August 2013

Essays on Mechanism Design and the Informed Principal Problem

Nicholas C. Bedard
The University of Western Ontario

Supervisor
Charles Z. Zheng
The University of Western Ontario

Graduate Program in Economics

A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy

Follow this and additional works at: https://ir.lib.uwo.ca/etd

Part of the Economic Theory Commons

Essays on Mechanism Design and the Informed Principal Problem

(Thesis Format: Integrated Article)

By

Nicholas C <u>Bedard</u>

Graduate Program in Economics

A thesis submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy.

The School of Graduate and Postdoctoral Studies
The University of Western Ontario
London, Ontario, Canada

Abstract

Three models of a privately informed contract designer (a principal) are examined. In the first, I study how much private information the principal wants to acquire before offering a contract to an agent. Despite allowing her to acquire all information for free, I prove in a general environment that there is a nontrivial set of parameters for which it is strictly suboptimal for the principal to be completely informed, regardless of the continuation equilibrium following any information acquisition choice. This result holds even when the principal is able to employ the most general mechanisms available and, in particular, when she can choose her most favourable full-information continuation equilibria. Further, in a specialized environment I characterize the principal's optimal information choice.

The second is a two-state principal-agent model with moral hazard in which the principal knows the state but the agent does not. This model is relevant to situations where an employer has private information about the productivity of a worker in a particular task while the worker has private information about the effort she exerts on the job. Much of the literature on this subject restricts the employer to offer contracts that leave her no discretion once a contract is accepted, while more general contracts may allow the employer to exercise discretion after acceptance; such contracts are called menu-contracts. I show when the employer can obtain strictly higher expected payoffs by offering menu-contracts than by offering the restricted contracts used in

the literature.

The final model studies the ability of a bidder in an auction to organize collusion among her rival bidders and the resulting impact of this collusion on the seller. Bidders valuations are private information. I show that in a two bidder, discrete, independent private-value auction, the seller earns less when a bidder can offer her rival a collusion proposal than in the absence of collusion. This contrasts with a celebrated result by Che and Kim [1] stating that for such auctions there is a mechanism that eliminates all the effects of collusion. Che and Kim and much of the literature assume an uninformed third-party organizes collusion.

References

[1] Yeon-Koo Che and Jinwoo Kim. Robustly collusion-proof implementation. *Econometrica*, 74(4):1063–1107, July 2006. (document)

Acknowledgements

Table of Contents

Introduction

The problem of an individual bargaining with private information is well known to be relevant to countless economic circumstances such as franchising, vertical contracting, public procurement, auctions and managerial compensation (see Maskin and Tirole [2] and Segal and Whinston [4]). On the other hand, such problems are technically challenging; even in its simplest manifestation, where one player has all the bargaining power, difficult issues arise (Myerson [3]). While a few notable works have tackled this problem, there remains significant gaps in the theoretical literature about how privately informed individuals behave when they have the power to shape their trading environment.

This thesis fills in some of these gaps. In the first chapter I consider a principal-agent model in which the principal decides how much private information to acquire before making an offer to the agent. I prove that for non-trivial parameters of the model it is strictly suboptimal for the principal to be completely informed. The intuition is that to convince the agent that she is contracting honestly given her private information, the principal may need to distort the allocation. This distortion can be very costly ex ante. Choosing to be partially ignorant frees the principal from these incentive constraints and partially mitigates the damage to her ex ante payoff. In much of the relevant literature the principal is assumed to be endowed with a fixed set of information; this chapter demonstrates that this assumption may

be undesirable. Within the small literature that does study the principal's incentives to acquire information, this chapter is the first work to take a mechanism design approach to the problem in a general environment. This generality is important since it allows the principal to make full strategic use of any information she acquires.

In the next chapter, I expand on this last point and in so doing highlight a potential oversight in the literature. In particular, I present a model where an employer has private information about the potential productivity of a worker in a specific task, who in turn has private information about the effort she exerts on the job. For example, consider a law firm which has advanced knowledge about the likelihood of winning a trial and needs to assign an attorney to the case. Suppose further that the law firm cannot observe the effort the attorney exerts for the case. Much of the literature on this subject restricts the employer to offer contracts that leave her no discretion once a contract is accepted, while more general mechanisms may allow the employer to exercise discretion *after* acceptance. Such contracts are called menu-contracts. For example, the law firm may be restricted to paying a wage based on the only observable outcome: whether the trial is won or lost. On the other hand, she could in addition specify bonuses to be paid that depend on the difficulty of the case. In this chapter I describe the advantages to the employer of presenting the worker with a set of potential contracts from which the employer will choose after the worker has accepted the offer. Specifically, in a two-state principal-agent model with moral hazard, I characterize environments in which the employer can obtain strictly higher expected payoffs by offering menu-contracts than the restricted contracts of the literature.

In the final chapter I study collusion by bidders in an auction. I depart from most of the literature by supposing that one of the bidders can propose the collusion contract. The standard approach to modelling collusion is to assume the collusion

mechanism is designed by an uninformed third party whose mandate is to maximize a weighted sum of the bidder's expected surplus. This construction avoids the informed principal problem but obscures a number of issues that are important to the modelling of collusion; in particular, the strategic consideration of the bidder who proposes collusion as well as the limitations or advantages that are present for the proposer due to her private information (i.e. her valuation of the good). Within the third party collusion framework, the literature has shown that if bidders can collude only after agreeing to participate in the auction, then the seller can design the auction such that her payoff is no less than if the bidders could not collude at all (see Che and Kim [1]). In contrast, in this chapter I present an example in a bidder-led collusion framework such that any appropriately refined equilibrium results in the seller receiving strictly less than the payoff she would expect if bidders could not collude. Further, I develop a framework to study the general mechanism design problem of the seller who faces bidders who can self-organize collusion.

References

[1] Yeon-Koo Che and Jinwoo Kim. Robustly collusion-proof implementation. *Econometrica*, 74(4):1063–1107, July 2006. (document)

[2] Eric Maskin and Jean Tirole. The Principal-Agent Relationship with an Informed Principal II: Common Values. *Econometrica*, 60(1):1–42, 1992. (document)

[3] Roger B Myerson. Mechanism design by an informed principal. *Econometrica*, 51(6):1767–1797, December 1983. (document)

[4] Ilya Segal and Michael D. Whinston. Robust Predictions for Bilateral Constracting With Externalities. *Econometrica*, 71(3):757–791, 2003. (document)

Chapter 1

The Strategically Ignorant

Principal

1.1 Introduction

The problem of a privately informed principal contracting with an agent is known
to be relevant to many real world situations, as noted by Akerlof [1], Myerson [13],
Maskin and Tirole [10, 11] and Segal and Whinston [17]. For example, an insurer
may know more than the client about the risks she faces, or a franchiser may have
private access to data about demand in the territory of a franchisee. As observed in
this literature, a privately informed principal's payoff can be constrained by her need
to convince the agent that she is contracting honestly, which can require inefficient
contracts (c.f. Akerlof [1] and Maskin and Tirole [11]).

This chapter studies the advantages to the principal of bypassing these constraints
by making the *strategic* choice to be ignorant. I consider a standard principal-agent
model and extend it by allowing the principal to costlessly learn about the state

before making an offer to the agent.[1] Importantly, I allow the principal to offer a menu of contracts from which she chooses one to implement *after* the agent has accepted (à la Segal and Whinston [17]).[2] This approach favours the acquisition of information; by contrast, the simpler alternative of the point-conrtact leaves the principal no discretion once a contract is accepted and thus subjects her to the agent's arbitrary off-path posterior beliefs which can deter her from exploiting her private information. For example, very inefficient contracts can be supported in equilibrium by punishing deviations from said contracts with agent's beliefs that put probability 1 on the worst possible state. Despite giving the principal full strategic flexibility to exploit her information, I prove that there are nontrivial sets of preferences and prior beliefs such that it is strictly suboptimal for the principal to acquire full information. This holds even if an informed principal can choose the continuation equilibrium she most desires.

In my framework, both the principal and the agent care about the state of the world and all choices, including the information acquisition choice, are observable. Formally, I study a static adverse selection model with common values as in Maskin and Tirole [11]. I relax the observability of the information choice in Section 1.6.

My first result, Theorem 1.1, proves under general conditions that there are always preferences such that the distortions required to make the menu offer incentive compatible are so severe that for nontrivial priors the principal finds it strictly suboptimal to be fully informed, regardless of continuation equilibria following any information acquisition choice.[3] This results holds despite the fact that information is free in our

---

[1]I use the terminology of the literature by naming the actor that makes offers the *principal* while the actor who responds the *agent*. The principal is labelled as such because she controls mechanism to be played and the opposing party must accept this choice passively. A more informative, though less standard, label for the agent may be the *subordinate* as used in Myerson [13].

[2]Menu contracts are fully general trading mechanisms due to the revelation principle.

[3]My strategic ignorance result is not to be confused with Myerson's [13] *inscrutability principle.* Myerson notes that the principal can never be worse off by not revealing private information when

model and would thus hold *a fortiori* under the more realistic assumption that it is costly to acquire information.

While the proof of this strategic ignorance result is technically complicated, the intuition is straightforward. I choose preferences for the principal such that the difference in payoff functions between two adjacent states is small. This creates an incentive for the principal to lie in one of these states, requiring distortion in the menu-contract to maintain the principal's incentive compatibility. The principal prefers to be uninformed in order to avoid this distortion ex ante. For tractability, this theorem is based on a set of priors under which the equilibrium payoff of the fully informed principal's continuation game is uniquely the lower bound equilibrium payoff of the game. Its formal proof and those of subsequent results are presented in the Appendix.

I next ask the question of whether ignorance can be an optimal strategy when there are other equilibria in the fully informed principal's continuation game that deliver payoffs greater than the lower bound. In the general case, only these lower bound payoffs can be computed; to establish the entire set of expected equilibrium payoffs I specialize to a quasilinear, binary state environment in Section 1.4. I go beyond Theorem 1.1 to prove not only that the answer to this next question is yes, but that ignorance is optimal for nontrivial set of parameters of the model even when the principal expects to attain her highest ex ante payoff conditional on becoming informed. Thus, ignorance can be optimal even when the principal has a nontrivial opportunity to choose which equilibrium is played, à la Myerson [13]. Moreover, I prove that the restrictions on preferences needed for Theorem 1.1 to hold are compatible with quasilinearity, and provide more precise restrictions on the preferences and priors for which the ignorance result holds.

offering her menu of contracts (thus remaining inscrutable to the agents at this stage), whereas our result claims that foregoing the acquisition of private information can strictly improve payoffs.

In Section 1.5 I consider the three state case to examine the subtleties of the model when the principal is no longer restricted to being either fully informed or completely uninformed. I prove that complete ignorance is optimal for the principal in a nonempty open set of priors for nontrivial preferences when there are three states of the world.[4] More generally, I characterize the optimal information acquisition choice depending on preferences and priors.

Finally, I show that when the information choice of the principal is not observed by the agent, there is still a nontrivial set of parameters of the model under which ignorance is chosen with positive probability in equilibrium.

1.1.1 Related Literature

The seminal work on the informed principal problem asks whether and how the principal can exploit her informational asymmetry (Myerson [13]; Maskin and Tirole [10, 11]). These papers endow the principal with information and do not consider her decision to acquire it.

Since, a handful of papers have looked at the principal's information acquisition problem. Nosal [15] and Crémer [5] study finite horizon principal-agent problems in which a principal can acquire information before offering a contract. In both papers, the information acquired by the principal becomes public before the contract is implemented; the principal therefore does not face the same distortionary incentive compatibility constraints that drive our results. Finkle [7] also studies the information acquisition decision of a principal. His principal covertly acquires private information for a cost *after* a contract has been signed but before the contract is implemented. Finkle considers only contracts that induce full information acquisition. My focus

---

[4]This result is nontrivial since there is always a nonempty open set of priors such that complete knowledge is optimal for any preferences and there are always preferences such that complete knowledge is optimal for all priors.

is different since I am concerned about how distortionary contracts can be improved upon by acquiring less than perfect information.

A number of recent papers study the informed principal problem in other environments. With multiple agents with stochastically dependant (privately known) types, Severinov [18] provides a construction that allows a privately informed principal to extract all social surplus. Thus, in this environment, the principal always wishes to obtain as much information as possible. Mylovanov and Tröger [14] focus on a linear, independent private values environment. In contrast to our common values environment, the principal can never lose by having private information but Mylovanov and Tröger determine when the principal is not strictly better off than when her information is public.

Particularly related to the current chapter, Silvers [19], Kaya [8], Chade and Silvers [4], and Beaudry [2] study the value of the principal's private information in games with moral hazard and identify parameters when the principal prefers to be ignorant. While I focus on pure adverse selection and do not consider moral hazard, I provide a deeper consideration of the incentives for a principal to acquire information. In each case, these authors restrict the principal to offer only point-contracts to the agent, which leaves her no discretion once a contract is accepted. In contrast I allow the principal to offer menu-contracts, which are fully general trading mechanisms in our framework. By Myerson's inscrutability principle, menu-contracts allow the principal to reveal no information until the agent has accepted the contract then reveal the state afterwards. This is more than a matter of technical generality. Menu-contracts preserve the strategic nature of the informed principal problem captured in the seminal work by Myerson [13] and Maskin and Tirole [10, 11], giving the principal the best opportunity to capitalize on her informational advantage. Moreover, a restriction to point-contracts can be used to exploit mistrust in the trading relationship by

using pessimistic posteriors to support very inefficient equilibria, thus increasing the relative value of ignorance. Allowing the principal to offer menu-contracts eliminates these mistrustful equilibria from the game.[5]

1.2 An Example

The following example illustrates the main results of this chapter as well as demonstrates the importance of considering fully general *menu-contracts* instead of simpler point-contracts.

Consider a car manufacturer (the principal) who is negotiating the sale of cars produced via a new production process to a dealership (the agent) who then resells the cars to consumers. Suppose there is some uncertainty in the new production process about how effectively paint can be applied to the cars: in state 1, the standard paint does not adhere properly and requires an additive that is only effective with black paint; in state 2, the standard paint can be applied successfully in the manufacturing process, allowing cars to be painted in any colour. Using the additive raises the cost of painting each car and the lack of variety reduces the demand for the car. Formally, in state i the cost to the manufacturer of producing y units of the good is $C^i(y) := c_i y$ with $0 < c_2 < c_1 < 8$; in state 1, the downstream inverse demand for the car is $P^1(y) := 8 - y$ while in state 2 it is $P^2(y) := 9 - y$. Thus, given contract (y, t), the payoff to the manufacturer in state i is $V^i(y, t) = t - c_i y$ while the payoff to the dealership is $W^i(y, t) = P^i(y)y - t$. Let π be the common prior belief that the state

---

[5]The importance of allowing more general mechanisms here is analogous to the work of Segal and Whinston [17]. By generalizing offers in a family of bilateral contracting games to allow for menu contracts, these authors are able to make robust predictions about the game in the sense that they must be satisfied by all equilibria in all such games. Whether restricting the principal to point-contracts has bite depends on the specific environment. In Chapter 2 of this dissertation I characterize moral hazard environments where the principal can get strictly higher ex ante payoffs when allowed to use more general mechanisms.

of the world is 1.

Consider the case where the manufacturer is informed of the state of the world and suppose the manufacturer can only offer a point-contract: a single pair (y,t). A restriction to point contracts admits very low payoff equilibria for the manufacturer in the contracting game described above. Consider first an extreme example where the dealership is highly mistrustful of the manufacturer and rejects any offer that would give her negative payoff in at least one state of the world. Formally, she believes the state is 1 with probability 1 for any offer (y,t) such that $P^1(y)y - t < 0$ and maintains her prior belief π otherwise. The optimal equilibrium point-contract for the manufacturer given these beliefs is $\left(y^{PC}, t^{PC}\right) = \left(\frac{8-c_1}{2}, \left(8 - \frac{8-c_1}{2}\right)\frac{8-c_1}{2}\right)$ regardless of the state. Note that $\left(y^{PC}, t^{PC}\right)$ gives the dealership zero payoff in state 1 and strictly positive payoff in state 2.

The game where the manufacturer can offer point-contracts has other equilibria, some of which are better for her than the one described above. For example, there is an equilibrium where the manufacturer offers $\left(y_1^{LCS}, t_1^{LCS}\right) := \left(\frac{8-c_1}{2}, \left(8 - \frac{8-c_1}{2}\right)\frac{8-c_1}{2}\right)$ in state 1 and

$$\left(y_2^{LCS}, t_2^{LCS}\right) := \operatorname*{argmax}_{(y,t)} \left\{ t - c_2 y : V^1(y_1^{LCS}, t_1^{LCS}) \geq V^1(y,t), P^2(y)y = t \right\} \quad (1.1)$$

in state 2. This is the least-cost separating equilibrium and is the best equilibrium for the manufacturer when she can only offer point-contracts.

Now consider the case where the manufacturer can offer menu-contracts. A menu-contract is a list of point-contracts offered to the dealership that gives the manufacturer the discretion to choose which contract to implement after the dealership has accepted. I will show that the ability to offer menu-contracts eliminates highly inefficient outcomes such as (y^{PC}, t^{PC}). In fact, menu-contracts guarantee that the

manufacturer's payoff is at least as high as in the least-cost separating equilibrium.

To see this, suppose the manufacturer offers the menu $\left\{ \left(y_1^{LCS}, t_1^{LCS} \right), \left(y_2^{LCS}, t_2^{LCS} \right) \right\}$ in both states the world. This menu is acceptable to the manufacturer *regardless of her belief*: it gives her non-negative payoff in each state of the world, assuming the manufacturer chooses optimally from the menu. Since we have imposed an incentive compatibility constraint for the manufacturer, this assumption is valid.[6] Thus, the manufacturer can always offer this menu-contract and obtain its payoff. It therefore provides a lower bound on the payoff the manufacturer expects to earn when she is able to offer menu-contracts. This menu-contract is called the Rothchilds-Stiglitz-Wilson (RSW) menu-contract.[7] It is introduced by Maskin and Tirole [11, p11] and it plays a important role in our analysis below. I present its technical definition and discuss its significance in Section 1.3.

I will now determine when the manufacturer prefers to learn the state of her production process and when she would rather be uninformed. Let $c_1 = 4$ and $c_2 = 2.9$. First note that the production efficient level of the good (i.e. the level that equates marginal revenue with marginal cost) is 2 in state 1 and 3.05 in state 2.

The informed manufacturer's problem potentially has multiple equilibria depending on priors which can give her higher payoffs than the RSW menu. Nevertheless, we will start with the RSW lower bound menu. The RSW menu is given by $\{(2, 12), (4, 20)\}$ and gives expected payoff

$$4\pi + 8.4(1 - \pi). \tag{1.2}$$

Notice that the production level of 4 in state 2 is inefficiently high: marginal cost

---

[6]See problem (1.1).

[7]Rothchild-Stiglitz-Wilson is a reference to the similar least cost separating contracts developed in the insurance models of Rothschild and Stiglitz [16] and Wilson [21].

is greater than marginal revenue; because the manufacturer's incentive constraint is violated at the efficient state 2 production level, production in this state must be increased so that the constraint just binds. State 1 production is always efficient because the manufacturer will never want to pretend to be in state 1 when it is state 2 (i.e. the downward incentive constraint for the manufacturer will never bind). Since the dealership gets zero rents regardless of how much information the manufacturer has acquired, the value of information for the manufacture, given the RSW payoff is earned when the state is learned, is decreasing in the production distortion of state 2. A smaller difference between the marginal cost of production in the two states generates bigger distortions in state 2 and therefore reduces the value of information for the manufacturer.

Meanwhile, the uninformed manufacturer solves the problem

$$\max_{(y,t)}\{t - (4\pi + 2.9(1 - \pi))y \mid (8\pi + 9(1 - \pi) - y)y - t \geq 0\}.$$

The value of this problem is

$$\left(\frac{4\pi + 6.1(1 - \pi)}{2}\right)^2. \tag{1.3}$$

Expression (1.3) is strictly greater than (1.2) if and only if $\pi < 0.82$. So ignorance is preferred when the manufacturer expects the RSW menu to be played in equilibrium as long as the prior is below a cut-off value. This is because the inefficiency in the RSW menu occurs only in state 2; the manufacturer has to expect that state 2 is sufficiency likely to occur to prefer ignorance.

Depending on priors, other menu-contract equilibria can exist that give higher payoffs to the informed manufacturer ex ante. In particular, in Section 1.4 we characterize the highest payoff the informed manufacturer can expect. Although the details

are beyond the scope of this section, one can show that being ignorant of the state delivers strictly higher payoffs for the manufacturer ex ante than any equilibrium menu-contract if and only if $\pi \in (0.62, 0.82)$.

I have discussed why this interval has an upper cut-off. To understand the lower bound on this interval consider that for low π the manufacturer can mitigate the inefficiency in state 2. Myerson's [13] inscrutability principle states that we can assume without loss of generality that the manufacturer offers the same menu in both states of the world. This implies that dealership evaluates the menu offer using her prior belief: i.e. she accepts the offer if and only if her participation constraint is satisfied on average:

$$\pi[(8 - y_1)y_1 - t_1] + (1 - \pi)[(9 - y_2)y_2 - t_2] \geq 0. \tag{1.4}$$

Now suppose we set y_2 to be efficient and at the same time increase t_1 and decrease t_2 until the manufacturer's incentive constraint is just satisfied. When we do this, the first term of (1.4) becomes negative but the second term becomes positive. For small enough π, (1.4) will be satisfied and the dealership will accept the menu. Meanwhile, the manufacturer earns the full expected trade surplus at this prior and therefore chooses to become informed.[8] As π increases, eventually full efficiency will not be attainable. In this example, when $\pi = 0.62$, it is just low enough that the closest the manufacturer can get to the efficient y_2 generates ex ante payoffs that are equal to the uninformed manufacturer's payoff.

Finally, in this example the highest payoff the informed manufacturer can achieve under point-contracts is the RSW payoff, by definition the least cost separating equi-

---

[8]While it is true that for small π the uninformed level of production is close to efficient, it will never reach full efficiency as long as π is positive. Since full efficiency is possible through the menu-contract constructed as described, being informed always dominates not being informed at these low levels of π.

librium.[9] For $\pi < 0.82$, even this payoff is less than the uninformed equilibrium payoff (1.3). In contrast, we can show that if $c_2 < 2.73$ (with $c_1 = 4$), there exists at least one menu-contract for any prior such that it is better to be informed. Thus, if we were only to look at point-contracts in this case (with $c_2 < 2.73$ and low enough π) we would conclude that the principal has a negative value of information whereas this value can be positive when menu-contracts are allowed.

1.3 The Model and Suboptimality of Full Information

The state space is $N = \{1, \ldots, n\}$ for $n < \infty$. The game proceeds in four stages. First, the principal makes an *information acquisition choice*: a partition of the state space. This choice is observable and verifiable and the principal privately observes the partition cell to which the state belongs. There is no cost associated with the information choice. Second, she offers a menu of contracts. Third, the agent accepts or rejects the offer. Rejection leaves all parties with zero payoff. Acceptance leads to the final stage where the principal chooses a contract from the menu and said contract is implemented. The principal and agent can commit to the menu-contract which the agent accepted.

A contract specifies an action-transfer pair $(y, t) \in \mathbb{R}^2$. In state $i \in N$, when contract (y, t) is implemented, the principal earns payoff $V^i(y, t)$ and the agent earns payoff $W^i(y, t)$. I follow the notational convention of Maskin and Tirole [11] by having superscripts on payoff functions indicate the state. Both functions V^i and W^i are continuously differentiable and concave in (y, t). Function V^i is increasing in t and

---

[9]It can be shown that no pooling equilibrium can ever be sustained: the state 1 manufacturer will always wish to deviate.

decreasing in y while W^i is increasing in y and decreasing in t. In addition, W^i is increasing in state i for almost all (y,t). I make no explicit assumptions about the principal's preferences over states although item (iii) in Assumption 1.1 below puts some structure over how the principal's marginal rates of substitution varies by state. Both parties are expected utility maximizers.

I adopt the following standard sorting assumption on preferences from Maskin and Tirole [11]. Subscripts on payoff functions denote partial derivatives: $V_y^i(y,t) = \partial V^i(y,t)/\partial y$, $V_t^i(y,t) = \partial V^i(y,t)/\partial t$ with agent's marginal payoffs defined analogously.

Assumption 1.1 (Sorting) *(i) $W_y^i(y,t) \geq 0$ for all $(y,t) \in \mathbb{R}^2$ and there is an $\epsilon > 0$ such that $V_y^i(y,t) < -\epsilon$, $V_t^i(y,t) > \epsilon$, $W_t^i(y,t) < -\epsilon$ for all $i \in N$ and all $(y,t) \in \mathbb{R}^2$;*

(ii) for all numbers $\bar{w}$ and $\bar{v}$ there exists a finite solution to the problem $\max V^i(y,t)$ subject to $\bar{v} \geq V^i(y,t)$ and $W^i(y,t) \geq \bar{w}$.

(iii) $-V_y^i(y,t)/V_t^i(y,t) > -V_y^j(y,t)/V_t^j(y,t)$ for all $i < j \in N$ and all $(y,t) \in \mathbb{R}^2$.

In this framework, the menu contracts described above are direct revelation mechanisms: a list of n contracts $\{(y_i, t_i)\}_{i=1}^n$ such that the principal offers the menu-contract in stage two of the game and chooses a contract from the menu to implement in stage four of the game. Due to the revelation principle, menu-contracts are fully general trading mechanisms.

An important menu-contract in the informed principal game is the RSW menu.[10] Introduced by Maskin and Tirole [11, p11], it generates the lower bound payoff for the informed principal and it plays a large role in our analysis below. I now present its technical definition then provide intuition about why it is the principal's lower

---

[10]RSW is an acronym for Rothchild-Stiglitz-Wilson. See footnote 7.

bound payoff.

Definition 1.1 *The RSW payoff for the principal in state j is the principal's lower bound payoff in that state. It is attained by solving the problem*

$$V_r^j := \max_{\{(y_k, t_k)\}_{k \in N}} V^j(y_j, t_j)$$

$$s.t. \ (RSW\text{-}IC[l,k]) \quad V^l(y_l, t_l) \geq V^l(y_k, t_k) \ for \ all \ l, k \in N; \ and$$

$$(RSW\text{-}IR[k]) \quad W^k(y_k, t_k) \geq 0 \ for \ all \ k \in N.$$

Denote by (y_j^r, t_j^r) the state j principal's contract in her solution to this problem. Let $\{(y_k^r, t_k^r)\}_{k \in N}$ denote the menu such that each (y_k^r, t_k^r) solves the RSW problem for all $k \in N$.

The RSW problem generates lower bound payoffs for the principal in state j since the agent will accept any RSW menu *regardless of her belief about the state of the world*.[11] To see this, note first that the RSW problem for the principal in state j specifies an entire menu: a contract for each state $k \in N$. This menu must be incentive compatible in *every* state $k \in N$, not just state j. Finally, this menu must guarantee the agent her reservation payoff *ex post* in *every* state. Thus, the agent will always accept an RSW menu. The principal in any state $j \in N$ can always deviate to her RSW menu and get payoff V_r^j.[12]

Theorem 1.1 *Suppose Assumption 1.1 holds. Then, for any set of payoffs $(W^1, \ldots, W^n)$ for the agent, there are payoffs functions $(V^1, \ldots, V^n)$ for the principal and a nonempty open set of priors such that for any priors in this set, the principal finds it strictly*

---

[11]In terms of Myerson [13], any feasible solution to the RSW problem is *safe*. The RSW menu for the principal in state j is the best safe menu in state j.

[12]For further discussion of RSW menus and a general characterization of equilibrium menus in this framework, see Maskin and Tirole [11]

suboptimal to be fully informed regardless of the continuation equilibria (in pure strate-
gies) following information acquisition.

The formal proof of this theorem and all subsequent results appear in Section 1.8. To prove this theorem, we restrict priors such that within the restricted set the equilibrium payoff of the fully informed principal's continuation game is uniquely the RSW payoff. That is, under the set of priors referred to in the theorem, the principal's payoff when fully informed is unique and is her lower bound payoff for the fully informed continuation game. In the next section we show that the strategic ignorance result holds when there are multiple equilibria with payoffs that are greater than the RSW payoff for the principal in all states.

1.4 Strategic Ignorance Despite Multiple Equilibria

In this section we specialize to the quasilinear, binary state environment. Here, we are able to characterize the entire set of equilibrium payoffs. I therefore go beyond Theorem 1.1 to prove that ignorance can be optimal even when there exist equilibrium payoffs higher than the RSW lower bound, and in particular that ignorance is optimal for nontrivial parameters of the model even when the principal expects to attain her highest ex ante payoff conditional on becoming informed. This is shown in Theorem 1.2. Thus ignorance can be optimal even when principal can choose from among multiple equilibria, conditional on being informed, via persuasion over the agent's beliefs (à la Myerson [13]).

1.4.1 Preferences and Supplemental Assumptions

Let $n = 2$. Given contract (y, t), the principal gets payoff $V^i(y, t) = t - C^i(y)$ for $i \in \{1, 2\}$ and the agent gets payoff $W^i(y, t) = U^i(y) - t$. Let $MC^i := dC^i/dy$ and $MU^i := dU^i/dy$ for all $i \in \{1, 2\}$. I will refer to C^i as the principal's *cost* in state i and U^i as the agent's *revenue* in state i.

I assume these payoff functions have the same properties as defined in the Introduction and satisfy Assumption 1.1. I make the following further assumptions on the principal's cost function.

Assumption 1.2 *For all states $i \in \{1, 2\}$: (i) C^i is strictly decreasing in i for all $y \neq 0$; and (ii) $dMC^i(\cdot)/dy$ is nondecreasing in i.*

Item (i) says that the principal and the agent agree about which state is the good state.[13] Item (ii) ensures that the RSW contract is unique and deterministic. For example, $C^i(y) = y^2 - iy + 2 - i$ satisfies all our assumptions for $y > 0$.

