Summer 2016

Control of rigid formations for agents with passive nonlinear dynamics

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CONTROL OF RIGID FORMATIONS FOR AGENTS WITH PASSIVE NONLINEAR DYNAMICS

by

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A thesis submitted in partial fulfillment of the requirements for the Master of Science degree in Electrical and Computer Engineering in the Graduate College of The University of Iowa

August 2016

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's thesis of

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has been approved by the Examining Committee for the thesis requirement for the Master of Science degree in Electrical and Computer Engineering at the August 2016 graduation.

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ABSTRACT

This thesis concerns the decentralized formation shape control of a set of homogeneous agents in the plane whose actuation dynamics are nonlinear and passive. The formation shape is specified by a subset of interagent distances. The formation is modeled as an undirected graph, with vertices representing the agents. An edge exists between two vertices if the specification provides the distance between them. Enough distances are assumed to have been specified to make the underlying graph rigid. Each agent executes its control law by measuring its relative positions from its neighbor and by knowing its absolute velocity. The control law is the same as previously proposed for a network where the agents have linear time invariant (LTI) passive dynamics. Despite the nonlinearity we show local convergence of this same law. The stability proof is in fact simpler than given in the LTI case through a redefinition of the state space. The results are verified by simulations, which show that the control law can indeed stabilize under wider ranges of dynamics than previously perceived.

PUBLIC ABSTRACT

This thesis concerns multiple individual entities called agents working together in a multiagent system to create formation shapes using relative distances amongst each other. In modeling the agents, the relationship between the input and the output of the system is nonlinear and passive. It is required for the formation to be rigid, meaning the relative distances among all agents are constant for all continuous motions in which certain specified distances between some agents are unchanged. Each agent is responsible for achieving and maintaining the formation shape by measuring its relative positions from its neighbors and by knowing its absolute velocity. The control law to do so is the same as previously proposed for a network where the agents have an input to output relationship that is linear time invariant (LTI) passive. Despite the nonlinearity we show local convergence of this same law. The stability proof is in fact simpler than given in the LTI case through a redefinition of the state space. The results are verified by simulations, which show that the control law can indeed stabilize under wider ranges of dynamics than previously perceived.

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CHAPTER 1
INTRODUCTION

This thesis considers the distributed control of rigid formations for agents whose actuation to velocity dynamics are passive but nonlinear. In the sequel, we will say that such agents have passive, nonlinear dynamics. In doing so we extend the work of [1] that considers the same problem with linear time invariant (LTI) passive dynamics.

1.1 Motivation and Applications

Recent years have witnessed intense activity in the distributed control of multiagent systems, where groups of agents organize to perform tasks with limited centralized intervention and mutual exchange of information. This area of research is currently motivated by applications such as, but not limited to, high resolution imaging from space, coordinated search and rescue missions, and optimization of sensor networks, [16], [45].

The modern incarnation of multiagent systems, [1]-[37], was initially for achieving consensus, where all agents talk to their neighbors to achieve a common state. These literatures also include gossip algorithms, collision avoidances, and formation shape controls. What sets the literature on multiagent control apart from traditional control is the emphasis on the communication architecture that defines which agents share their state information with which other agents. Such an information exchange architecture is often modeled as a graph. Each agent represents a node. An edge exists between agent pairs that exchange information. The edge could be directed, where only one agent among the pair at the end points receives information from the other, or undirected if the information exchange is mutual.

The particular emphasis of this thesis is on *formation shape control*, [18]-[29]. Applications include self organizing sensor networks where particular agent configurations are optimal for surveillance and localization tasks. Most of this literature assumes that formation shape is specified by prescribing a subset of interagent distances. The resulting *formation topology* is also modeled as a graph. An edge exists between two nodes if the distance between them has been specified. If enough such distances are specified, then as discussed in Chapter 2, the graph is said to be *rigid*, [40]. As explained further in Chapter 2, rigid graphs represent shapes that are unique to within rotations, translations and flip ambiguities. The goal of this body of research is to devise decentralized schemes that achieve this desired formation shape with information exchange restricted between subsets of pairs of agents. Section 1.2 describes the current state of the art in formation shape control.

1.2 Background and Literature Review

We now review the state of the art on achieving formation shapes specified by interagent distances. Before doing that we acknowledge other types of formation specifications such as attitude control [38]. Such papers are beyond the scope of this review.

There are two types of control laws that have been proposed on formation shape control. The first involves laws that have a unidirectional communication architecture, [18]-[24]. In this class, of the two agents A and B at the end points of each specified interagent distance, only one (e.g. A) is responsible for achieving this distance. In the underlying graph representing the formation topology, the edge corresponding to this distance is directed from A to B. In the communication topolgy for achieving this control objective, A is permitted to sense B's position, but not vice versa. Thus, in these problems, the formation topology is modeled as a directed graph, and the information exchange topology mirrors this architecture. Much work in this area was spurred by the work of Bailleul and Suri, [18], who noted that when the graphs modeling the communication and the formation topologies have cycles, instabilities may occur due to the intuitive phenomenon of *chasing one's own tail.* The pair of papers [19] and [22] propose control laws that ameliorate instabilities caused by such cycles. Other related papers that study the effect of and combat cycles are [20], [21] and [23].

The class of problems of direct interest to this thesis was first formulated in [26]. The setting concerns formation graphs that are undirected, i.e. both agents at the end points of an edge are responsible for maintaining the distance specified. Additionally, each agent has a single integrator dynamics, i.e. with p_i the two dimensional vector of position of agent i, and u_i its control input, there holds:

$$\dot{p}_i = u_i. \tag{1.1}$$

Further, the desired formation represents a rigid graph. Suppose the formation topology is represented by the graph $G = (V, E)$, where V is the set of vertices and E the set of edges. The specified distances defining the formation topology are d_{ij} for all edges $(i,j) \in E$ between nodes $i \in V$ and $j \in V$. Then [26] formulates the obvious cost function,

$$J(p) = \sum_{(i,j)\in E} \left(d_{ij}^2 - \|p_i - p_j\|^2\right)^2, \tag{1.2}$$

where $p = [p_1, \cdots, p_n]^T$, and proposes the gradient descent control law

$$\dot{p}_i = -\frac{\partial J(p)}{\partial p_i}. \tag{1.3}$$

The resulting control law is decentralized in that to execute (1.3), the i-th agent only needs to know its relative position with respect to its *neighbors*, i.e. the nodes it shares edges with in E. Using center manifold theory, [42], [26] proves local asymptotic convergence of p_i to a point where the distance requirements are met. It also shows that in principle (1.3) may have false stationary points, and presents a simulation that shows *apparent convergence* to such a false stationary point.

The nature of these false stationary points are investigated at length in [27]-[29]. In fact [28] shows that the false stationary point example in [26] actually represents a saddle point of (1.2), i.e. it is locally unstable with respect to (1.3). In other words, if at all attained, it cannot be sustained. Simulations in [27] verify this fact. They show for example, that after a false stationary point is fleetingly acquired, the numerical errors in the Matlab simulation are enough to drive the trajectories off it, to eventual convergence of a desired formation. For formation topologies represented by K4 rectangles, [29] shows that all false stationary points of (1.3) are in fact locally unstable, i.e. (1.3) is in practical terms globally asymptotically stable.

