Modeling Exotic Options with Maturity Extensions by Stochastic Dynamic Programming

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PhD

The University of Edinburgh

2009

Declaration

I declare that this thesis has been composed by me and that the work is my own. Further, I declare that this work has not been submitted for any other degree or professional qualification.

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Abstract

The exotic options that are examined in this thesis have a combination of non-standard characteristics which can be found in shout, multi-callable, path-dependent and Bermudan options. These options are called reset options. A reset option is an option which allows the holder to reset, one or more times, certain terms of the contract based on pre-specified rules during the life of the option.

Overall in this thesis, an attempt has been made to tackle the modeling challenges that arise from the exotic properties of the reset option embedded in segregated funds. Initially, the relevant literature was reviewed and the lack of published work, advanced enough to deal with the complexities of the reset option, was identified. Hence, there appears to be a clear and urgent need to have more sophisticated approaches which will model the reset option.

The reset option on the maturity guarantee of segregated funds is formulated as a non-stationary finite horizon Markov Decision Process. The returns from the underlying asset are modeled using a discrete time approximation of the lognormal model. An Optimal Exercise Boundary of the reset option is derived where a threshold value is depicted such that if the value of the underlying asset price exceeds it then it is optimal for the policyholder to reset his maturity guarantee. Otherwise, it is optimal for the policyholder to rollover his maturity guarantee. It is noteworthy that the model is able to depict the Optimal Exercise Boundary of not just the first but of all the segregated fund contracts which can be issued throughout the planning horizon of the policyholder.

The main finding of the model is that as the segregated fund contract approaches its maturity, the threshold value in the Optimal Exercise Boundary

increases. However, in the last period before the maturity of the segregated fund, the threshold value decreases. The reason for this is that if the reset option is not exercised it will expire worthless.

The model is then extended to reflect on the characteristics of the range of products which are traded in the market. Firstly, the issuer of the segregated fund contract is allowed to charge a management fee to the policyholder. The effect from incorporating this fee is that the policyholder requires a higher return in order to optimally reset his maturity guarantee while the total value of the segregated fund is diminished. Secondly, the maturity guarantee becomes a function of the number of times that the reset option has been exercised. The effect is that the policyholder requires a higher return in order to choose to reset his maturity guarantee while the total value of the segregated fund is diminished. Thirdly, the policyholder is allowed to reset the maturity guarantee at any point in time within each year from the start of the planning horizon, but only once. The effect is that the total value of the segregated fund is increased since the policyholder may lock in higher market gains as he has more reset decision points.

In response to the well documented deficiencies of the lognormal model to capture the jumps experienced by stock markets, extensions were built which incorporate such jumps in the original model. The effect from incorporating such jumps is that the policyholder requires a higher return in order to choose to reset his maturity guarantee while the total value of the segregated fund is diminished due to the adverse effect of the negative jumps on the value of the underlying asset.

Acknowlegments

I would like to thank the supervisor of this thesis, Prof. T. W. Archibald, for all of his precious help and support throughout these last few years. He was the one who had the original research idea, the one who could always find what was wrong with my Fortran files, and even more importantly the one who found ways to improve the efficiency of our algorithms and thus increased the scope of this thesis. Without his enthusiasm and support this thesis would not be possible. I would further like to thank him for his sincere understanding and the support he provided me during the extremely difficult family circumstances that I went through. I will always be indebted to him.

I would like to thank my parents for their help and support throughout all these years that I have been studying abroad. They always respected the targets I was setting and supported me in achieving them. They stood by me in the good days and in the bad days.

I would also like to thank EPSRC for financial support.

Last but not least, I would like to thank my wife, Christina. The reasons why I would like to thank her could easily be another thesis itself - so I will focus on the main things! I would like to thank her for believing in me and my abilities. For the past six years, whenever there was a problem she was the one to help me come back on track. She knows how to motivate me and how to console me. She showed me that one can combine the striving for professional success with having a nice family. But, the reason for which I will always admire her is the courage and determination with which she faced the challenges of her pregnancy. In the end she gave me the best gift I could have ever wished for: an absolutely adorable and healthy son!

Dedication

It is not whether you get knocked down. It is whether you get back up.

Vince Lombardi

This thesis is dedicated to Christina for being all of my reasons

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Abbreviations

- CIA - Canadian Institute of Actuaries
- d - Percentage decrease of underlying asset
- DRM-C - Double Regime Model (crash)
- DRM-J - Double Regime Model (jumps)
- G - Maturity Guarantee
- MCCSR - Minimum Continuing Capital and Surplus Requirements
- MER - Management Expense Ratio Model
- N - Duration of segregated fund
- OEB - Optimal Exercise Boundary
- OSFI - Office of the Superintendent of Financial Institutions
- q - Number of periods in a year
- r - Risk free rate of interest
- S&P - Standard and Poor's
- SCM - Stochastic Crash Model
- SRM - Single Regime Model
- T - Duration of planning horizon
- TimRO - Variable Timing of Exercising of the Reset Option Model
- TSE - Toronto Stock Exchange
- $V^T_{1,Nq}$ - Total value of segregated fund contract
- VarG - Variable Maturity Guarantee Model
- u - Percentage increase of underlying asset

1 Introduction

Options give the holder the right but not the obligation to buy or sell an asset by a certain date for a certain price. Options that have standard well-defined properties and trade actively are called plain vanilla options [Hull, 2006]. Intense competition in the market place has led financial engineers to create non-standard products which are called exotic options [Clewlow and Strickland, 1997]. The family of exotic options that are examined in this thesis has a combination of non-standard characteristics which can be found in shout, multi-callable, path-dependent and Bermudan options.

The family of exotic options with the aforementioned characteristics are the so-called reset options. A reset option is an option which allows the holder to reset (alter), one or more times, certain terms of the contract based on pre-specified rules during the life of the option (before or at maturity). Reset options can be traded independently or more commonly are embedded in complex financial products [Hull, 2006]. Examples of such financial products are the extendible, retractable and convertible bonds, geared equity investment, executive stock options, energy commodity derivatives and segregated funds.

Segregated fund insurance contracts allow the holder to periodically reset the guaranteed amount and the maturity date. In contrast to the other types of reset options, the one embedded in segregated funds has an extra non-standard (exotic) characteristic: when the policyholder exercises his reset option, the

maturity of the contract is extended to its original duration. This maturity extension feature makes their modeling very challenging.

Chapter 2 expands on and critically analyses the nature of the reset option embedded in segregated funds and the reasons why it is worth modeling it. Further, a range of models relative to the valuation of the segregated fund investment guarantees adopted by regulatory authorities, practitioners and academics are critically reviewed.

Motivated by the importance of the reset option of segregated funds and the limitations of the relevant literature, an attempt is made to model the reset option of segregated funds in Chapter 3. The reset option on the maturity guarantee of segregated funds is formulated as a non-stationary finite horizon Markov Decision Process. The aim of this model is to depict the optimal exercise boundary of the reset option. In particular, given the model parameters, the aim to depict a threshold value such that if the asset value exceeds it then it is optimal for the policyholder to reset his maturity guarantee. Otherwise, it is optimal for the policyholder to rollover his maturity guarantee.

The rest of chapter 3 is organised as follows: section 3.2 formulates the reset option as a Markov Decision Process and states the assumptions that underpin the formulation. Then section 3.3, critically analyses ways in which the state space of the formulation can be reduced in order to make it more efficient. The revised formulation of the reset option is then provided in section

3.4. Section 3.5 succinctly analyses the structure of the model of segregated funds and highlights the main options and the corresponding decisions that the policyholder faces at each stage and state of the model.

Further, in section 3.6 the mathematical formulation is converted into a flowchart through pseudo-coding. The intuitive advantage of pseudo-coding is that its architecture and rationale are not framed by the syntax of any particular programming language, thus rendering the structure of the model generalisable. Then, in sections 3.7 and 3.8 the pseudo-coding is "translated" into Fortran Code, where the commercially available Salford software is used to compile and run the code. The aim is to extract the values of the underlying asset for which the policyholder will be optimally exercising his reset option.

Moreover, sections 3.9 and 3.10 highlight the main results of the original model as well as perform sensitivity analysis in order to examine the robustness of the model, gain insights into the behaviour of the segregated fund contract and make recommendations to both the policyholder and the issuer of the segregated funds.

In an attempt to reflect on the variety of segregated fund contracts which are traded in the market, an attempt is made to extend the original model. In particular, in Chapter 4 three of the extensions are examined.

Firstly, in the original model there is no provision for the issuer of the segregated fund contract to charge a management expense fee. This issue is

addressed in section 4.2 where the Management Expense Ratio model extends the original model in that it allows the issuer of the segregated fund to charge a management fee to the policyholder. In particular, it is assumed that the issuer charges the policyholder a fixed annual fee. This assumption is in line with guidance provided by the Canadian Institute of Actuaries (CIA) who suggests that this is a common approach used in practice. The model is analysed in section 4.2.1 and the main results as well as the sensitivity analysis are provided in sections 4.2.2 to 4.2.6.

Secondly, the original model assumes that the level of the maturity guarantee is fixed throughout the planning horizon regardless of the number of times that the policyholder has exercised his reset option. However, CIA has suggested that one way to reduce the risk of offering reset options is to diminish the level of maturity guarantee every time that the policyholder exercised his reset option. The idea is that if the policyholder takes advantage of favourable market conditions and locks in the relevant market gain, he should compensate the issuer by accepting a lower maturity guarantee. If, on the other hand, a policyholder does not exercise his reset option, thus, not causing any potential extra costs to the issuer, he should have the benefit of the full level of the maturity guarantee, as it was set at the beginning of the contract. Therefore, the level of the maturity guarantee should be directly related to the extent that the reset option is exercised by the policyholder, rather than a fixed percentage

of the original investment, as was originally used in the market.

This issue is addressed in section 4.3, where the Variable Maturity Guarantee model extends the original model in that the maturity guarantee becomes a function of the number of times that the reset option has been exercised since the maturity of the last segregated fund or the start of the planning horizon (whichever is most recent). In particular, every time the policyholder exercises his reset option the maturity guarantee is reduced by a pre-determined discount factor. The model is analysed in section 4.3.2 and the main results as well as the sensitivity analysis are provided in sections 4.3.3 to 4.3.7.

Thirdly, the original model assumes that the policyholder can exercise his reset option only at the end of each policy year. A policy year can be defined as the set of 365 days which start either when the segregated fund is issued or when the previous policy year ended. However, increased competition in the market place has led some of the issuers of segregated fund contracts to offer to policyholders more reset decision dates, but keeping the total number of reset options constant. In other words, the policyholder still has the standard one reset per policy year, but can decide whether to reset his maturity guarantee more often, than at the anniversary of the contract. CIA recommends that one should examine cases where the policyholder can decide whether to reset his maturity guarantee at least every quarter of the policy year, assuming one

reset option every policy year.

This issue is addressed in section 4.4, where the Variable Timing of Exercising the Reset Option model extends the original model in that it lifts the restriction that the policyholder can only exercise his reset option at the end of each policy year. In particular, under the new model the policyholder is allowed to reset the maturity guarantee at any point in time within each policy year from the start of the planning horizon, but only once. The model is analysed in section 4.4.2 and the main results as well as the sensitivity analysis are provided in sections 4.4.3 to 4.4.6.

For all three extensions to the original model a flowchart analysis is provided. As the logic and architecture of the various models share some common ground with the original model, rather than analysing the flowcharts in full, only the differences with the flowchart of the original model are highlighted. All other parts can be assumed to be the same.

In the SRM model built in chapter 3 the returns from the investment in a segregated fund are modeled using a discrete-time approximation of the lognormal model. While the lognormal model underpins the well known and widely used Black Scholes model it has been criticised, among other reasons, because empirical data of stock markets returns do not seem to follow the lognormal random walk [Bates, 1991, Heston, 1993, Wilmott, 1998]. As a matter of fact several empirical studies have demonstrated the existence of

jumps (both negative and positive) in the stock markets [Bates, 1996, Jorion, 1988, Carr et al., 2002]. In order to incorporate shocks in the model but to also preserve the comparability of the model's results with the results of previous chapters it has been decided to sustain the lognormal model but overlay it with stochastic jumps. This is the theme of Chapter 5.

Section 5.2 extends the original model in that it allows for instantaneous stochastic crashes to occur within the single regime of the SRM model, namely through the Stochastic Crash Model. In reality the evolution of the possible values of the underlying asset price is the same as with the original model. However, at every time period there is a small probability of a crash occurring. When a crash occurs, the residual value of the fund after the crash is equal to a fixed percentage of its original value.

Following that, in section 5.3, the Double Regime Model is built which provides alternative means to incorporate jumps into the original model. In contrast to the Stochastic Crash Model, the Double Regime Model is able to incorporate both negative (crash) and positive (surge) jumps as well as a combination of the two. In particular, it allows the underlying asset to switch between two distinct regimes. The market characteristics of the first regime are defined by the relevant scenario under examination and are equivalent to the ones used under the original model in order to facilitate comparisons. The second regime is intended to model periods of high volatility in the markets

and can be used to incorporate the jumps. Two distinct applications of the Double Regime Model are presented in sections 5.4 and 5.5 respectively.

In the first application the second regime, models the case where there is a large probability that the value of the underlying asset will marginally increase or a very small probability that it will drop by a substantial fixed percentage, thus essentially allowing only crashes like the Stochastic Crash Model. The parameters and transitions probabilities have been set so that a crash is as likely to happen and of the same magnitude, as in the Stochastic Crash Model, in order to facilitate comparisons. In the second application the second regime models the case where the stock market can exhibit variable jumps (i.e. both crashes and surges) with equal probability of occurrence. In particular, there is an equal probability that the value of the underlying asset will either increase by a large fixed percentage or it will drop by an equal in magnitude fixed percentage. Essentially, it is modeling a highly unstable market environment.

Finally, Chapter 6 discusses the main conclusions of the thesis and provides recommendations for future research.

2 Exotic Options

2.1 Introduction

Options give the policyholder the right but not the obligation to buy (sell) an underlying asset by a certain time for a certain price. Two basic types of options are the call and put options. A call option gives the policyholder the right to buy an asset by a certain date, the maturity of the option, for a certain price, the strike price. In contrast, a put option gives the holder the right to sell an asset [Hull, 2006]. Options can either be American or European. American options can be exercised at any time up to the maturity date, whereas European options can only be exercised at the maturity date itself. Options that have standard characteristics and have a high trade volume are called plain vanilla options and have their prices or implied volatilities quoted on regular basis by traders [Hull, 2006].

Intense competition in the market place has led financial engineers to create non-standard products which are called exotic options [Clewlow and Strickland, 1997]. It is beyond the scope of this thesis to review or categorize exotic options. However, interested readers can consult the seminal paper of Rubenstein and Reiner [1991]. The family of exotic options that will be examined in this thesis has a combination of non-standard characteristics which can be found in shout, multi-callable, path-dependent and Bermudan options.

A shout option is an option where the policyholder has the right to lock

in a minimum payoff at one point in time during its life [Zhang, 1998]. A multi-callable option is an option where the holder has, as the name suggests, multiple exercise rights. A path-dependent option is an option whose payoff at an exercise or maturity date depends on both the price of the asset at that date and the history of prices of the underlying asset [Goldman et al., 1979, Ritchken et al., 1993]. A Bermudan option is an option which allows the policyholder to exercise it before the maturity, but restricts the early exercise to predefined discrete dates [Zhang, 1998].

The family of exotic options with the aforementioned characteristics are the so-called reset options. A reset option is an option which allows the policyholder to alter, one or more times, some terms of the contract based on predetermined rules during the life of the option. Reset options can be traded independently or, more commonly, are embedded in complicated financial instruments [Hull, 2006]. For example the policyholder of some reset options has the right to reset its strike on predetermined dates, or time periods, if the underlying portfolio is lower (for reset calls) or higher (for reset puts) than the originally agreed strike price.

2.2 Types of Reset Options

Independently traded reset options have been issued since the mid-1990's [Liao and Wang, 2003]. Pertinent literature on independently traded reset options includes Cheuk and Vorst [1997], Gray and Whaley [1997], Gray and Whaley

[1999], Cheng and Zang [2000], Liao and Wang [2002], Liao and Wang [2003], Dai et al. [2004] Dai and Kwok [2005].

Apart from the independently traded reset options, five financial products have been identified which have embedded reset option(s). These are extendible, retractable and convertible bonds, geared equity investment, executive stock options, energy commodity derivatives and segregated funds.

An extendible bond is as a short dated bond with an embedded call option to buy a longer dated bond at the original value of the bond up to the extension date. A retractable bond is as a long dated bond with an embedded put option to sell the bond at the original value on the retraction date. Such types of bonds were first issued in Canada in 1959. For both types of bonds the term that can be reset is the maturity date and the reset can be triggered voluntarily by the policyholder [Ananthanarayanan and Schwartz, 1980].

A convertible bond is a bond that can be converted into shares of a company who issues them, commonly at some predetermined ratio. Convertible bonds were used in Japan by banks during the 1990's in their atttempt to incentivise investors in a declining stock market. In particular, "the bond issuers added an automatic reset option on the ratio at which the bond would be converted should the underlying asset price fall below the preset threshold on the prespecified date" [Lau and Kwok, 2003, Kimura and Shinahara, 2006].

Geared Equity Investments have been traded in Australia from Macquarie

Bank with embedded reset put options. The issuer provides liquidity to an investor to buy some Australian shares. In addition, the issuer guarantees the final payoff to the investor should the underlying asset drop from its original value. They achieve that by embedding a reset put option. The put option has a further feature in that it automatically resets the strike to the current value of the underlying asset on a predefined reset date should the underlying asset exceed the original strike [Gray and Whaley, 1999].

Executive stock options, essentially, are call options to buy an underlying asset whose strike is less than its market value and whose maturity is predetermined. In an attempt to reincentivise the policyholders, the issuer may elect to reset the strike price [Brenner et al., 2000]. Further studies on executive stock options with embedded reset options include those of Acharaya et al. [1998], Chance et al. [2000] and Corrado et al. [2001].

In the energy markets (electricity and gas) many contracts incorporate flexible delivery arrangements, known as swing or take-or-pay options. Subject to constraints, these contracts allow the policyholder to voluntarily reset the level of energy that he will purchase. Studies on energy commodity derivatives with embedded reset/swing options include those of Jaillet et al. [2004], Ibanez [2004] and Keppo [2004].

Segregated funds allow the policyholder to periodically alter (reset) the guaranteed amount and the maturity date. Indicatively, studies on segregated

funds with embedded reset options include those of Armstrong [2001], Hardy [2001] and Windcliff et al. [2001a,b,c]. In contrast to the other types of reset options, the one embedded in segregated funds has an extra non-standard (exotic) characteristic: when the policyholder exercises his reset option, the maturity of the contract is extended to its original duration. This maturity extension feature makes modeling very challenging. The following sections of this chapter expand on and critically analyse the nature of the reset option embedded in segregated funds and the reasons why it is worth modeling it.

2.3 Segregated Funds

Segregated funds are "variable annuity contracts distributed by Canadian insurance companies which are primarily used for the investment of contributions to group pension plans. The assets in each such fund, though owned by the life insurer, are segregated from its other assets. As such, they are defined very similarly to mutual funds — pools of investments in which an investor can acquire an interest by purchasing units" [Brizeli, 1998]. The difference with mutual funds is that segregated funds have additional features that provide a guarantee on the initial investment after a predetermined time.

The rest of this chapter will focus on the main feature of segregated funds namely the reset option of the maturity guarantee. The maturity guarantee is a long term (usually 10 years) put option on the underlying asset with an strike equal to the guaranteed amount. Segregated funds offer the policyholder the

option to reset the maturity guarantee from the original level to the current value of the underlying asset at predetermined times (e.g. at the anniversary of the contract). Upon exercising the reset option the maturity of the fund is extended to the original duration (e.g. 10 years).

Essentially, the policyholder is faced with the following trade-off: "at each available reset point a decision has to be made whether to keep the existing option or to substitute it with another one whose maturity is further in the future and whose exercise price is higher than the previous one" [Armstrong, 2001]. Therefore, "the reset option on the maturity guarantee embedded in segregated funds offers the policyholder the upside potential of the equity market while at the same time providing a protective floor should the market fall" [Windcliff et al., 2001b].

2.3.1 Reasons for Increased Attention

With the popularity and marketability of segregated funds on the ascendancy during the 1990's, issuing companies and actuaries realised that these guarantees are very complex and difficult to value. Further attention was drawn from the financial authority regulators: at the June 1998 Annual meeting of the Canadian Institute of Actuaries (CIA), the Superintendent of Financial Institutions, John Palmer stated [Canadian Institute of Actuaries, 2002]:

> *"Another important issue is that of the segregated fund's guarantees. Some institutions now offer segregated funds that are*

not only 100% guaranteed, but they also allow the purchaser to reset the base value upwards at certain intervals. ...By no means do we think that these guarantees are risk-free. They must be considered carefully, and actuaries pricing and reserving for them must remember their unique nature."

Despite the increased attention by end of the 1990's there were still no industry guidelines relative to the valuation of the segregated funds. Moreover, there "were no prescribed minimum capital requirements for offering this product. Overall, there was a wide variety of practices in the market ranging from doing nothing to doing some modeling" [Hancock, 2001].

2.3.2 Responses of the Canadian Regulatory Authorities

The aforementioned developments drew the attention of the financial industry in Canada. In 1998, the Office of the Superintendent of Financial Institutions (OSFI) in cooperation with CIA provided for the first time a deterministic valuation approach in order to tackle the valuation of segregated funds. The outcome of their work was prescribed scenarios with the remit to set a minimum liability (see section 2.3.3). At the same time stochastic techniques was suggested to test the adequacy of these liabilities, as they were only meant to be indicative minimums.

In June 1999 OSFI proposed a methodology with factors for the segregated funds. In the end of 1999 CIA appointed the Segregated Fund's Task Force with the aim to develop techniques for assessing the valuation of segregated funds.

Finally, the Segregated Fund's Task Force produced a long term valuation approach (see section 2.3.4 for more details) as well as an interim short term valuation approach (see section 2.3.5 for more details).

Since the reviewed literature (sections 2.3.1, and 2.3.2) raised concerns on the effect of the reset option on the cost of providing segregated fund contracts, in sections 2.3.3, 2.3.4, and 2.3.5 particular attention is drawn on how each valuation method deals with the reset option.

2.3.3 OSFI's Deterministic Approach

"The deterministic valuation approach which was prescribed by OSFI in order to set minimum liabilities, valued the guarantees by projecting net asset and liability cash flows, first with investment guarantees and then without investment guarantees. The liability for the guarantees was the additional assets needed to fund the incremental cash flow stream associated with the guarantees" [OSFI, 1999].

With respect to the reset option the deterministic scenario approach adopted the following heuristic. "Where voluntary resets of the guaranteed amount were available, not less than 75% (of those cases where such a reset would have caused merely an increase of the guaranteed amount) were assumed to reset at the valuation date immediately before the one time correction. Resets after the one time correction were ignored" [OSFI, 1999].

It is worth noting that this first approach raised a lot of criticism, especially

from Miles and Miles [2000]. One of the main comments against this approach was that the assumed heuristic regarding the reset decision was considered to be naive [of Actuaries, 2001]. Therefore, it is of great interest to examine in the following section (2.3.4) the recommendations of CIA relative to the valuation of the liabilities of segregated funds in light of the MCCSR requirement developed by OSFI for segregated fund guarantees in 1999.

2.3.4 CIA's Long Term Approach

CIA's segregated fund's task force highlighted the importance that "any model used to value financial guarantees, which are often deeply out-of-money, accurately captures the risk of the guarantee moving into the money" [Canadian Institute of Actuaries, 2002]. For this purpose they set out "a calibration method for the investment models that emphasizes the left tail of the asset return distribution over three different time periods; 1 year, 5 years and 10 years. An issuer of segregated funds can use any stochastic model that, when fitted to the baseline data (Toronto Stock Exchange (TSE) 300 total return index, 1956-1999) generates left tail probabilities at least as large as those prescribed by CIA, as summarised in table" 1. CIA further underlined the fact that the model must generate a mean 1-year accumulation factor close to the true mean of the data (in the range of 1.10 to 1.12), and the standard deviation of the 1-year accumulation factor must be at least 17.5% [Canadian

Institute of Actuaries, 2002].

Accumulation Period	2.5th percentile	5th percentile	10th percentile
1 year	0.76	0.82	0.90
5 year	0.75	0.85	1.05
10 year	0.85	1.05	1.35

Table 1: ***CIA's Calibration Table***

Source: [Canadian Institute of Actuaries, 2002]

Moreover, CIA argued that some factors which may complicate this process are the management expense ratio (MER) and the reset options. In particular, they advised actuaries that "their liability models should not assume any change in MERs unless there is clear evidence for doing so, as they expect a considerable competitive pressure not to increase MERs".

With respect to the reset option, CIA advised actuaries that "their liability models should assume that some policyholders would exercise this option". However, they argued that actuaries should not necessarily assume that policyholders behave with 100% efficiency, *"even if we knew what 100% efficient behaviour looked like"*, as a member of the CIA task force group highlights [Hardy, 2001].

Recognising their lack of knowledge on what would constitute optimal reset behaviour, CIA then advised actuaries to use an ad hoc method for simulating resets. In particular they recommended the use of the following heuristic:

"where elective resets of the guaranteed amount are available, not less than 75% of the cohort of policyholders eligible to reset should be assumed to reset each year where such a reset would cause a material increase in the guaranteed amount. A material increase in the guaranteed amount would be 15% or greater" [Canadian Institute of Actuaries, 2002].

Overall, CIA's long term approach is significantly more advanced compared to OSFI's deterministic approach. As matter of fact when it was completed CIA's task force realised that it would take some time for companies to develop models for full stochastic simulation. Therefore, an approach which was more short-term in nature was then developed for immediate use. This is examined in section (2.3.5). However, it should be noted that despite the fact that most of the long term approach uses some advanced techniques, the issue of the reset option was thoroughly neglected, as the Segregated Fund's Task Force used a heuristic similar in nature to the one used in the 1999 OSFI's deterministic approach.

2.3.5 CIA's Short Term Approach

The interim solution of CIA was to provide a set of tables that were mandatory for the issuers of segregated funds to use [Canadian Institute of Actuaries, 2002]. With respect to the reset option they explicitly mentioned in their guidelines that "the with resets rows of the tables assumed a 100% rate of

utilisation of the elective reset option (i.e. the proportion of policyholders expected to exercise the reset option given favourable investment performance). In developing the factor tables, favourable investment performance (i.e. prompting option exercise) was deemed to have occurred whenever the market value-to-guarantee value ratio equals or exceeds 115%. If the product offered an elective reset feature and the company had reliable experience regarding the rate of utilisation, the company should have interpolated between the corresponding no resets and with resets rows according to the proportion of business that exercised the reset option" [Canadian Institute of Actuaries, 2002].

Overall, they proposed that their approach should be used until the regulator and the industry is confident that the insurers have stochastic models and systems that are capable of adequately predicting the risk profile of the business. This approach was prescribed in all Canadian financial institutions who offer segregated funds. Therefore, given its high importance, it is noteworthy that among other assumptions and heuristics used, CIA still made no attempt to model optimal reset behaviour. Instead, they used a similar heuristic as the one adopted in their long term approach.

Having identified this consistent use of naive heuristics relative to the reset option on behalf of the regulating and advisory authorities of Canada, it is now of interest to see how practitioners have handled this issue.

2.3.6 Symposium on Stochastic Modeling on Segregated Funds

In 1999 CIA organised a symposium in order to address the shortage of research in the area of stochastic modeling for segregated fund investment guarantees. It is noteworthy that the vast majority of the authors, on the one hand, admitted the high importance of the reset feature on the valuation of the segregated funds, but on the other hand, chose to *ignore it for the sake of simplicity*. In the following subsections, four presentations of financial companies and practitioners which attempted to model the reset option are critically and succinctly presented.

RGA Financial defined and modeled what they call a "rational policyholder behaviour". In particular, they defined rational reset as one that occurs the moment the value of the guarantee to the policyholder after resetting is merely greater than the value of the guarantee by continuing the existing policy.

Manulife suggested the use of a simple set of rules in order to decide when to exercise a reset option: (i) resets only occur if the market value of the fund is at least 110% of the guaranteed value, (ii) the maximum percentage of all contracts at time t that resets is 30%.

Thompson Financial assumed that the reset option is utilised by 20% of the population per annum whenever the account value exceeds 120% of the guaranteed value. Interestingly, they noted that the assumption that a flat percentage of the population utilises the reset option is unrealistic. They then

argued that it is more likely that the utilisation will increase with the account value.

Lastly, FSA Insurances assumed that: 100% of policyholders reset if the fund is 10% above the guaranteed value. Overall, what becomes apparent from the four presentations examined is that in line with the regulatory and advisory authorities in Canada, practitioners use similarly naive heuristics in their attempt to model the reset decision. Therefore it is now of interest to review the academic literature on the reset option.

2.4 Research on the Reset Option Embedded in Segregated Funds

Gerber and Shiu [1999] examined the valuation of reset guarantees in the context of a mutual fund. In particular, they constructed a model "where the mutual fund prices moved in discrete jumps, and the reset feature of the guarantee considered of resetting the strike price back to its guaranteed amount immediately after each loss". However, the limitations of their model were that this reset was automatic without any decision being made by the policyholder.

Bilodeau [1997] examined a potential decision problem of segregated funds policyholders. In his paper he considers a case where a policyholder can choose on the maturity to either exercise the maturity guarantee and withdraw his investment or roll-over the guarantee for an extra time period. The author argued that what the policyholder should do at maturity, is to compare the

expected payoff by exercising the maturity guarantee with the expected value of renewing the maturity guarantee. However, the limitations of his model were that it does not take into account horizons lasting longer than two time periods and that it does allow for the maturity guarantee to be reset prior to the maturity date.

Armstrong [2001] attempted to address the trade-off faced by polyholders in terms of either resetting the maturity guarantee or allowing it to reach its maturity. What made his paper even more interesting was that he considered the problem from the policyholder's viewpoint. In particular, the author proposed "the use of a return threshold decision rule, such that whenever the return for a period exceeded the threshold, the policyholder would reset the guarantee". His results suggest that "while extreme strategies such as never resetting can lead to quite poor performance, many reasonable choices of return thresholds can lead to near-optimal performance even if the chosen threshold is not particularly close to the optimal value". However, the limitation of his model is that it is restricted to just two-period guarantees with a single intermediate decision point. As a matter of fact he highlights that "a more realistic model of segregated fund products would feature guarantees covering at least 10 periods and nine intermediate reset points, as well as more complex reset strategies".

Finally, Windcliff et al. [2001a,b,c] have done one of the most advanced

models relative to the valuation of segregated funds and the reset feature in particular. What makes their model of special importance to this thesis is that they have attempted to model the optimal reset behaviour of policyholders of segregated funds. In particular, they have computed an "optimal exercise boundary" for a sample segregated fund contract that allowed the investor 1 reset per annum over an infinite planning horizon. It is worthwhile highlighting an interesting finding of their model, namely that *"the location of the exercise boundary depends on the current maturity date of the contract. We can see that there is a trade-off between getting a higher guarantee level by resetting and deferring the maturity date of the contract by another 10 years"*. It is noteworthy that the optimal exercise boundary that they compute exhibits some jumps at the beginning of each policy year. As the authors suggest "this is attributed to the fact that the policyholder receives a new reset opportunity each year. Therefore near the end of a policy year, the investor may choose to exercise his reset option to lock-in even a relatively small gain, rather than losing the opportunity to reset the maturity guarantee altogether".

However, the limitation of this model is that, as Windcliff et al. [2001a,c] note, the computed optimal exercise boundary applies only to the initial contract sold to the policyholder. Once the policyholder resets the maturity guarantee the exercise boundary changes. Therefore it becomes apparent that in order to generate a comprehensive optimal reset strategy, the optimal

exercise boundaries for all of the segregated funds should be examined.

2.5 Discussion: Need to Model the Reset Option

Throughout sections 2.3.3, to 2.3.5 a wide range of models relative to the valuation of the segregated fund investment guarantees have been critically reviewed. The methods examined vary from the deterministic approach to the more elaborate and advanced CIAs long term approach. However, what all of the models seem to have in common is a consistent use of naive heuristics relative to the modeling of the reset decision. A side effect is that practitioners who are advised by the regulatory authorities which approach to use, make the exact same misleading and naive assumptions about the reset option (see section 2.3.6). Therefore, it becomes apparent that all the benefits that could accrue through the use of advanced stochastic modeling can be more than negated by the inherent naive assumption about the reset option.

In section 2.4 the academic literature on the modeling of the reset decision has been critically reviewed. The conclusion from this subsection is that there does not appear to be any published work which is advanced enough to deal with the complexities of the reset option faced by the policyholder of segregated funds.

Hence, there appears to be a clear and urgent need to have more sophisticated approaches which will model the reset option. In particular, it is

worthwhile to examine a segregated fund which allows the policyholder to reset the level of their maturity guarantee at least once every policy year. Also the maturity of the fund and the planning horizon should be long enough to take into account that segregated funds are primarily used as pension products. In chapter 3 an attempt is made to develop a model for the reset option with the aim to derive a comprehensive optimal reset strategy for the policyholder.

3 Modeling the Reset Option

3.1 Introduction

Motivated by the importance of the reset option embedded in segregated funds, the limitations of the relevant academic literature, as well as the inadequate approaches used by practitioners (as illustrated in chapter 2) an attempt is made to model the reset option of segregated funds.

The reset option on the maturity guarantee of segregated funds is formulated as a non-stationary finite horizon Markov Decision Process. The aim of this model is to depict the optimal exercise boundary (OEB) of the reset option. In particular, given the model parameters, the aim is to depict a threshold value such that if the value of the underlying asset exceeds it then it is optimal for the policyholder to reset his maturity guarantee. Otherwise, it is optimal for the policyholder to rollover his maturity guarantee.

The rest of the chapter is organised as follows: section 3.2 formulates the reset option as a Markov Decision Process and state the assumptions that underpin the formulation. Then, section 3.3 critically analyses ways in which the state space of the formulation can be reduced, thus making it more efficient. The revised formulation of the reset option is then provided in section 3.4. Section 3.5 succinctly analyses the structure of the model of segregated funds and highlights the main options and the corresponding decisions that

the policyholder faces at each stage and state of the model.

Further, in section 3.6 the formulation is converted into a flowchart through pseudo-coding. The intuitive advantage of pseudo-coding is that its architecture and rationale are not framed by the syntax of any particular programming language, thus rendering the structure of the model generalisable. Then, in sections 3.7 and 3.8 the pseudo-coding is "translated" into Fortran Code, where the commercially available Salford software is used to compile and run the code. The aim is to extract the values of the underlying asset for which the policyholder will be optimally exercising his reset option.

Moreover, sections 3.9 and 3.10 highlight the main results of the model as well as perform sensitivity analysis in order to examine the robustness of the model, gain insights into the behaviour of the segregated fund contract and make recommendations to both the policyholder and the issuer of the segregated funds. Lastly, section 3.11 concludes.

3.2 Formulating the reset option as a Markov Decision Process

In this section, the reset option on the maturity guarantee of segregated funds is modeled as a non-stationary finite horizon Markov decision process which is characterised by the following five elements:

Stage (denoted by t) which is the number of steps until the end of the planning

horizon

State Space (denoted by S^t) which is the set of possible states at stage t

Decision Space (denoted by D_i^t) for each $i \in S^t$, which is the set of decisions that can be taken in state i at stage t

Immediate Reward for each $i \in S^t$ and $d \in D_i^t$, $R_{i,d}^t$ = reward received when process is in state i at stage t and action d is chosen

State Transitions for $i \in S^t$, $d \in D_i^t$ and $j \in S^{t-1}$, $p_{i,j,d}^t$ = probability of process making a transition to state j at stage $t-1$ when process is in state i at stage t and action d is chosen

Lastly, let the maximum expected reward over the final t steps starting from state i at stage t, for each $i \in S^t$, be:

$$V_i^t = \max_{d \in D_i^t} \{R_{i,d}^t + \sum_{j \in S^{t-1}} p_{i,j,d}^t V_j^{t-1}\} \tag{1}$$

For further details on Markov decision processes see Puterman [1994]. The returns from the investment in a segregated fund are modeled using a discrete-time approximation of the Lognormal Model, namely the "Binomial Tree Method". For further details on the Binomial Tree Method see Hull [2006]. The Binomial Tree Method has been chosen as it models the underlying asset over time, as opposed to at a particular point in time and thus is able to handle a variety of conditions for which other models cannot easily be applied. For

example the reset option can be exercised before the maturity of the segregated fund. However, for such options under the Black-Scholes method there can be no analytical solution [Cox and Ross, 1976].

The assumptions underlying the model are the following. Firstly, the aim of the investor is to maximise the expected value of his initial investment (denoted by X) after a fixed time period (the planning horizon). Secondly, the investor can invest either in a bond with return equal to the risk-free rate of interest (denoted by r) or in a segregated fund. Thirdly, segregated funds allow the investor to reset the level of the maturity guarantee at pre-determined fixed points in time which are regularly spaced out in the planning horizon (reset decision points). Also, the maturity guarantee of the segregated fund states that at maturity, the investor will receive the maximum of a predetermined percentage (denoted by G) of the original investment and the prevailing value of the fund. Moreover, the value of the underlying asset is a random variable which is modeled by a discrete time approximation of the lognormal model, namely the binomial distribution. It can either change by $(1+u)\%$ or change by $(1+d)\%$ at each time period. The probability of changing by $(1+u)\%$ is p while the probability of changing by $(1+d)\%$ is $1-p$. The probability p is set in such a way that the expected return of the underlying asset is equal to the risk-free rate of interest, denoted by r ([Hull, 2006]). In particular, $(1+r) = (1+u) * p + (1+d) * (1-p)$. Further, there are q time periods in

a policy year. In total there are Nq periods until the maturity of a segregated fund while the policyholder's planning horizon has Tq time periods, where $T \geq N$. Lastly, the underlying asset does not pay any dividends and therefore there are no rewards during the planning horizon.

3.2.1 Formulation

Under the above assumptions the problem can be formulated as follows:

Stage (denoted by t) which is the number of periods until the end of the planning horizon, where $0 \leq t \leq Tq$.

State Space (denoted by S^t) which is the set of possible states at stage t. The defining characteristics of the possible states are the following.

The first state variable is the current value of the underlying asset (denoted by a). For the purposes of this thesis the underlying asset can be defined as whatever was bought by the initial investment (e.g. 100 ounces of gold, 500 shares of XYZ corporation etc.). This state variable is of the form $a = XG^k(1+u)^w(1+d)^m$, where $0 \leq w \leq Tq - t$, $0 \leq w + m \leq Tq - t$, and $0 \leq k \leq \frac{Tq-t}{Nq}$. The term $Tq - t$ in the boundaries of w and $w + m$ represents the number of time periods since the start of the planning horizon. Therefore, it is an upper bound on the times that the underlying asset price may have changed by $(1+u)\%$. The possible values of $w+m$ are $\leq Tq - t$ despite the fact that the total

number of times that the underlying asset price has changed in value by either $(1+u)\%$ or $(1+d)\%$ during the planning horizon is equal to $Tq-t$. The rationale behind this argument is that some of the times that the asset has changed by $(1+u)\%$ or $(1+d)\%$ may have been wiped out at the maturity of the segregated fund if the maturity guarantee has been applied. Further, the term $\frac{Tq-t}{Nq}$ in the boundary of k represents the maximum number of times that a segregated fund may have reached its maturity within the planning horizon. Thus it represents the maximum number of times that the maturity guarantee may have been applied.

The second state variable is the number of periods until the maturity of the current segregated fund contract. This is denoted by n and must satisfy the following conditions. Firstly, by the definition of n, $0 \leq n \leq Nq$. Secondly, $n \leq t$ in order to provide enough time for the fund to mature before the end of the planning horizon. Thirdly, $n \geq Nq-(Tq-t)$ in order to allow the initial investment in the fund to fall within the limits of the planning horizon. Lastly, $n = t - kq$ for some integer k, because opportunities to invest in a new segregated fund occur only once every q time periods.

The third, and final, state variable is the current value of the maturity guarantee, denoted by g, where $g \geq 0$. This is determined by the value of the initial investment in the segregated fund and is of the form $g =$

$XG^{k'}(1+u)^{w'}(1+d)^{m'}$, where $1 \leq k' \leq \frac{Tq-t-(Nq-n)}{Nq}+1$, $0 \leq w' \leq Tq-t-(Nq-n)$, and $0 \leq w'+m' \leq Tq-t-(Nq-n)$. The maturity guarantee of a segregated fund protects $G\%$ of the investment in the fund, so the value of the maturity guarantee is equal to G times the value invested in the fund. The term $Tq-t-(Nq-n)$ represents the stage when the investment in the current fund was made. Finally, $XG^{k'-1}(1+u)^{w'}(1+d)^{m'}$ represents the initial investment in the segregated fund and the boundaries of k', w' and $w'+m'$ follow using a similar argument as above.

Decision Space (denoted by $D^t_{a,n,g}$) which is the set of possible decisions that can be taken in state $[a,n,g]$ at stage t. There are four possible decisions that the policyholder can take. Firstly, initially or at the maturity of the segregated fund, to invest in a bond yielding the risk-free rate of interest abbreviated as **risk free**. Secondly, initially or at the maturity of the segregated fund, to re-invest in a new segregated fund contract abbreviated as **reinvest**. Thirdly, during the lifetime of the fund, to rollover the maturity guarantee by not exercising the reset option abbreviated as **rollover**. The last possible action, which is available only at regularly spaced decision points during the lifetime of the segregated fund, is to reset the maturity guarantee abbreviated as **reset**.

In particular, if the time until the end of the planning horizon is less than the duration of a segregated fund, and the current segregated fund

has reached its maturity (i.e. $t < Nq$ and $n = 0$) then $D^t_{a,0,g} = \{$risk free$\}$. Moreover, if the time until the end of the planning horizon is greater than or equal to the duration of a segregated fund, and the current segregated fund has reached its maturity (i.e. $t \geq Nq$ and $n = 0$) then $D^t_{a,0,g} = \{$reinvest, risk free$\}$. Further, if the time until the end of the planning horizon is more than the duration of the segregated fund, and it is a reset decision point then $D^t_{a,n,g} = \{$reset, rollover$\}$. Formally, state $[a, n, g]$, at stage t can be defined as a decision point if $(n, t) \in E$ where E=$\{(n, t) : 0 < n < Nq$, $t \geq Nq$ and $n = kq$ for some $k \in Z\}$. Lastly, if the current segregated fund has not reached its maturity (i.e. $n > 0$), and it is not a reset decision point (i.e. $(n, t) \notin E$) then $D^t_{a,n,g} = \{$rollover$\}$.

State transitions define the probability of the process making a transition from one state to another depending on the action which has been taken. In particular, in state $[a, 0, g]$ at stage t the action risk free determines the final value of the investment by multiplying the current value of the investment by $(1 + r)^t$. Further, in state $[a, 0, g]$ at stage t the action reinvest causes an instantaneous transition to state $[\max(a, g), Nq, G \max(a, g)]$. Also, in state $[a, n, g]$ at stage t the action rollover causes a transition to state $[a(1 + u), n - 1, g]$ at stage $t - 1$ with probability p or state $[a(1+d), n-1, g]$ at stage $t-1$ with probability $1-p$. Lastly, in state $[a, n, g]$ at stage t the action reset causes an instantaneous

transition to state $[a, Nq, Ga]$.

Overall, the aim of the policyholder is to maximise the expected return from the investment at the end of the planning horizon, i.e. after Tq time periods. Let $V^t_{a,n,g}$ be the maximum expected return from the investment when there are t periods until the end of the planning horizon and the investment is currently in a fund with n time periods to go to maturity, a current value of a and maturity guarantee of g. Therefore, the aim is to find $V^{Tq}_{X,0,0}$ where:

$$v^t_{a,n,g} = \begin{cases} \max(a,g)(1+r)^t \text{ if } t < Nq \text{ and } n = 0 \\ \\ \max(\max(a,g)(1+r)^t, V^t_{\max(a,g),Nq,G\max(a,g)}) \\ \text{ if } t \geq Nq \text{ and } n = 0 \\ \\ pV^{t-1}_{a(1+u),n-1,g} + (1-p)V^{t-1}_{a(1+d),n-1,g} \\ \text{ if } n > 0 \text{ and } (n,t) \notin E \\ \\ \max(pV^{t-1}_{a(1+u),n-1,g} + (1-p)V^{t-1}_{a(1+d),n-1,g}, V^t_{a,Nq,Ga}) \\ \text{ if } (n,t) \in E \end{cases} \tag{2}$$

Note that the initial state is defined to be the maturity of a fund with value X and maturity guarantee 0 to allow for an initial choice between investment in risk free and investment in a fund within the general framework of the model.

3.3 Model Refinement: Reducing the State Space

Of crucial importance has been the observation that in the calculation of the value of the portfolio at any point in time it is not necessary to know the value

of the underlying asset, only how this value has changed since the start of the relevant segregated fund. This result is formally stated as Proposition 1 below. A consequence of this is that state a can now be altered to represent the current value of the portfolio relative to the value of the portfolio at the start of the relevant segregated fund rather than the current value of the portfolio. Further, state g can be eliminated altogether as the only information that needs to be stored is the percentage of the original investment that is guaranteed upon maturity (which is assumed to be constant throughout the planning horizon) rather than the value of the maturity guarantee, which depends on the initial investment in the relevant segregated fund, and thus could be different for each fund.

The repercussions of this observation are significant as the number of states within the planning horizon is reduced greatly. For indicative purposes, if $T = 29$, $N = 10$, and $q = 1000$ then the number of states within the planning horizon under the original formulation is in the magnitude of approximately 1750 trillions whereas with the aforementioned simplification it is in the magnitude of approximately 1.5 billions. This amounts to a reduction of 99.99% which enables models with a significantly increased frequency of changes to the underlying asset price to be analysed. This not only offers more stable results but also improves the convergence of the binomial model of the value of the underlying asset to the lognormal distribution.

3.3.1 Proposition 1

$$\text{For } x > 0,\ V^t_{ax,n,gx} = xV^t_{a,n,g} \tag{3}$$

Proof

The proof is by induction on t. For $t = 0$ the only possible value of n is 0.

$$\begin{aligned} V^0_{ax,0,gx} &= \max\{ax, gx\} \\ &= x\max\{a, g\} \\ &= xV^0_{a,0,g} \end{aligned}$$

Therefore, equation 3 has been proven to hold true for $t = 0$. Assume that equation 3 holds true for $t - 1$, so the aim is to prove that equation 3 holds true for t. There are 3 cases to consider:

Case 1 For $0 < t < Nq$.

For this case, $0 \leq n \leq t$ and $(n, t) \notin E$. Due to differences in the decision spaces there are two cases that need to be considered separately.

Firstly, if $n = 0$ then the only decision is risk free, so:

$$\begin{aligned} V^t_{ax,0,gx} &= \max\{ax, gx\}(1+r)^t \\ &= x\max\{a, g\}(1+r)^t \\ &= xV^t_{a,0,g} \end{aligned}$$

Secondly, if $n > 0$ then the only decision is rollover, so:

$$\begin{aligned} V^t_{ax,n,gx} &= pV^{t-1}_{a(1+u)x,n-1,gx} + (1-p)V^{t-1}_{a(1+d)x,n-1,gx} \\ &= xpV^{t-1}_{a(1+u),n-1,g} + x(1-p)V^{t-1}_{a(1+d),n-1,g} \\ &\quad \text{by the inductive hypothesis} \\ &= xV^t_{a,n,g} \end{aligned}$$

Therefore, equation 3 has been proven to hold true for $0 < t < Nq$ provided equation 3 holds true for $t-1$.

Case 2 For $Nq \leq t \leq Tq$ and t is a multiple of q.

For this case, the fund can only have reached maturity ($n = 0$) if the number of periods since the start of the planning horizon ($Tq - t$) is at least Nq. Otherwise there must be at least $Nq - (Tq - t)$ steps until the maturity of the fund. Hence $\max(0, Nq - (Tq - t)) \leq n \leq Nq$. Further, since t is a multiple of q, it follows that n is also a multiple of q. Due to differences in the decision spaces there are three cases that need to be considered separately.

