The Islamic University of Gaza
Electrical Engineering Department

Optimal PLL loop Filter Design for Mobile WiMax Via LMI

By

Ayman Akram AlQuqa

Supervisors

Dr. Hatem Elaydi **Dr. Hala J. El-Khozondar**

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Dedicated to

My Father,Mother, Wife and sweet sons: Mohamed, Abd Allah and Ahmad.

Abstract

Achieving optimal design of phase-locked loop (PLL) is a major challenge in WiMax technology in order to improve system behavior against noise and to enhance Quality of Service (QOS). A new loop filter design method for phase locked loop (PLLs) is introduced taking into consideration various design objectives: small settling time, small overshoot and meeting Mobile WiMax requirements. Optimizing conflicting objectives is accomplished via linear programming and semidefinite programming (especially Linear Matrix Inequality (LMI)) in conjunction with appropriate adjustment of certain design parameters. Digital filters, Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) are designed using linear programming and convex programming.

Simulations show that IIR digital lowpass filter with narrow transition band could not work properly with mobile WiMax system. Simulations show that FIR digital lowpass filter utilizing linear programming managed to improve the transient behavior. The FIR digital lowpass filter utilizing semidefinite programming (LMI) will much improve the transient behavior; therefore it is recommended for mobile WiMax systems.

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1. Introduction

1.1 Motivation

The number of telecommunications innovations grew rapidly during the last half of the 20^{th} century [1]. Currently, there is widespread and growing use of cellular phones, cordless phones, digital satellite systems, and personal mobile radio networks. Over the past decade, there has been a tremendous growth in the popularity of wireless networking technologies [2]. Schools and universities are providing wireless access to students and faculty members. Malls are providing customers with wireless connectivity to allow them to search products, virtually navigate shops, and interact with services. Tourist sites are providing wireless devices to aid tourists in navigating, exploring and learning about attractions. Moreover, the emergence of new computing paradigms such as pervasiveness, ubiquity, and mobility has necessitated the rapid deployment of wireless networks as an infrastructure underneath such technologies.

One of the main technology that can lay the foundation for the next generation ,fourth generation (4G), of mobile broadband networks is popularly known as "WiMAX." WiMAX, *Worldwide Interoperability for Microwave Access,* is designed to deliver wireless broadband bitrates, with *Quality of Service* (QoS) guarantees for different traffic classes, robust security, and mobility.

WiMAX is a set of specifications created by the WiMAX Forum [3] and is based on standards developed by the IEEE 802.16 Working Group (WG) [4]. There are two standards of note here. The first is IEEE 802.16-2004, sometimes called 802.16d, which specifies a common air interface for fixed (both ends stationary) microwave equipment. But since the high-growth market opportunity for wireless of any form today is in mobile systems, the IEEE 802.16 WG subsequently issued IEEE 802.16e-2005, which specifies a mobile broadband technology. The 802.16e, as it is commonly known, is now seeing significant product development and production deployments on a global basis. Farpoint Group expects that effective per-user throughput of 2-4 Mbps will become common on carrier WiMAX networks over the next few years, with monthly pricing perhaps below that currently charged for mobile broadband services with far less throughput. WiMAX will also become a platform for application deployment and could even be catalytic in the broad availability of Web services and software as a service (SaaS), which Farpoint Group believes will become the dominant model for IT in the future – mobile or not. A significant part in WiMax system is the phase-locked loop.

Phase Locked Loop has become one of the most versatile building blocks in electronics [5]. They are at the heart of circuits and systems ranging from clock recovery blocks in data communications to the local oscillators that power the ubiquitous cellular phones. The property of making its output frequency an exact multiple of the reference frequency makes the Phase Locked Loop (PLL) the circuit of choice for frequency synthesizers. PLL is also used for aligning various clocks in synchronous systems and for a myriad of applications ranging from tracking satellite Doppler shift to sensing minute reactance changes in industrial proximity sensors.

However, phase-locked loop (PLL), is an electronic circuit that controls an oscillator so that it maintains a constant phase angle relative to a reference signal [6]. In communications, the oscillator is usually at the receiver, and the reference signal is extracted from the signal received from the remote transmitter.

1.2 Linear Matrix Inequalities

A wide variety of problems arising in system and control theory can be reduced to a few standard convex or quasi-convex optimization problems involving linear matrix inequalities (LMIs), that is constraints of the form

$$\boldsymbol{F}(\boldsymbol{x}) \triangleq \boldsymbol{F_0} + \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{m}} \boldsymbol{X_i} \boldsymbol{F_i} > 0, \tag{1.1}$$

Where $x \in R^m$ is the variable and $F_i = F_i^T \in R^{n \times n}, i = 0, \ldots, m,$ are given. Although the LMI form appears very specialized, it is widely encountered in system and control theory. Lists of many comprehensive examples are found in Boyd et al [7].

Since these resulting optimization problems can be solved numerically very efficiently as showed in [7], there are special cases with few analytical solutions to LMI optimization problems. Indeed, the recent popularity of LMI optimization for control can be directly traced to the recent breakthroughs in interior point methods for LMI optimization [8]. The growing popularity of LMI methods for control is also evidenced by the large number of publications in recent control conferences. Much of the research effort in the application of LMI optimization has been directed towards problem from control theory while, many of the underlying techniques extend to problems from other areas of engineering as well, for instant, truss topology design and VLSI design.

1.2.1 Linear Programming (LP)

A linear programming problem may be defined as the problem of *maximizing or minimizing a linear function subject to linear constraints*. The constraints may be equalities or inequalities. Here is a simple example[9].

Find numbers x_1 and x_2 that maximize the sum $x_1 + x_2$ subject to the constraints $x_1 \geq 0$, $x_2 \geq 0$, and

$$\begin{aligned} x_1 + 2x_2 &\leq 4 \\ 4x_1 + 2x_2 &\leq 12 \\ -x_1 + x_2 &\leq 1 \end{aligned} \tag{1.2}$$

In this problem there are two unknowns, and five constraints. All the constraints are inequalities and they are all linear in the sense that each involves an inequality in some linear function of the variables. The first two constraints, $x_1 \geq 0$ and $x_2 \geq 0$, are special. These are called *nonnegativity constraints* and are often found in linear programming problems. The other constraints are then called the *main constraints*. The function to be maximized (or minimized) is called the *objective function*. Here, the objective function is $x_1 + x_2$. Since there are only two variables, we can solve this problem by graphing the set of points in the plane that satisfies all the constraints (called the constraint set) and then finding which point of this set maximizes the value of the objective function. Each inequality constraint is satisfied by a half-plane of points, and the constraint set is the intersection of all the half-planes. In the present example, the constraint set is the five sided figure shaded in Figure 1-1.

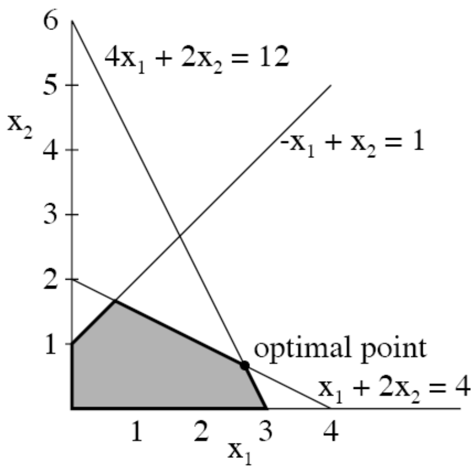

Figure 1-1 Linear Programming Example

We seek the point (x_1, x_2), that achieves the maximum of $x_1 + x_2$ as (x_1, x_2) ranges over this constraint set. The function $x_1 + x_2$ is constant on lines with slope −1, for example the line $x_1 + x_2 = 1$, and as we move this line further from the origin up and to the right, the value of $x_1 + x_2$ increases. Therefore, we seek the line of slope −1 that is farthest from the origin and still touches the constraint set. This occurs at the intersection of the lines $x_1 + 2x_2 = 4$ and $4x_1 + 2x_2 = 12$, namely, $(x_1, x_2) = (8/3, 2/3)$. The value of the objective function there is $(8/3) + (2/3) = 10/3$.

It is easy to see in general that the objective function, being linear, always takes on its maximum (or minimum) value at a corner point of the constraint set, provided the constraint set is bounded. Occasionally, the maximum occurs along an entire edge or face of the constraint set, but then the maximum occurs at a corner point as well. Not all linear programming problems are so easily solved. There may be many variables and many constraints. Some variables may be constrained to be nonnegative and others unconstrained. Some of the main constraints may be equalities and others inequalities. However, two classes of problems, called here the *standard maximum problem* and the *standard minimum problem*, play a special role. In these problems, all variables are constrained to be nonnegative, and all main constraints are inequalities.

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following three parts:

- A linear function to be maximized or minimized, e.g. maximize $c_1x_1 + c_2x_2$
- Problem constraints of the following form, e.g.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 &\leq b_1 \\
a_{21}x_1 + a_{22}x_2 &\leq b_2 \\
a_{31}x_1 + a_{32}x_2 &\leq b_3
\end{aligned} \tag{1.3}
$$

- Non-negative variables, e.g. $x_1 \geq 0,\ x_2 \geq 0$.

The problem is usually expressed in *matrix form*, and then becomes:

$$
\begin{aligned}
&\text{maximize } c^T x \\
&\text{subject to } Ax \leq b, x \geq 0
\end{aligned} \tag{1.4}
$$

Other forms, such as minimization problems, problems with constraints on alternative forms, as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form.

1.2.2 Semi-definite Programming (SDP)

The (linear) semidefinite programming problem (SDP) is essentially an ordinary linear program where the nonnegativity constraint is replaced by a semidefinite constraint on matrix variables.

SDP has many applications, ranging from control theory to structural design. In particular, many hard optimization problems (with integer constraints) can be relaxed to a problem with convex quadratic constraints which, in turn, can be formulated as an SDP. This SDP provides a polynomial time approximation to the original, hard problem. Usually, approximations from SDP relaxations are better than those from linear programming.

A semidefinite program is an optimization problem of the following form[10]:

$$
\begin{aligned}
&\text{minimize } c^T x \\
&\text{subject to } F(x) \geq 0
\end{aligned} \tag{1.5}
$$

Where

$$F(x) \triangleq F_0 + \sum_{i=0}^{m} x_i F_i$$

The problem data are the vector $c \in \boldsymbol{R}^m$ and $m + 1$ symmetric matrices $F_0, \cdots, F_m \in \boldsymbol{R}^{n \times n}$. The inequality sign in $F(x) \geq 0$ means that $F(x)$ is positive semidefinite, i.e., $z^T F(x) z \geq 0$ for all $z \in \boldsymbol{R}^n$. We call the inequality $F(x) \geq 0$ a linear matrix inequality and the problem (1.5) a semidefinite program.

A semidefinite program is a convex optimization problem since its objective and constraint are convex: if $F(x) \geq 0$ and $F(y) \geq 0$, then, for all $\lambda, 0 \leq \lambda \leq 1$, $F(\lambda x + (1 - \lambda y)) = \lambda F(x) + (1 - \lambda) F(y) \geq 0$. Figure 1-2 depicts a simple example with $x \in \boldsymbol{R}^2$ and $F_i \in \boldsymbol{R}^{7\times 7}$. Our goal here is to give the reader a generic picture that shows some of the features of semidefinite programs, so the specific values of the data are not relevant. The boundary of the feasible region is shown as the dark curve.

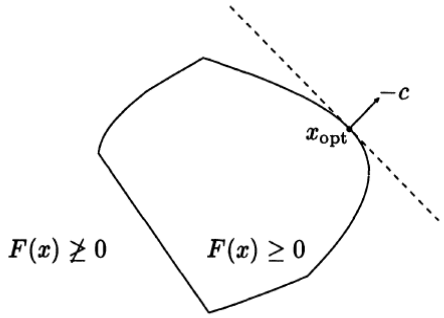

Figure 1-2 A simple semidefinite program with $x \in R^2$ and $F_i \in R^{7\times 7}$

The feasible region, i.e.,$\{x | F(x) \geq 0\}$ consists of this boundary curve along with the region it encloses. Very roughly speaking, the semidefinite programming problem is to move as far as possible in the direction -c, while staying in the feasible region. For this semidefinite program there is one optimal point, X_{opt}.

This simple example demonstrates several general features of semidefinite programs. We have already mentioned that the feasible set is convex. Note that the optimal solution X_{opt} is on the boundary of the feasible set, i.e., $F(x_{opt})$ is singular; in the general case there is always an optimal point on the boundary (provided the problem is feasible). In this example, the boundary of the feasible set is not smooth. It is piecewise smooth: it consists of two line segments and two smooth curved segments. In the general case the boundary consists of piecewise algebraic surfaces. Skipping some technicalities, the idea is as follows. At a point where the boundary is smooth, it is defined locally by some specific minors of the matrix $F(x)$ vanishing. Thus the boundary is locally the zero set of some polynomials in $x_1, \cdots, x_m$, i.e., an algebraic surface.

1.3 Phase Locked Loop Fundamentals

A Phase Locked Loop or a PLL is a feedback control circuit. As the name suggests, the phase locked loop operates by trying to lock to the phase of a very accurate input signal through the use of its negative feedback path. A basic form of a PLL consists of three fundamental functional blocks namely [11]

- A Phase Detector (PD)
- A Loop Filter (LF)
- A voltage controlled oscillator (VCO)

with the circuit configuration shown in Figure 1-3

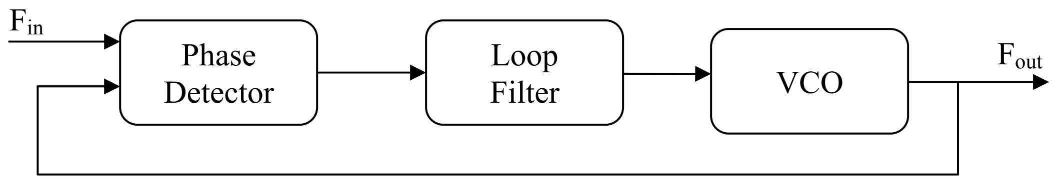

Figure 1-3 A basic PLL Block

1.3.1 Phase Detector (PD)

The phase detector (PD) compares the phase of the output signal to the phase of the reference signal. If there is a phase difference between the two signals, it generates an output voltage, which is proportional to the phase error of the two signals. This output voltage passes through the loop filter and then as an input to the voltage controlled oscillator (VCO) controls the output frequency. Due to this self correcting technique, the output signal will be in phase with the reference signal. When both signals are synchronized the PLL is said to be in lock condition. The phase error between the two signals is zero or almost zero. As long as the initial difference between the input signal and the VCO is not too big, the PLL eventually locks onto the input signal. This period of frequency acquisition, is referred as pull-in time, this can be very long or very short, depending on the bandwidth of the PLL. The bandwidth of a PLL depends on the characteristics of the phase detector (PD), voltage controlled oscillator and on the loop filter.

1.3.2 Loop Filter (LF)

The filtering operation of the error voltage (coming out from the Phase Detector) is performed by the loop filter. The output of PD consists of a dc component superimposed with an ac component. The ac part is undesired as an input to the VCO, hence a low pass filter is used to filter out the ac component. Loop filter is one of the most important functional blocks in determining the performance of the loop. A loop filter introduces poles to the PLL transfer function, which in turn is a parameter in determining the bandwidth of the PLL. Since higher order loop filters offer better noise cancelation, a loop filter of order 2 or more are used in most of the critical application PLL circuits.

1.3.3 Voltage Controlled Oscillator (VCO)

VCO is an electronic oscillator (nonlinear device) designed to be controlled in oscillation frequency by a voltage input. The frequency of oscillation is varied by the applied DC voltage, while modulating signals may also be fed into the VCO to cause frequency modulation (FM) or phase modulation (PM); a VCO with digital pulse output may similarly have its repetition rate (FSK, PSK) or pulse width modulated (PWM).

1.4 PLL Application: Frequency Synthesizer

One of the most common uses of a PLL is in Frequency synthesizers of Wireless systems. A frequency synthesizer generates a range of output frequencies from a single stable reference frequency of a crystal oscillator [11]. Many applications in communication require a range of frequencies or a multiplication of a periodic signal. For example, in most of the FM radios, a phase-locked loop frequency synthesizer technique is used to generate 101 different frequencies. Also most of the wireless transceiver designs employ a frequency synthesizer to generate highly accurate frequencies, varying in precise steps, such as from 600 MHz to 800 MHz in steps of 200 KHz. Frequency Synthesizers are also widely used in signal generators and in instrumentation systems, such as spectrum analyzers and modulation analyzers.

A basic configuration of a frequency synthesizer is shown in Figure 1-4 [11]. Besides a PLL, it also includes a very stable crystal oscillator with a divide by N-programmable divider in the feedback loop. The programmable divider divides the output of the VCO by N and locks to the reference frequency generated by a crystal oscillator.

