إقــــــرار

أنا الموقع أدناه مقدم الرسالة التي تحمل العنوان:

Designing Two-Dimensional Magnetic Levitation Control System

تصميم نظام ثنائي الأبعاد لتعليق جسم معدني بين مغناطيسيين

DECLARATION

The work provided in this thesis, unless otherwise referenced, is the researcher's own work, and has not been submitted elsewhere for any other degree or qualification

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Designing Two-Dimensional Magnetic Levitation Control System

تصميم نظام ثنائي الأبعاد لتعليق جسم معدني بين مغناطيسيين

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نتيجة الحكم على أطروحة ماجستير

بناءً على موافقة شئون البحث العلمي والدراسات العليا بالجامعة الإسلامية بغزة على تشكيل لجنة الحكم على أطروحة الباحث/ **محمد اسماعيل حسين سرور** لنيل درجة الماجستير في **كلية *الهندسة*** قسم **الهندسة الكهربائية – أنظمة التحكم** وموضوعها:

تصميم نظام ثنائي الأبعاد لتعليق جسم معدني بين مغناطيسيين

Designing Two Dimensional Magnetic Levitation Control System

وبعد المناقشة العلنية التي تمت اليوم الأربعاء 12 جمادى الآخر 1436هـ، الموافق 2015/04/01م الساعة العاشرة صباحاً بمبنى القدس، اجتمعت لجنة الحكم على الأطروحة والمكونة من:

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وبعد المداولة أوصت اللجنة بمنح الباحث درجة الماجستير في **كلية *الهندسة*/** قسم **الهندسة الكهربائية – أنظمة التحكم**.

واللجنة إذ تمنحه هذه الدرجة فإنها توصيه بتقوى الله ولزوم طاعته وأن يسخر علمه في خدمة دينه ووطنه.

والله ولي التوفيق،،،

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DEDICATION

With all love in my heart I dedicate this thesis to:

My lovely parents, brothers and sisters, for unlimited and endless love.

My lovely wife for Full support at all stages of the implementation of the thesis.

My daughters.

To the stars light up our lives, the martyrs and the wounded and prisoners of freedom.

Thank you ALL

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I acknowledge all Faculty members and administrative staff who have made a lot of effort and giving to overcome the obstacles and provide an ideal environment study.

ABSTRACT

This thesis presents the design of two-dimensional axis magnetic lavitation control system. An object should be able to levitate purely with magnetics fields. In order to produce a stable output and allwo movement of the levitated object between two solenoids around, a two-dimensional classical feedback control may be utilized.

This is accomplished by developingstate space equations to model one- and two-dimensional magnetic lavitation. Since the system is nonlinear and open-loop unstable, it should be linearized around a set point.The stability is analyzed around possible equilibrium point in the sense of the qualitative behavior of the system and Lyapunov stability.

A control startagy is developed for levitating the object purely in the vertical components with magnetic fields and allows movement in the horizontal components between two solenoids around a two-dimensional space. Negative feedback and lead compensators based on the linearized model of the two dimensional magnetic levitation system are designed to stablize the system. MATLAB is used in designing and simulating the system.

ملخص الدراسة

تقدم هذه الأطروحة تصميم نظام تحكم مغناطيسي ثنائي الأبعاد. جسم معدني يجب أن يكون قادر على التحليق في الهواء فقط بالقوة المغناطيسية. حتى يكون النظام مستقر و يستطيع الجسم المعدني و هو معلق في الهواء أن يتحرك في مجال مغناطيسي ثنائي الأبعاد بين مغناطيسيين, يمكن استخدام نظام التحكم الكلاسيكي ذو التغذية الراجعة.

يتم ذلك عن طريق إيجاد المعادلات التفاضلية لنظام تعليق مغناطيسي أحادى الأبعاد و ثنائي الأبعاد. و حيث أن النظام غير خطى و غير مستقر و من دون تغذية راجعة فتم تقريب النظام إلى خطى عند نقطة تشغيل اختيارية. لقد تم تحليل استقرار النظام عند نقطة توازن ممكنة عن طريق دراسة السلوك النوعي للنظام و باستخدام دالة ليبانوف.

تم تطوير إستراتيجية بحيث يستطيع الجسم المعدني و هو معلق في الهواء ان يتحرك في مجال مغناطيسي ثنائي الأبعاد بين مغناطيسيين. لجعل نظام التعليق المغناطيسي ثنائي الأبعاد مستقر, تم استخدام التغذية الراجعة السالبة و المتحكم(lead compensators). لمحاكاة النظام تم استخدام برنامج الماتلاب.

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Chapter 1: Introduction

1.1. Introduction

The easiest way to levitate an object electromagnetically (from a control perspective) is via magnetic suspension. The object that is to be levitated is placed below an electromagnet (only one is required), and the strength of the magnetic field produced by the electromagnet is controlled to exactly cancel out the downward force on the object caused by its weight. This method circumvents Earnshaw's theorem by making use of feedback [1].

Thus, the system only has to contend with one force, the levitating object's weight. This system works via the force of attraction between the electromagnet and the object. Because of this, the levitating object does not need to be a magnet. It can be any ferrous material. This further simplifies the design consideration.

To prevent the object from immediately attaching itself to the electromagnet, the object's position has to be sensed and this information fed back into the control circuit regulating the current in the electromagnet. This produces the basic feedback arrangement depicted below in figure (1-1)

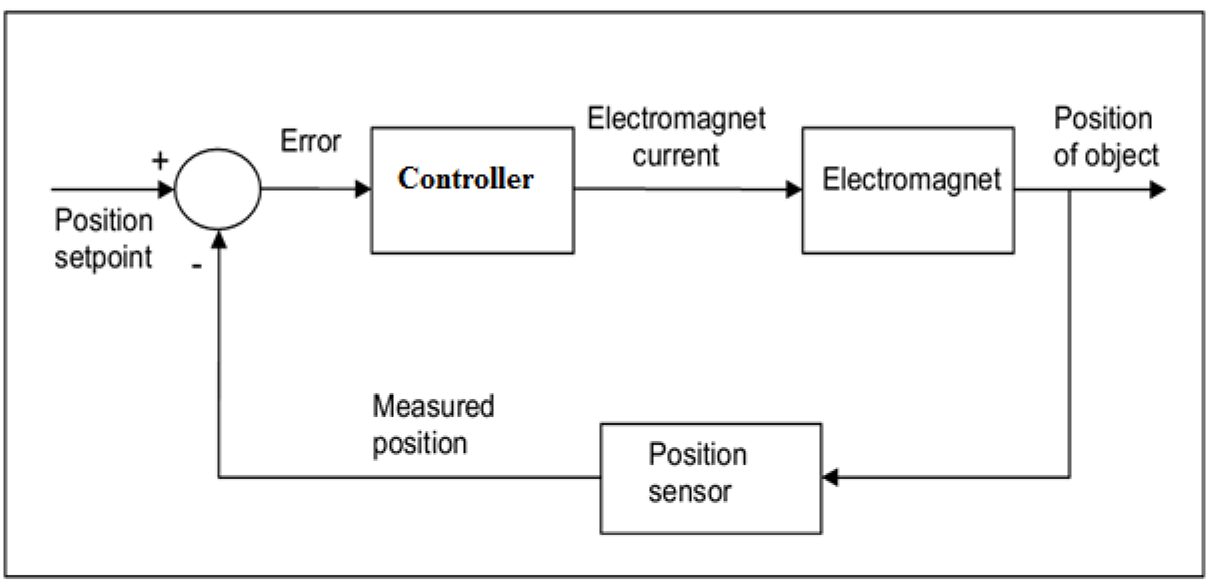

Figure 1-1: The basic control arrangement of a magnetic levitation system.

If the object gets too close to the electromagnet, the current in the electromagnet must be reduced. If the object gets too far, the current to the electromagnet must be increased. A possible physical arrangement is shown below in figure (1-2). Controller

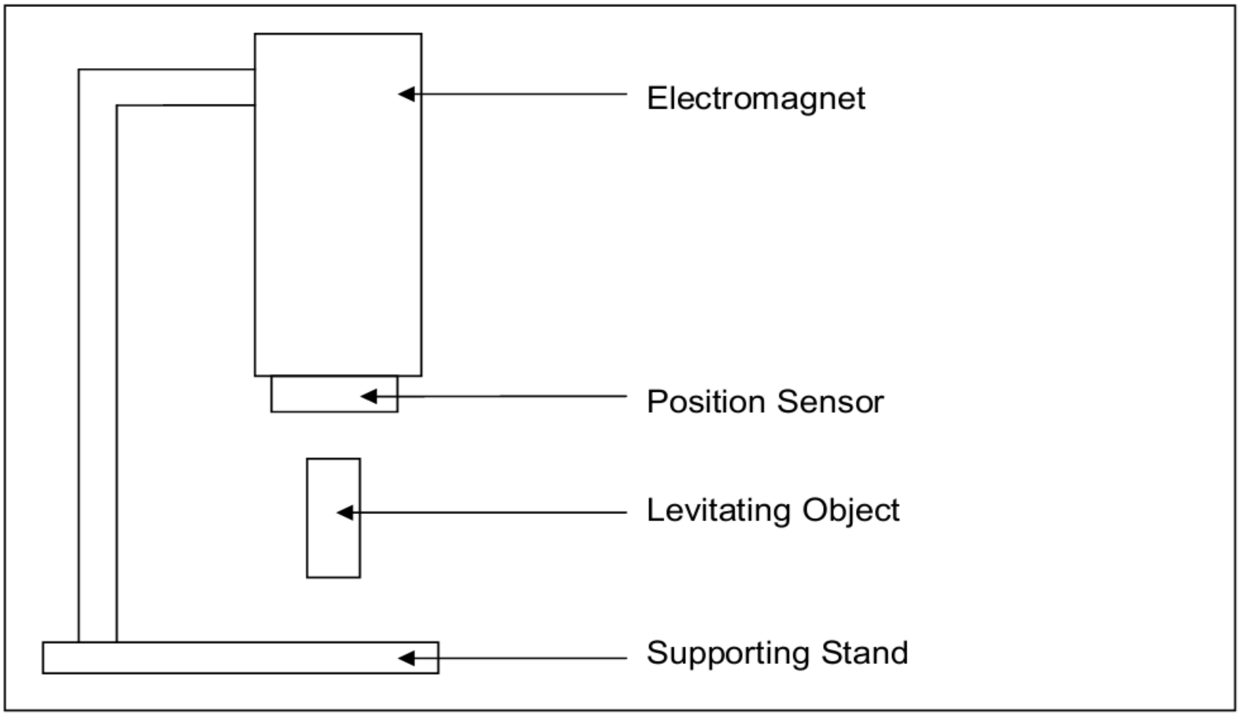

Figure 1-2: The physical model of a magnetic levitation system.

1.2. Motivation and Objectives

The control of magnetic levitation technology is on the rise and the technology has been utilized in space, automotive, and train industries. Controlling the magnetic ball vertically is a simple task; however, controlling it in two-dimensions is not quite that simple but a challenging task. Using classical control techniques may not do the job; thus, there is a need to utilize nonlinear control for the design and analysis of the magnetic ball problem.

The main objectives of this research are:

1. Develop the state equations of one-dimensional magnetic lavitation system.
2. Develop the state equations for the vertical components and the horizontal components; the two-dimensional magnetic lavitation system.
3. Analyze the stability of a possible equilibrium point in the open-loop in the sense of the qualitative behavior of the system and Lyapunov stability .
4. Develop a control startagy for levitating the object purely in the vertical components with magnetic fields, and allow movement in the horizontal components between two solenoids around an two-dimensional space
5. Design a lead compensators based on the linearized model of the two-dimensional magnetic levitation system to stablize the system.

6. Simulate the system using MATLAB and implement a modular cell of the the two-dimensional levitator.

1.3. Statement of problem

The two-dimensional levitator could be built with two solenoids; unfortunately, the equations for the one-dimensional levitator do not readily apply to the horizontal balancing that needs to happen in the two-dimensional levitator. We have to develop the mathematical model for the horizontal components. However the system is nonlinear and unstable; therefore, it should be linearized at a set point and analyze its stability. The forces that act on the horizontal and vertical axes are related and sometimes coupled which make it harder to model. Therefore, obtaining a two-dimensional model for the magnetic levitation and analyzing the stability in the sense of Lyapunov is still a complex problem; moreover, controlling the ball to move according to a reference signal horizontally and vertically is challenging.

1.4. Literature Review

- W. Barie and J. Chiasson (2001) [1], described and verifiedthe design and the possibility of implementing one dimensional magnetic levitation system using phase lead compensation technique. The theoretical background of the magnetic levitation was studied from mathematical perspective that led to deriving the model of the main system and the associated controller. The design was carried out using MATLAB, following Root Locus method, with a primary target of levitating a steel ball of 21.6 g at a distance of 1 cm below the coil tip. The system was practically implemented and tested with actual mass. It was observed that the system has achieved the goal by levitating the object at predetermined distance. Moreover, it was been tested for several other masses and the system was capable of levitating all of them which makes it a robust system.

- Gerardus (2012) [2], modelled the dynamics of n-dimensional levitation system for a particular system with certain dimensions so that the developed model cannot be generalized. The transfer function is developed for a system with characteristic length. The stability in the sense of lead copmensator was analyzed.

- Charara(2008) [3], designed a one dimensional magnetic levitation controllers using Jacobian linearization, feedback linearization and sliding mode control. A controller based on the Jacobian linearization about a nominal operating point was designed, despite the fact that magnetic levitation system was described by nonlinear differential equations. Through the use of feedback linearization, the author transformed the system dynamics from complicated nonlinear ones to more simple linear dynamics to be controlled through the use of linear state feedback. This method has advantage over Jacobian linearization by allowing the control of a simple linear plant without neglecting the nonlinear dynamic of the system. The feedback linearization version was extremely sensitive to parameter variation. Thus, paper used a sliding mode controller to produce robust feedback system.

- Katon, and Kidance (2010) [4], developed a nonlinear robust controller for the magnetic levitation system based on Nakamura's inverse optimal controller is developed. First, a controller which compensate for gravity by feedforward input is proposed and its effectives by the experiment confirmed. However, the controller lacks robustness, thus an improved controller via adaptive control technique to recover robustness is developed.

- Al-Muthairi and Zribi (2008) [5], proposed a sliding mode control schemes of the static and dynamic types are proposed for the control of a magnetic levitation system. The proposed controllers guarantee the asymptotic regulation of the states of the system to their desired values. Simulation results of the proposed controllers were given to illustrate the effectiveness of them. Robustness of the control schemes to changes in the parameters of the system was also investigated.

1.5 Methodology

In order to model and control a one- and two-dimensional magnetic levitation, the following steps are followed:

1. Developingstate space equations to model one- and two-dimensional magnetic lavitation.

2. Linearizing the system linearized around a set point.

3. Analyzing the stability using:

    i. The qualitative behavior of the system.

    ii. Lyapunov stability.

4. Controlling the system using:

    i. A lead compensators based on the linearized model.

    ii. Simulation the controlled system using MATLAB.

1.6 Contribution

This thesis presents a general mathematical model for the 2-dimmensional axis maglev system. The thesis also presents a control strategy using lead control. Finally, the thesis performs a stability analysis on the developed 2-dimensional maglev system.

1.7 Outlines of Thesis

This thesis is broken into several different themes:

Chapter two: In this chapter we develop the theory behind one-dimensional and 2-dimensional levitation. We will derive the state equations for one-dimensional and 2-dimensional levitation.

Chapter three: In this chapter we study mainly the stability of the equilibrium point in the sense of Lyapunov. Before we discuss the lyapunov stability for the two dimensional levitation system, we will look at the qualitative behavior of the system at an equilibrium point, the eigenvalue and type of the equilibrium point

Chapter four: In this chapter we develop the theory of a control algorithm for levitating the object purely with magnetic fields, and allow movement between two solenoids around an two dimensional space. Compensate the systems, and develop feedback diagrams to represent, the closed loop systems. Simulation of the two Dimensional Levitation System using SIMULINK

Chapter five: Conclusion and future work

Chapter 2: Modelling of Magnetic Levitation

2.1. Introduction

In this section, we develop the theory behind one -dimensional and 2-dimensional levitation. We will derive the state equations, we begin with the theory of levitation in one dimension, then follow on with the 2-dimensional.

2.2. Nonlinear system

Nonlinear system representation means the characterization of nonlinear systems using nonlinear mathematical models. In fact, nonlinear models may be considered as a tool for explaining the nonlinear behavior patterns in terms of a set of easily understood elements.
In nature, most practical systems used for control are essentially nonlinear, and in many applications, particular in the area of chaos, it is the nonlinear rather than the linear characteristics that are most used. Signals found in the physical world are also far from conforming to linear models.
Indeed, the complex structure of dynamic systems makes it almost impossible to use linear models to represent them accurately.
Nonlinear models are designed to provide a better mathematical way to characterize the inherent nonlinearity in real dynamic systems, although we may not be able to consider all their physical properties [5].

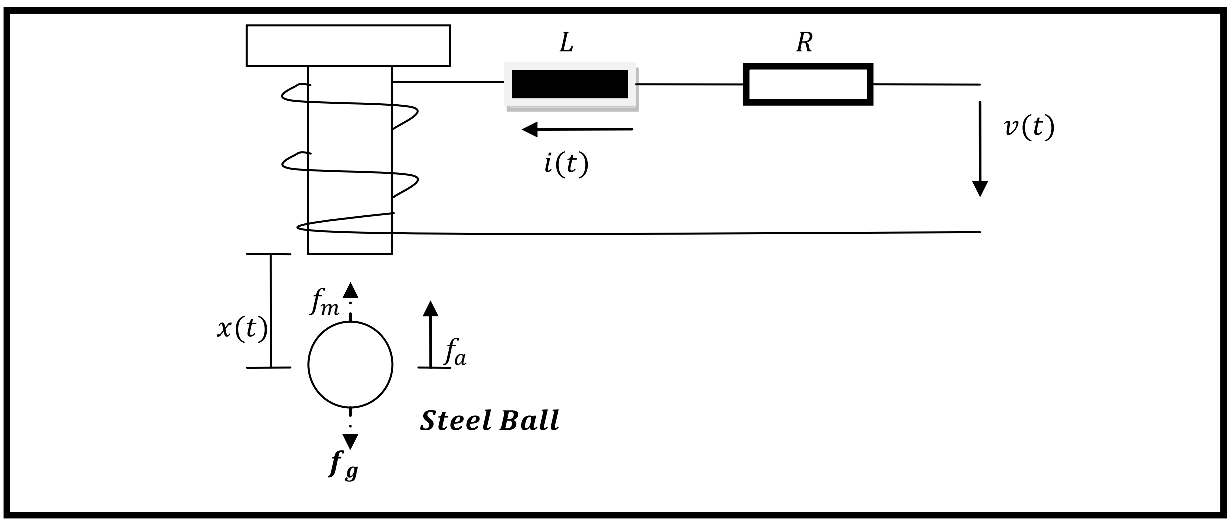

Figure 2-1: One-Dimensional Levitation Basic Setup

2.3. One Dimensional Magnetic Ball Levitation State Equation

The basic setup of a one-dimensional magnetic levitator is detailed in Figure 2-1. The direction of the current through the inductor $i(t)$ and the direction of the position of the object $x(t)$ are marked. The motion equation is based on the balance of all forces acting on the Ball. We have three forces: gravity force f_g, electromagnetic force f_m and the acceleration force f_a. The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object [5].

