The Islamic University of Gaza
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Fuzzy Gain Scheduling Control For Non-Linear Systems

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This Thesis is Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Electrical Engineering

May, 2013

بسم الله الرحمن الرحيم

الجامعة الإسلامية – غزة
The Islamic University - Gaza

عمادة الدراسات العليا

هاتف داخلي: 1150

الرقم: ج س غ/35/ Ref

التاريخ: 2013/05/27م Date

نتيجة الحكم على أطروحة ماجستير

بناءً على موافقة عمادة الدراسات العليا بالجامعة الإسلامية بغزة على تشكيل لجنة الحكم على أطروحة الباحث/ **محمد سالم محمد أبو نصر** لنيل درجة الماجستير في **كلية *الهندسة*** قسم/ **الهندسة الكهربائية– أنظمة التحكم** وموضوعها:

Fuzzy Gain Scheduling Control For Nonlinear Systems

وبعد المناقشة التي تمت اليوم الاثنين 17 رجب 1434هـ، الموافق 2013/05/27م الساعة الحادية عشرة صباحاً ، اجتمعت لجنة الحكم على الأطروحة والمكونة من:

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وبعد المداولة أوصت اللجنة بمنح الباحث درجة الماجستير في **كلية *الهندسة*/** قسم **الهندسة الكهربائية- أنظمة التحكم.**

واللجنة إذ تمنحه هذه الدرجة فإنها توصيه بتقوى الله ولزوم طاعته وأن يسخر علمه في خدمة دينه ووطنه.

والله ولي التوفيق ،،،

عميد الدراسات العليا

أ.د. فؤاد علي العاجز

DEDICATION

I dedicate this work to my Father and my Mother who encourages me to follow up my studies, who taught me when I was child, who gave me their time, love, and attention.

I also dedicate this thesis to my darling sisters and my Brother Ahmad.

ACKNOWLEDGMENTS

At the beginning, I thank ALLAH for giving me the strength and health to let this work see the light.

I thank my supervisor Prof. Dr. Mohammed T. Hussein for his time, consideration, suggestions, ideas and advice during this thesis. Special thanks go to Dr. Hatem Elaydi and Dr. Mohammed Alhanjouri -thesis examiners- for their patience, guidance, and generous supports during this research.

I would also like to extend my gratitude to my family for their patience and encouragement during my thesis. Finally, thanks go to my best friends for their help.

ABSTRACT

In this thesis, a new control methodology is proposed for a class of nonlinear control systems. The proposed methodology combines the advantages of both gain scheduling technique and fuzzy control technique to obtain an advanced tracking control structure. This methodology is applicable for tracking a special case of 2^{nd} order nonlinear systems that represented in a general form similar to many electromechanical systems model representations, such as the magnetic ball levitation and the inverted pendulum.

A continuous gain scheduled tracking controller for a special system is designed based on the use of state feedback. A relationship between state feedback and PI+D control is derived, and then the state feedback controller is transformed into its equivalent PI+D controller with a pre-filter added on the reference input.

At the first stage of employing fuzzy logic approach, a discrete model of the designed gain scheduled PI+D controller is obtained and then Fuzzy PI+D controller is implemented by using simple analytical equations with preserving the same linear structure of the discretized controller. Fuzzy logic is employed only for the design; the resulting controller does not need to execute any fuzzy rule base.

Finally, a simulation is performed by applying the programmed fuzzy control approach to the simulation model of the magnetic ball levitation CE152. Simulation results showed that the proposed technique has achieved better performance compared to the PI +D controller without employing fuzzy logic control.

ملخص الدراسة:

في هذه الأطروحة, تم تقديم طريقة جديدة للتحكم بأحد أنواع الأنظمة غير الخطية. تجمع الطريقة المقترحة بين مزايا كل من طريقة المعاملات المجدولة والتحكم الضبابي لاستنباط نظام تحكم تتبعي مطور. يمكن تطبيق هذه الطريقة على حالات خاصة من النظم اللاخطية ذات الدرجة الثانية, مثل الأنظمة الكهروميكانيكية كالبندول المعكوس ونظام التعليق المغناطيسي.

المتحكم ذو المعاملات المجدولة لبعض النظم الخاصة يتم تصميمه بناءً على التحكم بالتغذية الراجعة لحالات النظام ; لذا فقد تم اشتقاق العلاقة بين نظام التحكم بالتغذية الراجعة للحالات و نظام التحكم الطردي التكاملي + التغييري لحالة خاصة من النظم اللاخطية, ومن ثَم تم تحويل صورة المتحكم بالحالات الراجعة إلى متحكم مكافئ للمتحكم الطردي التكاملي + التغييري بإضافة مرشح مسبق إلى إشارة الدخل الأساسية.

في بداية تصميم المتحكم بالمنطق الضبابي, تم اشتقاق الصورة المتقطعة المكافئة لمتحكم المعاملات المجدولة مع المتحكم الطردي التكاملي + التغييري, و من ثَم تم تمثيل المتحكم الضبابي باستخدام معادلات تحليلية مبسطة تحافظ على نفس التركيب الخطي للمتحكم المتقطع. مع ملاحظة أن المتحكم المنطقي الضبابي تم اشتقاقه واستخراج المعادلات المكافئة له في جميع حالاته; لذا فإن المتحكم النهائي لم يعد يحتاج إلى المرور بجميع القواعد المنطقية عند التنفيذ.

ختاماً, تم عمل محاكاة للنظام بتطبيق طريقة التحكم الضبابي ذو المعاملات المجدولة على نموذج التعليق المغناطيسي من نوع CE152 وقد أظهرت النتائج أن استجابة النظام المتحكَم به من خلال الطريقة المقترحة قد أعطت نتائج أفضل مقارنةً باستخدام المتحكم الطردي التكاملي + التغييري.

Table of Contents

List of Figures

List of Tables

List of Abbreviations

A/D	Analog to Digital Converter
CSTR	Continuous Stirred Tank Reactor
D/A	Digital to Analog Converter
FGS	Fuzzy Gain Scheduling
FGSC	Fuzzy Gain Scheduling Controller
FLC	Fuzzy Logic Control
GA	Genetic Algorithm
GS	Gain Scheduling
IC	Input Combination
IOFL	Input Output Feedback Linearization
Maglev	Magnetic Levitation
Maglev CE152	Magnetic Ball Levitation CE152
PID	Proportional-Integral-Derivative
ZOH	Zero Order Hold

List of Symbols

ρ	Damping ratio
α	Scheduling variable
ω_n	Natural frequency
$e.n$	Error negative
ep	Error positive
F_a	Acceleration force
F_g	Gravity force
F_m	Electromagnetic force
i	Coil current
k_c	Coil constant
k_{fv}	Damping constant
$o.n$	Output negative
$o.p$	Output positive
$O.S$	Over Shoot
$o.z$	Output zero
r	Reference Signal
t_r	Rising Time
t_s	Settling Time
u	Control signal
$v.n$	Rate negative
$v.p$	Rate positive
x	State variable
x_{L0}	Position offset
y	Output signal

CHAPTER 1

1. INTRODUCTION

1.1 General Introduction

Control systems are an integral part of modern society. Practically some types of control systems affect every aspect of our day-to-day activities. Control systems are found in abundance in all sectors of industry, such as quality control of manufactured products, automatic assembly line, machine-tool control, computer control and many others. A control system consists of subsystems and processes (or plants) assembled for the purpose of controlling the outputs of the processes [1].

All physical processes are non-linear in reality and there is a high complexity to control them. Gain scheduling design is one of the most popular methods for designing controllers for non-linear plants, especially during the last decade. It has special features that make it easy to apply compared with others design methods for non-linear plants. Among those features, the most attractive is that gain scheduling employs linear design tools in the design stage [2].

Indeed, control design via gain scheduling approach has a number of limitations: The controllers operate only in a neighborhood of the equilibrium points; it assumes the existence of an explicit model of the controlled process. To overcome these limitations, fuzzy logic tool can be employed in order to design an advanced gain scheduling controller [3].

1.2 Feedback Control Systems

The feedback control system (or closed loop control system) compensates the disturbances by measuring the output response, feeding those measurements back through a feedback path, and comparing that response to the input at the summing junction. If there is any difference between the two responses, the system derives the plant, via the actuating signal, to make a correction. If there is no difference, the system does not derive, since the plants response is already the desired response [1].

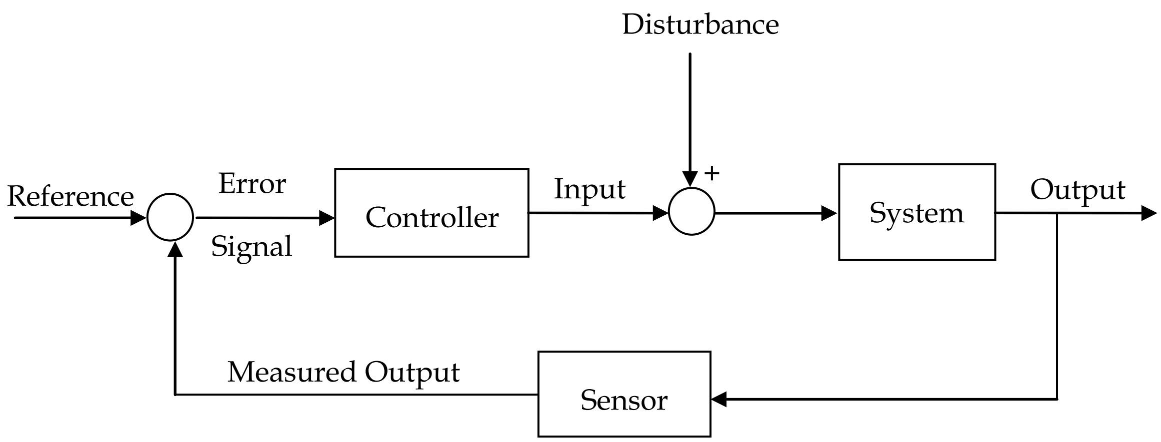

Figure 1.1: Closed loop control system

When compared with the open loop systems, the closed loop systems have the obvious advantage of great accuracy than open loop systems. They are less sensitive to noise, disturbances, and changes in the environment. Transient response and steady-state error can be controlled more conventionally and with greater flexibility in closed loop systems.

1.3 Computer Controlled Systems

In many modern systems, the controller (or compensator) is a digital computer. The advantage of using a computer is that many loops can be controlled by the same computer through time sharing due to using sampling time which plays a big role in the digital control. Furthermore, any adjustment of the controller parameters required to yield a desired response can be made by changes in software rather than hardware. The computer can also perform supervisory functions, such as scheduling many required applications. The digital computers give other important advantages such as reducing the cost and noise immunity [1].

1.4 Statement of The Problem

In the face of the tracking problem for the nonlinear systems, the controller with fixed gains may fail to provide satisfactory control performance or the system goes unstable. One of the most common tools that used for industrial applications to overcome the nonlinear process characteristics is the gain scheduling tool. Gain scheduling technique can extend the validity of the linearization tool by selecting a defined number of operating points belonging to the reference state trajectory by allowing the gains to be tuned or scheduled as a function of operating points. Since the gain scheduling technique is based on linearization, it guarantees only local stabilization about the selected operating points and the region of attraction for each point cannot be prescribed.

Because of stability reasons, the tracking task using the gain scheduling technique can be achieved by setting the reference trajectory input to be varied slowly or transitioned gradually. The performance of gain scheduling approach decreases especially when the system dynamic is very fast; thus, the fuzzy logic tool is used in order to design an advanced gain scheduling tracking controller to maximize the guaranteed stability regions around the operating points and to ensure stable and fast transition between successive operating points.

1.5 Thesis Assumptions

The study of the system in this thesis is done based on the following assumptions:

- The controlled process should be a special case of 2^{nd} order nonlinear system:

$$
\begin{aligned}
\dot{x_1} &= x_2 \\
\dot{x_2} &= f_2(x_1, x_2, u) \\
y &= h(x_1)
\end{aligned}
\tag{1.1}
$$

and its linearization about the equilibrium point using Taylor series can be found in the form:

$$
\begin{aligned}
\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} u \\
y &= \begin{bmatrix} c_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\end{aligned}
\tag{1.2}
$$

It can be noticed that the nominator of the transfer function of this linearized model is constant, and the second state of this model is the derivative of its first state, this model describes many physical systems, especially systems in which the position and velocity are the state variables; for example, the magnetic ball levitation and the inverted pendulum.

- The full state variables are available for measurements; thus, the state feedback controller can be designed.
- In the discrete time model, the plant and the zero-order hold (ZOH) are discretized.

1.6 Literature Review

Some researches of fuzzy gain scheduling are mentioned below:

- Y. C. Kim and K. H. Kim, (1994), proposed a gain scheduling approach for the suspension control of a nonlinear MAGLEV System. They improved the performance of the operational disturbances originating aerodynamic force and the robustness of the uncertainty of payload. They showed by simulations that the gain scheduling control system performed very well compared with other methods which used a nonlinear feedback linearization or a fixed gain linear feedback [4]. However, they did not use any type of fuzzy logic control.

- S. H. Lee and J. T. Lim, (1998), proposed the fuzzy logic-based fast gain scheduling (FFGS) controller for regulation problem in nonlinear systems. They utilized the fuzzy scheduling variable which reflected the derivative information on the original scheduling variable in order to achieve better performance in nonlinear suspension system [5]. However, they did not use state feedback control.

- M. A. Brdy´s and J. J. Littler, (2002), proposed the use of gain scheduling as a method of controlling a servo system with hard non-linear elements. The servo controlled two elements of a tracker mounted on a ship at sea. There was restriction at the zero velocity point and non-linear friction against the motion of each tracker axis. A dual feedback loop control structure was employed. Fuzzy logic was used to provide smoothly varying non-linear scheduling functions to map the velocity of the servo relevant to the deck of the ship onto the rate loop controller parameters. Consideration was given to the use of a derivative signal as a secondary input to the fuzzy inference system [6]. However, fuzzy logic control employed was more complicated.

- KH. Jouili and H. Jerbi, (2010), proposed a novel fuzzy control approach developed for a class of nonlinear continuous systems. They combined an input–output feedback linearization (IOFL) method and a gain scheduling (GS) approach to obtain a tracking control. They proposed fuzzy logic controller (FLC) in order to determine the intermediate operating points which allowed to online implementing the tracking control for nonlinear systems. The effectiveness of the fuzzy gain scheduling schema was demonstrated through a simulation to a temperature control problem in CSTR [7]. However, fuzzy logic control employed was more complicated.

- H. Elaydi and M. Elamassie, (2012), proposed a new design methodology for deadbeat control of nonlinear systems in discrete-time based on partitioning the solution into two components; each with different sampling time. The proposed control can be divided into two sub-controllers: one uses state feedback and the other uses the Diophantine equations. The complete nonlinear design guaranteed the convergence to a neighborhood of origin from any initial state in finite time; thus, providing a stable deadbeat performance. Results showed that the ripple-free deadbeat controller was able to track the input signal and the error decays to zero in a finite number of sampling times [8]. Therefore, in this thesis we propose another technique to control the same application that they used.

- H. Abu Alreesh, (2011), discussed the Magnetic Levitation (Maglev) models as an example of nonlinear systems. He showed the design of fuzzy logic controllers for this model to prove that the fuzzy controller can work properly with nonlinear system. Genetic Algorithm (GA) was used to optimize the membership, output gain and inputs gain of the fuzzy controllers. The result of fuzzy controller with GA optimization was compared with H_2 controller which is one of optimal control techniques, he proved that fuzzy controller with GA optimization gave better performance over H_2 controller [9]. Therefore, in this thesis we propose another technique to control the same application that he used.

1.7 Thesis Contribution

1. Designing a continuous state feedback with integral gain scheduled controller for a special case of 2^{nd} order nonlinear systems, then mapping the total controller to the PID model.
2. Implementing a digital gain scheduled PI+D controller for a special case of 2^{nd} order nonlinear systems from its continuous version.

3. Combining the fuzzy logic control with the digital gain scheduled PI+D controller to form a new developed approach which known as Fuzzy Gain Scheduling.

1.8 Outline of the Thesis

This thesis consists of seven chapters to report the whole research activities and to analyze and discuss the results. Each of the following paragraphs generally describes the contents of each chapter. The second chapter talks about the magnetic ball levitation CE152 in which its mathematical model is derived. In the third chapter, important concepts needed in the design of gain scheduling controller are presented, such as linearization tool, stabilization problem, tracking problem, integral control, and also a relation between state feedback control and PID control is illustrated. The fourth chapter covers some concepts of digital control and derives a method of implementing a digital PI+D controller from its continuous version. The fifth chapter gives an overview of the fuzzy logic control, and demonstrates an approach for implementing fuzzy PI+D controller. The sixth chapter shows the simulation results using MATLAB Program and contains a discussion of these results, and the final chapter concludes this thesis.

CHAPTER 2

2. MAGNETIC BALL LEVITATION CE 152

2.1 Introduction

Levitation (from Latin levitas "lightness") is the process by which an object is suspended by a physical force against gravity, in a stable position without solid physical contact [10]. Magnetic Levitation (Maglev) is becoming an attractive technology for application areas such as high-speed trains, vibration isolation systems, magnetic bearings and photolithography steppers [11]. Many researches in the field of control systems have been prepared to control the Magnetic Levitation which has a range of products designed for the theoretical study and practical investigation of control engineering principles.

2.2 Magnetic Ball Levitation CE 152

The Magnetic Levitation CE 152 (Maglev CE 152) Model shown in figure 2.1 is one of the ranges of educational scale models offered by Humusoft for teaching system dynamics and control engineering principles. The Magnetic Levitation CE 152 Apparatus shows control problems with nonlinear, unstable systems. The apparatus consists of a steel ball held in a magnetic field produced by a current-carrying coil. At equilibrium, the downward force on the ball due to gravity (its weight) is balanced by the upward magnetic force of attraction of the ball towards the coil. Any imbalance, the ball will move away from the set-point position. The basic control task is to control the vertical position of the freely levitating ball in the magnetic field of the coil [12].

Figure 2.1: CE152 magnetic ball levitation.

2.3 System Modeling

The magnetic ball suspension system can be categorized into two systems: a mechanical system and an electrical system. The ball position in the mechanical system can be controlled by adjusting the current through the electromagnet where the current through the electromagnet in the electrical system can be controlled by applying controlled voltage across the electromagnet terminals.

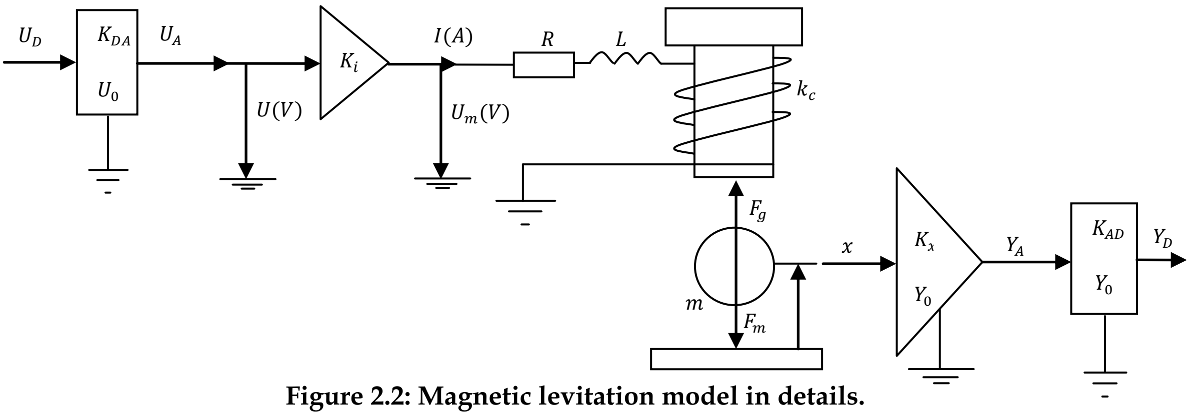

Figure 2.2: Magnetic levitation model in details.

The magnetic levitation CE 152 model shown in Figure 2.2 consists of D/A converter, Power amplifier, Ball & coil subsystem, Position sensor and A/D converter and can be illustrated in details as follows [13]:

2.3.1 D/A Converter

Digital to Analog Converter (D/A), shown in Figure 2.3 has model output voltage U_A, D/A converter input U_D, Digital to Analog converter gain K_{DA}, and the D/A converter offset U_0. The output is defined in equation (2.1).

$$U_A = K_{DA} * U_D + U_0 \tag{2.1}$$

From the table (2.1) $U_0 = 0$.

Then:

$$U_A = K_{DA} * U_D \tag{2.2}$$

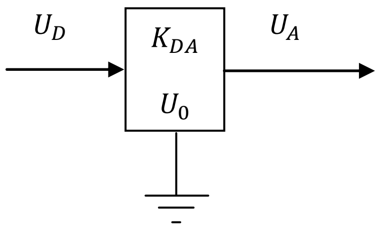

Figure 2.3: D/A Converter.

2.3.2 Power Amplifier

The approximation model of power amplifier shown in figure 2.4 is designed as a source of constant current. Relation between input current to output voltage from power amplifier is defined in equation (2.3):

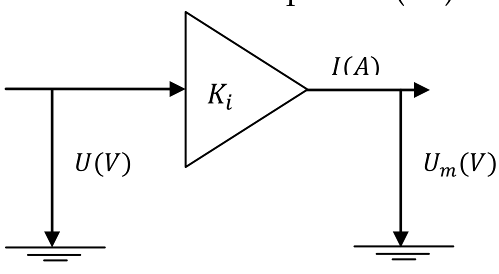

Figure 2.4: Power amplifier (Approximation Model)

$$I = K_i * U_A \tag{2.3}$$

2.3.3 Ball and Coil Subsystem

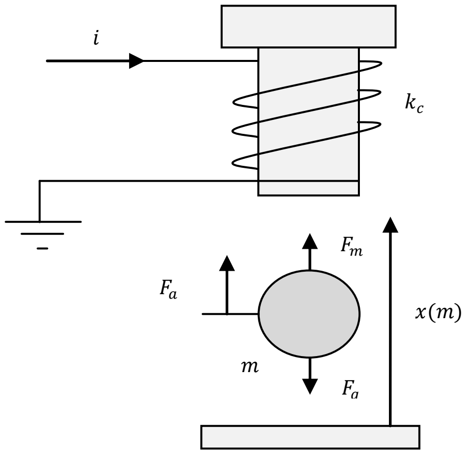

Figure 2.5: Free diagram of the ball and coil subsystem.

