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# Survivable multipath provisioning in OFDM-based exible optical networks

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**Survivable multipath provisioning in OFDM-based flexible optical networks**

by

Nan Xiao

A thesis submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of  
MASTER OF SCIENCE

Major: Computer Science

Program of Study Committee:

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Ames, Iowa

2012

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## DEDICATION

I would like to dedicate this thesis to my husband Ying Xia without whose support I would not have been able to complete this work.

## TABLE OF CONTENTS

<b>LIST OF TABLES</b> . . . . .	v
<b>LIST OF FIGURES</b> . . . . .	vi
<b>ACKNOWLEDGEMENTS</b> . . . . .	vii
<b>ABSTRACT</b> . . . . .	viii
<b>CHAPTER 1. INTRODUCTION</b> . . . . .	1
1.1 Background and Literature Review . . . . .	1
1.2 Outline of This Thesis . . . . .	2
<b>CHAPTER 2. PROBLEM DEFINITION</b> . . . . .	4
2.1 The Survivable Multipath Provisioning Scheme . . . . .	4
2.2 The Static Survivable Multipath Routing and Spectrum Allocation Problem . .	5
<b>CHAPTER 3. ILP FORMULATIONS FOR STATIC SM-RSA AND SS-</b> <b>RSA PROBLEMS</b> . . . . .	7
3.1 ILP Formulation For the Static SM-RSA Problem . . . . .	7
3.2 ILP Formulation For the Static SS-RSA Problem . . . . .	9
<b>CHAPTER 4. HEURISTIC ALGORITHM FOR THE STATIC SM-RSA</b> <b>PROBLEM</b> . . . . .	11
4.1 Single Path Allocation . . . . .	11
4.2 Single Request Allocation . . . . .	12
4.3 Ordering of Requests . . . . .	12
4.4 Path Reconfiguration . . . . .	13

<b>CHAPTER 5. NUMERICAL RESULTS</b> . . . . .	<b>14</b>
5.1 Optimal Solutions of ILP Formulations . . . . .	14
5.1.1 A Sample Optimal Solution . . . . .	15
5.1.2 Comparison of Different Protection Levels . . . . .	16
5.1.3 Effect of Different Demand's Bandwidth . . . . .	17
5.2 Comparison of Optimal and Heuristic Solutions . . . . .	18
5.3 Results of Heuristic Algorithm . . . . .	19
5.3.1 Comparison between SPP and MPP . . . . .	21
5.3.2 Effectiveness of Path Reconfiguration . . . . .	21
5.3.3 Comparison between LPF and LDF . . . . .	22
5.3.4 Number of Paths used in MPP . . . . .	23
5.3.5 Effect of Number of Guard Subcarriers . . . . .	24
<b>CHAPTER 6. CONCLUSION</b> . . . . .	<b>26</b>

## LIST OF TABLES

Table 2.1	Capacity requirement of MPP for request $r = \langle s, d, 1, 0.8 \rangle$ . . . . .	5
Table 5.1	Optimal MPP and SPP accommodation of a sample demand set . . . .	15
Table 5.2	The effect of path reconfiguration on maximum occupied subcarrier index and total occupied subcarriers . . . . .	22
Table 5.3	Number of link-disjoint paths used by demands in MPP when $G = 2$ .	23

## LIST OF FIGURES

Figure 5.1	A sample 6 node network topology . . . . .	14
Figure 5.2	Maximum occupied subcarrier index of SPP and MPP schemes for ILP formulation . . . . .	17
Figure 5.3	Maximum occupied subcarrier index of SPP and MPP schemes for ILP and heuristic algorithms. . . . .	18
Figure 5.4	A sample US network topology . . . . .	19
Figure 5.5	Maximum occupied subcarrier index of SPP and MPP schemes for dif- ferent protection levels in heuristic algorithm . . . . .	20
Figure 5.6	Maximum occupied subcarrier index of SPP and Heuristic schemes using different number of guard subcarriers . . . . .	25

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**ABSTRACT**

Compared with traditional WDM network, OFDM-based flexible optical networks are able to provide better spectral efficiency due to its flexible allocation of requests on finer granularity subcarriers. Survivability is a crucial issue in OFDM-based networks, although little work has been done in this topic. In this thesis, a survivable multipath provisioning scheme is presented, which provides flexible protection levels to individual demands in OFDM-based flexible optical networks. We also define the static Survivable Multipath Routing and Spectrum Allocation (SM-RSA) problem which aims to accommodate a given set of demands with minimum spectral utilization. We show that the static SM-RSA problem is NP-hard and provide ILP formulation for it. Also, an efficient heuristic algorithm is given to solve the problem. Our simulation results of both ILP solution and heuristic method show that the proposed multipath provisioning scheme achieves better spectral efficiency than the traditional single path provisioning scheme.

## CHAPTER 1. INTRODUCTION

### 1.1 Background and Literature Review

The rigid and coarse granularity of conventional WDM optical networks require full wavelength capacity to support a connection. When the bandwidth of a demand is less than the capacity of a wavelength, WDM networks may waste network capacity. Flexible optical networks with finer granularity are preferred for better spectral efficiency. Orthogonal frequency division multiplexing (OFDM), which is a widely used modulation technique in broadband wired and wireless communication systems, has high spectral efficiency, flexibility and tolerance to impairments. Due to all these advantages, OFDM is also a promising technology for optical network (??). A data stream in optical OFDM is split into lower rate data streams and modulated onto separate subcarriers. In optical OFDM network, a demand is allocated an appropriate number of subcarriers, as opposed to a whole wavelength in WDM network. A novel network architecture called spectrum-slice elastic optical path network (SLICE), based on OFDM technology, is proposed in (?). In SLICE network, just enough bandwidth is allocated to an end-to-end optical path, leading to efficient accommodation of sub-wavelength and super-wavelength traffic. In (????), the authors demonstrated the advantages of OFDM optical network compared with traditional WDM network.

An important problem in design and operation of OFDM-based networks is the routing and spectrum allocation (RSA) problem. The goal of RSA is to select a path and allocate a set of contiguous subcarriers for a demand while minimizing utilized spectrum. Dynamic RSA problem has been discussed in (??). In (?), RSA algorithm assigned each connection a route and allocated a flexible reference frequency to match the source rate fluctuations. A nonlinear programming model and its decomposition are proposed in (?). The static RSA problem has

been proved to be NP-hard in ??), which also developed optimal ILP formulation and heuristic algorithms.

Survivability is a critical issue in optical networks, because tremendous data can be lost upon a link failure. However there isn't much study on survivable OFDM network. A heuristic algorithm for survivable flexible WDM network design has been proposed in ?). Researchers also developed two backup sharing policies for OFDM-based optical networks ?). Both ?) and ?) consider single path provisioning with full protection. Which means a demand is provisioned on a single working path and a link-disjoint backup path is used to provide full protection against any single link failure.

MultiPath provisioning scheme (MPP) is able to support both full and partial protection levels with higher spectral efficiency compared with Single Path Provisioning (SPP) scheme. In MPP, a traffic demand is accommodated on multiple paths with lower bandwidth on each path. When single link failure occurs, MPP is able to provide partial protection naturally, since the unaffected paths are still able to carry the traffic. Multipath provisioning schemes providing full and partial protection in next-generation SONET/SDH networks with virtual concatenation are studied in ??). Researchers also have studied partial protection using MPP in general mesh networks. To fulfill the bandwidth and partial protection requirement of a demand, ?) proposed a linear program to find multipath routing and capacity allocation strategy. Also a online multipath provisioning was developed in ?) to enable maximum possible partial-protection. In ?), multipath provisioning problem with differential delay constraint was studied.

