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Three Essays on Macroeconomic Implications of Contemporary Financial Intermediation

Hyun Woong Park

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**THREE ESSAYS ON MACROECONOMIC
IMPLICATIONS OF
CONTEMPORARY FINANCIAL INTERMEDIATION**

A Dissertation Presented

by

HYUN WOONG PARK

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

September 2015

Department of Economics

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ABSTRACT

THREE ESSAYS ON MACROECONOMIC IMPLICATIONS OF CONTEMPORARY FINANCIAL INTERMEDIATION

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This dissertation contributes to the growing literature on macroeconomic models with a financial intermediary sector. The first two chapters use the circuit of capital modeling methodology to study the relation between growth and profitability in capitalist economy where credit is essential, and the third uses a more standard macrodynamic model to investigate how securitized banking, which relies on short-term collateralized borrowing, as opposed to traditional commercial banking, generates procyclical bank leverage, which in turn leads to supply-led fluctuation in credits and ultimately to a boom-bust cycle of asset prices.

In chapter 1, I extend the baseline model of circuit of capital to incorporate a profit-making financial capitalist sector and the associated financial variables. The extended model is examined to see how the main findings of the existing literature regarding growth, profitability, and credits are modified. It is shown that once finance

is explicitly incorporated, the Cambridge equation–type result is modified in a way that relates growth to net return on equity of a firm sector, not to the latter’s gross profit rate; hence, leverage ratio of the firm sector and the bank profitability, which is determined in line with interest rates, become crucial variables.

In chapter 2, by relying on the extended circuit of capital model, I propose a new categorization for growth theory that characterizes financial aspects of growth, i.e. firm leverage–led vs. bank leverage–led growth. When the growth is led by firm leverage, on the one hand, growth does not face any upper bound while it stimulates excess demand for bank credits and hence is accompanied by a rise in the interest rates. On the other hand, when the growth is led by bank leverage, growth faces some upper bound but it stimulates excess supply of bank credits and consequently is accompanied by a fall in the interest rate.

Chapter 3 draws upon the recent empirical finding on procyclical bank leverage and makes a contribution to the related literature in two directions. First, I build a one–period banking model of repurchase agreement to show that repo transactions motivate borrowers to manage their leverage procyclically due to counter–party risks and collateral value risks involved in the transactions. Second, with this result as a microfoundation, I build a macrodynamic model, which reveals the logic underlying persistent boom–bust cycles observed in the securitized banking system.

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INTRODUCTION

Mark Gertler wrote in 1988 that “Most of macroeconomic theory presumes that the financial system functions smoothly – and smoothly enough to justify abstracting from financial considerations”(Gertler, 1988). And in 1995 Marc Lavoie wrote that “one of the surprising aspects of the evolution of Cambridge theory is that, starting from Keynes’s *General Theory* which described a monetary production economy, the Cambridge post–Keynesians have failed to incorporate money [and finance] into their models of growth and distribution” (Lavoie, 1995). The status of finance in macroeconomic theory summarized in these statements did not changed substantially until very recently despite of a few exceptions such as financial accelerator models by Bernanke and his various coauthors (Bernanke and Gertler, 1989; Bernanke et al., 1996; Bernanke, 1999) and Minsky’s financial instability hypothesis (Minsky, 1975, 1982, 1986).

The 2008 financial crisis has changed this and now macro models that include a financial sector are now exponentially growing in number. Most notable examples are New Keynesian dynamic stochastic general equilibrium (DSGE) models with a banking sector and financial frictions (Gerali et al., 2010; Gertler and Kiyotaki, 2010) and the Post Keynesian stock–flow consistent (SFC) modeling framework (Pilkington, 2008; Passarella, 2012; Le Heron and Mouakil, 2008). In broad, the present dissertation lies in this trend and contributes to building a macro model that incorporates a financial intermediary.

Alternatively to a DSGE model and SFC model, I use the circuit of capital macroeconomic model first developed in Foley (1982, 1986a,b). The circuit of capital model is based on the analysis in Volume II of *Capital* and rigorously formalizes the diagram

$M-C-M'$. It is a macroeconomic representation of the essential aspects of Marxian economic theory. In comparison to the existing macro models, the circuit of capital model is distinctive in several ways.

First, having the labor theory of value as its theoretical foundation, the circuit of capital model describes capitalist economy from the perspective of circular flows of value and valorizing movement of capital. Note that at the core of the DSGE model lies households' consumption and saving behavior aimed at utility maximization. In terms of the $M-C-M'$ diagram, households' decision-making is oriented towards larger C , i.e. more consumption. In this sense, the DSGE model describes capitalist economy as $C-M-C'$ with the initial C representing endowments and the last C' representing consumption. The model is ultimately driven by consumption determined in a way that satisfies Euler equation, and hence one of the deep parameters of the model is time preference of the representative household.

In contrast, the central economic agent in the circuit of capital model is a capitalist firm that engages in production and accumulation motivated by a pursuit for profits, i.e. $M'(>M)$, and as will be seen below, the model's deep parameters include exploitation rate and organic composition of capital, both of which reflecting class struggle and technical condition in the production process.

Second, related to the first, the circuit of capital model is based on a distinctive theory of profit and growth where a central determinant is the creation of surplus value through exploitation of workers. The class-centered approach is unique to the circuit of capital model. On the one hand, Post Keynesian theory of growth and distribution and SFC framework do not provide a substantive explanation of the source of profits, but are concerned only about distribution between profits and wage and about how it affects growth as can be seen in the literature on wage-led versus

profit-led growth.¹ In DSGE models, on the other hand, even the income distribution is not on the agenda in the first place as earnings of each economic agent merely reflect its contribution to production.²

Third, the circuit of capital model is a dynamic model and so are DSGE model and SFC model. However, there is a significant difference on how to model the dynamics. While the standard dynamic macroeconomic models primarily concern the volume of variables of the model at each period *synchronously*, a central focus of the circuit of capital model is to trace a movement of monetary value in historical time from one form — e.g. investment — to the other — e.g. outputs — *diachronically*. In this framework, the question how long it takes for the monetary value to traverse, to use the same example, from investment to outputs becomes crucial.

The diachronic approach of the circuit of capital model is represented by lag variables, which are a mathematical formalization of the concept of turnover time discussed in the second volume of *Capital*. Turnover time is a very unique concept in economic theories which allows to distinctively characterize economic processes as inherently dynamic. Similarly, lag variables are one of the major innovations of the circuit of capital model, enabling the latter to describe the capitalist economy in historical time in a way that is not possible in the other standard models.

Fourth, by tracing an evolution of balance sheet and income statement of each sector of the model over time, the circuit of capital model allows to formalize the capitalist economy in a stock-flow consistent way. While the stock-flow consistency features crucial in the other macroeconomic models as well — e.g. Godley and Lavoie (2007) — the circuit of capital model is distinctive in modeling the formation of stock variables in relation to the lag variables. Stocks emerge as a logical consequence of

¹See Dos Santos (2013) for a critique of the Post Keynesian theory of wage-led versus profit-led growth from the perspective of the circuit of capital model.

²This explains why the mainstream literature hardly paid attention to this topic before the publication of Piketty (2014)

lag variables. As it takes time for the monetary value to proceed from one form to the other, e.g. from inputs to outputs, it accumulates into a stock, i.e. inventories, to use the same example, during that time period.

Lastly, related to the fourth, the structure of the circuit of capital model makes it open to various behavioral stories, which enhances the model's applicability. Equations constituting the circuit of capital model are accounting identities that necessarily hold by definition. A substantive character of the system is captured by parameters of the model including exploitation rate, organic composition of capital, recapitalization rate, and lag variables, which are all treated as a fixed constant. Adding behavioral specifications for these deep parameters, which summarize institutional, structural, and socioeconomic character of the system, will make the content of the model analytically richer.

Despite of these merits, however, there are some issues that need to be addressed in order to make the model more widely applicable and practically accessible. One of them regards the lag variables.³ While the lag variables are what make the circuit of capital model innovative and distinctive, they at the same time render the model very difficult to deal with in a mathematical sense. For this reason, in many cases, simplifying assumptions are adopted regarding the lag variables as is done in this dissertation. The existence of the lag variables is a major challenge facing the circuit of capital model. As the DSGE modelers adopt log linearization as a way to solve the model which otherwise would have been almost impossible mathematically, the circuit of capital model needs a technical tactic to address this issue.

Having the above-discussed merits and demerits in mind, probably the most important contribution of the circuit of capital model in the macro-finance literature

³There are four different ways to mathematically define the lag variables, i.e. fixed time lag, variable time lag, finite distributed lag, infinite distributed lag (Basu, 2013). For the sake of simplicity, I adopt fixed time lag, which is the simplest one.

lies in proving the necessity of credit for growth of the capitalist economy by showing that the capitalist economy without exponentially growing net credits will not be able to stay on a positive steady-state balanced growth path. This result makes it essential to explicitly incorporate credits and the associated financial variables such as financial assets and liabilities and interest earnings and payments whenever modeling the capitalist economy, rather than treating finance as a secondary that may or may not be added at a later stage of analysis.

Duncan Foley's work on the circuit of capital model has been followed by a small number of papers. First, there are papers that develop the model's proposition on the necessity of credit in various directions. Using a different modeling approach, Kotz (1991) derives a stronger result that the new capital investment must be financed *entirely* by credit. Basu (2013) shows that the steady-state growth rate is negatively related to the share of consumption credit in total net credit. Dos Santos (2011, 2014) further demonstrates that consumption credit contributes to intensifying credit risk by lowering total wage and profit incomes relative to interest payments.

Second, there are papers that tackle the widely discussed topic of wage-led growth versus profit-led growth in the recent Post Keynesian literature from the perspective of the circuit of capital model. Basu (2013), on one hand, shows that the circuit of capital model allows those two types of growth regime in relation to income distribution. On the other hand, Dos Santos (2013) criticizes the Post Keynesian growth theory of wage-led vs. profit-led categorization from the perspective of the circuit of capital model.

Third, the financial circuit that can additionally emerge in relation to, and as opposed to, the industrial circuit is another topic in the literature. Loranger (1989) analyzes a divergence between capital investment and financial investment and studies an inflationary effect of the latter. Satoh (2012) adds the circuit of *bank capital* to the baseline model, which can be characterized as the circuit of *industrial capital*, and

examines how interest rate is determined in the model. Satoh (2012) is an exception compared to the rest of the papers in explicitly formalizing the banking sector and the associated financial variables.

First two essays of this dissertation contribute to the above-discussed literature on the circuit of capital model by further specifying the banking sector and examining its profitability and the impact of its leverage behavior on growth and interest rate. In chapter 1, in order to introduce the basic methodology of the model, I start with presenting the baseline model, which then is extended to incorporate a banking sector with the related financial variables. I use the extended model to study the steady-state growth rate of the system and profit rates of both industrial capital and financial capital by relying on comparative static analysis and simulation exercise. In chapter 2, I use the extended circuit of capital model to propose a new categorization of growth regimes that characterizes financial aspects of growth, i.e. firm leverage-led vs. bank leverage-led growth. On the other hand, chapter 3 uses a more standard approach to present a macro model with a securitized banking system and examines how procyclical bank leverage aggravates an asset price cycle driven by supply-led credit expansion and collapse.

Chapter 1 examines how the main findings of the existing literature of the circuit of capital model regarding growth, profitability, and credits are extended and modified when a banking sector and the associated financial variables are explicitly accounted for. The existing papers show how the steady-state growth rate and the rate of profit are determined by the parameters of the model such as exploitation rate, organic composition of capital, and investment and consumption behaviors of firms and households with the relation between growth rate and profit rate being established in the Cambridge equation-type result. In addition, it has been proved that the economy without exponentially growing net credits will not be able to attain a positive steady-state growth rate. However, these findings are derived from

the model without finance. In chapter 1, I present an extended model of circuit of capital that includes a banking sector and use it to see how the above findings are extended and modified when financial variables, such as financial assets and liabilities and interest payments and earnings, are explicitly incorporated.

The main results are the following. First, the extended model allows to investigate the growth impact of financial variables. It is shown that while the bank lending rate negatively affects the steady-state growth rate, borrowing rate of the firm has a positive impact on the growth rate when the lending rate is low and a negative impact when the rate is high. Second, once the rate of profit is measured to reflect both asset side and liability side of balance sheet and the related financial variables, thereby being redefined as net return on equity, the Cambridge equation-type result continues to hold even when the baseline circuit of capital model is extended to include a banking sector. The extended model further allows to measure profitability of banks as well as that of nonbank firms and to show how both are related to the other financial variables such as leverage ratio and margin of safety.

Third, a simulation of the extended model shows that the established result on the necessity of credits for growth is modified when finance is explicitly considered. In contrast to the finding of the existing literature that the exponentially growth net bank credits allow the system to be on a positive steady-state growth path, the simulation result in this chapter exhibits that the system could possibly collapse to zero growth state even when bank credits grow exponentially under the condition that the consumption lag of the banker household is larger than any other expenditure lags in the model and that the interest rate is sufficiently large.

A theoretical interest in the relation between growth rate and profit rate has recently been revived by Piketty (2014). Note that the latter points as the main determinant of an inequality of wealth and income the fact that the profit rate is larger than the growth rate. However, $r > g$ is suggested as a historically observed

fact rather than being supported by a rigorous theoretical foundation. A contribution of this chapter, which provides a theoretical framework for understanding growth and profitability in a generalized model of circuit of capital with finance, can be appreciated against this background.

The model in chapter 1 considers only the goods market equilibrium condition and interest rates are taken as given. On the other hand, chapter 2 examines equilibrium condition in bank credit market along with that in the goods market, thereby endogenizing both growth rate and the interest rate. In this setup, chapter 2 studies growth in relation to demand and supply of credit, which allows to identify distinctive growth impact of leverage of the firm sector and leverage of the banking sector.

It is found that while the firm leverage-led growth is unbounded, the growth in this case takes place through facilitating credit demand, thereby tightening the bank credit market and hence raising the interest rate spread alongside the growth rate. Bank leverage-led growth, on the other hand, has an upper bound, but under the condition that the banking system has attained a developed financial technology, the growth could possibly take place by facilitating credit supply, which creates the slack in the bank credit market, consequently enabling the system to experience high growth and low interest rate.

The main focus of Post Keynesian growth theory has been on income distribution and its growth impact. See the literature on wage-led vs. profit-led growth (Bhaduri, 2008). And more recent papers in this tradition, especially after the recent financial turmoil, pay attention to financial aspects of growth; e.g. debt-led vs. debt-burdened growth (Hein, 2007; Nishi, 2012). In most cases, debts refers to liabilities of nonfinancial sectors such as nonfinancial firms or households. However, historical data show that leverage ratio of the financial sector is much higher and more volatile than that of the nonfinancial sectors (see figure 2.1). By explicitly incorporating the leverage ratio of the bank, chapter 2 allows to see that growth led by the bank leverage can

differ from growth led by the nonbank firm leverage in two respects: i) existence of upper bound of growth, ii) correlation between growth and interest rate.

While the financial intermediary modeled in the first two chapters is highly simplified, the intermediary sector in chapter 3 is dealt with in a more concrete detail; it is divided into three sub-sectors, i.e. commercial banks, investment banks, and asset-managing firms. Chapter 3 is motivated by Adrian and Shin (2010)'s empirical finding that the leverage ratio of an investment banking sector is procyclical. Two main purposes of the chapter are to provide a theoretical explanation of the procyclical leverage of the investment bank and to examine a macroeconomic consequence of it.

First, using a banking model of repo transaction, it is shown that an optimal leverage of the investment bank is positively related to asset prices. The main mechanism underlying this relation is a movement of repo rate, which reflects counter-party risks and collateral value risks repo lenders are facing. Second, based on this result, I develop a dynamic macroeconomic balance sheet model with a securitized banking system characterized by loan securitization and collateralized short-term borrowing. The main analytical result suggests that under plausible condition the model exhibits a limit cycle behavior. A vulnerability of the repo market to asset price fluctuations leads to procyclical bank leverage, which aggravates asset price cycles driven by the supply-led credit expansion and collapse. It is also found that as the procyclicality of bank leverage becomes stronger the financial cycles also get more intense and severe.

The model in chapter 3 allows to see how different leverage behaviors between nonfinancial sectors and a financial sector can have an aggravating effect on an asset price cycle. Since the nonfinancial sectors demand credits while the financial sector supplies credits, the fact that the financial sector borrows more in the case of asset price appreciation and less in the case of asset price depreciation in a more responsive way than the borrowings of the nonfinancial sectors implies that during upturns of

cycles with asset price appreciation there will be excess supply of credits and during downturns of cycles with asset price depreciation there will be excess demand of credits. This ultimately will aggravate the financial cycle.

CHAPTER 1

GROWTH AND PROFITABILITY IN THE CIRCUIT OF CAPITAL WITH A FINANCIAL INTERMEDIARY

1.1 Introduction

The existing literature on the circuit of capital model produced two fundamental macroeconomic results regarding growth, profitability, and credit (see Foley, 1982, 1986a,b; Kotz, 1991; Basu, 2013). First, it has been demonstrated that an equality exists between the rate of profit and the growth rate divided by recapitalization rate; formally,

$$r = \frac{g}{p} \tag{1.1}$$

where r is the rate of profit, g growth rate, and p recapitalization rate. Second, it has been shown that exponentially growing net bank credits resolve the problem of insufficient demand, which would have plagued the system without bank credits, and guarantee that a system expands on a positive steady-state growth path. That is, without exponentially growing net credits the economy will not be able to be on a positive steady-state balanced growth path.

Unless we are interested in a simple reproduction with zero growth, the latter result regarding the necessity of credits for growth implies that the credits and the associated financial variables, such as financial assets and liabilities and interest payments and earnings, are not something that *may* or *may not* be added at a later stage of analysis depending on authors' interests but something that *must* be explicitly accounted for from the very start. However, most of the existing models in the literature ignore them. In order to fill this gap, in this chapter, I extend the baseline model of

the circuit of capital to incorporate a profit-earning banking sector and examine how the above-mentioned two fundamental results are modified when finance is explicitly considered.

First of all, in the circuit of capital model g in equation (1.1) is endogenously determined by parameters of the model such as exploitation rate, organic composition of capital, and investment and consumption behaviors of firms and households (see section 1.3.1.1). By using the extended model presented in this chapter, I further examine how financial variables such as interest rates and leverage behaviors affect growth. It is shown that while the bank loan rate has a negative effect on the steady-state growth rate, borrowing rate of the firm has a positive effect on the growth rate when the loan rate is low and a negative effect when the rate is high.

Regarding the rate of profit, on the other hand, I suggest that its definition needs to be reconsidered in two respects once it is recognized that finance is an essential character of a capitalist economy. First, in measuring capital in the denominator of the profit rate, financial assets need to be included since financial capital is one of the essential forms capital takes during the process of the circuit of capital $M-C-M'$ and since, as a consequence, at any moment in time there always exists financial capital along with various forms of nonfinancial capital.

Second, not only the asset side of the balance sheet of capitalist firms but also the liability side of it needs to be considered regarding both numerator and denominator of the rate of profit. Accordingly, the denominator is measured by the difference between assets and liabilities, i.e. equity, and the numerator is measured by the difference between the rate of aggregate profits and the net interest payments. What results is net return on equity.

In this context, it is shown that equation (1.1) continues to hold even when the baseline model is extended to incorporate financial variables as long as profitability is measured by net return on equity. That is, while in the model without finance

an equality exists between the rate of gross profit and the steady-state growth rate divided by recapitalization rate, in the extended model with finance the same equality holds with the rate of gross profit replaced by the net return on equity.

Furthermore, the extended model with a banking sector allows to measure both profitability of industrial capitalists and that of financial capitalists.¹ A decomposition of the net return on equity of industrial capitalists demonstrates that it is positively related to gross profit rate, leverage ratio of the firm, and margin of safety whereas a decomposition of the net return on equity of financial capitalists shows that it is positively related to interest rate and leverage ratio of the bank.

Lastly, by simulating the extended model, I show how the established result on the necessity of credits for growth is modified when financial variables are considered. The extended model presented in this chapter exhibits that even with exponentially growing net bank credits, the system could possibly be unable to overcome the problem of insufficient demand and thus collapse to zero growth state even when bank credits grow exponentially.

An important factor driving this result is a consumption behavior of the banker household, which earns dividends out of bank profits, which are interests paid by firms to banks. As the banker household's consumption expenditure constitutes an additional source of effective demand, if the banker household's marginal propensity to save is sufficiently small, a sufficient portion of the interest payments will return back to, instead of being drained from, the circuit. If not, however, the interest payments will be a drainage of the value from the system, which will definitely undermine the growth capacity of the system. In this context, a simulation result shows that when the consumption lag of the banker household is larger than any other expenditure

¹While there are a small number of papers that study measuring the nonfinancial firm sector's profit rate in relation to finance (Duménil and Lévy, 2004; Bakir and Campbell, 2013; Norfield, 2013), a theoretical work on the bank's the rate of profit is almost nonexistent.

lags in the model and when interest rate is sufficiently large, both growth rate and profit rate collapse to zero.

The rest of the chapter is organized as follows. Section 1.2 presents the baseline model, introducing the circuit of capital model methodology, which is then extended to incorporate a banking sector and the related financial variables. In section 1.3, steady-state growth rate, profit rate of industrial capitalists, and profit rate of financial capitalists in the extended model of circuit of capital are examined. In section 1.4, the extended model is simulated to examine the necessity of the bank credits. Section 1.5 is a conclusion.

1.2 A model of the circuit of capital

In this section, the baseline circuit of capital model without finance is presented first and then it is extended to include financial variables. The model is in a discrete-time framework. For the sake of simplicity, time description is omitted from the notation for an endogenous variable in time t . Most of the variables will have a subscript in their notation and therefore time description is added as a superscript in order to avoid notational confusion. For example, net bank credits of nonfinancial firms, denoted by B_k , in time t and those in time $t + \tau$ are each denoted by B_k and $B_k^{+\tau}$ instead of B_k^t and $B_k^{t+\tau}$. A list of the notations can be found in appendix A.

The model is a two-class economy. Capitalist class is subdivided into industrial capitalist (nonfinancial firms) and financial capitalist (banks). Each group of capitalists consists of entrepreneurs or bankers and their households. Overall, the model consists of entrepreneur (k), entrepreneur's household (s), banker (b), banker household (m), and worker household (w). Bankers and their households do not appear in the baseline model but only in the extended model.

1.2.1 Baseline model

The circuit of capital macroeconomic model is based on the analysis in Volume II of *Capital* and formalizes the diagram

$$\dots - M - C(LP, MP) - P - C' - M' - \dots$$

The circuit starts with money capital (M) which is advanced to purchase inputs (C) — labour power (LP) and means of production (MP) — and the process of production (P) begins. Output commodities (C') embodying surplus value are produced and are sold in the market to realize the surplus value as profits ($M' - M$). Part of these profits is recommitted to finance capital outlays and the entire process starts anew. Each of these processes — production, realization, and finance processes — takes time during which flows of value are accumulated and hence stocks of money capital, productive capital, and commodity capital are built up before they can proceed to the next process.

The circular movement of value just described can be formalized by the following stock–flow consistent relations:

$$P = Z^{-\tau_P} \tag{1.2}$$

$$R = P^{-\tau_R} \tag{1.3}$$

$$Z = (1 + p_k q) R^{-\tau_F} \tag{1.4}$$

$$\Delta U^{+1} = Z - P \tag{1.5}$$

$$\Delta X^{+1} = P - R \tag{1.6}$$

$$\Delta F_k^{+1} = (1 + p_k q) R - Z \tag{1.7}$$

where Δ refers to an increment of the variable from the previous period to the current one. Z denotes the flow of capital outlays on constant capital and variable capital.

Share of variable capital out of total capital outlays is a with $0 < a < 1$. Hence aZ is used in paying wages while $(1 - a)Z$ in purchasing non-wage inputs. P is the flow of finished commodities measured at cost. τ_P is a time delay for production. Equation (1.2) demonstrates that it takes τ_P periods of time for the capital outlay to emerge as finished product.

R is the flow of sales revenue measured at cost. τ_R is a time lag for realization of finished commodities. Equation (1.3) states that the finished products are realized into final sales after τ_R periods. Since final sales are financed by effective demand, τ_R reflects the conditions of effective demand. In this sense, taking τ_R as constant implies presupposing that there always emerges a sufficient volume of effective demand so as to maintain the realization lag at a certain level. On the other hand, an approach alternative to taking τ_R as exogenous is to account for effective demand by explicitly identifying it within the model. In this case, the realization lag would be endogenized and resolved into lags associated with spending behaviors of firms and households. This will be discussed more in detail below.

With q denoting markup, total sales revenue can be expressed as $(1 + q)R$, and aggregate profits, which are realized surplus value, would be qR . q is determined as a product between a organic composition of capital — or, more precisely, its inverse — and exploitation rate, denoted by e , i.e. $q = ae$. This reflects that profits are created by surplus labor. Aggregate profits qR are divided into retained earnings and dividends to capitalist household according to recapitalization (retention) ratio, denoted by p_k . Consequently, $p_k qR$ expresses retained earnings of firms. τ_F is the finance lag which describes time delay during which internal funds lie idle before being invested on capital outlays. Equation (1.4) shows that investments are financed by the retained earnings $p_k qR$ and revolving funds R of τ_F periods ago.

U denotes the stock of productive capital and equation (1.5) states that it increases by the inflow of capital outlays and decrease by the outflow of finished products. X is

the stock of commodity capital and, by the same logic, equation (1.6) describes that it increases by the inflow of finished products and decreases by the outflows of sales measured at cost. F_k is the stock of financial capital which, as stated in equation (1.7), increases by the inflow of revolving funds and retained earnings while decreasing by the outflow of investments on capital outlays.

As mentioned earlier, the realization lag τ_R can be endogenized by explicitly formalizing effective demand. In the baseline model, there are three sources of demand: capitalist firms' investment expenditure on constant capital $(1 - a)Z$, and worker households' consumption expenditure financed by wage revenues aZ , and capitalist households' consumption expenditure financed by dividend earnings $(1 - p_k)qR$. Similarly to modeling capitalist firms' financing process, households' financing process can be formalized as taking time. Use τ_w and τ_s to denote time delay for consumption financing by revenues of worker household and capitalist household, respectively. Then aggregate demand is expressed as

$$D = (1 - a)Z + aZ^{-\tau_w} + (1 - p_k)qR^{-\tau_s} \quad (1.8)$$

One of the major results that emerges from the circuit of capital model outlined above is that in the steady-state balanced growth setting, the system will not be able to achieve a positive steady-state growth rate without exponentially growing credit (see section 1.3.1.1). Accordingly, it is suggested that the existence of credit is a necessary condition for growth in capitalist economy. Therefore, in order to further understand the implications and consequences of credit, the model needs to be extended to include a banking sector and associated financial variables.

1.2.2 Extended model

In this section, the baseline circuit of capital model is extended to add the banking sector. Accordingly, a business sector includes industrial capitalists (entrepreneurs

or nonbank firms) and financial capitalists (banks) while household sector includes entrepreneur household, banker household, and worker household. For the sake of simplicity, it is assumed that only nonbank firms and worker households borrow from banks.

Let us define net bank credits as aggregate bank credits minus principal repayments.² Then net bank credits of nonbank firms and those of worker households are denoted by B_k and B_w , respectively. Furthermore, denoting by L_k and L_w stock of liabilities of each of these sectors, the following will hold by accounting identity.

$$\begin{aligned}\Delta L_k &= B_k \\ \Delta L_w &= B_w\end{aligned}\tag{1.9}$$

Suppose nonbank firms finance a share of capital outlays by net bank credit. Denoting this share by b_k with $(0 < b_k < 1)$ ³ yields

$$B_k = b_k Z\tag{1.10}$$

Similarly, worker households finance a share of their consumption expenditure by net bank credits. Denote this share by b_w with $(0 < b_w < 1)$ and the worker household's consumption expenditure by D_w . Then it yields

$$B_w = b_w D_w\tag{1.11}$$

Interests are paid on liabilities and earned on financial assets at the bank loan interest rate i_L and deposit rate i_D , respectively. Accordingly, net profits of nonbank

²Net bank credit appears as one of the central variables in Foley (1982, 1986a,b) and Basu (2013), but the definition is ambiguous in these papers.

³If the assumption that borrowed funds are used only for real investments is relaxed, then $b_k > 1$ would be of possibility with an implication that some portion of the borrowed funds is used in financial investments.

firms are obtained by subtracting net interest payments from gross profits,

$$\Pi_k = qR - (i_L L_k - i_D F_k) \quad (1.12)$$

When the net interest payments are taken into account, capital outlays in equation (1.4) will be modified into

$$Z = R^{-\tau_F} + p_k \Pi_k^{-\tau_F} + B_k \quad (1.13)$$

which states that capital outlays are financed by a combination of past flows of revolving funds and retained earnings — expressed in the first two terms on the right hand side — and the current net bank credits. As a consequence, the law of motion of the nonbank firm's financial assets in equation (1.7) is also modified as follows.

$$\Delta F_k^{+1} = R + p_k \Pi_k - (Z - B_k) \quad (1.14)$$

which states that financial assets increase by the inflow of revolving funds and retained earnings — expressed in the first two terms on the RHS — and decrease by the outflow of capital outlays net of net bank credit.

Unless the worker household spends wage income immediately in the contemporaneous period it will build up financial assets and earn interests. Since workers also pay interests on their outstanding debts, their net income, denoted by Y_w , will be expressed as

$$Y_w = aZ + i_D F_w - i_L L_w \quad (1.15)$$

The worker household's consumption expenditure, denoted by D_w , is financed by a combination of past net income flows and the current net bank credits,

$$D_w = Y_w^{-\tau_w} + B_w \quad (1.16)$$

As a consequence, the worker household's financial assets will evolve by the following law of motion,

$$\Delta F_w^{+1} = Y_w - (D_w - B_w) \quad (1.17)$$

which states that the financial assets increase by the inflow of wage income net of net interest payments and decrease by the outflow of consumption expenditure net of net bank credit.

The same logic can be applied to the entrepreneur household's income Y_s , consumption D_s , and financial assets F_s . This group of households does not borrow and hence its consumption expenditure is financed solely by past income,

$$D_s = Y_s^{-\tau_s} \quad (1.18)$$

As a result, the entrepreneur household's income amounts to

$$Y_s = (1 - p_k)\Pi_k + i_D F_s \quad (1.19)$$

i.e. a sum of dividend income from the nonbank firms and interest earnings on financial assets. The entrepreneur household's financial assets increases by the inflows of income and decrease by the outflows of consumption expenditure,

$$\Delta F_s^{+1} = Y_s - D_s \quad (1.20)$$

Now let us incorporate the banking sector consisting of bankers and their households. While only nonbank firms and worker households incur debts, nonbank firms and all the three types of the household accumulate financial assets through saving,

which is reflected in their consumption lags. In this setup, bank profits, defined as net interest earnings, is

$$\Pi_b = i_L(L_k + L_w) - i_D(F_k + F_w + F_s + F_m) \quad (1.21)$$

with F_m denoting financial assets of banker household. Similarly to the nonbank firms, banks make dividend payments out of the profits to their owners, i.e. banker households, and keep the rest as retained earnings. Analogously to the firm's recapitalization rate p_k , let us denote the bank's recapitalization rate by p_b . Since the retained earnings accumulate into net worth according to accounting principle, if bank equity is denoted by E_b , $\Delta E_b = p_b \Pi_b$ holds.⁴ The bank uses equity, along with debts, i.e. deposit funds, in financing loans to firms and worker households, which constitute its assets. Hence, $E_b = L_k + L_w - F_k - F_w - F_s - F_m$ holds as an accounting identity.

The banker household's income Y_m , consumption D_m , and financial assets F_m can be obtained in the following way. The banker household does not borrow and hence its consumption is financed solely by past income,

$$D_m = Y_m^{-\tau_m} \quad (1.22)$$

where τ_m denotes the banker household's consumption lag. As a result, its revenues is determined as a sum of dividend income from banks and interest earnings,

$$Y_m = (1 - p_b)\Pi_b + i_D F_m \quad (1.23)$$

⁴Throughout this dissertation, I will use net worth and equity interchangeably. To this, the literature adds capital as well and uses the three interchangeably.

remembering that p_b denotes retention ratio of the bank. The banker household's financial assets increase by the inflows of income and decrease by the outflows of consumption expenditure,

$$\Delta F_m^{+1} = Y_m - D_m \quad (1.24)$$

Now, the banker household's consumption expenditure is an additional source of demand in this extended model. Consequently, the aggregate demand in equation (1.8) is modified into

$$D = (1 - a)Z + D_w + D_s + D_m \quad (1.25)$$

where D_w , D_s , and D_m are specified in equations (1.16), (1.18), and (1.22), respectively.

Lastly, for the sake of simplicity, let us take productive capital U and commodity capital X combined together, representing nonfinancial, or real, capital as opposed to financial assets. From equations (1.5) and (1.6), it readily follows that the nonfinancial capital increases by the inflow of capital outlays Z and decreases by the outflow of produced commodities P . Denoting the nonfinancial capital by Q , its law of motion will be expressed as

$$\Delta Q^{+1} = Z - R \quad (1.26)$$

which replaces equations (1.5) and (1.6).

1.2.3 Summary of the model

The equations constituting the extended model are collected here. An assumption is added that all the variables grow at the same constant rate, g .