Moving beyond single integrator dynamics [2] considers double integrator models i.e. where the control input at the i-th agent, u_i, interacts with system through

$$\dot{p}_i = v_i, \ \dot{v}_i = u_i, \tag{1.4}$$

v_i being the 2-vector of velocities in two dimensions. As attaining the formation objective also requires velocity consensus among agents, [2] has an added layer of a velocity consensus algorithm. As a result it requires that each agent sense both its relative positions and relative velocities with respect to its neighbors. This thesis on its part is most directly related to [1]. The actuation model of [2] can be viewed as a special case of that in [1], where the input to velocity relationship is more general LTI and *passive*, in other words there are $K_1 \geq 0$ and K_2 such that for all $u_i(\cdot)$, initial conditions and $t_0 < t$, [42]

$$\int_{t_0}^{t} u_i^T(\tau)v_i(\tau)d\tau \geq K_1 \int_{t_0}^{t} \|u_i(\tau)\|^2 d\tau + K_2. \tag{1.5}$$

Physical and mathematical significance of passivity is discussed in Chapter 2. As an integrator is also passive, the setting of [2] is automatically covered. However, unlike [2], the control law of Dasgupta and Anderson in [1] does not require agents to measure their relative velocities with their neighbors. Instead each agent needs its own absolute velocity.

1.3 Approach and Contribution

This thesis directly extends the work of Dasgupta and Anderson in [1] by removing the LTI restriction on the u_i to v_i dynamics, to permit it to be nonlinear but still passive. We show that the control law of [1] suffices despite the relaxation of linearity.

A notable feature of our proof is that it is in fact simpler than what was given in [1]. Specifically, both proofs invoke Lasalle's invariance principle, [42]. Among other things, this requires one to first demonstrate the compactness of the underlying state space. To that end, the proof in [1] has an intermediate step that demonstrates initial boundedness of the centroid of the formation. By redefining the state space, we are able to sidestep this additional step.

We note that passivity based ideas are also found in the context of multi-agent systems in [30]-[38]. All but [37], however, do not deal with controlling the shape of formations defined by interagent distances. Arcak in [37] does lay out a very general framework for the passive control of multiagent systems to meet general objectives that require the relative positions to converge to specified compact sets. As such, our objective may be viewed as a special case. However, the design in [37] requires the selection of certain objective dependent storage functions and memoryless nonlinearities satisfying a variety of conditions. Finding these storage functions and memoryless nonlinearities are highly nontrivial. Our control law requires no such additional design and is vastly simpler.

1.4 Outline of the Thesis

Chapter 2 reviews background material on graph rigidity and passivity. Chapter 3 gives the precise problem formulation, including the system model, actuation dynamics, other assumptions that underlie our work, and the control law. Chapter 4 has the stability analysis. Chapter 5 provides simulations. Chapter 6 concludes.

CHAPTER 2
BACKGROUND ON RIGIDITY AND PASSIVITY

There are two overarching assumptions that underlie the work of this thesis. First, the formation specification corresponds to a *rigid undirected graph*. Second, the actuation dynamics relating the control input of an agent to its velocity is *passive*. This chapter explains these concepts and provides some important mathematical characterizations that are used in later chapters. Section 2.1 describes the graphical framework used to represent formations. Section 2.2 defines and characterizes rigid graphs. Section 2.3 explains the notion of passivity.

2.1 Graphical Representation of Formations

We consider an n-agent formation in the plane with the i-th agent having two dimensional position vector $p_i = [x_i, y_i]^T \in \mathbb{R}^2$. The agents must organize themselves into a formation specified by a subset of interagent distances. In the sequel, define $p \in \mathbb{R}^{2n}$ as

$$p = [p_1^\top, \cdots, p_n^\top]^\top. \tag{2.1}$$

The formation will be associated with an *undirected* graph $G = (V, E)$. We will call the formation $F = (G, p)$. An edge exists in G between vertices $i, j \in V$ whenever the desired distance $d_{ij} = \|p_i - p_j\|$ is provided in the formation specification. The edge set E also represents the information architecture of the control law to be formulated here in the sense that, in its decentralized execution, an agent i can access the position of agent j iff j is its neighbor, i.e. there is an edge in G between the vertices i and

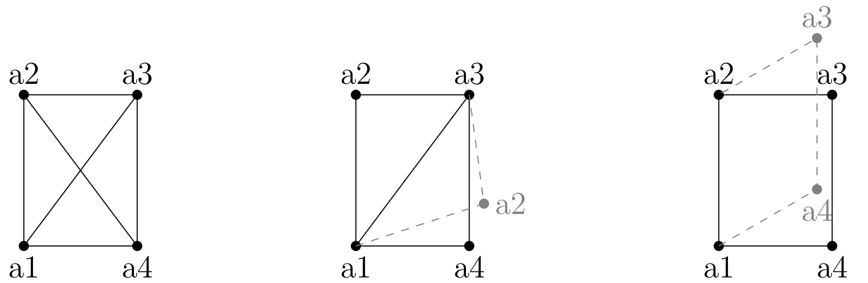
j. Unlike [2], agents have access only to their own velocities rather than that of their neighbors. Observe that $p : V \to \mathbb{R}^{2n}$ is known as a *representation* of the graph G in the sense that the vertex $i \in V$ has the position p_i in the formation $F = (G, p)$.

Figure 2.1 depicts several examples of four agent formations and the underlying graphs. Thus, in the specifications underlying Figure 2.1a, all six possible distances are specified; in Figure 2.1b, five are specified; and in Figure 2.1c, only four are specified.

2.2 Graph Rigidity

An undirected graph $G = (V, E)$ is rigid if the distances among all vertices in G are constant for all continuous motions in which the distances in E are unchanged. If the condition also holds for all discontinuous motions, the graph is said to be globally rigid. Since the formation F embodies the graph G, F is called a rigid formation if G is rigid.

Thus the graph in Figure 2.1c is not rigid as one can continuously move the nodes a_3 and a_4 in a manner that preserves the specified distances d_{12}, d_{14}, d_{23} and d_{34} while changing the distance d_{13}. On the other hand the graph in Figure 2.1b has the additional distance d_{13}. It is not globally rigid as flipping node a_2 to the depicted position maintains all five distances d_{12}, d_{14}, d_{23}, d_{13} and d_{34}, but changes the distance d_{24}. However, this is rigid as a continuous motion since moving a_2 to the new position fails to preserve all the distances d_{12}, d_{14}, d_{23}, d_{13} and d_{34}. Figure 2.1a on the other hand is globally rigid.



(a) a rigid formation that is also globally rigid

(b) a rigid but not globally rigid formation

(c) a non-rigid formation

Figure 2.1: Three formations with identical node locations. One shows global rigidity; whereas another shows the edge lengths preserved through a discontinuous motion ambiguity (i.e. local but not globally rigid); and a third formation shows all specified edge lengths preserved with continuous motion ambiguity.

Figure 2.1b is in fact also *minimally rigid* as removing any of the five specified edges removes rigidity. Observe with $n = 4$, $2n - 3 = 5$ the number of edges in this minimally rigid graph. Indeed it has been known since the days of Maxwell that a necessary condition for an n-node graph to be rigid is that it has at least $2n-3$ edges. This is however not a sufficient condition. Consider for example the five vertex graph in Figure 2.2, which does have $2\times5-3 = 7$ edges. The problem is that 6 of these edges are incident at only four nodes between them. In fact, rigidity requires that there be at least $2n - 3$ *well distributed edges*. A characterization of "well distributed" has been provided relatively recently by the celebrated Laman's theorem. This theorem refers to an induced subgraph that is defined as follows: Consider $G = (V, E)$ and

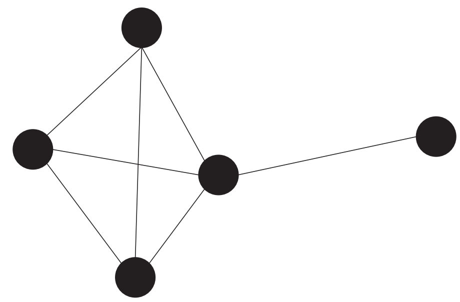


Figure 2.2: Example of a graph with $2n-3$ edges that is not rigid.