## 1.2 Outline of This Thesis

In this thesis, we propose a survivable multipath provisioning scheme (MPP) for OFDM-based optical networks supporting user-defined protection level. To the best of our knowledge, no prior work has been done on MPP in OFDM network. We define static Survivable Multipath Routing and Spectrum Allocation(SM-RSA) problem. The aim of this problem is to accommodate a given set of static demands using multipath provisioning scheme such that the utilized spectrum is minimized. We develop optimal ILP formulation to solve static SM-RSA problem. To compare spectral efficiency of MPP and SPP scheme in OFDM-based network,

we also provide the ILP formulation to solve the static Survivable Single-path RSA(SS-RSA) problem on OFDM-based network. An efficient heuristic algorithm for static SM-RSA problem is also developed. Numerical results demonstrate that the proposed multipath provisioning scheme achieves significant spectrum saving over the single path provisioning scheme.

The rest of the thesis is organized as follows. In Chapter 2 multipath provisioning scheme is explained and its advantage over single path provisioning scheme is discussed. Then the static SM-RSA problem is defined. ILP formulation and heuristic algorithm of static SM-RSA are discussed in Chapter 3 and Chapter 4 respectively. In Chapter 5 we present the numerical results of ILP formulation and heuristic algorithm. Finally, we conclude the thesis in Chapter 6.

## CHAPTER 2. PROBLEM DEFINITION

This chapter explains the proposed survivable multipath provisioning scheme and demonstrates its advantages over single path provisioning scheme. Then static survivable multipath routing and spectrum allocation problem is defined.

### 2.1 The Survivable Multipath Provisioning Scheme

OFDM-based optical networks are able to support flexible protection levels due to its flexible bandwidth allocation capability. In this work, we assume a connection request has both bandwidth and protection level requirement. Specifically, a request is represented by  $r = \langle s, d, B, q \rangle$ , where  $s$  and  $d$  are the source and destination nodes,  $B$  is the bandwidth requirement, and  $q$  ( $0 \leq q \leq 1$ ) is the protection level requirement, which means after single link failure,  $qB$  bandwidth must be available.  $q = 1$  indicates full protection,  $q = 0$  indicates no protection and  $0 < q < 1$  indicates partial protection.

To accommodate a connection request  $r = \langle s, d, B, q \rangle$  using multipath provisioning scheme (MPP),  $N \geq 2$  link-disjoint paths are chosen between  $s$  and  $d$ . Working and backup capacity are allocated on each of these paths such that the total working capacity on  $N$  paths is  $B$  and the total working and backup capacity on any group of  $N - 1$  paths is greater than or equal to  $qB$ . If only one demand is considered, reserving the same amount of bandwidth on each path minimizes the utilized spectrum. Specifically, we allocate  $\frac{B}{N}$  working capacity on each path. If  $N \geq \frac{1}{1-q}$ , no backup capacity needs to be allocated because each path carries less than  $(1 - q)B$  working capacity. If  $N < \frac{1}{1-q}$ , we allocate at least  $\frac{qB}{N-1} - \frac{B}{N}$  backup capacity on each path. This ensures that any group of  $N - 1$  paths has total capacity  $qB$  so that the protection level requirement is satisfied. The total working and backup capacity allocated on

$N$  paths is  $B$  when  $N \geq \frac{1}{1-q}$  and is  $\frac{qNB}{N-1}$  when  $N < \frac{1}{1-q}$ . If multiple demands are considered, the optimal solution may reserve uneven bandwidth on each path, which will be discussed in Section 5.1.1.

Table 2.1 Capacity requirement of MPP for request  $r = \langle s, d, 1, 0.8 \rangle$

N	Working Capacity Per Path	Backup Capacity Per Path	Total Capacity on $N$ Paths
2	0.5	0.3	1.6
3	0.333	0.067	1.2
4	0.25	0.0167	1.067
5	0.2	0	1

Table 2.1 shows the capacity requirement of multipath provisioning (MPP) for request  $r = \langle s, d, 1, 0.8 \rangle$  (bandwidth requirement is 1 and protection level requirement is 0.8) when different number of link-disjoint paths are used. It can be seen that the total capacity allocation decreases as  $N$  increases. When  $N = 5$ , no backup capacity allocation is needed.

In single-path provisioning (SPP),  $r = \langle s, d, B, q \rangle$  can be accommodated by allocating a working path with capacity  $B$  and a backup path with capacity  $qB$ . So the total capacity required is  $(1 + q)B$ . On the other hand, MPP with  $N = 2$  requires  $2qB$  total capacity ( $qB$  on each path). Since  $1 + q \geq 2q$ , MPP with  $N = 2$  is more efficient than SPP even though both approaches use two link-disjoint paths. For the example request  $r = \langle s, d, 1, 0.8 \rangle$ , SPP requires 1 unit capacity on the working path and 0.8 unit capacity on the backup path, giving a total capacity of 1.8 units. This is more than the 1.6 units required in the case of MPP with  $N = 2$ . It can be seen from the above analysis that MPP is more efficient than SPP and the efficiency gap between the two schemes becomes bigger as the number of link-disjoint paths used in MPP increases.

## 2.2 The Static Survivable Multipath Routing and Spectrum Allocation Problem

In OFDM-based flexible optical networks, the frequency spectrum is divided into a number of subcarriers or slots with equal frequency. Accommodating a demand requires selecting

a route and allocating contiguous subcarriers on each link on the route. This is called the routing and spectrum allocation (RSA) problem. Accommodating a given set of demands while minimizing the utilized spectrum is called the static RSA problem, which is proved to be NP-hard in [?]. Since demands can be accommodated more efficiently using MPP, we define a new problem, the static Survivable Multipath RSA (SM-RSA) problem, as follows: Given a set of traffic demands, each represented by  $r = \langle s, d, B, q \rangle$ , accommodate all the demands using multipath provisioning such that the maximum occupied subcarrier index is minimized. In this problem, we need to determine two or more link-disjoint paths for each demand and allocate spectrum on each path so that the bandwidth and protection requirements of each demand are satisfied and the utilized spectrum is minimized.

The static SM-RSA problem requires the following constraints to be satisfied.

- *Working and backup capacity constraint:* For each request  $r = \langle s, d, B, q \rangle$ , the total working capacity allocated to all its paths is  $B$  and the total working and backup capacity remaining after any single link failure is at least  $qB$ .
- *Spectrum contiguity constraint:* A set of contiguous subcarriers must be allocated to a spectrum path.
- *Non-overlapping spectrum constraint:* A subcarrier on a link can only be allocated to at most one spectrum path routed over the link.
- *Guard subcarrier constraint:* When two adjacent spectrum paths share a link, they must be separated by  $GS$  guard subcarriers.

The static SM-RSA problem is significantly more complicated than the static RSA problem. In fact, the NP-hard static RSA problem is a special case of the static SM-RSA problem where each demand is provisioned on a single path and no protection is required. Thus, the static SM-RSA problem is also NP-hard and it is impossible to efficiently solve the static SM-RSA problem for large networks. In Chapter 3 we develop ILP formulation for the static SM-RSA. And then in Chapter 4, we present an efficient heuristic algorithm to solve the static SM-RSA problem.

## CHAPTER 3. ILP FORMULATIONS FOR STATIC SM-RSA AND SS-RSA PROBLEMS

In this chapter, we describe Integer Linear Programming (ILP) formulation of static SM-RSA and static SS-RSA problems for OFDM optical networks. We develop path based ILP formulation for both problems to fairly compare the SPP and MPP schemes.