The motion equation is based on the balance of all forces acting on the ball. There are three forces: gravity force F_g, electromagnetic force F_m and the acceleration force F_a, as shown in Figure 2.5, equation of free body diagram will be derived where i is the coil current, k_c is coil constant, x_{L0} is a position offset, and k_{fv} is a damping constant. According to Newton's second law of motion, the acceleration of an object produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.

The net force:

$$F_a = F_m - F_g \quad (2.4)$$

Where, the magnetic force:

$$F_m = \frac{i^2 k_c}{(x - x_{L0})^2} \quad (2.5)$$

The gravitational force:

$$F_g = mg \quad (2.6)$$

And the acceleration force:

$$F_a = m\ddot{x} \quad (2.7)$$

Substituting equations (2.5), (2.6) and (2.7) into (2.4) then:

$$m\ddot{x} = \frac{i^2 k_c}{(x - x_{L0})^2} - mg \quad (2.8)$$

When the ball damping is taken into account, the term ($k_{fv}\dot{x}$) is introduced into equation (2.8) which becomes,

$$\ddot{x} + k_{fv}\dot{x} = \frac{i^2 k_c}{(x - x_{L0})^2} - mg \qquad (2.9)$$

Since $i = k_i U_A$, then

$$\ddot{x} = \frac{k_c {k_i}^2 U_A^2}{m(x - x_{L0})^2} - \frac{k_{fv}\dot{x}}{m} - g \qquad (2.10)$$

2.3.4 Position Sensor

The position sensor, shown in Figure 2.6 which used to measure the ball position has model output voltage Y_A, the ball position x , the position sensor gain K_x, and the position sensor offset Y_0. The output is defined in equation (2.11).

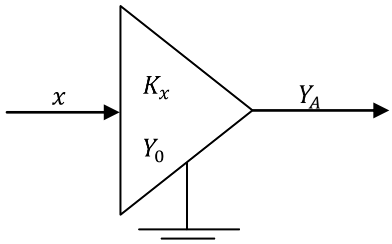

Figure 2.6: Position sensor subsystem.

$$Y_A = K_x x + Y_0 \qquad (2.11)$$

2.3.5 A/D Converter

The Analog to Digital Converter (A/D), shown if Figure 2.7 has model output voltage Y_D, A/D converter input Y_A , analog to digital converter gain K_{AD}, and the A/D converter offset Y_0. The output is defined in equation (2.12).

$$Y_D = K_{AD} Y_A + Y_0 \qquad (2.12)$$

From the table $Y_0 = 0$.

$$Y_D = K_{AD} Y_A \qquad (2.13)$$

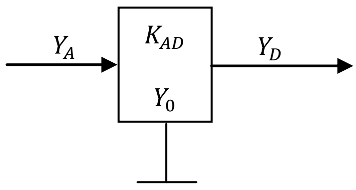

Figure 2.7: D/A converter

2.3.6 Complete Modeling

To make a simulation for the magnetic ball levitation CE 152 according to its parameters, we built the complete model on the MATLAB SIMULINK program and the final block diagram is shown in Figure 2.8:

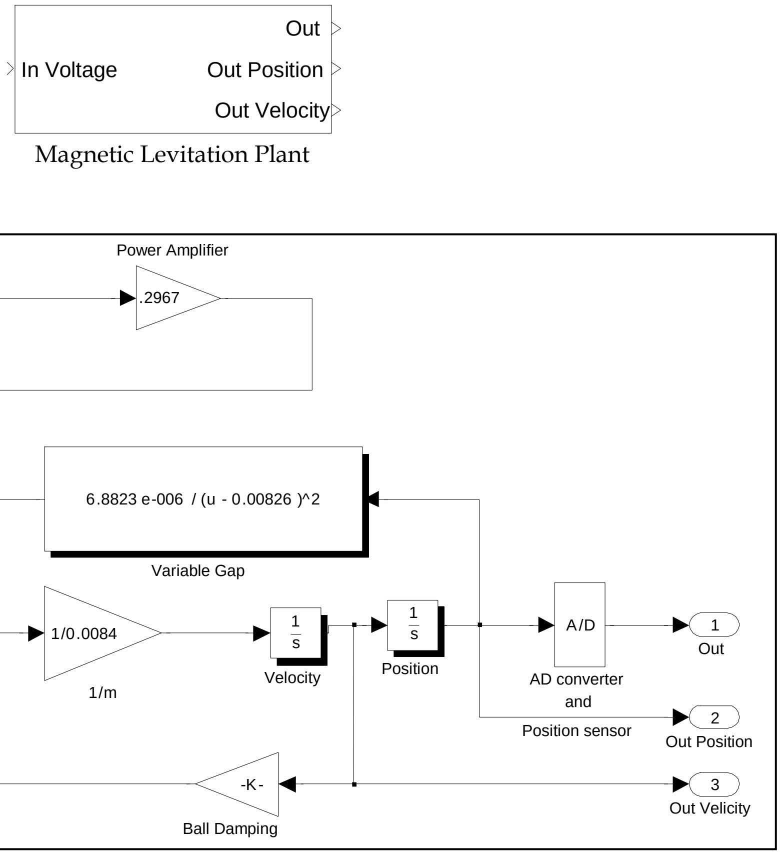

Figure 2.8: The complete model and final block diagram of maglev CE152

2.4 Mathematical Model of Maglev CE 152

From the equation of motion (2.10), we conclude that the system is a non-linear system.

Define $x_1 = x \; and \; x_2 = \dot{x_1}$ as state variables, and then the state space model is derived from equation (2.10) as follows:

$$\begin{bmatrix} \dot{x_1} = x_2 \\ \dot{x_2} = \dfrac{k_c {k_i}^2 U_A^2}{m(x_1 - x_{L0})^2} - \dfrac{k_{fv} x_2}{m} - g \end{bmatrix} \qquad (2.14)$$

$Let\ k_c {k_i}^2 = k_F$, then

$$\begin{bmatrix} \dot{x_1} = x_2 \\ \dot{x_2} = \frac{k_F U_A^2}{m(x_1 - x_{L0})^2} - \frac{k_{fv} x_2}{m} - g \end{bmatrix} \quad (2.15)$$

Now, we want to linearize the above system about the equilibrium point(x_{ss}, u_{ss}), Where $x_{ss} = \begin{pmatrix} x_{1ss} \\ x_{2ss} \end{pmatrix}$

At equilibrium, the following equations must be satisfied:

$$\dot{x_1} = x_2 = 0 \quad (2.16)$$

$$\dot{x_2} = \frac{k_F U_A^2}{m(x_1 - x_{L0})^2} - \frac{k_{fv}}{m} x_2 - g = 0 \quad (2.17)$$

From equations (2.16) and (2.17), we get:

$$x_{2ss} = 0 \quad (2.18)$$

$$\left(\frac{k_F {u_{ss}}^2}{(x_{1ss} - x_{L0})^2} - mg \right) = 0 \quad (2.19)$$

From equation (2.19), then

$$u_{ss} = \sqrt{\frac{mg}{k_F}} (x_{1ss} - x_{L0}) \quad (2.20)$$

By using Taylor series, the linearization about (x_{ss}, u_{ss}) can be written as:

$$f(x, u) = f(x_{ss}, u_{ss}) + \left(\frac{\partial\big(f(x_{ss}, u_{ss})\big)}{\partial(x_{ss}) * 1!} x(t) + \frac{\partial\big(f(x_{ss}, u_{ss})\big)}{\partial(u_{ss}) * 1!} u(t) \right) \quad (2.21)$$

By applying equation (2.21) to the nonlinear term $\frac{k_F U_A^2}{m(x_1 - x_{L0})^2}$ that found in equation (2.17), we have:

$$k_F \left(\frac{U_A}{(x_{1ss} - x_{L0})} \right)^2 = \left(\frac{k_F {u_{ss}}^2}{(x_{1ss} - x_{L0})^2} \right) + \left(\frac{-2 k_F {u_{ss}}^2}{(x_{1ss} - x_{L0})^3} \right) x_1(t) + \left(\frac{2 k_F u_{ss}}{(x_{1ss} - x_{L0})^2} \right) u(t) \quad (2.22)$$

Substitute from equation (2.19) into (2.22), we get

$$k_F \left(\frac{U_A}{(x_{1ss} - x_{L0})} \right)^2 = mg + \left(\frac{-2 k_F {u_{ss}}^2}{(x_{1ss} - x_{L0})^3} \right) x_1(t) + \left(\frac{2 k_F u_{ss}}{(x_{1ss} - x_{L0})^2} \right) u(t) \quad (2.23)$$

Substituting equation (2.23) into (2.17), then

$$\dot{x}_2 = \left(\frac{-2k_F {u_{ss}}^2}{m(x_{1ss} - x_{L0})^3}\right) x_1 - \left(\frac{k_{fv}}{m}\right) x_2 + \left(\frac{2k_F u_{ss}}{m(x_{1ss} - x_{L0})^2}\right) u(t) \qquad (2.24)$$

Since from equation (2.2) that $U_A = K_{DA} * U_D$ and from equation (2.15) and (2.24), the linearized state space model can be written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-2k_F {u_{ss}}^2}{m(x_{1ss} - x_{L0})^3} & \frac{-k_{fv}}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2k_F K_{DA} u_{ss}}{m(x_{1ss} - x_{L0})^2} \end{bmatrix} U_D$$

$$y = [k_x * K_{AD} \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (2.25)$$

After substituting the value of u_{ss} from equation (2.20) into the state space model (2.25), the linearized state space model becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2g}{m(x_{L0} - x_{1ss})^3} & \frac{-k_{fv}}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2\sqrt{\frac{k_F g}{m}} \frac{K_{DA}}{(x_{L0} - x_{1ss})} \end{bmatrix} U_D$$

$$y = [K_x * K_{AD} \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (2.26)$$

The values of parameters for the magnetic ball levitation CE152 are shown in Table 2.1[12]:

Table 2.1: Parameters of magnetic ball levitation CE 152.

Parameter	Symbol	Value
ball diameter	D_k	12.7x10-3 m
ball mass	m_k	0.0084 kg
distance from the ground and the edge of the magnetic coil	T_d	0.019 m
distance of limits= 0.019 – Dk	L	0.0063 m
gravity acceleration constant	g	9.81 m/s^2
maximum DA converter output voltage	U_{DAm}	5 V
coil resistance	Rc	3.5 Ω
coil inductance	Lc	30 $x10^{-3}$ H
current sensor resistance	Rs	0.25 Ω
current sensor gain	Ks	13.33
power amplifier gain	Kam	100
maximum power amplifier output current	I_am	1.2 A
amplifier time constant= Lc/((Rc+Rs)+Rs*Ks*K_am)	Ta	1.8694 $x10^{-5}$ s
amplifier gain= K_am / ((Rc+Rs)+Rs*Ks*K_am)	k_i	0.2967
viscose friction	k_{FV}	0.02 N.s/m
converter gain	K_{DA}	10
Digital to Analog converter offset	U_0	0 V
Analog to Digital converter gain	K_{AD}	0.2
Analog to Digital converter offset	Y_0	0 V
position sensor constant	Kx	797.4603
coil bias	x_{LO}	8.26 $x10^{-3}$ m
Aggregated coil constant	k_F	0.606 $x10^{-6}$ N/V
coil constant =k_f/(k_i)^2	k_C	6.8823 $x10^{-6}$ N/V

After substituting the values of parameters of the CE 152 magnetic ball levitation from Table (2.1) in the state space model (2.26) becomes,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{19.62}{(0.00826 - x_{1ss})} & -2.381 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{0.53206}{(0.00826 - x_{1ss})} \end{bmatrix} U_D \qquad (2.27)$$
$$y = [159.49206 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2.5 MATLAB Work

From the state space model for the magnetic ball levitation described in equation (2.27), the matrices A and B are parameterized by the value of x_{1ss} which means that the state space model is varying according to the ball displacement.

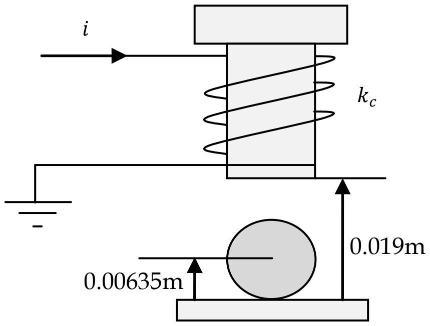

Figure 2.9: The allowable ball displacement and the radius of the ball.

From Figure 2.10, we note that the range of the ball displacement that allowable for moving is$[0 - (0.019 - ball\ diameter)] = [0 - 0.0063]m$.
If we multiply this interval by the element $K_x * K_{AD}$ of the output equation, the interval of the output is $[0 - 1]\ volts$. so, if the control task is tracking a reference signal which is a unit step response, then the ball will move to the complete allowed displacement. If the ball moved to one half the allowed displacement as described in Figure 2.11, then $x_{1ss} = \frac{[0-0.0063]}{2} = 0.00315m$, at which the value of the output$y = 0.5$.

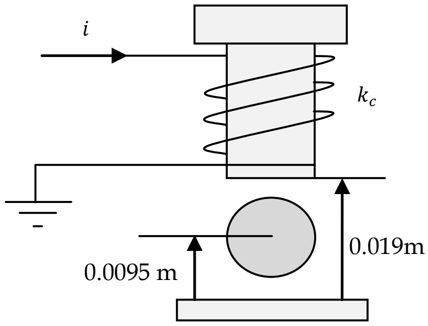

Figure 2.10: One half displacement of overall displacement for the ball.

Substituting by x_{1ss} in the parameterized state space model (2.27),the resulting model becomes,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3839.53 & -2.381 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 104.12 \end{bmatrix} U_D$$
$$y \quad = [159.49206 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2.28}$$

The open Loop Transfer function $G(s)$is defined as: $G(s) = C(SI - A)^{-1}B$,then

$$G(s) = [159.49206 \quad 0] \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3839.53 & -2.381 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 104.12 \end{bmatrix}$$

$$G(s) = \frac{16606.653}{s^2 + 2.381s - 3839.53} = \frac{N(s)}{D(s)} \tag{2.29}$$

The closed loop Transfer function without controller is given by:

$$TF = \frac{G(s)}{1 + G(s)} = \frac{\frac{N(s)}{D(s)}}{1 + \frac{N(s)}{D(s)}} = \frac{N(s)}{N(s) + D(s)} \tag{2.30}$$

Then, the transfer function (2.30) for the magnetic ball levitation is evaluated as:

$$TF = \frac{16606.653}{s^2 + 2.381s + 12767.123} \tag{2.31}$$

The step response of the closed loop system (2.31) is applied on MATLAB program and its step response is shown in Figure 2.11:

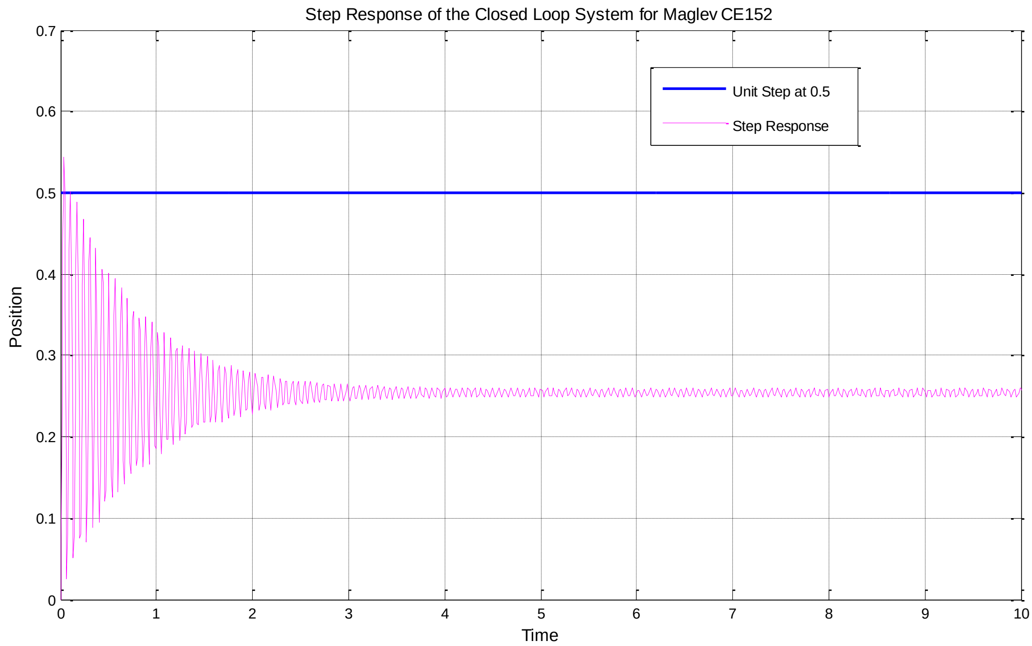

Figure 2.11: Step response of the magnetic ball levitation.

CHAPTER 3

3. GAIN SCHDULING

3.1 Introduction

In nature, most practical systems used for control purpose are essentially nonlinear, and in many applications, the nonlinear rather than the linear characteristics that are most used [14]. Various tools for designing controllers of the nonlinear systems are available in the control area such as linearization, gain scheduling, feedback linearization, sliding mode control, and high gain observers. Linearization tool is used to simplify the analysis and design of controllers for the nonlinear systems, but the basic limitation of using the linearization tool is that the controller is available to work well only for a certain region around the operating (equilibrium) point. Gain scheduling technique can extend the validity of the linearization tool to a range of operating points which makes the design of controllers for nonlinear systems is more practical in real applications. First, the linearization tool is presented since the gain scheduling technique extends the linearization tool concept.

3.2 Linearization Tool

Linearization makes it possible to use the tools for linear systems to analyze the behavior of a nonlinear system near an operating (equilibrium) point and to design controllers in order to achieve many control tasks such as stabilization, tracking, and disturbance rejection. The linearization of a nonlinear system function is the first order term of its Taylor series expansion around the interesting operating point.

Consider a second nonlinear system defined by the following equation:

$$\dot{x} = f(x, u) \tag{3.1}$$

Let $p = (x_0, u_0)$ be an equilibrium point of the nonlinear system (3.1). By expanding the right hand side of system (3.1) into its Taylor series about the operating point (x_0, u_0) and dropping the higher order terms, the resulting approximate linear state equation is:

$$\dot{x} = Ax + Bu \tag{3.2}$$

Where:

$$A = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} \quad and \quad B = \left.\frac{\partial f}{\partial u}\right|_{u=u_0} \tag{3.3}$$

In this chapter, linear state feedback control is used as a tool for designing a feedback control system to achieve the stabilization and tracking tasks in which the transient response specifications is met.

3.3 Stability problem

The stability problem is the problem of keeping all state variables at their equilibrium value (zero) in the face of disturbance which acting on the plant. To stabilize the nonlinear system (3.1) at zero equilibrium point, a linear state feedback control must be designed such that:

$$u = -K_x x \tag{3.4}$$

After applying the designed controller (3.4) to the nonlinear system (3.1) as shown in figure 3.1, the linearized state equation of the closed loop system becomes,

$$\dot{x} = (A - BK_x)x \tag{3.5}$$

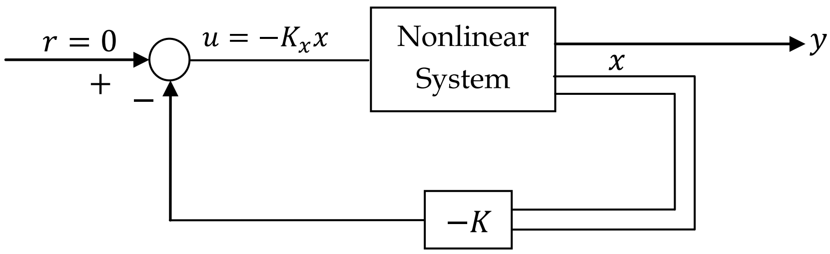

Figure 3.1: State feedback controller for a nonlinear system.

The origin of the closed loop system (3.5) is said to be asymptotically stable if and only if the closed loop matrix $(A - BK_x)$ is Hurwitz, which means that the stability problem is reduced to a problem of finding a feedback gains vector K_x that assigns the eigenvalues of $(A - BK_x)$ at the specified locations which depend on the performance requirements. The stability of the closed loop system (3.5) requires the controllability of the pair(A, B).

3.3.1 Controllability

Controllability describes the ability of an external input to move the internal state of a system from any initial state to any other final state in a finite time interval [15]. Several methods are used for testing the condition of the controllability of a system using matrices A and B by the form:

$$CM = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

Where CM is called the controllability matrix. The system is said to be completely controllable if the matrix CM has a full rank.