Firstly if $n = Nq$ then the only decision is rollover, so:

$$\begin{aligned} V^t_{ax,Nq,gx} &= pV^{t-1}_{a(1+u)x,Nq-1,gx} + (1-p)V^{t-1}_{a(1+d)x,Nq-1,gx} \\ &= xpV^{t-1}_{a(1+u),Nq-1,g} + x(1-p)V^{t-1}_{a(1+d),Nq-1,g} \\ &\quad \text{by the inductive hypothesis} \end{aligned}$$

$$= xV^t_{a,Nq,g} \tag{4}$$

Secondly if $0 < n < Nq$ then $(n,t) \in E$ and the investor may choose between rollover and reset, so:

$$\begin{aligned}
V^t_{ax,n,gx} &= \max\{pV^{t-1}_{a(1+u)x,n-1,gx} + (1-p)V^{t-1}_{a(1+d)x,n-1,gx}, V^t_{ax,Nq,Gax}\} \\
&= \max\{xpV^{t-1}_{a(1+u),n-1,g} + x(1-p)V^{t-1}_{a(1+d),n-1,g}, V^t_{ax,Nq,gx}\} \\
&\quad \text{by the inductive hypothesis} \\
&= \max\{xpV^{t-1}_{a(1+u),n-1,g} + x(1-p)V^{t-1}_{a(1+d),n-1,g}, xV^t_{a,Nq,g}\} \\
&\quad \text{by equation 4} \\
&= x\max\{pV^{t-1}_{a(1+u),n-1,g} + (1-p)V^{t-1}_{a(1+d),n-1,g}, V^t_{a,Nq,g}\} \\
&= xV^t_{a,n,g}
\end{aligned}$$

Finally, if $n = 0$ then the investor may choose between risk free and re-invest, so:

$$\begin{aligned}
V^t_{ax,0,gx} &= \max\{\max(ax,gx)(1+r)^t, V^t_{\max(ax,gx),Nq,G\max(ax,gx)}\} \\
&= \max\{x\max(a,g)(1+r)^t, V^t_{x\max(a,g),Nq,xG\max(a,g)}\} \\
&= \max\{x\max(a,g)(1+r)^t, xV^t_{\max(a,g),Nq,G\max(a,g)}\} \text{ by equation 4} \\
&= x\max\{\max(a,g)(1+r)^t, V^t_{\max(a,g),Nq,G\max(a,g)}\} \\
&= xV^t_{a,0,g}
\end{aligned}$$

Therefore, equation 3 has been proven to hold true for t a multiple of q satisfying $Nq \leq t \leq Tq$ provided equation 3 holds true for $t-1$.

Case 3 For $Nq \leq t \leq Tq$ and t not a multiple of q.

For this case, $\max(0, Nq-(Tq-t)) \leq n \leq Nq$ and n is not a multiple of q. Hence $n \neq 0$, $n \neq Nq$ and $(n,t) \notin E$. It follows that the only decision is rollover and:

$$\begin{aligned} V^t_{ax,n,gx} &= pV^{t-1}_{a(1+u)x,n-1,gx} + (1-p)V^{t-1}_{a(1+d)x,n-1,gx} \\ &= xpV^{t-1}_{a(1+u),n-1,g} + x(1-p)V^{t-1}_{a(1+d),n-1,g} \\ &\quad \text{by the inductive hypothesis} \\ &= xV^t_{a,n,g} \end{aligned}$$

Therefore, equation 3 has been proven to hold true for t not a multiple of q satisfying $Nq \leq t \leq Tq$ provided equation 3 holds true for $t-1$.

Combining cases 1, 2 and 3, it has been proven that $V^t_{n,ax,gx} = xV^t_{n,a,g}$ for all values of t by induction.

3.4 Revised Formulation

Applying equation 3 in the optimality equation of the original formulation leads to the following:

$$v^t_{a,n,g} = \begin{cases} \max(a,g)(1+r)^t \text{ if } t < Nq \text{ and } n = 0 \\ \max(a,g)\max((1+r)^t, V^t_{1,Nq,G}) \text{ if } t \geq Nq \text{ and } n = 0 \\ pV^{t-1}_{a(1+u),n-1,g} + (1-p)V^{t-1}_{a(1+d),n-1,g} \text{ if } n > 0 \text{ and } (n,t) \notin E \\ \max(pV^{t-1}_{a(1+u),n-1,g} + (1-p)V^{t-1}_{a(1+d),n-1,g}, aV^t_{1,Nq,G}) \text{ if } (n,t) \in E \end{cases} \tag{5}$$

With this simplification, it is only necessary to consider a unit investment in a new segregated fund in order to evaluate the reset and reinvest decisions. Hence, it is only necessary to consider segregated fund contracts in which the value of the maturity guarantee is G. These observations lead to the following revised formulation of the problem.

Stage (denoted by t) which is the number of periods until the end of the planning horizon, where $0 \leq t \leq Tq$.

State Space (denoted by S^t) which is the set of possible states at stage t. The defining characteristics of the possible states are the following.

The first state variable is the current value of the underlying asset relative to its value at the time of the investment in the current segregated fund (denoted by a) which is of the form $a = (1+u)^i(1+d)^{Nq-n-i}$, where $0 \leq i \leq Nq - n$. The term $Nq - n$ in the boundary of i represents the number of time periods since the start of the current segregated fund. Therefore it represents the maximum number of times that the underlying asset may have changed by $(1+u)\%$. If i represents the number of periods since the start of the current segregated fund that the underlying asset has changed by $(1+u)\%$, then $Nq - n - i$ must be the number of times since the start of the current fund that the underlying

asset has changed by $(1+d)\%$.

The second state variable is the number of periods until the maturity of the current segregated fund contract. This variable is denoted by n and must satisfy the same conditions as set out in the original formulation.

Decision Space (denoted by $D^t_{a,n}$) which is the set of possible decisions that can be taken in state $[a,n]$ at stage t. The decision space in the original formulation depends only on the time until the end of the planning horizon and the time until the maturity of the fund, not the current values of the underlying asset and the maturity guarantee. Hence, the decision space remains essentially the same and is formally stated as:

$$D^t_{a,n} = \begin{cases} \{\text{risk free}\} \text{ if } t < Nq \text{ and } n = 0 \\ \{\text{reinvest, risk free}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ \{\text{rollover}\} \text{ if } n > 0 \text{ and } (n,t) \notin E \\ \{\text{rollover, reset}\} \text{ if } (n,t) \in E \end{cases}$$

State transitions define the probability of the process making a transition from one state to another depending on the action which has been taken. In particular: in state $[a,0]$ at stage t the action risk free determines the final value of the investment by multiplying the current value of the investment by $(1+r)^t$. Further, in state $[a,0]$ at stage t the action reinvest causes an instantaneous transition to state $[1,Nq]$. Also, in state $[a,n]$ at stage t the action rollover causes a transition to state $[a(1+u),n-1]$ at stage $t-1$ with probability p or state $[a(1+d),n-1]$ at stage $t-1$

with probability $1-p$. Lastly, in state $[a,n]$ at stage t the action reset causes an instantaneous transition to state $[1,Nq]$.

The aim of the policyholder is to maximise the expected return from the investment at the end of the planning horizon, i.e. after Tq time periods. Let $V^t_{a,n}$ be the maximum expected return from the investment at the end of the planning horizon, after t time periods, when investment is currently in a fund with n time periods to go to maturity, a current relative value of a and a maturity guarantee of G. Therefore, the aim is to find $X \max\{(1+r)^{Tq}, V^{Tq}_{1,Nq}\}$ where:

$$V^t_{a,n} = \begin{cases} \max(a,G)(1+r)^t \text{ if } t < Nq \text{ and } n = 0 \\ \max(a,G)\max\{(1+r)^t, V^t_{1,Nq}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ pV^{t-1}_{a(1+u),n-1} + (1-p)V^{t-1}_{a(1+d),n-1} \text{ if } n > 0 \text{ and } (n,t) \notin E \\ \max(pV^{t-1}_{a(1+u),n-1} + (1-p)V^{t-1}_{a(1+d),n-1}, aV^t_{1,Nq}) \text{ if } (n,t) \in E \end{cases} \tag{6}$$

It seems intuitive that if it is optimal to reset the maturity guarantee when the value of the underlying asset at a particular decision point is a, then it would be optimal to reset the maturity guarantee when the value of the underlying asset is greater than a. Proposition 2 proves that this property holds for the formulation of the problem derived in this thesis.

3.4.1 Proposition 2

If $a' > a > 0$ and the optimal action in state $[a, n]$ at stage t is to reset the level of the maturity guarantee, then it is optimal to reset the maturity guarantee in state $[a', n]$ at stage t.

Proof

Assume that it is optimal to reset the level of the maturity guarantee in state $[a, n]$ at stage t. It follows that $aV^t_{1,Nq} > pV^{t-1}_{a(1+u),n-1} + (1-p)V^{t-1}_{a(1+d),n-1}$. The aim is to prove that it is also optimal to reset the level of the maturity guarantee in state $[a', n]$ at stage t. Hence it is required to prove that:

$a'V^t_{1,Nq} > pV^{t-1}_{a'(1+u),n-1} + (1-p)V^{t-1}_{a'(1+d),n-1}$

which is equivalent to:

$aV^t_{1,Nq} > \frac{a}{a'}(pV^{t-1}_{a'(1+u),n} + (1-p)V^{t-1}_{a'(1+d),n})$ since $\frac{a}{a'} > 0$.

Hence it is sufficient to show that:

$$pV^t_{a(1+u),n} + (1-p)V^t_{a(1+d),n} \geq \frac{a}{a'}(pV^t_{a'(1+u),n} + (1-p)V^t_{a'(1+d),n}) \tag{7}$$

The proof is by induction on t. For $t = 0$ the only possible value of n is 0.

$$\begin{aligned}
&\frac{a}{a'}\{pV^0_{a'(1+u),0} + (1-p)V^0_{a'(1+d),0}\} \\
&= \frac{a}{a'}\{p\max(G, a'(1+u)) + (1-p)\max(G, a'(1+d))\} \\
&\quad \text{by the optimality equation} \\
&= p\max(\frac{a}{a'}G, a(1+u)) + (1-p)\max(\frac{a}{a'}G, a(1+d)) \text{ since} \frac{a}{a'} > 0
\end{aligned}$$

$$\leq \quad p\max(G, a(1+u)) + (1-p)\max(G, a(1+d)) \text{ since}\frac{a}{a'} < 1$$

$$= \quad pV^0_{a(1+u),0} + (1-p)V^0_{a(1+d),0}$$

Thus equation 7 has been proven to hold true for $t = 0$.

Assume that equation 7 holds true for $t-1$. Thus, the aim is to prove that equation 7 holds true for t. There are 3 cases to consider:

Case 1 For $0 < t < Nq$ then:

For this case, $0 \leq n \leq t$ and $(n, t) \notin E$. Due to differences in the decision spaces there are two cases that need to be considered separately.

Firstly, if $n = 0$ then the only decision is risk free. Note that $V^t_{a,0} = \max(a, G)C(t)$ where $C(t)$ depends only on t and constant parameters.

$$\frac{a}{a'}\{pV^t_{a'(1+u),0} + (1-p)V^t_{a'(1+d),0}\}$$

$$= \quad \frac{a}{a'}\{p\max(G, a'(1+u))C(t) + (1-p)\max(G, a'(1+d))C(t)\}$$

by the optimality equation

The result follows as in the case of $t = 0$ and $n = 0$ above.

Secondly, if $n > 0$ then the only decision is rollover, so:

$$pV^t_{a(1+u),n} + (1-p)V^t_{a(1+d),n}$$

$$= \quad p\{pV^{t-1}_{a(1+u)^2,n-1} + (1-p)V^{t-1}_{a(1+u)(1+d),n-1}\}$$

$$+(1-p)\{pV^{t-1}_{a(1+u)(1+d),n-1} + (1-p)V^{t-1}_{a(1+d)^2,n-1}\}$$

by the optimality equation

$$
\begin{aligned}
&\geq p\frac{a(1+u)}{a'(1+u)}\{pV^{t-1}_{a'(1+u)^2,n-1}+(1-p)V^{t-1}_{a'(1+u)(1+d),n-1}\}\\
&\quad +(1-p)\frac{a(1+d)}{a'(1+d)}\{pV^{t-1}_{a'(1+u)(1+d),n-1}+(1-p)V^{t-1}_{a'(1+d)^2,n-1}\}\\
&\quad \text{by the inductive hypothesis}\\
&= \frac{a}{a'}\{pV^{t}_{a'(1+u),n}+(1-p)V^{t}_{a'(1+d),n}\} \text{ by the optimality equation}
\end{aligned}
$$

Therefore, equation 7 has been proven to hold true for $0 < t < Nq$ provided equation 7 holds true for $t-1$.

Case 2 For $Nq \leq t < Tq$ and t a multiple of q then:

For this case, $\max(0, Nq-(Tq-t)) \leq n \leq Nq$ and n is a multiple of q. Due to differences in the decision spaces there are three cases that need to be considered separately.

Firstly, if $n = Nq$ then the only decision is rollover and the result follows as in the case of $0 < t < Nq$ and $n > 0$.

Secondly, if $n = 0$ then the decision can either be reinvest or risk free. Note that $V^t_{a,0} = \max(a, G)C(t)$ where $C(t)$ depends only on t and constant parameters.

$$
\begin{aligned}
&\frac{a}{a'}\{pV^{t}_{a'(1+u),0}+(1-p)V^{t}_{a'(1+d),0}\}\\
&= \frac{a}{a'}\{p\max(G, a'(1+u))C(t)+(1-p)\max(G, a'(1+d))C(t)\}\\
&\quad \text{by the optimality equation}
\end{aligned}
$$

The result follows as in the case of $t = 0$ and $n = 0$.

Finally, if $0 < n < Nq$ then $(n,t) \in E$ so the decision can either be reset or rollover:

$$
\begin{aligned}
& pV^t_{a(1+u),n} + (1-p)V^t_{a(1+d),n} \\
&= p\max\{pV^{t-1}_{a(1+u)^2,n-1} + (1-p)V^{t-1}_{a(1+u)(1+d),n-1}, a(1+u)V^t_{1,Nq}\} \\
&\quad +(1-p)\max\{pV^{t-1}_{a(1+u)(1+d),n-1} + (1-p)V^{t-1}_{a(1+d)^2,n-1}, a(1+d)V^t_{1,Nq}\} \\
&\quad \text{by the optimality equation} \\
&\geq p\max\{\frac{a(1+u)}{a'(1+u)}\{pV^{t-1}_{a'(1+u)^2,n-1} + (1-p)V^{t-1}_{a'(1+u)(1+d),n-1}\}, \\
&\quad a(1+u)V^t_{1,Nq}\} + (1-p)\max\{\frac{a(1+d)}{a'(1+d)}\{pV^{t-1}_{a'(1+u)(1+d),n-1} \\
&\quad +(1-p)V^{t-1}_{a'(1+d)^2,n-1}\}, a(1+d)V^t_{1,Nq}\} \\
&\quad \text{by the inductive hypothesis} \\
&\geq \frac{a}{a'}\{p\max\{pV^{t-1}_{a'(1+u)^2,n-1} + (1-p)V^{t-1}_{a'(1+u)(1+d),n-1}, \\
&\quad a(1+u)V^t_{1,Nq}\} + (1-p)\max\{pV^{t-1}_{a'(1+u)(1+d),n-1} \\
&\quad +(1-p)V^{t-1}_{a'(1+d)^2,n-1}, a(1+d)V^t_{1,Nq}\}\} \\
&= \frac{a}{a'}\{pV^t_{a'(1+u),n} + (1-p)V^t_{a'(1+d),n}\} \text{ by the optimality equation}
\end{aligned}
$$

Therefore, equation 7 has been proven to hold true for t a multiple of q satisfying $Nq \leq t \leq Tq$ provided equation 7 holds true for $t-1$.

Case 3 For $Nq \leq t \leq Tq$ and t not a multiple of q then:

For this case, $\max(0, Nq - (Tq - t)) \leq n \leq Nq$ and n is not a multiple of q. Hence, $n \neq 0$ $n \neq Nq$ and $(n,t) \notin E$. It follows that the only

decision is rollover. The proof is the same as $0 < t < Nq$ and $n > 0$. Therefore, equation 7 has been proven to hold true for t not a multiple of q satisfying $Nq \leq t \leq Tq$ provided equation 7 holds true for $t-1$.

Combining cases 1, 2 and 3, equation 7 has been proven to hold true for all values of t by induction.

As a consequence of Proposition 2, it is possible to define an optimal reset strategy by specifying a set of threshold values, one for each of the possible segregated funds at each reset decision point in the planning horizon. The decision to reset the maturity guarantee of a segregated fund at a decision point is optimal if and only if the current value of the underlying asset (relative to its value at the initial investment in the current fund) is greater than the threshold for that fund at that decision point. So we can formally define an optimal exercise boundary (OEB) for a segregated fund to be the sequence of up to $N-1$ threshold values corresponding to the decision points for the fund.

Further, a transformation can be applied such that $V^t_{a,n}$ is defined as equal to $(1+r)^t * V'^t_{a,n}$, and as with the earlier formulation, it is the maximum expected return from the investment at the end of the planning horizon, after t time periods, when investment is currently in a fund with n time periods to go to maturity, a current relative value of a and a maturity guarantee of G. Therefore, the aim is to find $X \max\{(1+r)^{Tq}, (1+r)^{Tq} * V'^{Tq}_{1,Nq}\}$ which is equivalent to $X * (1+r)^{Tq} \max\{1, V'^{Tq}_{1,Nq}\}$. With this transformation one can

show that the decision to reinvest is better than risk free initially and at the maturity of a fund, provided reinvest is feasible at that time. This is formally proven in Proposition 3.

3.4.2 Proposition 3

If $G \geq 1$ and $r \geq 0$ then the decision to reinvest is better than risk free initially and at the maturity of a fund, provided reinvest is feasible at that time.

Proof

If $G \geq 1$, the value of the investment in a segregated fund is never less than the amount invested. If $r \geq 0$, then $u > 0$ and so it is possible for the value of the investment in the fund to increase. Hence, the expected value of reinvestment in the fund is greater than the amount invested ($V'^{t}_{1,Nq} > 1$).

3.5 A sample model of segregated funds

Having examined the formulation of the mathematical model it is now of interest to better understand the structure of the binomial model of segregated funds (see figure 1) and to highlight the main options and the corresponding decisions that the policyholder faces at each stage and state of the model. In order to achieve this the following simple segregated fund is examined: the time to maturity is $N = 3$ years, the planning horizon is $T = 5$ years, the price

of the underlying asset fluctuates every six months and the policyholder has the option to reset his maturity guarantee every year (i.e. $q = 2$).

In terms of notation there are 11 possible values for the stage ranging from 0 to 10 and 7 possible values for n ranging from 0 to 6 (i.e. in increments of six months). Also, given the range of values of n, the possible values of the underlying asset relative to its value at the time of investment are numbered in order of increasing value from 0 to $Nq - n$ where i at stage n corresponds to a change in asset value of $a_{i,n} = (1+u)^i(1+d)^{Nq-n-i}$ since the initial investment in the fund.

Given all possible combinations of t, n, and i, at stage 10, there is only 1 possible state, the state [0,6]. At stage 10 and state [0,6] the policyholder will always decide to rollover, as there is no incentive to reset at this stage since the maturity guarantee of his segregated fund will remain unchanged (in other words as no year has elapsed, there is no possible gain in the fund value that the policyholder could lock-in through resetting his maturity guarantee). Therefore there are only two outcomes: the segregated fund will either move to stage 9 and state [1,5] (in which case the value of the fund has changed by $u\%$) or move to stage 9 and state [0,5] (in which case the value of the fund has changed by $d\%$). The expected return to the policyholder at this state is equal to the sum of: $V^{10}_{0,6} = pV^9_{1,5} + (1-p)V^9_{0,5}$. The same rational applies in stage 9 and states [5,0:1] as there is no reset option available the segregated fund can

only rollover.

At stage 8 there are 4 possible states, the state [0:2,4] and [0,6]. At stage 8 and state [0,4] the policyholder may either decide to rollover or to reset the maturity guarantee of his segregated fund to the current asset value ($a_{0,4}$). If the policyholder decides to rollover then his expected return will be equal to: $V^8_{0,4} = pV^7_{1,3} + (1-p)V^7_{0,3}$. If on the other hand the policyholder decides to reset then his expected return will be equal to: $V^8_{0,4} = a_{0,4}V^8_{0,6}$, in essence the policyholder will be investing, in a new segregated fund starting at stage 8, a lump sum of money equal to the $a_{0,4}$. Therefore the expected return to the policyholder is equal to: $V^8_{0,4} = \max(pV^7_{1,3} + (1-p)V^7_{0,3}, a_{0,4}V^8_{0,6})$. The same rational applies at stage 8 and states [1:2,4].

At stage 8 and state [0,6] the policyholder will always decide to rollover, as there is no incentive to reset at this stage since the maturity guarantee of his segregated fund will remain unchanged (the same rationale as in stage 10 and state [0,6] applies). Therefore there are only two outcomes: the segregated fund will either move to stage 7 and state [1,5] (in which case the value of the fund has changed by $u\%$) or move to stage 7 and state [0,5] (in which case the value of the fund has changed by $d\%$). The expected return to the policyholder at this state is equal to: $V^8_{0,6} = pV^7_{1,5} + (1-p)V^7_{0,5}$.

The process continues to evolve in this way until in stage 4 the first segregated fund examined (see first tree from the left in figure 1) reaches its

maturity. Therefore, at stage 4 and state [0,0] the expected return to the policyholder is the maximum of the current asset value $a_{0,0}$ and the guarantee value G. As there is no reset option available the policyholder has to invest $\max(a_{0,0}, G)$ in a bond yielding the risk-free interest rate for a number of periods equal to the time remaining to reach the end of the planning horizon. Therefore, his expected return will be equal to: $V^4_{0,0} = \max(a_{0,0}, G)(1+r)^4$. The same rational applies to the stage 4 and states [1:6,0].

The process continues to evolve in a similar way until stage 0 which is the last stage of the model where the last segregated fund reaches its maturity. The material difference in this stage is that the policyholder does not have to make any decision as we have reached then end of the planning horizon. In stage 0 and state [0,0] the expected return to the policyholder is equal to the maximum of the current asset value $a_{0,0}$ and the guarantee value G. The same rational applies to stage 0 and states [1:6,0].

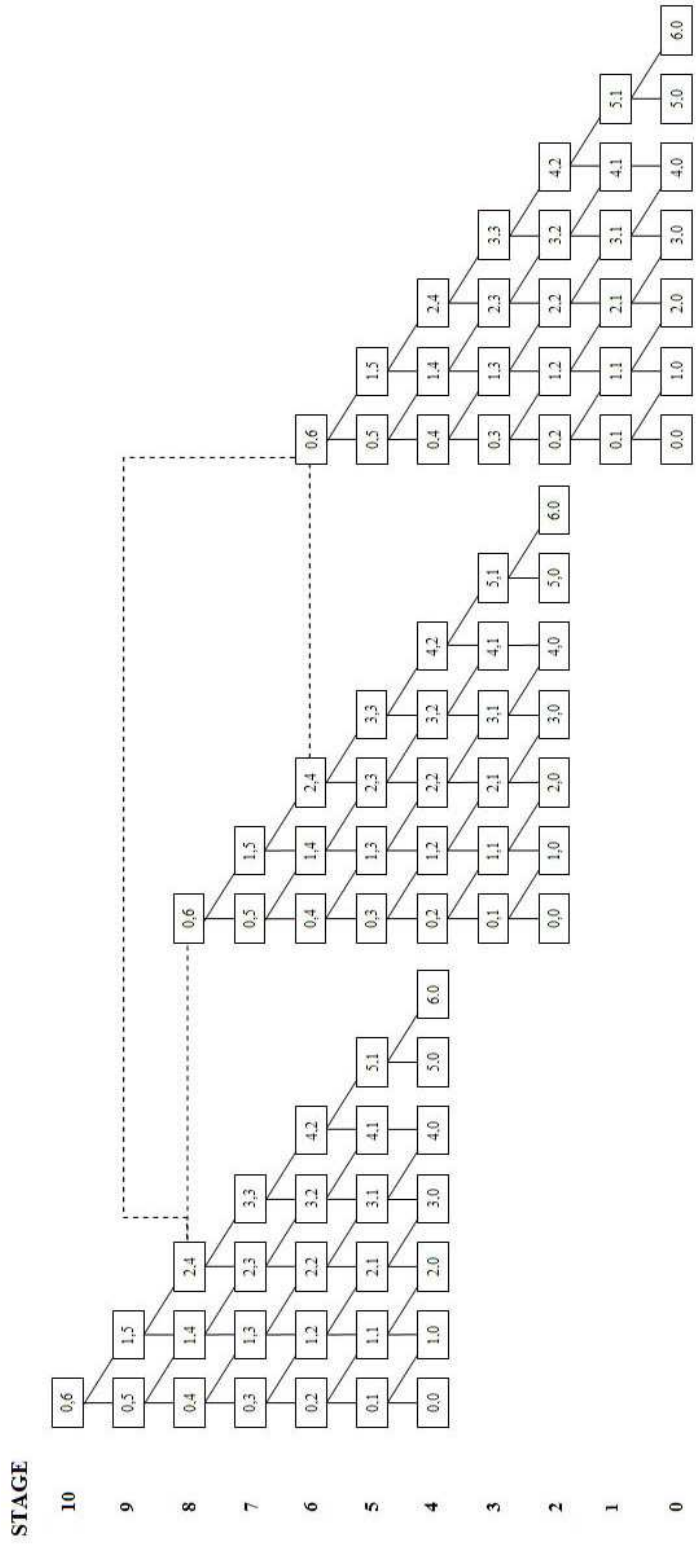

Figure 1: ***Layout of Sample Segregated Fund Contract***

3.6 Flowchart Analysis

Having examined the formulation of the mathematical model as well as a sample segregated fund contract, it is now of interest to examine the way in which the mathematical model can be converted into a flowchart through pseudo-coding. The intuitive advantage of pseudo-coding is that its architecture and rationale are not framed by the syntax of any particular programming language. The pseudo-coding used is illustrated in figure 2.

Relative to notation it should be highlighted that the terms appearing in superscript format (referring to the stage) in the mathematical formulation are absent in the notation used in the flowchart. The reason for that is that while the code makes all the necessary computations for each and every period in time, only a few are actually going to be needed at a later time period. In particular, as the code progresses values which had been calculated in previous steps and are no longer needed are overwritten by the values of new computations. In essence, while all calculations are performed only a snapshot in time is ever kept in memory. The purpose of that is to decrease the memory requirements and, thus, increase the efficiency of the code. A further note on notation relates to the terms appearing on the flowchart after the underscore which are equivalent to the terms appearing on the formulation in subscript format.

The model of a general segregated fund analysed in this section will

be called the Single Regime Model, abbreviated as SRM. The first process (**Parameters**) of the SRM sets the values of the model's parameters. In particular the values of a, U, D, Y, G, T, N, q are set and then the value of p can be calculated. In the flowchart the Y is equivalent to the $1 + r$ in the mathematical formulation. A similar convention applies for U and D. The second process (**Asset's price distribution**) calculates the possible values of the underlying asset price from the start until the maturity of a fund.

The third process (**Initialisation**) sets $t = 0$ as the Markov Decision Process which underpins the model uses a backward iteration algorithm. Further, the process sets the initialising values for V_{res}, N_L, and N_U. V_{res} is used to store the value of investing on a segregated fund at that point. Further, N_U and N_L are pointers to the set of funds which store the possible values that the portfolio can have during its lifetime. At any one time there is a set of funds each with a different number of years to maturity. N_L is the pointer to the first fund while N_U is the pointer to the last fund. At the end of the planning horizon there can only be one fund the $(N-1)^{th}$ fund.

The next step is the check of the first condition, whether it is possible for a fund to mature at this step. If the first condition is true then the fourth process (**Fund Maturing**) defines $V_{N_U,i}$ to represent a fund which has reached its maturity. If there is no fund maturing at this point, the pointers N_U and N_L are updated to reflect that there is one fewer fund. Subsequently, the fifth

process (**Go backward in time**) moves the model backward in time. Then, the sixth process (**Rollover 1 period**) calculates the value of the segregated fund if it is rolled over for 1 time period. By checking the second condition the fifth process is repeated for q time periods until the next decision point is reached. Each time the rollover process is applied to a fund, the number of values that need to be calculated falls by 1. This explains why $t - t_0$ is subtracted from the boundary of i each time. During the q repetitions of the rollover process of fund n, the number of values falls from $q(n+1)$ to qn.

Then, the model checks the third condition, whether it has reached the end of the planning horizon. If the third condition is false then the model checks the fourth condition, whether the policyholder has the option to reset his maturity guarantee. If the fourth condition is false then the seventh process (**Adjust pointers**) adjusts the pointers to allow for an extra fund which will be active. However, if the fourth condition is true it is a reset decision point. The eighth process (**New reset point**) notes the values of investing in a segregated fund at that point (V_{res}). This value is then used in the ninth process (**Reset vs. rollover**) which compares the value of resetting the maturity guarantee with the value of rolling it over. Then the first condition is repeated along with the subsequent steps until the end of the planning horizon is reached.

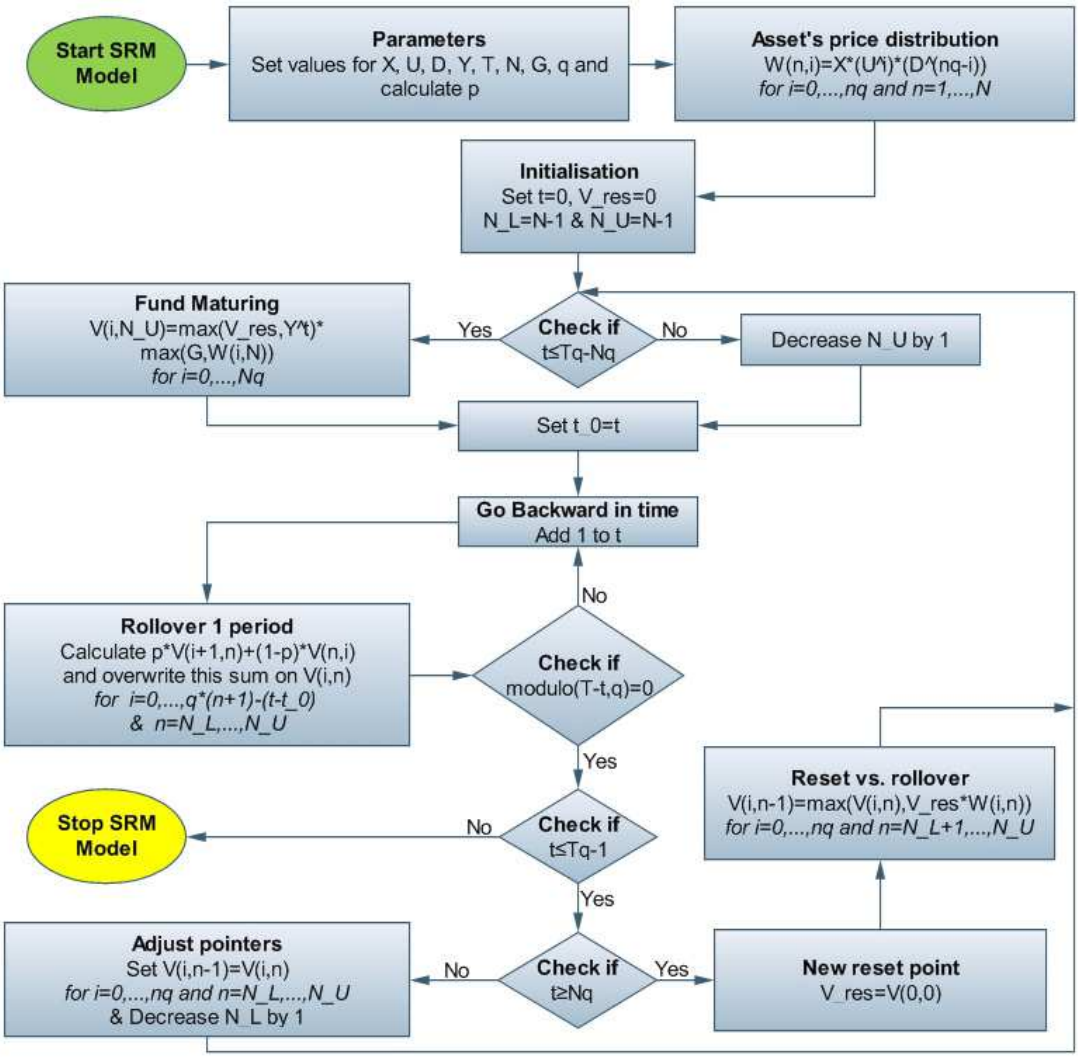

Figure 2: ***Flowchart of SRM model***

3.7 Depicting an Optimal Exercise Boundary of the Reset Option

The aim of the model described in the previous sections is to depict an optimal reset strategy to the policyholder. In particular, given the model parameters the aim is to depict the maximum value of the underlying asset for which the policyholder would choose to rollover his maturity guarantee as well as the minimum value for which the policyholder would choose to reset his maturity guarantee. Taking these two boundaries into consideration the ultimate aim is to create an optimal reset strategy for the policyholder i.e. to determine the value of the underlying asset required to trigger the reset of the maturity guarantee.

3.7.1 Characteristics of the standard segregated fund examined

There can be a plethora of segregated funds, with different numbers of reset options, various levels of maturity guarantee and investors can have different planning horizons. Therefore, it has been decided to use the characteristics of a standard segregated fund as defined by the CIA Canadian Institute of Actuaries [2002]. In particular, the segregated fund under examination offers one reset option at the end of each policy year. A policy year can be defined as the set of 365 days which start either when the segregated fund is issued or when the previous policy year ended. Further, a standard segregated fund has duration of 10 years and offers a maturity guarantee of 100% of the original sum

invested. Regarding the planning horizon of the policyholder, a 29 year period was chosen because CIA considers that the average policyholder purchases a segregated fund contract at the age of 50 and the last time that an investor is allowed to reset his maturity guarantee is before his 70th birthday [Miles and Miles, 2000]. Therefore, taking into account that the standard time to maturity of a segregated fund is 10 years, if the investor exercises his last available reset option at the age of 69, the maximum total planning horizon that should be examined is 29 years.

The next issue to consider is which values of u, d and r to use in the model. It has been decided to use a set of values of u, d, and r that satisfy the criteria for the return from the underlying asset recommended by the CIA based on the stock market returns of the Toronto Stock Exchange total return index from 1956 to 1999 [of Actuaries, 2001]. Based on the conditions set by CIA it was derived that these values are $u = 19\%$ per annum, $d = -28\%$ per annum, and $r = 11\%$ per annum. The last issue to consider is the number of periods in a year (i.e. q). This is issue is important as it inherently determines the number of times that the value of the underlying asset is allowed to fluctuate, assuming that at every time period the underlying asset changes by the equivalent of either u or d. It has been decided to allow the value of the underlying asset to fluctuate 1000 times within a year. This value of q has been chosen because the maximum expected return from the original investment does not change

significantly when q is increased from this level. Higher values of q increase the computational complexity of the model, but have no significant impact on the results. This is illustrated in figure 3.

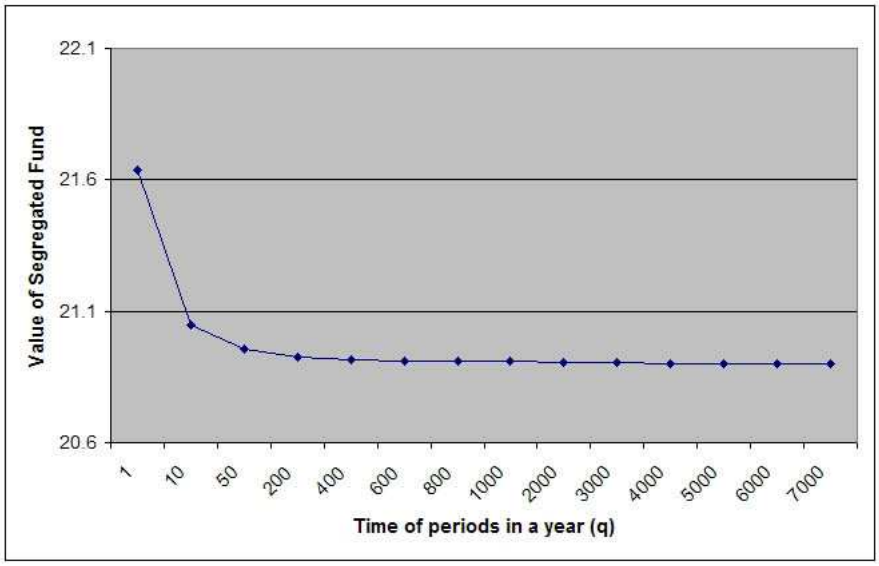

Figure 3: ***Stabilisation of segregated fund's value as q increases***

3.7.2 Optimal Exercise Boundaries

Applying the formulation of section 3.4 and taking into consideration the standard segregated fund of section 3.5, the Optimal Exercise Boundaries of the reset option can be depicted. In particular, as illustrated on figure 4 there are 2 boundaries: the higher one represents the minimum value of the underlying asset for which it is optimal for the policyholder to reset his maturity guarantee whereas the lower one represents the maximum value for which it is optimal for the policyholder to rollover his maturity guarantee. The following figure

illustrates a possible example of Optimal Exercise Boundaries (OEB) given the values of u, d, r, and q chosen in section 3.7.1.

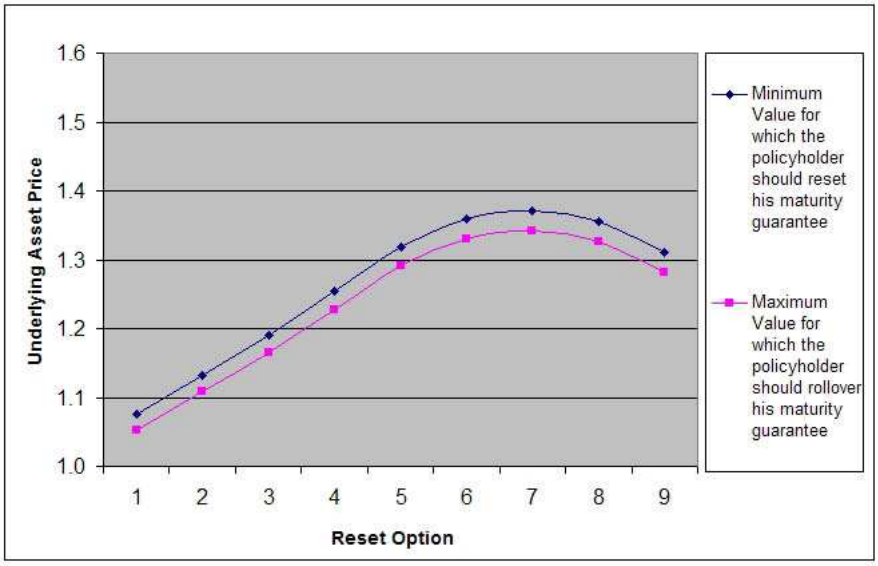

Figure 4: ***Optimal Exercise Boundaries of segregated fund***

It should be highlighted that figure 4 illustrates the OEB for the first segregated fund issued in the planning horizon where the time to retirement is $t = 29$ years and the time to maturity of the current fund is $n = 10$ years. As section 3.10 will illustrate the OEBs for subsequent segregated funds are not necessarily the same.

Examining figure 4, what becomes apparent is that there is a considerable gap between the two boundaries, i.e. the minimum value of the underlying asset for which it is optimal for the policyholder to reset his maturity guarantee and the maximum value for which it is optimal for the policyholder to rollover his maturity guarantee. This gap is equal to $(\frac{1+u}{1+d}\%)$.

The existence of such a gap decreases the precision of a potential optimal reset strategy. It is noteworthy that the gap between the two boundaries depends on the coefficient of variation of the model (thus, on the selection of u, d, and r) and most importantly on the frequency of the segregated fund's stock price fluctuation (i.e. q): the higher the frequency the smaller the gap. However, it is reasonable to assume that regardless of q there will always be a gap between the two boundaries.

Therefore, the aim from the model is now to depict a threshold value such that if the asset value exceeds the threshold value then it will be optimal for the policyholder to reset his maturity guarantee, otherwise, it will be optimal for the policyholder to rollover his maturity guarantee. In order to achieve that, the gap between the two boundaries has to be eliminated.

3.7.3 Eliminating the gap of the two boundaries

In an attempt to decrease this gap, a new model was used which created a hypothetical case. For each pair of points of the OEBs it calculated the midpoint and run the original model to determine whether the policyholder would choose to reset or rollover his maturity guarantee. Then, if the decision on the midpoint was to reset the maturity guarantee the minimum reset value would be replaced with the midpoint value.

Likewise, if the decision was to rollover the maturity guarantee the maximum rollover value would be replaced with the midpoint value. This

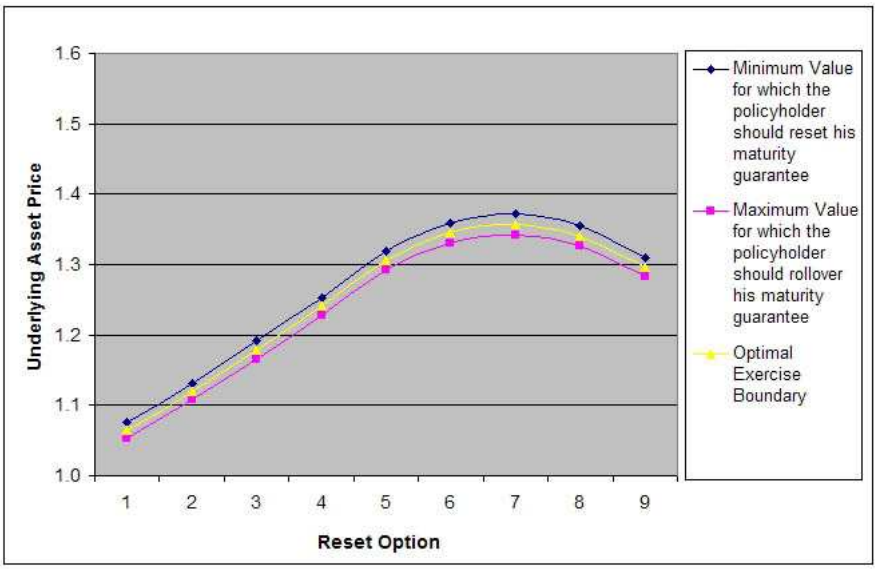

Figure 5: ***Eliminating the gap between the OEBs***

procedure would continue until the gap between the maximum rollover value and the minimum reset value was negligible (set as < 0.00001). Again, this procedure would be repeated for all the reset options (nine in the standard segregated fund) until the gap between the two boundaries was effectively eliminated.

In this way a unique OEB can be depicted. Figure 5 illustrates the unique OEB in relation to the previously derived set of boundaries. The unique OEB is lot more efficient in depicting the optimal reset strategy for the policyholder. In particular, for each of the nine reset decision points a threshold value has been computed such that if the value of the underlying asset exceeds the threshold then it will be optimal for the investor to reset his maturity guarantee; otherwise it will be optimal to rollover the investment.

It should be highlighted that a wide range of different sets of values for u, d, and r were tested, and it was observed that all the resulting OEBs (with increased precision) followed the same pattern: for the first 8 decision points the policyholder requires a progressively larger increase in the value of the underlying asset in order to optimally reset the level of his maturity guarantee. The rationale behind this argument is that if the policyholder resets at the first decision point the maturity will be extended by 1 year. In contrast if he resets in the eighth decision point the maturity will be extended by 8 years. Therefore as the "time penalty" increases, the return that the policyholder requires in order to choose to exercise his reset option increases. At the last decision point the increase in the value of the underlying asset that the investor requires in order to optimally reset is lower compared to the eighth decision point. The rationale behind this argument is that if the investor does not exercise his reset at that point it will expire worthless.

Overall, figure 5 illustrates that the OEB is a function of the time remaining until the maturity of the fund, a feature which is altogether neglected in the heuristic prescribed by the CIA. Thus, figure 5 can be thought of as a close approximation of what the generic OEB should look like.

3.8 Parameters of the model

3.8.1 Determining the Level of Asset Price Fluctuation

Chapter 2 highlighted that the binomial model provides a discrete approximation to the continuous process underlying the Black-Scholes model. In particular, the binomial model converges to the Black-Scholes formula as the number of binomial calculation steps increases. Therefore, a critical issue to be addressed is how fine the discretisation should be. In the context of the current model the issue is how often should the segregated fund's stock price be allowed to fluctuate within a time period (e.g. a policy year). The trade-off is that the finer the discretization the higher the amount of time required to run the model on a desktop PC. Figure 6 illustrates this point.

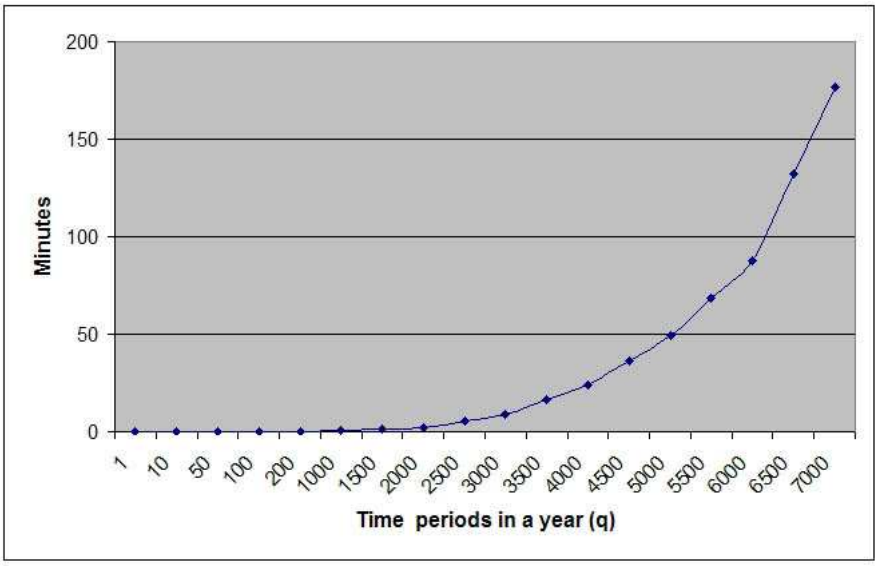

Figure 6: ***Time required to run the model***

It should be noted that the non-linear relationship between the time required and the value of q is probably due to the memory requirement. If the computer used had more memory, then the flat portion of the graph would be likely to extend further, but eventually one would expect to see the same behaviour as q increases.

For the model under consideration the higher the value of q the more precise the model becomes and, thus, the closer it gets to converging to the lognormal distribution. Therefore, there is not an optimal value of q as such, as it appears to be an unbounded problem and, thus, it tends to infinity.

Hence, the aim is to find a "technically optimal" q for which the trade-off between extra computational effort and increased precision is taken into account. In order to decide what the technically optimal q is, two criteria have been set: (i) the stability of the total value of segregated fund contract ($V_{1,Nq}^{Tq}$) and (ii) the consistency of the pattern of the OEB, as q increases.

3.8.2 Comparability of Results

In order to decide what the technically optimal q is, the model should be run for different values of q and the stability of $V_{1,Nq}^{Tq}$ as well as the consistency of the pattern of the OEB should be monitored. Therefore, it becomes apparent that there is a need to make sure that the results across different values of q are comparable. In order to achieve that it has been decided that regardless of q the mean and variance of the annual return from the underlying asset, as well

as the probability p of the various models should be constant to five decimal places. Thus, as the value of q is increased appropriate values of u, d and r have to be chosen so as to keep the basic model's characteristics constant.

A technical difficulty with this issue has been the calculation of the binomial distribution for very large values of q. It should be noted that the implementation of the model does not require the explicit calculation of the binomial probabilities, it is only the calculation of the mean and variance that requires these. Commercially available software such as Microsoft Excel when calculating the binomial distribution require the calculation of some very large numbers which as the value of q increases eventually exceed their limit thus rendering the function "undefined". In order to overcome this limitation a customised code was written which enables the calculation of the binomial distribution for practically any value of q. Overall, the comparability of the model's results has been ensured for different values of q.

3.8.3 Values of u, d, and r

The next issue to consider was for which values of u, d, and r to run the model. This issue arises because these values ultimately set the coefficient of variation of the model which affects both the consistency of the pattern of the OEB as well as the stability of the $V^{Tq}_{1,Nq}$. Relative to this issue CIA has set some conditions that the model of the underlying asset must satisfy in order to fit the stock market returns of the Toronto Stock Exchange (TSE) over a 30 year

period [of Actuaries, 2001]. Based on these conditions values for u, d, and r can be derived.

It has been decided to adopt these values as one possible scenario (scenario 1 in the following table) but to also create five further scenarios to cover a reasonable range of values of coefficients of variation. For the determination of these values, the risk-free rate of interest has been assumed to be constant to the Bank of England's interest rate (which at the time of the analysis was 4.5%). In terms of the u and d values, it has been assumed that a reasonable range of the variance of the underlying asset price should be from 5% to 60% as suggested in Hull [2006]. Overall, the six scenarios which were chosen to be run are summarised in the following table. It should be noted that these values apply to $q = 1$, in other words they are annual rates.

