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On Developing Distributed Differential Space-Time Codes

By

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DEDICATION

To all my family members for their continuous support

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First and foremost I thank Allah, without his help and guidance this thesis would not have been come out.

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Abstract

Multiple-input-multiple-output (MIMO) communication systems are used to improve the signal quality, increase the data throughput and suppress interference. Unfortunately, multiple antennas cannot be mounted on small size terminals. To overcome this limitation, cooperative communication was developed. In this technique, all user terminals act as relays to assist a source terminal in transmitting information to the destination terminal, to form virtual multiple-input multiple-output (MIMO) systems. Applying space-time codes with cooperative approach, the resultant system is called distributed space-time code (DSTC). If the channel response change very fast, the well known differential space-time coding is needed. This eliminate the need for the channel state knowledge information at neither the relay nor the receiver, where each transmitted block acts as a reference for the next one.

In this thesis, developing new low decoding complexity distributed differential space-time codes is considered. To achieve this, the idea of MIMO communication systems, differential space-time codes, low decoding complexity, cooperative communications and distributed differential space time codes will be studied. The developed codes are designed using circulant space-time codes. They work for networks with multiple number of two of relays, and have two-group decodable maximum likelihood receiver. The performance of the new code is analyzed via Matlab simulation which demonstrates that they outperform both Cyclic codes and Circulant codes.

الملخص

تستخدم أ نظمة الاتصالات متعدد ة المد اخل والمخارج (MIMO systems) لتحسين جودة الإشارة المرسلة ، وزيادة سرعة نقل البيانات واخماد التداخل والتشويش ، ولكن لسوء الحظ لا يمكن أن يتم تركيب أكثر من هوائي على الاجهزة الطرفية صغيرة الحجم. أنظمة الاتصالات التعاونية (Cooperative systems) بمقدورها التغلب على هذه المشكلة ، حيث تقوم هذه التقنية بتحويل جميع الاجهزة الطرفية الموجودة في محيط الجهاز المصدر (The transmitter) الى محطات مساعدة (Relays) في نقل المعلومات والبيانات إلى الجهاز المقصد (The receiver) ، وبذلك تتكون أنظمة افتراضية متعددة المداخل والمخارج (Virtual MIMO systems) ، عند تطبيق الترميز الزمكاني (space-time codes) على الانظمة التعاونية الناتج يسمى بالانظمة الزمكانية الموزعة (Distributed space-time code). تبرز الحاجة الى استخدام الترميز الزمكاني التفاضلي عندما يكون التغير في استجابة القناة سريع جدا، هذا يزيل الحاجة لمعرفة معلومات القناة في المحطات المساعدة والمصدر، حيث تعمل كل كتلة (Block) كمرجع للكتلة التالية.

في هذه الأطروحة، سنقوم بتطوير أنظمة زمكانية موزعة تفاضلية منخفضة التعقيد، وللوصول الى ذلك سيتم دراسة أنظمة الاتصالات متعددة المداخل والمخارج (MIMO systems) والترميز الزمكاني التفاضلي والتعقيد المنخفض والاتصالات التعاونية و الأنظمة الزمكانية الموزعة التفاضلية ، تم تصميم الانظمة الجديدة باستخدام الترميز الزمكاني الدائري (circulant space-time code) التي تعمل لشبكات لاسلكية مع عدد زوجي من المحطات المساعدة ، وتفك باستخدام مجموعتين (Two-group decodable) ، تم تحليل أداء النظام الجديد عن طريق المحاكاة حيث تبين انها تتفوق على الترميز الزمكاني الدوري والدائري على حد سواء.

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NOTATIONS

A	**Any matrix**
A*	**Conjugate of A**
$\mathbf{A^T}$	**Transpose of A**
$\mathbf{A^H}$	**Conjugate transpose of A**
detA	**Determinant of A**
rankA	**Rank of A**
trA	**Trace of A**
$\mathbf{A_1 \times A_2}$	**Cartesian product of** $\mathbf{A_1}$ **and** $\mathbf{A_2}$
CN(0, Ω)	**Complex Gaussian vector with zero mean and covariance matrix Ω**
Log	**Natural logarithm**
$\|K\|_F$	**Frobenius norm**

TABLE OF ABBREVIATIONS

STBC	**Space-Time Block Code**
OSTBC	**Orthogonal Space-Time Code**
QOSTBC	**Quasi-Orthogonal Space-Time Code**
SNR	**Signal-to-Noise Ratio**
AWGN	**Additive White Gaussian Noise**
AF	**Amplify and Forward**
DF	**Decode and Forward**
CSI	**Channel State Information**
M_t	**Transmit Antenna**
M_r	**Receive Antenna**
DPSK	**Differential Phase Shift Keying**
QPSK	**Quadrature Phase Shift Keying**
PAM	**Pulse Amplitude Modulation**
QAM	**Quadreture Amplitude Modulation**
ML	**Maximum Likelihood**
SISO	**Single Input-Single Output**
MIMO	**Multiple Input-Multiple Output**
DSTC	**Distributed Space-Time Code**
DDSTC	**Distributed differential Space-time code**
MRC	**Maximum Ratio Combining**
SC	**Selection Combining**
SAST	**Semi-orthogonal space time code**
Bpcu	**bit per channel use**

Chapter 1

Introduction

1.1 Introduction:

The need for transmission data between people forces them to invent means to communicate with each other such as flags, fire, mirror, …etc. Hills also used as observation point to retransmit and relay data. Therefore, we can say that communication systems appeared long time ago. These early communication systems do not satisfy human needs, so they were replaced by telegraph and telephone in 1838 and 1895 respectively; the born of the first radio communication was in 1897 when Marconi managed to communicate through radio with a tugboat 18 miles away. Radio broadcasting may be considered as the preliminary successful wireless communication application followed by TV broadcasting.

The design of the first and the second cellular systems attract researchers to wireless communication applications. They are trying to improve its performance and expand its use from speech only to other sources of information with high data rate such as multimedia, wireless broadband Internet, games, and etc. After that, the wireless communication grew rapidly to satisfy these demands. Ultra Mobile broadband (UMB), Long Term Evolution (LTE), and IEEE 802.16e (WiMAX) are examples. These applications face similar challenges such as high data rate, mobility, quality of service, interference from other users and others. However, each system has different priorities. Nevertheless, we can say that the goal of any wireless communication is to transmit data from anywhere to anywhere at anytime for anything.

Through this chapter, a motivation and problem definition of the research will be illustrated. Furthermore, the main concept of channel fading and diversity techniques are pointed up. At the end, we present the contributions and organization of this research.

1.2 Motivation:

People want wireless communication to be just the same as wire communication in case of high data rate and quality of service. These are considered as the most important challenges for wireless communication. In the late of 1940s, Claude Shannon managed to determine the maximum data rate (capacity) can be transmitted through channel with negligible probability of error, using mathematical theory of communication. Any data rate exceeds that capacity, probability of error will increase, which known after that as Shannon's theory [4].

While the demand for data rates in wireless communications increased exponentially, it's significant to determine the capacity of their channel to find the maximum possible data rate that can be transmitted over a wireless channel. Multipath propagation takes the main responsibility of the poor performance of wireless communications; it prevents them to reach Shannon's capacity. Researchers worked hardly to reach this limit without bandwidth expansion. The idea of multiple antennas might be used to overcome this limitation [4] and [5]. For multiple antennas system, multiple antennas built up at transmitter and receiver; these systems commonly referred as multiple input multiple output (MIMO) systems. MIMO systems can be used to enhance the performance of wireless system by increasing capacity through multiplexing and/or decreasing probability of error through diversity [6-8] and [10]. The cost of this improvement is paid throughout adding more than one antenna, and increasing the complexity of the receiver. The spectral efficiency of MIMO systems can be increased if we exploit the time diversity via transmitting multiple symbols, and we will have space-time code, which first created by Alamouti [9]. More information about MIMO systems and space-time codes can be found in chapter 2.

Since MIMO system cannot be applied physically to small nodes due to size and cost limitations, cooperative communications can be used to overcome this problem by implementing virtual antenna array converting the single-input single-output (SISO) system into a virtual multiple-input multiple-output (MIMO). We can say that cooperative system exploits the spatial diversity between relay nodes to form a MIMO system.

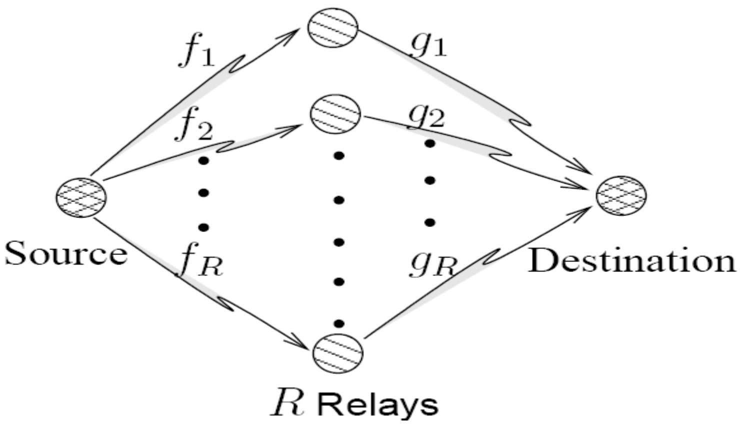

Figure (1.1): Typical cooperative system

There are many cooperative protocols, the most widely used are amplify and forward and decode and forward. In amplify and forward the source transmits data signal to the relays, and the relays amplify these signals and re-transmit them again to the destination, but in decode and forward the relays have to decode, re-modulate and re-transmit these signals, we will work on the first one in our research. Distributed space-time coding (DSTC) is a new strategy that applies a space-time code on cooperative systems. Cooperative protocols and distributed space-time code are both explained in details in chapter 3. Before we begin, the behavior of wireless channel and diversity should be understood. This is the main subject of the next section.

1.3 Wireless Communication:

Most of the wireless communication systems face the same limitations came from the medium signal pass through. Wireless transmitted signal is affected by many factors. One of them is the Additive White Gaussian Noise (AWGN) where the noise is added to the transmitted signal, and modeled as a random variable with a Gaussian distribution. Another factor that impairs the transmitted signal is the propagation way of the radio waves. Figure (1.2) shows different paths in wireless channel. In fact, the existence of different paths results in receiving different versions of the transmitted signal at the receiver. At the receiver, all received signals added together, this addition may reduce the received signal's power.

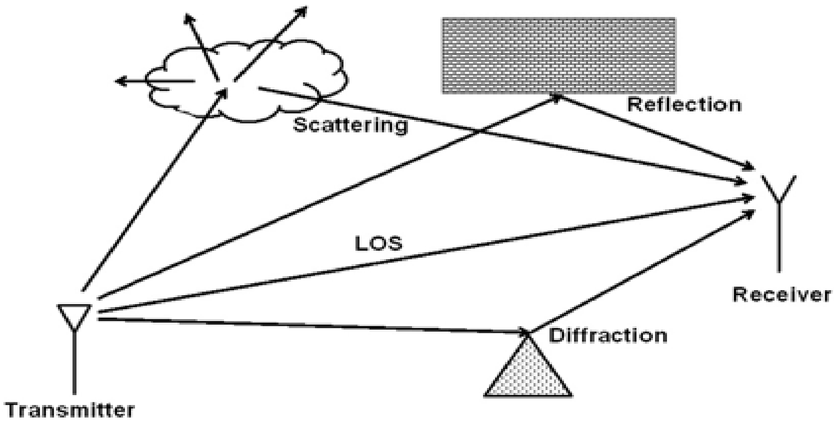

Figure (1.2): Different examples of paths in wireless channel

There are two general kinds of power reduction: large scale fading (path loss) and small scale fading (fading). In next section, a brief explanation for small scale fading is given, reader can go to [1-3] for more detail. Also diversity will be explained as a way to compensate these effects.

1.3.1 Small Scale Fading:

Small scale fading (fading) is used to describe the fluctuation of the amplitude of a radio signal over a short period of time or travel distance, so that large-scale path loss effects may be ignored. Multipath waves are the versions of the transmitted signal received at the receiver. These versions cause fading. Receiver combines them together resulting in a new signal that varies widely in amplitude and phase from the original one.

Fading channels are classified based on their multipath time delay into flat and frequency selective. In flat fading, the spectral characteristics of the transmitted signal are preserved (narrowband channel). While as, in frequency selective fading, ISI exists and the received signal is distorted. Based on Doppler spread caused by mobility, the fading can be classified into slow and fast. In slow fading, the channel is assumed to be static over one or several reciprocal bandwidth intervals (the effect is negligible). In fast fading, the channel changes rapidly within a symbol duration. Based on these two independent phenomena fading channels are classified into four types as follows:

- *Flat Slow Fading:* The bandwidth of the signal is smaller than the coherence bandwidth of the channel and the signal duration is smaller than the coherence time of the channel.

- *Flat Fast Fading:* The bandwidth of the signal is smaller than the coherence bandwidth of the channel and the signal duration is larger than the coherence time of the channel.

- *Frequency Selective Slow Fading:* The bandwidth of the signal is larger than the coherence bandwidth of the channel and the signal duration is smaller than the coherence time of the channel.

- *Frequency Selective Fast Fading:* The bandwidth of the signal is larger than the coherence bandwidth of the channel and the signal duration is larger than the coherence time of the channel

Fading channel can be modeled by time varying impulse response, one delta function for flat fading and multiple delta functions for frequency selective [1]. Due to the nature of multipath, the amplitude of these deltas will vary randomly. Statistical models are needed to represent the behavior of the amplitude and power of the received signal. One of the most famous models is the *Rayleigh* distribution fading model. It is used to describe the statistical time varying nature of the received envelope of a flat fading signal. It is also used to model fading channels in this thesis, where the real and imaginary parts of the faded coefficient (channel gain) are zero-mean Gaussian random variables with unity variance.

1.3.2 Diversity:

Diversity is considered as one of the techniques used to compensate faded channel, and is usually implemented by using two or more receiving antennas [1].Through this way multi-copy of the transmitted signal received over different channels. If one of them undergoes deep fading, another independent channel may have a strong signal. In diversity, the link performance is improved without increasing the transmitted power or bandwidth. In wireless communications, diversity can be achieved through using three main forms listed as follows [1], [4] and [5]:

- *Time diversity:* The transmitted signal copies are provided across time. The channel must provide sufficient variations in time to be effective, to have two independent faded channels the two time intervals separated more than the coherence time of the channel.

- *Frequency diversity:* The transmitted signal copies carried to different carrier frequencies. These carrier frequencies must be separated by more than the coherence bandwidth of the channel. This guarantees independent faded channels.

- *Space diversity:* also called antenna diversity and is an effective method for combating multipath fading. The transmitted signal copies are provided across different antennas for the receiver. Antenna spacing at the receiver must be larger than the coherent distance to obtain independent faded channels.

If both Space and time diversity applied the result will be space-time codes. These codes will be explained in the next chapter. After arrival of the independent faded copies to the receiver they need to be combined. Combining can be done by several methods. There are two main combining methods, which are Maximum Ratio

Combining (MRC) and Selection Combining (SC). These methods vary in complexity and performance. More explanations and details of these combining methods can be found at [1], [4] and [5].

1.4 Problem Definition:

Correctly full channel information at the receiver (destination) is required for any SISO, MIMO and cooperative systems, in order to detect the transmitted signal. this work, the cooperative systems use amplify and forward protocol, the channel information is needed only at destination. The relay nodes don't decode the received signal, so they don’t use any channel state information. Both channels from source to relays f_i and channels from relays to destination g_i must be known to the receiver as seen in figure (1.1). In most practical systems, the transmitter sends pilot signals every time interval, and the receiver uses them to estimate the channel, and coherently decode data in that interval. The cost of coherent detection paid in time, power and computation complexity. This high cost is not desired sometimes, especially when the channel change rapidly. Hence, there is a need for developing differential distributed space-time coding for wireless networks where there is no need for channel information at neither the relays nor the receiver. This will be the main goal of this research.

1.5 Thesis Contribution:

The main contributions of this thesis can be summarized as following:

- A new Distributed Differential Space-time code (DDSTC) have been developed, which based on circulant code. The new code has low decoding complexity, 2- group decodable, and it outperforms cyclic code and circulant code.

- This thesis contains many topics in wireless communication, so it can be considered as a reference or starting point for whom want to study in one of these topics. Also it groups many subject in one.

1.6 Thesis Organization:

In this thesis, we study Distributed Differential Space Time Code (DDSTC). Therefore, MIMO systems, space-time code, and cooperative systems are the main pillars of this thesis, and they will be explained in the following three chapters:

- *Chapter 2* is a theoretical background of multiple antenna systems and space-time codes, where MIMO systems, types of space- time codes, orthogonal and quasi-orthogonal codes and others codes that may be used in this thesis are explained. Differential transmission for MIMO system is explained, and the last section gives an overview of code complexity and how we can generate space-time code with low decoding complexity.

- *Chapter 3* introduces cooperative communication systems and describes the benefits that they hold for wireless networks. Many protocols are described, fixed and adaptive protocols. Comparison of these protocols is also made. To overcome the low spectrum efficiency space-time codes are used with cooperative systems to generate what called Distributed Space-Time Code (DSTC).
- *Chapter 4* consider as the main goal of this thesis, in which we analyze Distributed Differential Space-Time Code (DDSTC), showing its constrains came from differential space time code and distributed space time code, based on paper [42],[43] and [45] the general model is introduced and an extension will be made to let this system has low decoding complexity. New code for distributed space time code is introduced**.**

- *In Chapter 5* we conclude the thesis work and suggest future works.

Chapter 2

Multiple Antennas and Space-Time Code

2.1 Introduction:

No one can deny that high speed communication systems such as internet have become essential part of our daily life. Although wired communications serve this demand, the desire to communicate from anywhere to anywhere drive the need for high data rate wireless systems. This can be achieved if we overcome the bottle neck in the wireless communications, which is the wireless channel. One of the main challenges of the wireless channel is fading, the received signal affected hardly and the performance of the wireless system degraded. Diversity techniques used repeatedly to outfight the multipath fading, thus compensating the performance of the wireless communication links [1]. In time diversity information transmitted in different time slots, but in space diversity we need multiple antennas at transmitter or at receiver or at both. This chapter explains the main concept of MIMO system which exploits spatial diversity, and space-time code which exploits both space and time diversity.

2.2 The MIMO System

In mid of 1990s, Foschini [6], and Telatar [7] [8] triggered the basic idea of MIMO systems, simply, they suggest that multiple antenna can be mounted at both transmitter and receiver. Ever since, this system known's as Multiple Input Multiple

Output (MIMO) systems. The multiple path between any pair of transmit-receive antenna can be used to transmit the same data, achieving diversity gain, resulting in decreasing the probability of bit error rate (BER). Or it can be used to transmit independent data through independent channel paths, achieving multiplexing gain and increase transmitted data rate. It can be said that MIMO systems objectives are to increase data rates through multiplexing and/or to improve performance through diversity [4]. A question rises to surface is when multiplexing or diversity gains can be used and when both of them can be used. The answer to this question is that there should be a tradeoff between multiplexing and diversity, more information about multiplexing gain, diversity gains and the tradeoff between them can be found in chapter 10 from [4]. The price for enhancing the system performance is paid out at the receiver complexity and implementation of more antennas.

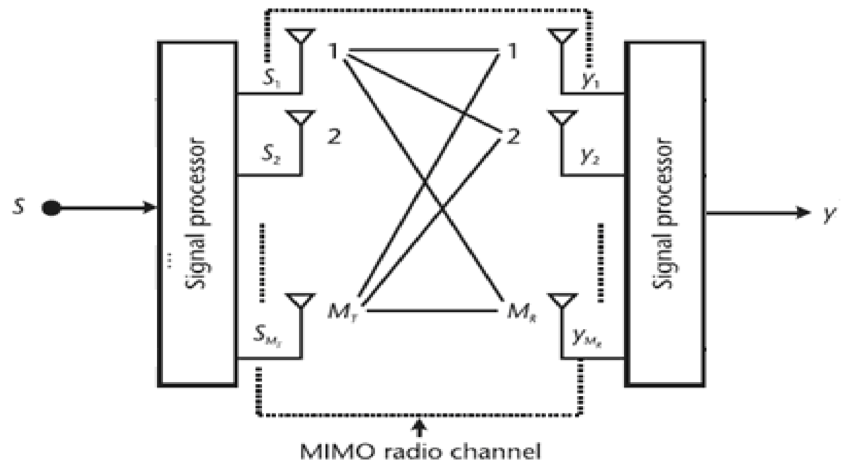

Figure (2.1): Typical MIMO system [5]

2.2.1 MIMO System Model:

Suppose that there exist two wireless communication systems want to communicate with each other, one terminal acts as a transmitter and the other one acts as a receiver. The transmitter occupied with M_t transmit antenna, and the receiver occupied with M_r antenna, see Figure (2.1). The channel between M_t antenna and M_r antenna denote as h_{MtMr}, whose statistical model is Rayleigh which described in chapter one. To transmit information $S = [\, s_1, s_2, s_3, \ldots\ldots s_L]$, where L is the cardinality of symbols. Transmitter should supply a set of symbol consist of $s_1, s_2, s_3, \ldots\, s_{Mt}$ to M_t antenna every transmission time, and all these antennas transmit their symbol at the same time. Every M_r antenna receives this set, each symbol pass through different channel path. As mentioned in chapter one, the received signal effected by Additive White Gaussian Noise (AWGN). If we denote the noise at M_r antenna as N_{Mr}, then the received signal at M_r antenna can be written as:

$$y_{Mr} = \sqrt{\frac{\rho}{M_t}} \sum_{i=1}^{M_t} h_{iMr} S_i + N_{Mr}\,, \tag{2.1}$$

where ρ is expected signal to noise ratio (SNR) at receiver. Also the system can be rewritten in matrix form as:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{Mr} \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1Mt} \\ \vdots & \ddots & \vdots \\ h_{Mr1} & \cdots & h_{MrMt} \end{bmatrix} \begin{bmatrix} S_1 \\ \vdots \\ S_{Mt} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_{Mr} \end{bmatrix} \tag{2.2}$$

Simply the input-output relationship of MIMO system expressed as $Y = HS + N$. Recovering process of the transmitted symbol S needs acknowledgment of the channel gain matrix H, which is known as the channel state information at the transmitter (CSIT) and the channel state information at the receiver (CSIR). Moreover, transmitted symbol S can be recovered differentially, without have any information about channel gain; this will be discussed later in this chapter.