Real capital (stock)

$$gQ = Z - R \quad (1.26)$$

Financial assets (stock)

$$gF_k = R + p_k \Pi_k - (Z - B_k) \quad (1.14)$$

$$gF_w = Y_w - (D_w - B_w) \quad (1.17)$$

$$gF_s = Y_s - D_s \quad (1.20)$$

$$gF_m = Y_m - D_m \quad (1.24)$$

Credits (flow) and outstanding loans (stock)

$$B_k = b_k Z \quad (1.10)$$

$$B_w = b_w D_w \quad (1.11)$$

$$gL_k = B_k \quad (1.9)$$

$$gL_w = B_w \quad (1.9)$$

Net profits (flow)

$$\Pi_k = qR - (i_L L_k - i_D F_k) \quad (1.12)$$

$$\Pi_b = i_L (L_k + L_w) - i_D (F_k + F_w + F_s + F_m) \quad (1.21)$$

Household income (flow)

$$Y_w = aZ + i_D F_w - i_L L_w \quad (1.15)$$

$$Y_s = (1 - p_k) \Pi_k + i_D F_s \quad (1.19)$$

$$Y_m = (1 - p_b) \Pi_b + i_D F_m \quad (1.23)$$

Investment and consumption (flow)

$$Z = R^{-\tau_F} + p_k \Pi_k^{-\tau_F} + B_k \quad (1.13)$$

$$D_w = Y_w^{-\tau_w} + B_w \quad (1.16)$$

$$D_s = Y_s^{-\tau_s} \quad (1.18)$$

$$D_m = Y_m^{-\tau_m} \quad (1.22)$$

Aggregate demand (flow)

$$D = (1 - a)Z + D_w + D_s + D_m \quad (1.25)$$

The model consists of nineteen variables and nineteen equations. The nineteen endogenous variables appear on the left-hand side of the nineteen equations presented above. While g is introduced now as a constant, two approaches to endogenizing it will be discussed later. The other constant parameters include e , a , p_k , p_b , τ_F , τ_w , τ_s , τ_m , i_D , and i_L with $q = ae$.⁵

The model is well-determined and solutions for the nineteen variables can be obtained by solving the above nineteen equations simultaneously.⁶ These solutions can be used in measuring various variables of interest such as rate of profit, income distribution, leverage ratio, etc. The rest of this chapter will focus on the determination of profit rate and growth rate in the circuit of capital model.

1.3 Growth and profitability

In examining growth and profitability, let us first consider the case of baseline model as a reference point. It will be shown that in the model without bank credits both growth rate and profit rate collapse to zero. On the other hand, regarding the fully extended model presented in section 1.2.2 and summarized in section 1.2.3, the solution is complex and analytically intractable. Hence, in order to simplify, in the

⁵Notice that equations (1.2) and (1.3) and the associated variables and parameters, which describe the lag processes of production and realization, are omitted. Since I am consolidating productive capital and commodity capital into nonfinancial, real capital Q — see equation (1.26) — equation (1.2) is unnecessary. On the other hand, as just mentioned, there are two approaches to obtaining g endogenously from within the model. One is to presuppose the existence of sufficient volume of effective demand which guarantees a constant realization lag and the other is to explicitly account for the effective demand within the model thereby endogenizing the realization lag, in which case the latter is resolved into the spending lags such as finance lag of firms and consumption lags of households. As will be discussed below more in detail, the second approach is analytically richer in content. Hence, this chapter adopts the second approach and this makes equation (1.3) also unnecessary.

⁶As it is almost impossible to solve it manually, I relied on *Mathematica*. Interested readers can obtain *Mathematica* code for solving the model from the author upon request.

second part of this section, the extended model will be analyzed under the Classical assumption that capitalists do not consume and workers do not save.

1.3.1 Baseline model

The extended model reduces to the baseline model without finance under a set of assumptions that

Assumption 1. $b_k = 0$; $b_w = 0$; $i_D = 0$

That is, net bank credits are zero and interest earnings on financial assets are zero as well. In particular, since loans are the only asset of the banking sector in the model, the assumption of zero net bank credits effectively excludes the banking sector from the model. The solution of the model under assumption 1 is presented in appendix B.1.⁷

1.3.1.1 Growth

There are two ways to endogenously determine the steady-state growth rate in the circuit of capital model. The first approach is to presuppose that in each and every period the volume of effective demand emerges sufficiently to guarantee equilibrium in a way that maintains the realization lag at a constant level along with given production lag and finance lag. That is, the growth rate is determined by the lag structure of the circuit of capital, which, for the case of baseline model, is expressed in equations (1.2), (1.3), and (1.4).

A consecutive substitution of the three equations yields

$$(\tau_F + \tau_P + \tau_R) \ln(1 + g) = \ln(1 + p_k q)$$

⁷Throughout this chapter the model is solved with the help of software program *Mathematica*. The code can be obtained from the author upon request.

which uses the fact $Z(t) = Z(0)(1 + g)^t$. Since $\ln(1 + \alpha) \approx \alpha$ when α is sufficiently small, the above equation can be simplified into

$$g = \frac{p_k q}{\tau_F + \tau_P + \tau_R} \quad (1.27)$$

This result shows that when the existence of sufficient volume of effective demand is presupposed, the steady-state growth rate of the system is determined by the markup, recapitalization rate of the firm, and the lag structure of the circuit of capital involving the processes of production, realization, and finance.

On the other hand, the second approach to determine the endogenous growth rate is to directly measure the effective demand within the model and to impose the goods market equilibrium explicitly. Accordingly, the steady-state growth rate is determined by the requirement that the effective demand that emerges at each and every period is large enough to realize the final sales, i.e. $D = (1 + q)R$. This equilibrium condition requires that aggregate demand is at the level which realizes the final outputs at a markup q . A normalization by R rearranges the equation into $D' = 1 + q$ where D' is D normalized by R . Substituting the solution for D' presented in appendix B.1 into the equilibrium condition yields

$$\frac{(1 - p_k)q}{(1 + g)^{\tau_s}} + \frac{(1 - a)(1 + p_k q)}{(1 + g)^{\tau_F}} + \frac{a(1 + p_k q)}{(1 + g)^{(\tau_F + \tau_w)}} = 1 + q \quad (1.28)$$

Note that the effective demand in the two-class model of circuit of capital consists of the firm's investment expenditure and households' consumption expenditure and that the realization lag τ_R reflects these investment and consumption behaviors. Consequently, explicitly considering the goods market equilibrium by measuring the effective demand has an effect of resolving τ_R into the finance lag τ_F and the consumption lag of the worker household and entrepreneur household, i.e. τ_w and τ_s in the case of baseline model, and additionally into the consumption lag τ_m of banker

households in the case of extended model as will be discussed below. Consequently, τ_R is endogenized. For this reason, while τ_R appears in equation (1.27), it does not in equation (1.28); instead, τ_s and τ_w appear along with τ_F in the latter equation.

The second approach summarized in equation (1.28) does not generate the steady–state growth rate in a closed form as in the case of the first approach in equation (1.27). However, it allows to explicitly see how investment behavior of firms and consumption behaviors of the two types of households, reflected in τ_F , τ_s , and τ_w , affect growth in a way that is not possible for the first approach. Furthermore, since spending behaviors of all of those three different types of agents are different from one another, as would be reflected in the difference among the values of τ_F , τ_s , and τ_w , it matters how the capital outlays are divided between constant capital and variable capital. This division is captured by the organic composition of capital, which in the model is reflected in a . Hence, a appears only in equation (1.28). In these respects, the analysis of the steady–state growth rate by the second approach that endogenizes the realization lag is richer in content.

Most importantly, it enables to see that the central assumption of the first approach that there exists a sufficient volume of effective demand is invalid when net credits are zero. With given ranges of the parameters, i.e. $0 < q < 1$, $0 < a < 1$, $0 < p_k < 1$, $\tau_F > 1$, $\tau_w > 1$, and $g > 0$, a closer inspection of the equilibrium condition in equation (1.28) reveals that the equality holds only when $g = 0$ or when all the expenditure lags, i.e. τ_F , τ_w , and τ_s , are zero; otherwise, it will hold that

$$\frac{(1 - p_k)q}{(1 + g)^{\tau_s}} + \frac{(1 - a)(1 + p_k q)}{(1 + g)^{\tau_F}} + \frac{a(1 + p_k q)}{(1 + g)^{(\tau_F + \tau_w)}} < 1 + q \quad (1.29)$$

which implies $D < (1 + q)R$, i.e. the aggregate demand is insufficient to realize final outputs at any level of markup. Since the condition that all the expenditure lags are zero is unrealistic as it implies that revenues are spent instantaneously, it is concluded

that in the baseline model without zero net bank credits the steady–state growth rate collapses to zero, i.e. $g = 0$.

1.3.1.2 Profitability

One of the crucial issues in the study of the profit rate regards its definition. When the literature on either theoretical and empirical studies of the rate of profit defines it as a ratio between the flow of profits and the stock capital, in most cases the latter refers to real capital, or, more precisely, productive capital, which is constant fixed capital (Duménil and Lévy, 1993; Weisskopf, 1979). As highlighted in the circuit of capital model, however, it is an essence of capital that it constantly changes its form throughout its circular movement of self–valorization. Due to the lag structure of the circuit of capital, capital necessarily takes the form of financial assets and inventories as well as the form of productive capital. Hence, at any moment in time capital always exists in all of these three forms. Consequently, a more proper way to measure capital as a stock is to include them all.

In this context, using the notations of the model, the definition of the rate of aggregate profit, denoted by r , is expressed as gross profit divided by a sum of productive *and* commercial capital *and* financial capital:

$$r \equiv \frac{qR}{Q + F_k} \tag{1.30}$$

Dividing both numerator and denominator of the right hand side by R and substituting the solution for the normalized Q and F_k in appendix B.1 yields

$$r \equiv \frac{q}{Q' + F'_k} = \frac{g}{p_k} \tag{1.31}$$

The result in equation (1.31) demonstrates the rate of profit of the economy on a steady–state balanced growth path. Since the circuit of capital model is based on the

labor theory of value, equation (1.31) can be interpreted as stating that if the system is to expand at a steady–state balanced rate of g with a given recapitalization rate of p_k , the rate of profit has to be determined at a level equal to g divided by p_k . In this sense, r in (1.31) is an equilibrium rate of profit.

Note that g itself is an endogenous variable which is determined as a function of the parameters of the model as in (1.27) when the realization lag is constant or as in (1.28) when the realization lag is endogenous. Hence, a complete illustration of the determination of the rate of profit is obtained by combining (1.31) with either (1.27) or (1.28) depending on the assumption regarding the endogeneity of the realization lag. In the case of exogenous realization lag, on the one hand, the rate of profit is determined by substituting the expression of g in (1.27) into (1.31), thereby yielding

$$r = \frac{g}{\tau_F + \tau_P + \tau_R} \quad (1.32)$$

In the case of endogenous realization lag, on the other, the rate of profit cannot be obtained in a closed form as in (1.32); but by comparative statics with respect to equations (1.28) and (1.31), how r will change in response to changes in the parameters can be known. Since $g = 0$ is the case from equation (1.28), $r = 0$ follows according to equation (1.31). This leads to a conclusion that in the baseline model with zero net bank credits the growth rate and profit rate collapse to zero, i.e. $g = 0$ and $r = 0$. Again, the fact that not only the growth rate but also the profit rate is zero when net bank credits are zero is revealed only in the second approach with endogenous realization lag.

The discussion so far on profitability and growth in the circuit of capital model can be compared to a Post Keynesian model of growth and distribution. First of all, it can be readily seen that the expression in equation (1.31) is identical to the key result of the Post Keynesian model as summarized in the Cambridge equation,

which states that the rate of profit is determined by growth rate divided by the saving propensity of capitalists, i.e. $r = \frac{g}{s}$ with s denoting the saving propensity of capitalists (Pasinetti, 1962)⁸. The existing literature of the circuit of capital model approvingly quotes the Cambridge equation and highlights that the latter can also be derived from the circuit of capital model. However, the difference between the two model is insufficiently recognized and particularly the causality between r and g in the circuit of capital model literature is not always clear.⁹

First of all, both $r = \frac{g}{s}$ of the Cambridge model and $r = \frac{g}{p_k}$ of the circuit of capital model hold as an equilibrium condition. However, in the Cambridge model, on the one hand, the rate of accumulation is exogenously determined by the population growth and technical progress. g as well as s is exogenous and hence r is an endogenous variable that is determined by both g and s . Hence, the causation runs from g to r . On the other hand, in the circuit of capital model g is not a constant parameter but is an endogenous variable that is determined as in equation (1.27) in the case of exogenous realization lag or as in equation (1.28) in the case of endogenous realization lag. In either case, g is determined as a function of the model parameters.

For this reason, the causality from g to r that holds in the Cambridge model does not hold in the circuit of capital model. Rather, the Cambridge equation–*type* result of the circuit of capital model, along with the endogenous determination of g , reveals

⁸According to Pasinetti’s original interpretation, the Cambridge equation analysis is “a logical framework to answer interesting questions about what *ought* to happen if full employment is to be kept over time, more than as a behavioral theory expressing what actually happens” (Pasinetti, 1962). For an extensive study of the Cambridge equation, see Bortis (1993). After identifying three characteristics of classical growth theory as opposed to neoclassical models, Michl (2009, p.7) writes “In particular, the Cambridge theorem is a *cynosure* that provides the classical approach with distinctive explanatory powers”.

⁹For instance, an inconsistent understanding on the causality between r and g in the circuit of capital model literature is most notable in the fact that the relation between the two variables is presented as $r = \frac{g}{p}$ in one case and as $g = pr$ in the other without a clear discussion on the difference of the two different expositions.

how r and g are determined by the deep parameters of the model along an equilibrium steady-state growth path.

1.3.2 Extended model under the Classical assumption

In reference to the above discussion on growth and profitability in the baseline model without finance, let us now consider the case of the extended model with finance. To begin with, it turns out that the solution of the extended model is too complex. In order to make the rate of profit and the steady-state growth rate of the extended model more analytically tractable, the model — summarized in section 1.2.3 — is considered under a set of assumptions that

Assumption 2. $\tau_w = 0$; $p_k = 1$; $p_b = 1$; $b_w = 0$; $i_D = 0$

The first three imply the Classical assumption that capitalists do not consume and workers do not save; $\tau_w = 0$ implies that workers spend their wage income immediately in the contemporaneous period; $p_k = 1$ and $p_b = 1$ imply industrial capitalist household and financial capitalist household do not earn any revenues and hence cannot consume. In order to further simplify the model, it is supposed that only the nonbank firms borrow for production purpose; hence $b_w = 0$. Since this chapter does not aim to compare the two different types of credits, i.e. production credits and consumption credits, it is enough to consider one type of credit only. On the other hand, as bank liability is the only type of money in the model economy, it is effectively cash and hence riskless. So it is reasonable to assume that interest rate on it is zero; hence $i_D = 0$.

Eventually, parameters of the extended model under assumption 2 include q , a , τ_F , b_k , and i_L . Notice that since the two types of capitalist households, i.e. entrepreneurs' and bankers, all do not earn any revenues, their consumption lags τ_s and τ_b are effectively excluded. The solution of the extended model under assumption 2 is presented in appendix B.2. Using the solution, profit rate and growth rate of the

extended model of circuit of capital can be examined in a way similar to the case of baseline model as discussed above.

1.3.2.1 Growth

In examining growth in the extended model, I will focus on the second approach with an endogenous realization rate as it allows to specify components of the effective demand and see their growth impact. When τ_R is endogenous, the steady-state growth rate is obtained by the requirement that the volume of aggregate demand is sufficient to realize total final sales, i.e. $D = (1 + q)R$ which, by a normalization by R , is rearranged into $D' = 1 + q$, where D' denotes D normalized by R . Substituting the solution for D' presented in appendix B.2 into the equilibrium condition yields g as an implicit function Γ of the four parameters, q , τ_F , b_k , and i_L .

$$\Gamma(g; q, \tau_F, b_k, i_L) = (1 + q) \left(\frac{g}{g(1 - b_k)(1 + g)^{\tau_F} + b_k \beta_L} - 1 \right) = 0 \quad (1.33)$$

Using the Implicit Function Theorem, $\frac{\partial g}{\partial \alpha} = -\frac{\frac{\partial \Gamma}{\partial \alpha}}{\frac{\partial \Gamma}{\partial g}}$ where $\frac{\partial \Gamma}{\partial g} \neq 0$, comparative static analysis generates the following results, which are all mathematically clear and economically intuitive. First, $\frac{\partial g}{\partial i_L} < 0$: a larger interest rate leads to a lower growth rate. This result corresponds to the IS relation of the Keynesian IS-LM model.¹⁰ Second, $\frac{\partial g}{\partial \tau_F} < 0$: a faster use of own funds by industrial capitalists in financing capital outlays leads to a higher growth rate. This result is based on the assumption that capital outlays are used only for productive investment, not for financial investment. Relaxing this assumption could produce a different result which might highlight a negative impact of financial investment on growth.¹¹

¹⁰While the interest rate is taken as a constant parameter in this chapter, it will be endogenized and hence determined simultaneously with the growth rate in chapter 2.

¹¹In the early draft of this paper, I included an additional parameter for a share of financial investment in total capital outlays. A comparative statics result shows that the larger the parameter the lower the growth rate, implying a negative growth impact of financial investment.

Third, $\frac{\partial g}{\partial b_k} > 0$ if i_L is sufficiently small and $\frac{\partial g}{\partial b_k} < 0$ if i_L is sufficiently large: a larger borrowing ratio of the firm is conducive to growth only when the interest rate is sufficiently low. A more interesting result would be obtained if an intensification of financial fragility due to a rise in the borrowing ratio is explicitly modeled and an investment function that negatively responds to a rise in the financial fragility is introduced. In this case, b_k will have a nonlinear impact on g , and in order to properly capture this result, the model will need to be modified within a non-steady-state growth setting.

On the other hand, the above result regarding the sign of $\frac{\partial g}{\partial b_k}$ can be interpreted as reflecting, in some sense, such dynamics. The loan interest rate can be considered as being determined in a way that reflects financial status of a borrower, an increase in i_L reflecting an intensification of the firm's financial fragility while a fall in i_L reflecting an enhancement of the firm's financial health. In this context, when the firm is in a good financial status — reflected in a sufficiently low i_L — a more borrowing will lead to a higher growth — $\frac{\partial g}{\partial b_k} > 0$ — and when the outstanding debt of the firm is already too large, which makes it financially fragile — reflected in a sufficiently high i_L — a more borrowing will lead to a lower growth — $\frac{\partial g}{\partial b_k} < 0$.

Fourth, $\frac{\partial g}{\partial q} > 0$ if τ_F is sufficiently small and $\frac{\partial g}{\partial q} < 0$ if τ_F is sufficiently large: if own funds of the firm are used in capital outlays sufficiently fast, a larger markup is conducive to growth; otherwise, it undermines the system's rate of expansion. Since q is a product of e exploitation rate and a organic composition of capital, with a given, the former case corresponds to profit-led growth regime while the latter case to wage-led growth regime.¹²

¹²Basu (2013) also shows that the circuit of capital model allows both profit-led and wage-led growth regimes. In contrast, Dos Santos (2013) criticizes the literature on profit-led vs. wage-led growth from the perspective of the circuit of capital model.

1.3.2.2 Profitability of industrial capitalists

When finance is explicitly considered, the appropriate definition of profitability needs to be reconsidered further than merely adding inventories and financial assets in measuring capital as done in section 1.3.1.2. While such broader measure of capital is closer to the concept of capital as defined in the circuit of capital than is the conventional measure that only includes fixed capital, both are, from an accounting perspective, insufficient as they relate only to the asset side of the balance sheet and not to the liability side. If the question of how capital is financed is crucial in understanding capital, then a proper measure should account for the liability structure of the balance sheet.

Compare the following two cases; first, capital consisting of fixed capital of \$70 and inventories of \$30 financed exclusively by own funds of \$100, and second, capital consisting of the same combination of fixed capital and inventories financed by a mixture of own funds of \$50 and debt of \$50. Are the capitals in these two cases the same capital? More conventional approaches are treating the two as the same and, if not, they are at least implicitly assuming that capital is financed entirely by own funds, thereby excluding the possibility of the second case and ignoring the issue of finance. In this respect, there is an ironic similarity with the neoclassical financial theory *à la* Modigliani–Miller theorem, which states that capital structure does not matter for the value of the firm (Modigliani and Miller, 1958).

However, the recent New Keynesian literature is producing theoretical and empirical works that depart from Modigliani–Miller theorem, by drawing primarily upon the relatively recent experience of the financial turmoil in 2008 (e.g. Shin and Shin, 2011). Accordingly, leverage ratio has become a central variable in the recent macro–finance literature. Moreover, the Post Keynesian literature has a long tradition which highlights that how economic activities are financed do matter for macroeconomic performance of the economy. This tradition, in particular, has generated two related

approaches. One is monetary circuit theory by Graziani (2003) according to which production is initiated by, and hence cannot be examined without a consideration of, firms' borrowing; the other is the stock–flow consistent modeling methodology by Godley and Lavoie (2007), which sheds light on the source of funds vis-à-vis the use of funds.

Most importantly, one of the central results of the circuit of capital model, i.e. that in a steady–state setting capital accumulation could not possibly take place when net credits do not grow, makes a strong case that debts are an essential aspect of capital. Informed by these various approaches, I suggest to account for capital structure when measuring capital, i.e. subtracting liabilities from total assets.

On the other hand, regarding profits that appear in the numerator of the rate of profit, Marxian economic theory recognizes a division of capitalist class into industrial capitalists and financial capitalists and a consequent division of aggregate profits into profit of enterprise and interests between the two groups of capitalists. Furthermore, once financial assets are considered as an essential part of capital as is done in this chapter according to the logic of the circuit of capital, interest earnings need to be taken into account when measuring profit of enterprise.

Reflecting the above discussions on capital and profits, an alternative measure of the rate of profit would be as follows:

$$r_k \equiv \frac{qR - i_L L_k}{Q + F_k - L_k} \quad (1.34)$$

The denominator is equity or net worth as own funds which is defined as a difference between total assets and liabilities. The numerator is profit of enterprise which is obtained from subtracting net interest payments from aggregate profits.¹³ The resulting

¹³Notice that while net interest payments of the firm would be expressed as $i_L L_k - i_D F_k$, since $i_D = 0$ is assumed in this section $i_L L_k$ becomes net interest payments.

ratio is net return on equity, denoted by r_k , while r in equation (1.30) is the rate of aggregate profits. Norfield (2013), one of the few recent theoretical works on the rate of profit in relation to finance, suggests that return on equity is a more appropriate measure of profitability.

As a comparison, let us first measure the rate of aggregate profit of the extended model under assumption 2, using the solution for Q' and F'_k presented in appendix B.2.

$$r \equiv \frac{q}{Q' + F'_k} = \frac{g(g(1+g)^{\tau_F}(1-b_k) + i_L b_k)q}{g(1+g)^{\tau_F}(1-b_k)q + b_k(g(1+g) - i_L)} \quad (1.35)$$

The net return on equity r_k can be obtained in the same way, first by normalizing the numerator and denominator of the definition of r_k by R and substituting the solution for Q' , F'_k , and L'_k :

$$r_k \equiv \frac{q - i_L L'_k}{Q' + F'_k - L'_k} = \frac{g}{p_k} \quad (1.36)$$

While the expression for r is quite complicated, the expression for r_k is much simpler and exactly the same as the Cambridge equation-type result in (1.31) except that the rate of gross profit is replaced by the net return on equity.

In order to see the relation between the two different measures of profitability, i.e. r and r_k , the definition of r_k can be decomposed in the following way:

$$r_k \equiv \underbrace{\frac{qR}{Q + F_k}}_r \underbrace{\frac{Q + F_k}{Q + F_k - L_k}}_{\lambda_k} \underbrace{\frac{qR - i_L L_k}{qR}}_{1 - \frac{1}{\eta}} \quad (1.37)$$

where λ_k is the industrial capitalist sector's leverage defined as a ratio between total assets and equity, and $\eta = \frac{qR}{i_L L_k}$ is a Minskyan concept of margin of safety, which measures cash inflows of the firm against its cash outflows for debt service. Both leverage ratio and margin of safety are a proxy for the financial health of an entity. On the other hand, r reflects aggregate profitability of capital with the liability side of the balance sheet assumed away. Similarly to Duménil and Lévy (2004), which is

one of the first papers that studies the rate of profit with a consideration of financial variables, the above decomposition describes the net return on equity in relation to real and financial components.

Notice that r , λ_k , and η as well as r_k are all endogenous variables; they are a function of model parameters. A change in any of the parameters will bring about a change in some or all of these variables. Hence, the decomposition gives us an important insight that a change in the model parameters is transmitted to the net return on equity of industrial capitalist sector through affecting both real and financial components of profitability, i.e. the sector's gross profit rate, leverage ratio, and margin of safety.

For instance, Duménil and Lévy (2004) identify two opposing effects of debt on the rate of profit; first, negative effect by raising interest costs, and second, positive effect by decreasing net worth, i.e. denominator of the profit rate. This can be verified from the decomposition in equation (1.37). A change in the parameters that raises L_k leads to a rise in the leverage ratio λ_k , which is a positive effect, and a fall in the margin of safety, which is a negative effect operating through the third component. An ultimate result of this depends on the relative strength of the two opposing effects which is reflected in the relative change of the second and third ratios of the decomposition.¹⁴

In this context, it would be helpful to obtain an expression for λ_k and η in the same way as r in (1.35). Substituting the solution of the model presented in appendix B.2 into the definition of λ_k yields

$$\lambda_k \equiv \frac{Q' + F'_k}{Q' + F'_k - L'_k} = 1 + \frac{g(1+q)b_k}{g(1+g)^{\tau_F}(1-b_k)q - i_L b_k} \quad (1.38)$$

¹⁴For a future empirical study of the profit rate in relation to financial variables, the decomposition in (1.37) can be rearranged using the log differentiation as follows:

$$g_{r_k} = g_r + g_{\lambda_k} + g_{1-\frac{1}{\eta}}$$

where g_x is a growth rate of x .

and, similarly, an expression for η can be obtained as

$$\eta \equiv \frac{q}{i_L L'_k} = \frac{g(1+g)^{\tau_F}(1-b_k)q + i_L b_k q}{i_L b_k (1+q)} \quad (1.39)$$

where it should be reminded that in both of the above two expressions g is endogenously determined by equation (1.33).

Now, by using the expressions for r , λ_k , and η in (1.35), (1.38), and (1.39), it can be more concretely investigated how a change in the model parameters is transmitted to the net return on equity through affecting these three real and financial components.

1.3.2.3 Profitability of financial capitalists

Explicitly incorporating the banking sector and the associated financial variables allows us to examine profitability of the bank as well. The profit rate of the bank is defined as a ratio between net interest earnings, which are bank profits, and net worth of the banking sector. Since in the extended model under assumption 2, only the nonbank firms incur debts and accumulate financial assets in the form of bank deposits,¹⁵ the banking sector's total assets are L_k and its liabilities are F_k . Hence by an accounting identity, the bank equity is $L_k - F_k$. Consequently, the profit rate of the bank is obtained as

$$r_b \equiv \frac{i_L L_k}{L_k - F_k} \equiv i_L \underbrace{\frac{L_k}{L_k - F_k}}_{\lambda_b} \quad (1.40)$$

which is the net return on equity of the bank. λ_b is the financial capitalist sector's leverage ratio, i.e. bank leverage, defined as a ratio between total bank loans (total assets of the bank) and equity (bank capital). The above expression demonstrates

¹⁵More precisely, the bank also incurs debts by accepting deposits and, due to the assumption of $p_b = 1$, accumulates financial assets. But the undistributed profits of the bank are accumulated into the equity, which funds its asset holdings in loans.

that the bank profit rate is determined as the interest rate multiplied by bank leverage. For a given i_L , a change in the parameters that increases (decreases) the bank leverage will enhance (undermine) the bank profitability.

Normalizing the variables in the leverage ratio by R and substituting the solution for L'_k and F'_k presented in B.2 into the definition of the bank leverage gives

$$\lambda_b \equiv \frac{L'_k}{L'_k - F'_k} = \frac{b_k}{1 - (1 - b_k)(1 + g)^{\tau_F}} \quad (1.41)$$

where g is endogenously determined by equation (1.33). In this way, the bank profit rate can be obtained as a function of the model parameters. Particularly, it can be concretely examined how a change in the parameters is transmitted to the bank profit rate through affecting the bank leverage ratio.

1.4 The necessity of credits

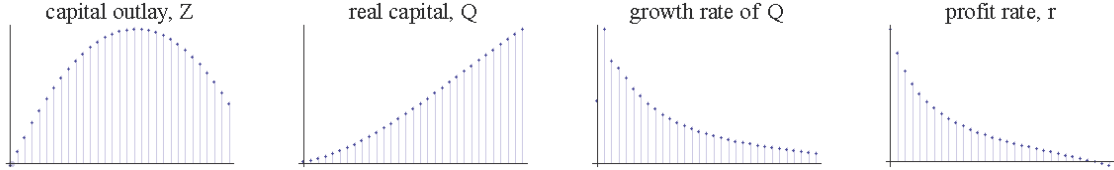
Lastly, this section investigates how the existing literature's finding on the necessity of the bank credits for growth is modified when finance is explicitly considered. For this purpose, I rely on simulation exercises which enable to examine the extended model presented in section 1.2.2 and summarized in section 1.2.3 without any simplifying assumption. Two distinct cases regarding the net bank credits will be considered. In the first case, net bank credits for both nonbank firms and worker households are taken as exogenously given. In the second case, they are endogenously determined as in equations (1.10) and (1.11).

1.4.1 Extended model with exogenous bank credits

Let us take the net bank credits for nonbank firms and worker households, i.e. B_k and B_w , as a constant parameter, replacing equations (1.10) and (1.11). First, figure 1.1 displays the simulation result for the model under assumption 1 as discussed in section 1.3.1, i.e. the baseline model with zero net bank credits, $B_k=0$ and $B_w=0$,

and no interest earnings on financial assets, $i_D=0$. The result confirms that in these conditions the growth rate and the profit rate all collapse to zero.

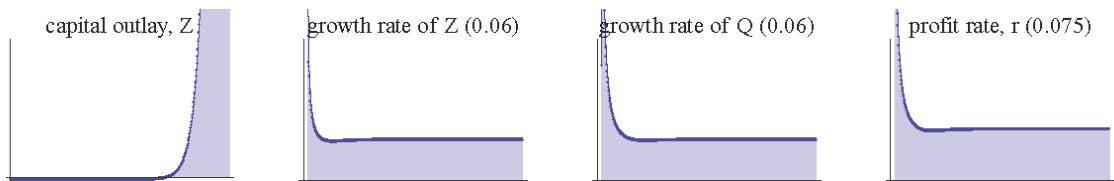
Figure 1.1: Baseline model: the model under assumption 1



(Parameter values: $a=0.3$, $q=0.3$, $p_k=0.8$, $\tau_F=2$, $\tau_w=1$, $\tau_s=3$)

Now relax the assumptions $B_k=0$, $B_w=0$, and $i_D=0$, and consider the extended model with B_k and B_w growing exponentially at 6%. Figure 1.2 shows that the steady-state growth rate of the system converges to the growth rate of net bank credit, i.e. 6%. The profit rate also converges to a positive value at 7.5% (see parentheses in the figure). These results support the main result established in the literature that exponentially growing net credits guarantee the system to be on a positive steady-state growth path. In particular, it does so in a more generalized setting with finance explicitly considered.

Figure 1.2: The extended model with bank credits exponentially growing at 6%



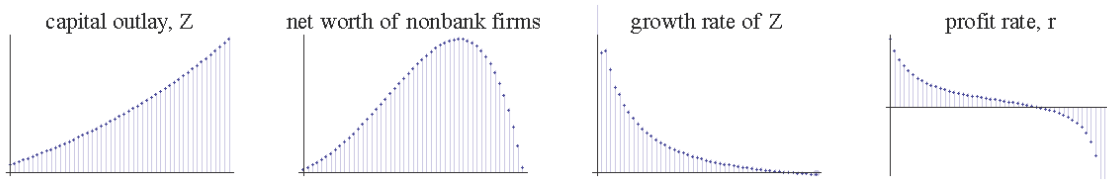
(Parameter values: $a=0.3$, $q=0.3$, $p_k=0.8$, $p_b=0.9$, $\tau_F=2$, $\tau_w=1$, $\tau_s=3$, $\tau_m=5$, $i_L=0.04$, $i_D=0.02$; numbers in parentheses are the steady-state value the variable converges to.)

On the other hand, the extended model also allows to see that under certain conditions the exponentially growing credits are not enough to guarantee a positive steady-state growth rate. Figure 1.3 illustrates that when the interest rate spread is

large at 500 basis point compared to 200 basis point in figure 1.2, both the growth rate and the profit rate collapse to zero.¹⁶

This result can be explained by the consumption lag, τ_m , of the banker household sector being large compared to the expenditure lags of the nonbank sectors. Net interest payments to the banking sector are distributed as dividends to banker households according to banks' recapitalization rate p_b which are then used for banker households' consumption expenditure according to τ_m . Consequently, when τ_m is larger than any other expenditure lags, as more values are flowed into the banking sector due to a higher interest rate spread, aggregate demand will be pressed down, which could possible generate insufficient demand and realization problem.