### 3.1 ILP Formulation For the Static SM-RSA Problem

We present an ILP formulation for the SM-RSA problem stated in Section 2.2. The purpose of our ILP formulation is to minimize the utilized spectrum while satisfying the constraints stated in section 2.2. Our network topology is represented as  $G = (V, E)$ . Here  $V$  and  $E$  denote sets of vertices and edges in graph. For each request  $r = \langle s, d, B, q \rangle$  from node  $s$  to  $d$  we precomputed a set of candidate link-disjoint paths  $\mathbf{P}_{s,d}$  ( $|\mathbf{P}_{s,d}| \geq 2$ ) by using Bhandari's link-disjoint paths algorithm (?). Bhandari's algorithm computes largest number of link-disjoint paths with least total cost for each request.

#### Notations

$B_{s,d}$ : Amount of traffic demands from  $s$  to  $d$

$p_{s,d,k}$ : The  $k$ -th link-disjoint path from  $s$  to  $d$

$X_{s,d,k}^l$ : Equals to 1 if path  $p_{s,d,k}$  uses link  $l$ , 0 otherwise

$K_{s,d}$ : Number of link disjoint paths from  $s$  to  $d$

$P$ : Number of total paths in path set  $\mathbf{P} = \bigcup_{(s,d)} \mathbf{P}_{s,d}$ ,  $P = |\mathbf{P}|$

$\mathbf{D}$ : Demand set

$\phi$ : Number of subcarriers for each link

$GS$ : Guard subcarriers



## Variables

$c_{s,d,k}^w$ : Boolean variable denotes if path  $p_{s,d,k}$  uses subcarrier  $w$ . 1 if the path  $p_{s,d,k}$  uses subcarrier with index  $w$  and 0 otherwise

$u^w$ : Equals to 1 if exists a light-path using subcarrier  $w$

$MS$ : Index of maximum utilized subcarrier

### MPP ILP formulation:

minimize  $MS$

subject to the following constrains:

- Cost function:

$$MS \geq wu^w \quad \forall w \quad (3.1)$$

$$\sum_{(s,d) \in \mathbf{D}} \sum_{k \in [1, K_{s,d}]} c_{s,d,k}^w \leq u^w P \quad \forall w \quad (3.2)$$

Equation 3.1 obtains the index of maximum occupied subcarrier. When no path utilizes  $w$ , left hand side of Equation 3.2 equals to 0, so  $u^w$  also equals to 0. Otherwise,  $u^w$  equals to 1 when at least a path occupies  $w$ .

- Traffic demand constrains:

$$\sum_{k \in [1, K_{s,d}]} \sum_{w \in [1, \phi]} c_{s,d,k}^w \geq B_{s,d} \quad \forall (s, d) \in \mathbf{D} \quad (3.3)$$

$$\sum_{k \in [1, K_{s,d}], k \neq m} \sum_{w \in [1, \phi]} c_{s,d,k}^w \geq qB_{s,d} \quad \forall (s, d) \in \mathbf{D}, 1 \leq m \leq K_{s,d} \quad (3.4)$$

Equation 3.3 denotes the working demands between nodes  $s$  and  $d$ . The summation of used subcarriers on all candidate paths between  $s$  and  $d$  should be larger than or equal to requested  $B_{s,d}$  subcarriers. When a link fails, the summation of occupied subcarriers on all other uninfluenced paths need to be larger than or equal to  $qB_{s,d}$ . Equation 3.4 guarantees this requirement.

- Subcarrier capacity constrains:

$$\sum_{(s,d) \in \mathbf{D}} \sum_{k \in [1, K_{s,d}]} c_{s,d,k}^w X_{s,d,k}^l \leq 1 \quad \forall l \in \mathbf{L}, w \quad (3.5)$$

Equation 3.4 ensures each subcarrier can only be utilized by one path.

- Guard subcarrier constrains:

$$(c_{s,d,k}^w \cdot x_{s,d,k}^l - 1) \cdot 2GS \cdot P + \sum_{w' \in [\max(1, w-GS), \min(\phi, w+GS)]}^{s', d', k' \neq s, d, k} c_{s', d', k'}^{w'} \cdot x_{s', d', k'}^l \leq 0 \quad (3.6)$$

$$\forall w, l, p_{s,d,k} | x_{s,d,k}^l = 1$$

When two spectrum paths share the same link, their occupied subcarriers have to be separated by  $GS$  guard subcarriers. If path  $p_{s,d,k}$  uses subcarrier  $w$ , subcarriers  $[w - GS, w + GS]$  on all links in  $p_{s,d,k}$  can't be occupied by other paths. If a path  $p_{s,d,k}$  contains link  $l$  and uses slot  $w$ ,  $c_{s,d,k}^w \cdot x_{s,d,k}^l = 1$ . Then all other paths, which also use link  $l$ , are not able to occupy subcarriers  $[w - GS, w + GS]$  in the limit of Equation 3.6.

- Spectrum continuous constrain:

$$(c_{s,d,k}^w - c_{s,d,k}^{w+1} - 1)(-\phi) \geq \sum_{w' \in [w+2, \phi]} c_{s,d,k}^{w'} \quad \forall w, p_{s,d,k} \quad (3.7)$$

Equation 3.7 represents the spectrum contiguity constrain. In this constrain, if a path  $p_{s,d,k}$  utilizes subcarrier  $w$  and doesn't utilize  $w + 1$ , all  $[w + 2, \phi]$  subcarriers can't be occupied by the path.

### 3.2 ILP Formulation For the Static SS-RSA Problem

In order to fairly compare spectrum efficiency, we also develop ILP formulation of SPP scheme. Same as MPP, For a graph  $G = (V, E)$  and a given request  $r = \langle s, d, B, q \rangle$ , a set of candidate link-disjoint paths  $\mathbf{P}_{s,d}$  are calculated. For SPP scheme, a working and backup path are selected from candidate path set. The working path requires  $B$  bandwidth and  $qB$  bandwidth are saved for backup path. This formulation also minimize index of maximum utilized subcarrier.

#### Variables

$\lambda_{s,d,k}^{wp}$ : Boolean variable denotes if path  $p_{s,d,k}$  is a working path.

$\lambda_{s,d,k}^{bp}$ : Boolean variable denotes if path  $p_{s,d,k}$  is a backup path.

All other variables are same as MPP ILP formulation.

#### SPP ILP formulation:

minimize  $MS$

subject to the following constrains:

- Cost function:

$$MS \geq wu^w \quad \forall w \quad (3.8)$$

$$\sum_{(s,d) \in \mathbf{D}} \sum_{k \in [1, K_{s,d}]} c_{s,d,k}^w \leq u^w P \quad \forall w \quad (3.9)$$

- Working and backup paths constrains:

$$\sum_{k \in [1, K_{s,d}]} \lambda_{s,d,k}^{wp} = 1 \quad \forall (s, d) \in \mathbf{D} \quad (3.10)$$

$$\sum_{k \in [1, K_{s,d}]} \lambda_{s,d,k}^{bp} = 1 \quad \forall (s, d) \in \mathbf{D} \quad (3.11)$$

$$\lambda_{s,d,k}^{wp} + \lambda_{s,d,k}^{bp} \leq 1 \quad \forall (s, d) \in \mathbf{D}, 1 \leq k \leq K_{s,d} \quad (3.12)$$

In SPP scheme, one demand only has one working and one backup path. Equation 3.10 and 3.11 guarantees a demand  $(s, d)$  has only one working and one backup path. Equation 3.12 ensures that one path can't be working and backup path at the same time.