2.3 Space-Time Block Codes:

Severe attenuation in multipath wireless channel increases the difficulty for the receiver to determine the transmitted signal unless diversity is used. Exploiting all available diversity schemes in one transmission technique was a question that occupied many researchers mind. In October 1998, Alamouti [9] managed to create a scheme that exploits space and time diversities. It called later as Space-Time code and become the core idea of MIMO systems.

Most space-time codes, including all codes discussed in this section, are designed for quasi-static Rayleigh fading channels where the channel is constant over a block of T symbol times, and the channel is assumed unknown at the transmitter. The transmitted signal S is redefined to be a $T \times M$ matrix with s_{tM_t} the signal sent by M_t-th transmit antenna at time t. that's mean the rows of S represent the spatial domain and the columns of S represent the temporal domain, so redundancy signals are added in both domains. The system equation can be written as:

$$\mathrm{Y} = \sqrt{\frac{\rho T}{M}}\, \mathrm{HS} + \mathrm{N} \tag{2.3}$$

where

$$Y = \begin{bmatrix} y_{11} & \cdots & y_{1M_r} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TM_r} \end{bmatrix} \qquad S = \begin{bmatrix} s_{11} & \cdots & s_{1M_t} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TM_t} \end{bmatrix}$$

$$H = \begin{bmatrix} h_{11} & \ldots & h_{1Mr} \\ \vdots & \ddots & \vdots \\ h_{Mt1} & \cdots & h_{MtMr} \end{bmatrix} \qquad N = \begin{bmatrix} n_{11} & \cdots & n_{1M_r} \\ \vdots & \ddots & \vdots \\ n_{T1} & \cdots & n_{TM_r} \end{bmatrix}$$

To ensure that the average expected power at each antenna is constant over-all channel use, the transmitted signal should be normalized as,

$$\frac{1}{M}\sum_{m=1}^{M_r} E\left|s_{tm}\right|^2 = \frac{1}{T}, \text{ for } \text{t=1,2,... T,} \tag{2.4}$$

And the expected received power at any received antenna at t-th transmission will be as follows,

$$E\,\frac{\rho T}{M}\left|\sum_{m=1}^{M_r} s_{tm} h_{M_r M_t}\right|^2 = \frac{\rho T}{M}\sum_{m=1}^{M_r} E\left|s_{tm}\right|^2 E\left|h_{M_r M_t}\right|^2 = \rho \tag{2.5}$$

The expected noise power at every received antenna will be as,

$$E\left|n_{tM_r}\right|^2 = 1, \tag{2.6}$$

Which guaranteed that’s ρ is expected SNR at each receiver antenna.

This chapter expose a brief explanation of some kind of space-time codes that may be used in our thesis, which includes orthogonal, quasi-orthogonal, cyclic, linear dispersion and differential codes; these codes and others can be found at [5], [10] and [11]. But before that some important design criteria should be mentioned, that result in achieving codes with good performance. Let's say that we transmit code S_i, and while decoding mistake occurs and receiver decides code S_j was sent. These criteria are:

1- *Diversity gain (Rank criterion):*

   In MIMO system maximum diversity obtained through coherent combining of M_t transmits and M_r receives antennas, and its equal to M_tM_r. Thus, to obtain this maximum diversity gain, the space-time code must be designed such that the difference matrix between any two code words $D(S_i,S_j)$ has a full rank, equal to M_t [5].

2- *Coding gain (determinant criterion):*

   A high coding gain is achieved by maximizing the minimum of the determinant of $\det(S_i,S_j) = D(S_i,S_j)\, D(S_i,S_j)^H$ over all input matrix pairs S_i and S_j, where $i \neq j$, and 'H' refers to transpose complex conjugate [5].

Therefore, to design a good performance space-time code firstly we have to make the code achieve the full diversity criterion after that trying to apply code gain criteria. The full diversity criteria determine the slope of the probability of error that's why it has the priority [5] and [10]. Another important criterion is the rate of the space-time block code (R) which defined as the ratio between the number of symbols (k) the encoder takes as its input and the number of time slots (T). It is given by,

$$R = \frac{k}{T} \qquad \text{Symol/channel} \tag{2.7}$$

Also the spectral efficiency (η) of the space-time block code is given by

$$\eta = \frac{\log_2 L}{M_t} \quad \text{bit/s/Hz} \tag{2.8}$$

where L is the cardinality of the codebook and M_t is the number of transmit antenna.

2.3.1 Orthogonal Space-Time block Code (OSTBC):

We start our demonstration of orthogonal space-time block code (OSTBC) by the first existed one which is the Alamouti's code. Alamouti introduced two approaches, the first one suggests that the transmitter has two antennas and the receiver has one antenna. The second approach suggested there are two antennas at the receiver. We will present the first approach and the second will be straight forward. Alamouti transmission matrix is defined as below:

$$S = \begin{bmatrix} X_1 & X_2 \\ -X_2^* & X_1^* \end{bmatrix} \tag{2.9}$$

Block diagram of Alamouti space-time code encoder is shown in Figure (2.2). Transmitter divides the data stream into b bit, and use modulation scheme that map every b bit into one symbol from a constellation of size 2^b , for example, PAM, QAM, PSK and so on. Then symbols are divided into blocks with two in each one X_1 and X_2, at first time slot antenna one sends X_1 and antenna two sends X_2 , and at the second- time slot antenna one sends $-X_2^*$ and X_1^* from second antenna.

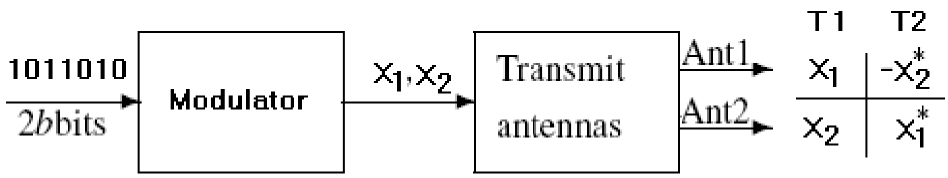

Figure (2.2): Alamouti space-time code encoder [5].

This scheme proposed interesting property that is for every complex symbol X_1 and X_2 the columns of S are orthogonal to each other; that's mean:

$$S^H S = \left(|X_1|^2 + |X_2|^2\right) I_2 \quad (2.10)$$

Now let us go to the other side , to the receiver, let the path gains from antenna one and two are h_1 and h_2 respectively, as shown in figure (2.3), and base on *MIMO* input-output relationship mentioned early the received signals at the decoder can be written as:

$$\begin{aligned} y_1 &= h_1 X_1 + h_2 X_2 + n_1 \\ y_2 &= -h_1 X_2^* + h_2 X_1^* + n_2 \end{aligned} \quad (2.11)$$

where n_1 and n_2 are Additive White Gaussian Noise (*AWGN*) at the first and second time slot respectively. If the receiver has channel state information, coherent detection can be done. Combiner combines the received signal as follows:

$$\begin{aligned} \tilde{X}_1 &= h_1^* y_1 + h_2 y_2^* = \left(|h_1|^2 + |h_2|^2\right) X_1 + h_1^* n_1 + h_2 n_2^* \\ \tilde{X}_2 &= h_2^* y_1^* - h_1 y_2^* = \left(|h_1|^2 + |h_2|^2\right) X_2 + h_1 n_2^* + h_2^* n_1 \end{aligned} \quad (2.12)$$

And sends them to the maximum likelihood detector, which minimizes the following decision metric over all possible values of X_1 and X_2,

$$|y_1 - h_1 X_1 - h_2 X_2|^2 + |y_2 + h_1 X_2^* - h_2 X_1^*|^2 \quad (2.13)$$

Expanding the above formula and delete term the minimization will separated one for detection X_1 as:

$$|y_1 h_1^* + y_2^* h_2 - X_1|^2 + (|h_1|^2 + |h_2|^2 - 1)|X_1|^2 \quad (2.14)$$

and the second to detect X_2 as:

$$|y_1 h_2^* - y_2^* h_1 - X_2|^2 + (|h_1|^2 + |h_2|^2 - 1)|X_2|^2 \quad (2.15)$$

For equal energy constellation such as *PSK* , the *ML* detector should minimize

$$|X_1 - y_1 h_1^* - y_2^* h_2|^2 \quad (2.16)$$

And

$$\left|X_2 - y_1 h_2^* + y_2^* h_1\right|^2 \tag{2.17}$$

To detect X_1 and X_2 respectively.

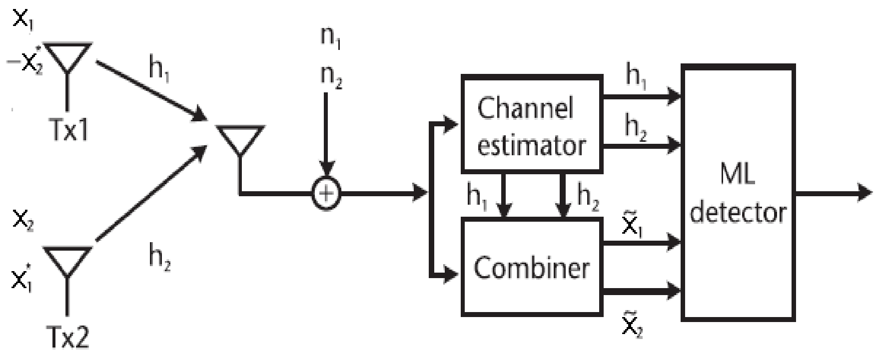

Figure (2.3): Alamouti space-time code decoder [5].

We can see that Alamouti code has two features:

- Fast maximum likelihood detection.
- Have full diversity since it satisfies the rank criteria [5].

Alamouti scheme works only when the number of transmit antenna is two. An extension of this code to include more than two antennas preserving Alamouti's features was done by applying theory of orthogonal design, the generalization is referred to as space-time block codes (STBCs) [12]. The decoding complexity increases linearly, not exponentially, with the code size. There are two classes of orthogonal design real and complex where the symbol taken from real and complex constellation respectively.

1- *Complex orthogonal design*

Complex orthogonal design is defined as $T\times M_t$ orthogonal matrix with entries $0, \pm X_1, \pm X_1^*, \pm X_2, \pm X_2^*, \ldots, \pm X_k, \pm X_k^*$. All symbols are chosen from complex constellation like PSK and QAM. The only space-time code that has a full rate, *R=1*, is

Alamouti code [14]. A systematic way of design complex orthogonal code for $M_t= 2^k$ (k=1, 2, 3,…) is shown as follows [9]:

$$S_{2^k}(X_1,\ldots,X_{k+1}) = \begin{bmatrix} S_{2^{k-1}}(X_1,\ldots,X_k) & X_{k+1} I_{2^{k-1}} \\ -X_{k+1}^* I_{2^{k-1}} & S_{2^k}^H(X_1,\ldots,X_{k+1}) \end{bmatrix}, \tag{2.18}$$

where

$$S_1(X_1) = X_1 I_1$$

The code for $M_t= 2$ transmit antenna will be alamouti code ($R=1$),

$$S = \begin{bmatrix} X_1 & X_2 \\ -X_2^* & X_1^* \end{bmatrix} \tag{2.19}$$

Codes for $M_t = 4$ with rate 3/4 will by as,

$$S = \begin{bmatrix} X_1 & X_2 & X_3 & 0 \\ -X_2^* & X_1^* & 0 & X_3 \\ -X_3^* & 0 & X_1^* & -X_2 \\ 0 & -X_3^* & X_2^* & X_1 \end{bmatrix} \tag{2.20}$$

Codes for $M_t= 8$ with rate 1/2 will by as,

$$S = \left[\begin{array}{cccc|cccc} X_1 & X_2 & X_3 & 0 & X_4 & 0 & 0 & 0 \\ -X_2^* & X_1^* & 0 & X_3 & 0 & X_4 & 0 & 0 \\ -X_3^* & 0 & X_1^* & -X_2 & 0 & 0 & X_4 & 0 \\ 0 & -X_3^* & X_2^* & X_1 & 0 & 0 & 0 & X_4 \\ \hline -X_4^* & 0 & 0 & 0 & X_1^* & -X_2 & -X_3 & 0 \\ 0 & -X_4^* & 0 & 0 & X_2^* & X_1 & 0 & -X_3 \\ 0 & 0 & -X_4^* & 0 & X_3^* & 0 & X_1 & X_2 \\ 0 & 0 & 0 & -X_4^* & 0 & X_3^* & -X_2^* & X_1^* \end{array}\right] \tag{2.21}$$

These codes are square with size $2^k \times 2^k$ and with rate $(k+1)/2^k$. Other codes can be obtained by deleting some columns from a larger space-time code with a number of transmit antenna that is a power of two. For system with M_t=3 we can take the first three column of S from (2.20) code with rate R=3/4 as,

$$S = \begin{bmatrix} X_1 & X_2 & X_3 \\ -X_2^* & X_1^* & 0 \\ -X_3^* & 0 & X_1^* \\ 0 & -X_3^* & X_2^* \end{bmatrix} \tag{2.22}$$

This is not the only structure of complex orthogonal codes, for complex orthogonal design it's not necessary to be square, constructions of such codes are found in [12]. Equation (2.23) illustrates an example code for M_t=4.

$$S = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 \\ -X_2 & X_1 & -X_4 & X_3 \\ -X_3 & X_4 & X_1 & -X_2 \\ -X_4 & -X_3 & X_2 & X_1 \\ X_1^* & X_2^* & X_3^* & X_4^* \\ -X_2^* & X_1^* & -X_4^* & X_3^* \\ -X_3^* & X_4^* & X_1^* & -X_2^* \\ -X_4^* & -X_3^* & X_2^* & X_1^* \end{bmatrix} \tag{2.23}$$

In 2004[16] Su, Xia, and Liu succeeded in developing a systematic design of high rate complex orthogonal space-time code with non-square size, these code rates are $(n_0 + 1)/(2n_0)$ if the number of transmit antennas is $M_t= 2n_0$ or $n = 2n_0 - 1$. For example, for $M_t = 4$ transmitter antennas, an orthogonal space-time code is given by:

$$S = \begin{bmatrix} X_1 & X_2 & X_3 & 0 \\ -X_2^* & X_1^* & 0 & X_4^* \\ -X_3^* & 0 & X_1^* & X_5^* \\ 0 & -X_3^* & X_2^* & X_6^* \\ 0 & -X_4 & -X_5 & X_1 \\ X_4 & 0 & -X_6 & X_2 \\ X_5 & X_6 & 0 & X_3 \\ -X_6^* & -X_5^* & -X_4^* & 0 \end{bmatrix} \tag{2.24}$$

The existence of complex orthogonal designs for space-time block codes can be summarized as follows:

- For 2 transmit antennas, space-time block code exists with the maximum symbol transmission rate 1, Alamouti’s scheme [9].

- For 3 and 4 transmit antennas, space-time block codes exist with symbol transmission rate 3/4 [12].
- For any number of transmit antennas, space-time block codes exist with symbol transmission rate 1/2 [6].
- In 2004 a systematic design of high rate complex orthogonal space-time block codes exists with symbol transmission rates greater than 1/2 and below 3/4. [16].

2- Real orthogonal designs

Real orthogonal design [12] is defined as $T\times M_t$ orthogonal matrix with entries $0,\pm X_1,\pm X_2,\ldots,\pm X_k$, where all these symbols are chosen from real signal constellation such as *PAM*. Square real orthogonal design exists only for M_t=2, 4 and 8, All mentioned codes have rate one or full diversity. Examples of orthogonal design are 2×2 designs:

$$S=\begin{bmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{bmatrix} \tag{2.25}$$

And 4×4 design

$$S=\begin{bmatrix} X_1 & X_2 & X_3 & X_4 \\ -X_2 & X_1 & -X_4 & X_3 \\ -X_3 & X_4 & X_1 & -X_2 \\ -X_4 & -X_3 & X_2 & X_1 \end{bmatrix} \tag{2.26}$$

And 8×8 design

$$S=\begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 \\ -X_2 & X_1 & X_4 & -X_3 & X_6 & -X_5 & -X_8 & X_7 \\ -X_3 & -X_4 & X_1 & X_2 & X_7 & X_8 & -X_5 & -X_6 \\ -X_4 & X_3 & -X_2 & X_1 & X_8 & -X_7 & -X_6 & -X_5 \\ -X_5 & -X_6 & -X_7 & -X_8 & X_1 & X_2 & X_3 & X_4 \\ -X_6 & X_5 & -X_8 & X_7 & -X_2 & X_1 & -X_4 & X_3 \\ -X_7 & X_8 & X_5 & -X_6 & -X_3 & X_4 & X_1 & -X_2 \\ -X_8 & -X_7 & X_6 & X_5 & -X_4 & -X_3 & X_2 & X_1 \end{bmatrix} \tag{2.27}$$

Other codes less than eight (M_t = 3, 5, 6 and 7) are non square and delay optimal, which mean that T has minimum possible value [9]. Real orthogonal design for M_t = 3 is shown in equation (2.28).

$$S = \begin{bmatrix} X_1 & X_2 & X_3 \\ -X_2 & X_1 & -X_4 \\ -X_3 & X_4 & X_1 \\ -X_4 & -X_3 & X_2 \end{bmatrix} \tag{2.28}$$

Codes for M_t = 9 or greater with a full rate will have a delay in time slot, as an example for M_t = 9 number of time slots T=16, therefore the delay will be seven-time slots. We can conclude that the maximum rate 1 can be reached for real orthogonal designs for any number of transmit antennas not like complex orthogonal space-time code [5].

2.3.2 Quasi-Orthogonal Space-Time Codes (QOSTBC):

Orthogonal design gives full data rate only for Alamouti scheme, rate 3/4 for codes with 3 and 4 transmit antennas, and rate between 1/2 and 3/4 for system with transmit antennas greater than four. Sacrificing orthogonality can increase data rate [5, 9], the resulting codes become Quasi-Orthogonal Space-Time Block Codes (QOSTBC). The encoding process of QOSTBC is very similar to that for OSTBC, but the *ML* decoding of quasi-orthogonal done by search pairs of symbol, this mean that the complexity of the space-time code will increase exponentially, not linearly as OSTBC. The encoding process of QOSTBCs is similar to the encoding of orthogonal STBCs, so we don't need to re-clarification it. We can construct 4×4 QOSTBC from alamouti Alamouti code [13, 19], and this code may be having the following form:

$$S = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix} = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 \\ -X_2^* & X_1^* & -X_4^* & X_3^* \\ -X_3^* & -X_4^* & X_1^* & X_2^* \\ X_4 & -X_3 & -X_2 & X_1 \end{bmatrix} \tag{2.29}$$

Checking the orthogonality of (2.29) code gives us:

$$S^H S = \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} \tag{2.30}$$

where

$$a = |x_1|^2 + |x_2|^2 + |x_3|^2 + |x_4|^2$$
$$and$$
$$b = x_1 x_4^* + x_4 x_1^* - x_2 x_3^* - x_3 x_2^*$$

The *ML* decision metric of this code can be written in two separated term, the first depend on X_1 and X_3 and the second one depend on X_2 and X_4, which are:

$$\begin{aligned} f_{14}(x_1, x_4) = \sum_{m=1}^{M_r} \Bigg[& \left(|x_1|^2 + |x_4|^2 \right) \left(\sum_{n=1}^{M_t} |h_{n,m}|^2 \right) \\ & + 2\Re\Big\{ \left(-h_{1,m} y_{1,m}^* - h_{2,m}^* y_{1,m} - h_{3,m}^* y_{3,m} - h_{4,m} y_{4,m}^* \right) x_1 \\ & + \left(-h_{4,m} y_{1,m}^* + h_{3,m}^* y_{1,m} - h_{2,m}^* y_{3,m} - h_{1,m} y_{4,m}^* \right) x_4 \Big\} \\ & + 4\Re\left\{ h_{1,m} h_{4,m}^* - h_{2,m}^* h_{3,m} \right\} \Re\left\{ x_1 x_4^* \right\} \Bigg] \end{aligned} \tag{2.31}$$

and the second

$$\begin{aligned} f_{23}(x_2, x_3) = \sum_{m=1}^{M_r} \Bigg[& \left(|x_2|^2 + |x_3|^2 \right) \left(\sum_{n=1}^{M_t} |h_{n,m}|^2 \right) \\ & + 2\Re\Big\{ \left(-h_{2,m} y_{1,m}^* + h_{1,m}^* y_{2,m} - h_{4,m}^* y_{3,m} + h_{3,m} y_{4,m}^* \right) x_2 \\ & + \left(-h_{3,m} y_{1,m}^* + h_{4,m}^* y_{2,m} - h_{1,m}^* y_{3,m} - h_{2,m} y_{4,m}^* \right) x_3 \Big\} \\ & + 4\Re\left\{ h_{2,m} h_{3,m}^* - h_{1,m}^* h_{4,m} \right\} \Re\left\{ x_2 x_3^* \right\} \Bigg] \end{aligned} \tag{2.32}$$

Therefore, the *ML* decoding is to minimize the term f_{14} and f_{23} overall value of x_1, x_4 and x_2, $x3$ respectively. If the term of $\Re\{x_2 x_3^*\}$ and $\Re\{x_1 x_4^*\}$ equal zero, then we can decode all symbols of the code separately. The rank of this code is 2, so it's not full diversity. To achieve full diversity part of the symbol must be chosen from a rotated constellation, and the rotation must be optimized. Optimum signal constellation rotation for *BPSK, QPSK, 8-PSK,* and *QAM* are π/2, π/4, π/8, and π/4, respectively, more information about constellation rotation will be found in [5]. Combining any two OSTBC, square or non-square, with each other will result QOSTBC with the same rate, removing columns from QOSTBC result also QOSTBC for transmit antenna less one. For example, if we omit column number four form equation (2.29) we get QOSTBC for system with M_t=3, which illustrate as follows:

$$S = \begin{bmatrix} X_1 & X_2 & X_3 \\ -X_2^* & X_1^* & -X_4^* \\ -X_3^* & -X_4^* & X_1^* \\ X_4 & -X_3 & -X_2 \end{bmatrix} \tag{2.33}$$

and

$$S^H S = \begin{bmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{bmatrix} \tag{2.34}$$

Where $a = |x_1|^2 + |x_2|^2 + |x_3|^2 + |x_4|^2$ and $b = 2\Re\,(x_1 x_4^* - x_2 x_3^*)$, the conditions of the full diversity is apply here.

