represents the planner's comparative statics and the blue lines do the same for the competitive equilibrium.

As figure 3 indicates, the planner delivers a higher utility with lower consumption than the market in both young and old age. The higher utility in the planner's solution comes from two sources, viz., the higher discount factor for old age consumption and higher levels of warm-glow satisfaction on account of greater mitigation expenditure. In the standard



Diamond model the only source of inefficiency arises from the the failure of atomistic agents to internalize the effect of their savings on the prevailing interest rate in the economy. In the steady state of the standard model we have

$$\begin{aligned} u' (c_1^M) &= \rho R u' (c_2^M) \\ u' (c_1^P) &= \rho f' (k^P) u' (c_2^P) \end{aligned}$$

where the superscripts  $M$  and  $P$  stand for the market and the planner, respectively,  $\rho$  stands for the discount factor for old age consumption,  $R$  represents the market rate of interest, and  $c_1$  and  $c_2$  stand for young and old consumption, respectively. The market solution is different from the planner's when  $R \neq f' (k^P)$ . In my model, in addition to this, we have two more sources of inefficiency, the first of which relates to capital accumulation.

$$\begin{aligned} u' (c_1^M) &= \varphi R u' (c_2^M) \\ u' (c_1^P) &= \varphi (P^P) f' (k^P) u' (c_2^P) \end{aligned}$$

where  $P^P$  is the planner's steady state pollution. Now, in addition to the rate of return on capital, the planner's discount factor might be different from the markets. This discount factor depends on pollutions levels which in turn depend partly on the level of capital. Thus from the planner's perspective, the market is overaccumulating capital. The second factor which affects this discount factor is the level of mitigation. To isolate the effect of mitigation on the differences between the planner's and the market's solutions, we conduct a thought experiment. We ask what happens when we force the level of capital to be the same for the market and the planner and allow the other variables to adjust optimally. To do this we take the optimal levels of capital in the market and set the planner's capital to equal the market's capital for different values of  $\hat{P}$ . Figure 4 represents the comparative statics for the planner and the market under these conditions. Note, the parameter values are identical to the ones used for generating figure 3.

With capital and hence the output equalized between the planning and competitive equilibria, the only source of suboptimality in the market solution comes from allocation of that output to consumption and mitigation. Here again, we find that the planner allocates greater resources to mitigation at the cost of young and old consumption. The differences in the discount factor in this case reflect differences in pollution solely on account of differences in mitigation. Internalizing the impact of lower pollution on marginal utility of old consumption through the discount factor causes the planner to mitigate much more than the market even when the output is the same for both.

### 3.5 Conclusion

This paper sought to study the role of environment as public good in an overlapping generation growth model. The main body of the literature in this area has focused on the effect of human activity on the environment through the emission of pollutants. Additive separability of pollution and consumption in these models ensures that levels of pollution do not affect human choices. This paper added to the small but growing line of studies that seek to study the impact of long-term pollution on human choices, by relaxing the assumption of additive separability of pollution and consumption in the utility function. A second contribution of this paper has been to incorporate green lifestyle choices and corporate environmentalism as a driver of pollution abatement. This is the first such attempt to model a growing trend in developed economies in the western world that is catching on in the fast growing economies like India and China as well. Using green choices as the primary driver of pollution abatement is an improvement on the standard literature because it gets around the problematic assumption that earlier studies make whereby mitigation investment is either decided by a central authority or by agents in a provision of a public good game.

As expected, I find that an economy starts to expend on mitigation of the environment only when it is rich enough in terms of having high enough capital and consumption. Among economies that are not abating pollution, capital and pollution are both higher for the economies with higher tolerance for pollution. But in such economies capital stock is smaller for those with dirtier industries, i.e. having greater emissions per unit capital used for production. Pollution is also higher for these economies. This supports the traditional view that transfer of cleaner technology to these economies will bring down pollution in the long run. Multiple steady states are possible for economies that mitigate the environment. In particular some of these steady states may involve cyclical fluctuations in pollution. Thus even though an economy is investing in the abatement of pollution, it may experience alternating periods of high and low pollution. Unless mitigation is sufficiently inefficient, this economy will eventually converge to the steady state. In case of multiple steady states, the one with the lower pollution is stable while the one with the higher pollution is not. This implies that when an economy is engaging in pollution abatement, it will not converge to a long run equilibrium with high pollution levels. Comparing with the benchmark planner's solution we find that because atomistic agents do not internalize the polluting impact of their savings decisions, the market will overaccumulate capital and usually under-mitigate the environment in the long run.

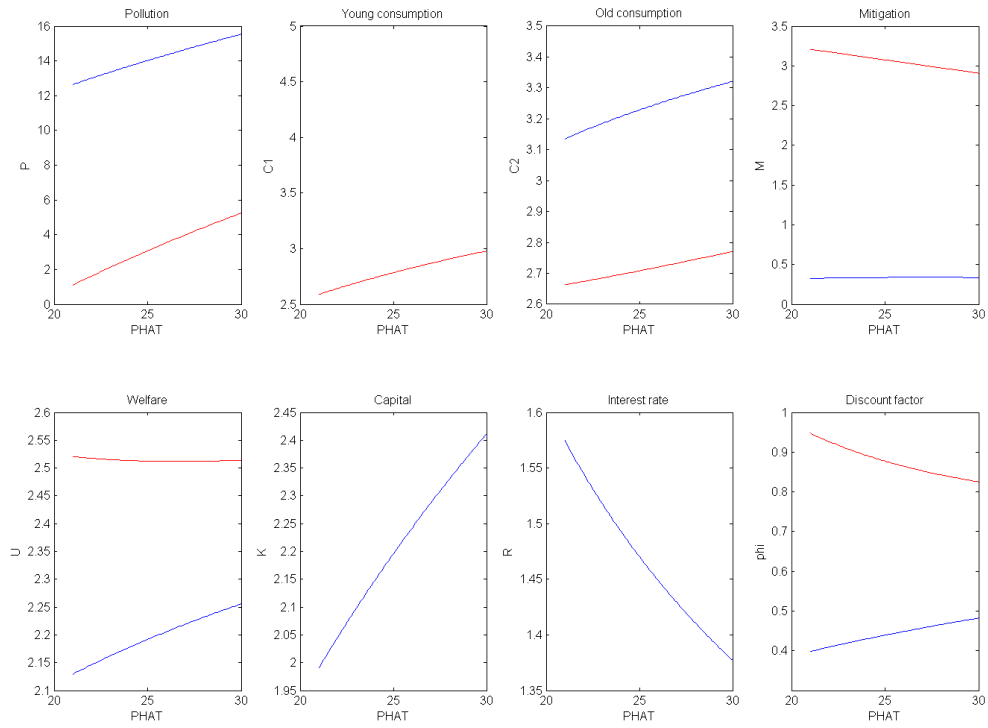


Figure 3.4 Comparative Statics of the Planner and Market with respect to  $\hat{P}^*$ : Planner's Capital Equals the Market's

\*The planning solutions are indicated in red and the competitive equilibria in blue