Since an information choice is a partition of the state space, for $n = 2$ the principal is either fully informed or completely ignorant. If the principal chooses not to learn the state, the offer in stage two is a single contract. Define $\pi := \pi_1$ as the probability that the state is 1. In this case, the contract is the solution to the uninformed principal's problem:

$$V_u(\pi) := \max_{(y,t)} \left\{ t - \pi C^1(y) - (1 - \pi)C^2(y) \mid \pi U^1(y) + (1 - \pi)U^2(y) - t \geq 0 \right\}. \quad (1.5)$$

An equilibrium consists of an information acquisition choice (either ignorance or knowledge) together with a contract for each known state and a list of accept/reject decisions from the agent corresponding to any information choice and menu offered

---

[13] This eases incentive compatibility requirements relative to the case where they disagree. I therefore expect the results to carry over to the latter case.

such that the information strategy, the offer, and list of the agent's decisions constitute a perfect Bayesian Nash equilibrium.

Define

$$\kappa := \sup_{y} MC^1(y)/MC^2(y) > 1.$$

The parameter κ measures the severity of distortions needed in an informed principal's menu to maintain incentive compatibility as a function of preferences.

1.4.2 Ignorance and the Best Ex Ante Informed Payoff

The following problem delivers the highest equilibrium payoff the principal can expect ex ante conditional on becoming informed. The **ex ante optimal informed principal's problem** is

$$V^*(\pi) := \max_{\{(y_i, t_i)\}_{i \in \{1,2\}}} \sum_{i \in \{1,2\}} \pi_i \left(t_i - C^i(y_i) \right) \tag{1.6}$$

$$s.t. \ (\text{IC}[i,j]) \quad t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in \{1,2\}$$

$$(\text{IR}) \quad \sum_{i \in \{1,2\}} \pi_i \left(U^i(y_i) - t_i \right) \geq 0$$

$$(\text{NB}[i]) \quad t_i - C^i(y_i) \geq V_i^r \text{ for all } i \in \{1,2\}.$$

The constraints NB[i] for $i \in N$ are the *non-blocking* constraints. They state that the informed principal cannot commit to a contract that gives her a payoff lower than her RSW payoff in any state. Maskin and Tirole's [11, p19] Theorem 1 proves that these constraints form sufficient and necessary conditions for a menu-contract to be an equilibrium.

Next, we define an ordering for menus among the principal in different states. One menu is superior to another if it delivers strictly higher payoff to the principal in at

least one state and at least as high a payoff in the other.

Definition 1.2 *A menu* $\{(y_i, t_i)\}_{i \in N}$ *is* **superior** *to another menu* $\{(y_i', t_i')\}_{i \in \{1,2\}}$ *if* $t_i - C^i(y_i) \geq t_i' - C^i(y_i')$ *for all* $i \in \{1, 2\}$ *and there exists* $j \in \{1, 2\}$ *such that* $t_j - C^j(y_j) > t_i' - C^j(y_j')$.

Our main result of this section says that there exist preferences such that even when the principal expects to earn V^* *and* that payoff is superior to her RSW payoff, she will still wish to remain ignorant of the state for a nontrivial set of priors.

Theorem 1.2 *Suppose Assumptions 1.1 and 1.2 hold. If κ is sufficiently close to 1, there exists a nonempty, open interval of priors such that, for any priors in this interval, the principal is uninformed regardless of the continuation equilibrium played following information acquisition* and *there are multiple continuation equilibria following information acquisition that are superior to the informed principal's RSW lower bound.*

In particular, for any preferences and priors π specified in the theorem, choosing to be ignorant delivers strictly higher payoff than becoming informed and earning payoff $V^*(\pi)$.

To discuss the intuition of Theorem 1.2 we define the first best menu of contracts.

Definition 1.3 *Let action y_i^E be called efficient in state $i \in \{1, 2\}$ if $MC(y_i^E) = MU(y_i^E)$. A menu is* **first best** *if it is efficient in both states. Define*

$$V^{FB}(\pi) := \pi \left(U^1(y_1^E) - C^1(y_1^E) \right) + (1 - \pi) \left(U^2(y_2^E) - C^2(y_2^E) \right)$$

to be the value of the first best menu to the principal ex ante.

Figure 1.1 illustrates the following intuition behind Theorem 1.2. I show in Lemma 1.9 in Section 1.8.2 that when κ is close to 1, RSW-IC[1,2] binds and as a result $y_2^r > y_2^E$. The RSW menu in the continuation game following full information acquisition is thus distorted away from the first best. For low π (lower than π^{FB} in Figure 1.1), the menu that solves problem (1.6) can completely mitigate this inefficiency and the principal can attain the first best payoff ex ante. As π increases, however, this become impossible to do and V^* eventually settles to the RSW lower bound payoff $V_r(\pi) := \pi V_r^1 + (1 - \pi)V_r^2$. I label this point π^r.

In Proposition 1.1 (to follow), we show that there exists preferences and $\pi^* \leq 1$ such that $V_u(\pi) > V_r(\pi)$ for all priors $\pi \in (0, \pi^*)$: ignorance generates a higher payoff than the expected RSW payoff for the principal. This can be seen in Figure 1.1. Further, in Proposition 1.2 we show that that V^* is continuous and that there exists preferences such that $\pi^r < \pi^*$. Thus, $V_u(\pi) - V^*(\pi) < 0$ for $\pi \in (0, \pi^{FB}]$ and $V_u(\pi) - V^*(\pi) > 0$ for $\pi \in [\pi^r, \pi^*)$. Since both V_u and V^* are continuous, the intermediate value theorem states there must be some $\pi' \in (\pi^{FB}, \pi^r)$ such that $V_u(\pi') = V^*(\pi')$.[14] Thus, for $\pi \in (\pi', \pi^r)$, we have $V_u(\pi) > V^*(\pi) > V_r(\pi)$: the statements of Theorem 1.2 hold.

The next proposition establishes the value of the ignorant principal's problem (1.5) relative to the ex ante RSW payoff and characterizes this relative value in terms of preferences and priors.

Proposition 1.1 *Suppose Assumptions 1.1 and 1.2 hold. If κ is sufficiently close to 1 then there exists $\pi^* \in (0, 1]$ such that for any priors $\pi \in (0, \pi^*)$, $V_u(\pi) > V_r(\pi)$: the principal strictly prefers her ignorant payoff to her informed ex ante RSW payoff; if $\pi \in (\pi^*, 1)$, then $V_u(\pi) < V_r(\pi)$. Moreover, there exists $\underline{\kappa}$ such that $\pi^* = 1$ if $\kappa < \underline{\kappa}$.*

Figure 2.1 illustrates the following intuition behind Proposition 1.1. Figure 1.2(a)

---

[14]If there are multiple such π', choose the largest.
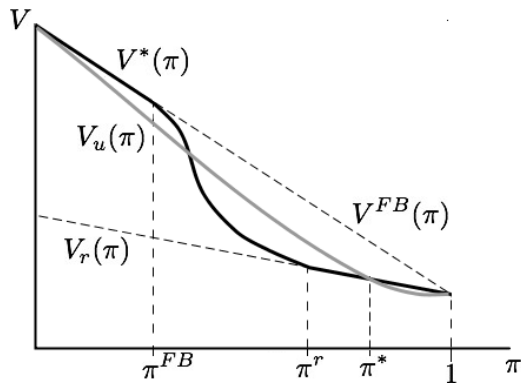
Figure 1.1: This figure illustrates Theorem 1.2. Note the nonempty, open set of priors such that $V_u(\pi) > V^*(\pi) > V_r(\pi)$.

plots, in (y, t)-space, the informed RSW solution when the informed principal is constrained by incentive compatibility. It illustrates how the RSW contract entails inefficiently high y in state 2 and efficient y in state 1. To see why the RSW action is efficient in state 1, note that the principal can offer the menu-contract $\left\{\left(y^E, U(y^E)\right), \left(y^E, U(y^E)\right)\right\}$. It is straight forward to check that this menu is ex post incentive compatible (i.e. satisfies RSW-IC[1,2] and RSW-IC[2,1]) and is individually rational for the agent in both states. Thus, $\left\{\left(y^E, U(y^E)\right), \left(y^E, U(y^E)\right)\right\}$ is an RSW menu for the principal in state 1. Since $\left(y^E, U(y^E)\right)$ is a tangency point on the agent's indifference curve at her reservation utility, it is the unique state contract that gives the state 1 principal her efficient payoff $U^1(y^E) - C^1(y^E)$ and therefore the unique state 1 contract in the RSW menu. The state 2 contract in the RSW menu is then the least cost separating equilibrium, as plotted in the figure. The Figure 1.2(b) plots the functions V_u and V_r when κ is sufficiently close to 1 that the state 2 RSW contract is inefficient.

Notice that the state i RSW problem is independent of priors; this implies that, even as the probability of state 2 approaches 1, the value of the RSW problem for the state 2 principal will be less than the value of the first-best menu. Meanwhile, the
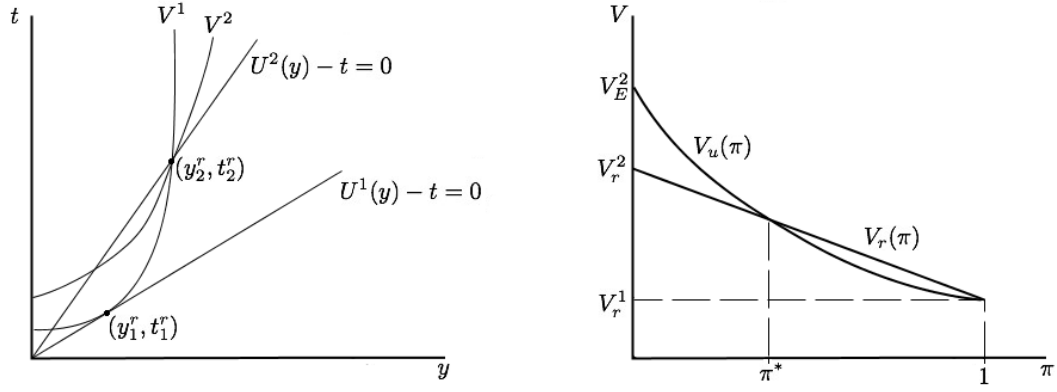
(a) V^i indicates the RSW indifference curve for the principal in state i.

(b) $V_2^E := U^2(y_2^E) - C^2(y_2^E)$ is the value of the efficient contract payoff to the principal in state 2.

Figure 1.2: Example of informed principal RSW solution and value function and uninfored value function when the principal is constrained by incentive compatility

uninformed principal is unburdened by incentive compatibility constraints and her ex post payoff approaches efficient levels as π approach 0 and 1. Further, the uninformed value function is convex in π. Since V_r is linear in π, these value functions must intersect at most twice as a function of π: once at $\pi = 1$, since the state 1 contract is always efficient when the principal is informed, and once at some $\pi \geq 0$. Denote the first intersection as π increases from 0 to 1 by π^*. As Proposition 1.1 asserts, $\pi^* > 0$ for κ close enough to 1. For all priors $\pi < \pi^*$, the uninformed principal's payoff will be higher ex ante than the informed principal RSW payoff.

Our next proposition states that there exists preferences and priors such that the optimal ex ante equilibrium payoff is achieved by being ignorant of the state, even when the principal expects to attain V^* upon becoming informed.

Proposition 1.2 *Suppose Assumptions 1.1 and 1.2 hold. If κ is sufficiently close to 1 then there exists $\pi^r < \pi^*$ such that for any priors $\pi \in (\pi^r, \pi^*)$, $V_u(\pi) > V^*(\pi) = V_r(\pi)$: the unique ex ante optimal informed payoff is the RSW payoff and the uninformed principal's payoff is strictly larger.*

24

Remark 1 While Propositions 1.1 and 1.2 may appear to be corollaries of Theorem 1.1, they are making stronger statements than such a corollary could make. First, our assumptions on preferences (i.e. that κ is sufficiently close to 1) restrict only the second order properties of the payoff functions rather than the entire function as in Theorem 1.1. Moreover, Theorem 1.1 could not be specific about which priors admit ignorance as an optimal strategy whereas the results in this section can.

The main task in the proof of Proposition 1.2 is to characterize the ex ante optimal informed principal problem (1.6). This allows us to prove the existence of π^r and, importantly, that it is strictly less than 1. Further, we show that V^* is continuous.

The existence of π^r is proved by demonstrating that for high enough π the state 2 RSW contract cannot be altered at all without violating either the state 1 principal's incentive compatibility constraint or the agent's individual rationality constraint. Thus, V^* must equal the ex ante RSW payoff for such priors. To see this, note that to improve on the RSW payoff we must reduce y_2^r closer to its efficient level: since the principal gets all gains from trade in the RSW payoff, the only way to increase her payoff is to increase the gains from trade. Decreasing y_2 requires that we deliver a higher payoff to the state 1 principal to maintain incentive compatibility. Since y_1^r is efficient, however, U^1 is tangent to C^1 at (y_1^r, t_1^r). This implies that the agent's payoff must be less than her reservation value in state 1. I can give the agent a payoff higher than her reservation value in state 2 as we move y_2 closer to y_2^E to balance out this state 1 deficit ex ante; if π is too large, however, we cannot give the agent a high enough surplus in state 2 to make up for the deficit in state 1 that is required to maintain incentive compatibility. I label π^r as the prior at which this point is just hit as π increases from 0 to 1 and we note that $\pi^r < 1$ since the state 1 indifference curve is everywhere steeper than the state 2 indifference curve. Hence, for $\pi \in [\pi^r, 1)$, we have $V^*(\pi) = V_r(\pi)$. Finally, we can appeal to Proposition 1.1 and choose κ close enough

to 1 such that $\pi^* > \pi_r$. Then for $\pi \in (\pi^r, \pi^*)$, we have $V^*(\pi) = V_r(\pi) < V_u(\pi)$.

The results in this section have so far used the distortionary effects of the incentive constraints conditional on the principal being informed as a sufficient condition for ignorance of the state to be of strategic advantage. The final proposition of this section shows that binding incentive constraints in the menu offered by the informed principal are also necessary.

Proposition 1.3 *If RSW-IC[1,2] does not bind, then ignorance will never be chosen in equilibrium. Moreover, the informed RSW problem generates the first best menu and the unique equilibrium payoff for all priors.*

1.5 Optimal Information Structure: Three States

In this section we consider the three state case to examine the subtleties of the model when the principal no longer faces a binary choice of information acquisition. She can now choose *how* informed or *how* ignorant she wishes to be. I show that complete ignorance of the state is optimal for the principal in a nonempty open set of priors for nontrivial preferences. More generally, we characterize optimal information acquisition choice depending on preferences and priors. Further, we find that if the principal is exogenously restricted to choosing between complete knowledge of the state or complete ignorance, there are preferences and a nonempty open set of priors such that complete ignorance is preferred.

1.5.1 General Information Structures

An information choice by the principal consists of any partition of the set N. Let $\mathscr{P}$ be the set of all partitions of N. I will refer to $p \in \mathscr{P}$ as an information acquisition option; the ith cell of p is denoted p_i and is referred to as an information set. Given

information acquisition option p, the state space becomes p in a new informed principal problem with typical state p_i. A choice of information option p generates payoff functions

$$C^{p_i}(y) := \left(\frac{1}{\sum_{j \in p_i} \pi_j} \right) \sum_{j \in p_i} \pi_j C^j(y)$$

$$U^{p_i}(y) := \left(\frac{1}{\sum_{j \in p_i} \pi_j} \right) \sum_{j \in p_i} \pi_j U^j(y)$$

for each information set $p_i \in p$. Associated with each $p \in \mathscr{P}$ there is an RSW menu which we denote the p-RSW menu.[15]

Our goal is to analyze the optimal information acquisition options in this environment. As in the case of two states, we use the closeness of the relative marginal costs between states to measure the severity of the distortions introduced by the incentive constraints. Since the information acquisition choice is no longer binary, however, we require a second parameter. The second measures the *separateness* of the relative marginal costs between states. Whereas the first provided us with sufficient conditions for ignorance between two states, the second will provide sufficient conditions for the principal to be informed of the two states. Define the following

$$\kappa_i^S := \sup_y \frac{MC^i(y)}{MC^{i+1}(y)}; \text{and} \tag{1.7}$$

$$\kappa_i^I := \inf_{y > y_i^E} \frac{MC^i(y)}{MC^{i+1}(y)} \tag{1.8}$$

for all $i \in N \setminus \{n\}$ where y_i^E satisfies $MU^i(y_i^E) = MC^i(y_i^E)$.

---

[15]See Section 1.8.3 for a formal description of the p-RSW menu.

1.5.2 Three states of the world

Our result in this section identifies sufficient conditions for certain information acquisition strategies to be optimal. Figure 1.3, panels (a) to (c) indicate (the shaded areas) the priors under which Proposition 1.4 parts (A) to (C) apply respectively in a 3 dimensional simplex.

Proposition 1.4 *Suppose Assumptions 1.1 and 1.2 hold. Let $\Delta_o^3 := \{\pi \in (0,1)^3 | \sum_i \pi_i = 1\}$ be the set of non-degenerate priors and $p^1 = \{\{1,2\},\{3\}\}, p^2 = \{\{1\},\{2,3\}\},$ $p^3 = \{\{1,2,3\}\}, p^4 = \{\{1\},\{2\},\{3\}\},$ and $p^5 = \{\{1,3\},\{2\}\}.$*

(A) There exists $\bar{\kappa}_1^S > 1$ and $\underline{\kappa}_2^I$ such that for $\kappa_1^S < \bar{\kappa}_1^S$ and $\kappa_2^I > \underline{\kappa}_2^I$, there exists $\hat{\pi} \in \Delta_o^3$ such that for any

$$
\pi \in \left\{ \pi' \in \Delta^3 \;\middle|\; \begin{array}{c} \pi_1 \in (\pi_1', 1), \frac{\pi_1'}{\pi_1'+\pi_2'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1+\hat{\pi}_2}, 1\right), \frac{\pi_2'}{\pi_2'+\pi_3'} \in \left(\frac{\hat{\pi}_2}{\hat{\pi}_2+\hat{\pi}_3}, 1\right) \\ \pi_1' + \pi_3' \in (\hat{\pi}_1 + \hat{\pi}_3, 1), \frac{\pi_1'}{\pi_1'+\pi_3'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1+\hat{\pi}_3}, 1\right) \end{array} \right\}
$$

the optimal information acquisition option is p^1;

(B) There exists $\bar{\kappa}_2^S > 1$ and $\underline{\kappa}_1^I$ such that for $\kappa_2^S > \bar{\kappa}_2^S$ and $\kappa_1^I < \underline{\kappa}_2^I$, then there exists $\hat{\pi} \in \Delta_o^3$ such that for any

$$
\pi \in \left\{ \pi' \in \Delta^3 \;\middle|\; \begin{array}{c} \pi_2' \in (\hat{\pi}_2, 1), \frac{\pi_2'}{\pi_2'+\pi_3'} \in \left(\frac{\hat{\pi}_2}{\hat{\pi}_2+\hat{\pi}_3}, 1\right), \\ \frac{\pi_1'}{\pi_1'+\pi_2'} \in \left(0, \frac{\hat{\pi}_1}{\hat{\pi}_1+\hat{\pi}_2}\right), \frac{\pi_1'}{\pi_1'+\pi_3'} \in \left(0, \frac{\hat{\pi}_1}{\hat{\pi}_1+\hat{\pi}_3}\right) \end{array} \right\}
$$

the optimal information acquisition option is p^2; and

(C) There exists $\underline{\kappa}_1^S > 1$ and $\underline{\kappa}_2^S > 1$ such that if $\kappa_1^S < \underline{\kappa}_1^S$ and $\kappa_2^S < \underline{\kappa}_2^S$, then there

exists $\hat{\pi} \in \Delta_o^3$ such that for any

$$\pi \in \left\{ \pi' \in \Delta^3 \;\middle|\; \begin{array}{c} \pi_1' + \pi_2' \in (\hat{\pi}_1 + \hat{\pi}_2, 1), \, \pi_1' \in (\hat{\pi}_1, 1), \\ \frac{\pi_1'}{\pi_1' + \pi_2'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_2}, 1 \right), \, \pi_1' \in (\hat{\pi}_1, 1), \, \frac{\pi_1'}{\pi_1' + \pi_3'} \in \left(\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_3}, 1 \right) \end{array} \right\}$$

the optimal information acquisition option is p^3.

(D) *There exists $\bar{\kappa}_1^I > 1$ and $\bar{\kappa}_2^I > 1$ such that if $\kappa_1^I > \bar{\kappa}_1^I$ and $\kappa_2^I > \bar{\kappa}_2^I$ then the optimal information acquisition option is p^4. Moreover, there exists $\hat{\pi} \in \Delta_o^3$ such that if $\pi_3 \in (\hat{\pi}_3, 1)$ then the optimal information acquisition option is p^4.*

(a) p^1 optimal information acquisition option (κ_1^S close to 1, κ_2^I large).

(b) p^2 optimal information acquisition option (κ_2^S close to 1, κ_1^I large).

(c) p^3 optimal information acquisition option (both κ_1^S and κ_2^S close to 1).

(d) Full ignorance preferred to full knowledge in restricted game (κ_1^S close to 1, κ_2^I large).
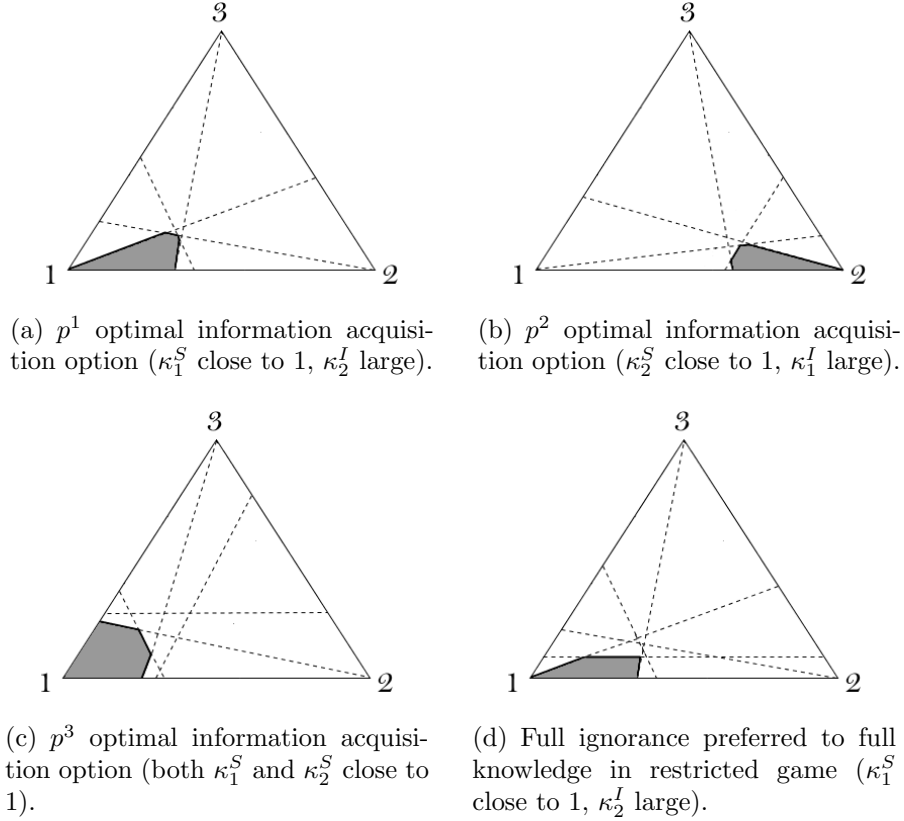
Figure 1.3: Proposition 1.4 and Corollary 1 are illustrated in this figure. The labels on the vertices indicate the probability-one state. The dashed lines represent the restrictions on priors stipulated in the propositions. Panels (a) to(c) demonstrate the priors under which Proposition 1.4 (A), (B), and (C) apply respectively. Panel (d), indicate priors under which Corollary 1 applies if κ_1^S close to 1 and κ_2^I large.

Notice that information acquisition options p^1, p^2 and p^5 are two-cell partitions. In the proof of Proposition 1.4, we treat these as two state informed principal problems to which we can apply Proposition 1.2 to compare their values to the fully ignorant information acquisition strategy p^3 and characterize the priors and preferences under which they are preferred to p^3 or vice versa. This is straightforward for p^1 and p^2 – they induce preferences that conform to Assumptions 1.1 and 1.2 – but to use Proposition 1.2 on p^5 we must first ensure that the payoff functions it generates conform to Assumptions 1.1 and 1.2. For parts (A) and (C) this is done by restricting priors such that π_1 is large relative to π_3 so that event $\{1,3\}$ is analogous to state 1 in Section 1.4 and for part (B) we restrict priors such that π_1 is small relative to π_3 so that event $\{1,3\}$ is analogous to state 2. Comparing the values from these two-cell partitions to the fully informed information acquisition strategy p^4 and characterizing the priors and preferences under which they are preferred to p^4 or vice versa uses techniques similar to those used to prove Theorem 1.1.

I have no theory to directly compare the value of the two-cell partitions to each other, or to directly compare the fully informed payoff to the fully ignorant payoff. To characterize the priors and preferences under which one is preferred to the other in each case, we use indirect comparisons over which Proposition 1.2 can be used.

Take for example item (A) of Proposition 1.4. I first note that in the continuation game following information acquisition option p^1 is a two state informed principal game and the p^1-RSW menu is first best, given the information acquisition option. Therefore, by Proposition 1.3, the principal must prefer p^1 to the fully ignorant option p^3. Next we characterize priors under which the p^4-RSW payoff is the unique payoff following information acquisition option p^4 *and* the p^4-RSW payoff is strictly lower than any p^1 equilibrium payoff using Proposition 1.2; this requires that π_1 is sufficiently close to 1 and sufficiently larger than π_2 respectively.

The next two steps compare the value of choosing information acquisition strategy p^1 to p^2 and p^5 *indirectly* by comparing the latter values to information acquisition option p^3. The p^2-RSW payoff is the unique payoff following information acquisition option p^2 *and* the p^2-RSW payoff is strictly lower than any p^3 equilibrium payoff if π_1 is sufficiently close to 1 and π_2 sufficiently larger than π_3. To use Proposition 1.2 to compare p^5 to p^3 we need to ensure that p^5 conforms to Assumptions 1.1 and 1.2. This is so if π_1 is sufficiently larger than π_3. Then, applying Proposition 1.2, the p^5-RSW payoff is the unique payoff following information acquisition option p^5 *and* the p^5-RSW payoff is strictly lower than the p^3 payoff if $\pi_1 + \pi_3$ is sufficiently close to 1.

Thus, we have developed a set of restriction on priors such that within this set of priors, ex ante, the principal knows that if she chooses any information acquisition option other than p^1, she will attain her RSW payoff for that information acquisition option and this payoff is necessarily less than the payoff to choosing information acquisition option p^1. I note that this intersection is open and nonempty, since any priors such that π_1 is sufficiently large (but less than 1) and π_2 is sufficiently larger than π_3 is in this intersection.

In the final result in this section, we present a corollary to Proposition 1.4 where we consider an environment in which it is technologically infeasible for the principal to choose *any* partition of N. In particular, we suppose that she is restricted to choosing either to acquire full information or no information.

Corollary 1 *Suppose the principal was restricted to choose between complete knowledge and complete ignorance. If either κ_1^S or κ_2^S is close to 1, (so some ignorance is desired in the unrestricted game) there is a nonempty set of priors for which the principal prefers complete ignorance.*

Figure 1.3, panel (d) indicate the priors under which Corollary 1 applies in a 3 dimensional simplex if κ_1^S close to 1 and κ_2^I large.

1.6 Information Acquisition as Hidden Action

In this section we examine the case where the principal's information acquisition decision is her private information. The problem becomes one of an informed principal with three states in which one of the states is endogenously chosen by the principal: the informed principal in each of the two states and the uninformed state of the principal.

A menu-contract is a list $\{(y_0, t_0), (y_1, t_1), (y_2, t_2)\}$ where state 0 is the uninformed state. Let $\alpha \in [0, 1]$ denote the probability that the principal becomes informed. Thus, α is the principal's information acquisition strategy. Finally, define $C^0(y) := \pi C^1(y) + (1-\pi)C^2(y)$ to be the expected cost of implementing effort y for the principal and $U^0(y) := \pi U^1(y) + (1 - \pi)U^2(y)$ to be the expected revenue of effort y for the agent.

Our first result shows that there is always an equilibrium where the principal is informed with zero probability.

Lemma 1.1 *There always exists an equilibrium with $\alpha = 0$.*

On the other hand, we assert in our next proposition that a payoff equivalent equilibrium exists in which the principal is uninformed with strictly positive probability if κ is close enough to 1.

Proposition 1.5 *Suppose Assumptions 1.1 and 1.2 hold. If κ is sufficiently close to 1, then there exists a nonempty open interval of priors such that the principal remains ignorant with positive probability.*

As shown in the proof of Lemma 1.1, incentive compatibility ensures that the payoff to the uninformed principal will never be larger than that of the informed principal in expectation. To prove Proposition 1.5, we start with an equilibrium where the principal acquires information with zero probability and construct a payoff equivalent equilibrium where she acquires information with strictly positive probability. As long as κ is sufficiently small, there is an interval of priors such that the contract is inefficient in at least one state. This allows us to increase the agent's payoff while maintaining the principal's payoff, thus creating a surplus for the agent in this state. By choosing a sufficiently low but positive α, we can leverage this surplus to increase the payoff to the uninformed principal sufficiently high to make her indifferent between being informed and being ignorant while maintaining the individual rationality constraint. The formal construction of the payoff equivalent contract is demonstrated in the proof.

1.7   Conclusion

I have studied a principal-agent problem where the principal can decide how much private information to (costlessly) acquire before offering a contract to an uninformed agent. Importantly, the state is directly payoff relevant to both the principal and the agent. In this setting I have found that the principal will not choose to be completely informed of the state for some priors as long as her payoffs between at least two states of the world are sufficiently close. Indeed, this result holds regardless of the continuation equilibrium played following any information acquisition choice and is robust to the existence of multiple equilibria in the informed principal continuation game. I show further, in a three state, quasilinear environment, that the principal chooses to be completely ignorant of the state for nontrivial parameters of the model. Notably,

these results were obtained in a full mechanism design framework: the principal was given full strategic flexibility to make use of whatever information she decides to acquire.

1.8 Proofs

I assume that incentive compatibility constraints are still imposed at degenerate priors.

