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**Volume-Change Restraining Effects in Continuous  
Precast/Prestressed Bridge Girders**

By

Athul Abraham Alex

A Thesis

Presented to the faculty of

The Graduate College at the University of Nebraska

In partial Fulfillment of Requirements

For the Degree of Master of Science

Major: Civil Engineering

Under the Supervision of Professor Christopher Y. Tuan

Lincoln, Nebraska

May, 2016

# **Volume-Change Restraining Effects in Continuous Precast/Prestressed Bridge Girders**

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University of Nebraska, 2016

Advisor: Christopher Y. Tuan

A variety of design and construction practices are feasible when building precast concrete continuous bridges with long spans. Precast, prestressed concrete continuous bridges have been implemented by countries around the world. Although these bridges have been in service for many years, there has been limited verification of the ability of connection to provide the predicted continuity. Subsequently many states in the United States design the girders as simple spans for both dead and live loads without considering any moments developed by the connection. The effect of thermal expansion and contraction is hardly considered in the analysis, even though it is found to have significant effects on continuity.

The objective of this study is to evaluate the current state of the art practices relevant to continuous precast concrete bridges and to recommend the most suitable design methods of analyzing the continuity behavior. This research focuses on providing detailed analysis to evaluate the restraining effects in a continuous bridge system. Detailed analysis was performed using the specifications of the NU-girder system, which has been a widely adopted solution in the State of Nebraska.

This research consisted of two phases:

Phase 1: Conduct an extensive literature survey to find information regarding existing continuity behavior as investigated by various researchers.

Phase 2: Propose the most suitable method for analyzing connection design. Discuss advantages, construction time and cost comparisons of the NU-girder system with other systems adopted in the United States.

## **Acknowledgements**

First of all I would like to thank God for giving me this opportunity to fulfill my ambition of developing into a more knowledgeable engineer by completing my graduate studies. I would like to thank my advisors Professor Maher K. Tadros, Professor Christopher Y. Tuan and Dr. Amgad Girgis for their valuable technical input as well as Professor George Morcous for serving on my advisory committee

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# Chapter 1

## Introduction

### 1.1 Background information

Precast, prestressed concrete continuous bridges have been implemented by many countries around the world. One of the primary advantages of using continuity with prestressed concrete girders is the elimination of the maintenance cost associated with expansion joints as well as the deck drainage onto the substructure. Apart from enhancing the riding qualities of the bridge, continuity also helps in improving the aesthetics of the bridge. There is also significant reduction in mid-span bending moment and deflections. If the load capacity is exceeded for a particular girder, a continuous bridge will help redistribute the moments. This research focuses on composite bridge system in which the deck and girders are connected together so that the system strains and deflects as a single unit. Figure 1.1 depicts a continuity diaphragm with a cast-in-place deck.

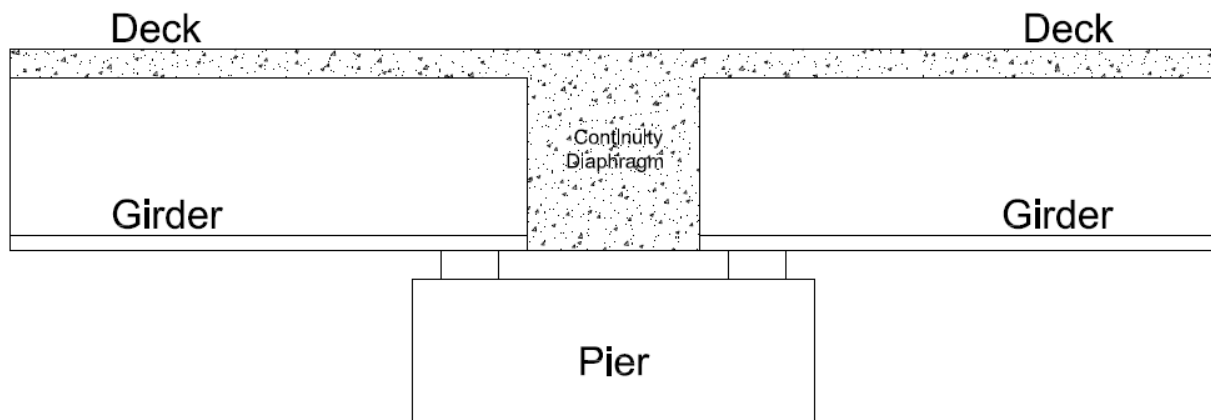
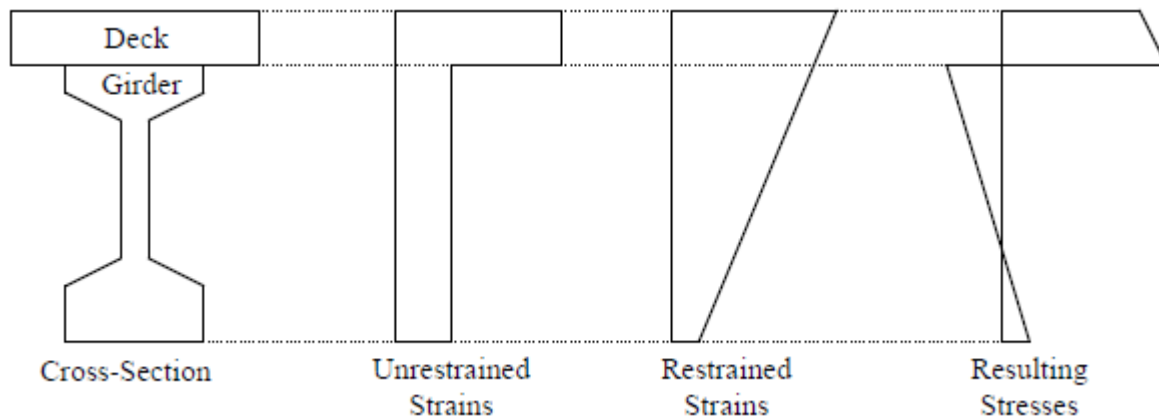


Figure 1.1 Illustration of Continuity Diaphragm

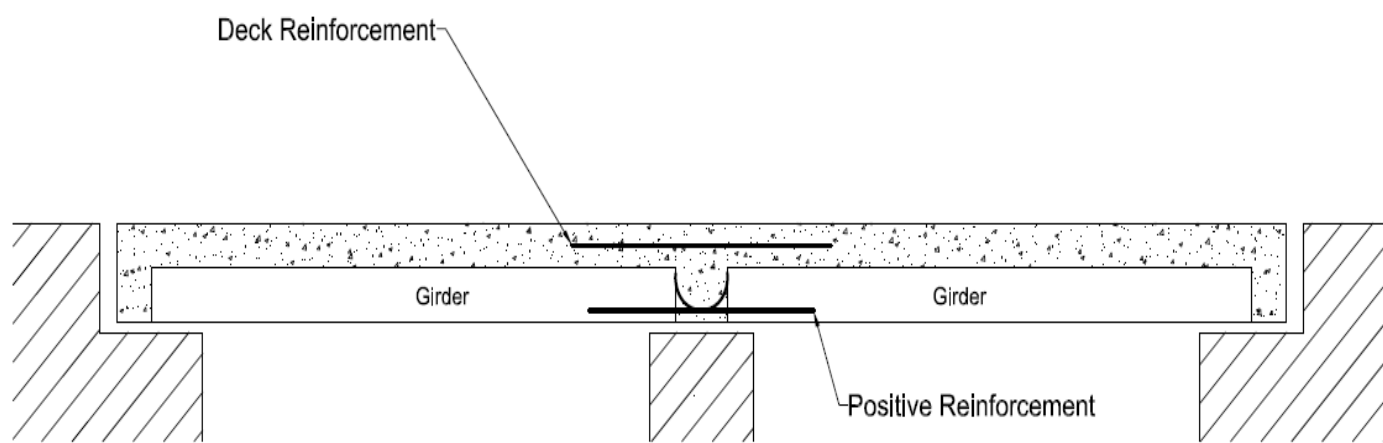
Continuity is established in two steps, the first is by placing the precast girders on abutments or piers and casting a composite deck. The next step is to pour concrete between the girder ends which upon hardening is referred to as the diaphragm. In this process continuity for live load is achieved. As for the girder and slab dead load, the girder behaves as a simple span as they are not connected until the deck hardens. Once the concrete deck and the diaphragm harden, they connect the girders together and make the entire structure continuous under all the additional dead and live loads. Compression develops in the top of the girder and tension develops at the bottom of the deck before the composite action becomes effective. Since the bridge is continuous, these forces will cause the development of restraint moments in the continuity diaphragm. These restraint moments help in nullifying the moments that would cause the ends of the girder to rotate if they were unrestrained. Figure 1.2 shows the stresses and strains developed in the composite cross-section.



**Figure 1.2 Stresses and Strains developed in the composite cross-section.**

Negative moment continuity is accomplished by placing reinforcements on the deck above the connection. Further studies and research showed that although using a reinforced deck served as an adequate connection to resist moments over the piers, cracks were developed in the

diaphragm due to the formation of positive moments. These positive moments are formed due to time dependent effects, mainly due to creep and shrinkage. The established continuity tends to keep the girders ends from rotating which results in positive restraining moments over the piers. This positive moment causes cracks to develop at the bottom of the diaphragms. These cracks not only impair bridge aesthetics, but also cause corrosion of the reinforcement in the diaphragms, leading to high maintenance cost. If no positive moment connection is supplied, the joint usually cracks and continuity may be lost. Positive connections are usually made either by extending the prestressing strand from the girder into the diaphragm or by embedding reinforcing bar from the end of the girder into the diaphragm.



**Figure 1.3 Continuous two-span precast bridge girder system**

Apart from having numerous advantages and being implemented by various States in the United States, there is no consensus on the best method to calculate restraining moments that develop in the continuity diaphragm or how to detail positive moment connections. This research is based on the AASHTO LRFD Bridge Design Specifications, for the analysis of the positive moments in the continuity diaphragm. Finite element analysis are also used to verify the results of the design which are then validated against test data.



**Figure 1.4** Continuous bridge with precast I-girder, courtesy D.H. Ordonez



**Figure 1.5** Continuous bridge with precast I-girder, courtesy D.H. Ordonez



**Figure 1.6 Severely Cracked Beam ends  
at continuity diaphragm**



**Figure 1.7 Cracking at the beam and  
Diaphragm intersection**

## **1.2 Problem Statement**

Most prestressed concrete slab-on girder bridges are simply supported with pretensioned girders and a cast-in-place deck. Generally the spans are limited to about 150 ft. due to the weight and length restrictions on transporting the precast girders from the precast plant to the bridge site. Although economical from an initial cost point of view, it becomes limiting when longer spans are needed.

Continuity connections have their own cost, construction and maintenance drawbacks as continuity is achieved by extending and bending reinforcements into the diaphragm, it creates congestion thereby making the construction labor intensive, time consuming and expensive. Many details called for several bars or strands extending from girder ends to be meshed into the diaphragm area thereby placing a large number of strands in a small space without adequate clearance between the bars. Questions were raised as to whether this congestion would limit the



capacity of the connection due to bar interactions and the inability to consolidate the concrete in the diaphragm.

In a National Cooperative Highway Research Program (NCHRP) study, (Oesterle et al. 1989) concluded that the positive moment connection provided no structural benefit as the positive connection restrains the girder ends, creating restraining moments in addition to the live load moments. They also pointed out that the positive moment in the span was virtually the same whether it was designed as a simple or a continuous span with both live load and restraining moments. There is a lot of discrepancy as to which method should be used to calculate the restraining moments developed due to the time- dependent effects of creep and shrinkage. Most States do not consider the effects of a temperature gradient which can create substantial moments at the piers. This study proposes the most viable continuity details for continuous precast concrete bridge girders and standard design procedures for this type of long span bridges in the United States.

### **1.3 Research Objective and Scope**

The main objective of this research is to provide detailed analysis in order to evaluate the restraining effects caused in a continuous bridge system, using the NU-girder system developed in the State of Nebraska as a design example. This NU-girder system achieves continuity by extending 8 strands into the diaphragm, bending them at 6 in. from end face of the girder and bent up at least 18 in. The strands are embedded in a 24 in. wide cast-in-place diaphragm. From the centerline of the pier, the diaphragm width is 12 inches with girder embedded into it for about 8 in. The 8 in.-gap between the girder ends is filled with cast-in-place concrete.

The Age Adjusted Effective Modulus method was used to calculate the restraining moments caused by creep and shrinkage. Since thermal analysis are often overlooked in detailing for continuity, this study considers the thermal effects on the continuity behavior and on the connection design. The thermal effects are calculated using the Initial strain theory and the AASHTO-LRFD specifications. A two-span continuous bridge system is evaluated using the NU-girder system as an example.

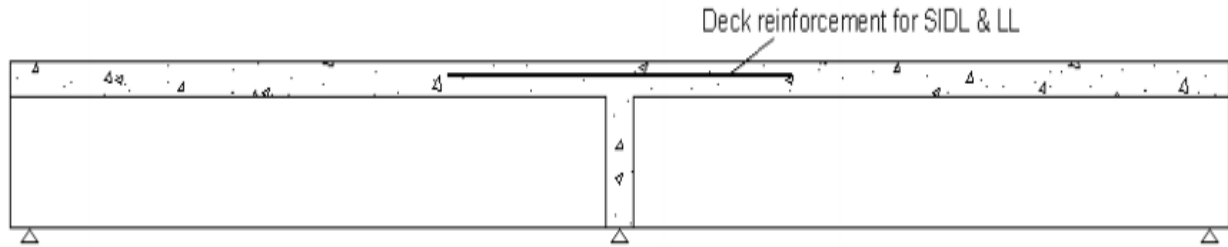
The literature review provided us valuable information about the continuity behavior studied by various researchers over the years. This study also compares the advantages and disadvantages of the various methods adopted by different States in the United States to achieve continuity. A cost comparison is also presented and correlated with the proposed NU-girder sections. The findings from this study may be included in the AASHTO-LRFD Specifications as an optional method of analysis. The findings also suggest that the NU-girder system is a durable and efficient bridge system with optimum continuity behavior.

### **1.3.1 Types of continuity systems**

A wide variety of designs for achieving continuity have been developed over the years. A few of them are listed below:

#### **1.3.1.1 Conventional Deck reinforcement**

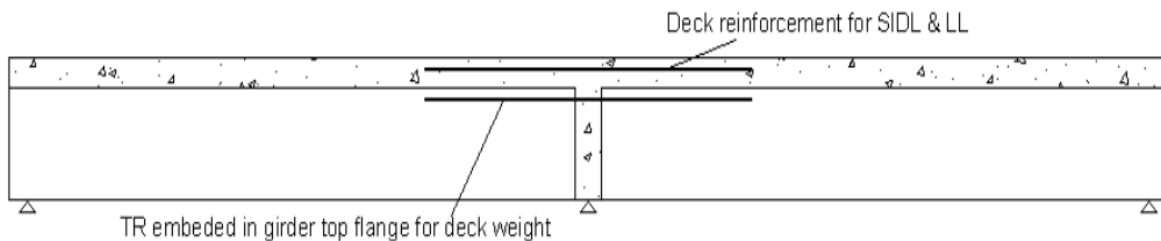
The conventional design used deformed reinforcement in the cast-in-place deck slab over the girders to provide continuity design for resisting live loads (Kaar et al. 1960). The connection detail had deformed rebar in the deck slab which were made continuous over the supports and casting a diaphragm over the piers extending laterally between the girders on either side.



**Figure 1.8 Continuity system using conventional deck reinforcement**

### **1.3.1.2 Threaded Rod Continuity System**

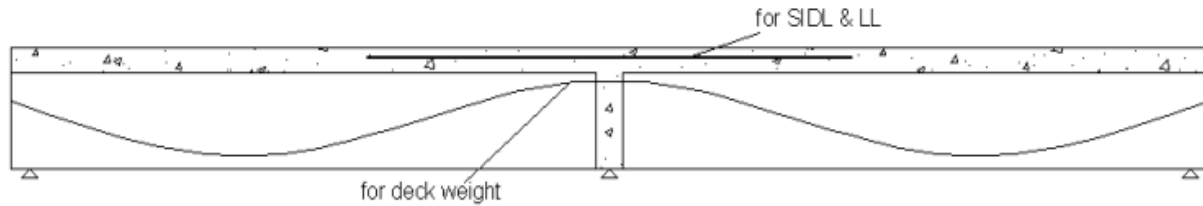
Tadros et al. (1998) at the University of Nebraska-Lincoln developed the threaded continuity system for the Nebraska Department of Roads (NDOR). Continuity was achieved by first embedding high strength threaded rods in girder ends followed by coupling the girder over piers. The diaphragm is then cast and the deck is placed with continuity deck reinforcement in it.



**Figure 1.9 Continuity system using threaded rod system**

### **1.3.1.3 Post-tensioning continuity system**

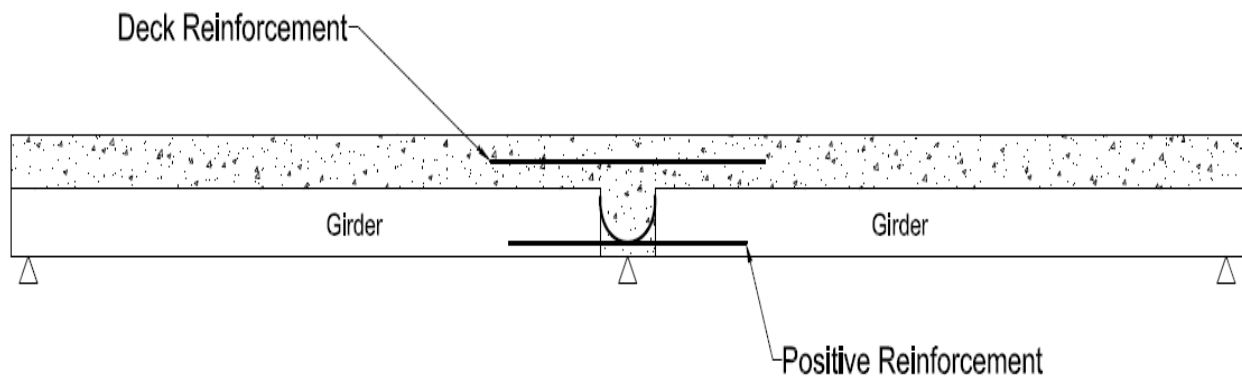
A new girder system was developed by Ficene et al. (1993). The girder segments were made continuous by splicing, coupling, and post-tensioning strand extensions at the adjacent ends of the girder segments.



**Figure 1.10 Continuity system using post tensioning**

#### **1.3.1.4 Positive Moment Connections**

The NCHRP Report 322 (Oesterle et al. 1989) presented a new continuity system. Prestressed bridge girders were made continuous by extending prestressed strands or embedding bent bars into the diaphragm and then casting the deck with conventional reinforcement. The deck and the diaphragm are cast together and form a continuous girder for live loads and time-dependent effects.



**Figure 1.11 Continuity system using positive moment reinforcements**

The NCHRP Report 519 (Miller et al. 2004) presented a research on the connection of simple span precast concrete girders for continuity. Continuity was achieved by providing positive moment connections between the bottom of the girders and the diaphragms. This was done by either extending or bending the bars or strands from each ends of the girder into the diaphragm.



**Figure 1.12 Continuity system using bent- bar & strand positive moment connection**

Newhouse et al. (2005) at the Virginia Polytechnic and State University developed a continuity system using positive moment reinforcements. The connection developed consisted U-bars bent into a 180 degree hook extending out form the face of the girders.



**Figure 1.13 Continuity system using U-bars bent at 180°**

## 1.4 RESEARCH ORGANIZATION

This thesis is organized as follows:

- Chapter 1 provides all the background information of continuity behavior of bridge system, problem statement and research objectives.
- Chapter 2 summarizes a comprehensive literature review of various studies performed by researchers on precast/prestressed continuous bridges. The method of analysis used by the researchers to determine the restraining moments and the construction sequences are discussed herein.
- Chapter 3 discusses in detail the method used to calculate restraining moments. The Age Adjusted Effective modulus approach is used to determine the restraining moments due to time-dependent effects. Thermal analysis of the bridge system is performed using the Initial strain theory and the AASHTO-LRFD Specifications. This methodology is also validated by analyzing a two- span continuous structure using a NU-girder section. The Finite element method is also utilized to analyze the indeterminate structure. A numerical design example is provided in the Appendix.
- Chapter 4 compares the cost of the proposed NU- girder system to other systems adopted by various States. The advantages and disadvantages of these systems are correlated with the proposed NU-girder system.
- Chapter 5 provides the conclusions and recommendations to be considered when analyzing and evaluating connection details for achieving continuity for long-span precast/prestressed girder bridges.

## Chapter 2

### Literature Review

#### 2.1 Background

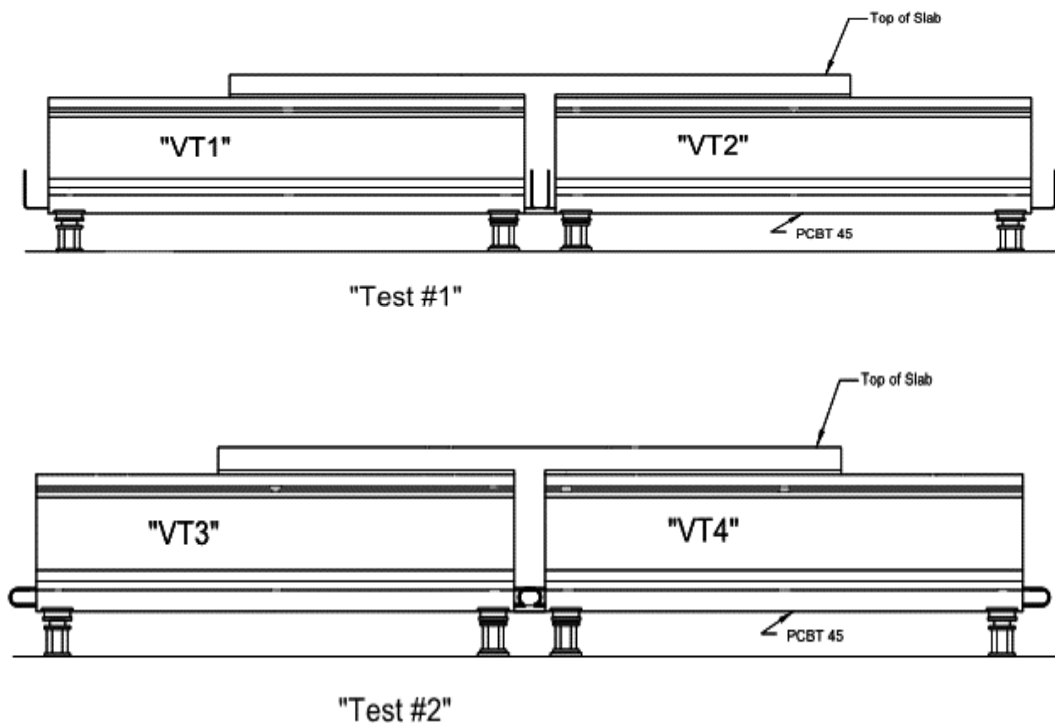
Approximately one-third of the bridges built in the United States are of the standard I-shape and bulb-tee precast concrete girder sections of lengths up to 160 ft. The use of precast, prestressed concrete girders has facilitated long-span bridge construction that can be efficiently transported and erected with minimal maintenance. Some of the earliest long-span continuous highway bridges were built in the United States in the early 1960's, including the Big Sandy River Bridge in Tennessee and the Los Penasquitos Bridge in California. These aesthetic bridges displayed excellent performance, and subsequently many states researched, designed and implemented their own continuous bridge systems.

Even though there is a consensus about the many advantages of the continuous prestressed concrete bridges, there are discrepancies in the methods used for the design of these systems and the associated reinforcement details. Detailed Analysis of the different methodologies for providing continuity is vital to construct economical precast, prestressed concrete bridges.

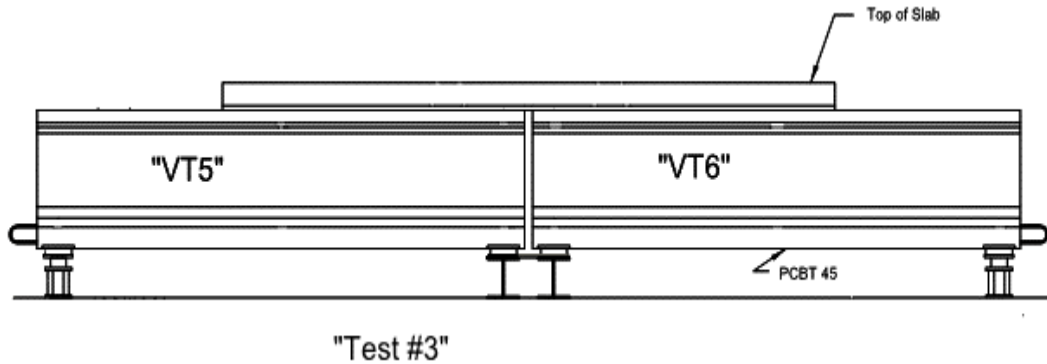
The current state-of-the-art practices for continuous bridges made of precast, prestressed concrete girders are reviewed herein. This study will focus on the benefits and drawbacks of various connections details to recommend the most suitable design methodology for the design of a continuous bridge system.

## 2.2 Previous Studies on the Continuity Behavior

**2.2.1 Newhouse et al. (2005)** Studies were carried out at the Virginia Polytechnic and State University on continuity connections over the bridge piers. This research focused on appropriate continuity details for the precast concrete bulb-tee (PBCT) girder sections. Three continuity details using PBCT-45 girder sections were developed and tested. The first two consisted of a full continuity diaphragm with a cast-in-place deck. Test #1 was carried out on specimens with prestressing strands extending out from the ends of the girder and bent to form a 90-degree hook. Test #2 involved specimens with #6 U bars bent into a 180-degree hook extending out from the bottom of the girder. Test #3 consisted of the slab only which was cast continuous over girders. Refer figure 2.1 for the details of the test specimens.







**Figure 2.1 Details for test specimens**

Five different methods were used to predict the restraining moments in a typical slab bridge system using the PCBT sections as follows:

1. **PCA Method** –The Portland Cement Association released in 1969 an engineering bulletin which was primarily based on the research by Alan Mattock (Mattock 1961). The engineering bulletin titled “Design of Continuous highway bridges with Precast, Prestressed Concrete Girders”, became the standard for continuous bridge design and is still used today by many designers. (Freyermuth 1969).

The PCA method determines the magnitude of any restraining moments that may develop over an interior support due to creep and differential shrinkage. The ratio of creep strain to the elastic strain,  $\phi$ , is determined for the girder. To obtain this value the specific creep value for loading at 28 days is obtained from a graph using the elastic modulus of the girder concrete at the time of loading, assuming that the ultimate creep occurs at 20 years. This specific creep value is then adjusted for the age when the loading actually takes place. For a prestressed girder, this is the age of release of the strands usually 1 or 2 days. The value is also adjusted for the actual volume to surface area ratio

of the girder. The amount of creep that has taken place is determined by entering a graph with the age that the continuity connection is made and determining the proportion of creep that has taken place. Mattock (1961) suggested that the uniform differential shrinkage moment in a composite concrete section at any time is given by:

$$M_s = \varepsilon_s E_b A_b (e'_2 + t/2) \quad (2.1)$$

where:

$\varepsilon_s$  = Differential shrinkage

$E_b$  = Modulus of elasticity for concrete in the deck

$A_b$  = Area of the deck

$e'_2$  = Distance from the centroid of the composite section to the bottom of the deck

$t$  = deck thickness

the restraining moment at the center support of a two span continuous bridge is calculated as:

$$M_r = \left(\frac{3}{2} M_p - M_d\right) (1 - e^{-\phi}) - \frac{3}{2} M_s \left(\frac{1 - e^{-\phi}}{\phi}\right) \quad (2.2)$$

where:

$M_p$  = Moment caused by prestressing force about centroid of the composite member

$M_d$  = Midspan moment due to dead load

$M_s$  = Moment caused due to differential shrinkage between girder and deck concrete

$e$  = base of Naperian logarithm

$\phi$  = Creep coefficient, ratio of creep strain to elastic strain at time of investigation

The most common method of determining the differential shrinkage is to use an ultimate shrinkage value of  $0.6 \times 10^{-3}$  for an exposure of 50 % relative humidity. This value is then adjusted for the actual relative humidity expected by applying a humidity correction factor. The adjusted ultimate shrinkage value is then multiplied by a factor accounting for the proportion of shrinkage that has taken place in the girder from the time the girder was cast to the time the deck was cast. This factor comes from the same graph used to determine the proportion of creep which has taken place. The PCA method assumes that the girder and the deck will have the same ultimate shrinkage values as well as similar creep coefficients. The influence of prestress losses was not accounted for directly. Instead, the final force after all losses is used in the calculation of the restraining moment due to prestress force.

Once the shrinkage restraining moment is determined for a given span, the basic or unadjusted restraining moments due to shrinkage, dead load creep and creep caused by the prestressing force can be determined. The moment distribution method is used to determine the resulting moments in multiple span situations. The equivalent simple span moments are applied to each span and the resulting restraining moments are determined using the moment distribution method. The resulting moments are then adjusted due to time-dependent effects and are used for the design of the diaphragm. Equation (2.2) shows that the total restraining moment at the pier is equal to the sum of three components of shrinkage, dead load creep, and creep due to prestress. In this equation,

the shrinkage moment and the moment caused by the eccentric prestressing force are multiplied by the factor  $\left(\frac{3}{2}\right)$ . This is the multiplier used to obtain the moment at center support of a two span continuous beam due to an applied uniform moment. The moment due to dead load on the other hand has a multiplier of 1. This is because the moment at the center support of a two span beam with uniform loading in both spans is equal to the midspan moment of a uniformly loaded simply supported beam of equal span length.

Loads that are applied at initial time and do not change, such as dead load and the prestress force, are multiplied by  $(1-e^{-\phi})$ . Loads that are initially zero but increase slowly over time, such as the differential shrinkage, are multiplied by the quantity  $(1-e^{-\phi})/\phi$ . The restraining moments due to dead loads, prestress force, and differential shrinkage are then summed up to determine the total restraint moment. For a typical structure, the dead load and shrinkage will cause a negative restraining moment to develop while the creep due to the prestressing force will cause a positive restraining moment to develop over the interior supports.

- 2. RM Calculation Method** - It is an algorithm developed by Michael McDonough of Entranco Inc. (McDonough 2001). The program determines restraining moments in a continuous girder system due to creep and shrinkage. To determine the restraining moments, an incremental time step solution is performed. The program uses ACI -209 (1982) creep and shrinkage models published in 1982. The influence of the reinforcing in the deck on the shrinkage of the deck is also considered. This program considers the

actual length of the continuity diaphragm in the direction of the span as a small interior span.

**3. Comparison method-1-** It was developed by Newhouse et al. (2005) and is a modified version of the PCA method. The ultimate creep and shrinkage values of the concrete for the girder and the deck were obtained separately. Final restraining moments are determined by multiplying the instantaneous restraining moments by the time dependent factors which include the influence of concrete ageing. An ageing coefficient  $X$  is considered. For loads applied instantaneously, such as the initial prestressing force and the dead loads, the moments are multiplied by  $\phi / (1+X\phi)$ . For moments applied slowly over time, such as the shrinkage restraining moments and prestress losses, the moments are multiplied by  $1 / (1+X\phi)$ , where  $\phi$  is the creep coefficient.

**4. Comparison method 2 -** This method was also developed by Newhouse et al. (2005) and is based on the CEB-FIB, Model Code for concrete structures which predicts the time effects of temperature, shrinkage and creep. The code was intended for concrete having compressive strength ranging from 1.74 ksi. to 11.6 ksi. This method uses the mean compressive strength  $f_{cm}$  which is calculated as follows:

$$f_{cm} = f_{ck} + 1.16 \text{ ksi} \quad (2.3)$$

This method is based on a design example presented by Ghali and Favre (Ghali et al., 1994) where a flexibility-based approach is used for the moment distribution. The change in rotation over a restrained joint,  $\Delta D$ , is first determined with the restraint removed. If the load is slowly applied, then the change in rotation is determined using the age-adjusted modulus of elasticity:

$$E_{\text{adj}} = \frac{E}{1+X\phi} \quad (2.4)$$

where  $E$  is the modulus of Elasticity at 28 days,  $X$  is the aging coefficient and  $\phi$  is the creep coefficient. An age-adjusted flexibility coefficient,  $f$ , is then determined for all the loads.

$$f = \frac{1}{E_{\text{adj}}} \left( \frac{l}{a} + \frac{l}{b} \right) \quad (2.5)$$

where  $l$  is the span length,  $I$  is the moment of inertia of the system and  $\mathbf{a}$  and  $\mathbf{b}$  are coefficients depending on the geometry of the continuous system. The ultimate restraint moment  $\Delta F$ , is determined by:

$$\Delta F = \frac{\Delta D}{f} \quad (2.6)$$

**5. Thermal Gradients** – AASHTO –LRFD specifications is used to find the suitable thermal gradient. The structure is first made determinate by removing a sufficient number of internal redundancies. After the internal redundancies are removed, the self-equilibrating stresses are determined. The redundancies are then reapplied, producing the continuity stresses. Assuming that the structure is totally restrained, the longitudinal stresses  $\sigma_t(Y)$  are determined at a distance  $Y$  from the center of gravity and are given

$$\sigma_t(Y) = E \alpha T(Y) \quad (2.7)$$

where  $E$  is the modulus of elasticity,  $\alpha$  is the coefficient of thermal expansion and  $T(Y)$  is the temperature at the given distance  $Y$  from the center of the gravity of the system. The restraining axial force  $P$  is determined by integrating over the depth of the structure.

$$P = \int \sigma_t(Y)b(Y)dY \quad (2.8)$$

$b(Y)$  is the section width at location  $Y$ . The restraining moment is determined by integrating the product of the stress, the width, and the distance from the centroid over the height of the structure.

$$M = \int \sigma_t(Y)b(Y)YdY \quad (2.9)$$

The self-equilibrating stresses,  $\sigma(Y)$  is given by

$$\sigma(Y) = \sigma_t(Y) - \frac{P}{A} - \frac{M}{I} \quad (2.10)$$

where  $A$  is the area of the section and  $I$  is the moment of inertia of the section.

Any redundancies that were removed to make the structure determinate are then reapplied. The self-equilibrating stresses  $\sigma(Y)$ , or self-equilibrating forces ( $P$  and  $M$ ) are then redistributed to produce the continuity stresses and forces.

From the results of the experimental tests, some of the advantages of this continuity system developed by Newhouse et al. (2005) are:

- The connection was able to transfer service loads effectively and the bent bars were designed for maximum factored service loads.
- The diaphragms were designed for thermal restraining moments
- Continuity diaphragm was cast flush with girder ends. No embedment of girders in the diaphragm

Several disadvantages of this continuity detail are listed below:

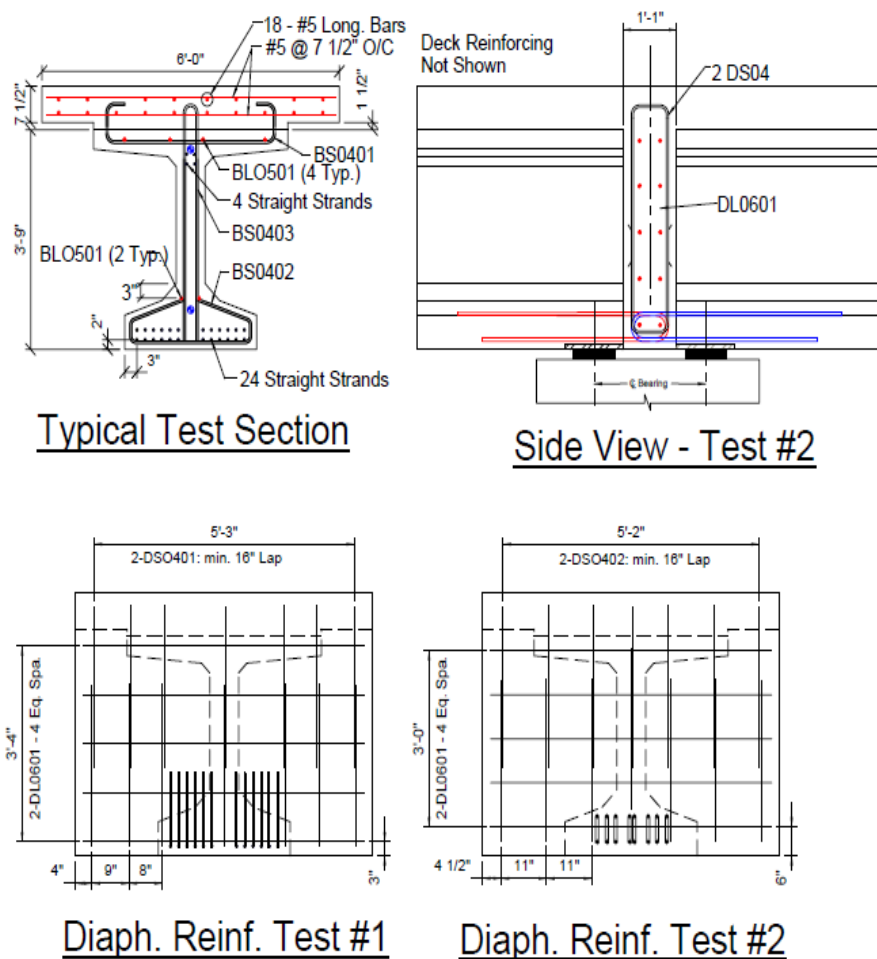
- Initial cracking occurred at tensile stress lower than the modulus of rupture of concrete at the girder-diaphragm interface.
- Cracking was expected at the girder-diaphragm interface. Interface edges were required to be sealed during the initial construction phase.
- The girders should be stored for 90 days before continuity was established. There is significant increase in the initial cost of the construction detail.

