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Triangle counting in graph streams: Power of multi-sampling

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by

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ABSTRACT

Some of the well known streaming algorithms to estimate number of triangles in a graph stream work as follows: Sample a *single* triangle with high enough probability and repeat this basic step to obtain a global triangle count. For example, the algorithm due to Buriol *et al.* [6] uniformly at random picks a *single vertex v* and a *single edge e* and checks whether the two *cross edges* that connect v to e appear in the stream. This basic sampling step is repeated multiple times to obtain an estimate for the global triangle count in the input graph stream.

This work, proposes a *multi-sampling variant* of this algorithm: In case of Buriol *et al*'s algorithm, instead of randomly choosing a single vertex and edge, randomly sample *multiple vertices and multiple edges* and and collect cross edges that connect sampled vertices to the sampled edges. We provide a theoretical analysis of the algorithm and prove that this simple modification yields improves upon the known space and accuracy bounds. We experimentally show that the algorithm out performs several well known triangle counting streaming algorithms.

CHAPTER 1. INTRODUCTION

With the current scale of web based networks, it is often extremely difficult to calculate the exact characteristic of graphs, the main reason being their humongous size which makes it inefficient to apply traditional algorithms. It is often sufficient to obtain an approximate estimation of these characteristic quantities. One such quantity is the number of triangles in the graph.

Triangle is the smallest non-trivial clique of a graph. Understanding coarser and finer properties of triangle structures in a network has found applications in various areas such as social network analysis, recommendation systems, spam and fraud detection, and understanding network structure and evolution [29, 4, 8, 9, 20, 3, 7, 26]. For example, the frequently used notions of *transitivity coefficient* and *clustering coefficient* critically depend on the number of triangles present in a social network. Many of the applications involve estimating these co-coefficients. With the advent and presence of very large scale graphs such as online social networks and web graphs, the computational complexity of computing the number of triangles of a graph has received a lot of attention. Since exact triangle computation is expensive on large scale graphs, considerable emphasis has been placed on designing algorithms that will *estimate* the number of triangles of a graph with *provable guarantees* on the quality of the estimation.

1.1 The streaming model

One of the practical and well accepted models of computation for large scale data analytics is that of *data stream model*. In this model the data items arrive as a stream and the goal is to compute a function over the entire stream. Often it is the case that the stream processor does not have enough resources to store the entire stream in its memory and perform an off-line computation. Thus the processor can not access an individual item of the stream multiple times unless it is stored in the memory. This streaming model is applicable in scenarios where the volume and velocity of the data is huge. In the data stream model for graphs, the stream processor receives a stream $(e_1, e_2, \cdots, e_m)$ as input, where each e_i is an edge of an underlying graph $G = (V, E)$. No assumption is made on the order in which the edges arrive. The goal is to compute a function of interest by observing the graph stream while using as little memory as possible. Due to the constraints on the memory, it is not possible to compute exact answers for most of the functions. Thus it is desirable to compute an approximate value of the underlying function.

1.2 Contribution

This thesis presents a new algorithm to estimate the number of triangles of a graph in the data stream model. The algorithms is based on the algorithm due to Buriol *et al.* [6]. A basic step in both the algorithms is that they sample a single triangle. Firstly, a high-level description of the triangle sampling procedures is provided. The streaming algorithm of Buriol *et al.* is as follows: Uniformly at random sample a vertex v and an edge $\langle a, b\rangle$ of the graph. The uniform sampling is done using the classical reservoir sampling technique. Once sampled, if the both *cross edges* $\langle a, v\rangle$ and $\langle b, v\rangle$ arrive then output 1 otherwise output 0. Run R *independent* of copies of this basic procedure and take the average output as an estimate for the the number of triangles. Let T denote the number of triangles of the graph and m, n denote the number of edges and vertices of the graph. They showed that if R is $O(\frac{mn}{T}\frac{1}{\epsilon^2}\log(1/\delta))$, then with probability at least $(1-\delta)$, the estimate is accurate within a relative error of ϵ, i.e, the estimate differs from correct value by at most ϵT. From now we will refer to this algorithm as *Edge-Vertex Single Sampling algorithm*, EVSS algorithm for short.

The EVSS algorithm attempts to sample a *single triangle.* Simply stated the main contribution in the thesis is the following: By Modifying the above procedures to sample multiple triangles we obtain better bounds on the memory and space used. Consider EVSS algorithm. Instead of picking a single vertex and a single edge, pick *multiple vertices and multiple edges* , store the *cross edges* that connect the sampled vertices to the sampled edges, and count the number of triangles formed by sampled edges and cross edges and scale the number by an appropriate factor. Surprisingly, it is shown that this simple variant, not only outperforms the original algorithms but has better accuracy, space bound and runtimes compared to some state of the art triangle counting algorithms. A theoretical analysis of the algorithm is provided and the algorithm is experimentally evaluated. It is proved that the multi-sampling version of the EVSS algorithm uses $O(\frac{m\sqrt{\Delta_V}}{\sqrt{T}}\frac{1}{\epsilon}\log(\Delta_V/\delta))$ memory to compute an (ϵ, δ)-approximation of number of triangles, here Δ_V denotes the maximum triangle degree of the graph (maximum number of triangles a vertex can participate in). The new variant has smaller factors in terms of ϵ—$\frac{1}{\epsilon}$ instead of $\frac{1}{\epsilon^2}$. The algorithm is validated by performing an extensive set of experiments, where the proposed algorithm is compared to a few other well known algorithms. We empirically show that the new algorithms performs better than several other triangle counting streaming algorithms from the literature [23, 18, 19, 6]. The experiments show that even on very large graphs such as Orkut (containing more than 100 million edges), the estimates produced by our algorithm have very high accuracy ($> 99\%$) while storing only 2% of the edges. Impressively, the run time is less than a minute.

CHAPTER 2. REVIEW OF LITERATURE

2.1 Non-Streaming algorithms

A naive method to count triangles is to consider all triplets in a graph and verify if they form a triangle. This takes $O(n^3)$ time, where n is the number of vertices of the graph. The algorithm proposed by Alon *et al.* [1] reduces the triangle counting problem into a matrix multiplication problem which runs in $O(m^{1.4})$. Other simple counting algorithms are node and edge iterator algorithms. The node iterator counts for each node, the number of triangles contained in it. The edge iterator in turn counts the number of common neighbors that the vertices of each edge have. Though a lot of minor improvements have been suggested using special data structures, hashing and sorting techniques these algorithms run in $O(mn)$ time with worst case run time of $O(n^3)$. The above algorithms are suitable for smaller graphs, but are practically in-feasible to work on larger real time graphs.

2.2 Streaming algorithms

There has been an extensive body of research on the problem of exact and approximate triangle counts in various computational models [21, 25, 26, 27, 28, 24, 18, 14]. Here we restrict our attention to the prior in the data stream model. Bar-Yosseff *et al.* were the first to formally study the problem of triangle counting the data stream model [2]. They reduced the triangle counting problem on graphs to estimating zeroth and second frequency moments over numeric data.