2.3.3 Cyclic codes:

Now we present space-time code with group structure, *Cyclic space-time code*, this code characterizes by full diversity, unitary, simplicity and has the following structure [17, 18]:

$$S_l = V_1^l \qquad , l = 0,1,\ldots,L-1, \tag{2.35}$$

where $V_1$is the generator matrix with diagonal entries, and can be written as,

$$V_1 = \begin{bmatrix} e^{j(2\pi/L)u_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & e^{j(2\pi/L)u_M} \end{bmatrix}$$

$$\text{where} \quad u_m \in \{0,\ldots,L-1\}; \quad m = 1,\ldots,M \tag{2.36}$$

We can notice that the term V_1^L equal V_1^0 which have has a cyclic structure. The terms $u_1, u_2, \ldots, u_M$ should by optimized to guarantee full diversity. Example 2.1 from [10] demonstrate such code for M_t=2, L=4 (R= 1 b/cu) and u_1, u_2 equal one,

$$S_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad S_1 = \begin{bmatrix} j & 0 \\ 0 & j \end{bmatrix}$$
$$S_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad S_3 = \begin{bmatrix} -j & 0 \\ 0 & -j \end{bmatrix} \tag{2.37}$$

Cyclic space-time code with non-zero off-diagonal can be design through Fourier transforms or by using unitary matrices with some special structures. Such design has larger diversity product, example 2.2 from [10] show it:

$$S_0 = \begin{bmatrix} j & 1-j \\ -1-j & -j \end{bmatrix} \quad S_1 = \begin{bmatrix} -j & -1-j \\ 1-j & j \end{bmatrix}$$
$$S_2 = \begin{bmatrix} -j & 1+j \\ -1+j & j \end{bmatrix} \quad S_3 = \begin{bmatrix} j & -1+j \\ 1+j & -j \end{bmatrix} \tag{2.38}$$

Table (2.1) summarizes some good cyclic codes. The codes for three and six transmit antennas have been found in [18], where cyclic codes with maximized coding gain are available until dimension 6 only.

M_t	L	Rate	Diversity product	u
3	8	1	0.5164	(1,1,3)
3	63	1.99	0.3301	(1,17,26)
4	255	1	0.4527	(1,11,67,101)
6	64	1	0.3792	(1,7,15,23,25,31)
6	4096	2	0.1428	(1,599,623,1445,1527,1715)
9	57	0.65	0.361	(1,4,16,7,28,55,49,25,43)

Table (2.1): Cyclic code with good diversity product

The code for nine transmit antennas is the diagonal component based on a fixed-point free group code [20], some cyclic codes in higher dimensions have been proposed in [21].

2.4 Differential Space-time Block Code:

2.4.1 Introduction:

In a typical multiple-input multiple-output (MIMO) system the channel assumed to be changing slowly over an entire frame, which means that the change in channel characteristics is negligible. If the receiver has a perfect acknowledgment of the channel state information (CSI) the system performance will be enhanced. However, in a real communication environment, the receiver has no prior knowledge of the realization of H and has to estimate it. A standard technique to estimate the channel matrix H consists of transmitting training symbols (pilot symbols) among the data, whose composition is known to the receiver. Let the transmitter send a pilot matrix S_p and a data matrix S, both affected by the same channel matrix H, using the input-output relationship of MIMO at the receiver we will have:

$$
\begin{aligned}
Y_p &= HS_p + N_p \\
\text{and} \qquad & \\
Y &= HS + N
\end{aligned}
\tag{2.39}
$$

where N_p and N are additive white Gaussian noise for pilots and data symbols respectively. The receiver estimates the channel matrix H from Y_p and S_p and uses the result H matrix to coherently decode the data symbols during the same frame. However, in some circumstances, we may not be able to use this technique due to high cost and complexity of the handset, or due to the rapidly change of channel gains in high-mobility situations, which make channel estimation is difficult or requires too many training symbols (pilots). So non-coherent communication system is needed, which consider a new modulation technique that does not need any acknowledge of the channel state information (CSI) neither at the transmitter nor at the receiver. For single input single output system (SISO), differential phase-shift keying (DPSK) can be demodulated without channel estimates. Also this technique was extended to include multiple input multiple output systems (MIMO). This section will explain both.

2.4.2 Differential phase-shift keying (DPSK):

Differential phase-shift keying (DPSK) has been used when the channel change slowly from one time sample to the next, and affect only on symbol's phase. Transmitter encodes the data symbol into phase differences from symbol to symbol. Figure (2.4) present transmitter of differential phase-shift keying, the transmitter picks *b* bits and modulates it from any *L*-PSK constellation ($L=2^b$), at any time *t* the transmitted symbol will be as $S_t = X_t S_{t-1} \ where \ t = 1,2,.....(S_0 = 1)$. The initial symbol S_o does not carry any information and can be thought of as a training symbol.

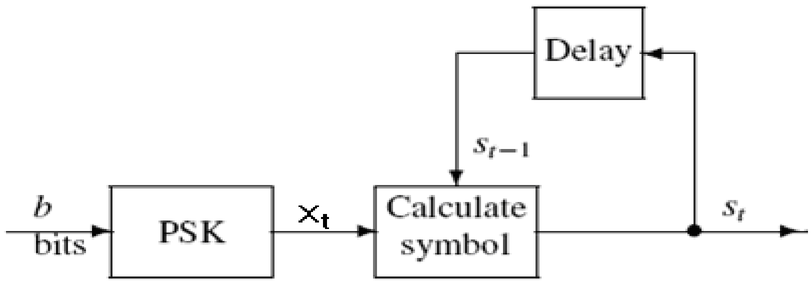

Figure (2.4): Decoder of differential phase-shift keying [5].

The received signal will have the form $Y_t = \sqrt{\rho}\, h_t S_t + n_t$, where h_t is channel gain and n_t is AWGN . To detect the transmitted data let h_t equal to h_{t-1} and compute the phase difference between two consecutive symbols $Y_{t-1}^{*} Y_t$, after that find the closest symbol from *L*-PSK constellation to the value $Y_{t-1}^{*} Y_t$. The whole process represented mathematically as follows:

$$\begin{aligned} Y_{t-1}^{*} Y_t &= \left(H^{*} S_{t-1}^{*} + N_{t-1}^{*}\right)\left(H S_t + N_t\right) \\ &= |H|^2 S_{t-1}^{*} S_t + H^{*} S_{t-1}^{*} N_t + H S_t N_{t-1}^{*} + N_t N_{t-1}^{*} \\ &\approx |H|^2 S_{t-1}^{*} X_t S_{t-1} + \mathrm{N} = |H|^2 X_t + \mathrm{N} \end{aligned} \tag{2.40}$$

where the term *N* is Gaussian noise, the term $N_t N_{t-1}^{*}$ ignored. The estimated symbol calculated from the ML detector:

$$\hat{X}_t = \arg\min \left| Y_{t-1}^{*} Y_t - X_t \right|^2 \tag{2.41}$$

Notice that decoder doesn't depend on channel fading, and its compute the data in the current symbol by comparing its phase to the phase of the previous symbol. Additionally, we can notice that the term $N = H^{*}S_{t-1}^{*}N_{t} + HS_{t}N_{t-1}^{*}$ has two noise terms, which mean the performance of DPSK is worse than coherent-PSK by 3-dB. Differential phase-shift keying (DPSK) demodulation requires two successive symbol at the decoder, which mean we have to transmit signal of length two, coherent time *T*=2, contains both the previous and the current symbol. The transmitted signal will have the form:

$$S_t = \begin{bmatrix} X_{t-1} \\ X_t \end{bmatrix} \tag{2.42}$$

Since each symbol acts as a reference for the next symbol, so we have signals that occupy two symbols but overlap by one symbol, these overlapping symbols can be seen in figure (2.5).

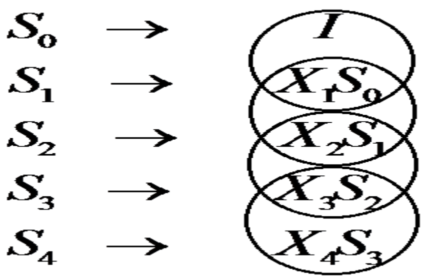

Figure (2.5): Overlapping signal in DPSK

The receiver groups received symbols in (overlapping) vectors of length two and compute the non-coherent maximum-likelihood demodulation. The properties of a DPSK modulation are:

- The information is embedded between successive symbols.
- To find the closest symbol, we do not need to know the fade.
- 3-dB is lost due to non-coherent detection for a Rayleigh fading channel.
- Receiver structure is simplified since channel estimation and carrier or phase tracking are omitted.

2.4.3 Differential modulation for MIMO systems:

DPSK modulation and demodulation can be fitted into a multiple-antenna system, if we suppose that DSPK act as MIMO system with only one transmitter and one receiver. For multiple antennas, similar to DPSK, we need a block of $M_t \times M_t$ space–time symbols to act as a reference for the next block. Hence, a signal of size $2M_t \times M_t$ is considered that overlap by M_t samples. So unitary differential modulation technique can be used for non-coherent MIMO channels, by asking the transmitter to send at each time a codeword multiplied by what was sent at time [17, 18]. At time t the system equation for differential unitary space-time code can be written as,

$$\mathrm{Y}_t = \sqrt{\rho}\,\mathrm{H}_t \mathrm{S}_t + \mathrm{N}_t \tag{2.43}$$

Where H_t ,Y_t and N_t are $M_t \times M_r$, S_t is $M_t \times M_t$ matrix equals the product of unitary data matrix X_{Zt} with the previous code S_{t-1}, where $\mathrm{z}_t \in \{0,\ldots,L-1\}$ is the data to be transmitted, and X_{Zt} taken from our designed signal set, in other words,

$$S_t = X_{Zt} S_{t-1}, \tag{2.44}$$

where $S_0 = I_{Mt}$, and X_{Zt} must be unitary, otherwise transmitted signal will vanish or blow up to infinity. If the channel kept constant for $2M_t$ consecutive channel uses, that is, $H_t \approx H_{t-1}$, then from equation (2.43) and (2.44) we can get the differential received equation for MIMO system as,

$$\begin{aligned} \mathrm{Y}_t &= \sqrt{\rho}\, X_{Zt} S_{t-1} \mathrm{H}_{t-1} + \mathrm{N}_t \\ &= X_{Zt}(Y_{t-1} - \mathrm{N}_{t-1}) + \mathrm{N}_t \\ &= X_{Zt} Y_{t-1} + \mathrm{N}'_t \end{aligned} \tag{2.45}$$

where $N'_t = N_t - X_{Zt} N_{t-1}$, Since the channel matrix H does not appear in the last equation, this means decoding in differential transmission can be done without knowledge of the channel at receiver.Therefore, the maximum-likelihood decoding of Z_t can be written as

$$\hat{Z}_t = \arg \max_{l=0,\ldots,L-1} \left\| Y_t - X_l Y_{t-1} \right\| \tag{2.46}$$

If the number of unitary matrices in the signal set of X_{Zt} is quite large, the decoding process at the receiver will do via an exhaustive search. To design code that can be decoded in real time, some structure should be imposed upon the signal set, that's what we will see in the next section.

2.5 Low decoding complexity:

In addition to the full rate and full diversity, the decoding complexity of maximum likelihood (ML) must be minimized as possible as we can for space-time code. As seen previously, OSTBC characterized by full diversity and linear decoding complexity, but its rate is not more than 3/4 for more than two transmit antennas. On another hand, quasi-orthogonal STBC (QSTBC) have has a full rate with exponential decoding complexity, several works on it have been done to make its complexity low. Recently, many research try to design STBC with higher rate while keeping the complexity low, [22],[23] introduce algebraic structure of STBC with single complex symbol decoding for 4 transmit antenna only, several rate-one STBC for any number of transmit antennas have been proposed [52-55], in which the transmitted symbols can be completely separated into two groups for ML detection. This section will present low decoding complexity based on group structure, where the transmitted codeword can be divided into *g*-groups for ML detection, where *g* greater than one. Before explaining the *g*-group encodable and decodable first linear STBC must be defined, suppose that we have *K* symbols $\left(x_1,x_2,...,x_K\right)$, a linear design $\mathrm{S}\left(x_1,x_2,...,x_K\right)$ matrix of size $n\times n$ is a linear combination of these symbols, which can be written as follows[45],

$$S\left(x_1,x_2,...,x_K\right)=\sum_{\imath=1}^{K}x_{\imath}Bi \tag{2.47}$$

where $\mathrm{B_i}\in\mathbb{C}^{n\times n}$ and called the weight matrices. A linear design STBC $\ell=\left\{S\left(X\right)\middle|X\in A\right\}$ is said to be g-group encodable if:

- G divides K
- $\mathrm{A}=\mathrm{A}_1\times\mathrm{A}_2\times\mathrm{A}_3,\ldots\times\mathrm{A}_g$ where each $\mathrm{A_i}\ ,i=1,2,\ldots,g\subset\mathbb{C}^{k/g}$, A represents the whole signal set and $\mathrm{A_i}$ are subset signal for each group.

A linear design STBC $\ell=\left\{S\left(X\right)\middle|X\in A\right\}$ is said to be *g*-group decodable if:

- Its *g*-group encodable.
- For any two weight matrices B_i and B_j that's belong to different groups the following condition satisfied,

$$B_i^H B_j + B_j^H B_i = 0 \tag{2.48}$$

The last condition in equation (2.48) means that the decoding metric of the ML detection,

$$\left\| y_t - S(X) y_{t-1} \right\|^2 \tag{2.49}$$

Can be minimize to be,

$$\left\| y_t - S_k(X_k) y_{t-1} \right\|^2 \tag{2.50}$$

Where $S_K(X_K) = \sum_{i=\frac{(k-1)K}{g}+1}^{\frac{kK}{g}} x_i B_i$ for each $1 \le k \le g$, which mean every group can be decoded separately, so instead of search in the whole space we can do it in reduced space. The following example demonstrate a linear STBC for 4 antenna,

$$S = \begin{bmatrix} x_1 + jx_2 & x_3 + jx_4 & -x_5 + jx_6 & -x_7 + jx_8 \\ x_3 + jx_4 & x_1 + jx_2 & -x_7 + jx_8 & -x_5 + jx_6 \\ x_5 + jx_6 & x_7 + jx_8 & x_1 - jx_2 & x_3 - jx_4 \\ x_7 + jx_8 & x_5 + jx_6 & x_3 - jx_4 & x_1 - jx_2 \end{bmatrix} = \sum_{i=1}^{K=8} X_i B_i \tag{2.51}$$

The weight matrices can be generated straight forward, and the four groups are (x_1,x_3), (x_2,x_4), (x_5,x_7) and (x_6,x_8).

Chapter 3

Cooperative and Distributed STBC Systems

3.1 Introduction:

As mentioned in chapter one that people get unconvinced with voice communication only, and they demand high data rate such as multimedia, wireless broadband Internet and other applications become huge which push to create a new one to satisfy these demands. MIMO systems discussed in the previous chapter are used to achieve these high data rates. In MIMO systems, there are only two users, transmitter and receiver, so it is considered as a point-to-point communication system, The multiple antennas produce independent channel gains which can be combined together to have high SNR, and mitigating faded channel.

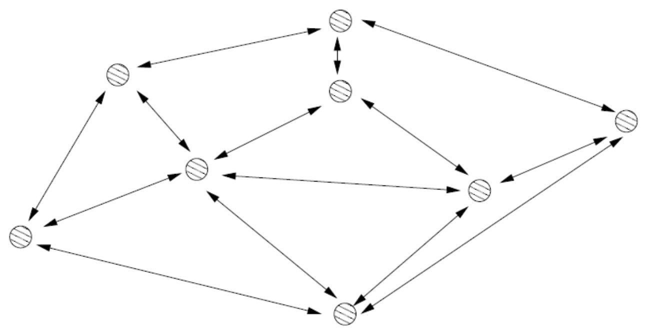

Figure (3.1): Ad hoc network

It is well known that wireless networks are the milestone in our modern life, and it can be separated into two main categories. The first one is the networks that have a master node, such as cellular phone system and satellite communication system, in these networks if any node wants to communicate with another one it must take permission from the master node, and all transmitted data pass through the master node and controlled by it. The second type is the ad hoc networks or sensory networks, in this type a group of wireless nodes form the network without the existence of a master node, and all nodes communications are peer to peer. Figure (3.1) represents a simple ad hoc network. In practice, these nodes have only one antenna, so they cannot get the benefits of MIMO system such as high rates due to the size, cost, or hardware limitation of their devices. Also nodes may be not in the range of communication of every other node want to communicate with.

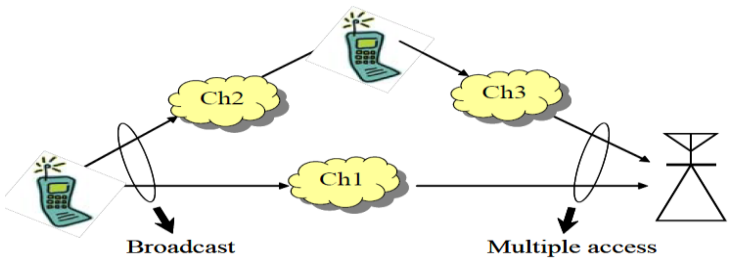

Figure (3.2): Illustration of Cooperative Communication

To overcome these limitations, a new technique of communication created. Since all nearby nodes overhear the transmission between transmitter (source) and receiver (destination). Engineer thought that why they did not cooperate with each other and exploit characteristic of broadcasting to processing and forwarding these messages to the intended destination; all nodes help each other. The cooperated nodes can be thought as a set of distributed antennas forming virtual antenna array in the wireless system, and they are acting as relay nodes to the source node, this guarantees independent channel paths to get some benefit of MIMO system and achieving reliable communication, as seen in figure (3.2). If nodes are out of range, they can cooperate in routing each other's data. Therefore, transmissions may be completed by one-hop routing or even multiple-hop routing. The whole system called user cooperative diversity or simply cooperative communication. In this chapter, a brief explanation of cooperative communication will be discussed. Besides, we will apply space- time code to the cooperative system to form a distributed space-time code.

3.2 Cooperative Communication:

The basic idea of cooperative communication was appeared in 1970s [24,25] when van der Meulen introduced and studied a basic three communication node model, after that T. Cover and A. E. Gamal study the capacity of the relay channel in [26]. Recently, many efforts and researches have been introduced focusing on cooperative diversity protocols. In [27,28] J. N. Laneman, D. N. C. Tse, and G. W. Wornell propose different cooperative protocols for wireless communication and the performance of these protocols was shown. Cooperative protocols classified as fixed and adaptive protocols [10]. In the following sections, we give brief description of some of these protocols before explaining that the main concept of relay channel.

3.2.1 Relay Channel

As mentioned in the introduction that cooperative systems use other nodes as relays of source. So the relay channel can be thought of as an auxiliary channel to the direct channel between the source and the destination. The relayed information from relays are combined at the destination using one of the combining method such as selection combining, equal gain combining or maximal ratio combining so as to create spatial diversity.

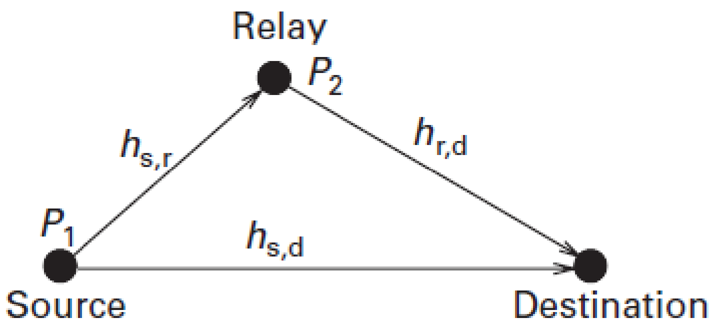

Figure (3.3): Basic model of relay channel.

Since the relay channel forms the basis of cooperative communication, we should study how it works. Figure (3.3) shows basic model element of relay channel and how transmission occurs in two steps (phases), 1- *Transmission from source to relay and destination (broadcasting).*

The received signals $y_{s,d}$ and $y_{s,r}$ at the destination and the relay, respectively, can be written as:

$$y_{s,d} = \sqrt{P} h_{s,d} s + n_{s,d} \quad , \tag{3.1}$$

and

$$y_{s,r} = \sqrt{P} h_{s,r} s + n_{s,r} \, , \tag{3.2}$$

where

P: is the transmitted power from the source,

S: is the transmitted information,

$n_{s,d}$ and $n_{s,r}$: are Additive White Gaussian Noise (AWGN).

2- *Transmission from relay to destination (relaying).*

This can be written as

$$y_{r,d} = h_{r,d} \, q(y_{s,r}) + n_{r,d} \tag{3.3}$$

Where:

$h_{r,d}$: channel gain from relay to destination.

q (·): depends on which processing is implemented at the relay node which will be discuss in the next section.

$n_{r,d}$: is additive white Gaussian noise.

These two transmission phases must be orthogonal (in TDMA or FDMA) to prevent interference between them because the relay cannot transmit and receive at the same time. Also synchronization between the three terminals is required.

If there are many nodes ready to cooperate with each other, we will have four scenarios, as listed in table (3.1). The first one will be explained and the other will be more or less the same. At the first phase, the source transmits to the relays and destination, and at the second phase, both the source and the relays transmit to the destination. In our work, we will use only the fourth scenario only.