3.3.2 Designing of a Stabilizing Controller for a Special Case of 2nd Order Non-Linear System

Consider a special case of 2nd order nonlinear system,

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= f_2(x_1, x_2, u) \\ y &= h(x_1) \end{aligned} \qquad (3.6)$$

The linearization of system (3.6) is the first order term of its Taylor series about the origin yields in the linear form:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} u$$
$$y = \begin{bmatrix} c_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3.7}$$

Where

$$a_1 = \frac{\partial f_2}{\partial x_1}, \qquad a_2 = \frac{\partial f_2}{\partial x_2}, \qquad b_1 = \frac{\partial f_2}{\partial u} \quad and \quad c_1 = \frac{\partial h}{\partial x_1}$$

The designed linear state feedback control can be written as

$$u = -K_x x = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -k_1 x_1 - k_2 x_2 \tag{3.8}$$

After applying the designed feedback controller, the closed loop system becomes:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 0 \\ b_1 \end{bmatrix} - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a_1 - b_1 k_1 & a_2 - b_1 k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3.9}$$

The characteristic equation of the closed loop system can be evaluated as:

$$\det[SI - (A - BK)] = \begin{bmatrix} S & -1 \\ -(a_1 - b_1 k_1) & S - (a_2 - b_1 k_2) \end{bmatrix}$$
$$= S^2 + (b_1 k_2 - a_2) S + (b_1 k_1 - a_1) \tag{3.10}$$

To meet the performance requirements: damping ratio ρ and natural frequency ω_n, define the desired characteristic equation as:

$$\Delta(S) = S^2 + 2\rho\omega_n S + \omega_n^2 \tag{3.11}$$

And equating the coefficients of equation (3.11) and (3.10), we obtain

$$b_1 k_1 - a_1 = \omega_n^2 \rightarrow k_1 = \frac{\omega_n^2 + a_1}{b_1}$$
$$b_1 k_2 - a_2 = 2\rho\omega_n \rightarrow k_2 = \frac{2\rho\omega_n + a_2}{b_1} \tag{3.12}$$

From equation (3.12) we get the state feedback gains vector K_x as:

$$K_x = [k_1 \quad k_2] = \begin{bmatrix} \dfrac{\omega_n^2 + a_1}{b_1} & \dfrac{2\rho\omega_n + a_2}{b_1} \end{bmatrix} \tag{3.13}$$

3.4 Tracking Problem

In the tracking problem, the controlled output, y, should tracks the reference input r. In other words, the tracking problem is a problem of driving the nonlinear system to a sequence of nonzero operating points. The tracking problem can be reduced to a stabilizing problem by shifting the equilibrium point to the origin. For that purpose a feedforward input u_{ss} must be applied to maintain the equilibrium at point x_{ss}

At this point the following equation must be satisfied:

$$0 = f(x_{ss}, u_{ss}) \tag{3.14}$$

Let,

$$x_\delta = x - x_{ss}\,, \qquad u_\delta = u - u_{ss}$$

The state space model of the transformed system becomes,

$$\begin{aligned} \dot{x}_\delta &= f_\delta(x_\delta, u_\delta) \\ y_\delta &= h_\delta(x_\delta, u_\delta) \end{aligned} \tag{3.15}$$

Where:

$$\begin{aligned} f_\delta(0,0) &= 0 \\ h_\delta(0,0) &= 0 \end{aligned} \tag{3.16}$$

The overall control input, u , can be written as :

$$u = u_\delta + u_{ss} \tag{3.17}$$

Where u_δ is the feedback control and u_{ss} is a feedforward control.

3.4.1 Integral Control

The state feedback control with constant gain feedback is generally useful only for stabilized systems for which the system does not track inputs. In general, most control systems must track inputs to achieve zero steady state error. One solution to this problem is to introduce integral control together with constant gain state feedback [16].

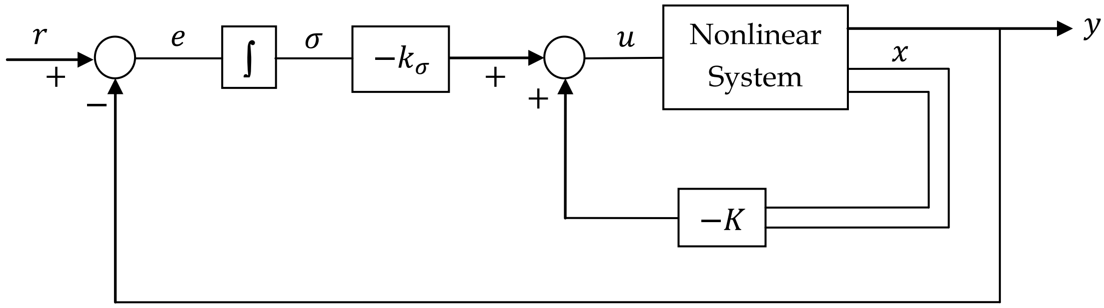

Figure 3.2: State feedback with integral control.

When including the integral action together with a system as shown in figure 3.2, a new additional state variable σ is produced, and then the nonlinear augmented state model can be written as:

$$\begin{aligned} \dot{x} &= f(x,u) \\ \dot{\sigma} &= h(x) - r \end{aligned} \tag{3.18}$$

At the operating point (x_{ss}, σ_{ss}), the following equation should be satisfied:

$$\begin{aligned} 0 &= f(x_{ss}, \sigma_{ss}) \\ r &= h(x_{ss}) \end{aligned} \tag{3.19}$$

The linearization of the augmented system (3.19) about (x_{ss}, σ_{ss}) is obtained as:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\sigma} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ \sigma \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \end{bmatrix} r \\ y &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \sigma \end{bmatrix} \end{aligned} \tag{3.20}$$

The designed linear tracking controller takes the form

$$u = -K_x x + k_\sigma \sigma \tag{3.21}$$

Substituting for u from equation (3.21) into system (3.20) and making some simplifications, the closed loop system can be written as:

$$\begin{bmatrix} \dot{x} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} (A - BK_x) & Bk_\sigma \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} r$$
$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \sigma \end{bmatrix} \tag{3.22}$$

From (3.22), the tracking problem now is reduced to design the gains vectors$K_x$and$k_\sigma$to place the closed loop poles at the desired locations.

3.4.2 Designing of a Tracking Controller for a Special case of 2nd Order Nonlinear System

Consider a special case of 2nd order nonlinear system described in equation (3.6) whose linearization about (x_{ss}, u_{ss})found in equation (3.7). By adding the integral control, the augmented state space model becomes,

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & 0 \\ c_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} r \tag{3.23}$$

From (3.23), the feed forward control input can be evaluated as:

$$u_{ss} = -\frac{a_1}{b_1} x_{ss}$$

The designed feedback control law takes the form,

$$u = -k_1 x_1 - k_2 x_2 - k_\sigma \sigma = -\begin{bmatrix} k_1 & k_2 & k_\sigma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \sigma \end{bmatrix} \tag{3.24}$$

Substituting for u from (3.24) in (3.23) the closed loop system as shown in figure 3.3 becomes,

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & 0 \\ c_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -b_1 k_1 & -b_1 k_2 & -b_1 k_\sigma \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \sigma \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} r \tag{3.25}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ a_1 - b_1 k_1 & a_2 - b_1 k_2 & -b_1 k_\sigma \\ c_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} r$$

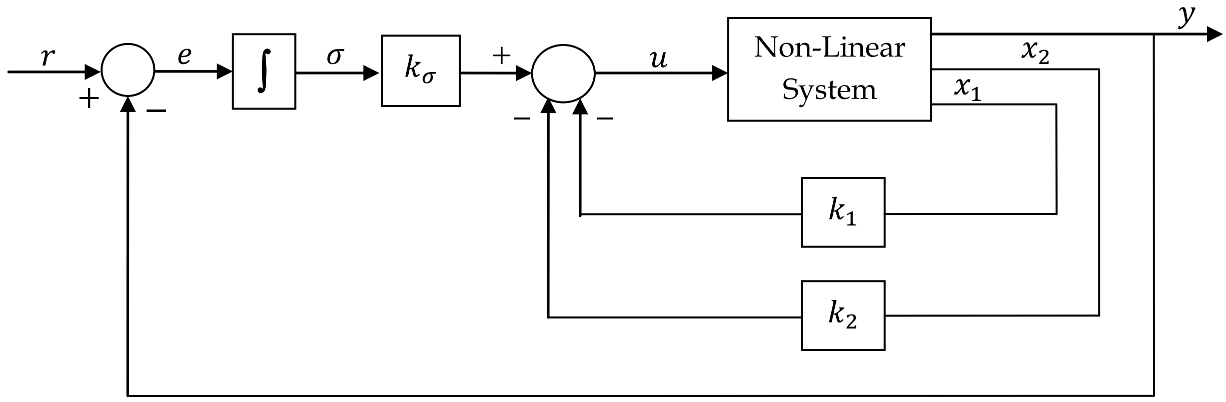

Figure 3.3: Tracking controller for 2nd order nonlinear system.

The characteristic equation of the closed loop system (3.25) is:

$$\det[SI - (A - BK)] = \left\| \begin{bmatrix} S & -1 & 0 \\ b_1 k_1 - a_1 & b_1 k_2 - a_2 & b_1 k_\sigma \\ -c_1 & 0 & S \end{bmatrix} \right\| \tag{3.26}$$

$$= S^3 + (b_1 k_2 - a_2) S^2 + (b_1 k_1 - a_1) S + b_1 k_\sigma c_1$$

The selection of the desired characteristic polynomial is similar to the desired equation in (3.11) with selecting an addition real pole that locates far enough from the real part of the desired poles in the same equation.

The additional pole can be selected as:

$$S_3 = -10 \rho \omega_n$$

Then, the desired characteristic polynomial becomes,

$$\Delta(S) = (S + 10\rho\omega_n)(S^2 + 2\rho\omega_n S + \omega_n^2)$$
$$(S^3 + 12\rho\omega_n S^2 + (1 + 20\rho^2){\omega_n}^2 + 10\rho\omega_n^3) \tag{3.27}$$

The matching of coefficients method can be used to design the state feedback controller by equating the coefficient of (3.27) with (3.26) the results are:

$$b_1k_1 - a_1 = (1 + 20\rho^2){\omega_n}^2 \rightarrow k_1 = \frac{(1 + 20\rho^2)\omega_n^2 + a_1}{b_1}$$
$$b_1k_2 - a_2 = 12\rho\omega_n \rightarrow k_2 = \frac{12\rho\omega_n + a_2}{b_1} \quad (3.28)$$
$$b_1k_\sigma c_1 = 10\rho\omega_n^3 \rightarrow k_\sigma = \frac{10\rho\omega_n^3}{c_1b_1}$$

From (3.28), the designed vector of gains is:

$$K = [k_1 \quad k_2 \quad k_\sigma] = \left[\frac{(1 + 20\rho^2)\omega_n^2 + a_1}{b_1} \quad \frac{12\rho\omega_n + a_2}{b_1} \quad \frac{10\rho\omega_n^3}{c_1b_1}\right] \quad (3.29)$$

Finally, the procedure of designing the tracking controller for a special case of 2nd order nonlinear system can be summarized in two steps as follows:

1- Specify the values of two parameters of the performance requirements: settling time, t_s and the overshoot, $O.S$, thus, evaluate the values of damping ratio ρ and natural frequency ω_n,
Where:

$$\rho = \sqrt{\frac{(ln(O.S))^2}{(ln(O.S))^2 + \pi^2}} \ , \qquad \omega_n = \frac{4}{\rho * t_s} \quad (3.30)$$

2- Substitute the values of ρ and ω_n from step (1) and the values of coefficients $a_1, b_1, and\ c_1$ of the linearized state space model (3.7) all into (3.29) to evaluate the designed vector of gains K.

3.5 Gain Scheduled Controller

The design of a stabilizing feedback controller via linearization achieves local stabilization, and the region of attraction cannot be estimated. The gain scheduling is a technique aims to extend the region of validity of linearization by solving the stability problem at different operating points and allowing the controller to move from one design to another in a smooth or abrupt way.

To achieve the gain scheduling goal, we may linearize the system at several equilibrium points, design a linear feedback controller at each point, and implement the resulting family of linear controllers as a single controller whose parameters are changed by monitoring the scheduling variables.

To design a gain scheduled controller, the following steps can be used [2].

1- Linearizing the nonlinear model about the family of operating points, parameterized by the scheduling variables.
2- Designing a parameterized family of linear controllers to achieve the specified performance for the parameterized family of linear systems at each operating point.
3- Constructing a gain scheduled controller such that, at each constant operating point, the controller provides a constant control value yielding zero error, the linearization of the closed-loop nonlinear system at each operating point is the same as the feedback connection of the parameterized linear system and the corresponding linear controller.
4- Checking nonlocal performance of the gain scheduled controller for the nonlinear model by simulation.

Designing of a Gain Scheduled Tracking Controller for a Special Case of 2nd Order Nonlinear System:

Consider the nonlinear system described in equation (3.6),

$$\dot{x_1} = x_2$$
$$\dot{x_2} = f_2(x_1, x_2, u)$$
$$y = h(x_1)$$

In the tracking problem, the controller stabilizes the system at multi operating points; these points can be formulated as a function of scheduling variables. Let α be the scheduling variable, the family of parameterized operating points can be written as:

$$\left(x_{ss}(\alpha), u_{ss}(\alpha)\right)$$

Linearization of the nonlinear system (3.6) about $(x_{ss}(\alpha), u_{ss}(\alpha))$can be written as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a_1(\alpha) & a_2(\alpha) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_1(\alpha) \end{bmatrix} u$$
$$y = [c_1(\alpha) \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{3.31}$$

When$r = \alpha = constant$, the following equations must be satisfied:

$$\alpha = c_1(\alpha) x_{1ss}(\alpha)$$
$$u_{ss}(\alpha) = -\frac{a_1(\alpha)}{b_1(\alpha)} x_{1ss}(\alpha) = -\frac{a_1(\alpha)}{c_1(\alpha) b_1(\alpha)} \alpha \tag{3.32}$$

When the integral control is combined with the system, the scheduled augmented model becomes,

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_1(\alpha) & a_2(\alpha) & 0 \\ c_1(\alpha) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 \\ b_1(\alpha) \\ 0 \end{bmatrix} u$$
$$y = [c_1(\alpha) \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ \sigma \end{bmatrix} \tag{3.33}$$

The designed gain scheduled state feedback controller is given by:

$$u = -K_x(\alpha) x + k_\sigma(\alpha) \sigma \tag{3.34}$$

From the above analysis, the designed gain scheduled gains vector $K(\alpha)$ for the family of parameterized operating points $(x_{ss}(\alpha), u_{ss}(\alpha))$ can be computed as the result of gains vector obtained in (3.29) with the allowance of its gains to be parameterized by the scheduling variable α, thus, $K(\alpha)$ can be written as:

$$K(\alpha) = [k_1(\alpha) \quad k_2(\alpha) \quad k_\sigma(\alpha)]$$
$$= \begin{bmatrix} \frac{(1+20\rho^2)\omega_n^2 + a_1(\alpha)}{b_1(\alpha)} & \frac{12\rho\omega_n + a_2(\alpha)}{b_1(\alpha)} & \frac{10\rho\omega_n^3}{c_1(\alpha) b_1(\alpha)} \end{bmatrix} \tag{3.35}$$

3.6 Relationship between State Feedback and PID Control

The PID controller is the most common controller that used in the design of the continuous data control systems because of its simple structure and robust performance in a wide range of operating condition.

In time domain, the PID controller can be written as:

$$\begin{aligned} u(t) &= \left(K_P\, e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t) \right) \\ &= u_P(t) + u_I(t) + u_D(t) \end{aligned} \qquad (3.36)$$

Where $e(t)$ is the error signal or the difference?

$$e(t) = r(t) - y(t) \qquad (3.37)$$

Between the reference signal (set-point)$r(t)$and the output$y(t)$of the controlled system.

The transfer function of PID controller in the S-domain is:

$$G_{PID}(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} \qquad (3.38)$$

As shown in figure (3.4) , the PID controller has three parts: the proportional part $u_P(t)$ is proportional to the error signal, the integral part $u_I(t)$ removes the steady-state error, and the derivative part $u_D(t)$ reduces the overshoot.

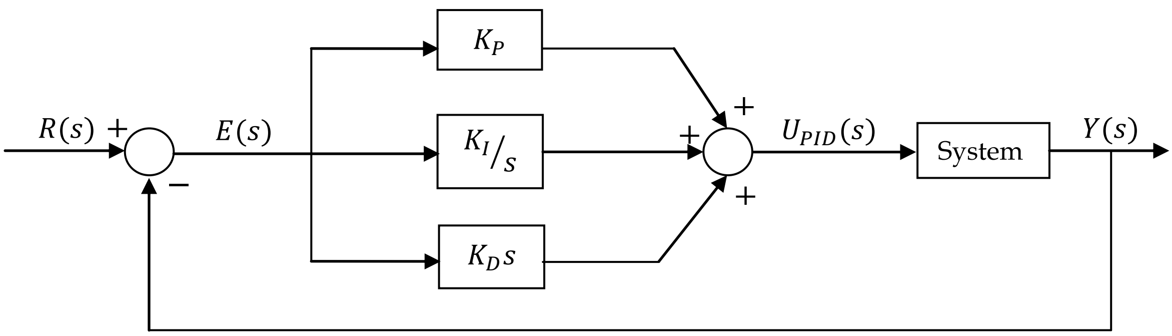

Figure 3.4: A continuous data PID controller.

The designed gain scheduling state feedback controller of a special case 2nd order non-linear system as shown in figure 3.5 can be transformed into a scheduled PID controller with a pre-filter applied on the reference input, such that the gains of the state feedback controller are equivalent to the gains of the

PID controller. The transformation between the two types of controllers can be described as follows:

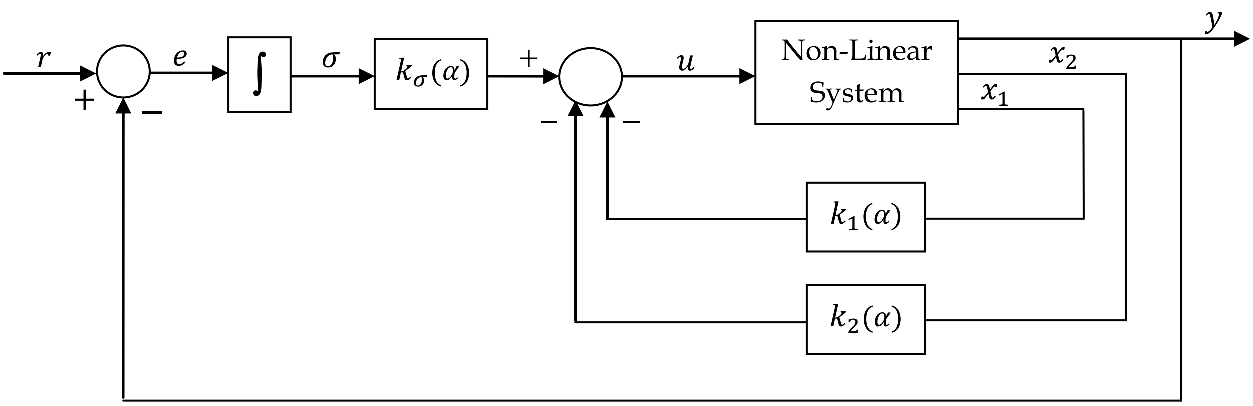

Figure 3.5: Gain scheduling state feedback controller

Since from system (3.7) that $x_2 = \dot{x}_1$, then by taking the Laplace transform, we get

$$x_2 = sx_1 \tag{3.39}$$

Let the feedback controller from the states only can be written as

$$u_x = -k_1(\alpha)x_1 - k_2(\alpha)x_2 \tag{3.40}$$

Substituting forx_2 form (3.39) into (3.40), the feedback control from the states can be expressed as:

$$u_x = -(k_1(\alpha) + k_2(\alpha)s)x_1 \tag{3.41}$$

Then the overall feedback control is:

$$u = u_x - k_\sigma(\alpha)\sigma \tag{3.42}$$

And figure 3.5 becomes:

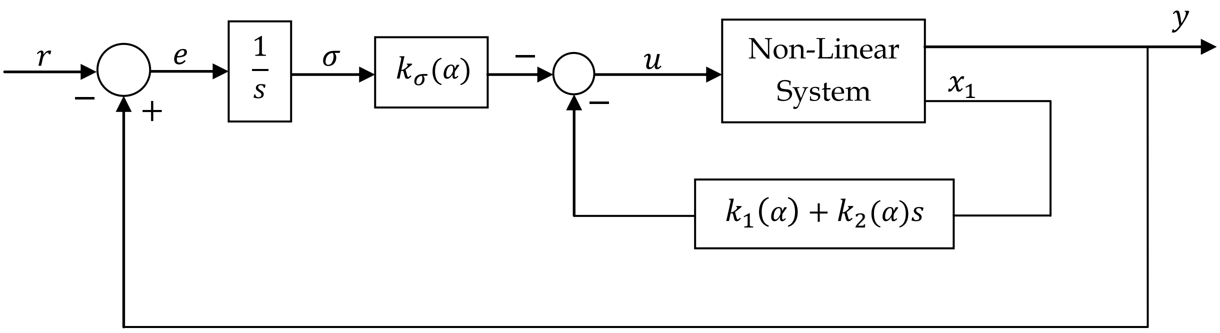

Figure 3.6: State feedback from σ and x_1.