Figure 1.3: The extended model with bank credits exponentially growing at 6% and a large interest rate spread



(Parameter values: $a=0.3$, $q=0.2$, $p_k=0.8$, $p_b=0.9$, $\tau_F=2$, $\tau_w=1$, $\tau_s=3$, $\tau_b=5$, $i_L=0.07$, $i_D=0.02$)

1.4.2 Extended model with endogenous bank credits

Now let us consider the extended model with endogenous bank credit. This is the most generalized model in this chapter without any simplifying assumptions and it is summarized in section 1.2.3. B_k and B_w are now determined as in equations (1.10) and (1.11) instead of being given exogenously. The first interesting result that emerges from the simulation results displayed in figure 1.4 is that even when net

¹⁶The simulation results show that they even turn negative and continue to fall thereafter. This is because of the model setup that the net bank credits continue to grow at a constant rate even when the economy is suffering from a sluggish growth. In real economy, however, bank loans will stop flowing into the economy when the borrowers' balance sheet is under water with their net worth wiped out, which is happening in the second diagram in figure 1.3. This will prevent g and r exploding to negative infinity and will confine the system to a zero growth state.

bank credits are endogenous, the system smoothly converges to a steady-state. It shows that with the parameter values given below the figure, the system converges to a steady-state growth rate at 7%. By comparing this number with the steady-state value of the nonbank firm's net return on equity r_k (8.8%) along with a given parameter value of p_k (0.8), we can see that the Cambridge equation-type relation for the extended model, expressed in equation (1.36), holds; $8.8 \approx 7/0.8$.

Furthermore, using the steady-state values for the other variables, the relation between profitability and leverage for both industrial capitalists and financial capitalists as expressed in equations (1.37) and (1.40) can be confirmed. For instance, since the nonbank firm's net return on equity is expressed as $r_k = r\lambda_k(1 - \frac{1}{\eta})$, from the convergence values we can see $0.088 \approx 0.047 \times 3.76 \times (1 - \frac{1}{1.3})$.

Lastly, the existing literature's finding that exponentially growing net bank credits bring the system to a positive steady-state growth path is modified in the case of extended model with endogenous credits as well. By the logic similar to the case of extended model with exogenous credits simulated in figure 1.3, a combination of sufficiently large interest rate spread and sufficiently large spending lag of banker households undermine growth capacity of the system, moving it to zero growth. This is simulated in figure 1.5. Growth rates, profit rates, and margin of safety all keep falling while the nonbank firms' leverage explodes.

1.5 Conclusion

Growth rate and profit rate are two central variables in economic theory as highlighted in Piketty (2014)'s recent discussion, which has revived an interest in the relation between the two variables. A few digressing comments: Note that Piketty points out the fact that profit rate is larger than growth rate as the main determinant of inequality of wealth and income. In this sense, $r > g$ is at the core of Piketty's theory. However, this is suggested as a historically observed fact and there is no the-

Figure 1.4: Generalized model with endogenous bank credit

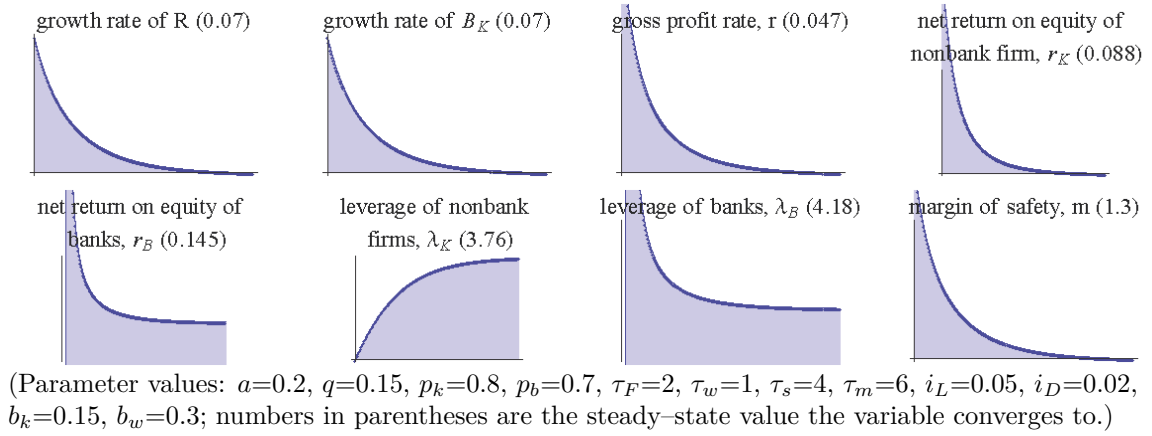
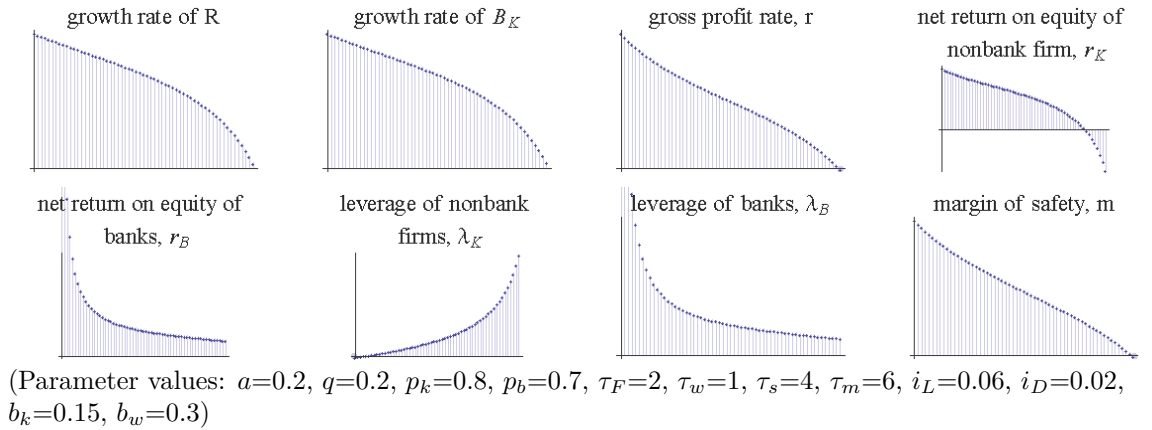


Figure 1.5: Extended model with endogenous bank credits and a large interest rate spread



oretical explanation on how r is determined and on why the inequality holds between the two variables.

A contribution of this chapter can be appreciated against this context. First of all, in this chapter the definition of the rate of profit is reconsidered in two regards. On one hand, it is suggested that when measuring capital as a denominator in the profit rate, financial assets need to be included since financial capital is one of the essential forms capital takes during the process of $M-C-M'$ the circuit of capital

and since for this reason financial capital, along with the other two forms of capital, is observed at any moment in time.

On the other hand, it is further suggested that not only the asset side but also the liability side of the balance sheet of capitalist firms needs to be considered when measuring both numerator and denominator in the rate of profit. Accordingly, the denominator is measured by equity as a difference between assets and liabilities, and the numerator is measured by gross profits minus net interest payments. This yields net return on equity. With this result, it is shown that when the gross profit rate is replaced by the net return on equity, the Cambridge equation–type result continues to hold, i.e. there is an equality between the net return on equity and the steady–state growth rate divided by recapitalization rate. Since the recapitalization rate is bounded by zero and one by definition, $r > g$ automatically follows.¹⁷

Second, profitability of nonbank firms (industrial capitalists) and that of banks (financial capitalists) are examined in the extended circuit of capital model with a banking sector explicitly incorporated. A decomposition of the net return on equity of nonbank firms illustrates that it is positively related to the gross profit rate, leverage ratio of the firm, and margin of safety. A decomposition of the net return on equity of financial capitalists shows that it is positively related to interest rate and the bank leverage.

Simulation results of the model regard the necessity of credits in the capitalist economy. First, it is shown that with zero net bank credits, both growth rate and profit rate collapse to zero, which confirms the finding in the existing literature that

¹⁷In this sense, in both the circuit of capital model and the Cambridge model $r > g$ necessarily holds by definition, and hence there is no causal relation between $r > g$ and income inequality. On the other hand, Piketty characterizes $r > g$ as the main reason for the inequality of wealth and income. Furthermore, $r > g$ is explained as associated with the nature of capitalism — it is dubbed as the ‘fundamental contradiction of capitalism’ — but also as something that can be contained and reversed by external disturbances such as wars and tax policies, which, according to Piketty, were actually the case from early 20th century until neoliberal period.

the existence of bank credits is a necessary for accumulation and growth in the capitalist economy. Second, however, it is also shown that when the model is generalized this result is modified; i.e. under the condition that interest rate is sufficiently high and the saving propensity of banker households is sufficiently large, the exponentially growing net bank credits could be not enough to guarantee a positive steady-state growth rate.

While the model explicitly formalizes a banking sector and its profit rate, it treats interest rates as exogenous. Since economic behaviors of the banking sector, as a financial intermediary, are reflected in the movement of interest rates, constant interest rates imply that the model highly simplifies the bank behavior. This issue is addressed in the next chapter where I identify demand and supply of bank credit and examine the equilibrium condition of the bank credit market along with that of the goods market, thereby endogenously obtaining the steady-state growth rate and the interest rate spread.

CHAPTER 2

LEVERAGE-LED GROWTH IN THE CIRCUIT OF CAPITAL MODEL WITH A BANKING SECTOR

2.1 Introduction

One of the notable contributions of Duncan Foley's circuit of capital model (Foley, 1982, 1986a,b) in relation to macro-finance literature lies in providing a formal proof of the necessity of credit for economic growth. It is shown that exponentially growing credit is necessary for the system to stay on a positive steady-state growth path; otherwise, the economy will systematically suffer from the lack of effective demand and realization problem. In this sense, growth in the capitalist economy described by the circuit of capital model is essentially *credit-led* or *leverage-led*. However, most of the related papers treat the credit creation simply as exogenous. This chapter is an attempt to fill this gap by presenting an extended circuit of capital model with a banking sector, which supplies credit by accommodating demand.

The extended model distinguishes demand and supply of credit within a steady-state balanced growth setting. This allows to identify growth that takes place through facilitating credit demand, which thus imposes a pressure on the credit market and hence raises interest rate, and growth that takes place through facilitating credit supply, which consequently creates the slack in the market, thereby decreasing the interest rate. Furthermore, by specifying a firm sector as the main entity that demands the credit and a banking sector as an intermediary that borrows in order to lend, the model also allows to compare between a growth led by the firm's leverage and a growth led by the bank's leverage. In this setting, the firm leverage-led growth

and the bank leverage-led growth are compared in regard to two respects; i) whether they face some growth limit and ii) whether they take place through stimulating credit demand or credit supply. The main findings are the following.

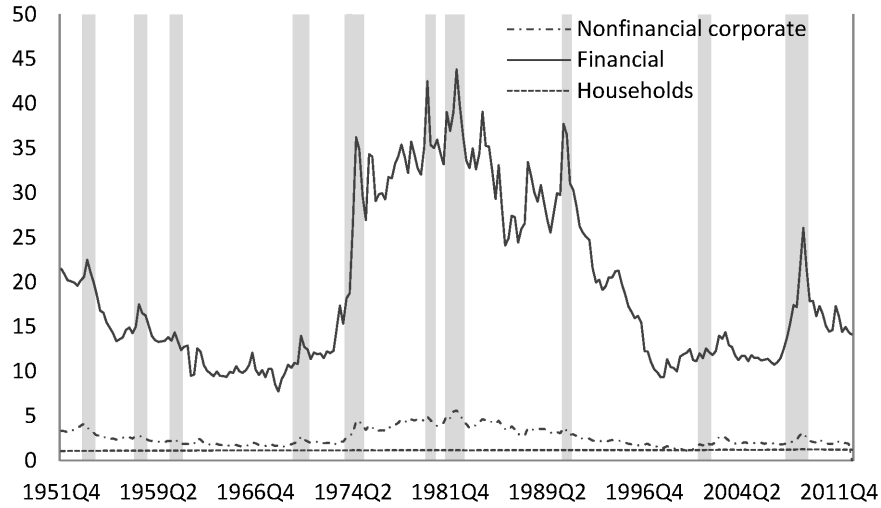
On the one hand, while growth is unbounded when led by the firm sector leverage, it takes place by stimulating excess demand for bank credit, thereby tightening the credit market and thus pushing up the interest rate alongside the growth rate. On the other hand, when growth is led by the banking sector leverage, it faces some limit. But the credit market does not always get tightened as is the case for the firm leverage-led growth; it depends on the status of the banking system. When leverage ratio of the banking sector is high, reflecting a developed financial technology, the bank leverage-led growth produces the slack in the market, thereby pressing down the interest rate, although this is not the case when the banking sector leverage is low, reflecting an absence of developed financial technology.

This chapter compares to two strands in post-Keynesian literature that study various types of growth regime. First, there are papers that identify wage-led vs. profit-led growth regimes by focusing on the relation between income distribution and growth (Bhaduri, 2008).¹ In the wage-led growth, the economy grows as the wage share increases whereas in the profit-led growth the economy grows as the profit share increases. However, this categorization omits financial aspects of growth. The second strand of papers, on the other hand, focuses on the role of debt in growth (Hein, 2007; Nishi, 2012). They distinguish between debt-led growth regime, where debt stimulates capital accumulation, and debt-burdened growth regime, where debt restrains capital accumulation.

Relying on Minsky's works (Minsky, 1975, 1982, 1986) that focus on the nonbank firm's indebtedness or addressing the issue of stagnant trend of real wage, which trig-

¹In Marxian tradition, (Basu, 2013) demonstrates that the circuit of capital model can also possibly allow the wage-led and profit-led growth regimes.

Figure 2.1: Sectoral leverage ratio



(Data source: the Flow of Funds Z.1 Release of the FRB.)

ged a spike in consumption debt, the main focus in the second categorization is the indebtedness of the nonbank sectors, either nonbank firms or households. However, historical data in figure 2.1 show that leverage of the banking sector is not only incomparably higher but also much more volatile than that of the other nonbank sectors; leverage ratio is defined as total assets divided by equity.

Moreover, there is an ongoing debate on whether credit growth and credit crunch during the periods before and after the recent financial crisis were due to demand-side or supply-side factors, and a number of papers have found that the supply-side factors were the main driver (Holton et al., 2012; Balke and Zeng, 2013), although some papers have also found a mixed evidence that depends on whether it be normal times or crisis times (Jiménez et al., 2012). In any case, this debate as well as the the above flow of funds data highlight the importance of considering financial behavior of the bank and its leverage in relation to those of the other sectors. Motivated by this, I extend the baseline circuit of capital model to explicitly incorporate the banking sector, and introduce bank leverage as a new parameter.

Some of the developments made in the circuit of capital model literature regarding the necessity of credit in the macro economy are also worth noting. With a different modeling approach, Kotz (1991) derives a stronger result according to which the new capital investment must be financed entirely by credit. Basu (2013) shows that the steady-state growth rate is negatively related to the share of consumption credit in total net credit. Dos Santos (2011) further demonstrates that consumption credit contributes to intensifying credit risk by lowering total wage and profit incomes relative to interest payments. Loranger (1989) analyzes a divergence between capital investment and financial investment, which can have an inflationary effect. Park and Basu (2012) add debt and interest into the baseline circuit of capital model and derives an extended Cambridge equation with finance, which establishes a negative correlation between steady-state growth rate and interest rate.

While examining macroeconomic consequences of credit in various respects, these papers leave out factors that determine credit creation, simply by taking it as exogenous. On the other hand, Satoh (2012) is a notable exception that adds the circuit of bank capital to the baseline model, which is the circuit of industrial capital. A negative relation between growth rate and interest rate is derived from the circuit of industrial capital and a positive relation from the circuit of bank capital. From these, both the growth rate and the interest rate are endogenized. My model closely resembles this part of Satoh's.

However, there are important differences between the two. First, contrary to Satoh's model, my model is demand-led and derives the two growth rate-interest rate relations explicitly from the equilibrium condition in goods market and that in credit market. Second, which follows from the first, while the growth rate-interest rate relation derived from the industrial capital circuit is negative in Satoh's model, in my model, it is inverted-U shaped, thereby generating nonlinearity. Consequently, in Satoh's model, financial innovation, for example, can possibly enhance growth and

at the same time reduce interest rate, whereas in my model the financial innovation will have that result only in the economy with a developed financial system; otherwise growth will be accompanied by a rise in the interest rate.

The remainder of the chapter is organized as follows. Section 2.2 presents an extended circuit of capital model. In section 2.3, two relations between growth rate and interest rate are derived from the equilibrium condition in the goods market and that in the credit market. In section 2.4, firm leverage-led growth and bank leverage-led growth are examined with comparative dynamic analyses. Section 2.5 summarizes the main results of the model and applies them in understanding the contrast between two historical regimes, i.e. the so-called 'golden age' and neoliberal regimes.

2.2 Extended model of the circuit of capital

The model economy consists of four sectors, i.e. firms, households, private banks, and central bank. However, the classical assumption will be adopted that workers do not save and capitalists do not consume. Accordingly, the household sector, either worker or capitalist, does not play a big role in the model. The central bank will also be excluded later by making a set of simplifying assumptions when conducting an equilibrium analysis in section 2.3 and 2.4.

The model is in a discrete-time framework. Similarly to chapter 1, time description appears as a superscript — instead of subscript — of notations and, for simplicity, t is omitted. For instance, with Π_K denoting net profits of the nonfinancial firm sector, the variable in time t and that in time $t - \tau$ are each denoted by Π_K and $\Pi_K^{-\tau}$ instead of Π_K^t and $\Pi_K^{t-\tau}$. All the endogenous variables are denoted by upper case letters; exceptions include profit rate of firms r_K , profit rate of banks r_B , leverage ratio of firms λ_K , steady-state growth rate g , and interest rate spread ω . In particular, the latter two will be first introduced as a constant parameter and will later be endogenized. A list of the notations can be found in appendix C.

2.2.1 The firm sector and its profitability

Z denotes the flow of investment on capital outlays, that include constant capital and variable capital. a denotes organic composition of capital defined as a ratio of variable capital over capital outlays. Accordingly, $(1 - a)Z$ is invested on constant capital (raw materials, equipment, and factory, etc.) and aZ on variable capital (workers), which is the source of wage income of worker households. All of wage income is spent in the contemporaneous period without saving. The flow of sales revenue measured at cost is denoted by R .

The flow of investment on capital outlays undergoes processes of production and realization. Time taken for these processes is called turnover time as a sum of production time and realization time. As a consequence, during the turnover time, the stock of inventories builds up, consisting of means of production and finished/unfinished commodities. It increases by the inflow of capital outlays and decreases by the outflow of sales revenue measured at cost. Denoting the inventories stock by Q , its law of motion is obtained as

$$\Delta Q^{+1} = Z - R \quad (2.1)$$

where Δ refers to an increment of a variable from the previous period to the current one.

Besides capital assets, the firm has financial assets and liabilities. In correspondence, interest earnings and payments are made, and net interest payments are subtracted from aggregate profits. What results is net profit, or profit of enterprise, denoted by Π_K . There is no dividend payments to capitalist household. Hence net profits are identical to retained earnings and are a source of funding for investment on capital outlays. Similarly to the case of production and realization processes, investment takes time, i.e. it takes time for capitalists to make investment decisions. This is investment time, or investment lag, denoted by τ . Firms have an additional source of investment funding, i.e. net bank credit, which is used in a contemporaneous

ous period for a capital investment purpose. Let us denote the net bank credit by B . In this setting, the flow of investment can be expressed as

$$Z = R^{-\tau} + \Pi_K^{-\tau} + B \quad (2.2)$$

which states that investment is financed by the past flows of net revenues, i.e. R and Π_K of τ periods ago, and the current net borrowing.

By the logic similar to the case of inventories stock in equation (1.26), the law of motion for the stock of financial assets and liabilities can be established. On the asset side of the balance sheet, due to the investment lag the stock of financial assets build up, increasing by the inflow of net revenues while decreasing by the outflow of investment net of bank credit. F denotes the financial asset stock of the firm sector. Then its law of motion be can expressed as

$$\Delta F^{+1} = R + \Pi_K - (Z - B) \quad (2.3)$$

Suppose firms make financial asset portfolio decisions between non-interest-bearing central bank liability and interest-bearing private bank liabilities. A fraction of financial asset is held in the central bank liability. With this fraction denoted by δ ($0 \leq \delta \leq 1$), δF and $(1 - \delta)F$ express, respectively, bank deposit and cash balance of the firm sector.

On the liability side, firms have the stock of loans which accumulates by the net bank credit:

$$\Delta L^{+1} = B \quad (2.4)$$

Whereas B is treated as exogenous in the literature (Foley, 1982; Basu, 2013), this chapter attempts to endogenize it. In doing so, I follow Dos Santos (2014) in determining B as a share, denoted by b , of investment,

$$B = bZ \tag{2.5}$$

b , a borrowing ratio, is different from the conventional measure of leverage as a ratio between total assets and equity. But it will be seen below, in proposition 2, that b is positively related to the firm sector's leverage. At any rate, B can be conceived as the *realized* net bank credit, given the firms' borrowing ratio b . In section 3.4.5, the determination of B is analyzed at a more concrete level by identifying demand and supply of bank credit.

Regarding the profitability of the firm sector, aggregate profits, on the one hand, are determined as a markup over production cost. Denoting markup by q , profits can be expressed as qR . By definition, markup is determined as a produce of the rate of exploitation, denoted by e , and by the organic composition of capital k ; hence $q = ek$ holds. On the other hand, earnings and costs involved in the financial assets and liabilities are measured according to the interest rate for bank liabilities and the interest rate for bank loans. Suppose the bank loan interest rate is determined as a spread over the rate on bank liabilities.² Denoting the spread and the interest rate on bank liabilities by ω and i , respectively, the net interest payments of the firm sector will be expressed as $(i + \omega)L - i(1 - \delta)F$. Consequently, its net profits are determined as

$$\Pi_K = qR - \left((i + \omega)L - i(1 - \delta)F \right) \tag{2.6}$$

When finance is explicitly considered as in this model, an appropriate measure of profitability is return on equity, defined as a ratio between net profits and net worth or equity (Norfield, 2013). The firm sector's return on equity, denoted by r_K , is obtained as

²In the recent New Keynesian literature, various attempts are made to include financial frictions into a DSGE model. One approach is to introduce multiple interest rates and hence interest rate spread, which reflects costs of intermediation (Woodford, 2010).

$$r_K = \frac{\Pi_K}{Q + F - L} = g \quad (2.7)$$

where the last equality easily results from substituting the expression for Q , F , and L — obtained by equations (2.1), (2.3), and (2.4) — into the definition of r_K . The result in (2.7) corresponds to the Cambridge equation–type result derived in the circuit of capital model literature.³ However, equation (2.7) is more generalized as it is obtained with a consideration of finance. Hence, the difference is that the gross profit rate in the original result is replaced by the net return on equity. In either case, the main thrust of the Cambridge equation–type result in the circuit of capital model is that a positive correlation between growth and profitability emerges in long–run.

2.2.2 The banking sector and its profitability

Banks supply credit by issuing liabilities.⁴ There are two types of private bank liabilities; deposits which are subject to reserve requirement and publicly insured and non–deposits which are not, such as money market mutual funds. It is assumed that no interest is paid on the central bank liabilities. Hence, the private banks have an incentive to increase the share of non–deposit liabilities in their total liabilities through financial innovations. This share is denoted by n .

On the asset side of balance sheet, the banks hold loans extended to firms and required reserves, which are obtained in the open market transactions with the central bank by selling a portion of the outstanding loans. In this way, some of the bank loans are shifted to the central bank balance sheet. This volume is denoted by L_C . Consequently, with L denoting the total outstanding bank loans, the loans held in the private bank balance sheet would be $L_B = L - L_C$. On the other hand, since the

³Note that recapitalization rate is assumed to be one.

⁴Basically, the banking sector combines private banks and central bank. But later, for the simplification, the central bank will be assumed away. Therefore, to avoid confusions, it will be convenient to conceive ‘the bank’ or ‘the banking sector’ without any qualification as referring to the private bank.

deposits are $(1 - n)(1 - \delta)F$, denoting the reserve requirement ratio by γ , the amount of required reserves is $\gamma(1 - n)(1 - \delta)F$.

Two simplifying assumptions for the central bank balance sheet are made. Since the central bank keeps part of the banks loans as a result of open market operation, some of the borrowers' interest payments accrues to it. First, it is assumed that the values transferred to the central bank as interests are permanently parked in its net worth rather than being distributed or spent somehow. Second assumption is that the central bank's earnings are all used in financing its position in international reserves, which entails that the central bank net worth equals some type of international reserve such as gold or a key currency. Consequently, from the balance sheet identity for the central bank it follows

$$L_C = \gamma(1 - n)(1 - \delta)F + \delta F \quad (2.8)$$

the right hand side of which expresses the monetary base, a sum of reserves and currency held by the firms.⁵

Equation (2.8) allows to compute L_B from its definition $L_B = L - L_C$ as

$$L_B = L - \gamma(1 - n)(1 - \delta)F - \delta F \quad (2.9)$$

Furthermore, net worth or equity of the private banking sector, denoted by E_B , is, by definition, $E_B = L_B + \gamma(1 - n)(1 - \delta)F - (1 - \delta)F$, remembering that the second and third term on the RHS expresses reserves and bank liabilities, respectively. Using (2.9), it can be shown that the expression for the bank equity is reduced to

$$E_B = L - F \quad (2.10)$$

⁵Denoting interest reserves by R_I and the central bank equity by E_C , the balance sheet identity of the central bank is expressed as $L_C + R_I = \gamma(1 - n)(1 - \delta)F + \delta F + E_C$. Equation (2.8) is obtained from this identity under the second assumption $R_I = E_C$.

In order to avoid a degenerate case of nonpositive bank equity the following assumption is made.

Assumption 3. $L > F$

Banks earn interests on the outstanding loans held in their balance sheet and pay interests on their liabilities. The net interest earnings of the private banking sector constitute bank profits, which are denoted by Π_B .

$$\Pi_B = (i + \omega)L_B - i(1 - \delta)F \quad (2.11)$$

Note that while the total interest payments by the firms are $(i + \omega)L$ the private banking sector obtains only $(i + \omega)L_B$. The rest accrues to the central bank.

Similarly to the case of the firm sector, suppose that the entire bank profits are retained without any dividend payment. In this context, as a measure of bank profitability, the banking sector's return on equity can be obtained as $r_B = \frac{\Pi_B(t)}{E_B}$, which then, using equations (2.9), (2.10), and (2.11), can be expressed as

$$r_B = \frac{L(i + \omega) - F(\gamma(1 - n)(1 - \delta)(i + \omega) + i + \delta\omega)}{L - F} \quad (2.12)$$

A complete solution for r_B would be obtained only when the solution for L and F is computed first, as will be discussed below, and then substituted into equation (2.12). However, under assumption 3, equation (2.12) itself is useful in examining how key financial variables of the model affect bank profitability. For instance, $\frac{\partial r_B}{\partial \omega} > 0$ and $\frac{\partial r_B}{\partial n} > 0$ can be easily verified which imply that the interest rate spread and financial innovation have a positive impact on bank profitability. In particular, financial innovation in this model enhances the bank's return on equity by helping increase the share of its non-deposit liabilities that are not subject to regulatory constraints.⁶

⁶Note that the regulatory constraint in the model is reflected the reserve requirement.

2.3 Equilibrium condition

In examining an equilibrium of the model, so as to make a model analysis tractable, it is assumed that

Assumption 4. $\gamma = 0$; $i = 0$; $n = 0$; $\delta = 0$

Assumption 4 has an effect of assuming the central bank away from the model. Accordingly, the model economy becomes a pure credit economy without outside money, i.e. the private bank liabilities become effectively cash. Hence, banks do not have to pay interest on their liabilities, $i = 0$; the public's preference for outside money is zero, $\delta = 0$; so there is no reasons for the bank to hold reserves and thus the reserve requirement ratio is zero, $\gamma = 0$; consequently, there is no distinction between reservable and non-reservable bank deposits, and hence $n = 0$. Under these assumptions, there is no transfer of bank loans to the central bank, hence $L_C = 0$ follows. The total outstanding loans will be held in the private bank balance sheet, $L_B = L$, which reflects the size of the bank balance sheet since the bank does not hold reserves anymore. On the other hand, the entire financial assets, F , of the firm sector are held in bank liability.

It is assumed that all variables exponentially grow at the same and constant rate g . In this steady-state balanced growth framework, the reduced model that emerges under assumption 4 consists of seven equations, i.e. (2.1), (2.2), (2.3), (2.4), (2.5), (2.6), and (2.11), and eight endogenous variables, i.e. R , Z , Q , F , L , B , Π_K , and Π_B with constant parameters including steady-state growth rate g , markup q , finance (investment) lag τ , the firm sector's borrowing ratio b , interest rate spread ω ; later, bank leverage will be introduced and g and ω will be endogenized. Normalizing the variables by R , the flow of final sales at cost, renders the model well-determined. The solution of the model is presented in appendix D.⁷

⁷The solution is obtained by simultaneously solving the seven equations with the help of *Mathematica*. Interested readers can obtain *Mathematica* code from the author upon request.

In this section, goods market and bank credit market are analyzed in turn, and from the equilibrium condition of each, two parameters g and ω are endogenized. In the analysis of goods market, ω is taken as exogenous while in the credit market g is taken as given. In this sense, the equilibrium of each of these markets is partial. A general equilibrium of the model is obtained by combining the two. Substituting the consequent solution for g and ω into the solution in appendix D yields a complete solution of the model.

2.3.1 Goods market equilibrium

The model economy is demand-led; the flow of final sales is financed by the aggregate demand. Denoting the aggregate demand by D , an equilibrium in goods market is obtained as

$$(1 + q)R = D \tag{2.13}$$

remembering that $(1 + q)R$ is total sales revenue. Due to the classical assumption that workers do not save and capitalists do not consume, the aggregate demand in this economy consists of firms' investment on means of production and worker households' consumption expenditure, which equals wage income. Hence,

$$D = (1 - k)Z + kZ = Z \tag{2.14}$$

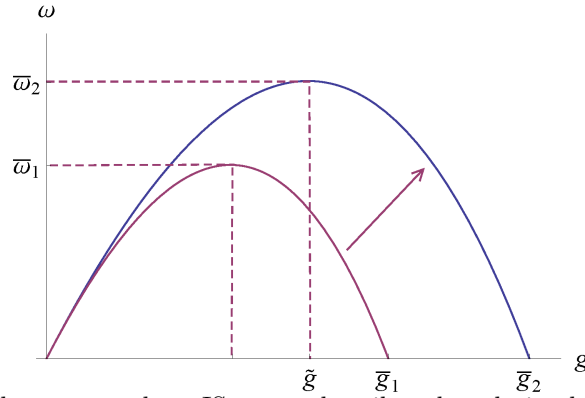
Finally, it follows from the above two equations that

$$(1 + q)R = Z \tag{2.15}$$

Using the solution for Z (see appendix D), equation (2.15) can be expressed as an implicit function Γ with g chosen as an endogenous variable,

$$\Gamma(g; \omega, b, q, \tau) = (1 + q) \left(1 - \frac{g}{g(1 - b)(g + 1)^\tau + b\omega} \right) = 0 \tag{2.16}$$

Figure 2.2: GM curve



(Note: GM curve, which corresponds to IS curve, describes the relation between growth rate g and spread ω as an equilibrium condition of goods market. It is shown that the GM curve is inverted-U shaped. It scales out as τ decreases or b increases.)

which can be solved for ω ,

$$\omega = \frac{g - g(1 - b)(1 + g)^\tau}{b} \quad (2.17)$$

Equation (2.16) or (2.17) can be called GM curve analogously to IS curve in the textbook Keynesian model (GM indicating goods market equilibrium). In the $g-\omega$ space, the GM curve exhibits an inverted-U shape in a plausible range for g , i.e. $0 < g < 1$, so that the curve has a unique local maximum within that range. This is displayed in figure 2.2. The equilibrium relation between g and ω is positive when the growth rate lies below g that maximizes ω , after which it turns negative.⁸

The nonlinearity of the GM curve can be understood in the following way. First, using the solution for B' , which is net bank credit normalized by R , it can be seen that B' is negatively related to ω , i.e. $\frac{\partial B'}{\partial \omega} < 0$. Second, credit has two opposite effects on the firms' profits, which are the source of growth (see the equality between growth and

⁸The slope of the GM curve is obtained as $\frac{\partial \omega}{\partial g} = \frac{1 - (1 - b)(1 + g(\tau + 1))(1 + g)^{\tau - 1}}{b}$. From this, it is easy to see that $\frac{1}{(1 + g(\tau + 1))(1 + g)^{\tau - 1}} > 1 - b$, which is identical to $\frac{\partial \omega}{\partial g} > 0$, requires, for any given b and τ , $g < \tilde{g}$ where \tilde{g} satisfies $\frac{1}{(1 + \tilde{g}(\tau + 1))(1 + \tilde{g})^{\tau - 1}} = 1 - b$. On the contrary, $\frac{1}{(1 + g(\tau + 1))(1 + g)^{\tau - 1}} < 1 - b$, i.e. $\frac{\partial \omega}{\partial g} < 0$, requires $g > \tilde{g}$ for any given b and τ .

profitability in equation (2.7)). On the one hand, credit boosts investment (equation (2.2)), which leads to a rise in growth and hence profitability (equation (2.7)). On the other hand, an increase in the credit elevates debt accumulation (equation (2.4)) and hence financial burden, which negatively affects profitability (equation (2.6)).

Now, in the economy characterized by weak profitability and sluggish growth which corresponds to the left half of the GM curve, a rise in the interest payment caused by an increase in the outstanding loan will be of an excessive financial burden, which would offset the positive effect of taking more bank credits. In this circumstance, a less bank credit, which should result from a higher interest rate spread, will have an effect of facilitating demand, and in order to maintain an equilibrium, therefore, the growth rate of the system has to be higher; hence, the higher the spread, the higher the growth.

In contrast, in the economy characterized by robust profitability and strong growth, which corresponds to the right half of the GM curve, the positive effect of borrowing dominates the negative effect. In this case, a lower interest rate spread, through facilitating the firm's borrowing, will have an effect of stimulating demand, and in order to maintain in equilibrium the growth rate of the system has to be higher; hence, the lower the spread, the higher the growth. Overall, in a weak growth regime the equilibrium relation between growth and interest rate spread is positive and in a strong growth regime the relation is negative; hence the inverted-U shaped GM curve.

