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Dynamic survivable multipath provisioning in OFDM-based flexible optical networks

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Dynamic survivable multipath provisioning in OFDM-based flexible optical networks

by

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A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Computer Science

Program of Study Committee:
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Iowa State University

Ames, Iowa

2013

DEDICATION

I would like to dedicate this thesis to my parents Xuehua and Yinhui and to my sister Yanling without whose support I would not have been able to complete this work. I would also like to thank my friends and family for their loving guidance and financial assistance during the writing of this work.

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ACKNOWLEDGEMENTS

I would like to take this opportunity to express my thanks to those who helped me with various aspects of conducting research and the writing of this thesis.

First and foremost, I would like to thank my major Professor, Dr. Lu Ruan for her guidance, patience and support throughout this research and the writing of this thesis. Her insights have often inspired me and her patience have made me more confident during my pursuit of degree.

I would like to thank Dr. Shashi K. Gadia and Dr. Wensheng Zhang to be my POS committee.

I would like to thank my dear friends, Luica, Angela, Jia, Keji, Jingya, Thelma, Ray and many other friends who provide me emotional support when I get discouraged.

I would like to thank my parents and my sister for providing financial and emotional supports. Without them, I would not be able to continue my degree.

ABSTRACT

Compared with traditional WDM network, OFDM-based flexible optical networks are able to provide better spectral efficiency due to its flexible allocation of requests on fine granularity subcarrirers. Survivability is a crucial issue in OFDM-based flexible optical networks. In (19), Ruan and Xiao propose a new survivable multipath provisioning scheme (MPP) that provides flexible protection levels in OFDM-based flexible optical networks. They also studies the static Survivable Multipath Routing and Spectrum Allocation (SM-RSA) problem which aims to accommodate a given set of demands with minimum utilized spectrum. It is shown that the MPP scheme achieves higher spectral efficiency than the traditional single-path provisioning (SPP) scheme. In this thesis, we study the dynamic SM-RSA problem, which allocates multiple routes and spectrum for a given demand as it arrives at the network. We develop an ILP model for the problem as well as a heuristic algorithm. We conduct simulations to study the advantage of MPP over SPP for dynamic traffic scenario in terms of blocking performance and fairness. We also compare the performance of the MPP heuristic algorithm and the ILP model.

CHAPTER 1.   Introduction

1.1   Background and Literature Review

In conventional WDM optical networks, a connection is supported by a wavelength channel occupying 50GHz spectrum. This rigid and coarse granularity leads to waste of spectrum when the traffic between the end nodes is less than the capacity of a wavelength channel. To address this issue, optical networks capable of flexible bandwidth allocation with fine granularity are needed. Orthogonal frequency division multiplexing (OFDM) is a promising modulation technology for optical communications because of its good spectral efficiency, flexibility, and tolerance to impairments (1; 2). In optical OFDM, a data stream is split into multiple lower rate data streams, each modulated onto a separate subcarrier. By allocating an appropriate number of subcarriers, optical OFDM can use just enough bandwidth to serve a connection request. A novel OFDM-based optical transport network architecture called spectrum-sliced elastic optical path network (SLICE) is proposed in (3). SLICE network can efficiently accommodate sub-wavelength and super-wavelength traffic by allocating just enough spectral resource to an end-to-end optical path according to the user demand. The performance superiority of OFDM-based exible optical networks over conventional WDM optical networks has been demonstrated in (4; 5; 6; 7)

An important problem in the design and operation of OFDM-based exible optical networks is the routing and spectrum allocation (RSA) problem. The RSA problem for static demands is studied in (8; 9). In (10; 11), dynamic RSA algorithms are pro-

posed to efficiently accommodate connection requests as they arrive at the network. In (12), the authors propose a split spectrum approach that splits a bulky demand into multiple spectrum channels, all of which are routed over the same path. This approach relaxes the constraint of transmission impairment over long distance and also makes more efficient use of discontinued spectrum fragments. A similar approach called light-path fragmentation is proposed in (13). A dynamic multipath provisioning algorithm with differential delay constraint for OFDM-based elastic optical networks is proposed in (14). Here a demand is split over multiple routing paths. In (15), the authors propose several dynamic routing, modulation, and spectrum assignment algorithms in elastic optical networks with hybrid single-/multi-path routing. These algorithms achieve lower bandwidth blocking probability than the conventional single-path routing and the split spectrum approaches.

Survivability is a crucial requirement in optical transport networks. The authors in (16) propose a heuristic algorithm for survivable flexible WDM network design. In (17), two backup sharing policies for OFDM-based optical networks are proposed. A single-path provisioning multi-path recovery scheme in flexgrid optical networks is presented in (18). Recently, the authors in (19) propose a survivable multipath provisioning (MPP) scheme for OFDM-based flexible optical networks that can support full and partial protection with higher efficiency than conventional single-path provisioning (SPP) scheme. In the survivable MPP scheme, a demand is routed over multiple link-disjoint paths and subcarriers are allocated on these paths to satisfy the bandwidth requirement and the protection requirement of the demand. The static Survivable Multipath Routing and Spectrum Allocation (SM-RSA) problem for accommodating a given set of demands has been studied in (19).

1.2   Outline

In this thesis, we define the Dynamic Survivable Multipath Routing and Spectrum Allocation (dynamic SM-RSA) problem and propose an ILP model and a heuristic algorithm for the dynamic SM-RSA problem. The goal of the dynamic SM-RSA problem is to accommodate a coming request with multipath provisioning. We conduct simulations to demonstrate the advantage of MPP over SPP in dynamic traffic scenario and evaluate the performance of the ILP and the heuristic algorithms.

The rest of the thesis is organized as follows:

- *Chapter 2:* Discusses the advantage of MPP over SPP and defines the SM-RSA problem.

- *Chapter 3:* Formulates an ILP model for the dynamic SM-RSA problem.

- *Chapter 4:* Describes the heuristic algorithm for dynamic SM-RSA.

- *Chapter 5:* Analyses the numerical results.

- *Chapter 6:* Concludes the thesis.

CHAPTER 2.   The Dynamic SM-RSA Problem Definition

This chapter explains the proposed survivable multipath provisioning scheme and demonstrates the advantage of multipath provisioning scheme over single path provisioning scheme. Then dynamic Survivable Multipath Routing and Spectrum Allocation (SM-RSA) problem is defined.

2.1   The Survivable Multipath Provisioning Scheme

With flexible bandwidth allocation capability, OFDM-based optical networks are able to support flexible protection levels. We assume that a request arrives with a bandwidth and protection level requirement. In this work, a request is represented as $r = < s, d, B, q >$ where s and d are the source and destination nodes, B is the bandwidth requirement, and q ($0 \leq q \leq 1$) is the protection level requirement. Protection level requirement indicates the percentage of bandwidth B that must be available after single link failure. Specifically, $q=1$ indicates full protection, $q = 0$ implies no protection, and $0 < q < 1$ means partial protection.