From the output equation of system (3.7) we get

$$x_1 = \frac{y}{c_1(\alpha)} \tag{3.43}$$

Substituting for x_1 in equation (3.41), then

$$u_x = -\left(\frac{k_1(\alpha)}{c_1(\alpha)} + \frac{k_2(\alpha)}{c_1(\alpha)}s\right)y \tag{3.44}$$

And figure 3.6 becomes:

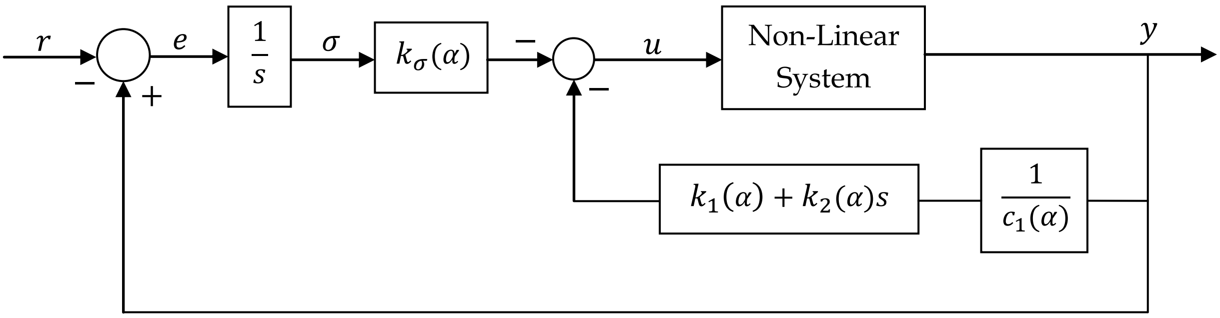

Figure 3.7: State feedback from σ and y

Now, by moving the integral control block left past the summing junction, we get:

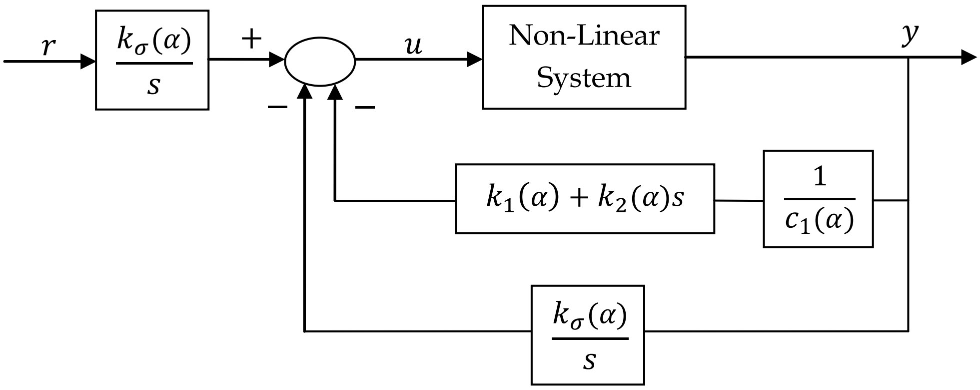

Figure 3.8: System with integral control block left past the summing junction

Then, by summing all the feedback blocks from output together, we have:

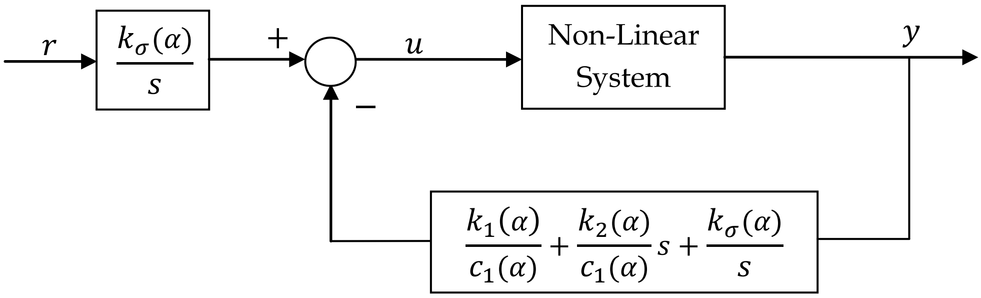

Figure 3.9: System with all feedback blocks from $y$are summed together

By moving the overall feedback block right past the summing junction, we get:

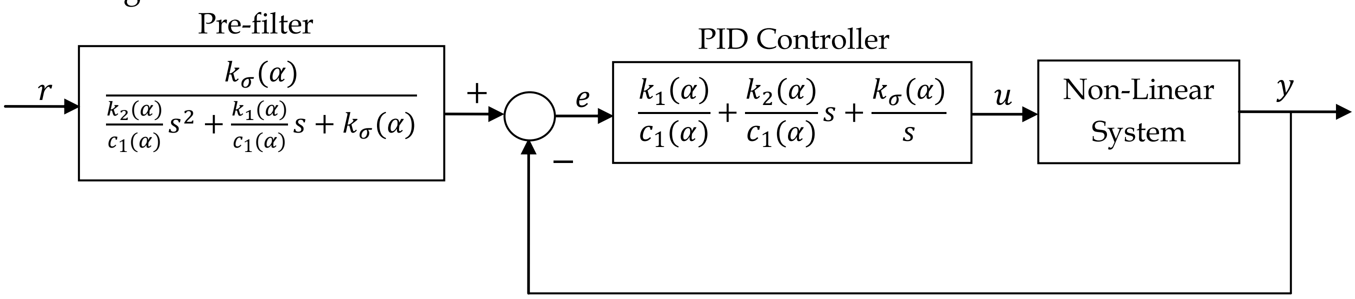

Figure 3.10: Gain Scheduled PID controller with a pre-filter.

Let:

$$K_P(\alpha) = \frac{k_1(\alpha)}{c_1(\alpha)} \tag{3.45}$$

$$K_D(\alpha) = \frac{k_2(\alpha)}{c_1(\alpha)} \tag{3.46}$$

$$K_I(\alpha) = k_\sigma(\alpha) \tag{3.47}$$

Finally, the scheduled PID controller with a pre-filter applied on the reference input produced from moving the blocks is shown as follows:

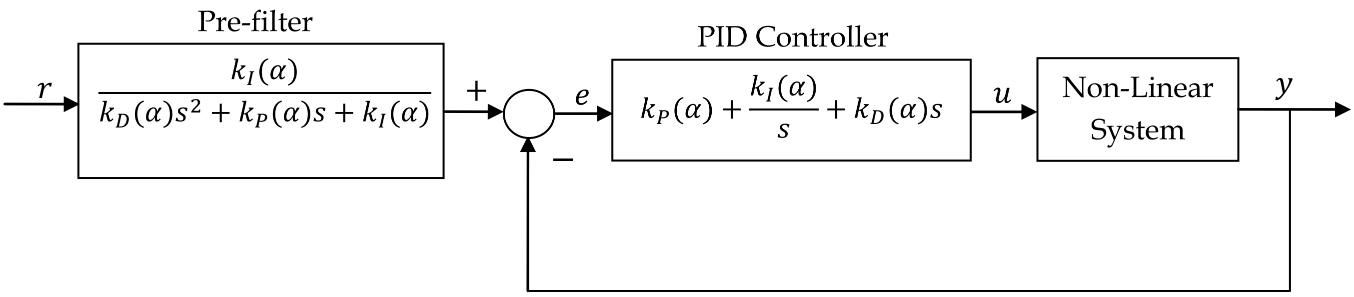

Figure 3.11: Final diagram of gain scheduled PID controller with a pre-filter.

Practical PID Structure:

The PID controller shown in figure 3.4 is not a good combination of the three controllers in practice, since if the error signal e(t) has discontinuities then the D-controller will produce very bad (even unbounded) responses. A practical combination of the three controllers is the PI+D controller shown in figure 3.11, where the system output signal *y(t)* is usually smoother than the error signal [17].

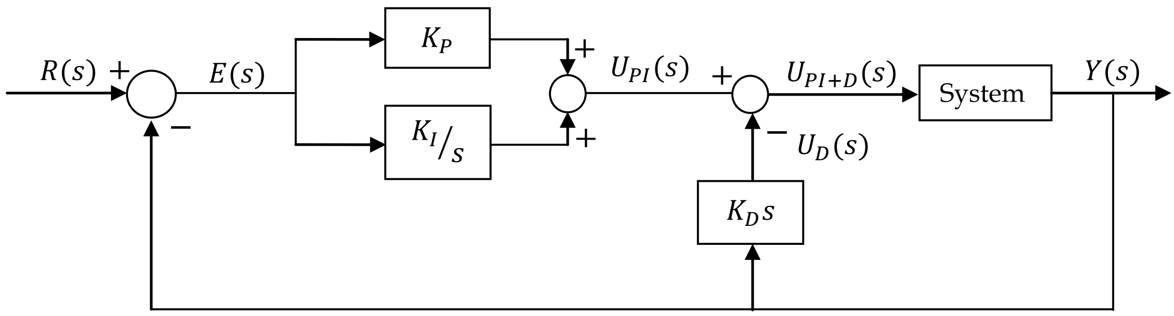

Figure 3.12: A continuous data PI+D controller.

The resulted scheduled PID controller shown in figure 3.10 can be reconstructed as shown in figure 3.12.

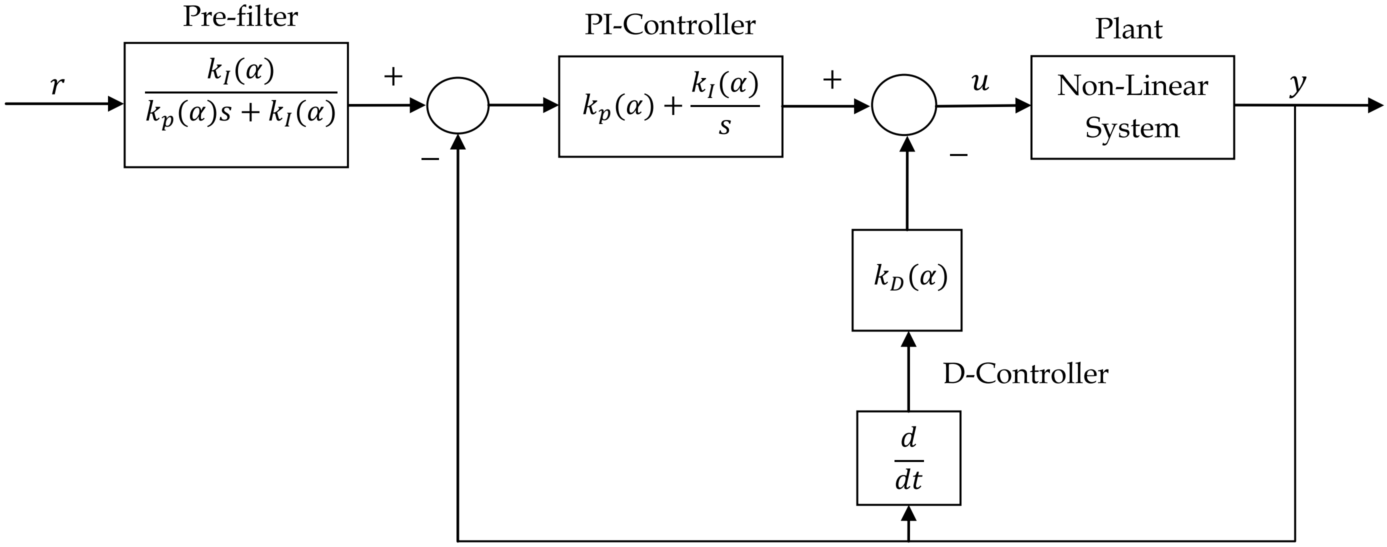

Figure 3.13: Scheduled PI+D controller with a pre-filter.

CHAPTER 4

4. DIGITALCONTROL

4.1 Introduction

Control systems in continuous-time setting in the past are usually implemented using analog devices such resistors, capacitors, inductors and operational amplifiers together with some necessary mechanical components. These devices are neither economical nor durable [18].

The revolutionary advances in computer technology today have made it possible to replace conventional controllers with digital computers. Digital control thus refers to the control scheme in which the controller is a digital device, practically a digital computer [19]. The glaring difference between analog feedback control systems and digital feedback control systems is the effect that the sampling rate has on the transient response. Changes in the sampling rate not only change the nature of the response from overdamped to underdamped, but also can turn a stable system into unstable one [1]. In general, if the sampling rate is too slow, the closed loop digital system will be unstable. According to the Nyquist criterion, the sampling frequency should at least be twice as high as the bandwidth of the error signal. This bandwidth is bounded by the system bandwidth, hence $f_s \geq 2f_B$. However, in order to guarantee satisfactory response, a factor of 10 to 20 may be required. In order to deal with a nonlinear system by a reconstructed control signal, the sampling rate may require to be increased to $40f_B$ to compensate the approximation produced from linearization and digitization processes.

Digital control offers some of the following advantages over the analog control [20]:

1- Digital components are less susceptible to aging and environmental variations.
2- They are less sensitive to noise and increase stability.
3- Changing a controller does not require an alteration in the hardware.
4- They provide improved sensitivity to parameter variations.

4.2 Digital Control System Structure

A digital control system can be described schematically as in figure 4.1. The system contains four basic elements: The plant, A/D converter, D/A converter, and the digital computer. The digital control system is a hybrid system which contains both continuous-time signals and discrete time signals. The output from the plant *y(t)* is a continuous-time signal. The output is converted into a sequence of values $\{y(kT)\}$by the analog-to-digital (A/D) converter. The digital controller processes the converted signal, and produces anew sequence of control values $\{u(kT)\}$ using an algorithm. This sequence is converted to an analog signal by a digital-to-analog (D/A) converter.

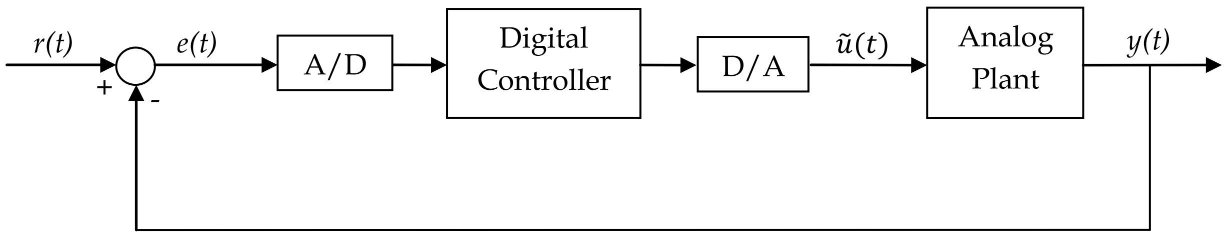

Figure 4.1: Digital control system.

4.2.1 Analog to Digital Conversion

The discrete signal is obtained by sampling the continuous time signal at regular interval. In the sampled data form, the A/D device is approximated by an ideal sampler which may be considered as a switch. When the switch is closed for a short duration of time, a signal is available at the output and zero otherwise. As shown in figure 4.2, a continuous signal $e(t)$ is sampled by a sampler and the output is a sequence of values $e(KT)$.

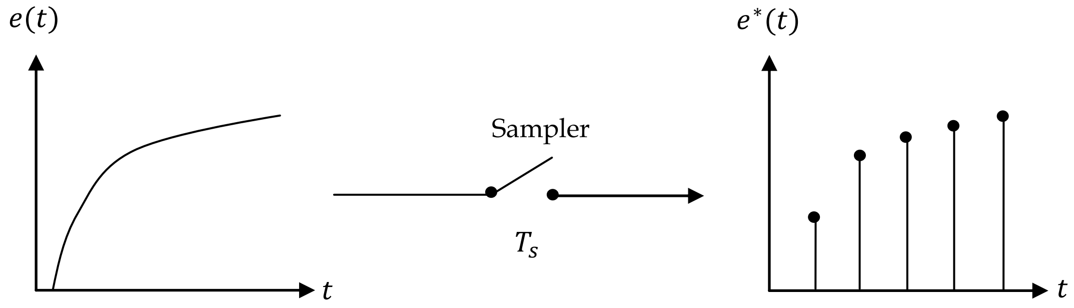

Figure 4.2: Continuous signal sampled by a sampler.

4.2.2 Digital to Analog Conversion

Digital to Analog Conversion is a reconstruction process that converts the digital samples to an analog output. The most common of reconstruction forms used in practice is the zero-order-hold (ZOH). The ZOH device holds the sampled value of $f(kT)$ for $(kT) \leq t \leq (k+1)T$ until the next sample value $f[(k+1)T]$ arrives. From figure 4.3, we can derive the transfer function of the (ZOH) as:

$$g_{zoh}(t) = u(t) - u(t-T) \tag{4.1}$$

By taking the Laplace transform, the result is:

$$G_{zoh}(s) = \frac{1 - e^{-sT}}{s} \tag{4.2}$$

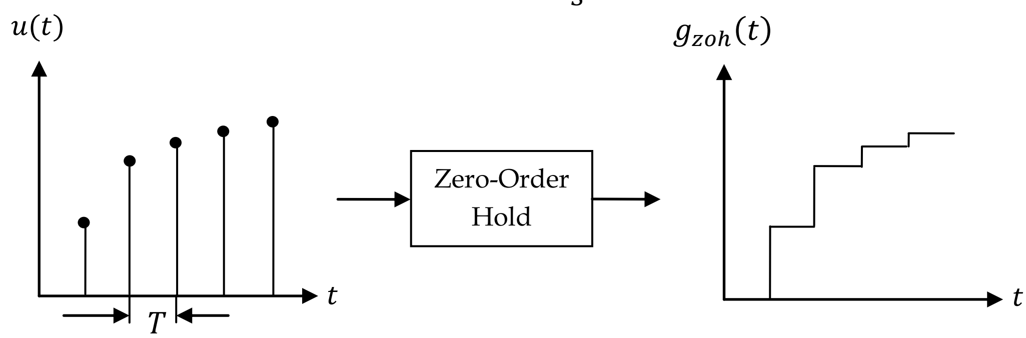

Figure 4.3: Sampled signal as an input to the ZOH.

4.3 Digital Controller Design

A large number of control systems in operation in industry are continuous data systems. However, as the technology of digital computers becomes more advanced, refitting these systems with digital controller is often desirable. Rather than carrying out a completely new design using digital control theory, it is possible to apply the digital redesign technique to arrive at an equivalent digital system [21]. Digital redesign is the procedure of finding the digital equivalent of an analog controller.

The digital equivalent of a continuous data system as shown in figure 4.4 and figure 4.5 can be found by using the following three steps:

1- Insert sampler and zero order hold device in the continuous data system.
2- Select a small sampling time T, so that the dynamic characteristics of the analog controller are not lost through the digitization.
3- Approximate the analog controller by a transfer function in the z domain.

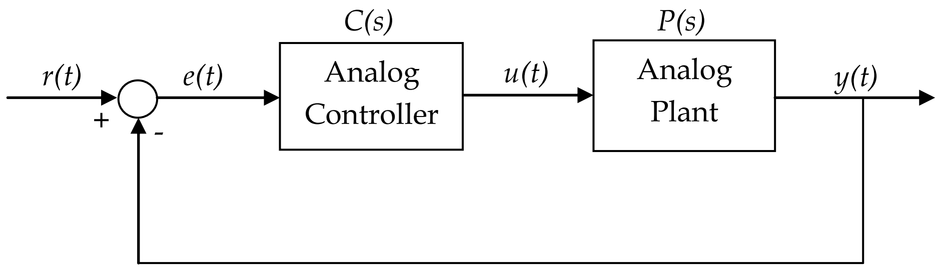

Figure 4.4: Analog control system.

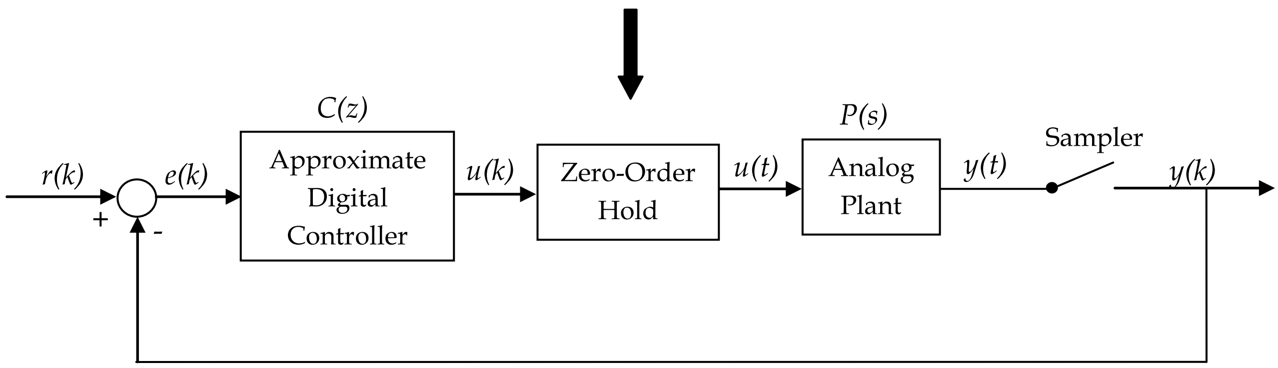

Figure 4.5: Equivalent digital control system.

4.3.1 Digital Implementation of PID Controller

First, we design a continuous-time PID controller that meets all design specifications and then discretize it using one of discretization technique to obtain an equivalent digital controller.
The PID controller in the continuous data system is described by:

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s \tag{4.3}$$

The proportional component K_P is implemented in digital form by the same constant gain K_P, there are a number of discretization formulas that can be used to implement the integral $U_I(s)$ and derivative $U_D(s)$ controllers by the approximate digital controllers $U_I(z)$ and $U_D(z)$, a three basic formulas are:

1- The forward divided difference :

$$s = \frac{z-1}{T}$$

This formula maps the left halfs-plane onto more than just the unit Circle. Therefore, Controller C(s), with high frequency or lightly damped poles will give unstable C(z), thus; this formula does not guarantee stability.

2- The backward divided difference:

$$s = \frac{1-z^{-1}}{T}$$

This formula maps the left halfs-plane onto the region as a part inside the unit circle. So stable C(s) implies stable C(z). But, C(z) cannot have lightly damped poles, even if C(s) had lightly damped poles.

3- The trapezoidal formula:

$$s = \frac{2(z-1)}{T(z+1)}$$

This formula maps the entire left-half (right-half) s-plane into the entire inner (outer) unit circle in the z-plane in such a way that mapping is one-to-one.

Hence, this formula is one of the most used formulas and also known as the bilinear transform [17].

By applying the bilinear transform to the continuous PID controller, the result in digital controller is:

$$U(z) = \frac{K_D\left(\frac{2(z-1)}{T(z+1)}\right)^2 + K_P\left(\frac{2(z-1)}{T(z+1)}\right) + K_I}{\left(\frac{2(z-1)}{T(z+1)}\right)} E(z) \tag{4.4}$$

After making some simplifications the result becomes,

$$U(z) = \frac{(K_I T^2 + 2K_P T + 4K_D)z^2 + 2(K_I T^2 - 4K_D)z + (K_I T^2 - 2K_P T + 4K_D)}{2T(z^2 - 1)} E(z) \tag{4.5}$$

The discrete-data control system with digital PID controller is shown figure 5.5

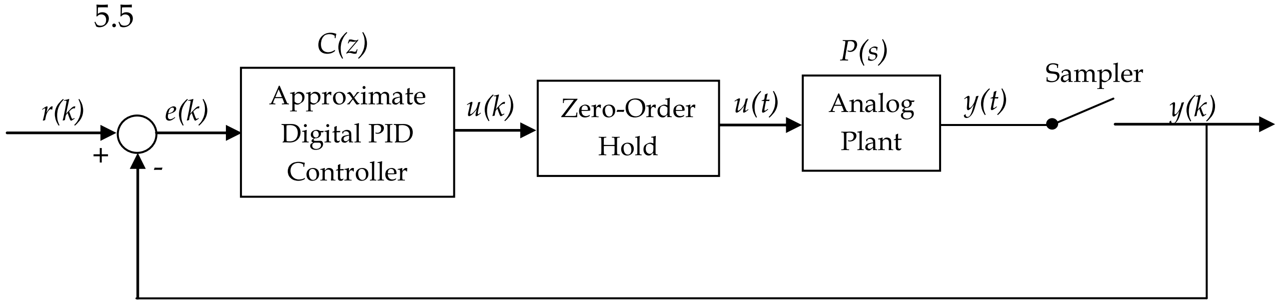

Figure 4.6: Approximate digital PID control system.