Figure 2.2 also displays the effect of changes in the parameters. Using equation (2.17) it can be easily shown that the curve scales out to the upper-right direction with the origin as a pivot when the firms increase the borrowing ratio, raising b , or when they invest more swiftly, decreasing τ .⁹ On the other hand, markup q does not have any impact on the GM curve. This is because of the classical assumption adopted

⁹Formally, it can be shown from (2.17) $\frac{\partial \omega}{\partial b} = \frac{g((1+g)^\tau - 1)}{b^2} > 0$ and $\frac{\partial \omega}{\partial \tau} = -\frac{(1-b)g(1+g)^\tau \log(1+g)}{b} < 0$.

in the model that worker households do not save. Due to this assumption, in this demand-led model, the realization problem associated with consumption spending does not emerge.

2.3.2 Bank credit market equilibrium

In analyzing the bank credit market, as a first approximation, financial frictions documented in the finance literature of the last two decades or so will be assumed away. Accordingly, let us suppose the market clears through a change in the interest rate spread. In this context, the equilibrium condition of the bank credit market provides a closure of the model to endogenize ω . Recall from section 2.2.1 that net bank credit is determined as $B = bZ$. Suppose the demand for bank credit is fully accommodated by banks. Denoting the demand for bank credit by B_D , it holds $B = B_D$ and consequently

$$B_D = bZ \tag{2.18}$$

Normalizing both sides of the equation by R and substituting the solution for Z' yields

$$B'_D = \frac{bg(1+q)}{g(1-b)(1+g)^\tau + b\omega} \tag{2.19}$$

where prime implies the normalization by R . In the B' - ω space, the demand curve is downward sloping as in figure 2.3.¹⁰ From the solution for Z' , it is known that $\frac{\partial Z'}{\partial \omega} < 0$, i.e. a larger borrowing cost lowers investment. When this result is combined with equation (2.18), it is readily seen that a higher ω lowers Z' , which in turn leads to a smaller B'_D ; hence the downward sloping demand curve.

In correspondence to the demand for bank credit B_D , the supply of bank credit can be identified both in flow and stock terms, denoted by B_S and L_S , respectively. By definition, it follows

¹⁰The demand curve can be represented by an inverse function of (2.19) as $\omega = \frac{g(1+q)}{B'_D} - \frac{g(1-b)(1+g)^\tau}{b}$.

$$\Delta L_S^{+1} = B_S \tag{2.20}$$

In order to specify the supply of bank credit, let us consider bank capitalization and introduce the banking sector's leverage ratio as a new parameter. These will reflect lending capacity of the banking system and credit availability in the economy.

First, bank equity, or bank capital, is a basis for the bank's lending capacity. Since it was assumed that banks do not distribute dividends, by the accounting principle, the bank equity grows by the inflow of bank profits.

$$\Delta E_B^{+1} = \Pi_B \tag{2.21}$$

Second, the banking sector's leverage ratio is measured as a ratio between the supply of loan stock over the bank equity and is denoted by $\lambda = \frac{L_S}{E_B}$. Note that in this definition L_S replaces its realized counterpart L . In this way, the determination of the supply of loan stock is highlighted,

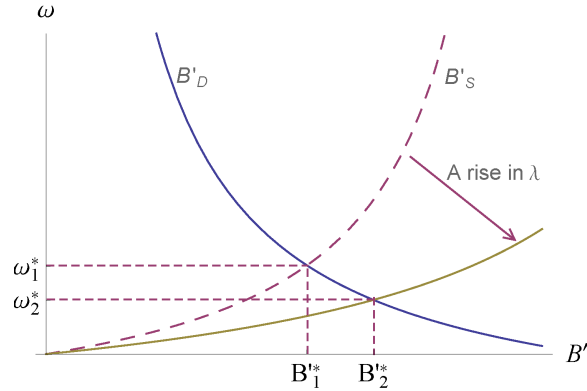
$$L_S = \lambda E_B \tag{2.22}$$

i.e. by the bank equity multiplied by bank leverage. By implication, λ is a measure of the leverage ratio the banking sector is *willing*, and *able*, to achieve rather than the leverage actually *realized*. Hence, λ is a parameter that reflects a *potential* lending capacity of the banking sector. In equilibrium, however, the potential bank leverage and realized bank leverage become identical with each other, i.e. $\lambda = \frac{L}{L-F}$, which can be verified by using the solution for L and F .

In the steady-state balanced growth framework, using the accounting identities in (2.20) and (2.22), the expression for bank capitalization in (2.21) is rearranged into

$$B_S = \lambda \Pi_B \tag{2.23}$$

Figure 2.3: Bank credit market equilibrium



(Note: B'_D and B'_S are demand and supply of bank credit normalized by the flow of final sales at cost R . The supply curve pivots out when the bank leverage rises. As a consequence the equilibrium spread falls and the equilibrium bank credit rises.)

which is one of the key equations of the model. It states that the bank credit supply is determined by bank profits multiplied by a bank leverage. Normalizing both sides of equation (2.23) and substituting the solution for Π_B in appendix D into the equation yields

$$B'_S = \frac{b\lambda(1+q)\omega}{g(1-b)(1+g)^T + b\omega} \quad (2.24)$$

In the $B'-\omega$ space, the supply curve is positively sloped as in figure 2.3.¹¹ From the solution for Π'_B in appendix D, it is verified that $\frac{\partial \Pi'_B}{\partial \omega} > 0$ holds, i.e. the interest rate spread has a positive impact on bank profits. When this result is combined with equation (2.23), it is readily seen that a higher ω raises Π'_B , which leads to a larger B'_S ; hence the upward sloping supply curve.

The demand and supply of normalized bank credit in (2.19) and (2.24) determine ω and B' in equilibrium, with g given along with the other parameters, q , τ , b , λ . Proposition 1 summarizes the analysis of this partial equilibrium.

¹¹The supply curve can be represented by an inverse function of equation (2.24) as $\omega = \frac{g(1-b)(1+g)^T B'_S}{b(\lambda(1+q) - B'_S)}$.

Proposition 1. A partial equilibrium and comparative static analysis of the bank credit market:

- i) $B'^* = \frac{\lambda b(1+g)}{\lambda(1-b)(1+g)^{\tau+b}}, \omega^* = \frac{g}{\lambda}$
- ii) $\frac{\partial B'^*}{\partial g} > 0, \frac{\partial B'^*}{\partial q} > 0, \frac{\partial B'^*}{\partial \tau} < 0, \frac{\partial B'^*}{\partial b} > 0, \frac{\partial B'^*}{\partial \lambda} > 0$
- iii) $\frac{\partial \omega^*}{\partial g} > 0, \frac{\partial \omega^*}{\partial q} = 0, \frac{\partial \omega^*}{\partial \tau} = 0, \frac{\partial \omega^*}{\partial b} = 0, \frac{\partial \omega^*}{\partial \lambda} < 0$

First of all, the comparative static analysis results in (ii) and (iii) are mathematically evident and economically intuitive.¹² For instance, as shown in figure 2.3, a rise in λ results in the supply curve pivoting clockwise. A higher bank leverage implies an enhancement of the banking system's lending capacity which allows to supply a given amount of credit at a lower interest rate. A consequence of this is a rise in the normalized bank credit, $\frac{\partial B'^*}{\partial \lambda} > 0$, and a decrease in the spread, $\frac{\partial \omega^*}{\partial \lambda} < 0$.

The other comparative static analysis results can be understood in the similar way. The results in (ii) are intuitively clear. Higher growth, larger markup, faster turnover, higher borrowing ratio of firms, and higher leverage of banks all increase the amount of net bank credit. The results in (iii) show that a change in the firm sector's parameters, i.e. g , τ , and b , has no impact on ω^* while they affect B'^* . This is because these parameters impart an impact on both the demand and supply of bank credit in the same degree so that there is no price effect but only quantity effect.

Most importantly, the expression for the equilibrium spread in (i) describes the relation between g and ω as an equilibrium condition for the bank credit market. First, note that ω^* depends only on λ , given g , i.e. an equilibrium spread depends on the leverage ratio the banking sector is willing, and able, to achieve. Second, in correspondence to the GM relation which describes the g - ω relation as an equilibrium condition for the goods market, let us call the equation $\omega = \frac{g}{\lambda}$ the BM relation

¹²These are comparative *static* analysis in the sense that the growth rate is taken as constant. Later, in a comparative *dynamic* analysis in proposition 1 g will be endogenized, and some of the results in (ii) and (iii) will be modified accordingly.

analogously to LM curve in the textbook Keynesian model (with BM standing for bank credit market equilibrium).

Lastly, in comparison to the bank leverage being a constant parameter, the firm sector's leverage is obtained endogenously as $\lambda_K = \frac{Q+F}{Q+F-L}$ by definition. It will be informative to see how the firm sector leverage is related to the model parameters. See proposition 2

Proposition 2. With $\lambda_K = 1 + \frac{gb(1+g)}{gq(1-b)(1+g)^\tau - b\omega}$, $\frac{\partial \lambda_K}{\partial g} < 0$, $\frac{\partial \lambda_K}{\partial q} < 0$, $\frac{\partial \lambda_K}{\partial \tau} < 0$, $\frac{\partial \lambda_K}{\partial b} > 0$, and $\frac{\partial \lambda_K}{\partial \omega} > 0$ hold.

The expression for λ_K is yield by using the solution for Q' , F' , and L' in appendix D. On the other hand, the partial derivative results can be easily proved and intuitively evident. A stronger growth of the economy and a larger markup lead to a smaller leverage of the firms, whereas a faster investment (smaller τ) and a more reliance on debt financing in investment (larger b) raises the firm leverage.

2.3.3 General equilibrium

The GM relation and the BM relation, which are reproduced below, constitute two characteristic equations of the model, thereby allowing g and ω to be determined endogenously.

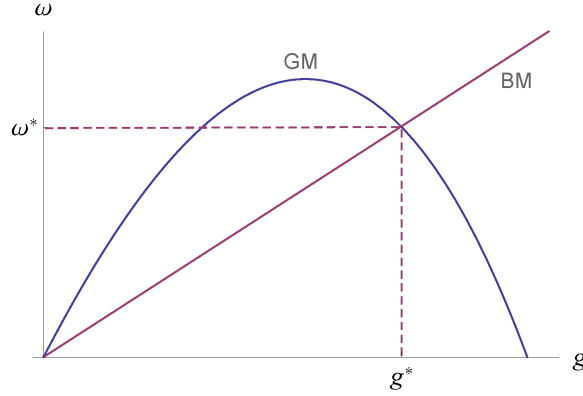
$$\omega = \frac{g - g(1-b)(1+g)^\tau}{b} \quad (\text{GM relation})$$

$$\omega = \frac{g}{\lambda} \quad (\text{BM relation})$$

While the GM curve is inverted-U shaped, the BM curve is linear and positively sloped. An equilibrium of the model is obtained at the intersection of the two curves.¹³ This is stated in proposition 3 and illustrated in figure 2.4.

¹³The number of general equilibrium of the model is $\tau + 1$. But given the boundaries of the parameters, i.e. $0 \leq b \leq 1$, $0 \leq \tau$, $1 \leq \lambda$, $0 \leq g \leq 1$, $0 \leq \omega \leq 1$, the number of non-trivial solution is two, one of which is $(0, 0)$. The rest is either negative or complex.

Figure 2.4: General equilibrium of the model



Proposition 3. General equilibrium of the model:

$$g^* = \left(\frac{\lambda - b}{\lambda(1 - b)} \right)^{\frac{1}{\tau}} - 1, \quad \omega^* = \frac{1}{\lambda} \left[\left(\frac{\lambda - b}{\lambda(1 - b)} \right)^{\frac{1}{\tau}} - 1 \right]$$

Substituting these endogenously determined g and ω into the solution in appendix D yields a complete solution of the model.

2.4 Comparative dynamic analysis

The extended circuit of capital model, simplified under classical assumptions and assumption 4, reduces to a model which is solved to generate the steady-state growth rate g , the interest rate spread ω , and the stock and flow variables as a function four parameters, i.e. markup q (as a product between exploitation rate and an inverse of organic composition of capital), investment lag τ , firms' borrowing ratio b , and bank leverage λ .

In this section, with comparative dynamic analyses two types of growth, i.e. firm leverage-led and bank leverage-led, are identified and their distinctive impact on the interest rate spread is examined.

2.4.1 Bank leverage–led growth

In essence, the bank borrows in order to lend and hence the banking sector’s indebtedness, reflected in its leverage ratio, implies credit flows into the real economy. Proposition 4 summarizes how the leverage behavior of the banking sector imparts an impact on growth and interest rate spread in equilibrium.

Proposition 4. Impact of bank leverage:

- i) if $\lambda = 1$, then $g^* = 0$ and $\omega^* = 0$,
- ii) $\frac{\partial g^*}{\partial \lambda} > 0$,
- iii-a) $\frac{\partial \omega^*}{\partial \lambda} > 0$ if $1 < \lambda < \bar{\lambda}$,
- iii-b) $\frac{\partial \omega^*}{\partial \lambda} < 0$ if $\lambda > \bar{\lambda}$,

where $\bar{\lambda}$ is λ that maximizes ω^* given q , τ , and b .¹⁴

The first two statements (i) and (ii) of proposition 4 can be easily proved by examining the expression for g^* and ω^* in proposition 3. First of all, note that if a negative net worth is ruled out then $\lambda \in [1, \infty)$ would be the case by definition. Now, (i) in proposition 4 suggests that when the bank leverage is at its lower bound the growth rate and the interest rate spread in equilibrium will be at their lowest as well, i.e. zero; here, negative growth rate and negative spread are ruled out as well. This situation is depicted in figure 2.5a.

The bank leverage being unity implies the banking system is not issuing any liabilities, which in turn implies that in the model economy, which is a pure credit

¹⁴Formally, $\bar{\lambda}$ is such that

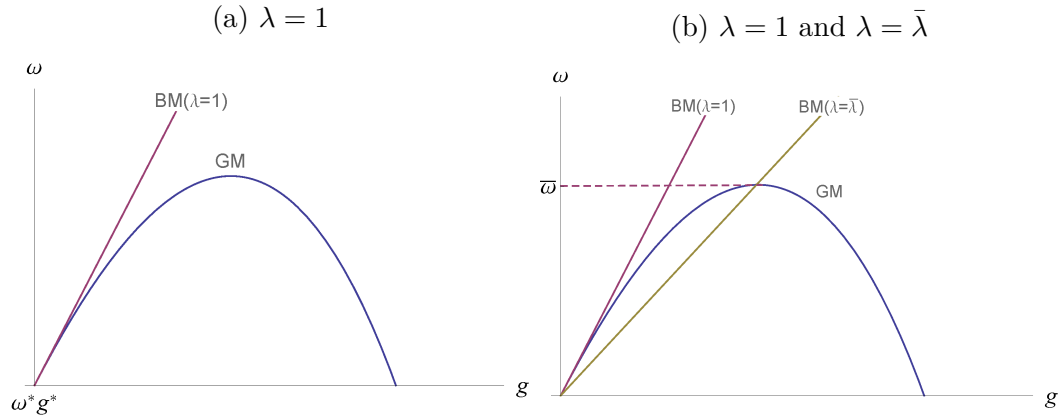
$$\max_{\lambda} \frac{1}{\lambda} \left[\left(\frac{\lambda - b}{\lambda(1 - b)} \right)^{\frac{1}{\tau}} - 1 \right]$$

The first–order condition is obtained as

$$\left(\frac{\lambda - b}{\lambda(1 - b)} \right)^{\frac{1}{\tau}} - \frac{(\lambda - b)\tau}{(\lambda - b)\tau - b} = 0$$

$\bar{\lambda}$ is implicitly determined in a way that satisfies the above F.O.C. For graphical representation of $\bar{\lambda}$, see figure E.1 in appendix E.

Figure 2.5: General equilibrium and the bank leverage



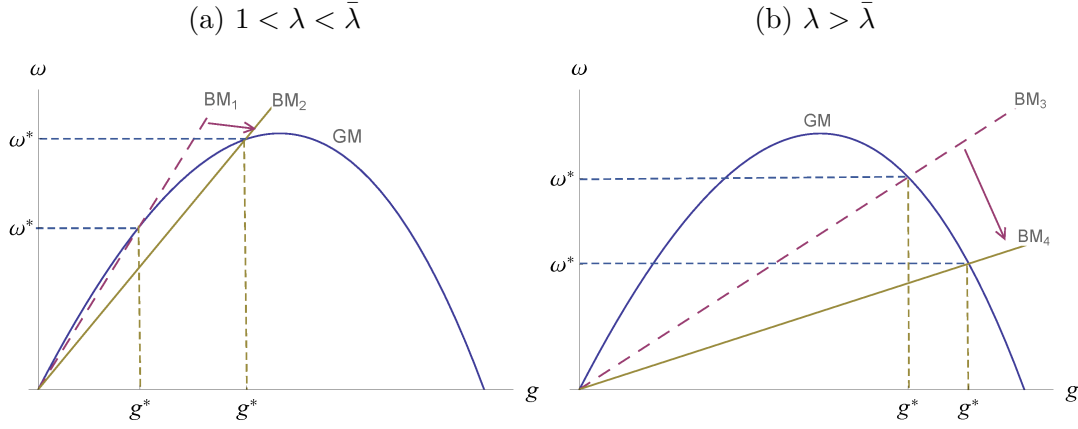
(Note: (a) The bank leverage being one, i.e. zero bank indebtedness, yields $g^* = 0$ and $\omega^* = 0$. (b) There exists $\lambda = \bar{\lambda}$ where the equilibrium spread is maximized at $\bar{\omega}$.)

economy under assumption 4, there is no money supply and hence the nonbank sector does not hold any money balance. No economic activity can possibly take place and consequently the economy cannot grow. Furthermore, there is no bank lending and hence no interest earnings, which are the source of bank profits, since in the pure credit economy banks can extend loans only by issuing liabilities. Both the demand and supply of bank credit are zero and hence the interest rate is zero as well.

As the bank leverage gradually grows from its lower bound, the equilibrium growth rate also rises as (ii) in proposition 4 suggests. Graphically, a rise in λ lowers the slope of the BM curve, which thus pivots clockwise thereby pushing up g^* . Growth in this case can be characterized as *bank leverage-led*. On the other hand, the response of ω^* is more complicated. This is stated in (iii) in proposition 4. A formal proof of this is moved to appendix E, and here let us explain it graphically.

First see figure 2.5b. Due to the inverted-U shape of the GM curve, there exists a unique value of λ , denoted by $\bar{\lambda}$, that corresponds to the maximum level of the equilibrium spread denoted by $\bar{\omega}$. Depending on whether λ is smaller or larger than $\bar{\lambda}$, a change in the bank leverage will have a distinctive impact on ω^* . Figure 2.6

Figure 2.6: Differential effects of bank leverage on the model equilibrium depending on the level of bank leverage



illustrates this. In the case of $1 < \lambda < \bar{\lambda}$ which is reflected in the line BM_1 in figure 2.6a, the BM curve crosses the upward-sloping segment of the GM curve, and consequently a rise in λ will shift BM_1 to BM_2 , and ω^* will increase. The reverse is true in the case of $\lambda > \bar{\lambda}$, which is reflected in the line BM_3 in the figure 2.6b. The BM curve crosses the downward-sloping segment of the GM curve. Consequently, an increase in λ will shift BM_3 to BM_4 and thus ω^* will be reduced. Hence (iii) in proposition 4 is confirmed. Note that g^* keeps rising in both cases for the range of λ .

The differential response of ω^* to a change in λ can be explained by looking at the bank credit market, i.e. by examining how a change in λ is transmitted through affecting the demand and supply of bank credit. In the partial equilibrium analysis of the bank credit market in section 3.4.5 the bank leverage is a parameter that affects the supply of bank credit only (compare equations (2.19) and (2.24)). However, it is already known from the general equilibrium analysis in section 2.3.3 that a change in λ will bring out a change in g , which was taken as exogenous in the bank credit market analysis. Since g affects both demand and supply of bank credit, once the analysis moves from a partial equilibrium to a general equilibrium, thereby the endogenous

change in g being considered, a change in λ will affect both the demand and supply curve.

To formally understand this full dynamics of the bank credit market, the demand and supply function of bank credit in equations (2.19) and (2.24) should be expanded by substituting g^* , which yields

$$\begin{aligned} B'_D &= \frac{\lambda b(\Omega - 1)(q + 1)}{(\Omega - 1)(\lambda - b) + \lambda b\omega} \\ B'_S &= \frac{\lambda^2 b(1 + q)\omega}{(\Omega - 1)(\lambda - b) + \lambda b\omega} \end{aligned} \quad (2.25)$$

where $\Omega = \left(\frac{\lambda - b}{\lambda(1 - b)}\right)^{\frac{1}{\tau}}$. These expanded functions of B'_D and B'_S then can be taken a derivative with respect to λ . The consequent partial derivative results would reflect a *dynamic* impact of a change in the bank leverage on the demand and supply curve of the normalized bank credit; dynamic in the sense that it incorporates an endogenous change of g .¹⁵ See lemma 1.

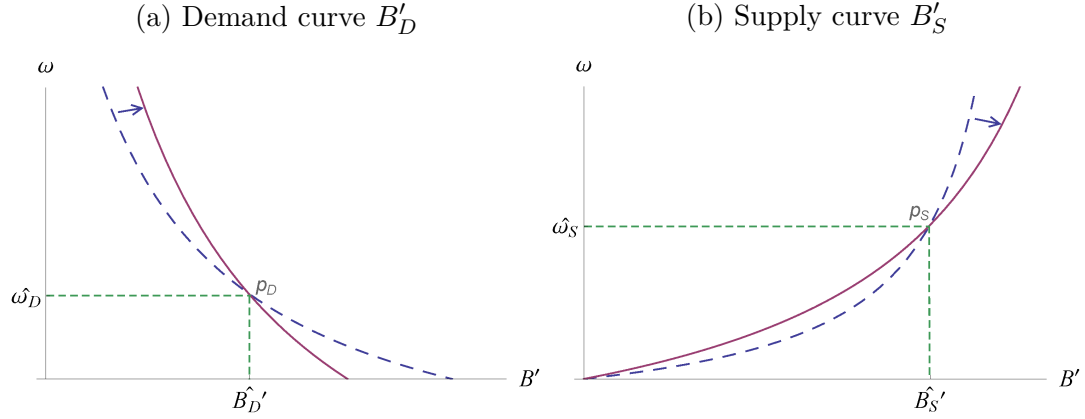
Lemma 1. A sign of $\frac{\partial B'_D}{\partial \lambda}$ and $\frac{\partial B'_S}{\partial \lambda}$ is determined as

- i) $\frac{\partial B'_D}{\partial \lambda} \geq 0$ if $\omega \geq \hat{\omega}_D$ where $\hat{\omega}_D = \frac{(\Omega - 1)^2(\lambda - b)\tau}{\Omega \lambda b}$,
- ii) $\frac{\partial B'_S}{\partial \lambda} \geq 0$ if $\omega \geq \hat{\omega}_S$ where $\hat{\omega}_S = \frac{1}{\Omega \lambda \tau} + \frac{(\Omega - 1)(\lambda - 2b)}{\Omega \lambda b}$.

which can be easily proved by examining the sign of $\frac{\partial B'_D}{\partial \lambda}$ and $\frac{\partial B'_S}{\partial \lambda}$. Lemma 1 states that, for a given ω , the sign of the dynamic impact of a bank leverage on the demand and supply of the normalized bank credit changes at some threshold point $\hat{\omega}_D$ and $\hat{\omega}_S$, respectively. As displayed in figure 2.7, in response to a rise in λ , the demand and supply curve would pivot clockwise with $p_D(\hat{B}'_D, \hat{\omega}_D)$ and $p_S(\hat{B}'_S, \hat{\omega}_S)$, respectively, as a pivot point.

¹⁵Accordingly, the results will differ from the comparative *static* analysis of the bank credit market examined in proposition 1.

Figure 2.7: A dynamic impact of a rise in bank leverage on the demand and supply curves of normalized bank credit

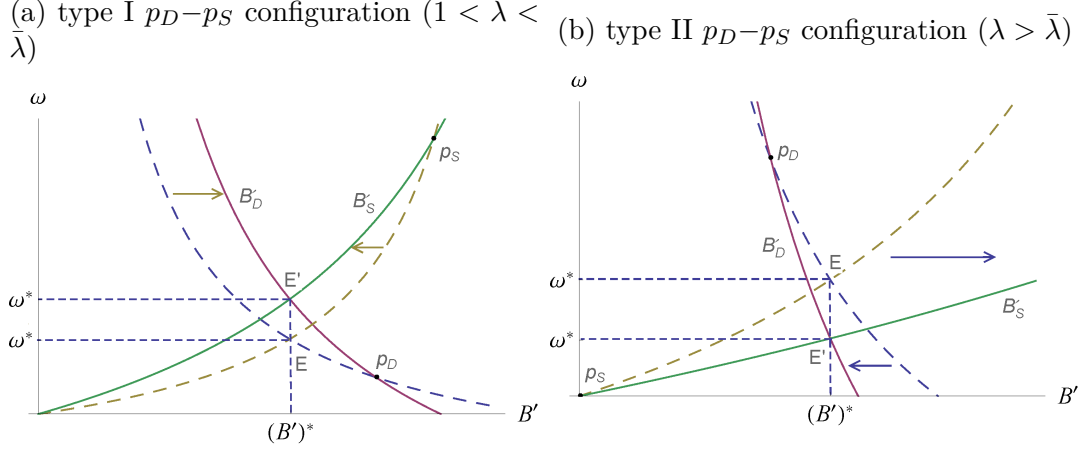


In which direction the demand and supply curve will actually move and hence how an equilibrium will subsequently change depend on the configuration of the two pivot points. Figure 2.8 illustrates two possible cases. First, figure 2.8a displays a p_D-p_S configuration where the segment of the demand and supply curve on the left of the pivot points are intersected with each other at E to the effect that a rise in λ ultimately generates an rightward shift of the demand curve and a leftward shift of the supply. As a result, the equilibrium moves to E' , obtaining a higher ω^* while B'^* remains the same.¹⁶ Let us conveniently call the p_D-p_S configuration described here type I.

On the other hand, figure 2.8b illustrates a p_D-p_S configuration that produces the opposite result regarding the change in ω^* . The segments of the demand and supply curves on the right of the pivots cross each other at E so that a rise in λ effectively generates an leftward shift of the demand curve and a rightward shift of the supply curve. As a consequence, the equilibrium shifts to E' , thereby obtaining

¹⁶To formally see the result regarding B'^* , substitute the expression for g^* in proposition 3 into the one for B'^* in proposition 1 to obtain $B'^* = b(1 + q)$, with which $\frac{\partial B'^*}{\partial \lambda} = 0$ can be verified.

Figure 2.8: Comparative dynamic analysis of the bank credit market with respect to λ



a lower ω^* while B'^* remains the same. In comparison to type I configuration, let us conveniently call the p_D-p_S configuration described here type II.

In sum, in the case of type I p_D-p_S configuration, on the one hand, an increase in λ , which enhances growth, eventually expands the demand curve and shrinks the supply curve of the bank credit market, thereby raising ω along with g . On the other, in the case of type II p_D-p_S configuration, a rise in λ strengthens growth by shrinking the demand curve while expanding the supply curve. Consequently, ω falls although g rises.

Proposition 5 states the analytical result regarding the condition under which each of the p_D-p_S configurations will take place.

Proposition 5. Conditions for the two types of p_D-p_S configuration:

- i) if $1 < \lambda < \bar{\lambda}$, type I p_D-p_S configuration will be the case (hence the bank leverage-led growth emerges through facilitating credit demand);
- ii) if $\lambda > \bar{\lambda}$, type II p_D-p_S configuration will be the case (hence the bank leverage-led growth emerges through facilitating credit supply).

where $\bar{\lambda}$ is λ that maximizes ω given a GM curve.

Graphically, proposition 5 suggests a correspondence between figure 2.6a and figure 2.8a, and that between figure 2.6b figure 2.8b. This implies that the growth initiated by a rise in the bank leverage emerges through facilitating credit demand when the bank leverage is low, whereas it is through facilitating credit supply when the bank leverage is high.

Then what explains the shifts of the B'_D and B'_S curve in response to a parameter change, in this case a change in λ ? This question can be answered by investigating the behavioral specification of the demand and supply of the normalized bank credit, expressed in (2.18) and (2.23) and reproduced below.

$$\begin{aligned} B'_D &= bZ' = b(1 + \Pi'_K)(1 + g)^{-\tau} \\ B'_S &= \lambda\Pi'_B \end{aligned} \tag{2.26}$$

Regarding the demand, its behavioral specification $B'_D = bZ'$ is further expanded by using the expression for Z' in (2.2). In this way, it is highlighted that, *ceteris paribus*, the bank credit demand is linked, through investment Z , to the firm sector's net profit Π_K , while the bank credit supply is linked to the banking sector's profits Π_B .

Under assumption 4 the expression for Π'_K and Π'_B , specified in (2.6) and (2.11), is reduced to, respectively,

$$\begin{aligned} \Pi'_K &= q - \Pi'_B \\ \Pi'_B &= \omega L' \end{aligned} \tag{2.27}$$

according to which bank profits are interest earnings from the outstanding stock of loans L , which in this model reflects a size of the banking sector balance sheet, and the firm sector's net profits are total profits qR net of bank profits.

From equations in (2.27), a dynamic impact of λ on Π'_K and Π'_B is obtained as

$$\begin{aligned}\frac{\partial \Pi'_K}{\partial \lambda} &= \underbrace{\frac{\partial \Pi'_K}{\partial g}}_{\oplus} \underbrace{\frac{\partial g}{\partial \lambda}}_{\oplus} > 0 \\ \frac{\partial \Pi'_B}{\partial \lambda} &= \underbrace{\frac{\partial \Pi'_B}{\partial g}}_{\ominus} \underbrace{\frac{\partial g}{\partial \lambda}}_{\oplus} < 0\end{aligned}\tag{2.28}$$

$\frac{\partial \Pi'_K}{\partial g} > 0$ and $\frac{\partial \Pi'_B}{\partial g} < 0$ can be easily verified by using equations in (2.27) and $\frac{\partial L'}{\partial g} < 0$, which in turn directly follows from the solution for L' . These partial derivatives with respect to g reflect that, compared to the economy on a lower steady-state growth path, in the economy on a higher steady-state growth path the weight of the banking sector, measured by its balance sheet size and its net revenue, will be smaller while the weight of the firm sector, measured by its net revenue, will be larger.

Eventually, with the help of equations in (2.26) and inequalities in (2.28), a dynamic impact of λ on the B'_D curve and the B'_S curve can be established in the following way. First, regarding the B'_D curve, since a rise in λ will have both a positive impact through raising Π'_K and a negative impact through raising g , an overall result depends on the relative magnitude of the two opposing forces. Second, regarding the B'_S curve, since a rise in λ has a directly positive impact while it also has an indirectly negative impact through lowering Π'_B , an overall result depends on the bank leverage elasticity of the normalized bank profits. These are summarized in proposition 6.

Proposition 6. For a given ω ,

- i) if a dynamic impact of the bank leverage on the firm sector's normalized net profits is sufficiently large (sufficiently small), i.e. $\frac{\partial \Pi'_K}{\partial \lambda} > \theta$ ($\frac{\partial \Pi'_K}{\partial \lambda} < \theta$) where $\theta = \frac{\tau(1+\Pi'_K)}{1+g} \frac{\partial g}{\partial \lambda} - 1$, then a rise in λ will dynamically shift the B'_D curve rightward (leftward), i.e. $\frac{\partial B'_D}{\partial \lambda} > 0$ ($\frac{\partial B'_D}{\partial \lambda} < 0$);

- ii) if a response of the banking sector's normalized net profits to the bank leverage is elastic (inelastic), i.e. $\xi_{\Pi_B}^\lambda > 1$ ($\xi_{\Pi_B}^\lambda < 1$) where $\xi_{\Pi_B}^\lambda = -\frac{\partial \Pi'_B}{\partial \lambda} \frac{\lambda}{\Pi'_B}$, then a rise in λ will dynamically shift the B'_S curve leftward (rightward), i.e. $\frac{\partial B'_S}{\partial \lambda} < 0$ ($\frac{\partial B'_S}{\partial \lambda} > 0$).

These statements can be easily proved by verifying the sign of $\frac{\partial B'_S}{\partial \lambda}$ and $\frac{\partial B'_D}{\partial \lambda}$, using (2.26), (2.27), and (2.28). With ω given, while B'_D and B'_S depend on Π'_K and Π'_B , respectively, and a rise in λ leads to, a rise in Π'_K and a fall in Π'_B , proposition 6 shows that the ultimate impact of λ on the B'_D and B'_S curve depends on how strong the response of these profits are. By combining proposition 5 and 6, the mechanism underlying the two different cases of the bank leverage-led growth can be clarified as follows.

If $1 < \lambda < \bar{\lambda}$, for a given ω , the response of Π'_K to a rise in λ is sufficiently strong so as to raise B'_D while the response of Π'_B is elastic so that B'_S falls. The bank leverage-led growth in this case takes place through facilitating credit demand and consequently the spread rises along with the growth rate. This is depicted in the combination of figure 2.6a and 2.8a. If $\lambda > \bar{\lambda}$, the response of Π'_K is sufficiently weak so as to decrease B'_D while the response of Π'_B is inelastic so that B'_S rises. The bank leverage-led growth in this case takes place through facilitating credit supply, and hence the spread is pushed down even when growth rate rises. A combination of figure 2.6b and 2.8b illustrates this.