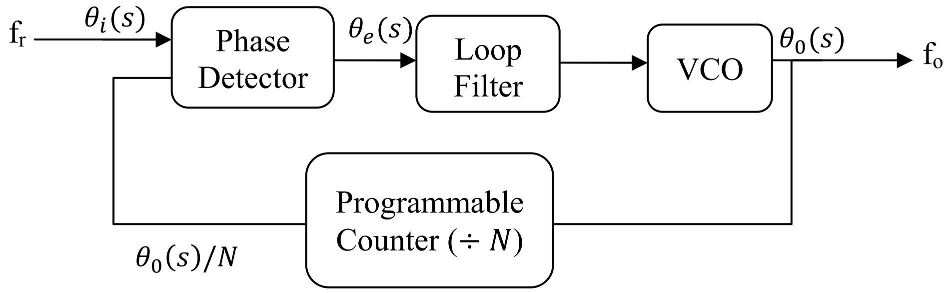

Figure 1-4 basic Frequency Synthesizer

The output frequency of VCO is a function of the control voltage generated by the PD. The Output of the phase comparator, which is proportional to the phase difference between the signals applied at its two inputs, control the frequency of the VCO. So the phase comparator input from the VCO through the programmable divider remains in phase with the reference input of crystal oscillator. The VCO frequency is thus maintained at Nf_r. This relation can be expressed as

$$f_r = \frac{f_0}{N} \tag{1.6}$$

This implies that the output frequency is equal to

$$f_0 = Nf_r \tag{1.7}$$

Using this technique one can produce a number of frequencies separated by f_r and a multiple of *N*. For example if the input frequency is 24KHz and the *N* is selected to be 32 (a single integer) then the output frequency will be 0.768 MHz In the same way, if *N* is a range of numbers, the output frequencies will be in the proportional range. This basic technique can be used to develop a frequency synthesizer from a single reference frequency. This is the most basic form of a frequency synthesizer using phase locked loop technique.

Return to Figure 1-4, note that $\theta_i(s)$ is the phase input, $\theta_e(s)$ the phase error, and $\theta_0(s)$ output phase. Phase error (phase detector output) can be calculated from

$$\theta_e(s) = \frac{1}{1 + G(s)H(s)}\theta_i(s) \tag{2.3}$$

and the VCO output can be calculated from

$$\theta_0(s) = \frac{G(s)}{1 + G(s)H(s)}\theta_i(s) \tag{2.4}$$

where $G(s)$ is the product of the individual feed forward transfer functions, and $H(s)$ is the product of the individual feedback transfer functions

1.5 The Problem

My goal is to design PLL loop filter that accomplish all requirements to work properly and efficiently with Mobile WiMax system. The design is based on optimization particularly Linear Programming and Linear Matrix Inequalities (LMI) techniques. Much of the research effort in the application of LMI optimization has been directed towards problems from control theory while, many of the underlying techniques extend to problems from other areas of engineering as well, for instant, wireless applications. The problem to design and optimize the loop filter using LMI is new in that it is used for Mobile WiMax system.

Mobile WiMax systems use digital low-pass filter (IIR or FIR) as main component of frequency systhesizer. In other world, the problem is to design IIR Filter or FIR Filter and replace it instead of loop filter in PLL block.

The designed loop filter will be stable and compatible with:

1. Frequency range used for Mobile WiMax systems (2.3 – 2.7) GHz.
2. 15 Mbps capacity up to 3km per channel.
3. Channel resolution (125 KHz).
4. Non line-of-site Requirement.
5. Low settling time, to lower the lock-in range.
6. Low overshoot.

In this thesis, we will utilize the Chou's approach [12] which implement simple PLL architecture to design loop filter by transforming the problem to convex optimization particularly LMI. The implemented VCO was replaced with an integrator multiplied by gain and their designed filter was used specially for GPS system. Trying to use their technique to design loop filter for mobile WiMax is not good idea because in wireless systems, we do not use *simple PLL architecture*, instead we can use *integer-N frequency synthesizer* and for more modern systems we use *N-Fractional synthesizer*.

In our design, the VCO noise is assumed to be white Gaussian noise and it is neglected. *N-Fractional* Synthesizer is used instead of *N-Integer* Synthesizer to reduce noise resulted from the factor *N*.

Traditional frequency synthesizers use low-pass analog filter to eliminate the high frequency components, but in the new design we will use IIR or FIR digital low-pass filter to eliminate the high frequency components resulting in more immunity to noise.

1.6 Thesis Organization

Chapter 2 will cover literature review of PLL loop filter designs and optimizations. Theoretical background on mobile WiMax standard will be presented and outlined in chapter 3. The design of *fractional-N* frequency synthesizer using IIR and FIR digital filters will be discussed in chapter 4. Chapter 5 will display the solution of the problem using lmitool (cvx) and linear programming with comparison with other techniques. Finally the conclusion and suggestions for future work will be given in chapter 6.

2. Literature Review

A wide variety of problems arising in system and control theory can be reduced to a handful of standard convex and quasi-convex optimization problems that involve matrix inequalities [7]. For a few special cases there are “analytic solutions” to these problems, but their main point is that they can be solved numerically in all cases. These standard problems can be solved in polynomial-time, and so are tractable, at least in a theoretical sense. Recently developed interior-point methods for these standard problems had been found to be extremely efficient in practice. Therefore, they considered the original problems from system and control theory as solved.

Abbas-Turki, et al. [13], presented an LMI formulations for designing controllers according to time response and stability margin constants. Convex mathematical translations of both kinds of objectives (time and frequency-domain) were proposed using Linear Matrix Inequalities (LMI). The application of Youla parameterization allows restoring the linearity in the compensator parameters, but a huge state space representation of the system was induced. Showing that the Cutting Plane Algorithm (CPA) was efficiently used to overcome the problem of having a huge number of added variables, which often occurs in Semi-Definite Programming (SDP) particularly when used in conjunction with Youla Parameterization. The application of the CPA leads to prevent the introduction of additional decision variables, which implies that a high order of the Youla parameter can be considered without numerical difficulties. So the feasibility of the problem can be easily checked by increasing gradually the order of Youla parameter to be determined. The simplicity of using the CPA made it attractive, although some numerical improvements can be a subject of forthcoming works. The stability margins constrained were considered for MISO (Multi Input Single Output) or SIMO (Single Input Multi Output) plants, the extension to the MIMO (Multi Input Multi Output) case was being under investigation (up to the authors [13]).

Focusing our attention to the new filter design methods, we start with the Henrion, et al. [14] simultaneous Optimization over the numerator and denominator polynomials in the Youla-Kurcera (YK) parameterization. They showed that optimizing simultaneously over the numerator and denominator polynomials of the rational YK parameter provides the designer with a great flexibility. Stability of the denominator polynomial was ensured with the sufficient condition, meaning that the key ingredient in the design procedure was the choice of the so-called central polynomial around which the closed-loop dynamics must be optimized. With the help of numerical examples, it was shown that the approach is suitable for fixed-order and H_∞ controller design. Stability of the denominator polynomial, as well as fixed-order controller design with H_∞ performance were ensured via notation of a central polynomial and LMI conditions for polynomial positivity. It was therefore possible to control at will the growth in the controller order, hence overcoming an often mentioned difficulty of the YK parameterization.

Since the theoretical description of phase-lock loop (PLL) was well established, there have been several approaches to design a loop filter; In 1988, Abramovitch [15] proved that Lyapunov stability techniques were adequate for analyzing the stability of a third order phase-lock loop and was not substantially more difficult than that of a second order loop. He treated the third order PLL as a nonlinear control system: first examine the small signal (linear) operation and then extending the analysis to the nonlinear region. To accomplish his work he used the second method of Lyapunouv and LaSall’s Theorem. In 1990 [16], Abramovitch solved the problem of stability and tracking analysis for the nonlinear model of analog phase-locked loops using Lyapunouv redesign [17]. However, there was a difficulty in applying the proposed method to high order loops.

After that a concise review of the PLL technique, which is applicable to communication and servo control system is illustrated in Phase-locked loop Techniques Survey presented by Hisch and Hung, 1996[18]. As a result, it is expected that PLL will contribute to improvement in performance and reliability for future communication systems. It will also contribute to the development of higher accuracy and higher reliability servo control systems, such as those involved in machine tools. The status of the PLL technology and its applications has been discussed, and a summary of the PLL technology and its development trends are also included. It is pointed out that the development of better PLL technology and the associated modular IC’s is continuing. The PLL-based servo control system has become important and popular in the development of mechatronics.

After that, Suplin and Shaked [19] introduced a simple systematic procedure to design PLLs that entail parameter uncertainty and time delay. The procedure was based on translating the resulting output-feedback control problem into a problem of designing a state-feedback on an augmented non minimal system. A mixed H_2/H_∞, was used to obtain the required stability margin for the PLL while minimizing the effect of the measurement and the phase noise inputs. One of the main drawbacks of the above method of translating the output-feedback problem into a state-feedback one was that it was tuned to the exact model one assumes for the phase noise spectrum.

Yaniv and Raphael [20] presented a design method for near-optimal PLL taking into consideration the phase noise, the thermal noise, the undesired but unavoidable loop delay caused by delayed decisions, and margins for protection from gain uncertainty and insuring good step response. Unlike the conventional designs, their design incorporated a possible large decision delay and S-curve slope uncertainty. Large decision delays frequently existed in modern receivers due to, for example, a convolutional decoder or an equalizer. The new design also applied to coherent optical communications where delays in the loop limit the laser linewidth. They provided an easy-to-use complete design procedure for second-order loops, and also introduced a design procedure for higher order loops for near-optimal performance. They also showed that using the traditional second-order loop was suboptimal when there was a delay in the loop, and also showed large improvements, either in the amount of allowed delay, or the phase error variance in the presence of delay.

To show the stability problem, an analog phase-lock loop design using Popov Criterion [21] was presented by N. Eva Wu, 2002. The phase-lock criterion, when combined with some straightforward numerical search, can be used to design the low-pass filter in a phase-lock loop with a guaranteed lock range.

A PLL with digitally-controllable loop parameters was designed to optimize jitter performance, by Mansuri and Ken Yang, 2002 [22]. They developed an intuition for designing low-jitter PLLs both by deriving a closed-form solution for a second-order loop and by plotting the sensitivity to various loop parameters for higher order loops. Furthermore, the loop served as a test bench to verify their analysis. The analysis showed a simple expression for long term jitter due to VCO and buffering noise to the damping factor and natural frequency. They derived an expression that relates the jitter contribution of clock buffering (in the feedback) and VCO to the same parameters. They also validated the common design practice of using high loop bandwidth to reduce VCO-induced jitter. However to minimize jitter, they found that accounting for the loop delay in the phase margin is critical. Interestingly, this minimum is very insensitive to Pressure Volume Temperature (PVT) and parameter variations making such a design robust. For applications that require small short-term jitter (i.e., short distance links and block to block interconnect), an underdamped loop can result in much higher short-term rms jitter. For applications that filters input jitter, their modeling showed that very low bandwidths (0.002% f_{osc}) were necessary to reduce noise by a factor of 10 while a damping factor greater than 2 was sufficient.

However, Fahim and Elmansury [23] introduced fast lock digital phase-locked loop architecture for wireless applications. The main advantages of this architecture include small area and digitally selectable frequency resolution. Also, a fully digital solution to reduce the phase-lock time was introduced. In their study, techniques for fast lock PLL design were reviewed. A rigorous time-domain based approach was adopted in order to better understand the frequency locking phenomenon in PLLs. Bounds on expressions of frequency lock time were developed. Phase-locking phenomena were also investigated. It was determined that there seems to be an optimal trajectory in which the phase lock time of the PLL is minimized. Based on this study, an effective method of reducing the lock time was demonstrated. It was shown that the lock time and frequency resolution trade-off can be performed in the digital domain, as opposed to the analog domain. The advantage of this was that in the digital domain, the tradeoff deals with the area and power penalties incurred for extra performance gains. The digital PLL architecture was shown to be effective for low frequency resolution standards such as cordless and wireless LAN standards. For high frequency resolution standards, such as GSM, the architecture could only be used as a frequency aid circuit. The cost of fine resolution became excessively high in terms of area penalty as well as lock time. A simple, yet effective method of reducing the phase lock time was also introduced.

An efficient, systematic, and robust method for the optimization of PLL circuits using geometric programming was illustrated in [24], where PLL circuits with VCO frequencies ranging from 814MHz to 1.9GHz were automatically generated. The optimization returns a clean design rule checking (DRC) and layout versus schematic (LVS) of PLL. Power consumption predictions down to 3mW and accumulated jitter predictions below 6ps agreed with silicon measurements. Using this technique, the PLL design cycle was reduced from a matter of weeks to a matter of hours. To the authors' knowledge [24], this was the first example of fully automated PLL design.

In 2004, A Magnitude/Phase-Locked Loop System Based on Estimation of Frequency and In-Phase/Quadrature-Phase Amplitudes was presented by M. Karimi-Ghartemani, et al. [25]. A new PLL system that provides the dominant frequency component of the input signal and estimates its frequency was presented. The QPLL (Quadrature PLL) is advantageous for communication system applications such as QPSK, QAM, and DSB-SC which deals with quadrature modulation techniques. The proposed system provided the dominant frequency component of the input signal and estimated its frequency. The mechanism of the proposed PLL was based on estimating in-phase and quadrature-phase amplitudes of the desired signal and, hence, had application advantages for communication systems which employed quadrature modulation techniques. The studies demonstrated that the proposed PLL also provided a superior performance for power system applications. Derivation of the mathematical model and theoretical stability analysis of the proposed PLL were carried out using dynamical systems theory. Advantages of the proposed PLL over the conventional PLLs were its capability of providing the fundamental component of the input signal which is not only locked in phase but also in amplitude to the actual signal while providing an estimate of its frequency. Computer simulation was used to evaluate its performance. Evaluations confirmed structural robustness of the proposed PLL with respect to noise and distortions.

All-pole phase-locked loop: calculating lock-in range by using Evan's root-locus was presented by Piqueira, and Monteiro [26] in 2006. By studying the problem of synchronizing the PLLs with all-pole filter by using an equivalent linear feedback control system they calculated the lock-in range, for any order $n+1$ of the loop. This contribution had, mainly, a theoretical point of view showing the conditions for the existence of the lock-in range in a higher-order PLL with all-pole filter but giving practical conditions to calculate it when designing circuits. Analysis was performed by detecting a H_{opf} bifurcation on the synchronous state by using the root-locus method combined with the dynamical system theory. The lock-in range was calculated by applying the classical control tools defining an equivalent feedback control system.

In 2007, PLL Equivalent Augmented System Incorporated with State Feedback Designed by LQR was presented by Wanchana, Benjanarasuth et al [27]. They derived the optimal value of filter time constant of loop filter (LF) in the phase-locked loop control system and the optimal state feedback gain designed by using linear quadratic regulator approach. In designing, the structure of phase-locked loop control system rearranged to be a phase-locked loop equivalent augmented system by including the structure of loop filter into the process and by considering the voltage control oscillator as an additional integrator. The designed controller consisting of state-feedback gain matrix K and integral gain K_I is an optimal controller. The integral gain K_I related to weighing matrices q and R was an optimal value for assigning the filter time constant of loop filter. The experimental results in controlling the second-order lag pressure process using two types of loop filters showed that the system response was fast without steady-state error, the output disturbance effect rejection was fast and the tracking to step changes was good. It can be concluded that the proposed technique allowed the designer to easily assign the filter time constant of the loop filter (LF) from the gain K_dK_o and the weighting matrices q and R.

Now, we want to focus on PLL frequency Synthesizer; one of the drawbacks of a traditional frequency synthesizer [28], also known as an Integer-N frequency synthesizer, is that the output frequency is constrained to be N times the reference frequency. If the output frequency is to be adjusted by changing N, which is constrained by the divider to be an integer, then the output frequency resolution is equal to the reference frequency. If fine frequency resolution is desired, then the reference frequency must be small. This in turn limits the loop bandwidth as set by the loop filter, which must be at least 10 times smaller than the reference frequency to prevent signal components at the reference frequency from reaching the input of the VCO and modulating the output frequency, creating spurs or sidebands at an offset equal to the reference frequency and its harmonics. A low loop bandwidth is undesirable because it limits the response time of the synthesizer to changes in N. In addition, the loop acts to suppress the phase noise in the VCO at offset frequencies within its bandwidth, so reducing the loop bandwidth acts to increase the total phase noise at the output of the

VCO. The constraint on the loop bandwidth imposed by the required frequency resolution is eliminated if the divide ratio N is not limited to be an integer. This is the idea behind fractional-N synthesis.

'SWRA029' is technical brief published by Texas Instruments and edited by Curtis Barrett [29] described -in depth- Fractional/Integer-N PLL basics. This document detailed basic loop transfer functions, loop dynamics, noise sources and their effect on signal noise profile, phase noise theory, loop components (VCO, crystal oscillators, dividers and phase detectors) and principles of integer-N and fractional-N technology. The approach was mainly heuristic, with many design examples.

A new methodology for designing fractional-N frequency synthesizers and other phase locked loop (PLL) circuits is presented by Lau, and Perrott, "Fractional-N frequency synthesizer design at the transfer function level using a direct closed loop realization algorithm"[30]. The approach achieved direct realization of the desired closed loop PLL transfer function given a set of user-specified parameters and automatically calculated the corresponding open loop PLL parameters. The algorithm also accommodated nonidealities such as parasitic poles and zeros. The entire methodology was implemented in a GUI-based software package which was used to verify the approach through comparison of the calculated and simulated dynamic and noise performance of a third-order fractional-N frequency synthesizer.

Furthermore, Design and Simulation of Fractional-N PLL Frequency Synthesizers is presented by Kozak and Friedman [31] where a behavioral level simulation environment was developed for Fractional-N PLL frequency synthesizers on a mixed MATLAB™ and CMEX™ platform. A uniform simulation time step was allowed by appropriately modeling the continuous-time average current-to-voltage loop filter transfer function as a discrete-time charge difference-to-voltage transfer function. The simulator enabled the exhaustive behavioral level simulation of Fractional-N PLL frequency synthesizers in a fast and accurate manner. The simulation results demonstrated the effectiveness of the "1" LSB initial condition imposed on the first integrator of the $\Delta\sum$ modulator in rejecting fractional spurs.

