Three Essays in Public Economics: Theory and Experiments

Marco Faravelli

Doctor of Philosophy in Economics

The University of Edinburgh

2007

Acknowledgements

I am profoundly indebted to my supervisor, Ed Hopkins, for all he has taught me, for his constant attention and encouragement. I am very grateful to all the Edinburgh School of Economics for providing me with an extremely stimulating environment and particularly to Stuart Sayer for giving me the possibility of doing a PhD. I am also grateful to Tatiana Kornienko, John Moore, József Sákovics, Santiago Sánchez Pagés, Bruce Teubes, Jonathan Thomas and Duncan Whitehead for all their comments and suggestions.

Last but not least, I would like to thank my family, for not thinking that I was wasting my time; all my friends, for the patience shown on the innumerable occasions when I have declined invitations to the pub for the sake of economics; and especially Emma, for tolerating my horrible mood.

Declaration

I certify that this thesis does not incorporate any material previously submitted for a degree or diploma in any University; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person where due reference is not made in the text.

I also certify that the thesis has been composed by myself and that all the work is my own.

The first chapter of this thesis is published in the *Journal of Public Economics*, 91, 1399-1422 (2007).

Marco Faravelli, June 2007

Abstract

This thesis is composed of three chapters, which can be read independently.

In the first chapter we explore distributive justice and perception of fairness using survey data from freshmen and senior students of economics and sociology. We analyse the impact of context and education on their preferences over a hypothetical distribution of resources between individuals which presents a trade off between efficiency and equality. With context giving minimal information, economics students are less likely to favour equality; studying economics influences the preferences of the subjects, increasing this difference. However, when the same problem is inserted into a meaningful context, the difference disappears. Four distribution mechanisms are analysed: egalitarianism, maximin, utilitarianism and utilitarianism with a floor constraint.

The second chapter considers a public good game with heterogeneous endowments and incomplete information affected by extreme free-riding. We overcome this problem through the implementation of a contest in which several prizes may be awarded. We identify a monotone equilibrium, in which the contribution is strictly increasing in the endowment. We prove that it is optimal for the social planner to set the last prize equal to zero, but otherwise total expected contribution is invariant to the prize structure. Finally, we show that private provision via a contest Pareto-dominates public provision and is higher than the total contribution raised through a lottery.

The third chapter investigates fund-raising mechanisms based on a prize as a way to overcome free riding in the private provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. We compare experimentally the performance of a lottery, an all-pay auction and a benchmark voluntary

contribution mechanism. We find that prize-based mechanisms perform better than voluntary contribution in terms of public good provision after accounting for the cost of the prize. Comparing the prize-based mechanisms, total contributions are significantly higher in the lottery than in the all-pay auction. Focusing on individual income types, the lottery outperforms voluntary contributions and the all-pay auction throughout the income distribution.

Contents

3 A Prize To Give for: An Experiment on Public Good Funding Mechanisms 70

Chapter 1

How Context Matters: A Survey Based Experiment on Distributive Justice

1.1 Introduction

One of the most interesting results that arises from dictator and ultimatum experiments is that fairness seems to be a strong concern. Experimental results on the ultimatum game show that a large fraction of players offer a "fair" allocation and that "unfair" offers are systematically rejected. Furthermore, while economists tend to evaluate allocations purely quantitatively, these experimental data also suggest that whether an allocation is seen as "fair" can depend on qualitative factors, the context in which it is presented and the way it is framed. In addition, the data from dictator experiments suggest that there is significant heterogeneity in what people consider fair, with many people giving nothing as well as many splitting the available resources equally.

The main motivation for the present study is the further investigation of this evidence. However, we will not employ a game theoretical approach. This paper is a

survey based investigation of perception of fairness and attitudes towards distributive justice. We focus on unbiased justice, or justice from the viewpoint of an "impartial spectator", in comparison to many other studies that involve stakes and analyse the effect of fairness on the stakeholders.

Our aim is twofold. First, we wish to analyse how context influences preferences over a hypothetical distribution of resources and in what way adding justice-related information to the context affects judgements. Second, we explore the impact of education on perception of fairness. We surveyed both first and last year undergraduate students of economics and sociology (hereafter referred to as freshmen and seniors). We submitted to them different versions of a problem involving the distribution of resources between two individuals in which we asked them to choose the distribution that they considered the most fair. The two individuals obtain a different utility from the resources and there is, therefore, a trade off between efficiency, which involves handing more resources to the more productive individual, and equality, which might demand an equal division even if that would not maximise total output. We will refer to this problem as the distribution problem. We found that, with a context giving minimal information, economics students were less likely to favour equality than sociology students and this difference was more marked in senior students. Thus, studying economics seemed to have influenced the preferences of the subjects over the distribution of resources, while we found no significant difference between the choices of sociology freshmen and seniors. This evidence suggests that previous survey studies carried out on economics students (see for example Engelmann and Strobel, 2004) might have obtained different results with a different subject population. However, when the same question was rephrased to give a meaningful context, there was now significant agreement over which allocation was fairest and there was no significant difference between economics and sociology students.

Let us consider the first of our aims. It is well known that perception of fairness and behaviour related to fairness judgements are context dependent. The set of individuals being compared, the type of good being distributed, the historical terms

4

of transactions or the framing of information are all examples of contextual elements. Probably the most cited study of justice in economics that emphisises the variation of views of fairness with context is that of Kahneman, Knetsch, and Thaler (1986). We are interested on how such judgements are related to various classes of context. One of the first examples of studies in this direction is that of Yaari and Bar-Hillel (1984), who analysed how judgements are affected by context when it specifies whether individuals need the goods to be distributed or they simply like them. There exists by now a large economic literature on this topic (see Schokkaert and Overlaet, 1989; Gaertner, 1994; Gaertner, Jungeilges, and Neck, 2001; Gaertner and Jungeilges, 2002; Schokkaert and Devooght, 2003; Gaertner and Schwettmann, 2005, among others) as well as a lot of evidence from the psychological and sociological literature (for an overview see Konow 2003).

As we will explain in Section 1.2, we consider the specific problem of how intuitions of fairness vary with contextual factors which determine whether or not individuals are responsible for the outcomes of their actions. Several papers from the social choice literature have addressed this issue (e.g., see Schokkaert and Lagrou, 1983; Schokkaert and Overlaet, 1989; Schokkaert and Devooght, 2003; Cappelen, Sørensen, and Tungodden, 2005; Gaertner and Schwettmann, 2005), and a survey of the economic and the psychological literature on this topic can be found in Konow (2003). We wish to examine how the relative importance of equality and efficiency can depend on these factors, and which distributions are considered fair given the type of context. Further, we are interested in assessing whether clearly specifying the type of context helps overcome the (possible) differences in judgments between economics and sociology students and between freshmen and seniors. The evidence that additional information facilitates the attainment of a more widespread consensus on what is fair is an important finding for public debates[1]. Given that in-

---

[1]Interestingly Babcock et al. (1995) report the opposite result, i.e. adding information increases bias. The difference is that the participants in their study have a monetary incentive, so information is employed in a biased way.

dividuals' judgements may vary according to education, profession and age, amongst other categories, this finding should hopefully allow progress in controversial policy debates.

We investigated four versions of the distribution problem. The versions are all formally identical, but each of them is characterised by a different context. In the first version no explanation of the difference between the two individuals is provided. The second and the third versions present two distinct explanations: in one, the second individual is less productive because he is handicapped, in the other, because he works less hard. The same possible allocations are present in all versions of the problem: an egalitarian, a utilitarian and a maximin allocation. Context had a significant effect on what the subjects thought was fair and led to greater level of agreement between the parties: the maximin allocation was preferred when the difference was due to a handicap; in case of a different effort the utilitarian distribution was chosen. People tend to favour the less productive individual when different outcomes are due to external causes (for instance a handicap), but they will punish him if the cause is internal (for instance for exerting less effort). The two situations are perceived differently and imply distinct reasons for allocating resources. People tend to distribute according to need when abilities are different, and according to efficiency when there is difference in effort. The fourth version presents no explanation of the difference between the two individuals, but a floor is introduced in terms of minimum utility necessary for each individual. The tension here is that the efficient allocation does not give the minimum survival utility to the less efficient individual. As well as the previous allocations, a fourth allocation is permitted, deriving from the application of utilitarianism with a floor.

Our hypothesis is that when no explanation of the difference between the individuals is provided, the subjects involuntarily insert the distribution problem into a determined context, filling the lack of information according to their personal attitudes and background. The preference for a particular allocation under this condition will reveal the relative concern of the subject for either the efficiency or the

equality of the distribution. We will refer to such a preference as the "ideology" of the subject.

Our second purpose is to investigate the influence of education on perception of fairness. While a few studies have focused on differences in judgement (e.g., see Marwell and Ames, 1981; Amiel and Cowell, 1999, for a summary of their findings), most of the literature on the differences between economists and non-economists has concentrated on their behaviour. Several experiments have been conducted to ascertain and to analyse any different behaviours in terms of propensity to co-operate (through prisoner's dilemma games), to free ride (for instance, in the provision of public goods) or in the degree of selfishness.

Both experiments with monetary incentives and surveys are valuable instruments to explore fairness, according to the purpose of the analysis. Differences in judgement can be as relevant as differences in behaviour, depending on the situation and the circumstances. In daily life people's intuitions of fairness determine not only their behaviour but also their judgements of situations in which they are not directly involved. People often act on unbiased views when their stakes are low or negligible, for example as voters or in the case of juries. Further, even when personal stakes are relatively high and agents trade-off self-interest and social preferences, it is interesting to examine the fairness point against which the self-interest point is being traded off. Economists participate in boards, are members of councils, vote and legislate. It is important to analyse whether their judgements differ from other people's and to what extent this is context-related. When the purpose is to explore intuitions of fairness, eliminating monetary stakes reduces self-interest bias and presents the advantage of encouraging "participants to prescind and abstract from personal stakes" (Konow 2003, p. 1191)[2].

In conducting our analysis, we will proceed as follows. Having found that an ideological difference does exist we will show that a significant agreement can be

_____

[2]See also Fong (2001).

reached clarifying the context of the distribution. Further, we have to investigate the reasons for this difference. The literature that compares economists and non-economists suggests that the differences could be due to two causes. They may be the result of a self-selection process or of training in economics. These two conjectures have been called the selection and the learning hypothesis (Carter and Irons, 1991). Comparing the answers of freshmen students of the two courses will show the existence of a selection effect. Comparing the answers given by freshmen and senior students of the same course, we will find that a learning effect only exists for economics students.

In Section 1.2 we will discuss how context relates to responsibility concerns and present the distribution mechanisms examined in our analysis. Section 1.3 reports the results of the most prominent experiments directed to compare economists and non-economists. In Section 1.4 we will discuss the design of the questionnaire and the hypotheses that we are going to test. Section 1.5 displays the results. Section 1.6 concludes.

1.2   Equality, Efficiency and Responsibility

In any discussion of fairness and justice theories a fundamental issue is the conflict between equality and efficiency. In an excellent survey on positive and normative theories of justice James Konow (2003) examines this contrast describing the principles that are behind these concepts: the Need Principle and the Efficiency Principle. The former invokes the equal satisfaction of basic needs and inspires theories such as egalitarianism and Rawls' theory of justice (1971). The latter advocates maximising surplus and is most closely associated with utilitarianism.

We examine the problem of how the balance between efficiency and equality is influenced by responsibility considerations. Further we wish to verify whether inserting the problem into a meaningful context, which clearly states who can be held accountable for a certain outcome and who cannot, leads to a greater consensus

on what is considered *just*.

The issue of responsibility has been investigated by philosophers as well as by economists and psychologists, focusing mainly on two questions. First, how to characterise responsibility and what does make a person accountable for an outcome. The second question is related to the critique of the welfarist interpretation (see Fleurbaey, 1998, for a survey of this literature). Welfarist theories take utilities as the only relevant personal features. The insufficiency of the welfarist framework to capture the complexity of the distribution problem was already the main conclusion reached by Yaari and Bar-Hillel (1984) and several subsequent questionnaire-studies abandoned the welfarist perspective in their analysis (see Schokkaert and Overlaet, 1989; Gaertner, 1994; Gaertner, Jungeilges, and Neck, 2001; Gaertner and Jungeilges, 2002; Schokkaert and Devooght, 2003; Gaertner and Schwettmann, 2005, among others). Further, there exists by now a large literature in social choice about how to incorporate ideas of responsibility in the social evaluations (for an overview see Fleurbaey and Maniquet, in press). Although we appreciate the importance of this literature, we are mainly concerned with the first question, that is the issue of characterising responsibility. Given the goals of our analysis, we chose to adopt a welfarist interpretation. We will therefore explore whether welfarist mechanisms may lead to higher consensus on what is fair when the distribution problem is inserted into a meaningful context. Further, we are interested in analysing which welfarist distribution is preferred in different contexts.

Although a concern for individual responsibility was present in Rawls (1971) and Sen (1980), with regards to primary goods and functionings respectively, Dworkin (1981) has been the first among political philosophers to explicitly focus on this issue. Dworkin considers that a person must be held responsible for her preferences whether or not they are voluntarily cultivated, as long as she identifies with them, but she cannot be held accountable for her resources. He also makes a helpful distinction between "option luck", which is the output of a gamble explicitly taken, and "brute luck", which is an output in which no gamble was entered into. Thus

9

option luck is a matter of choice and hence fair, while brute luck is morally arbitrary and therefore unfair. Arneson (1989, 1990) and Cohen (1989) proposed a revision of what Cohen (1989) called "Dworkin's cut", that is the division between preferences and resources. Although they articulate their positions differently, both these authors agree that the "right cut is between responsibility and bad luck, not between preferences and resources" (Cohen, 1989, p. 921). The debate is far from being closed and Roemer (1993) has introduced a relativistic position, proposing that the cut between responsibility and compensation can be seen as cultural-dependent.

Economists have analysed the importance of responsibility considerations in perception of fairness through several questionnaire-studies and experiments and have tried to single out which characteristics are considered to be within the control of an individual and which are not (see Schokkaert and Lagrou, 1983; Schokkaert and Overlaet, 1989; Schokkaert and Devooght, 2003; Cappelen, Sørensen, and Tungodden, 2005; Gaertner and Schwettmann, 2005, among others). A thorough review of this literature and its relations with social psychology can be found in Konow (2003). The author reports the results of several experiments and surveys in economics and in psychology that indicate that individuals are held accountable for their effort and choices that affect their contribution, but they are not considered responsible for their birth, brute luck and choices that do not affect their productivity. Such results seem to confirm the predictions of attribution theory. Attribution theory is a social psychology theory initiated by Heider (1958) that explains behaviour on the basis of causal attributions of responsibility. According to this theory, people infer causes of events and evaluate to what extent an agent has contributed to the outcome, holding the agent responsible only for those factors that the agent can influence. As Konow (2003) suggests "attribution theory provides a powerful criterion for describing desert according to the views of most people" (p. 1214).

In our empirical investigation, we build a distribution problem such that the theories inspired by the Need Principle advocate allocations of resources substantially different from the distribution that would result if the Efficiency Principle was ap-

10

plied. In the base treatment, where the context of the problem is not clarified, it is not clear what the different productivity of the two individuals depends on. We then examine two classes of context, which fill the gap of information and enable the subjects to establish whether the agents can or cannot be held accountable for their productivity. In one the different productivity is explained in terms of brute luck, a factor that is out of the control of the individuals; in the other it depends on effort, an element that they can control. Following Konow (2000) we will refer to "exogenous differences" in the first case and "discretionary differences" in the second. Finally we consider a version of the distribution problem in which "need" considerations are involved; this treatment is identical to the first one except for the introduction of a threshold under which the individuals do not survive.

Let us consider the distribution mechanisms that will be examined in our analysis. Besides the Egalitarian solution we are going to consider three other distribution mechanisms, whose application may determine particular departures from equality. Let us examine these distribution principles.

Many different forms of utilitarianism exist, but we will refer to it as the principle that advocates the maximisation of the sum of individual utilities.

Rawls' (1971) theory of justice was conceived as an alternative to utilitarianism, in all of its forms, and has become a powerful contestant to utilitarian theory in recent years. Rawls proposes two principles of justice that are meant to rule the basic structure of society and determine the division of advantages of social cooperation. The above principles would result from a social contract made by rational individuals behind a "veil of ignorance", which would guarantee the impartiality of the parties. While the first principle rules the scheme of liberties each person has the right to, the second principle determines which social and economic inequalities are acceptable. The distribution mechanism we are interested in is what Rawls refers to as the maximin equity criterion, which came subsequently to be known as the difference principle. It is identifiable with the first part of the second principle, which is defined as following: "Social and economic inequalities are to meet two conditions: they

11

must be (a) to the greatest expected benefit of the least advantaged members of society (the maximin equity criterion) and (b) attached to offices and positions open to all under conditions of fair equality of opportunity." (Rawls, 1974, p. 142). This criterion is clearly opposed to the utilitarian that only cares about maximising utility regardless of its distribution.

Finally, utilitarianism with a floor is a mechanism that prescribes the maximisation of the average utility with a floor constraint. Preferences for distributions prescribed by the application of this principle have been tested in several experiments (e.g. Frohlich, Oppenheimer, and Eavy, 1987a, 1987b; Lissowsky, Tyszka, and Okrasa, 1991).

Rawls' theory of justice's informational basis does not coincide with the utilitarian. In the utilitarian theory the informational basis consists only of the utilities of the individuals in the states of affairs under evaluation. Rawls' theory, on the contrary, ranks the different states of affairs according to the distribution of primary goods, that are defined as anything any rational person wants and will want regardless of his plan of life or his place in the social scheme. As we mentioned above we are going to consider all of the distribution mechanisms discussed, including the maximin criterion, from a welfarist perspective. Among the welfarist theories, Sen (1992) recalls the utility-based maximin as that distribution mechanism that prescribes to maximise the utility of the least advantaged individual. This is the interpretation that we are going to assume in the course of our analysis.

1.3   Are Economists Different?

Most of the literature on the differences between economists and non-economists has concentrated on differences in behaviour. Amiel and Cowell (1999) summarise some of their findings on the different moral intuitions of economists and non-economists. They focused mainly on the acceptance of the monotonicity axiom, a concept which

is very close to the Pareto principle[3]. Their results show that the monotonicity axiom was not a very popular concept. However, it was generally accepted more favourably by economics students than by their sociologist colleagues.

Marwell and Ames (1981) conducted the first study that compared economists and non-economists, through an experiment that called for private contributions to public goods. They found that first-year graduate students in economics are much more likely than others to free ride. They conjectured that there might be two reasons for why economists might behave differently, defined by Carter and Irons (1991) as the selection and the learning hypothesis. However, Marwell and Ames did not check the extent to which this difference is due to one hypothesis or the other (or to both of them). Further, they collected a wide range of information regarding the different perceptions, expectations and explanations for the behaviour of the subjects. Two questions were asked. First, what is a fair investment in the public good? 75 percent of the non-economists answered "half or more" of the endowment, and 25 percent answered "all". The other question asked whether they were concerned about fairness in making their own investment decision. Almost all non-economists answered "yes". The answers of the economics students were more difficult to analyse. More than one-third of them either refused to answer the first question or gave uncodable responses. As Marwell and Ames wrote, "it seems that the meaning of 'fairness' in this context was somewhat alien for this group" (Marwell and Ames, 1981, p. 309). Those who did answer found that little or no contribution was fair. With regard to the second question, economics students were much less concerned with fairness when making their decisions.

Carter and Irons (1991) investigated the behaviour of students of economics compared to students of other disciplines in an ultimatum bargaining game. They found that economics students behaved more self-interestedly than other students. They tested the selection and the learning hypothesis, finding that "economists are born,

---

[3]While the Pareto criterion is expressed in terms of utility "the monotonicity axiom is usually put in terms of persons' incomes" (Amiel and Cowell, 1999, p. 64).

not made" (Carter and Irons, 1991, p. 174). They claimed that studying economics does not create rational, self-interested homines economici, but subjects who are particularly concerned with economic incentives self-select into economics. Using a prisoner's dilemma game, Frank, Gilovich, and Regan (1993) found that economists behave in more self-interested ways and are much more likely to defect from coalitions. Further, their results support the learning hypothesis. According to them, "exposure to the self-interest model does in fact encourage self-interested behavior" (Frank, Gilovich, and Regan, 1993, p. 159) and inhibit co-operation.

Yezer, Goldfarb, and Poppen (1996) strongly criticised the results obtained by Frank and his coauthors (1993) from a methodological point of view. They claimed that the evidence of that paper only implies that economics students display uncooperative behaviour in specialised games. They conducted a "lost-letter" experiment, in which envelopes containing currency are dropped in classrooms before the beginning of the lectures. The return rate on lost letters is used as a measure of co-operation. According to their results, the "real life" behaviour of economics students is actually more co-operative than that of subjects studying other disciplines. Similarly, Frey and Meier (2003) claim that "students may play the equilibrium learned in their economics classes, but they do not apply it to real life situations" (Frey and Meier, 2003, p. 448). Further, their results indicate that the particular behaviour of economists is only due to self-selection. On the basis of Yezer's results, Zsolnai (2003) suggests that there might be no contradiction between honesty and co-operation, which are two different qualities, and claims that economists' behaviour is characterised by respect for property rights and self-interest motivation simultaneously. Finally, Hu and Liu (2003) found evidence that economics students are more likely to co-operate in prisoner's dilemma games.

In sum, the results are inconclusive and depend on the different settings. Further, most of these studies are aimed to test whether economics students behave more in accordance with predictions of the rational/self-interest model of economics. However, despite the different approach assumed in this work, it will be useful, in the

course of our analysis, to compare the above results with ours, taking into account the different perspectives assumed.

1.4   Methods and Hypotheses

In March 2002, a total of 1333 students of the University of Milan took part in the survey. 661 of them were sociology students, 345 freshmen and 316 seniors. The remaining 672 were economics students, 354 freshmen and 318 seniors. In each of the four groups women and men were present in approximately equal number. Regarding these groups there are two important points to make. First, nobody, among the freshmen, had studied economics in the previous years of school. Second, the freshmen of economics had not started economic topics yet, having only studied mathematics, statistics and law courses. Participation was voluntary and all those asked to participate agreed to do so. There was no show-up fee paid. Each student was given a sheet containing on the front the base problem and on the back, at random, one of the remaining three problems. Thus, each of the four groups was divided into three classes, according to the nature of the second question. Students were asked to read the question on the front and only after answering that they could read and answer the one on the back. It was not possible to change the answer to the first problem after reading the second one. The total time for conducting the questionnaire, including our instructions, varied between 20 and 25 minutes, due to the difference in class sizes.

The four questions are reported in Appendix 1A. We will refer to them as question 1 (base treatment), question 2a (exogenous difference treatment), question 2b (discretionary difference treatment) and question 2c (need treatment). All of the respondents answered question 1. 464 students answered question 2a; of them, 124 were economics freshmen, 115 economics seniors, 134 sociology freshmen and 91 sociology seniors. Question 2b was submitted to 451 respondents: 129 economics freshmen, 109 economics seniors, 95 sociology freshmen and 118 sociology seniors.

15

Finally, a total of 418 students answered question 2c; of them, 101 were economics freshmen, 94 economics seniors, 116 sociology freshmen and 107 sociology seniors. As previously noted, the four problems are formally identical. Resources are to be distributed between two individuals. Robinson and Friday live on two different islands. Robinson lives on island A and Friday lives on island B. On each island one can till 12 plants. Utility deriving from the goods is increasing and marginal utility is constant[4]. The two characters obtain different levels of utility from the goods, and are only interested in the utility they get.

"The only reason why both Robinson and Friday would like to cultivate these plants is because they produce fruit, and the higher amount of fruit they obtain the more their welfare would be; every additional fruit produces an equal value, which is identical for both people."

The respondents are asked to choose a solution among the ones that are provided so that the distribution is just, recalling that there is no possibility of redistributing the plants after the allocation. In question 1 no explanation of the difference between the individuals is provided.

"Friday obtains 120 fruits per year from every plant on island B, **but he cannot obtain** any fruit from island A's plants.
On both islands Robinson obtains 20 fruits per plant."

In question 2a and question 2b the difference between the individuals is explained. In the former the two individuals differ in their physical abilities, an exogenous difference.

"**Both Robinson and Friday put the same amount of work** into tilling the plants; the only way to move from one island to the other is to swim.

---

[4]With an assumption of constant marginal utility, utilitarianism amounts to maximising the sum of resources. It is worth noting that this is only a simplified form of welfarism.

Friday can obtain 120 fruits per year from every plant of island B, **but he cannot swim** and he cannot till any plant on island A.

Robinson is a perfect swimmer and he can therefore till plants on both islands, **but due to a wound** caused by the shipwreck he cannot obtain more than 20 fruits per year from every plant of island A and island B."

In question 2b Robinson and Friday put in different efforts in tilling their plants, which is a discretionary difference.

"Robinson and Friday can till plants and move from one island to the other in the same way, **but they do not put the same amount of work** into tilling the plants.

Friday can obtain 120 fruits per year from every plant of island B, **but he doesn't want to go** on island A and he will not produce fruits on this island.

To Robinson moving from one island to the other is all the same, **but he does not put as much amount of work into tilling his plants as Friday** and he doesn't produce more than 20 fruits per year from every plant, both on island A and B."[5]

Question 2c introduces need considerations. No explanation is provided, but a minimum level of utility is introduced.

"The **minimum quantity** needed by every one of them in order to survive is 300 fruits per year."

The distributions are provided in terms of resources as well as in terms of utility; the sum of utility obtained by the individuals is shown too. Three solutions are provided to question 1, 2a and 2b.

---

[5]The idea of the two islands and, in particular, the fact that Friday works harder but does not want to go to the other island is an unnecessary complication, which we realise could have been avoided.