From chapter 3, a practical combination of the three components of the PID controller is the PI+D controller .The equation of PI+D Controller in S-domain is:

$$U_{PI+D}(s) = U_{PI}(s) + U_D(s) = \left(K_P + \frac{K_I}{s}\right)E(s) + (K_D s)Y(s) \tag{4.6}$$

To implement the equivalent digital PI+D controller as shown in figure 4.7, the bilinear transformations is applied to each controller separately:

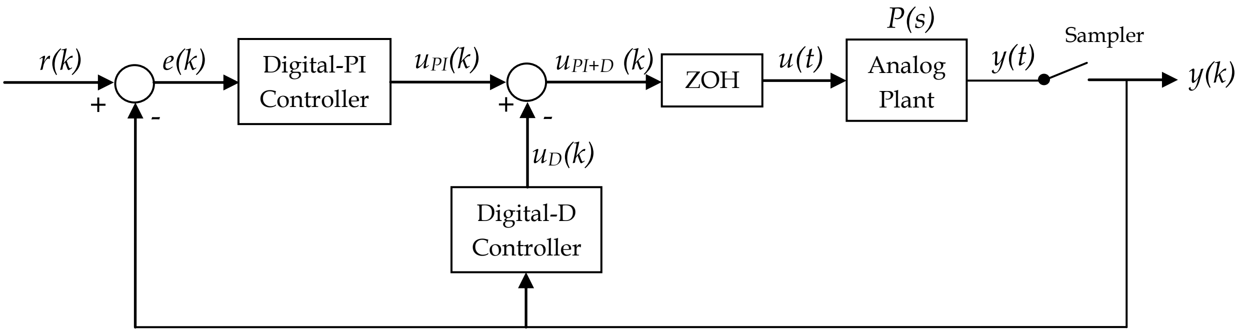

Figure 4.7: Approximate digital PI+D control system.

First, for Digital-PI Controller, we have

$$U_{PI}(z) = K_P E(z) + \frac{K_I}{\frac{2(z-1)}{T(z+1)}} E(z) \tag{4.7}$$

Then,

$$U_{PI}(z) = \frac{(2K_P + K_I T)z + (K_I T - 2K_P)}{2(z-1)} E(z) \tag{4.8}$$

Let

$$\widetilde{K}_P = K_P - \frac{K_I T}{2} \tag{4.9}$$

And

$$\widetilde{K}_I = K_I T \tag{4.10}$$

Where $\widetilde{K}_P$ and $\widetilde{K}_I$ are the gains of the digital PI controller, substituting from equation (4.9) and (4.10) into (4.8) and making some simplifications, then

$$(1 - z^{-1})\widetilde{U}_{PI}(z) = \widetilde{K}_P (1 - z^{-1}) E(z) + \widetilde{K}_I E(z) \tag{4.11}$$

From the inverse z-transform, we obtain

$$u_{PI}(kT) - u_{PI}(kT - T) = \widetilde{K}_P [e(kT) - e(kT - T)] + \widetilde{K}_I e(kT) \tag{4.12}$$

Equation (4.12) can be implemented in a digital computer as shown in figure 4.8

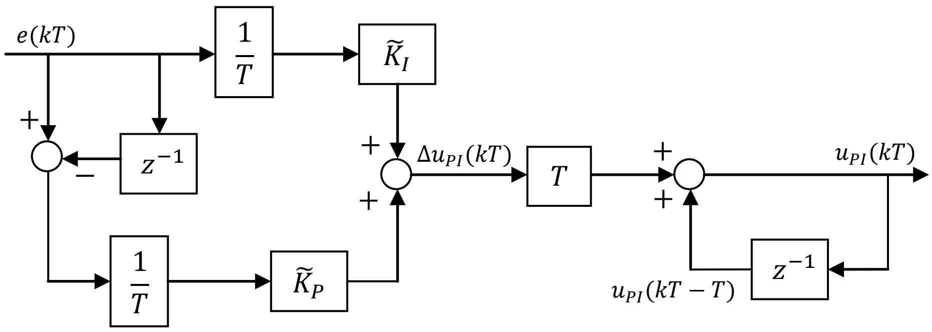

Figure 4.8: Block diagram of the digital-PI control system.

Second, for Digital-D controller

$$U_D(z) = K_D \frac{2(z-1)}{T(z+1)} E(z) \tag{4.13}$$

Let

$$\widetilde{K}_D = 2K_D \tag{4.14}$$

Where $\widetilde{K}_D$ is the gain for the digital D controller, substituting from equation (4.14) into (4.13), then?

$$(1+z^{-1})U_D(z) = \frac{\widetilde{K}_D}{T}(1-z^{-1})Y(z) \tag{4.15}$$

The inverse z-transform gives:

$$u_D(kT) + u_D(kT-T) = \frac{\widetilde{K}_D}{T}[y(kT) - y(kT-T)] \tag{4.16}$$

Equation (4.16) can be implemented in a digital computer as shown in figure 4.9

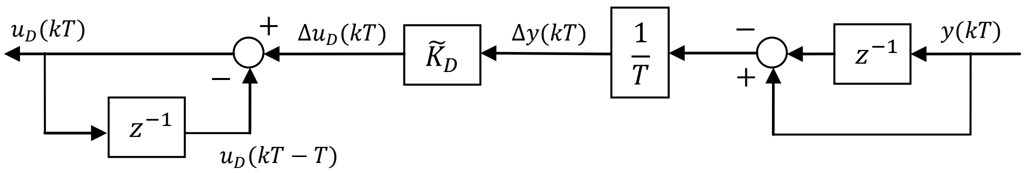

Figure 4.9: Block diagram of the digital-D control system.

4.3.2 Digital Gain Scheduled PI+D Tracking Controller

As illustrated in chapter 3, the tracking problem is a problem of stabilizing the nonlinear system at a sequence of operating points. To extend the region of validity for the digital PI+D controller to work well at each point of the sequence, the gains $\widetilde{K}_P, \widetilde{K}_I, and\ \widetilde{K}_D$of the digital PI+D controller must be parameterized by the scheduling variableα, and then the scheduled digital-PI controller can be written as:

$$u_{PI}(kT) - u_{PI}(kT-T) = \widetilde{K}_P(\alpha)[e(kT) - e(kT-T)] + \widetilde{K}_I(\alpha)e(kT) \tag{4.17}$$

And the scheduled digital-D controller is:

$$u_D(kT) + u_D(kT-T) = \frac{\widetilde{K}_D(\alpha)}{T}[y(kT) - y(kT-T)] \tag{4.18}$$

CHAPTER 5

5. FUZZY LOGIC CONTROL

5.1 Introduction

In Language, the word “fuzzy” means “not clear”, but in the technical sense, fuzzy systems are precisely defined systems, and fuzzy control is a precisely defined method of non-linear control. The main objective of fuzzy logic is to make computers think like humans and to enable computing with words [22]. The key components of fuzzy system’s knowledge base are a set of IF-THEN rules obtained from human knowledge and expertise.

In 1965, Professor L.A. Zadeh of University California, Berkley, presented his first paper on fuzzy set theory [23]. He developed a mathematical way of looking at vagueness. Mamdaniand Takagi with Sugeno had published their first papers for fuzzy applications in 1974 and 1985 respectively.

The applications of Fuzzy Logic Controllers (FLCs) have dramatically increased since 1970 to now[24], these applications includes performance evaluation, medical diagnosis, product quality control, commercial trading, and industrial applications, the main intension of using fuzzy logic in these applications is to provide some efficient methodologies for computer based diagnosis, information processing, decision making, as well as approximate reasoning in designing practical computational schemes using imprecise criteria and inaccurate data to solve some real applications problems.

5.2 Why Using Fuzzy Logic Control?

Due to the need of modeling the human thinking process, models that attempt to emulate the natural language should be used. These models can be implemented by using fuzzy logic control.

All physical processes are non-linear and to model them, a reasonable amount of approximation is necessary. For simple systems, mathematical expressions give precise descriptions of the system behavior. But for complex systems where not much numerical data exists, fuzzy logic control can be used without making analysis for these systems.

The strength of fuzzy logic control is that it makes use of linguistic variables rather than numerical variables to represent imprecise data. The flexibility of using fuzzy logic control is also a good benefit especially when changes occurred in the system.

5.3 Fuzzy Sets

In the classical set theory, an element in the universe has well defined membership or non-membership to a given set. By contrast, in fuzzy set theory, an entity may have a certain degree of membership belonging to a set, and in the meantime it has a certain degree of non-membership. These membership and non-membership with respect to a particular set do not have to sum-up to one (or 100%) since it is not probability as shown in figure 5.1. For more detailed description of fuzzy sets and the set operations that can be performed on them, see references [17, 25, 26].

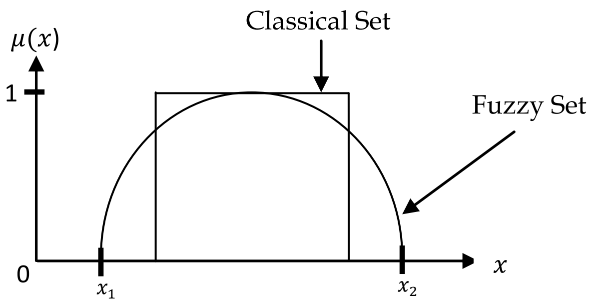

Figure 5.1: Classical and fuzzy sets

5.4 Membership Functions

A membership function is a continuous function in the range of 0 to 1, it is used to characterize a fuzzy set. It consists of members with varying degrees of membership based on the values of the membership function. In mathematical terms, the fuzzy set S in the universe U can be represented as a set of ordered pairs of an element x and its membership function $\mu_S(x)$. Formally we have:

$$S = \{(x, \mu_S(x)) | x \in U, \text{where } U \text{ is continuous}\} \quad (5.1)$$

Also, the membership function is a nonnegative-valued function. It differs from the probability density functions in that the area under the curve of a membership function need not be equal to unity (it can be any value between 0 and ∞). In general, a fuzzy membership function can have various shapes, as shown in figure 5.2 [17], depending on the concerned application.

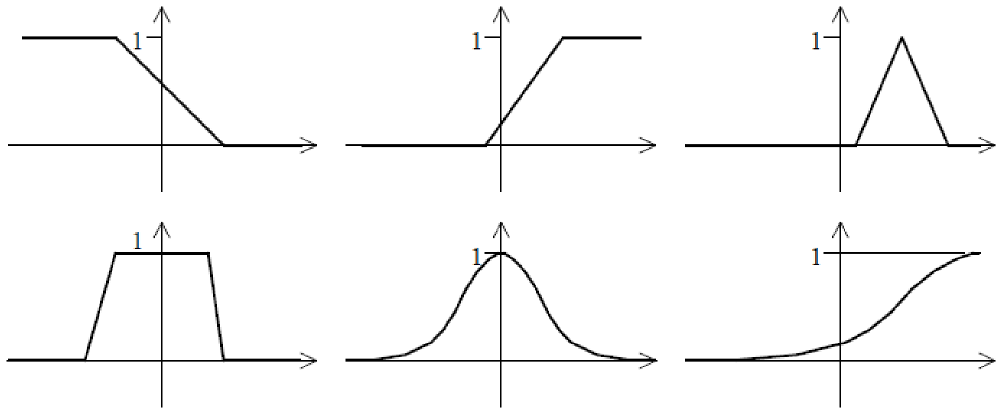

Figure 5.2: Some typical membership functions

A fuzzy set may assume two or more (even conflicting) membership values. For example, in figure 5.3, a number like s = 0.7 may be considered as "positive" with a certain (high) degree of confidence and, in the meantime, also considered as "negative" with a certain (low) degree of confidence.

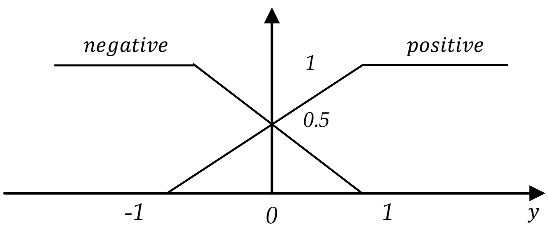

Figure 5.3: A number can be considered both "positive" and "negative"

5.5 Fuzzy Logic Controller Structure

The general continuous time, set point tracking system with a fuzzy logic controller is shown in figure 5.4. The basic parts of every fuzzy controller are shown in figure 5.5.The fuzzy logic controller (FLC) is composed of a fuzzification module, fuzzy rule base, fuzzy inference engine, and defuzzification module.

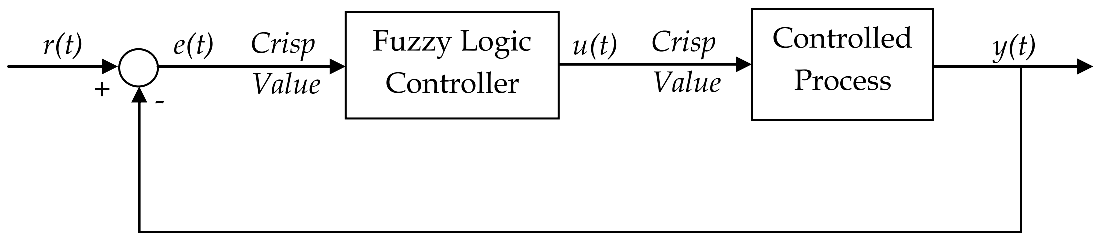

Figure 5.4: Set point tracking system with a fuzzy logic controller.

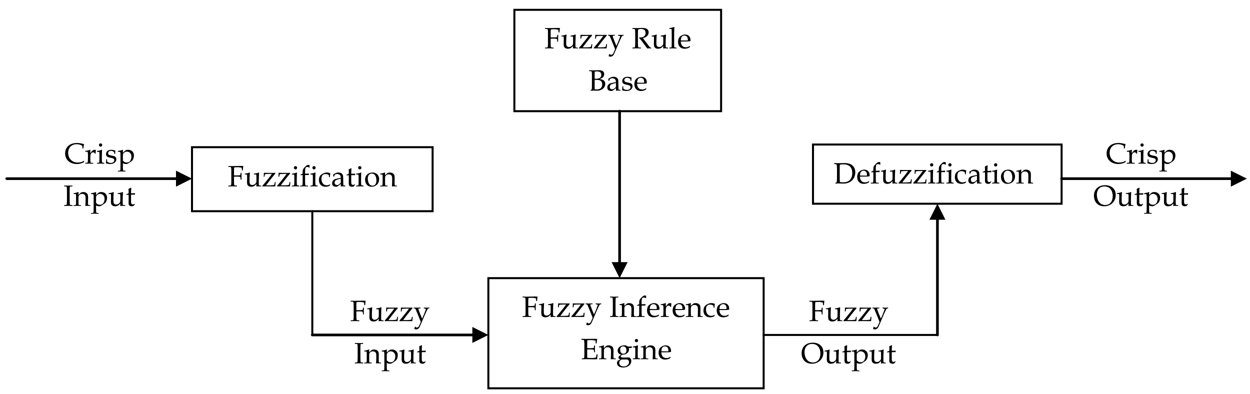

Figure 5.5: Fuzzy Logic Controller Structure

5.5.1 Fuzzification

The fuzzification module performs the following functions [17]:

1- It transforms the physical values (position, voltage, degree, etc.) of the process signal, the error signal shown in Figure 5.4 which is an input to the fuzzy logic controller, into a normalized fuzzy subset consisting of a subset (interval) for the range of the input values and a normalized membership function describing the degree of confidence of the input belonging to this range.

2- It selects reasonable and good, ideally optimal, membership functions under certain convenient criteria meaningful to the application.

In the fuzzification module, the input is a crisp physical signal (e.g., velocity) of the real process and the output is a fuzzy subset consisting of intervals and membership functions.

5.5.2 Rule Base (IF-THEN Rules)

Fuzzy logic control depends on linguistic variables as its rule base. Examples of these linguistic variables are small, medium, large, week, good and excellent. There could be a combination of these variables too, i.e. "small – week product". These IF-THEN rule statements are used to formulate the conditional statements that comprise fuzzy logic. A single fuzzy if-then rule assumes the form:

IF x is A THEN y is B

Where A and B are linguistic values defined by fuzzy sets on the ranges X and Y, respectively. The IF-part of the rule "x is A" is called the *antecedent* or premise, while the THEN-part of the rule "y is B" is called the *consequent* or conclusion. An example of such a rule might be:

IF service is good THEN tip is average

5.5.3 Fuzzy Inference Engine

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The mapping then provides a basis from which decisions can be made. The process of fuzzy inference involves all of membership functions, fuzzy logic operators, and if-then rules. There are a lot of inference methods which deals with fuzzy inference such as Mamdani method, Takagi-Sugeno method, Larsen method, Tsukamoto method. The most important and widely used methods that can be implemented in the fuzzy logic controller are the Mamdani and Takagi-Sugeno methods.

5.5.3.1 Mamdani Method

This fuzzy inference method is the most commonly used. In 1974, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators [27]. The Mamdani style fuzzy inference process is performed in four steps:

- Fuzzification of the input variable.
- Rule evaluation: By taking the fuzzified input, and applying them to the antecedents of the fuzzy rules. If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represent the result of the antecedent evaluation.
- Aggregation of the rule output: Is the process of unification of the outputs of all rules, we take the membership functions of all rule consequents and combine them into a single fuzzy set.
- Defuzzification: It will be described later.

5.5.3.2 Sugeno Method

Most fuzzy controllers have been designed, based on human operator experience and/or control engineer knowledge. It is often the case that an operator cannot tell linguistically what kind of action he takes in a particular situation. In this situation, it is useful to provide a method of modeling the control actions using numerical data [28]. In 1985 Takagi-Sugeno-Kang suggested to use a single spike, a singleton, as the membership function of the rule consequent, and they suggested another approach that using equation consequent in place off singleton consequent. Sugeno style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent, instead of a fuzzy set, he used a mathematical function of the input variable. The format of the Sugeno style fuzzy rule is

$$If\ \ X\ \ is\ \ A\ \ AND\ \ Y\ \ is\ \ B\, THEN\ \ Z\ \ is\ \ f(x,y)$$

Where X,Y and Z are linguistic variables; A and B are fuzzy sets on universe of discourses X and Y, respectively; and $f(x, y)$is a mathematical function.

5.5.4 Defuzzification

The defuzzification module is in a sense the reverse of the fuzzification module: it converts all the fuzzy terms created by the fuzzy inference engine of the controller to crisp terms (numerical values) and then sends them to the controlled process, so as to execute the control of the system. The defuzzification module performs the following functions [17]:

1- It creates a crisp, overall control signal, u, by combining all possible control outputs from the inference engine into a weighted average formula.
2- Just like the first step of the fuzzification module, this step of the defuzzification module transforms the after all control output, u, obtained in the previous step, to the corresponding physicalvalues (position, voltage, degree, etc.) that the controlled process can accept. This converts the fuzzy logic controller's numerical output to a physical means that can actually drive the given plant (process) to produce the expected outputs.

There are several commonly used defuzzification formulas such as:

1- The "center-of-gravity" formula:

$$u(t) = \frac{\int_U \mu_U(u) \cdot u du}{\int_U \mu_U(u) du} \tag{5.2}$$

Where U is the value range (interval) of the control u in the inference engine

2- The "center-of-sums" formula:

$$u(t) = \frac{\int_U u \cdot \sum_{j=1}^{n} \mu_{Uj}(u) du}{\int_U \sum_{j=1}^{n} \mu_{Uj}(u) du} \tag{5.3}$$

3- The “mean-of-maxima” formula:

$$u(t) = \frac{1}{2}\{\inf(u \in U|\mu_U(u) = max) + \sup(u \in U|\mu_U(u) = max)\} \qquad (5.4)$$

5.6 Fuzzy PID Controllers

Conventional PID controllers are the most prevailing and applicable controllers in modern industries. Simplicity, high reliability and high efficiency are the most significant advantages of PID controllers [29]. Fuzzy PID controllers are generally superior to the conventional ones, particularly for higher-order, time-delayed, nonlinear systems, uncertain systems, and for those systems that have only vague mathematical models which the conventional PID controllers are difficult, if not impossible, to handle.

In this thesis, fuzzy PID controller is used as a natural extension of the conventional one to make an enhancement for treating the nonlinearity of the system and have the following features:

1. It has the same linear structure as the conventional PID controller, but has scheduled control gains: the proportional, integral, and derivative gains are functions of scheduling parameters (input signals).
2. The controller is designed based on the classical discrete PID controller (redesigned from its continuous version using bilinear transformations), from which the fuzzy control law is derived.
3. At the end of the designing procedure, there will be no need to any of the fuzzy if-then rules, membership functions or defuzzification methods and just some simple analytical equations remain to be implemented. Since these analytical equations are simple and easy to process, there will be no need to any look-up tables, and the control procedure can be operated in real time.

In this section, fuzzy logic control concepts is used to make an enhancement for the digital PI+D gain scheduling controller that designed in chapter 4. A fuzzy logic approach proposed in [17, 30] is used.

At the first stage of designing process, we consider the digital PI+D gain scheduling controller as shown in figure 5.6 with internal structures of PI-controller and D-controller as shown in figure 5.7 and 5.8 respectively.