Much of the latter work is is based on various sampling strategies in the sense that they attempt to sample a sub-graph of the original graph [17, 13, 5, 22, 11, 10, 12, 6, 19, 16, 23]. In the algorithm defined by R. Pagh *et al*[18], each vertex of the graph is colored with $N = \frac{1}{p}$ colors randomly, the triangles having the same colored vertex are then counted. They claim that when

$p \geq max(\frac{\delta logn}{t}, \sqrt{\frac{logn}{t}})$, the triangle estimate is concentrated around its expectation. The work of Buriol *et al.* [6], randomly picks a vertex and an edge and looks for the two cross edges that connect the sampled vertex and the sampled edge, and this process is repeated. The work [19], randomly picks an edge e_1 and a random neighbor e_2 and looks for a cross edge that connects e_1 with e_2. Both these works use reservoir sampling to randomly picks vertices/edges. The work of Lim and Kang [16] uses a fixed probability to sample the edges (as opposed to reservoir sampling). They fix a parameter p and pick each edge with probability p and count the number of triangles formed by the sampled sub-graph. The recent work of Stefani *et al.* [23] uses reservoir sampling to sample multiple edges and counts the number of triangles in the sampled-subgraph. Their algorithm also handles dynamic streams (where edges can be deleted) and can be used to obtain local-triangle counts. The memory used by these algorithms depend on various graph parameters such as number of vertices (n), number of edges (m), total number of triangles (T), maximum-degree (Δ), maximum number of triangles an vertex participates (Δ_V) etc. We describe the EVSS sampling strategy in Algorithm 1 from Buriol et al [6] as our multi-sampling algorithm is built on it's base sampling strategy. This algorithm is repeated $\frac{1}{\epsilon^2}$ times and the average is returned to get a triangle estimate.

```
Input: Edge Stream $E$
Initialize: $i = 1, \beta = 0$;
for $\forall e = \langle u, w \rangle$ from edge stream $E$ do
    if $coin(\frac{1}{i}) == head$ then
        $a \leftarrow u; b \leftarrow w$;
        $v \leftarrow$ vertex uniformly chosen from $V \backslash \{a, b\}$
        $x \leftarrow false; y \leftarrow false$
    end
    if $e == (a,v)$ then
        $x \leftarrow true$
    end
    if $e == (b,v)$ then
        $y \leftarrow true$
    end
    $i \leftarrow i + 1$
end
if $x \leftarrow true$ and $y \leftarrow true$ then
    $\beta = 1$
end
return $\beta$;
```

Algorithm 1: EVSS algorithm

CHAPTER 3. MULTI-SAMPLING ALGORITHMS

This section describes the EVMS algorithm proposed as part of the thesis. The algorithms is an extension to Buriol *et al.*[6] inspired by the multi sampling approach used by Stefani *et al.* [23].

3.1 Algorithm variations

The EVMS algorithm in turn has two variations based on the sampling approach used. The sampling of edges and vertices can be done based on a fixed probability p, $0 < p < 1$ or based on the reservoir sampling approach. One main difference between the two is the certainty of the sample size after the sampling process. In the reservoir sampling approach, the size of the sample is fixed whereas the sample size using the fixed sampling greatly depends on the value of p. The reservoir sampling works as follows: Let's assume we have a graph stream where edge e_i arrives at time i. We need to maintain a reservoir R of size M. We sample the edge e_i if $|R| < M$ else we sample e_i by randomly replacing an element from R with probability $\frac{M}{i}$. Another variation in the algorithms is the number of passes (multi-pass or single-pass) made on the graph stream. The thesis makes extensive theoretical and experimental analysis only on the single pass, fixed sampling version.

3.2 Edge-vertex multi-sampling (EVMS)

Before the algorithm is presented, we provide an intuitive reasoning behind as why multi-sampling is a better strategy. Consider a two triangle $t_1 = \{a, b, c\}$ and $t_2 = \{a, d, e\}$ that share a vertex a. Assume that the order in which the edge appear in the stream is bc, ab, ca, ad, ae, de. The EVSS algorithm samples t_1 if vertex a is sampled and the edge ab is sampled which happens with probability $1/(mn)$. If we were two sample both the triangles, then the sampling step of EVSS need to be repeated twice and thus probability of sampling both the triangles in $1/(mn)^2$. Suppose that we sample a single vertex and *two edges* (and look for cross edges), what is the probability that both t_1 and t_2 are sampled? This happens if the sampled vertex is a and the sampled edges are bc and ad. This probability is proportional to $1/m^2n$ which is higher than $1/(mn)^2$. Thus the second strategy has a better chance of entangled triangle (triangles that share a vertex/edge) than the first strategy. This is the rationale behind the multi-sampling algorithm.

Our algorithm works as follows: We fix two sampling probabilities p_v and p_e. For simplicity, we assume that the vertex set V is known in advance. Later, we discuss how this assumption can be removed. We place each $v \in V$ into a set S_v with probability p_v. We then process the graph stream—when an edge e arrives place it in a set S_e with probability p_e. In addition, if e connects a vertex from S_v to an edge from R_e, then we place e in a set B_e. We call the edges from R_e as red edges and edges from B_e as black edges. Note that an edge can be both red edge and black edge.Let us denote the time at which an edge $\langle u, v\rangle$ appears in the stream as $t_{\langle u,v\rangle}$. We count only the sampled triangles such that each triangle has exactly two black edges and a red edge, and the two black edges appear after the red edge in the stream. We scale this count with an appropriate factor to get a global estimate.

The reason we check for triangles in which black edges arrive after the red edges is, if we sample edges in the stream in order of arrival $\langle a, b\rangle$ as red,$\langle b, d\rangle$ as black and $\langle b, c\rangle$ as red while the vertex d is a sampled vertex, the edge $\langle c, d\rangle$ can be sampled as a black edge afterwards. Then the triangle $\langle b, c, d\rangle$ should be counted as a sampled triangle. But we are not counting such triangles.

3.3 Theoretical Preliminaries

This section presents certain results used later in the theoretical analysis of the algorithms.

Definition 3.3.1. Random Variable: A random variable X is a function from a sample space Ω, to the real numbers, $X : \Omega \rightarrow R$.

Definition 3.3.2. Expectation of a random variable is defined as

$$E[X] = \sum_{\alpha} Pr(X = \alpha)\alpha$$

Lemma 3.3.1. *Chebyshev's Inequality: Let X be a random variable. Then,*

$$Pr[|X - E(X)| \geq \delta] \leq \frac{Var(X)}{\delta^2}$$

Lemma 3.3.2. *Chernoff Inequality: Let $X_1, X_2 ...., X_m$ be independent random variables that take values between 0 and 1. Let $E(X_1) = E(X_2) = ... = E(X_m) = p$. Let $X = X_1 + X_2 + ... + X_m$, then*

$$Pr\left[\left|X/m - p\right| \geq p\delta\right] \leq 2e^{\frac{-\delta^2 mp}{2}}$$

Lemma 3.3.3. *Hajnal-Szemerédi Theorem: Every graph with n vertices and maximum vertex degree at most k is k + 1 colorable with all color classes of size at least $\frac{n}{k+1}$.*

3.4 EVMS Algorithm

3.4.1 Algorithm

As mentioned above, in the fixed sampling approach, the vertices and edges are sampled with a fixed probability. Let us denote these probabilities as p_v and p_e respectively. Let G be input graph with V as the vertex set and E as the edge stream of the graph G. Let S_v denote the sampled vertex set, the set of all sampled triangles - Y, the set of all red edges - R_e, and the set of all black edges - B_e. The algorithm can be implemented as a multi-pass algorithm, where initially we sample vertices with probability p_v, in the first pass we sample edges with probability p_e. On the second pass, we check if each edge can be sampled as black edge. This approach can be slow if the graph is too large or in real-world scenarios where it is often not practical to read the stream multiple times. The two passes can be easily combined to a single pass with a slight modification to the algorithm involving time stamps of the edges. The passes are combined such that every edge in the stream is considered to be sampled as a red edge as well as a black edge. A formal description of the EVMS algorithm can be found in Algorithm 2. Here, Y is a random variable that denotes the number of triangles sampled by the algorithm. The algorithm outputs $\frac{|Y|}{p_v p_e}$ whose expected value is the number of triangles in the entire graph.

```
Input: Edge Stream $E$, vertex set $V, p_v, p_e$
Initialize: $S_v = \emptyset$, $Y = \emptyset$, $R_e = \emptyset$, $B_e = \emptyset$;
for every $v \in V$ do
    if coin($p_v$) == head then
        $S_v \leftarrow S_v \cup \{v\}$
    end
end
for $\forall e_i = \langle a, b \rangle$ from edge stream $E$ do
    if coin($p_e$) == head then
        $R_e \leftarrow R_e \cup \{e\}$
    end
    if ($a \in S_v$ and $R_e$ has an edge with vertex $b$) OR ($b \in S_v$ and $R_e$ has an edge with vertex $a$) then
        $B_e \leftarrow B_e \cup \{e_i\}$
    end
    for every $(e_j, e_k)$, $e_j \in R_e$, $e_k \in B_e$ such that $(t_{e_j} < t_{e_k})$ and $(t_{e_j} < t_e)$ and $\langle e, e_j, e_k \rangle$ forms a triangle do
        $Y \leftarrow Y \cup \langle e_i, e_j, e_k \rangle$
    end
end
Output: $\tau = |Y|/(p_v p_e)$
```

Algorithm 2: EVMS - Fixed sampling

3.4.2 Theoretical Analysis

Let $\tau(G)$ be set of all triangles in graph G, and $|\tau(G)| = T$.