1	Plants island A	Plants island B	Fruits
Robinson	12	0	240
Friday	0	12	1440
	Total production of fruits	**1680**	

2	Plants island A	Plants island B	Fruits
Robinson	12	8	400
Friday	0	4	480
	Total production of fruits	**880**	

3	Plants island A	Plants island B	Fruits
Robinson	9	9	360
Friday	3	3	360
	Total production of fruits	**720**	

The first solution derives from the application of the utilitarian principle, the second one is the maximin solution, while the third one is the Egalitarian. The Utilitarian solution is the fairest in terms of resources, (R: 12-0; F: 0-12): each individual receives all of the plants of his island. However, this distribution is the most unequal in terms of utility: (R: 240; F: 1440). Social welfare, though, is maximised. The Rawlsian distribution is much more unequal in terms of resources, (R: 12-8; F: 0-4): Robinson receives 8 of the 12 plants of island B, besides the 12 plants of island A. Welfare distribution is much more equal, though, (R: 400; F: 480). The cost of this greater equity is a much lower total welfare. Finally, the Egalitarian distribution gives every individual a utility of 360, distributing the plants as follows: (R: 9-9; F: 3-3). Total welfare is much less than according to the other allocations. Only the Utilitarian and the Rawlsian solutions are Pareto-efficient.

Applying the maximin solution both the individuals are better off than under the Egalitarian distribution, which is not Pareto-efficient. Besides these distributions a fourth solution[6], corresponding to utilitarianism with a floor, is provided to question 2c.

4	Plants island A	Plants island B	Fruits
Robinson	12	3	300
Friday	0	9	1080
	Total production of fruits		**1380**

Plants are divided as follows: (R: 12-3; F: 0-9). Robinson gets 300 fruits, just enough to survive, and Friday gets a utility of 1080. This distribution is also Pareto-efficient and, in terms of utility, stands between the Rawlsian and the Utilitarian. The latter is the only one that does not guarantee the survival of both the individuals.

We explore five hypotheses.

1. *Selection hypothesis.* We are interested in testing whether students choosing to study economics and students choosing to study sociology differ in their ideology. In order to test this hypothesis we are going to compare the answers of freshmen of economics and sociology to question 1.

2. *Learning hypothesis.* We wish to test whether education influences ideology. To test this hypothesis we are going to compare the answers to question 1 given by freshmen and seniors of the same course.

3. *Transformation of ideological differences.* We are going to test whether ideological differences increase with the seniority of the subjects. This hypothesis

---

[6]An extension to this study might be the inclusion of the utilitarian solution with a floor in the first three treatments. It would also be interesting to combine the idea of a minimum quantity with the explanations from questions 2a and 2b. However, we believe these not to be crucial points of our analysis.

will be tested by comparing the answers to question 1 given by senior students of economics and sociology.

4. *Relevance of exogenous differences / discretionary differences / need.* We are going to test whether contexts affect the preferences of the respondents. We will consider each class separately and test the hypothesis of no change in the answer to the first and the second question.

5. *Agreement hypothesis.* Finally, we will compare the answers of the four groups to questions 2a, 2b and 2c, and test whether clarifying the context or introducing a minimum utility allows reaching an agreement between the groups.

1.5   Results

In presenting our results we will proceed as follows. First, we will focus on the difference between economics and sociology students, testing the selection and learning hypotheses. Then, we will analyse the effects of clarifying the context of the distribution and the extent to which this facilitates an agreement between the parties.[7]

---

[7]Before proceeding to test the above hypotheses we have to make sure that in each group the three different versions of the questionnaire have been randomly distributed among the respondents. For each group, we have to check that the answers to the first question follow the same distribution in everyone of the three classes. For each one of the four groups, we apply the Chi-square test to test the following hypothesis

$$H_0 \quad : \quad \text{the proportion of subjects in each of the option}$$
$$\text{categories is the same in each of the three classes.}$$

The value of the test statistic is $\chi^2 = 0.55$ for economics freshmen ($p = 0.9685$), $\chi^2 = 3.38$ for economics seniors ($p = 0.4963$), $\chi^2 = 0.72$ for sociology freshmen ($p = 0.9488$) and $\chi^2 = 2.98$ for sociology seniors ($p = 0.5612$). For every group we cannot reject the null hypothesis. This allows us to proceed to any type of inferential analysis of the data and to test the hypotheses presented above.

1.5.1 Ideology

Table 1.1 reports the answers to question 1. In the tables and figures we will present, E, R, U and UF indicate, respectively, Egalitarian, Rawlsian, Utilitarian and Utilitarian with a floor.

Table 1.1: Question 1 (Percentage responses)

Solutions	Economics Freshmen	Economics Seniors	Sociology Freshmen	Sociology Seniors
E	35	22	47	47
R	38	50	37	34
U	27	28	16	19

Let us consider the answers to question 1 given by the freshmen of economics and sociology first. This will allow us to test the selection hypothesis. We can easily notice a consistent difference between the two distributions, the preferences of the economics students being more equally distributed between the three options. In both the groups there are a similar percentage of subjects choosing the Rawlsian principle, 38 percent of the economics students and 37 percent of the sociology ones. However, while 47 percent of the sociologists prefer the Egalitarian solution and only 16 percent the Utilitarian, these percentages are much closer among the economists, respectively 35 percent and 27 percent.

$$Selection\ hypothesis \tag{1.1}$$

$$H_0 \quad : \quad \text{the choice of a particular option}$$

$$\text{is unrelated to the university course.}$$

Given the value of the test statistic, $\chi^2 = 15.57$, we reject the null hypothesis ($p = 0.0004$). This leads to the following result.

21

Result 1.1: A selection effect does exist. Sociology students are more concerned with equality than economics students and prefer the Egalitarian distribution despite its inefficiency.

1.5.2 Education: Equity and Efficiency

We are going to analyse whether education influences ideology by comparing the answers to question 1 given by freshmen and seniors of the same discipline. Table 1.1 shows that the answers of the economics seniors are much more differentiated than those of their younger colleagues. The percentage of preferences for the Utilitarian allocation is almost identical in both the groups (27 percent among the freshmen and 28 percent among the seniors). However the preferences for the Egalitarian distribution diminish from the freshmen (35 percent) to the seniors (22 percent) to the advantage of the maximin principle. On the other hand, interestingly, we notice that the distributions of preferences of sociology freshmen and seniors are almost identical.

$$\textit{Learning hypothesis} \tag{1.2}$$

$$H_0 \quad : \quad \text{the choice of a particular option is}$$

$$\text{unrelated to the university year.}$$

We reject the null hypothesis ($p = 0.0003$) with respect to the economics students ($\chi^2 = 15.88$), but we cannot reject it ($p = 0.61569$) for the sociology ones ($\chi^2 = 0.97$). This leads to the next result.

Result 1.2: A learning effect only exists for the economics students. The Egalitarian solution is less popular with economics seniors, who instead prefer the Rawlsian distribution.

Unlike Carter and Irons (1991), we can therefore conclude that economists *are*

not only born, but also made. Our results also seem to contradict the results obtained by Frank, Gilovich, and Regan (1993). They found that economics students appear to be more prone than others to defect, that is to go for the Pareto-inferior solution, and this trend increases with the seniority of the subjects. They claimed that training in economics has, amongst others, negative consequences, i.e. anti-social behaviour (Frank, Gilovich, and Regan, 1996). Our results indicate the presence of a learning effect that reflects an increasing appreciation for the maximin principle. The latter does satisfy the Pareto criterion[8], which is not the case for the Egalitarian distribution. However, the shift in preferences is not at all in the direction of the Utilitarian solution, which suggests that senior students are no more concerned with the maximisation of output than their younger colleagues. Given that senior students of economics are more likely to favour inequality *only if* this implies making both the individuals better off, training in economics *does not seem to have negative consequences.*

However, we have to bear in mind the differences between the experiments we discussed in Section 1.3 and our study. Carter and Irons (1991) and Frank and his colleagues (1993) were interested in finding whether exposure to the rational model of economics makes subjects *behave* in a more self-interested way. Our approach differs in two ways. First, we concentrate our analysis on the perception of fairness of the subjects rather than on their behaviour. Second, self-interest bias is drastically reduced by the elimination of monetary incentives[9].

---

[8]It is interesting to notice that several studies show that the Pareto principle is not a very popular concept with economics and business students (e.g. McClelland and Rohrbaugh, 1978; Amiel and Cowell, 1999). However, non-economists seem to believe in it even less (Amiel and Cowell, 1999).

[9]Note that self-interest bias cannot be completely eliminated. Although the respondents have to divide the resources between two hypothetical individuals, they might still act as vicarious stakeholders.

1.5.3 Does Education Increase Ideological Differences?

We want to check whether ideological differences between the students of the two courses increase with the different education or remain stable. Looking at Table 1.1 we can see that the difference between the two distributions seems to have increased from the first to the last year. Let us test the hypothesis of *transformation of ideological differences*. The null hypothesis is the same as Hypothesis (1.1). The value of the test statistic is $\chi^2 = 44.5$ and we reject the null hypothesis ($p = 2.177$e-10). Moreover, the Cramer coefficient[10] shows that the ideological difference between the seniors of the two courses is much greater than between their younger colleagues.

> **Result 1.3:** The ideological difference between senior students of economics and sociology is greater than between the freshmen of the two courses.

1.5.4 The Impact of Context

Table 1.2 reports the answers to the three different versions of question 2. Unlike the base version, in the exogenous difference treatment the absolute majority of each group prefer the Rawlsian solution. This trend can be understood if we consider that both the individuals exert the same effort, but they differ in their physical abilities, a characteristic they cannot be held responsible for. Maximising the utility of the more disadvantaged is considered fair by most of the individuals, whatever group they belong to.

In the case of discretionary differences, the Utilitarian solution is the most preferred by each one of the four groups. This result is even more striking considering that utilitarianism was the least preferred solution to the first problem by three of

---

[10]The Cramer coefficient measures the degree of relation between two sets of variables. The value of this coefficient almost doubles passing from the freshermen's sample to the seniors' one, from 0.15 to 0.27, indicating a much stronger relation between the preferences of the senior students and the course attended.

the four groups and only the second choice of the economics seniors. The two individuals are held accountable for their outcomes when they differ in their effort. The Utilitarian solution, which rewards the character that puts in more effort, is preferred to the other allocations in each group. The answers to this problem are even more homogenous then those to question 2a. The four classes present the same order of preferences: the maximin solution is the second choice, followed by the Egalitarian.

Table 1.2: Question 2a / 2b / 2c (Percentage responses)

Questions	Solutions	Economics Freshmen	Economics Seniors	Sociology Freshmen	Sociology Seniors
2a	E	15	7	25	32
(exogenous)	R	63	76	60	60
	U	22	17	15	8
2b	E	12	6	13	14
(discretionary)	R	34	28	38	36
	U	54	66	49	50
2c	E	30	19	41	36
(need)	R	41	44	44	32
	U	4	1	0	3
	UF	26	36	15	30

Need seems the most obvious force driving the results on question 2c. Economics freshmen and seniors and sociology freshmen prefer the maximin solution, while among the sociology seniors egalitarianism is still the first choice. The four groups share the same scepticism for the Utilitarian solution, which does not guarantee the survival of the more disadvantaged individual. Egalitarianism still proves to be more appreciated by the sociology students; however, even among them, the preferences for the Egalitarian solution decrease from freshmen to seniors in favour of utilitarianism

with a floor. The latter seems to exert a particular attraction on more mature students, whatever course they attend.

In order to analyse the relevance of context we are going to compare the answers given by each student to the first and the second question, anlysing first problem 2a and 2b and then problem 2c. The results of all the tests of differences are reported in Appendix 1B.

Concerning those subjects who received question 2a and those who received question 2b, we can test the significance of the change in their preferences by applying the Stuart-Maxwell test[11].

$$Relevance\ of\ exogenous\ /\ discretionary\ differences \qquad (1.3)$$

$$H_0 \quad : \quad \text{there is no change in the preferences of the}$$

$$\text{subjects passing from the first to the second problem.}$$

In each case we can reject the null hypothesis. This leads to the following result:

Result 1.4: Introducing either exogenous or discretionary differences has changed the preferences of the subjects, whatever group they belong to.

With reference to both classes of each of the four groups, we can look for those single categories for which the differences are significant. We can collapse the original 3x3 tables into 2x2 tables[12] and apply the McNemar test to three different

---

[11]The Stuart-Maxwell test is a variation of McNemar's test appropriate for case-control comparisons involving 3x3 contingency tables. It can be used to test marginal homogeneity between two raters across all categories simultaneously (for a general discussion see Fleiss, 1981).

[12]In these 2x2 tables the answers to the first and to the second question will be categorised respectively as Egalitarian and not Egalitarian, Rawlsian and not Rawlsian, Utilitarian and not Utilitarian.

hypotheses.

$$H_0 \quad : \quad \text{among the respondents who change their} \qquad (1.4)$$

preference, the probability that a respondent

will switch from E (R / U) to not E (not R /

not U) will be the same as the probability that

a respondent will change from not E (not R /

not U) to E (R / U).

In the case of exogenous differences, for each of the four groups we reject the null hypothesis concerning the Egalitarian and the maximin allocation. Once it is explained that the lower productivity of one of the two characters is due to a handicap the consent for the Egalitarian solution diminishes, while more subjects are in favour of the Rawlsian allocation. Furthermore, considering the economics seniors, we can also reject the null hypothesis concerning the Utilitarian distribution, which is preferred by fewer subjects as a solution to problem 2a.

Result 1.5: With the introduction of exogenous differences, the preferences of each group for the Egalitarian distribution have decreased in favour of the maximin solution. The preferences of the economics seniors for both the Egalitarian and Utilitarian solutions have diminished in favour of the maximin one.

Let us consider the discretionary difference treatment. For each of the four groups we reject the null hypothesis concerning the Egalitarian and Utilitarian solutions. When the different productivity is explained in terms of effort the preferences for the Egalitarian distribution decrease, while more people prefer the Utilitarian allocation. Considering the economics seniors, we reject the null concerning each of the three options: the preferences for the maximin and the Egalitarian distribution diminish from the first to the second problem, in favour of the Utilitarian allocation.

Result 1.6: With the introduction of discretionary differences, the preferences of each group for the Egalitarian distribution have decreased in favour of the Utilitarian solution. The preferences of the economics seniors for both the Egalitarian and maximin solutions have diminished in favour of the Utilitarian one.

As revealed by these results, passing from the first to the second question has significantly lowered the consent for the Egalitarian solution among all the groups. This leads to an important result:

Result 1.7: The inefficient allocation seems to be an inadequate solution to the problem, once the circumstances of the distribution are clear.

Let us finally test the hypothesis of *relevance of need* with respect to the subjects who received question 2c[13]. For each group we can reject the null hypothesis with reference to the Utilitarian solution. Need considerations have driven the preferences for the latter to drastically fall in every group. In addition, considering the senior students we reject the null concerning two other mechanisms: the egalitarian with respect to the sociology seniors, and the maximin with reference to the seniors of economics. The two groups show much less consent for these principles when answering question 2c.

Result 1.8: With the introduction of a threshold, the preferences of each group for the Utilitarian distribution have drastically fallen. Moreover, the preferences of the sociology seniors for the Egalitarian allocation have decreased, while the maximin solution is preferred by fewer economics seniors.

---

[13]While the first problem presents three solutions, four distinct distributions are provided as possible solutions to question 2c. The Stuart-Maxwell test cannot be used to test marginal homogeneity in this case and, furthermore, no appropriate test for case-control comparisons involving 3x4 contingency tables exists. It will only be possible to test Hypothesis (1.4) by applying the McNemar test.
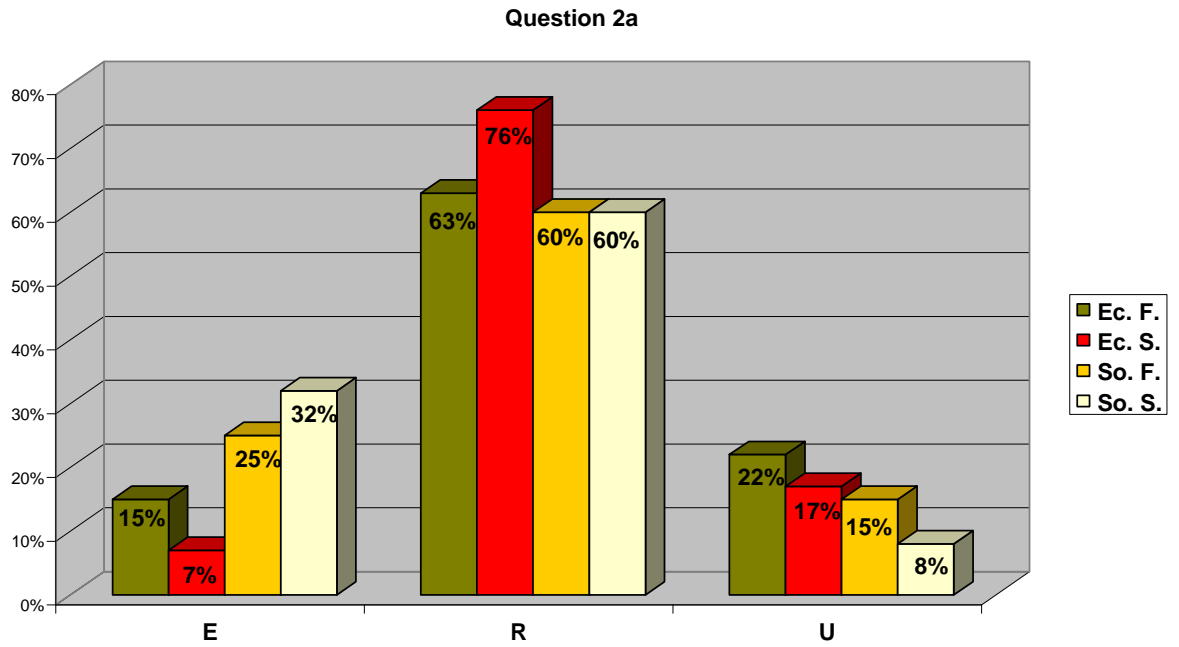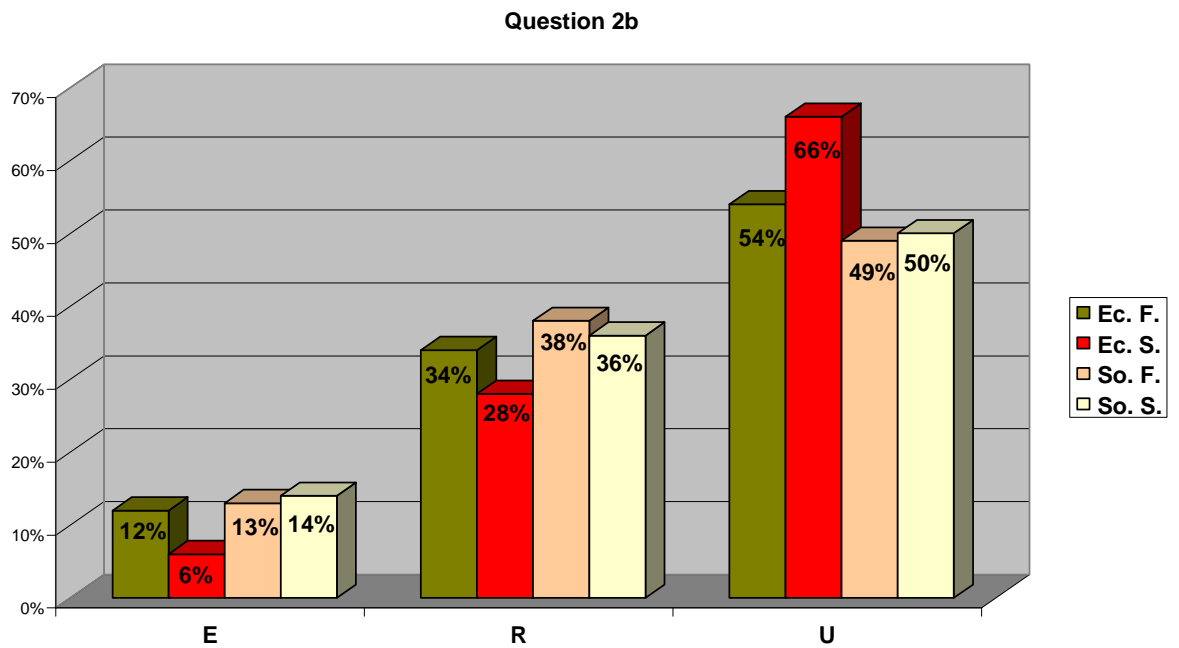
Question 2a

Figure 1.1

Question 2b

Figure 1.2

Comparing the results reported in Table 1.1 with Figures 1.1 and 1.2, we can intuitively infer that when either exogenous or discretionary differences are introduced the answers of the four groups are much more similar. In the base treatment sociology students preferred the Egalitarian solution to the Rawlsian, and the latter to the Utilitarian. Economics seniors strongly preferred the Rawlsian distribution to the Utilitarian, and the Utilitarian to the Egalitarian. Finally, among economics freshmen the percentage of preferences for the maximin allocation was slightly higher than for the Egalitarian, and the latter was preferred to the Utilitarian distribution. Let us conclude our analysis investigating whether clarifying the context of the distribution leads to a common solution accepted by the parties.

$$
\begin{aligned}
&\textit{Agreement hypothesis} \qquad\qquad\qquad (1.5)\\
&H_0 \quad : \quad \text{the proportion of subjects in each of}\\
&\qquad\qquad \text{the option categories is the same in}\\
&\qquad\qquad \text{each group}
\end{aligned}
$$

Let us first consider the case of exogenous differences. The value of the test statistic is $\chi^2 = 29.21$ and we reject the null hypothesis ($p = 0.00005$): although in each of the four groups the absolute majority prefer the Rawlsian solution, the preferences of the four groups do not follow the same distribution. We can test Hypothesis (1.5) with reference to four different cases: economics and sociology freshmen, economics and sociology seniors, economics freshmen and seniors, sociology freshmen and seniors. We reject the null ($p = 0.00001$) only concerning economics and sociology seniors, given the high Chi-square value, $\chi^2 = 22.9$. We cannot reject it in the other cases. Testing the hypothesis with economics and sociology freshmen we have $\chi^2 = 4.49$ ($p = 0.1059$). Testing it with economics freshmen and seniors the Chi-square value is $\chi^2 = 5.68$ ($p = 0.0584$). In the case of sociology freshmen and seniors the result is $\chi^2 = 3.39$ ($p = 0.1836$). Let us make two interesting points.

In comparison to the answers to question 1, the differences between economics and sociology freshmen and between economics freshmen and seniors have drastically diminished[14]. Let us summarise these results.

> **Result 1.9:** When exogenous differences are introduced, the absolute majority of every group prefer the maximin principle as a solution to the problem. In addition, the difference due to the selection effect as well as the disagreement between economics freshmen and seniors has decreased.

In the case of discretionary differences, testing Hypothesis (1.5) with respect to all the groups the value of the test statistic is $\chi^2 = 8.5$. We cannot reject the hypothesis according to which the preferences of the four groups follow the same distribution ($p = 0.2037$). The answers to this question confirm the results obtained in the literature on differences in effort (see Konow, 2003, for an overview).

> **Result 1.10:** The preferences of the four groups with respect to the problem with discretionary differences follow the same distribution. The Utilitarian allocation is the most preferred, the maximin solution is the second choice and the Egalitarian is the least preferred.

Finally, we consider the need version of the distribution problem. Testing Hypothesis (1.5) with reference to the four groups the value of the test statistic is $\chi^2 = 26.5$ and we have to reject the null hypothesis ($p = 0.0017$). The only case in which we cannot reject it ($p = 0.1316$) is comparing the answers of the economics freshmen and seniors, $\chi^2 = 5.62$. With respect to them, we can conclude that their preferences follow the same distribution. Unlike other experiments (Frohlich, Oppenheimer, and Eavy, 1987a, 1987b; Lissowsky, Tyszka, and Okrasa, 1991), the introduction of a floor did not enable a solution accepted by all of the parties. However, we notice

---

[14]Note that although the p-values are higher than 0.05, they are still close (especially in the case of economics freshmen and seniors).

that an agreement looks closer now than in the first problem. Three groups prefer the Rawlsian solution and even among the sociology seniors the percentage of preferences for the maximin allocation is very close to the first choice. The solution we proposed according to utilitarianism with a floor only guarantees the *survival* of the least advantaged character. That did not look enough to most of the subjects, who instead preferred maximising his utility. An interesting extension would be to test whether solutions with an intermediate floor would lead to greater consensus. The following summarises these results.

> **Result 1.11:** In the need treatment all of the groups, except for the sociology seniors, prefer the maximin allocation as a solution to the question, while the utilitarian distribution is by far the least preferred by each group. Utilitarianism with a floor is more preferred by senior students.

1.6 Conclusions

Several studies have been conducted to analyse whether judgments of fairness are context dependent. To our knowledge, though, this is the first attempt to explore distributive justice analysing the extent to which clarifying the context of a distribution favours greater consensus on what is just. We surveyed first and last year undergraduate students of economics and sociology, focusing on contextual factors related to responsibility and need considerations. Let us summarise the main results we obtained.

A selection effect does exist. The ideology of students choosing to study sociology differs from that of students who choose to study economics, the former group being much more concerned about equality. Further, the differences in ideology increase with the seniority of the subjects. Economics seniors show a higher appreciation of the Pareto principle than their younger colleagues, which is not the case with sociology students (with reference to this point see also Amiel and Cowell, 1999, although

the authors do not distinguish between selection and learning effects). However, training in economics does not seem to influence the concern for the maximisation of total output. These results cannot be directly compared to most of the literature about the differences between economists and non-economists, these studies being aimed to analyse whether students of economics behave in a more self-interested way than students of other disciplines. In any case, unlike the results obtained by Marwell and Ames (1981), our results indicate that even economics students have an interest in fairness. This may be due to the different design of the question.

Clarifying the context of the distribution, by either explaining the differences between the individuals or introducing a minimum survival utility, significantly influences perception of fairness of the subjects, whatever group they belong to. Whenever it is not efficient, the egalitarian principle seems to provide inadequate solutions if the circumstances of the distributions are clear. Need considerations seem to drive the results when a floor is introduced: the utilitarian principle is abandoned, if the solution it prescribes does not enable every individual to reach the minimum utility. Interestingly sociology seniors develop an increased preference for utilitarianism with a floor relative to sociology freshmen.