$V' \subset V$. Then the subgraph induced by V' is obtained by removing the vertices in $V \setminus V'$ and the edges incident on any of these vertices. Thus the graph in Figure 2.3 is an induced subgraph of the graph in Figure 2.2.

Laman's Theorem: Almost all representations of a graph $G = (V, E)$ is rigid iff there is an edge set $E_1 \subset E$ such that $G = (V, E_1)$ is minimally rigid, i.e. $|E_1| = 2|V| - 3$ and for all $V' \subset V$, the subgraph induced by V' has no more than $2|V'| - 3$ edges.

More pertinent to our purposes, an n-node graph is rigid iff its *rigidity matrix* $R(p)$ has rank $2n-3$ for almost all representations. The rigidity matrix of the representation $F(G, p)$ for an n-node graph $G = (V, E)$ is defined as follows: $R(p)\colon p \to \mathbb{R}^{\|E\| \times 2n}$ has one row for every edge in G. If the edge is between vertices i

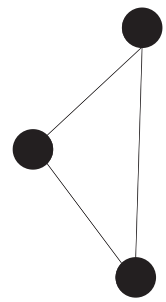


Figure 2.3: An induced subgraph of the graph in Figure 2.2.

and j, then the corresponding row has $(p_i - p_j)^T$ and $(p_j - p_i)^T$ in the $2i-1, 2i$ and $2j-1, 2j$ locations, respectively. The remaining elements of the row are zero. The rigidity matrix from Figure 2.1a, which is used for simulation in chapter 6, can be written as follows:

$$\begin{bmatrix} (p_1 - p_2)^\top & (p_2 - p_1)^\top & 0 & 0 \\ (p_1 - p_3)^\top & 0 & (p_3 - p_1)^\top & 0 \\ (p_1 - p_4)^\top & 0 & 0 & (p_4 - p_1)^\top \\ 0 & (p_2 - p_3)^\top & (p_3 - p_2)^\top & 0 \\ 0 & (p_2 - p_4)^\top & 0 & (p_4 - p_2)^\top \\ 0 & 0 & (p_3 - p_4)^\top & (p_4 - p_3)^\top \end{bmatrix}$$

Finally, a graph is minimally rigid if it is rigid and has precisely $2n-3$ edges.

2.3 Passivity

We now turn to the notion of passivity. Consider a system with an input vector $u(\cdot)$ and output vector $y(\cdot)$ with the same dimensions. Then the system is passive if there exists a $K_1 \geq 0$ and K_2 such that for all $u(\cdot)$, and initial time t_0, there holds

$$\int_{t_0}^{t} y^T(\tau)u(\tau)d\tau \geq K_1 \int_{t_0}^{t} \|u(\tau)\|^2 d\tau + K_2, \tag{2.2}$$

where K_2 may depend on initial conditions but K_1 may not. In systems theory, this notion of passivity has its origins in passive circuits. In a multi-port passive circuit, suppose the elements of u and y are matching port voltages and currents, then with $K_2 = 0$, the non-negativity of K_1 and (2.2) together reflect the fact that the circuit cannot generate power. Nonzero K_2 helps capture initial condition effects. Many mechanical systems, like those with masses, springs, and dashpots modeling viscous friction, are also passive with elements of u being forces acting on masses and the corresponding elements of y as the displacements of those masses.

An LTI system with transfer function matrix $G(\cdot) : \mathbb{C} \to \mathbb{C}^{m\times m}$ is passive iff it is *Positive Real (PR)*, i.e. it satisfies the following properties:

- $G(s)$ is analytic for $\text{Re}[s] > 0$.
- It is real for positive real s.
- For all $\text{Re}[s] \geq 0$

  $$G^H(s) + G(s) \geq 0$$

  i.e. $G(s)$ is positive semidefinite.

Observe, all admittance and impedance matrices of m-port passive circuits are PR. The celebrated Kalman-Yakubovic-Popov (KYP) Lemma, [44], [43] characterizes PR in terms of properties of the state variable realization (SVR) of PR. Specifically, suppose with $x : \mathbb{R} \to \mathbb{R}^N$, $A \in \mathbb{R}^{N\times N}$, $B \in \mathbb{R}^{N\times N}$, $C \in \mathbb{R}^{m\times N}$ and $D \in \mathbb{R}^{m\times m}$, $G(s)$

has the SVR

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.3}$$

$$y(t) = Cx(t) + Du(t). \tag{2.4}$$

The the KYP lemma asserts that $G(s)$ is PR iff there exists a positive definite symmetric P and a matrices Q_i such that:

$$\begin{bmatrix} A^T & -C^T \\ B^T & -D^T \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A & B \\ -C & -D \end{bmatrix} = - \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \begin{bmatrix} Q_1^T & Q_2^T \end{bmatrix}. \tag{2.5}$$

In other words, there obtains

$$A^T P + PA = -Q_1 Q_1^T \tag{2.6}$$

$$PB - C^T = -Q_1 Q_2^T \tag{2.7}$$

$$D + D^T = Q_2 Q_2^T. \tag{2.8}$$

Then as $P = P^T$, from (2.6-2.8) we get

$$\begin{aligned} 2x^T P\dot{x} - 2u^T y &= x^T P\dot{x} + \dot{x}^T Px - 2u^T y \\ &= x^T (PAx + PBu) + (PAx + PBu)^T x - 2u^T y \\ &= x^T (PA + A^T P)x + 2x^T PBu - 2y^T u \\ &= -x^T Q_1 Q_1^T x + 2\left(x^T PB - x^T C^T\right) u - u^T (D + D^T) u \\ &= -x^T Q_1 Q_1^T x - 2x^T Q_1 Q_2^T u - u^T Q_2 Q_2^T u \\ &= -x^T Q_1 Q_1^T x - x^T Q_1 Q_2^T u - u^T Q_2 Q_1^T x - u^T Q_2 Q_2^T u \\ &= -\left\| Q_1^T x + Q_2^T u \right\|^2 \\ &\leq 0. \end{aligned}$$

It turns out that this last inequality is enough to prove (2.2). In particular consider the positive definite *storage function*

$$S(x) = x^T P x. \tag{2.9}$$

Observe that

$$\begin{aligned} \dot{S}(x) &= x^T P \dot{x} + \dot{x}^T P x \\ &= 2x^T P \dot{x} \\ &\leq 2u^T y \end{aligned} \tag{2.10}$$

Consequently, as $S(x) \geq 0$ for all x,

$$\begin{aligned} \int_{t_0}^{t} u^T(\tau) y(\tau) d\tau &\geq \frac{S(x(t)) - S(x(t_0))}{2} \\ &\geq -\frac{S(x(t_0))}{2}, \end{aligned}$$

i.e. (2.2) holds with $K_1 = 0$ and $K_2 = -\frac{S(x(t_0))}{2}$.

What about nonlinear passive systems? From the seminal work of Hill and Moylan, [39] provides a comparable characterization: That a nonlinear system

$$\dot{x} = f(x, u); \quad y = h(x, u) \tag{2.11}$$

is passive if there exists a *positive definite storage function* $S(x)$ such that (2.10) holds. Indeed, when we talk of nonlinear passive systems in subsequent chapters, we will assume that each has such a storage function whose derivative is upper bounded by the inner product of the input and output vectors. Whereas this storage function is quadratic in the state for LTI passive systems, it need not be so for nonlinear systems.

2.4 Conclusion

As groundwork for future chapters, in this chapter, we have provided a review of graph rigidity and nonlinear system passivity. These concepts are crucial to this thesis. The formations we will seek to attain will be rigid. The actuation dynamics of the agents will be possibly nonlinear but passive.