Scenario	u	d	r	p	σ^2	σ	$\frac{\sigma}{\mu}$
1	19.0%	-28.0%	11.0%	0.82979	0.03120	0.17664	0.15914
2	7.0%	-6.0%	4.5%	0.80769	0.00263	0.05123	0.04902
3	14.0%	-19.5%	4.5%	0.71642	0.02280	0.15100	0.14450
4	20.0%	-36.0%	4.5%	0.72321	0.06278	0.25055	0.23976
5	45.4%	-45.0%	4.5%	0.54757	0.20250	0.45000	0.43062
6	65.0%	-55.0%	4.5%	0.49583	0.36000	0.60000	0.57416

Table 2: ***Basic Characteristics of the 6 scenarios examined***

3.8.4 Criterion 1: Stability of the total value of segregated fund

The purpose of this section is to monitor the stability of $V_{1,Nq}^{Tq}$ as q increases and to try to deduce what the technically optimal value of q is for the different scenarios. The values of $V_{1,Nq}^{Tq}$ for the different scenarios across different values of q are highlighted in figure 7.

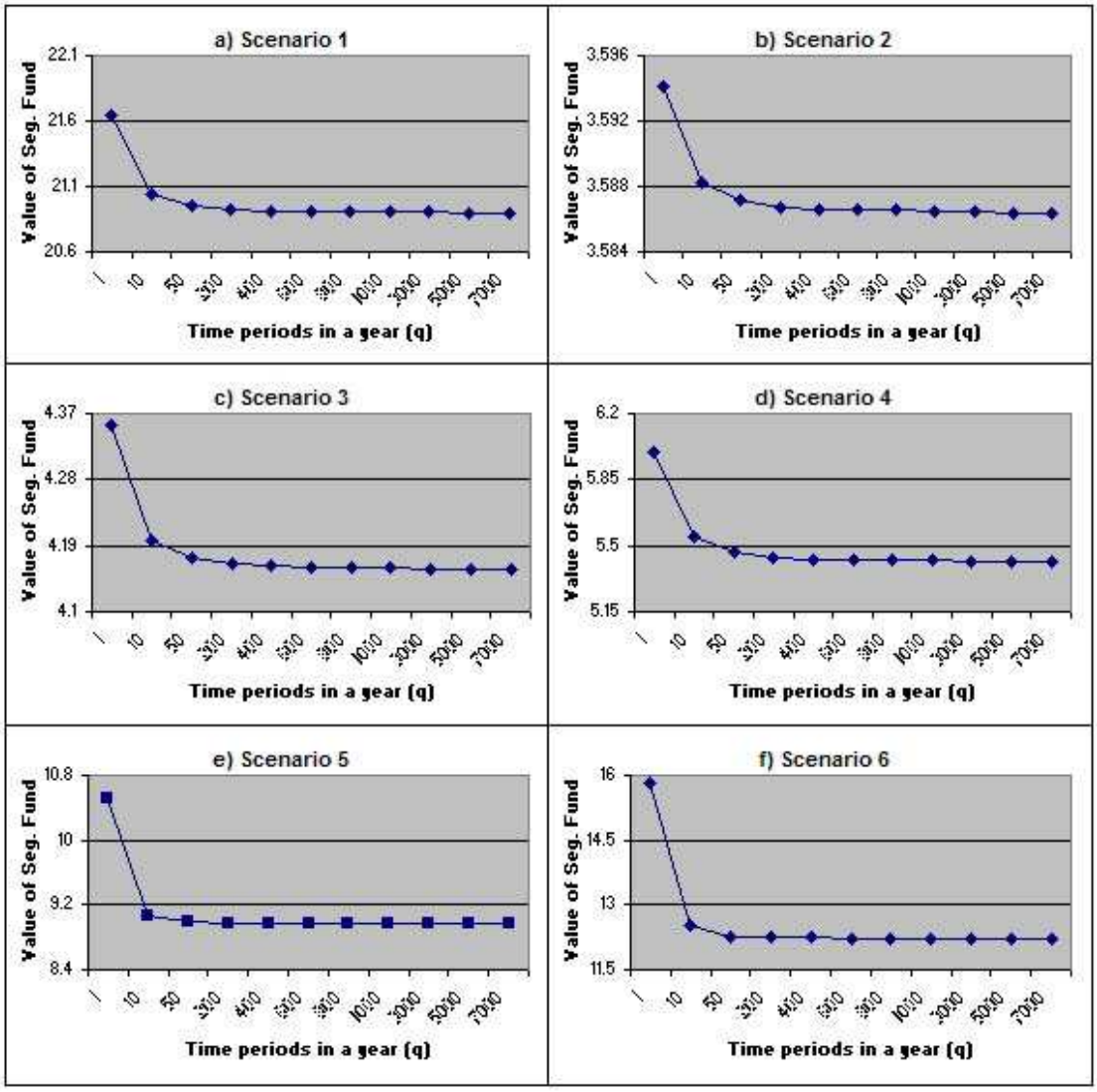

Figure 7: ***Stabilisation of*** $V_{1,Nq}^{Tq}$

Overall what becomes apparent from the criterion on the stability of $V_{1,Nq}^{Tq}$ is that it is not easy to decide what the value of the "technically" optimal q should be as for different scenarios it is different (ranging from 200 to 1000). Overall, the lower the coefficient of variation of the scenario the smaller the level of q required to approach stabilisation of $V_{1,Nq}^{Tq}$.

However, in order to ensure the comparability of the results of the model across scenarios it has been decided to use the same value of a "technically" optimal q for all scenarios, the value of which will be decided in conjunction with the results from the examination of criterion 2.

3.8.5 Criterion 2: Consistency of the pattern of the OEB

The purpose of this section is to fluctuate the level of q and monitor the value for which the pattern of the OEB stabilises. This experiment is performed in turn for all six scenarios. Figures 8 to 13 illustrate the results. Figure 8 illustrates that for scenario 1 the pattern of OEB appears to stabilise in $q = 600$.

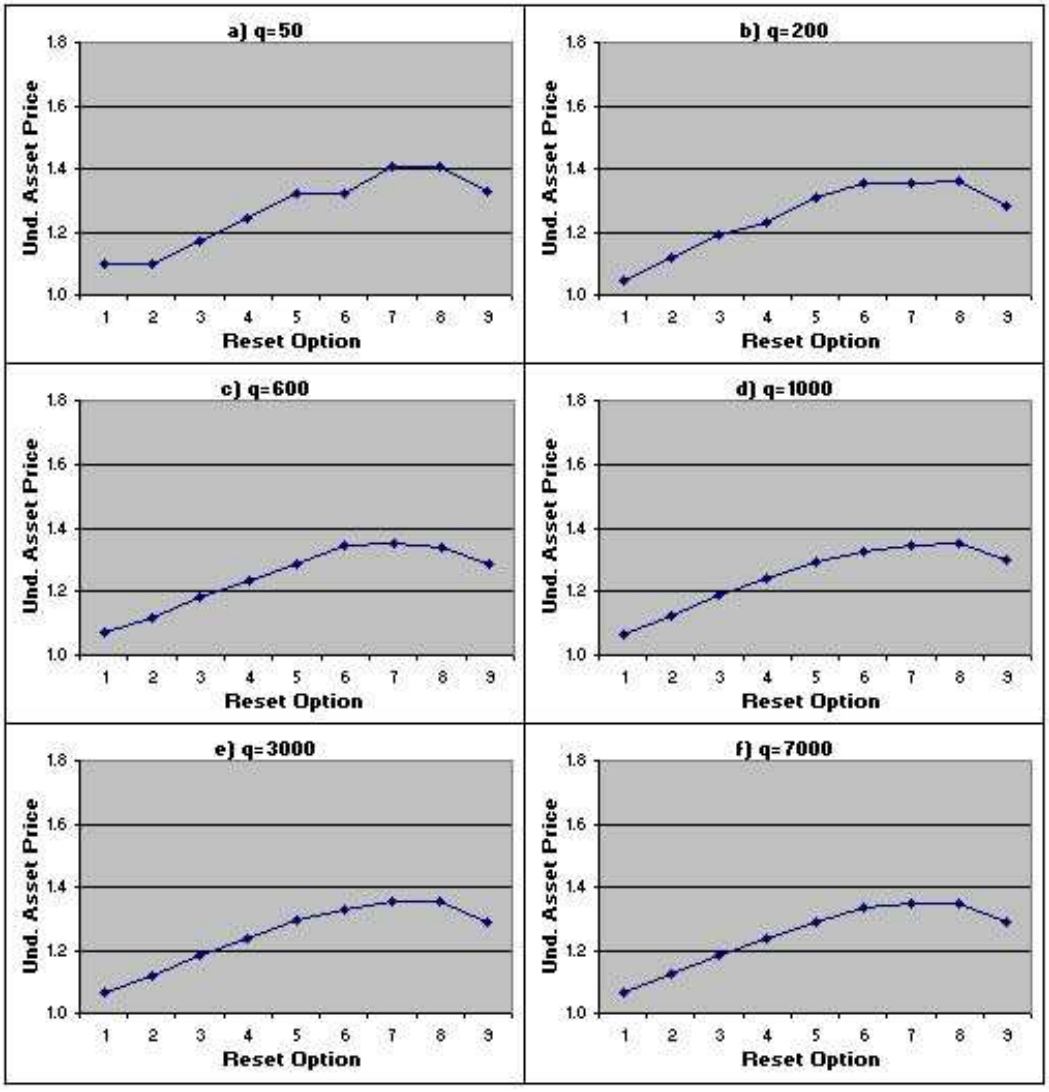

Figure 8: ***Scenario 1 Stabilisation of OEB***

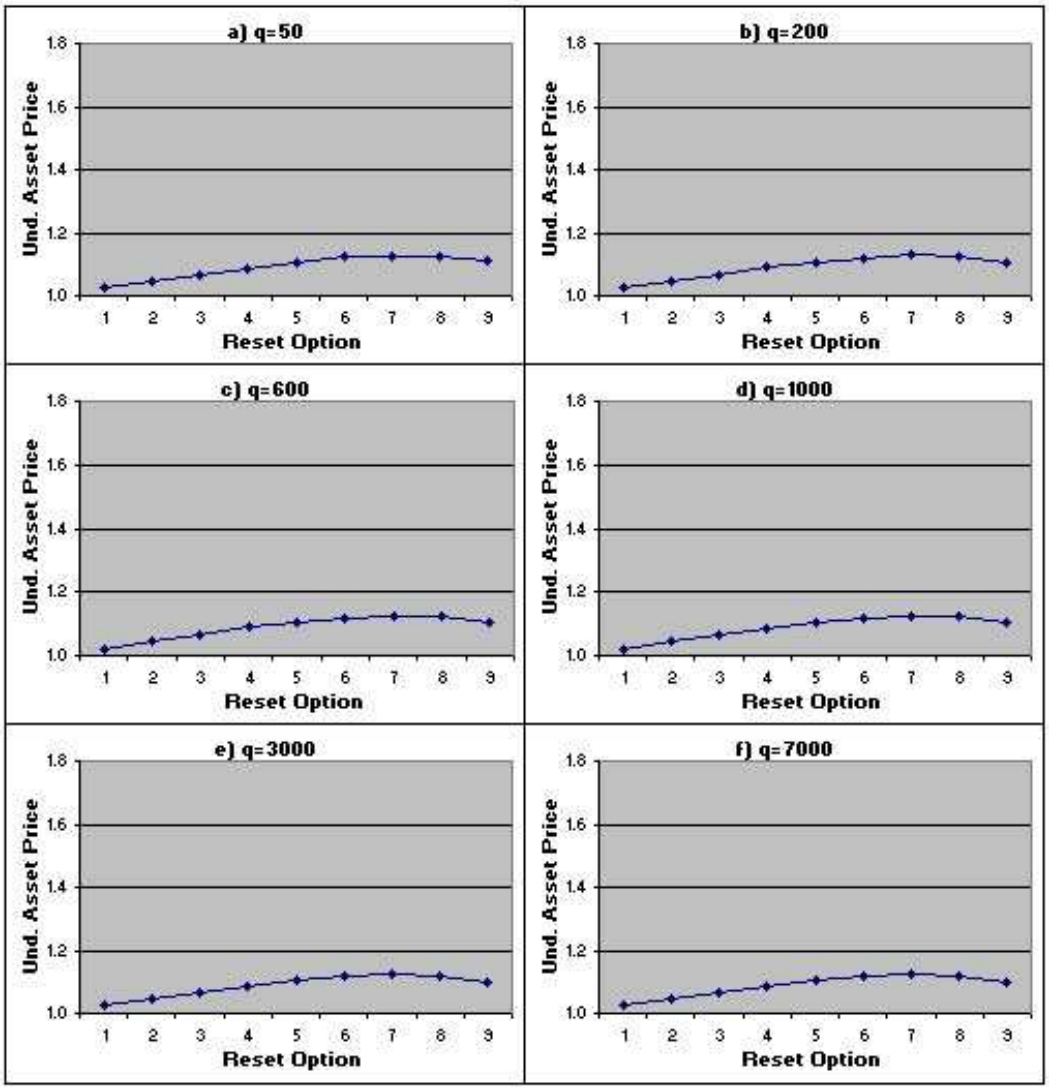

Figure 9: ***Scenario 2 Stabilisation of OEB***

Figure 9 illustrates that for scenario 2 the pattern of OEB appears to be stable in $q = 50$.

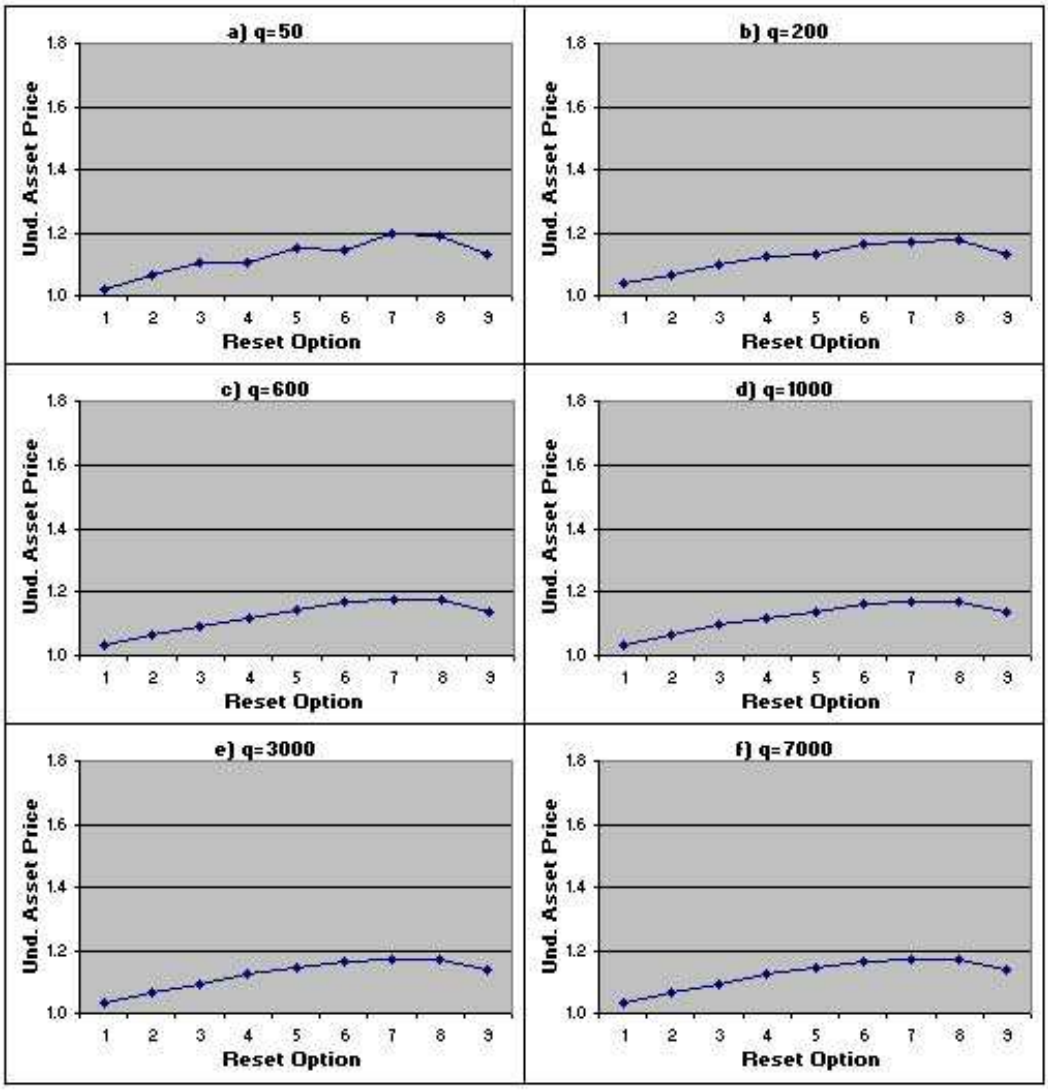

Figure 10: ***Scenario 3 Stabilisation of OEB***

Figure 10 illustrates that for scenario 3 the pattern of OEB appears to stabilise in $q = 600$.

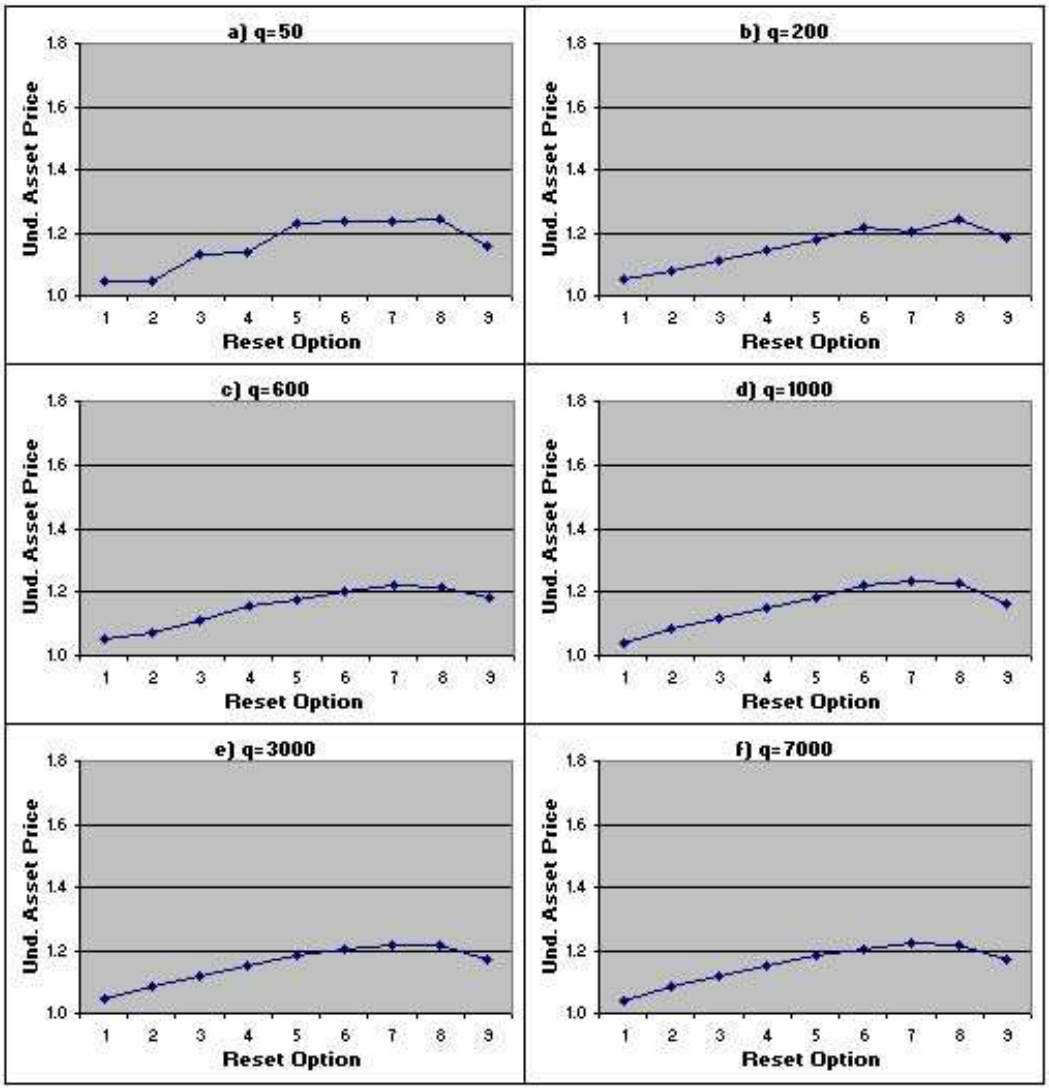

Figure 11: ***Scenario 4 Stabilisation of OEB***

Figure 11 illustrates that for scenario 4 the pattern of OEB appears to stabilise in $q = 1000$.

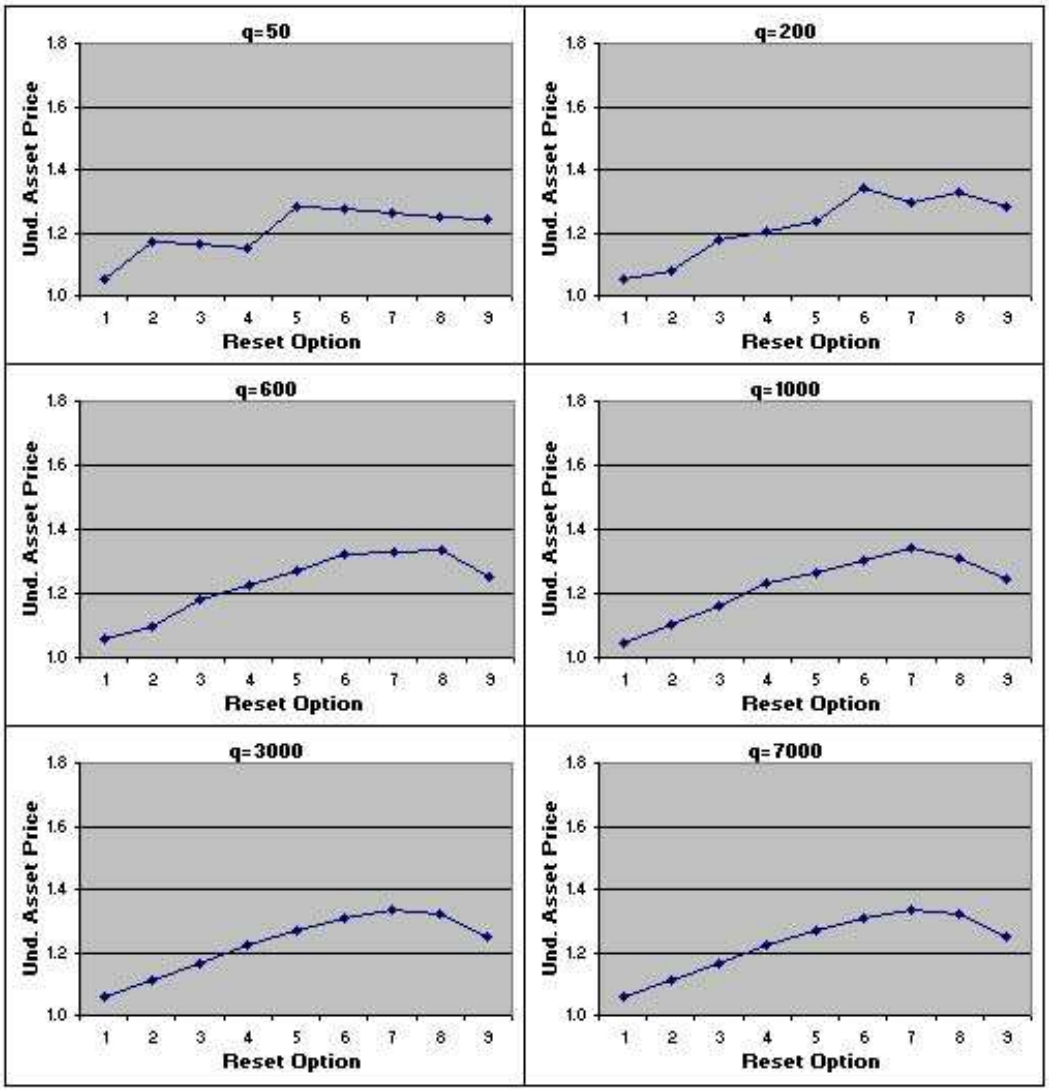

Figure 12: ***Scenario 5 Stabilisation of OEB***

Figure 12 illustrates that for scenario 5 the pattern of OEB appears to stabilise in $q = 1000$.

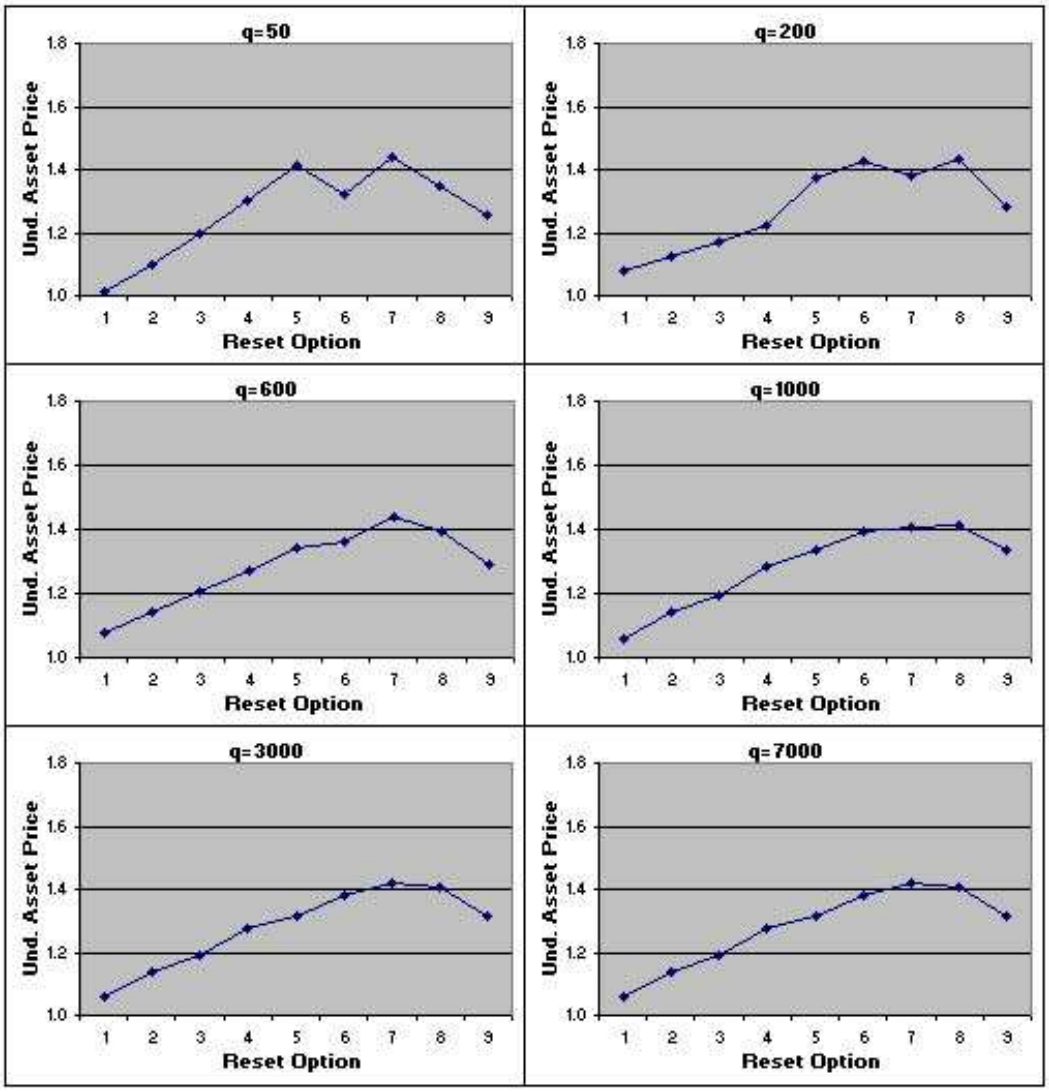

Figure 13: ***Scenario 6 Stabilisation of OEB***

Figure 13 illustrates that for scenario 6 the pattern of OEB appears to stabilise in $q = 1000$.

In figures 8 to 13 it can be observed that for small values of q the pattern of the OEB is distorted. The reason for the distortion is clearly the crudeness of the model due to the large time interval between the fluctuations of the value of the underlying asset. However, as q increases the pattern seems to be remarkably consistent regardless of the market conditions assumed.

3.8.6 Choice of technically optimal q

As it was observed in both criteria 1 and 2, it is not easy to determine what the value of the technically optimal q should be as for different scenarios it appears to be different. However, in order to ensure the comparability of the model's results across scenarios it has been decided to use the smallest value of q which stabilises all scenarios. Overall, $q = 1000$ has been selected.

This value ensures that the results from the model will not be distorted by crudeness as well as that the binomial model converges to a great extent to the lognormal distribution. At the same time with $q = 1000$ the model needs approximately 30 seconds to run on a desktop PC and, therefore, it enables an extensive sensitivity analysis to be undertaken. The results of the sensitivity analysis are presented in the following sections.

3.9 Single Regime Model's Main Results

The main finding of the Single Regime Model (SRM), as illustrated in figure 14, is that as the segregated fund approaches its maturity, a proportionately larger percentage increase in the value of the underlying asset will be necessary to trigger an optimal reset of the segregated fund's maturity guarantee. The

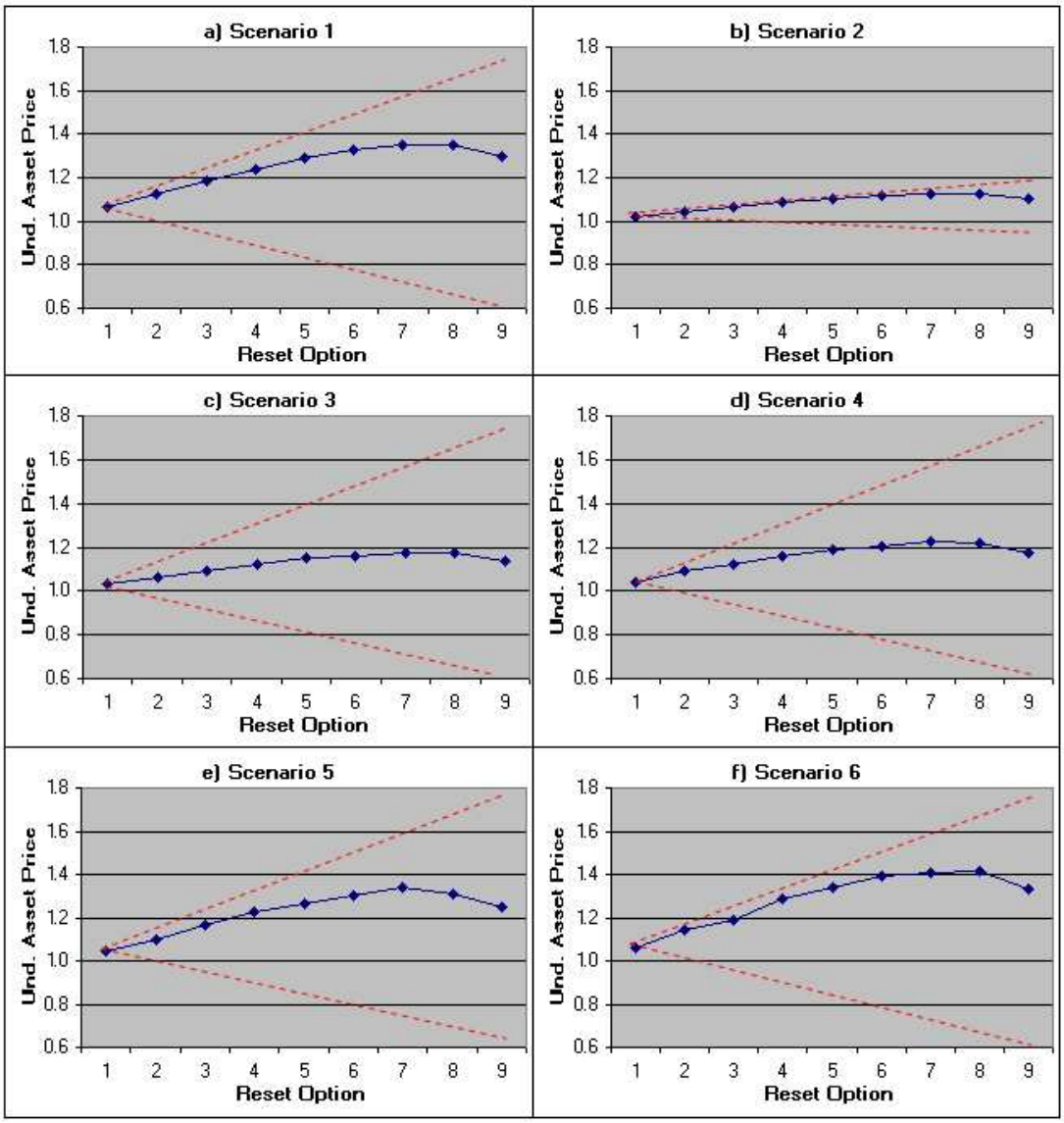

Figure 14: ***Single Regime Model Main Results***

rationale behind this argument is that if the policyholder resets at the first decision point the maturity will be extended by 1 year. In contrast if he resets in the eighth decision point the maturity will be extended by 8 years. Therefore as the "time penalty" increases, the return that the policyholder requires in order to choose to exercise his reset option increases. However, in the last period before the maturity of the segregated fund, the return that the policyholder requires in order to optimally exercise his reset option decreases. The reason for this is that if the option is not exercised it will expire worthless. This finding is in line with the findings of section 3.7.3.

Further, in figure 14, for each scenario two boundaries have been provided (colour coded red) which represent an approximation of the 95% confidence interval for the values of the underlying asset. The purpose of this illustration is to highlight that the values of the underlying asset for which the policuholder will optimally exercise his reset option are indeed obtainable (not extreme) for each of the scenarios under examination.

In terms of the $V_{1,Nq}^{Tq}$, scenario 1 has the largest value (20.90836) and this is due to the fact that the annual mean of this scenario is 11% whereas for the other scenarios it is 4.5%. Further, for the other scenarios it holds that the higher the coefficient of variation, the higher the value of the $V_{1,Nq}^{Tq}$. In particular, the $V_{1,Nq}^{Tq}$ of scenarios 2 to 6 are 3.58652, 4.15905, 5.42369, 8.96388 and 12.22753, respectively.

3.10 SRM Sensitivity Analysis

Sensitivity analysis has been undertaken in order to examine the robustness of the model, to gain insights into the behaviour of the segregated fund contract and to make recommendations to both the policyholder as well as the issuer of the segregated funds.

The sensitivity analysis has been performed for all the six scenarios built and can be categorised into three areas: (i) fluctuating the level of the maturity guarantee (G), (ii) examining the value of offering the reset options under different cases, and (iii) examining the pattern of the OEBs for all the segregated funds generated within the original planning horizon of 29 years.

Relative to the last area of sensitivity analysis, in section 3.7.2 it was highlighted that the OEB depicted related to the segregated fund with $n = Nq$ when $t = Tq$. However, this is just the first segregated fund generated during the planning horizon of the policyholder. As observed in section 3.5 in the sample of a segregated fund contract, every time the policyholder has the opportunity to reset the level of his maturity guarantee, a new segregated fund can be generated (marked as a new binomial tree in figure 1). As a matter of fact given the characteristics of the standard segregated fund contract ($T = 29$ and $N = 10$, see section 3.7.1) a total of 19 segregated funds can be generated. For the first 11 of them their OEB has 9 reset opportunities, one for each year until the maturity of the fund. For the 12th onwards their corresponding OEB

has one fewer reset opportunity than the preceding segregated fund due to the end of the planning horizon. Finally the 19th segregated fund has no reset opportunities, following the same rationale as above.

In order to clarify this issue, consider the $T - (N - 2) = 12$th segregated fund which reaches maturity when there are $(N-2)q$ periods remaining in the planning horizon. This segregated fund allows the policyholder a reset option when q periods before the fund reaches maturity. However, the policyholder is unable to exercise this option because there is insufficient time for the resulting segregated fund to reach maturity. Hence, the 8th available reset option, which arises when there are Nq periods remaining in the planning horizon, will be the last one in this contract that the policyholder can exercise.

Thus, the OEB of the 12th segregated fund includes the value of the underlying asset which would optimally trigger a reset option for just 8 reset decision points. Likewise the 13th segregated fund includes the value of the underlying asset which would optimally trigger a reset option for just 7 reset decision points, so on and so forth.

Overall, it has been decided to examine the OEB of the first 11 segregated funds as they include all 9 reset options and, thus, are comparable.

3.10.1 SRM Experiment 1: Fluctuating the maturity guarantee

In this experiment the aim is to examine the effect of the fluctuation of G on both the OEB and the $V_{1,Nq}^{Tq}$. In particular, G is fluctuated from its original value of 100% to the range 80% to 120% at increments of 5%.

Figure 15 illustrates that across all scenarios as G increases, the OEB shifts

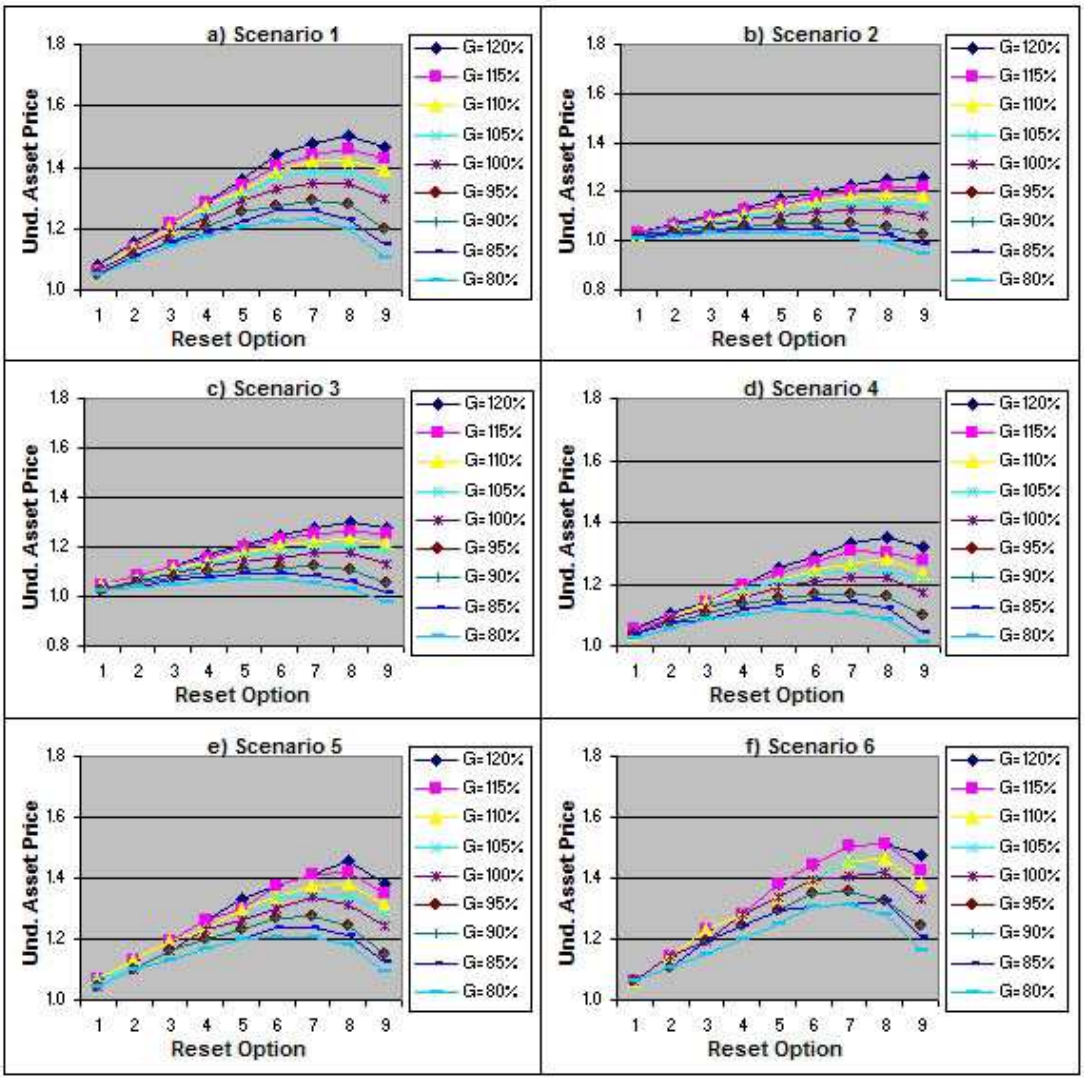

Figure 15: ***SRM Experiment 1 - Fluctuating the maturity guarantee***

upwards with a larger increase towards the maturity of the contract. Likewise, figure 15 illustrates that across all scenarios as G decreases, the OEB shifts downwards with a larger decrease towards the maturity of the contract.

For the purposes of stress testing extreme values of G were tested to observe the behaviour of the model. The finding from this analysis is that as G tends to infinity the OEB increases exponentially. In terms of the pattern of the OEB the boundary becomes a straight line. On the other hand, as G tends to nill the OEB shifts downwards while broadly maintaining its pattern. It is of interest to note that decreasing G to values lower than 50% (although the exact number varies with the scenario under examination) does not seem to materially affect the OEB. A possible explanation for that is that given the distribution of values of the underlying asset, it is not very likely that the underlying will ever fall materially below 50%, hence rendering such a level of maturity guarantee not valuable to the policuholder. Thus, further reducing the value of a guarantee that the policyholder would already not make use of does not affect the OEB.

Further, table 3 illustrates that decreasing the value of G causes the $V_{1,Nq}^{Tq}$ to decrease across all scenarios. Similarly, increasing the value of G causes the $V_{1,Nq}^{Tq}$ to increase across all scenarios.

The rationale of this observation is that by decreasing the value of G, the protection offered by the maturity guarantee against adverse stock market

G	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
80%	-0.99%	-0.07%	-8.74%	-15.11%	-20.03%	-21.67%
85%	-0.82%	-0.07%	-7.05%	-11.76%	-15.37%	-16.58%
90%	-0.60%	-0.06%	-5.04%	-8.12%	-10.47%	-11.27%
95%	-0.33%	-0.05%	-2.69%	-4.20%	-5.35%	-5.74%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
105%	0.40%	0.11%	3.04%	4.49%	5.58%	5.96%
110%	0.88%	0.34%	6.45%	9.26%	11.40%	12.15%
115%	1.43%	0.76%	10.22%	14.32%	17.45%	18.55%
120%	2.08%	1.49%	14.36%	19.66%	23.72%	25.17%

Table 3: ***SRM Experiment 1 - Fluctuating the maturity guarantee***

conditions is weakened thus ultimately decreasing the total value of the segregated fund contract. On the other hand, by increasing the value of G, the protection offered by the maturity guarantee against adverse stock market conditions is increased thus ultimately increasing the total value of the segregated fund contract.

The higher the increase in the value of G, the higher the percentage increase of $V_{1,Nq}^{Tq}$. Further, the $V_{1,Nq}^{Tq}$ of scenarios 1 and 2 is least affected by the increase in the value of G. For the other scenarios, it holds that the $V_{1,Nq}^{Tq}$ of the scenarios with higher coefficient of variation is more increased as G is increased.

Furthermore, the higher the decrease in the value of G, the higher the percentage decrease of $V_{1,Nq}^{Tq}$. Again, the $V_{1,Nq}^{Tq}$ of scenarios 1 and 2 is least affected by the decrease in the value of G. For the other scenarios, it holds that the $V_{1,Nq}^{Tq}$ of the scenarios with higher coefficient of variation is more decreased as G is decreased.

3.10.2 SRM Experiment 2: Value of the reset option

The aim of this experiment is to estimate the value of offering reset options on a segregated fund contract under different sets of values of the model parameters. In particular, the value of offering reset options is calculated for the original set of values of the model parameters and then the parameter of interest (G) is fluctuated, while keeping the rest of the parameters constant.

The value of the reset options is the percentage increase in the $V^{Tq}_{1,Nq}$ of the segregated fund from embedding the reset options and is calculated as follows: $\frac{V^{Tq}_{1,Nq} - V^{*Tq}_{1,Nq}}{V^{*Tq}_{1,Nq}}$, where $V^{*Tq}_{1,Nq}$ stands for the value of the fund without reset options. The results of this experiment are summarised in table 4.

G	***Sc.1***	***Sc.2***	***Sc.3***	***Sc.4***	***Sc.5***	***Sc.6***
100%	0.97%	0.05%	8.98%	25.10%	62.60%	93.11%
90%	0.55%	0.01%	6.08%	20.36%	55.22%	83.77%
80%	0.28%	0.01%	3.73%	15.79%	47.61%	74.03%

Table 4: ***SRM Experiment 2 - Value of the reset option***

The first row of table 4 shows the value of the reset options under the original value for the parameter of interest. In particular for scenarios 1 and 2 the reset options add a small value to the segregated fund (0.97% and 0.05% respectively). However, for the other scenarios, the higher the coefficient of variation, the higher the value of the reset options (8.98%, 25.10%, 62.60% and 93.11% for scenarios 3 to 6 respectively).

Rows 2 and 3 summarise the sub-experiment where the G is fluctuated from its original value of 100% to the range 80% to 100% at increments of 10% while all other parameters of interest are kept constant. As the value of G is decreased to 80%, the value of the reset options is decreased across all scenarios. The rationale for this observation is that as G decreases the protection that it offers against adverse market conditions worsens. Subsequently, a reduced value of the maturity guarantee leads to a reduced value to an option to reset it.

Overall, for scenarios 1 and 2 embedding the reset options adds only a small value to the segregated fund (0.28% and 0.01% respectively). However, for the other scenarios, the higher the coefficient of variation, the higher the value of the reset options (3.73%, 15.79%, 47.61% and 74.03% for scenarios 3 to 6 respectively).

A further interesting finding, from the point of view of the issuer, is that having an estimate of the value of a segregated fund contract without reset options (denoted by $V_{1,Nq}^{*Tq}$ above), one can estimate the cost of offering segregated funds with reset options allowed under the assumption that only a percentage of the policyholders would actually choose to reset their maturity guarantee when it was indeed optimal to do so. The rest of the policyholders would simply choose to rollover, hence behaving as if a reset option was not allowed. If one assumes that 75% of the cohort of policyholders eligible to reset

would actually choose to reset each year, as suggested by CIA, (see section 2.3.4) then the total cost of offering a segregated fund with reset options would be equal to : $0.75 * V_{1,Nq}^{Tq} + 0.25 * V_{1,Nq}^{*Tq}$.

3.10.3 SRM Experiment 3: OEB of all segregated funds

The aim of this experiment is to depict the OEB of all the segregated funds generated during the planning horizon of the policyholder, as explained in section 3.10. Section 3.8 highlighted that the OEB is inherently dependent on the prevailing market conditions (i.e. u, d and r). Indeed in the current experiment it was observed that the OEB is strongly affected by the type of investment that is preferred for the last Nq time periods. The relationship of the risk-free rate of interest and the expected return on investment from the rolling over of the segregated fund contract until the end of the planning horizon largely determines this effect.

In particular, it has been observed that the OEB of all the segregated funds can be categorised into three main types. In **Type A** the market conditions are such that the policyholder has a strong preference in the last Nq time periods to switch his portfolio to a bond yielding the risk-free rate of interest. In **Type B** the market conditions are such that the policyholder has a strong preference in the last Nq time periods to keep his segregated fund contract and roll it over until the end of the planning horizon. Lastly, in **Type C** the

market conditions are such that the policyholder is practically indifferent from the two above options.

Before explaining in detail the defining characteristics of each type it is of interest to depict the OEB of all the segregated funds generated during the planning horizon of the policyholder and to classify them under the three different types. Then, an in depth analysis of each type follows. As illustrated in figure 16, scenarios 1, 2 and 3 are of type C, while scenarios 4, 5 and 6 are of type B.

The preference of the policyholder for the last Nq time periods greatly affects his behaviour (and thus the OEB) of the segregated funds 2 to 10. The rationale of this argument is as follows. Under Type B the policyholder has a strong preference to remain within his segregated fund contract for as long as possible, ideally until the end of the planning horizon. In order to achieve this he has to make sure that when $t = 2Nq$ he has just re-invested (case of segregated fund 1) or reset (case of segregated funds 2 to 10) to a new segregated fund (which is segregated fund 11). Under segregated fund 11 the policyholder can last exercise his reset option at $t = Nq$ which is the last time period in the entire planning horizon that the policyholder has a reset decision. A segregated fund which starts at $t = Nq$ will mature at $t = 0$, thus ensuring that the policyholder has remained within a segregated fund contract until the end of the planning horizon. The reverse behaviour applies for Type A.