1.8.1 The Suboptimality of Full Information

Proof of Theorem 1.1   Before we prove Theorem 1.1, we first describe how Assumption 1.1 simplifies the computation of an RSW menu according to Proposition 2 in Maskin and Tirole [11, p12].

Proposition 2, Maskin and Tirole [11] *Suppose Assumption 1.1 holds. The RSW allocation (within the class of deterministic solutions) is obtained by successively solving the following programs:*

$$\max_{(y_1,t_1)} V^1(y_1,t_1) \tag{RSW^1}$$

$$s.t. \ (RSW\text{-}IR[1]) \ W^1(y_1,t_1) = 0$$

and for all $k = 2,\ldots,n,$ *given* $(y_1,t_1),\ldots,(y_{k-1},t_{k-1})$

$$\max_{(y_k,t_k)} V^k(y_k,t_k) \tag{RSW^k}$$

$$s.t. \ (RSW\text{-}IC[k\text{-}1,k]) \ V^k(y_{k-1},t_{k-1}) \geq V^{k-1}(y_k,t_k); \ and$$

$$(RSW\text{-}IR[k]) \quad W^k(y_k,t_k) = 0$$

Further, $y_{k-1} < y_k$ *and* $t_{k-1} < t_k$ *for all* $k = 2, \ldots, n$.

Remark 2 Note that, (i) the RSW individual rationality constraints in each state always bind; (ii) of all the incentive compatibility constraints, only those of the form RSW-IC$[j, j + 1]$ for all $j \in \{1, \ldots, n - 1\}$ can possibly bind; (iii) the constraint RSW-IC$[j - 1, j]$ only shows up in the RSW problem of the principal in state j; (iv) the choice variable in each state is now a single contract rather than a full menu; and (v) (y_j^r, t_j^r) is strictly increasing in the state j.

Let $\pi \in \Delta^n := \left\{ \hat{\pi} \in [0, 1]^n : \sum_{i \in N} \hat{\pi}_i = 1 \right\}$ be the common prior belief over the state space N. I begin by defining two information acquisition options for the principal (one partially ignorant, one fully informed) and their payoffs. Choose any $i \in N$ and consider:

(a) FI: The *full information option* reveals the precise state before the contract is offered;

(b) PI: The *partial ignorance option* reveals all states precisely unless that state is either i or $i + 1$; if the state is either i or $i + 1$, it is only revealed that the state is in $\{i, i + 1\}$.

I refer to the continuation game following the information acquisition option FI as the *original game* and the continuation game following the information acquisition option PI as the *modified game*. Our goal is to compare the ex ante RSW payoffs for each game.

Consider the principal in the interim stage who knows that the state is in $\{i, i+1\}$; call her the $\{i, i + 1\}$-state principal. Let

$$\alpha = \frac{\pi_i}{\pi_i + \pi_{i+1}}.$$

The $\{i, i+1\}$-state principal's interim expected payoff from choosing FI is

$V_{FI}^{\{i,i+1\}}(\alpha) := \alpha V_r^i + (1-\alpha)V_r^{i+1}$.

Consider the modified game that treats $\{i, i+1\}$ as a single state: the state space is $\hat{N} = \{1, \ldots, i-1, \{i, i+1\}, i+2, \ldots, n\}$, the principal has payoff $V^j(y, t)$ and the agent has payoff $W^j(y, t)$ in all states $j = 1, \ldots, i-1, i+2, \ldots, n$ and payoffs $V^{\{i,i+1\}}(y, t) := \alpha V^i(y, t) + (1-\alpha)V^{i+1}(y, t)$ and $W^{\{i,i+1\}}(y, t) := \alpha W^i(y, t) + (1-\alpha)W^{i+1}(y, t)$ respectively in state $\{i, i+1\}$, given contract (y, t).

The following lemma establishes the state $\{i, i+1\}$ RSW problem for the principal who chooses PI.

Lemma 1.2 *The interim expected payoff for the principal from playing PI is represented by the problem*

$$V_{PI}^{i,i+1}(\alpha) := \max_{(y,t)} \ \alpha V^i(y, t) + (1-\alpha)V^{i+1}(y, t) \tag{1.9}$$

$$s.t \ \ \alpha W^i(y, t) + (1-\alpha)W^{i+1}(y, t) = 0$$

$$V^{i-1}(y_{i-1}^r, t_{i-1}^r) \geq V^{i-1}(y, t).$$

Proof The result follows from Proposition 2 of Maskin and Tirole [11] if we can show that the modified game with state space $\hat{N} = \{1, \ldots, \{i, i+1\}, \ldots, n\}$ satisfies the associated Sorting Assumption 1.1. In the modified game, we treat the combined states $\{i, i+1\}$ as a single state.

By inspection, items (i) and (ii) of Sorting Assumption 1.1 are satisfied in the modified game. For item (iii) we need to show that $-\frac{V_y^{i+2}}{V_t^{i+2}} < -\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} <$

$-\frac{V_y^{i-1}}{V_t^{i-1}}$.[16] Recall that $V_t > 0$ and $V_y < 0$. Assumption 1.1 for the original game has

$$-\frac{V_y^i}{V_t^i} > -\frac{V_y^{i+1}}{V_t^{i+1}} \Leftrightarrow -V_y^i V_t^{i+1} > -V_y^{i+1} V_t^i. \tag{1.10}$$

Then

$$
\begin{aligned}
-\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} &= -\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} \cdot \frac{V_t^{i+1}}{V_t^{i+1}} \\
&> \frac{-\alpha V_y^{i+1}V_t^i - (1-\alpha)V_y^{i+1}V_t^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} \cdot \frac{1}{V_t^{i+1}} \\
&= -\frac{V_y^{i+1}}{V_t^{i+1}} > -\frac{V_y^{i+2}}{V_t^{i+2}} \tag{1.11}
\end{aligned}
$$

where the first inequality follows from inequality (1.10) and the second results from the Sorting Assumption 1.1. And, by a symmetric argument $-\frac{\alpha V_y^i + (1-\alpha)V_y^{i+1}}{\alpha V_t^i + (1-\alpha)V_t^{i+1}} < -\frac{V_y^{i-1}}{V_t^{i-1}}$ as needed. ∎

Denote by $(y(\alpha), t(\alpha))$ the solution to this problem. The following four lemmas characterize $V_{PI}^{\{i,i+1\}}$ and bound it from below.

Lemma 1.3 $V_{PI}^{\{i,i+1\}}(1) = V_r^i$.

Proof By Proposition 2 of Maskin and Tirole [11],

$$V_r^i = \max_{(y_i, t_i)} \left\{ V^i(y_i, t_i) : V^{i-1}(y_{i-1}^r, t_{i-1}^r) \geq V^{i-1}(y_i, t_i) \text{ and } W^i(y_i, t_i) = 0 \right\}.$$

Problem (1.9) at $\alpha = 1$ is

$$V_{PI}^{\{i,i+1\}}(1) = \max_{(y,t)} \left\{ V^i(y, t) : V^{i-1}(y_{i-1}^r, t_{i-1}^r) \geq V^i(y, t) \text{ and } W^i(y, t) = 0 \right\}$$

due to the previous lemma. These problems are equivalent. ∎

---

[16]Recall that the subscripts on the payoff functions indicate partial derivatives.

Lemma 1.4 *The payoff to the information acquisition option PI can be expressed as*

$$V_{PI}^{i,i+1}(\alpha) = V_r^i - \int_\alpha^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da$$

$$- \int_\alpha^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da.$$

Proof Consider the optimization problem (1.9). By the integral form of the envelope theorem (Milgrom and Segal, Corollary 5, [12]), its value is

$$V_{PI}^{\{i,i+1\}}(\alpha) = V^{i+1}(y(0), t(0)) + \int_0^\alpha \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da \quad (1.12)$$

$$+ \int_0^\alpha \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da$$

where λ is the multiplier on the first constraint. Simple algebra on equation (1.12) shows that

$$V_{PI}^{\{i,i+1\}}(\alpha) = V^{i+1}(y(0), t(0)) + \int_0^\alpha \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da \quad (1.13)$$

$$+ \int_0^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da$$

$$- \int_\alpha^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da.$$

By Lemma 1.3 we can plug V_r^i in for $V_{PI}^{\{i,i+1\}}(1)$ in equation (1.13) evaluated at $\alpha = 1$ and rearrange to get

$$\int_0^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da = \quad (1.14)$$

$$V_r^i - V^{i+1}(y(0), t(0)) - \int_0^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da$$

Now plug (1.14) into (1.13) to get

$$V_{PI}^{\{i,i+1\}}(\alpha) = V_r^i - \int_\alpha^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a)) \right) da$$
$$- \int_\alpha^1 \lambda(a) \left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a)) \right) da.$$

as needed. ∎

Lemma 1.5 *Let $\mathcal{V}$ denote the set of payoff functions for the principal that satisfy all our assumptions with typical element $V = (V^1, \ldots, V^n)$. For any $V \in \mathcal{V}$, define*

$$M(\alpha; V) := \frac{-V_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \left[\frac{V_y^i(y(\alpha), t(\alpha))}{V_t^i(y(\alpha), t(\alpha))} - \frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha))} \right]}{-\frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha))} W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + W_y^{\{i,i+1\}}(y(\alpha), t(\alpha))},$$

$$M(V) := \min_{\alpha \in [0,1]} M(\alpha; V).$$

Choose small $\delta > 0$ such that $\bar{\mathcal{V}} := \{V \in \mathcal{V} : M(V) > \delta\} \neq \varnothing$. Then for all $V \in \bar{\mathcal{V}}$, $\lambda(\alpha) > \delta$ for any $\alpha \in [0, 1]$.

Proof I claim that $M(\alpha; V)$ is well defined and strictly positive for all α and V. To see this, note first that by the Sorting Assumption 1.1 the numerator in $M(\alpha; V)$ is strictly negative for all $\alpha \in [0, 1]$ and $V \in \mathcal{V}$. Define

$$Z(\alpha; V) = -\frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha))} W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + W_y^{\{i,i+1\}}(y(\alpha), t(\alpha))$$

To see that the $Z(\alpha; V) < 0$ for all $\alpha \in [0, 1]$ and all $V \in \mathcal{V}$ suppose by contradiction that there is some $\alpha \in [0, 1]$ and some $V \in \mathcal{V}$ such that $Z(\alpha; V) \geq 0$. Let μ be the Lagrange multiplier on the second constraint in problem (1.9). To demonstrate a

contradiction, consider the Lagrangian for problem (1.9) evaluated at the maximum

$$\mathscr{L} = V^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \lambda(\alpha)W^{\{i,i+1\}}(y(\alpha), t(\alpha))$$

$$+ \mu(\alpha)\left(V^{i-1}(y_{i-1}^r, t_{i-1}^r) - V^{i-1}(y(\alpha), t(\alpha))\right)$$

and the following deviation from the optimal contract $(y(\alpha), t(\alpha))$: $(\hat{y}, \hat{t}) := (y(\alpha) + \delta_y, t(\alpha) + \delta_t)$ for small $\delta_y, \delta_t > 0$ such that $V_y^{i-1}(y(\alpha), t(\alpha))\delta_y + V_t^{i-1}(y(\alpha), t(\alpha))\delta_t = 0$.

By the Sorting Assumption 1.1, part (iii)

$$-\frac{V_y^{i+1}(y(\alpha), t(\alpha))}{V_t^{i+1}(y(\alpha), t(\alpha))} < -\frac{V_y^i(y(\alpha), t(\alpha))}{V_t^i(y(\alpha), t(\alpha))} < -\frac{V_y^{i-1}(y(\alpha), t(\alpha))}{V_t^{i-1}(y(\alpha), t(\alpha)} = \frac{\delta_t}{\delta_y} \tag{1.15}$$

Cross multiplying and rearranging the (1.15) gives $\delta_y V_y^i(y(\alpha), t(\alpha)) + \delta_t V_t^i(y(\alpha), t(\alpha)) > 0$ and $\delta_y V_y^{i+1}(y(\alpha), t(\alpha)) + \delta_t V_t^{i+1}(y(\alpha), t(\alpha)) > 0$. Taking a convex combination of these expressions gives (weighting by α and $1 - \alpha$)

$$dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) := \delta_y V_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t V_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) > 0$$

Let $\hat{\mathscr{L}}$ denote the value of the Lagrangian at the deviation $(\hat{y}, \hat{t})$. The net gain from the deviation is

$$\hat{\mathscr{L}} - \mathscr{L} = dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \lambda(\alpha)\left[\delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha))\right]$$

$$= dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y\lambda(\alpha)\left[W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \frac{\delta_t}{\delta_y} \cdot W_t^{\{i,i+1\}}(y(\alpha), t(\alpha))\right]$$

$$= dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y\lambda(\alpha)Z(\alpha; V) > 0$$

where the third equality follows from the equality in (1.15) and the definition of Z and the inequality follows since we have assumed $Z(\alpha; V) \geq 0$. If δ_y, δ_t are sufficiently small, the deviation contract $(\hat{y}, \hat{t})$ strictly increases the Lagrangian which contradicts

the supposition that $(y(\alpha), t(\alpha))$ is an optimum. Thus, $Z(\alpha; V) < 0$ so $M(\alpha; V) > 0$ for all $\alpha \in [0, 1]$ and $V \in \mathcal{V}$ and so $M(V) > 0$ for all $V \in \mathcal{V}$. Thus, $\bar{\mathcal{V}}$ is a nonempty for sufficiently small $\delta > 0$.

To see that $\lambda(\alpha) > \delta$ for any $V \in \bar{\mathcal{V}}$, suppose there exists $V \in \bar{\mathcal{V}}$ such that $\lambda(\alpha) \leq \delta$ and consider the same deviation proposed above. Choose any $\alpha \in [0, 1]$. From equality in (1.15)

$$\delta_y V_y^i(y(\alpha), t(\alpha)) + \delta_t V_t^i(y(\alpha), t(\alpha)) = M(\alpha; V) \cdot Z(\alpha; V) \cdot \left[-\frac{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}{\delta_y V_t^i(y(\alpha), t(\alpha))} \right]^{-1}$$

$$= -\delta_y V_t^i(y(\alpha), t(\alpha)) \cdot M(\alpha; V) \frac{Z(\alpha; V)}{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}$$

and due to the first inequality and the equality in (1.15)

$$\delta_y V_y^{i+1}(y(\alpha), t(\alpha)) + \delta_t V_t^{i+1}(y(\alpha), t(\alpha)) > M(\alpha; V) \cdot Z(\alpha; V) \cdot \left[-\frac{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}{\delta_y V_t^{i+1}(y(\alpha), t(\alpha))} \right]^{-1}$$

$$= -\delta_y V_t^{i+1}(y(\alpha), t(\alpha)) \cdot M(\alpha; V) \frac{Z(\alpha; V)}{V_t^{\{i,i+1\}}(y(\alpha), t(\alpha))}.$$

Summing the last two expressions (weighted by α and $1 - \alpha$) we get

$$dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) > -\delta_y M(\alpha; V) Z(\alpha; V)$$

$$= -M(\alpha; V) \left[\delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right]$$

$$> -M(V) \left[\delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right]$$

$$> -\delta \left[\delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right].$$

The equality follows from the definition of Z and the equality in (1.15). The last two inequalities follow since the term in the square brackets is negative.[17] Using this last

---

[17]Otherwise, the deviation is strictly better for the $\{i, i+1\}$ principal, at least a good for the agent and maintains the incentive compatibility constraint, a contradiction that $(y(\alpha), t(\alpha))$ is an

inequality, the gain from deviation is

$$\hat{\mathscr{L}} - \mathscr{L} = dV^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \lambda(\alpha) \left[\delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right]$$

$$> -(\delta - \lambda(\alpha)) \left[\delta_y W_y^{\{i,i+1\}}(y(\alpha), t(\alpha)) + \delta_t W_t^{\{i,i+1\}}(y(\alpha), t(\alpha)) \right] \geq 0.$$

where the final inequality is due to our assumption that $\delta \geq \lambda(\alpha)$. If δ_y, δ_t are sufficiently small, the deviation contract $(\hat{y}, \hat{t})$ strictly increases the Lagrangian which contradicts the supposition that $(y(\alpha), t(\alpha))$ is an optimum. Thus, $\lambda(\alpha) > \delta$ for any $V \in \bar{\mathscr{V}}$. Since α was chosen arbitrarily, this holds for all $\alpha \in [0, 1]$. ∎

Lemma 1.6 *There exists a $\xi > 0$ such that for any specification of preferences $W^i(y(\alpha), t(\alpha)) - W^{i+1}(y(\alpha), t(\alpha)) < -\xi$ for any $\alpha \in [0, 1]$.*

Proof Since $0 < y_i^r < y(\alpha) < y_{i+1}^r$ and $0 < t_i^r < t(\alpha) < t_{i+1}^r$ and we assume that W^i is strictly increasing in i[18]

$$W^i(y(\alpha), t(\alpha)) - W^{i+1}(y(\alpha), t(\alpha)) \leq \max_{(y,t) \in [y_i^r, y_{i+1}^r] \times [t_i^r, t_{i+1}^r]} W^i(y, t) - W^{i+1}(y, t) < -\xi$$

as needed. ∎

Lemma 1.7 *For any $V \in \bar{\mathscr{V}}$ such that $\|V^i - V^{i+1}\|_\infty < \frac{1}{2}\delta\xi$, we have for any $\alpha \in [0, 1)$, $V_{PI}^{\{i,i+1\}}(\alpha) > V_{FI}^{\{i,i+1\}}(\alpha)$.*

---

optimum.

[18] The ordering of RSW actions and transfers is stated in Proposition 2 of Maskin and Tirole [11].

Proof Using Lemma 1.4

$$V_{PI}^{i,i+1}(\alpha) - V_{FI}^{i,i+1}(\alpha) \geq (1-\alpha)\left(V^i(y_{i+1}^r, t_{i+1}^r) - V^{i+1}(y_{i+1}^r, t_{i+1}^r)\right) \quad (1.16)$$

$$-\int_\alpha^1 \left(V^i(y(a), t(a)) - V^{i+1}(y(a), t(a))\right) da$$

$$-\int_\alpha^1 \lambda(a)\left(W^i(y(a), t(a)) - W^{i+1}(y(a), t(a))\right) da$$

$$> -(1-\alpha)\delta\xi + (1-\alpha)\delta\xi = 0$$

where the first inequality due to the RSW-IC[$i,i+1$] constraint, the second holds due to Lemmas 1.5 and 1.6 and since $\|V^i - V^{i+1}\| < \frac{1}{2}\delta\xi$. ∎

Lemma 1.8 *Let $V_r^j(PI)$ denote the RSW payoff of the principal in state $j \neq \{i, i+1\}$ in the continuation game following information acquisition option PI and let $(y_j^r(PI), t_j^r(PI))$ be the associated RSW contract.[19] Then $V_r^j(PI) \geq V_r^j$ for all $j \neq \{i, i+1\}$.*

Proof Take $j \neq \{i, i+1\}$. For $j < i$, $V_r^j(PI) = V_r^j$ due to item (iii) in Remark 2.

I claim that the incentive compatibility constraint in the state $j \geq i+2$ PI-RSW problem is weaker than in the state j FI-RSW problem. The argument is illustrated in Figure 1.4.

If $j = i + 2$, then the incentive compatibility constraint is weaker. To see this, define the indifference curve of any principal in state $l \in N \cup \{i, i+1\}$ at payoff K to be $\bar{t}^l(y; K)$ such that $V^l(y, \bar{t}^l(y; K)) = K$ for any y.[20] Note that the slope of the curve $\bar{t}^l(y; K)$ with respect to y is $-V_y^l(y, t)/V_t^l(y, t)$ and $\bar{t}^l(y; K)$ is strictly increasing in K since V^l is strictly increasing in t for all $l \in N \cup \{i, i+1\}$.

---

[19]I have suppressed the dependence of these objects on α for clarity.

[20]The existence of such a $\bar{t}$ is guaranteed by the implicit function theorem.
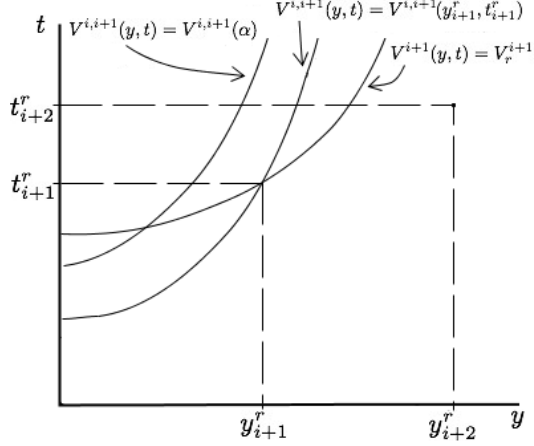
Figure 1.4: If the original game is incentive compatible, we can replace states i and $i+1$ with $\{i, i+1\}$ and maintain incentive compatibility.

By inequality (1.11) we have

$$\bar{t}^{\{i,i+1\}}(y; V^{\{i,i+1\}}(y^r_{i+1}, t^r_{i+1})) \begin{cases} = \bar{t}^{i+1}(y; V^{i+1}_r) \text{ if } y = y^r_{i+1} \\ > \bar{t}^{i+1}(y; V^{i+1}_r) \text{ if } y > y^r_{i+1} \\ < \bar{t}^{i+1}(y; V^{i+1}_r) \text{ if } y < y^r_{i+1} \end{cases} \tag{1.17}$$

Since $y^r_{i+2} > y^r_{i+1}$, by the middle line of (1.17) we have

$$\bar{t}^{\{i,i+1\}}(y^r_{i+2}; V^{\{i,i+1\}}(y^r_{i+1}, t^r_{i+1})) > \bar{t}^{i+1}(y^r_{i+2}; V^{i+1}_r)) \geq t^r_{i+2} \tag{1.18}$$

where the last inequality follows since $V^{i+1}(y^r_{i+1}, t^r_{i+1}) \geq V^{i+1}(y^r_{i+2}, t^r_{i+2})$ by the definition of the RSW menu.

Finally, note that $V^{\{i,i+1\}}(\alpha) \geq V^{\{i,i+1\}}(y^r_{i+1}, t^r_{i+1})$ since (y^r_{i+1}, t^r_{i+1}) is a feasible solution for problem (1.9) for all $\alpha \in (0,1)$. Then, by (1.18)

$$\bar{t}^{\{i,i+1\}}(y^r_{i+2}; V^{\{i,i+1\}}(\alpha)) > t^r_{i+2}$$

44

so that the principal in state $\{i, i+1\}$ will not misrepresent the state as $i+2$ when the state $i+2$ principal gets her RSW contract $\left(y_{i+2}^r, t_{i+2}^r\right)$:

$$V^{\{i,i+1\}}(\alpha) > \alpha V^i(y_{i+2}^r, t_{i+2}^r) + (1-\alpha)V^{i+1}(y_{i+2}^r, t_{i+2}^r).$$

Moreover, by the Sorting Assumption 1.1, for $j = i+2, \ldots, n$, if we assign to the state j principal (y_j^r, t_j^r), the state $\{i, i+1\}$ principal will not misrepresent the state as j.

Thus, if $\{(y_k^r, t_k^r)\}_{k \in N}$ is an RSW menu for the continuation game following the full information acquisition option, then

$$\left\{ (y_1^r, t_1^r), \ldots, (y_{i-1}^r, t_{i-1}^r), (y(\alpha), t(\alpha)), (y_{i+2}^r, t_{i+2}^r), \ldots, (y_n^r, t_n^r) \right\}$$

is a safe menu: it is incentive compatible and the agent will accept it regardless of her beliefs.[21] Therefore, the RSW payoff in each state for the modified game is at least as high as that in the original game. ∎

Maskin and Tirole [11] show that there is a nonempty set of priors such that the RSW payoff $\sum_j \pi_j V_r^j$ is the unique equilibrium payoff when the principal is perfectly informed of the state. Choose any prior π' in this set; π' determines some α'. Then, by Lemma 1.7, for $\|V^i - V^{i+1}\|_\infty$ sufficiently small

$$\sum_j \pi_j' V_r^j - \left[\sum_{j \neq i, i+1} \pi_j' V_r^j(PI) + (\pi_i' + \pi_{i+1}')V_{PI}^{\{i,i+1\}}(\alpha') \right]$$

$$\leq \sum_{j \neq i, i+1} \pi_j' V_r^j + (\pi_i' + \pi_{i+1}')V_{FI}^{\{i,i+1\}}(\alpha') - \left[\sum_{j \neq i, i+1} \pi_j' V_r^j + (\pi_i' + \pi_{i+1}')V_{PI}^{\{i,i+1\}}(\alpha') \right]$$

$$= (\pi_i' + \pi_{i+1}') \left(V_{PI}^{\{i,i+1\}}(\alpha') - V_{FI}^{\{i,i+1\}}(\alpha') \right) < 0.$$

---

[21]Recall that the solution to problem (1.9) requires that the state $i-1$ principal not wish to misrepresent the state as $\{i, i+1\}$.

The first term in both lines is the expected (unique) equilibrium payoff for the fully informed principal. The second term is expected equilibrium payoff if she confounds states i and $i + 1$. The first inequality follows from Lemma 1.8, the second from Lemma 1.7. ∎

1.8.2 Strategic Ignorance Despite Multiple Equilibria

Proof of Proposition 1.1

I begin by showing that if κ is small, the RSW-IC[1,2] binds.

Lemma 1.9 *If κ is sufficiently close to 1, then $t_1^r - C^1(y_1^r) = t_2^r - C^1(y_2^r)$.*

Proof By way of contradiction, assume that $t_1^r - C^1(y_1^r) > t_2^r - C^1(y_2^r)$. Let $(\hat{y}_1, \hat{t}_1)$ denote the optimal contract for the principal in state 1 when she has convinced the agent that she is in state 2:

$$(\hat{y}_1, \hat{t}_1) = \arg \max_{y_1, t_1} \{t_1 - C^1(y_1) | t_1 = U^2(y_1)\} \tag{1.19}$$

This solution is uniquely characterized by $MC^1(\hat{y}_1) = MU^2(\hat{y}_1)$ and $\hat{t}_1 = U^2(\hat{y}_1$. Further, the state 1 principal's RSW contract is characterized by $MC^1(y_1^r) = MU^1(y_1^r)$ and $t_1^r = U^1(y_1^r)$ and since RSW-IC[1,2] does not bind, the state 2 principal's RSW contract is characterized by $MC^2(y_2^r) = MU^2(y_2^r)$ and $t_2^r = U^2(y_2^r)$. Now,

$$MC^2(\hat{y}_1) < MC^1(\hat{y}_1) = MU^2(\hat{y}_1) = MC^1(\hat{y}_1) \left(\frac{MC^2(\hat{y}_1)}{MC^2(\hat{y}_1)} \right) \leq \kappa MC^2(\hat{y}_1).$$

So, if κ is close enough to 1, since costs are convex, we can bound the difference between the two maximizers for some small $\delta_a > 0$: $y_2^r - \hat{y}_1 < \delta_a$. In a similar way, we can show there exists small $\delta_y^b > 0$ such that $\hat{y}_1 - y_1^r < \delta_b$. Thus, we can choose κ sufficiently close to 1 such that $y_2^r - y_r^1 < \delta_a + \delta_b$ and hence $C^1(y_2^r) - C^1(y_1^r) < \delta_c :=$

$\min_y \{U^2(y) - U^1(y)\}$. Then

$$0 > U^2(y_2^r) - C^1(y_2^r) - [U^1(y_1^r) - C^1(y_1^r)] = U^2(y_2^r) - U^1(y_1^r) - [C^1(y_2^r) - C^1(y_1^r)]$$

$$> \delta_c - [C^1(y_2^r) - C^1(y_1^r)] > 0$$

where the first inequality follows from the fact that we have assumed RSW-IC[1,2] does not bind. This is a contradiction so we must have RSW-IC[1,2] bind for κ close to 1. ∎

Now we characterize payoffs for the ignorant strategy and the informed strategy. Note that the uninformed principal's problem (1.5) can be expressed as $V_u(\pi) = \max_y \pi[U^1(y) - C^1(y)] + (1 - \pi)[U^2(y) - C^2(y)]$ since IG-IR constraint always binds.

Fix κ such that RSW-IC[1,2] binds. The first statement of Proposition 1.1 results from the following properties of the payoff functions: (a) $V_r^1 = V_u(1)$ since the maximand and constraints are identical in the RSW and uninformed problems at $\pi = 1$; (b) $V_2^r < V_u(0)$ since the state 2 RSW problem is more constrained (i.e. by RSW-IC[1,2]) than the uninformed principal's problem at $\pi = 0$ by our choice of κ; (c) $V_u(\pi)$ is convex and downward sloping in π since the maximand is linear in π; and (d) $V_r(\pi)$ is linear and downward sloping in π.

Properties (c) and (d) imply that the equation $V_u(\pi) = V_r(\pi)$ has at most two solution. Clearly, one solution is always $\pi = 1$. Due to properties (b) - (d), a second solution $\pi^* > 0$ exists and $V_u(\pi) > V_r(\pi)$ for all $\pi \in (0, \min(1, \pi^*))$.

The following lemma completes the proof of Proposition 1.1.

Lemma 1.10 *Fix C^1 and U^i for $i \in \{1, 2\}$. There exists κ^* such that if $\kappa < \kappa^*$, then $V_u(\pi) > V_r(\pi)$ for all $\pi \in (0, 1)$.*

Proof Define $S^i(y) := U^i(y) - C^i(y)$. By the integral form of the envelope theorem (Milgrom and Segal, [12]) $V_u(\pi) = V_u(0) + \int_0^\pi (S^1(y(\pi)) - S^2(y(\pi)))d\pi$. As in Lemma

47

1.4 we can write $V_u(\pi) = V_r^1 - \int_\pi^1 (S^1(y(\pi)) - S^2(y(\pi)))d\pi$ and so

$$V_u(\pi) - V_r(\pi) = (1 - \pi)(V_r^1 - V_r^2) - \int_\pi^1 (S^1(y(\pi)) - S^2(y(\pi)))d\pi. \qquad (1.20)$$

Note that

$$S^1(y(\pi)) - S^2(y(\pi)) < S^1(y_1^r) - S^2(y_1^r) \qquad (1.21)$$

$$= V_r^1 - (U^2(y_1^r) - C^2(y_1^r))$$

$$= -(U^2(y_1^r) - U^1(y_1^r)) - (C^1(y_1^r) - C^2(y_1^r))$$

$$\leq -(U^2(y_1^r) - U^1(y_1^r)) - \Delta C(0).$$

for all $\pi \in (0,1)$ where $\Delta C(0) = C^1(0) - C^2(0)$ is the difference in fixed costs between states.