The Nonlinear Phase-Locked Loop Design using Semidefinite Programming is illustrated by Wang, et al. [12]. This design approach was based on the polynomial nonlinear model of the PLL system. This approach started with the linear design of the controller and then estimated the domain-of-attraction of the linear designed system to get the suitable local Lyapunov function for the system. The Lyapunov function was then used as the performance constraints to further refine the performance of the system outside the linear region. In otherworld, a Lyapunov function was searched as a certificate of the lock-in region of the PLL system. Moreover, the polynomial design technique was used to further refine the controller parameters for system response away from the equilibrium point. Various simulation results were provided to show the effectiveness of the approach.

Chou, et al. 2006 [32], presented a new filter design method for PLL taking into consideration the various design objectives such as small noise bandwidth, good transient response (small settling time, small overshoot), and larger gain and phase margins. This method is simple and applicable to PLL of any order. Particularly, it allows one to specify the filter poles to desired locations in advance (including the special case of PI form filters which have all the poles at the origin). Numerical simulation of a *GPS (Global Positioning System)* application was performed using nonlinear PLL model. It was observed that this method yields much better performance, when compared with the traditional GPS PLL design. Trade-off among the conflicting objectives was made via recently developed convex optimization skill in conjunction with appropriate adjustment of certain design parameters.

An LMI approach to H_∞ Optimal Filter Design for GPS Receiver Tracking Loop was presented by Phi Long, et al. [33]. They investigated a new approach for GPS receiver tracking loop, using H_∞ theory, where a closed-loop system was designed with objective to minimize the worst case amplification from the input noise power to the output. The main drawback of this approach was the much higher complexity compared with the H_∞ controller design.

Wasim Al-Baroudi in his thesis titled "Digital Filter Design using LMI Based Techniques" in 1997 [34] formulated the FIR filter design problem as a Linear Objective Optimization problem with LMI constraints, and developed a necessary software for solving it. He also formulated the IIR filter design problem as a Linear Objective Optimization problem with LMI constraint and with iterative scheme to overcome the nonlinear term, and developed the necessary software for solving it. Another contribution was the introduction of the Frequency Selection Algorithm that reduced the number of LMI's solved to reach the optimal solution. He also introduced the Formulation of the FIR/IIR Optimal Power Spectrum Approximation Problem as Linear Objective Optimization problem with LMI constraint, and developed the necessary software for solving it. Finally he introduced an LMI-based Model Reduction Technique.

FIR Filter Design via Semidefinite Programming and Spectral Factorization was presented by Wu, et al. [35]. They presented a new semidefinite programming approach to FIR filter design with arbitrary upper and lower bounds on the frequency response magnitude. It was shown that the constraints can be expressed as linear matrix inequalities (LMIs), and hence they could be easily handled by recent interior-point methods. Using this LMI formulation, we can cast several interesting filter design problems as convex or quasi-convex optimization problems, e.g., minimizing the length of the FIR filter and computing the Chebychev approximation of a desired power spectrum or a desired frequency response magnitude on a logarithmic scale. Many other extensions that were not discussed in the paper can be handled in the same framework, such as, maximum stopband attenuation or minimum transition-band width FIR design given magnitude bounds, or even linear array beam-forming. Recent interior-point methods for semidefinite programming can solve each of these problems very efficiently.

Linear programming design of IIR digital filters with arbitrary magnitude function was presented by Rabiner, et al. [36]. This paper discussed the use of linear programming techniques for the design of Infinite Impulse Response (IIR) digital filters. In particular, it was shown that, in theory, a weighted equiripple approximation to an arbitrary magnitude function can be obtained in a predictable number of applications of the simplex algorithm of linear programming. When one implements the design algorithm, certain practical difficulties (e.g., coefficient sensitivity) limit the range of filters which can be designed using this technique. However, a fairly large number of IIR filters were successfully designed and several examples were presented to illustrate the range of problems for which they found this technique to be useful.

Previous studies mentioned above have made important contributions to this challenging problem; however, none of the published studies appear to have completely accounted of the optimization of the Phase Locked Loop (PLL) using the Linear Matrix Inequality (LMI) method for Mobile WiMax application.

3. Mobile WiMax

The term ‘WiMAX’ has been used generically to describe wireless systems that are based on the WiMAX certification profiles of the IEEE 802.16-2004 Air Interface standard. With additional profiles pending that are based on the IEEE 802.16e-2005 Mobile Amendment, it is necessary to differentiate between the two WiMAX systems. ‘Fixed’ WiMAX is used to describe 802.16-2004-based systems while ‘Mobile’ WiMAX is used to describe 802.16e-2005-based systems [37].

Mobile WiMAX [38] is a broadband wireless solution that enables convergence of mobile and fixed broadband networks through a common wide area broadband radio access technology and flexible network architecture. The Mobile WiMAX Air Interface adopts Orthogonal Frequency Division Multiple Access (OFDMA) for improved multi-path performance in non-line-of-sight environments.

Mobile WiMAX systems offer scalability in both radio access technology and network architecture, thus providing a great deal of flexibility in network deployment options and service offerings. Some of the salient features supported by Mobile WiMAX are:

- ***High Data Rates*** : The inclusion of MIMO antenna techniques along with flexible sub-channelization schemes, Advanced Coding and Modulation all enable the Mobile WiMAX technology to support peak DL (Down Link) data rates up to 63 Mbps per sector and peak UL (Up Link) data rates up to 28 Mbps per sector in a 10 MHz channel.

- ***Quality of Service (QoS)*** : The fundamental premise of the IEEE 802.16 MAC architecture is QoS. It defines Service Flows which can map to DiffServ code points or MPLS flow labels that enable end-to-end IP based QoS. Additionally, sub-channelization and MAP-based signaling schemes provide a flexible mechanism for optimal scheduling of space, frequency and time resources over the air interface on a frame-by-frame basis.

- ***Scalability*** : Despite an increasingly globalized economy, spectrum resources for wireless broadband worldwide are still quite disparate in its allocations. Mobile WiMAX technology therefore, is designed to be able to scale to work in different channelizations from 1.25 to 20 MHz to comply with varied worldwide requirements as efforts proceed to achieve spectrum harmonization in the longer term. This also allows diverse economies to realize the multifaceted benefits of the Mobile WiMAX technology for their specific geographic needs such as providing affordable internet access in rural settings versus enhancing the capacity of mobile broadband access in metro and suburban areas.

- ***Security*** : The features provided for Mobile WiMAX security aspects are best in class with AP based authentication, AES-CCM-based authenticated encryption, and CMAC and HMAC based control message protection schemes. Support for a diverse set of user credentials exists including; SIM/USIM cards, Smart Cards, Digital Certificates, and Username/Password schemes based on the relevant EAP methods for the credential type.

- ***Mobility*** : Mobile WiMAX supports optimized handover schemes with latencies less than 50 milliseconds to ensure real-time applications such as VoIP perform without service degradation. Flexible key management schemes assure that security is maintained during handover.

802.16m is the next generation standard beyond 802.16e-2005 and will be adopted by the WiMAX Forum once the standard is completed in the 2009 timeframe. 802.16m is considered to be a strong candidate for a 4G technology. The IEEE has defined its expected parameters for 802.16m, which can be found on their Web site [37].

3.1 Fixed WiMAX vs. Mobile WiMAX

The most useful resource along this section is taken from Mobile WiMAX Handbook [38]. WiMAX is also called Mobile WiMAX as it can serve all usage models from fixed to mobile with the same infrastructure. Table 3-1 shows a comparison between fixed and mobile WiMax technologies. Based on the IEEE 802.16e-2005 standard, Mobile WiMAX offers fixed, nomadic, portable, mobile capabilities and:

- It does not rely on line-of-sight transmissions in lower frequency bands (2 to 11 GHz).
- It provides enhanced performance, even in fixed and nomadic environments.
- It is currently uses Time Division Duplexing (TDD).
- It's system bandwidth is scalable to adapt to capacity and coverage needs.

Table 3-1 Fixed WiMAX vs. Mobile WiMAX

	Fixed WiMAX	**Mobile WiMAX**
Frequency(GHz)	3.5, 5.8	2.3, 2.5, 3.5, etc
Channel (MHz)	3.5, 7, 10, 14	3.5, 7, 8.75, 10, 14, etc
Duplexing	TDD/FDD	TDD/FDD
Multiple Access	TDMA	OFDMA

3.2 WiMAX Working

The most useful resource along this section is taken from 'The Business of WiMAX [39]'. WiMAX has been designed to address challenges associated with traditional wired and wireless access deployments [39]. A WiMAX network has a number of base stations and associated antennas communicating by wireless to a much larger number of client devices (or subscriber stations).

The WiMax MAN is schematically similar to the point-to-multipoint layout of a cellular network. The original 802.16 specification paved the way for fixed wireless-access coverage, which requires a mounted outdoor antenna at the customer's access point. This fixed wireless-access coverage enables clients to communicate with their respective base station, but the 802.16e 'mobility' extension enables seamless communication from station to station (roaming property).

A WiMax base station is connected to public networks using optical fiber, cable, microwave link or any other high-speed point-to-point connectivity, referred to as a backhaul. In a few cases, like mesh networks, a point-to-multipoint WiMAX link to other base stations is used as a backhaul. Ideally WiMAX should use point-to-point antennas as a backhaul to connect aggregate subscriber sites to each other and to base stations across long distances.

The base station serves subscriber stations (also called customer premise equipment) using non-line-of-sight or line-of-sight point-to multipoint connectivity referred to as 'last mile'. Ideally, WiMAX should use non-line-of-sight point-to-multipoint antennas to connect residential or business subscribers to the base station (See Figure 3-1).

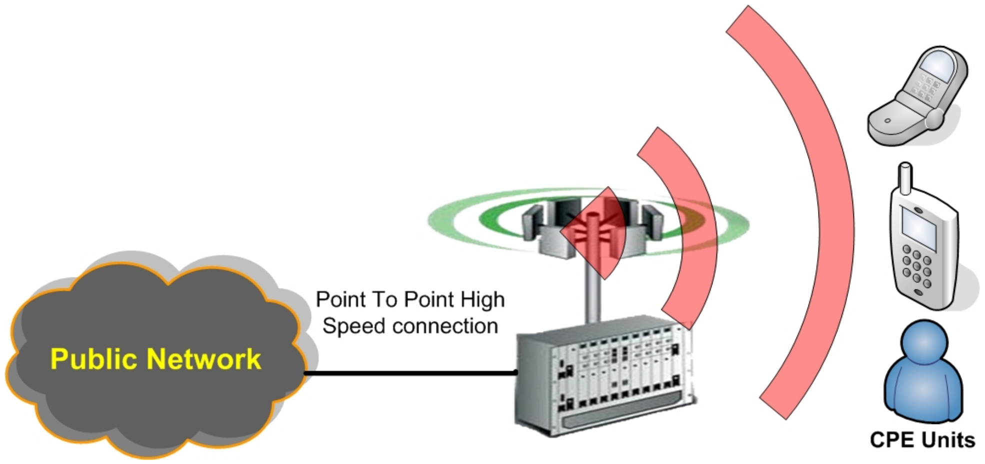

Figure 3-1 WiMAX Working

3.3 WiMAX: Building Blocks

The most useful resource along this section is taken from 'The Business of WiMAX [39]'. Typically, a WiMAX system consists of two parts: a WiMAX base station and a WiMAX receiver also referred as customer premise equipment (CPE). While the backhaul connects the system to the core network, it is not the integrated part of WiMAX system as such.

3.3.1 WiMAX Base Station

A WiMAX base station consists of indoor electronics and a WiMAX tower. Typically, a base station can cover up to a radius of 30 miles or 50 km -theoretically; however, any wireless node within the coverage area would be able to access the Internet (Figure 3-2).

Figure 3-2 WiMAX Base Station

The WiMAX base stations would use the MAC layer defined in the standard – a common interface that makes the networks interoperable – and would allocate uplink and downlink bandwidth to subscribers according to their needs, on an essentially real-time basis. Each base station provides wireless coverage over an area called a cell. The maximum radius of a cell is theoretically 50 km (depending on the frequency band chosen). As with conventional cellular mobile networks, the base-station antennas can be Omni-directional, giving a circular cell shape, or directional to give a range of linear or sectoral shapes for point-to-point use or for increasing the network's capacity by effectively dividing large cells into several smaller sectoral areas.

3.3.2 WiMAX Receiver (CPE)

A WiMAX receiver (Figure 3-3) may have a separate antenna (i.e. receiver electronics and antenna are separate modules) or could be a stand-alone box or a PCMCIA card that sits in your laptop or computer or Impeded CPE used inside Laptop. Access to a WiMAX base station is similar to accessing a wireless access point in a WiFi network, but the coverage is greater. So far one of the biggest deterrents to the widespread acceptance of BWA has been the cost of CPE. This is not only the cost of the CPE itself, but also the installation cost.

Figure 3-3 WiMAX Receivers

3.3.3 Backhaul

Backhaul refers both to the connection from the access point back to the provider and to the connection from the provider to the core network. A backhaul can deploy any technology and media provided it connects the system to the backbone. In most of the WiMAX deployment scenarios, it is also possible to connect several base stations to one another using high-speed backhaul microwave links. This would also allow for roaming by a WiMAX subscriber from one base station coverage area to another, similar to the roaming enabled by cell phones (Figure 3-4).

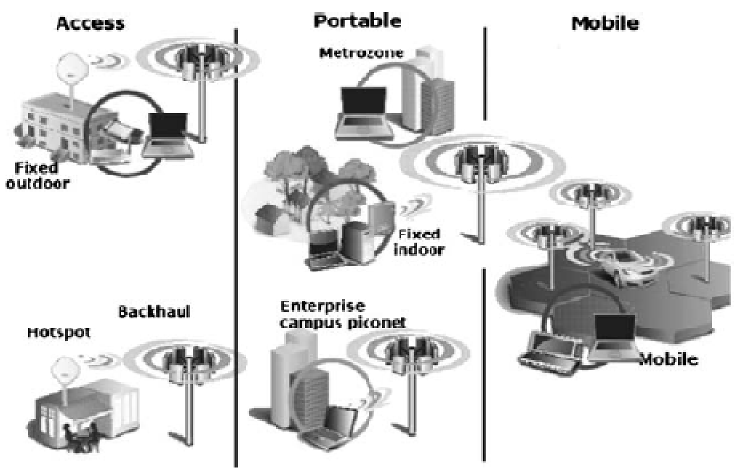

Figure 3-4 WiMAX Technology

3.4 WiMAX Application

The most useful resource along this section is taken from 'The Business of WiMAX [39]'. The 802.16 standard will help the industry provide solutions across multiple broadband segments. WiMAX was developed to become a last mile access technology comparable to DSL, cable and T1 technologies. It is a rapidly growing technology that is most viable for backhauling the rapidly increasing volumes of traffic being generated by Wi-Fi hotspots.

WiMAX is a Metropolitan Area Network (MAN) technology that fits between wireless LANs, such as 802.11, and wireless wide-area networks (WANs), such as the cellular networks. Bandwidth generally diminishes as range increases across these classes of networks. Proponents believe that WiMAX can serve in applications such as cellular backhaul systems, in which microwave technologies dominate, backhaul systems for Wi-Fi hotspots and most prominently as residential and business broadband services.

WiMAX was developed to provide high-quality, flexible, reduce cost, BWA using certified, compatible and interoperable equipments from multiple vendors. There are many application of WiMAX that discussed in followed subsections.

3.4.1 Metropolitan Area Network (MAN)

What makes WiMAX so attractive is its potential to provide broadband wireless access to entire sections of metropolitan areas as well as small and remote locales throughout the world. People who could not afford broadband will now be able to get it, and in places where it may not previously have been available. WiMAX enables coverage of a large area very quickly (Figure 3-5).

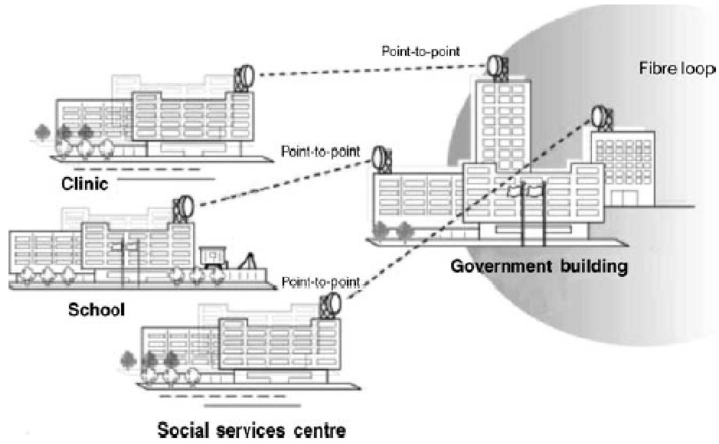

Figure 3-5 Metropolitan Area Network

Today, MANs are being implemented by a wide variety of innovative techniques such as running fiber cables through subway tunnels or using broadband over power lines (BPL). In response to these new techniques, there has been a growing interest in the development of wireless technologies that achieve the same results as traditional MANs without the difficulty of supplying the actual physical medium for transmission, such as copper or fiber lines.

Undeniably, wireless MANs (WMANs) are emerging as a viable solution for broadband access. MANs are intended to serve an area approximately the size of a large city; MANs serve as the intermediary network between LANs and WANs. WMANs consist of a fixed wireless installation that interconnects locations within a large geographic region.