The most notable result is that clarifying the context favours greater consensus on what is fair. The maximin criterion proves to be an adequate solution to the distribution problem when exogenous differences, for which the individuals cannot be held responsible, are introduced. The utilitarian principle, on the contrary, meets a great success when the individuals put in different efforts (discretionary differences). The introduction of a floor does not seem to be an equally successful way to achieve a social agreement. However, only a particular solution corresponding to the utilitarian principle with a floor constraint has been explored. There are several interesting extensions for future research and a series of social experiments should be conducted in order to point out a plausible threshold that would appear just to most people.

Appendix 1A

Question 1

After a shipwreck Robinson and Friday have landed on two different islands divided by a narrow but deep channel.

On each of the two islands one can till **12 plants**. The only reason why both Robinson and Friday would like to cultivate these plants is because they produce fruit and the higher amount of fruit they obtain, the more their welfare would be; every additional fruit produces an equal value, which is identical for both people.

It has been decided that you are the one who will chose how to distribute the plants between Robinson and Friday. You've been given the following information, which the two survivors also know:

Robinson lives on island **A** and Friday lives on island **B**.

All plants of one island are identical to the ones of the other island. How much fruit they produce depends on the way they are cultivated.

Friday obtains **120 fruits** per year from every plant on island B, but he cannot obtain any fruit from island A's plants.

On both islands Robinson obtains **20 fruits** per plant.

There's no possibility of redistributing the plants after the allocation and there's also no chance to exchange any fruit, which is produced.

How would you divide the 12 plants of island A and the 12 plants of island B so that, from your point of view, the distribution would be just?

Choose:

1	Plants island A	Plants island B	Fruits
Robinson	12	0	240
Friday	0	12	1440
	Total production of fruits	1680	

2	Plants island A	Plants island B	Fruits
Robinson	12	8	400
Friday	0	4	480
	Total production of fruits	880	

3	Plants island A	Plants island B	Fruits
Robinson	9	9	360
Friday	3	3	360
	Total production of fruits	720	

Question 2a

After a shipwreck Robinson and Friday have landed on two different islands divided by a narrow but deep channel.

On each of the two islands one can till **12 plants**. The only reason why both Robinson and Friday would like to cultivate these plants is because they produce fruit and the higher amount of fruit they obtain, the more their welfare would be; every additional fruit produces an equal value, which is identical for both people.

It has been decided that you are the one who will chose how to distribute the plants between Robinson and Friday.

You've been given the following information, which the two survivors also know:

Robinson lives on island **A** and Friday lives on island **B**.

All plants of one island are identical to the ones of the other island. How much fruit they produce depends on the way they are cultivated.

Both Robinson and Friday put the same amount of work into tilling the plants; the only way to move from one island to the other is to swim.

Friday can obtain **120 fruits** per year from every plant of island B, but he cannot swim and he cannot till any plant on island A.

Robinson is a perfect swimmer and he can therefore till plants on both islands, but due to a wound caused by the shipwreck he cannot obtain more than **20 fruits** per year from every plant of island A and island B.

There's no possibility of redistributing the plants after the allocation and there's also no chance to exchange any fruit, which is produced.

How would you divide the 12 plants of island A and the 12 plants of island B so that, from your point of view, the distribution would be just?

Choose:

1	Plants island A	Plants island B	Fruits
Robinson	12	0	240
Friday	0	12	1440
	Total production of fruits	**1680**	

2	Plants island A	Plants island B	Fruits
Robinson	12	8	400
Friday	0	4	480
	Total production of fruits	**880**	

3	Plants island A	Plants island B	Fruits
Robinson	9	9	360
Friday	3	3	360
	Total production of fruits	**720**	

Question 2b

After a shipwreck Robinson and Friday have landed on two different islands divided by a narrow but deep channel.

On each of the two islands one can till **12 plants**. The only reason why both Robinson and Friday would like to cultivate these plants is because they produce fruit and the higher amount of fruit they obtain, the more their welfare would be; every additional fruit produces an equal value, which is identical for both people.

It has been decided that you are the one who will chose how to distribute the plants between Robinson and Friday.

You've been given the following information, which the two survivors also know:

Robinson lives on island **A** and Friday lives on island **B**.

All plants of one island are identical to the ones of the other island. How much fruit they produce depends on the way they are cultivated.

Robinson and Friday can till plants and move from one island to the other in the same way, but they do not put the same amount of work into tilling the plants.

Friday can obtain **120 fruits** per year from every plant of island B, but he doesn't want to go on island A and he will not produce fruits on this island.

To Robinson moving from one island to the other is all the same, but he does not put as much amount of work into tilling his plants as Friday and he doesn't produce more than **20 fruits** per year from every plant, both on island A and B.

There's no possibility of redistributing the plants after the allocation and there's also no chance to exchange any fruit, which is produced.

How would you divide the 12 plants of island A and the 12 plants of island B so that, from your point of view, the distribution would be just?

Choose:

1	Plants island A	Plants island B	Fruits
Robinson	12	0	240
Friday	0	12	1440
	Total production of fruits	**1680**	

2	Plants island A	Plants island B	Fruits
Robinson	12	8	400
Friday	0	4	480
	Total production of fruits	**880**	

3	Plants island A	Plants island B	Fruits
Robinson	9	9	360
Friday	3	3	360
	Total production of fruits	**720**	

Question 2c

After a shipwreck Robinson and Friday have landed on two different islands divided by a narrow but deep channel.

On each of the two islands one can till **12 plants**. The only reason why both Robinson and Friday would like to cultivate these plants is because they produce fruit and the higher amount of fruit they obtain, the more their welfare would be; every additional fruit produces an equal value, which is identical for both people.

It has been decided that you are the one who will chose how to distribute the plants between Robinson and Friday.

You've been given the following information, which the two survivors also know:

The minimum quantity needed by every one of them in order to survive is 300 fruits per year.

Robinson lives on island **A** and Friday lives on island **B**

All plants of one island are identical to the ones of the other island. How much fruit they produce depends on the way they are cultivated.

Friday obtains **120 fruits** per year from every plant on island B, but he cannot obtain any fruit from island A's plants.

On both islands Robinson obtains **20 fruits** per plant.

There's no possibility of redistributing the plants after the allocation and there's also no chance to exchange any fruit, which is produced.

How would you divide the 12 plants of island A and the 12 plants of island B so that, from your point of view, the distribution would be just?

Choose:

1	Plants island A	Plants island B	Fruits
Robinson	12	0	240
Friday	0	12	1440
	Total production of fruits	**1680**	

2	Plants island A	Plants island B	Fruits
Robinson	12	8	400
Friday	0	4	480
	Total production of fruits	**880**	

3	Plants island A	Plants island B	Fruits
Robinson	9	9	360
Friday	3	3	360
	Total production of fruits	**720**	

4	Plants island A	Plants island B	Fruits
Robinson	12	3	300
Friday	0	9	1080
	Total production of fruits	**1380**	

Appendix 1B

We report the Chi-square values resulting from testing Hypotheses (1.3) and (1.4). The numbers in brackets represent the p-values.

Table 1.3 reports the results of the application of the Stuart-Maxwell tests to Hypothesis (1.3).

Table 1.3: Stuart-Maxwell tests

	Economics Freshmen	Economics Seniors	Sociology Freshmen	Sociology Seniors
Question 1/2a	19.8	21.89	20.21	19.53
	(0.0005)	(0.0002)	(0.0004)	(0.0006)
Question 1/2b	25.32	35.36	40.65	48.17
	(0.00004)	(3.918e-7)	(3.176e-8)	(8.698e-10)

The following tables report the results of the application of the McNemar tests to Hypothesis (1.4) with respect to economics freshmen, economics seniors, sociology freshmen and sociology seniors.

Table 1.4: Economics Freshmen (McNemar tests)

	E / not E	R / not R	U / not U
Question 1/2a	14.38	15.8	0.24
	(0.0001)	(0.00007)	(0.6242)
Question 1/2b	16.68	0.5	19.59
	(0.0004)	(0.4795)	(9.597e-06)
Question 1/2c	0.45	0.11	19.05
	(0.5023)	(0.7401)	(0.00001)

Table 1.5: Economics Seniors (McNemar tests)

	E / not E	R / not R	U / not U
Question 1/2a	8.45	20.1	7.26
	(0.0036)	(7.350e-6)	(0.0070)
Question 1/2b	12	8.76	32.59
	(0.0005)	(0.0031)	(1.138e-8)
Question 1/2c	0.36	5.26	18.05
	(0.5485)	(0.0218)	(0.00002)

Table 1.6: Sociology Freshmen (McNemar tests)

	E / not E	R / not R	U / not U
Question 1/2a	17.42	16.17	0
	(0.00002)	(0.00005)	(1)
Question 1/2b	27.84	0.21	28.26
	(1.318e-7	(0.6468)	(1.061e-7)
Question 1/2c	1.07	2.23	18.05
	(0.3009)	(0.1353)	(0.0002)

Table 1.7. Sociology Seniors (McNemar tests)

	E / not E	R / not R	U / not U
Question 1/2a	9.48	16.96	2.28
	(0.0021)	(0.00003)	(0.1310)
Question 1/2b	30.62	0.08	31.03
	(3.138e-8)	(0.7773)	(2.541e-8)
Question 1/2c	7.56	0	17.05
	(0.0060)	(1)	(0.0003)

Chapter 2

The Important Thing Is not (Always) Winning but Taking Part: Funding Public Goods with Contests

2.1 Introduction

This paper looks at multiple prize contests as a way to overcome the free-riding problem. It is well known that the public good provision resulting from individual voluntary contributions is generally sub-optimal, because of the incentive to free-ride associated with positive externalities (e.g. see Bergstrom et al., 1986; Andreoni, 1988a). While fund-raising mechanisms based on tax rewards and penalties can be designed to solve this problem (e.g. Groves and Ledyard, 1977; Walker, 1981), they are not available to private organisations with no coercive power, such as charities or civic groups. Contests as incentive mechanisms are different from the above solutions because no power to enforce sanctions is required on the part of the institution conducting the tournament. Contests are competitions in which agents spend resources

in order to win one or more prizes. The main characteristic is that, independently of success, all participants bear some costs.

A number of recent studies have explored different incentive based fund-raising mechanisms both theoretically and through laboratory experiments. These studies have focused on the use of lotteries (e.g. Morgan, 2000; Morgan and Sefton, 2000), the comparison between winner-pay and all-pay auctions with one prize (e.g. Goeree et al., 2005) and between lotteries and one prize all-pay auctions (e.g. Orzen, 2005).

Morgan (2000) studies a lottery mechanism where one prize is awarded as a way to overcome the free-riding problem. Contributions to the public good entitle to lottery tickets, one ticket is randomly drawn and the holder wins the prize. The public good consists of the revenue of the lottery net of the value of the prize. He considers a model with quasilinear preferences where all players contribute the same amount in equilibrium, independently of their income.

Goeree et al. (2005) compare the results of a winner-pay and an all-pay auction with one prize with a lottery. They study a public good game with a linear production function where agents are unconstrained and have heterogeneous preferences which are private information. The authors show that the all-pay auction dominates both the other mechanisms.

The puzzle with the equilibrium defined in Morgan (2000) is that it predicts that agents with different incomes contribute the same amount. This pattern does not seem to properly describe reality, while it seems more plausible that individuals with higher incomes contribute more than what poorer ones do[1]. On the other hand, Goeree et al. (2005) do not explore the behaviour of agents with heterogeneous endowments. Contrary to these analyses, in the present study we identify an equilibrium in which the contribution is strictly increasing in the endowment. Furthermore, although there exists a large literature which analyses the use of mul-

---

[1]Interestingly, so far experiments on lotteries as a way to finance public goods have only focused on the case of subjects with homogeneous endowments (see, for instance, Morgan and Sefton, 2000; Orzen, 2005).

tiple prize contests and tournaments as incentive schemes[2], their use as fund-raising mechanisms has not been explored.

Moldovanu and Sela (2001) study a multiple prize contest where unconstrained agents differ in the ability to exert effort, and the ability is private information. They show that when costs are either linear or concave allocating only one prize maximises the total expected effort exerted by the bidders, while when costs are convex more prizes could be optimal. However, the paper on multiple prize contests most closely related to ours is the one by Barut and Kovenock (1998), who study symmetric multiple prize all-pay auctions with complete information. They extend the analysis of first price all-pay auctions with complete information[3] and show that, when players are not constrained, only mixed strategy equilibria exist. Further, expected expenditures are maximised by driving the value of the lowest prize to zero, but are invariant across all configurations leaving the lowest value fixed and the sum of the values constant.

In this paper we consider a public good game with a linear production function (as in Goeree et al., 2005) where agents have heterogenous endowments which are private information. Such a game is a modified version of the game with complete information which is typically employed in public good experiments. Each agent chooses how much of her wealth to allocate to the public good; this money is multiplied by a parameter, which takes a value between one and the number of players, and shared equally among all the agents. The unique Nash equilibrium is to contribute nothing, although it is socially optimal to contribute all the wealth. We overcome this extreme free-riding via a contest where several prizes may be awarded. We as-

---

[2]Applications have been made to promotions in labour markets (Lazear and Rosen, 1981), technological and research races (Wright, 1983; Dasgupta, 1986; Taylor, 1995; Fullerton and McAfee, 1999; Windham, 1999), credit markets (Breocker, 1990), and rent seeking (Tullock, 1980) among others.

[3]The first price all-pay auction with complete information has been utilised extensively in the literature (Dasgupta, 1986; Hillman and Samet, 1987; Hillman and Riley, 1989; Ellingsen, 1991; Baye et al., 1993). There exists no pure strategy Nash equilibrium and a complete characterisation of its equilibria appears in Baye et al. (1996).

sume that the social planner has access to a small[4] budget, which can be allocated in form of prizes. The first prize is awarded to the player who contributes the most, the second prize to the player with the second highest contribution and so on until all prizes are awarded. The social planner determines the prize structure in order to maximise expected total welfare net of the value of the total prize sum.

Heterogeneity and incomplete information enable us to characterise a monotone equilibrium, in which the contribution is strictly increasing in the endowment. Such an equilibrium is a purification of the mixed strategy equilibrium identified by Barut and Kovenock (1998) in a symmetric setting with complete information and unconstrained agents. Our prediction seems more plausible than a completely symmetric equilibrium, either in mixed or in pure strategies. We prove that it is optimal for the social planner to set the last prize equal to zero, but otherwise total expected contribution is independent of the distribution of the total prize sum among the prizes. We show that the contest is a budget balanced incentive mechanism: expected total contribution is higher than the value of the total prize sum. There exists a critical level of the budget under which the monotone equilibrium exists independently of the prize structure. For any possible distribution of the endowments we identify necessary and sufficient conditions for the total prize sum to be below this critical level. Finally we prove that private provision via a contest Pareto-dominates public provision and is higher than total contribution in a lottery.

The paper is structured as follows. In Section 2.2 we introduce a linear public good game with complete information, as it is usually employed in public good experiments. In Section 2.3 we present the model and identify the Nash equilibrium. In Section 2.4 we solve the designer's problem and discuss the existence of the equilibrium. Section 2.5 compares private provision via a contest with both public provision and private provision in a lottery. Section 2.6 concludes.

---

[4]We focus on cases where fund-raisers auction prizes of relatively low value. This seems to be the main source of revenue for most charities. See for example Goeree et al. (2005), who provide data showing that the vast majority of fund-raisers seek small donations from a large number of donors.

2.2 A Linear Public Good Game with Complete Information

In this section we present the game which is typically used in public good experiments. n subjects take part in the experiment. Each subject is endowed with the same amount of money z and simultaneously chooses how much of her wealth to allocate to the public good; this money is multiplied by a parameter α and shared equally among all the subjects. Agent i's payoff can be described by

$$U_i = z - g_i + \alpha \frac{G}{n}$$

where g_i is i's contribution to the public good and $G = \sum_{i=1}^{n} g_i$ is the total level of public good. If $\alpha \in (1, n)$ an individual's opportunity cost of contributing to the public good exceeds the marginal return of investing in the public good. Thus, the unique Nash equilibrium of the game is to contribute nothing, while it is efficient to contribute all the wealth.

2.3 The Model

Let us consider n players. Each player i is assumed to have endowment z_i, which is private information. Endowments are drawn independently of each other from the interval $[0, 1]$ according to the distribution function $F(z)$, which is common knowledge, with mean $E[z]$. We assume that $F(z)$ has a continuous and bounded density $F'(z) > 0$. Players play a public good game in which each individual has to choose how much to contribute to the public good. At the same time they take part in a contest in which n prizes are awarded such that $\pi_1 \geq \cdots \geq \pi_{m-1} > \pi_m = \cdots = \pi_n \geq 0$, $1 < m \leq n$ and $\sum_{j=1}^{n} \pi_j = \Pi$. This assumption rules out the possibility of awarding n equal prizes and will enable us to find an equilibrium. We will call

$\pi = (\pi_1, \cdots, \pi_n) \in \mathbb{R}^n$ the vector of prizes. The player with the highest contribution wins π_1, the player with the second highest contribution wins π_2, and so on until all the prizes are allocated. For each player, a strategy $g(z)$ will be the contribution to the public good as a function of the player's endowment and the action space for player i will be the interval $[0, z_i]$. If player i, who has endowment z_i and contributes g_i, wins prize j her payoff is

$$U_i = z_i - g_i + \alpha \frac{G}{n} + \pi_j$$

where $\alpha \in (1, n)$. Given the value assumed by α, notice that the equilibrium in the absence of a contest is the same as in the game described in Section 2.2.

Each player i chooses her contribution in order to maximise expected utility (given the other players' contributions and given the values of the different prizes). We will assume that Π is exogenously determined. For a given value of Π, the social planner determines the number of prizes having positive value and the distribution of the total prize sum among the different prizes in order to maximise the expected value of total welfare net of the value of Π (given the players' equilibrium strategy functions).

We will focus on the case in which the equilibrium strategy $g(z)$ is less than z for any type z on the interval $[0, 1]$. In order to find the equilibrium of the game it is useful to introduce the function

$$K(F(z)) = \sum_{i=1}^{n} \pi_i \binom{n-1}{i-1} (F(z))^{n-i} (1 - F(z))^{i-1} \tag{2.1}$$

Given a vector of prizes π, $K(F(z))$ is a linear combination of n order statistics with weights equal to the prizes. If all agents adopt the same strictly increasing strategy $g(z)$, $K(F(z))$ represents the expected prize of the player with endowment z. The following result will help us identify the equilibrium of the game.

Lemma 2.1 *The function $K(F(z))$ is strictly monotonic increasing in z.*

Proof. Let's consider z_i and z_j such that $0 \leq z_i < z_j \leq 1$. Given that $F(z_i) < F(z_j)$, and given the assumption that $\pi_1 \geq \cdots \geq \pi_{m-1} > \pi_m = \cdots = \pi_n \geq 0$, $1 < m \leq n$, $K(F(z_j))$ assigns higher weights than $K(F(z_i))$ to higher prizes and lower weights than $K(F(z_i))$ to lower prizes. Therefore $K(F(z_i)) < K(F(z_j))$. ∎

Given Lemma (2.1), at interior solutions for all players we are able to characterise a monotone equilibrium, in which the contribution is strictly increasing in the endowment. Later on we will identify necessary and sufficient conditions for its existence independently of the prize structure.

Proposition 2.1 *Given a vector π of prizes, at an interior solution for all players the game has a symmetric pure strategy equilibrium given by*

$$g(z) = \frac{n}{n - \alpha}(K(F(z)) - \pi_n)$$

Proof. The expected utility of a player from a choice g can be calculated as

$$E[U(z - g, \pi) \mid g, g_{-i}] =$$
$$z - g + \alpha\frac{G}{n} + (\Pr[1 \mid g, g_{-i}]\pi_1 + \Pr[2 \mid g, g_{-i}]\pi_2 + \cdots + \Pr[n \mid g, g_{-i}]\pi_n)$$

where $\Pr[j \mid g, g_{-i}]$ is the probability of a choice g being j-th highest conditional on the other strategies g_{-i}. If all agents adopt the same strictly increasing strategy $g(z)$, then the probability that a candidate with endowment z_i is higher ranked than another randomly chosen candidate is $\Pr[g(z_i) > g(z)] = \Pr[z_i > z] = F(z_i)$. Therefore

$$(\Pr[1 \mid g, g_{-i}]\pi_1 + \Pr[2 \mid g, g_{-i}]\pi_2 + ... + \Pr[n \mid g, g_{-i}]\pi_n) =$$
$$K(F(z)) = \sum_{i=1}^{n} \pi_i \binom{n-1}{i-1}(F(z))^{n-i}(1 - F(z))^{i-1}$$

Now, given the common strategy $g(z)$, we suppose that an individual with endow-

ment z chooses $g(\hat{z})$ for some $\hat{z}$, then her expected utility will be

$$z - g(\hat{z}) + \alpha \frac{G_{-i} + g(\hat{z})}{n} + K(F(\hat{z}))$$

where G_{-i} is the sum of the contributions of all the other players. Differentiating with respect to $\hat{z}$ we obtain

$$\frac{\alpha - n}{n} g'(\hat{z}) + K'(F(\hat{z}))F'(\hat{z})$$

In equilibrium the individual with endowment z should choose $g(z)$ so that the above will be equal to zero when $\hat{z} = z$, and we have

$$g'(z) = \frac{n}{n - \alpha} K'(F(z))F'(z)$$

A player with the lowest possible endowment $z = 0$ does not contribute to the public good and wins the last prize. This yields the boundary condition $g(0) = 0$. Hence, the solution is

$$g(z) = \frac{n}{n - \alpha}(K(F(z)) - \pi_n)$$

From Lemma (2.1) we know that the candidate equilibrium function g is strictly monotonic increasing.

Assuming that all players rather than i play according to g, we finally need to show that, for any type z of player i, the contribution $g(z)$ maximises the expected utility of that type. Let us consider an individual with endowment z. If she plays $g(z) = \frac{n}{n-\alpha}(K(F(z)) - \pi_n)$ her expected utility is given by

$$E[U(z, g(z)) \mid g_{-i}] = z - \frac{\alpha}{n - \alpha}K(F(z)) + \frac{n}{n - \alpha}\pi_n + \frac{\alpha}{n}G$$

If she deviates and plays $\frac{n}{n-\alpha}(K(F(\hat{z})) - \pi_n)$ for some $\hat{z} \neq z$ her expected utility will

be

$$E[U(z, g(\hat{z})) \mid g_{-i}] =$$

$$z - \frac{n}{n-\alpha}K(F(\hat{z})) + \frac{n}{n-\alpha}\pi_n + \frac{\alpha}{n}(G - \frac{n}{n-\alpha}K(F(z)))$$

$$+\frac{n}{n-\alpha}\pi_n + \frac{n}{n-\alpha}K(F(\hat{z})) - \frac{n}{n-\alpha}\pi_n) + K(F(\hat{z}))$$

$$= z - \frac{\alpha}{n-\alpha}K(F(z)) + \frac{n}{n-\alpha}\pi_n + \frac{\alpha}{n}G$$

Therefore she is indifferent to play any other strategy $\frac{n}{n-\alpha}(K(F(\hat{z})) - \pi_n)$. If her action space $[0, z]$ is a subset of the set g_{-i} this rules out the possibility that she might be better off deviating from $g(z)$. If $z > g(1)$ it is easy to show that she would be worse off playing any strategy greater than $g(1)$. In fact, playing $g(1)$ would already guarantee π_1 and any higher contribution would result in a lower expected utility. ∎

Notice that the equilibrium strategy function defined in Proposition (2.1) can be rearranged as $g(z) = \frac{1}{1-\frac{\alpha}{n}}(K(F(z)) - \pi_n)$. The latter is the sum of a convergent series with reason $\frac{\alpha}{n}$ and can be expressed as

$$g(z) = (K(F(z)) - \pi_n) \sum_{m=0}^{\infty} (\frac{\alpha}{n})^m$$

The first part of the above equation represents the expected prize, in equilibrium, of a player with endowment z, net of the value of the last prize. This is because a contribution equal to zero would guarantee the agent to win the lowest prize. The multiplier $\sum_{m=0}^{\infty} (\frac{\alpha}{n})^m$ reflects the return to investment in the public good. In standard all-pay auctions an agent bids her expected prize in equilibrium. In our model, if an agent contributes up to her expected prize (net of the last prize) she receives back $\frac{\alpha}{n}$ times the value of her bid because of the public good. This implies that she will add to her contribution this remaining value, from which she will get back an equal proportion, and so on.

2.4 Designer's Problem and Revenue Equivalence

In this section we consider the maximisation problem faced by the designer, assuming that wealth constraints are non-binding for all players. We will then discuss the conditions which guarantee the existence of the equilibrium independently of the allocation of Π across the prizes.

Recall that the social planner determines the number of prizes having positive value and the distribution of the total prize sum among the different prizes in order to maximise expected total welfare net of the value of Π (given the players' equilibrium strategy functions). In order to analyse the maximisation problem we let the vector of prizes π be variable, maintaining the assumptions that $\pi_1 \geq \cdots \geq \pi_{m-1} > \pi_m = \cdots = \pi_n \geq 0$, $1 < m \leq n$ and $\sum_{j=1}^{n} \pi_j = \Pi$, and we study the family of functions

$$\phi(F(z), \pi \mid \sum_{j=1}^{n} \pi_j = \Pi, \pi_1 \geq \cdots \geq \pi_{m-1} > \pi_m = \tag{2.2}$$

$$\cdots = \pi_n \geq 0, 1 < m \leq n) =$$

$$\sum_{i=1}^{n} \pi_i \binom{n-1}{i-1} (F(z))^{n-i}(1 - F(z))^{i-1}$$

Notice that, if π were fixed expression (2.2) would reduce to $K(F(z))$, as presented in equation (2.1). Letting the vector of prizes π be variable, at an interior solution for all players, the equilibrium strategy is represented by the following[5]

$$g(z, \pi) = \frac{n}{n-\alpha}(\phi(F(z), \pi) - \pi_n) =$$

$$\frac{n}{n-\alpha} \sum_{i=1}^{n} \pi_i \binom{n-1}{i-1} (F(z))^{n-i}(1 - F(z))^{i-1} - \frac{n}{n-\alpha}\pi_n$$

---

[5] For simplicity of notation, unless differently specified, from now on we will refer to $\phi(F(z), \pi \mid \sum_{j=1}^{n} \pi_j = \Pi, \pi_1 \geq \cdots \geq \pi_{m-1} > \pi_m = \cdots = \pi_n \geq 0, 1 < m \leq n)$ as $\phi(F(z), \pi)$.