The major findings of this research are as follows:

1. Bent bar connection was efficient compared to the extended prestressing strands bent at 90 degrees in the diaphragm with regard to cracks developing under service loads. Cracking at the girder-diaphragm interface could be controlled by providing additional reinforcements.
2. The predicted positive moments due to thermal restraint can be significant for common girder spacing and span lengths when compared with actual cracking moment capacity of the section at the continuity diaphragm. This moment was found to be in the range of 0.7 -1.3 times the cracking moment capacity.
3. When compared to the most commonly used current methods, the PCA method generally gives the most conservative positive restraining moments due to time dependent effects, such as creep and shrinkage.
4. For typically used span and strand arrangements, as the span length decreases, the positive restraining moment due to creep and shrinkage generally also decreases.



5. At early ages of continuity, when concrete is less than 15 days old, it was predicted that the positive restraining moments due to creep and shrinkage are greater than 1.2 times the cracking moment capacity. For ages of continuity greater than approximately 90 days, the most current methods predict that no positive restraining moment will develop due to the time dependent effects.



**Figure 2.2 Reinforcing details for test specimens**

**2.2.2 Miller et al. (2004)** presented research findings in the NCHRP report 519 (Miller et al., 2004). A literature survey was conducted on the continuity details commonly used by the different States in the U.S. The survey helped to identify the types of negative and positive

moment connections at the supports, the age at which continuity was established, design techniques, construction sequence and the associated issues. The objectives of the research were to determine what connections can be used for continuous live loads, to develop design methods and to propose changes to the AASHTO-LRFD specifications.

In the first phase, six positive moment connection details were selected and subjected to testing. The connection details included:

- Extended mild steel bars
- Extended prestressing strands
- Extended bar with girder ends embedded in the diaphragm
- Extended strand with girder ends embedded in the diaphragm
- Extended bars with girder ends embedded in the diaphragm with additional stirrups near the bottom of the girder.
- Extended strand with girder ends embedded in the diaphragm with horizontal bars placed through the web of the girder.

These details were tested using short 16 ft. Type II AASHTO girders, with a composite slab attached to a diaphragm. In a second phase, 50-ft.-long Type III AASHTO I-girders were assembled into two spans, 100-ft long each, continuous-for-live load specimens. The first specimen used a reinforced concrete deck for the negative moment connection and an extended bar for the positive moment connection. A part of the diaphragm was cast 28 days before the slab was cast. The specimen was monitored for 120 days and then loaded to test for continuity. A second 100-ft long, continuous-for-live load specimen was subsequently cast. This specimen used an extended strand connection. Similar to the first specimen, a post-tensioning system was

used to develop positive moment at the connection, and additional positive moment was induced by jacking up the ends of the specimen. This specimen was tested for negative moment capacity.

In order to perform analytical studies, a standard spreadsheet program called RESTRAINT was developed. This program helped in calculating the restraining moments. The program modeled a two-span continuous structure with supports at each end of the girder. The program used flexibility-based analysis by discretizing the span and the diaphragm into several elements. Prior to using the RESTRAINT program, moment curvature relationships are developed for the cross-section. For this study, the program RESPONSE was used to find the relationship to be used in the spreadsheet. Basic material properties as well as the time the diaphragm and deck are cast are used as input into the RESTRAINT spreadsheet. The program calculates the internal moments that would result from creep of the girder and the shrinkage of both the girder and the deck. It also accounts for the loss of prestressing force using the method given in the PCI Design Handbook (1999).

Once the internal moments are determined, the program adds the dead-load moments. The program then divides each span into 10 or more elements which can be defined by the user. After determining the curvature of each element from the moment-curvature relationship. The program performs a consistent deflection analysis. The center reactions are removed to make the system statically determinate.

Using the curvature, the deflection at the center supports can be found. The reactions required to remove the center support deflection are found. The other reactions are found from

equilibrium and are used to calculate the continuity moments. The continuity moments are then added to the primary moments caused due to dead and live loads and the entire analysis is repeated until the answer converges. This program was verified with the PCA method to be accurate.

Some of the main advantages of the continuity details developed in this study are listed below:

- Controlled cracking found in the diaphragm was due to positive moments. The structure was deemed safe even after cracking at the girder-diaphragm interface, but was at the expense of the elimination of continuity action.
- Ductility of the connection could be improved by providing additional stirrups in the diaphragm close to the outside edge of the bottom flange of the girder. These stirrups could replace some of the extended bent bars and minimize congestion.
- The bent strand connection was easy to fabricate and erect as the strands were flexible and easy to place.

Some of the disadvantages of this continuity system are as follows:

- Spalling of the diaphragm concrete was observed when the girder end was embedded in the diaphragm.
- Increasing the amount of positive moment reinforcement tends to increase the positive restraining moment, which should be accounted for in the design.

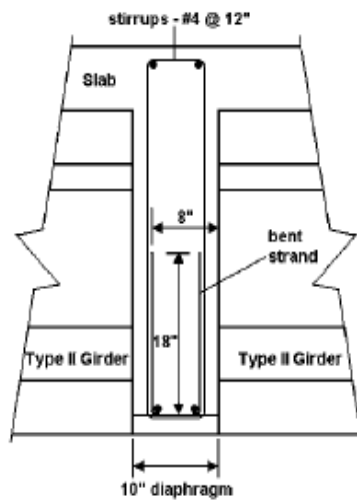
The main conclusions of the research are as follows:

1. The most significant conclusion of the study was that the end reactions varied to about  $\pm 20\%$  per day, depending on the daily temperature variations. The temperature effects on

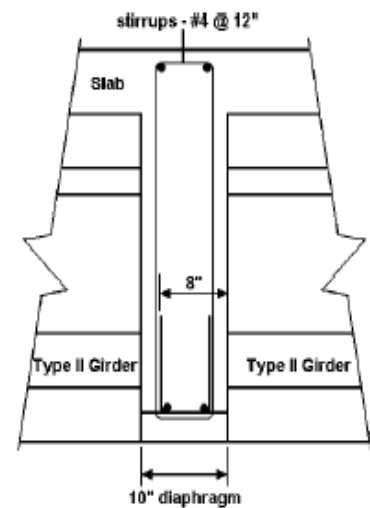
the system can be as significant as Live Load effects. However the thermal effects did not reduce the strength of the continuity connection in the laboratory tests.

2. All the details were designed for  $1.2 M_{cr}$ , which is the positive moment calculated using the non-transformed composite cross-section and concrete strength of the diaphragm. It was found that all the details achieved the design cracking moment and the last two details also achieved additional ductility.
3. Large positive moments are developed due to creep if continuity is established when girders have not aged. In the case the girders have aged, moments are caused by shrinkage. The maximum positive moment due to Live Load decreased by increasing the amount of positive reinforcement at the connection, which also reduced the cracking at the connections. However, by increasing the amount of positive moment reinforcement, the positive restraining moment would be increased.
4. The formation of negative moments, including the downward deflection of the girders is caused by the differential shrinkage between the deck slab and the girder. If the negative moment does not form, the models may underestimate the positive moment. If the negative moment is ignored, the models may unrealistically overestimate the positive moment.

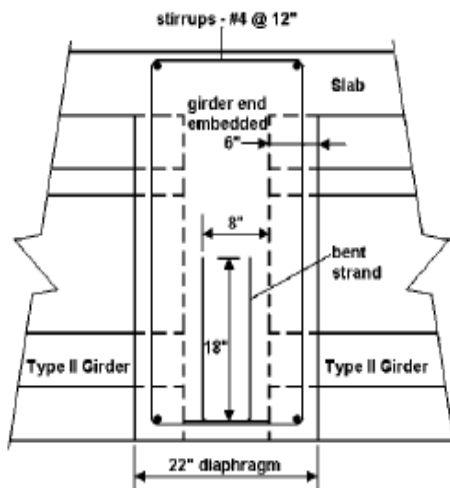
5. The negative cracking moment capacity is reduced if the positive moment cracking extends into the slab. Otherwise, the presence of positive moment cracking does not affect negative moment capacity of the connection.
6. The additional cost for providing continuity for live loads was about \$200 per girder.



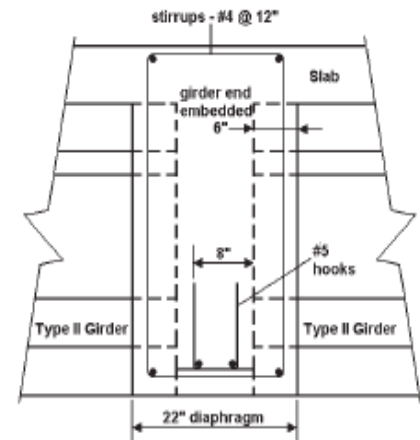
Specimen #1 - Bent Strand



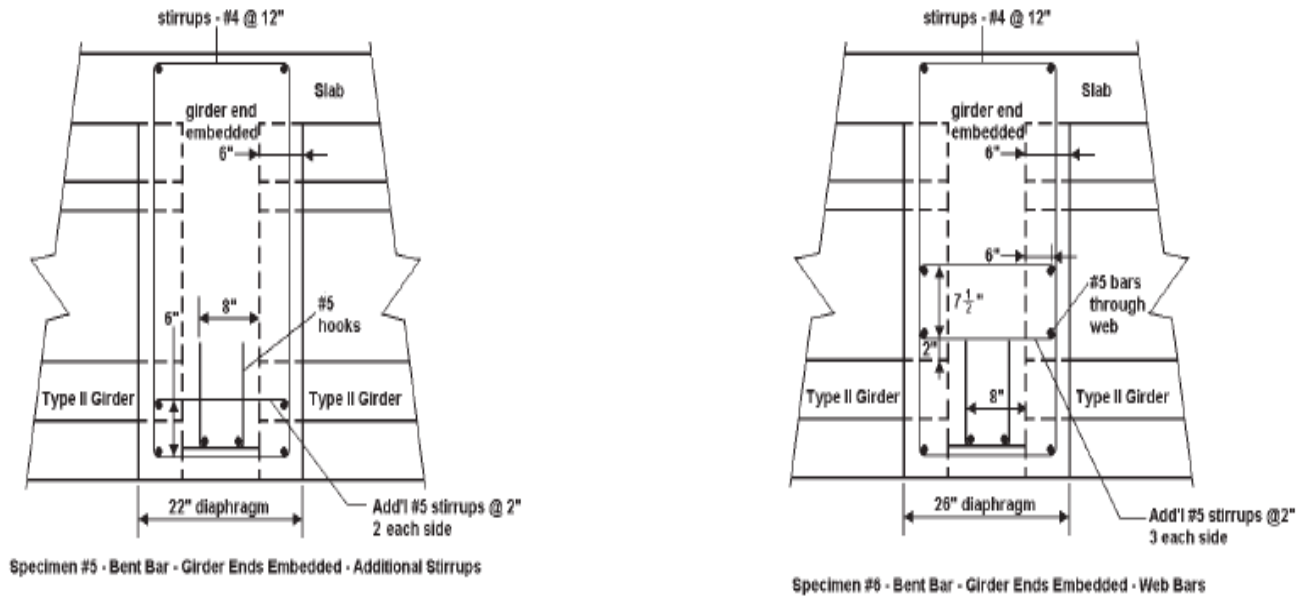
Specimen #2 - Bent Bar



Specimen #3 - Bent Strand - Girder Ends Embedded



Specimen #4 - Bent Bar - Girder Ends Embedded



**Figure 2.3 Details of the connection reinforcement.**

**2.2.3 Mirmiran et al. (2001)** performed an analytical study to understand the performance of continuity connections for precast, prestressed concrete girders with cast-in-place decks affected by positive moment reinforcement diaphragms.

A flexibility based analytical tool was developed to predict the time-dependent restraining moment and the effectiveness of the continuity connection under service loads. The model considers the different nonlinear stress-strain responses of the continuity diaphragm and the girder/deck composite sections, as well as the change in the stiffness of the structure under time dependent effects. This tool can also be used to evaluate how effectively the connection maintains continuity, should it crack under time-dependent effects.

The flexibility analysis comprises of two modules: a time-dependent analysis for a given time period, and a live load continuity analysis at any time after continuity is established. The program RESPONSE was used to calculate the moment-curvature relationships of the

girder/deck section and the continuity diaphragm section. RESPONSE develops the entire moment-curvature relationship of a prestressed or reinforced concrete section subject to moment, axial load and shear. The reinforcement in the positive moment connection was assumed to be fully effective across the diaphragm, and no tension-stiffening is included in the analysis.

The analysis is performed incrementally over a specified time period with each of the span and the diaphragm divided into a number of segments. The moments due to the differential shrinkage and creep effects of prestressing are assumed to be constant in the span and zero in the diaphragm, respectively. The moment due to the creep effect of dead load is parabolic in the span and zero in the diaphragm.

The flexibility analysis is carried out using the interior support reaction as redundant by making the structure determinate. The total moment at each section is then calculated by adding moments due to time dependent effects, the moment due to dead load of the composite section in the span and the moment due to the assumed redundant actions at the interior support. Once the total moment is established at each section, the program finds the corresponding curvature from the appropriate moment–curvature relationship. The total moment at each section is given by the following equation:

$$M_r = (M_p - M_d) (1 - e^{-\phi}) - M_s \left( \frac{1 - e^{-\phi}}{\phi} \right) \quad (2.11)$$

where:

$M_p$  = Moment caused by prestressing force about centroid of the composite member

$M_d$  = Midspan moment due to dead load



- $M_s$  = Moment caused due to differential shrinkage between girder and deck concrete
- $e$  = Natural logarithm
- $\phi$  = Creep coefficient, ratio of creep strain to elastic strain at time of investigation

In this study performed by Mirmiran et al.,(2001) both creep and shrinkage strain are estimated using the ACI 209 (1982) method, including correction factors for relative humidity, volume to surface ratio and age at loading for creep.

From the total moment and total curvature, the moment and curvature due to dead loads are subtracted to arrive the moments and curvatures due to time dependent effects. This procedure is carried out at different sections along the span and diaphragm to obtain the curvature diagram for each step. The curvature of any section can be calculated by the following equation

$$\Phi = \frac{M}{EI} \quad (2.12)$$

where:

$M$  = Bending moment

$I$  = Moment of Inertia

$E$  = Modulus of Elasticity

$\phi$  = Curvature

The deflection at the interior support is then calculated by the moment-area method using the curvature diagram. The program reiterates on the interior support reaction to eliminate the

deflection at the support. The moment thus obtained in the diaphragm section is the time-dependent restraint moment at the interior support.

This method was adopted in conjunction with the time-dependent analysis in order to determine the degree of continuity for live loads. At a specified age, live load is applied in the form of two equal point loads acting at the center of each span of a two span bridge. The live load is then normalized with respect to equivalent service loads.

The flexibility based analysis described in the previous section is also used herein, except that the live load moments and curvatures are now added to the dead load and time dependent moments and curvatures from the last step of the time dependent analysis. The live loads are applied incrementally, until the maximum curvature in a girder/deck section along the bridge or in the diaphragm section is exceeded, causing failure. The program calculates live load moments at interior supports and midspan, and compares them with the respective theoretical elastic moments based on a full continuity assumption. A continuity index, is defined as the ratio of live load moment at center support or midspan to the elastic moment at that location assuming full continuity. An index of one represents full continuity. The index is less than one at the interior support, and greater than one at the midspan. Degree of continuity is given by the following equation:

$$\text{D.O.C} = (M_s - M_r) / (M_s - M_c) \quad (2.13)$$

where:

D.O.C = Degree of continuity

$M_s$  = Midspan Moment assuming fully supported

$M_r$  = Midspan moment considering time dependent effects and concrete cracking

$M_c$  = Midspan moment assuming full continuity

To verify the accuracy of the proposed method, the results were first compared to linear elastic models such as the CTL method (Oesterle et al. 1989) by ignoring the difference in stiffness between the diaphragm and the girder/deck sections and the existence of cracking. The same uniform linear moment-curvature relationship was assumed for all sections. The method was subject to further validation using the PCA tests and was found to be satisfactory.

The following are the conclusions of the study:

- The age of the girder when continuity is established is a major factor that influences the time-dependent restraining moments and continuity for live loads. If girders are older than 90 days when continuity is established, the predominant effect is the differential shrinkage, which may prevent the development of positive restraining moment or uplift at the center support.
- When continuity is established early, at 7 days, the continuity diaphragm may crack if no positive moment reinforcement is provided. The cracks in the diaphragm can be limited by providing sufficient reinforcement, however, the reinforcement will in return develop higher positive restraining moments.
- The continuity behavior of bridges are generally better when continuity is established in the girder at 90 days as compared to an early age of less than 15 days. In such cases, the continuity behavior is also independent of the amount of positive moment reinforcement provided in the diaphragm.
- A minimum amount of positive moment reinforcement equivalent to  $1.2 M_{cr}$  is recommended to address durability and structural integrity. As the total midspan moments are independent of the amount of positive moment reinforcement, an

additional reinforcement above  $1.2 M_{cr}$  does not appreciably improve continuity for live loads.

- The cracking in the continuity diaphragms has been attributed to the thermal effects in some states. Therefore, the effect of thermal gradients on restraining moment should be considered.

**2.2.4 Tadros et al. (1998)** developed a threaded rod continuity system for precast concrete I-girders. The continuity detail used 1 in. high strength threaded bars of 92 and 150 ksi, embedded in the top flange of the girder and connected using steel block and nuts. The construction sequence for this continuity is explained as follows:

- The precast girders are fabricated with high strength threaded rods placed in the top flange of the girder as required by design and are connected in the field using two steel bars. The gap needed for the connection is 10 to 12 in.
- The girders are erected and then aligned.
- The threaded rods of the two adjacent girders are connected
- Form and place the concrete diaphragms to the underside of the beam's top flange.
- Conventional longitudinal reinforcement is placed in the deck.
- Place the deck concrete.

The longest span achieved using this system was 151 ft. on a four span unit 50-in. deep NU 1100 I-girders.

Approximate methods as well as rigorous methods are available to the designer to analyze the time dependent effects. These methods are essentially the same as conventional elastic analysis of a prestressed concrete cross-section, using transformed section properties. However, an age-adjusted effective modulus is used to replace the conventional modulus of elasticity for all the concrete elements. .

Initial strains, which are defined as strains not caused directly by an applied stress, are considered in a step-by-step method. The initial strains normally considered are:

- Free shrinkage of concrete occurring during the interval being considered.
- Creep strains of concrete, occurring during the interval being considered, that are due to the previously applied load.
- The apparent steel strain due to relaxation of prestressing steel during the time of interval being considered.

These initial strains were incorporated into cross-section analysis by using a fictitious restraining load to restrain the initial strain described above. The restraining load is then subtracted from any real loads applied to the section. Using the net load, an analysis is performed in a similar manner to conventional transformed section analysis. Finally, internal forces are calculated using two components. First, the internal forces associated with the net load applied to the entire composite section are calculated. These are then added to individual element restraining forces to give the total forces on an individual element of the cross-section.

A computer program CREEP3 was developed to execute the steps described above. In this program, the analysis time is divided into many intervals. The stresses and deformation at the end of each time interval were calculated in terms of the stress applied in the first interval and the stress increments that occurred in the preceding intervals. Linear creep growth is assumed

along with that plane cross –sections remain plane, the axial strain  $\varepsilon$  at any cross-section can be related to the axial force,  $N$ . During the interval  $i$ , an increment axial strain  $\Delta\varepsilon (i)$ , occurs:

$$\Delta\varepsilon (i) = \frac{\Delta N(i)}{AE_{ce}(i)} + \Delta\varepsilon' (i) \quad (2.14)$$

where

$E_{ce} (i)$  = effective modulus of elasticity of concrete at middle of interval  $i$

$\Delta\varepsilon (i)$  = initial strain in the  $i^{\text{th}}$  interval as defined by the equation provided below

$$\Delta\varepsilon' (i) = \sum_{j=1}^{i-1} \frac{\Delta N(j)}{AE_c(j)} \left( C(i + \frac{1}{2}, j) - C(i - \frac{1}{2}, j) \right) + \Delta\varepsilon_{sh} (i) \quad (2.15)$$

where:

$J$  = time interval

$(i - \frac{1}{2})$  = time interval at beginning of the  $i^{\text{th}}$  interval

$(i + \frac{1}{2})$  = time interval at end of the  $i^{\text{th}}$  interval  $i$

$\Delta N (j)$  = axial force increment at middle of  $j$  interval

$C$  = Creep coefficient

$\Delta\varepsilon_{sh} (i)$  = Free shrinkage strain during interval

$E_c (j)$  = Modulus of Elasticity of concrete at middle of interval  $j$

$A$  = Cross-sectional area of concrete

Some of the advantages and disadvantages of the system are as follows:

- NU-I girder had wide top and bottom flanges that improved strand capacity at both positive and negative moment locations. These girders facilitated shorter deck slab spans and served as better working platforms. The bulky bottom flange of the NU I-beam is found to have at least 1.5 times the required ultimate negative moment capacity.
- Girders were able to share some of the negative moment. Diaphragm bottom was precompressed to balance the tension at the top of the beam ends and it also mitigated the tension due to time dependent positive moments.
- The proposed connection details are relatively simple to construct without the need of any specialty contractors. However, one potential problem with this design is that the bulky steel hardware may aggravate the reinforcement congestion in the diaphragm.

Based on the analysis and experimental results the following conclusions were drawn:

1. Time dependent restraining positive moments, which develop after a rigid diaphragm is cast, may cause section to crack. It is not recommended that the diaphragm be cast earlier than 14 days of precast beam age but preferably at 28 days.
2. Placing some continuity reinforcement in the top flange of the I- beam not only increases the composite action between the deck slab and the precast I –beam, but also lengthens the span capacity by 20%. However, placing all the continuity reinforcement in the deck slab is not recommended.

3. When the diaphragm and deck are cast simultaneously, the girders would have to be designed as simple spans to allow the ends to rotate at the time of deck placement. This is done by casting the diaphragm with unbonded joints that will allow the girders to rotate freely while the deck concrete is placed. Continuity is achieved when rigid joints are formed over the piers as the deck is placed.
4. If a rigid diaphragm is cast ahead of the deck without the negative moment reinforcement, the girder-diaphragm joint may crack and spall due to the deck weight.

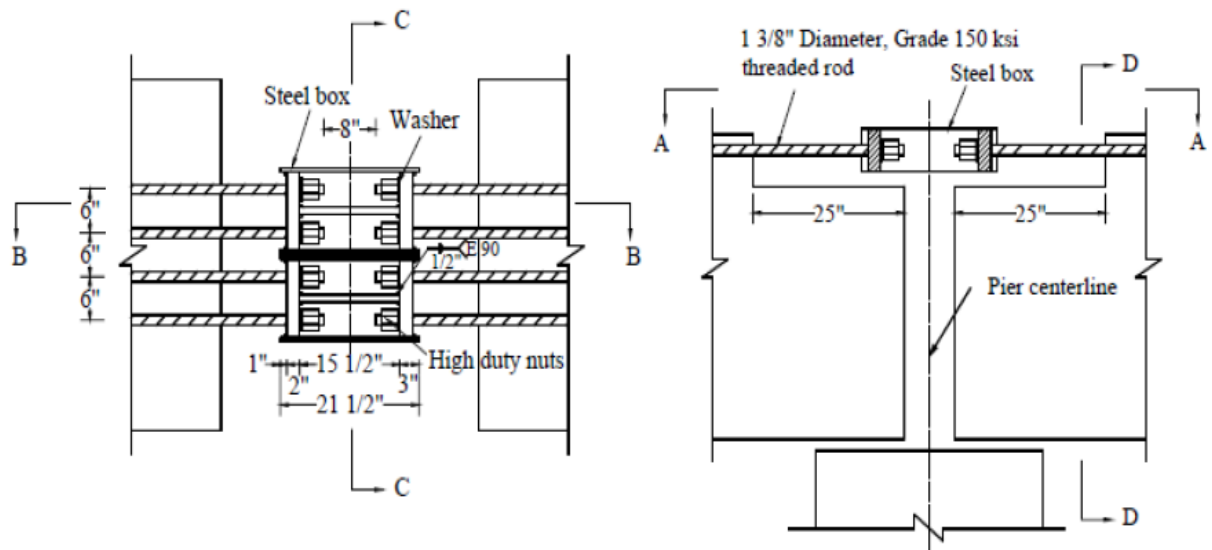


Figure 2.4 Details for high strength threaded rods

**2.2.5 Oesterle et al. (1989).** This study was conducted by the Construction Technology Laboratories in Illinois and was released in 1989 as the NCHRP Report 322 (Oesterle et al., 1989). The project's first task was to survey the current state of practice throughout the country. This was done by providing a questionnaire to provide information on typical bridge



configuration, material properties, and positive and negative moment reinforcement details for connection at piers, design procedures for connection details, bridge construction timing and sequence. Results from the questionnaire indicated that the respondents primarily used the PCA method for design of positive moment connection. The questionnaire also provided a listing of bridge performance-related problems as follows:

- Positive moment reinforcing requiring field adjustment
- Extended Strands accidentally cut off
- Cracking of the diaphragm due to long term creep and shrinkage
- Cracking and spalling of the continuity diaphragm when cast prior to deck
- Incorrect construction sequencing

An extensive literature review was conducted focusing on the following items: a) creep and shrinkage data for concrete; b) Data on camber; c) Mathematical formulations to predict creep and shrinkage; and d) Analytical techniques to account for the time dependent effects of creep, shrinkage, relaxation of strands, and construction sequence on the behavior of continuous prestressed girders. The literature review provided very little information regarding the prediction of creep and shrinkage, specifically for the steam cured concrete. Creep testing was done in accordance with ASTM C 512 and drying shrinkage was measured from a control cylinder stored in the same environment as the cylinders used for the creep tests. The creep strain was determined by measuring total strain of loaded cylinders and subtracting shrinkage strains. The creep coefficient was measured as the creep strain divided by the initial elastic strain. The results found were compared to the predicted results of ACI -209 (1982), "Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures". For creep coefficient the standard equation is

$$V_t = V_u \frac{t^\psi}{d + t^\psi} \quad (2.16)$$

where

$V_t$  = creep coefficient at time  $t$  days ( $V_t = 1.30$  to  $4.15$ )

$V_u$  = ultimate creep coefficient ( $V_t = 1.30$  to  $4.15$ )

$\psi, d$  = parameters defining the hyperbolic time function ( $\psi = 0.40$  to  $0.80$ ,  $d = 6$  to  $30$ )

For shrinkage strain the standard equation is

$$(\epsilon_s)_t = (\epsilon_s)_u \frac{t^\alpha}{f + t^\alpha} \quad (2.17)$$

where

$(\epsilon_s)_t$  = shrinkage strain at time  $t$  days

$(\epsilon_s)_u$  = ultimate shrinkage strain ( $(\epsilon_s)_u = 415$  to  $1,070$ )

$\alpha, f$  = parameters defining the hyperbolic time function ( $\alpha = 0.90$  to  $1.10$ ,  $f = 20$  to  $130$ )

For the later stages of concrete, more than 90 days, it was found that the actual creep and shrinkage values measured were within the ranges predicted by ACI-209 (1982). However, for early stages of concrete less than 15 days old, the actual shrinkage strains and the ultimate creep coefficients for all five specimens were greater than the ACI-209 (1982) recommended upper bound. The results were also compared to the simplified Bazant-Panula (Bazant 1975) prediction model. This model which is not intended to be used for concretes loaded earlier than seven days, resulted in large errors in the predicted values than the ACI-209 (1982) model. Therefore, the

ACI-209 (1982) model was used for the remainder of the NCHRP Report 322 (Oesterle et al., 1989).

Parametric studies were performed using existing computer programs and two newly created programs to determine the degree of continuity and the moments resulting from dead loads, live loads and time dependent effects. The computer programs used can analyze composite prestressed concrete structures of any cross-sectional shape with one axis of symmetry. The program accounts for the effects of non-linearity of stress-strain responses of materials and time-varying strength, stiffness, creep and shrinkage of concrete, and stress relaxation in steel. The program also allows flexibility in analyzing various construction sequence and live load applications. The primary factor used to evaluate and compare results was the response to live load applied at various stages of service life. Time-dependent support restraining moments and live load service moments at supports and midspan were also evaluated.

The program PBEAM (Suttikan 1978) was used to analyze the continuous structures. The program uses a step-by-step analysis method to account for the non-linear stress-strain response of the concrete. The ACI -209 (1982) model is used by PBEAM (Suttikan 1978) to estimate the time dependent factors such as strength, creep and shrinkage. The analysis accounts for construction sequence and the casting of the deck and diaphragm can be done at any girder age. The program also modelled crack development and was able to track whether a crack in the concrete was open or closed. With increasing live load and rotation at the diaphragm, the bottom crack closes and negative moment continuity becomes effective. The amount of rotation needed to close the crack is dependent on the creep and shrinkage properties of both the girder and the deck concrete, the age of the concrete at the time of live load, the girder type, span length and spacing. The degree of negative moment continuity is dependent upon all these parameters.

PBEAM (Suttikan 1978) correctly models the change in negative moment stiffness that accompanies closing of the diaphragm cracks, thereby providing an analytical tool to evaluate the effects of these parameters. To confirm the PBEAM (Suttikan 1978) analytical methods of predicting the time-dependent response of precast, prestressed bridges, results of computer analyses were compared to the PCA method and the results were almost identical.

A new program called BRIDGERM (Oesterle et al., 1989) was developed primarily to help determine the restraining moments that may develop in a continuous member as the PBEAM (Suttikan 1978) program was found to be complex. The program's restraining moment calculation method is based on the PCA method with some modifications. The program carries out an incremental time-step solution with the capability to output the complete time-history of the restraining moments rather than just one restraining moment at a particular age. The time dependent material properties for concrete are determined using the ACI-209 (1982) including separate shrinkage functions for the deck and girder concrete, and time dependent functions for the strength and stiffness of the deck concrete. Prestress losses are determined at each time step. The restraining effects of reinforcement on deck shrinkage are also considered. The analysis is carried out on a simplified model that considers the finite length of the support regions.

A second program WALL\_HINGE (Oesterle 1986) originally developed to analyze concrete shear walls was also used to model the behavior of the structure at the continuity connections at their failure loads. WALL\_HINGE (Oesterle 1986) considers the influence of strength and inelastic deformation capacity over the hinge region under combined loads. In conjunction with this program, another program BEAM BUSTER (Oesterle et al., 1989) was used to model the moment- curvature relationship of the system.

Yet, two other computer programs were developed to determine the service moments at supports of continuous bridges. The program BRIDGERM (Oesterle et al., 1989) is an improved version of the PCA procedure for calculating time-dependent restraining moments. BRIDGELL (Oesterle et al., 1989) was developed to calculate the live load moments in a continuous bridge under AASHTO HS loading.

The conclusions from the CTL study are as follows:

- Current practice for analysis, design, and construction of this type of bridge varies widely within the United States. Although most states use the PCA design procedure for their design of continuity connection, this procedure has many uncertainties regarding the construction timing and sequence.
- Continuity performance is highly dependent on the age of the girder at the time the diaphragm and deck are cast. When continuity is established at late girder ages of more than 90 days old, negative restraining moments occur at the support connections, preventing the diaphragm from cracking. However, by delaying the casting of deck and diaphragm may cause a delay in bridge construction. Casting the deck prior to diaphragms increases resultant positive moments at the midspan. It was concluded that simultaneous casting of the deck and the diaphragm is the simplest construction procedure.
- When continuity is established at early age of less than 15 days, time-dependent positive restraining moment generally induces a crack in the bottom of the diaphragm. When live load is applied, the positive moment cracks must close prior to inducing negative moment

at the continuity connection. The presence of positive reinforcement helps to maintain relatively small cracks, thereby increasing the live load continuity. The positive restraining moment resulting from the reinforcement in the support connection increases the positive mid-span resultant moment.

- When negative restraining moments develop, positive reinforcement is in the compression zone and thus offers no structural advantage. The resultant mid-span moments which includes moments caused by dead loads, restraining moment due to creep and shrinkage, and live load plus impact moments are virtually independent of the area of positive reinforcement in the diaphragm at the supports.

### **2.3 Summary of Literature Review**

Significant findings from the studies on the continuity behavior are summarized as follows:

1. The age of the girders at the time continuity is established was the most important factor on the behavior. If girders are more than 90 days old when continuity is established, the predominant effect is differential shrinkage which may prevent the development of positive restraining moment or uplift at the center support. If continuity is established at an earlier age of less than 15 days, the continuity diaphragm may crack if no positive reinforcement is provided due to the formation of positive restraining moment.

2. Temperature variations through-out the cross-section created a thermal gradient which created significant restraint moments. These effect should be considered when designing positive reinforcements
3. The negative cracking moment capacity gets reduced if the positive moment cracking extends into the slab. Otherwise the presence of a positive moment cracking does not affect negative moment capacity of the connection.
4. Many researchers recommended that the positive moment connection at the diaphragm has a maximum capacity of  $1.2 M_{cr}$ , where  $M_{cr}$  is the positive cracking moment.
5. The widely adopted PCA method overestimated the restraining moments and offers a conservative design approach.

## Chapter 3

### Analysis of Restraining Moments

#### 3.1 Time-Dependent Effects in Prestressed concrete

There is little or no change in the distribution of forces and moments in simple-span bridges from time dependent deformations. Multi-span bridges, made continuous for live loads and superimposed dead loads, become statically indeterminate after the deck is cured. As a result, any time-dependent deformations that occur after the deck is cured will induce forces and moments in the beams that are restrained at the ends. Apart from the time dependent effects, thermal effects also cause additional restraining moments which should be accounted for in the design.

##### 3.1.1 Creep

Creep of concrete results from the sustained load of prestressing and the dead weight of the bridge. Creep is influenced by the following factors:

- Magnitude and duration of the stress
- Maturity of the concrete at the time of application of load
- Temperature of concrete

The center of the prestressing force usually lies below the neutral axis of the section and causes the members to camber due to the eccentricity of the force, resulting in the formation of positive moments. This camber generally increases with time due to the creep of concrete under the sustained eccentric prestressing force. When the members are made continuous, the end



rotations due to creep are restrained which causes positive moments to develop at the interior piers. The creep due to dead loads results in negative restraining moments, thereby partially counteracting the effects caused by creep. The age at which continuity is established plays a significant role in determining the relative magnitudes of these two opposing forces.

### **3.1.2 Shrinkage**

Shrinkage is a reduction in the volume of concrete due to loss in moisture. Shrinkage is affected by the following:

- Aggregate characteristics and proportions
- Average humidity at the bridge site
- W/C ratio
- Type of curing
- Volume to surface area ratio of member
- Duration of drying period.

Since the girders are cast before the deck, most of the girder shrinkage would have already occurred before the deck concrete is placed. This causes the girder to restrain the shrinkage of the deck concrete. Due to the difference in the age and type of concrete, differential shrinkage occurs between the girder and deck concrete. This generally causes a downward deflection, however, if the girders are made continuous by a cast-in-place diaphragm. The end rotations of the girders will be restrained, causing negative moments in the diaphragm.

### **3.2 Methods for Creep and Shrinkage Analysis**

Several different methods have been used to analyze the effects of creep and shrinkage over time, such as the rate of creep method, effective modulus and age adjusted effective modulus method. If the positive moment predicted turns out to be excessive, the designer must resort to other alternatives such as construction sequence restrictions, special pier details and beam design modifications.

The Age Adjusted Effective Modulus method is used in this study to take into account of the effects of creep and shrinkage. The method of analysis is essentially the same as a conventional elastic analysis of a prestressed concrete cross-section, using the transformed section properties. Instead of a conventional modulus of elasticity, the age adjusted effective modulus is used for all the concrete elements in the section. In addition, the initial strains are also considered.

#### **3.2.1 Initial Strain**

An initial strain is defined as a strain that is not directly caused by an applied load. The initial strains normally considered in a time-dependent analysis of concrete members include:

- Free shrinkage of the concrete occurring during the interval being considered
- Creep strains of the concrete, occurring during the interval being considered, that are due to previously applied loads.
- The apparent steel strain due to relaxation of prestressing steel during the interval being considered.

It is necessary to consider the entire history of the cross-section to determine its time-dependent behavior. The history is usually composed of time intervals of varying lengths. The beginning and the end of each interval are marked by events such as the release of prestressing strands, the addition of the weight of a cast-in-place topping.

During the time between these events, there is continual creep, shrinkage and relaxation, as well as redistribution of internal stresses. Each event is treated as to have occurred during a time interval of zero length. Table 3.1 summarizes the significant time intervals during the life of a simple span girder.

**Table 3.1 Summary of time intervals during the life of a typical simple span girder.**

<b>INTERVAL</b>	<b>EVENT</b>	<b>TYPICAL DURATION</b>
1	Strand relaxation before transfer	12 to 24 hours
2	Transfer of prestress	0
3	Creep, shrinkage and relaxation of beam after transfer	30 days to 1 year
4	Placement of cast- in-place deck	0
5	Creep Shrinkage and relaxation of composite deck and beam	7 days to 6 months
6	Application of superimposed dead load on the composite deck and beam	0
7	Creep, shrinkage and relaxation of composite deck and beam	25 years or more

For a given time interval, the cross-section is analyzed by an elastic analysis with initial strains. Transformed composite section properties are recalculated for the analysis in each time interval since the properties of the concrete are time-dependent. A unique set of initial strains, dependent upon all the stress increments applied during the history of the member, are calculated for each time interval. If the time history is divided into many small steps, the accuracy of the analysis will be improved.