3.4.2 Last-Mile: High-Speed Internet Access or Wireless DSL

DSL operators, who initially focused their deployments in densely populated urban and metropolitan areas, are now faced with the challenge to provide broadband services in suburban and rural areas where new markets are quickly taking root. Governments are prioritizing broadband as a key political objective for all citizens to overcome the ‘broadband gap’ also known as the ‘digital divide’ (Figure 3-6).

WiMAX is also a natural choice for underserved rural and outlying areas with low population density.

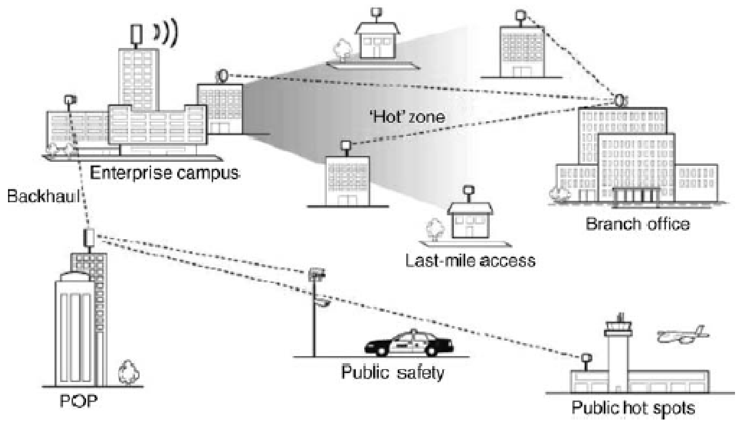

Figure 3-6 Last Mile

3.4.3 Broadband on Demand

One aspect of the existing IEEE 802.16a standard that will make it attractive to service providers and end customers alike is its provision for multiple service levels. Thus, for example, the shared data rate of up to 75 Mbps that is provided by a single base station can support the 'committed information rate' to business customers of a guaranteed 2 Mbps (equivalent to a E1), as well as 'best-effort' non-guaranteed 128 kbps service to residential customers.

The key parameters of WiMAX receiving attention are concerned with its capability to provide differential services. Quality of service enables NLOS operation without severe distortion of the signal from buildings, weather and vehicles. It also supports intelligent prioritization of different forms of traffic according to its urgency.

3.4.4 Cellular Backhaul

The robust bandwidth of IEEE 802.16 makes it an excellent choice for backhaul for commercial enterprises such as hotspots as well as point-to-point backhaul applications (Figure 3-7). Also, with the WiMAX technology, cellular operators will have the opportunity to lessen their independence on backhaul facilities leased from their competitors. Here the use of point-to-point microwave is more prevalent for mobile backhaul, but WiMAX can still play a role in enabling mobile operators to cost-effectively increase backhaul capacity using WiMAX as an overlay network.

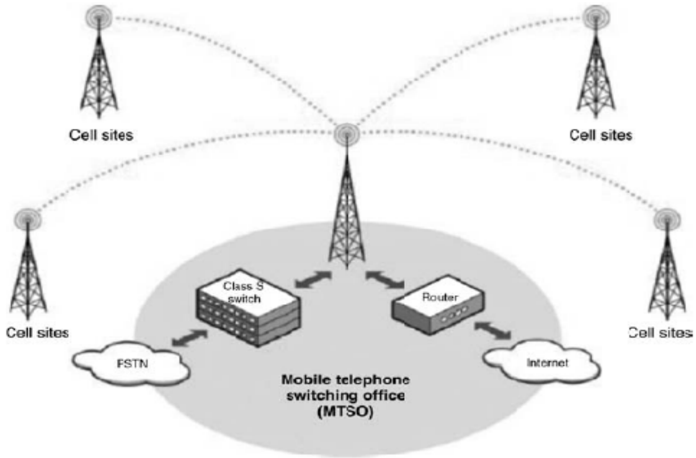

Figure 3-7 Cellular Backhaul

Some salient points about WiMAX use as cellular backhaul are:

- High-capacity backhaul.
- Multiple cell sites are served.
- There is capacity to expand for future mobile services.
- It is a lower cost solution than traditional landline backhaul.

3.4.5 Residential Broadband: filling the gaps in cable & DSL coverage

The range, absence of a LOS requirement, high BW, flexibility, and low cost help to overcome the limitations of traditional wired and proprietary wireless technologies.

3.4.6 Wireless VoIP

Wireless VoIP is a simple and cost-effective service which allows a subscriber to use VoIP services while on the move. This is possible because of WiMAX which can provide carrier-grade connectivity while being wireless. WiMAX using a dynamic resource allocation protocol (DRAP) that provides admission & congestion control within the wireless domain of the network.

3.4.7 Mobility

IEEE 802.16e allows users to connect to a WISP even when they roam outside their home or business, or go to another city that also has a WISP.

3.5 WiMAX versus WiFi

The main and most useful resource for these comparisons is taken from [40].

3.5.1 Scalability

Table 3-2 shows a comparison between the IEEE 802.11 & 802.16a

Table 3-2 Scalability Comparison

802.11	802.16a
• Wide (20MHz) frequency channels. • MAC designed to support 10's of users	• MHz to 20 MHz width channels. Channel bandwidths can be chosen by operator. • MAC designed to support thousands of users.

3.5.2 Relative Performance

Table 3-3 shows a relative performance comparison between IEEE 802.11 & 802.16a.

Table 3-3 Relative Performance Comparison

Standard	802.11	802.16a
Channel Bandwidth	20 MHz	Selectable channel bandwidths between 1.25 and 20 MHz
MAX. Data Rate	54 Mb/s	75 Mb/s

3.5.3 QoS

Table 3-4 shows a Quality of Service comparison between IEEE 802.11 & 802.16a.

Table 3-4 QoS Comparison

802.11	802.16a
• Contention-based MAC (CSMA/CA) => no guaranteed QoS • Standard cannot currently guarantee latency for Voice, Video. • TDD only – asymmetric • 802.11e (proposed) QoS is prioritization only.	• Grant-request MAC • Designed to support Voice and Video. • TDD/FDD – symmetric or asymmetric

3.5.4 Range

Table 3-5 shows a range comparison between IEEE 802.11 & 802.16a.

Table 3-5 Range Comparison

802.11	802.16a
• Up to100 meters • Optimized for indoor performance.	• Up to 50 Km • Optimized for outdoor NLOS performance.

3.5.5 Security

Table 3-6 shows security comparison between IEEE 802.11 & 802.16a.

Table 3-6 Security Comparison

802.11	802.16a
• Standard WEP + WPA • 802.11i in process of addressing security.	• Triple-DES (128-bit) and RSA (1024-bit).

4. Design of PLL Filter

One of the most common uses of a PLL is in Frequency synthesizers. A frequency synthesizer generates a range of output frequencies from a single stable reference frequency of a crystal oscillator.

Basic configuration of a frequency synthesizer is shown in Figure 4-1. Besides a PLL it also includes a very stable crystal oscillator with a divide by N-programmable

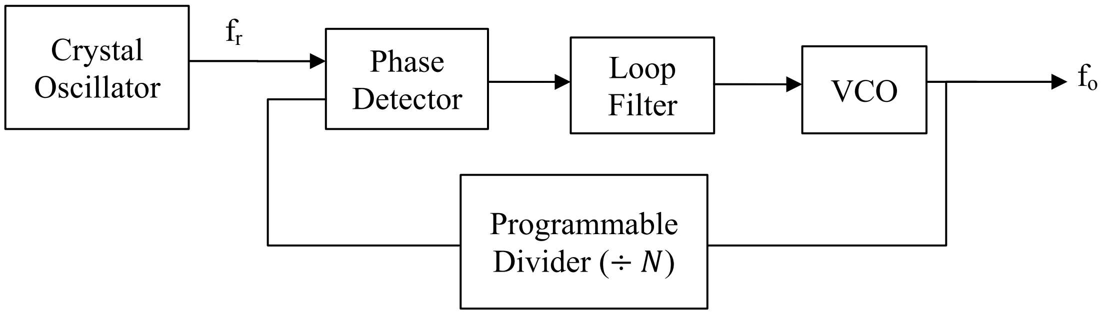

Figure 4-1 Basic configuration of a frequency synthesizer

divider in the feedback loop. The programmable divider divides the output of the VCO by N and locks to the reference frequency generated by a crystal oscillator.

To gain some design experience and some more insight, consider a real design problem of a frequency synthesizer loop filter with the following specifications as shown in Table 4-1.

Table 4-1 Design Specifications

DESIGN SPECIFICATIONS [41]	
PARAMETER	**SPECIFICATION**
Frequency Range	2.3 GHz – 2.7 GHz
Resolution	125 KHz
Overshoot	Less than 20%
Settling time	Less than $40\mu s$
Directivity	Non Line-of-Sight

The design process is divided into several stages. We first present the overall block of frequency synthesizer, then select the integer value of N according to reference frequency and resolution. The next step is the design of digital IIR low-pass loop filter. IIR low-pass loop filter design is carried out using linear programming (LP) technique.

After that, FIR low-pass loop filter will be designed. FIR low-pass loop Filter design is carried out using linear programming (LP) and then using Semi-Definite programming (SDP) utilizing Linear Matrix Inequalities formulation (LMI).

In the next chapter, we will implement the algorithm and simulate the designed filters using our generated MATLAB codes.

4.1 Fractional-N PLL block diagram

The fractional-N PLL block diagram is showed in Figure 4-2,

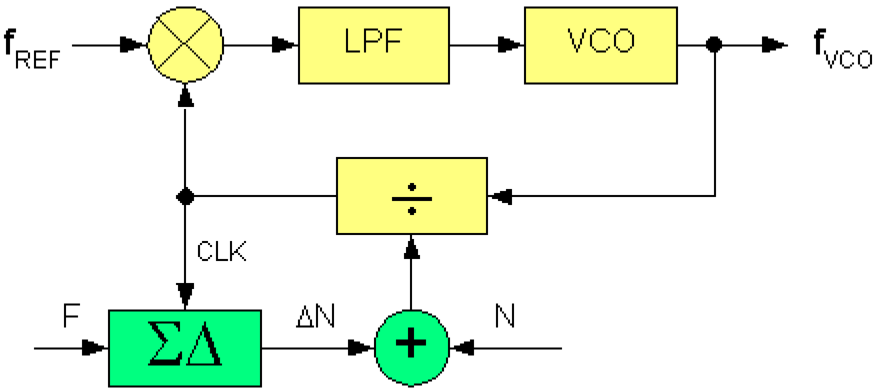

Figure 4-2 Basic Fractional-N PLL Block Diagram

The fractional-N PLL consists of:

1. Phase/Frequency detector which is assumed XOR type.
2. Loop Filter which is the objective of our designe, is a low-pass filter (LPF).
3. Voltage Control Oscillator (VCO).

We first begin the design with Integer-*N* PLL: 125 kHz Reference pushes *N* from 18400 to 21600 (2700/.125). As a result the loop filter cutoff (<12.5 KHz) produces long settling time and VCO phase noise increased by $20*\log_{10}(N) \approx 87$dB.

To overcome the previous drawback, we use the Multi-Modulus Fractional PLL with these properties:

- Fractional value between *N* and *2N-1* (64-127).
- Sigma Delta Modulator (Programmable resolution).
- Large Reference (20MHz) for good tradeoff with settling time.
- Reduced N impact on phase noise by 45dB over Integer *N*.

Example 4.1:

To produce 2300MHz, we produce 1533MHz (from VCO) and then upconvert it to 2300MHz (1533MHz * 1.5 ≈ 2300MHz). The 1533MHz can be produced with N = 76 and a fraction = 0.65 (means that 20MHz * (76 + 0.65) = 1533MHz).

As a result, for *N* = (76 ~ 90), it can produce frequency range (1533MHz ~ 1800MHz), which can be upconverted to (2300MHz ~ 2700MHz).

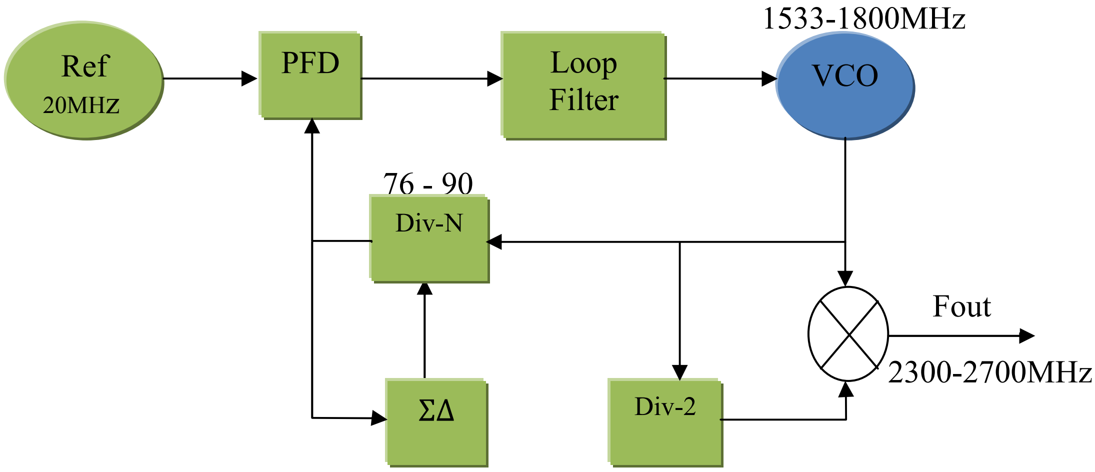

Figure 4-3 Designed Fractional-*N* Synthesizer Block

Figure 4-3 shows the Fractional-*N* PLL design block diagram, with *N* range from 76 to 90 and $\Sigma\Delta$ is the fraction.

Our goal now is to design the low pass loop filter in order to meet the previously mentioned requirements.

4.2 IIR Low-Pass Filter Design

We consider the design of infinite impulse response (IIR) filters subject to upper and lower bounds on the frequency response magnitude. The associated optimization problems, with the filter coefficients as the variables and the frequency response bounds as constraints, are in general non-convex. Using a change of variables and spectral factorization, we can pose such problems as linear or nonlinear convex optimization problems. As a result we can solve them efficiently (and globally) by recently developed interior-point methods. The design procedure follows [36].

Let *H(z)* be the transfer function of an IIR digital filter. Assume *H(z)* has the form

$$H(z) = \frac{N(z)}{D(z)} = \sum_{i=0}^{m} b_i z^{-i} \Big/ \sum_{i=0}^{n} a_i z^{-i} \; . \tag{4.1}$$

where the numerator polynomial N(z) is of m^{th} degree, and the denominator polynomial D(z) is of n^{th} degree. The a_0 term in (4.1) can be set to 1.0 without any loss in generality. The magnitude response of the filter is obtained by evaluating (4.1) on the unit circle (i.e., for $z = exp\,[\,j\omega])$, and taking its magnitude, thus giving

$$|H(\exp[j\omega])| = \left|\frac{N(\exp[j\omega])}{D(\exp[j\omega])}\right| = \left|\sum_{i=0}^{m} b_i \exp[-j\omega_i] \Big/ \sum_{i=0}^{n} a_i \exp[-j\omega_i]\right| . \tag{4.2}$$

In many frequency domain filter design problems, the magnitude required of the resulting filter can be approximated to a given magnitude function $M(exp\,[\,j\omega])$, with a tolerance $G(\omega,\delta)$, where $G(\omega,\delta)$ is a monotically increasing function of δ for fixed ω. Thus, the resultant approximation problem is selecting the filter coefficients (the a_i's and b_i's) to minimize the quantity δ consistent with that constraint inequality

$$\left|\left|\frac{N(\exp[j\omega])}{D(\exp[j\omega])}\right| - M(\exp[j\omega])\right| \le G(\omega,\delta)\,. \tag{4.3}$$

Inequality (4.3) is generally evaluated over a union of disjoint subintervals of the band $0 \le \omega \le \pi$.

The above approximation problem is a nonlinear one in that the filter coefficients enter into the constraint equation nonlinearity. There are several methods to solve the previous problem where a linear approximation problem can be defined by considering the magnitude squared function of the filter. The derivations are presented in reference [36].

The resultant magnitude squared function of the filter is

$$|H(\exp[j\omega])|^2 = H(z)H(z^{-1})|_{z=\exp[j\omega]} = \frac{\widehat{N}(\omega)}{\widehat{D}(\omega)}$$
$$= {[c_0 + \sum_{i=0}^{m} 2c_i \cos(\omega_i)]}\Big/{[d_0 + \sum_{i=0}^{n} 2d_i \cos(\omega_i)]} \cdot \qquad (4.4)$$

Equation (4.4) shows that the magnitude squared function of the filter is a ratio of trigonometric polynomials. It is also seen that both $\widehat{N}(\omega)$**,** the numerator polynomial, and $\widehat{D}(\omega)$**,** the denominator polynomial, are linear in the unknown filter coefficients $\{c_i\}$ and $\{d_i\}$. Now, linear programming technique can be used to determine c_i's and d_i's such that $|H(\exp\,[j\omega])|$ approximates a given magnitude squared characteristic $F(\omega)$ where the peak weighted error of approximation is minimized.

If we let $F(\omega)$ be the desired magnitude squared characteristic, then the approximation problem consists of finding the filter coefficients such that

$$-\epsilon(\omega) \le \frac{\widehat{N}(\omega)}{\widehat{D}(\omega)} - F(\omega) \le \epsilon(\omega), \qquad (4.5)$$

where $\epsilon(\omega)$ is a tolerance function on the error which allows for unequal weighting of errors as a function of frequency.

