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# An Analysis of Factor Extraction Strategies: A Comparison of the Relative Strengths of Principal Axis, Ordinary Least Squares, and Maximum Likelihood in Research Contexts that Include both Categorical and Continuous Variables

Kevin Barry Coughlin

University of South Florida, kevinbcc1@gmail.com

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An Analysis of Factor Extraction Strategies: A Comparison of the Relative Strengths of  
Principal Axis, Ordinary Least Squares, and Maximum Likelihood in Research Contexts  
that Include both Categorical and Continuous Variables

by

Kevin B. Coughlin

A dissertation submitted in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
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College of Education  
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Major Professor: Jeffrey D. Kromrey, Ph.D.  
Robert F. Dedrick, Ph.D.  
John M. Ferron, Ph.D.  
Constance V. Hines, Ph.D.  
Wayne James, Ed.D.

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Matrices of Association, Multivariate Normality, Overdetermination, Repeated Measures,  
Sample Size, Scale Coarseness

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## **Dedication**

This dissertation is dedicated to my wife, Deanna, and my children: Melia, Imriel, and Jeremiah. Without their patience, tolerance, and support, I would not have completed my course of studies or this dissertation. My wife and children have taught me how strong and resilient a family can be, and, for this, I will always be thankful.

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## **Abstract**

This study is intended to provide researchers with empirically derived guidelines for conducting factor analytic studies in research contexts that include dichotomous and continuous levels of measurement. This study is based on the hypotheses that ordinary least squares (OLS) factor analysis will yield more accurate parameter estimates than maximum likelihood (ML) and principal axis factor analysis (PAF); the level of improvement in estimates will be related to the proportion of observed variables that are dichotomized and the strength of communalities within the data sets.

To achieve this study's objective, maximum likelihood, ordinary least squares, and principal axis factor extraction models were subjected to various research contexts. A Monte Carlo method was used to simulate data under 540 different conditions; specifically, this study is a four (sample size) by three (number of variables) by three (initial communality levels) by three (number of common factors) by five (ratios of categorical to continuous variables) design. Factor loading matrices derived through the tested factor extraction methods were evaluated through four measures of factor pattern agreement and three measures of congruence.

To varying degrees, all of the design factors, as main effects, yielded significant differences in measures of factor loading sensitivity, agreement between sample and population, and congruence. However, in all cases, the main effects were components of interactions that yielded differences in values of these measures that were at least medium in effect size. The number of factors imbedded in the population was a component in six interactions that resulted in medium effect size differences in measures of agreement between population and sample factor loading matrices. of factor loading sensitivity, general pattern agreement, per element agreement, congruence, factor score correlations, and factor loading bias; in terms of the number of interactions that yielded at least medium effect size differences in measures of sensitivity, agreement, and congruence. The number of factors design factor exerted a larger influence than any of the other design factors. The level of communality interacted with the number of factors, number of observed variables, and sample size main effects to yield at least medium effect size differences in factor loading sensitivity, general pattern agreement, per element agreement, congruence, factor score correlations, factor loading bias, and RMSE; in terms of the number of factors that included communality as a component, this design factor exerted the second largest amount of influence on the measures of sensitivity, agreement, and congruence. The level of dichotomization, sample size, and number of observed variables were included in smaller numbers of interactions; however, these interactions yielded differences in all of the outcome variables that were at least medium in effect size.

Across the majority of interactions among the manipulated research contexts, the ordinary least squares factor extraction method yielded factor loading matrices that were in better agreement with the population than either the maximum likelihood or the principal axis methods. In three of the four measures of congruence, the ordinary least squares method yielded factor loading matrices that exhibited less bias and error than the other two tested factor extraction methods. In general, the ordinary least squares method yielded factor loading matrices that correlated more strongly with the population than either of the other two tested methods.

The suggested use of ordinary least squares factor analytic techniques represents the major, empirically derived recommendation derived from the results of this study. In all tested conditions, the ordinary least squares factor extraction method identified common factors with a high degree of efficacy. Suggested studies for future would incorporate the limiting constraints associated with this dissertation into methodological studies to extend the generalizability of conclusions and recommendations into areas that are beyond the scope of this dissertation.

## **Chapter One**

### **Introduction**

The existence of internal attributes, or factors, is the central assertion in factor analytic theory (Tucker & MacCallum, 1997). While these factors are not directly observable, much of the variation in the phenomena that researchers witness and measure is attributable to these underlying traits (Bartholomew, 1984; Cureton & D'Agostino, 1983; Stevens, 2002; Tucker & MacCallum, 1997). Moreover, factor analytic theory asserts that these hypothetical, internal attributes are more “fundamental” than the surface attributes which we observe (Tucker & MacCallum, 1997, p. 2).

In the most basic sense, factor analysis is a set of procedures that researchers employ to analyze relationships among variables (Cureton & D'Agostino, 1983). The objective of this set of procedures is to account for complex patterns of covariation among observed random variables with a set of common factors. The central goals of exploratory factor analysis include the identification of theoretical constructs that underlie a set of observations and the quantification of the extent to which these constructs “represent the original variables” (Henson & Roberts, 2006, p. 396).

The method through which factors are extracted from data represents one of the central procedures associated with exploratory factor analysis (Cureton & D'Agostino, 1983; Stevens, 2002; Tucker & MacCallum, 1997). The goal of this study is to provide researchers with empirical data regarding the most common methods of factor extraction;

the result of this study will be a performance assessment of specific factor extraction techniques applied to a variety of research conditions. To simulate many samples within a large spectrum of research contexts, this study will employ a Monte Carlo simulation (Metropolis & Ulam, 1949).

### **Statement of Problem**

Although exploratory factor analysis is useful in “both measurement and substantive research contexts” (Henson & Roberts, 2006, p. 396), it has been subjected to persistent criticism. Many of these objections are based on the subjectivity of the decisions that researchers must make when conducting their factor analyses (Henson & Roberts, 2006). Another important source of criticism is related to the “indeterminacy of factor solutions” (Harman, 1976, p. 27) which implies that, for a given matrix of correlations, an infinite number of uncorrelated factors can be selected.

Attempts to incorporate non-normal data types into factor analyses represent another potential area of criticism (Yuan, Marshall, & Bentler, 2002). Guiding principles are well established for contexts that include continuous variables that exhibit multivariate normality. However, “no firm guidelines have as yet emerged concerning situations in qualitative and quantitative variables are mixed together” (Krzanowski, 1983, p. 235).

The lack of guidelines associated with incorporating non-normal data sets into factor analytic studies represents the problem to be addressed by this dissertation. The absence of these guidelines becomes especially salient as researchers attempt to incorporate mixtures of continuous and categorical data into factor analytic studies. Without empirically derived guidelines for conducting factor analytic studies with these

types of complex data sets, factor analytic design choices can be criticized as subjective which yield results that can be considered problematic.

### **Theoretical Framework**

While the work of Pearson represents the mathematical foundation for classical factor analysis, Spearman provided the initial description of statistical treatments associated with principal axis analyses (Harman, 1976). Spearman's (1904) work in identifying the strength and direction of relationships among intellectual abilities describes a method for extracting "something of substance" (Spearman, 1904, p. 258) from correlations. This study of intelligence and its advocacy for the use correlation studies in the field of psychology serve as the theoretical basis for this dissertation.

In a series of four experiments, Spearman collected demographic and performance data from 123 students. These subjects were sampled from both a set of village schools and a preparatory academy. In terms of chronological age, students from the preparatory academy were drawn from the highest class. Students from the village schools were selected based on their relative position with respect to age; these students were the oldest children in their families (Spearman, 1904).

After gathering information regarding height, weight, and age, Spearman subjected the students to a variety of perceptual acuity, or discernment, tests. Spearman also assessed the students' performance in a variety of academic skills which included Latin, Mathematics, French, English, Music, and Greek. To examine each student's outward expression of common sense and general intelligence, Spearman included interviews of the test subjects' classmates, siblings, and teachers.

Spearman's initial discussion regarding the experimental results focused on defining the method through which true correlations could be derived from observed correlations (Spearman, 1904). The development of this true correlation required the following four steps:

1. Determine the strength of the observed correlations.
2. Estimate the amount of errors included in correlations between two sets of variables.
3. Identify any spurious correlations or "any factors irrelevantly admitted" (Spearman, 1904, p. 257).
4. Critically examine the experimental design and theory supporting the design to identify any disturbing factors.

Based on the observed correlations, Spearman attempted to measure the extent to which two series of observations had something of substance in common. The existence of this common substance is asserted when the observed correlation is at least four to five times greater than the amount of error associated with the estimate (Spearman, 1904).

The study yielded a hierarchy of intelligences that Spearman summarized through a table of correlations among observed variables. This matrix contained corrected correlations which represent estimates of association after observational errors were eliminated (Spearman, 1904); the main diagonal of this matrix contained values less than one. This hierarchy of intelligences is presented through a table containing correlations between sensory discernment and school subject performance observations, a factor of general intelligence, and a column presenting ratios of the common factor to specific factors (Spearman, 1904).



Spearman concluded that all branches of intellectual capacity had in common “one fundamental function” (Spearman, 1904, p. 284). The remaining elements of intellectual activity are unique for each observation. Spearman’s description of observations in terms of common and specific factors represents a fundamental premise in common or classic factor analytic theory.

The common factor to which Spearman referred is an unobservable, internal attribute that influences the value of observed variables (Tucker & MacCallum, 1997). These hypothetical constructs, in conjunction with researchers’ theories, can be used to explain “the variation and covariation across a wide range of surface attributes” (Tucker & MacCallum, 1997, pp. 2-3). Factor analytic theory includes two types of internal attributes; these are *common factors* and *specific factors*.

Formally, a common factor is “an internal attribute which affects more than one of the surface attributes in the selected set, or battery” (Tucker & MacCallum, 1997, pp. 2-3). While still being an internal attribute, a specific factor influences only one variable in a given data set. A third type of influence includes measurement error; this influence is neither internal nor systematic (Tucker & MacCallum, 1997).

The “traditional” or “classical” factor analysis model is defined by the following equation:

$$Z_j = a_{j1}F_1 + a_{j2}F_2 + \dots + a_{jm}F_m + u_jY_j \quad (j = 1, 2, \dots, n)$$

In this model, a variable  $Z_j$  is described by a linear combination of common factors ( $F_1, F_2, \dots, F_m$ ), and a unique factor  $u_jY_j$ . The  $a$ ’s represent coefficients or loadings for the common factors; the number of common factors ( $m$ ) is normally smaller than the

number of observed variables,  $n$  (Harman, 1976). When considering the value of a specific variable,  $j$ , for a given individual,  $i$ , the factor model can be written as:

$$Z_{ji} = \sum_{p=1}^m a_{jp} f_{pi} + u_j Y_{ji}$$

Where  $f_{pi}$  is the common factor  $p$  for individual  $i$ ;  $a_{jp} f_{pi}$  represents the contribution of the factor on the linear composite. The residual error is given by  $u_j Y_{ji}$  (Harman, 1976).

The factor analytic model provides estimates for the values of loadings on common factors (Harman, 1976). Classical factor analysis includes many strategies for extracting these factor loadings from data. The manner in which researchers select specific methods for factor extraction represents the focus of this study.

Recent developments in structural equation modeling define formative factors as a category of latent variables that are neither common nor unique. As opposed to factor analytic theory, in which observed variables are attributable to factors, formative factors are generated by observed variables (Bollen & Lennox, 1991; Kim, Shin, & Grover, 2010; Treiblmaier, Bentler, & Mair, 2011). In formative measurement models, indicators cause latent variables in the following manner (Bollen & Lennox, 1991):

$$\eta_1 = \gamma_{11}\chi_1 + \gamma_{12}\chi_2 + \dots + \gamma_{1q}\chi_q + \xi_1$$

In this expression,  $\gamma$ 's are coefficients,  $\chi$ 's are "explanatory or observed variables" (Bollen & Lennox, 1991, p. 306), and the dependent variable is the latent construct,  $\eta_1$ .

In the field of information sciences, recent reviews of literature highlight the increased importance of structural equation modeling and the use of formative measurement models (Kim, Shin, and Grover, 2010). For example, searches of MIS Quarterly and Information Systems Research, yielded 24 articles published since 2001

that focused on formative latent variables. In addition to noting the prevalence of formative constructs in the published literature, Kim, Shin, and Grover (2010) examined the differing perspectives regarding the utility of formative latent variables, design issues associated formative measurement models, and the impact of these models on the “quality of IS research . . .” (Kim, Shin, & Grover, 2010, p. 363). They identified “interpretational confounding” and “external consistency” as central issues in the controversy among information scientists over the “viability of formative indicators” (Kim, Shin, & Grover, 2010, p. 347).

Interpretational confounding occurs when empirical meaning is assigned to an unobserved variable in a manner that does not agree with the *a priori* meaning given to the variable by the researcher (Kim, Shin, & Grover, 2010). Researchers who oppose the use of formative constructs cite the requirement of additional endogenous variables to estimate “formative indicator weights” (Kim, Shin, & Grover, 2010, p. 347) as a major factor contributing the prevalence of interpretational confounding in formative measurement models. Moreover, the few solutions to these identification problems include the expansion of formative measurement models to include reflective indicators (Kim, Shin, & Grover, 2010; Treiblmaier, Bentler, & Mair, 2011).

External consistency is achieved “when the measures of a construct correlate with the measures of other constructs” (Kim, Shin, & Grover, 2010, p. 347). Unlike reflective measurement models, the constructs associated with formative models do not maintain linkages with “antecedents and consequences of a construct” (Kim, Shin, & Grover, 2010, p. 347). Proponents of formative measurement models assert that external

consistency and interpretational confounding are not salient issues when models are correctly specified (Kim, Shin, & Grover, 2010).

To examine the relative strengths of these competing perspectives concerning formative measurement models and their associated constructs, Kim, Shin, and Grover (2010) studied an existing dataset derived from an information technology survey. The survey included 243 respondents and two exogenous variables. These two variables include: “IT infrastructure flexibility as formatively theorized construct” and “relational knowledge as a reflectively theorized construct” (Kim, Shin, & Grover, 2010, p. 350). The constructs were measured via index and conventional scale development procedures (Kim, Shin, & Grover, 2010). The endogenous variables included financial performance, information technology performance, business process performance, effectiveness in information technology planning, and effectiveness in information technology coordination. The variables were analyzed through both correctly and incorrectly specified models (Kim, Shin, & Grover, 2010).

The researchers’ results indicated that formative measurement models posed “fundamental problems” in estimating weights (Kim, Shin, & Grover, 2010, p. 359). They found substantial interpretational confounding; moreover, the lack of external consistency leads to “unpredictability of model fit . . .” (Kim, Shin, & Grover, 2010, p. 363). Ultimately the researchers concluded that the use of formative measure models has substantial and negative impact on the quality of information systems research (Kim, Shin, & Grover, 2010).

Researchers concede that, under limited conditions, a formative construct may be scientifically meaningful. However, these same researchers assert that a thoughtfully

developed reflective measurement approach is the most practical (Treiblmaier, Bentler, & Mair, 2011). The reflective (common factor) approach to construct development is the focus of this dissertation.

### **Purpose of the Study**

**Objective.** This study is intended to provide researchers with empirically derived guidelines for conducting factor analytic studies in complex research contexts.

Specifically, the scope of this study includes the evaluation of factor extraction methods when applied to data sets that contain mixtures of categorical and continuous variables.

To enhance the potential utility of this study, the research focused on factor extraction methods commonly employed by social scientists; these methods include principal axis factor analysis, ordinary least squares factoring, and standard maximum likelihood method.

To meet the goal of this study, factor extraction models were subjected to several research conditions. These contexts differed in sample sizes, number of variables, communalities, number of common factors, and ratios of categorical to continuous variables. Data were simulated under 540 different conditions; specifically, this study employed a four (sample size) by three (number of variables) by three (initial communality levels) by three (number of common factors) by five (ratios of categorical to continuous variables) design.

**Rationale.** One measure of the prevalence of factor analytic research designs in contemporary literature can be found in a recent analysis of literature published in PsycInfo over a two-year period. By focusing on this published research, the survey yielded more than 1700 articles that involved exploratory factor analytic research

(Costello & Osborne, 2005). The variety of purposes to which these factor analyses are applied also highlights the importance of this statistical tool. As Conway and Huffcutt noted (2003), social scientists employ exploratory factor analysis to refine measurement tools, establish construct validity, and test hypotheses. The proliferation of exploratory factor analysis in social science research has served as justification for a number of studies that either attempt to establish research design guidelines or contrast typical practices with ideal reporting procedures (Costello & Osborne, 2005; Henson & Roberts, 2006; Krzanowski, 1983).

Although exploratory factor analysis is useful in “both measurement and substantive research contexts (Henson & Roberts, 2006, p. 396),” it has been the focus of methodological criticism. According to Henson and Roberts (2006), many of the objections to the use of exploratory factor analysis are based on the “inherent subjectivity of the decisions” ( p. 396) that researchers must make when conducting their analyses. For example, without referring to criterion variables, researchers select matrices of association, factor extraction methods, criteria for retaining factors, factor rotation strategies, and coefficients for interpretation (Henson & Roberts, 2006).

As social scientists attempt to incorporate data sets that contain diverse preference, socio-economic, and quality of life measures into their multivariate analyses, they cannot rely on empirically developed guidelines. While these types of guidelines are well established for contexts that include continuous variables that exhibit multivariate normality, they are not as useful when applied to contexts that include mixtures of qualitative and quantitative data (Krzanowski, 1983). This difficulty in applying exploratory factor analysis becomes especially pronounced when the research context

includes categorical variables (Krzanowski, 1983). The lack of methodological guidelines contributes to the rationale for this study.

### **Supporting Examples.**

*First supporting example.* To identify the factor structure of risky sexual behavior and substance use, the Vanzile-Tamsen, Testa, Harlow, and Livingston (2006) recruited 1,014 college women to participate in a study of sexual risk taking behavior. The levels and types of risk taking behavior, alcohol use, and drug use were assessed via a “computer assisted self interview” (VanZile-Tamsen et al. 2006, p. 250). As part of the study’s design, the researchers included observed variables from two domains: Sexual risk taking and Substance Use (VanZile-Tamsen et al., 2006).

The sexual risk taking behaviors include eight variables measured at a variety of levels. The authors assessed age at first sexual encounter as a ratio level measure. A subject’s number of life time partners was measured at an interval level; this included seven levels that ranged from 0 to 10 or more (VanZile-Tamsen et al., 2006). The time between meeting a new partner and having sex with him is measured through a six-level, Likert-type scale; a score of one indicates the first day that a subject meets a new partner, and a score of six means a year or more after meeting a new partner.

Also as part of the sexual risk taking domain, the researchers developed a complex indicator of alcohol use associated with sexual encounters; this continuous indicator represents composite of two, Likert-type variables. The first part of the index measured the number of times that alcohol use occurred prior to or during intercourse; this measure had five levels ranging from 1 (once in a while) to 5 (all the time). The second part of this index measured the level of intoxication during sex; this Likert-type

response scale ranged from 1 (not at all intoxicated) to 7 (very intoxicated). Neither portion of this composite variable defined the distances between the levels as equivalent. Without this information, the variables could (at most) be considered ordinal.

The last set of variables associated with sexual risk taking involved the perception of sexually transmitted infection (STI) risk of each partner. This variable is an index based on the subject's estimate of life-time sexual partners that a new partner had and the subject's belief that a new partner has had sex with men, ever had or transmitted an STI, and ever injected drugs (Vanzile-Tamsen et al., 2006).

Substance use was measured via four variables. The number of drinks during a typical drinking occasion is measured at the ratio level. *Frequency of binge drinking* was measured on a 6 point Likert-type scale. Frequency of drinking occasions contains a Likert-type response set with eight levels. A drug use index was derived from the number of illicit drugs used, the frequency of drug use (a Likert-type response scale), and the results of a drug abuse screening test.

Impulsivity, sensation seeking, and anxiety were measured through 38 yes/no items. These dichotomously scored items were all highly correlated and used to construct three latent variables that would be included in a more general model. Negative affect consisted of counts of 10 depressive symptoms and 21 trauma symptoms from the DSM IV.

The researchers employed a maximum likelihood confirmatory factor analysis to compare three models of risk behavior. The comparisons were based on three fit indices, root mean squared error of approximation (RMSEA), and a "chi-squared difference test" (Vanzile-Tamsen et al., 2006, p. 250). Based on the results of these comparisons, the



authors proposed a four latent factor model to account for their observations; this model contained two higher order factors (Vanzile-Tamsen et al., 2006).

The researchers highlighted several limitations associated with their study. For example, the model includes direct influences from personality factors to risk-taking behaviors; however, these influences “represent partial correlations . . .” (Vanzile-Tamsen et al., 2006, p. 253). Due to the researchers’ efforts to identify a model that best fit their data, “no conclusions should be made with regards to the model’s usefulness outside of community samples” (Vanzile-Tamsen et al., 2006, p. 2006). The researchers attributed the complexity of their accepted factor model to the use of multiple indicators for each domain.

*Second supporting example.* To improve the quality of evaluative research in civics education, Finkel and Ernst (2005) presented the findings of a study conducted in 1998. This study examined the “impact of civic education” (Finkel & Ernst, 2005, p. 335) on South African high school students. The sample included 600 students; 385 of these students were exposed to formal civics education, and 261 of whom participated in this education through the U.S. Agency for International Development (USAID) “Democracy for All” (Finkel & Ernst, 2005, p. 335) program. The remaining 215 students did not receive formal civics education.

The researchers used a battery of items to determine students’ “political knowledge, civic duty, tolerance, institutional trust, civic skills, and approval of legal forms of political participation” (Finkel & Ernst, 2005, p. 335). Binary response and correct/incorrect items were used to assess knowledge. Items measured on a Likert-type scale were used to assess indices of civic duty, political tolerance, trust in political

institutions, and approval of political participation, and interval level measures were used to assess students' perceptions of their own civic skills (Finkel & Ernst, 2005).

Civic education was assessed through measures of "students perceptions of teacher quality" (Finkel & Ernst, 2005, p. 347), frequency of exposure to civic education, and teaching methods. Frequency of education was measured through a single Likert-type item, ratio scale index scores were used to measure perceptions of teacher quality. Binary items were used to obtain information concerning active teaching methods (Finkel & Ernst, 2005).

In total, this study included 53 observed variables. Of these, 25 items were measured on a binary scale (either yes/no or correct/incorrect); four items were measured on an ordinal scale, and 24 items were measured at the interval level. The items measured at the interval level included 19 variables which yielded Likert-type observations. The researchers presented their results through a table containing two sets of factor loading coefficients; one set of coefficients were associated with students who received civics education, and another set was associated with students who received no civics education. Through comparing the strengths of loading coefficients, the researchers highlighted slight differences in loadings between the groups. The results of this study did not yield a comprehensive model of political engagement that could be subjected to a confirmatory analysis.

Through empirically derived guidelines for incorporating differing scales of measurement into a single exploratory factor analysis, the authors would have been in a better position to incorporate all of their observations into a single, parsimonious factor model. This model would have demonstrated the manner in which demographic

variables and instructional characteristics interact with behavioral outcomes to yield a comprehensive model of political engagement. Such an analysis would be more amenable to replication and more easily subjected to confirmatory analysis.

### **Research Questions**

The agreement between factor pattern matrices in a simulated population and matrices developed through selected exploratory factor analytic techniques is the primary comparison associated with this study. This agreement was assessed through the proportion of variables that load on the same factors, total factor loading agreement, and factor loading congruence coefficients (MacCallum et al., 1999). Measures of agreement, correlations between population and sample factor score matrices, root mean square error, statistical bias, and solution variability were considered as measures of factor pattern agreement.

The measures of congruence and agreement among population and sample matrices were used to answer the following research questions:

1. How do varying ratios of categorical to continuous variables influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
2. How does the number of variables in a correlation matrix influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
3. How does sample size influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?

4. How does communality influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
5. How does the number of common factors influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
6. How do all of the independent variables interact to influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?

### **Hypotheses**

This study focused on comparisons among three factor extraction methods; these methods include principal axis factor analysis, ordinary least squares factor analysis, and maximum likelihood factor analysis. Each factor extraction method was subjected to the same variety of research conditions. The hypotheses associated with this study were evaluated through the measures defined in the research questions.

**First hypothesis.** The first hypothesis asserted that ordinary least squares (OLS) factor analysis will perform better than maximum likelihood factor analysis as the number of dichotomously scored variables increases. Research into factor analytic techniques indicate that iterative, principal axis factor extraction methods perform better than maximum likelihood methods when the assumption of multivariate normality is not met (Bartholomew, 1980). Based on the findings of earlier research, this study also asserts that, when the research context includes dichotomously scored variables,

maximum likelihood factor analysis will yield factor structures that are less like those simulated in the population than principal axis factor analysis.

**Second hypothesis.** The second hypothesis asserted that, when the factor structure in the population is not strongly defined, ordinary least squares (OLS) factor analysis will identify common factors that maximum likelihood factor analytic methods fail to identify. According to this hypothesis, OLS's relative advantage in identifying common factors will be negatively related to communality and positively related to the number of dichotomous variables. This hypothesis is based on two complimentary studies that highlight OLS's insensitivity to error and maximum likelihood's reliance on the assumption of multivariate normality (Briggs & MacCallum, 2003; Mislevy, 1986).

### **Significance of the Study**

As a tool for generating theory, exploratory factor analysis can play a vital role in developing a knowledge base for social scientists (Stevens, 2002). However, the quality of this knowledge base will be directly related to the quality of the decisions that researchers make as they implement their factor analysis procedures (Bartholomew, 1984; Henson & Roberts, 2006; Tucker & MacCallum, 1997). As one of the primary decisions that a researcher must make, the selection of a factor extraction strategy has a tremendous impact on the quality of conclusions derived from an exploratory factor analytic design. This study can provide researchers with useful guidance in this selection process.

Several recent studies have proposed a variety of Bayesian latent trait models to be employed by researchers when they encounter discrete and mixed data research contexts (Merkle, 2005; Sammel, Ryan, & Legler, 1997; Song & Lee, 2001). However,

these types of exploratory factor analysis strategies are not often used by social science researchers (Henson & Roberts, 2006). Because this study included principal axis, maximum likelihood, and ordinary least squares factor extraction techniques, the results can provide researchers with empirical information concerning the factor extraction methods that they are likely to use.

### **Definition of Terms**

Categorical Level of Data--or nominal data is the result of assigning numbers to categories; this is the “most rudimentary” level of measurement in which all individuals assigned to a given group are the same in terms of a characteristic or set of characteristic (Glass & Hopkins, 1996, p. 7).

Congruence - among factor pattern matrices simulated in the population and the sample pattern matrices is measured through a congruence coefficient. As defined by MaCallum (et al., 1999), the phi coefficient is the cosine of the angle between the sample and population factor solutions “when plotted on the same space” (p. 93). To assess the congruence across all factor loading matrices, an average of the phi coefficient was calculated for each factor extraction method.

Continuous Level Data--consist of measures that can be any value within a specified range (Glass & Hopkins, 1996).

Factor Loadings--are the coefficients of the factors in the basic factor model. The ambiguous use of the term is problematic when the factors are correlated. To interpret the resulting factor solutions, correlated factor solutions undergo oblique rotations and yield two distinct factor matrices: factor pattern and factor structure (Harman, 1976).

Factor Loading Bias – is one measure of a factor extraction methods performance; it is a number of observed variables by number of factors matrix which is populated by estimates of statistical bias for each factor loading; in this study these bias estimates were averaged across all samples (Hogarty et al., 2005).

Factor Loading Root Mean Squared Error – is an indicator of congruence between sample and population factor loading matrices. This outcome variable is a number of observed variables by number of factors matrix containing root mean squared error (RMSE) estimates for each factor loading. To provide an overall index for each factor solution, the RMSE estimates are averaged across all samples in each research context (Hogarty et al., 2005).

Factor Loading Sensitivity- is one of the measures of agreement between sample and population factor loading matrices. This is the count of variables that meet a .30 factor loading threshold for at least one factor in both the sample and the population divided by the count of variables that meet the .30 loading threshold in the population (Hogarty et al., 2005).

Factor Score Correlations - is a number of factors by 1 column vector of correlations between factor scores derived from the sample and those derived from the population. These score estimates will be linear combinations of variables; however, as opposed to using factor score coefficients, these estimates will be computed via the following process: A positive one scoring coefficient is assigned when the observed structure coefficient is  $\geq .30$ ; a negative one scoring coefficient is assigned when the observed structure coefficient is  $\leq -.30$ ; a scoring coefficient of zero is assigned when the structure coefficient is between .30 and -.30. Once factor scores estimates are

computed for both the population and sample matrices, a correlation among the scores will be used to measure how closely factor scores derived from each of the factor extraction strategies approximates that factor score pattern that is imbedded in the population (Hogarty et al., 2005).

General Pattern Agreement-- is one of the measures of agreement between sample and population factor loading matrices. This variable is based on the proportion of variables that load on both the sample and population factors in a similar fashion at least once. This variable is a number of observed variables by one column vector. When the absolute value of the variable loading on the same factor in both the sample and the population is greater than or equal to .30, then a one is assigned to the row associated with the variable. For this measure a variable can meet this threshold for multiple factors and still contribute to general pattern agreement (Hogarty et al., 2005).

Monte Carlo Experiments--refer to a class of experiments in which researchers employ computer programs to generate “vectors of random variates” that are analogous to samples of data (Robey, 1990, p. 278). These samples are derived from a population model containing characteristics of interest to the researcher. Monte Carlo simulations can be used to demonstrate the influence that a variety of research contexts can have on an experiment (Robey, 1990).

Per Element Agreement – is based on a number of observed variables by number of factors matrix. Elements of each matrix contain ones, indicating agreement, where the corresponding factor loading has an absolute value of .30 or greater in both the sample and population factor pattern matrices; elements of the matrix also contained ones when the absolute value of the loading for a variable was less than .30 in both the sample and



the population. When these criteria are not met, the element is assigned a zero. The resulting outcome measure is the proportion of samples in which the agreement criteria were met for each observed variable by factor combination (Hogarty et al., 2005).

Total Pattern Agreement - is measured through a scalar. This scalar is populated with a one when the mean of the per element agreement matrix is one. If any element of the per element agreement matrix is less than one, then the mean of that matrix will be less than one, and the total pattern agreement scalar associated with that matrix is set to zero. This scalar represents the proportion of sample matrices in which all factor loadings are in agreement with the population in terms of the absolute value of .30 loading criterion (Hogarty et al., 2005).

### **Delimitations**

The objective of this study was to provide researchers with empirically derived guidelines for conducting factor analytic studies in complex research contexts. Specifically, the scope of this study included the evaluation of factor extraction methods when applied to data sets containing mixtures of categorical and continuous variables. The extent to which the results of this study can be generalized is limited by certain characteristics associated with the simulation of sample data and the selection of factor analytic studies that are the focus of this dissertation.

The correlation matrices through which data were simulated were based on factor models in which the influences of minor factors are constrained to zero. These factor models were included uncorrelated factors exclusively. When extracting factors from sample data, the number of factors to be retained was equal to the number of factors simulated in the population. In addition to these research design decisions, the selection

of only three commonly employed factor analytic strategies imposed a limit to which the potential guidelines derived from this study can be generalized.

### **Organization of Remaining Chapters**

The next chapters include a review of literature and a description of the methods through which the research questions were addressed. Chapter two contains a general review of methodological research associated with exploratory factor analytic techniques and a more focused analysis of the three factor extraction methods that represent the primary focus of this study. Chapter three describes the manner in which correlation matrices were generated, the method for simulating specific research conditions, measures employed to compare sample and population factor patterns, and measures by which factor extraction strategies were evaluated.

Chapters four and five present the results of the study and a discussion of the results. Chapter four provides results divided into specific sections associated with each outcome variable; the chapter includes a summary of results, and responses to each of the research questions. Chapter five begins with a restatement of the study's purpose, its research questions, and a summary of the research methods. In addition to a general discussion of results, chapter five includes a discussion of the results in terms of each hypothesis and its applications to follow-up studies.

## **Chapter Two**

### **Literature Review**

The sequence of decisions that researchers make when designing and implementing an exploratory factor analytic study provides this dissertation with a three-part structure around which the literature review was organized. The first section of this literature review extends the theoretical framework to include a general examination of the design choices associated with factor analytic research. This overview includes model selection, samples of subjects, samples of variables, multivariate normality, data types, matrices of association, factor retention, and rotation strategies. The second section of this review focuses on descriptive literature regarding principal axis factoring, ordinary least squares factor extraction, and maximum likelihood factor analysis; technical descriptions of these factor analytic methods are provided in Appendix A. The last section of this literature review summarizes several simulation studies conducted on various aspects of factor analysis.

#### **Exploratory Factor Analysis Design Considerations**

**Model selection.** In general, social science researchers employ factor analysis to pursue three objectives: data reduction, identification of influences on overt behavior, and confirmation of hypotheses regarding influences on behavior (Cureton & D'Agostino, 1983; Merrifield, 1974; Stevens, 2002). When the researcher's intent is data reduction only, then exploratory or common factor analysis is not the appropriate design; in this

case, the researcher should conduct a principal component analysis (Conway & Huffcutt, 2003; Fabrigar, MacCallum, Wegener, & Strahan, 1999, p. 273; Henson & Roberts, 2006; Merrifield, 1974; Stevens, 2002). When a researcher is confirming an existing or well understood structural or measurement model, then confirmatory factor analysis would be a more appropriate design than exploratory factor analysis. If a researcher believes that the relationships among observations can be accounted for by underlying characteristics but is unsure of the number or the organization of these characteristics, then the decision to conduct an exploratory factor analysis (EFA) is justifiable (Cureton & D'Agostino, 1983; Harman, 1976; Stevens, 2002).

The interactions among researchers' intentions and the state of knowledge in given fields represent critical considerations in the design of an exploratory factor analysis (Raykov & Marcoulides, 2006; Stevens, 2002). Exploratory factor analysis provides researchers with information concerning the number of common factors and matrices of factor loadings (Harman, 1976). In this context, exploratory factor analysis is a useful set of tools for generating new theories or modifying existing ones (Cureton & D'Agostino, 1983; Harman, 1976; Stevens, 2002).

Conway and Huffcutt (2003) examined the quality of factor analytic research published between 1985 and 1999. Their study indicated that preliminary evaluations of ad-hoc instruments represented the most prevalent purpose behind factor analytic research; assessing the performance of existing measures represented the next most commonly reported purpose. Testing unidimensionality represented one of the least reported purposes (Conway & Huffcutt, 2003).

According to the Conway and Huffcutt (2003) study, principal components analysis (PCA) was the most frequently cited method for analyzing data. When factor analysis represented the “important goal” of the study, common factor analysis was more frequently cited than PCA (Conway & Huffcutt, 2003, p. 160). However, in 28% of the articles, researchers failed to describe either their intention behind their analyses or methods for extracting factors (Conway & Huffcutt, 2003).

Henson and Roberts (2006) examined the information reported in 60 exploratory factor analyses published before 1999. The authors focused on studies that employed at least one exploratory factor analysis strategy. Although Henson and Roberts noted that most of the articles reported researcher objectives that warranted an EFA design, nearly 57% of the researchers engaged in principal components analysis. As a suggested rationale for this problematic model selection, the authors noted that principal components analysis was the “default option for most statistical software packages” (Henson & Roberts, 2006, p. 403).

**Samples of subjects.** In exploratory factor analysis, common prescriptions regarding specific subject per measured variable ratios are simplistic and problematic (Fabrigar et al., 1999). These types of guidelines fail to account for levels of over-determination and communalities among measured variables. Moreover, if the sample is more homogeneous on the common factors than the population, high sample to measured variable ratios will not ameliorate the restriction of range in observations and the resulting attenuation in the correlations among variables (Fabrigar et al., 1999).

When each factor is represented by three to four measured variables and the communalities exceed .70, relatively small sample sizes will allow researchers to make

accurate estimates about population parameters (Fabrigar et al., 1999). If researchers believe that a sample of convenience may be inappropriate, methodological studies highlight a number of sampling strategies that can be employed. For example, to avoid distortions derived from sample characteristics, researchers can select a sample that maximizes variance on measured variables that are not relevant to the construct of interest (Fabrigar et al., 1999).

Recent assessments of published factor analyses indicate that many researchers either do not have sample sizes large enough to support their study designs or do not provide sufficient information concerning sampling strategies. In Conway and Huffcutt's (2003) assessment of 371 articles published between 1985 and 1999, nearly half the studies included sample sizes of 200 or less, and 40% of the studies had sample to variable ratios of 10:1 or less (Conway & Huffcutt, 2003). In a similar study completed in 2006, Henson and Roberts found that the median sample to variable ratio was 11:1; they concluded that "most samples sizes were marginal to sufficient, depending on component saturation" (Henson & Roberts, 2006, p. 402).

**Samples of variables.** The selection of variables represents another important element in the set of decisions that comprise a factor analytic research design. When researchers include too few variables, they may not have a sufficient sample of variables from the domain to identify important common factors; however, by including many irrelevant measured variables, researchers may include "spurious common factors" that obscure the "true" factors (Fabrigar et al., 1999, p. 273). Although prescriptions regarding the total number of variables required to conduct factor analysis are not prevalent in the literature, methodological research highlights the positive relationship

between overdetermination and factor accuracy (Fabrigar et al., 1999; Guadagnoli & Velicer, 1988; Hogarty et al., 2005). Moreover, methodological research suggests that as the number of factors increases with respect to the number of measured variables, the accuracy and reliability of factor solutions decreases (Guadagnoli & Velicer, 1988).

Because communality and over-determination exert a strong influence on the quality of factor solutions, these facets of factor analytic research warrant reporting. However, assessments of common research practices indicate that researchers fail to describe their measured variables comprehensively (Conway & Huffcutt, 2003; Henson & Roberts, 2006). For example, in their 2006 survey of factor analytic research, Henson and Roberts found that fewer than 17% of the articles that they examined reported communalities among measured variables.

**Scale coarseness and dichotomization.** A response scale is considered coarse when a continuous trait is measured in such a way that a variety of “true scores are collapsed into the same category” (Aguinis, Pierce, & Culpepper, 2009, p. 625). In exploratory factor analysis, continuous constructs observed through Likert type responses represent commonly encountered occurrences of scale coarseness (Aguinis, Pierce, & Culpepper, 2009). Research into the effects of scale coarseness indicates that, in many research contexts, scale coarseness leads to downward bias in correlations (Aguinis, Pierce, & Culpepper, 2009).

Scale coarseness is an artifact of a study’s design; it is the result of the measurement instrument (Aguinis, Pierce, & Culpepper, 2009). Because dichotomization introduces error after data are collected, it does not contribute to scale coarseness (Aguinis, Pierce, & Culpepper, 2009; Cohen, 1983). However, the downward bias that it

causes in correlation is “centrally a measurement issue;” because the loss of information is systematic, the “dichotomization drop will occur for both true and observed scores” (Cohen, 1983, p. 252).

Cohen identifies four common rationales used by researchers as they dichotomize their data. First, they employ dichotomization to use loglinear models; this practice is similar to dichotomizing on a control variable in analyses of variance designs (Cohen, 1983). Second, researchers will dichotomize a set of scaled items in preparation for a factor analysis. This rationale for dichotomization is problematic for a number of reasons; for example, the resulting phi coefficients and factor loadings are only two-thirds as large as the product-moment correlations on the original data (Cohen, 1983). Moreover, regardless of estimation method, the “communalities were less than half as large” (Cohen, 1983, p. 253). As an example of a third rationale for dichotomization, market researchers will often dichotomize attitude scales; specifically, a top box is segregated from the remaining response categories. A fourth rationale includes dichotomization in psychiatric research in which a behavioral scale is dichotomized to ensure that a symptom is clear cut (Cohen, 1983).

Recent methodological research highlights the importance of binary data in the social sciences, medical research, ecology, and engineering (Lin & Clayton, 2005; Mislevy, 1986; Osh & Lee, 2001; Sammel, Ryan, & Legler, 1997). For example, by including dichotomous data sets into exploratory factor analyses, researchers reduce large contingency tables into more interpretable tables of fewer dimensions (Bartholomew, 1980). In their efforts to develop new instruments and validate existing tests, researchers also incorporate dichotomous data in exploratory factor analyses and meta-analyses



(Aguinis, Pierce, & Culpepper, 2009; Cohen, 1983; Crocker & Algina, 1986; Mislevy, 1986).

However, when “testing and estimating the reliability of a measured trait,” dichotomously measured variables provide researchers with a variety of difficulties (Donner & Eliaszew, 1994, p. 550). Specifically, when researchers attempt to determine the sample size required to achieve a specified reliability coefficient, contexts that include “truly dichotomous outcome variables and continuous constructs that have been dichotomized both have less power than contexts that contain continuous outcome variables” (Donner & Eliaszew, 1994, p. 552). This loss in power was especially pronounced when inherently continuous traits were dichotomized (Donner & Eliaszew, 1994).

**Non-normal models.** In addition to the assumption that the variation in observed variables can be explained by a set of common factors, several models of exploratory factor analysis also require that the distribution of observed variables exhibits multivariate normality (Yalcin & Amemiya, 2001). To meet this assumption, all the constituent variables in a multivariate analysis must be normally distributed. Additionally, any linear combination of the variables must also have distributions that are normal, and any bivariate subset of the constituent variables must be “bivariate normal” (Stevens, 2002, p. 262).

The assumption of multivariate normality cannot be maintained when studies rely on dichotomous data (Krzanowski, 1980; Mislevy, 1986). As part of an effort to incorporate categorical data into exploratory factor analyses, several researchers have introduced strategies for contending with data sets that are more complicated than those

containing dichotomous data exclusively (Mislevy, 1986; Song & Lee, 2001; Yalcin & Amemiya, 2001). For example, Mislevy (1986) examined the comparative advantages of unweighted least squares, generalized least squares, and maximum likelihood factor analysis in contending with nonlinear factor models. While all three methods provided consistent estimates of factor loading matrices and residuals (Mislevy, 1986), the results highlighted the comparative strength of maximum likelihood factor analysis when researchers are working with many measured variables and anticipate extracting relatively few factors. Generalized least squares becomes the preferred method with fewer variables and more common factors (Mislevy, 1986). To facilitate the analysis of mixed polytomous and continuous data, Song and Lee (2001) introduced a Bayesian method for conducting factor analytic studies. Their research focused on the development of computation procedures that would yield “Bayesian estimates of thresholds, latent factor scores and structural parameters” (Song & Lee, 2001, p. 256).

In response to a perceived need for factor analytic methods that are appropriate for categorical variables, Bartholomew (1980) focused on conditional probability functions as applied to contingency tables. The relationship between observed variables and latent factors is stochastic and expressed as a conditional probability function. This function includes a probability density when the observed variables are continuous and a probability mass when observed variables are categorical (Bartholomew, 1980). Specifically, this research demonstrated the manner in which a probit function leads directly to the minimal residual (MINRES), or ordinary least squares, factor analysis method. The MINRES factor analysis model is based on a solution that minimizes the following:

$$\sum_{i=1}^p \sum_{\substack{j=1 \\ j \neq i}}^p (\hat{c}_{ij} - \alpha_i \alpha_j)^2$$

The  $c_{ij}$ 's are covariances, and the  $\alpha$ 's are factor loadings (Bartholomew, 1980).

According to Bartholomew's conclusions, researchers can treat the indicators associated with a  $2^p$  table, where  $p$  is the number of manifest variables, as if they were derived from data that were normally distributed; and they can perform "a factor analysis on the estimated covariance matrix" (Bartholomew, 1980, p. 310).

**Matrices of association.** In their review of common practices in exploratory factor analysis, Henson and Roberts (2006) highlighted the need for researchers to report the matrices of association included in their studies more frequently. When conducting factor analytic studies, researchers can select from a variety of correlation measures; these include Pearson Product Moment correlation coefficients, matrices of Spearman rank order coefficients, phi-coefficients, tetrachoric correlations, point-biserial coefficients, polychoric correlations, polyserial correlations, and polychoric-polyserial correlations (Corten et al., 2002; Edwards & Allenby, 2003; Folwer, 1987; Gilbert & Hilton, 1992; Greer, Dunlap, & Beatty, 2003; Harman, 1976; Song & Lee, 2003). The relative advantages associated with these matrices are based on the distributional characteristics of data being studied.

Because the majority of measurements in social science research are in arbitrary units, the Pearson product moment correlation represents a better choice than covariance in the general description of the relationship between two variables (Glass & Hopkins, 1996). When researchers report the matrix of association that they analyze, they report using correlation matrices nearly twice as often as covariance matrices (Henson &

Roberts, 2006). In terms of parameters, the formula for Pearson Product Moment Correlation Coefficient is described in Appendix A.

According to Glass and Hopkins (1996), the “Spearman rank correlation is . . . tailor-made to fit situation in which both variables are expressed as ranks” (Glass & Hopkins, 1996, p.129). To employ  $r_{ranks}$ , data must be in the form of ranks; however, the number of ranks will not impact the calculation of the correlation. When two sets of ranks being correlated contain no ties, the Spearman Rank and Pearson Product Moment coefficients are equivalent (Glass & Hopkins, 1996). A mathematical description of the Spearman rank correlation is provided in Appendix A.

When encountering data sets that contain dichotomous variables exclusively, social science researchers often employ phi-coefficients to quantify the relationship among variables (Merrifield, 1974). Researchers interpret a positive phi-coefficient as indicating that 1’s on a variable X will imply a higher likelihood of having 1’s on a related variable Y. Just as Pearson product moment correlation coefficients, a matrix of phi coefficients will have ones on the main diagonals and measures of bivariate association on the off-diagonal elements (Harman, 1976). The formula for the phi-coefficient is described in Appendix A.

When researchers dichotomize variables that have normal distributions, they can select the tetrachoric coefficient as an estimate of correlation (Glass & Hopkins, 1996; Harman, 1976; Merrifield, 1974). Tetrachoric correlations are interpreted as estimates of product moment correlations if the variables were measured more accurately, normally distributed, and linearly related (Glass & Hopkins, 1996; Greer, Dunlap, & Beatty, 2003). In spite of their apparent utility, when variables exhibit bivariate normality, tetrachoric

correlations can appear to provide more explanatory information than is possible (Nunnally, 1978). In Appendix A, the mathematical description of the tetrachoric correlation coefficient is presented.

When researchers correlate a dichotomous variable with a continuous variable, they can employ a *point-biserial* coefficient (Glass & Hopkins, 1996). Test developers employ this correlation coefficient when they estimate the relationship between performance on a dichotomously scored item and performance on the total test score (Crocker & Algina, 1986). The formula for the *point-biserial* correlation is included in Appendix A.

When contending with attitude items and performance ratings, social scientists increasingly encounter opportunities to correlate interval level observations with polytomous item responses (Lee, Poon, & Bentler, 1994). By extending the polyserial correlation model to incorporate polytomous, as well as bivariate, data, Lee, Poon, and Bentler (1994) developed and tested a partition maximum likelihood process for estimating correlation coefficients (Lee, Poon, & Bentler, 1994; Song & Lee 2003). A technical description of the partition maximum likelihood model for estimating polyserial correlation coefficients is included in Appendix A.

The computational difficulty associated with direct minimization of the likelihood function led Lee, Poon, and Bentler to develop a two-stage process for modeling covariance structure. The first stage of this process includes a more simple process for obtaining polyserial and polychoric associations among variables (Lee, Poon, Bentler, 1994). These correlations comprise the elements of  $\Sigma$ ; the polyserial correlations between  $X$  and  $Y_a$  are identified by  $\rho_a$ , and polychoric correlations between  $Y_a$  and  $Y_b$  are

identified by  $\rho_{ab}$  (Lee, Poon, & Bentler, 1994). In Appendix A, a description of the algorithms associated with these correlation estimates is presented.

Fowler (1987) compared the relative performance of Pearson product moment correlation coefficients, Spearman rank order coefficients, point-biserial coefficients, and phi-coefficients under a variety of research conditions. These research conditions included sample sizes that ranged from 10 to 100 observations that were drawn from distributions of differing shapes. Type I, Type II, and Type III error rates represent the basis of for comparisons (Fowler, 1987); type III error is the condition in which the researcher rejects a false null hypothesis but draws false conclusions regarding the ranking of parameters.

According to Fowler's results, phi coefficients had the least amount of power across all data conditions. Although point-biserial coefficients maintained good relative power when sample sizes were large and when one variable was dichotomized at the median, this measure of association experienced substantial loss of power when sample sizes were small (Fowler, 1987). Spearman rank order coefficients maintain a high degree of power in most data conditions, and, when Kurtosis is greater than three, Spearman coefficients are more powerful than Pearson product moment coefficients. However, in conditions containing small sample sizes, Pearson product moment correlations are more powerful than rank order coefficients regardless of distribution shapes (Fowler, 1987).

Under most research conditions, simulation studies indicate that use of the phi coefficient is not justified (Fowler, 1987). When contending with distributions that are highly skewed or leptokurtic, point-biserial correlation coefficients offer some

advantages over Pearson product moment correlations; however, this advantage is only present when one of the variables is dichotomized at the median (Fowler, 1987). While the Spearman rank-order correlation can be an alternative to the Pearson product moment correlation when the data conform to bivariate normality and sample sizes are small, the complications associated with ties and computational difficulty seem to highlight the comparative strength of Pearson product moment correlations under most research conditions (Fowler, 1987). Ultimately, Fowler's study indicates that the "Pearson product moment correlation is remarkably robust . . . under even extreme violations of distributional assumptions" (Fowler, 1987, p. 427).

Greer, Dunlap, and Beatty (2003) examined tetrachoric correlations under a variety of research conditions. The results of their Monte Carlo study indicated that, when scores are dichotomized at the median, tetrachoric coefficients yield estimates that are close to the population correlations; significant skewness does not substantially alter the quality of these estimates. The bias associated with tetrachoric correlations decreases as sample size increases; this bias increases as the strength of the correlation in the population increases (Greer, Dunlap, & Beatty, 2003). From their results, Greer, Dunlap, and Beatty concluded that, due to insensitivity to the shape of marginal distributions, tetrachoric correlation coefficients approximate Pearson correlations if the "distributions were transformed to normality" (Greer, Dunlap & Beatt, 2003, p. 948).

In exploring multitrait-multimethod models, Corten, Saris, Coenders, van der Veld, Aalberts, and Kornelis (2002) conducted a study that highlighted the relative strengths and weaknesses of Pearson product moment coefficients and Polyserial-polychoric associations. This study examined matrices of association in a confirmatory

factor analysis research context. The authors incorporated 87 data sets from previously conducted studies; these data sets included categorical and continuous variables. The data were derived from personal interviews, mail questionnaires, and computer assisted interviews (Corten et al., 2002).

The authors compared correlation matrices in terms of percentage of confirmatory factor models that failed to converge, the models that yielded improper solutions, and standardized root mean squared residuals (RMSR) (Corten et al., 2002). The Pearson correlations yielded the smallest percentage of nonconvergence and improper solutions. The polyserial-polychoric correlations yielded substantially smaller RMSR's. In spite of the greater precision associated with models that include polyserial-polychoric correlations, the increased likelihood of nonconvergence and number of improper solutions rendered these matrices less useful than Pearson correlations (Corten et al., 2002).

**Number of factors retained.** In selecting the number of factors to be retained, researchers balance the desire for parsimony against the need for plausibility (Fabrigar et al., 1999). To achieve this balance, researchers can select from a number of techniques and rules of thumb. Recommendations based on reviews of factor analytic studies include the use of multiple, simultaneous considerations when selecting factors to be retained; these include fit indices derived from maximum likelihood strategies, scree plots, eigenvalues greater than one, and parallel analysis (Fabrigar et al., 1999).

According to the Kaiser criterion, all factors with eigenvalues greater than one should be retained in a study. In reviews of factor analytic literature, the Kaiser rule is the most commonly employed criteria for selecting factors (Conway & Huffcutt, 2003;



Costello & Osborne, 2005; Henson & Roberts, 2006). However, methodological literature suggests that the eigenvalues greater than one rule is among the least accurate methods for retaining factors (Conway & Huffcutt, 2003; Costello & Osborne, 2005).

In exploratory factor analysis, the Kaiser rule is “often misapplied” by referring to the eigenvalues of a reduced correlation matrix (Fabrigar et al., 1999, p. 278). To apply the rule correctly, researchers compute the eigenvalues for a correlation matrix to determine how many exceed one; this number represents the number of components to be retained in an analysis. This process is appropriately applied to correlation matrices with ones on the main diagonal; in other words, this process is appropriately applied when researchers are conducting principal component analyses as opposed to factor analyses (Fabrigar et al., 1999). Applying the Kaiser rule to correlation matrices with communality estimates that are less than one on the main diagonal is an “erroneous procedure” (Fabrigar et al., 1999, p. 278).

A second rule uses a scree plot; according to this method, factors are retained until the diminishing eigenvalues stop declining (Cureton & D’Agostino, 1983; Harman, 1976). Parallel analysis represents a third type of guideline for factor selection; when employing parallel analysis, researchers compare eigenvalues from sample data to eigenvalues that one would expect to obtain from completely random data. The resulting factor model is based on the number of eigenvalues that are larger than their corresponding eigenvalues from the random data (Fabrigar et al., 1999).

The goal for factor retention guidelines is to identify the necessary number of factors to account for the correlations among measured variables. Empirical research suggests that under-factoring, retaining too few factors, is more problematic than over-

factoring (Fabrigar et al., 1999). However, over-factoring is not ideal; for example, when over-factoring, researchers may postulate the existence of factors with no theoretical basis which can “accentuate poor decision made at other steps in factor analysis” (Fabrigar et al., 1999, p. 278).

**Rotation.** Factor solutions are considered to have simple structure when observed variables load, primarily, on one factor (Stevens, 2002). Simple structure will yield factors that are easily interpretable, psychologically meaningful, and replicable (Fabrigar et al., 1999). A system of factors is considered parsimonious when factors in a system are distinct from one another and all factors are required to explain a phenomenon of interest (Merrifield, 1974).

The parsimony associated with a factor solution is directly related to the “linear independence” among the factors (Merrifield, 1974, p. 397). The degree to which linear independence is achieved is measured by correlation coefficients. Factors that are linearly dependent suggest the existence of higher order factors (Merrifield, 1974). The extreme case of “linear independence is called orthogonality” (Merrifield, 1974, p. 397). The most appropriate rotation strategy for this type of factor matrix would be orthogonal.

Orthogonal rotation algorithms yield uncorrelated factors (Fabrigar et al., 1999; Harman, 1976; Stevens, 2002). The most commonly employed type of orthogonal rotation is varimax (Fabrigar et al., 1999; Merrifield, 1974). Although orthogonal rotation yields simple structure, the use of orthogonal rotation when the factors are correlated in the population results in the loss of important data (Costello & Osborne, 2005).

When researchers have reason to suspect the existence of second-order factors or that their factors should be correlated, they can employ oblique rotation methods (Costello & Osborne, 2005; Cureton & D'Agostino, 1983; Harman, 1976; Merrifield, 1974). Oblique rotation strategies provide researchers with more information regarding the interpretation of factors; for example, these rotation strategies yield correlation coefficients among the factors. Common oblique rotation procedures include: direct quartimin rotation, promax, and Harris Kaiser. Unlike orthogonal rotation, no one method of oblique rotation is dominant in the literature (Costello & Osborne, 2005; Fabrigar et al., 1999).

Procrustean rotation is a less commonly cited rotation strategy. Procrustean rotation can be based, in part, on substantive considerations as opposed to the exclusive concern for simple structure. Moreover, methodological evidence suggests that Procrustean rotation yields rotated factor patterns that are very similar to their target matrices (Raykov & Little, 1999).

In published literature, orthogonal strategies appear to be the most commonly cited rotation procedure. According to Conway and Huffcutt's (2003) findings, researchers reported using an orthogonal rotation strategy most frequently (40%); in only 18% of the studies, researchers reported the use of an oblique rotation strategy. More recently, in their study of common practices in factor analytic research, Henson and Roberts (2006) found that 55% of the articles included orthogonal rotation strategies; researchers reported the use of oblique rotation strategies in 38.3% of the articles, and, in 1.7% of the articles, researchers failed to report any factor rotation method.

The “conventional wisdom advises researchers to use orthogonal rotation because it produces more easily interpretable results . . .” (Costello & Osborne, 2005, p. 3). However, this argument is flawed in two areas. Firstly, social science researchers “generally expect some correlation among factors” (Costello & Osborne, 2005, p. 3); therefore, the use of orthogonal rotation results in the loss of information concerning the correlations among factors. Secondly, output associated with oblique rotation is “only slightly more complex” than orthogonal rotation output and yield substantive interpretations that “are essentially the same” (Costello & Osborne, 2005, p. 3).

Reviews of published factor analytic studies indicate that researchers are not employing ideal practices as they report the choices that define their analyses. Many studies did not provide information concerning the psychometric properties associated with measured variables. One author found the frequent use of the “largely discredited” eigenvalue greater than one rule to be discouraging (Fabrigar et al., 1999, p. 292). These reviews indicated that researchers were not likely to report their matrices of association and the amount of variance accounted for by their retained factors (Conway & Huffcutt, 2003). In some cases, the reviewers highlighted the lack of researcher training in exploratory factor analysis as a reason for the problematic designs and reporting practices (Conway & Huffcutt, 2003).

### **Factor Extraction Methods**

According to Merrifield (1974), dimensional options include the methods that social science researchers employ to extract factors from “person by task matrices” (Merrifield, 1974, p. 395). The most commonly used methods include maximum likelihood, principal axis factors with prior estimates of communalities, and “iterative

principal factors” (Fabrigar et al., 1999, p. 277). The relative utility of each method is dependent on the researchers’ intentions and the distributions of observed data (Fabrigar et al., 1999).

Maximum likelihood factoring allows the researcher to test for statistical significance in terms of correlation among factors and the factor loadings, but this method for estimating factor models can yield distorted results when observed data are not multivariate normal (Costello & Osborne, 2005; Fabrigar et al., 1999). Principal axis factoring does not rely on distributional assumptions and is more likely than maximum likelihood to converge on a solution. However, principal axis factoring does not provide the variety of fit indices associated with maximum likelihood methods, and this method does not lend itself to the computation of confidence intervals and tests of significance (Fabrigar et al., 1999).

In addition to measurement error, the covariation among surface attributes is influenced by two types of factors: common and specific (Cureton & D’Agostino, 1983; Spearman, 1904; Stevens, 2002; Tucker & MacCallum, 1997). Common factors are internal attributes that influence more than one of the observed attributes in a test, battery of tests, or a survey. Specific, or unique, factors affect the observed values of only one variable in a matrix.

In general, the covariance among observed variables can be expressed in the following relationship among common and unique factors (Mislevy, 1986):

$$\Sigma = \Lambda\Phi\Lambda' + \Psi$$

Where,  $\Sigma$  is the covariance matrix of manifest variables ( $y$ );  $\Lambda$  is a matrix of factor loadings;  $\Phi$  is the covariance matrix of the elements in  $\theta$ ; the elements of  $\theta$  comprise an

“m-dimensional” latent variable, and  $\Psi$  represents the covariance matrix of the residuals,  $e$ ; the diagonal elements of this matrix are referred to as the unique variances of the  $y$ 's (Mislevy, 1986, p. 5).

**Principal axis factor analysis.** Principal axis factor analysis and principal components analysis are computationally similar (Stevens, 2002). In the case of principal components analysis (PCA), the linear combination of variables results in components that account for all of the variance in the original data. Principal axis factor analysis yields factors that account for the common variance in the original data (Cureton & D'Agostino, 1983; Harman, 1976; Stevens, 2002).

Principal components analysis employs a correlation matrix as the matrix of association. When conducting a principal axis factor analysis, researchers focus on a reduced correlation matrix as the matrix of association. This reduced correlation matrix contains communality estimates on the main diagonal as opposed to ones (Cureton & D'Agostino, 1983; Harman, 1976; Stevens, 2002). A frequently used prior communality estimate of a variable,  $z_t$ , is its squared multiple correlation with all of the other variables in an instrument (Cureton & D'Agostino, 1983; Harman, 1976):

$$SMC_t = 1 - \frac{1}{r^{tt}}$$

Where  $r^{tt}$  is the diagonal element of an inverse correlation matrix that corresponds to the variable  $z_t$  (Harman, 1976).

When the number of observed variables is at least moderately large and arranged “so that every rotated common factor has at least four or five substantial loadings” (Cureton & D'Agostino, 1983, p. 138), the squared multiple correlation approximates the lower bounds of the communality. When factors are underspecified, squared multiple

correlations may underestimate the item's communality (Cureton & D'Agostino, 1983; Harman, 1976).

When the influence of unique factors is omitted, the relationship between common factors and the predicted value of an observed variable is summarized in the following expression:

$$\tilde{Z}_j = a_{j1}F_1 + a_{j2}F_2 + \cdots + a_{jm}F_m \quad (j = 1, 2, \dots, n)$$

The sum of squared factor coefficients gives the communality of a particular variable; as highlighted by the expression above,  $a_{j1}^2$  quantifies the first factor's contribution to the communality of variable  $Z_j$  (Harman, 1976). The coefficients for the first factor are selected to maximize the factor's contribution to the total communality,  $V_1$ ; this sum is given by (Harman, 1976):

$$V_1 = a_{11}^2 + a_{21}^2 + \cdots + a_{n1}^2$$

Lagrange multipliers are used to develop a system of "characteristic equations" (Harman, 1976, p. 137). The roots of a characteristic equation are eigenvalues, and the largest root is the maximum value of the first factors contribution to the total communality (Cureton & D'Agostino, 1983; Harman, 1976).

The next step in the principal axis factor method includes determining the coefficients for the second factor. These coefficients are selected to maximize the factor's contribution to the remaining, or residual, communality. In a fashion similar to finding the coefficients for the first factor, the second largest root is equivalent to the root, or eigenvalue, of the first factor's residual communality matrix. This root of this residual matrix is equivalent to the second largest eigenvalues of the original, reduced

correlation matrix (Cureton & D'Agostino, 1983; Harman, 1976). This procedure proceeds until the entire matrix of factor coefficients is developed.

Appendix A contains a technical description of the Principal Axis Factor Analysis method. This description includes sections that focus on the relationship between eigenvalues and reduced correlation matrices, the relationship between reduced correlation matrices and matrices of factor loadings, and the relationship between matrices of factor loadings and eigenvalues.

**Ordinary least squares.** The primary goal of ordinary least squares (OLS), also known as the Unweighted Least Squares or MINRES (Obenchain, 1975), method for obtaining factor solutions is to minimize the sum of squared differences between the observed and implied covariance matrices (Briggs & MacCallum, 2003). The OLS method for extracting factors assigns weights to residuals of large and small factors equally; unlike maximum likelihood methods for extracting factors, OLS does not rely on assumptions about the distribution of observed variables (Briggs & MacCallum, 2003). OLS solutions are derived through an iterative principal axis computational method (Briggs & MacCallum, 2003; Cureton & D'Agostino, 1983; Harman, 1976).

The ordinary least squares procedure begins with a matrix of factor coefficients that, when multiplied by its transpose, yields a matrix of reproduced correlations with communalities on the principal diagonal (Cureton & D'Agostino, 1983; Harman, 1976). The next phase in the procedure involves fitting the reproduced correlation matrix to the matrix of observed correlations. This is achieved through minimizing an objective function that is based on the sum of squares of the off-diagonal residuals between the observed and reproduced correlation matrices (Harman, 1976).



Through an iterative process of adding “displacements” to each element of the original factor loading matrix, a new factor matrix is developed (Harman, 1976, p. 177). The values of these displacements are selected so that the resulting matrix minimizes the objective function. Appendix A provides a comprehensive description of the method through which these “displacements” are calculated (Harman, 1976, p. 179). Appendix A also describes the constraint employed to ensure that the OLS procedure yields factor models that imply communalities that are less than or equal to one (Harman, 1976).

**Maximum Likelihood.** Based on the assumption that a specified number of factors exists in a population, maximum likelihood factor analysis yields estimates of factor loadings for a given sample size and number of observed variables (Harman, 1976). When the observed variables exhibit multivariate normality and the sample size is large, maximum likelihood strategies facilitate the calculation of confidence intervals for the estimated loadings (Chen, 2003).

The basic factor model defined in the theoretical framework can be restated as

$$x = \mu + \Lambda f + \varepsilon$$

As defined above,  $x$  is a column of observed variables,  $\mu$  is the mean vector of observed variables,  $f$  is a column vector of common factors,  $\Lambda$  is a matrix of factor loadings, and  $\varepsilon$  is a vector of unique factors (Chen, 2003). The population covariance matrix,  $\Sigma$ , is given by the following expression:

$$\Sigma = \Lambda\Lambda' + \Psi$$

Where  $\Psi$  is a diagonal matrix of unique variances (Chen, 2003). Because the observed covariance matrix,  $S$ , can be calculated from the sample and is “an unbiased estimate of  $\Sigma$ ,” the factor loadings and unique variances are all that must be estimated from the

sample (Chen, 2003, p. 310). If the observed variables exhibit multivariate normality, maximum likelihood strategies can yield estimators that utilize all the information from the sample and contain small limiting variances (Chen, 2003; Harman, 1976). Moreover, as sample sizes increase, “the estimators will converge (in a probabilistic sense) to the true parameters” (Harman, 1976, p. 198).

From a maximum likelihood perspective, the parameters are estimated by maximizing a likelihood function or minimizing a corresponding function (Chen, 2003; Cureton & D’Agostino, 1983; Harman, 1976). These expressions are functions of the elements of  $\Lambda$  and  $\Psi$  matrices (Chen, 2003; Harman, 1976). Appendix A contains a comprehensive description of the maximum likelihood algorithms through which factors parameters estimated.

Maximum likelihood strategies are scale-invariant; both covariance matrices and their corresponding correlation matrices will yield the same factor patterns (Cureton & D’Agostino, 1983). Maximum likelihood strategies are dependent on the assumptions that, in addition to the observed variables, the common factors exhibit multivariate normality. Maximum likelihood techniques provide researchers with both parameter estimates and statistical indicators of model adequacy (Conway & Huffcutt, 2003; Harman, 1976; Mislevy, 1986).

### **Simulation Studies in Exploratory Factor Analysis**

Briggs and MacCallum (2003) compared maximum likelihood (ML) and ordinary least squares (OLS) factor analysis methods in their capacities to recover “relatively weak common factors” (p. 26). To make this comparison, the researchers simulated two types of data sets. The first data set contained 12 measured variables and three major domain

factors; the second data set contained 16 measured variables and four major domain factors. Factors with loadings of .45 or lower were defined as weak. The three factor model contained one weak factor, and the four factor model contained two weak factors; the researchers embedded multiple levels of measurement and sample error in their simulations (Briggs & MacCallum, 2003). For each research condition, the researchers generated 1,000 correlation matrices.

To assess the accuracy of estimates derived by the tested factor analysis methods, the researchers employed a coefficient of congruence between the population and sample factor loadings for weak factors and a root mean square deviation calculated for weak factors only (Briggs & MacCallum, 2003). The results of their study indicated that, under a majority of conditions, OLS outperformed ML factor analysis in recovering weak factors. The advantage of OLS over ML became especially pronounced in the two-weak factor design; in these conditions, ML did not recover the fourth factor at all (Briggs & MacCallum, 2003).

The interpretation of factor loading matrices and the scores derived from them depend on a variety of sampling consideration (Velicer & Fava, 1998). Velicer and Fava (1998) examined the impact of subject and variable sampling on the quality of factor analysis solutions. The researchers' stated intent is "to determine the conditions that are likely to produce patterns that closely approximate the population patterns" (Velicer & Fava, 1998, p. 233). To achieve this aim, the researchers produced an extensive review of existing literature and conducted two simulation studies.

In the first simulation study, the researchers generated population correlation matrices that contained varying levels of factor pattern complexity. Simplicity implied

equal loadings, and increased complexity involved unequal loadings. The researchers also manipulated the variable to factor ratios and sample sizes. The analytical methods included in this study were principal components analysis, image component analysis, and maximum likelihood factor analysis (Velicer & Fava, 1998).

The comparisons between sample and population factor loading matrices were evaluated through a root mean squared error ( $g$ ). The mean values of  $g$  and the standard deviations of  $g$  were subjected to an analysis of variance (ANOVA); this analysis had six levels of subject sample size, three levels of loadings, five levels of sample correlation matrices, three levels of number of variables, and three levels of factor analytic methods (Velicer & Fava, 1998). The factor loading main effect had the greatest impact on the ANOVA's associated with the mean  $g$ ; the higher loading values had significantly smaller mean root mean squared errors than the lower loading values. As demonstrated by the ANOVA's, sample size and the number of values also had substantial impact on the mean values of  $g$ . However, the main effects and interaction effects associated with method of analysis did not have a significant impact on the mean value of the root mean squared error (Velicer & Fava, 1998). When the ANOVA's were conducted on the standard deviations of  $g$ , the results also indicated that the main effects and interaction effects associated with method of analysis "produced a small effect" (Velicer & Fava, 1998, p. 239).

Because the authors understood that constraining the loadings to be equal on all salient variables was "highly artificial" (Velicer & Fava, 1998, p. 241), they conducted a second study in which the loadings varied in the population. The methods, analyses, and comparison criteria for the second study were nearly identical as those associated with

the first study included in the article. The primary change included the number of variables per factor and the manner in which the variables loaded on each factor (Velicer & Fava, 1998).

In the second study, eight variables loaded on each factor. The levels of loading included .40, .40, .40, .60, .60, .80, .80 and .80; the average loading was .60 which allowed the authors to compare the results of the second study with those of the first. When the ANOVA's were conducted with the mean value of  $g$ , the method of analysis, subject sample size, and variable sample size yielded results that were large enough to warrant interpretation (Velicer & Fava, 1998). This pattern of interpretability was replicated when the ANOVA's were conducted on the standard deviations of  $g$ .

In terms of the size of subject samples, the authors confirmed the results of early studies: "the most critical conditions for determining the degree of similarity between a sample pattern and the corresponding population pattern are the square root of the sample size and the average loading" (Velicer & Fava, 1998, p. 243). The results indicated that ICA "was clearly inferior to both MLFA and PCA" (Velicer & Fava, 1998, p. 245), and, when samples were either very large or very small, PCA was superior to MLFA.

Ogasawara (2000) explored methods for evaluating the variability in parameter estimates derived from factor analysis and component analysis. This exploration included "asymptotic correlations" between parameter estimates in factor analysis and principal component analysis, correlations for standardized variables, and "mean squared canonical correlation between factors and components. . ." (Ogasawara, 2000, p. 168); the author also included asymptotic standard errors for the estimates of the correlations

between factors and principal components. Through a Monte Carlo simulation study and the use of existing data sets, the author tested the accuracy of these measures.

The simulation consisted of eight manifest variables and one factor; the sample size was 300; the simulation procedure was repeated 1000 times. The previously developed data set consisted of a correlation matrix of for six variables with a sample size of 220 (Ogasawara, 2000). The results from the simulation study and the study conducted on the existing data yielded large asymptotic correlations between estimates for factor analysis and the corresponding components analysis; the results yielded small standard errors associated with the canonical correlations. When the variances of the unique factors approach sphericity, “the two sets of results give similar interpretations for factors and components” (Ogasawara, 2000, p. 182).