- Traffic demand constrains:

$$\lambda_{s,d,k}^{wp} B_{s,d} \leq \sum_{w \in [1, \phi]} c_{s,d,k}^w \quad \forall (s, d) \in \mathbf{D}, 1 \leq k \leq K_{s,d} \quad (3.13)$$

$$\lambda_{s,d,k}^{bp} \cdot q B_{s,d} \leq \sum_{w \in [1, \phi]} c_{s,d,k}^w \quad \forall (s, d) \in \mathbf{D}, 1 \leq k \leq K_{s,d} \quad (3.14)$$

$$(\lambda_{s,d,k}^{wp} + \lambda_{s,d,k}^{bp}) B_{s,d} \geq \sum_{w \in [1, \phi]} c_{s,d,k}^w \quad \forall (s, d) \in \mathbf{D}, 1 \leq k \leq K_{s,d} \quad (3.15)$$

Equation 3.13 states capacity of working path in SPP equals to requested bandwidth  $B_{s,d}$ . For backup path  $qB_{s,d}$  subcarriers are enough, which is limited in Equation 3.14. Equation 3.15 guarantees no subcarrier is reserved for unutilized path. When both  $\lambda_{s,d,k}^{wp}$  and  $\lambda_{s,d,k}^{bp}$  equals to 0,  $c_{s,d,k}^w$  for all possible subcarrier index  $w$  should also be 0.

Subcarrier capacity constrains, guard subcarrier constrains and spectrum continuous constrains are same as MPP ILP formulation.

## CHAPTER 4. HEURISTIC ALGORITHM FOR THE STATIC SM-RSA PROBLEM

Our heuristic algorithm contains three steps. In the first step, for each request  $r = \langle s, d, B, q \rangle$ , a set of candidate link-disjoint paths  $P_{s,d}$  ( $|P_{s,d}| \geq 2$ ) is computed between  $s$  and  $d$ . Here Bhandari's link-disjoint paths algorithm (?) is used to compute the largest number of link-disjoint paths with the least total cost for each request. In the second step, we sort the requests in some order and then serve them one-by-one. For each request, we select a number of routing paths from its candidate path set and allocate an appropriate number of subcarriers on these paths. The constraints given in Section 2.2 are taken into account when serving each request. In the third step, we reconfigure some paths to reduce the maximum occupied subcarrier index. The details of the algorithm are given in the following sections.

### 4.1 Single Path Allocation

We associate each link  $e$  in the network with a boolean array  $\overline{o}_e = (o_{e1}, o_{e2}, \dots, o_{e\phi})$  to represent the availability of each subcarrier in  $e$ . Here,  $\phi$  represents the maximum subcarrier index in the link.  $o_{ew}$  equals 1 if the  $w$ th subcarrier is available in link  $e$ . Suppose  $n$  subcarriers need to be allocated for path  $p$ . First, the availability array of path  $p$  is calculated based on the following equation:  $\overline{o}_p = \&_{e \in p} \overline{o}_e$ . In this equation,  $\&$  donates boolean AND operation. Then, vector  $\overline{o}_p$  is checked from low index to high index, the first  $n$  contiguous available subcarriers are allocated to path  $p$ . Finally, for each link  $e$  in path  $p$ , the allocated subcarriers are marked as unavailable in  $\overline{o}_e$ .

## 4.2 Single Request Allocation

To accommodate request  $r = \langle s, d, B, q \rangle$ , we first select  $N$  paths from  $P_{s,d}$  ( $2 \leq N \leq |P_{s,d}|$ ), and then calculate the working and backup capacity to be allocated on each path using the method given in Section 2.1. The total number of subcarriers required on each path, denoted by  $n$ , can be calculated by  $n = \lceil \frac{A}{C} \rceil + GS$ . Here  $A$  is the total working and backup capacity to be allocated on each path,  $C$  is the capacity of a subcarrier, and  $GS$  is the number of guard subcarriers.

To determine the value of  $N$ , we note that  $N = |P_{s,d}|$  may not be the best choice although Table 2.1 shows that the total path capacity decreases with increasing number of paths. This is because the guard subcarrier overhead increases if more paths are used. Also, a path with more links will occupy more subcarriers than a path with fewer links, so shorter paths are preferable. To determine the number of paths for request  $r$ , we order the paths in  $P_{s,d}$  in increasing order of path cost (i.e., number of links in the path). Our goal is to satisfy  $r$  with minimum number of occupied subcarriers. Thus, we calculate the total number of subcarriers required when using the first two, first three, ..., first  $|P_{s,d}| - 1$ , and all candidate paths in  $P_{s,d}$ . Out of these  $|P_{s,d}| - 1$  path set choices, the path set that occupies the least subcarriers is selected. Once the path set is determined, we use the method described in Section 4.1 to allocate  $n$  contiguous subcarriers for each path.

## 4.3 Ordering of Requests

To satisfy a given set of requests, our algorithm sorts these requests and serves each request one-by-one by using the method in section 4.2. We propose two ordering strategies as follows:

- *Largest Demand First (LDF)*: We order the requests in decreasing order of bandwidth requirement and serves the request with the largest demand first. If two or more requests have the same demand, we compare their shortest paths in their candidate path sets. The request with the longest shortest path is served first.
- *Longest Path First (LPF)*: We order the requests in decreasing order of the shortest path length in the candidate path set of each request. The request with the longest shortest

path is served first. If two or more requests have the same shortest path length, we compare their requested bandwidth and serve the request with the largest demand first.

#### 4.4 Path Reconfiguration

After serving all requests in LDF or LPF order, we employ a path reconfiguration step to reduce the maximum occupied subcarrier index. The idea is to iteratively reroute the path that currently occupies the largest subcarrier index so that it can be allocated with the lowest available subcarriers. This idea is similar to the defragmentation technique proposed in [?], which applies to dynamic traffic scenario where the connection setup and teardown processes lead to fragmentation of spectral resources. The defragmentation algorithms in [?] can be applied periodically to consolidate the available network resources, bringing the network to its optimal state. Our path reconfiguration procedure is different from the defragmentation algorithms in that it applies to static traffic to reduce the utilized spectrum. Also, we consider multipath provisioning instead of single-path provisioning as in [?].

The path reconfiguration procedure works as follows. First, we sort allocated spectrum paths in decreasing order of their largest occupied subcarrier index. Then the first spectrum path from the ordered list, denoted by  $p$ , is selected. Suppose  $p$  is allocated  $n$  subcarriers. Set  $i$  to the lowest subcarrier index and construct an auxiliary graph  $G$  in which an edge between a pair of nodes exists if starting from subcarrier  $i$ ,  $n$  contiguous subcarriers are available on the link connecting the two nodes. Let  $S_p$  be the set of all other link-disjoint paths that belong to the same request's path set as  $p$ . Delete all edges used by the paths in  $S_p$  from  $G$  and then find the shortest path between the source and destination of the request. If a path exists, then reconfigure  $p$  using the found path and allocate  $n$  contiguous subcarriers starting from subcarrier  $i$ . If a path does not exist, we increment  $i$  to  $i + 1$  and construct a new auxiliary graph until a new path for  $p$  is found or  $i$  equals the current start subcarrier index of  $p$ . We keep reconfiguring paths from the ordered list until we reach a path that cannot be reconfigured. That means the maximum occupied subcarrier index of the network cannot be further reduced.