Further,

$$V_r^1 - V_r^2 = t_2^r - C^1(y_2^r) - (t_2^r - C^2(y_2^r)) \qquad (1.22)$$

$$= -(C^1(y_2^r) - C^2(y_2^r)))$$

$$= -\int_0^{y_2^r} \left[MC^1(y) - MC^2(y) \right] dy - (C^1(0) - C^2(0))$$

$$\geq -(\kappa - 1) \left[C^2(y_2^r) - C^2(0) \right] - \Delta C(0)$$

The first equality follows from the fact that $t_1^r - C^1(y_1^r) = t_2^r - C^1(y_1^r)$.

Now, applying inequalities (1.21) and (1.22) to equation (1.20) we have

$$V_u(\pi) - V_r(\pi) \geq -(\kappa - 1)\left[C^2(y_2^r) - C^2(0)\right] - \Delta C(0) + U^2(y_1^r) - U^1(y_1^r) + \Delta C(0)$$

$$= \underbrace{-(\kappa - 1)\left[C^2(y_2^r) - C^2(0)\right]}_{A} + U^2(y_1^r) - U^1(y_1^r) \qquad (1.23)$$

48

The RSW actions y_i^r for all $i = 1, 2$ will be the same for all κ: both are determined solely by the cost function of the state 1 principal. The term A in (1.23) can be made arbitrarily small by taking κ close to 1 since $C^2(y_2^r) < C^1(y_2^r)$. Moreover, the term $U^2(y_1^r) - U^1(y_1^r) > 0$ and does not change with κ. Therefore 3 for $\kappa > 1$ sufficiently close to 1, we have $V_u(\pi) > V_r(\pi)$ for all $\pi \in (0, 1)$. Define $\underline{\kappa} := \sup\{\kappa | V_u(\pi) > V_r(\pi) \text{ for all } \pi \in (0, 1)\} > 1.$ ∎ ∎

Proof of Proposition 1.2   I begin by proving three useful lemmas. Then, in Lemma 1.14, we characterize V^*. The important fact derived in this lemma is that the RSW payoff is the unique equilibrium payoff for all $\pi \in [\pi^r, 1)$ for some $\pi^r < 1$.

Lemma 1.11 *If RSW-IC[i, i + 1] is strictly binding, for any $i = 1, \ldots, n - 1$ (i.e. $t_i^r - C^i(y_i^r) = t_{i+1}^r - C^i(y_{i+1}^r)$) then the state $i + 1$ RSW contract is inefficient: $MC^{i+1}(y_{i+1}^r) > MU^{i+1}(y_{i+1}^r).$*

Proof   Suppose $MC^{i+1}(y_{i+1}^r) < MU^{i+1}(y_{i+1}^r)$ and consider the following deviation for type 2 in the RSW problem: $y' = y_{i+1}^r + \varepsilon$; and

$$t' \in \left(t_{i+1}^r + \varepsilon MC^2(y_{i+1}^r), t_{i+1}^r + \varepsilon \min\left\{MU^2(y_{i+1}^r), MC^i(y_{i+1}^r)\right\}\right).$$

Then for sufficiently small $\varepsilon > 0$, this deviation is profitable and feasible:

$$t' - C^i(y') < t_{i+1}^r - C^i(y_{i+1}^r) = t_i^r - C^i(y_i^r);$$
$$t' - C^{i+1}(y') > t_{i+1}^r - C^{i+1}(y_{i+1}^r); \text{ and}$$
$$U^{i+1}(y') - t' > U^{i+1}(y_{i+1}^r) - t_{i+1}^r.$$

If $MC^{i+1}(y_{i+1}^r) = MU^{i+1}(y_{i+1}^r)$, then RSW-IC[i, i + 1] is not strictly binding. ∎

Lemma 1.12 *If $\{(y_i, t_i)\}_{\in N}$ is an equilibrium of the informed principal problem, then $y_2 \le y_2^r$.*

Proof Suppose $y_2 > y_2^r$. If RSW-IC[1,2] is not binding, then the first best contract is possible and $y_2 = y_2^r$; this is a contradiction.

If RSW-IC[1,2] is binding, first note that since $C^1(y_1^r)$ is tangent to $U^1(y_1^r)$ (so that $t_1^r = U^1(y_1^r)$), any state 1 contract that satisfies NB^1 must have $t_1^r \geq U^1(y_1^r)$. By Lemma 1.11, $MC^2(y_2^r) > MU^2(y_2^r)$. This implies, that since C^2 is convex and increasing and U^2 is concave and increasing, if $t_2 - C^2(y_2) \geq V_r^1 = U^2(y_2^r) - C^2(y_2^r)$ then $t_2 > U^2(y_2)$ for $y_2 > y_2^r$. But this violates the individual rationality constraint of the agent, a contradiction that $y_2 > y_2^r$ can occur in equilibrium. ∎

Lemma 1.13 $V^*(\pi)$ *is continuous.*

Proof Consider the ex ante optimal informed principal's problem (1.6) and its value function $V^*(\pi)$. Let $\mathbf{y} = (y_1, y_2)$ and $\mathbf{t} = (t_1, t_2)$. I will show that the feasibility correspondence

$$
\Gamma(\pi) = \left\{ (\mathbf{y}, \mathbf{t}) \in \mathbb{R}^4 \, \middle| \,
\begin{array}{ll}
(\text{IC}[i,j]) & t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in N \\
(\text{IR}) & \sum_i \pi_i \left(U^i(y_i) - t_i \right) \geq 0 \\
(\text{NB}[i]) & t_i - C^i(y_i) \geq V_i^r \text{ for all } i \in N
\end{array}
\right\}
$$

is both upper and lower hemi-continuous in π.

Due to Lemma 1.12 and Assumption 1.2, without lost of generality we can restrict the feasibility correspondence to

$$
\Gamma'(\pi) = \left\{ (\mathbf{y}, \mathbf{t}) \in [0, y_2^r]^2 \times [0, T]^2 \, \middle| \,
\begin{array}{ll}
(\text{IC}[i,j]) & t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in N \\
(\text{IR}) & \sum_i \pi_i \left(U^i(y_i) - t_i \right) \geq 0 \\
(\text{NB}[i]) & t_i - C^i(y_i) \geq V_i^r \text{ for all } i \in N
\end{array}
\right\}
$$

for some large finite T. Then the graph of Γ'

$$Gr(\Gamma') = \left\{ (\pi, \{(y_i, t_i)\}_{i=1}^2) \in [0,1] \times [0, y_2^r]^2 \times [0, T]^2 : \{(y_i, t_i)\}_{i=1}^2 \in \Gamma'(\pi) \right\}$$

is closed. Moreover, for any closed interval $\Pi \subseteq [0,1]$, $\Gamma'(\Pi)$ is bounded. So by Theorem 3.4 in Stokey and Lucas [20], Γ' is upper hemi-continuous.

As for lower-hemicontinuity, we first note, that of the five possible constraints, at most four will bind. To see this, suppose there is $(\mathbf{y}, \mathbf{t}) \in \Gamma(\pi)$ for some π such that all five constraints bind. Then we have the following series of implications

(a) NB$[i]$ binds for $i = 1, 2$ implies that state i contract is on the state i principal's RSW indifference curve;

(b) IC[1,2] binds implies that (y_2, t_2) is on the state 1 principal's indifference;

(c) IC[21] binds implies that (y_1, t_1) is on the state 1 principal's indifference;

(d) Items (b) and (c) imply that $y_1 = y_2 =: y'$ and $t_1 = t_2 =: t'$ since the indifference curves cross only once due to item (a.iii) of Assumption 1.2

(e) Items (a), (b) and (d) imply that $y' = y_2^r$, $t' = U^2(y_2^r)$ since y_2^r is defined such that $U^2(y_2^r) - C^1(y_2^r) = V_r^1$;

(f) $U^1(y') - t' < 0$ since $U^1(\cdot)$ is tangent to $C^1(\cdot)$ at y_r^1 and therefore any $y \neq y_1^r$ results in $U^1(y) - t < 0$;

(g) Items (e) and (f) imply IR is violated: $\pi(U^1(y) - t) + (1 - \pi)(U^2(y) - t) = \pi(U^1(y) - t) < 0$.

The final item contradicts the assumption that $(\mathbf{y}, \mathbf{t}) \in \Gamma(\pi)$. Thus, for any π at most four constraints are active.

The following argument is due to Duggan and Kalandrakis [6]. Suppose four constraints bind at $\pi^0 \in (0,1)$. Take any $(\mathbf{y}^0, \mathbf{t}^0) \in \Gamma(\pi^0)$. Let $f_s(\mathbf{y}, \mathbf{t}, \pi)$ for $s = 1, \ldots, 4$ denote the four binding constraints. Then the Jacobian matrix of $F(\mathbf{y}, \mathbf{t}, \pi) := (f_s(\mathbf{y}, \mathbf{t}, \pi))_{s=1}^4$ is invertible at $(\mathbf{y}^0, \mathbf{t}^0, \pi^0)$. So, by the implicit function theorem there exists a continuous function $h(\pi)$ such that $h(\pi^0) = (\mathbf{y}^0, \mathbf{t}^0)$ and

51

$F(h(\pi), \pi) = \mathbf{0}$ in an open neighbourhood around π^0. Since the remaining constraint is slack at π^0, it is also slack in an open neighbourhood around π^0. Thus, there is an open neighbourhood of π^0 such that $h(\pi) \in \Gamma(\pi)$ for all π in this neighbourhood and we conclude that Γ is lower hemi-continuous at π^0.

If only $d < 4$ constraints bind at π^0, then let $f_s(\mathbf{y}, \mathbf{t}, \pi)$, for $s = 1, \dots, d$ denote the d binding constraints and define $g_s(\mathbf{y}, \mathbf{t}, \pi)$, for $s = d+1, \dots, 4$ as affine linear functions that are constant in π, satisfy $g_s(\mathbf{y}^0, \mathbf{t}^0, \pi^0) = 0$ for all $s = d+1, \dots, 4$, and have total derivative $D_{(\mathbf{y}, \mathbf{t})} g_s(\mathbf{y}, \mathbf{t}, \pi) = \mathbf{v}_s$ such that the matrix

$$\left(\left(D_{(\mathbf{y}, \mathbf{t})} f_s(\mathbf{y}^0, \mathbf{t}^0, \pi^0) \right)_{s=1}^d, (\mathbf{v}_s)_{s=d+1}^4 \right)$$

has full rank and is invertible. As above, we can apply the implicit function theorem to conclude that Γ is lower hemi-continuous at π^0.

So by the Theorem of the Maximum (Stokey and Lucas [20, Theorem 3.6]), $V^*(\pi)$ is continuous in π. ∎

Lemma 1.14 *If RSW-IC[1,2] binds, there are two cutoff points $0 < \pi^{FB} < \pi^r < 1$ such that $V^*(\pi)$ is the first best payoff if $\pi \leq \pi^{FB}$ and the ex ante RSW payoff if $\pi \geq \pi^r$.*

Proof   Claim 1 *If π is close enough to 1, $V^*(\pi) = V_r(\pi)$.* This holds by Lemma 1.15 below: $i^* = 1$. □

Claim 2 *If π is sufficiently small, then $V^*(\pi) = V_{FB}(\pi)$.* Recall that the superscript E indicates the efficient action. To see that the first best solution is attainable for small π set $y_i^* = y_i^E$, set t_2^* such that $t_2^* - C^2(y_2^E) = V_r^2$ and set t_1^* sufficiently high such that IC[1,2] is satisfied. To see that we can do this last step while satisfying the IR constraint, note that, by Lemma 1.11, $y_2^E < y_2^r$ which implies that $t_2^* < t_2^r$. Finally,

since $U^2(y_2^r) = t_2^r$, Lemma 1.11 implies that $U^2(y_2^E) - t_2^* > 0$. Thus, we can find small enough π such that $\pi(U^1(y_1^E) - t_1^*) + (1 - \pi)(U^2(y_2^E) - t_2^*) = 0$. $\square$

Define $\pi^r := \inf \{\pi \in [0, 1] : V^*(\pi) = V_r(\pi)\}$. This infimum is attained in $[0, 1)$ due to Claim 1 above and Proposition 4 of Maskin and Tirole [11] which says that the set of beliefs relative to which the RSW payoff is the unique equilibrium payoff consists entirely of strictly positive vectors. As a result, $\pi^r < 1$ regardless of κ so $[\pi^r, 1)$ is always well defined and nonempty. By definition, $V^*(\pi) = V_r(\pi)$ if and only if $\pi \in [\pi^r, 1)$.

Further, by assumption, RSW-IC[1,2] binds which implies, by Lemma 1.11, that the state 2 contract is inefficient. Thus, $V^{FB}(\pi) > V_r(\pi)$ for all $\pi \in (0, 1)$. Given Claim 2, we must have $\pi^r > 0$; otherwise, $V^{FB}(\pi)$ and $V_r(\pi)$ must coincide, which is a contradiction.

Define $\pi^{FB} := \sup \{\pi \in [0, 1] : V^*(\pi) = V^{FB}(\pi)\}$. This supremum is attained in $(0, 1)$ by Claim 2. By definition $V^*(\pi) = V_{FB}(\pi)$ if and only if $\pi \in (0, \pi^{FB}]$. This point exists and is strictly greater than 0 by Claim 3. Further, $\pi^{FB} < \pi^r$. To see this, suppose $\pi^{FB} \geq \pi^r$. Then there exists $\tilde{\pi} \in [\pi^r, \pi^{FB}]$. But, by the definitions of π^r, π^{FB} this implies
$V^*(\tilde{\pi}) = V_{FB}(\tilde{\pi}) = V_r(\tilde{\pi})$ a contradiction, since by Lemma 1.11 the state 2 contract is inefficient. Thus, $0 < \pi^{FB} < \pi^r < 1$. Figure 1.1 plots V^*, V_u, V_r and V^{FB}. $\blacksquare$

Since $\pi^r < 1$, due to Proposition 1.1 (item (ii) of the second statement) there is κ close enough to 1 such that $\pi^r < \pi^*$ and for all $\pi \in (\pi^r, \pi^*)$ Proposition 1.2 holds. $\blacksquare$

Proof of Theorem 1.2 By Lemma 1.9 RSW-IC[1,2] binds since κ is assumed to be sufficiently close to 1. Recall that for such κ, $0 < \pi^{FB} < \pi^r < \pi^* \leq 1$ (see proof of Proposition 1.2).

Consider the following facts

(a) V^* and V_u are continuous: the former is proved in Lemma 1.14 (Claim 1), the

latter is immediate by inspection of problem (1.5);

(b) $V^*(\pi) > V_u(\pi)$ for all $\pi \in (0, \pi^{FB}]$: this holds since $V_u(\pi)$ cannot be efficient in both states where as $V^*(\pi)$ is first best by definition in this domain;

(c) $V^*(\pi) < V_u(\pi)$ for all $\pi \in [\pi^r, \pi^*)$: established by Proposition 1.2;

(d) $V^*(\pi) > V_r(\pi)$ for all $\pi \in (0, \pi^r)$: by definition of π^r in Lemma 1.14 (Claim 2).

Due to items (a) through (c), the intermediate value theorem guarantees the existence of a $\hat{\pi} \in (\pi^{FB}, \pi^r)$ such that for all $\pi \in (\hat{\pi}, \pi^r)$, $V_u(\pi) > V^*(\pi)$. This confirms the first statement of Theorem 1.2. Since $\hat{\pi} \in (0, \pi^r)$, by item (d) we also have that $V^*(\pi) > V_r(\pi)$ thus confirming the second statement of Theorem 1.2. ∎

Proof of Proposition 1.3 When RSW-IC[1,2] does not bind, the RSW contract in both states is efficient. To see this, recall that the state 1 contract is always efficient and note that, according to Proposition 2 of Maskin and Tirole [11], the problem of the state 2 principal in this case is $\max_{(y_2, t_2)} \{t_2 - C^2(y_2) : U^2(y_2) - t_2 = 0\}$. So $V_u(0) = V_r^2$ and $V_u(1) = V_r^1$. Since V_u is convex and V_r is linear (see the proof of Proposition 1.1, items (c) and (d)), $V_u(\pi) < V_r(\pi)$ for all $\pi \in (0, 1)$. ∎

1.8.3 Optimal Information Structure: Three States

Before proving the results of this section, we define the principal's problems and strategies relative to p. For this, we need some additional notation.

The RSW problem relative to information strategy p for principal in p-state i is to choose $\{(y_i, t_i)\}_{i \in I(p)}$ to solve

$$\max \ t_i - C^{p_i}(y_i)$$

$$s.t. \quad (p\text{-RSW-IC}[i,j]) \quad t_j - C^{p_j}(y_j) \ \geq t_k - C^{p_j}(y_k) \quad \text{for all } j, k \in I(p); \text{ and}$$

$$(p\text{-RSW-IR}[j]) \qquad\qquad U^{p_j}(y_j) \ \geq t_j \qquad\qquad\qquad \text{for all } j \in I(p).$$

I will refer to this problem as the p-RSW problem for p-state i or the p_i-RSW problem. Let $V_r^{p_i}(\pi; p)$ denote the p_i-RSW given priors π.

Our first lemma in this section characterizes the priors under which the RSW payoff is unique for the fully informed principal problem.

Lemma 1.15 *Consider the problem of the fully informed principal when there are either two or three states. Let $E \subset N$ denote the set of states for which the RSW contracts are efficient. Define $I := N/E$ to be the set of states with inefficient RSW contracts and let $i^* = \max\{i \in E | i < \min I\}$.*

Then: (i) If $I = \varnothing$, the RSW payoff is the unique payoff for all priors; (ii) if $|I| = 1$, then if π_{i^*} is sufficiently large, the RSW payoff is the unique equilibrium payoff; and (iii) if $|I| = 2$, then if π_1 and $\pi_2/(\pi_2 + \pi_3)$ are sufficiently close to 1, the RSW payoff is the unique equilibrium payoff. Moreover, all of these bounds on priors are strictly less than 1.

Proof First note that $1 \in \{i \in E | i < \min I\}$ since state 1 is always efficient. Therefore, i^* is always well defined.

If $I = \varnothing$, then all states are efficient and the RSW contract is first best (see Proposition 1.3). The RSW payoff is therefore the unique payoff for all priors.

Now suppose $I \neq \varnothing$. By Theorem 1 in Maskin and Tirole [11] $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ is an equilibrium menu if and only if it satisfies the following conditions

$$
\begin{cases}
(\text{IC}[i,j]) & t_i - C^i(y_i) \geq t_j - C^j(y_j) \text{ for all } i \neq j \in N \\
(\text{IR}) & \sum_i \pi_i \left(U^i(y_i) - t_i \right) \geq 0 \\
(\text{NB}[i]) & t_i - C^i(y_i) \geq V_i^r \text{ for all } i \in N.
\end{cases}
$$

Suppose there exists a menu $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ gives payoff strictly higher than the RSW menu in equilibrium. For each $i \in I$, the action in the state i RSW contract is higher

55

than the efficient level (see Lemma 1.11). For all $i \in I$, define $\delta_i = U^i(\hat{y}_i) - \hat{t}_i$. This is the surplus given to the agent in state i by the proposed menu.

If the proposed menu delivers strictly higher payoff than the RSW menu, there must exist at least one $i \in I$ such that $\delta_i > 0$. To see this, suppose not: for all $i \in I$, $U^i(\hat{y}_i) - \hat{t}_i = 0$. Call this assumption $(\star)$. Note that for all $k \in E$, C^k is tangent to U^k at (y_k^r, t_k^r). This implies that for all (y_k', t_k') such that $t_k' - C^k(y_k') > V_k^r$, $U^k(y_k') - t_k' < 0$. This last implication, along with $(\star)$ and the equilibrium condition IR implies that $(\hat{y}_k, \hat{t}_k) = (y_k^r, t_k^r)$ for all $k \in E$. So, we have that $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ satisfies IC$[i, j]$ for all $i, j \in N$ and $U^i(\hat{y}_i) - \hat{t}_i = 0$ for all $i \in N$. But the RSW menu is the best of all menus that satisfy these assumptions so that the menu $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ cannot give a strictly higher payoff than the RSW menu.

Thus, there exist at least one state $i \in I$ such that $\delta_i > 0$. Note that this implies that $\hat{y}_i < y_i^r$ by Lemma 1.11. For each $i \in I$, define $\bar{\delta}_i := \max_{(y_i, t_i)} \{U^i(y_i) - t_i | t_i - C^i(y_i) \geq V_r^i\}$. So, $\bar{\delta}_i$ is the largest surplus we can assign to the state i agent for $i \in I$. Note that this maximum is achieved at the efficient state i action along the state i RSW indifference curve.

Now consider two cases.

Case 1: $|I| = 1$. Then $i^* + 1 \in I$ by definition and $\delta_{i^*+1} > 0$. I claim that if the principal receives her RSW contract in state i^* she will have strict incentive to lie given state i^*+1 contract $(\hat{y}_{i^*+1}, \hat{t}_{i^*+1})$. To see this first note that, by Proposition 1.3, RSW-IC$[i^*, i^*+1]$ must bind: $t_{i^*}^r - C^{i^*}(y_{i^*}^r) = t_{i^*+1}^r - C^{i^*}(y_{i^*}^r)$. Since $MC^{i^*} > MC^{i^*+1}$ the indifference curves of the principal in states i^* and $i^* + 1$ cross only once and the latter crosses the former from below. Consider the indifference curves that pass

through the RSW contracts

$$V_r^{i^*} + C^{i^*}(y) - \left(V_r^{i^*+1} + C^{i^*+1}(y)\right) \begin{cases} < 0 & \text{if } y < y_{i^*+1}^r \\ = 0 & \text{if } y = y_{i^*+1}^r \\ > 0 & \text{if } y > y_{i^*+1}^r \end{cases} \tag{1.24}$$

Note that

$$V_r^{i^*} + C^{i^*}(\hat{y}_{i^*+1}) - \left(V_r^{i^*+1} + C^{i^*+1}(\hat{y}_{i^*+1})\right) \geq V_r^{i^*} + C^{i^*}(\hat{y}_{i^*+1}) - \left(\hat{t}_{i^*+1} - C^{i^*+1}(\hat{y}_{i^*+1})\right.$$

$$\left. + C^{i^*+1}(\hat{y}_{i^*+1})\right) \tag{1.25}$$

$$= V_r^{i^*} - \left(\hat{t}_{i^*+1} - C^{i^*}(\hat{y}_{i^*+1})\right) \tag{1.26}$$

where the inequality follows from the NB$[i^*+1]$ condition. Since $\hat{y}_{i^*+1} < y_{i^*+1}^r$, by the first line of expression (1.24) we have that $0 > V_r^{i^*} + C^{i^*}(\hat{y}_{i^*+1}) - \left(V_r^{i^*+1} + C^{i^*+1}(\hat{y}_{i^*+1})\right)$ which, given (1.26), implies that $V_r^{i^*} < \hat{t}_{i^*+1} - C^{i^*}(\hat{y}_{i^*+1})$.

Thus, given the state i^*+1 contract $\left(\hat{y}_{i^*+1}, \hat{t}_{i^*+1}\right)$, to satisfy incentive compatibility we must give the state i^* principal payoff that is strictly higher than her RSW payoff.

Since $i^* \in E$, C^{i^*} is tangent to U^{i^*} at the RSW contract; thus, any contract that increases the payoff to the principal in this state necessarily assigns a strictly positive deficit to the agent. Denote this deficit by $\underline{\delta}_{i^*} := \hat{t}_{i^*} - U^{i^*}(\hat{y}_{i^*}) > 0$.

Without loss of generality, set $(\hat{y}_i, \hat{t}_i) = (y_i^r, t_i^r)$ for all $i \in E/\{i^*\}$ and assume the resulting contract is incentive compatible. Then, if π_{i^*} is close enough to 1

$$\sum_i \pi_i \left(U^i(y_i) - t_i\right) = \sum_{i \in E/\{i^*\}} \pi_i \left(U^i(\hat{y}_i) - \hat{t}_i\right) - \pi_{i^*}\underline{\delta}_{i^*} + \pi_{i^*+1}\delta_{i^*+1}$$

$$= -\pi_{i^*}\underline{\delta}_{i^*} + \pi_{i^*+1}\delta_{i^*+1} \leq -\pi_{i^*}\underline{\delta}_{i^*} + \pi_{i^*+1}\bar{\delta}_{i^*+1} < 0$$

where the first equality follows since there is zero surplus for the agent in states

57

$E/\{i^*\}$. This contradicts the assumption that $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ is an equilibrium.

Case 2: $|I| = 2$. The state 2 and three contracts are inefficient. Then RSW-IC[1,2] and RSW-IC[2,3] bind by Proposition 1.3. If $\delta_2 > 0$ or $\delta_2 = 0$ and $\delta_3 > 0$ then the argument in Case 1 can be applied in much the same way; if π_1 is sufficiently large, IR cannot hold and $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ cannot be an equilibrium.

Now suppose that $\delta_3 > 0$ and $\delta_2 < 0$. Without loss of generality, set $(\hat{y}_1, \hat{t}_1) = (y_1^r, t_1^r)$. As above, if the state 2 principal receives her RSW contract she will have a strict incentive to lie given the state three contract $(\hat{y}_3, \hat{t}_3)$. Thus, the menu $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ must give the state 2 principal strictly higher payoff than her RSW contract. Define $\underline{\delta}_2 := -\delta_2 > 0$. If $\pi_2/(\pi_2 + \pi_3)$ is close enough to 1, then

$$\sum_i \pi_i \left(U^i(y_i) - t_i \right) = \pi_1 \left(U^1(\hat{y}_1) - \hat{t}_1 \right) - \pi_2 \underline{\delta}_2 + \pi_3 \delta_3$$

$$= -\pi_2 \underline{\delta}_2 + \pi_3 \delta_3 \leq (\pi_2 + \pi_3) \left[-\frac{\pi_2}{\pi_2 + \pi_3} \underline{\delta}_2 + \frac{\pi_3}{\pi_2 + \pi_3} \bar{\delta}_3 \right] < 0$$

where the first equality follows since there is zero surplus for the agent in state 1. This contradicts the assumption that $\{\hat{y}_i, \hat{t}_i\}_{i \in N}$ is an equilibrium.

Finally note that Maskin and Tirole [11, Proposition 4] asserts that the set of beliefs relative to which the RSW payoff is unique consists of strictly positive vectors. Thus, the bounds we have placed on priors in this lemma are strictly less than one.

∎

Proof of Proposition 1.4 Note that α is used below to denote conditional priors. Be aware that α is redefined in subsequent lemmas. Further, any priors (conditional or unconditional) superscripted with r are meant to be analogous to those in Propositions 1.1 and 1.2.

This proof proceeds by applying Proposition 1.2 to the various subgames associated with choosing different information acquisition options. Recall that in Proposi-

tion 1.2, as long as $\kappa < \underline{\kappa}$, $\pi^* = 1$. To ease exposition, when we apply Proposition 1.2 we sacrifice its generality (i.e. allowing the upper bound on priors to be less than 1) and simply assume all the the starred priors (conditional or unconditional) are 1.

I first characterize priors such that p^5 conforms to the Assumption 1.2.

Lemma 1.16 *Let*

$$C^{\{1,3\}}(\cdot) := \frac{\pi_1}{\pi_1 + \pi_3} C^1(\cdot) + \frac{\pi_3}{\pi_1 + \pi_3} C^3(\cdot)$$

and define $U^{\{1,3\}}$ in the same way. There exists priors $\hat{\pi} \in \Delta^3$ such that for all

$$\frac{\pi_1}{\pi_1 + \pi_3} \in \left[\frac{\hat{\pi}_1}{\hat{\pi}_1 + \hat{\pi}_3}, 1 \right)$$

Assumptions 1.1 and 1.2 are satisfied for the two state informed principal game with principal payoff functions ordered $(V^{\{1,3\}}, V^2) = (t - C^{\{1,3\}}, t - C^2)$ and agent payoff functions ordered $(W^{\{1,3\}}, W^2) = (U^{\{1,3\}} - t, U^2 - t)$.

Proof I will prove that part (iii) from Assumption 1.1 holds. Parts (i) and (ii) of Assumption 1.1 are immediate. Both parts of Assumption 1.2 are proved in a similar manner.

Let $\alpha := \pi_1/(\pi_1 + \pi_3)$. By Assumption 1.1, there exists $\delta > 0$ such that

$$\frac{MC^1(y)}{MC^2(y)}, \frac{MC^2(y)}{MC^3(y)} > \delta + 1.$$

Define

$$\hat{\alpha} := \frac{\kappa_2^S - 1}{\delta(\delta + 1) + \kappa_2^S - 1} < 1. \tag{1.27}$$

Then for all $\alpha \in [\hat{\alpha}, 1)$ we have

$$
\begin{aligned}
MC^{\{1,3\}}(y) - MC^2(y) &= \alpha \left(MC^1(y) - MC^2(y) \right) + (1 - \alpha) \left(MC^3(y) - MC^2(y) \right) \\
&> MC^3(y) \left[\alpha \left(\frac{MC^1(y)}{MC^3(y)} - \frac{MC^2(y)}{MC^3(y)} \right) + (1 - \alpha) \left(1 - \kappa_2^S \right) \right] \\
&= MC^3(y) \left[\alpha \left(\frac{MC^1(y)}{MC^2(y)} - 1 \right) \frac{MC^2(y)}{MC^3(y)} + (1 - \alpha) \left(1 - \kappa_2^S \right) \right] \\
&> MC^3(y) \left[\alpha \delta(\delta + 1) + (1 - \alpha)(1 - \kappa_2^S) \right] > 0
\end{aligned}
$$

where the first inequality follows from the definition of κ_2^S in (1.7) and the second follows from the definition of $\hat{\alpha}$. The lemma is proved. $\blacksquare$

(A) The proof of the statement is in the form of a series of claims, each describing conditions on priors such that the principal prefers information acquisition strategy p^1 to each of the others. First, we prove that the state three RSW action is efficient given either information acquisition strategy p^1 or p^4, under the assumptions of claim (A).