Lastly, the growth led by the bank leverage is not without bound. Increasing the bank leverage forever does not lead to an infinite growth. This can be easily verified with figure 2.4. It can be seen that, given τ and b and hence given GM curve, g^* will be maximized when λ is maximized, which makes the BM curve flat. Proposition 7 states this result formally.

Proposition 7. The bank leverage-led growth has a limit, i.e. $\lim_{\lambda \rightarrow \infty} g^* = \left(\frac{1}{1-b}\right)^{\frac{1}{\tau}} -$

See appendix E for a proof.

The comparative dynamics with respect to λ as discussed so far is informative about the role of bank leverage in the economy on a steady-state growth path. In particular, since the essence of financial innovations lies in liability management of the bank, the bank leverage can be broadly considered as reflecting the status of development of financial technology.¹⁷ In this respect, in the economy without developed financial technologies, reflected in a low bank leverage, a growth initiated by bank borrowing affects the structure of profitability of the firm sector and the banking sector in a way that ultimately boosts the excess demand for bank credit and hence tightens the market.

However, such pressure on the bank credit market does not take place in the economy with the banking system characterized by developed financial technologies, reflected in a high bank leverage. Rather, as the banks borrow more, the structure of profitability is affected in a way that favors the banking sector and consequently the bank credit market becomes slack. In this way, financial innovations, which allow the banking sector to maintain its leverage high, make it possible to achieve a regime of high growth and low interest rate.

On the other hand, the growth led by financial innovations and bank leverage is limited by an upper bound as suggested by proposition 7.

2.4.2 Firm leverage-led growth

In comparison to the nonlinearity of the impact of bank leverage in equilibrium, the rest of the parameters affect the model equilibrium in a unilateral way. This can be very easily understood by looking at figure 2.4 and recalling that only λ shifts

¹⁷The parameter n denoting financial innovation has been excluded from the model under assumption 4. It can be restored in the future research and be examined together with bank leverage.

BM curve while the other parameters shift GM curve. The consequent comparative dynamic analysis results are summarized as follows.

Proposition 8. Comparative dynamic analysis with respect to q , τ , b

$$\text{i) } \frac{\partial g^*}{\partial q} = 0, \frac{\partial g^*}{\partial \tau} < 0, \frac{\partial g^*}{\partial b} > 0$$

$$\text{ii) } \frac{\partial \omega^*}{\partial q} = 0, \frac{\partial \omega^*}{\partial \tau} < 0, \frac{\partial \omega^*}{\partial b} > 0$$

These results can be easily proved by using the expression for g^* and ω^* . Statement (i) suggests that a faster investment, reflected in a lower τ , and a higher borrowing ratio of the firm, reflected in a larger b , generate a stronger growth, which is economically intuitive. Since these investment and borrowing behaviors increase the firm sector's leverage ratio as shown in proposition 2, growth initiated by decreasing τ and increasing b can be characterized as *firm leverage-led*. On the other hand, since q does not shift the GM curve because of the absence of realization problem associated with worker household consumption (which is due to the classical assumption of immediate consumption of wage income), the markup has no impact on growth in equilibrium.

The parameters' impact on the equilibrium spread suggested in statement (ii) can be understood by looking at the bank credit market dynamics, similarly to the case of λ as done in the previous subsection, especially in lemma 1.¹⁸ And depending on how the demand and supply curve of bank credit are shifted, the firm leverage-led growth in statement (i) can be either a growth that takes place through facilitating credit demand, which raises the spread, or a growth that takes place through facilitating credit supply, which lowers the spread. Rather than presenting the whole process which is similar to the case of λ , here only the result is reported as follows.

¹⁸Caution: the results regarding ω^* in (ii) of proposition 8 are different from those in (iii) of proposition 1 as the former is the general equilibrium analysis with g being endogenous while the latter is a partial equilibrium analysis with g being exogenous.

Proposition 9. The firm leverage–led growth takes place through stimulating credit demand–led in the following ways:

- i) In the case of growth initiated by a rise in markup q , B'_D and B'_S in equilibrium increase equally, i.e. $\frac{\partial B'_D}{\partial q} = \frac{\partial B'_S}{\partial q}$ when $\omega = \omega^*$, and consequently there is only quantity effect but no price effect; hence no change in ω^* .
- ii) In the case of growth initiated by a fall in investment lag τ , the B'_D curve shifts right while the B'_S curve shifts left, i.e. $\frac{\partial B'_D}{\partial \tau} < 0$ and $\frac{\partial B'_S}{\partial \tau} > 0$; hence ω^* rise.
- iii) In the case of growth initiated by a rise in the firm sector's borrowing ratio b , the B'_D curve shifts right while the B'_S curve shifts left under the condition that b is sufficiently high, i.e. $\frac{\partial B'_D}{\partial b} < 0$ and $\frac{\partial B'_S}{\partial b} > 0$ as long as $b > \frac{\tau}{\tau+1}$; hence ω^* rise.

These results can be easily proved by verifying the sign of the partial derivative of B'_D and B'_S , expressed in (2.25), with respect to each of the three parameters in question.

It is already known from the discussion in proposition 6 that a dynamic impact of λ is transmitted onto the demand and supply curve of the bank credit through affecting the profits of the firm and the banking sector. Corresponding cases regarding each of the three firm sector parameters, i.e. q , τ , and b , can be analyzed in the similarly way, which is omitted here due to page limit.

On the other hand, contrarily to the bank leverage–led growth which has a limit as stated in proposition 7, the firm leverage–led growth is unbounded.

Proposition 10. The firm leverage–led growth faces no limit, i.e. $\lim_{\tau \rightarrow 0} g^* = \infty$ and $\lim_{b \rightarrow \infty} g^* = \infty$.

Using the fact that $g^* = \left(\frac{\lambda - b}{\lambda(1-b)} \right)^{\frac{1}{\tau}} - 1$, the above result can be easily proved algebraically.

2.5 Conclusion

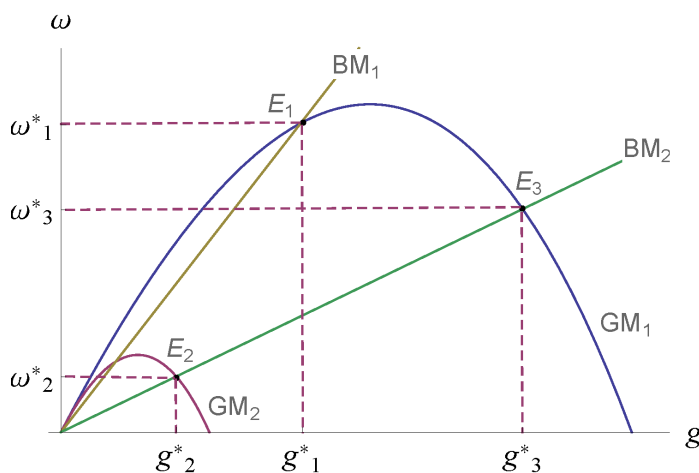
In this chapter, an extended circuit of capital model with the banking sector has been developed to endogenize credit creation by identifying demand and supply of bank credits. This allowed to introduce a taxonomy of firm leverage-led vs. bank leverage-led growth in a way that highlights financial aspects of growth. The main findings of the model analysis can be summarized as follows:

(1) The firm leverage-led growth, taking place either by a decrease in the investment lag or by an increase in the firm's borrowing in financing investment, is unbounded. (2) However, it emerges through stimulating the excess demand for bank credits, thereby tightening the market and thus increasing the interest rate spread alongside the growth rate. (3) The bank leverage-led growth is bounded by an upper limit. (4) However, the bank leverage-led growth can possibly takes place through stimulating excess supply of bank credits, thereby enable to achieve a regime of high growth *and* low interest rate under the condition that the banking sector leverage is in a high range, reflecting a developed financial technology; otherwise, the bank leverage-led growth takes place through facilitating excess demand for bank credits, in which case a regime of high growth accompanied by high interest rate is unavoidable as in the case of the firm leverage-led growth.

The extended circuit of capital model provides a theoretical framework to understand economic regimes distinguished by firms' investment and the status of banking system, and to make a prediction for each of the regimes regarding growth rate and interest rate. First of all, a distinction can be made between an economic regime characterized by strong borrowing and investment by firms with a less developed banking system and an economic regime where the banking system is more developed but borrowing and investment by firms are weak.

In figure 2.9, the first case is depicted by GM_1 with a low τ and BM_1 with a low λ achieving an equilibrium at E_1 with g_1^* and ω_1^* . And the second case is depicted

Figure 2.9: A comparison among three distinctive economic regimes



(Note: E_1 describes an economic regime characterized by strong borrowing and investment by firms with a less developed banking system; E_2 describes an economic regime characterized by weak borrowing and investment by firms with a developed banking system; E_3 describes an economic regime characterized by strong borrowing and investment by firms with a developed banking system.)

by GM_2 with a large τ and BM_2 with a large λ achieving an equilibrium at E_2 with g_2^* and ω_2^* . According to the framework of the model presented in this chapter, the growth in the first economic regime can be characterized as firm leverage-led. Hence, it is predicted that this economy will experience high interest rates alongside strong growth; high growth-high interest rate regime. On the other hand, the second regime is characterized by a bank leverage-led growth coupled with a large λ and a shrank GM curve due to a very large τ . Accordingly, it is predicted that this economy will suffer from a sluggish growth despite low interest rates; low growth-low interest rate regime.

The contrast between these two hypothetical regimes is analogous to a widely-discussed contrast between two historical regimes, i.e. the so-called Golden Age regime in the 1950s until the early 1970s and the neoliberal regime that followed. It is now an established fact that the Golden Age regime experienced strong growth, which was supported by robust capital investment by the firm sector while its banking system was mainly characterized by traditional commercial banking with strong

regulation. Contrarily, the neoliberal regime is marked by financial innovations and weak capital investment by the firms.

Note that the correlation between growth rate and interest rate predicted by the contrast between the economy at E_1 and the economy at E_2 corresponds to the standard economic theory, which demonstrates that the two variables will converge to each other in a steady-state (von Neumann, 1945, e.g.). However, historical data suggest otherwise. Even though the Golden Age period experienced strong growth and the neoliberal regime underwent sluggish growth as predicted by the model, data show that the prediction regarding interest rates of the two economic regimes does not hold true. Even by a quick observation, it is hard to miss that regardless what interest rates we look at the interest rate on average was lower in the Golden Age regime than that in the neoliberal period. Furthermore, recent empirical research finds no strong correlation between growth rate and interest rates; rather, a negative correlation is reported in some cases (Bosworth, 2014; Hansen and Seshadri, 2013). On the other hand, however, these empirical works do not directly address the issue regarding Golden Age vs. neoliberal periods, and the correlation between growth rate and interest rates in these two economic regimes is an area where more works need to be done.

On a positive side, the model presented in this chapter does not always predict a positive correlation between growth rate and interest rates. Consider the third economic regime with strong borrowing and investment by firms, reflected in GM_1 , coupled with a developed banking system, reflected in BM_2 . An equilibrium is established at E_3 with g_3^* and ω_3^* . A transition from E_1 to E_3 involves a rise in the growth rate and a fall in the interest rate spread; a negative correlation. In this way, the model of this chapter allows various possibilities regarding the correlation between growth rate and interest rates in a way that would be not possible in a model that does not incorporate a banking sector.

The model of this chapter can also be assessed in comparison to some of the recent growth theories. For instance, from the perspective of the categorization of debt-led vs. debt-burdened growth regimes in Nishi (2012), the extended circuit of capital model would belong to the debt-led regime as it has growth positively related to both the firm's and the bank's leverage. This is due to technical details of the model. Recall that a fall in τ , which reflects a faster investment, increases not only growth (proposition 8) but also the firm sector leverage (proposition 2), and this conforms to a Minskian idea that with capital accumulation the firm's financial position will become fragile. In this circumstance, a debt-burdened growth regime could possibly emerge if the rising firm leverage negatively affects investment (raising τ) by reducing the difference between the firm's profits and interest payment, which is the margin of safety.

However, in the model τ is a constant parameter and hence does not respond to changes in the other parameters or variables. This is why the debt-burdened growth regime cannot emerge in the model of this chapter. Therefore, in order to account for economic problems arising from intensification of financial fragility, it would be desirable to endogenize τ . A possible behavioral specification along the line of Minskian proposition would be to make τ as a negative function of the margin of safety. The latter could be measured within the model by the net profits of the firm sector Π_K since it is already defined by netting interest payments from the aggregate profits.¹⁹ This would be a major extension of the model that would contribute to developing a macro-finance model within the circuit of capital framework.

¹⁹A theoretical issue involved in modeling the margin of safety is whether to compare the profits and interest payments either as a ratio or as a difference.

CHAPTER 3

SECURITIZED BANKING, PROCYCLICAL BANK LEVERAGE, AND FINANCIAL INSTABILITY

3.1 Introduction

Adrian and Shin (2010) have recently documented an empirical observation that banking sector leverage is procyclical; the financial intermediary leverages up its balance sheet during upturns of cycles marked by asset price appreciation while shedding risk exposures by deleveraging during downturns. As the banking sector's total balance sheet size implies credit flows through the real economy, a direct consequence of the procyclical bank leverage is an amplification of an asset price cycle and hence an aggravation of financial instability. Motivated by this finding, a literature is now growing in two directions. The first is to provide a theoretical explanation for procyclical bank leverage and the second is to examine its macroeconomic consequences.¹ The main purpose of this chapter is to contribute to this emerging literature by addressing both of these issues.

Underlying the procyclical pattern of financial intermediaries is the transformation of banking to the 'originate and distribute' model, which characterizes the run-up period to the 2007-2008 disruption and has decisively aggravated the financial fragility of the system (Brunnermeier, 2009, e.g.). Contrary to the traditional banking model where bank loan creation is financed primarily by FDIC-insured deposits and the issuing banks hold the loans until maturity, in the new intermediation model, banks offload risks via securitization and the security broker-dealers finance their portfolio

¹I will discuss this literature in greater detail below.

through collateralized short-term borrowing such as repurchase agreements (repo). Gorton and Metrick (2012) label this type of banking business model ‘securitized banking’. While macro-models incorporating the banking sector have been growing in number since the recent financial turmoil, those which distinguish between commercial banks and investment banks are relatively few. As a way to fill this gap, my model explicitly incorporates the securitized banking system.

In particular, I explore the logic underpinning the endogenous development of boom–bust cycles of asset price, bank leverage, and liquidity which have become more severe in the securitized banking system. A central aspect of this intermediation system is the change in the bank funding source from deposits to repos. In repo transactions, collateral replaces the government insurance. As the collateral value is directly affected by the price of underlying assets, repo creditors are typically exposed to counter–party risk and collateral value risk. On the other hand, repo maturities are in most cases overnight and this creates liquidity risk as borrowers have to continuously roll–over debts. When asset prices are on the rise, repo investors have little concern about these risks and allow broker–dealers to borrow not only at a lower rate but also with a lower haircut,² which enables the borrowers to leverage up further. However, in times of stress the investors quickly pull back from the market as the probability of default increases. Both the repo rate and haircut surge, which forces securitized banks to deleverage sharply.³

This chapter reveals the connection between such an unstable repo market dynamics and the procyclicality of the securitized bank’s leverage. For this purpose, I first provide a theoretical explanation for the procyclical leverage of the securitized

²Haircut is the value of equity measured against the market value of an asset that is being used as collateral; it is ‘skin in the game’. Haircut is an inverse of leverage ratio.

³This is a situation comparable to the tradition bank run. Repo investors’ rejection to rollover is analogous to retail depositors’ cash withdrawal. When a herd behavior emerges, each of these can put commercial banks and investment banks, respectively, into a liquidity crisis. This is the run on repo as analyzed in Gorton and Metrick (2012) and Martin et al. (2014).

bank by using a banking model of repo transaction. It compares to, among others, Adrian and Shin (2014)'s contracting model where banks follow the Value-at-Risk rule and the main friction is risk-shifting moral hazard. A distinctive aspect of the Adrian and Shin model is that it adopts Extreme Value Theory to study extreme and rare events that match the repo market disruption. In contrast, my approach adopts a more generalized framework where the bank is simply characterized by standard profit-maximizing behavior without necessarily invoking the extreme outcome specification as in the Adrian and Shin model.

Particularly, my model focuses on repo rate, instead of haircuts as in Geanakoplos (2009) and Adrian and Shin (2014), as a channel through which an asset price cycle is transmitted to the bank leverage cycle. This reflects Smith (2012)'s finding that for the 2008 crisis episode the repo rate is a superior measure of market stress in the form of collateral value risk. Accordingly, in my model the repo rate moves in a way that reflects risks faced by repo lenders, who are risk-neutral. That is, when asset prices rise, which strengthens the collateral value, counter-party risk and collateral value risk of the repo lenders will fall and consequently the repo rate will fall as well. In this environment the borrowers, i.e. security broker-dealers, will be able to achieve maximum profitability by raising leverage. On the contrary, in times of market stress when assets serving as collateral experience loss the repo rate will rise. As a consequence, the profit-maximizing repo borrowers will have to lower the leverage. In this respect, optimal leverage of the securitized bank is shown to be positively related to asset prices, i.e. procyclical.

In order to examine macroeconomic implications of the procyclical bank leverage, I then build a macrodynamic balance sheet model consisting of a nonfinancial sector and a financial intermediary sector. The intermediary sector is subdivided into three subsectors including traditional banks that extend loans to the real economy and two other nonbank financial firms, i.e. broker-dealers that purchase the securitized bank

loans by issuing repos and asset managing firms that provide funds to the repo market. The main analytical result of the model derives from the Hopf bifurcation theorem, and accordingly it is shown that under plausible conditions the model exhibits a limit cycle behavior, generating a persistent cycle of asset price and bank leverage.

More specifically, on the one hand, from balance sheet identities demand and supply of bank loans are derived as a positive function of the nonfinancial sector's leverage and the securitized bank's leverage, respectively. On the other hand, the leverage of each sector is specified as a positive function of asset prices. In this setup, procyclical bank leverage is defined as emerging when the coefficient that captures asset prices' impact on the securitized bank' leverage is sufficiently large so that in effect the supply curve of bank loans is more responsive than the demand curve is to a asset price fluctuation. This definition differs from the one suggested in Adrian and Shin (2010) where leverage is said to be procyclical when growth rate of total assets and that of leverage are the same.

The analytical definition of procyclical bank leverage presented in this chapter is theoretically rewarding in two ways. First, it allows to see that underlying the dynamics of leverage cycle and asset price cycle is the supply-led credit expansion and collapse. In the case of asset price appreciation, the bank loan will be in excess supply, which will lead to a fall in interest rates, which in turn will further boost the asset price rise; the same process will take place in the opposite direction in the case of asset price depreciation. Second, it also allows to quantify the degree of procyclicality of leverage by using the marginal impact of asset prices on leverage as a proxy. Accordingly, it is shown that the stronger the procyclicality of the securitized bank's leverage, the more severe and more intense will be cycles of asset prices and bank leverage.

This chapter relates to two strands of the contemporary macro-finance literature. First, there are a group of papers that incorporate the financial intermediary sector

in the dynamic stochastic general equilibrium model (Christiano et al., 2010; Gerali et al., 2010; Gertler and Karadi, 2011, e.g.). While in these papers leverage constraints the bank face develop endogenously due to various forms of agency problems, they require exogenous shocks to actually generate leverage cycles. For instance, one of the main findings of Nuño and Thomas (2012) — which is one of the few recent papers that explicitly formalizes procyclical bank leverage in the DSGE framework — is that cross-sectional volatility shocks, instead of the standard total factor productivity shocks, are responsible for the bank leverage fluctuations. Additionally, in most of the models in this literature the household sector does not borrow but only lends to the firms through intermediation; and there is no securitization. On the contrary, my model does not require exogenous disturbances to generate a cycle; it is an entirely endogenous phenomenon. Also, the firms and households both borrow and lend via the banking sector, which securitizes the loans.

Second, there is a small but growing literature that studies bank behavior in the macrodynamic modeling and stock-flow consistent approach (Chiarella et al., 2012; Hartmann and Flaschel, 2013; Nikolaidi, 2014; Ryoo, 2013, e.g.). A key result commonly emerging from these papers is that the endogenous development of the bank's financial position tends to destabilize the system. However, in most cases little attention is paid to the liability constraint facing banks due to the change in their funding sources. In contrast, my model looks carefully into the changing liability structure of the intermediaries and therein the government insured debts versus collateralized short-term borrowing plays an important role.

The rest of the chapter is organized as follows. Section 3.2 presents empirical data on sectoral leverage to see in what sense the securitized banking sector's leverage is procyclical. In section 3.3, by relying on a model of repo transaction, a theoretical explanation of the procyclical bank leverage is provided. Using the result of section 3.3 as a microfoundation, section 3.4 develops a macrodynamic model to examine the

consequence of procyclical bank leverage within a broader macroeconomic context. The model is analyzed in in section 3.5. I conclude the chapter in section 3.6.

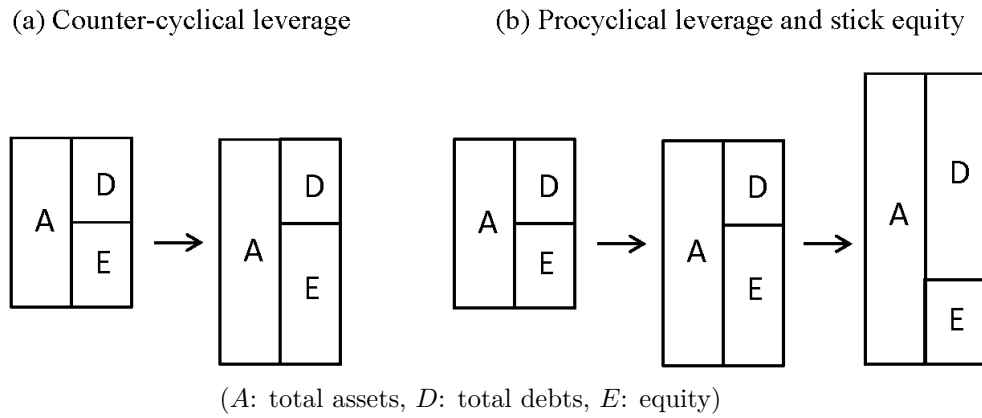
3.2 Empirical observation on leverage and asset prices

The main empirical finding in Adrian and Shin (2010) is that in the case of the broker–dealer sector, its asset and leverage grow in step while equity is constant. Asset growth is entirely driven by growth of debt, not of equity; hence the procyclical leverage and what they call ‘sticky’ equity. Following this approach, leverage is measured as a ratio of total assets over equity throughout this chapter. Figure 3.1 visualizes the mechanics underlying Adrian and Shin (2010)’s observation. It shows how, in marking–to–market accounting system, balance sheet variables change in response to asset price developments. In panel (a), a rise in asset prices initially increases the book value of equity along with the total asset size, thereby lowering the leverage ratio. The leverage is counter–cyclical. However, in panel (b) the consequent financial slack is used to issue more debts, thereby purchasing more assets and hence increasing an exposure to risks. Moreover, the equity is maintained at its initial level through capital policies such as dividend payments and share buybacks, etc. Consequently, the leverage ratio more than restores its initial level. In this case, the leverage is procyclical and equity is sticky.

Adrian and Shin (2010) illustrate the relation between the total asset and leverage in growth terms. From a slightly different angle, I define cyclicity of leverage in regard to asset prices in level terms. The data collected below motivate this approach.⁴ Figure 3.2 plots the asset price–leverage relation for the broker–dealer sector. As mortgage–related securities constitute a substantial part of the broker–dealer’s assets, I used a housing price. It is clear that the leverage rises with the asset price. The case

⁴Data presented in the figures below are collected from Z.1 Financial Accounts of the United States released by the Federal Reserves.

Figure 3.1: Distinctive leverage management



for the commercial bank is depicted in figure 3.3. It plots the leverage in relation to both a housing price and a stock market index. In either case, while the asset prices change widely the leverage trend is quite stable despite a few outliers.

Figure 3.4a describes the positive relation between the asset price and leverage for the household sector. However, when compared with the case of securitized banks as shown in figure 3.4b, we can see that changes in leverage are much less pronounced, almost negligible. The case for nonfinancial corporate business is presented in figure 3.5. The first panel covers the entire period of 1975–2013. It shows that the leverage was procyclical when the stock market price was low and, when it has become higher the leverage has changed to be counter-cyclical. The turning point is around 1990. The asset price–leverage relation for the period of 1990–2013 is separately plotted in the second panel and the counter–cyclical leverage is more manifest.

An interesting observation emerging from these data is that for the broker–dealer sector the positive correlation between asset prices and leverage is quite consistent over time, while for the other sectors the correlation is either weak or inconsistent. By implication, this sector borrows more in the case of asset price appreciation and deleverages when asset prices fall. This is what is implied by ‘procyclical leverage’ in this chapter.

Figure 3.2: Asset prices and leverage of the securitized bank (1975–2013)

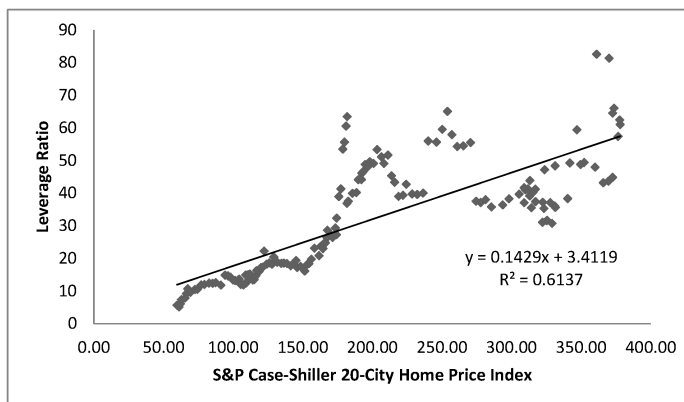
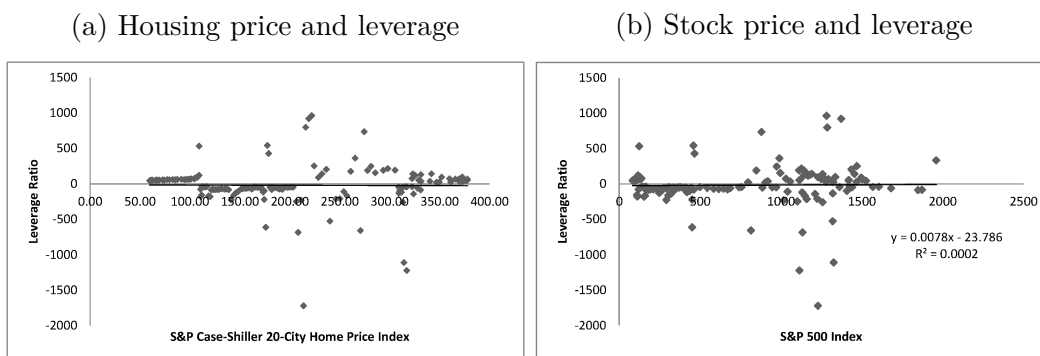


Figure 3.3: Asset prices and leverage of commercial banks (1975–2012)



In the next section, I will provide a theory that explains procyclical leverage of the securitized bank in relation to its repo financing.

3.3 A model of repo transaction

A repo contract stipulates that a borrower sells security to lenders with an agreement that she will repurchase it at a later date. In effect, the difference between the repurchase price and original price constitutes interest, called repo rate, and the borrower’s security acts as collateral. In the event of the borrower’s default on her obligation, the lender keeps the security and liquidates it to recover her initially lent cash. However, the liquidation will be difficult, possible only at a fire-sale price, if the

Figure 3.4: Asset prices and leverage of households (1975–2012)

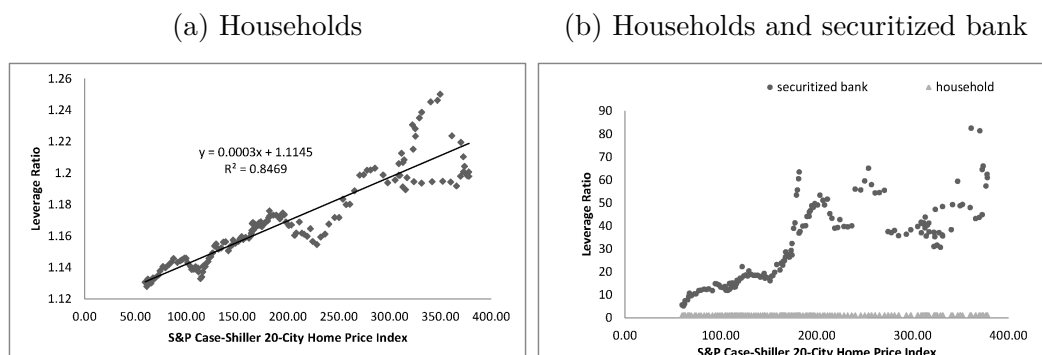
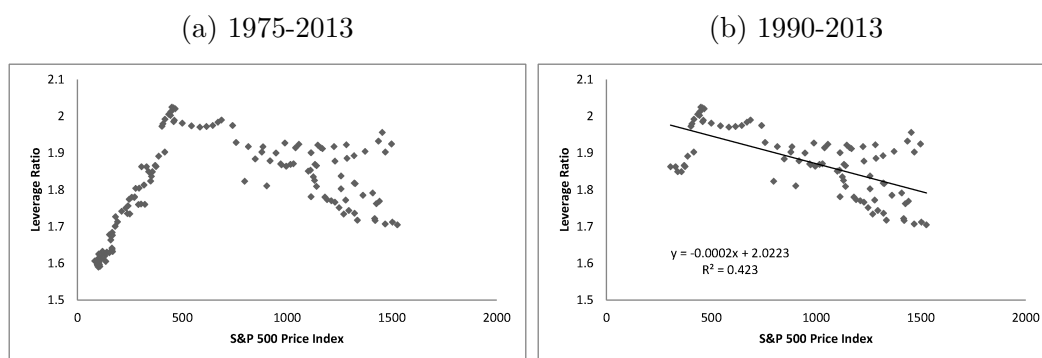


Figure 3.5: Asset prices and leverage of nonfinancial firm (1975-2013)



securities market is under stress. In order to mitigate such credit risk, repos are often overcollateralized the degree of which is dictated by haircut; in addition, the repo rate includes a credit risk premium over and above the interest rate for insured deposits.

However, empirical research on the movement of haircut during the recent financial crisis produces mixed evidence depending on the types of repo markets and quality of collateral. In the bi-lateral repo market for low-grade collateral, haircuts surged during the financial crisis (Gorton and Metrick, 2012). In the tri-party repo market, however, haircuts changed very little during the same period (Krishnamurthy et al., 2014). On the other hand, Smith (2012) finds that the repo rate gives a consistent

measure of market stress in the form of collateral value risk. This chapter adheres to Smith's finding and, accordingly, the repo market dynamics will be reflected in the movement of the repo rate. Haircut is only implicit in the determination of the securitized bank's leverage.

Consider a securitized bank, as a leveraged investor, that finances its asset position in securities, denoted by A , through own equity E and repo borrowing D at a repo rate r^p , using the securities as collateral. The associated credit risk is formalized as follows: At the repo maturity date, the rate of return on A is high at r^H with a probability θ , satisfying $0 < \theta < 1$, and the bank is able to honor the contract by paying $(1 + r^p)D$. With a probability $1 - \theta$ the return on A is r^L which is too low for the bank to carry out the obligation. The bank defaults and the lender keeps the collateral. The repo rate includes a risk-premium over risk-free rate r^f and is lower than the high yield on risky investment. Accordingly, the order of magnitude of interest rates is the following:

$$r^L < 0 < r^f < r^p < r^H < 1 \quad (3.1)$$

Regarding the asset value, a distinction is made between notional value, which is computed by the expected, or average, return, and liquidated value, which is a realized price when liquidated. First, the average return on the securitized bank's total asset is $\theta r^H + (1 - \theta)r^L$. Thus, its notional value would be $(1 + \theta r^H + (1 - \theta)r^L)A$. On the other hand, when the asset is liquidated, its notional value may or may not be realized in full depending on the market conditions. I use q , with $0 \leq q \leq 1$, to denote the degree of realization of the notional value in case of asset liquidation. Accordingly, the liquidated value would be expressed $q(1 + \theta r^H + (1 - \theta)r^L)A$. By examining the derivative of the asset value with respect to θ , it is easy to see that the probability of high asset return positively affects the asset value whether it be notional or liquidated.

Since the securitized bank's assets are posted as collateral, when it defaults on its repo contract the collateral is obtained by the repo lender, who will most probably attempt to sell it to recover the initially invested cash. However, when the default rate is high in the repo market, the number of repo lenders attempting to liquidate collateral will be large and consequently the liquidation can take place only at a loss. This circumstance will be reflected in q being small. More specifically, in the case of the borrower's default on repo contract due to the asset return being low, the notional value of the collateral the lender obtains would be $(1 + r^L)A$ and its liquidated value $q(1 + r^L)A$ where it holds that $q(1 + r^L)A < (1 + r^p)D$.⁵ A greater indebtedness of borrowers in the economy increases their financial burden and therefore raises the probability of debt default. Hence q can be specified as a decreasing function of the borrower's leverage ratio, which is defined as $\lambda = A/E$ as before.

$$q = q(\lambda), \quad q' < 0 \tag{3.2}$$

The repo lender's return, denoted by r^c (superscript c indicating 'creditor'), can be measured as

$$r^c = \frac{\theta(1 + r^p)D + (1 - \theta)(1 + r^L)Aq - D}{D} \tag{3.3}$$

The first term of the numerator is the repo lender's payoff for lending D in case of the higher asset return. The lender's payoff in case of the low asset return is the liquidated value, not the notional value, of the borrower's asset as reflected in the second term of the numerator. The lender is risk-neutral and therefore it is required that r^c equals the alternative risk-free rate r^f . The repo rate r^p should be determined at a level that guarantees $r^c = r^f$, which is a participation constraint of the repo lender. As a

⁵If $q(1 + r^L)A = (1 + r^p)D$ holds, the defaulted borrower would not have had to default in the first place.

consequence, using the definition of leverage ratio, the required rate repo is obtained from $r^c = r^f$ as

$$1 + r^p = \frac{1 + r^f}{\theta} - \frac{1 - \theta}{\theta} \frac{\lambda}{\lambda - 1} (1 + r^L)q \quad (3.4)$$

Equation (3.4) regards the determination of repo rate. It demonstrates that, other things being equal, the repo rate depends on θ and λ . Proposition 11 summarizes the relations.