Lemma 3.4.1. *Let $t \in \tau(G)$. The probability that t is sampled is,*

$$Pr[t \in Y] = p_v p_e$$

Proof. A triangle, $\langle a, b, c \rangle$ is sampled when one of its vertex and its opposite edge is sampled as a red edge. Since every vertex and edge is sampled with a fixed probability, p_v and p_e respectively, and the two events are independent, the probability that a triangle , $t \in \tau(G)$ gets sampled is $p_v p_e$. □

To estimate the number of triangles, we consider the algorithm's output into a random variable such that the expectation of the random variable gives the triangle estimate.

Lemma 3.4.2. *The algorithm EVMS-fixed-sampling outputs τ whose expectation is the number of triangles in the graph G.*

Proof. Let us define a random variable X, which denotes the number of triangles sampled by the algorithm. Let us calculate the $E[X]$. To calculate $E[X]$, we split the random variable X into smaller random variables X_i such that $X = \sum X_i, 1 \leq i \leq T$. We define,

$$\begin{aligned}
X_i &= 1,\ if\ the\ i^{th}\ triangle\ of\ the\ graph\ gets\ sampled \\
&= 0,\ else \\
E[X_i] &= Pr[X_i = 1].1 + Pr[X_i = 0].0 \\
&= p_v p_e
\end{aligned}$$

Since X_i is a 0,1 random variable, $|Y| = \sum\limits_{i=1}^{T} X_i$

$$\begin{aligned}
E[\tau] &= E[\frac{\sum\limits_{i=1}^{T} X_i}{p_v p_e}] \\
&= \frac{1}{p_v p_e}\sum_{i=1}^{T} E[X_i] \\
&= T
\end{aligned}$$

□

Lemma 3.4.3. *The EVMS algorithm returns a triangle estimate τ, such that*

$$Var[\tau] \leq \frac{T}{p_e}(\frac{1}{p_v} + 3\Delta_V T)$$

.

Proof.

$$\begin{aligned} Var[\tau] &= Var[\frac{1}{p_v p_e}\sum_{i=1}^{T} X_i] \\ &= \frac{1}{{p_v}^2{p_e}^2}\sum_{i=1}^{T}\sum_{j=1}^{T} Cov[X_i, X_j] \\ &= \frac{1}{{p_v}^2{p_e}^2}(\sum_{i=1}^{T} Var(X_i) + \sum_{i=1}^{T}\sum_{\substack{i=1\\ i\neq j}}^{T} Cov[X_i, X_j]) \end{aligned}$$

From the expression for Variance, $Var[X_i] = p_v p_e - {p_v}^2{p_e}^2$. If triangles i and j do not share a vertex then $Cov[X_i, X_j] = 0$. Triangles i and j can be dependent when they share the red edge or the sampled vertex. If they share a vertex, $Cov[X_i, X_j] = p_e{p_v}^2 - {p_v}^2{p_e}^2$.

Let us calculate an upper bound for $\sum_{i=1}^{T}\sum_{\substack{i=1\\ i\neq j}}^{T} Cov[X_i, X_j]$. If T_v is the number of triangles incident on vertex v, then,

$$\begin{aligned} \sum_{i=1}^{T}\sum_{\substack{i=1\\ i\neq j}}^{T} Cov[X_i, X_j] &= \sum_{i=1}^{T}\sum_{\substack{i=1\\ i\neq j}}^{T}(p_e{p_v}^2 - {p_v}^2{p_e}^2) \\ &\leq \sum_{i=1}^{T}\sum_{\substack{i=1\\ i\neq j}}^{T}(p_e{p_v}^2) \\ &\leq p_e{p_v}^2\sum_{v\in G}{T_v}^2 \\ &\leq p_e{p_v}^2\sum_{v\in G}T_v\Delta_V \\ &\leq p_e{p_v}^2\Delta_V\sum_{v\in G}T_v \\ &\leq 3p_e{p_v}^2\Delta_V T \end{aligned}$$

Hence we have,

$$\begin{aligned} Var[\tau] &\leq \frac{1}{{p_v}^2{p_e}^2}(\sum_{i=1}^{T}(p_vp_e - {p_v}^2{p_e}^2) + 3p_e{p_v}^2\Delta_V T) \\ &\leq \frac{1}{{p_v}^2{p_e}^2}(\sum_{i=1}^{T}(p_vp_e) + 3p_e{p_v}^2\Delta_V T) \\ &= \frac{1}{{p_v}^2{p_e}^2}(T(p_vp_e) + 3p_e{p_v}^2\Delta_V T) \\ Var[\tau] &\leq \frac{T}{p_e}(\frac{1}{p_v} + 3\Delta_V T) \end{aligned}$$

□

Theorem 3.4.4. *Concentration using Chebyshev's inequality: For any $\epsilon, \delta \in (0,1)$, and $p_v = p_e = q$, the EVMS fixed sampling algorithm returns a triangle estimate τ such that when*

$$q \geq \frac{-3\Delta_V + \sqrt{q\Delta_V^2 + 4\epsilon^2\delta}}{2\delta\epsilon^2}$$

$$Pr[|\tau - T| \geq \epsilon T] \leq \delta$$

Proof. The proof follows by simple application of Chebyshev's inequality using the variance obtained from Lemma 3.4.3. Recall that the output of the algorithm is $\dfrac{X}{p_ep_v}$.

$$Pr[|\tfrac{X}{p_ep_v} - T| \geq \epsilon T] \leq \tfrac{Tp_vp_e(1+3P\Delta_V)}{p_v^2p_e^2\epsilon^2T^2} \leq \delta$$

For simplicity, if set $p_v = p_e = q$, after solving this inequality we get $q \geq \dfrac{-3\Delta_V + \sqrt{q\Delta_V^2 + 4\epsilon^2\delta}}{2\delta\epsilon^2}$. Thus the above algorithm outputs (ϵ, δ) approximation if $p_v = p_e = q$ and the above inequality holds. □

Theorem 3.4.5. *For any $\epsilon, \delta \in (0,1)$, the EVMS fixed sampling algorithm returns a triangle estimate τ such that when*

$$p_ep_v \geq \frac{2}{\epsilon^2}\frac{3\Delta_V + 1}{T}\ln(\frac{2(3\Delta_V + 1)}{\delta})$$

$$Pr[|\tau - T| \leq \epsilon T] > 1 - \delta$$

Proof. We can use Chernoff bounds to find the concentration bounds for the random variable X, since it can be split into smaller random variables with equal expectation. But one cavaet to apply Chernoff bounds is that the random variables need to be independent. In this case, random variables, X_i and X_j are not independent when the triangles they represent share a vertex. One way to deal with this is to group the triangles into groups, such that no triangle in a group share a vertex. Hence all random variables within a group are independent, enabling us to apply Chernoff bounds on each of these group. But how do we know how many such groups exist ?