Phase\Protocol	I	II	III	IV
1	s→r,d	s→r,d	s→r	s→r
2	s→d, r→d	r→d	s→d, r→d	r→d

Table (3.1): possible transmission from Source-to-destination

3.2.2 Cooperation protocols:

Since cooperative communication is a network, protocols must be placed in relay to coordinate transmission. In general, these protocols can be classified into fixed relaying schemes and adaptive relaying schemes. In fixed protocols, the channel resources are divided between source and relay each one has fifty percent of the resources, this lead to low bandwidth efficiency and reduces the overall rate especially if the link between the source and destination is good enough, this is considered as a disadvantage, but it's advantage is easy to implement, the protocols in fixed scheme are as follows:

- Amplify-and-forward (AF) relaying protocol
- Decode-and-forward (DF) relaying protocol.
- Compress-and-forward relaying protocol.
- Coded relaying protocol.

Adaptive relaying schemes overcome the disadvantage of fixed relay, and it includes:

- Selective relaying protocol.
- Incremental relaying protocol.

The following section discusses briefly these protocols considering only one relay channel.

3.2.2.1 Fixed cooperation schemes:

The behavior of the relay depends on the employed protocol, amplify and forward and decode and forward are the most common protocol in fixed cooperation schemes so they will discuss in this thesis

.

1- Amplify-and-Forward relaying protocol

In this protocol, the relay nodes amplify the received signal from the source and retransmit it to the destination; figure 3.4 makes a clear illustration of this protocol. The amplification factor equalizes the effect of channel fading between the source and destination.

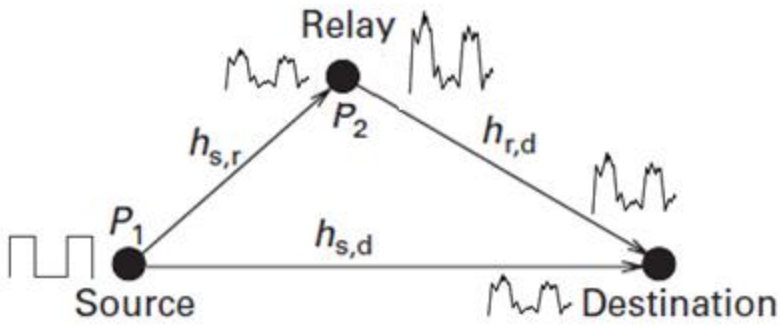

Figure (3.4): Amplify and Forward protocol.

As mentioned previously the received signal at relay is:

$$y_{s,r} = \sqrt{P} h_{s,r} + n_{s,r} \tag{3.4}$$

So the amplification factor is equal to:

$$\beta_r = \frac{\sqrt{P}}{\sqrt{P \left| h_{s,r} \right|^2 + N_0}} \tag{3.5}$$

Thus the transmitted signal form the relay is equal to β_r multiplied by $y_{s,r}$ and its power equals to that transmitted from the source and can be modeled as:

$$y_{r,d} = \frac{\sqrt{P}}{\sqrt{P \left| h_{s,r} \right|^2 + N_0}} h_{r,d}\, y_{s,r} + n_{r,d} \tag{3.6}$$

Although the noise is amplified along with the signal in this technique, we still gain spatial diversity by transmitting the signal over two spatially independent channels. The destination receives two copies of the signal and combining them using one of the combining methods to maximize the signal to noise ratio (SNR). The performance of this protocol compared with direct transmission is shown in figure (3.5) depicting the outage probability versus SNR.

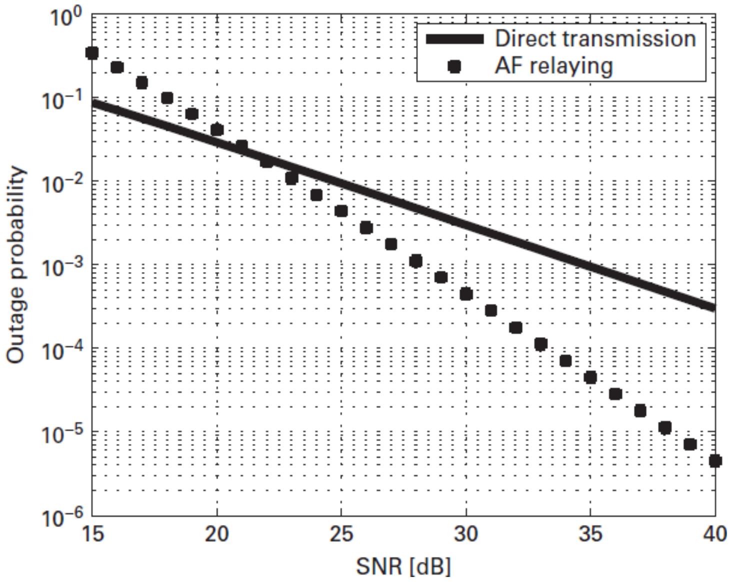

Figure (3.5): Outage probability versus SNR, [10]

From the figure we note that amplify and forward protocol is better than the direct transmission (have less outage probability) especially at high SNR.

2- Decode-and-Forward relaying protocol

Simply, the relay decodes the received signal from the source, re-encodes it, and retransmits it to the destination, figure (3.6) illustrate this protocol. If the relay decodes signal incorrectly and retransmits it to the destination. Only the link from source to destination works and the cooperation diversity become one.

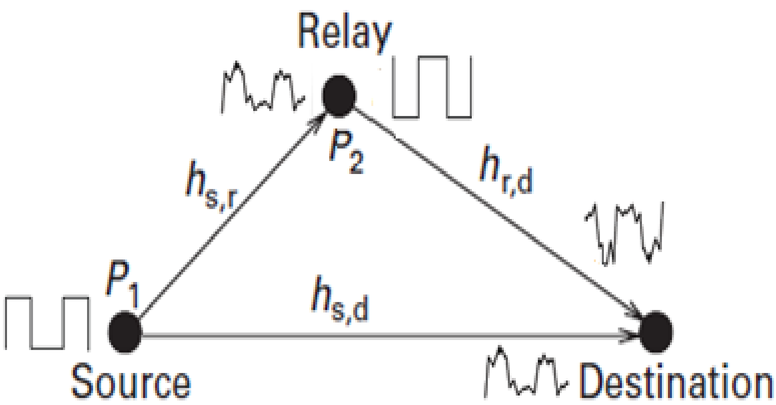

Figure (3.6): Decode and Forward protocol.

This protocol is considered better than the previous one because it reduces the effect of AWGN of the channel, but propagation in the error due to the error in decoding reduces the overall performance of the protocol.

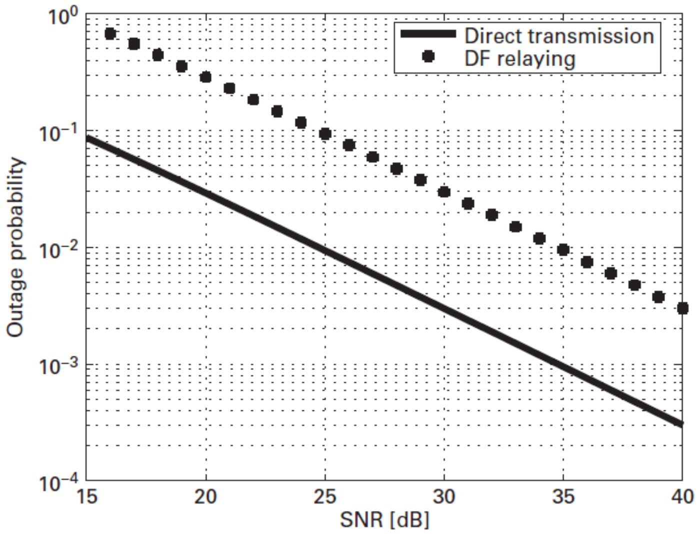

Figure (3.7): Outage probability versus SNR, [10]

The outage probability of this protocol versus SNR is shown in figure (3.7), it's clear that direct transmission is better than Decode and forward relaying at a fixed data rate.

3.2.2.2 Adaptive cooperation schemes:

Adaptive cooperative protocol can be applied in relay node to overcome the disadvantage of fixed cooperative protocol and increase the overall performance, there are two protocols selective Decode and forward relaying and incremental relaying.

1- Selective Decode and Forward relaying

In this protocol, the relay computes the signal to noise ratio (SNR) of the transmitted signal from the source, if this SNR is greater than a specified threshold, the relay decodes it and retransmits it to the destination after re-encoding, but if this SNR is below that threshold which means that the channel between the source and relay suffers from multipath fading, the relay idles. So the error propagation in fixed decode and forward due to the error in decoding is not included here.

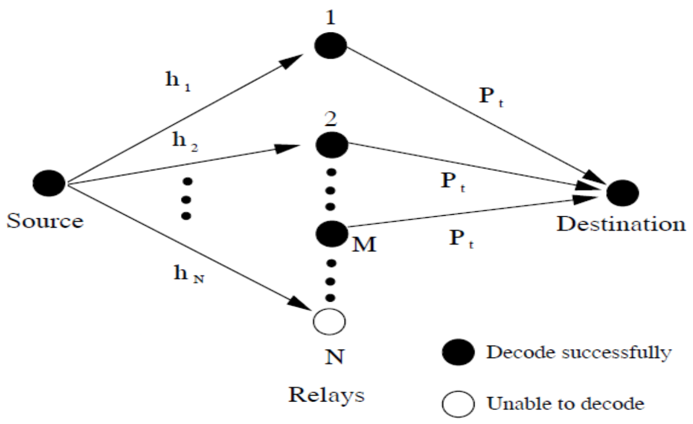

Figure (3.8): Selection Decode and Forward protocol.

At the destination, the received SNR (in case of source – relay SNR greater than the threshold) is a combination of source – destination SNR and relay-destination SNR which can be computed any combination method such as maximal ratio combining (MRC). We can say that the selective relaying scheme can achieve diversity of order two just like Fixed amplify and forward protocol (both of them have the same diversity gain).

2- Incremental relaying

We have seen that in both fixed and selection relay node retransmit the received signal from source to destination all the time, what if the information has been received from the source in the first step is correct, which mean the transmission from relay to destination will be meaningless. So we need a feedback from the destination to the relay indicating the success or failure of the direct transmission, the advantage of this protocol will appear in the spectral efficiency.

Let us take an example of this protocol, we have three nodes source, destination and relay, the relay node will use incremental and amplify and forward. First, the source will transmit to the destination if SNR between the source –destination channel is acceptable the feedback indicates success of the direct transmission, and the relay do nothing, but if SNR is low, the feedback asks the relay to retransmit the information to him via amplify and forward protocol. The relay retransmits in an attempt to exploit spatial diversity.

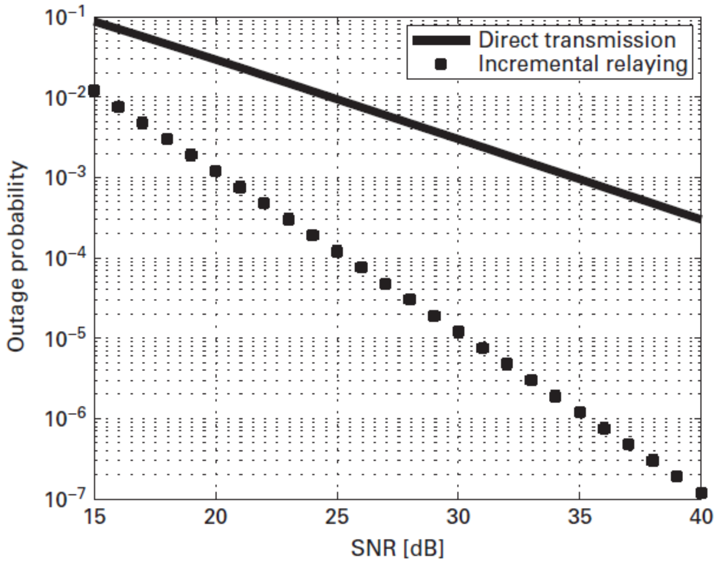

Figure (3.9): Outage probability versus SNR, [10]

The performance of the incremental relaying and direct transmission in term of outage probability is shown in figure (3.9), clearly, using this protocol is better than the direct transmission because it will have adversity gain of two.

3.2.2.3 Comparison between different protocols:

In this section, we will compare the different protocol discussed early, figure (3.10) shows the performance of these protocols, they are arranged from best to worse as follows:

- Incremental relaying.
- Amplify and forward & selective decode and forward relaying
- Direct transmission
- Decode and forward relaying

The same thing is done for these protocol but now outage probability against data rate, from figure (3.11) we can arrange them from best to the worse as :

- Incremental relaying.
- Amplify and forward & selective decode and forward relaying
- Direct transmission
- Decode and forward relaying

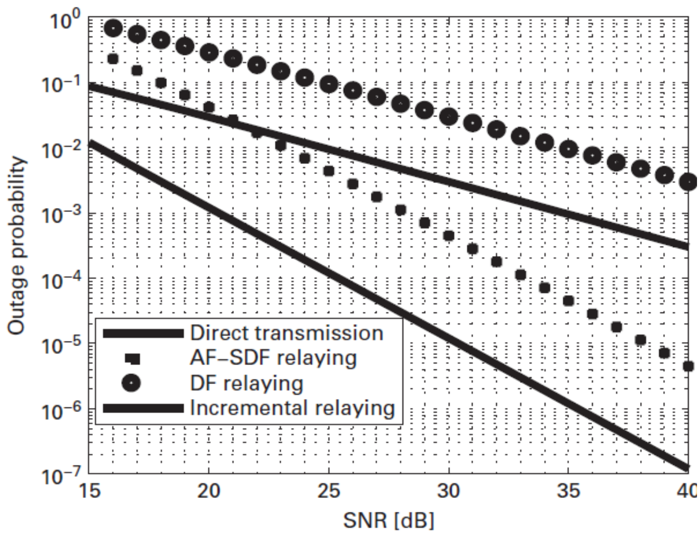

Figure (3.10): Outage probability versus SNR for different protocol, [10]

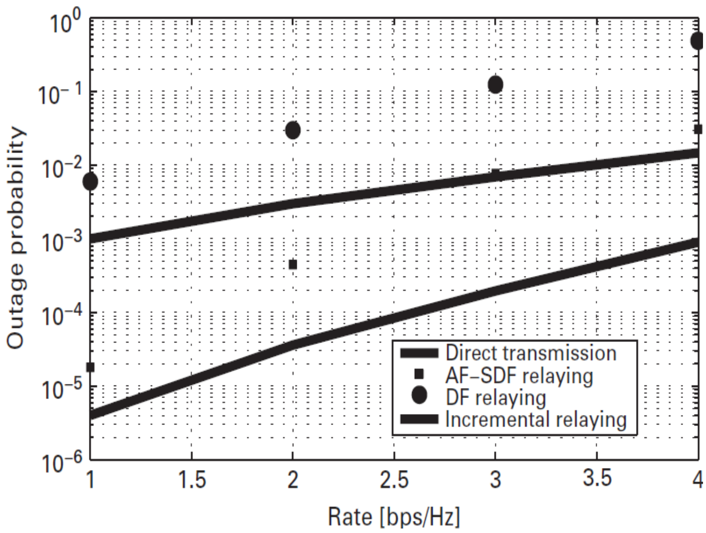

Figure (3.11): Outage probability versus data rate for different protocol [10].

Incremental relaying position in the first rank in terms of the overall performance because the diversity gain of this protocol always two. Finally, and in simple words, Cooperative diversity is achieved by several single-antenna terminals cooperate and form a virtual multiple antenna system and we can summaries the previous work of cooperative by these two points:

1. Benefits of cooperation
   - Higher spatial diversity
   - Resistance to large and small scale fading

  - Lower total transmitted energy which reduces interference and extends the battery life
2. Detriments of cooperation
  - Lowering of spectral efficiency which is solved using space-time code.

3.3 Distributed Space-Time Code (DSTC):

3.3.1 Introduction:

The use of fixed protocol, repetition-base protocol, such as the decode-and-forward (DF) protocol and the amplify-and-forward (AF) protocol, leading to a highly loss in the system bandwidth efficiency, which considered as a result of using orthogonal sub-channels for the relay node transmissions, either through TDMA or FDMA. Although adaptive protocol used to overcome this problem but they are hard to implement because feedback required from destination to relay [10]. Since, using space-time code in multiple input multiple output system (MIMO) can greatly increase the capacity and reliability of a wireless communication link in a fading environment [6],[7],[12],[29]. So space-time code was applied to cooperative systems which can improve the bandwidth efficiency without the need of feedback [30], [31], [32]. Such systems known as distributed Space-time code (DSTC), where relay nodes are allowed to simultaneously transmit over the same channel. The word distributed comes from the fact that the virtual multi-antenna transmitter is distributed between randomly located relay nodes.

In practical implementation, there are a number of challenges associated with distributed space-time code, some of which are shared with conventional space-time coding used in MIMO systems, such as the knowledge of channel fading between the relays and the destination must be known to the destination to be able to decode coherently. And other are unique to distributed space-time codes systems such as the synchronization challenge, where received signals at destination suffers from offsets in time, these offsets happened because the transmitters are widely separated and have different time references, and due to differences in the propagation delay between the relays and the destination. This problem can be handled with delay diversity, delay-tolerant distributed space-time codes [56], [57], or space-time spreading [58].

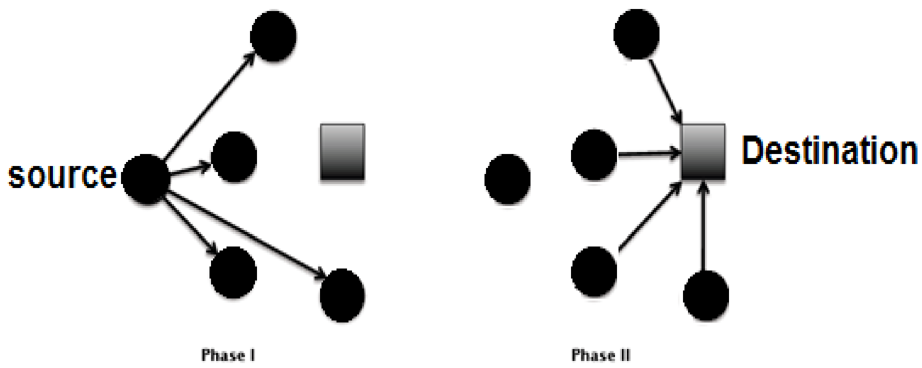

Figure (3.12): Two-hop distributed space-time code

Some works have considered the design of distributed space–time codes. In [30] the authors exploit spatial diversity using the repetition and space-time algorithms, and the relays need to decode their received signals. In addition, an outage analysis was analyzed for the system. In [33], distributed space–time coding based on the Alamouti scheme and amplify-and-forward cooperation protocol was analyzed. In [34], a new distributed space-time coding was proposed the performance which depends on linear dispersion (LD) space–time codes of [35]. In this scheme, relay did not need to know the channel fading, and don't decode the received signal. In [36], the authors proposed the use of space-time codes based on Hurwitz-Radon matrices in wireless relay networks. In [37], author proposed distributed space-time codes using real orthogonal, complex orthogonal, and quasi-orthogonal designs, performance and low decoding complexity of these codes was presented. Distributed space-time code designs based on cyclotomic field theory can be found in [38] and designs using commuting sets and doubling construction can be found in [39].

The following sections, study the design of distributed space–time codes for wireless relay networks based on papers [34], [37]. Where a two-hop relay network model is considered as in Figure (3.12), it consists of two phases broadcasting and relaying and there is no direct link from the source to destination. Furthermore, a brief explanation of decode and forward with DSTC will be presented.

3.3.2 Decode and forward DSTC [10]:

Decode and forward cooperation protocol with DSTC will be explained in this section, suppose that there is R+2 node as in figure (3.13), one source, one destination and R relays. The system has two phases, in the first phase the source broadcasts the data to the R relays. Since these data corrupted by multipath fading and noise, there is no guarantee that any relay will receive the transmission correctly. So channel coding is done at the source, and relay can participate into the second phase if and only if they correctly decode the data. This can be achieved through using cyclic redundancy check (CRC) codes, also by setting a signal to noise ratio(SNR) threshold at the relays, if the value of SNR is up to the threshold, the relay will forward the source data otherwise the relay will remain idle [9].

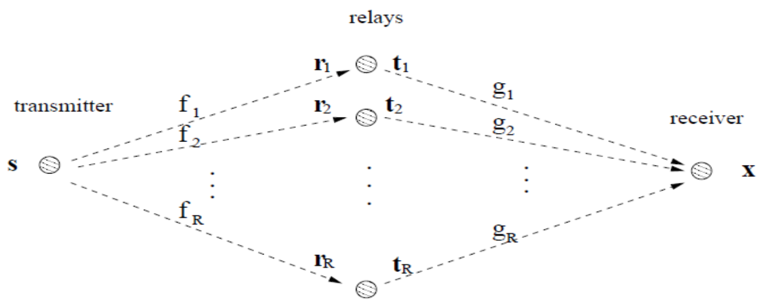

Figure (3.13): DSTC with decode and forward

If the transmitted power from the source equal to P_1 and the channels from source to relay are f_R , which are assumed to be constant for one transmission block, then the received signal at any relay will be as,

$$Y = \sqrt{P_1} f_R S + n \, , \tag{3.7}$$

where n is additive white Gaussian noise (AWGN) and S is $L\times1$ transmitted data vector. In phase two the relay nodes that have correctly decoded data vector S re-encode it with a pre-signed space-time code. The space-time code is distributed through a different relay so everyone acts as a single antenna in the multiple antennas system. This can be represented as,

$$Y_d = \sqrt{P_2} X H + N \tag{3.8}$$

Where P_2 power transmitted from each relay, X is space-time code, H is $R\times1$ channel gain from relay to destination and N is $R\times1$ additive white Gaussian noise.