Initial strains can be incorporated into the cross-section analysis by calculating a fictitious load to restrain the initial strains due to shrinkage, creep and relaxation of steel. This restraining load is then subtracted from the loads applied to the section. The internal forces associated with this net load applied to the composite section are calculated. These forces are then added to the individual element restraining forces to give the actual forces on an individual element in the cross-section. A detailed description of this procedure is given in the next section.

### **3.2.2 Stress-Strain-Time relationship**

The time dependent analysis is carried out by establishing a stress-strain-time relationship for the concrete material. The stress-strain-time relationship for the concrete material is used to predict the total strain,  $\epsilon$ , at a future time,  $t$  that results from a stress increment applied at time,  $t_0$ . The total concrete strain at any time,  $t$ , can be separated into three components:

$\epsilon_f$  = the immediate strain due to the applied stress,  $f$

$\epsilon_{cr}$  = the time-dependent creep strain

$\epsilon_{sh}$  = free shrinkage strain

It is important to recognize that both the modulus of elasticity,  $E$ , and the creep coefficient,  $\psi$ , are functions of time. In addition, because concrete is an aging material,  $\psi$  also depends on the loading age,  $t_0$ .

### Constant Stress

Total concrete strain is  $(\varepsilon_f + \varepsilon_{cr} + \varepsilon_{sh})$ , which is usually expressed as:

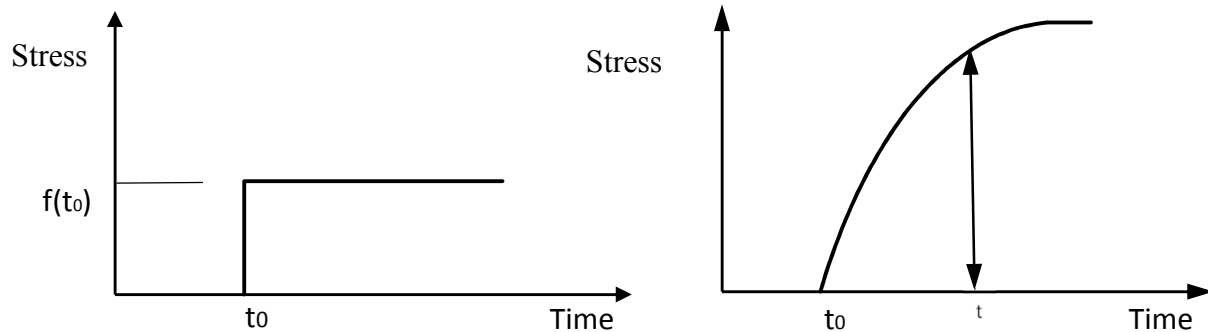
$$\varepsilon = \frac{f(t_0)}{E_c(t_0)} [1 + \psi(t, t_0)] + \varepsilon_{sh} \quad (3.1)$$

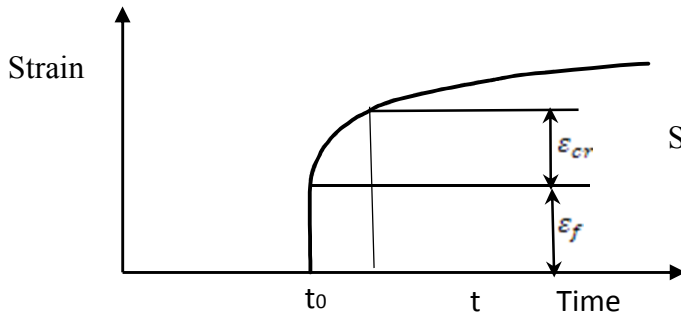
where

$E_c(t_0)$  = modulus of elasticity at time,  $t_0$ , the beginning of the interval

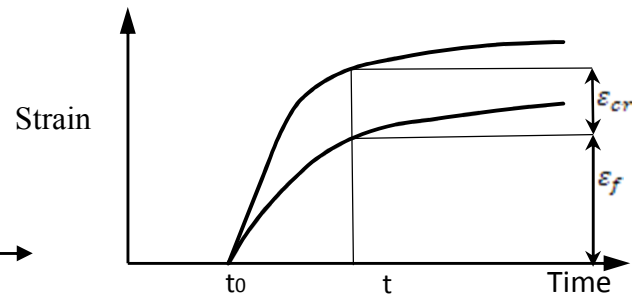
$\psi(t, t_0)$  = creep coefficient during a time interval from  $t_0$  to  $t$  for stress applied at time  $t_0$  and kept constant.

Eq. 3.1 is valid as long as stress,  $f$ , is a constant, sustained stress. Figure 3.1 shows the gradual development of creep strains with time under a constant stress.





**Figure 3.1 Concrete Strain vs. Time under Constant stress, shrinkage included**



**Figure 3.2 Concrete Strain vs. Time under variable stress**

### Variable Stress

When the applied stress,  $f$ , is variable, Equation 3.1 cannot be used directly. Figure 3.2 depicts the development of creep strains under the effects of an increasing applied stress. Using the principle of superposition, the effects of a series of applied stress increments can be determined individually, using Equation 3.1, and then combined to give the total time-dependent concrete strain. This approach is often called time-step method and is suitable for numerical modeling.

#### 3.2.3 Age Adjusted Effective Modulus Method

In the age adjusted effective modulus method, an “aging factor” is applied to the creep coefficient to account for the effect that the stress is gradually applied to an aging concrete with gradually increasing modulus of elasticity and decreasing creep effect (Bazant 1972). The aging coefficient  $\chi(t, t_0)$  is a function of the age of concrete at the time of initial load introduction. The total strain is represented by Equation 3.2:

$$\varepsilon = \frac{f(t)}{E_c(t_0)} [1 + \chi(t, t_0) \psi(t, t_0)] \quad (3.2)$$

The aging coefficient  $\chi(t, t_0)$  accounts for three separate effects:

1. When the applied stress,  $f(t)$ , is increasing, the concrete experiences the maximum force for only an instant at the end of the time-interval  $(t_0, t)$ . At all other times, the concrete experiences a load that is less than the maximum.
  2. The concrete is gaining strength and therefore the modulus is increasing with time, at an earlier age, when the concrete is less than 15 days old, the time varying loads acts on concrete that are less stiff. As the concrete ages the loads are larger and the concrete is also more stiffer when compared to concrete that is less than 15 days old,
  3. The total creep potential for load applied to concrete which is less than 15 days old is larger than for the same loads applied to the concrete that is more than 90 days old.
- A pseudo-elastic analysis may be performed using a reduced modulus of elasticity to account for the creep effects. The age-adjusted effective modulus of elasticity of concrete is defined as follows:

For sustained constant stress:

$$E_{ctc}^*(t, t_0) = \frac{E_c(t_0)}{1 + \psi(t, t_0)} \quad (3.3)$$

Eq. (3.1) can be rewritten to take advantage of the effective-modulus concept

$$\varepsilon = \frac{f(t)}{E_c^*(t, t_0)} + \varepsilon_{sh} \quad (3.4)$$

For gradually developing stress, the age-adjusted effective modulus is:

$$E_{ctv}^*(t, t_0) = \frac{E_c(t_0)}{1 + \chi(t, t_0)\psi(t, t_0)} \quad (3.5)$$

From here on the effective-modulus will be referred to as defined by Eq. (3.5), with the understanding that Eq. (3.3) represents the special case of an instantaneously applied load for

which  $\chi = 1$ . Further simplification is introduced in this study to assume  $\chi = 0.7$ , which has been shown to be reasonable by Dilger (1982) and by Tadros and Ghali (1985) for the type of “loads” acting on precast prestressed beams at a relatively young age of concrete.

### **3.2.4 Understanding Creep Restraint**

Only loads introduced before continuity can cause restraining moment due to creep. Typically, these are the pretensioning force, member self-weight and the deck weight. Each of these loads is considered separately. The total effect is obtained by superposition.

The following assumptions are made:

- The self-weight and the prestressing force are assumed to be introduced at time  $t_0$ .
- The modulus of elasticity of concrete at that time is  $E(t_0)$ .
- The continuity is made at time  $t_1$  and the modulus of elasticity of the concrete at that time is  $E(t_1)$ .

To explain the difference in behavior due to loads prior to continuity and loads after continuity, a two-span bridge is used as an example. The prestressing force and the beam weight will cause the simple spans to camber. . Let the rotation at the center support of the left beam be denoted by  $\theta_{el}$ . The creep due to the prestress and the beam weight causes the rotation to grow by an increment equal to  $\theta_{el} * \psi_1$ . If the two beam ends are joined with a rigid connection, a restraining moment develops gradually and causes the end to have an equal and opposite rotation  $= \theta_{r, el} * (1 + \chi * \psi_2)$ . By setting these two rotations equal to each other, it can be shown that the restraining moment = the elastic moment \*  $\psi_1 / (1 + \chi * \psi_2)$ .



On the other hand, a load that is introduced right after continuity is made would have a free rotation of  $\theta_{el}*(1+\psi_2)$  and a restraining rotation of  $\theta_{r, el}*(1+\chi*\psi_2)$ . For this case, the restraining moment = the elastic moment \*  $(1+\psi_2)/(1+\chi*\psi_2)$ , which would be equal to the elastic moment if  $\chi$  is approximated at 1.0. If the “loading” is gradually introduced at the same rate as creep develops after continuity is made, the restraining moment would be exactly equal to the elastic moment.

Therefore, it is reasonably accurate to assume that there is no creep restraining moment due to loads that are introduced after continuity is made. In a design, the restraining moment would consist of creep moment due to prestress, beam weight and deck weight, and elastic moment due to superimposed dead load, live load, and daily temperature gradients.

### **3.2.5 Coefficients of Creep and Shrinkage in the AASHTO-LRFD Specifications**

The coefficients of creep and shrinkage are calculated according to the AASHTO-LRFD Specifications Section 5.4.2.3, and are based on the work by Huo et al. (2001), Al-Omaishi (2001), Tadros (2003), and Collins and Mitchell (1991). These methods are based on the recommendation of ACI Committee 209 (1982) modified by additional published data.

The ultimate creep coefficient and the ultimate shrinkage coefficient of the girder and deck concrete are directly related to the restraining moments developed at the pier. Typically these coefficients are based on a 20-year loading period and mainly depend upon the concrete composition, girder and deck geometry and ambient relative humidity during the life of the girder. The creep coefficient is given by the following equation:

$$\Psi(t, t_i) = 1.9 k_s k_f k_{hc} k_{td} t^{-0.118} \quad (5.4.2.3.2-1)$$

in which

$$k_s = 1.45 - 0.13(V/S) \geq 1.0$$

$$k_{hc} = 1.56 - 0.008 H$$

$$k_f = \frac{5}{1 + f_{ci}}$$

$$k_{td} = \left( \frac{t}{61 - 4f_{ci} + t} \right)$$

Shrinkage of concrete can vary over a wide range from nearly zero if continually immersed in water to in excess of 0.0008 strain, for thin sections made with high shrinkage aggregates and sections not properly cured. The strain due to shrinkage  $\epsilon_{sh}$  at time,  $t$ , is given by:

$$\epsilon_{sh} = k_s k_f k_{hs} k_{td} 0.48 \times 10^{-3} \quad (5.4.2.3.3-1)$$

in which

$$k_{hs} = (2.00 - 0.014H) \quad (5.4.2.3.3-2)$$

where

H: Relative humidity (%) = 70 %

$k_s$ : Factor for the effect of the volume -to-surface ratio of the beam

$k_f$ : Factor for the effect of concrete strength

$k_{hc}$  = Humidity factor for creep

$k_{hc}$  = Humidity factor for shrinkage

$k_{td}$  = Time development factor

$f_{ci}$  = specified compressive strength of concrete at time of prestressing for pretensioned members and at time of initial loading for non-prestressed members

$V/S$  = Volume-to-surface ratio

$t_i$  = 1 day prestress release, (loading time)

$t_d$  = 28 days: time of casting the deck, (continuity starts)

$t$  = 75 years = 27375 days = final time.

These ultimate creep coefficient and shrinkage strain obtained from the calculation above are utilized in determining the positive restraining moment. Detailed procedure is explained below.

### 3.2.6 Analysis of Restraining Moments due to Creep and Shrinkage

#### 3.2.6.1 Restraining moment due to creep

Specifically, the following procedure is used for each load:

1. Calculate time-dependent material properties:

$\psi(t, t_0)$  is creep at time,  $t$ , for concrete loaded under prestress and beam weight at time,  $t_0$

$\psi(t, t_1)$  is creep at time,  $t$ , for concrete loaded under deck weight and the restraining moment at time,  $t_1$

$\psi(t_1, t_0)$  is creep at time,  $t_1$  for concrete loaded at time,  $t_0$

Time  $t$  is generally assumed equal to 75 years, or 27,000 days. Several researchers have assumed 2000 days and 20,000 days to represent time at which creep growth becomes nearly zero.

Age- adjusted effective modulus of concrete subjected to gradual loading at time  $t_1$  with creep developing in the period  $(t-t_1)$  is given by:

$$E_{ctv}^*(t, t_1) = \frac{E_c(t_1)}{1+0.7\psi(t, t_1)} \quad (3.6)$$

Age- adjusted effective modulus of concrete subjected to constant stress introduced at time  $t_0$  with creep developing in the period  $(t-t_1)$  is given by:

$$E_{ctc}^*(t, t_0) = \frac{E_c(t_0)}{\psi(t, t_0) - \psi(t_1, t_0)} \quad (3.7)$$

2. Perform elastic analysis, assuming as if the load were introduced to a continuous member. Determine the fictitious elastic restraining moments at the supports,  $M_{el}$
3. Determine the time-dependent multiplier,  $\delta_c$ , corresponding to the load:

$$\delta_c = \frac{E_{ctv}^*(t, t_1)}{E_{ctc}^*(t, t_0)} \quad (3.8)$$

4. Determine the restraining moment,

$$M_{cr}(t) = \delta_c M_{el} \quad (3.9)$$

5. Add the creep restraining moments due to all the loads applied before the continuity becomes effective to the elastic continuity moments after the continuity becomes effective to get the total moments. Both the maximum and the minimum values are needed for design. For example, maximum positive moment should not include negative moment due to the live load. Even though future wearing surface load is considered dead load, its negative moment should not be included as the time of its application may be many years after the bridge has been constructed.

### **3.2.6.2 Restraining Moment due to Differential Shrinkage**

1. Calculate the cross-sectional properties of the transformed section and other parameters such as modulus of elasticity of the deck, deck thickness required to calculate the shrinkage. Calculate the strain caused due to shrinkage on the deck using the equation (5.4.2.3.3-1) as provided in the AASHTO-LRFD Specification.
2. Calculate the compressive forces acting on the deck due to the effects of differential shrinkage by multiplying the area of the deck with the modulus of elasticity of the deck.
3. The restraining force required to keep the structure from deforming is equal to the compressive force calculated above. In order to calculate the restraint moments at the pier, these forces are applied as fixed end moments on the structure. By solving the indeterminate structure using a finite element analysis, the total restraining moment due to shrinkage is calculated.

### **3.2.7 Calculation of Restraining Moment due to Differential Shrinkage –PCI-BDM (1997)**

The restraining moments due to differential shrinkage are calculated based on the following assumptions:

- The curing of the beam concludes at time,  $t_2$
- The curing of the deck ends at time,  $t_3$

The following procedure is used for calculating the restraining moment due to differential shrinkage:

1. Calculate the time-dependent material properties

Deck:

- a).  $C_d(t, t_3)$  is the creep at time,  $t$ , for deck concrete loaded at time,  $t_3$

- b).  $\epsilon_{shd}(t, t_3)$  is the shrinkage strain of the deck from time  $t_3$  to time,  $t$   
 c).  $E_{cd}(t_3)$  is the modulus of elasticity for deck concrete at time,  $t_3$

Beam:

- a)  $C_b(t, t_3)$  is the creep at time,  $t$ , for beam concrete loaded at time,  $t_3$   
 b).  $\epsilon_{shb}(t, t_2)$  is the shrinkage strain of the deck from time  $t_2$  to time,  $t$   
 c).  $\epsilon_{shb}(t_3, t_2)$  is the shrinkage strain of the deck from time  $t_2$  to time,  $t_3$   
 d).  $E_{cb}(t_3)$  is the modulus of elasticity for beam concrete at time,  $t_3$ .

2. Calculate age adjusted, effective modulus for concrete subjected to gradual loading

$$a). E_{cd}^* = \frac{E_{cd}(t_3)}{1+0.7C_d(t, t_3)}$$

$$b). E_{cb}^* = \frac{E_{cb}(t_3)}{1+0.7C_b(t, t_3)}$$

3. Calculate the shrinkage moment,  $M_{sh}$  :

$$M_{sh} = S h_d E_{cd}^* \epsilon_{shd}(t, t_3) [y_{tc} - h_d/2] - A E_{cb}^* [\epsilon_{shb}(t, t_2) - \epsilon_{shb}(t, t_2) - \epsilon_{shb}(t_3 - t_2)](y_{bc} - y_b)$$

where

$S$  = Beam spacing

$H_d$  = deck thickness

$y_{tc}$  = Distance from the centroidal axis of the composite section to the top of the deck.

$A$  = Gross Area of the non-composite beam

$y_{bc}$  = Distance from the centroidal axis of the non-composite section to the bottom of the beam.

$y_b$  = Distance from centroidal axis of non-composite section to the bottom of the beam.

4. Perform moment distribution analysis for the continuous structure, using the shrinkage moments as the fixed end moments and the stiffness properties calculated from the composite section. The moment at the supports after moment distribution is the restraining moment,  $M_{sr}(t)$ , due to differential shrinkage.
5. In order to eliminate this compressive force, equal and opposite forces are applied at the fixed ends of the composite section. The statically indeterminate structure is solved using finite element analysis from which the restraining moments at center supports can be obtained.

### **3.3 Calculation of Restraining Moments according to the AASHTO-LRFD Specifications**

AASHTO-LRFD article 5.14.1.4 gives the provisions for bridges composed of simple span precast girder made continuous. These bridges are made by erecting single-span, precast concrete girders and then connecting them over the supports with a cast-in-place concrete diaphragm and deck slab to establish full-depth positive and negative moment connections. The girders carry their own dead load and the slab dead load as simple spans, but all the subsequent loads are carried as continuous spans. Deck reinforcement provides the negative moment resistance.

The main drawback of this design is that the girders will camber upward due to creep and shrinkage. In contrast, differential shrinkage between the deck and the girders causes the girders to deflect downward. Temperature gradients also affect the camber. If the net camber is positive,

a positive moment develops and the connection cracks. For this reason, Article 5.14.1.4.9 requires a positive moment connection at the joint by providing:

- Mild reinforcement embedded in the precast girders and developed into the continuity diaphragm
- Pretensioning strands extended beyond the end of the girder and anchored into the continuity diaphragm. These strands shall not be debonded at the end of the girder

In effect, the provisions and commentary of Article 5.14.1.4 give the designer five options:

- 1) Provide a positive moment connection with strength of  $1.2 M_{cr}$  and require the girders to be at least 90 days old at the time continuity is established. The reasoning given in the commentary is that by 90 days, 60 % of the creep and 70% of the shrinkage in the girder is theoretically complete. The behavior of the system will be dominated by differential shrinkage of the deck so the possibility for positive moment cracking to affect continuity is very low.
- 2) Provide a positive moment connection with a strength of  $1.2 M_{cr}$  and use the provisions of Article 5.4.2.3, with  $k_{td} = 0.7$ , to establish the minimum age at which continuity can be established.
- 3) If the contract documents specify a minimum girder age of 90 days is required when continuity is established, computation of restraining moments is not required.
- 4) Use the provisions of Article 5.14.4.4.5 and consider the bridge continuous if the net stress at the bottom of the diaphragm from superimposed permanent loads, settlement, creep, shrinkage, temperature gradient, and 50 % of live load is compressive.
- 5) Calculate the actual restraining moments and determine the degree of continuity from the analyses (Article 5.14.1.4.2).



If the connection does not provide full continuity, the effect of partial continuity must be considered per Article 5.14.1.4.5.

### **3.4 Thermal Effects**

Solar radiation acting on the surfaces is partly absorbed and partly reflected. The absorbed energy heats the surface and produces a temperature rise through the deck. A bridge deck continuously gains and loses heat from thermal radiation, re-radiation to the sky, and convection to or from the surrounding atmosphere. Temperature variations induced by these sources depend on geometry, location, and orientation of the bridge, climatological conditions, and thermal properties of the material and exposed surfaces. Thermal effects on the bridge are caused by both the short term daily temperature changes and the long term seasonal temperature changes.

The material properties which affect the magnitude of the gradient are the conductivity, density, absorptivity and specific heat. Temperature gradients occur because the top and bottom of a member are exposed to a change in temperature and absorb heat rapidly while the middle portion is insulated from these effects as the heat is not transferred quickly through the depth of the member due to the non-conductive nature of concrete. A positive thermal gradient is formed when the deck is warmer than the girder webs, the top surface of the structure expands more than the bottom surface which causes the structure to deflect upwards. A negative thermal gradient is formed in which the deck is cooler than the girder which causes high tensile stresses to develop at deck. The effects of temperature gradient on a continuous concrete structure should be analyzed as they develop bending moments which must added to the restraining moment in the continuity diaphragm.

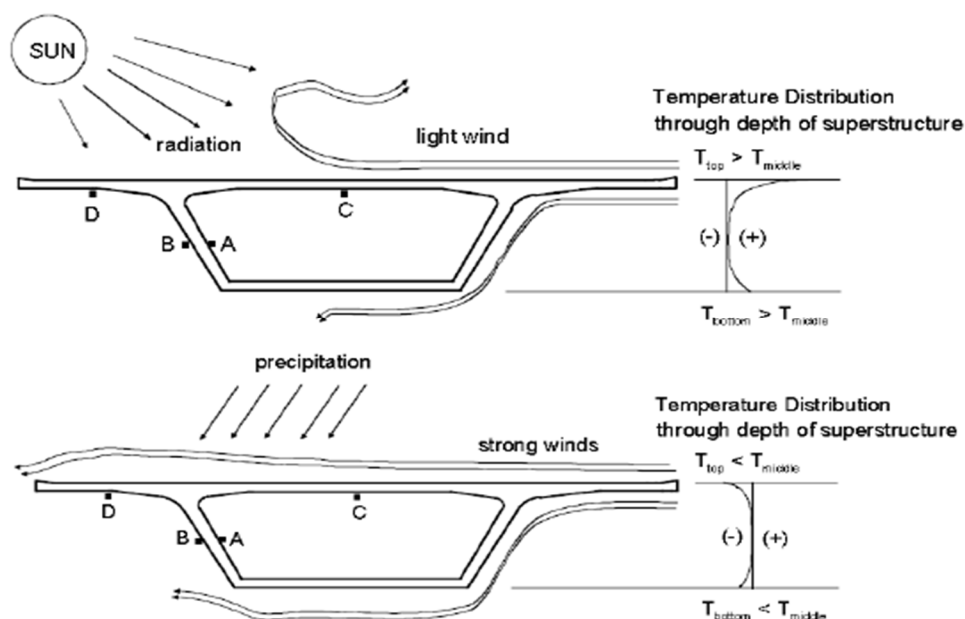


Figure 3.3 Conditions for the development of a). Positive b). Negative thermal gradients.

### 3.4.1 Analysis of Thermal Effects

The main factors which affect structural response are the linearity of the gradient and the determinacy of the structure. Consider a statically determinant beam which is subjected to a positive linear temperature gradient, it will not experience any stresses induced by temperature but will elongate and camber upwards. Whereas if the same beam is subjected to a non-linear gradient, it will experience self-equilibrating stresses because plane sections must remain plane.

Since there is no shear deformation, stresses will develop because of the difference between the strains the structure wants to develop and the strains it is forced to develop to keep plane sections plane. The stress developed in the member due to restraint of elongation and rotation is given by:

$$\sigma_{\text{temp}} = E \times \alpha \times T(Y) \quad (3.10)$$

The restraining axial force P is calculated as:

$$P = \int_V E \alpha T(Y) b(Y) dY \quad (3.11)$$

The restraining force is compressive if the temperature gradient is positive. The restraint moment acting on the section is:

$$M = \int_V E \alpha T(Y) b(Y) Y dy \quad (3.12)$$

The magnitude of the self-equilibrating stress is given by:

$$\sigma_{sc}(Y) = E \alpha T(Y) - P/A - MY/I \quad (3.13)$$

The net force on the section due to self-equilibrating stress is zero

Where:

Y = Distance from the center of gravity of the cross-section.

T(Y) = Temperature at a depth Y,

b(Y) = Net section width at a depth Y,

$\sigma_{sc}$  = self- equilibrating stress at depth Y,

A = Cross-Sectional Area,

I = Moment of Inertia of the section

E = Modulus of Elasticity

$\alpha$  = Coefficient of thermal expansion

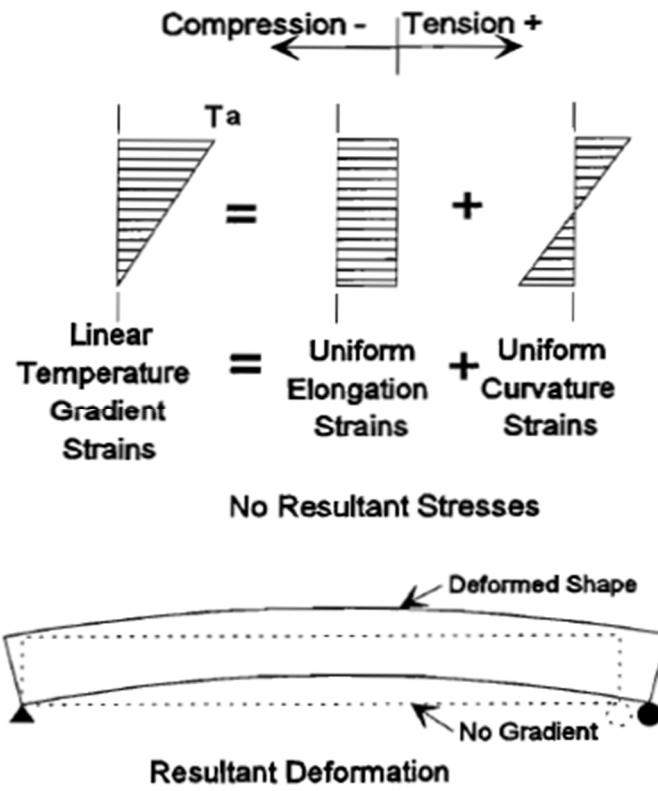


Figure 3.4 Determinant beam subjected to linear gradient

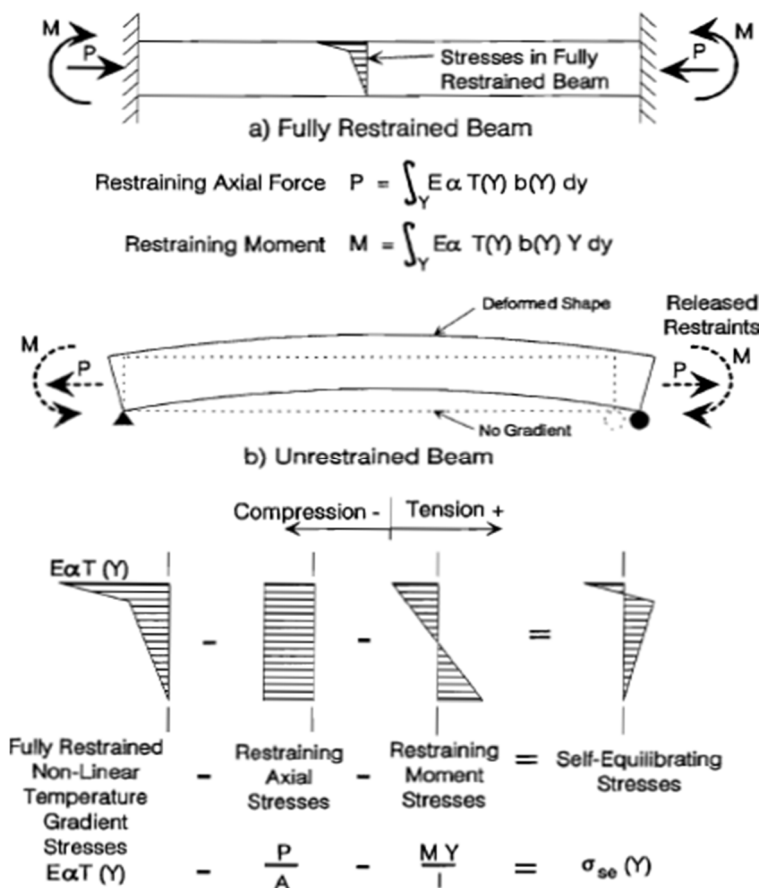


Figure 3.5 Determinant beam subjected to non-linear gradient

### 3.4.2 Analysis for Thermal Restraining Moment in an Indeterminate Structure

An indeterminate structure subjected to a linear or nonlinear gradient will develop restraining moments at the interior piers. For example, under a positive thermal gradient, the top fibers of the deck will undergo greater elongation than the middle and bottom fibers. Therefore, bending moments are caused by the temperature gradient similar to the secondary moments caused by a prestressing force.

The calculation of the stress distribution through the deck under a variation of temperature starts from the assumption that the deck is rigidly restrained and then calculate the effects of removing the artificial restraints. The detailed procedure is given below:

- Selection of the most appropriate temperature gradient using the AASHTO-LRFD specifications and calculation of the cross-sectional properties of the structure.
- The restrained stress diagram is divided into sufficient sections of depths ( $Y$ ). The primary restraining force is calculated by multiplying the restrained stress with the area of the section. (As shown in equation 3.11).
- The primary restraining moment is found by summing the force on each section multiplied by the distance of its centroid to the neutral axis. The restrained stress diagram is divided into rectangles and triangles as the position of the centroid of these shapes are known.
- The primary restraining axial forces and bending moments calculated in the above step are applied as fixed end moments on the entire structure. The restraining moment at the interior pier is calculated by using these inputs into a finite element program.
- Stresses computed from the structural analysis are then superimposed on stresses due to the primary restraining axial force and bending moments to give the total restraining moments and the stresses developed due to continuity.

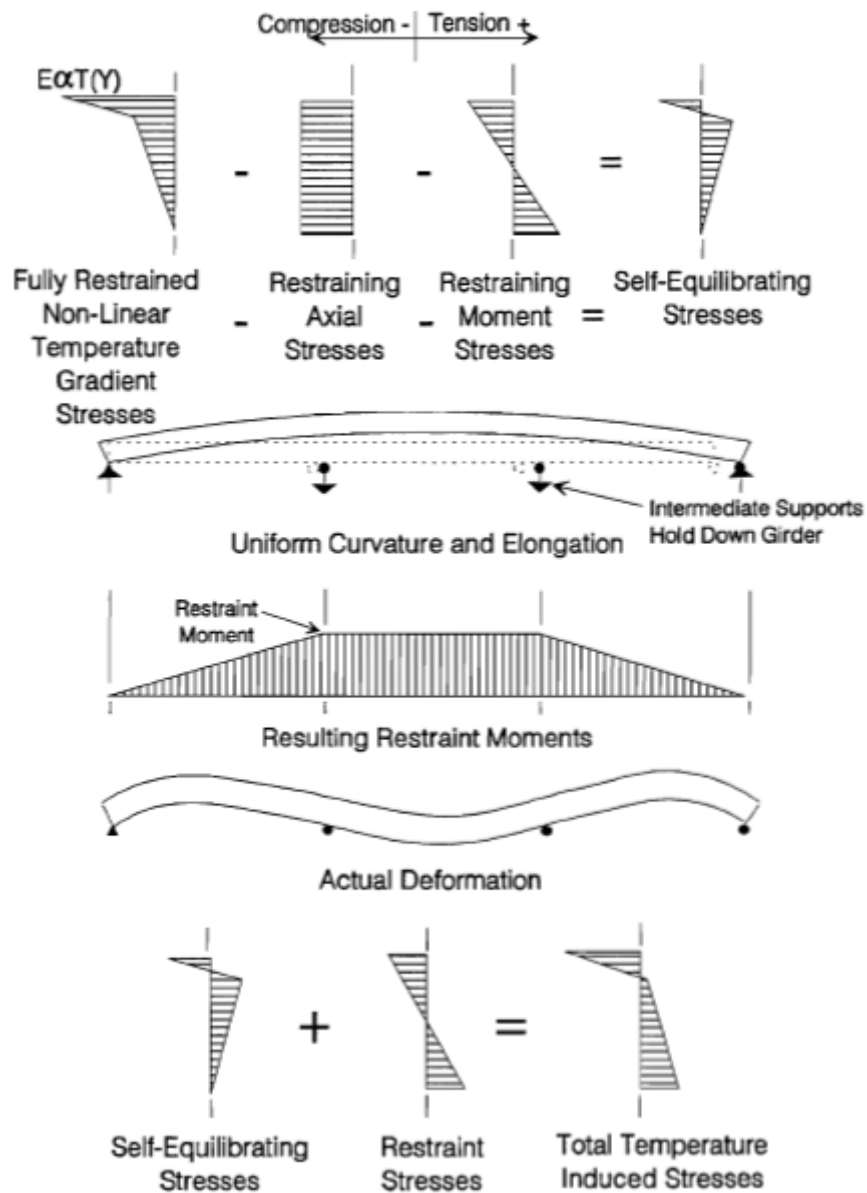


Figure 3.6 Indeterminate beam subjected to non-linear gradient

### 3.4.3 Thermal Analysis using the AASHTO LRFD Specifications

The AASHTO LRFD Bridge Design Specifications Section 3.12.3 outlines the current temperature gradient that should be used to calculate thermal effects that occur through a cross-section of a bridge system. The standard temperature gradient is portrayed in the Figure 3.12.3-2 in the AASHTO Specifications. Section 3.12.3 defines the value of dimension A in Figure 2.6 as 12.0 in. for concrete superstructures that have a depth of 16 in. or more. Section 2.5.1 states that the United States is divided into 4 zones based on climate. From Figure 3.12.3-1 in the specification Nebraska falls under Zone 2 and from Table 3.12.3-1 the temperatures associated with Zone 2 are  $T_1 = 46^\circ\text{F}$  and  $T_2 = 12^\circ\text{F}$ .  $T_3$  is taken as  $0^\circ\text{F}$  unless a site study indicates otherwise and the maximum value that can be used for  $T_3$  is  $5^\circ\text{F}$ .

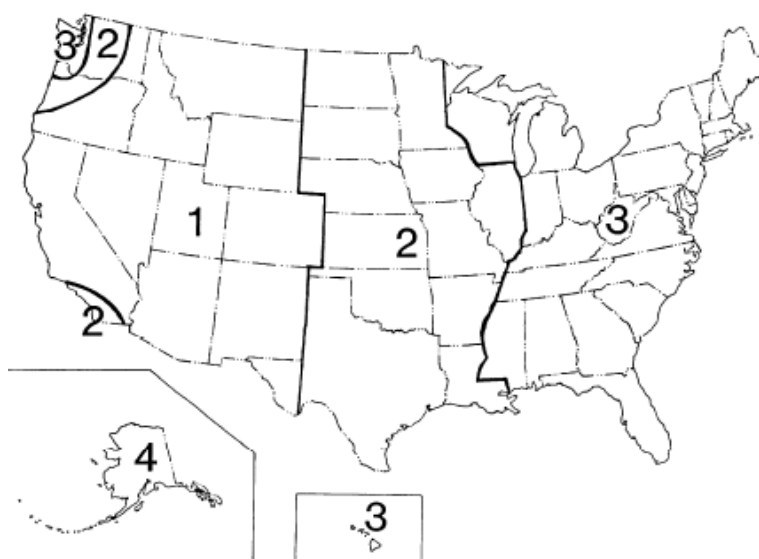
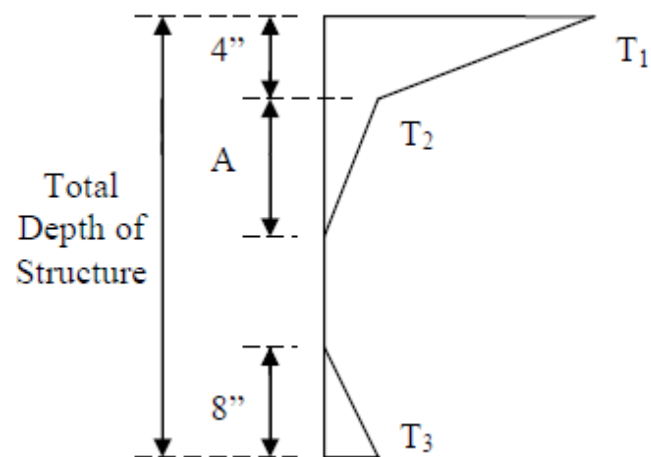


Figure 3.7 Solar radiation Zones for the United States



Zone	$T_1$ (°F)	$T_2$ (°F)
1	54	14
2	46	12
3	41	11
4	38	9

**Table 3.2 – Basis for Temperature Gradients**



**Figure 3.8 Positive Temperature Gradient throughout the Cross-section**

The standard temperature gradient is from Figure 3.12.3-2 in the AASHTO-LRFD Specifications and is shown in the Figure 3.8. Section 3.12.3 defines the value of dimension A in Figure 2.6 as 12.0 in. for concrete superstructures that have a depth of 16 in. or more.