The social planner faces the following maximisation problem

$$\max_{\pi} W = n \int_0^1 (z - g(z,\pi) + \frac{\alpha}{n}G + \phi(F(z),\pi))F'(z)dz - \Pi \tag{2.3}$$

Recall that in equilibrium $\phi(F(z),\pi)$ represents the expected prize of a player with endowment z. This means that independently of the distribution of the total prize sum across the prizes the following holds

$$n \int_0^1 \phi(F(z),\pi)F'(z)dz = \Pi \tag{2.4}$$

Therefore the expected total contribution in equilibrium is equal to

$$G = n \int_0^1 g(z,\pi)F'(z)dz = \frac{n}{n-\alpha}\Pi - \frac{n^2}{n-\alpha}\pi_n \tag{2.5}$$

Note that the above expression depends only on the total prize sum and on the value of the last prize. Finally we can rearrange expression (2.3) as

$$\max_{\pi} W = n \int_0^1 (z - g(z,\pi) + \frac{\alpha}{n-\alpha}\Pi - \frac{\alpha n}{n-\alpha}\pi_n)F'(z)dz \tag{2.6}$$

and we can state the following result.

Proposition 2.2 *At an interior solution for all players the social planner optimally sets the last prize equal to zero, but otherwise expected total contribution is $G = \frac{n}{n-\alpha}\Pi$ independently of the distribution of the total prize sum among the prizes.*

Proof. Expression (2.6) can be rewritten as

$$
\max_{\pi} W = n \int_0^1 (z - \frac{n}{n-\alpha} \phi(F(z), \pi) + \tag{2.7}
$$

$$
\frac{n}{n-\alpha} \pi_n + \frac{\alpha}{n-\alpha} \Pi -
$$

$$
\frac{\alpha n}{n-\alpha} \pi_n) F'(z) dz = nE[z] + \frac{n(\alpha-1)}{n-\alpha}(\Pi - n\pi_n)
$$

It is obvious that $\pi_n = 0$ maximises the above expression. Further, from equation (2.5) we know that total expected contribution equals $\frac{n}{n-\alpha}\Pi$ when $\pi_n = 0$. ∎

Total expected contribution is higher than the total prize sum. While the standard result in all-pay auctions is the total dissipation of rent, in our model over-dissipation occurs because of the marginal return of investing in the public good. Furthermore, from expression (2.7) we can see that in equilibrium total expected welfare net of the value of Π equals $nE[z] + \frac{n(\alpha-1)}{n-\alpha}\Pi$, where $nE[z]$ represents initial welfare. This implies that the contest is a budget balanced incentive mechanism: the social planner does not need to possess Π in the first place, but can simply detract it from the total contribution.

Corollary 2.1 *The contest is a budget balanced incentive mechanism. At an interior solution for all players, provided that the social planner optimally sets the last prize equal to zero, total expected welfare net of the value of Π is higher than initial welfare and equals $nE[z] + \frac{n(\alpha-1)}{n-\alpha}\Pi$.*

So far we have assumed that wealth constraints are non-binding for all agents. In order for the revenue equivalence to hold the solution must be interior for all players for any possible distribution of the total prize sum among the prizes. Continuity together with the assumption that $F(z)$ has a bounded density guarantee the existence of the equilibrium, independently of the allocation of prizes, if Π is small enough.

Proposition 2.3 *There exists a critical level $\bar{\Pi}$ such that the equilibrium strategy*

function is interior for all players independently of the distribution of the total prize sum across the first $n-1$ prizes if and only if $\Pi \leq \bar{\Pi}$.

Proposition (2.8) in Appendix 2 provides necessary and sufficient conditions for Π to be below such a critical value independently of the distribution of Π among the prizes, provided that the social planner optimally sets the last prize equal to zero.

2.5   Contest versus Public Provision and Lottery

We are going to compare the result obtained through a contest with both the welfare generated by public provision and the total contribution resulting from the use of a lottery.

When socially desirable public goods are not privately provided the obvious alternative is to publicly provide them. Suppose that the social planner has access to a budget equal to $\Pi \leq \bar{\Pi}$. Instead of allocating this sum in form of prizes he provides an amount of public good equal to Π. We want to analyse how the total expected welfare generated by public provision compares with that resulting form the use of a contest as an incentive scheme.

Proposition 2.4 *Private provision of public good via a contest, in which the total sum prize $\Pi \leq \bar{\Pi}$ is distributed among the $n-1$ players who contribute the most, Pareto-dominates public provision. If the social planner uses $\Pi \leq \bar{\Pi}$ to publicly provide the public good the expected total welfare net of the value of Π is $W^P = nE[z] + (\alpha - 1)\Pi$.*

Proof. If the social planner uses $\Pi \leq \bar{\Pi}$ to provide the public good the expected total welfare net of the value of Π is given by

$$W^P = n\int_0^1 (z + \frac{\alpha}{n}\Pi)F'(z)dz - \Pi = \tag{2.8}$$
$$nE[z] + (\alpha - 1)\Pi$$

From Corollary (2.1) we know that, if the last prize is equal to zero, the expected total welfare generated by a contest is equal to

$$W = nE[z] + \frac{n(\alpha - 1)}{n - \alpha}\Pi$$

which is strictly greater than expression (2.8). ∎

We now consider the case where the social planner resorts to a lottery to encourage contribution to the public good. In order to be able to compare the lottery mechanism with a contest we will restrict the analysis to interior solutions. To do this let us assume n players whose endowments are drawn independently of each other from the interval $[\underline{z}, \bar{z}]$, with $\underline{z}$ strictly positive, according to the distribution function $F(z)$, which is common knowledge. Assume that the social planner decides to award the sum Π in a lottery with the following properties. If player i with endowment z_i contributes $g_i \in [0, z_i]$ she wins Π with probability $\frac{g_i}{g_i + G_{-i}}$, where G_{-i} is the sum of the contributions of all the other agents. Player i's expected utility is given by

$$E[U(z_i - g_i, \Pi) \mid g_i, G_{-i}] = z_i - g_i + \alpha \frac{G_{-i} + g_i}{n} + \frac{g_i}{g_i + G_{-i}}\Pi$$

Differentiating with respect to g_i and setting this equal to zero we obtain the following

$$\frac{\alpha - n}{n} + \frac{G_{-i}}{(g_i + G_{-i})^2}\Pi = 0$$

Assuming that total contribution is different from zero[6] and rearranging we obtain player i's best response function, given by the following expression

$$g_i^* = -G_{-i} + \sqrt[2]{\frac{n}{n - \alpha}\Pi G_{-i}} \tag{2.9}$$

---

[6]Notice that in equilibrium the total contribution will not be zero. In fact, if any other player different from i contributes zero, player i will contribute ε arbitrarily close to zero and win the prize.

Based on equation (2.9) we can write an expression for the total contribution when player i plays according to her best response function

$$G(g_i^* \mid G_{-i}) = \sqrt[2]{\frac{n}{n-\alpha}\Pi G_{-i}}$$

Although endowments are private information, notice that z does not enter the first order condition. Each player will have the same best response function and the contribution in equilibrium will be the same for any z. Therefore we know that g_i^* will be $\frac{G(g_i^*|G_{-i})}{n}$ and we can express it as follows

$$g_i^* = \frac{\sqrt[2]{\frac{n}{n-\alpha}\Pi G_{-i}}}{n} \tag{2.10}$$

Setting equations (2.9) and (2.10) equal we obtain an expression for G_{-i} when all players play according to the best response function. This is represented by

$$G_{-i}^* = \frac{(n-1)^2}{n(n-\alpha)}\Pi$$

Therefore, assuming that wealth constraints are non-binding, in equilibrium all agents will play according to the following

$$g^L = \frac{n-1}{n(n-\alpha)}\Pi$$

It is easy to see that $\Pi \le \frac{n(n-\alpha)}{n-1}\underline{z}$ guarantees that the solution will be interior for all players. Contrary to the equilibrium we identified for the case of a contest, in a lottery all players contribute the same amount (as in Morgan, 2000). Total contribution in equilibrium is given by

$$G^L = \frac{n-1}{n-\alpha}\Pi$$

These results are summarised in the following proposition.

58

Proposition 2.5 *Assume n players whose endowments are drawn independently of each other from the interval $[\underline{z}, \bar{z}]$, with $\underline{z}$ strictly positive, according to the distribution function $F(z)$, which is common knowledge. Assume that z is private information. If $\Pi \leq \frac{n(n-\alpha)}{n-1} \underline{z}$ the lottery has a symmetric pure strategy equilibrium in which all players contribute $g^L = \frac{n-1}{n(n-\alpha)} \Pi$ and total contribution is $G^L = \frac{n-1}{n-\alpha} \Pi$.*

Note that in order to prove Proposition (2.7) in Appendix 2 we have not resorted to the support of z. The same conditions guarantee the existence of the equilibrium described in Proposition (2.1) also when endowments are drawn from the interval $[\underline{z}, \bar{z}]$, with $\underline{z}$ strictly positive, according to the distribution function $F(z)$, which is common knowledge, with a continuous and bounded density $F'(z) > 0$. In this case, provided that the social planner optimally sets the last prize equal to zero, the expected total contribution generated by a contest is given by

$$
\begin{aligned}
G &= n \int_{\underline{z}}^{\bar{z}} \frac{n}{n-\alpha} \phi(F(z), \pi \mid \sum_{j=1}^{n} \pi_j = \Pi, \pi_1 \geq \\
\cdots &\geq \pi_n, \pi_n = 0) F'(z) dz = \frac{n}{n-\alpha} \Pi
\end{aligned}
$$

We can conclude that for any finite n, when Π guarantees interior solutions for all players in both mechanisms, the expected total contribution raised with a contest is greater than that obtained through a lottery. The intuition behind this result is that a lottery can be thought of as a stochastic contest (see Tullock, 1980): the higher level of noise results in lower total revenue.

Proposition 2.6 *Assume n players whose endowments are drawn independently of each other from the interval $[\underline{z}, \bar{z}]$, with $\underline{z}$ strictly positive, according to the distribution function $F(z)$, which is common knowledge. Assume that z is private information and that $F(z)$ has a continuous and bounded density $F'(z) > 0$. If $\Pi \leq \min[\frac{n(n-\alpha)}{n-1} \underline{z}, \bar{\Pi}]$, the expected total contribution in a contest, where Π is allocated among the $n-1$ players who contribute the most, is greater than the total provision generated by a lottery.*

2.6 Conclusions

Exploring effective ways to fund public goods is a policy question of great importance, given the fundamental role they play in society. There exists an extensive literature on fund-raising mechanisms based on taxes and penalties. However, solutions to the free-riding problem which do not require coercive power have only recently started to be studied. In the case of institutions which are unable to enforce sanctions, such as charities, this difference may be extremely important. To our knowledge, this is the first attempt to analyse multiple prize contests as incentive schemes to finance public goods. Further, this recent literature has either focused on cases where agents are unconstrained or have homogeneous endowments (e.g. Morgan and Sefton, 2000; Goeree et al. 2005; Orzen, 2005) or predicts that players with different incomes would contribute the same amount (Morgan, 2000). Contrary to these studies we identified an equilibrium in which the contribution is strictly increasing in the endowment.

We considered a linear public good game as it is often employed in laboratory experiments. The main characteristics of the model are the possibility of awarding multiple prizes on the one side, and heterogeneity of the endowments and incomplete information on the other. We assumed that the social planner has access to a small budget and uses it to implement a contest. The first prize is awarded to the player who contributes the most, the second prize to the player with the second highest contribution and so on until all prizes are awarded. The social planner's objective function is represented by the expected total welfare net of the total prize sum.

We concentrated our analysis on interior solutions. We found that there exists a critical level of budget under which wealth constraints are non-binding for all agents. For any possible distribution of wealth we identified necessary and sufficient conditions for the budget to be below this critical level. We found that it is optimal for the social planner to set the last prize equal to zero, but otherwise total expected contribution is invariant to all configurations leaving the lowest value fixed. Further, a contest is a budget balanced mechanism: the revenue generated is higher than

the total prize sum. Provided interior solutions, we proved that a contest Pareto-dominates public provision of the public good and performs better than a lottery.

Heterogeneity of the endowments and incomplete information about income levels allowed us to characterise a monotone equilibrium, in which the higher the endowment of a player the higher her contribution. On the contrary, in the case of a lottery, a symmetric equilibrium arises (as in Morgan, 2000). This is an interesting difference which makes the equilibrium of a contest look more realistic than the latter. Indeed it does seem generally more plausible that richer people contribute more than individuals with lower incomes.

An interesting extension to the present work would be to test experimentally the main results of the model. An important question would be to check whether a contest actually generates a higher contribution than a lottery, and whether the revenue of a contest is independent of the prize structure. Further, it would be interesting to test whether a monotone equilibrium would arise, both in a contest and in a lottery.

Appendix 2

We want to find necessary and sufficient conditions for the value of Π such that $g(z)$ is interior for any z on the interval $[0, 1]$ for any possible allocation of Π among the first $n - 1$ prizes. In fact, assuming interior solutions, Proposition (2.2) assures us that the social planner will set $\pi_n = 0$.

If we let the vector of prizes π be variable, provided that the last prize is equal to zero and that the sum of the first $n - 1$ prizes is equal to Π, $g(z)$ is represented by the following[7]

$$\frac{n}{n - \alpha} \phi(F(z), \pi \mid \sum_{j=1}^{n} \pi_j = \Pi, \pi_1 \geq \cdots \geq \pi_n, \pi_n = 0) =$$

$$\frac{n}{n - \alpha} \sum_{i=1}^{n} \pi_i \binom{n - 1}{i - 1} (F(z))^{n-i} (1 - F(z))^{i-1}$$

Let us define the following object.

Definition 2.1 *Define the envelope function*

$$V(z) = \max_{\pi} \{ \phi(F(z), \pi) \mid \sum_{j=1}^{n} \pi_j = \Pi, \pi_1 \geq \cdots \geq \pi_n, \pi_n = 0 \}$$

for any z on the interval $[0, 1]$.

If we are able to provide necessary and sufficient conditions for $V(z)$ to be weakly less than z for any z on the interval $[0, 1]$, it will be easy to extend the result to $g(z)$. In order to do this we will define some useful concepts that will help us in the course of our analysis.

Definition 2.2 *For any i such that $1 \leq i \leq n - 1$:*

---

[7]Hereafter, unlike the rest of the paper, when writing $\phi(F(z), \pi)$ we will refer to $\phi(F(z), \pi \mid \sum_{j=1}^{n} \pi_j = \Pi, \pi_1 \geq \cdots \geq \pi_n, \pi_n = 0)$.

1) define the set $Q^i \subset \mathbb{R}^n$ such that for every $\pi \in Q^i$ it holds that $\pi_1 \geq \cdots \geq \pi_i > \pi_{i+1} = \cdots = \pi_n = 0$ and $\sum_{l=1}^{i} \pi_l = \Pi$.

2) call $\bar{\pi}^i$ the vector $\pi \in Q^i$ such that $\pi_1 = \cdots = \pi_i = \frac{\Pi}{i}$.

Definition 2.3 *For any i such that $2 \leq i \leq n-1$ define the set $\tilde{Q}^i \subset Q^i$ such that for every $\pi \in \tilde{Q}^i$ it holds that $\pi_1 > \pi_i$.*

Obviously $\bar{\pi}^1 \in Q^1$, characterised by $\pi_1^1 = \Pi, \pi_l^1 = 0$ for $2 \leq l \leq n$, is the only element of the set Q^1 and $\phi(F(z), \bar{\pi}^1) = \Pi(F(z))^{n-1}$.

The next Proposition presents necessary and sufficient conditions for $V(z)$ to be weakly less than z on the interval $[0, 1]$.

Proposition 2.7 $\phi(F(z), \bar{\pi}^i) \leq z$ *on the interval $[0, 1]$ for $1 \leq i \leq n-1$ are necessary and sufficient conditions for $V(z) \leq z$.*

Proof. The necessity of these conditions is obvious. In order to prove sufficiency we will have to present some technical results.

Lemma 2.2 *Given a vector $\pi^R \in \mathbb{R}^n$ such that $\sum_{j=1}^{n} \pi_j^R = \Pi$ and $\pi_1^R \geq \cdots \geq \pi_n^R$, $\pi_n^R = 0$, consider a redistribution of the type $-\Delta\pi_i^R = \Delta\pi_{i+1}^R$, with $1 \leq i \leq n-1$ and $\Delta\pi_i^R > 0$, and call the resulting vector π^S. Then, $\phi(F(z), \pi^S) > \phi(F(z), \pi^R)$ for any z such that $F(z) < \frac{n-i}{n}$ and $\phi(F(z), \pi^S) < \phi(F(z), \pi^R)$ for any z such that $F(z) > \frac{n-i}{n}$.*

Proof. Notice that $\frac{\partial\phi(F(z),\pi)}{\partial\pi_i} = \binom{n-1}{i-1}(F(z))^{n-i}(1 - F(z))^{i-1}$. To see how a redistribution of the type $-\Delta\pi_i = \Delta\pi_{i+1}$ affects $\phi(F(z), \pi)$ we have to study the sign of

$$-\frac{\partial\phi(F(z), \pi)}{\partial\pi_i} + \frac{\partial\phi(F(z), \pi)}{\partial\pi_{i+1}} \tag{2.11}$$

$$= (F(z))^{n-i}(1 - F(z))^1\left(-\binom{n-1}{i-1}(F(z))\right)$$

$$+ \binom{n-1}{i}(1 - F(z)))$$

63

It is the case that expression (2.11) > 0 for any z such that $F(z) < \frac{\binom{n-1}{i}}{\binom{n-1}{i-1}+\binom{n-1}{i}}$ and (2.11) < 0 for any z such that $F(z) > \frac{\binom{n-1}{i}}{\binom{n-1}{i-1}+\binom{n-1}{i}}$. Further, it is easy to show that

$$\frac{\binom{n-1}{i}}{\binom{n-1}{i-1}+\binom{n-1}{i}} = \frac{\frac{(n-1)!}{i!(n-1-i)!}}{\frac{(n-1)!}{(i-1)!(n-i)!}+\frac{(n-1)!}{i!(n-1-i)!}} = \frac{n-i}{n}$$

∎

Lemma 2.3 *Assume* $1 \le i \le n-2$. *Consider a vector* $\pi^B \in \tilde{Q}^{i+1}$. *If* $2 \le i \le n-2$ *then* $\phi(F(z), \bar{\pi}^{i+1}) > \phi(F(z), \pi^B)$ *for any* z *such that* $F(z) \le \frac{n-i}{n}$. *If* $i = 1$ *then* $\phi(F(z), \bar{\pi}^2) > \phi(F(z), \pi^B)$ *for any* z *such that* $F(z) < \frac{n-1}{n}$ *and* $\phi(\frac{n-1}{n}, \pi \mid \pi \in Q^2) = \phi(\frac{n-1}{n}, \bar{\pi}^1) = \Pi(\frac{n-1}{n})^{n-1}$.

Proof. *Let us first consider the case in which* $2 \le i \le n-2$. *The vector* $\bar{\pi}^{i+1}$ *can be obtained from vector* π^B *applying the following algorithm in* i *steps.*

Algorithm 2.1 *Step* 1. *From vector* π^B *construct vector* π^{B1} *such that* $\pi_1^{B1} = \frac{\Pi}{i+1}, \pi_2^{B1} = \pi_2^B + \pi_1^B - \frac{\Pi}{i+1}, \pi_j^{B1} = \pi_j^B, 3 \le j \le i+1$. *Given that* $\pi_2^B \ge \pi_3^B$ *it will now be the case that* $\pi_2^{B1} > \pi_3^{B1} \ge \cdots \ge \pi_{i+1}^{B1}$. *Therefore* $\frac{\Pi}{i+1} + i\pi_2^{B1} > \Pi$. *The last inequality can be rewritten as* $\pi_2^{B1} > \frac{\Pi}{i+1}$, *therefore we can move to the next step and repeat the process.*

Step j, *with* $2 \le j \le i-1$. *From vector* π^{Bj-1} *construct vector* π^{Bj} *such that* $\pi_j^{Bj} = \frac{\Pi}{i+1}, \pi_{j+1}^{Bj} = \pi_{j+1}^{Bj-1} + \pi_j^{Bj-1} - \frac{\Pi}{i+1}, \pi_l^{Bj} = \pi_l^{Bj-1}$ *for* $1 \le l \le j-1$ *and* $j+1 \le l \le i+1$. *Given that* $\pi_{j+1}^{Bj-1} \ge \pi_{j+2}^{Bj-1}$ *it will now be the case that* $\pi_{j+1}^{Bj} > \pi_{j+2}^{Bj} \ge \cdots \ge \pi_{i+1}^{Bj}$. *Therefore it is the case that* $j\frac{\Pi}{i+1} + (i+1-j)\,\pi_{j+1}^{Bj} > \Pi$. *Rearranging the last inequality we obtain* $\pi_{j+1}^{Bj} > \frac{\Pi}{i+1}$. *This means that we can move to the next step and repeat the process.*

Step i. *From vector* π^{Bi-1} *construct vector* π^{Bi} *such that* $\pi_i^{Bi} = \frac{\Pi}{i+1}, \pi_{i+1}^{Bi} = \pi_{i+1}^{Bi-1} + \pi_i^{Bi-1} - \frac{\Pi}{i+1}, \pi_l^{Bi} = \pi_l^{Bi-1}$ *for* $1 \le l \le i-1$. *Notice that* $\pi_l^{Bi-1} = \frac{\Pi}{i+1}$ *for* $1 \le l \le i-1$. *Therefore* $\pi^{Bi} = \bar{\pi}^{i+1}$.

Notice that from Lemma 2.2 we know that $\phi(F(z), \pi^{Bj}) > \phi(F(z), \pi^{Bj-1})$ *for any*

64

z such that $F(z) < \frac{n-j}{n}$ for $1 \le j \le i$. Therefore $\phi(F(z), \bar{\pi}^{i+1}) > \phi(F(z), \pi^B)$ for any z such that $F(z) \le \frac{n-i}{n}$, as we wished to show.

Consider now the case in which $i = 1$. Notice that $\bar{\pi}_1^2 < \pi_1^B$ and $\bar{\pi}_2^2 > \pi_2^B$. Applying the same algorithm as above from π^B we will obtain $\bar{\pi}_2^2$ after the first step. Applying Lemma 2.2 we know that $\phi(F(z), \bar{\pi}^2) > \phi(F(z), \pi^B)$ for any z such that $F(z) < \frac{n-1}{n}$. Further, from Lemma 2.2 we also know that $\phi(F(z), \pi \mid \pi \in Q^2) > \phi(F(z), \bar{\pi}^1)$ for any z such that $F(z) < \frac{n-1}{n}$ and $\phi(F(z), \pi \mid \pi \in Q^2) < \phi(F(z), \bar{\pi}^1)$ for any z such that $F(z) > \frac{n-1}{n}$. Therefore, by continuity, we can conclude that $\phi(\frac{n-1}{n}, \pi \mid \pi \in Q^2) = \phi(\frac{n-1}{n}, \bar{\pi}^1) = \Pi(\frac{n-1}{n})^{n-1}$. $\blacksquare$

Lemma 2.4 Assume $2 \le i \le n - 2$. $\phi(F(z), \bar{\pi}^{i+1}) > \phi(F(z), \pi \mid \pi \in Q^j)$ for any z such that $F(z) \le \frac{n-i}{n}$ and for $1 \le j \le i$.

Proof. The structure of this proof is in three parts.

First of all, from Lemma 2.3 we know that $\phi(F(z), \bar{\pi}^j) > \phi(F(z), \pi \mid \pi \in \tilde{Q}^j)$ for any z such that $F(z) \le \frac{n-j+1}{n}$ and, given that $2 \le j \le i$, for any z such that $F(z) \le \frac{n-i}{n}$.

For the second part of the proof, let us firs assume $j = 1$. Consider a vector $\pi^B \in \tilde{Q}^{i+1}$. We want to show that $\phi(F(z), \pi^B) > \phi(F(z), \bar{\pi}^1)$ for any z such that $F(z) \le \frac{n-i}{n}$.

If $2 \le j \le i$, consider a vector $\pi^B \in \tilde{Q}^{i+1}$ such that $\pi_l^B = \bar{\pi}_l^j$ for $1 \le l \le j - 1$. Notice that, obviously, $\pi_j^B < \bar{\pi}_l^j$. We want to show that $\phi(F(z), \pi^B) > \phi(F(z), \bar{\pi}^j)$ for any z such that $F(z) \le \frac{n-i}{n}$ if $2 \le j \le i-1$ and for any z such that $F(z) < \frac{n-i}{n}$ if $j = i$.

Vector π^B can be obtained from $\bar{\pi}^j$ through the following algorithm in $i + 1 - j$ steps.

Algorithm 2.2 Step 1. If $j = 1$, from vector $\bar{\pi}^1$ construct vector $\pi^{A1} \in \tilde{Q}^2$ such that $\pi_1^{A1} = \pi_1^B$ and $\pi_2^{A1} = \Pi - \pi_1^B$. If $2 \le j \le i$, from vector $\bar{\pi}^j$ construct vector $\pi^{A1} \in \tilde{Q}^{j+1}$ such that $\pi_l^{A1} = \bar{\pi}_l^j = \frac{\Pi}{j}$ for $1 \le l \le j - 1$, $\pi_j^{A1} = \pi_j^B$ and $\pi_{j+1}^{A1} = \bar{\pi}_j^j - \pi_j^B = \frac{\Pi}{j} - \pi_j^B$.

Step k, with $2 \leq k \leq i - j$. From vector π^{Ak-1} construct vector $\pi^{Ak} \in \tilde{Q}^{j+k}$ such that $\pi_l^{Ak} = \pi_l^{Ak-1}$ for $1 \leq l \leq j = k-2$, $\pi_{j+k-1}^{Ak} = \pi_{j+k-1}^{B}$ and $\pi_{j+k}^{Ak} = \pi_{j+k-1}^{Ak-1} - \pi_{j+k-1}^{B}$.