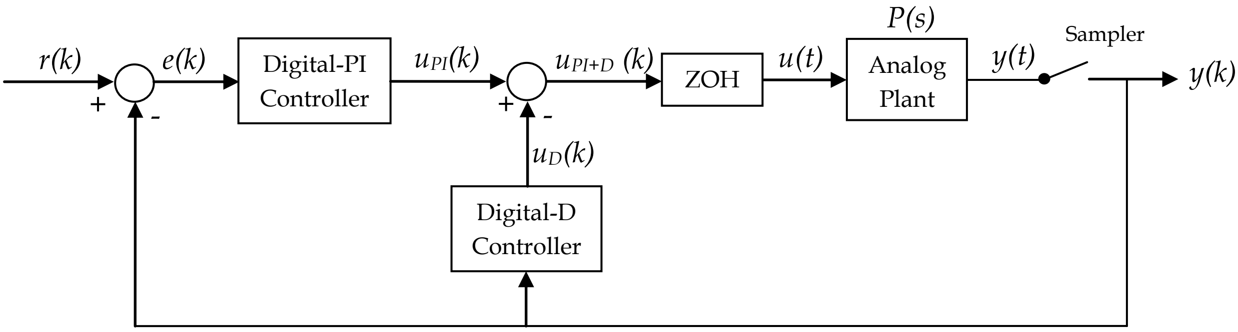

Figure 5.6: A approximate digital PI+D controller.

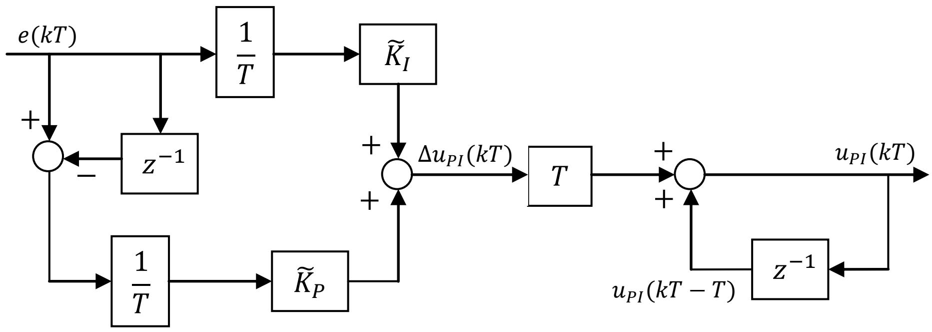

Figure 5.7: Internal structure of the digital-PI control system

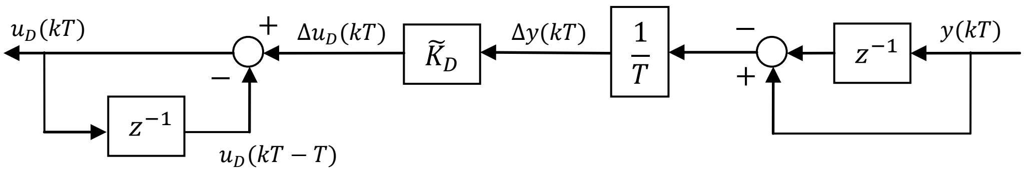

Figure 5.8: Internal structure of the digital-D control system.

The total system with the internal structures of the digital PI-controller and digital D-controller combined together is shown in figure 5.9. In this structure, There are two incremental control signals: $\Delta u_{PI}(kT)$ and $\Delta u_D(kT)$ that their analytical equations were presented in chapter 4.

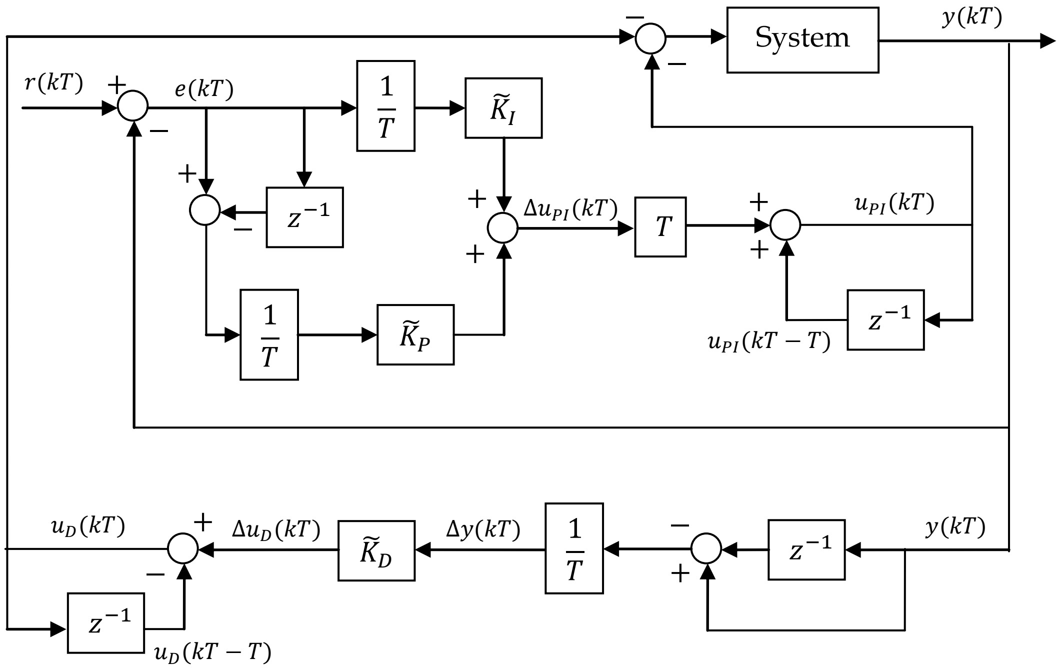

Figure 5.9: The overall block diagram of the digital PI+D control system.

To this end, the fuzzy PI and fuzzy D controllers will be inserted into figure 5.9, resulting the configuration shown in figure 5.10:

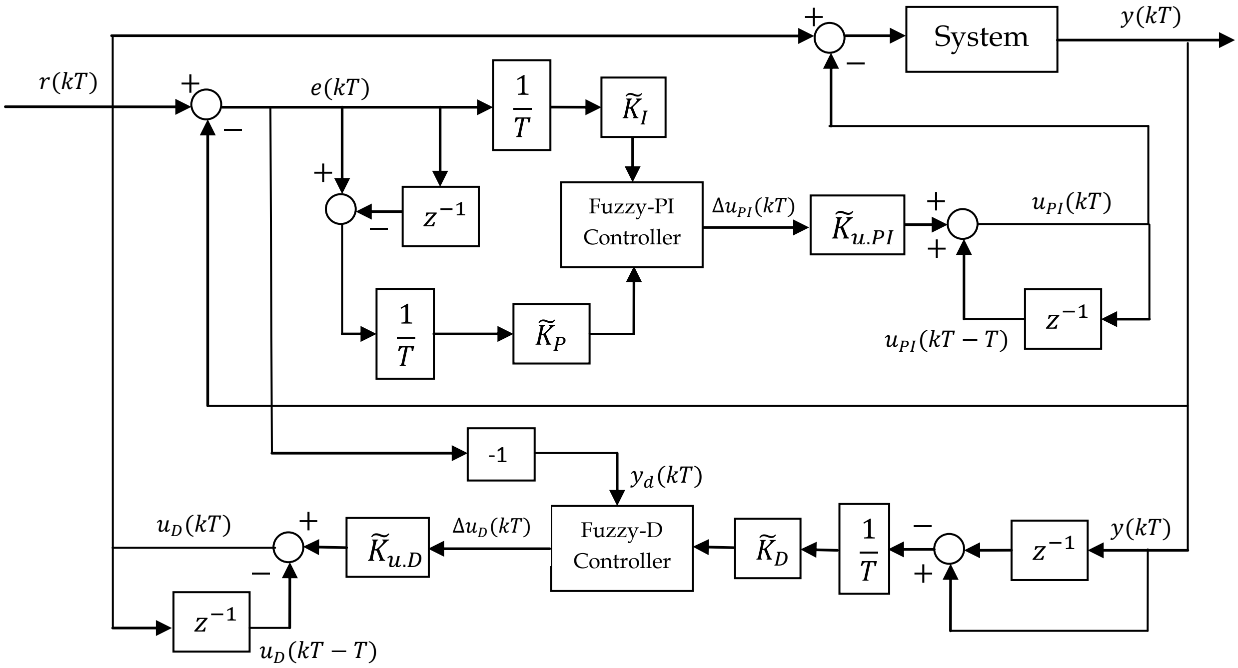

Figure 5.10: The overall block diagram of the Fuzzy PI+D control system.

The fuzzy PI+D controller is implemented by figure 5.10, in which there are five control gains: $\widetilde{K}_I, \widetilde{K}_P, \widetilde{K}_D, \widetilde{K}_{u,PI}, \widetilde{K}_{u,D}$ where $\widetilde{K}_I, \widetilde{K}_P, \widetilde{K}_D$ are scheduled gains (self-tuned) and $\widetilde{K}_{u,PI}, \widetilde{K}_{u,D}$ can be tuned or one may set $\widetilde{K}_{u,PI} = \widetilde{K}_{u,D} = T$ for simplification.

The procedure of designing fuzzy PI+D controller consists of three basic parts: fuzzification, control rule base with fuzzy inference, and defuzzification.

5.6.1 Fuzzification

We fuzzify the PI and D components of the PI+D control system individually and then combine the desired fuzzy control rules for each of them, taking into consideration the overall PI+D fuzzy control law:

$$u_{PI+D}(kT) = u_{PI}(kT - T) + \widetilde{K}_{u,PI}\Delta u_{PI}(kT) + u_D(kT - T) - \widetilde{K}_{u,D}\Delta u_D(kT) \quad (5.5)$$

The fuzzy PI-controller has two inputs: the error signal $e(kT)$ andthe rate of change of the error signal $v(kT)$ with one control output $u(kT)$ where the membership functions for inputs and output are shown in figure 5.11. Similarly, the fuzzy D-controller has also two inputs: $y_d(kT)$and$\Delta y(kT)$, and one control output $\Delta u_D(kT)$, the membership functions for input and output are shown in figure 5.12.

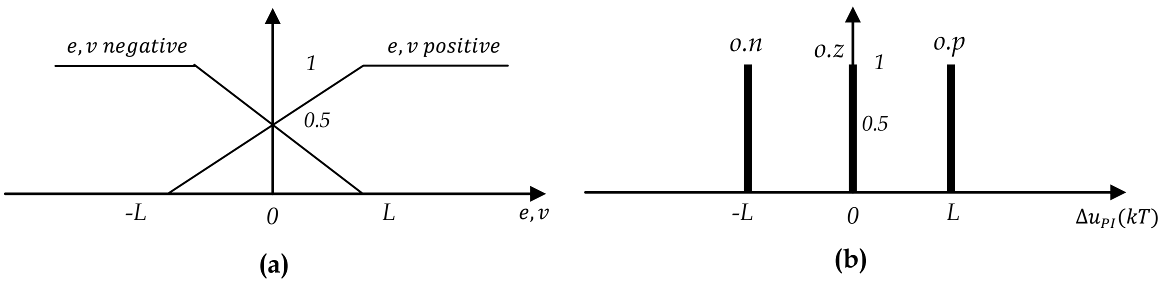

Figure 5.11: Membership functions for the PI-component, (a) Input, (b) output.

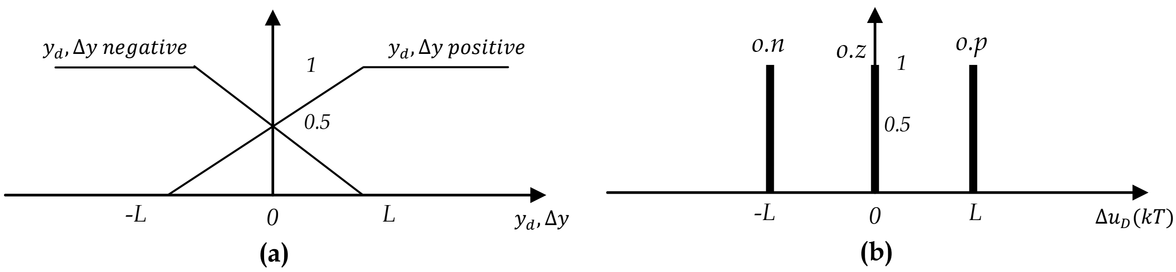

Figure 5.12: Membership functions for the D-component, (a) Input, (b) output.

Both the error and the rate have two membership values: positive and negative, while the output has three values: positive, negative, and zero. The constant $L > 0$ used in the definition of the membership functions is chosen according to the application.

5.6.2 Control Rule Base with Fuzzy Inference

Based on the aforementioned membership functions, the fuzzy control rules that used for the fuzzy PI-controller are the following:

$$R(1): IF\ error\ =\ en\ AND\ rate\ =\ vn\ THEN\ output\ =\ o.n.$$

$$R(2): IF\ error\ =\ en\ AND\ rate\ =\ vp\ THEN\ output\ =\ o.z.$$

$$R(3): IF\ error\ =\ ep\ AND\ rate\ =\ vn\ THEN\ output\ =\ o.z.$$

$$R(4): IF\ error\ =\ ep\ AND\ rate\ =\ vp\ THEN\ output\ =\ o.p.$$

Similarly, from the membership functions of the fuzzy D-controller, the fuzzy control rules that used for the fuzzy D-controller are the following:

$$R(5):\ IF\ y_d\ =\ y_dp\ AND\ \varDelta y\ =\ \varDelta_y p\ \ THEN\ D - output\ = o.z.$$

$$R(6):\ IF\ y_d\ =\ y_dp\ AND\ \varDelta y\ =\ \varDelta_y n\ \ THEN\ D - output\ = o.p.$$

$$R(7):\ IF\ y_d\ =\ y_dn\ AND\ \varDelta y\ =\ \varDelta_y p\ THEN\ D - output\ = o.n.$$

$$R(8):\ IF\ y_d\ =\ y_dn\ AND\ \varDelta y\ =\ \varDelta_y n\ THEN\ D - output\ = o.z.$$

Here, “output” is the fuzzy control action $\varDelta u(kT)$, “ep” means “errorpositive,” “oz” means “output zero,” etc. The “AND” is the logical AND defined by $\mu_A\ AND\ \mu_B\ =\ min\{\ \mu_A, \mu_B\ \}$ for any two membership values μ_A and μ_B on the fuzzy subsets A and B, respectively.

The reason for establishing the rules in such formulation can be understood as follows: First, it is important to observe that since the error signal is defined to be e = r – y, where r is the reference (set-point) and y is the system output, we have $\dot{e} = \dot{r} - \dot{y} = -\dot{y}$ in the case that the set-point r is constant. As an example, for Rule 1: condition en (the error is negative, $e < 0$) implies that $r < y$ and condition vn($\dot{e} < 0$) impliesthat ($\dot{y} > 0$), from these two conditions we note that the system output is at position $\boldsymbol{b}$ as shown in figure 5.13. In this case, the controller at the previous step is driving the system output, y, to move upward. Since the $\Delta u_{PI}(kT)$component of equation (5.5) contains more control terms with gain parameters than the D-controller, we set this term to be negative and set the $\Delta u_D(kT)$ component to be zero. Thus, the combined control action willdrive the system output downward by Rules ($R1$) and ($R5$) ofboth controllers. Rules 2, 3, and 4 are similarly determined.

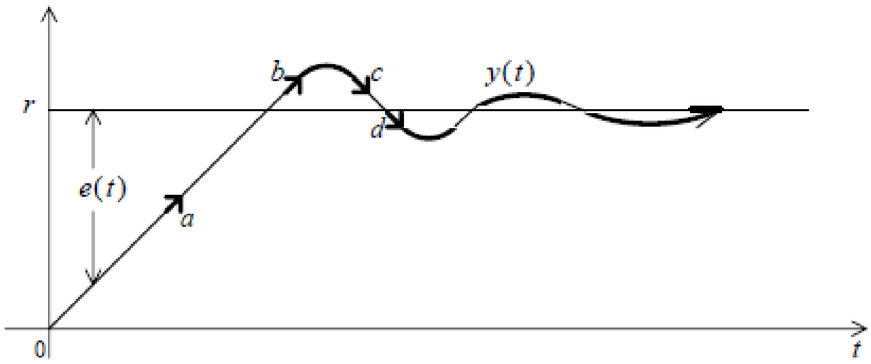

Figure 5.13: Set point tracking.

5.6.3 Defuzzification

In the defuzzification step, the centroid formula is employed for both fuzzy PI and D controllers to defuzzify the incremental control of the fuzzy control law (5.5) as:

$$\Delta u(kT) = \frac{\sum(membership\ value\ of\ input\ \times corresponding\ value\ of\ output\ )}{\sum(membership\ value\ of\ input)} \tag{5.6}$$

The overlap of the membership functions for the fuzzy PI-controller decompose their value ranges for$K_I e(kT)$, $K_P v(kT)$into twenty adjacent input-combination regions (IC1 to IC20) as shown in figure 5.14.

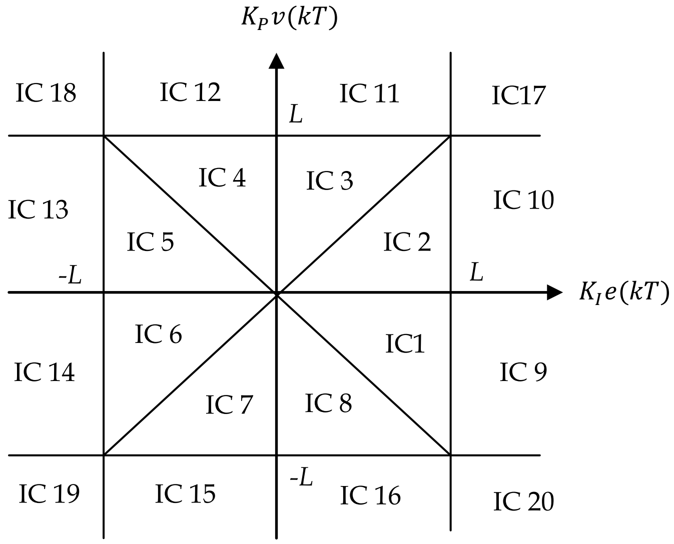

Figure 5.14: Regions for the fuzzy PI-controller, input combination values.

To understand how the twenty input combination regions are formulated, we put the membership function of the error signal (given by the curves for ep in figure 5.11(a)) over the horizontal $K_I e(kT)$ axis in figure 5.14, and put the membership function of the rate of change of the error signal (given by the curves for ev in figure 5.11(a))over the vertical $K_P v(kT)$ axis in Figure 5.14.These two membership functions then overlap and form the third-dimensional picture as shown in figure 5.15:

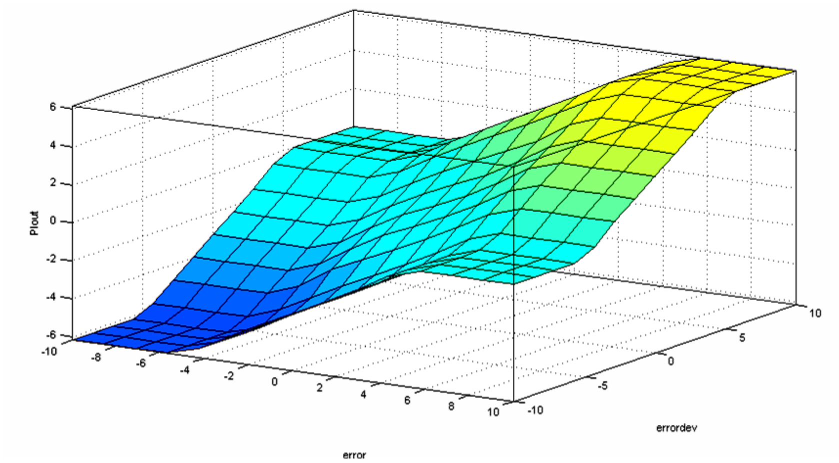

Figure 5.15: Regions for the fuzzy PI-controller, input combination values.