The response of a structure to a temperature gradient is categorized into the following three effects:

- **Axial Expansion** – bridges are generally designed for an assumed uniform temperature change. Lateral thermal forces cause the bridge to expand radially as well as

longitudinally. The axial expansion is due to the uniform component of temperature distribution which is calculated as follows.

$$T_{UG} = \frac{1}{A_c} \iint T_G dw dz \quad (C4.6.6-1)$$

the corresponding axial expansion is given by:

$$\epsilon_u = \alpha (T_{UG} + T_U) \quad (C4.6.6-2)$$

- **Flexural Deformation** – A curvature is imposed on the superstructure to accommodate the linearly variable component of the temperature gradient. The rotation per unit length corresponding to this curvature is determined as:

$$\phi = \frac{\alpha}{I_c} \iint T_G z dw dz \quad (C4.6.6-3)$$

- **Internal Stress**- Internal Stresses in addition to those corresponding to the restrained axial expansion or rotation may be calculated as:

$$\sigma_e = E [\alpha T_G - \alpha T_{UG} - \phi z] \quad (C4.6.6-4)$$

For a two-span structure with span length L in ft. A restraining moment is developed at the pier which forces the beam to eliminate the deflection caused in an unrestrained beam and is given by:

$$M_c = \frac{3}{2} E I_c \phi \quad (C4.6.6-5)$$

where

$T_G$  = temperature gradient ( $\Delta^\circ\text{F}$ )

$T_{UG}$  = temperature averaged across the cross-section ( $^\circ\text{F}$ )

$T_u$  = uniform specified temperature ( $^\circ\text{F}$ )

$A_c$  = cross-section area ( $\text{in}^2$ )

$I_c$  = inertia of cross-section ( $\text{in}^4$ )

- $\alpha$  = coefficient of thermal expansion (in./in./°F)
- E = Modulus of Elasticity (ksi)
- R = Radius of curvature
- w = width of the element in cross-section. (in.)
- z = vertical distance from center of gravity of cross-section (in.)

A detailed explanation of the procedure is provided below and a numerical example is provided in the Appendix

**Analysis Steps:**

1. Select the most appropriate temperature gradient based on (Table 3.12.3-1) which gives us the values of gradients for all the states as they are divided into zones (Figure 2.12.3-1) based on annual solar radiation.
2. Calculate the cross-sectional properties, modulus of elasticity of the transformed section. The coefficient of linear expansion is taken as  $6 \times 10^{-6}$  in/in°F for normal weight concrete (Section 5.4.2.2)
3. The next step is to integrate the non-linear temperature gradient through the cross-section in order to determine the curvature at each section. The equation mentioned above (C4.6.6-3) is utilized.
4. Once the curvature is calculated for the sections the restraint moment at the interior support is calculated using the equation provided in the previous section (C4.6.6-3).

### 3.5 Effect of Construction Sequencing on Continuity

Several factors influence the total restraining moment in a bridge and the corresponding degree of continuity. These factors include girder age at continuity, girder geometry, prestressing strand-layout, girder and deck concrete properties, and bridge geometry.

The age of the girders when the bridge is made continuous determines how much girder creep and shrinkage have already occurred in an unrestrained state, and how much remains after continuity in a restrained state. This is a condition over which a designer has little control and which has a significant effect on the restraining moments. The girder age depends on the precast plant production schedule, the size of the bridge and the resulting construction schedule and the timing between placing the deck over the span and placing the deck over the piers.

The positive restraining moments will be relatively low if continuity is established when concrete is more than 90 days old, as there would be less creep and shrinkage remaining to develop in the girder. Less remaining creep results in lower positive restraining moments due to creep. Less remaining girder shrinkage results in larger differential shrinkage between deck and girder concrete, which translates to larger negative restraining moments due to shrinkage. The combined effect is a lower positive restraining moment which prevents the diaphragm from cracking and maintains the continuity.

Since it is difficult to accurately predict the construction timing, the engineers are entitled to make reasonable assumptions to arrive at reasonable values for restraining moments and degree of continuity.

To demonstrate the impact of construction timing, restraining moments were computed for a NU900 section using girder ages of 7, 28, 42, 60, 90 and 120 days at the time of continuity. From the analysis it is seen that there are primarily two main effects on the restraining moments:

1. Negative moment due to differential shrinkage between the deck slab and the girder.
2. Positive moment due to prestressing, and temperature gradient.

The results of the analysis are given in Table 3.2 and shown in Figure 3.9. It shows that if the deck is cast when the girder is more than 90 days old, the magnitude of positive restraining moment developed in the diaphragm is relatively low when compared to a system wherein the deck is cast at an early girder age. It can be observed that there is no significant difference in the restraining moment developed when the girder is aged 60 days and when the girder is 90 days. If we compare the magnitude of positive moments formed when the girder is 90 days, there is a reduction of positive moment of about 20% when the girders are 60 days old as compared to reduction of 45 % which is observed when girders are 28 days old.

The AASHTO-LRFD Specifications article 5.14.1.4 states that a positive moment connection with strength of  $1.2 M_{cr}$  along with the girders to be 90 days old at the time of continuity is the recommended way to design for positive restraining moment. According to the AASHTO-LRFD Specifications the connections are designed based on the strength limits. The continuity diaphragms are not prestressed concrete so the stress limits for the service limit states do not apply. However cracking is a serviceability issue. From the results of the analysis carried out in this research, it is recommended that the connection be designed for the positive restraining moment at the face of the diaphragm. For example, if it is desirable to use 28 days as the age of girder concrete at the time connection was made, analysis should reveal the amount of steel required to control cracking. It is expected that the results would show only some of the

existing bottom strands would need to be extended into diaphragm and that additional rebar are not necessary. Calculations verifying these results are provided in the Appendix.

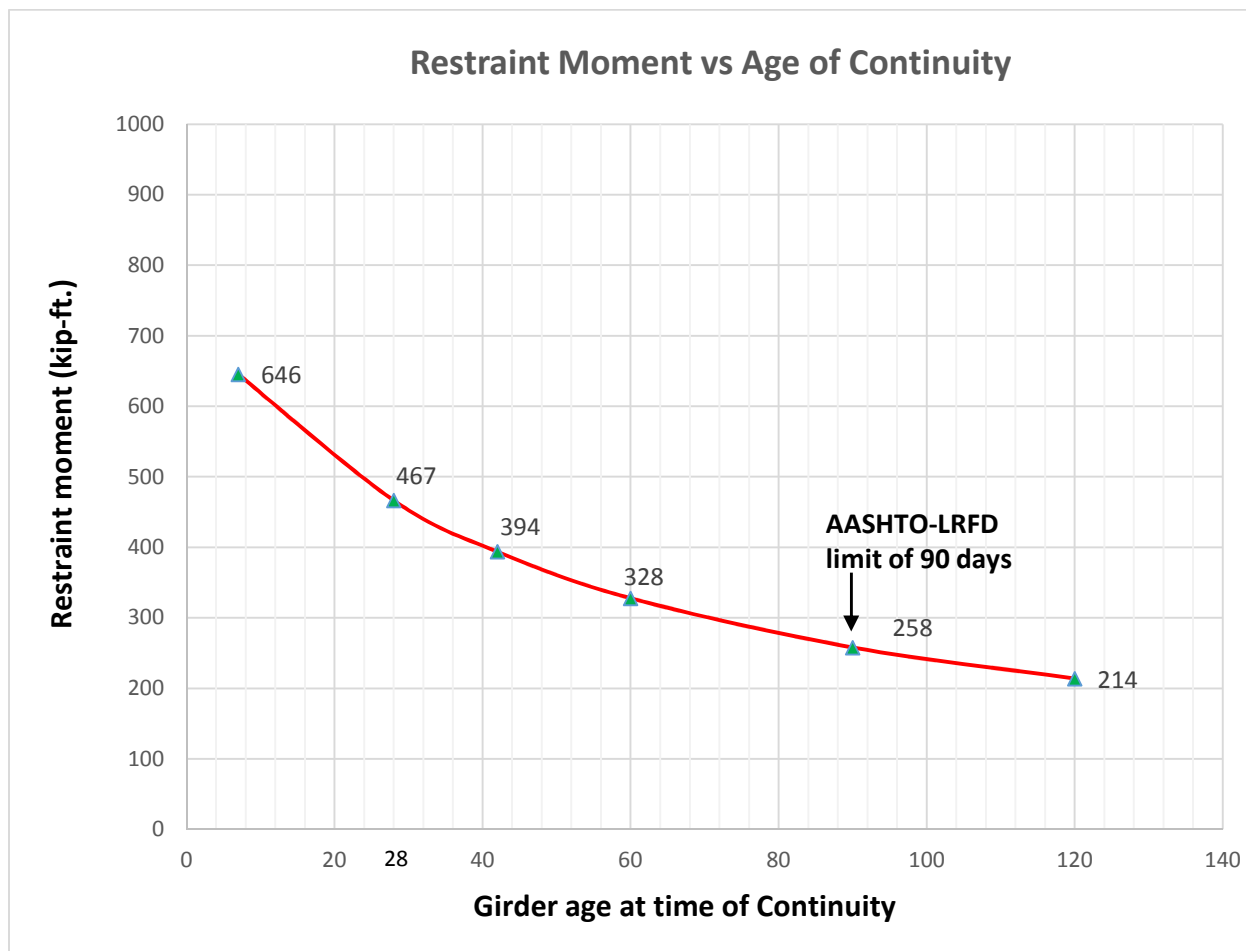


Figure 3.9 Restraint Moment vs Age continuity

S.No.	RESTRAINT MOMENTS (kip-ft.)	GIRDER AGE AT TIME OF CONTINUITY					
		7 days	28 days	42 days	60 days	90 days	120 days
1).	Restraint moment due to girder weight	-590.0	-431.2	-362.8	-301.2	-234.8	-192.3
2).	Restraint moment due to prestressing	1985.7	1451.3	1220.9	1013.5	790.1	647.3
3).	Restraint moment due to deck weight	-765.4	-568.8	-479.5	-399.7	-312.5	-256.2
4).	Restraint moment due to temperature	570.1	570.1	570.1	570.1	570.1	570.1
5).	Restraint Moment due to deck shrinkage	-352.4	-352.4	-352.4	-352.4	-352.4	-352.4
6).	Elastic moment due to barrier weight	-202.5	-202.5	-202.5	-202.5	-202.5	-202.5
7).	Total Net Moment	645.5	466.5	393.8	328.7	258.0	214.0
8).	Diaphragm reinforcements (nos.)	26	19	16	13	10	9

**Table 3.3 Comparison between the positive moments formed depending on the age of girder**

Reversing the order of deck placement, by placing the concrete over piers first would ultimately lead to lower positive restraining moments thereby preventing any cracks from developing in the diaphragm and maintaining continuity. . However, it would require more negative moment reinforcement to prevent negative restraining moment cracking. The results from the analysis show that the order of placing the deck can have a significant effect on the development of positive restraining moment. It can be observed that if the deck is cast after the girder achieves continuity there is hardly any positive restraint moment formed. This is because when the deck is placed, the time dependent effects diminishes as the girder ages. This causes a reduction in the magnitude of positive moments and an increase in the magnitude of the negative moment Figure 3.10 and Table 3.3 shown below gives a comparison between the two continuity systems. A numerical example is given in the Appendix-A.

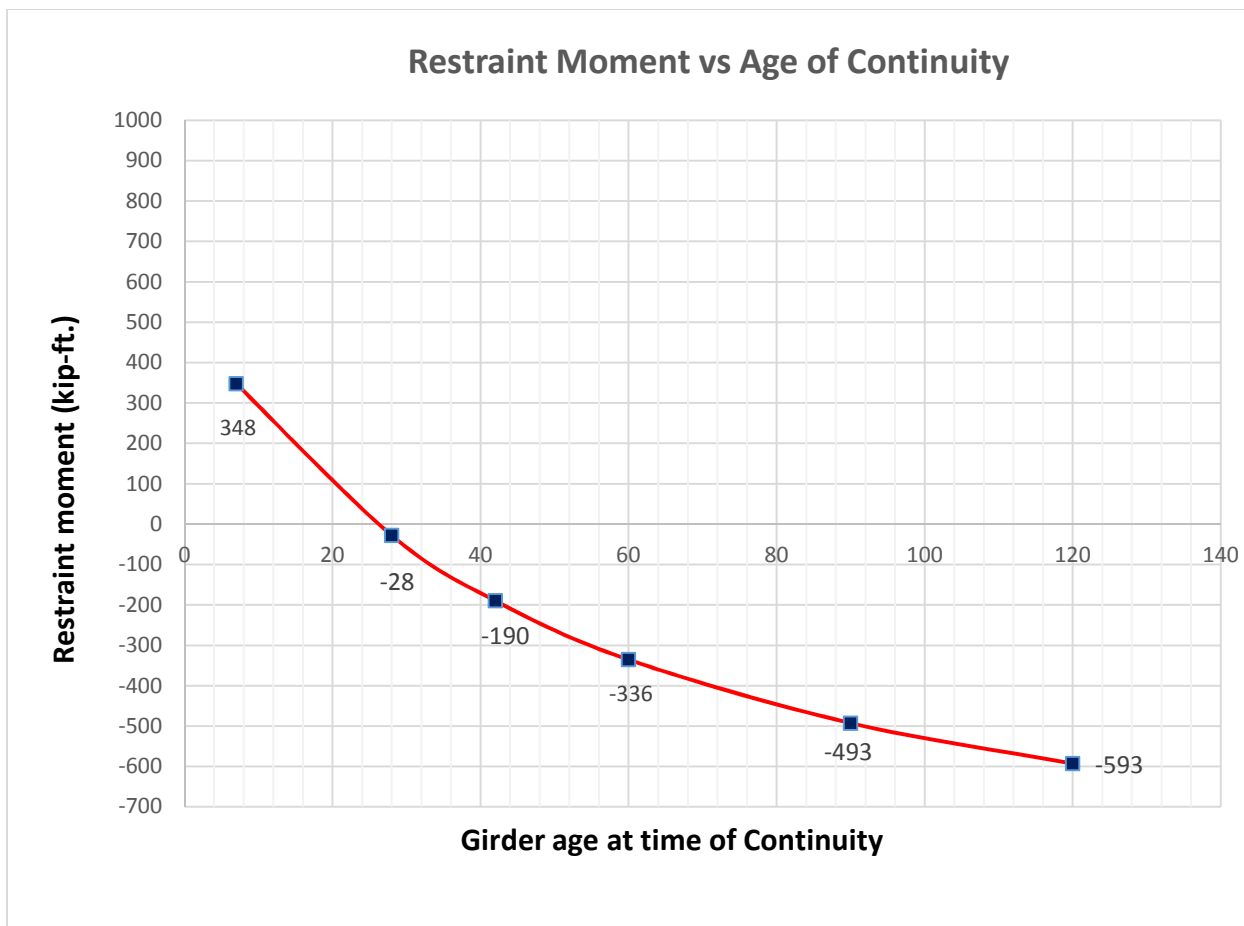


Figure 3.10 Restraint Moment vs Age of continuity when deck is placed after girder continuity is achieved

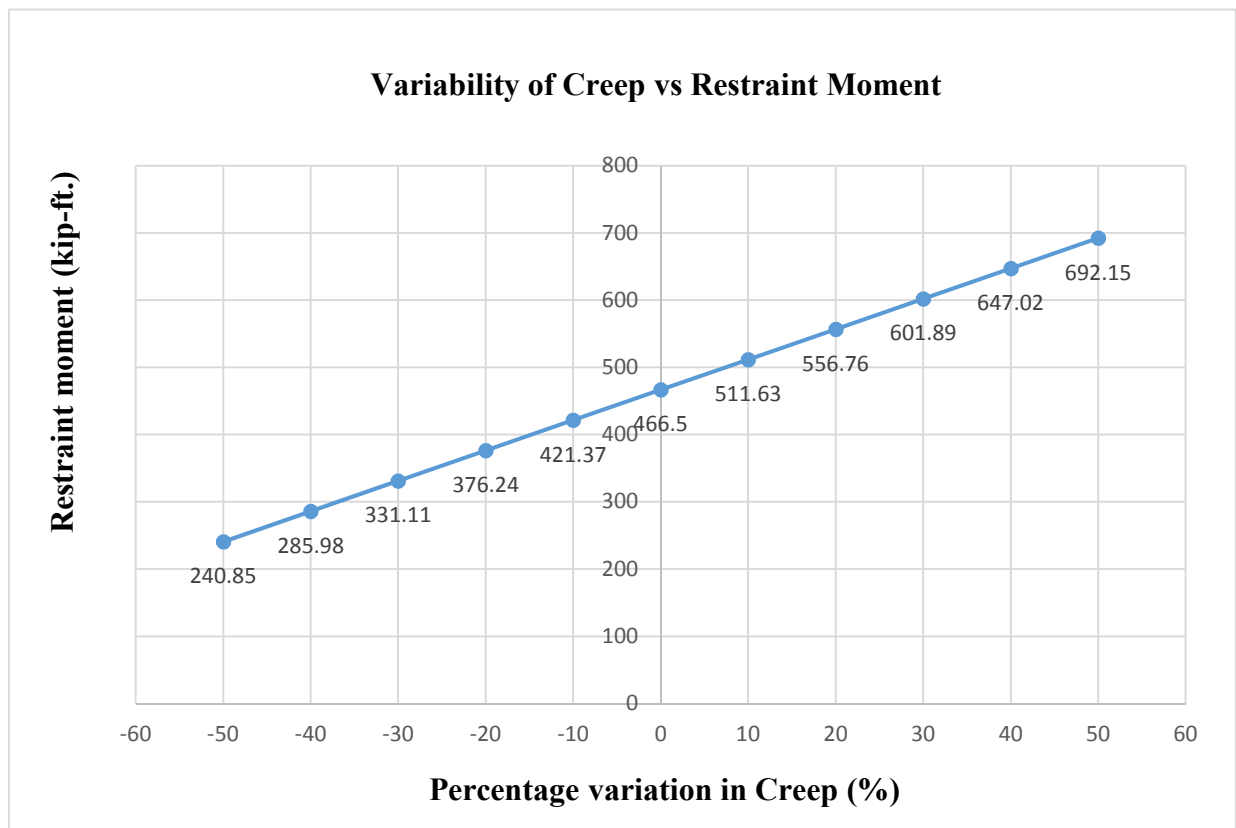


S.NO		CONTINUITY SYSTEM	
		Deck placed when age of girder is 28 days	Deck placed after continuity is achieved
	<b>RESTRAINING MOMENTS</b>		
1)	Restraint Moment due to girder weight	-431.2 ft.-kips	-431.2 ft.-kips
2)	Restraint Moment due to prestressing	1451.3 ft.-kips	1451.3 ft.-kips
3)	Restraint Moment due to temperature	570.1 ft.-kips	570.1 ft.-kips
4)	Restraint Moment due to deck weight	-568.8 ft.-kips	0 ft.-kips
5)	Restraint moment due to deck shrinkage	-352.4 ft. kips	-352.4 ft. Kips
	<b>ELASTIC MOMENTS</b>		
6)	Elastic Moment due to deck weight	0 ft.-kips	-1063.1 ft.-kips
7)	Elastic Moment due to barrier weight	-202.5 ft.-kips	-202.5 ft.-kips
8)	<b>Total Net Moment</b>	466.5 ft.-kips Positive moment reinforcement is required	-27.8 ft.-kips No positive moment reinforcement is required

**Table 3.4 Comparison between the positive moments formed depending on the construction sequencing**

### 3.6 Variability of Creep with Positive Restraining Moment.

To better understand the effect of creep on a continuous bridge system, a variability analysis was carried out by varying the creep-causing effects due to prestressing, girder weight and other parameters that occur before continuity is established. From the results of the analysis it was observed that there is a linear relationship between the variation in creep and the magnitude of positive restraining moment. As the percentage of creep-causing effects is increased, the magnitude of positive restraining moment also increase. Figure 3.11 depicts this trend.



**Figure 3.11** Variability of Creep causing effects vs Magnitude of Positive restraint moment.

### **3.7 Variability of Allowable Stress in Steel Reinforcement and the Effect of Restraining Moments on Crack Control**

All reinforced concrete members are subject to cracking under loading, including thermal effects and restraint of deformations. Crack width is influenced by shrinkage and other time dependent effects and steps should be taken in detailing the reinforcement to control flexural cracking.

Excessive positive moment at the diaphragm may cause the joints to crack if no positive moment connection is provided, which may eventually lead to the loss in continuity. Although tests showed that a bridge system could maintain continuity (Miller et al.2004) even if positive moment cracking occurred at the joint. Loss of continuity does not occur until the slab and diaphragm crack and the connection is near failure.

Improved crack control is obtained when the steel reinforcement is well distributed over the maximum tension zone in concrete. The crack width model developed by Frosch (1999) illustrates that the crack spacing and width are functions of the distance between the reinforcing steel. Maximum bar spacing can be determined by limiting the crack widths to acceptable limits. The limiting crack width was selected as 0.017 in. with Class 1 Exposure and 0.013 in. with Class 2 Exposure (AASHTO-LRFD 2012).The latest AASHTO-LRFD Specifications states that the crack width is directly proportional to the exposure factor  $\gamma_e$ , which ranges from 1 to 0.75.

The stress in the reinforcement at service level in the diaphragm can be computed by taking the total restraining moment divided by the steel area and the internal moment arm. It can also be taken as 60% of the specified yield strength. Several analysis were carried out by varying the stress in steel (24, 36, 48 ksi) to better understand the behavior of limiting crack width on

continuity. The next section provides information about the methodology used to calculate the maximum crack width.

### **3.7.1 Calculation of Maximum Crack Width and Spacing of Reinforcement.**

Robert. J. Frosch (1999) developed an equation for the calculation of maximum crack width:

$$w_c = 2 \frac{f_s}{E_s} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \quad (3.14)$$

where

s = maximum bar spacing

w<sub>c</sub> = limiting crack width

E<sub>s</sub> = Modulus of elasticity of steel E<sub>s</sub>

f<sub>s</sub> = allowable stress in steel (24, 36, 48 ksi)

β = 1.0 + 0.08 d<sub>c</sub>

d<sub>c</sub> = bottom cover.

The maximum spacing of the reinforcement is calculated using the AASHTO-LRFD

Specifications article 5.7.3.4

$$s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c \quad (3.15)$$

$$\beta_s = 1 + \frac{d_c}{0.7(h-d_c)} \quad (3.16)$$

where

γ<sub>e</sub> = Exposure factor

d<sub>c</sub> = Thickness of concrete cover measured from extreme tension fiber to the center of the flexural reinforcement located closest.

$f_{ss}$  = Tensile stress in steel reinforcement at the service limit state (ksi)

$h$  = Overall thickness or depth of the component.

The maximum spacing of the reinforcement is calculated using the AASHTO –LRFD Specification. This value obtained is then incorporated onto equation 3.14 to get the limiting crack width. From the analysis carried out it was found that as the allowable stress increased the spacing between the reinforcements decreased. As the spacing between the reinforcements decreased the magnitude of crack width also reduced. Therefore crack control is achieved by limiting the spacing of reinforcements. A numerical design example is given in Appendix A to show the calculation of crack width by varying the allowable stress. Table 3.4 gives the various values of the allowable steel stress.

**Table 3.5 Variability of allowable stress in the diaphragm**

S.No.	Allowable Steel Stress (ksi.)	Crack width (in.)	Spacing of reinforcement provided (in).	Maximum spacing of reinforcement (in).
1).	24	0.0069	6	16
2).	36	0.0103	6	10
3).	48	0.0138	6	6.31

## Chapter 4

### Cost Analysis of Various Continuity Systems in the United States

#### 4.1 Design, Detailing Consideration and Practices:

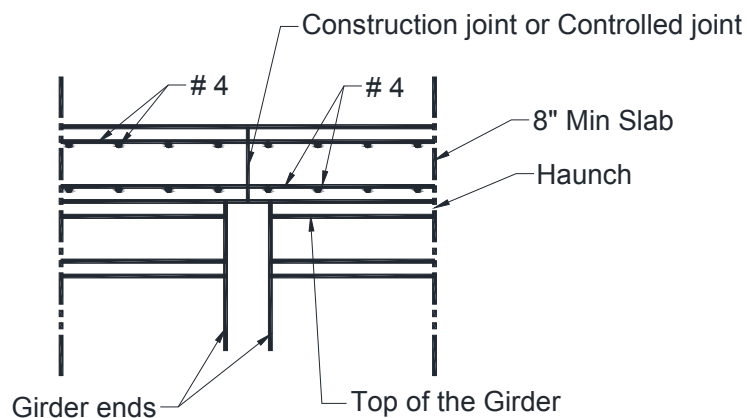
##### 4.1.1 Deck Slabs for Continuity

Based on their past experience, the States of Florida, Georgia, Texas and Minnesota are not using continuity in the design for the precast prestressed concrete girder bridges. Florida and Texas with long history of using prestressed concrete girders and has a reputation of low bridge construction costs. Most of these superstructures are designed as simply supported girders supporting a longitudinally continuous reinforced concrete over one to three interior supports. Of the 32,547 bridges in the Texas Bridge inspection database, 325 have superstructures listed as reinforced concrete slab on precast, prestressed concrete beams made continuous for live load. This type of structure is designed and detailed to resist non-composite loads by simple span action and composite loads by continuous beam action.

After experiencing difficulty in attaining continuity, minor cracking and spalling of concrete at the pier diaphragm, the Texas Highway Department has abandoned the practice of continuity and mainly designs simple span girder bridge. This is because the reduction of expansion joints and the associated reduction in structural deterioration, is more easily achieved by making the slab continuous and designing the precast concrete beams as simple spans. The only significant effect of the interior diaphragms is to distribute the load more evenly across the bridge and there are no appreciable reduction in the governing design moment found. Based on the results, it was recommended that interior diaphragms should not be provided in simple

supported prestressed concrete girder and slab bridges. Since the majority of the Texas precast concrete girders are modified AASHTO Type IV girders up to 45 m (150 feet), High Performance Concrete (HPC) is used. The continuity design was performed in accordance with the AASHTO-LRFD Specifications. The main advantages of this system is that it was easy to construct and relatively economical. However, cracks developed at the bottom of the diaphragm due to positive restraint moment over the piers resulting from creep.

Figure 4.1 shows the details of Texas continuous slab with the prestressed girders.



**Figure 4.1 Texas Department of Transportation, Continuous Slab over Pier**

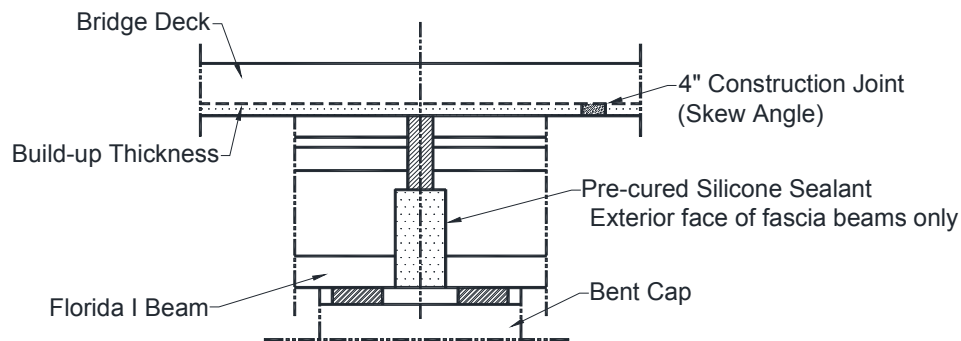
In Florida, decks of the bridges are designed using the traditional design method of AASHTO-LRFD, while the empirical design method is not permitted due to the potential for future widening or phased construction and associated traffic impacts. When the cast-in-slabs are made composite with simple span concrete beams, and are cast continuous over intermediate

piers or bents, a supplemental longitudinal reinforcing should be placed in the top of slabs. Size, space, and place reinforcing are in accordance with the following criteria:

- No. 5 Bars placed between the continuous, longitudinal reinforcing bars.
- A minimum of 35 feet in length or  $2/3$  of the average span length whichever is less.
- Placed symmetrically about the centerline of the pier or bent, with alternating bars staggered 5 feet. (FDOT, Structures Detailing Manual, Volume 2 January 2014.).

A sequence and direction of each deck concrete pour should be planned to minimize cracking in the continuous slab and girders superstructures. This sequence should result in construction joints spaced approximately at locations of the inflection points of the dead load moments.

Design details are shown in Figure 4.2:



**Figure 4.2 Florida Department of Transportation Design Details for continuous Deck over pier**

#### **4.1.2 Diaphragm over Piers to Resist Live Load and Superimposed Dead Load**

The States of Alabama, Colorado, Illinois, Iowa, Michigan, Nebraska, Pennsylvania, South Carolina, Tennessee, and Washington, Wisconsin have the most experience with this type of bridge. The construction of this bridge system includes the following steps:



- Erecting and aligning precast prestressed girders.
- Connecting positive moment reinforcement.
- Installing diaphragm and deck reinforcement.
- Casting diaphragm and deck concrete.

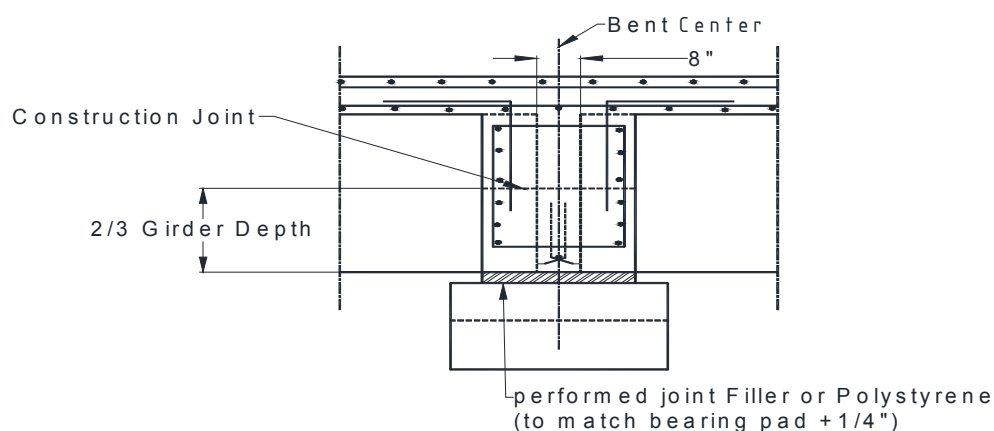
The advantage of this kind of construction is that it achieves continuity under live load and secondary dead loads. It is still simply supported under girder, deck self-weight and construction loads. From a maintenance perspective, continuous spans are more advantageous than simple spans since they eliminate expansion joints. If designed properly, continuous concrete bridges can be maintenance free, while bridges composed of simple spans need regular inspection and maintenance. From a structural point of view, it is desirable to achieve continuity not only for live loads, but also for girder and slab dead loads. More continuity means shallower sections or longer spans, which in turn will reduce the total cost of the bridge. The continuity of such bridges range from 0 percent to 100 percent, depending on the loading condition, construction sequence, material properties of the concrete and reinforcement, and structural parameters such as span length, girder geometry, etc.

Continuity connections also have their own structural, construction, and maintenance shortcomings. Due to time dependent effects the girders tend to camber upward even after continuity is established, the established continuity tends to keep the girder ends from rotating, which results in positive moment and cracks usually develop at the bottom of the diaphragms. These cracks may cause corrosion of the reinforcement in diaphragms, leading to maintenance problems.

Michigan and Utah do not use positive moment reinforcement in their continuity joints at all and are satisfied with their performance. However, both Michigan and Utah design their bridges as simply supported for all loads.

The State of Iowa extends and bends the top reinforcement in order to improve the integrity of the structure. However, the beams are designed as simply supported for all loads because of cracking problem. Tennessee uses wider diaphragm in order to prevent overlap of the positive moment reinforcement and not to embed the girder. The anchor bolts are designed for seismic loads, however, the bolts are placed in sheath to prevent bonding with diaphragm concrete and to allow girder end rotation.

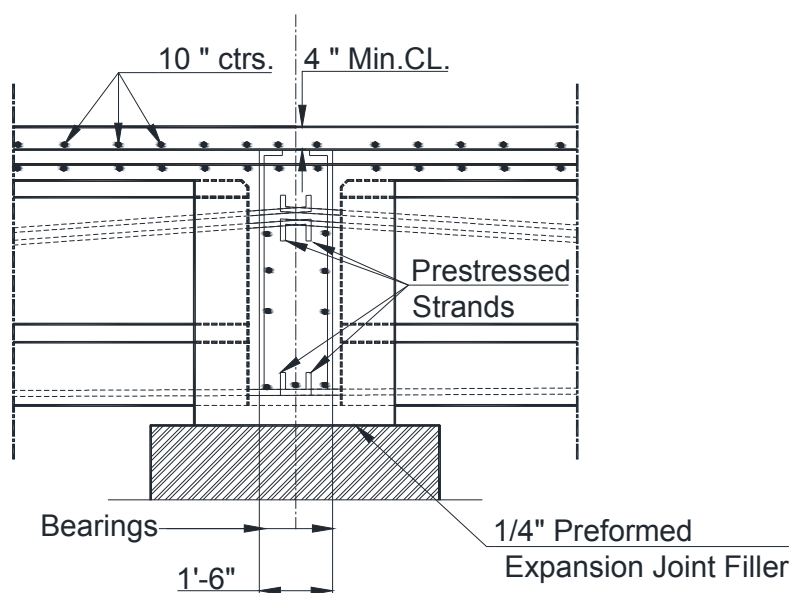
In Nebraska, diaphragms at the pier (or bent) require a mandatory construction joint at a point  $2/3$  of the girder height measured from the bottom of the girder as shown in Figure 4.3. Details shown are the minimum reinforcement, and designers should calculate the required reinforcement on a case-by-case basis. (Nebraska Department of Roads, Bridge Office Policies and Procedures 2014).



**Figure 4.3 Nebraska Department of Transportation Connection Details**

Iowa DOT design policy is to design beams using simple span condition for all strength and services stress checks and add longitudinal slab reinforcement to the concrete deck above continuous pier supports to avoid deck joint and to control tension cracking. However, the longitudinal reinforcement and continuity diaphragms will cause the superstructure to behave approximately as a continuous structure for deflections and abutment and pier loads.

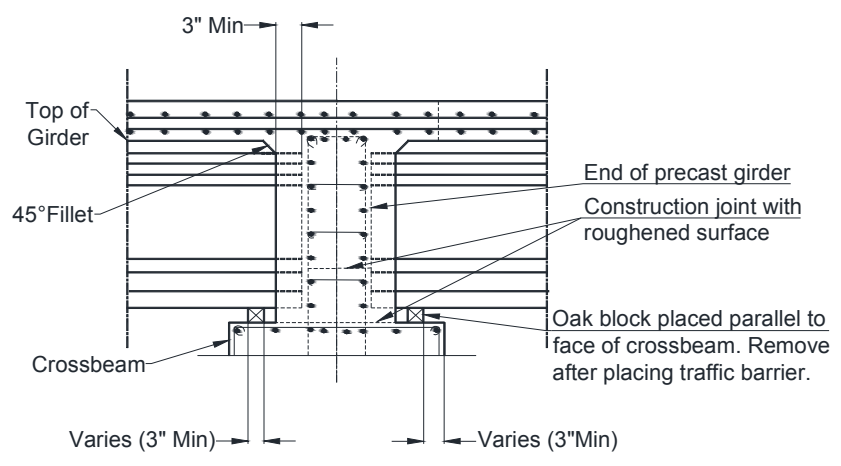
Generally the beams and decks were adequate for all continuity checks near and at a pier. With the development of longer beams, however, service checks at the transfer points and compression checks for negative moment at continuity diaphragms begin to fail under some conditions. As the result, the office has decide not to check the continuous condition for concrete compression. (Iowa Department of Transportation, LRFD Bridge Design Manual).



**Figure 4.4 Iowa Department of Transportation, Continuity beam Standard Details**

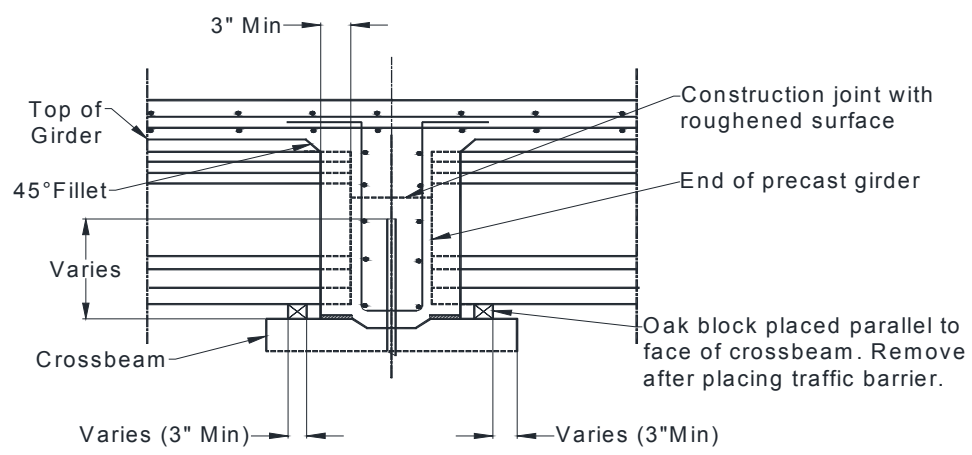
Washington Department of Transportation (WSDOT) allows bridges composed of simple span precast girders to have some degree of continuity for loads applied on the bridge, after the continuity diaphragms have been cast and cured. This assumption is based on the age of the girder when continuity is established, and the degree of continuity at various limit states. The envelope of simple span and continuous spans for applicable permanent and transient loads is used to design these bridges by WSDOT and it has yielded good results. Loads applied before establishing continuity (typically before placement of continuity diaphragms) need only be applied as a simple span loading. Continuity reinforcement is provided at supports for loads applied after establishing continuity.