Step $i + 1 - j$. From vector π^{Ai-j} construct vector $\pi^{Ai+1-j} \in \tilde{Q}^{i+1}$ such that $\pi_l^{Ai+1-j} = \pi_l^{Ai-j}$ for $1 \leq l \leq i - 2$, $\pi_i^{Ai+1-j} = \pi_i^{B}$ and $\pi_{i+1}^{Ai+1-j} = \pi_i^{Ai-j} - \pi_i^{B}$. Notice that $\pi_{i+1}^{Ai+1-j} = \pi_{i+1}^{B}$ and $\pi^{Ai+1-j} = \pi^{B}$ by construction.

From Lemma 2.2 we know that $\phi(F(z), \pi^{Ak}) > \phi(F(z), \pi^{Ak-1})$ for any z such that $F(z) < \frac{n+1-k}{n}$. Therefore if $1 \leq j \leq i - 1$ then $\phi(F(z), \pi^{B}) > \phi(F(z), \bar{\pi}^j)$ for any z such that $F(z) \leq \frac{n-i}{n}$. If $j = i$ then $\phi(F(z), \pi^{B}) > \phi(F(z), \bar{\pi}^j)$ for any z such that $F(z) < \frac{n-i}{n}$ and $\phi(\frac{n-i}{n}, \pi^{B}) = \phi(\frac{n-i}{n}, \bar{\pi}^j)$.

Finally, from Lemma 2.3 we know that $\phi(F(z), \bar{\pi}^{i+1}) > \phi(F(z), \pi^{B})$ for any z such that $F(z) \leq \frac{n-i}{n}$. Therefore $\phi(F(z), \bar{\pi}^{i+1}) > \phi(F(z), \pi \mid \pi \in \tilde{Q}^j)$ for any z such that $F(z) \leq \frac{n-1}{n}$. ∎

Lemma 2.5 *Assume $2 \leq i \leq n - 2$. Consider a vector $\pi^B \in \tilde{Q}^{i+1}$ such that $\pi_1^B > \pi_j^B$, with $2 \leq j \leq i$. Assume a vector $\pi^C \in \tilde{Q}^{i+1}$ such that $\pi_l^C = \pi_l^B$ for $j+1 \leq l \leq i+$
1 and $\pi_1^C = \cdots = \pi_j^C = \dfrac{\Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B}{j}$. If $3 \leq j \leq n - 2$ then $\phi(F(z), \pi^C) > \phi(F(z), \pi^B)$ for any z such that $F(z) \leq \frac{n-j+1}{n}$. If $j = 2$ then $\phi(F(z), \pi^C) > \phi(F(z), \pi^B)$ for any z such that $F(z) < \frac{n-1}{n}$ and $\phi(\frac{n-1}{n}, \pi^C) > \phi(\frac{n-1}{n}, \pi^B)$.*

Proof. *Notice that $\pi_1^B > \pi_1^C$ and $\pi_j^B < \pi_j^C$. Vector π^C can be obtained from vector π^B applying the following algorithm in $j - 1$ steps.*

Algorithm 2.3 *Step 1. From vector π^B construct vector π^{B1} such that $\pi_1^{B1} = \pi_1^C, \pi_2^{B1} = \pi_2^B + \pi_1^B - \pi_1^C, \pi_l^{B1} = \pi_l^B$ for $3 \leq l \leq i + 1$. Given that $\pi_2^B \geq \pi_3^B$ it will now be the case that $\pi_2^{B1} > \pi_3^{B1} \geq \cdots \geq \pi_{i+1}^{B1}$. Therefore $\pi_1^{B1} + (j - 1)\pi_2^{B1} >$
$\Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B$. Since $\pi_1^{B1} = \pi_1^C = \dfrac{\Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B}{j}$, the last inequality can be rearranged as
$\pi_2^{B1} > \dfrac{\Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B}{j}$. Therefore we can move to the next step and repeat the process.*

Step k, $2 \leq k \leq j-2$. From vector π^{Bk-1} construct vector π^{Bk} such that
$\pi_k^{Bk} = \pi_k^C$, $\pi_{k+1}^{Bk} = \pi_{k+1}^{Bk-1} + \pi_k^{Bk-1} - \pi_k^C$, $\pi_l^{Bk} = \pi_l^{Bk-1}$ *for $1 \leq l \leq k-1$ and*

$k+2 \leq l \leq i+1$. *Notice that, by construction* $\pi_l^{Bk} = \dfrac{\Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B}{j}$ *for $1 \leq l \leq k$ and*

$\pi_l^{Bk} = \pi_l^B$ *for $k+2 \leq l \leq i+1$. Given that $\pi_{k+1}^{Bk-1} \geq \pi_{k+2}^{Bk-1}$ it will now be the case that*

$\pi_{k+1}^{Bk} > \pi_{k+1}^{Bk} \geq \cdots \geq \pi_{i+1}^{Bk}$. *Therefore $\dfrac{k}{j}(\Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B) + (j-k)\pi_{k+1}^{Bk} > \Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B$.*

The last inequality can be rearranged as $\pi_{k+1}^{Bk} > \dfrac{\Pi - \sum\limits_{l=j+1}^{i+1} \pi_l^B}{j}$. Therefore we can move to the next step and repeat the process.

Step $j-1$. From vector π^{Bj-2} construct vector π^{Bj-1} such that $\pi_{j-1}^{Bj-1} = \pi_{j-1}^C$, $\pi_j^{Bj-1} =$
$\pi_j^{Bj-2} + \pi_{j-1}^{Bj-2} - \pi_{j-1}^C$, $\pi_l^{Bj-1} = \pi_l^{Bj-2}$ *for $1 \leq l \leq j-2$ and $j+1 \leq l \leq i+1$. Notice that $\pi^{Bj-1} = \pi^C$ by construction.*

From Lemma 2.2 we know that $\phi(F(z), \pi^{Bk}) > \phi(F(z), \pi^{Bk-1})$ for any z such that $F(z) < \dfrac{n-k}{n}$ for $1 \leq k \leq j-1$. This means that if $3 \leq i \leq n-3$ then, by construction, we will have $\phi(F(z), \pi^C) > \phi(F(z), \pi^B)$ for any z such that $F(z) \leq \dfrac{n-j+1}{n}$. If $i = 2$ then j will necessarily be equal to 3 and, by construction, we will have $\phi(F(z), \pi^C) > \phi(F(z), \pi^B)$ for any z such that $F(z) < \dfrac{n-1}{n}$. Further it will be the case that $\phi(\dfrac{n-1}{n}, \pi^C) > \phi(\dfrac{n-1}{n}, \pi^B)$. ∎

Lemma 2.6 *Consider a vector $\pi^C \in \tilde{Q}^{i+1}$ such that $\pi_{i+1}^C = x, \pi_j^C = \dfrac{\Pi - x}{i}$ with $0 < x < \dfrac{\Pi}{i+1}$ for $1 \leq j \leq i$ and $2 \leq i \leq n-2$. If $\phi(F(z), \pi^C) > \phi(F(z), \bar{\pi}^{i+1})$ then $\phi(F(z), \bar{\pi}^i) > \phi(F(z), \pi^C)$.*

Proof. *The inequality $\phi(F(z), \pi^C) > \phi(F(z), \bar{\pi}^{i+1})$ can be rewritten as*

$$\frac{\Pi - x}{i}((F(z))^{n-1} + \cdots +$$
$$\binom{n-1}{i-1}(F(z))^{n-i}(1 - F(z))^{i-1}) +$$
$$x\binom{n-i}{i}(F(z))^{n-i-1}(1 - F(z))^i -$$
$$\frac{\Pi}{i+1}((F(z))^{n-1} + \cdots + (F(z))^{n-i-1}(1 - F(z))^i) > 0$$

The above expression can be rearranged as

$$(\frac{\Pi - x}{i} - \frac{\Pi}{i+1})((F(z))^{n-1} + \cdots + \tag{2.12}$$
$$\binom{n-1}{i-1}(F(z))^{n-i}(1 - F(z))^{i-1}) >$$
$$(\frac{\Pi}{i+1} - x)(F(z))^{n-i-1}(1 - F(z))^i$$

Call A the expression $((F(z))^{n-1} + \cdots + \binom{n-1}{i-1}(F(z))^{n-i}(1 - F(z))^{i-1})$ and call B the expression $(F(z))^{n-i-1}(1 - F(z))^i$. Inequality (2.12) is satisfied for $\frac{A}{B} > i$.

The inequality $\phi(F(z), \bar{\pi}^i) > \phi(F(z), \pi^C)$ can be rewritten as

$$\frac{\Pi}{i}A - \frac{\Pi - x}{i}A - xB > 0 \tag{2.13}$$

Inequality (2.13) is satisfied for $\frac{A}{B} > i$. ■

From Lemma (2.4) we know that $\phi(F(z), \bar{\pi}^{i+1}) > \phi(F(z), \pi \mid \pi \in Q^j)$ for any z such that $F(z) \leq \frac{n-i}{n}$ and for $2 \leq i \leq n - 2$ and $1 \leq j \leq i$. In particular, this means that $V(z)$ will be equal to $\phi(F(z), \bar{\pi}^{n-1})$ for any z such that $0 \leq F(z) \leq \frac{2}{n}$. For those z such that $\frac{2}{n} \leq F(z) \leq \frac{3}{n}$ we will have to check the family of functions $\phi(F(z), \pi \mid \pi \in Q^{n-1})$ and $\phi(F(z), \bar{\pi}^{n-2})$. In general, assuming $0 \leq i \leq n - 3$, in order to find $V(z)$ for those z such that $\frac{n-i-1}{n} \leq F(z) \leq \frac{n-i}{n}$ we will have to check the families of functions $\phi(F(z), \pi \mid \pi \in Q^j)$ for $i + 2 \leq j \leq n - 1$ and the function

68

$\phi(F(z), \bar\pi^{i+1})$.

Consider now a vector $\pi^C \in Q^{i+1}$ such that $\pi_1^C = \cdots = \pi_i^C > \pi_{i+1}^C$, for $2 \le i \le n-2$. From Lemma (2.5) we know that, for those z such that $\frac{n-i}{n} < F(z) \le \frac{n-i+1}{n}$, the function $\phi(F(z), \pi^C)$ is greater than any other function of the family $\phi(F(z), \pi \mid \pi \in Q^{i+1})$ excluding $\phi(F(z), \bar\pi^{i+1})$.

From Lemma (2.6) though, we know that if $\phi(F(z), \pi^C) > \phi(F(z), \bar\pi^{i+1})$ then it is the case that $\phi(F(z), \bar\pi^i) > \phi(F(z), \pi^C)$.

Therefore, in order to find the envelope function $V(z)$ for those z such that $\frac{2}{n} \le F(z) \le \frac{3}{n}$, it will be sufficient to check the two functions $\phi(F(z), \bar\pi^{n-1})$ and $\phi(F(z), \bar\pi^{n-2})$. In general, assuming $0 \le i \le n-3$, in order to find $V(z)$ for those z such that $\frac{n-i-1}{n} \le F(z) \le \frac{n-i}{n}$ we will have to check the functions $\phi(F(z), \bar\pi^j)$ for $i+1 \le j \le n-1$.

From this follows that $\phi(F(z), \bar\pi^i) \le z$ for $1 \le i \le n-1$ are sufficient conditions for $V(z) \le z$ on the interval $[0, 1]$. ∎

Finally, given Proposition (2.7), by continuity we can establish the following result.

Proposition 2.8 *Provided that the last prize is equal to zero, $g(z)$ is interior for any z on the interval $[0, 1]$ independently of the distribution of Π among the first $n-1$ prizes if and only if $\frac{n}{n-\alpha}\phi(F(z), \bar\pi^i \mid \sum_{j=1}^{n} \bar\pi_j^i = \Pi) \le z$ on the interval $[0, 1]$ for $1 \le i \le n-1$.*

Chapter 3

A Prize To Give for: An Experiment on Public Good Funding Mechanisms

3.1 Introduction

Finding effective fund-raising mechanisms for the private provision of public goods is an important policy question. Voluntary contributions to public goods are typically well below socially optimal levels, given the incentive to free ride associated with positive externalities.[1] While fund-raising mechanisms based on tax rewards and penalties can be designed to overcome the incentive to free ride, they are not available to fund-raisers in the private sector who cannot enforce sanctions. A number of recent studies have examined, both theoretically and empirically, the performance of incentive-based funding mechanisms for the private provision of public goods, focusing in particular on lotteries (or raffles) and different types of auctions (e.g.

---

[1]Voluntary contributions to public goods are generally found to be greater than theoretical predictions, both in naturally occurring situations and in laboratory experiments, but nevertheless sub-optimal. See Ledyard (1995) for a survey of the experimental literature on the provision of public goods. See also e.g. Keser (1996), Laurie and Holt (1998), and Saijo (2003) for alternative explanations of over-contribution in the voluntary provision of public goods.

Morgan, 2000; Morgan and Sefton, 2000; Goeree et al., 2005; Orzen, 2005; Schram and Onderstal, 2006).[2]

In this paper we investigate with a laboratory experiment the performance of prize-based mechanisms for the private provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. We focus on a voluntary contribution mechanism, used as a benchmark, and two incentive-based mechanisms where a single prize is awarded: a lottery and an all-pay auction.

The experimental literature on incentive-based fund-raising mechanisms has focused on the case of income homogeneity (e.g. Morgan and Sefton, 2000; Orzen, 2005; Schram and Onderstal, 2006). However, actual contribution to public goods is generally characterised by heterogeneous incomes which are private information. Although several experimental studies have investigated public good provision when incomes are heterogeneous, this literature has only explored the voluntary contribution mechanism.[3] The performance of incentive-based fund-raising mechanisms when subjects have different incomes remains empirically unexplored.

Morgan (2000) provides a theoretical analysis of lotteries as a way to finance public goods. Players buy tickets of a lottery in which one prize is awarded. One ticket is randomly drawn and the holder wins the prize. Public good provision consists of the revenue of the lottery net of the prize. The author considers agents with heterogeneous preferences and endowments who have quasi-linear utility functions. Public good provision is shown to be strictly higher than with voluntary contributions. The solution identified by Morgan (2000) predicts that agents with different incomes contribute the same amount in equilibrium. Such an equilibrium does not seem realistic, while it appears more plausible that the contribution would be in-

---

[2]See also e.g. Isaac and Walker (1988), Bagnoli, and Lipman (1989), Bagnoli and McKee (1991) for earlier studies on mechanisms for improving economic efficiency in the voluntary provision of public goods.

[3]Research has examined the effects of income heterogeneity on either overall public good provision (Anderson et al., 2004; Chan et al., 1996; Chan et al., 1999; Rapoport and Suleiman, 1993) or contributions of individual income types (Buckley and Croson, 2005).

creasing in the endowment. Morgan and Sefton (2000) investigate experimentally the performance of a linear version of Morgan's model, finding that, as predicted, public good provision via a lottery is higher than through voluntary contributions. However they only consider homogeneous endowments, without testing the validity of a completely symmetric equilibrium.

Orzen (2005) compares in a laboratory experiment the performance of a lottery and different all-pay auctions as fund-raising mechanisms, under the assumptions of homogeneous preferences and endowments. Public good provision generated by the incentive-based mechanisms is higher than voluntary contributions. Interestingly, although theory predicts that the first price all-pay auction raises a higher revenue than the lottery, no significant difference is found between the two treatments. Finally, Schram and Onderstal (2006) present an experimental study that compares a winner-pay auction, an all-pay auction and a lottery in the case of heterogeneous preferences, but homogeneous endowments. They find that the all-pay auction performs better than the lottery mechanism, as predicted by the theory. In sum, all of these studies focus on the case of homogeneous endowments.

Our analysis is based on a theoretical framework where prizes are used as a means to finance public goods when agents have heterogeneous endowments which are private information. In this setting, an all-pay auction generates a higher expected total contribution than a lottery with an equal prize. In the all-pay auction, it is possible to identify a monotone equilibrium such that contributions are strictly increasing in the endowment. The equilibrium of the lottery is instead completely symmetric, as in Morgan (2000), with all agents contributing the same amount independently of their endowment.

In our experiment, we test the following theoretical predictions. First, incentive-based mechanisms should outperform the voluntary contribution mechanism in terms of net contributions (after taking into account the cost of prizes). Second, the total revenue of the all-pay auction should be higher than that of a lottery with an equal prize. Third, absolute contributions should not depend on income in the lottery,

whereas they should rise with income in the all-pay auction. As a consequence, individual contributions should be higher in the lottery than in the all-pay auction at the lower end of the income distribution, while the opposite should hold at the upper end of the income distribution.

The main findings of the analysis can be summarised as follows. In all mechanisms average contributions are generally higher than theoretical predictions and tend to converge towards the predicted values over successive rounds. The voluntary contribution mechanism replicates the behavioural patterns observed in similar experiments under income homogeneity or heterogenous incomes and complete information. The introduction of a prize as an incentive has significant effects on contributions: the lottery and, to a lesser extent, the all-pay auction perform better than voluntary contribution in terms of public good provision after accounting for the cost of prizes. Comparing the prize-based mechanisms, contributions are significantly higher in the lottery than in the all-pay auction, contrary to the theoretical predictions. Focusing on the behaviour of individual income types, absolute contributions rise with income in all treatments, although more steeply in the prize-based mechanisms, so that relative contributions are generally similar across income types. In terms of relative performance, the lottery does better than the other mechanisms for all income types. At the individual level, subjects choose zero contributions in the all-pay auction about three times as often as in the lottery.

The paper is structured as follows. Section 3.2 presents the theoretical background of the analysis. Section 3.3 describes the experimental design and the theoretical predictions to be tested. Section 3.4 presents the results. Section 3.5 concludes with a discussion of the main findings and implications of the analysis.

3.2   Theoretical Background

In this section we present the main theoretical predictions for the performance of single-prize all-pay auctions and lotteries, as fund-raising mechanisms for the private

provision of public goods, under the assumptions of income heterogeneity and incomplete information about income levels. The analysis is based on the framework presented in Faravelli (2007).

Consider a public good game with n risk neutral players. Each player i is assumed to have income z_i, which is private information. Incomes are drawn independently of each other from the distribution function $F(z)$ on the interval $[\underline{z}, \bar{z}]$. $F(z)$ is common knowledge and has a continuous and bounded density $F'(z)$. Each player has to decide how much of his endowment to contribute to the public good, knowing that total contributions to the public good are multiplied by a parameter $\alpha \in (1, n)$ and shared equally among all the agents. The cost of contributing to the public good exceeds the marginal return of investing in it. Therefore, the unique Nash equilibrium is to contribute nothing, although it is socially optimal to contribute all the endowment.

Suppose that the fund-raiser has access to an amount Π. The fund-raiser moves first. He can either use Π to provide the public good or organise either a lottery or an all-pay auction in which the winner is awarded a prize equal to Π. Then the agents choose their contributions in order to maximise their utility (expected utility in the case of an all-pay auction), given the other players' contributions and the value of Π. If the fund-raiser spends all his budget Π to provide the public good then the payoff of of player i with endowment z_i who contributes g_i is given by

$$z_i - g_i + \frac{\alpha}{n}(\Pi + g_i + G_{-i})$$

where G_{-i} is the sum of the contributions of all the other players. If the fund-raisers uses Π as a prize, player i's payoff will be

$$z_i - g_i + E[\Pi, g_i, g_{-i}] + \frac{\alpha}{n}(g_i + G_{-i})$$

where g_{-i} represents the vector of the individual contributions of all the other players.

$E[\Pi, g_i, g_{-i}]$ is the expected prize of player i given all the other players' contributions. In the lottery, player i wins the prize with probability $\frac{g_i}{g_i+G_{-i}}$, which is the number of tickets he bought divided by the total number of tickets. In the all-pay auction he wins if and only if his contribution is higher than all the other agents' contributions.

The main results for the different contribution mechanisms can be summarised as follows.[4] In the voluntary contribution mechanism the Nash equilibrium is to contribute nothing. At interior solutions,[5] the lottery has a symmetric pure strategy equilibrium (as in Morgan, 2000) where every player contributes the same amount

$$g^{LOT} = \frac{n-1}{n(n-\alpha)}\Pi \tag{3.1}$$

and the total contribution is

$$G^{LOT} = \frac{n-1}{n-\alpha}\Pi$$

Total contribution is higher than the cost of the prize. If public good provision is socially desirable the lottery provides positive net revenues. The all-pay auction, at an interior solution for all players, has a symmetric pure strategy equilibrium given by

$$g^{APA}(z) = \frac{n}{n-\alpha}F(z)^{n-1}\Pi \tag{3.2}$$

Note that contributions in equilibrium are strictly increasing in the endowment. $F(z)^{n-1}\Pi$ represents the expected prize of a player with endowment z, when all players play according to the same strictly increasing strategy. Total expected contribution in the all-pay auction is given by

$$E[G^{APA}] = \frac{n}{n-\alpha}\Pi$$

The above expression is strictly greater than both the cost of the prize and the total contribution under the lottery. A lottery can be thought of as a stochastic all-pay

---

[4]See Faravelli (2007) for the proofs of the following results.
[5]We focus on the case in which constraints are non-binding for all agents.

auction, where the higher noise results in lower revenue (see Tullock, 1980).

Note that the equilibrium strategy function described by expression (3.2) can be rearranged as $g^{APA}(z) = \frac{1}{1-\frac{\alpha}{n}}F(z)^{n-1}\Pi$. This is the sum of a convergent series with reason $\frac{\alpha}{n}$ and can be written as

$$g^{APA}(z) = F(z)^{n-1}\Pi\sum_{m=0}^{\infty}(\frac{\alpha}{n})^m$$

The standard result in all-pay auctions is the total dissipation of the rent. In this case each individual bids more than the expected prize because of the marginal return of investing in the public good, which is equal to $\frac{\alpha}{n}$.

3.3   Experimental Design

Our experimental design follows Morgan and Sefton (2005) and, more closely, Orzen (2005), while introducing income heterogeneity and incomplete information about the income of other subjects. We considered three different treatments: a voluntary contribution mechanism (VCM), a lottery (LOT) and an all-pay auction (APA). We ran three sessions for each treatment, with sixteen subjects participating in each session, for a total of 144 subjects. Each session consisted of 20 rounds.

3.3.1   The Baseline Game

In each round, every subject had to allocate entirely a given endowment between two accounts. The language used in the instructions did not refer to contributions or public goods, but asked subjects to allocate tokens to either an "individual account" or a "group account". A subject received 2 points for each token he allocated to the individual account, while he received 1 point for each token allocated by him or by any other member of his group to the group account.

At the beginning of the session the sixteen subjects were randomly and anonymously assigned an endowment of either 120, 160, 200, or 240 tokens. Concentrating

on a small number of possible endowments allowed us to analyse the effects of income heterogeneity in a controlled and simple setting, while providing several observations on the same income type. The subjects were informed that in each round they would receive the same endowment as determined at the beginning of the session.

Incomplete information about incomes was introduced by using a matching procedure similar to the *strangers* condition used in Andreoni (1988b). At the beginning of each round, subjects were randomly and anonymously rematched in groups of four people. Therefore, in each round subjects did not know the identity and the endowment of the other three members of their group. They only knew that the endowment of each of the other group members could be either 120, 160, 200, or 240 tokens with equal probabilities.

Group matching for each of the twenty rounds was determined randomly before the beginning of the experiment in the following way. Four pools of four subjects were formed, each containing the four different income types (120, 160, 200, 240). Each of the four groups was formed by randomly drawing one subject from each pool. As a consequence, within every group each member could have an endowment of 120, 160, 200, or 240 tokens with equal probability.[6] Having formed the four groups for each round in this way, the same sequence of group matchings for the twenty rounds was used in each session of all three treatments.

3.3.2 Treatments

The three treatments differed in the way prizes (extra points) could be earned by the subjects. In VCM, 120 tokens were exogenously allocated by the experimenter to the group account in each round, independently of the subjects' choices, thus implying that each member of the group received 120 points as a bonus. In LOT, a subject received a lottery ticket for each token he allocated to the group account.

_____

[6]Note that in every round there were four subjects for each of the four possible endowments, so that the average endowment was 180 tokens.

At the end of each round the computer randomly selected one ticket among all those purchased by the members of the group, and the owner of the selected ticket won the prize of 240 points. In case no tokens were allocated to the group account, the winner of the prize was selected randomly among the four members of the group. In APA, in each round the member of the group who allocated the highest amount to the group account won the prize of 240 points. In case of ties between two or more group members, the winner was determined randomly by the computer. Note that the four mechanisms imply the same financial commitment for the fund-raiser: allocating 120 tokens to the group account in VCM is equivalent to paying a prize of 240 points in APA or in LOT.

3.3.3 Procedures

In each session, the subjects were randomly assigned to a computer terminal at their arrival. To ensure public knowledge, instructions were distributed and read aloud (see Appendix 3A for the instructions). Moreover, to ensure understanding of the experimental design, sample questions were distributed and the answers privately checked and, if necessary, individually explained to the subjects.

At the end of each round, the subjects were informed about their payoffs from the group account, the individual account and the prize (or bonus in VCM). At the end of the last round, subjects were informed about their total payoff for the twenty rounds expressed in points and euros. They were then asked to answer a short questionnaire on the understanding of the experiment and socio-demographic information, and were then paid in private using an exchange rate of 1000 points per euro. Subjects earned 12.25 euros on average for sessions lasting about 50 minutes, including the time for instructions. Participants were mainly undergraduate students of Economics and were recruited through an online system. The experiment took place in May 2006 at the Experimental Lab of the University of Milan Bicocca. The experiment was computerized using the zTree software (Fischbacher, 1999).

3.3.4 Predictions

In this experimental design, within each group every subject can have one of four possible endowments with equal probability. Compared with the model described in Section 3.2, where each player's endowment is drawn from a continuous distribution function, it is easy to see that the equilibrium in VCM will still be to contribute nothing. Similarly, LOT is characterised by the same equilibrium as described by equation (3.1), and the same total contribution. This is because the best response function of a player in the lottery game is independent of income, as in Morgan (2000). The equilibrium is instead slightly modified in APA, although qualitatively unchanged, given that the pure strategy equilibrium, as described by equation (3.2), depends on the continuity of endowment distribution.