Proposition 11. Repo rate movements:

- i) r^p rises (falls) as λ increases (decreases).
- ii) r^p rises (falls) as θ decreases (increases).
- iii) The impact λ has on r^p as described in (i) becomes weaker (stronger) as θ gets larger (smaller).

Proof is moved in H. On the one hand, since borrowers' leverage λ incurs risks on the part of lenders, its rise will increase the repo rate. This is stated in (i). On the other hand, θ affects the repo rate in two ways. First, it is positively related to the value, both notional and liquidated, of the asset that serves as collateral. Consequently, when θ is higher the repo lender will allow the securitized bank to borrow at a lower rate. This is stated in (ii). Second, while a rise in λ will lead to an increase in r^p as in (i), the increase will be smaller when the asset value improves with a rise in θ . This is because the enhancement of collateral value weakens the lender's counter-party risk. This is stated in (iii).

In the case of the low asset return r^L , the borrower's loss would be her asset's notional value while the lender's payoff would be the liquidated value of collateral, which is the borrower's asset. In this respect, the securitized bank's expected net revenue can be measured as

$$\Pi^b = \theta(r^H A - r^p D) - (1 - \theta)(1 + r^L)A \quad (3.5)$$

The first term is the bank's net revenue in the case of the high asset return, and the second term is the loss in the case of default. Then the bank's expected return on equity, denoted by $r^b = \frac{\Pi^b}{E}$ (superscript b indicating borrower), can be obtained as

$$r^b = \theta(r^H\lambda - r^p(\lambda - 1)) - (1 - \theta)(1 + r^L)\lambda \quad (3.6)$$

Substituting the incentive constraint, expressed in equation (3.4), into (3.6) yields

$$r^b = \theta r^H \lambda - (1 + r^f - \theta)(\lambda - 1) - (1 - \theta)(1 + r^L)(1 - q)\lambda \quad (3.7)$$

Now, let us take λ as an endogenous variable, chosen by the bank to maximize r^b . Accordingly, it solves the following optimization problem:

$$\max_{\lambda} \theta r^H \lambda - (1 + r^f - \theta)(\lambda - 1) - (1 - \theta)(1 + r^L)(1 - q)\lambda$$

Optimal leverage is obtained by the first-order condition as:

$$\lambda^* = \frac{1}{q'} \left(1 - q + \frac{1 + r^f - \theta(1 + r^H)}{(1 - \theta)(1 + r^L)} \right) \quad (3.8)$$

Since $q' < 0$ is assumed, in order to avoid a degenerate case of negative leverage, an additional condition is adopted that requires the expression in the bracket to be negative. The result yields

$$1 + r^H > \frac{1 - \theta}{\theta}(1 - q)(1 + r^L) + \frac{1 + r^f}{\theta} \quad (3.9)$$

which requires r^H to be sufficiently large. Under this condition, it can be verified how the optimal leverage changes along with a variation of asset prices reflected in θ . Proposition 11 states the result.

Proposition 12. The optimal leverage λ^* rises (falls) as θ rises (falls).

Proof. It is enough to verify the sign of the partial derivative of λ^* , expressed in equation (3.8), w.r.t. θ .

$$\frac{\partial \lambda^*}{\partial \theta} = -\frac{(1+r^L)(r^H-r^f)}{q'(1-\theta)^2(1+r^L)^2} > 0 \quad (3.10)$$

due to $r^H > r^f$ and $q' < 0$. □

Proposition 12 implies that the securitized bank's leverage is procyclical. A change in θ , which determines the asset value, imparts an impact on the bank's leverage behavior in two ways. First, a rise in θ , for instance, lowers r^p as demonstrated in (i) of proposition 11. In the environment of a higher average asset return and a lower borrowing cost, a debt-financed investment would be more profitable; hence, λ will rise. Second, the consequent rise in the leverage ratio pushes the repo rate up as shown in (ii) of proposition 11. However, this feedback effect of λ on r^p becomes smaller as the asset value strengthens, as in (iii) of proposition 11, so that it does not fully offset the initial reduction of r^p . Through these two processes, a rise (fall) in θ leads to a fall (rise) in r^p , which in turn increases (decreases) λ ; hence, procyclical behavior of the securitized bank's leverage.

3.4 A balance sheet model

In this section, I build a macrodynamic model and examine the consequence of procyclical bank leverage within a macro context. The model is built by first specifying distinctive leverage behaviors of financial and nonfinancial sectors in response to asset price developments, then, from these, deriving the demand and supply of bank loans; notice that the nonfinancial sector's leverage shapes the demand for credit while the financial sector's leverage shapes the supply of credit since financial firms

borrow in order to lend. Lastly, the model describes how the procyclical bank leverage generates supply-led boom and bust of credit and, consequently, determines the credit market interest rate in a way that amplifies asset price cycles. A central behavioral specification of the model is founded upon the empirical and theoretical findings, presented in section 3.2 and 3.3 respectively, that the securitized bank's leverage has a sufficiently strong correlation with asset prices.

Nonfinancial sector includes nonfinancial firms and households which are consolidated into one sector, which I call real sector. Financial sector is divided into three subsectors, i.e. commercial bank, investment bank, interchangeably called securitized bank, and asset managing firm. Notations for the real sector and the three financial sectors in order are R , B , X , and N . Notations for the balance sheet variables are used in the same way as in the previous section; A for asset, D debt, E equity, and λ leverage ratio. The sectors associated with these balance sheet variables will be indicated as a subscript. For instance, leverage ratio of the securitized banking sector is denoted by λ_X . A list of the notations can be found in appendix F.

3.4.1 Real sector: firms and households

The real sector's asset, A_R , consists of both real and financial assets. First, there are two types of financial assets; deposit accounts, denoted by M^B , at the commercial bank – cash is included in this category – and nondeposit accounts, denoted by M^N , at insurance company, pension funds, mutual funds, etc., which are managed by asset managers. The real asset, denoted by K , includes the firms' capital and the households' housing, and P_R is some consolidated price index of these real assets.

The real sector's assets are financed by a mix of own funds, i.e. equity, E_R , and debts, D_R . It is assumed that the real sector's debt exclusively consists of bank loan, denoted by L , i.e. $D_R = L$, and particularly that the bank loan finances this sector's positions in the real assets only. In sum, the real sector's balance sheet identity

($A_R \equiv D_R + E_R$) is expressed as

$$P_R K + M^B + M^N \equiv L + E_R \quad (3.11)$$

The first behavioral assumption of the model regards the real sector's leverage ratio in relation to asset prices. Reflecting the empirical observations in section 3.2 where the real sector's leverage is almost flat in relation to housing price and stock market index, it can be specified as a simple linear function of P_R .

$$\lambda_R = \alpha_0 + \alpha_1 P_R, \quad \alpha_0, \alpha_1 > 0 \quad (3.12)$$

where α_1 is close to zero. For the sake of convenience, α_1 will be set to zero, which implies that the real sector's leverage is constant at α_0 . On the other hand, the real sector's equity value will vary, by the accounting definition, along with asset price fluctuations. Hence, when the real sector's equity is specified similarly as a simple linear function of P_R as

$$E_R = \delta_0 + \delta_1 P_R, \quad \delta_0, \delta_1 > 0 \quad (3.13)$$

Asset price determination is modeled with regard to two channels, i.e. expectations channel and funding cost channel. First, when asset prices are expected to rise, asset investors will increase their demand for the assets while the supply will fall since, from the perspective of sellers, the assets can be sold at a higher price in the subsequent periods. Second, obviously a funding cost negatively affects the asset demand through while the asset supply is not affected significantly. In this setting, P_R is an increasing function of its expected future value, denoted by P_R^e , and a decreasing function of the bank lending interest rate, denoted by r^l .⁶

⁶An alternative way to motivate equation (3.14) is to rely on the present valuation asset pricing, where interest rate is negatively correlated to asset price while expected asset price has a positive impact on the latter.

$$P_R = \epsilon_0 + \epsilon_1 P_R^e - \epsilon_2 r^l, \quad \epsilon_0, \epsilon_1, \epsilon_2 > 0 \quad (3.14)$$

Regarding the determination of P_R^e , suppose the case where investors reckon that asset prices cannot rise permanently. There is a certain level of the asset price which is perceived by the market as *too high*. When the current asset price is below this level, investors expect its future value to rise, but as the current price reaches this level the market sharply adjusts its expectation downward. This type of nonlinear relation between the expected asset price and current asset price can be captured by formalizing a change in the former as a nonlinear function of the latter. A possible shape of this function, named g , is depicted in figure 3.6. P_R^* is the level of current asset price which is perceived by the market as unsustainable. When P_R is below this level, investors expect it to increase further but at a slower pace as P_R becomes higher.⁷ As P_R reaches the threshold level, investors expect it to undergo a free fall in the near future.⁸ Formally,⁹

$$\dot{P}_R^e = g(P_R), \quad g' < 0, \quad P_R \leq P_R^* \Leftrightarrow g \geq 0 \quad (3.15)$$

This specification of asset price expectations, along with equation (3.14), excludes a degenerate case where asset prices permanently explode. The dynamics of asset price expectation channel is a stabilizing factor of the model. On the other hand, as will be shown in detail below, the contribution of the interest rate channel to the asset

⁷Reflecting this, the two flat segments of the curve is slightly downward sloping.

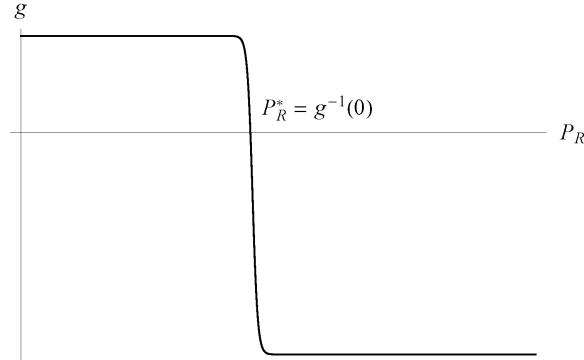
⁸In modeling the euphoria and pessimism in the asset market, it is more appropriate to endogenize the threshold P_R^* . In this chapter, P_R^* is treated as constant only as a first approximation.

⁹The nonlinear curve in figure 3.6 can be approximated by the following function, which is adopted from Ryoo (2010) with a slight change:

$$g(x) = -(g_0 + g_4) + \frac{g_1 + g_0}{1 + e^{-g_2(x-g_3)}}$$

where $g_0, g_1, g_2, g_3, g_4 > 0$.

Figure 3.6: Changes in the expected asset price as a nonlinear function of the current asset price



price stability depends on a specify type of the banking system. More specifically, the interest rate channel operates as a stabilizing factor in the traditional banking system but as a destabilizing factor in the securitized banking system. Consequently, in the securitized banking system, the model will avoid a degenerate case of permanent explosion only when the destabilizing channel, reflected in ϵ_2 , is sufficiently small.

3.4.2 Commercial bank

The commercial bank's total asset, A_B , includes reserves, denoted by H_B , and loans, denoted by L . Bank loans are extended to the real sector. Part of them is taken off from the loan book and packaged into asset-backed securities (ABS). This share is denoted by ω , a securitization rate, which satisfies $0 < \omega < 1$. It is assumed that the ABS is purchased only by the securitized banking sector. Therefore, ω is determined by the securitized bank's activity, which will be discussed shortly.¹⁰ Bank loans mainly finance real assets such as housing and firms' capital and, consequently, the securitized loans will be backed by the performance of these real assets. Hence, the real assets are, in this model, underlying assets for the ABS.

¹⁰The unspecified assumption here is that ω positively affects profitability of the commercial bank and the latter, therefore, is willing to securitize its loan as much as possible.

Commercial banks' debt, D_B , is FDIC-insured deposit held by the real sector; hence $D_B \equiv M^B$. Its balance sheet identity ($A_B \equiv D_B + E_B$) would be expressed

$$L(1 - \omega) + H_B \equiv M^B + E_B \quad (3.16)$$

Liquidity ratio, or reserve ratio, denoted by h_B , is measured as $h_B \equiv \frac{H_B}{D_B}$. Reflecting the fact that the commercial bank is highly regulated, its liquidity ratio and leverage ratio, λ_B , are taken as fixed at a certain policy level.¹¹

3.4.3 Securitized bank

For simplicity, it is assumed that ABS is the only asset of the securitized banking sector. The number of the security and its price are denoted by X and P_X , respectively. The securitized bank's asset position is financed by issuing debts in the repo market, using the ABS as collateral. Accordingly, the securitized bank's debt, D_X , consists of repos, denoted by Q . Its equity is denoted by E_X and is taken as constant, reflecting Adrian and Shin (2010)'s finding of 'sticky equity'. The balance sheet identity, ($A_X \equiv D_X + E_X$), can be expressed as

$$P_X X \equiv Q + E_X \quad (3.17)$$

As the investment bank operates outside of regulatory control, it is supposed that it does not hold reserves. Therefore, the total asset size of this sector equals the total

¹¹Standard credit supply curve is increasing with respect to the lending interest rate: $\frac{\partial L}{\partial r^l} > 0$. But in the case of the 'originate and distribute' model, bank revenues are produced mostly out of fees incurred in various stages across the loan securitization process rather than interest earnings. Consequently, the more the loans are securitized, as measured by a higher ω , the less concerned the banks would be about the level of loan interest rate. The standard credit supply curve will become flatter as securitization rate rises: $\frac{\partial^2 L}{\partial \omega \partial r^l} < 0$.

volume of outstanding bank loans that are packaged into ABS, i.e. $\lambda_X E_X = \omega L$. From this, an expression for ω follows as

$$\omega = \frac{\lambda_X E_X}{L} \quad (3.18)$$

which shows that securitization rate is determined by, given total bank loans, balance sheet policy of the securitized bank, i.e. the latter's equity and leverage. Intuitively, a rise in the securitized bank's leverage, given its sticky equity, enables the commercial bank to securitize more of its existing loans.¹²

In parallel to the real sector leverage in equation (3.12), the leverage behavior of the securitized banking sector can be specified as a simple linear function of the ABS price.

$$\lambda_X = \beta_0 + \beta_1 P_X, \quad \beta_0, \beta_1 > 0 \quad (3.19)$$

The central behavioral assumption underpinning the main analytical result of the model regards the value of β_1 which will be discussed shortly.

Determining the value of ABS is not an easy task. The initial aim of designing these derivatives so as to spread and mitigate risks through pooling and tranching has made it extremely difficult to see the properties of underlying assets. This is particularly true in the case of mortgage-backed securities and even more so in the case of subprime segment (Fender and Scheicher, 2008). I take a simpler and intuitive approach to specify the price of ABS as an increasing function of the underlying asset value¹³ and a decreasing function of the repo rate:

$$P_X = \mu_0 + \mu_1 P_R - \mu_2 r^p, \quad \mu_0, \mu_1, \mu_2 > 0 \quad (3.20)$$

¹²As will be shown in section 3.4.5, ω has a positive impact on the commercial bank's lending capacity.

¹³This assumption is based on the recent experience of the correlation between housing price and price of mortgage-related securities during the subprime mortgage turmoil.

Equation (3.20) reflects the recent observation regarding the housing market boom that the economic environment with low rates of interest led to an appreciation in the underlying assets market and subsequently in the ABS market by drastically increasing the demand for these assets while the asset supply was relatively stable (Perraudin, 2008).

Leverage–asset price correlation for the securitized banking sector can be examined with either type of asset, i.e. the ABS or the underlying assets. The securitized bank’s leverage behavior in response to the ABS price is reflected in β_1 while that in response to the underlying asset price is reflected in $\beta_1\mu_1$ due to $\frac{\partial\lambda_X}{\partial P_R} = \frac{\partial\lambda_X}{\partial P_X} \frac{\partial P_X}{\partial P_R}$ from equations (3.19) and (3.20). In this setup, the securitized bank’s procyclical leverage will be reflected in the value of $\beta_1\mu_1$ being sufficiently large or, if μ_1 is taken as given, β_1 being sufficiently large. A specific level of these parameters will be discussed more rigorously in section 3.5.1.

3.4.4 Asset managing firm

An asset managing firm is an unleveraged entity. It manages the real sector’s nondeposit funds at insurance companies, pension funds, mutual funds, hedge funds, etc. These institutional investors’ funds are voluminous much beyond the deposit insurance limit. Thus, they are invested in the repo market where funds are secured by collateral. Asset managers hold cash as well. Suppose a share, denoted by h_N , of the asset manager’s total asset, A_N , is held in cash and the rest is held in reverse–repo, Q .¹⁴ Accordingly, this sector’s balance sheet identity ($A_N \equiv E_N$) is expressed as

$$Q + h_N A_N \equiv E_N \tag{3.21}$$

Asset managers set repo rate. The repo rate determination is modeled as a combined effect of two sets of dynamics as follows:

¹⁴In this model, the run on repo will be reflected in a surge in h_N .

$$\dot{r}^p = \kappa_1(r^p - r^{p*}) - \kappa_2\dot{P}_R, \quad \kappa_1, \kappa_2 > 0 \quad (3.22)$$

The second term reflects a collateral value risk. Recall the second statement of proposition 11, according to which asset price appreciation, by enhancing collateral value, leads to a reduction in repo rate. In line with this result, the second term formalizes a change in the repo rate as a decreasing function of a change in the underlying asset price. Let us further consider a feedback effect a change in the repo rate might possibly have on asset prices. A lower repo rate relaxes a funding cost burden on the part of the securitized bank and this will raise the demand for assets and hence their prices.

Combining these two mechanisms, it follows that a fall (rise) in r^p will increase (decrease) the securities price, which in turn will reduce (raise) r^p further, and so on. Such destabilizing tendency of the repo rate dynamics is captured by the first term of equation (3.22). It is supposed that there is some normal level of repo rate, denoted by r^{p*} . When the repo rate is lower than this, it tends to fall further, while when it is higher there is a tendency to increase.

This type of unstable dynamics of the repo rate can be justified from the recent disruption of the repo market — so-called the repo run — especially related to low quality collateral. Before the subprime mortgage crisis emerged the spread in the repo market over the federal funds rate was close to 1% point even when the leverage of the borrowers, i.e. broker–dealers, was exceptionally high. However, with the rising mortgage default rate and the collapse of Lehman Brothers, the spread surged and liquidity dried, which pushed up the repo rate even further.

In sum, the repo rate movement is a combined result of these two dynamics, reflected in the two terms of the above equation. While the first component, when considered alone, renders the repo rate unstable, the second component acts as a stabilizing factor that prevents a permanent explosion of the repo rate. In this frame-

work, a sudden spike of r^p from its record low, as observed in the recent episode of repo run, can be explained by a sharp drop in P_R , causing $\dot{P}_R < 0$ sufficiently enough to bring r^p over and above r^{p*} , which in turn, along with $\dot{P}_R < 0$, will intensify $\dot{r}^p > 0$.

For this scenario to be effective, the response of the repo rate to the underlying asset price fluctuations should be sufficiently sensitive so as to reverse the course of the destabilizing force. That is, κ_2 should be sufficiently large. This condition will play an important role in characterizing the full dynamics of the model (see section 3.5.2).

3.4.5 Bank credit market

Now let us illustrate bank credit market, determining quantities and prices in this market. First, from the balance sheet specifications of each sector presented so far, the demand and supply of bank loans can be easily derived. Second, the lending interest rate and the interest rate spread will be determined by the demand and supply rule.

In the model, the real sector's outstanding debt constitutes the total demand for bank loan, which is denoted by L^D ; hence $D_R \equiv L^D$ ¹⁵ Using the balance sheet identity of the real sector, $A_R \equiv D_R + E_R$, and the definition of leverage ratio, the following can be easily obtained.

$$L^D \equiv (\lambda_R - 1)E_R \quad (3.23)$$

On the other hand, the outstanding loans in the balance sheet of the commercial bank shape the total supply of bank credit, which is denoted by L^S . It can be derived from the balance sheet identity of the commercial bank in equation (3.16) and the definition of its leverage and liquidity ratios as follows:

$$L^S \equiv \left(\frac{\frac{\lambda_B}{\lambda_B - 1} - h_B}{1 - \omega} \right) M^B \quad (3.24)$$

¹⁵Note that the demand and supply of bank lending are considered in stock terms.

The above expression presents a couple of interesting aspects of the commercial bank's lending capacity. First, deposit fund, M^B , obtained from the real sector, is a funding source for the bank in supplying loans. It is determined by portfolio behavior of the real sector. The bank, therefore, cannot directly control it. The multiplier involved in this loan creation process is captured by the bracketed term, which is denoted by m .¹⁶ Its determination can be easily verified. Most interestingly, $\frac{\partial m}{\partial \omega} > 0$ reflects that securitization definitely enhances banks' lending capacity, which is what financial innovations are all about. The traditional commercial bank without securitization will have $\omega = 0$.¹⁷

Let us introduce a new parameter v_B which denotes a portfolio coefficient of the real sector, reflecting the share of its total asset held in bank deposits. Then by definition it holds $v_B A_R \equiv M^B$. Using this and the expression of ω in equation (3.18) and the identity $A_R \equiv \lambda_R E_R$, equation (3.24) can be rearranged into

$$L^S = m v_B \lambda_R E_R + \lambda_X E_X \quad (3.25)$$

where $m = \frac{\lambda_B}{\lambda_B - 1} - h_B$. In comparison to equation (3.24), the above equation shows how the total outstanding bank loans, originally extended by the commercial banks, are divided between commercial banks and securitized banks through loan securitization. The two terms in the equation represent each of these.¹⁸

By definition, the excess demand for bank loans, denoted by Φ , is

¹⁶It is different from the conventional money multiplier, which converts central bank money into commercial bank money.

¹⁷A higher liquidity ratio has the opposite effect, i.e. $\frac{\partial m}{\partial h_B} < 0$. But $\frac{\partial m}{\partial \lambda_B} < 0$ appears to contradict what we know about bank leverage. With given equity, a higher bank leverage ratio should imply larger bank asset, which is nothing but more bank lending. However, since what is given in equation (3.24) is bank liability, i.e. deposits, a higher λ_B has the opposite effects.

¹⁸In obtaining this expression, the securitization rate ω in the money multiplier m has been resolved into the second term.

$$\Phi \equiv L^D - L^S \quad (3.26)$$

Suppose that the loan rate, r^l , is determined by interest rate spread over the repo rate.¹⁹

$$r^l = r^p + \xi \quad (3.27)$$

where ξ denotes the interest rate spread. The spread is determined by an interaction between the demand and supply forces in the bank loan market. Hence it is an increasing function of the excess demand for bank credit.

$$\xi = \tau_0 + \tau_1 \Phi, \quad \tau_0, \tau_1 > 0 \quad (3.28)$$

τ_0 is a constant equilibrium level of the spread where the demand and supply equilibrate with each other. When the bank credit market is in excess demand, i.e. $\Phi > 0$, the spread will rise above τ_0 and in case of excess supply it will fall below τ_0 . The spread will be at its equilibrium level, τ_0 , only when $\Phi = 0$. Introducing a constant level of equilibrium spread can be justified within the context of this chapter which describes a short-run fluctuation without a long-run trend.²⁰

3.4.6 Summary of the model

The model consists of twelve endogenous variables and twelve equations, which are reproduced in appendix G. As the basic aim of the model is to explain a persistent boom–bust cycle of asset prices, the underlying asset price P_R is modeled as a function of its expected future value P_R^e and a borrowing cost r^l . More importantly, P_R feeds

¹⁹The repo rate acts a short-term interest rate in this model.

²⁰A more economically intuitive way to model the spread would be to have $\dot{\xi} = \tau_2 + \tau_3 \Phi$ rather than equation (3.28). In this case, the model will be reduced to a three-dimensional differential equations system. As will be seen below, by adopting equation (3.28), it becomes possible to reduce the model to a two-dimensional differential equations system, which is much more convenient to deal with. My approach can be compared to the existing models of cycles where the bench mark rate such as the federal funds rate is taken as constant.

back into both P_R^e and r^l , thereby generating endogenous loops as depicted in figure 3.7a. The $P_R \rightarrow P_R^e$ link is modeled simply as a nonlinear relation as in figure 3.6.

On the other hand, the other feedback mechanism, the $P_R \rightarrow r^l$ link, which is the heart of the model, is much more complicated. Figure 3.7b illustrates the key causal relations of that link. The main thrust is the underlying asset price affecting the leverage behavior of the real sector and that of the securitized banking sector, which in turn shape, respectively, demand and supply of bank loans, which, finally, affect the loan rate. Section 3.5.1 discusses this in great detail.

The model's full dynamics as a combined result of all of the four links in figure 3.7a is presented in section 3.5.2. The twelve equations and identities of the model are reduced to two differential equations in P_R^e and r^p , which are two state variables of the model. Once the model is solved, therefore, all the variables will be expressed as a function of P_R^e and r^p , which reflect the two channels of asset price determination, i.e. expectations channel and interest rate channel. The full solution of the model is presented in appendix G.²¹ The main analytical result of the model, discussed in section 3.5.2, regards the conditions under which the system exhibits a limit cycle behavior.

3.5 Model analysis

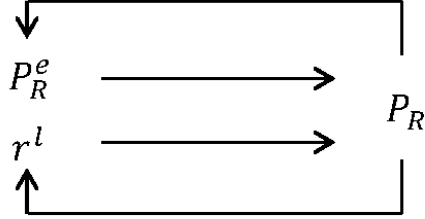
3.5.1 Analytics of procyclical bank leverage

Let us formalize the $P_R \rightarrow r^l$ link. It will be shown that this link exhibits a peculiar pattern depending on the procyclicality of bank leverage. The key mechanism here is that each of the nonfinancial and financial sector's leverage behavior shapes the demand and supply of bank credit, respectively. To see this, expand the identities

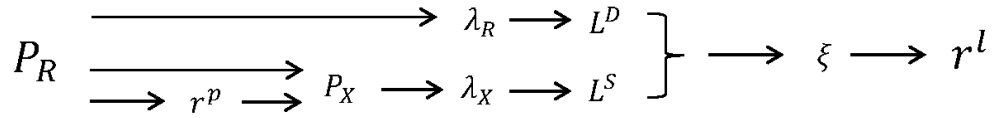
²¹All the mathematics involved in solving and analyzing the model in the rest of the chapter are obtained with *Mathematica* as they are somewhat messy to be obtained simply by hand. *Mathematica* codes will be available directly from the author upon request.

Figure 3.7: Causal links of the model

(a) Dynamics of the underlying asset price P_R in relation to its expected future value P_R^e and the bank lending interest rate (funding cost) r^l



(b) Details of the $P_R \rightarrow r^l$ link



of L^D and L^S in equations (3.23) and (3.25) by using behavioral equations (3.12), (3.13), (3.19), and (3.20).

$$\begin{aligned}
 L^D &= \delta_0(\alpha_0 - 1) + \delta_1(\alpha_0 - 1)P_R \\
 L^S &= v_B m \alpha_0 \delta_0 + \beta_0 + \beta_1 \mu_0 - \beta_1 \mu_2 r^p + (v_B m \alpha_0 \delta_1 + \beta_1 \mu_1)P_R
 \end{aligned}
 \tag{3.29}$$

The demand and supply of bank credit are now expressed as a linear function of P_R . By comparing the demand response to the asset price, i.e. $\frac{\partial L^D}{\partial P_R} = \delta_1(\alpha_0 - 1)$, and the supply response, $\frac{\partial L^S}{\partial P_R} = v_B m \alpha_0 \delta_1 + \beta_1 \mu_1$, an interesting analytical result regarding the procyclicality of the securitized bank leverage is obtained as demonstrated in lemma 2. Remember β_1 is the marginal response of the securitized bank's leverage to the ABS price.

Lemma 2. The level of β_1 and the relative response of the demand and supply of bank loan to the asset price are related in the following way:

$$\begin{aligned}\frac{\partial L^D}{\partial P_R} < \frac{\partial L^S}{\partial P_R} &\iff \beta_1 > \frac{(\alpha_0(1 - v_B m) - 1)\delta_1}{\mu_1} \iff W_1 > 0 \\ \frac{\partial L^D}{\partial P_R} > \frac{\partial L^S}{\partial P_R} &\iff \beta_1 < \frac{(\alpha_0(1 - v_B m) - 1)\delta_1}{\mu_1} \iff W_1 < 0\end{aligned}$$

where $W_1 = \beta_1\mu_1 - (\alpha_0(1 - v_B m) - 1)\delta_1$.

Lemma 2, which can be proved by a simple algebra, states that β_1 being sufficiently large so as to have $W_1 > 0$ is analytically equivalent to the supply response to the asset price being stronger than the demand response; if β_1 is not so large that $W_1 < 0$ is the case, then this is analytically equivalent to the demand response being stronger. This result provides a useful gauge to rigorously determine how large β_1 should be in order to say the leverage is procyclical. Accordingly, the following definition is adopted.

The leverage ratio is said to be procyclical when $\beta_1 > \frac{(\alpha_0(1 - v_B m) - 1)\delta_1}{\mu_1}$ holds so as to generate $W_1 > 0$. The securitized banking system is characterized by having this property and therefore, due to lemma 2, by $\frac{\partial L^D}{\partial P_R} < \frac{\partial L^S}{\partial P_R}$. On the contrary, the traditional banking system is characterized by the opposite, i.e. $\beta_1 < \frac{(\alpha_0(1 - v_B m) - 1)\delta_1}{\mu_1}$ so that $W_1 < 0$ holds, and hence $\frac{\partial L^D}{\partial P_R} > \frac{\partial L^S}{\partial P_R}$.

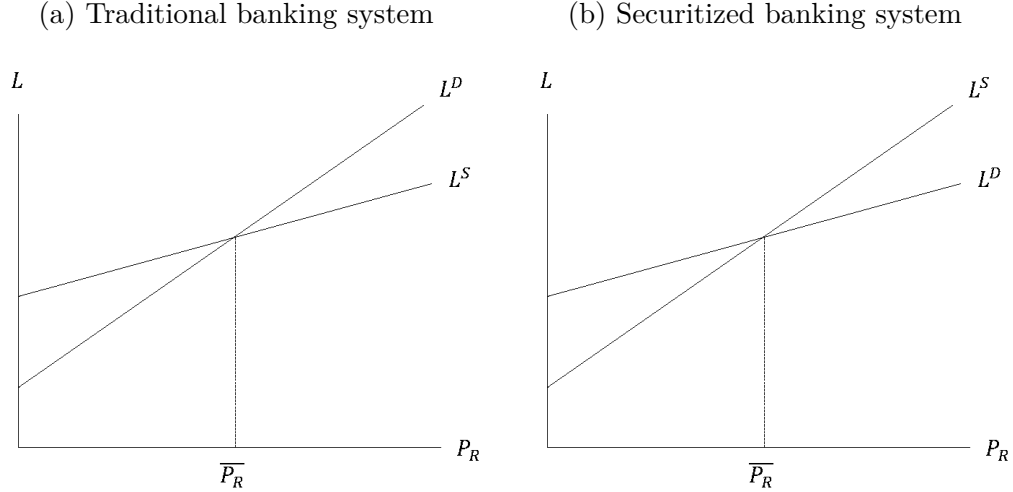
Figure 3.8 illustrates the difference between the two distinctive banking systems in terms of the demand and supply responses to the underlying asset price. In order to see how this difference affects the bank lending interest rate, let us use the expressions for demand and supply of bank loan in (3.29) to obtain the excess demand as a function of the underlying asset.

$$\Phi = -\beta_0 + (\alpha_0(1 - v_B m) - 1)\delta_0 - \beta_1\mu_0 + \beta_1\mu_2r^p - W_1P_R \quad (3.30)$$

By examining the response of the excess bank credit demand to the asset price, i.e.

$\frac{\partial \Phi}{\partial P_R} = -W_1$, it easily follows that

Figure 3.8: Demand and supply of bank loan as a function of the underlying asset price in the two different banking systems



Lemma 3. If $W_1 > 0$, then $\frac{\partial \Phi}{\partial P_R} < 0$; if $W_1 < 0$, then $\frac{\partial \Phi}{\partial P_R} > 0$

According to lemma 3, in the traditional banking system the excess bank loan demand rises (falls) when the underlying asset price rises (falls), which is normally expected, and the opposite is true in the securitized banking system; that is, when the asset price rises (falls) the excess bank loan demand falls (rises).

The impact the excess loan demand has on the lending rate can be obtained from equations (3.27) and (3.28) as $\frac{\partial r^l}{\partial \Phi} = \tau_1 > 0$. Combining this result with lemma 3, it follows that, using $\frac{\partial r^l}{\partial P_R} = \frac{\partial r^l}{\partial \Phi} \frac{\partial \Phi}{\partial P_R}$,

Lemma 4. If $W_1 > 0$, then $\frac{\partial r^l}{\partial P_R} < 0$; if $W_1 < 0$, then $\frac{\partial r^l}{\partial P_R} > 0$.

Lemma 4 characterizes the $P_R \rightarrow r^l$ link in figure 3.7a. Similarly to lemma 3, it states that while in the traditional banking system the lending interest rate moves in the same direction with the underlying asset price movement, which is normally expected, in the securitized banking system it moves in the opposite direction; i.e.

the lending rate fluctuates counter-cyclically, rising in case of asset price depreciation and falling in case of appreciation.