Lemma 1.17 *There exists $\underline{\kappa}_2^I$ such that for all $\kappa_2^I > \underline{\kappa}_2^I$, the p^4-RSW and p^1-RSW state 3 actions are efficient.*

Proof I first prove the statement for the p^4-RSW state 3 action. Let $V_r^3(\pi; p)$ denote the state three p-RSW payoff. Note that

$$
V_r^3(\pi; p^4) + C^2(y) = U^3(y) \tag{1.28}
$$

has two solutions since C^2 is convex, U^i is concave and $V_r^3(\pi; p^4) + C^2(y_2^r) = U^2(y_2^r) < U^3(y_2^r)$. Define $\hat{y}$ as the larger solution to (1.28). As y increases in a neighbourhood around $\hat{y}$, the left hand side of (1.28) crosses the right hand side from below. Since $MC^2(\hat{y}) > MU^3(\hat{y})$ we have $V_r^3(\pi; p^4) + C^2(y) > U^3(y)$ for all $y \geq \hat{y}$.

Since $MU^3(y_3^E) = MC^3(y_3^E)$, as we increase κ_2^I, y_3^E increases towards infinity. Thus, there exists a $\underline{\kappa}_2^I$ such that for all $\kappa_2^I > \underline{\kappa}_2^I$, $y_3^E > \hat{y}$. Thus, by the previous paragraph, $V_r^3(\pi; p^4) + C^2(y_3^E) > U^3(y_3^E)$ and therefore RSW-IC[1,2] does not bind and the lemma holds.

To see that this holds for the p^1-RSW state 3 action, define $\alpha := \pi_1/(\pi_1 + \pi_2)$ and replace C^2 and U^2 above with $\alpha C^1(\cdot) + (1 - \alpha)C^2(\cdot)$ and $\alpha U^1(\cdot) + (1 - \alpha)U^2(\cdot)$ respectively. ∎

The next lemma characterizes priors such that information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^4.

Lemma 1.18 *Define $\alpha = \pi_1/(\pi_1 + \pi_2)$. There exists $\underline{\pi}_1 < 1$, $\alpha^r(p^4) < 1$ such that for all $\pi_1 \in (\underline{\pi}_1, 1)$ and $\alpha \in (\alpha^r(p^4), 1)$ the unique payoff following information acquisition strategy p^4 is the p^4-RSW payoff and any continuation payoff following information acquisition strategy p^1 is strictly larger.*

Proof From Lemma 1.9, the state 2 p^4-RSW contract is inefficient for sufficiently small κ_1^S and from Lemma 1.17 we know that the state three p^4-RSW contract is efficient. Thus, from Lemma 1.15 part (ii), $i^* = 1$ so there exists $\underline{\pi}_1$ such that the RSW payoff is the unique payoff following information strategy p^4 for $\pi \in (\underline{\pi}_1, 1)$.

By Lemma 1.17 the state 3 p^1-RSW contract is efficient. Thus, $V_r^3(\pi; p^4) = V_r^3(\pi; p^1)$.

Now, consider the RSW problem of the state $\{1, 2\}$ principal

$$V_r^{\{1,2\}}(\pi; p^1) := \max_{(y_{12}, t_{12})} \left\{ t_{12} - \alpha C^1(y_{12}) - (1 - \alpha)C^2(y_{12}) : \alpha U^1(y_{12}) + (1 - \alpha)U^2(y_{12}) = t_{12} \right\}$$

(1.29)

Since κ_1^S is small, we can apply Proposition 1.2 to conclude that there exists $\alpha^r(p^4)$ such that $\alpha^r(p^4) < 1$ and for all $\alpha \in (\alpha^r(p^4), 1)$ we have $V_r^{\{1,2\}}(\pi; p^1) > \alpha V_r^1(\pi; p^4) +$

$(1 - \alpha)V_r^2(\pi; p^4)$. So $(\pi_1 + \pi_2)V^{\{1,2\}}(\pi; p^1) + \pi_3 V_r^3(\pi; p^1) > \sum_i \pi_i V_r^i(\pi; p^4)$. ∎

The next lemma characterizes priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^2.

Lemma 1.19 *Define $\alpha := \pi_2/(\pi_2 + \pi_3)$. There exists $\pi^r(p^2) < 1$ and $\underline{\alpha} < 1$ such that for all $\pi_1 \in (\pi^r(p^2), 1)$ and $\alpha \in (\underline{\alpha}, 1)$ the unique payoff following information acquisition strategy p^2 is the p^2-RSW payoff and the p^3 payoff is strictly larger.*

Proof The continuation game following information strategy p^2 is a two state game with priors $(\pi_1, \pi_2 + \pi_3)$. Define $\kappa(p^2) := \sup_y MC^1(y)/\left(\alpha MC^2(y) + (1 - \alpha)MC^3(y)\right)$. According to Proposition 1.2, if $\kappa(p^2)$ is sufficiently small, there exists $\pi^r(p^2)$ such that $\pi^r(p^2) < 1$, the unique payoff following information acquisition strategy p^2 is the p^2-RSW payoff for all $\pi_1 \in (\pi^r(p^2), 1)$ and the p^3 payoff is strictly larger.

I now show that $\kappa(p^2)$ can be made sufficiently small given the hypotheses of the proposition $\kappa(p^2) < \kappa_1^S/\left(\alpha + (1 - \alpha)/\kappa_2^S\right)$. For fixed κ_2^S, if we take α and κ_1^S close enough to 1, $\kappa(p^2)$ can be made sufficiently small to apply Proposition 1.2. The lemma is proved. ∎

The next lemma characterizes priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^5.

Lemma 1.20 *Let $\alpha = \pi_1/(\pi_1 + \pi_3)$. There exists $\underline{\alpha} < 1$ and $\pi^r(p^5) < 1$ such that for all $\alpha \in (\underline{\alpha}, 1)$ and $\pi_1 \in (\pi^r(p^5), 1)$ the unique payoff following information acquisition strategy p^5 is the p^5-RSW payoff and the p^3 payoff is strictly larger.*

Proof By Lemma 1.16, there exists an $\hat{\alpha}$ such that for $\alpha \in (\hat{\alpha}, 1)$ the problem for the principal who chooses information acquisition strategy $p^5 = \{\{1, 3\}, \{2\}\}$ is a two state informed principal problem with priors $(\pi_1 + \pi_3, \pi_2)$ that satisfies Assumption 1.2.

Define $\kappa(p^5) := \sup_y MC^{13}(y)/MC^2(y)$. Proposition 1.2 applies and the claim is proved if $\kappa(p^5)$ is sufficiently close to 1. I now check whether $\kappa(p^5)$ can be sufficiently close to 1. Note that $\kappa(p^5) \le \alpha\kappa_1^S + (1-\alpha)/\kappa_2^S < \alpha\kappa_1^S + (1-\alpha)$ where the first inequality follows from the convexity of the supremum operator. Choosing α less than but close to 1 and small κ_1^S, we can make $\kappa(p^5)$ small and Proposition 1.2 applies. ∎

Finally, note that for the informed game with state space p^1, the first best payoff has been achieved since each p^1-state principal is producing her efficient output. It follows from Proposition 1.3 that introducing further ignorance (i.e. an information strategy of p^3) will not improve payoffs. Thus, information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^3 for any priors.

By Lemma 1.19, the principal prefers p^3 to p^2 for appropriately restricted priors for any equilibrium following the choice of p^2; thus, she prefers p^1 to p^2 on these priors as well. Moreover, by Lemma 1.20 the principal prefers p^3 to p^5 for appropriately restricted priors for any equilibrium following the choice of p^5; thus, she prefers p^1 to p^5 on these priors as well.

To see that the intersection of the sets characterized in Lemmas 1.18 to 1.20 is open and nonempty, note that any priors such that π_1 is sufficiently large (but less than 1) and π_2 is sufficiently larger than π_3 is in this intersection.

(B) Follows same procedure as part **(A)**.

(C) As in part **(A)**, this part is shown in a series of lemmas each characterizing the set of priors such that ignorance is better than each of the other information acquisition options. The first lemma characterizes the set of priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^1.

Lemma 1.21 *There exists $\pi^r(p^1)$ such that $\pi^r(p^1) < 1$ and for any $\pi_1 + \pi_2 \in (\pi^r(p^1), 1)$ the unique payoff following information acquisition strategy p^1 is the p^1-*

RSW payoff and the completely uninformed principal's payoff is strictly larger.

Proof Define $\alpha := \pi_1/(\pi_1 + \pi_2)$ and $\kappa(p^1) := \sup_y \left(\alpha MC^1(y) + (1-\alpha)MC^2(y)\right)/MC^3(y)$. The game following information strategy p^1 is a two state informed principal problem with priors $(\pi_1 + \pi_2, \pi_3)$. Since the supremum operator is convex $\kappa(p^1) < \alpha\kappa_1^S\kappa_2^S + (1-\alpha)\kappa_2^S$. Thus, we choose κ_1^S, κ_2^S sufficiently small to apply Proposition 1.2 and our claim follows. ∎

The next lemma characterizes the set of priors such that information acquisition strategy p^3 is strictly preferred to information acquisition strategy p^2.

Lemma 1.22 *There exists $\pi^r(p^2)$ such that $\pi^r(p^2) < 1$ and for any $\pi_1 \in (\pi_r(p^2), 1)$ the unique payoff following information acquisition strategy p^2 is the p^2-RSW payoff and the uninformed principals payoff is strictly larger.*

Proof This proof is analogous to that of Lemma 1.19. Since $\kappa(p^2) < \sup_y \frac{MC^1(y)}{MC^3(y)} \leq \kappa_1^S\kappa_2^S$ we can choose κ_1^S, κ_2^S sufficiently small to apply Proposition 1.2. ∎

The next lemma characterizes the set of priors such that information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^4.

Lemma 1.23 *Define $\alpha = \pi_1/(\pi_1 + \pi_2)$. There exists $\alpha^r(p^4) < 1$ and $\underline{\pi}_1$ such that for any $\alpha \in (\alpha^r(p^4), 1)$ and $\pi_1 \in (\underline{\pi}_1, 1)$ the unique payoff following information acquisition strategy p^4 is the p^4-RSW payoff and the p^1-RSW payoff is strictly larger.*

Proof From Lemma 1.9, we know that the state 2 and 3 p^4-RSW contracts are inefficient for sufficiently small κ_1^S and κ_2^S respectively. Thus, from Lemma 1.15 item (iii), there exists priors $\underline{\pi}$ the p^4-RSW payoff is the unique payoff following information acquisition strategy p^4 for $\pi_1 \in (\underline{\pi}_1, 1)$ and any $\pi_2/(\pi_2 + \pi_3) \in (\underline{\pi}_2/(\underline{\pi}_2 + \underline{\pi}_3), 1)$.

This remainder analogous to Lemma 1.18 except we appeal to Lemma 1.8 to ensure that $V_r^3(\alpha; p^1) \geq V_r^3(\alpha; p^4)$ instead of Lemma 1.9.[22] ∎

---

[22] The p^4 payoffs are constant in α so the statement trivially holds for all $\alpha \in [0, 1]$.

Our final lemma characterizes the set of priors such that information acquisition strategy p^1 is strictly preferred to information acquisition strategy p^5.

Lemma 1.24 *Let* $\alpha = \pi_1/(\pi_1 + \pi_3)$. *There exists,* $\underline{\alpha} < 1$ *and* $\pi^r(p^5)$ *such that* $\pi^r(p^5) < 1$ *and for any* $\pi_1 + \pi_3 \in (\pi_r(p^5), 1)$ *and* $\alpha \in (\underline{\alpha}, 1)$ *the unique payoff following information acquisition strategy* p^5 *is the* p^5-*RSW payoff and the uninformed principals payoff is strictly larger.*

Proof This follows immediately from Lemma 1.20. Although Lemma 1.20 is proved under the assumptions of claim (A), only the hypothesis that k_1^S is sufficiently small was used in the proof. Since claim (C) shares this hypothesis, the lemma applies here as well. ∎

To see that the intersection of the sets characterized in Lemmas 1.21 to 1.24 is nonempty and open, note any priors with π_1 sufficiently large (but less than 1) is in this intersection. ∎

(D) If π_1 is small enough, we can achieve the first best ex ante payoff using the same technique as in Claim 2 of Lemma 1.14. If κ_1^I and κ_2^I are large enough, we can show that the p^4-RSW menu is efficient and therefore achieves the first best ex ante payoff using the same technique as in Lemma 1.17.

Proof of Corollary 1 If both κ_1^S and κ_2^S are close to 1, simply apply Proposition 1.4 (C).

Suppose κ_2^S is close to 1 and κ_2^I is large so that p^1 is optimal on the set of priors described in Proposition 1.4 (A): $V(\pi; p^1) := (\pi_1 + \pi_2)V_r^{\{1,2\}}(\alpha; p^1) + \pi_3 V_r^3 > \sum \pi_i V_r^i$. Since $V(\pi; p^1)$ is continuous in π, $V(\pi; p^1) \to V(\pi; p^3) := V_r^{\{1,2,3\}}(\pi, p^3)$ as $\pi_3 \to 0$. So for small π_3 there exists $\delta > 0$ such that $V(\pi; p^1) - V(\pi; p^3) = \delta$ and $V(\pi; p^3) - \sum_i \pi_i V_r^i = V(\pi; p^1) - \delta - \sum_i \pi_i V_r^i > 0$.

For κ_1^I large and κ_2^S small close to 1, the proof is similar. ∎

1.8.4 Information Acquisition as Hidden Action

Denote the value of the principal's RSW problem in state $k \in \{0, 1, 2\}$ by

$$V_k^r := \max_{\{(y_i, t_i)\}_{i \in \{0,1,2\}}} t_k - C^k(y_k)$$

$$s.t. \quad (\text{IC}[i,j]) \quad t_i - C^i(y_i) \geq t_j - C^i(y_j) \quad \text{for all } i, j \in \{0, 1, 2\} \text{ and}$$

$$(\text{RSW-IR}[i]) \quad U^i(y_i) = t_i \qquad \text{for all } i \in \{0, 1, 2\}$$

Let (y_i^r, t_i^r) denote the RSW contract for the state $i \in \{0, 1, 2\}$ principal.[23]

The following lemma gives the necessary and sufficient conditions for equilibrium in this environment.

Lemma 1.25 *The contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ and the information acquisition strategy α is an equilibrium if and only if*

$$(MIX) \quad \alpha \in \text{argmax} \left\{ \alpha \left[t_0^* - C^0(y_0^*) \right] + (1 - \alpha) \sum_{i=1,2} \pi_i \left(t_i^* - C^i(y_i^*) \right) \right\}$$

$$(IR) \quad \alpha \sum_{i=1,2} \pi_i \left(U^i(y_0^*) - t_0^* \right) + (1 - \alpha) \sum_{i=1,2} \pi_i \left(U^i(y_i^*) - t_i^* \right) \geq 0$$

$$(IC) \quad t_i^* - C^i(y_i^*) \geq t_j^* - C^i(y_j^*) \text{ for all } i, j \in \{0, 1, 2\}$$

$$(NB) \quad t_i^* - C^i(y_i^*) \geq V_i^r \text{ for all } i \in \{0, 1, 2\}$$

Proof of Lemma 1.25 Sufficiency: Suppose the contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ and the information acquisition strategy α satisfy MIX, IR, IC, and NB. Then, the contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ is an equilibrium contract given α by Theorem 1 in Maskin and Tirole [11]. Moreover, given, the contract $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$, the MIX condition ensures that the principal cannot deviate profitably by choosing a different α.

Necessity: Suppose, IR, IC, or NB is violated. Then by Theorem 1 in Maskin

---

[23]I have suppressed the dependance of the uninformed principal's RSW strategies and payoffs on priors.

and Tirole [11] $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ cannot be an equilibrium given α. If MIX is violated, then the principal has a profitable deviation to another α. ∎

Proof of Lemma 1.1 Due to the IC conditions of the equilibrium $t_2^r - C^2(y_2^r) \geq t_0^r - C^2(y_0^r)$ and $t_1^r - C^1(y_1^r) \geq t_0^r - C^1(y_0^r)$. Weighting each of these by the appropriate prior we have

$$\sum_{i=1}^{2} \pi_i^r(t_i^r - C^i(y_i^r)) \geq t_0^r - C^0(y_0^r).$$

∎

Proof of Proposition 1.5 Let $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ be an equilibrium contract with information strategy α^*. Due to lemma 1.1, without loss of generality we can set $\alpha^* = 0$. I are therefore considering an equilibrium in a 2 state informed principal problem (while still respecting the extra incentive compatibility constraint of the uninformed principal). Thus, by Proposition 1.2, since κ is close to 1, we know that there exists an interval of priors such that the action is inefficient in at least one state.

Suppose the inefficient state is state 2. Let (y_2', t_2') be a contract for the state 2 principal that lies on the same indifference curve as the contract (y_2^*, t_2^*) but is closer to the efficient level of y. Then the agent receives a higher payoff at (y_2', t_2') in state 2 than at (y_2^*, t_2^*).[24]

Choose (y_0', t_0') to be the (unique) intersection between the state 1 and state 2 indifference curves passing through the points (y_1^*, t_1^*) and (y_2^*, t_2^*) respectively. Then $t_1^* - C^1(y_1^*) = t_0' - C^1(y_0')$ and $t_2' - C^2(y_1') = t_0' - C^2(y_0')$. Weighting by the appropriate prior and summing these two equations we get

$$t_0' - C^0(y_0') = \pi \left(t_1^* - C^1(y_1^*)\right) + (1 - \pi)\left(t_2' - C^2(y_1')\right) \tag{1.30}$$

---

[24]To see this note that, fixing the payoff to the principal, the greatest payoff to the agent is at the efficient level of y: $\max_{(y,t)}\{U^i(y) - t : t - C^i(y) = \bar{V}\} = \max\{U^i(y) - C^i(y) - \bar{V}\} = U^i(y_i^E) - C^i(y_i^E)$ where $\bar{V}$ is a constant.

Now we check the agents IR constraint. First note that

$$\pi(U^1(y_1^*) - t_1^*) + (1 - \pi)(U^2(y_2') - t_2') > \pi(U^1(y_1^*) - t_1^*) + (1 - \pi)(U^2(y_2^*) - t_2^*) \geq 0$$

where the first inequality follows by our choice of (y_2', t_2') and the second follows since $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ is assumed to be an equilibrium and $\alpha^* = 0$. Thus, there exists $\alpha' > 0$ such that

$$\alpha' \left(U^0(y_0') - t_0'\right) + (1 - \alpha') \left[\pi(U^1(y_1^*) - t_1^*) + (1 - \pi)(U^2(y_2') - t_2')\right] = 0.$$

Since (y_2', t_2') is on the same indifference curve as (y_2^*, t_2^*)

$$\pi\left(t_1^* - C^1(y_1^*)\right) + (1 - \pi)\left(t_2^* - C^2(y_1^*)\right) = \pi\left(t_1^* - C^1(y_1^*)\right) + (1 - \pi)\left(t_2' - C^2(y_1')\right)$$

and due to equation (1.30)

$$\alpha'\left(t_0' - C^0(y_0')\right) + (1 - \alpha')\left[\pi\left(t_1^* - C^1(y_1^*)\right) + (1 - \pi)\left(t_2' - C^2(y_1')\right)\right] =$$
$$\pi\left(t_1^* - C^1(y_1^*)\right) + (1 - \pi)\left(t_2^* - C^2(y_1^*)\right).$$

Thus, the expected payoff to the principal from offering contract $\{(y_0', t_0'), (y_1^*, t_1^*), (y_2', t_2')\}$ with α' is equal to the expected payoff from offering $\{(y_0^*, t_0^*), (y_1^*, t_1^*), (y_2^*, t_2^*)\}$ with α^*.

If the state 1 contract is inefficient, we can similarly find a payoff equivalent menu with positive probability of being ignorant. ∎

68

References

[1] George A. Akerlof. The market for "Lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, 84(3):488–500, 1970. 1.1

[2] Paual Beaudry. Why an informed principal may leave rents to an agent. *International Economic Review*, 35(4):821–832, November 1994. 1.1.1

[3] Kenneth. Black, Jr and Harold D. Skipper, Jr. *Life and Health Insurance*. Upper Saddle River, N.J. : Prentice Hall, 13th edition, 2000.

[4] Hector Chade and Randy Silvers. Informed principal, moral hazard, and the value of a more informative technology. *Economics Letters*, 74:291–300, 2002. 1.1.1

[5] Jacques Crémer. Arm's Length Relationships. *The Quarterly Journal of Economics*, 110(2):275–295, May 1995. 1.1.1

[6] John Duggan and Tasos Kalandrakis. A Note on Strict Lower Hemi-continuity of Mixed Constraint Correspondences. `http://www.johnduggan.net/misc/LHC.pdf`, 2007. 1.8.2

[7] Aaron Finkle. Relying on information acquired by a principal. *International Journal of Industrial Organization*, 23(3-4):263–278, April 2005. 1.1.1

[8] Ayça Kaya. When Does It Pay To Get Informed? *International Economic Review*, 51(2):533–551, May 2010. 1.1.1

[9] Lionel Macedo. The Role of the Underwriter in Insurance. *Primer Series on Insurance*, (8), 2009.

[10] Eric Maskin and Jean Tirole. The Principal-Agent Relationship with an Informed Principal: The Case of Private Values. *Econometrica*, 58(2):379–409, 1990. 1.1, 1.1.1

[11] Eric Maskin and Jean Tirole. The Principal-Agent Relationship with an Informed Principal II: Common Values. *Econometrica*, 60(1):1–42, 1992. 1.1, 1.1.1, 1.2, 1.3, 1.3, 12, 1.4.2, 1.8.1, 1.8.1, 1.8.1, 18, 1.8.1, 1.8.2, 1.8.2, 1.8.3, 1.8.3, 1.8.4

[12] Paul R Milgrom and Ilya Segal. Envelope Theorems for Arbitrary Choice Sets. *Econometrica*, 70(2):583–601, 2002. 1.8.1, 1.8.2

[13] Roger B Myerson. Mechanism design by an informed principal. *Econometrica*, 51(6):1767–1797, December 1983. 1.1, 1, 3, 1.1.1, 1.2, 11, 1.4

[14] Tymofiy Mylovanov and Thomas Tröger. Irrelevancy of the principal's private information in independent private values environments. Working Paper, 2012. 1.1.1

[15] Ed Nosal. Information Gathering By a Principal. *International Economic Review*, 47(4):1093–1111, November 2006. 1.1.1

[16] Michael Rothschild and Joseph Stiglitz. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *The Quarterly Journal of Economics*, 90(4):629–649, 1976. 7

[17] Ilya Segal and Michael D. Whinston. Robust Predictions for Bilateral Constracting With Externalities. *Econometrica*, 71(3):757–791, 2003. 1.1, 5

[18] Sergei Severinov. An efficient solution to the informed principal problem. *Journal of Economic Theory*, 141:114 – 133, 2008. 1.1.1

[19] Randy Silvers. The value of information in a principal-agent model with moral hazard: the ex post contracting case. *Games and Economic Behavior*, 74(1):352–365, January 2012. 1.1.1

[20] Nancy L. Stokey and Robert E. Lucas. *Recursive methods in economic dynamics*. Harvard University Press, 1989. 1.8.2, 1.8.2

[21] Charles Wilson. A model of insurance markets with incomplete information. *Journal of Economic Theory*, 16(2):167–207, 1977. 7

Chapter 2

When Should an Employer Offer a Menu Contract?

2.1 Introduction

When a privately informed employer is contracting with a worker, the employer needs to design a contract that releases the right information at the right time to optimally exploit her information asymmetry. For example, consider a law firm which has advanced knowledge about the likelihood of winning a trial and needs to assign an attorney to the case. In addition, the law firm cannot observe the effort the attorney exerts for the case. A number of papers have analyzed such environments but have restricted the contracts proposed by the principal to be point-contracts: contracts that leave the principal no discretion once a contract is accepted (see for example Beaudry [1]; Inderst [3]; Chade and Silvers [2][1]; Kaya [4]; Silvers [10]). For example, the law firm may be restricted to paying a wage based on the only observable outcome: whether the trial is won or lost. On the other hand, she could in addition specify

---

[1]Chade and Silvers [2] do consider more general contracts as a robustness check but mainly focus on point contracts. See below for more details.

bonuses to be paid that depend on the difficulty of the case. In this paper I allow the principal to propose contracts that allow her further discretion after the contract has been accepted. Such contracts are called menu-contracts in the literature since they take the form of a list of contracts that is offered to the agent from which the principal chooses after the agent has accepted.

First, I prove that allowing menu-contracts removes implausible inefficient equilibria that exist when only point-contracts are considered without needing to make ad-hoc assumptions on the set of equilibria studied; typically, these inefficient equilibria are avoided in the literature via equilibrium refinements or by assuming the principal will separate in the least costly way. These equilibria exists because point-contracts can subject the principal to the agent's arbitrary off-path posterior beliefs that deter her from exploiting her private information. For example, very inefficient contracts can be supported in equilibrium by punishing deviations from said contracts with agent's beliefs that put probability 1 on the worst state. I characterize the lower bound payoff for each principal type in a two-type principal-agent model with moral hazard when the principal is unrestricted in her contract choice. The contract that achieves this lower bound is belief free: it is acceptable to the agent regardless of her beliefs about the principal's type and is therefore not susceptible to the punishing off-path beliefs mentioned above. Moreover, I show that this lower bound is always strictly higher than the lower bound when the principal is restricted to offering point-contracts. In particular, the set of menu-contract equilibrium payoffs is higher than the set of point-contract equilibrium payoffs in the strong set order.

Further, depending on the specific environment, the restriction to point-contracts can strictly reduce the informed principal's ex ante payoff, and thus the value of information to the principal. While Myerson's [7] *inscrutability principle* implies that the principal cannot lose by remaining inscrutable via menu-contracts, I characterize

precisely when this ability to remain inscrutable and retain discretion after the contract is accepted is strictly beneficial to the principal. If one wants to understand the principal's incentives to gather information prior to entering a contract, restricting the space of contracts to point-contracts can skew results towards ignorance.

Myerson [7] defined the general problem of the privately informed principal while, in a specialized environment, Maskin and Tirole [5] characterize the equilibria in the model and derive their properties. In particular, Maskin and Tirole characterize a lower bound in informed principal problems in environments with pure adverse selection (i.e. environments with no hidden actions). Segal and Whinston [9] successfully apply Maskin and Tirole's methodology to reduce the indeterminacy of equilibria in a class of bilateral contracting problems. Like our work, Segal and Winston use the concept of menu-contracts to eliminate very inefficient equilibria. Their work differs from the current paper in that the private information of the principal is her own hidden effort, in particular her profile of trade with other agents, and therefore endogenous in their model.

To my knowledge, there has not been a complete characterization in the literature of the equilibria in a model of moral hazard when the principal has private information of the productivity of the agent's effort and is not restricted in her contract choice. Notably, Chade and Silvers [2] consider menu-contracts in a robustness check of one of their main results. They show that the equilibrium payoff in their result, the least-cost separating payoff, can be supported by a menu-contract equilibrium. I show that the principal can never do worse than under the menu-contract equilibrium they construct and depending on the parameters of the model there may exist menu-contract equilibria under which the principal can do strictly better.

2.2 Model

2.2.1 Preferences and Technologies

I adopt a model similar to Silvers [10]. The agent (e.g. the potential worker) is (weakly) risk averse and maximizes expected utility. Her von Neumann-Morgenstern utility function over wage income, w, and effort, a, is given by $U(w) - a$ with $U'(w) > 0$ and $U''(w) \leq 0$. The agent chooses an effort a from a set $A := \{a_1, a_2\} \subset \mathbb{R}_+^2$ with $0 \leq a_1 < a_2 < \infty$. Let $h := U^{-1}$ denote the inverse of U.

The effort chosen induces a conditional probability distribution over the set of possible outcomes $Q := \{q_s, q_f\} \subset \mathbb{R}_{++}^2$ where $q_f < q_s < \infty$ (the subscript f denotes failure while s denotes success). These outcome are the principal's revenues. Thus, if she pays wage w and outcome q_n is realized her payoff is $q_n - w$.

The principal (e.g. the employer) is a risk neutral expected profit maximizer who needs to hire the agent to complete a task. Task productivity can either be high or low. In particular, each type of task is associated with a set of conditional probability distributions that determines the probability of the task being successful given a particular effort level. The task of type $i \in \{H, L\}$ has conditional probability distribution $\Pi_i = \{\pi_i(a_1), \pi_i(a_2)\}$ where $\pi_i(a) = (\pi_{is}(a), \pi_{if}(a))$ denotes the conditional probability distribution across Q when the effort is $a \in A$. Set $\pi_{Hs}(a_2) > \pi_{Ls}(a_2)$ and $\pi_{Hs}(a_1) \geq \pi_{Ls}(a_1)$ so that the type-H task is more productive that the type-L task. Let $\lambda \in (0, 1)$ be the common prior probability that the task is of high probability; i.e. has conditional probability distribution Π_H. I will refer to a principal who has task type i as a type-i principal.

I assume that the probability distributions satisfy the monotone likelihood ratio property (MLRP): the relative likelihood of a higher outcome to a lower outcome is

increasing in the effort. Formally, for both $i \in \{H, L\}$

$$\frac{\pi_{is}(a_1)}{\pi_{if}(a_1)} < \frac{\pi_{is}(a_2)}{\pi_{if}(a_2)}.$$

2.2.2   Contracts

A point-contract is a set of payments from the principal to the agent $\mathbf{w} \in [\underline{w}, \infty)^2$ such that the principal pays the agent w_n when outcome q_n is realized where $\underline{w} > -\infty$ for $n \in \{s, f\}$.

A menu-contract is a direct revelation mechanism that specifies a set of point-contracts $\{\mathbf{w}^H, \mathbf{w}^L\}$ such that $\mathbf{w}^i \in [\underline{w}, \infty)^2$ for all $i \in \{H, L\}$ and allows the principal to choose from amongst this set after the agent has accepted the offer.[2] Note that a point-contract can be seen as a degenerate menu-contract where $\mathbf{w}^H = \mathbf{w}^L$.