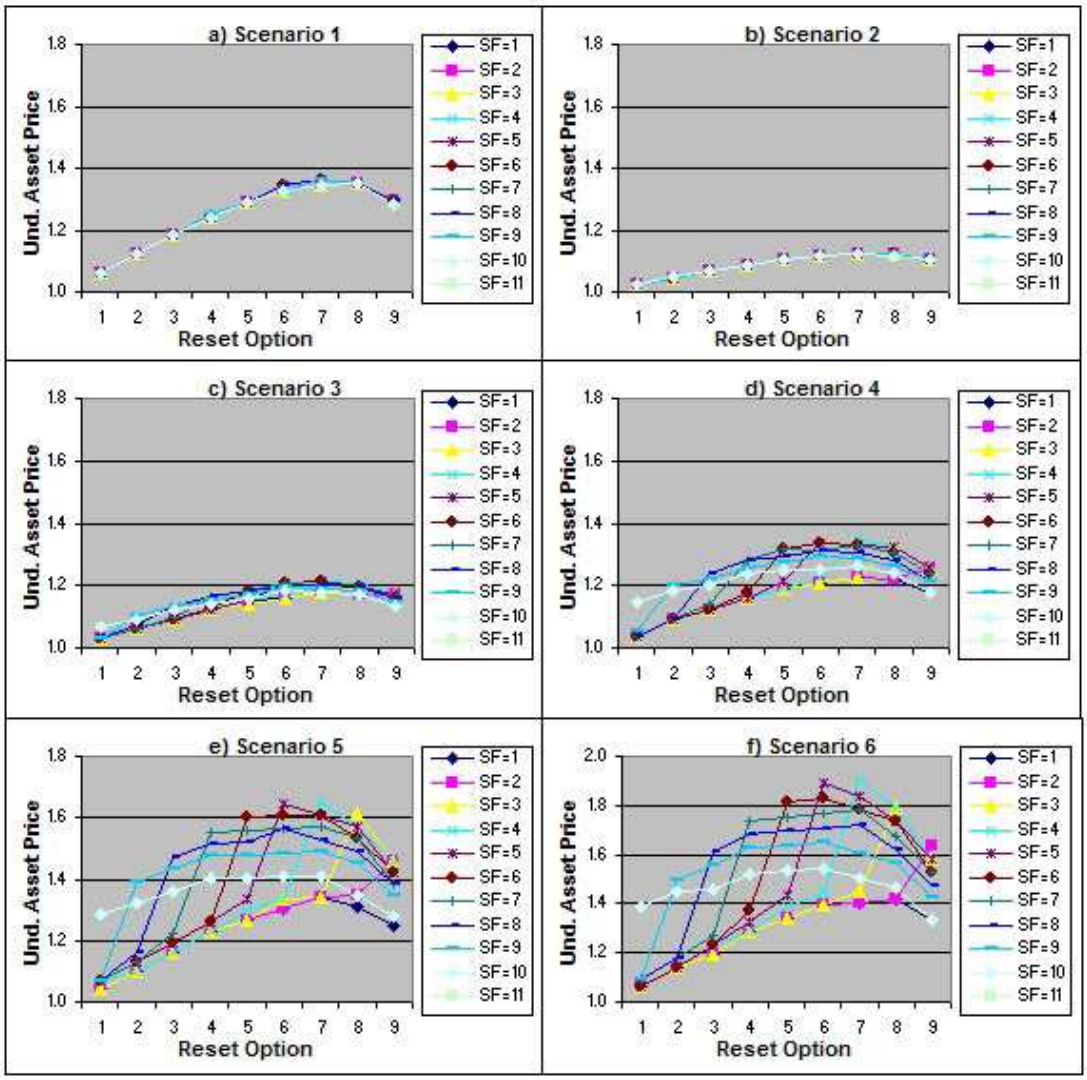

Figure 16: ***SRM Experiment 3 - Depicting the OEB of all segregated funds***

For segregated fund 1 the preference of the policyholder for the last Nq time periods (as described by the aforementioned types) does not affect the OEB as it matures at time $t = 2Nq$ when the policyholder can freely make the choice to re-invest in a segregated fund (segregated fund 11) and thus to have the option to remain within segregated fund contracts until the end of the

planning horizon or to invest in a bond yielding the risk-free rate of interest.

However, for segregated funds 2 to 10 the preference of the policyholder for the last Nq time units (as described by the aforementioned types) affects their OEB significantly. On the one hand, under type A, a jump down in the OEB occurs after the last reset decision point which would allow the policyholder to switch to risk free at the last Nq time units. On the other hand, under type B, a jump up in the OEB occurs after the last reset decision point which would allow the policyholder to be in a segregated fund for the last Nq time units. In contrast for type C the OEB looks like the OEB of segregated fund 1 as the policyholder is indifferent from the above two types.

3.11 Discussion

In this chapter an attempt has been made to formulate the reset option on the maturity guarantee of segregated funds as a non-stationary finite horizon Markov Decision Process. The efficient formulation allows the value of the underlying asset to be fluctuated up to 7000 times in every policy year, thus enabling the distribution of the underlying asset price to converge towards the lognormal distribution.

An important feature of the Single Regime Model, developed in this chapter, is the ability to derive the Optimal Exercise Boundary of the reset option, where given the model parameters, a threshold value is depicted such that if the value

of the underlying asset price exceeds it then it is optimal for the policyholder to reset his maturity guarantee. Otherwise, it is optimal for the policyholder to rollover his maturity guarantee.

It is noteworthy that the SRM model is able to depict the OEB of not just the first but of all the segregated fund contracts which can be issued throughout the planning horizon of the policyholder. The reason why this is of great importance is that once the investor resets the maturity guarantee the OEB changes. Therefore, it becomes apparent that in order to generate a comprehensive optimal reset strategy, the optimal OEB for all of the segregated funds has to be derived and examined. In this way the model has managed to address one of the significant deficiencies in the existing literature as highlighted in section 2.4.

The main finding of the SRM model has been that as the segregated fund approaches its maturity, a proportionately larger percentage increase in the value of the underlying asset will be necessary to trigger an optimal reset of the segregated fund's maturity guarantee. The rationale behind this argument is that if the policyholder resets at the first decision point the maturity will be extended by 1 year. In contrast if he resets in the eighth decision point the maturity will be extended by 8 years. Therefore as the "time penalty" increases, the return that the policyholder requires in order to choose to exercise his reset option increases. However, in the last period before the maturity of the

segregated fund, the return that the policyholder requires in order to optimally exercise his reset option decreases. The reason for this is that if the option is not exercised it will expire worthless.

It should be underlined that the aim of the model has not been to prescribe any particular reset strategy as this is highly dependent on the parameters and assumptions of the model, but rather to further our understanding on what constitutes an optimal reset strategy and how it is affected by the fluctuation of the main variables of the model. However, it should be highlighted that the findings of the SRM model suggest that a single heuristic such as the one prescribed by Canadian Institute of Actuaries (as analysed in sections 2.3.4 and 2.3.5), independent of the parameters and assumptions of the model and most importantly of the years remaining to maturity can prove to be a misleading approximation of the optimal reset strategy.

Overall, given the importance of the SRM findings it is interesting to alter some of its assumptions in order to reflect on the characteristics of the wide range of segregated fund contracts which are traded in the market. The methodology and results of this analysis are included in chapter 4.

4 Extending the Single Regime Model

4.1 Introduction

In this chapter an attempt has been made to enhance and extend the SRM model in order to reflect on the characteristics of the wide range of segregated fund contracts which are traded in the market. For this end, three different extensions have been added to the model.

In the SRM model there is no provision for the issuer of the segregated fund contract to charge a management expense fee. This issue is addressed in section 4.2 where the issuer is allowed to charge the policyholder a fixed fee per policy year. The model is analysed in section 4.2.1 and the main results as well as the sensitivity analysis are provided in sections 4.2.2 to 4.2.6.

Further the SRM model assumes that the level of G is fixed throughout the planning horizon regardless of the number of times that the policyholder has exercised his reset option. However, section 4.3 extends the SRM in that G becomes a function of the number of times that the reset has been exercised. In particular, every time the policyholder exercises his reset option, G is instantaneously reduced by a pre-determined amount. The model is analysed in sections 4.3.1 and 4.3.2 and the main results as well as the sensitivity analysis are provided in sections 4.3.3 to 4.3.7.

Lastly, the SRM model assumes that the policyholder can exercise his reset option only at the end of each policy year. However, section 4.4 extends the

SRM in that it lifts this restriction and allows the policyholder to reset the maturity guarantee at any point in time within each policy year, but only once. The model is analysed in sections 4.4.1 and 4.4.2 and the main results as well as the sensitivity analysis are provided in sections 4.4.3 to 4.4.6.

For all three extensions to the SRM model a flowchart analysis has been provided. As the logic and architecture of the various models share some common ground with the SRM, rather than analysing the flowcharts in full, only the differences with the flowchart of the SRM will be highlighted. All other parts can be assumed to be the same. For this purpose the flowcharts have been designed to facilitate comparisons: the structure is consistent and wherever there is a difference it is colour-coded in red. If a process or a condition has a significant difference then only the title of the process is colour-coded red.

4.2 Management Expense Ratio

The Management Expense Ratio (MER) model extends the SRM in that it allows the issuer of the segregated fund to charge a management fee to the policyholder. In particular, it has been assumed that the issuer charges the policyholder a fee equal to a fixed proportion of the value of the fund at the end of each policy year. This assumption is in line with guidance provided by CIA who suggests that this is a common approach used in practice [Canadian Institute of Actuaries, 2002]. The fixed proportion is referred to as the Management Expense Ratio and is denoted by L in the model.

This extension is not formally presented as a Markov Decision Process because its formulation so closely resembles that of the original model. In particular, the formulations of the two models only differ in that at each time period the value of the underlying asset is discounted by the management expense ratio. Section 4.2.1 provides a flowchart analysis of the model, while section 4.2.2 highlights the main results of the MER model.

As with the SRM, it is of interest to experiment with the values of several parameters and observe the effect of their fluctuation on both the OEB and $V_{1,Nq}^{Tq}$. The aim of these experiments is to check the robustness of the model as well as to depict interesting trends and causalities. In particular, section 4.2.3 examines the effect of the fluctuation of the management expense ratio (L), while section 4.2.4 examines the effect of the fluctuation of the level of G offered

to the policyholder. Further, section 4.2.5 examines the value of offering the reset option under different model parameters. In this section an analysis is included which highlights the level of management expense ratio that the issuer should charge in order to cancel out the cost / risk of embedding reset options to the segregated fund contract. Lastly, section 4.2.6 depicts the OEB of all the segregated funds generated during the planning horizon of the policyholder.

4.2.1 Flowchart Analysis

As illustrated in figure 17 the only differences between MER and SRM are in the first two processes. The first process (**Parameters**) of the MER differs from the SRM in that it has a new parameter: L, which denotes the management fee that the issuer charges the policyholder for the provision of the segregated fund contract. Then, the second process (**Asset's price distribution**) differs in that the value of the underlying asset is multiplied by $(1 - L)$ which essentially discounts the asset's value by the management expense ratio.

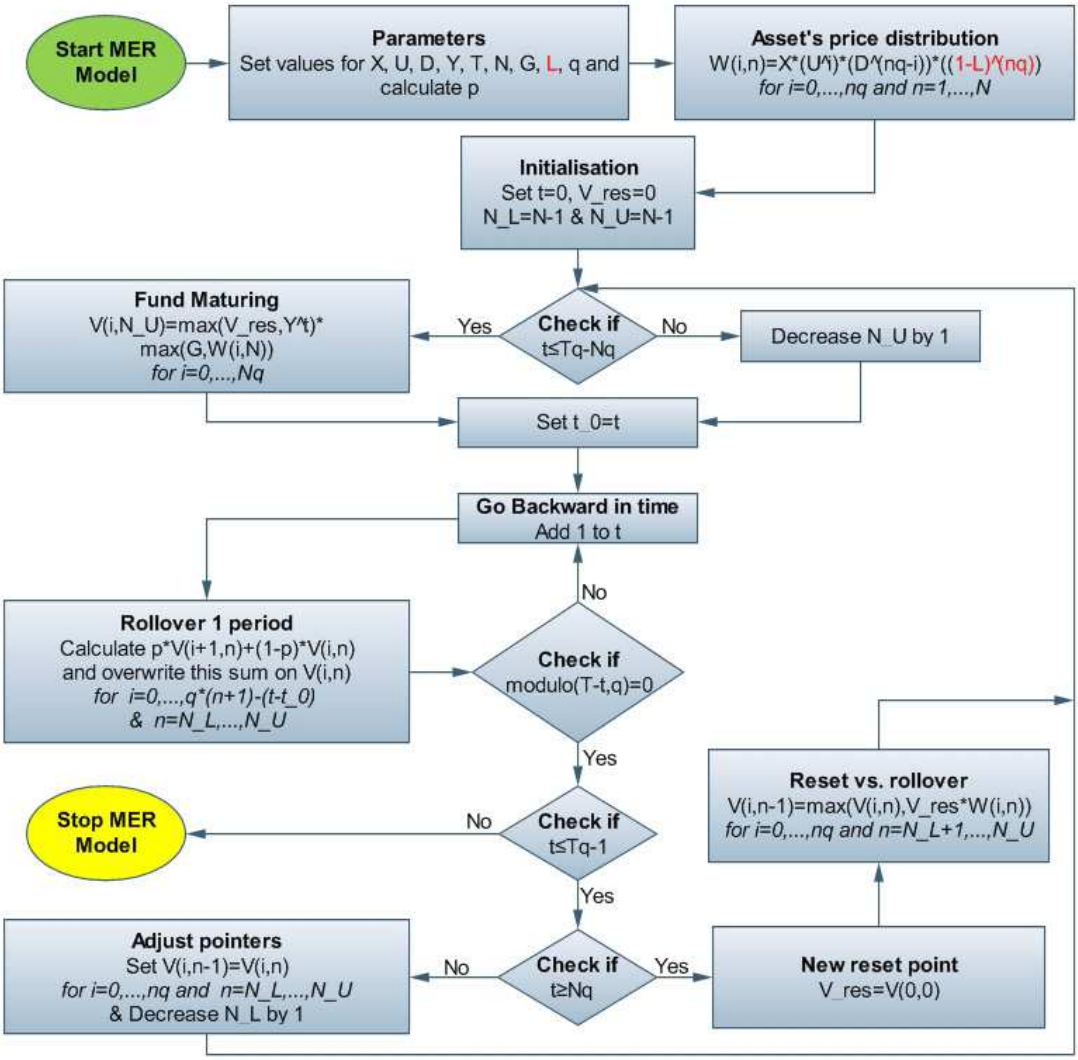

Figure 17: ***Flowchart of MER model***

4.2.2 Main Results

The management expense ratio has been assumed to be 1% of the segregated fund's value for each policy year. The main findings from incorporating this fee is that the policyholder requires a higher return before it is optimal to reset his maturity guarantee while the segregated fund's value is diminished due to

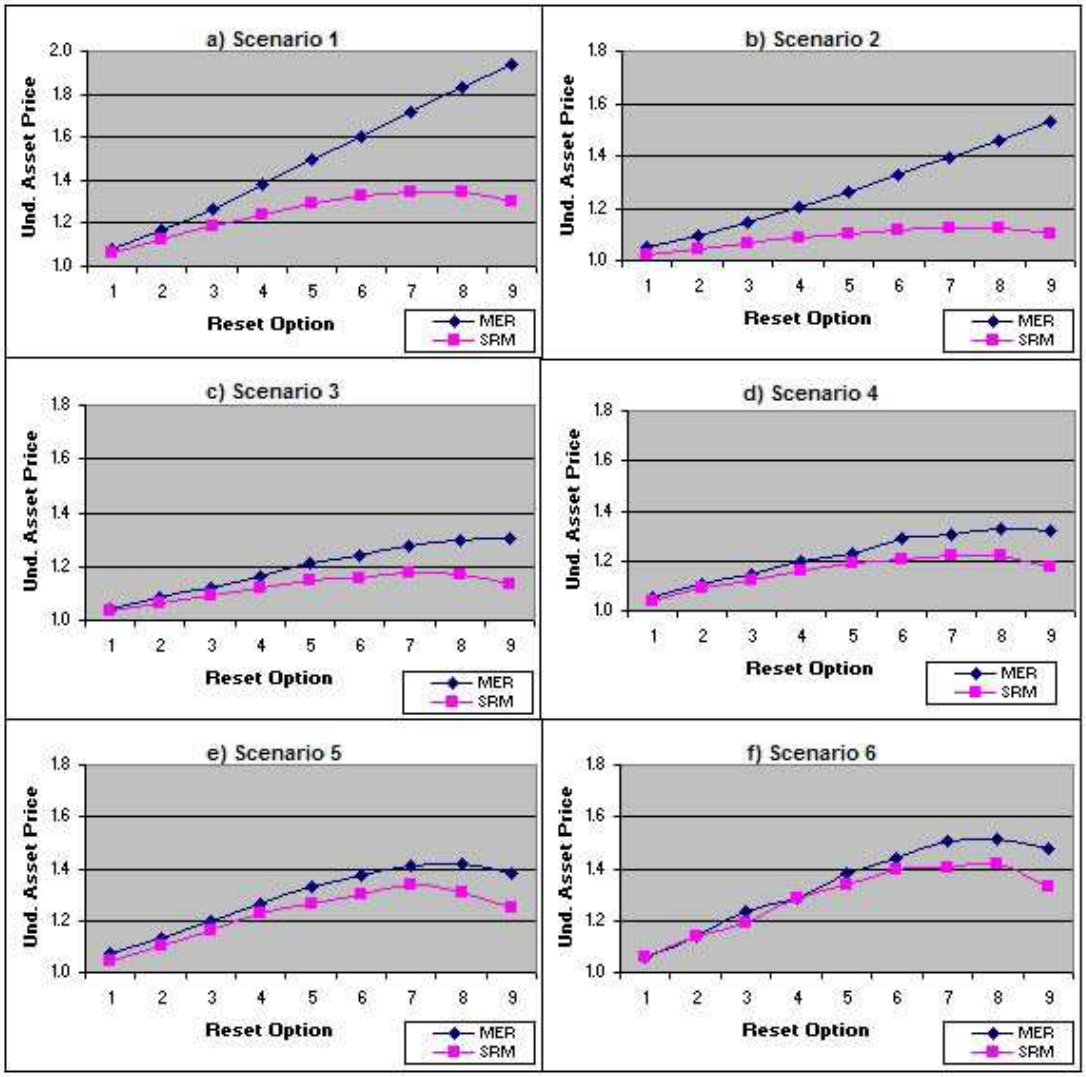

Figure 18: ***MER - Effect on OEB***

the negative effect of the fee.

In particular, as illustrated in figure 18, the introduction of the management expense ratio causes the OEB to shift upwards with a greater increase towards the maturity of the segregated fund. This finding is more acute on scenarios 1 to 3. A possible reason for this is that the relative gain for the policyholder from switching to the risk free rate of interest is greater as the introduction of the fee invariably reduces the potential return from staying in the segregated fund. This observation can be confirmed by the comparison of the return to the policyholder from investing in a segregated fund at the beginning of the planning horizon with investing in risk free for the entire planning horizon. The results of this comparison are presented in the following table.

Return	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
MER=0%	20.90836	3.58652	4.15905	5.42369	8.96388	12.22753
MER=1%	17.06998	2.94173	3.42705	4.57133	7.71522	10.58641
$(1+r)^{Tq}$	20.62369	3.58404	3.58404	3.58404	3.58404	3.58404

Table 5: ***Investing in a segregated fund vs. risk free***

The first row of table 5 highlights the maximum expected return from the original investment if the policyholder chooses to invest, at the beginning of his planning horizon, in a segregated fund with MER=0%. The second row is the equivalent return if the segregated fund charges an MER of 1%. The last row is the equivalent return if the policyholder invests in risk free for the entire planning horizon. The comparison of these values depicts that the policyholder

would prefer to invest his original investment in risk free for the entire planning horizon given the market conditions assumed in scenarios 1 to 3. This suggests that, given the market conditions of scenarios 1 to 3, if the investor has started in a segregated fund then the only way for him to want to continue in such a fund (i.e. to reset and extend its maturity) is if he can lock in very high returns, otherwise he has a strong preference for switching to risk free. This is illustrated in the sharp increase in the OEB of scenarios 1 to 3.

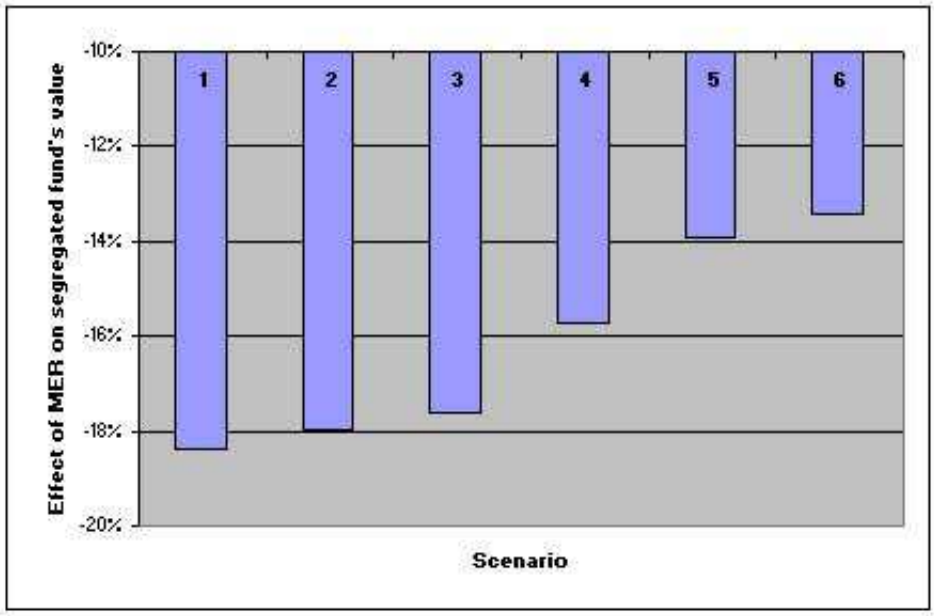

Figure 19: ***MER - Effect on*** $V_{1,Nq}^{Tq}$

In terms of the $V_{1,Nq}^{Tq}$, as illustrated in figure 19, scenario 1 is most affected by the introduction of L (decrease of 18.36%). For the other scenarios it holds that the lower the coefficient of variation, the greater the percentage decrease in the $V_{1,Nq}^{Tq}$ from the application of the management expense ratio (17.98%, 17.60%, 15.72%, 13.93% and 13.42% for scenarios 2 to 6 respectively)

4.2.3 Experiment 1: Fluctuating the management expense ratio

In this experiment the aim is to examine the effect of fluctuating the management expense ratio (L) on both the OEB and the $V_{1,Nq}^{Tq}$. In particular, L is fluctuated from its original value of 1% to the range 0% to 2% at increments of 0.25%.

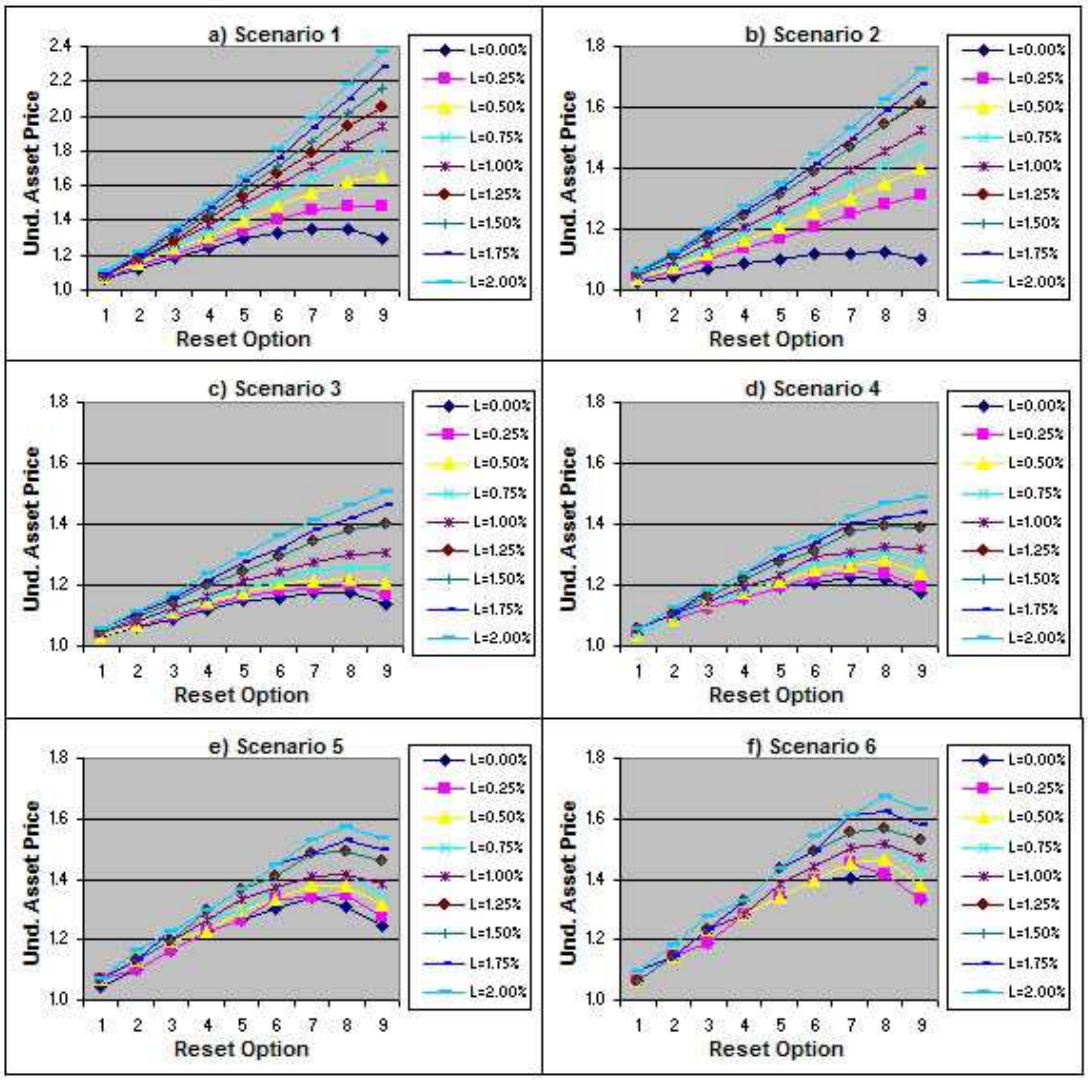

Figure 20: ***MER Experiment 1 - Fluctuating the Management Expense Ratio***

Figure 20 illustrates that across all scenarios as L increases, the OEB shifts upwards with a larger increase towards the maturity of the contract. Likewise, figure 20 illustrates that across all scenarios as L decreases the OEB shifts downwards with a larger decrease towards the maturity of the contract. Overall, as figure 20 illustrates, the fluctuation of L has a more acute effect on scenarios 1 to 3.

The justification for this observation is that for scenarios 1 to 3 the maximum expected return to the policyholder from investing in risk free throughout the planning horizon is higher than from investing in a segregated fund. Therefore, unless the policyholder sees exceptional growth in the fund, he will look for the quickest way out of the fund, which is by essentially not resetting (or more precisely requiring a very high return in order to optimally reset).

A further observation, from figure 20, is that the maturity guarantee repays not only the potential losses from the fund, but also the management fees. Therefore, the maturity guarantee is essentially worth more from the introduction of the management expense ratio. This observation may help to understand why, while the OEB shifts up, it still falls towards the end. In other words, due to the increased value of the maturity guarantee, the fund may still be attractive to the investor, despite the introduction of the management expense ratio. Therefore, the typical OEB shape is observed.

However, the curve is shifted upwards due to the increase in the value of the maturity guarantee that the management expense ratio causes. Essentially, as L increases the value of the maturity guarantee increases. As already illustrated and analysed in section 3.10.1, as the value of the maturity guarantee increases, the OEB shifts upwards.

Further, table 6 illustrates that decreasing the value of L causes the $V_{1,Nq}^{Tq}$ to increase across all scenarios. Similarly, increasing the value of L causes the $V_{1,Nq}^{Tq}$ to decrease across all scenarios.

L	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
0.00%	22.32%	21.75%	21.21%	18.51%	16.18%	15.39%
0.25%	15.74%	15.85%	15.11%	13.38%	11.78%	11.23%
0.50%	10.20%	10.29%	9.56%	8.60%	7.63%	7.29%
0.75%	4.97%	5.01%	4.54%	4.15%	3.71%	3.55%
1.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1.25%	-4.72%	-4.74%	-4.13%	-3.86%	-3.51%	-3.37%
1.50%	-9.19%	-9.23%	-7.91%	-7.47%	-6.82%	-6.58%
1.75%	-13.44%	-13.46%	-11.38%	-10.82%	-9.96%	-9.62%
2.00%	-17.48%	-17.44%	-14.59%	-13.96%	-12.94%	-12.52%

Table 6: ***MER Experiment 1 - Fluctuating the Management Expense Ratio***

The rationale of this observation is that by decreasing the value of L, the value of the underlying asset increases, thus, ultimately increasing the value of the segregated fund contract. On the other hand, by increasing the value of L, the value of the underlying asset decreases, thus, ultimately reducing the value of the segregated fund contract.

The higher the increase in the value of L, the higher the percentage decrease of $V_{1,Nq}^{Tq}$. Further, scenario 1 is the most affected by the increase of L. For the other scenarios it holds that the $V_{1,Nq}^{Tq}$ of the scenarios with lower coefficient of variation is more decreased as L is increased.

On the other hand, the higher the decrease in the value of L, the higher the percentage increase of $V_{1,Nq}^{Tq}$. Again, scenario 1 is the most affected by the decrease of L. For the other scenarios it holds that the $V_{1,Nq}^{Tq}$ of the scenarios with lower coefficient of variation is more increased as L is decreased.

4.2.4 Experiment 2: Fluctuating the maturity guarantee

In this experiment the aim is to examine the effect of the fluctuation of G on both the OEB and the $V_{1,Nq}^{Tq}$. In particular, G is fluctuated from its original value of 100% to the range 80% to 120% at increments of 5%.

Figure 21 illustrates that across all scenarios as G increases, the OEB shifts upwards with a larger increase towards the maturity of the contract. Likewise, figure 21 illustrates that across all scenarios as G decreases, the OEB shifts downwards with a larger decrease towards the maturity of the contract. Overall, as figure 21 illustrates, the fluctuation of G has a more acute effect on scenarios 3 to 6. The reason why scenarios 1 and 2 are not affected by the fluctuation of G could be because the return to the policyholder from investing in risk free is higher compared to investing in a segregated fund. Therefore, the

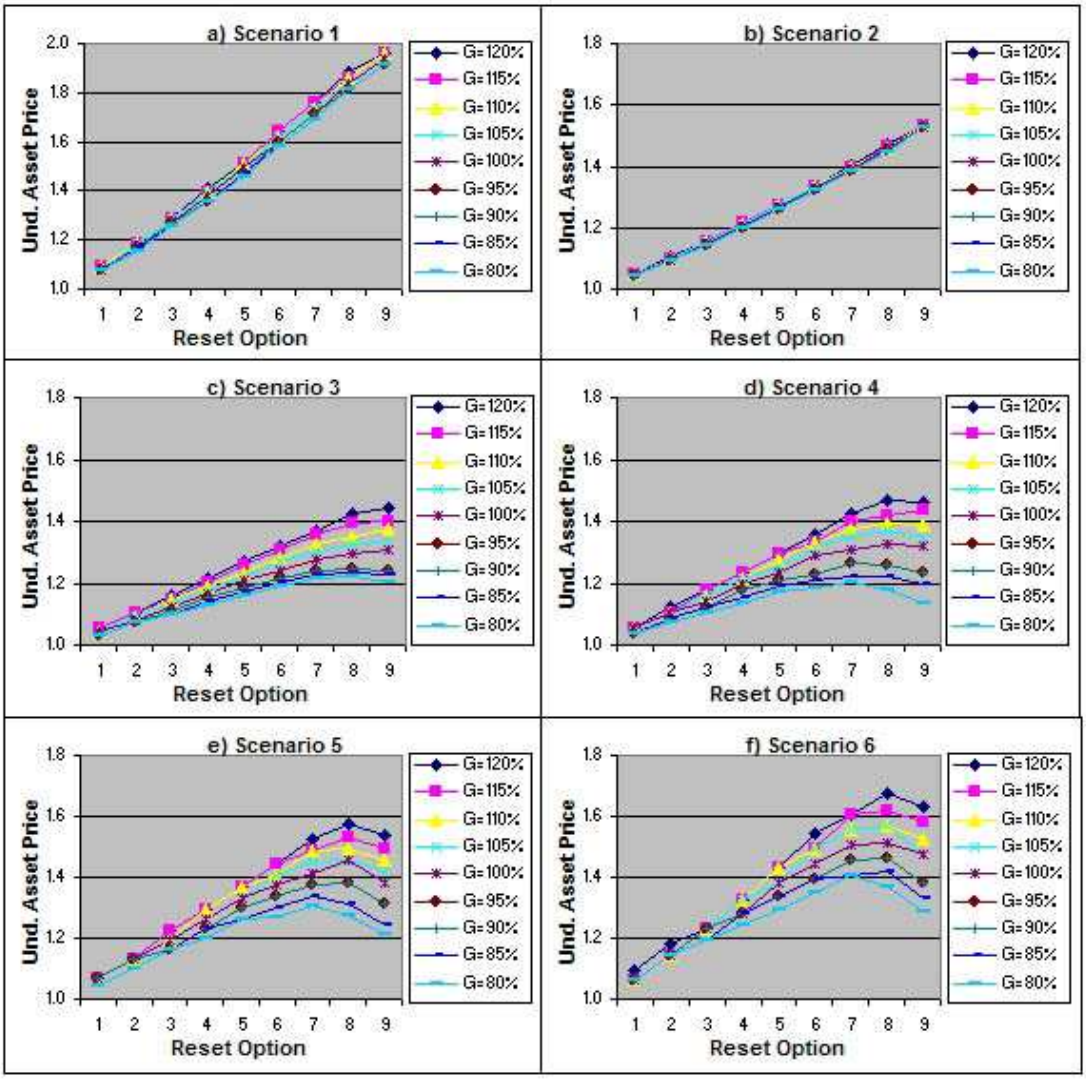

Figure 21: ***MER Experiment 2 - Fluctuating the maturity guarantee***

policyholder may choose to switch to risk free at the first available opportunity, hence, not really needing or making use of the maturity guarantee.

Further, table 7 illustrates that decreasing the value of G causes the $V_{1,Nq}^{Tq}$ to decrease across all scenarios. Similarly, increasing the value of G causes the $V_{1,Nq}^{Tq}$ to increase across all scenarios. The rationale of this observation is equivalent to the one provided in section 3.10.1.

G	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
80%	-0.73%	-0.15%	-10.02%	-17.36%	-21.55%	-22.91%
85%	-0.61%	-0.15%	-8.32%	-13.51%	-16.54%	-17.54%
90%	-0.45%	-0.13%	-6.10%	-9.33%	-11.28%	-11.93%
95%	-0.25%	-0.10%	-3.32%	-4.83%	-5.77%	-6.09%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
105%	0.31%	0.21%	3.82%	5.15%	6.02%	6.32%
110%	0.69%	0.63%	8.12%	10.62%	12.30%	12.89%
115%	1.14%	1.36%	12.88%	16.41%	18.33%	19.70%
120%	1.66%	2.55%	18.09%	22.53%	25.61%	26.75%

Table 7: ***MER Experiment 2 - Fluctuating the maturity guarantee***

4.2.5 Experiment 3: Value of the reset option

The aim of this experiment is to estimate the value of offering reset options on a segregated fund contract under different sets of values of the model parameters. In particular, the value of offering reset options is calculated for the original set of values of the model parameters and then one of the parameters of interest (G, E) is fluctuated at a time, while keeping the rest of the parameters constant. The value of the reset options is calculated as in section 3.10.2 The results of this experiment are summarised in table 8.

G	L	***Sc.1***	***Sc.2***	***Sc.3***	***Sc.4***	***Sc.5***	***Sc.6***
100%	1.00%	0.24%	0.02%	5.88%	21.97%	59.34%	89.71%
90%	1.00%	0.14%	0.01%	3.06%	16.98%	51.88%	80.41%
80%	1.00%	0.06%	0.01%	1.40%	12.11%	44.17%	70.62%
100%	0.75%	0.25%	0.03%	6.36%	22.63%	60.24%	90.50%
100%	0.50%	0.26%	0.04%	7.04%	23.37%	60.99%	91.33%
100%	0.25%	0.29%	0.05%	7.91%	24.19%	61.78%	92.19%

Table 8: ***MER Experiment 3 - Value of the reset option***

The first row of table 8 shows the value of the reset options under the

original values for the parameters of interest. In particular, for scenarios 1 and 2 the reset options add a small value to the segregated fund (0.24% and 0.02% respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (5.88%, 21.97%, 59.34% and 89.71% for scenarios 3 to 6 respectively). A possible reason for the small value added by embedding reset options on segregated funds under the market condition assumed in scenarios 1 and 2 is that the return to the policyholder from investing in risk free throughout the planning horizon is higher compared to investing in a segregated fund. Thus, the policyholder will switch to risk free at the first available opportunity, thus not really needing or making use of the reset options.

Rows 2 and 3 summarise the first sub-experiment where G is fluctuated from its original value of 100% to the range 80% to 100% at increments of 10% while all other parameters of interest are kept constant. As the value of G is decreased to 80% the value of the reset options is decreased across all scenarios. The rationale for this observation is the same as the provided in section 3.10.2. Overall, for scenarios 1 to 3 embedding the reset options adds only a small value to the segregated fund (0.06%, 0.01% and 1.40% respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (12.11%, 44.17% and 70.62% for scenarios 4 to 6 respectively). The reason for this observation is equivalent to the one analysed

above.

Rows 4 to 6 summarise the second sub-experiment where L, is fluctuated from its original value of 1% to the range 0.25% to 1% at increments of 0.25% while all other parameters of interest are kept constant. As L is decreased to 0.25% the value of the reset options increases (0.29%, 0.05%, 7.91%, 24.19%, 61.78% and 92.19% for scenarios 1 to 6 respectively). The rationale for this observation is that a lower in magnitude level of L will lead to a higher value of the underlying asset, therefore, increasing the probability of the asset value being greater than the maturity guarantee, thus, increasing the likelihood of the policyholder choosing to reset his maturity guarantee. In turn, this increases the value of the reset option. The comparatively lower values of the reset options for scenarios 1 to 3 can be explained by the rationale analysed above.

It is further interesting to depict the values of L which can negate the cost of embedding reset options to a segregated fund contract. In order to achieve this a sensitivity analysis can be performed which increases the level of L until the return from investing in a segregated fund is reduced to the equivalent return from investing in a fund governed by the same market conditions but where the policyholder can not reset his maturity guarantee. These "break-even" values of L are 0.01%, 0.01%, 0.42%, 1.34%, 3.61% and 5.40% for scenarios 1 to 6 respectively. This finding confirms the observation that for scenarios 1 to 3 the policyholder would rather invest in risk free and will eventually

choose to switch to risk free at the first available opportunity. For the other scenarios the finding is that the higher the coefficient of variation, the higher the management expense ratio that the issuer should charge in order to break even the cost of embedding reset options in segregated funds.

This result should be very interesting and useful for both the policyholders and issuers. From the point of view of the issuer, it should help with risk management policies as well as to convince the relevant regulatory authorities that they indeed take appropriate cover for the risk that they face from issuing segregated funds with embedded reset options. From the point of view of the policyholder, it should signal that if the underlying asset is not very risky, thus has a low coefficient of variation, they should not be willing to pay a relatively high management expense ratio. The opposite should hold for a risky underlying asset.

4.2.6 Experiment 4: OEB of all segregated funds

The aim of this experiment is to depict and classify, in accordance to section 3.10.3, the OEB of all the segregated funds generated during the planning horizon of the policyholder. As illustrated in figure 22, scenarios 1 to 3 are of type A while scenario 4 is of type C and scenarios 5 and 6 are of type B.

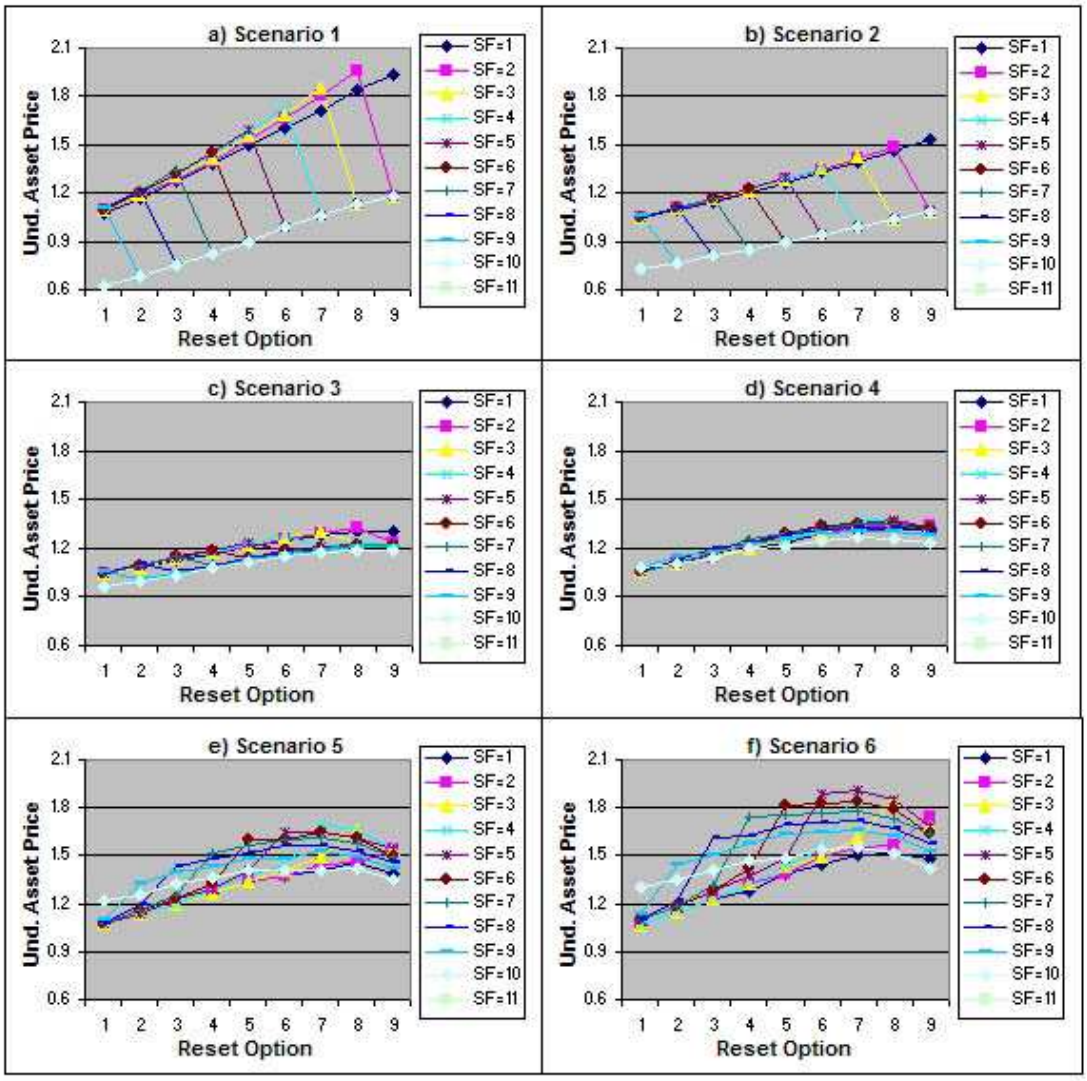

Figure 22: ***MER Experiment 4 - Depicting the OEB of all segregated funds***

4.3 Variable Maturity Guarantee

In the symposium that CIA organised in order to address the shortage in research in the area of modeling for segregated fund investment guarantees (see section 2.3.6), the regulators suggested to practitioners that one way to reduce the risk of offering reset options was to diminish the level of maturity guarantee every time that the policyholder exercised his reset option. The idea is that if the policyholder takes advantage of favourable market conditions and locks in the relevant market gain, he should compensate the issuer by accepting a lower maturity guarantee. If, on the other hand, a policyholder does not exercise his reset option, thus, not causing any potential extra costs to the issuer, he should have the benefit of the full level of the maturity guarantee, as it was set at the beginning of the contract. Therefore, the level of the maturity guarantee should be directly related to the extent that the reset option is exercised by the policyholder, rather than a fixed percentage of the original investment, as was originally used in the market.

In order to address this issue, the Variable Maturity Guarantee (VarG) model extends the SRM in that G becomes a function of the number of times that the reset option has been exercised since the maturity of the last segregated fund or the start of the planning horizon (whichever is most recent), denoted by R. In particular, every time the policyholder exercises his reset option the maturity guarantee is reduced by a pre-determined discount factor, denoted by

β.

Section 4.3.1 offers the formulation of the model while section 4.3.2 provides a flowchart analysis of the model. Then section 4.3.3 highlights the main results of the VarG model.

As with previous models in this thesis, it is of interest to experiment with the values of several parameters and observe the effect of their fluctuation on both the OEB and the maximum expected return from the investment. The aim of these experiments is to check the robustness of the model as well as to depict interesting trends and causalities.

In particular, section 4.3.4 examines the effect of the fluctuation of the discount factor (β), while section 4.3.5 examines the effect of the fluctuation of the level of G offered to the policyholder. Further, section 4.3.6 examines the value of offering the reset option under different model parameters and lastly section 4.3.7 depicts the OEB of all the segregated funds generated during the planning horizon of the policyholder.

4.3.1 Formulation

The VarG model can be formulated to comprise the following four elements:

Stage (denoted by t) which is the number of periods until the end of the planning horizon, where $0 \leq t \leq Tq$.

State Space (denoted by S^t) which is the set of possible states at stage t. The defining characteristics of the possible states are the following. The first two are the same as with the SRM model whereas the third is new.

The first state variable is the current value of the underlying asset relative to its value at the time of the investment in the current segregated fund (denoted by a) which is of the form $a = (1+u)^i(1+d)^{Nq-n-i}$, where $0 \leq i \leq Nq - n$.

The second state variable is the number of periods until the maturity of the current segregated fund contract denoted by n. This variable must satisfy the same conditions as set out in the formulation of SRM.

The third, and last, state variable is the number of times that the policyholder has exercised his reset option since the maturity of the last segregated fund or the start of the planning horizon (whichever is most recent), denoted by R, where $0 \leq R \leq \frac{Tq-t-Nq+n}{q}$. The term $\frac{Tq-t-Nq+n}{q}$ in the boundary of R represents the number of the reset decision points before the investment in the current fund. Therefore, it represents the maximum number of times that the reset option may have been exercised.

Decision Space (denoted by $D^t_{a,n,R}$) - which is the set of possible decisions that can be taken in state $[a, n, R]$ at stage t. As the decisions only depend on n and t the decision space of the VarG model is essentially the same as in the SRM model and is formally stated as:

$$D^t_{a,n,R} = \begin{cases} \{\text{risk free}\} \text{ if } t < Nq \text{ and } n = 0 \\ \{\text{reinvest, risk free}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ \{\text{rollover}\} \text{ if } n > 0 \text{ and } (n,t) \notin E \\ \{\text{rollover, reset}\} \text{ if } (n,t) \in E \end{cases}$$

State transitions in state $[a, 0, R]$ at stage t the action risk free determines the final value of the investment by multiplying the current value of the investment by $(1 + r)^t$. Further, in state $[a, 0, R]$ at stage t the action reinvest causes an instantaneous transition to state $[1, Nq, 0]$. Also, in state $[a, n, R]$ at stage t the action rollover causes a transition to state $[a(1 + u), n - 1, R]$ at stage $t - 1$ with probability p and state $[a(1 + d), n - 1, R]$ at stage $t - 1$ with probability $1 - p$. Lastly, in state $[a, n, R]$ at stage t the action reset causes an instantaneous transition to state $[1, Nq, R + 1]$.