Figure 4.5 shows a type of girder end is used for continuous spans and an intermediate hinge diaphragm at an intermediate pier. There is no bearing recess and the girder is temporarily supported on oak blocks. This detail is generally used only in low seismic areas. The designer should check the edge distance and provide a dimension that prevents edge failure. The designer should also check to prevent spalling at the top corner of the supporting cross beam for load from the oak block, including dead loads from girder, deck slab, and construction loads. In addition, the prestressed girders should be checked for the size and minimum embedment hinge bars in diaphragm and for the interface shear friction at girder end.



**Figure 4.5 Washington State DOT, Type (D) Intermediate pier connection for continuous spans fully fixed to columns**

Figure 4.6 shows another type of girder end used for continuous spans fully fixed to columns at intermediate piers. There is no bearing recess and the girder is temporarily supported on oak blocks. (Washington Department of Transportation, LRFD Bridge Design Manual)



**Figure 4.6 Washington State DOT, End Type (c) Intermediate Hinge Diaphragm**

## 4.2 Advantages and Disadvantages of various systems made continuous for Live Load

The following table summarizes the advantages and disadvantages of the various continuity systems discussed in the previous section.

S.No.	Continuity System	Practicing States	Advantages	Disadvantages
1).	Deck Slabs for continuity	<ul style="list-style-type: none"> <li>• Texas</li> <li>• Florida</li> <li>• Georgia</li> <li>• Minnesota</li> </ul>	<ul style="list-style-type: none"> <li>• Simple to construct and relatively economical</li> <li>• Reduction in the number of expansion joints</li> </ul>	<ul style="list-style-type: none"> <li>• As the girder deflects under live load. Lateral cracking is caused on the surface of the deck. This allows the water to leak through the cracks and damages the bearing area as well as corrodes the reinforcement.</li> <li>• Maximum span length was restricted. Increasing span length makes the transportation difficult and expensive</li> </ul>
2).	Diaphragm over piers	<ul style="list-style-type: none"> <li>• Washington</li> <li>• Virginia</li> <li>• Illinois</li> <li>• Nebraska</li> <li>• Colorado</li> <li>• New Jersey</li> <li>• Missouri</li> <li>• Vermont</li> <li>• Utah</li> <li>• Kansas</li> <li>• Ohio</li> <li>• Oregon</li> <li>• Pennsylvania</li> <li>• Alaska</li> <li>• Idaho</li> <li>• New York</li> <li>• Delaware</li> </ul>	<ul style="list-style-type: none"> <li>• Connection was easy to fabricate and erect.</li> <li>• Cracking at the girder-diaphragm interface could be controlled by providing additional reinforcement.</li> <li>• Reduced maintenance costs when compared to simple span bridges. Elimination of expansion joints ensures smooth riding.</li> </ul>	<ul style="list-style-type: none"> <li>• Discrepancies in the design procedures for determining the number and embedment length of the prestressing strands.</li> <li>• If positive moment reinforcements are not provided in the diaphragm, crack develop at the bottom of the diaphragm.</li> <li>• The system that used bent bars required the bars to be bent consistently in the field. Due to closure of forms this was difficult to achieve.</li> </ul>

**Table 4.1 Advantages and disadvantages of various continuity systems adopted.**

### 4.3 Cost Comparisons of various continuity systems

Most states only track cost data on a project basis, or at best, separate bridge and roadways cost. Detailed cost tabulations are difficult to gather once a bridge has been constructed. Items such as concrete and reinforcing steel, may be lumped together without regard to their function in the bridge. This makes it hard to distinguish superstructure costs from substructure cost. There are two main criteria to analyze the variation in cost among the different continuity systems are:

- Short Term Initial Construction Cost
- Long Term Performance Cost.

These criteria can be further classified as Low, Medium and High, with Low denoting the least cost and High denoting a large cost. The following Table 4.2 gives the cost comparison between various continuity systems adopted in the United States.

S.No.	Continuity systems	Short Term Initial Construction cost	Long Term performance cost
1).	Deck Slabs for continuity	High – due to large number simple span girders and large number of strands. Cost of concreting and cost of steel is high.	Medium – As the girder deflects under live load. Lateral cracking is caused on the surface of the deck. This allows the water to leak through the cracks and damages the bearing area as well as corrodes the reinforcement. Substantial maintenance cost. Expansion joints should be provided
2).	Diaphragm over piers		
	a) Bent Strands	Low- Fabrication of girders is fairly simple. There is no need to	Low –No maintenance, repairs, or expansion joints are

		modify forms and by extending the strands there is no congestion in the diaphragm.	required. No cracking or spalling of diaphragm.
	b) Bent Bars	High – Initial cost is high as the forms have to be modified to include the holes. Large reinforcements have larger bend diameter.	Medium- Large number of bars in the diaphragm can cause congestion. This leads to concentration of stress over a small area and causes the cracking of member

**Table 4.2 Cost Comparisons of various continuity systems**

From Table 4.2 and 4.3 it can be observed that most cost effective approach for achieving continuity in precast/prestressed bridges would be to utilize the extended or bent strand continuity system.



## Chapter 5

### Conclusions and Recommendations

#### 5.1 Conclusions

This study focused on the methods of calculating the positive restraining moment developed in a continuous precast/prestressed concrete girder bridge system. The following conclusions were drawn:

- The age of the girder when continuity is established is vital in establishing the magnitude of the positive restraining moment. It was found that the magnitude of the net positive moment decreased with an increase in the age of the girder. It is recommended that the connection should be designed for the positive restraining moment at the face of the diaphragm.
- Construction sequence of deck and pier diaphragms affects the development of positive restraining moments. Casting the deck prior to the construction of the diaphragm increases the resultant positive moments, while casting the deck after the diaphragms are constructed creates minimal or no positive moment at all.
- From the variability analysis it was found that creep was the major contributor to the formation of positive moments.
- The thermal analysis showed that the temperature effects on the system are significant. At present, few design methods account for the temperature effects, but the moment induced can be as significant as caused by the live load.

## 5.2 Recommendations

The following are some of the recommendations that may be considered for adoption in the AASHTO-LRFD Specifications: The current specifications does not contain a method for designing the positive moment connection. A cost effective method to make a positive moment connection is by extending the prestressing strand from the end of the girder, bending it at 90° and then embedding the bent strand into the continuity diaphragm.

- The existing specifications do not address detailed methods of analysis for determining time-dependent material properties. Based on the information gained in the literature review and the analyses conducted in this research, suggested time dependent material properties and analysis methods are presented.
- The current specifications stipulate that a positive moment connection with a strength of  $1.2 M_{cr}$  and the girders must be at least 90 days old at the time continuity is established. From the analysis, it is recommended that connection should be designed for the maximum positive restraining moment at the face of the diaphragm.

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## Appendix

## APPENDIX -A

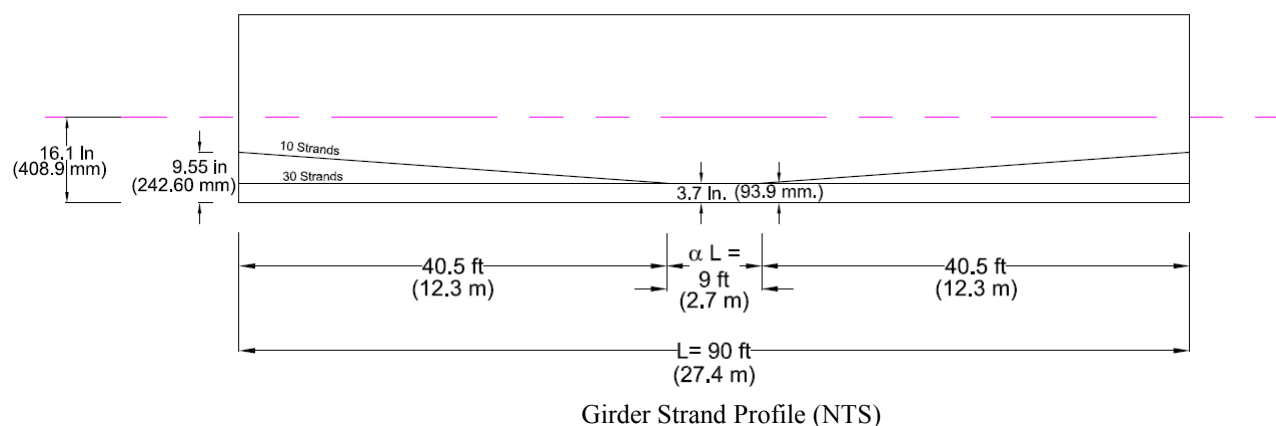
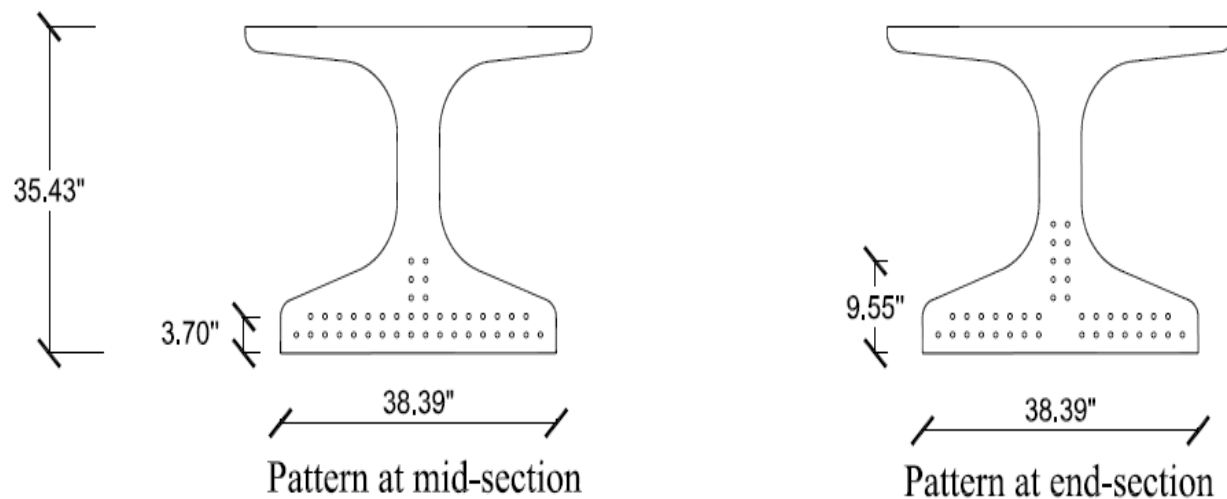
### SENSITIVITY ANALYSIS

#### Numerical Hand calculated example of Restraint Moments in a Two-Span Bridge

##### Case 1. When deck is cast 7 days after the girder construction

##### Bridge Data

Geometry	
Bridge width	50 ft. (15.240 m)
Bridged length	2 spans x 90 ft. = 180 ft. (54.864 m)
Bridge Skew:	0 degree angle
Girder:	
Girder Spacing:	10 ft. (3.048 m)
Girder strength:	5.5 ksi at release (37.9 MPa, cylinder strength. Cube may be 1.15 times cylinder)
Girder strength:	8 ksi at 28 days (55.2 MPa, cylinder strength)
Girder Prestress:	40-0.6" low relaxation strands [Bottom cover to strand C.L. = 2" (50 mm)]
Deck thickness:	8 in. CIP concrete (203 mm), plus a minimum 1 in. (25 mm) haunch
Deck strength:	4 ksi at 28 days (27.6 MPa, cylinder)
Loads:	
Barrier:	20 psf. (0.96 kN/m <sup>2</sup> )
Future Wearing surface:	25 psf. (1.12 kN/m <sup>2</sup> )
The design live load:	HL-93
Relative Humidity	70%
Construction girder	
Prestressing strand released:	1 day
Diaphragm and Deck construction:	7 days
End of Girder life	20000 days
Design Specifications:	AASHTO LRFD Bridge Design Specifications 2012



### Girder Section properties:

Moment of Inertia	$I = 110,444 \text{ in}^4 (0.458 \text{ m}^4)$
Area	$A = 649 \text{ in}^2 (0.419 \text{ m}^2)$
Height	$h = 35.43 \text{ in. } (0.900 \text{ m})$
Centroid to bottom fiber	$y_b = 16.10 \text{ in. } (0.409 \text{ m})$
Girder Weight	$w = 0.676 \text{ k/ft. } (9.863 \text{ KN/m})$

### Material Properties

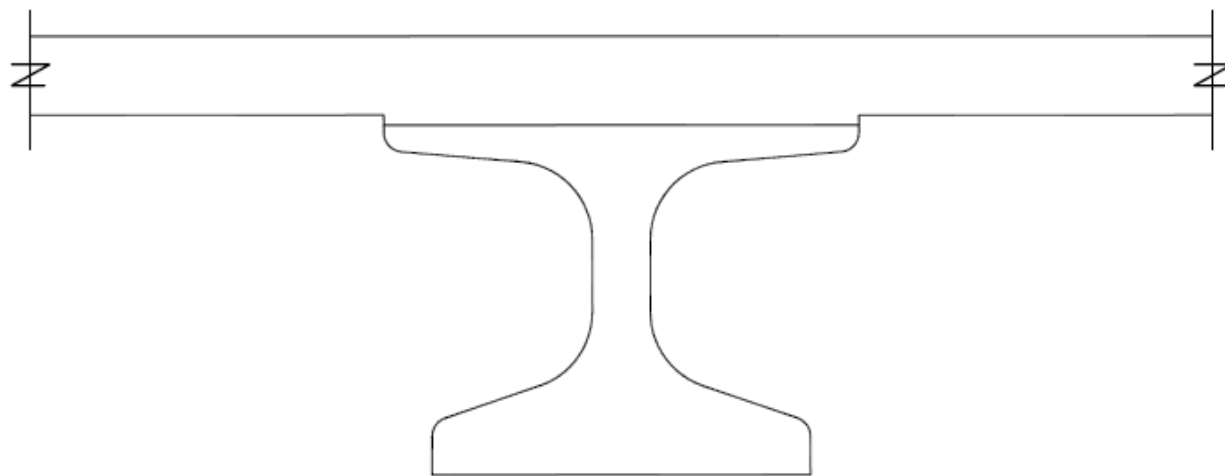
Modulus of Elasticity			
Girder	Initial	$E_{ci} =$	4,406 ksi (30,400 MPa)
	At deck placement	$E_c =$	5,314 ksi (36,600 MPa)
Deck	At 28 days	$E_{cd} =$	3,607 ksi (24,900 MPa)



## Shrinkage Strains

Girder	Initial to final	$\epsilon_{bif}$	0.000393
	Initial to deck placement	$\epsilon_{bid}$	0.000053
	Deck placement to final	$\epsilon_{bdf}$	0.000341
Deck	Deck placement to final	$\epsilon_{ddf}$	0.000274
Creep coefficients			
Girder	Initial to final	$\psi_{bif}$	1.526
	Initial to deck placement	$\psi_{bid}$	0.204
	Deck placement to final	$\psi_{bdf}$	1.213
Deck	Deck placement to final	$\psi_{ddf}$	2.126

Note: creep coefficient = creep strain/initial strain for a constant sustained stress.



Girder cross section

### Restraint moment due to Time dependent effects according to the Bridge Design manual 8.13.4.3.2.1

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, there are pretensioning forces, member self-weight and possibly deck weight. Each loading case is considered separately. The total effect is obtained by simple superposition.

Perform elastic analysis, assuming that the load was introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports,  $M_{el}$  :

$M_o$ : elastic moment due to effect of self weight of the girder:

$$M_o = \frac{wl^2}{8} = 0.676 \times (90)^2/8 = 684.5 \text{ ft-kips (928.1 KN-m)}$$

w: weight of the girder = 0.676 k/ft. (9.865 KN/m)

L: length of the girder = 90 ft. (27.4 m)

$M_d$ : elastic moment due to effect of weight of the deck, (load applied before continuity is made):

$$M_d = \frac{w_d l^2}{8} = 1,063.1 \text{ ft-kips (1,441.4 KN-m)}$$

$w_d$ : weight of the deck = 1.05 k/ft. (15.323 KN/m)

$M_p$ : elastic moment due to prestress release, assuming as if the beam was continuous before load applied:

$$M_p = \frac{3P}{4} [2e_e + (1+\alpha)(e_c - e_e)] = 3 \times 1,581.9 / 4 [2 \times 10.9 + (1+0.1) (12.4-10.9)] / 12$$

$$= 2,303.6 \text{ ft-kips. (3,123.3 KN-m)}$$

$$e_e = 16.1 - (10 \times 9.55 + 30 \times 3.7)/40 = 10.9 \text{ in. (276.9 mm)}$$

$$e_c = 16.1 - 3.7 = 12.4 \text{ in. (315.0 mm)}$$

P: 90% of the strand prestressing force = 0.90 x 40 x 0.217 x 202.5 = 1,581.9 kips (7,036.6 KN)

$M_{Barrier}$ : elastic moment due to effect of self-weight of the barrier:

$$M_{Barrier} = \frac{wl^2}{8} = 0.20 \times (90)^2/8 = 202.5 \text{ ft-kips (274.6 KN-m)}$$

w: Barrier and Wearing Surface = 0.020 x 10 = 0.20 k/ft. (2.918 KN/m)

L: length of the girder = 90 ft. (27.4 m)

Age-adjusted effective modulus for concrete subjected to gradually introduced restraining moment from time of deck placement to time.

$$E_{ctv}^*(t, t_d) = \frac{E_c(t_d)}{1+0.7\psi(t, t_d)} = \frac{5,314}{1+0.7(1.213)} = 2,873 \text{ ksi. (20 GPa)}$$

Age-Adjusted Effect Modulus for concrete subjected to constant stress introduced at  $t_i$ , with creep determined to the period  $(t-t_d)$

$$E_{ctc}^*(t, t_i) = \frac{E_c(t_i)}{\Psi_{bif} - \Psi_{bid}} = \frac{4,406}{1.526 - 0.204} = 3,333 \text{ ksi. (23 GPa)}$$

Determine the time-dependent multiplier,  $\delta_1$  corresponding to prestressing and girder self-weight:

$$\delta_1 = \frac{E_{ctv}^*(t, t_d)}{E_{ctc}^*(t, t_0)} = \frac{2,873}{3,333} = 0.862$$

Determine the time-dependent multiplier,  $\delta_2$  due to deck weight:

$E_{cd}$ : age-adjusted effective modulus of elasticity for beam concrete due deck weight:

$$E_{cd} = \frac{E_c(t_d)}{1+1.0\psi(t, t_d)} = \frac{5,314}{1+1.0(1.213)} = 2,401 \text{ ksi (17 GPa)}$$

Determine the time-dependent multiplier,  $\delta_2$  corresponding to deck load:

$$\delta_2 = \frac{2,401}{3,333} = 0.720$$

**Determine the restraint moment  $M_r$ :**

Restraint moment due girder weight:

$$M_{r1} = \delta_1 \times M_o = 0.862 \times -684.5 = -590.0 \text{ ft.-kips. (799.9 KN-m)}$$

Restraint moment due to prestressing

$$M_{r2} = \delta_1 \times M_p = 0.862 \times 2,303.6 = 1,985.7 \text{ ft.-kips. (2,692.2 KN-m)}$$

Note: the moment due to prestressing is positive and the moment due self-weight of the beam is negative.

Restraint moment due to deck weight:

$$M_{r3} = 0.720 \times -1,063.1 = -765.4 \text{ ft.-kips. (1025.5 KN-m)}$$

**Total restraint moment**

Restraint moment due to girder weight	=	-590.0 ft.-kips.
Restraint moment due to prestressing	=	1,985.7 ft.-kips.
Restraint moment due to deck weight	=	-765.4 ft.-kips.
Restraint moment due to temperature	=	570.2 ft.-kips
Restraint moment due to deck shrinkage	=	-354.2 ft.-kips
<u>Elastic moment due to barrier weight</u>	=	<u>-202.5 ft.-kips.</u>

**Total Net moment** = **645.5 ft.-kips. (875.2KN-m)**

Elastic restraint moment for future wearing surface and live loads were not accounted for in this example. The reason these loads create negative moments causing compression at the bottom of the connection and are not permanently present throughout the lifetime of the girder. Therefore, this is a reasonable design practice.

**Diaphragm reinforcement:**

H = 35.4 in: (899.2 mm)

Z = 35.4 (899.2 mm) (beam height) + 4 in. (101.6 mm) (half of the deck slab thickness) + 1 in. (25.4 mm) (Haunch) – 2 in (50.8 mm) (distance from the beam soffit to the bottom reinforcement) = 38.4 in. (975.4mm)

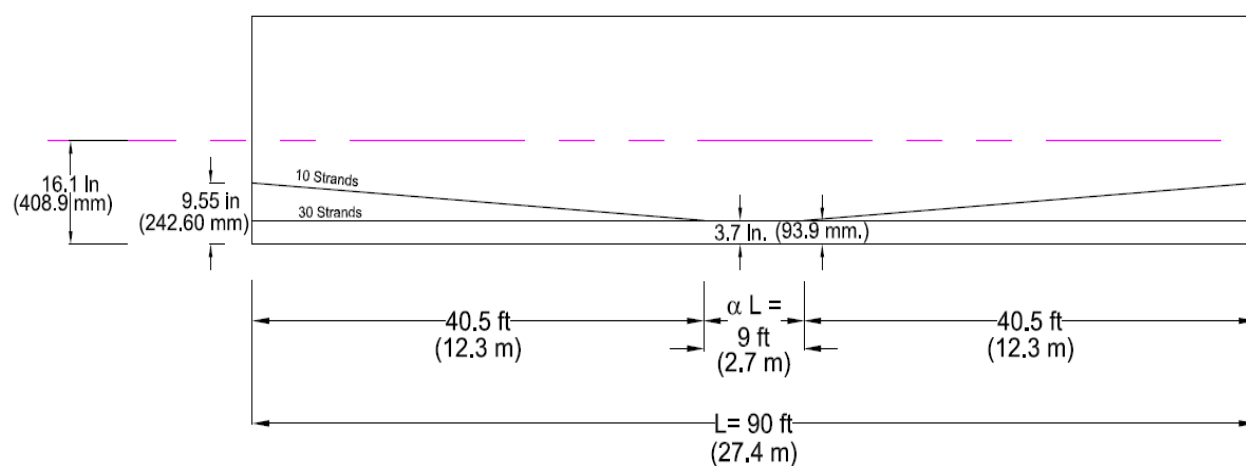
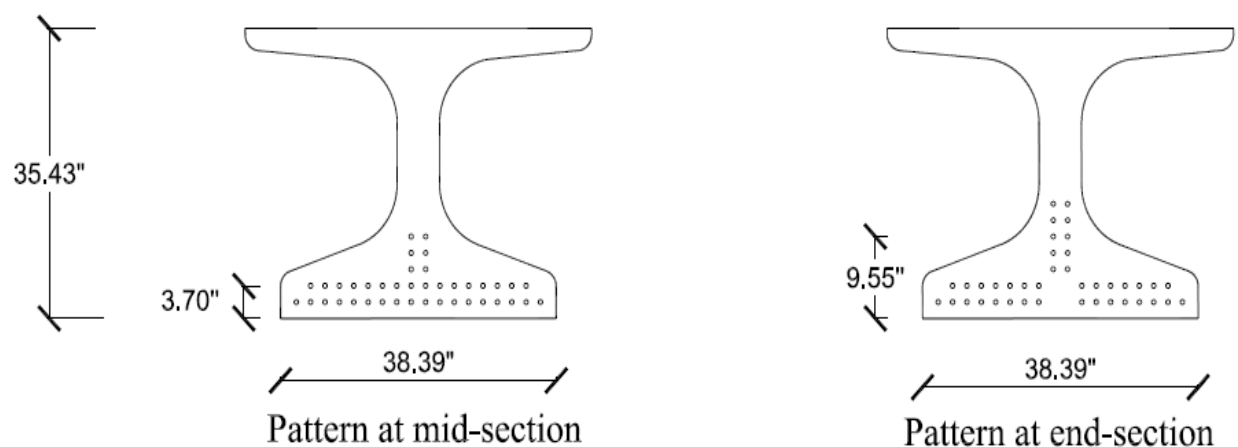
$$A_s = \frac{M_r}{zf_s} = \frac{645.5 \times 12}{38.4 \times 36} = 5.603 \text{ in}^2 (3615 \text{ mm}^2)$$

Use 26 strands from the bottom row flange.

## Case 2. When deck is cast 28 days after the girder construction

### Bridge Data

Geometry	
Bridge width	50 ft. (15.240 m)
Bridged length	2 spans x 90 ft. = 180 ft. (54.864 m)
Bridge Skew:	0 degree angle
Girder:	
Girder Spacing:	10 ft. (3.048 m)
Girder strength:	5.5 ksi at release (37.9 MPa, cylinder strength. Cube may be 1.15 times cylinder)
Girder strength:	8 ksi at 28 days (55.2 MPa, cylinder strength)
Girder Prestress:	40-0.6" low relaxation strands [Bottom cover to strand C.L. = 2" (50 mm)]
Deck thickness:	8 in. CIP concrete (203 mm), plus a minimum 1 in. (25 mm) haunch
Deck strength:	4 ksi at 28 days (27.6 MPa, cylinder)
Loads:	
Barrier:	20 psf. (0.96 kN/m <sup>2</sup> )
Future Wearing surface:	25 psf. (1.12 kN/m <sup>2</sup> )
The design live load:	HL-93
Relative Humidity	70%
Construction girder	
Prestressing strand released:	1 day
Diaphragm and Deck construction:	28 days
End of Girder life	20000 days
Design Specifications:	AASHTO LRFD Bridge Design Specifications 2012



Girder Strand Profile (NTS)

### Girder Section properties:

Moment of Inertia	$I = 110,444 \text{ in}^4 (0.458 \text{ m}^4)$
Area	$A = 649 \text{ in}^2 (0.419 \text{ m}^2)$
Height	$h = 35.43 \text{ in. } (0.900 \text{ m})$
Centroid to bottom fiber	$y_b = 16.10 \text{ in. } (0.409 \text{ m})$
Girder Weight	$w = 0.676 \text{ k/ft. } (9.863 \text{ KN/m})$

### Material Properties

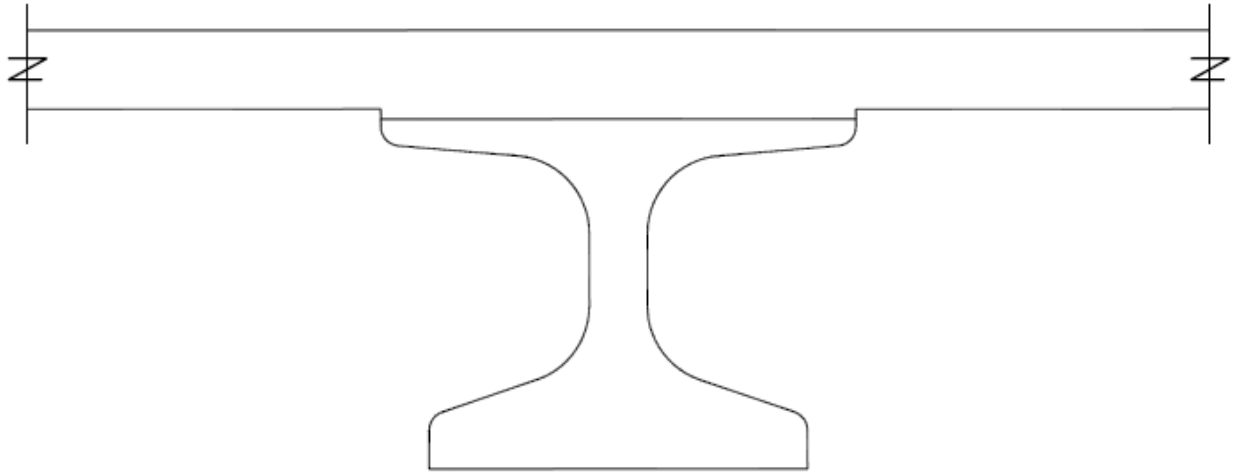
#### Modulus of Elasticity

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	At deck placement	$E_c = 5,314 \text{ ksi } (36,600 \text{ MPa})$
Deck	At 28 days	$E_{cd} = 3,607 \text{ ksi } (24,900 \text{ MPa})$

## Shrinkage Strains

Girder	Initial to final	$\epsilon_{bif}$	0.000393
	Initial to deck placement	$\epsilon_{bid}$	0.000161
	Deck placement to final	$\epsilon_{bdf}$	0.000232
Deck	Deck placement to final	$\epsilon_{ddf}$	0.000274
Creep coefficients			
Girder	Initial to final	$\psi_{bif}$	1.526
	Initial to deck placement	$\psi_{bid}$	0.626
	Deck placement to final	$\psi_{bdf}$	1.030
Deck	Deck placement to final	$\psi_{ddf}$	2.126

Note: creep coefficient = creep strain/initial strain for a constant sustained stress.



Girder cross section

### Restraint moment due to Time dependent effects according to the Bridge Design manual 8.13.4.3.2.1

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, there are pretensioning forces, member self-weight and possibly deck weight. Each loading case is considered separately. The total effect is obtained by simple superposition.

Perform elastic analysis, assuming that the load was introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports,  $M_{el}$  :

$M_o$ : elastic moment due to effect of self weight of the girder:

$$M_o = \frac{wl^2}{8} = 0.676 \times (90)^2/8 = 684.5 \text{ ft-kips (928.1 KN-m)}$$

w: weight of the girder = 0.676 k/ft. (9.865 KN/m)

L: length of the girder = 90 ft. (27.4 m)

$M_d$ : elastic moment due to effect of weight of the deck, (load applied before continuity is made):

$$M_d = \frac{w_d l^2}{8} = 1,063.1 \text{ ft-kips (1,441.4 KN-m)}$$

$w_d$ : weight of the deck = 1.05 k/ft. (15.323 KN/m)

$M_p$ : elastic moment due to prestress release, assuming as if the beam was continuous before load applied:

$$M_p = \frac{3P}{4} [2e_e + (1+\alpha)(e_c - e_e)] = 3 \times 1,581.9 / 4 [2 \times 10.9 + (1+0.1) (12.4-10.9)] / 12$$

$$= 2,303.6 \text{ ft-kips. (3,123.3 KN-m)}$$

$$e_e = 16.1 - (10 \times 9.55 + 30 \times 3.7)/40 = 10.9 \text{ in. (276.9 mm)}$$

$$e_c = 16.1 - 3.7 = 12.4 \text{ in. (315.0 mm)}$$

P: 90% of the strand prestressing force = 0.90 x 40 x 0.217 x 202.5 = 1,581.9 kips (7,036.6 KN)

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w: Barrier and Wearing Surface = 0.020 x 10 = 0.20 k/ft. (2.918 KN/m)

L: length of the girder = 90 ft. (27.4 m)



Age –adjusted effective modulus for concrete subjected to gradually introduced restraining moment from time of deck placement to time.

$$E_{ctv}^*(t, t_d) = \frac{E_c(t_d)}{1+0.7\psi(t, t_d)} = \frac{5,314}{1+0.7(1.030)} = 3,088 \text{ ksi. (21 GPa)}$$

Age-Adjusted Effect Modulus for concrete subjected to constant stress introduced at  $t_i$ , with creep determined to the period (t- $t_d$ )

$$E_{ctc}^*(t, t_i) = \frac{E_c(t_i)}{\psi_{bif} - \psi_{bid}} = \frac{4,406}{1.526 - 0.626} = 4,896 \text{ ksi. (34 GPa)}$$

Determine the time-dependent multiplier,  $\delta_1$  corresponding to prestressing and girder self-weight:

$$\delta_1 = \frac{E_{ctv}^*(t, t_d)}{E_{ctc}^*(t, t_0)} = \frac{3,088}{4,896} = 0.630$$

Determine the time-dependent multiplier,  $\delta_2$  due to deck weight:

$E_{cd}$ : age-adjusted effective modulus of elasticity for beam concrete due deck weight:

$$E_{cd} = \frac{E_c(t_d)}{1+1.0\psi(t, t_d)} = \frac{5,314}{1+1.0(1.030)} = 2,618 \text{ ksi (18 GPa)}$$

Determine the time-dependent multiplier,  $\delta_2$  corresponding to deck load:

$$\delta_2 = \frac{2,618}{4,896} = 0.535$$

Determine the restraint moment  $M_r$ :

Restraint moment due girder weight:

$$M_{r1} = \delta_1 \times M_o = 0.630 \times -684.5 = -431.2 \text{ ft-kips. (584.6 KN-m)}$$

Restraint moment due to prestressing

$$M_{r2} = \delta_1 \times M_p = 0.630 \times 2,303.6 = 1,451.3 \text{ ft-kips. (1,967.7 KN-m)}$$

Note: the moment due to prestressing is positive and the moment due self-weight of the beam is negative.

Restraint moment due to deck weight:

$$M_{r3} = 0.535 \times -1,063.1 = -568.8 \text{ ft.-kips. (771.2 KN-m)}$$

**Total restraint moment**

Restraint moment due to girder weight	=	-431.2 ft.-kips.
Restraint moment due to prestressing	=	1,451.3 ft.-kips.
Restraint moment due to deck weight	=	-568.8 ft.-kips.
Restraint moment due to temperature	=	570.2 ft.-kips
Restraint moment due to deck shrinkage	=	-352.4 ft.-kips
<u>Elastic moment due to barrier weight</u>	=	<u>-202.5 ft.-kips.</u>

**Total Net moment** = **466.5 ft.-kips. (632.5 KN-m)**

Elastic restraint moment for future wearing surface and live loads were not accounted for in this example. The reason these loads create negative moments causing compression at the bottom of the connection and are not permanently present throughout the lifetime of the girder. Therefore, this is a reasonable design practice.

**Diaphragm reinforcement:**

H = 35.4 in: (899.2 mm)

Z = 35.4 (899.2 mm) (beam height) + 4 in. (101.6 mm) (half of the deck slab thickness) + 1 in. (25.4 mm) (Haunch) – 2 in (50.8 mm) (distance from the beam soffit to the bottom reinforcement) = 38.4 in. (975.4mm)

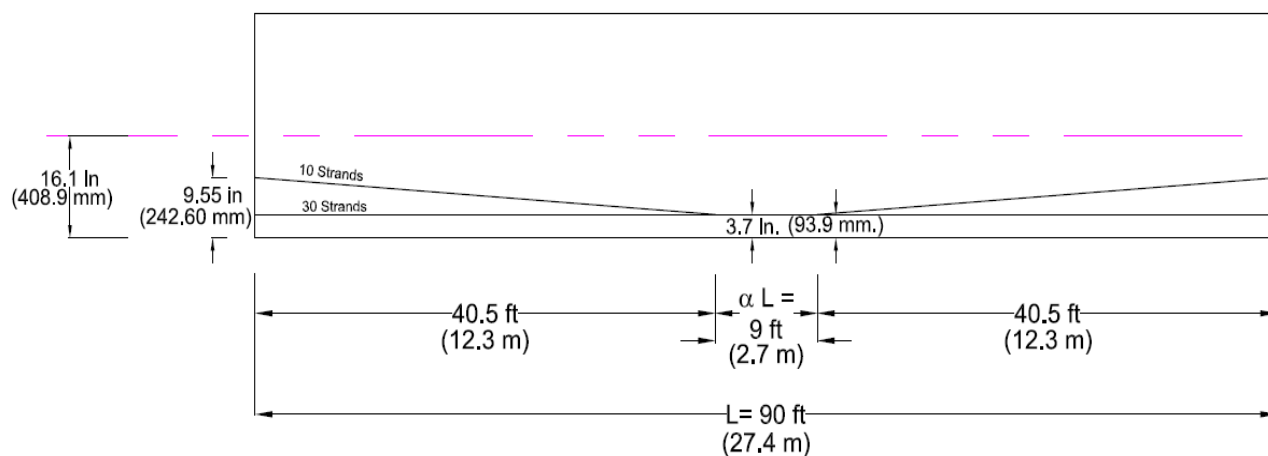
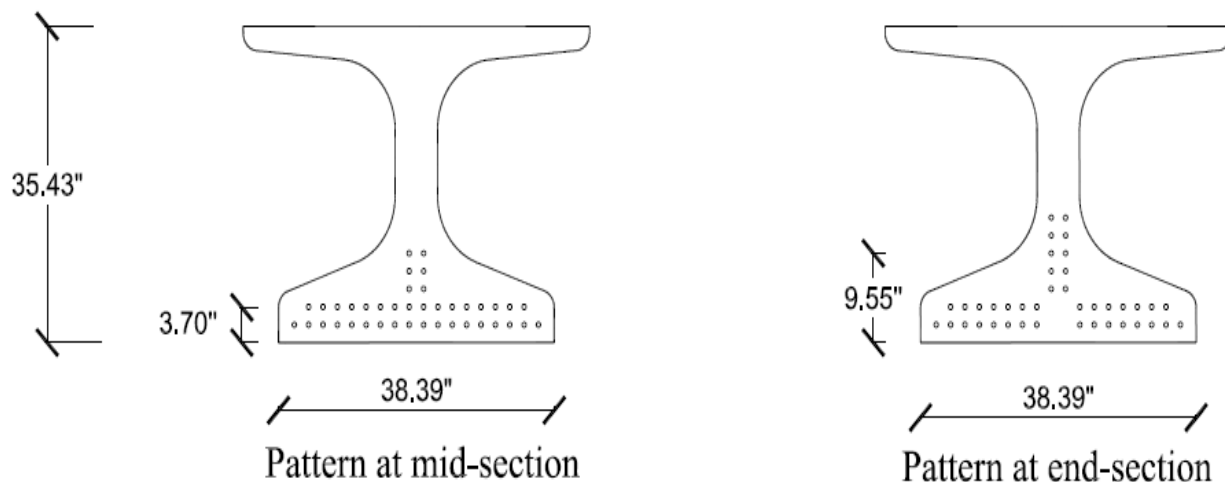
$$A_s = \frac{M_r}{z f_s} = \frac{466.5 \times 12}{38.4 \times 36} = 4.049 \text{ in}^2 (2612 \text{ mm}^2)$$

Use 19 strands from the bottom row flange.