In their investigation of the relationship between sample size and the accuracy of factor analytic solutions, Hogarty, Hines, Kromrey, Ferron, and Mumford (2005) conducted a Monte Carlo study to evaluate the minimum sample size requirements suggested in the methodological literature. The researchers compared factor pattern matrices extracted through principal axis factor analysis from generated correlation matrices with known factor pattern loadings. The researchers manipulated levels of communality, number of variables, number of common factors, and sample size; the researchers also examined the interactions among all independent variables. The factor solutions were evaluated in terms of “factor loading sensitivity,” “pattern accuracy” (general pattern accuracy, total pattern accuracy, and per element pattern accuracy), congruence coefficients, “factor score estimate accuracy,” and Root Mean Squared Error (RMSE) (Hogarty et al., 2005, p. 206).

The researchers found that, under nearly all research conditions, factor solution sensitivity was excellent. In terms of general pattern accuracy, the conditions associated with high communality yielded the best solutions. Although the researchers found low total pattern accuracy in nearly all research conditions, their results indicated that this accuracy increased as sample size and communality increased (Hogarty et al., 2005). Their results indicated that per element accuracy was positively correlated with sample size, communality, and variable to factor ratio (Hogarty et al., 2005). The results also highlighted a negative correlation between sample size and RMSE (Hogarty et al., 2005). The researchers concluded that no minimum sample size or subject to variable ratio ensured “good factor recovery;” moreover, their results indicated that the quality of factor solutions was more strongly related with communality than sample size (Hogarty et al., 2005, p. 223).

Factor scores provide researchers with information concerning a subject’s “relative spacing” on the latent variable (Grice, 2001, p. 67). In their attempts to use factor scores in regression analyses or ANOVA’s, researchers must contend with factor score indeterminacy and the selection of units for factor score coefficients (Grice, 2001). Through a Monte Carlo simulation study, Grice (2001) examined six methods for computing factor scores. These methods included exact regression ( $F_{\text{exact}}$ ), unit regression ( $F_{1/3}$ ), unit pattern ( $P_{.30}$ ), unit-structure ( $S_{.30}$ ), unique pattern ( $P_{\text{unique}}$ ), and unique-unit-structure ( $S_{\text{unique}}$ ) (Grice, 2001).

Grice developed two populations of z scores; each population contained identical factor structure. Factor scores were created, and 35 samples were randomly selected from the first population; these samples ranged in size from 100 to 700. Factors were

extracted through an iterated principal axis analysis procedure. The factor scoring methods that represented the focus of this article “were then derived, evaluated, and applied to random samples of equal size drawn from the second population” (Grice, 2000, p. 70). These factor score estimation methods were evaluated, primarily, through correlations among the six factor score estimates and the known factor scores. Estimates of statistical bias in these correlations and variability in the correlation matrices were also calculated (Grice, 2000).

The results indicated that the unit-regression scores were “generally superior” (Grice, 2000, p. 77) to the other methods of factor score estimation. These results also demonstrated that the pair-wise comparisons between the unit-regression estimates and the other methods were moderated by sample size. In most cases, the unit-regression estimates were more valid than the other methods under consideration; however, in the larger sample size conditions, the exact regression estimation procedure proved to be more valid than the unit-regression scores (Grice, 2000).

To explore methods for identifying the dimension of item pools, Knol and Berger (1991) compared full-information models with models that included pairwise information exclusively. To accomplish this comparison, they generated data matrices that contained known item discrimination and item difficulty parameters. The researchers drew multidimensional ability,  $\theta$ , from a multivariate normal distribution; for the binary response items, zeros are assigned when the value of the characteristic function,  $p_i(\theta_v)$ , is less than a value “randomly drawn from the uniform [0, 1] and a one when it is greater than or equal to the value” (Knol & Berger, 1991, p. 462). The researchers included three



levels of sample size in their study. For each combination of study conditions, the researchers generated 10 replications (Knol & Berger, 1991).

All models included in the comparisons were assessed in terms of both IRT and factor analytic parameters. The evaluation criteria included mean squared differences between known and estimated parameters. The IRT models included in this study were TESTFACT, NOHARM, and MAXLOG; the factor analytic models include ordinary least squares (MINRES), unweighted least squares (ULS), generalized least squares (GLS), maximum likelihood (ML), alpha factor analysis (ALPHA), and iterated principal factor analysis (IPFA) (Knol & Berger, 1991).

The researchers' "most striking" (Knol & Berger, 1991, p. 471) result highlighted the advantageous nature of the factor analytic models and the NOHARM item response theory model when compared to TESTFACT and MAXLOG. For multidimensional datasets, the results of the Knol and Berger study indicate that the factor analysis methods performed slightly better than TESTFACT in terms of IRT criteria. The researchers also found that IPFA, ULS, and MINRES outperformed ML and GLS factor analytic models in nearly all research conditions (Knol & Berger, 1991).

### **Summary**

This review of literature outlines common practices and problems with the manner in which social scientists conduct factor analyses and report their results. The review of literature presents the results of methodological research in factor extraction strategies, selection of matrices of association, sample size requirements, and the impact of violations of the assumption of multivariate normality. For researchers who intend to work with mixtures of polytomous and continuous level data, several methodological

studies propose probabilistic and Bayesian factor analytic strategies; however, these proposed strategies rarely appear in factor analytic literature. By exploring the manner in which complex data contexts and commonly employed factor analytic strategies interact, the results of this study provide social scientists with information concerning research methods that they will frequently use. The results of this study will extend the scope of the methodological research associated with exploratory factor analysis.

## **Chapter Three**

### **Methods**

#### **Purpose of Study**

The intention underlying this study is to provide researchers with empirically derived information concerning the interaction of factor extraction methods and types of data. The scope of this study includes the evaluation of factor extraction methods when applied to data sets that contain a mixture of categorical and continuous variables. To enhance the potential usefulness of this study's results, this research focused on methods commonly employed by social scientists; these include principal axis factor analysis, ordinary least squares factoring, and standard maximum likelihood method.

#### **Research Questions**

The agreement between factor pattern matrices in a simulated population and matrices developed through selected exploratory factor analytic techniques is the primary comparison employed in this study. This agreement was assessed through the proportion of variables that load on the same factors, total factor loading agreement, and factor loading congruence coefficients (MacCallum et al., 1999). Measures of agreement, correlations between population and sample factor score matrices, root mean square error, statistical bias, and solution variability were considered as measures of factor pattern agreement.

The measures of congruence and agreement among population and sample matrices will be used to answer the following research questions:

1. How do varying ratios of categorical to continuous variables influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
2. How does the number of variables in a correlation matrix influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
3. How does sample size influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
4. How does communality influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
5. How does the number of common factors influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
6. How do all of the independent variables interact to influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?

### **Introduction to Methods**

The research methods and Monte Carlo design included in this study are based on previous methodological research in the field of common factor analysis. The strategies used to generate correlation matrices in this study are derived from Tucker, Koopman,

and Linn's (1969) examination of factor analytic methods. The sampling methods are based on the strategies employed by Hogarty, et al. (2005).

To address the research questions, this study incorporated samples simulated through a variety of research contexts. These contexts differed in number of variables, the number of common factors, communalities, sample sizes, and ratios of categorical to continuous variables. Data were generated under 540 different conditions; specifically, this study was a three (number of variables) by three (number of common factors) by three (communality levels) by four (sample size) by five (ratios of categorical to continuous variables) design.

In the simulation procedure, ten correlation matrices were generated for each combination of data conditions. For each correlation matrix, 1000 samples were generated. These samples varied in the combinations of sample size and ratio of categorical to continuous variables. In total, this simulation process yielded 5,400,000 samples; each combination of data conditions accounted for 10,000 samples.

## **Procedures**

Figure 1 provides a graphical presentation of the procedures used in this study; each facet of the manipulated conditions is addressed specifically in separate sections of this chapter. The procedures employed in this study can be summarized as follows:

1. Develop one population factor loading matrix for each of the 27 possible combinations of the levels of observed variables, common factors, and communalities.

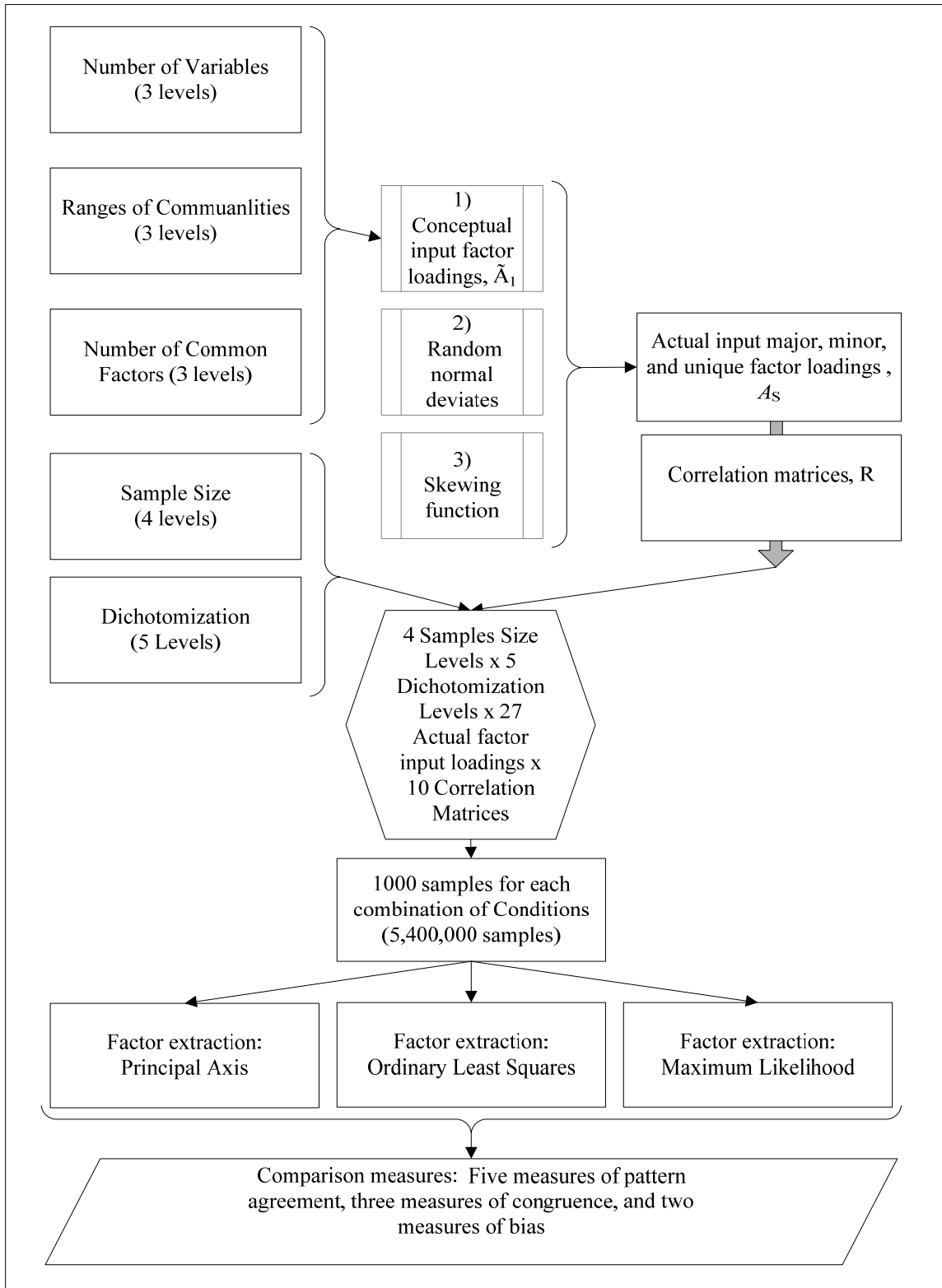


Figure 1. Flowchart summarizing the generation of population and sample matrices

2. Generate 10 correlation matrices for each combination of sample size, level of dichotomization, and factor loading matrix; this results in 5,400 correlation matrices; the number of correlation matrices per design condition facilitate “major comparisons between cells by reduction of between cell variation due to random effects” (Tucker, Kiipman, & Linn, 1969, p. 435).
3. Employ Monte Carlo design to simulate 1,000 samples for each combination of correlation matrix, level of dichotomization, and level of sample size; this results in 1,000 samples for each combination of manipulated variables (5,400,000 samples).
4. Extract factor loadings from the simulated data via principal axis, ordinary least squares, and maximum likelihood methods.
5. Through indicators of agreement between population and sample factor pattern matrices, compare the factor patterns derived from each factor extraction method to known population characteristics.

### **Generation of Population Matrices**

This study employed a method for generating population correlation matrices that is described in Tucker, Koopman, and Linn’s (1969) study of factor analytic methods. The simulation process includes a “mathematical, probabilistic model” and presumes the existence of major, minor, and unique factors. The major factors represent the “influences on observed scores of individuals for the phenomena which the experimenter wishes to study” (Tucker, Koopman, & Linn, 1969, p. 424); minor factors exert systematic influence on the value of observations but are not within the experimenters’ control, and unique factors represent error. Major factors are identified by a subscript

value of one; minor factors are given a subscript value of two, and a subscript of three indicates a unique factor. The number of each type of factor is designated by  $M_s$  (Tucker, Koopman, & Linn, 1969).

The generation of correlation matrices begins with a matrix  $A_s$  of “actual input factor loadings” (Tucker, Koopman, & Linn, 1969, p. 425). Through a three-step process, these actual input loadings are derived from a matrix of conceptual input loadings,  $\tilde{A}$ . Conceptual input factor loadings represent the researcher’s expectations concerning the “factorial composition of the variables” (Tucker, Koopman, & Linn, 1969, p. 426).

The first step in the development of conceptual input loadings involved the creation of “relative conceptual input loadings” for each variable. For a three-factor domain, the loadings conform to the following guidelines:

1. A zero, one, or two is chosen at random and is assigned to the first factor.
2. The sum of the loadings for any one variable is limited to two; this limit implies that if the first loading is two, then other two must be zero; if the first loading is one, then the other two have an equal probability of being a zero or a one.
3. The loading of the third factor is chosen so that the sum of all three will be two (Tucker, Koopman, & Linn, 1969).

Translating conceptual input factor loading matrices into matrices of actual input factor loadings is accomplished through a three-step process. Through the first step, the conceptual input factor loadings are combined with “random normal deviates;” these deviates represent the natural “discrepancies” that occur in the construction of



instruments (Tucker, Koopman, & Linn, 1969, p. 428). The output of this step,  $(y_1)_{im_1}$ , is defined by:

$$(y_1)_{jm_1} = (\tilde{a}_1)_{jm_1}c_{m_1} + d_{1j}x_{1m_1}(1 - c_{m_1}^2)^{1/2}$$

Where:

1.  $(\tilde{a}_1)_{jm_1}$  is the entry in row j and column  $m_1$  of matrix  $\tilde{A}_1$
2.  $x_{jm_1}$  is a random, normal deviate ( $\mu = 0, \sigma = 1$ )
3.  $c_{m_1}$  is a constant for each factor  $m_1$ ; the possible values range from zero to one; the constants represent the “general control an experimenter has on the loading of actual variables on the factors” (Tucker, Koopman, & Linn, 1969, p. 429)
4.  $d_{1j}$  is a constant for each variable j; this constant normalizes each row of  $x_{1m_1}$  to a unit length vector; it is defined as:  $d_{1j} = (\sum_{m_1} x_{jm_1}^2)^{-1/2}$  (Tucker, Koopman, & Linn, 1969, p. 429)

The second step in this translation process includes a skewing function that reduces negativity in factor loadings. This function yields coefficients,  $(z_1)_{jm_1}$ , according to the following equality:

$$(z_1)_{jm_1} = \frac{(1+k)}{(2+k)} \frac{(y_1)_{jm_1} [(y_1)_{jm_1} + |(y_1)_{jm_1}| + k]}{[|(y_1)_{jm_1}| + k]}$$

In this expression, k is a parameter that can range from zero to infinity. Each vector of  $(z_1)_{jm_1}$  coefficients is reduced to unit length by the following:

$$(a_1^*)_{jm_1} = g_{1j}(z_1)_{im_1}$$

$$\text{where } g_{1j} = [\sum_{m_1} (z_1)_{jm_1}^2]^{-1/2}$$

The third step in this process includes scaling the matrix “to ensure desired levels of communality” (Hogarty et al., 2005, p. 207).

The matrix of actual input factor loadings,  $A_s$  is a  $J \times M_s$  matrix that contains a row for each variable  $J$  and a column for each major, minor, and unique factor (Tucker, Koopman, & Linn, 1969, p. 425). For each matrix  $A_s$ , a matrix  $A_s^*$  can be defined by adjusting the rows of  $A_s$  to unit length vectors.  $P$  is a square, symmetric matrix of order  $J$ ;  $P$  is positive and semi-definite; it is defined by:

$$P_s = A_s^* A_s^{*'} \\ \text{Diag}(P_s) = I.$$

The simulated correlation matrix is given by:

$$R = B_1 P_1 B_1 + B_2 P_2 B_2 + B_3 P_3 B_3$$

$B_s$  are diagonal matrices that include  $b_{1i}$ ,  $b_{2i}$ , and  $b_{3i}$  as entries. These entries are real, positive numbers that have the following property:

$$b_{1i}^2 + b_{2i}^2 + b_{3i}^2 = 1$$

These considerations imply the following equalities:

$$r_{ii} = 1 \\ \text{Diag}(R) = I$$

Matrix  $A_s$  is now defined as:

$$A_s = B_s A_s^*$$

The correlation matrix is given by:

$$R = A_1 A_1' + A_2 A_2' + A_3 A_3' = (A_1, A_2, A_3)(A_1, A_2, A_3)'$$

The supermatrix  $(A_1, A_2, A_3)$  contains the matrices  $A_1$ ,  $A_2$ , and  $A_3$  as horizontal sections (Tucker, Koopman, & Linn. 1969).

The coefficients in the  $B_s$  matrices “regulate” the amount of variability in the variables that is related to the major, minor, and unique factors. The  $B_1^2$  matrix contains communalities, and the  $B_3^2$  contain values for uniqueness. When  $B_2$  matrix is zero, the “simulation model” equals the “formal model” (Tucker, Koopman, & Linn, 1969, p. 426).

This study included the formal model as the simulation model. The  $B_2$  matrix is set to zero, and by implication, the input factor loadings for minor factors were zero. This forced the “data generation model” to match “a factor analytic model” with the number of common factors equal to the levels specified for each combination of research contexts that was examined in this study. This study included two, four, and eight common factors.

### **Illustrative Example for Generating a Correlation Matrix**

This illustration of the process for generating population correlation matrices includes nine variables and three common factors. In this simulated factor model, the communalities range from 0.6 to 0.8. Table 1 presents the matrix of conceptual factor input loadings,  $\tilde{A}$ , that is employed in this illustration. Note that the first conceptual factor loading for each observed variable was assigned randomly; the sum of the conceptual loadings for each variable is two.

As described in the previous section, translating this matrix of conceptual factor input loadings into a matrix of actual input loadings requires the incorporation of variation and discrepancies that mimic realistic instruments and studies. This process includes random normal deviates, constants for each factor, and a constant for each

variable. Tables 2, 3, 4, and 5 illustrate the components associated with the second step in developing the matrix of actual input loadings.

Table 1  
*Matrix of Relative Conceptual Factor Input Loadings,  $\tilde{A}$*

Variable	Factor 1	Factor 2	Factor 3
1	0	1	1
2	2	0	0
3	0	2	0
4	2	0	0
5	0	0	2
6	0	1	1
7	2	0	0
8	1	0	1
9	2	0	0

Table 2 contains a matrix of random normal deviates,  $x_{jm_1}$ . When combined with the vector of normalizing constants for each factor,  $d_{1j}$ , the 9 x 3 matrix of random normal deviates is reduced to a unit length vector.

The normalizing coefficients,  $d_{1j}$ , for the random deviates included in this illustration are given in Table 3. Tucker, Koopman, and Linn's (1969) procedure for simulating natural variation includes a matrix of constants,  $c_{m_1}$ , for each major factor; this formula for  $(y_1)_{im_1}$  also includes the squared values of these constants.

Table 2

*Matrix of Random Normal Deviates  $\chi_{jm_1}$* 

Variable	Factor 1	Factor 2	Factor 3
1	0.699	1.078	0.043
2	1.056	0.950	1.669
3	0.385	0.151	-0.971
4	-0.687	-0.590	-1.156
5	0.040	1.016	-1.560
6	-1.133	1.162	1.269
7	1.208	1.356	-0.129
8	1.266	-0.932	-0.201
9	1.603	0.088	04.435

Table 3

*Values for the  $d_{1j}$  Coefficients*

Variable	Value
1	0.778
2	0.456
3	0.947
4	0.681
5	0.537
6	0.485
7	0.549
8	0.631
9	0.601

Table 4 highlights the distribution of  $c_{m_1}$  values among the three common factors. The step in the process for incorporating naturally occurring discrepancies into the matrix of actual input factor loadings results in a  $(y_1)_{im_1}$  value for each variable of each of the three factors. Table 5 represents this matrix.

Table 4  
*Matrix of Constants  $c_{m_1}$*

Variable	Factor 1	Factor 2	Factor 3
1	0.8	0.7	0.8
2	0.8	0.7	0.8
3	0.8	0.7	0.8
4	0.8	0.7	0.8
5	0.8	0.7	0.8
6	0.8	0.7	0.8
7	0.8	0.7	0.8
8	0.8	0.7	0.8
9	0.8	0.7	0.8

The second step in the process of translating the conceptual input factor loadings into a matrix of actual input loadings involves a skewing function which reduces negativity in the resulting factor loadings. Table 6 contains a matrix of values,  $(z_1)_{jm_1}$ , associated with this step. Through the equation for  $g_{1j}$  given in the previous section, this matrix is reduced to unit length to yield the matrix  $(a_1^*)_{jm_1}$  which is illustrated in Table 7.

Table 5  
*Matrix of Values for  $(y_1)_{im_1}$*

Variable	Factor 1	Factor 2	Factor 3
1	0.326	1.299	0.820
2	1.889	0.309	0.457
3	0.218	1.502	-0.552
4	1.319	-0.287	-0.472
5	0.013	0.390	1.097
6	-0.330	1.103	1.169
7	1.998	0.532	-0.042
8	1.280	-0.420	0.724
9	2.178	0.038	0.157

Table 6  
*Matrix of Values for  $(z_1)_{jm_1}$*

Variable	Factor 1	Factor 2	Factor 3
1	0.288	1.322	0.807
2	1.962	0.271	0.422
3	0.182	1.542	-0.080
4	1.344	-0.064	-0.077
5	0.008	0.353	1.105
6	-0.068	1.11	1.183
7	2.081	0.501	-0.019
8	1.301	-0.074	0.704
9	2.276	0.239	0.123

Table 7  
*Matrix of Values for  $(a_1^*)_{jm_1}$*

Variable	Factor 1	Factor 2	Factor 3
1	0.183	0.839	0.512
2	0.987	0.134	0.209
3	0.117	0.992	-0.051
4	0.997	-0.048	-0.568
5	0.006	0.304	0.952
6	-0.042	0.684	0.728
7	0.972	0.234	-0.009
8	0.878	-0.050	0.475
9	0.998	0.011	0.054

The third (and final) step in the process for transforming conceptual input factor loadings into actual input factor loadings involves  $b_s^2$  matrices which regulate the amount of variability in the variables that is driven by the major and unique factors. The vector  $b_1^2$  contains communalities, and the vector  $b_3^2$  is associated with the unique factors.

Table 8 provides the values for these vectors; the vector of zeros associated with  $b_2^2$  highlights this studies focus on the influence of major factors. The product of  $(a_1^*)_{jm_1}$  and  $b_1^2$  yields a matrix of actual input factor loadings for the major domain. Table 9 provides these loadings for each variable and factor. Table 10 illustrates the manner in which the unique factors influence the variability in the nine variables.



Table 8  
*Values of  $b^2$  Coefficients,  $b_s^2$*

Variable	$b_1^2$	$b_2^2$	$b_3^2$
1	0.7	0.0	0.3
2	0.6	0.0	0.4
3	0.7	0.0	0.3
4	0.7	0.0	0.3
5	0.8	0.0	0.2
6	0.8	0.0	0.2
7	0.8	0.0	0.2
8	0.6	0.0	0.4
9	0.6	0.0	0.4

Table 9  
*Matrix of Actual Factor Input Loadings for the Major Domain,  $A_1$*

Variable	Factor 1	Factor 2	Factor 3
1	0.153	0.702	0.428
2	0.750	0.104	0.162
3	0.098	0.830	-0.043
4	0.834	-0.040	-0.047
5	0.005	0.272	0.852
6	-0.037	0.612	0.651
7	0.869	0.209	-0.008
8	0.680	-0.039	0.368
9	0.773	0.008	0.042

Table 10  
*Values for Factor Input Loadings for the Unique Factors, A<sub>3</sub>*

Variable	Value
1	0.548
2	0.632
3	0.548
4	0.548
5	0.447
6	0.447
7	0.447
8	0.632
9	0.632

Table 11 presents the correlation matrix that is a result of the sum of products of the A<sub>1</sub> matrix with its inverse and the A<sub>3</sub> with its inverse.

Table 11  
*Simulated Correlations*

Variable	1	2	3	4	5	6	7	8	9
1		.257	.579	.080	.557	.703	.277	.235	.142
2			.152	.614	.170	.141	.673	.566	.588
3				.500	.190	.476	.259	.186	.080
4					-.465	-.087	.717	.552	.643
5						.721	.055	.307	.191
6							.090	.191	.003
7								.581	.674
8									.541
9									

This simulation was developed in SAS 9.2. The IML code was adapted from a study conducted by Hogarty, Hines, Kromrey, Ferron, and Mumford (2005). The program associated with this design example can be found in Appendix B.

### **Monte Carlo Simulation Design**

With few modifications, the simulation strategy included in this study is derived from Hogarty, Hines, Kromrey, Ferron, & Mumford's (2005) investigation of the relationship between sample size and factor solutions. The primary modification includes the addition of various proportions of categorical variables to the simulated data sets. This study also controlled for the number of variables, communality levels, and sample size.

**Proportion of categorical variables.** The ratios of categorical to continuous variables are selected to provide insight into a variety of research contexts. The samples developed through the simulated correlation matrices included distributions that contained the following percentages of categorical variables: 5%, 25%, 50%, 75%, and 95%. While simulating a broad range of data contexts, these percentages resulted in whole numbers of variables in data sets that contain 20, 40, and 60 observed variables.

During the data generation phase of this design, differing percentages of each sample of observed variables were dichotomized to yield the appropriate number of categorical observations. When data were intended to represent categorical variables, the resulting, simulated value was dichotomized at 0.5. When the value was less than 0.5, the value of the categorical variable was set to zero; when the value was greater than or equal to 0.5, the value of the categorical value was set to one.

**Number of variables.** To enhance this project's generalizability, this study included simulations of results from data sets of varying size. When considered in concert with the differing sample sizes, this study incorporated participant ( $N$ ) to variable ( $p$ ) ratios that range from “insufficient to those considered more than acceptable” (MacCallum, Widaman, Zhang, & Hong, 1999, p. 92). This study simulated data sets containing 20, 40, and 60 variables. The simulated combinations of sample sizes and numbers of observed variables yielded  $N:p$  ratios that range from 1.67:1 to 30:1.

**Number of common factors.** This study simulated research contexts that involve varying ratios of observed variables to common factors. The number of factors condition consisted of three levels: two, four, and eight common factors. Because factors defined by fewer than two variables would “contradict the basic idea of a factor as a latent construct” (Henson & Roberts, p. 408), the upper limit on the number of common factors simulated in this study was constrained by the simulated context which includes 20 observed variables. In this study, the ratio of observed variables to common factors ranged from 2.5:1, or insufficient (Fabrigar et al., 1999), to 30:1.

**Communality levels.** The simulated levels of communality included in this study are also derived from existing methodological research in factor analysis. This study simulated three community levels: high, wide, and low communality ( $h^2$ ) (Hogarty et al., 2005; MacCallum, Widaman, Zhang, & Hong, 1999). These communality levels correspond to the following:

1. High-- $h^2$  for each variable are randomly drawn from values of .6, .7, and .8
2. Wide-- $h^2$  for each variable are randomly drawn from values of .2, .3, .4, .5, .6, .7, and .8

3. Low-- $h^2$  for each variable are randomly drawn from values of .2, .3, and .4

**Sample size.** The four sample size ( $N$ ) conditions included in this study were 100, 200, 300, and 1000 simulated subjects. Although many studies provide suggested minimum values for the ratio of participants to variables when employing factor analytic techniques (Stevens, 2002), recent simulation studies do not find a minimum sample size to variable ratio that can be associated with good factor recovery (Hogarty et al., 2005; MacCallum, Widaman, Zhang, & Hong, 1999). Therefore, the sample size levels included in this study are based on ranges of sample sizes that are commonly found in recent factor analytic research in organizational management and social science research (Conway & Huffcutt, 2003).

**Rotation.** The initial factor solutions from each model in each condition was subjected to an orthogonal rotation strategy. Specifically, this study employed a varimax rotation in all simulated contexts. Because the intent of this study is to address methodological issues that are frequently encountered in social science literature, varimax rotation was considered to be a more appropriate choice than the methodologically sound procustean rotation (Fabrigar et al., 1999; Merrifield, 1974; Raykov & Little, 1999). The parsimony associated with uncorrelated factors represents one of the strongest rationales for employing orthogonal rotation strategies (Fabrigar et al., 1999; Raykov & Little, 1999).

### **Evaluation of Factor Extraction Methods**

Principal axis, ordinary least squares, and maximum likelihood factor extraction methods were evaluated in terms of five general sets of criteria. These criteria include: population and sample factor pattern agreement, congruence, and correlations between

population and sample factor score matrices. (Hogarty et al., 2005; MacCallum, Widaman, Zhang, & Hong, 1999). The following sections address each of these evaluation criteria.

**Population and sample factor pattern agreement.** This study includes assessments of agreement between the population, or true, factor characteristics and those characteristics implied by the factor analytic techniques under investigation, the sample. These measures were also described by Hogarty, Hines, Kromrey, Ferron, and Mumford (2005), and they include:

1. Loading sensitivity or agreement between sample and population in terms of the proportion of variables that have factor pattern coefficients that are greater than or equal to .30 on at least one factor; this indicator measures the level of agreement between sample and population matrices in terms of variables that load on at least one factor (Hogarty et al., 2005).
2. General pattern agreement as measured by the proportion of samples in which all of the variables loaded on the same factors in both the sample and the population. This measure does not take into account the magnitude of variable loadings on other factors in the sample matrices. “This index represents the extent of agreement for all variables that load on at least on factor” (Hogarty et al., 2005, p. 209).
3. Total agreement between sample and population is expressed in terms of the proportion of samples in which all variables loaded on sample and a population factors to the same extent. Factor loadings will be assessed as agreeing if both

sample and population loadings are greater than or equal to .30 or less than .30 (Hogarty et al., 2005).

4. “Per element accuracy” (Hogarty et al., 2005, p. 209) is reflected in the proportion of variables that load correctly in each sample; these proportions will be averaged across all samples.

**Congruence.** In addition to the above criteria, the capacity for each of the examined factor analytic approaches to extract the appropriate pattern of loadings for each factor will be measured by a congruence coefficient as described by MacCallum et al. (1999). This coefficient  $k$  is defined as (MacCallum et al., 1999):

$$\phi_k = \frac{\sum_{j=1}^p f_{jk(s)} f_{jk(t)}}{\sqrt{\left( \sum_{j=1}^p f_{jk(s)}^2 \right) \left( \sum_{j=1}^p f_{jk(t)}^2 \right)}}$$

Where  $f_{jk(t)}$  is the true population factor loading for variable  $j$  on factor  $k$  (in the population), and  $f_{jk(s)}$  is the sample population factor loading for variable  $j$  on factor  $k$ .

To provide a summary of each method’s effectiveness across all factors, an average of this congruence coefficient was computed. This summary statistic is denoted by  $K$  and is defined as (MacCallum et al., 1999):

$$K = \frac{\sum_{k=1}^r \phi_k}{r}$$

A second indicator of congruence included in this study is a measure of statistical bias. This estimate for the  $j$ th coefficient of the  $k$ th factor is given by:

$$Bias(\hat{\lambda}_{jk}) = \frac{1}{M} \sum_M (\hat{\lambda}_{jkm} - \lambda_{jk})$$

Where  $\hat{\lambda}_{jkm}$  is the coefficient obtained via the  $m$ th sample,  $\lambda_{jk}$  is the population coefficient, and  $M$  is the number of samples included in the study. The absolute values of bias will be used in the computation of an average across the  $M$  samples (Hogarty et al., 2005).

The third indicator of congruence included in this study was the root mean square error (RMSE). The estimate of RMSE is given by:

$$RMSE(\hat{\lambda}_{jk}) = \sqrt{\frac{1}{M} \sum_M (\hat{\lambda}_{jkm} - \lambda_{jk})^2}$$

The elements included in the estimate of RMSE are identical to those used to estimate a value for bias.

**Correlations between population and sample factor score matrices.** The first step in developing this evaluation measure includes the development of factor score estimates. These score estimates are linear combinations of variables; however, as opposed to using factor score coefficients, these estimates were computed using the following process:

1. A positive one scoring coefficient is assigned when the observed structure coefficient is  $\geq .30$ ;
2. A negative one scoring coefficient is assigned when the observed structure coefficient is  $\leq -.30$ ;
3. A scoring coefficient of zero is assigned when the structure coefficient is between  $.30$  and  $-.30$ .

Ater factor scores estimates werere computed for both the population and sample matrices, a correlation among the scores was used to measure how closely factor scores



derived from each of the factor extraction strategies approximates that factor score pattern that is imbedded in the population (Hogarty et al., 2005).

### **Analysis of Results**

The results of this study were examined through a repeated measures analysis of variance with a five-between and one-within group design (Stevens, 2007). The between groups factors are the manipulated variables that comprise the set of simulated research contexts; these variables include ratio of categorical to continuous variables, sample size, number of observed variables, communality level, and number of common factors. The within groups portion of the analysis includes the three factor extraction methods under consideration in this study.

One repeated measures analysis of variance will be conducted for each of eight dependent variables included in this study. These dependent variables are the four measures of agreement among population and sample pattern matrices, a coefficient for congruence, factor score correlations, factor loading bias, and root mean squared error (RMSE).

To identify “practical differences” when statistical differences are found, effect sizes were examined (Stevens, 2007, p. 127). Specifically, this study employed a generalized eta-squared; this effect size parameter is defined by (Bakeman, 2005, p. 380)

$$\eta_G^2 = \frac{\sigma_{effect}^2}{\delta \times \sigma_{effect}^2 + \sigma_{measured}^2}$$

where:

1.  $\sigma_{effect}^2$  is variance due to “manipulated factors,” or between group variance (Bakeman, 2005, p. 380);

2.  $\delta$  is equal to one if the effect includes only factors that are manipulated by the investigator;
3.  $\sigma_{measured}^2$  includes variance due to individual differences (Bakeman, 2005);

In addition to the summary tables associated with balanced, between groups, factorial analyses of variance, results of this study are presented graphically. When statistically significant differences among factor extraction methods were identified, the Tukey procedure was employed to identify the sources of these significant differences.

## **Chapter Four**

### **Results**

The intent of this study is to explore the extent to which characteristics of factor models derived through the tested factor analytic methods agree with the true characteristics simulated in the population. Because of the explorative nature of this study, it will include both multivariate and univariate approaches to the repeated measures analysis; this approach can identify sets of significant treatment effects that neither approach may resolve on its own (Stevens, 2002, 2007). To accommodate for increased experiment-wise, type I error rates, tests of significance are based on .025 alpha level

Table 12 provides means and standard deviations of all performance measures by factor extraction method; these measures include factor loading sensitivity, general pattern agreement, per element agreement, total pattern agreement of agreement, congruence, factor score correlations, bias, and root mean squared error (RMSE). As the table indicates, mean values of total agreement are near zero for all tested factor extraction methods. In the two factor by 60 observed variable condition, principal axis and maximum likelihood factor extraction methods yielded factor score estimates of zero; therefore factor score correlations for these conditions could not be calculated

Table 12  
*Means and Standard Deviations of all Performance Measures by Factor Extraction Method*

Performance Measure	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Factor Loading Sensitivity	.414	0.326	.993	0.008	.391	0.300
General Pattern Agreement	.222	0.168	.826	0.167	.238	0.173
Per Element Agreement	.593	0.180	.750	0.078	.605	0.185
Total Pattern Agreement	.000	0.000	<.001	0.001	<.001	<0.001
Congruence	.454	0.240	.496	0.215	.556	0.236
Factor Score Correlations*	.326	0.196	.511	0.208	.382	0.213
Factor Loading Bias	-.121	0.064	.028	0.022	-.122	0.063
RMSE	.113	0.053	.092	0.046	.105	0.052

*N* = 540

\* For principal axis and maximum likelihood methods, factor score correlations *N* = 480

Table 13 provides Pearson product moment correlation coefficients for all performance measures associated with factor loading matrices derived through the principal axis factor extraction method. As the table demonstrates, factor loading sensitivity, general pattern agreement, and per element agreement are significantly and positively related. These three performance measures have significantly negative relationship with RMSE. In the case of principal axis factor extraction, all total pattern

agreement values were zero; therefore, pairwise correlations with other performance measures were not estimated.

Table 13  
*Pearson Product Moment Correlations among Outcome Variables for Principal Axis Factor Analysis*

	1	2	3	4	5	6	7	8
Factor Loading Sensitivity (1)		.720*	.616*	-	-.693*	.002	.524*	-.276*
General Pattern Agreement (2)			.400*	-	-.123*	.657*	.268*	-.276*
Per Element Agreement (3)				-	-.707*	.036	.923*	-.814*
Total Agreement (4)					-	-	-	-
Congruence (5)						.559*	-.663*	.431*
Factor Score Correlations (6)							-.105*	.011
Factor Loading Bias (7)								-.924*
RMSE (8)								

*N = 480*

*\* Significant at the alpha = .05 Level*

Table 14 provides Pearson product moment correlation coefficients for all performance measures associated with factor loading matrices derived through the ordinary least squares factor extraction method. As the table demonstrates, factor loading sensitivity and general pattern agreement are positively and significantly related. However, unlike the correlations found in outcome measures associated with principal axis factor extraction, neither of these measures are significantly related to per element agreement.

Table 14  
*Pearson Product Moment Correlations Among Outcome Variables for Ordinary Least Squares Factor Analysis*

	1	2	3	4	5	6	7	8
Factor Loading Sensitivity (1)		.667*	-.049	.172*	.281*	.231*	.350*	.305*
General Pattern Agreement (2)			-.074	.213*	.723*	.710*	.757*	.283*
Per Element Agreement (3)				-.129*	-.419*	-.396*	-.007	-.832*
Total Agreement (4)					.361*	.337*	-.068	.183*
Congruence (5)						.989*	.630*	.358*
Factor Score Correlations (6)							.642*	.329*
Factor Loading Bias (7)								.128*
RMSE (8)								

*N = 540*

*\* Significant at the alpha = .05 Level*

Table 15 provides Pearson product moment correlation coefficients for all performance measures associated with factor loading matrices derived through the maximum likelihood factor extraction method. As Table 15 demonstrates, factor loading sensitivity, general pattern agreement, and per element agreement are significantly and positively related. These three performance measures have significantly negative relationship with RMSE.

For each outcome variable, these analyses include information concerning the assumptions associated with repeated measures analysis. Specifically, the results include

tests of multivariate normality and sphericity (Stevens, 2002, 2007). When the assumptions cannot be maintained, repeated measures analyses of variance tests of significance are based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002, 2007).

Table 15  
*Pearson Product Moment Correlations Among Outcome Variables for Maximum Likelihood Factor Analysis*

	1	2	3	4	5	6	7	8
Factor Loading Sensitivity (1)		.756*	.639*	.052	-.696*	.006	.517*	-.257*
General Pattern Agreement (2)			.455*	.058	-.170*	.623*	.308*	-.166*
Per Element Agreement (3)				.028	-.675*	.076	.927*	-.787*
Total Agreement (4)					-.004	.022	.045	-.031
Congruence (5)						.542*	-.586*	.313*
Factor Score Correlations (6)							-.039	-.078
Factor Loading Bias (7)								-.915*
RMSE (8)								

*N* = 480

\* Significant at the  $\alpha = .05$  Level

In addition to tests of statistical significance, the repeated measures analyses of variance include generalized eta-squared,  $\eta_G^2$ , effect size estimates (Bakeman, 2005). When statistical significance is accompanied by a medium effect size, an  $\eta_G^2$  value of .0588, or greater, the means of the outcome measure for each factor extraction method are compared (Cohen, 1988). However, if any interactions among main effects exhibit

the same pattern of statistical significance and effect size, then only the interaction are interpreted. Comparisons among means are considered through tables, graphs of means, and box and whisker plots.

### **Factor Loading Sensitivity**

Factor loading sensitivity is expressed in terms of agreement between sample and population in terms of the proportion of variables that have factor pattern coefficients that are greater than or equal to .30 on at least one factor. This is the count of variables that meet a .30 factor loading threshold for at least one factor in both the sample and the population divided by the count of variables that meet the .30 loading threshold in the population. Table 16 presents descriptive statistics concerning the univariate distribution of this loading sensitivity measure by each of the tested factor extraction methods.

Table 16  
*Descriptive Statistics for Distribution of Factor Loading Sensitivity Values*

Factor Extraction Method	<i>N</i>	<i>M</i>	<i>SD</i>	Skewness	Kurtosis
Principal Axis	540	0.414	0.326	0.305	-1.130
Ordinary Least Squares	540	0.993	0.008	-1.122	0.158
Maximum Likelihood	540	0.391	0.300	0.254	-1.120

When combined with the observed levels of skewness and Kurtosis, the Shapiro-Wilk tests of normality provide evidence of significant non-normality in the univariate



distributions of factor loading sensitivity measures for all three factor extraction methods. Moreover, Mauchly's test of transformed variables,  $\chi^2 (2) = 337.428, p < .0001$ , indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To account for these violations of the assumptions associated with the univariate, repeated measures analysis of variance, the Greenhouse-Geisser adjustment to degrees of freedom was used to develop  $p$  values for the within subjects tests of significance.

The multivariate analysis of variance indicate that factor loading sensitivity differed significantly by factor extraction method,  $\Lambda = .0001, F (2, 495) = 342901, p < .0001$ . In addition to the factor extraction method main effect, all interaction among factor extraction method and manipulated research characteristics yielded significant differences in factor loading sensitivity. Eight of the 10 first order interactions among main effects are also significant at the  $\alpha = 0.025$  level. The results of these analyses can be found in see Table C1 (see appendix C).

Results of the repeated analysis of variance yield a similar pattern of significance as those associated with the multivariate analysis. For example, the within-subjects portion of the ANOVA indicate that mean values of factor loading sensitivity differ significantly by factor extraction method,  $F (2, 920) = 600815, p < .0001, \eta_G^2 = .999$ . The model including main effects and first-order interactions accounted for 99.9% of the variability associated with factor loading sensitivity.

In addition to the method effect, the between subjects analysis yielded evidence that the sensitivity measure differed significantly for all of the main effects: Number of factors (K), number of observed variables (P), sample size (N), communality range (H),

and dichotomization (D). The results also indicate that mean values of this measure were significantly related to five interactions among these main effects; these interactions include number of factors by number of observed variables ( $K \times P$ ), number of factors by communality range ( $K \times H$ ), number of factors by level of dichotomization ( $K \times D$ ), number of observed variables by communality range ( $P \times H$ ), and communality range by level of dichotomization ( $H \times D$ ). The results of the univariate, repeated measures ANOVA can be found in C2 (see appendix C).

Agreement between the multivariate and univariate approaches was not universal. The multivariate approach yielded evidence that factor loading sensitivity is significantly related to interactions between number of observed variables by sample size ( $P \times N$ ) and number of observed variables by level of dichotomization ( $P \times D$ ) (see Table C1 in appendix C). However, the univariate, repeated measures analysis failed to yield evidence of significant differences in mean levels of factor loading sensitivity associated with these interactions.

Because the generalized eta-squared values for the  $P \times N$  and  $P \times D$  interactions indicated that their effect sizes were less than medium (Cohen, 1988), comparisons among the mean values for factor loading sensitivity based on these interactions were not analyzed. Although each main effect is also included in a first-order interaction that yielded a medium or greater effect size, this study includes interpretations of the interaction effects only. The two interactions with factor extraction methods that met both the statistical significance and the effect size requirements for further analyses include number of factors by number of observed variables,  $F(8, 920) = 1086.34, p < .0001, \eta_G^2 = .862$ , and number of observed variables by communality range,  $F(8, 920) =$

20.47,  $p < .0001$ ,  $\eta_G^2 = .059$ . The between subjects analyses indicate that number of factors by communality range,  $F(4, 460) = 11.81$ ,  $p < .0001$ ,  $\eta_G^2 = .062$ , and the number of factors by level of dichotomization,  $F(8, 460) = 5.43$ ,  $p < .0001$ ,  $\eta_G^2 = .574$ , also warrant follow-up comparisons. Tables 17 through 20 provide means and standard deviations for factor loading sensitivity measures for all four of these interactions.

Table 17  
*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Factors by Observed Variables Interaction (K x P)*

Interaction (K x P)	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 20	.478	0.030	.995	0.007	.475	0.031
2 × 40	.012	0.012	.995	0.006	.140	0.013
2 × 60	.000	0.000	.995	0.006	.000	0.00
4 × 20	.794	0.025	.991	0.009	.753	0.036
4 × 40	.317	0.016	.992	0.008	.316	0.016
4 × 60	.138	0.018	.992	0.008	.141	0.015
8 × 20	.980	0.004	.993	0.009	.900	0.016
8 × 40	.659	0.027	.993	0.008	.583	0.037
8 × 60	.345	0.034	.993	0.007	.334	0.037

$N = 540$

At all levels of the interaction between number of factors and number of observed variables, the ordinary least squares factor extraction method yields the highest mean proportions of variables that load on at least one factor in both the population and the sample; this mean proportion exceeds .99. For the principal axis and maximum likelihood strategies, factor loading sensitivity appears to be positively related to the number of factors and negatively related to the number of observed variables.

As the graph of means in Figure 2 highlights, the differences in factor loading sensitivity values among the three factor extraction methods are the smallest when the

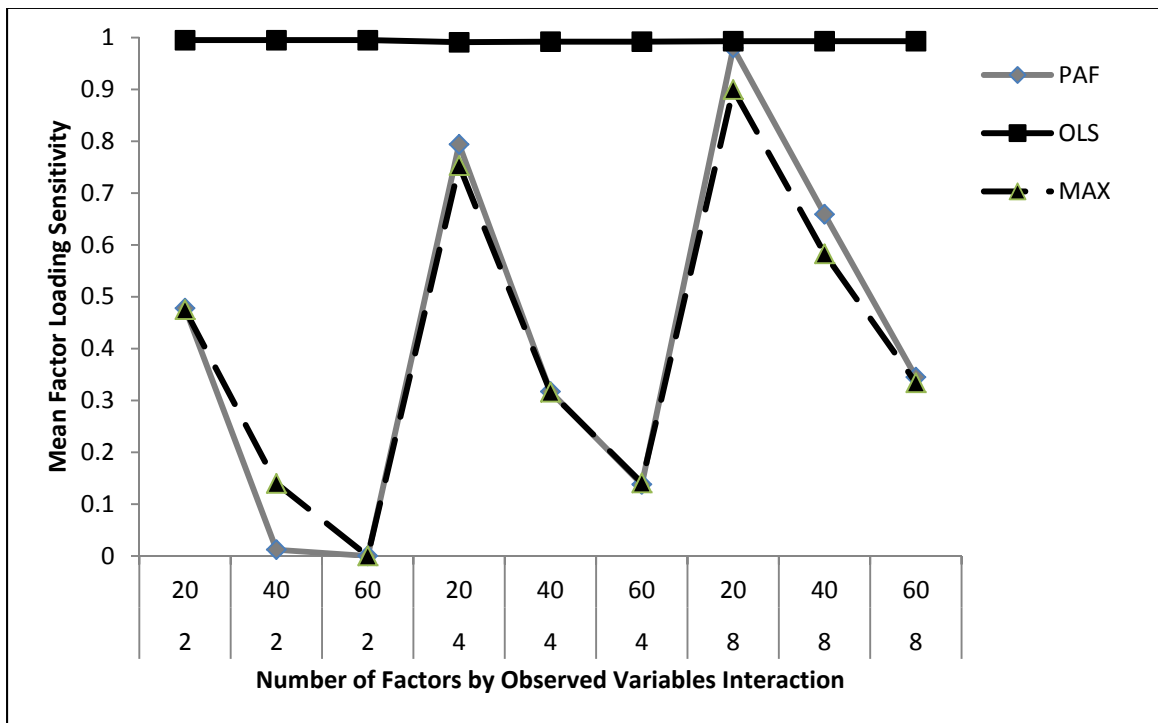


Figure 2. Mean values of factor loading sensitivity by interactions between number of factors and observed variables.

observed variable to factor ratio is the smallest. Box and whisker plots of these mean proportions indicate that factor loading sensitivity values associated with the least squares have smaller ranges and semi-interquartile ranges than the distributions associated with maximum likelihood and principal axis factor extraction methods (see Figure D1 in appendix D). For the ordinary least squares factor extraction method, measures of both central tendency and dispersion in the distributions of factor loading sensitivity values appear to be independent of observed variable to number of factor ratios.

Table 18 provides means and standard deviations of factor loading sensitivity values based on the number of factors by communality range interaction. The ordinary least squares factor extraction method yields the highest mean proportions of variables that load on at least one factor in both the population and the sample. For the principal axis and maximum likelihood strategies, the measure of factor loading sensitivity is positively related to both the number of factors and community level. Mean values of factor loading sensitivity associated with the Ordinary Least Squares factor extraction method appeared to be independent of a  $K \times H$  interaction effect values. As the graph of means in Figure 3 indicates, the differences in factor loading sensitivity decrease as number of factors and communality range increase.

Box and whisker plots indicate that factor loading sensitivity values associated with the least squares have smaller ranges and semi-interquartile ranges than the distributions associated with maximum likelihood and principal axis factor extraction methods (see Figure D2 in appendix D). For the ordinary least squares factor extraction method, measures of both central tendency and dispersion in the distributions of factor

loading sensitivity values appear to be independent of levels of the number of factors by communality interaction.

Table 18  
*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Factors by Communality Range Interaction (K x H)*

Interaction ( <i>K x H</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × Low	.155	0.214	.999	0.007	.154	0.212
2 × Wide	.165	0.217	.996	0.003	.165	0.214
2 × High	.170	0.243	1.000	0.000	.170	0.242
4 × Low	.409	0.289	.982	0.006	.390	0.259
4 × Wide	.412	0.266	.993	0.002	.398	0.246
4 × High	.429	0.284	.999	0.000	.422	0.276
8 × Low	.651	0.273	.982	0.005	.584	0.242
8 × Wide	.655	0.264	.996	0.002	.600	0.233
8 × High	.678	0.250	.999	0.000	.635	0.231

*N* = 540

Note:

Table 19 presents means and standard deviations of factor loading sensitivity values based on the number of factors by level of dichotomization interaction (*K × D*).

Across all levels of the interaction, factor loading sensitivity is highest for the Ordinary Least Squares factor extraction method.

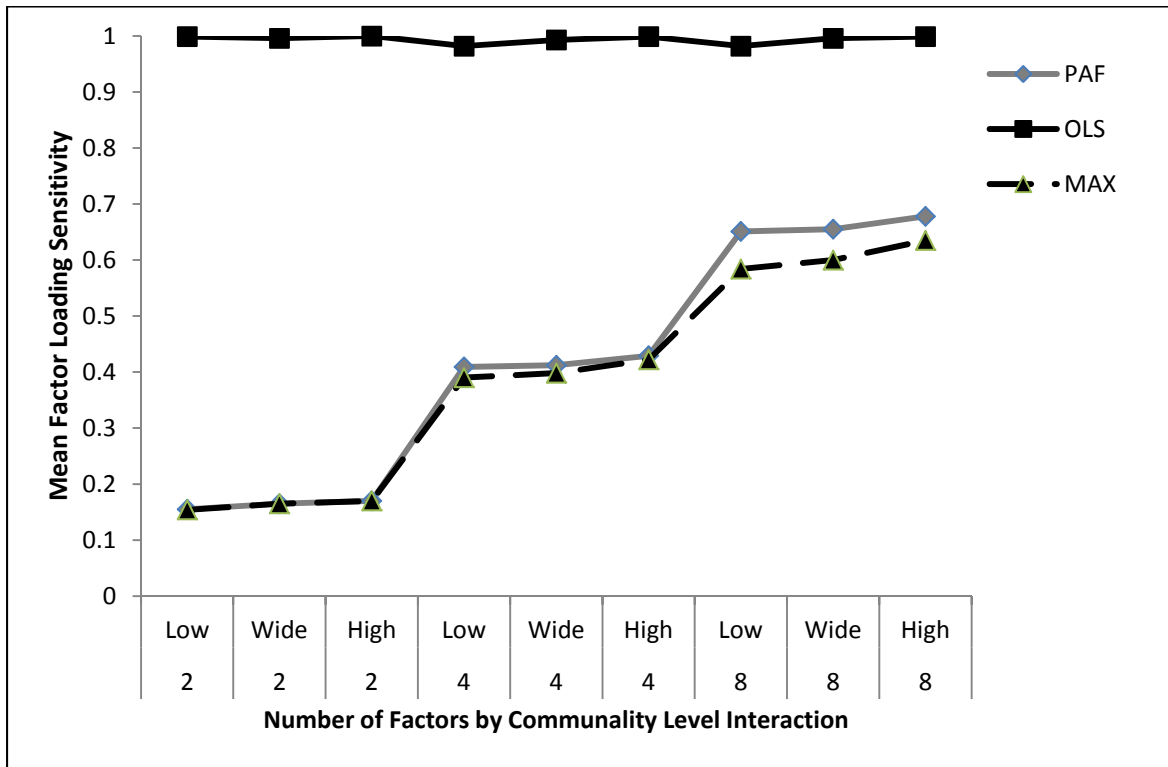


Figure 3. Mean values of factor loading sensitivity by interactions between number of factors and communality.

For the principal axis and maximum likelihood strategies, values of the factor loading sensitivity measure are positively related to the number of factors. For each number of factors level, the mean values for factor loading sensitivity increase in value between the .05 and .75 levels of dichotomization. However, for each combination of effects, the mean factor loading sensitivity value decreases between the .75 and .95 dichotomization levels.

Table 19  
*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Factors by Level of Dichotomization Interaction (K x D)*

Interaction (K x D)	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × .05	.155	0.213	.994	0.007	.155	0.211
2 × .25	.160	0.223	.995	0.006	.160	0.222
2 × .50	.165	0.228	.995	0.006	.165	0.226
2 × .75	.170	0.237	.995	0.006	.170	0.235
2 × .95	.167	0.230	.996	0.006	.167	0.229
4 × .05	.413	0.288	.992	0.008	.398	0.266
4 × .25	.418	0.285	.991	0.009	.405	0.267
4 × .50	.423	0.287	.992	0.007	.412	0.271
4 × .75	.414	0.274	.992	0.008	.402	0.254
4 × .95	.413	0.274	.991	0.009	.399	0.253
8 × .05	.646	0.271	.993	0.008	.584	0.247
8 × .25	.656	0.267	.993	0.007	.597	0.240
8 × .50	.666	0.263	.992	0.008	.613	0.238
8 × .75	.673	0.260	.992	0.009	.621	0.230
8 × .95	.667	0.259	.993	0.008	.616	0.231

*N* = 540



Box and whisker plots indicate that factor loading sensitivity values associated with the least squares methods have smaller ranges and semi-interquartile ranges than the distributions associated with maximum likelihood and principal axis factor extraction methods (see Figure D3 in appendix D). For ordinary least squares samples, measures of both central tendency and dispersion in the distributions of factor loading sensitivity values appear to be independent of levels of the number of factors by dichotomization interaction.

As the graphed means in Figure 4 indicates, the differences among mean factor loading sensitivity levels diminish (generally) as the number of factors increases.

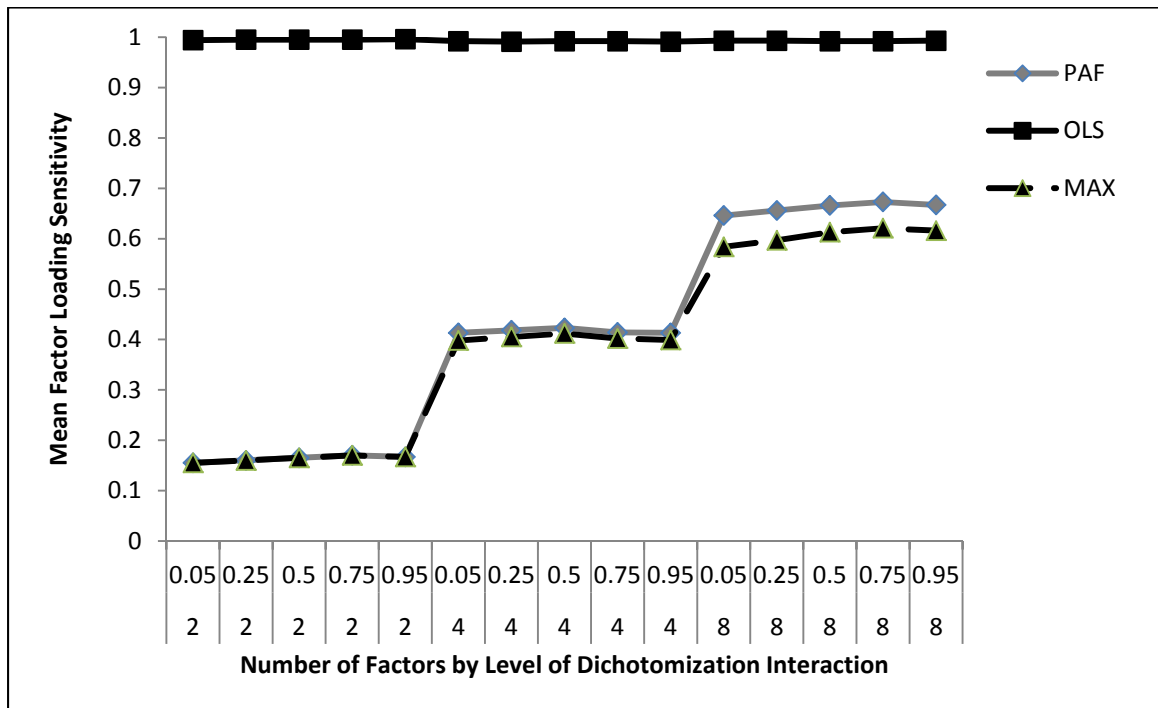


Figure 4. Mean values of factor loading sensitivity by interactions between number of factors and level of dichotomization.

The number of factors by level of dichotomization interaction effect does not appear to have a substantial impact on the mean values of factor loading sensitivity associated with the Ordinary Least Squares factor extraction method.

Mean values for factor loading sensitivity associated with the interaction among numbers of observed variables and communality ( $P \times H$ ) are presented in Table 20.

Table 20  
*Means and Standard Deviations of Factor Loading Sensitivity by Factor Extraction Method and Number of Observed Variables by Communality Range Interaction ( $P \times H$ )*

Interaction ( $P \times H$ )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
20 × Low	.745	0.220	.983	0.008	.693	0.184
20 × Wide	.739	0.211	.995	0.002	.696	0.179
20 × High	.767	0.198	.999	0.001	.741	0.175
40 × Low	.322	0.266	.985	0.007	.292	0.226
40 × Wide	.329	0.256	.995	0.003	.307	0.225
40 × High	.336	0.78	.999	0.000	.315	0.255
60 × Low	.147	0.133	.986	0.006	.144	0.126
60 × Wide	.162	0.140	.995	0.003	.160	0.138
60 × High	.173	0.158	1.000	0.000	.170	0.155

$n = 540$

As was the case for all significant interactions, ordinary least squares yields the highest mean proportion of variables that load on at least one factor in both the sample and the population. For principal axis and maximum likelihood factor extraction strategies, the mean values for the factor loading sensitivity measure are negatively related to the number of observed variables.

As the graph in Figure 5 demonstrates, differences in mean factor loading sensitivity values increase as the communality range increase.

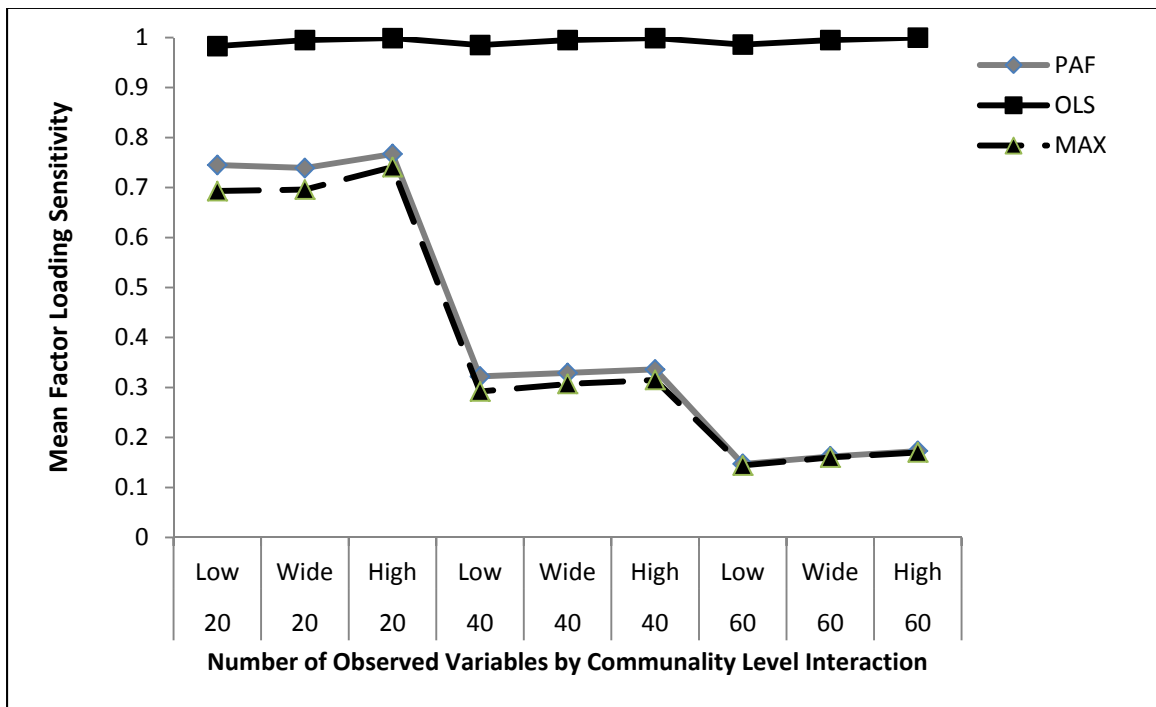


Figure 5. Mean values of factor loading sensitivity by interactions between number of observed variables and communality.

Box and whisker plots indicate that factor loading sensitivity values associated with ordinary least squares have smaller ranges and semi-interquartile ranges than the distributions associated with maximum likelihood and principal axis factor extraction methods (see Figure D4 in appendix D). For ordinary least squares samples, measures of both central tendency and dispersion in the distributions of factor loading sensitivity values appear to be independent of levels of the number of observed variables by communality interaction.

Across all significant interactions among the manipulated research characteristics, the ordinary least squares factor extraction method exhibited greater mean levels of factor loading sensitivity than the principal axis and the maximum likelihood factor extraction methods. Ordinary least squares factor extraction also resulted in distributions of factor loading sensitivity that were less variable than the distributions associated with principal axis and maximum likelihood. In interactions that included the number of factors main effect, the differences in factor loading sensitivity between ordinary least squares and the other two factor extraction methods were moderated by the number of factors; these differences decreased as the number of factors increased.

### **General Pattern Agreement**

General Pattern Agreement is based on the proportion of variables that load on both the sample and population factors in a similar fashion at least once. This variable is a  $P \times 1$  column vector. When the absolute value of a variable loading on the same factor in both the sample and the population is greater than or equal to .30 at least once; then a one is assigned to the row associated with the variable.

The proportion of variables that meet .30 loading criterion contributes to the general pattern agreement. For this measure the variable can load on multiple factors and still contribute to the pattern of agreement. Table 21 presents descriptive statistics concerning the univariate distribution of the general pattern agreement measure by each of the tested factor extraction methods.

Table 21  
*Descriptive Statistics for Distribution of General Pattern Agreement*

Factor Extraction Method	<i>N</i>	<i>M</i>	<i>SD</i>	Skewness	Kurtosis
Principal Axis	540	.222	0.168	0.561	-0.443
Ordinary Least Squares	540	.826	0.167	-1.389	1.211
Maximum Likelihood	540	.238	0.173	0.470	-0.533

For all tested factor extraction methods, Shapiro Wilks' tests of normality yield evidence that the univariate distributions of general pattern accuracy values are not normally distributed. Mauchly's test of transformed variables,  $\chi^2(2) = 600.964$ ;  $p < .0001$ , indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type I error rate, tests of significance associated with the repeated measures analysis of variance will be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

The results of the multivariate analyses of variance indicate that factor extraction method is a significant source of variation in general pattern agreement,  $\Lambda = .002$ ,  $F(2, 460) = 109246$ ,  $p < .0001$ . In addition to the method main effect, all interactions between manipulated research characteristics and factor extraction methods accounted for significant variability in mean values of general pattern agreement. General pattern agreement differed significantly by seven of the ten first order interactions. The results of these analyses are presented in Table C3 (see appendix C).

The univariate, repeated measures analysis of variance also indicate that values of general pattern agreement differ significantly by factor extraction method,  $F(2, 920) = 192576$ ,  $p < .0001$ ,  $\eta_G^2 = .991$ . The model including main effects and first-order interactions accounted for 99.4% of the variability associated with values of general pattern agreement. The summary table for these analyses is provided in Table C4 (see appendix C).

In nearly all cases, the multivariate and univariate approaches agreed in their identifications of four main effects and seven interaction effects that yielded significant differences in mean values for general pattern agreement. Specifically, these analyses identified number of factors by observed variables ( $K \times P$ ), number of factors by sample size ( $K \times N$ ), number of factors by communality range ( $K \times H$ ), number of factors by dichotomization level ( $K \times D$ ), observed variables by samples size ( $P \times N$ ), observed variables by communality range ( $P \times H$ ), and sample size by communality range ( $N \times H$ ) as sources of significant differences among mean general pattern agreement values. The between subjects portion of the univariate, repeated measures analysis of variance

identified dichotomization level as a significant main effect; however, the effect size for this effect was less than medium (Cohen, 1988).

To qualify for further interpretation, interactions among main effects had to be statistically significant and have effect sizes of medium or greater. Based on the within subjects analysis, the following interactions with factor extraction method met these criteria:

1. Number of factors by number of observed variables,  $F(8, 920) = 284.98, p < .0001, \eta_G^2 = .422,$
2. Number of factors by communality range,  $F(8, 920) = 189.60, p < .0001, \eta_G^2 = .327,$  and
3. Sample size by communality range,  $F(12, 920) = 57.33, p < .0001, \eta_G^2 = .180.$

Because the number of observed variables by communality range,  $F(4, 460) = 91.50, p < .0001, \eta_G^2 = .359,$  met the criteria when considered from a between subjects perspective, mean values for general pattern agreement based on this interaction were also compared.

Table 22 provides means and standard deviations for general pattern agreement for each of the factor extraction methods by each level of the number of factors by observed variable interaction. As Table 22 highlights, the mean values for general pattern agreement were highest for Ordinary Least Squares factor analysis in all levels of the  $K \times P$  interaction. Within each level of the interaction, the differences in mean values of general pattern agreement between Ordinary Least Squares and the other two factor extraction methods appear to be the smallest in the levels that include 20 observed variables.

Table 22  
*Means and Standard Deviations of General Pattern Agreement by Factor Extraction Method and Number of Factors by Observed Variables Interaction (K x P)*

Interaction ( <i>K x P</i> )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 20	.450	0.038	.921	0.062	.467	0.031
2 × 40	.021	0.018	.914	0.065	.023	0.020
2 × 60	.009	0.011	.913	0.064	.009	0.011
4 × 20	.497	0.105	.793	0.160	.533	0.085
4 × 40	.249	0.038	.877	0.095	.280	0.023
4 × 60	.132	0.014	.906	0.073	.146	0.011
8 × 20	.285	0.049	.597	0.187	.294	0.042
8 × 40	.214	0.053	.710	0.196	.225	0.038
8 × 60	.143	0.033	.807	0.161	.164	0.023

*n* = 540

Figure 6 demonstrates the relationship among factor extraction methods and the  $K \times P$  interaction graphically. Box and whisker plots indicate that factor loading sensitivity values associated with the least squares have larger ranges and semi-interquartile ranges than the distributions associated with maximum likelihood and principal axis factor extraction methods (see Figure D5 in appendix D). For the ordinary least squares factor extraction method, ranges and semi-interquartile ranges of general pattern agreement



values appear to be positively related to the number of factors and negatively related to the number observed variables. For maximum likelihood and principal axis factor extraction methods, the levels of dispersion are negatively related to both the number of factors and the number of observed variables.

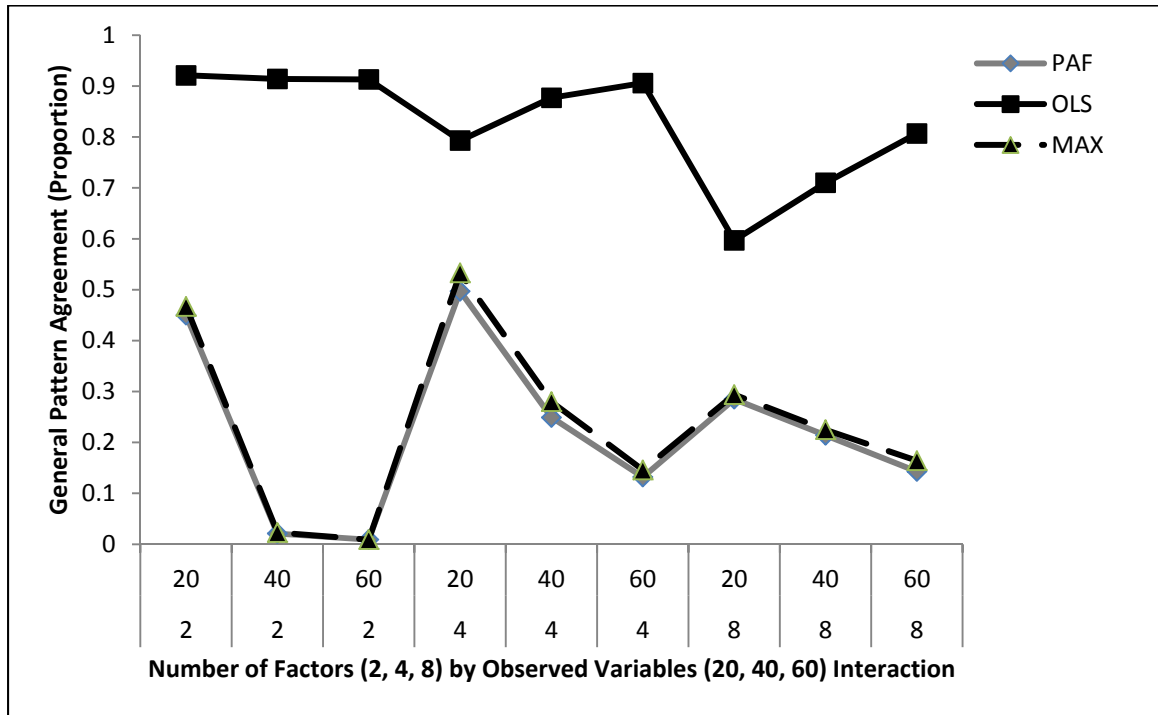


Figure 6. Mean values for general pattern agreement by levels of interaction between number of factors and number of observed variables.