2.3.1 Nonlinear Dynamic Equation

The net forces

$$f_a = f_g - f_m \qquad (2.1)$$

Where;

$$f_m = \frac{i^2 K_c}{x^2} \qquad (2.2)$$

K_c= Coil constant

Gravitational force

$$f_g = mg \qquad (2.3)$$

Accelerations force

$$f_a = m\frac{d^2x}{dt^2} \qquad (2.4)$$

The simplified equation of motion for the ball is given by

$$m\frac{d^2x}{dt^2} = mg - f_m (2.5)$$

Substituting (2.2) in (2.5)

$$\mathrm{m}\frac{d^2x}{dt^2} = mg - \frac{i^2K_c}{x^2} \Rightarrow \frac{d^2x}{dt^2} = g - \frac{i^2K_c}{mx^2} = f_1(\ddot{x}, \dot{x}, x, i) \qquad (2.6)$$

While the equation for the circuit shown in figure (2.1) is given by

$$L\frac{di}{dt} = -Ri + v(t) \Rightarrow \frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v = f_2(i, v) \qquad (2.7)$$

It is usually desirable to operate a nonlinear system about some equilibrium point. To find the equilibrium condition, we must have

$$\ddot{x} = \dot{x} = 0, \frac{di}{dt} = 0, \qquad (2.8)$$

which implies that $f_1(\ddot{x_e}, \dot{x_e}, x_e, i_e) = 0, \quad f_2(i_e, v_e) = 0$

Where;

$\ddot{x_e}$=Ball acceleration

$\dot{x_e}$=Ball Velocity at equilibrium

$x_{e=}$Equilibrium position

i_e=Equilibrium current

v_e= Equilibrium input volt

At Equilibrium position the accelerations $\frac{d^2x}{dt^2} = 0$ and$\frac{di}{dt} = 0$. This means, the magnetic force and the gravity force are equal, therefore the object balanced at equilibrium position.

Refer to equation (2.6)

$$0 = g - \frac{i^2 K_c}{mx^2} \Rightarrow i_e^2 = (\frac{mg}{K_c}) x_e^2 \Rightarrow i_e = \left[\sqrt{\frac{mg}{K_c}}\right] x_e \quad (2.9)$$

We will need this equation later in this section to calculate the equilibrium current at selected equilibrium point.

We may assume, without loss of generally, that $i_e > 0$, the equilibrium voltage obtain from equation (2.7)

$$v_e = Ri_e = R\left[\sqrt{\frac{mg}{K_c}}\right] x_e \qquad (2.10)$$

Equations (2.9) and (2.10) tell us that in order to keep the system at given equilibrium position, the equilibrium current i_e is directly proportional to x_e.

2.3.2 Linearization

In order to obtain a linear model of the system, author used Taylor Series Expansion of the first two terms [5].

$$\frac{d^2x}{dt^2} = a_1(x - x_e) + a_2(\dot{x} - \dot{x_e}) + a_3(i - i_e) + b_1(v - v_e)$$

We may do this if we let

$$a_1 = \frac{\partial f_1}{\partial x}|_{equil} = \frac{\partial}{\partial x}\left(g - \frac{K_c i^2}{mx^2}\right)|_{equil} = \frac{2K_c i_e^2}{mx^3} \qquad (2.11)$$

$$a_2 = \frac{\partial f_1}{\partial \dot{x}}|_{equil} = \frac{\partial}{\partial x}\left(g - \frac{K_c i^2}{mx^2}\right)|_{equil} = 0 \qquad (2.12)$$

a_2 = Equilibrium velocity at equilibrium point=0

$$a_3 = \frac{\partial f_1}{\partial i}|_{equil} = \frac{\partial}{\partial i}\left(g - \frac{K_c i^2}{mx^2}\right)|_{equil} = -\frac{2K_c i}{mx^2}|_{equil} = -\frac{2K_c i_e}{m\,x_e^2} \quad (2.13)$$

$$b_1 = \frac{\partial f_1}{\partial v}|_{equil} = \frac{\partial}{\partial v}\left(g - \frac{K_c i}{mx^2}\right)|_{equil} = 0 \quad (2.14)$$

We note that the equation for the electrical part of the system is already linear.

Now define the following variables

$$x_1 = x - x_e\,, x_2 = \dot{x} - \dot{x_e}\,, x_3 = i - i_e\,, u = v - v_e$$

Where:

x_1 = Ball position

x_2=Ball velocity

x_3=Drive current

u = Input voltage

Refer to equation (2.7)

$$\dot{x_3} = \frac{di_e}{dt} = -\frac{R}{L}i_e + \frac{1}{L}v$$

Thus, state-space model is:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ a_1 & 0 & a_3\\ 0 & 0 & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{L}\end{bmatrix}u(t)$$

$$x_e = x_1 = [1 \quad 0 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad (2.15)$$

Hence, state space model of Linearized model around certain operating point (x_e) is written as:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ \frac{2K_c i_e^2}{mx^3} & 0 & -\frac{2K_c i_e}{m\,x_e^2}\\ 0 & 0 & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{L}\end{bmatrix}u(t)$$

$$x_e = x_1 = [1 \quad 0 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad (2.16)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2K_c i_e^2}{mx^3} & 0 & -\frac{2K_c i_e}{m\, x_e^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \text{, B} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad \text{D=[0]} \qquad (2.17)$$

We specify the design parameters [6].

$$R = 3.5\Omega, m = 0.0084kg,\ k_c = 68.823 * 10^{-6} \frac{Nm^2}{A}$$

$$L = 0.01H, \qquad g = 9.81 \frac{m}{sec^2}$$

Compute the equilibrium current i_e at the equilibrium point $x_e = 0.01m$ from equation (2.9)

$$i_e = \left[\sqrt{\frac{mg}{K_c}}\right] x_e$$

$$i_e = \left[\sqrt{\frac{0.0084*9.81}{68.823*10^{-6}}}\right] * 0.01 = 0.346024896\ A$$

The corresponding equilibrium voltage, refer to equation (2.10)

$$v_e = Ri_e = 3.5 * 0.346024896 = 1.211087136 \text{ V}$$

The resulting state space model of linearized magnetic ball levitation around ($x_{v_e} = 0.01m$) is:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1962 & 0 & -56.7011 \\ 0 & 0 & -350 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} u(t)$$

$$x_e = x_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1962 & 0 & -56.7011 \\ 0 & 0 & -350 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$D = [0] \qquad (2.18)$$

Hence, from Eq.(2.18), transfer function expression $G(s)$ of the linearized model is given by using MATLAB:

Thus

$$G(s) = \frac{-5670}{s^3+350s^2-1962s-686700} \qquad (2.19)$$

Calculating the open loop poles of the system computed with MATLAB, we get three poles

1. -350 (stable pole)
2. -44.2945 (stable pole)
3. 44.2945 (unstable pole)

The system is unstable because it has positive pole (44.11).

In order to determine the controllability matrix and find its rank, we use MATLAB and we get.

Controllability matrix=$[\, B \ \ |AB \ \ |A^2B]$

$$Co = \begin{bmatrix} 0 & 0 & -0.006e7 \\ 0 & -0.0006e7 & 0.1985e7 \\ 0 & -0.0035e7 & 1.2250e7 \end{bmatrix}$$

The Rank =3 = states number, therefore the system is controllable.

2.4. Two Dimensional Magnetic Ball Levitation

Figure 2-2 shows the basic setup for the two-dimensional levitation system. There are two solenoids *(solinoids1, soinoids2)*, each of which exerts a force (f_{m_1}, f_{m_2}) on the metallic ball. Each of the solenoids is run at the same or different current $(i_1(t), i_2(t))$.Although, they may have different inductances because of imperfect matching. The ball can levitate purely in the vertical components$x_v(t)$, and in the horizontal components$x_h(t)$, between two solenoids around two-dimensional space. The length of the horizontal components equals b. The two solenoids should be driven with the same or different voltage sources $(v_1(t), v_2(t))$.

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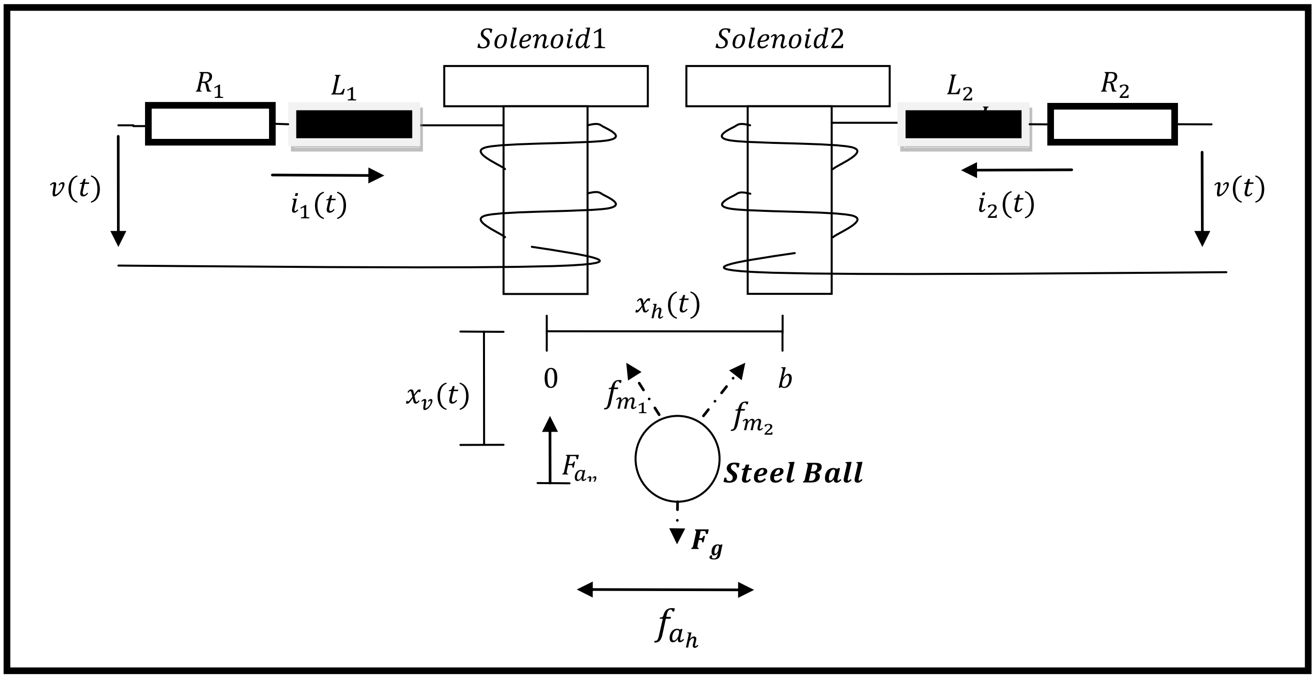


Figure 2-2: Basic Setup for Two Dimensional Levitation.

2.4.1 Vertical Component State Equation

The forces that act on the horizontal and vertical axis are related and sometimes coupled which make it harder to model. Figure (2-2) and (2-3) show the total force which exerts on the ball.

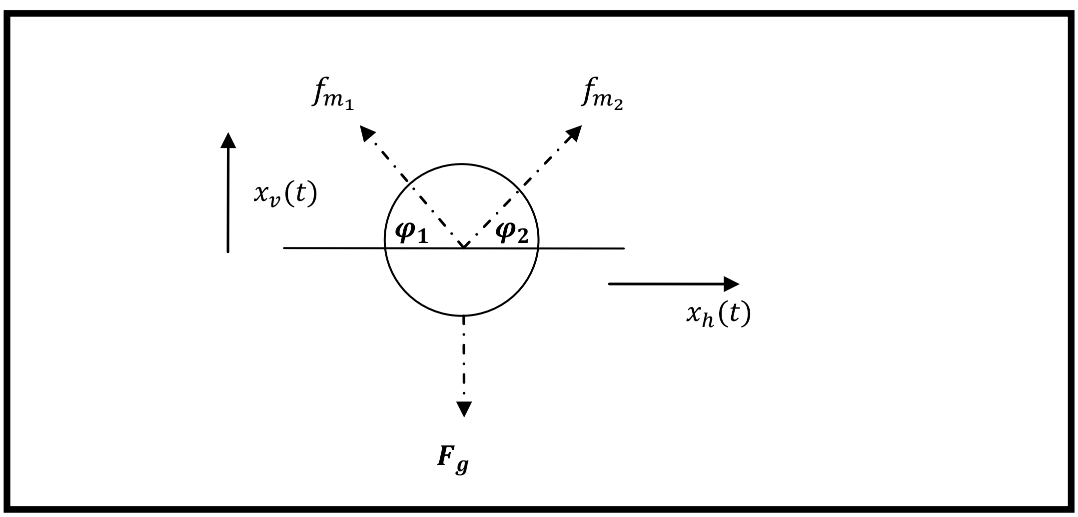


Figure 0-3: Forces on the Object in the two-Dimensional Case

The total force $f_{M_{net},v}$in the vertical direction is the sum of the vertical components of f_{m_1} and f_{m_2}.

$$f_{M_{net},v} = f_{M1}\sin(\varphi_1(t)) + f_{M2}\sin(\varphi_2(t)) \quad (2.20)$$

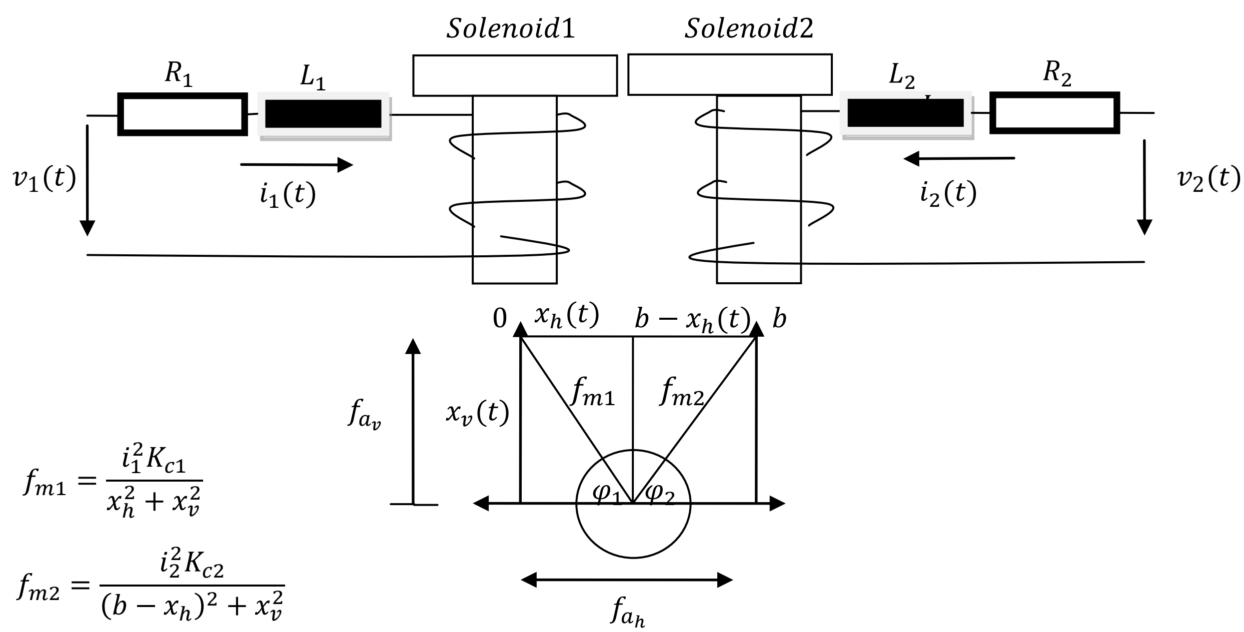
If $f_{M_{net},v} + mg = 0$, then our levitated object is stable at equilibrium point.

To obtain the $f_{M_{net},v}$, there are several possibilities:

1. Both solenoids are driven with different currents $(i_1(t), i_2(t))$ and different voltage sources $(v_1(t), v_2(t))$.

2. Both solenoids are driven with the same current $i(t) = i_1(t) = i_2(t)$ and the same voltage sources $v(t) = v_1(t) = v_2(t)$.

The disadvantage of the first possibility is that, the order and the complexity of the system will increase. In order to simplify our model, the second possibility is a good candidate for our model. We justify this approximation by the fact that the solenoids will be as close as we can make them, the object will be as far away as we can allow, and the change in the angels will be very small compared to the change in the current and voltage passing through the electromagnetic. This means that the ball will be levitate in the vertical components in homogenous magnetic field.



$$f_{m1} = \frac{i_1^2 K_{c1}}{x_h^2 + x_v^2}$$

$$f_{m2} = \frac{i_2^2 K_{c2}}{(b - x_h)^2 + x_v^2}$$

$$f_{M_{net},v} = f_{M1}\sin(\varphi_1(t)) + f_{M2}\sin(\varphi_2(t))$$

$$f_{M_{net},h} = f_{M1}\cos(\varphi_1(t)) + f_{M2}\cos(\varphi_2(t))$$

Figure 0-4: Detail analysis of the forces on the vertical and horizontal component

2.4.1.1 Nonlinear Dynamic Equation of the Vertical component

Unfortunately, the equations from the one-dimensional levitator do not readily apply to the vertical balancing that needs to happen in the two dimensional levitator. We have to substituting for $f_{M_{net},v}$ in the state equation from the one dimensional levitator from section (2.3.1).

Also, we are no longer dealing with one solenoids. Therefore, the inductances and the inductivity of the solenoids can be different, so we will have $(K_{c1,}K_{c2})$ and $(L_{1,}L_2)$.

We will try to develop our model by using the first possibility. (*Both solenoids are driven with different currents ($i_1(t), i_2(t)$) and different voltage sources ($v_1(t), v_2(t)$).*

To simplify the model the second possibility will be used.

Refer to equation (2.12)

$$f_{M_{net},v} = f_{M1}\sin(\varphi_1(t)) + f_{M2}\sin(\varphi_2(t))$$

Hence, from Figure (2-4) and using the first possibility

$$f_{m1} = \frac{i_1^2 K_{c1}}{x_h^2 + x_v^2} \qquad (2.21)$$

$$f_{m2} = \frac{i_2^2 K_{c2}}{(b - x_h)^2 + x_v^2} \qquad (2.22)$$

The net forces f_{a_v} in the vertical components

$$f_{a_v} = f_g - f_{M_{net},v} \qquad (2.23)$$

Substituting (2.21) and (2.22) in (2.23)

$$f_{a_v} = f_g - \left[\frac{i_1^2 K_{c1}}{x_h^2 + x_v^2}\sin(\varphi_1(t)) + \frac{i_2^2 K_{c2}}{(b - x_h)^2 + x_v^2}\sin(\varphi_2(t))\right] \qquad (2.24)$$

This term coupled vertical and horizontal axis and dependent on six variables.