### Case 3. When deck is cast 42 days after the girder construction

#### Bridge Data

Geometry	
Bridge width	50 ft. (15.240 m)
Bridged length	2 spans x 90 ft. = 180 ft. (54.864 m)
Bridge Skew:	0 degree angle
Girder:	
Girder Spacing:	10 ft. (3.048 m)
Girder strength:	5.5 ksi at release (37.9 MPa, cylinder strength. Cube may be 1.15 times cylinder)
Girder strength:	8 ksi at 28 days (55.2 MPa, cylinder strength)
Girder Prestress:	40-0.6" low relaxation strands [Bottom cover to strand C.L. = 2" (50 mm)]
Deck thickness:	8 in. CIP concrete (203 mm), plus a minimum 1 in. (25 mm) haunch
Deck strength:	4 ksi at 28 days (27.6 MPa, cylinder)
Loads:	
Barrier:	20 psf. (0.96 kN/m <sup>2</sup> )
Future Wearing surface:	25 psf. (1.12 kN/m <sup>2</sup> )
The design live load:	HL-93
Relative Humidity	70%
Construction girder	
Prestressing strand released:	1 day
Diaphragm and Deck construction:	42 days
End of Girder life	20000 days
Design Specifications:	AASHTO LRFD Bridge Design Specifications latest Edition



Girder Strand Profile (NTS)

**Girder Section properties:**

Moment of Inertia	$I = 110,444 \text{ in}^4 (0.458 \text{ m}^4)$
Area	$A = 649 \text{ in}^2 (0.419 \text{ m}^2)$
Height	$h = 35.43 \text{ in.} (0.900 \text{ m})$
Centroid to bottom fiber	$y_b = 16.10 \text{ in.} (0.409 \text{ m})$
Girder Weight	$w = 0.676 \text{ k/ft.} (9.863 \text{ KN/m})$

**Material Properties**

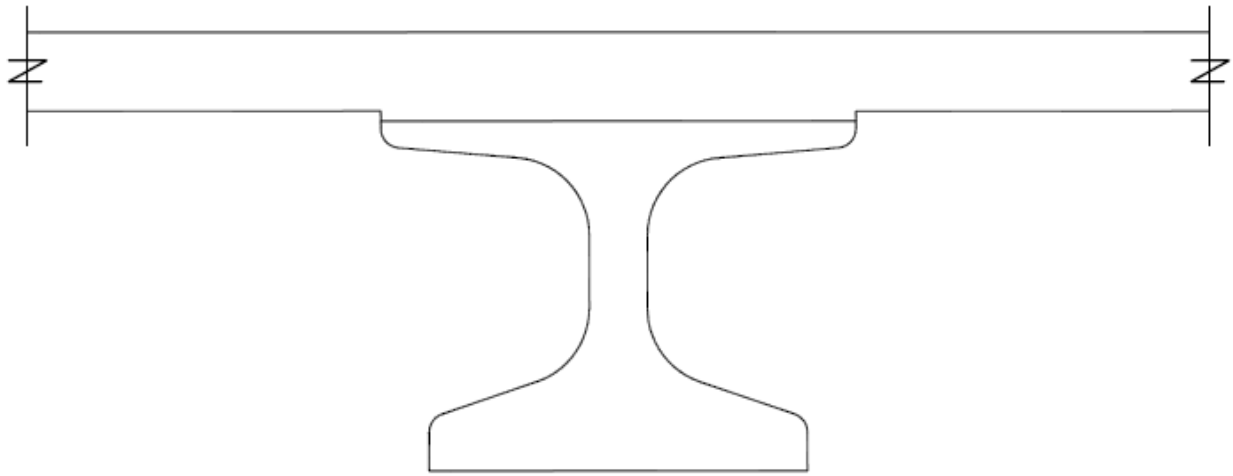
## Modulus of Elasticity

Girder	Initial	$E_{ci} = 4,406 \text{ ksi} (30,400 \text{ MPa})$
	At deck placement	$E_c = 5,314 \text{ ksi} (36,600 \text{ MPa})$
Deck	At 28 days	$E_{cd} = 3,607 \text{ ksi} (24,900 \text{ MPa})$

## Shrinkage Strains

Girder	Initial to final	$\epsilon_{bif}$	0.000393
	Initial to deck placement	$\epsilon_{bid}$	0.000202
	Deck placement to final	$\epsilon_{bdf}$	0.000191
Deck	Deck placement to final	$\epsilon_{ddf}$	0.000274
Creep coefficients			
Girder	Initial to final	$\psi_{bif}$	1.526
	Initial to deck placement	$\psi_{bid}$	0.784
	Deck placement to final	$\psi_{bdf}$	0.982
Deck	Deck placement to final	$\psi_{ddf}$	2.126

Note: creep coefficient = creep strain/initial strain for a constant sustained stress.



Girder cross section

### Restraint moment due to Time dependent effects according to the Bridge Design manual 8.13.4.3.2.1

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, there are pretensioning forces, member self-weight and possibly deck weight. Each loading case is considered separately. The total effect is obtained by simple superposition.

Perform elastic analysis, assuming that the load was introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports,  $M_{el}$  :

$M_o$ : elastic moment due to effect of self weight of the girder:

$$M_o = \frac{wl^2}{8} = 0.676 \times (90)^2/8 = 684.5 \text{ ft-kips (928.1 KN-m)}$$

w: weight of the girder = 0.676 k/ft. (9.865 KN/m)

L: length of the girder = 90 ft. (27.4 m)

$M_d$ : elastic moment due to effect of weight of the deck, (load applied before continuity is made):

$$M_d = \frac{w_d l^2}{8} = 1,063.1 \text{ ft-kips (1,441.4 KN-m)}$$

$w_d$ : weight of the deck = 1.05 k/ft. (15.323 KN/m)

$M_p$ : elastic moment due to prestress release, assuming as if the beam was continuous before load applied:

$$M_p = \frac{3P}{4} [2e_e + (1+\alpha)(e_c - e_e)] = 3 \times 1,581.9 / 4 [2 \times 10.9 + (1+0.1) (12.4-10.9)] / 12$$

$$= 2,303.6 \text{ ft-kips. (3,123.3 KN-m)}$$

$$e_e = 16.1 - (10 \times 9.55 + 30 \times 3.7)/40 = 10.9 \text{ in. (276.9 mm)}$$

$$e_c = 16.1 - 3.7 = 12.4 \text{ in. (315.0 mm)}$$

P: 90% of the strand prestressing force = 0.90 x 40 x 0.217 x 202.5 = 1,581.9 kips (7,036.6 KN)

$M_{Barrier}$ : elastic moment due to effect of self-weight of the barrier:

$$M_{Barrier} = \frac{wl^2}{8} = 0.20 \times (90)^2/8 = 202.5 \text{ ft.-kips (274.6 KN-m)}$$

w: Barrier and Wearing Surface = 0.020 x 10 = 0.20 k/ft. (2.918 KN/m)

L: length of the girder = 90 ft. (27.4 m)

Age –adjusted effective modulus for concrete subjected to gradually introduced restraining moment from time of deck placement to time.

$$E_{ctv}^*(t, t_d) = \frac{E_c(t_d)}{1+0.7\psi(t, t_d)} = \frac{5,314}{1+0.7(0.982)} = 3,149 \text{ ksi. (22 GPa)}$$

Age-Adjusted Effect Modulus for concrete subjected to constant stress introduced at  $t_i$ , with creep determined to the period (t- $t_d$ )

$$E_{ctc}^*(t, t_i) = \frac{E_c(t_i)}{\Psi_{bif} - \Psi_{bid}} = \frac{4,406}{1.526 - 0.784} = 5,938 \text{ ksi. (41 GPa)}$$

Determine the time-dependent multiplier,  $\delta_1$  corresponding to prestressing and girder self-weight:

$$\delta_1 = \frac{E_{ctv}^*(t, t_d)}{E_{ctc}^*(t, t_0)} = \frac{3,149}{5,938} = 0.530$$

Determine the time-dependent multiplier,  $\delta_2$  due to deck weight:

$E_{cd}$ : age-adjusted effective modulus of elasticity for beam concrete due deck weight:

$$E_{cd} = \frac{E_c(t_d)}{1+1.0\psi(t, t_d)} = \frac{5,314}{1+1.0(0.982)} = 2,681 \text{ ksi (18 GPa)}$$

Determine the time-dependent multiplier,  $\delta_2$  corresponding to deck load:

$$\delta_2 = \frac{2,681}{5,938} = 0.451$$

Determine the restraint moment  $M_r$ :

Restraint moment due girder weight:

$$M_{r1} = \delta_1 \times M_o = 0.530 \times -684.5 = -362.8 \text{ ft.-kips. (491.9 KN-m)}$$

Restraint moment due to prestressing

$$M_{r2} = \delta_1 \times M_p = 0.530 \times 2,303.6 = 1,220.9 \text{ ft.-kips. (1,655.3 KN-m)}$$

Note: the moment due to prestressing is positive and the moment due self-weight of the beam is negative.

Restraint moment due to deck weight:

$$M_{r3} = 0.451 \times -1,063.1 = -479.5 \text{ ft.-kips. (645.2 KN-m)}$$

**Total restraint moment**

Restraint moment due to girder weight	=	-362.8 ft.-kips.
Restraint moment due to prestressing	=	1,220.9 ft.-kips.
Restraint moment due to deck weight	=	-479.5 ft.-kips.
Restraint moment due to Temperature	=	570.1 ft.-kips.
Restraint moment due to deck shrinkage	=	-354.2 ft.-kips.
<u>Elastic moment due to barrier weight</u>	=	<u>-202.5 ft.-kips.</u>

**Total Net moment** = **393.8 ft.-kips.** (533.9 KN-m)

Elastic restraint moment for future wearing surface and live loads were not accounted for in this example. The reason these loads create negative moments causing compression at the bottom of the connection and are not permanently present throughout the lifetime of the girder. Therefore, this is a reasonable design practice.

**Diaphragm reinforcement:**

H = 35.4 in: (899.2 mm)

Z = 35.4 (899.2 mm) (beam height) + 4 in. (101.6 mm) (half of the deck slab thickness) + 1 in. (25.4 mm) (Haunch) – 2 in (50.8 mm) (distance from the beam soffit to the bottom reinforcement) = 38.4 in. (975.4mm)

$$A_s = \frac{M_r}{zf_s} = \frac{393.8 \times 12}{38.4 \times 36} = 3.457 \text{ in}^2 \text{ (2230 mm}^2\text{)}$$

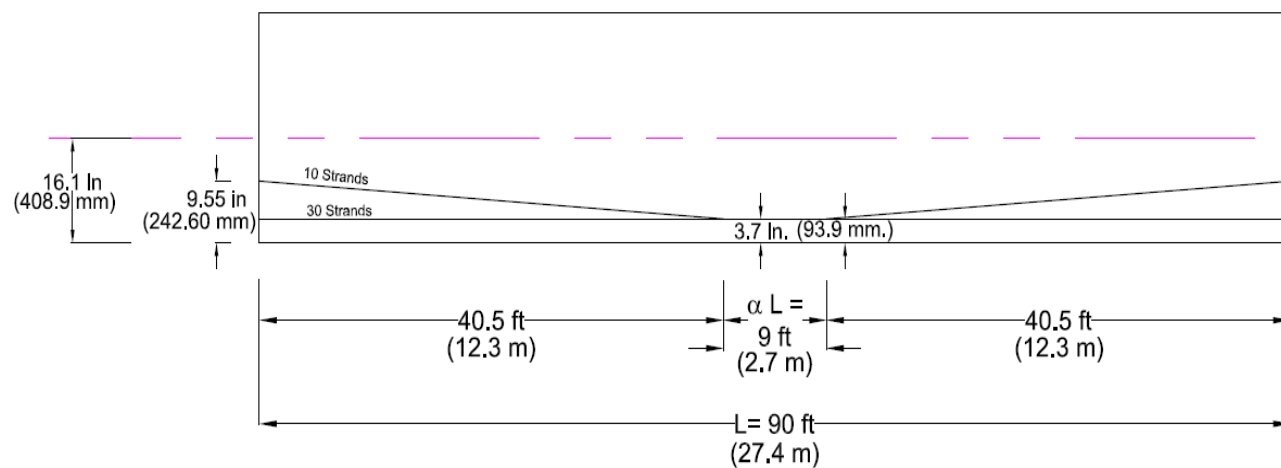
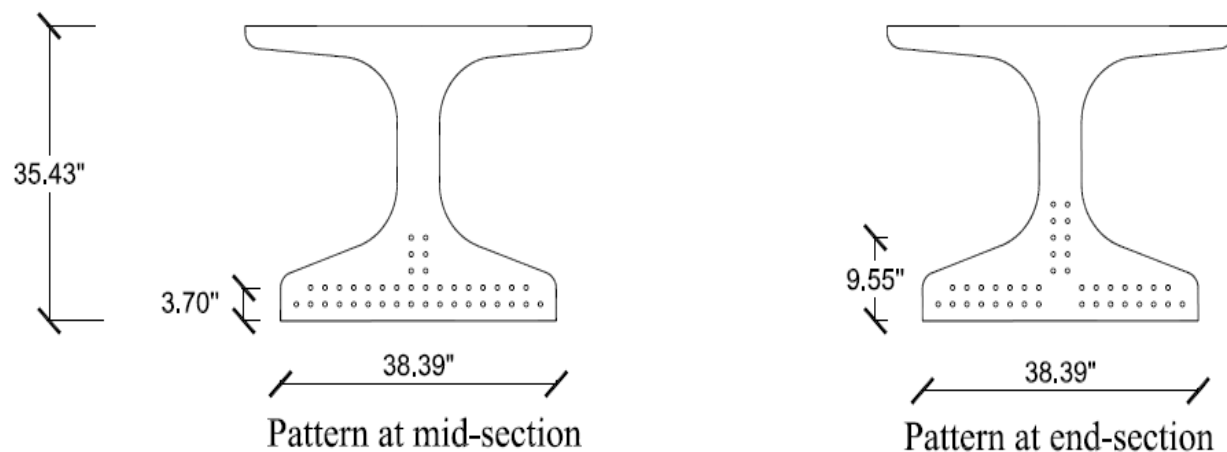
Use 16 strands from the bottom row flange.



#### Case 4. When deck is cast 60 days after the girder construction

##### Bridge Data

Geometry	
Bridge width	50 ft. (15.240 m)
Bridged length	2 spans x 90 ft. = 180 ft. (54.864 m)
Bridge Skew:	0 degree angle
Girder:	
Girder Spacing:	10 ft. (3.048 m)
Girder strength:	5.5 ksi at release (37.9 MPa, cylinder strength. Cube may be 1.15 times cylinder)
Girder strength:	8 ksi at 28 days (55.2 MPa, cylinder strength)
Girder Prestress:	40-0.6" low relaxation strands [Bottom cover to strand C.L. = 2" (50 mm)]
Deck thickness:	8 in. CIP concrete (203 mm), plus a minimum 1 in. (25 mm) haunch
Deck strength:	4 ksi at 28 days (27.6 MPa, cylinder)
Loads:	
Barrier:	20 psf. (0.96 kN/m <sup>2</sup> )
Future Wearing surface:	25 psf. (1.12 kN/m <sup>2</sup> )
The design live load:	HL-93
Relative Humidity	70%
Construction girder	
Prestressing strand released:	1 day
Diaphragm and Deck construction:	60 days
End of Girder life	20000 days
Design Specifications:	AASHTO LRFD Bridge Design Specifications latest Edition



Girder Strand Profile (NTS)

### Girder Section properties:

Moment of Inertia	$I = 110,444 \text{ in}^4 (0.458 \text{ m}^4)$
Area	$A = 649 \text{ in}^2 (0.419 \text{ m}^2)$
Height	$h = 35.43 \text{ in.} (0.900 \text{ m})$
Centroid to bottom fiber	$y_b = 16.10 \text{ in.} (0.409 \text{ m})$
Girder Weight	$w = 0.676 \text{ k/ft.} (9.863 \text{ KN/m})$

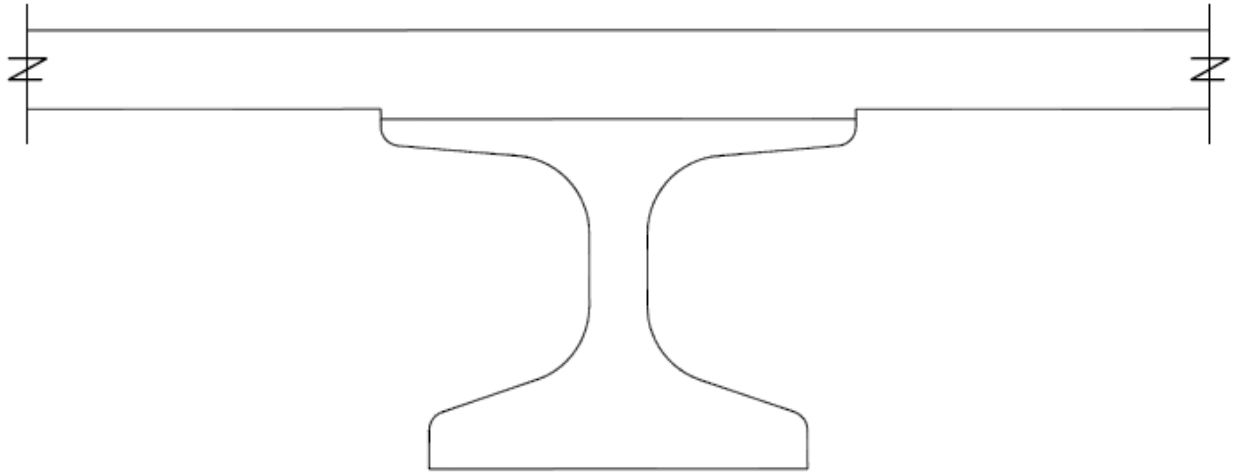
### Material Properties

Modulus of Elasticity	Girder	Initial	$E_{ci} = 4,406 \text{ ksi} (30,400 \text{ MPa})$
		At deck placement	$E_c = 5,314 \text{ ksi} (36,600 \text{ MPa})$
	Deck	At 28 days	$E_{cd} = 3,607 \text{ ksi} (24,900 \text{ MPa})$

## Shrinkage Strains

Girder	Initial to final	$\epsilon_{bif}$	0.000393
	Initial to deck placement	$\epsilon_{bid}$	0.000237
	Deck placement to final	$\epsilon_{bdf}$	0.000156
Deck	Deck placement to final	$\epsilon_{ddf}$	0.000274
Creep coefficients			
Girder	Initial to final	$\psi_{bif}$	1.526
	Initial to deck placement	$\psi_{bid}$	0.921
	Deck placement to final	$\psi_{bdf}$	0.941
Deck	Deck placement to final	$\psi_{ddf}$	2.126

Note: creep coefficient = creep strain/initial strain for a constant sustained stress.



Girder cross section

### Restraint moment due to Time dependent effects according to the Bridge Design manual 8.13.4.3.2.1

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, there are pretensioning forces, member self-weight and possibly deck weight. Each loading case is considered separately. The total effect is obtained by simple superposition.

Perform elastic analysis, assuming that the load was introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports,  $M_{el}$  :

$M_o$ : elastic moment due to effect of self weight of the girder:

$$M_o = \frac{wl^2}{8} = 0.676 \times (90)^2/8 = 684.5 \text{ ft.-kips (928.1 KN-m)}$$

w: weight of the girder = 0.676 k/ft. (9.865 KN/m)

L: length of the girder = 90 ft. (27.4 m)

$M_d$ : elastic moment due to effect of weight of the deck, (load applied before continuity is made):

$$M_d = \frac{w_d l^2}{8} = 1,063.1 \text{ ft-kips (1,441.4 KN-m)}$$

$w_d$ : weight of the deck = 1.05 k/ft. (15.323 KN/m)

$M_p$ : elastic moment due to prestress release, assuming as if the beam was continuous before load applied:

$$M_p = \frac{3P}{4} [2e_e + (1+\alpha)(e_c - e_e)] = 3 \times 1,581.9 / 4 [2 \times 10.9 + (1+0.1) (12.4-10.9)] / 12$$

$$= 2,303.6 \text{ ft-kips. (3,123.3 KN-m)}$$

$$e_e = 16.1 - (10 \times 9.55 + 30 \times 3.7)/40 = 10.9 \text{ in. (276.9 mm)}$$

$$e_c = 16.1 - 3.7 = 12.4 \text{ in. (315.0 mm)}$$

P: 90% of the strand prestressing force = 0.90 x 40 x 0.217 x 202.5 = 1,581.9 kips (7,036.6 KN)

$M_{Barrier}$ : elastic moment due to effect of self-weight of the barrier:

$$M_{Barrier} = \frac{wl^2}{8} = 0.20 \times (90)^2/8 = 202.5 \text{ ft-kips (274.6 KN-m)}$$

w: Barrier and Wearing Surface = 0.020 x 10 = 0.20 k/ft. (2.918 KN/m)

L: length of the girder = 90 ft. (27.4 m)

Age –adjusted effective modulus for concrete subjected to gradually introduced restraining moment from time of deck placement to time.

$$E_{ctv}^*(t, t_d) = \frac{E_c(t_d)}{1+0.7\psi(t, t_d)} = \frac{5,314}{1+0.7(0.941)} = 3,204 \text{ ksi. (22 GPa)}$$

Age-Adjusted Effect Modulus for concrete subjected to constant stress introduced at  $t_i$ , with creep determined to the period (t- $t_d$ )

$$E_{ctc}^*(t, t_i) = \frac{E_c(t_i)}{\psi_{bif} - \psi_{bid}} = \frac{4,406}{1.526 - 0.921} = 7,283 \text{ ksi. (50 GPa)}$$

Determine the time-dependent multiplier,  $\delta_1$  corresponding to prestressing and girder self-weight:

$$\delta_1 = \frac{E_{ctv}^*(t, t_d)}{E_{ctc}^*(t, t_0)} = \frac{3,204}{7,283} = 0.440$$

Determine the time-dependent multiplier,  $\delta_2$  due to deck weight:

$E_{cd}$ : age-adjusted effective modulus of elasticity for beam concrete due deck weight:

$$E_{cd} = \frac{E_c(t_d)}{1+1.0\psi(t, t_d)} = \frac{5,314}{1+1.0(0.941)} = 2,738 \text{ ksi (19 GPa)}$$

Determine the time-dependent multiplier,  $\delta_2$  corresponding to deck load:

$$\delta_2 = \frac{2,738}{7,283} = 0.376$$

Determine the restraint moment  $M_r$ :

Restraint moment due girder weight:

$$M_{r1} = \delta_1 \times M_o = 0.440 \times -684.5 = -301.2 \text{ ft-kips. (408.4 KN-m)}$$

Restraint moment due to prestressing

$$M_{r2} = \delta_1 \times M_p = 0.440 \times 2,303.6 = 1,013.5 \text{ ft-kips. (1,374.1 KN-m)}$$

Note: the moment due to prestressing is positive and the moment due self-weight of the beam is negative.

Restraint moment due to deck weight:

$$M_{r3} = 0.376 \times -1,063.1 = -399.7 \text{ ft-kips. (634.2 KN-m)}$$

**Total restraint moment**

Restraint moment due to girder weight	=	-301.2 ft.-kips.
Restraint moment due to prestressing	=	1,013.5 ft.-kips.
Restraint moment due to deck weight	=	-399.7 ft.-kips.
Restraint moment due to Temperature	=	570.1 ft.-kips.
Restraint moment due to deck shrinkage	=	-352.4 ft.-kips.
<u>Elastic moment due to barrier weight</u>	=	<u>-202.5 ft.-kips.</u>

**Total restraint moment** = **327.8 ft.-kips.** (352.2 KN-m)

Elastic restraint moment for future wearing surface and live loads were not accounted for in this example. The reason these loads create negative moments causing compression at the bottom of the connection and are not permanently present throughout the lifetime of the girder. Therefore, this is a reasonable design practice.

**Diaphragm reinforcement:**

$$H = 35.4 \text{ in. (899.2 mm)}$$

$$Z = 35.4 \text{ (899.2 mm) (beam height) + 4 in. (101.6 mm) (half of the deck slab thickness) + 1 in. (25.4 mm) (Haunch) - 2 in (50.8 mm) (distance from the beam soffit to the bottom reinforcement) = 38.4 in. (975.4mm)}$$

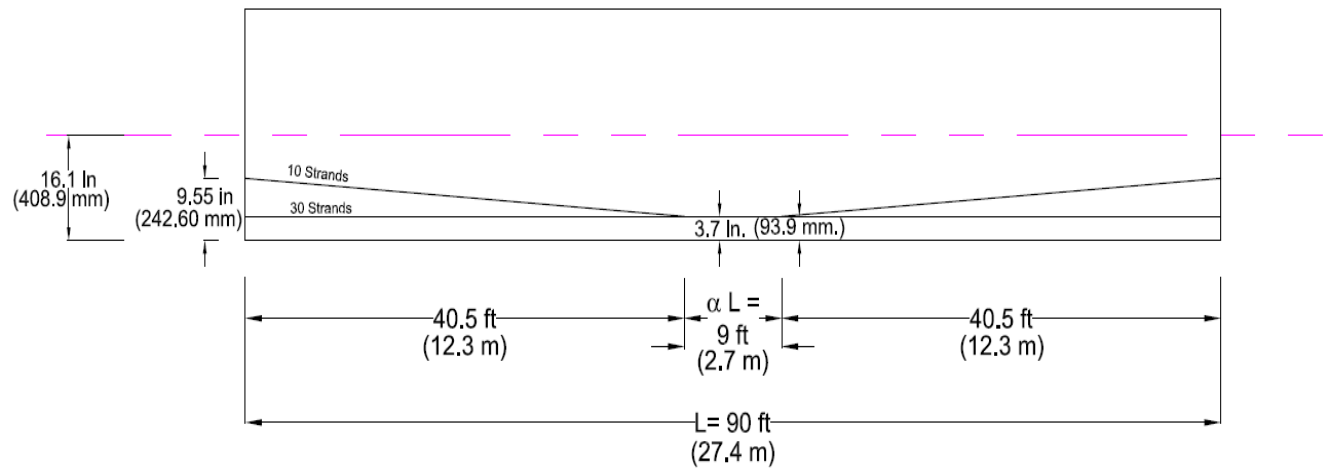
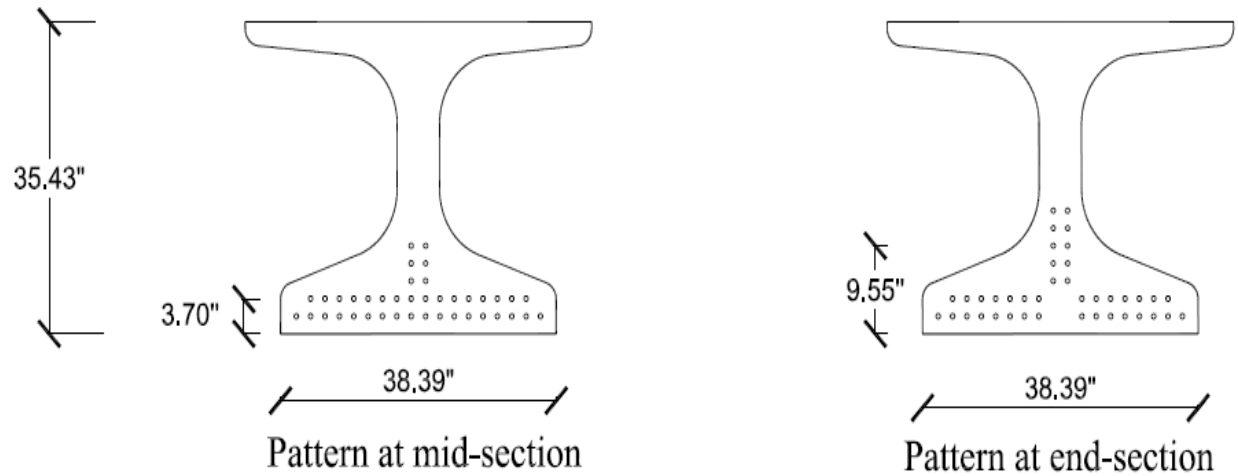
$$A_s = \frac{M_r}{z f_s} = \frac{327.8 \times 12}{38.4 \times 36} = 2.845 \text{ in}^2 \text{ (1455 mm}^2\text{)}$$

Use 10 strands from the bottom row flange.

### Case 5. When deck is cast 90 days after the girder construction

#### Bridge Data

Geometry	
Bridge width	50 ft. (15.240 m)
Bridged length	2 spans x 90 ft. = 180 ft. (54.864 m)
Bridge Skew:	0 degree angle
Girder:	
Girder Spacing:	10 ft. (3.048 m)
Girder strength:	5.5 ksi at release (37.9 MPa, cylinder strength. Cube may be 1.15 times cylinder)
Girder strength:	8 ksi at 28 days (55.2 MPa, cylinder strength)
Girder Prestress:	40-0.6" low relaxation strands [Bottom cover to strand C.L. = 2" (50 mm)]
Deck thickness:	8 in. CIP concrete (203 mm), plus a minimum 1 in. (25 mm) haunch
Deck strength:	4 ksi at 28 days (27.6 MPa, cylinder)
Loads:	
Barrier:	20 psf. (0.96 kN/m <sup>2</sup> )
Future Wearing surface:	25 psf. (1.12 kN/m <sup>2</sup> )
The design live load:	HL-93
Relative Humidity	70%
Construction girder	
Prestressing strand released:	1 day
Diaphragm and Deck construction:	90 days
End of Girder life	20000 days
Design Specifications:	AASHTO LRFD Bridge Design Specifications latest Edition



Girder Strand Profile (NTS)

**Girder Section properties:**

Moment of Inertia	$I = 110,444 \text{ in}^4 (0.458 \text{ m}^4)$
Area	$A = 649 \text{ in}^2 (0.419 \text{ m}^2)$
Height	$h = 35.43 \text{ in.} (0.900 \text{ m})$
Centroid to bottom fiber	$y_b = 16.10 \text{ in.} (0.409 \text{ m})$
Girder Weight	$w = 0.676 \text{ k/ft.} (9.863 \text{ KN/m})$



## Material Properties

### Modulus of Elasticity

Girder	Initial	$E_{ci}$	= 4,406 ksi (30,400 MPa)
	At deck placement	$E_c$	= 5,314 ksi (36,600 MPa)
Deck	At 28 days	$E_{cd}$	= 3,607 ksi (24,900 MPa)

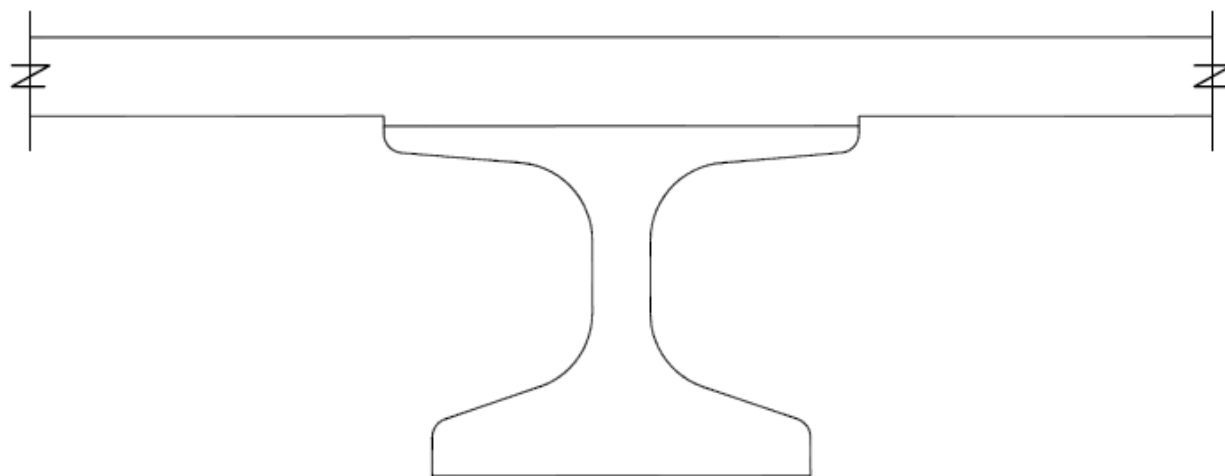
### Shrinkage Strains

Girder	Initial to final	$\epsilon_{bif}$	0.000393
	Initial to deck placement	$\epsilon_{bid}$	0.000274
	Deck placement to final	$\epsilon_{bdf}$	0.000119
Deck	Deck placement to final	$\epsilon_{ddf}$	0.000274

### Creep coefficients

Girder	Initial to final	$\Psi_{bif}$	1.526
	Initial to deck placement	$\Psi_{bid}$	1.063
	Deck placement to final	$\Psi_{bdf}$	0.897
Deck	Deck placement to final	$\Psi_{ddf}$	2.126

Note: creep coefficient = creep strain/initial strain for a constant sustained stress.



Girder cross section

### Restraint moment due to Time dependent effects according to the Bridge Design manual 8.13.4.3.2.1

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, there are pretensioning forces, member self-weight and possibly deck weight. Each loading case is considered separately. The total effect is obtained by simple superposition.

Perform elastic analysis, assuming that the load was introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports,  $M_{el}$  :

$M_o$ : elastic moment due to effect of self weight of the girder:

$$M_o = \frac{wl^2}{8} = 0.676 \times (90)^2/8 = 684.5 \text{ ft-kips (928.1 KN-m)}$$

w: weight of the girder = 0.676 k/ft. (9.865 KN/m)

L: length of the girder = 90 ft. (27.4 m)

$M_d$ : elastic moment due to effect of weight of the deck, (load applied before continuity is made):

$$M_d = \frac{w_d l^2}{8} = 1,063.1 \text{ ft-kips (1,441.4 KN-m)}$$

$w_d$ : weight of the deck = 1.05 k/ft. (15.323 KN/m)

$M_p$ : elastic moment due to prestress release, assuming as if the beam was continuous before load applied:

$$M_p = \frac{3P}{4} [2e_e + (1+\alpha)(e_c - e_e)] = 3 \times 1,581.9 / 4 [2 \times 10.9 + (1+0.1) (12.4-10.9)] / 12$$

$$= 2,303.6 \text{ ft-kips. (3,123.3 KN-m)}$$

$$e_e = 16.1 - (10 \times 9.55 + 30 \times 3.7)/40 = 10.9 \text{ in. (276.9 mm)}$$

$$e_c = 16.1 - 3.7 = 12.4 \text{ in. (315.0 mm)}$$

P: 90% of the strand prestressing force = 0.90 x 40 x 0.217 x 202.5 = 1,581.9 kips (7,036.6 KN)

$M_{Barrier}$ : elastic moment due to effect of self-weight of the barrier:

$$M_{Barrier} = \frac{wl^2}{8} = 0.20 \times (90)^2/8 = 202.5 \text{ ft-kips (274.6 KN-m)}$$

w: Barrier and Wearing Surface = 0.020 x 10 = 0.20 k/ft. (2.918 KN/m)

L: length of the girder = 90 ft. (27.4 m)

Age –adjusted effective modulus for concrete subjected to gradually introduced restraining moment from time of deck placement to time.

$$E_{ctv}^*(t, t_d) = \frac{E_c(t_d)}{1+0.7\psi(t, t_d)} = \frac{5,314}{1+0.7(0.897)} = 3,264 \text{ ksi. (23 GPa)}$$

Age-Adjusted Effect Modulus for concrete subjected to constant stress introduced at  $t_i$ , with creep determined to the period (t- $t_d$ )

$$E_{ctc}^*(t, t_i) = \frac{E_c(t_i)}{\psi_{bif} - \psi_{bid}} = \frac{4,406}{1.526 - 1.063} = 9,516 \text{ ksi. (66 GPa)}$$

Determine the time-dependent multiplier,  $\delta_1$  corresponding to prestressing and girder self-weight:

$$\delta_1 = \frac{E_{ctv}^*(t, t_d)}{E_{ctc}^*(t, t_0)} = \frac{3,264}{9,516} = 0.343$$

Determine the time-dependent multiplier,  $\delta_2$  due to deck weight:

$E_{cd}$ : age-adjusted effective modulus of elasticity for beam concrete due deck weight:

$$E_{cd} = \frac{E_c(t_d)}{1+1.0\psi(t, t_d)} = \frac{5,314}{1+1.0(0.897)} = 2,801 \text{ ksi (19 GPa)}$$

Determine the time-dependent multiplier,  $\delta_2$  corresponding to deck load:

$$\delta_2 = \frac{2,801}{9,516} = 0.294$$

Determine the restraint moment  $M_r$ :

Restraint moment due girder weight:

$$M_{r1} = \delta_1 \times M_o = 0.343 \times -684.5 = -234.8 \text{ ft-kips. (318.34 KN-m)}$$

Restraint moment due to prestressing

$$M_{r2} = \delta_1 \times M_p = 0.343 \times 2,303.6 = 790.1 \text{ ft-kips. (1,071.2 KN-m)}$$

Note: the moment due to prestressing is positive and the moment due self-weight of the beam is negative.