3.3.2 Amplify and forward DSTC [34][37]:

Unlike DSTC base on decode-and-forward protocols, amplify-and-forward DSTC needs no channel information at relays, because the relay does not decode the received data vectors from the source. Therefore, it saves both power and time at relays. In the following sections which based mainly on [34], a distributed space–time coding based on a linear dispersion (LD) space-time code [35] is introduced. Where transmission from source to destination done through two steps, in the first step the source broadcasts data, the relays encode their received signals into a LD code, and then re-transmit them to the destination. This method guarantees optimum diversity in a network [34]. The design of practical distributed space-time codes (DSTCs) of paper [37] using orthogonal and quasi-orthogonal space-time code explained.

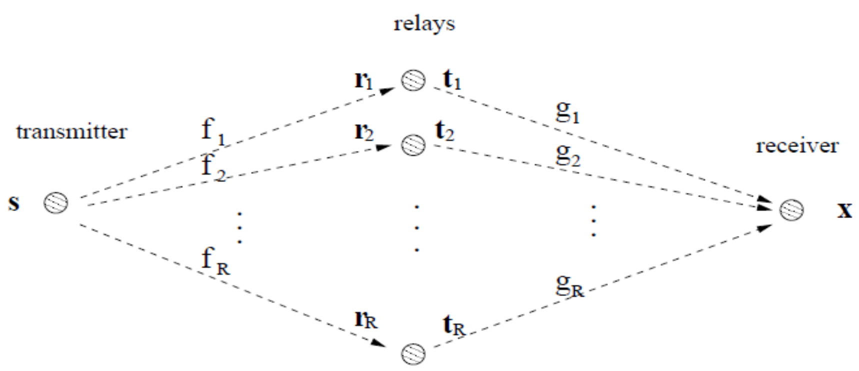

Figure (3.14): DSTC with amplify and forward

First, we illustrate the system model, suppose a wireless network with $R+2$ nodes, as in figure (3.14). Each of them is placed randomly and independently, one node act as a source other one act as destination, rest of the nodes act as a relay. Let the channel from source to i-th relay denoted as f_i, and the channel from i-th relay to the destination as g_i, both channel f_i and g_i assumed Rayleigh flat fading, i.e., independent complex Gaussian random variables with zero-mean and unit variance. These channel coefficients are known to the destination through using training symbols from the source and relay. A block-fading model is used with a coherence interval T, i.e., f_i and g_i keep constant for a block of T transmissions and jump to other independent values for next T transmissions. Every node has a single antenna, so it can't transmit and received at the same time. Also, synchronization between relays is expected.

Before transmission information bits are encoded into symbols to form a codebook $\{x_1, x_2, \ldots\ldots, x_L\}$, where L is the cardinality of the codebook, after that this codebook divided into groups each with T element, $s=\{x_1,x_2,\ldots.,x_T\}^T$, s normalized so $Es^*s=1$, the two transmission steps will be explained separately bellow.

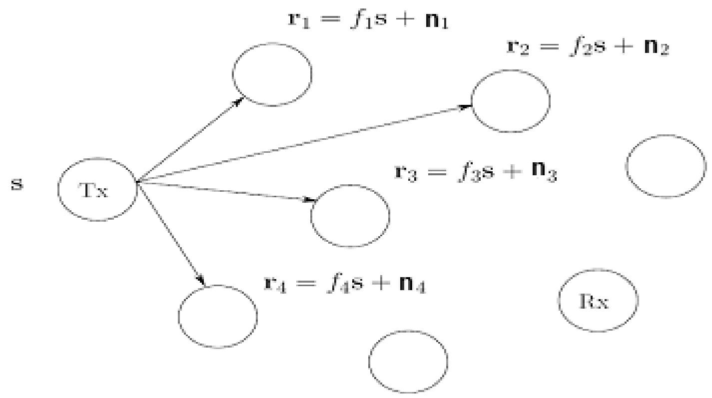

Figure (3.15): First step of DSTC.

1- *First step (broadcasting phase):*

The source multiplies every group by $\sqrt{P_1T}$ and broadcasts them to each relay one by one, P_1 is the transmitted power all this takes place from time 1 to T. Since each group s is normalized, the average power used for transmission is P_1. As demonstrated in figure (3.15), the received signal at the i-th relay is,

$$r_i = \sqrt{P_1T} f_i s + n_i , \qquad (3.9)$$

where $r_i=[r_{i,1},\ldots\ldots.,r_{i,T}]^T$, $r_{i,t}$ is the received signal at the i-th relay at time t, f_i is the channel fading coefficient, and $n_i =[n_{i,1},\ldots\ldots.,n_{i,T}]^t$ is the additive white Gaussian noise, which assume i.i.d with zero mean and unit variance *CN(0,1)*, because $Es^*s=1$ and f_i and n_i is CN(0,1), the average transmitted power at relay i is equal to

$$Er_i^* r_i = E(P_1T\,|f_i|^2 s^* s + n_i^* n_i) = (P_1+1)T \qquad (3.10)$$

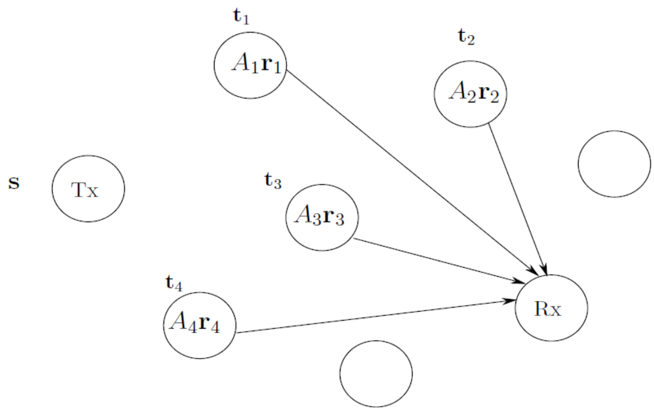

Figure (3.16): Second step of DSTC.

2- Second step (relaying phase):

From time T+1 to 2T relays want forward these signal to the destination, to achieve maximum diversity relay does not need to decode them, and the idea of linear dispersion space-time code is used, so the transmitted signal t_i will be a linear combination of the received signal r_i and its conjugate.

$$t_i = \sqrt{\frac{P_2}{P_1+1}}(A_i r_i + B_i r_i^*), \qquad i = 1,2,....,R, \tag{3.11}$$

where t_i=[$t_{i,1}$,….., $t_{i,T}$]^t, A_i and B_i are $T\times T$ complex matrices where

$$\begin{bmatrix} A_{i,\mathrm{Re}} + B_{i,\mathrm{Re}} & -A_{i,\mathrm{Im}} + B_{i,\mathrm{Im}} \\ A_{i,\mathrm{Im}} + B_{i,\mathrm{Im}} & A_{i,\mathrm{Re}} - B_{i,\mathrm{Re}} \end{bmatrix} \tag{3.12}$$

is a $2T\times 2T$ orthogonal matrix, if A_i is zero matrix then B_i should be unitary and vice versa. The average transmitted power at every relay should be P_2 because

$$Et_i^* t_i = \frac{P_2}{P_1+1} E\left((A_i r_i + B_i \overline{r_i})^* (A_i r_i + B_i \overline{r_i})\right) = P_2 T \tag{3.13}$$

That's why the orthogonaltiy of equation (3.12) and normalization of equation (3.11) is done. Let us go now to the destination side, the received signal x = [x_1,….,x_T]^T considered as a summation of different signal, which came from different paths (g_i) as,

$$x = \sum_{i=1}^{R} g_i t_i + w \;, \tag{3.14}$$

where w = [w_1,……,w_T]^T is additive white Gaussian noise, substitute equation (3.9) and (3.11) in (3.14) the received signal can be calculated to be,

$$Y = \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} SH + W \ , \qquad (3.15)$$

where

$$S = [\hat{A}_1 \hat{s} \ .... \ \hat{A}_R \hat{s} \ ],$$
$$H = [\hat{f}_1 g_1 \ ..... \ \hat{f}_1 g_1]^t ,$$
$$W = \sqrt{\frac{P_2}{P_1 + 1}} \sum_{i=1}^{R} g_i \hat{A}_i \hat{n}_i + w \ ,$$

and

$$\begin{cases} \hat{A}_i = A_i , \hat{f}_i = f_i , \hat{v}_i = v_i , \hat{s}_i = s & if \ B_i = 0 \\ \hat{A}_i = B_i , \hat{f}_i = \overline{f}_i , \hat{v}_i = \overline{v}_i , \hat{s}_i = \overline{s} & if \ A_i = 0 \end{cases}$$

That means, if B_i = 0 the column of the code matrix S will contain the information symbols $s_1,\ldots,s_T$, and if A_i = 0 the column matrix of S will contain the conjugate of information symbol $s_1,\ldots,s_T$ only. Thus, the relays generate a space–time codeword distributively at the receiver. H is the equivalent channel and W is the equivalent noise. If H is known to the receiver then the maximum likelihood (ML) decoding is,

$$\arg\min_{s} \left\| Y - \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} SH \right\| \qquad (3.16)$$

If the total power per symbol transmission used in the whole network is fixed as P, the optimal power allocation that maximizes the expected receive SNR is

$$P_1 = \frac{P}{2} \quad and \quad P_2 = \frac{P}{2R} \qquad (3.17)$$

To make it clear, two examples of space-time code will be declared, one of them represents orthogonal code and the other will be quasi- orthogonal code, and both of them have a rate equal to one, which mean the coherence time T and number of relay R are equal. Alamouti orthogonal code needs T=R=2, in the first phase the transmitter picks up randomly two information symbol to form the vector $s = [x_1 \ x_2]^t$ and broadcasts them, the two relays receive,

$$r_1 = \begin{bmatrix} r_{1,1} \\ r_{1,2} \end{bmatrix} \qquad and \qquad r_2 = \begin{bmatrix} r_{2,1} \\ r_{2,2} \end{bmatrix} \qquad (3.18)$$

The linear dispersion matrices which exist at both relay are given by,

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ B_1 = 0,\ A_2 = 0, B_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{3.19}$$

During the second phase, the first relay sends,

$$t_1 = A_1 r_1 + B_1 \bar{r}_1 = \begin{bmatrix} r_{1,1} \\ r_{1,2} \end{bmatrix} \tag{3.20}$$

And the second one sends

$$t_2 = A_2 r_2 + B_2 \bar{r}_2 = \begin{bmatrix} -\bar{r}_{2,2} \\ r_{2,1} \end{bmatrix} \tag{3.21}$$

After simple calculation the at the receiver,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \begin{bmatrix} f_1 g_1 \\ \bar{f}_2 g_2 \end{bmatrix} + \sqrt{\frac{P_2}{P_1 + 1}} \left(g_1 n_1 + g_2 n_2 \right) + W \tag{3.22}$$

where n_1, n_2 and W are additive noise at first relay, second relay and receiver respectively. They have the following form,

$$n_1 = \begin{bmatrix} n_{1,1} \\ n_{1,2} \end{bmatrix},\ n_2 = \begin{bmatrix} -\bar{n}_{2,2} \\ \bar{n}_{2,1} \end{bmatrix} \quad and\ W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{3.23}$$

We notice that the Alamouti code is formed at the receiver like,

$$S = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \tag{3.24}$$

The maximum likelihood at receiver is calculated as

$$\arg\min_{x_1, x_2} \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \begin{bmatrix} f_1 g_1 \\ \bar{f}_2 g_2 \end{bmatrix} \right\|_F^2 \tag{3.25}$$

This decoding can be decoupled as

$$\begin{aligned} &\arg\min_{x_1} \left| x_1 - \sqrt{\frac{P_1 + 1}{P_1 P_2 T}} \frac{\bar{f}_1 \bar{g}_2 y_1 + \bar{f}_2 g_2 y_2^*}{|f_1 g_1|^2 + |f_2 g_2|^2} \right|^2 \\ &+ \arg\min_{x_2} \left| x_2 + \sqrt{\frac{P_1 + 1}{P_1 P_2 T}} \frac{\bar{f}_2 g_2 y_1^* + \bar{f}_1 \bar{g}_1 y_2}{|f_1 g_1|^2 + |f_2 g_2|^2} \right|^2 \end{aligned} \tag{3.26}$$

The special structure of the code S makes the decoding complexity linear. As early mentioned that two codes will be explained, the turn now to the quasi-orthogonal code, bellow the code will be written with its linear dispersion metrics and the rest will be straight forward as the orthogonal code,

$$\begin{bmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{bmatrix}, \tag{3.27}$$

where

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = A_3 = B_1 = B_4 = 0$$

$$B_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The previous two assume that $T = R$, the time interval at both phases are the same, what if this is not true, a general approach must be made to permit unequal time interval. Suppose the time interval is T_1 at first phase and T_2 at the second one, the transmitter at the first step sends $\sqrt{P_1 T}\, s$, the coherence time for f_i and g_i should be not less than T_1 and T_2 respectively. s, r_i and n_i now are a 1× T_1 vectors, whereas t_i, y and w are a 1× T_2 vectors, in the second phase the signal sent by the i-th relay is,

$$t_i = \sqrt{\frac{P_2 T_2}{(P_1+1)T_1}} \left(A_i r_i + B_i \overline{r_i} \right), \tag{3.28}$$

where A_i and B_i are $T_2 \times T_1$ matrices. The system equation can be written as,

$$x = \sqrt{\frac{P_1 P_2 T_2}{P_1+1}} SH + W \ , \tag{3.29}$$

where S is a $T_2 \times R$ matrix represents a space-time code, the following example illustrates a 3/4 orthogonal space-time code, T_1=3 and T_2=R=4.

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_3^* & -x_2^* & -x_1 \end{bmatrix} \tag{3.30}$$

The linear dispersion matrices are

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3.31}$$
$$B_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, B_4 = 0_{4\times 3}.$$

Now we wish to see the performance of DSTCs, figure (3.17) shows the BERs with BPSK symbols for networks of two and four relays. As we can see the performance separated into two parts, low transmit powers and high transmit powers part. When the power is low, the bit errors of DSTC with selection DF is lower than DSTC with amplify and forward, because noises at the relay are also forwarded to the receiver. Also the normalization has done at the relays scales down the signal power when power is not high enough. However, at high transmit power, selection DF can only achieve diversity one, while DSTC with amplify and forward the maximal diversity is achieved, therefore, they have lower BERs. This is because in selection DF mistakes made by the relays rule the communication errors. Also figure (3.17) show the performance of the quasi-orthogonal DSTC of equation (3.27) with the second column deleted, that's mean the second relay is idle, and the network became with only three relays. Figure (3.17) shows that the diversity is about three. If two of the four relays are idle, we will have Alamouti design. Therefore, the quasi-orthogonal DSTC has good performance, since its can achieve the maximal diversity.

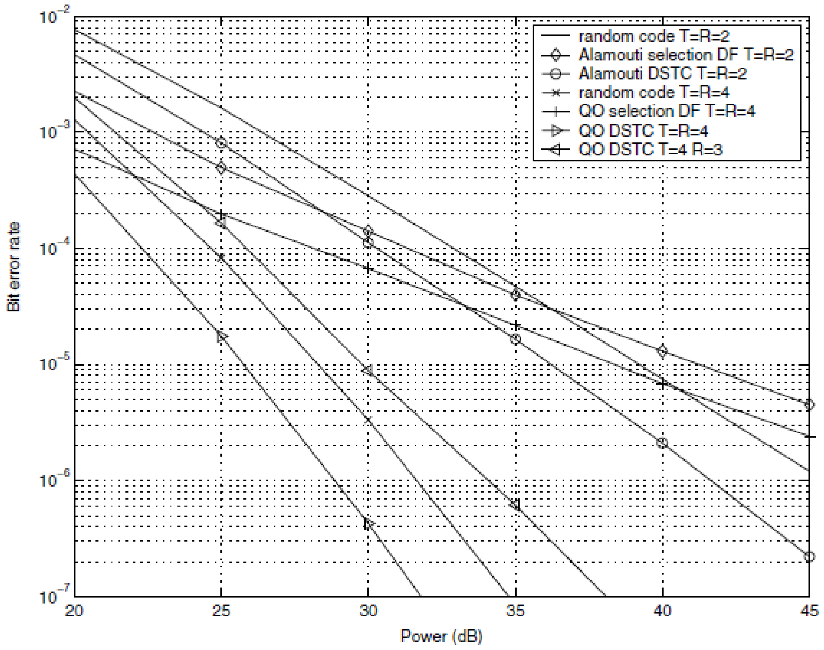

Figure (3.17): Performance of relay network [37].

However, a good DSTC should be *scale-free*, which mean if one of relay or more is inactive or equivalently, when some columns of the original code matrices are deleted. The code should have large diversity products as well as the original code. At the end, the following four points clarify the main difference between DF and AF distributed space time code:

- In AF, all R relays transmit during the second phase, but in DF just those can decode the source’s data correctly.
- In AF, the channel vector consists of the both the source-relay and relay-destination channel gains, but in DF just the relay-destination channel gains are existed.
- In AF, codewords are a linear combination of the received vector and its conjugates, but in DF it's a linear combination of the re-modulated symbols in the data vector and its conjugates.
- In AF, the additive white Gaussian noise from the first phase will be amplified and retransmit, contrasting DF, where the noise will be disappeared.

Chapter 4

Distributed Differential STC

4.1 Introduction:

Distributed space-time codes are considered as the counterpart of space-time code in MIMO networks, in which receiver requires full channel information from the transmitter to relays and from relays to the receiver to decode the transmitted signal coherently. However, there is usually no need of channel information at relay nodes depending on the cooperating scheme. Coherent detection needs training symbols that increase the cost and complexity of the receiver. It is also not valid in fast fading situation. So it is useful to develop transmission schemes that require no channel information at both relays and receiver. Inspiring by differential *space-time coding* a new technique for relay network was introduced, which called *Distributed Differential Space-Time Code* (DDSTC). It is worth noting that the differential scheme suffers from a 3-dB loss in effective SNR compared to the coherent one, because the receiver detect the current block using the previous transmitted one which corrupted by noise (AWGN).

Several authors independently suggested differential encoding/decoding for wireless networks in [40-45]. In [40], differential schemes for relay network have been adapted for a decode-and-forward protocol. An amplify-and-forward based differential scheme using the single-antenna DPSK technique can be found in [41]. In [43], a partially coherent distributed space-time code is proposed that does not require the knowledge of fading coefficients between the relay and destination. In [44], Cyclic distributed space time code that does not require the knowledge of any of the fading coefficients is presented. The authors of [42] make a general approach for DDSTC and they have also provided few code constructions. In [45], differential distributed space-time coding with low decoding complexity is considered using scaled unitary matrices, and codes are constructed using Clifford's algebras.

All the authors concluded that designing problem of DDSTCs is more challenging than that of differential space time code for MIMO systems, since in this scenario additional constraints are needed. For DDSTC suitable differential codes are families of unitary commuting matrices this will be shown later. In this chapter, papers [42], [44], [45] will be explained in details, also we will see how we can use circulant code to produce low decoding complexity DDSTC.

4.2 Generalize DDSTC [42]:

As in coherent distributed space time coding, a network consisting of R+2 nodes is considered: source node, destination node and R relay nodes. The wireless channels between the terminals are assumed to quasi-static flat fading. f_i and g_i are the channel fading gains from the source to the i-th relay and from the i-th relay to the destination respectively, see figure (4.1), all assumed to be independent and identically distributed complex Gaussian random variables with zero mean and unit variance. All nodes cannot receive and transmit at the same time, Moreover, the system needs to be synchronized at the symbol level.

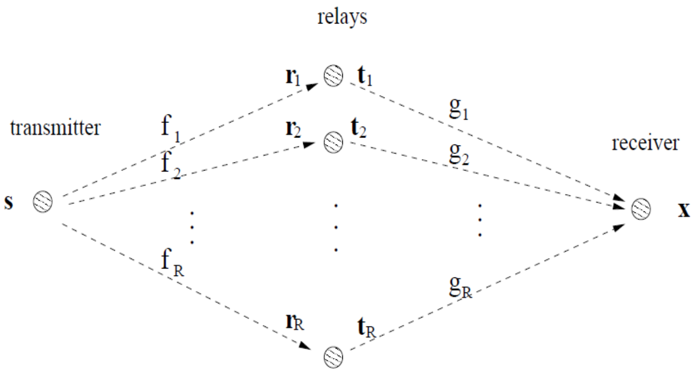

Figure 4.1: System model for DDSTC.

The transmission of T symbols called a block, where T = R, every transmission contains 2T channel uses, T channel uses for each step. Our differential scheme uses two blocks that overlap by one block. One block acts as a reference for the next similar to the differential space-time coding mentioned in chapter 2.