To accommodate a request $r = < s, d, B, q >$ using multipath provisioning (MPP) scheme, $N \geq 2$ link-disjoint paths are chosen between s and d. We need to allocate capacity on the N paths such that the total capacity on these N paths is at least B while total capacity on any $N - 1$ paths is at least qB. On the other hand, the single path provisioning (SPP) requires 2 paths with one path allocated B capacity and qB capacity on the other path. Consider a simple network with 2 nodes A and node B and link

capacity is 10. There are two links from A to B. Let $r_1 = r_2 = < A, B, 10, 0.5 >$. With SPP, only one of r_1 and r_2 can be accommodated, and the other will be blocked. While with MPP, r_1 and r_2 can both be accepted with r_1 being allocated 5 capacity units on link 1, 5 capacity units on link 2 and r_2 being allocated 5 capacity units on link 1 and 5 capacity units on link 2. From the example, it can be seen that MPP is more capacity efficient than SPP.

2.2 The Dynamic Survivable Multipath Routing and Spectrum Allocation Problem

In OFDM-based flexible optical networks, the frequency spectrum is divided into subcarriers with equal frequency. The routing and spectrum allocation (RSA) problem is to accommodate a request by selecting a route and allocating contiguous subcarriers on each link on the route. Note that the definition uses SPP scheme. Since MPP performs better on subcarrier usage, we define the dynamic Survivable Multipath RSA (SM-RSA) problem as: Given a request $r = < s, d, B, q >$, accommodate the request with MPP scheme such that the total subcarrier allocated is minimized. In this problem, we need to determine two or more link-disjoint paths from source to destination and allocate subcarriers on these paths such that the bandwidth requirement and the protection requirement are satisfied and the total number of subcarriers used is minimized.

The dynamic SM-RSA problem requires the following constraints to be satisfied:

- *Bandwidth constraint:* For each request $r = < s, d, B, q >$, the total number of subcarriers allocated to all its paths must be equal to or greater than B.

- *Protection constraint:* For each request $r = < s, d, B, q >$, if N paths are assigned to r, then the sum of allocated subcarriers of any $N - 1$ paths must be equal to or greater than qB.

- *Spectrum contiguity constraint:* A set of contiguous subcarriers must be allocated to a spectrum path.

- *Non-overlapping spectrum constraint:* A subcarrier on a link can be allocated to at most one spectrum path routed over the link.

- *Guard subcarrier constraint:* When two adjacent spectrum paths share a link, they must be separated by G guard subcarriers.

CHAPTER 3. An ILP Model for the Dynamic SM-RSA Problem

In this chapter, we present an Integer Linear Programming (ILP) formulation for dynamic SM-RSA problem.

The purpose of the ILP formulation is to minimize the cost for a given demand while satisfying the constraints stated in Chapter 2. The cost is represented as the sum of all products of number of subcarriers used and the length of path. For each pair of s, d in network we pre-compute a set of candidate link-disjoint paths $\mathbf{P}_{s,d}$ ($|\mathbf{P}_{s,d}| \geq 2$) from s to d using Bhandari's link-disjoint paths algorithm (20). We also keep track of the availability of each subcarrier with boolean parameter U_k^w where k is the path number and w is the subcarrier index. U_k^w is updated whenever a reqeust is accommodated or a demand terminates.

The ILP model for a request $r = < s, d, B, q >$ is shown below:

Notations

K: The number of link disjoint path in $\mathbf{P}_{s,d}$. $K = |\mathbf{P}_{s,d}|$

p_k: The kth link-disjoint path in $\mathbf{P}_{s,d}$, $1 \leq k \leq K$.

L_k: length of path p_k in hops

n: The total number of subcarriers in each link.

U_k^w: Boolean parameter that equals 1 if subcarrier w ($1 \leq w \leq n$) is not available on path p_k (i.e., subcarrier w on at least one link of p_k is occupied), and equals 0 if subcarrier w is available on p_k (i.e.,subcarrier w is available on every link of p_k).

Variables

S_k^w: Boolean variable that denotes if path $p_{s,d,k}$ uses subcarrier w. 1 if path $p_{s,d,k}$ uses subcarrier w $(1 \leq w \leq n)$ and 0 otherwise.

X_k: Boolean variable that equals 1 if path k from s to d is used, 0 otherwise.

MPP ILP formulation:

$$\text{minimize} \sum_{w \in [1,n]} \sum_{k \in [1,K]} S_k^w * L_k$$

subject to the following constrains:

- Capacity allocation constraints:

$$\sum_{k \in [1,K]} \sum_{w \in [1,n]} S_k^w \geq \sum_{k \in [1,K]} X_k * G + B \tag{3.1}$$

$$\sum_{k \in [1,K], k \neq m} \sum_{w \in [1,n]} S_k^w \geq qB + \left(\sum_{k \in [1,K]} X_k - 1 \right) * G \tag{3.2}$$

$$\forall m \in [1, K]$$

Equation 3.1 ensures that the total number of subcarriers allocated on all the paths of a demand (s, d) is larger than or equal to the requested number of subcarriers B. When a link failure affects one of the routing paths, Equation 3.2 guarantees that the total number of subcarriers allocated on remaining path is at least qB. Note that the right hand side of both equations takes into account G guard subcarriers on each routing path.

- Per path guard subcarrier constraint:

$$\sum_{w \in [1,n]} S_k^w > G * X_k \forall k \tag{3.3}$$

Equation 3.3 ensures that on every selected path, G guard subcarriers are allocated.

- Number of path constraints:

$$\sum_{k \in [1,K]} X_k \leq 3 \tag{3.4}$$

$$\sum_{k \in [1,K]} X_k \geq 2 \tag{3.5}$$

Equation 3.4 and 3.5 limit the number of paths used to be either 2 or 3. We choose to route a demand over 2 or 3 paths because the numerical results for the static SM-RSA problem in (19) show that no more than 3 candidate paths are used in optimal and heuristic solutions. Although using more routing paths results in more backup capacity saving, it is not cost effective to use more than 3 paths since the overhead of guard subcarriers and the longer paths generally outweigh the saving in backup capacity (19).

- Spectrum contiguity constraint:

$$(S_k^w - S_k^{w+1} - 1)(-n) \geq \sum_{w' \in [w+2,n]} S_k^{w'} \quad \forall w, p_k \tag{3.6}$$

Equation 3.6 ensures that contiguous subcarriers are alloccated to a path. If path k uses subcarrier w and does not use subcarrier $w + 1$, then it can not use any subcarrier with index within $[w + 2, n]$.

.

- Non-overlapping spectrum constraints:

$$U_k^w * S_k^w \leq 0 \ \forall k, w \tag{3.7}$$

Equation 3.7 ensures subcarrier w cannot be allocated on path k if it is not available.

- Path selection constraints:

$$\sum_{w \in [1,n]} S_k^w \leq X_k * n \quad \forall k \tag{3.8}$$

$$X_k \leq \sum_{w \in [1,n]} S_k^w \quad \forall k \tag{3.9}$$

Equation 3.8 and 3.9 ensures the correctness of X_k. Equation 3.8 ensures that if one or more subcarriers are allocated on path k, then path k is marked as used. Equation 3.9 ensures that if no subcarrier is allocated on path k, then path k is marked as unused.