The defuzzification process uses the control rules for the fuzzy PI-controller (R1)–(R4), with membership functions and IC regions together to evaluate the analytical control laws formulas for each region. To find the defuzzification formula for region IC2 for example, in this region $K_I e(kT)$ and$K_P v(kT)$ are located in the domain $[0, L]$, but $K_I e(kT) > K_P v(kT)$. The membership functions for the error and rate for the range $[0, L]$ are:

$$\begin{aligned} ep &= \frac{K_I e(kT) + L}{2L} & \qquad en &= \frac{-K_I e(kT) + L}{2L} \\ vp &= \frac{K_P v(kT) + L}{2L} & \qquad vn &= \frac{-K_P v(kT) + L}{2L} \end{aligned} \tag{5.7}$$

For rule (1):

$$\{\text{“}error \;=\; en\; AND\; rate \;=\; vn\text{”}\} \;=\; min\{\, en, vn\,\} \;=\; en,$$

$$R(1): \begin{cases} the\ selected\ input\ membership\ value\ is\ en \\ \quad the\ corresponding\ output\ value\ is\ o.n \end{cases}$$

For rule (2):

$$\{\text{“}error \;=\; en\; AND\; rate \;=\; vp\text{”}\} \;=\; min\{\, en, vp\,\} \;=\; en,$$

$$R(2): \begin{cases} the\ selected\ input\ membership\ value\ is\ en \\ \quad the\ corresponding\ output\ value\ is\ o.z \end{cases}$$

For rule (3):

$$\{\text{“}error \;=\; ep\; AND\; rate \;=\; vn\text{”}\} \;=\; min\{\, en, vn\,\} \;=\; vn,$$

$$R(3): \begin{cases} the\ selected\ input\ membership\ value\ is\ en \\ \quad the\ corresponding\ output\ value\ is\ o.z \end{cases}$$

For rule (4):

$$\{\text{“}error \;=\; ep\; AND\; rate \;=\; vp\text{”}\} \;=\; min\{\, ep, vp\,\} \;=\; vp,$$

$$R(4): \begin{cases} the\ selected\ input\ membership\ value\ is\ vp \\ \quad the\ corresponding\ output\ value\ is\ o.p \end{cases}$$

Substituting from the above results in the centroid defuzzification formula (5.6), then:

$$\Delta u_{PI}(kT) \;= \frac{en \times o.n + en \times o.z + vn \;\times o.z + vp \;\times o.p}{en + en + vn + vp} \tag{5.8}$$

To this end, by applying $o.p = L$, $o.n = -L$, $o.z = 0$, obtained from figure 5.11(b) and substituting from equations (5.7) into equation (5.8), we obtain:

$$\Delta u_{PI}(kT) = \frac{L}{2\big(2L - K_I e(kT)\big)}[K_I e(kT) + K_P v(kT)] \qquad (5.9)$$

Here, we note that$e(kT) \geq 0$ in regions IC1 and IC2. In thesame way, one can verify that in regions IC5 and IC6 we have:

$$\Delta u_{PI}(kT) = \frac{L}{2\big(2L + K_I e(kT)\big)}[K_I e(kT) + K_P v(kT)] \qquad (5.10)$$

Where it should be noted that $e(kT) \leq 0$in regions IC5 andIC6. Hence, by combining the above two formulas we arrive atthe following result for the four regions IC1, IC2, IC5, and IC6:

$$\Delta u_{PI}(kT) = \frac{L[K_I e(kT) + K_P v(kT)]}{2(2L - K_I|e(kT)|)},$$

Working through all regions in the same way, we obtain the following formulas for the 20 IC regions:

$$\begin{aligned}
\Delta u_{PI}(kT) \quad &= \frac{L[K_I e(kT) + K_P v(kT)]}{2(2L - K_I|e(kT)|)}, && \text{in IC1, IC2, IC5, IC6} && (5.11)\\
&= \frac{L[K_I e(kT) + K_P v(kT)]}{2(2L - K_P|e(kT)|)}, && \text{in IC3, IC4, IC7, IC8} && (5.12)\\
&= \frac{1}{2}[K_P v(kT) + L], && \text{in IC9, IC10} && (5.13)\\
&= \frac{1}{2}[K_I e(kT) + L], && \text{in IC11, IC12} && (5.14)\\
&= \frac{1}{2}[K_P v(kT) - L], && \text{in IC13, IC14} && (5.15)\\
&= \frac{1}{2}[K_I e(kT) - L], && \text{in IC15, IC16} && (5.16)\\
&= 0, && \text{in IC18, IC20} && (5.17)\\
&= -L, && \text{in IC17} && (5.18)\\
&= L, && \text{in IC19} && (5.19)
\end{aligned}$$

Similarly, defuzzification of the fuzzy D-controller follows the same procedure as described above for the PI component, except that the input signals in this case are different. The IC combinations of these two inputs are decomposed into twenty similar regions, as shown in figure 5.15.

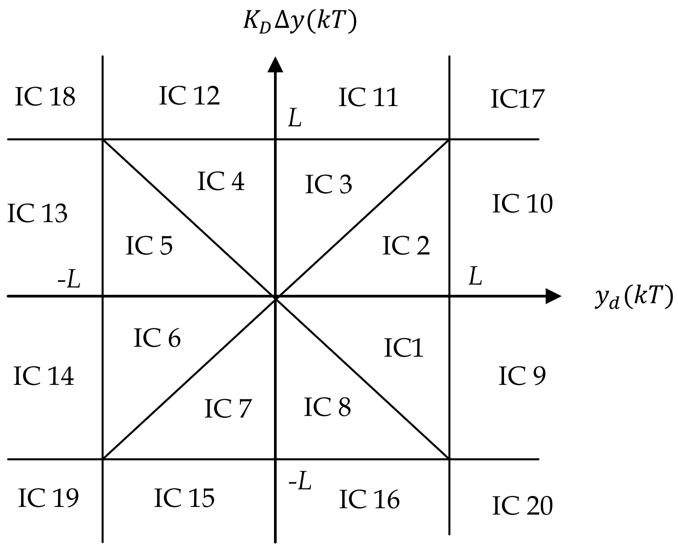

Figure 5.16: Regions for the fuzzy D-controller, input combination values.

Similarly, we obtain the following formulas for the D controller in the 20 IC regions:

$$\Delta u_D(kT) = \frac{L[y_d(kT) + K_D\Delta y(kT)]}{2(2L - K_I|y_d(kT)|)}, \quad \text{in IC1, IC2, IC5, IC6} \tag{5.20}$$

$$= \frac{L[y_d(kT) + K_D\Delta y(kT)]}{2(2L - K_I|\Delta y(kT)|)}, \quad \text{in IC3, IC4, IC7, IC8} \tag{5.21}$$

$$= \frac{1}{2}[-K_D\Delta y(kT) + L], \quad \text{in IC9, IC10} \tag{5.22}$$

$$= \frac{1}{2}[y_d(kT) - L], \quad \text{in IC11, IC12} \tag{5.23}$$

$$= \frac{1}{2}[-K_D\Delta y(kT) - L], \quad \text{in IC13, IC14} \tag{5.24}$$

$$= \frac{1}{2}[y_d(kT) + L], \quad \text{in IC15, IC16} \tag{5.25}$$

$$= 0, \quad \text{in IC17, IC19} \tag{5.26}$$

$$= -L, \quad \text{in IC18} \tag{5.27}$$

$$= L, \quad \text{in IC20} \tag{5.28}$$

CHAPTER 6

6. SIMULATION AND RESULTS

This chapter presents simulations and results for the fuzzy gain scheduling technique methodology that combines between gain scheduling and fuzzy control techniques that used to solve the tracking control problem for a special case of 2^{nd} order nonlinear system. The performance of the approached methodology will be verified by applying each step of that methodology to the magnetic ball levitation CE 152 as an application of the proposed case. The simulations are performed using the Matlab/Simulink software package ver. 2007b, by PC, Ram 4GB, Intel(R) Core(TM)2 Quad CPU 2.66 GHz.

6.1 Introduction

The problem of tracking a reference signal using fuzzy gain scheduling technique for 2^{nd} order nonlinear systems is considered. In chapter 3, a continuous state feedback gain scheduling controller for the nonlinear system was designed, also a relation between state feedback and PID controllers for a special case of 2^{nd} order nonlinear systems was introduced, thus, the whole continuous state feedback gain scheduled controller was converted into a continuous gain scheduled PI+D controller. In chapter 4, a digital gain scheduled PI+D controller was implemented from its continuous version using the redesign technique, so that the fuzzy control technique can be applied. In chapter 5, Fuzzy Proportional- Integral- Derivative (Fuzzy PI+D) controller was described since the employment of this controller will make an enhancement of the tracking. In this chapter, we apply the previous theoretical results for the continuous state feedback, PID, PI+D gain scheduled controllers and the digital PI+D gain scheduled controller to the magnetic ball levitation CE 152 and finally employ the fuzzy control technique on the digital PI+D gain scheduled controller to examine improvement of performance that occurred.

6.2 Continuous Gain Scheduled Tracking Controller

Now, we want to apply the continuous state feedback, PID, PI+D gain scheduled controllers that designed in chapter 3 to the magnetic ball levitation CE 152.

From chapter 2 and 3, the linearization of the magnetic ball levitation CE 152 is given by:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ \dfrac{19.62}{(0.00826 - x_{1ss})} & -2.381\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{0.53206}{(0.00826 - x_{1ss})}\end{bmatrix}U_D \qquad (6.1)$$
$$y = \begin{bmatrix}159.492 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

To extend the region of validity for the linearization tool to a range of operating points, the operating point must be parameterized by the scheduling variable α as $x_{1ss}(\alpha)$, where $\alpha = r$, and then the extended state space model for the magnetic ball levitation CE 152 can be written as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ \dfrac{19.62}{(0.00826 - x_{1ss}(\alpha))} & -2.381\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{0.53206}{(0.00826 - x_{1ss}(\alpha))}\end{bmatrix}U_D \qquad (6.2)$$
$$y = \begin{bmatrix}159.492 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

When $\alpha = r = constant$, then the output follows the reference value, thus

$$x_{1ss}(\alpha) = \frac{\alpha}{159.49206} \qquad (6.3)$$

Substituting for $x_{1ss}(\alpha)$ from equation (6.3) into system (6.2), the result is:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ \dfrac{3129.234}{(1.3174 - \alpha)} & -2.381\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \dfrac{84.86}{(1.3174 - \alpha)}\end{bmatrix}U_D \qquad (6.4)$$
$$y = \begin{bmatrix}159.492 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

The desired natural frequency ω_n and damping ratio ρ can be obtained from specifying the performance requirements: overshoot $O.S = 5\%$ and the settling time $t_s = 0.1\ second$ then:

$$\rho = \sqrt{\frac{(\ln(O.S))^2}{(\ln(O.S))^2 + \pi^2}} = \sqrt{\frac{(\ln(0.05))^2}{(\ln(0.05))^2 + \pi^2}} = 0.6901 \tag{6.5}$$

$$\omega_n = \frac{4}{\rho * t_s} = -\frac{4}{0.6901 * 0.1} = 57.9626 \tag{6.6}$$

Substituting for equation (6.5), (6.6), and the matrices coefficients of system (6.4) into equation (3.35) in chapter 3, we get

$$\begin{aligned} k_1(\alpha) &= 416.686(1.4059 - \alpha) \\ k_2(\alpha) &= 5.62836(1.3174 - \alpha) \\ k_\sigma(\alpha) &= 99.2927(1.3174 - \alpha) \end{aligned} \tag{6.7}$$

Where $k_\sigma(\alpha)$ is the scheduled gain for an extra state variable that generated from including the integral action to achieve zero steady state error as described in chapter 3.

6.2.1 Unit Step Response

As described in chapter 2, reaching a maximum value of the position can be done by applying a unit step response. The unit step response can be built as a sequence of five step changes in the reference signal to allow enough time for the system to settle down after each step change.

The gain scheduled controller for state feedback, PID, and PI+D controllers for the magnetic ball levitation are built on MATLAB SIMULINK Tool as shown in the figures below:

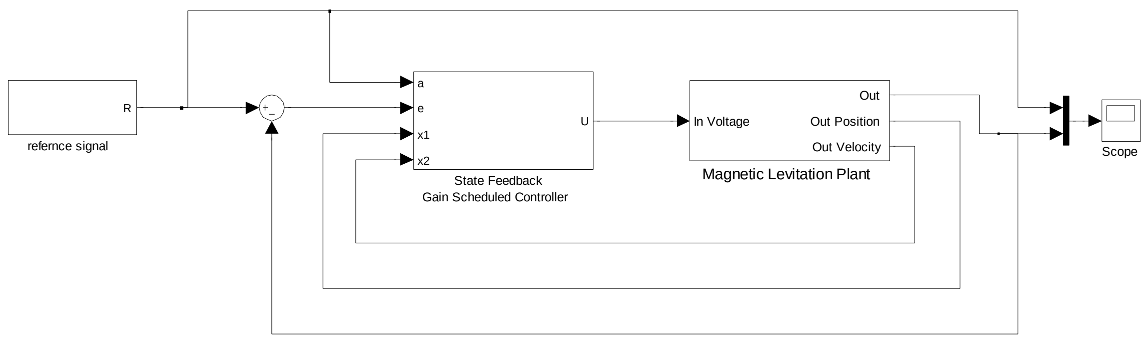

Figure 6.1: State feedback gain scheduled controller applied on SIMULINK

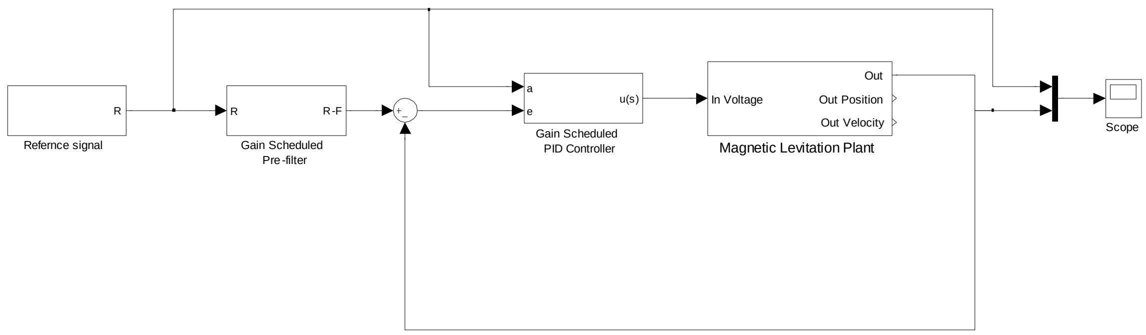

Figure 6.2: PID gain scheduled controller applied on SIMULINK

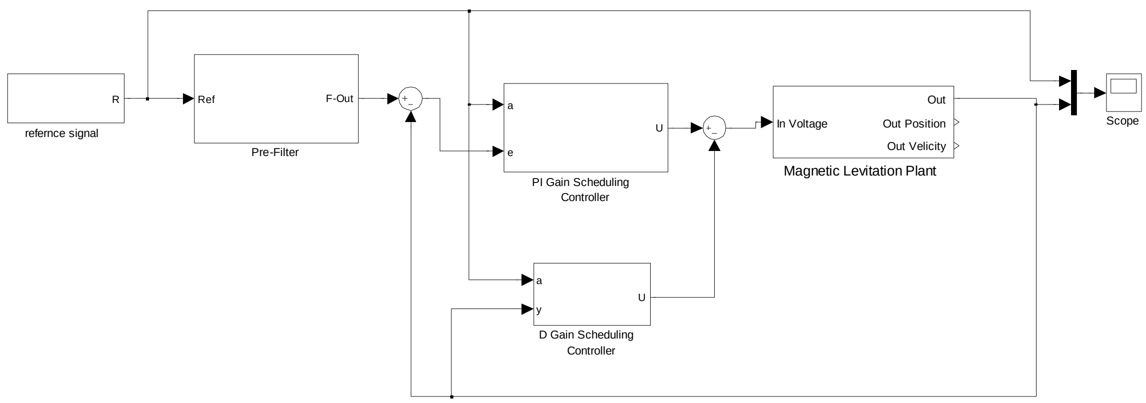

Figure 6.3: PI+D gain scheduled controller applied on SIMULINK

The closed loop transfer functions of the magnetic ball levitation when applying any one of the three controllers are equivalent, so the responses of the three controllers for the sequence of step changes are the same and shown in figure 6.4.

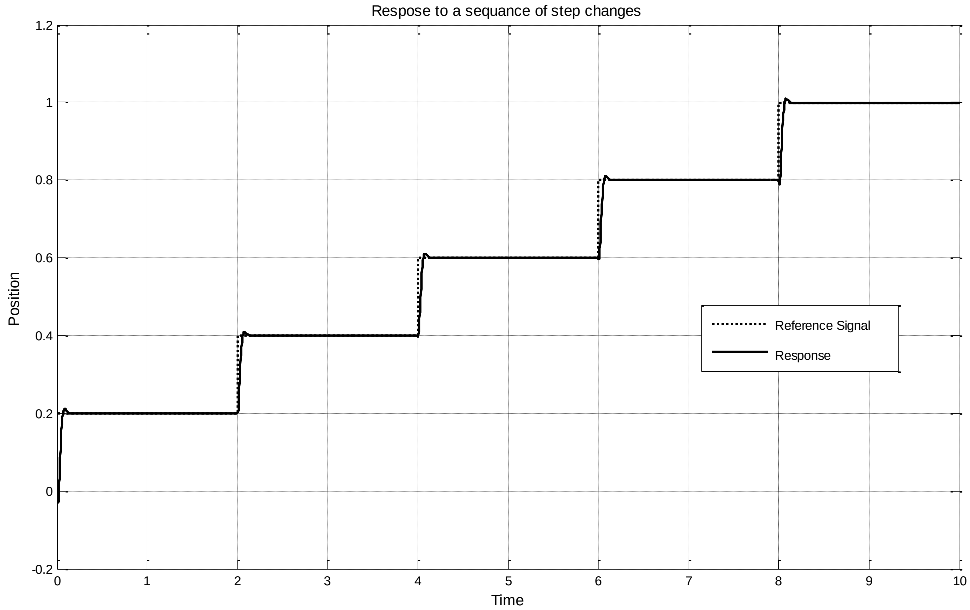

Figure 6.4: Response of the closed loop system to a sequence of step changes in the reference signal when continuous state feedback controller is applied

6.2.2 Ramp Response

Another method to reach the maximum position for the ball is done by applying slow ramp input with slope of 0.1, the response of the closed loop system is shown in figure 6.5.

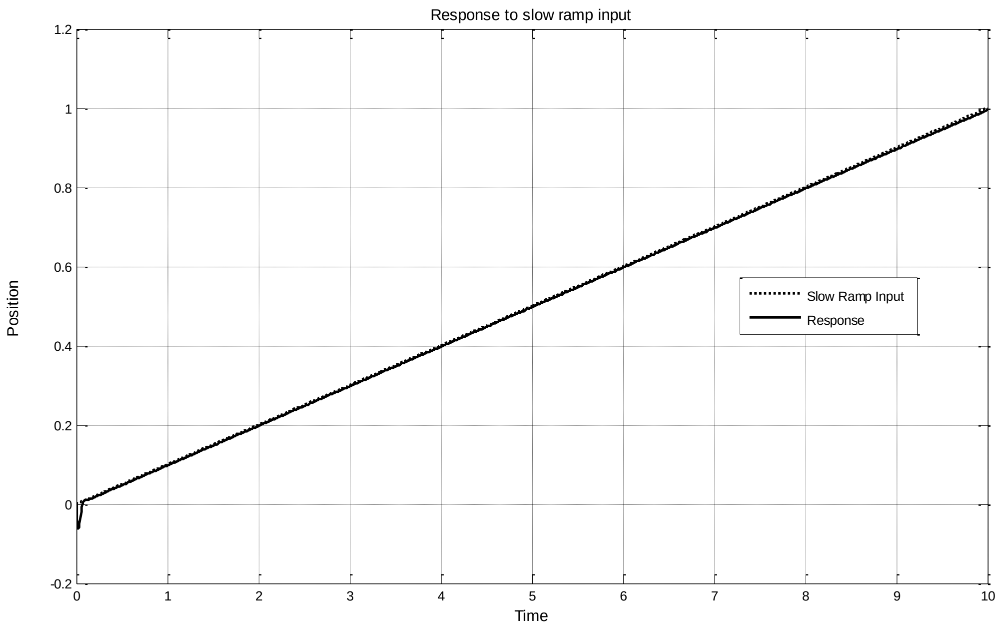

Figure 6.5: Response of the closed loop system to a slow ramp input when continuous state feedback controller is applied

6.3 Digital Gain Scheduled PI+D Tracking Controller

Now, we want to apply the digital PI+D gain scheduled controller that designed in chapter 4 to the magnetic ball levitation CE 152.

From equation (6.7), and substituting into equations (3.45-3.47) in chapter 3, the scheduled gains k_P ,k_I and k_D for the analog PI+D controller are evaluated by:

$$K_P(\alpha) = \frac{k_1(\alpha)}{c_1(\alpha)} = \frac{416.686(1.4059-\alpha)}{159.49206} = 2.6126(1.4059-\alpha) \quad (6.8)$$

$$K_D(\alpha) = \frac{k_2(\alpha)}{c_1(\alpha)} = \frac{5.62836(1.3174-\alpha)}{159.49206} = 0.0353(1.3174-\alpha) \quad (6.9)$$

$$K_I(\alpha) = k_\sigma(\alpha) = 99.2927(1.3174-\alpha) \quad (6.10)$$

From equations (6.8-6.10), and choosing a sampling time of $T = 0.01\ second$, and then substituting into equations (4.9-4.10) and(4.14) in chapter 4, the scheduled gains $\widetilde{K}_P, \widetilde{K}_I, and \widetilde{K}_D$ for the digital PI+D controller can be evaluated as:

$$\widetilde{K}_P(\alpha) = K_P(\alpha) - \frac{K_I(\alpha)T}{2} = 2.1162(1.4265-\alpha) \quad (6.11)$$

$$\widetilde{K}_I(\alpha) = 0.992927 = K_I(\alpha)T = (1.3174-\alpha) \quad (6.12)$$

$$\widetilde{K}_D(\alpha) = 2K_D(\alpha) = 0.0706(1.3174-\alpha) \quad (6.13)$$

The gain scheduled digital PI+D tracking controller for the magnetic ball levitation is built on MATLAB SIMULINK Tool as shown in the figure 6.6 below:

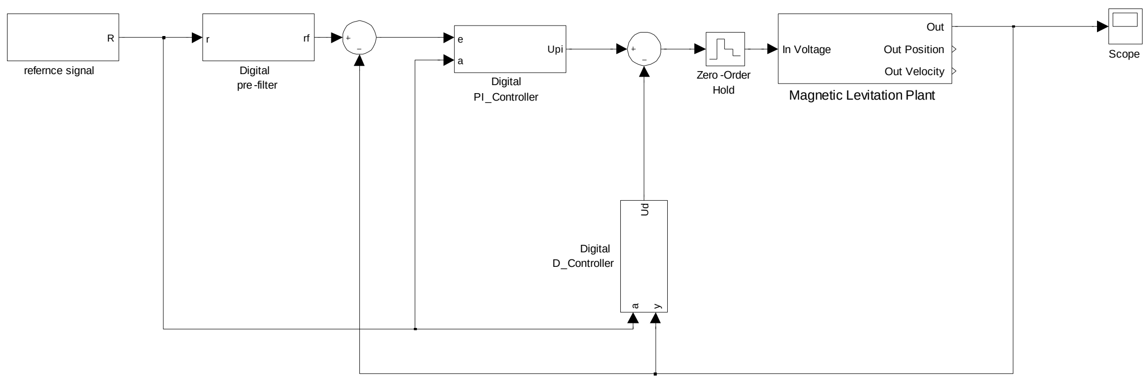

Figure 6.6: Digital PI+D gain scheduled tracking controller applied on SIMULINK

The internal structures of the gain scheduled digital PI-Controller and digital D-Controller for the magnetic ball levitation are built on MATLAB SIMULINK Tool according to the block diagrams of figures(4.8-4.9) in chapter 4, and shown in the figures 6.7 and 6.8 below:

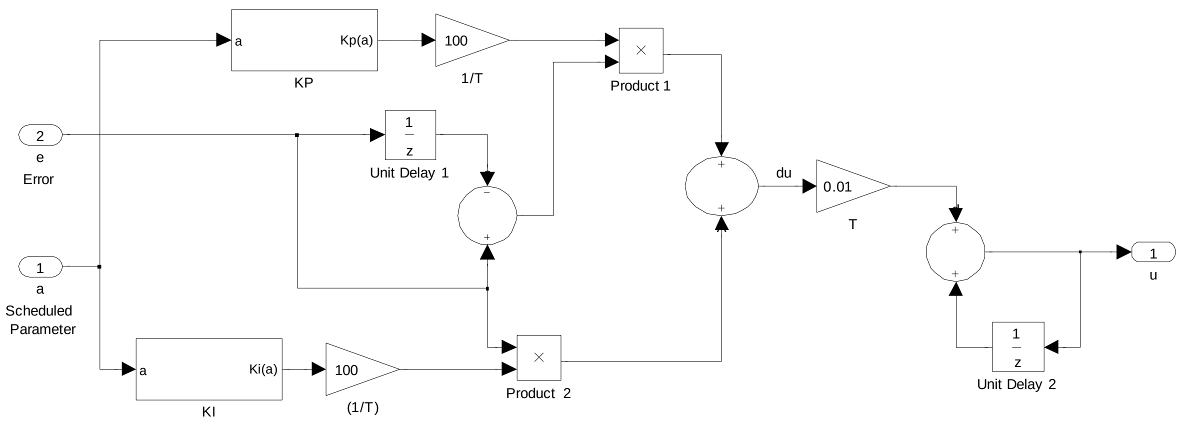

Figure 6.7: Internal structure of the digital PI-controller applied on SIMULINK.