CHAPTER 3
PROBLEM FORMULATION AND THE CONTROL LAW

This chapter provides the precise problem formulation and the distributed control law that meets the control objective. As noted in earlier chapters, our overarching goal is to formulate a distributed control law that induces a collection of n agents to organize themselves into a specified rigid formation, with information exchange limited to that between nearest neighbors. Like [1] the control input to velocity dynamics, known henceforth as *actuation dynamics*, of each agent is assumed to be passive, integral action being a special case. Unlike [1], the actuation dynamics is permitted to be nonlinear. Section 3.1 defines the system dynamics. Section 3.2 formally defines the problem and states the assumptions. Section 3.3 provides the control law. Section 3.4 summarizes the contributions of this chapter.

3.1 System Model

We assume that there are n agents in the plane with position vector of the i-th agent $p_i \in \mathbb{R}^2$. We will call $v_i \in \mathbb{R}^2$ the velocity vector for agent i. In particular each agent obeys:

$$\dot{p}_i = v_i. \tag{3.1}$$

Equivalently, with $p = [p_1^\top, p_2^\top, ..., p_n^\top]^\top \in \mathbb{R}^{2n}$,

$$\dot{p} = v \tag{3.2}$$

where $v = [v_1^\top, v_2^\top, ..., v_n^\top]^\top \in \mathbb{R}^{2n}$. We assume $u_i \in \mathbb{R}^2$ as the control input to the i-th agent, and that $u = [u_1^\top, u_2^\top, ..., u_n^\top]^\top \in \mathbb{R}^{2n}$. We also assume that the agents are homogeneous. Furthermore, with z_i the state vector for the i-th agent, for suitable functions $f(.,.)$ and $h(.,.)$, the u_i to v_i dynamics is given by:

$$\dot{z}_i = f(z_i, u_i), \;\; f(0,0) = 0. \tag{3.3}$$

$$v_i = h(z_i), \;\; h(0) = 0. \tag{3.4}$$

We assume that this dynamics is passive. More precisely, in view of the discussion in Section 2.3, we make the following assumption.

Assumption 3.1. The state space representation in (3.3, 3.4) are such that there exists a positive definite storage function $S(\cdot)$, such that for all $i \in \{1, \cdots, n\}$,

$$\dot{S}(z_i) \leq u_i^T v_i. \tag{3.5}$$

We also make an observability and controllability assumption.

Assumption 3.2. In the state space representation in (3.3, 3.4) $v_i \equiv 0$ implies $z_i \equiv 0$. Further $z_i \equiv 0$ implies $u_i \equiv 0$. Finally, z_i is bounded if v_i is bounded.

We contrast this system model with those of [1],[2] and [26], and argue that all three are special cases. All three obey (3.2). In [26], $v_i = u_i$, which is trivially a passive system. In [2], $\dot{v}_i = u_i$, which is passive as $\dfrac{1}{s}$ is PR. In [1] the transfer function from u_i to v_i is LTI and PR and thus obeys Assumption 3.1.

3.2 Formation Specification and Control Objective

The desired rigid formation $F(\bar{p}, G)$ involves a rigid undirected graph $G = (V, E)$, with $V = \{1, \cdots, n\}$. Define

$$d_{ij} = \|p_i - p_j\|, \;\; \forall \{i,j\} \in E. \tag{3.6}$$

Define the vector $D(p) \in \mathbb{R}_+^{|E|}$, as

$$D(p) = \left\{d_{ij}^2\right\}_{\{i,j\}\in E}. \tag{3.7}$$

The desired representation $p = \bar{p}$ has the property that for every edge $\{i, j\} \in E$ and desired distance $\bar{d}_{ij}$ between i and j, $\bar{p}$ obeys

$$\bar{d}_{ij} = \|\bar{p}_i - \bar{p}_j\|,\ \forall\ \{i, j\} \in E. \tag{3.8}$$

Since rotating and translating a rigid formation does not change the interagent distances, there exists a noncompact manifold of any member which is a valid formation:

$$\mathcal{M}(\bar{p}) = \{p \mid D(p) = D(\bar{p})\}. \tag{3.9}$$

More formally, the control objective is as follows. Call the set of neighbors of i,

$$\mathcal{N}(i) = \{j|\{i, j\} \in E\}. \tag{3.10}$$

Define $P_i \in \mathbb{R}^{2|\mathcal{N}(i)|}$ to be the vector of position vectors of the neighbors of i, i.e.

$$P_i = \{p_j\}_{j\in\mathcal{N}(i)}. \tag{3.11}$$

Then the control goal is to choose $u_i : \mathbb{R} \to \mathbb{R}^2$ of the form

$$u_i = f(P_i, v_i) \tag{3.12}$$

so that

$$\lim_{t\to\infty} p(t) \in \mathcal{M}(\bar{p}), \tag{3.13}$$

or equivalently

$$\lim_{t\to\infty} D(p(t)) = D(\bar{p}). \tag{3.14}$$

This must be done under the following rigidity assumption.

Assumption 3.3. The rigidity matrix $R(\bar{p})$ has rank greater or equal to $2n-3$.

A few remarks are in order. First, (3.12) sets the information exchange architecture. To execute its control law, each agent can only use the positions of its neighbors and its own velocity. This contrasts with [2] which additionally requires all its neighbor's velocities. Second, there is no unique $\bar{p}$ to which convergence is needed; rather, any member of the manifold $\mathcal{M}(\bar{p})$ suffices. However, we demand that convergence be to a point in this manifold. Further, the fact that all elements of this manifold suffice to meet the control goal, precludes global stability. This is also the case with [1],[2] and [26].

3.3 The Control Algorithm

As in [1],[2] and [26], define the cost function

$$J(p) = \frac{\|D(p) - D(\bar{p})\|^2}{4}. \tag{3.15}$$

Clearly $J(p) = 0$ iff $p \in S(\bar{p})$. Moreover, the gradient $\nabla J(p)$ can be written as

$$\nabla J(p) = R^\top(p)(D(p) - D(\bar{p})). \tag{3.16}$$

Then the proposed control law for the nonlinear system is borrowed from [1] where

$$u = -v - R^\top(p)(D(p) - D(\bar{p})). \tag{3.17}$$

Also restated here from [1],

$$u_i = -v_i - \sum_{j \in N(i)} (d_{ij}^2 - \bar{d}_{ij}^2)(p_i - p_j). \tag{3.18}$$

Clearly, this law respects the form in (3.12).

3.4 Conclusion

In this chapter the formation architecture has been provided and the nonlinear dynamics of the agents have been presented. We have specified a control objective and enunciated a decentralized control law for meeting this objective. The next chapter formally proves that the control law does indeed achieve the desired objective.

CHAPTER 4
STABILITY ANALYSIS

Having presented the system model, the control objective and the control law in the previous chapter, we now formally prove its local uniform asymptotic stability. A key device to be used in our proof is Lasalle's invariance principle, [42]. Thus, in Section 4.1 we summarize the version of this principle with relevance to our proof. In Section 4.2 we explain why the proof of [1] for the LTI case does not go through here. Section 4.3 provides the formal proof; which, because of a redefinition of the state space, turns out to be simpler than that in [1]. Section 4.4 is the conclusion for this chapter.

4.1 Lasalle's Invariance Principle

Lasalle's invariance principle, also known as Lasalle's theorem, is a powerful stability analysis tool for time invariant systems of the form

$$\dot{x} = F(x). \tag{4.1}$$

Note in particular, time dependence in the right hand side is not explicit. Dependence on time only comes from the time dependence of $x(\cdot)$. Suppose the following conditions are obtained.

(A) The state trajectories of (4.1) lie in a compact set.

(B) There is a function $V(x)$ that is bounded from above and below.