1. Position of the horizontal placement x_h
2. Position of the vertical placement x_v
3. The angel $\varphi_1(t)$ as function of time
4. The angel $\varphi_2(t)$ as function of time
5. The current in the first solenoid $i_1(t)$
6. The current in the second solenoid $i_2(t)$

Now we will simplify the last term by using the second possibility.(*Both solenoids are driven with the same current*$i(t) = i_1(t) = i_2(t)$ *and the same voltage source* $v(t) = v_1(t) = v(t)$*).*When we assume that the placement of the solenoids are very close to each other such they produce a homogeneous magnetic field.

Then (2.24) becomes: $f_{a_v} = f_g - \left[\frac{i^2 K_{c1}}{x_h^2 + x_v^2} \sin(\varphi_1(t)) + \frac{i^2 K_{c2}}{(b - x_h)^2 + x_v^2} \sin(\varphi_2(t))\right]$ (2.25)

The ball should be levitated in the vertical component in homogenous magnetic fields, therefore $(\varphi_1, \varphi_2) = 90^{\circ}$ and excluded the horizontal component,

Then (2.25) becomes

$$f_{a_v} = f_g - \left[\frac{i^2 K_{c1}}{x_h^2 + x_v^2} \sin(90^{\circ}) + \frac{i^2 K_{c2}}{(b - x_h)^2 + x_v^2} \sin(90^{\circ})\right] \quad (2.26)$$

$$= f_g - \left[\frac{i^2 K_{c1}}{x_v^2} + \frac{i^2 K_{c2}}{x_v^2}\right] (2.27)$$

The relationships between Gravitational forcef_g and the Accelerations force f_{a_v} are obtained from figure (2-4).

Gravitational force

$f_g = mg$ (2.28)

Accelerations force

$f_{a_v} = m \frac{d^2 x_v}{dt^2}$ (2.29)

The simplified equation of motion for the object in the vertical components is given by substituting (2.28) and (2.29) in (2.27)

$m \frac{d^2 x_v}{dt^2} = mg - \left[\frac{i^2 K_{c1}}{x_v^2} + \frac{i^2 K_{c2}}{x_v^2}\right]$(2.30)

Then

$$\frac{d^2 x_v}{dt^2} = g - \frac{i^2 (K_{c1} + K_{c2})}{m x_v^2} = f_1(\ddot{x}_v, \dot{x}_v, x_v, i) \quad (2.31)$$

Where;

$\ddot{x}_v$= Ball acceleration

$\dot{x}_v$= Ball Velocity

x_v = Ball position

i = Drive current

Since the both solenoids should be operated with the same current the equation for the circuit show in figure (2.4)is given by

$$v(t) = R_1 i + R_2\, i + \; L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \quad (2.32)$$

$$= i(R_1 + R_2\,) + \; \frac{di}{dt}(L_1 + L_2)$$

$$\Rightarrow (L_1 + L_2)\frac{di}{dt} = -i(R_1 + R_2\,) + \; v(t)$$

$$\Rightarrow \frac{di}{dt} = -\frac{i(R_1 + R_2\,)}{(L_1 + L_2)} + \; \frac{1}{(L_1 + L_2)} v(t) = f_2(\,i, v) \qquad (2.33)$$

where v is the input voltage.

At equilibrium point. We have the following condition:

$$\ddot{x} = \dot{x} = 0, \frac{di}{dt} = 0, \quad (2.34)$$

Which implies that $f_1\left(\; \ddot{x}_{v_e}\,, \dot{x}_{v_e,} x_{v_e}\right) = 0, \quad f_2(i_e, v_e) = 0,$

Where;

$\ddot{x}_{v_e}$=Ball acceleration at equilibrium = 0

$\dot{x}_{v_e}$=Ball Velocity at equilibrium = 0

x_{v_e} =Equilibrium position

i_e =Equilibrium current

v_e= Equilibrium input voltage

At Equilibrium position the accelerations $\frac{d^2 x_v}{dt^2} = 0$ and $\frac{di}{dt} = 0$

Refer to equation (2.31)

$$0 = \; g - \frac{i^2(K_{c1} + K_{c2})}{m x_v^2}$$

$$\Rightarrow i_e^2 = \; \left(\frac{mg}{(K_{c1} + K_{c2})}\right) x_{v_e}^2$$

$$\Rightarrow \frac{i_e}{x_{v_e}} = \; \sqrt{\frac{mg}{(K_{c1} + K_{c2})}} \qquad (2.35)$$

$$\Rightarrow i_e = \left[\sqrt{\frac{mg}{(K_{c1}+K_{c2})}}\right] x_{v_e} \qquad (2.36)$$

We will need the last terms later in this section to calculate the equilibrium current at selected equilibrium point.

We may assume, without loss of generally, that $i_e > 0$, the equilibrium voltage obtain from equation (2.33)

$$0 = -i(R_1 + R_2) + v(t)$$

$$v_e = (R_1 + R_2)i_e = (R_1 + R_2)\left[\sqrt{\left(\frac{mg}{K_{c1}+K_{c2}}\right)}\right] x_{v_e} \qquad (2.37)$$

Equations (2.36) and(2.37) tell us that in order to keep the system at given equilibrium position, the equilibrium current $i_e$is directly proportional to x_{v_e}.

Now, we linearrized the nonlinear equation (2.26) at equilibrium point

$$\frac{d^2x_v}{dt^2} = a_1\left(x - x_{v_e}\right) + a_2\left(\dot{x} - \dot{x_{v_e}}\right) + a_3(i - i_e) + b_1(v - v_e)$$

We may do this if we let

$$a_1 = \frac{\partial f_1}{\partial x_v}|_{equil} = \frac{\partial}{\partial x}\left(g - \frac{i^2(K_{c1}+K_{c2})}{mx_v^2}\right)|_{equil} = \frac{2(K_{c1}+K_{c2})i_e^2}{mx_{v_e}^3} \qquad (2.38)$$

$$a_2 = \frac{\partial f_1}{\partial \dot{x_v}}|_{equil} = \frac{\partial}{\partial x_v}\left(g - \frac{i^2(K_{c1}+K_{c2})}{mx_v^2}\right)|_{equil} = 0 \qquad (2.39)$$

a_2 = Equilibrium velocity at equilibrium point=0

$$a_3 = \frac{\partial f_1}{\partial i}|_{equil} = \frac{\partial}{\partial i}\left(g - \frac{i^2(K_{c1}+K_{c2})}{mx_v^2}\right)|_{equil} = -\frac{2(K_{c1}+K_{c2})i_e}{mx_{v_e}^2} \qquad (2.40)$$

$$b_1 = \frac{\partial f_1}{\partial v}|_{equil} = \frac{\partial}{\partial v}\left(g - \frac{i^2(K_{c1}+K_{c2})}{mx_v^2}\right)|_{equil} = 0 \qquad (2.41)$$

We note that the equation for the electrical part of the system is already linear.

Define the variables

$$x_1 = x - x_e\ , x_2 = \dot{x} - \dot{x_e}\ , x_3 = i - i_e\ , u = v - v_e$$

Where:

x_1 = Ball position

x_2 =Ball velocity

x_3 =Drive current

u = Input voltage

This implies:

$$\dot{x_1} = \dot{x} = x_2,\ \dot{x}_3 = \frac{di}{dt}$$

Refer to equation (2.33)

$$\dot{x_3} = \frac{di_e}{dt} = -\frac{i(R_1+R_2)}{(L_1+L_2)} + \frac{1}{(L_1+L_2)} v(t)$$

and obtain the linearized state-space model

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_1 & 0 & a_3 \\ 0 & 0 & -\frac{R_1+R_2}{(L_1+L_2)} \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{1}{(L_1+L_2)} \end{bmatrix} u(t)$$

$$x_v = x_1 = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (2.42)$$

Hence, state space model of Linearized model around certain operating point (x_{v_e}) is written as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2(K_{c1}+K_{c2})i_e^2}{mx_{v_e}^3} & 0 & -\frac{2(K_{c1}+K_{c2})i_e}{mx_{v_e}^2} \\ 0 & 0 & -\frac{R_1+R_2}{(L_1+L_2)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{(L_1+L_2)} \end{bmatrix} u(t)$$

$$x_{v_e} = x_1 = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (2.43)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2(K_{c1}+K_{c2})i^2}{mx_{v_e}^3} & 0 & -\frac{2(K_{c1}+K_{c2})i_e}{mx_{v_e}^2} \\ 0 & 0 & -\frac{R_1+R_2}{(L_1+L_2)} \end{bmatrix},= \begin{bmatrix} 0 \\ 0 \\ \frac{1}{(L_1+L_2)} \end{bmatrix}, C = [1 \quad 0 \quad 0] D = [0]$$

(2.44)

We specify the design parameters [6]:

$$R_1 = 3.5\Omega\,, R_2 = 3.5\Omega\,, m = 0.0084kg\,, k_{c1} = 6.8.823 * 10^{-6}\frac{Nm^2}{A},$$

$$k_{c2} = 6.8.823 * 10^{-6}\frac{Nm^2}{A}, L_1 = 0.01H\,, L_2 = 0.01H\,, g = 9.81\frac{m}{sec^2}$$

To compute the equilibrium current i_e at this equilibrium point , $x_{v_e} = 0.01m$.

From equation (2.36)

$$i_e = \left[\sqrt{\frac{mg}{(K_{c1}+K_{c2})}}\right] x_{v_e}$$

$$i_e = \left[\sqrt{\frac{0.0084*9.81}{(6.8.823*10^{-6}+6.8.823*10^{-6})}}\right] * 0.01 = 0.24467655\ A$$

The corresponding equilibrium voltage, refer to equation (2.37)

$$v_e = (R_1 + R_2)i_e = (3.5 + 3.5) * 0.24467655 = 1.712736\ V$$

The resulting state space model of linearized magnetic ball levitation around ($x_{v_e} = 0.01m$) is:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1962 & 0 & -80.1875 \\ 0 & 0 & -350 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix} u(t)$$

$$x_{v_e} = x_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1962 & 0 & -80.1875 \\ 0 & 0 & -350 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 50 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$D = [0] \qquad (2.45)$$

Hence, from Eq.(2..45), transfer function expression $G_v(s)$ of the linearized model is given by using MATLAB:

$$G_v(s) = \frac{X_v(s)}{I_v(s)} = \frac{-4009.37}{s^3+350-1962s-686700} \qquad (2.46)$$

Calculating the open loop poles of the system computed with MATLAB, we get three poles

4. -350 (stable pole)
5. -44.2945 (stable pole)
6. 44.2945 (unstable pole)

The system is unstable because it has positive pole (44.42).

In order to determine the controllability matrix and find its rank, we use MATLAB and we get.

Controllability matrix *Co* =[B $|AB$ $|A^2B$]

$$Co = \begin{bmatrix} 0 & 0 & -0.0040\text{e}6 \\ 0 & -0.0040\text{e}6 & 1.4033\text{e}6 \\ 0.0001e6 & -0.0175\text{e}6 & 6.1250\text{e}6 \end{bmatrix}$$

The rank of the controllability matrix =3

The system is **controllable** because it has full rank (i.e. if rank(CO) = n where n is the number of states). Our system has three states.

2.4.2 Horizontal Component State Equation

In the last section (2.4.1), we analyzed the forces that act on the vertical axis and developed the mathematical model for the vertical component. As we can see in figure (2-4), the vertical and the horizontal components are geometrically coupled which make it harder to model. If $f_{M_{net},h} = 0$, then our levitated object is stable at equilibrium point in the horizontal component.

To develop our dynamic model for the horizontal components, we assume that the ball moves stably between the two solenoids, There are several possibilities to obtain $f_{M_{net},h}$.

1. Drive the both solenoids with different currents ($i_1(t), i_2(t)$) and different dependence voltage source ($v_1(t), v_2(t)$).

2. Drive the both solenoids with different currents ($i_1(t), i_2(t)$) and different independence voltage source ($v_1(t), v_2(t)$).

3. Both solenoids have the same or different inductance driving with the same current $i(t) = i_1(t) = i_2(t)$and different independence voltage source ($v_1(t), v_2(t)$).*(special case of the second possibility)*

4. Both solenoids are driven with different currents ($I_{constant}, i(t)$) and different independence voltage source ($v_1(t), v_2(t)$)..The ball should be put at start point under solenoid1 and move horizontal to solenoids2. Figure (2-4) shows the movement.

Now, we will discuss all possibilities

- ❖ The disadvantage of the first possibility is that the order and complexity of the system will be increase. The system is hard to model.

- ❖ As mentioned in the first possibility, order and complexity of the system will be increased however; the control theories and model building of vertical and horizontal allow the movement between two solenoids.

- The advantage of the third possibility is that the order and the complexity of the system will be minimum however, the disadvantage that the ball will be balanced or has an equilibrium point in horizontal components between the solenoids dependent on geometric coupled of the vertical and horizontal components and independent on the current $i(t)$.In the next section this theorem will be proved.

- The last possibility in our system could be have one equilibrium points, Our main objective that the object should be moved stable between two solenoids, therefore we have to linearized our horizontal model at selected equilibrium point.

In the next section, we will try to develop our model by using the second possibility. The third possibility will be discussed and proved as special case of the second possibility. According the disadvantages of special case of the second possibility, the last possibilities can simplify the model.

2.4.2.1 Nonlinear Dynamic Equation of the Horizontal component

Refer to figure (2-4), the total force$f_{M_{net},h}$in the horizontal direction is the sum of the vertical components of f_{m1} and f_{m2}.

$$f_{M_{net},h} = f_{M1}\cos(\varphi_1(t)) + f_{M2}\cos(\varphi_2(t))$$

The net acceleration f_{a_h}forces in the horizontal components

$$f_{a_h} = f_{M1}\cos(\varphi_1(t) - f_{M2}\cos(\varphi_2(t) \qquad (2.47)$$

Now, we will develop our model by using the second possibility in the last section"*Drive the both solenoids with different currents ($i_1(t), i_2(t)$) and different independence voltage source ($v_1(t), v_2(t)$)"* refer to figure (2.4)

$$f_{m1} = \frac{i_{1h}^2 K_{c1}}{x_h^2 + x_v^2} \qquad (2.48)$$

$$f_{m2} = \frac{i_{2h}^2 K_{c2}}{(b - x_h)^2 + x_v^2} \qquad (2.49)$$

K_{c1}= Coil constant of the first solenoids.

K_{c2}= Coil constant of the second solenoids.

Substituting (2.48) and (2.49) in (2.47)

$$f_{a_h} = \left[\frac{i_{1h}^2 K_{c1}}{x_h^2 + x_v^2}\cos(\varphi_1(t)) - \frac{i_{2h}^2 K_{c2}}{(b - x_h)^2 + x_v^2}\cos(\varphi_2(t))\right] \qquad (2.50)$$

Where

$$\cos(\varphi_1(t)) = \frac{x_h}{\sqrt{(x_h^2+x_v^2}} \quad (2.51)$$

$$\cos(\varphi_1(t)) = \frac{(b-x_h)}{\sqrt{(b-x_h)^2+x_v^2}} \qquad (2.52)$$

The accelerations force $f_{a_h} = \frac{d^2x_h}{dt^2}$ (2.53)

Substituting in (2.50)

$$m\frac{d^2x_h}{dt^2} = \left[\frac{i_{1h}^2 K_{c1}}{x_h^2+x_v^2}\cos(\varphi_1(t)) - \frac{i_{2h}^2 K_{c2}}{(b-x_h)^2+x_v^2}\cos(\varphi_2(t))\right] \qquad (2.54)$$

This term coupled vertical and horizontal axis and dependent on six variables.

1. Position of the horizontal placement x_h
2. Position of the vertical placement x_v
3. The angel $\varphi_1(t)$ as function of time
4. The angel $\varphi_2(t)$ as function of time
5. The current in the first solenoid $i_1(t)$
6. The current in the second solenoid $i_2(t)$

Before go toward and try to simplify the last term, we want to prove the disadvantage by using the third possibility "*Both solenoids have the same or different inductance driving with the same current $i(t) = i_1(t) = i_2(t)$and different independence voltage sources ($v_1(t), v_2(t)$)".*

Rewrite (2.54) according the third possibility

$$m\frac{d^2x_h}{dt^2} = \left[\frac{i_h^2 K_{c1}}{x_h^2+x_v^2}cos(\varphi_1(t)) - \frac{i_h^2 K_{c2}}{(b-x_h)^2+x_v^2}\cos(\varphi_2(t))\right] \qquad (2.55)$$

Assume that the ball has an equilibrium point in the horizontal components. This can be when the total forces which exerts on the ball$f_{M_{net},h} = 0$.

Substituting in (2.55)

$$0 = \left[\frac{i_h^2 K_{c1}}{x_h^2+x_v^2}cos(\varphi_1(t)) - \frac{i_h^2 K_{c2}}{(b-x_h)^2+x_v^2}\cos(\varphi_2(t))\right] \qquad (2.56)$$

$$\Rightarrow \frac{i_h^2 K_{c1}}{x_h^2+x_v^2}cos(\varphi_1(t)) = \frac{i_h^2 K_{c2}}{(b-x_h)^2+x_v^2}\cos(\varphi_2(t)) \qquad (2.57)$$

$\Rightarrow \frac{K_{c1}}{x_h^2+x_v^2} cos(\varphi_1(t)) = \frac{K_{c2}}{(b-x_h)^2+x_v^2} \cos(\varphi_2(t))$ (2.58)

As we seen in last term the equilibrium point is only dependent on the inductance of solenoids and the geometries of the system and not more dependent on the current. But the current will be needed to control the motion of the ball in the horizontal component; therefore, we will use the last possibility. This demonstrates and proves the useless and the disadvantage by using the third possibility.

In order to simplify the term (2.54), we will get the last possibility in consideration.*(Both solenoids are driven with different currents (*$I_{constant}, i(t)$*) and different independence voltage sources(*$v_1(t), v_2(t)$*).*

From equation (2.54), we have

$$m\frac{d^2x_h}{dt^2} = \left[\frac{i_1^2K_{c1}}{x_h^2+x_v^2}\cos(\varphi_1(t) - \frac{i_2^2K_{c2}}{(b-x_h)^2+x_v^2}\cos(\varphi_2(t)\right]$$

Rewrite equation (2.54) according the last possibility and drive the solinoid1 with constant current I_{h_c}

$$m\frac{d^2x_h}{dt^2} = \left[\frac{I_h^2K_{c1}}{x_h^2+x_v^2}\cos(\varphi_1(t) - \frac{i^2K_{c2}}{(b-x_h)^2+x_v^2}\cos(\varphi_2(t)\right] \qquad (2.59)$$

We simplify the last equation if we assume $\cos(\varphi_1(t) \approx 0°) \approx 1$ *and* $\cos(\varphi_2(t) \approx 0° \approx 1$. We justify this approximation by the fact that the solenoids will be as close as we can make them and φ_1 and φ_2 do not change appreciably during the course of the horizontal transits.