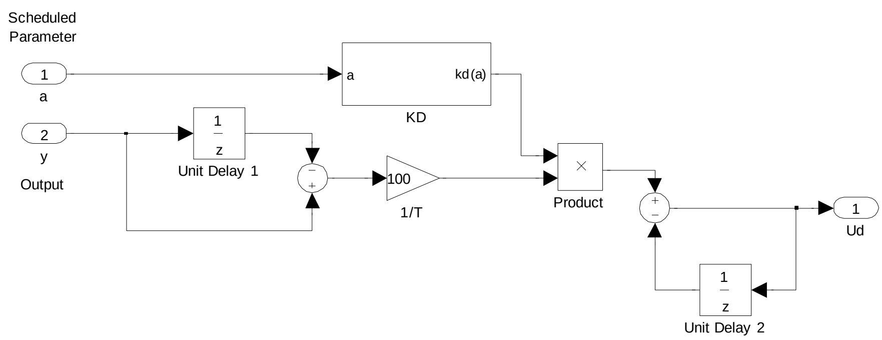

Figure 6.8: Internal structure of the digital D-controller applied on SIMULINK.

The response of the digital PI+D controller for the sequence of step changes is shown in figure 6.9

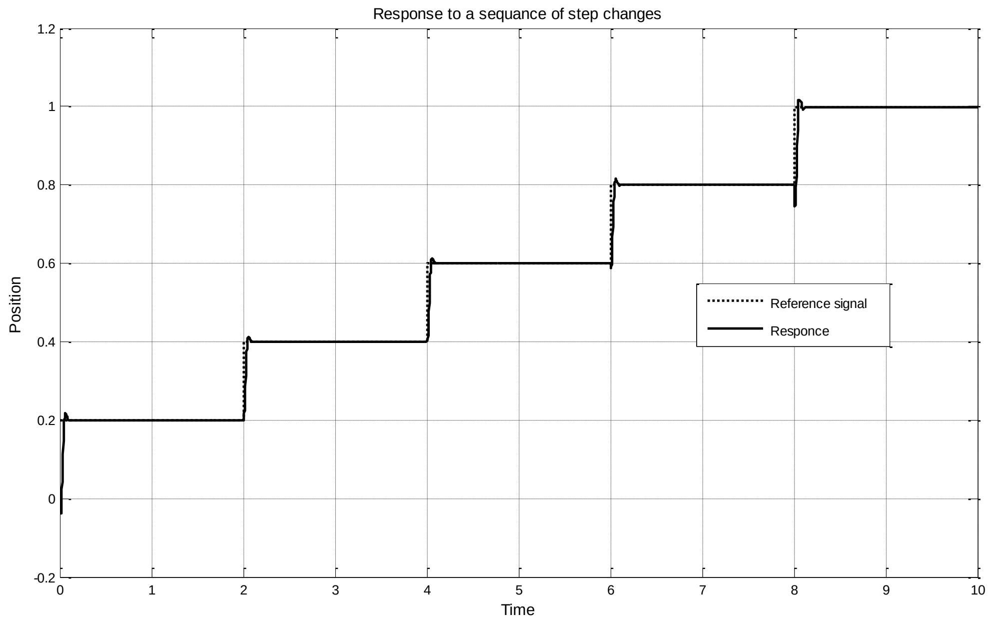

Figure 6.9: Response of the closed loop system to a sequence of step changes in the reference signal when digital PI+D controller is applied

And the response to a slow ramp input with slope of 0.1 is shown in figure 6.10

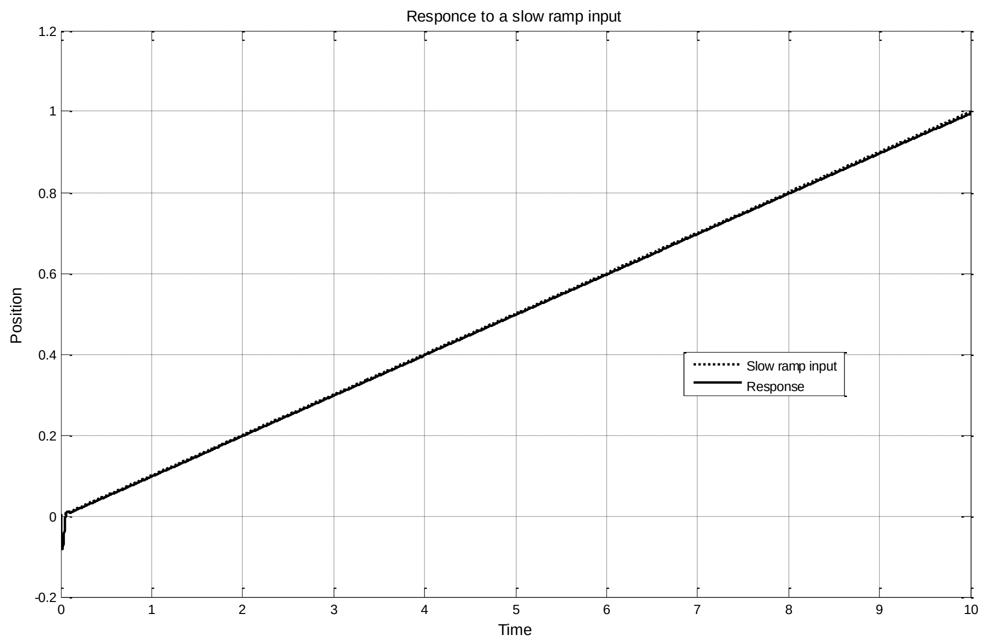

Figure 6.10: Response of the closed loop system to a slow ramp input when digital PI+D controller is applied

6.4 Fuzzy Gain Scheduled Tracking Controller

Now, we want to employ the Fuzzy PI+D controller that described in chapter-5 to the magnetic ball levitation CE 152 by implementing the derived analytical formulas on MATLAB Program for the fuzzy PI and D components and then employing these formulas in the digital PI+D model that described in figure 6.6. This fuzzy PI+D controller have six control gain coefficients which are: $\widetilde{K}_I, \widetilde{K}_P, \widetilde{K}_D, \widetilde{K}_{u,PI}, \widetilde{K}_{u,D}, L$ where $\widetilde{K}_I, \widetilde{K}_P, \widetilde{K}_D$ are self tuned gains due to utilizing the gain scheduling technique, and $\widetilde{K}_{u,PI}, \widetilde{K}_{u,D}, L$ can be tuned and we set $\widetilde{K}_{u,PI} = 0.048$ and $\widetilde{K}_{u,D} = 0.01$ and $L = 120$.

The fuzzy PI+D gain scheduled tracking controller for the magnetic ball levitation is built on MATLAB SIMULINK Tool as shown in figure 6.11 below:

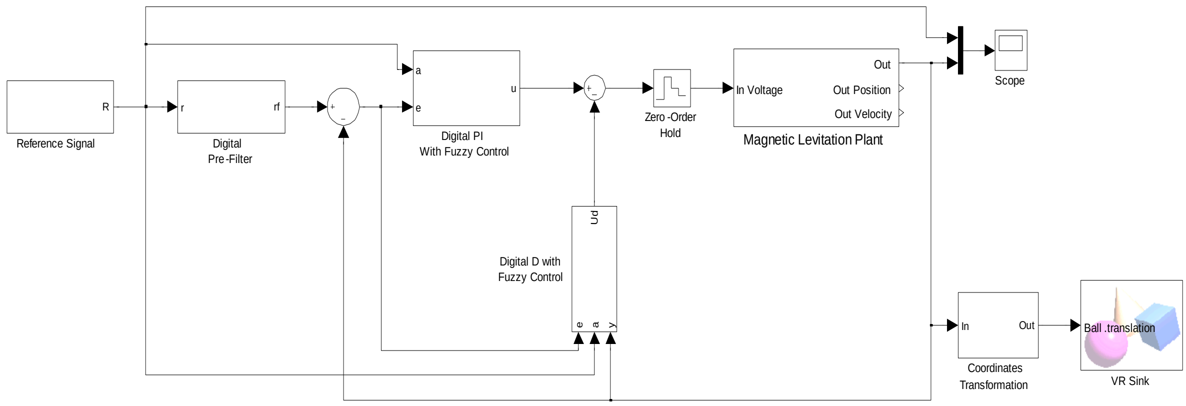

Figure 6.11: Fuzzy PI+D gain scheduled tracking controller applied on SIMULINK

The internal structure components of the gain scheduled fuzzy PI-Controller and fuzzy D-Controller for the magnetic ball levitation CE152 are built on MATLAB SIMULINK Tool according to the block diagram of figure (5.10) in chapter 5, which includes the programming of two Matlab embedded functions based on the analytical formulas that derived in equations (5.11-5.19) and equations (5.20-5.28) for fuzzy PI and D controllers respectively and shown in the figures 6.12 and 6.13:

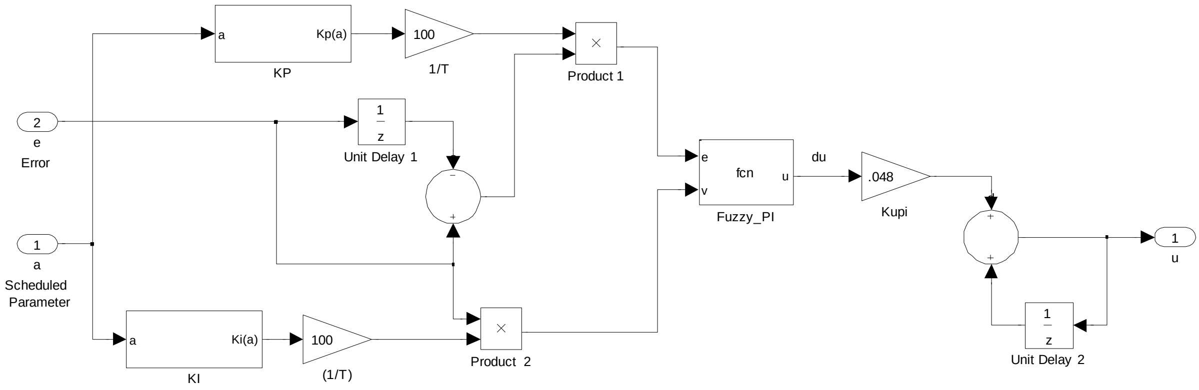

Figure 6.12: Internal structure of the fuzzy PI-controller applied on SIMULINK.

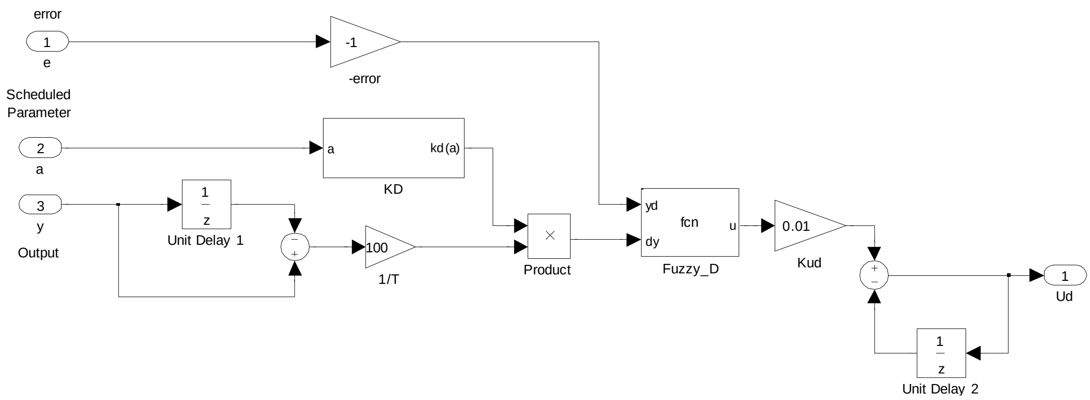

Figure 6.13: Internal structure of the fuzzy D-controller applied on SIMULINK.

The response of the fuzzy PI+D controller for the sequence of step changes is shown in figure 6.14

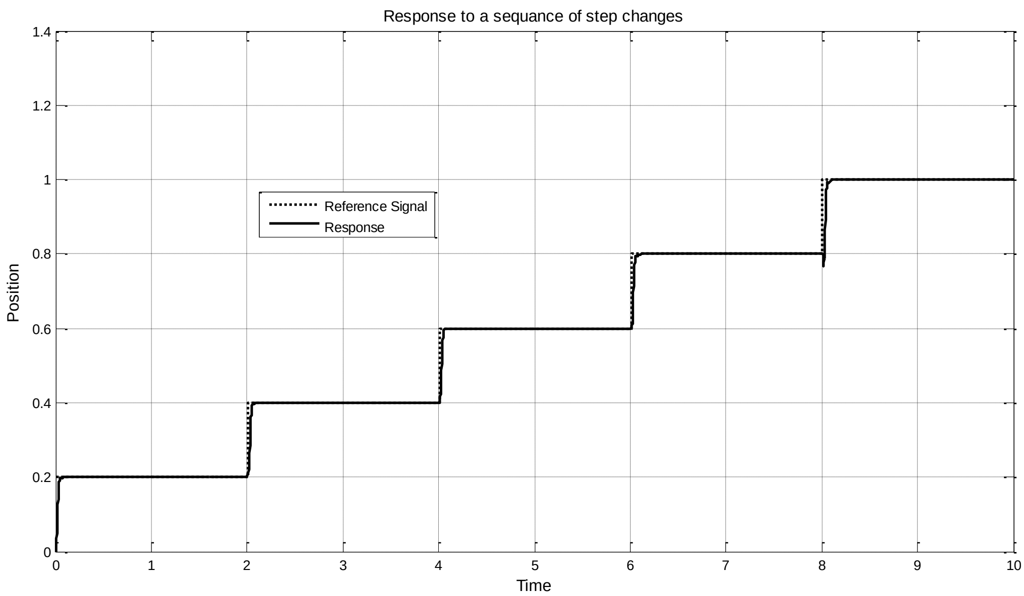

Figure 6.14: Response of the closed loop system to a sequence of step changes in the reference signal when fuzzy PI+D controller is applied

And the response to a slow ramp input with slope of 0.1 is shown in figure 6.15

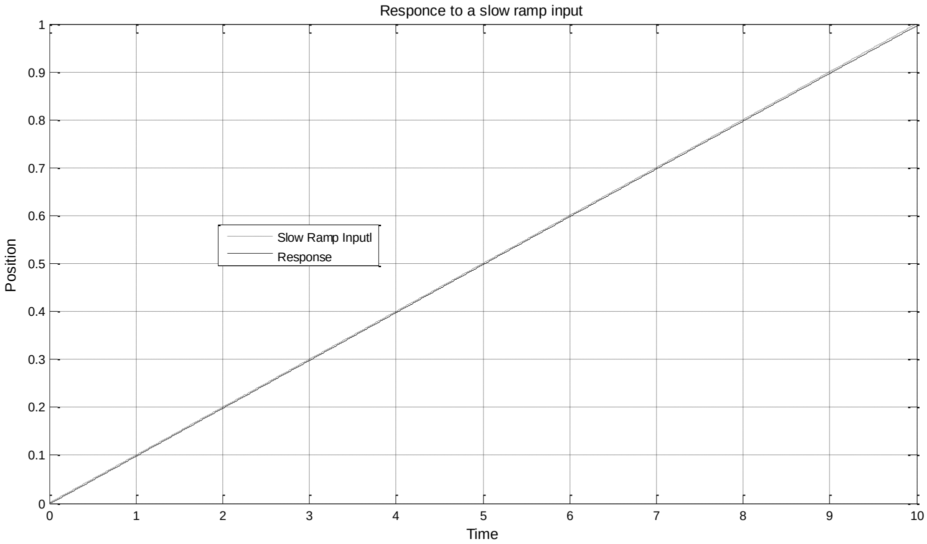

Figure 6.15: Response of the closed loop system to a slow ramp input when fuzzy PI+D controller is applied

6.5 Discussion of the Results

In order to analyze the simulation results clearly and effectively, we present a comparison between the system performance with/without employment of fuzzy approach in the digital PI+D gain scheduled controller as shown in figure 6.16, we select one step change from position 0.2 to 0.4 to show how the performance has improved due to employing the fuzzy logic control approach.

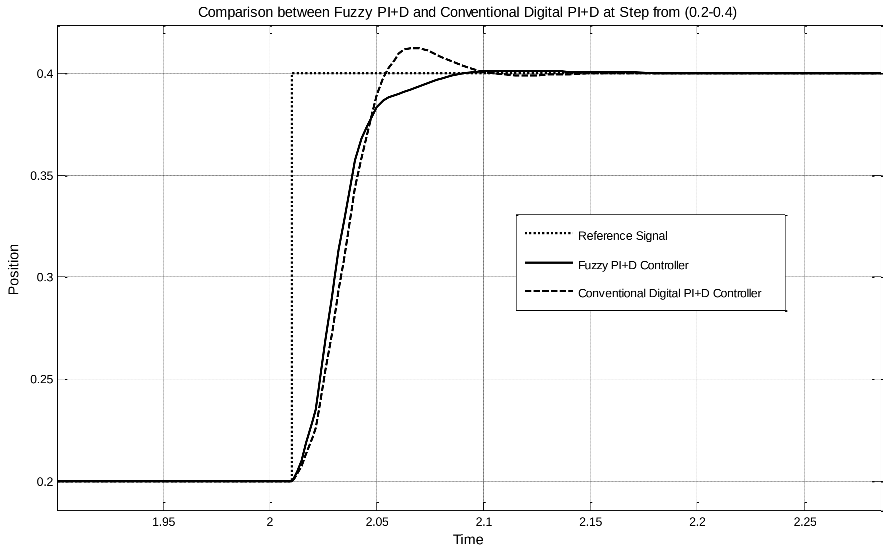

Figure 6.16: System performance with/without employment of fuzzy approach for step (0.2-0.4).

In the figure above, the dashed line represents the response of the system without employing the fuzzy logic approach, while the solid line represents response of the system with employing the fuzzy logic approach.

Also, from figure 6.16, it can be seen that the performance when employing the fuzzy logic approach has generally better transient response than without that employing according to table 6.1, in which at the same rising time the settling time has been improved, and also the overshoot has been canceled . This benefit is applicable to the magnetic ball levitation CE152 especially when reaching a maximum value of position is necessary. We can also compare the results of this thesis with the work proposed in [9, 13].

Table 6.1: Comparison of some controllers for Maglev CE 152

Type of Controller	Overshoot	Settling Time
Conventional Gain Scheduled PI+D	6.25%	0.1 sec
Fuzzy Gain Scheduled PI+D	0%	0.09 sec
Fuzzy Controller With GA Optimization [9]	6%	0.046 sec
Deadbeat Controller [13]	1.65%	0.1 sec

CHAPTER 7

7. CONCLUSION AND FUTURE WORK

The purpose of this work is developing and employing a new technique for handling the tracking problem for a special case type of 2nd order nonlinear systems. The tracking control structure was implemented by using a fuzzy gain scheduling approach that combines between classical gain scheduling techniques and fuzzy logic control. The developed technique was applied on a simulation model of the magnetic ball levitation CE152.

The procedures of the research methodology employed a combination of control tools such as linearization, state feedback, integral control, gain scheduling, and fuzzy logic to derive an approach for tracking a special case of 2nd order nonlinear systems. The continuous state feedback gain scheduling technique was first considered, then a relation between state feedback control and PID control was demonstrated for a special case of 2nd order nonlinear systems, an implementation of a digital PI+D controller from its continuous version then was performed and last a combination of fuzzy logic control with PI+D gain scheduling technique were used, which known as Fuzzy gain scheduling. All designing procedures tackled in this thesis were programmed using the MATLAB code and SIMULINK and applied to the magnetic ball levitation CE152 application. From the simulation results, it has been demonstrated that the proposed technique has achieved better performance when compared with gain scheduled PI+D without employing fuzzy Logic. Also, the simplest configuration of the controller shows the flexibility, and applicability of the Fuzzy Gain Scheduling technique. Therefore, the implementation of fuzzy gain scheduled controller for a special case of 2nd order nonlinear systems is a contribution to the modern heuristics research in the control systems engineering area.

A general recommendation for future work is to apply Fuzzy Gain Scheduling to same special case of 2^{nd} order nonlinear systems including plant model uncertainties, systems with time delayed problems especially when the controller is located far from the plant, and to another cases of nonlinear problems.

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