(C) Along the trajectories of (4.1),

$$\dot{V}(t) \leq 0. \tag{4.2}$$

Then all trajectories converge to the largest invariant set of (4.1) on which

$$\dot{V}(t) \equiv 0. \tag{4.3}$$

Much effort in [1] is expended in proving (A); specifically, by first showing that the centroid of the formations generated by the control law in [1] are bounded and that the interagent distances are also bounded. Even though [26] uses center manifold theory, it too relied on proving *a priori* that the centroid of the formation, its law generates, is constant. As explained in Section 4.2, the nonlinear nature of our setup makes proving the *a priori* boundedness of the centroid difficult. Instead in Section 4.3 we use a redefined state space to circumvent this problem.

4.2 Departure from [1]

The device used in [1] to show (A) in Lasalle's invariance principle heavily relied on the agent dynamics being linear. In particular, the rigidity matrix $R(p)$ has the following property. With $\mathbf{1}$ the vector of all ones,

$$R(p)\mathbf{1} = 0. \tag{4.4}$$

Thus from (3.17) one gets

$$\mathbf{1}^T u = -\mathbf{1}^T v. \tag{4.5}$$

If, as is the case with [1], for each i the u_i to v_i relationship is LTI and passive then so is the relationship from $\mathbf{1}^T u$ to $\mathbf{1}^T v$. Consequently, (4.5) boils down to the closed loop relationship depicted as in Figure 4.1.

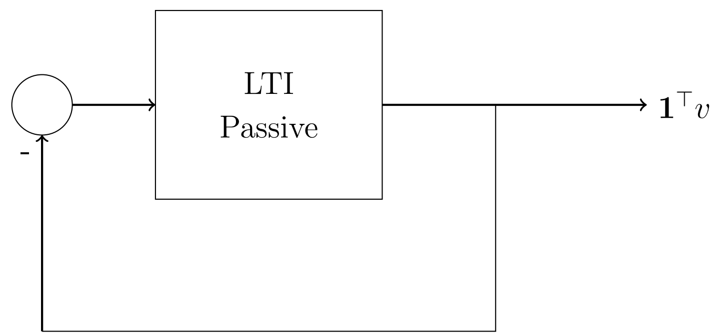


Figure 4.1: A closed loop used to show boundedness in [1].

Then as the forward path is passive and the reverse strictly passive, from the passivity theorem [42], $\mathbf{1}^T v$ asymptotically approaches zero. As the closed loop in Figure 4.1 is LTI, this implies that $\mathbf{1}^T v$ converges *exponentially* to zero. Because of (3.2), this implies that the centroid of the formation

$$\frac{\mathbf{1}^T v}{n}$$

is bounded.

This argument fails in our setup despite the fact that (4.5) still holds. This is so, as in our case, the u_i to v_i blocks are nonlinear. Thus, their individual passivity does not mean that the relationship from $\mathbf{1}^T u$ to $\mathbf{1}^T v$ is also passive. Further, for a nonlinear system, asymptotic stability need not imply exponential stability. Thus, from *a priori* it is difficult to conclude the boundedness of the centroid and hence the conclusion of (A).

4.3 Convergence Proof

We first begin with a redefinition of the state space. Consider the closed loop comprising the equations (3.2-3.4) and (3.18). Observe, first that the closed loop is time invariant. Further, beyond (3.2) the p_i only appears in the form of its difference with other position vectors, through (3.18). Thus, one can redefine the state space as represented by a vector x that beyond the v_i and z_i *comprises not of* p_i *per se, but rather* $p_i - p_j$. The closed loop can then be expressed as in (4.1) for a suitably defined $F(\cdot)$ whose precise form is immaterial. Thus, if one can show that v_i, z_i, and $J(p)$ defined in (3.15) is bounded, then x is bounded, i.e. the state space is compact. This is so, as again, $J(p)$ comprises of summands involving $\|p_i - p_j\|^2$. This key observation permits us to avoid having to prove the *a priori* boundedness of the centroid, in order to prove the compactness of the state space.

Indeed, in the sequel the role of $V(\cdot)$ in Section 4.1 will be played by the Lyapunov like function:

$$L(p, z, v) = J(p) + \int_{t_0}^{t} v^{\top}(\tau) u(\tau) d\tau. \tag{4.6}$$

Using this, we will prove the boundedness of $J(p)$, v, and z and hence the compactness of the redefined state space.

As a first step, we now establish that the integral in (4.6) is bounded from below. Thus $\dot{L}(p, z, v) \leq 0$ will imply the boundedness of $J(p)$.

Lemma 4.1. *Consider (3.3,3.4) under Assumption 3.1. Then for every* t_0 *and finite* $z_i(t_0)$*, the integral in (4.6) is bounded from below.*

Proof. Because of Assumption 3.1 there is a positive definite $S(z_i)$ such that (3.5) holds. Thus, as $S(z_i(t)) \geq 0$ for all t_0,

$$\begin{aligned}\int_{t_0}^{t} v^\top(\tau)u(\tau)d\tau &\geq S(z_i(t)) - S(z_i(t_0)) \\ &\geq -S(z_i(t_0)).\end{aligned} \tag{4.7}$$

The result follows.

We now prove the uniform convergence of the $\nabla J(p)$ to zero.

Theorem 4.2. *Consider the system described by (3.2-3.4) and (3.17). Suppose Assumptions 3.1-3.3 hold. Then all trajectories converge uniform asymptotically to the set:*

$$\nabla J(p) \equiv 0,\ \ v \equiv 0,\ \ \textit{and}\ z_i \equiv 0. \tag{4.8}$$

Proof. As the closed loop is time invariant, asymptotic convergence must be uniform with respect to the initial time. Observe, as in [1] that

$$\begin{aligned}\dot{L}(p,z,v) &= [\nabla J(p)]^\top \dot{p} + v^\top u \\ &= [\nabla J(p)]^\top v + v^\top u \\ &= -\|v\|^2 \\ &\leq 0.\end{aligned} \tag{4.9}$$

Thus $L(p,z,v)$ is bounded from above. As $J(p) \geq 0$, from Lemma 4.1, $L(p,z,v)$ is also bounded from below. Thus from (4.9) v is bounded and from Assumption 3.2, the z_i are bounded. Also $J(p)$ is bounded. Thus, for all $\{i,j\} \in E$, $\|p_i - p_j\|$ are bounded. As from Assumption 3.3 the formation is rigid, at one p, then at all p, the formation is connected. Thus $p_i - p_j$ are bounded for all i, j and the state vector x comprising of $p_i - p_j$, v, and z_i is bounded. As the closed loop can be written as in (4.1); by Lasalle's theorem, uniform asymptotic convergence to $v \equiv 0$ must occur. From Assumption 3.2, this leads to $u \equiv 0$. From (3.17), this in turn implies that $\nabla J(p) \equiv 0$.

Thus, uniform convergence occurs to a critical point of $J(p)$. As at the limit point $v \equiv 0$, convergence is to a point. As shown in [26] some of these critical points may not correspond to the global minimum of $J(\cdot)$. Thus global convergence may not be possible, though [29] did show that at least when the desired formation is a K-4 rectangle, these false stationary points are unstable at least for the algorithm in [26]. Whether that is the case here is an open issue. Nonetheless, all that Theorem 4.2 shows is convergence to a critical point and that global convergence to the global minimum may not be possible. The next theorem does show that local convergence is indeed guaranteed.

The proof of the theorem is identical to that in [1] but is repeated for the sake of completeness.

Theorem 4.1. Under the conditions of Theorem 4.2, there is a neighborhood $U(\bar{p})$ of $\mathcal{M}(\bar{p})$ and a ball $B(\epsilon) = \{z_i | \|z_i\| \leq \epsilon\}$ such that (3.13) holds for all $p(t_0) \in U(\bar{p})$ and $z_i(t_0) \in B(\epsilon)$.