CHAPTER 4.   A Heuristic Algorithm for the Dynamic SM-RSA Problem

Our heuristic algorithm contains two main steps. First, for each pair of s and d, a set of candidate link-disjoint paths $\mathbf{P}_{s,d}$ ($|\mathbf{P}_{s,d}| \geq 2$) between s and d is pre-computed using Bhandaris link-disjoint paths algorithm (20). $\mathbf{P}_{s,d}$ is sorted in increasing order of path length. Then, depending on the protection level q, different algorithms will be used to determine paths and number of subcarriers allocated to r. Algorithm 1 presents the pseudo code for the algorithms mentioned above. When $q \leq 0.5$ algorithm 2 will be called. When $q > 0.5$ it calls algorithm 3 to get a 2-path solution S_2 and it also calls algorithm 4 to get a 3-path solution. It then compares the two solutions in terms of the number of subcarriers allocated and returns the better solution. The output of all algorithms are the routing paths for r and the number of subcarrirers to be allocated on each path.

Algorithm 1 Heuristic algorithm for dynamic SM-RSA

1: **if** $q <= 0.5$ **then**
2:     call Algorithm 2 and return its solution
3: **else**
4:     call Algorithm 3 and save its solution in S_2
5:     call Algorithm 4 and save its solution in S_3
6:     **if** total allocated subcarriers in $S_2 <=$ total allocated subcarriers in S_3 **then**
7:       retrun S_2
8:     **else**
9:       return S_3
10:    **end if**
11: **end if**

4.1   Maximum Contiguous Subcarriers

In algorithm 2-4, maximum contiguous subcarriers (MCS) play an important role. For each link e, e is associated with a Boolean array $A_e = (a_{e1}, a_{e2}, \cdots, a_{en})$ to represent the availability of each subcarrier in e. n is the maximum subcarrier index in link e. a_{ex} equals 1 if the xth subcarrier in link e is available. For a path p, the availability array Ap will then be the result of AND operation on all of the Boolean arrays of its edges. For $A_p = (a_{p1}, a_{p2}, \cdots, a_{pn})$, if a_{px} to a_{py} are all available($x, y \in [1, n]$), we define it as a $(y - x + 1)$ contiguous available subcarriers. A maximum contiguous subcarrier is then the longest contiguous available subcarriers in path p. In the following algorithms, when a candidate path p is chosen, the algorithm will get an array of array that contains all the available contiguous subcarriers of p. The array will be sorted increasingly by the length of available contiguous subcarriers. For example, let n be 20 and a path $A - B - C - D$ is chosen with u, v, and w be the edges along the path. If the available contiguous subcarriers are $a_{u1}/a_{v1}/a_{w1}$ to $a_{u4}/a_{v4}/a_{w4}$, and $a_{u10}/a_{v10}/a_{w10}$ to $a_{u15}/a_{v15}/a_{w15}$ then the array of array returned will be $\{\{1, 2, 3, 4\}, \{10, 11, 12, 13, 14, 15\}\}$. Thus, the last element will be the MCS. MCS will be used to determine if path p can be used or not. By the end of the algorithm, subcarriers will be allocated to the first available contiguous subcarrier in the array that can fit the allocation to reduce fragmentation. For instance, if the algorithm determines that 3 subcarriers will be assigned to path $A - B - C - D$ in the previous example, then contiguous subcarriers of $1, 2, 3, 4$ will be used because it can cover the allocation and leave the longer contiguous subcarriers for other requests with bigger demand.

4.2   Algorithm for $q \leq 0.5$

Algorithm 2 computes a SM-RSA solution for $r = < s, d, B, q >$ when $q <= 0.5$. From line 1 to line 14, the algorithm first tries to find two candidate paths such that MCS on

---

Algorithm 2 Algorithm for a request with $q <= 0.5$.

1: **for** each path i in $\mathbf{P}_{s,d}$ **do**
2:    **if** MCS of path i is greater than G **then**
3:      $mcs1 = \text{MCS(path } i)$
4:    **else**
5:      continue to next i
6:    **end if**
7:    **for** each path j in $\mathbf{P}_{s,d}$, $j > i$ **do**
8:      **if** MCS of path j is greater than G **then**
9:        $mcs2 = \text{MCS(path } j)$
10:      **else**
11:        continue to next j
12:      **end if**
13:      **if** $mcs1 + mcs2 < qB + 2G$ **then**
14:        continue to next j
15:      **else**
16:        $alloc1 = min(B - qB + G, mcs1)$
17:        $alloc2 = min(B - alloc1 + 2G, mcs2)$
18:        **if** $alloc2 > B - qB + G$ **then**
19:          $alloc2 = B - qB + G$
20:        **end if**
21:        **if** $alloc1 + alloc2 < B + 2G$ **or** $alloc1 < qB + G$ **or** $alloc2 < qB + G$ **then**
22:          **for** each path k in $\mathbf{P}_{s,d}$, $k > j$ **do**
23:            **if** MCS of path k is greater than G **then**
24:              $mcs3 = \text{MCS(path } k)$
25:            **else**
26:              continue to next k
27:            **end if**
28:            $alloc3 = B - alloc1 - alloc2 + 3G$
29:            **if** $alloc3 <= mcs3$ **then**
30:              return path i, j, k and $alloc1, alloc2, alloc3$
31:            **else**
32:              continue to next path k.
33:            **end if**
34:          **end for**
35:        **else**
36:          **return** path i, j and $alloc1, alloc2$
37:        **end if**
38:      **end if**
39:    **end for**
40: **end for**

---

each path is more than G. Line 13 ensures that the sum of the MCS of the two candidate paths can satisfy the protection requirement qB and the guard band requirement. Then, from line 16 to line 20, it tries to find a 2-path solution by allocating subcarriers on the two candidate paths. Line 16 and line 18 ensures that the number of subcarriers allocated on each path is no more than $B - qB + G$ as the other path must have at least qB+G. The resulting paths will be checked in Line 21 to make sure bandwidth and protection requirements are satisfied. If satisfied, then this 2-path solution will be returned in line 36. Otherwise, a third path is needed. To obtain the third path, line 22 to line 27 find a candidate path with MCS more than G. Line 28 allocates subcarriers on the third path to satisfy the bandwidth requirement. Line 29 checks if the path can accommodate the allocation and returns the solution if the answer is yes in line 30. Otherwise, the algorithm will go back to line 22 to find another candidate path.