Equation (4.5) can be expressed as a set of linear inequalities in the c_i's and d_i's as follows

$$\widehat{N}(\omega)-\widehat{D}(\omega)F(\omega)\leq\epsilon(\omega)\widehat{D}(\omega),$$

$$-\widehat{N}(\omega)+\widehat{D}(\omega)F(\omega)\leq\ \epsilon(\omega)\widehat{D}(\omega), \tag{4.6}$$

Or

$$\widehat{N}(\omega)-\widehat{D}(\omega)[F(\omega)+\epsilon(\omega)]\leq 0, \tag{4.7}$$

$$-\widehat{N}(\omega)+\widehat{D}(\omega)[F(\omega)-\epsilon(\omega)]<0. \tag{4.8}$$

The additional linear inequalities

$$-\widehat{N}(\omega)\leq 0, \tag{4.9}$$

$$-\widehat{D}(\omega)\leq 0. \tag{4.10}$$

completely define the approximation problem.

Thus, the question of whether or not there exists a digital filter with magnitude squared characteristic *F(w)* and tolerance function $\epsilon(\omega)$ is equivalent to the question of whether or not there exists a set of filter coefficients satisfying the system of constraints defined by (4.6)-(4.10). The question can be answered by using linear programming techniques. First, an auxiliary variable v is subtracted from the left side of each constraint, forming the new set of constraints

$$\widehat{N}(\omega)-\widehat{D}(\omega)[F(\omega)+\epsilon(\omega)]-v\leq 0, \tag{4.11}$$

$$-\widehat{N}(\omega)+\widehat{D}(\omega)[F(\omega)-\epsilon(\omega)]-v\leq 0, \tag{4.12}$$

$$-\widehat{N}(\omega)-v\leq 0, \tag{4.13}$$

$$-\widehat{D}(\omega)-v\leq 0. \tag{4.14}$$

The objective function $z=v$ is chosen to be minimized under the constraints of (4.11) –(4.14). Clearly a solution to constraints (4.6) - (4.10) exists if and only if the minimum value of z under constraints (4.11) - (4.14) is zero. If the minimum value of v is 0, then a solution exists to the approximation problem and the filter coefficients may be obtained directly as the output of the linear programming routine. If $v\ >\ 0$, then no solution to the approximation problem exists, and either $F(\omega)$, or $\epsilon(\omega)$, or both must be modified in order to obtain a solution.

4.3 FIR Low-Pass Filter Design

We consider the problem of designing a finite impulse response (FIR) filter with upper and lower bounds on its frequency response magnitude [35]: given filter length N, find filter tap coefficients $h \epsilon R^N,\ h = \big(h(0), \ldots., h(N-1)\big)$, such that the frequency response

$H(\omega) = \sum_{i=0}^{N-1} h(n)e^{-j\omega n}$ satifies the magnitude bounds

$$L(\omega) \le |H(\omega)| \le U(w), \quad \omega\, \epsilon\, \Omega\ \subseteq [0, \pi]. \tag{4.13}$$

over the frequency range Ω of interest.

One conventional approach to FIR filter design is Chebychev approximation of a desired filter response $D(\omega)$, i.e., one minimizes the maximum approximation error over Ω.

We present a new way of solving the proposed class of FIR filter design problems, based on magnitude design i.e., instead of designing the frequency response $H(\omega)$ of the filter directly, we design its power spectrum $|H(\omega)|^2$ to satisfy the magnitude bounds [35].

Let autocorrelation function $r(n)$ denote

$$r(n) = \sum_{k=-\infty}^{\infty} h(k)h(k+n), \tag{4.14}$$

where we take $h(k) = 0\ for\ k < 0\ or\ k > N-1$. The sequence $r(n)$ is symmetric around $n = 0, zero\ \ for\ n \le -N\ \ or\ n \ge N, and\ \ r(0) \ge 0$. Note that the Fourier transform of $r(n)$,

$$R(\omega) = \sum_{n=-\infty}^{\infty} r(n)e^{-j\omega n} = |H(\omega)|^2,$$

Is the power spectrum of $h(n)$. If we use *r* as our design variables, we can reformulate the FIR design problem in R^N as

$$
\begin{aligned}
&\text{find} && r = \big(r(0), \ldots\ldots, r(N-1)\big) \\
&\text{subject to} && L^2(\omega) \leq R(\omega) \leq U^2(\omega), \quad \omega \in \Omega \\
& && R(\omega) \geq 0, \quad \omega \in [0, \pi]
\end{aligned} \tag{4.15}
$$

The non-negativity constraint $R(\omega) \geq 0$ is a necessary and sufficient condition for the existence of x satisfying (4.14) by the Fejér-Riesz theorem (see § 4 in [35]). Once a solution of (4.15) is found, an FIR filter can be obtained via spectral factorization. An efficient method of minimum-phase spectral factorization is given in Section 4 in [35].

4.3.1 LP formulation

A common practice of relaxing the semi-definite program (4.15) is to solve a discretized version of it, *i.e.,* impose the constraints only on a finite subset of the [0,π] interval and the problem becomes

$$
\begin{aligned}
&\text{find} && r = \big(r(0), \ldots\ldots, r(N-1)\big) \\
&\text{subject to} && L^2(\omega_i) \leq R(\omega_i) \leq U^2(\omega_i), \quad \omega_i \in \Omega \\
& && R(\omega_i) \geq 0, \quad i = 1, \ldots\ldots, M,
\end{aligned} \tag{4.16}
$$

where $0 \leq \omega_1 < \omega_2 < \cdots < \omega_M \leq \pi$. Since $R(\omega_i)$ is a linear function in r for each i, (4.16) is in fact a linear program and can be efficiently solved. When M is sufficiently large, the LP formulation gives very good approximations of (4.15) in practice. A rule of thumb of choosing M, $M \approx 15N$, is recommended in [42].

4.3.2 SDP formulation

We will show that the non-negativity of $R(\omega_i)$ for all $\omega \in [0, \pi]$ can be cast as an LMI constraint and imposed exactly at the cost of $N(N-1)/2$ auxiliary variables. We will use the following theorem.

Theorem 1 *Given a discrete-time linear system* (A, B, C, D)*, A stable,* (A, B, C) *minimal and* $D + D^T \geq 0$*. The transfer function* $H(z) = C(zI - A)^{-1}B + D$ *satisfies*

$$H(e^{j\omega}) + H^*(e^{j\omega}) \geq 0 \quad for \ all \ \ \omega \in [0,2\pi]$$

if and only if there exists real symmetric matrix P such that the matrix inequality

$$\begin{bmatrix} P - A^T PA & C^T - A^T PB \\ C - B^T PA & D + D^T - B^T PB \end{bmatrix} \geq 0. \qquad (4.17)$$

is satisfied.

For more information, proof of this theorem can be found in [35].

In order to apply Theorem 1, we would like to define (A, B, C, D) in terms of r such that

$$C(zI - A)^{-1}B + D = \frac{1}{2}r(0) + r(1)z^{-1} + \cdots + r(N-1)z^{-(N-1)} \qquad (4.18)$$

An obvious choice is the controllability canonical form:

$$A = \begin{bmatrix} 0 & 0 & & \cdots & 0 \\ 1 & 0 & & \cdots & 0 \\ & 1 & & & \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & & & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$C = [r(1) \quad r(2) \quad \cdots \quad r(N-1)], \quad D = \frac{1}{2}r(0) \qquad (4.19)$$

It can be easily checked that (A, B, C, D) given by (4.19) satisfies (4.18) and all the hypotheses of Theorem 1. Therefore the existence of r and symmetric P that satisfy the matrix inequality (4.17) is the necessary and sufficient condition for $R(\omega) \geq 0$, $for \ all \ \omega \in [0, \pi]$ by Theorem 1.

Note that (4.17) depends affinely on r and P. Thus we can formulate the SDP feasibility problem:

$$\begin{aligned} &\text{Find} && r \in R^N \ \ and \ \ P = P^T \in R^{N-1 \times N-1} \\ &\text{Subject to} && L^2(\omega_i) \leq R(\omega_i) \leq U^2(\omega_i),\ \omega_i \in \Omega \\ &&& \begin{bmatrix} P - A^T PA & C^T - A^T PB \\ C - B^T PA & D + D^T - B^T PB \end{bmatrix} \geq 0 \end{aligned} \tag{4.20}$$

with (A, B, C, D) given by (4.19). The SDP feasibility problem (4.20) can be cast as an ordinary SDP and solved efficiently.

5. Results Analysis

The design process is divided into three setps presented as follows: The first step is to design IIR digital low pass filter using LP MATLAB-based algorithm. Second, FIR digital low-pass filter design using LP and SDP algorithms. The final step is to simulate the designed filters, discuss the results, and compare it with others. To simulate the designed filters we constructed simulation module shown in Figure 5-1.

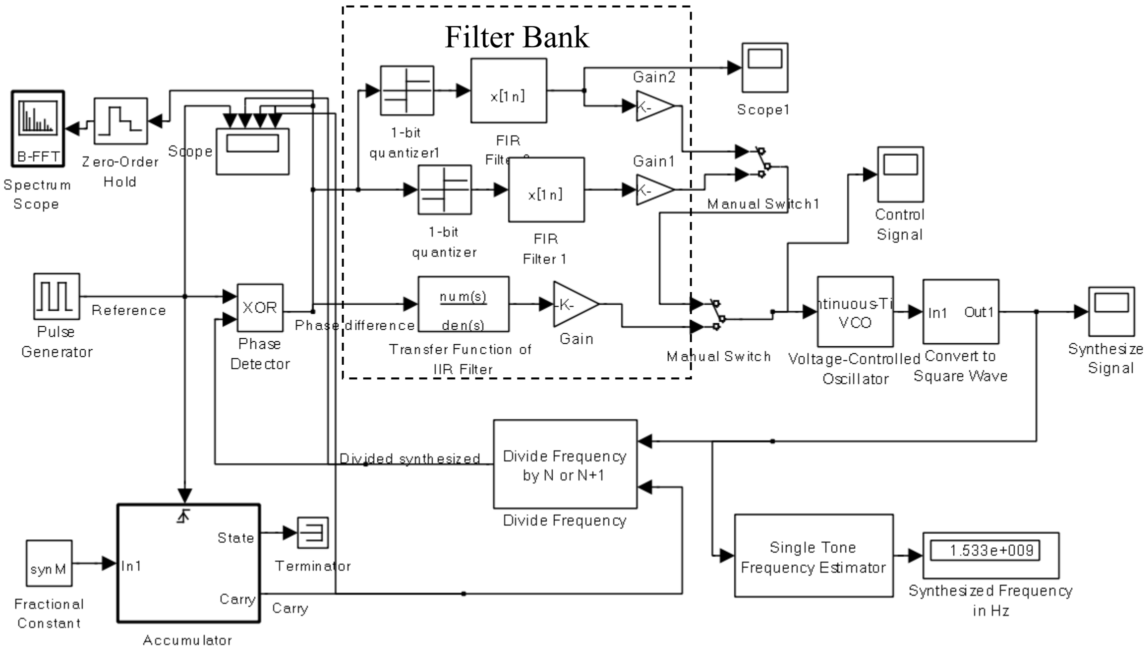

Figure 5-1 PLL Frequency Synthesizer Simulation Model

The simulation module consists of:

1. Reference Frequency: Pulse generator is chosen to produce 20MHz reference frequency.
2. Filter Bank: three filters are designed and separated by two manual switches as shown in Figure 5-1.
3. Voltage Controlled Oscillator (VCO) with output signal amplitude equal to 1V, quiescent frequency equal to 1.511 GHz, and input sensitivity equal to 10MHz/V.
4. Phase Detector: XOR type selected.
5. Frequency Divider which produces (synN + synM) values used to divide the output of VCO. Where synN is an integer and synM is the fraction.
6. Sigma/Delta Modulator: to produce the required fraction synM.

7. The Gain formula $K = K_x * \frac{(synFr*synN-synFq)}{synSen}$, where $K_x = 2.27,\ 2.5$ after the output of IIR filter, FIR filter respectively.

Note that the output synthesized frequency from the previous simulation module (1.533GHz – 1.8GHz) must be multiplied with 1.5 to obtain the required range (2.3GHz – 2.7GHz), see Figure 4-3.

5.1 IIR low pass Filter

We begin the design process by using CVX (convex optimization tool) software, which is a powerful tool developed by Michael Grant and Stephen Boyd to work under MatLab environment. Such as any language, CVX has its own syntax, for more information refer to CVX webpage [43]. IIR low pass Filter can be designed by implementing the procedure described in the previous chapter.

We begin IIR filter design process using the following values:

M = 4; % nominator

N = 4; % denominator, order = N-1

w_{pass} *= .006*pi; % end of the passband*

w_{stop} *= .2*pi; % start of the stopband*

delta = 1; % maximum passband ripple in dB (+/- around 0 dB)

where, w_{pass} value complies with mobile WiMax system specifications as follows:

$$w_{pass} = \Omega_{\text{pass}} * T \ where\ T = 0.02449\mu s, \qquad and\ \Omega_{\text{pass}} = 2\pi(125)Krad/s.$$

$$\therefore w_{pass} = 2\pi(125K) * 2.449 * 10^{-8} = 0.0061\pi\ rad/s.$$

The maximum passband ripple was chosen very low to lower the noise in the overall system. Note that, the stopband attenuation is not constrained here. Using IIR1 code listed in the appendix, the resultant IIR filter Transfer Function (transformed to s-domain using zero-order hold):

$$T(s) = \frac{0.00108\, s^3 \ +\ 9.066 * 10^4\, s^2 \ +\ 9.611 * 10^{12}\, s \ +\ 2.168 * 10^{20}}{s^3 \ +\ 5.638 * 10^7\, s^2 \ +\ 8.907 * 10^{15}\, s \ +\ 2.176 * 10^{20}}$$

note, 3rd order filter obtained.

The filter magnitude/Phase response is shown in Figure 5-2:

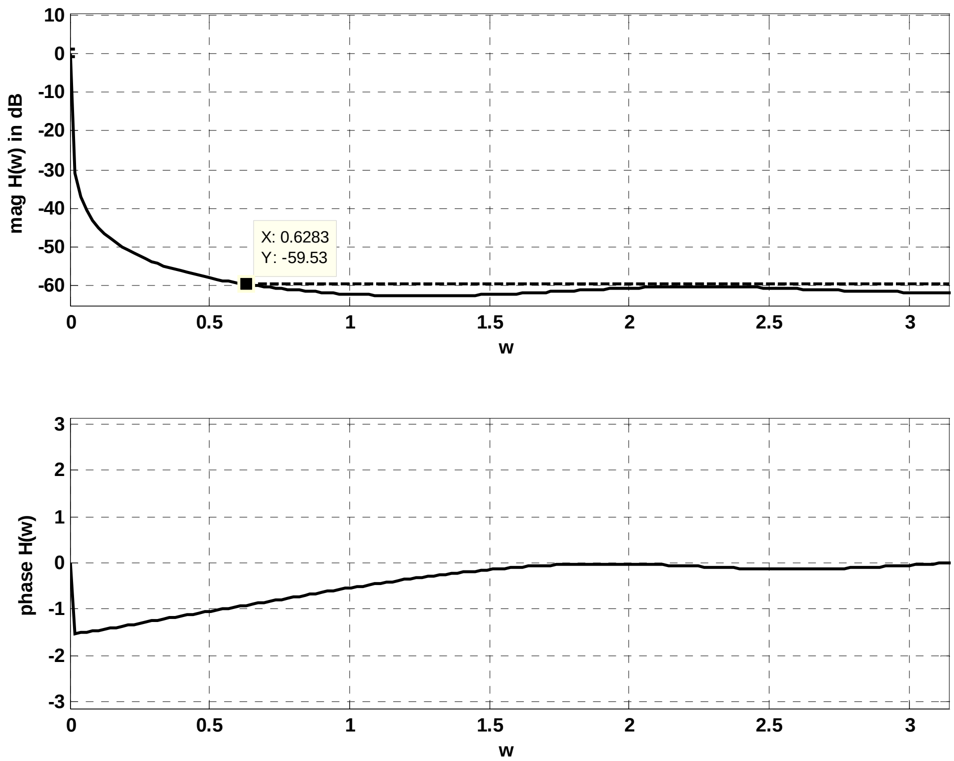

Figure 5-2 3rd order IIR Filter Magnitude/Phase Response

It is clear from Figure 5-2 that we can summarize the designed IIR filter specifications as follow:

- The maximum pass band ripple = 1dB with $w_{pass} = 0.0061\pi\ rad/s$.
- Stop band attenuation below -59.53 dB with $w_{stop} = 0.2\pi\ rad/s$.

This means that our obtained IIR filter is in complement with passband stage and stopband stage. However, we are encouraged to simulate the obtained IIR filter with mobile WiMax simulation block diagram shown in Figure 5-1.

For our bad luck, the simulation is very slow and does not work properly, as a result we cannot obtain our expected output frequency. Yes the designed IIR filter is ok but not for mobile WiMax system. Anyway, let us try to design another IIR filter with much more stop band frequency ($w_{stop} = 0.7\pi\ rad/s$, chosen after several iterations).