We modify the equation (2.59)

$$m\frac{d^2x_h}{dt^2} = \left(\frac{I_{h_c}^2\,K_{c1}}{x_h^2} - \frac{i^2K_{c2}}{(b-x_h)^2}\right) \Rightarrow \quad \frac{d^2x_h}{dt^2} = \frac{1}{m}\left(\frac{I_{h_c}^2\,K_{c1}}{x_h^2} - \frac{i^2K_{c2}}{(b-x_h)^2}\right)$$

$$\Rightarrow \frac{d^2x_h}{dt^2} = \left(\frac{I_{h_c}^2\,K_{c1}}{mx_h^2} - \frac{i^2K_{c2}}{m(b-x_h)^2}\right) \quad = f_1(\ddot{x}_h, \dot{x}_{h,} x_h, i) \qquad (2.60)$$

Where

$\ddot{x}_h$=Ball acceleration

$\dot{x}_h$=Ball Velocity

x_h = Ball position

i =Drive current

The both solenoids should be operated with different currents, however the current I_{h_c} of solenoid-1 is constant, therefore we get only the current of solenoid-2 in consideration. The equation for the circuit shown in figure (2.5)is given by

$$v(t) = R_2\, i + L_2 \frac{di}{dt} \Rightarrow \quad L_2 \frac{di}{dt} = -R_2\, i + \quad v(t) \qquad (2.61)$$

$$\Rightarrow \quad \frac{di}{dt} = -\frac{R_2}{L_2} i + \frac{v(t)}{L_2} = f_2(\,i, v)$$

Where v is the input voltage.

At equilibrium point, we have the following condition:

$$\ddot{x} = \dot{x} = 0, \frac{di}{dt} = 0. \qquad (2.62)$$

This implies:

$\dot{x_{h_e}}$=Ball Velocity at equilibrium

x_{h_e} = Equilibrium position

i_e=Equilibrium current = constant

v_e= Equilibrium input volt =constant

At Equilibrium position the accelerations $\frac{d^2 x_h}{dt^2}$ =0

$$0 = \left(\frac{I_{h_c}^2 K_{c1}}{m x_h^2} - \frac{i^2 K_{c2}}{m(b-x_h)^2}\right) \quad \Rightarrow \quad \frac{i_e^2 K_{c2}}{\left(b-x_{h_e}\right)^2} = \frac{I_{h_c}^2\, K_{c1}}{x_{h_e}^2} \qquad (2.63)$$

In the special case $K_{c1} = K_{c2}$ and $I_{h_c} = i$

Substituting (2.63)

$$x_h^2 = (b - x_h)^2 \ \Rightarrow x_h = b - x_h \Rightarrow \ x_h = \frac{1}{2} b \qquad (2.64)$$

That means, when the both solenoids have the same inductance and driven with same current, the ball will balance in the middle between the two solenoids.

From equation (2.63) the equilibrium current i_e

$$i_e^2 = \frac{I_{h_c}^2\, K_{c1}\left(b-x_{h_e}\right)^2}{x_{h_e}^2 K_{c2}} \Rightarrow \quad i_e = \frac{(b-x_{h_e}) I_{h_c}}{x_{h_e}} \sqrt{\frac{K_{c1}}{K_{c2}}} \qquad (2.65)$$

We will need this equation later in this section to calculate the equilibrium current at selected equilibrium point.

The equilibrium voltage obtain from equation (2.61)

$$v_e = R_2 i_e = R_2 \frac{(b - x_{h_e}) I_{h_c}}{x_{h_e}} \sqrt{\frac{K_{c1}}{K_{c2}}} \qquad (2.66)$$

2.4.2.2 Linearization

Now, we linearrized the nonlinear equation (2.60) at equilibrium point

$$\frac{d^2 x_h}{dt^2} = a_1\left(x - x_{h_e}\right) + a_2\left(\dot{x} - \dot{x_{h_e}}\right) + a_3\left(i - i_{h_e}\right) + b_1\left(v - v_{h_e}\right)$$

Refer to equation (2.60)

$$\frac{d^2 x_h}{dt^2} = \left(\frac{I_{h_c}^2 K_{c1}}{m x_h^2} - \frac{i^2 K_{c2}}{m(b - x_h)^2}\right)$$

We may do this if we let

$$a_1 = \frac{\partial f_1}{\partial x_h}\Big|_{equil} = \frac{\partial}{\partial x}\left(\frac{I_{h_c}^2 K_{c1}}{m x_h^2} - \frac{i^2 K_{c2}}{m(b - x_h)^2}\right)\Big|_{equil} = -\frac{2 I_{h_c}^2 K_{c1}}{m x_{h_e}^3} + \frac{2 i_e^2 K_{c2}}{m\left(b - x_{h_e}\right)^3}$$

(2.67)

$$a_2 = \frac{\partial f_1}{\partial \dot{x_h}}\Big|_{equil} = \frac{\partial}{\partial x_v}\left(\frac{I_{h_c}^2 K_{c1}}{m x_h^2} - \frac{i^2 K_{c2}}{m(b - x_h)^2}\right)\Big|_{equil} = 0 \qquad (2.68)$$

a_2 = Equilibrium velocity at equilibrium point=0

$$a_3 = \frac{\partial f_1}{\partial i_h}\Big|_{equil} = \frac{\partial}{\partial i_h}\left(\frac{I_{h_c}^2 K_{c1}}{m x_h^2} - \frac{i_h^2 K_{c2}}{m(b - x_h)^2}\right)\Big|_{equil} = -\frac{2 i_e K_{c2}}{m\left(b - x_{h_e}\right)^2} \qquad (2.69)$$

$$b_1 = \frac{\partial f_1}{\partial v_h}\Big|_{equil} = \frac{\partial}{\partial v}\left(\frac{I_{h_c}^2 K_{c1}}{m x_h^2} - \frac{i_h^2 K_{c2}}{m(b - x_h)^2}\right)\Big|_{equil} = 0 \qquad (2.70)$$

We note that the equation for the electrical part of the system is already linear.

Define the variables

$$x_1 = x - x_e \,, x_2 = \dot{x} - \dot{x_e} \,, x_3 = i - i_e \,, u = v - v_e$$

Where:

x_1 = Ball position

x_2=Ball velocity

x_3=Drive current

u = Input voltage

This implies:

$$\dot{x_1} = \dot{x} = x_2,\ \dot{x_3} = \frac{di}{dt}$$

Refer to equation (2.6)

$$\dot{x_3} = \frac{di_e}{dt} = -\frac{R_2}{L_2}i + \frac{v(t)}{L_2}$$

Thus, state-space model is:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ a_1 & 0 & a_3\\ 0 & 0 & -\frac{R_2}{L_2}\end{bmatrix}.\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\begin{bmatrix}0\\ 0\\ \frac{1}{L_2}\end{bmatrix}u(t)$$

$$x_v = x_1 = [1 \quad 0 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \qquad (2.71)$$

Hence, state space model of Linearized model around certain operating point (x_{v_e}) is written as:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -\frac{2I_{h_c}^2\,K_{c1}}{mx_{h_e}^3} + \frac{2i_e^2K_{c2}}{m\left(b-x_{h_e}\right)^3} & 0 & -\frac{2i_eK_{c2}}{m\left(b-x_{h_e}\right)^2}\\ 0 & 0 & -\frac{R_2}{L_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{L_2}\end{bmatrix}u(t)$$

$$x_{v_e} = x_1 = [1 \quad 0 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \qquad (2.72)$$

$$A = \begin{bmatrix}0 & 1 & 0\\ -\frac{2I_{h_c}^2\,K_{c1}}{mx_{h_e}^3} + \frac{2i_e^2K_{c2}}{m\left(b-x_{h_e}\right)^3} & 0 & -\frac{2i_eK_{c2}}{m\left(b-x_{h_e}\right)^2}\\ 0 & 0 & -\frac{R_2}{L_2}\end{bmatrix}\ , B = \begin{bmatrix}0\\ 0\\ \frac{1}{L_2}\end{bmatrix}, C = [1 \quad 0 \quad 0]$$

$$D = [0] \qquad (2.73)$$

We specify the design parameters [6]:

$$R_1 = 3.5\Omega\,, R_2 = 3.5\Omega\,, m = 0.0084kg\,, k_{c1} = 68.823 * 10^{-6}\frac{Nm^2}{A},$$

$$k_{c2} = 68.823 * 10^{-6} \frac{Nm^2}{A}, L_1 = 0.01H, L_2 = 0.01H, g = 9.81 \frac{m}{sec^2}$$

The first solenoid should be driven with constant current; therefore we get the calculated equilibrium current of the vertical component in the last section as constant current, $I_{h_c} = 0.24467655A$.

Hence, the equilibrium current i_e at this equilibrium point, $x_{h_e} = 0.01m \ and \ b = 0.04m$

from equation (2.65)

$$i_e = \frac{(b - x_{h_e}) I_{h_c}}{x_{h_e}} \sqrt{\frac{K_{c1}}{K_{c2}}}$$

$$i_e = \frac{(0.04 - 0.01) * 0.24467655}{0.01} \sqrt{\frac{68.823 * 10^{-6}}{68.823 * 10^{-6}}} = 0.734029651A$$

The corresponding equilibrium voltage, refer to equation (2.66)

$$v_e = R_2 * i_e = 3.5 * 0.734029651 = 2.56910378\,V$$

The resulting state space model of linearized magnetic ball levitation around ($x_{h_e} = 0.01m$) is:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 654 & 0 & -13.3646 \\ 0 & 0 & -350 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} u(t)$$

$$x_{v_e} = x_1 = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 654 & 0 & -13.3646 \\ 0 & 0 & -350 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}, C = [1 \quad 0 \quad 0]$$

$$D = [0] \tag{2.74}$$

Hence, from Eq.(2.45), transfer function expression $G_h(s)$ of the linearized model is given by using MATLAB:

Thus, $$G_h(s) = \frac{X_h(s)}{I_h(s)} = \frac{-1336.46}{s^3 + 350s^2 - 654s - 228900} \tag{2.75}$$

Calculating the open loop poles of the system computed with MATLAB, we get three poles

1. -350 (stable pole)
2. -25.5734(stable pole)

3. 25.5734 (unstable pole)

The system is unstable because it has positive pole (36).

In order to determine the controllability matrix and find its rank, we use MATLAB and we get.

Controllability matrix $Co = [\,B \;\; |AB \;\; |A^2B]$

$$Co = \begin{bmatrix} 0 & 0 & -0.0001e7 \\ 0 & -0.0001e7 & 0.0468e7 \\ 0 & -0.0035e7 & 1.2250e7 \end{bmatrix}$$

The rank of the controllability matrix =3

The system is **controllable** because it has full rank (i.e. if rank (CO) = n where n is the number of states). Our system has three states.

Chapter 3: Qualitative Behaviour and Lyapunov Stability

3.1.Introduction

Stability theory plays a central role in system theory and engineering. The two dimensional levitation systems is open-loop unstable and there is a nonlinear relationship between force, current, and the distance between the poles of the solenoids and the object. Equilibrium is reached when the magnetic force balances the gravitational force. The instability arises because a slight deviation from this equilibrium drives the ball further from the equilibrium point. As shown in chapter 2 we traditionally solve this nonlinear controls problem by linearzing about an equilibrium g point.

This chapter is concerned mainly with stability of the equilibrium point in the sense of lyapunov. Before we discuss the lyapunov stability for the two dimensional levitation system, we will look at the qualitative behavior of the system at an equilibrium point, the eigenvalue and type of the equilibrium point.

3.1.1 Basic stability theorem of Lyapunov

Lyapunov theory is used to make conclusions about trajectories of a system without finding the trajectories, that means without solving the differential equation [7].

A positive definite function $V: R^n \to R$ is positive *definite* if

- $V(x,t) \geq 0$ for all $x(t)$
- $V(0,t) = 0$ if and only if $x = 0$
- $V(x,t) \to \infty \; as \; t \; \to \; \infty$

A continuous function $V: R^n \to R$ is *decrescent* if for some $\varepsilon > 0$ and some continues, strictly increasing function $\beta: R^n \to R$

$$V(x,t) \leq \beta\|x\|$$

The function $V(x,t)$ is defined as generalized *energy function*. The time derivate $\dot{V}(x,t)$ of $V(x,t)$ is taken along the trajectories of the system and associated generalized dissipation function.

$$\dot{V}(x,t) = \frac{\partial V}{\partial x_1} f_1 + \frac{\partial V}{\partial x_2} f_2 + \frac{\partial V}{\partial x_3} f_3$$

Let $V(x,t)$ be a *positive definite* function with derivative $\dot{V}$ along the trajectories of the system.

1. If $V(x,t)$ is locally positive definite and $\dot{V}(x,t) \leq 0$ locally in x and for all t, then the origin of the system is locally stable (in the sense of Lyapunov).

2. If $V(x,t)$ is locally positive definite and decrescent, and $-\dot{V}(x,t) \leq 0$ locally in x and for all t, then the origin of the system is uniformly locally stable (in the sense of Lyapunov).

3. If $V(x,t)$ is locally positive definite and decrescent, and $-\dot{V}(x,t) \leq 0$ is locally positive definite, then the origin of the system is uniformly locally stable (in the sense of Lyapunov).

4. If $V(x,t)$ is positive definite and decrescent, and $-\dot{V}(x,t) \leq 0$ is positive definite, then the origin of the system is globally uniformly asymptotically stable (in the sense of Lyapunov).

The conditions of Lyapunov's theorem are only sufficient. Failure of a Lyapunov function candidate to satisfy the conditions for stability or asymptotic stability does not mean that the equilibrium point is not stable or asymptotically stable. It only means that such stability property cannot be established by using this Lyapunov function candidate.

3.1.2 Vertical Components Qualitative Behaviour

To classify the equilibrium point, first calculate the eigenvalues of the matrix the of the linearized *A-Matrix* of the vertical component, refer to equation (2.43), the linearized state space dynamic model of vertical components at equilibrium point x_{v_e}=0.01m is:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1962 & 0 & -80.1875 \\ 0 & 0 & -350 \end{bmatrix}$$

Compute the eigenvalues of A-Matrix with MATLAB

$\lambda_1 = 44.29, \lambda_2 = -44.29, \lambda_3 = -350$

That implies the system is unstable, because the eigenvalues are of opposite sign, the equilibrium point is a saddle; trajectories approach asymptotically the eigenvector associated with the positive eigenvalue [7].

3.2 Horizontal Components Qualitative Behaviour

We do the same as the last section. Classify the equilibrium point, first calculate the eigenvalues of the matrix the of the linearized *A-Matrix* of the horizontal component, refer to equation (2.43), the linearized state space dynamic model of vertical components at equilibrium point x_{v_e}=0.04m is:

Refer to (2.610), the operating point equation for horizontal component

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 654 & 0 & -13.3646 \\ 0 & 0 & -350 \end{bmatrix}$$

Compute the eigenvalues of A-Matrix with MATLAB

$\lambda_1 = 25.57, \lambda_2 = -25.57, \lambda_3 = -350.$

The equilibrium point is a saddle point, so the system is unstable *see last section*.

3.3 Lypanuov stability for the vertical components

Refer to (2.43) equation, the linearized state space dynamic model of vertical components is:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2(K_{c1}+K_{c2})i_e^2}{mx_{v_e}^3} & 0 & -\frac{2(K_{c1}+K_{c2})i_e}{mx_{v_e}^2} \\ 0 & 0 & -\frac{R_1+R_2}{(L_1+L_2)} \end{bmatrix}$$

A possible definite generalized energy Lypanuov function candidate is:

$V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2$, where $V(x)$ is a positive definite function.

This implies a according the basic stability theorem of Lyapunov section (3.1.3)

$$\dot{V} = \frac{\partial V}{\partial x_1} f_1 + \frac{\partial V}{\partial x_2} f_2 + \frac{\partial V}{\partial x_3} f_3$$

$$\dot{V} = x_1 x_2 + x_2 \left(\frac{2(K_{c1}+K_{c2})i^2}{mx_{v_e}^3} - \frac{2(K_{c1}+K_{c2})i_e}{mx_{v_e}^2} \right) + x_3 \left(-\frac{R_1+R_2}{(L_1+L_2)} \right) \quad (3.2)$$

However, at the equilibrium point the acceleration $x_2 = 0$, substituting into (3.1)

$$\dot{V} = 0 + 0 + x_3 \left(-\frac{R_1+R_2}{(L_1+L_2)} \right)$$

$$= -\frac{R_1+R_2}{(L_1+L_2)} x_3 \le 0$$

As can be seen from above equation the $\dot{V} \le 0$ and $\dot{V}$ is not decresent function. Refer to first basic stability theorem of Lyapunov section (3.1.3), we conclude that the above system is at the equilibrium point stable in the sense of Lyapunov.

3.2 Lypanuov stability for the Horizontal components

Refer to (2.72) equation, the linearized state space dynamic model of horizontal components is:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2I_{h_c}^2\ K_{c1}}{mx_{h_e}^3} - \frac{2i_e^2 K_{c2}}{m(b-x_{h_e})^3} & 0 & -\frac{2i_e K_{c2}}{m(b-x_{h_e})^2} \\ 0 & 0 & -\frac{R_2}{L_2} \end{bmatrix}$$

A possible Lypanuov candidate is $V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2,$ where $V(x)$ is a positive definite function

$$\dot{V} = \frac{\partial V}{\partial x_1} f_1 + \frac{\partial V}{\partial x_2} f_2 + \frac{\partial V}{\partial x_3} f_3$$

This implies a according the basic stability theorem of Lyapunov section (3.1.3)

$$\dot{V} = x_1 x_2 + \ x_2 \left(\frac{2I_{h_c}^2\ K_{c1}}{mx_h^3} - \frac{2i_h^2 K_{c2}}{mx_h^3} \right) + x_3 \left(-\frac{R_2}{L_2} \right) \quad (3.2)$$

Therefore, at the equilibrium point the acceleration$x_2 = 0$, substituting into (3.2

$$\dot{V} = 0 + \ 0 + x_3 \left(-\frac{R_2}{L_2} \right) = -\frac{R_2}{L_2} x_3 \leq 0$$

As can be seen by above equation the $\dot{V} \leq 0$ and $\dot{V}$ is not decresent function. By applying the first basic stability theorem of Lyapunov section (3.1.3), we conclude the above system is at the equilibrium point stable in the sense of Lyapunov.

Chapter 4: Controller Design

4.1 Introduction

In this chapter, we develop the control strategy for levitating the ball purely with magnetic fields, and allow movement between two solenoids around an two dimensional space. We will find ways to compensate the systems, and develop feedback diagrams to represent, the closed loop systems. We begin with the theory of the control strategy, and then follow on with the 2-dimensional compensation.

4.2 Control Strategy of two dimensional levitation system

Each of the solenoids is run at different current, although they may have different inductances because of imperfect matching, our algorithm for moving the object from $X_H = 0$ to $X_H = X_s$ has two parts:

1. Stabilize the sum of the vertical components of $f_{M1}$and f_{M2} against the force of gravity *mg* and drive the both solids equally, We justify this approximation by the fact that the solenoids will be as close as we can make them, the object will be as far away as we can allow from the solenoids.