To accommodate request $r =< s, d, B, q >$, we need to prove that bandwidth and protection requirements are fulfilled by Algorithm 2. First, the algorithm searches for 2 candidate paths from $\mathbf{P}_{s,d}$. Each of the MCS must be greater than G and the sum of MCS of these two paths must be greater or equal to $qB + 2G$ (line 1-12 and line 13). This guarantees that the first two paths at least have the protection level required by q. Next, the algorithm tries to put $B - qb + G$ subcarriers on the first path. Because $q \leq 0.5$ and the first path is shorter than second path, the final cost will be minimized. If path 1 cannot handle $B - qB + G$, the algorithm will then use MCS as the allocation. The allocation on path 1 is denoted as alloc1 (line 16). If MCS of path 2 is greater than $B - alloc1 + 2G$, $B - alloc1 + 2G$ subcarriers will be allocated to path 2, else MCS of path 2 will be used as alloc2 (line 17). If alloc2 is greater than $B - qB + G$, we reduce it to $B - qB + G$ (line 18-19). It is clear that if path 1 can handle $B - qB + G$, then path2 will take over $qB + G$ if mcs2 $\geq qB + G$. If the sum of allocation on the two paths is equal to B, then a two path solution will be returned. However, it is possible that the sum of alloc1 and alloc2 is smaller than B, and it is also possible that alloc1 or

alloc2 is less than qB (line 21). In this case, a third path is necessary. The allocation of path 3 will be $B - alloc1 - alloc2 + 3G$ in all cases. It is clear that the allocation of path 3 guarantees the sum of three paths will be equal to B. Due to the fact that alloc1 and alloc2 is at most $B - qB + G$, the sum of alloc3 with either alloc2 or alloc1 is at least $qB + 2G$. In line 13-14, $alloc1 + alloc2 \geq qB + 2G$ is guaranteed. Together, the protection requirement is met. If alloc3 is greater than MCS of path 3, that means a new candidate path should be tried.

4.3   Algorithm for $q > 0.5$

Algorithm 3 Algorithm for computing a 2-path solution when $q > 0.5$.

1: **for** each path i in $\mathbf{P}_{s,d}$ **do**
2:    **if** mcs of path $i \geq qB + G$ **then**
3:       $alloc1 = qB + G$
4:    **else**
5:       continue to next i
6:    **end if**
7:    **for** each path j in $\mathbf{P}_{s,d}$, $j > i$ **do**
8:       **if** MCS of path $j \geq qB + G$ **then**
9:          $alloc2 = qB + G$
10:          return path i, j and $alloc1, alloc2$
11:       **else**
12:          continue to next j
13:       **end if**
14:    **end for**
15: **end for**

The algorithm for $q > 0.5$ compares the cost of two sub-functions. Algorithm 3 simply finds a 2-path solution with each path containing qB subcarriers. Since $q > 0.5$, the sum of the allocations must be greater than B.

Algorithm 4 computes a 3-path dynamic SM-RSA solution for r when $q > 0.5$. It tries to find 3 candidate routing path $i, j, k (k > j > i)$ for r from line 1 to line 24. Then it ensures that the 3 candidate paths have enough free contiguous subcarriers to satisfy the bandwidth and protection requirement in line 13 and line 22. From line 25 to line 40,

---

Algorithm 4 Algorithm for computing a 3-path solution when $q > 0.5$.

1: **for** each path i in $\mathbf{P}_{s,d}$ **do**
2:    **if** MCS of path i is greater than G **then**
3:      $mcs1 = \text{MCS}(\text{path } i)$
4:    **else**
5:      continue to next path i
6:    **end if**
7:    **for** each path j in $\mathbf{P}_{s,d}$, $j > i$ **do**
8:      **if** MCS of path j is greater than G **then**
9:        $mcs2 = \text{MCS}(\text{path } j)$
10:      **else**
11:        continue to next j
12:      **end if**
13:      **if** $mcs1 + mcs2 < qB + 2G$ **then**
14:        continue to next j
15:      **else**
16:        **for** each path k in $\mathbf{P}_{s,d}$, $k > j$ **do**
17:          **if** MCS of path k is greater than G **then**
18:            $mcs3 = \text{MCS}(\text{path } k)$
19:          **else**
20:            continue to next k
21:          **end if**
22:          **if** $mcs1 + mcs3 < qB + 2G$ **or** $mcs3 + mcs2 < qB + 2G$ **or** $mcs1 + mcs2 + mcs3 < B + 3G$ **then**
23:            continue to next k
24:          **end if**
25:          $alloc1 = min(qB/2 + G, mcs1)$
26:          $alloc2 = qB + 2G - alloc1$
27:          **if** $alloc2 > mcs2$ **then**
28:            $alloc2 = mcs2$
29:            $alloc1 = alloc1 + (alloc2 - mcs2)$
30:          **end if**
31:          $alloc3 = qB - min(alloc1, alloc2) + 2G$
32:          **if** $alloc3 > mcs3$ **then**
33:            $alloc3 = mcs3$
34:            **if** $alloc + alloc3 < qB + 2G$ **then**
35:              $alloc1 = qB + 2G - alloc3$
36:            **end if**
37:            **if** $alloc3 + alloc2 < qB + 2G$ **then**
38:              $alloc2 = qB + 2G - alloc3$
39:            **end if**
40:          **end if**
41:          **if** $alloc1 + alloc2 + alloc3 < B + 3G$ **then**
42:            $diff = B + 3G - alloc1 - alloc2 - alloc3$
43:            Sequentially increase $alloc1$ up to $mcs1$, $alloc2$ up to $mcs2$, $alloc3$ up to $mcs3$ until total increment is equal to $diff$
44:          **end if**
45:          **return** paths i, j, k and $alloc1, alloc2, alloc3$
46:        **end for**
47:      **end if**
48:    **end for**
49: **end for**

the algorithm computes the subcarrier allocation on the three candidate routing paths to satisfy the protection requirement. Finally, From line 41 to the end, the algorithm checks if the solution meets the bandwidth requirement. If not, it will adjust the allocation such that the requirement will be met and then return the solution.

We now show that Algorithm 4 provides a solution that meets the protection and bandwidth requirements. First of all, the algorithm pick first three path from $\mathbf{P}_{s,d}$ that $MCS1 + MCS2$, $MCS1 + MCS3$ and $MCS2 + MCS3$ each is at least $qB + 2G$(line 13 and line 22) and $MCS1 + MCS2 + MCS3$ is at least $B + 3G$(line 22). We try to allocate $qB/2 + G$ to path 1 (line 25). If it does not have enough free subcarrriers, use MCS of path 1 instead. We denote allocation on path 1 as alloc1. For path 2, we try $qB - alloc1 + 2G$ subcarriers and denote it as alloc2 (line 26). It is possible that alloc2 > MCS of path 2 (line 27). While the algorithm picks path 1 and path 2, it checked that the sum of MCSs is greater or equal to $qB + 2G$. We can thus safely move $alloc2 - MCS2$ subcarriers to alloc1, namely, $alloc2 = MCS2$ and $alloc1 = alloc1 + (alloc2 - mcs2)$ (line 28-29). Now, alloc1 and alloc2 meets the q protection level. Next, we try to allocate $qB - min(alloc1, alloc2) + 2G$ subcarriers on path 3 (line 31). Clearly, if the sum of alloc3 with minimum of alloc1 and alloc2 is greater than qB, the sum of alloc3 with the maximum must also meet the requirement. Similarly, alloc3 may be greater than MCS3 (line 32). In this case, the algorithm will set $alloc3 = MCS3$, and modifies $alloc1$ and $alloc2$ accordingly. If the sum of $alloc3$ with $alloc1$ or $alloc2$ is less than $qB + 2G$, then $alloc1$ or/and $alloc2$ will be set to $qB + 2G - alloc3$ to make sure that the allocation on path k meets the protection requirement with the other two paths (line 34-39). This modification is safe because line 22-23 ensures that path i and path j have enough free subcarriers to satisfy the protection requirement when $alloc3$ is set to $mcs3$. By this step, the algorithm makes sure that the sum of any two paths is at least qB.