Table 23 provides means and standard deviations for the values of the general pattern accuracy measure by levels of the number of factors by communality range ( $K \times H$ ) interaction.

Table 23  
*Means and Standard Deviations of General Pattern Agreement for Factor Extraction Method and Number of Factors by Community Range Interaction (K x H)*

Interaction (K x H)	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	M	SD	M	SD	M	SD
2 × Low	.153	0.186	.848	0.050	.166	0.201
2 × Wide	.163	0.203	.922	0.030	.166	0.205
2 × High	.164	0.233	.977	0.018	.167	0.239
4 × Low	.242	0.118	.737	0.122	.288	0.126
4 × Wide	.292	0.147	.868	0.062	.312	0.152
4 × High	.344	0.206	.971	0.022	.359	0.210
8 × Low	.170	0.048	.493	0.149	.195	0.045
8 × Wide	.213	0.067	.727	0.112	.223	0.054
8 × High	.260	0.075	.894	0.068	.261	0.073

*N* = 540

Mean values for the general pattern accuracy are highest for ordinary least squares across all levels of the interaction effect. In the case of ordinary least squares, the mean values for the general pattern accuracy measure appear to be positively related to the level of communality and negatively related to the number of factors; this general trend is not apparent in mean values of general pattern agreement for principal axis and maximum likelihood factor extraction methods. As the graphed means in Figure 7

indicate, the differences in mean levels of general pattern agreement are the smallest at the four factors by high communality condition.

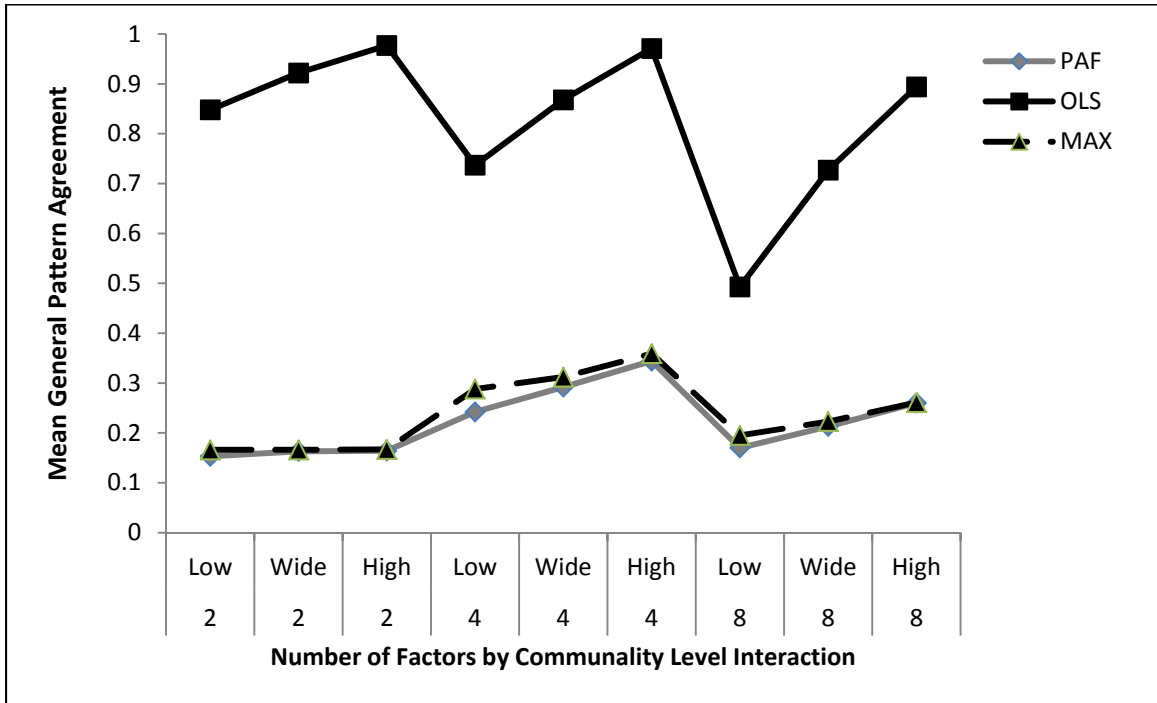


Figure 7. Mean values of general pattern agreement for levels of interaction between number of factors and communality.

For all levels of the number of factors by communality interaction, box and whisker plots indicate that factor loading sensitivity values associated with the least squares have smaller ranges and semi-interquartile ranges than the distributions associated with maximum likelihood and principal axis factor extraction methods (see Figure D6 in appendix D). For the ordinary least squares factor extraction method, communality range in the distributions of general pattern agreement values appear to be

positively related to the number of factors and negatively related to the level of communality. For maximum likelihood and principal axis factor extraction methods, the levels of dispersion are negatively related to the number of factors and positively related to the level of communality.

Table 24 provides means and standard deviations for the values of the general pattern agreement measure by number of observed variables and communality range ( $P \times H$ ) interaction. The mean values for the general pattern agreement are highest for Ordinary Least Squares across all levels of the interaction effect. In the case of ordinary least squares, the mean values for the general pattern agreement appear to be positively related to both number of observed variables and the level of communality. Principal Axis Factor and Maximum Likelihood factor extractions yield similar mean values for general pattern agreement across all levels of the  $P \times H$  interaction. As the number of observed variables increases, the mean values of general pattern agreement associated with Ordinary Least Squares improves to a greater extent over the other two factor extraction methods. These relationships are summarized graphically in Figure 8.

Box and whisker plots of general pattern agreement by levels of the  $P \times H$  interaction highlight several relationships among communality range, factor extraction method, and level of interaction. In conditions that include 60 observed variables, several outliers are present in distributions associated with the ordinary least squares factor extraction method. In these distributions of general pattern agreement, ranges and semi-interquartile ranges are negatively related to both number of observed variables and level of communality.

Table 24  
*Means and Standard Deviations of General Pattern Agreement for Factor Extraction Method and Number of Observed Variables and Community Range Interaction (P x H)*

Interaction ( <i>P x H</i> )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
20 × Low	.343	0.343	.614	0.214	.383	0.097
20 × Wide	.408	0.408	.780	0.143	.417	0.097
20 × High	.481	0.481	.916	0.076	.493	0.125
40 × Low	.134	0.134	.696	0.178	.162	0.099
40 × Wide	.166	0.166	.850	0.091	.177	0.103
40 × High	.185	0.184	.953	0.042	.189	0.138
60 × Low	.088	0.088	.768	0.132	.107	0.063
60 × Wide	.090	0.094	.885	0.056	.106	0.071
60 × High	.100	0.101	.972	0.025	.106	0.080

*N* = 540

In distributions associated with maximum likelihood and principal axis factor extraction methods, measures of dispersion are positively related to communality and negatively related to the number of observed variables.

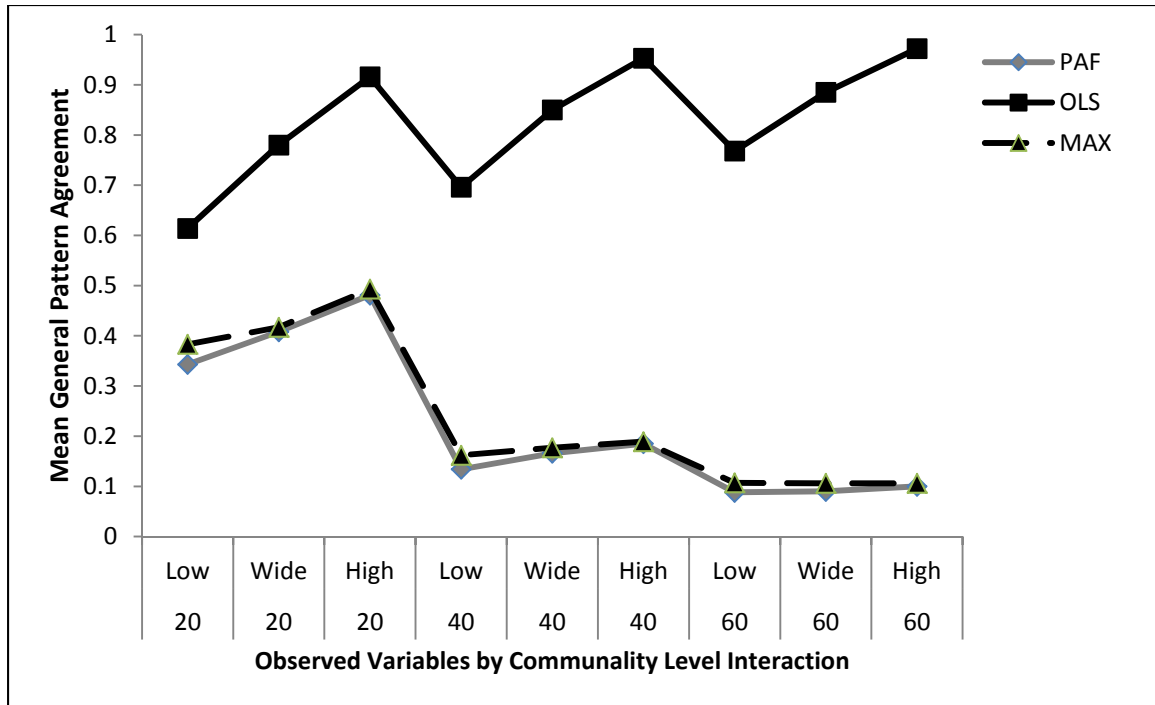


Figure 8. Mean values of general pattern agreement for levels of interaction between number of observed variables and communality.

Table 25 provides means and standard deviations for the values of the general pattern agreement measure by levels of sample size by communality range ( $N \times H$ ) interaction. The mean values for the general pattern accuracy measure are highest for ordinary least squares across all levels of the interaction effect. In the case of ordinary least squares, the mean values for the general pattern accuracy measure appear to be positively related to both sample size and the level of communality; this general trend in mean, general pattern agreement values is present in the principal axis and maximum likelihood factor extraction method; however, the trend is less pronounced.

Box and whisker plots of general pattern agreement values indicate that semi-interquartile ranges are negatively related to both communality level and sample size in

distributions associated with ordinary least squares (see Figure D8 in appendix D). In distributions associated with maximum likelihood and principal axis factor extraction methods, these communality range appear to be positively related to communality.

Table 25  
*Means and Standard Deviations of General Pattern Agreement for Factor Extraction Method and Sample Size by Communality Range Interaction (N x H)*

Interaction ( <i>N x H</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
100 × Low	.178	0.116	.598	0.183	.224	0.140
100 × Wide	.211	0.152	.789	0.114	.229	0.157
100 × High	.233	0.188	.918	0.058	.245	0.196
200 × Low	.183	0.130	.672	0.186	.218	0.147
200 × Wide	.220	0.158	.833	0.110	.233	0.162
200 × High	.252	0.198	.945	0.055	.260	0.203
300 × Low	.188	0.138	.714	0.176	.2145	0.150
300 × Wide	.225	0.161	.851	0.107	.234	0.164
300 × High	.261	0.202	.955	0.054	.267	0.206
1000 × Low	.204	.0157	.787	0.159	.213	0.160
1000 × Wide	.235	0.168	.882	0.098	.238	0.169
1000 × High	.276	0.210	.971	0.048	.278	0.212

*N* = 540

The relationships among mean values of the general pattern agreement measure, factor extraction method, and the  $N \times H$  interaction are presented graphically in Figure 9.

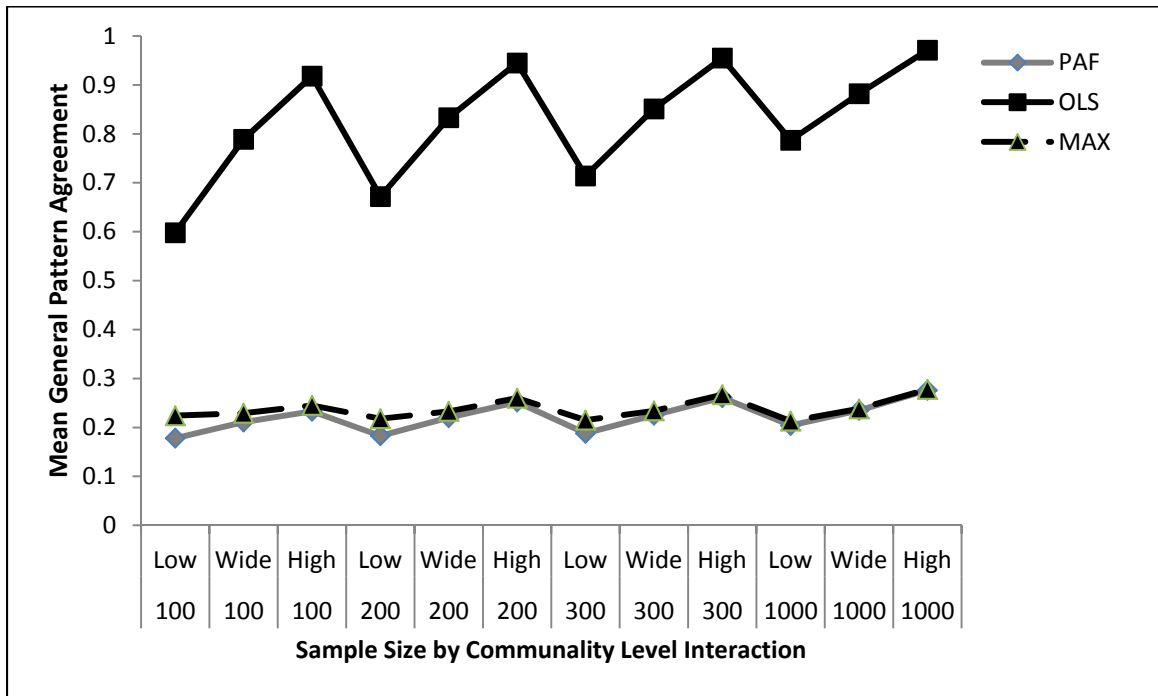


Figure 9. Mean values of general pattern agreement for levels of interaction between sample size and communality.

Mean values of general pattern agreement were higher for the ordinary least squares factor extraction method than they were for the principal axis and the maximum likelihood methods; this relationship among factor extraction methods was apparent across all significant interactions. For distributions associated with the ordinary least squares factor extraction method, general pattern agreement values exhibited greater levels of dispersion than the distributions of factor loading sensitivity values. Moreover, mean values of general pattern agreement for ordinary least squares appeared to be more



closely related to the values of the interaction effects than was the case with factor loading sensitivity.

### **Per Element Pattern Agreement**

The measure for per element pattern agreement is based on a  $P \times K$  matrix. Elements of each matrix contained ones, indicating agreement, where the corresponding factor loading had an absolute value of .30 or greater in both the sample and population factor pattern matrices; elements of the matrix also contained ones when the variable had loadings of less than the absolute value of .30 on the corresponding factor in both the sample and population. When these criteria are not met, the element had a value of zero. The resulting measure of per element pattern agreement is the proportion of samples in which the agreement criteria were met for each observed variable by factor combination. Table 26 presents descriptive statistics concerning the distribution of the per element pattern agreement measure by each of the tested factor extraction methods.

Shapiro-Wilks' tests of normality for per element pattern agreement measures associated with all three factor extraction methods yield evidence that the three distributions do not conform to assumptions regarding univariate normality. Mauchly's test of transformed variables,  $\chi^2(2) = 494.79, p < .0001$ , indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type I error rate, tests of significance associated with the repeated measures analysis of variance will be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

Table 26  
*Descriptive Statistics for Distribution of Per Element Pattern Agreement Values*

Factor Extraction Method	N	M	SD	Skewness	Kurtosis
Principal Axis	540	.593	0.180	-0.842	-0.465
Ordinary Least Squares	540	.750	0.750	0.005	-0.819
Maximum Likelihood	540	.605	0.185	-0.892	-0.445

The results of the multivariate analyses of variance indicate that per element agreement differed significantly by factor extraction method,  $\Lambda = .008$ ,  $F(2, 459) = 26828.4$ ,  $p < .0001$ . In addition to the method main effect, per element agreement values differed significantly by all main effects. The multivariate analysis of variance indicated that per element agreement differed significantly across eight of the first-order interactions. The results of this analysis are presented in Table C4 (see Appendix C).

The univariate, repeated measures analysis of variance also indicated that per element pattern agreement differed significantly based on factor extraction method,  $F(2, 920) = 45801.9$ ,  $p < .0001$ ,  $\eta_G^2 = .934$ . Results of the between subject portion of the analysis indicated that all manipulated research characteristics, main effects, were associated with significant differences in per element pattern agreement. Moreover, per element pattern agreement differed significantly by all interactions between factor extraction method and main effects. The model that included main effects and first-order interactions among main effects accounted for 98.7% of the variability in the observed data (see Table C6 in appendix C).

The between and within sections of the univariate, repeated measures analysis identified number of factors by observed variables ( $K \times P$ ), number of factors by sample size ( $K \times N$ ), number of factors by level of communality ( $K \times H$ ), number of factors by level of dichotomization ( $K \times D$ ), number of observed variables by level of communality ( $P \times H$ ), number of observed variables by level of dichotomization ( $P \times D$ ), sample size by communality ( $N \times H$ ), and level of communality by level of dichotomization ( $H \times D$ ) as sources of significant differences in per element pattern agreement. In the between groups analysis, per element agreement differed significantly by levels of the interaction between number of variables and sample size ( $P \times N$ ),  $F(6, 460) = 4.16, p = .0004, \eta_G^2 = .044$ ; however, when factor extraction method is included, the within groups analysis, this significant effect is not present. From both perspectives, the sources of variation that included the  $P \times N$  interaction failed to yield a medium effect size (Cohen, 1988).

According to the results of within subjects analyses, the number of factors by observed variables,  $F(8, 920) = 724.05, p < .0001, \eta_G^2 = .474$ , and the samples size by communality range,  $F(12, 920) = 44.65, p < .0001, \eta_G^2 = .077$ , were both statistically significant and yielded effect sizes that were medium or greater. The between subjects analysis indicated that the following interactions met the criteria for continued analyses:

1. Number of factors by sample size,  $F(6, 460) = 18.540, p < .0001, \eta_G^2 = .172$
2. Number of factors by communality range,  $F(4, 460) = 96.70, p < .0001, \eta_G^2 = .419$
3. Number of observed variables by communality range,  $F(4, 460) = 25.54, p < .0001, \eta_G^2 = .155$ .

Table 27 contains means and standard deviations for the per element pattern agreement measure associated with the number of factors by observed variable interaction.

Table 27  
*Means and Standard Deviations of Per Element Pattern Agreement for Factor Extraction Methods by the Number of Factors and Number of Observed Variables Interaction (K x P)*

Interaction ( <i>K x P</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 20	.521	0.052	.704	0.053	.539	0.057
2 × 40	.302	0.075	.666	0.040	.303	0.076
2 × 60	.296	0.071	.659	0.041	.296	0.071
4 × 20	.660	0.035	.718	0.057	.686	0.034
4 × 40	.628	0.046	.771	0.040	.673	0.049
4 × 60	.646	0.057	.789	0.043	.652	0.060
8 × 20	.724	0.021	.766	0.032	.719	0.025
8 × 40	.776	0.026	.821	0.050	.775	0.028
8 × 60	.796	0.025	.855	0.044	.801	0.026

*N* = 540

In all levels of the interaction, the Ordinary Least Squares factor extraction strategy yielded the highest mean levels of per element pattern agreement. For all factor extraction methods, mean values of the per element agreement measure are positively related to the number of factors. As Figure 10 highlights, the differences among the three factor extraction methods, in terms of per element agreement, decreases as the number of factors increases. For maximum likelihood and principal axis factor extraction methods, box and whisker plots for the distributions of per element agreement indicate that communality range are negatively related to the number of factors imbedded in the population (see Figure D9 in appendix D).

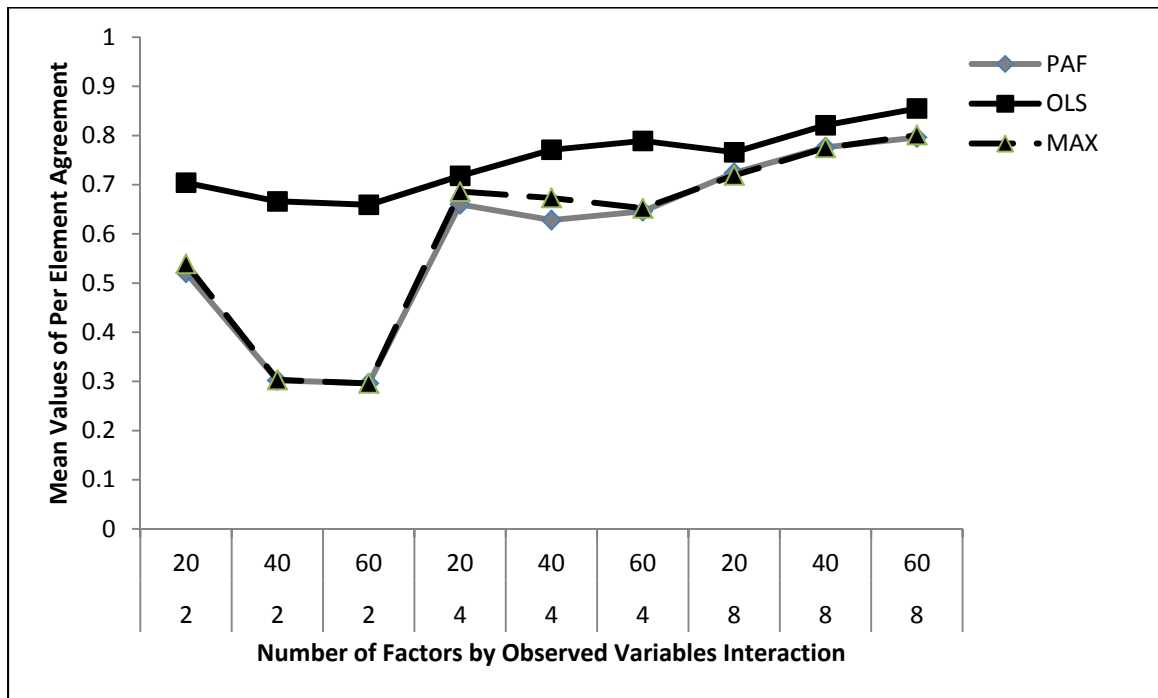


Figure 10. Mean values of per element agreement by interactions between the number of factors by number of observed variables.

Table 28 presents means and standard deviations in per element accuracy for levels of the number of factors by sample size interaction.

Table 28  
*Means and Standard Deviations of Per Element Agreement by Factor Extraction Method and Number of Factors by Sample Size Interaction (K x N)*

Interaction ( <i>K x N</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 100	.375	0.112	.654	0.037	.388	0.128
2 × 200	.373	0.123	.675	0.044	.380	0.132
2 × 300	.372	0.128	.683	0.048	.377	0.134
2 × 1000	.371	0.137	.694	0.057	.372	0.138
4 × 100	.631	0.038	.712	0.052	.654	0.042
4 × 200	.652	0.042	.756	0.046	.669	0.048
4 × 300	.660	0.045	.772	0.043	.675	0.051
4 × 1000	.675	0.052	.798	0.045	.683	0.056
8 × 100	.731	0.041	.760	0.043	.743	0.039
8 × 200	.750	0.047	.808	0.044	.763	0.042
8 × 300	.758	0.048	.829	0.043	.770	0.043
8 × 1000	.770	0.047	.860	0.042	.782	0.041

*N* = 540

Ordinary Least Squares yields the highest mean values of the per element pattern agreement measure across all levels of the  $K \times N$  interaction. For all three factor extraction methods the mean values of the per element pattern agreement measure is positively related to both the number of factors and the sample size levels; differences in this measure among the three tested factor extraction strategies diminishes as the number of factors increases. As the graph of means in Figure 11 and the box and whisker plots indicate (see Figure D10 in appendix D), mean and median values of per element factor agreement appear to converge at the interaction level that includes eight factors and a sample size of 100.

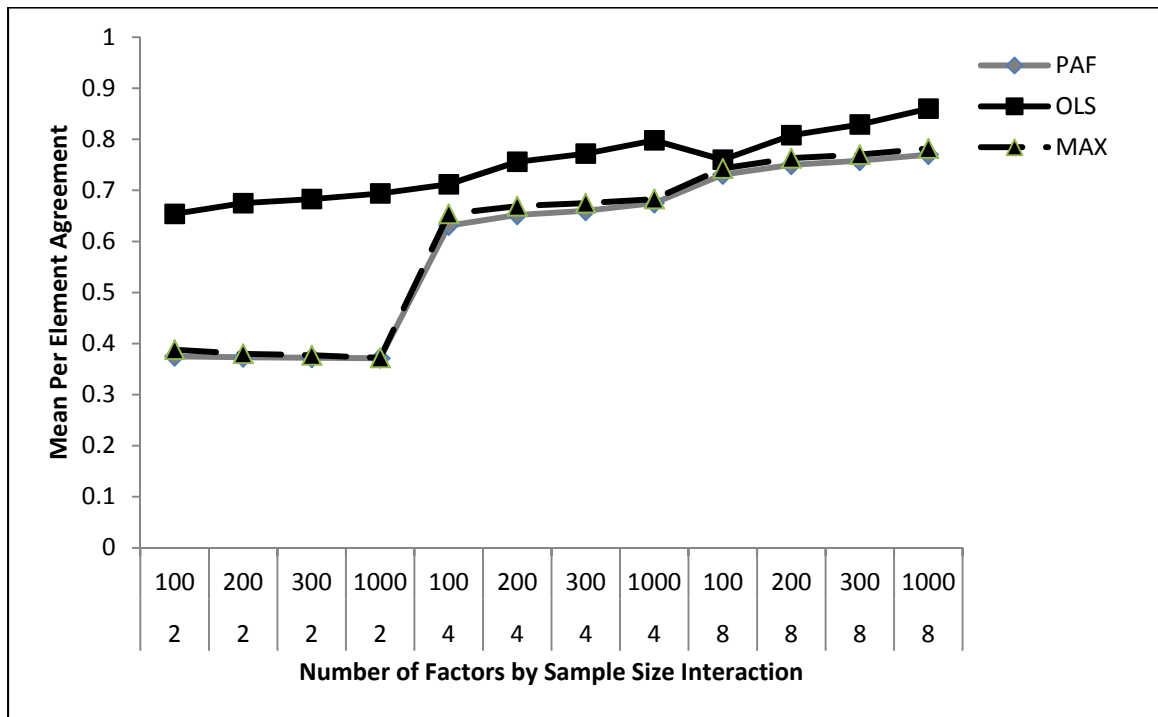


Figure 11. Mean values of per element agreement by interactions between the number of factors and sample size.

Table 29 provides means and standard deviations for these values of per element agreement for the number factors by level of communality ( $K \times H$ ) interaction. The mean per element pattern agreement is highest for Ordinary Least Squares in all levels of the interaction. The mean values for per element pattern agreement are positively related to the number of factors and negatively related to the communality range.

Table 29  
*Means and Standard Deviations of Per Element Pattern Agreement for Factor Extraction Method and Number of Factors by Communality Range Interaction ( $K \times H$ )*

Interaction ( $K \times H$ )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × Low	.444	0.092	.690	0.053	.456	0.107
2 × Wide	.379	0.108	.690	0.047	.382	0.111
2 × High	.296	0.125	.648	0.033	.300	0.130
4 × Low	.697	0.032	.775	0.075	.721	0.021
4 × Wide	.654	0.022	.758	0.047	.667	0.021
4 × High	.612	0.040	.745	0.033	.622	0.042
8 × Low	.775	0.047	.797	0.065	.792	0.038
8 × Wide	.750	0.043	.815	0.054	.761	0.038
8 × High	.732	0.044	.830	0.043	.741	0.038

$N = 540$



As the graph of means in Figure 12 and box and whisker plots (See Figure D11 in appendix D) demonstrate, the relationship between per element pattern agreement and the  $K \times H$  interaction appears to be more pronounced in the principal axis and the maximum likelihood factor extraction methods than in the ordinary least squares method.

Differences in mean per element agreement diminishes as the number of factors increases; however, because of the negative relationship between levels communality and per element agreement that is present in the principal axis and maximum likelihood factor extraction methods, the smallest difference among the three factor extraction methods occurs at the eight factors by low communality condition.

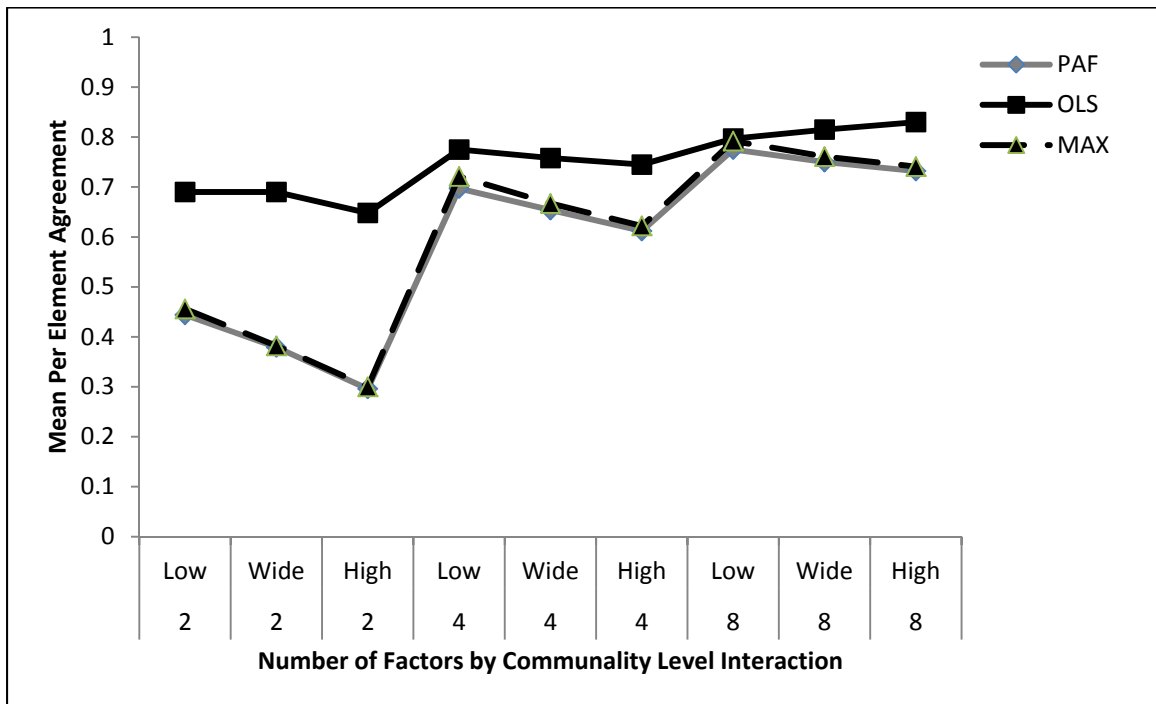


Figure 12. Mean values of per element agreement by interactions between the number of factors and communality.

Table 30 presents means and standard deviations for the per element pattern agreement measures associated with the number of observed variables by level of communality ( $P \times H$ ) interaction.

Table 30  
*Means and Standard Deviations of Per Element Agreement for Factor Extraction Methods and Number of Observed Variables by Community Range Interaction ( $P \times H$ )*

Interaction ( $P \times H$ )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
20 × Low	.654	0.070	.728	0.062	.688	0.065
20 × Wide	.625	0.077	.733	0.049	.642	0.082
20 × High	.599	0.096	.727	0.055	.614	0.099
40 × Low	.624	0.176	.752	0.075	.638	0.184
40 × Wide	.581	0.193	.763	0.067	.588	0.196
40 × High	.520	0.232	.744	0.090	.525	0.235
60 × Low	.638	0.191	.782	0.091	.644	0.194
60 × Wide	.577	0.212	.768	0.087	.580	0.214
60 × High	.521	0.234	.753	0.096	.524	0.236

$n = 540$

In all levels of the interaction, ordinary least squares factor extraction method has the highest mean value for the per element agreement measure. For the principal axis and

the maximum likelihood factor extraction methods, the relationship between per element agreement and level of communality is negative. As the Figure 13 highlights, a negative relationship between per element agreement and the number of observed variables also exists for the principal axis and maximum likelihood factor extraction methods. The negative relationship between the manipulated research conditions and per element agreement is not associated with the ordinary least squares method.

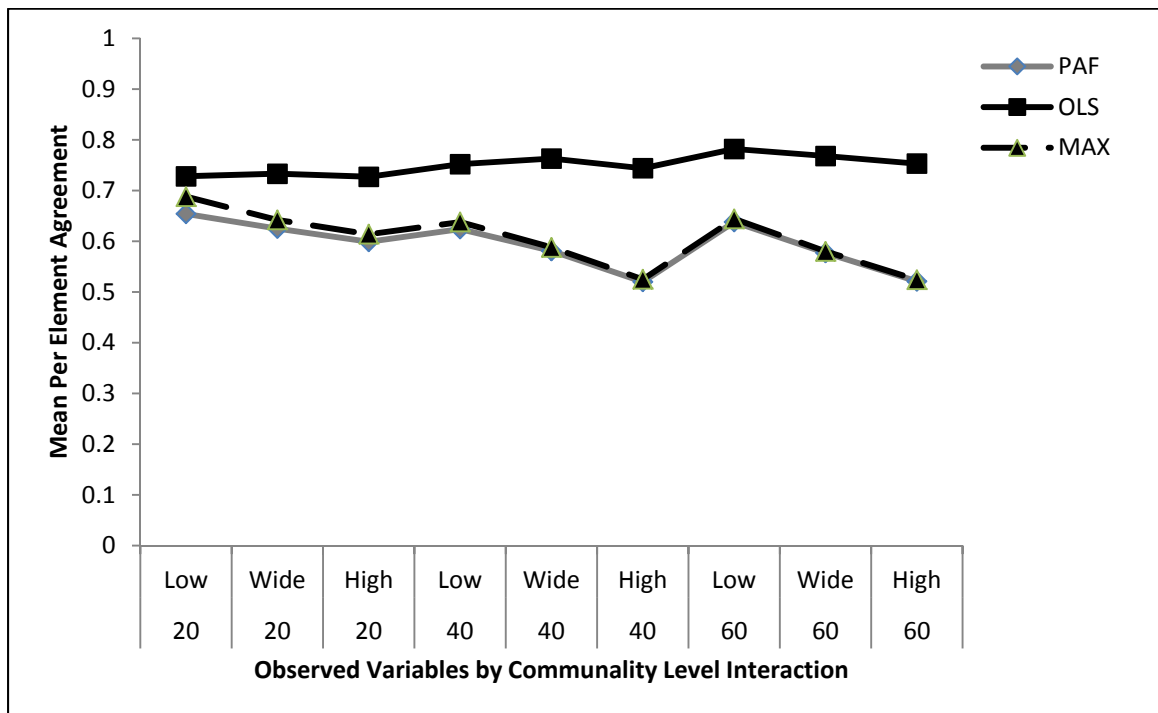


Figure 13. Mean values of per element agreement by interactions between the number of observed variables and communality.

The patterns of central tendency and dispersion in distributions of per element agreement for maximum likelihood and principal axis factor extraction methods are quite

nearly identical (see Figure D12 in appendix D). While the generally negative relationship between median and mean values of per element agreement and the level of communality is present in all factor extraction methods, the pattern is not as strongly expressed in distributions associated with ordinary least squares. In all conditions, ordinary least squares factor extraction method yields distributions that have smaller ranges and semi-interquartile ranges.

The mean values of per element pattern agreement associated with the sample size by communality interaction ( $N \times H$ ) are presented in Table 31. For all levels of the interaction, ordinary least squares yields the highest values of the per element factor agreement measure. For all factor extraction methods, the relationship between sample size and per element agreement is positive. For maximum likelihood and principal axis factor extraction methods, the relationship between per element agreement and communality is negative.

Figure 14 highlights the relationship among mean values of per element pattern agreement measures, factor extraction methods, and levels of the  $N \times H$  interaction. For maximum likelihood and principal axis factor extraction methods, box and whisker plots in Figure D13 (see appendix D) indicate that major changes in median, mean and semi-interquartile range occur within the sample size conditions as opposed to across them; the ranges in per element agreement increase as communality ranges from low through wide to high. These patterns are not apparent in distributions associated with the ordinary least squares factor extraction method.

Table 31  
*Means and Standard Deviations of Per Element Agreement for Factor Extraction Method and Sample Size by Communality Range Interaction (N x H)*

Interaction ( <i>N x H</i> )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
100 × Low	.619	0.140	.687	0.054	.644	0.144
100 × Wide	.581	0.160	.717	0.057	.594	0.164
100 × High	.537	0.189	.722	0.069	.547	0.192
200 × Low	.636	0.153	.745	0.066	.656	0.159
200 × Wide	.593	0.171	.752	0.066	.603	0.176
200 × High	.546	0.201	.741	0.081	.554	0.204
300 × Low	.643	0.159	.772	0.069	.660	0.164
300 × Wide	.598	0.176	.766	0.070	.606	0.180
300 × High	.549	0.205	.747	0.086	.556	0.209
1000 × Low	.657	0.169	.812	0.074	.664	0.173.
1000 × Wide	.605	0.184	.784	0.076	.611	0.187
1000 × High	.554	0.213	.755	0.093	.559	0.216

*N* = 540

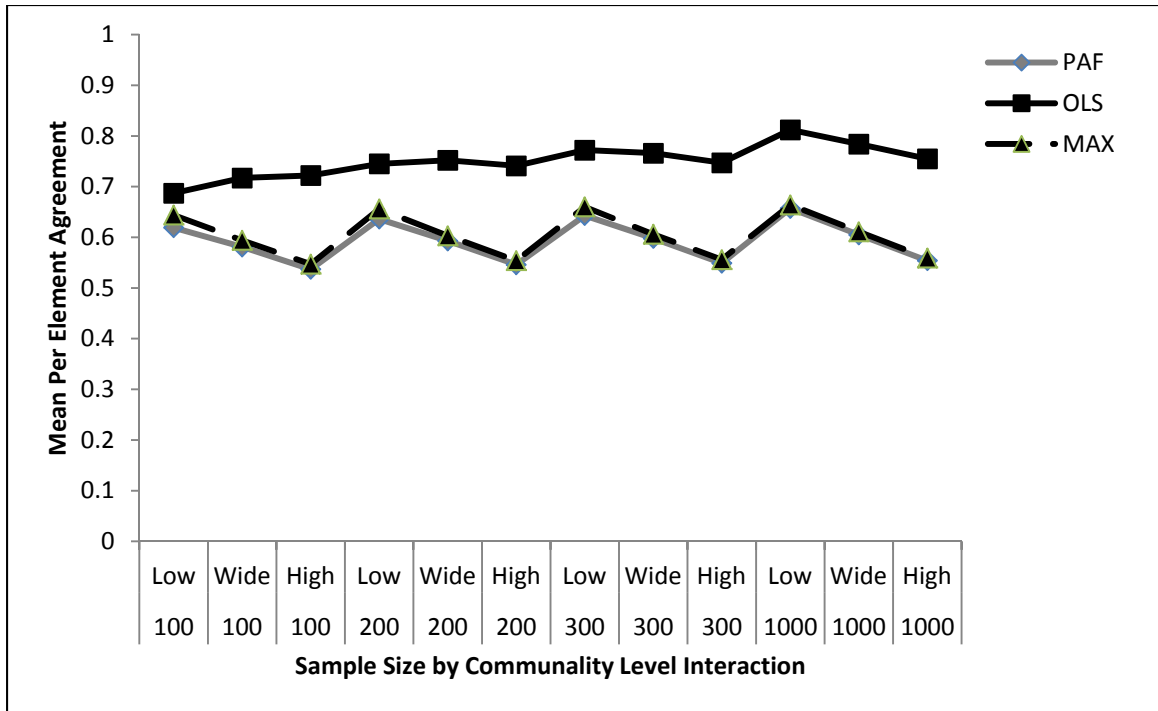


Figure 14. Mean values of per element agreement by interactions between sample size and communality.

Across all interactions among main effects, mean per element agreement values for the ordinary least squares factor extraction exceeded the mean values associated with principal axis and maximum likelihood methods. For interactions that included the number of factors imbedded in the population main effect, the differences among the three factor extraction methods, in terms of mean per element agreement, diminished as the number of factors increased. In all interactions that included communality, the relationship between mean per element agreement values and communality was negative; this relationship was apparent for all factor extraction methods.

### Total Pattern Agreement

Total pattern agreement is measured through a scalar. This scalar is populated with a one when the mean of the per element agreement matrix is one. If any element of

the per element agreement matrix is less than one, then the mean of that matrix will be less than one, and the total pattern agreement scalar associated with that matrix is set to zero. This scalar represents the proportion of sample matrices in which all factor loadings are in agreement with the population in terms of the absolute value of .30 loading criterion.

Table 32 provides descriptive statistics concerning the univariate distribution of total pattern agreement (perfect accuracy) values. For principal axis factor analysis, the minimum and maximum value for the total pattern agreement is zero. In this case, testing distributional assumptions regarding skewness and Kurtosis was not possible. For both the ordinary least squares and the maximum likelihood factor extraction methods, Shapiro Wilk's tests for normality yielded significant evidence that, in a univariate sense, the perfect accuracy values were not normally distributed.

Table 32  
*Descriptive Statistics for Distribution of Total Pattern Agreement Values*

Factor Extraction Method	<i>N</i>	<i>M</i>	<i>SD</i>	Skewness	Kurtosis
Principal Axis	540	0	0	-	-
Ordinary Least Squares	540	.0003	0.0012	4.500	23.201
Maximum Likelihood	540	<.0001	<0.0001	23.238	540.000

In addition to the univariate, non-normality, Mauchly's test of transformed variables,  $\chi^2(2) = 3922.4504$ ,  $p < .0001$ , indicate that the data do not conform to the

sphericity assumption associated with repeated measures ANOVA. The results of the multivariate analyses of variance indicate that total pattern agreement differs significantly by factor extraction method,  $\Lambda = .7920$ ,  $F(2, 459) = 85.36$ ,  $p < .0001$ . In addition to the method main effect, total pattern accuracy differed significantly by interactions between factor extraction method and the number of factors, observed variables, sample size, and level of dichotomization. However, the mean values of total pattern agreement were not substantially greater than zero in any of the studied conditions. While statistically significant differences among means were found, comparisons among zero values were not informative and are not included in this study.

### **Congruence**

The congruence among factor pattern matrices simulated in the population and the sample pattern matrices is measured through a congruence coefficient. As defined by MaCallum (et al., 1999), the phi coefficient is the cosine of the angle between the sample and population factor solutions “when plotted on the same space” (p. 93). To assess the congruence across all factor loading matrices, an average of the phi coefficient was calculated for each factor extraction method. Table 33 provides descriptive statistics concerning the univariate distribution of the mean phi coefficients (Congruence) for each of the tested factor extraction methods.

Shapiro-Wilks’ tests of normality for mean congruence associated with all three factor extraction methods yield evidence that the three distributions do not conform to the assumptions of univariate normality. Mauchly’s test of transformed variables  $\chi^2(2) = 989.341$ ,  $p < .0001$ , indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type



I error rate, tests of significance associated with the repeated measures analysis of variance will be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

Table 33  
*Descriptive Statistics for Distribution of Congruence*

Factor Extraction Method	<i>N</i>	<i>M</i>	<i>SD</i>	Skewness	Kurtosis
Principal Axis	540	.454	0.240	-0.230	-1.312
Ordinary Least Squares	540	.496	0.215	-0.219	-1.274
Maximum Likelihood	540	0.556	0.236	-0.487	-1.166

Results of the multivariate analysis of variance indicate that the factor extraction method yield significant differences in mean values of congruence,  $\Lambda = .0262$ ,  $F(2, 459) = 8533.92$ ,  $p < .0001$ . In addition to the method main effect, all of the manipulated research characteristics yielded significantly different congruence levels (see Table C9 in appendix C). The univariate, repeated measures analysis of variance also indicates that level of congruence between sample and population loading matrices differ significantly by factor extraction method,  $F(2, 920) = 7026.19$ ,  $p < .0001$ ,  $\eta_G^2 = .966$  (see Table C 10 in appendix C). The model including main effects and first-order interactions accounted for 97.1% of the variability associated with the measure of congruence between population and sample factor pattern matrices.

As results of the between subjects analyses indicate, all main effects and six of the ten first-order interaction effects are associated with significant differences among

congruence values; the significant interaction effects include number of factors by observed variables ( $K \times P$ ), number of factors by sample size ( $K \times N$ ), number of factors by communality range ( $K \times H$ ), number of factors by dichotomization level ( $K \times D$ ), sample size by level of communality ( $N \times H$ ), and level of communality by level of dichotomization ( $H \times D$ ). Results of the within subjects analysis indicate that the interactions of factor extraction methods with the number of factors, number of observed variables, sample size, and level of communality are associated with significant differences in congruence between sample and population factor loading matrices. The level of dichotomization does not yield significant results.

In addition to yielding statistically significant differences in average congruence between population and sample factor pattern matrices, the between subjects analyses yielded at least medium effect sizes for the following interactions:

1. Number of factors by observed variables,  $F(4, 460) = 85.49, p < .0001, \eta_G^2 = .405$
2. Number of factors by sample size,  $F(6, 460) = 10.89, p < .0001, \eta_G^2 = .115$
3. Number of factors by level of dichotomization,  $F(8, 460) = 5.94, p < .0001, \eta_G^2 = .086$
4. Sample size by level of communality,  $F(6, 460) = 7.46, p < .0001, \eta_G^2 = .067$
5. Level of communality by dichotomization,  $F(8, 460) = 5.17, p < .0001, \eta_G^2 = .076$

Comparisons among means were conducted for these interactions only.

Table 34 presents means and standard deviations for the mean congruence measure associated with the number of factors by observed variable interaction.

Table 34  
*Means and Standard Deviations of Congruence Values for Factor Extraction Method and Number of Factors by Observed Variables Interaction (K x P)*

Interaction ( <i>K x P</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 20	.712	0.093	.728	0.082	.789	0.079
2 × 40	.690	0.073	.710	0.069	.784	0.040
2 × 60	.692	0.076	.711	0.072	.784	0.045
4 × 20	.409	0.119	.460	0.104	.522	0.094
4 × 40	.513	0.092	.540	0.081	.638	0.050
4 × 60	.573	0.084	.596	0.075	.690	0.049
8 × 20	.088	0.022	.171	0.033	.179	0.054
8 × 40	.172	0.067	.249	0.064	.264	0.095
8 × 60	.242	0.090	.303	0.083	.351	0.104

*N* = 540

Across all interactions between the number of factors and observed variables, maximum likelihood factor extraction yielded higher mean levels of congruence between sample and population factor pattern matrices. The smallest, mean congruence values are associated with principal axis factor analysis. The graph of means in Figure 15 highlights the negative relationship among mean phi values and the number of factors.

As box and whisker plots indicate, ranges and semi-interquartile ranges for values of congruence decrease as the number of factors increase (see Figure D17 in appendix

D). However, in the eight factor conditions for all three factor extraction methods, these semi-interquartile ranges increase in value as the number of observed variables increase. In the two and four factor conditions, the semi-interquartile range decreases as the number of observed variables increase.

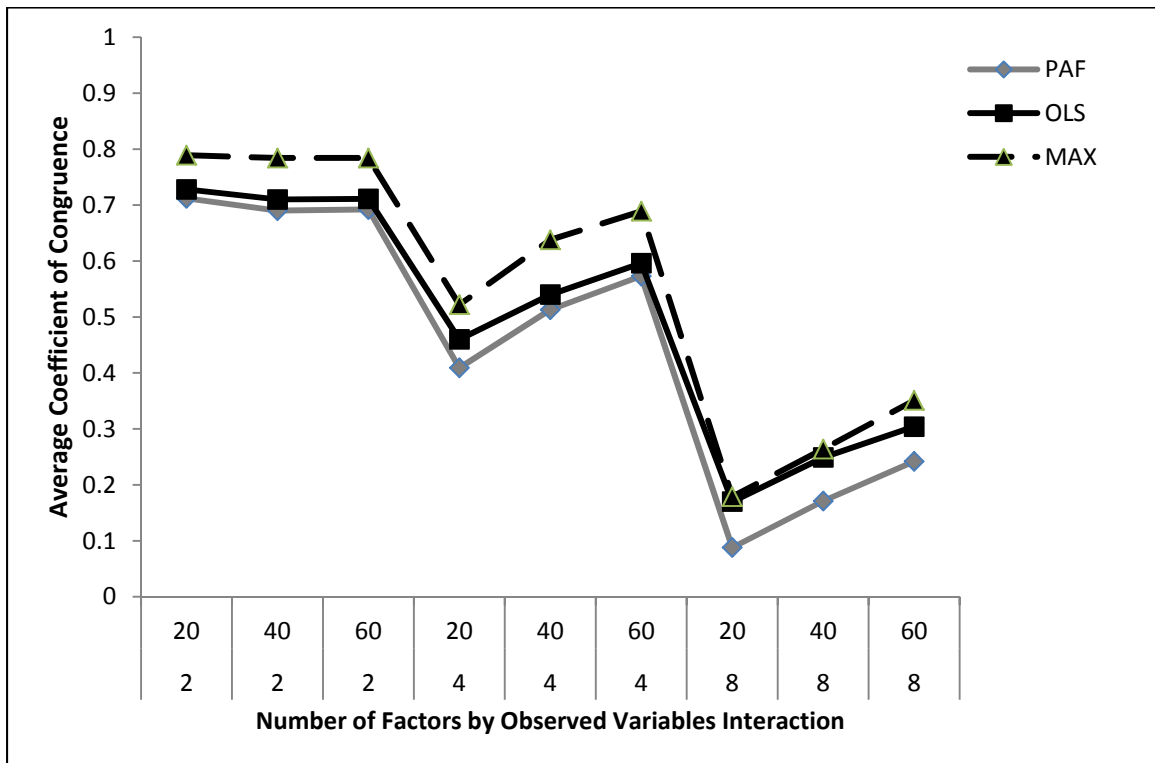


Figure 15. Mean values of congruence by interactions between the number of factors and number observed variables.

Means and standard deviations for congruence by interactions among the number of factors and sample size are presented in Table 35.

Table 35  
*Means and Standard Deviations of Congruence Values for Factor Extraction Method and Number of Factors by Sample Size Interaction (K x N)*

Interaction (K x N)	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 100	.636	0.068	.659	0.063	.783	0.050
2 × 200	.689	0.070	.708	0.064	.786	0.056
2 × 300	.712	0.070	.729	0.064	.787	0.058
2 × 1000	.755	0.071	.768	.065	.788	0.065
4 × 100	.392	0.099	.439	0.086	.557	0.110
4 × 200	.477	0.098	.514	0.083	.612	0.090
4 × 300	.519	0.094	.550	0.080	.632	0.082
4 × 1000	.605	0.081	.625	0.067	.664	0.073
8 × 100	.113	0.046	.187	0.049	.165	0.052
8 × 200	.150	0.069	.225	0.063	.241	0.078
8 × 300	.172	0.081	.247	0.072	.281	0.090
8 × 1000	.234	0.110	.305	0.097	.370	0.106

*N* = 540

The values indicate that the mean congruence values decreases as the number of factors increase. In general, the level of congruence increases as the sample size increases for all three factor extraction methods. With the exception of the eight-factor

by sample size of 100 level of the interaction, the maximum likelihood factor extraction method yields the highest mean values of congruence. In this exceptional case, the ordinary least squares factor extraction method yields a mean congruence value that is higher than mean congruence values associated maximum likelihood and principal axis factor extraction methods. As the graphed means in Figure 16 and the box and whisker plots (see Figure D18 in appendix D) indicate, values of central tendency and dispersion measures are positively associated with sample sizes.

Means and standard deviations of congruence by levels of interaction between the number of factors and level of dichotomization are presented in Table 36.



Figure 16. Mean values of congruence by interactions between the number of factors and sample size.

Table 36  
*Means and Standard Deviations of Congruence Values by Factor Extraction Method and Number of Factors by Level of Dichotomization Interaction (K x D)*

Interaction ( <i>K x D</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × .05	.712	0.089	.730	0.081	.792	0.060
2 × .25	.731	0.065	.746	0.059	.806	0.044
2 × .50	.709	0.064	.723	0.060	.798	0.042
2 × .75	.691	0.092	.707	0.087	.791	0.043
2 × .95	.646	0.070	.673	0.065	.742	0.069
4 × .05	.490	0.130	.523	0.109	.606	0.108
4 × .25	.500	0.122	.533	0.107	.613	0.099
4 × .50	.496	0.109	.529	0.095	.613	0.091
4 × .75	.517	0.119	.548	0.103	.636	0.091
4 × .95	.488	0.123	.526	0.108	.613	0.098
8 × .05	.159	0.075	.231	0.068	.263	0.103
8 × .25	.168	0.094	.240	0.085	.271	0.115
8 × .50	.171	0.097	.246	0.089	.267	0.118
8 × .75	.171	0.094	.246	0.087	.264	0.113
8 × .95	.168	0.097	.242	0.090	.257	0.115

*N* = 540

As with the other interactions that included the number of factors main effect, the mean congruence values diminish as the number of factors increase. In all interactions, maximum likelihood factor extraction yielded highest levels of congruence and principal axis yielded the lowest.

The relationships among mean values of congruence and the number of factors by level of dichotomization interaction are highlighted by the graphed means in Figure 17 and the box and whisker plots in Figure D19 (see appendix D).

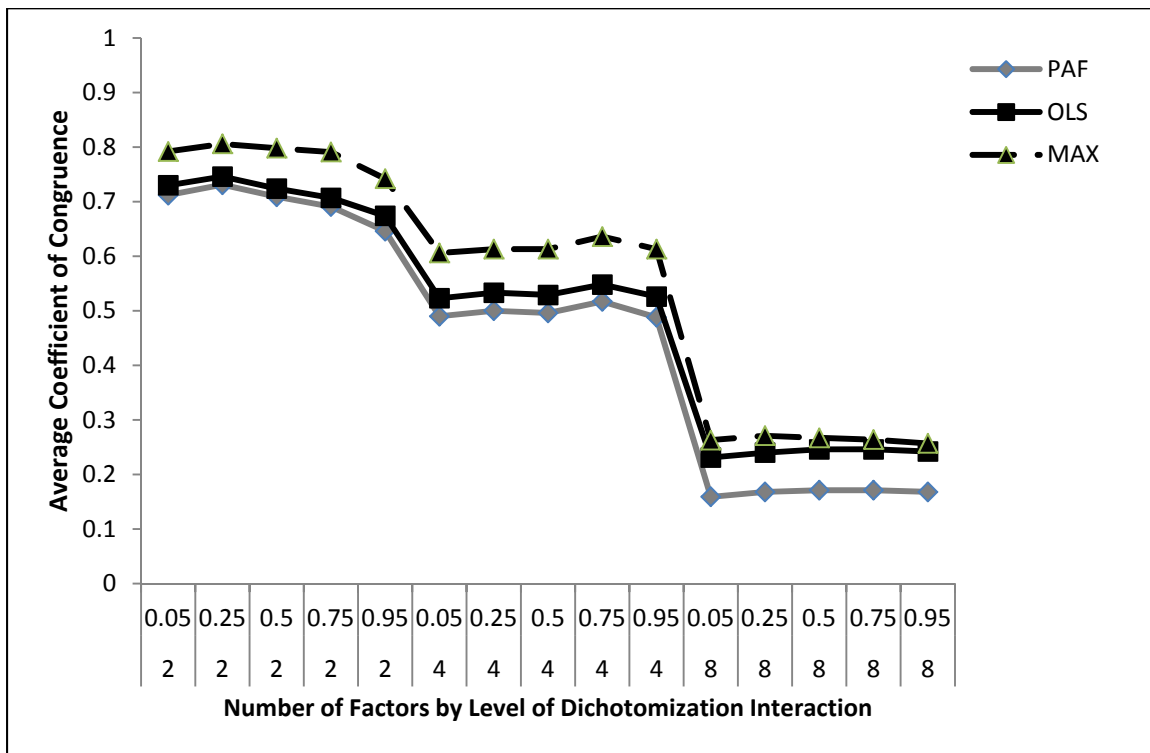


Figure 17. Mean values of congruence by interactions between the number of factors and level of dichotomization interaction.

Means and standard deviations of mean values of congruence for the samples size by level communality interaction are presented in Table 37.



Table 37  
*Means and Standard Deviations of Congruence Values by Factor Extraction Method and Sample Size by Community Range Interaction (N x H)*

Interaction ( <i>N x H</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
100 × Low	.332	0.226	.378	0.206	.503	0.291
100 × Wide	.396	0.235	.444	0.213	.487	0.262
100 × High	.414	0.216	.463	0.190	.515	0.251
200 × Low	.400	0.245	.445	.0218	.566	0.255
200 × Wide	.446	0.245	.490	0.220	.520	0.244
200 × High	.470	0.218	.512	.192	.555	0.223
300 × Low	.436	0.251	.479	0.223	.594	0.234
300 × Wide	.469	0.246	.510	0.222	.534	0.234
300 × High	.498	0.217	.537	0.190	.572	0.209
1000 × Low	.520	0.257	.554	.228	.652	.187
1000× Wide	.514	0.243	.550	0.220	.564	0.208
1000 × High	.559	0.209	.594	0.178	.606	0.182

*N* = 540

Across all levels of the interaction, maximum likelihood factor analysis yields the greatest congruence values, and principal axis factor analysis maintains the lowest levels

of congruence. Congruence values for all three factor extraction methods increase as both samples size and levels of communality increase. The graphed means in Figure 18 and box and whisker plots (see Figure D20 in appendix D) highlight a pattern in which mean values of congruence for the three factor extraction methods appear to converge as sample size increases.

Table 38 provides means and standard deviations for congruence levels as they are associated with interactions between levels of communality and dichotomization.

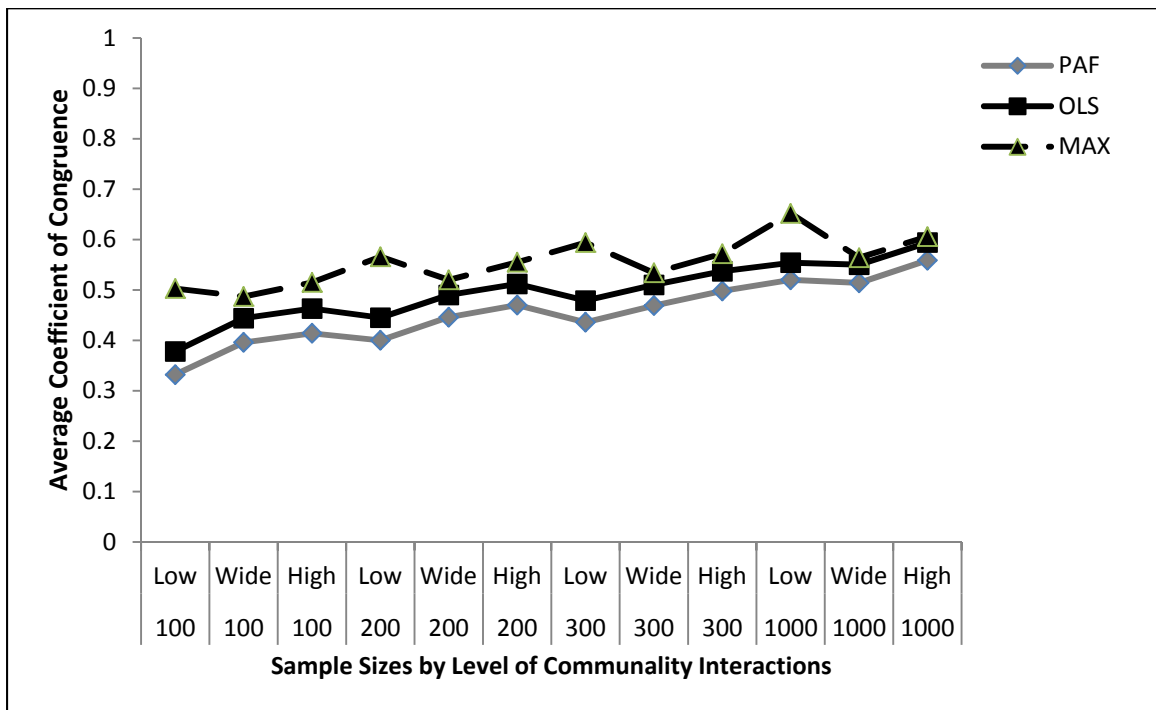


Figure 18. Mean values of congruence by interactions between Sample Size and Communality.

Table 38  
*Means and Standard Deviations of Congruence Values by Factor Extraction Method and  
 Communality Range by Level of Dichotomization Interaction*

Interaction ( <i>H x D</i> )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Low × .05	.427	0.262	0.466	0.234	.579	0.253
Low × .25	.425	0.256	0.468	0.230	.583	0.252
Low × .50	.428	0.260	0.470	0.232	.584	0.256
Low × .75	.407	0.234	0.449	0.207	.575	0.245
Low × .95	.423	0.262	0.467	0.239	.571	0.251
Wide × .05	.459	0.265	0.500	0.240	.530	0.246
Wide × .25	.459	0.247	0.499	0.225	.528	0.238
Wide × .50	.462	0.244	0.505	0.218	.534	0.244
Wide × .75	.486	0.266	0.526	0.240	.552	0.257
Wide × .95	.415	0.201	0.463	0.181	.488	0.209
High × .05	.475	0.223	0.518	0.194	.552	0.220
High × .25	.515	0.248	0.553	0.218	.578	0.233
High × .50	.486	0.218	0.524	0.191	.561	0.216
High × .75	.486	0.213	0.525	0.185	.565	0.217
High × .95	.465	0.202	0.511	0.177	.554	0.216

*N* = 540

As with all previous interactions, maximum likelihood yields the highest levels of congruence and principal axis yielded the lowest. However, as the level of communality increase, the differences in congruence between maximum likelihood and principal axis diminish. The graphed means in Figure 19 and box and whisker plots (see Figure D21 in appendix D) highlight the relationship between levels of the interaction and mean congruence values.

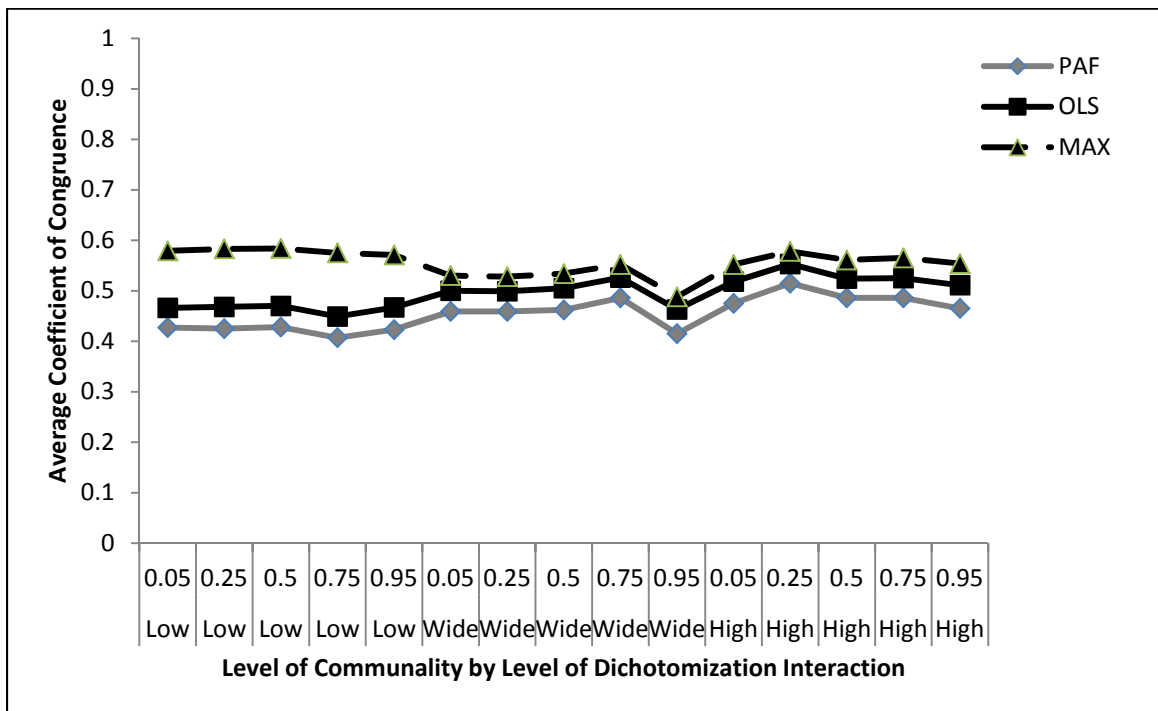


Figure 19. Mean values of congruence by interactions between communality and level of dichotomization interaction.

## Factor Score Correlations

This measure of agreement is a  $k$  by 1 column vector of correlations between factor scores derived from the sample and those derived from the population. These score estimates will be linear combinations of variables; however, as opposed to using factor score coefficients, these estimates will be computed via the following process:

1. A positive one scoring coefficient is assigned when the observed structure coefficient is  $\geq .30$ ;
2. A negative one scoring coefficient is assigned when the observed structure coefficient is  $\leq -.30$ ;
3. A scoring coefficient of zero is assigned when the structure coefficient is between  $.30$  and  $-.30$ .

Once factor scores estimates are computed for both the population and sample matrices, a correlation among the scores will be used to measure how closely factor scores derived from each of the factor extraction strategies approximates that factor score pattern that is imbedded in the population (Hogarty et al., 2005).

Table 39 provides descriptive statistics concerning the distribution of these correlations. As the table highlights, correlations could not be estimated for the principal axis and maximum likelihood factor extraction methods in all samples. In the two factor, 60 observed variable condition, principal axis and maximum likelihood factor extraction strategies yielded factor score estimates of zero; therefore, correlations for these conditions could not be calculated.

Table 39

*Descriptive Statistics for Distributions of Factor Score Correlations*

Factor Extraction Method	<i>N</i>	<i>M</i>	<i>SD</i>	Skewness	Kurtosis
Principal Axis	480	.326	0.196	0.449	-0.619
Ordinary Least Squares	540	.511	0.208	-0.336	-1.256
Maximum Likelihood	480	.382	0.213	0.208	-0.962

Shapiro-Wilks' tests of normality of the distribution of factor score correlations associated with all three factor extraction methods yield evidence of significant non-normality. Mauchly's test of transformed variables  $\chi^2 (2) = 348.154, p < .0001$ , indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type I error rate, tests of significance associated with the repeated measures analysis of variance will be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

Results of the multivariate analysis of variance indicate that factor score correlations differ significantly by factor extraction method,  $\Lambda = .0218, F(2, 459) = 8978.7, p < .0001$ . In addition to factor extraction method, the number of factors, number of observed variables, sample size, and communality range are associated with significant differences in factor score correlations. Results from these analyses are presented in Table C11 (see appendix C).

The univariate, repeated measures analysis of variance also indicate that factor score correlations differ significantly by factor extraction method,  $F(2, 802) = 7246.31, p < .0001, \eta_G^2 = .783$ . The model including main effects and first-order interactions

accounted for 96.8% of the variability associated with values of general pattern agreement. The summary table for these analyses is provided in Table C12 (see appendix C).

According to the between subjects portion of the repeated measures analysis of variance, factor score correlations differed significantly across levels of the number of factors by observed variables ( $K \times P$ ), number of factors by sample size ( $K \times N$ ), number of factors by communality range ( $K \times H$ ), number of factors by dichotomization level ( $K \times D$ ), observed variables by communality range ( $P \times H$ ), sample size by communality range ( $N \times H$ ), and communality range by level of dichotomization ( $H \times D$ ) interactions.

The within subjects analysis, highlighted the number of factors by number of observed variables interaction,  $F(6, 802) = 1147.13, p < .0001, \eta_G^2 = .632$ , as yielding an effect size that was at least medium. The between subjects highlighted the following interactions among manipulated research characteristics as yielding effect sizes that warranted further analyses (Cohen, 1988):

1. Number of factors by samples size,  $F(6, 401) = 8.78, p < .0001, \eta_G^2 = .095$
2. Number of factors by communality range,  $F(4, 401) = 60.09, p < .0001, \eta_G^2 = .324$ ,
3. Number of observed variables by communality range,  $F(4, 401) = 21.99, p < .0001, \eta_G^2 = .149$ .

Although the main effects that contribute to these interactions also result in medium (or larger) effect sizes, only means based on the interactions will be compared.

Table 40 provides means and standard deviations of correlations between sample and population factor scores for interactions between the number of factors and the

number of observed variables. The only interaction level in which ordinary least squares does not yield the highest levels of correlation is the level that includes four factors and 20 observed variables; in this case, maximum likelihood yields a higher correlation. The strongest correlation is associated with ordinary least squares in the two-factor by 20 observed variable condition.

Table 40  
*Means and Standard Deviations of Factor Score Correlations for Factor Extraction Methods and the Number of Factors by Observed Variables Interaction (K x P)*

Interaction (K x P)	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 20	.661	0.080	.726	0.070	.725	0.066
2 × 40	.091	0.083	.706	0.053	.112	0.096
2 × 60	-	-	.705	0.059	-	-
4 × 20	.435	0.106	.473	0.094	.505	0.088
4 × 40	.417	0.075	.578	0.072	.516	0.047
4 × 60	.416	0.124	.641	0.073	.515	0.108
8 × 20	.137	0.040	.178	0.042	.163	0.027
8 × 40	.216	0.064	.265	0.050	.239	0.060
8 × 60	.234	0.074	.321	0.058	.278	0.064

*N* = 480



As the graph of means in Figure 20 highlights, the factor score correlations associated with the three tested factor extraction methods are the most similar in interaction levels that include 20 observed variables. Where the observed variable to number of factor ratios are the highest, the differences in factor score correlations among the three factor extraction methods are the greatest; this is especially true in the two factor condition in which the principal axis and maximum likelihood methods failed to yield non-zero factor score estimates.