## CHAPTER 5. NUMERICAL RESULTS

In this chapter, we first present optimal solutions of SM-RSA and SS-RSA ILP formulations to compare multipath (MPP) and single path (SPP) provisioning schemes. Then simulation results are described to illustrate the performance of our heuristic algorithm.

### 5.1 Optimal Solutions of ILP Formulations

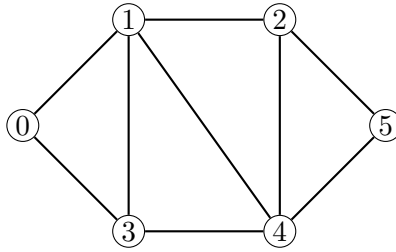


Figure 5.1 A sample 6 node network topology

Here, we compare the optimal solutions of multipath (MPP) and single path (SPP) provisioning schemes to demonstrate the advantages of MPP on spectral efficiency. We use a simple 6 nodes network in our experiments (shown in Fig. 5.1). Each edge in the figure represents two directed optical links with opposite direction. In this simple network, there are two to three paths between any pair of nodes. We consider two demand sets representing demands with low bandwidth and high bandwidth. In both cases, demands are randomly generated between two nodes until total number of requested subcarriers reaches 40. For low bandwidth situation, the number of requested subcarriers by a single demand should be a random number between 1 and 5. On the other hand, for high bandwidth situation, number of subcarriers is between 1 and 10. For both demand sets, we tested three protection levels: 0.5, 0.75 and 1. The results shown in this section are average of 10 randomly generated demand sets. And guard subcarrier

in this part is 1.

Firstly, we use a sample accommodation of a set of demands to demonstrate the correctness of our ILP formulations. Then we compare optimal solution of maximum occupied subcarrier in SPP and MPP schemes under different protection levels and demand's requested bandwidths. For a given request  $r = \langle s, d, B, q \rangle$ , SPP reserves a working path with  $B$  capacity and backup path with  $qB$  capacity. And MPP allocates both working and backup capacity on multiple paths.

### 5.1.1 A Sample Optimal Solution

Table 5.1 Optimal MPP and SPP accommodation of a sample demand set

s	d	B	Path	SPP Subcarrier	MPP Subcarrier
0	1	9	0-1	B1-9	3-11
			0-3-1	W1-9	1-9
1	3	9	1-3	W1-9	1-8
			1-0-3	NA	19
			1-4-3	B10-18	7-14
2	3	9	2-1-3	B11-19	11-15
			2-4-3	NA	16-19
			2-5-4-1-3	W11-19	11-15
3	1	7	3-1	W12-18	11-16
			3-0-1	NA	13-14
			3-4-1	B1-7	1-9
4	3	1	4-3	W1	NA
			4-1-3	B21	18
			4-2-1-0-3	NA	17
5	0	5	5-2-1-0	W2-6	4-8
			5-4-3-0	B3-7	1-5

Table 5.1 lists an optimal solution to a high bandwidth demand set with full protection ( $q = 1$ ). The index of maximum occupied subcarrier of MPP and SPP scheme are 19 and 21 respectively. Solution for this particular demand set demonstrates that, for full protection, when three paths are employed in MPP scheme, the total number of utilized subcarriers in MPP



scheme may be smaller than SPP scheme. For example, SPP scheme requires 18 subcarriers for demand from node 2 to node 3. While MPP scheme only needs 14 subcarriers in total on three paths. But when only two paths are used under full protection condition, MPP and SPP schemes have same spectral efficiency. In Section 2.1, we analyzed MPP scheme by only considering one demand, in which same amount of bandwidth are allocated to all candidate paths to minimize utilized spectrum. But in optimal solution of a given set of demand, the bandwidth may be allocated unevenly. For example, demand 1 to 3 only reserve one slot (19) on the second path and 8 slots on the other two paths. We also notice that in optimal solution the shortest path may not be utilized. For example, from node 4 to 3, in MPP scheme, the path 4-3 is not utilized because link between 4 and 3 is relatively busier than links in path 4-2-1-0-3. Sometimes, the optimal solution may reserve more subcarriers than required. For demand from 3 to 1, MPP scheme reserves 6 and 9 subcarriers for the first and third path respectively. But only 5 subcarriers are needed for each path when 2 subcarriers are allocated for the second path. That is because our ILP formulation tries to minimize the index of maximum occupied subcarrier. If reserving more subcarriers than necessary doesn't increase the maximum occupied subcarrier, the optimal solution provided by the solver may reserve more than enough. But extra reservation can be removed in linear time by checking each demand.

### 5.1.2 Comparison of Different Protection Levels

As shown in Fig. 5.2, under different demands' requested bandwidth, MPP demonstrates higher spectra efficiency than SPP with all 0.5, 0.75 and 1 protection levels. Also, the performance gap between MPP and SPP are bigger when protection level is low. When  $q = 0.5$ , SPP scheme requires  $B$  subcarriers for working path and  $0.5B$  for backup path. But if two paths are utilized in MPP scheme, only  $0.5B$  for each path is enough. So MPP is able to save  $0.5B$  in total by using same number of paths as SPP. But when protection level increases, the difference between MPP and SPP gets smaller. For example, when  $q = 1$ , if only two paths are employed in MPP, each path needs  $B$  subcarriers, which is same as SPP. To reduce occupancy of subcarriers, if possible more than two paths may be utilized in MPP. When more paths are used, more guard subcarriers are required for OFDM network. Also, normally paths of

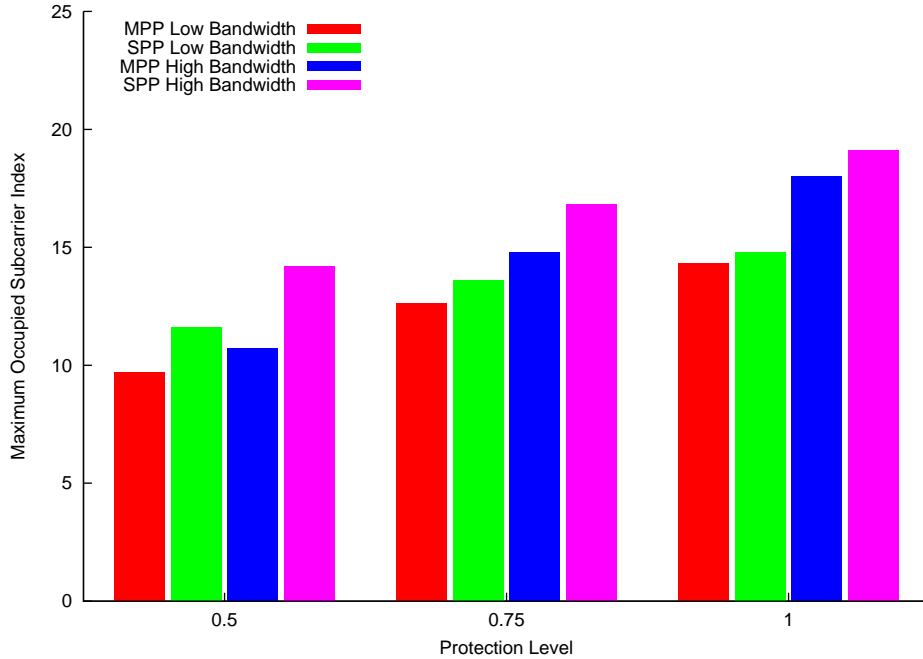


Figure 5.2 Maximum occupied subcarrier index of SPP and MPP schemes for ILP formulation

a demand have different number of links, which means longer path occupies more subcarriers than shorter path. With these two overheads, when protection level is high, performance gap between MPP and SPP are smaller.