The last step is to check if the sum of alloc1, alloc2 and alloc3 is at least B (line 41). Let deficit be $B - alloc1 - alloc2 - alloc3 + 3G$. When the algorithm picks the third

path, it not only checks if the path meets the protection requirement, it also checks if the sum of the 3 MCSs is at least B. This means that $deficit <= (MCS1 - alloc1) + (MCS2 - alloc2) + (MCS3 - alloc3)$ and can be distributed to the three paths. We start from path 1, and increase alloc1 by deficit if deficit is smaller than $MCS1 - alloc1$. Else, alloc1 is increased by $MCS1 - alloc1$, and $deficit = deficit - (MCS1 - alloc1)$. Continue the same process to the second and third path. Since $deficit <= (MCS1 - alloc1) + (MCS2 - alloc2) + (MCS3 - alloc3)$, by the third path, deficit must be 0 (line 42-43).

CHAPTER 5.   Numerical Results

Figure 5.1   A sample US network topology.
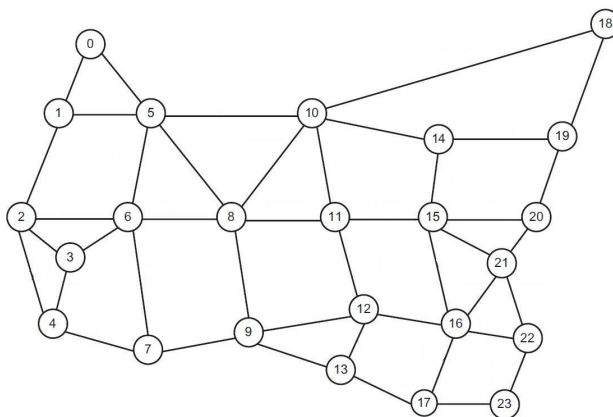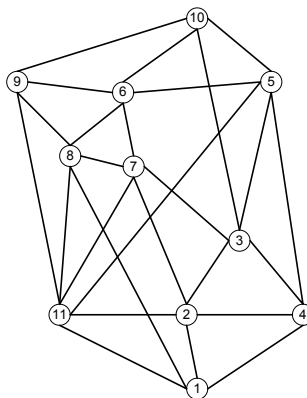

Figure 5.2   COST 239 European Optical Network.

In this section, we evaluate the performance of the ILP model and the heuristic algorithm for dynamic traffic scenario. We also show the results of an SPP algorithm to demonstrate the advantage of MPP over SPP. The SPP algorithm works as follows. For a given demand $r = < s, d, B, q >$, we use Bhandari's algorithm (20) to compute

(a) q=0.5

(b) q=0.75

(c) q=1.0

(d) mixed q
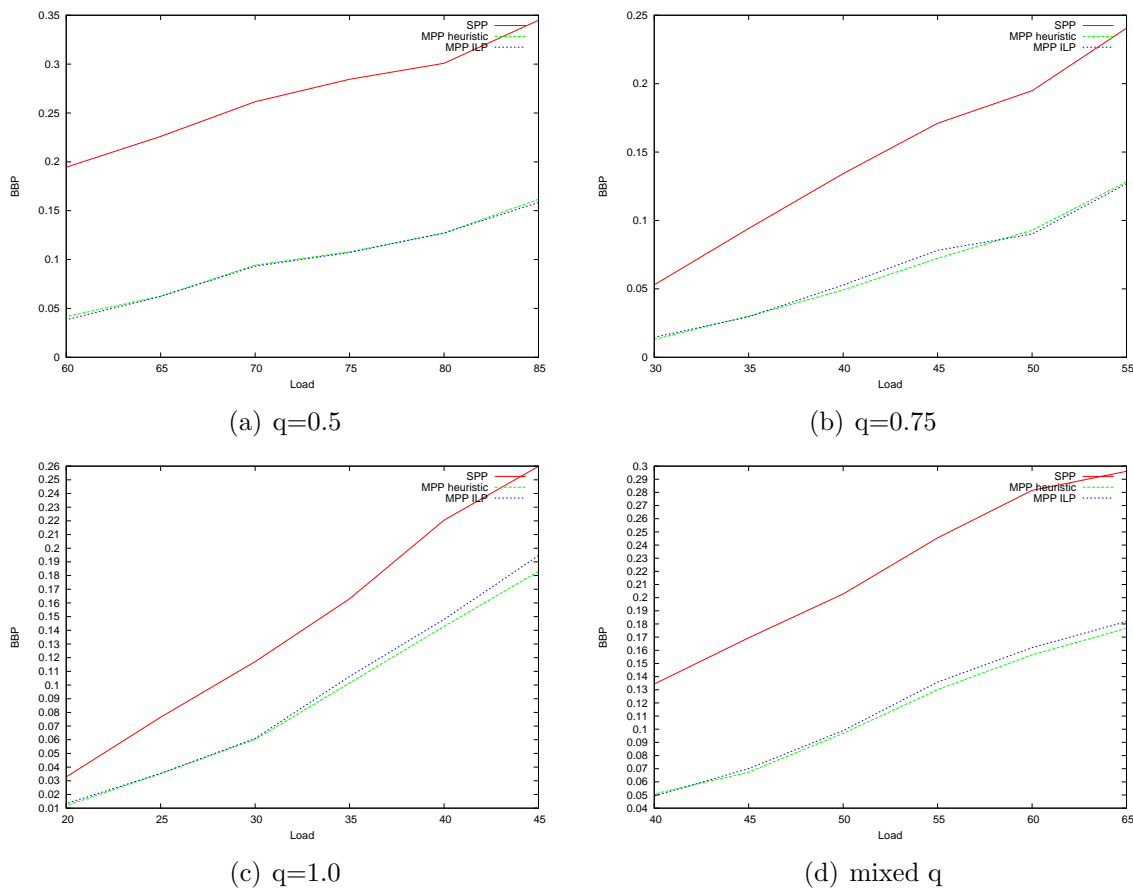
Figure 5.3   BBP of SPP, MPP Heuristic, and MPP ILP as a function of load under different q values for US topology.

a set of link-disjoint candidate paths for r and sort the candidate paths in increasing order of path length in hops. We find the first candidate path that has at least $B + G$ contiguous available subcarriers. This path is chosen as the working path for r with the first $B + G$ contiguous available subcarriers allocated to it. We then remove the working path from the candidate path set and find the first remaining candidate path that has at least $qB + G$ contiguous available subcarriers. This path is chosen as the backup path for r with the first $qB + G$ contiguous available subcarriers allocated to it.