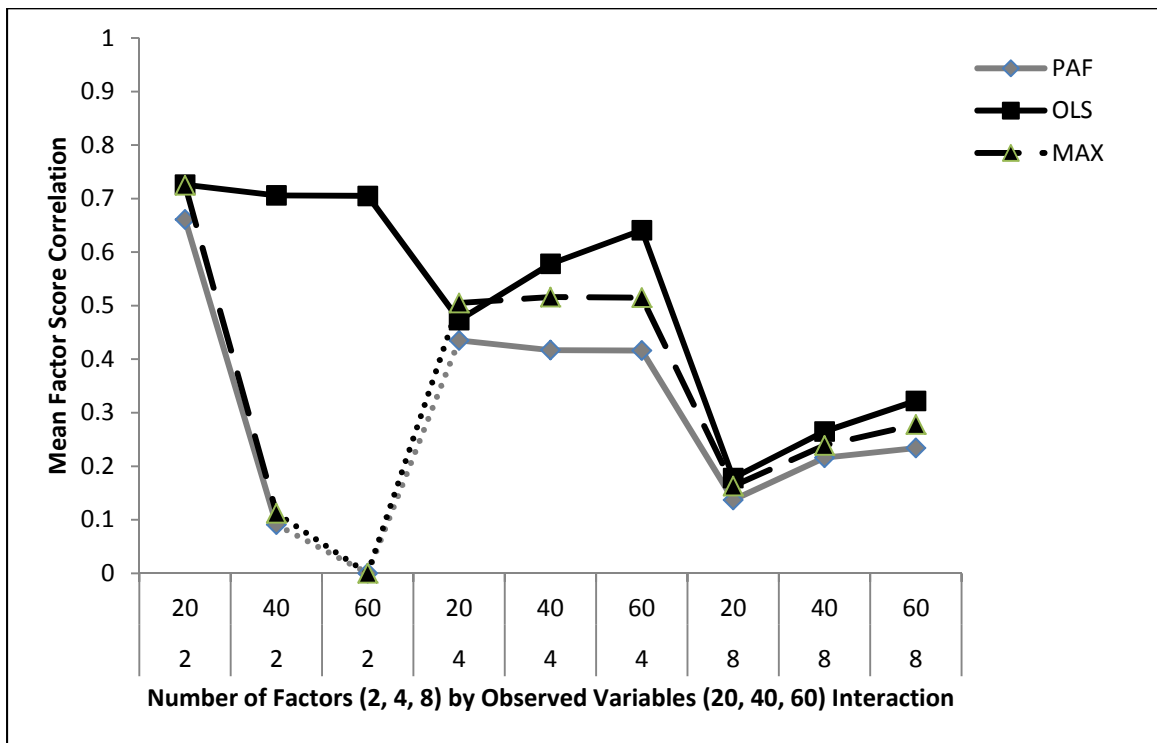


Figure 20. Mean values of factor score correlations by interactions between the number of factors and number of observed variables.

The negative relationship between factor score correlation and the number of factors is the only relationship that is common to all three factor extraction methods at all levels of the interaction. Within the eight factor conditions of the interaction, both the graphed means and the box plots (see Figure D22 in appendix D) indicate that the mean and median factor score correlations increase as the number of observed variables increase.

Table 41 provides means and standard deviations for factor score correlations by factor extraction method and levels of the number of factors by sample size interaction. In all levels of this interaction effect, ordinary least squares yielded the highest levels of correlation between the sample and population in terms of factor scores. The strongest correlation between sample and population is associated with the two factor condition in which sample size equals 1000.

Across all factor extraction methods and levels of the interaction, box and whisker plots (see Figure D23 in appendix D) indicate the median factor score correlations are positively related to sample size within each number of factors condition and, for the ordinary least square condition, negatively related to the number of factors. Except for the conditions that include two factors, this negative relationship is also apparent in the maximum likelihood and the principal axis factor extraction methods. In maximum likelihood and principal axis factor extraction methods, the ranges and interquartile ranges shrink substantially between the two and four factor conditions of the interaction; this effect may be related to the diminished sample sizes in the two factor conditions associated with both maximum likelihood and principal axis factor extraction methods.

Figure 21 demonstrates that the differences among the three factor extraction methods in terms of factor correlations diminish as the number of factors increases. The largest difference among the factor analytic strategies is associated with the  $K = 2, N = 100$  condition.

Table 41  
*Means and Standard Deviations of Factor Score Correlations for Factor Extraction Method and Number of Factors by Sample Size Interaction ( $K \times N$ )*

Interaction ( $K \times N$ )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 100	.347	0.276	.680	0.053	.420	0.309
2 × 200	.371	0.296	.711	0.056	.417	0.322
2 × 300	.382	0.305	.725	0.058	.417	0.326
2 × 1000	.404	0.323	.751	0.063	.419	0.332
4 × 100	.337	0.094	.483	0.089	.461	0.087
4 × 200	.405	0.082	.548	0.091	.509	0.076
4 × 300	.440	0.080	.581	0.090	.525	0.073
4 × 1000	.510	0.076	.644	0.087	.553	0.077
8 × 100	.149	0.049	.218	0.061	.175	0.048
8 × 200	.184	0.063	.246	0.069	.220	0.061
8 × 300	.203	0.069	.261	0.074	.238	0.066
8 × 1000	.247	0.078	.296	0.086	.274	0.071

$N = 480$

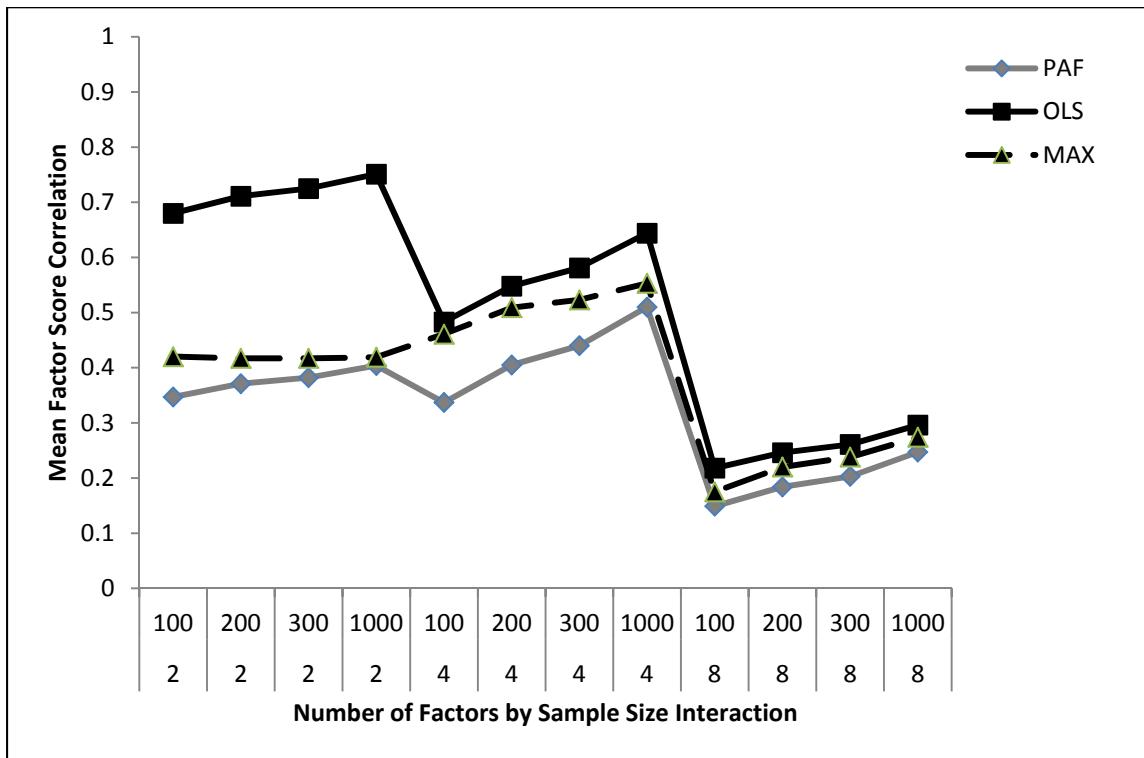


Figure 21. Mean values of factor score correlations by interactions between number of factors and sample size.

Table 42 provides means and standard deviations for factor score correlations by levels of the number of factors by communality interaction. As the table highlights, ordinary least squares factor analysis results in the strongest correlations at each level of the interaction. The strongest correlation is associated with the two factors by wide (2) communality range interaction.

Figure 22 highlights the differences among the three factor extraction methods in terms of factor score correlations is the greatest in the two factor condition. The differences reach their smallest levels in the eight-factor, high communality range of the

interaction. The graphed means and box and whisker plots (see Figure D24 in appendix D) indicate that, mean and median factor score correlations for the ordinary least squares factor extraction method are positively related to communality range and negatively related to the number of factors; with the exception of the conditions containing two factors, this relationship is also apparent in distributions of correlations yielded by maximum likelihood and principal axis factor extraction methods.

Table 42  
*Means and Standard Deviations of Factor Score Correlations for Factor Extraction Method and Number of Factors by Communality Range Interaction (K x H)*

Interaction ( <i>K x H</i> )	Principal Axis		Ordinary		Maximum	
			Least Squares		Likelihood	
	M	SD	M	SD	M	SD
2 × Low	.367	0.300	.720	0.062	.437	0.336
2 × Wide	.443	0.443	.746	0.059	.474	0.263
2 × High	.318	0.318	.683	0.051	.344	0.343
4 × Low	.358	0.358	.539	0.140	.491	0.070
4 × Wide	.419	0.419	.564	0.088	.478	0.059
4 × High	.492	0.492	.589	0.074	.569	0.092
8 × Low	.146	0.146	.212	0.080	.194	0.064
8 × Wide	.192	0.192	.253	0.056	.220	0.052
8 × High	.250	0.249	.299	0.071	.267	0.076

*N* = 480

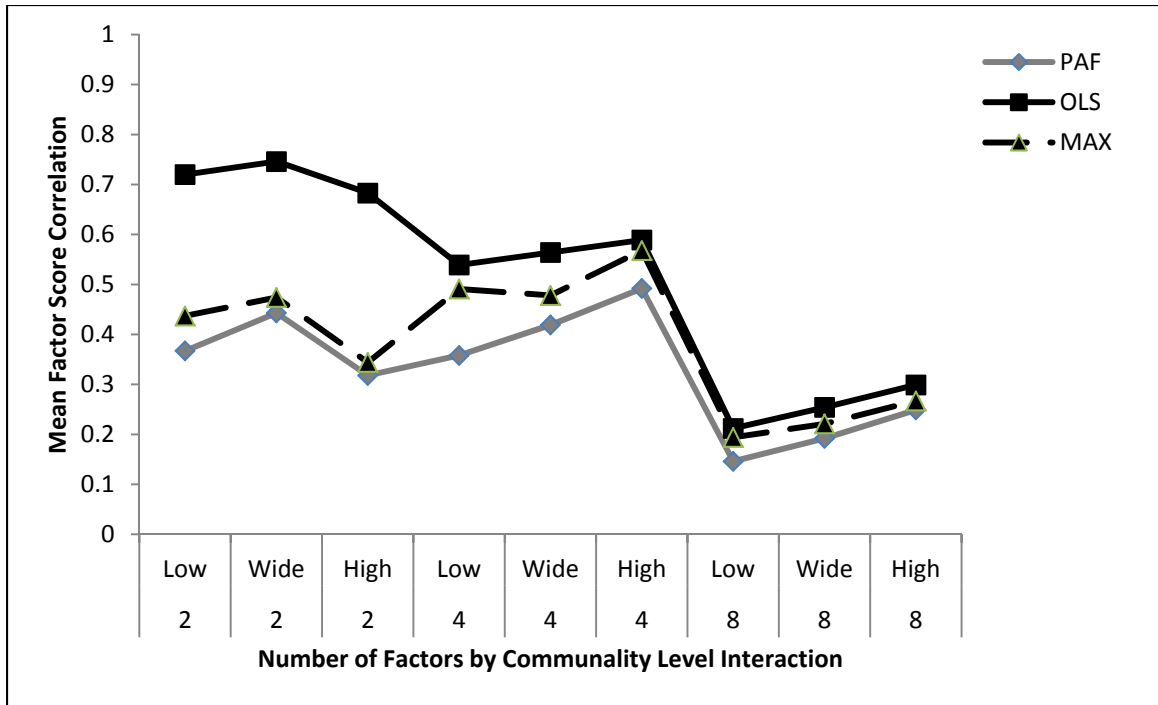


Figure 22. Mean values of factor score correlations by interactions between number of factors and communality.

Table 43 provides means and standard deviations of factor score correlations for factor extraction method by the number of variables and communality range interaction effect. Except for the low 20 observed variable by low communality range condition, ordinary least squares yields the highest levels of correlation between factor scores associated with the population and the sample. The strongest correlation is associated with the ordinary least squares factor extraction method at the  $P = 40$ ,  $H = 2$  (wide) interaction level.

Table 43  
*Means and Standard Deviations of Factor Score Correlations for Factor Extraction Method and Number of Observed Variables by Communality Range Interaction (P x H)*

Interaction Levels	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	M	SD	M	SD	M	SD
20 × Low	.368	0.243	.422	0.257	.458	0.263
20 × Wide	.421	0.234	.470	0.240	.454	0.237
20 × High	.444	0.206	.485	0.207	.481	0.224
40 × Low	.206	0.141	.493	0.216	.281	0.193
40 × Wide	.290	0.134	.530	0.200	.330	0.151
40 × High	.228	0.172	.527	0.166	.257	0.197
60 × Low	.262	0.108	.473	0.205	.355	0.126
60 × Wide	.293	0.099	.473	0.173	.345	0.094
60 × High	.421	0.144	.498	0.138	.491	0.169

*N* = 480

As the graphed means in Figure 23 and the box and whisker plots (see Figure D24 in appendix D) demonstrate, the mean and median correlations associated with the ordinary least squares factor extraction method are negatively associated with the number of factors; with the exception of the two factor conditions, this relationships is also apparent in distributions associated with maximum likelihood and principal axis factor extraction methods. Distributions of correlations associated with maximum likelihood

and principal axis factor extraction methods exhibited sharp declines in range and semi-interquartile range between the two and four-factor conditions.

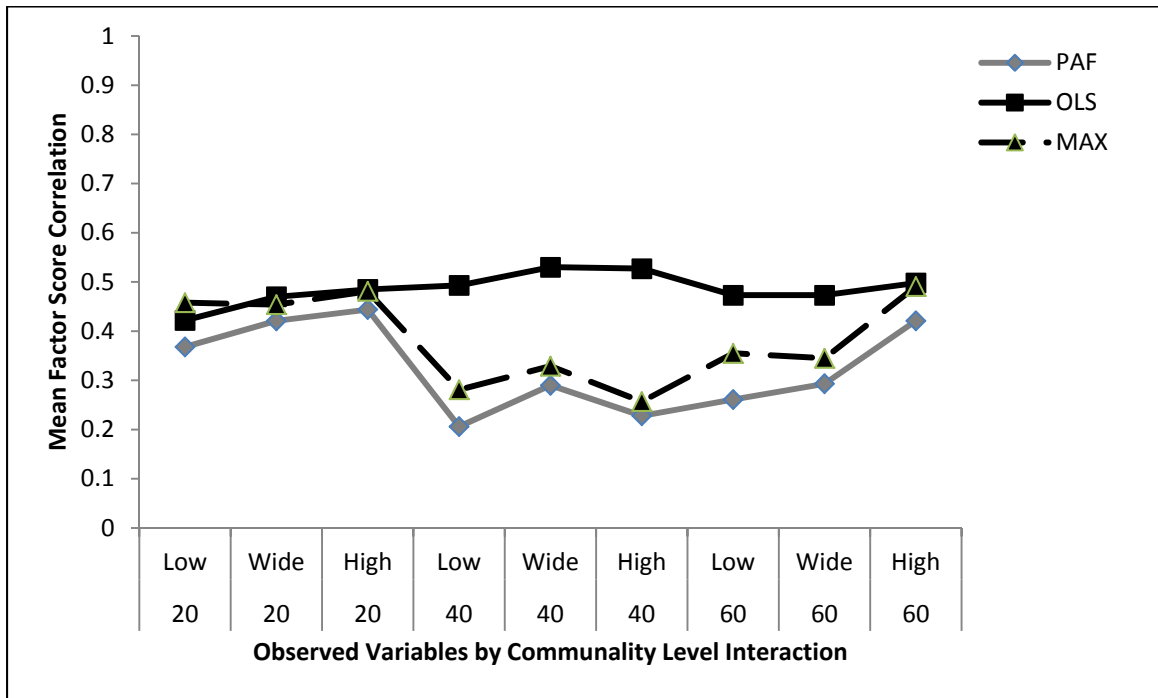


Figure 23. Mean values of factor score correlations by interactions between number of observed variables and community.

### Factor Loading Bias

A second indicator of congruence between sample and population factor loadings is statistical bias. The  $P \times K$  matrix is populated by estimates of statistical bias for each factor loading; these bias estimates were averaged across all samples. Table 44 provides describes the univariate distribution of factor loading bias estimates for the three tested factor extraction methods.



Shapiro-Wilks' tests of normality yielded evidence of non-normality in the distributions of bias for all factor extraction methods. Mauchly's test of transformed variables  $\chi^2(2) = 1544.3812, p < .0001$ , indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type I error rate, tests of significance associated with the repeated measures analysis of variance will be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

Table 44  
*Descriptive Statistics for Distribution of Loading Bias Estimates*

Factor Extraction Method	N	M	SD	Skewness	Kurtosis
Principal Axis	540	-0.121	0.064	-1.016	-0.084
Ordinary Least Squares	540	0.027	0.022	-0.191	-0.439
Maximum Likelihood	540	-0.122	0.063	-1.023	-0.069

Results of the multivariate analysis of variance indicate that levels of factor loading bias differed significantly across the tested factor extraction methods,  $\Lambda = .0018, F(2, 459) = 127099, p < .0001$  (see Table C13 in appendix C); the multivariate analyses highlighted significant differences in mean bias by number of factors, number of observed variables, communality range, and level of dichotomization. The univariate, repeated measures analysis of variance also indicates that factor loading bias differ significantly by factor extraction method,  $F(2, 920) = 25232, p < .0001, \eta_G^2 = .992$  (see Table C14 in appendix C). The model including main effects and first-order interactions

accounted for 99.49% of the variability associated with statistical bias in factor pattern matrices.

In addition to the method main effects, the multivariate and repeated measures analyses of variance identified significant differences in bias associated with six of the ten first-order interactions. However, only four of these interactions yielded effect sizes that were medium or greater. These interactions included

1. Number of factors by observed variables,  $F(4, 460) = 244.87, p < .0001, \eta_G^2 = .621$
2. Number of factors by sample size,  $F(6, 460) = 15.62, p < .0001, \eta_G^2 = .136$
3. Number of factors by level of communalities,  $F(4, 460) = 454.50, p < .0001, \eta_G^2 = .753$
4. Number of factors by level of dichotomization,  $F(8, 460) = 4.91, p < .0001, \eta_G^2 = .062$

Comparisons among means were conducted for these interactions only.

Table 45 presents mean factor loading biases by for each factor extraction method by each level of the interaction between number of factors and number of observed variables. For each level of the interaction, the comparisons indicate that mean bias values for ordinary least squares are closer to zero than they are for either the maximum likelihood or the principal axis methods. Biases for principal axis and maximum likelihood are negative. As both Table 42 and Figure 24 highlight, bias levels decrease as the number of factors increase. This relationship is more strongly expressed in the distributions associated with principal axis and maximum likelihood factor extraction methods than it is in distributions associated with ordinary least squares.

Table 45  
*Means and Standard Deviations of Factor Loading Bias for Factor Extraction Method and Number of Factors by Number of Observed Variables Interaction (K x P)*

Interaction ( <i>K x P</i> )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	M	SD	M	SD	M	SD
2 × 20	-.170	0.052	.022	0.010	-.171	0.053
2 × 40	-.200	0.050	.035	0.013	-.200	0.051
2 × 60	-.213	0.048	.037	0.011	-.213	0.048
4 × 20	-.089	0.023	.022	0.024	-.090	0.023
4 × 40	-.093	0.025	.046	0.015	-.094	0.025
4 × 60	-.102	0.025	.053	0.011	-.101	0.026
8 × 20	-.085	0.017	-.007	0.009	-.085	0.017
8 × 40	-.073	0.017	.011	0.014	-.074	0.017
8 × 60	-.065	0.015	.026	0.013	-.067	0.015

*n* = 540

The ranges and semi-interquartile ranges in distributions associated with maximum likelihood and principal axis diminish as the number of factors increase. For these two factor extraction methods, the communality range are not strongly related to the number of observed variables within the number of factor conditions (See Figure D25 in

appendix D). In distributions of bias associated with the ordinary least squares factor extraction method, the mean and median levels of bias are positively related to the ratio of observed variables to factors across all levels of the interaction.

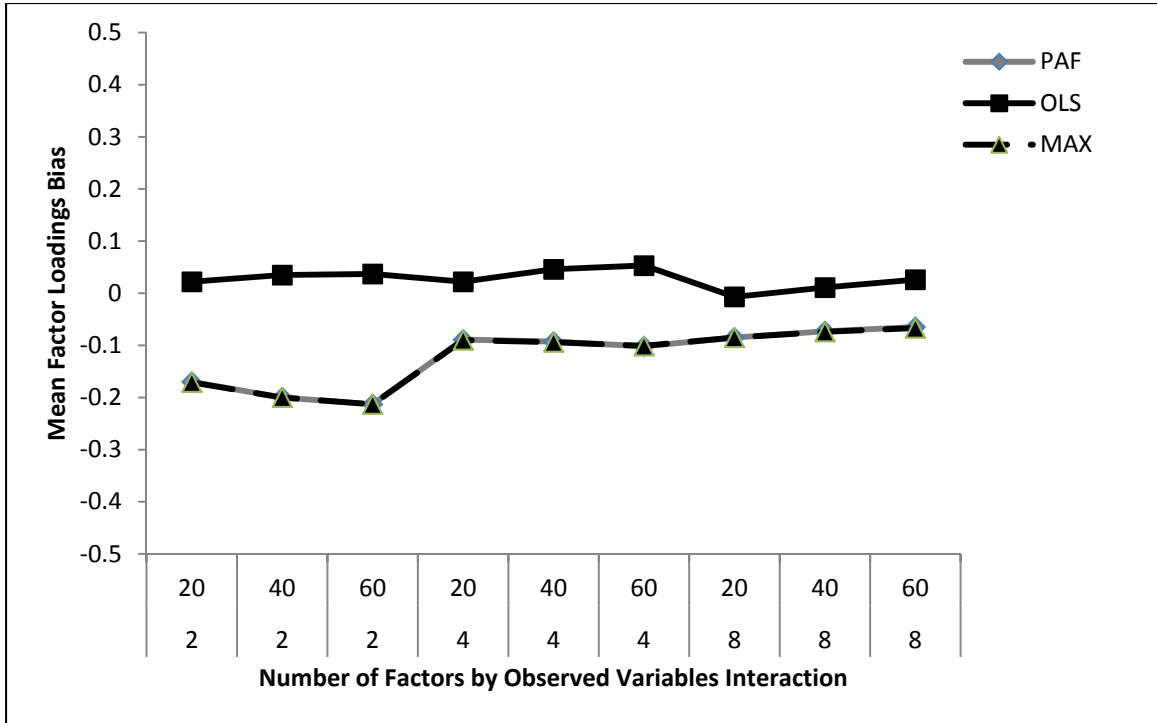


Figure 24. Mean value of factor loading bias by interactions between number of factors by number of observed variables.

Table 46 provides means and standard deviations for mean bias levels across factor extraction methods and levels of the number of factors by sample size interaction. As was the case with the previous set of comparisons, the levels of bias decrease as the number of factors increase. At the eight-factor by  $N = 100$  condition, ordinary least

squares yields a mean bias of -.002; this is the smallest average amount of bias across all factor methods and all interactions between number of factors and sample size.

Table 46  
*Means and Standard Deviations of Factor Loading Bias by Factor Extraction Method and Number of Factors by Sample Size Interaction (K x N)*

Interaction ( <i>K x N</i> )	<u>Principal Axis</u>		<u>Ordinary</u>		<u>Maximum</u>	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × 100	-.198	0.054	.030	0.011	-.200	0.064
2 × 200	-.195	0.054	.032	0.013	-.200	0.054
2 × 300	-.194	0.054	.032	0.013	-.194	0.054
2 × 1000	-.192	0.054	.033	0.016	-.193	0.054
4 × 100	-.103	0.022	.029	0.024	-.103	0.022
4 × 200	-.095	0.024	.040	0.021	-.095	0.024
4 × 300	-.093	0.025	.043	0.020	-.092	0.025
4 × 1000	-.088	0.028	.048	0.018	-.089	0.027
8 × 100	-.085	0.016	-.002	0.015	-.085	0.015
8 × 200	-.076	0.016	.008	0.017	-.077	0.016
8 × 300	-.072	0.017	.013	0.017	-.073	0.017
8 × 1000	-.065	0.017	.020	0.017	-.066	0.017

*N* = 540

As the graphed mean bias levels in Figure 25 demonstrate, maximum likelihood and principal axis factor extraction methods yield factor pattern matrices that exhibit similar patterns of negative bias. In distributions associated with maximum likelihood and principal axis factor extraction methods, the mean and median levels of bias decrease (in absolute value) as the number of factors and the sample sizes increase; ranges and semi-interquartile ranges, for these distributions, are also negatively related to both number of factors and sample size (see Figure D26 in appendix D).

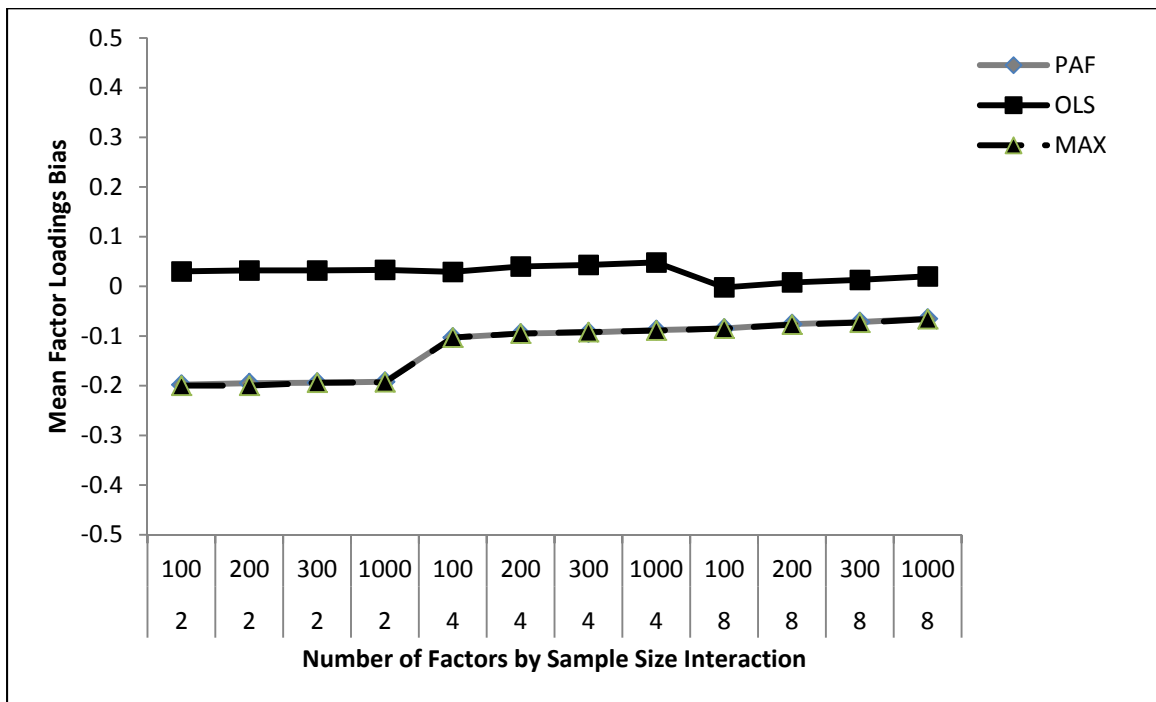


Figure 25. Mean value of factor loading bias by interactions between number of factors and sample size.

Table 47 provides means and standard deviations of loading bias estimates for the three factor extraction methods by all levels of the number of factors by communality range interaction. For maximum likelihood and principal axis factor extraction methods, all mean levels of factor loading bias are negative. Ordinary least squares yields factor loading bias estimates that are closer to zero than the other two factor extraction methods. The smallest mean level of factor loading bias is associated with the eight-factor by low communality condition ( $<.001$  and  $> 0$ ).

Table 47  
*Means and Standard Deviations of Factor Loading Bias for Factor Extraction Method and Number of Factors by Communality Range Interaction (K x H)*

Interaction (K x H)	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × Low	-.132	0.021	.027	0.010	-.132	0.022
2 × Wide	-.200	0.018	.030	0.010	-.200	0.018
2 × High	-.253	0.019	.037	0.017	-.253	0.019
4 × Low	-.065	0.009	.025	0.020	-.066	0.009
4 × Wide	-.097	0.009	.039	0.019	-.098	0.009
4 × High	-.121	0.011	.056	0.014	-.121	0.011
8 × Low	-.058	0.011	<.001	0.017	-.058	0.011
8 × Wide	-.073	0.010	.011	0.015	-.075	0.009
8 × High	-.092	0.013	.019	0.018	-.092	0.013

*N* = 540

In the distributions of bias associated with maximum likelihood and principal axis factor extraction methods, the graphed means in Figure 26 and box and whisker plots (see Figure D27 in appendix D) indicate that the absolute values of bias are positively related to communality and negatively related to the number of factors imbedded in the population. This pattern is not expressed in the distribution of bias values for ordinary least squares.

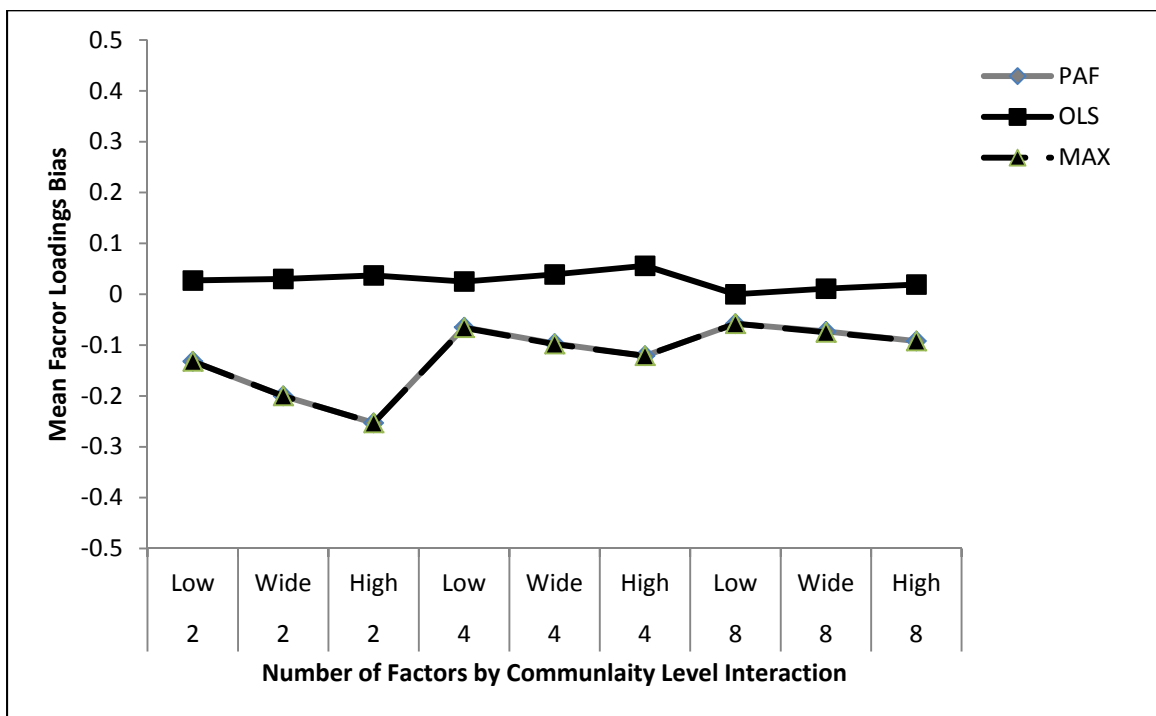


Figure 26. Mean value of factor loading bias by interactions between number of factors and communality.



In the distributions of bias associated with maximum likelihood and principal axis factor extraction methods, the graphed means in Figure 26 and box and whisker plots (see Figure D27 in appendix D) indicate that the absolute values of bias are positively related to communality and negatively related to the number of factors imbedded in the population. This pattern is not expressed in the distribution of bias values for ordinary least squares.

Table 48 includes means and standard deviations of factor loading bias values for each level of the interaction between number of factors and level of dichotomization interaction. The mean bias values for principal axis and maximum likelihood factor extraction methods are negative for each level of the interaction. Across all levels of the interaction, ordinary least squares resulted in the smallest mean amount of factor loading bias.

The graphed means in Figure 27 and box and whisker plots (see Figure D28 in appendix D) indicate that the absolute values of means and median levels of statistical bias in the factor loading matrices diminish as the number of factors increase. For maximum likelihood and principal axis factor extraction methods, the ranges and semi-interquartile ranges also diminish as the number of factors increase. These patterns in measures of central tendency and dispersion are not apparent in the distributions of loading bias associated with the ordinary least squares factor extraction method.

Across all significant interactions, ordinary least squares factor extraction method yielded factor loading matrices that exhibited smaller mean values of statistical bias than matrices derived from the principal axis and the maximum likelihood methods.

Table 48  
*Means and Standard Deviations of Factor Loading Bias for Factor Extraction Method  
and Number of Factors by Level of Dichotomization Interaction (K x D)*

Interaction ( <i>K x D</i> )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × .05	-.194	0.059	.033	0.012	-.194	0.059
2 × .25	-.194	0.055	.031	0.014	-.194	0.055
2 × .50	-.190	0.050	.038	0.014	-.190	0.050
2 × .75	-.197	0.053	.024	0.013	-.197	0.052
2 × .95	-.200	0.053	.031	0.011	-.200	0.053
4 × .05	-.096	0.025	.040	0.022	-.096	0.025
4 × .25	-.092	0.025	.043	0.022	-.093	0.025
4 × .50	-.093	0.025	.041	0.023	-.094	0.025
4 × .75	-.094	0.025	.040	0.020	-.095	0.025
4 × .95	-.097	0.026	.038	0.023	-.098	0.026
8 × .05	-.073	0.017	.011	0.017	-.074	0.017
8 × .25	-.073	0.018	.012	0.019	-.073	0.018
8 × .50	-.074	0.018	.010	0.019	-.075	0.018
8 × .75	-.075	0.018	.009	0.017	-.075	0.018
8 × .95	-.078	0.019	.007	0.020	-.079	0.018

*N* = 540

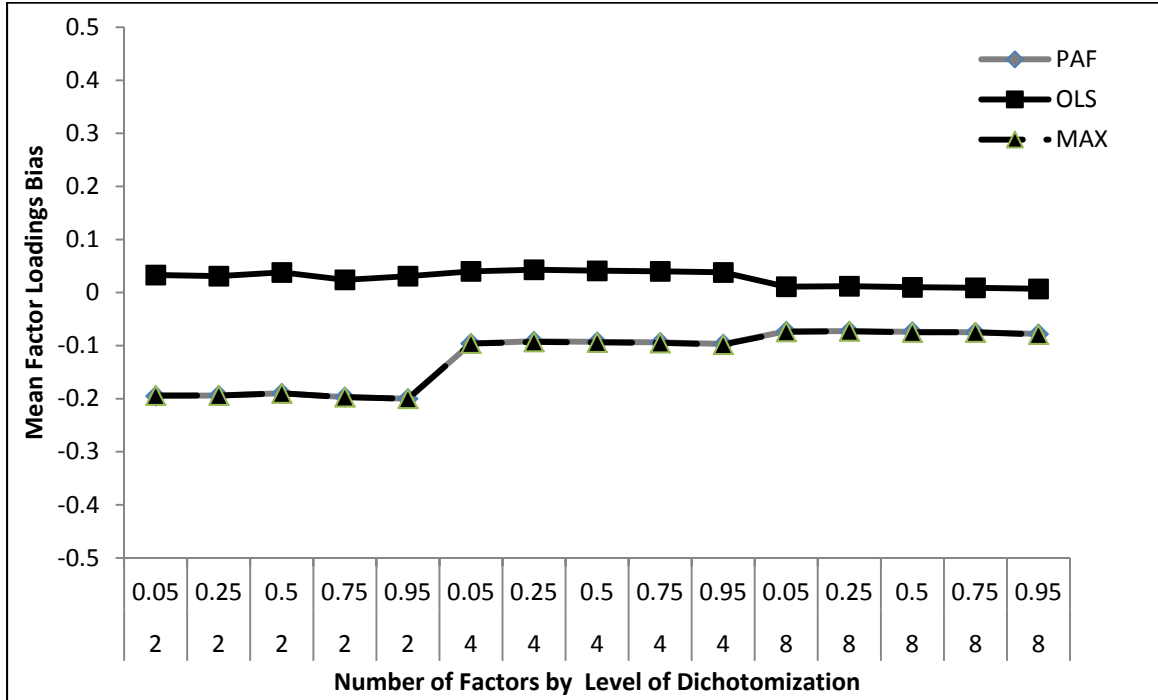


Figure 27. Mean value of factor loading bias estimates for factor extraction methods and number of factors by level of dichotomization.

All of the interactions that yielded effect sizes that warranted interpretation included the number of factors main effect; as the number of factors increased, the mean values of statistical bias (in terms of their absolute values) decreased; this trend was apparent in mean values of statistical bias for all factor extraction methods. As the box and whisker plots highlight, all bias values associated with principal axis and maximum likelihood methods were negative.

### Root Mean Squared Error

Root mean squared error (RMSE) for each factor loading represents the last indicator of congruence. To provide an overall index for each factor solution, the RMSE

estimates are averaged across all samples in each research context. This congruence measure is expressed in a  $P \times K$  matrix. Table 49 provides means and standard deviations of RMSE for all samples in the study.

Table 49  
*Descriptive Statistics for Distribution of Factor Loading RMSE*

Factor Extraction Method	<i>N</i>	<i>M</i>	<i>SD</i>	Skewness	Kurtosis
Principal Axis	540	.113	0.053	1.224	0.907
Ordinary Least Squares	540	.091	0.046	1.344	1.802
Maximum Likelihood	540	.105	0.052	1.174	0.773

Shapiro-Wilks' tests of normality yielded evidence of non-normality in distributions of RMSE for all factor extraction methods. Mauchly's test of transformed variables  $\chi^2(2) = 1081.891, p < .0001$ , indicate that the data do not conform to the sphericity assumption associated with repeated measures ANOVA. To address the potential for increased Type I error rate, tests of significance associated with the repeated measures analysis of variance will be based on the Greenhouse-Geisser adjustment to degrees of freedom (Stevens, 2002).

Results of the multivariate analysis of variance indicate that RMSE differed significantly across the tested factor extraction methods,  $\Lambda = .0269, F(2, 459) = 8274.65, p < .0001$  (see Table C15 in appendix C). Results of the multivariate analyses also indicated the RMSE differed significantly by all main effects. According to the results of the univariate, repeated measures analysis of variance, mean values of RMSE

differed significantly by factor extraction method,  $F(2, 920) = 1559.4, p < .0001, \eta_G^2 = .458$  (see Table C16 in appendix C). The model including main effects and first-order interactions accounted for 96.26% of the variability associated with statistical bias in factor pattern matrices.

In addition to the method effect, the multivariate and repeated measures analyses of variance identified significant differences in RMSE associated with seven of the ten first-order interactions. However, according to the within subjects and the between subjects portions of the repeated measures analysis of variance, only four of these interactions yield effect sizes that were medium or greater. These interaction include

1. Number of factors by number of observed variables,  $F(4, 460) = 209.92, p < .0001, \eta_G^2 = .578$
2. Number of factors by level of communality,  $F(4, 460) = 548.08, p < .0001, \eta_G^2 = .781$
3. Number of factors by level of dichotomization,  $F(8, 460) = 9.99, p < .0001, \eta_G^2 = .115$
4. Level of communality by level of dichotomization,  $F(8, 460) = 6.73, p < .0001, \eta_G^2 = .081$

In addition to these interactions, the sample size main effect yielded an effect size of at least medium strength,  $F(3, 460) = 172.94, p < .0001, \eta_G^2 = .458$ . Because sample size is not included in interactions that met the effect size criteria for further analyses, it will also be included in the comparisons among means.

Table 47 presents means and standard deviations of RMSE for each factor extraction method by each level of sample size.

Table 50  
*Means and Standard Deviations of Factor Loading RMSE for Factor Extraction Method by Sample Size*

Effect Levels	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	M	SD	M	SD	M	SD
100	.124	0.052	.114	0.046	.112	0.049
200	.115	0.053	.094	0.044	.105	0.052
300	.111	0.053	.085	0.043	.103	0.052
1000	.104	0.053	.071	0.041	.099	0.054

*N = 540*

As the table highlights, RMSE is negatively related to sample size. Results of the comparison indicate that principal axis factor analysis yields the highest values of RMSE. Moreover, except for the  $N = 100$  condition, ordinary least squares yields the lowest levels of RMSE.

The relationship among sample size, factor extraction method, and mean RMSE is demonstrated graphically in Figure 28. As this graph highlights, the negative relationship between sample size and RMSE is slightly more apparent in the trend associated with the ordinary least squares factor extraction method.

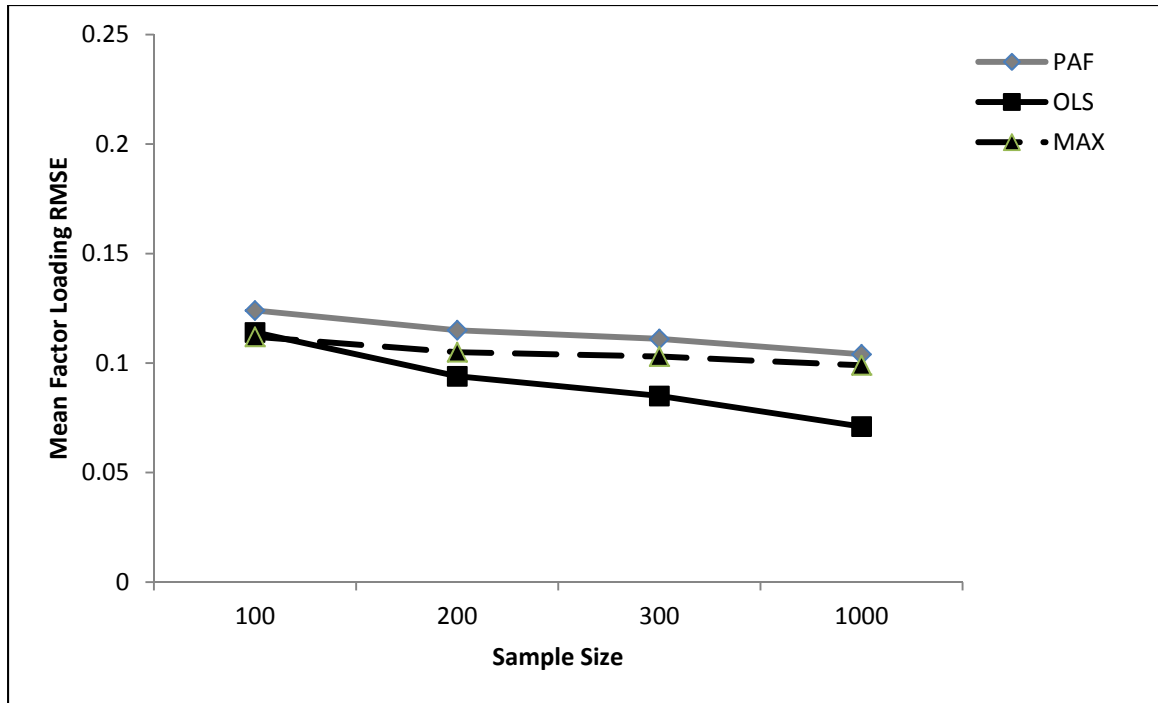


Figure 28. Mean value of RMSE by sample size main effect.

Table 51 presents means and standard deviation of mean RMSE values for each factor extraction method by levels of the number of factors and observed variables interaction. The mean RMSE values do not have a simple relationship with either effect included in the interaction. However, the ordinary least squares factor extraction method yields slightly smaller values of RMSE than either principal axis or maximum likelihood factor extraction methods.

As the trend lines in Figure 29 highlight, the differences among factor extraction methods in mean values of RMSE do not appear to vary substantially as either the number of factors or the number of observed variables increases.

Table 51  
*Means and Standard Deviations of Factor Loading RMSE for Factor Extraction Method and Number of Factors by Observed Variables ( $K \times P$ )*

Interaction ( $K \times P$ )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
$2 \times 20$	.142	0.058	.124	0.056	.127	0.059
$2 \times 40$	.164	0.061	.132	0.054	.150	0.061
$2 \times 60$	.174	0.063	.133	0.052	.164	0.063
$4 \times 20$	.109	0.023	.095	0.026	.098	0.024
$4 \times 40$	.092	0.027	.079	0.022	.083	0.028
$4 \times 60$	.090	0.030	.070	0.023	.083	0.030
$8 \times 20$	.110	0.012	.079	0.014	.104	0.014
$8 \times 40$	.075	0.013	.059	0.016	.072	0.015
$8 \times 60$	.064	0.013	.048	0.015	.061	0.014

$N = 540$

For all factor extraction methods, box and whisker plots of RMSE values highlight a general decline in ranges and semi-interquartile ranges as the number of factors increases (See Figure D29 in appendix D). Within the four and eight-factor conditions, mean and median RMSE values decline as the ratio of observed variables to factors increases.



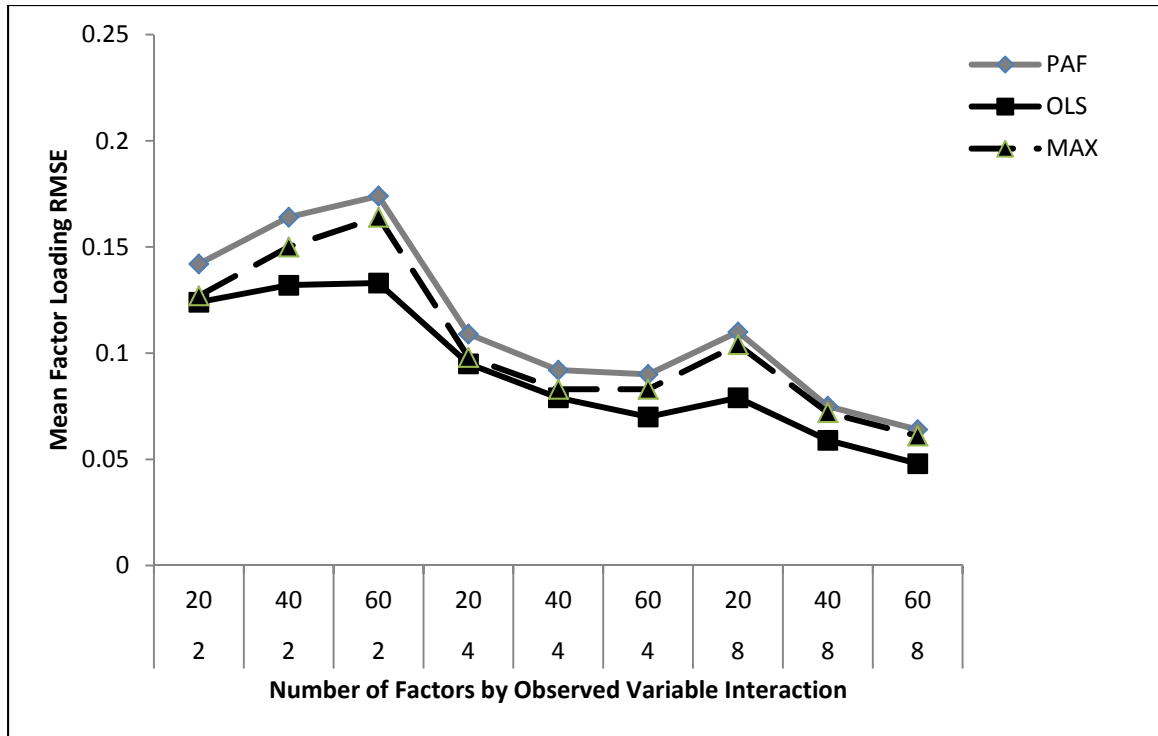


Figure 29. Mean value of factor loading RMSE by interactions between number of factors and number of observed variables.

Table 52 provides means and standard deviations of RMSE for factor extraction methods and levels of the number of factors by communality interaction. The results indicate that mean values of RMSE are positively related to the range of communality. For the ordinary least squares factor extraction method, the mean RMSE values are negatively associated with the number of factors. In all levels of the interaction, the principal axis factor extraction method yields the highest mean values of RMSE. The relationships summarized in Table 49 are highlighted in the graphed means found in Figure 30 and in the box and whisker plots (see Figure D30 in appendix D).

Table 52  
*Means and Standard Deviations of Factor Loading RMSE for Factor Extraction Method and Number of Factors by Communality Range Interaction (K x H)*

Interaction ( <i>K x H</i> )	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × Low	.090	0.015	.084	0.022	.074	0.014
2 × Wide	.155	0.022	.121	0.038	.147	0.020
2 × High	.233	0.022	.184	0.041	.221	0.021
4 × Low	.067	0.018	.061	0.023	.055	0.015
4 × Wide	.098	0.013	.085	0.020	.091	0.011
4 × High	.126	0.010	.098	0.019	.118	0.007
8 × Low	.070	0.022	.058	0.021	.063	0.020
8 × Wide	.082	0.020	.063	0.019	.080	0.019
8 × High	.097	0.020	.064	0.019	.094	0.019

*N* = 540

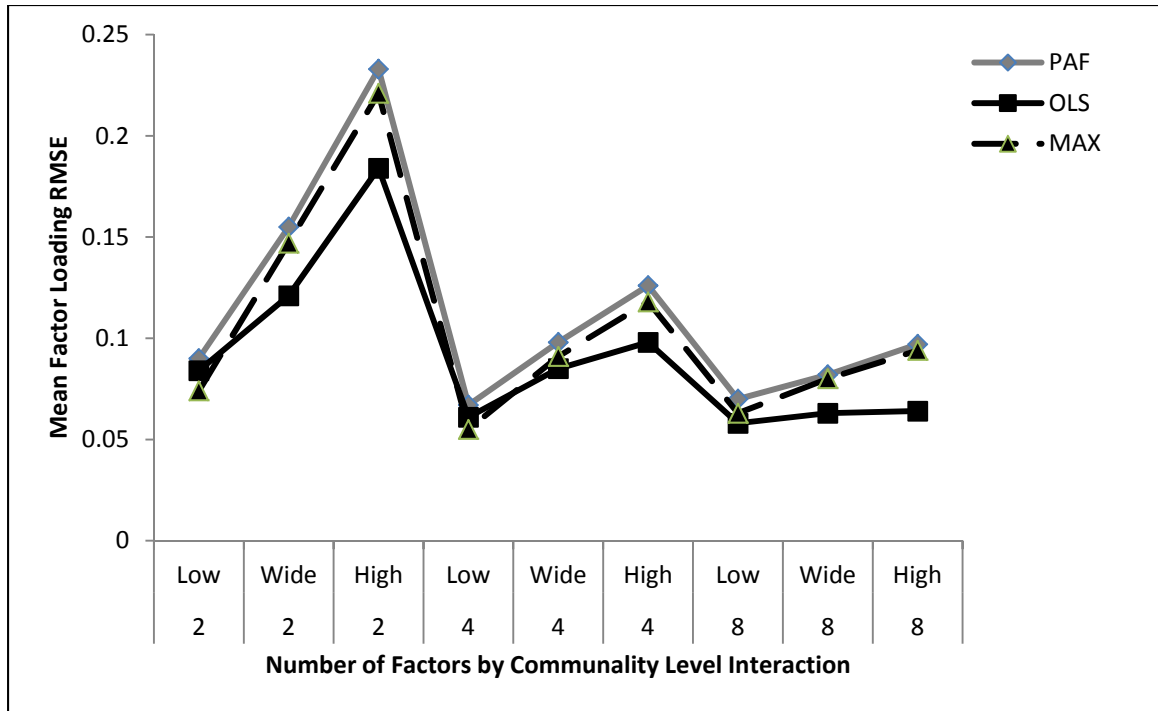


Figure 30. Mean value of factor loading by interactions between number of factors and communality.

As Table 53 presents highlights, the ordinary least squares method yielded the lowest mean RMSE's for all levels of the number of factors by dichotomization interaction. Moreover, for all factor extraction methods, RMSE is negatively related to the number of factors simulated in the population. In addition to mean values of RMSE, box and whisker plots indicate that ranges and semi-interquartile ranges, for all distributions, decrease as the number of factors increases (see Figure D31 in appendix D). Figure 31 demonstrates the relationships summarized in Table 50 graphically.

Table 53

*Means and Standard Deviations of Factor Loading RMSE for Factor Extraction Method and Number of Factors by Level of Dichotomization Interaction (K x D)*

Interaction (K x D)	Ordinary				Maximum	
	<u>Principal Axis</u>		<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
2 × .05	.158	0.069	.124	0.058	.147	0.068
2 × .25	.156	0.057	.114	0.034	.145	0.061
2 × .50	.156	0.061	.125	0.047	.143	0.062
2 × .75	.160	0.060	.133	0.057	.145	0.060
2 × .95	.169	0.064	.153	0.063	.155	0.060
4 × .05	.099	0.029	.087	0.026	.091	0.030
4 × .25	.097	0.028	.082	0.027	.089	0.030
4 × .50	.096	0.028	.078	0.026	.088	0.029
4 × .75	.095	0.027	.080	0.026	.086	0.028
4 × .95	.097	0.027	.081	0.024	.087	0.027
8 × .05	.084	0.023	.064	0.019	.080	0.023
8 × .25	.083	0.024	.062	0.020	.079	0.023
8 × .50	.083	0.024	.062	0.022	.079	0.024
8 × .75	.082	0.023	.061	0.020	.078	0.023
8 × .95	.083	0.024	.061	0.020	.080	0.024

*N* = 540

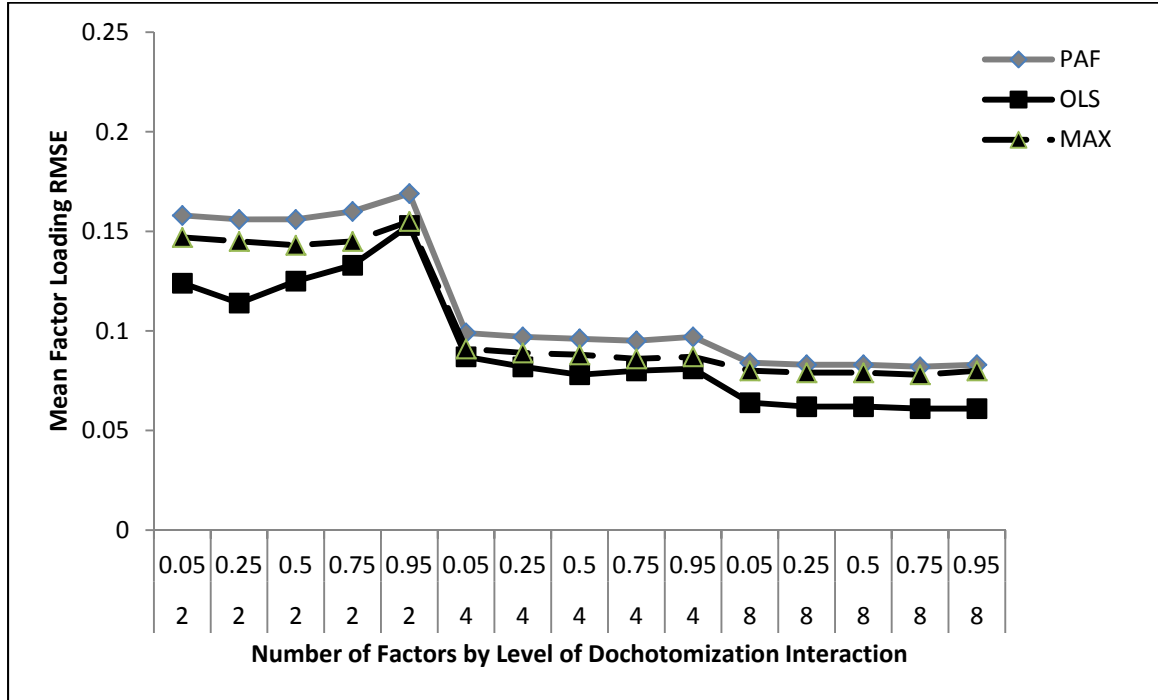


Figure 31. Mean value of factor loading RMSE by interactions between number of factors and level of dichotomization.

Table 54 provides means and standard deviations for RMSE for each factor extraction method by each level of the communality range and dichotomization interaction. The results indicate that RMSE is positively related to the range of communality. However, the rate at which RMSE increase with communality is the least for ordinary least squares factor analysis.

The relationships among levels of communality and mean values of RMSE are highlighted graphically in Figure 32. For distributions of RMSE associated with all factor extraction methods, box and whisker plots also highlight a positive relationship among communality ranges, ranges in mean RMSE values, and semi-interquartile range.

Table 54

*Means and Standard Deviations of Factor Loading RMSE by Factor Extraction Method and Community Range by Level of Dichotomization Interaction (H x D)*

Interaction ( <i>H x D</i> )	<u>Principal Axis</u>		Ordinary		Maximum	
			<u>Least Squares</u>		<u>Likelihood</u>	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Low × .05	.075	0.021	.067	0.023	.063	0.019
Low × .25	.075	0.020	.067	0.022	.063	0.017
Low × .50	.075	0.022	.067	0.026	.063	0.019
Low × .75	.078	0.023	.072	0.030	.064	0.018
Low × .95	.077	0.021	.067	0.024	.065	0.018
Wide × .05	.110	0.035	.087	0.036	.105	0.032
Wide × .25	.113	0.038	.091	0.035	.108	0.035
Wide × .50	.110	0.035	.086	0.030	.103	0.032
Wide × .75	.108	0.032	.082	0.025	.103	0.031
Wide × .95	.118	0.043	.104	0.048	.112	0.039
High × .05	.157	0.064	.120	0.055	.149	0.060
High × .25	.148	0.056	.102	0.036	.142	0.055
High × .50	.151	0.059	.113	0.054	.142	0.056
High × .75	.151	0.064	.119	0.066	.142	0.059
High × .95	.154	0.066	.125	0.072	.145	0.061

*N* = 540

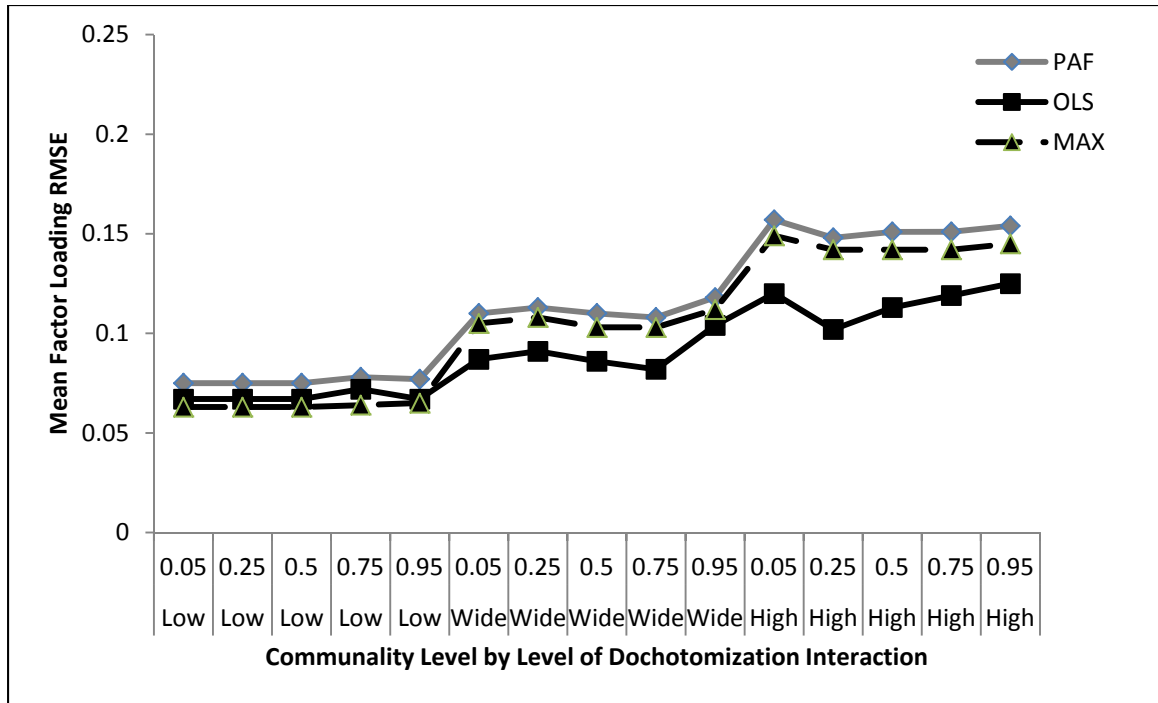


Figure 32. Mean value of factor loading RMSE by interactions between communality and level of dichotomization.

### Summary of Results

Factor extraction method accounted for statistically significant differences in all measures of agreement between sample and population factor loading matrices. As indicated by the generalized eta-squared values, factor extraction method accounted for more than 90% of the variability in factor loading sensitivity ( $\eta_G^2 = .998$ ), general factor loading pattern agreement ( $\eta_G^2 = .991$ ), and per element agreement ( $\eta_G^2 = .934$ ). Although values of total factor pattern agreement differed significantly by factor extraction method, the method effect accounted for less than 20% of the variance in this measure ( $\eta_G^2 = .198$ ).

The interaction between factor extraction method and the ratio of categorical (dichotomous) variables to the total number of observed variables accounted for statistically significant differences in only three measures of agreement between the sample and the population. In the cases where statistical differences were observed, the effect sizes were small (Cohen, 1988); these measures included factor loading sensitivity ( $\eta_G^2 = .049$ ), per element agreement ( $\eta_G^2 = .003$ ), and total pattern agreement ( $\eta_G^2 = .025$ ). However, as part of a between subjects, first-order interaction with the number of factors imbedded in the population, the level of dichotomization accounted for more than 57% ( $\eta_G^2 = .574$ ) of the variance in factor loading sensitivity. The comparison of means by factor extraction method indicated that, for every level of this interaction, ordinary least squares yielded higher mean values of factor loading sensitivity.

Interactions between the number of observed variables and factor extraction method accounted for statistically and practically significant differences in all measures of agreement between sample and population factor loading matrices. Within subjects analyses indicated that the ratio of observed variables to number of factors imbedded in the population yielded large effect sizes in loading sensitivity ( $\eta_G^2 = .770$ ), general pattern agreement ( $\eta_G^2 = .442$ ), per element agreement ( $\eta_G^2 = .474$ ), and total pattern agreement ( $\eta_G^2 = .436$ ). These same analyses indicated that the number of observed variables interacted with communality to yield at least a medium effect size in factor loading sensitivity ( $\eta_G^2 = .059$ ). The between subject analysis indicated that the number of variables interacted with communality to yield at least medium effect sizes in general pattern agreement ( $\eta_G^2 = .359$ ) and per element agreement ( $\eta_G^2 = .155$ ). For all levels of interactions among these main effects, ordinary least squares yielded significantly higher



mean values in all measures of agreement than either maximum likelihood or principal axis factor extraction methods.

Sample size interacted with factor extraction method to yield significant differences in mean values for all measures of agreement between sample and population factor loading matrices. Between subjects analyses indicated that sample size interacted with the number of factors imbedded in the population to yield a medium effect size in per element agreement ( $\eta_G^2 = .172$ ); a within subjects analysis indicated that this first order interaction yielded a medium effect size in total pattern agreement ( $\eta_G^2 = .094$ ). The within subjects analysis also indicated that sample size and communality interacted to yield medium effect sizes in general pattern agreement ( $\eta_G^2 = .180$ ) and per element agreement ( $\eta_G^2 = .077$ ). For all levels of interactions among these main effects, ordinary least squares yielded significantly higher mean values in all measures of agreement than either maximum likelihood or principal axis factor extraction methods.

The levels of communality interacted with factor extraction method to yield statistically significant differences in factor loading sensitivity, general pattern agreement, and per element agreement. Between subjects analyses indicated that interactions between level of communality and the number of factors imbedded in the population yielded at least medium effects in factor loading sensitivity ( $\eta_G^2 = .062$ ), general pattern agreement ( $\eta_G^2 = .458$ ), and per element agreement ( $\eta_G^2 = .077$ ). For all of these measures, ordinary least squares yielded the highest mean values across all levels of the interaction.

Factor extraction method accounted for statistically significant differences in all measures of congruence. As indicated by the generalized eta-squared values, factor

extraction method accounted for more than 50% of the variability in three of the four measures associated with sample and population congruence; these included mean phi coefficient ( $\eta_G^2 = .521$ ), factor score correlations ( $\eta_G^2 = .783$ ), and factor loading bias ( $\eta_G^2 = .992$ ). Although values of factor loading RMSE differed significantly by factor extraction method, the method effect accounted for less than 50% of the variance in this measure ( $\eta_G^2 = .458$ ).

As a main effect, the ratio of categorical variables to the total number of observed variables accounted for statistically significant differences in factor loading bias and RMSE. Between subjects analyses indicated that interactions between level of dichotomization and the number of factors imbedded in the population yielded at least medium effect sizes in the mean values of the phi coefficient ( $\eta_G^2 = .086$ ), factor loading bias ( $\eta_G^2 = .458$ ), and RMSE ( $\eta_G^2 = .115$ ). Between subjects analyses also indicated that dichotomization interacted with communality to yield at least medium effect sizes in mean phi coefficients ( $\eta_G^2 = .076$ ) and RMSE ( $\eta_G^2 = .081$ ). In general, maximum likelihood yielded higher mean phi coefficients than either ordinary least squares or maximum likelihood factor extraction methods. Across the majority of levels of the first-order interactions, ordinary least yielded values of bias and RMSE that were closer to zero than either maximum likelihood or principal axis factor extraction methods; however, differences among the three factor extraction methods were small.

The number of observed variables main effect for statistically significant differences in all of the congruence measures. Between subjects analyses indicated that ratios of observed variables to factor yielded at least medium effect sizes in mean phi coefficients ( $\eta_G^2 = .405$ ), factor score correlations ( $\eta_G^2 = .804$ ), factor loading bias ( $\eta_G^2 =$

.621), and RMSE ( $\eta_G^2 = .578$ ). These same analyses indicated that the interaction between communality and the number of observed variables yielded an effect size that was at least medium in factor score correlations ( $\eta_G^2 = .149$ ). Maximum likelihood yielded higher mean phi coefficients across all ratios of observed variables to number of factors than ordinary least squares or principal axis factor extraction methods. In the majority of the levels of this interaction, ordinary least squares factor extraction method resulted in stronger correlations between sample and population factor scores; in the four-factor by 20 variable condition, maximum likelihood yielded a slightly higher correlation. In the interaction between the number of observed variables and the level of communality, ordinary least squares also yielded higher factor correlations in nearly all levels of the interaction.

The sample size main effect yielded significant differences in mean values for three measures of congruence among sample and population factor loading matrices; these include mean phi coefficients, factor score correlations, RMSE. Between subjects analyses indicated that the sample size interacted with the number of factors in the population to yield at least medium effect sizes in mean phi coefficients ( $\eta_G^2 = .115$ ), factor score correlations ( $\eta_G^2 = .095$ ), and factor loading bias ( $\eta_G^2 = .136$ ). Sample size interacted with communality to yield an at least medium effect size in mean phi coefficients ( $\eta_G^2 = .067$ ). Across all ratios of sample size to number of factors, maximum likelihood yielded higher mean phi coefficients than ordinary least squares and principal axis factor extraction methods. For nearly all levels of the sample size by number of factors and sample size by communality interactions, ordinary least squares yielded stronger correlations among sample and population factor score correlations. In nearly

all levels of sample size by number of factors interaction, ordinary least squares yielded factor loading bias values that were closer to zero than either maximum likelihood or principal axis factor extraction methods.

The communality main effect yielded statistically significant differences in all congruence measures. Between subjects analyses indicated that interactions between level of communality and the number of factors imbedded in the population yielded at least medium effects in factor score correlations ( $\eta_G^2 = .324$ ), factor loading bias ( $\eta_G^2 = .753$ ), and RMSE ( $\eta_G^2 = .781$ ). In all levels of the interaction, ordinary least squares yielded stronger factor score correlations, factor score bias levels closer to zero, and smaller values of RMSE.

### **Research Questions**

*How do varying ratios of categorical to continuous variables influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?* Level of dichotomization resulted in significant differences in values of factor loading sensitivity, per element factor pattern agreement, total pattern agreement, congruence, factor score correlation, bias, and RMSE. As estimated by generalized eta squared, levels of dichotomization accounted for medium effect size differences in values of factor loading sensitivity and factor loading bias. However, in both cases, levels of dichotomization were components of interactions that also resulted in effect sizes that were at least medium. Therefore, the effects of dichotomization could only be interpreted as part of first order interactions.

Dichotomization interacted with the number of factors to result in statistically and practically significant differences factor loading sensitivity, congruence coefficients, factor loading bias, and RMSE. In general, comparisons among means of these variables based on the first order interactions indicated that the level of dichotomization did not influence the impact that the number of factors had an on agreement and congruence measures. However, within the two factor condition, mean values of the congruence coefficient did exhibit a slight negative relationship to the level of dichotomization. Levels of dichotomization and communality interacted to yield statistically and practically significant differences in congruence coefficients and RMSE; comparisons among means indicated that, in neither case, did the level of dichotomization exert influence on the impact associated with communality.

*How does the number of variables in a correlation matrix influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?* The number of observed variables resulted in significant differences in values of factor loading sensitivity, general pattern agreement, per element pattern agreement, total pattern agreement, congruence, factor score correlation, bias, and RMSE. As estimated by generalized eta squared, the observed variable main effect accounted for medium effect size differences in values of factor loading sensitivity, general pattern agreement, per element pattern agreement, total pattern agreement, congruence, factor score correlations, bias, and RMSE. However, in all cases, the numbers of observed variables were components of interactions that also resulted in effect sizes that were at least medium. Therefore, the impact of observed variables could only be interpreted as part of first order interactions.