### 5.1.3 Effect of Different Demand's Bandwidth

The index of maximum occupied subcarrier is larger for high bandwidth than low bandwidth demand set (Fig. 5.2). In our demand sets, the total number of requested subcarriers are same (40 subcarriers) for both high and low bandwidth. So high bandwidth demand set has less number of demands than low bandwidth set. That means to accommodate high bandwidth demand sets, some links in network are much busier than others. Normally, the index of maximum occupied subcarrier is limited by these links with heavier loads. But for low bandwidth demand set, subcarriers requirement are more evenly distributed. Thus, with the same total requested subcarriers for both demand sets, high bandwidth demand set requires larger maximum occupied subcarrier index.

In the figure, we are also able to tell the differences between maximum occupied subcarrier

index of SPP and MPP are smaller for low bandwidth demand set. In a low bandwidth demand set, each demand requests 1 to 5 subcarriers. The guard subcarrier for each path is 1, which means guard subcarrier overhead is large compared with demand. We've demonstrated in Section 2.1, for MPP scheme, more paths are employed more capacity per path is saved. The relatively large guard subcarrier make MPP scheme prefer to utilize less path, which limit the advantages of MPP in spectral efficiency.

## 5.2 Comparison of Optimal and Heuristic Solutions

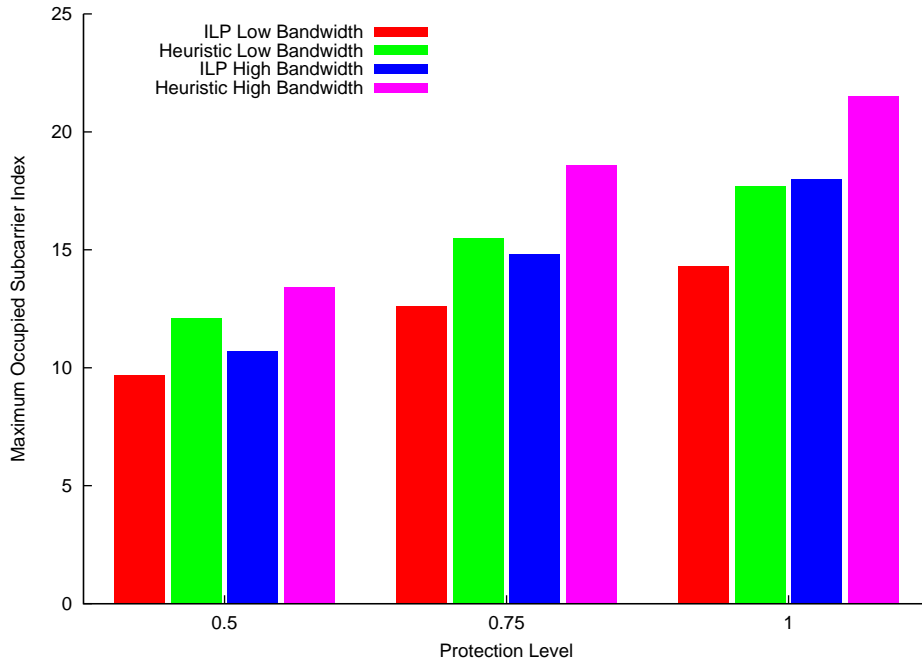


Figure 5.3 Maximum occupied subcarrier index of SPP and MPP schemes for ILP and heuristic algorithms.

We also compared maximum occupied subcarrier index of both low and high bandwidth request set between ILP optimal solution and our heuristic solution. As shown in Fig. 5.3, our heuristic algorithm requires around 20% to 25% more subcarriers than optimal ILP solution. But solving heuristic algorithm needs much less runtime for accommodating demand set than ILP formulation. We use CPLEX to solve ILP formulation described in Section 3, which takes up to 800 seconds. By using heuristic algorithm, only 0.07 second is enough to provide heuristic

solution. Also, the small 6 nodes network topology we used in this result limits the possibility of path reconfiguration in heuristic algorithm. In each demand set at most one or two paths are able to be reconfigured. But for a larger network topology (e.g. US network in Fig. 5.4), paths are more easier to be reconfigured with lower index subcarriers. So in larger network topology, heuristic algorithm may be more effective.

### 5.3 Results of Heuristic Algorithm

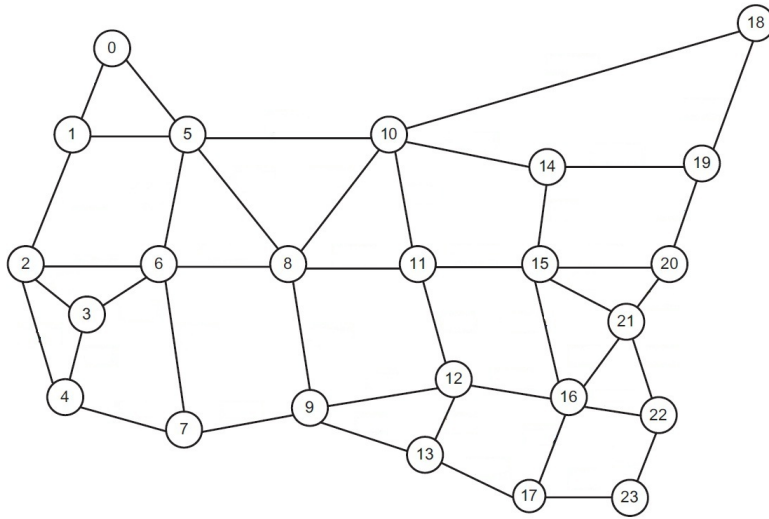


Figure 5.4 A sample US network topology

In this section, we present the simulation results to demonstrate the performance of our heuristic algorithm. We used a sample US network topology with 24 nodes and 43 links as shown in Fig. 5.4. We considered two demand sets representing low load and high load cases. In both cases, there is one demand for each ordered pair of nodes in the network, leading to a total of  $24 \times 23 = 552$  demands. In the low load demand set, the bandwidth requirement of a demand is a random number between 1 and 10 representing the number of subcarriers required. In the high load demand set, the bandwidth requirement of a demand is a random number between 1 and 40. For both demand sets, we tested three protection levels: 0.5, 0.75 and 1. Note that multipath provisioning offers at least 0.5 protection level due to the use of at least two paths.