We run simulations over a sample US network topology (Fig. 5.1) with 24 nodes and 43 links and a COST 239 European network topology with 11 nodes and 26 links (Fig. 5.2). Simulations are run with protection level 0.5, 0.75, 1 and a mixture of 0.5,
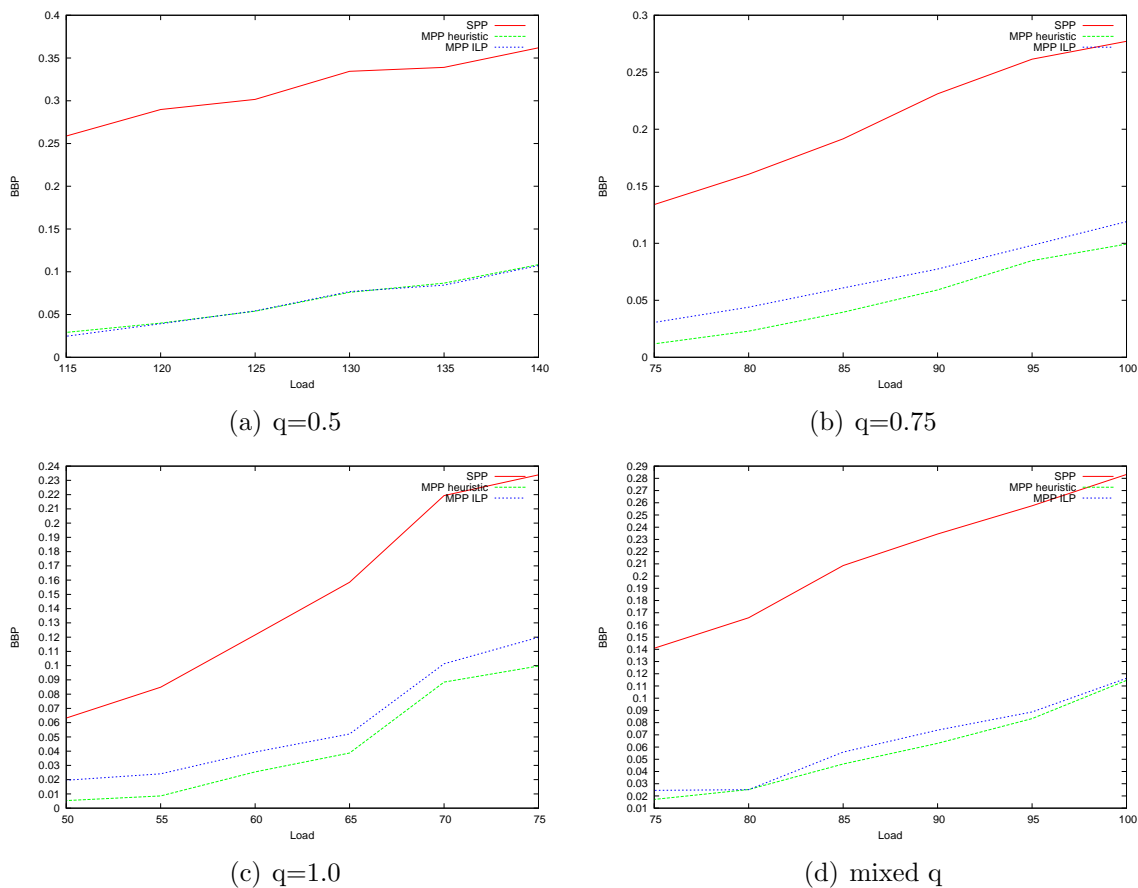
Figure 5.4   BBP of SPP, MPP Heuristic, and MPP ILP as a function of load under different q values for COST 239 topology.

0.75, and 1. The bandwidth requirement B is chosen from $10, 20, 30, 40$. The number of subcarriers on each link is set to 300 and the guard subcarrier G is 1. For each protection level, 10,000 requests are processed. Fig. 5.3 and Fig. 5.4 are results from the simulation for the US topology and the COST 239 topology. In the figures, the x-axis is the network load, and the y-axis is the Bandwidth Blocking Probability (BBP). In this simulation, the arrival event follows a passion distribution with λ requests per second. The holding time is exponentially distributed with a mean of $1/\mu$. Thus, the traffic load in Erlang is λ/μ. Bandwidth Blocking Probability (BBP) is the ratio of blocked bandwidth to total requested bandwidth.

The following sections will first compare SSP with MPP and then MPP heuristic

with MPP ILP.

5.1 Blocking Performance Comparison between SPP and MPP

Table 5.1   Ratio of SPP's BBP to MPP Heuristic's BBP with different q values for US topology.

q	Load 1	Load 2	Load 3	Load 4	Load 5	Load 6
0.5	4.67	3.62	2.78	2.64	2.37	2.13
0.75	4.06	3.13	2.73	2.36	2.10	1.88
1.0	2.77	2.16	1.95	1.61	1.54	1.42
mixed	2.64	2.52	2.09	1.89	1.80	1.68

Table 5.2   Ratio of SPP's BBP to MPP Heuristic's BBP with different q values for COST 239 topology.

q	Load 1	Load 2	Load 3	Load 4	Load 5	Load 6
0.5	8.89	7.26	5.60	4.41	3.91	3.34
0.75	11.34	6.99	4.85	3.91	3.084	2.79
1.0	11.83	9.92	4.78	4.095	2.48	2.35
mixed	8.22	6.57	4.52	3.72	3.09	2.47

Fig 5.3 and 5.4 both contain 4 subfigures that show the BBP of SPP, MPP ILP and MPP heuristic as a function of load for different q values. In Figure 5.3(a) and Figure 5.4(a), the q value is 0.5. In Figure 5.3(b) and 5.4(b), q value is 0.75. In Figure 5.3(c) and 5.4(c), q value is 1.0. Finally, q value is randomly chosen from $(0.5, 0.75, 1.0)$ in Figure 5.3(d) and Figure 5.4(d). Load value for each protection value q are chosen such that the BBP of MPP is within the range of 0.01-0.2 for US topology and 0.01-0.1 for COST 239 topology. Thus, the six load values are different for each q. When q=0.5, load value starts from 60 and increases by an interval of 5 and ends at 85 for the US topology. For the COST 239 topology, the load value vary from 115 to 140 with the same interval. When q=0.75, the range of load value is 30-55 for the US topology and 75-100 for COST 239 topology. And for q=1 and mixed q, the range is 20-45 and 40-65 for US topology. As for COST 239 topology, the range is 50-75 and 75-100 respectively.