In the first step and at t-th time , a data vector of T symbols is encoded into T×T unitary matrix $X_t \in L$, where L is the set of all possible codeword, the source sends the differentially encoded signal,

$$s_t = \sqrt{P_1 T}\, X_t\, s_{t-1} \tag{4.1}$$

Where P_1 is the average power transmitted from the source, s_t normalize such that it's satisfy E{$s^* s$} =1, the first block s_o is the initial vector known to the destination and with a unit-norm, $E s_o^* s_o = 1$. The assumption that X_t is unitary preserve the transmitted power from vanish or blow up, the same as differential space time code. The received vector at i-th relay is given by,

$$r_i = \sqrt{P_1 T}\, f_i s + n_i \tag{4.2}$$

The second step of transmission, all relays perform a linear operation on r_i vector or on its conjugate,

$$t_i = \sqrt{\frac{P_2}{P_1+1}}(A_i r_i + B_i r_i^*), \qquad i = 1, 2, ..., R, \tag{4.3}$$

Where the relay matrices A_i and B_i are T×T unitary matrices, the received vector at t-th time can be written as,

$$\begin{aligned} y_t &= \sqrt{\frac{P_1 P_2 T}{P_1+1}} S_t H_t + W_t \\ &= \sqrt{\frac{P_1 P_2 T}{P_1+1}} \left[\hat{A}_1 s_t \;\; \dots \;\; \hat{A}_R s_t \right] H_t + W_t \qquad , \\ &= \sqrt{\frac{P_1 P_2 T}{P_1+1}} \left[\hat{A}_1 \hat{X}_t s_{t-1} \;\; \dots \;\; \hat{A}_R \hat{X}_t s_{t-1} \right] H_t + W_t \end{aligned} \tag{4.4}$$

where

$$\hat{X}_t = \begin{cases} X_t & if\ B_i = 0 \\ \bar{X}_t & if\ A_i = 0 \end{cases}$$

If $X_t \hat{A}_i = \hat{A}_i \hat{X}_t$ or equivalently,

$$\begin{cases} X_t A_i = A_i X_t \\ X_t B_i = B_i X_t^* \end{cases} \tag{4.5}$$

If f_i and g_i are kept constant for two consecutive blocks, i.e. $H_t = H_{t-1}$, then equation (4.4) become,

$$\begin{aligned} y_t &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} \left[\hat{A}_1 \hat{X}_t s_{t-1} \ \ldots \ \hat{A}_R \hat{X}_t s_{t-1} \right] H_t + W_t \\ &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} X_t \left[\hat{A}_1 s_{t-1} \ \ldots \ \hat{A}_R s_{t-1} \right] H_{t-1} + W_t \\ &= \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} X_t S_{t-1} H_{t-1} + W_t \\ &= X_t y_{t-1} + W_t' \end{aligned} \tag{4.6}$$

where

$$W_t' = W_t - X_t W_{t-1}$$

Now to decode the codeword X_t maximum likelihood detector can be applied as,

$$\arg\max_X \left\| y_t - X y_{t-1} \right\|. \tag{4.7}$$

No need for any channel information f_i or g_i during the detection process, it's obviously that the construction of DDSTC suggests the relays cooperate to encode a unitary space–time code, where the differential encoding is actually happened at the transmitter. The design problem of DDSTC can be summarized as:

1- Design a family of unitary codewords (X_t) with full diversity, the same as the design problem from differential space-time code.

2- Designs relay unitary matrices (A_i):a new problem of distributed space-time code design.

3- Make every X_t commutate with every A_i:a design problem for DDSTC.

Distributed differential space-time coding is 3dB worse than distributed coherent space-time coding. To make it clear, a brief explanation for Alamouti code will be present, also real orthogonal code, *Sp* (2), circulant and cyclic codes are shown with their performances.

1) *Alamouti code [42]:*

Suppose that we have two blocks *(t-1)*-th and *t*-th, each one have two symbols (T=2), during the *(t-1)*-th block the transmitter sends,

$$s_{t-1} = \begin{bmatrix} s_{1,t-1} \\ s_{2,t-1} \end{bmatrix} \tag{4.8}$$

The received signal at destination can be written as,

$$y_{t-1} = \sqrt{\frac{P_1P_2T}{P_1+1}}\left[A_1 s_{t-1} \ \ B_2 s_{t-1}^*\right] H_{t-1} + W_{t-1} \tag{4.9}$$

Any massage encoded into a unitary matrix with alamouti structure, that is data are chosen from the set,

$$\mathcal{U} = \left\{ \frac{1}{\sqrt{|x_1|^2 + |x_2|^2}} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \middle| x_1 \in F_1, x_2 \in F_2. \right. \quad (4.10)$$

where F_1 and F_2 are some finite set, for example PSK or QAM modulation. In the next block, *t*-th block, the transmitter sends,

$$s_t = \sqrt{P_1T}\, X_t s_{t-1} \tag{4.11}$$

Which mean the data vector $X_t \in \mathcal{U}$ is encoded differentially before sending, also the normalization of s_t is satisfied, the received signal can be written as,

$$\begin{aligned} y_t &= \sqrt{\frac{P_1P_2T}{P_1+1}}\left[A_1 s_t \;\; B_2 s_t^*\right]H_t + W_t \\ &= \sqrt{\frac{P_1P_2T}{P_1+1}}\left[A_1 X_t s_{t-1} \;\; B_2 X_t^* s_{t-i}^*\right]H_t + W_t. \end{aligned} \tag{4.12}$$

By direct matrix multiplication, the commuting constrain is satisfied as

$$\begin{aligned} A_1X &= XA_1 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ &\mathit{and} \\ B_2X^* &= XB_2 \Rightarrow \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1^* & -x_2 \\ x_2^* & x_1 \end{bmatrix} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned} \tag{4.13}$$

Therefore, the received signal can be rewritten as

$$\begin{aligned} y_t &= \sqrt{\frac{P_1P_2T}{P_1+1}}X_t\left[A_1 s_{t-1} \;\; B_2 s_{t-1}^*\right]H_t + W_t \\ &= \sqrt{\frac{P_1P_2T}{P_1+1}}X_t S_{t-1} H_t + W_t \end{aligned} \tag{4.14}$$

If the channels f_i and g_i keep constant for two block, $H_t = H_{t-1}$, we have,

$$\begin{aligned} y_t &= X_t\left(y_{t-1} - W_{t-1}\right) + W_t \\ &= X_t y_{t-1} + W_t^{'} \end{aligned} \tag{4.15}$$

where

$$W_t^{'} = W_t - X_t W_{t-1}$$

The massage can be decode using the following maximum likelihood detector,

$$\arg\max_{x_1,x_2} \left\| y_t - X_t y_{t-1} \right\|_F^2 \tag{4.16}$$

This decoding does not need any channel information. The information symbols u_1 and u_2 can be decoupled at the receiver. Thus, the decoding complexity is linear having low decoding complexity. We notice that the distributed space-time codewords formed at the receiver have the same Alamouti structure.

Figure (4.2) represents DDSTC for two wireless relay networks, block error rate plotted against total transmitted power. Information symbols are chosen from BPSK and QPSK. Therefore, the transmission rates are 0.5 and 1 bit per channel use, respectively, BPSK has better performance than QPSK but it has lower rate. We can see that diversity two is achieved at high transmit powers. Compared with the corresponding coherent scheme, the differential scheme is about 3dB worse with the advantage of no need for channel state information.

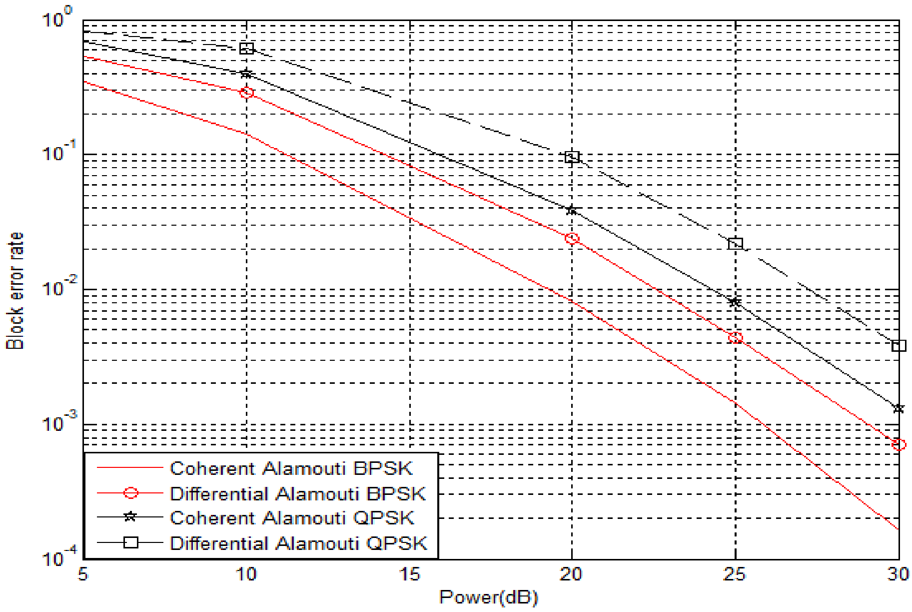

Figure (4.2): DDSTC for two relay network

2) *Square real orthogonal code [42]:*

In chapter two, we represented a square real orthogonal code for two, four and eight symbols. These codes are characterized by low decoding complexity and full diversity. Below, these codes are rewritten again and applied to DDSTC. Since all constellations are real, we have to design A_i matrices only, B_i equals zero always. For wireless network with two relay the code matrix is,

$$\begin{bmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{bmatrix} \tag{4.17}$$

This matrix is commutate with the following relay matrices,

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad and \quad A_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{4.18}$$

For network with four relays, the real orthogonal code is

$$\begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \tag{4.19}$$

and the corresponding relay matrices are

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.20}$$

The eight real orthogonal code and their relay matrices are shown in the next page. The commutating constrain are still held even if the constellations are complex, but if the data matrices are used, the code will not be unitary anymore. The same as Alamouti case, the distributed space-time codewords generated at the receiver have the same square real orthogonal structure as the data matrices.

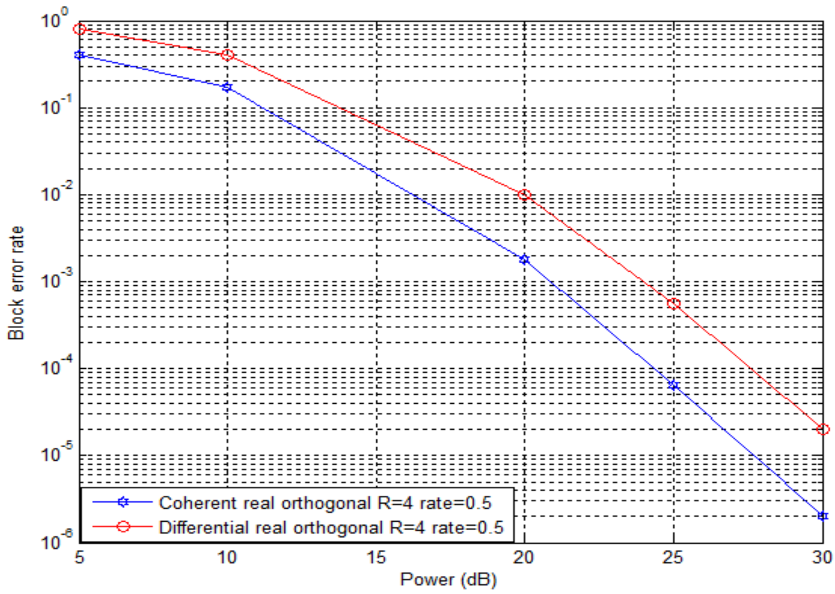

Figure (4.3): DDSTC for four real orthogonal code

In Figure (4.3), we show the performance of a wireless relay network with four relays using two 4×4 real orthogonal codes, the coherent and the differential curves are plotted using BPSK. Appling equation (2.8), the bit rate is 0.5 bit/s/Hz.

$$\begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 & -x_5 & -x_6 & -x_7 & -x_8 \\ x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_3 & x_4 & x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_4 & -x_3 & x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ x_5 & x_6 & x_7 & x_8 & x_1 & -x_2 & -x_3 & -x_4 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & x_1 & x_4 & -x_3 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & x_1 & x_2 \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & x_1 \end{bmatrix} \tag{4.21}$$

where the relay matrices are,

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, A_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, A_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3) Sp (2) code [42]:

Sp (2) code is considered as an extension of Alamouti code to dimension four. Its symbol rate is one, and it can be thought as a special kind of Quasi-orthogonal space-time code, and have the following format,

$$\mathcal{U} = \left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} V_1 V_2 & V_1 \overline{V}_2 \\ -\overline{V}_1 V_2 & \overline{V}_1 \overline{V}_2 \end{bmatrix} \right\}, \tag{4.22}$$

where

$$V_i = \frac{1}{\sqrt{|a_i|^2 + |b_i|^2}} \begin{bmatrix} a_i & b_i \\ -\overline{b}_i & \overline{a}_i \end{bmatrix}, \; a_i \in f_i, b_i \in g_i, \; for \;\; i = 1, 2$$

where f_i and g_i are any arbitrary constellation, real or complex. Author in [46], introduced condition for Sp (2) code to be full diverse with PSK signal by defining

$$\begin{aligned} x_1 &= \frac{a_1 a_2 - b_1 \overline{b}_2}{\sqrt{2} \prod_{i=1}^{2} \sqrt{|a_i|^2 + |b_i|^2}}, \\ x_2 &= -\frac{\overline{a}_1 \overline{b}_2 + \overline{b}_1 a_2}{\sqrt{2} \prod_{i=1}^{2} \sqrt{|a_i|^2 + |b_i|^2}}, \\ x_3 &= -\frac{\overline{a}_1 a_2 - \overline{b}_1 \overline{b}_2}{\sqrt{2} \prod_{i=1}^{2} \sqrt{|a_i|^2 + |b_i|^2}}, \\ &and \\ x_4 &= \frac{a_1 \overline{b}_2 + b_1 a_2}{\sqrt{2} \prod_{i=1}^{2} \sqrt{|a_i|^2 + |b_i|^2}}, \end{aligned} \tag{4.23}$$

The matrix can be rewritten as,

$$\begin{bmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{bmatrix} \tag{4.24}$$

The relationship between all the code elements makes it unitary. Therefore, the unitary condition is satisfied. The relay matrices are presented in equation (4.25), and by direct multiplication, it can be verified that these matrices satisfied the commutating constrain.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_1 = A_2 = A_3 = B_4 = 0 \tag{4.25}$$

$$B_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Decoding of *Sp* (2) code can be done by a sphere decoder or pairwisely [46]. The distributed space-time codewords formed at the receiver also have the quasi-orthogonal structure of the data matrix.

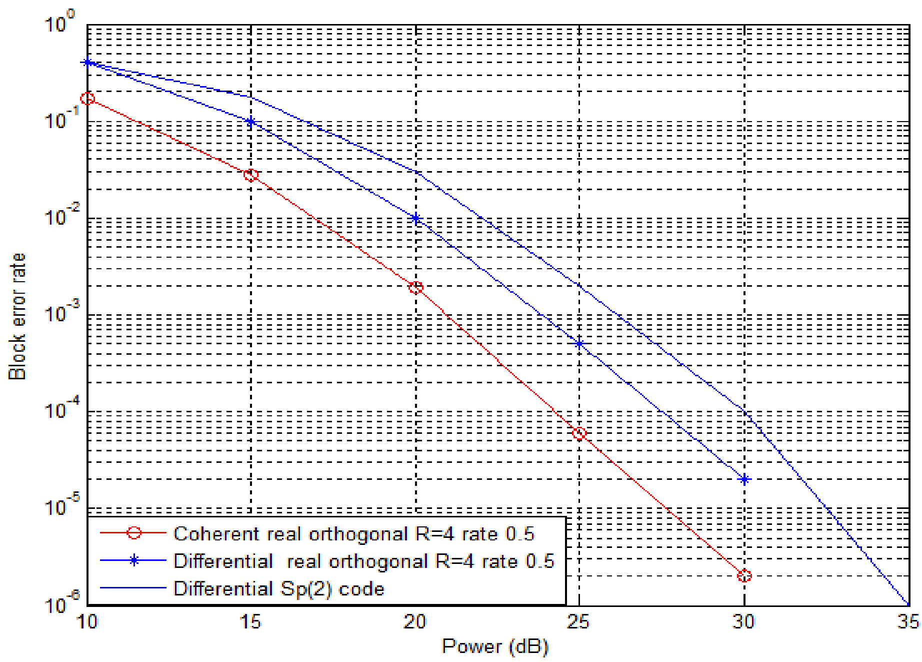

Figure (4.4): DDSTC for Sp (2) code

According to paper [46], to achieve full diversity the element of Sp(2) must be chosen for *L*-PSK, where *L* is a prime number, so the elements of Sp(2) code, a_1, b_1 are chosen as BPSK and a_2, b_2 are chosen as 3-PSK. The bit rate of the network is 0.6462 bit/s/Hz. As seen in figure (4.3), the differential Sp|(2) code is worse than differential real orthogonal space-time code.

4) Circulant code [42]:

The previously mentioned codes are restricted to two, four and eight relay networks. So there is a need for code valid for any number of relay. To do this the commutating condition must be beaten, data matrix must commutate with the relay matrices. Commuting matrix theory employ to solve such a problem, which state that a matrix B commutes with all matrices that commute with A if and only if B is a polynomial of A [47]. Therefore, the relay matrices can be design as $A^i = A^{i-1}$, where A is R-th primitive root of $T\times T$ identity matrix. Matrix A will have the following design:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{4.26}$$

and the matrix relay at the *i*-th relay found as,

$$A_i = A^{i-1} \tag{4.27}$$

Any matrix commutes with A will also commutes with the set $\{A_1, \ldots, A_R\}$. The data set u that commutes with A matrix if it has circulant form as,

$$\begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_R \\ x_R & x_1 & x_2 & \cdots & x_{R-1} \\ x_{R-1} & x_R & x_1 & \cdots & x_{R-2} \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ x_2 & x_3 & x_4 & \cdots & x_1 \end{bmatrix} \tag{4.28}$$

The commutating problem is solved, but we have another problem that is the circulant matrix is not unitary. To make it unitary, we use the following set,

$$u = \{x_1A_1, x_2A_2, \ldots, x_RA_R \mid x_i \in K_i, i = 1,2,\ldots,R.\} \tag{4.29}$$

where K_i is an arbitrary finite set with unit-norm elements. To make this clear, we give an example for a network with three relays. The data set and relay matrices are shown as follow,

The data sets are,

$$u = \left\{ \begin{bmatrix} x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & x_1 \end{bmatrix}, \begin{bmatrix} 0 & x_2 & 0 \\ 0 & 0 & x_2 \\ u_2 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & x_3 \\ x_3 & 0 & 0 \\ 0 & x_3 & 0 \end{bmatrix} \middle| x_i \in K_i, i = 1,2,3. \right\} \quad (4.30)$$

The three relay matrices are,

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \; and \; A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (4.31)$$

The discussion of the diversity product of circulant codes with K_i chosen as M-PSK rotated by an angle θ_i, found in [42]. Some results are given in table (4.1). The first angle θ is always equal zero.

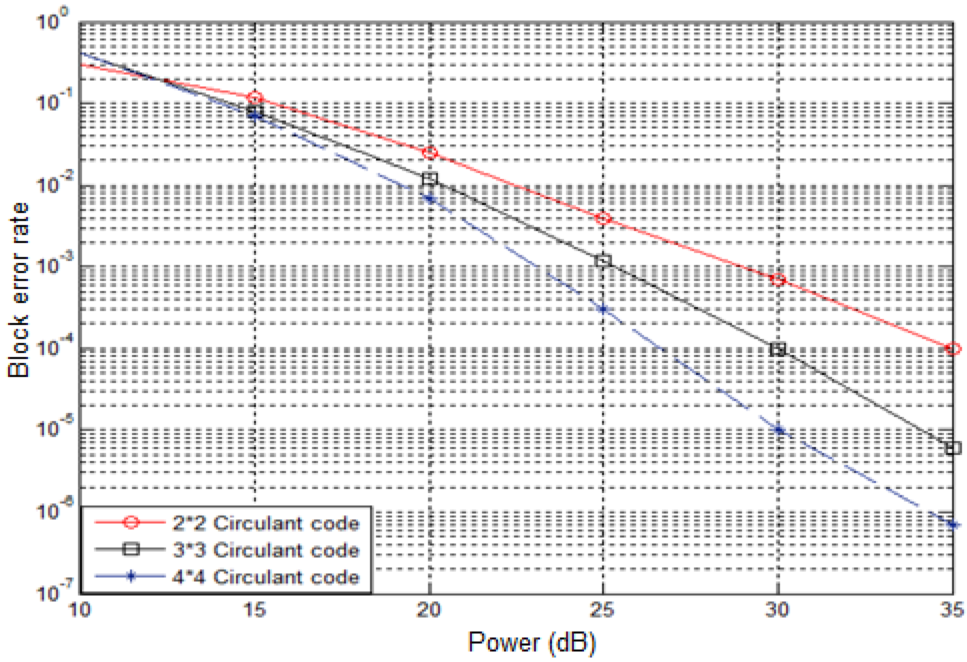

Figure (4.5): Performance of circulant code for R=2, 3 and 4

Figure (4.5) illustrate the performance of circulant codes in networks with three different values of relays. For networks with two and three relays, the bit rates are 0.5 and 0.4308 respectively, and the information symbols are modulated as rotated BPSK. While for the network with four relays, the bit rate 0.3322, and the information symbols are modulated as rotated QPSK. From the figure we seen that the differential space time code are effective at high SNR which is mentioned in [20].