Proof. As the system is time invariant, we will assume that $t_0 = 0$. Observe $J(p)$ is analytic. Thus the Lojasiewicz inequality, [46], shows that there is a neighborhood

$\Omega(\bar{p})$ of $\mathcal{M}(\bar{p})$, and constants $K > 0$ and $\lambda \in (0, 1)$ such that for all $p \in \Omega(\bar{p})$

$$J(p) \leq K\|\nabla J(p)\|^{\lambda}. \tag{4.10}$$

Thus, $\nabla J(p) = 0$ implies $J(p) = 0$ for all $p \in \Omega(\bar{p})$.

Call $\partial\Omega(\bar{p})$ to be the boundary of $\Omega(\bar{p})$ and

$$J^* = \min_{p \in \partial\Omega(\bar{p})} J(p). \tag{4.11}$$

By continuity, there exists an ϵ and $U(\bar{p}) \subset \Omega(\bar{p})$ such that for all $z_i(0) \in B(\epsilon)$ and $p \in U(\bar{p})$, because of (4.7)

$$L(p, z, v) < J^* - \sum_{i=1}^{n} S(z_i(0)) \text{ and } J(p) \leq J^*. \tag{4.12}$$

Choose, $z_i(0) \in B(\epsilon)$ and $p(0) \in U(\bar{p})$. As $L(p, z, v)$ is non-increasing, for all $t \geq 0$ there holds:

$$\begin{aligned} J^* - \sum_{i=1}^{n} S(z_i(0)) &> L(p(t), z(t)) \\ &\geq J(p(t)) - \sum_{i=1}^{n} S(z_i(0)) \end{aligned}$$

Thus for all $t \geq 0$, $J(p(t)) < J^*$. Because of (4.11) and continuity, $p(t) \in \Omega(\bar{p})$ for all $t \geq 0$. Thus the result holds from Theorem 4.2 and (4.10).

4.4 Conclusion

We have shown that despite nonlinearity manifest in the actuation dynamics, the control law of [1], formulated for LTI dynamics, still works as long as the nonlinear dynamics are passive. Further, the stability proof is in fact simpler than that in [1] where we do not need to prove the boundedness of the centroid. The next chapter provides simulations.

CHAPTER 5
SIMULATIONS

This section presents some simulations. We assume that for each agent the control input to velocity dynamics are decoupled in the two dimensions. For simplicity, call the control input in a given direction to be w, the sate vector $[\eta_1, \eta_2]^T$, and the velocity y. Then the dynamics are given as

$$\begin{aligned}\dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= -(1+\eta_1^2)\eta_1 - \eta_2 + w \\ y &= \eta_2.\end{aligned} \tag{5.1}$$

This is in fact a mass-spring-dashpot system with a nonlinear spring.

We now show that this system is passive. Indeed choose:

$$S(\eta_1, \eta_2) = \int_0^{\eta_1} (1+s^2)s ds + \frac{\eta_2^2}{2}$$

Clearly this is positive definite. Further

$$\begin{aligned}\dot{S}(\eta_1, \eta_2) &= \dot{\eta}_1(1+\eta_1^2)\eta_1 + \dot{\eta}_2\eta_2 \\ &= \eta_2(1+\eta_1^2)\eta_1 + \eta_2\left(-(1+\eta_1^2)\eta_1 - \eta_2 + w\right) \\ &= -\eta_2^2 + \eta_2 w \\ &\leq yw.\end{aligned}$$

The formation is as in Figure 2.1a: the specified $(a1, a3)$ and $(a2, a4)$ edges are of length five, $(a1, a2)$ and $(a3, a4)$ edges are of length four, and $(a2, a3)$ and $(a1, a4)$ edges are of length three.

In the first simulation, we set $[a_1 = (2,2), a_2 = (0,4), a_3 = (3,4), a_4 = (3,0)]$ as the initial (x, y) positions of the agents. The results are shown in Figure 5.1 and 5.2. From a qualitative perspective, the initial position of agent a_1 has edge distances shorter to its nearest neighbors than the specified length. Initially in Figure 5.2, the agent's position moves out towards the position $(0,0)$. As expected, the velocity decreases over time as the error distances, as seen in Figure 5.1, between the agents' actual and specified edge-lengths approaches zero.

In the second simulation, we consider the opposite scenario with the agents' initial positions set at $[a_1 = (0,0), a_2 = (0,4), a_3 = (5,5), a_4 = (3,0)]$. In this case, the agent a_4 initial distances to its nearest neighbors are greater than the specified edge-lengths. The results from Figure 5.3 and 5.4 show the change of velocity over time mapped to the positions of the agents as the differences between actual and specified edge-length distances decreases to zero. An interesting difference between the first simulation and the second simulation is the rate of change on the agents position. Intuitively, from (3.15) the cost function is based on the difference between the edge-lengths squared. Thus, with the second simulation where the $(a1, a3)$ edge-length difference is much greater than in the first simulation, the velocity of the agents seen in Figure 5.4 is also greater than that of Figure 5.2.

Finally, in the third simulation, we observe an initial formation where all four agents' neighboring edge-lengths differ from the specified formation's edge-lengths. The initial positions of the agents in Figure 5.6 are $[a_1 = (1,2), a_2 = (1,4), a_3 = (2,2), a_4 = (2,1)]$. The results shown in Figure 5.5 and 5.6. Compared to the first simulation, we see similar motion trajectory dynamics as agent a_1 except seen across all four agents in the third simulation. Additionally, the final positions of the agents exhibits the rotational ambiguity that exists with formation control.

To this end, we have shown through three simulations the results of the control algorithm and the convergence to a specified formation. In the first simulation, we show that an initial formation with smaller edge-length distances than the specified formation converges to the desired formation. In the second simulation, an initial formation with greater edge-length than the specified formation also converges to the desired formation. Lastly, the final simulation starts with an initial formation much smaller of agents' neighboring distances than the specified formation, which also converges to the desired formation.

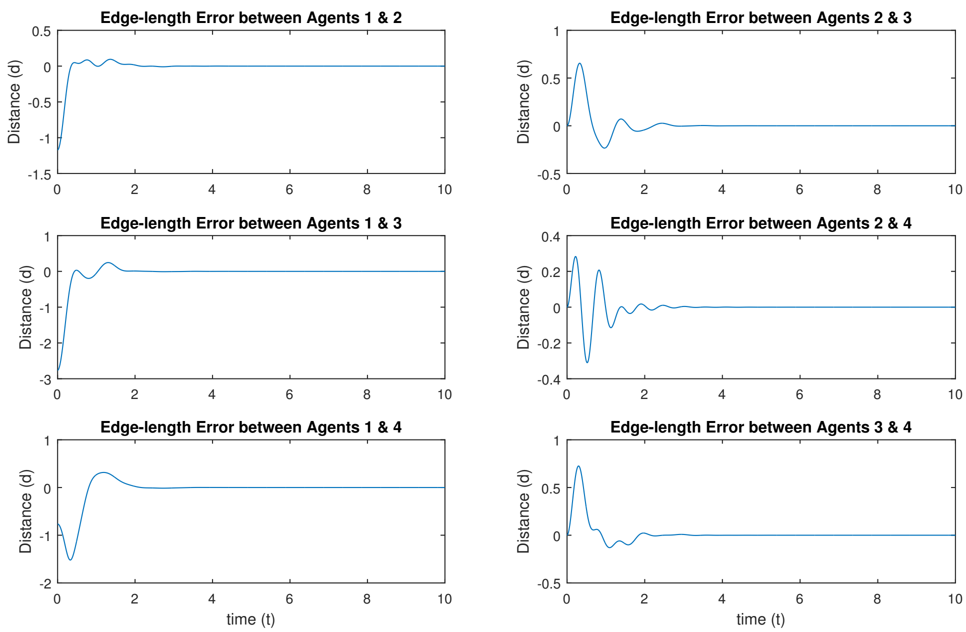


Figure 5.1: The individual edges of $\|D(p) - D(\bar{p})\|$ for the case where $D(p)$ at $t = 0$ is derived from the initial positions of the agents at $a_1 = (2, 2)$, $a_2 = (0, 4)$, $a_3 = (3, 4)$, and $a_4 = (3, 0)$ and $D(\bar{p}(t) \forall t \geq 0$ can be represented by a graph: $a_{1f} = (0, 0)$, $a_{2f} = (0, 4)$, $a_{3f} = (3, 4)$, and $a_{4f} = (3, 0)$