Applying IIR1 code again with new stopband frequency, this will diverge from our required optimal filter design specifications. The designed IIR filter transfer function (transformed to s-domain using zero-order hold) is:

$$T(s) = \frac{0.0105\, s^3 \; + \; 1.494 * 10^6\, s^2 \; + \; 1.032 * 10^{14}\, s \; + \; 8.863 * 10^{21}}{s^3 \; + \; 4.612 * 10^7\, s^2 \; + \; 5.258 * 10^{15}\, s \; + \; 8.199 * 10^{21}}$$

The filter magnitude/Phase response is shown in Figure 5-3:

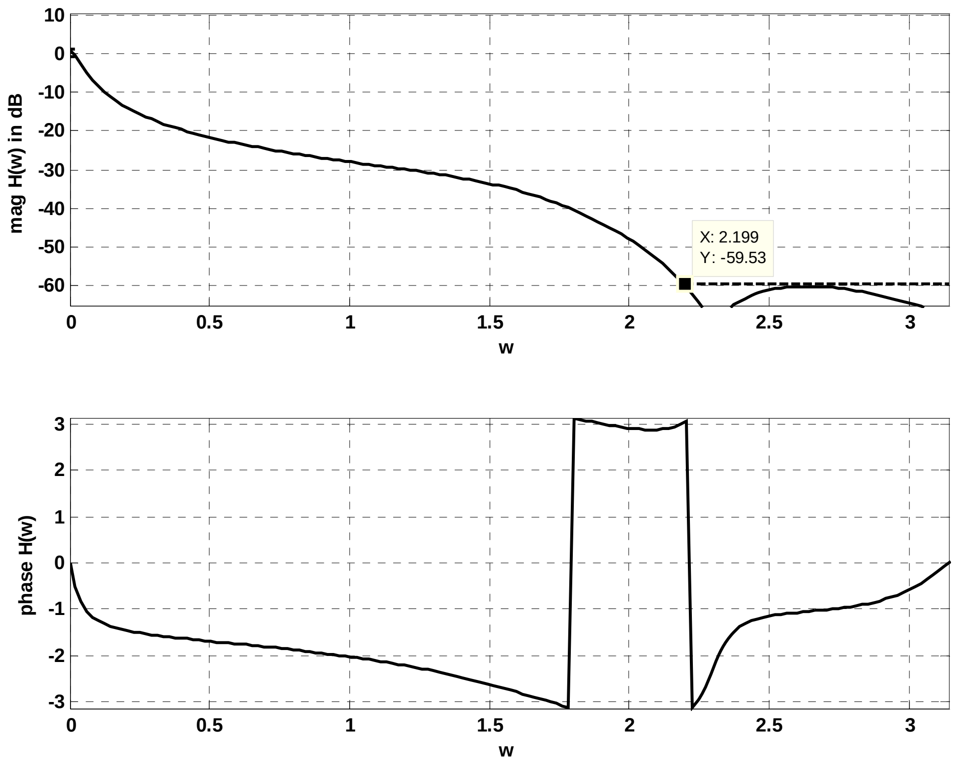

Figure 5-3 3rd order IIR Filter Magnitude/Phase Response with much higher stopband frequency

By looking at Figure 5-3 we conclude that increasing w_{stop} will result in a filter with much wider transition band. Anyway, let us try to simulate it with Mobile WiMax Simulation block diagram shown in Figure 5-1.

The simulation run properly and the correct output frequency obtained. Figure 5-4 shows the control signal of VCO input using current designed IIR filter.

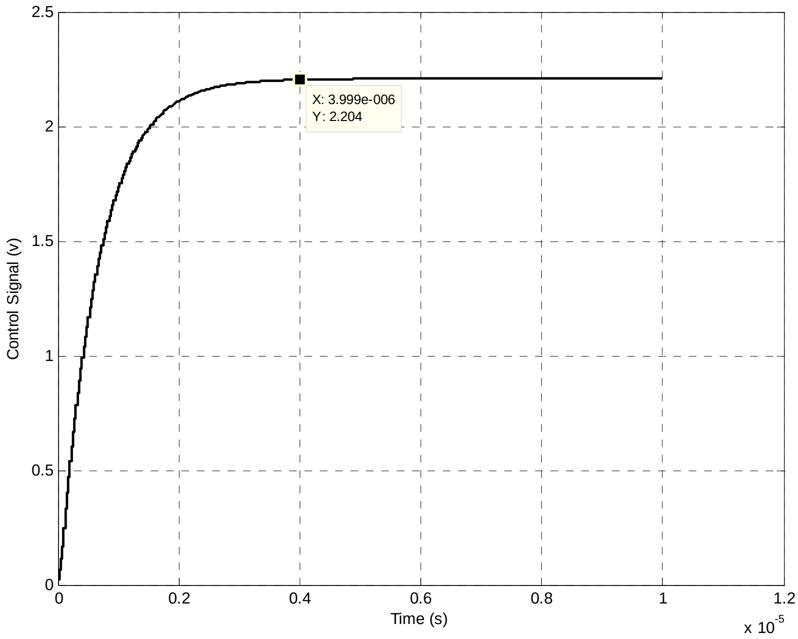

Figure 5-4 The Control Signal of VCO input using Designed IIR Filter

Figure 5-4 shows that

- The settling time is about $4\mu s$.
- The rise time is very low ($1.35\ \mu s$).
- The overshoot is eliminated (zero) which agrees with our design specifications listed in Table 4-1.

As a result we conclude that designing IIR digital filter with narrow transition band using LP is not a good idea because it does not work properly with mobile WiMax simulation. One reason for this draw back is that IIR phase does not taken into consideration (IIR phase consideration is not with in our thesis goal). Another way to overcome this draw back is to use FIR filter instead of IIR filter.

5.2 FIR low-pass Filter

The design of FIR filter is implemented using two different methods described in the previous chapter. Linear programming is used first in order to check if it is the best way for optimal FIR filter design or we need to use Semi-definite programming technique.

5.2.1 Linear Programming

In this section, we design FIR low-pass filter using Linear Programming (LP) to replace IIR lowpass filter. After several iterations, the best outcome is obtained with these parameters:

max_order = 22; % the proposed filter order (2n + 1)

w_{pass} *= 0.006*pi; % passband cutoff freq (in radians)*

w_{stop} *= 0.2*pi; % stopband start freq (in radians)*

delta = 0.4; % max (+/-) passband ripple in dB

atten_level = -44.5; % stopband attenuation level in dB

Note that, we added constraints on the passband and stopband regions and not between them (transition region). By applying FIR1 code listed in the appendix, the resultant FIR filter Impulse Response is shown in Figure 5-5:

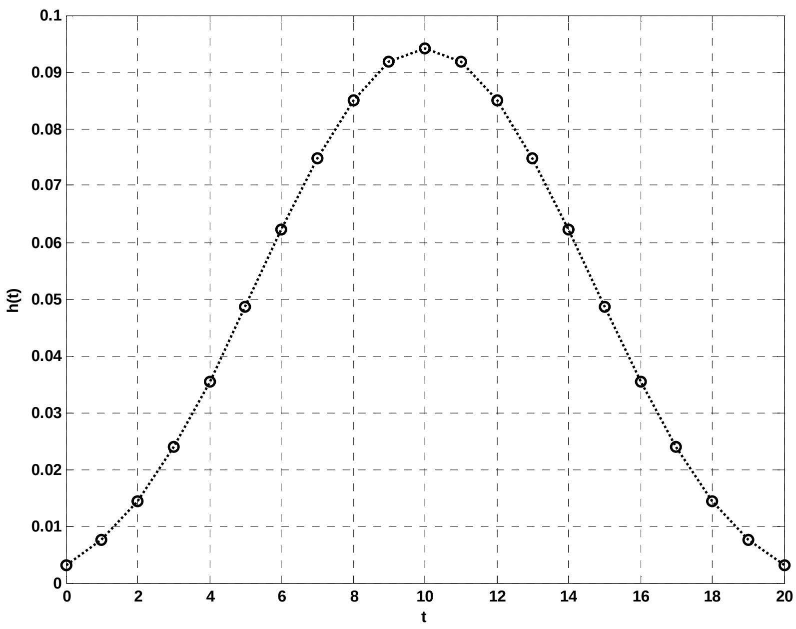

Figure 5-5 FIR Impulse Response (LP)

And the FIR filter magnitude/Phase response is shown in Figure 5-6:

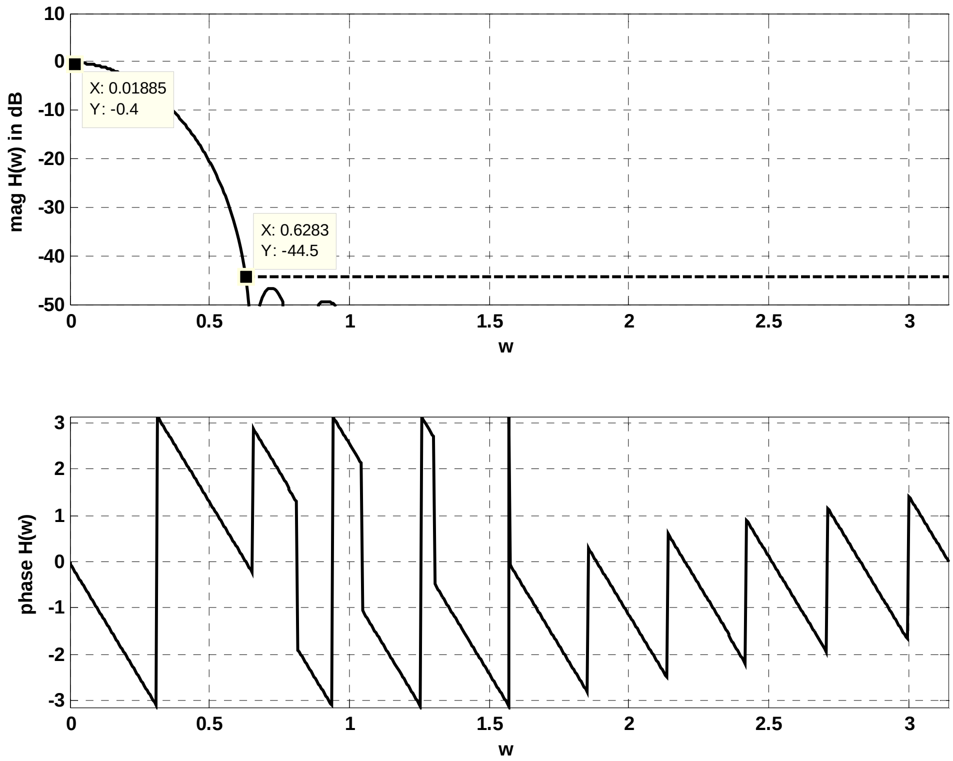

Figure 5-6 FIR Filter Magnitude/Phase Response (LP)

It is clear from Figure 5-6 that the designed FIR filter length is 21 taps where, filter order equal to $2n + 1$, and n = 10. Figure 5-6 shows that:

- The maximum passband ripple does not exceed 0.4 dB with $w_{pass} = 0.01885 = 0.006\pi\ rad/s$.
- Stopband attenuation below -44.5 dB with $w_{stop} = 0.6283 = 0.2\pi\ rad/s$.

Let us try to simulate this FIR filter with Mobile WiMax Simulation block diagram shown in Figure 5-1. The simulation work properly and correct output frequency obtained. The simulation result of the control signal using this FIR filter is shown in Figure 5-7.

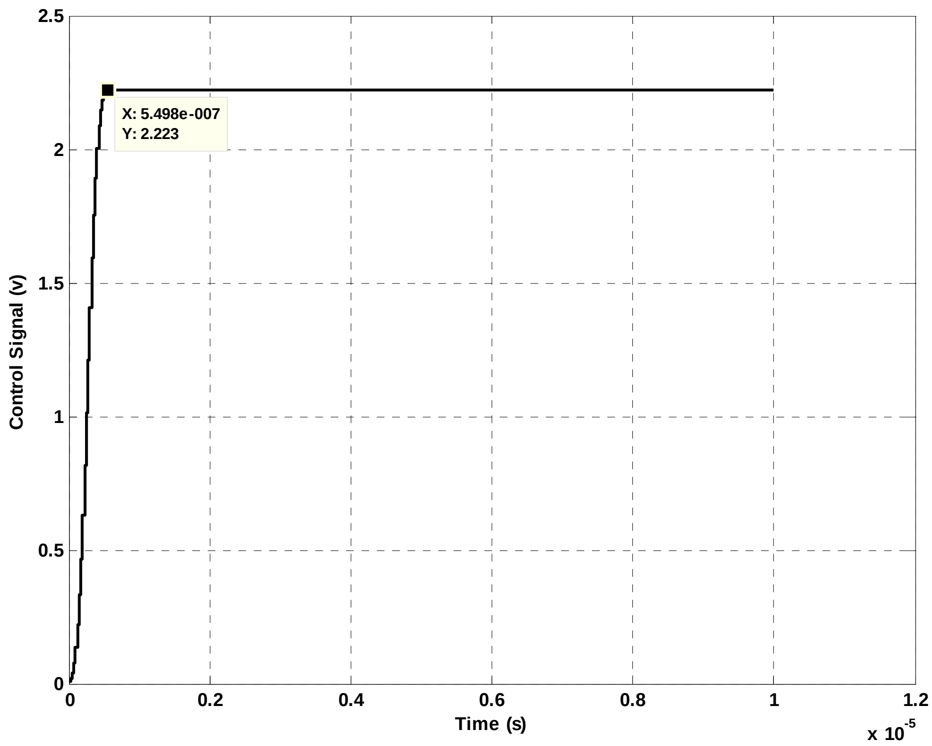

Figure 5-7 The Control Signal of VCO input using Designed FIR Filter

We see from Figure 5-7 that:

- The overshoot equal zero (eliminated).
- The settling time is about $0.5498\mu s$.
- The rise time equal $0.2749\mu s$.

It is clear from the results that using LP to design FIR filter is a good idea that results FIR filter in complement with all mobile WiMax system requirements presented in Table 4-1. In other words, we conclude that using LP to design FIR digital filter will improve settling time, rise time and eliminate overshoot. In the next section, we will use SDP programming technique to answer the next question. Is SDP better than LP or not?

5.2.2 SDP Programming (LMI)

In this section, we use SDP method to design FIR low-pass filter. We implement the SDP algorithm with these parameters:

n = 19; % proposed filter length (order)

*wpass = 0.006*pi; % end of the passband*

*wstop = 0.2*pi;   % start of the stopband*

delta = 0.4;   % maximum passband ripple in dB (+/- around 0 dB)

Using FIR2 code listed in the appendix, the resultant FIR filter Impulse Response shown in Figure 5-8:

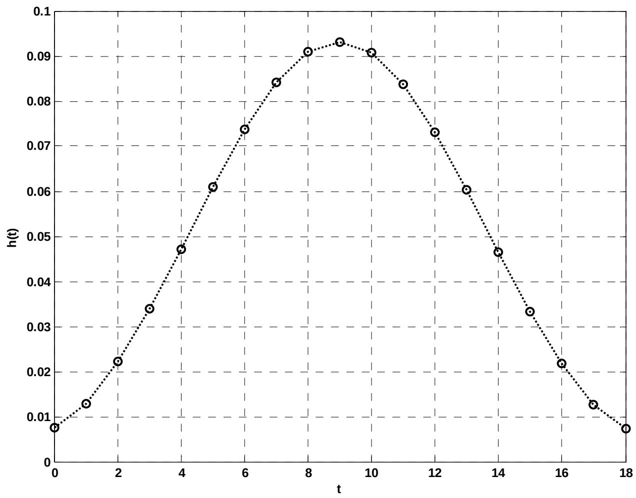

Figure 5-8 FIR Impulse Response (LMI)

And the FIR filter magnitude/Phase response is shown in Figure 5-8:

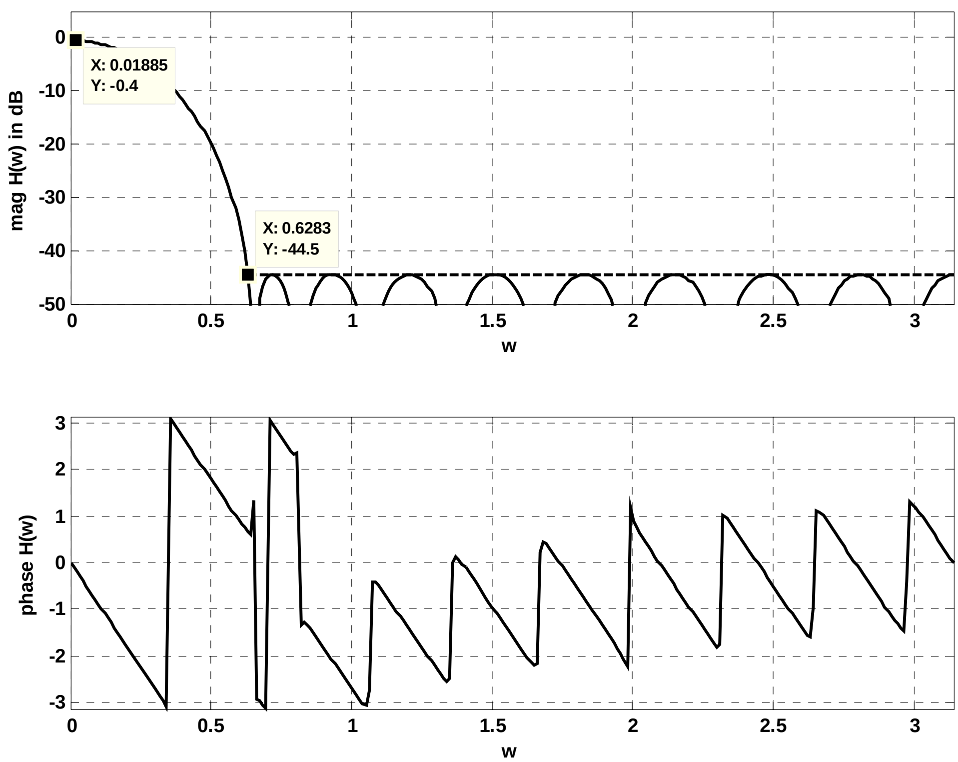

Figure 5-9 FIR Filter Magnitude/Phase Response (LMI)

It is clear from Figure 5-9 that we obtained 19 taps FIR filter order (fewer than LP FIR langth) with:

- Maximum passband ripple does not exceed 0.4 dB with $w_{pass} = 0.01885 = 0.006\pi\ rad/s$.
- Stopband attenuation below -44.5 dB with $w_{stop} = 0.6283 = 0.2\pi\ rad/s$.