2. Stabilize the horizontal components of $f_{M1}$and f_{M2} against the each other and drive the both solenoids differently.
3. Switch between the vertical component and the horizontal components. if the object should be have movement between the solenoids

The two dimensional levitator could be built with any number of solenoids within reason. To design an arbitrarily sized system, it suffices to show how to create stable levitation and movement between two solenoids. This is because we can use a circuit to switch between pairs of solenoids: Using one pair to transfer the object from point A to B and another to transfer it from point B to C.

4.3 Control problem description

As shown in chapter 2 and 3, open loop nonlinear system is unstable and its specification is not adequate. Thus, stability and performance criteria will be solved. Then, an equivalent, linear expression of the resulting system will be driven and used at the next chapter.

Hence, the requirements of the controller is to be able to position the ball at any arbitrary location stable in the vertical components with magnetic fields and allows movement smoothly in the horizontal components between two solenoids around a two-dimensional space.

In order to make the system stable and according the control strategy of two dimensional levitation system (section 4.2), two dimensional controllers are needed, one to control the most important component the fastest, namely the vertical component and the other slowly change the object so that the horizontal position changes.

From equation (2.46), $G_v(s) = \frac{X_v(s)}{I_v(s)} = \frac{-4009.37}{s^3+350-1962s-686700}$ and

From equation (2.75), $G_h(s) = \frac{X_h(s)}{I_h(s)} = \frac{-1336}{s^3+350s^2-654s-228900}$

Figure (4-1) shows the step response of the vertical component without applying any controller, therefore when no controller applied to the system the ball will fall down or attract to the solenoid.

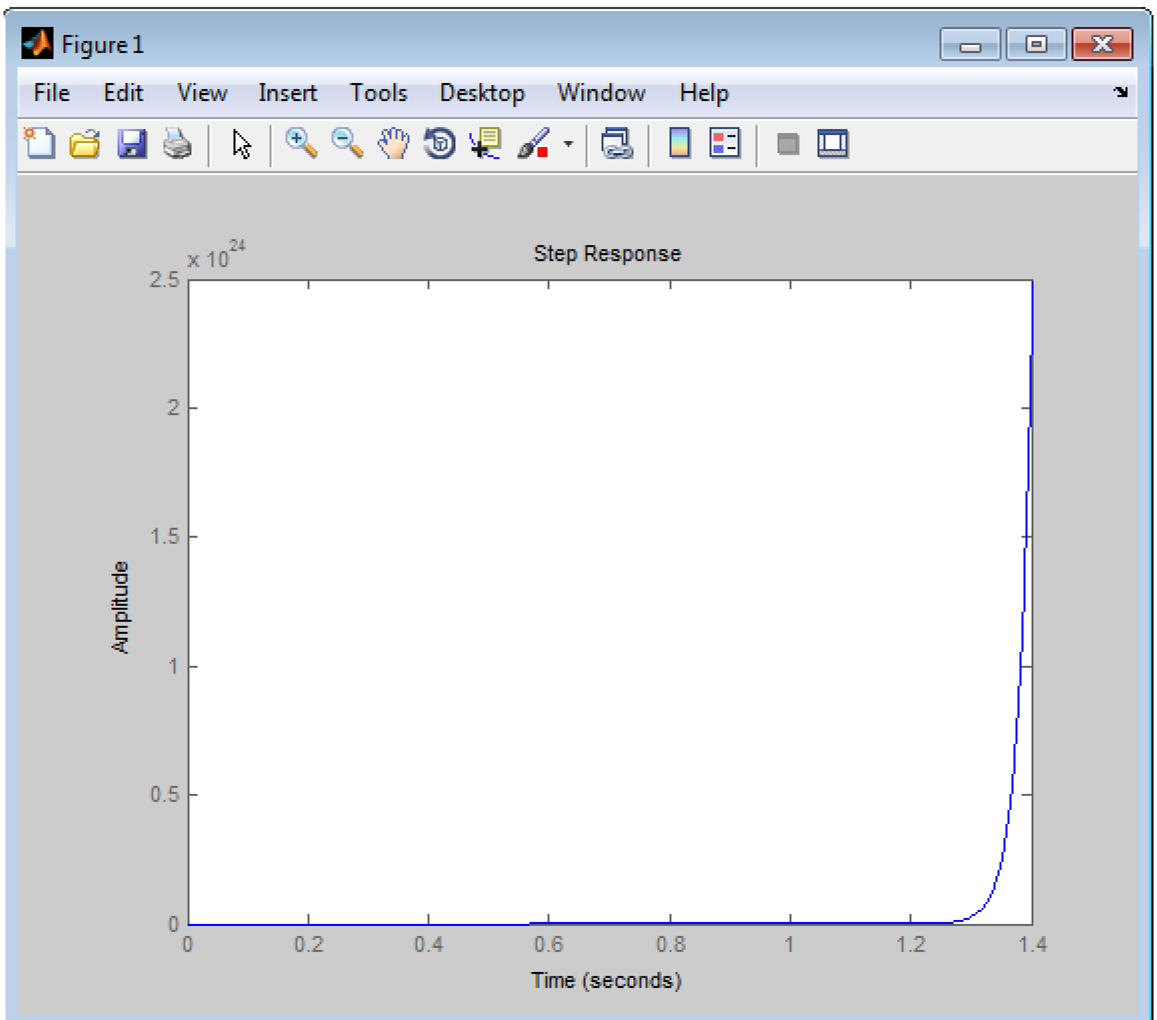


Figure 4-1: The step response of the vertical component without applying any controller

Figure shows (4-2) the step response of the horizontal component without applying any controller, therefore the ball will attract from one of the two solenoids.

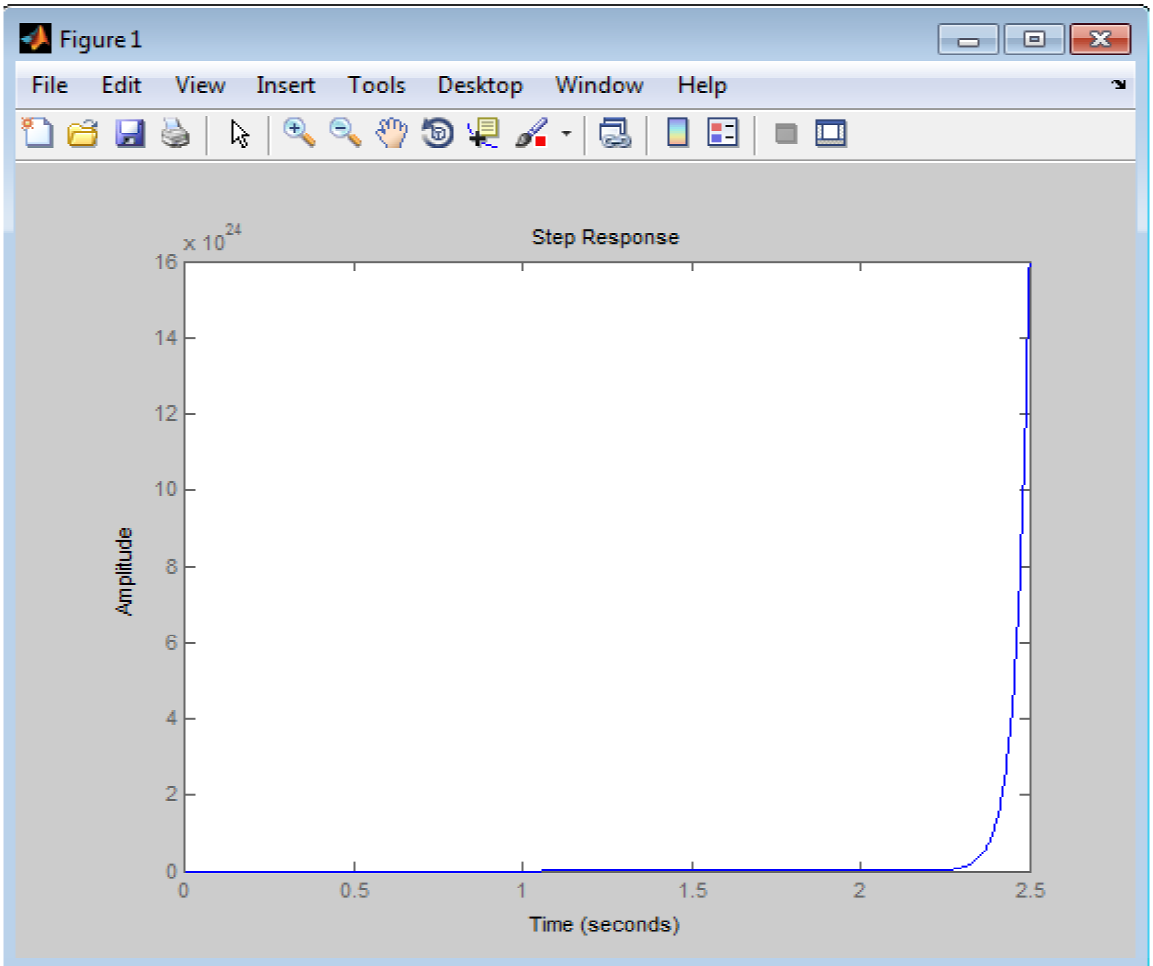


Figure 4-2: The step response of the horizontal component without applying any controller

A root locus plot using MATLAB is shown in figures (4-3) and (4-4) for the uncompensated transfer functions $G_v(s)$ of vertical component and of the uncompensated transfer function $G_h(s)$ of vertical component.

As can seen from figures (4-3) and (4-4), the vertical component have the positive pole (44.2945) and the horizontal components the positive pole (25) in the right half plan and no value of system gain can nullify the effect of this poles to stabilize the system.

Therefore, insertion of a phase lead compensator is a must to pull the root locus into the left half plan. The pole of the lead compensator should be introduced as such that it is in deeper location than the deepest left hand pole of the system. If the gain of the lead compensators large, all the poles of the system are in the left half plane and the system is stable. A phase lead controller is chosen because it is simplest method to achieve stability of a magnetic levitation system. A root locus method is followed for the fact that it offers the clear advantage of giving the designer the ability of choosing the pole and zero location thus impose control over the transit response. However the phase lead controller is not the optimal controller to achieve any transit response because its control gain is limited and have only one pole and one zero at the real axis.

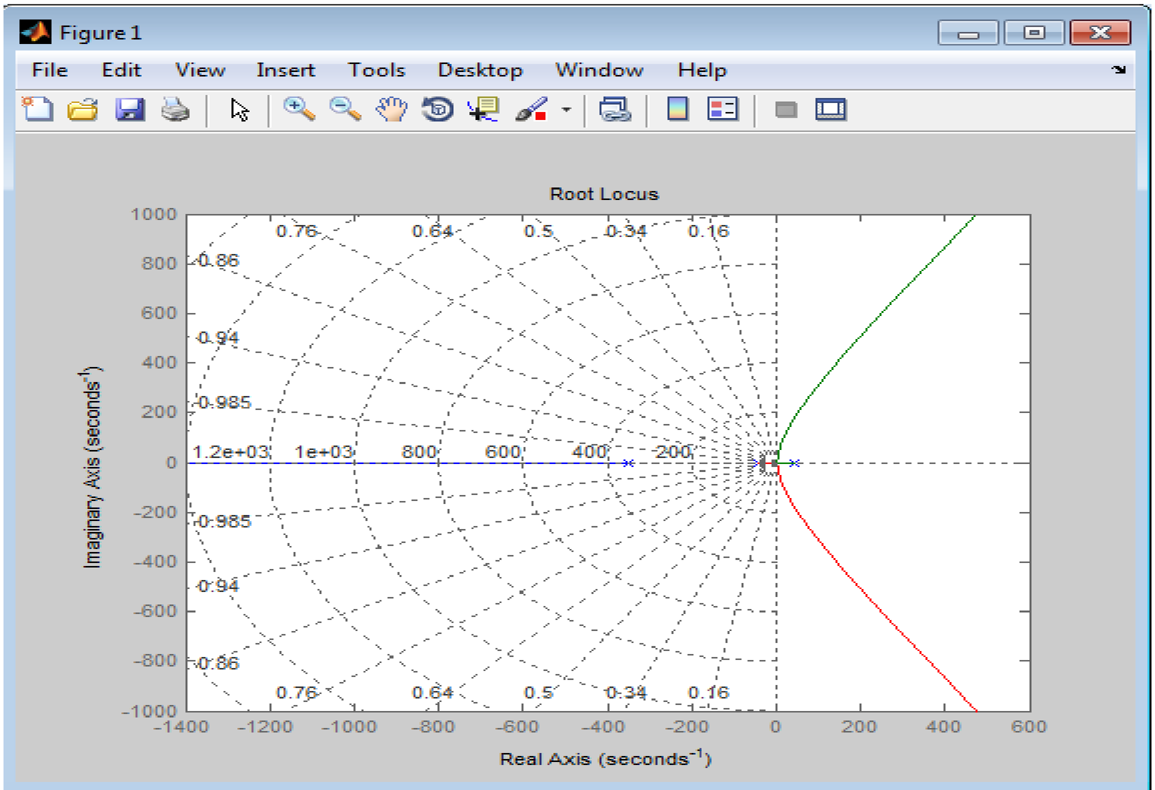


Figure 4-3: Root locus plot for uncompensated vertical component

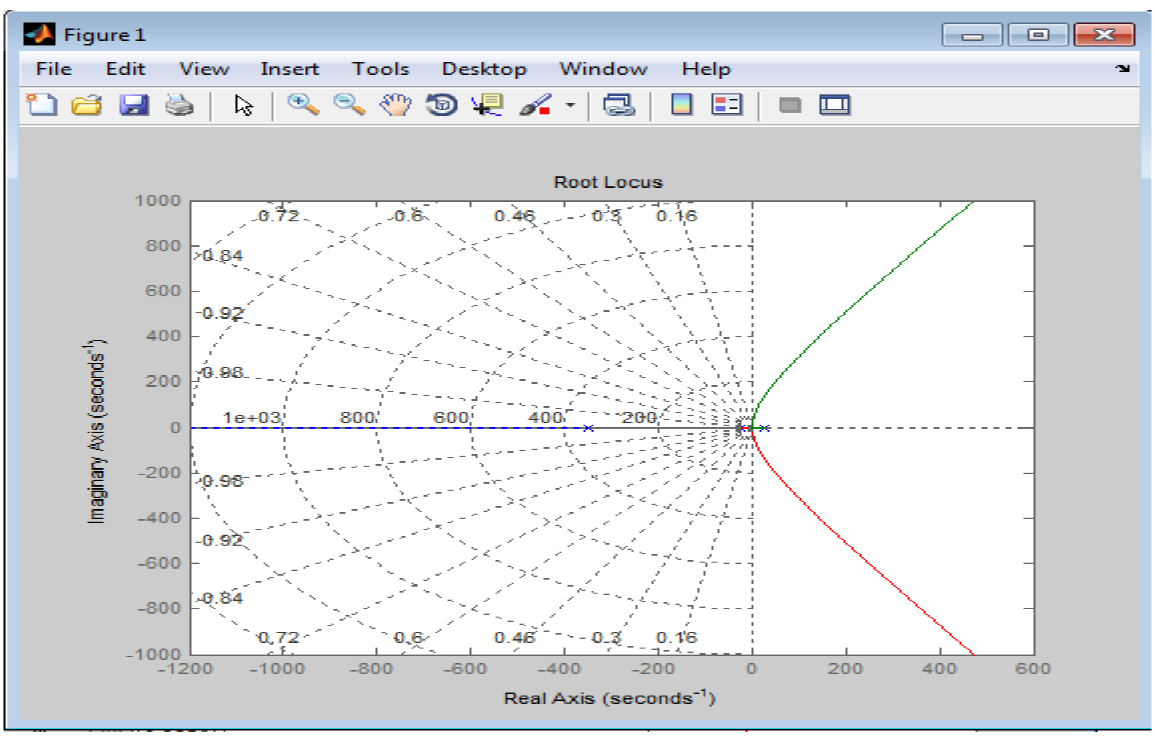


Figure 4-4: Root locus plot for uncompensated horizontal component.

The transfer function for a lead-compensator is $C_c = k\frac{s+z}{s+p}$, where k, z, and p are the compensator gain, zero, and pole respectively. To make the compensator work correctly, $z < p$ must be satisfied and the zero and the pole should be located on the real axis. In the next section two lead compensators will be designed, one to stabilize the vertical component and the other to stabilize the horizontal component according a specified transfer function [3].

4.3.1 Design Phase Lead Compensator for the vertical component

In order the controller to be able to position the ball at an equilibrium point in the homogenous magnetic field of the vertical component and move the ball smoothly to desired position and hold it at equilibrium point horizontally, we define the following specification [6]:

- Overshoot, OS= 5% (4.1)
- Settling Time,T_s=0.08 sec (4.2)

The second order prototype is $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ (4.3)

To obtain the desired pole, we need to translate the system specification to ζ and ω_n such as:

$$\zeta = \sqrt{\frac{\ln(OS)^2}{\ln(OS)^2+\pi^2}} = \sqrt{\frac{\ln(0.05)^2}{\ln(0.05)^2+\pi^2}} = 0.6901 \qquad (4.4)$$

$$T_s \approx \frac{4}{\omega_n\zeta} \quad , \; T_s \text{ is } \pm 2\% \qquad (4.5)$$

$$\omega_n = \frac{4}{\zeta T_s} = \frac{4}{0.6901*0.08} = 72.453 \qquad (4.6)$$

$$\omega_{d=}\omega_n\sqrt{1-\zeta^2} = 72.453 * \sqrt{1-(0.6901)^2} = 52.435 \qquad (4.7)$$

The desired poles are:

$$\mathbf{s_1} = -\zeta\,\omega_n + j\omega_d = -0.6901 * 72.453 + \mathrm{j} * 52.435$$
$$= -50 + 52.435\mathrm{j}$$

$$\mathbf{s_2} = -\zeta\,\omega_n - j\omega_d = -0.6901 * 72.453 - \mathrm{j} * 52.435$$
$$= -50 - 52.435\mathrm{j}$$

(4.8)

Substituting (4.4) and (4.5) into (4.3)

$$= \frac{(72.45)^2}{s^2+2*0.6901*72.453*\mathrm{s}+(72.45)^2} = \frac{5249.476}{s^2+100*\mathrm{s}+5249.476}$$

With characteristic equation $s^2 + 100 * \mathrm{s} + 5249.476$ (4.9) and the poles $[(-50\ +52.435j), (-50-52.435j\ )]$

A step response plot using MATLAB is shown in figures (4-5) for the estimated compensated transfer functions $G_v(s)$ of vertical component and of the compensated transfer function $G_h(s)$ of vertical component. Figure (4-6) show the schematic diagram of compensated vertical component.