# Relay	Modulation	Optimal angle (Θ_1=0)	Diversity product
R=2	BPSK	Θ_2=π/2	0.7071
	QPSK	Θ_2= π/4	0.5930
R=3	BPSK	Θ_2= π/9, Θ_3= 2π/9	0.4992
	QPSK	Θ_2= π/9, Θ_3= 2π/9	0.4008
R=4	BPSK	Θ_2= π/7, Θ_3= π/2, Θ_4= 5π/14	0.5572
	QPSK	Θ_2= π/8, Θ_3= π/4, Θ_4= 3π/8	0.5445
R=5	BPSK	Θ_2= π/25, Θ_3= 3π/25,Θ_4= 7π/25,Θ_5=π/25	0.4513
	QPSK	Θ_2= 3π/50,Θ_3= 6/50,Θ_4=14π/50,Θ_5=17π/25	0.3949

Table (4.1): Diversity product for circulant code

5) *Cyclic code :*

In [44] a coding strategy for wireless networks with no channel information pioneered, this DDSTC can be used for any number of relays, it is inspired by unitary differential modulation of [17, 18], which is based mainly on cyclic codes. These codes are discussed in chapter two and will be summarized below. Additionally, the relay matrices which are constructed using Generalized Butson–Hadamard (GBH), will be explained. Cyclic codes are diagonal unitary code, it was designed for MIMO system, and the codebook has the form,

$$X_l = V_1^{\,l} \qquad , l = 0,1,\dots,L-1 \tag{4.32}$$

where

$$V_1 = \begin{bmatrix} e^{j(2\pi/L)u_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & e^{j(2\pi/L)u_R} \end{bmatrix}$$

where $u_m \in \{0,\dots,L-1\}; \quad m = 1,\dots,R$

To achieve full diversity parameters L and u must be optimized. Table (2.1) summarizes some good cyclic codes, in that table M_t represent the number of relays. To satisfy the commutating constrain, the $T\times T$ relay matrices should be design to be diagonal and unitary. As X_l are diagonal unitary matrices, such as matrices can be generated through using *A Generalized Butson–Hadamard (GBH). GBH matrices* are $T\times T$ matrices with coefficients in a ring [48]. For this work, the GBH matrices coefficients are chosen to be roots of unity. Such that the inverse of M equals the conjugate,

$$MM^{*} = M^{*}M = TI_T \tag{4.33}$$

Then the relay matrices A_i are chosen to be

$$A_i = diag(M_i), \quad i = 1,....,R \tag{4.34}$$

where M_i indicate the column of M, the following example will make this clear. For wireless network with three relays, let $\zeta_3 = \exp(2i\pi/3)$ a primitive third root of unity. Then we will have

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \zeta_3 & \zeta_3^2 \\ 1 & \zeta_3^2 & \zeta_3 \end{pmatrix} \Rightarrow M^{*}M = 3I_3 \tag{4.35}$$

The unitary relay matrices can be written as,

$$\begin{aligned} A_1 &= diag\left(M_1\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ A_2 &= diag\left(M_2\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta_3 & 0 \\ 0 & 0 & \zeta_3^2 \end{pmatrix} and \\ A_3 &= diag\left(M_3\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta_3^2 & 0 \\ 0 & 0 & \zeta_3 \end{pmatrix} \end{aligned} \tag{4.36}$$

The tensor product of two GBH matrices also GBH matrix, so for nine relay network we can use the following GBH

$$M \otimes M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \zeta_3 & \zeta_3^2 \\ 1 & \zeta_3^2 & \zeta_3 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 & 1 \\ 1 & \zeta_3 & \zeta_3^2 \\ 1 & \zeta_3^2 & \zeta_3 \end{pmatrix}$$

Unlike the previous DDSTC, the relay matrices A_i can be used for any diagonal unitary code X_l, the design of A_i and X_l are independent.

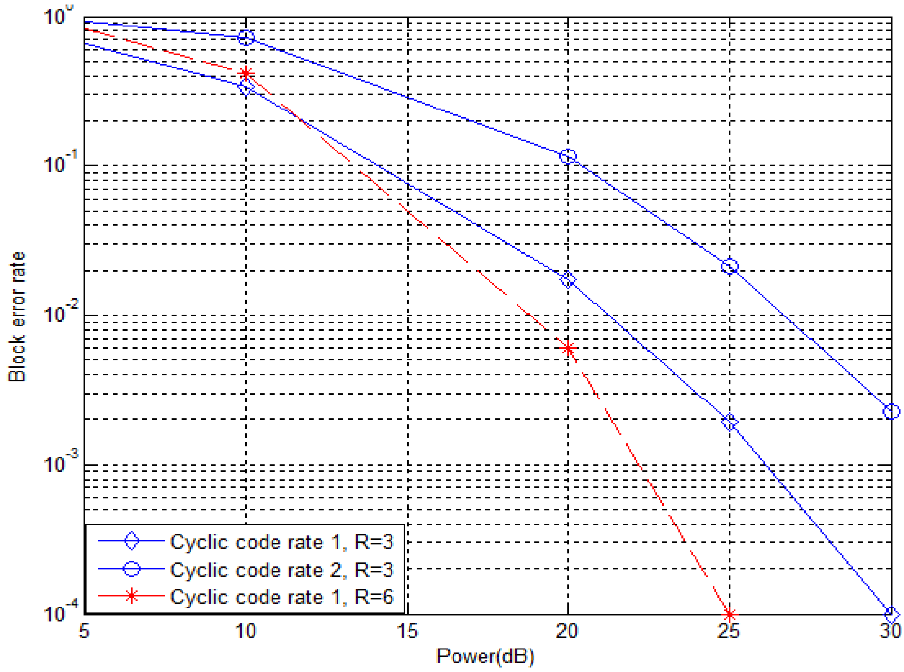

Figure (4.5): Performance of cyclic code for R= 3 and 6

The performance in terms of block error rate for cyclic code is shown in figure (4.5) for three and six wireless networks; the bit rates for three relay network are 0.5 and 1, whereas the bit rate for six relay network is 0.5 bpcu.

4.3 DDSTC with low decoding complexity [45]:

Construction of DDSTC for any number of relays using cyclic codes or circulant codes become difficult, considering full diversity and decoding complexity, especially for a large number of relays. Thus, a general code characterize with full rate, full diversity and low decoding complexity are needed, the author of [45] present a new class of DDSTC satisfying these conditions for power of two number of relays, also these codes can be used in differential space-time code in MIMO systems. The same system model of DDSTC explained earlier will be used with a slight modification. As DDSTC uses unitary codebooks which extended to be scaled unitary codebooks, the transmitted signal become,

$$s_t = \frac{\sqrt{P_1 T}\, X_t s_{t-1}}{a_{t-1}} \tag{4.37}$$

where, X_t is any codeword containing the information at the *t*-th block, which satisfies $X_t^H X_t = a_t^2 I_T$, $a_t \in R$. In the previous work a_t is forced to equal one for all codewords. The received signal at destination will be,

$$y_t = \frac{1}{a_{t-1}} X_t y_{t-1} + W_t' \tag{4.38}$$

and the decoding metric become

$$\arg\max_{x_t} \left\| y_t - \frac{1}{a_{t-1}} X_t y_{t-1} \right\|^2 \tag{4.39}$$

where a_{t-1} is estimated from the decision at the *(t-1)*-th block, the use of scaled unitary matrices provides an opportunity to low encoding/decoding complexity. Linear designs space-time block codes, discussed in chapter two, have been used to construct code with low decoding complexity. Author constructed a linear designs code with four-group decodable/encodable using extended Clifford's algebras. These space-time block codes are not orthogonal and do not achieve full diversity if an arbitrary signal set is used. Hence, a signal set must be created to mitigate these problems. In the following, section we will present code construction and the design of signal set.

4.3.1 Construction of 4-group linear design:

In [49] a linear design distributed coherent space time code obtained using extended Clifford's algebras. This code has a rate of one and four group decodable. To create such linear design code for $R = 2^{\lambda},\ \lambda = 2,3,\ldots$ we follow the two steps below:

1. *ABBA construction*: ABBA linear design *D* have the following form:

$$D = \begin{bmatrix} A\left(x_1, x_2, \ldots, x_L\right) & B\left(x_{L+1}, x_{L+2}, \ldots, x_{2L}\right) \\ B\left(x_{L+1}, x_{L+2}, \ldots, x_{2L}\right) & A\left(x_1, x_2, \ldots, x_L\right) \end{bmatrix} \tag{4.40}$$

where D is a $2n\times 2n$ matrix, A is $n\times n$ matrix with L complex variables, and B is the same as A but with new label. We can start by one complex variable x_1, keep applying ABBA construction until a $2^{\lambda-1}\times\ 2^{\lambda-1}$ linear design D is obtained. The ABBA construction for R = 4 is,

$$D=\begin{bmatrix} x_1 & x_2 \\ x_2 & x_1 \end{bmatrix} \tag{4.41}$$

2. *Doubling construction*: now we apply doubling construction on D and multiply it by $1/\sqrt{R}$ to obtain the linear design S, where S is a $2n\times 2n$ linear design have the following form:

$$S=\begin{bmatrix} A\left(x_1,x_2,\ldots,x_L\right) & -B^H\left(x_{L+1},x_{L+2},\ldots,x_{2L}\right) \\ B\left(x_{L+1},x_{L+2},\ldots,x_{2L}\right) & A^H\left(x_1,x_2,\ldots,x_L\right) \end{bmatrix} \tag{4.42}$$

A is $n\times n$ matrix with L complex variables, and B is the same as A but with new label. The doubling construction for R = 4 is,

$$S=\frac{1}{\sqrt{4}}\begin{bmatrix} x_1 & x_2 & -x_3^* & -x_4^* \\ x_2 & x_1 & -x_4^* & -x_3^* \\ x_3 & x_4 & x_1^* & x_2^* \\ x_4 & x_3 & x_2^* & x_1^* \end{bmatrix} \tag{4.43}$$

Wireless network with eight relay, the codewords have the following form,

$$S=\frac{1}{\sqrt{8}}\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & -x_5^* & -x_6^* & -x_7^* & -x_8^* \\ x_2 & x_1 & x_4 & x_3 & -x_6^* & -x_5^* & -x_8^* & -x_7^* \\ x_3 & x_4 & x_1 & x_2 & -x_7^* & -x_8^* & -x_5^* & -x_6^* \\ x_4 & x_3 & x_2 & x_1 & -x_8^* & -x_7^* & -x_6^* & -x_5^* \\ x_5 & x_6 & x_7 & x_8 & x_1^* & x_2^* & x_3^* & x_4^* \\ x_6 & x_5 & x_8 & x_7 & x_2^* & x_1^* & x_4^* & x_3^* \\ x_7 & x_8 & x_5 & x_6 & x_3^* & x_4^* & x_1^* & x_2^* \\ x_8 & x_7 & x_6 & x_5 & x_4^* & x_3^* & x_2^* & x_1^* \end{bmatrix} \tag{4.44}$$

The relay matrices can easily obtained of this construction, equation (4.45) show such matrices for four relays.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$B_1 = B_2 = A_3 = A_4 = 0, \tag{4.45}$$
$$B_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, B_4 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

4.3.2 Construction of signal set:

In this subsection, signal set design conditions for 4 relays will be derived. Then, a generalization constructing signal sets for any $R = 2^{\lambda}$ relays will be done. The design for 4 relays can be rewritten as,

$$S = \frac{1}{\sqrt{4}} \begin{bmatrix} x_{1I} + jx_{1Q} & x_{2I} + jx_{2Q} & -x_{4I} + jx_{4Q} & -x_{4I} + jx_{4Q} \\ x_{2I} + jx_{2Q} & x_{1I} + jx_{1Q} & -x_{4I} + jx_{4Q} & -x_{4I} + jx_{4Q} \\ x_{3I} + jx_{3Q} & x_{4I} + jx_{4Q} & x_{1I} - jx_{1Q} & x_{2I} - jx_{2Q} \\ x_{4I} + jx_{4Q} & x_{3I} + jx_{3Q} & x_{2I} - jx_{2Q} & x_{1I} - jx_{1Q} \end{bmatrix} \tag{4.46}$$

In general, the signal sets for DDSTC with low decoding complexity should be designed to meet the following conditions:

1. *Four-group encodable and Four-group decodable:*

During grouping process care must be taken to the low decoding complexity constrain are satisfied, the weight matrices of the different group, also signal sets must not contain any joint constraints on variables from different groups, that's every group encode/decode separately. The four groups are:

- First group $\{x_{1I}, x_{2I}\}$
- Second group $\{x_{1Q}, x_{2Q}\}$,

- Third group $\{x_{3I}, x_{4I}\}$
- Fourth group $\{x_{3Q}, x_{4Q}\}$.

2. Scaled unitary codeword matrices meeting power constraint.

To satisfy the encode/decode constrain, a clear sight must be taken to equation (4.47), which represent the multiplication of codeword of four relay network with its transposes conjugate, this equation clarified the required conditions for all signal points taken from the signal set.

$$S^H S = \frac{1}{4}\begin{bmatrix} a & b & 0 & 0 \\ b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & b & a \end{bmatrix} \tag{4.47}$$

where

$$a = \sum_{i=1}^{4} |x_i|^2 \ \ and \ \ b = x_1^* x_2 + x_2^* x_1 + x_3^* x_4 + x_4^* x_3$$

The scaled unitary codewords requirement satisfied if element b in equation (4.47) equal zero, so without disturbing 4- group encodability all the signal points must satisfy the conditions,

$$\begin{aligned} x_{1I}x_{2I} &= -x_{1Q}x_{2Q} = c_1, \\ x_{3I}x_{4I} &= -x_{3Q}x_{4Q} = c_2. \end{aligned} \tag{4.48}$$

where, c_1 and c_2 are positive real constants. Then, the average power constraint requirement can be met by satisfying the conditions,

$$\begin{aligned} E(x_{1I}^2 + x_{2I}^2) = 1, E(x_{1Q}^2 + x_{2Q}^2) = 1, \\ E(x_{3I}^2 + x_{4I}^2) = 1, E(x_{3Q}^2 + x_{4Q}^2) = 1. \end{aligned} \tag{4.49}$$

3. Achieving full diversity.

To guarantee full diversity the following conditions must be satisfied (derivation found in [45]):

$$\Delta x_{1I} \neq \pm\Delta x_{2I}\ ,\ \Delta x_{1Q} \neq \pm\Delta x_{2Q1},$$
$$\Delta x_{3I} \neq \pm\Delta x_{4I}\ ,\ \Delta x_{3Q} \neq \pm\Delta x_{4Q2}. \qquad (4.50)$$

To design signal set, three condition need to be met, equations (4.48), (4.49) and (4.50). Condition (4.48) represents hyperbola xy = *c*, while condition (4.49) represents a unit circle $x^2 + y^2 = 1$, the intersection of these two conditions yield four different points, where $c < 1$ on the two dimensional *xy* plane. After enforcing full diversity condition $\Delta x = \pm\Delta y$ only two points remain. The solution can be either the set of points marked *A* or the set of points marked *B*, Thus we have obtained a signal set containing 2 points. This is illustrated in figure (4.6).

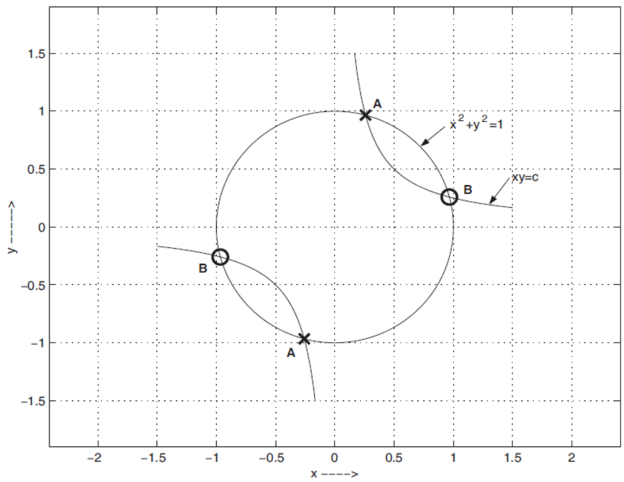

Figure (4.6): Signal set structure in two dimensions [45].

If more than two points needed, more circles (centered at origin) must be drawn and then find those points intersecting with the hyperbola. The radii of the drawn circles must meet the average power constraint. To make this clear, suppose that m points are wanted, *m*/2 circles are drawn with increasing radii $r_1, r_2, ..., r_{m/2}$ such that:

$$\sum_{i=1}^{\frac{m}{2}} r_i^2 = \frac{m}{2}.$$

Then we find those points intersecting with the hyperbola xy = c, to make sure hyperbola intersect all circles c must be a positive number less than r_1^2, considering a different hyperbola figure (4.7) demonstrate the signal set for the variables x_{1I}, x_{2I} and x_{3I}, x_{4I}. The signal set for the variables x_{1Q}, x_{2Q} and x_{3Q}, x_{4Q}.

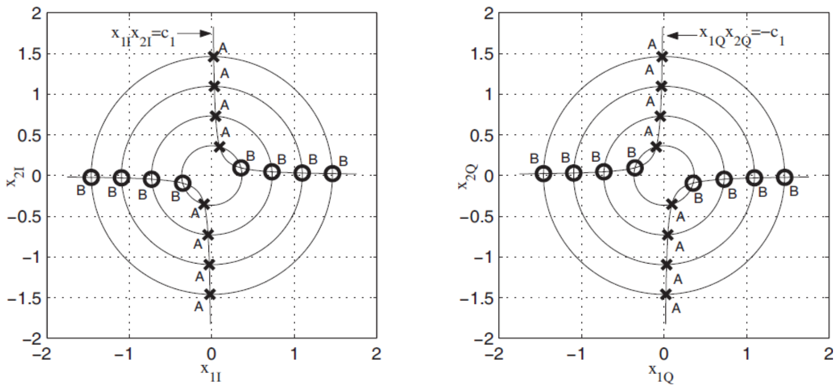

Figure (4.7): General signal set for four relay [45].

Based on the 4 relay signal design conditions, a generalization can be made for higher dimensions, which given in Construction 4.4 in paper [45] as follows,

***Signal set generalization*:** For four group encodable, we need four identical signal set each one contains $\sqrt[4]{Q}$ points, where Q is the total number of required points. The resulting signal set $\subset \mathbb{R}^{2^{\lambda+1}}$ should be a Cartesian product of 4 signal sets in $\mathbb{R}^{2^{\lambda-1}}$. Let the signal points in $\mathbb{R}^{2^{\lambda-1}}$ be labeled as $P_i,\ i=1,\dots,\sqrt[4]{Q}$ If $i=2q+r$ for some integers q and r where $1 \le r \le 2$, then P_i is given by:

$$
\begin{aligned}
&P_i\left[j\right]=0 \quad \forall j \neq (q \bmod 2^{\lambda-1})+1\\
&P_i[(q \bmod 2^{\lambda-1}+1)]=+r_{q+1}, \quad if \quad r=1\\
&P_i[(q \bmod 2^{\lambda-1}+1)]=-r_{q+1}, \quad if \quad r=2
\end{aligned}
\tag{4.51}
$$

where $r_i,\ i=1,\dots,\frac{\sqrt[4]{Q}}{2}$ are positive real numbers such that $r_{i+1} \rangle r_i,\ \forall_i=1,\dots,\frac{\sqrt[4]{Q}}{2}-1$ and $\sum_{i=1}^{\frac{\sqrt[4]{Q}}{2}} r_i^2=\frac{\sqrt[4]{Q}}{2}$.

As an example, a four dimensional signal set for R = 2^3 = 8 and Q = 164. The circles radii will be $r_1 = 0.3235, r_2 = \sqrt{3}r_1, r_5 = 3r_1, r_3 = r_2 + \frac{r_5 - r_2}{3}, r_4 = r_2 + 2\frac{r_5 - r_2}{3}$, $r_6 = (2+\sqrt{3})r_1, r_7 = r_3 + 2r_1$ *and* $r_8 = r_4 + 2r_1$. The signal points are shown in the next page.

$$
\begin{aligned}
&P_1 = [r_1 \quad 0 \quad 0 \quad 0]^T, P_2 = [-r_1 \quad 0 \quad 0 \quad 0]^T, \\
&P_3 = [0 \quad r_2 \quad 0 \quad 0]^T, P_4 = [0 \quad -r_2 \quad 0 \quad 0]^T, \\
&P_5 = [0 \quad 0 \quad r_3 \quad 0]^T, P_6 = [0 \quad 0 \quad -r_3 \quad 0]^T, \\
&P_7 = [0 \quad 0 \quad 0 \quad r_4]^T, P_8 = [0 \quad 0 \quad 0 \quad -r_4]^T, \\
&P_9 = [r_5 \quad 0 \quad 0 \quad 0]^T, P_{10} = [-r_5 \quad 0 \quad 0 \quad 0]^T, \\
&P_{11} = [0 \quad r_6 \quad 0 \quad 0]^T, P_{12} = [0 \quad -r_6 \quad 0 \quad 0]^T, \\
&P_{13} = [0 \quad 0 \quad r_7 \quad 0]^T, P_{14} = [0 \quad 0 \quad -r_7 \quad 0]^T, \\
&P_{15} = [0 \quad 0 \quad 0 \quad r_8]^T, P_{16} = [0 \quad 0 \quad 0 \quad -r_8]^T
\end{aligned}
$$

The two dimensional projections of the signal points is graphically shown as,

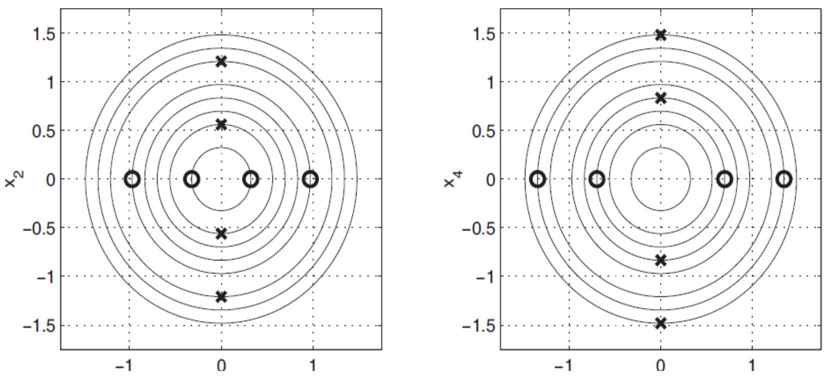

Figure (4.8): Four signal set for eight relay [45]

The performance of the proposed code for four relay network is shown in figure (4.9), where the value of $r_1 = 1/\sqrt{3}$ *and* $r_1 = \sqrt{5/3}$ for rate 1.