In Appendix 3B we consider an all-pay auction in a linear public good game under the assumption of *complete* information. We solve the game for N players, who can have any possible endowment, and for any positive level of prize. We show that when the prize is not too "high", only mixed strategy equilibria exist. The equilibrium for a subject under *incomplete* information about the incomes of other players consists of a randomisation over the mixed strategies he would play in all the possible group matchings he faces, according to their corresponding probabilities.[7]

The total expected contribution in APA for the mixed strategy equilibrium under a discrete income distribution is lower than that for the pure strategy equilibrium under a continuous income distribution. This loss of revenue results from the discontinuity in the possible endowments: a subject with an endowment higher than the lowest one will face opponents with a strictly lower endowment with positive probability. In this case he will not have any incentive to bid more than the highest of his opponents' endowments. Nevertheless, despite the lower expected revenue, all the theoretical predictions described in Section 3.2 under a continuous distribution

---

[7]For instance, a subject endowed with 120 tokens could be grouped with three other subjects with his same endowment, or with one subject with 160 and two others with 200 tokens, and so on.

of endowments are qualitatively unchanged.

Table 3.1 presents the predicted contributions for the experimental design, both in absolute and relative terms, for each income type and on average. Average contributions for both prize-based mechanisms are higher than the predicted contribution in VCM, which is zero. They are also higher than the average provision in VCM, where an amount equivalent to the cost of the prize is used to directly finance the public good, resulting in an average provision of 30 tokens per subject. The average absolute contribution in APA (51 tokens) is higher than in LOT (45 tokens). Note also that in LOT the predicted absolute contributions are independent of income levels (25% in relative terms on average). Predicted contributions are instead steeply increasing in the endowment in APA, both in absolute and relative terms.

Table 3.1: Theoretical predictions: absolute and relative contributions

	Incomes				Average
Treatments	120	160	200	240	180
	Absolute contributions				
VCM	0	0	0	0	0
LOT	45	45	45	45	45
APA	5	28	68	102	51
	Relative contributions				
VCM	0	0	0	0	0
LOT	38	28	23	19	25
APA	4	18	34	43	28

Note: contributions are rounded to the nearest integer. Relative contributions are expressed as a percentage of the endowment.

Summing up, the main hypotheses to be tested are as follows:

Hypothesis 1 (Absolute Efficiency): Both LOT and APA outperform VCM not only in terms of gross contributions, but also after taking into account the cost of the prize.

Hypothesis 2 (Relative Efficiency): The total contribution to the public good is higher in APA than in LOT.

Hypothesis 3 (Individual income types): Individual contributions do not depend on income in LOT, while they increase with income in APA. Contributions are therefore higher in LOT than in APA for low-income types, while the opposite holds for high-income types.

3.4 Results

This section presents the experimental results. We start with a descriptive analysis of the main features of the data for the three treatments. Next, we examine the replicability of sessions within each treatment, the effects of repetition over rounds, and the dependence of individual observations within sessions. We then present formal tests of the theoretical predictions, considering first average contributions over all subjects and then contributions by individual income types.

3.4.1 Overview

Figures 3.1-3.3 present an overview of average relative contributions (as a percentage of the endowment) over rounds for each session of the three treatments. Table 3.2 reports relative contributions obtained by averaging over all subjects within each session for all 20 rounds and for the following sub-sets of rounds: 1st, first 10, last 10, and 20th.

The results for VCM sessions are similar to those generally obtained in public good experiments with homogeneous incomes. Average contributions to the group account are substantially higher than the equilibrium prediction of zero throughout the twenty rounds, but display a clear downward trend over successive rounds (Figure 3.1). Averaging over all sessions and subjects, individual contributions are 21.6% over the 20 rounds, falling from 35.1% in the first round to 8.2% in the last round, and from 26.1% in rounds 1-10 to 17.1% in rounds 11-20. The same pattern of positive but declining contributions is observed in each of the three VCM sessions, and a

Table 3.2: Average individual contributions: by session and rounds

			Rounds		
Session	1-20	1	1-10	11-20	20
VCM 1	18.6	25.7	20.6	16.6	7.7
VCM 2	26.3	32.4	28.9	23.7	11.2
VCM 3	19.9	47.1	28.9	10.9	5.7
Average	21.6	35.1	26.1	17.1	8.2
LOT 1	42.1	38.5	45.3	38.9	39.9
LOT 2	46.2	48.5	50.0	42.3	39.3
LOT 3	52.7	45.7	52.8	52.5	41.8
Average	47.0	44.2	49.4	44.6	40.3
APA 1	41.8	51.8	46.0	37.7	52.4
APA 2	40.7	46.8	45.3	36.1	29.2
APA 3	36.2	45.3	38.5	33.9	30.3
Average	39.6	48.0	43.2	35.9	37.3

Note: contributions to the public good are expressed as a percentage of the endowment.

clear tendency to converge to a common level is observed in the final rounds.

Average contributions for LOT sessions are also systematically higher than the predicted contribution of 25%, and remain virtually constant over time, except for a slight decline in the final rounds (Figure 3.2). Average contributions over all sessions, subjects and rounds are 47%, falling from 44.2% to 40.3% over the 20 rounds, and from 49.4% to 44.6% between the first and the second half of the session. The declining pattern is not observed in all sessions and, although the three sessions converge to a common level in the final round, a relatively high variability across sessions is observed in rounds 11-20.

In APA sessions, average contributions are larger than the predicted contribution of 28%, and display a slight decline over rounds (Figure 3.3). All sessions start with relatively high contributions, but tend to converge to the theoretical prediction within the first ten rounds. The average contribution is 39.6% over the twenty rounds, falling from 43.2% in rounds 1-10 to 35.9% in rounds 11-20. Each of the three APA sessions displays the same pattern of declining contributions, and the
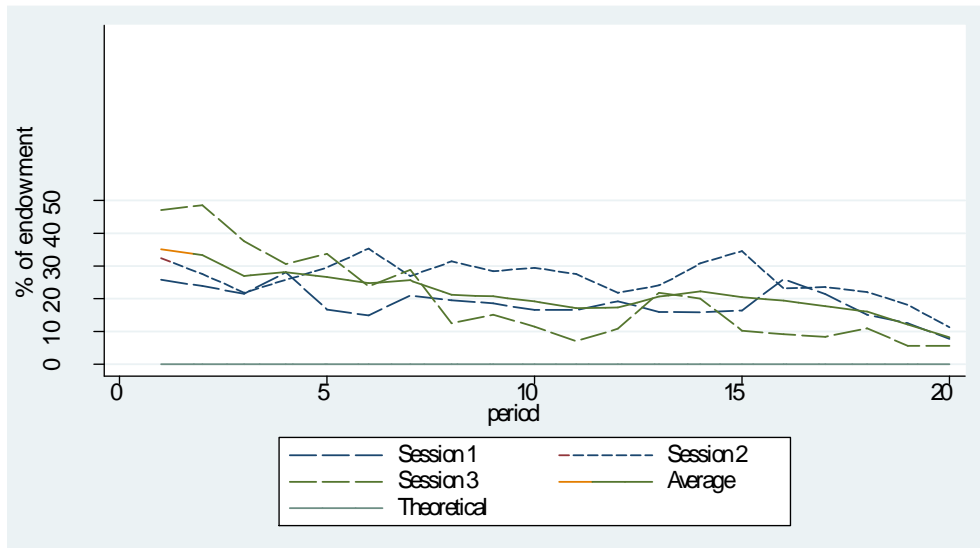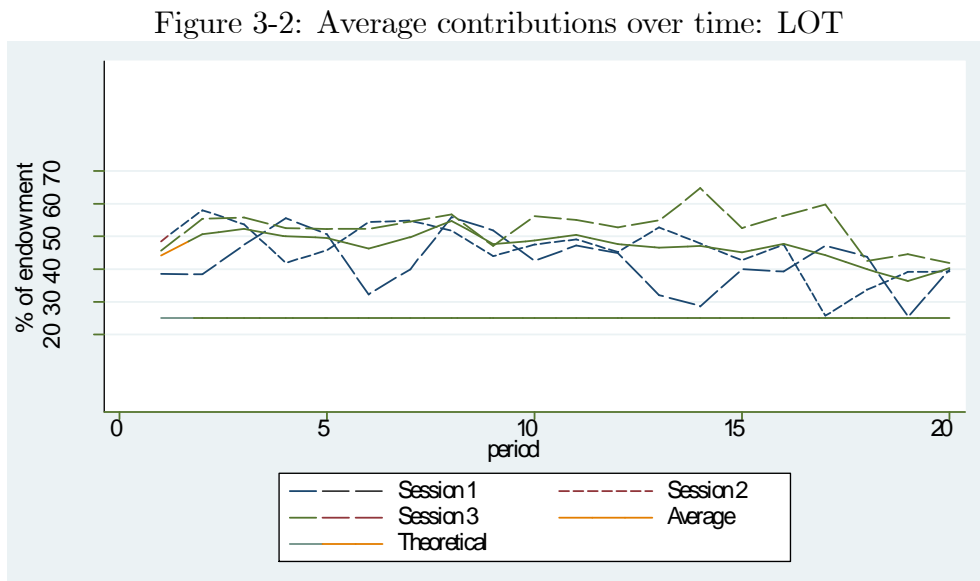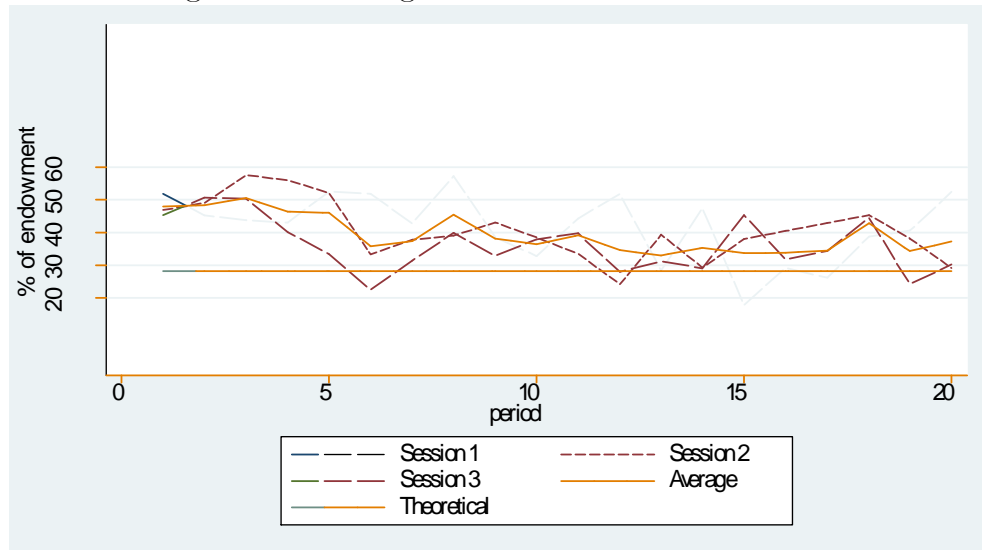
Figure 3-1: Average contributions over time: VCM

Figure 3-2: Average contributions over time: LOT

Figure 3-3: Average contributions over time: APA

profiles are very similar.

3.4.2 Replicability, Repetition, Sectional Dependence

The descriptive analysis of session-level data indicates that contributions for individual sessions within each treatment are qualitatively similar in terms of both average levels and dynamics over rounds. We provide formal tests for the hypothesis of replicability of session results within treatments. Table 3.3 presents Kruskal-Wallis test statistics for the null hypothesis that median contributions are equal across the three sessions within each treatment, focusing on the same sub-sets of rounds as in Table 3.2. Focusing on the whole session (rounds 1-20) or the last round, the results indicate that the null hypothesis cannot be rejected for all treatments. We therefore conclude that the three sessions can be pooled and the analysis is carried out on observations for 48 individuals for each of the three treatments.

The analysis of session-level data also indicates that there are substantial changes in contributions over successive rounds (repetition effects). Contributions tend to fall over rounds, generally converging towards theoretical predictions, in all treatments

Table 3.3: Tests for replicability of sessions

Treatment	Rounds				
	1-20	1	1 - 10	11-20	20
VCM	4.93	5.83	4.21	6.43	0.75
	(0.08)	(0.05)	(0.12)	(0.04)	(0.69)
LOT	2.16	1.08	0.89	3.68	0.26
	(0.34)	(0.58)	(0.64)	(0.16)	(0.88)
APA	1.90	0.50	1.67	0.99	2.83
	(0.39)	(0.78)	(0.43)	(0.61)	(0.24)

Note: the table reports Kruskal-Wallis test statistics for the null hypothesis that median contributions are equal across the three sessions within each treatment. P-values (in brackets) are based on the χ^2 distribution with 2 degrees of freedom.

except for LOT, where the tendency to converge is less marked. This could suggest that excessive contributions may be due to the fact that subjects are learning how to behave rationally. We provide formal tests for the effects of repetition on contributions. Table 3.4 presents results of Wilcoxon signed-rank tests for the hypothesis that median contributions are the same across selected pairs of rounds within each treatment.

Table 3.4: Tests for repetition effects

Treatment	Rounds				
	1 vs 10	10 vs 20	1 vs 15	5 vs 20	1 vs 20
VCM	3.04	2.96	3.26	4.11	5.54
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LOT	-1.42	1.65	-0.04	1.23	0.99
	(0.16)	(0.10)	(0.97)	(0.22)	(0.32)
APA	2.04	0.16	1.95	-0.31	1.76
	(0.04)	(0.87)	(0.05)	(0.76)	(0.08)

Note: the table reports normalized Wilcoxon signed-rank test statistics for the hypothesis that median contributions are the same in the two rounds indicated in the column headings. P-values (in brackets) refer to two-sided tests based on the standard normal distribution.

Irrespective of the time horizon considered, decreases in contributions are significant in VCM. LOT does not display any significant round effect, whereas in APA the differences are significant between rounds 1 and 10 (p-value 0.04) and marginally significant between rounds 1 and 20 (p-value 0.08). We conclude that repetition

affects different mechanisms in different ways, so that comparisons between treatments, and between actual and predicted contributions within treatments, cannot focus on a single round but should consider alternative sub-sets of rounds in order to take into account the different role played by repetition in each treatment.

In order to go beyond descriptive analysis and provide formal tests of the theoretical predictions we need to define the appropriate unit of analysis (subject, group, session). It is important to note that, because of repetition, subject-level observations within each session and round might be dependent, given that (in rounds beyond the first) subjects have interacted in previous rounds. In addition, because of the random rematching mechanism (at the beginning of each round subjects are randomly and anonymously rematched in groups of four people), independence could also be violated for group-level observations. If the dependence of subject-level observations due to interactions in earlier rounds was relevant, inference would have to be based on session-level observations (see e.g. Orzen, 2005).

However, the characteristics of the experimental design are such that the dependence across individual observations can be considered negligible. First, at the end of each round subjects only learn about the total contribution of other group members, so that it is difficult for them to infer individual absolute contributions. Second, since subjects do not know the endowments of other group members, it is even more difficult for them to infer other subjects' relative contributions (e.g. an absolute contribution of 120 could be a relative contribution of 50% as well as 100%, depending on the endowment of the other subject). Third, the number of players within each session (sixteen) is sufficiently large, so that subjects know that there is a relatively small probability of interacting with the same subjects as in the previous round. This further reduces the motivation to reciprocate in successive rounds, thus weakening the possible dependence across individual observations.

We also investigated the issue at the empirical level, by considering Spearman rank correlation tests for the null hypothesis of independence between the contributions of each subject and the average contributions of the subjects who were in

86

his group in the previous round. The test statistics, based on 16 individual observations for each session and each round, are significant at the 5 per cent level in only about 15 percent of the cases. In addition, within the significant cases, 30% of the correlation coefficients are positive and 70% are negative, indicating that there is no systematic pattern in the relationship between each subject's contribution and those of his past group members. As a result, considering both the features of the experimental design and the results of the Spearman tests, we conclude that the dependence across individuals can be considered negligible. Hence, in the following, we use subjects as the unit of analysis (see Morgan and Sefton, 2000, for a similar approach).

3.4.3   Comparison between Treatments: Total Contributions

In order to compare the relative performance of the different funding mechanisms, Figure 3.4 displays average individual contributions over rounds for each treatment.[8] The introduction of prizes has a substantial effect on individual contributions. Average contributions in LOT are more than twice as large as in VCM over the 20 rounds, and about five times as large in the final round. Average contributions in APA are almost twice as large as in VCM over the 20 rounds and almost five times as large in the final round.

It is important to observe, however, that VCM is not directly comparable with the incentive-based treatments in terms of individual contributions. In order to make the results comparable we must either consider contributions *net of the cost of prizes* in the incentive-based mechanisms or, equivalently, refer to overall *provision* (i.e. also take into account public provision in VCM). Public provision in VCM

---

[8]The graphs confirm that repetition strongly affects each treatment in different ways. Both incentive-based mechanisms display high contribution levels in initial rounds, but their dynamics in the following rounds differ. While in the first ten rounds of APA contributions decline markedly, in the lottery they remain virtually unchanged. Thereafter, contributions fall somewhat in LOT, while they remain constant in APA. As a consequence, focusing on final rounds only, LOT and APA converge to very similar contribution levels. VCM contributions are on a downward trend and are systematically lower than those of the other two mechanisms.
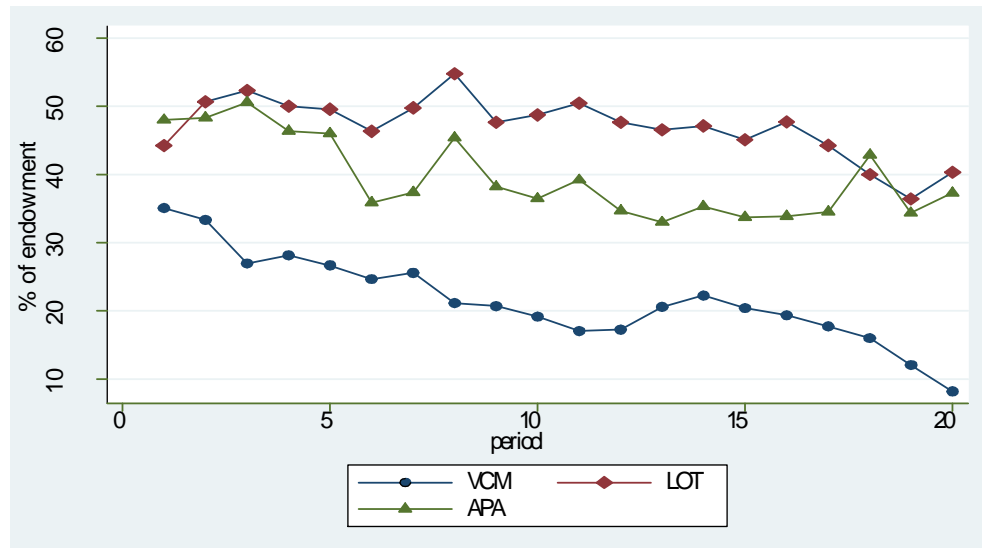
Figure 3-4: Average individual contributions: all treatments

accounts for 120 tokens per group, corresponding to about 17 percentage points per subject in terms of relative contributions. Figure 3.5 displays average public good *provision* over rounds, providing the appropriate reference for comparing incentive-based mechanisms with the benchmark VCM.

Interestingly, when we compare the treatments in terms of overall provision (or, equivalently, contributions *net* of the cost of prizes), the comparison of the incentive-based mechanisms relative to VCM is not as clear-cut as before. While LOT systematically outperforms VCM (with the only exception of the first round), APA provision is very close to VCM, except for the last rounds where the two profiles diverge. Averaging over all rounds, relative to VCM, public good provision in LOT is about 20 per cent higher and 3.5 per cent higher in APA.

The informal evidence presented in Figure 3.5 is examined further in Table 3.5, presenting results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across treatments. The first two tests compare each of the incentive-based mechanisms with the benchmark VCM. The next compares APA with LOT. Given that our model predicts the direction of departure from the

Figure 3-5: Average public good provision: all treatments
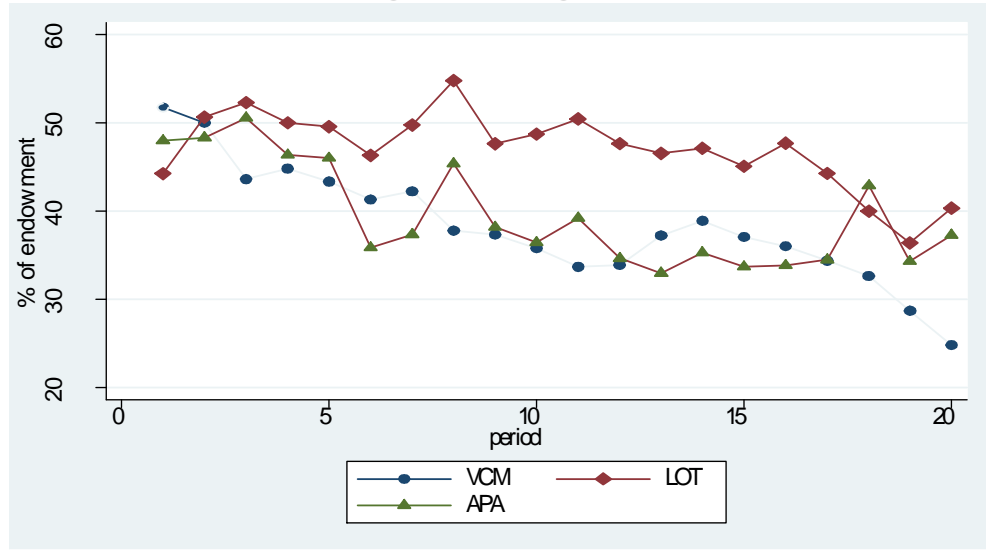
null hypothesis, we use the relevant one sided-tests.

Table 3.5: Tests of equality between treatments: All subjects

	Rounds				
Treatments	1-20	1	1 - 10	11-20	20
LOT - VCM	4.55	-3.10	3.43	4.75	5.22
	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)
APA - VCM	1.88	-1.55	1.12	1.52	4.75
	(0.03)	(0.94)	(0.13)	(0.06)	(0.00)
APA - LOT	-4.09	1.52	-2.84	-4.39	-0.92
	(1.00)	(0.06)	(1.00)	(1.00)	(0.82)

Note: the table reports Wilcoxon rank-sum tests (normalized z-statistics) for the hypothesis that the median of the difference between individual relative provisions to the public good in the given two treatments is zero. P-values (in brackets), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

The test statistics are positive and highly significant at all time horizons (except for round 1) in the comparison between LOT and VCM. APA provision also significantly outperforms VCM, although the significance level is quite variable across sub-samples, owing to the different effects of repetition in the two treatments.

Result 3.1: Both the lottery and the all-pay auction are more effec-

tive than voluntary contribution in funding public goods.

The comparison between LOT and APA indicates that not only APA does not perform better than LOT, but also public good provision is statistically higher in LOT than in APA, contrary to the model's predictions.

Result 3.2: The lottery is more effective than the all-pay auction in funding public goods.

It is interesting to observe that incentive-based funding mechanisms are generally efficient in covering the cost of the prize. Averaging over all sessions and rounds for each treatment, group contributions cover the cost of the prize in 96.3 per cent of the cases in LOT and 88.3 per cent of the cases in APA. Thus the lottery outperforms the all-pay auction also in terms of financial efficiency.

3.4.4 Comparison between Treatments by Income Level

So far we have considered contributions by taking averages over all subjects, thus abstracting from differences across individuals characterized by different income levels. The theory, however, provides predictions for the contributions of each income type (see Table 3.1). In this section we focus explicitly on income heterogeneity. We first examine whether individuals with different incomes behave as predicted by the theory and how over-contribution is related to income levels. Next, we consider how different funding mechanisms compare at different ends of the income distribution.

Table 3.6 and Figures 3.6 and 3.7 provide a description of the relationship between contributions and income levels. In all treatments absolute provisions rise with income levels, so that relative provisions are generally relatively similar across income types. Over-contributions in VCM are observed for all income types, and rise slightly with income in absolute terms. In LOT all income types over-contribute and, contrary to the theoretical predictions, absolute contributions rise almost linearly with income. In APA, absolute contributions rise with income, although not as

steeply as predicted by the theory. As a consequence, the three lowest-income types over-contribute, while the contributions of subjects with the highest income (240) are very close to the theoretical prediction.

Result 3.3: Absolute contributions are weakly related to income in VCM, and steeply increasing in income in both LOT and APA.

Table 3.6: Average relative contributions: by endowment and rounds

Endowment	Predicted	Rounds				
		1 - 20	1	1 - 10	11 - 20	20
VCM-120	0.0	28.4	37.8	36.5	20.2	9.2
VCM-160	0.0	15.3	31.5	16.3	14.4	8.6
VCM-200	0.0	23.8	35.8	27.2	20.3	10.8
VCM-240	0.0	19.0	35.1	24.6	13.4	4.1
LOT-120	37.5	47.1	36.6	52.0	42.3	40.3
LOT-160	28.1	48.2	46.4	49.8	46.6	45.6
LOT-200	22.5	48.5	46.3	49.8	47.1	43.1
LOT-240	18.7	44.1	47.7	46.0	42.2	32.3
APA-120	4.2	35.6	49.3	35.7	35.6	37.6
APA-160	17.5	40.7	35.7	42.3	39.1	41.5
APA-200	34.0	40.8	53.8	46.4	35.1	36.9
APA-240	42.5	41.1	53.1	48.5	33.7	33.2

Note: contributions are expressed as a percentage of the endowment.

Figures 3.8 and 3.9 provide a comparison of absolute and relative provision in the three treatments by income levels.[9] Interestingly, the lottery outperforms both other mechanisms for all income types. Public good provision in APA is higher than in VCM for income types 160 and 240, but the opposite holds for the lowest income type, indicating that prizes in contests provide relatively less effective incentives for poorer individuals.

Table 3.7 presents results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across treatments, when considering sepa-

---

[9]Note that in order to ensure comparability we assumed that in VCM the 120 tokens of public provision can be attributed to each income-type on the basis of equal income shares (about 17%).

rately each income type. The results indicate that LOT performs significantly better than VCM for all income types except the lowest. LOT also performs significantly better than APA for incomes 120 and 200. APA does significantly better than VCM only for income type 160.

> **Result 3.4**: The lottery outperforms the other funding mechanisms for
> all income types.