When this result is combined with the $r^l \rightarrow P_R$ link, it can be seen that the interest rate channel operates as a stabilizing factor in the traditional banking system while as a destabilizing factor in the securitized banking system. On the other hand, the specifications for the $P_R \rightarrow P_R^e$ link and the $P_R^e \rightarrow P_R$ link ensure that the expectations channel operates as a stabilizing factor, regardless of a specific type of banking system. Consequently, the stability properties of the securitized banking system will depend on the effectiveness of these channels while the traditional banking system will always be stable.

3.5.2 Full dynamics: limit cycle

In order to understand the full dynamics of the model, let us reproduce the model in 2D system.

$$\begin{aligned}\dot{P}_R^e &= g(P_R) \\ \dot{r}^p &= \frac{g\epsilon_1\kappa_2 + (r^p - r^{p*})\kappa_1 W_2}{W_3}\end{aligned}\tag{3.31}$$

where from appendix ?? we know that P_R is solved as

$$P_R = -\frac{U_2 + \epsilon_1 P_R^e - (\epsilon_2 + \beta_1 \epsilon_2 \mu_2 \tau_1) r^p}{W_2}$$

with²²

$$\begin{aligned}W_1 &= \beta_1 \mu_1 - (\alpha_0 (1 - v_B m) - 1) \delta_1 \\ W_2 &= -1 + \epsilon_2 \tau_1 W_1 \\ W_3 &= W_2 + \epsilon_2 \kappa_2 (1 + \beta_1 \mu_2 \tau_1)\end{aligned}\tag{3.32}$$

²²See appendix ?? for the expression of U_2 , which is omitted here since it is inessential for the following discussion.

The Jacobian matrix evaluated at the steady-state, denoted with an upper bar, is obtained as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial P_R^e}{\partial P_R^e} & \frac{\partial P_R^e}{\partial r^p} \\ \frac{\partial r^p}{\partial P_R^e} & \frac{\partial r^p}{\partial r^p} \end{bmatrix} = \begin{bmatrix} -\frac{\epsilon_1 \bar{g}'}{W_2} & \frac{\epsilon_2(1+\beta_1\mu_2\tau_1)\bar{g}'}{W_2} \\ -\frac{\epsilon_1^2 \kappa_2 \bar{g}'}{W_2 W_3} & \kappa_1 \frac{W_2}{W_3} + \epsilon_1 \bar{g}' \left(\frac{1}{W_2} - \frac{1}{W_3} \right) \end{bmatrix}$$

where $\bar{g}' = \frac{\partial g}{\partial P_R} \Big|_{P_R=\bar{P}_R} < 0$, the sign of which is due to the shape of the function g , i.e. $g' < 0$.

These expressions show that the stability of the system depends on the signs of W_1 , W_2 , and W_3 . It is already known from lemma 2 that W_1 is related to the securitized bank's leverage behavior. It will be further shown that W_2 is related to the effectiveness of the interest rate channel and that W_3 regards the repo market behavior. By implication, the existence of limit cycle in the model of securitized banking system depends on the conditions regarding these three aspects.

In studying the limit cycle behavior of the model, I rely on Hopf bifurcation theorem (Gandolfo, 2010). According to this theorem, a bifurcation occurs when the system shifts from stable fixed point to stable cycle as a system parameter gradually changes. The change in the system parameter underpins the shift in the dynamic stability properties of the model. In the model κ_1 will be used as the bifurcation parameter. Remember κ_1 is the coefficient that captures the destabilizing force in the repo market expressed (see equation (3.22)). The central result of the model is presented in proposition 13.

Proposition 13. Under the conditions of $W_2 < 0$ and $W_3 > 0$, the system undergoes a bifurcation at $\kappa_1 = \kappa_1^* = \frac{\epsilon_1 \bar{g}'}{W_2}$. When the bifurcation parameter κ_1 passes through its critical value κ_1^* , a limit cycle emerges by way of Hopf bifurcation theorem.

Proof of proposition 13 is moved to appendix H. Here, let us consider economic implications of each of the three conditions for the existence of limit cycle. Regarding the first condition $W_2 < 0$, consider the following sign relations.

Lemma 5. Since $W_2 = -1 + \epsilon_2 \tau_1 W_1$,

1. in the case of $W_1 > 0$,
 - (a) $W_2 > 0$ holds as long as $\epsilon_2 > \frac{1}{\tau_1 W_1}$ and
 - (b) $W_2 < 0$ holds as long as $\epsilon_2 < \frac{1}{\tau_1 W_1}$.
2. In the case of $W_1 < 0$, $W_2 < 0$ holds always.

Remember ϵ_2 is the marginal response of the asset price to the bank lending rate, thus reflecting the strength of the interest rate channel. According to lemma 5, in the securitized banking system ($W_1 > 0$) the sign of W_2 depends on the strength of the interest rate channel, while it is always negative in the traditional banking system ($W_1 < 0$).

As the focus of this chapter is to understand a more recent banking system characterized by securitized banking, the analysis below will be confined to the case of $W_1 > 0$. In this context, the first condition, $W_2 < 0$, states that for the existence of the limit cycle in the securitized banking system the interest rate channel should be sufficiently weak. Since in the securitized banking system the interest rate channel has a destabilizing tendency as discussed in the previous subsection, limiting its strength to a certain level helps the system avoid a permanent explosion.

The other two conditions of proposition 13 regard the two elements in the repo rate dynamics. Let us reproduce the equation for the repo rate movement:

$$\dot{r}^p = \kappa_1(r^p - r^{p*}) - \kappa_2 \dot{P}_R$$

Remember the first term reflects a destabilizing force characteristic in the repo rate movement and the second term captures the sensitivity of the repo investors to the underlying asset price development.

By using the expression for W_3 in (3.32) it can be seen that the second condition, $W_3 > 0$, imposes, given the other parameters, a certain restriction on κ_2 :

$$\kappa_2 > -\frac{W_2}{\epsilon_2(1 + \beta_1\mu_2\tau_1)}$$

This requirement demonstrates that for the existence of limit cycle the repo investors have to be sufficiently sensitive to changes in the underlying asset price, adjusting the repo rate quite drastically. It is in line with the second result of proposition 11 according to which the repo rate adjusts in the opposite direction from the value of assets that serve as collateral. On the other hand, the last condition $\kappa_1 = \frac{\epsilon_1\bar{g}'}{W_2}$, which regards Hopf bifurcation point, requires that in order for the system to generate a limit cycle, the destabilizing force in the repo market has to be neither too small nor too large but be near the neighborhood of a critical value $\kappa_1^* = \frac{\epsilon_1\bar{g}'}{W_2}$.

These two conditions require κ_1 and κ_2 to be at a proper range so that they together shape the repo market dynamics in a way that induces the system to generate a limit cycle. For instance, a sufficiently large κ_2 will ensure that in the environment of low repo rate — being below the normal rate r^{p*} so that it tends to keep falling and stay lower — a substantial drop in P_R will change the course of r^p by pushing it up above r^{p*} , which will now force r^p to rise further. The similar drastic change in the direction of the repo rate movement will take place when there is a rise in P_R in the high repo rate environment.

If, on the contrary, κ_2 is relatively small and, accordingly, the repo investors are not sufficiently sensitive to asset price developments, i.e. $W_3 < 0$, then there would be no such mechanism that reverses the destabilizing tendency in the repo market, and thus the repo will keep rising or keep falling (in which case the stability property of the system will be an unstable node). Similarly, if κ_1 is too larger or too smaller compared to the critical value $\kappa_1^* = \frac{\epsilon_1\bar{g}'}{W_2}$, the system will cyclically converge to the steady-state (stable focus) or cyclically diverge (unstable focus), respectively.

In all, proposition 13 states that the securitized banking system will experience persistent boom–bust cycles of asset prices, bank leverage, and liquidity under the

three conditions: i) underlying asset market is driven primarily by self-realizing expectations rather than being constrained by funding cost conditions, ii) in setting the repo rate, lenders are sufficiently sensitive to the asset price developments regarding the collateral value, and iii) there is a certain degree of destabilizing tendency in the repo rate movement.

With the signs of W_1 , W_2 , and W_3 required for the existence of limit cycle and the critical value of the bifurcation point $\kappa_1^* = \frac{\epsilon_1 g'}{W_2}$, the Jacobian matrix of the system now obtains definite signs:

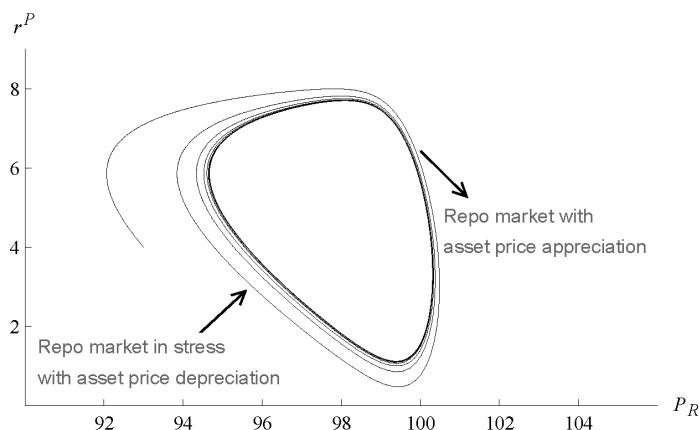
$$\mathbf{J} = \begin{bmatrix} \frac{\partial \dot{P}_R^e}{\partial P_R^e} & \frac{\partial \dot{P}_R^e}{\partial r^p} \\ \frac{\partial \dot{r}^p}{\partial P_R^e} & \frac{\partial \dot{r}^p}{\partial r^p} \end{bmatrix} = \begin{bmatrix} - & + \\ - & + \end{bmatrix}$$

These signs clearly reflect that while the repo rate is behaving in a destabilizing way $\frac{\partial \dot{r}^p}{\partial r^p} > 0$, the dynamics of asset price expectation operates as a stabilizing mechanism in the system $\frac{\partial \dot{P}_R^e}{\partial P_R^e} < 0$. Due to the risk in collateral value, the repo market is susceptible to widely fluctuating asset price expectations, thus lowering (raising) the change in the repo rate when expectations enhance (deteriorate); $\frac{\partial \dot{r}^p}{\partial P_R^e} < 0$. On the other hand, the asset price appreciation reinforced by a fall in the repo rate²³ cannot continue without bound due to the nonlinearity in the asset price expectation dynamics. The decrease in the repo rate will ultimately lower the rise in the expected asset price; $\frac{\partial \dot{P}_R^e}{\partial r^p} > 0$.

Figure 3.9 depicts the limit cycle behavior of the full model linearized around the steady-state. While the 2D system is in P_R^e and r^p , since P_R is what is actually observed in the market, I have presented the limit cycle in P_R and r^p . Starting from some arbitrary initial values for P_R and r^l near the steady-state, the model eventually generates a limit cycle. The two episodes that are characteristic to the repo market

²³This can be seen by verifying from the solution for P_R that $\frac{\partial P_R}{\partial r^p} < 0$ holds.

Figure 3.9: The limit cycle of the underlying asset price and repo rate



cycle is immediately observable; the one where the repo rate falls with asset price appreciation and the other where the repo rate rises with asset price depreciation.

Since the two state variables P_R^e and r^P exhibit cyclical behavior, all the other variables of the model, which are solved as a function of the state variables, will also evolve cyclically. Their complete descriptions are illustrated in appendix ???. To clarify the intuition, let us use the solution of the model listed in appendix ?? and examine the partial derivative of each with respect to the two state variables. With the conditions $W_2 < 0$ and $W_3 > 0$ which are required for the existence of a limit cycle, these partial derivatives obtain definite signs which otherwise would have been ambiguous. Therefore, the relations that emerge from this exercise provide important insights regarding the mechanism underlying the limit cycle. The result is demonstrated in table 3.1. The first two columns correspond to the signs of the Jacobian matrix. It can be easily verified that the rest of the signs are exactly as expected.

A central element that drives these results is the leverage behavior of the securitized banking sector. A rise in the expected asset price, through raising the current underlying asset price ($\frac{\partial P_R}{\partial P_R^e} > 0$) and hence asset-backed securities price ($\frac{\partial P_X}{\partial P_R^e} > 0$), ultimately induces the securitized banks to increase their leverage ($\frac{\partial \lambda_X}{\partial P_R^e} > 0$). This

Table 3.1: Comparative dynamic analysis: partial derivatives of the endogenous variables with respect to the two state variables, P_R^e and r^p .

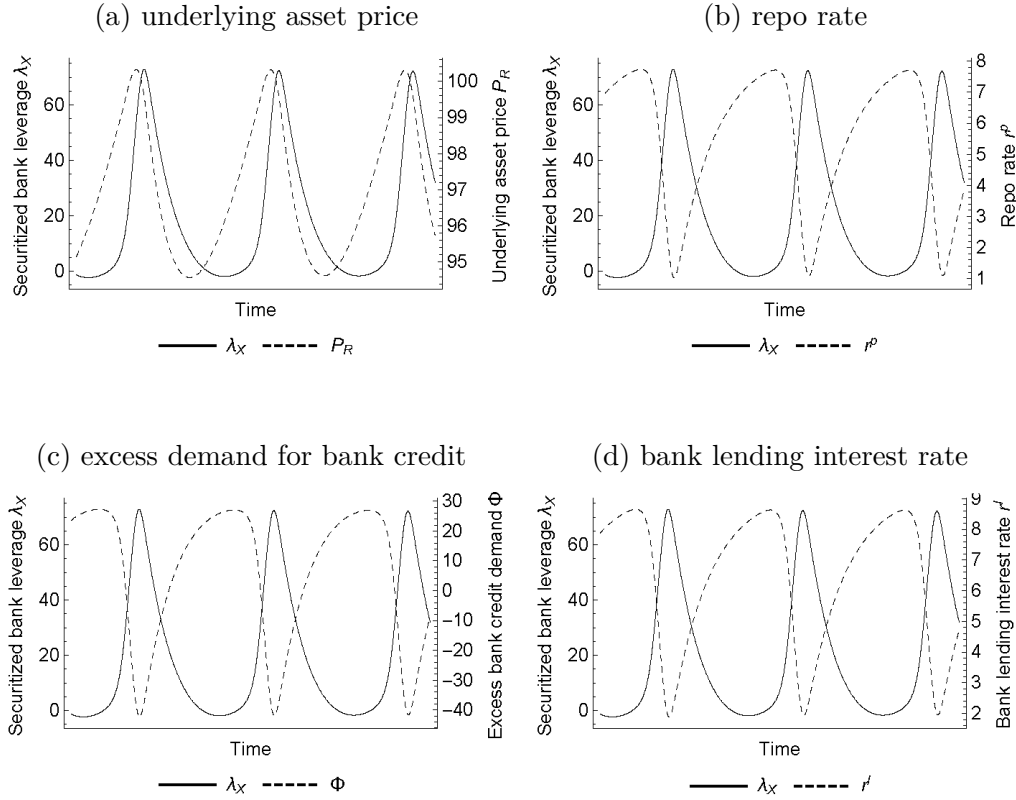
	\dot{P}_R^e	\dot{r}^p	P_R	P_X	λ_X	Φ	ξ	r^l
P_R^e	-	-	+	+	+	-	-	-
r^p	+	+	-	-	-	+	+	+

type of procyclical movement of bank leverage shapes the total bank credit supply to be more responsive to asset market developments than the total bank credit demand is. As a consequence, the improvement in the market expectation will increase the bank credit supply more than the demand, resulting in a fall in the excess demand ($\frac{\partial \Phi}{\partial P_R^e} < 0$), which in turn will suppress the interest rate spread ($\frac{\partial \xi}{\partial P_R^e} < 0$) and thus the lending interest rate ($\frac{\partial r^l}{\partial P_R^e} < 0$).

This mechanism is visualized in simulations in figure 3.10. Panel 3.10a displays that the underlying asset price drives and hence leads the bank leverage with the two variables moving in close step. However, the bank leverage moves in the opposite direction from the funding cost measured by the repo rate as shown in the second panel. Impacts of the procyclicality of bank leverage on the bank credit market are depicted in the lower two panels. First, a rise in the bank leverage enhances the market condition by increasing the supply while its fall decreases the supply. As a result, the excess demand for bank credit moves in the opposite direction from the bank leverage as depicted in panel 3.10c. Such changes in the demand and supply condition would be reflected in changes in the bank lending interest rate and consequently the latter will also move in the opposite direction from the bank leverage as shown in panel 3.10d. Ultimately, the changes in the lending rate will impart an impact on the asset price in a way that reinforces its cyclical movement.

This is in contrast to the traditional banking system where the banking sector has little, or less, incentive to manage its balance sheet in step with asset price fluctuations. Consequently, the total supply of bank credit will be less sensitive to

Figure 3.10: The relation of the securitized bank leverage with

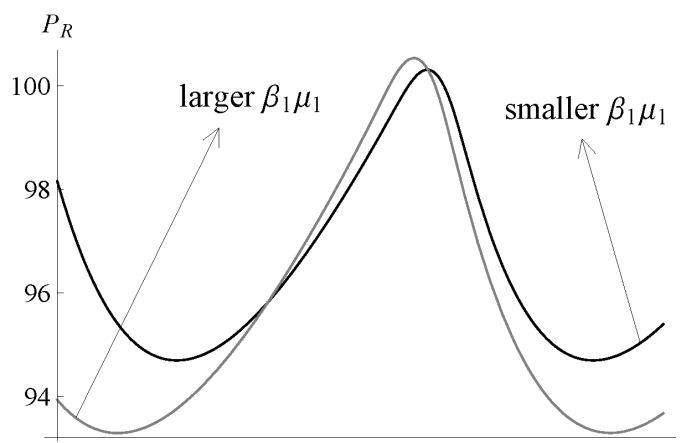


asset prices than the total demand is. Asset price appreciation will thus raise the excess demand for bank loans, which in turn will increase the spread and the market interest rate. The initial asset value appreciation will eventually be suppressed.

The key difference is that in the securitized banking system the interest rate does not develop procyclically and hence in a way that stabilizes the credit market as it does in the traditional banking system; rather, the interest rate falls with asset price appreciation and rises with depreciation, i.e. counter-cyclically, thereby amplifying the asset price fluctuation further. The main thrust of the discussion thus far has been that this is due to the procyclical leverage of the securitized banking sector.

However, recall from proposition 13 that $W_1 > 0$, which implies sufficiently large β_1 and thus procyclical bank leverage, is not a necessary condition for the *existence*

Figure 3.11: Amplifying effect of procyclical bank leverage



of the limit cycle. Then what is the role of the procyclical bank leverage regarding the limit cycle?

A close look at the specific expressions of the solution of the model laid out in appendix G shows that an increase in β_1 reinforces the effect of interactions among the endogenous variables thereby amplifying the ups and downs of the cycle.²⁴ Numerically, this will be reflected in an intensification of the result of the comparative dynamic analysis in table 3.1. For instance, from the solution for P_R we can easily see that the impact P_R^e has on P_R becomes stronger when β_1 is larger, by verifying that the absolute value of $\frac{\partial P_R}{\partial P_R^e} = -\frac{\epsilon_1}{W_2}$ increases when β_1 rises.

Figure 3.11 visualizes the amplifying effect of procyclical bank leverage on asset prices. Note that here the procyclical bank leverage is measured by the marginal impact of P_R on λ_X , i.e. $\beta_1 \mu_1$, instead of by that of P_X on λ_X , i.e. β_1 , which are two alternative measures adopted in this chapter. It is clear that the procyclicality of securitized bank's leverage with larger $\beta_1 \mu_1$ magnifies the asset price cycle.

²⁴This result resonates with the main thesis of financial accelerator model where financial frictions amplify external shocks and thus aggravate the impact on the real economy (Bernanke and Gertler, 1989, e.g.).

3.6 Conclusion

I have presented a macrodynamic balance sheet model that replicates the high drama of 2008 marked by securitization, housing market trouble, the run on money markets, etc. The main drivers of persistent cycles of asset prices, bank leverage, and liquidity are the dynamics of asset price expectation and repo rate behavior. First, while the market for underlying assets exhibits an inflationary spiral led by self-realizing expectations, asset prices do not explode without bound but finally collapse as investors at some point reckon that the current asset price is *too high* thereby sharply adjusting their expectations.

Second, as the repo transactions are secured by collateral and as the collateral value is directly affected by the underlying asset price, repo investors are typically exposed to counter-party risk and collateral value risk. Consequently, repo rate is set in a highly unstable way, falling when asset prices appreciate and rising when they depreciate. More importantly, this provides an incentive for the securitized bank to manage its leverage procyclically in a way that ultimately aggravates the asset price cycle.

While the model contains a number of crucial features of the recent financial crisis and replicates them analytically, it simplifies some other related aspects. First, the real sector dynamics has not been dealt with sufficiently. Nonfinancial firms and households demand bank loans to finance their portfolios and hence shape the total demand for bank loans. However, income generation of the real sector is not considered in the model. Second, while the asset price expectation changes endogenously to the current asset price, a specific degree of the change is exogenous to the model, given by the specific form of function g (see figure 3.6). This makes the asset price expectation not endogenous enough.

One possible way to address these two issues simultaneously is to make the expectation formation as a function not only of the current price but also of profitability

and cash flows in the real sector. As for the specific shape of the function g , the parameters g_3 and g_4 are particularly crucial.²⁵ The former determines P_R^* which is the level of a current price perceived by the market as unsustainable. On the other hand, g_4 shapes the vertical segment of the curve, which determines the speeds of expectation adjustment. In order to highlight the connection between the financial cycle and the economic performance of the real sector, it may be desirable to endogenize these two parameters in relation to, for instance, income share, debt-to-income ratio, and corporate profitability.

Regarding the repo market dynamics, my result suggests that its vulnerability to asset price fluctuations makes it a key causal factor to the severity of the financial cycle. Since repo funds are not supported by the government guarantee, investors in this market tend to pull back in times of market stress, either requiring higher returns or even withdrawing funds. It raises a question on regulating this segment of financial markets.

Recently-implemented macroprudential regulations such as minimum capital and liquidity requirements have made short-term funding more expensive. As a result, tri-party repo contracts in the U.S. have fallen from its peak of \$2,800bn in early 2008 to about \$1,600bn in October 2014. However, not only is repo borrowing still the largest source of funding for broker-dealers but the core vulnerability has not gone away. The key problem of repo transaction is that there is no third-party that guarantees the deal credible during the market stress. Accordingly, the possibility of investors not being able to receive their cash back or to sell collateral even at a firesale price still lingers. Hence, it is highly probable that lenders will panic again and run on repo at the first sign of disaster thereby bringing about another crisis.

²⁵See footnote 9 for the mathematical form of g .

A fundamental way to address the problem, other than directly limiting repo activity, would be to extend implicit and explicit public backstop such as FDIC insurance or low-cost loans from the Fed discount window to the short-term money market. In fact, these measures were what the Fed had adopted temporarily in 2008. In terms of my model, they will lower κ_2 which captures the degree of the repo market's susceptibility to asset price movements. Consequently it becomes possible that the system exhibits stable focus, which is a stable dynamics, instead of limit cycle.

While the significant role played by repo transactions in the fall-down of two giant investment banks in 2008 has been well recognized, Rosengren the Boston Fed president has recently commented “[u]nfortunately that potential for problems has not been fully addressed since the crisis” (Rosengren, 2014). The key result of my model echoes this observation.

CONCLUSION

The current status of macroeconomic theory especially on the topic of finance and banking has evolved from those evidenced in the statements by Gertler (1988) and Lavoie (1995) quoted at the beginning of this dissertation. Remarkable attempts are taking place from both New Keynesian and Post Keynesian traditions to explicitly incorporate a financial system in macroeconomics models, and this dissertation has aimed to contribute to this notable trend.

Then what is an explanatory power of the macro models presented in this dissertation that makes them distinctive from those that do not include finance? What have we gained from them? The extended model of circuit of capital in chapter 1 enables us to see how the relation between growth and profitability captured in the Cambridge equation changes when profit-making financial capitalists are incorporated. Among others, it allows to examine profitability of capitalist economy with a distinction not only between gross profit rate and net return on equity but also between profit rate of industrial capital and profit rate of financial capital.

From this we see that when finance is considered, the Cambridge equation holds in regard to the net return on equity of industrial capital rather than to the gross profit rate. An implication is that what matters for economic growth is not gross rate of profit but the net return on equity. In particular, the model demonstrates that when the interest payments burden is heavy, growth capacity of the system could be undermined. These findings contrast to the baseline model of circuit of capital and the original Cambridge model which were derived by abstracting from a profit-making financial intermediary sector and by assuming an equality between the rate of profit and the rate of interest.

On the other hand, the extended circuit of capital model in chapter 2 shows how growth rate and interest rate are determined in equilibrium in relation to demand-side and supply-side factors of the bank lending market. By identifying nonbank firms that demand for bank credit and the banking firms that supply loans, the model allows to distinguish and compare between growth led by the leverage of nonbank firms and growth led by the leverage of banks. In particular, this approach provides a distinctive theoretical framework in explaining distinctive economic regimes with various combinations between growth rate and interest rate where the correlation between the two could be either positive or negative.

The categorization of firm leverage-led vs. bank leverage-led growth regimes presented in chapter 2 compares to that of profit-led vs. wage-led growth which relates growth to income distribution. It also compares to the categorization of debt-led vs. debt-burden growth that relates growth to indebtedness. While the latter approach that focuses on debt examines a financial aspect of growth which is absent in the former approach that focuses on income distribution, it does not pay attention to an important distinction between indebtedness of nonbank sectors and that of the bank. On the other hand, by explicitly formalizing the banking sector in the model, chapter 2 enables us to see distinctive growth impacts of firm leverage and bank leverage.

A distinction between demand and supply of bank lending is an important setup in the model in chapter 3 as well. In addition to this, chapter 3 divides the banking sector into three subsectors and formalizes nontraditional banking such as loan securitization and collateralized short-term borrowing. While the number of macro models that include a banking sector is growing, those that distinguishes between commercial banks and investment banks are only a few in number. By specifying three different financial firms, the model in chapter 3 allows to theorize the changed

nature of financial intermediation and explain why the financial system has become more unstable compared to the traditional commercial banking system .

There are several points to be made for further development of this dissertation and for the future research. There are three ways chapter 1 can be further developed. First, simulation exercises in chapter show the convergence of the model to a steady-state for given parameter values. It would be interesting to add exogenous shocks and see how the model response over time. Second, we can check if the modified version of the Cambridge equation-type result, i.e. $r_k = \frac{g}{p_k}$, and the decomposition of the net return on equity of nonfinancial firms, i.e. $r_k = r\lambda_k(1 - \frac{1}{\eta})$, and the decomposition of the bank profit rate, i.e. $r_b = i_L$, match empirical data. Third, while the extended model with a banking sector reveals that the bank profit rate is positively related to interest rate and the bank leverage, the interest rate is taken as constant. It would be interesting to see how the Cambridge equation would be modified when the interest rate is endogenized as in chapter 2.

Regarding chapter 2, while it provides a novel perspective of growth in terms of leverages of those who demand credit and those who supply credit, which is absent in the debt-burden vs. debt-led growth framework, it does not capture negative consequences of financial fragility associated with an increasing indebtedness. This is because the finance lag is taken constant, which does not allow the nonbank firm sector's investment behavior to respond positively or negatively to the changing fragility of the system. In order to address this issue, it would be desirable to add a behavioral specification of the finance lag to the model. Minsky's works on financial instability and fragility could be a helpful theoretical resource for this.

The model in chapter 3 faces a similar issue. In analyzing the contemporary financial system by providing a macro model that includes a three-tier banking system and in focusing on the financial behavior of the banking sector, I had to highly simplify investment and financial behaviors of the nonbank sectors. In this sense, financial

cycles of asset prices and bank leverage derived in the model are financial sector-driven. This makes it difficult to sufficiently investigate the role of the nonbank sector, either nonfinancial firms or households, in driving the boom–bust financial cycles. Therefore, it would be desirable to formalize, at least, investment and consumption behaviors in response to financial fluctuations.

In all, it turns out that establishing a realistic investment function that incorporates an influence of financial variables such as leverage ratio and margin of safety as well as those variables that are considered in the existing models such as profit rate and capital utilization rate seems to be one of the most pressing issues in building a macroeconomic model with finance. This is among the top in the list of topics for the future research.

APPENDIX A

LIST OF NOTATIONS IN CHAPTER 1

Notation	Definition	Notation	Definition
P	final output measured at cost	i_L	loan interest rate
R	sales measured at cost	i_D	deposit interest rate
Z	capital outlay	η	margin of safety
U	productive capital	a	share of variable capital in capital outlays
X	commercial capital	e	exploitation rate
Q	productive and commercial capital	q	markup
F_k	financial capital of firms	p_k	recapitalization rate of firms
F_s	financial assets of entrepreneur households	p_b	recapitalization rate of banks
F_w	financial assets of worker households	τ_F	finance lag
F_m	financial assets of banker households	τ_P	production lag
B_k	net bank credits for firms	τ_R	realization lag
B_w	net bank credits for worker households	τ_w	consumption lag of worker households
L_k	liabilities of firms	τ_s	consumption lag of entrepreneur households
L_w	liabilities of worker households	τ_m	consumption lag of banker households
Π_k	net profits of firms	b_k	share of capital outlays financed by net bank credits
Π_b	net profits of banks	b_w	share of worker households' consumption financed by net bank credits
Y_s	income of entrepreneur households	λ_k	leverage ratio of firms
Y_w	income of worker households	λ_b	leverage ratio of banks
Y_m	income of banker households	r_k	net profit rate of firms
D	aggregate demand	r_b	net profit rate of banks
D_s	consumption of entrepreneur households	r	gross profit rate
D_w	consumption of worker households		
D_m	consumption of banker households		

APPENDIX B

SOLUTIONS OF THE MODEL IN CHAPTER 1

B.1 Solution of the baseline model in section 1.3.1

The model, summarized in section 1.2.3, becomes the baseline model of circuit of capital under assumption 1. The simultaneous equations system, normalized by R , is solved with the help of *Mathematica*. Interested readers can acquire the *Mathematica* code from the author upon request. A single prime implies a normalization by R .

$$\begin{aligned}
 Q' &= \frac{(1+g)^{-\tau_F}(1+p_kq) - 1}{g} \\
 Z' &= (1+g)^{-\tau_F}(1+p_kq) \\
 F'_k &= \frac{(1 - (1+g)^{-\tau_F})(1+p_kq)}{g} \\
 F'_s &= \frac{(1 - (1+g)^{-\tau_s})(1-p_k)q}{g} \\
 F'_w &= \frac{a(1+g)^{-\tau_F}(1 - (1+g)^{-\tau_w})(1+p_kq)}{g} \\
 \Pi'_k &= q \\
 Y'_s &= (1-p_k)q \\
 Y'_w &= a(1+g)^{-\tau_F}(1+p_kq) \\
 D'_s &= (1+g)^{-\tau_s}(1-p_k)q \\
 D'_w &= a(1+g)^{-\tau_F-\tau_s}(1+p_kq) \\
 D' &= \frac{(1-p_k)q}{(1+g)^{\tau_s}} + \frac{(1-a)(1+p_kq)}{(1+g)^{\tau_F}} + \frac{a(1+p_kq)}{(1+g)^{(\tau_F+\tau_w)}}
 \end{aligned} \tag{B.1}$$

B.2 Solution of the extended model with the Classical assumption in section 1.3.2

The model, summarized in section 1.2.3, is solved under assumption 2 with the help of *Mathematica*. Interested readers can acquire the *Mathematica* code from the author upon request. Variables are normalized by R and are denoted with a single prime.

$$\begin{aligned}
Z' &= \frac{g(1+q)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
Q' &= \frac{(1+q)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} - \frac{1}{g} \\
F'_k &= \frac{(1-b_k)(1+q)((1+g)^{\tau_F} - 1)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
B'_k &= \frac{g b_k (1+q)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
L'_k &= \frac{b_k (1+q)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
\Pi_k &= q - \frac{b_k (1+q) i_L}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
\Pi_b &= \frac{b_k (1+q) i_L}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
Y'_w &= \frac{g a (1+q)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
D'_w &= \frac{g a (1+q)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L} \\
D' &= \frac{g(1+q)}{g(1+g)^{\tau_F}(1-b_k) + b_k i_L}
\end{aligned} \tag{B.2}$$

APPENDIX C

LIST OF NOTATIONS IN CHAPTER 2

Notation	Definition	Notation	Definition
Q	productive and commercial capital	a	share of variable capital in capital outlays
F	financial capital of firms	e	exploitation rate
R	final sales measured at cost	q	markup
Z	capital outlay	p	recapitalization rate of firms
B	equilibrium net bank credits	τ	finance lag
B_S	supply of net bank credits	r_k	net profit rate of firms
B_D	demand for net bank credits	r_b	net profit rate of banks
L	liabilities of firms	i_L	loan interest rate
L_S	supply of bank loans in stock term	i_D	deposit interest rate
L_b	loans held in private banks' balance sheet	ω	interest rate spread
L_c	loans held in central bank's balance sheet	γ	reserve requirement ratio
E_b	equity of banks	n	share of nondeposit liabilities in banks' total liabilities
D	aggregate demand	δ	share of bank deposits in firms' total financial assets
Π_k	net profits of firms	b	share of capital outlays financed by net bank credits
Π_b	net profits of banks	λ	leverage ratio of banks

APPENDIX D

SOLUTIONS OF THE MODEL IN CHAPTER 2

The model, presented in 2.2, is reduced under assumption 4 to a simplified model consisting of seven equations (2.1), (2.2), (2.3), (2.4), (2.5), (2.6), and (2.11) and eight endogenous variables R , Z , Q , F , L , B , Π_K , and Π_B . The simultaneous equations system, normalized by R , is solved with the help of *Mathematica* and its solution is as follows. Interested readers can acquire the *Mathematica* code from the author upon request. A single prime implies a normalization by R .

$$\begin{aligned}
 Z' &= \frac{g(1+q)}{g(1-b)(1+g)^\tau + b\omega} \\
 \Pi'_K &= \frac{gq(1-b)(1+g)^\tau - b\omega}{g(1-b)(1+g)^\tau + b\omega} \\
 B' &= \frac{bg(1+q)}{g(1-b)(1+g)^\tau + b\omega} \\
 Q' &= \frac{1+q}{g(1-b)(1+g)^\tau + b\omega} - \frac{1}{g} \\
 F' &= \frac{(1-b)(1+q)\left((1+g)^\tau - 1\right)}{g(1-b)(1+g)^\tau + b\omega} \\
 L' &= \frac{b(1+q)}{g(1-b)(1+g)^\tau + b\omega} \\
 \Pi'_B &= \frac{b(1+q)\omega}{g(1-b)(1+g)^\tau + b\omega}
 \end{aligned} \tag{D.1}$$

In section 2.3, g and ω are endogenized. Substituting the consequent solution for g and ω , expressed in proposition 3, into the above solution will generate a complete solution.