The aim of the policyholder is to maximise the expected payoff of investment at the end of the planning horizon, after Tq time periods. Let $V^t_{a,n,R}$ be the maximum expected payoff of the investment at the end of the planning horizon, after t time periods, when investment is currently in a fund with n time periods to go to maturity, a current relative value of a and a maturity guarantee of $G\beta^R$. Therefore, the aim is to find $X \max\{(1 + r)^{Tq}, V^{Tq}_{1,Nq,0}\}$ where:

$$V^t_{a,n,R} = \begin{cases} \max(a, G\beta^R)(1+r)^t \text{ if } t < Nq \text{ and } n = 0 \\ \max(a, G\beta^R)\max\{(1+r)^t, V^t_{1,Nq,0}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ pV^{t-1}_{a(1+u),n-1,R} + (1-p)V^{t-1}_{a(1+d),n-1,R} \text{ if } n > 0 \text{ and } (n,t) \notin E \\ \max(pV^{t-1}_{a(1+u),n-1,R} + (1-p)V^{t-1}_{a(1+d),n-1,R}, aV^t_{1,Nq,R+1}) \text{ if } (n,t) \in E \end{cases}$$

4.3.2 Flowchart Analysis

As illustrated in figure 23, the main difference between VarG and SRM models is that in VarG there is one extra state denoted by R, which is the number of times that the policyholder has exercised his reset option since the maturity of the last segregated fund or the start of the planning horizon (whichever is most recent). This increases the dimensions of array $V(i, n, R)$ and makes the value of investing on a segregated fund at any given point in time an array indexed by R, denoted as $V_{res}(R)$. Also the level of the maturity guarantee becomes an array indexed by R, denoted as G_R. The value of G_R is calculated in the first process and is a factor of R and two new parameters: G_0 (which is the original level of the maturity guarantee) and β (which is the factor by which the level of the maturity guarantee is reduced every time the policyholder exercises his reset option).

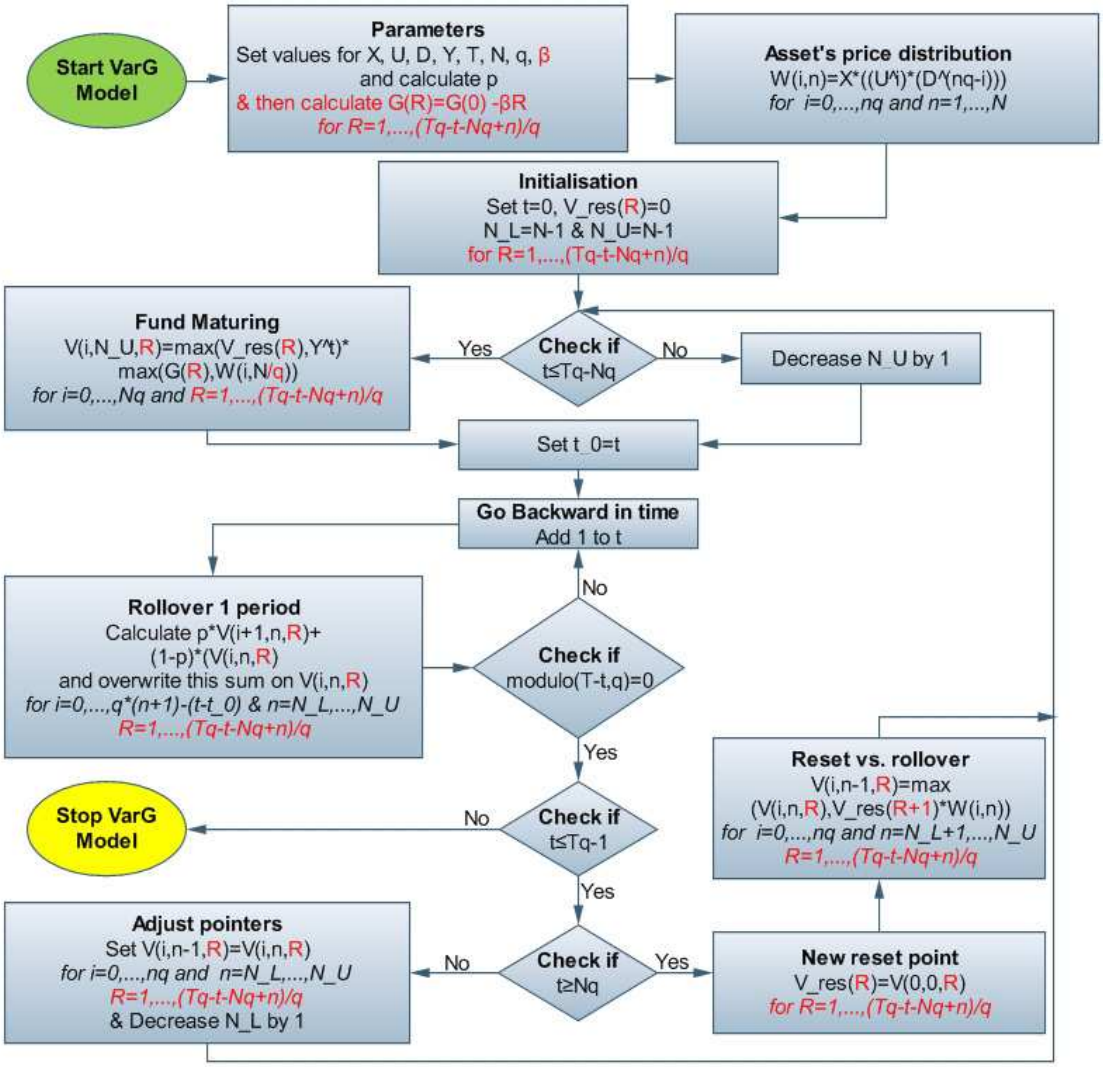

Figure 23: ***Flowchart of VarG model***

4.3.3 Main Results

The parameters of the model have been set so that every time the policyholder exercises his reset option the level of his maturity guarantee is reduced by 5%. The main findings from incorporating the discount factor (β) to the maturity guarantee is that the policyholder requires a higher return before it is optimal

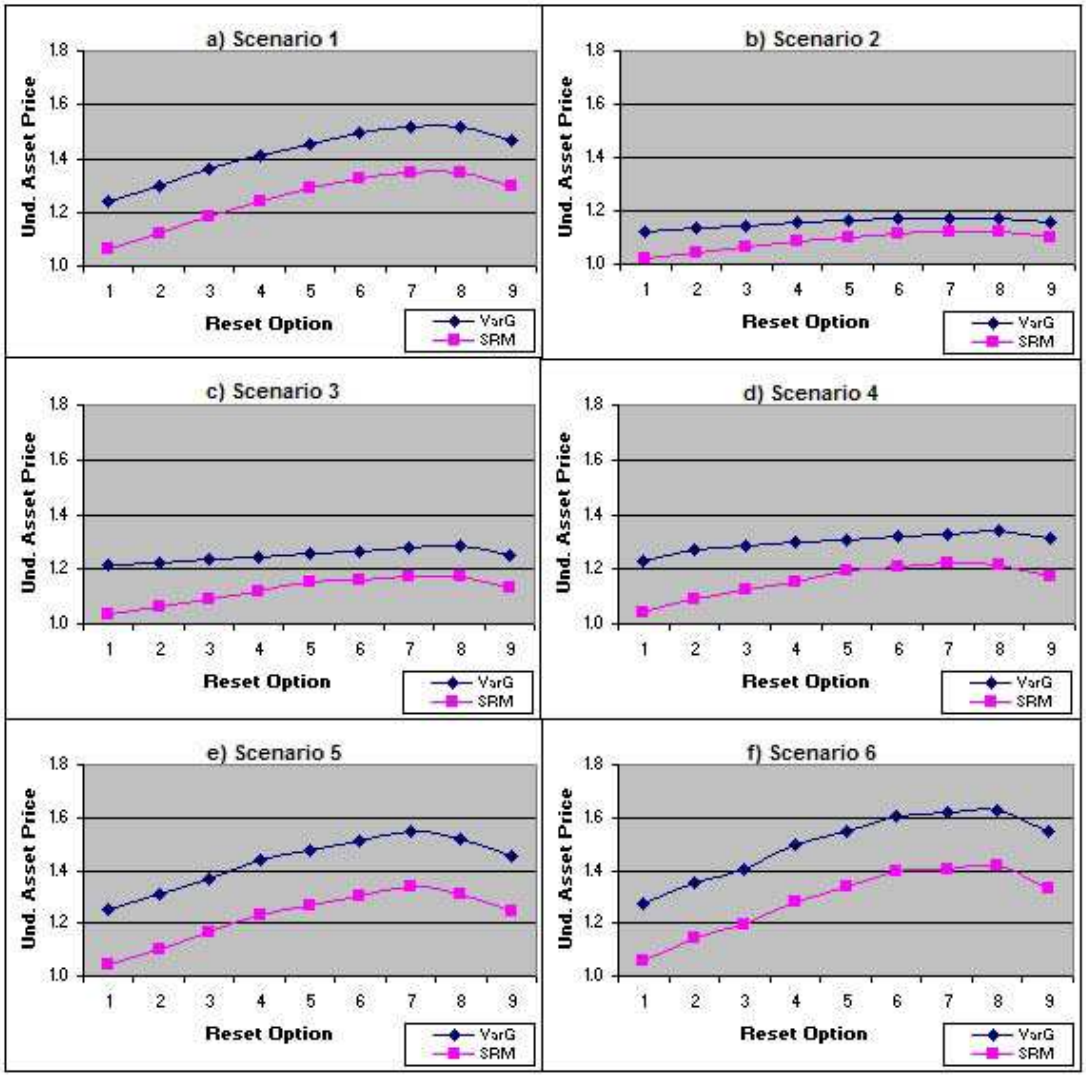

Figure 24: ***VarG - Effect on OEB***

to reset his maturity guarantee (i.e. OEB shifts upwards) while the total value of the segregated fund is diminished due to the negative effect of the discount factor on the segregated fund's value. In particular, as illustrated in figure 24 the incorporation of the discount factor causes the OEB to shift upwards with a larger increase towards the maturity of the fund. Further, as illustrated in figure 24, this finding is more acute as the coefficient of variation of the assumed market conditions increases.

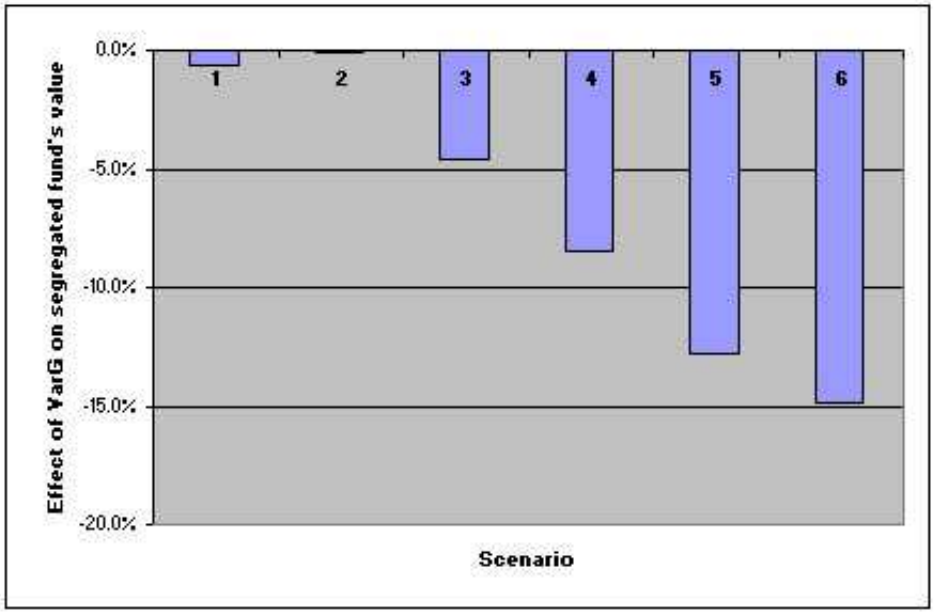

Figure 25: ***VarG - Effect on*** $V_{1,Nq}^{Tq}$

In terms of the $V_{1,Nq}^{Tq}$, as illustrated in figure 25, scenarios 1 and 2 are least affected by the incorporation of the discount factor (decrease of 0.06% and 0.05% respectively). A possible reason for this observation is that under the assumed market conditions of scenarios 1 and 2 the policyholder would prefer to invest in risk free throughout the planning horizon, compared to investing

in a segregated fund. Therefore, the policyholder will choose to switch to risk free at the first available opportunity. In order to do that, he will choose not to reset, so as not to extend the maturity of the contract. If the policyholder is unlikely to reset, the maximum expected return from his investment is not expected to be highly affected by a discount factor applied to the maturity guarantee if a reset is exercised.

For the other scenarios it holds that the higher the coefficient of variation, the greater the percentage decrease in the $V_{1,Nq}^{Tq}$ from the application of the discount factor (4.56%, 8.48%, 12.77% and 14.82% for scenarios 3 to 6 respectively). The reason for this is that given the assumed market conditions the policyholder can find opportunities to lock in potential market gain by resetting his maturity guarantee. This will in turn lead to the discount factor being applied which will reduce the level of his maturity guarantee. A reduced level of maturity guarantee offers a lower protection against potential adverse market conditions. This lower protection propagates to a decrease in the value of the segregated fund contract.

4.3.4 Experiment 1: Fluctuating the discount factor

In this experiment the aim is to examine the effect of fluctuating the discount factor (β) applied to the maturity guarantee every time the policyholder exercises his reset option on both the OEB and the $V_{1,Nq}^{Tq}$. In particular, β is

fluctuated from its original value of 5% to the range 0% to 10% at increments of 2.5%.

Figure 26 illustrates that across all scenarios as β increases, the OEB shifts upwards, while the opposite holds if β decreases. The rational behind this finding is that a higher level of β will lead to a lower value of the maturity guarantee. Clearly, this is unattractive to the policyholder who in turn requires

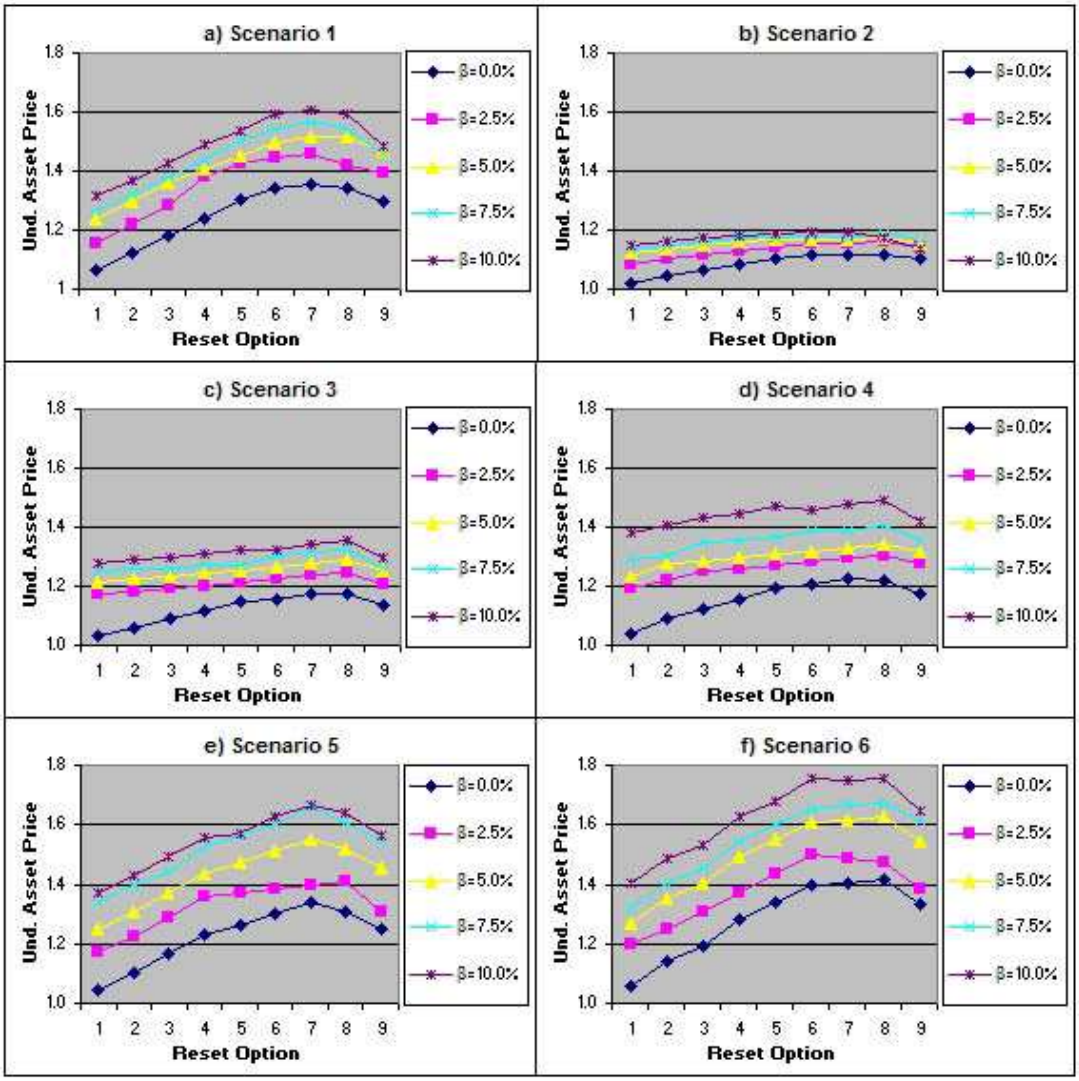

Figure 26: ***VarG Experiment 1 - Fluctuating the discount factor***

a higher return in order to exercise his reset option optimally. Therefore, leading the OEB to shift upwards. The opposite holds for a lower values of β.

Further, table 9 illustrates that decreasing the value of β causes the $V_{1,Nq}^{Tq}$ to increase across all scenarios. The opposite holds when the value of β is increased.

β	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
0.0%	0.61%	0.05%	4.78%	9.27%	14.64%	17.40%
2.5%	0.12%	0.01%	1.36%	3.21%	5.96%	7.46%
5.0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7.5%	-0.13%	0.00%	-0.86%	-2.12%	-4.09%	-5.25%
10.0%	-0.14%	0.00%	-1.48%	-3.72%	-7.25%	-9.29%

Table 9: ***VarG Experiment 1 - Fluctuating the discount factor***

The rationale of this observation is that a reduced value of β leads to an increased value of G which in turn translates to a better protection against adverse market conditions. Subsequently, the increased value of the maturity guarantee leads to an overall increased value of the segregated fund contract. The higher the decrease in the value of β the higher the increase of $V_{1,Nq}^{Tq}$. Further, the $V_{1,Nq}^{Tq}$ of scenarios 1 and 2 is least affected by the decrease in the value of β. For the other scenarios, it holds that the $V_{1,Nq}^{Tq}$ of the scenarios with higher coefficient of variation is more increased as β is decreased. Possible reasons behind these observations are equivalent to ones analysed in the previous section. The opposite holds when the value of β is increased.

4.3.5 Experiment 2: Fluctuating the maturity guarantee

In this experiment the aim is to examine the effect of the fluctuation of G on both the OEB and the $V_{1,Nq}^{Tq}$. In particular, G is fluctuated from its original value of 100% to the range 80% to 120% at increments of 5%.

Figure 27 illustrates that across all scenarios as G increases, the OEB shifts

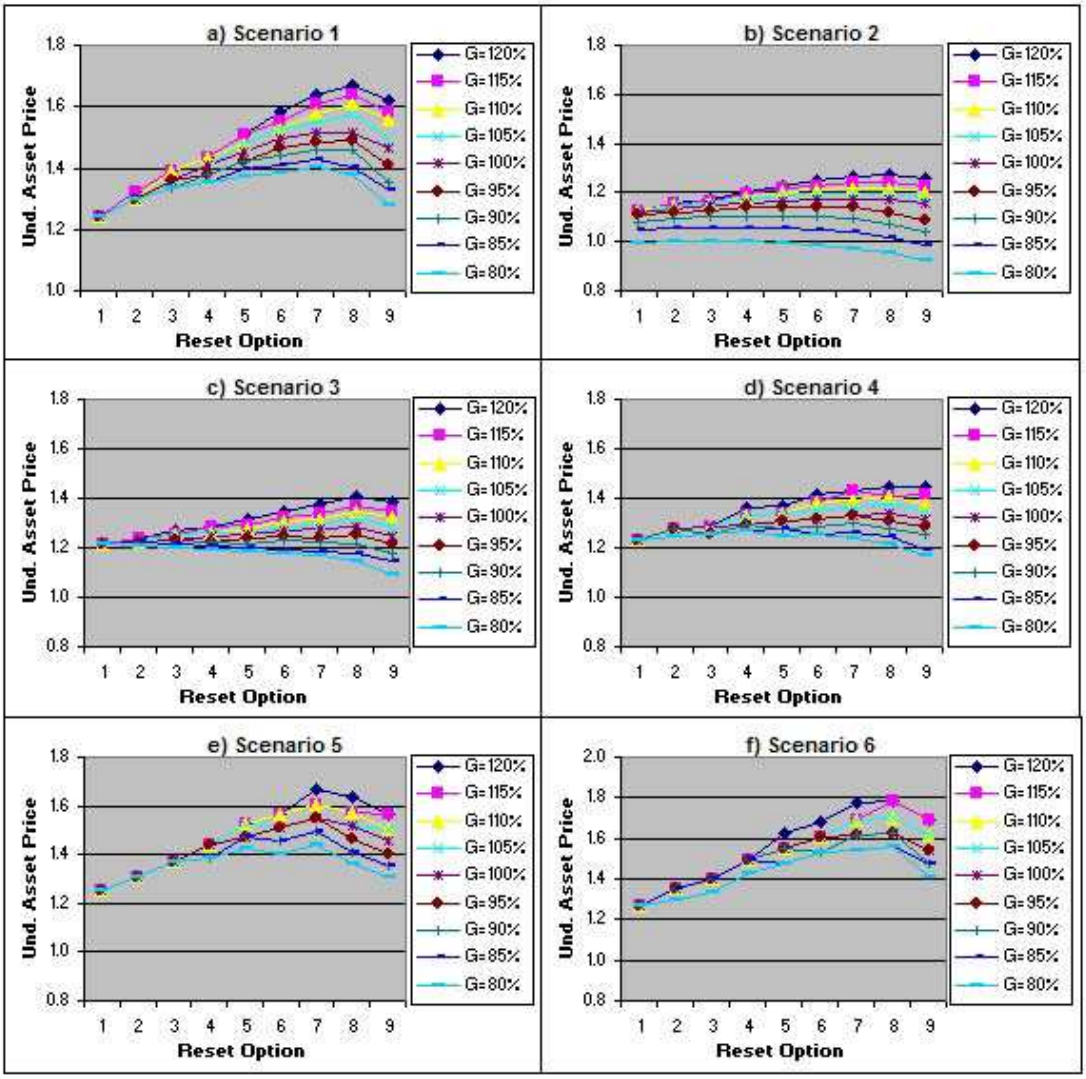

Figure 27: ***VarG Experiment 2 - Fluctuating the maturity guarantee***

upwards with a larger increase towards the maturity of the contract. Likewise, figure 27 illustrates that across all scenarios, as G decreases the OEB shifts downwards with a larger decrease towards the maturity of the contract.

Further, table 10 illustrates that decreasing the value of G causes the $V_{1,Nq}^{Tq}$ to decrease across all scenarios. Similarly, increasing the value of G causes the $V_{1,Nq}^{Tq}$ to increase across all scenarios. The rationale of this observation is equivalent to the one provided in section 3.10.1.

The higher the coefficient of variation of the assumed market conditions, the higher the impact from the fluctuation of G. It is noteworthy that for scenarios 1 and 2, which are least affected by the fluctuation of G, the policyholder would most probably prefer to switch to risk free at the first available opportunity. Therefore, he will not be making much use of the maturity guarantee.

G	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
80%	-0.90%	-0.05%	-8.46%	-15.08%	-20.22%	-21.92%
85%	-0.75%	-0.05%	-6.85%	-11.75%	-15.52%	-16.77%
90%	-0.56%	-0.04%	-4.91%	-8.13%	-10.58%	-11.41%
95%	-0.31%	-0.03%	-2.63%	-4.21%	-5.41%	-5.82%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
105%	0.38%	0.08%	2.99%	4.50%	5.64%	6.04%
110%	0.83%	0.26%	6.36%	9.29%	11.52%	12.31%
115%	1.36%	0.61%	10.10%	14.37%	17.62%	18.79%
120%	1.97%	1.22%	14.21%	19.75%	23.98%	25.50%

Table 10: ***VarG Experiment 2 - Fluctuating the maturity guarantee***

4.3.6 Experiment 3: Value of the reset option

The aim of this experiment is to estimate the value of offering reset options on a segregated fund contract under different sets of values of the model parameters. In particular, the value of offering reset options is calculated for the original set of values of the model parameters and then one of the parameters of interest (G, β) is fluctuated at a time, while keeping the rest of the parameters constant. The value of the reset options is calculated as in section 3.10.2. The results of this experiment are summarised in table 11.

G	β	***Sc.1***	***Sc.2***	***Sc.3***	***Sc.4***	***Sc.5***	***Sc.6***
100%	5.00%	0.38%	0.01%	4.05%	14.55%	44.77%	70.26%
80%	5.00%	0.22%	0.00%	3.08%	14.27%	41.87%	64.50%
100%	7.50%	0.25%	0.00%	3.15%	12.10%	36.07%	55.86%
100%	2.50%	0.49%	0.01%	5.47%	18.20%	50.33%	76.78%

Table 11: ***VarG Experiment 3 - Value of the reset option***

The first row of table 11 shows the value of the reset options under the original values for the parameters of interest. In particular, for scenarios 1 and 2 the reset options have a minimal effect on the value to the segregated fund (0.38% and 0.01% respectively). The reason for this, as already analysed in earlier sections, is that the policyholder has a preference to invest in risk free throughout the planning horizon, rather than in a segregated fund. However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (4.05%, 14.55%, 44.77% and 70.26% for scenarios 3 to 6 respectively).

Row 2 summarise the first sub-experiment where G is reduced from its original value of 100% to 80%, while all other parameters of interest are kept constant. The effect is that the value of the reset options is decreased across all scenarios. The rationale for this observation is the same as in section 3.10.2. Overall, for scenarios 1 and 2 embedding the reset options adds only a small value to the segregated fund (0.22% and 0.00% respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (3.08%, 14.27%, 41.87% and 64.50% for scenarios 3 to 6 respectively).

Rows 3 to 4 summarise the second sub-experiment where the discount factor (β) applied to the maturity guarantee every time the policyholder exercises his reset option is fluctuated from its original value of 5% to the range 2.5% to 7.5% at increments of 2.5% while all other parameters of interest are kept constant. As β is decreased to 2.5% the value of offering the reset options increases across all scenarios. (0.49%, 0.01%, 5.47%, 18.20%, 50.33% and 76.78% for scenarios 1 to 6 respectively). The rationale for this observation is that a lower in magnitude level of β will lead to an increased value of G which in turn translates to a better protection against adverse market conditions. Subsequently, the increased value of the maturity guarantee leads to a increased value of an option to reset it. The opposite behaviour can be observed when β is increased.

4.3.7 Experiment 4: OEB of all segregated funds

The aim of this experiment is to depict and classify, in accordance to section 3.10.3, the OEB of all the segregated funds generated during the planning horizon of the policyholder. As illustrated in figure 28, scenario 1 and 2 are of type C while scenarios 3 to 6 of type B.

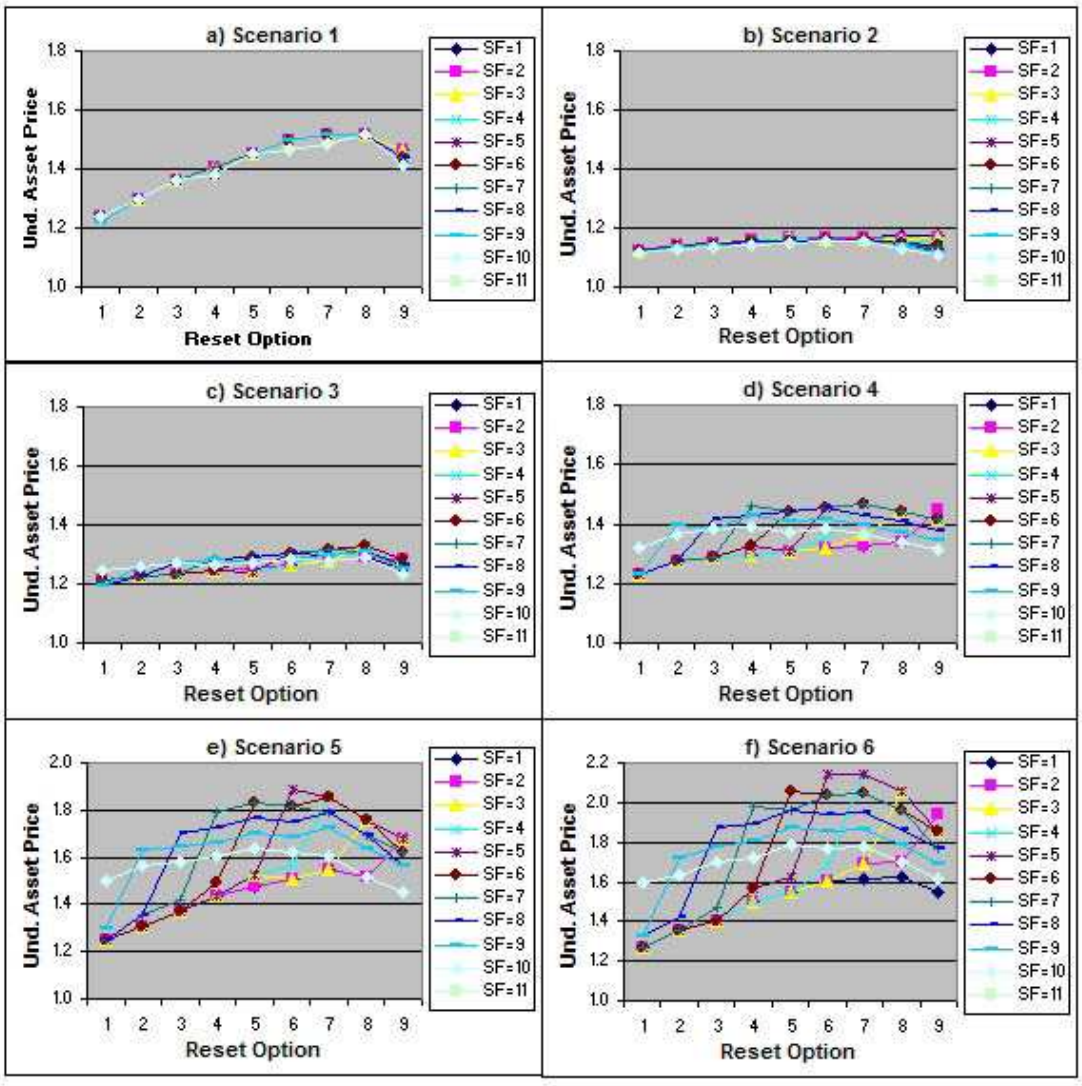

Figure 28: ***VarG Experiment 4 - Depicting the OEB of all segregated funds***

4.4 Variable Timing of Exercising the Reset Option

Increased competition in the market place has led some of the issuers of segregated fund contracts to offer to policyholders more reset decision dates, while keeping the total number of reset options constant. In other words, the policyholder still has the standard 1 reset per policy year, but can decide whether to reset his maturity guarantee more often, than at the anniversary of the contract. CIA recommends that one should examine cases where the policyholder can decide whether to reset his maturity guarantee at least every quarter of the policy year, assuming one reset option every policy year [Canadian Institute of Actuaries, 2002].

In order to address this issue, the Variable Timing of Exercising the Reset Option model (TimRO) has been built, which extends the SRM in that it lifts the restriction that the policyholder can only exercise his reset option at the end of each policy year. In particular, under the TimRO model the policyholder is allowed to reset the maturity guarantee at any point in time within each policy year from the start of the planning horizon, but only once.

To facilitate this extension, to the SRM model, a new parameter H is introduced which represents the number of the reset decision points in a policy year. These are regularly spaced with the last one falling at the end of the policy year. Crucially, the total number of periods in a policy year (q) has to be a multiple of H, so that it is possible to determine the value of the

underlying asset at each reset decision point. If q was not a multiple of H the model would generate possible values of the underlying asset for points in time which do not cover all reset decision points. It follows that q/H represents the number of periods between two consecutive reset decision points, and is denoted by h.

A further issue to consider is the level of q and H. When H is large, this would be very demanding computationally and arguably unnecessary as the underlying asset values are only changing slightly from period to period. It is also arguably impractical to consider the reset decision at every period (e.g. the standard $q = 1000$ would mean 3 times a day). Therefore, the aim is to choose H in such a way as to balance the computational complexity and the extra flexibility offered to the investor.

Section 4.4.1 offers the formulation of the model while section 4.4.2 provides a flowchart analysis of the model. Then section 4.4.3 highlights the main results of the TimRO model. As with previous models in this thesis, it is of interest to experiment with the values of several parameters and observe the effect of their fluctuation on both the OEB and the maximum expected return from the investment. The aim of these experiments is to check the robustness of the model as well as to depict interesting trends and causalities. In particular, section 4.4.4 examines the effect of the fluctuation of the level of G offered to the policyholder. Section 4.4.5 examines the value of offering the reset option

under different model parameters and lastly section 4.4.6 depicts the OEB of all the segregated funds generated during the planning horizon of the policyholder.

4.4.1 Formulation

The TimRO can be formulated similarly to the SRM model. The main difference lies in the definition of the reset decision point. In the SRM model at the end of each of the first $T-N$ years of the planning horizon a reset decision point exists. Under the TimRO model a reset decision point exists at a number discrete points within each of the first $T-N$ years of the planning horizon with the restriction that the policyholder cannot exercise the reset option on the maturity guarantee more than once in any of the $T-N$ years. Essentially, under the SRM model the policyholder can exercise his reset option at the end of each of the first $T-N$ years of the planning horizon, whereas under the TimRO model the policyholder can exercise his reset option several times in each of the first $T-N$ years but only once within each year. Hence, there are significantly more reset decision points for examination which leads to a considerable increase in the computational complexity of the model.

Stage (denoted by t) which is the number of periods until the end of the planning horizon, where $0 \leq t \leq Tq$.

State Space (denoted by S^t) which is the set of possible states at stage t. The defining characteristics of the possible states are the same as in the

SRM model.

The first state variable is the current value of the underlying asset relative to its value at the time of the investment in the current segregated fund (denoted by a) which is of the form $a = (1+u)^i(1+d)^{Nq-n-i}$, where $0 \leq i \leq Nq - n$.

The second state variable is the number of periods until the maturity of the current segregated fund contract denoted by n. This variable must satisfy the same conditions as set out in the formulation of the SRM model.

The third state variable represents whether the policyholder is allowed to reset his maturity guarantee at the first policy year or not. This variable is denoted by b and can either be equal to 1, when a reset is allowed, or 0, when a reset is not allowed in the first policy year.

Decision Space (denoted by $D^t_{a,n,b}$) - which is the set of possible decisions that can be taken in state $[a, n, b]$ at stage t. The main difference between the decision spaces of the SRM model and of the TimRO model is the definition of the decision point. In the TimRO model there are more decision points as a reset may be exercised at any time period within the policy year rather than solely at the anniversary of the contract. Formally, state $[a, n, b]$, at stage t can be defined as a decision point if $(n, t) \in E'$ where $E' = \{(n, t) : 0 < n < Nq,\ t \geq Nq,\ n = kh$ for

some $k \in Z$ and $\lfloor \frac{t+Nq-n}{q} \rfloor \neq \lfloor \frac{t}{q} \rfloor$. The first three conditions in the definition of a decision point are the same as in the original model. The fourth condition is the one which ensures that while the policyholder can consider whether to reset his maturity guarantee at any point in time within the policy year, he can only exercise it once.

The term t is the number of periods until the end of the planning horizon, so the $\lfloor \frac{t}{q} \rfloor$ (which means the smallest integer smaller than or equal to $\frac{t}{q}$) represents the current year. The term $t + Nq - n$ is the number of periods until the end of the planning horizon at the time of the initial investment in this fund. Assuming this is the time of the last reset decision, the floor of this term divided by q represents the year in which the fund was last reset. If $\lfloor \frac{t+Nq-n}{q} \rfloor = \lfloor \frac{t}{q} \rfloor\}$ then the policyholder cannot exercise his reset option because of the restriction on one reset per year. Hence, the decision space is formally stated as:

$$D^t_{a,n,b} = \begin{cases} \{\text{risk free}\} \text{ if } t < Nq \text{ and } n = 0 \\ \{\text{reinvest, risk free}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ \{\text{rollover}\} \text{ if } n > 0 \text{ and } (n, t) \notin E' \\ \{\text{reset, rollover}\} \text{ if } (n, t) \in E' \end{cases}$$

State transitions in state $[a, 0, b]$ at stage t the action risk free determines the final value of the investment by multiplying the current value of the investment by $(1 + r)^t$. Further, in state $[a, 0, b]$ at stage t the action reinvest causes an instantaneous transition to state $[1, Nq, 1]$. Also, in

state $[a, n, b]$ at stage t the action rollover causes a transition to state $[a(1+u), n-1, b]$ at stage $t-1$ with probability p or state $[a(1+d), n-1, b]$ at stage $t-1$ with probability $1-p$. Lastly, in state $[a, n, b]$ at stage t the action reset causes an instantaneous transition to state $[1, Nq, 0]$.

The aim of the policyholder is to maximise the expected payoff of investment at the end of the planning horizon, after Tq time periods. Let $V^t_{a,n,b}$ be the maximum expected payoff of the investment at the end of the planning horizon, after t time periods, when investment is currently in a fund with n time periods to go to maturity, a current relative value of a and a maturity guarantee of G. Therefore, the aim is to find $X \max\{(1+r)^{Tq}, V^{Tq}_{1,Nq,1}\}$ where:

$$V^t_{a,n,b} = \begin{cases} \max(a, G)(1+r)^t \text{ if } t < Nq \text{ and } n = 0 \\ \max(a, G)\max\{(1+r)^t, V^t_{1,Nq,b}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ pV^{t-1}_{a(1+u),n-1,b} + (1-p)V^{t-1}_{a(1+d),n-1,b} \text{ if } (n,t) \notin E' \\ \max(pV^{t-1}_{a(1+u),n-1,b} + (1-p)V^{t-1}_{a(1+d),n-1,b}, aV^t_{1,Nq,b}) \text{ if } (n,t) \in E' \end{cases}$$

4.4.2 Flowchart Analysis

As illustrated in figure 29, the main difference of theTimRO and the SRM models is the introduction of the new parameter H which represents the number of the reset decision points in a year. This parameter has been introduced to

facilitate the introduction of allowing the policyholder to have the option to reset his maturity guarantee at any point in time within each year rather than only at the end of the year as per the SRM model. However, there is a restriction that the policyholder can, still, only exercise his reset option once per year. This is achieved by the new condition: $\lfloor \frac{t+Nq-n}{q} \rfloor \neq \lfloor \frac{t}{q} \rfloor \}$.

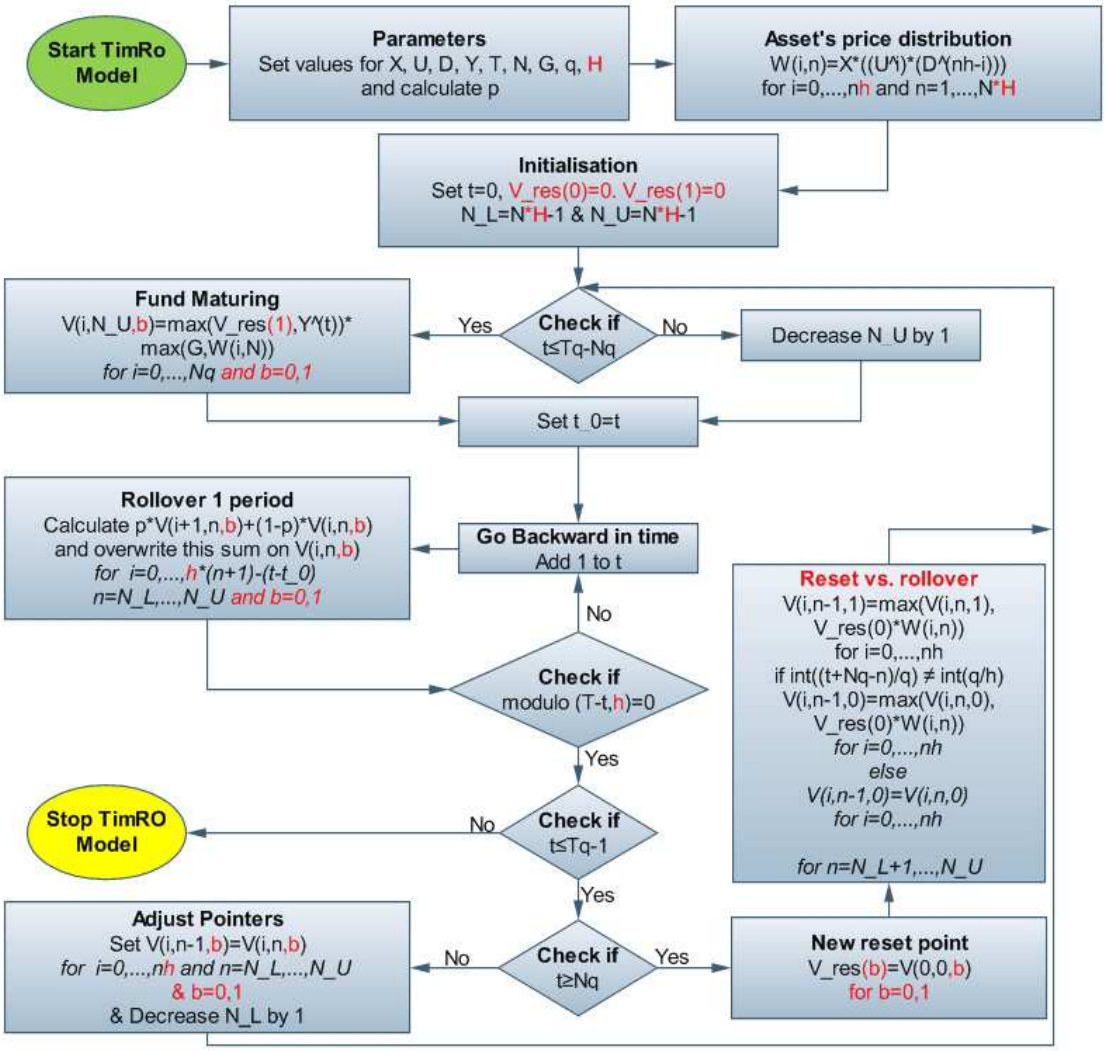

Figure 29: ***Flowchart of TimRO model***

4.4.3 Main Results

The parameters of the model have been set so that each policy year has been split in 20 discrete time periods (i.e. $H = 20$). Therefore, the total number of reset decision points (i.e. points in the OEB) is not 9 like the SRM, but 180. The main findings from allowing the policyholder to reset his maturity

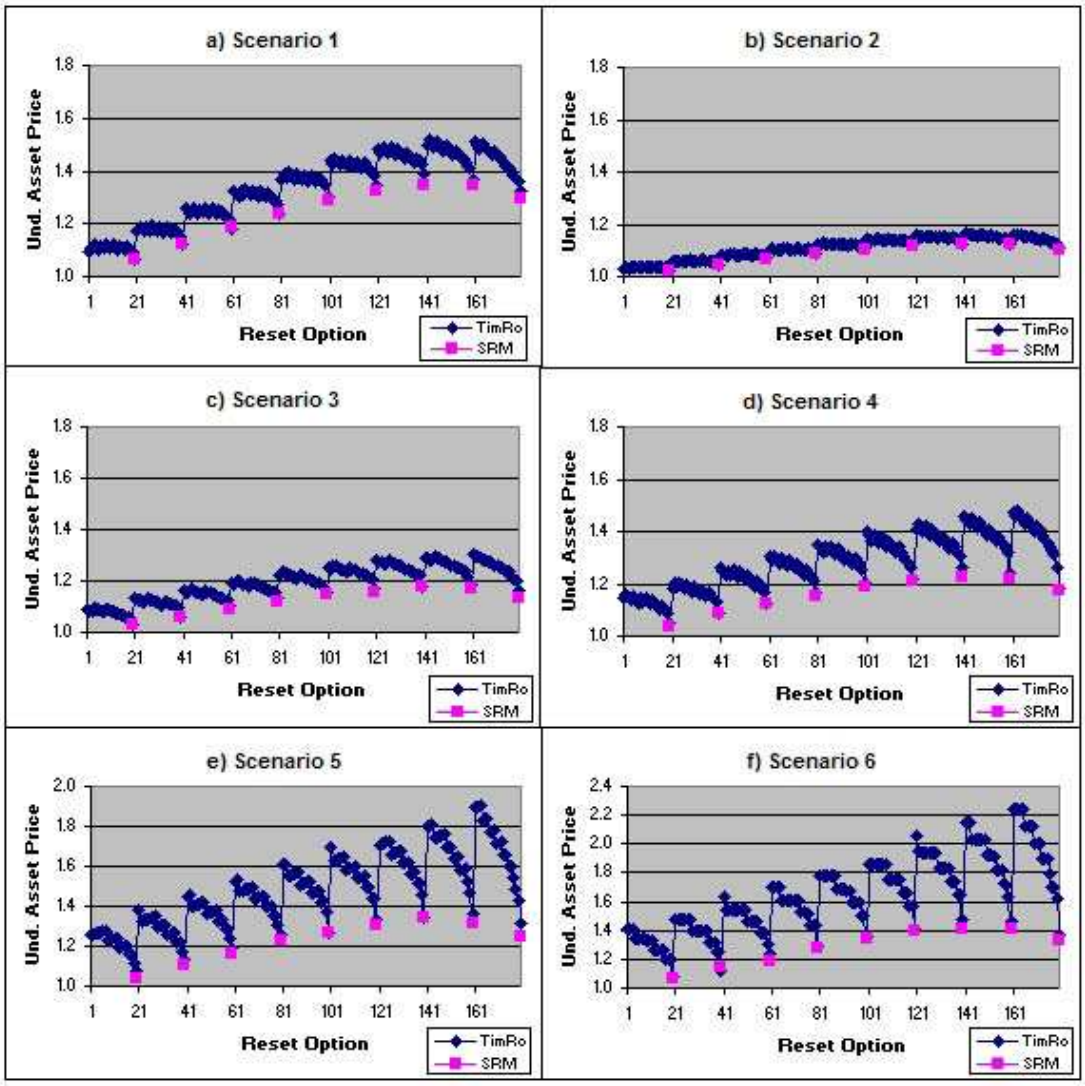

Figure 30: ***TimRO - Effect on OEB***

guarantee at any point within each policy year (but only once) are the following. At the beginning of each policy year (i.e. every 20 reset decision points) the OEB exhibits a jump. The reason for this is that the policyholder receives a new reset option at that date. Also figure 30 illustrates that as the end of each policy year is approached the OEB steadily drops. The reason for this is that unless the policyholder exercises his reset option before the end of the policy year, the option will expire worthless. The jumps and the drops are more acute towards the maturity of the fund.

A further observation, from figure 30, is that the values in the OEB which correspond to the twentieth reset decision point in any of the policy years is very similar to the values in the OEB of the SRM model. This is indeed a good verification of the model's results as the twentieth reset decision date in each policy year represents the reset decision date at the anniversary of the contract, which is the time when the policyholder is allowed to reset under the assumptions of SRM. This observation holds true regardless of the assumed market conditions of the scenarios examined.