Restraint moment due to deck weight:

$$M_{r3} = 0.294 \times -1,063.1 = -312.55 \text{ ft-kips. (423.8 KN-m)}$$

**Total restraint moment**

Restraint moment due to girder weight	=	-234.8 ft.-kips.
Restraint moment due to prestressing	=	790.1 ft.-kips.
Restraint moment due to deck weight	=	-312.5 ft.-kips.
Restraint moment due to temperature	=	570.1 ft.-kips.
Restraint moment due to deck shrinkage	=	-352.4 ft.-kips.
<u>Elastic moment due to barrier weight</u>	=	<u>-202.5 ft.-kips.</u>

**Total Net moment** = **258.0 ft-kips.** (349.8 KN-m)

Elastic restraint moment for future wearing surface and live loads were not accounted for in this example. The reason these loads create negative moments causing compression at the bottom of the connection and are not permanently present throughout the lifetime of the girder. Therefore, this is a reasonable design practice.

**Joint reinforcement:**

H = 35.4 in. (899.2 mm)

Z = 35.4 (899.2 mm) (beam height) + 4 in. (101.6 mm) (half of the deck slab thickness) + 1 in. (25.4 mm) (Haunch) – 2 in (50.8 mm) (distance from the beam soffit to the bottom reinforcement) = 38.4 in. (975.4mm)

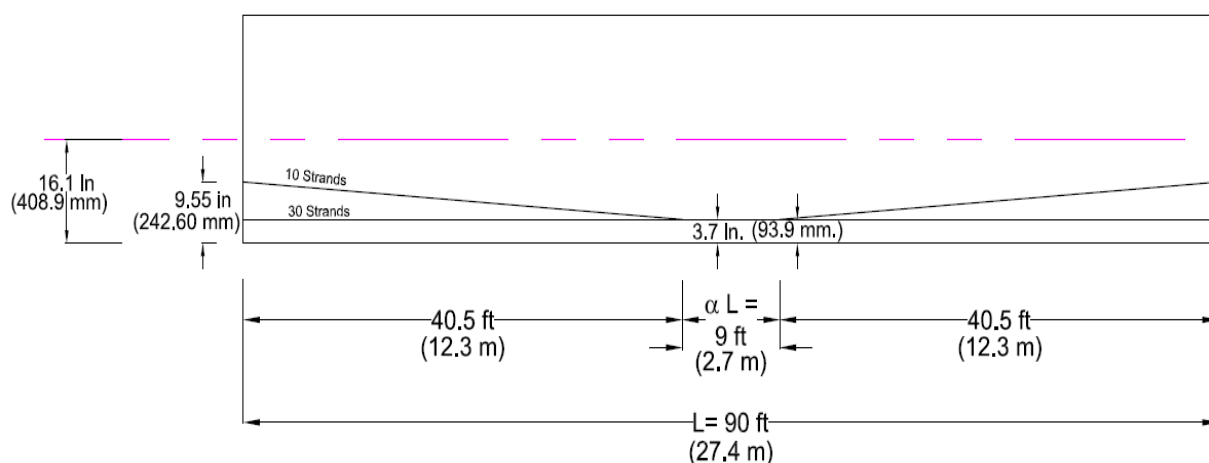
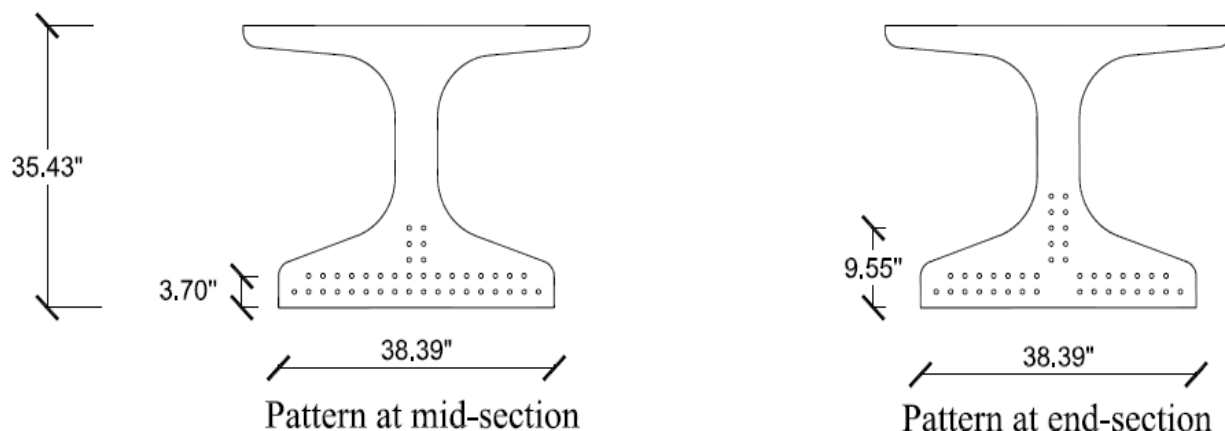
$$A_s = \frac{M_r}{zf_s} = \frac{258.0 \times 12}{38.4 \times 36} = 2.239 \text{ in}^2 (1445 \text{ mm}^2)$$

Use 10 strands from the bottom row flange.

## Case 6. When deck is cast 120 days after the girder construction

### Bridge Data

Geometry	
Bridge width	50 ft. (15.240 m)
Bridged length	2 spans x 90 ft. = 180 ft. (54.864 m)
Bridge Skew:	0 degree angle
Girder:	
Girder Spacing:	10 ft. (3.048 m)
Girder strength:	5.5 ksi at release (37.9 MPa, cylinder strength. Cube may be 1.15 times cylinder)
Girder strength:	8 ksi at 28 days (55.2 MPa, cylinder strength)
Girder Prestress:	40-0.6" low relaxation strands [Bottom cover to strand C.L. = 2" (50 mm)]
Deck thickness:	8 in. CIP concrete (203 mm), plus a minimum 1 in. (25 mm) haunch
Deck strength:	4 ksi at 28 days (27.6 MPa, cylinder)
Loads:	
Barrier:	20 psf. (0.96 kN/m <sup>2</sup> )
Future Wearing surface:	25 psf. (1.12 kN/m <sup>2</sup> )
The design live load:	HL-93
Relative Humidity	70%
Construction girder	
Prestressing strand released:	1 day
Diaphragm and Deck construction:	120 days
End of Girder life	20000 days
Design Specifications:	AASHTO LRFD Bridge Design Specifications latest Edition



Girder Strand Profile (NTS)

### Girder Section properties:

Moment of Inertia	$I = 110,444 \text{ in}^4 (0.458 \text{ m}^4)$
Area	$A = 649 \text{ in}^2 (0.419 \text{ m}^2)$
Height	$h = 35.43 \text{ in.} (0.900 \text{ m})$
Centroid to bottom fiber	$y_b = 16.10 \text{ in.} (0.409 \text{ m})$
Girder Weight	$w = 0.676 \text{ k/ft.} (9.863 \text{ KN/m})$

### Material Properties

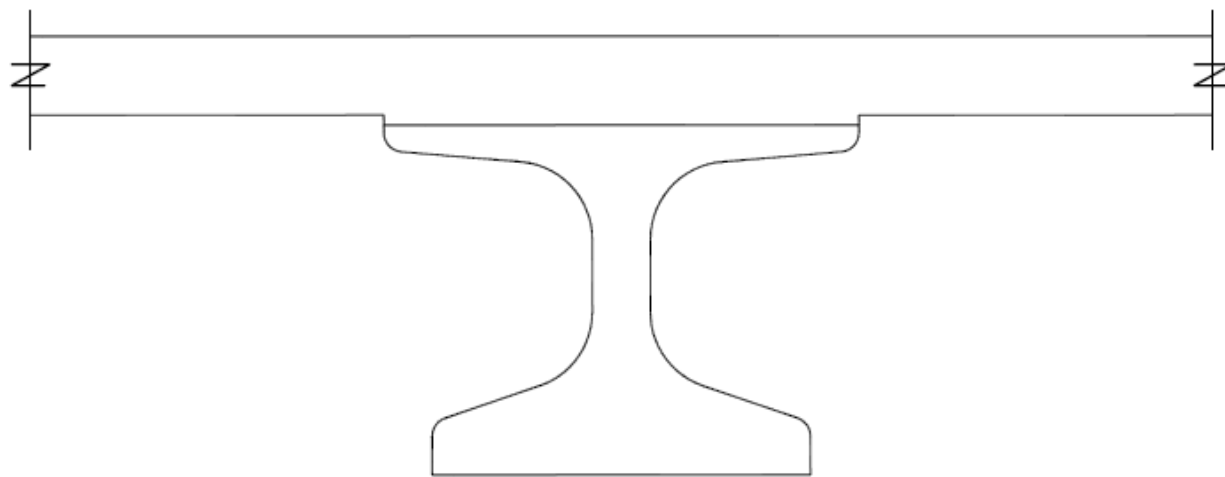
#### Modulus of Elasticity

Girder	Initial	$E_{ci} = 4,406 \text{ ksi} (30,400 \text{ MPa})$
	At deck placement	$E_c = 5,314 \text{ ksi} (36,600 \text{ MPa})$
Deck	At 28 days	$E_{cd} = 3,607 \text{ ksi} (24,900 \text{ MPa})$

## Shrinkage Strains

Girder	Initial to final	$\epsilon_{bif}$	0.000393
	Initial to deck placement	$\epsilon_{bid}$	0.000297
	Deck placement to final	$\epsilon_{bdf}$	0.000097
Deck	Deck placement to final	$\epsilon_{ddf}$	0.000274
Creep coefficients			
Girder	Initial to final	$\psi_{bif}$	1.526
	Initial to deck placement	$\psi_{bid}$	1.152
	Deck placement to final	$\psi_{bdf}$	0.868
Deck	Deck placement to final	$\psi_{ddf}$	2.126

Note: creep coefficient = creep strain/initial strain for a constant sustained stress.



Girder cross section

### Restraint moment due to Time dependent effects according to the Bridge Design manual 8.13.4.3.2.1

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, there are pretensioning forces, member self-weight and possibly deck weight. Each loading case is considered separately. The total effect is obtained by simple superposition.

Perform elastic analysis, assuming that the load was introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports,  $M_{el}$  :

$M_o$ : elastic moment due to effect of self weight of the girder:

$$M_o = \frac{wl^2}{8} = 0.676 \times (90)^2/8 = 684.5 \text{ ft-kips (928.1 KN-m)}$$

w: weight of the girder = 0.676 k/ft. (9.865 KN/m)

L: length of the girder = 90 ft. (27.4 m)

$M_d$ : elastic moment due to effect of weight of the deck, (load applied before continuity is made):

$$M_d = \frac{w_d l^2}{8} = 1,063.1 \text{ ft-kips (1,441.4 KN-m)}$$

$w_d$ : weight of the deck = 1.05 k/ft. (15.323 KN/m)

$M_p$ : elastic moment due to prestress release, assuming as if the beam was continuous before load applied:

$$M_p = \frac{3P}{4} [2e_e + (1+\alpha)(e_c - e_e)] = 3 \times 1,581.9 / 4 [2 \times 10.9 + (1+0.1)(12.4-10.9)] / 12$$

$$= 2,303.6 \text{ ft-kips. (3,123.3 KN-m)}$$

$$e_e = 16.1 - (10 \times 9.55 + 30 \times 3.7)/40 = 10.9 \text{ in. (276.9 mm)}$$

$$e_c = 16.1 - 3.7 = 12.4 \text{ in. (315.0 mm)}$$

P: 90% of the strand prestressing force = 0.90 x 40 x 0.217 x 202.5 = 1,581.9 kips (7,036.6 KN)

$M_{Barrier}$ : elastic moment due to effect of self-weight of the barrier:

$$M_{Barrier} = \frac{wl^2}{8} = 0.20 \times (90)^2/8 = 202.5 \text{ ft-kips (274.6 KN-m)}$$

w: Barrier and Wearing Surface = 0.020 x 10 = 0.20 k/ft. (2.918 KN/m)

L: length of the girder = 90 ft. (27.4 m)



Age –adjusted effective modulus for concrete subjected to gradually introduced restraining moment from time of deck placement to time.

$$E_{ctv}^*(t, t_d) = \frac{E_c(t_d)}{1+0.7\psi(t, t_d)} = \frac{5,314}{1+0.7(0.868)} = 3,306 \text{ ksi. (23 GPa)}$$

Age-Adjusted Effect Modulus for concrete subjected to constant stress introduced at  $t_i$ , with creep determined to the period (t- $t_d$ )

$$E_{ctc}^*(t, t_i) = \frac{E_c(t_i)}{\psi_{bif} - \psi_{bid}} = \frac{4,406}{1.526 - 1.152} = 11,781 \text{ ksi. (81 GPa)}$$

Determine the time-dependent multiplier,  $\delta_1$  corresponding to prestressing and girder self-weight:

$$\delta_1 = \frac{E_{ctv}^*(t, t_d)}{E_{ctc}^*(t, t_0)} = \frac{3,306}{11,781} = 0.281$$

Determine the time-dependent multiplier,  $\delta_2$  due to deck weight:

$E_{cd}$ : age-adjusted effective modulus of elasticity for beam concrete due deck weight:

$$E_{cd} = \frac{E_c(t_d)}{1+1.0\psi(t, t_d)} = \frac{5,314}{1+1.0(0.868)} = 2,845 \text{ ksi (20 GPa)}$$

Determine the time-dependent multiplier,  $\delta_2$  corresponding to deck load:

$$\delta_2 = \frac{2,845}{11,781} = 0.241$$

Determine the restraint moment  $M_r$ :

Restraint moment due girder weight:

$$M_{r1} = \delta_1 \times M_o = 0.281 \times -684.5 = -192.3 \text{ ft-kips. (260.7 KN-m)}$$

Restraint moment due to prestressing

$$M_{r2} = \delta_1 \times M_p = 0.281 \times 2,303.6 = 647.3 \text{ ft-kips. (872.7 KN-m)}$$

Note: the moment due to prestressing is positive and the moment due self-weight of the beam is negative.

Restraint moment due to deck weight:

$$M_{r3} = 0.241 \times -1,063.1 = -256.2 \text{ ft-kips. (347.4 KN-m)}$$

**Total restraint moment**

Restraint moment due to girder weight	=	-192.3 ft.-kips.
Restraint moment due to prestressing	=	647.3 ft.-kips.
Restraint moment due to deck weight	=	-256.2 ft.-kips.
Restraint moment due to temperature	=	570.1 ft.-kips.
Restraint moment due to deck shrinkage	=	-352.4 ft.-kips.
<u>Elastic moment due to barrier weight</u>	=	<u>-202.5 ft.-kips.</u>

**Total Net moment** = **214.0 ft.-kips.** (290.1 KN-m)

Elastic restraint moment for future wearing surface and live loads were not accounted for in this example. The reason these loads create negative moments causing compression at the bottom of the connection and are not permanently present throughout the lifetime of the girder. Therefore, this is a reasonable design practice.

**Diaphragm reinforcement:**

H = 35.4 in. (899.2 mm)

Z = 35.4 (899.2 mm) (beam height) + 4 in. (101.6 mm) (half of the deck slab thickness) + 1 in. (25.4 mm) (Haunch) – 2 in (50.8 mm) (distance from the beam soffit to the bottom reinforcement) = 38.4 in. (975.4mm)

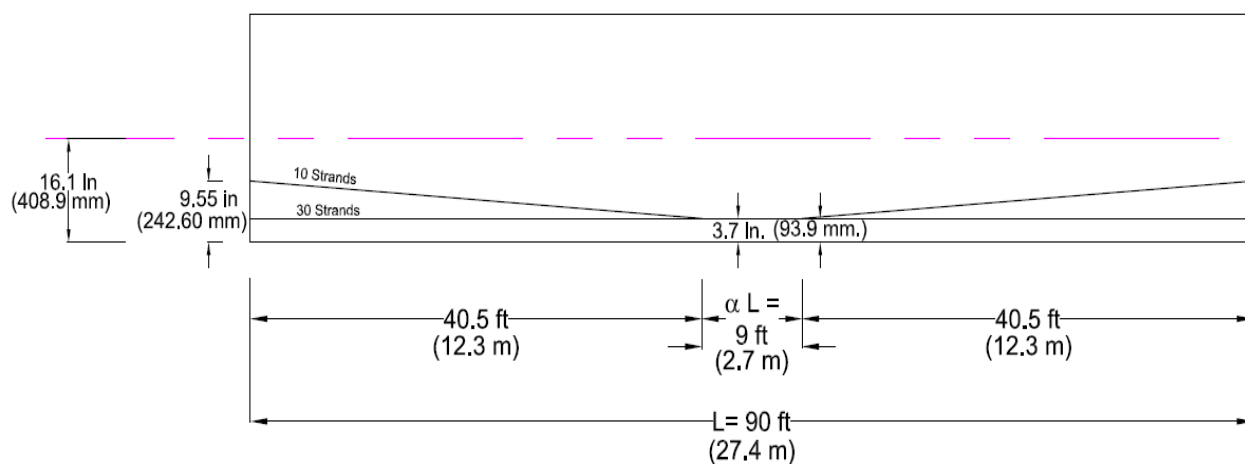
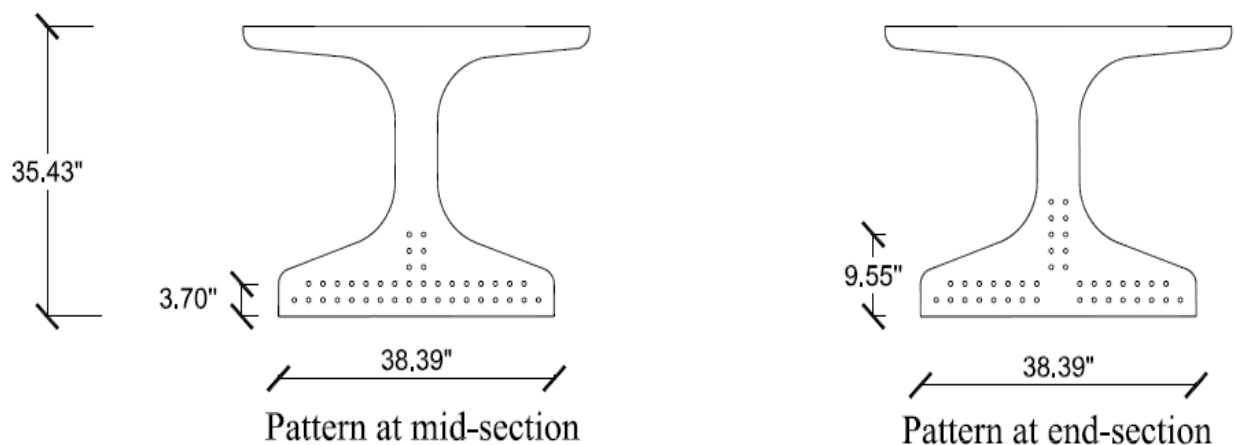
$$A_s = \frac{M_r}{zf_s} = \frac{214.0 \times 12}{38.4 \times 36} = 1.857 \text{ in}^2 \text{ (1198 mm}^2\text{)}$$

Use 9 strands from the bottom row flange.

## Case 7. When deck is cast after the girder achieves continuity

### Bridge Data

Geometry	
Bridge width	50 ft. (15.240 m)
Bridged length	2 spans x 90 ft. = 180 ft. (54.864 m)
Bridge Skew:	0 degree angle
Girder:	
Girder Spacing:	10 ft. (3.048 m)
Girder strength:	5.5 ksi at release (37.9 MPa, cylinder strength. Cube may be 1.15 times cylinder)
Girder strength:	8 ksi at 28 days (55.2 MPa, cylinder strength)
Girder Prestress:	40-0.6" low relaxation strands [Bottom cover to strand C.L. = 2" (50 mm)]
Deck thickness:	8 in. CIP concrete (203 mm), plus a minimum 1 in. (25 mm) haunch
Deck strength:	4 ksi at 28 days (27.6 MPa, cylinder)
Loads:	
Barrier:	20 psf. (0.96 kN/m <sup>2</sup> )
Future Wearing surface:	25 psf. (1.12 kN/m <sup>2</sup> )
The design live load:	HL-93
Relative Humidity	70%
Construction girder	
Prestressing strand released:	1 day
Diaphragm and Deck construction:	28 days
End of Girder life	20000 days
Design Specifications:	AASHTO LRFD Bridge Design Specifications 2012



Girder Strand Profile (NTS)

### Girder Section properties:

Moment of Inertia	$I = 110,444 \text{ in}^4 (0.458 \text{ m}^4)$
Area	$A = 649 \text{ in}^2 (0.419 \text{ m}^2)$
Height	$h = 35.43 \text{ in. } (0.900 \text{ m})$
Centroid to bottom fiber	$y_b = 16.10 \text{ in. } (0.409 \text{ m})$
Girder Weight	$w = 0.676 \text{ k/ft. } (9.863 \text{ KN/m})$

### Material Properties

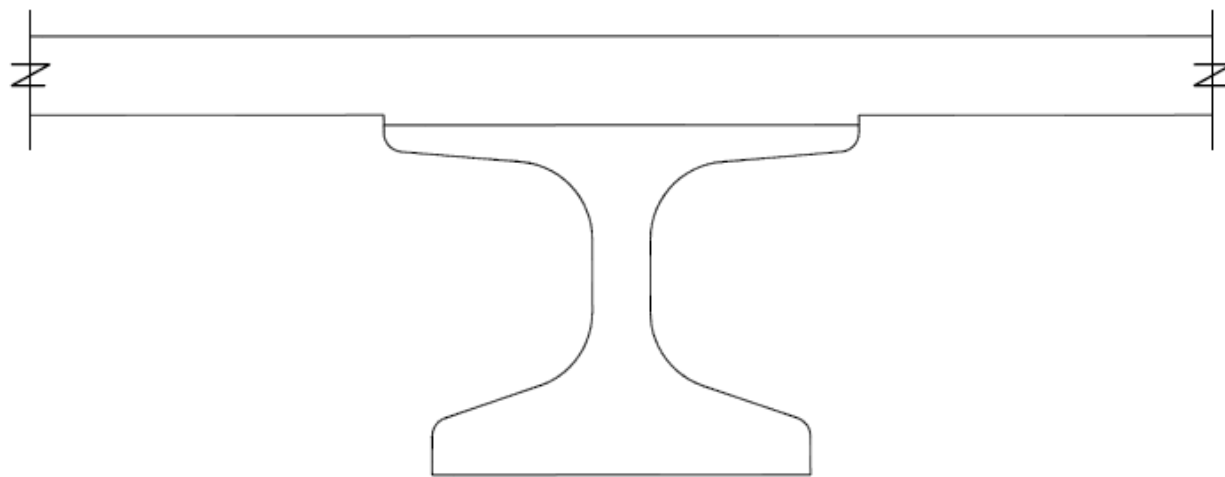
#### Modulus of Elasticity

Girder	Initial	$E_{ci} = 4,406 \text{ ksi } (30,400 \text{ MPa})$
	At deck placement	$E_c = 5,314 \text{ ksi } (36,600 \text{ MPa})$
Deck	At 28 days	$E_{cd} = 3,607 \text{ ksi } (24,900 \text{ MPa})$

## Shrinkage Strains

Girder	Initial to final	$\epsilon_{bif}$	0.000393
	Initial to deck placement	$\epsilon_{bid}$	0.000161
	Deck placement to final	$\epsilon_{bdf}$	0.000232
Deck	Deck placement to final	$\epsilon_{ddf}$	0.000274
Creep coefficients			
Girder	Initial to final	$\psi_{bif}$	1.526
	Initial to deck placement	$\psi_{bid}$	0.626
	Deck placement to final	$\psi_{bdf}$	1.030
Deck	Deck placement to final	$\psi_{ddf}$	2.126

Note: creep coefficient = creep strain/initial strain for a constant sustained stress.



Girder cross section

### Restraint moment due to Time dependent effects according to the Bridge Design manual 8.13.4.3.2.1

Only loads introduced before continuity can cause time-dependent restraint moment due to creep. Typically, there are pretensioning forces, member self-weight and possibly deck weight. Each loading case is considered separately. The total effect is obtained by simple superposition.

Perform elastic analysis, assuming that the load was introduced to a continuous member. Determine the fictitious elastic restraint moments at the supports,  $M_{el}$  :

$M_o$ : elastic moment due to effect of self weight of the girder:

$$M_o = \frac{wl^2}{8} = 0.676 \times (90)^2/8 = 684.5 \text{ ft-kips (928.1 KN-m)}$$

w: weight of the girder = 0.676 k/ft. (9.865 KN/m)

L: length of the girder = 90 ft. (27.4 m)

$M_d$ : elastic moment due to effect of weight of the deck, (load applied before continuity is made):

$$M_d = \frac{w_d l^2}{8} = 1,063.1 \text{ ft-kips (1,441.4 KN-m)}$$

$w_d$ : weight of the deck = 1.05 k/ft. (15.323 KN/m)

$M_p$ : elastic moment due to prestress release, assuming as if the beam was continuous before load applied:

$$M_p = \frac{3P}{4} [2e_e + (1+\alpha)(e_c - e_e)] = 3 \times 1,581.9 / 4 [2 \times 10.9 + (1+0.1)(12.4-10.9)] / 12$$

$$= 2,303.6 \text{ ft-kips. (3,123.3 KN-m)}$$

$$e_e = 16.1 - (10 \times 9.55 + 30 \times 3.7)/40 = 10.9 \text{ in. (276.9 mm)}$$

$$e_c = 16.1 - 3.7 = 12.4 \text{ in. (315.0 mm)}$$

P: 90% of the strand prestressing force = 0.90 x 40 x 0.217 x 202.5 = 1,581.9 kips (7,036.6 KN)

$M_{Barrier}$ : elastic moment due to effect of self-weight of the barrier:

$$M_{Barrier} = \frac{wl^2}{8} = 0.20 \times (90)^2/8 = 202.5 \text{ ft-kips (274.6 KN-m)}$$

w: Barrier and Wearing Surface = 0.020 x 10 = 0.20 k/ft. (2.918 KN/m)

L: length of the girder = 90 ft. (27.4 m)

Age –adjusted effective modulus for concrete subjected to gradually introduced restraining moment from time of deck placement to time.

$$E_{ctv}^*(t, t_d) = \frac{E_c(t_d)}{1+0.7\psi(t, t_d)} = \frac{5,314}{1+0.7(1.030)} = 3,088 \text{ ksi. (21 GPa)}$$

Age-Adjusted Effect Modulus for concrete subjected to constant stress introduced at  $t_i$ , with creep determined to the period (t- $t_d$ )

$$E_{ctc}^*(t, t_i) = \frac{E_c(t_i)}{\psi_{bif} - \psi_{bid}} = \frac{4,406}{1.526 - 0.626} = 4,896 \text{ ksi. (34 GPa)}$$

Determine the time-dependent multiplier,  $\delta_1$  corresponding to prestressing and girder self-weight:

$$\delta_1 = \frac{E_{ctv}^*(t, t_d)}{E_{ctc}^*(t, t_0)} = \frac{3,088}{4,896} = 0.630$$

Determine the time-dependent multiplier,  $\delta_2$  due to deck weight:

$E_{cd}$ : age-adjusted effective modulus of elasticity for beam concrete due deck weight:

$$E_{cd} = \frac{E_c(t_d)}{1+1.0\psi(t, t_d)} = \frac{5,314}{1+1.0(1.030)} = 2,618 \text{ ksi (18 GPa)}$$

Determine the time-dependent multiplier,  $\delta_2$  corresponding to deck load:

$$\delta_2 = \frac{2,618}{4,896} = 0.535$$

Determine the restraint moment  $M_r$ :

Restraint moment due girder weight:

$$M_{r1} = \delta_1 \times M_o = 0.630 \times -684.5 = -431.2 \text{ ft-kips. (584.6 KN-m)}$$

Restraint moment due to prestressing

$$M_{r2} = \delta_1 \times M_p = 0.630 \times 2,303.6 = 1,451.3 \text{ ft-kips. (1,967.7 KN-m)}$$

Note: the moment due to prestressing is positive and the moment due self-weight of the beam is negative.

Restraint moment due to deck weight:

$$M_{r3} = -1,063.1 \text{ (1441.4 KN-m)}$$

**Total restraint moment**

Restraint moment due to girder weight = -431.2 ft.-kips.

Restraint moment due to prestressing = 1,451.3 ft.-kips.

Restraint moment due to deck weight = -1063.1 ft.-kips.

Restraint moment due to temperature = 570.2 ft.-kips

Restraint moment due to deck shrinkage = -352.4 ft.-kips

Elastic moment due to barrier weight = -202.5 ft.-kips.

**Total Net moment = -27.7 ft.-kips. (37.5 KN-m)**

No positive moment reinforcement is required.



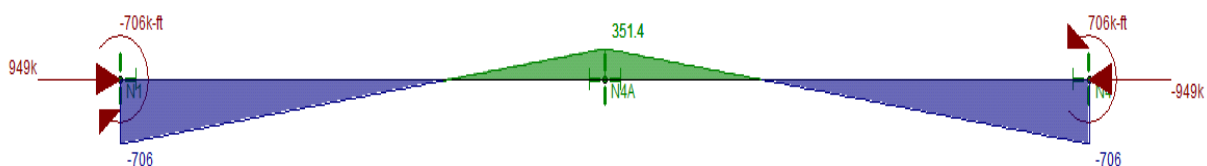
## Calculation of restraint moments caused by Differential Shrinkage

### Cross-Sectional Properties

- Modulus of Elasticity- girder = 5314 ksi.
- Modulus of Elasticity- deck-  $E_{cd}$  = 3607 ksi.
- Modular ratio = 3607/5314
- = 0.68
- Girder spacing = 10ft.
- = 120 in.
- Span = 90 ft.
- Deck thickness = 8 in.
- Area of the Deck-  $A_d$  = 960 in<sup>2</sup>
- Moment of Inertia of cross-section = 235,962 in<sup>4</sup>
- Centroid of the section from the top = 12.927 in.
- Total depth of the cross-section = 44.43 in.
- Deck Shrinkage Strain = 0.00274 in
- Compressive force due to  
shrinkage –  $\epsilon_s \times A_d \times E_{cd}$  = 0.00274 x 960 x 3607
- = 949 kips
- Moment caused due to shrinkage-  $M_{sh}$  =  $(949 \times (12.927 - 8/2))/12$
- = 705.82 kip-ft.

This moment calculated is applied as fixed end moment on the girders, by using a finite element software (RISA- 3D). The restraining moment caused at the interior support can be found out.

## RISA- 3D RESULTS



- **Total restraint moment = 351.4 kip-ft.**

### Calculation of restraint moments caused by Thermal loads – AASHTO-LRFD

#### Cross-sectional properties

- Coefficient of linear expansion =  $6 \times 10^{-6}$  in/in<sup>°</sup>F
- Modulus of Elasticity = 5422 ksi.
- Moment of Inertia of cross-section = 235,962 in<sup>4</sup>
- Centroid of the section from the top = 12.927 in.
- Total depth of the cross-section = 44.43 in.

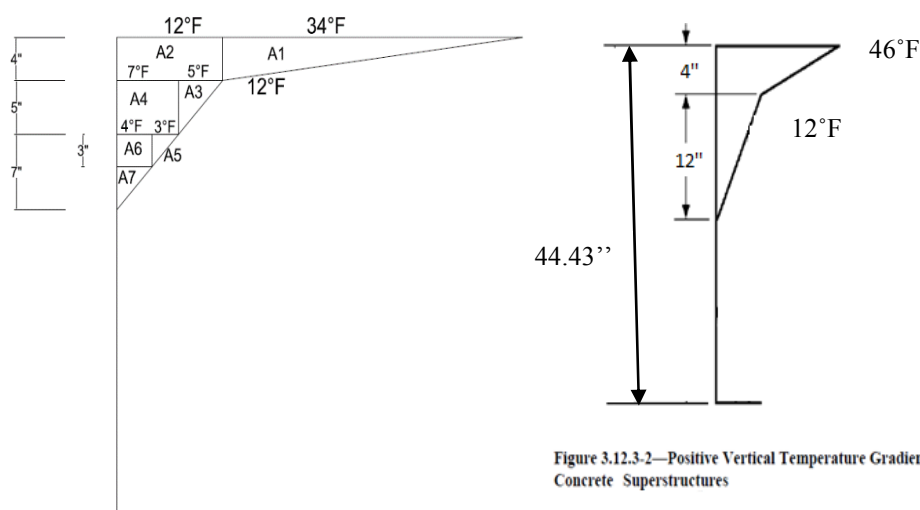
S.NO.	WIDTH OF SECTION (w)	DEPTH OF THE SECTION (d)	DISTANCE FROM C.G OF THE CROSS-SECTION (z)	TEMPERATURE GRADIENT (°F)	CURVATURE $\phi$
1).	84.85	4	10.93	12	1.13168E-06
2).	84.85	4	11.59	17	1.70103E-06
3).	84.85	5	7.26	2.5	1.95817E-07
4).	84.85	5	6.43	7	4.85355E-07
5).	48.23	3	2.43	4	3.57172E-08
6).	48.23	3	2.93	1.5	1.61533E-08
7).	6	4	1.54	1	9.37676E-10

**Total  $\phi$  = 3.5667E-06**

- **Total Restraint Moment =  $3/2EI\phi$  = 6844.7727 kip-in = 570.40 kip-ft.**

### Calculation of restraint moments caused by Thermal loads – Initial Strain theory

- Coefficient of linear expansion =  $6 \times 10^{-6}$  in/in $^{\circ}$ F
- Modulus of Elasticity = 5422 ksi.
- Moment of Inertia of cross-section = 235,962 in $^4$
- Centroid of the section from the top = 12.927 in.
- Total depth of the cross-section = 44.43 in



S.No.	Modulus of Elasticity (ksi)	Width of the section (in)	$\Delta T$ ( $^{\circ}$ F)	Depth of each section (in)	Force (kip)	Distance from NA to the centroid (in)	Moment (kip-in)
A1	5422.25	84.85	17	4	187.71	11.59	2176.267903
A2	5422.25	84.85	12	4	132.50	10.93	1447.854149
A3	5422.25	84.85	2.5	5	34.51	7.26	250.5239253
A4	5422.25	84.85	7	5	96.62	6.43	620.9533562
A5	5422.25	48.43	1.5	3	7.09	2.93	20.75298122
A6	5422.25	48.43	4	3	18.91	2.427	45.88769882
A7	5422.25	6	2	4	-1.56	0.4063	-0.63448133

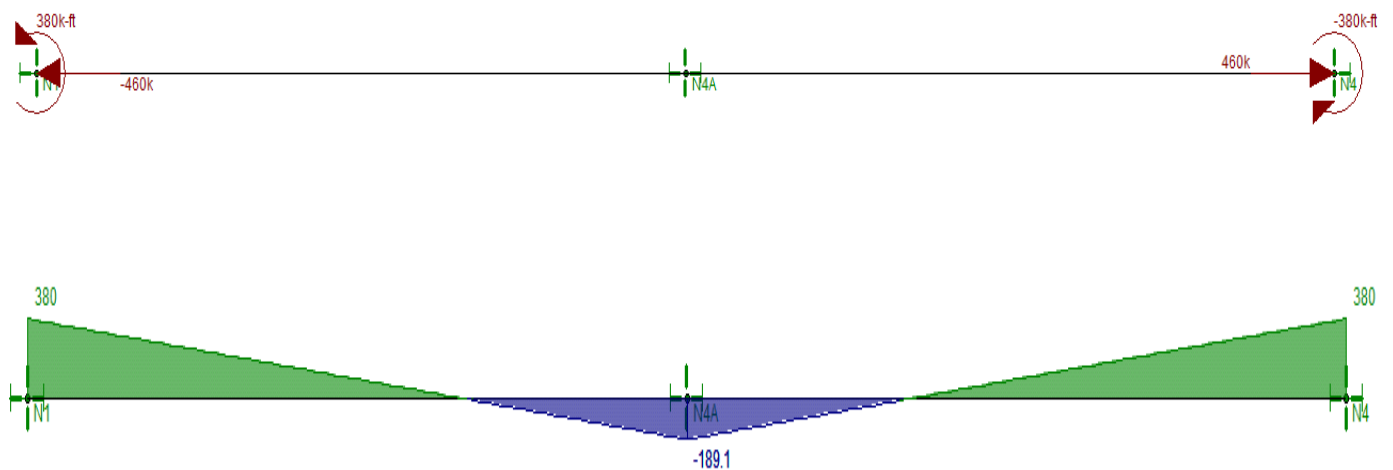
**Total 475.77**

**4561.61 kip-in.**  
**380.13 kip-ft.**

- Primary restraint force = 475.77 kips
- Primary restraint moments = 380.13 kip-ft.

After finding the primary effects, the secondary effects can be calculated by applying the primary force and moments to a finite element analysis program (RISA-3D). Stresses computed from this structural analysis are then superimposed on stresses due to the primary restraining axial force and bending moment to give the stresses due to continuity.

### RISA Results



- Total restrain moment caused by temperature = 189.1 + 380  
= 569.1 kip-ft.