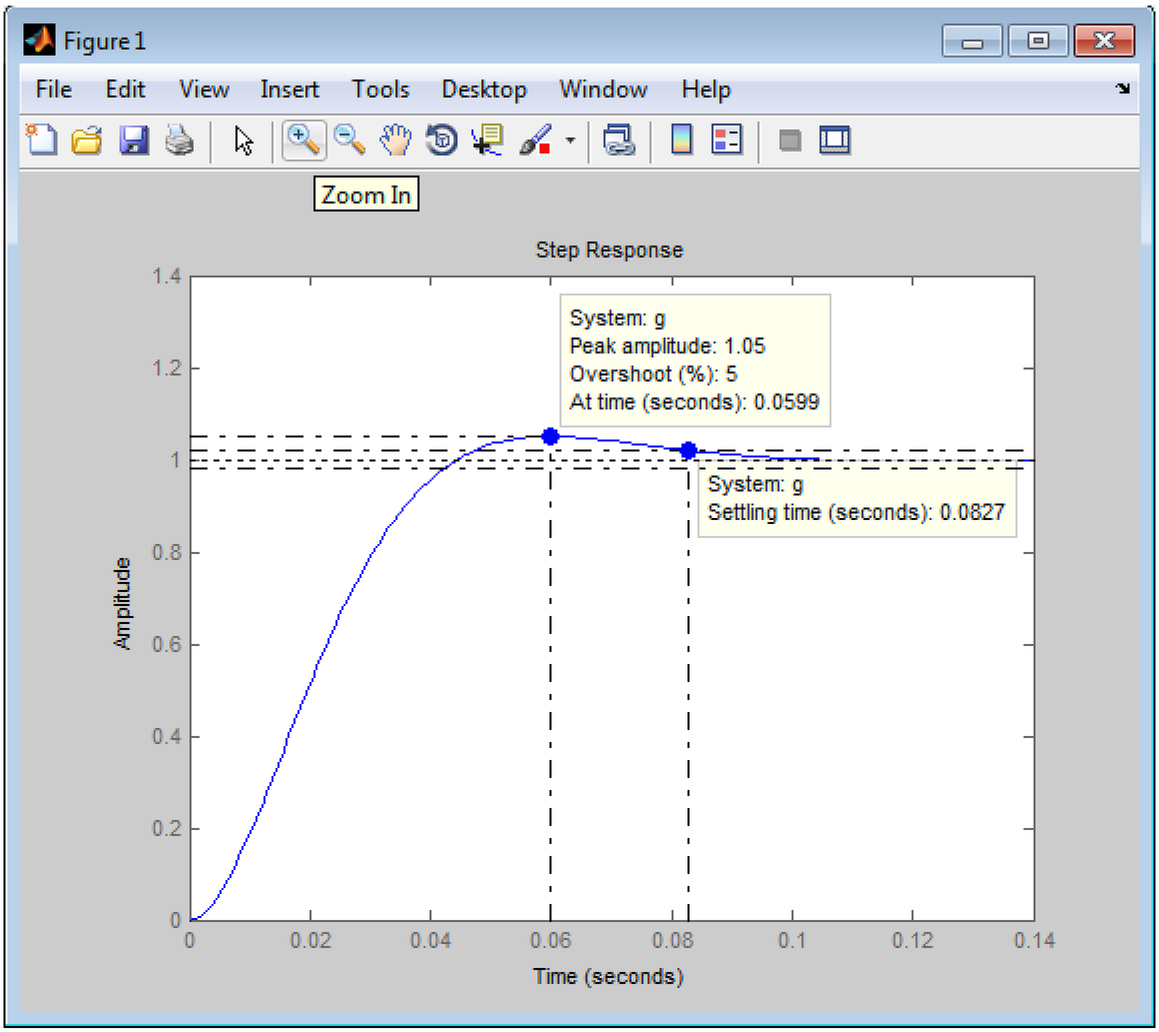


Figure 4-5: Output signal of estimated compensated two dimensional levitator.

In order to obtain the compensator gain = k, zero = z, and pole = p according the design constrain ζ= 0.6901 T_s = 0.08 sec, **a SISO Design Tool** by MATLAB approach can be employed to find the appropriate values of k, z, and p [11].

Refer to equation (2.46), $G_v(s) = \frac{X_v(s)}{I_v(s)} = \frac{-4009.37}{s^3+350-1962s-686700}$

From the denominator of the transfer function, the open loop poles of the system are calculated as -350, 44.2945 and -44.2945.

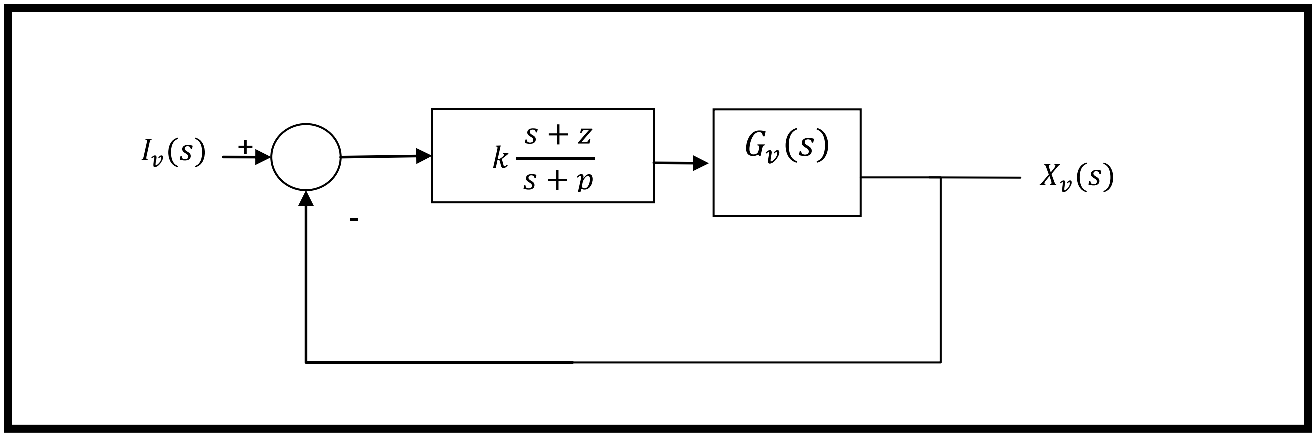


Figure 4-6: Schematic diagram showing compensated vertical component.

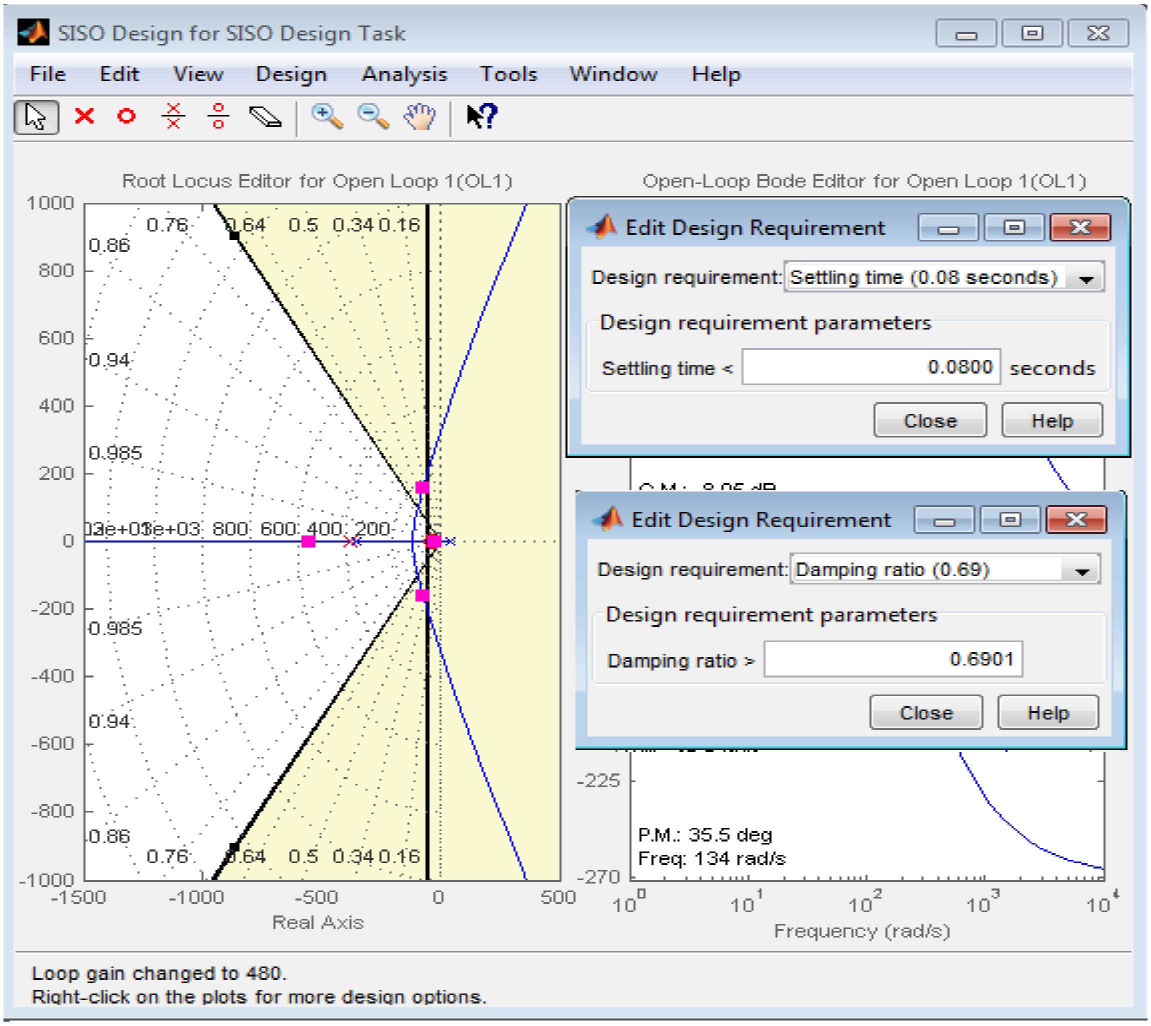


Figure 4-7: Design approach showing lead compensator design by using SISOTOOL.

A zero has to be added between the first left hand plane and the origin i.e., between -44.2945 and origin. The pole of the compensator should be introduced as such that it is deeper location than the deepest left hand pole -350 of the system. This approach is showing in figure 4-7 to design the lead compensator of the vertical component with the given constraint for ζ= 0.6901 T_s = 0.08 sec. We obtained the lead compensator transfer function $C_v = 4690\frac{s+34.71}{s+354.44}$, where *k*=4690, *z* =34.71, and *p*= 354.44.We note that the controller have a big gain *K* because the sensor gain is neglected in the feedback path and the transfer function is dependent from the input current and not the voltage[5].

The closed loop transfer function = $\frac{1.88e07\ s+6.527e08}{s^4+s^3 704.4+s^2 122092+4.093e08}$ (4.10)

with the poles 1.0e+02 *(-5.3427, -0.7099 ± 1.4877i, -0.2819)

Figure (4-8) show the step response of the compensated vertical component with $OS \approx 5\%\ and\ T_s \approx 0.08\ sec$.

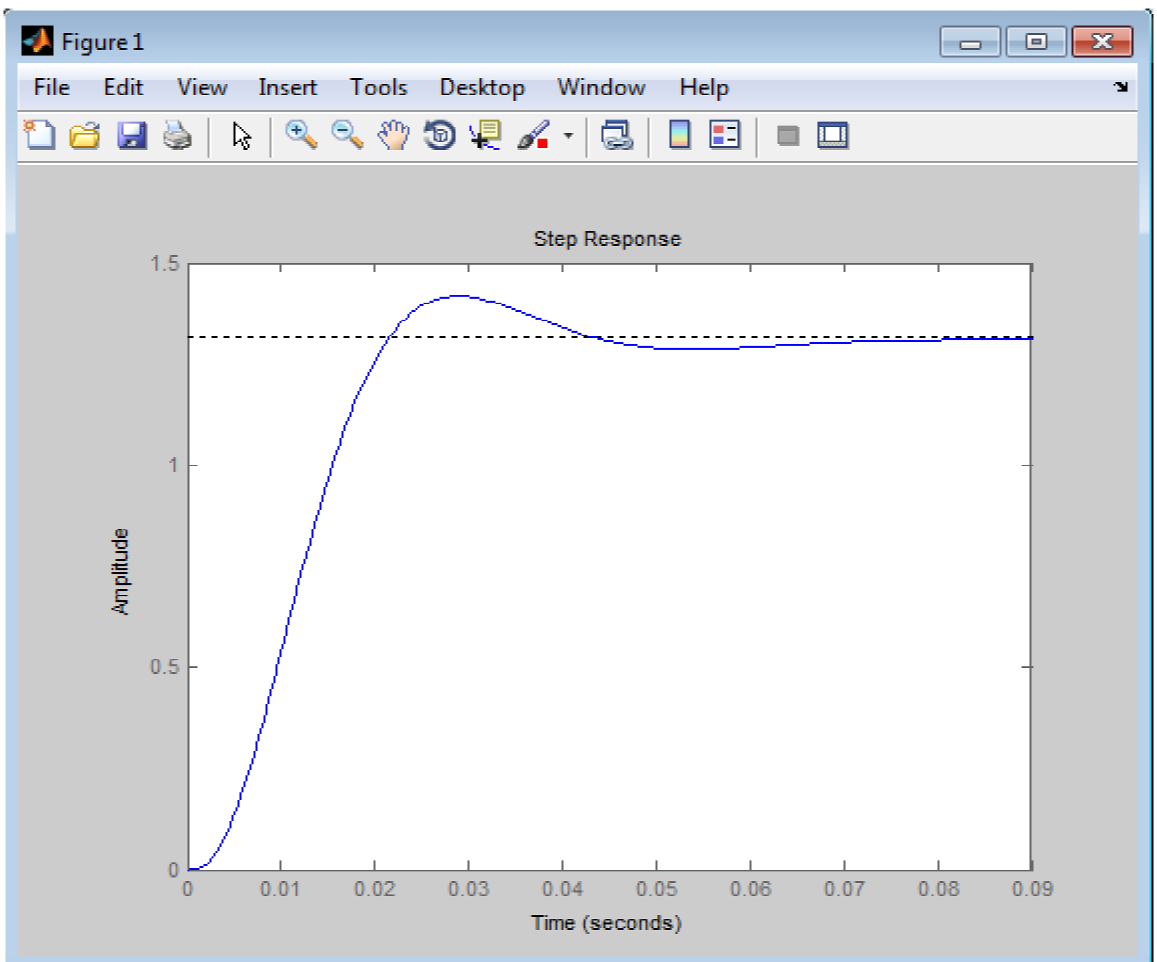


Figure 4-8: Step response of the compensated vertical component

4.3.2 Design Phase Lead Compensator for the horizontal component

Figure (4-9) show the schematic diagram of compensated vertical component.

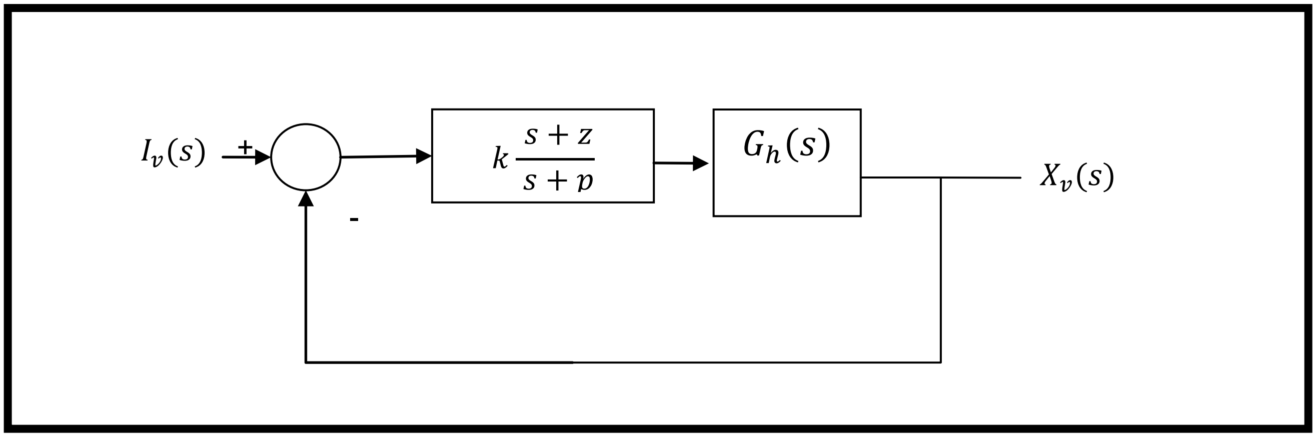


Figure 4-9: Schematic diagram showing compensated horizontal component.

We get the same specification from the last section ζ= 0.6901 T_s = 0.08 sec. refer to equation (2.75)

$$G_h(s) = \frac{X_h(s)}{I_h(s)} = \frac{-1336}{s^3 + 350s^2 - 654s - 228900}$$

From the denominator of the transfer function, the open loop poles of the system are calculated as -350, 25.5734 and -25.5734.

Like the last section **a SISO Design Tool** by MATLAB approach can be employed to find the appropriate values of *k*, *z*, and *p* of the lead compensator.

We obtained the lead compensator transfer function $C_v = 10590\frac{s+24}{s+360}$, where *k*=10590, *z* =24, and *p*= 360

The closed loop transfer function$= \dfrac{1.415e07\ s+3.396e08}{s^4+s^3 710+s^2 125346+s 1.368e07+2.572e08}$ (4.11)

with the poles

1.0e+02 *(-5.1684, -0.8506 ± 1.1983i, -0.2304)

Figure (4-10) show the step response of the compensated vertical component with

$OS \approx 5\%\ and\ T_s \approx 0.08\ sec.$

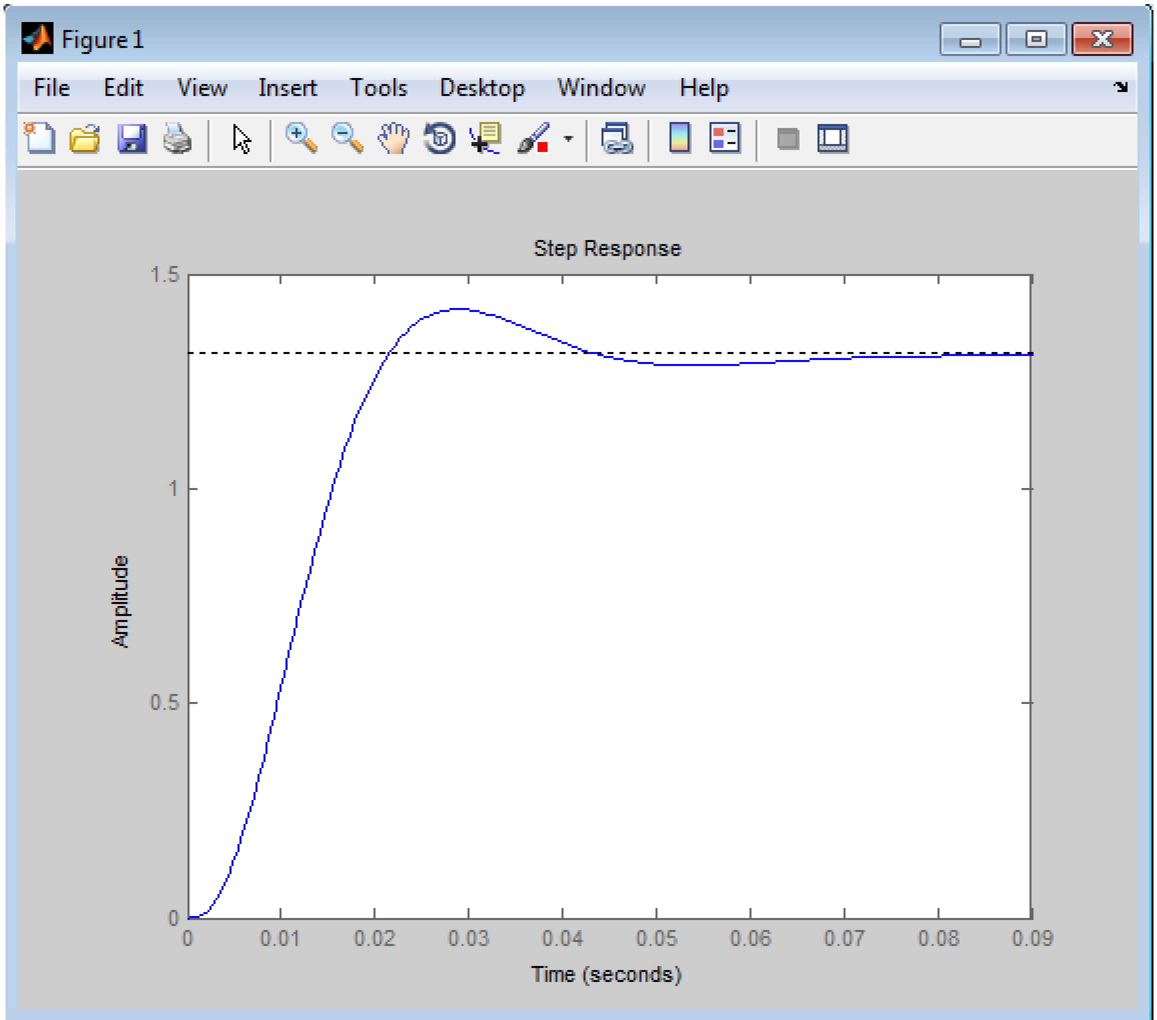


Figure 4-10: Step response of the compensated horizontal component.