The number of observed variables interacted with the number of factors imbedded in the population to yield differences in all measures of agreement and congruence that reached effect sizes that warranted interpretation. For maximum likelihood and principal axis factor extraction methods, values of factor loading sensitivity and general pattern agreement were negatively related to the number of variables within each level of the number of factors condition; for factor loading matrices associated with ordinary least squares, these values were not influenced by the number of observed variables. The direction of the relationship between the number of observed variables and values of per element agreement changed with each level of the number of factors imbedded in the population. Within each level of the number of common factors, values of congruence were positively associated with the number of observed variables. In the eight factor condition, this positive relationship with the number of observed variables is also present in the correlations between factor scores in the sample and factor scores in the population. However, in the two factor condition, the relationship between factor score correlations and number of observed variables is negative for all factor extraction methods. For both bias and RMSE, the relationship with number of observed variables changed direction (positive versus negative) for each level of the number of factors effect.

The number of observed variables interacted with the level of communality to yield differences in all factor loading sensitivity, general pattern agreement, per element agreement, and factor score correlation that were at least medium in effect sizes. While values of factor loading sensitivity associated with the ordinary least squares method were not influenced by the interaction between number of observed variables and

communality; the values of these measures were negatively related to the number of variables for both maximum likelihood and principal axis factor extraction methods. For the 40 and 60 observed variable conditions, factor score correlation appears to be positively related to the number of observed variables within each level of communality; however, this relationship is only apparent in maximum likelihood and principal axis factor extraction methods.

*How does sample size influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?* Sample size resulted in significant differences in values of factor loading sensitivity, general pattern agreement, per element factor pattern agreement, total pattern agreement, congruence, factor score correlation, and RMSE. As estimated by generalized eta squared, the between subjects analysis indicated that sample size accounted for medium effect size differences in values of general pattern agreement, per element agreement, congruence coefficients, sample and population factor score correlations, and RMSE. In the case of RMSE, the sample size main effect yielded an effect size that was at least moderate without being a component of a substantial interaction; for all three factor extraction methods, RMSE diminished as sample size increased. In all other measures of agreement and congruence, sample size was a part of interactions that met the effect size threshold for interpretation.

Sample size interacted with the number of factors imbedded in the population to yield statistically and practically significant differences in per element agreement, total agreement, congruence, correlations between sample and population factor scores, and bias. In all measures of agreement between sample and population, the levels of

agreement were positively associated with sample size within each number of factors condition. Because maximum likelihood and principal axis factor extraction methods failed to yield non-zero factor scores in all conditions, a universally applicable trend is difficult to identify; however, when the two factor conditions are ignored, correlations among sample and population factor scores are positively related to sample size. Within each number of factors condition, the amount of negative bias in solutions associated with maximum likelihood and principal axis methods diminishes as the sample size increases; the amount of bias associated with the ordinary least squares method is not influenced by sample size.

Differences in mean values for general pattern agreement, per element agreement, and congruence of at least medium effect are associated with interactions between sample size and communality. For all three methods, values of general pattern agreement and congruence were positively related to sample size. Values of per element agreement were positively associated with sample size in pattern matrices derived from the ordinary least squares factor extraction method; this relationship was not apparent in maximum likelihood or principal axis methods.

*How does communality influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?* The level of communality (low, wide, and high) resulted in significant differences in values of factor loading sensitivity, general pattern agreement, per element pattern agreement, congruence, factor score correlation, bias, and RMSE. As estimated by generalized eta squared, the communality main effect accounted for medium effect size differences in values of factor loading sensitivity, general pattern agreement,



per element pattern agreement, congruence, factor score correlations, bias, and RMSE. However, in all cases, the levels of communality were components of interactions that also resulted in effect sizes that were at least medium. Therefore, the impact of communality could only be interpreted as part of first order interactions.

The number of factors imbedded in the population interacts with communality to yield differences in factor loading sensitivity, general pattern agreement, per element agreement, factor score correlations, bias, and RMSE that are at least medium in effect size. For maximum likelihood and principal axis factor extraction methods, values of factor loading sensitivity are positively related to the level of communality within each level of the interaction; values of factor loading sensitivity associated with ordinary least squares are not influenced by communality within its interaction with the number of factors. Within all levels of the interaction, values of general pattern agreement are positively related to level of communality; this is true across all factor extraction methods. In nearly all levels of the interaction, values of per element agreement are negatively related to level of communality; however, for the ordinary least squares factor extraction method, this relationship is slightly positive in the eight factor condition. In the four and eight factor conditions, factor score correlations exhibit a generally positive relationship with level of communality; however, this trend is not universal and not apparent in the two factor conditions. For all three factor extraction methods, absolute values of bias and RMSE are positively related to the level of communality within each level of the number of factors condition.

The number of observed variables interacted with level of communality to yield substantial differences in factor loading sensitivity, general pattern agreement and factor

score correlation. For maximum likelihood and principal axis factor extraction methods, factor loading sensitivity is positively associated with the level of communality in all levels of the interaction. However, this relationship is not apparent in loading sensitivity values associated with ordinary least squares. In most levels of the interaction, values of general pattern agreement are positively related to level of communality; however, for the maximum likelihood factor extraction method, a slightly negative relationship between general agreement and communality is apparent in the 60 observed variable condition. The direction of the relationships among factor score correlations and communality levels appears to vary by factor extraction method and level of the observed variables effect.

The sample size effect interacts with communality to yield differences in general pattern agreement, per element agreement, and congruence. Within all levels of the interaction and for all factor extraction methods, values of general pattern agreement are positively related to level of communality. For the maximum likelihood and principal axis factor extraction methods, values of per element agreement are negatively related to communality; for the ordinary least squares factor extraction method, this relationship holds in the 300 and 1000 sample size conditions only. With the exception of matrices associated with the ordinary least squares factor extraction method, congruence values are positively related to the level of communality in all sample size conditions. For ordinary least squares, the highest values of congruence are associated with the low communality conditions and the lowest values are associated with the wide conditions.

The communality main effect interacted with dichotomization to yield differences in congruence and RMSE that were at least medium in effect size. With the exception of the wide communality condition in which 95% of the variables were dichotomous, values

of congruence associated with maximum likelihood and principal axis factor extraction methods were positively related to level of communality; values of congruence associated with ordinary least squares factor extraction appear not to be influenced by level of communality in this interaction. Within each level of the interaction, RMSE is positively related to the level of communality

*How does the number of common factors influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?* The number of factors main effect is associated with statistically significant differences in all measures of agreement and congruence. As estimated by generalized eta squared, differing numbers of factors imbedded in the population accounted for medium effect size differences in values of factor loading sensitivity, general pattern agreement, per element pattern agreement, total pattern agreement, congruence, factor score correlations, bias, and RMSE. However, in all cases, the number of factors main effect is a component of interactions that also resulted in effect sizes that were at least medium. Therefore, the impact of this effect could only be interpreted as part of first order interactions.

The number of factors imbedded in the population interacted with the number of observed variables, level of communality, and level of dichotomization to yield differences in factor loading sensitivity that were at least medium in effect size. Across all of these interactions, factor loading sensitivity values associated with maximum likelihood and principal axis factor extraction methods were positively related to the number of factors. However, in all cases, factor loading sensitivity values associated

with the ordinary least squares factor extraction method were not influenced by the number of factors imbedded in the population.

The number of factors imbedded in the population interacted with the number of observed variables and level of communality to yield differences in general factor agreement that were at least medium in effect size. In both interactions, general pattern agreement was negatively related to the number of factors for matrices associated with ordinary least squares. For maximum likelihood and principal axis factor extraction methods, this negative trend was not apparent.

The number of factors imbedded in the population interacted with the number of observed variables, sample size, and level of communality to yield differences in per element agreement that were at least medium in effect size. In each interaction, values of per element agreement were positively related to the number of factors. This relationship was apparent in matrices associated with all factor extraction methods.

The number of factors imbedded in the population interacted with the number of observed variables, sample size, and level of dichotomization to yield differences in values of congruence that were at least medium in effect size. For all levels of these interactions, congruence values were negatively related to the number of factors. This relationship was apparent in the three tested factor extraction methods.

The number of factors imbedded in the population interacted with the number of observed variables, sample size, and communality to yield differences in correlations among factor scores derived from the sample and those derived from the population that met the effect size requirement for follow up analyses. In each of the cases, factor score correlations were negatively related to the number of factors. While this relationship

could be found in correlations associated with all three factor extraction methods, the relationship was complicated by the zero factor scores (and the subsequent missing factor score correlations) for the maximum likelihood and principal axis factor extraction method in several of the two factor conditions.

The number of factors interacted with the number of observed variables, sample size, level of communality, and level of dichotomization to yield differences in factor loading bias that were at least medium in effect size. Across all levels of these interactions, the absolute values of bias were negatively related to the number of factors. The number of factors interacted with the number of observed variables, level of communality, and level of dichotomization to yield differences in RMSE that met the effect size requirement for follow-up analyses. As was the case in the bias comparisons, values of RMSE were negatively related to the number of factors in all levels of the interactions.

## **Chapter Five**

### **Discussion**

The intention underlying this study was to provide researchers with empirically derived guidelines concerning the interaction among factor extraction methods and types of data. The scope of this study included the evaluation of factor extraction methods when applied to data sets that contained a mixture of categorical and continuous variables. To enhance the potential usefulness of this study's results, this research focused on methods commonly employed by social scientists; these included principal axis factor analysis, ordinary least squares factoring, and standard maximum likelihood method.

### **Research Questions**

The agreement between factor pattern matrices in a simulated population and matrices developed through selected exploratory factor analytic techniques is the primary comparison associated with this study. This agreement was assessed through the proportion of variables that loaded on the same factors, total factor loading agreement, and factor loading congruence coefficients (MacCallum et al., 1999). Measures of agreement, correlations between population and sample factor score matrices, root mean square error, statistical bias, and solution variability were considered as measures of factor pattern agreement.

The measures of congruence and agreement among population and sample matrices addressed the following research questions:

1. How do varying ratios of categorical to continuous variables influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
2. How does the number of variables in a correlation matrix influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
3. How does sample size influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
4. How does communality influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
5. How does the number of common factors influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?
6. How do all of the independent variables interact to influence the agreement between factor pattern matrices extracted through the examined factor analysis strategies and factor pattern matrices simulated in the population?

## **Summary of Methods**

The research methods and Monte Carlo design included in this study were based on previous methodological research in the field of common factor analysis. The strategies used to generate correlation matrices in this study were derived from Tucker, Koopman, and Linn's (1969) examination of factor analytic methods. The sampling methods were based on the strategies employed by Hogarty et al. (2005).

To address the research questions, this study incorporated samples simulated through a variety of research contexts. These contexts differed in number of variables, the number of common factors, communalities, sample sizes, and ratios of categorical to continuous variables. Data were generated under 540 different conditions; specifically, this study was a three (number of variables) by three (number of common factors) by three (communality levels) by four (sample size) by five (ratios of categorical to continuous variables) design.

In the simulation procedure, ten correlation matrices were generated for each combination of data conditions. For each correlation matrix, 1000 samples were generated. These samples varied in the combinations of sample size and ratio of categorical to continuous variables. In total, this simulation process yielded 5,400,000 samples; each combination of data conditions accounted for 10,000 samples.

## **Conclusions**

Across the majority of interactions among the manipulated research contexts that accounted for statistically significant differences and moderate effect sizes, the ordinary least squares factor extraction method yielded factor loading matrices that were in better agreement with the population than either the maximum likelihood or the principal axis



methods. The ordinary least squares method yielded factor loading matrices that exhibited less bias and error than the other two tested factor extraction methods. In general, ordinary least squares loading matrices resulted in factor scores that correlated more strongly with population factor scores than the other tested methods.

In only one set of comparisons, those associated with mean phi coefficients, the maximum likelihood method resulted in factor loading matrices that exhibited slightly greater congruence than ordinary least squares across several interactions. This measure of congruence represents the only measure that excludes the influence of the .30 factor loading threshold. One influence of this threshold effect can be seen in the factor loading bias analyses. In each case, principal axis and maximum likelihood yielded factor loading matrices that exhibited negative bias; the impact of this bias can be found in the depressed the factor loading sensitivity measures for both principal axis and maximum likelihood. Without the impact of this threshold requirement, the congruence values for all three factor extraction methods were very similar.

The first hypothesis included in this study asserted that ordinary least squares factor analysis will perform better than maximum likelihood factor analysis as the number of dichotomously scored variables increase. This assertion was derived from research into factor analytic techniques that indicated that iterative, principal axis factor extraction methods perform better than maximum likelihood methods when the assumption of multivariate normality is not met (Bartholomew, 1980). In interactions that included the level of dichotomization main effect, ordinary least squares yielded factor loading matrices that were in better agreement (and more congruent) with population matrices; however, the differences among the three factor extraction methods

tended to be unaffected by the level of dichotomization. Therefore, this hypothesis is not supported by the findings in this study.

The second hypothesis asserts that, when the factor structure in the population is not strongly defined, ordinary least squares (OLS) factor analysis will identify common factors that maximum likelihood factor analytic methods fail to identify. According to this hypothesis, OLS's relative advantage in identifying common factors will be negatively related to communality and positively related to the number of dichotomous variables. This hypothesis is based on two complimentary studies that highlight OLS's insensitivity to error and maximum likelihood's reliance on the assumption of multivariate normality (Briggs & MacCallum, 2003; Mislevy, 1986). Factor loading sensitivity, or the proportion of variables that have factor pattern coefficients that are greater than or equal to .30 on at least one factor, is the primary measures associated with this hypothesis.

In so far that the ordinary least squares method resulted in higher levels of factor loading sensitivity than the other tested factor extraction methods, the results of this study are supportive of the second hypothesis. However, in the only interaction that included communality and resulted in a medium effect size associated with differences in factor loading sensitivity, the interaction between number of factors and communality level, the differences among the three factor extraction methods were not influenced by the level of communality. Therefore, the results of this study only partially support the assertions made in the second hypothesis.

This study included samples that were simulated under 540 different research contexts. The factor loading matrices were assessed through eight measures of sample-

population agreement and congruence. The analyses focused on all first-order interactions among five manipulated research characteristic. Results of analyses led to several conclusions that were not directly related to the two hypotheses; these include

1. As the number of factors in the population increases, the factor loading values become more representative of the population parameters. This conclusion is well supported by the measures of factor loading sensitivity, general pattern agreement, loading bias, and root mean squared error. However, this effect is not universal; increased numbers of factors is negatively associated with values of per element pattern agreement, average congruence ( $\phi$ ) coefficients, and correlations between sample and population factor scores.
2. Increased ratios of observed variables to factors are not necessarily associated with improved agreement between sample and the population factor loading matrices; moreover, increases in these ratios do not appear to positively influence measures of congruence. This conclusion appears to contradict previous methodological research associated with over-determination (Fabrigar et al., 1999; Guadagnoli & Velicer, 1988; Hogarty et al., 2005). However, the results of this study may merely identify an upper limit to existing rules of thumb. Mean values of agreement and congruence diminish as the ratios exceeded 15:1; however, in interactions that included ratios of 15:1 and less (with the exception of factor loading sensitivity), increased levels of over-determination had either little effect on the agreement and congruence measures or, where the effect was present, it was positive.

3. In terms of sample agreement with the population, the results associated with interactions that include communality are contradictory. For all factor extraction methods included in this study, factor loading sensitivity and levels of general pattern agreement tend to improve as the level of communality increases. However, per element agreement diminishes as the level of communality increases; moreover, the amount of bias in factor loadings appears to be positively associated with the level of communality.
4. The proportion of variables that were dichotomous interacted with the number of observed variables and the level of communality to yield significant differences in two measures of congruence and one measure of agreement. The level of dichotomization did not exert a generally positive or negative influence on any of the relevant outcome measures.

### **Recommendations for Research Practices**

The suggested use of ordinary least squares factor analytic techniques represents the major, empirically derived recommendation derived from the results of this study. In all tested conditions, the ordinary least squares factor extraction method identified common factors with a high degree of efficacy. This capacity to identify common factors is independent of over-determination, ratio of sample size to observed variables, communality, and the ratio of dichotomous variables to the total number of observed variables in a data set.

Results of this study also highlight a positive relationship between sample-population agreement and sample size; this relationship is apparent in all measures of factor loading sensitivity and agreement; in the factor loading matrices derived through

principal axis and maximum likelihood methods, the amount of bias tended to diminish as sample size increases. Therefore, results of this study support a secondary recommendation for researchers to increase sample sizes as they attempt to improve estimates of factor loading parameters in the population. However, this recommendation is not applicable when ordinary least squares methods are employed.

The generalizability of this study's results and recommendations derived from them are limited by constraints imbedded in the research design. These constraints include

1. The consideration of uncorrelated factors.
2. Equating the number of factors in the sample to the number of factors in the population.
3. Selection of Pearson product moment correlation coefficients for the matrix of association.
4. The limitation of measurement levels to include continuous and dichotomous levels only.
5. The exclusive use of the varimax rotation strategy
6. The application of a .30 loading threshold to be met before an observed variable is assigned to a factor.

### **Recommendations for Future Research**

Researchers can incorporate the limiting constraints associated with this dissertation into methodological studies that extend the generalizability of conclusions and recommendations into areas that are beyond the scope of this study. For example:

1. Through incorporating varying levels of correlation among the factors as a manipulated research context, researchers can explore the impact that varying strengths of relationships among factors have on the relative advantages of tested factor extraction methods; this type of study would also yield results and recommendations that are applicable when oblique rotation methods are employed.
2. By not forcing the number of factors in the sample to be equal to the number of factors in the population, researchers can explore the influence that interactions among factor retention and factor extraction methods have on the quality of factor loading, parameter estimates.
3. As opposed to assigning variables to factors based on a .30 factor loading threshold, researchers can explore the impact that different thresholds and strategies have on the accuracy of factor loading parameter estimates.
4. Although the use of Pearson Product Moment Correlation Coefficients (PPMCC) as a matrix of association in exploratory factor analysis is well supported by empirical research (Fowler, 1987), tetrachoric correlations can be applicable when sample sizes are small (Greer, Dunlap, & Beatty, 2003). In data contexts that include mixtures of polytomous response items and continuous level variables, polyserial correlations may be more appropriate than PPMCC's (Lee, Poon, & Bentler, 1994). Through incorporating these and other matrices of association into simulation studies, researchers can explore the impact that interactions among tested factor extraction methods and matrices of association have on the quality of parameter estimates.

5. By dichotomizing variables in locations other than the mid-point of possible ranges, means, or medians, researchers can explore the manner in which differing types of dichotomies influence the quality of parameter estimates derived through tested factor extraction methods.

While maintaining many design characteristics of this dissertation, these example follow-up studies can provide a more comprehensive picture of exploratory factor analysis through which guidelines and recommendations can be derived that are applicable to a broader range of research context

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## Appendix A

### Technical Descriptions

#### Pearson Product Moment Correlation Coefficient

The Pearson product moment correlation is given by (Glass & Hopkins, 1996):

$$\rho = \sigma_{xy} / \sigma_x \sigma_y \quad (1)$$

Where:

- a)  $\sigma_{xy}$  is the covariance between variables  $x$  and  $y$ ;
- b)  $\sigma_x$  is the variance associated with  $x$ ;
- c)  $\sigma_y$  is the variance of  $y$ .

#### Spearman Rank Correlation Coefficient

Spearman rank correlation is calculated through the following equation:

$$r_{ranks} = 1 - \frac{6 \sum_i D_i^2}{n(n^2-1)} \quad (2)$$

Where  $n$  is the number of pairs, and  $D_i$  is the difference between two ranks for the  $i$ th case (Glass & Hopkins, 1996).

#### Phi Coefficient

In the simplest form, a phi coefficient for a two by two contingency table is given by (Glass & Hopkins, 1996; Greer, Dunlap, & Beatty, 2003):

$$phi = \frac{bc-ad}{\sqrt{(a+b)(c+d)(a+c)(b+d)}} \quad (3)$$

Where  $a$ ,  $b$ ,  $c$ , and  $d$  are cell counts in the contingency table.

## Appendix A (Continued)

### Tetrachoric Correlation Coefficient

The tetrachoric coefficient,  $r_{tet}$ , is defined by the following (Glass & Hopkins, 1996; Greer, Dunlap, & Beatty, 2003):

$$r_{tet} = \frac{bc-ad}{u_x u_y n^2} \quad (4)$$

Where:

- a)  $a$ ,  $b$ ,  $c$ , and  $d$  are cell frequencies;
- b) the terms  $u_x$  and  $u_y$  are “ordinates of the unit normal distribution at  $p_x$  and  $p_y$ ” (Glass & Hopkins, 1996, p. 136);
- c) the  $p$  terms represent the proportion of  $X$  and  $Y$  observations that become ones after dichotomization;
- d)  $n$  is the total number of observations (Glass & Hopkins, 1996; Greer, Dunlap, & Beatty, 2003).

### Point-Biserial Coefficient

The point-biserial correlation coefficient is given by the following formula (Glass & Hopkins, 1996):

$$r_{pb} = \frac{\bar{Y}_1 - \bar{Y}_0}{s_y} \sqrt{\frac{n_1 n_0}{n(n-1)}} \quad (5)$$

Where:

- a)  $\bar{Y}_1$  is the mean score on the continuous variable,  $X$ , for those subjects that score a 1 on the dichotomously scored variable;

### Appendix A (Continued)

- b)  $\bar{Y}_0$  is the mean value on the  $X$  variable for those who score 0 on the dichotomous variable;
- c)  $S_Y$  is the standard deviation of all  $Y$  scores;
- d) The terms  $n_1$  and  $n_0$  are the number of subjects that scored one and the number of subjects that scored zero (respectively) on the dichotomously measured variable;
- e)  $n$  is the sum of  $n_0$  and  $n_1$

### Partition Maximum Likelihood Model for Estimating Polyserial Correlation Coefficient

Let the following considerations be given:

- a)  $\Sigma_{xx}$ ,  $\Sigma_{yy}$ , and  $\Sigma_{yx}$  are functions of  $\theta$  corresponding to correlation matrices of  $(X, Y)$  and  $(Y, X)$  respectively;
- b)  $X$  is a  $r \times 1$  vector and  $Y$  is an  $s \times 1$  vector;
- c) Each vector is continuous and random with a joint distribution of  $N[0, \Sigma_0]$ ;
- d)  $Y$  is not observable; instead, values of  $Y$  are given by a random, polytomous vector,  $Z$ ; the  $a$ th element of  $Z$  is equal to  $k(a)$ ;
- e)  $\text{Pr}(k)$  is the probability of the cell  $k$ .

This probability is given by (Lee, Poon, & Bentler, 1994):

$$\text{Pr}(k) = (-1)^s \sum_{j(1)=0}^1 \cdots \sum_{j(s)=0}^1 (-1)^{\sum_{u=1}^s j(u)} \Phi_s(\alpha_{1,v(1)}, \dots, \alpha_{s,v(s)}, \Sigma_{yy}) \quad (6)$$

## Appendix A (Continued)

“The maximum likelihood estimate of  $\theta$  can be obtained by minimizing the following negative logarithm of the likelihood function of  $\theta$  (Lee, Poon, & Bentler, 1994):

$$- \sum_{k(1)=1}^{m(1)} \cdots \sum_{k(s)=1}^{m(s)} \sum_{i=1}^{n_k} \{ \log[p_1(x_{k,1})] + \log[p_x(k|x_{k,1})] \} \quad (7)$$

Where:

- a)  $m(a)$  is the number of categories associated with the  $a^{\text{th}}$  variable;
- b)  $\alpha_{a,k(a)}$  is the threshold parameter for  $\alpha_{a,1}$  and  $\alpha_{a,m(a)+1}$ ;
- c)  $p_1(x_{k,1})$  is the  $r$ -dimensional multivariate normal density function.

The multivariate normal density function is defined as:

$$p_1(x_{k,1}) = (2\pi)^{-r/2} |\Sigma_{xx}|^{-1/2} \exp\{-x'_{k,1} \Sigma_{xx}^{-1} x_{k,1} / 2\} \quad (8)$$

The conditional density function of  $Z$  given  $x_{k,1}$  is  $p_2(k|x_{k,1})$ ; this function is given by:

$$p_2(k|x_{k,1}) = (-1)^s \sum_{i(1)=0}^1 \cdots \sum_{i(s)=0}^1 (-1)^{\sum_{u=1}^s i(u)} \Phi_s(\alpha_1^*, \dots, \alpha_s^*, R^*) \quad (9)$$

Where  $R^*$  is the correlation matrix of  $Y|X$ , and  $\alpha_a^*$  is given by (Lee, Poon, & Bentler, 1994):

$$\alpha_a^* = (\alpha_{a,v(a)} - \sigma'_a \Sigma_{xx}^{-1} x_{k,1}) (\sigma_a^2 - \sigma'_a \Sigma_{xx}^{-1} \sigma_a)^{-1/2} \quad (10)$$

### Two Stage Estimation of Polyserial and Polychoric Correlation

The polyserial correlation is based on a random sample  $(X', Z_a)$  with only one polytomous variable  $Z_a$ . In this procedure, the following equalities are assumed (Lee, Poon, & Bentler, 1994):

- a)  $\alpha_a = (\alpha_{a,2}, \dots, \alpha_{a,m(a)})'$ ;



## Appendix A (Continued)

- b) the number of observations are  $n_{k(a),k(b)} \cdots n_{k\cdot}$ ;
- c) The numbers of observations correspond to  $Z_a = k(a), Z_b = k(b), Z = k$ ;
- d)  $s = 1$ .

Given the above, the negative log likelihood function is given by (Lee, Poon, & Bentler, 1994, p. 352):

$$\begin{aligned}
 &L_a(\rho_x, \alpha_a, \rho_a) \\
 &= \frac{rT}{2} \log(2\pi) + \frac{T}{2} \log |\Sigma_{xx}| \\
 &+ \frac{1}{2} \sum_{k(a)=1}^{m(a)} \sum_{j=1}^{n_{k(a)}} x'_{k(a),j} \Sigma_{xx}^{-1} x_{k(a),j} \\
 &- \sum_{k(a)=1}^{m(a)} \sum_{j=1}^{n_{k(a)}} \log \left\{ \Phi \left( \frac{\alpha_{a,k(a)+1} - \rho_a \Sigma_{xx}^{-1} x_{k(a),j}}{(1 - \rho'_a \Sigma_{xx}^{-1} \rho_a)^{1/2}} \right) \right. \\
 &\left. - \Phi \left( \frac{\alpha_{a,k(a)} - \rho_a \Sigma_{xx}^{-1} x_{k(a),j}}{(1 - \rho'_a \Sigma_{xx}^{-1} \rho_a)^{1/2}} \right) \right\} \tag{11}
 \end{aligned}$$

“ $\rho_x$  is column vector from  $r(r-1)/2$  non-duplicated lower triangle of elements  $\Sigma_{xx}$  sequentially row by row, and  $\Phi$  is the standard univariate normal distribution function” (Lee, Poon, & Bentler, p. 352).

The polychoric correlation,  $\rho_{ab}$ , of the “bivariate submodel corresponding to  $Y_a$  and  $Y_b$  can be determined through minimizing the following “negative logarithm likelihood function:”

## Appendix A (Continued)

$$\begin{aligned}
 F_{ab}(\alpha_a, \alpha_b, \rho_{ab}) &= - \sum_{k(1)=1}^{m(1)} \sum_{k(b)=1}^{m(b)} n_{k(a),k(b)} \log\{Pr(Z_a = k(a), Z_b \\
 &= k(b))\} \tag{12}
 \end{aligned}$$

Where “. . . {k(a), k(b)} are respectively the ath and bth element of an observation  $z_k$  in the s dimensional sample space” (Lee, Poon, & Bentler, 1994, p. 353).

Because each random observation in the bivariate submodel corresponds to an observation in the multivariate model, the negative logarithm likelihood function can be simplified to:

$$\begin{aligned}
 F_{ab}(\alpha_a, \alpha_b, \rho_{ab}) &= - \sum_{k(1)=1}^{m(1)} \sum_{k(b)=1}^{m(b)} n_k \log\{Pr(Z_a = k(a), Z_b = k(b))\} \tag{13}
 \end{aligned}$$

In this maximum likelihood model,  $(\tilde{\alpha}_a, \tilde{\alpha}_b, \tilde{\rho}_{ab})'$  is the vector that minimizes  $F_{ab}(\alpha_a, \alpha_b, \rho_{ab})$  (Poon, Lee, & Bentler, 1994).

### Principal Axis Factor Analysis

If R is a correlation matrix with either communality estimates or ones on the main diagonal, the following relationship can be defined (Cureton & D'Agostino, 1983):

$$EDE' = R \tag{14}$$

Where:

- a) The matrix E satisfies the condition that  $E'E = I$ ;
- b) the columns of E are eigenvectors for R;

## Appendix A (Continued)

c) D is a diagonal matrix; the elements of the main diagonal of D are eigenvalues of R

Because the above relationship holds “under permutations of the diagonal elements of D, with corresponding permutations of the columns of E and the rows of E’,” the elements of D are arranged in descending order (Cureton & D’Agostino, 1983, p. 143).

If R is of rank  $m < n$ , then D is a diagonal matrix containing zeroes in the last  $n - m$  elements; the last  $n - m$  rows of E also consists of zeroes (Cureton & D’Agostino, 1983). The symmetric, matrix D can be written as:

$$\left(D^{1/2}\right)\left(D^{1/2}\right) = D \quad (15)$$

Because  $D^{1/2}$  contains diagonal elements  $\sqrt{d_j}$ , R can be written as (Cureton & D’Agostino, 1983; Harman, 1976):

$$\left(ED^{1/2}\right)\left(ED^{1/2}\right)' = R \quad (16)$$

A Gramian matrix is symmetric and includes only real numbers among its elements. A Gramian (B) matrix must also have another real matrix (A) that when multiplied by its transpose yields the original matrix. Formally, this relationship is defined as (Cureton & D’Agostino, 1983; Harman, 1976):

$$AA' = B \quad (17)$$

When R is a Gramian, correlation matrix with communality estimates in the main diagonal, then the trace of the matrix is equal to the sum of its eigenvalues. The relationship between the principal-axes of factor loadings (F) and the reduced correlation matrix is defined as (Cureton & D’Agostino, 1983):

$$FF' = R \quad (18)$$

## Appendix A (Continued)

From the above consideration, then the matrix  $F$  can be defined as (Cureton & D'Agostino, 1983):

$$F = ED^{1/2} \quad (19)$$

The trace of  $R$  is equal to the sum of its eigenvalues. The sum of the squared elements of  $F$  is equal to the eigenvalues for that column. The relationship between the matrix of factor loadings and eigenvalues can be written as (Cureton & D'Agostino, 1983):

$$F'F = D^{1/2}E'ED^{1/2} = D^{1/2}ID^{1/2} = D \quad (20)$$

According to the basic factor analysis model, the predicted value of a given variable, in standardized form, can be given by (Cureton & D'Agostino, 1983; Harman, 1976):

$$z_j = a_{j1}F_1 + \dots + a_{jp}F_p + \dots + a_{jm}F_m \quad j = (1, 2, \dots, n) \quad (21)$$

Where the observed variables,  $j$ , are described in terms of  $m$  common factors,  $F$ . The value of a given variable  $j$  for an individual  $i$  is given by (Cureton & D'Agostino, 1983; Harman, 1976):

$$z_{ji} = \sum_{p=1}^m a_{jp}F_{pi} + u_jy_i \quad (i = 1, 2, \dots, N; j = 1, 2, \dots, n) \quad (22)$$

In this equation,

- a)  $F_{pi}$  is the value of the common factor  $p$  for individual  $i$ ;
- b)  $u_jy_i$  represents the residual error;
- c)  $a_{jp}F_{pi}$  represents the contribution of a specific factor,  $p$ , to the value of  $z_{ji}$

(Harman, 1976, p. 15).

## Appendix A (Continued)

The first step in principle axis factor analysis involves the selection of a set of factor coefficients,  $a_{j1}$ , so that the sum of the squares of these coefficients maximizes the contribution of the factor to the communality among the original variables. This relationship is given by (Harman, 1976):

$$V_1 = a_{11}^2 + a_{21}^2 + \dots + a_{n1}^2 \quad (23)$$

Where the coefficients  $a_{ji}$  are chosen to maximize the value of  $V_1$  under the following conditions (Harman, 1975, p. 136):

$$r_{jk} = \sum_{p=1}^m a_{jp} a_{kp} \quad (j, k = 1, 2, \dots, n) \quad (24)$$

$$r_{jk} = r_{kj} \quad (25)$$

$$r_{jj} = h_j^2 \quad (26)$$

In this model,  $h_j^2$  is the communality estimate for the variable  $j$  (Cureton & D'Agostino, 1983; Harman, 1976).

Under the conditions described above, Lagrange multipliers can be used to maximize the value of  $V_1$ . The following relationship highlights the manner in which these multipliers are employed (Harman, 1976):

$$2T = V_1 - \sum_{j,k=1}^n u_{jk} r_{jk} = V_1 - \sum_{j,k=1}^n \sum_{p=1}^m u_{jk} a_{jp} a_{kp} \quad (27)$$

In the above relationship,  $u_{jk}$  are the Lagrange multipliers (Harman, 1976).

By setting the partial derivative of the function  $T$  to zero for any variable  $a_{j1}$  and for the coefficients  $a_{jp}$  ( $p \neq 1$ ), then the following equation is obtained:

### Appendix A (Continued)

$$\frac{\partial T}{\partial a_{jp}} = \delta_{1p} a_{j1} - \sum_{k=1}^n u_{jk} a_{kp} = 0 \quad (28)$$

Where  $\delta_{1p} = 1$  if  $p = 1$  and  $\delta_{1p} = 0$  if  $p \neq 1$  (Harman, 1976, p. 136).

When the above equation is multiplied by  $a_{j1}$  and summed over  $j$ , the following relationship results (Harman, 1976):

$$\delta_{1p} \sum_{j=1}^n a_{j1}^2 - \sum_{j=1}^n \sum_{k=1}^n u_{jk} a_{j1} a_{kp} = 0 \quad (29)$$

According to the above relationship:

$$\sum_{j=1}^n u_{jk} a_{j1} = a_{k1} \quad (30)$$

Therefore, by setting

$$\sum_{j=1}^n a_{j1}^2 = \lambda_1 \quad (31)$$

the above equation becomes

$$\delta_{1p} \lambda_1 - \sum_{k=1}^n a_{k1} a_{kp} = 0 \quad (32)$$

Under the condition that

$$r_{jk} = \sum_{p=1}^m a_{jp} a_{kp} \quad (j, k = 1, 2, \dots, n) \quad (33)$$

the last equation becomes (Harman, 1976):

$$\sum_{k=1}^n r_{jk} a_{k1} - \lambda_1 a_{j1} = 0 \quad (34)$$

### Appendix A (Continued)

The above relationship leads to a set of equations for each value of  $j$ ; this system of equations can be written as (Harman, 1976):

$$\begin{array}{cccc}
 (h_1^2 - \lambda)a_{11} + & r_{12}a_{21} + & r_{13}a_{31} + \cdots + & r_{1n}a_{n1} = 0 \\
 r_{21}a_{11} + & (h_2^2 - \lambda)a_{21} + & r_{23}a_{31} + \cdots + & r_{2n}a_{n1} = 0 \\
 r_{31}a_{11} + & r_{32}a_{21} + & (h_3^2 - \lambda)a_{31} + \cdots + & r_{3n}a_{n1} = 0 \\
 \cdots & \cdots & \cdots & \cdots \\
 r_{n1}a_{11} + & r_{n2}a_{21} + & r_{n3}a_{31} + \cdots + & (h_n^2 - \lambda)a_{n1} = 0
 \end{array} \tag{35}$$

The maximization of  $V$  “. . . leads to the system of equations for the solution of the  $n$  unknowns  $a_{j1}$ .” For this system of equations to have a “non-trivial solution,” the determinant of the coefficients of the  $a_{j1}$  must vanish (Harman, 1976, p. 137). The following relationship illustrates this condition:

$$\begin{vmatrix}
 (h_1^2 - \lambda) & r_{12} & r_{13} & \cdots & r_{1n} \\
 r_{21} & (h_2^2 - \lambda) & r_{23} & \cdots & r_{2n} \\
 r_{31} & r_{32} & (h_3^2 - \lambda) & \cdots & r_{3n} \\
 \cdots & \cdots & \cdots & \cdots & \cdots \\
 r_{n1} & r_{n2} & r_{n3} & \cdots & (h_n^2 - \lambda)
 \end{vmatrix} = 0 \tag{36}$$

The above equation, in “determinantal form,” is a characteristic equation. The roots of this equation are real, and, when substituted for  $\lambda$ , these roots reduce the rank of the determinant to  $(n - q)$ . From these considerations, the value of  $V$  can be defined as:

$$V_1 = \sum_{j=1}^n a_{j1}^2 = \lambda_1 \tag{37}$$

Therefore,  $\lambda_1$  is the largest root of the characteristic equation (Cureton & D’Agostino, 1983; Harman, 1976). The characteristic root  $\lambda_1$  is also referred to as an eigenvalue, and this allows for the identification of coefficients of the first factor that account for the

## Appendix A (Continued)

largest amount of the total communality possible (Cureton & D'Agostino, 1983; Harman, 1976).

The pattern of factor coefficients for the first factor is determined by:

$$a_{j1} = \alpha_{j1}\sqrt{\lambda_1} / \sqrt{(\alpha_{11}^2 + \alpha_{21}^2 + \dots + \alpha_{n1}^2)} \quad (j = 1, 2, \dots, n) \quad (38)$$

The set  $a$ 's are solutions to the equations for values of  $j$ . This vector is called an eigenvector (Cureton & D'Agostino, 1983; Harman, 1976).

Once the set of factor coefficients for the first factor  $F_1$  are determined, finding a factor that accounts for the maximum amount of residual communality becomes the next step in the factor analytic process (Cureton & D'Agostino, 1983; Harman, 1976). The matrix of first factor residuals is defined as:

$$R_1 = R - \hat{R}_1 \quad (39)$$

Where

$$\hat{R}_1 = a_1 a_1' \quad (40)$$

This is an  $n \times n$  matrix of the products of the first factor coefficients (Harman, 1976).

By maximizing the value of  $V_2$ , the the coefficients for the second factor,  $F_2$ , are determined. This value is given by:

$$V_2 = a_{12}^2 + a_{22}^2 + \dots + a_{n2}^2 \quad (41)$$

The maximum value of the root of the first factor residuals is the second largest root of the original correlation matrix (Cureton & D'Agostino, 1983; Harman, 1976). By this process, the eigenvalues and associated eigenvectors are derived from the original



## Appendix A (Continued)

correlation matrix until  $m$  factors are extracted (Cureton & D'Agostino, 1983; Harman, 1976).

### Ordinary Least Squares Factor Analysis

In Ordinary Least Squares factor analysis, the relationship between factor pattern matrices,  $A$ , and implied correlation matrices,  $\hat{R}$ , is given by (Cureton & D'Agostino, 1983; Harman, 1976):

$$\hat{R} = AA' \quad (42)$$

The least squares solution can be found by “fitting  $(R - I)$  by  $(\hat{R} - H^2)$ ” (Harman, 1976, p. 176).

Where

$$H^2 = I - U^2 = \text{diag}(AA') \quad (43)$$

The diagonal matrix described in equation 43 contains communalities. Minimizing the off-diagonal residuals results in the ordinary least squares method for developing factor solutions (Cureton & D'Agostino, 1983; Harman, 1976). This minimization is given by the following expression (Harman, 1976):

$$\min_A \{ \{ [R - I] - [AA' - \text{diag}(AA')] \} \} \quad (44)$$

The function to be minimized can be written algebraically as:

$$f(A) = \sum_{k=j+1}^n \sum_{j=1}^{n-1} \left( r_{jk} - \sum_{p=1}^m a_{jp} a_{kp} \right)^2 \quad (45)$$

Through varying the values of factor loadings, the objective is to minimize this function for a specified number of factors,  $m$  (Harman, 1976). To develop a function that is

### Appendix A (Continued)

independent of the number of variables in the sample correlation matrix, Harman (1976) suggests the minimization of the root-mean-square deviation (*rms*); this is given by:

$$rms = \sqrt{2f(a)/n(n-1)} \quad (46)$$

In addition to ensuring that the fitting function is independent of the order of the correlation matrix, the ordinary least squares method requires communality estimates to be less than or equal to one; communality estimates are restricted to values between zero and one via the following condition (Harman, 1976):

$$h_j^2 = \sum_{p=1}^m a_{jp}^2 \leq 1 \quad (47)$$

The iterative process through which  $f(A)$  is minimized involves small changes in the variables, and the resulting variables replace the original ones (Harman, 1976).

Specifically, “for any row  $j$  in  $A$  an increment  $\epsilon_p$  ( $p = 1, 2, \dots, m$ ) is added to each element:

$a_{j1} + \epsilon_1, a_{j2} + \epsilon_2, \dots, a_{jp} + \epsilon_p, \dots, a_{jm} + \epsilon_m$ ” (Harman, 1976, pp. 177-178). The new factor loadings are described in the following form:

$$b_{jp} = a_{jp} + \epsilon_p \quad (j = 1, 2, \dots, m) \quad (48)$$

The reproduced correlation matrix of a given variable  $j$  with any other variable  $k$  is given by (Harman, 1976):

$$\hat{r}_{jk} = \sum_{p=1}^m a_{kp} b_{jp} \quad (49)$$

The sum of squared residuals correlations is (Harman, 1976):

### Appendix A (Continued)

$$f_j = \sum_{\substack{k=1 \\ k \neq j}}^n \left( r_{jk} - \sum_{p=1}^m a_{kp} b_{jp} \right)^2 \quad (50)$$

When the original factor loading is “separated from the incremental change,” the above equation becomes (Harman, 1976, p. 178):

$$f_j = \sum_{\substack{k=1 \\ k \neq j}}^n \left( r_{jk}^* - \sum_{p=1}^m a_{kp} \epsilon_p \right)^2 \quad (51)$$

With the incremental changes in factor loadings removed, the original residual correlations,  $r_{jk}^*$ , of variables  $k$  with a fixed  $j$  are given by (Harman, 1976):

$$r_{jk}^* = r_{jk} - \sum_{p=1}^m a_{kp} a_{jp} \quad (k = 1, 2, \dots, n; k \neq j) \quad (52)$$

The first step in determining the values of  $\epsilon$  that minimize the function  $f$  involves taking the partial derivatives of the sum of squared residual correlations with the original factor loadings separated from the incremental changes; this expression becomes:

$$\frac{\partial f_j}{\partial \epsilon_q} = 2 \sum_{\substack{k=1 \\ k \neq j}}^m \left( r_{jk}^* - \sum_{p=1}^m a_{kp} \epsilon_p \right) (-a_{kq}) \quad (q = 1, 2, \dots, m) \quad (53)$$

In the second step, these equations are set to zero; this leads to the following “implicit equations” (Harman, 1976, p. 178):

$$\sum_{p=1}^m \left( \sum_{\substack{k=1 \\ k \neq j}}^n a_{kp} a_{kq} \right) \epsilon_p = \sum_{\substack{k=1 \\ k \neq j}}^m r_{jk}^* a_{kq} \quad (q = 1, 2, \dots, m) \quad (54)$$

### Appendix A (Continued)

$$\epsilon_j A'_{j(A)_{j(\cdot)}} = r_j^0 A \quad (\epsilon_j = \epsilon_1, \epsilon_2, \dots, \epsilon_m) \quad (55)$$

In this expression,

- a)  $\epsilon_j$  is a row vector of incremental changes of the factor loadings for variable  $j$ ;
- b)  $A_{j(\cdot)}$  is the factor loading matrix with the elements in row  $j$  replaced with zeros;
- c)  $r_j^0$  is the row vector of residual correlations of  $j$  with all other variables.

The solution for the values of  $\epsilon_j$  that minimize the function  $f$  is given by (Harman, 1976):

$$\epsilon_j = r_j^0 A (A'_{j(A)_{j(\cdot)}})^{-1} \quad (56)$$

In ordinary least squares, Heywood cases, or factor solutions that imply communalities greater than one, are not considered proper solutions. Therefore, when solutions that minimize the function  $f$  result in communalities that are greater than one, the following constraint is applied (Harman, 1976):

$$\sum_{p=1}^m b_{jp}^2 \leq 1 \quad (57)$$

This constraint ensures that minimum values of the fitting function yield proper factor solutions.

### Maximum Likelihood Factor Analysis

Before using a maximum likelihood strategy to develop estimates of common factor loadings, a researcher must first use sample data to create a distribution of the elements of a covariance matrix. When samples of observations are drawn from multivariate normal distributions, the distribution function of the elements of the covariance matrix can be defined as (Harman, 1976):

### Appendix A (Continued)

$$dF = K|\Sigma|^{-\frac{1}{2}(N-1)}|S|^{-\frac{1}{2}(N-n-2)} \exp -\frac{N-1}{2} \sum_{j,k=1}^n \sigma^{jk} S_{jk} \prod_{j < k=1}^n dS_{jk} \quad (58)$$

In this expression,

- a) K is a constant involving only  $N$  and  $n$ ;
- b)  $\Sigma$  is the population covariance matrix;
- c)  $S$  is the sample covariance matrix;
- d) the elements of the inverse matrix  $\Sigma^{-1}$  are represented by  $\sigma^{jk}$  (Harman, 1976).

The  $\sigma$  in the above equation serves as the likelihood function,  $L$ , for the sample.

Given this relationship, the next portion of the process involves estimating values for  $\hat{A}$  and  $\hat{U}^2$  that satisfy the following relationship (Harman, 1976):

$$\Sigma = AA^1 + U^2 \quad (59)$$

The objective is to maximize the value of  $L$ . Where  $A$  is the matrix of common factor coefficients, and  $U^2$  is a diagonal matrix of “uniqueness” (Harman, 1976, p. 201).

This distribution function serves as a basis for a likelihood function ( $L$ ); this function is given by (Harman, 1976, p. 201):

$$\log L = -\frac{N-1}{2} (\log |\Sigma|) + \sum_{j,k=1}^n \sigma^{jk} S_{jk} + \text{function independent of } \Sigma \quad (60)$$

The maximization of  $L$  implies the minimization of the following expression (Harman, 1976):

$$-\frac{2}{N-1} \log L = \log |\Sigma| + \sum_{j,k=1}^n \sigma^{jk} S_{jk} + \text{function independent of } \Sigma \quad (61)$$

## Appendix A (Continued)

The next step in the maximum likelihood estimation process involves finding the partial derivatives with respect to  $a_{jp}$  and  $u_j$  and equating these expressions to zero; these calculations include “ $nm + n$  variables in all.” Because the “estimated factor loadings for each variable are proportional the standard deviation of that variable,” the estimation equations are scale independent (Harman, 1976, p. 201). The following equations present the results of the estimation procedures in matrix form (Harman, 1976):

$$\hat{P} = \hat{A}\hat{A}' + \hat{U}^2 \quad (62)$$

$$\hat{A} = \hat{P}R^{-1}\hat{A} \quad (63)$$

$$\hat{U}^2 = I - \text{diag}\hat{A}\hat{A}' \quad (64)$$

$$\hat{A}'R^{-1}\hat{A} = \text{a diagonal matrix} \quad (65)$$

Where  $P$  is the population correlation matrix,  $\hat{P}$  is an estimator of the population correlation matrix, and  $R$  is the sample correlation matrix with ones on the main diagonal (Harman, 1976).

Although the process described above will provide a basis for developing maximum likelihood estimates of factor loadings, Harman (1976) suggests assuming an equivalency between the sample correlation matrix and the estimator of the population correlation matrix. This assumption yields a simpler process for obtaining factor loading estimates. The expression for  $\hat{P}$  can be rewritten as (Harman, 1976):

$$AA' + U^2 = R \quad (66)$$

The following expression results from premultiplying both sides of the equation by  $A'U^{-2}$ :

$$(A'U^{-2}A + I)A' = A'U^{-2}R \quad (67)$$

## Appendix A (Continued)

We define a matrix  $J$  as:

$$J = A'U^{-2}A \quad (68)$$

With this definition, the following equation can be formed:

$$(I + J)A' = A'U^{-2}R \quad (69)$$

By simplifying the above equation, Harman (1976) describes the following expression as “amenable to an iterative method of solution” (p. 203):

$$JA' = AU^{-2}R - A' \quad (70)$$

The vector of factor loadings is given by:

$$A = (a_1 \ a_2 \ \dots \ a_m) \quad (71)$$

Where each column vector,  $m$ , can be defined as:

$$a_p = (a_{1p} \ a_{2p} \ \dots \ a_{np}) \quad (p = 1, 2, \dots, m) \quad (72)$$

The iterative process for determining the matrix of factor loadings begins with trial values of  $a_p$ . As described by Harman (1976), the products of this process are termed  $b_p$ ;  $B$  represents the complete pattern matrix, and  $V^2$  is the resultant matrix of residuals. The following are the iteration equations that yield trial values of  $a_p$ :

$$b_1 = (RU^{-2}a_1 - a_1) / \sqrt{a_1'U^{-2}(RU^{-2}a_1 - a_1)} \quad (73)$$

$$b_2 = (RU^{-2}a_2 - a_2 - b_1b_1'U^{-2}a_2) / \sqrt{a_2'U^{-2}(RU^{-2}a_2 - a_2 - b_1b_1'U^{-2}a_2)} \quad (74)$$

$$b_3 = \frac{(RU^{-2}a_3 - a_3 - b_1b_1'U^{-2}a_3 - b_2b_2'U^{-2}a_3)}{\sqrt{a_3'U^{-2}(RU^{-2}a_3 - a_3 - b_1b_1'U^{-2}a_3 - b_2b_2'U^{-2}a_3)}} \quad (75)$$

## Appendix A (Continued)

$$V^2 = I - \text{diag}BB' \quad (76)$$

While the above equations illustrate a three factor pattern, the process can be generalized to any number of factors (Harman, 1976). This process is repeated until the algorithm converges on a solution, a matrix that fulfills a prescribed degree of closeness. The resulting matrix, A, contains maximum likelihood estimates of factor loadings (Harman, 1976).



## Appendix B

### SAS IML Code for Illustrative Example

```
proc iml;

nvar = 9;
nfact = nvar - 6;
step1_A_tilde = j(nvar,nfact,0);
step2_A_tilde = round((uniform(step1_A_tilde)*(nfact-.0000001))-.5);

see_step2 = round((uniform(step1_A_tilde)*(nfact-.0000001))-.5);

test0 = round((uniform(step1_A_tilde)*(nfact-.0000001))-.5);
test1 = (uniform(step1_A_tilde)*(nfact-.0000001))-.5;
test2 = ((step1_A_tilde)*(nfact-.0000001))-.5;
test3 = round((step1_A_tilde)*(nfact-.0000001))-.5;

Altilde=step2_A_tilde;
do j=2 to nfact;
  do i=1 to nvar;
    if j<nfact then do;
      Altilde[i,j]=round(((nfact-.0000001-sum(Altilde[i,1:j-1]))
        *uniform(0))-.5);
    end;
    if j=nfact then do;
      Altilde[i,nfact]=nfact-sum(Altilde[i,1:nfact-1])-1;
    end;
  end;
end;

x=normal(Altilde);
x2=x##2;
d=j(nvar,nfact,0);
do j=1 to nfact;
  do i=1 to nvar;
    d[i,j]=(sum(x2[i,1:nfact]))##-.5;
  end;
end;

cvec=j(1,nfact,0);
do j=1 to nfact;
  cvec[1,j]=round((uniform(0)*.2999999)+.65,.1);
end;
c=j(nvar,1,1)*cvec;
c2=c##2;

bp1=j(nvar,nvar,0);
bp2=uniform(bp1);
```

## Appendix B (Continued)

```
bp3=(bp2*.2999999)+.55;
blsquare=round(diag(bp3),.1);
B1=blsquare##.5;
b3square=I(nvar)-blsquare;
B3=b3square##.5;

ones=j(nvar,nfact,1);
y=Altilde#c + d#x#((ones-c2)##.5);

k=.2;
z=j(nvar,nfact,0);
do j=1 to nfact;
  do i=1 to nvar;
    z[i,j]=((1+k)*y[i,j]*(y[i,j]+abs(y[i,j])+k))/((2+k)*(abs(y[i,j])+k));
  end;
end;

z2=z##2;
g=j(nvar,nfact,0);
do j=1 to nfact;
  do i=1 to nvar;
    g[i,j]=(sum(z2[i,1:nfact]))##-.5;
  end;
end;

Alstar=g#z;

A1=B1*Alstar;

A3star=I(nvar);
A3=B3*A3star;

R=A1*A1`+A3*A3`;

print nvar;
print nfact;
print step1_A_tilde;
print test0;
print test1;
print test2;
print test3;
print see_step2;
print Altilde;
print x;
print x2;
print bp1;
print B1;
print blsquare;
print bp3;
print B3;
print b3square;
print d;
print c;
print c2;
```

## Appendix B (Continued)

```
print y;  
print z;  
print Alstar;  
print A1;  
print A3star;  
print A3;  
print R;  
  
quit;
```

## Appendix C

### Multivariate Analyses of Variance Summary Tables

Table C1

*Multivariate Analysis of Variance for Factor Loading Sensitivity*

	<i>df</i>	$\Lambda$	<i>f</i>	$P > f$
Factor Extraction Method (M)	2 (459)	.0001	342901*	<.0001
M × No. of Factors (K)	4 (918)	.0026	4277.32*	<.0001
M × No. Observed Variables (P)	4 (918)	.0024	4454.70*	<.0001
M × Sample Size (N)	6 (918)	.8863	9.51*	<.0001
M × Community Level (H)	4 (918)	.5980	67.26*	<.0001
M × Dichotomization (D)	8 (918)	.8226	11.77*	<.0001
M × K × P	8 (918)	.0270	582.84*	<.0001
M × K × N	12 (918)	.8134	8.32*	<.0001
M × K × H	8 (918)	.6907	22.32*	<.0001
M × K × D	16 (918)	.8521	4.78*	<.0001
M × P × N	12 (918)	.8775	5.16*	<.0001
M × P × H	8 (918)	.6109	32.06*	<.0001
M × P × D	16 (918)	.9381	1.86*	.0204
M × N × H	12 (918)	.9735	1.04	.4134
M × N × D	24 (918)	.9856	0.28	.9998
M × H × D	16 (918)	.9134	2.66*	.0004

\* Significant at  $\alpha = .025$  level

**Appendix C (Continued)**

Table C2

*Repeated Measures Analysis of Variance for Factor Loading Sensitivity*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	26.477	13.239	34780.8*	<.0001	.999
Number of Variables (P)	2	41.680	20.840	54751.0*	<.0001	.993
Sample Size (N)	3	0.012	0.004	10.96*	<.0001	.044
Communality (H)	2	0.142	0.071	186.45*	<.0001	.343
Dichotomization (D)	4	0.027	0.007	17.86*	<.0001	.091
K × P	4	1.693	0.423	1111.93*	<.0001	.862
K × N	6	0.003	0.0005	1.42	.205	.011
K × H	4	0.018	0.004	11.81*	<.0001	.062
K × D	8	0.016	0.002	5.43*	<.0001	.574
P × N	6	0.003	0.0006	1.51	.174	.012
P × H	4	0.216	0.005	14.21*	<.0001	.074
P × D	8	0.004	0.0005	1.38	.204	.015
N × H	6	0.001	0.0002	0.55	.770	.004
N × D	12	0.002	0.0001	0.38	.969	.006
H × D	8	0.10	0.001	3.41*	.001	.037
Error	460	0.175	0.0003			

**Appendix C (Continued)**

Table C2

*Repeated Measures Analysis of Variance for Factor Loading Sensitivity (Continued)*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Within Subjects (with Greenhouse-Geisser asjusted Pr > F)						
Factor Extraction Method (M)	2	125.806	62.903	600815*	<.0001	.998
M × K	4	13.599	3.400	32474.6*	<.0001	.980
M × P	4	20.986	5.246	50111.1*	<.0001	.987
M × N	6	0.003	0.0005	4.42*	.0019	.010
M × H	4	0.018	0.005	44.39*	<.0001	.064
M × D	8	0.014	0.0018	16.81*	<.0001	.049
M × K × P	8	0.910	0.114	1086.34*	<.0001	.770
M × K × N	12	0.009	0.0008	7.48*	<.0001	.033
M × K × H	8	0.011	0.0014	13.09*	<.0001	.038
M × K × D	16	0.010	0.0006	5.75*	<.0001	.034
M × P × N	12	0.003	0.0002	2.21	.0269	.010
M × P × H	8	0.0171	0.0020	20.47*	<.0001	.059
M × P × D	16	0.003	0.0002	1.74	.0663	.011
M × N × H	12	0.001	0.0001	1.16	.325	.005

**Appendix C (Continued)**

Table C3

*Multivariate Analysis of Variance for General Pattern Agreement Values*

	<i>df</i>	$\Lambda$	<i>f</i>	$P > f$
Factor Extraction Method (M)	2 (459)	.0020	109246*	<.0001
M × No. of Factors (K)	4 (918)	.0349	997.42*	<.0001
M × No. Observed Variables (P)	4 (918)	.0235	1267.08*	<.0001
M × Sample Size (N)	6 (918)	.3531	104.48*	<.0001
M × Community Level (H)	4 (918)	.0962	540.34*	<.0001
M × Dichotomization (D)	8 (918)	.9779	1.29	.2451
M × K × P	8 (918)	.2009	141.24*	<.0001
M × K × N	12 (918)	.7233	13.45*	<.0001
M × K × H	8 (918)	.3087	91.78*	<.0001
M × K × D	16 (918)	.9242	2.31*	.0025
M × P × N	12 (918)	.9455	2.17*	.0113
M × P × H	8 (918)	.7637	16.56*	<.0001
M × P × D	16 (918)	.9718	0.15	.6554
M × N × H	12 (918)	.5289	28.99*	<.0001
M × N × D	24 (918)	.9920	0.15	1.0000
M × H × D	16 (918)	.9819	0.53	.9343

\* Significant at alpha = .025 level

**Appendix C (Continued)**

Table C4

*Repeated Measures Analysis of Variance for General Pattern Agreement*

<i>Source</i>	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	3.356	1.678	1053.97*	<.0001	.764
Number of Variables (P)	2	9.830	4.915	3087.36*	<.0001	.904
Sample Size (N)	3	0.547	0.182	114.48*	<.0001	.345
Communality (H)	2	4.040	2.020	1268.82*	<.0001	.795
Dichotomization (D)	4	0.018	0.005	2.98*	.0191	.018
K × P	4	5.067	1.267	795.69*	<.0001	.830
K × N	6	0.100	0.0167	10.51*	<.0001	.088
K × H	4	0.877	0.219	137.80*	<.0001	.458
K × D	8	.034	0.004	2.67*	.0072	.032
P × N	6	0.006	0.001	0.66	.6793	.006
P × H	4	0.583	0.146	91.50*	<.0001	.359
P × D	8	0.018	0.002	1.43	.1799	.017
N × H	6	0.033	0.005	3.51*	.0021	.031
N × D	12	0.002	0.000	0.09	1.00	.001
H × D	8	0.004	0.005	0.33	.9549	.004
Error	460	0.732	0.001			



**Appendix C (Continued)**

Table C4

*Repeated Measures Analysis of Variance for General Pattern Agreement (Continued)*

<i>Source</i>	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta^2_G$
Within Subjects (with Greenhouse-Geisser adjusted Pr > F)						
Factor Extraction Method (M)	2	128.078	64.039	192576*	<.0001	.991
M × K	4	4.695	1.174	3529.54*	<.0001	.819
M × P	4	11.741	2.935	8826.69*	<.0001	.919
M × N	6	0.403	0.067	201.90*	<.0001	.280
M × H	4	2.414	0.603	1814.89*	<.0001	.699
M × D	8	0.003	0.000	0.99	.4209	.002
M × K × P	8	0.758	0.095	284.98*	<.0001	.422
M × K × N	12	0.026	0.002	6.50*	<.0001	.024
M × K × H	8	0.504	0.063	189.60*	<.0001	.327
M × K × D	16	0.184	0.001	3.47*	.0003	.017
M × P × N	12	0.011	0.001	2.80*	.0078	.011
M × P × H	8	0.022	0.003	8.49*	<.0001	.021
M × P × D	16	0.007	0.000	1.38	.1950	.007
M × N × H	12	0.229	0.019	57.33*	<.0001	.180
M × N × D	24	0.001	0.000	0.10	1.000	.001
M × H × D	16	0.004	0.000	0.71	.6973	.003
Error (Method)	920	0.306	0.000			

\* Significant at the alpha = .025 level

**Appendix C (Continued)**

Table C5

*Multivariate Analysis of Variance for Per Element Pattern Agreement*

	df	$\Lambda$	f	P > f
Factor Extraction Method (M)	2 (459)	.0085	26828.4*	<.0001
M × No. of Factors (K)	4 (918)	.0157	1599.46*	<.0001
M × No. Observed Variables (P)	4 (918)	.0888	540.51*	<.0001
M × Sample Size (N)	6 (918)	.3154	119.42*	<.0001
M × Community Level (H)	4 (918)	.1318	402.65*	<.0001
M × Dichotomization (D)	8 (918)	.9611	2.30*	.0192
M × K × P	8 (918)	.1220	213.70*	<.0001
M × K × N	12 (918)	.7942	9.34*	<.0001
M × K × H	8 (918)	.6730	25.12*	<.0001
M × K × D	16 (918)	.8355	5.39*	<.0001
M × P × N	12 (918)	.8618	5.91*	<.0001
M × P × H	8 (918)	.6102	32.14*	<.0001
M × P × D	16 (918)	.9431	1.71	.0403
M × N × H	12 (918)	.5823	23.75*	<.0001
M × N × D	24 (918)	.9940	0.12	1.0000
M × H × D	16 (918)	0.9336	2.00*	.0107

\* Significant at alpha = .025 level

**Appendix C (Continued)**

Table C6

*Repeated Measures Analysis of Variance for Per Element Pattern Agreement*

<i>Source</i>	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	26.148	13.074	12137.8*	<.0001	.978
Number of Variables (P)	2	0.285	0.142	132.18*	<.0001	.330
Sample Size (N)	3	0.338	0.113	104.67*	<.0001	.370
Communality (H)	2	1.293	0.646	600.08*	<.0001	.691
Dichotomization (D)	4	0.017	0.004	4.01*	.0033	.029
K × P	4	3.874	0.968	899.10*	<.0001	.870
K × N	6	0.120	0.020	18.540*	<.0001	.172
K × H	4	0.417	0.104	96.70*	<.0001	.419
K × D	8	0.035	0.004	4.07*	.0001	.057
P × N	6	0.027	0.004	4.16*	.0004	.044
P × H	4	0.106	0.026	25.54*	<.0001	.155
P × D	8	0.030	0.004	3.43*	.0007	.049
N × H	6	0.059	0.010	9.10*	<.0001	.092
N × D	12	0.001	0.000	0.05	1.000	.001
H × D	8	0.020	0.002	2.33*	0.019	.033
Error	460	0.495	0.001			

**Appendix C (Continued)**

Table C6

*Repeated Measures Analysis of Variance for Per Element Pattern Agreement (Continued)*

<i>Source</i>	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta^2_G$
Within Subjects (with Greenhouse-Geisser adjusted Pr > F)						
Factor Extraction Method (M)	2	8.235	4.118	45801.9*	<.0001	.934
M × K	4	4.122	1.030	11461.9*	<.0001	.877
M × P	4	0.641	0.160	1781.58*	<.0001	.526
M × N	6	0.140	0.233	259.35*	<.0001	.195
M × H	4	0.438	0.109	1219.6*	<.0001	.431
M × D	8	0.002	0.000	2.86*	.0178	.003
M × K × P	8	0.521	0.065	724.05*	<.0001	.474
M × K × N	12	0.003	0.000	2.67*	.0100	.004
M × K × H	8	0.028	0.003	38.25*	<.0001	.045
M × K × D	16	0.121	0.001	8.43*	<.0001	.020
M × P × N	12	0.002	0.000	1.96	.0622	.004
M × P × H	8	0.018	0.002	24.36*	<.0001	.029
M × P × D	16	0.003	0.000	2.14*	.0226	.005
M × N × H	12	0.048	0.004	44.65*	<.0001	.077
M × N × D	24	0.000	0.000	0.09	1.0000	.000
M × H × D	16	0.004	0.000	3.13*	.0009	.007
Error (Method)	920	0.083	0.000			

\* Significant at the alpha = .025 level

**Appendix C (Continued)**

Table C7

*Multivariate Analysis of Variance Summary for Total Pattern Agreement*

	<i>Df</i>	<i>Λ</i>	<i>f</i>	<i>P &gt; f</i>
Factor Extraction Method (M)	2 (459)	.7290	85.36*	<.0001
M × No. of Factors (K)	4 (918)	.5764	72.79*	<.0001
M × No. Observed Variables (P)	4 (918)	.6269	60.35*	<.0001
M × Sample Size (N)	6 (918)	.9184	6.65*	<.0001
M × Communality Level (H)	4 (918)	.9806	2.26	.0609
M × Dichotomization (D)	8 (918)	.9541	2.73*	.0057
M × K × P	8 (918)	.4592	54.59*	<.0001
M × K × N	12 (918)	.8543	6.27*	<.0001
M × K × H	8 (918)	.9592	2.41	.0141
M × K × D	16 (918)	.9094	2.79*	.0002
M × P × N	12 (918)	.8778	5.15*	<.0001
M × P × H	8 (918)	.9683	1.86	.0626
M × P × D	16 (918)	.9250	2.28*	.0028
M × N × H	12 (918)	.9856	0.56	.8782
M × N × D	24 (918)	.9679	0.63	.9165
M × H × D	16 (918)	.9455	1.63	.0552

\* Significant at alpha = .025 level

**Appendix C (Continued)**

Table C8

*Repeated Measures Analysis of Variance for Total Pattern Agreement*

<i>Source</i>	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	<.0001	<.0001	167.05*	<.0001	.195
Number of Variables (P)	2	<.0001	<.0001	136.36*	<.0001	.165
Sample Size (N)	3	<.0001	<.0001	12.55*	<.0001	.026
Communality (H)	2	<.0001	<.0001	3.12	.0321	.005
Dichotomization (D)	4	<.0001	<.0001	4.49*	.0015	.013
K × P	4	<.0001	<.0001	133.20*	<.0001	.278
K × N	6	<.0001	<.0001	11.90*	<.0001	.049
K × H	4	<.0001	<.0001	3.90*	.0040	.011
K × D	8	<.0001	<.0001	4.64*	<.0001	.026
P × N	6	<.0001	<.0001	9.56*	<.0001	.040
P × H	4	<.0001	<.0001	2.68	.0311	.008
P × D	8	<.0001	<.0001	3.59*	.0005	.020
N × H	6	<.0001	<.0001	0.12	.9943	.000
N × D	12	<.0001	<.0001	0.25	.9945	.002
H × D	8	<.0001	<.0001	2.27*	.0220	.013
Error	460	<.0001	<.0001			

**Appendix C (Continued)**

Table C8

*Repeated Measures Analysis of Variance for Total Pattern Agreement (Continued)*

<i>Source</i>	<i>Df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta^2_G$
Within Subjects (with Greenhouse-Geisser adjusted Pr > F)						
Factor Extraction Method (M)	2	<.0001	<.0001	170.43*	<.0001	.198
M × K	4	<.0001	<.0001	167.25*	<.0001	.326
M × P	4	<.0001	<.0001	136.17*	<.0001	.283
M × N	6	<.0001	<.0001	12.55*	<.0001	.052
M × H	4	<.0001	<.0001	3.50	.0310	.010
M × D	8	<.0001	<.0001	4.49*	.0014	.025
M × K × P	8	<.0001	<.0001	133.37*	<.0001	.436
M × K × N	12	<.0001	<.0001	11.91*	<.0001	.094
M × K × H	8	<.0001	<.0001	3.89*	.0041	.022
M × K × D	16	<.0001	<.0001	4.64*	<.0001	.051
M × P × N	12	<.0001	<.0001	9.56*	<.0001	.077
M × P × H	8	<.0001	<.0001	2.71	.0295	.015
M × P × D	16	<.0001	<.0001	3.60*	.0005	.040
M × N × H	12	<.0001	<.0001	0.12	.9944	.001
M × N × D	24	<.0001	<.0001	0.26	.9944	.004
M × H × D	16	<.0001	<.0001	2.27*	.0217	.025
Error (Method)	920	.0001	<.0001			

\* Significant at the alpha = .025 level

**Appendix C (Continued)**

Table C9

*Multivariate Analysis of Variance for Congruence*

	<i>df</i>	<i>Λ</i>	<i>f</i>	<i>P &gt; f</i>
Factor Extraction Method (M)	2 (459)	0.0262	8533.92*	<.0001
M × No. of Factors (K)	4 (918)	0.0818	572.86*	<.0001
M × No. Observed Variables (P)	4 (918)	0.5824	71.24*	<.0001
M × Sample Size (N)	6 (918)	0.5961	45.17*	<.0001
M × Community Level (H)	4 (918)	0.2569	223.29*	<.0001
M × Dichotomization (D)	8 (918)	0.9199	4.89*	<.0001
M × K × P	8 (918)	0.6105	32.11*	<.0001
M × K × N	12 (918)	0.3077	61.40*	<.0001
M × K × H	8 (918)	0.8003	13.52*	<.0001
M × K × D	16 (918)	0.8792	3.81*	<.0001
M × P × N	12 (918)	0.8819	4.96*	<.0001
M × P × H	8 (918)	0.9306	4.20*	<.0001
M × P × D	16 (918)	0.9180	2.51*	.0009
M × N × H	12 (918)	0.9592	1.61	.0835
M × N × D	24 (918)	0.9954	0.09	1.0000
M × H × D	16 (918)	0.9171	2.54*	.0008

\* Significant at alpha = .025 level



**Appendix C (Continued)**

Table C10

*Repeated Measures Analysis of Variance for Congruence*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	71.713	35.856	7026.19*	<.0001	.966
Number of Variables (P)	2	2.618	1.309	256.53*	<.0001	.506
Sample Size (N)	3	3.626	1.209	236.82*	<.0001	.586
Communality (H)	2	0.417	0.208	40.83*	<.0001	.140
Dichotomization (D)	4	0.152	0.038	7.46*	<.0001	.056
K × P	4	1.745	0.436	85.49*	<.0001	.405
K × N	6	0.333	0.055	10.89*	<.0001	.115
K × H	4	0.563	0.141	27.60*	<.0001	.180
K × D	8	0.242	0.030	5.94*	<.0001	.086
P × N	6	0.012	0.002	0.40	.8799	.005
P × H	4	0.419	0.010	2.05	.0859	.016
P × D	8	0.077	0.009	1.89	.0594	.029
N × H	6	0.185	0.030	7.46*	<.0001	.067
N × D	12	0.006	0.000	0.10	1.0000	.002
H × D	8	0.211	0.026	5.17*	<.0001	.076
Error	460	2.347	0.005			