I will generically refer to a contract offered for corresponding to outcome q_n with the notation C_n with $C_n = \{C_{Hn}, C_{Ln}\}$. A contract C_n, for $n \in \{s, f\}$ could be a menu-contract (i.e. $C_n = \{w_n^H, w_n^L\}$) or a point-contract (i.e. $C_n = w_n$).

2.2.3   Information

I assume that the principal perfectly observes the task productivity while the agent does not. Let $\rho(C) := \{\rho(C; H), \rho(C; L)\}$ denote the agent posterior belief over the type of the principal after observing the proposed contract C. Further, let

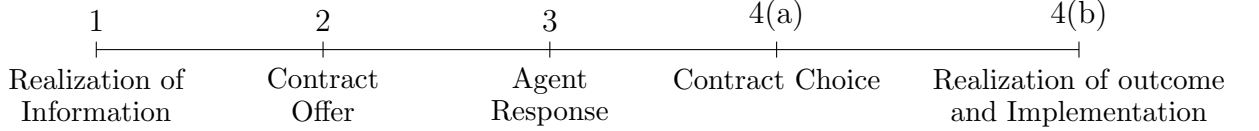$$p(a; \rho(C)) := \rho(C; H)\pi_H(a) + \rho(C; L)\pi_2(a)$$

denote the agent's expected probability distribution over Q conditional on having chose effort $a \in A$ where $p_n(a; \rho(C))$ is her expected probability of outcome q_n con-

---

[2]The formal timing of the game is outlined in Section 2.2.4.

75

ditional on having chosen effort $a \in A$ for $n \in \{s, f\}$.

2.2.4 Timing

The timing of the game is as follows:

1	2	3	4(a)	4(b)
Realization of Information	Contract Offer	Agent Response	Contract Choice	Realization of outcome and Implementation

In stage 1, nature chooses the principal's technology and this is observed by the principal. In stage 2, the principal offers the agent a contract. The agent accepts or rejects the contract in stage 3. If the agent rejects the contract, the principal receives zero utility and the agent receives utility $\bar{U}$. In stage 4, if a point-contract was offered, the game skips to stage 4(b); the agent chooses her effort level, the outcome is realized and the corresponding wage is paid. If a menu-contract was offered, the principal chooses from the menu of contracts in stage 4(a). Then stage 4(b) follows as before: the agent chooses her effort level, the outcome is realized and the corresponding wage is paid.

2.2.5 Constraints

Point Contracts

A contract C will implement effort a if it is incentive compatible and individually rational for the agent. A point-contract is incentive compatible for the agent for effort a_k if

$$\sum_{n \in \{s,f\}} \left[p_n(a_k; \rho(C)) - p_n(a_{k-1}; \rho(C)) \right] U(C_n) \geq a_k - a_{k-1} \qquad (AIC(a_k; \rho(C)))$$

76

where $a_0 = 0$ and is individual rationality for effort a_k for $k \in \{1, 2\}$ if

$$\sum_{n \in \{s,f\}} p_n(a_k; \rho(C))U(C_n) - a_k \geq \bar{U}. \qquad (AIR(a_k; \rho(C)))$$

A type-i principal who implements effort $a \in A$ with contract C^i incurs expected cost $\sum_{n \in \{s,f\}} \pi_{in}(a)w_n^i$ where, again with some abuse of notation, $\{w_s^i, w_f^i\}$ is either a point-contract offered by type-i or the type-i part of the menu contact offered, and reaps expected benefit $\sum_{n \in \{s,f\}} \pi_{in}(a)q_n$. Suppose each type $i \in \{H, L\}$ principal implements effort $a(i)$. The contract C^i that implements $a \in A$ is incentive compatible for the type-i principal if it satisfies for all $i, j \in \{H, L\}$

$$\sum_{n \in \{s,f\}} \pi_{in}(a(i))(q_n - w_n^i) \geq \sum_{n \in \{s,f\}} \pi_{in}(a(j))(q_n - w_n^j). \qquad (PIC(a; i, j))$$

Menu Contracts

Due to Myerson's [7] inscrutability principle, the agent's individual rationality constraint need only be satisfied in expectation. A feasible menu-contract must be individually rational for the agent and incentive compatible for both the agent and the principal. A menu-contract C is individually rational if

$$\lambda \left[\sum_{n \in \{s,f\}} \pi_{Hn}(a(1))U(w_n^H) - a(H) \right] + (1 - \lambda) \left[\sum_{n \in \{s,f\}} \pi_{Ln}(a(L))U(w_n^L) - a(L) \right] \geq \bar{U}.$$

A separating menu-contract C (i.e. a menu-contract $\{\mathbf{w}^H, \mathbf{w}^L\}$ such that $\mathbf{w}^H \neq \mathbf{w}^L$) is incentive compatible for the agent if $AIC(a(H); \{1, 0\})$ and $AIC(a(L); \{0, 1\})$ are satisfied for each $a(i)$ for $i \in \{H, L\}$. A pooling menu-contract C, like a pooling point-contract, is incentive compatible if $a(H) = a(L) = a$ and it satisfies $AIC(a; \{\lambda, 1 - \lambda\})$. If $a(i) = a_1$ for any $i \in \{H, L\}$, the principal does not have

to incentivize the agent to take an effort since she has already accepted the menu-contract.

Finally, a menu-contract C that implements a is incentive compatible for the principal if it satisfies $PIC(a; i, j)$ for $i, j \in \{H, L\}$.

2.3 Preliminary Analysis

In this section we establish a number of benchmark equilibrium contracts and their payoffs. I use these payoffs to establish upper and lower bounds on payoffs that are supported in equilibrium for the game where only point-contracts are allowed and the game where menu-contracts can be offered.

I first define the contracts that would result if the task type were common knowledge; the payoff from these contracts is called the public information benchmark. It is first best ex ante payoff for the principal. Further, we define the lowest payoffs supported by an equilibrium in the game where only point-contracts can be offered and the game where menu-contracts can be offered.

2.3.1 The Public Information Benchmark

Let $\mathbf{w} = \{w_s, w_f\}$ denote a generic point-contract. When the principal's type is public information, for each effort a the principal of type $i \in \{H, L\}$ solves

$$\min_{\mathbf{w}} \sum_{n \in \{s, f\}} \pi_{in}(a) w_n \quad \text{s.t. } AIC(a; \rho(\mathbf{w})) \text{ and } AIR(a; \rho(\mathbf{w}))$$

where $\rho(\mathbf{w}; i) = 1$ for all w. Denote the solution to this problem by $\mathbf{w}^*(a; i)$. The principal then choose the effort a that maximizes

$$\sum_{n \in \{s,f\}} \pi_{in}(a)\left(q_n - w_n^*(a; i)\right).$$

Denote this effort by $a^*(i)$.

2.3.2 Lower Bound Equilibrium Point-Contract Payoff

If

$$\pi_{Hs}(a_2) - \pi_{Hs}(a_1) > \pi_{Ls}(a_2) - \pi_{Ls}(a_1)$$

then the lower bound point contract payoff is obtained when both principal types offer $\mathbf{w}^*(a^*(L); L)$. It is straightforward to check that this equilibrium is supported by agent's beliefs that put probability 1 on the principle being type-L whenever any contract $\mathbf{w}' \neq \mathbf{w}^*(a^*(L); L)$ is offered.

2.3.3 Lower Bound Equilibrium Menu-Contract Payoff

Let $a(i)$ denote the effort implemented by the principal when she has a type-i task for $i \in \{H, L\}$. In the spirit of Maskin and Tirole [5] a menu-contract plus induced effort

set $\left\{\left(\mathbf{w}_r^H, a(H)\right), \left(\mathbf{w}_r^L, a(L)\right)\right\}$ is RSW for $i \in \{H, L\}$ if the type-i principal solves[3]

$$\underline{U}(i) = \begin{cases} \max\limits_{\{(\mathbf{w}^H, a(H)), (\mathbf{w}^L, a(L))\}} \sum_n \pi_{in}(a(i))\left(q_n^i - w_n^i\right) \quad \text{subject to} \\ \\ \sum\limits_{n \in \{s,f\}} \pi_{in}(a(H))U(w_n^i) - a(i) \geq \bar{U} \qquad\qquad\qquad \text{for } i \in \{H, L\} \\ \\ (\pi_{is}(a(i)) - \pi_{is}(a_1))\left[U(w_s^i) - U(w_f^i)\right] \geq a(i) - a_1 \qquad \text{for } i \in \{H, L\} \\ \\ \sum\limits_{n \in \{s,f\}} \pi_{in}(a(i))\left(q_n - w_n^i\right) \geq \sum\limits_{n \in \{s,f\}} \pi_{in}(a(j))\left(q_n - w_n^j\right) \quad \text{for } i, j \in \{H, L\} \end{cases} .$$

When menu-contracts are allowed, the RSW problem generates lower bound pay-offs for the type-i principal since the agent will accept any RSW menu *regardless of her belief about the type of the principal.*[4] To see this, note first that the RSW problem for the type-i principal specifies an entire menu: a contract for each task type $j \in \{H, L\}$. This menu must be incentive compatible for *every* principal type $j \in \{H, L\}$, not just type-i. Finally, this menu must guarantee the agent her reservation payoff *ex post* and induces the appropriate effort level regardless of the type of task the principal has. Thus, the agent will always accept an RSW menu and perform the correct effort. The type-i principal can always deviate to her RSW menu and obtain the associated payoff.[5]

2.4 The Deficiency of Point-Contracts

Let $\mathscr{P}^M$ be the set of ex ante equilibrium payoffs for the principal when menu-contracts are allowed and $\mathscr{P}^P$ be the set of ex ante equilibrium payoffs for the principal when only point-contracts are allowed. In our first result, we prove that $\mathscr{P}^M$ is higher

---

[3]RSW is an acronym for Rothchild-Stiglitz-Wilson, a reference to the similar least cost separating contracts developed in the insurance models of Rothschild and Stiglitz [8] and Wilson [11].

[4]In terms of Myerson [7], any feasible solution to the RSW problem is *safe*. The RSW menu for the type-i principal is her best safe menu.

[5]For further discussion of RSW menus see Maskin and Tirole [5].

than $\mathscr{P}^P$ in the strong set ordering (and the converse is not true).

Proposition 2.1 *The set $\mathscr{P}^M$ is higher in the strong set order than the set $\mathscr{P}^P$. Further, if*

$$\pi_{Hs}(a_2) - \pi_{Hs}(a_1) > \pi_{Ls}(a_2) - \pi_{Ls}(a_1), \tag{2.1}$$

then there exists $v \in \mathscr{P}^P$ such that $v < \min \mathscr{P}^M$.

Condition (2.1) ensures that a separating equilibrium exists. The value of the RSW contract for the type-i principal provides a lower bound payoff on the principal's problem *if the principal can offer menu-contracts.* As we show in Lemma 2.1 below, the RSW payoff *can* be obtained using point-contracts: it is the least-cost separating equilibrium. However, $\mathscr{P}^P$ also contains strictly lower payoffs. Allowing menu-contracts eliminates these low value equilibria.

Proof The first statement follows since menu-contracts are generalizations of point-contracts.

To prove the second statement I characterize the RSW menu-contract and show that it gives a strictly higher ex ante payoff to the principal than the lower bound point-contract. Due to the linearity of the principal's indifference curves (in particular, the fact that this the linearity endows a single crossing property on the indifference curves between principal's types) computing the RSW can be simplified, as we show in the following lemma.

Lemma 2.1 *The RSW allocation is the least-cost separating equilibrium that has type-L principal offering $\mathbf{w}^*(a^*(L); L)$ and the type-H principal offering the solution*

to

$$I^{RSW} \begin{cases} \max_{(\mathbf{w}^H, a(H))} \sum_n \pi_{Hn}(a(H)) \left(q_n - w_n^H \right) \quad subject\ to \\[2ex] \sum_{n \in \{s,f\}} \pi_{Hn}(a(H)) U(w_n^H) - a(H) \geq \bar{U} \\[1ex] (AIR(a(H); \{1,0\})) \\[2ex] (\pi_{Hs}(a(H)) - \pi_{Hs}(a_1)) \left[U(w_s^H) - U(w_f^H) \right] = a(H) - a_1 \\[1ex] (AIC(a(H), \{1,0\})) \\[2ex] \sum_{n \in \{s,f\}} \pi_{Ln}(a^*(L)) \left(q_n - w_n(a^*(L)) \right) \geq \sum_{n \in \{s,f\}} \pi_{Ln}(a(H)) \left(q_n - w_n^H \right) \\[1ex] (PIC(a^*(L); L)) \end{cases} .$$

The RSW menu always exists.

Proof This proof essentially follows that of Proposition 2 in Maskin and Tirole [5]. Let $(\hat{\mathbf{w}}(a(H); H), \hat{a}(H))$ be a solution to I^{RSW}.

First, we claim that the constraint $AIC(a(H); \{1,0\})$ in problem I^{RSW} must bind. Suppose $AIC(a(H); \{1,0\})$ holds with strict inequality and let $\pi_{Hs} \geq \pi_{Hf}$. Then, decrease w_s^H and increase w_f^H slightly to $(w_s^H - \epsilon_s, w_f^H + \epsilon_f)$ for small $\epsilon_s, \epsilon_f > 0$ so that $AIR(a(H); \{1,0\})$ and $AIC(a(H); \{1,0\})$ still hold. Since $\pi_{Hs} > \pi_{Ls}$, (ϵ_s, ϵ_f) can be chosen such that the right hand side of $PIC(a^*(L); L, H)$ (possibly weakly) decreases while the objective function strictly increases. If $\pi_{Hs} < \pi_{Hf}$ we can increase w_s^H and decrease w_f^H and arrive at a similar result.

Second, we claim that $\{(\hat{\mathbf{w}}(a(H); H), \hat{a}(H)), (\mathbf{w}^*(a^*(L); L), a^*(L))\}$ is incentive compatible. This is vacuously true for the type-L principal since $PIC(a^*(L); L, H)$ is imposed in problem I^{RSW} and $\mathbf{w}^*(a^*(L); L)$ is incentive compatible for the agent by construction. Further, $AIC(H, \{1,0\})$ is imposed in problem I^{RSW}. It remains to

show that

$$\sum_{n \in \{s,f\}} \pi_{Hn}(\hat{a}(H)) \left(q_n - \hat{w}_n^H(\hat{a}(H))\right) \geq \sum_{n \in \{s,f\}} \pi_{Hn}(a^*(L)) \left(q_n - w_n(a^*(L))\right). \quad (2.2)$$

I claim that (2.2) holds with strict inequality. Note that the curve in (w_s, w_f) space implicitly defined by the agents RSW incentive compatibility constraint for the type-H principal,

$$(\pi_{Hs}(a(H)) - \pi_{Hs}(a_1)) \left[U(w_s^H) - U(w_f^H)\right] = a(H) - a_1,$$

is strictly above that of the type-L principal,

$$(\pi_{Ls}(a(H)) - \pi_{Ls}(a_H)) \left[U(w_s^H) - U(w_f^H)\right] = a(L) - a_1$$

due to inequality (2.1). Further, the indifference curves the type-H principal's indifference curves are steeper than the type-L principal's. Therefore, the indifference curves possess the single crossing property. If $PIC(a^*(L); L, H)$ holds with equality, $\hat{\mathbf{w}}_H$ lies to the north-west of $\mathbf{w}_2^*$ in (w_s, w_f) space which implies that (2.2) strictly holds. Otherwise, $\hat{\mathbf{w}}_H = \mathbf{w}_H^*$ and (2.2) strictly holds since $\pi_s(a^*(H)) > \pi_s(a^*(L))$.

The RSW problem for the type-H principal is more constrained than I^{RSW} but $(\hat{\mathbf{w}}(a(H); H), a(H))$ solves the latter problem and satisfies all the constraints of the former. Therefore it solves the RSW problem for the type-H principal. Similarly, the RSW problem for the type-H principal is more constrained than the public information problem, but $(\mathbf{w}^*(a^*(L); L), a^*(L))$ solves the latter problem and satisfies all the constraints of the former. Therefore it solves the RSW problem for the type-L principal.

To see that this menu-contract exists we first claim that $\mathbf{w}^*(a; L)$ exists for any a. For $a = a_1$, $\mathbf{w}^*(a; L) = (h(\bar{U} + a_1), h(\bar{U} + a_1))$. For $a = a_2$, the constraints $AIC(a_2; \{0, 1\})$ and $AIR(a_2; \{0, 1\})$ will be satisfied with equality and therefore define the implicit functions

$$w_{f,AIR}(w_s) = h\left(\frac{\bar{U} + a_2 - \pi_{Ls}(a_2)U(w_s)}{\pi_{Lf}(a_2)}\right), \text{ and}$$

$$w_{f,AIC}(w_s) = h\left(U(w_s) - \frac{a_2 - a_1}{(\pi_{Ls}(a_2) - \pi_{Ls}(a_1))}\right).$$

Since $w_{f,AIR}$ is strictly decreasing and $w_{f,AIC}$ is strictly increasing, they must intersect exactly once in $\mathbb{R}^2$. Denote this intersection point (w'_s, w'_f). If this $(w'_s, w'_f) \in [\underline{w}, \infty)^2$ we are done: $\mathbf{w}^*(a; L) = (w'_s, w'_f)$. Otherwise, the solution is $\mathbf{w}^*(a; L) = (w''_s, \underline{w})$ where w''_s satisfies $w_{f,AIC}(w''_s) = \underline{w}$. If the type-$L$ principal is indifferent between a_1 and a_2, set $a^*(L) = a_2$.

I can break I^{RSW} down into separate problems of minimizing the cost of implementing each effort then choosing most profitable effort. Note that $a^*(H) = a_1$ implies that $a^*(L) = a_1$ since the expected payoff from the agent's effort is strictly higher for they type-H principal. Thus if $a^*(H) = a_1$, $\hat{\mathbf{w}}(a(H); H) = (h(\bar{U} + a_1), h(\bar{U} + a_1))$ which satisfies all the constraints of I^{RSW} given our previous statement.

If $a^*(H) = a_2$ and $PIC(a^*(L); L)$ does not bind, the solution to I^{RSW} is simply $\mathbf{w}^*(a^*(H); H)$ which exists by our previous argument. Otherwise, the solution to I^{RSW} is defined by

$$(\pi_{Hs}(a_2) - \pi_{Hs}(a_1))\left[U(w_s^H) - U(w_f^H)\right] = a_2 - a_1, \text{ and} \tag{2.3}$$

$$\sum_{n \in \{s,f\}} \pi_{Ln}\left(a^*(L)\right)\left[q_n - w_n\left(a^*(L)\right)\right] = \sum_{n \in \{s,f\}} \pi_{Ln}(a_2)\left(q_n - w_n^H\right). \tag{2.4}$$

Equation (2.3) implicitly defines a strictly increase line in (w_s, w_f)-space while equation (2.4) defines a strictly decreasing line in (w_s, w_f)-space. These lines therefore intersect exactly once in $\mathbb{R}^2$. Denote this intersection point (w'_s, w'_f). If this $(w'_s, w'_f) \in [\underline{w}, \infty)^2$ we are done: $\hat{\mathbf{w}}(a_2; H) = (w'_s, w'_f)$. Otherwise, the solution is $\mathbf{w}^*(a; L) = (w''_s, \underline{w})$ where w''_s satisfies equation (2.3) with $w_f^H = \underline{w}$. ∎

Since

$$\sum_n \pi_{Hn}(\hat{a}(H))[q_n - \hat{w}_n(\hat{a}(H); H)] > \sum_n \pi_{Hn}(a^*(L))[q_n - w_n(a^*(L); L)]$$

the expected payoff to the type-H principal is strictly higher under the RSW menu-contract than lower bound point contract. The type-L principal is just as well off. Therefore, the ex ante payoff to the principal of the RSW menu is strictly greater than the ex ante point-contract lower bound payoff equilibrium payoff. ∎

Let $\mathscr{P}_S^P$ be the set of ex ante equilibrium payoffs for the principal that can be earned via separating separating point contracts. Define $\mathscr{P}_S^M$ similarly. Our next result characterizes environment such that $\mathscr{P}_S^M$ is strictly higher than $\mathscr{P}_S^P$ in the strong set ordering except at the RSW payoff. I first make the following assumptions.

Assumption 2.1 *(a)* $\sum_{n \in \{s,f\}} \pi_{Hn}(a_1)q_n \geq h(\bar{U} + a_1)$ *for both* $i \in \{H, L\}$;

(b) $\sum_{n \in \{s,f\}} \pi_{Hn}(a_2)(q_n - \hat{w}(a_2; H)) \geq \sum_{n \in \{s,f\}} \pi_{Hn}(a_1)(q_n - h(\bar{U} + a_1))$;

(c) $PIC(a^*(L); L, H)$ *does not hold if the type-H principal implements* $a^*(H)$ *with* $\mathbf{w}^*(a^*(H); H)$; *and*

(d) $\pi_{Hs}(a_2) - \pi_{Hs}(a_1) > \pi_{Ls}(a_2) - \pi_{Ls}(a_1)$.

Part (a) ensures that revenues are such that it is always (weakly) profitable for the principal to hire the agent. Part (b) ensures that the type-H principal will prefer

85

to implement a_2 at the least-cost separating equilibrium. Parts (a) and (b) are only necessary to exclude uninteresting equilibria in which the agent is not hired or she is hired to exert minimal effort. Part (c) ensures that the first best (i.e. full information) contract cannot be implemented; in particular, in trying to do so, the type-L principal would try to mimic the type-H principal. Thus, under this assumption, there is inefficiency in the least-cost separating equilibrium. Part (d) ensures a separating equilibrium exists.

2.4.1 Separating Equilibria

Our next proposition gives a necessary and sufficient condition such that under Assumption 2.1, equilibrium payoffs can be obtained using menu-contracts which are strictly higher than any separating equilibrium payoff using point contracts.

Proposition 2.2 *If Assumption 1 holds, then $\mathscr{P}_S^P$ and $\mathscr{P}_S^M$ intersect only at the RSW (or least-cost separating) equilibrium payoff and there exist a payoff $v \in \mathscr{P}_S^M$ such that $v > \lambda \underline{U}(H) + (1 - \lambda)\underline{U}(L)$ if and only if*

$$h\left(\bar{U} + a(L) - \frac{(a(L) - a_1)\pi_{Ls}(a_2)}{\pi_{Ls}(a_2) - \pi_{Ls}(a_1)}\right) > \underline{w}. \tag{2.5}$$

Proof The proposition is a result of the following lemma that proves that (2.5) is a necessary and sufficient condition for menu-contracts to deliver higher ex ante payoff to the principal than any separating point-contract. ∎

Lemma 2.2 *If Assumption 1 holds, there exists an equilibrium in menu-contracts that gives higher payoffs to both principal types than the least-cost separating equilibrium in point-contracts if and only if (2.5) holds.*

To see why this lemma holds, note that

$$w_f^*(a^*(L); L) = h\left(\bar{U} + a_2 - \frac{(a_2 - a_1)\pi_{Ls}(a_2)}{\pi_{Ls}(a_2) - \pi_{Ls}(a_1)}\right)$$

and observe Figure 2.1 (note that a contract that implements effort $a^*(L)$ must be below the curve $w_{f,AIC}$ and above the curve $w_{f,IR}$). In panel (a), the least costly contract that implements $a^*(L)$ is strictly interior. As we have noted, in the least cost separating equilibrium the type-H principal gives the agent utility strictly greater than her reservation utility. Moreover, separation requires the type-H principal to increase the cost of her contract to dissuade the type-L principal from mimicking her. Using menu-contracts and the inscrutability principle, we can transfer some of the rents ceded to the agent by the type-H principal to the type-L principal, effectively shifting her individual rationality constraint down, allowing the type-L principal to offer a less costly contract and earn more profits. This eases the incentive compatibility constraint between the principals, allowing the type-H principal to make her contract less costly and earn more profits herself.

On the other hand, if condition (2.5) fails, as in panel (b) of Figure 2.1, relaxing the type-L principal's individual rationality constraint does not generate a less costly contract for her to offer.

Proof *Sufficiency.* Suppose the $a^*(L) = a_2$. The contract $\mathbf{w}^*(a_2; L)$ is the unique solution to

$$\pi_{Lf}(a_2)U(w_f) + \pi_{Ls}(a_2)U(w_s) = \bar{U} + a_2 \qquad (2.6)$$

$$(\pi_{Lf}(a_2) - \pi_{Lf}(a_1))U(w_f) + (\pi_{Ls}(a_2) - \pi_{Ls}(a_1))U(w_s) = a_2 - a_1. \qquad (2.7)$$

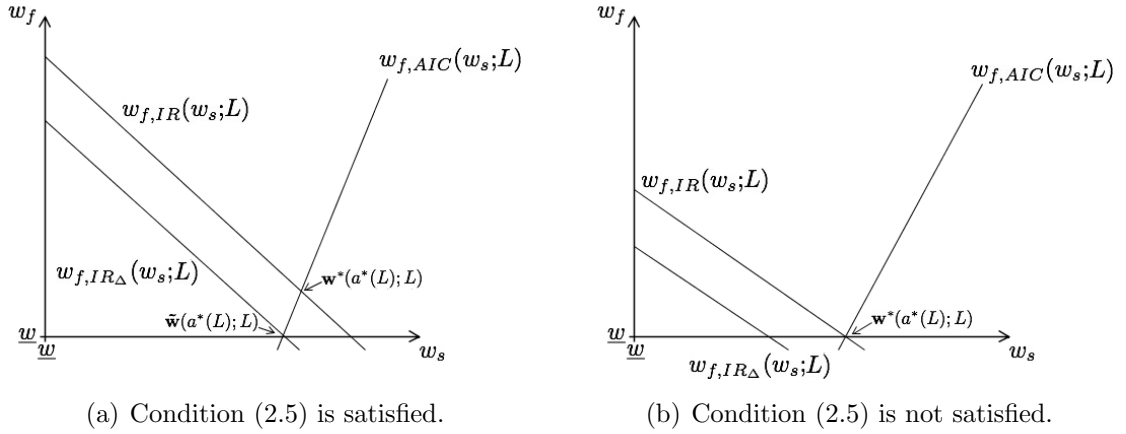(a) Condition (2.5) is satisfied.   (b) Condition (2.5) is not satisfied.

Figure 2.1: Examples of when condition (2.5) is and isn't satisfied.

Solving (3.7) and (2.7) we get

$$w_f^*(a_2; L) = h\left(\bar{U} + a_2 - \frac{(a_2 - a_1)\pi_{Ls}(a_2)}{\pi_{Ls}(a_2) - \pi_{Ls}(a_1)}\right) > \underline{w}$$

by assumption.

By hypothesis

$$\frac{\pi_{Hs}(a_2)}{\pi_{Hf}(a_2)} > \frac{\pi_{Ls}(a_2)}{\pi_{Lf}(a_2)}$$

and it is clear that $w_s^*(a_2; H) > w_f^*(a_2; H)$. Thus we can apply Lemma 1 of Silvers [10] to conclude that $IR(a_2; \{1, 0\})$ is satisfied with strict inequality: when the principal's type is her private information, in the least cost separating equilibrium the principal of type-H cedes rents to the agent ex ante. Thus,

$$\Delta := \frac{\lambda}{2(1 - \lambda)}\left[\sum_n \pi_{Hn}U(\hat{w}_n) - a_2 - \bar{U}\right] > 0.$$

Since for menu-contracts individual rationality only needs to be satisfied in expectation, we can transfer the half of rents ceded to the agent from the principal of type-H to the principal of type-L, essentially relaxing her individual rationality constraint by

88

Δ. The type-L principal's contract can then be solved as the unique solution to (2.7) and

$$\pi_{Lf}(a^*(L))U(w_f) + \pi_{Ls}(a^*(L)U(w_s) = \bar{U} + a^*(L) - \Delta. \tag{2.8}$$

I can solve (3.7), (2.7) and (2.8) for w_f as a function of w_s:

$$w_{f,AIR}(w_s) = h\left(\frac{\bar{U} + a_2 - \pi_{Ls}(a_2)U(w_s)}{\pi_{Lf}(a_2)}\right)$$

$$w_{f,AIC}(w_s) = h\left(U(w_s) - \frac{a_2 - a_1}{(\pi_{Ls}(a_2) - \pi_{Ls}(a_1))}\right)$$

$$w_{f,IR_\Delta}(w_s) = h\left(\frac{\bar{U} + a_2 - \Delta - \pi_{Ls}(a_2)U(w_s)}{\pi_{Lf}(a_2)}\right).$$

Taking the derivative of $w_{f,AIC}(w_s)$:

$$w'_{f,AIC}(w_s) = h'\left(U(w_s) - \frac{a_2 - a_1}{(\pi_{Ls}(a_2) - \pi_{Ls}(a_1))}\right)U'(w_s) > 0.$$

Note that $\mathbf{w}^*(a_2; L)$ is the solution to $w_{f,IR}(w_s) = w_{f,AIC}(w_s)$; that is,

$$w_{f,IR_\Delta}(w_s^*(a_2; L)) = w_{f,AIC}(w_s^*(a_2; L))$$

$$= w_f^*(a_2; L).$$

Let $\tilde{\mathbf{w}}(a_2; L)$ be the solution to $w_{f,IR_\Delta}(w_s) = w_{f,AIC}(w_s)$ if $\tilde{w}_f(a_2; L) \geq \underline{w}$ and $\tilde{\mathbf{w}}(a_2; L) := (\underline{w}, w_{f,IR_\Delta}(\underline{w}))$ otherwise. Since $w_{f,AIC}(w_s)$ is decreasing and $w_{f,IR_\Delta} < w_{f,IR}$, we have $\tilde{\mathbf{w}}(a_2; L) \ll \mathbf{w}^*(a_2; L)$.[6] Thus, $\tilde{\mathbf{w}}(a_2; L)$ implements a_2 at a lower cost than $\mathbf{w}^*(a_2; L)$.

---

[6]If $\mathbf{a}$ and $\mathbf{b}$ are two vectors of the same size, $\mathbf{a} \ll \mathbf{b}$ indicates that each element of $\mathbf{a}$ is strictly less than each element of $\mathbf{b}$.