One way to find a lower bound on the number of groups is to build an auxiliary graph H and apply Hajanal-Szeméredi therorem on the graph H. We build H, so that for every triangle in $\tau(G)$, we have a corresponding vertex in H. An example of this construction is shown in Figure 3.1. There is an edge between any two vertices in H if, the corresponding triangles in $\tau(G)$ share a vertex. Let the maximum number of triangles shared by a vertex in $\tau(G)$ be Δ_V. By Hajanal-Szeméredi theorem, H can be properly colored with at most $3\Delta_V + 1$ colors, with atleast $k = \frac{|\tau(G)|}{3\Delta_V+1}$ vertices in each color.

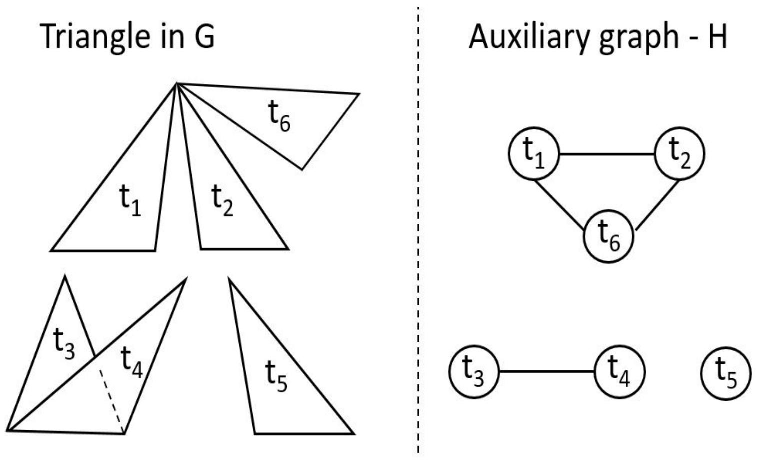


Figure 3.1: Construction of auxiliary graph from the sampled triangles

Let us number each of the triangles in $\tau(G)$ by a arbitrary number i, so that the random variable X_i corresponds to that particular triangle i. Let c be one of the colors in the coloring of H. Let Y_c

be the set of random variables X_i, such that the node i in H has color c. The random variables in Y_c are independent. Also it is to be noted that $\sum\limits_{\forall c}[\sum\limits_{X_i \ inY_c} X_i] = \sum\limits_{\forall X_i}$ and $\sum\limits_{\forall c} k = T$.

Applying Chernoff bounds to the color set c, we have,

$$Pr[|\frac{\sum\limits_{X_i \in c} X_i}{k} - p_e p_v| \geq \epsilon p_e p_v] \leq 2e^{\frac{-\epsilon^2 k p_e p_v}{2}}$$

$$Pr[|\frac{\sum\limits_{X_i \in c} X_i}{p_e p_v} - k| \geq \epsilon k] \leq 2e^{\frac{-\epsilon^2 k p_e p_v}{2}}$$

The above equation shows the probability bound for a single color class. Hence, the probability that at least one of the class has $Pr[|\frac{\sum\limits_{X_i \in c} X_i}{k} - p_e p_v|] \geq \epsilon p_e p_v$ can be obtained by union over all color sets. Let us denote this by δ.

$$\sum_{\forall c} Pr[|\frac{\sum\limits_{X_i \in c} X_i}{k} - p_e p_v| \geq \epsilon p_e p_v] \leq \sum_{\forall c} 2e^{\frac{-\epsilon^2 k p_e p_v}{2}}$$

$$Pr[|\frac{\sum\limits_{\forall c}\sum\limits_{X_i \in c} X_i}{p_e p_v} - T| \geq \epsilon T] \leq (3\Delta_V + 1) 2e^{\frac{-\epsilon^2 k p_e p_v}{2}}$$

$$Pr[|\frac{\sum X_i}{p_e p_v} - T| \geq \epsilon T] \leq (3\Delta_V + 1) 2e^{\frac{-\epsilon^2 k p_e p_v}{2}}$$

$$Pr[|\tau - T| \geq \epsilon T] \leq (3\Delta_V + 1) 2e^{\frac{-\epsilon^2 k p_e p_v}{2}}$$

$$Pr[|\tau - T| \geq \epsilon T] \leq \delta$$

Therefore,

$$e^{\frac{-\epsilon^2 k p_e p_v}{2}} \leq \frac{\delta}{2(3\Delta_V + 1)}$$

$$p_e p_v \geq \frac{2}{\epsilon^2} \frac{3\Delta_V + 1}{T} \ln(\frac{2(3\Delta_V + 1)}{\delta}) \simeq \frac{\Delta_V}{\epsilon^2 T} ln \frac{\Delta_V}{\delta}$$

From the above expression, we infer that when $p_e p_v \geq \frac{2}{\epsilon^2} \frac{3\Delta_V + 1}{T} \ln(\frac{2(3\Delta_V + 1)}{\delta})$, the EVMS fixed sampling algorithm provides a (ϵ, δ) approximation for the number of triangles in the graph G. $\square$

3.4.3 Memory bounds

We now analyze the memory used by the algorithm. The memory used for this algorithm consists of the memory used to store the black and red edges. The expected number of red edges in total is $p_e m$. We can compute the expected number of black edges as follows: Let ab be an edge of the graph. This becomes a black edge if a is in S_v and an edge incident on b is sampled as a red edge. The other case is when b is in S_v and an edge incident on a was is sampled as a red edge . Let's look at the former case. The vertex $a \in S_v$ with probability p_v. Let d be the number of edges that are incident on vertex b and arrive before edge ab. Probability that at least one of these edges being a red edge is $(1-(1-p_e)^d)$. Thus ab becomes black edge with probability $(1-(1-p_e)^d)p_v$. In general let $d(e)$ denote the number of edges that arrive before e and is incident on e. The expected number of black edges is at most

$$\begin{aligned}
&\sum_{e \in G}[1-(1-p_e)^{d(e)}]p_v \\
&= p_v \sum_{e \in G} 1 - \sum_{e \in G}(1-p_e)^{d(e)} \\
&= p_v(m - \sum_{e \in G}(1-p_e)^{d(e)}) \leq mp_v
\end{aligned}$$

So the expected number of red and black edges is bounded by $m(p_e + p_v)$. Subject to the constraint from Theorem 3.4.5, $p_e p_v \geq \frac{\Delta_V}{\epsilon^2 T} log \frac{\Delta_V}{\delta}$ the quantity $p_e + p_v$ is minimized when $p_v = p_e$ equals $\sqrt{\frac{\Delta_V}{\epsilon^2 T} log \frac{\Delta_V}{\delta}}$. Thus the expected number of edges stored by the algorithm is $O(mp_e)$ and the expected number of vertices stored is $O(np_e)$ which is $O(mp_e)$. Putting together we obtain the following.

Theorem 3.4.6. *There is a streaming algorithm that computes (ϵ, δ)-approximation of number of triangles in a graph by using expected space $O(mp)$ where p equals $\sqrt{\frac{\Delta_V}{T}}\frac{1}{\epsilon}\sqrt{log\frac{\Delta_V}{\delta}}$.*

3.4.4 Time Complexity

The running time of the algorithm depends only on the step that computes the exact count of the triangles from the vertex and edge samples. We use the edge iterator algorithm to find the exact count. The Edge Iterator algorithm computes for each edge the number of triangles it a part of. It does this by find the common neighbors of its end vertices. The time complexity of this algorithm is $O(\sum_{v \in S_v} d_v^2)$.