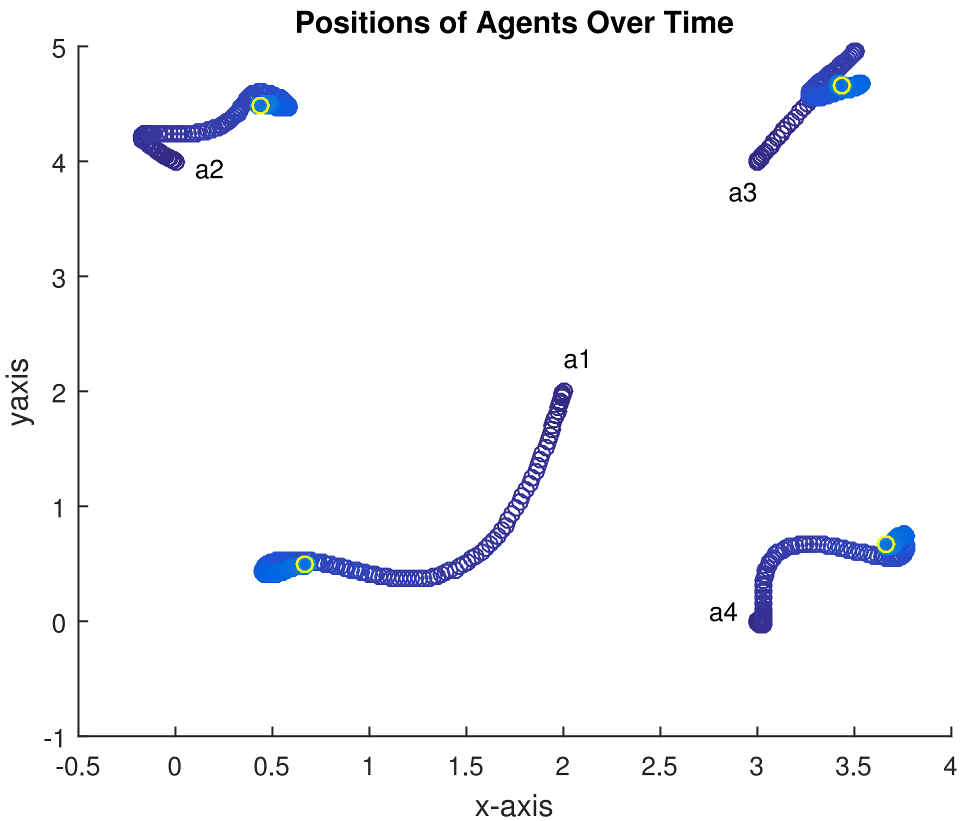


Figure 5.2: Agents trajectory in the xy-plane with initial positions $a_1 = (2, 2)$, $a_2 = (0, 4)$, $a_3 = (3, 4)$, and $a_4 = (3, 0)$.

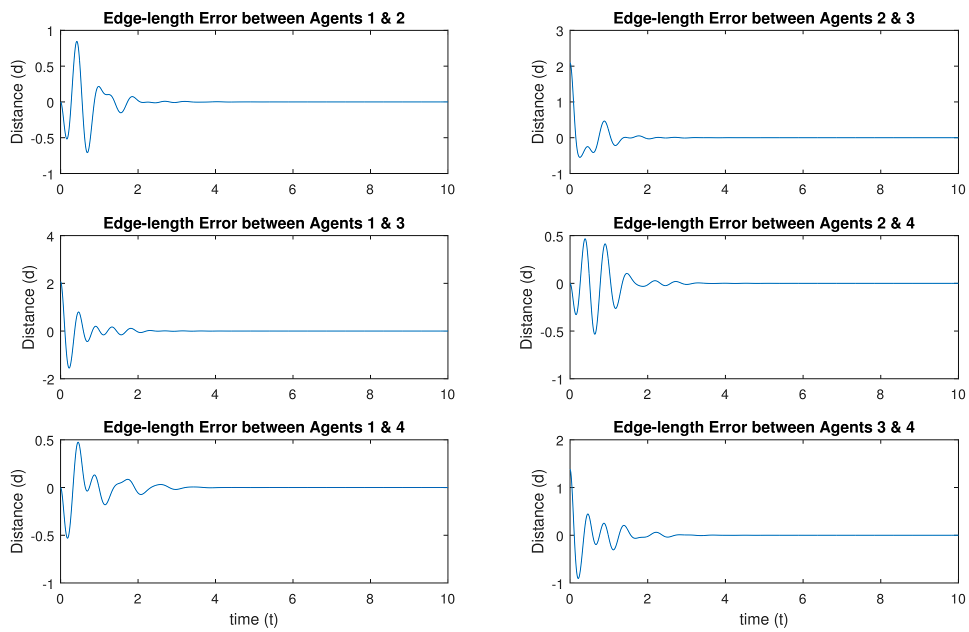


Figure 5.3: The individual edges of $\|D(p) - D(\bar{p})\|$ for the case where $D(p)$ at $t = 0$ is derived from the initial positions of the agents at $a_1 = (0, 0)$, $a_2 = (0, 4)$, $a_3 = (5, 5)$, and $a_4 = (3, 0)$ and $D(\bar{p})$ is the same as in Figure 5.1