Let us try to simulate it with Mobile WiMax Simulation block diagram shown in figure 5-1. The simulation work properly and we got the correct output frequency. The control signal using this FIR filter is shown in Figure 5-10.

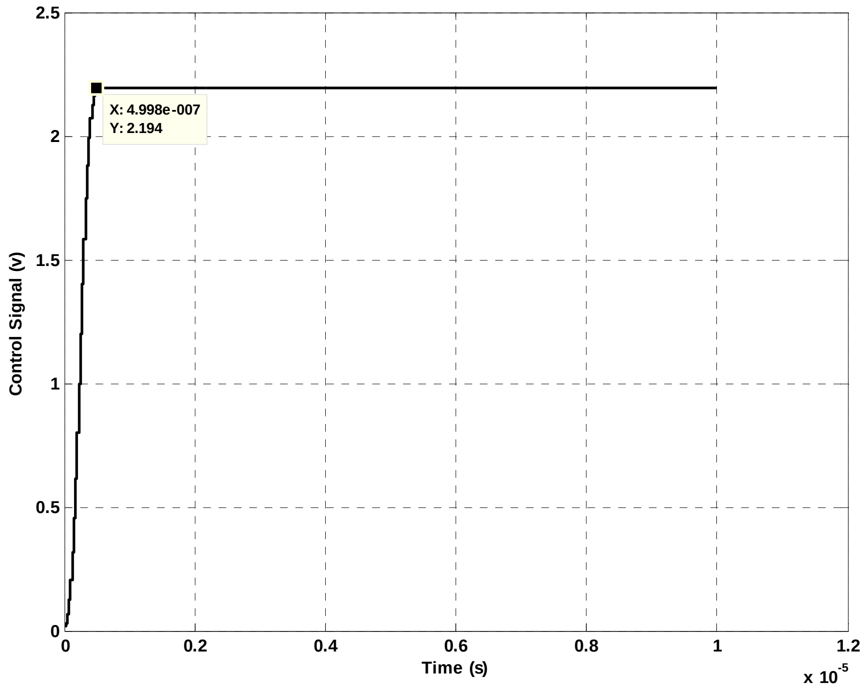

Figure 5-10 The Control Signal of VCO input using Designed FIR Filter (LMI)

Figure 5-10 shows that:

- Overshoot is also eliminated.
- The rise time =0.25μs (agree with FIR filter designed by LP).
- The settling time = 0.4998μs (better than value obtained using LP).

It is clear that using SDP technique to design FIR filter is better than using LP technique as result of the following:

1. Filter order is reduced from 21 taps to 19 taps.

2. Settling time is enhanced (lowered) from $0.5498\mu s$ with LP to $0.4998\mu s$ with SDP.
3. Rise time is enhanced (lowered) from $0.275\mu s$ with LP to $0.25\mu s$ with SDP.

The next section discusses results in more detail and compares it with others.

5.3 Discussion

Simple PLL contains passive low pass loop filter to eliminate high frequency components and pass low frequency components into VCO in order to generate much higher frequency based on the input dc voltage. To eliminate the VCO noise and external noise we must take care of the loop filter design. Modern communication need to corporate much faster PLL as frequency synthesizer to produce many high frequencies. Designers of the first GPS system incorporated simple PLL architecture. The main challenge was how to improve the performance and reduce noise. Low pass loop filter was the major key for their designs in order to reduce noise and improve the behavior of the overall system. Chou et al [32] introduced the design procedure for the simple GPS PLL, the loop filter was designed using LMI method, and the design is a good choice for reducing bandwidth noise. We tried to use their procedure for Integer-N frequency synthesizer however, the design introduced very high bandwidth noise up to thousands resulted from the N divider. The noise was increased by a value equal $10 \log N$ which can be reduced (about 45dB) by replacing the Integer-N with N-fractional frequency synthesizer. In other words, the Chou's procedure is not a good choice for N-fractional frequency synthesizer design. More immunity to noise is the main advantage of Digital over analog filter. For this reason we replaced loop filter with digital low pass IIR or FIR filter. IIR digital lowpass filter was designed using Linear Programming (LP) technique. From the design results, we concluded that increasing the filter order have somehow negligible effect on the overall filter performance, instead, it will increase the complexity of the filter. To this end, the most good low pass behavior can be obtained with 3rd order IIR filter. The designed IIR filter with narrow transition band does not work properly with mobile WiMax systems. IIR filter with much wider transition band works properly with mobile WiMax system with much degradation in performance compared to FIR filter. Linear programming was used for FIR filter design with filter length (order) increased to about 21 taps. Much high FIR taps provide the system with large delay and more complexity to implement. Semi-Definite Programming SDP (especially LMI) is the best way required to obtain optimal FIR filter design with lower filter length. The design problem of loop filter for mobile WiMax system using LP and SDP did not discussed previously. To our best knowledge this study is the first focused on the usage of LP and SDP (LMI) to design loop filter that incorporated into frequency synthesizer. As a result, we will present our design significance and benefits in comparison to other relative methods such as for GPS and other systems.

Beginning with IIR digital lowpass filter, we conclude, from Figure 5-3, that the settling time is about 4 μs which is very low compared with Chou's [32] result (40 μs). Chou designed loop filter for GPS using LMI technique. On the other hand, the rise time obtained from our design (1.35 μs) is higher than Chou's result (0.07313 μs). Moreover, the maximum overshoot obtained by IIR filter design (eliminated) is good compared to Chou's value (about 21%). This elmination of overshoot is good for GPS as well as for mobile WiMax requirements. The standard 3rd order loop filter for mobile WiMax system designed by Staggs [41] had lower overshoot (about 25 %) than our design. On the other hand, the settling time is about (40 μs) which is not better than our obtained value (about 4 μs).

Design and simulation of fractional-N PLL frequency synthesizers presented by Kozak's [31] had larger overshoot (about 29.6 %) than our design. On the other hand, the rise time is better than our obtained values. In contrast, our obtained settling time is better than Kozak's value. Table 5-1 present the comparison between IIR digital low pass filter design and others.

Table 5-1 Comparison between IIR digital filter design using LP and other designs

	IIR filter design	**Chou's**	**Staggs**	**Kozak**
Settling time	4 μs	40 μs	40 μs	$\approx \mathbf{10\mu s}$
Rise time	$\approx$ 1.35 μs	$\approx$ 0.07313 μs	$\approx$ 0.1μs	$\approx \mathbf{1\mu s}$
Maximum overshoot	**Zero**	≈ 21%	≈ 25%	≈ 29.6%

Because IIR digital lowpass filter designed with narrow transition band does not work properly with mobile WiMax system, we designed FIR digital low pass filter by using LP technique in order to work properly with mobile Wimax system. FIR digital filter results showed that the maximum overshoot (eliminated) and the settling time ($0.5498\ \mu s$) are much better than IIR digital filter. In addition, they were better than Chou's, Staggs's, and Kozak's values. Unfortunately, rise time was degraded from that obtained by Chou's, Staggs's values. On the other hand, it is better than Kozak's and our IIR result. Table 5-2 shows the comparison between FIR digital filter design using LP and other designs.

Table 5-2 Comparison between FIR digital filter design using LP and other designs

	FIR filter design (LP)	**Chou's**	**Staggs**	**Kozak**
Settling time	$0.5498\ \mu s$	$40\ \mu s$	$40\ \mu s$	$\approx \mathbf{10\mu s}$
Rise time	$\approx 0.2749\ \mu s$	$\approx 0.07313\ \mu s$	$\approx 0.1\mu s$	$\approx \mathbf{1\mu s}$
Maximum overshoot	Zero	≈ 21%	≈ 25%	≈ 29.6%

To obtain much better filter performance, we designed another FIR digital lowpass filter using SDP (LMI). The maximum overshoot (eliminated) is better than previously mentioned designs and is the same compared with IIR/FIR digital filter designed using LP. The settling time ($0.4998\ \mu s$) and the rise time ($0.25\ \mu s$) are better than FIR filter designed by LP. To this end, the FIR filter designed by SDP is the best for mobile WiMax system. Table 5-3 shows the comparison between FIR digital filter design using SDP and other designs.

Table 5-3 Comparison between FIR digital filter design using SDP and other designs

	FIR filter design (SDP)	**Chou's**	**Staggs**	**Kozak**
Settling time	$0.4998\ \mu s$	$40\ \mu s$	$40\ \mu s$	$\approx \mathbf{10\mu s}$
Rise time	$\approx 0.25\ \mu s$	$\approx 0.07313\ \mu s$	$\approx 0.1\mu s$	$\approx \mathbf{1\mu s}$
Maximum overshoot	**zero**	≈ 21%	≈ 25%	≈ 29.6%

From the previous discussion, we conclude that using FIR digital lowpass filter designed with LP will improve the transient behavior of the overall system. Much better transient performance with can be achieved with FIR lowpass filter designed using SDP.

6. Conclusion and Future Work

Phase locked loop remained an interesting topic for the research, as it covered many discipline of electrical engineering such as communication theory, control theory, signal analysis, design with transistors and op amps, digital circuit design and non-linear analysis.

6.1 Conclusion

A new loop filter design method for frequency synthesizer was introduced taking into consideration various design objectives: small settling time, small overshoot and meeting mobile WiMax requirements. IIR and FIR digital low pass filters were designed using linear programming and semi-definite programming.

Simulations showed that IIR digital lowpass filter with narrow transition band is not good choice for mobile WiMax system. On the other hand, simulations showed that FIR digital lowpass filter utilizing linear programming managed to improve the transient behavior A FIR digital lowpass filter utilizing semi-definite programming (LMI) much improve transient performance and fit best mobile WiMax systems.

6.2 Future work

We can summarize our suggestions for future works as follows:

It is recommended for IIR low pass digital filter design to extend the constrained region to include the transition band between pass band and stop band, and to take into consideration the filter phase. Although, IIR digital filter was designed using traditional Linear Programming, we can expand the design and add LMI-based constraints.

The designed low-pass digital filters were compatible with Mobile WiMax systems working for (2.300 GHz – 2.700 GHz); however, it can be extended to include much higher frequency bands. On other hand, the proposed procedure can be examined with Fixed WiMax system.

7. References

[1] Bystrom, M. (2001). Maja Byston. Retrieved April 21, 2008, Boston University : http://iss.bu.edu/bystrom/WirelessCommunications.doc

[2] Al-Bin-Ali, F. (2004). Inside Wireless Evolution. Retrieved Octobar 31, 2008, from The Association for Computing Machinery: http://www.acm.org/crossroads/xrds9-4/wirelessevolution.html

[3] *Wiax Forum.* (n.d.). Retrieved Octobar 31, 2008, from WiMax Forum: http://www.Wimaxforum.org/home/

[4] *IEEE groups.* (n.d.). Retrieved Octobar 31, 2008, from IEEE Group: http://grouper.ieee.org/groups/802/16/

[5] Tektronix. (n.d.). *Characterizing Phase Locked Loops Using Tektronix Real-Time Spectrum Analyzers*. Retrieved April 21, 2008, tektronix: http://www.tek.com/Measurement/App_Notes/37_18170/eng/37W_18170_1.pdf

[6] *Digital PLL*. (n.d.). Retrieved April 21, 2008, University of Aberdeen : http://www.erg.abdn.ac.uk/users/gorry/course/phy-pages/dpll.html

[7] Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: SIAM.

[8] Vandenberghe, L., & Balakrishnan, V. (1998). Linear matrix inequalities for signal processing : an overview. *32nd Annual Conference on Information Sciences and Systems. Princeton*: CISS.

[9] Ferguson, T. (n.d.). Ferguson homepage. Retrieved MAY 01, 2008, UCLA Department of Mathematics: http://www.math.ucla.edu/~tom/LP.pdf

[10] Vandenberghe, L., & Boyd, S. (1996). Semidefinite programming. Society for Industrial and Applied Mathematics , 38, pp. 49-95.

[11] Best, R. E. (2007). *Phase-Locked Loops: Design, Simulation, and Applications, Sixth Edition*. New York: McGraw Hill International.

[12] T. Wang, T. Chiou, and S. Lall. (2008). Nonlinear Phase-Locked Loop Design using Semidefinite Programming. *Proceedings of the Mediterranean Conference on Control and Automation* , 1640 - 1645.

[13] Abbas-Turki, M., Duc, G., & Clement, B. (2005). LMI formulation for designing controllers according to time response and stability margin constraints. *44th IEEE Conference on Decision and Control, 2005 and 2005 European Control Conference*, (pp. 5740 - 5745). Seville, Splain ,France.

[14] Henrion, D., Kucera, V., & Molina-Cristobal, A. (2004). Optimizing simultaneously over the numerator and denominator polynomials in the Youla-Kučera paramertrization. *43rd IEEE Conference on Decision and Control*, *2*, pp. 2177 - 2181. Nassau.

[15] Abramovitch, D. (1988). Analysis and design of a third order phase-lock loop. *Military Communications Conference. 2*, pp. 455-459. San Diego, CA, USA: Ford Aerosp. Corp., Palo Alto, CA.

[16] Abramovitch, D. (1990). Lyapunov redesign of analog phase-locked loops. *IEEE Trans. on Communication*, *38*, pp. 2197-2202.

[17] Parks, P. (1966). Lyapunov redesign of model reference adaptive control systems. *IEEE Transaction Automatic Control*, *11*, pp. 362- 367.

[18] Hsieh, G.-C., & Hung, J. (1996). Phase-locked loop techniques : a survey. *IEEE Transaction On Industrial Electronics*, *43*, pp. 609-615.

[19] Suplin, V., & Shaked, U. (2000). Mixed H∞ /H2 design of digital phase locked loops with polytopic type uncertainties. *Proceedings of the 39th IEEE Conference on Decision and Control*, *2*, pp. 1930 - 1935.

[20] Raphaeli, D., & Yaniv, O. (1999). Near-optimal PLL design for decision feedback carrier and timing recovery. *IEEE International Conference on Communications*, *3*, pp. 1515 - 1520.

[21] Wu, N. (2002). Analog phaselock loop design using Popov criterion. *Proceedings of the American Control Conference*, *1*, pp. 16 - 18.

[22] Mansuri, M., & Ken, C.-K. (2002). Jitter optimization based on phase-locked loop design parameters. *IEEE Journal of Solid-State Circuits*, *37*, pp. 1375 - 1382.

[23] Fahim, A., & Elmasry, M. (2003). A fast lock digital phase-locked-loop architecture for wireless applications. *IEEE Trans. on circuits and systems-II: Analog and Digital Signal Processing*, *50*, pp. 63 - 72.

[24] Colleran, D., Portmann, C., Hassibi, A., Crusius, C., Mohan, S., Boyd, S., et al. (2003). Optimization of phase-locked loop circuits via geometric programming. *Proceedings of the Custom Integrated Circuits Conference (CICC)*, (pp. 377-380).

[25] Karimi-Ghartemani, M., Karimi, H., & Iravani, M. (2004). A magnitude/phase-locked loop system based on estimation of frequency and in-phase/quadrature-phase amplitudes. *IEEE Trans. On Industrial Electronics* , *51*, pp. 511- 517.

[26] Piqueira, J. R., & Monteiro, L. H. (2006). All-pole phase-locked loops: calculating lock-in range by using Evan's root-locus. *IEEE International Journal of Control*, *79*, pp. 822 - 829.

[27] Wanchana, S., Benjanarasuth, T., Komine, N., & Ngamwiwit, J. (2007). PLL Equivalent augmented system incorporated with state feedback designed by LQR. *International Journal of Control, Automation, and Systems*, *5*, pp. 161-169.

[28] Kundert, K. (2006, August). *Predicting the phase noise and jitter of PLL-based frequency synthesizers.* Retrieved Octobar 01, 2008, The Designer's Guide Community: http://www.designers-guide.org/

[29] Barrett, C. (1999, August). *Fractional/Integer-N PLL Basics.* Retrieved June 03, 2008, Texas Instruments, Wireless Communication Business Unit: http://focus.ti.com/lit/an/swra029/swra029.pdf

[30] Lau, C. Y., & Perrott, M. H. (2003). Fractional-N frequency synthesizer design at the transfer function level using a direct closed loop realization algorithm. *Proceedings Design Automation Conference* (pp. 526 - 531). New York: ACM.

[31] Kozak, M., & Friedman, E. (2004). Design and simulation of fractional-N PLL frequency synthesizers. *Proceedings of the 2004 International Symposium on Circuits and Systems*, *4*, pp. IV- 780-3.

[32] Chou, Y., Mao, W., Chen, Y., & Chang, F. (2006). A novel loop filter design for phase-locked loops. *IEEE International Conference on Systems*, *4*, pp. 2932 - 2938.