3.5 EVMS - Reservoir sampling

3.5.1 Algorithm

The thesis gives a description of the reservoir sampling version of the algorithm, but does not discuss the theoretical and experimental analysis. In the reservoir sampling version of the EVMS algorithm, the vertices and red edges are sampled using a reservoir of fixed size. Let us denote the vertex reservoir memory as N and edge reservoir memory as M. Let n and m be the total number of vertices and edges respectively in the original graph. Let $V = \{v_i, 1 \leq i \leq n\}$ and $E = \{e_i, 1 \leq i \leq m\}$ be the vertex set and edge stream respectively. The formal description of the EVMS-reservoir sampling algorithm is defined in Algorithm 3.

```
Input: Edge Stream $E$, vertex set $V, p_v, p_e$
Initialize: $S_v = \emptyset$, $Y = \emptyset$, $R_e = \emptyset$, $B_e = \emptyset$;
for every $v_i \in V$ do
    if $|S_v| < N$ then
        $S_v \leftarrow S_v \cup \{v_i\}$ ;
    end
    else if coin($\frac{M}{i}$) == head then
        $t \leftarrow$ random vertex from $S_v$ ;
        $S_v \leftarrow S_v \setminus \{t\}$;
        $S_v \leftarrow S_v \cup \{v_i\}$ ;
    end
end
for $\forall e_i = \langle a, b \rangle$ from edge stream $E$ do
    if $|R_e| < M$ then
        $R_e \leftarrow R_e \cup \{e_i\}$ ;
    end
    else if coin($\frac{N}{i}$) == head then
        $x \leftarrow$ random vertex from $R_e$ ;
        $R_e \leftarrow R_e \setminus \{x\}$ ;
        $R_e \leftarrow R_e \cup \{e_i\}$ ;
    end
    if ($a \in S_v$ and $R_e$ has an edge with vertex $b$) OR ($b \in S_v$ and $R_e$ has an edge with vertex $a$) then
        $B_e \leftarrow B_e \cup \{e_i\}$ ;
    end
    for every $(e_j, e_k)$, $e_j \in R_e$, $e_k \in B_e$ such that $(t_{e_j} < t_{e_k})$ and $(t_{e_j} < t_{e_i})$ and $\langle e_i, e_j, e_k \rangle$ forms a
     triangle do
        $Y \leftarrow Y \cup \langle e_i, e_j, e_k \rangle$
    end
end
Output: $|Y| \frac{mn}{MN}$
```

Algorithm 3: EVMS - Reservoir sampling

CHAPTER 4. EXPERIMENTAL EVALUATION

The experiments were performed to analyze the memory, accuracy, and speed of both the algorithms. The aim is to verify, for a given value of p_v and p_e the theoretical bounds hold true. We also analyze how the parameters like maximum triangle degree, number of edges and transitivity co-efficient of the graph affect the accuracy of the algorithms. We run the algorithms with the probabilities set to $0.01, 0.05, 0.1, 0.15$ and 0.2. Hence for each graph, we have 25 possible experiments.

Experiments were conducted on several graphs from the Stanford Large Network Dataset Collection [15]. The implementation was done in Java 8, and run on a PC with Core i7 2.5 Ghz, 16GB RAM with Windows 10 operating system. The experiments were run on insertion only streams, where each edge is a new edge with no repetitions. The datasets were chosen in such a way that they have varying topologies. The graphs have varying densities ranging from around 900,000 edges (Amazon) to more than 100,000,000 edges (Orkut). The graph was pre-processed so that the vertices are numbered form 0 to $n-1$, where n is the total number of vertices of the graph. In the implementations, the graph is represented as an adjacency list with a map of all the vertices and corresponding values as the list of all neighbors of that vertex.

The performance of the proposed algorithms were compared against EVSS algorithm from Buriol *et al*'s [6], Triest-Base [23], neighborhood sampling algorithm [19], and colorful sampling algorithm due to Pagh and Tsourakakis [18]. The rational for this choice of algorithms is as follows: Naturally, we would like to compare EVMS and NMS against their counter parts. Since, Triest-Base [23], the authors experimentally showed that Triest-Base algorithm outperforms several other algorithms from the literature, we chose Triest-Base algorithm. We chose the colorful

Table 4.1: Memory functions for the various algorithms

	EVSS	NS	Triest	Colorful	EVMS
$f(G)$	$\frac{mn}{T}\ \frac{1}{\epsilon^2}log\frac{1}{\delta}$	$\frac{m\Delta}{T}\ \frac{1}{\epsilon^2}log\frac{1}{\delta}$	$m(\frac{\Delta_E}{T})^{\frac{1}{3}}\ \frac{1}{(\epsilon)^{\frac{2}{3}}}\sqrt{log\frac{\Delta_E}{\delta}}$	$\frac{m\Delta_V}{T}\ \frac{1}{\epsilon^2}$	$m\sqrt{\frac{\Delta_V}{T}}\ \frac{1}{\epsilon}\sqrt{log\frac{\Delta_V}{\delta}}$

triangle counting algorithm because as far as our knowledge its performance on graph streams has not been tested before. The theoretical memory bounds can be expressed as a function of the different parameters of the graph like number of vertices, edges and actual triangle count. Let us denote this as $f(G)$. The $f(G)$ for all the above algorithms are show in Table 4.1

All the above algorithms were implemented in java. The implementation is available at *https : //github.com/kpneeraj/triangle_counting*. The experiments were first run on the EVMS-fixed sampling algorithm with different values for p_v and p_e. To get a fair comparison, the other algorithms were run using the total memory, T_m used by the both the algorithms. The total memory is the sum of the red and black edges for the EVMS algorithm.

4.1 Datasets used

The following datasets [15] were used whose vertex and edge counts are shown in Table 4.2.

DBLP Collaboration network : A co-authorship network of computer science bibliography where two authors are connected if they publish at least one paper together. Authors who published to a certain journal or conference form a community.

Skitter : An Internet topology graph from trace routes run daily in 2005 from several scattered sources to million destinations.

Live-journal: A friendship graph of a free on-line blogging community where users declare friendship among each other. Users are vertices and the presence of a edge represent friendship between two users.

Amazon co-purchasing network: It is a graph built by collecting items that were purchased along with each other. The vertices are products and the presence of an edge between two vertices

Table 4.2: Properties of the various datasets considered for the experiments

Graph	Nodes	Edges	Triangle count
DBLP	317,080	1,049,866	2,224,385
As-Skitter	1,696,415	11,095,298	28,769,868
Live Journal	3,997,962	34,681,189	177,820,130
Amazon	334,863	925,872	667,129
Berk-Stanford	685,230	7,600,595	64,690,980
Orkut	3,072,441	117,185,083	627,584,181

represent the fact that the products are frequently purchased with each other.

Berkeley-Stanford web graph : Nodes represent pages from berekely.edu and stanford.edu domains and edges represent hyperlinks between them. The graph was initially directed, which was converted to undirected by removing redundant edges that represent direction.

Orkut web graph : A friendship graph of the Orkut social network where each vertex is a person and each edge represents friendship between two people.

4.2 EVMS sampling

Since the vertices are numbered from 0 to $n-1$, the first step of vertex sampling is replaced by sampling integers from 0 to $n-1$ with probability p_v. The implementation of the EVMS algorithm maintains two adjacency lists one for the red edges and the other for the black edges. Along with the neighbors for each vertex, the time stamp for the corresponding neighbors are stored as well. The actual count of the number of triangles is done using the edge-iterator algorithm by iterating through all the red edges. For every red edge x, we find two black edges, y, z such that y and z share common vertices and $t_x < t_y$ and $t_x < t_z$.