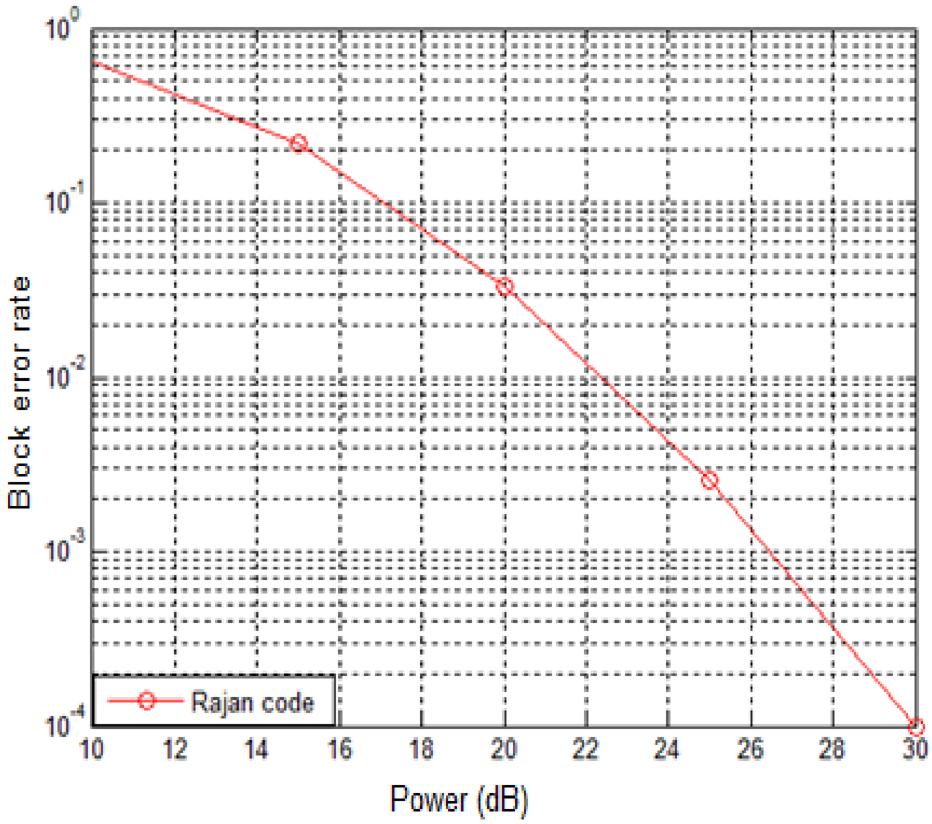

Figure (4.9): DDSTC for 4-group space-time code

4.4 New DDSTC based on circulant codes:

Low decoding complexity issue becomes importance, especially if the number of cooperating terminals are large, which is expected in applications such as wireless sensor networks. Although the four-group decodable DDSTC satisfies the low decoding complexity demand, a limitation of this method is that it is available only for power of two number of relays. Circulant code can be used to defeat this limitation, which allow to construct STBCs for relay network equal double of circulant code, such code have rate one and called semi-orthogonal algebraic space-time (SAST) codes which have been proposed in [50]. The SAST codes allow decoding the transmitted symbols into two- groups. In this section, we will see how to implement these codes and applying them to cooperative network to get DDSTC.

4.4.1 Code construction:

The circulant code mentioned in section (4.2) is rewritten in equation (4.52), the semi-orthogonal algebraic space-time (SAST) code matrix is constructed using two circulant code each of length L to be employing in networks of length $2L$ relay.

$$A = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_L \\ x_L & x_1 & x_2 & \cdots & x_{L-1} \\ x_{L-1} & x_L & x_1 & \cdots & x_{L-2} \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ x_2 & x_3 & x_4 & \cdots & x_1 \end{bmatrix} \tag{4.52}$$

The SAST code has construction from as

$$S = \begin{bmatrix} A\left(x_1, x_2, \ldots, x_L\right) & -B^H\left(x_{L+1}, x_{L+2}, \ldots, x_{2L}\right) \\ B\left(x_{L+1}, x_{L+2}, \ldots, x_{2L}\right) & A^H\left(x_1, x_2, \ldots, x_L\right) \end{bmatrix} \tag{4.53}$$

For example, the SAST code for 4, 6 and 8 transmit antennas is

$$S = \begin{bmatrix} x_1 & x_2 & -x_3^* & -x_4^* \\ x_2 & x_1 & -x_4^* & -x_3^* \\ x_3 & x_4 & x_1^* & x_2^* \\ x_4 & x_3 & x_2^* & x_1^* \end{bmatrix} \tag{4.54}$$

$$S = \begin{bmatrix} x_1 & x_2 & x_3 & -x_4^* & -x_6^* & -x_5^* \\ x_3 & x_1 & x_2 & -x_5^* & -x_4^* & -x_6^* \\ x_2 & x_3 & x_1 & -x_6^* & -x_5^* & -x_4^* \\ x_4 & x_5 & x_6 & x_1^* & x_3^* & x_2^* \\ x_6 & x_4 & x_5 & x_2^* & x_1^* & x_3^* \\ x_5 & x_6 & x_4 & x_3^* & x_2^* & x_1^* \end{bmatrix}$$

$$S = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & -x_5^* & -x_8^* & -x_7^* & -x_6^* \\ x_4 & x_1 & x_2 & x_3 & -x_6^* & -x_5^* & -x_8^* & -x_7^* \\ x_3 & x_4 & x_1 & x_2 & -x_7^* & -x_6^* & -x_5^* & -x_8^* \\ x_2 & x_3 & x_4 & x_1 & -x_8^* & -x_7^* & -x_6^* & -x_5^* \\ x_5 & x_6 & x_7 & x_8 & x_1^* & x_4^* & x_3^* & x_2^* \\ x_8 & x_5 & x_6 & x_7 & x_2^* & x_1^* & x_4^* & x_3^* \\ x_7 & x_8 & x_5 & x_6 & x_3^* & x_2^* & x_1^* & x_4^* \\ x_6 & x_7 & x_8 & x_5 & x_4^* & x_3^* & x_2^* & x_1^* \end{bmatrix}$$

These codes have unitary relay matrices A_i , and by direct multiplication we can see that any code matrix commutate with all relay matrices A_i , so two out of the three conditions that require to construct DDSTC are satisfied. The next section demonstrates how these codes can be made unitary.

4.4.2 Signal set design:

All the previous space-time codes used in DDSTC are unitary also this code should be unitary too. To make this happen, the idea of joint modulation mentioned in [59] will be used in this section and extension to more than two symbols will be easy. We will take two examples of wireless network,wireless network with four and six relays.

Case 1: Four relays networks

We can start by compute $S_4^H S_4$ as follows,

$$S^H S = \begin{bmatrix} a & b & 0 & 0 \\ b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & b & a \end{bmatrix} \tag{4.55}$$

where

$$a = \sum_{i=1}^{4} |x_i|^2 \ \ and \ \ b = x_1^* x_2 + x_2^* x_1 + x_3^* x_4 + x_4^* x_3$$

Equation (4.55) is unitary if and only if $a=1$ and $b=0$, so some constrains are required between element of first group, symbols X_1 and X_2 , and between element of second group, symbols X_3 and X_4 , and no joint constrain between these two-groups are required in order to obtain two group decodable. Authors of [59] suggest that we have a joint constellation set M consist of L complex- valued constellation pair $\{a_k, b_k\}$ where $1 \le k \le L$. Every constellation pair is mapped to one group, the proposed constellation M has the form,

$$\begin{cases} a_k = \exp[j(2k\pi / M)] / \sqrt{2} \\ b_k = 0 \end{cases} \qquad for\ 1 \le k \le \frac{L}{2}$$
$$\begin{cases} a_k = 0 \\ b_k = \exp[j(2(k - L/2)\pi / M + \theta)] / \sqrt{2} \end{cases} \qquad for\ \frac{L}{2} \le k \le L \tag{4.56}$$

where $M = L/2$ is an integer and θ is a constellation rotation angle between 0 and $2\pi / M$. Applying this constellation set to SAST code will satisfy the unitary condition of DDSTC. To achieve the full diversity and the maximum coding gain, the rotation angle θ equal π / M if M is even and equal to $\pi / 2M$ or $3\pi / 2M$ if M is odd, the prove can be found in [59]. Figure (4.10) shows the performance of SAST code combined the signal set of [59], the rate is one bit per channel use (bpcu) that's mean the constellation set have 16 pairs.

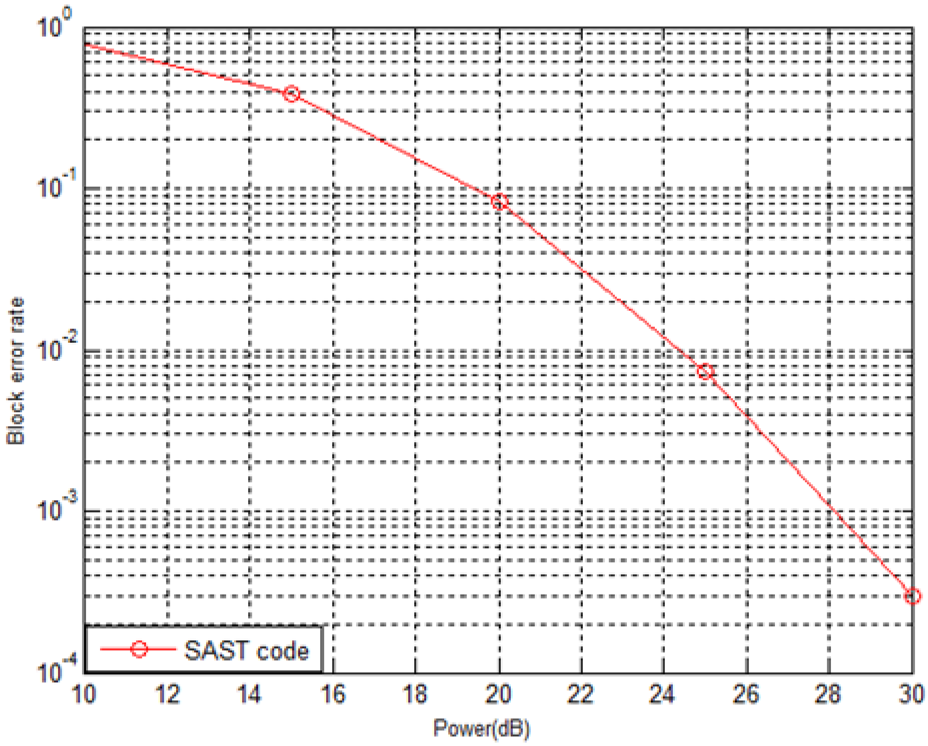

Figure (4.10): DDSTC based on circulant code for 4 relay

Case 2: Six relays networks

$$S_6{}^H S_6 = \begin{bmatrix} a & b & c & 0 & 0 & 0 \\ c & a & b & 0 & 0 & 0 \\ b & c & a & 0 & 0 & 0 \\ 0 & 0 & 0 & a & b & c \\ 0 & 0 & 0 & c & a & b \\ 0 & 0 & 0 & b & c & a \end{bmatrix} \tag{4.57}$$

where

$$\begin{aligned} a &= \sum_{i=1}^{6} |x_i|^2 , \\ b &= x_1^* x_2 + x_3^* x_1 + x_2^* x_3 + x_4^* x_5 + x_6^* x_4 + x_5^* x_6 \\ c &= x_2^* x_1 + x_1^* x_3 + x_3^* x_2 + x_5^* x_4 + x_4^* x_6 + x_6^* x_5 \end{aligned}$$

The same as four relays network, let the first group contains symbols X_1, X_2 and X_3 and the second group contians symbols X_4, X_5 and X_6. We have M joint constellation sets consist of L complex- valued constellation groups $\{a_k, b_k, c_k\}$ where $1 \leq k \leq L$. Every constellation group is mapped to one symbol group. Figure (4.11) shows the performance of SAST code. the rate is 0.5 bit per channel use (bpcu) that's mean the constellation set have 8 groups. The proposed constellation has the form,

$$\begin{aligned} &\begin{cases} a_k = \exp[j(2k\pi/3)]/\sqrt{2} \\ b_k = c_k = 0 \end{cases} && \text{for } 1 \leq k \leq 3 \\ &\begin{cases} b_k = \exp[j(2k\pi/3 + \theta_1)]/\sqrt{2} \\ a_k = c_k = 0 \end{cases} && \text{for } 4 \leq k \leq 6 \\ &\begin{cases} c_k = \exp[j(2k\pi/2 + \theta_2)]/\sqrt{2} \\ a_k = b_k = 0 \end{cases} && \text{for } 7 \leq k \leq 8 \end{aligned} \tag{4.58}$$

where θ_1 and θ_2 is a constellation rotation angle have value $2\pi/3$ and $\pi/2$, respectivily . Applying this constellation set to SAST code will satisfy the unitary condition of DDSTC.

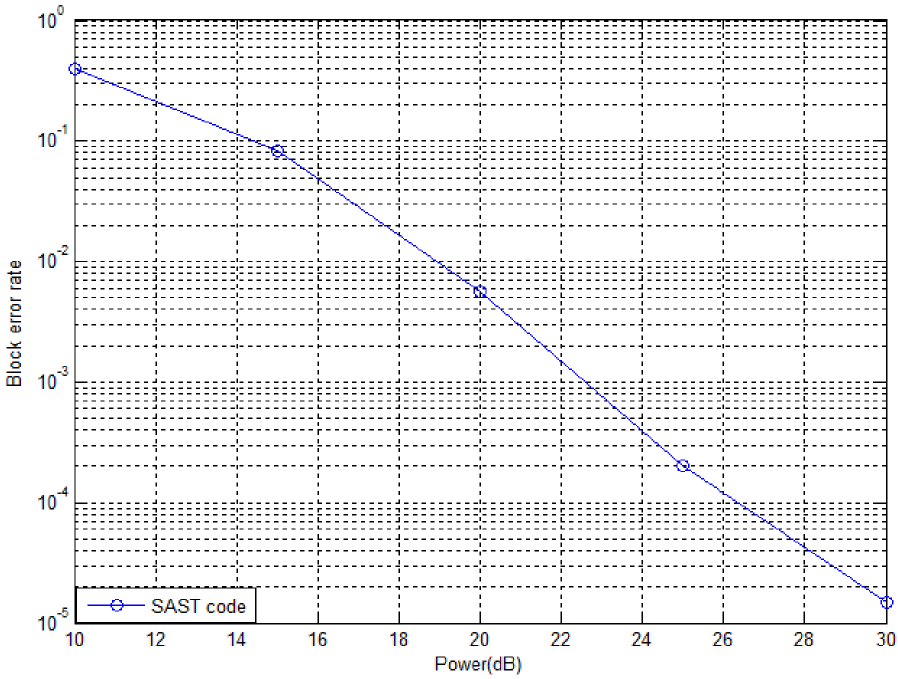

Figure (4.11): DDSTC based on circulant code for 6 relay

4.5 Comparison:

We now compare the performance of all the mentioned DDSTC against each other, all curves are plotted in terms of block error rate. It is noticed that among all these codes, the algebraic distributed differential space-time code with low decoding complexity presented by Rajan outperforms all other codes both in error performance as well as in encoding and decoding complexity. The low decoding complexity advantages come from relaxing the unitary matrix codebook to scaled unitary matrix codebook. But this code works only for power of two number of relays, I get around this limitation by using SAST codes which perform no more than 1.5 dB worse. But work for multiple of two number of relays. however its decoding complexity increase by two to become 2-group decoding instead of 4-group. Figure (4.11) compares the block error rate (BLER) performance of these codes for four relay network, which can be arranged from better to worse as follows:

- Rajan code
- SAST code
- Cyclic code

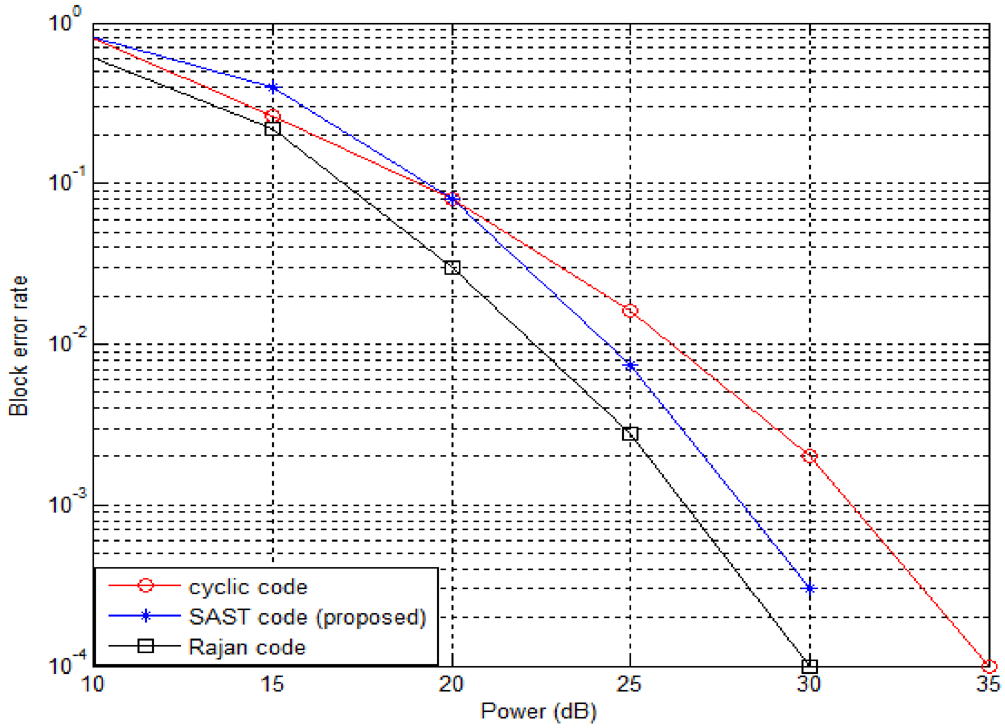

Figure (4.12): Performance of different DDSTC for 4 relay network

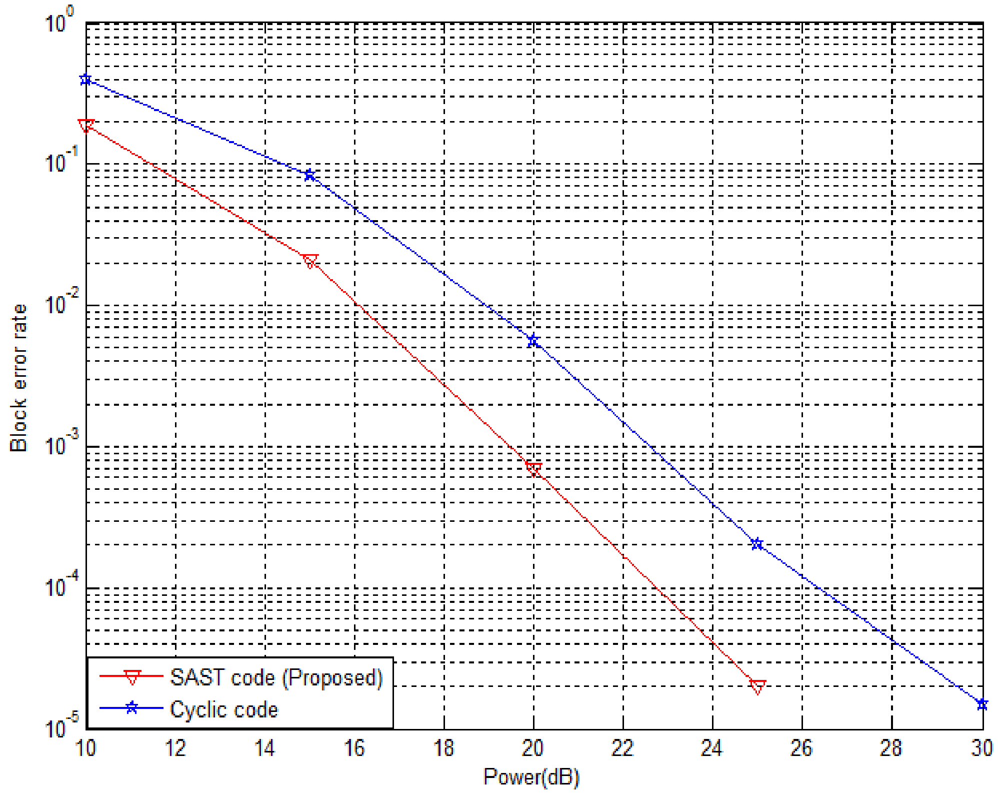

Figure (4.13): Performance of different DDSTC for 6 relay network

DDSTC	Relay number	Constellation	ML search space	Group number
Alamouti	2	BPSK	4	1
Circulant	2	BPSK	4	1
Circulant	3	BPSK	6	1
Cyclic	3	Eq.(4.32)	63	1
Real orthogonal	4	BPSK	16	1
Sp(2)	4	BPSK & 3-PSK	24	1
Circulant	4	QPSK	256	1
Cyclic	4	Eq.(4.32)	256	1
Rajan code	4	Eq.(4.51)	4	4
SAST	4	Eq.(4.56)	8	2

Table (4.2): Comparison of the decoding complexity for different DDSTC

Table (4.2) clarifies the decoding complexity of all DDSTC for different number of relays. It can be noted that these codes have only one ML decoder works at a time except the last two codes, which have four and two parallel ML decoders working at the same time. These parallel decoders reduce the search space of each ML decode, and so reduce the overall complexity. In the other codes, the searching space increase exponentially while the transmission rate increase, which mean its need large time to compute the error performance for high rate. However, the Alamouti and real orthogonal codes have linear growth search space because of their orthogonality.

Chapter 5

Conclusion and Future Work

5.1 Conclusion:

The goal of this research was to develop a differential transmission scheme for cooperative networks with low decoding complexity, using distributed differential space-time cod. To achieve this goal many topics dealing with cooperative diversity and space-time code have been explored. At the beginning, space-time codes and MIMO systems have been studied, then a study of the main concept of cooperative networks and the application of space-time code into it has been done. Finally, many distributed differential space-time codes are investigated and studied such as Alamouti, square real orthogonal code, Sp (2) code, circulant, cyclic code and 4-group decodable DDSTC based on extended Clifford's algebra were investigated. Theoretical analysis and numerical simulation have showen that compared with the corresponding coherent scheme, distributed differential space-time coding performs 3dB worse. The performances of these codes was compared with a new DDSTC based on circulant code with 2-group decodable. Which appeared that the new code outperformed the cyclic code by 3dB and Rajan code is 1.5dB better than the new code.

5.2 Future Work:

There are several possible future works on this as follows:

- Designing single-symbol decodable differential codes, if it's possible, for networks with any number of relays.

- In order to reduce the 3-dB gap, decoding more than one block at the same time can be considered as a future work
- Studying DDSTC when the relays are allowed to co-operate with each other before sending to the destination.
- DDSTC can be studied with other cooperative protocols such as relay selection protocol.
- Our work suggests that all relay transmit at the same time in the second phase, so we can study DDSTC when synchronization is not assumed among all the relays.

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