Meanwhile, at $\tilde{\mathbf{w}}(a_2; L)$, the constraint $PIC(a^*(L); L)$ is relaxed (since the type-L principal now implements $a^*(L)$ at a lower cost and therefore receives a higher payoff) and therefore the type-H principal can implement her effort at a lower cost. Thus, both types of the principal are strictly better off.

If $a^*(L) = a_1$, then $w_f^*(a_1; L) = w_s^*(a_1; L) = h(\bar{U} + a_1)$. Let $z = \min\{\bar{U} + a_1, \underline{w}\}$. Then $\tilde{w}_f(a_1; L) = \tilde{w}_s(a_1; L) = h(z)$ implements a_1 in menu-contracts. As before at $\tilde{\mathbf{w}}(a_1; L)$, the constraint $PIC(a^*(L); L)$ is relaxed (since the type-L principal now implements $a^*(L)$ at a lower cost and therefore receives a higher payoff) and therefore the type-H principal can implement her effort at a lower cost. Thus, both types of the principal are strictly better off.

Necessity. To show necessity, suppose that $w_f^*(a^*(L), 2) = \underline{w}$. By definition,

$$w_{f,AIC}(w_s^*(a^*(L); L)) = \underline{w}.$$

But then, since $w_{f,AIC}$ is strictly increasing, there is no $\tilde{w}_s$ that implements $a^*(L)$ such that $\tilde{w}_s < w_s^*(a^*(L); L)$, even for the relaxed individual rationality constraint. So the least cost contract that implements $a^*(L)$ remains $(\underline{w}, w_s^*(a^*(L); L))$. ∎

2.4.2 Pooling Equilibria

In any complete pooling equilibrium with contract C, the agent's posterior is her prior: $\rho(C) = (\lambda, 1 - \lambda)$. The high effort complete pooling contract $\mathbf{w}^p$ is the defined by

$$(\lambda\delta_H + (1 - \lambda)\delta_L) \left[U(w_s^p) - U(w_f^p) \right] = a_2 - a_1 \text{ and}$$

$$\sum_{n \in \{s,f\}} \left[\left(\lambda\pi_{Hn}(a_2) + (1 - \lambda)\pi_{Ln}(a_2) \right) w_n^p - \left(\bar{U} + a_2 \right) \right].$$

Contract $\mathbf{w}^p$ exists since the first equation is strictly increasing and the second is strictly decreasing in (w_s, w_f) space (possibly needing to enforce the lower bound wage $\underline{w}$). Define $\delta_i := \pi_{is}(a_2) - \pi_{is}(a_1)$ for $i \in \{H, L\}$.

Assumption 2.2 *(a)* $\mathbf{w}^p \in (\underline{w}, \infty)$;

(b) $\sum_{n \in \{s, f\}} \pi_{Ln}(a_2)(q_n - w_n^p) \geq \sum_{n \in \{s, f\}} \pi_{Ln}(a_2) \left(q_n - h\left(\bar{U} + a_1\right)\right)$;

(c) U *is strictly concave.*

Part (a) of Assumption 2 requires the solution to be interior. This is guaranteed if δ_H and δ_L are sufficiently large (i.e. if high effort is sufficiently worthwhile). Part (b) ensures that the type-L (and hence the type-H) principal prefers to implement high effort in the pooling contract. This is guaranteed if q_s is sufficiently large.

Proposition 2.3 *Suppose Assumption 1 parts (a) and (c) and Assumption 2 hold. There exists $\bar{\delta}$ such that if $\delta_L - \delta_H \in (0, \bar{\delta})$, then there exists a menu-contract that is more profitable for the principal ex-ante than the complete pooling contract for any $\lambda \in (0, 1)$.*

The condition for the proposition are satisfied, for example, for $U(\cdot) = \ln(\cdot)$ or $U(\cdot) = \sqrt{\cdot}$,

$$\pi_{Hs} = 0.9$$
$$\pi_{Ls} = 0.8$$
$$\delta_H = 0.45$$
$$\delta_L = 0.6$$

and any costs of effort $a_1 < a_2$ and any reservation $\bar{U}$ such that revenues satisfy Part (b) of Assumption 2.

Proof Let $\mathbf{w}^p$ denote the complete pooling contract. Define

$$K_p^i := \pi_{is}(a_2)w_s^p + \pi_{if}(a_2)w_f^p.$$

Define a menu-contract $\{\mathbf{w}^H, \mathbf{w}^L\}$ that satisfies

$$\pi_{is}(a_2)w_s^i + \pi_{if}(a_2)w_f^i = K_p^i \text{ and} \tag{2.9}$$

$$(\pi_{is}(a_2) - \pi_{is}(a_1)) \left[U(w_s^i) - U(w_f^i)\right] = a_2 - a_1. \tag{2.10}$$

I claim that

$$\lambda \left(\pi_{Hs}(a_2)U(w_s^H) + \pi_{Hf}(a_2)U(w_f^H)\right) + (1-\lambda) \left(\pi_{Ls}(a_2)U(w_s^L) + \pi_{Lf}(a_2)U(w_f^L)\right) > \bar{U} + a_2 \tag{2.11}$$

if the conditions in the proposition are met. Thus, $\{\mathbf{w}^H, \mathbf{w}^L\}$ constitutes a menu-contract that provides the agent with more (ex ante) utility than is needed for her to agree to the contract. I can reduce wages $\mathbf{w}^H$ by an amount that is small enough such that the principal's incentive compatibility constraints are satisfied. This new contract is thus strictly preferred to $\mathbf{w}^p$.

The pooling equilibrium solves the following problem:

$$K_p^i(\lambda) = \begin{cases} \min_{\mathbf{w}} & \sum_{n \in \{s,f\}} \pi_{in}(a)w_n \\ \text{subject to} & AIC(a; \{\lambda, 1-\lambda\}) \\ & AIR(a; \{\lambda, 1-\lambda\}) \end{cases} . \tag{P}$$

Since both constraints are linear in λ, K_p^i must be concave in λ.

I will use the following lemma below.

Lemma 2.3 $w_s^H > w_s^L > w_f^L > w_s^H.$

92

Proof Since both constraints in problem (P) bind

$$\sum_{n\in\{s,f\}} \left[\left(\lambda\left(\pi_{Hn}(a_2) - \pi_{Hn}(a_1)\right) + (1-\lambda)\left(\pi_{Ln}(a_2) - \pi_{Ln}(a_1)\right)\right)U(w_n) - (a_2 - a_1)\right]$$

$$= (\lambda\delta_H + (1-\lambda)\delta_L)\left[U(w_s^p) - U(w_f^p)\right] - (a_2 - a_1).$$

Since $\delta_H < \lambda\delta_H + (1-\lambda)\delta_L$ either $w_s^H > w_s^p$, $w_f^H < w_f^p$ or both. By equation (2.9), $w_s^H > w_s^p$ if and only if $w_f^H > w_f^p$. Thus, we have both $w_s^H > w_s^p$ and $w_f^H < w_f^p$. Similarly, since $\delta_L > \lambda\delta_H + (1-\lambda)\delta_L$ we have both $w_s^H < w_s^p$ and $w_f^H > w_f^p$. ∎

Using (2.9) and (2.10), we can rewrite the left hand side of (2.11) as follows

$$\lambda\left(\pi_{Hs}(a_2)U(w_s^H) + \pi_{Hf}(a_2)U(w_f^H)\right) + (1-\lambda)\left(\pi_{Ls}(a_2)U(w_s^L) + \pi_{Lf}(a_2)U(w_f^L)\right)$$

$$= (a_2 - a_1)\left(\frac{\lambda\pi_{Hs}}{\Delta_H} + \frac{(1-\lambda)\pi_{Ls}}{\Delta_2}\right) + \lambda U(w_f^H) + (1-\lambda)U(w_f^L)$$

$$= \bar{U} + a_2 - \left[\bar{U} + a_2 - (a_2 - a_1)\left(\frac{\lambda\pi_{Hs}}{\Delta_H} + \frac{(1-\lambda)\pi_{Ls}}{\Delta_2}\right)\right] + \lambda U(w_f^H) + (1-\lambda)U(w_f^L)$$

$$= \bar{U} + a_2 - \left[\lambda\left(\bar{U} + a_2 - (a_2 - a_1)\frac{\pi_{Hs}}{\Delta_H}\right) + (1-\lambda)\left(\bar{U} + a_2 - (a_2 - a_1)\frac{\pi_{Ls}}{\Delta_2}\right)\right]$$

$$+ \lambda U(w_f^H) + (1-\lambda)U(w_f^L)$$

$$= \bar{U} + a_2 + \lambda U(w_f^H) + (1-\lambda)U(w_f^L) - \lambda U(w_f^*(a_2; H)) - (1-\lambda)U(w_f^*(a_2; L))$$

$$\tag{2.12}$$

$$=: S(\lambda).$$

I claim that $S(\lambda)$ is a concave function. To see this note

$$S''(\lambda) = 2\left(\frac{\partial U(w_f^H(\lambda))}{\partial\lambda} - \frac{\partial U(w_f^L(\lambda))}{\partial\lambda}\right) + \lambda\frac{\partial^2 U(w_f^H(\lambda))}{\partial\lambda^2} + (1-\lambda)\frac{\partial^2 U(w_f^L(\lambda))}{\partial\lambda^2}.$$

$$\tag{2.13}$$

The following lemma shows that the first term in (2.13) is negative.

Lemma 2.4 $\frac{dU\left(w_f^i(\lambda)\right)}{d\lambda} < \frac{dU\left(w_f^i(\lambda)\right)}{d\lambda}$.

Proof The Lagrangean of problem (P) is

$$
\sum_{n \in \{s,f\}} \pi_{in}(a) w_n + \theta_{AIC} \sum_{n \in \{s,f\}} \Big[\big(\lambda \left(\pi_{Hn}(a_2) - \pi_{Hs}(a_1) \right)
$$
$$
+ (1 - \lambda) \left(\pi_{Ln}(a_2) - \pi_{Ln}(a_1) \right) U(w_n) - (a_2 - a_1) \Big]
$$
$$
+ \theta_{IR} \sum_{n \in \{s,f\}} \Big[\big(\lambda \pi_{Hn}(a_2) + (1 - \lambda) \pi_{Ln}(a_2) \big) w_n - \bar{U} \Big].
$$

where θ_{AIC} and θ_{IR} are (non-positive) Lagrange multipliers for the first and second constraints in problem (P) respectively. By the envelope theorem (see Milgrom and Segal [6]),

$$
\frac{dK_p^i(\lambda)}{d\lambda} = \theta_{AIC} \left(\delta_H - \delta_L \right) \big(U(w_s^p(\lambda)) - U(w_f^p(\lambda)) \big)
$$
$$
+ \theta_{IR} \big(\pi_{Hs}(a_2) - \pi_{Ls}(a_2) \big) \big(U(w_s^p(\lambda)) - U(w_f^p(\lambda)) \big). \qquad (2.14)
$$

Since both constraints are binding, $\theta_{AIC}, \theta_{IR} < 0$. Thus, if $\delta_H - \delta_L$ is not too negative, $K_p^i(\lambda)$ is decreasing. Further, by equation (2.10) the sign of $\frac{dw_n^i(\lambda)}{d\lambda}$ must be the same for all $n \in \{s, f\}$ and all $i \in \{H, L\}$. Therefore, since $K_p^i(\lambda)$ is decreasing, $\frac{dw_n^i(\lambda)}{d\lambda} < 0$ for all $n \in \{s, f\}$ and all $i \in \{H, L\}$. Note that (2.14) does not depend on i.

Taking the total derivative of (2.10) and rearranging we get

$$
\frac{dU(w_s^i(\lambda))}{d\lambda} - \frac{dU(w_f^i(\lambda))}{d\lambda} = U'(w_s^i) \frac{dw_s^i(\lambda)}{d\lambda} - U'(w_f^i) \frac{dw_f^i(\lambda)}{d\lambda} = 0.
$$

Since $w_s^i > w_f^i$ and U is concave, it must be that

$$
\frac{dw_s^i(\lambda)}{d\lambda} > \frac{dw_f^i(\lambda)}{d\lambda}.
$$

94

Further

$$U'(w_s^H)\frac{dw_s^H(\lambda)}{d\lambda} - U'(w_f^H)\frac{dw_f^H(\lambda)}{d\lambda} = U'(w_s^L)\frac{dw_s^L(\lambda)}{d\lambda} - U'(w_f^L)\frac{dw_f^L(\lambda)}{d\lambda} \Leftrightarrow$$

$$U'(w_s^H)\frac{dw_s^H(\lambda)}{d\lambda} - U'(w_s^L)\frac{dw_s^L(\lambda)}{d\lambda} = U'(w_f^H)\frac{dw_f^H(\lambda)}{d\lambda} - U'(w_f^L)\frac{dw_f^L(\lambda)}{d\lambda} \qquad (2.15)$$

$$U'(w_s^L)\left(\frac{dw_s^H(\lambda)}{d\lambda} - \frac{dw_s^L(\lambda)}{d\lambda}\right) < U'(w_f^H)\left(\frac{dw_f^H(\lambda)}{d\lambda} - \frac{dw_f^L(\lambda)}{d\lambda}\right) \qquad (2.16)$$

where the inequality follows since U is concave and $w_s^H > w_s^L$ and $w_f^H < w_f^L$.

Now, taking the total derivative of (2.9) and applying the observation that

$$\frac{dK_p^H(\lambda)}{d\lambda} = \frac{dK_p^L(\lambda)}{d\lambda} =: \bar{K}$$

we get

$$\pi_{Hs}(a_2)\frac{dw_s^H(\lambda)}{d\lambda} + \pi_{Hf}(a_2)\frac{dw_f^H(\lambda)}{d\lambda} = \pi_{Ls}(a_2)\frac{dw_s^L(\lambda)}{d\lambda} + \pi_{Lf}(a_2)\frac{dw_f^L(\lambda)}{d\lambda}.$$

Thus, either $\frac{dw_s^H(\lambda)}{d\lambda} < \frac{dw_s^L(\lambda)}{d\lambda}$ or $\frac{dw_f^H(\lambda)}{d\lambda} < \frac{dw_f^L(\lambda)}{d\lambda}$ or both. From inequality (2.16), we conclude that $\frac{dw_s^H(\lambda)}{d\lambda} < \frac{dw_s^L}{d\lambda}$ (otherwise we contradict the previous statement). By equation (2.15)

$$U'(w_f^H)\frac{dw_f^H(\lambda)}{d\lambda} - U'(w_f^L)\frac{dw_f^L(\lambda)}{d\lambda} < U'(w_s^H)\frac{w_s^H(\lambda)}{d\lambda} - U'(w_f^H)\frac{w_s^L(\lambda)}{d\lambda}$$

$$< U'(w_s^H)\frac{w_s^H(\lambda)}{d\lambda} - U'(w_f^H)\frac{w_f^H(\lambda)}{d\lambda} = 0$$

where the first inequality follows since $w_f^H < w_s^L$ (see Lemma 2.3) and the second follows since

$$\frac{dw_f^H(\lambda)}{d\lambda} < \frac{dw_s^H(\lambda)}{d\lambda} < \frac{dw_s^L(\lambda)}{d\lambda}.$$

Thus,

$$\frac{dU\left(w_f^i(\lambda)\right)}{d\lambda} < \frac{dU\left(w_f^i(\lambda)\right)}{d\lambda}$$

as needed. ∎

Lemma 2.5 $U\left(w_f^i(\lambda)\right)$ *is concave.*

Proof Suppose not: $U\left(w_f^i(\lambda)\right)$ is convex. Then, since U is concave, $w_f^i(\lambda)$ must be convex. Further, rearranging (2.10) we have

$$U\left(w_s^i(\lambda)\right) = \frac{a_2 - a_1}{\pi_{is}(a_2) - \pi_{if}(a_1)} + U\left(w_f^i(\lambda)\right)$$

so that $U\left(w_s^i(\lambda)\right)$ must also be convex. Again, since U is concave $w_s^i(\lambda)$ must be convex. Thus $\pi_{is}(a_2)w_s^i(\lambda) + \pi_{if}(a_2)w_f^i(\lambda)$ is convex. But

$$\pi_{is}(a_2)w_s^i(\lambda) + \pi_{if}(a_2)w_f^i(\lambda) = K_p^i$$

and, as we noted above, K_p^i is concave. Thus we have a contradiction. ∎

By the previous two lemmata $S(\lambda)$ is concave. Further, since

$$S(0) = S(1) = \bar{U} + a_2,$$

inequality (2.11) holds for all $\lambda \in (0, 1)$. ∎

2.5 Conclusion

I have shown that allowing menu-contracts instead of just point-contracts increases the set of equilibrium payoffs in the strong set ordering and that allowing menu-contract eliminates many poor equilibria for the principal; in particular, the principal

will never obtain less than her least-cost separating equilibrium payoff when offering menu-contracts. Additionally, I characterize environments where equilibrium payoffs can be obtained using menu-contracts which are strictly higher than any separating equilibrium payoff using point-contracts. Thus, in a labour market environment with moral hazard where the employer has private information about the productivity of the worker, this paper shows that it can be strictly beneficial for the employer to maintain discretion over the particulars of the contract after the employer has accepted the job; moreover, the employer can never do worse by maintaining this discretion.

Extending the space of efforts and outcomes to any finitely countable set would be straight forward since none of my results depend on there being only two efforts and two outcomes.

References

[1] Paual Beaudry. Why an informed principal may leave rents to an agent. *International Economic Review*, 35(4):821–832, November 1994. 2.1

[2] Hector Chade and Randy Silvers. Informed principal, moral hazard, and the value of a more informative technology. *Economics Letters*, 74:291–300, 2002. 2.1, 1

[3] Roman Inderst. Incentive schemes as a signaling device. *Journal of Economic Behavior & Organization*, 44(4):455–465, April 2001. 2.1

[4] Ayça Kaya. When Does It Pay To Get Informed? *International Economic Review*, 51(2):533–551, May 2010. 2.1

[5] Eric Maskin and Jean Tirole. The Principal-Agent Relationship with an Informed Principal II: Common Values. *Econometrica*, 60(1):1–42, 1992. 2.1, 2.3.3, 5, 2.4

[6] Paul R Milgrom and Ilya Segal. Envelope Theorems for Arbitrary Choice Sets. *Econometrica*, 70(2):583–601, 2002. 2.4.2

[7] Roger B Myerson. Mechanism design by an informed principal. *Econometrica*, 51(6):1767–1797, December 1983. 2.1, 2.2.5, 4

[8] Michael Rothschild and Joseph Stiglitz. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *The Quarterly Journal of Economics*, 90(4):629–649, 1976. 3

[9] Ilya Segal and Michael D. Whinston. Robust Predictions for Bilateral Constracting With Externalities. *Econometrica*, 71(3):757–791, 2003. 2.1

[10] Randy Silvers. The value of information in a principal-agent model with moral hazard: the ex post contracting case. *Games and Economic Behavior*, 74(1):352–365, January 2012. 2.1, 2.2.1, 2.4.1

[11] Charles Wilson. A model of insurance markets with incomplete information. *Journal of Economic Theory*, 16(2):167–207, 1977. 3

Chapter 3

A Note on Bidder-Led Collusion

3.1   Introduction

In this chapter I present an example of a discrete, independent private-value auction
in which a bidder (the proposer) can offer her rival (the receiver) a collusion contract
after agreeing to the seller's mechanism. A celebrated result by Che and Kim [5]
states that for such auctions, there is a mechanism that eliminates all the effects
of collusion. The example demonstrates that the mechanism developed by Che and
Kim [5] fails to raise the seller her Myerson payoff; that is, the payoff the seller
would earn if collusion were impossible. The Che and Kim mechanism essentially
calls for the seller to charge entrance fees to the bidders that sum to the Myerson
payoff and leave the task of allocating the good to the colluding coalition. In my
example, the expected entrance fee is higher than the expected value of the good for
some lower type bidders; to ensure participation of these types in the Che and Kim
mechanism, higher type bidders must provide sufficient compensation. The proposal
my bidder offers refuses to pay this compensation when she is a higher type and
therefore excludes the lower types of her rivals from participating in the mechanism

and reduces the seller's mechanism. This proposal, and the off equilibrium behaviour that supports it, is shown to dominate the proposer's strategy of playing the Che and Kim mechanism truthfully.

I show further that there is no symmetric mechanism that achieves the Myerson payoff in a discrete, independent, private-value auction in which a bidder can offer a collusion contract to her rival. To see why this is true, note that any mechanism that collects more than the low valuation of the good when both bidders have low valuations is susceptible to the same type of collusion as Che and Kim's mechanism. I then show that any mechanism that collects less than the low valuation of the good when both bidders have low valuations is susceptible to collusion whereby the bidders always announce they have low valuations.

The model differs from Che and Kim's in two ways. First, the offer of collusion is made by a bidder rather than a third party. Second, the reservation utility of the bidder who receives the collusion offer is determined endogenously, i.e. via equilibrium play in the seller's mechanism following rejection of the collusion proposal; this is in contrast to the assumption of Che and Kim that the lowest level of utility the collusion contract must offer to ensure participation cannot be lower than the receiver's initial reservation utility level – that is, the level of utility that the *seller's* mechanism must deliver to ensure participation.

Once we rid ourselves of the assumption that a third party is organizing collusion, we are able to consider how the proposer determines her off-path play in order to maximize her payoff within the seller's mechanism. This allows me to pin down the most reasonable equilibrium play in the seller's mechanism. I find that one type of the proposer is indifferent between truth-telling and lying in the off-path subgame following rejection of her collusion proposal, but that she strictly prefers the equilibrium outcome of the mechanism when she lies. Thus, if dominated strategies are

ruled out, any equilibrium we consider should break the indifference in this subgame in favour of lying. This highlights the importance of the second difference between the current chapter and Che and Kim's model. The credible threat of lying to the seller's mechanism reduces the utility of one of the receiver's types below her initial reservation value. Anticipating the actions of the proposer before agreeing to the seller's mechanism, this type of the receiver will refuse to participate and therefore reduce the expected payoff of the seller.

Finally, I outline the bidder's general problem for designing an optimal collusion contract.

3.2 Literature Review

There is a large theoretical literature studying collusion. The current chapter belongs to a strand of this literature that studies collusion that is explicitly agreed to by the relevant parties via an enforceable contract. This literature has its genesis in McAfee and McMillan [16] who characterize the optimal collusion mechanism, organized by a benevolent and uninformed third-party who maximizes the sum of bidder's payoffs, when the seller's mechanism is fixed to be a first-price auction.

Laffont and Martimort [11, 12] extend this analysis by endogenizing the principal's mechanism, allowing her to respond optimally to any potential collusion contract. In problems of regulating firms and providing public goods, Laffont and Martimort show that the principal is able to exploit the constraints imposed on the agents' collusion contract given their private information to minimize the effects of collusion on revenue. Che and Kim [5] significantly extend the framework of Laffont and Martimort.

Quesada [15] studies agent-led collusion in a procurement/public goods setting where the principal has a Leontief production function. She finds that the principal

can achieve her Myerson payoff when the agents' types are uncorrelated. However, it is unclear how much the result depends on the principal's production technology. The Leontief function implies that the agents' actions are perfect complements, whereas in an auction setting, bidders' actions are perfect substitutes.

Laffont and Martimort [11, 12] and Che and Kim [5] assume, as in this chapter, that the collusion contract is designed only after all bidders have agreed to participate in principal's mechanism. A number of more recent papers (see for example, Dequiedt [7]; Pavlov [14]; and Che and Kim [6]) have studied the collusion problem when agents can collude prior to entering the principal's mechanism. Notably, the ability to collude on participation decision strengthens the ability of the colluders to extract rents from the principal. Relatedly, Eső and Schummer [8] and Rachmilevitch [17] look at a particular collusive mechanism: the ability to bribe rivals to abstain from a second-price and a first-price auction respectively.

A second strand of the literature studies how collusion can be sustained via repeated games, both with and without tacit communication between bidders (see for example Fudenberg, Lavine and Maskin [9]; Athey and Bagwell [2]; Aoyagi [1]; Athey, Bagwell and Sanchirico [3]; and Skrzypacz and Hopenhayn [18]). In a similar vein, Garratt, Tröger and Zheng [10] examine how bidders are able to collude in auctions by participating in resale markets following the initial auction.

Finally, the bidder-led organization of collusion studied this chapter is similar to the process of reciprocal contracting studied by Celik and Peters [4]. Their contracting procedure has all players of the game offering contracts that are each conditional on the contracts offered by the others; if all contracts agree, a cooperative action is implemented in the default game (e.g. an auction); otherwise, all contracts are void and the default game is played non-cooperatively. Their work is mostly concerned with characterizing the outcomes that can be supported as perfect Bayesian equilibria

of the reciprocal contracting game. While they do suggest how the procedure can be used to model collusion, much of the analysis of the problem is left for future research. The reciprocal contracting approach to modelling collusion is best viewed as complementary to the bidder-led collusion studied in this chapter since it too dispenses with the assumption that a third-party organizes collusion.

3.3   The Model

Consider a seller facing two potential bidders of one unit of a good. Buyers have private information about their valuation of the good; valuations for bidder i's valuation v_i is drawn independently from some arbitrary measurable set T_i. Valuations for bidder i are distributed according to some distribution $F_i \in \Delta T_i$. The seller offers mechanism $M = \left\{ (q_i(\cdot), t_i(\cdot))_{i=1}^2 \right\}$ where $q_i : T_1 \times T_2 \to [0, 1]$, such that $q_1(v_1, v_2) + q_2(v_1, v_2) \leq 1$ for all $(v_1, v_2) \in T_1 \times T_2$, maps valuations into the probability that bidder i obtains the good and $t_i : T_1 \times T_2 \to \mathbb{R}$ maps valuations into the payment to be made from bidder i to the seller.

An allocation (q_1, q_2, t_1, t_2) with draw $(v_1, v_2) \in T_1 \times T_2$ gives utility $u_i(\theta_1, \theta_2) :=$ $v_i q_i - t_i$ to bidder i and $w(\theta_1, \theta_2) := t_1 + t_2$ to the seller. The reservation payoff for each player is 0.

After both players have accepted the seller's mechanism, we allow bidder 1 to offer a collusion contract to bidder 2. Formally, a collusion contract is a set $P = \left\{ m, (\phi_i(\cdot))_{i=1}^2, y(\cdot) \right\}$ where $m \subseteq T_1 \times T_2$ is a set of messages bidder 1 allows to be exchanged between bidder 1 and bidder 2; $\phi_i : T_1 \times T_2 \to T_1 \times T_2$ maps the types of the bidders into a report to the seller for bidder i and $y : T_1 \times T_2 \to \mathbb{R}$ maps the types of the bidders into transfers from bidder 2 to bidder 1.[1]

---

[1]Due to the specification of preferences, any reallocation of the good can be replicated via transfers between the bidders. There is therefore no loss of generality in not allowing the collusion

The timeline of the full game is as follows:

1. Seller offers mechanism M.

2. Bidders accept or reject seller's mechanism.

3. If both reject, all players receive reservation utility and the game ends; if one rejects, the seller's mechanism is played with the bidder who accepted.

4. If both accept, bidder 1 offers bidder 2 collusion contract P.

5. Bidder 2 accepts or rejects.

6. If bidder 2 rejects, bidders play seller's mechanism non-cooperatively.

7. If bidder 2 accepts:

   (a) the bidders simultaneously make announcements from m to each other;

   (b) the bidders announce type to seller (i.e. play seller's mechanism); and

   (c) the transfer is made between bidders.

Before outlining the general contract design problem of bidder 1, I present a discrete type example to demonstrate how Che and Kim's [5] mechanism is susceptible to collusion and develop some of the central problems to studying bidder-led collusion.

3.4   Discrete Type Example

Let $T_1 = T_2 = \{v_L, v_H\}$ with $v_H > v_L$. For convenience, I say buyer i with valuation v_{θ_i} has *type* θ_i where $\theta_i \in \{L, H\}$. Each bidder draws valuation v_H with probability α. Define $\Delta v := v_H - v_L$.

---

contract to reallocate the good.

Assume that $v_L - \frac{\alpha}{1-\alpha}\Delta v > 0$; then the optimal allocation for the seller in the absence of collusion is to sell in all states of the world. In this case, the seller earns payoff $\alpha \cdot v_H + (1 - \alpha) \cdot v_L$. This is the seller's Myerson payoff (i.e. the highest payoff she can achieve given her lack of knowledge of the bidder's types).

Suppose the bidders can collude via a benevolent third party who maximizes a weighted sum of their payoffs. Buyers reveal their types to the third party who can then manipulate these reports to the seller, reallocate q assigned by the seller and exchange transfers among the bidders in a budget balanced way. Che and Kim [5] show that the seller can still achieve her Myerson payoff and implement the Myerson allocation.

Proposition 3.1 *Let* $\rho := (\alpha \cdot v_H + (1 - \alpha) \cdot v_L)/2$. *The following quantity and transfer schedules achieve the Myerson allocation:*

$$q(H, H) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right), \qquad\qquad q(H, L) = (1, 0),$$
$$q(L, H) = (0, 1), \qquad\qquad q(L, L) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right),$$
$$t(H, H) = (\rho, \rho), \qquad\qquad t(H, L) = \left(\rho + \tfrac{1}{2}v_H, \rho - \tfrac{1}{2}v_H\right),$$
$$t(L, H) = \left(\rho - \tfrac{1}{2}v_H, \rho + \tfrac{1}{2}v_H\right), \quad t(L, L) = (\rho, \rho).$$

The resulting expected payoffs are

$$U_i(H) := \mathbb{E}_{\theta_j}\left(u_i(100, \theta_j)\right) = v_H/2 - \rho$$
$$U_i(L) := \mathbb{E}_{\theta_j}\left(u_i(50, \theta_j)\right) = 0.$$

for $j \neq i; i, j \in \{1, 2\}$. *The seller achieves her Myerson payoff of* $\alpha \cdot v_H + (1 - \alpha) \cdot v_L$.

I will refer to the mechanism in Proposition 3.1 as the Che-Kim mechanism and its outcome as the Che-Kim outcome.