**Appendix C (Continued)**

Table C10

*Repeated Measures Analysis of Variance for Congruence (Continued)*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta^2_G$
Within Subjects (with Greenhouse-Geisser adjusted Pr > F)						
Factor Extraction Method (M)	2	2.788	1.393	6061.00*	<.0001	.521
M × K	4	0.247	0.062	268.64*	<.0001	.088
M × P	4	0.034	0.009	37.61*	<.0001	.013
M × N	6	0.076	0.013	55.10*	<.0001	.029
M × H	4	0.557	0.139	605.91*	<.0001	.179
M × D	8	0.003	0.000	1.71	.1352	.001
M × K × P	8	0.021	0.003	11.66*	<.0001	.008
M × K × N	12	0.368	0.031	133.46*	<.0001	.126
M × K × H	8	0.015	0.002	8.28*	<.0001	.006
M × K × D	16	0.014	0.001	3.86*	<.0001	.005
M × P × N	12	0.018	0.001	6.47*	<.0001	.007
M × P × H	8	0.006	0.001	3.19*	.0088	.002
M × P × D	16	0.006	0.000	1.76	.0694	.002
M × N × H	12	0.004	0.000	1.38	.2097	.001
M × N × D	24	0.001	0.00	0.13	1.0000	.000
M × H × D	16	0.010	0.001	2.85*	.0023	.004
Error (Method)	920	0.212	0.000			

\* Significant at the alpha = .025 level

**Appendix C (Continued)**

Table C11

*Multivariate Analysis of Variance for Factor Score Correlations*

	<i>df</i>	<i>Λ</i>	<i>F</i>	<i>P &gt; f</i>
Factor Extraction Method (M)	2 (400)	.0218	8978.7*	<.0001
M × No. of Factors (K)	4 (800)	.0226	1129.49*	<.0001
M × No. Observed Variables (P)	4 (800)	.0698	556.90*	<.0001
M × Sample Size (N)	6 (800)	.5146	52.54*	<.0001
M × Communality Level (H)	4 (800)	.3978	117.08*	<.0001
M × Dichotomization (D)	8 (800)	.9486	2.67*	.0067
M × K × P	6 (800)	.0712	366.49*	<.0001
M × K × N	12 (800)	.6312	17.24*	<.0001
M × K × H	8 (800)	.6205	26.95*	<.0001
M × K × D	16 (800)	.9029	2.62*	.0005
M × P × N	12 (800)	.8982	3.67*	<.0001
M × P × H	8 (800)	.5600	33.63*	<.0001
M × P × D	16 (800)	.9544	1.18	.2777
M × N × H	12 (800)	.9209	2.80*	.0009
M × N × D	24 (800)	.9917	0.14	1.0000
M × H × D	16 (800)	.9704	0.76	.7346

\* Significant at alpha = .025 level

**Appendix C (Continued)**

Table C12

*Repeated Measures Analysis of Variance for Factor Score Correlations*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	25.397	19.198	3046.54*	<.0001	.924
Number of Variables (P)	2	3.278	1.639	378.36*	<.0001	.601
Sample Size (N)	3	1.498	0.499	115.29*	<.0001	.408
Communality (H)	2	0.414	0.207	47.75*	<.0001	.160
Dichotomization (D)	4	0.019	0.005	1.08	.3666	.008
K × P	3	12.752	4.251	981.17*	<.0001	.854
K × N	6	0.228	0.038	8.78*	<.0001	.095
K × H	4	1.041	0.260	60.09*	<.0001	.324
K × D	8	0.090	0.113	2.60*	.0088	.040
P × N	6	0.019	0.003	0.74	.6208	.009
P × H	4	0.381	0.095	21.99*	<.0001	.149
P × D	8	0.082	0.010	2.38*	.0162	.037
N × H	6	0.113	0.019	4.34*	.0003	.049
N × D	12	0.003	0.000	0.06	1.0000	.001
H × D	8	0.120	0.015	3.48*	.0007	.053
Error	401	1.737	0.004			

**Appendix C (Continued)**

Table C12

*Repeated Measures Analysis of Variance for Factor Score Correlations (Continued)*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta^2_G$
Within Subjects (with Greenhouse-Geisser adjusted Pr > F)						
Factor Extraction Method (M)	2	7.841	3.921	7246.31*	<.0001	.783
M × K	4	4.341	1.085	2005.77*	<.0001	.666
M × P	4	3.493	0.873	1613.95*	<.0001	.617
M × N	6	0.085	0.014	26.17*	<.0001	.038
M × H	4	0.110	0.027	50.68*	<.0001	.048
M × D	8	0.010	0.001	2.45	.0291	.005
M × K × P	6	3.724	0.621	1147.13*	<.0001	.632
M × K × N	12	0.077	0.006	11.83*	<.0001	.034
M × K × H	8	0.089	0.011	20.57*	<.0001	.039
M × K × D	16	0.021	0.001	2.40*	.0069	.009
M × P × N	12	0.013	0.001	2.01	.0425	.006
M × P × H	8	0.275	0.034	63.62*	<.0001	.112
M × P × D	16	0.006	0.000	0.72	1.0000	.002
M × N × H	12	0.023	0.002	3.61*	.0004	.011
M × N × D	24	0.001	0.000	0.08	1.0000	.000
M × H × D	16	0.006	0.00	0.76	.6801	.003
Error (Method)	802	0.434	0.001			

\* Significant at the alpha = .025 level

**Appendix C (Continued)**

Table C13

*Multivariate Analysis of Variance for Estimates of Loading Bias*

	df	$\Lambda$	F	P > f
Factor Extraction Method (M)	2 (459)	.0018	127099*	<.0001
M $\times$ No. of Factors (K)	4 (918)	.0114	1923.23*	<.0001
M $\times$ No. Observed Variables (P)	4 (918)	.1339	397.61*	<.0001
M $\times$ Sample Size (N)	6 (918)	.9765	1.83	.0900
M $\times$ Communality Level (H)	4 (918)	.0182	1469.72*	<.0001
M $\times$ Dichotomization (D)	8 (918)	.9366	3.82*	.0002
M $\times$ K $\times$ P	8 (918)	.3271	85.88*	<.0001
M $\times$ K $\times$ N	12 (918)	.9207	3.22*	.0001
M $\times$ K $\times$ H	8 (918)	.1148	223.97*	<.0001
M $\times$ K $\times$ D	16 (918)	.9424	1.73	.0369
M $\times$ P $\times$ N	12 (918)	.9775	0.88	.5709
M $\times$ P $\times$ H	8 (918)	.8786	7.67*	<.0001
M $\times$ P $\times$ D	16 (918)	.9147	2.67*	.0005
M $\times$ N $\times$ H	12 (918)	.9888	0.43	.9516
M $\times$ N $\times$ D	24 (918)	.9786	0.42	.9944
M $\times$ H $\times$ D	16 (918)	.9116	2.72*	.0003

\* Significant at alpha = .025 level

**Appendix C (Continued)**

Table C14

*Repeated Measures Analysis of Variance for Estimates of Loading Bias*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	1.826	0.913	8596.94*	<.0001	.966
Number of Variables (P)	2	< 0.001	< 0.001	1.48	.2290	.005
Sample Size (N)	3	0.039	0.013	122.94*	<.0001	.382
Communality (H)	2	0.441	0.220	2076.21*	<.0001	.874
Dichotomization (D)	4	0.006	0.002	15.63*	<.0001	.095
K × P	4	0.104	0.026	244.87*	<.0001	.621
K × N	6	0.010	0.002	15.62*	<.0001	.136
K × H	4	0.193	0.048	454.50*	<.0001	.753
K × D	8	0.004	< 0.001	4.91*	<.0001	.062
P × N	6	< 0.001	< 0.001	0.85	.5323	.008
P × H	4	< 0.001	< 0.001	0.61	.6567	.004
P × D	8	0.002	< 0.001	2.03	.0410	.026
N × H	6	0.001	< 0.001	1.74	.1098	.017
N × D	12	< 0.001	< 0.001	0.09	1.000	.002
H × D	8	0.003	< 0.001	3.58*	.0005	.046
Error	460	0.049	< 0.001			

**Appendix C (Continued)**

Table C14

*Repeated Measures Analysis of Variance for Estimates of Loading Bias (Continued)*

<i>Source</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>P</i>	$\eta^2_G$
Within Subjects (with Greenhouse-Geisser adjusted Pr > F)						
Factor Extraction Method (M)	2	7.964	3.982	252532*	<.0001	.992
M × K	4	1.237	0.309	19610.8*	<.0001	.951
M × P	4	.0925	0.231	1466.67*	<.0001	.594
M × N	6	.0001	< 0.001	1.64	.1778	.002
M × H	4	0.492	0.123	7807.54*	<.0001	.886
M × D	8	< 0.001	< 0.001	4.61*	.0011	.009
M × K × P	8	0.022	0.003	176.98*	<.0001	.261
M × K × N	12	< 0.001	< 0.001	1.98	.0666	.006
M × K × H	8	0.063	0.008	501.09*	<.0001	.499
M × K × D	16	0.001	< 0.001	2.38*	.0157	.009
M × P × N	12	< 0.001	< 0.001	0.36	.9033	.001
M × P × H	8	< 0.001	< 0.001	4.01*	.0032	.008
M × P × D	16	0.001	< 0.001	3.40*	.0008	.013
M × N × H	12	< 0.001	< 0.001	0.42	.8693	.001
M × N × D	24	< 0.001	< 0.001	0.17	.9994	.001
M × H × D	16	< 0.001	< 0.001	1.85	.0644	.007
Error (Method)	920	0.014	< 0.001			

\* Significant at the alpha = .025 level



**Appendix C (Continued)**

Table C15

*Multivariate Analysis of Variance for Factor Loading RMSE*

	<i>df</i>	$\Lambda$	<i>F</i>	<i>P &gt; f</i>
Factor Extraction Method (M)	2 (459)	.0269	8274.65*	<.0001
M × No. of Factors (K)	4 (918)	.1677	330.94*	<.0001
M × No. Observed Variables (P)	4 (918)	.6742	50.01*	<.0001
M × Sample Size (N)	6 (918)	.3964	90.00*	<.0001
M × Community Level (H)	4 (918)	.2156	264.72*	<.0001
M × Dichotomization (D)	8 (918)	.9128	5.35*	<.0001
M × K × P	8 (918)	.6137	31.73*	<.0001
M × K × N	12 (918)	.3581	51.33*	<.0001
M × K × H	8 (918)	.6972	22.68*	<.0001
M × K × D	16 (918)	.7844	7.41*	<.0001
M × P × N	12 (918)	.9955	0.17	.9993
M × P × H	8 (918)	.7143	21.02*	<.0001
M × P × D	16 (918)	.8908	3.42*	<.0001
M × N × H	12 (918)	.9641	1.41	.1543
M × N × D	24 (918)	.9932	0.13	1.0000
M × H × D	16 (918)	.8451	5.04*	<.0001

\* Significant at alpha = .025 level

**Appendix C (Continued)**

Table C16

*Repeated Measures Analysis of Variance for Factor Loading RMSE*

Source	df	SS	MS	F	P	$\eta_G^2$
Between Subjects						
Number of Factors (K)	2	1.521	0.760	2932.62*	<.0001	.905
Number of Variables (P)	2	0.037	0.019	71.76*	<.0001	.190
Sample Size (N)	3	0.134	0.045	172.94*	<.0001	.458
Communality (H)	2	1.256	0.628	2422.57*	<.0001	.888
Dichotomization (D)	4	0.009	0.002	9.05*	<.0001	.056
K × P	4	0.218	0.054	209.92*	<.0001	.578
K × N	6	< 0.001	< 0.001	0.13	.9929	.001
K × H	4	0.568	0.142	548.08*	<.0001	.781
K × D	8	0.021	0.002	9.99*	<.0001	.115
P × N	6	0.003	< 0.001	1.88	.0824	.018
P × H	4	0.007	0.002	6.69*	<.0001	.042
P × D	8	0.005	< 0.001	2.51	.0112	.032
N × H	6	0.003	< 0.001	1.95	.0713	.019
N × D	12	< 0.001	< 0.001	0.06	1.0000	.001
H × D	8	0.014	0.002	6.73*	<.0001	.081
Error	460	0.119	< 0.001			

**Appendix C (Continued)**

Table C16

*Repeated Measures Analysis of Variance for Factor Loading RMSE (Continued)*

Source	df	SS	MS	F	P	$\eta^2_G$
Within Subjects (with Greenhouse-Geisser adjusted Pr > F)						
Factor Extraction Method (M)	2	0.134	0.067	1559.4*	<.0001	.458
M × K	4	0.013	0.003	76.32*	<.0001	.076
M × P	4	0.004	0.001	25.47*	<.0001	.027
M × N	6	0.035	0.006	137.08*	<.0001	.182
M × H	4	0.058	0.014	335.37*	<.0001	.267
M × D	8	0.003	< 0.001	8.49*	<.0001	.018
M × K × P	8	0.016	0.002	46.71*	<.0001	.092
M × K × N	12	0.005	< 0.001	8.90*	<.0001	.028
M × K × H	8	0.009	0.001	25.08*	<.0001	.052
M × K × D	16	0.008	<0.001	12.00*	<.0001	.049
M × P × N	12	< 0.001	< 0.001	0.13	.9939	<.001
M × P × H	8	0.003	< 0.001	8.41*	<.0001	.018
M × P × D	16	0.003	< 0.001	4.13*	<.0001	.018
M × N × H	12	< 0.001	< 0.001	0.52	.8004	.002
M × N × D	24	< 0.001	< 0.001	0.07	1.0000	<.001
M × H × D	16	0.005	< 0.001	7.27*	<.0001	.031
Error (Method)	920	0.040	< 0.001			

\* Significant at the alpha = .025 level

## Appendix D

### Box and Whisker Plots

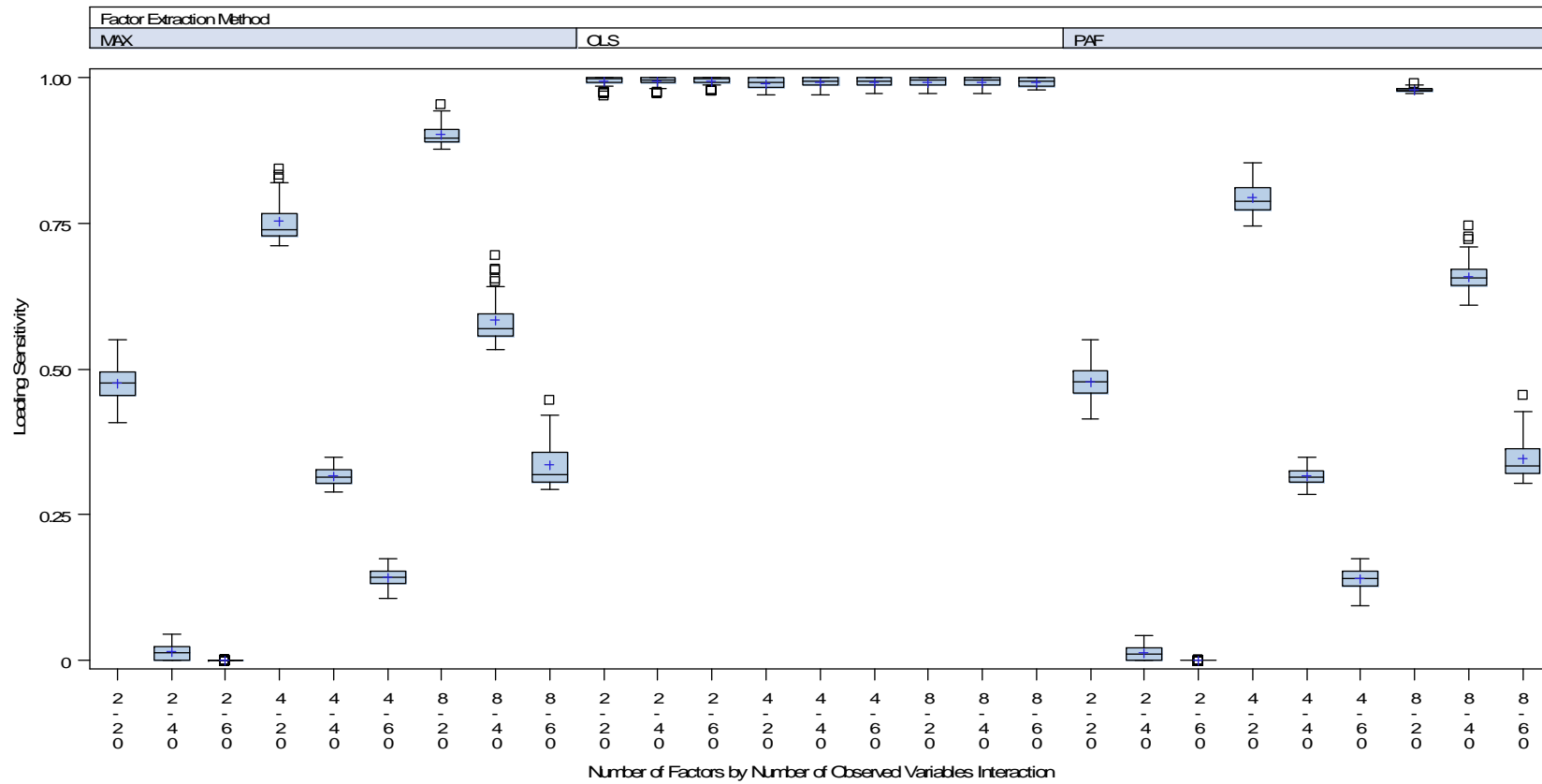


Figure D1. Factor loading sensitivity by the interaction between the number of factors and number of observed variables

### Appendix D (Continued)

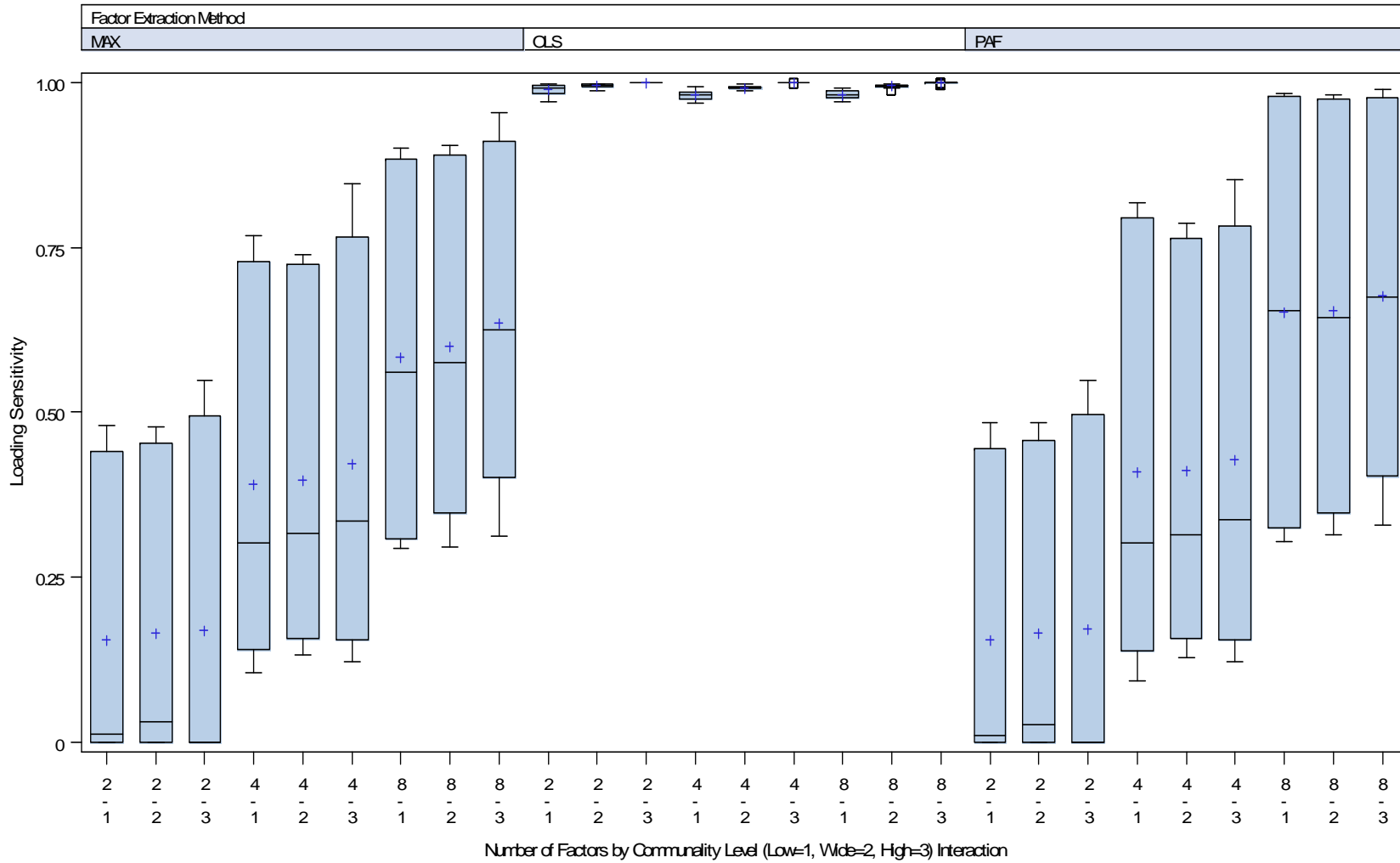
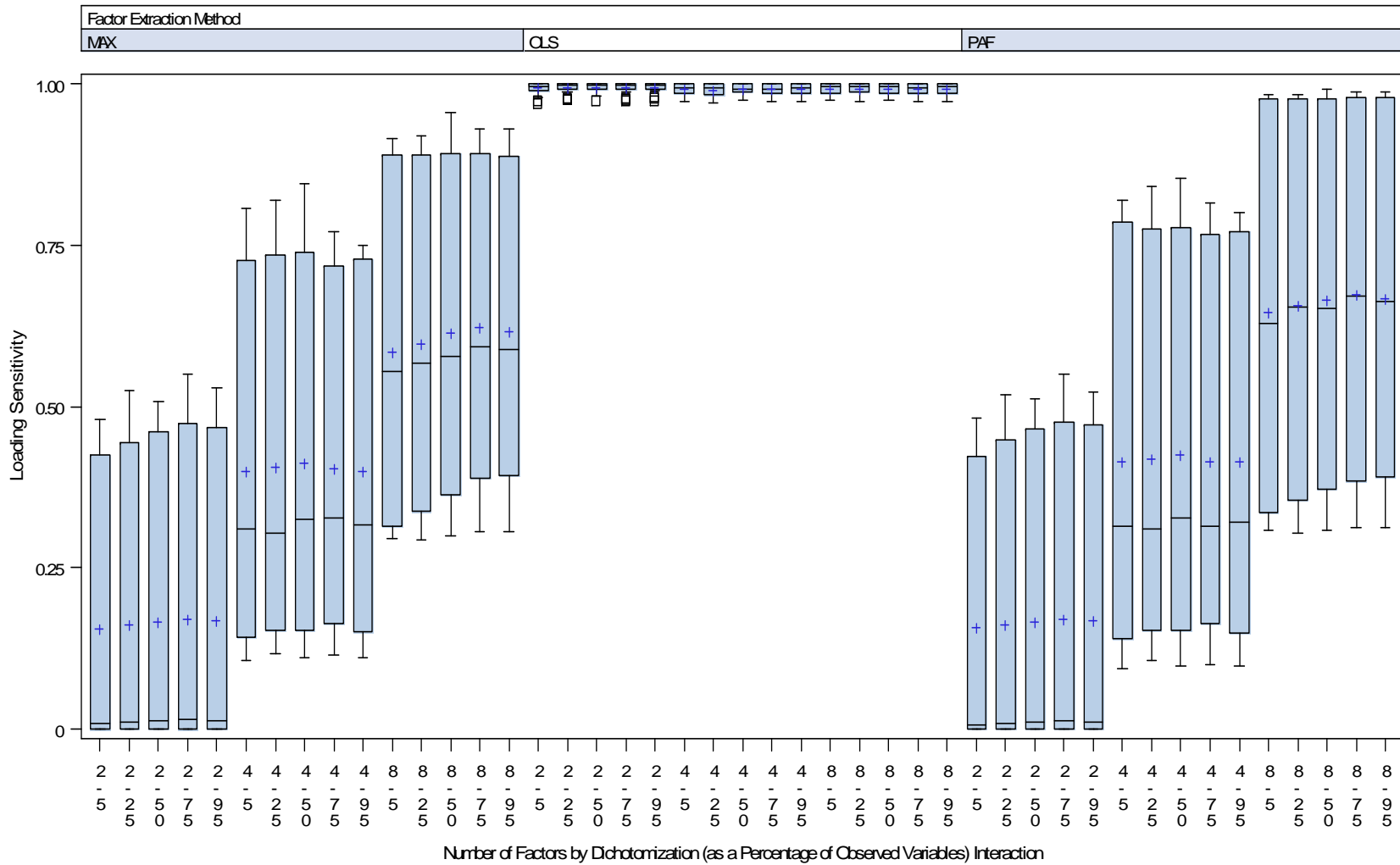


Figure D2. Factor loading sensitivity by the interaction between the number of factors and communality level

### Appendix D (Continued)



*Figure D3.* Factor loading sensitivity by the interaction between the number of factors and dichotomization

### Appendix D (Continued)

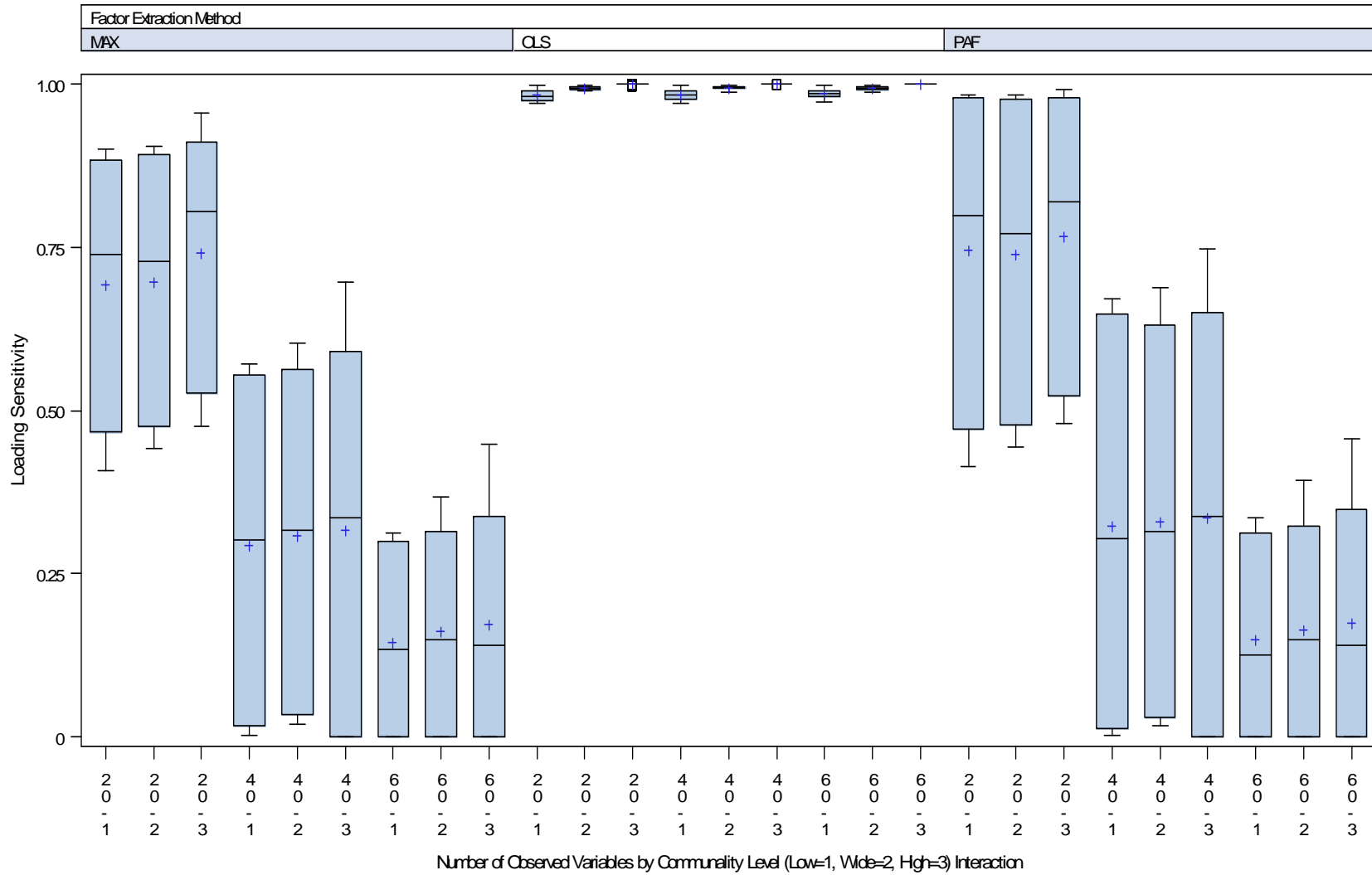


Figure D4. Factor loading sensitivity by the interaction between the number of observed variables by community level

### Appendix D (Continued)

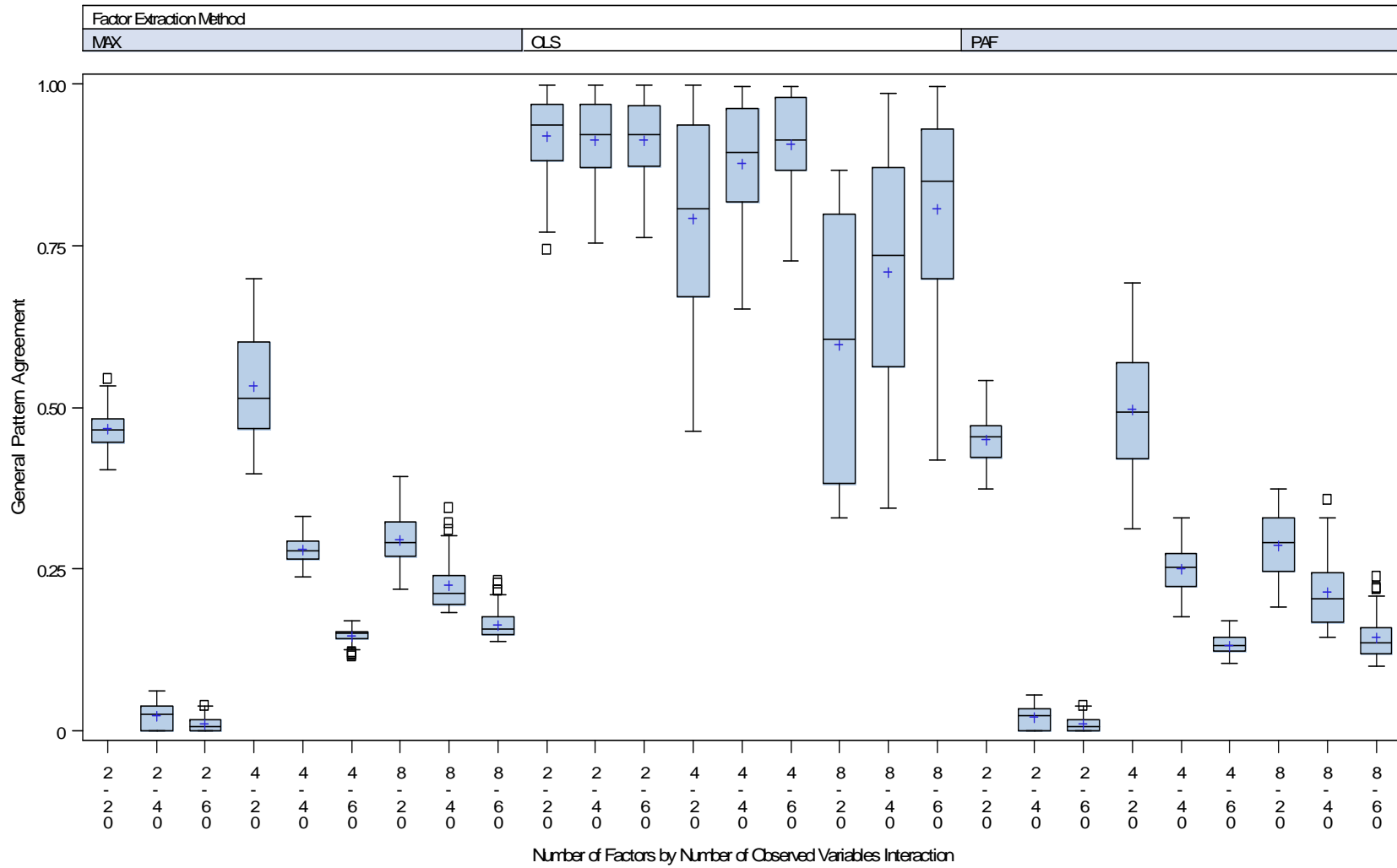


Figure D5. General factor pattern agreement by the interaction between the number of factors and observed variables



### Appendix D (Continued)

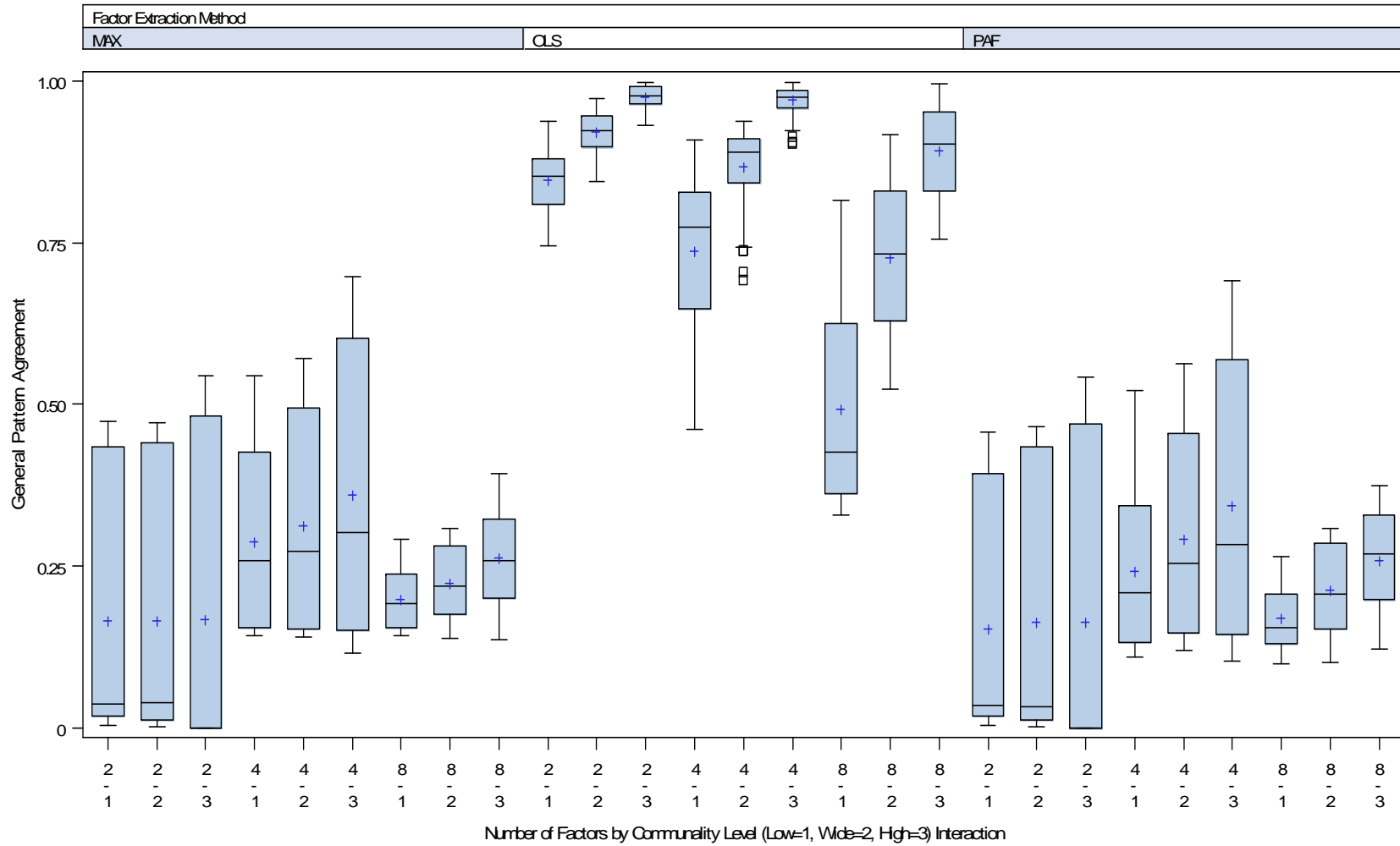


Figure D6. General factor pattern agreement by the interaction between the number of factors and communality level

### Appendix D (Continued)

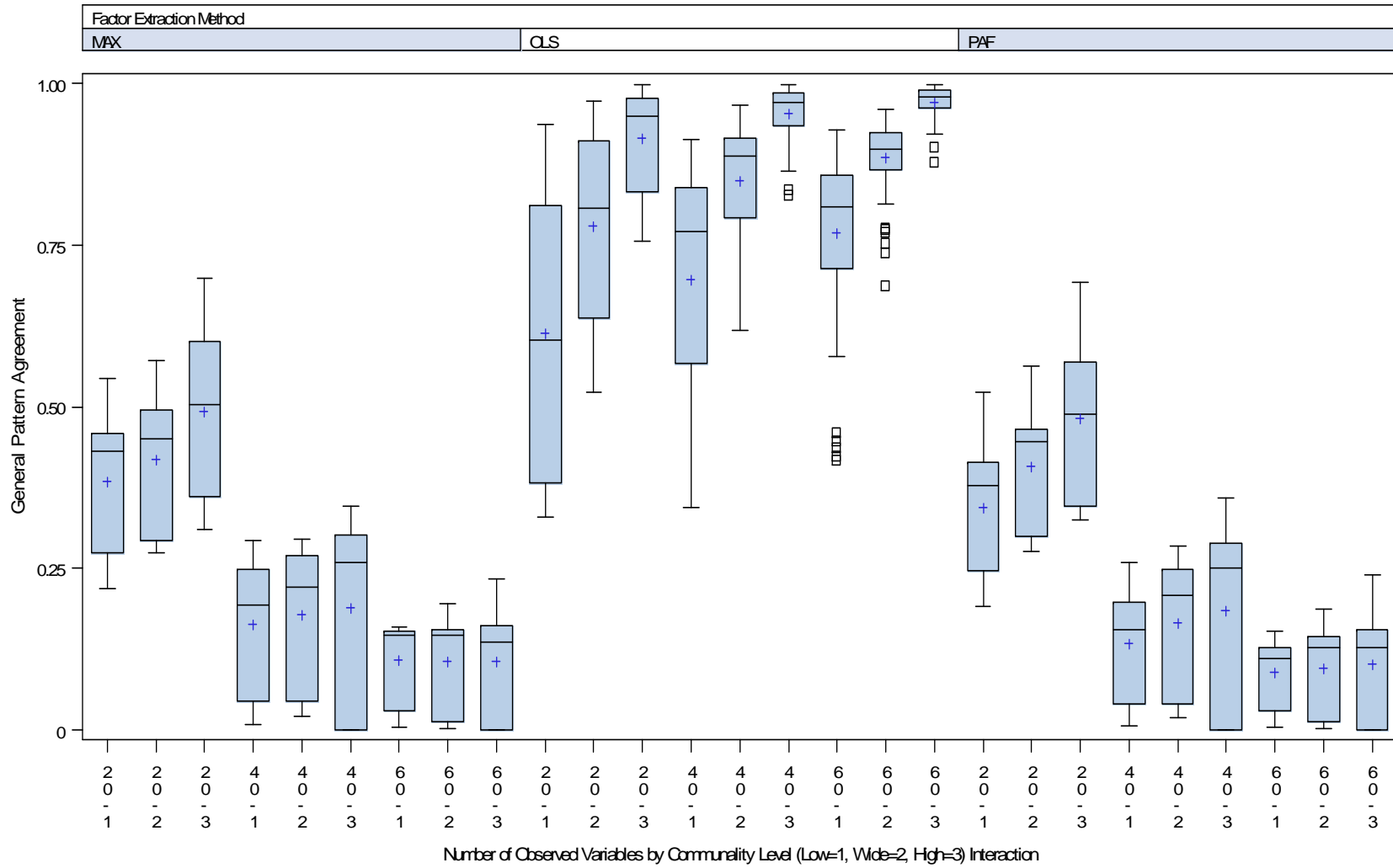


Figure D7. General factor pattern agreement by the interaction between the number of observed variables and community level

### Appendix D (Continued)

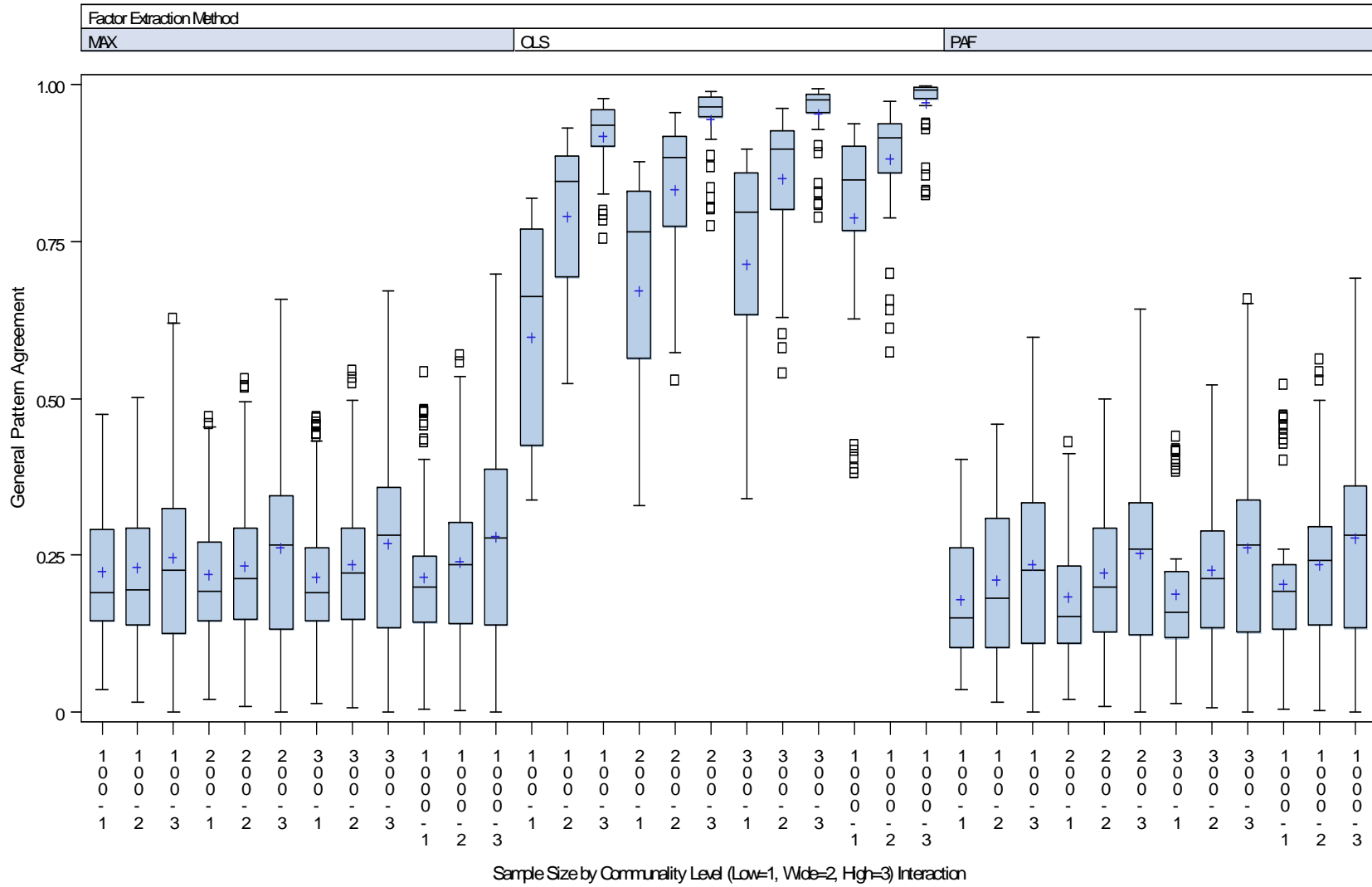
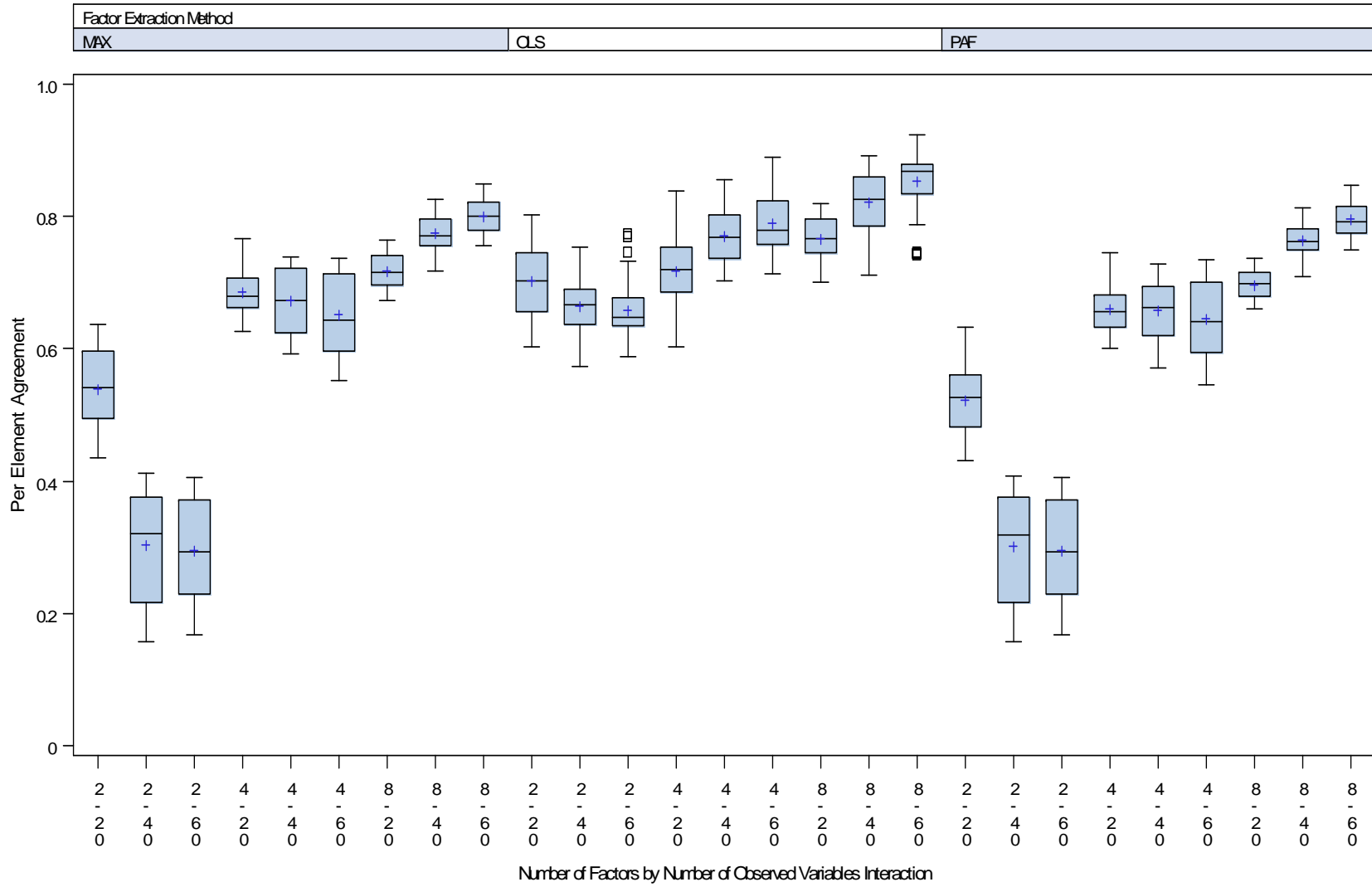


Figure D8. General factor pattern agreement by the interaction between sample size and communality level

### Appendix D (Continued)



*Figure D9.* Per element factor pattern agreement by the interaction between number of factors and number of observed variables

### Appendix D (Continued)

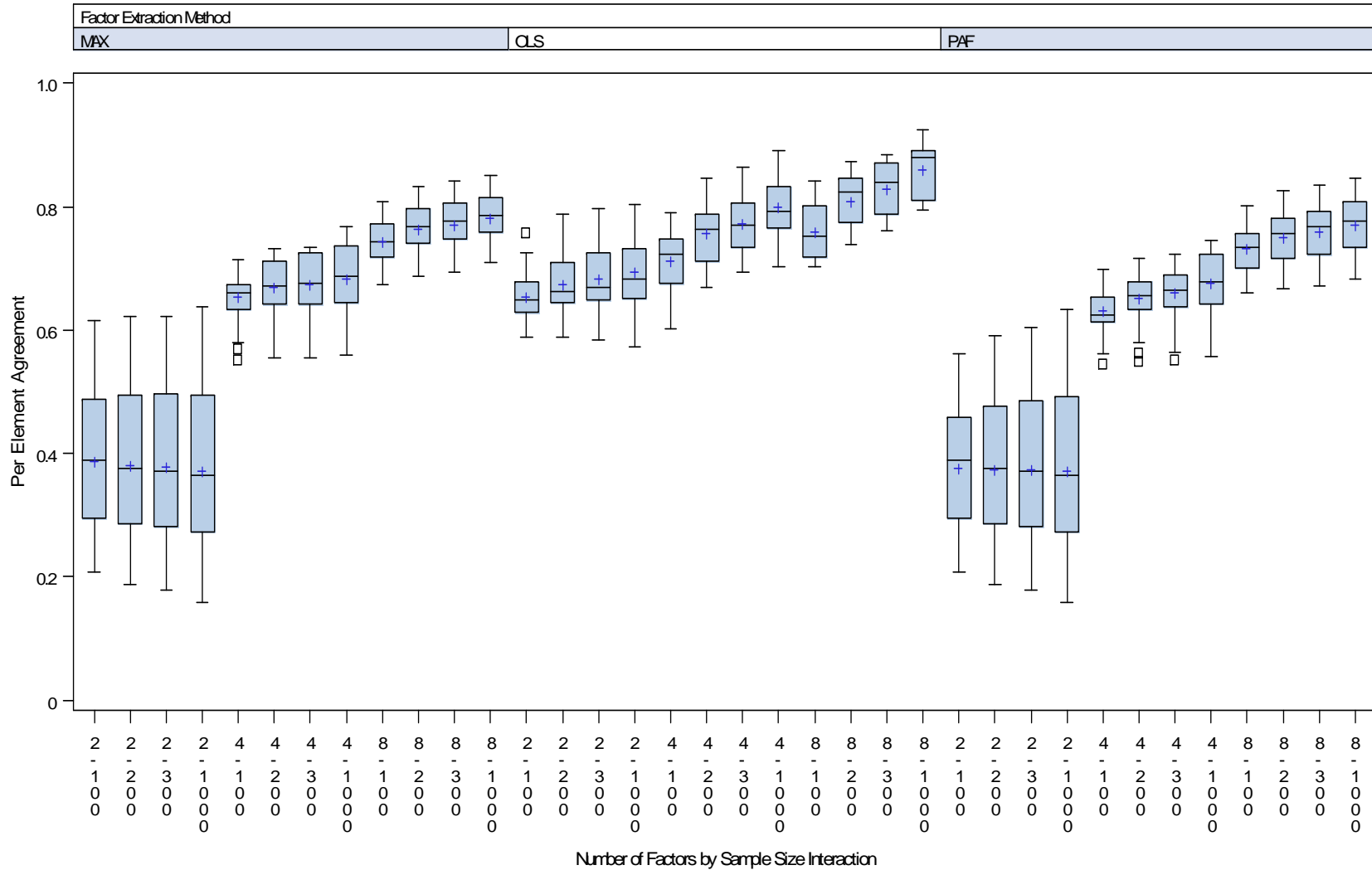


Figure D10. Per element factor pattern agreement by the interaction between number of factors and sample size

### Appendix D (Continued)

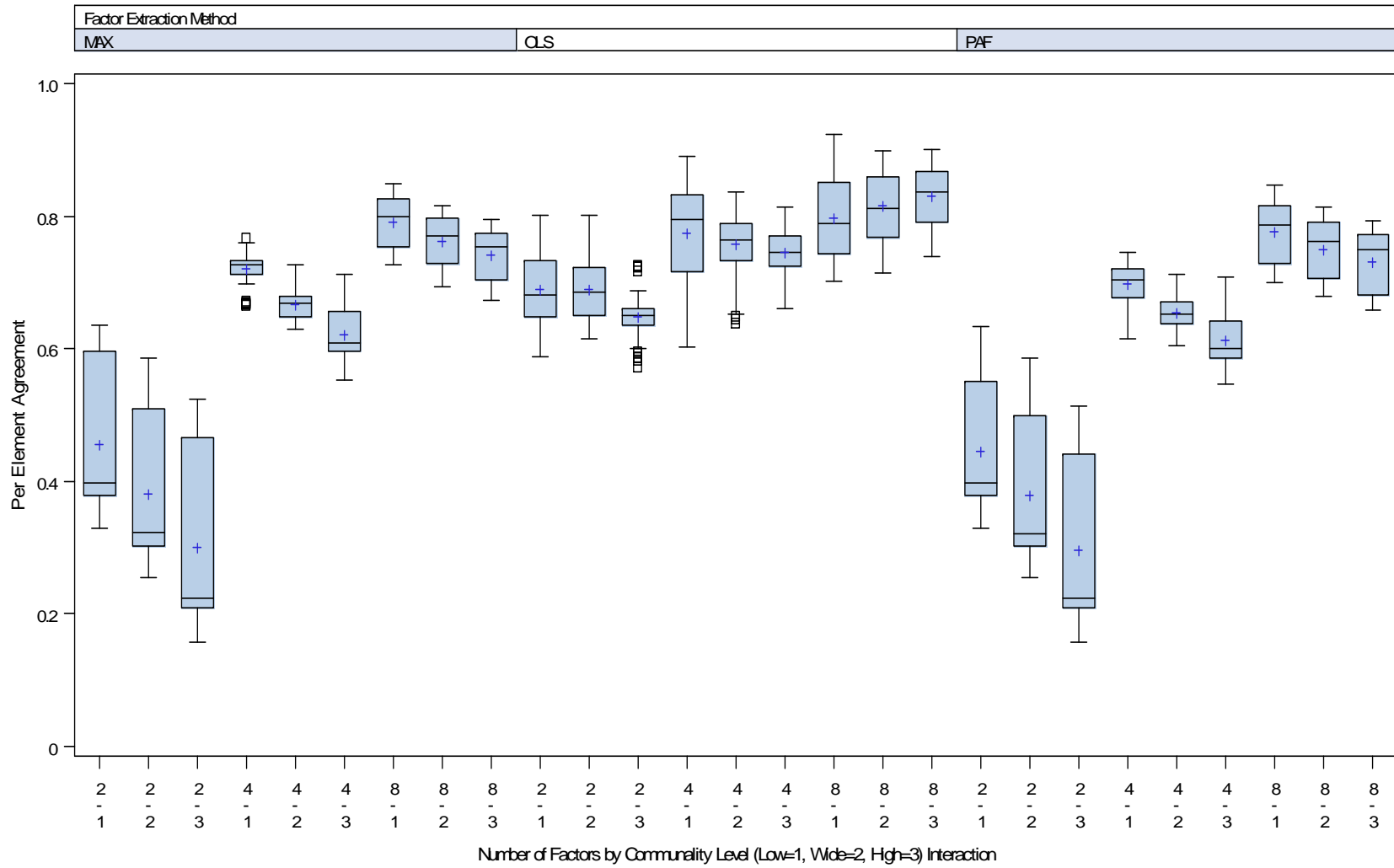


Figure D11. Per element factor pattern agreement by the interaction between number of factors and level of communality

### Appendix D (Continued)

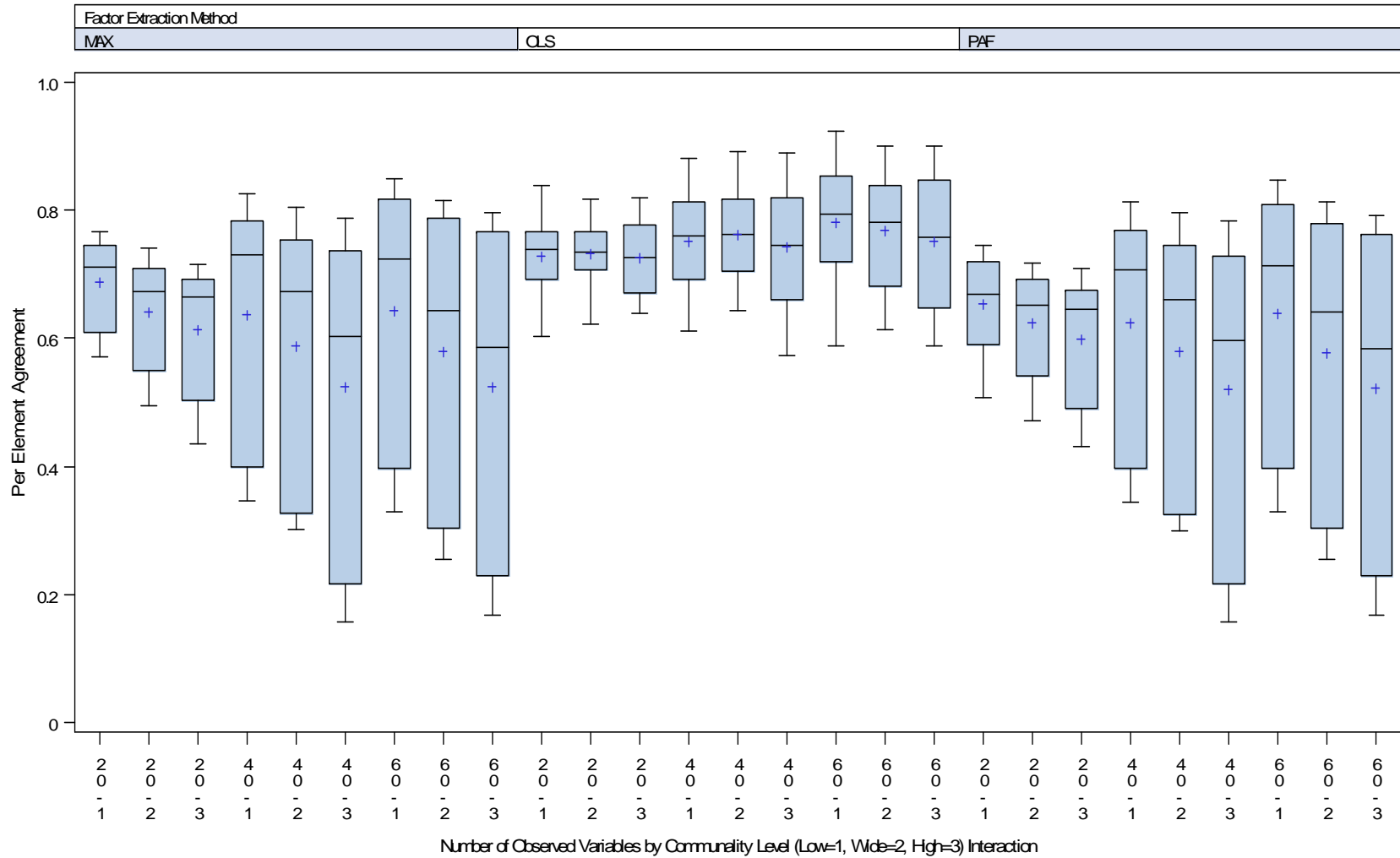


Figure D12. Per element factor pattern agreement by the interaction between number of observed variables and level of communality

### Appendix D (Continued)

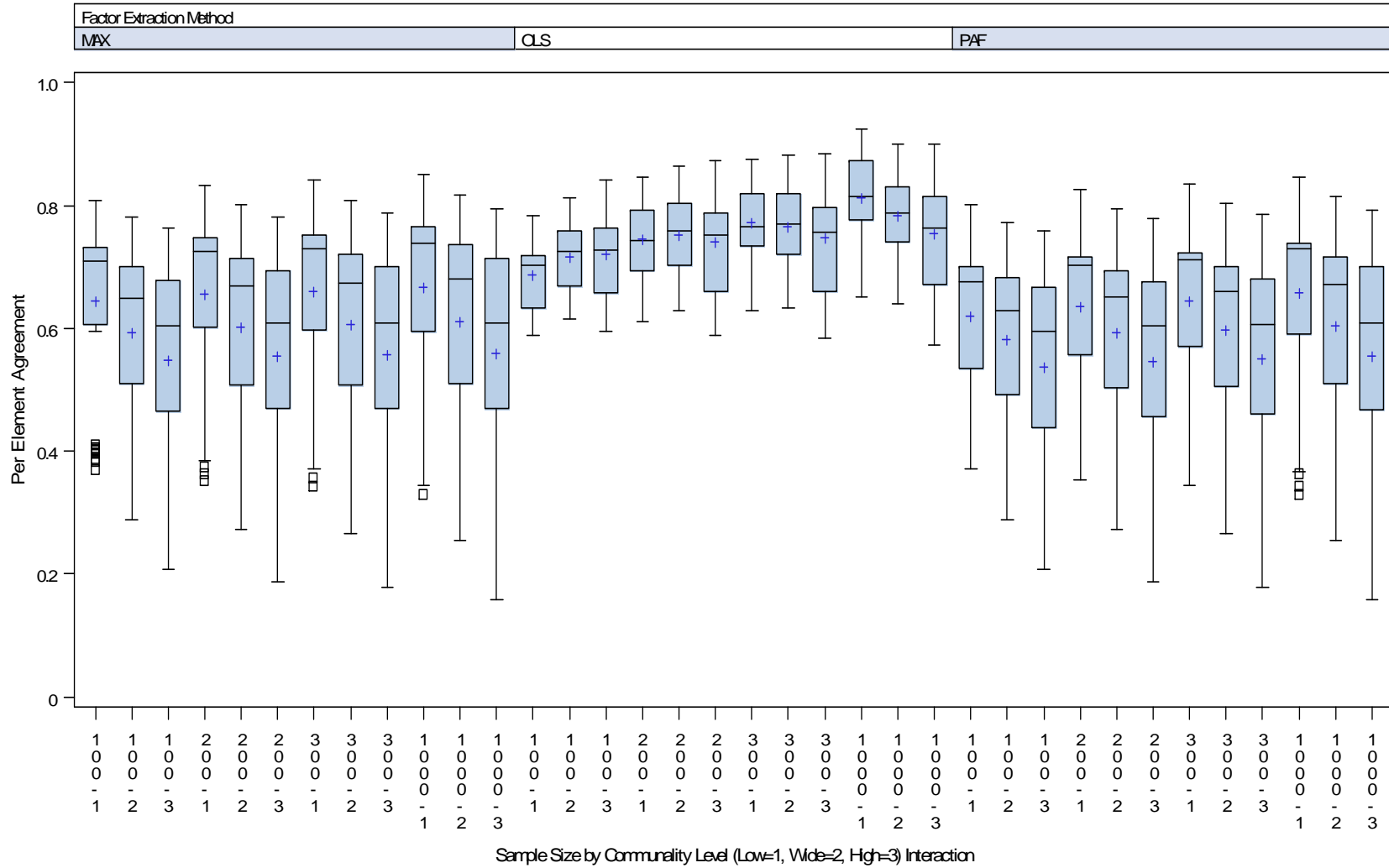


Figure D13. Per element factor pattern agreement by the interaction between sample size and level of communality



### Appendix D (Continued)

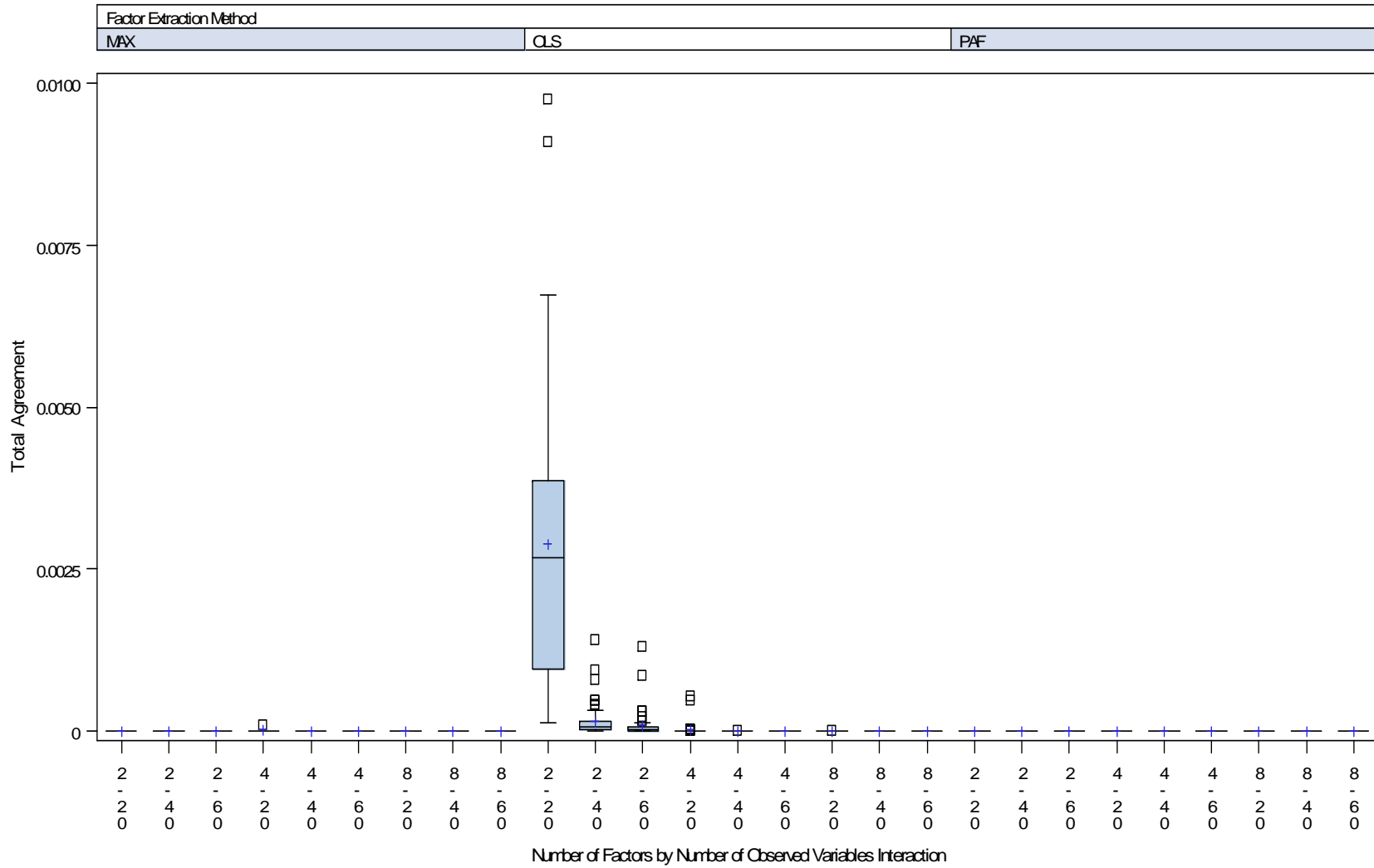


Figure D14. Total factor pattern agreement by the interaction between the number of factors and number of observed variables

### Appendix D (Continued)

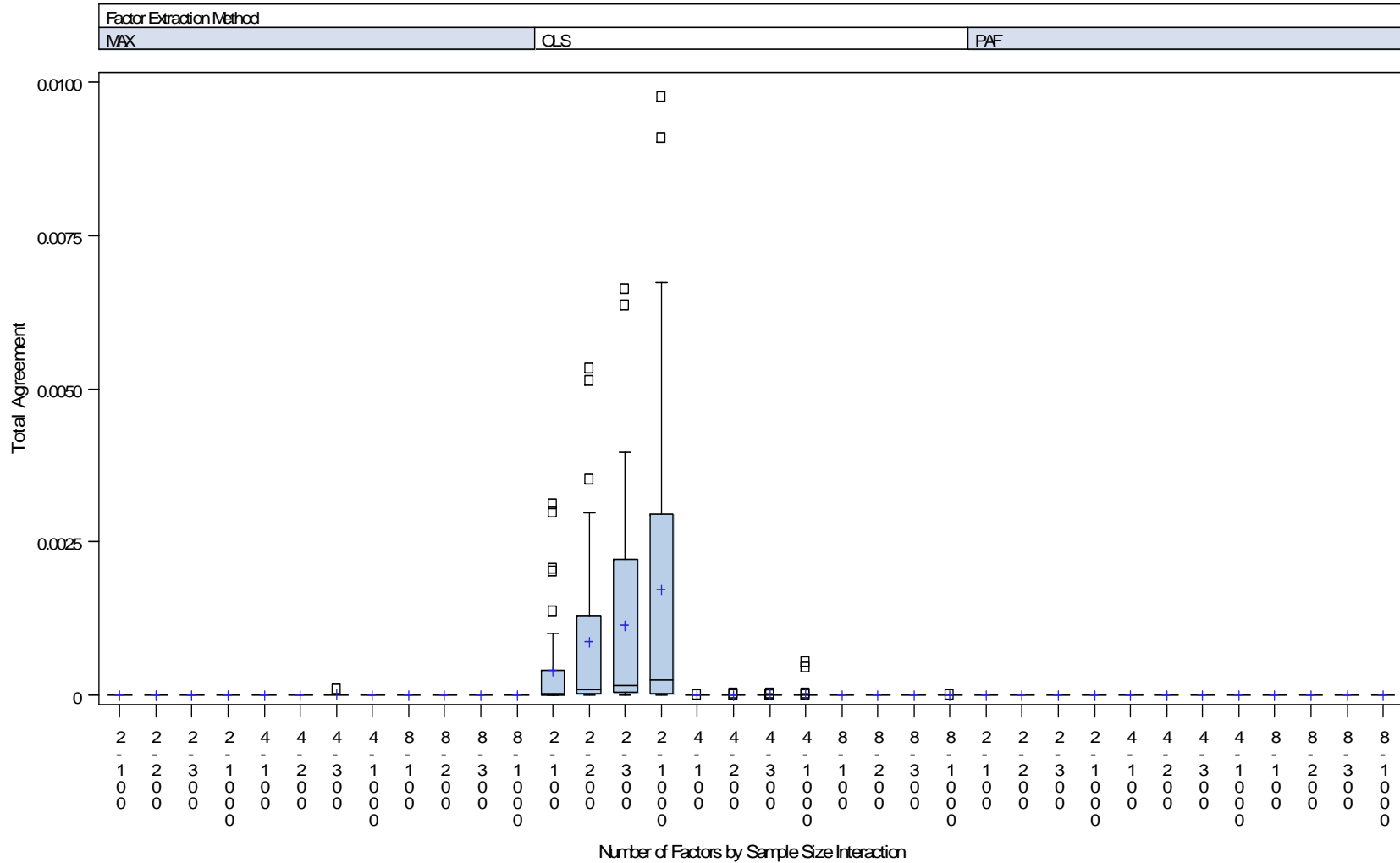


Figure D15. Total factor pattern agreement by the interaction between the number of factors and sample size