4.4 Simulation and Result

4.4.1 Modelling and Simulation of the Vertical Component

In order to obtain the complete linearized SIMULINK model of the vertical component, the output gain and the input gain should be considered. The output gain P is defined as [3]:

$$P = \frac{x_v}{S_{ss}} \qquad (4.12)$$

Where

- x_v is the ball position in the vertical component.
- S_{ss} is the steady state value of the system by defined input.

Hence, the model will be build with the linearized state space, therefore output gain can be obtained from the steady state of the closed loop transfer function of the compensated vertical component. Refer to section (2.4.1), the calculated equilibrium current i_e= 0.244 A and equilibrium voltage V_e= 1.7127V.

Refer to equation (4.10)

$$\frac{X_v(s)}{I_v(s)} = \frac{1.88e07\ s+6.527e08}{s^4+s^3 704.4+s^2 122092+4.093e08}$$

$$S_{ss} = X_v(s) = s * I_e(s) \lim_{s\to 0} \frac{1.88e07\ s+6.527e08}{s^4+s^3 704.4+s^2 122092+4.093e08} \quad (4.13)$$

Taking Laplace transform of $i_e = I_e(s) = \frac{0.244}{s}$ (4.14)

Substituting (4.14) and (4.13) into

$$S_{ss} = X_v(s) = s * \frac{0.244}{s} \lim_{s\to 0} \frac{1.88e07\ s+6.527e08}{s^4+s^3 704.4+s^2 122092+4.093e08} = 0.244 * \frac{6.527e08}{4.093e08} = 0.39$$

Substituting the above value and the equilibrium point $x_{v_e} = 0.01m$ of the vertical component in equation (4.12)

$$P = \frac{x_v}{S_{ss}} = \frac{x_{v_e}}{S_{ss}} = \frac{0.01}{0.39} = 0.025641$$

Refer to equation (2.37)

$v_e = (R_1 + R_2)i_e$, Thus, the input gain $= \frac{i_e}{v_e} = \frac{1}{R_1+R_2} = \frac{1}{3.5+3.5} = 0.1428$

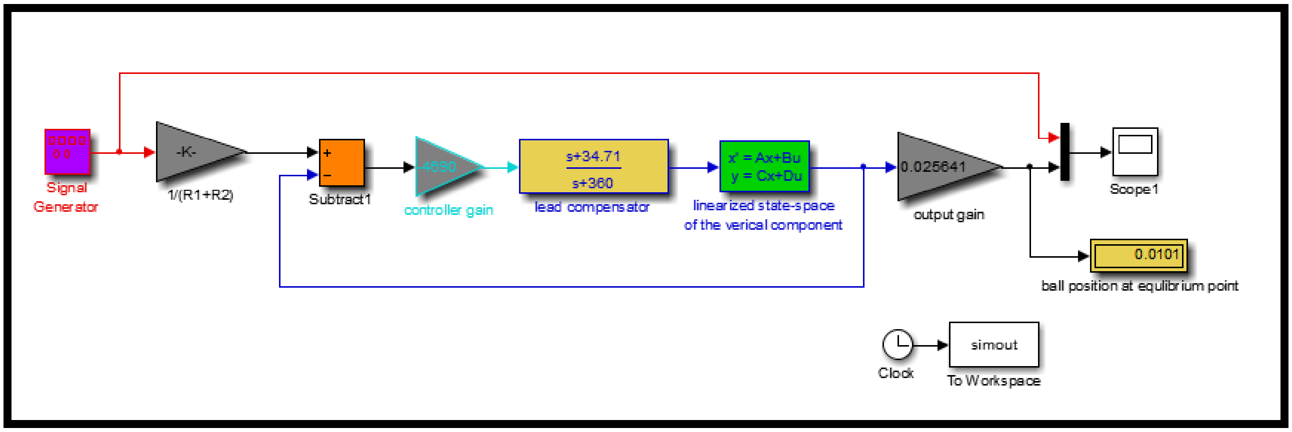


Figure 4-11: SIMULINK linearized state space model of vertical component.

SIMULINK model for the compensated closed loop control system of the linearized state space model of vertical component is shown in Figure (4-11).

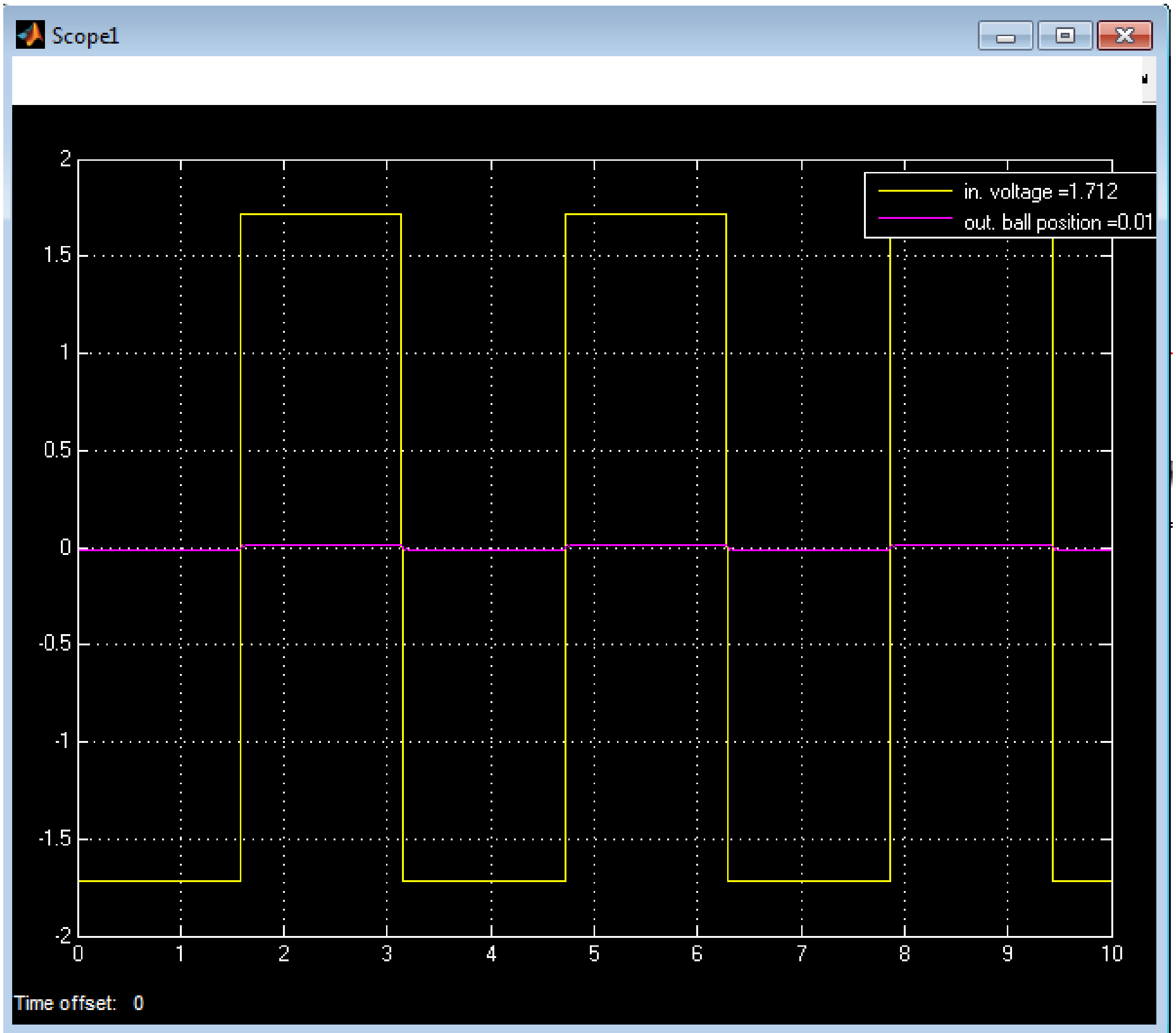


Figure 4-12: The ball levitates in the vertical component exactly at the equilibrium point $0.01m$ with equilibrium voltage $1.712V$ as square wave set point.

Previous figures show that our developed lead compensator in the last section work properly and can work with different set points. The ball levitates in the vertical component exactly at the equilibrium point $0.01m$ with equilibrium voltage $1.712V$ as square wave set point.

4.4.2 Modelling and Simulation of the Horizontal Component

It is the same as the vertical component, the model will be build with the linearized state space, therefore output gain can be obtained from the steady state of the closed loop transfer function of the compensated horizontal component. Refer to section (2.4.2), the calculated equilibrium current i_e= $0.734\ A$ and equilibrium voltage V_e= $2.5691V$.

Refer to equation (4.10)

$$\frac{X_v(s)}{I_v(s)} = \frac{1.415e07\ s+3.396e08}{s^4+s^3 710+s^2 125346+s1.368e07+2.572e08}$$

$$S_{ss} = X_v(s) = s * I_e(s) \lim_{s\to 0} \frac{1.415e07\ s+3.396e08}{s^4+s^3 710+s^2 125346+s1.368e07+2.572e08} \quad (4.15)$$

Taking Laplace transform of $i_e = I_e(s) = \frac{0.734}{s}$ (4.16)

Substituting (4.14) and (4.13) into

$$S_{ss} = X_h(s) = s * \frac{0.734}{s} \lim_{s\to 0} \frac{1.415e07\ s+3.396e08}{s^4+s^3 710+s^2 125346+s1.368e07+2.572e08}$$

$$= 0.734 * \frac{3.396e08}{2.572e08} = 0.9691$$

Substituting the above value and the equilibrium point $x_{h_e} = 0.04m$ of the vertical component in equation (4.16)

$$P = \frac{x_h}{S_{ss}} = \frac{x_{he}}{S_{ss}} = \frac{0.04}{0.9691} = 0.04127$$

Refer to equation (2.66)

$v_e = R_2 i_e$, Thus, the input gain $= \frac{i_e}{v_e} = \frac{1}{R_2} = \frac{1}{3.5} = 0.2857$

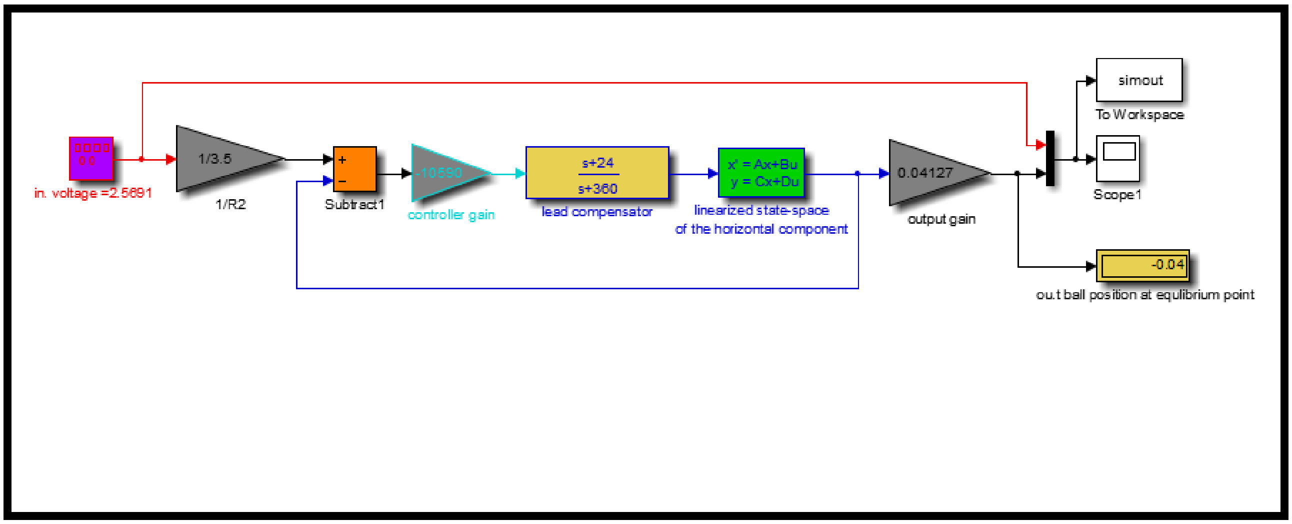


Figure 4-13: SIMULINK linearized state space model of horizontal component

We can see from figure (4-14) that the developed lead compensator of the horizontal component in the last section works properly. The ball stands exactly at *0.04m* between the two solenoids horizontally with equilibrium voltage *2.5691V* as square wave set point.

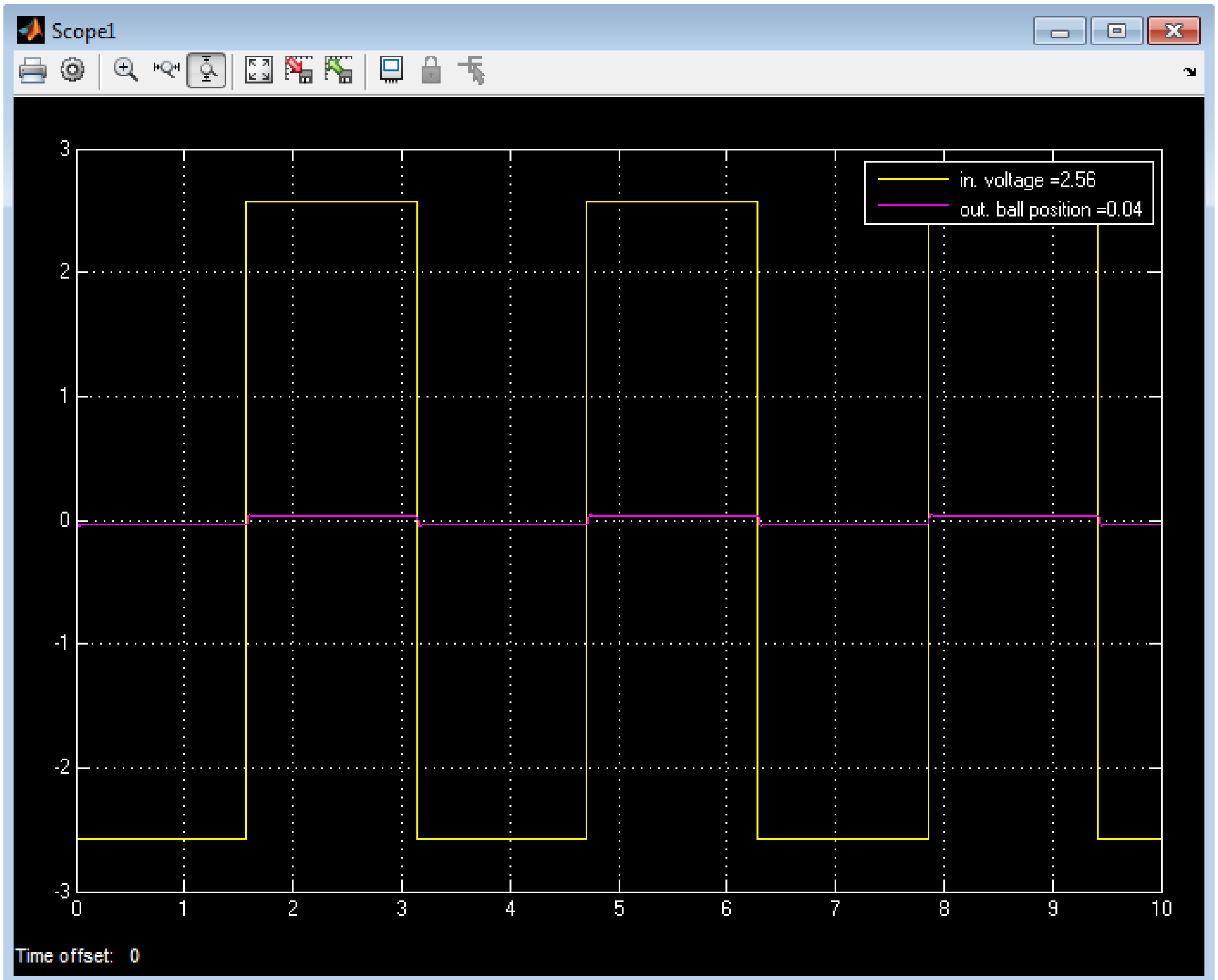


Figure 4-14: The ball stands exactly at *0.04m* between the two solenoids horizontally with equilibrium voltage *2.5691V* as square wave set point

4.4.3 Overall Modelling of Vertical and Horizontal Component

Figure (4-15) shows the overall nonlinear model. As we can be seen both components the vertical and horizontal are driven with different currents ($I_{constant}, i(t)$) and different independence voltage sources ($v_1(t), v_2(t)$). Voltage changing at the second solenoids let the ball go in movement because the first solenoids should be driven with constant voltage or with the vertical current if the vertical component is off. One way to make the system stable is to control the most important component, therefore making sure the ball is stable in the vertical dimension, slowly change the ball so that the horizontal position changes but all the time we have to switch between the vertical component and the horizontal components, if the ball should be moved between the solenoids.