3.4.5 Comparison between Treatments at Individual Level

We finally consider the relative performance of the three mechanisms at the individual level. Figure 3.10 compares the cumulative distribution functions of relative contributions for the three treatments. The main difference between the two prize-based mechanism is that in APA subjects choose zero contributions about three times as often as in LOT (20.8 and 5.83 per cent, respectively). The cumulative distribution for APA lies above that for LOT only up to a relative contribution of 50 per cent, while the two distributions are virtually identical thereafter. Note also that, although average contributions in APA and VCM are relatively similar, the distributions of individual contributions in the two treatments are similar only for low relative contributions.

> **Result 3.5**: At the individual level, APA is characterized by a much
> higher fraction of zero contributions than LOT.

Figure 3.11 compares the cumulative distribution functions of relative contributions for the three treatments, considering individual income types. It is interesting to observe that the difference between APA and LOT in the frequency of low contributions is very pronounced for low income types, but it becomes less and less evident for higher income types. While in LOT subjects contribute almost uniformly irrespective of their income type, in APA low-income individuals contribute zero much more often, as predicted by the theory. Nevertheless, low relative contributions are much more frequent than predicted by the theory for high-income individuals.

Table 3.7: Tests of equality between treatments by income level

	Incomes			
Treatments	120	160	200	240
LOT - VCM	1.05	2.62	2.09	3.14
	(0.15)	(0.00)	(0.02)	(0.00)
APA - VCM	-1.05	2.62	0.00	1.32
	(0.85)	(0.00)	(0.50)	(0.09)
APA - LOT	-3.14	-1.57	-2.09	-0.52
	(0.00)	(0.06)	(0.98)	(0.70)

Note: the table reports Wilcoxon rank-sum tests (normalized z-statistics) for the hypothesis that the median of the difference between individual contributions in the given two treatments is zero. P-values (in brackets), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

Figure 3-6: Average absolute contributions by endowment: all treatments
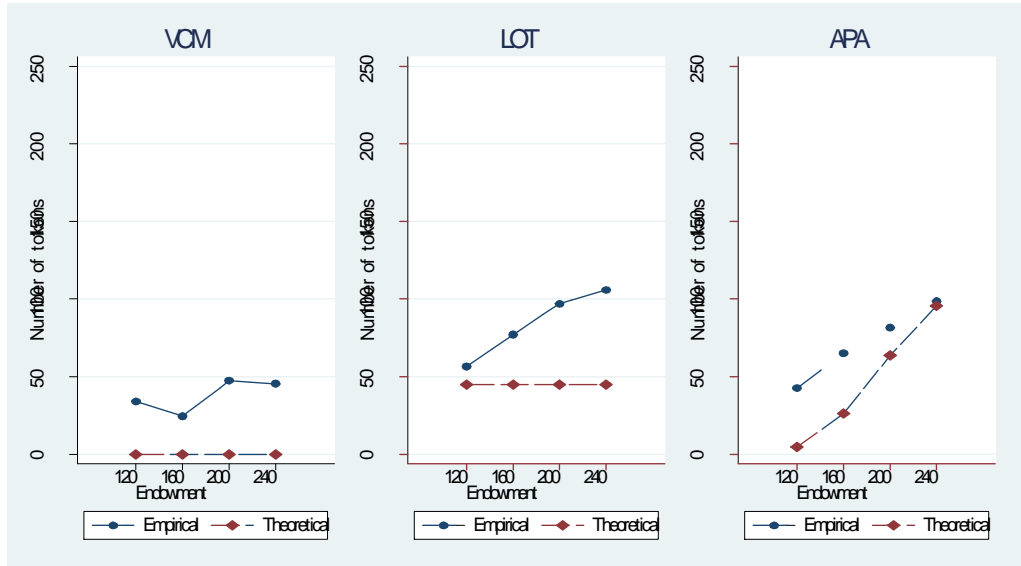
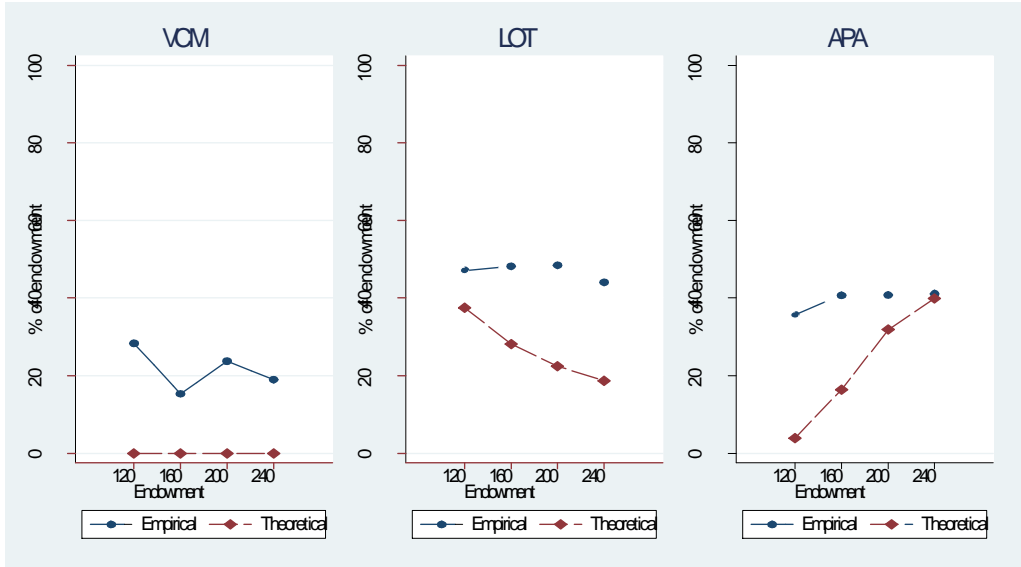Figure 3-7: Average relative contributions by endowment: all treatments

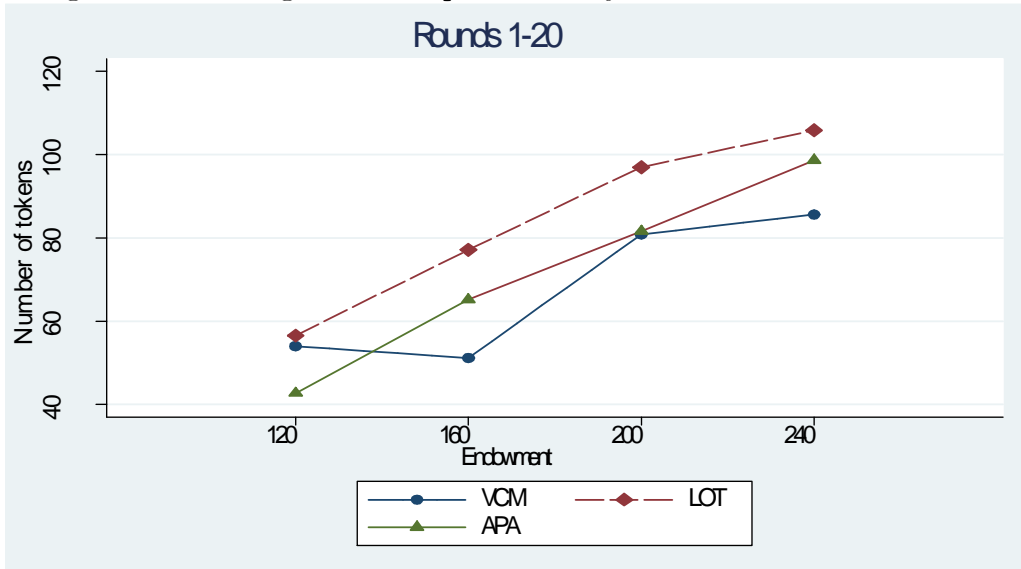Figure 3-8: Average absolute provisions by endowment: all treatments

Figure 3-9: Average relative provisions by endowment: all treatments
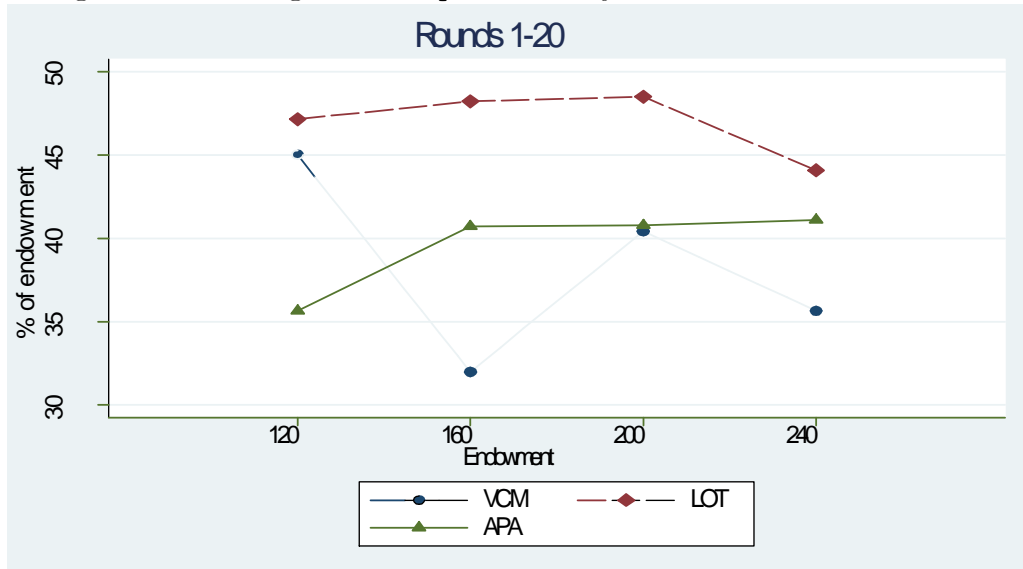

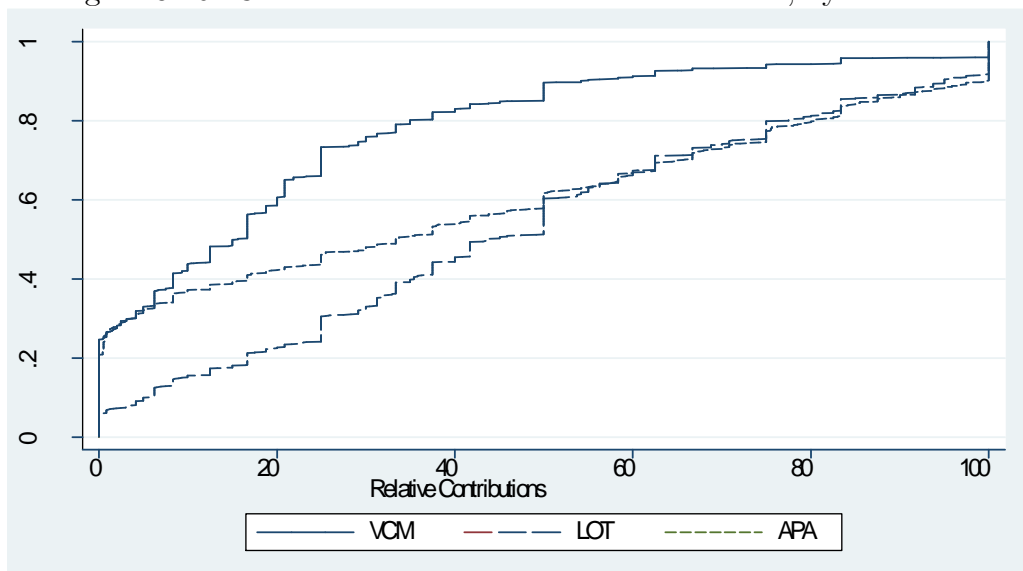Figure 3-10: Cumulative distribution of contributions, by treatment
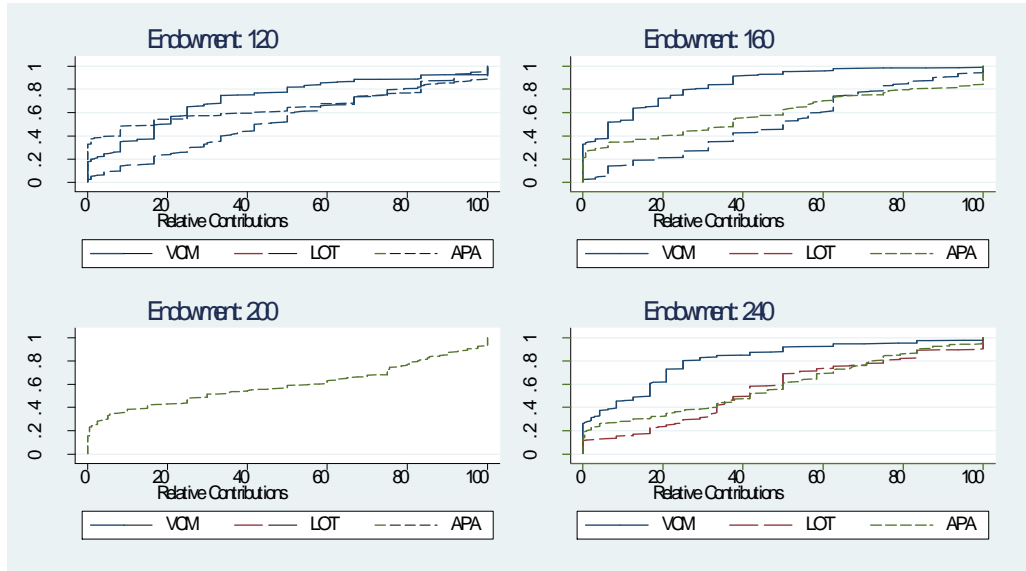
Figure 3-11: Cumulative distribution of contributions, by treatment and endowment

3.5   Conclusions

A number of experimental papers have analysed public good provision when incomes are heterogeneous. However, these studies have only explored voluntary contributions. The experimental literature on fund-raising mechanisms based on prizes has focused on the case of income homogeneity. To our knowledge, this paper is the first experimental investigation of the performance of incentive-based fund-raising mechanisms when subjects have heterogenous endowments which are private information. We compared a lottery and an all-pay auction, while also considering a voluntary contribution mechanism as a benchmark.

The results indicate that the introduction of a prize has sizeable effects on individual contributions relative to the VMC. Both the lottery and the all-pay auction outperform voluntary contributions after taking into account the cost of the prize. The lottery, in particular, systematically and significantly outperforms VCM. Averaging over all rounds, public good provision is 20 per cent higher in the lottery than in VCM. Provision in the all-pay auction is also higher than in VCM, but the

difference is not significant in the earlier rounds of the sessions. This is an important result. It indicates that, in a setting where agents have heterogeneous incomes which are private information, prize-based fund-raising mechanisms can be an effective way of overcoming free riding.

The comparison between the incentive-based mechanisms indicates that, contrary to the theoretical predictions, contributions to the public good are significantly higher in the lottery than in the all-pay auction. This result suggests a number of possible interpretations. It could be argued that subjects are more familiar with lotteries than with all-pay auctions. As a consequence, they might tend to bid more conservatively in the latter. This intuition is supported by the finding that, at the individual level, subjects choose zero contributions in APA three times as often as in LOT. It could also be argued that subjects perceive the lottery as more fair than the all-pay auction. However, such arguments would not help explain the differences between our result and those in Orzen (2005) and in Schram and Onderstal (2006). The first study found no significant difference between the two mechanisms, focusing on homogeneous endowments and complete information. On the other hand, Schram and Onderstal (2006) focused on the case of symmetric endowments but heterogeneous preferences which are private information, finding that the all-pay auction raises higher revenues, as predicted by the theory.

Focusing on income heterogeneity, over-contributions are observed for all income types in VCM, and are slightly increasing with income in absolute terms. In the all-pay auction, absolute contributions rise with income, even though not as steeply as predicted by the theory. In the lottery, all income types over-contribute and, contrary to the theoretical predictions, absolute contributions rise linearly with income. The comparison of contributions across treatments indicates that the lottery outperforms both other mechanisms for all income types. This result indicates that, from a theoretical perspective, the completely symmetric equilibrium of a lottery game does not seem to properly describe the actual behaviour of subjects. Further, experiments on lotteries focusing on homogeneous endowments may be missing a crucial trait of

97

the subject's behaviour.

There are several extensions for future research. A crucial point would be to investigate whether and how the specific features of the experimental designs can explain the differences between our results and the findings of other studies (see Orzen, 2005; Schram and Onderstal, 2006). Further, it would be interesting to develop a model that predicts a positive correlation between contributions and endowments in a lottery.

Appendix 3A: Instructions

[ALL TREATMENTS]

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully and make good decisions you can earn an amount of money that will be paid to you in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with other participants. If you have any questions raise your hand and one of the assistants will come to you to answer it. The rules that you are reading are the same for all participants.

General rules

There are 16 people participating in this experiment. At the beginning of the experiment each participant will be assigned randomly and anonymously an endowment of either 120, 160, 200, or 240 tokens with equal probabilities.

The experiment will consist of 20 rounds. In each round you will have the same endowment that has been assigned to you at the beginning of the experiment. In each round you will be assigned randomly and anonymously to a group of four people. Therefore, of the other three people in your group you will not know the identity and the endowment, that could be 120, 160, 200, or 240 tokens with equal probabilities.

How your earnings are determined

In each round you have to decide how to allocate your endowment between an INDIVIDUAL ACCOUNT and a GROUP ACCOUNT, considering the following information:

- for each token that you allocate to the INDIVIDUAL ACCOUNT you will receive 2 points.

- for each token allocated to the GROUP ACCOUNT (by you or by any other of the members of your group), every group member will receive 1 point.

[VCM]

In each round you will receive 120 bonus points.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

[LOT]

In each round you can win a prize of **240 points** on the basis of the following rules. For each token allocated to the GROUP ACCOUNT you will receive a lottery ticket. At the end of each round the computer selects randomly the winning ticket among all the tickets purchased by the members of your group. The owner of the winning ticket wins the prize of 240 points. Thus, your probability of winning is given by the number of tokens you place in the GROUP ACCOUNT divided by the total number of tokens placed in the GROUP ACCOUNT by members of your group. In case no tokens are placed in the GROUP ACCOUNT, the winner of the prize is selected randomly among the four members of the group.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

[APA]

In each round you can win a prize of **240 points** on the basis of the following rules. The member of your group who allocates the highest amount to the GROUP ACCOUNT is the winner of the prize. In case of ties among one or more group members, the winner is determined randomly.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of

the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

Appendix 3B

In this appendix we study a linear public good game financed through an all-pay auction in which one prize is awarded, assuming that players have homogeneous preferences but heterogeneous endowments and information is complete. We solve the game for N players, who can have any possible endowment, and for any positive level of prize.

Consider N players and the set of endowments $Z = (z_1, ..., z_S)$ such that $0 < z_1 < \ldots < z_S$. Each player has an endowment which assumes a value from the set Z. Call $n[z_i]$ the number of players with endowment $z_i \in Z$ such that $\sum_{i=1}^{S} n[z_i] = N$. The players' endowments and their number are common knowledge. With no loss of generality, assume that $n[z_i] \geq 0$ for $1 \leq i \leq S - 1$ and $n[z_S] \geq 1$. Players play a public good game in which each individual has to choose how much to contribute to the public good. At the same time they take part in an all-pay auction in which a prize is awarded to the agent who contributes the most. The bidders are risk-neutral and they all value the prize at $\Pi > 0$.

The payoff for a player with endowment z_i who contributes g_i is given by

$$\beta(z_i - g_i) + \beta E[\Pi, g_i, g_{-i}] + g_i + G_{-i}$$

where G_{-i} represents the sum of all other players' contributions and $1 < \beta < N$.[10]

We divide our analysis in two parts: the case where $n[z_S] > 1$ and where $n[z_S] = 1$.

More than One Player with the Highest Endowment

We study first the case in which $n[z_S] > 1$. Let us define the following equilibrium.

Definition 3.1 *Call type-symmetric equilibrium an equilibrium in which agents with the same endowment play according to the same strategy.*

---

[10]In the experimental design presented in Section 3.3 $\beta = 2$.

There exist three possible scenarios: the prize level can be "high", "medium" or "low". In the next two propositions, we show that if and only if the prize level is "high" there exists a type-symmetric pure strategy equilibrium.

Proposition 3.1 *When $n[z_S] > 1$ and $z_S \leq \frac{\beta \Pi}{n[z_S](\beta-1)}$, there exists a type-symmetric pure strategy equilibrium in which players with endowment z_S contribute their full endowment, while if there are other agents with lower endowments they all contribute 0.*

Proof. If all players with endowment z_S contribute their full endowment each of them has an expected payoff of

$$\beta z_s + \frac{\beta \Pi}{n[z_S]} - (\beta - 1)z_S + G_{-i}$$

which is greater or equal than the payoff he could get from any other choice $g \in [0, z_S)$.[11]

If there are other players with lower endowments it is equally obvious that contributing 0 is for them a dominant strategy. ∎

Proposition 3.2 *When $n[z_S] > 1$ and $\frac{\beta \Pi}{n[z_S](\beta-1)} < z_S$, there exist no type-symmetric pure strategy equilibria.*

Proof. In order to prove this it is enough to show that there exist no equilibria in which players with endowment z_S play according to the same pure strategy. The proof is in two parts.

i) Consider first the case in which $\frac{\beta \Pi}{n[z_S](\beta-1)} < z_S \leq \frac{\beta \Pi}{\beta-1}$. Suppose that players with endowment z_S contribute $g \in [0, z_S)$, then player i has an incentive to raise his own bid by an amount ε and win the prize. Equally, if all of them contribute z_S, then player i has an incentive to contribute 0.

---

[11] In all the proofs G_{-i} represents the sum of all other agents' contributions.

103

ii) Consider now the case in which $z_S > \frac{\beta\Pi}{\beta-1}$. Notice first that any contribution $g > \frac{\beta\Pi}{\beta-1}$ is dominated by $g = 0$. Suppose that players with endowment z_S contribute $g \in [0, \frac{\beta\Pi}{\beta-1})$. Player i has an incentive to raise his own bid by an amount ε and win the prize. On the other hand if all of them contribute $g = \frac{\beta\Pi}{\beta-1}$, then player i has an incentive to deviate and contribute nothing. ∎

If the prize level is "medium" only the agents with the highest endowment will submit non-zero bids.

Proposition 3.3 *When $\frac{\beta\Pi}{n[z_S](\beta-1)} < z_S < \frac{\beta\Pi}{\beta-1}$ and $n[z_S] > 1$ there exists a mixed strategy equilibrium in which:*

- *players with endowment z_S contribute their full endowment with probability p and with probability $1 - p$ they choose their contribution from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{n[z_S]-1}}$ on the interval $[0, a]$, such that $F(a) = 1 - p$, where $a < z_S$ and p is the unique solution to the following equation $\frac{1-(1-p)^{n[z_S]}}{n[z_S]p} = \frac{(\beta-1)z_S}{\beta\Pi}$;*

- *players with endowments lower than z_S contribute 0.*

Proof. The proof is in five parts.

Let us first focus on the players with endowment z_S and show that, when they are the only active bidders, the candidate equilibrium is indeed an equilibrium.

i) Assume that all but one of the $n[z_S]$ players with endowment z_S choose their contribution from the distribution function $F(g) = (\frac{(\beta-1)g}{\beta\Pi})^{\frac{1}{n[z_S]-1}}$ on the interval $[0, a]$, where $0 < a < z_S$. Then the expected payoff of the remaining player i from contributing $g \in [0, a]$ is given by

$$\beta z_S + \beta\Pi(F(g))^{n[z_S]-1} - (\beta - 1)g + G_{-i}$$
$$= \beta z_S + G_{-i}$$

which is independent of g.

Assume now that $n[z_S] - 1$ players contribute their full endowment with probability p. Player i's expected prize from contributing z_S is given by

$$\beta \Pi \sum_{j=0}^{n[z_S]-1} \frac{1}{j+1} \binom{n[z_S]-1}{j} p^j (1-p)^{n[z_S]-j-1} \tag{3.3}$$

where $\binom{n[z_S]-1}{j} p^j (1-p)^{n[z_S]-j-1}$ represents i's probability of tying with j other players, while $\frac{\beta \Pi}{j+1}$ is his expected prize when he ties with j others. Applying binomial rules expression (3.3) can be rewritten as

$$\beta \Pi \frac{1 - (1-p)^{n[z_S]}}{n[z_S]p}$$

and therefore player i's expected payoff from playing z_S is given by

$$\beta z_S + \beta \Pi \frac{1 - (1-p)^{n[z_S]}}{n[z_S]p} - (\beta - 1)z_S + G_{-i}$$

For this to be an equilibrium player i's expected payoff from contributing z_S must be equal to his expected payoff from choosing any $g \in [0, a]$, which means that

$$\beta z_S + \beta \Pi \frac{1 - (1-p)^{n[z_S]}}{n[z_S]p} - (\beta - 1)z_S + G_{-i}$$
$$= \quad \beta z_S + G_{-i}$$

Therefore p must satisfy the following

$$\frac{1 - (1-p)^{n[z_S]}}{n[z_S]p} = \frac{(\beta - 1)z_S}{\beta \Pi} \tag{3.4}$$

ii) We are going to prove that there is a unique solution to equation (3.4). This equation can be rewritten as

$$1 - (1-p)^{n[z_S]} = \frac{n[z_S](\beta - 1)z_S}{\beta \Pi} p$$

105

Notice that the left hand side is concave while the right hand side is linear. Further, given the restrictions on z_S, it is the case that $1 < \frac{n[z_S](\beta-1)z_S}{\beta\Pi} < n[z_S]$. When $p = 0$ both sides of the equation are equal to zero. When $p = 1$ the left hand side is equal to 1 while the left hand side is strictly greater than 1. Finally, notice that the slope of the left hand side when $p = 0$ is $n[z_S]$, which is steeper than the right hand side. Therefore there must be a unique solution for $p \in (0, 1]$.

iii) We want to show that a, such that $F(a) = 1 - p$, is strictly less than z_S. We will prove it by contradiction. Assume the opposite, then it should be the case that $F(z_S) \le 1 - p$. Given equation (3.4), the latter can be rearranged as

$$1 - (1 - p)^{n[z_S]} \le n[z_S]p(1 - p)^{n[z_S]-1} \tag{3.5}$$

When $p = 0$ both sides are equal to 0. The first derivative of the left hand side is equal to $n[z_S](1 - p)^{n[z_S]-1}$, while the first derivative of the right hand side is $n[z_S](1 - p)^{n[z_S]-1} - (n[z_S] - 1)n[z_S]p(1 - p)^{n[z_S]-2}$. Notice that the former is strictly greater than the latter for any p on the interval $(0, 1]$. Therefore the left hand side of inequality (3.5) is strictly greater than the right hand side for any positive probability, which contradicts our assumption.

iv) What we have just shown means that the players will not choose any contribution from the interval (a, z_S). Let us check that this is the case. Assume that all other players play according to the candidate equilibrium while player i contributes $g \in (a, z_S)$. Then i wins the prize with probability $(1 - p)^{n[z_S]-1} = \frac{(\beta-1)a}{\beta\Pi}$ and his expected payoff is

$$\beta z_S + \beta\Pi\left(\left(\frac{(\beta-1)a}{\beta\Pi}\right)^{\frac{1}{n[z_S]-1}}\right)^{n[z_S]-1} - (\beta-1)g + G_{-i}$$
$$= \beta z_S + (\beta-1)a - (\beta-1)g + G_{-i}$$

which is strictly less than $\beta z_S + G_{-i}$. Therefore contributing 0 dominates any choice $g \in (a, z_S)$.

v) Let us now show that, when players with endowment z_S play according to the equilibrium candidate, it is a dominant strategy for all the other players to contribute nothing. Suppose that $z_{S-1} > a$. Point iv) proves that contributing 0 dominates any $g \in (a, z_{S-1}]$. On the other hand, if a player i with endowment $z_i < z_S$ contributes $g_i \in (0, a]$ then his expected payoff is

$$\beta z_i + \beta \Pi \left(\frac{(\beta - 1)g_i}{\beta \Pi} \right)^{\frac{n[z_S]}{n[z_S]-1}} - (\beta - 1)g_i + G_{-i}$$

Given that

$$\left(\frac{(\beta - 1)g_i}{\beta \Pi} \right)^{\frac{n[z_S]}{n[z_S]-1}} < \frac{(\beta - 1)g_i}{\beta \Pi}$$

it must be the case that contributing 0 is a dominant strategy for all players with endowment lower than z_S.