We discuss the results of the EVMS algorithm and later compare them with the results of Buriol [6], Triest [23], Pavan et al [19] and Pagh *et al.*'s [18] colorful triangle counting algorithms in terms of accuracy and time taken. A subset of the results of the fixed sampling algorithm for various values of p_v and p_e is shown in Table 4.2. For every p_e and p_v the results were taken by running the algorithm 5 times and taking the median. The average of the runs are also reported. We can see that for even large graphs like Orkut with more than 100 million edges, we get considerable accuracy (98%) in the triangle estimate with a total memory of just 2% of the entire graph with time just over a minute. We define percentage error, $pE = (T - \tau)/T$, where T is the actual number of triangles in the graph and τ is the estimated triangle count by our algorithm. A negative pE means that the experiment under estimated the number of triangles in the graph. We can observe that $|pE| < 1\%$ in majority of the cases even for very small p_e and p_v.

Though the number of black edges increase consistently with the increase in p_e, the accuracy does not. This is because the number of vertices or red edges sampled will be low when p_v or p_e is low correspondingly. This results in fewer number of sampled triangles that affect the accuracy of the estimates. We can observe that though the accuracy drops, it stays well within the theoretical bound stated in Theorem 3.4.5.

In the next few sections, we compare the results of the single pass fixed sampling algorithm with Buriol , Triest and Pavan et al [19]. To get a fair comparison, the other algorithms were re-implemented in Java. The experiments were first run on the EVMS fixed sampling algorithm with different values of p_v and p_e. The total memory, T_m (sum of red and black edges) is recorded and is used as the input to the other algorithms.

4.2.1 EVMS vs EVSS algorithm

The optimized version of EVSS algorithm was implemented and its results were compared with the EVMS algorithm. The algorithm was run such that the number of memory consumed by both the algorithms are similar. In most cases for very small repetitions, r, the EVSS algorithm fails to find any triangle, resulting in low-quality estimates whereas EVMS shows very good accuracy. The EVSS algorithm could not be run for large graphs like Orkut due to the low performance of the algorithm. The results of the comparison is shown in Table 4.4.

4.2.2 EVMS vs Triest-Base algorithm

The Triest-Base algorithm works by sampling M edges using reservoir sampling and counts the number of triangles in the sampled subgraph. Similar to the EVSS's experimental setup, the Triest algorithm was run with $M = T_m$. Similar to the Buriol's experimental setup, the Triest algorithm was run with $M = T_m$. Triest shows comparable performance and accuracy to the EVMS algorithm. Triest shows comparable performance and accuracy to the EVMS algorithm. Triest-base shows better accuracy for higher M, but tends to require more memory when the graph is sparse or when the number of edges of the graph is high. For example in the Amazon graph, with $M = T_m = 14,336$, $pE = 1.03\%$ for EVMS, where as Triest shows a pE of 59.61%. The EVMS performs better than the Triest-Base and EVSS in terms of time as well. The reason is that in general, reservoir sampling takes more time than fixed probability sampling (especially when the reservoir size is high). As an example let us consider the the Orkut graph with more than 170 million edges. The EVMS algorithm gives an estimate with $pE = 4\%$ in over a minute with $T_m = 145000$, where as Triest-Base takes over 5 minutes and over 800,000 units of memory for the achieving the same accuracy. Figure 4.2 shows the time taken by both the algorithms for the amazon graph and DBLP graphs. Figure 4.1 shows the time taken by the EVSS,EVMS and Triest-Base algorithms for the As-Skitter graph. The EVMS algorithm takes less than $4s$ in all cases, whereas the time increases linearly with T_m for the Triest-Base and EVSS algorithms.

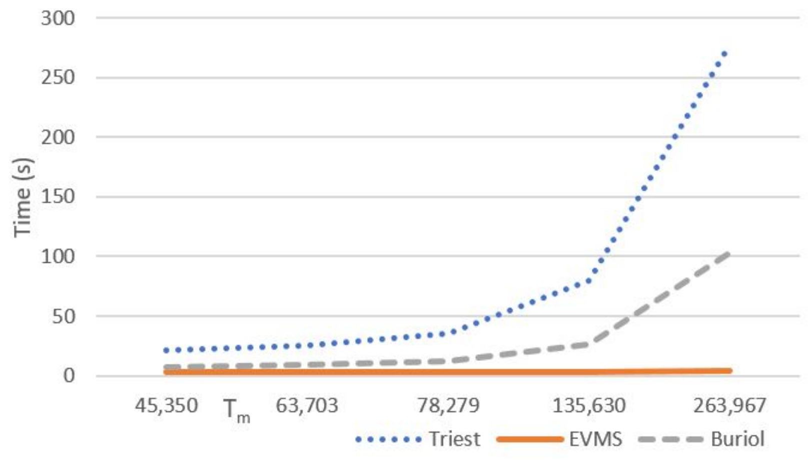
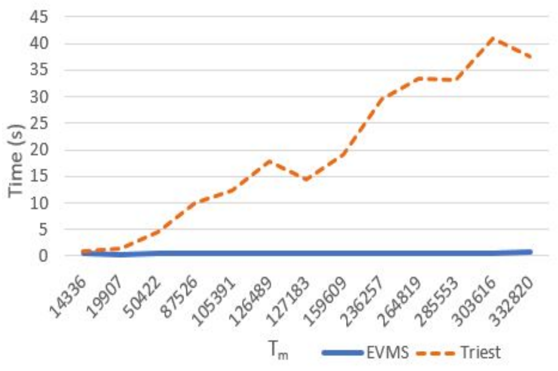
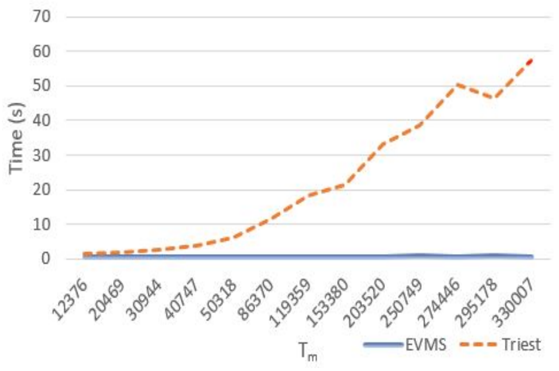


Figure 4.1: T_m vs $Time$ for Triest-base, EVSS and EVMS algorithms



(a) Amazon



(b) DBLP

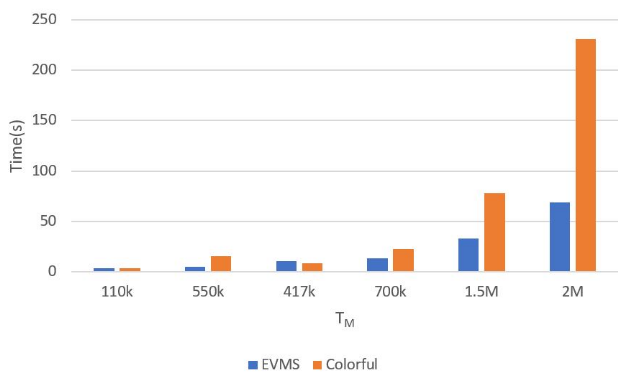
Figure 4.2: T_m vs $Time$ for Triest-base and EVMS algorithms

4.2.3 EVMS vs Neighborhood Sampling

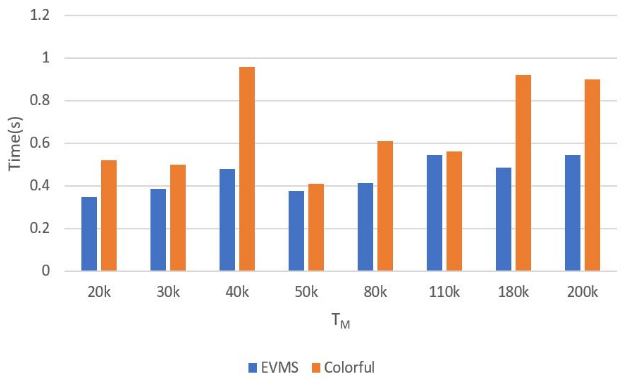
The base version of neighborhood sampling was implemented. We observed that EVMS algorithm has better accuracy compared to the neighborhood sampling algorithm. Moreover, the run-time of the base implementation of the neighborhood sampling algorithm is high. In their work they optimized the run time by doing a batch processing. However batch processing has the disadvantage that we have to wait till the entire stream is processed to obtain a triangle count. More specifically, triangle count can not be monitored continuously. Due to very high running times further analysis and comparison with this algorithm are not reported.