### Appendix D (Continued)

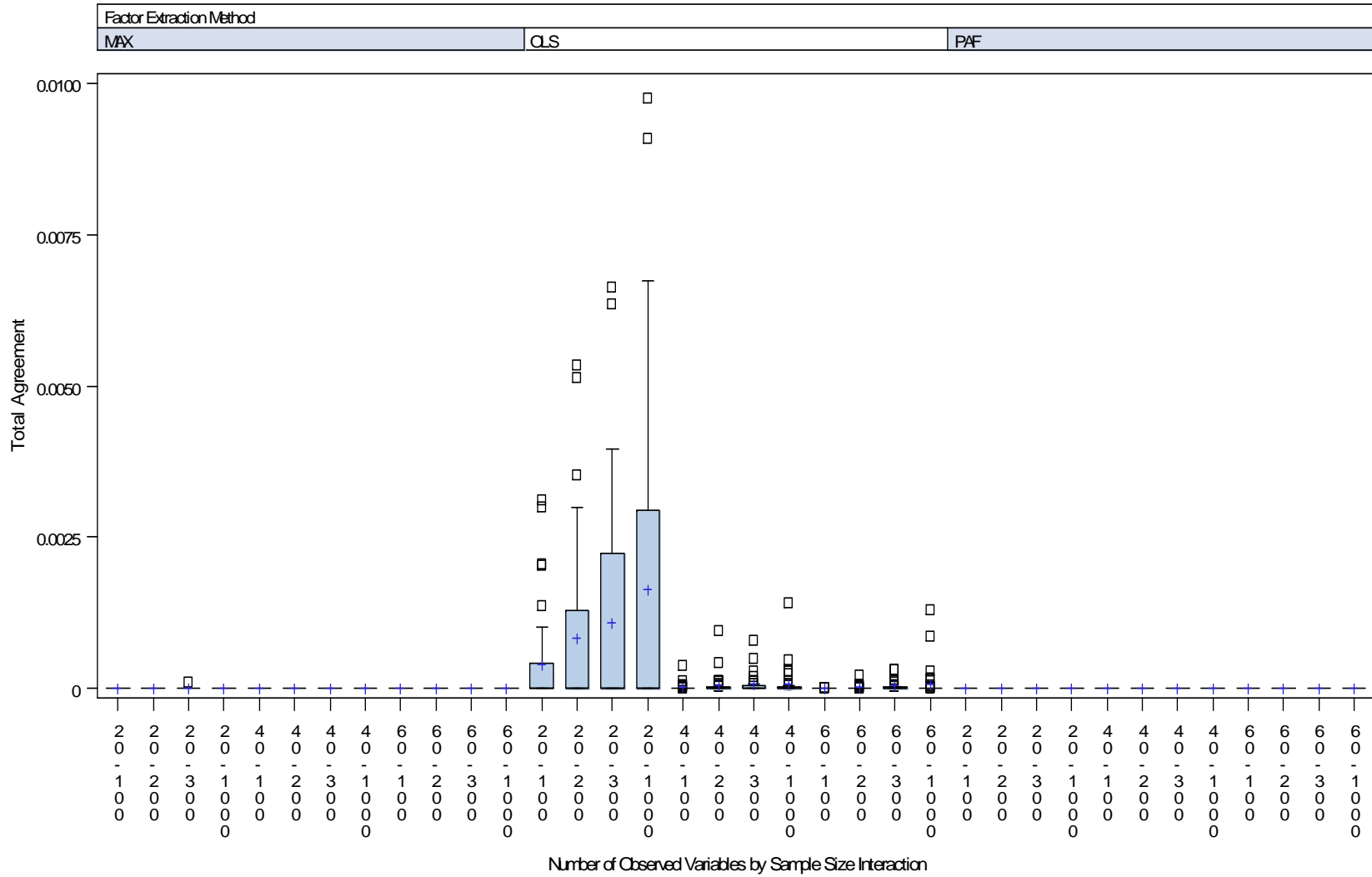


Figure D16. Total factor pattern agreement by the interaction between the number of observed variables and sample size

### Appendix D (Continued)

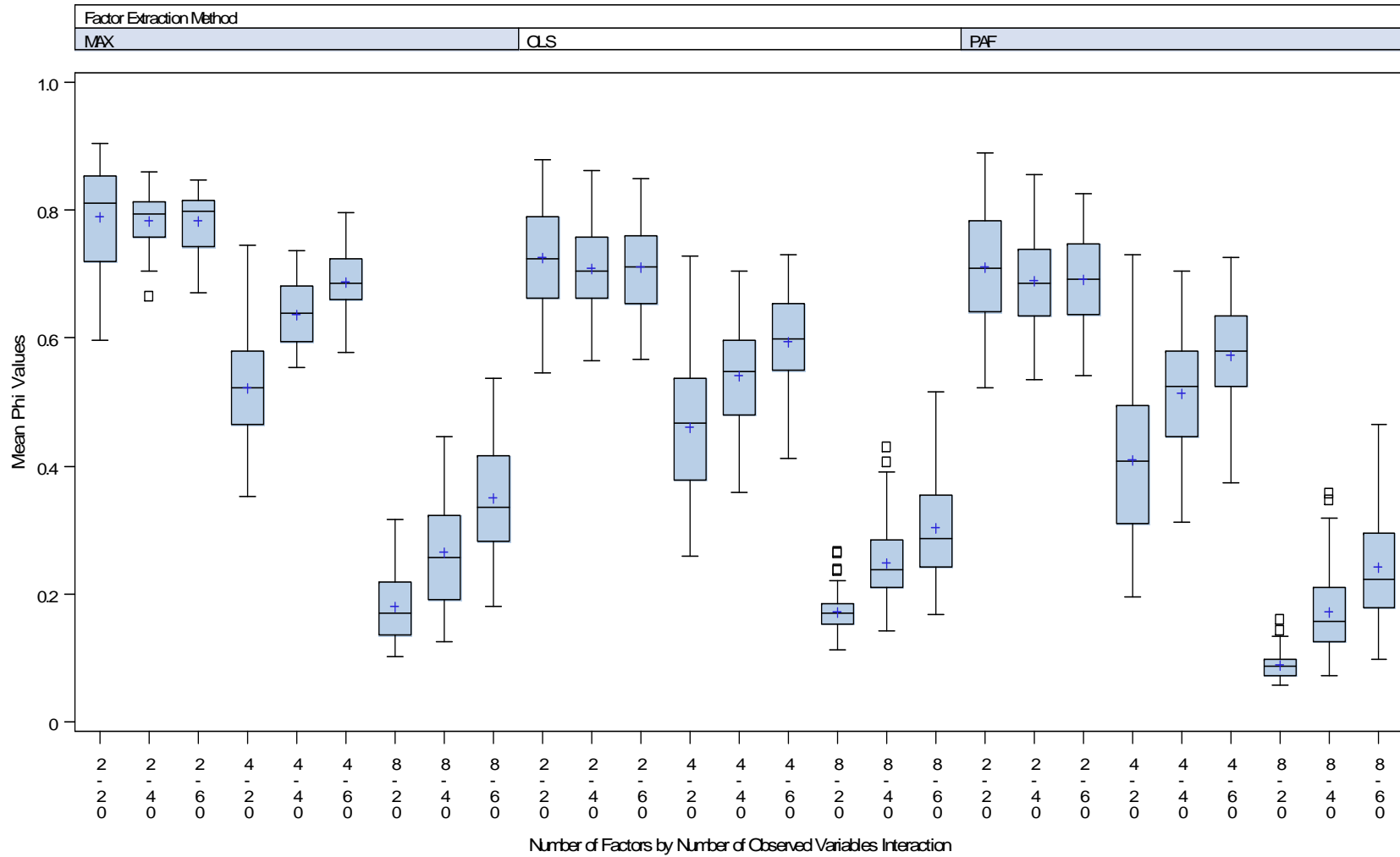


Figure D17. Mean Phi Values by the interaction between the number of factors and number of observed variables

### Appendix D (Continued)

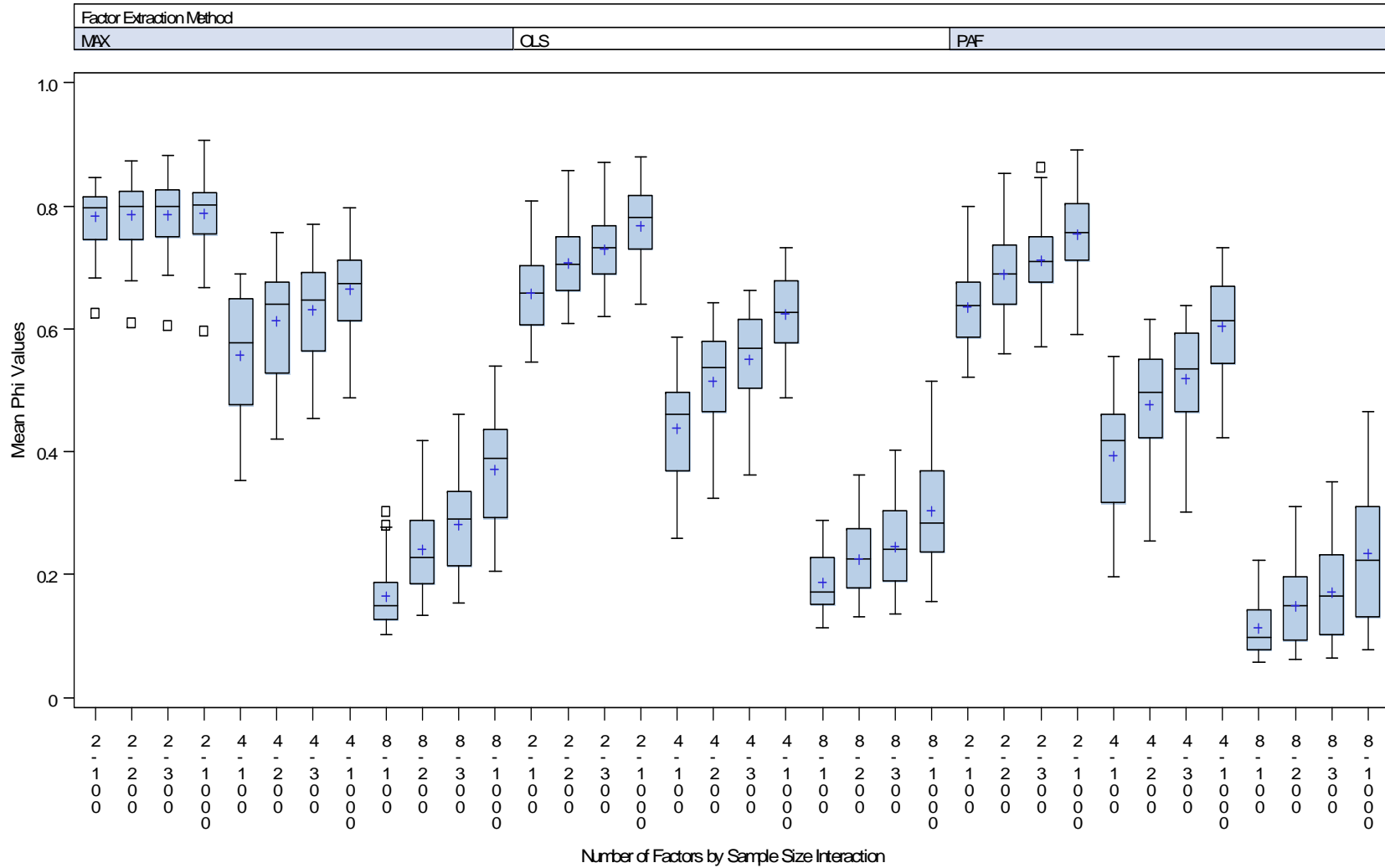


Figure D18. Mean Phi Values by the interaction between the number of factors and sample size

Appendix D (Continued)

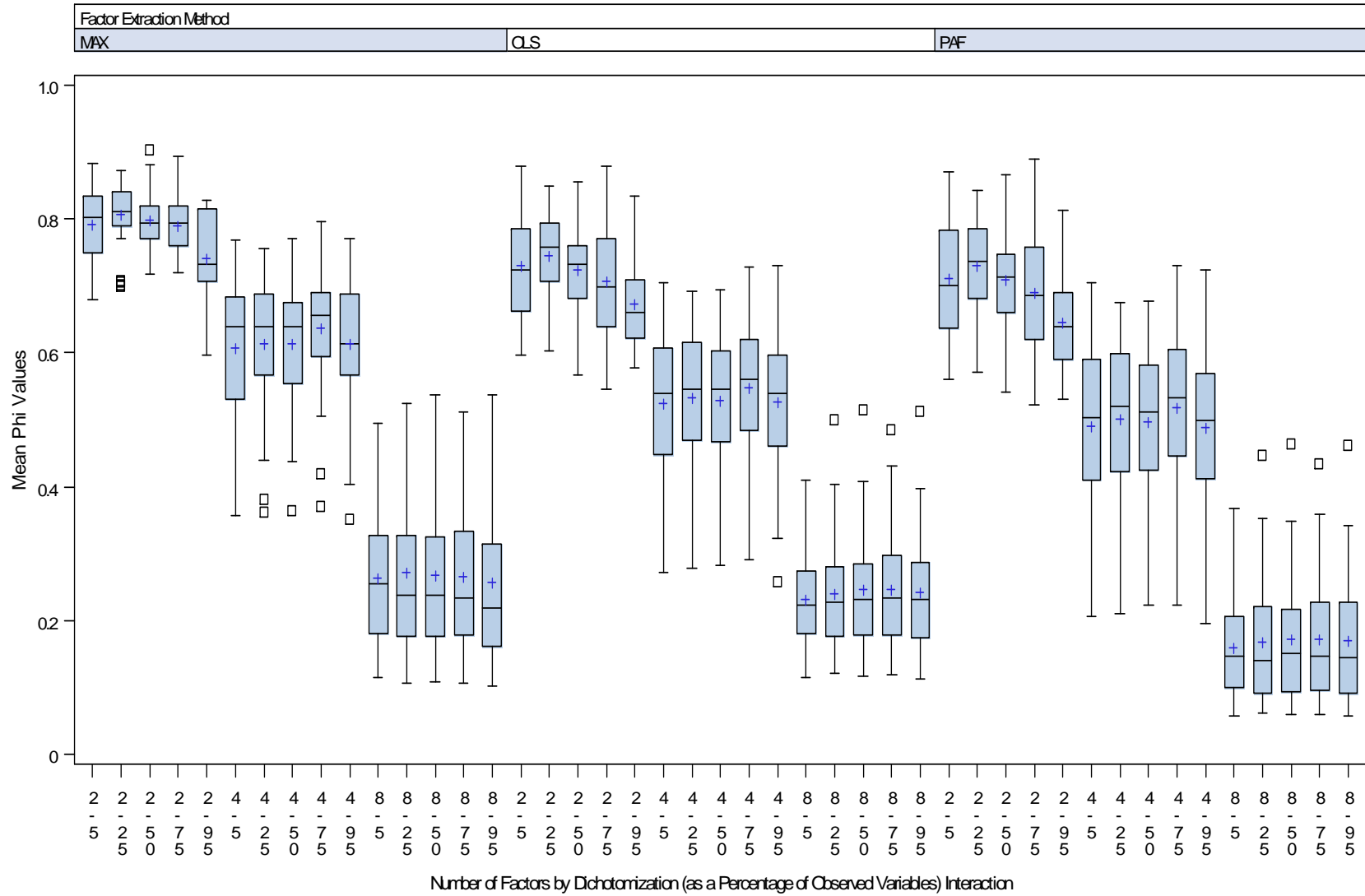


Figure D19. Mean Phi Values by the interaction between the number of factors and level of dichotomization

### Appendix D (Continued)

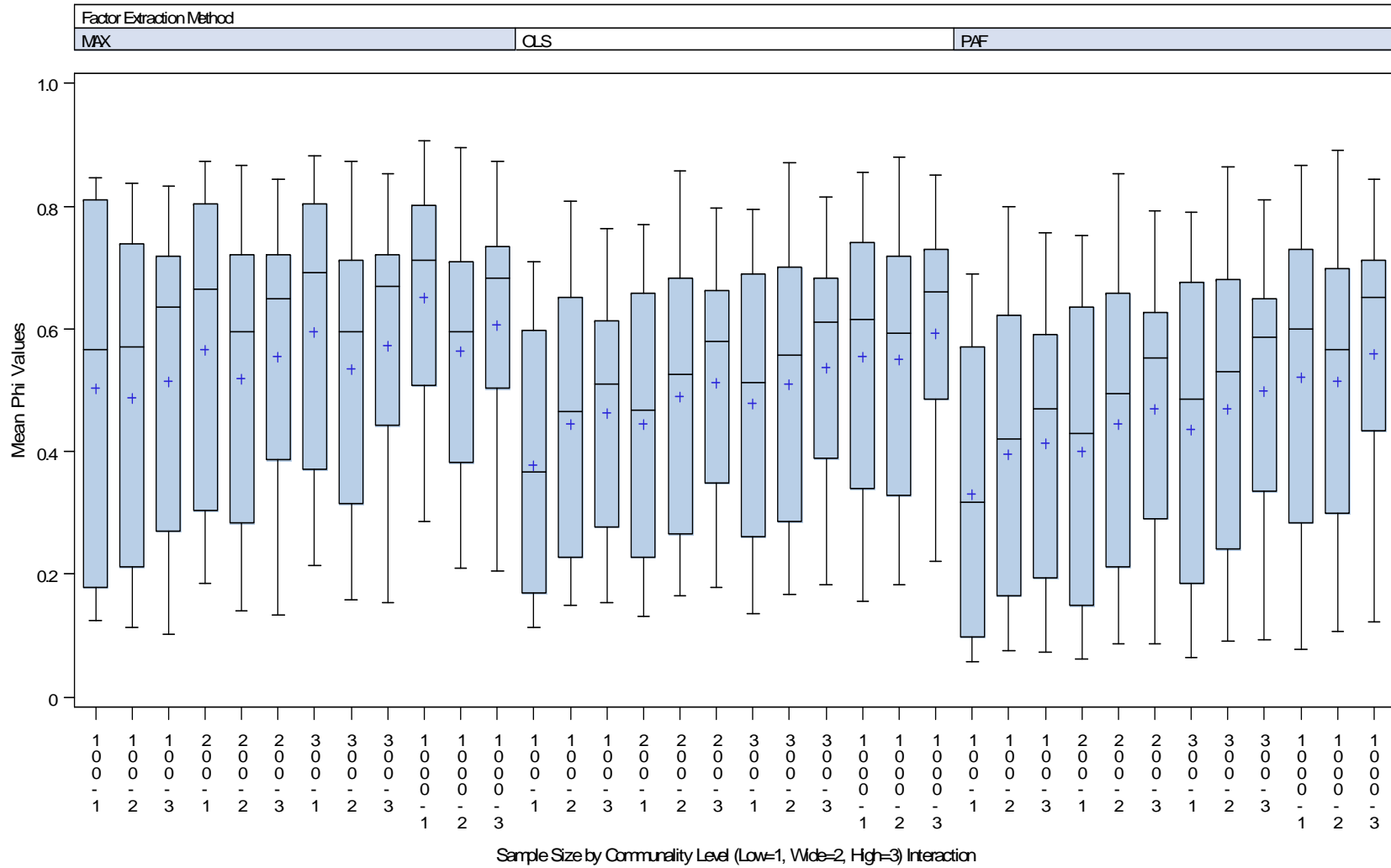


Figure D20. Mean Phi Values by the interaction between sample size and communality level

### Appendix D (Continued)

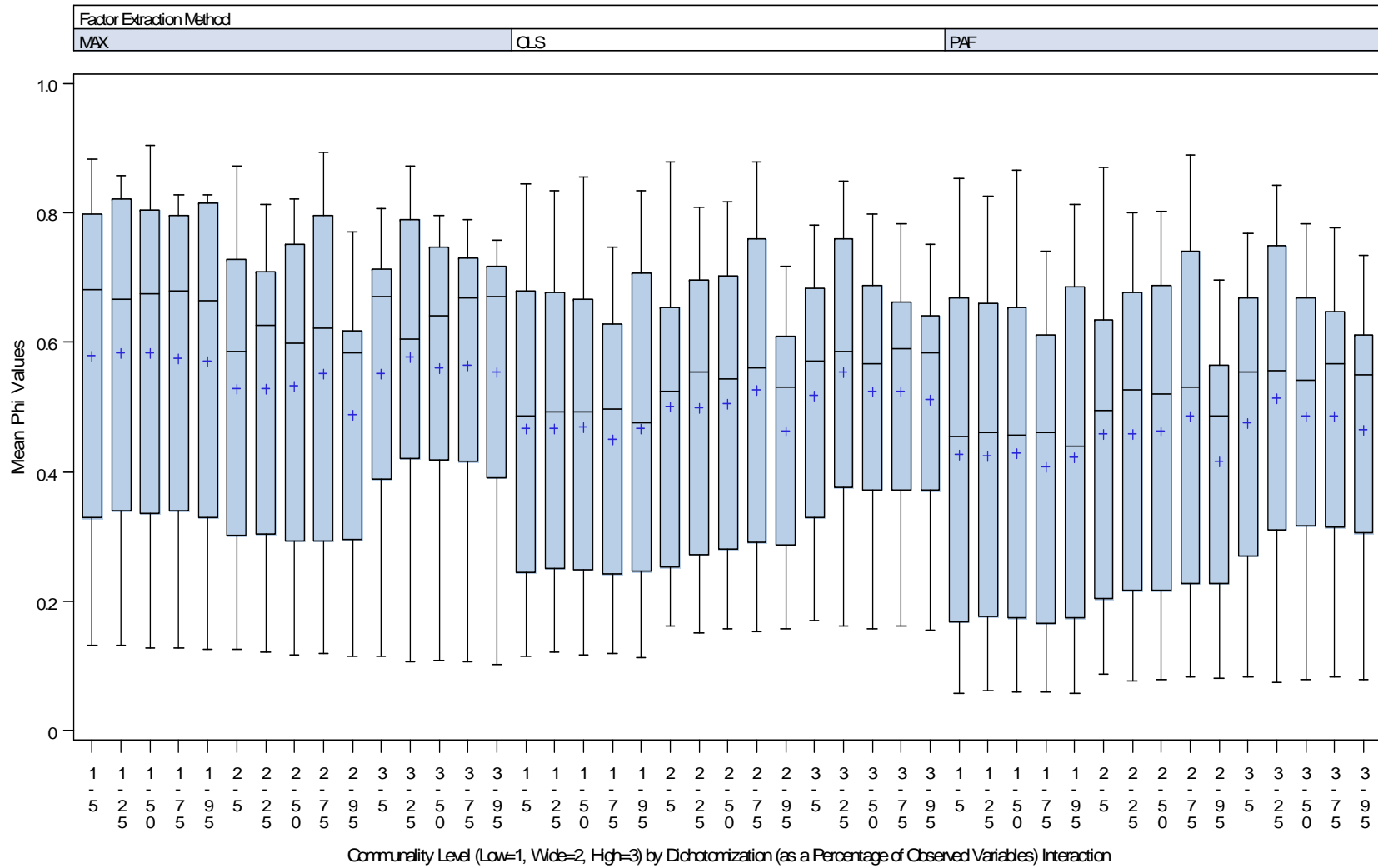


Figure D21. Mean Phi Values by the interaction between levels of communality and dichotomization



### Appendix D (Continued)

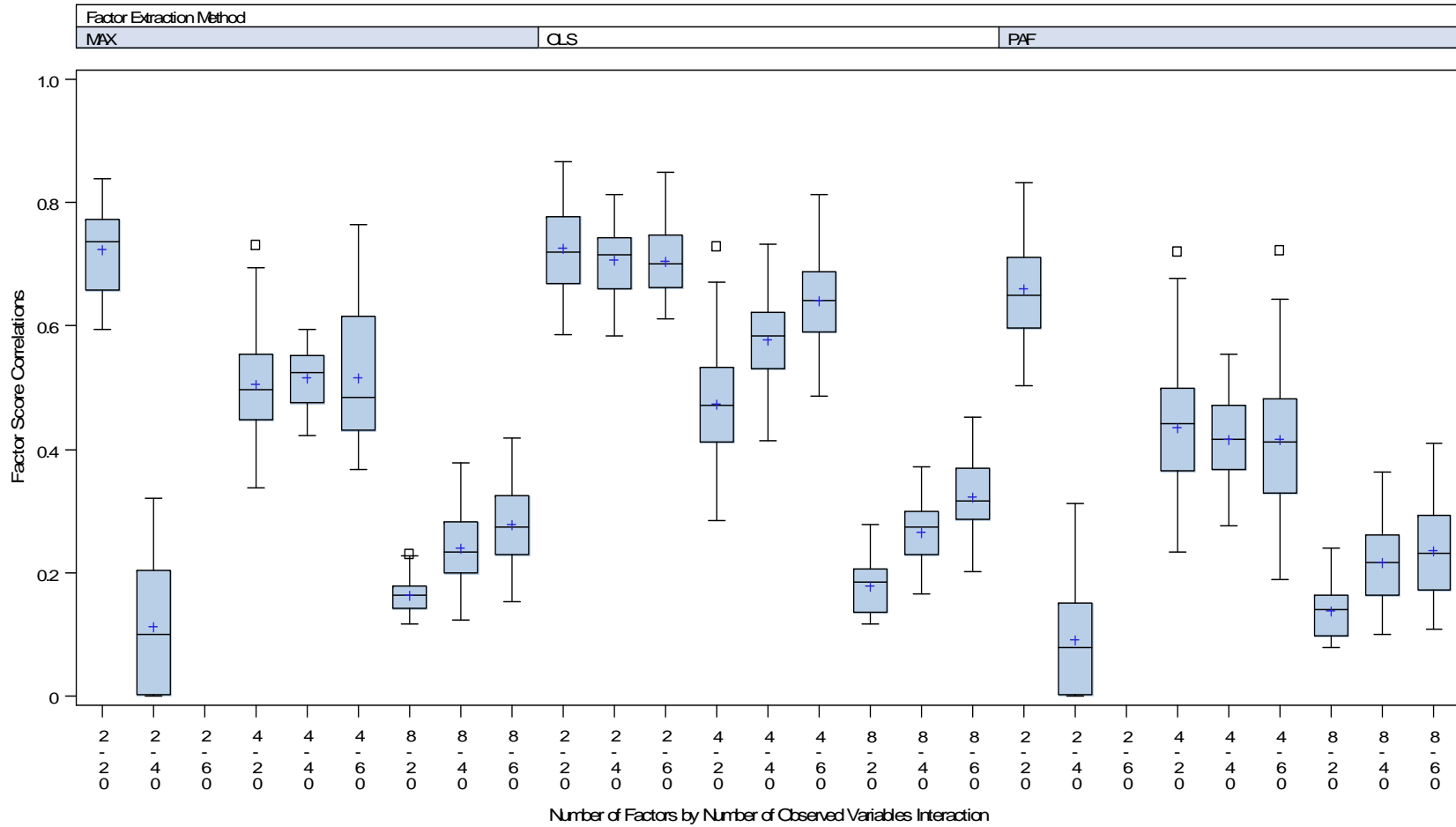


Figure D22. Factor score correlations by the interaction between number of factors and number of observed variables

### Appendix D (Continued)

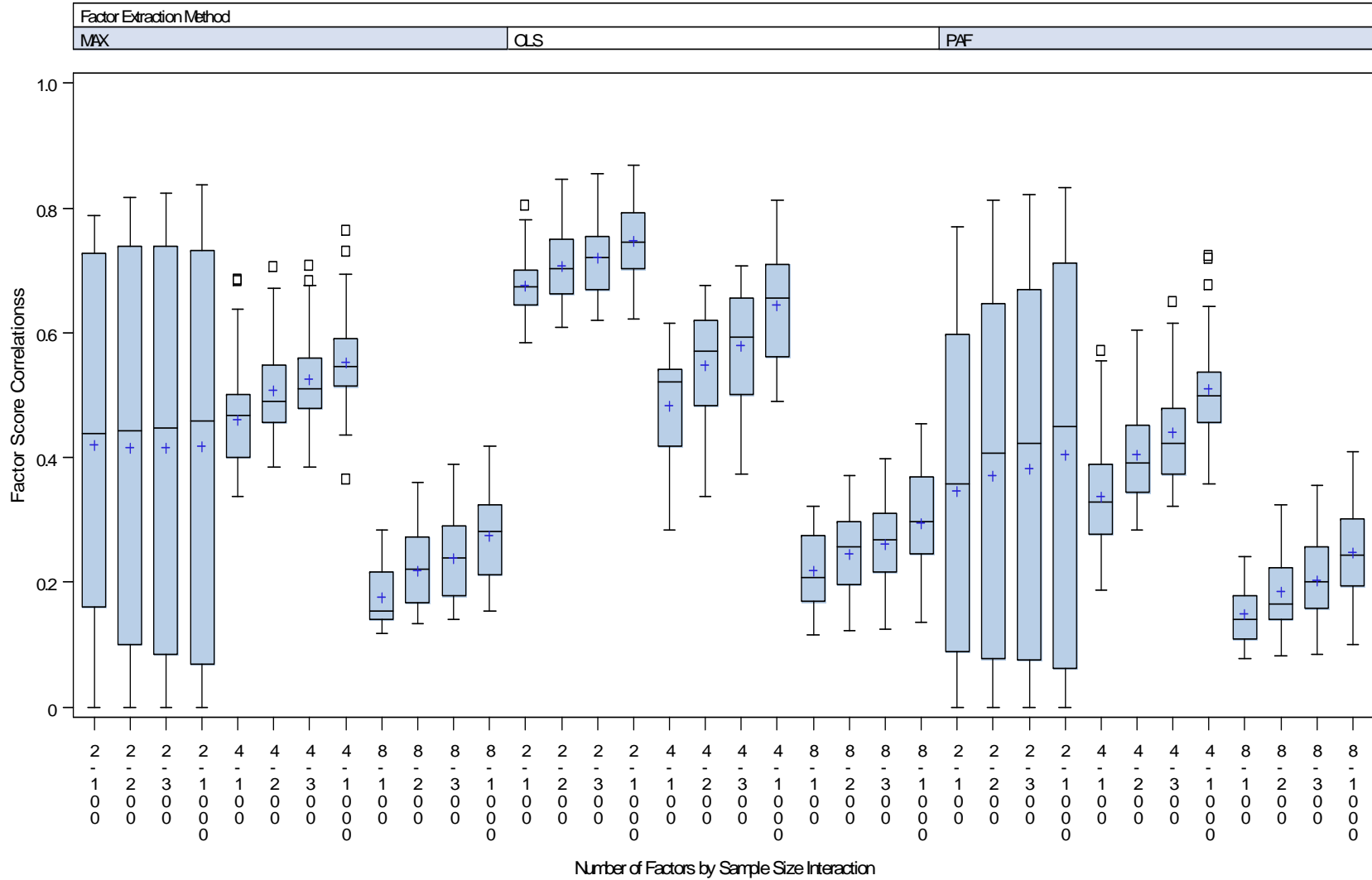


Figure D23. Factor score correlations by the interaction between number of factors and sample size

### Appendix D (Continued)

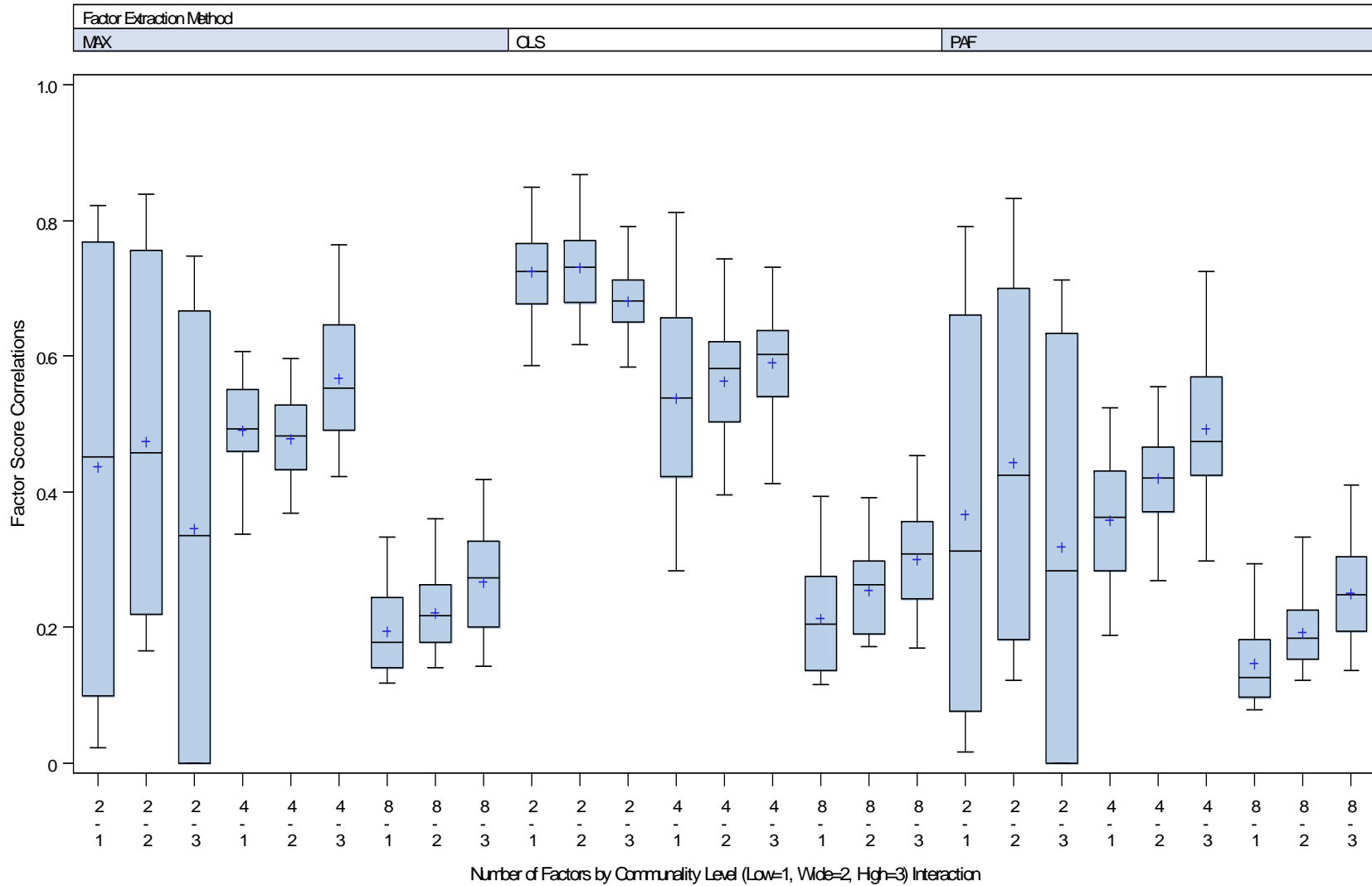


Figure D24. Factor score correlations by the interaction between number of factors and communality level

### Appendix D (Continued)

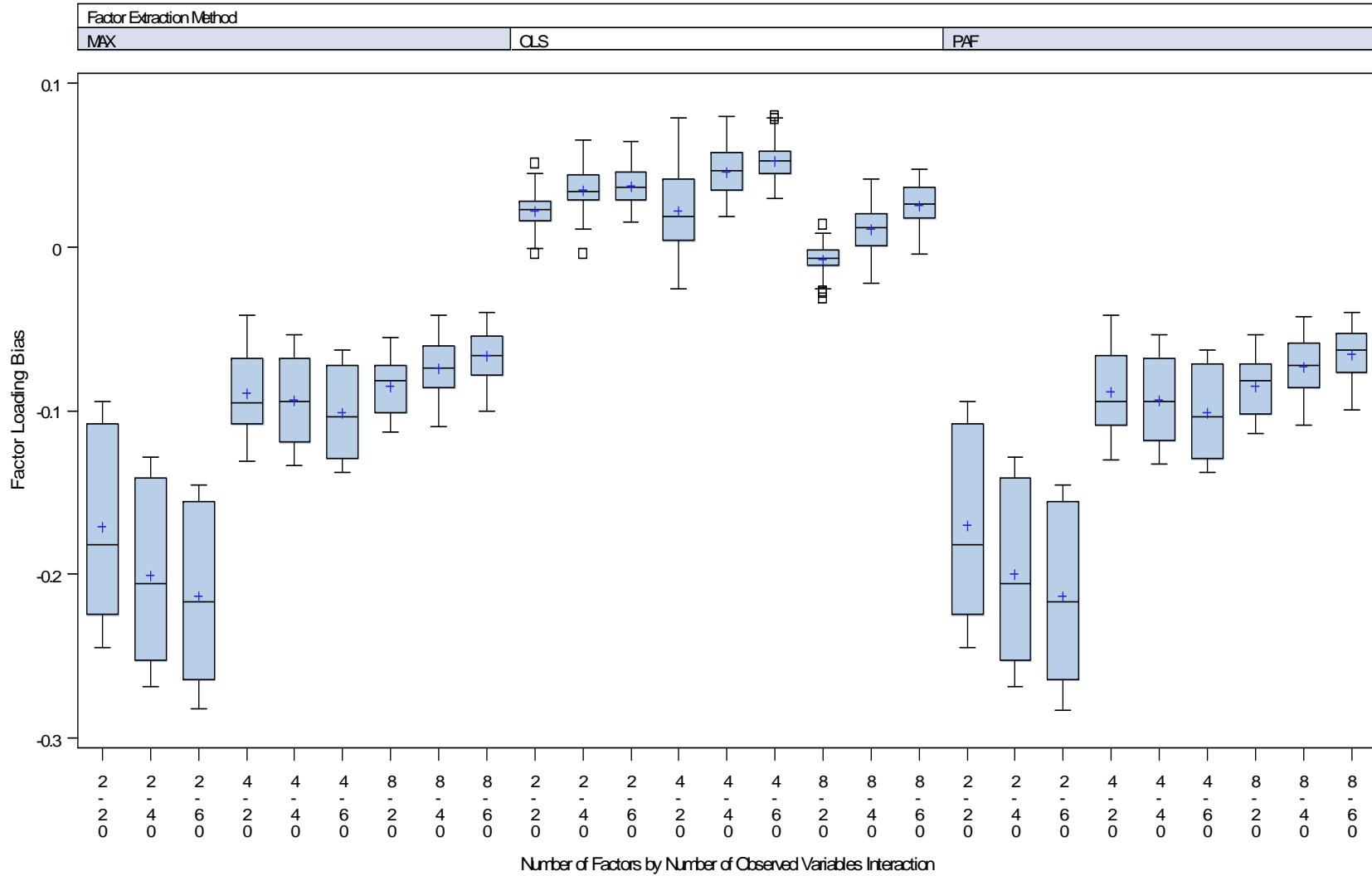


Figure D25. Factor loading bias by the interaction between the number of factors and number of observed variables

### Appendix D (Continued)

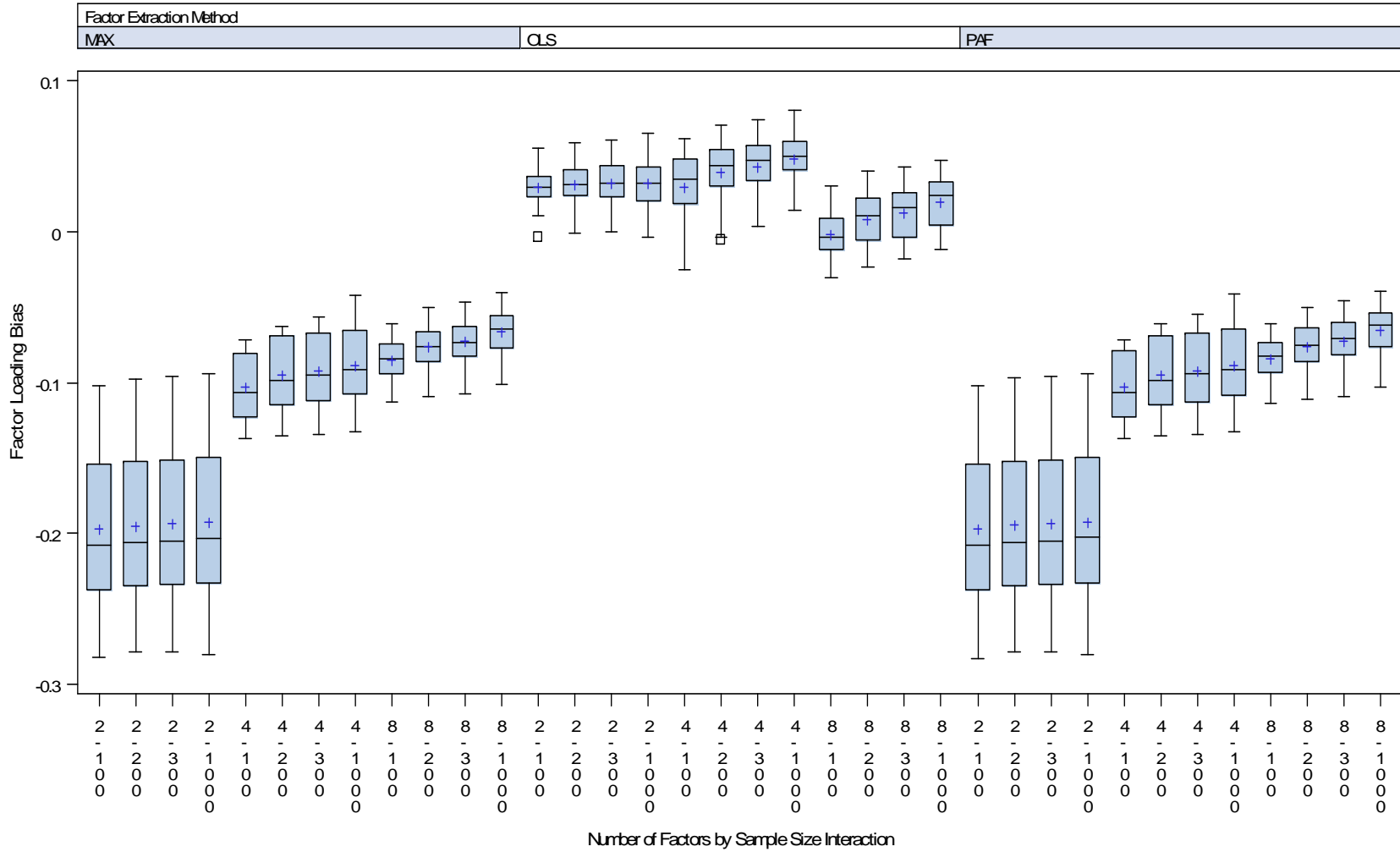


Figure D26. Factor loading bias by the interaction between the number of factors and sample size

### Appendix D (Continued)

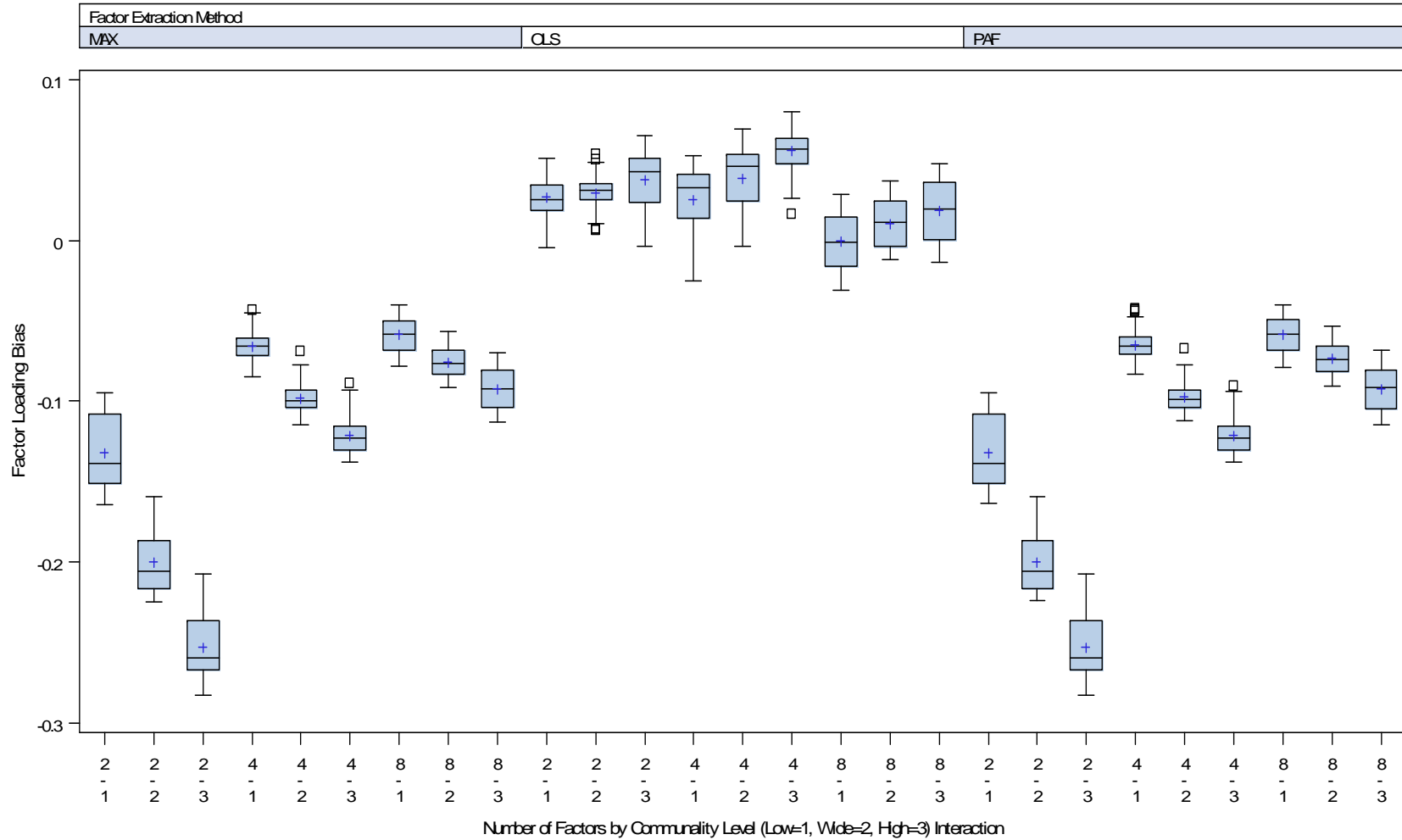


Figure D27. Factor loading bias by the interaction between the number of factors and community

### Appendix D (Continued)

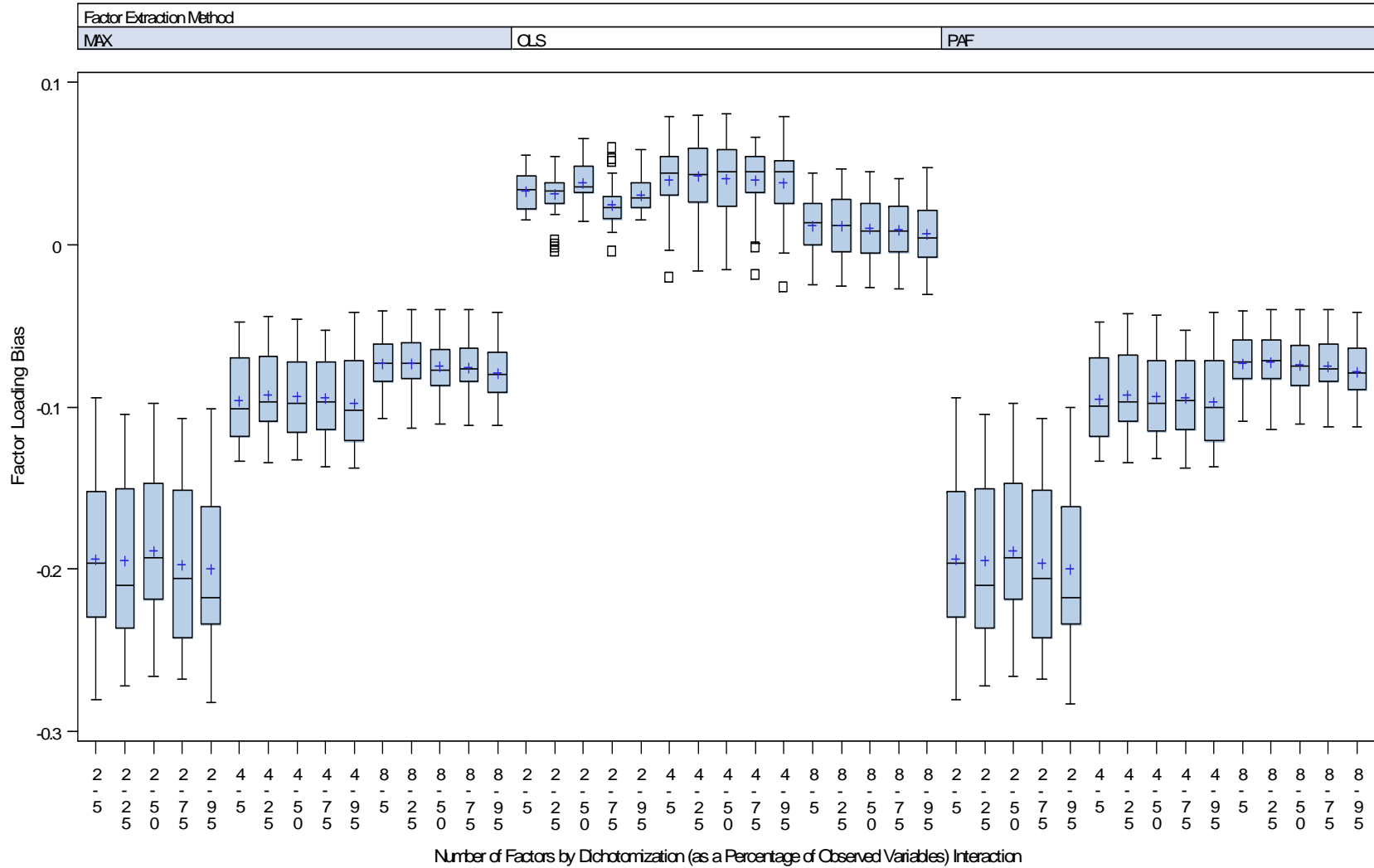


Figure D28. Factor loading bias by the interaction between the number of factors and level of dichotomization

### Appendix D (Continued)

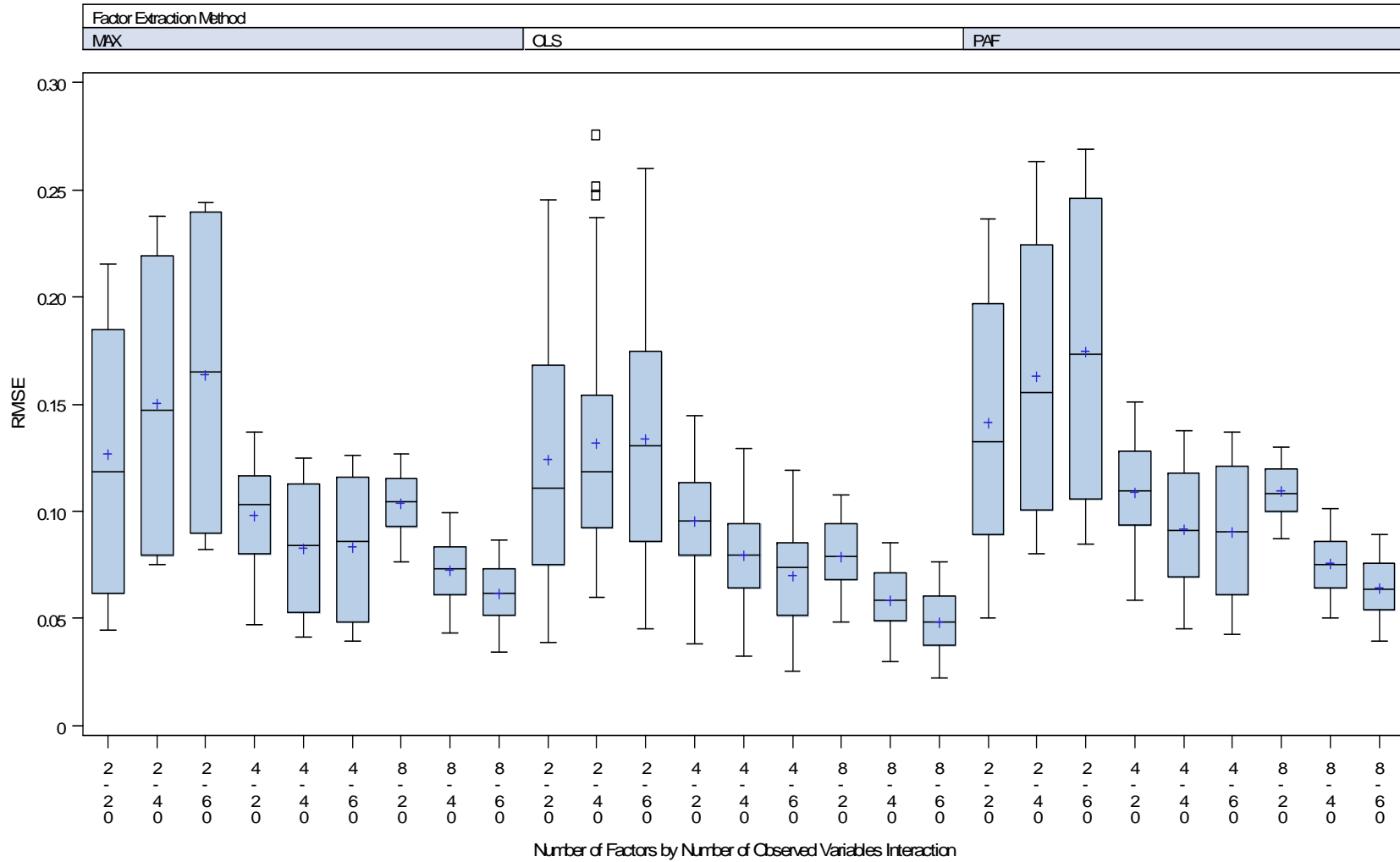


Figure D29. RMSE by the interaction between number of factors and number of observed variables



### Appendix D (Continued)

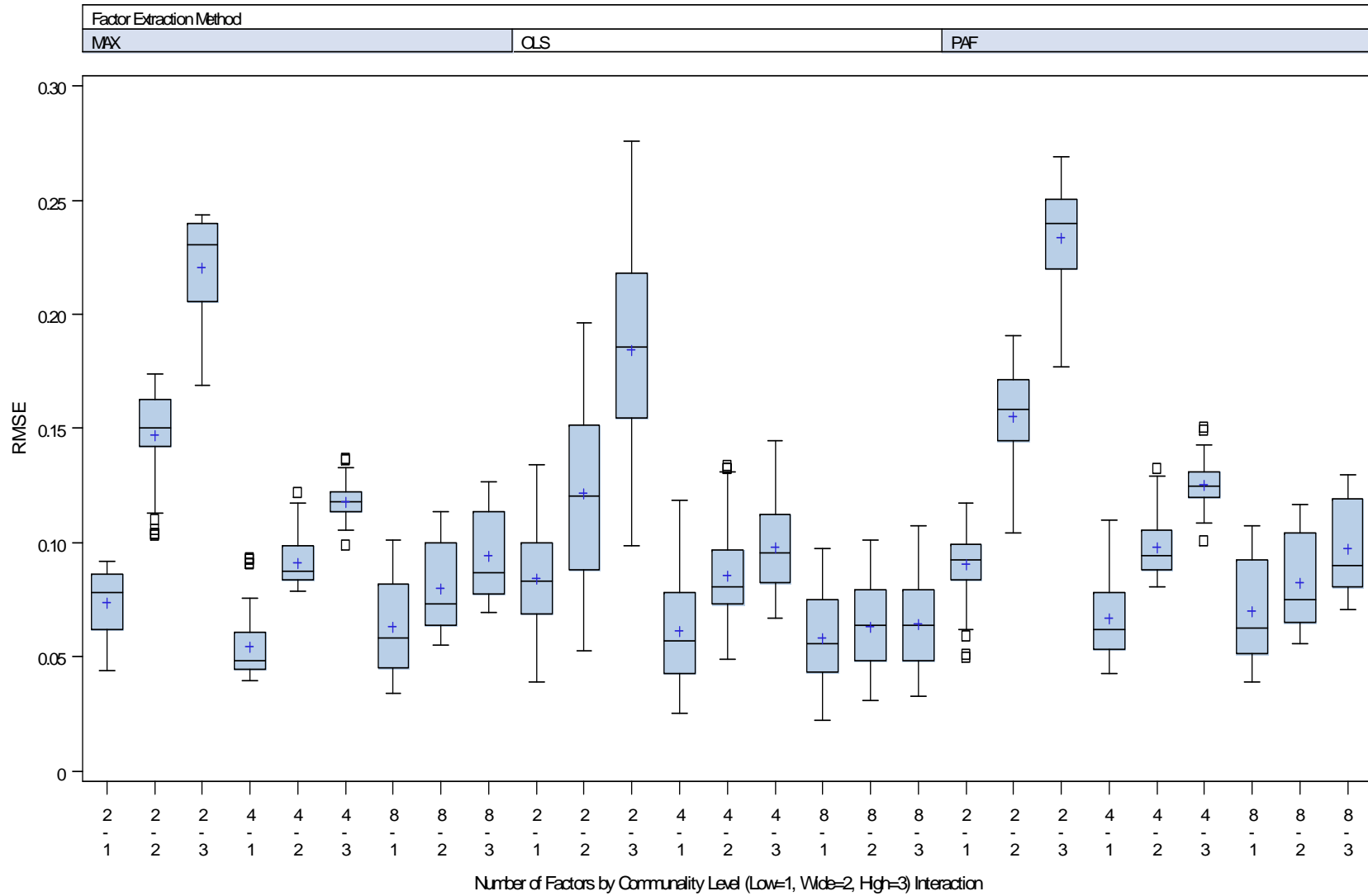


Figure D30. RMSE by the interaction between number of factors and communality level

### Appendix D (Continued)

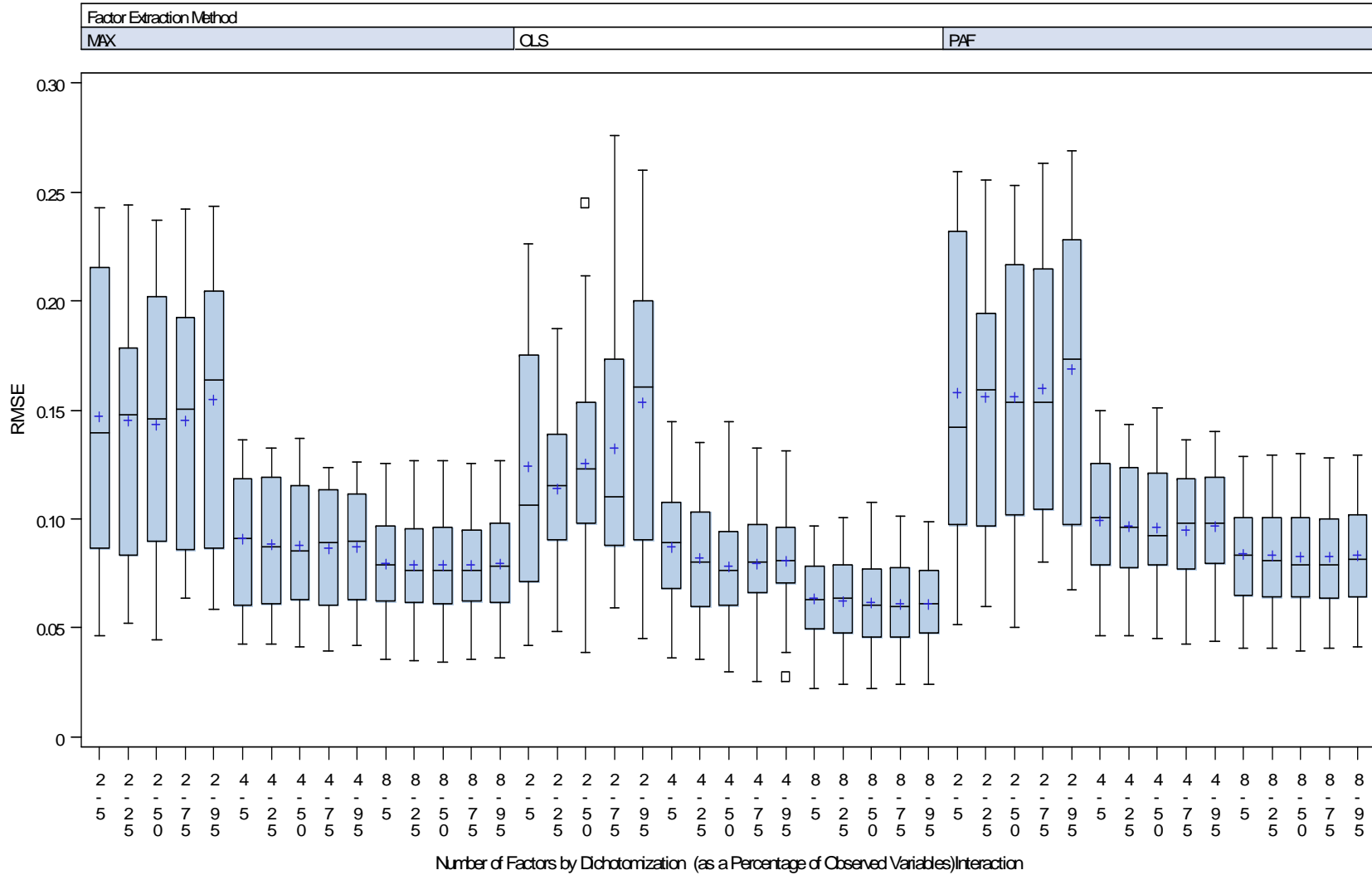


Figure D31. RMSE by the interaction between number of factors and level of dichotomization

### Appendix D (Continued)

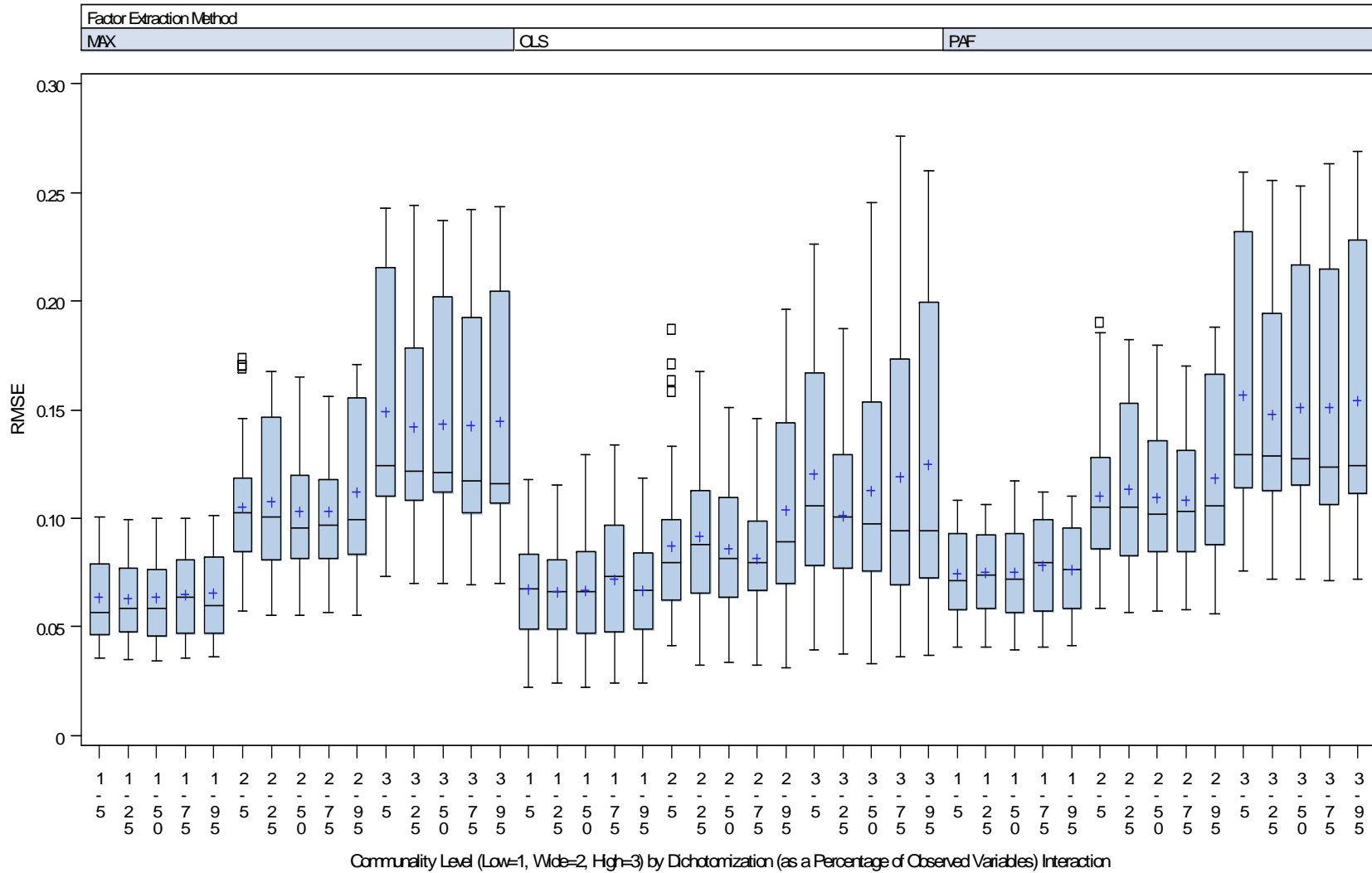


Figure D32. RMSE by the interaction between level of communality and dichotomization

## Appendix E

### Simulation Program: SAS, IML©

```
option ls = 256 ps = max nonumber nodate nocenter;
proc printto print='c:\EFA_full.lst';
proc iml;

*****;
***** Begin Specification of Data Conditions *****;
*****;

p = 60; *20;    *40;    *60;
k = 2;        *2;        *4;        *8;
d_frac = .50; *.05; *.25; *.50; *.75; *.95;
Commun_type = 1;
    * commun_type =1, low communalities, elements = .2, .3 or .4;
    * commun_type =2, wide communalities, elements = .2, .3, .4 ,.5, .6, .7 or .8;
    * commun_type =3, high communalities, elements = .6, .7 or .8;

N_pops = 10; * N of populations to generate;
replicat=1000; * N of samples from each population;
*replicat=10; * N of samples from each population for testing;

/*Inputs for Gendata2 A and B modules*/
nn1 = 100000;
means=j(1,p,0);
variance = j(1,p,1);

*****;
***** End Specification of Data Conditions *****;

***** Begin Subroutines for the Program *****;
*****;

start Make_PopR(nvars,nfactors,commun_type,A1,R);

* program to generate the population covariance matrices
  based on Tucker, Koopman, & Linn, 1969, Psychometrica;

* Inputs are nvars = number of variables in the matrix
  nfactors = number of factors
  commun = Type of communality

  Output is R = population correlation matrix;

* construct B1square such that
  commun=1, low communalities, elements = .2, .3 or .4
```

## Appendix E (Continued)

commun=2, wide communalities, elements = .2, .3, .4, .5, .6, .7 or .8  
commun=3, high communalities, elements = .6, .7 or .8;

```
bp1=j(nvars,nvars,0);
bp2=uniform(bp1);
if commun_type=3 then do;
  bp3=(bp2*.2999999)+.55;
  b1square=round(diag(bp3),.1);
end;
if commun_type=1 then do;
  bp3=(bp2*.2999999)+.15;
  b1square=round(diag(bp3),.1);
end;
if commun_type=2 then do;
  bp3=(bp2*.6999999)+.15;
  b1square=round(diag(bp3),.1);
end;

B1=b1square##.5;

b3square=I(nvars)-b1square;
B3=b3square##.5;

* construct A1tilde the matrix of conceptual input factor loadings
  so that each element is a whole number between 0 and nfactor and
  so the sum of each row equals nfactor-1;

A1tilde1=j(nvars,nfactors,0);
A1tilde2=round((uniform(A1tilde1)*(nfactors-.0000001))-.5);
A1tilde=A1tilde2;
do j=2 to nfactors;
  do i=1 to nvars;
    if j<nfactors then do;
      A1tilde[i,j]=round(((nfactors-.0000001-sum(A1tilde[i,1:j-1]))
        *uniform(0))-.5);
    end;
    if j=nfactors then do;
      A1tilde[i,nfactors]=nfactors-sum(A1tilde[i,1:nfactors-1])-1;
    end;
  end;
end;

* construct A1 the matrix of actual input factor loadings from A1tilde;

x=normal(A1tilde);
x2=x##2;
d=j(nvars,nfactors,0);
do j=1 to nfactors;
  do i=1 to nvars;
    d[i,j]=(sum(x2[i,1:nfactors]))##-.5;
  end;
end;

cvec=j(1,nfactors,0);
do j=1 to nfactors;
  cvec[1,j]=round((uniform(0)*.2999999)+.65,.1);
end;
```

## Appendix E (Continued)

```

c=j(nvars,1,1)*cvec;
c2=c##2;

ones=j(nvars,nfactors,1);
y=A1tilde#c + d#x#((ones-c2)##.5);

k=.2;
z=j(nvars,nfactors,0);
do j=1 to nfactors;
  do i=1 to nvars;
    z[i,j]=((1+k)*y[i,j]*(y[i,j]+abs(y[i,j])+k))/((2+k)*(abs(y[i,j])+k));
  end;
end;

z2=z##2;
g=j(nvars,nfactors,0);
do j=1 to nfactors;
  do i=1 to nvars;
    g[i,j]=(sum(z2[i,1:nfactors]))##-.5;
  end;
end;

A1star=g#z;

A1=B1*A1star;

A3star=I(nvars);
A3=B3*A3star;

R=A1*A1`+A3*A3`;

Finish;

start gendata2a(NN1,seed1,variance,bb,cc,dd,mu,r_matrix,YY,p,d_frac);
  L = eigval(r_matrix);
  neg_eigval = 0;
  do r = 1 to nrow(L);
    if L[r,1] < 0 then neg_eigval = 1;
  end;

  if neg_eigval = 0 then do; * matrix is positive definite, so use the Cholesky root approach;
    COLS = NCOL(r_matrix);
    G = ROOT(r_matrix);
    YY=rannor(repeat(seed1,nn1,COLS));
    YY = YY*G;
    do r = 1 to NN1;
      do c = 1 to COLS;
        YY[r,c] = (-1*cc) + (bb*YY[r,c]) + (cc*YY[r,c]##2) + (dd*YY[r,c]##3);
        YY[r,c] = (YY[r,c] * SQRT(variance[1,c])) + mu[1,c];
      end;
    end;
  end;

  if neg_eigval = 1 then do; * matrix is not positive definite, so use the PCA approach;
    COLS = NCOL(r_matrix);
    V = eigvec(r_matrix);
    do i = 1 to nrow(L);
      do j = 1 to ncol(V);