The same is true when $z_{S-1} \leq a$. ∎

Finally, if the prize level is "low" only the players with endowments higher than $\frac{\beta \Pi}{\beta - 1}$ will contribute positive amounts.

Proposition 3.4 *When $z_S \geq \frac{\beta \Pi}{\beta - 1}$ and $n[z_S] > 1$, there exists a mixed strategy equilibrium in which:*

- *players with endowment $z_i \geq \frac{\beta \Pi}{\beta - 1}$ choose their contributions from the distribution function $F(g) = \left(\frac{(\beta - 1)g}{\beta \Pi} \right)^{\frac{1}{m-1}}$ on the interval $[0, \frac{\beta \Pi}{\beta - 1}]$, where m is the number of players with endowment greater or equal than $\frac{\beta \Pi}{\beta - 1}$;*

- *all other players contribute 0.*

Proof. Suppose that $z_{l-1} < \frac{\beta \Pi}{\beta - 1}$ while $z_l \geq \frac{\beta \Pi}{\beta - 1}$, with $1 \leq l \leq S$, and call $m = \sum_{i=l}^{S} n[z_i]$ the number of players with endowment greater or equal than $\frac{\beta \Pi}{\beta - 1}$. If $l = 1$ then consider z_{l-1} to be zero. The proof is in four parts.

i) Notice first that any strategy above $\frac{\beta \Pi}{\beta - 1}$ is dominated by contributing 0.

ii) Let us focus on the interval $(z_{l-1}, \frac{\beta \Pi}{\beta - 1}]$ where only m players are active. Assume that all but one of the m players choose their contribution from the distribution

107

function $F(g)$ on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$. In order for this to be an equilibrium the remaining player i must be indifferent to play any $g \in (z_{l-1}, \frac{\beta\Pi}{\beta-1}]$. Hence his expected payoff from playing g must be

$$\beta z_i + \beta\Pi(F(g))^{m-1} - (\beta - 1)g + G_{-i} = \beta z_i + G_{-i} + c$$

where $c \geq 0$.

This means that on the interval $(z_{l-1}, \frac{\beta\Pi}{\beta-1}]$ any player with endowment greater than z_{l-1} randomises according to the following distribution function

$$F(g) = (\frac{(\beta - 1)g + c}{\beta\Pi})^{\frac{1}{m-1}}$$

Note that $F(\frac{\beta\Pi}{\beta-1}) \leq 1$ implies that c must be equal to 0 and therefore we have a unique solution

$$F(g) = (\frac{(\beta - 1)g}{\beta\Pi})^{\frac{1}{m-1}} \tag{3.6}$$

iii) Suppose that $l = 1$. When the other $N - 1$ players choose their contribution from $F(g)$ on the interval $[0, \frac{\beta\Pi}{\beta-1}]$, then player i's expected payoff is equal to

$$\beta z_i + G_{-i}$$

independently of his contribution on the same interval.

iv) If $l > 1$ then point v) of the proof of Proposition (3.3) shows that contributing 0 is a dominant strategy for all players with endowment less than $\frac{\beta\Pi}{\beta-1}$, while players with higher endowments will randomise according to $F(g)$ from the interval $[0, \frac{\beta\Pi}{\beta-1}]$.
∎

Only One Player with the Highest Endowment

We look now at the case where $n[z_S] = 1$. First we will prove that only mixed strategy equilibria exist.

Proposition 3.5 *When $n[z_S] = 1$ there exist no pure strategy equilibria.*

Proof. The proof is in two parts.

i) Consider the case $z_{S-1} < \frac{\beta\Pi}{\beta-1}$. Suppose that there exists a pure strategy equilibrium characterised by the strategy profile $[g_1, \ldots, g_i, \ldots, g_N]$, where g_i is the contribution chosen by the generic player i. Call g_h the highest contribution. If $g_h > z_{S-1}$ then the player with endowment z_S could marginally lower his bid and increase his payoff. If $g_h < z_{S-1}$ then there is at least one player who could deviate and contribute $g_h + \varepsilon$, winning the prize and making a positive profit. If $g_h = z_{S-1}$ and a player with endowment z_{S-1} is contributing g_h, then the player with the highest endowment has an incentive to deviate and contribute $z_{S-1} + \varepsilon$. If $g_h = z_{S-1}$ and the players with endowment z_{S-1} are contributing strictly less than g_h, then the player with endowment z_S could lower his bid increasing his payoff.

ii) Consider the case $z_{S-1} \geq \frac{\beta\Pi}{\beta-1}$. Notice that any strategy $g > \frac{\beta\Pi}{\beta-1}$ is dominated by $g = 0$. As we have done above, suppose that there exists a pure strategy equilibrium characterised by the strategy profile $[g_1, \ldots, g_i, \ldots, g_N]$ and call g_h the highest contribution. If $g_h < \frac{\beta\Pi}{\beta-1}$ then there is at least one player who has an incentive to deviate and contribute $g_h + \varepsilon$. If $g_h = \frac{\beta\Pi}{\beta-1}$ and only one player is contributing g_h, then he could lower his bid. If $g_h = \frac{\beta\Pi}{\beta-1}$ and two or more players are bidding g_h, then each one of them would be better off by contributing zero. ∎

There exist two possible cases: when the prize level is "low" and when it is "high". Let us start focusing on the first scenario.

Proposition 3.6 *When $z_{S-1} \geq \frac{\beta\Pi}{\beta-1}$ and $n[z_S] = 1$, there exists a mixed strategy equilibrium in which:*

- *players with endowment $z_i \geq \frac{\beta\Pi}{\beta-1}$ choose their contributions from the distribution function $F(g) = \left(\frac{(\beta-1)g}{\beta\Pi}\right)^{\frac{1}{m-1}}$ on the interval $[0, \frac{\beta\Pi}{\beta-1}]$, where m is the number of players with endowment greater or equal than $\frac{\beta\Pi}{\beta-1}$;*

- *all other players contribute 0.*

Proof. Proof as in Proposition (3.4). ∎

When the prize level is "high", specifically $z_{S-1} < \frac{\beta\Pi}{\beta-1}$, if the strategy space is continuous, and ties are broken by randomly assigning the prize to one player, then no equilibrium exists. In order to avoid this problem, given that we are interested in the theoretical predictions of an experiment, where the strategy space is discrete, we will assume that there exists a smallest currency unit strictly above z_{S-1} (see Che and Gale, 1997).[12]

Proposition 3.7 *When $z_{S-1} < \frac{\beta\Pi}{\beta-1}$ and $n[z_S] = 1$, there exists a mixed strategy equilibrium in which:*

- *the player with endowment z_S chooses his contribution from the distribution function $H(g) = \dfrac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}(\beta\Pi-(\beta-1)(z_{S-1}-g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}}$ on the interval $[0, z_{S-1}]$ and puts a mass equal to $\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}$ on the smallest currency unit strictly above z_{S-1};*

- *players with endowment z_{S-1} contribute zero with probability*

  $(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}}$ and choose their contribution from the distribution function

  $$L(g) = (\frac{\beta\Pi-(\beta-1)(z_{S-1}-g)}{\beta\Pi})^{\frac{1}{n[z_{S-1}]}} \text{ on the interval } (0, z_{S-1}];$$

- *all other players contribute zero.*

Proof. Assuming that the players with budgets z_{S-1} and z_S are the only ones who submit positive bids, we show that by playing according to the equilibrium candidate they make each others indifferent between any possible choice. We then go on to prove that if they play in such a way it is a dominant strategy for all other players to contribute zero. The proof is in three parts.

---

[12] The non-existence of the equilibrium is due to a discontinuity in the payoffs. Another way to avoid this problem would be to always break ties in favour of the player with the higher budget.

i) Let us start supposing that the players with endowments strictly lower than z_{S-1} contribute zero. Note first that the player of type z_S can guarantee himself a positive surplus by submitting a bid above z_{S-1}. We want to show that if players with endowment z_{S-1} choose their contribution from $L(g)$, and play zero with probability $\left(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}\right)^{\frac{1}{n[z_{S-1}]}}$, then the agent with the highest endowment is indifferent between any choice on the interval $(0, z_{S-1}]$. His payoff from playing $g \in (0, z_{S-1}]$ will be

$$
\begin{aligned}
&\beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]} - (\beta-1)g + G_{-i} \\
=\ &\beta z_S + \beta\Pi\left(\frac{\beta\Pi - (\beta-1)(z_{S-1}-g)}{\beta\Pi}\right) - (\beta-1)g + G_{-i} \\
=\ &\beta z_S + \beta\Pi - (\beta-1)z_{S-1} + G_{-i}
\end{aligned}
$$

which indeed does not depend on g.

ii) Suppose now that the player with endowment z_S randomises according to $H(g)$ on the interval $[0, z_{S-1}]$ and puts a mass equal to $\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}$ on the smallest currency unit strictly above z_{S-1}.[13] If all other agents of type z_{S-1} play according to $L(g)$, and contribute zero with probability $\left(\frac{\beta\Pi-(\beta-1)z_{S-1}}{\beta\Pi}\right)^{\frac{1}{n[z_{S-1}]}}$, then the payoff of a player with z_{S-1} from a choice $g \in [0, z_{S-1}]$ is given by

$$
\begin{aligned}
&\beta z_S + \beta\Pi(L(g))^{n[z_{S-1}]-1} H(g) - (\beta-1)g + G_{-i} \\
=\ &\beta z_S + \beta\Pi\left(\frac{\beta\Pi-(\beta-1)(z_{S-1}-g)}{\beta\Pi}\right)^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}} \\
&\frac{(\beta-1)g}{(\beta\Pi)^{\frac{1}{n[z_{S-1}]}}\left(\beta\Pi-(\beta-1)(z_{S-1}-g)\right)^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}} \\
&-(\beta-1)g + G_{-i} \\
=\ &\beta z_S + G_{-i}
\end{aligned}
$$

_____

[13] Note that, acording to $H(g)$, player z_S's bid is strictly positive and therefore no ties are possibile at zero.

which again is independent of g. It should be clear now why it is necessary to assume that there exists a smallest unit strictly above z_{S-1}. If this was not the case the player with the highest endowment would have a mass point at z_{S-1}. But then, if ties are broken by randomly assigning the prize to one player, an agent of type z_{S-1} would have an incentive to deviate and bid all his endowment.

iii) Finally, we want to show that if the agents of type z_{S-1} and z_S play as we described then it is a dominant strategy for all other players to contribute zero. If a player i with endowment $z_i < z_{S-1}$ contributes $g \in (0, z_i]$ his payoff is represented by

$$
\begin{aligned}
& \beta z_S + \beta \Pi (L(g))^{n[z_{S-1}]} H(g) - (\beta - 1)g + G_{-i} \quad\quad\quad\quad (3.7) \\
= \; & \beta z_S + \beta \Pi \left(\frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi} \right) \\
& \frac{(\beta - 1)g}{(\beta \Pi)^{\frac{1}{n[z_{S-1}]}} (\beta \Pi - (\beta - 1)(z_{S-1} - g))^{\frac{n[z_{S-1}]-1}{n[z_{S-1}]}}} - (\beta - 1)g + G_{-i} \\
= \; & \beta z_S + \left(\frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi} \right)^{\frac{1}{n[z_{S-1}]}} (\beta - 1)g - (\beta - 1)g + G_{-i} \\
= \; & \beta z_S + (\beta - 1)g \left(\left(\frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi} \right)^{\frac{1}{n[z_{S-1}]}} - 1 \right) + G_{-i}
\end{aligned}
$$

On the other hand, if he plays $g = 0$ he gets a payoff equal to $\beta z_S + G_{-i}$. Note that $\left(\frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi} \right)^{\frac{1}{n[z_{S-1}]}} < 1$ and we conclude that expression (3.7) is strictly lower than $\beta z_S + G_{-i}$. $\blacksquare$

References

Amiel, Y., Cowell, F. A., 1999. *Thinking About Inequality*, Cambridge University Press, Cambridge.

Anderson, L., Mellor, J., Milyo, J., 2004. Inequality and public good provision: An experimental analysis, College of William and Mary, Department of Economics, Working paper Number 12.

Andreoni, J., 1988a. Privately Provided Public Goods in a Large Economy: the Limits of Altruism, *Journal of Public Economics*, 35, 57-73.

Andreoni, J., 1988b. Why free ride?, *Journal of Public Economics*, 37, 291-304.

Arneson, R., 1989. Equality of opportunity for welfare, *Philosophical Studies*, 56, 77-93.

Arneson, R., 1990. Liberalism, distributive subjectivism, and equal opportunity for welfare, *Philosophy and Public Affairs*, 19, 159-194.

Babcock, L., Loewenstein, G., Issacharoff, S., Camerer, C., 1995. Biased judgments of fairness in bargaining, *American Economic Review*, 85, 1337-1343.

Bagnoli, M., Lipman, B. L., 1989. Provision of Public Goods: Fully Implementing the Core through Private Contributions, *Review of Economic Studies*, 56, 583-601.

Bagnoli, M., McKee, M., 1991. Voluntary Contribution Games: Efficient Private Provision of Public Goods, *Economic Inquiry*, 29, 351-66.

Barut, Y., Kovenock, D., 1998. The Symmetric Multiple Prize All-pay Auction with Complete Information, *European Journal of Political Economy*, 14, 627-644.

Baye, M., Kovenock, D., de Vries, C., 1993. Rigging the Lobbying Process: An Application of the All-pay Auction, *American Economic Review*, 83, 289-294.

Baye, M., Kovenock, D., de Vries, C., 1996. The All-pay Auction with Complete Information, *Economic Theory*, 8, 291-305.

Bergstrom, T., Blume, L., Varian H., 1986. On the Private Provision of Public Goods, *Journal of Public Economics*, 29, 25-49.

Breocker, T., 1990. Credit-Worthiness Tests and Interbank Competition, *Econometrica*, 58, 429-452.

Buckley, E., Croson, R., 2006. Income and wealth heterogeneity in the voluntary provision of linear public goods, *Journal of Public Economics*, 90, 935-955.

Cappelen, A. W., Sørensen, E. Ø., Tungodden, B., 2005. Responsibility for what? An experimental approach to fairness and responsibility, University of Oslo: Mimeo.

Carter, J. R., Irons, M. D., 1991. Are economists different, and if so, why?, *Journal of Economic Perspectives*, 5, 171-177.

Chan, K. S., Mestelman, S., Moir, R., Muller, R. A., 1996. The Voluntary Provision of Public Goods under Varying Income Distributions, *Canadian Journal of Economics*, 29, 54-69.

Chan, K. S., Mestelman, S., Moir, R., Muller, R. A., 1999. Heterogeneity and the voluntary provision of public goods, *Experimental Economics*, 2, 5-30.

Che, Y., Gale, I., 1997. Rent dissipation when rent seekers are budget constrained, *Public Choice*, 92, 109-126.

Cohen, G. A., 1989. On the currency of egalitarian justice, *Ethics*, 99, 906-944.

Dasgupta, P., 1986. The Theory of Technological Competition, in: Stiglitz, J., Mathweson, G. (Eds.), *New developments in the analysis of market structure*, Cambridge, MA: MIT Press.

Dworkin, R., 1981. What is equality? Part 2: equality of resources, *Philosophy and Public Affairs*, 10, 283-345.

Ellingsen, T., 1991. Strategic Buyers and the Social Cost of Monopoly, *American Economic Review*, 81, 648-657.

Engelmann, D., Strobel, M., 2004. Inequality aversion, efficiency, and maximin preferences in simple distribution experiments, *American Economic Review*, 94, 857-869.

Faravelli, M., 2007. The Important Thing Is not (Always) Winning but Taking Part: Funding Public Goods with Contests, ESE Discussion Paper 156, University of Edinburgh.

Fischbacher, U., 1999. Zurich Toolbox for Readymade Economic Experiments, Working Paper No. 21, University of Zurich.

Fleiss, J. L., 1981. *Statistical Methods for Rates and Proportions*, John Wiley & Sons, USA.

Fleurbaey, M., 1998. Equality among responsible individuals, in: Laslier, J-Fr., Fleurbaey, M., Trannoy, A., Gravel, N. (Eds.), *Freedom in Economics*, Routledge, London.

Fleurbaey, M., Maniquet, F., in press. Compensation and Responsibility, in: Suzumura, K., Arrow, K., Sen, A. K. (Eds.), *Handbook of Social Choice and Welfare*, vol. 2. North-Holland.

Fong, C., 2001. Social preferences, self-interest, and the demand for redistributions, *Journal of Public Economics*, 82, 225-246.

Frank, R. H., Gilovich, T. D., Regan, D. T., 1993. Does studying economics inhibit cooperation?, *Journal of Economic Perspectives*, 7, 159-171.

Frank, R. H., Gilovich, T. D., Regan, D. T., 1996. Do economists make bad citizens?, *Journal of Economic Perspectives*, 10, 187-192.

Frey, B. S., Meier, S., 2003. Are political economists selfish and indoctrinated? Evidence from a natural experiment, *Economic Inquiry*, 41, 448-462.

Frohlich, N., Oppenheimer, J., Eavy, C., 1987a. Choices of principles of distributive justice in experimental groups, *American Journal of Political Science*, 606-636.

Frohlich, N., Oppenheimer, J., Eavy, C., 1987b. Laboratory results on Rawls's distributive justice, *British Journal of Political Science*, 17, 1-21.

Fullerton, D., McAfee, P., 1999. Auctioning Entry into Tournaments, *Journal of Political Economy*, 107, 573-605.

Gaertner, W., 1994. Distributive justice: theoretical foundations and empirical findings, *European Economic Review*, 38, 711-720.

Gaertner, W., Jungeilges, J., Neck, R., 2001. Cross-cultural equity evaluations: a questionnaire-experimental approach, *European Economic Review*, 45, 953-963.

Gaertner, W., Jungeilges, J., 2002. Evaluation via extended orderings: empirical findings from Western and Eastern Europe, *Social Choice and Welfare*, 19, 29-55.

Gaertner, W., Schwettmann, L., 2005. Equity, responsibility and the cultural dimension, University of Osnabrück: Mimeo.

Goeree, J., Maasland, A., Onderstal, S., Turner, J., 2005. How (Not) to Raise Money, *Journal of Political Economy*, 113, 897-918.

Groves, T., Ledyard, J., 1977. Optimal Allocation of Public Goods: A Solution to the Free Rider Problem, *Econometrica*, 45, 783-809.

Heider, F., 1958. *The Psychology of Interpersonal Relations*, John Wiley & Sons, New York.

Hillman, A., Riley, J., 1989. Politically Contestable Rents and Transfers, *Economics and Politics*, 1, 17-40.

Hillman, A., Samet, D., 1987. Dissipation of Contestable Rents by Small Numbers of Contenders, *Public Choice*, 54, 63-82.

Hu, Y.-A., Liu, D.-Y., 2003. Altruism versus egoism in human behavior of mixed motives, *American Journal of Economics and Sociology*, 62, 677-705.

Isaac, R. M., Walker, J. M., 1988. Communication and Free-Riding Behavior: The Voluntary Contributions Mechanism, *Economic Inquiry*, 26, 585-608.

Kahneman, D., Knetsch, J. L., Thaler, R., 1986. Fairness as a constraint on profit seeking: entitlements in the market, *American Economic Review*, 76, 728-741.

Keser, C., 1996. Voluntary Contributions to a Public Good when Partial Contribution is a Dominant Strategy, *Economics Letters*, 50, 359-366.

Konow, J., 2000. Fair shares: accountability and cognitive dissonance in allocation decisions, *American Economic Review*, 90, 1072-1091.

Konow, J., 2003. Which is the fairest one of all? A positive analysis of justice theories, *Journal of Economic Literature*, 11, 1188-1239.

Laury, S. K., Holt, C. A., 1998. Voluntary Provision of Public Goods: Experimental Results with Interior Nash Equilibria, in: Plott, C., Smith, V. (Eds.), *Handbook of Experimental Economics Results*, New York: Elsevier Press.

Lazear, E. P., Sherwin, R., 1981. Rank-Order Tournaments as Optimum Labor Contracts, *Journal of Political Economy*, 89, 841-864.

Ledyard, J. O., 1995. Public Goods: A Survey of Experimental Research, in: Roth, A., Kagel, J. (Eds.), *Handbook of Experimental Economics*, Princeton: Princeton University Press.

Lissowsky, G., Tyszka, T., Okrasa, W., 1991. Principles of distributive justice: experiments in Poland and America, *The Journal of Conflict Resolution*, 35, 98-119.

Marwell, G., Ames, R. E., 1981. Economists free ride, does anyone else?, *Journal of Public Economics*, 15, 295-310.

McClelland, G., Rohrbaugh, J., 1978. Who accepts the Pareto axiom? The role of utility and equity in arbitration decisions, *Behavioural Science*, 23, 446-456.

Moldovanu, B., Sela, A. (2001). The Optimal Allocation of Prizes in Contests, *American Economic Review*, 91, 542-558.

Morgan, J., 2000. Financing Public Goods by Means of Lotteries, *Review of Economic Studies*, 67, 761-784.

Morgan, J., Sefton, M., 2000. Funding Public Goods with Lotteries: Experimental Evidence, *Review of Economic Studies*, 67, 785-810.

Orzen, H., 2005. Fundraising through Competition: Evidence from the Laboratory, Discussion Papers 2005-04, The Centre for Decision Research and Experimental Economics, School of Economics, University of Nottingham.

Rapoport, A., Suleiman, R., 1993. Incremental contribution in step-level public goods games with asymmetric players, *Organizational Behavior and Human Decision Processes*, 55, 171-194.

Rawls, J., 1971. *A Theory of Justice*, Harvard University Press, Cambridge.

Rawls, J., 1974. Some reasons for the maximin criterion, *The American Economic Association*, 64, 141-146.

Roemer, J. E., 1993. A pragmatic theory of responsibility for the egalitarian planner, *Philosophy and Public Affairs*, 22, 146-166.

Saijo, T., 2003. Spiteful Behaviour in Voluntary Contribution Mechanism Experiments, Institute of Social and Economic Research, Osaka University: Mimeo.

Schram, A., Onderstal, S., 2006. Bidding to Give: An Experimental Comparison of Auctions for Charity, CREED, Amsterdam School of Economics: Mimeo.

Sen, A., 1980. Equality of what?, in: McMurrin, S. (Ed.), *The Tanner Lectures on Human Values, Vol. 1*, University of Utah Press, Salt Lake City.

Sen, A., 1992. *Inequality Reexamined*, Oxford University Press, Oxford.

Schokkaert, E., Lagrou, L., 1983. An empirical approach to distributive justice, *Journal of Public Economics*, 21, 33-52

Schokkaert, E., Overlaet, B., 1989. Moral intuitions and economic models of distributive justice, *Social Choice and Welfare*, 6, 19-31.

Schokkaert, E., Devooght, K., 2003. Responsibility-sensitive fair compensation in different cultures, *Social Choice and Welfare*, 21, 207-242.

Taylor, C.R., 1995. Digging for Golden Carrots: An Analysis of Research Tournaments, *American Economic Review*, 85, 873-90.

Tullock, G., 1980. Efficient Rent-Seeking, in: Buchanan, J., Tollison, R., Tullock, G. (Eds.), *Towards a theory of the rent-seeking society*, College Station, Tx: Texas A&M University Press.

Walker, M., 1981. A Simple Incentive Compatible Scheme for Attaining Lindhal Allocations, *Econometrica*, 49, 65-71.

Windham, P., 1999. A Taxonomy of Technology Prizes and Contests, in *Concerning federally sponsored inducement prizes in engineering and science*, Washington, DC: National Academy Press, 21-34.

Wright, B.D., 1983. The Economics of Invention Incentives: Patents, Prizes, and Research Contracts, *American Economic Review*, 73, 691-707.

Yaari, M. E., Bar-Hillel, M., 1984. On dividing justly, *Social Choice and Welfare*, 1, 1-24.

Yezer, A. M., Goldfarb, R. S., Poppen, P. J., 1996. Does studying economics discourage cooperation? Watch what we do, not what we say or how we play, *Journal of Economic Perspectives*, 10, 177-186.

Zsolnai, L., 2003. Honesty versus cooperation, *American Journal of Economics and Sociology*, 62, 707-712.