From Figures with $q = 0.5, q = 0.75$ and $q = 1$ we can see that the network is able to carry more traffic when q value decreases. For example, BBP=0.1 is achieved at load 50 when q=0.75 and at load 75 when q=0.5 in the US topology. For same q, COST 239 network can carry more traffic than the US network. For example, when $q = 0.5$, COST 239 network achieves a BBP of 0.1 with load about 130 while the US network reaches the same BBP at load 75.

In the figures, it is clear that with any given protection level and load, SPP results in a higher probability of bandwidth blocking than MPP. Table 5.1 and Table 5.2 contain the ratio of SPP's BBP to MPP heuristic's BBP for different q and load values. We can see that the ratio is over 2 in most of the cases and can be as high as 4.67 in US topology. That is, MPP heuristic's BBP is less than half of SPP's BBP in most of the cases in US network. In case of COST 239 topology, the ratio is over 3 in most cases and reaches 11.83 in the extreme case. Even the smallest ratio is over 2 in COST 239 network indicating bigger performance gap between SPP and MPP heuristic algorithm. This shows that, in denser network, advantage of MPP over SPP is greater. We also observe from the two tables that the ratio decreases as the load increases for each q value. This means that the performance advantage of MPP over SPP is bigger when the load is smaller. In practice, the network should operate with low BBP (e.g, under 5%); this corresponds to the low load value in Figure 5.3 and 5.4 when the advantage of MPP over SPP is the greatest.

It can also be observed from Table 5.1 that the performance gap between SPP and MPP gets smaller as q increases. Specifically, when q = 0.5, the ratio of SPP's BBP to MPP heuristic's BBP is in the range 2.13-4.67; when $q = 0.75$, the range of the ratio decreases to 1.88-4.06; when $q = 1$, the range of the ratio further decreases to 1.42-2.77. This can be explained as follows, when q=0.5, SPP will require $1.5B$ subcarriers while MPP only needs B subcarriers. When $q > 0.5$, algorithm 3 and algorithm 4 will be called to calculate solutions which the better one will be returned as the final solution.

Algorithm 3 returns a solution with $2qB$ allocation while if we look at algorithm 4 line 25-31, we can see that the algorithm tries to give a solution close to $qB/2$ on each path. Roughly, algorithm 4 gives a solution close to $1.5qB$. On the other hand, SPP gives a solution with $(1 + q)B$ allocation. The ratio of SPP/MPP when q=0.5 is 1.5 while the ratio for $q > 0.5$ is $(1 + q)/2q$ and $(1 + q)/1.5q$. In either case, the ratio decreases when q goes from 0.5 to 0.75. This is also true when q is originally greater than 0.5. For example, when q increases from 0.75 to 1, the ratio for algorithm 3 decreases from 1.17 to 1 and from 1.56 to 1.33 for algorithm 4. Thus, the gap between SPP and MPP gets smaller as the protection level increases. However, this conclusion does not seem to hold in Table 5.2 for load 1 and load 2. This is because the load values for COST 239 network do not overlap as in the US topology. In US topology, the load values have a high percentage of overlapping except when q=0.5. In the COST 239 topology, the load values do not overlap, and the value for q=0.75 and q=1.0 is much smaller than that for q=0.5. With our last conclusion that "performance advantage of MPP over SPP is bigger when the load is smaller", It is reasonable to see such an increase in this case. In addition, starting from load 3, the gap between MPP and SPP again begin to shrink as the protection level increases.

5.2   Fairness Comparison between SPP and MPP

Table 5.3   Drop rate of SPP and MPP heuristic with B=10 and B=40 for US topology.

q	Load	SPP B=40	MPP B=40	SPP B=10	MPP B=10
0.5	85	0.54	0.26	0.02	0.02
0.5	60	0.34	0.067	0.003	0.0023
0.75	55	0.38	0.20	0.011	0.012
0.75	30	0.099	0.021	0.0012	0.0008
1	45	0.43	0.28	0.01	0.022
1	20	0.063	0.021	0	0
Mixed	65	0.46	0.27	0.019	0.018
Mixed	40	0.23	0.082	0.0044	0.002

It has been shown in (21) that a high degree of unfairness in call blocking may

Table 5.4   Drop rate of SPP and MPP heuristic with B=10 and B=40 for COST 239
topology.

q	Load	SPP B=40	MPP B=40	SPP B=10	MPP B=10
0.5	140	0.598	0.20	0.0024	0.0008
0.5	115	0.45	0.055	0.0016	0.0004
0.75	100	0.49	0.18	0.00039	0.0012
0.75	75	0.26	0.024	0	0
1	75	0.43	0.19	0.00078	0.00078
1	50	0.12	0.01	0	0
Mixed	100	0.49	0.21	0.0012	0.002
Mixed	75	0.27	0.037	0.00041	0

arise in multi-rate flexible optical networks where high bandwidth demanding services experience much higher call blocking than low bandwidth demanding services. In (21), the authors consider different services sharing a given optical link and the services do not have protection requirement. In this section, we evaluate the fairness of SPP and MPP in the USA network and COST 239 network for requests with both bandwidth and protection requirements.

Table 5.3 and 5.4 show the drop rates of SPP and MPP heuristic for each protection level with bandwidth request being 10 (i.e.,low bandwidth requests) and 40 (i.e.,high bandwidth requests). Only the lowest load and the highest load are listed since other load values show the same tendency. From these two tables, we can see that for a given q value and a given load, SPP's drop rate for B=40 is much higher than that for B=10. MPP has a similar situation. This means that both SPP and MPP give significant advantage to low bandwidth requests.

Table 5.5   Ratio of drop rate of maximum B and minimum B for US topology.

q	Load	SPP40/SPP10	MPP40/MPP10
0.5	85	34.77	13.22
0.5	60	120.66	28.19
0.75	55	35.20	15.86
0.75	30	81.94	26.37
1	45	42.6	12.79
1	20	N/A	N/A
Mixed	65	23.88	15.05
Mixed	40	52.38	40.71

Table 5.6   Ratio of drop rate of maximum B and minimum B for COST 239 topology.

q	Load	SPP40/SPP10	MPP40/MPP10
0.5	140	248.98	251.53
0.5	115	281.06	138.49
0.75	100	1269.53	158.65
0.75	75	N/A	N/A
1	75	550.95	245.88
1	50	N/A	N/A
Mixed	100	403.95	104.80
Mixed	75	664.59	N/A

Table 5.5 and 5.6 indicate the ratio of drop rate with high B and low B for SPP and MPP. The ratio of BPP's drop rate for B=40 to drop rate for B=10 is labelled as SPP40/SPP10 while the ratio for MPP is labelled as MPP40/MPP10. In the US network, when q=1 and load=20, the ratio is not available because as table 5.3 shows, the drop rate for SPP10 and MPP10 is 0. Same reason for COST 239 network, some values are labelled as N/A. This ratio indicates the relation of high bandwidth requests with low ones. A larger ratio means that more higher bandwidth requests are dropped. From Table 5.5, we can see that both SPP and MPP have high ratio: for SPP, the ratio is between 23.88 and 120.66 and for MPP the ratio is between 12.79 and 40.71. Similarly, Table 5.6 shows a ratio between 248.98 and 1269.53 for SPP, and 104.80-251.53 for MPP. This indicates that both SPP and MPP favor low bandwidth requests, i.e., low bandwidth requests have much lower drop rate than high bandwidth requests. However, if we compare the ratio of SPP and MPP for a given q and load value, it can be seen that the ratio of MPP is almost always much smaller than ratio of SPP. The only exception is when q=0.5 and load=140 in the COST 239 network which has a very close ratio. This implies that SPP results in more dramatic difference in the drop rate between low bandwidth requests and high bandwidth requests. Thus, MPP is relatively fairer than SPP.