As illustrated in figure 31, allowing the reset to be exercised at any point within the policy year does not affect much the $V_{1,Nq}^{Tq}$ of scenarios 1 and 2 (0.17% and 0.05% increase respectively). A possible reason for this finding is that given the market conditions assumed in scenarios 1 and 2 the policyholder would prefer to invest in risk free, compared to segregated funds, throughout

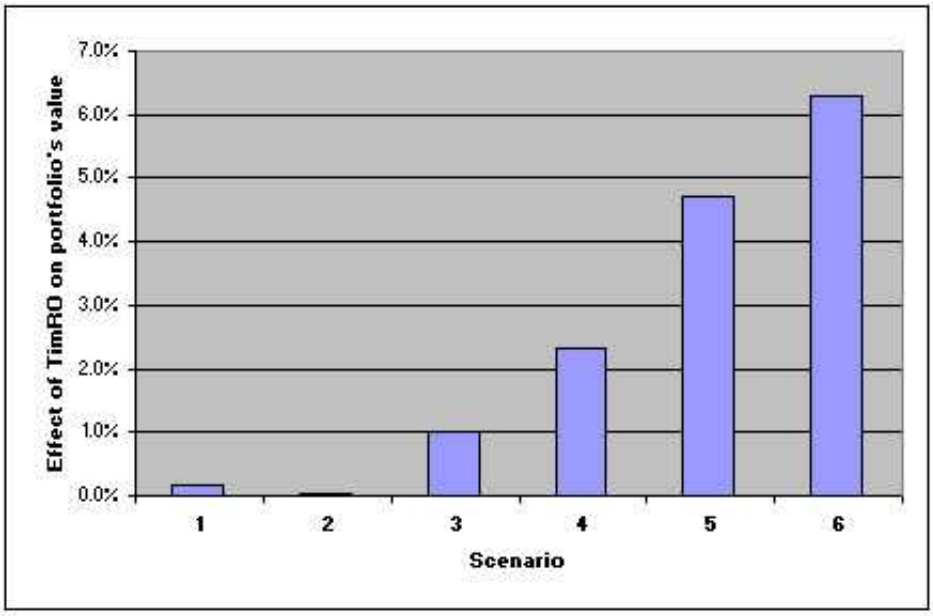

Figure 31: ***TimRO - Effect on*** $V_{1,Nq}^{Tq}$

the planning horizon. Therefore, the policyholder will choose to switch to risk free at the first available opportunity. In order to do that he will choose not to reset so as not to extend the maturity of the contract. Thus, if he is unlikely to reset his maturity guarantee, having more reset decision dates should not have a great effect in the total value of his contract. For the other scenarios it holds that the greater the coefficient of variation the greater the increase in the $V_{1,Nq}^{Tq}$ (1.02%, 2.32%, 4.71% and 6.29% for scenarios 3 to 6 respectively).

4.4.4 Experiment 1: Fluctuating the maturity guarantee

In this experiment the aim is to examine the effect of the fluctuation of G on both the OEB and the $V_{1,Nq}^{Tq}$. In particular, G is fluctuated from its original value of 100% to the range 80% to 120% at increments of 10%. The increments of 10% were chosen, rather than the increments of 5% used in previous similar

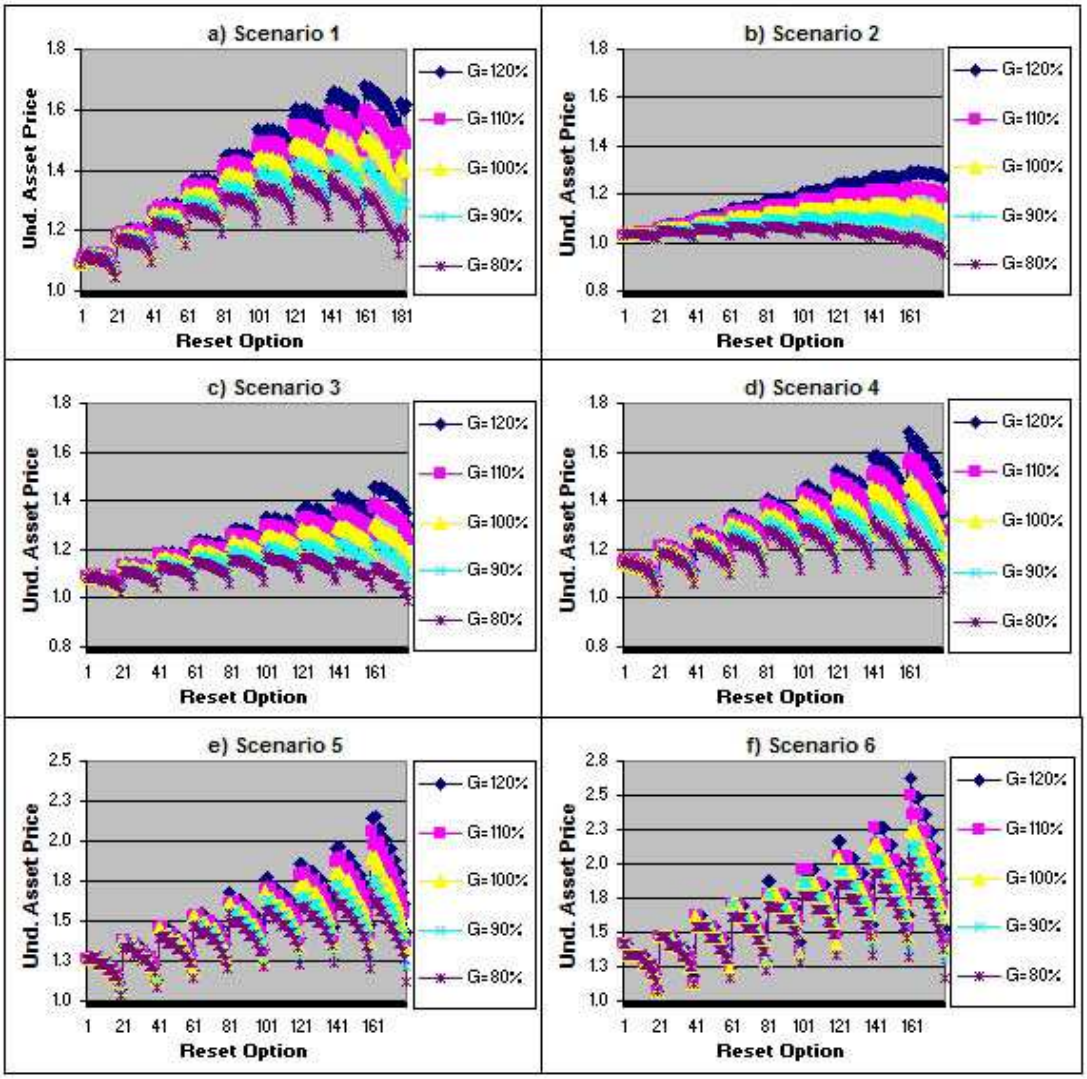

Figure 32: ***TimRO Experiment 1 - Fluctuating the maturity guarantee***

examples, in order to enhance the clarity of the figure.

Figure 32 illustrates that across all scenarios as G increases, the OEB shifts upwards with a larger increase towards the maturity of the contract. Likewise, figure 32 illustrates that across all scenarios as G decreases, the OEB shifts downwards with a larger decrease towards the maturity of the contract.

Further, table 12 illustrates that decreasing the value of G causes the $V_{1,Nq}^{Tq}$ to decrease across all scenarios. Similarly, increasing the value of G causes the $V_{1,Nq}^{Tq}$ to increase across all scenarios. The rationale of this observation is equivalent to the one provided in section 3.10.1.

G	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
80%	-1.09%	-0.08%	-9.20%	-15.62%	-20.55%	-22.18%
90%	-0.67%	-0.07%	-5.28%	-8.39%	-10.75%	-11.54%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
110%	0.96%	0.38%	6.72%	9.54%	11.69%	12.44%
120%	2.27%	1.63%	14.90%	20.23%	24.33%	25.79%

Table 12: ***TimRO Experiment 1 - Fluctuating the maturity guarantee***

4.4.5 Experiment 2: Value of the reset option

The aim of this experiment is to estimate the value of offering reset options on a segregated fund contract under different sets of values of the model parameters. In particular, the value of offering reset options is calculated for the original set of values of the model parameters and then the parameter of interest (G) is fluctuated, while keeping the rest of the parameters constant. The value of the reset options is calculated as in section 3.10.2 The results of this experiment are summarised in table 13.

G	***Sc.1***	***Sc.2***	***Sc.3***	***Sc.4***	***Sc.5***	***Sc.6***
100%	1.13%	0.06%	10.04%	27.94%	70.22%	105.26%
90%	0.65%	0.02%	6.85%	22.73%	62.00%	94.73%
80%	0.34%	0.01%	4.23%	17.70%	53.53%	83.75%

Table 13: ***TimRO Experiment 2 - Value of the reset option***

The first row of table 13 shows the value of the reset options under the original values for the parameters of interest. In particular, for scenarios 1 and 2 the reset options adds a small value to the segregated fund (1.13% and 0.06% respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (10.04%, 27.94%, 70.22% and 105.26% for scenarios 3 to 6 respectively). A possible reason for the small value added by embedding reset options on segregated funds under the market condition assumed in scenarios 1 and 2 is that the return to the policyholder

from investing in risk free throughout the planning horizon is higher compared to investing in a segregated fund. Thus, the policyholder will switch to risk free at the first available opportunity, thus not really needing or making use of the reset options.

Rows 2 and 3 summarise the sub-experiment where G is fluctuated from its original value of 100% to the range 80% to 100% at increments of 10% while all other parameters of interest are kept constant. As the value of G is decreased to 80% the value of the reset options is decreased across all scenarios. The rationale for this observation is the same as the one provided in section 3.10.2. Overall, for scenarios 1 and 2 embedding the reset options adds only a small value to the segregated fund (0.34% and 0.01% respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (4.23%, 17.70%, 53.53% and 83.75% for scenario 3 to 6 respectively).

4.4.6 Experiment 3: OEB of all segregated funds

The aim of this experiment is to depict and classify, in accordance to section 3.10.3, the OEB of all the segregated funds generated during the planning horizon of the policyholder. As illustrated in figure 33, all scenarios are of type C.

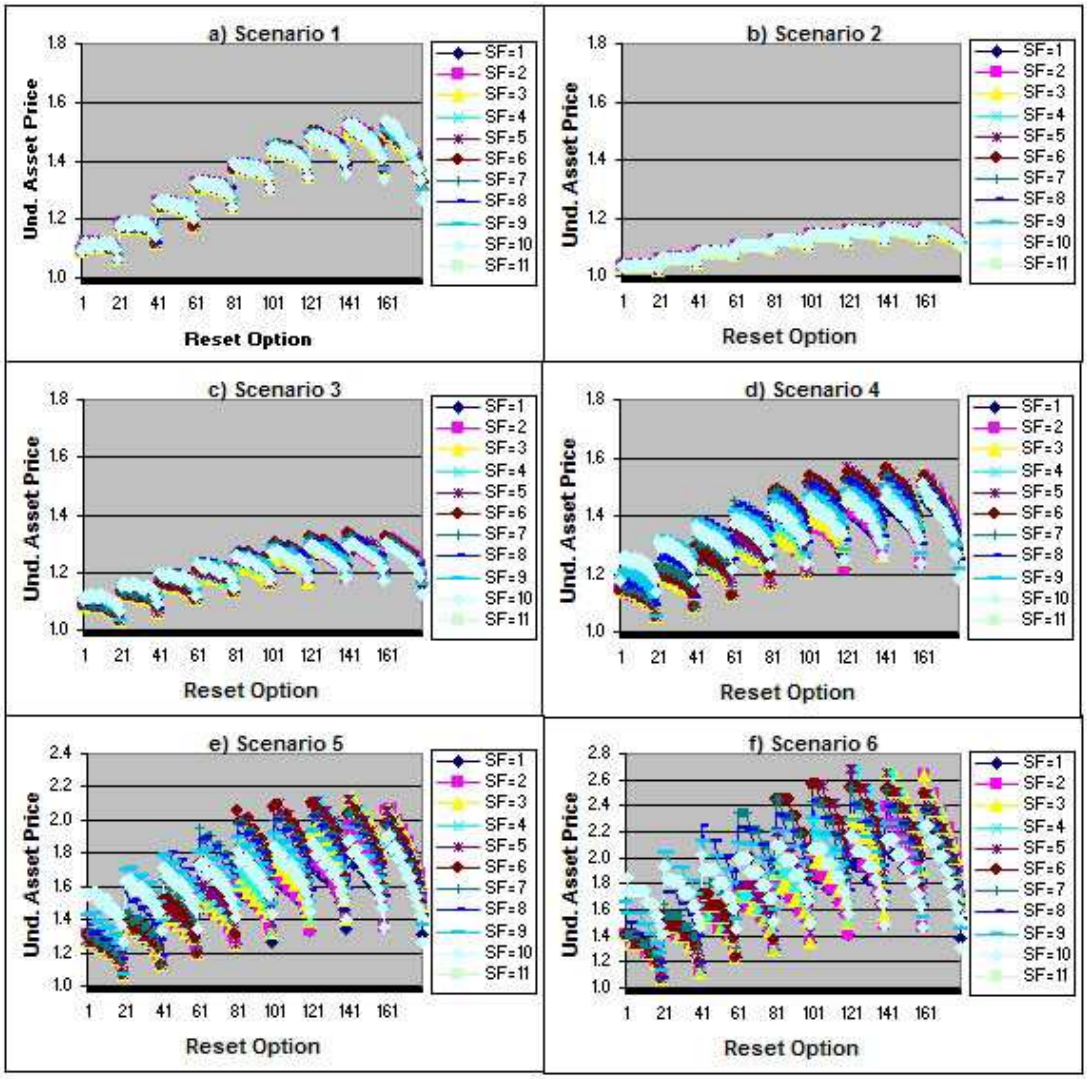

Figure 33: ***TimRO Experiment 3 - Depicting the OEB of all segregated funds***

4.5 Discussion

In this chapter an attempt has been made to enhance and extend the SRM model in order to reflect on the characteristics of the wide range of segregated fund contracts which are traded in the market. To this end, three different extensions have been added to the model.

The Management Expense Ratio (MER) model extends the SRM in that it allows the issuer of the segregated fund contract to charge a management fee to the policyholder. In particular, it has been assumed that the issuer charges the policyholder a fee equal to a fixed proportion of the value of the fund at the end of each policy year.

The main findings from incorporating this fee is that the policyholder requires a higher return in order to optimally reset his maturity guarantee (i.e. OEB shifts upwards) while the total value of the segregated fund contract is diminished due to the negative effect of the fee. This finding is more acute when the assumed market conditions are such that the maximum expected return from investing in a segregated fund is less than the return from investing in risk free throughout the planning horizon. Under such market conditions, the policyholder will choose to switch to risk free at the first available opportunity. A possible reason for this is that the relative gain for the policyholder from switching to the risk free rate of interest is greater as the introduction of the fee invariably reduces the potential return from staying in the segregated fund.

Further, the level of management expense ratio which can negate the cost of embedding reset options to a segregated fund contract was depicted. These "break-even" values of the management expense ratio increase as the coefficient of variation of the assumed market conditions increases. This finding is in line with the observation made above. Depicting the "break-even" values can be very interesting and useful for both the policyholders and issuers. From the point of view of the issuer, it should help with risk management policies as well as to convince the relevant regulatory authorities that they indeed take appropriate cover for the risk that they face from issuing segregated funds with embedded reset options. From the point of view of the policyholder, it should signal that if the underlying asset is not very risky, thus has a low coefficient of variation, they should not be willing to pay a relatively high management expense ratio. The opposite should hold for a risky underlying asset.

The Variable Maturity Guarantee (VarG) model extends the SRM in that the maturity guarantee becomes a function of the number of times that the reset option has been exercised since the maturity of the last segregated fund or the start of the planning horizon (whichever is most recent). In particular, every time the policyholder exercises his reset option the maturity guarantee is reduced by a pre-determined discount factor.

The main findings from incorporating the discount factor to the maturity guarantee is that the policyholder requires a higher return in order to optimally

reset his maturity guarantee (i.e. OEB shifts upwards) while the total value of the segregated fund is diminished due to the negative effect of the discount factor on the total value of the segregated fund contract. This observation holds true, if under the assumed market conditions the policyholder prefers to invest in a segregated fund compared to investing in risk free throughout the planning horizon.

The Variable Timing of Exercising the Reset Option Model (TimRO) extends the SRM in that it lifts the restriction that the policyholder can only exercise his reset option at the end of each policy year. In particular, under the TimRO model the policyholder is allowed to reset the maturity guarantee at any point in time within each policy year from the start of the planning horizon, but only once.

The main findings from the TimRO model are the following. Firstly, at the beginning of each policy year the OEB exhibits a jump. The reason for this is that the policyholder receives a new reset option at that date. Secondly, as the end of each policy year is approached the OEB steadily drops. The reason for this is that unless the policyholder exercises his reset option before the end of the policy year, the option will expire worthless. The jumps and the drops are more acute towards the maturity of the segregated fund contract.

Further, the total value of the fund is increased, compared to the SRM model, since the policyholder may lock in higher market gains as he has more

reset decision points (but the same number of total reset options). This finding, however, does not hold true if the assumed market conditions are such that the policyholder would prefer to invest in risk free throughout the planning horizon, compared to investing in a segregated fund. The rationale is that under such market conditions, the policyholder will choose to switch to risk free at the first available opportunity. In order to do that he will choose not to reset so as not to extend the maturity of the contract. Thus, if he is unlikely to reset his maturity guarantee, having more reset decision dates should not have a great effect in the total value of his contract.

Lastly, it was derived that the values in the OEB which correspond to the last reset decision point in any of the policy years is very similar to the values in the OEB of the SRM model. This is indeed a good verification of the model's results as the last reset decision date in each policy year represents the reset decision date at the anniversary of the contract, which is the time when the policyholder is allowed to reset under the assumptions of SRM. This observation holds true regardless of the assumed market conditions of the scenarios examined.

5 Incorporating Stock Market Jumps

5.1 Introduction

As mentioned in section 3.2, so far in this thesis the returns from the investment in a segregated fund have been modeled using a discrete-time approximation of the lognormal model, namely the "Binomial Tree Method". While the lognormal model underpins the well known and widely used Black Scholes model it has been criticised, among other reasons, because empirical data of stock markets returns do not seem to follow the lognormal random walk [Bates, 1991, Heston, 1993, Wilmott, 1998].

As a matter of fact, several empirical studies have demonstrated the existence of jumps (both negative and positive) in the stock markets, the foreign exchange markets and the bond markets [Bates, 1996, Jorion, 1988, Carr et al., 2002]. If a negative jump, of similar level to 1987, was to occur simultaneously on the stock markets the result would be a loss of trillions of British pounds.

It is beyond the scope of this thesis to comprehensively review and analyse alternative models (to the lognormal model) that have been proposed in the literature. However, the proposed models can be briefly split in three categories [Hull, 2006]. Firstly, one can retain the property of the lognormal model that the asset price changes continuously, but assume an alternative process to the Geometric Brownian motion. These models are known as diffusion models

[Heston, 1993, Hull and White, 1987]. Secondly, one can overlay continuous asset price changes with jumps. These models are known as mixed jump-diffusion models [Merton, 1976, Bjork et al., 1997, Duffie et al., 2000, Kou, 2002]. Thirdly, one can assume a process where all the asset prices changes that take place are jumps. These models are known as pure jump models [Madan et al., 1998].

In order to incorporate shocks in the model but to also preserve the comparability of the model's results with the results of previous chapters it has been decided to keep the lognormal model but overlay it with stochastic negative jumps (crash). Section 5.2 extends the SRM model in that it allows for instantaneous stochastic crashes to occur within the single regime of the SRM model, namely through the Stochastic Crash Model (SCM). In reality the evolution of the possible values of the underlying asset price is the same as with the SRM model. However, at every time period there is a small probability of a crash occurring. When a crash occurs, the residual value of the fund after the crash is equal to a fixed percentage of its original value. The aim of this model is to update the OEB of the SRM model in order to advise both the policyholders and the issuers when faced with the risk of stock market crashes.

Following that, in section 5.3, the Double Regime Model (DRM) is built which provides alternative means to incorporate jumps into the SRM model. In contrast to the SCM model, the DRM model is able to incorporate both

negative (crash) and positive (surge) jumps as well as a combination of the two. In particular, it allows the underlying asset to switch between two distinct regimes. The market characteristics of the first regime are defined by the relevant scenario under examination and are equivalent to the ones used under the SRM in order to facilitate comparisons. The second regime is intended to model periods of high volatility in the markets and can be used to incorporate the jumps. Two distinct applications of the DRM are presented in sections 5.4 and 5.5 respectively.

In the first application the second regime, models the case where there is a large probability that the value of the underlying asset will marginally increase or a very small probability that it will drop by a substantial fixed percentage, thus essentially allowing only crashes like the SCM. The parameters and transitions probabilities have been set so that a crash is as likely to happen and of the same magnitude as in the SCM, in order to facilitate comparisons. In the second application the second regime models the case where the stock market can exhibit variable jumps (i.e. both crashes and surges) with equal probability of occurrence. In particular, there is an equal probability that the value of the underlying asset will either increase by a large fixed percentage or it will drop by an equal in magnitude fixed percentage. Essentially, it is modeling a highly unstable market environment.

5.2 Stochastic Crash Model (SCM)

The Stochastic Crash Model (SCM) extends the SRM model in that it allows for random instantaneous crashes (negative jumps) to occur within the single regime of the SRM model. The market characteristics of the scenarios examined in the SCM model have been selected to be the same six scenarios which were defined in section 3.8.3 in order to facilitate interesting comparisons of results.

It is noteworthy that the evolution of the possible values of the underlying asset is the same as with the SRM model. However, at every time period there is a small probability (p_c) that a crash may occur. In case of the crash materialising, the residual value of the underlying asset is equal to a fixed percentage of its original value (C).

The aim of this model is to update the optimal reset strategy derived under the SRM model in order to advise both the policyholders and the issuers when faced with the risk of a sudden large decrease in the value of their underlying asset due to a stock market crash. The model is analysed in sections 5.2.1 and 5.2.2 and the main results as well as the sensitivity analysis are provided in sections 5.2.3 to 5.2.7.

5.2.1 Formulation

The Stochastic Crash Model (SCM) can be formulated to comprise the following four elements:

Stage (denoted by t) which is the number of periods until the end of the planning horizon, where $0 \leq t \leq Tq$.

State Space (denoted by S^t) which is the set of possible states at stage t. The defining characteristics of the possible states are the following. The first two are the same as with the SRM model whereas the third is new.

The first state variable is the current value of the underlying asset relative to its value at the time of the investment in the current segregated fund, ignoring any crashes, (denoted by a) which is of the form $a = (1+u)^i(1+d)^{Nq-n-i}$, where $0 \leq i \leq Nq - n$.

The second state variable is the number of periods until the maturity of the current segregated fund contract denoted by n. This variable must satisfy the same conditions as set out in the original formulation.

The third, and last, state variable is the number of times that the stock market has experienced a crash during the lifetime of the segregated fund contract, denoted by f where $0 \leq f \leq Nq - n$.

Decision Space (denoted by $D^t_{a,n,f}$) - which is the set of possible decisions that can be taken in state $[a, n, f]$ at stage t. As the decisions only

depend on n and t the decision space of the SCM model is essentially the same as in the SRM model and is formally stated as:

$$D^t_{a,n,f} = \begin{cases} \text{risk free if } t < Nq \text{ and } n = 0 \\ \text{reinvest, risk free if } t \geq Nq \text{ and } n = 0 \\ \text{rollover if } n > 0 \text{ and } (n,t) \notin E \\ \text{rollover, reset if } (n,t) \in E \end{cases}$$

State transitions in state $[a, 0, f]$ at stage t the action risk free determines the final value of the investment by multiplying the current value of the investment by $(1 + r)^t$. Further, in state $[a, 0, f]$ at stage t the action reinvest causes an instantaneous transition to state $[1, Nq, 0]$. Also, in state $[a, n, f]$ at stage t if $f < Nq - n$ the action rollover causes a transition to state $[a(1 + u), n - 1, f]$ at stage $t - 1$ with probability $(1 - p_c)p$, or state $[a(1 + d), n - 1, f]$ at stage $t - 1$ with probability $(1 - p_c)(1 - p)$, or state $[a(1 + u), n - 1, f + 1]$ at stage $t - 1$ with probability $p_c p$, or state $[a(1 + d), n - 1, f + 1]$ at stage $t - 1$ with probability $p_c(1 - p)$, where p_c denotes the probability of a stock market crash occurring. Lastly, in state $[a, n, f]$ at stage t the action reset causes an instantaneous transition to state $[1, Nq, 0]$.

The aim of the policyholder is to maximise the expected payoff of investment at the end of the planning horizon, after Tq time periods. Let $V^t_{a,n,f}$ be the maximum expected payoff of the investment at the end of the

planning horizon, after t time periods, when investment is currently in a fund with n time periods to go to maturity, a current relative value of aC^f and a maturity guarantee of G. Therefore, the aim is to find $X \max\{(1+r)^{Tq}, V^{Tq}_{1,Nq,0}\}$ where:

$$V^t_{a,n} = \begin{cases} \max(aC^f, G)(1+r)^t \text{ if } t < Nq \text{ and } n = 0 \\ \\ \max(aC^f, G)\max\{(1+r)^t, V^t_{1,Nq,0}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ \\ (1-p_c)(pV^{t-1}_{a(1+u),n-1,f} + (1-p)V^{t-1}_{a(1+d),n-1,f}) \\ +p_c(pV^{t-1}_{a(1+u),n-1,f+1} + (1-p)V^{t-1}_{a(1+d),n-1,f+1}) \\ \text{ if } n > 0 \text{ and } (n,t) \notin E \\ \\ \max\{(1-p_c)(pV^{t-1}_{a(1+u),n-1,f} + (1-p)V^{t-1}_{a(1+d),n-1,f})+ \\ p_c(pV^{t-1}_{a(1+u),n-1,f+1} + (1-p)V^{t-1}_{a(1+d),n-1,f+1}), V^t_{1,Nq,0}\} \\ \text{ if } (n,t) \in E \end{cases}$$

5.2.2 Flowchart Analysis

As illustrated in figure 34 the main difference between SCM and SRM is that SCM models overlays the continuous asset price changes with jumps. In order to achieve that an extra state has been introduced, denoted by f, which is the number of times that the stock market crashes during the lifetime of the segregated fund contract. This increases the dimensions of array $V(i,n,f)$ and makes the value of investing on a segregated fund at any given point in time an array indexed by f, denoted as $V_{res}(f)$. Moreover, there is a new parameter, denoted by C, which is the residual value of the underlying asset after the crash occurs. This affects the value of array $V(i,n,f)$, which is now a function

of C.

A further new parameter denoted by p_c, is the probability of a crash occurring. This new parameter is used in the calculation of the value of the segregated fund when the maturity guarantee is rolled over for one time period.

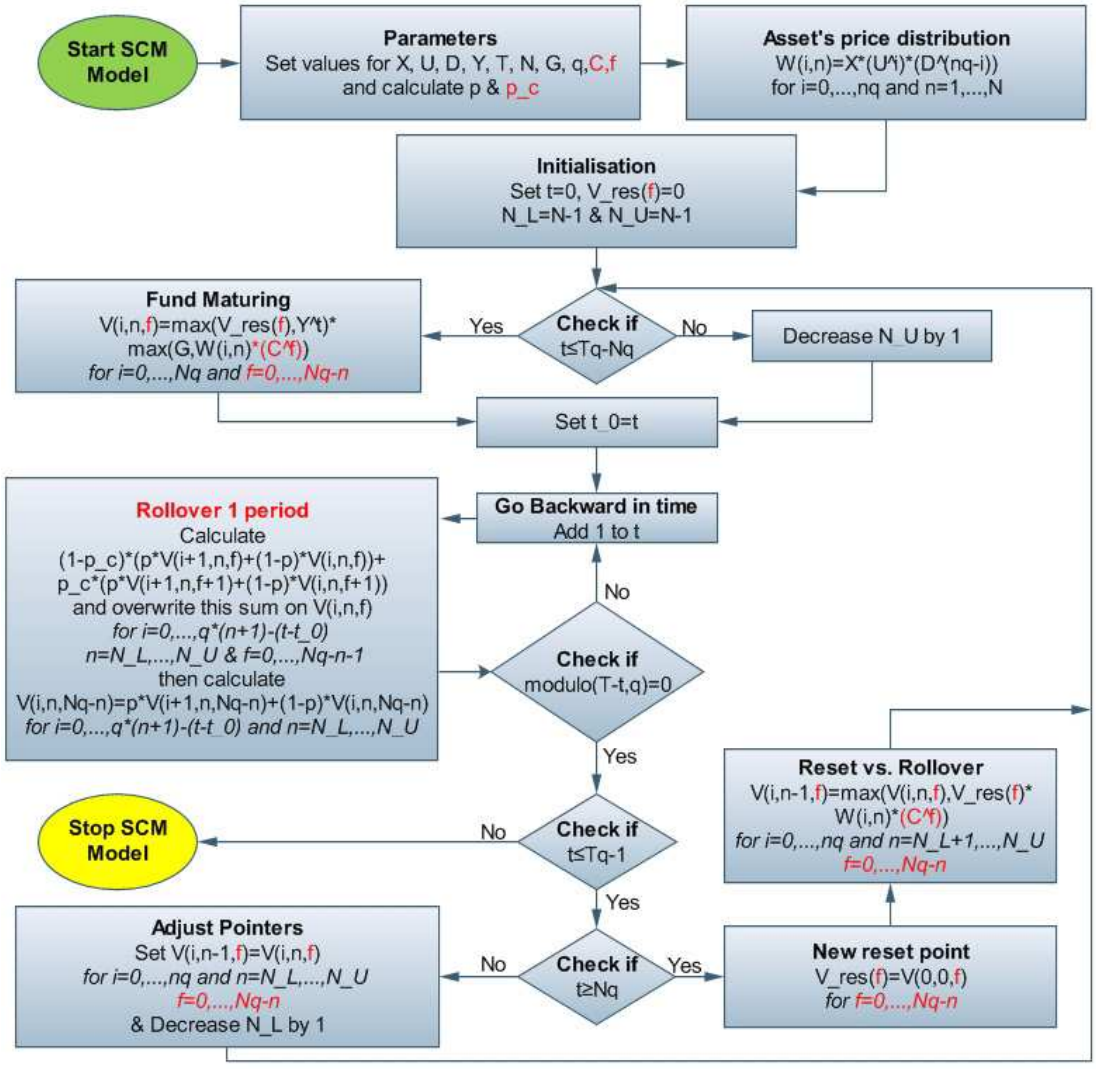

Figure 34: ***Flowchart of SCM model***

5.2.3 Main Results

In the formulation of the SCM model, up to $Nq - n$ crashes are allowed to take place during the lifetime of any one segregated fund contract. However, for computational purposes, a tighter upper bound on f has to be introduced, denoted by F. The intuitive explanation is that after F crashes during the life of any one segregated fund, the effect of any further crashes can be ignored as the value of the underlying asset relative to its original value at the time of investment, is highly likely to be less than the maturity guarantee. Therefore, under such conditions the policyholder will not reset the level of his maturity guarantee and the fund will be worth G at maturity. Thus, even if a further crash was to take place, during the lifetime of the same segregated fund, the policyholder would still choose not to reset, hence the value of the fund would be the same as before, i.e. it would be worth G at maturity.

This was tested empirically by considering increasing F from 0,1,2,... to as high as the personal computer used allowed. The assumption was that the value of the fund would converge to a limit very quickly as F increases. The finding of this test was that the value of the fund converges with $F = 1$. Increasing the value of F from 1, had negligible effects on both the value of the fund and OEB. It should be highlighted that F is the limit of crashes which can occur during the lifetime of one segregated fund. Thus, if the policyholder resets or reinvests in a new segregated fund, a further crash is allowed to happen.

A further issue to consider was the frequency of the crashes that should be allowed. As Hull [2006] highlights, approximately every decade, there is one major shock in the stock markets. Therefore, the probability of a crash occurring (p_c) was set so that a crash occurs every 10 policy years. Lastly, it was assumed that the residual value of the fund after the crash is equal to 80% of its original value.

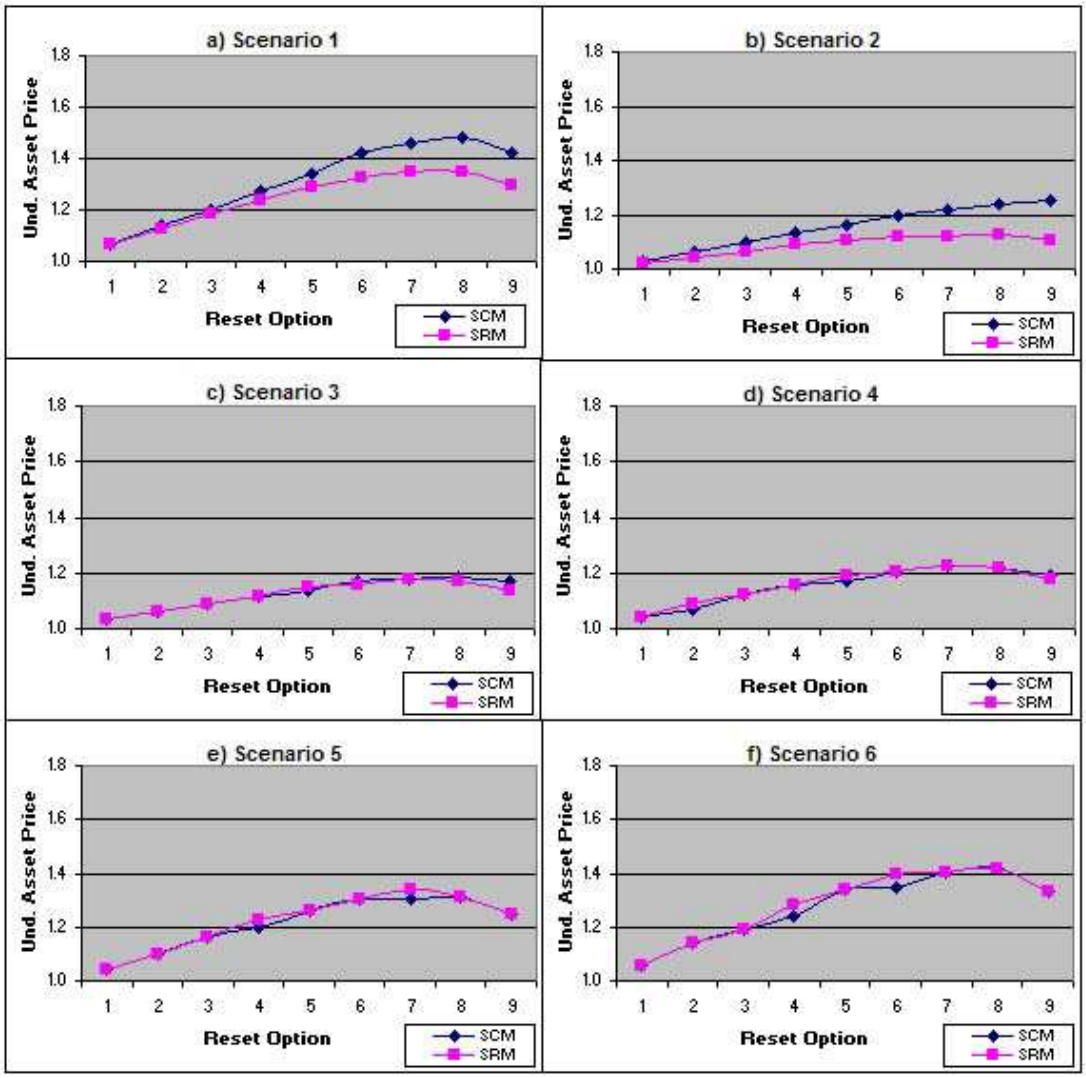

Figure 35: ***SCM - Effect on OEB***

The main finding from incorporating such crashes is that, under certain market conditions, the policyholder requires a higher return in order to optimally reset his maturity guarantee (i.e. OEB shifts upwards) while the total value of the segregated fund is diminished due to the negative effect of the crashes on the underlying asset value. In particular, as illustrated in figure 35 the incorporation of crashes causes the OEB of scenarios 1 and 2 to shift upwards with a larger increase towards the maturity of the contract, while the OEB of scenarios 3 to 6 is practically unaffected.

The justification for this observation is that for scenarios 1 and 2 the maximum expected return to the policyholder from investing in risk free throughout the planning horizon is higher than from investing in a segregated fund. Therefore, unless the policyholder sees exceptional growth in the fund, he will look for the quickest way out of the fund, which is by essentially not resetting or, more precisely, requiring a very high return in order to optimally reset. For scenarios 3 to 6 it holds that the maturity guarantee acts as a safety net which protects the maximum expected return to the policyholder. Despite the crash, if the value of the underlying asset relative to its original value at investment, is less than the maturity guarantee, the policyholder's expected return will be equal to at least the maturity guarantee.

In terms of the $V^{Tq}_{1,Nq,0}$, as illustrated in figure 36, scenario 1 is most affected by the incorporation of the crash (decrease of 10.98%). For the other scenarios

it holds that the lower the coefficient of variation, the greater the percentage decrease in the $V^{Tq}_{1,Nq,0}$ from the incorporation of the shock (10.38%, 9.94%, 9.06%, 8.36% and 8.10% for scenarios 2 to 6 respectively).

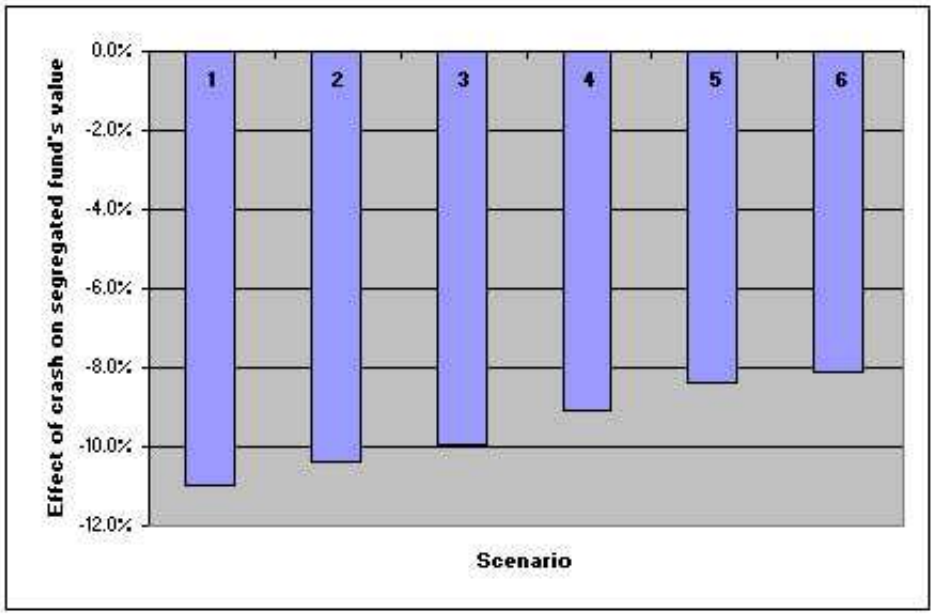

Figure 36: ***SCM - Effect on*** $V^{Tq}_{1,Nq,0}$

As with the previous models analysed in this thesis, it is of interest to experiment with the values of several parameters and observe the effect of their fluctuation on both the OEB and $V^{Tq}_{1,Nq,0}$. The aim of these experiments is to check the robustness of the model as well as to depict interesting trends and causalities. In particular, section 5.2.4 examines the effect of the fluctuation of the residual value of the portfolio after a crash (C), section 5.2.5 examines the effect of the fluctuation of the level of G offered to the policyholder, section 5.2.6 examines the value of offering the reset option under different model parameters and lastly section 5.2.7 depicts the OEB of all the segregated funds

generated during the planning horizon of the policyholder.

5.2.4 Experiment 1: Fluctuating the residual value of the portfolio after the crash

In this experiment the aim is to examine the effect of fluctuating the level of the portfolio's residual value after the crash (C) on both the OEB and

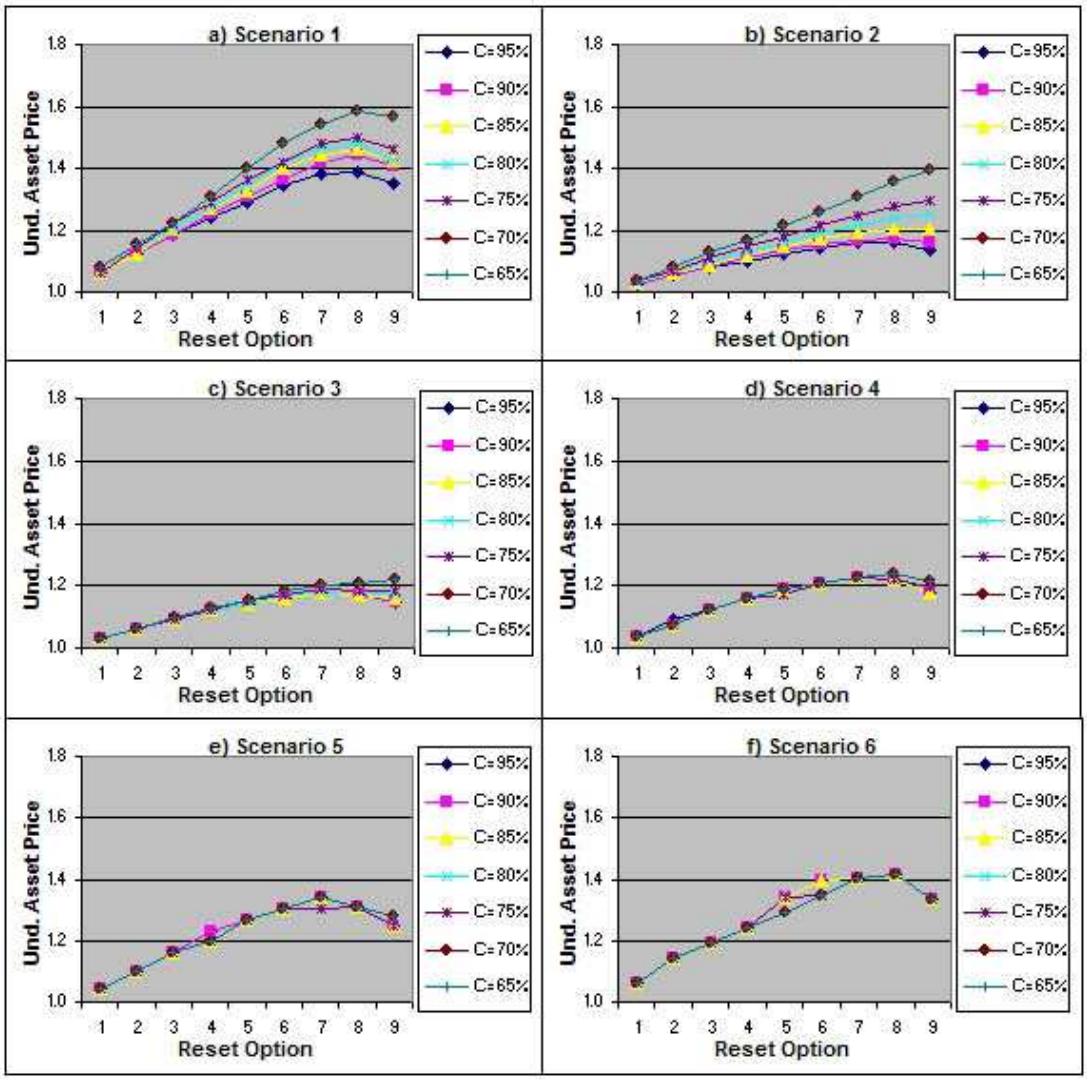

Figure 37: ***SCM Experiment 1 - Fluctuating the residual value of the portfolio after the crash***

the $V^{Tq}_{1,Nq,0}$. In particular, C is fluctuated from its original value of 80% to the range 65% to 95% at increments of 5%. Figure 37 illustrates that the fluctuation of C practically does not affect the OEB of scenarios 3 to 6 while it has a more significant effect on the OEB of scenarios 1 and 2. In particular, in scenarios 1 and 2 as C increases, the OEB shifts downwards with a larger decrease towards the maturity of the contract, whereas as C decreases the OEB shifts upwards with a larger increase towards the maturity of the contract. The rationale for the observation on scenarios 1 and 2 is the same as the one provided in the previous section. For scenarios 3 to 6 it holds that the maturity guarantee acts as a safety net which protects the maximum expected return to the policyholder. Regardless of the level of the crash, if the value of the underlying asset relative to its original value at investment, is less than the maturity guarantee, the policyholder's expected return will be equal to at least the maturity guarantee.

Further, table 14 illustrates that increasing the value of C causes the $V^{Tq}_{1,Nq,0}$ to increase across all scenarios whereas decreasing the value of C causes the $V^{Tq}_{1,Nq,0}$ to decrease across all scenarios.

The rationale of this observation is that by decreasing the value of C the value of the underlying asset price decreases thus ultimately reducing the total value of the segregated fund contract. On the other hand, by increasing the value of C the value of the underlying asset price increases, thus, ultimately

C	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
65%	-7.92%	-6.23%	-5.71%	-5.65%	-5.62%	-5.60%
70%	-5.37%	-4.59%	-3.92%	-3.86%	-3.84%	-3.83%
75%	-2.73%	-2.47%	-2.02%	-1.99%	-1.97%	-1.96%
80%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
85%	2.80%	2.71%	2.28%	2.18%	2.09%	2.06%
90%	5.68%	5.57%	4.86%	4.56%	4.31%	4.21%
95%	8.67%	8.52%	7.77%	7.15%	6.65%	6.46%

Table 14: ***SCM Experiment 1 - Fluctuating the residual value of the portfolio after the crash***

increasing the total value of the segregated fund contract. The higher the increase in the value of C the higher the percentage increase of $V^{Tq}_{1,Nq,0}$. Further, scenario 1 is the most affected by the increase of C. For the other scenarios it holds that the $V^{Tq}_{1,Nq,0}$ of the scenarios with lower coefficient of variation is more increased as C is increased. On the other hand, the higher the decrease in the value of C, the higher the percentage decrease of $V^{Tq}_{1,Nq,0}$. Again, scenario 1 is the most affected by the decrease of C. For the other scenarios it holds that the $V^{Tq}_{1,Nq,0}$ of the scenarios with lower coefficient of variation is more decreased as C is decreased.

5.2.5 Experiment 2: Fluctuating the maturity guarantee

In this experiment the aim is to examine the effect of the fluctuation of G on both the OEB and the $V^{Tq}_{1,Nq,0}$. In particular, G is fluctuated from its original value of 100% to the range 80% to 120% at increments of 5%.

Figure 38 illustrates that across all scenarios as G increases, the OEB shifts

upwards with a larger increase towards the maturity of the contract. Likewise, figure 38 illustrates that across all scenarios as G decreases, the OEB shifts downwards with a larger decrease towards the maturity of the contract.

Further, table 15 illustrates that decreasing the value of G causes the $V^{Tq}_{1,Nq,0}$ to decrease across all scenarios. Similarly, increasing the value of G causes the $V^{Tq}_{1,Nq,0}$ to increase across all scenarios. The rationale of this observation is

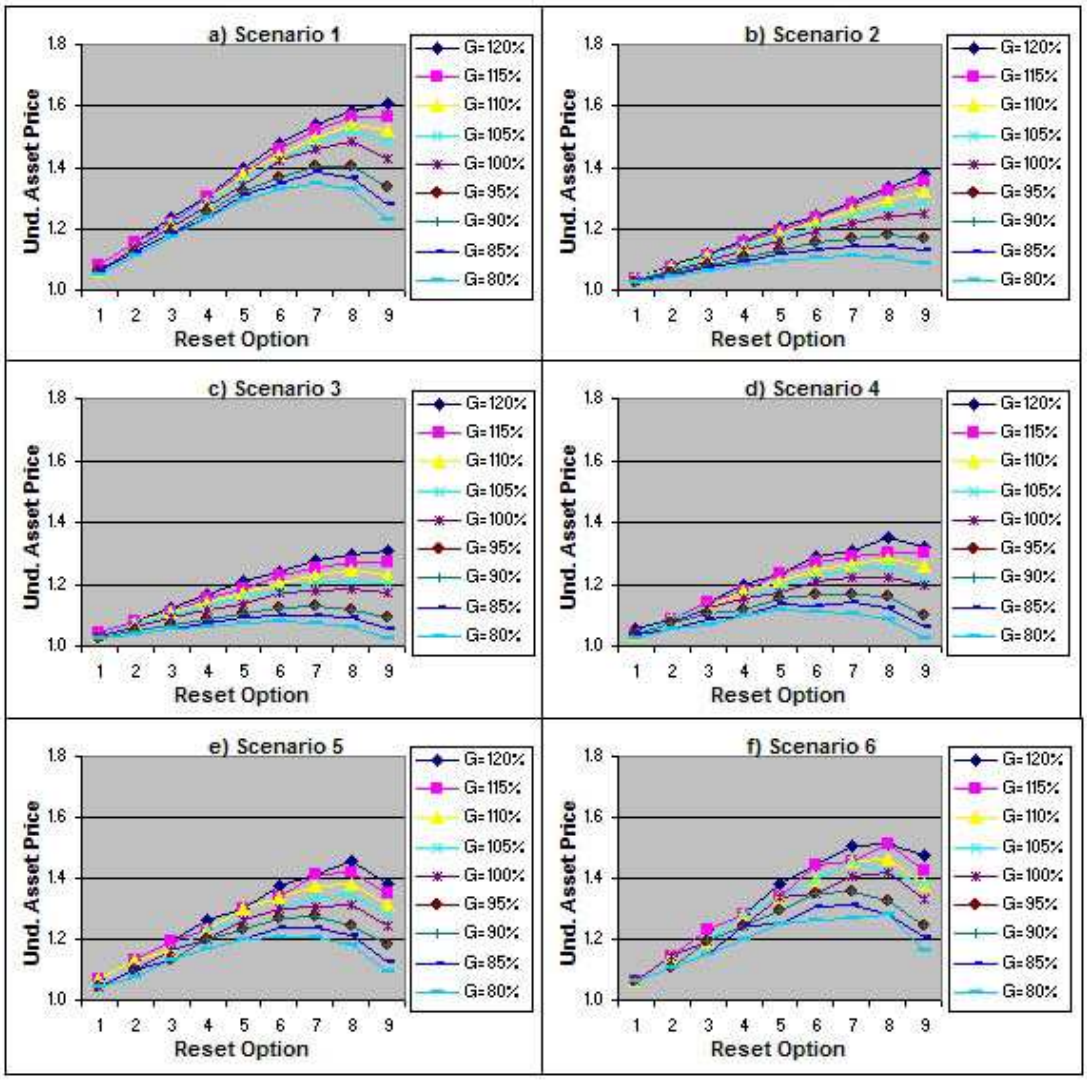

Figure 38: ***SCM Experiment 2 - Fluctuating the maturity guarantee***

equivalent to the one provided in section 3.10.1.