Now suppose that, instead of having a third-party propose a collusion contract, bidder 1 can offer bidder 2 a collusion contract after both have accepted the seller's mechanism. It is natural to consider the *forward looking* incentives of bidder 1 when determining off-path decisions and pinning down a reasonable equilibrium; in particular, we are interested in how bidder 1 plays in the seller's mechanism in the event that bidder 2 rejects her collusion proposal. An equilibrium satisfies the *forward induction criterion* if the associated equilibrium of the normal form representation of the game is composed of undominated strategies.

Theorem 3.1 *No equilibrium of the Che-Kim mechanism satisfies the forward induction criterion and earns the seller her Myerson payoff when bidder 1 can make a collusion offer to bidder 2.*

I first determine the reservation payoff of bidder 2 in the collusion contract game.

Lemma 3.1 *There exists an equilibrium in the Che-Kim mechanism such that type H of bidder 2 earns $\bar{u}_2(v_H) := \frac{1}{2}v_H - \rho$ and type L of bidder 2 earns $\bar{u}_2(v_L) := \frac{1}{2}v_L - \rho$.*

Proof Note that when bidder 2 is being truthful, type H of bidder 1 is indifferent between lying and telling the truth in the Che-Kim mechanism. Further, given that bidder 1 is lying, being truthful remains a best response for bidder 2. Thus, we have an equilibrium in the Che-Kim mechanism where bidder 1 always announces that she is type L and bidder 2 plays a truthful strategy. In case bidder 2 is type H, her utility is $v_H - \frac{1}{2}v_H - \rho = \frac{1}{2}v_H - \rho$; in case she is type L her utility is $\frac{1}{2} \cdot v_L - \rho$. ∎

Thus, upon rejection of the collusion contract, if the equilibrium of Lemma 3.1 is played, the low type of bidder 2 obtains payoff $\frac{1}{2} \cdot v_L - \rho$. Supposing this equilibrium is played upon rejection then the collusion contract offered by bidder 1 must provide at least $\frac{1}{2} \cdot v_L - \rho$ for the low type of bidder 2 to be acceptable to her. Note that

$\frac{1}{2} \cdot v_L - \rho < 0$ and therefore delivers to type L of bidder 2 less than her initial reservation utility. Che and Kim [5] explicitly rule out any collusive mechanism that delivers to the bidders utility less than their utility from rejecting the seller's mechanism. Since the collusive contract generates a payoff for bidder 2 that is less than this outside option, it cannot be considered in Che and Kim's analysis. This restriction implicitly rules out the equilibrium where bidder 1 follows the strategy described in the previous lemma.

I now present a collusion contract that bidder 1 can offer bidder 2. Next, I show that the strategy of making this offer and always announcing L in the seller's mechanism following bidder 2's rejection of the offer dominates the truthful strategy leading to the Che-Kim outcome for bidder 1.

Lemma 3.2 *There is an equilibrium in the Che-Kim mechanism where Buyer 1 offers a collusion contract such that both bidders tell the truth and the low type of bidder 2 transfers $\frac{1}{2}\Delta v$ to the high type of bidder 1. In case of rejection, bidder 1 always announces L to the seller and bidder 2 is truthful.*

The resulting expected payoffs are

$$U_1(H) = \alpha \tfrac{1}{2} v_H + (1 - \alpha)\left(v_H - \tfrac{1}{2}v_L\right) - \rho, \quad U_1(L) = 0,$$
$$U_2(H) = v_H/2 - \rho, \qquad\qquad\qquad\qquad U_2(L) = v_L/2 - \rho.$$

Proof *Incentive Compatibility* For bidder 1 of type H:

$$\alpha \cdot v_H \cdot \frac{1}{2} + (1 - \alpha)\left(v_H - \frac{1}{2}v_L\right)$$
$$\geq \alpha \cdot v_H \cdot \frac{1}{2} + (1 - \alpha) \cdot v_H \cdot \frac{1}{2} = \frac{1}{2}v_H;$$

for bidder 1 of type L:

$$\alpha \frac{1}{2} v_H + (1 - \alpha) \frac{1}{2} v_L$$
$$\geq \alpha \left(v_L - \frac{1}{2} v_L \right) + (1 - \alpha) \frac{1}{2} v_L = \frac{1}{2} v_L.$$

For bidder 2 of type H:

$$\alpha \frac{1}{2} v_H + (1 - \alpha) \left(v_H - \frac{1}{2} v_H \right) = \frac{1}{2} v_H$$
$$\geq \alpha \frac{1}{2} v_L + (1 - \alpha) \frac{1}{2} v_H.$$

for bidder 2 of type L:

$$\alpha \frac{1}{2} v_L + (1 - \alpha) v_L = \frac{1}{2} v_L$$
$$\geq \alpha \frac{1}{2} v_L + (1 - \alpha) \left(v_L - \frac{1}{2} v_H \right).$$

It is clear that the offer is individually rational for bidder 2 given that the equilibrium from Lemma 3.1 is played following rejection. ∎

Lemma 3.3 *Any equilibrium where bidder 1 tells the truth following the rejection of any collusion proposal is dominated by the strategy of offering the collusion contract of Lemma 3.2 and announcing type L in the seller's mechanism following rejection of her offer.*

Proof For bidder 1, any strategy that prescribes being truthful in the Che-Kim mechanism is weakly dominated by offering the collusive contract and lying (i.e. always announcing she is L) following rejection of the collusive contract. To see this, note that the latter earns type H of bidder 1 utility of $\alpha \frac{1}{2} v_H + (1 - \alpha) \left(v_H - \frac{1}{2} v_L \right) - \rho > \frac{1}{2} v_H - \rho$ (where $\frac{1}{2} v_H - \rho$ is her utility in the Che-Kim outcome) if type L of bidder

2 accepts the collusion contract and $\frac{1}{2}v_H - \rho$ otherwise. A truthful strategy for type H of bidder 1 following rejection earns her utility $\frac{1}{2}v_H - \rho$. Type L of bidder 1 earns the entrance fee in both cases. ∎

Finally, I can show that there is no equilibrium of the Che-Kim mechanism that satisfies the forward induction criterion and achieves the Myerson payoff for the seller. We know from the previous lemma that any equilibrium that satisfies the forward induction criterion in the Che-Kim mechanism will give the low type of bidder 2 strictly less than her reservation payoff; thus the low type of bidder 2 will not participate in the seller's mechanism. In the two states in which bidder 2 is of the low type the seller only obtains the entrance fee from bidder 1. An upper bound on expected revenue for the seller is

$$\alpha^2 \left(\alpha \cdot v_H + (1-\alpha) \cdot v_L\right) + \alpha(1-\alpha)\left((\alpha \cdot v_H + (1-\alpha) \cdot v_L)/2\right)$$
$$+ (1-\alpha)\alpha\left(\alpha \cdot v_H + (1-\alpha) \cdot v_L\right) + (1-\alpha)^2\left((\alpha \cdot v_H + (1-\alpha) \cdot v_L)/2\right)$$
$$< \alpha \cdot v_H + (1-\alpha) \cdot v_L.$$

A mechanism M offered by bidder 1 is **safe** if for every type of bidder 1, M is incentive compatible and individually rational if bidder 2 knew the principal's type. See Myerson [13]. A safe mechanism has the advantage that regardless of the inferences made by bidder 2 about bidder 1 when bidder 1 offers a mechanism, that mechanism will be accepted by bidder 2.

Proposition 3.2 *Buyer 1's collusion offer (from Proposition 3.2) is a safe mechanism.*

Proof Suppose that bidder 2 believes with probability $\beta \in [0,1]$ that bidder 1 is of the high type. The high type of bidder 2 expects payoff of $\beta\frac{1}{2}v_H + (1-\beta)\frac{1}{2}v_L - \rho = \frac{1}{2}v_L - \rho$ while the low type expects a payoff of $\beta\frac{1}{2}v_L + (1-\beta)\frac{1}{2}v_L - \rho = \frac{1}{2}v_L - \rho$. Truth telling

remains a best response for bidder 2: for bidder 2 of type H:

$$v_H \frac{1}{2} - \rho \geq \frac{1}{2} v_L \beta + \frac{1}{2} v_H (1 - \beta) - \rho;$$

for bidder 2 of type L:

$$\frac{1}{2} v_L - \rho \geq \frac{1}{2} v_L \beta + (v_L - \frac{1}{2} v_H) - \rho.$$

∎

Since the collusion offer is safe, it cannot be ruled out by any refinement of equilibria that restricts β following acceptance of the collusive offer. Moreover, if the high type of bidder 1 augments his offer with a small acceptance bonus of $\epsilon > 0$ for bidder 2, for any beliefs $\beta \in (0, 1]$, acceptance of the offer is strictly optimal (for $\beta = 0$ bidder 2 is indifferent between accepting and rejecting the offer). Further, note that the proposal requires no exchange of information between the bidders: bidder 1 can determine bidder 2's report to the seller, and therefore her type, through the allocation of the good and the payment made to the seller; thus knowing bidder 2's type, bidder 1 can extract the appropriate payment from bidder 2.

I have noted that the equilibrium proposed in the Che-Kim mechanism satisfies the forward induction criterion; in particular, the strategy of announcing L following rejection is maximal for bidder 1 regardless of bidder 2's action. This relies on the fact that type H of bidder 1 is indifferent between announcing H and L following rejection. The seller could then break this indifference with a small extra payment to any bidder who announces H. The equilibrium no longer satisfies the forward induction criterion and our argument that it is a likely outcome breaks down. Thus, one could argue that the seller can get arbitrarily close to her Myerson payoff with such a scheme. However, the collusion equilibrium could just as reasonably be said

to be arbitrarily close to satisfying the forward induction criterion, thus restoring the argument for choosing such an equilibrium.

Finally, note that the game could be modified such that the bidder who is able to offer the collusion contract is chosen at random after having entered the grand mechanism. The low type of bidder i will obtain payoff 0 if she is chosen to make the offer and $\frac{1}{2}v_L - \rho$ otherwise. Any expectation over these payoffs is less than the entrance fee so low types of both bidders will refuse to participate, therefore maintaining the statement of the corollary.

3.4.1 Impossibility of Earning Myerson Payoff with Symmetric Mechanisms

In this section I show that there is no symmetric mechanism that guarantees the seller earns her Myerson payoff in expectation. We define a *symmetric mechanism* such that $\bar{t} := t_1(H, H) = t_2(H, H)$; $\underline{t} := t_1(L, L) = t_2(L, L)$; $t_{LH} := t_1(L, H) = t_2(H, L)$; $t_{HL} := t_1(L, H) = t_2(H, L)$ and similarly with $q(\cdot, \cdot)$.

Proposition 3.3 *No symmetric grand mechanism with full participation will earn the seller her Myerson payoff in any equilibrium that satisfies the forward induction criterion.*

Proof Since the seller sells the good in all states of the world, in a symmetric mechanism

$$\bar{q} = \underline{q} = \frac{1}{2} \tag{3.1}$$

$$q_{HL} = 1 - q_{LH}. \tag{3.2}$$

I first show that the high type of bidder 1 never strictly prefers truth telling to announcing she is the low type when the seller earns her Myerson payoff.

Lemma 3.4 *In any symmetric grand mechanism with full participation that earns the seller her Myerson payoff, type H of either bidder is indifferent between announcing H and announcing L.*

Proof To ensure participation by the low type of bidder 2 we need

$$\alpha(q_{LH}v_L - t_{LH}) + (1 - \alpha)(\underline{q}v_L - \underline{t}) \geq 0$$

or equivalently that

$$\alpha t_{LH} + (1 - \alpha)\underline{t} \leq \alpha q_{LH}v_L + (1 - \alpha)\frac{1}{2}v_L. \tag{3.3}$$

Suppose that the high type of bidder 1 strictly prefers to tell the truth in the seller's mechanism (i.e. following rejection of her collusion contract):

$$\alpha\left(\frac{1}{2}v_H - \bar{t}\right) + (1 - \alpha)(q_{HL}v_H - t_{HL}) > \alpha(q_{LH}v_H - t_{LH}) + (1 - \alpha)\left(\frac{1}{2}v_H - \underline{t}\right)$$

$$\geq \alpha q_{LH}\Delta v + \frac{1}{2}(1 - \alpha)\Delta v \tag{3.4}$$

where the second inequality is due to (3.3).

Now note that the maximum expected surplus from the trade is equal to

$$S := \alpha^2 v_H + 2\alpha(1 - \alpha)v_H + (1 - \alpha)^2 v_L = \alpha v_H + (1 - \alpha)v_L + \alpha(1 - \alpha)\Delta V. \tag{3.5}$$

Define

$$W := \alpha(q_{LH}v_L - t_{LH}) + (1 - \alpha)\left(\frac{1}{2}v_L - \underline{t}\right).$$

The expected surplus accruing to the bidders is

$$2\left[\alpha\left(\alpha\left(\frac{1}{2}v_H - \bar{t}\right) + (1-\alpha)\left(q_{HL}v_H - t_{HL}\right)\right) + (1-\alpha)W\right]$$

$$> 2\left[\alpha\left(\alpha q_{LH}v_L + \frac{1}{2}(1-\alpha)v_L\right) + (1-\alpha)W\right]$$

$$= 2\alpha^2 q_{LH}v_L + \alpha(1-\alpha)\Delta v + 2(1-\alpha)W \tag{3.6}$$

where the inequality follows from (3.4). So the maximum payoff accruing to the seller is

$$S - 2\alpha^2 q_{LH}v_L - \alpha(1-\alpha)\Delta v - 2(1-\alpha)W < \alpha v_H + (1-\alpha)v_L - 2\alpha^2 q_{LH}v_L - 2(1-\alpha)W$$

$$\leq \alpha v_H + (1-\alpha)v_L$$

where the strict inequality follows from (3.6) and the second inequality follows since $2\alpha^2 q_{LH}v_L \geq 0$ and $W \geq 0$ by type L's individual rationality. ∎

Thus, the seller can only earn her Myerson payoff if type H of bidder 1 is indifferent between announcing H and L in the seller's mechanism. But when this is the case we can construct a collusion contract as in Proposition 3.2 and define a strategy such that bidder 1 always announces L following the rejection of her contract. Further, this strategy, along with truth telling by bidder 2, constitutes a forward induction equilibrium within the seller's mechanism. Type L of bidder 2 is thus excluded from any mechanism when we apply the forward induction criterion and the seller's expected payoff falls below the Myerson payoff.

Suppose a mechanism earns the seller her Myerson payoff. Since type H of bidder 1 is indifferent between announcing H and L in the seller's mechanism, $\underline{t} \leq \frac{1}{2}v_L$; otherwise one can construct a collusion contract as in Proposition 3.2 and define a strategy such that bidder 1 always announces L following the rejection of her contract. This

strategy, along with truth telling by bidder 2, constitutes a forward induction equilibrium within the seller's mechanism. Type L of bidder 2 is thus excluded from any mechanism when we apply the forward induction criterion and the seller's expected payoff falls below the Myerson payoff, a contradiction.

Consider the collusion contract that specifies that both bidders always announce L. If their true types match, each is awarded the good with equal probability and no additional transfers are made. Otherwise, the type H bidder is awarded the good with probability 1 and pays her type L rival $\frac{1}{2}v_H$. To show that this contract is incentive compatible for both bidders, we will need the following lemma.

Lemma 3.5 *If the seller's mechanism earns her Myerson payoff, then*

$$\frac{1}{2}v_H - \underline{t} \geq \alpha \left(\frac{1}{2}v_H - \bar{t} \right) + (1 - \alpha) \left(q_{HL}v_H - t_{HL} \right).$$

Proof Suppose not: $\frac{1}{2}v_H < q_{LH}v_L - t_{LH} \leq \frac{1}{2}v_L - t_{LH}$ where the last inequality follows from the monotonicity of q. Then

$$t_{LH} < \frac{1}{2}v_L - \frac{1}{2}v_H.$$

From the type H individual rationality constraint

$$\bar{t} \leq \frac{1}{2}v_H + \frac{1 - \alpha}{\alpha}(q_{HL}v_H - t_{HL}).$$

The seller's expected payoff is

$$\alpha^2 2\bar{t} + (1-\alpha)^2 2\underline{t} + 2\alpha(1-\alpha)(t_{HL} + t_{LH})$$

$$< \alpha^2 2\left(\frac{1}{2}v_H + \frac{1-\alpha}{\alpha}(q_{HL}v_H - t_{HL})\right) + (1-\alpha)^2 2\underline{t} + 2\alpha(1-\alpha)\left(t_{HL} + \frac{1}{2}v_L - \frac{1}{2}v_H\right)$$

$$= \alpha^2 v_H + 2\alpha(1-\alpha)\left(q_{HL}v_H + \frac{1}{2}v_L - \frac{1}{2}v_H\right) + (1-\alpha)^2 v_L$$

$$\leq \alpha^2 v_H + 2\alpha(1-\alpha)\left(\frac{1}{2}v_H + \frac{1}{2}v_L\right) + (1-\alpha)^2 v_L$$

$$= \alpha^2 v_H + \alpha v_H - \alpha^2 v_H + \alpha v_L - \alpha^2 v_L + v_L + \alpha^2 v_L - 2\alpha v_L$$

$$= \alpha v_H + (1-\alpha)v_L$$

a contradiction. ■

Now we show that the collusion contract is incentive compatible (after the seller's mechanism is resolved):

For type H

$$\alpha\frac{1}{2}v_H + (1-\alpha)\left(v_H - \frac{1}{2}v_H\right) \geq \alpha\frac{1}{2}v_H + (1-\alpha)\frac{1}{2}v_H$$

where the inequality follows since $q_{LH}v_L - t_{LH} \leq \frac{1}{2}v_H$ by the previous lemma.

For type L

$$\alpha\frac{1}{2}v_H + (1-\alpha)\frac{1}{2}v_L \geq \alpha\frac{1}{2}v_L + (1-\alpha)\left(v_L - \frac{1}{2}v_H\right).$$

■

3.5   The Colluder's Problem

In this section I write down bidder 1's collusion proposal problem, given the seller's mechanism.

As seen above, an important feature of modelling collusion as a proposal from

one of the bidders is choosing the appropriate reservation utility for the receiver to use in the proposer's mechanism design problem. Here we formally incorporate the proposer's posterior following rejection of the proposal. Let β_2^r represent bidder 1's belief over bidder 2's types following the rejection of the collusion proposal. In general there are no restrictions on β_2^r but applying such refinements as the intuitive criterion may be used to focus on particular rejection beliefs. Note that due to Myerson's [13] inscrutability principle, we can assume without loss of generality that bidder 1's proposal reveals no information to bidder 2. The receiver's reservation utility is derived as an equilibrium payoff in the seller's mechanism, given β_2^r. Let $\mathcal{E}(M, \beta_2^r)$ be the set of equilibrium strategies in the seller's mechanism M (played non-cooperatively) given bidder 1's beliefs β_2^r. Suppose the seller offers the mechanism $M = \{q_1(\cdot), q_2(\cdot), t_1(\cdot), t_2(\cdot)\}$. Let

$$\underline{U}_2(v_2, \beta_2^r) \in \left\{ \mathbb{E}_{v_1} \left[v_2 q_2(\sigma_1(v_1), \sigma_2(v_2)) - t_2(\sigma_1(v_1), \sigma_2(v_2)) \right] \big| (\sigma_1(v_1), \sigma_2(v_2)) \in \mathcal{E}(M, \beta_2^r) \right\}.$$

$\underline{U}_2(v_2, \beta_2^r)$ is the reservation utility for bidder 2 given bidder 1's rejection beliefs β_2^r. If the set on the left hand side is not singleton, bidder 1 would be afforded the greatest scope for collusion by choosing the infimum element of the set; i.e. imposing the worst possible equilibrium for bidder 2 following rejection. In general, this can be any equilibrium. Fix β_2^r, and $\underline{U}_2(v_2, \beta_2^r)$. Let $A_i(M) \subseteq T_i$ be the set of bidder i's types that participate in the seller's mechanism M. A collusion proposal P is individually rational if

$$\mathbb{E}_{v_1} \left[v_2 q_2(\phi(v_1, v_2)) - t_2(\phi(v_1, v_2)) - y(v_1, v_2) \right] \geq \underline{U}_2(v_2, \beta_2^r) \tag{3.7}$$

for all $v_2 \in A_2$. The notation $\mathbb{E}_{v_1}$ is the expectation operator taken over the random variable v_1. A crucial difference from Che and Kim's [5] model of collusion is that

their notion of individual rationality requires that the right hand side of (3.7) is 0, or more generally, equal to the reservation payoff of bidder 2.

Let $\mathscr{I}_i$ represent the information bidder i has following the the announcements made from m and any information revealed by the seller's mechanism. A collusion proposal P is incentive compatible if, given $\mathscr{I}_i$, each type of each bidder prefers to take the actions prescribed P. Formally,

$$
\mathbb{E}_{v_i}\left[v_i q_i(\phi(v_i, v_{-i})) - t_i(\phi(v_i, v_{-i})) + (-1)^{i+1} y(v_i, v_{-i}) | \mathscr{I}_i\right] \geq
$$
$$
\mathbb{E}_{v_i}\left[v_i q_i(\phi(v_i', v_{-i})) - t_i(\phi(v_i', v_{-i})) + (-1)^{i+1} y(v_i', v_{-i}) | \mathscr{I}_i\right]
$$

for all i and all $v_i, v_i' \in A_i$.

Finally, bidder 1's collusion problem is to choose P to maximize

$$
\mathbb{E}_{v_2}\left[v_1 q_1(\phi(v_1, v_2)) - t_1(\phi(v_1, v_2)) + y(v_1, v_2)\right]
$$

such that P is individually rational and incentive compatible.

3.6 Conclusion

In this chapter I have illustrated the limitations of studying the problem of collusion in auctions as being managed by a disinterested third party. In particular, I have shown that Che and Kim's [5] robustly collusion proof mechanism is susceptible to collusion when a bidder proposes collusion and equilibria are refined to satisfy the forward induction criterion.

In future work I will build on the framework outlined in Section 3.5 to study the general mechanism design problem of the seller who faces bidders who can self-organize collusion.

References

[1] Masaki Aoyagi. Bid rotation and collusion in repeated auctions. *Journal of Economic Theory*, 112(1):79–105, September 2003. 3.2

[2] Susan Athey and Kyle Bagwell. Optimal collusion with private information. *RAND Journal of Economics*, 32(3):428–465, Autumn 2001. 3.2

[3] Susan Athey, Kyle Bagwell, and Chris Sanchirico. Collusion and price rigidity. *The Review of Economic Studies*, 71(2):317–349, April 2004. 3.2

[4] Gorkem Celik and Michael Peters. Reciprocal relationships and mechanism design. September 2012. 3.2

[5] Yeon-Koo Che and Jinwoo Kim. Robustly collusion-proof implementation. *Econometrica*, 74(4):1063–1107, July 2006. 3.1, 3.2, 3.3, 3.4, 3.4, 3.5, 3.6

[6] Yeon-Koo Che and Jinwoo Kim. Optimal collusion proof auctions. *Journal of Economic Theory*, 144:565–603, 2009. 3.2

[7] Vianney Dequiedt. Efficient collusion in optimal auctions. *Journal of Economic Theory*, 136:302–323, 2007. 3.2

[8] Péter Eső and James Schummer. Bribing and signaling in second price auctions. *Games and Economic Behaviour*, 47(2):299–324, 2004. 3.2

[9] Drew Fudenberg, David Lavine, and Eric Maskin. The folk theorem with imperfect public information. *Econometrica*, 62(5):997–1039, September 1994. 3.2

[10] Rodney J. Garratt, Thomas Tröger, and Charles Z. Zheng. Collusion via resale. *Econometrica*, 77(4):1095–1136, July 2009. 3.2

[11] Jean-Jacques Laffont and David Martimort. Collusion under asymmetric information. *Econometrica*, 65(4):875–911, July 1997. 3.2

[12] Jean-Jacques Laffont and David Martimort. Mechanism design with collusion and correlation. *Econometrica*, 68(2):309–342, March 2000. 3.2

[13] Roger B Myerson. Mechanism design by an informed principal. *Econometrica*, 51(6):1767–1797, December 1983. 3.4, 3.5

[14] Gregory Pavlov. Auction design in the presence of collusion. *Theoretical Economics*, 3(3):409–455, 2008. 3.2

[15] Lucía Quesada. Collusion as an informed principal problem. October 2005. 3.2

[16] McAfee. Preston R. and John McMillan. Bidding rings. *American Economic Review*, 82(3):579–599, June 1992. 3.2

[17] Shiran Rachmilevitch. Bribing in first-price auctions. *Games and Economic Behaviour*, 77(1):299–324, 2013. 3.2

[18] Andrzej Skrzypacz and Hugo Hopenhayn. Tacit collusion in repeated auctions. *Journal of Economic Theory*, 114(1):153–169, January 2004. 3.2

Conclusion

Despite a wealth of important examples of informed parties designing and implementing mechanisms, or instruments of trade, relatively little theoretical work studies the problem. As I have shown throughout this dissertation, striking differences appear relative to cases where the mechanism designer is uninformed or the informed player is not the mechanism designer.

From the technical side, in the first chapter I demonstrate that the unique incentive constraints faced by the informed principal can lead her to choose to be ignorant in order to maintain a strategic advantage when offering a contract to an agent. That the principal refuses to acquire full information despite: (a) it being costless to do so; (b) being able to employ the most general mechanisms available; and (c) being able to choose her most favourable full-information continuation equilibria, makes this result particularly notable.

From a more applied side, I show that considering the informed principal's problem forces the modeller to change perspective when considering such details as the space of mechanisms available to the principal or how to properly refine the set of equilibria. More specifically, in a moral-hazard environment in the second chapter, I demonstrate that it can be insufficient to consider only the typical point-contracts that pay a fixed wage associated with each observable outcome when the principal has private information about the productivity of the worker; I show when more general *menu-*

contracts can improve the principal's payoff. Further, in the final chapter, I show that considering *bidder-led* collusion in auctions forces the modeller to take into account the proposing bidder's strategic incentives when deciding on the appropriate set of equilibria to consider; specifically, the modeller should pay particular attention to the proposer's undominated strategies. Such a change in perspective lays bare the restrictiveness of seemingly innocuous assumption such as forcing collusion contracts to deliver to the receivers their reservation utility fixed from before entering the seller's mechanism. I show that deriving the receivers' participation constraints via equilibrium play in the seller's mechanism and allowing beliefs to change in response to actions in the collusion contracting subgame can dramatically alter how we expect collusion to affect the seller's revenue.

Curriculum Vitae

Nicholas C Bedard

Education

- Ph. D. Economics, University of Western Ontario, 2013.

- M.A. Economics, University of Western Ontario, 2008.

- B.A. Honours Economics, University of Western Ontario, 2007.

Dissertation

- *Essays on Mechanism Design and the Informed Principal Problem*

- Committee Members: Professors Charles Zheng (supervisor), Al Slivinski, Maria Goltsman, Gregory Pavlov

Teaching/Research Interests

- Microeconomics, game theory, information acquisition, contract theory, auction theory

Presentations

- **The Strategically Ignorant Principal**

  - University of Technology, Sydney, May 2013

  - ESEI, University of Zurich, April 2013

  - University of Maastricht, April 2013

  - HECER, February 2013

  - Fall Midwest Economic Theory Meeting, October 2012; *Washington University in St. Louis*

  - Western Economics Alumni Conference in Honour of Yvonne Adams, September 2012; *University of Western Ontario*

  - PhD Workshop in Economics, August 2012; *University of Guelph*

  - International Conference on Game Theory, July 2012; *Stony Brook University*

  - Annual Conference of the Canadian Economics Association (CEA), June 2012; *University of Calgary*

Professional Activities

- **Teaching Development**

  - Teaching Workshop for Subjects Heavy in Mathematics, UWO April 2013; attendee

- **Discussions**

- Conference of the Canadian Economics Association (CEA), June 2012; *University of Calgary*

    * Discussant for "Marketing versus Engineering: Who Should Decide?" by Stefen Ambec and Michel Poitevin

- **Refereeing**

  - Theory and Decision

Scholarships and Awards

- Ontario Graduate Scholarship, 2011, $15 000 annually

- Western Graduate Research Scholarship, 2007-2012, currently $6804 annually

- Social Sciences and Humanities Research Council of Canada, Canadian Graduate Scholarship (Joseph-Armand Bombardier CGS), Doctoral, 2008-2011, $35 000 annually

- Ontario Graduate Scholarship, 2008, $15 000 annually

  - Declined: the Ontario Graduate Scholarship and the Canadian Graduate Scholarship cannot be held concurrently.

- Borwein Memorial Prize (Mathematics), 2007, $100

- Four Year Continuing Admission Scholarships (Western), 2003-2006, $2000 annually

- Shaw Foundation Scholarship, 2006, $1500

- Dr. Mark K. Inman Scholarship, 2006, $1000

- Dr. W. Glenn Campbell and Dr. Mark K. Inman Scholarships in Economics, 2005, $2000

- Shaw Family Foundation Scholarship, 2005, $1000

Research Experience

- Research Assistant, Professor Charles Zheng, Department of Economics, University of Western Ontario, Fall 2012 - Spring 2013

- Research Assistant, Professor Ben Lester, Department of Economics, University of Western Ontario, Summer 2010

Teaching Experience

- Instructor, The University of Western Ontario

  - Intermediate Microeconomic Theory I - Summer 2012

- Graduate Teaching Assistant, The University of Western Ontario

  - Intermediate Honours Microeconomics II - Winter 2010, 2012

  - Intermediate Honours Microeconomics I - Fall 2009, 2011

    * Responsibilities included occasional lectures.

  - Senior Research Seminar in Economics - Fall/Winter 2010/2011

    * Responsibilities running occasional tutorials and counselling students on their research.

  - Microeconomics II - (Ph. D Level) Winter 2009.

             * Responsibilities included running weekly tutorials.

      – Macroeconomics I - (Ph. D Level) Fall 2008.

             * Responsibilities included running weekly tutorials.

      – Introductory Economics - Fall/Winter 2007/2008

- Tutor, Intermediate Macroeconomics (honors undergraduate), 2005

Service

- Graduate Representative, Committee for Academic Policy, Department of Economics, University of Western Ontario, 2009/2010

- Editor, Western Undergraduate Economic Review, 2006/2007

- Vice President of Communications, Economics Students' Association, 2005-2007