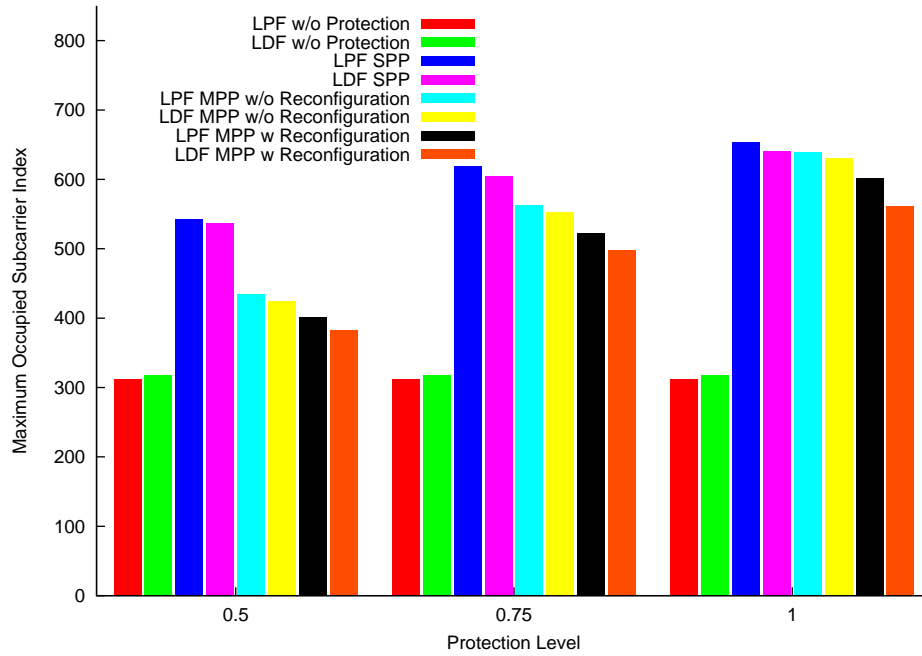
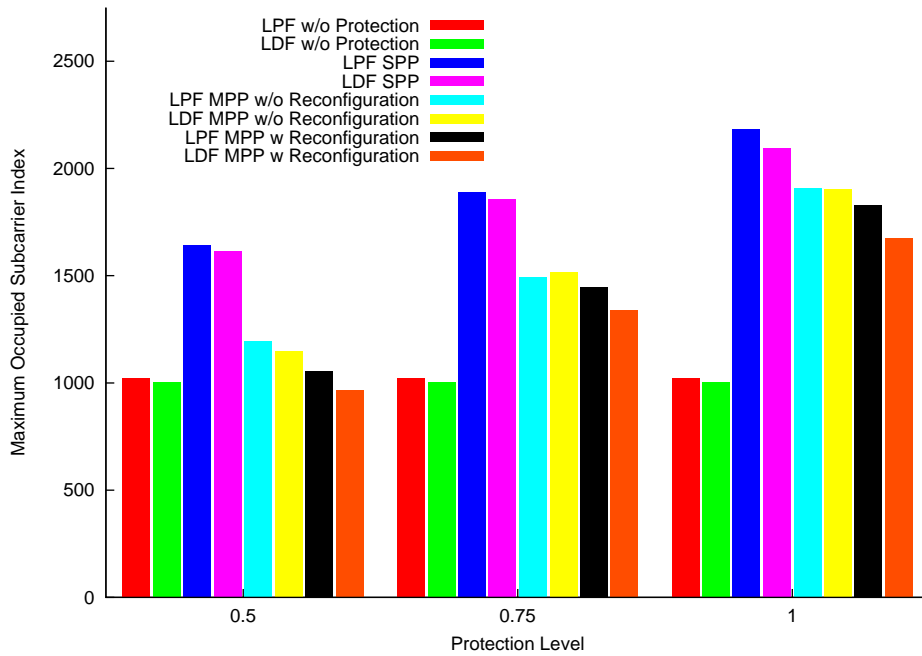
(a) Low load,  $G = 2$ (b) High load,  $G = 2$ 

Figure 5.5 Maximum occupied subcarrier index of SPP and MPP schemes for different protection levels in heuristic algorithm

### 5.3.1 Comparison between SPP and MPP

First, we compare the spectrum requirement of SPP and MPP schemes. For a request  $r = \langle s, d, B, q \rangle$ , SPP allocates a working path with capacity  $B$  and a backup path with capacity  $qB$  while MPP employs two or more paths and allocates working and backup capacity on each path. Fig. 5.5 shows the maximum occupied subcarrier index of SPP schemes and MPP schemes under different protection levels for low load case and high load case with  $G = 2$  (i.e., 2 guard subcarriers). The no protection case is also shown for reference. In the no protection case, each demand is provisioned on a single working path with required capacity. The figure shows that MPP requires less spectrum than SPP in all cases. Also, the performance gap between SPP and MPP is bigger when the protection level is lower. When  $q = 0.5$ , MPP without path reconfiguration achieves a spectrum saving of about 20% and 28% over SPP in the low load case and high load case respectively. For the high load case with  $q = 0.5$ , MPP without path reconfiguration only requires about 12% more spectrum than the no protection case. Note that when  $q = 0.5$  MPP requires no backup capacity due to the splitting of traffic over two paths. However, MPP still requires more spectrum than the no protection case for two reasons. First, only one path is allocated for a demand in the no protection case while two paths are allocated to a demand in MPP. As a result, MPP has higher overhead of guard subcarriers than the no protection case. Second, in MPP the second path used by a demand is generally longer than the shortest path. This means that MPP requires more subcarriers to satisfy a demand than the no protection case. When  $q$  increases, MPP tends to use more paths for a demand, so the guard subcarrier overhead and the total occupied subcarriers both increase. This explains why the performance gap between SPP and MPP becomes smaller when  $q$  becomes bigger.

### 5.3.2 Effectiveness of Path Reconfiguration

Our heuristic algorithm employs a path reconfiguration procedure after serving all demands one-by-one to reduce the utilized spectrum. In our simulation, out of 552 demands around 80 paths are able to be reconfigured. Fig. 5.5 shows the maximum occupied subcarrier index with

and without path reconfiguration under different protection levels. It can be seen that path reconfiguration is effective in reducing the utilized spectrum in all cases. Interestingly, in the high load  $q = 0.5$  case, LDF MPP with path configuration even requires less spectrum than the no protection case.

A side effect of path reconfiguration is that it increases the total number of occupied subcarriers. This can be explained as follows. The candidate paths for a given demand are computed using Bhandari’s algorithm, which finds the maximum number of link-disjoint paths with the least total cost. The new path computed in the reconfiguration step for a given path is based on the availability of contiguous subcarriers in the network links, so the new path contains more links than the old one and occupies more subcarriers. In Table. 5.2, we show the percentage decrease of maximum occupied subcarrier index and percentage increase of total occupied subcarriers after path reconfiguration. We see from the table that under all cases the percentage decrease of utilized spectrum is larger than the percentage increase of total occupied subcarriers. That is, the reduction in utilized spectrum is achieved with relatively small increase in total occupied subcarriers.

Table 5.2 The effect of path reconfiguration on maximum occupied subcarrier index and total occupied subcarriers

Method	Load	$q = 0.5$		$q = 0.75$		$q = 1$	
		Max <sup>1</sup>	Total <sup>2</sup>	Max	Total	Max	Total
LPF	low	7.8	3.3	6.9	2.6	9.9	3.1
LDF	low	9.9	3.2	9.9	3.1	11.0	3.1
LPF	high	11.9	4.5	3.1	1.8	4.2	2.3
LDF	high	16.1	4.0	11.7	2.9	12.0	2.8

<sup>1</sup> Percentage decrease of maximum occupied subcarrier index

<sup>2</sup> Percentage increase of total occupied subcarriers

### 5.3.3 Comparison between LPF and LDF

We consider two ordering strategies in our heuristic algorithm: longest path first (LPF) and largest demand first (LDF). From Fig. 5.5, we can see that LDF MPP always performs better than LPF MPP with or without path reconfiguration, except in the case of high load and  $q =$

0.75. Furthermore, path reconfiguration is more effective when LDF is used. This is supported by the data in Table. 5.2, which shows that in all cases LDF achieves higher percentage decrease of maximum occupied subcarrier index than LPF. This can be explained as follows. In the path reconfiguration procedure, we reconfigure paths in decreasing order of largest occupied subcarrier index. If we encounter a path that cannot be reconfigured, the maximum occupied subcarrier index cannot be reduced anymore. In LDF, the requests with smaller demands are allocated later than the requests with higher demands in the sequential allocation phase. So in the path reconfiguration phase, the paths with fewer allocated subcarriers are at the beginning of the list. Since paths that require fewer subcarriers are easier to be reconfigured, LDF is able to reconfigure more paths than LPF.