However, the horizontal component controller job is that keeping the movement ball stand at the equilibrium point between the two solenoids. The closed loop control system shown in figure (4-16) was simulated using SIMULINK and the results closely analyzed. The objective of this simulation was to get a better understanding of control strategy of two dimensional levitation system and the individual effects of switch strategy between the vertical and the horizontal components. The movement of the ball from point *0* to point *b* between the two solenoids is not appearing in this simulation see figure (2-4). However, the horizontal component controller job is that keeping the movement ball stand at the equilibrium point between the two solenoids.

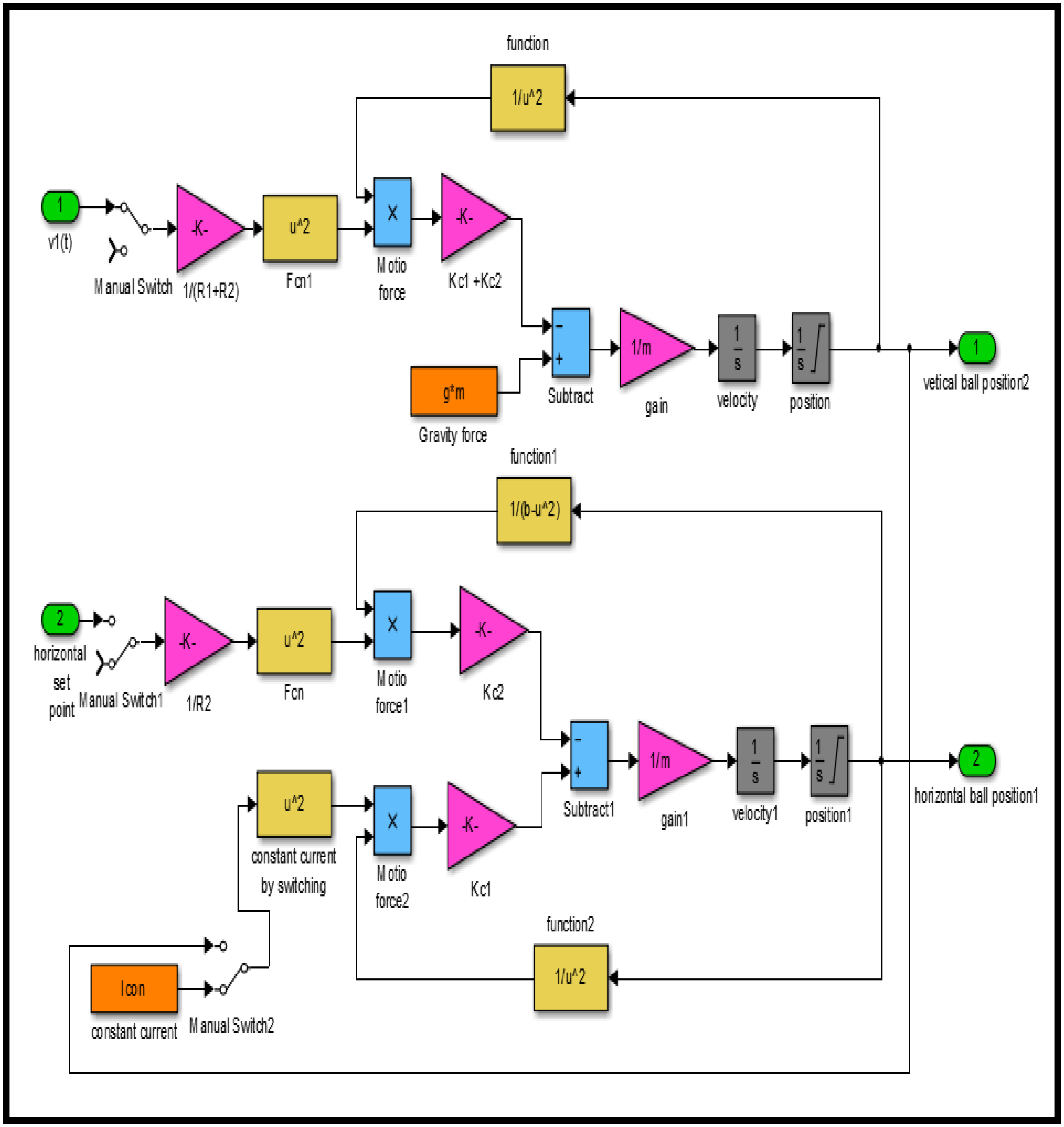


Figure 4-15: SIMULINK Nonlinear Model of the overall system

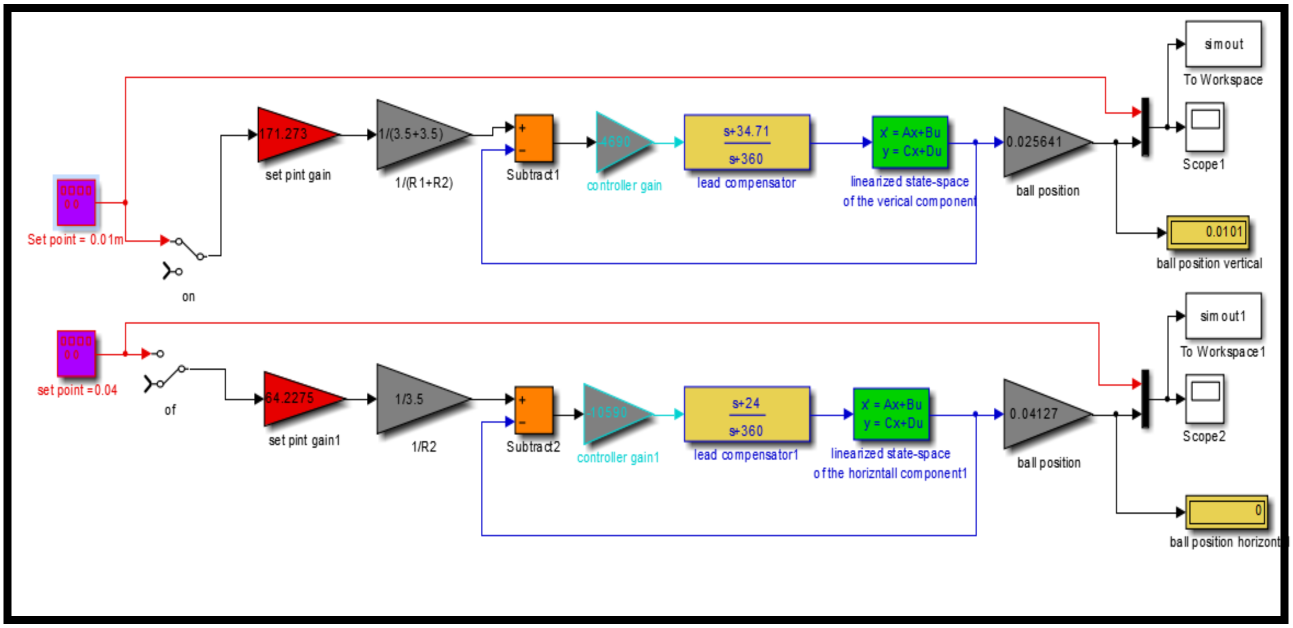


Figure 4-16: SIMULINK Model of the overall system .

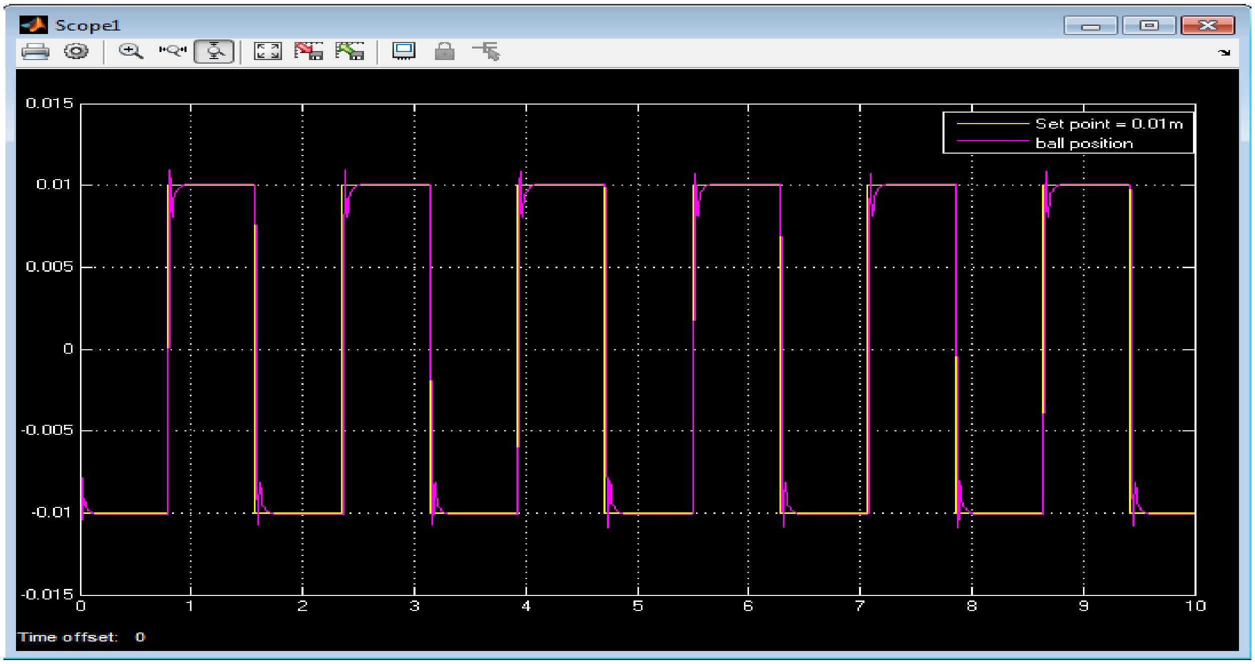


Figure 4-17: The ball levitates in the vertical component at an equilibrium point 0.01*m* with square wave set point 0.01**.**

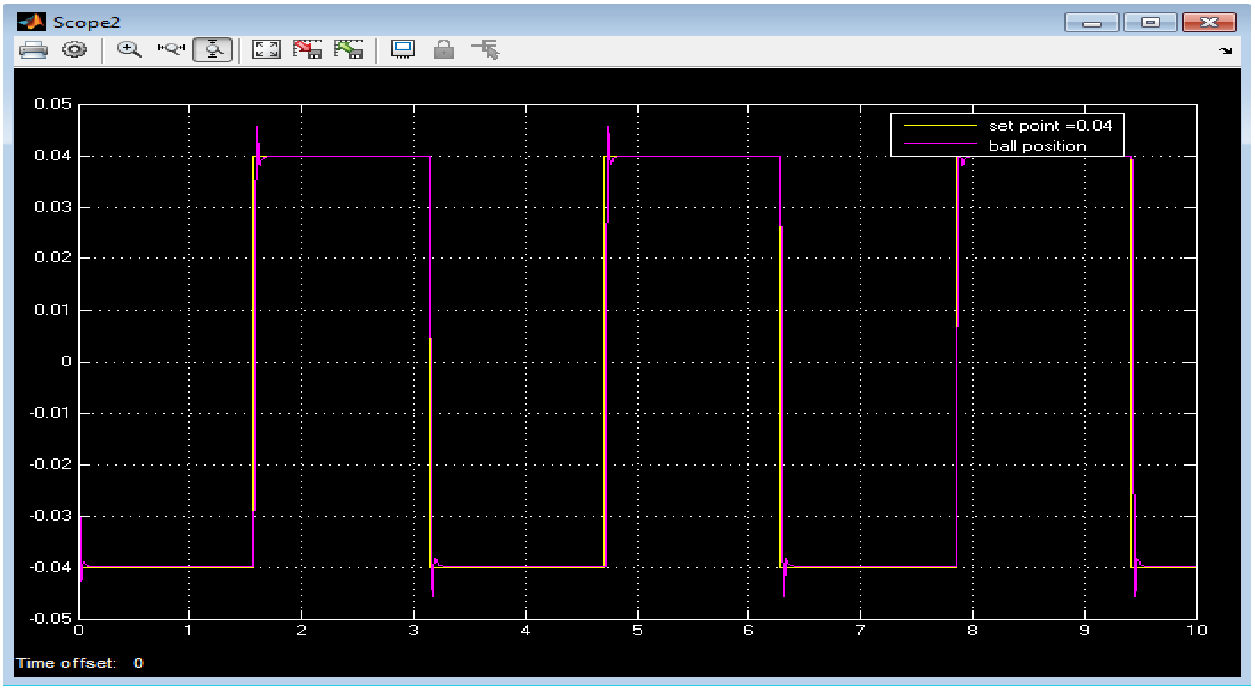


Figure 4-18: The ball stands at *0.04m* between the two solenoids horizontally with square wave set point 0.04**.**

Figure 4-17 present real time simulation results of the compensated vertical component of two dimensional levitation systems. Only the vertical component turned on, horizontal component is turned off. The gain values have been selected. to simulation the ball position (*set point= 0.01*), function generator is used. As can be seen the ball will move up and down smoothly and stable, overshoot is nearly 5%.

Finally figure 4-18 show the simulation only the horizontal component turned on, vertical component is turned off. The ball stands between the two solenoids stable but the overshoot is more as 5% but although is acceptable.

Chapter 5: Conclusion and Future Work

In this thesis, we have developed the state space equations and the transfer functions to model one- and two-dimensional magnetic lavitation system. The system is linearized around a set point.The syetem unstability is proved around possible equilibrium point in the sense of the qualitative behavior of the system. A Lyapunov based stability analysis was performed to prove the stability the system.

A control startagy is developed for levitating the object purely in the vertical components with magnetic fields, and allow movement in the horizontal components between two solenoids around an two-dimensional space.

Negative feedback and lead compensators based on the approximatelinearized model of the two dimensional magnetic levitation system are designed to stablize the system. We consider two linear compensators, one for the verical components and the other for the horizonatl components and show that two dimensional magnetic lavitation system can be stabilized by an appropriate selection of the parameters of the compensators.The simulation show that lead compensation for the verical components and the for the horizonatl components will suffice to stablize the system, as long the gain is larg enough. finally a real time simulation using SIMULINK of the compensated two dimensional levitation system was implemented. The result shows, the object in the vertical component will move up and down smoothly and stable and stand stable between two solenoids.

Future work includes:

1. Find the switch frequency to swith between pairs of solinoids, to stablize the vertical and the horizontal components
2. Implement the controller practically on a digital platform.
3. Toimprove the syetem perfromance,several strategies can be employed, such as sliding mode control, adaptive control and optimal control.

References

[1] W. Barie and J. Chiasson, *Linear and nonlinear state-space controllers for magnetic levitation*, International Journal of Systems Science **27**, no. 11, 1153–1163 (2001).

[2] Paul GerardusHlebowitsh, *The Road to Multi-dimensional Magnetic Levitation*: Realizing Two-Dimensional Control in Classical Feedback, Proc. 2012 American Control Conference, (Virginia), vol. 5, pp. 3731–3736 (2012).

[3]A. Charara, J. De Miras, and B. Caron, *Design one dimensional magnetic levitation controllers using Jacobian linearization, feedback linearization and sliding mode control*, IEEE Transactions on Control Systems Technology **4** , no. 5, 513–14 (2008).

[4]K.Katoh, N.Kidane, *Robust Nonlinear Adaptive Control for the Magnetic Levitation System,* Proc. Of 14th IEEE CCA,pp1373-1378(2010).

[5]R.A.PAZ, *Computer Controlled Systems*, New Mexico State University. 3rdedtion,(2005)

[6] El M.Elamassie, *Multi-Rate Ripple-Free Deadbeat controller for nonlinear systems*, Master Thesis, , Islamic University of Gaza,(2011).

[7]H.K.Khalil, Nonlinear Systems,3rdedtion, Prentice Hall (2002).

[8] El Hajjaji and M. Ouladsine, *Modeling and nonlinear control of magnetic levitation systems*, IEEE Transactions on Industrial Electronics **48**, no. 4, 831–838 (2001).

[10]S. A. Green and K. C. Craig, *Robust, design, nonlinear control of magnetic-levitation systems*, Journal of Dynamics, Measurement, and Control **120** , no. 4, 488–495 (1998).

[11]Dr.Hatem Elaydi,PID&Lead-Lag via Root Locus Control System Design, Islamic University of Gaza,(2011)

Appendix

1. Computation the transfer function, poles, controllability matrix and it's rank of one dimensional magnetic levitation system.

```
%Computation the transfer function, poles, uncompensated root locus,
% controllability matrix and its rank of one dimensional magnetic levitation system

A=[0 1 0;1962 0 -56.7011;0 0 -350]
B=[0;0;100]
C=[1,0,0]
D=0;
G=ss(A,B,C,D)
[num,den]=ss2tf(A,B,C,D)
G=tf(num,den)
pole(G)
Co=ctrb(A,B)
rank(Co)
```

2 .Computation the transfer function, poles, uncompensated root locus, controllability matrix and its rank for vertical component of two dimensional magnetic levitation system.

```
%Computation the transfer function, poles, uncompensated root locus,
% controllability matrix and its rank of vertical
% component of two dimensional magnetic levitation system


A=[0 1 0;1962 0 -80.1875;0 0 -350]
B=[0;0;50]
C=[1,0,0]
D=0;
G=ss(A,B,C,D)
[num,den]=ss2tf(A,B,C,D)
G=tf(num,den)
pole(G)
Co=ctrb(A,B)
rank(Co)
step(-G)
```

3. Computation the transfer function, poles, uncompensated root locus, controllability matrix and its rank for horizontal component of two dimensional magnetic levitation system.

```
Computation the transfer function, poles, uncompensated root locus,
% controllability matrix and its rank of horizontal
% component of two dimensional magnetic levitation system

A=[0 1 0;654 0 -13.3646;0 0 -350]
B=[0;0;100]
C=[1,0,0]
D=0;
G=ss(A,B,C,D)
[num,den]=ss2tf(A,B,C,D)
G=tf(num,den)
pole(G)
rlocus(-G)
sgrid
Co=ctrb(A,B)
rank(Co)
```

4. Computation the compensated vertical component with Lead compensator

```
% Computation the compensated vertical component with Lead compensator
a=34.71
b=354.44
Kc=4690
sys2=1
num1=4009.37;
den1=1;
den2= 350;
den3=-1962;
den4=-686700;
G=tf([num1],[den1 den2 den3 den4])
sysnum=Kc*num1*[0  0 0 1 a];
sysden=conv([1 b],[den1 den2 den3 den4]);
sys1=tf(sysnum,sysden)
sysfun=feedback(sys1,sys2)
pole(sysfun)
figure(1)
step(sysfun)
```

5. Computation the compensated horizontal component with Lead compensator

```
% Computation the compensated vertical component with Lead compensator

a=24.3
b=360.2
Kc=10590
sys2=1
num1=1336;
den1=1;
den2= 350;
den3=-654;
den4=-228900;
G=tf([num1],[den1 den2 den3 den4])
sysnum=Kc*num1*[0  0 0 1 a];
sysden=conv([1 b],[den1 den2 den3 den4]);
sys1=tf(sysnum,sysden)
sysfun=feedback(sys1,sys2)
pole(sysfun)
figure(1)
step(sysfun)
```