APPENDIX E

PROOFS IN CHAPTER 2

Proof of (iii) in proposition 4. This will be proved with the help of lemma 6 and lemma 7. The derivative of ω^* with respect to λ is obtained as

$$\frac{\partial \omega^*}{\partial \lambda} = \frac{(\lambda - b)\tau - ((\lambda - b)\tau - b) \left(\frac{\lambda - b}{\lambda(1-b)} \right)^{\frac{1}{\tau}}}{\lambda^2(\lambda - b)\tau}$$

Since $\lambda > 1$ and $0 < b < 1$ by definition, a sign of $\frac{\partial \omega^*}{\partial \lambda}$ depends on a sign of the numerator. Accordingly, four possible cases will emerge as summarized in lemma 6. For a convenience, let $\Omega = \left(\frac{\lambda - b}{\lambda(1-b)} \right)^{\frac{1}{\tau}}$ and $\Phi = \frac{(\lambda - b)\tau}{(\lambda - b)\tau - b}$.

Lemma 6. A sign of $\frac{\partial \omega^*}{\partial \lambda}$ is determined as follows in four different cases:

- i) in case $\lambda > b \left(\frac{\tau + 1}{\tau} \right)$: if $\Omega < \Phi$, $\frac{\partial \omega^*}{\partial \lambda} > 0$, and if $\Omega > \Phi$, $\frac{\partial \omega^*}{\partial \lambda} < 0$
- ii) in case $\lambda < b \left(\frac{\tau + 1}{\tau} \right)$: if $\Omega > \Phi$, $\frac{\partial \omega^*}{\partial \lambda} > 0$, and if $\Omega < \Phi$, $\frac{\partial \omega^*}{\partial \lambda} < 0$

which logically follows from the expression for $\frac{\partial \omega^*}{\partial \lambda}$ (hence does not require a separate proof).

Now, in order to verify which of the above four cases regarding the sign of $\frac{\partial \omega^*}{\partial \lambda}$ holds, relative value of the two functions Ω and Φ — as a function of λ — needs to be compared with each other, i.e. we have to check whether $\Omega(\lambda) < \Phi(\lambda)$ or $\Omega(\lambda) > \Phi(\lambda)$. This is done in lemma 7.

Lemma 7. With $b < b \left(\frac{\tau + 1}{\tau} \right) < \bar{\lambda}$ holding true, $\Omega(\lambda)$ and $\Phi(\lambda)$ can be compared as follows.

- i) in case $b < \lambda < b\left(\frac{\tau+1}{\tau}\right)$: $\Omega > \Phi$,
- ii) in case $b\left(\frac{\tau+1}{\tau}\right) < \lambda < \bar{\lambda}$: $\Omega < \Phi$,
- iii) in case $\lambda > \bar{\lambda}$: $\Omega > \Phi$,

Proof of lemma 7. Lemma 7 can be easily proved by verifying the diagram of the two functions $\Omega(\lambda)$ and $\Phi(\lambda)$.

First, $\Omega(\lambda)$ is a function monotonically increasing at a decreasing rate starting at x -intercept $(b,0)$ as in figure E.1. This can be verified by confirming the following:

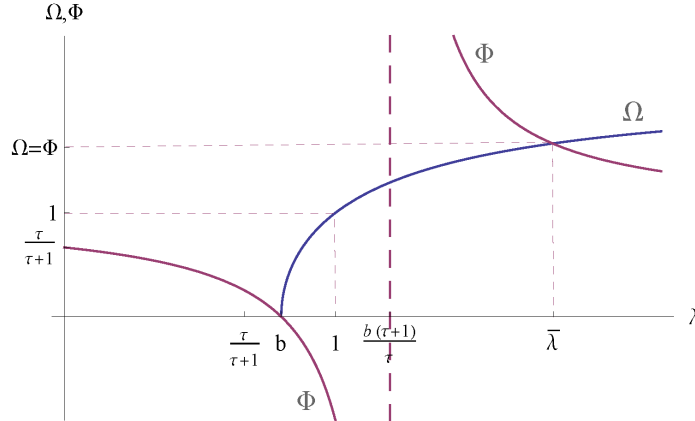
- a) $\Omega(\lambda) > 0$
- b) $\Omega(b) = 0$
- c) First derivative of Ω is positive, i.e. $\frac{\partial\Omega}{\partial\lambda} = \frac{b\Omega}{\lambda\tau(\lambda-b)} > 0$
- d) Second derivative of Ω is negative, i.e. $\frac{\partial^2\Omega}{\partial\lambda^2} = \frac{b\left(b(1+\tau)-2\lambda\tau\right)\Omega}{(\lambda^2\tau^2(\lambda-b)^2)} < 0$

which all follow from the given conditions that $0 < b < 1$, $\tau > 1$, and $\lambda > 1$.

Second, $\Phi(\lambda)$ is a hyperbolic function with asymptotes $\lambda = b\left(\frac{\tau+1}{\tau}\right)$ and $\Phi = 1$ with x -intercept at $(b,0)$ and y -intercept at $(0, \frac{\tau}{1+\tau})$ as in figure E.1. This can be verified by confirming the following:

- a) $\lim_{\lambda \rightarrow b\left(\frac{\tau+1}{\tau}\right)_+} \Phi = +\infty$ and $\lim_{\lambda \rightarrow b\left(\frac{\tau+1}{\tau}\right)_-} \Phi = -\infty$ yield an asymptote $\lambda = b\left(\frac{\tau+1}{\tau}\right)$.
- b) By l'Hôpital's rule, it holds that $\lim_{\lambda \rightarrow \infty+} \Phi = \lim_{\lambda \rightarrow \infty+} \frac{(\lambda-b)\tau}{(\lambda-b)\tau-b} = \lim_{\lambda \rightarrow \infty+} \frac{\tau}{\tau} = 1$ and $\lim_{\lambda \rightarrow \infty-} \Phi = \lim_{\lambda \rightarrow \infty-} \frac{(\lambda-b)\tau}{(\lambda-b)\tau-b} = \lim_{\lambda \rightarrow \infty-} \frac{\tau}{\tau} = 1$, which consequently yields an asymptote $\Phi = 1$.
- c) Two intercepts are obtained by $\Phi(0) = \frac{\tau}{1+\tau}$ and $\Phi(b) = 0$.

Figure E.1: Diagram for a proof of proposition 4 and lemma 9



(Note: With Ω and Φ as a function of λ , this diagram displays the conditions of λ under which either $\Omega > \Phi$ or $\Omega < \Phi$ holds, which matters in proving both proposition 4 and lemma 9.)

which all follow from the given conditions that $0 < b < 1$, $\tau > 1$, and $\lambda > 1$.

Now, from the diagram of the two functions $\Omega(\lambda)$ and $\Phi(\lambda)$ in figure E.1 derived as above, it can be seen that there are two solutions for $\Omega(\lambda) = \Phi(\lambda)$, i.e. b and $\bar{\lambda}$ where $\bar{\lambda}$ is λ that satisfies $\Omega(\lambda) = \Phi(\lambda)$ other than b . That is, one of the two solutions cannot be obtained as closed-form but only as an implicit function.

From these, it can be seen that $b < \bar{\lambda}$ as confirmed by the diagram. Furthermore, from the fact that $\tau > 1$ and with an assumption that $b > \frac{\tau}{\tau+1}$, it holds that $b < b^{\frac{(1+\tau)}{\tau}} < \bar{\lambda}$.¹

Finally, the diagram of the two functions $\Omega(\lambda)$ and $\Phi(\lambda)$ with an order of magnitude $\frac{\tau}{1+\tau} < b < 1 < b^{\frac{(1+\tau)}{\tau}} < \bar{\lambda}$ is obtained as in figure E.1.

The results in (i), (ii), and (iii) of lemma 7 can be proved by the diagram in figure E.1 □

Lemmas 6 and 7 combined together prove (iii) in proposition 4. □

¹The reverse case, i.e. $b < \frac{\tau}{\tau+1}$, does not change the result since an economically meaningful range of λ is the right-hand side of 1 due to the definition of leverage ratio being larger than unity.

Proof of proposition 5. This proposition can be proved in two steps. Each of these is in lemma 8 and lemma 9.

Lemma 8. The condition for type I and type II p_D-p_S configuration:

- i) type I configuration of $p_D(\hat{B}'_D, \hat{\omega}_D)$ and $p_S(\hat{B}'_S, \hat{\omega}_S)$ is obtained if $\hat{\omega}_S > \hat{\omega}_D$
- ii) type II configuration of $p_D(\hat{B}'_D, \hat{\omega}_D)$ and $p_S(\hat{B}'_S, \hat{\omega}_S)$ is obtained if $\hat{\omega}_S < \hat{\omega}_D$.

Proof of lemma 8. In addition to the two pivots $p_D(\hat{B}'_D, \hat{\omega}_D)$ and $p_S(\hat{B}'_S, \hat{\omega}_S)$, consider B'_S that corresponds to $\hat{\omega}_D$ and denote it by \tilde{B}'_S , and by the same manner, consider B'_D that corresponds to $\hat{\omega}_S$ and denote it by \tilde{B}'_D .

It can be graphically verified with figure E.2 that type I p_D-p_S configuration is obtained as long as the following three conditions are met simultaneously, i.e. a) $\hat{\omega}_S > \hat{\omega}_D$; b) $\hat{B}'_S > \tilde{B}'_D$; c) $\hat{B}'_D > \tilde{B}'_S$. Using the equations in (2.25), it can be shown that the mathematical condition for each of the three inequalities is exactly identical to one other. To prove this, we can verify the following.

a') Using the expression for $\hat{\omega}_S$ and $\hat{\omega}_D$ in lemma 1 yields

$$\hat{\omega}_S - \hat{\omega}_D > 0 \iff \frac{\tau}{1 + \tau} < \Omega < \Phi$$

b') Substituting $\hat{\omega}_S$ into expressions for B'_D and B'_S in (2.25) yields $\hat{B}'_S = \frac{\lambda(1+q)(b\Phi+\tau(\Phi-1)(2b-\lambda))}{b(\Phi+\tau\Phi-\tau)}$ and $\tilde{B}'_D = \frac{\lambda(1+q)(\Phi-1)\tau}{\Phi+\tau\Phi-\tau}$. From these it can be verified that

$$\hat{B}'_S - \tilde{B}'_D > 0 \iff \frac{\tau}{1 + \tau} < \Omega < \Phi$$

c') Substituting $\hat{\omega}_D$ into expressions for B'_D and B'_S in (2.25) yields $\hat{B}'_D = \frac{b\lambda(1+q)\Phi}{(\lambda-b)(\Phi+\tau\Phi-\tau)}$ and $\tilde{B}'_S = \frac{\lambda(1+q)(\Phi-1)\tau}{\Phi+\tau\Phi-\tau}$. From these it can be verified that

$$\hat{B}'_D - \tilde{B}'_S > 0 \iff \frac{\tau}{1 + \tau} < \Omega < \Phi$$

It is readily seen that the results in (a'), (b'), and (c') prove that the mathematical conditions for each of (a), (b), and (c) are all identical with each other. This implies that under the condition where $\hat{\omega}_S > \hat{\omega}_D$ holds — whatever that condition might be — the other two conditions, i.e. those in (b) and (c) will also hold.² This proves statement (i) of lemma 8.

The condition for type II p_D-p_S configuration stated in (ii) of lemma 8 can be verified in the similar way. It can be shown that type II p_D-p_S configuration is obtained as long as the following three conditions are met simultaneously, i.e. d) $\hat{\omega}_S < \hat{\omega}_D$; e) $\hat{B}'_S < \tilde{B}'_D$; f) $\hat{B}'_D < \tilde{B}'_S$. Using the equations in (2.25), it can be shown that the mathematical condition for each of the three inequalities is exactly identical to one other. To prove this, we can verify the following.

d') Using the expression for $\hat{\omega}_S$ and $\hat{\omega}_D$ in lemma 1 yields

$$\hat{\omega}_D - \hat{\omega}_S > 0 \iff \Omega > \Phi$$

e') Substituting $\hat{\omega}_S$ into expressions for B'_D and B'_S in (2.25) yields $\hat{B}'_S = \frac{\lambda(1+q)(b\Phi+\tau(\Phi-1)(2b-\lambda))}{b(\Phi+\tau\Phi-\tau)}$ and $\tilde{B}'_D = \frac{\lambda(1+q)(\Phi-1)\tau}{\Phi+\tau\Phi-\tau}$. From these it can be verified that

$$\tilde{B}'_D - \hat{B}'_S > 0 \iff \Omega > \Phi$$

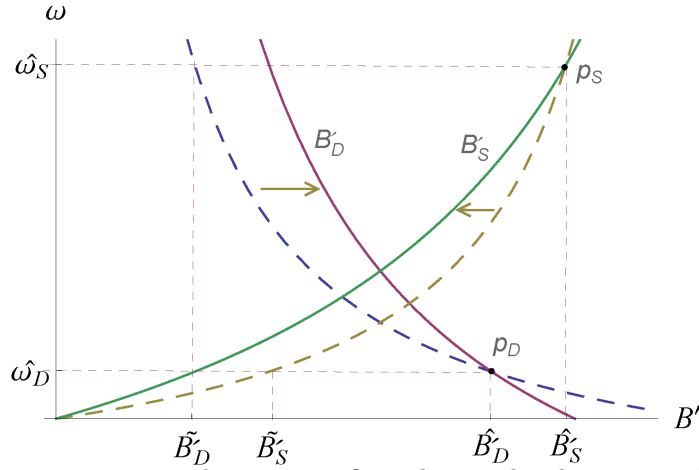
f') Substituting $\hat{\omega}_D$ into expressions for B'_D and B'_S in (2.25) yields $\hat{B}'_D = \frac{b\lambda(1+q)\Phi}{(\lambda-b)(\Phi+\tau\Phi-\tau)}$ and $\tilde{B}'_S = \frac{\lambda(1+q)(\Phi-1)\tau}{\Phi+\tau\Phi-\tau}$. From these it can be verified that

$$\tilde{B}'_S - \hat{B}'_D > 0 \iff \Omega > \Phi$$

It is readily seen that the results in (d'), (e'), and (f') prove that the mathematical conditions for each of (d), (e), and (f) are all identical with each other. This implies

²Lemma 9 discusses the condition for $\hat{\omega}_S > \hat{\omega}_D$.

Figure E.2: Diagram for a proof of lemma 8



(Note: The two pivot points, p_D and p_S , are configured as in this diagram so that, effectively, the demand curve shifts right and the supply curve shifts left in response to a rise in λ as long as the following three conditions are met: i) $\hat{\omega}_S > \hat{\omega}_D$; ii) $\hat{B}'_S > \hat{B}'_D$; iii) $\hat{B}'_D > \hat{B}'_S$.)

that under the condition where $\hat{\omega}_D > \hat{\omega}_S$ holds — whatever that condition might be — the other two conditions, i.e. those in (e) and (f) will also hold.³ This proves statement (ii) of lemma 8. □

Lemma 9. The condition of λ for an inequality between $\hat{\omega}_D$ and $\hat{\omega}_S$:

- i) $\hat{\omega}_S > \hat{\omega}_D$ if $1 < \lambda < \bar{\lambda}$ and
- ii) $\hat{\omega}_S < \hat{\omega}_D$ if $\lambda > \bar{\lambda}$,

where footnote 14 can be referred to for the expression for $\bar{\lambda}$.

Proof. Case (i): Using the expressions for $\hat{\omega}_D$ and $\hat{\omega}_S$ in lemma 1, the condition for $\hat{\omega}_S > \hat{\omega}_D$ to hold is obtained as follows:

- in case $\lambda > b\left(\frac{\tau+1}{\tau}\right)$: $\frac{\tau}{\tau+1} < \Omega < \Phi$
- in case $\lambda < b\left(\frac{\tau+1}{\tau}\right)$: $\Omega < \Phi$ or $\Omega > \frac{\tau}{\tau+1}$

³Lemma 9 discusses the condition for $\hat{\omega}_D > \hat{\omega}_S$.

where $\Omega = \left(\frac{\lambda-b}{\lambda(1-b)}\right)^{\frac{1}{\tau}}$ and $\Phi = \frac{(\lambda-b)\tau}{(\lambda-b)\tau-b}$. By examining the above two cases with figure E.1, it follows that $\hat{\omega}_S > \hat{\omega}_D$ holds when $1 < \lambda < \bar{\lambda}$.

Case (ii): Similarly, using the expressions for $\hat{\omega}_D$ and $\hat{\omega}_S$ in lemma 1, the condition for $\hat{\omega}_S > \hat{\omega}_D$ to hold is obtained as follows:

- in case $\lambda > b\left(\frac{\tau+1}{\tau}\right)$: $\Omega < \frac{\tau}{\tau+1}$ or $\Omega > \Phi$ and
- in case $\lambda < b\left(\frac{\tau+1}{\tau}\right)$: $\Phi < \Omega < \frac{\tau}{\tau+1}$.

By examining the these two cases with figure E.1, it follows that $\hat{\omega}_S < \hat{\omega}_D$ holds when $\lambda > \bar{\lambda}$. □

A combination of lemma 8 and lemma 9 logically leads to proposition 5. □

Proof of proposition 7. With $g^* = \left(\frac{\lambda-b}{\lambda(1-b)}\right)^{\frac{1}{\tau}} - 1$, since g^* is monotonically increasing in $\frac{\lambda-b}{\lambda(1-b)}$, $\lim_{\lambda \rightarrow \infty} g^*$ is obtained when $\frac{\lambda-b}{\lambda(1-b)}$ is at its limit. By l'Hôpital's rule,

$$\lim_{\lambda \rightarrow \infty} \frac{\lambda-b}{\lambda(1-b)} = \lim_{\lambda \rightarrow \infty} \frac{1}{1-b} = \frac{1}{1-b}$$

Hence, it follows $\lim_{\lambda \rightarrow \infty} g^* = \left(\frac{1}{1-b}\right)^{\frac{1}{\tau}} - 1$. □

APPENDIX F

LIST OF NOTATIONS IN CHAPTER 3

Notation	Definition	Notation	Definition
K	consolidated real index for housing and firms' capital	P_R	consolidated index of housing price and stock market index
A_R	assets of the real sector	P_R^e	expected P_R
D_R	debts of the real sector	P_X	price of asset-backed securities
E_R	equity of the real sector	λ_R	leverage ratio of the real sector
M^B	deposits of the real sector	λ_B	leverage ratio of commercial banks
M^N	nondeposit funds of the real sector	λ_X	leverage ratio of securitized banks
L	bank loans	h_B	liquidity ratio of commercial banks
L_D	demand for bank loans	h_N	liquidity ratio of asset managing firms
L_S	supply of bank loans	v_B	share of bank deposits in the total assets of the real sector
Φ	excess demand for bank loans	ω	securitization rate
A_B	assets of commercial banks	r^l	loan interest rate
D_B	debts of commercial banks	r^p	repo rate
E_B	equity of commercial banks	r^{p*}	normal repo rate
H	reserves	r^f	risk-free rate
A_X	assets of securitized banks	r^c	required return of repo lenders
D_X	debts of securitized banks	r^b	securitized banks' return on equity
E_X	equity of securitized banks	r^L	low return
Q	total volume of repos	r^H	high return
X	total quantity of asset-backed securities	ϵ	interest rate spread
A_N	assets of asset managing firms	θ	probability of high return
D_N	debts of asset managing firms	q	degree of realization of notional value of assets when liquidated
E_N	equity of asset managing firms		

(Note: (a) $m \equiv \frac{\lambda_B}{\lambda_B - 1} - h_B$

(b) For the coefficients $\alpha_0, \alpha_1, \delta_0, \delta_1, \beta_0, \beta_1, \epsilon_0, \epsilon_1, \epsilon_2, \mu_0, \mu_1, \mu_2, \tau_0, \tau_1, \kappa_0, \kappa_1$, see appendix G.)

APPENDIX G

A SUMMARY AND SOLUTION OF THE MODEL IN CHAPTER 3

In reproducing the equations here, all the balance sheet variables are normalized by the securitized banking sector's sticky equity E_X . But the same notations are used in order to avoid complication in the notations. For example, Φ , which is the excess bank loan demand, actually represents $\frac{\Phi}{E_X}$.

G.1 Summary

The model consists of twelve variables and twelve equations. The twelve variables are L^D , L^S , Φ , λ_R , λ_X , E_R , P_R , P_X , P_R^e , r^l , r^p , ξ with P_R^e and r^p being two state variables of the model. The behavioral specification of the two state variables is represented as a differential equation.

<Identities and equations>

$$\begin{aligned}
L^D &\equiv (\lambda_R - 1)E_R \\
L^S &\equiv mv_B \lambda_R E_R + \lambda_X \\
\Phi &\equiv L^D - L^S \\
\lambda_R &= \alpha_0 + \alpha_1 P_R \\
E_R &= \delta_0 + \delta_1 P_R \\
\lambda_X &= \beta_0 + \beta_1 P_X \\
P_R &= \epsilon_0 + \epsilon_1 P_R^e - \epsilon_2 r^l \\
P_X &= \mu_0 + \mu_1 P_R - \mu_2 r^p \\
r^l &= r^p + \xi \\
\xi &= \tau_0 + \tau_1 \Phi
\end{aligned} \tag{G.1}$$

<Differential equations>

$$\begin{aligned}
\dot{P}_R^e &= g(P_R) \\
\dot{r}^p &= \kappa_1 (r^p - r^{p*}) - \kappa_2 \dot{P}_R
\end{aligned} \tag{G.2}$$

G.2 Solution

The twelve equations system summarized above is solved under the assumption that $\alpha_1 = 0$ with the help of *Mathematica*. Interested readers can acquire the *Mathematica* code from the author upon request. The resulting solution of the model is presented below.

As can be seen, all the solution is expressed as a function of the constant parameters and the two state variables, i.e. P_R^e and r^p . Since the evolution of the two state variables is given by the two differential equations, once their initial value is known along with the parameter values, concrete values for the entire twelve variables over time will be obtained.

$$\begin{aligned}
\lambda_R &= \alpha_0 \\
E_R &= \delta_0 - \frac{\delta_1(U_1 + \epsilon_1 P_R^e - (\epsilon_2 + \beta_1 \epsilon_2 \mu_2 \tau_1) r^p)}{W_2} \\
\lambda_X &= \beta_0 + \beta_1 \mu_0 - \frac{\beta_1(U_1 \mu_1 + \epsilon_1 \mu_1 P_R^e - (\epsilon_2 \mu_1 - W_2 \mu_2 + \beta_1 \epsilon_2 \mu_1 \mu_2 \tau_1) r^p)}{W_2} \\
P_R &= -\frac{U_2 + \epsilon_1 P_R^e - (\epsilon_2 + \beta_1 \epsilon_2 \mu_2 \tau_1) r^p}{W_2} \\
P_X &= \mu_0 - \frac{U_1 \mu_1 + \epsilon_1 \mu_1 P_R^e - (\epsilon_2 \mu_1 - W_2 \mu_2 + \beta_1 \epsilon_2 \mu_1 \mu_2 \tau_1) r^p}{W_2} \\
r^l &= \tau_0 + \frac{U_2 \tau_1 + W_1 \epsilon_1 \tau_1 P_R^e + (W_2 - (W_1 \epsilon_2 + \beta_1 \mu_2) \tau_1) r^p}{W_2} \\
\xi &= \tau_0 + \frac{\tau_1(U_2 + W_1 \epsilon_1 P_R^e - (W_1 \epsilon_2 + \beta_1 \mu_2) r^p)}{W_2} \\
\Phi &= \frac{U_1 + \epsilon_1 W_1 P_R^e - (\epsilon_2 W_1 + \beta_1 \mu_2) r^p}{W_2} \\
\dot{P}_R^e &= g(P_R), \quad g' < 0 \\
r^p &= \frac{g \epsilon_1 \kappa_2 + (r^p - r^{p*}) \kappa_1 W_2}{W_3}
\end{aligned} \tag{G.3}$$

where

$$\begin{aligned}
W_1 &= \beta_1 \mu_1 - (\alpha_0(1 - v_B m) - 1) \delta_1 \\
W_2 &= -1 + \epsilon_2 \tau_1 W_1 \\
W_3 &= W_2 + \epsilon_2 \kappa_2 (1 + \beta_1 \mu_2 \tau_1) \\
U_1 &= (1 - \alpha_0 + \alpha_0 v_B m)(\delta_0 + \epsilon_0 \delta_1 - \epsilon_2 \delta_1 \tau_0) + \beta_0 + \beta_1(\mu_0 + \epsilon_0 \mu_1 - \epsilon_2 \mu_1 \tau_0) \\
U_2 &= \delta_0 + \epsilon_0 \delta_1 - \epsilon_2 \delta_1 \tau_0 + \epsilon_2(\beta_0 \delta_1 + \beta_1 \delta_1 \mu_0 - \beta_1 \delta_0 \mu_1) \tau_1
\end{aligned} \tag{G.4}$$

APPENDIX H

PROOFS IN CHAPTER 3

Proof of Proposition 11. Each of the three statements can be proved by examining the relevant partial derivative as follows:

1. $\frac{\partial r^p}{\partial \lambda} = -\frac{1-\theta}{\theta}(1+r^L)\left(q'\frac{\lambda}{\lambda-1} - \frac{q}{(\lambda-1)^2}\right) > 0$ due to $0 < \theta < 1$, $q' < 0$, and $\lambda > 1$. The last condition reflects a simple fact that the broker-dealer sector is leveraged.
2. $\frac{\partial r^p}{\partial \theta} = \frac{1}{\theta^2}\left(\frac{\lambda}{\lambda-1}(1+r^L)q - (1+r^f)\right) < 0$ due to $0 \leq q \leq 1$, $r^L < r^f$, and $\frac{\lambda}{\lambda-1} \approx 1$ with λ being substantially large.
3. $\frac{\partial^2 r^p}{\partial \theta \partial \lambda} = \frac{(1+r^L)}{\theta^2}\left(q'\frac{\lambda}{\lambda-1} - \frac{q}{(\lambda-1)^2}\right) < 0$.

□

Proof of Proposition 13. According to Hopf bifurcation theorem, if the differential equations system with a system parameter ρ obtains the Jacobian matrix, which, evaluated at the steady state, has the following properties: i), it possesses a pair of simple complex conjugate eigenvalues $x(\rho) \pm y(\rho)i$ that become pure imaginary at the critical value ρ^* of the parameter — i.e. $x(\rho^*) = 0$, while $y(\rho^*) > 0$ — and no other eigenvalues with zero real part exist at the steady state at ρ^* ; ii) $\left.\frac{dx(\rho)}{d\rho}\right|_{\rho=\rho^*} \neq 0$; then the system has a family of periodic solutions.

Regarding the first property, on the one hand, it needs to be shown that there exists $\rho^* > 0$ such that $J|_{\rho=\rho^*} = 0$ and $\det J|_{\rho=\rho^*} > 0$. Trace and determinant of the Jacobian matrix are:

$$J = -\frac{\epsilon_1 g'}{W_2} + \kappa_1 \frac{W_2}{W_3} + \epsilon_1 \bar{g}' \left(\frac{1}{W_2} - \frac{1}{W_3} \right)$$

$$\det J = -\frac{\epsilon_1 \kappa_1 g'}{W_3}$$

κ_1 is the system parameter. Its critical value can be obtained from setting $J = 0$ and solving it for κ_1 . The result is $\kappa_1^* = \frac{g' \epsilon_1}{W_2}$ which is the Hopf bifurcation point. In this way, we have $J|_{\kappa_1=\kappa_1^*} = 0$ automatically. Using κ_1^* generates $\det J|_{\kappa_1=\kappa_1^*} = -\frac{g'^2 \epsilon_1^2}{W_2 W_3}$. Since $W_3 > W_2$ by definition of W_3 , the only conditions for $\det J|_{\kappa_1=\kappa_1^*} > 0$ to hold are $W_2 < 0$ and $W_3 > 0$. This addresses the first property of Hopf bifurcation theorem. Moreover, this result generates $\kappa_1^* > 0$, remembering $g' < 0$, which satisfies the basic assumption of the model that all the parameters are positive.

Regarding the second property, on the other hand, it can be easily verified that $\frac{\partial J}{\partial \kappa_1} \Big|_{\kappa_1=\kappa_1^*} = \frac{W_2}{W_3} \neq 0$ as long as $W_2 \neq 0$ and $W_3 \neq 0$.

Consequently, $W_2 < 0$ and $W_3 > 0$ guarantee that the two conditions of Hopf bifurcation theorem are satisfied. □

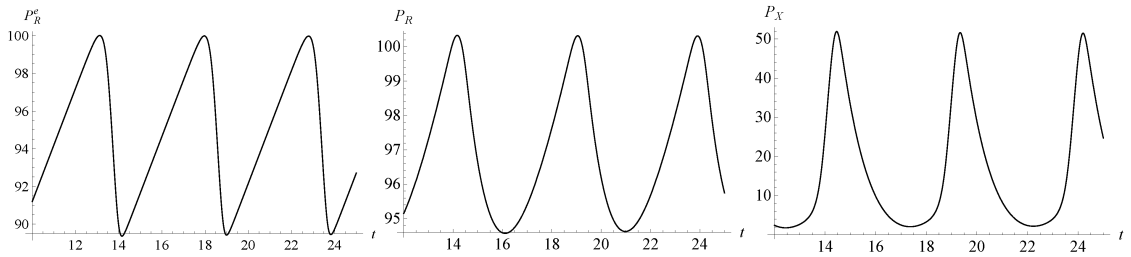
APPENDIX I
PARAMETER VALUES IN CHAPTER 3

Table I.1: Parameter values

α_0	α_1	δ_0	δ_1	β_0	β_1	h_B	λ_B	v_B	ϵ_0	ϵ_1	ϵ_2	μ_0
1.5	0	600	5.4	-5	1	0.1	10	0.1	-5	1.2	2	75
μ_1	μ_2	κ_1	κ_2	τ_0	τ_1	r^{p^*}	g_0	g_1	g_2	g_3	g_4	
2.3	6.66	5	4	1	0.001	5	-35	2	2	100	32	

APPENDIX J

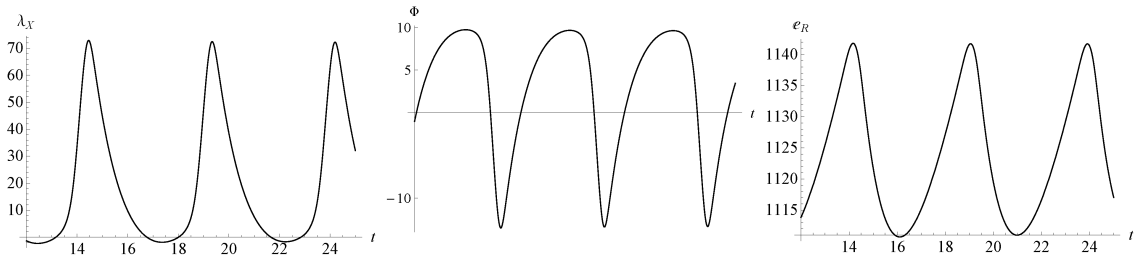
CYCLES OF THE ENDOGENOUS VARIABLES IN CHAPTER 3



(a) expected asset price

(b) current asset price

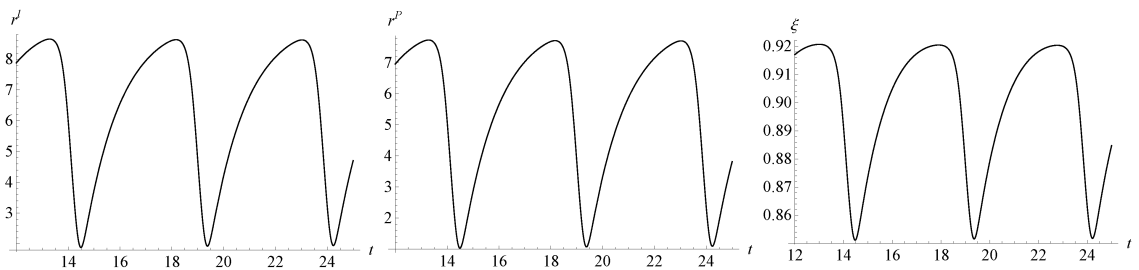
(c) ABS price



(d) securitized bank leverage

(e) excess bank credit demand

(f) equity of the real sector



(g) bank lending interest rate

(h) repo rate

(i) interest rate spread

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