4.2.4 EVMS vs Colorful triangle Counting

The colorful triangle counting algorithm first chooses $1/p$ many colors and randomly assigns a color to each vertex. Since the vertices are numbered from 0 to $n-1$, $1/p$ colors were randomly assigned to integers from 0 to $n-1$. Then edges whose end points have the same color are placed in the sample. The colorful triangle counting was run by setting the p with values 0.01, 0.03, 0.05, 0.08, 0.10, 0.17, 0.20. The total memory, T_c used was measured as the total number of edges sampled in each color. The comparison is done by approximately matching the total memory T_m used by the EVMS algorithm and the total memory used by the colorful triangle counting. We observe that though the time taken is more than EVMS algorithm, *the colorful triangle counting gives better accuracy when compared to EVMS*. Figure 4.3 shows the time taken by both the algorithms for the Berkeley Stanford and Amazon graphs.



(a) Berkeley Stanford



(b) DBLP

Figure 4.3: T_m vs $Time$ for Colorful Triangle and EVMS algorithms

Table 4.3: Results of the EVMS algorithm for various graphs

DBLP social network

p_v	p_e	V	E	B_e	T_m	τ	pE	Time (s)
0.05	0.01	15780	10380	10089	20469	2154000.00	-3.16	0.34
0.05	0.05	15692	52561	33809	86370	2241200.00	0.76	0.41
0.15	0.15	47670	157062	172945	330007	2209466.67	-0.67	0.667

Amazon Product Graph

p_v	p_e	V	E	B_e	T_m	τ	pE	Time (s)
0.15	0.01	50189	9424	15761	25185	691333.33	3.63	0.28
0.05	0.05	16675	46089	20210	66299	648800.00	-2.75	0.33
0.05	0.1	17025	92809	34374	127183	668000.00	0.13	0.40

Berkeley-Stanford web graph

p_v	p_e	V	E	B_e	T_m	τ	pE	Time (s)
0.05	0.01	34220	66984	249093	316077	67424000.00	4.22	3.17
0.15	0.01	103180	66359	735434	96542	64361333.33	-0.51	6.52
0.05	0.05	34094	332610	402696	735306	64628800.00	-0.10	13.74

As-skitter

p_v	p_e	V	E	B_e	T_m	τ	pE	Time (s)
0.1	0.01	169712	110627	753437	27417	27417000.00	-4.70	4.48
0.01	0.05	16982	555164	121783	676947	28384000.00	-1.34	4.19
0.05	0.1	84387	1108978	748970	1857948	28740000.00	-0.10	14.75

Live-journal

p_v	p_e	V	E	B_e	T_m	τ	pE	Time (s)
0.01	0.01	39834	347209	192482	18577	185770000.00	4.47	15.28
0.15	0.01	599880	347082	2816486	3163568	172709333.33	-2.87	15.67
0.1	0.05	400063	1735031	4005049	887176	177435200.00	-0.22	20.46

Orkut-social network

p_v	p_e	V	E	B_e	T_m	τ	pE	Time (s)
0.01	0.01	30856	1171375	1112385	2283760	622330000.00	-0.84	38.10
0.15	0.01	460448	1171998	16168020	17340018	629705333.33	0.34	66.34
0.1	0.05	306699	5857609	18158724	24016333	627276600.00	-0.05	106.96

Table 4.4: EVMS vs EVSS sampling

	EVSS		EVMS	
T_m or Repetitions	pE	Time (s)	pE	Time (s)
DBLP				
203520	100.00	25.49	1.56	0.544
224229	-127.79	33.03	-7.37	0.492
275683	42.88	32.42	3.40	0.693
295178	-70.31	34.01	-1.26	0.789
330007	-13.65	45.16	-0.67	0.667
340952	-14.00	59.01	-5.57	1.03
Amazon				
10415	100.00	1.33	-8.56	0.37
50422	100.00	3.40	-0.77	0.40
66299	100.00	7.87	-2.75	0.33
159609	100.00	15.25	0.12	0.50
332820	-100.51	42.60	0.03	0.69
379469	100.00	54.83	-0.62	0.73
Berkley-Stanford				
116,033.00	-35.46	26.11	-10.13	3.75
316,077.00	-1.79	122.01	4.22	3.17
559,491.00	-18.57	343.02	-1.87	5.30
754,190.00	15.37	592.31	-4.90	19.03
801,793.00	-18.60	734.91	-0.51	6.52
1,044,961.00	6.74	996.58	-0.86	7.71
1,563,453.00	-1.56	2271.79	1.94	33.20
As-Skitter				
188,576.00	100.00	64.59	1.70	2.79
864,064.00	-89.02	876.19	-4.70	4.48
1,189,096.00	5.79	1672.59	-2.26	8.22
1,227,522.00	100.00	2150.96	5.04	5.09
1,603,202.00	-41.53	2749.18	3.13	7.40

Table 4.5: EVMS vs Triest Base

	Triest		EVMS	
T_m or M	pE	Time (s)	pE	Time (s)
DBLP				
12376	-9.803146769	1.329	-23.12	0.667
20469	-9.20353482	1.879	-3.16	0.346
30944	-7.110734777	2.67	-1.37	0.387
40747	-0.7411185107	3.841	-0.35	0.479
50318	2.401285592	6.063	-3.05	0.375
86370	3.913206082	11.448	0.76	0.412
119359	0.3248504283	18.375	-0.16	0.543
153380	-0.04366580563	21.281	1.08	0.46
203520	-0.1907962044	33.106	1.56	0.544
250749	-0.2391002385	38.61	-2.12	0.752
274446	-0.2964278279	50.219	-1.24	0.62
295178	1.06512402	46.328	-1.26	0.789
330007	-0.2668924484	57.3	-0.67	0.667
Amazon				
14336	59.612	0.946	1.03	0.53
19907	-35.74	1.444	-1.67	0.30
50422	4.40	4.577	-0.77	0.40
87526	-2.55	10.121	-1.07	0.40
105391	0.80	12.395	-0.07	0.46
126489	-1.23	17.761	-0.06	0.45
127183	-0.85	14.476	0.13	0.40
159609	-0.36	19.155	0.12	0.50
236257	0.62	29.455	-0.06	0.55
285553	0.31	33.138	0.35	0.63
332820	-0.12	37.663	0.03	0.69
Berkley-Stanford				
116,033	4.87	82.42	-10.13	3.75
417,102	1.83	901.08	7.89	10.35
559,491	0.55	1,560.59	-1.87	5.30
801,793	-2.98	2,804.86	-0.51	6.52
1,044,961	-3.75	3,903.47	-0.86	7.71
As-Skitter				
188,576.00	100.00	64.59	1.70	2.79
864,064.00	-89.02	876.19	-4.70	4.48
1,189,096.00	5.79	1672.59	-2.26	8.22
1,227,522.00	100.00	2150.96	5.04	5.09
1,603,202.00	-41.53	2749.18	3.13	7.40