G	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
80%	-0.73%	-0.37%	-10.05%	-16.50%	-20.93%	-22.41%
85%	-0.61%	-0.35%	-8.18%	-12.83%	-16.06%	-17.15%
90%	-0.45%	-0.30%	-5.87%	-8.86%	-10.95%	-11.66%
95%	-0.25%	-0.20%	-3.15%	-4.58%	-5.60%	-5.95%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
105%	0.30%	0.35%	3.57%	4.88%	5.84%	6.18%
110%	0.67%	0.91%	7.55%	10.07%	11.93%	12.59%
115%	1.11%	1.76%	11.94%	15.56%	18.26%	19.23%
120%	1.66%	3.05%	16.74%	21.36%	24.84%	26.11%

Table 15: ***SCM Experiment 2 - Fluctuating the maturity guarantee***

5.2.6 Experiment 3: Value of the reset option

The aim of this experiment is to estimate the value of offering reset options on a segregated fund contract under different sets of values of the model parameters. In particular, the value of offering reset options is calculated for the original set of values of the model parameters and then one of the parameters of interest (G, C) is fluctuated at a time, while keeping the rest of the parameters constant. The value of the reset options is calculated as in section 3.10.2 The results of this experiment are summarised in table 16.

The first row of table 16 shows the value of the reset options under the original values for the parameters of interest. In particular, for scenarios 1 and 2 the reset options add a small value to the segregated fund (0.35% and 0.07%

G	C	***Sc.1***	***Sc.2***	***Sc.3***	***Sc.4***	***Sc.5***	***Sc.6***
100%	80%	0.35%	0.07%	8.04%	24.19%	61.55%	91.97%
90%	80%	0.19%	0.02%	4.95%	19.33%	54.14%	82.62%
80%	80%	0.09%	0.01%	2.48%	14.61%	46.47%	72.85%
100%	**85%**	0.38%	0.08%	8.16%	24.34%	61.75%	92.20%
100%	**75%**	0.31%	0.05%	7.98%	24.09%	61.39%	91.78%

Table 16: ***SCM Experiment 3 - Value of the reset option***

respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (8.04%, 24.19%, 61.55% and 91.97% for scenarios 3 to 6 respectively). A possible reason for the small value added by embedding reset options on segregated funds under the market condition assumed in scenarios 1 and 2 is that the return to the policyholder from investing in risk free throughout the planning horizon is higher compared to investing in a segregated fund. Thus, the policyholder will switch to risk free at the first available opportunity, thus not really needing or making use of the reset options.

Rows 2 and 3 summarise the first sub-experiment where G is fluctuated from its original value of 100% to the range 80% to 100% at increments of 10% while all other parameters of interest are kept constant. As the value of G is decreased to 80% the value of the reset options is decreased across all scenarios. The rationale for this observation is the same as the one provided in section 3.10.2. Overall, for scenarios 1 and 2 embedding the reset options adds only a small value to the segregated fund (0.09% and 0.01% respectively). However,

for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (2.48%, 14.61%, 46.47% and 72.85% for scenarios 3 to 6 respectively).

Rows 4 and 5 summarise the second sub-experiment where the portfolio's residual value after a crash (C) is fluctuated from its original value of 80% to the range 75% to 85% at increments of 5% while all other parameters of interest are kept constant. As C is increased to 85% (i.e. lower in magnitude crash occurring) the value of offering the reset options increases across all scenarios. (0.38%, 0.08%, 8.16%, 24.34%, 61.75% and 92.20% for scenarios 1 to 6 respectively). The rationale for this observation is that a lower in magnitude level of crash will lead to a higher value of the underlying asset therefore increasing the probability of the value of the underlying asset relative to its original value at investment being greater than the maturity guarantee, thus increasing the likelihood of the policyholder choosing to reset his maturity guarantee. In turn, this increases the value of the reset option. On the other hand, as C is decreased to 75% (i.e. a higher in magnitude level of crash occurring) the value of offering the reset options is decreased across all scenarios (0.31%, 0.05%, 7.98%, 24.09%, 61.39% and 91.78% for scenarios 1 to 6 respectively). The rationale for this observation is equivalent to the one offered above.

5.2.7 Experiment 4: OEB of all segregated funds

The aim of this experiment is to depict and classify, in accordance to section 3.10.3, the OEB of all the segregated funds generated during the planning horizon of the policyholder. As illustrated in figure 39, scenario 1 and 2 are of type A while scenario 3 is of type C and scenarios 4 to 6 are of type B.

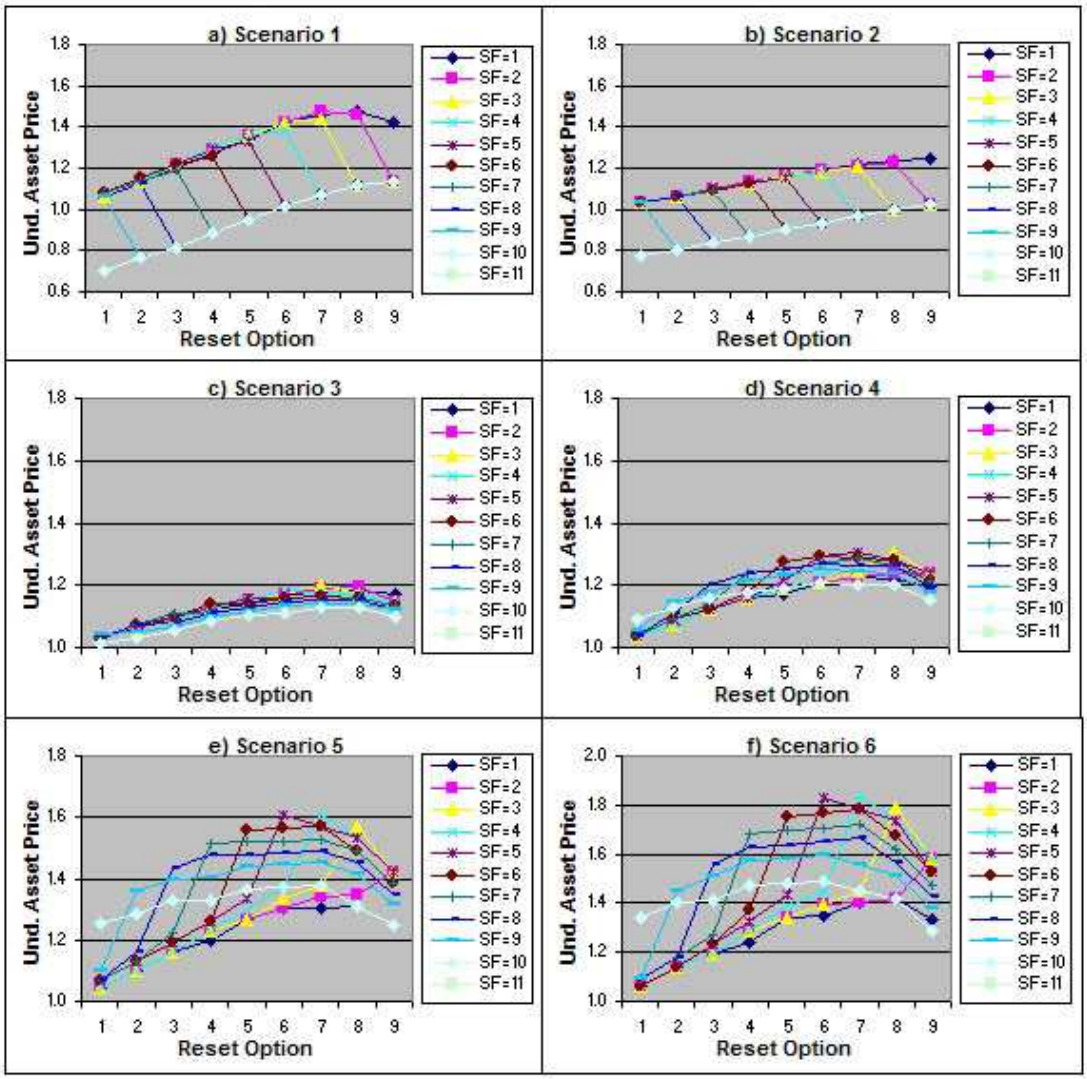

Figure 39: ***SCM Experiment 4 - Depicting the OEB of all segregated funds***

5.3 Double Regime Model

The Double Regime Model (DRM) provides alternative means to incorporate jumps into the SRM model. In contrast to the SCM model, the DRM model is able to incorporate both negative (crash) and positive (surge) jumps as well as a combination of the two. In order to achieve this it allows the underlying asset to switch between two distinct regimes. The market characteristics of the first regime (u_1, d_1 and r) are defined by the relevant scenario under examination and are equivalent to the ones used under the SRM in order to facilitate comparisons. Thus, u_1 will equal the u and d_1 will equal the d of the respective scenario, while r remains the same. The second regime is intended to model periods of high volatility in the markets, and can be used to incorporate the jumps. The market characteristics of the second regime are denoted by u_2 and d_2, while r is considered to be the same as in the first regime.

Of crucial importance to the DRM model are the transition probabilities. At any one point in time there is probability of switching from the current regime to the other regime. In particular, the probability of switching from the regime s to regime s' is denoted by $p_{s,s'}$. Once the regime that the current state is in is determined, then the next step is to determine the movement of the underlying asset. In particular, p_1 denotes the probability of the underlying asset changing by u_1, while $1 - p_1$ denotes the probability of the underlying asset changing by d_1. Further, p_2 denotes the probability of the underlying

asset changing by u_2, while $1 - p_2$ denotes the probability of the underlying asset changing by d_2.

The formulation and flowchart analysis of the DRM model are provided in sections 5.3.1 and 5.3.2 respectively. Results and sensitivity analysis are provided for two distinct cases. In the first case the parameters of the second regime are set to incorporate crashes, like the SCM model. In the second case the parameters of the second regime are set to incorporate both crashes and surges.

In particular, in section 5.4 the second regime, models the case where there is a large probability (p_2) that the underlying asset price will marginally increase (u_2) or a very small probability ($1-p_2$) that it will drop by a substantial fixed percentage ($1 - d_2$), thus essentially allowing only crashes like the SCM. The parameters and transitions probabilities have been set so that a crash is as likely to happen and of the same magnitude as in the SCM, in order to facilitate comparisons. The main results and the sensitivity analysis of this application of the DRM model (denoted by DRM-C) are provided in section 5.4.

In contrast to the previous application of the DRM model (DRM-C), the second regime of this application of the DRM model, denoted by DRM-J, allows the stock market to exhibit variable jumps (i.e. both crashes and surges) with equal probability of occurrence. In particular, the second regime, models the

case where there is an equal probability ($p_2 = 1 - p_2$) that the underlying asset price will either increase by a large fixed percentage (u_2) or it will drop by an equal in magnitude fixed percentage ($1 - d_2$). Essentially, DRM-J is modeling a highly unstable market environment where both positive and negative jumps are equally likely with the aim to update the OEB in order to advise both the policyholders and the issuers when faced with such conditions. The main results and the sensitivity analysis of this application of the DRM model are provided in section 5.5.

5.3.1 Formulation

The Double Regime model can be formulated to comprise the following four elements:

Stage (denoted by t) which is the number of periods until the end of the planning horizon, where $0 \leq t \leq Tq$.

State Space (denoted by S^t) which is the set of possible states at stage t. The defining characteristics of the possible states are the following.

The first state variable is the current value of the underlying asset relative to its value at the time of the investment in the current segregated fund (denoted by a) which is of the form $a = (1+u_1)^i(1+d_1)^j(1+u_2)^k(1+d_2)^{Nq-n-i-j-k}$, where $0 \leq i \leq Nq-n$, $0 \leq j \leq Nq-n-i$, $0 \leq k \leq Nq-n-i-j$. The term $Nq-n$ in the boundary of i represents the number

of time periods since the start of the current segregated fund. Therefore it represents the maximum number of times that the underlying asset price may have changed by $(1 + u_1)\%$. Likewise, the term $Nq - n - i$ in the boundary of j represents the number of time periods since the start of the current segregated fund minus the number of time periods during which the value of the underlying asset has changed by $(1 + u_1)\%$. Therefore it represents the maximum number of times that the underlying asset price may have changed by $(1 + d_1)\%$. Lastly, the term $Nq - n - i - j$ in the boundary of k represents the number of time periods since the start of the current segregated fund minus the number of time periods during which the value of the underlying asset has changed by $(1 + u_1)\%$ and by $(1 + d_1)\%$. Therefore it represents the maximum number of times that the underlying asset price may have changed by $(1 + u_2)\%$.

The second state variable is the number of periods until the maturity of the current segregated fund contract denoted by n. This variable must satisfy the same conditions as set out in the original formulation.

The third, and last, state variable is the regime that the asset is in, denoted by s, which can either be equal to 1 or 2. Regime 1 represents the original regime that the model commences, while regime 2 represents an alternative regime that the model occasionally switches to. The market conditions of regime 2 allow for the turbulence in the market environment

through a significantly increased coefficient of variation.

Decision Space (denoted by $D^t_{a,n,s}$) - which is the set of possible decisions that can be taken in state $[a, n, s]$ at stage t. As the decisions only depend on n and t the decision space of the DRM model is essentially the same as in the SRM model and is formally stated as:

$$D^t_{a,n,s} = \begin{cases} \{\text{risk free}\} \text{ if } t < Nq \text{ and } n = 0 \\ \{\text{reinvest, risk free}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ \{\text{rollover}\} \text{ if } n > 0 \text{ and } (n, t) \notin E \\ \{\text{rollover, reset}\} \text{ if } (n, t) \in E \end{cases}$$

State transitions in state $[a, 0, s]$ at stage t the action risk free determines the final value of the investment by multiplying the current value of the investment by $(1 + r)^t$. Further, in state $[a, 0, s]$ at stage t the action reinvest causes an instantaneous transition to state $[a, Nq, s]$.

Also, in state $[a, n, s]$ at stage t the action rollover causes a transition to state $[a(1 + u_1), n - 1, 1]$ at stage $t - 1$ with probability $p_{s,1} * p_1$, or a transition to state $[a(1 + d_1), n - 1, 1]$ at stage $t - 1$ with probability $p_{s,1} * (1 - p_1)$, or a transition to state $[a(1 + u_2), n - 1, 2]$ at stage $t - 1$ with probability $p_{s,2} * p_2$, or a transition to state $[a(1 + d_2), n - 1, 2]$ at stage $t - 1$ with probability $p_{s,2} * (1 - p_2)$.

Lastly, in state $[a, n, s]$ at stage t the action reset causes an instantaneous transition to state $[a, Nq, s]$.

The aim of the policyholder is to maximise the expected payoff of investment at the end of the planning horizon, after Tq time periods. Let $V^t_{a,n,s}$ be the maximum expected payoff of the investment at the end of the planning horizon, after t time periods, when investment is currently in a fund with n time periods to go to maturity, a current relative value of a and a maturity guarantee of G. Therefore, the aim is to find $X \max\{(1+r)^{Tq}, V^{Tq}_{1,Nq,1}\}$ where:

$$V^t_{a,n,s} = \begin{cases} \max(a, G)(1+r)^t \text{ if } t < Nq \text{ and } n = 0 \\ \\ \max(a, G)\max\{(1+r)^t, V^t_{0,Nq,m}\} \text{ if } t \geq Nq \text{ and } n = 0 \\ \\ p_{s,1} * p_1 * V^{t-1}_{a(1+u_1),n-1,1} + p_{s,1} * (1-p_1) * V^{t-1}_{a(1+d_1),n-1,1} \\ +p_{s,2} * p_2 * V^{t-1}_{a(1+u_2),n-1,2} + p_{s,2} * (1-p_2) * V^{t-1}_{a(1+d_2),n-1,2} \\ \text{if } n > 0 \text{ and } (n,t) \notin E \\ \\ \max(p_{s,1} * p_1 * V^{t-1}_{a(1+u_1),n-1,1} + p_{s,1} * (1-p_1) * V^{t-1}_{a(1+d_1),n-1,1} \\ +p_{s,2} * p_2 * V^{t-1}_{a(1+u_2),n-1,2} + p_{s,2} * (1-p_2) * V^{t-1}_{a(1+d_2),n-1,2}, V^t_{0,Nq,1}) \\ \text{if } (n,t) \in E \end{cases}$$

5.3.2 Flowchart Analysis

As it is illustrated in figure 40 the main difference between DRM and SRM models is that DRM overlays the continuous asset price changes with jumps. In order to achieve that an extra state has been introduced, denoted by s, which is the regime that the underlying asset is in. This increases the dimensions of array $V(a, n, s)$ and makes the value of investing on a segregated fund at any given point in time an array indexed by s, denoted as $V_{res}(s)$. Moreover,

there are several new parameters in the calculation of the underlying asset's price distribution. Namely, U_1 is equivalent to $1 + u_1$ in the mathematical formulation. Similar conventions apply for U_2, D_1 and D_2. These new parameters drastically increase the dimensions of array $W(n, i, j, k)$ and make the value of the underlying asset a function of U_1, U_2, D_1 and D_2. Further, the state denoted by a in the formulation is replaced in the flowchart by i, j, k, as it is a function of them. Further, the sixth process (**Rollover 1 period**) is altered significantly due to the new set of transition probabilities used in the calculation of the value of the segregated fund when the maturity guarantee is rolled over for one time period.

Overall, due to the substantial increase in the computational complexity the maximum Q that the model can run is 16. While this is much less than the 1000 that was the case in the SRM it is more than adequate as it can produce smooth and consistent OEB (see section 5.4). In reality the DRM calculates many "parallel funds" and thus achieves a higher discretization. The explanation behind this argument lies in the fact that as Q is doubled in the SRM the number of possible values of the underlying asset doubles. However, in the DRM as Q is doubled the number of values of the underlying asset increases by an eight-fold. In reality the $Q = 1000$ of the SRM produces 1001 different values for the underlying asset while the $Q = 16$ of the DRM produces 969 possible values. As a matter of fact the stabilisation in the value

of the segregated fund that can be achieved with the DRM is possibly better than the corresponding of the SRM since the range of 969 values (DRM) is smaller compared to the range of the 1001 values (SRM). The cost of the higher discretization is a significantly higher running time of the algorithm: 3 minutes and 30 seconds for the DRM compared to 30 seconds of the SRM.

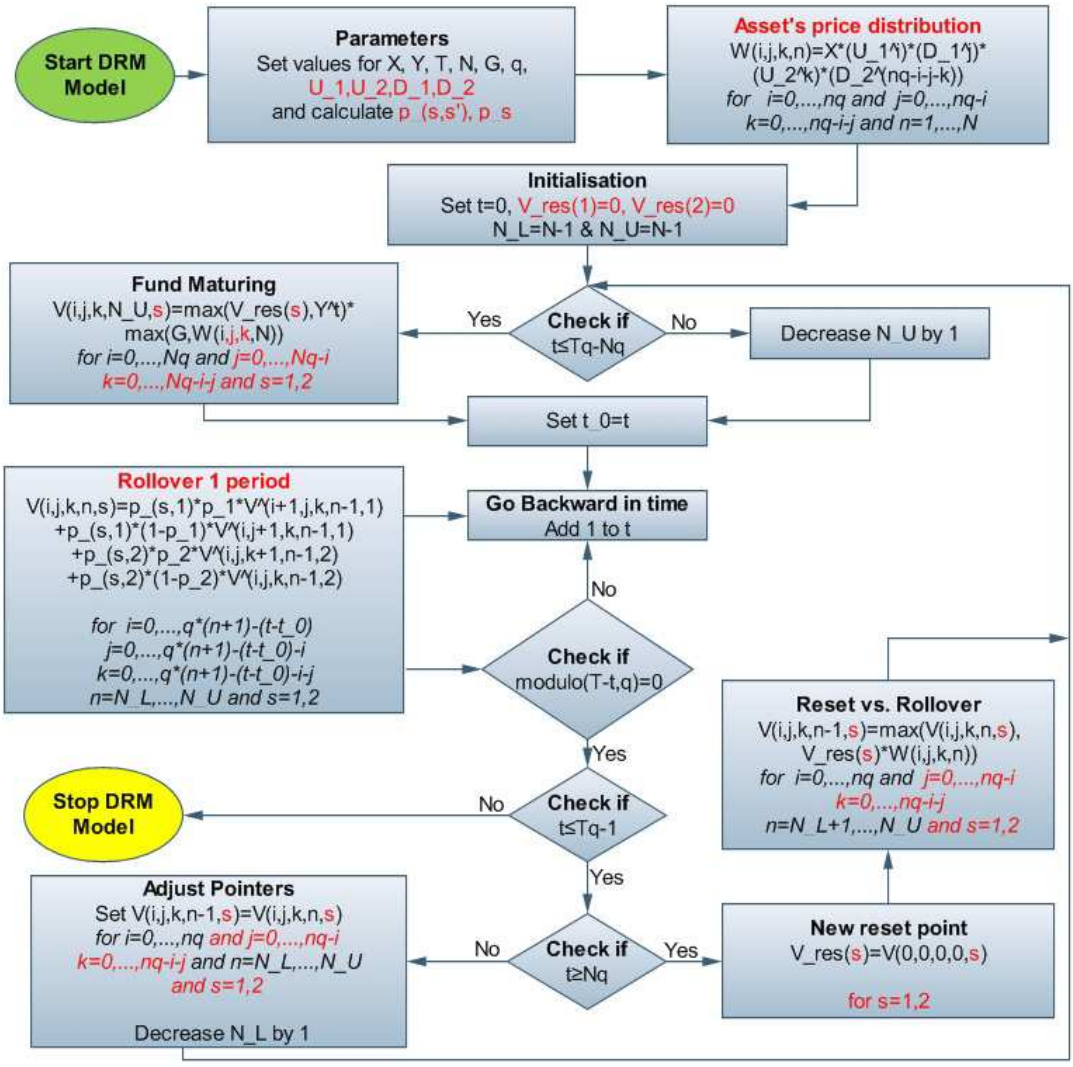

Figure 40: ***Flowchart of DRM model***

5.4 Main Results of Double Regime Model (crash)

The parameters of this application of the DRM model (denoted by DRM-C) have been set so that they are equivalent to the SCM model. Hence, on average every 10 policy years a crash occurs which reduces the value of the underlying asset by 20%. A good verification of the results of the SCM and DRM-C models is that they very similar. In particular, in line with SCM,

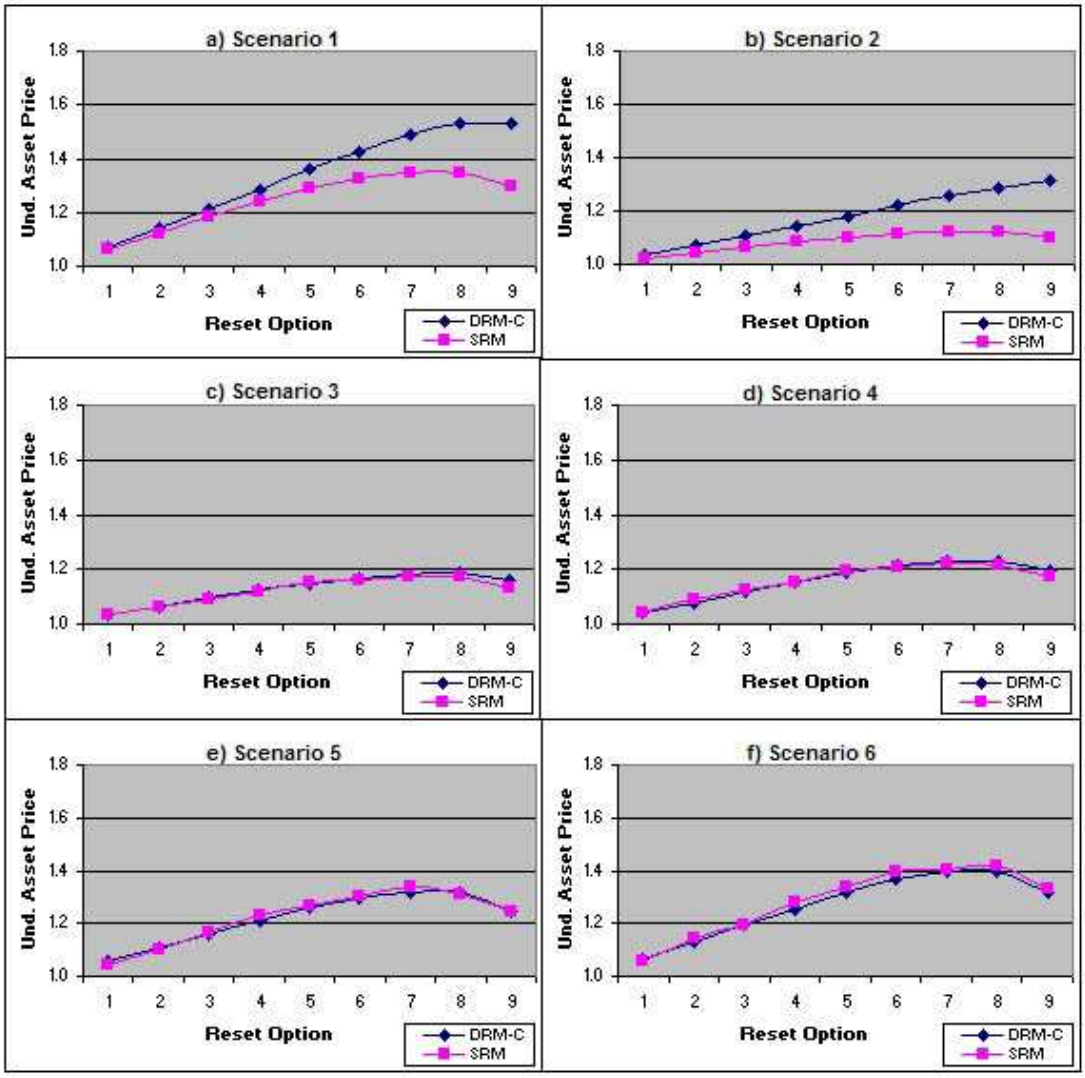

Figure 41: ***DRM-C - Effect on OEB***

the main findings from incorporating such crashes in DRM-C, is that under certain market conditions the policyholder requires a higher return in order to optimally reset his maturity guarantee (i.e. OEB shifts upwards) while the total value of the segregated fund is diminished due to the negative effect of the crashes on the underlying asset value. As illustrated in figure 41, the incorporation of crashes causes the OEB of scenarios 1 and 2 to shift upwards with a larger increase towards the maturity of the contract, while the OEB of scenarios 3 to 6 is practically unaffected.

The justification for this observation is that for scenarios 1 and 2 the maximum expected return to the policyholder from investing in risk free throughout the planning horizon is higher than from investing in a segregated fund. Therefore, unless the policyholder sees exceptional growth in the fund, he will look for the quickest way out of the fund, which is by essentially not resetting or, more precisely, requiring a very high return in order to optimally reset. For scenarios 3 to 6 it holds that the maturity guarantee acts as a safety net which protects the maximum expected return to the policyholder. Despite the crash, if the value of the underlying asset relative to its original value at investment, is less than the maturity guarantee, the policyholder's expected return will be equal to at least the maturity guarantee.

In terms of the $V^{Tq}_{1,Nq,1}$ scenarios 5 and 6 are more affected compared to scenarios 1 to 4. As illustrated in figure 42 the incorporation of the crashes

reduces the $V^{Tq}_{1,Nq,1}$ of scenarios 1 to 6 by 6.2%, 5.31%, 5.78%, 6.16%, 8.38% and 8.68% respectively.

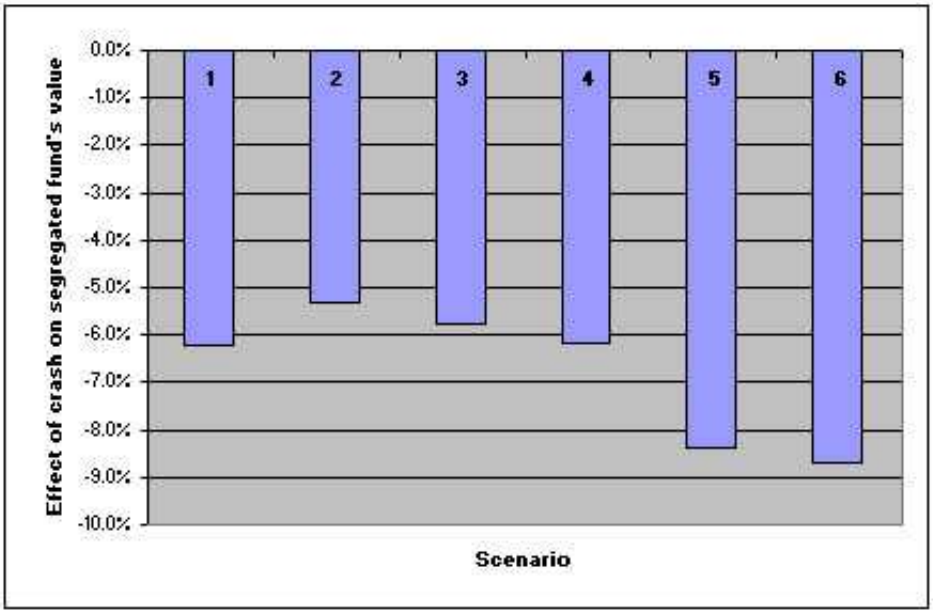

Figure 42: ***DRM-C - Effect on*** $V^{Tq}_{1,Nq,1}$

As with the previous models analysed in this thesis, it is of interest to experiment with the values of several parameters and observe the effect of their fluctuation on both the OEB and $V^{Tq}_{1,Nq,1}$. The aim of these experiments is to check the robustness of the model as well as to depict interesting trends and causalities. In particular, section 5.4.1 examines the effect of the fluctuation of the residual value of the portfolio after a crash (d_2), section 5.4.2 examines the effect of the fluctuation of the level of G offered to the policyholder, section 5.4.3 examines the value of offering the reset option under different model parameters and lastly section 5.4.4 depicts the OEB of all the segregated funds generated during the planning horizon of the policyholder.

5.4.1 Experiment 1: Fluctuating the residual value of the portfolio after the crash

In this experiment the aim is to examine the effect of fluctuating the level of the portfolio's residual value after the crash (d_2) on both the OEB and the $V^{Tq}_{1,Nq,1}$. In particular, d_2 is fluctuated from its original value of 80% to the

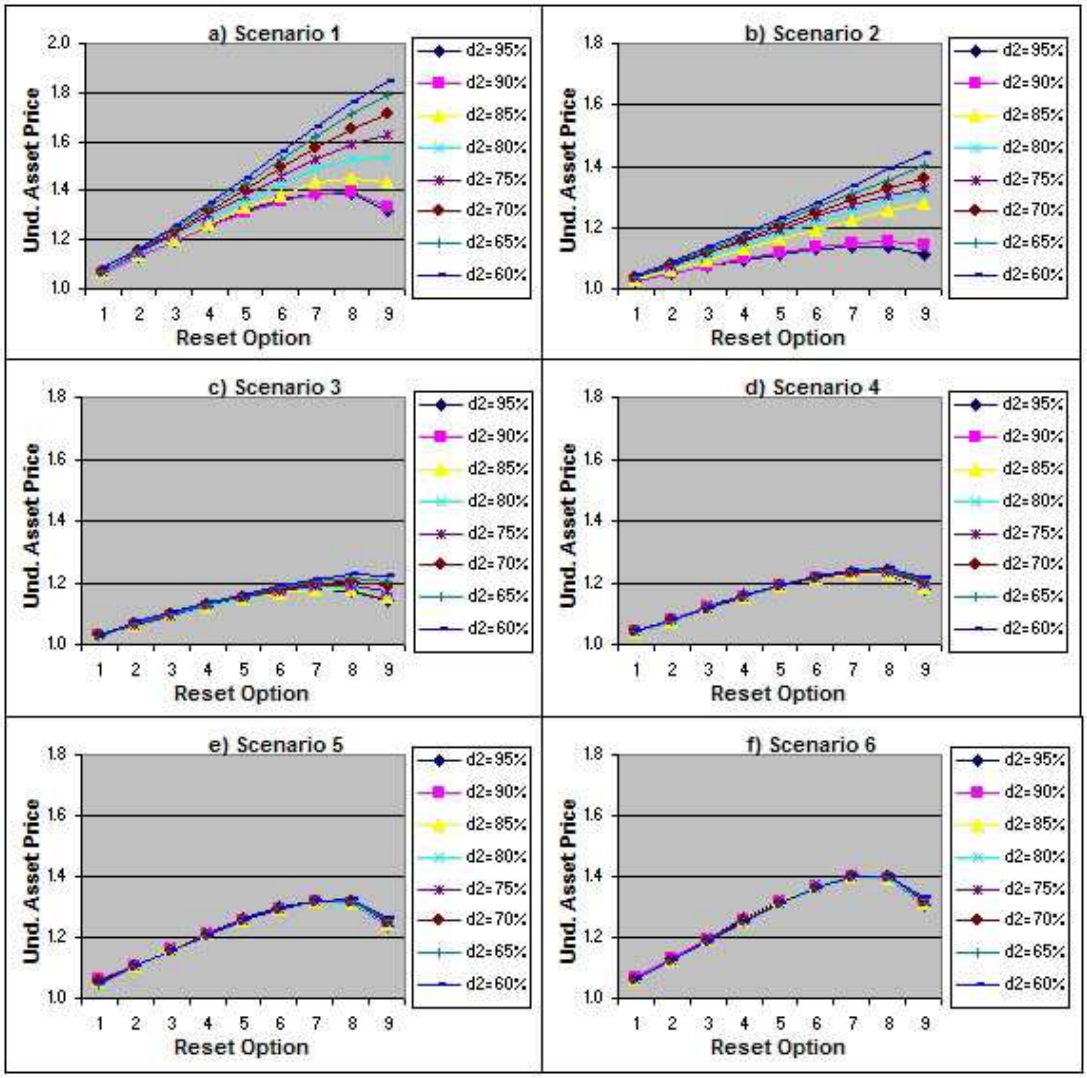

Figure 43: ***DRM-C Experiment 1 - Fluctuating the residual value of the portfolio after the crash***

range 65% to 95% at increments of 5%.

Figure 43 illustrates that the fluctuation of d_2 practically does not affect the OEB of scenarios 3 to 6 while it has a more significant effect on the OEB of scenarios 1 and 2. In particular, in scenarios 1 and 2 when d_2 increases, the OEB shifts downwards with a larger decrease towards the maturity of the contract, whereas when d_2 decreases, the OEB shifts upwards with a larger increase towards the maturity of the contract. The rationale for this observation is the same as the one provided in section 5.2.4.

Further, table 17 illustrates that increasing the value of d_2 causes the $V^{Tq}_{1,Nq,1}$ to increase across all scenarios whereas decreasing the value of d_2 causes the $V^{Tq}_{1,Nq,1}$ to decrease across all scenarios. The rationale for this observation is the same as the one provided in section 5.2.4.

d_2	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
60%	-9.90%	-6.52%	-6.50%	-6.80%	-6.99%	-7.05%
65%	-7.66%	-5.53%	-5.31%	-5.40%	-5.44%	-5.45%
70%	-5.26%	-4.09%	-3.85%	-3.82%	-3.76%	-3.74%
75%	-2.70%	-2.24%	-2.09%	-2.02%	-1.95%	-1.93%
80%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
85%	2.87%	2.59%	2.43%	2.24%	2.08%	2.03%
90%	6.38%	5.50%	5.22%	4.71%	4.30%	4.17%
95%	10.63%	9.89%	8.36%	7.40%	6.66%	6.42%

Table 17: ***DRM-C Experiment 1 - Fluctuating the residual value of the portfolio after the crash***

5.4.2 Experiment 2: Fluctuating the maturity guarantee

In this experiment the aim is to examine the effect of the fluctuation of G on both the OEB and the $V^{Tq}_{1,Nq,1}$. In particular, G is fluctuated from its original value of 100% to the range 80% to 120% at increments of 5%.

Figure 44 illustrates that across all scenarios as G increases, the OEB shifts

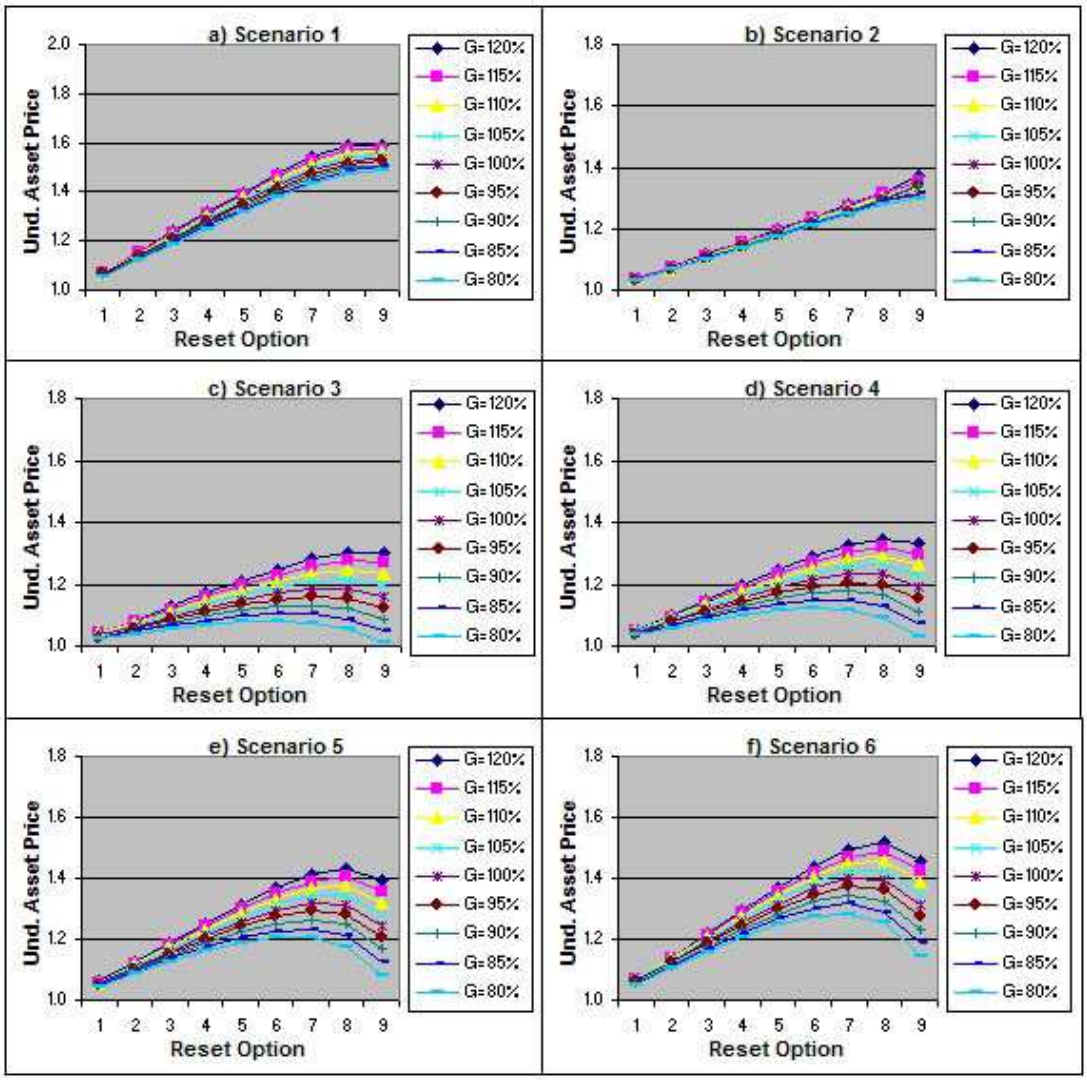

Figure 44: ***DRM-C Experiment 2 - Fluctuating the maturity guarantee***

upwards with a larger increase towards the maturity of the contract. Likewise, figure 44 illustrates that across all scenarios as G decreases, the OEB shifts downwards with a larger decrease towards the maturity of the contract. The effect of the fluctuation of G is more acute on scenarios 3,4,5 and 6 compared to scenarios 1 and 2.

A possible justification for this observation is that for scenarios 1 and 2 the maximum expected return to the policyholder from investing in risk free throughout the planning horizon is higher than from investing in a segregated fund. Therefore, unless the policyholder sees exceptional growth in the fund, he will look for the quickest way out of the fund, which is by essentially not resetting or, more precisely, requiring a very high return in order to optimally reset. Hence, if the policyholder chooses to switch to risk free at the first available opportunity, the maximum expected return from his original investment is not going to be highly affected by the fluctuations of G.

Further, table 18 illustrates that decreasing the value of G causes the $V^{Tq}_{1,Nq,1}$ to decrease across all scenarios. Similarly, increasing the value of G causes the $V^{Tq}_{1,Nq,1}$ to increase across all scenarios. The rationale of this observation is equivalent to the one provided in section 3.10.1.

G	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
80%	-0.72%	-0.51%	-9.45%	-15.48%	-20.25%	-21.82%
85%	-0.60%	-0.46%	-7.62%	-12.04%	-15.55%	-16.70%
90%	-0.44%	-0.38%	-5.44%	-8.32%	-10.60%	-11.36%
95%	-0.24%	-0.23%	-2.90%	-4.31%	-5.43%	-5.79%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
105%	0.30%	0.36%	3.27%	4.60%	5.64%	6.03%
110%	0.68%	0.91%	6.90%	9.49%	11.55%	12.28%
115%	1.15%	1.74%	10.92%	14.69%	17.68%	18.75%
120%	1.76%	3.02%	15.31%	20.17%	24.06%	25.47%

Table 18: ***DRM-C Experiment 2 - Fluctuating the maturity guarantee***

5.4.3 Experiment 3: Value of the reset option

The aim of this experiment is to estimate the value of offering reset options on a segregated fund contract under different sets of values of the model parameters. In particular, the value of offering reset options is calculated for the original set of values of the model parameters and then one of the parameters of interest (G, d_2) is fluctuated at a time, while keeping the rest of the parameters constant. The value of the reset options is calculated as in section 3.10.2. The results of this experiment are summarised in table 19.

G	d_2	***Sc.1***	***Sc.2***	***Sc.3***	***Sc.4***	***Sc.5***	***Sc.6***
100%	80%	0.38%	0.13%	7.81%	23.29%	58.12%	87.09%
90%	80%	0.21%	0.04%	4.81%	18.58%	50.93%	78.08%
80%	**80%**	0.10%	0.01%	2.39%	14.02%	43.51%	68.66%
100%	**85%**	0.42%	0.14%	8.08%	23.58%	58.45%	87.45%
100%	**75%**	0.36%	0.12%	7.69%	23.10%	57.86%	86.78%

Table 19: ***DRM-C Experiment 3 - Value of the reset option***

The first row of table 19 shows the value of the reset options under the original values for the parameters of interest. In particular, for scenarios 1 and 2 embedding the reset options adds only a small value to the segregated fund (0.38% and 0.13% respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (7.81%, 23.29%, 58.12% and 87.09% for scenarios 3 to 6 respectively). A possible reason for the small value added by embedding reset options on segregated funds under the market condition assumed in scenarios 1 and 2 is that the return to the policyholder from investing in risk free throughout the planning horizon is higher compared to investing in a segregated fund. Thus, the policyholder will switch to risk free at the first available opportunity, thus not really needing or making use of the reset options.

Rows 2 and 3 of table 19 summarise the first sub-experiment where G is fluctuated from its original value of 100% to the range 80% to 100% at increments of 10% while all other parameters of interest are kept constant. As the value of G is decreased to 80% the value of the reset options is decreased across all scenarios. The rationale for this observation is the same as the one provided in section 3.10.2. Overall, for scenarios 1 and 2 embedding the reset options adds only a small value to the segregated fund (0.10% and 0.01% respectively). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (2.39%, 14.02%, 43.51% and

68.66% for scenarios 3 to 6 respectively).

Rows 4 and 5 of table 19 summarise the second sub-experiment where the portfolio's residual value after a crash (d_2) is fluctuated from its original value of 80% to the range 75% to 85% at increments of 5% while all other parameters of interest are kept constant. As d_2 is increased to 85% (i.e. lower in magnitude crash occurring) the value of offering the reset options increases across all scenarios (0.42%, 0.14%, 8.08%, 23.58%, 8.45% and 87.45% for scenarios 1 to 6 respectively). The rationale for this observation is that a lower in magnitude level of crash will lead to a higher value of the underlying asset therefore increasing the probability of the value of the underlying asset relative to its original value at investment being greater than the maturity guarantee, thus increasing the likelihood of the policyholder choosing to reset his maturity guarantee. In turn this increases the value of the reset option. On the other hand, as d_2 is decreased to 75% (i.e. a higher in magnitude level of crash occurring) the value of offering the reset options is decreased across all scenarios (0.36%, 0.12%, 7.69%, 23.10%, 57.86% and 86.78% for scenarios 1 to 6 respectively). The rationale for this observation is equivalent to the one offered above.

5.4.4 Experiment 4: OEB of all segregated funds

The aim of this experiment is to depict and classify in accordance to section 3.10.3 the OEB of all the segregated funds generated during the planning horizon of the policyholder. As illustrated in figure 45 scenario 1 and 2 are of type C while scenarios 3 to 6 are of type B.