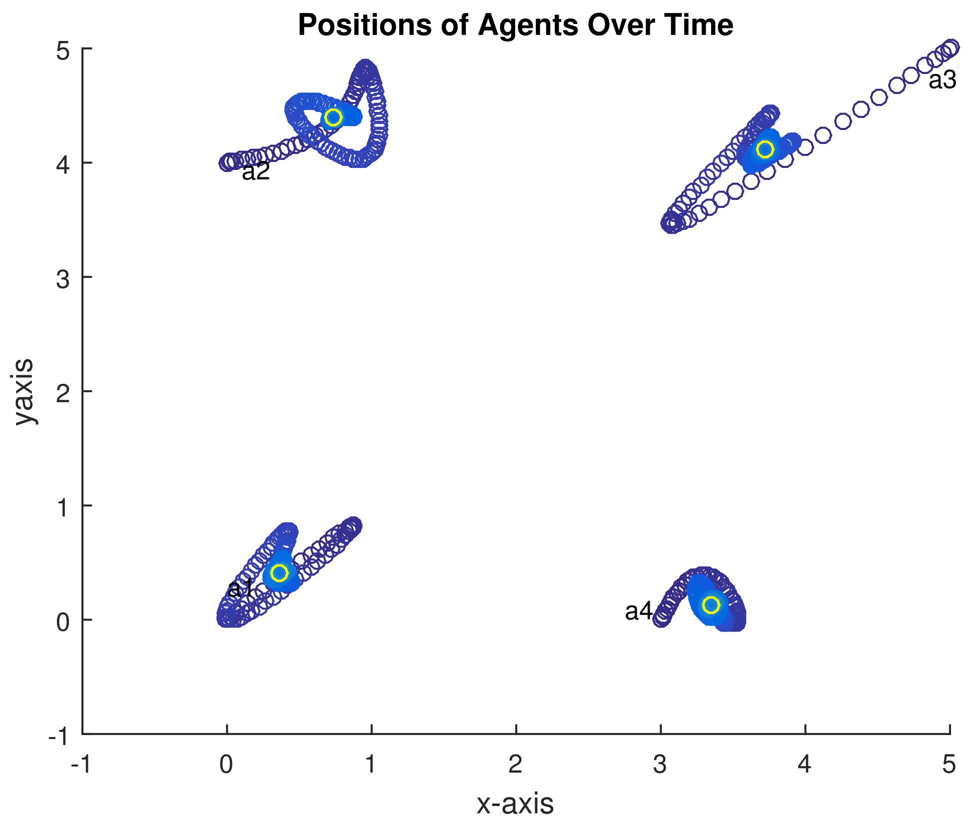


Figure 5.4: Agents trajectory in the xy-plane with initial positions $a_1 = (0, 0)$, $a_2 = (0, 4)$, $a_3 = (5, 5)$, and $a_4 = (3, 0)$.

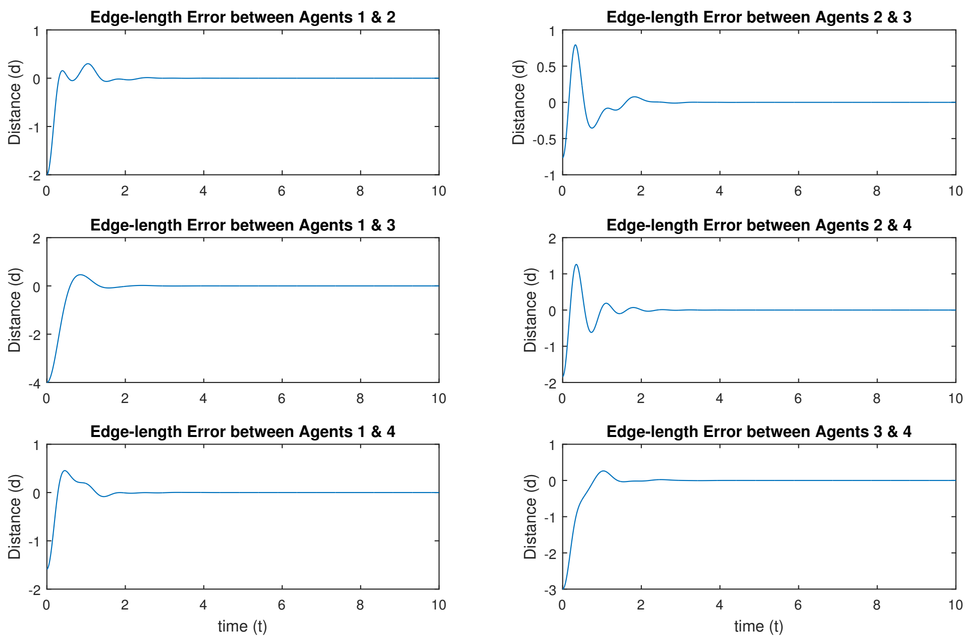


Figure 5.5: The individual edges of $\|D(p) - D(\bar{p})\|$ for the case where $D(p)$ at $t = 0$ is derived from the initial positions of the agents at $a_1 = (1, 2)$, $a_2 = (1, 4)$, $a_3 = (2, 2)$, and $a_4 = (2, 1)$ and $D(\bar{p})$ is the same as in Figure 5.1

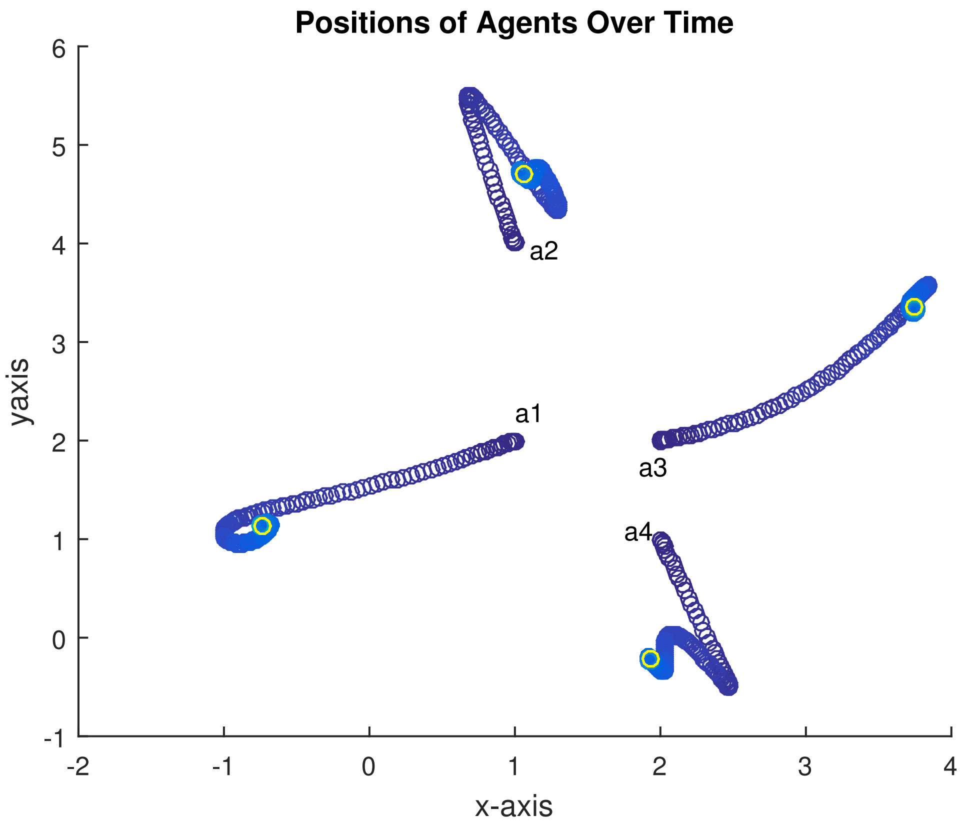


Figure 5.6: Agents trajectory in the xy-plane with initial positions $a_1 = (1, 2)$, $a_2 = (1, 4)$, $a_3 = (2, 2)$, and $a_4 = (2, 1)$.

CHAPTER 6
CONCLUSION

This thesis examined the decentralized formation shape control of a set of homogeneous agents in the plane whose control input to velocity dynamics are nonlinear and passive. The formation shape is specified by a subset of interagent distances and corresponds to a rigid formation. Each agent executes its control law by measuring its relative positions from its neighbor and by knowing its absolute velocity. The control law is the same as previously proposed for a network where the agents have linear time invariant (LTI) passive dynamics. Despite the nonlinearity, we show local convergence of this same law. The stability proof is in fact simpler than given in the LTI case through a redefinition of the state space. The results are verified by simulations where each agent's control to velocity dynamics represents a mass spring dashpot system with a nonlinear spring.

6.1 Future Work

There are several possible extensions to this work. First, global convergence is to a critical point of a cost function, which may not correspond to a desired formation. It was shown in [29] that, at least when the desired formation is a K-4 rectangle, all false stationary points are in fact saddle points and that, at least for the algorithm of [26], are locally unstable. In other words, the latter algorithm is for all practical purposes globally stable for a desired formation that is a K-4 rectangle. Whether that is also the case for the algorithm here is an open question.

Second, we have assumed that all agents have identical dynamics. How must one modify the control law to accommodate heterogeneous models is an open question. Finally, is passivity really needed or can the condition be relaxed?

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