Table 4.6: EVMS vs Colorful triangle counting

Colorful Triangle Counting			EVMS		
Total memory	pE	Time (s)	T_m	pE	Time (s)
DBLP					
10,544	5.65	0.52	12,376	-23.12	0.67
26,191	-0.81	0.50	30,944	-1.37	0.39
52,864	0.85	0.41	50,318	-3.05	0.38
80,930	5.35	0.61	86,370	0.76	0.41
104,707	-0.08	0.56	119,359	-0.16	0.54
174,509	2.80	0.92	183,740	-2.09	0.49
210,079	-0.90	1.08	203,520	1.56	0.54
Berkeley-Stanford Graph					
166,101	-6.31	3.81	116,033	-10.13	3.75
332,018	-0.30	8.68	316,077	4.22	3.17
511,439	6.04	15.41	559,491	-1.87	5.30
664,925	-0.78	22.27	801,793	-0.51	6.52
1,109,374	0.48	78.16	1,147,703	5.33	22.63
1,329,356	-0.12	96.97	1,563,453	1.94	33.20
Live-Journal					
346,764	1.04	16.16	539,691	4.47	15.28
867,710	-0.39	16.77	1,299,055	-2.65	13.43
2,667,720	4.58	23.86	2,246,382	-0.64	13.42
3,468,870	-0.62	31.33	3,163,568	-2.87	15.67
5,782,986	1.00	56.19	5,740,080	-0.22	20.46
6,940,331	0.02	65.18	7,707,350	0.36	25.42
Orkut					
2,929,852	-0.45	56.47	2,283,760	-0.84	38.10
5,856,111	-0.15	83.04	6,664,778	-0.46	49.68
11,717,612	0.08	166.33	11,717,812	1.33	113.89

CHAPTER 5. CONCLUSION AND FUTURE WORK

The thesis presents the Edge vertex Multi Sampling algorithm that has better theoretical bounds $O(\frac{m\sqrt{\Delta_V}}{\sqrt{T}}\frac{1}{\epsilon}\log(\frac{\Delta_V}{\delta}))$ than several state of art algorithms. We also verify if the gain seen in the theoretical analysis reflects in the experimental analysis. Based on the experiments we can make the following conclusions. The EVMS algorithm has better accuracy and run-times compared to EVSS , Triest-Base and neighborhood sampling, lower accuracy compared to colorful triangle counting. However, EVMS is faster the the colorful triangle counting. The algorithm can be further extended for dynamic streams, where the stream contains deletion edges. Also, the algorithm can be made even faster using the map-reduce technique suggested in R. Pagh *et al*[18]. Further, the multi-sampling strategy can also be applied to other single sampling techniques like the neighborhood sampling.

BIBLIOGRAPHY

[1] Alon, N., Matias, Y., and Szegedy, M. (1996). The space complexity of approximating the frequency moments. *ACM Symposium on Theory of Computing*, pages 20–29.

[2] Bar-Yossef, Z., Kumar, R., and Sivakumar, D. (2002). Reductions in streaming algorithms, with an application to counting triangles in graphs. In *SODA*, pages 623–632.

[3] Becchetti, L., Boldi, P., Castillo, C., and Gionis, A. (2008). Efficient semi-streaming algorithms for local triangle counting in massive graphs. In *KDD*, pages 16–24.

[4] Bott, E. (1957). *Family and Social Network: Roles, Norms and External Relationships in Ordinary Urban Families.*

[5] Braverman, V., Ostrovsky, R., and Vilenchik, D. (2013). How hard is counting triangles in the streaming model? In *ICALP*, pages 244–254.

[6] Buriol, L. S., Frahling, G., Leonardi, S., Marchetti-Spaccamela, A., and Sohler, C. (2006). Counting triangles in data streams. In *SIGMOD*, pages 253–262.

[7] Eckmann, J.-P. and Moses, E. (2002). Curvature of co-links uncovers hidden thematic layers in the world wide web. *PNAS*, 99(9):5825–5829.

[8] Foster, D. V., Foster, J. G., Grassberger, P., and Paczuski, M. (2011). Clustering drives assortativity and community structure in ensembles of networks. *Phys. Rev. E*, 84:066117.

[9] Huang, Z. (2010). Link prediction based on graph topology: The predictive value of generalized clustering coefficient. *Social Science Research Network.*

[10] Jha, M., Seshadhri, C., and Pinar, A. (2015a). Path sampling: A fast and provable method for estimating 4-vertex subgraph counts. In *WWW*, pages 495–505.

[11] Jha, M., Seshadhri, C., and Pinar, A. (2015b). A space-efficient streaming algorithm for estimating transitivity and triangle counts using the birthday paradox. *TKDD*, 9(3):15:1–15:21.

[12] Jowhari, H. and Ghodsi, M. (2005). New streaming algorithms for counting triangles in graphs. In *COCOON*, pages 710–716.

[13] Kane, D. M., Mehlhorn, K., Sauerwald, T., and Sun, H. (2012). Counting arbitrary subgraphs in data streams. In *ICALP*, pages 598–609.

[14] Latapy, M. (2008). Main-memory triangle computations for very large (sparse (power-law)) graphs. *Theor. Comput. Sci.*, 407(1-3):458–473.

[15] Leskovec, J. and Krevl, A. (2014). SNAP Datasets: Stanford large network dataset collection. http://snap.stanford.edu/data.

[16] Lim, Y. and Kang, U. (2015). MASCOT: memory-efficient and accurate sampling for counting local triangles in graph streams. In *Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Sydney, NSW, Australia, August 10-13, 2015*, pages 685–694.

[17] Manjunath, M., Mehlhorn, K., Panagiotou, K., and H. (2011). Approximate counting of cycles in streams. In *ESA*, pages 677–688.

[18] Pagh, R. and Tsourakakis, C. E. (2012). Colorful triangle counting and a MapReduce implementation. *Inf. Process. Lett.*, 112(7):277–281.

[19] Pavan, A., Tangwongsan, K., Tirthapura, S., and Wu, K.-L. (2013). Counting and sampling triangles from a graph stream. *PVLDB*, 6(14):1870–1881.

[20] Sala, A., Cao, L., Wilson, C., Zablit, R., Zheng, H., and Zhao, B. Y. (2010). Measurement-calibrated graph models for social network experiments. In *WWW*, pages 861–870.

[21] Schank, T. and Wagner, D. (2005). Finding, counting and listing all triangles in large graphs, an experimental study. In *Experimental and Efficient Algorithms, 4th InternationalWorkshop, WEA 2005, Santorini Island, Greece, May 10-13, 2005, Proceedings*, pages 606–609.

[22] Seshadhri, C., Pinar, A., Durak, N., and Kolda, T. G. (2013). The importance of directed triangles with reciprocity: patterns and algorithms. *CoRR*, abs/1302.6220.

[23] Stefani, L., Epasto, A., Riondato, M., and Upfal, E. (2016). Trièst: Counting local and global triangles in fully-dynamic streams with fixed memory size. In *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, San Francisco, CA, USA, August 13-17, 2016*, pages 825–834.

[24] Suri, S. and Vassilvitskii, S. (2011). Counting triangles and the curse of the last reducer. In *Proceedings of the 20th International Conference on World Wide Web, WWW 2011, Hyderabad, India, March 28 - April 1, 2011*, pages 607–614.

[25] Tsourakakis, C. E. (2008). Fast counting of triangles in large real networks without counting: Algorithms and laws. In *ICDM*, pages 608–617.

[26] Tsourakakis, C. E., Drineas, P., Michelakis, E., Koutis, I., and Faloutsos, C. (2011a). Spectral counting of triangles via element-wise sparsification and triangle-based link recommendation. *Social Netw. Analys. Mining*, 1(2):75–81.

[27] Tsourakakis, C. E., Kang, U., Miller, G., and Faloutsos, C. (2009). DOULION: counting triangles in massive graphs with a coin. In *Proceedings of the 15th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Paris, France, June 28 - July 1, 2009*, pages 837–846.

[28] Tsourakakis, C. E., Kolountzakis, M., and Miller, G. (2011b). Triangle sparsifiers. *J. Graph Algorithms Appl.*, 15(6):703–726.

[29] Wolff, K., editor (1950). *Selections from The Sociology of Georg Simmel.*