```

## Appendix E (Continued)

```

        if L[i,1] > 0 then V[j,i] = V[j,i] # sqrt(L[i,1]);
        if L[i,1] <= 0 then V[j,i] = V[j,i] # sqrt(.000000001);
    end;
end;
YY=rannor(repeat(seed1,nn1,COLS));
YY = V*YY`;
YY = YY`;
do r = 1 to NN1;
do c = 1 to COLS;
    YY[r,c] = (-1*cc) + (bb*YY[r,c]) + (cc*YY[r,c]##2) + (dd*YY[r,c]##3);
    YY[r,c] = (YY[r,c] * SQRT(variance[1,c])) + mu[1,c];
end;
end;
end;
do r = 1 to nn1;
do c = 1 to (p*d_frac);
    if yy[r,c] < 0 then yy[r,c] = 0;
    else if yy[r,c] = 0 then yy[r,c] = 1;
    else if yy[r,c] > 0 then yy[r,c] = 1;
    end;
end;
end;
finish;

start gendata2b(NN2,seed1,variance,bb,cc,dd,mu,r_matrix,YY,p,d_frac);
L = eigval(r_matrix);
neg_eigval = 0;
do r = 1 to nrow(L);
    if L[r,1] < 0 then neg_eigval = 1;
end;

if neg_eigval = 0 then do; * matrix is positive definite, so use the Cholesky root approach;
    COLS = NCOL(r_matrix);
    G = ROOT(r_matrix);
    YY=rannor(repeat(seed1,nn2,COLS));
    YY = YY*G;
    do r = 1 to NN2;
    do c = 1 to COLS;
        YY[r,c] = (-1*cc) + (bb*YY[r,c]) + (cc*YY[r,c]##2) + (dd*YY[r,c]##3);
        YY[r,c] = (YY[r,c] * SQRT(variance[1,c])) + mu[1,c];
    end;
    end;
end;

if neg_eigval = 1 then do; * matrix is not positive definite, so use the PCA approach;
    COLS = NCOL(r_matrix);
    V = eigvec(r_matrix);
    do i = 1 to nrow(L);
    do j = 1 to ncol(V);
        if L[i,1] > 0 then V[j,i] = V[j,i] # sqrt(L[i,1]);
        if L[i,1] <= 0 then V[j,i] = V[j,i] # sqrt(.000000001);
    end;
    end;
    YY=rannor(repeat(seed1,nn2,COLS));
    YY = V*YY`;
    YY = YY`;
    do r = 1 to NN2;
    do c = 1 to COLS;

```

## Appendix E (Continued)

```

YY[r,c] = (-1*cc) + (bb*YY[r,c]) + (cc*YY[r,c]**2) + (dd*YY[r,c]**3);
YY[r,c] = (YY[r,c] * SQRT(variance[1,c])) + mu[1,c];
end;
end;
end;
do r = 1 to nn2;
do c = 1 to (p*d_frac);
if yy[r,c] < 0 then yy[r,c] = 0;
else if yy[r,c] = 0 then yy[r,c] = 1;
else if yy[r,c] > 0 then yy[r,c] = 1;
end;
end;
finish;

start VMAX(e_vec,n,vec_new);
evec_tmp = e_vec;
u_vec = J(n,1,0);
v_vec = J(n,1,0);
c_vec = J(n,1,0);
d_vec = J(n,1,0);
do i = 1 to n;
u_vec[i] = evec_tmp[i,1]**2 - evec_tmp[i,2]**2;
v_vec[i] = 2#evec_tmp[i,1] # evec_tmp[i,2];
c_vec[i] = u_vec[i]**2 - v_vec[i]**2;
d_vec[i] = u_vec[i] * v_vec[i];
end;

I = J(1,n,1);
A = I * u_vec;
B = I * v_vec;
C = I * c_vec;
D= 2 * (I * d_vec);
E = D - ((2*A*B)/n);
F = C - ((A**2 - B**2)/n);
G = sqrt(E**2 + F**2);

*+ _____+
IF G=0 no rotation required
+ _____+;
if G ^= 0 then do;
cos4 = F/G;
cos2 = sqrt((1 + cos4)/2);
cos=sqrt((1 + cos2)/2);
sin=sqrt((1-cos2)/2);

*+ _____+
IF E < 0 then change sign of sin
+ _____+;

if e<0 then sin = -1#sin;
rotation=J(2,2);
rotation[1,1]=cos;
rotation[1,2]=(-1*sin);
rotation[2,1]=sin;
rotation[2,2]=cos;

```



## Appendix E (Continued)

```

    evec_new = evec_tmp * rotation;
    if G = 0 then evec_new = evec_tmp;
end;
finish;

start ROTATE(e_vec,h_vec);

n=nrow(e_vec);
n_fact = ncol(e_vec);

* +-----+
  Normalize the loadings
+-----+;
h_vec = J(n,1,0);

do i = 1 to n;
  do j = 1 to n_fact;
    h_vec[i] = h_vec[i] + e_vec[i,j]**2;
  end;
  h_vec[i] = sqrt(h_vec[i]);
  do j = 1 to n_fact;
    e_vec[i,j] = e_vec[i,j] / h_vec[i];
  end;
end;

* +-----+
  Compute the variance of the factor loadings
+-----+;
sum1 = 0;
sum2 = 0;
do i = 1 to n_fact;
  sum2a = 0;
  do j = 1 to n;
    sum1 = sum1 + e_vec[j,i]**4;
    sum2a = sum2a + e_vec[j,i]**2;
  end;
  sum2 = sum2+sum2a**2;
end;
V_old = (n*sum1) - sum2;

Change = 1;
do until (Change < .000001);
  do i = 1 to n_fact - 1;
    do j = i + 1 to n_fact;
      evec_tmp = e_vec[,i] || e_vec[,j];
      run VMAX(evec_tmp,n,evec_new);
      e_vec[,i] = evec_new[,1];
      e_vec[,j] = evec_new[,2];
    end;
  end;

  sum1 = 0;
  sum2 = 0;
  do i = 1 to n_fact;
    sum2a = 0;
    do j = 1 to n;
      sum1 = sum1 + e_vec[j,i]**4;
      sum2a = sum2a + e_vec[j,i]**2;
    end;
  end;
end;

```

## Appendix E (Continued)

```

    end;
    sum2 = sum2+sum2a##2;
end;
V = (n#sum1) - sum2;
Change = ABS(V_old - V);
* print V_old V Change;
V_old = V;
end;

do i = 1 to n;
do j = 1 to n_fact;
    e_vec[i,j] = e_vec[i,j] # h_vec[i];
end;
end;
* +-----+
  Compute final communalities
+-----+;
h_vec = J(n,1,0);
do i = 1 to n;
do j = 1 to n_fact;
    h_vec[i] = h_vec[i] + e_vec[i,j]##2;
end;
end;
finish;

* +-----+
  Subroutine computes bias, MSE and accuracy of factor loadings

  Inputs: L = matrix of sample pattern coefficients
          Lambda = matrix of population pattern coefficients

* +-----+;
START ACCURACY(L,Lambda,OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,
              pattern_accuracy,perfect_accuracy,
              pattern_accuracy30,perfect_accuracy30);

p = nrow(L);
k = ncol(L);

* +-----+
  Determine if each variable loads on at least
  one factor using the simple .30 rule of thumb
* +-----+;
pop_loaded = J(p,1,0);

do variable = 1 to p;
do factor = 1 to k;
    if abs(Lambda[variable,factor]) >= .30 then pop_loaded[variable,1] = 1;
end;
end;

* +-----+
  Determine if each variable loads on at least
  one factor using the simple .30 rule of thumb
* +-----+;

sample_loaded = J(p,1,0);
do variable = 1 to p;

```

## Appendix E (Continued)

```

do factor = 1 to k;
  if abs(L[variable,factor]) >= .30 then sample_loaded[variable,1] = 1;
end;
end;

* +-----+
  Count number of variables that load "somewhere"
  in both population and sample, and number that
  do not load "anywhere" in population and sample
* +-----+;
OK_Load = 0;
OK_NoLoad = 0;
do variable = 1 to p;
  if sample_loaded[variable,1] = 1 & pop_loaded[variable,1] = 1 then
    OK_Load = OK_Load + 1;
  if sample_loaded[variable,1] = 0 & pop_loaded[variable,1] = 0 then
    OK_NoLoad = OK_NoLoad + 1;
end;
OK_Load = OK_Load / sum(pop_loaded); * proportion of variables with loading agreement;

* +-----+
  Only compute OK_NoLoad if some variables do not
  load in the population
* +-----+;
  * +-----+
    25 Sept: Jeff changed these two statements, using p instead
    of k
  * +-----+;
if sum(pop_loaded) = p then OK_NoLoad = .;
if sum(pop_loaded) < p then OK_NoLoad = OK_NoLoad/(p-sum(pop_loaded));

* +-----+
  Compute bias and MS Error in the factor loadings
* +-----+;
bias_loadings=j(p,k,0);
MSE_loadings=j(p,k,0);
do row =1 to p;
  bias_loadings[row,]=L[row,]-lambda[row,];
  MSE_loadings[row,]=(L[row,]-lambda[row,])##2;
end;

* +-----+
  Compute pattern accuracy using the .30 thumb
* +-----+;

pattern_accuracy = j(p,k,0);
perfect_accuracy = 0;
pattern_accuracy30 = j(p,1,0);
perfect_accuracy30 = 0;

  * +-----+
    25 Sept: Jeff changed this loop to include the counter NoLoad
    If a variable does not load (abs>.30) on any factor
    in both population and sample, 'credit' is given for
    pattern accuracy
  * +-----+;
do variable = 1 to p;
  NoLoad = 0;

```

## Appendix E (Continued)

```

do factor = 1 to k;
  if (abs(Lambda[variable,factor]) >= .30 &
      abs(L[variable,factor]) >= .30)
    then do;
      pattern_accuracy[variable,factor] = 1;
      pattern_accuracy30[variable,1] = 1;
    end;
  if (abs(Lambda[variable,factor]) < .30 &
      abs(L[variable,factor]) < .30) then do;
    pattern_accuracy[variable,factor] = 1;
    NoLoad = NoLoad + 1;
  end;
end;
if NoLoad = k then pattern_accuracy30[variable,1] = 1;
end;
if mean(pattern_accuracy) = 1 then perfect_accuracy = 1;
if mean(pattern_accuracy30) = 1 then perfect_accuracy30 = 1;

finish;

*+ _____ +
Subroutine to calculate coefficient of congruence - mean phi
Inputs: pop = population factor loadings
        samp = sample factor loadings
Output: Meanphi
+ _____ +;

start get_phi(pop,samp,meanphi);
k = ncol(pop);
num = vecdiag(pop`samp);
den1 = vecdiag(pop`pop);
den2 = vecdiag(samp`samp);
phi_k = J(k,1,0);
meanphi = 0;
n_terms = 0;
do m = 1 to k;
  if (den1[m,1]>0 & den2[m,1]>0) then do;
    phi_k[m,1] = num[m,1] / sqrt(den1[m,1]#den2[m,1]);
    meanphi = meanphi + phi_k[m,1];
    n_terms = n_terms + 1;
  end;
  if (den1[m,1] = 0 | den2[m,1] = 0) then do;
    phi_k[m,1] = 999;
  end;
end;
meanphi = meanphi / n_terms;
finish;

start factor_scores(Lambda,L,sampdat,R_fscores,Non_zero);
*+ _____ +
Compute Factor Score Estimates
Create a matrix of scoring coefficients for the sample (SC) [p X k]

```

## Appendix E (Continued)

Create a matrix of scoring coefficients for the population (SCP)[p X k]  
Retrieve original sample data [n x p]  
Compute Factor Score estimates (FSE) using SC [p X k][n x p] = [p x p]  
Compute Factor Score estimates (FS) using SCP [p X k][n x p] = [p x p]  
Correlation between factor score estimates for population and sample  
Extract diagonal elements in lower left quadrant

Output: a vector of k correlations  
a vector of k flags to indicate if scores were possible for  
each factor (population) using the .30 criterion

```
+_____+;

p = nrow(L);
k = ncol(L);
SC =J(p,k,0);
SCP =J(p,k,0);
Non_zero = J(k,1,0);

do j = 1 to p;
  do kk = 1 to k;
    if L [j,kk] >= .3 then SC[j,kk]=1;
    if L [j,kk] <= -.3 then SC[j,kk]=-1;

    if Lambda [j,kk] >= .3 then do;
      SCP[j,kk]=1;
      Non_zero[kk,1] = 1;
    end;
    if Lambda [j,kk] <= -.3 then do;
      SCP[j,kk]=-1;
      Non_zero[kk,1] = 1;
    end;
  end;
end;

FSE = sampdat*SC;
FS = sampdat*SCP;
FS_FSE = FSE||FS;
R_matrix = corr(FS_FSE);
R_matrix2 = R_matrix[k+1:2#k,1:k];
R_fscores = vecdiag(R_matrix2);
finish;

start PAF_extract (R_samp, k, L, Lambda, pop, samp, sampdat, e_vec, e_val, h_vec,
OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,pattern_accuracy,
perfect_accuracy, pattern_accuracy30, perfect_accuracy30, meanphi, R_fscores, Non_zero,
ext_type);

ext_type = 1;

R = R_samp;
n_vars = nrow(R);
ones_mtx = j(n_vars,n_vars,1);
diag_ones = diag(ones_mtx);
smc_mtx = ones_mtx - (1/(inv(R)));
h2a = diag(smc_mtx);
```

## Appendix E (Continued)

```
h_vec = vecdiag(h2a);
R_star = R-diag_ones + h2a;
all_e_val = eigval(R_star);
e_val = all_e_val[1:k,];
abs_e_val = abs(e_val);
sqrt_e_val = sqrt(abs_e_val);
diag_e_val = diag(sqrt_e_val);
all_e_vec = eigvec(R_star);
e_vec = all_e_vec[,1:k];
Fpaf = e_vec*diag_e_val;
Fvar_mtx = Fpaf`*Fpaf;
Fpaf_var = vecdiag(Fvar_mtx);

run ROTATE(e_vec,h_vec);
L = e_vec;

run ACCURACY(L,Lambda,OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,
             pattern_accuracy,perfect_accuracy,
             pattern_accuracy30,perfect_accuracy30);

pop = Lambda;
samp = L;

run get_phi(pop,samp,meanphi);

run factor_scores(Lambda,L,sampdat,R_fscores,Non_zero);

Finish;

start OLS_extract (R_samp,k, L, Lambda, pop,samp,sampdat, e_vec
,e_val,h_vec,OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,pattern_accuracy,
perfect_accuracy, pattern_accuracy30, perfect_accuracy30, meanphi, R_fscores,
Non_zero,ext_type);
* +-----+
  Subroutine to extract common factors from a correlation matrix
  using SMCs on the diagonal.

  INPUTS: R_mtx  -- sample correlation matrix
          n_fact -- number of factors to retain

  OUTPUTS: e_Val  -- vector of eigenvalues
           e_Vec -- matrix of eigenvectors

  Note: Jeff modified this subroutine on 26 October 2000 to
  produce iterated principal factors (closer to ML results
  to check with Kano tables).

<See pages 175-179 in Harman, 1976>

* +-----+;

ext_type = 2;

R = R_samp;
n_fact = k;
n_vars = nrow(R);
inv_r = inv(R);
```

## Appendix E (Continued)

```

h2a = J(n_vars,1,0);
do i = 1 to n_vars;
    h2a[i,1] = 1-1/inv_r[i,i];
    r[i,i] = h2a[i,1];
end;
oldcomm = h2a;
change = 1;
do until (change < .0001);
    e_val = eigval(R);
    e_val = e_val[1:n_fact,];
    eVec_orig = eigvec(R);
    eVec_orig = eVec_orig[,1:n_fact];

    e_vec = J(n_vars,n_fact,0);
    do i = 1 to n_vars;
        do j = 1 to n_fact;
            if (e_val[j,1] < 0) then e_val[j,1] = 0;
            e_vec[i,j] = eVec_orig[i,j]*sqrt(e_val[j,1]);
        end;
    end;
    change = 0;
    h_vec = h2a;
end;

run ROTATE(e_vec,h_vec);
L = e_vec;

run ACCURACY(L,Lambda,OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,
    pattern_accuracy,perfect_accuracy,
    pattern_accuracy30,perfect_accuracy30);

pop = Lambda;
samp = L;

run get_phi(pop,samp,meanphi);

run factor_scores(Lambda,L,sampdat,R_fscores,Non_zero);

finish;

/*Maximum Likelihood method adapted from Chen, R. (2003)*/
start mlfa;

ext_type = 3;
s = r_samp;

/*    objective function module        */

start objfun (x) global(s, k, p, xpsy, sigma, lambda, omega, f);
    xpsy = diag(1/sqrt(x));
    A = xpsy*s*xpsy;
    Omega = (eigvec(A)) [,1:k];
    theta = (eigval(A)) [1:k];
    lambda = diag(sqrt(x))*omega*diag(sqrt(abs(theta-1)));
    sigma = lambda*t(lambda)+diag(x);
    f = log(det(sigma))+trace(s*inv(sigma))-log(det(s))-p;
    return(f);

```

## Appendix E (Continued)

```
finish;

/* Gradient function module */

start gradfun(x) global(s, sigma);
    invpsy = inv(diag(x));
    g = (vecdiag(invpsy*(sigma-s)*invpsy))`;
    return(g);
finish;

/* starting values for iteration */
p = ncol(s);
x = (1-k/(2#p))#(1/vecdiag(inv(s)));

/* Set up Options and Constraints */
option = {0 0 . 4};
con = j(1,p, 0.00001)//j(1,p,.);

call nlpqn(rc, xr, "objfun", x, option, con) grd = "gradfun";

r_star = lambda*lambda`;
h_vec = vecdiag(r_star);
all_e_vec = eigvec(R_star);
e_vec = all_e_vec[,1:k];
finish;

start mlfa_extract;

run mlfa;

if rc < 0 then m_rc0 = 1; else m_rc0 = 0;
if rc = 3 then m_rc3 = 1; else m_rc3 = 0;
if rc = 6 then m_rc6 = 1; else m_rc6 = 0;

run ROTATE(e_vec,h_vec);
L = e_vec;

run ACCURACY(L,Lambda,OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,
    pattern_accuracy,perfect_accuracy,
    pattern_accuracy30,perfect_accuracy30);

pop = Lambda;
samp = L;

run get_phi(pop,samp,meanphi);

run factor_scores(Lambda,L,sampdat,R_fscores,Non_zero);

finish;

Do pop_num = 1 to N_pops; * Loop for 10 populations;

run Make_PopR(p,k,commun_type,A1,R_pop);
Lambda = A1;

numr = r_pop[+,+] - p;
```



## Appendix E (Continued)

```

deno = r_pop[+,+];
ratio = numr/deno;
f2_pop = (p/(p-1))*ratio;
r2_pop = f2_pop/(1+f2_pop);
corr = r_pop;
seed1=round(1000000*ranran(0));
*nn = 100000;
chg = 1;
cycle = 0;
corr_tmp = corr;
do until (chg = 0);
    run gendata2a(NN1,seed1,variance,1,0,0,means,corr_tmp,sim_data,p,d_frac);
    sim_corr = corr(sim_data);
    resid_m = sim_corr - corr;
    tot_res = sum(abs(resid_m));
    if cycle = 0 then do;
        best_corr = corr_tmp;
        best_res = tot_res;
    end;
    if cycle > 0 then do;
        if tot_res < best_res then do;
            best_corr = corr_tmp;
            best_res = tot_res;
        end;
    end;
    if tot_res < (.005#(((p-1)#p)/2)) then CHG = 0; * Convergence!;
    if cycle > 30 then do;
    if tot_res < (.01#(((p-1)#p)/2)) then CHG = 0; * Convergence!;
    end;
    if cycle > 200 then CHG = 0;
    if CHG = 1 then corr_tmp = corr_tmp - resid_m; * adjust template and simulate another
large sample;
    cycle = cycle + 1;
    if CHG = 0 then do;
        end;
    end;
end;

Do S_Size = 1 to 4;      * Loop for sample sizes;

    if S_Size = 1 then Sampsize2=100;
    if S_Size = 2 then Sampsize2=200;
    if S_Size = 3 then Sampsize2=300;
    if S_Size = 4 then Sampsize2=1000;

Do rep=1 to replicat;    * Loop for 1000 Samples;

*****;
*   Begin data generation   *;
*****;

seed1=round(1000000*ranuni(0));
nn2 = sampsize2;
corr_tmp = best_corr;

r_sing = 0;

```

## Appendix E (Continued)

```
do until (det(r_samp) > 0);
  run gendata2b(NN2,seed1,variance,1,0,0,means,corr_tmp,sim_data,p,d_frac);
  sampdat = sim_data;
  r_samp=corr(sampdat);
  if det(r_samp)<=0 then do;
    r_sing = r_sing +1;
  end;
end;

if rep = 1 then _r_sing = r_sing;
if rep > 1 then _r_sing = _r_sing + r_sing;

run PAF_extract (R_samp, k, L, Lambda, pop, samp, sampdat, e_vec, e_val, h_vec,
OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,pattern_accuracy,
perfect_accuracy, pattern_accuracy30, perfect_accuracy30, meanphi, R_fscores, Non_zero,
ext_type);

ext_type_p = ext_type;

if rep = 1 then do;
  _ok_load_p = ok_load;
  _ok_NoLoad_p = ok_NoLoad;
  _Bias_Loadings_p = Bias_Loadings;
  _MSE_Loadings_p = MSE_Loadings;
  _Pattern_Accuracy_p = Pattern_Accuracy;
  _Perfect_Accuracy_p = Perfect_Accuracy;
  _Pattern_Accuracy30_p = Pattern_Accuracy30;
  _Perfect_Accuracy30_p = Perfect_Accuracy30;
  _meanphi_p = meanphi;
  _R_Fscores_p = R_Fscores;
  _Non_zero_p = Non_zero;
end;

if rep > 1 then do;
  _ok_load_p = _ok_load_p + ok_load;
  _ok_NoLoad_p = _ok_NoLoad_p + ok_NoLoad;
  _Bias_Loadings_p = _Bias_Loadings_p + Bias_Loadings;
  _MSE_Loadings_p = _MSE_Loadings_p + MSE_Loadings;
  _Pattern_Accuracy_p = _Pattern_Accuracy_p + Pattern_Accuracy;
  _Perfect_Accuracy_p = _Perfect_Accuracy_p + Perfect_Accuracy;
  _Pattern_Accuracy30_p = _Pattern_Accuracy30_p + Pattern_Accuracy30;
  _Perfect_Accuracy30_p = _Perfect_Accuracy30_p + Perfect_Accuracy30;
  _meanphi_p = _meanphi_p + meanphi;
  _R_Fscores_p = _R_Fscores_p + R_Fscores;
  _Non_zero_p = _Non_zero_p + Non_zero;
end;

run OLS_extract (R_samp,k, L, Lambda, pop,samp,sampdat, e_vec
,e_val,h_vec,OK_Load,OK_NoLoad,bias_loadings,MSE_loadings,pattern_accuracy,
perfect_accuracy, pattern_accuracy30, perfect_accuracy30, meanphi, R_fscores,
Non_zero,ext_type);

ext_type_o = ext_type;
```

## Appendix E (Continued)

```
if rep = 1 then do;
  _ok_load_o = ok_load;
  _ok_Noload_o = ok_Noload;
  _Bias_Loadings_o = Bias_Loadings;
  _MSE_Loadings_o = MSE_Loadings;
  _Pattern_Accuracy_o = Pattern_Accuracy;
  _Perfect_Accuracy_o = Perfect_Accuracy;
  _Pattern_Accuracy30_o = Pattern_Accuracy30;
  _Perfect_Accuracy30_o = Perfect_Accuracy30;
  _meanphi_o = meanphi;
  _R_Fscores_o = R_Fscores;
  _Non_zero_o = Non_zero;
end;

if rep > 1 then do;
  _ok_load_o = _ok_load_o + ok_load;
  _ok_Noload_o = _ok_Noload_o + ok_Noload;
  _Bias_Loadings_o = _Bias_Loadings_o + Bias_Loadings;
  _MSE_Loadings_o = _MSE_Loadings_o + MSE_Loadings;
  _Pattern_Accuracy_o = _Pattern_Accuracy_o + Pattern_Accuracy;
  _Perfect_Accuracy_o = _Perfect_Accuracy_o + Perfect_Accuracy;
  _Pattern_Accuracy30_o = _Pattern_Accuracy30_o + Pattern_Accuracy30;
  _Perfect_Accuracy30_o = _Perfect_Accuracy30_o + Perfect_Accuracy30;
  _meanphi_o = _meanphi_o + meanphi;
  _R_Fscores_o = _R_Fscores_o + R_Fscores;
  _Non_zero_o = _Non_zero_o + Non_zero;
end;

run mlfa_extract;

ext_type_m = ext_type;

if rep = 1 then do;
  _m_rc0 = m_rc0;
  _m_rc3 = m_rc3;
  _m_rc6 = m_rc6;
  _ok_load_m = ok_load;
  _ok_Noload_m = ok_Noload;
  _Bias_Loadings_m = Bias_Loadings;
  _MSE_Loadings_m = MSE_Loadings;
  _Pattern_Accuracy_m = Pattern_Accuracy;
  _Perfect_Accuracy_m = Perfect_Accuracy;
  _Pattern_Accuracy30_m = Pattern_Accuracy30;
  _Perfect_Accuracy30_m = Perfect_Accuracy30;
  _meanphi_m = meanphi;
  _R_Fscores_m = R_Fscores;
  _Non_zero_m = Non_zero;
end;

if rep > 1 then do;
  _m_rc0 = _m_rc0 + m_rc0;
  _m_rc3 = _m_rc3 + m_rc3;
  _m_rc6 = _m_rc6 + m_rc6;
  _ok_load_m = _ok_load_m + ok_load;
  _ok_Noload_m = _ok_Noload_m + ok_Noload;
  _Bias_Loadings_m = _Bias_Loadings_m + Bias_Loadings;
  _MSE_Loadings_m = _MSE_Loadings_m + MSE_Loadings;
```

## Appendix E (Continued)

```
_Pattern_Accuracy_m = _Pattern_Accuracy_m + Pattern_Accuracy;
_Perfect_Accuracy_m = _Perfect_Accuracy_m + Perfect_Accuracy;
  _Pattern_Accuracy30_m = _Pattern_Accuracy30_m + Pattern_Accuracy30;
_Perfect_Accuracy30_m = _Perfect_Accuracy30_m + Perfect_Accuracy30;
_meanphi_m = _meanphi_m + meanphi;
_R_Fscores_m = _R_Fscores_m + R_Fscores;
_Non_zero_m = _Non_zero_m + Non_zero;
end;

if rep = 1 then nsamples = 1;
if rep > 1 then nsamples = nsamples +1;

end; *End Replications Loop*;

_ok_load_p = _ok_load_p/NSamples;
_ok_Noload_p = _ok_Noload_p/NSamples;
_Bias_Loadings_p = _BIAs_Loadings_p/NSamples;
_MSE_Loadings_p = _MSE_Loadings_p/NSamples;
_Pattern_Accuracy_p = _Pattern_Accuracy_p/NSamples;
_Perfect_Accuracy_p = _Perfect_Accuracy_p/NSamples;
_Pattern_Accuracy30_p = _Pattern_Accuracy30_p/NSamples;
_Perfect_Accuracy30_p = _Perfect_Accuracy30_p/NSamples;
_meanphi_p = _meanphi_p/NSamples;
_R_Fscores_p = _R_Fscores_p/NSamples;
_Non_zero_p = _Non_zero_p/NSamples;

_ok_load_o = _ok_load_o/NSamples;
_ok_Noload_o = _ok_Noload_o/NSamples;
_Bias_Loadings_o = _BIAs_Loadings_o/NSamples;
_MSE_Loadings_o = _MSE_Loadings_o/NSamples;
_Pattern_Accuracy_o = _Pattern_Accuracy_o/NSamples;
_Perfect_Accuracy_o = _Perfect_Accuracy_o/NSamples;
_Pattern_Accuracy30_o = _Pattern_Accuracy30_o/NSamples;
_Perfect_Accuracy30_o = _Perfect_Accuracy30_o/NSamples;
_meanphi_o = _meanphi_o/NSamples;
_R_Fscores_o = _R_Fscores_o/NSamples;
_Non_zero_o = _Non_zero_o/NSamples;

_m_rc0 = _m_rc0;
_m_rc3 = _m_rc3;
_m_rc6 = _m_rc6;
_ok_load_m = _ok_load_m/NSamples;
_ok_Noload_m = _ok_Noload_m/NSamples;
_Bias_Loadings_m = _BIAs_Loadings_m/NSamples;
_MSE_Loadings_m = _MSE_Loadings_m/NSamples;
_Pattern_Accuracy_m = _Pattern_Accuracy_m/NSamples;
_Perfect_Accuracy_m = _Perfect_Accuracy_m/NSamples;
_Pattern_Accuracy30_m = _Pattern_Accuracy30_m/NSamples;
_Perfect_Accuracy30_m = _Perfect_Accuracy30_m/NSamples;
_meanphi_m = _meanphi_m/NSamples;
_R_Fscores_m = _R_Fscores_m/NSamples;
_Non_zero_m = _Non_zero_m/NSamples;

_r_sing = _r_sing;

print Sampsize2 commun_type d_frac k p;
print ' ';
```

## Appendix E (Continued)

```
print _OK_Load_p _OK_Load_o _OK_Load_m _OK_NoLoad_p _OK_NoLoad_o _OK_NoLoad_m
_Perfect_Accuracy30_p _Perfect_Accuracy30_o _Perfect_Accuracy30_m
_Perfect_Accuracy_p _Perfect_Accuracy_o _Perfect_Accuracy_m _Meanphi_p _Meanphi_o
_Meanphi_m;
print ' ';
print _Pattern_Accuracy30_p _Pattern_Accuracy30_o _Pattern_Accuracy30_m _R_Fscores_p
_R_Fscores_o _R_Fscores_m _Non_zero_p _Non_zero_o _Non_zero_m;
print ' ';
print _Bias_Loadings_p _Bias_Loadings_o _Bias_Loadings_m;
print ' ';
print _MSE_Loadings_p _MSE_Loadings_o _MSE_Loadings_m;
print ' ';
print _Pattern_Accuracy_p _Pattern_Accuracy_o _Pattern_Accuracy_m;
print ' ';
print _m_rc0 _m_rc3 _m_rc6 _r_sing;

    end; *End Sample Size Loop*;
end; *End loop for 10 populations*;

quit;
```

## Appendix F

### Analysis Program: SAS, IML©

```
data a;
infile 'C:\kcoughlin document\dissertation\data files\refined merged data 07242012.lst' lrecl
= 256 missover pad;

input test_var $ 1 - 256 @;
rec_num = _n_;
file = 'kcoughlin';

if index(test_var, 'SAMPSIZE2') ^=0 OR index(test_var, 'Sampsize2') ^=0 then record_type =
'Record 10';
if index(test_var, '_OK_LOAD_P') ^=0 OR index(test_var, '_ok_load_p') ^=0 then record_type =
'Record 20';
if index(test_var, '_PATTERN_ACCURACY30_P') ^=0 OR index(test_var, '_Pattern_Accuracy30_p') ^=0
then record_type = 'Record 30';
if index(test_var, '_BIAS_LOADINGS_P') ^=0 OR index(test_var, '_Bias_Loadings_p') ^=0 then
record_type = 'Record 40';
if index(test_var, '_BIAS_LOADINGS_M') ^=0 OR index(test_var, '_Bias_Loadings_m') ^=0 then
record_type = 'Record 42';
if index(test_var, '_MSE_LOADINGS_P') ^=0 OR index(test_var, '_MSE_Loadings_p') ^=0 then
record_type = 'Record 50';
if index(test_var, '_MSE_LOADINGS_M') ^=0 OR index(test_var, '_MSE_Loadings_m') ^=0 then
record_type = 'Record 52';
if index(test_var, '_PATTERN_ACCURACY_P') ^=0 OR index(test_var, '_Pattern_Accuracy_p') ^=0
then record_type = 'Record 60';
if index(test_var, '_PATTERN_ACCURACY_M') ^=0 OR index(test_var, '_Pattern_Accuracy_m') ^=0
then record_type = 'Record 62';
if index(test_var, '_M_RC0') ^=0 OR index(test_var, '_m_rc0') ^=0 then record_type = 'Record
70';

if record_type = 'Record 10' then do;
input @1 dummy 1. Sampsize2 Commun_type2 d_frac2 k2 p2;
if D_FRAC2 ne . then do;
P = P2;
K = K2;
COMMUN_TYPE = COMMUN_TYPE2;
D_FRAC = D_FRAC2;
SAMPSIZE = SAMPSIZE2;
end;
end;

* retain Sampsize2 Commun_type d_frac k p;

if record_type = 'Record 20' then do;
input @1 dummy 1. _ok_load_p _ok_load_o _ok_load_m _ok_Noload_p _ok_Noload_o
_ok_Noload_m _Perfect_Accuracy30_p _Perfect_Accuracy30_o _Perfect_Accuracy30_m
_Peact_Accuracy_p _Perfect_Accuracy_o _Perfect_Accuracy_m _meanphi_p _meanphi_o _meanphi_m;
end;
```

## Appendix F (Continued)

```
if record_type = 'Record 30' then do;
    input @1 dummy 1. _Pattern_Accuracy30_p _Pattern_Accuracy30_o _Pattern_Accuracy30_m
    _R_Fscores_p _R_Fscores_o _R_Fscores_m _Non_zero_p _Non_zero_o _Non_zero_m;
end;

if record_type = 'Record 40' and k = 8 then do;
    input @1 dummy 1.
    _Bias_Loadings_p1 _Bias_Loadings_p2 _Bias_Loadings_p3 _Bias_Loadings_p4 _Bias_Loadings_p5
    _Bias_Loadings_p6 _Bias_Loadings_p7 _Bias_Loadings_p8
    _Bias_Loadings_o1 _Bias_Loadings_o2 _Bias_Loadings_o3 _Bias_Loadings_o4;
end;

if record_type = 'Record 42' and k = 2 then do;
    input @1 dummy 1.
    _Bias_Loadings_p1 _Bias_Loadings_p2
    _Bias_Loadings_o1 _Bias_Loadings_o2
    _Bias_Loadings_m1 _Bias_Loadings_m2;
end;

if record_type = 'Record 42' and k = 4 then do;
    input @1 dummy 1.
    _Bias_Loadings_p1 _Bias_Loadings_p2 _Bias_Loadings_p3 _Bias_Loadings_p4
    _Bias_Loadings_o1 _Bias_Loadings_o2 _Bias_Loadings_o3 _Bias_Loadings_o4
    _Bias_Loadings_m1 _Bias_Loadings_m2 _Bias_Loadings_m3 _Bias_Loadings_m4;
end;

if record_type = 'Record 42' and k = 8 then do;
    input @1 dummy 1.
    _Bias_Loadings_o5 _Bias_Loadings_o6 _Bias_Loadings_o7 _Bias_Loadings_o8
    _Bias_Loadings_m1 _Bias_Loadings_m2 _Bias_Loadings_m3 _Bias_Loadings_m4 _Bias_Loadings_m5
    _Bias_Loadings_m6 _Bias_Loadings_m7 _Bias_Loadings_m8;
end;

if record_type = 'Record 50' and k = 8 then do;
    input @1 dummy 1.
    _MSE_Loadings_p1 _MSE_Loadings_p2 _MSE_Loadings_p3 _MSE_Loadings_p4 _MSE_Loadings_p5
    _MSE_Loadings_p6 _MSE_Loadings_p7 _MSE_Loadings_p8
    _MSE_Loadings_o1 _MSE_Loadings_o2 _MSE_Loadings_o3 _MSE_Loadings_o4;
end;

if record_type = 'Record 52' and k = 2 then do;
    input @1 dummy 1.
    _MSE_Loadings_p1 _MSE_Loadings_p2
    _MSE_Loadings_o1 _MSE_Loadings_o2
    _MSE_Loadings_m1 _MSE_Loadings_m2;
end;

if record_type = 'Record 52' and k = 4 then do;
    input @1 dummy 1.
    _MSE_Loadings_p1 _MSE_Loadings_p2 _MSE_Loadings_p3 _MSE_Loadings_p4
    _MSE_Loadings_o1 _MSE_Loadings_o2 _MSE_Loadings_o3 _MSE_Loadings_o4
    _MSE_Loadings_m1 _MSE_Loadings_m2 _MSE_Loadings_m3 _MSE_Loadings_m4;
end;

if record_type = 'Record 52' and k = 8 then do;
    input @1 dummy 1.
    _MSE_Loadings_o5 _MSE_Loadings_o6 _MSE_Loadings_o7 _MSE_Loadings_o8
```

## Appendix F (Continued)

```
_MSE_Loadings_m1 _MSE_Loadings_m2 _MSE_Loadings_m3 _MSE_Loadings_m4 _MSE_Loadings_m5
_MSE_Loadings_m6 _MSE_Loadings_m7 _MSE_Loadings_m8;
end;

if record_type = 'Record 60' and k = 8 then do;
    input @1 dummy 1.
    _Pattern_Accuracy_p1 _Pattern_Accuracy_p2 _Pattern_Accuracy_p3 _Pattern_Accuracy_p4
    _Pattern_Accuracy_p5 _Pattern_Accuracy_p6 _Pattern_Accuracy_p7 _Pattern_Accuracy_p8
    _Pattern_Accuracy_o1 _Pattern_Accuracy_o2 _Pattern_Accuracy_o3 _Pattern_Accuracy_o4;
end;

if record_type = 'Record 62' and k = 2 then do;
    input @1 dummy 1.
    _Pattern_Accuracy_p1 _Pattern_Accuracy_p2
    _Pattern_Accuracy_o1 _Pattern_Accuracy_o2
    _Pattern_Accuracy_m1 _Pattern_Accuracy_m2;
end;

if record_type = 'Record 62' and k = 4 then do;
    input @1 dummy 1.
    _Pattern_Accuracy_p1 _Pattern_Accuracy_p2 _Pattern_Accuracy_p3 _Pattern_Accuracy_p4
    _Pattern_Accuracy_o1 _Pattern_Accuracy_o2 _Pattern_Accuracy_o3 _Pattern_Accuracy_o4
    _Pattern_Accuracy_m1 _Pattern_Accuracy_m2 _Pattern_Accuracy_m3 _Pattern_Accuracy_m4;
end;

if record_type = 'Record 62' and k = 8 then do;
    input @1 dummy 1.
    _Pattern_Accuracy_o5 _Pattern_Accuracy_o6 _Pattern_Accuracy_o7 _Pattern_Accuracy_o8
    _Pattern_Accuracy_m1 _Pattern_Accuracy_m2 _Pattern_Accuracy_m3 _Pattern_Accuracy_m4
    _Pattern_Accuracy_m5 _Pattern_Accuracy_m6 _Pattern_Accuracy_m7 _Pattern_Accuracy_m8;
end;

if record_type = 'Record 70' then do;
    input @1 dummy 1. _m_rc0 _m_rc3 _m_rc6;
end;

retain record_type Sampsize Commun_type d_frac k p;
drop test_var;

if Sampsize = . and _ok_load_p = . and _Pattern_Accuracy30_p = . and _Bias_Loadings_p1 = . and
_MSE_Loadings_p1 = . and _Pattern_Accuracy_p1 = . and _m_rc0 = . then delete;

data a1;
set a;
_perfect_accuracy_o = _perfect_accuracy_o/10000;

proc sort data = a1;
by Sampsize Commun_type d_frac k p;
proc means noprint data = a1;
by Sampsize Commun_type d_frac k p;
var _ok_load_p _ok_load_o _ok_load_m _ok_Noload_p _ok_Noload_o _ok_Noload_m
_Perfect_Accuracy30_p _Perfect_Accuracy30_o _Perfect_Accuracy30_m
_Perfect_Accuracy_p _Perfect_Accuracy_o _Perfect_Accuracy_m _meanphi_p _meanphi_o _meanphi_m
_Pattern_Accuracy30_p _Pattern_Accuracy30_o
_Pattern_Accuracy30_m _R_Fscores_p _R_Fscores_o _R_Fscores_m _Non_zero_p _Non_zero_o
_Non_zero_m
```



## Appendix F (Continued)

```
_Bias_Loadings_p1 _Bias_Loadings_p2 _Bias_Loadings_p3 _Bias_Loadings_p4 _Bias_Loadings_p5
_Bias_Loadings_p6 _Bias_Loadings_p7 _Bias_Loadings_p8
_Bias_Loadings_o1 _Bias_Loadings_o2 _Bias_Loadings_o3 _Bias_Loadings_o4 _Bias_Loadings_o5
_Bias_Loadings_o6 _Bias_Loadings_o7 _Bias_Loadings_o8
_Bias_Loadings_m1 _Bias_Loadings_m2 _Bias_Loadings_m3 _Bias_Loadings_m4 _Bias_Loadings_m5
_Bias_Loadings_m6 _Bias_Loadings_m7 _Bias_Loadings_m8
_MSE_Loadings_p1 _MSE_Loadings_p2 _MSE_Loadings_p3 _MSE_Loadings_p4 _MSE_Loadings_p5
_MSE_Loadings_p6 _MSE_Loadings_p7 _MSE_Loadings_p8
_MSE_Loadings_o1 _MSE_Loadings_o2 _MSE_Loadings_o3 _MSE_Loadings_o4 _MSE_Loadings_o5
_MSE_Loadings_o6 _MSE_Loadings_o7 _MSE_Loadings_o8
_MSE_Loadings_m1 _MSE_Loadings_m2 _MSE_Loadings_m3 _MSE_Loadings_m4 _MSE_Loadings_m5
_MSE_Loadings_m6 _MSE_Loadings_m7 _MSE_Loadings_m8
_Pattern_Accuracy_p1 _Pattern_Accuracy_p2 _Pattern_Accuracy_p3 _Pattern_Accuracy_p4
_Pattern_Accuracy_p5 _Pattern_Accuracy_p6 _Pattern_Accuracy_p7 _Pattern_Accuracy_p8
_Pattern_Accuracy_o1 _Pattern_Accuracy_o2 _Pattern_Accuracy_o3 _Pattern_Accuracy_o4
_Pattern_Accuracy_o5 _Pattern_Accuracy_o6 _Pattern_Accuracy_o7 _Pattern_Accuracy_o8
_Pattern_Accuracy_m1 _Pattern_Accuracy_m2 _Pattern_Accuracy_m3 _Pattern_Accuracy_m4
_Pattern_Accuracy_m5 _Pattern_Accuracy_m6 _Pattern_Accuracy_m7 _Pattern_Accuracy_m8
_m_rc0 _m_rc3 _m_rc6;
output out = b mean = ;

/*descriptive statistics*/

data c;
set b;

_bias_loadings_p = mean (of _Bias_Loadings_p1 - _Bias_Loadings_p8);
_bias_loadings_o = mean (of _Bias_Loadings_o1 - _Bias_Loadings_o8);
_bias_loadings_m = mean (of _Bias_Loadings_m1 - _Bias_Loadings_m8);
_mse_leadings_p = mean (of _MSE_Loadings_p1 - _MSE_Loadings_p8);
_mse_leadings_o = mean (of _MSE_Loadings_o1 - _MSE_Loadings_o8);
_mse_leadings_m = mean (of _MSE_Loadings_m1 - _MSE_Loadings_m8);
_pattern_accuracy_p = mean (of _Pattern_Accuracy_p1 - _Pattern_Accuracy_p8);
_pattern_accuracy_o = mean (of _Pattern_Accuracy_o1 - _Pattern_Accuracy_o8);
_pattern_accuracy_m = mean (of _Pattern_Accuracy_m1 - _Pattern_Accuracy_m8);

proc means data = c;
var _ok_load_p _ok_load_o _ok_load_m _Perfect_Accuracy30_p _Perfect_Accuracy30_o
_Perfect_Accuracy30_m
_Perfect_Accuracy_p _Perfect_Accuracy_o _Perfect_Accuracy_m _meanphi_p _meanphi_o _meanphi_m
_Pattern_Accuracy30_p _Pattern_Accuracy30_o
_Pattern_Accuracy30_m _R_Fscores_p _R_Fscores_o _R_Fscores_m _Non_zero_p _Non_zero_o
_Non_zero_m
_bias_loadings_p _bias_loadings_o _bias_loadings_m
_mse_leadings_p _mse_leadings_o _mse_leadings_m
_pattern_accuracy_p _pattern_accuracy_o _pattern_accuracy_m
_m_rc0 _m_rc3 _m_rc6;

proc univariate normal data = c;
var _ok_load_p _ok_load_o _ok_load_m _Perfect_Accuracy30_p _Perfect_Accuracy30_o
_Perfect_Accuracy30_m
_Perfect_Accuracy_p _Perfect_Accuracy_o _Perfect_Accuracy_m _meanphi_p _meanphi_o _meanphi_m
_Pattern_Accuracy30_p _Pattern_Accuracy30_o
_Pattern_Accuracy30_m _R_Fscores_p _R_Fscores_o _R_Fscores_m _Non_zero_p _Non_zero_o
_Non_zero_m
_bias_loadings_p _bias_loadings_o _bias_loadings_m
_mse_leadings_p _mse_leadings_o _mse_leadings_m
_pattern_accuracy_p _pattern_accuracy_o _pattern_accuracy_m
```

## Appendix F (Continued)

```
_m_rc0 _m_rc3 _m_rc6;

/*repeated measures analyses*/

proc glm data = c;
class k p Sampsiz commun_type d_frac;
model _ok_load_p _ok_load_o _ok_load_m = k | p | Sampsiz | commun_type | d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c1;
means k*p p*commun_type k*d_frac k*commun_type;

proc iml;
use c1;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsize' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsize*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsize*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
```

## Appendix F (Continued)

```
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;
```

```
total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprt_n_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
```

## Appendix F (Continued)

```
print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

proc glm data = c;
class k p Sampsizе commun_type d_frac;
model _Perfect_Accuracy30_p _Perfect_Accuracy30_o _Perfect_Accuracy30_m = k | p | Sampsizе |
commun_type | d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c2;
means k*p k*commun_type Sampsizе*commun_type p*commun_type;

proc iml;
use c2;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPSIZE' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
```

## Appendix F (Continued)

```

read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsize*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsize*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprtn_var_expl = model_ss/total_ss;

```

## Appendix F (Continued)

```
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
```

## Appendix F (Continued)

```
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

proc glm data = c;
class k p Sampsiz commu_type d_frac;
model _Perfect_Accuracy_p _Perfect_Accuracy_o _Perfect_Accuracy_m = k | p | Sampsiz |
commu_type | d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c3;
means k*p k*sampsiz p*sampsiz;

proc iml;
use c3;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsize' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsize*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsize*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
```

## Appendix F (Continued)

```
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;
```

```
total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprt_n_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
```



## Appendix F (Continued)

```
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
```

```
print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;
```

```
proc glm data = c;
class k p Sampsiz commn_type d_frac;
model _meanphi_p _meanphi_o _meanphi_m = k | p | Sampsiz | commn_type | d_frac @2;
```

## Appendix F (Continued)

```
repeated method 3/printe;
ods output ModelANOVA = c4;
means k*p k*Sampsize k*d_frac commun_type*d_frac Sampsize*commun_type ;

proc iml;
use c4;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsize' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsize*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsize*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
```

## Appendix F (Continued)

```
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprtn_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
```

## Appendix F (Continued)

```
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

proc glm data = c;
class k p Sampsizе commun_type d_frac;
model _Pattern_Accuracy30_p _Pattern_Accuracy30_o _Pattern_Accuracy30_m = k | p | Sampsizе |
commun_type | d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c5;
means k*p k*commun_type sampsizе*commun_type p*commun_type;

proc iml;
use c5;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsize' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
```

## Appendix F (Continued)

```

read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPSIZE*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPSIZE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPSIZE' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPSIZE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPSIZE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPSIZE*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPSIZE*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprtn_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);

```

## Appendix F (Continued)

```
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
```

## Appendix F (Continued)

```
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

proc glm data = c;
class k p Sampsiz e commun_type d_frac;
model _R_Fscores_p _R_Fscores_o _R_Fscores_m = k | p | Sampsiz e | commun_type | d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c6;
means k*p p*commun_type k*sampsiz e k*commun_type;

proc iml;
use c6;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsiz e' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPsiz e' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsiz e' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsiz e*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsiz e*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsiz e' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsiz e' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
```

## Appendix F (Continued)

```
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprt_n_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
```



## Appendix F (Continued)

```
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

proc glm data = c;
class k p Sampsizе commun_type d_frac;
model _Non_zero_p _Non_zero_o _Non_zero_m = k | p | Sampsizе | commun_type | d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c7;
lsmeans k p Sampsizе commun_type d_frac;

proc iml;
use c7;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
```

## Appendix F (Continued)

```

read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPSIZE' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPSIZE' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPSIZE' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPSIZE*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPSIZE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPSIZE' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPSIZE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPSIZE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPSIZE*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPSIZE*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +

```

## Appendix F (Continued)

```
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn);
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprtn_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
```

## Appendix F (Continued)

```
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

proc glm data = c;
class k p Sampsiz commu_type d_frac;
model _bias_loadings_p _bias_loadings_o _bias_loadings_m = k | p | Sampsiz | commu_type |
d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c8;
means k*p k*commu_type k*d_frac k*Sampsiz;

proc iml;
use c8;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsize' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsize*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsize*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
```

## Appendix F (Continued)

```

read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

```

```

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprt_n_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);

```

## Appendix F (Continued)

```
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;
```

## Appendix F (Continued)

```
proc glm data = c;
class k p Sampsiz e comun_type d_frac;
model _mse_leadings_p _mse_leadings_o _mse_leadings_m = k | p | Sampsiz e | comun_type |
d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c9;
means k*comun_type k*p k*d_frac comun_type*d_frac Sampsiz e;

proc iml;
use c9;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsize' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsize*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsize*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
```

## Appendix F (Continued)

```
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprt_n_var_expl = model_ss/total_ss;
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
```



## Appendix F (Continued)

```
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

proc glm data = c;
class k p Sampsiz e commun_type d_frac;
model _pattern_accuracy_p _pattern_accuracy_o _pattern_accuracy_m = k | p | Sampsiz e |
commun_type | d_frac @2;
repeated method 3/printe;
ods output ModelANOVA = c10;
means k*p k*commun_type k*sampsiz e p*commun_type sampsiz e*commun_type;

proc iml;
use c10;

read all var {SS} where (Source = 'K' & DEPENDENT = 'BetweenSubjects') into SS_K;
read all var {SS} where (Source = 'P' & DEPENDENT = 'BetweenSubjects') into SS_P;
read all var {SS} where (Source = 'SAMPsize' & DEPENDENT = 'BetweenSubjects') into SS_SAMP;
read all var {SS} where (Source = 'COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into SS_COMM;
read all var {SS} where (Source = 'D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_DFRAC;
read all var {SS} where (Source = 'K*P' & DEPENDENT = 'BetweenSubjects') into SS_K_P;
```

## Appendix F (Continued)

```

read all var {SS} where (Source = 'K*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_K_SAMP;
read all var {SS} where (Source = 'K*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_K_COMM;
read all var {SS} where (Source = 'K*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_K_DFRAC;
read all var {SS} where (Source = 'P*SAMPsize' & DEPENDENT = 'BetweenSubjects') into
SS_P_SAMP;
read all var {SS} where (Source = 'P*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_P_COMM;
read all var {SS} where (Source = 'P*D_FRAC' & DEPENDENT = 'BetweenSubjects') into SS_P_DFRAC;
read all var {SS} where (Source = 'SAMPsize*COMMUN_TYPE' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_COMM;
read all var {SS} where (Source = 'SAMPsize*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_SAMP_DFRAC;
read all var {SS} where (Source = 'COMMUN_TYPE*D_FRAC' & DEPENDENT = 'BetweenSubjects') into
SS_COMM_DFRAC;
read all var {SS} where (Source = 'Error' & DEPENDENT = 'BetweenSubjects') into SS_Error_Btwn;
read all var {SS} where (Source = 'method' & DEPENDENT = 'WithinSubject') into SS_Method;
read all var {SS} where (Source = 'method*K' & DEPENDENT = 'WithinSubject') into SS_Method_K;
read all var {SS} where (Source = 'method*P' & DEPENDENT = 'WithinSubject') into SS_Method_P;
read all var {SS} where (Source = 'method*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP;
read all var {SS} where (Source = 'method*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_COMM;
read all var {SS} where (Source = 'method*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_DFRAC;
read all var {SS} where (Source = 'method*K*P' & DEPENDENT = 'WithinSubject') into
SS_Method_K_P;
read all var {SS} where (Source = 'method*K*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_K_SAMP;
read all var {SS} where (Source = 'method*K*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_K_COMM;
read all var {SS} where (Source = 'method*K*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_K_DFRAC;
read all var {SS} where (Source = 'method*P*SAMPsize' & DEPENDENT = 'WithinSubject') into
SS_Method_P_SAMP;
read all var {SS} where (Source = 'method*P*COMMUN_TYPE' & DEPENDENT = 'WithinSubject') into
SS_Method_P_COMM;
read all var {SS} where (Source = 'method*P*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_P_DFRAC;
read all var {SS} where (Source = 'method*SAMPsize*COMMUN_TYPE' & DEPENDENT = 'WithinSubject')
into SS_Method_SAMP_COMM;
read all var {SS} where (Source = 'method*SAMPsize*D_FRAC' & DEPENDENT = 'WithinSubject') into
SS_Method_SAMP_DFRAC;
read all var {SS} where (Source = 'method*COMMUN_TYPE*D_FRAC' & DEPENDENT = 'WithinSubject')
into SS_Method_COMM_DFRAC;
read all var {SS} where (Source = 'Error(method)' & DEPENDENT = 'WithinSubject') into
SS_Error_Wthn;

total_ss = SS_K + SS_P + SS_SAMP + SS_COMM + SS_DFRAC + SS_K_P + SS_K_SAMP + SS_K_COMM +
SS_K_DFRAC + SS_P_SAMP + SS_P_COMM + SS_P_DFRAC +
SS_SAMP_COMM + SS_SAMP_DFRAC + SS_COMM_DFRAC + SS_Error_Btwn + SS_Method + SS_Method_K +
SS_Method_P + SS_Method_SAMP + SS_Method_COMM +
SS_Method_DFRAC + SS_Method_K_P + SS_Method_K_SAMP + SS_Method_K_COMM + SS_Method_K_DFRAC +
SS_Method_P_SAMP + SS_Method_P_COMM + SS_Method_P_DFRAC +
SS_Method_SAMP_COMM + SS_Method_SAMP_DFRAC + SS_Method_COMM_DFRAC + SS_Error_Wthn;
model_ss = total_ss - (SS_Error_Btwn + SS_Error_Wthn);
prprtn_var_expl = model_ss/total_ss;

```

## Appendix F (Continued)

```
eta_sqr_g_method = SS_Method/(SS_Method + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k = SS_K/(SS_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p = SS_P/(SS_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp = SS_SAMP/(SS_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm = SS_COMM/(SS_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_dfrac = SS_DFRAC/(SS_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_p = SS_K_P/(SS_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_samp = SS_K_SAMP/(SS_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_comm = SS_K_COMM/(SS_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_k_dfrac = SS_K_DFRAC/(SS_K_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_samp = SS_P_SAMP/(SS_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_comm = SS_P_COMM/(SS_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_p_dfrac = SS_P_DFRAC/(SS_P_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_comm = SS_SAMP_COMM/(SS_SAMP_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_samp_dfrac = SS_SAMP_DFRAC/(SS_SAMP_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_comm_dfrac = SS_COMM_DFRAC/(SS_COMM_DFRAC + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k = SS_Method_K/(SS_Method_K + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p = SS_Method_P/(SS_Method_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_samp = SS_Method_Samp/(SS_Method_Samp + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_comm = SS_Method_Comm/(SS_Method_Comm + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_dfrac = SS_Method_Dfrac/(SS_Method_Dfrac + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_p = SS_Method_K_P/(SS_Method_K_P + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_samp = SS_Method_K_SAMP/(SS_Method_K_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_comm = SS_Method_K_COMM/(SS_Method_K_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_k_dfrac = SS_Method_K_DFRAC/(SS_Method_K_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_p_samp = SS_Method_P_SAMP/(SS_Method_P_SAMP + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_comm = SS_Method_P_COMM/(SS_Method_P_COMM + SS_Error_Btwn + SS_Error_Wthn);
eta_sqr_g_method_p_dfrac = SS_Method_P_DFRAC/(SS_Method_P_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_comm = SS_Method_SAMP_COMM/(SS_Method_SAMP_COMM + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_samp_dfrac = SS_Method_SAMP_DFRAC/(SS_Method_SAMP_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);
eta_sqr_g_method_comm_dfrac = SS_Method_COMM_DFRAC/(SS_Method_COMM_DFRAC + SS_Error_Btwn +
SS_Error_Wthn);

print eta_sqr_g_k;
print eta_sqr_g_p;
print eta_sqr_g_samp;
print eta_sqr_g_comm;
print eta_sqr_g_dfrac;
print eta_sqr_g_k_p;
print eta_sqr_g_k_samp;
print eta_sqr_g_k_comm;
print eta_sqr_g_k_dfrac;
print eta_sqr_g_p_samp;
print eta_sqr_g_p_comm;
print eta_sqr_g_p_dfrac;
print eta_sqr_g_samp_comm;
print eta_sqr_g_samp_dfrac;
print eta_sqr_g_comm_dfrac;
print eta_sqr_g_method;
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
```

## Appendix F (Continued)

```
print eta_sqr_g_method_k;
print eta_sqr_g_method_p;
print eta_sqr_g_method_samp;
print eta_sqr_g_method_comm;
print eta_sqr_g_method_dfrac;
print eta_sqr_g_method_k_p;
print eta_sqr_g_method_k_samp;
print eta_sqr_g_method_k_comm;
print eta_sqr_g_method_k_dfrac;
print eta_sqr_g_method_p_samp;
print eta_sqr_g_method_p_comm;
print eta_sqr_g_method_p_dfrac;
print eta_sqr_g_method_samp_comm;
print eta_sqr_g_method_samp_dfrac;
print eta_sqr_g_method_comm_dfrac;
print prprtn_var_expl;
quit;

/*Box and Whisker plots*/

data c1;
set c;
d_frac_prct = d_frac*100;

data d;
set c1;
K_x_P = k||'-'||p;
K_x_N = k||'-'||sampsize;
K_x_H = k||'-'||commun_type;
K_x_D = k||'-'||d_frac_prct;
P_x_H = p||'-'||commun_type;
P_x_N = p||'-'||sampsize;
N_x_H = sampsize||'-'||commun_type;
H_x_D = commun_type||'-'||d_frac_prct;

data d1;
set d;
Loading_sensitivity = _ok_load_p;
General_agreement = _Pattern_Accuracy30_p;
Per_element_agreement = _pattern_accuracy_p;
Total_agreement = _Perfect_Accuracy_p;
Mean_phi = _meanphi_p;
Factor_score_R = _R_Fscores_p;
Bias = _bias_loadings_p;
RMSE = _mse_leadings_p;
method = 'PAF';

data d2;
set d;
Loading_sensitivity = _ok_load_o;
General_agreement = _Pattern_Accuracy30_o;
Per_element_agreement = _pattern_accuracy_o;
Total_agreement = _Perfect_Accuracy_o;
Mean_phi = _meanphi_o;
Factor_score_R = _R_Fscores_o;
Bias = _bias_loadings_o;
```

## Appendix F (Continued)

```
RMSE = _mse_loadings_o;
method = 'OLS';

data d3;
set d;
Loading_sensitivity = _ok_load_m;
General_agreement = _Pattern_Accuracy30_m;
Per_element_agreement = _pattern_accuracy_m;
Total_agreement = _Perfect_Accuracy_m;
Mean_phi = _meanphi_m;
Factor_score_R = _R_Fscores_m;
Bias = _bias_loadings_m;
RMSE = _mse_loadings_m;
method = 'MAX';

data d_comb;
set d1;

proc append base = d_comb data = d2 force;

proc append base = d_comb data = d3 force;

proc sort data = d_comb;
by method;

proc means data = d_comb;
var Loading_sensitivity General_agreement Per_element_agreement Total_agreement Mean_phi
Factor_score_R Bias RMSE;
by method;

data d4;
set d_comb;
xK_x_P = compress(K_x_P);
xK_x_N = compress(K_x_N);
xK_x_H = compress(K_x_H);
xK_x_D = compress(K_x_D);
xP_x_N = compress(P_x_N);
xP_x_H = compress(P_x_H);
xN_x_H = compress(N_x_H);
xH_x_D = compress(H_x_D);

method_x_N = method||'-'||sampsiz;

/*K by P Interaction Boxplots*/

proc sort data = d4;
by Method K_x_P;

proc boxplot data = d4;
plot Loading_sensitivity*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1
boxstyle = schematic;
label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
label method = 'Factor Extraction Method';
label Loading_sensitivity = 'Loading Sensitivity';
```

## Appendix F (Continued)

```
proc boxplot data = d4;
  plot General_agreement*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle
= schematic;
  label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
  label method = 'Factor Extraction Method';
  label General_agreement = 'General Pattern Agreement';

proc boxplot data = d4;
  plot Per_element_agreement*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1
boxstyle = schematic;
  label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
  label method = 'Factor Extraction Method';
  label Per_element_agreement = 'Per Element Agreement';

proc boxplot data = d4;
  plot Total_agreement*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
  label method = 'Factor Extraction Method';
  label Total_agreement = 'Total Agreement';

proc boxplot data = d4;
  plot Mean_phi*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
  label method = 'Factor Extraction Method';
  label Mean_phi = 'Mean Phi Values';

proc boxplot data = d4;
  plot Factor_score_R*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
  label method = 'Factor Extraction Method';
  label Factor_score_R = 'Factor Score Correlations';

proc boxplot data = d4;
  plot Bias*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle = schematic;
  label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
  label method = 'Factor Extraction Method';
  label Bias = 'Factor Loading Bias';

proc boxplot data = d4;
  plot RMSE*xK_x_P (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle = schematic;
  label xK_x_P = 'Number of Factors by Number of Observed Variables Interaction';
  label method = 'Factor Extraction Method';
  label RMSE = 'RMSE';

/*K by N Interaction Boxplots*/

proc sort data = d4;
by Method K_x_N;

proc boxplot data = d4;
  plot Per_element_agreement*xK_x_N (Method)/npanelpos = 36 turnhlabels blockpos = 1
boxstyle = schematic;
  label xK_x_N = 'Number of Factors by Sample Size Interaction';
```

## Appendix F (Continued)

```
label method = 'Factor Extraction Method';
label Per_element_agreement = 'Per Element Agreement';

proc boxplot data = d4;
  plot Total_agreement*xK_x_N (Method)/npanelpos = 36 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xK_x_N = 'Number of Factors by Sample Size Interaction';
  label method = 'Factor Extraction Method';
  label Total_agreement = 'Total Agreement';

proc boxplot data = d4;
  plot Mean_phi*xK_x_N (Method)/npanelpos = 36 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xK_x_N = 'Number of Factors by Sample Size Interaction';
  label method = 'Factor Extraction Method';
  label Mean_phi = 'Mean Phi Values';

proc boxplot data = d4;
  plot Factor_score_R*xK_x_N (Method)/npanelpos = 36 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xK_x_N = 'Number of Factors by Sample Size Interaction';
  label method = 'Factor Extraction Method';
  label Factor_score_R = 'Factor Score Correlations';

proc boxplot data = d4;
  plot Bias*xK_x_N (Method)/npanelpos = 36 turnhlabels blockpos = 1 boxstyle = schematic;
  label xK_x_N = 'Number of Factors by Sample Size Interaction';
  label method = 'Factor Extraction Method';
  label Bias = 'Factor Loading Bias';

/*K by H Interaction Boxplots*/

proc sort data = d4;
by Method K_x_H;

proc boxplot data = d4;
  plot Loading_sensitivity*xK_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1
boxstyle = schematic;
  label xK_x_H = 'Number of Factors by Commuality Level (Low=1, Wide=2, High=3)
Interaction';
  label method = 'Factor Extraction Method';
  label Loading_sensitivity = 'Loading Sensitivity';

proc boxplot data = d4;
  plot General_agreement*xK_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle
= schematic;
  label xK_x_H = 'Number of Factors by Commuality Level (Low=1, Wide=2, High=3)
Interaction';
  label method = 'Factor Extraction Method';
  label General_agreement = 'General Pattern Agreement';

proc boxplot data = d4;
  plot Per_element_agreement*xK_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1
boxstyle = schematic;
```

## Appendix F (Continued)

```
label xK_x_H = 'Number of Factors by Commuality Level (Low=1, Wide=2, High=3)
Interaction';
label method = 'Factor Extraction Method';
label Per_element_agreement = 'Per Element Agreement';

proc boxplot data = d4;
plot Factor_score_R*xK_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle =
schematic;
label xK_x_H = 'Number of Factors by Commuality Level (Low=1, Wide=2, High=3)
Interaction';
label method = 'Factor Extraction Method';
label Factor_score_R = 'Factor Score Correlations';

proc boxplot data = d4;
plot Bias*xK_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle = schematic;
label xK_x_H = 'Number of Factors by Commuality Level (Low=1, Wide=2, High=3)
Interaction';
label method = 'Factor Extraction Method';
label Bias = 'Factor Loading Bias';

proc boxplot data = d4;
plot RMSE*xK_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle = schematic;
label xK_x_H = 'Number of Factors by Commuality Level (Low=1, Wide=2, High=3)
Interaction';
label method = 'Factor Extraction Method';
label RMSE = 'RMSE';

/*K by D Interaction Boxplots*/

proc sort data = d4;
by Method K_x_D;

proc boxplot data = d4;
plot Loading_sensitivity*xK_x_D (Method)/npanelpos = 45 turnhlabels blockpos = 1
boxstyle = schematic;
label xK_x_D = 'Number of Factors by Dichotomization (as a Percentage of Observed
Variables) Interaction';
label method = 'Factor Extraction Method';
label Loading_sensitivity = 'Loading Sensitivity';

proc boxplot data = d4;
plot Mean_phi*xK_x_D (Method)/npanelpos = 45 turnhlabels blockpos = 1 boxstyle =
schematic;
label xK_x_D = 'Number of Factors by Dichotomization (as a Percentage of Observed
Variables) Interaction';
label method = 'Factor Extraction Method';
label Mean_phi = 'Mean Phi Values';

proc boxplot data = d4;
plot Bias*xK_x_D (Method)/npanelpos = 45 turnhlabels blockpos = 1 boxstyle = schematic;
label xK_x_D = 'Number of Factors by Dichotomization (as a Percentage of Observed
Variables) Interaction';
label method = 'Factor Extraction Method';
label Bias = 'Factor Loading Bias';

proc boxplot data = d4;
plot RMSE*xK_x_D (Method)/npanelpos = 45 turnhlabels blockpos = 1 boxstyle = schematic;
```



## Appendix F (Continued)

```
label xK_x_D = 'Number of Factors by Dichotomization (as a Percentage of Observed
Variables)Interaction';
label method = 'Factor Extraction Method';
label RMSE = 'RMSE';

/*P by N Interaction Boxplots*/

proc sort data = d4;
by Method P_x_N;

proc boxplot data = d4;
plot Total_agreement*xP_x_N (Method)/npanelpos = 36 turnhlabels blockpos = 1 boxstyle =
schematic;
label xP_x_N = 'Number of Observed Variables by Sample Size Interaction';
label method = 'Factor Extraction Method';
label Total_agreement = 'Total Agreement';

/*P by H Interaction Boxplots*/

proc sort data = d4;
by Method P_x_H;

proc boxplot data = d4;
plot Loading_sensitivity*xP_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1
boxstyle = schematic;
label xP_x_H = 'Number of Observed Variables by Communality Level (Low=1, Wide=2,
High=3) Interaction';
label method = 'Factor Extraction Method';
label Loading_sensitivity = 'Loading Sensitivity';

proc boxplot data = d4;
plot General_agreement*xP_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle
= schematic;
label xP_x_H = 'Number of Observed Variables by Communality Level (Low=1, Wide=2,
High=3) Interaction';
label method = 'Factor Extraction Method';
label General_agreement = 'General Pattern Agreement';

proc boxplot data = d4;
plot Per_element_agreement*xP_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1
boxstyle = schematic;
label xP_x_H = 'Number of Observed Variables by Communality Level (Low=1, Wide=2,
High=3) Interaction';
label method = 'Factor Extraction Method';
label Per_element_agreement = 'Per Element Agreement';

proc boxplot data = d4;
plot Factor_score_R*xP_x_H (Method)/npanelpos = 27 turnhlabels blockpos = 1 boxstyle =
schematic;
label xP_x_H = 'Number of Observed Variables by Communality Level (Low=1, Wide=2,
High=3) Interaction';
label method = 'Factor Extraction Method';
label Factor_score_R = 'Factor Score Correlations';

/*N by H Interaction Boxplots*/

proc sort data = d4;
```

## Appendix F (Continued)

```
by Method N_x_H;

proc boxplot data = d4;
  plot General_agreement*xN_x_H (Method)/npanelpos = 36 turnhlabels blockpos = 1 boxstyle
= schematic;
  label xN_x_H = 'Sample Size by Commuality Level (Low=1, Wide=2, High=3) Interaction';
  label method = 'Factor Extraction Method';
  label General_agreement = 'General Pattern Agreement';

proc boxplot data = d4;
  plot Per_element_agreement*xN_x_H (Method)/npanelpos = 36 turnhlabels blockpos = 1
boxstyle = schematic;
  label xN_x_H = 'Sample Size by Commuality Level (Low=1, Wide=2, High=3) Interaction';
  label method = 'Factor Extraction Method';
  label Per_element_agreement = 'Per Element Agreement';

proc boxplot data = d4;
  plot Mean_phi*xN_x_H (Method)/npanelpos = 36 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xN_x_H = 'Sample Size by Commuality Level (Low=1, Wide=2, High=3) Interaction';
  label method = 'Factor Extraction Method';
  label Mean_phi = 'Mean Phi Values';

/*H by D Interaction Boxplots*/

proc sort data = d4;
by Method H_x_D;

proc boxplot data = d4;
  plot Mean_phi*xH_x_D (Method)/npanelpos = 45 turnhlabels blockpos = 1 boxstyle =
schematic;
  label xH_x_D = 'Commuality Level (Low=1, Wide=2, High=3) by Dichotomization (as a
Percentage of Observed Variables) Interaction';
  label method = 'Factor Extraction Method';
  label Mean_phi = 'Mean Phi Values';

proc boxplot data = d4;
  plot RMSE*xH_x_D (Method)/npanelpos = 45 turnhlabels blockpos = 1 boxstyle = schematic;
  label xH_x_D = 'Commuality Level (Low=1, Wide=2, High=3) by Dichotomization (as a
Percentage of Observed Variables) Interaction';
  label method = 'Factor Extraction Method';
  label RMSE = 'RMSE';

/*sample size main effect*/
data d5;
set d4;
xMethod_x_N = compress(Method_x_n);

proc sort data = d5;
by xMethod_x_N;
```

## Appendix F (Continued)

```
proc boxplot data = d5;  
  plot RMSE*xMethod_x_N/boxstyle = schematic;  
  label xMethod_x_N = 'Method by Sample Size Main Effect';  
  label RMSE = 'RMSE';  
  
run;  
  
quit;
```