### 5.3.4 Number of Paths used in MPP

Table 5.3 Number of link-disjoint paths used by demands in MPP when  $G = 2$

Number of Candidate Paths		2	3	4	5
Number of Demands		132	310	98	12
Load	$q$	Number of Demands			
low	0.5	552	0	0	0
low	0.75	539	13	0	0
low	1.0	529	23	0	0
high	0.5	552	0	0	0
high	0.75	430	122	0	0
high	1.0	408	144	0	0

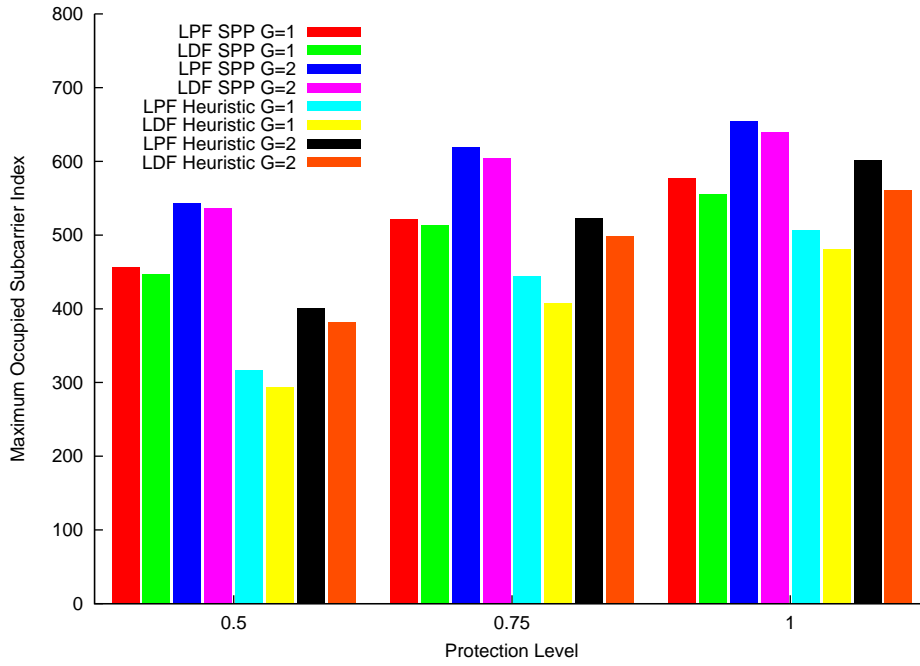
In our MPP scheme, we select a number of paths from the candidate path set by minimizing the total number of required subcarriers for a given demand. The top two rows of Table 5.3 show the number of demands with different candidate path set sizes. For example, there are 132 demands with 2 candidate paths and 12 demands with 5 candidate paths. The lower part of Table 5.3 shows the number of candidate paths used by demands in MPP under different loads and protection levels. We see from the table that all demands use only two paths when  $q = 0.5$ . Note that when  $q = 0.5$ , using two or more paths requires no backup capacity. Our scheme uses only two paths because more paths lead to more guard subcarrier overhead and



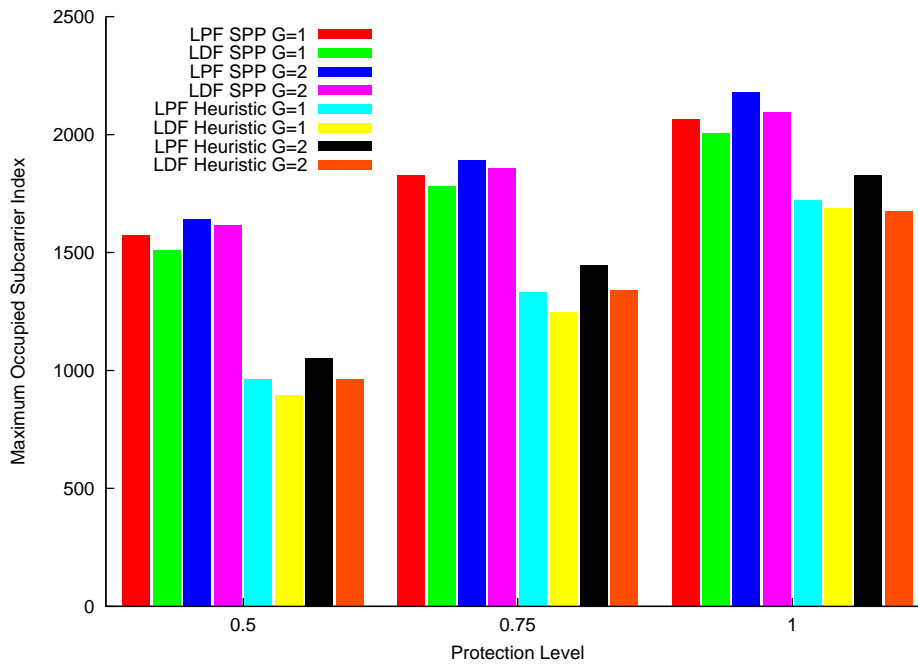
more occupied subcarriers. When  $q > 0.5$ , for some requests, employing more than two paths results in more backup capacity saving than the negative effects of more guard subcarriers and more occupied subcarriers. That is why some requests use three paths when  $q = 0.75$  and  $q = 1$ . No demands use more than three paths because the negative effects outweigh the saving in backup capacity. Also, we observe that for the same protection level, more demands use 3 paths in the high load case than in the low load case. This is because for the high load case, the overhead of 2 guard subcarriers is relatively small compared to the requested bandwidth of demands.

### 5.3.5 Effect of Number of Guard Subcarriers

Fig. 5.6 shows the maximum occupied subcarrier index of SPP schemes and our heuristics when  $G$  equals 1 and 2 for different protection levels. (Path configuration is used in the heuristics.) We see that our heuristic outperforms the SPP scheme in all cases and using 2 guard subcarriers leads to more utilized spectrum than using 1 guard subcarrier for any fixed scheme. Also, the difference in utilized spectrum between using one and two guard subcarriers is larger in the low load case than in the high load case. For example, in the low load  $q = 0.5$  case, the LDF heuristic requires 29.93% more spectrum when the number of guard subcarriers increases from 1 to 2; on the other hand, the increase is only 7.83% in the high load  $q = 0.5$  case for the LDF heuristic. The low load case is more sensitive to the increase in the number of guard subcarriers because the overhead of guard subcarriers is relatively high compared to the requested bandwidth of demands.



(a) Low load



(b) High load

Figure 5.6 Maximum occupied subcarrier index of SPP and Heuristic schemes using different number of guard subcarriers

## CHAPTER 6. CONCLUSION

To efficiently supports connections with bandwidth and protection requirements, we described a survivable multipath provisioning scheme for OFDM-based flexible optical networks. We also introduce the static Survivable Multipath Routing and Spectrum Allocation (SM-RSA) problem and show the problem is NP-hard. Then ILP formulation and heuristic algorithm for this problem were developed. To compare spectral efficiency of MPP and SPP schemes, we develop ILP formulation for the static SS-RSA problem and obtain optimal solutions for static SM-RSA and static SS-RSA problems on a small network. Simulation results of heuristic algorithm on US network were also presented. All our results demonstrated the higher spectral efficiency of MPP over SPP at different protection levels and network conditions.