5.3   Comparison of MPP Heuristic and MPP ILP

Figure 5.3 demonstrates a very close BBP/Load value for MPP heuristic and MPP ILP showing similar performance for the two. For $q = 0.5$, ILP performs slightly better than the heuristic. For $q = 0.75$, neither of the two algorithms is consistently better than the other. For $q = 1$, and the mixed q, the heuristic performs better than the ILP. On the other hand, a clear performance difference is shown in Figure 5.4. Same with the US case, when $q = 0.5$, ILP performs slightly better than the heuristic. Otherwise, when $q > 0.5$ and when q is mixed, the heuristic performs better than the ILP.
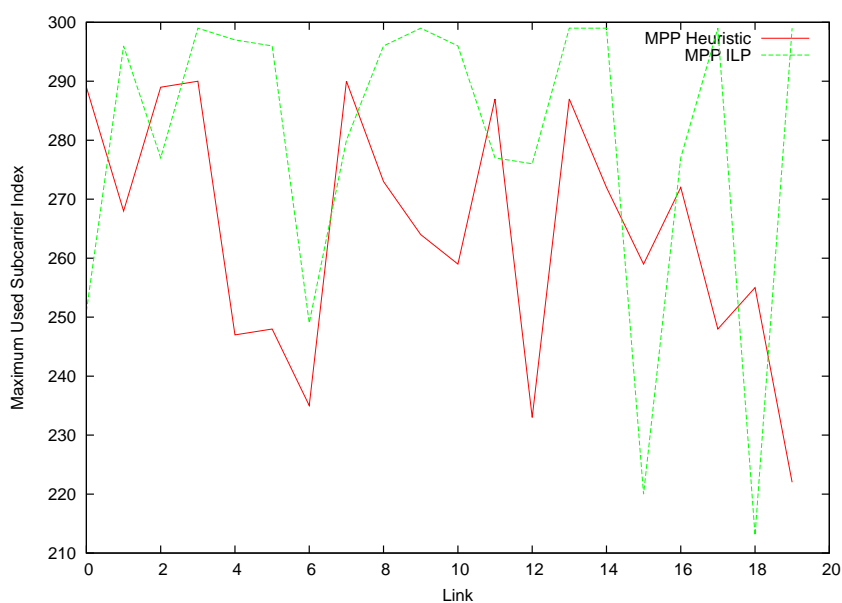

Figure 5.5   Maximum Used Subcarrier Index for MPP Heuristic and MPP ILP in COST 239 Network.

The MPP heuristic algorithm picks the routes in a different way compared to ILP. ILP aims at minimizing the total allocated subcarriers for a given demand and it accepts a request whenever there is enough spectrum resource to accommodate it. ILP does not guarantee a minimum BBP for dynamic demands. On the other hand, our heuristic algorithm may deny a request even with enough resource. Our algorithm allocates subcarriers with load balancing in algorithm 4, line 25. Figure 5.5 shows the maximum

used subcarrier index for both MPP heuristic and MPP ILP algorithm in the COST 239 network. The data is an average of 15 simulations with load being 90 and request number equals 100. We choose such a combination because a larger load or a larger request number will result in index reaching the maximum value in most cases in which the difference cannot be shown. From the figure we can see that most of the time, the maximum index used in our heuristic algorithm is smaller than that in the ILP case. In fact, the average of the maximum used sucarrier index for MPP heuristic is 264.35, and 279.75 for MPP ILP. The standard deviation is 21 and 26.57 respectively. This allows longer MCS for paths that have been partially used. Compared to a sparse network, there are more link disjoint paths for s and d in a denser network. With load balancing (i.e., longer MCS), these available subcarriers on a path have more chance to form a solution with other link disjoint path. Thus, for a request r, different paths may be chosen by the two algorithms, and subcarrier allocation may differ when the same set of paths are picked by the two algorithms. Different paths and subcarrier allocation on one request will result in distinct resource choice for later requests. This explains why the heuristic performs better than the ILP in some cases and why the performance gap between MPP ILP and MPP heuristic increases as the network gets denser as shown in Figure 5.3 and 5.4.

While MPP heuristic and ILP gives similar BBP, the time difference is huge. In our US network simulation, for MPP heuristic, processing 10,000 requests only takes around 45s while it takes about 7,000s to 12,000s for ILP to compute solutions for the same amount of requests. In the COST 239 simulation, MPP heuristic takes about 25s and MPP ILP takes 21000-28000s for different protection requirement. Specifically, ILP takes up to 1.2 seconds to compute a solution for one request in the US simulation, while MPP only take about 4.5ms to obtain a solution. In the COST 239 simulation, MPP ILP take up to 2.8s to compute a solution consuming a much longer time than the US network. On the other hand, MPP heuristic only takes about 2.5ms resulting in a shorter time than

the US simulation. The COST 239 network is a denser topology than the US network. MPP ILP is basically a brute force algorithm and thus will need more time to check all possibilities in a denser network. While for the MPP heuristic algorithm, the density does not affect it much. Although a denser topology results in more link disjoint paths, our heuristic algorithm will stop finding candidate path once it get one valid solution. Notice that, the COST 239 network has only half amount of nodes as in US topology. This greatly reduces the time for computing MCS for a candidate path which eventually reduces the totally time from 45s to 25s.

With similar BBP as ILP and dramatic time difference, MPP heuristic is the choice for practical networks.

CHAPTER 6.   Conclusion

In this thesis, we study the dynamic Survivable Multipath Routing and Spectrum Allocation (SM-RSA) problem. An ILP model for the problem is presented. Besides, a heuristic algorithm is developed for the MPP scheme. We run simulations on US topology and the European COST 239 topology with SPP scheme and the MPP scheme (the ILP version and the heuristic version). Simulation results shows that 1) MPP achieves lower blocking than SPP in dynamic traffic scenario; 2) our heuristic algorithm achieves similar results to the ILP model with dramatic lower amount of time; 3) Both SPP and MPP are unfair to large bandwidth requests, but MPP is relatively fairer than SPP. Moreover, with the comparison of the two networks, we conclude that in a denser network MPP is more advantageous than SPP in terms of BBP; and MPP heuristic is more advantageous than ILP in terms of BBP and time. A possible future work will be investigating technique to improve the fairness of the MPP scheme.

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