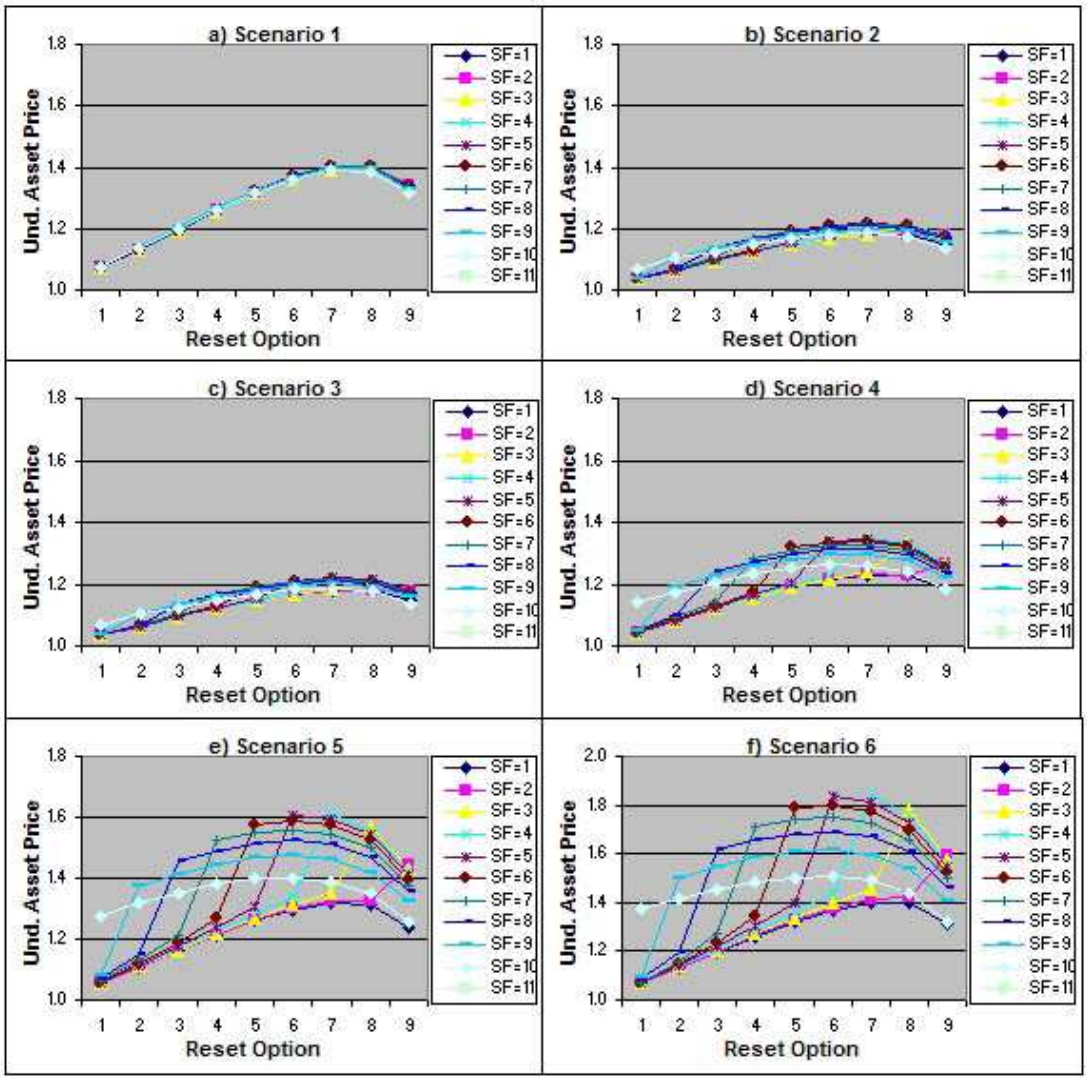

Figure 45: ***DRM-C Experiment 4 - Depicting the OEB of all segregated funds***

5.5 Main Results of Double Regime Model (jumps)

The parameters of this application of the DRM model (denoted by DRM-J) have been set so that on average every 10 policy years either a crash occurs which reduces the value of the underlying asset by 20% or a surge occurs which increases the value of the underlying asset by 20%. The main finding from incorporating such instability in the market environment is that the

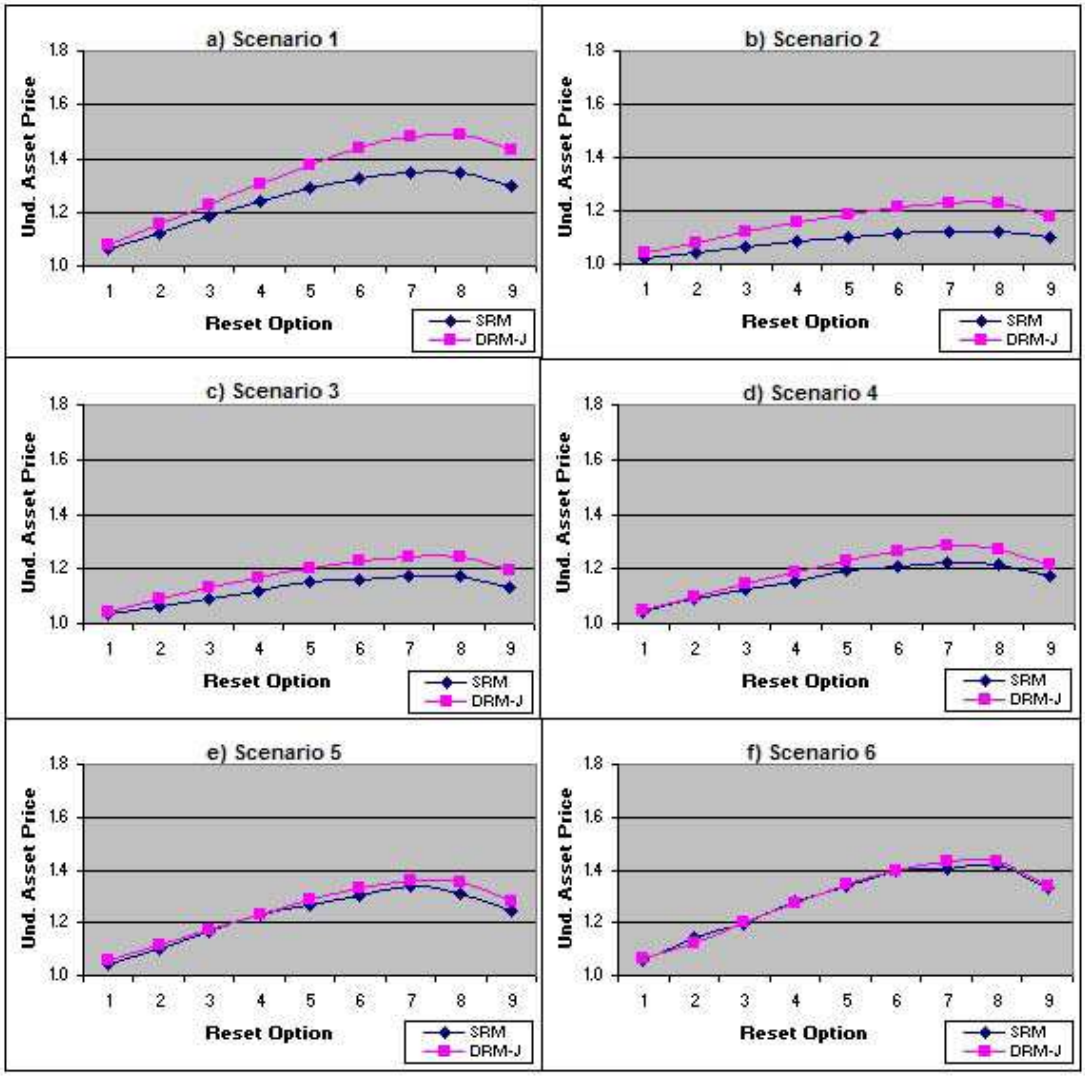

Figure 46: ***DRM-J: Effect on OEB***

policyholder requires a higher return in order to optimally reset his maturity guarantee (i.e. OEB shifts upwards). In particular, as illustrated in figure 46, the incorporation of the crashes and surges causes the OEB of scenarios 1 to 5 to shift upwards with a larger increase towards the maturity of the contract. In contrast, the OEB of scenario 6 is practically unaffected.

The justification behind the observation for scenarios 1 to 5 is that the policyholder requires some extra compensation (i.e. higher increase in the value of the underlying asset) in order to reset, and thus prolong his investment in segregated funds, due to the risks associated with switching to the second regime. Relative to scenario 6, the coefficient of variation is already rather high, so it is reasonable to assume that switching to the second regime will not prove to be a drastic increase in the volatility of the market returns. Hence, under the assumed market conditions of scenario 6, the policyholder does not require higher returns to the underlying asset in order to choose to reset his maturity guarantee.

Further, in contrast to both SCM and DRM-C the total value of the segregated fund is increased due to the cumulative effects of the surges and crashes on the underlying asset value, as modeled in DRM-J. The rationale behind this argument is based on the ability of the policyholder to lock in considerable market gains which accrue from stock market surges which more than cancel out the corresponding decreases in the value of the underlying asset

caused by the potential crashes. In other words, the policyholder can lock in the maximum of the potential market gains through optimally resetting the level of his maturity guarantee but does not have to suffer all losses due to stock market crashes as the maturity guarantee sets a lower bound and ultimately protects him from those.

Therefore, the model suggests that the cumulative effect of the potential crashes and surges is an increase in the value of the fund. In particular, the $V^{Tq}_{1,Nq,1}$ scenarios 2 to 4 is more affected compared to scenarios 1, 5 and 6. As illustrated in figure 42 the incorporation of the jumps increase the $V^{Tq}_{1,Nq,1}$ of scenarios 1 to 6 by 1.04%, 50.02%, 43.12%, 29.55%, 7.39% and 7.16% respectively.

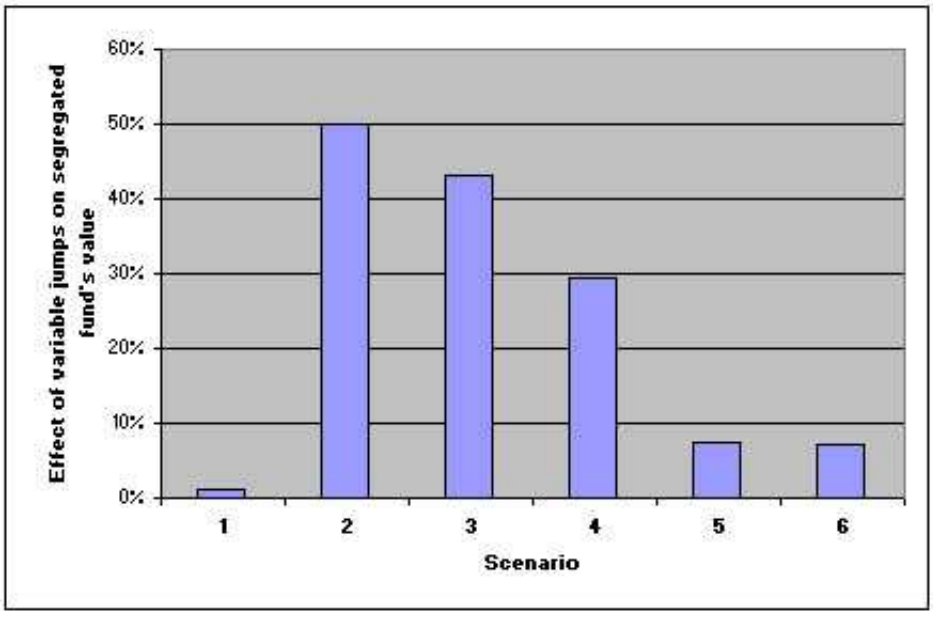

Figure 47: ***DRM-J: Effect on*** $V^{Tq}_{1,Nq,1}$

As with the previous models analysed in this thesis, it is of interest to experiment with the values of several parameters and observe the effect of their fluctuation on both the OEB and $V^{Tq}_{1,Nq,1}$. The aim of these experiments is to check the robustness of the model as well as to depict interesting trends and causalities. In particular, section 5.5.1 examines the effect of the fluctuation of the probability of switching from regime 1 to regime 2 (p_{12}), section 5.5.2 examines the effect of the fluctuation of the probability of switching from regime 2 to regime 1 (p_{21}). Section 5.5.3 examines the effect of the fluctuation of the level of G offered to the policyholder. Section 5.5.4 examines the value of offering the reset option under different model parameters and lastly section 5.5.5 depicts the OEB of all the segregated funds generated during the planning horizon of the policyholder.

5.5.1 Experiment 1: Fluctuating the probability of switching from regime 1 to regime 2

In this experiment the aim is to examine the effect of fluctuating the probability of switching from regime 1 to regime 2 (p_{12}) on both the OEB and the $V^{Tq}_{1,Nq,1}$. In particular, p_{12} is fluctuated from its original value of 10% to the range 10% to 90% at increments of 10%.

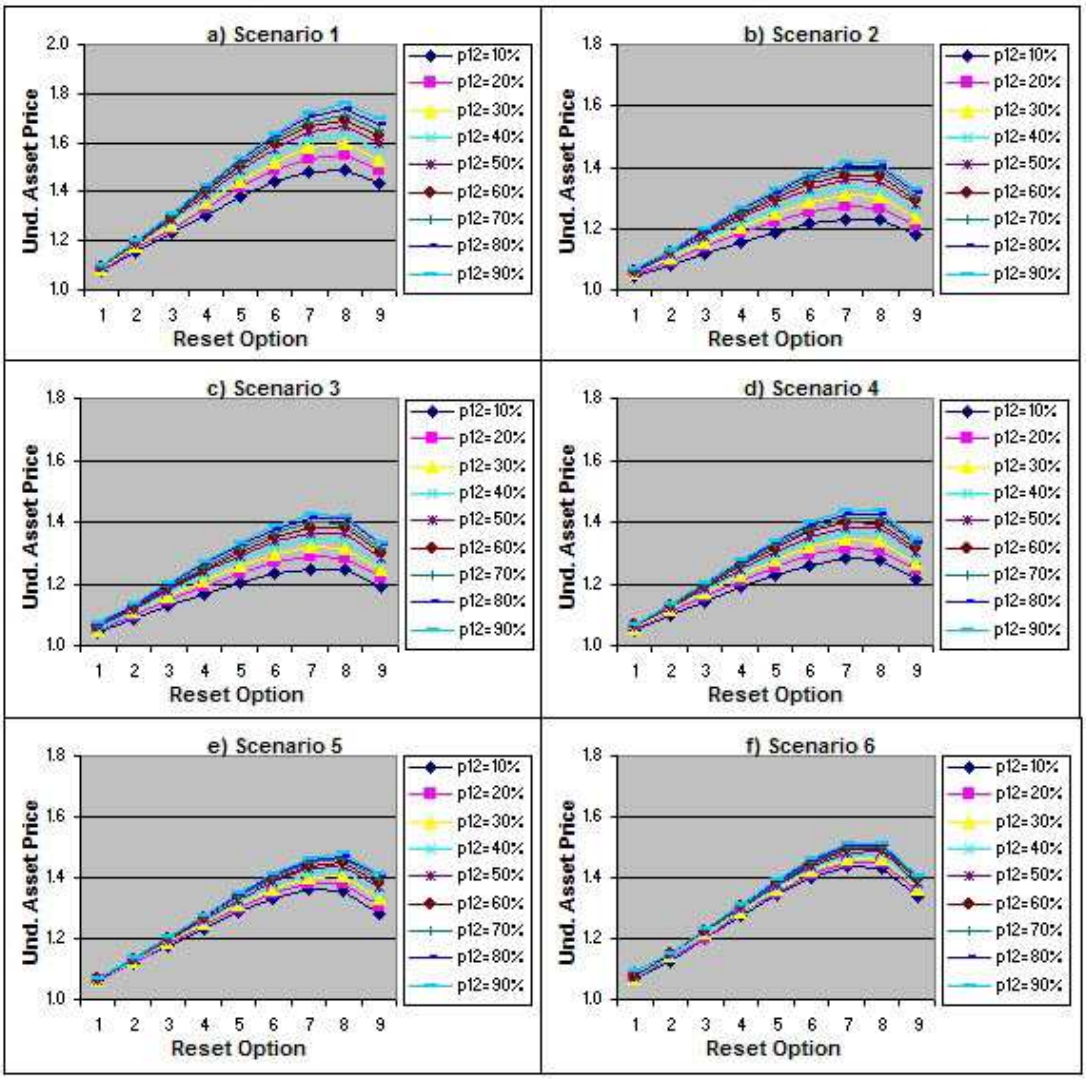

Figure 48: ***DRM-J Experiment 1 - Fluctuating the probability of switching from regime 1 to regime 2***

Figure 48 illustrates that across all scenarios as p_{12} increases the OEB shifts upwards with a larger increase towards the maturity of the contract. The rationale for this observation is that as the probability of switching to the second regime increases, the policyholder will require progressively higher returns from the underlying asset in order to be compensated for the increased volatility that he will have to face in the second regime.

p_{12}	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
10%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
20%	2.95%	26.66%	22.61%	17.16%	4.63%	4.5%
30%	5.38%	47.98%	40.94%	31.29%	8.31%	8.16%
40%	7.19%	65.63%	56.22%	43.15%	11.56%	10.95%
50%	8.53%	80.59%	69.20%	53.27%	14.37%	13.18%
60%	9.51%	93.42%	80.40%	62.02%	16.81%	15.01%
70%	10.24%	104.57%	90.16%	69.68%	18.97%	16.52%
80%	10.79%	114.46%	98.80%	76.42%	20.87%	17.83%
90%	11.20%	123.58%	106.54%	82.44%	22.58%	18.97%

Table 20: ***DRM-J Experiment 1 - Fluctuating the probability of switching from regime 1 to regime 2***

Further, table 20 illustrates that increasing the value of p_{12} causes the $V^{Tq}_{1,Nq,1}$ to increase across all scenarios. A reason for that, is that by increasing the value of p_{12} the chance of the market conditions switching to the second regime is increased proportionately. As already examined in section 5.5 the cumulative effect of the potential surges and crashes of the second regime is an increase in the value of the segregated fund. The higher the increase in the value of p_{12}, the higher the percentage increase of $V^{Tq}_{1,Nq,1}$. Lastly, the lower the coefficient of variation, the larger the percentage increase to $V^{Tq}_{1,Nq,1}$.

5.5.2 Experiment 2: Fluctuating the probability of switching from regime 2 to regime 1

In this experiment the aim is to examine the effect of fluctuating the probability of switching from regime 2 to regime 1 (p_{21}) on both the OEB and the $V^{Tq}_{1,Nq,1}$. In particular, p_{21} is fluctuated from its original value of 100% to the range 10% to 100% at increments of 10%. Figure 49 illustrates that across all scenarios

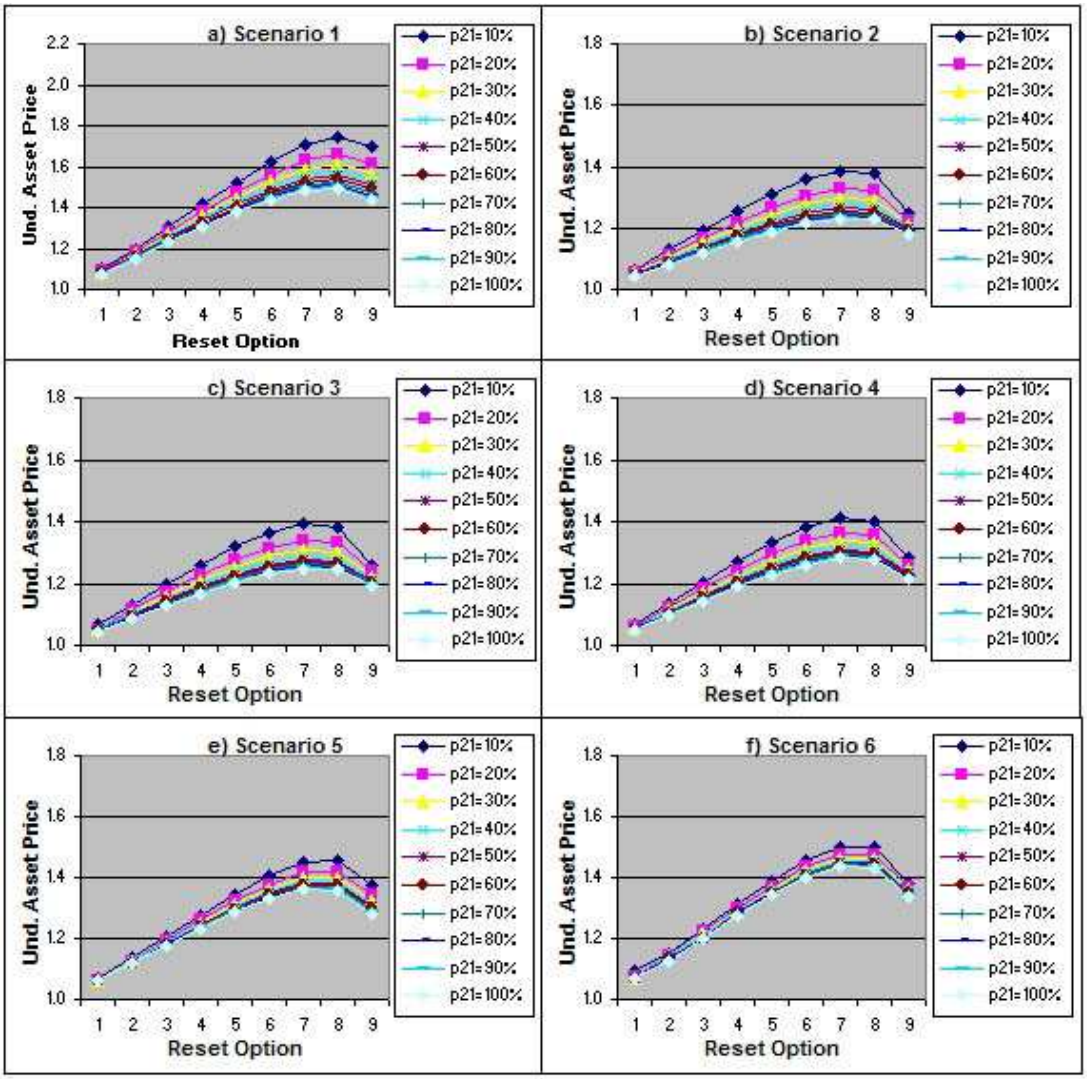

Figure 49: ***DRM-J Experiment 2 - Fluctuating the probability of switching from regime 2 to regime 1***

as p_{21} decreases, the OEB shifts upwards with a larger increase towards the maturity of the contract. The rationale for this observation is that as the probability of switching to the first regime decreases, the policyholder will require progressively higher returns from the underlying asset in order to be compensated for the increased volatility that he will have to face in the second regime.

p_{21}	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
10%	9.00%	120.67%	104.94%	82.01%	23.18%	18.24%
20%	6.56%	73.47%	63.80%	49.71%	13.93%	12.10%
30%	4.71%	49.29%	42.73%	33.18%	9.22%	8.48%
40%	3.36%	34.40%	29.77%	23.05%	6.36%	6.07%
50%	2.38%	24.25%	20.95%	16.17%	4.43%	4.34%
60%	1.63%	16.83%	14.52%	11.18%	3.05%	3.05%
70%	1.07%	11.16%	9.62%	7.39%	2.01%	2.04%
80%	0.63%	6.68%	5.75%	4.41%	1.19%	1.22%
90%	0.28%	3.03%	2.60%	1.99%	0.54%	0.56%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 21: ***DRM-J Experiment 2 - Fluctuating the probability of switching from regime 2 to regime 1***

Further, table 21 illustrates that decreasing the value of p_{21} causes the $V^{Tq}_{1,Nq,1}$ to increase across all scenarios. The rationale of this observation is equivalent to the one offered in the previous section. Further, the higher the decrease in the value of p_{21}, the higher the percentage increase of $V^{Tq}_{1,Nq,1}$. Lastly, the lower the coefficient of variation of a scenario, the larger the percentage increase to its $V^{Tq}_{1,Nq,1}$ from the increase of p_{12}.

5.5.3 Experiment 3: Fluctuating the maturity guarantee

In this experiment the aim is to examine the effect of the fluctuation of G on both the OEB and the $V^{Tq}_{1,Nq,1}$. In particular, G is fluctuated from its original value of 100% to the range 80% to 120% at increments of 5%. Figure 50 illustrates that across all scenarios as G increases, the OEB shifts upwards

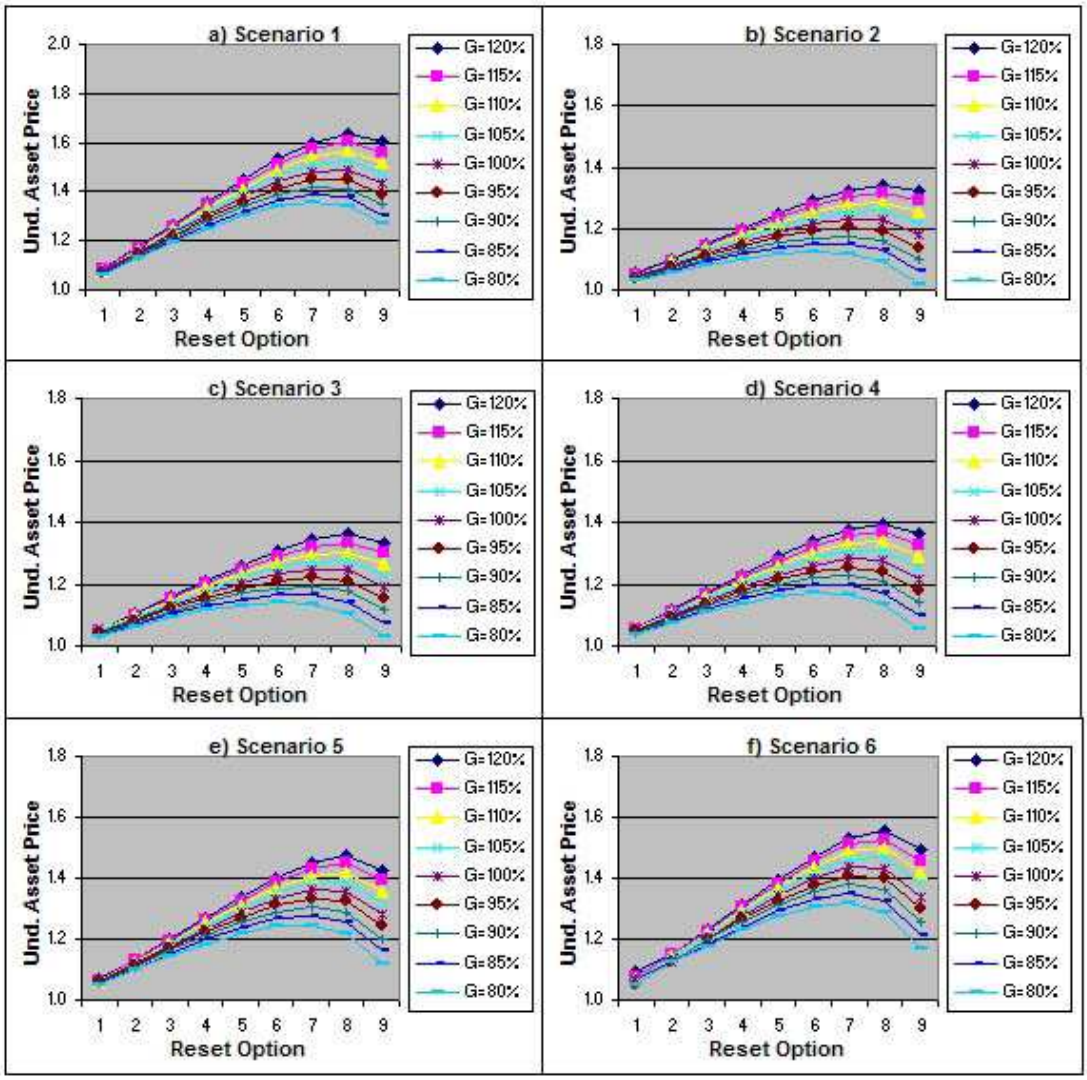

Figure 50: ***DRM-J Experiment 3 - Fluctuating the maturity guarantee***

with a larger increase towards the maturity of the contract. Likewise, figure 50 illustrates that across all scenarios as G decreases, the OEB shifts downwards with a larger decrease towards the maturity of the contract.

Further, table 22 illustrates that decreasing the value of G causes the $V^{Tq}_{1,Nq,1}$ to decrease across all scenarios. Similarly, increasing the value of G causes the $V^{Tq}_{1,Nq,1}$ to increase across all scenarios. The rationale of this observation is equivalent to the one provided in section 3.10.1.

G	***Scen.1***	***Scen.2***	***Scen.3***	***Scen.4***	***Scen.5***	***Scen.6***
80%	-6.65%	-14.82%	-16.30%	-18.06%	-21.22%	-21.96%
85%	-5.23%	-11.53%	-12.62%	-13.93%	-16.26%	-16.80%
90%	-3.64%	-7.97%	-8.68%	-9.51%	-11.07%	-11.42%
95%	-1.89%	-4.13%	-4.47%	-4.89%	-5.65%	-5.82%
100%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
105%	2.03%	4.40%	4.74%	5.14%	5.88%	6.03%
110%	4.20%	9.10%	9.74%	10.50%	12.00%	12.28%
115%	6.50%	14.07%	15.01%	16.13%	18.34%	18.75%
120%	8.94%	19.33%	20.55%	22.01%	24.93%	25.44%

Table 22: ***DRM-J Experiment 3 - Fluctuating the maturity guarantee***

5.5.4 Experiment 4: Value of the reset option

The aim of this experiment is to estimate the value of offering reset options on a segregated fund contract under different sets of values of the model parameters. In particular, the value of offering reset options is calculated for the original set of values of the model parameters and then one of the parameters of interest (G, p_{21}, p_{12}) is fluctuated at a time, while keeping the rest of the parameters

constant. The value of the reset options is calculated as in section 3.10.2. The results of this experiment are summarised in table 23.

G	p_{12}	p_{21}	***Sc.1***	***Sc.2***	***Sc.3***	***Sc.4***	***Sc.5***	***Sc.6***
100%	10%	100%	7.15%	24.42%	31.14%	42.91%	72.73%	101.15%
90%	10%	100%	4.94%	19.81%	25.95%	36.89%	64.61%	91.28%
80%	10%	100%	3.07%	15.37%	20.84%	30.79%	56.16%	80.96%
100%	10%	**80%**	8.67%	28.41%	34.82%	46.09%	74.70%	102.53%
100%	10%	**60%**	11.06%	34.28%	40.29%	50.87%	77.70%	104.67%
100%	**20%**	100%	13.52%	40.68%	45.79%	55.36%	80.33%	106.35%
100%	**30%**	100%	18.94%	52.69%	56.96%	65.16%	86.55%	110.74%

Table 23: ***DRM-J Experiment 4 - Value of the reset option***

The first row of table 23 shows the value of the reset options under the original values for the parameters of interest. In particular, for scenario 1 embedding the reset options adds a comparatively smaller value to the segregated fund (7.15%). However, for the other scenarios it holds that the higher their coefficient of variation, the higher the value of the reset options (24.42%, 31.14%, 42.91%, 72.73% and 101.15% for scenarios 2 to 6 respectively).

Rows 2 and 3 summarise the first sub-experiment where as the value of G is decreased to 80%, the value of the reset options is decreased across all scenarios. The rationale for this observation is the same as the one provided in section 3.10.2. Overall, for scenario 1 embedding the reset options adds a comparatively smaller value to the segregated fund (3.07%). However, for the other scenarios, the higher their coefficient of variation the higher the value of the reset options (15.37%, 20.84%, 30.79%, 56.16% and 80.96% for scenarios 2

to 6 respectively).

Rows 4 and 5 summarise the second sub-experiment where the probability of switching from regime 2 to regime 1 (p_{21}) is fluctuated from its original value of 100% to the range 60% to 100% at increments of 20% while all other parameters of interest are kept constant. As p_{21} is decreased to 60% the value of offering the reset options is increased across all scenarios (11.06%, 34.28%, 40.29%, 50.87%, 77.70% and 104.67% for scenarios 1 to 6 respectively). A possible reason for that is that by decreasing the value of p_{21} the likelihood of remaining in the second regime is increased proportionately. As already examined in section 5.5 the role of the reset option in the second regime is highly crucial, as its optimal use has the potential to lock in market gains during stock market surges which more than negate the losses incurred by the stock market crashes. Therefore the relative value of the reset option is higher when the probability of remaining on the second regime is higher.

Rows 6 and 7 summarise the third sub-experiment where the probability of switching from regime 1 to regime 2 (p_{12}) is fluctuated from its original value of 10% to the range 10% to 30% at increments of 10% while all other parameters of interest are kept constant. As p_{12} is increased to 30% the value of offering the reset options is increased across all scenarios (18.94%, 52.69%, 56.96%, 65.16%, 86.55% and 110.74% for scenarios 1 to 6 respectively). The rationale for this observation is equivalent to the one explained above.

5.5.5 Experiment 5: OEB of all segregated funds

The aim of this experiment is to depict and classify in accordance to section 3.10.3 the OEB of all the segregated funds generated during the planning horizon of the policyholder. As illustrated in figure 51 scenario 1 is of type C while scenarios 2 to 6 are of type B.

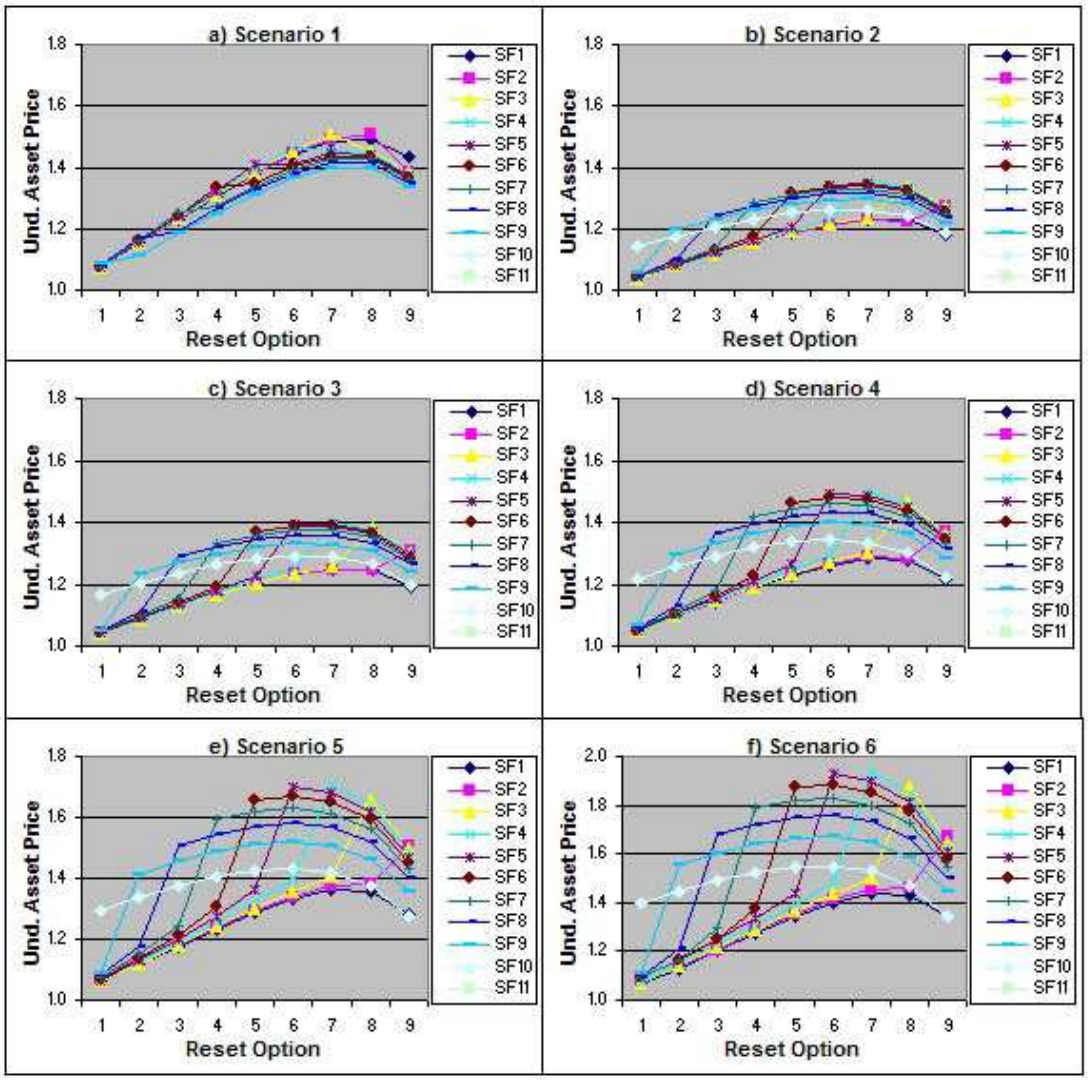

Figure 51: ***DRM-J Experiment 5 - Depicting the OEB of all segregated funds***

5.6 Discussion

In response to the well documented deficiencies of the lognormal model to properly capture the negative and positive jumps experienced by stock markets an attempt has been made to incorporate such jumps in the original model. In order to preserve the comparability of the model's results with the results of previous chapters it was decided to sustain the lognormal model but overlay it with stochastic negative and/or positive jumps.

Section 5.2 extended the SRM model in that it allows for instantaneous stochastic crashes to occur within the single regime of the SRM model, namely through the Stochastic Crash Model (SCM). In reality, the evolution of the possible values of the underlying asset price is the same as with the SRM model. However, at every time period there is a small probability of a crash occurring. When the crash materialises, the residual value of the fund after the crash is equal to a fixed percentage of its original value.

A finding from incorporating such crashes is that, if the assumed market conditions are such that the maximum expected return to the policyholder from investing in risk free throughout the planning horizon is higher than from investing in a segregated fund, the policyholder requires a higher return in order to optimally reset his maturity guarantee. Therefore, unless the policyholder sees exceptional growth in the fund, he will look for the quickest way out of the fund, which is by essentially not resetting or, more precisely, requiring a

very high return in order to optimally reset.

If, on the other hand, the maximum expected return to the policyholder from investing in a segregated fund is higher than from investing in risk free it holds that the maturity guarantee acts as a safety net which protects the maximum expected return to the policyholder. Despite the crash, if the value of the underlying asset relative to its original value at investment, is less than the maturity guarantee, the policyholder's expected return will be equal to at least the maturity guarantee. A further finding from incorporating such crashes is that the total value of the segregated fund diminishes due to the negative effect of the crashes on the underlying asset value.

Section 5.3 provided an alternative method to incorporate jumps into the SRM model, namely the Double Regime Model (DRM). In contrast to the SCM model, the DRM model is able to incorporate both negative (crash) and positive (surge) jumps as well as a combination of the two. In particular, it allows the underlying asset to switch between two distinct regimes. The market characteristics of the first regime are defined by the relevant scenario under examination and are equivalent to the ones used under the SRM in order to facilitate comparisons. The second regime is intended to model periods of high volatility in the markets and can be used to incorporate the jumps. Two distinct applications of the DRM were presented.

In the first application of the DRM model, denoted by DRM-C, the second

regime modeled the case where there is a large probability that the underlying asset price will marginally increase or a very small probability that it will drop by a substantial fixed percentage, thus essentially allowing only crashes like the SCM. The parameters and transitions probabilities were set so that a crash is as likely to happen and of the same magnitude as in the SCM, in order to facilitate comparisons. A good verification of the results of the SCM and DRM-C models is that they very similar.

In line with SCM, the main findings from incorporating such crashes in DRM-C, is that if the assumed market conditions are such that the policyholder would rather invest in risk free throughout the planning horizon compared to investing in segregated funds, then he will require a higher return in order to optimally reset his maturity guarantee while the total value of the segregated fund is diminished due to the negative effect of the crashes on the underlying asset value. The justification for this observation is equivalent to the one offered above.

If, on the other hand, he prefers to invest in segregated funds (compared to risk free), then it holds that the maturity guarantee acts as a safety net which protects the maximum expected return to the policyholder. Despite the crash, if the value of the underlying asset relative to its original value at investment, is less than the maturity guarantee, the policyholder's expected return will be equal to at least the maturity guarantee.

In the second application of the DRM model, denoted by DRM-J, the second regime modeled the case where the stock market can exhibit variable jumps (i.e. both crashes and surges) with equal probability of occurrence. The transition probabilities and parameters of the DRM-J model were set so that with the same frequency as in the previous models, either a crash occurs which reduces the value of the underlying asset or a surge occurs which increases the value of the underlying asset, by a fixed percentage equivalent in magnitude to the previous models.

The main finding from incorporating such instability in the market environment is that the policyholder requires a higher return in order to optimally reset his maturity guarantee. In particular, the incorporation of the crashes and surges causes the OEB to shift upwards with a larger increase towards the maturity of the contract. The justification behind this observation is that the policyholder requires some extra compensation (i.e. higher increase in the value of the underlying asset) in order to reset, and thus prolong his investment in segregated funds, due to the risks associated with switching to the second regime. This observation does not hold true if the assumed market conditions are such that in the first regime there is very high volatility. Then, since the coefficient of variation is already rather high, it is reasonable to assume that switching to the second regime will not prove to be a drastic increase in the volatility of the market returns. Hence, the policyholder does not require

higher returns to the underlying asset in order to choose to reset his maturity guarantee.

Further, in contrast to both SCM and DRM-C the total value of the segregated fund is increased due to the cumulative effects of the surges and crashes on the underlying asset value, as modeled in DRM-J. The rationale behind this argument is based on the ability of the policyholder to lock in considerable market gains which accrue from stock market surges which more than cancel out the corresponding decreases in the value of the underlying asset caused by the potential crashes. In other words, the policyholder can lock in the maximum of the potential market gains through optimally resetting the level of his maturity guarantee but does not have to suffer all losses due to stock market crashes as the maturity guarantee sets a lower bound and ultimately protects him from those. Therefore the model suggests that the cumulative effect of the potential crashes and surges is an increase in the maximum expected return from the original investment.

6 Conclusion and Future Research

Overall in this thesis, motivated by the importance of the reset option embedded in segregated funds, an attempt has been made to tackle the modeling challenges that arise from the non-standard (exotic) properties of the reset option. The first step was to review the relevant literature. In particular, in Chapter 2 a wide range of actuarial models relative to the valuation of the reset option embedded in segregated funds were critically examined and compared. The methods examined vary from the deterministic approach to the much more elaborate and advanced CIA's long term approach. However, what all of the models seem to have in common is a consistent use of naive heuristics relative to the modeling of the reset decision. A side effect is that practitioners who are prescribed which approach to use, make the exact same misleading and naive assumptions about the reset option. Therefore, it became apparent that all the benefits that could accrue through the use of advanced stochastic modeling can be more than negated by the inherent naive assumptions about the reset option.

Further, the academic literature on the modeling of the reset decision was critically reviewed. The conclusion from this subsection was that there does not appear to be any published work which is advanced enough to deal with the complexities of the reset option faced by the policyholder of segregated funds. Hence, a clear and urgent need to have more sophisticated approaches

which can model the reset option was identified.

In order to address this issue in Chapter 3 an attempt was made to develop a mathematical model for the reset option with the aim to derive a comprehensive optimal reset strategy for the policyholder. In particular, the reset option on the maturity guarantee of segregated funds was formulated as a non-stationary finite horizon Markov Decision Process. The efficient formulation allowed the values of the underlying asset price to fluctuate up to 7000 times in every policy year, thus enabling the distribution of the underlying asset price to converge towards the lognormal distribution. An important feature of the Single Regime Model, developed in this chapter, is the ability to derive the OEB of the reset option, where given the model parameters, a threshold value is depicted such that if the value of the underlying asset price exceeds it then it is optimal for the policyholder to reset his maturity guarantee. Otherwise, it is optimal for the policyholder to rollover his maturity guarantee.

It is noteworthy that the SRM model is able to depict the OEB of not just the first but of all the segregated fund contracts which can be issued throughout the planning horizon of the policyholder. The reason why this is of great importance is that once the investor resets the maturity guarantee the OEB changes. Therefore, it becomes apparent that in order to generate a comprehensive optimal reset strategy, the optimal OEB for all of the segregated funds has to be derived and examined. In this way the model has managed to

address one of the significant deficiencies in the existing literature.

The main finding of the SRM model has been that as the segregated fund approaches its maturity, a proportionately larger percentage increase in the value of the underlying asset will be necessary to trigger an optimal reset of the segregated fund's maturity guarantee. The rationale behind this argument is that if the policyholder resets at the first decision point the maturity will be extended by 1 year. In contrast, if he resets in the eighth decision point the maturity will be extended by 8 years. Therefore as the "time penalty" increases, the return that the policyholder requires in order to choose to exercise his reset option increases. However, in the last period before the maturity of the segregated fund, the return that the policyholder requires in order to optimally exercise his reset option decreases. The reason for this is that if the option is not exercised it will expire worthless.

It should be underlined that the aim of the model was not to prescribe any particular reset strategy as this is highly dependent on the parameters and assumptions of the model, but rather to further our understanding on what constitutes an optimal reset strategy and how it is affected by the fluctuation of the main variables of the model. However, it should be highlighted that the findings of the SRM model suggest that a single heuristic such as the one prescribed by CIA, independent of the parameters and assumptions of the model and most importantly of the time remaining to maturity, can prove to

be a misleading approximation of the optimal reset strategy.

Overall, given the importance of the SRM findings it was felt that it was interesting to alter some of its assumptions in order to reflect on the wide range of features of the segregated funds which are traded in the market. The methodology and results of this analysis were included in chapter 4. In particular, three different extensions were added to the model.

Firstly, the Management Expense Ratio model extended the SRM in that the issuer charges the policyholder a fee equal to a fixed proportion of the value of the fund at the end of each policy year. The main findings from incorporating this fee were that the policyholder requires a higher return in order to optimally reset his maturity guarantee while the fund's value is diminished due to the negative effect of the fee. This finding is more acute when the assumed market conditions are such that the maximum expected return from investing in a segregated fund is less than the return from investing in risk free throughout the planning horizon. Under such market conditions, the policyholder will choose to switch to risk free at the first available opportunity.

Further, the level of management expense ratio which can negate the cost of embedding reset options to a segregated fund contract was depicted. These "break-even" values of the management expense ratio increase as the coefficient of variation of the assumed market conditions increases. This finding is in line with the observation made above. Depicting the "break-even" values can be

very interesting and useful for both the policyholders and issuers. From the point of view of the issuer, it should help with risk management policies as well as to convince the relevant regulatory authorities that they indeed take appropriate cover for the risk that they face from issuing segregated funds with embedded reset options. From the point of view of the policyholder, it should signal that if the underlying asset is not very risky, thus has a low coefficient of variation, they should not be willing to pay a relatively high management expense ratio. The opposite should hold for a risky underlying asset.

Secondly, the Variable Maturity Guarantee model extended the SRM in that every time the policyholder exercises his reset option the maturity guarantee is reduced by a pre-determined discount factor. The main findings from incorporating the discount factor to the maturity guarantee were that the policyholder requires a higher return in order to optimally reset his maturity guarantee while the total value of the segregated fund is diminished due to the negative effect of the discount factor on the fund's value. This observation holds true, if under the assumed market conditions the policyholder prefers to invest in a segregated fund compared to investing in risk free throughout the planning horizon.

Thirdly, the Variable Timing of Exercising the Reset Option Model extended the SRM in that the policyholder is allowed to reset the maturity guarantee at any point in time within each year from the start of the planning

horizon, but only once. The main findings from the TimRO model were the following. Firstly, at the beginning of each policy year the OEB exhibits a jump. The reason for this is that the policyholder receives a new reset option at that date. Secondly, as the end of each policy year is approached the OEB steadily drops. The reason for this is that unless the policyholder exercises his reset option before the end of the policy year, the option will expire worthless. Further, the total value of the fund is increased, compared to the SRM model, since the policyholder may lock in higher market gains as he has more reset decision points (but the same number of total reset options). This finding, however, does not hold true if the assumed market conditions are such that the policyholder would prefer to invest in risk free throughout the planning horizon, compared to investing in a segregated fund. The rationale is that under such market conditions, the policyholder will choose to switch to risk free at the first available opportunity. In order to do that he will choose not to reset so as not to extend the maturity of the contract. Thus, if he is unlikely to reset his maturity guarantee, having more reset decision dates should not have a great effect in the total value of his contract.

In response to the well documented deficiencies of the lognormal model to properly capture the negative and positive jumps experienced by stock markets an attempt has been made to incorporate such jumps in the original model in chapter 5. In order to preserve the comparability of the model's results with

the results of previous chapters it was decided to sustain the lognormal model but overlay it with stochastic jumps.

Section 5.2 extended the SRM model in that while the evolution of the possible values of the underlying asset price is the same as with the SRM model, at every time period there is a small probability of a crash occurring. When the crash materialises, the residual value of the fund after the crash is equal to a fixed percentage of its original value. A finding from incorporating such crashes was that, if the assumed market conditions are such that the maximum expected return to the policyholder from investing in risk free throughout the planning horizon is higher than from investing in a segregated fund, the policyholder requires a higher return in order to optimally reset his maturity guarantee. Therefore, unless the policyholder sees exceptional growth in the fund, he will look for the quickest way out of the fund, which is by essentially not resetting or, more precisely, requiring a very high return in order to optimally reset. If, on the other hand, the maximum expected return to the policyholder from investing in a segregated fund is higher than from investing in risk free it holds that the maturity guarantee acts as a safety net which protects the maximum expected return to the policyholder. Despite the crash, if the value of the underlying asset relative to its original value at investment, is less than the maturity guarantee, the policyholder's expected return will be equal to at least the maturity guarantee. A further finding

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policyholder would rather invest in risk free throughout the planning horizon compared to investing in segregated funds, then he will require a higher return in order to optimally reset his maturity guarantee while the total value of the segregated fund is diminished due to the negative effect of the crashes on the underlying asset value. If on the other hand he prefers to invest in segregated funds (compared to risk free), then it holds that the maturity guarantee acts as a safety net which protects the maximum expected return to the policyholder. Despite the crash, if the value of the underlying asset relative to its original value at investment, is less than the maturity guarantee, the policyholder's expected return will be equal to at least the maturity guarantee.

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him from those. Therefore the model suggests that the cumulative effect of the potential crashes and surges is an increase in the maximum expected return from the original investment.

In future research it would be interesting to allow the investor more options in terms of the available investments. In particular, in addition to the segregated fund and the risk free rate it would be worth while to add other investment vehicles such as shares or commodities. One may expect to see in times of negative business climate an inclination from the investor to switch to less risky solutions as well as the opposite when the business climate is positive. Furthermore, it would be interesting to apply the knowledge gained from modeling the reset option embedded in segregated funds, to model reset options embedded in other financial products. One prominent candidate would be the swing option embedded in energy derivatives. Taking into account the high volatility of the energy markets, optimising its exercising could prove a very useful tool in the hands of both policyholders and issuers.

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