## Cracked Section Analysis

**Case 1: Assuming allowable stress in Steel is 24 ksi.**

### Bridge data and material properties

Diameter of the strand	:	0.6 in
Girder compressive strength	:	8000 psi
Deck compressive strength	:	4000 psi
Modulus of elasticity of concrete deck	:	$33 \times w_c^{1.5} \times \sqrt{f'_c}$ $(33 \times 150^{1.5} \times \sqrt{4000})/1000$ 3834.25 ksi.
Modulus of elasticity of concrete girder	:	$33 \times w_c^{1.5} \times \sqrt{f'_c}$ $(33 \times 150^{1.5} \times \sqrt{8000})/1000$ 5422.45 ksi.
Modulus of Elasticity of Steel	:	29000 ksi.
Maximum stress in steel	:	24 ksi.
Modular ratio $n_s$ - (steel)	:	$\frac{29000}{5422}$ 5.35
Modular ratio $n_c$ – (concrete)	:	$\frac{5422}{3834}$ 1.41
Girder Spacing	:	120 in.
Effective width of the cross –section	:	$\frac{120}{1.41}$ 84.85 in
Total Restraint moment $-M_r$	:	466.5 kip-ft. 5598.0 kip-in.
Area of Steel Reinforcement	:	$\frac{M_r}{jdf_s}$

$$\begin{aligned}
 & : \frac{0.9 \times 5598}{0.9 \times 38.4 \times 24} \\
 & : 6.07 \text{ in}^2 \\
 \text{Number of Strands} & : \frac{6.07}{0.217} \\
 & : 27.99 \sim 28 \text{ strands}
 \end{aligned}$$

### Calculation of Maximum Crack Width

Crack spacing decreases with increasing load and stabilizes after the reinforcement reaches a critical stress. Further stress increases act only to widen existing cracks. Crack spacing is controlled by the distance determined by the spacing of the reinforcement or the distance determined by the side cover.

Crack control is achieved by limiting the spacing of the reinforcing steel. Maximum bar spacing can be determined by limiting the crack widths to acceptable limits. Robert J. Frosch (1999) developed an equation for the calculation of maximum crack width as follows:

$$a). \quad w_c = 2 \frac{f_s}{E_s} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

$$\text{Bottom cover - } d_c \quad : 2 \text{ in.}$$

$$\beta = 1.0 + 0.08 d_c \quad : 1.16$$

$$\text{Spacing of reinforcement} \quad : 6 \text{ in.}$$

$$\begin{aligned}
 \text{Maximum Crack width-} w_c & : 2 \frac{f_s}{E_s} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \\
 & : 2 \frac{24}{29000} 1.16 \sqrt{2^2 + \left(\frac{6}{2}\right)^2} \\
 & : 0.0069 \text{ in.}
 \end{aligned}$$

### Check for Crack Control-(AASHTO-LRFD)

The spacing  $s$  of mild steel reinforcement in the layer closest to the tension face shall satisfy the following

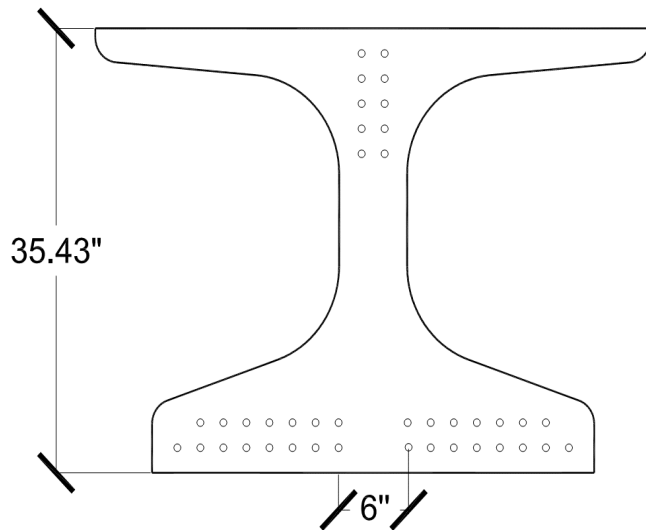
- $s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$
- $\beta_s = 1 + \frac{d_c}{0.7(h-d_c)}$

### Required Reinforcement Spacing

- $\gamma_e = 0.75$  (Class 2 exposure)
- $d_c = 2.0$  in.
- $f_{ss} = 24$  ksi.
- $h = 44.43$  in.
- $\beta_s = 1 + \frac{d_c}{0.7(h-d_c)} = 1 + \frac{2}{0.7(44.43-2)} = 1.06$
- $s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c = \frac{700 \times 0.75}{1.06 \times 24} - 2 \times 2 = 16$  in (maximum spacing)

The spacing of reinforcement used for the section is 6 in

### Spacing provided



- Maximum Spacing of reinforcement = 6.0 in < 16 in (Crack Control- O.K)

### Calculation for Cracked Moment of Inertia- $I_{cr}$ .

Several iterations were performed to calculate the depth of the neutral axis- $c$ , by equating the summation of moments about the neutral axis to be zero,  $\sum Ay = 0$ . The depth of the neutral axis was calculated to be 4.015 in. from the top of the section

Area of cross-section in compression	: $84.85 \times 4.015$ : $340.68 \text{ in}^2$
Distance from the top of the compression fiber to the centroid of the tension fiber $y_r$ .	: $38.4 - 4.015$ : $34.39 \text{ in}$
Modular ratio $n_s$ – steel	: $5.35$
Transformed Steel Area $nA_s$	: $5.35 \times 6.08$ : $32 \text{ in}^2$
Cracked Moment of Inertia- $I_{cr}$	: $((84.85 \times 4.015^3/12) + 340.68 \times (4.015/2)^2) +$ $(32 \times 34.39^2)$  : $40,251 \text{ in}^4$
Stress in Steel $f_s$	: $\frac{n \times M}{I_{cr}} \times y$  : $\frac{5.35 \times 5598}{40,251} \times 34.39$  : $25 \text{ ksi.}$

**Case 2: Assuming allowable stress in Steel is 36 ksi.**

### Bridge data and material properties

Diameter of the strand	: $0.6 \text{ in}$
Girder compressive strength	: $8000 \text{ psi}$
Deck compressive strength	: $4000 \text{ psi}$
Modulus of elasticity of concrete deck	: $33 \times w_c^{1.5} \times \sqrt{f'_c}$ : $(33 \times 150^{1.5} \times \sqrt{4000})/1000$ : $3834.25 \text{ ksi.}$



Modulus of elasticity of concrete girder	: $33 \times w_c^{1.5} \times \sqrt{f'_c}$ : $(33 \times 150^{1.5} \times \sqrt{8000}) / 1000$ : 5422.45 ksi.
Modulus of Elasticity of Steel	: 29000 ksi.
Maximum stress in steel	: 36 ksi.
Modular ratio $n_s$ - (steel)	: $\frac{29000}{5422}$ : 5.35
Modular ratio $n_c$ – (concrete)	: $\frac{5422}{3834}$ : 1.41
Girder Spacing	: 120 in.
Effective width of the cross –section	: $\frac{120}{1.41}$ : 84.85 in
Total Restraint moment $-M_r$	: 466.5 kip-ft. : 5598.0 kip-in.
Area of Steel Reinforcement	: $\frac{M_r}{j d f_s}$ : $\frac{0.9 \times 5598}{0.9 \times 38.43 \times 36}$ : 4.04 in <sup>2</sup>
Number of Strands	: $\frac{4.04}{0.217}$ : 18.61 ~ 19 strands

### Calculation of Maximum Crack Width

Crack spacing decreases with increasing load and stabilizes after the reinforcement reaches a critical stress. Further stress increases act only to widen existing cracks. Crack spacing is controlled by the distance determined by the spacing of the reinforcement or the distance

determined by the side cover. Crack control is achieved by limiting the spacing of the reinforcing steel. Maximum bar spacing can be determined by limiting the crack widths to acceptable limits.

Robert J. Frosch (1999) developed an equation for the calculation of maximum crack width as follows:

$$w_c = 2 \frac{f_s}{E_s} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

Bottom cover -  $d_c$  : 2 in.

$\beta = 1.0 + 0.08 d_c$  : 1.16

Spacing of reinforcement : 6 in.

Maximum Crack width- $w_c$  :  $2 \frac{f_s}{E_s} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$   
 :  $2 \frac{36}{29000} 1.16 \sqrt{2^2 + \left(\frac{6}{2}\right)^2}$   
 : 0.0103 in.

### Check for Crack Control-(AASHTO-LRFD)

The spacing  $s$  of mild steel reinforcement in the layer closest to the tension face shall satisfy the following

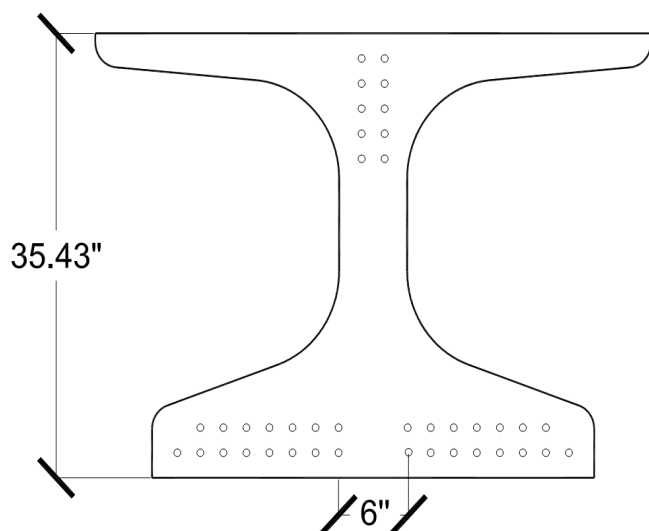
- $s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$
- $\beta_s = 1 + \frac{d_c}{0.7(h-d_c)}$

### Required Reinforcement Spacing

- $\gamma_e = 0.75$  (Class 2 exposure)
- $d_c = 2.0$  in.
- $f_{ss} = 25$  ksi.

- $h = 44.43$  in.
- $\beta_s = 1 + \frac{d_c}{0.7(h-d_c)} = 1 + \frac{2}{0.7(44.43-2)} = 1.06$
- $s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c = \frac{700 \times 0.75}{1.06 \times 36} - 2 \times 2 = 10$  in (maximum spacing)

### Spacing provided



- Maximum Spacing of reinforcement = 6.0 in < 10 in (Crack Control- O.K)

### Calculation for Cracked Moment of Inertia- $I_{cr}$

Several iterations were performed to calculate the depth of the neutral axis- $c$ , by equating the summation of moments about the neutral axis to be zero,  $\sum Ay = 0$ . The depth of the neutral axis was calculated to be 4.443 in. from the top of the section

Area of cross-section in compression :  $84.85 \times 4.443$   
 :  $376.98 \text{ in}^2$

Distance from the top of the compression fiber to the centroid of the tension fiber  $y_r$  :  $38.4 - 4.443$   
 :  $33.96$  in

Modular ratio  $n_s$  – steel : 5.35

Transformed Steel Area $nA_s$	: $5.35 \times 4.08$ : $22 \text{ in}^2$
Cracked Moment of Inertia- $I_{cr}$	: $((84.85 \times 4.443^3/12) + 376.98 \times (4.443/2)^2) +$ $(22 \times 33.96^2)$  : $27,906 \text{ in}^4$
Stress in Steel $f_s$	: $\frac{n \times M}{I_{cr}} \times y$  : $\frac{5.35 \times 5598}{27,906} \times 36.43$  : 36 ksi.

**Case 3: Assuming allowable stress in Steel is 48 ksi.**

**Bridge data and material properties**

Diameter of the strand	: 0.6 in
Girder compressive strength	: 8000 psi
Deck compressive strength	: 4000 psi
Modulus of elasticity of concrete deck	: $33 \times w_c^{1.5} \times \sqrt{f'_c}$ : $(33 \times 150^{1.5} \times \sqrt{4000})/1000$ : 3834.25 ksi.
Modulus of elasticity of concrete girder	: $33 \times w_c^{1.5} \times \sqrt{f'_c}$ : $(33 \times 150^{1.5} \times \sqrt{8000})/1000$ : 5422.45 ksi.
Modulus of Elasticity of Steel	: 29000 ksi.
Maximum stress in steel	: 48 ksi.
Modular ratio $n_s$ - (steel)	: $\frac{29000}{5422}$  : 5.35
Modular ratio $n_c$ – (concrete)	: $\frac{5422}{3834}$  : 1.41

Girder Spacing		: 120 in.
Effective width of the cross –section		: $\frac{120}{1.41}$ : 84.85 in
Total Restraint moment	$-M_r$	: 466.5 kip-ft. : 5598.0 kip-in.
Area of Steel Reinforcement		: $\frac{M_r}{jdf_s}$ : $\frac{0.9 \times 5598}{0.9 \times 38.43 \times 48}$ : 3.03 in <sup>2</sup>
Number of Strands		: $\frac{3.03}{0.217}$ : 13.98 ~ 14 strands

### Calculation of Maximum Crack Width

Crack spacing decreases with increasing load and stabilizes after the reinforcement reaches a critical stress. Further stress increases act only to widen existing cracks. Crack spacing is controlled by the distance determined by the spacing of the reinforcement or the distance determined by the side cover.

Crack control is achieved by limiting the spacing of the reinforcing steel. Maximum bar spacing can be determined by limiting the crack widths to acceptable limits. Robert J. Frosch (1999) developed an equation for the calculation of maximum crack width as follows:

$$w_c = 2 \frac{f_s}{E_s} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

Bottom cover - $d_c$		: 2 in.
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$$\beta = 1.0 + 0.08 d_c \quad : 1.16$$

$$\text{Spacing of reinforcement} \quad : 6 \text{ in.}$$

$$\begin{aligned} \text{Maximum Crack width-}w_c & : 2 \frac{f_s}{E_s} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \\ & : 2 \frac{48}{29000} 1.16 \sqrt{2^2 + \left(\frac{6}{2}\right)^2} \\ & : 0.0138 \text{ in.} \end{aligned}$$

### Check for Crack Control-(AASHTO-LRFD)

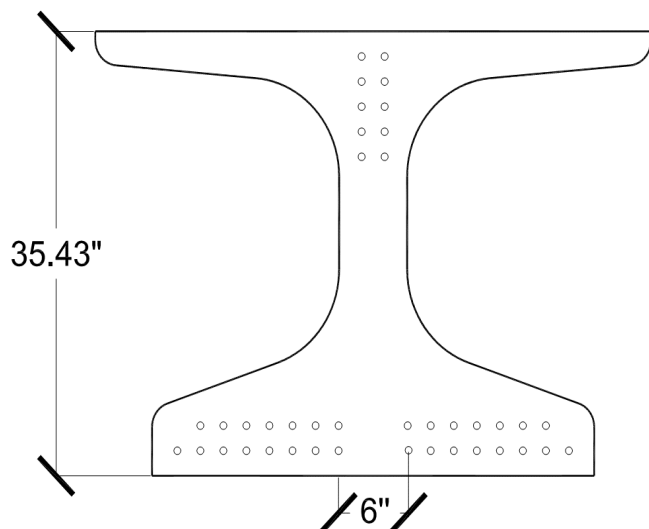
The spacing  $s$  of mild steel reinforcement in the layer closest to the tension face shall satisfy the following

- $s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$
- $\beta_s = 1 + \frac{d_c}{0.7(h-d_c)}$

### Required Reinforcement Spacing

- $\gamma_e = 0.75$  (Class 2 exposure)
- $d_c = 2.0$  in.
- $f_{ss} = 25$  ksi.
- $h = 44.43$  in.
- $\beta_s = 1 + \frac{d_c}{0.7(h-d_c)} = 1 + \frac{2}{0.7(44.43-2)} = 1.06$
- $s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c = \frac{700 \times 0.75}{1.06 \times 48} - 2 \times 2 = 6.31$  in (maximum spacing)

### Spacing provided



- Maximum Spacing of reinforcement = 6.0 in < 6.31 in (Crack Control- O.K)

### Calculation for Cracked Moment of Inertia- $I_{cr}$

Several iterations were performed to calculate the depth of the neutral axis- $c$ , by equating the summation of moments about the neutral axis to be zero,  $\sum Ay = 0$ . The depth of the neutral axis was calculated to be 4.675 in. from the top of the section

Area of cross-section in compression	: 84.85 x 4.675
	: 396.69 in <sup>2</sup>

Distance from the top of the compression fiber to the centroid of the tension fiber $y_r$ .	: 38.4 – 4.675
	: 33.73 in

Modular ratio $n_s$ – steel	: 5.35
-----------------------------	--------

Transformed Steel Area $nA_s$	: 5.35 x 3.04
	: 16.25 in <sup>2</sup>

Cracked Moment of Inertia- $I_{cr}$	: $((84.85 \times 4.675^3/12) + 396.69 \times (4.675/2)^2) +$ $(16.25 \times 33.73^2)$
	: 21,370 in <sup>4</sup>

Stress in Steel  $f_s$

$$: \frac{n \times M}{I_{cr}} \times y$$

$$: \frac{5.35 \times 5598}{21,370} \times 33.73$$

$$: 47 \text{ ksi.}$$



## APPENDIX -B

### Calculation of Material Properties

The modulus of elasticity of deck and girder concrete, the ultimate creep coefficient and shrinkage strain were all calculated using developed spreadsheets. The following are the extracts from the spread sheets.

### Spreadsheet –Input data

Beam		Materials		Service load moments		Beam Properties	
A	649.00 in <sup>2</sup>	H	70 %	Girder	7885 kip-in	Beam spacing	10.000 ft
I	110,444 in <sup>4</sup>	$f_{ci}$	5.5 ksi	Deck, haunch, diaphragms	12981 kip-in	Barrier	1.0 k/ft full bridge
$\phi_b$	16.1 in	$f'_c$	8.0 ksi	SI dead ld.	3374 kip-in	Barrier	0.150 k/ft EA beam
h	35.43 in	$f'_{cd}$	4.0 ksi	80% of HL93 plus impact*	16294 kip-in	FWS	0.250 k/ft EA beam
V/S	3.11 in	$t_i$ (release)	1 days	*The LL moment to be input here is the same used to check bottom fiber stress at final conditions (AASHTO LRFD Service III)			
$A_{ps}$	8.68 in <sup>2</sup>	$t$ (deck pour)	28 days				
$f_{pi}$	202.5 ksi	$t_f$ (final)	20000 days	M+                      M-		M+ per w= 1 (cont. spans)	703 k-ft
$E_s$	28500 ksi			Girder	657	M+ LL w/o DF	2247 k-ft
$\phi_{pb}$	3.70 in			Deck, haunch, diaphragms	1082	M- per w= 1 (cont. spans)	-1250 k-ft
span	90.00 ft			Barrier	105	M- LL w/o DF	-2302 k-ft
Deck				FWS	176	Mu- SID+LL+IM	-3746 k-ft
Width	120.0 in			100%LL	1697		
Thickness	8 in			Mu	-3746		
Haunch width	48.43 in						
Haunch thickness	1 in						
V/Sd	3.91						
NG	OK	OK	OK	OK	OK		
	-0.187						
	Sb 6860						
Fatigue LL				Interpretation			
DF	0.523			w	At 18/19 section	At CL pier	Face of Diaph.
IM	1.15			1	-935	-1250	-1151
Load factor	1.5			Barrier	0.15 -140	-188	-173
	0.90			FWS	0.25 -234	-313	-288
				LL w/o DF	-1767	-2302	-2134
				LL w/ DF	-1612	-1612	-1612
				Fatigue w/ DF	-517.9	-546.9	-538
							-3469

## 7-day Results

## Material Properties

Modulus of Elasticity in KSI (5.4.2.4)	$E=33000*w^{1.5}*K_1(f'_c)^{0.5}$ $w=0.140+f'_c/1000$	Beam	initial	$E_{ci}$	4406		
			at deck placement	$E_c$	5314		
		Deck		$E_{cd}$	3607		
Shrinkage (5.4.2.3.3)	$\epsilon=0.00048(5/(1+f_{ci}))k_s k_{hs} k_{td}$ $k_s=(1.45-0.13V/S)$ $k_{hs}=2.0-0.014H$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\epsilon_{bit}$	0.000393		
			initial to deck placam.	$\epsilon_{bid}$	0.000053		
			deck placem. to final	$\epsilon_{bdf}$	0.000341		
		Deck		$\epsilon_{dfr}$	0.000274		
Creep (5.4.2.3.2)	$\psi=1.90k_s k_{hs}(5/(1+f_{ci}))k_{td}t_i^{-0.118}$ $k_{hs}=(1.56-0.008*H)$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\Psi_{bit}$	1.526		
			initial to deck placement	$\Psi_{bid}$	0.204		
			Deck placem. to final	$\Psi_{bdf}$	1.213		
		Deck	Deck placem. to final	$\Psi_{dfr}$	2.126	Gross section	
Steel modular ratio " (at deck placement)	$n_i=E_s/E_{ci}$	6.4678	Transformed section factors, K		$K_{id}$	0.74103	0.74594
" (deck)	$n_d=E_s/E_c$	5.3628	$K=1/(1+n_i*\alpha_{net}*A_{ps}/A_{net}*(1+0.7*\Psi_{bid}))$ (5.9.5.4.2-2, 5.9.5.4.3-2)		$K_{df}$	0.75409	0.75853

## Section Properties

Section Properties	Precast Beam				Transformed Deck		Composite Bm, Deck		
	Gross	Net (-A <sub>ps</sub> )	Tr.-initial	Transformed-final	Deck	Haunch	Gross	Net	Tr.-final
A (in. <sup>2</sup> )	649.00	640.32	696.46	686.87	651.49	32.87	1333.36	1324.68	1371.23
y <sub>b</sub> (in.)	16.1	16.27	15.25	15.42	40.43	35.93	28.48	28.64	27.79
I (in. <sup>4</sup> )	110444	109091	117244	115946	3475	2.74	308248	302885	330854
e <sub>p</sub> (in.)	12.40	12.57	11.55	11.72			24.78	24.94	24.09
e <sub>d</sub> (in.)							11.74	11.57	
$\alpha=1+(A*e_p^2)/I$	1.9035	1.9271	1.7931	1.8132			3.6554	3.7201	3.4057
$\alpha_b=1+A*e_c*y_b/I$	2.1731	2.2001	2.0471	2.0700	1.0000		4.0519	4.1237	3.7751
$\alpha_t=1+A*e_c*(y_b-h)/I$	-0.4085	-0.4136	-0.3848	-0.3891	1.0000		0.2548	0.2593	0.2374
Area of deck incl. haunch =					1008.43				

## 28-day Results

## Material Properties

Modulus of Elasticity in KSI (5.4.2.4)	$E=33000*w^{1.5}*K_1(f_c)^{0.5}$ $w=0.140+f_c/1000$	Beam	initial	$E_{ci}$	4406	
			at deck placement	$E_c$	5314	
		Deck		$E_{cd}$	3607	
Shrinkage (5.4.2.3.3)	$\epsilon=0.00048(5/(1+f_{ci}))k_b k_{hd} k_{td}$ $k_s=(1.45-0.13V/S)$ $k_{hs}=2.0-0.014H$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\epsilon_{bit}$	0.000393	
			initial to deck placam.	$\epsilon_{bid}$	0.000161	
			deck placem. to final	$\epsilon_{bdf}$	0.000232	
		Deck		$\epsilon_{ddf}$	0.000274	
Creep (5.4.2.3.2)	$\psi=1.90k_b k_{hd} (5/(1+f_{ci}))k_{td} t_i^{-0.118}$ $k_{hc}=(1.56-0.008*H)$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\psi_{bit}$	1.526	
			initial to deck placement	$\psi_{bid}$	0.626	
			Deck placem. to final	$\psi_{bdf}$	1.030	
		Deck	Deck placem. to final	$\psi_{ddf}$	2.126	Gross section
Steel modular ratio	$n_i=E_c/E_{ci}$	6.4678	Transformed section factors, K	$K_{id}$	0.74103	0.74594
" (at deck placement)	$n=E_c/E_c$	5.3628	$K=1/(1+n_i*\alpha_{net}*A_{ps}/A_{net}*(1+0.7*\psi_{bid}))$	$K_{df}$	0.75409	0.75853
" (deck)	$n_d=E_d/E_c$	0.6786	(5.9.5.4.2-2, 5.9.5.4.3-2)			

## Section Properties

Section Properties	Precast Beam				Transformed Deck		Composite Bm, Deck		
	Gross	Net (-A <sub>ps</sub> )	Tr.-initial	Transformed-final	Deck	Haunch	Gross	Net	Tr.-final
A (in. <sup>2</sup> )	649.00	640.32	696.46	686.87	651.49	32.87	1333.36	1324.68	1371.23
y <sub>b</sub> (in.)	16.1	16.27	15.25	15.42	40.43	35.93	28.48	28.64	27.79
I (in. <sup>4</sup> )	110444	109091	117244	115946	3475	2.74	308248	302885	330854
e <sub>p</sub> (in.)	12.40	12.57	11.55	11.72			24.78	24.94	24.09
e <sub>d</sub> (in.)							11.74	11.57	
$\alpha=1+(A*e_p^2)/I$	1.9035	1.9271	1.7931	1.8132			3.6554	3.7201	3.4057
$\alpha_b=1+A*e_c*y_b/I$	2.1731	2.2001	2.0471	2.0700	1.0000		4.0519	4.1237	3.7751
$\alpha_t=1+A*e*(y_b-h)/I$	-0.4085	-0.4136	-0.3848	-0.3891	1.0000		0.2548	0.2593	0.2374
Area of deck incl. haunch =					1008.43				

## 42-day Results

## Material Properties

Modulus of Elasticity in KSI (5.4.2.4)	$E=33000*w^{1.5}*K_1(f'_c)^{0.5}$ $w=0.140+f'_c/1000$	Beam	initial	$E_{ci}$	4406		
			at deck placement	$E_c$	5314		
		Deck		$E_{cd}$	3607		
Shrinkage (5.4.2.3.3)	$\epsilon=0.00048(5/(1+f'_{ci}))k_{tr}k_{hd}$ $k_s=(1.45-0.13V/S)$ $k_{hs}=2.0-0.014H$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\epsilon_{bit}$	0.000393		
			initial to deck placem.	$\epsilon_{bid}$	0.000202		
			deck placem. to final	$\epsilon_{bdf}$	0.000191		
		Deck		$\epsilon_{ddf}$	0.000274		
Creep (5.4.2.3.2)	$\psi=1.90k_{tr}k_{hd}(5/(1+f'_{ci}))k_{td}t_i^{-0.118}$ $k_{hc}=(1.56-0.008*H)$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\psi_{bif}$	1.526		
			initial to deck placement	$\psi_{bid}$	0.784		
			Deck placem. to final	$\psi_{bdf}$	0.982		
		Deck	Deck placem. to final	$\psi_{ddf}$	2.126	Gross section	
Steel modular ratio	$n_i=E_s/E_{ci}$	6.4678	Transformed section factors, K		$K_{id}$	0.74103	0.74594
" (at deck placement)	$n=E_s/E_c$	5.3628	$K=1/(1+n_i*\alpha_{net}*A_{ps}/A_{net}*(1+0.7*\psi_{bit}))$		$K_{df}$	0.75409	0.75853
" (deck)	$n_d=E_s/E_c$	0.6786	(5.9.5.4.2-2, 5.9.5.4.3-2)				

## Section Properties

Section Properties	Precast Beam				Transformed Deck		Composite Bm, Deck		
	Gross	Net (-A <sub>ps</sub> )	Tr.-initial	Transformed-final	Deck	Haunch	Gross	Net	Tr.-final
A (in. <sup>2</sup> )	649.00	640.32	696.46	686.87	651.49	32.87	1333.36	1324.68	1371.23
y <sub>b</sub> (in.)	16.1	16.27	15.25	15.42	40.43	35.93	28.48	28.64	27.79
I (in. <sup>4</sup> )	110444	109091	117244	115946	3475	2.74	308248	302885	330854
e <sub>p</sub> (in.)	12.40	12.57	11.55	11.72			24.78	24.94	24.09
e <sub>d</sub> (in.)							11.74	11.57	
$\alpha=1+(A*e_p^2)/I$	1.9035	1.9271	1.7931	1.8132			3.6554	3.7201	3.4057
$\alpha_b=1+A*e_p*y_b/I$	2.1731	2.2001	2.0471	2.0700	1.0000		4.0519	4.1237	3.7751
$\alpha_t=1+A*e_p*(y_b-h)/I$	-0.4085	-0.4136	-0.3848	-0.3891	1.0000		0.2548	0.2593	0.2374
Area of deck incl. haunch =					1008.43				

## 60-day Results

## Material Properties

Modulus of Elasticity in KSI (5.4.2.4)	$E=33000*w^{1.5}*K_f(f'_c)^{0.5}$ $w=0.140+f'_c/1000$	Beam	initial	$E_{ci}$	4406		
			at deck placement	$E_c$	5314		
		Deck		$E_{cd}$	3607		
Shrinkage (5.4.2.3.3)	$\epsilon=0.00048(5/(1+f'_{ci}))k_{hs}k_{hd}$ $k_s=(1.45-0.13V/S)$ $k_{hs}=2.0-0.014H$ $k_{hd}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\epsilon_{bf}$	0.000393		
			initial to deck placem.	$\epsilon_{bid}$	0.000237		
			deck placem. to final	$\epsilon_{bdf}$	0.000156		
Deck			$\epsilon_{ddf}$	0.000274			
Creep (5.4.2.3.2)	$\psi=1.90k_{hs}k_{hd}(5/(1+f'_{ci}))k_{td}t_i^{-0.118}$ $k_{hs}=(1.56-0.008*H)$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\psi_{bf}$	1.526		
			initial to deck placement	$\psi_{bid}$	0.921		
			Deck placem. to final	$\psi_{bdf}$	0.941		
Deck		Deck placem. to final	$\psi_{ddf}$	2.126	Gross section		
Steel modular ratio " (at deck placement)	$n_t=E_f/E_{ci}$	6.4678	Transformed section factors, K		$K_{td}$	0.74103	0.74594
" (deck)	$n_d=E_f/E_c$	5.3628	$K=1/(1+n_t*\alpha_{net}*A_{ps}/A_{net}*(1+0.7*\psi_{bid}))$ (5.9.5.4.2-2, 5.9.5.4.3-2)		$K_{df}$	0.75409	0.75853
" (deck)	$n_d=E_f/E_c$	0.6786					

## Section Properties

Section Properties	Precast Beam				Transformed Deck		Composite Bm, Deck		
	Gross	Net (-A <sub>ps</sub> )	Tr.-initial	Transformed-final	Deck	Haunch	Gross	Net	Tr.-final
A (in. <sup>2</sup> )	649.00	640.32	696.46	686.87	651.49	32.87	1333.36	1324.68	1371.23
y <sub>b</sub> (in.)	16.1	16.27	15.25	15.42	40.43	35.93	28.48	28.64	27.79
I (in. <sup>4</sup> )	110444	109091	117244	115946	3475	2.74	308248	302885	330854
e <sub>p</sub> (in.)	12.40	12.57	11.55	11.72			24.78	24.94	24.09
e <sub>d</sub> (in.)							11.74	11.57	
$\alpha=1+(A*e_p^2)/I$	1.9035	1.9271	1.7931	1.8132			3.6554	3.7201	3.4057
$\alpha_b=1+A*e_p*y_b/I$	2.1731	2.2001	2.0471	2.0700	1.0000		4.0519	4.1237	3.7751
$\alpha_t=1+A*e_p*(y_b-h)/I$	-0.4085	-0.4136	-0.3848	-0.3891	1.0000		0.2548	0.2593	0.2374
Area of deck incl. haunch =					1008.43				

## 90-day Results

## Material Properties

Modulus of Elasticity in KSI (5.4.2.4)	$E=33000*w^{1.5}*K_1(f_c)^{0.5}$ $w=0.140+f_c/1000$	Beam	initial	$E_{ci}$	4406		
			at deck placement	$E_{cc}$	5314		
		Deck		$E_{cd}$	3607		
Shrinkage (5.4.2.3.3)	$\epsilon=0.00048(5/(1+f_{ci}))k_s k_{hs} k_{td}$ $k_s=(1.45-0.13V/S)$ $k_{hs}=2.0-0.014H$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\epsilon_{bif}$	0.000393		
			initial to deck placam.	$\epsilon_{bid}$	0.000274		
			deck placem. to final	$\epsilon_{bdf}$	0.000119		
		Deck		$\epsilon_{ddf}$	0.000274		
Creep (5.4.2.3.2)	$\psi=1.90k_s k_{bc}(5/(1+f_{ci}))k_{td}t_i^{-0.118}$ $k_{bc}=(1.56-0.008*H)$ $k_{td}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\psi_{bif}$	1.526		
			initial to deck placement	$\psi_{bid}$	1.063		
			Deck placem. to final	$\psi_{bdf}$	0.897		
		Deck	Deck placem. to final	$\psi_{ddf}$	2.126	Gross section	
Steel modular ratio " (at deck placement)	$n_i=E_s/E_{ci}$	6.4678	Transformed section factors, K		$K_{id}$	0.74103	0.74594
" (deck)	$n_d=E_s/E_c$	5.3628	$K=1/(1+n_i*\alpha_{net}*A_{ps}/A_{net}*(1+0.7*\psi_{bid}))$ (5.9.5.4.2-2, 5.9.5.4.3-2)		$K_{df}$	0.75409	0.75853
		0.6786					

## Section Properties

Section Properties	Precast Beam				Transformed Deck		Composite Bm, Deck		
	Gross	Net (-A <sub>ps</sub> )	Tr.-initial	Transformed-final	Deck	Haunch	Gross	Net	Tr.-final
A (in. <sup>2</sup> )	649.00	640.32	696.46	686.87	651.49	32.87	1333.36	1324.68	1371.23
y <sub>b</sub> (in.)	16.1	16.27	15.25	15.42	40.43	35.93	28.48	28.64	27.79
I (in. <sup>4</sup> )	110444	109091	117244	115946	3475	2.74	308248	302885	330854
e <sub>p</sub> (in.)	12.40	12.57	11.55	11.72			24.78	24.94	24.09
e <sub>d</sub> (in.)							11.74	11.57	
$\alpha=1+(A*c_p^2)/I$	1.9035	1.9271	1.7931	1.8132			3.6554	3.7201	3.4057
$\alpha_b=1+A*c*y_b/I$	2.1731	2.2001	2.0471	2.0700	1.0000		4.0519	4.1237	3.7751
$\alpha_t=1+A*c*(y_b-h)/I$	-0.4085	-0.4136	-0.3848	-0.3891	1.0000		0.2548	0.2593	0.2374
Area of deck incl. haunch =					1008.43				

## 120-day Results

## Material Properties

Modulus of Elasticity in KSI (5.4.2.4)	$E=33000*w^{1.5}*K_c(f_c)^{0.5}$ $w=0.140+f_c/1000$	Beam	initial	$E_{ci}$	4406		
			at deck placement	$E_c$	5314		
		Deck		$E_{cd}$	3607		
Shrinkage (5.4.2.3.3)	$\epsilon=0.00048(5/(1+f_{ci}))k_{hs}k_{sd}$ $k_s=(1.45-0.13V/S)$ $k_{hs}=2.0-0.014H$ $k_{sd}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\epsilon_{bif}$	0.000393		
			initial to deck placem.	$\epsilon_{bid}$	0.000297		
			deck placem. to final	$\epsilon_{bdf}$	0.000097		
		Deck		$\epsilon_{ddf}$	0.000274		
Creep (5.4.2.3.2)	$\psi=1.90k_{hs}k_{sd}(5/(1+f_{ci}))k_{df}t_i^{-0.118}$ $k_{hs}=(1.56-0.008*H)$ $k_{df}=t/(61-4f_{ci}+t)$	Beam	initial to final	$\psi_{bif}$	1.526		
			initial to deck placement	$\psi_{bid}$	1.152		
			Deck placem. to final	$\psi_{bdf}$	0.868		
		Deck	Deck placem. to final	$\psi_{ddf}$	2.126	Gross section	
Steel modular ratio	$n_i=E_s/E_{ci}$	6.4678	Transformed section factors, K		$K_{id}$	0.74103	0.74594
" (at deck placement)	$n=E_s/E_c$	5.3628	$K=1/(1+n_i*\alpha_{net}*A_{ps}/A_{net}*(1+0.7*\psi_{bid}))$		$K_{df}$	0.75409	0.75853
" (deck)	$n_d=E_s/E_c$	0.6786	(5.9.5.4.2-2, 5.9.5.4.3-2)				

## Section Properties

Section Properties	Precast Beam				Transformed Deck		Composite Bm, Deck		
	Gross	Net (-A <sub>ps</sub> )	Tr.-initial	Transformed-final	Deck	Haunch	Gross	Net	Tr.-final
A (in. <sup>2</sup> )	649.00	640.32	696.46	686.87	651.49	32.87	1333.36	1324.68	1371.23
y <sub>h</sub> (in.)	16.1	16.27	15.25	15.42	40.43	35.93	28.48	28.64	27.79
I (in. <sup>4</sup> )	110444	109091	117244	115946	3475	2.74	308248	302885	330854
e <sub>p</sub> (in.)	12.40	12.57	11.55	11.72			24.78	24.94	24.09
e <sub>d</sub> (in.)							11.74	11.57	
$\alpha=1+(A*e_p^2)/I$	1.9035	1.9271	1.7931	1.8132			3.6554	3.7201	3.4057
$\alpha_p=1+A*e_p*y_b/I$	2.1731	2.2001	2.0471	2.0700	1.0000		4.0519	4.1237	3.7751
$\alpha_\tau=1+A*e_p*(y_b-h)/I$	-0.4085	-0.4136	-0.3848	-0.3891	1.0000		0.2548	0.2593	0.2374
Area of deck incl. haunch =					1008.43				