[33] Long, N. P., Hyunsoo, K., Yang, E., & Hansil, K. (2007). LMI approach to H_∞ optimal filter design for GPS receiver tracking loop. *IEEE International Symposium on Electrical & Electronics Engineering.* HCM City, Vietnam.

[34] Al-Baroudi, W. (1997). *Digital Filter Design using LMI Based Techniques.* A thesis. Faculty of The Graduate Studies, King Fahd University of Petroleum & Minerals.

[35] Wu, S.-P., Boyd, S., & Vandenberghe, L. (1996). FIR filter design via semidefinite programming and spectral factorization. *Proceedings of the 35th IEEE Decision and Control*, *1*, pp. 271 - 276.

[36] Rabiner, L., Graham, N., & Helms, H. (1974). Linear programming design of IIR digital filters with arbitrary magnitude function. *IEEE Transactions on Acoustics, Speech and Signal Processing*, *22*, pp. 117 - 123.

[37] *WiMax Forum*. Retrieved May 1, 2008. WiMax Forum : http://www.wimaxforum.org/home.

[38] Telecoms of Korea. (2008). Mobile WiMAX Handbook. *Mobile Monday Global Summit.* Kuala Lumpur, Malaysia.

[39] Pareek, D. (2006). *The Business of WiMAX*. John Wiley & Sons, Ltd.

[40] Finneran, M. F. (2004). *WiMax versus WiFi: A comparison of technologies, markets, and business plans*. Retrieved June 03, 2008. dBrn Associates : http://searchmobilecomputing.techtarget.com/searchMobileComputing/downloads/Finneran.pdf

[41] Staggs, E. (n.d.). Mobile WiMax radio phase locked loop design. Retrieved March 2008, Ansoft: http://www.ansoft.com/converge/staggs_ansoft.pdf

[42] Alkhairy, A., Christian, K., & Lim, J. (1991). Design of FIR filters by complex Chebyshev approximation. *Proceedings of the Acoustics, Speech, and Signal Processing* (pp. 1985 - 1988). Washington, DC, USA: IEEE Computer Society.

[43] CVX: matlab software for disciplined convex programming. Retrieved May 1, 2008. Stanford university: http://www.stanford.edu/~boyd/cvx/.

8. Appendix

8.1 IIR1 MatLab Code

```
% Maximize stopband attenuation of a lowpass IIR filter
% "Linear Programming Design of IIR Digital Filters with Arbitrary
%  Magnitude Functions" by Ayman AlQouqa

%
% Designs a lowpass IIR filter using spectral factorization method
where we:
% - minimize maximum stopband attenuation
% - have a constraint on the maximum passband ripple
%
%   minimize   max |H(w)|                      for w in the stopband
%       s.t.   1/delta <= |H(w)| <= delta      for w in the passband
%
% where we now have a rational frequency response function:
%
%   H(w) = sum_{m=0}^{M-1} b_m exp{-jwm} / sum_{n=1}^{N-1} a_n exp{-
jwn}
%
% We change variables via spectral factorization method and get:
%
%   minimize   max R(w)                        for w in the stopband
%       s.t.   (1/delta)^2 <= R(w) <= delta^2  for w in the passband
%              R(w) >= 0                       for all w
%
% where R(w) is the squared magnited of the frequency response
% (and the Fourier transform of the autocorrelation coefficients r).
% We represent R(w) = N_hat(w)/D_hat(w), where now R(w) is a rational
% function since we deal with IIR filter (see the reference paper).
%
% Variables are coeffients of the numerator, denoted as c, and
% denominator, denoted as d. delta is the allowed passband ripple.
% This is a quasiconvex problem and can be solved using bisection.
%

clear;clc;
%*********************************************************************
% user's filter specs (for a low-pass filter example)
%*********************************************************************
% number of coefficients for the IIR filter (including the zeroth one)
% (also without loss of generality we can assume that d_0 = 1, which
%  is the zeroth coefficient of the autocorrelation denominator)
M = 4; % nominator
N = 4; % denominator

wpass = .006*pi;   % end of the passband
wstop = .7*pi;   % start of the stopband
delta = 1;         % maximum passband ripple in dB (+/- around 0 dB)

%*********************************************************************
% create optimization parameters
%*********************************************************************
% rule-of-thumb discretization (from Cheney's Approx. Theory book)
```

```
sample_order =30;
m = 15*(sample_order);
w = linspace(0,pi,m)'; % omega

% A's are maiir1trices used to compute the power spectrum
Anum = [ones(m,1) 2*cos(kron(w,[1:M-1]))];
Aden = [ones(m,1) 2*cos(kron(w,[1:N-1]))];

% passband 0 <= w <= w_pass
ind = find((0 <= w) & (w <= wpass));    % passband
Ap_num  = Anum(ind,:);
Ap_den  = Aden(ind,:);

% transition band is not constrained (w_pass <= w <= w_stop)

% stopband (w_stop <= w)
ind = find((wstop <= w) & (w <= pi));   % stopband
As_num  = Anum(ind,:);
As_den  = Aden(ind,:);

%*********************************************************************
% optimization
%*********************************************************************
cvx_quiet(true);

% use bisection (on the log of vars) to solve for the min stopband
atten
Us_top = 1e-0; % 0 dB
Us_bot = 1e-6; % -60 dB (in original variables)

while( 20*log10(Us_top/Us_bot) > 1)
  % try to find a feasible design for given specs
  Us_cur = sqrt(Us_top*Us_bot);

  % formulate and solve the magnitude design problem
  cvx_begin
    variable c(M,1)
    variable d(N-1,1)

    % feasibility problem
    % passband constraints
    (Ap_num*c) <= (10^(+delta/20))^2*(Ap_den*[1;d]); % upper constr
    (Ap_num*c) >= (10^(-delta/20))^2*(Ap_den*[1;d]); % lower constr
    % stopband constraint
    (As_num*c) <= (Us_cur)*(As_den*[1;d]); % upper constr
    % nonnegative-real constraint
    Anum*c >= 0;
    Aden*[1;d] >= 0;
  cvx_end

  % bisection
  if ~any(isnan(c)) % feasible
    fprintf(1,'Problem is feasible for stopband atten = %3.2f dB\n',
...
               10*log10(Us_cur));
    Us_top = Us_cur;
    b = spectral_fact(c);
    a = spectral_fact([1;d]);
  else % not feasible
```

```
    fprintf(1,'Problem not feasible for stopband atten = %3.2f dB\n',
...
               10*log10(Us_cur));
    Us_bot = Us_cur;
  end
end

% display the max attenuation in the stopband (convert to original
vars)
fprintf(1,'\nOptimum min stopband atten is between %3.2f and %3.2f
dB.\n',...
        10*log10(Us_bot),10*log10(Us_top));
disp('Optimal IIR filter coefficients are: ')
disp('Numerator: '), b
disp('Denominator: '), a

cvx_quiet(false);

%*********************************************************************
% plotting routines
%*********************************************************************
% frequency response of the designed filter, where j = sqrt(-1)
H = ([exp(-j*kron(w,[0:M-1]))]*b)./([exp(-j*kron(w,[0:N-1]))]*a);

% magnitude plot
figure(1)
subplot(2,1,1)

plot(w,20*log10(abs(H)), ...
     [0 wpass],[delta delta],'r--', ...
     [0 wpass],[-delta -delta],'r--', ...
     [wstop pi],[10*log10(Us_top) 10*log10(Us_top)],'r--')
xlabel('w')
ylabel('mag H(w) in dB')
axis([0 pi -65 10]);
grid

% phase plot
subplot(2,1,2)
plot(w,angle(H))
axis([0,pi,-pi,pi])
xlabel('w'), ylabel('phase H(w)')
grid

%convert to analog
Ts=1e-008;
F=tf(b',a',Ts);
Fc=d2c(F);
```

8.2 FIR1 MatLab Code

```
% Minimize order of a linear phase lowpass FIR filter
% "Filter design" by Ayman Alqouqa

%
% Designs a linear phase FIR lowpass filter such that it:
% - minimizes the filter order
% - has a constraint on the maximum passband ripple
% - has a constraint on the maximum stopband attenuation
%
% This is a quasiconvex problem and can be solved using a bisection.
%
%   minimize   filter order n
%       s.t.   1/delta <= H(w) <= delta     for w in the passband
%              |H(w)| <= atten_level        for w in the stopband
%
% where H is the frequency response function and variable is
% the filter impulse response h (and its order/length).
% Data is delta (max passband ripple) and atten_level (max stopband
% attenuation level).
%



%*******************************************************************
% user's filter specifications
%*******************************************************************
% filter order that is used to start the bisection (has to be
feasible)
max_order = 20;

wpass = 0.006*pi;        % passband cutoff freq (in radians)
wstop = 0.2*pi;        % stopband start freq (in radians)
delta = 0.4;              % max (+/-) passband ripple in dB
atten_level = -44.5;      % stopband attenuation level in dB

%*******************************************************************
% create optimization parameters
%*******************************************************************
m = 30*max_order; % freq samples (rule-of-thumb)
w = linspace(0,pi,m);

%********************************************************************
% use bisection algorithm to solve the problem
%********************************************************************
cvx_quiet(true);

n_bot = 1;
n_top = max_order;

disp('Rememeber that we are only considering filters with linear
phase, i.e.,')
disp('filters that are symmetric around their midpoint and have order
2*n+1.')
disp(' ')

while( n_top - n_bot > 1)
  % try to find a feasible design for given specs
```

```
  n_cur = ceil( (n_top + n_bot)/2 );

  % create optimization matrices (this is cosine matrix)
  A = [ones(m,1) 2*cos(kron(w',[1:n_cur]))];

  % passband 0 <= w <= w_pass
  ind = find((0 <= w) & (w <= wpass));    % passband
  Ap  = A(ind,:);

  % transition band is not constrained (w_pass <= w <= w_stop)

  % stopband (w_stop <= w)
  ind = find((wstop <= w) & (w <= pi));   % stopband
  As  = A(ind,:);

  % formulate and solve the feasibility linear-phase lp filter design
  cvx_begin
    variable h_cur(n_cur+1,1);
    % feasibility problem
    % passband bounds
    Ap*h_cur <= 10^(delta/20);
    Ap*h_cur >= 10^(-delta/20);
    % stopband bounds
    abs( As*h_cur ) <= 10^(atten_level/20);
  cvx_end

  % bisection
  if strfind(cvx_status,'Solved') % feasible
    fprintf(1,'Problem is feasible for n = %d taps\n',n_cur);
    n_top = n_cur;
    % construct the full impulse response
    h = [flipud(h_cur(2:end)); h_cur];
  else % not feasible
    fprintf(1,'Problem not feasible for n = %d taps\n',n_cur);
    n_bot = n_cur;
  end
end

n = n_top;
fprintf(1,'\nOptimum number of filter taps for given specs is 2n+1 =
%d.\n',...
           2*n+1);

cvx_quiet(false);

%*********************************************************************
% plots
%*********************************************************************
figure(1)
% FIR impulse response
plot([0:2*n],h','o',[0:2*n],h','b:')
xlabel('t'), ylabel('h(t)')
grid
figure(2)
% frequency response
H = exp(-j*kron(w',[0:2*n]))*h;
% magnitude
subplot(2,1,1)
plot(w,20*log10(abs(H)),...
```

```
     [wstop pi],[atten_level atten_level],'r--',...
     [0 wpass],[delta delta],'r--',...
     [0 wpass],[-delta -delta],'r--');
axis([0,pi,-50,10])
xlabel('w'), ylabel('mag H(w) in dB')
grid
% phase
subplot(2,1,2)
plot(w,angle(H))
axis([0,pi,-pi,pi])
xlabel('w'), ylabel('phase H(w)')
grid
```

8.3 FIR2 MatLab Code

```
% Maximize stopband attenuation of a lowpass FIR filter (magnitude
design)
% "FIR Filter Design via Spectral Factorization and Convex
Optimization"
% by Ayman AlQouqa

%
% Designs an FIR lowpass filter using spectral factorization method
where we:
% - minimize maximum stopband attenuation
% - have a constraint on the maximum passband ripple
%
%   minimize   max |H(w)|                      for w in the stopband
%       s.t.   1/delta <= |H(w)| <= delta      for w in the passband
%
% We change variables via spectral factorization method and get:
%
%   minimize   max R(w)                        for w in the stopband
%       s.t.   (1/delta)^2 <= R(w) <= delta^2  for w in the passband
%              R(w) >= 0                       for all w
%
% where R(w) is the squared magnited of the frequency response
% (and the Fourier transform of the autocorrelation coefficients r).
% Variables are coeffients r. delta is the allowed passband ripple.
% This is a convex problem (can be formulated as an LP after
sampling).
%



%*********************************************************************
% user's filter specs (for a low-pass filter example)
%*********************************************************************
% number of FIR coefficients (including the zeroth one)
n = 19;

wpass = 0.006*pi;   % end of the passband
wstop = 0.2*pi;   % start of the stopband
delta = 0.4;         % maximum passband ripple in dB (+/- around 0 dB)

%*********************************************************************
% create optimization parameters
%*********************************************************************
% rule-of-thumb discretization (from Cheney's Approx. Theory book)
m = 15*n;
w = linspace(0,pi,m)'; % omega

% A is the matrix used to compute the power spectrum
% A(w,:) = [1 2*cos(w) 2*cos(2*w) ... 2*cos(n*w)]
A = [ones(m,1) 2*cos(kron(w,[1:n-1]))];

% passband 0 <= w <= w_pass
ind = find((0 <= w) & (w <= wpass));    % passband
Lp  = 10^(-delta/20)*ones(length(ind),1);
Up  = 10^(+delta/20)*ones(length(ind),1);
Ap  = A(ind,:);

% transition band is not constrained (w_pass <= w <= w_stop)
```

```
% stopband (w_stop <= w)
ind = find((wstop <= w) & (w <= pi));   % stopband
As  = A(ind,:);
AA = [zeros(1,n-1);eye(n-2) zeros(n-2,1)];
BB = [1 ;zeros(n-2,1)];

%*******************************************************************
% optimization
%*******************************************************************
% formulate and solve the magnitude design problem
cvx_begin sdp
  variable r(n,1)
  CC = r(2:n)'
  DD = 0.5*r(1)
  variable P(n-1,n-1) symmetric;
  % this is a feasibility problem
  minimize( max( abs( As*r ) ) )
  subject to
    % passband constraints
    Ap*r >= (Lp.^2);
    Ap*r <= (Up.^2);
    % nonnegative-real constraint for all frequencies (a bit
redundant)
    A*r >= 0;
    [P-AA'*P*AA, CC'-AA'*P*BB;...
        CC-BB'*P*AA, DD+DD'-BB'*P*BB]>=0;
cvx_end

% check if problem was successfully solved
disp(['Problem is ' cvx_status])
if ~strfind(cvx_status,'Solved')
  return
end

% compute the spectral factorization
h_lmi = spectral_fact(r);

% compute the max attenuation in the stopband (convert to original
vars)
Ustop = 10*log10(cvx_optval);
fprintf(1,'The max attenuation in the stopband is %3.2f
dB.\n\n',Ustop);

%********************************************************************
% plotting routines
%********************************************************************
% frequency response of the designed filter, where j = sqrt(-1)
H = [exp(-j*kron(w,[0:n-1]))]*h_lmi;

figure(1)
% FIR impulse response
plot([0:n-1],h_lmi','o',[0:n-1],h_lmi','b:')
xlabel('t'), ylabel('h(t)')
grid
figure(2)
% magnitude
subplot(2,1,1)
plot(w,20*log10(abs(H)), ...
     [0 wpass],[delta delta],'r--', ...
```

```
     [0 wpass],[-delta -delta],'r--', ...
     [wstop pi],[Ustop Ustop],'r--')
xlabel('w')
ylabel('mag H(w) in dB')
axis([0 pi -50 5])
grid
% phase
subplot(2,1,2)
plot(w,angle(H))
axis([0,pi,-pi,pi])
xlabel('w'), ylabel('phase H(w)')
grid
```

8.4 List of Acronyms

Acronyms	Meaning
BPL	Broadband over Power Line
CPE	Customer Premises Equipment
CSMA	Carrier Sense Multiple Access
CVX	Convex software
DES	Data Encryption Standard
DL	Down Link
DSCP	Differentiated Services Code Point
DSL	Digital Subscriber Line
EAP	Extensible Authentication Protocol
FDD	Frequency Division Duplexing
FIR	Finite Impulse Response
GPS	Global Positioning System
IIR	Infinite Impulse Response
IP	Internet Protocol
LAN	Local Area Network
LF	Loop Filter
LMI	Linear Matrix Inequality
LOS	Line of Sight
LP	Linear Programming
MAC	Media Access Control
MAN	Metropolitan Area Network
MIMO	Multi-Input Multi-Output
MPLS	Multi Protocol Label Switching
NLOS	Non Line of Sight
OFDMA	Orthogonal Frequency Division Multiple Access

PFD	Phase/Frequency Detector
PLL	Phase Locked Loop
QOS	Quality of Service
RAS	Remote Access Service
SDP	Semi-Definite Programming
SIM	Subscriber Identity Module
TDD	Time Division Duplexing
TDMA	Time Division Multiple Access
UP	Up Link
USIM	Universal Subscriber Identity Module
VCO	Voltage Controlled Oscillator
VoIP	Voice Over IP
WEP	Wired Equivalent Privacy
WiFi	Wireless Fidelity
WiMax	Worldwide Interoperability for Microwave Access
WMAN	Wireless Metropolitan Area Network
WPA	Windows Privacy Activation
XOR	Exclusive OR